INTERRELATION OF PRODUCTION SYSTEM DESIGN, QUALITY AND PRODUCTIVITY IN HYBRID MANUFACTURING ENVIRONMENTS

by

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Dedicated to
my father

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ABSTRACT

INTERRELATION OF PRODUCTION SYSTEM DESIGN, QUALITY AND PRODUCTIVITY IN HYBRID MANUFACTURING ENVIRONMENTS

This study concentrates on the impact of lower quality of returned products for remanufacturing on machines that are shared by new and remanufactured products. The hybrid manufacturing shop floor with two part types where machines can have two different quality states are analyzed. The 2M1B line which is the essential element of manufacturing systems analysis is modeled as a Discrete State Continuous Time Markov Chain.

In order to perceive the behavior of the system, at first, a model has been developed and analyzed to evaluate the steady-state probabilities of a single machine of the hybrid production shop with quality and operational failures. Then by using the single machine model, extreme cases for 2M1B system are examined. A Model for finite buffer systems is constructed and an algebraic solution algorithm is developed for the finite buffer 2M1B model. Finally, results derived from numerical experimentation and sensitivity analysis are obtained, and comments on increasing quality and effective productivity are remarked.

ÖZET

MELEZ ÜRETİM SİSTEMLERİNDE ÜRETİM SİSTEMİ TASARIMI, KALİTE VE ÜRETKENLİK ETKİLEŞİMİ

Bu çalışma yeniden üretim için geri dönen ürünlerin düşük kalitelerinin, hem yeni hem de geri dönen ürünlerin üretildiği makinalar üzerindeki etkisi üzerine odaklanmıştır. İki farklı parça tipi işleyen ve makinelerin iki farklı kalitede üretim yapabildiği melez üretim sistemleri analiz edilmiştir. Aynı zamanda üretim sistemleri analizinin en temel elemanlarından birisi olan, 2 makine 1 arastok hattı, bu sistemde kesikli durumlu, sürekli zamanlı bir Markov Zinciri olarak modellenmiştir.

Sistem davranılarını algılayabilmek için ilk olarak, melez üretim atölyesindeki iki farklı parça işleyen, kalite ve operasyonel hatalar yapan bir makine modellenmiştir. Daha sonra bu tek makine modeli kullanılarak, sıfır ve sonsuz arastoklu 2 makine 1 arastok hatları değerlendirilmiştir. Sonlu arastok kapasiteli hatlar için bir model kurulmuş ve bu modelin çözümü için sayısal bir algoritma geliştirilmiştir. Son olarak da sayısal deney ve duyarlılık analizlerinden çıkarılan sonuçlar sunulmuş, bu sistemlerde etkin üretkenliği ve kaliteyi arttırmak için yorumlar geliştirilmiştir.

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LIST OF SYMBOLS /ABBREVATIONS

Pi	Steady state probability of the system at state i
P_T^S	Total production rate of Single Machine
P_E^S	Effective production rate of Single Machine
p_f^m	Probability that a manufacturing part (new part) will cause a persistent type quality failure at the machine during being processed
p_f^r	Probability that a remanufacturing part (returned part) will cause a persistent type quality failure at the machine during being processed
q_{ij}	Rate when in state i at which process makes a transition into state j
v_i	Rate at which process makes a transition when in state i
P_{ij}	The probability process makes its next transition from i to state j.
P_T^∞	Total production rate of the infinite buffer cae
$P_{bothworking}^0$	Total production rate of zero buffer case when no failures are considered
p_R	Proportion of remanufacturing parts in the system or probability that the next part enters system is a remanufacturing part
p_{f1}^m	Probability that a manufacturing part (new part) will cause a persistent type quality failure at the machine 1 during being processed
p_{f1}^r	Probability that a remanufacturing part (new part) will cause a persistent type quality failure at the machine 1 during being processed
p_{f2}^m	Probability that a remanufacturing part (new part) will cause a persistent type quality failure at the machine 2 during being processed
p_{f2}^r	Probability that a remanufacturing part (new part) will cause a persistent type quality failure at the machine 2 during being processed
P_T^N	Total production rate of the system when buffer size is N
P_E^N	Effective production rate of the system when buffer size is N

$\bar{n}$	Average inventory level
N	Buffer Capacity
p_b	Blockage probability
p_s	Starvation probability
μ_m	Manufacturing rate of the single machine
μ_r	Remanufacturing rate of the single machine
μ_{m1}	Manufacturing rate of machine 1 in the 2M1B system
μ_{r1}	Remanufacturing rate of machine 1 in the 2M1B system
λ_{f1}	Rate at which operational failure occurs at machine 2 in the 2M1B system
λ_{d1}	Rate at which inspection occurs at machine 1 in the 2M1B system
μ_{f1}	Rate at which machine 1 is repaired in the 2M1B system
μ_{m2}	Manufacturing rate of machine 2 in the 2M1B system
μ_{r2}	Remanufacturing rate of machine 1 in the 2M1B system
μ_{f2}	Rate at which machine 2 is repaired in the 2M1B system
λ_{d2}	Rate at which inspection occurs at machine 2 in the 2M1B system
λ_{f2}	Rate at which operational failure occurs at machine 2 in the 2M1B system
$\lfloor x \rfloor$	Function that returns largest integer which is lower than x. (floor function)

2M1B	2-machine-1-buffer
(M)	Manufacturing part (new part)
M1	Machine 1
M2	Machine 2
MTTD	Mean Time to Detect
MTQF	Mean Time to Quality Failure
(R)	Remanufacturing part (returned part)
OEM	Original Equipment Manufacturer

1. INTRODUCTION

Güngör and Gupta (1999) report many alarming facts about developed nations whose per capita consumption is especially acute: An average American consumes 20 tons of materials every year and every day the average American uses the equivalent of twenty seven years of stored solar energy in the form of fossil fuels. The products originating from renewable and non-renewable natural resources evolve into waste after their useful lives. According to The National Academy of Sciences, 94% of the substance that is pulled out of the earth enters the waste stream within months. In 1990 the amount of waste generated in the USA reached a whopping 196 million ton up from 88 million ton in the 1960s. About 12 billion ton of industrial waste is generated annually in the United States and over a third of this amount is hazardous waste. Same problems are seen in Europe. In Germany, it is reported that the amount of electronic waste had reached a volume of more than 800,000 ton annually in the early 90s.

The world’s resources are scarce. Mines, energy, water and air which are used to produce new products and the landfill sites where old products are disposed are limited. The society uses these resources to improve its living standard. However, it is also necessary to provide a sustainable environment for the next generations. To this end, as a consequence of both fast depletion of the raw materials and an increasing amount of different forms of waste, two commonly accepted primary objectives have been gaining momentum: (1) creating environmentally friendly products, and (2) developing techniques for product recovery and waste management.

This study is focused on the product recovery issue of the second objective which is carried out mainly due to hidden economic value of solid waste, market requirements and governmental regulations and perceived as a an environmentally and economically way to achieve many of the goals of sustainable development.

Since the aim of recovery creates a new material flow from the user back to the Original Equipment Manufacturer (OEM), the management of this material flow in addition to the conventional raw material flow generates problems in the shop floor that cannot be handled with traditional production control knowledge. Remanufacturing is one of the recovery options that attracts increasing attention. Remanufacturing is considered as an industrial process in which used products are restored to like-new condition. In a factory environment, a returned product is completely disassembled through a series of processes. Useable parts are tested, cleaned, refurbished, upgraded and sent to the parts inventory. Then a remanufactured product is reassembled using these refurbished parts and new parts when necessary (Guide et al., 1997). In most cases a remanufactured product is assumed to be as good as a new product in performance and expected lifetime.

Some OEM's prefer to carry out the remanufacturing process themselves. In some cases they are allowed to satisfy the demand with either new products from manufacturing lines or with remanufactured versions from the remanufacturing lines where a remanufactured product is assumed to be restored to like-new condition. These types of companies are referred to as hybrid companies (Korugan and Gupta, 2002).

Due to such uncertainties and coexistence of both new and returned items, hybrid manufacturing systems must be modeled in a different manner than traditional manufacturing systems. In this research we concentrate our efforts on the impact of lower quality of returned products on machines that are shared by new and remanufactured products. We analyzed the hybrid manufacturing shop floor with two part types where machines can have two different quality states

The rest of the study is organized as follows. In the next section, literature review and general background information on remanufacturing environment, quality issues and stochastic modeling of manufacturing systems are presented. Chapter 3 states the objectives of the study on larger perspective. Chapter 4 introduces terminology of manufacturing systems modeling and quality failure models, and presents fundamental modeling assumptions. After the explanation and analysis of single machine model, this section also includes the analysis and the validation of Single Machine and 2 Machine 1 Buffer (2M1B) systems with zero and infinite buffers. Developed Markov Chain Model for 2M1B systems with finite buffers and the algorithm that solves that model algebraically is described in section 5. Numerical experimentation and sensitivity analysis of the models are made and interpreted in section 6. Interpretations of sensitivity analysis and results are given in section 7. Finally, Section 8 concludes the study and points out the future research directions.

2. LITERATURE REVIEW AND BACKGROUND INFORMATION

2.1. Remanufacturing Literature

Since this study aims to investigate the production system design, quality and productivity relations in a hybrid shop floor, it will be valuable to have some background information about the remanufacturing environment. This section provides the reader general basis of product recovery environment and introduces foundations of remanufacturing shop floor.

2.1.1. Increasing Attention on Reuse

Reuse of products and materials is not a new experience in industry. Metal scrap brokers, waste paper recycling, and deposit systems for bottles are all examples that have been around for a long time. In these cases recovery of the used products is economically more attractive than disposal. Recently the growth of environmental concerns has given 'reuse' increasing attention (Fleischmann et al., 1997). Especially in the last decade, recovery has become an obligation to the environment and to the society itself, enforced primarily by governmental regulations and customer perspective on environmental issues (Güngör and Gupta, 1999). Thus, it can be concluded that material and product recovery are carried out mainly due to three reasons: (1) hidden economic value of solid waste, (2) market (customer) requirements and (3) governmental regulations.

Because of these reasons, firms started to be concerned with developing methods for manufacturing new products such that the environmental standards are satisfied, and with minimizing the amount of waste sent to landfills by recovering materials and parts from old products by means of recycling and remanufacturing.

2.1.2. Forms of Reuse

Fleischmann et al. (1997) noted that Thierry’s categorization of forms of reuse is widely accepted and have been adapted by many authors. Thierry classified forms of reuse into four categories: direct reuse, repair, recycling and remanufacturing.

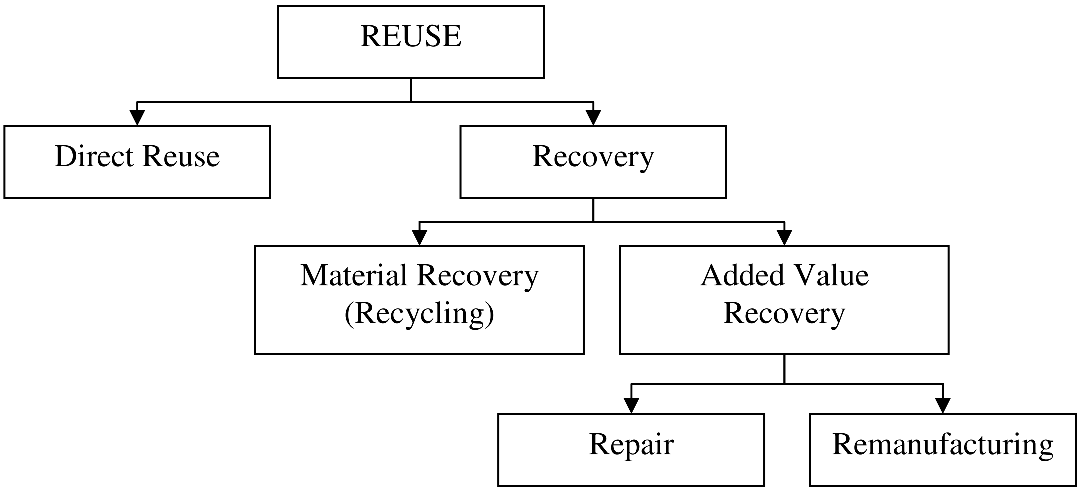

Figure 2.1 Forms of reuse

In the case of direct reuse, returned products can be used ‘as is’ possibly after cleaning or minor repair. On the other hand, 'recovery' is used to specify what is actually regained. Material recovery (recycling) and added value recovery (repair, remanufacturing) are two distinct types of recovery. Examples of items that may be reused directly are reusable packages such as bottles, pallets or containers.

Recycling denotes material recovery without conserving any product structures. The main purpose is to minimize the amount of disposal and maximize the amount of the materials returned back into the production cycle. It aims to recover the material content of retired products by performing the necessary disassembly, sorting and chemical operations. Examples are metal recycling from scrap, glass and paper recycling, and also plastic recycling. In USA the automobile industry is the most advanced industry in recycling among other industries. While just 20% of glass, 30% of paper products and 61% of aluminum cans are recycled, 95% of the 10 million cars and trucks that are retired each year go to the recycler and for each of those cars, 75% by weight is recovered for reuse (Güngör and Gupta, 1999). In Europe, according to the 1994 figures, the recovery rate (in percentage of total consumption) of paper products is relatively higher, about 43%. Also the recovery rate of electronic consumer products is developing (Fleischmann et al., 1997).

On the other hand, added value recovery conserves the product identity and seeks to bring the product back into an 'as good as new' condition by carrying out the necessary operations such as repair, disassembly, overhaul, and replacement. Added value recovery is classified into two classes: Repair and Remanufacturing. The aim of repair is to restore failed products to working order, though possibly with a loss of quality. There are numerous repair practice examples of durable products both in industrial and consumer markets such as domestic devices, industrial machines, and electronic equipment. Whereas remanufacturing is defined as a process of bringing the used products back to ‘as good as new’ condition by performing the necessary operations. Remanufacturing process encompasses the activities in which worn-out products are restored to like-new condition. Through a series of industrial processes in a factory environment, a discarded product is completely disassembled; usable parts are cleaned, refurbished and put into inventory. Then the product is reassembled from old parts (and where necessary new parts) to produce a unit fully equivalent or sometimes superior in performance and expected lifetime to the original new product (Fleischmann et al., 1997). Remanufacturing is distinctly different from repair operations, since products are disassembled completely and all parts are returned to like-new condition, which may include cosmetic operations (Guide, *et.all.* 1999).

2.1.3. Remanufacturing Environment

In his study, Guide (2000) states that remanufacturing is an environmentally and economically way to achieve many of the goals of sustainable development and it closes the materials use cycle and forms an essentially closed-loop manufacturing system. Additionally remanufacturing is a form of waste avoidance since products are reused rather than being discarded. These discarded products are usually landfilled, despite any residual value. Remanufacturing also captures value-added remaining in the product in the forms of materials, energy and labor.

Industries that apply remanufacturing typically include the automobile industry and the electronics industry. Traditional examples for remanufacturing are mechanical assemblies such as aircraft engines and machine tools. A more recent example is remanufactured copy machines. Remanufacturing has received growing attention especially in the USA. There are estimated to be in excess of 73,000 firms engaged in remanufacturing in the United States directly employing over 350,000 people. Remanufacturing operations account for total sales in excess of $53 billion per year. As a point of reference, consider that the US steel industry has annual sales of $56 billion per year and directly employs 241,000 people. In US's recent nation-wide assessment of remanufacturing, average profits margins of 20% are reported showing that remanufacturing is also profitable.

2.1.3.1. Hybrid Manufacturing Systems. The actors involved and their respective functions, including collection, testing, reprocessing, are another important aspect of remanufacturing activities. A major distinction can be made between remanufacturing by the original producer and remanufacturing by a third party. From an original producer's perspective the selection of the remanufacturing system functions to carry out in-house involves major strategic trade-offs.

Currently recycling is often carried out by specialized companies. By contrast, producers tend to perform remanufacturing in-house because of the specific product knowledge involved (Fleischmann et al., 1997). These Original Equipment Manufacturers (OEM's) which prefer to carry out the remanufacturing process themselves are allowed to satisfy the demand with either new products from manufacturing lines or with remanufactured versions from the remanufacturing lines where a remanufactured product is assumed to be restored to like-new condition. These types of companies are referred to as hybrid companies (Korugan and Gupta, 2002).

2.1.3.2. Remanufacturing Shop Floor. Guide et al. (1997a) state a remanufacturing facility may be thought of as three highly dependent subsystems. The first subsystem is the disassembly shop where m types of units are disassembled into n base components and parts. The second subsystem is the remanufacturing shop area where operations required for bringing the parts and components back to like-new condition are performed. The remanufacturing shop area also contains testing and evaluation operations where the parts are deemed useful and remanufactured or deemed useless and are discarded. The last subsystem is the reassembly area, which reassembles remanufactured and new parts (if required) into the final product. There is an additional flow to hybrid shop floor which includes raw materials for manufacturing, since this is the OEM performing hybrid production.

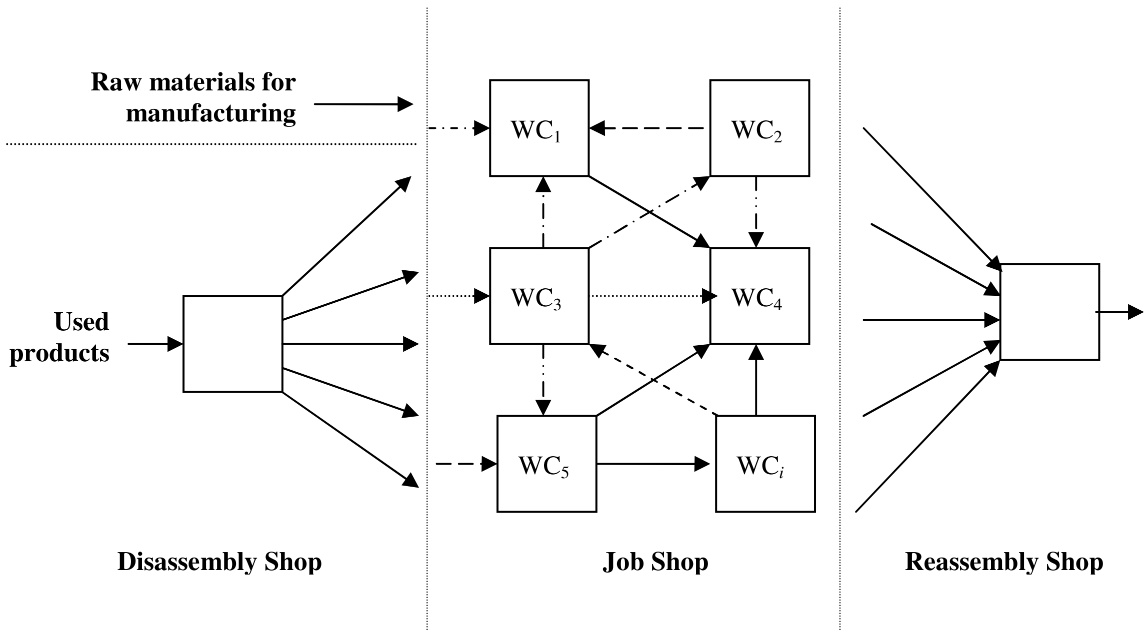

Figure 2.2. Hybrid Shop Floor (Adapted from Guide, 1997)

Remanufacturing shop floor layouts are most commonly in a job-shop form because of the use of general purpose equipment, and the need for flexibility. In Guide's study (2000), less than one-fifth (17.4%) of remanufacturers are reported using specialized CNC equipment or manufacturing cells. This may be, in part, from the diversity in products remanufactured and the low production volumes. Also the low level of technology is suspected to be because of a lack of specialized production and control systems.

2.1.3.3. Complexity of the Remanufacturing Shop Floor. Production planning and control is well understood for conventional manufacturing systems. However, available techniques for conventional manufacturing environment are not always transferable to a remanufacturing environment. Applicability of available techniques may vary from one system to another.

Although recycling involves new production processes, the difficulty of recycling lies in the technical conversion to usable raw materials rather than in managerial planning and control of these activities. From a production management point of view, these activities are no different from other production processes. Consequently, conventional production planning methods should suffice to plan and control recycling operations. On the other hand the management of production planning and control activities in remanufacturing systems can differ greatly from management activities in traditional manufacturing (Guide, 2000). Also the evidence suggests that production planning and control activities are inherently more complex and difficult for remanufacturers.

Remanufacturing firms have a more complex shop structure to plan, control and manage (Guide *et al*., 1997b). Individual repair requirements for every product returned, and coordination of several interdependent activities makes production planning a highly sophisticated task in this environment. The remanufacture operations needed to convert a returned product (also referred to as 'core') back to an 'as good as new' state depend on the actual condition of the product. This may vary from instance to instance and can in general only be decided after a number of testing and disassembly operations. Therefore, in contrast with traditional manufacturing no well-determined sequence of production steps are present in remanufacturing. This exposes planning in a remanufacturing environment to a much higher uncertainty. The articles point out this uncertainty with their simulation studies. (Guide, 1996), (Guide *et al.,* 1997a), (Guide *et al.,* 1997 b), (Guide *et al.,* 1997 c). Moreover the structure of the remanufacturing shop adds complexity to the control problem by containing used products which are less homogeneous and standardized input resource than traditional raw materials and new parts. Also stochastic product returns, disassembly operations, and highly variable material processing requirements are other sources of uncertainty in remanufacturing shop floor. (Guide and Srivastava, 1998). To handle this uncertainty adequately is one of the major tasks in the planning of remanufacturing activities.

Guide and Srivastava (1997a,b) list the following factors which induce complexity in a remanufacturing system:

- probabilistic recovery rates of parts from the inducted cores which implies a high degree of uncertainty in material planning,
- unknown conditions of the recovered parts until inspected, thus leading to stochastic routings and lead times,
- the part matching problem (units are often composed of serial number specific parts and components, along with common ones),
- the added complexity of a remanufacturing shop structure,
- the problem of imperfect correlation between supply of cores and demand for remanufactured units and
- uncertainties in the quantity and timing of returned products.

As mentioned above applicability of traditional production planning and scheduling methods to product recovery systems is very limited due to the previously highlighted differences. Thus, either new methodologies have to be developed or the necessary modifications have to be made to the traditional methods to handle the complications due to the recovery systems. Especially, stochastic interarrival times of the returned products, the uncertainty in materials recovered from returned items, stochastic routings for materials and highly variable processing times are the main uncertainty sources arising from remanufacturing environments related with the hybrid shop floor. Moreover, because of the coexistence of both remanufactured and newly manufactured versions of a certain product in the shop floor, complexity of the hybrid production line is much higher than a remanufacturing line.

2.2. Quality Literature

Quality has taken growing attention for the last two decades since it has been recognized as a key factor affecting the competitiveness of companies.

In the quality literature, two extreme kinds of quality failures based on the characteristics of variations that cause the failures are mentioned. These variations are called common (or chance or random) cause variations and assignable (or special or unusual) cause variations (Montgomery, 2001). Figure 2.3 shows the types of quality failures and variations.

Common cause failures are those in which the quality of each part is independent of the others. Such failures occur often when an operation is sensitive to external perturbations like defects in raw material or when the operation uses a new technology that is difficult to control. Such failures can be represented by independent Bernoulli random variables. The occurrence of a bad part implies nothing about the quality of future parts, so no permanent changes have occurred in the machine. In this case, if bad parts are destined to be scrapped, it is useful to catch them as soon as possible because the longer it takes them to be scrapped; the more they consume the capacity of downstream machines. However, there is no reason to stop a machine that has produced a bad part due to this kind of a failure.

The quality failures due to assignable cause variations (persistent-type quality failures) are those in which a quality failure only happens after a change occurs in the machine. In that case, it is very likely that once a bad part is produced, all subsequent parts will be bad until the machine is repaired. Such failures can also be called Markovian-type quality failures, because given the quality of the current part; the quality of the next part is independent from the past. Here, there is much more incentive to catch defective parts and to stop the machine quickly. In addition to minimizing the waste of downstream capacity, this strategy minimizes the further production of defective parts. For this kind of quality failure, there is no inherent measure of yield because the fractions of parts that are good and bad depend on how soon bad parts are detected and how quickly the machine is stopped for repair.

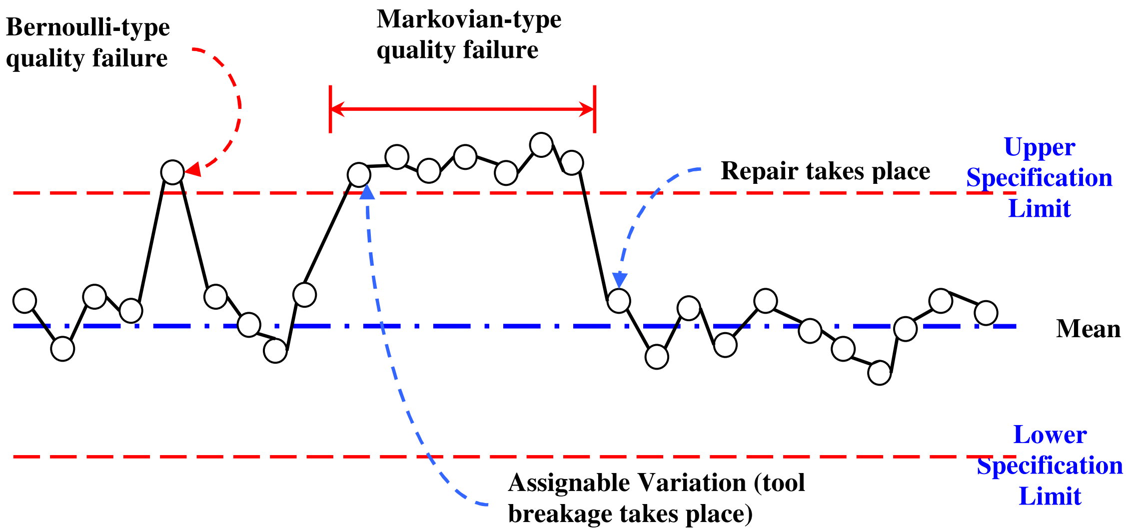

Figure 2.3. Types of Quality Failures as depicted in Kim and Gershwin (2005).

In his doctoral study, Kim (2005) reports most of the quantitative literature on inspection allocation assumes Bernoulli type of quality failures whereas most quantitative studies in Statistical Quality Control are dedicated to finding efficient inspection policies (sampling interval, sample size, and others) to detect persistent type of quality failures Also he argued that the quality strategy of the Toyota Production System, in which machines are stopped as soon as a defective part is detected, is implicitly based on the assumption of the persistent-type quality failure.

Kim and Gershwin (2005) note that quality failures are mixtures of Bernoulli-type quality failures and persistent-type quality failures, in reality. There may also be cases where failures occur independently but at different rates, depending on what state the machine is in. These are referred as multiple-yield quality failures. Specifically, the machine may produce defective parts with a certain small probability p when it is in good working order; when it is in need of adjustment, however, it might produce defective parts with a certain probability $q > p$.

2.3. Manufacturing Systems Engineering Literature

The earliest studies of stochastic models of transfer lines were reported in the former Soviet Union in the early 50's (Liberopoulos and Tsarouhas, 2002). Since then, a great deal of literature on transfer-line analysis has accumulated. A number of methods have been developed for analyzing production lines with unreliable machines and finite buffers. Buzacott and Gershwin have contributed much to the research in this area (Gershwin 2002). Dallery and Gershwin (1992) give a comprehensive review of stochastic models developed for transfer lines and flow lines.

In his book, Gershwin (2002) explores some important system problems in manufacturing. These are the problems that arise when several resources are used together to manufacture something. When manufacturing systems are created, a danger arises that these resources may interfere with one another. The main problem structure is summarized very well with the example below. If a part must pass through two machines before it is complete, and one of those machines is out of order then the other cannot be used. As a result, some capacity is lost because a perfectly good machine is forced to wait. This can be prevented (up to a point) if some parts have been stored for the operational machine to work on, and there is space to put the pieces it completes while the other is down. In designing such a system one must ask, how much space should be allocated for this purpose, and how much material storage (in-process inventory) should be allowed for this purpose. In a more complicated factory, where there are many part types and alternate production paths, a failure can be mitigated by using the remaining capacity for the production of parts that do not require the machine that is down. In designing the control policy for such a system one must ask, when a failure occurs, how should production be shifted so that the operational machines are well utilized without making unnecessary inventory. To answer these questions, Gershwin (2002) uses modern techniques of Markov process modeling and dynamic programming. Additionally, it is noticed that these problems can be extraordinarily complex, even for simple systems, because of the large number of different states the manufacturing system may be in, and because of the large number of design alternatives available. Systematic means must be found for dividing one enormous problem into many small conquerable problems, and then for reconstituting that solution.

Kim and Gershwin (2005) look at the interrelation of quality and productivity in pure manufacturing systems and note that productivity and quality have been extensively studied, but there is little research in their intersection. They argue that the Toyota production system and their suggestions in manufacturing systems engineering from a productivity-quality perspective. In TPS operators are equipped with means of stopping the production process whenever they note anything suspicious. TPS advocates argue that this prevents the waste that would result from producing a series of defective items. So it is a means to improve quality and increase productivity at the same time. On the other hand, quality failures are often those in which the quality of each part is independent of the others. Thus there is no reason to stop a machine that has made a bad part because there is no reason to believe that stopping it will reduce the number of bad parts in the future. In this case, stopping the operation does not influence quality but it does reduce productivity. Meanwhile, lean production advocates claim that reducing inventory on the factory floor reveals the problems in the production lines. Thus, it can help improve product quality. But it is also true that productivity would diminish significantly without stock due to machine failures and other unexpected production interruptions. Therefore when considering both quality and productivity, a balance for optimal work in stock levels process must be sought out.

We should note that we could not find out any study considering different quality states for machines and multiple part types in the manufacturing system engineering literature, confirming the claim of Kim (2005) about the lack of quantitative models in this area.

Last of all, studies of Ateş and Korugan (2005) and Korugan and Ateş (2005) are the previous work of this study presenting the single machine, zero and infinite buffer cases.

3. OBJECTIVES OF THE STUDY

General objective of this study is analysis of 2M1B transfer line with quality failures and two product types.

Hybrid manufacturing shop floor is chosen as the problem environment and fundamental characteristics of the problem environment is reflected to the models by making appropriate assumptions.

By assuming that the lower initial quality of returned products causes more wear and tear on machines than raw materials resulting in more frequent quality related machine failures, and processing times of returned products are lower than new raw materials, we expand a model presented by Kim and Gershwin (2005) to include two product types and different failure probabilities. We model the 2M1B line which is the essential element of manufacturing systems analysis as a Discrete State Continuous Time Markov Chain.

In order to perceive the behavior of the system, we analyzed:

- Single Machine Case
- 2M1B Zero Buffer Line
- 2M1B Infinite Buffer Line
- 2M1B Finite Buffer Line

Total and effective production rates, average inventory levels, and blockage and starvation probabilities of machines are determined as the performance measures of the system and responses of these performance measures to the changes in system parameters are observed via sensitivity analysis.

Gershwin (2002) states that to analyze long transfer lines, 2M1B models should be studied first. Then a decomposition technique, that divides a long transfer line into multiple 2-machine-1-buffer models, could be developed. Since analysis of 2M1B line is the first step in analyzing manufacturing systems, the main contribution of this study will be setting the first steps in manufacturing system analysis with two part types and different quality states of machines.

4. PROBLEM DEFINITION AND FUNDAMENTAL MODELS

4.1. Terminology and Modeling Assumptions

In this section, we specify terminology, and assumptions used in the thesis to model a hybrid production line with quality failures. These terminologies are derived from the literature review and all assumptions are harmonious with Gershwin's (2002) book.

- Blockage: A machine is blocked when the down stream buffer is full and the machine finishes a part.

- Starvation: A machine is starved when the upstream buffer is empty and the machine finishes a part

- Saturated system: A system where inexhaustible supply of workpieces is available upstream of the first machine, and an unlimited storage area is present downstream of the last machine is called a saturated system. Thus, the first machine is never starved, and the last machine is never blocked. This is a widespread assumption in the flow line literature (Dallery and Gershwin, 1992).

- Buffer transit time: Buffer transit time is the time from when a part enters an empty buffer that is not blocked by a downstream machine until that part is able to leave the buffer. Most of the flow line models in the literature as well as this study, assume a zero transit time in the buffer.

- Operational failure: Failures like motor burn-outs which cause machines to stop producing parts.

- Quality failures: The events that a defective part is produced. These may happen due to defective raw materials as well as failures like tool damage at the operation.

- Operation dependent failures: Machines fail only while processing workpieces. Thus, if a machine is operational but starved or blocked, it can not fail.

- Operation dependent inspection: Inspection is carried out only while a machine is processing workpieces. Thus, if a machine is operational but starved or blocked, inspection is not performed.

- Independent operational failures: Each machine's operational failure process is assumed to be independent of the state of the rest of the system. This excludes such event as a power failure that affects the whole line.

- Unlimited repair personnel: The repair process at each machine depends only on the characteristics of the machine, and not on any system-wide properties. For instance, there is no constraint for number of repair men.

- Non-self-correcting process: Once an either of operational failure or quality failure has occurred, the process can be returned to the good condition only by human involvement.

- Common (or chance or random) cause variation: Variation that is inherent in the design of the process and cannot be removed. Such variations occur often when an operation is sensitive to external perturbations like imperfect raw material.

- Assignable (or special or random) cause variation: Variation due to a specific, identifiable cause which changes the process mean or variance.

- Bernoulli quality failures: Quality failures due to common cause variations. Since no permanent changes have occurred in the machine, the occurrence of a bad part implies nothing about the quality of future parts.

- Persistent quality failures: Quality failures due to assignable cause variations. This kind of quality failures only happen after a change occurs in the machine or raw material. In that case, once a bad part is produced, all subsequent parts will be bad until the machine is repaired.

In this thesis we assume, stationary and saturated hybrid production systems where the buffer transit time is zero. Material flow is conserved and defective parts are reworked or scrapped later. No workpieces are destroyed in the line. Each machine can have operational and quality failures and these failures are operation dependent. That is, they occur only when the machine is processing a part. All the failures and repairs are independent which means that each machine works on a different feature. For example, two consecutive machines may be making two different holes. We do not consider cases where both machines work on the same hole, in which the first machine does a roughing operation and the second does a finishing operation. This allows us to assume that the quality failures of the two machines are independent. Also inspections are operation dependent and there are unlimited repair personnel.

4.2. Fundamental Models

4.2.1. Single Machine Model

In the single machine model, we have one machine in the shop where both manufactured and remanufactured items are being produced. We model that machine as a discrete state, continuous time Markov process and assume there are infinite storage buffers for raw and processed parts. Thus, during processing the machine never suffers from blockage or starvation. The processing times for manufacturing and remanufacturing parts are exponentially distributed random variables with rates μ_m and μ_r. Figure 4.1 shows the proposed state transitions of a single machine with persistent-type quality failures. In the model, the machine has five states:

State 0: The machine is operating and producing good manufacturing parts.

State 1: The machine is operating and producing bad manufacturing parts, but the operator does not know this yet.

State $\overline{0}$: The machine is operating and producing good remanufacturing parts.

State $\overline{1}$: The machine is operating and producing bad remanufacturing parts, but the operator does not know this yet.

State R: The machine is not operating.

The machine therefore has two different failure modes (i.e. transition to failure states from state 0 and $\overline{0}$):

Operational failures: transitions from states 0 or $\overline{0}$ to state R where the machine stops producing parts due to system failures, viz. engine failure.

Quality failures: transitions from states 0 or $\overline{0}$ to 1 or $\overline{1}$. The machine transitions from producing good parts (manufacturing or remanufacturing) to producing bad parts due to deterioration on processing tools, viz. cutting tool wear.

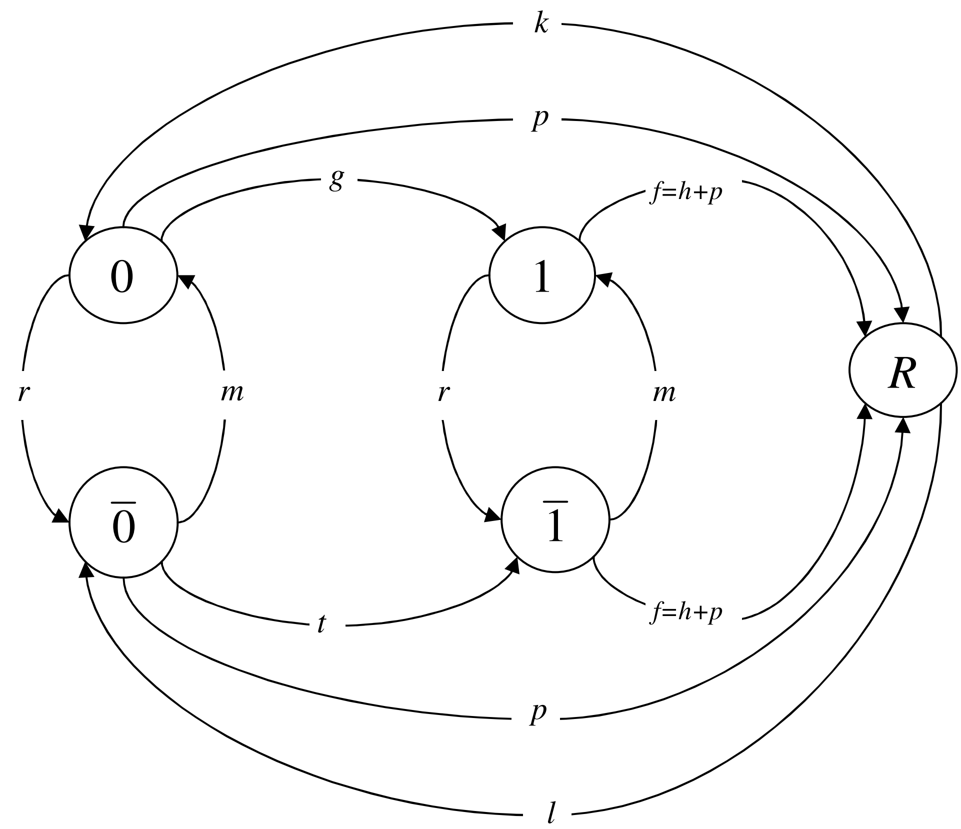

Figure 4.1. States and transitions of single machine model

When the machine is in states 0 or $\bar{0}$, it can fail due to two reasons (Operational failure and Quality failure):

1) It can fail due to a non-quality related event with transition rate *p* to go to state *R*. Then the machine is repaired with transition rates *k* and *l*, and it goes back to states 0 or $\bar{0}$ respectively;

2) Sometimes, due to deterioration in machining quality, the machine begins to produce bad parts, so there are transitions from states 0 and $\bar{0}$ to states 1 and $\bar{1}$ with rates *g* and *t* where $g < t$ since remanufactured parts cause more frequent failures. Here *g* and *t* are the reciprocal of the Mean Time to Quality Failure (*MTQF*) for manufacturing and remanufacturing parts. A more stable machine operation leads to a larger MTQF and a smaller *g* or *t*.

When it is in states 1 and $\bar{1}$ the machine can be stopped for two reasons:

1) It may experience the same kind of operational failure as it does when it is in states 0 and $\bar{0}$;

2) The operator may stop it for repair when he realizes that it is producing bad parts.

Transitions from states 1 or $\bar{1}$ to state R occur at probability rate $f = p + h$ where h is the reciprocal of the Mean Time to Detect (*MTTD*). A more reliable inspection leads to a shorter *MTTD* and a larger f. Note that this implies that $f > p$.

Here, for simplicity, it is assumed that whenever a machine is repaired, it goes back to states 0 or $\bar{0}$, like it is done by Kim (2005). All transitions are assumed to follow independent exponential distributions.

4.2.2. The Single Machine Analysis

In order to determine the production rate of a single machine, we first determine the steady-state probability distribution of the process. This is calculated using the balance equations generated from the transition rates given in Figure 2. We have,

$$P_0(r+g+p) = P_{\bar{0}}m + P_R k \tag{4.1}$$

$$P_{\bar{0}}(m+t+p) = P_0 r + P_R l \tag{4.2}$$

$$P_1(r+f) = P_0 g + P_{\bar{1}} m \tag{4.3}$$

$$P_{\bar{1}}(m+f) = P_{\bar{0}} t + P_1 r \tag{4.4}$$

$$P_R(k+l) = P_0 p + P_{\bar{0}} p + P_1 f + P_{\bar{1}} f \tag{4.5}$$

where

$$\begin{aligned} &g = p_f^m \mu_m, \quad t = p_f^r \mu_r, \quad p = \lambda_f, \quad f = h + p = \lambda_d + \lambda_f, \\ &r = \mu_m p_R, \quad m = \mu_r (1 - p_R), \quad k = \mu_f (1 - p_R), \quad l = \mu_f p_R \end{aligned} \tag{4.6}$$

The probabilities must also satisfy the normalization equation:

$$P_0 + P_{\bar{0}} + P_1 + P_{\bar{1}} + P_R = 1 \tag{4.7}$$

The solution of (4.1)-(4.6), with excluding one linearly dependent equation among (4.1)-(4.5), is

$$P_0 = \frac{(pk + mk + kt + ml)f}{gpf + pfm + ftg + fmk + prf + pkf + rfl + tfk + mfl + tfp + fp^2 + frk + trf + fgl + fpl + fmg + mgk + gtk + trl + gtl + trk + gpk + mgl + ptl} \tag{4.8}$$

$$P_{\bar{0}} = \frac{(pl + gl + rk + rl)f}{gpf + pfm + ftg + fmk + prf + pkf + rfl + tfk + mfl + tfp + fp^2 + frk + trf + fgl + fpl + fmg + mgk + gtk + trl + gtl + trk + gpk + mgl + ptl} \tag{4.9}$$

$$P_1 = \frac{(fgpk + tpml + gpmk + tfgk + mfgl + mgfk + gm^2l + gm^2k + tmgl + tmrk + tmrl + tmgk)}{\begin{aligned}&(m^2fg + tf^2k + tr^2l + fp^2m + mgfk + mgrk + f^2rk + mf^2k + r^2fk + 2mrfk + fprl + pmfl + f^2pl + mf^2g + tr^2k + gprk + gpmk + tfmg + tfpm \\ &+ tfmr + mrgf + trgf + prgf + pfmg + fpmk + fprk + fgpk + fp^2r + tf^2g + pf^2g + gm^2k + f^2pk + fm^2k + fm^2l + fm^2p + f^2p^2 + mgrl + 2pfmr \\ &+ 2mfgl + 2mfrl + grfl + pr^2f + prf^2 + pf^2m + rf^2l + r^2fl + mf^2l + f^2gl + gm^2l + r^2ft + rf^2t + tfgl + tgrl + tmgl + tmrl + trfl + tpf^2 + tfmk \\ &+ tprf + tgrk + tprl + tfgk + tfpl + tmgk + tpml + 2tfrk + tmrk)\end{aligned}} \tag{4.10}$$

$$P_{\bar{1}} = \frac{tfpl + gprk + tprl + tfgl + tfrk + trfl + tr^2l + tgrl + tgrk + mgrk + tr^2k + mgrl}{\begin{aligned}&(m^2fg + tf^2k + tr^2l + fp^2m + mgfk + mgrk + f^2rk + mf^2k + r^2fk + 2mrfk + fprl + pmfl + f^2pl + mf^2g + tr^2k + gprk + gpmk + tfmg + tfpm \\ &+ tfmr + mrgf + trgf + prgf + pfmg + fpmk + fprk + fgpk + fp^2r + tf^2g + pf^2g + gm^2k + f^2pk + fm^2k + fm^2l + fm^2p + f^2p^2 + mgrl + 2pfmr \\ &+ 2mfgl + 2mfrl + grfl + pr^2f + prf^2 + pf^2m + rf^2l + r^2fl + mf^2l + f^2gl + gm^2l + r^2ft + rf^2t + tfgl + tgrl + tmgl + tmrl + trfl + tpf^2 + tfmk \\ &+ tprf + tgrk + tprl + tfgk + tfpl + tmgk + tpml + 2tfrk + tmrk)\end{aligned}} \tag{4.11}$$

$$P_R = \frac{(tr + tg + tp + p^2 + pm + mg + pg + pr)f}{(gpf + pfm + ftg + fmk + prf + pkf + rfl + tfk + mfl + tfp + fp^2 + frk + trf + fgl + fpl + fmg + mgk + gtk + trl + gtl + trk + gpk + mgl + ptl} \tag{4.12}$$

These steady state probabilities will be used in the analysis of production rate of the system and the analysis of the two machine line case.

There are three performance measures for the single machine case. The total production rate represents the total number of parts produced –including the good and the bad parts- per unit time, besides effective production rate represents the number of good parts produced per unit time. The yield which is the proportion of effective production rate to total production rate stands for the ratio of good parts to the total parts produced during the same time period.

The total production rate including good and bad parts, is

$$P_T^S = \mu_m (P_0 + P_1) + \mu_r (P_{\bar{0}} + P_{\bar{1}}) \tag{4.13}$$

The effective production rate, the production rate of good parts only, is

$$P_E^S = \mu_m P_0 + \mu_r P_{\bar{0}} \tag{4.14}$$

The yield is

$$\frac{P_E^S}{P_T^S} = \frac{\mu_m P_0 + \mu_r P_{\bar{0}}}{\mu_m (P_0 + P_1) + \mu_r (P_{\bar{0}} + P_{\bar{1}})} \tag{4.15}$$

4.2.3. Validation of Single Machine Model

The steady state probabilities evaluated from (7)-(11) have been compared with a discrete-part simulation which considers the cross transitions (transitions from state 0 to $\bar{1}$ and from $\bar{0}$ to 1) developed by Arena 9.0. Table 4.1 shows satisfactory agreement between the mathematical and simulation models. All analytical results are in the confidence intervals of simulation runs with an α value of 0.1 at most. Figure 4.2. illustrates the per cent differences. The parameters and rates for the cases considered in numerical comparisons are given in the Appendix A.

Table 4.1 Validation of Single Machine Analysis

	Case #	1	2	3	4	5	6	7	8	9	10
P_T	**Analitical**	0.1417	0.1746	0.2054	0.2211	0.2714	0.1458	0.1634	0.1634	0.2618	0.2714
	Simulation	0.1365	0.1844	0.2035	0.2048	0.2632	0.1354	0.1738	0.1679	0.2687	0.2805
P_E	**Analitical**	0.1328	0.1636	0.1643	0.2010	0.2215	0.1191	0.1532	0.1532	0.2180	0.2215
	Simulation	0.1276	0.1720	0.1631	0.1858	0.2170	0.1102	0.1628	0.1562	0.2237	0.2257
% Differences	P_T	3.68%	-5.61%	0.93%	7.38%	3.02%	7.15%	-6.36%	-2.77%	-2.63%	-3.37%
	P_E	3.96%	-5.14%	0.69%	7.54%	2.02%	7.45%	-6.27%	-2.01%	-2.58%	-1.90%

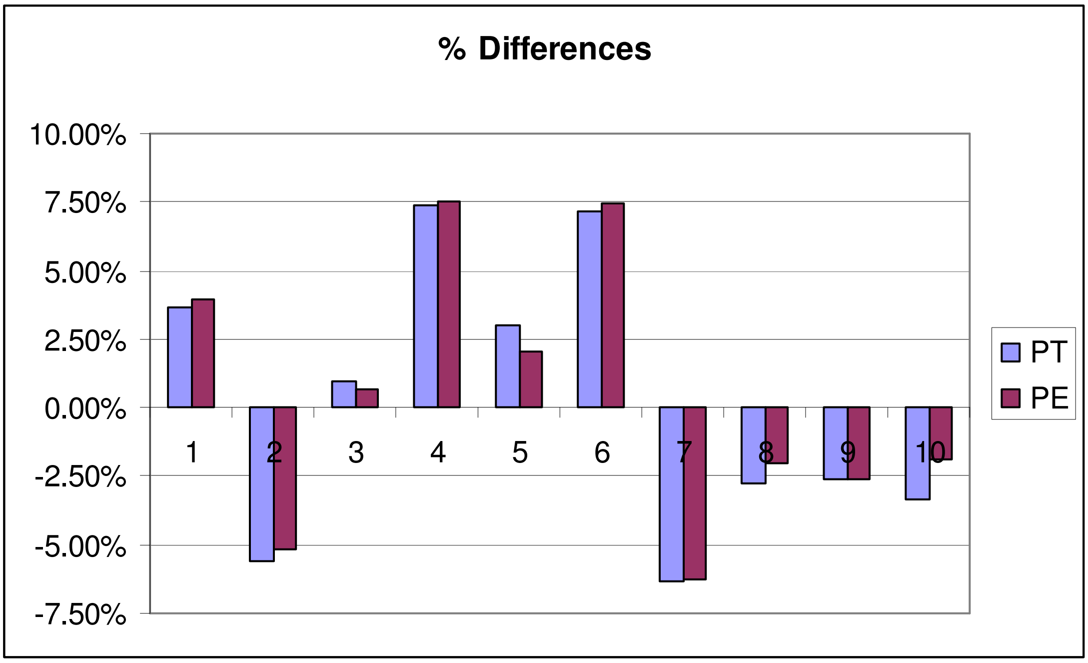

Figure 4.2. Per cent differences between analytic and simulation models for the zero buffer case

For validation of the single machine case, additionally, we compared our results with Kim and Gershwin's (2005) study about production lines with quality failures and single part type. First by setting the proportion of remanufacturing parts to zero, we obtained the same single part type problem that is analyzed by Kim and Gershwin. We compared our analytical findings of total production rate (Equation 4.13) and efficient production rate (Equation 4.14) with Kim and Gershwin's results which are

$$P_T^S = \mu \frac{1 + g / f}{1 + (p + g) / r + g / f} \qquad (4.16)$$

$$P_T^S = \mu \frac{1}{1+(p+g)/r+g/f} \tag{4.17}$$

where p is the reciprocal of *MTTF*, r is the reciprocal of *MTTR*, g is the reciprocal of *MTQF* and f is the reciprocal of sum of *MTTF* and *MTTD*.

Both for setting p_R to zero and to one –for the cases where only manufacturing parts or only remanufacturing parts are being produced- , we obtained exactly the same results with Kim and Gershwin's (2005) study on the single part type with two different types of quality failures.

4.2.4. Alternative Single Machine Model

In the previous single machine model, transitions from 0 to $\overline{1}$ and transitions from $\overline{0}$ to 1 are ignored for model simplicity. We developed another model which includes those transitions as shown in Figure 4.3.

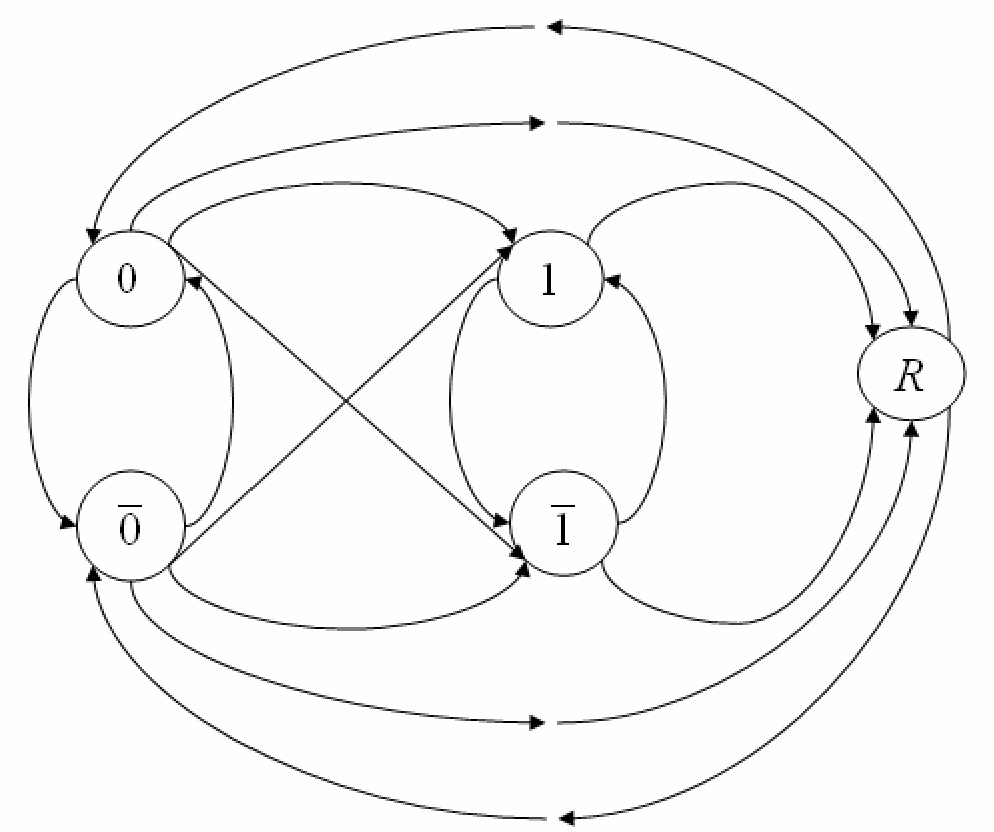

Figure 4.3. States and Transitions of Alternative Single Machine Model

Specifying the instantaneous transition rates determines the parameters of the Continuous Time Markov Chain. Table 4.2. shows the instantaneous transition rates (q_{ij} s) and the v_i rates for the alternative single machine model.

Table 4.2 Instantaneous transition rates and v_i rates of Single Machine Model

		q_{ij} s			
v_i	**States**	0	1	$\bar{0}$	$\bar{1}$
$\mu_m + \lambda_f - \mu_m(1-p_R)(1-p_f^m)$	0	0	$\mu_m(1-p_R)p_f^m$	$\mu_m p_R(1-p_f^r)$	$\mu_m p_R p_f^r$
$\mu_m + \lambda_f + \lambda_d - \mu_m(1-p_R)$	1	0	0	0	$\mu_m p_R$
$\mu_r + \lambda_f - \mu_r p_R(1-p_f^r)$	$\bar{0}$	$\mu_r(1-p_R)(1-p_f^m)$	$\mu_r(1-p_R)p_f^m$	0	$\mu_r p_R(1-p_f^r)$
$\mu_r + \lambda_f + \lambda_d - \mu_r p_R$	$\bar{1}$	0	$\mu_r(1-p_R)$	0	0
μ_f	R	$\mu_f(1-p_R)$	0	$\mu_f p_R$	0

Since all states communicate and all of them are positive recurrent (starting from any state mean time to return that state is finite) in our model, sufficient conditions for existence of limiting probabilities are satisfied. Limiting Probabilities are calculated with the formula below.

rate at which leaves = rate at which enters

$$\begin{aligned} v_j P_j &= \sum_{k \neq j} q_{kj} P_k \\ \sum_j P_j &= 1 \end{aligned} \tag{4.18}$$

where $v_i = \sum_j v_i P_{ij} = \sum_j q_{ij}$, $q_{ij} = v_i P_{ij}$, q_{ij} is the rate when in state *i* at which process makes a transition into state *j*, v_i is the rate at which process makes a transition when in state *i*, P_{ij} is the probability that transition is in to state *j*.

Limiting probabilities show that there is no significant difference in steady state probabilities of two models. Table 4.3 and Figure 4.4 show the per cent differences of the alternative single machine model and the previous model.

Table 4.3. Per cent differences of alternative single machine model with the previous model

Case #	1	2	3	4	5	6	7	8	9	10
P_0	0.004%	0.533%	0.239%	0.151%	-0.145%	0.000%	0.894%	-0.001%	0.172%	0.835%
P_1	-0.002%	-1.758%	-0.557%	-0.312%	0.684%	0.001%	-1.257%	-0.001%	-0.228%	-0.624%
$P_{\bar{0}}$	-0.067%	0.884%	0.321%	0.197%	-0.221%	0.000%	1.390%	-0.001%	0.210%	2.771%
$P_{\bar{1}}$	-0.143%	-2.137%	-0.659%	-0.363%	0.871%	-0.003%	-1.709%	0.000%	-0.254%	-1.633%
P_R	0.021%	-0.481%	-0.123%	-0.075%	0.160%	0.003%	-0.504%	-0.003%	-0.067%	-0.405%

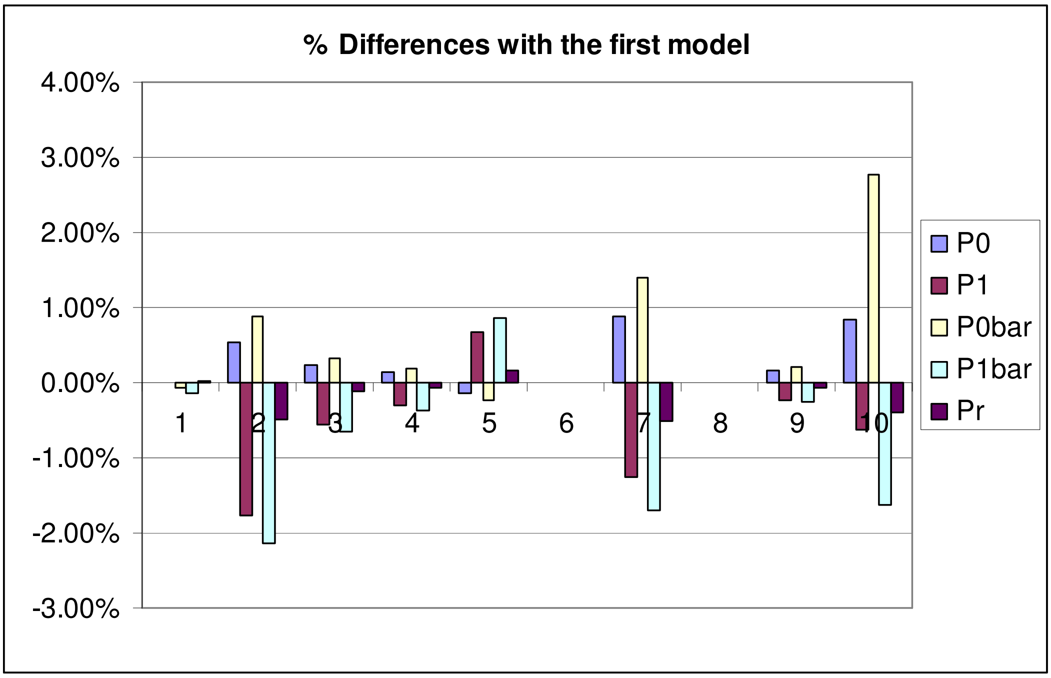

Figure 4.4. Per cent differences of alternative single machine model with the previous model

Since the previous model is simpler, and its rates are easier and more comprehensible in numerical analysis and qualitative interpretations, at the rest of the study, first single machine model with no cross transitions (transitions from 0 to $\bar{1}$ and transitions from $\bar{0}$ to 1) will be considered.

4.2.5. Special Two Machine One Buffer (2M1B) Models

A flow (or transfer) line is a manufacturing system with a very special structure. It is a linear network of service stations or machines (M_1, M_2, ...,M_k) separated by work-in-process buffers (B_1, B_2, ..., B_{k-1}). Material flows from outside the system to M_1, then to B_1, then to M_2, and so forth until it leaves the system (Gershwin, 2002). Figure 4.5 depicts a flow line with two machines. The rectangles represent machines and the circle represents the buffer.

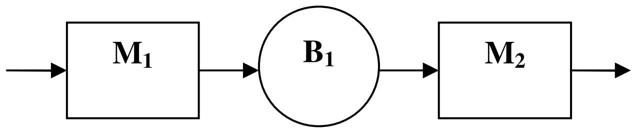

Figure 4.5 Two machine flow line

The case at which there is no buffer and the case where buffer capacity is infinite are the extreme cases of 2M1B transfer line. These extreme cases are important because they define the limits of buffer effectiveness and they are relatively easy to evaluate. They determine upper and lower bounds of production rates. If the lower bound is greater than the required production rate, there is no need for a buffer in the line; if the upper buffer is less than the required production rate, no amount of buffering will allow the line to achieve it.

Gershwin (2002) notes that Buzacott is the one who first shed light on the extreme cases by presenting the fundamentals of these extreme cases for the *k* machine lines where machines have equal and constant operation times in his 1967 study. The operation time is chosen to be the time unit and machines can only fail while they are working. Both up and down times are distributed geometrically. p_i and r_i are defined as the probability of failure and repair during a time unit respectively.

4.2.5.1. Infinite Buffer Case. An infinite buffer case is a special 2M1B line in which the size of the buffer is infinite. This is an extreme case in which the first machine (M_1) never suffers from blockage. To derive expressions for the total production rate and the effective production rate, we observe that when there is infinite buffer capacity between two machines (M_1, M_2), the total production rate of the 2M1B system is a minimum of the total production rates of M_1 and M_2. The total production rate of machine i is given equation (4.13) Hence the total production rate of the 2M1B system is

$$P_T^{\infty} = \min\left\{\mu_{m1}(P_0^1 + P_1^1) + \mu_{r1}(P_{\bar{0}}^1 + P_{\bar{1}}^1),\ \mu_{m2}(P_0^2 + P_1^2) + \mu_{r2}(P_{\bar{0}}^2 + P_{\bar{1}}^2)\right\} \tag{4.19}$$

where μ_{mi} and μ_{ri} denote production rates of machine i for manufacturing and remanufacturing respectively. P_n^i denotes steady state probability of machine i for state n (n = {0,1, $\bar{0}$, $\bar{1}$ and R}).

The probability that M_i does not add non-conformities to manufactured parts is

$$\frac{P_0^i}{P_0^i + P_1^i} \tag{4.20}$$

The probability that M_i does not add non-conformities to remanufactured parts is

$$\frac{P_{\bar{0}}^i}{P_{\bar{0}}^i + P_{\bar{1}}^i} \tag{4.21}$$

Since two machines work on different features, the probability that a manufactured part has no non-conformities is

$$\frac{P_0^1}{P_0^1 + P_1^1}\frac{P_0^2}{P_0^2 + P_1^2} \tag{4.22}$$

and the probability that a remanufactured part has no non-conformities is

$$\frac{P_{\bar{0}}^1}{P_{\bar{0}}^1 + P_{\bar{1}}^1}\frac{P_{\bar{0}}^2}{P_{\bar{0}}^2 + P_{\bar{1}}^2} \tag{4.23}$$

Thus the yield is found as $\left(p_R \dfrac{P_{\bar{0}}^1}{P_{\bar{0}}^1 + P_{\bar{1}}^1} \dfrac{P_{\bar{0}}^2}{P_{\bar{0}}^2 + P_{\bar{1}}^2} + (1 - p_R) \dfrac{P_0^1}{P_0^1 + P_1^1} \dfrac{P_0^2}{P_0^2 + P_1^2} \right)$ by conditioning on the part type. Similarly the yield can be obtained by conditioning on the machines as shown below.

$$Y = \left(\frac{P_0^1 + P_1^1}{1 - P_R^1} \frac{P_0^1}{P_0^1 + P_1^1} + \frac{P_{\bar{0}}^1 + P_{\bar{1}}^1}{1 - P_R^1} \frac{P_{\bar{0}}^1}{P_{\bar{0}}^1 + P_{\bar{1}}^1} \right) \left(\frac{P_0^2 + P_1^2}{1 - P_R^2} \frac{P_0^2}{P_0^2 + P_1^2} + \frac{P_{\bar{0}}^2 + P_{\bar{1}}^2}{1 - P_R^2} \frac{P_{\bar{0}}^2}{P_{\bar{0}}^2 + P_{\bar{1}}^2} \right) \quad (4.24)$$

Finally, the effective production rate is equal to

$$P_E^{\infty} = P_T^{\infty} \left(p_R \frac{P_{\bar{0}}^1}{P_{\bar{0}}^1 + P_{\bar{1}}^1} \frac{P_{\bar{0}}^2}{P_{\bar{0}}^2 + P_{\bar{1}}^2} + (1 - p_R) \frac{P_0^1}{P_0^1 + P_1^1} \frac{P_0^2}{P_0^2 + P_1^2} \right) \quad (4.25)$$

4.2.5.2. Validation of Infinite Buffer Case. The total and effective production rates have been compared with a discrete-event, discrete-part simulation. Table 4.4 and Figure 4.6 show the agreement. The parameters for these cases are shown in the Appendix A.

Table 4.4. Validation of Infinite Buffer Case

	Case #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
P_T	**Analitical**	0.0951	0.0927	0.0469	0.0356	0.0369	0.0480	0.0365	0.0352	0.0299	0.0475
	Simulation	0.0953	0.0919	0.0489	0.0389	0.0370	0.0487	0.0376	0.0351	0.0300	0.0511
P_E	**Analitical**	0.076	0.033	0.0326	0.0144	0.0263	0.024	0.0233	0.031	0.0157	0.0371
	Simulation	0.0747	0.0296	0.0325	0.0136	0.0229	0.0223	0.0214	0.03	0.0134	0.0377
% Differences	P_T	-0.18%	0.89%	-4.25%	-9.38%	-0.15%	-1.43%	-2.81%	0.35%	-0.32%	-7.75%
	P_E	1.71%	10.30%	0.31%	5.56%	12.93%	7.08%	8.15%	3.23%	14.65%	-1.62%

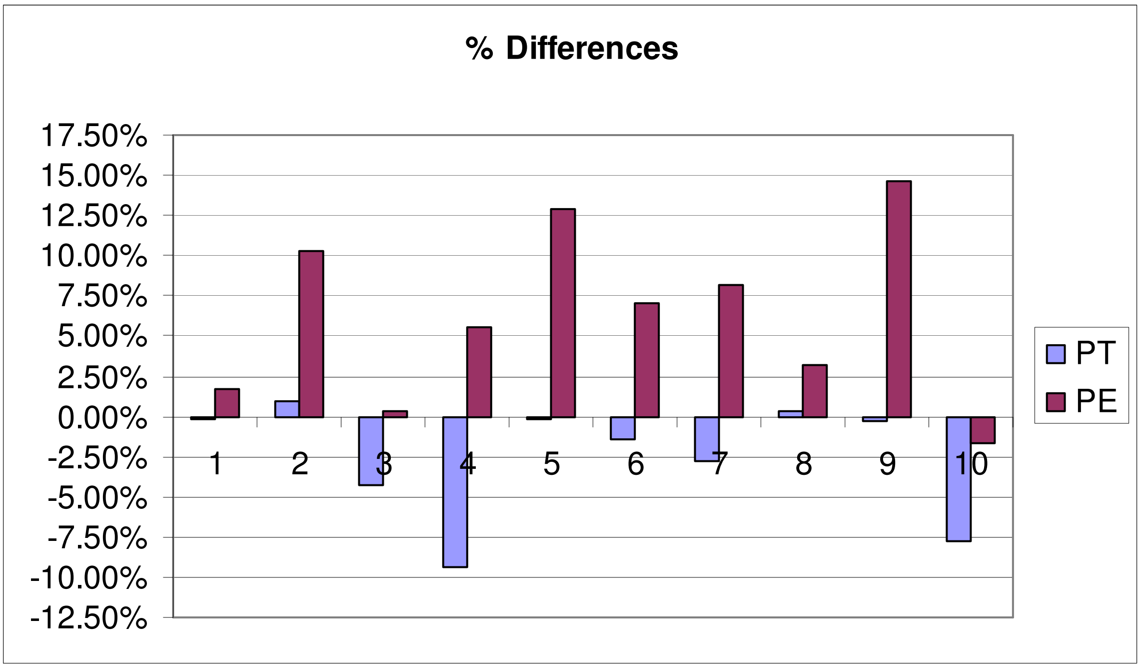

Figure 4.6Validation of Infinite Buffer Case

4.2.5.3. Zero Buffer Case. In the zero buffer case, there is no buffer space between the machines. This is the other extreme case where blockage and starvation take place most frequently. In the zero-buffer case whenever one of the machines stops, the other one is also stopped. When both of them are working, if the operation times of machines are different then there is inefficiency even though no machine is down. This is because the faster machine must wait for the slowest to complete its operations (Gershwin, 2002). In addition, when both of them are working, the part types they've been working on are dependent.

For this case two solution approaches which are performing very similar are developed. The first approach is based on the Gershwin's (2002) work. By using the Markov model developed for the first approach the second approach is developed with slight modifications. Details of the two approaches are explained below.

Consider a long time interval of length T during which M_1 fails m_{1m} times and M_2 fails m_{2m} times while machines were processing new parts. Also during T, M_1 fails m_{1r} times and M_2 fails m_{2r} times while machines were processing used parts. If we assume that the average time to repair M_1 is $1/(k_1+l_1)$ and the average time to repair M_2 is $1/(k_2+l_2)$, then the system down time during M_1 is manufacturing will be close to $m_{1m}\frac{1}{k_1+l_1}$ and system down time during M_2 is manufacturing will be $m_{2m}\frac{1}{k_2+l_2}$. Similarly $m_{1r}\frac{1}{k_1+l_1}$ and $m_{2r}\frac{1}{k_2+l_2}$ are the down times while M_1 and M_2 are remanufacturing, respectively.

So the total system down time will be

$$D=(m_{1m}+m_{1r})\frac{1}{k_1+l_1}+(m_{2m}+m_{2r})\frac{1}{k_2+l_2} \tag{4.26}$$

Consequently, the total up time will be approximately

$$U=T-D=T-\left((m_{1m}+m_{1r})\frac{1}{k_1+l_1}+(m_{2m}+m_{2r})\frac{1}{k_2+l_2}\right) \tag{4.27}$$

Since the total production rate can be written as in (4.25) we need to find the total production rate of machines when no failure is considered.

$$P_T^0=P_{bothworking}^0\frac{U}{T} \tag{4.28}$$

In order to obtain $P_{bothworking}^0$, a continuous time Markov chain model is developed only considering the working states of two machines with zero buffer. When we do not include the failures and quality states of the machines, it is relatively easy to model the process as a discrete state continuous time Markov chain with the intention of getting $P_{bothworking}^0$.

The reduced problem has ten states which are denoted by two terms where M stands for manufacturing, R for remanufacturing, S denotes starving and, bM and bR denotes the type of the blocked part -manufactured and remanufactured respectively-. Table 4.5. gives the instantaneous transition rates of the reduced problem.

Table 4.5. Instantaneous transition rates of the reduced problem.

v_i	States	q_{ij}									
		M-M	M-R	R-M	R-R	M-S	R-S	bM-M	bR-M	bM-R	bR-R
$\mu_{m1}+\mu_{m2}$	M-M	0	0	0	0	μ_{m2}	0	μ_{m1}	0	0	0
$\mu_{m1}+\mu_{r2}$	M-R	0	0	0	0	μ_{r2}	0	0	0	μ_{m1}	0
$\mu_{r1}+\mu_{m2}$	R-M	0	0	0	0	0	μ_{m2}	0	μ_{r1}	0	0
$\mu_{r1}+\mu_{r2}$	R-R	0	0	0	0	0	μ_{r2}	0	0	0	μ_{r1}
μ_{m1}	M-S	$\mu_{m1}(1-p_R)$	0	$\mu_{m1}\, p_R$	0	0	0	0	0	0	0
μ_{r1}	R-S	0	$\mu_{r1}(1-p_R)$	0	$\mu_{r1}\, p_R$	0	0	0	0	0	0
μ_{m2}	bM-M	$\mu_{m2}(1-p_R)$	0	$\mu_{m2}\, p_R$		0	0	0	0	0	0
μ_{m2}	bR-M	0	$\mu_{m2}(1-p_R)$	0	$\mu_{m2}\, p_R$	0	0	0	0	0	0
μ_{r2}	bM-R	$\mu_{r2}(1-p_R)$	0	$\mu_{r2}\, p_R$		0	0	0	0	0	0
μ_{r2}	bR-R	0	$\mu_{r2}(1-p_R)$	0	$\mu_{r2}\, p_R$	0	0	0	0	0	0

Since processing times for manufacturing and remanufacturing at M_1 and M_2 are exponentially distributed, the amount of time the process spends in any state before making a transition into another state is exponentially distributed with the total of respective rates (time passes until the first event occurs). Also one should note that for states which blocking and starvation exist, there is only one event that can cause a transition.

The production rate when no failure is considered is obtained by multiplying the respective steady state probabilities with μ_{m1} and μ_{r2}.

$$P^0_{bothworking} = (P_{M-M} + P_{R-M} + P_{bM-M} + P_{bR-M})\mu_m^2 + (P_{M-R} + P_{R-R} + P_{bM-R} + P_{bR-R})\mu_r^2 \quad (4.29)$$

Since we assume operation-dependent failures, the rates of failure are reduced for the faster machine whereas the repair rates do not change. The reduction of p_i is explained in detail in (Gershwin, 2002). Reductions of the other failure rates are done for the same reasons. Here there are four different sets of reduced rates: for both machines are manufacturing (*M-M*), both machines are remanufacturing (*R-R*), first machine is manufacturing-second is remanufacturing (*M-R*) and first machine is remanufacturing-second is manufacturing (*R-M*). Therefore,

$$p_i^b = p_i \frac{\min\{\mu_q^1,\mu_q^2\}}{\mu_q^i},\ \ g_i^b = g_i \frac{\min\{\mu_q^1,\mu_q^2\}}{\mu_q^i},\ \ t_i^b = t_i \frac{\min\{\mu_q^1,\mu_q^2\}}{\mu_q^i},\ \ f_i^b = f_i \frac{\min\{\mu_q^1,\mu_q^2\}}{\mu_q^i}$$
$$r_i^b = r_i \frac{\min\{\mu_q^1,\mu_q^2\}}{\mu_q^i},\ \ m_i^b = m_i \frac{\min\{\mu_q^1,\mu_q^2\}}{\mu_q^i} \quad (4.30)$$

where q denotes the production type of the corresponding machine at that time (q=m, r) and i denotes the machine (i=1,2)

Table 4.6 lists the possible working states α_1 and α_2 of M_1 and M_2. The third column is the probability of finding the system in the indicated state. That column is obtained by conditioning the desired event on the part types being processed by machines - Manufacturing or Remanufacturing – at that time. The fourth and fifth columns indicate the expected number of transitions to down states during the time interval from each of the states in column 1 and 2. Here P_n^{ib} denotes the steady state probability which is calculated with the corresponding reduced rates of machine i at state n (n=0, 1, $\bar{0}$, $\bar{1}$ and R), (i = 1, 2)

Table 4.6. Zero Buffer States, Probabilities, and Expected Number of Events

α_1	α_2	**Probability $\pi(\alpha_1, \alpha_2)$**	**E$m_1(\alpha_1, \alpha_2)$**	**E$m_2(\alpha_1, \alpha_2)$**
0	0	$\frac{P_0^{1b}}{P_0^{1b}+P_1^{1b}}\frac{P_0^{2b}}{P_0^{2b}+P_1^{2b}}P_{M-M}$	$U\,p_1^b\,\pi(0,0)$	$U\,p_2^b\,\pi(0,0)$
0	1	$\frac{P_0^{1b}}{P_0^{1b}+P_1^{1b}}\frac{P_1^{2b}}{P_0^{2b}+P_1^{2b}}P_{M-M}$	$U\,p_1^b\,\pi(0,1)$	$U\,f_2^b\,\pi(0,1)$
1	0	$\frac{P_1^{1b}}{P_0^{1b}+P_1^{1b}}\frac{P_0^{2b}}{P_0^{2b}+P_1^{2b}}P_{M-M}$	$U\,f_1^b\,\pi(1,0)$	$U\,p_2^b\,\pi(0,0)$
1	1	$\frac{P_1^{1b}}{P_0^{1b}+P_1^{1b}}\frac{P_1^{2b}}{P_0^{2b}+P_1^{2b}}P_{M-M}$	$U\,f_1^b\,\pi(1,0)$	$U\,f_2^b\,\pi(0,1)$
$\bar{0}$	$\bar{0}$	$\frac{P_{\bar{0}}^{1b}}{P_{\bar{0}}^{1b}+P_{\bar{1}}^{1b}}\frac{P_{\bar{0}}^{2b}}{P_{\bar{0}}^{2b}+P_{\bar{1}}^{2b}}P_{R-R}$	$U\,p_1^b\,\pi(\bar{0},\bar{0})$	$U\,p_2^b\,\pi(\bar{0},\bar{0})$
$\bar{0}$	$\bar{1}$	$\frac{P_{\bar{0}}^{1b}}{P_{\bar{0}}^{1b}+P_{\bar{1}}^{1b}}\frac{P_{\bar{1}}^{2b}}{P_{\bar{0}}^{2b}+P_{\bar{1}}^{2b}}P_{R-R}$	$U\,p_1^b\,\pi(\bar{0},\bar{1})$	$U\,f_2^b\,\pi(\bar{0},\bar{1})$
$\bar{1}$	$\bar{0}$	$\frac{P_{\bar{1}}^{1b}}{P_{\bar{0}}^{1b}+P_{\bar{1}}^{1b}}\frac{P_{\bar{0}}^{2b}}{P_{\bar{0}}^{2b}+P_{\bar{1}}^{2b}}P_{R-R}$	$U\,f_1^b\,\pi(\bar{1},\bar{0})$	$U\,p_2^b\,\pi(\bar{1},\bar{0})$
$\bar{1}$	$\bar{1}$	$\frac{P_{\bar{1}}^{1b}}{P_{\bar{0}}^{1b}+P_{\bar{1}}^{1b}}\frac{P_{\bar{1}}^{2b}}{P_{\bar{0}}^{2b}+P_{\bar{1}}^{2b}}P_{R-R}$	$U\,f_1^b\,\pi(\bar{1},\bar{1})$	$U\,f_2^b\,\pi(\bar{1},\bar{1})$
0	$\bar{0}$	$\frac{P_0^{1b}}{P_0^{1b}+P_1^{1b}}\frac{P_{\bar{0}}^{2b}}{P_{\bar{0}}^{2b}+P_{\bar{1}}^{2b}}P_{M-R}$	$U\,p_1^b\,\pi(0,\bar{0})$	$U\,p_2^b\,\pi(0,\bar{0})$
0	$\bar{1}$	$\frac{P_0^{1b}}{P_0^{1b}+P_1^{1b}}\frac{P_{\bar{1}}^{2b}}{P_{\bar{0}}^{2b}+P_{\bar{1}}^{2b}}P_{M-R}$	$U\,p_1^b\,\pi(0,\bar{1})$	$U\,f_2^b\,\pi(0,\bar{1})$
1	$\bar{0}$	$\frac{P_1^{1b}}{P_0^{1b}+P_1^{1b}}\frac{P_{\bar{0}}^{2b}}{P_{\bar{0}}^{2b}+P_{\bar{1}}^{2b}}P_{M-R}$	$U\,f_1^b\,\pi(1,\bar{0})$	$U\,p_2^b\,\pi(0,\bar{0})$
1	$\bar{1}$	$\frac{P_1^{1b}}{P_0^{1b}+P_1^{1b}}\frac{P_{\bar{1}}^{2b}}{P_{\bar{0}}^{2b}+P_{\bar{1}}^{2b}}P_{M-R}$	$U\,f_1^b\,\pi(1,\bar{1})$	$U\,f_2^b\,\pi(0,\bar{1})$
$\bar{0}$	0	$\frac{P_{\bar{0}}^{1b}}{P_{\bar{0}}^{1b}+P_{\bar{1}}^{1b}}\frac{P_0^{2b}}{P_0^{2b}+P_1^{2b}}P_{R-M}$	$U\,p_1^b\,\pi(\bar{0},0)$	$U\,p_2^b\,\pi(\bar{0},0)$
$\bar{0}$	1	$\frac{P_{\bar{0}}^{1b}}{P_{\bar{0}}^{1b}+P_{\bar{1}}^{1b}}\frac{P_1^{2b}}{P_0^{2b}+P_1^{2b}}P_{R-M}$	$U\,p_1^b\,\pi(\bar{0},1)$	$U\,f_2^b\,\pi(\bar{0},1)$
$\bar{1}$	0	$\frac{P_{\bar{1}}^{1b}}{P_{\bar{0}}^{1b}+P_{\bar{1}}^{1b}}\frac{P_0^{2b}}{P_0^{2b}+P_1^{2b}}P_{R-M}$	$U\,f_1^b\,\pi(\bar{1},0)$	$U\,p_2^b\,\pi(\bar{1},0)$
$\bar{1}$	1	$\frac{P_{\bar{1}}^{1b}}{P_{\bar{0}}^{1b}+P_{\bar{1}}^{1b}}\frac{P_1^{2b}}{P_0^{2b}+P_1^{2b}}P_{R-M}$	$U\,f_1^b\,\pi(\bar{1},1)$	$U\,f_2^b\,\pi(\bar{1},1)$

From Table 4.6, the expectations of m_1 and m_2 are

$$
\begin{aligned}
Em_{1m} &= \sum_{0,1}\sum_{S} Em_1(\alpha_1,\alpha_2) \\
Em_{2m} &= \sum_{S}\sum_{0,1} Em_2(\alpha_1,\alpha_2) \\
Em_{1r} &= \sum_{\bar{0},\bar{1}}\sum_{S} Em_1(\alpha_1,\alpha_2) \\
Em_{2r} &= \sum_{S}\sum_{\bar{0},\bar{1}} Em_2(\alpha_1,\alpha_2)
\end{aligned}
\tag{4.31}
$$

Finally the total production rate is calculated with

$$P_T^0 = P_{bothworking}^0 \frac{U}{T} = \frac{P_{bothworking}^0}{1+D} = \frac{P_{bothworking}^0}{1+(Em_{1m}+Em_{1r})\dfrac{1}{k_1+l_1}+(Em_{2m}+Em_{2r})\dfrac{1}{k_2+l_2}} \tag{4.32}$$

Then the effective production rate of the two machines zero buffer line is

$$P_E^0 = P_T^0\left(p_R \frac{P_{\bar{0}}^1}{P_{\bar{0}}^1+P_{\bar{1}}^1}\frac{P_{\bar{0}}^2}{P_{\bar{0}}^2+P_{\bar{1}}^2} + (1-p_R)\frac{P_0^1}{P_0^1+P_1^1}\frac{P_0^2}{P_0^2+P_1^2}\right) \tag{4.33}$$

4.2.5.4. Validation of Zero Buffer Case. The total and effective production rates have been compared with a discrete-event, discrete-part simulation. Table 4.7 shows the agreement. The parameters for these cases are shown in the Appendix A.

Table 4.7. Validation of zero buffer case

	Case #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
P_T^0	**Analitical**	0.0577	0.0506	0.0356	0.0262	0.0307	0.0313	0.0210	0.0199	0.0380
	Simulation	0.0519	0.0494	0.0324	0.0266	0.0275	0.0294	0.0217	0.0203	0.0339
P_E^0	**Analitical**	0.0461	0.0178	0.0247	0.0106	0.0219	0.0155	0.0133	0.0175	0.0297
	Simulation	0.0400	0.0161	0.0226	0.0093	0.0189	0.0145	0.0118	0.0171	0.0242
% Differences	P_T^0	10.07%	2.41%	9.16%	-1.32%	10.54%	6.09%	-3.31%	-2.15%	10.84%
	P_E^0	13.24%	9.76%	8.69%	12.01%	13.66%	6.62%	10.74%	2.32%	18.46%

	Case #	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
P_T^0	**Analitical**	0.0262	0.0268	0.0383	0.0391	0.0276	0.0279	0.0277	0.0278	0.0255
	Simulation	0.0253	0.0255	0.0367	0.0369	0.0268	0.0264	0.0266	0.0267	0.0251
P_E^0	**Analitical**	0.0213	0.0214	0.0280	0.0273	0.0222	0.0221	0.0222	0.0221	0.0227
	Simulation	0.0198	0.0195	0.0254	0.0245	0.0201	0.0196	0.0201	0.0205	0.0211
% Differences	P_T^0	3.68%	4.71%	4.22%	5.73%	3.12%	5.21%	4.01%	4.09%	1.49%
	P_E^0	7.31%	8.86%	9.38%	10.32%	9.38%	11.13%	9.43%	7.45%	6.66%

4.2.5.5. Alternative Zero Buffer Case. The model developed for the reduced problem where failures and quality states of the machines are not considered gives an opportunity to calculate expected values of the number of failures alternatively.

$$
\begin{aligned}
Em_{1m} &= P_M^1 \left(\frac{P_0^1}{P_0^1 + P_{\bar{0}}^1} p_1 + \frac{P_{\bar{0}}^1}{P_0^1 + P_{\bar{0}}^1} f_1\right) && where \quad P_M^1 = P_{M-M} + P_{M-R} + P_{M-S} \\
Em_{1r} &= P_R^1 \left(\frac{P_1^1}{P_1^1 + P_{\bar{1}}^1} p_1 + \frac{P_{\bar{1}}^1}{P_1^1 + P_{\bar{1}}^1} f_1\right) && where \quad P_R^1 = P_{R-M} + P_{R-R} + P_{R-S} \\
Em_{2m} &= P_M^2 \left(\frac{P_0^2}{P_0^2 + P_{\bar{0}}^2} p_2 + \frac{P_{\bar{0}}^2}{P_0^2 + P_{\bar{0}}^2} f_2\right) && where \quad P_M^2 = P_{M-M} + P_{R-M} + P_{bM-M} + P_{bR-M} \\
Em_{2r} &= P_R^2 \left(\frac{P_1^2}{P_1^2 + P_{\bar{1}}^2} p_2 + \frac{P_{\bar{1}}^2}{P_1^2 + P_{\bar{1}}^2} f_2\right) && where \quad P_R^2 = P_{M-R} + P_{R-R} + P_{bM-R} + P_{bR-R}
\end{aligned}
\qquad (4.34)
$$

With these quantities, the previously described approximation gives very similar results.

4.2.5.6. Validation of Alternative Zero Buffer Case. The total and effective production rates have been compared with the same simulation study done for previous zero buffer validation. Table 4.8 shows the agreement. The parameters for these cases are shown in the Appendix A.

Table 4.8. Validation of alternative zero buffer case

	Case #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**10**
P_T^0	**Analitical**	0.0603	0.0508	0.036	0.0252	0.03	0.0304	0.0195	0.0217	0.0353
	Simulation	0.0519	0.0494	0.0324	0.0266	0.0275	0.0294	0.0217	0.0203	0.0339
P_E^0	**Analitical**	0.0482	0.0181	0.0251	0.0102	0.0214	0.0152	0.0125	0.0192	0.0278
	Simulation	0.0400	0.0161	0.0226	0.0093	0.0189	0.0145	0.0118	0.0171	0.0242
% Differences	P_T^0	13.96%	2.77%	10.05%	-5.43%	8.40%	3.42%	-11.13%	6.49%	3.99%
	P_E^0	17.07%	11.03%	10.00%	8.77%	11.67%	4.79%	5.33%	10.77%	12.88%

	Case #	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
P_T^0	**Analitical**	0.0267	0.0265	0.0375	0.0373	0.0278	0.0278	0.0276	0.0279	0.0277
	Simulation	0.0253	0.0255	0.0367	0.0369	0.0268	0.0264	0.0266	0.0267	0.0251
P_E^0	**Analitical**	0.0217	0.0213	0.0275	0.0261	0.0224	0.0221	0.0223	0.0223	0.0246
	Simulation	0.0198	0.0195	0.0254	0.0245	0.0201	0.0196	0.0201	0.0205	0.0211
% Differences	P_T^0	5.31%	3.70%	2.08%	1.18%	3.70%	5.01%	3.61%	4.40%	9.34%
	P_E^0	8.92%	8.38%	7.60%	6.12%	10.36%	11.25%	9.77%	8.23%	14.03%

5. TWO MACHINE ONE FINITE BUFFER MODEL

5.1. Analytical Model of Two Machine One Finite Buffer Line

For accurate analytical solutions of the 2M1B finite buffer systems, an algorithm which constructs a continuous time discrete state Markov chain model of the 2M1B hybrid shop floor and algebraically solves it is developed and coded with Matlab 6.5. The coded algorithm algebraically produces total and effective production rates (P_T^N and P_E^N), average inventory level ($\bar{n}$), blockage and starvation probabilities (p_b and p_s) on the steady state given the initial values of 16 parameters (p_R, p_{f1}^m, p_{f1}^r, μ_{m1}, μ_{r1}, λ_{f1}, λ_{d1}, μ_{f1}, μ_{m2}, μ_{r2}, p_{f2}^m, p_{f2}^r, μ_{f2}, μ_{r2}, λ_{d2}, λ_{f2} and the buffer capacity N)

The main steps of the algorithm are given below and detailed explanation of each step is presented at the following part of this section

- Form the state space of the process given the buffer capacity.
- Calculate v_i vector.
- Determine the possible transitions and form the state transition matrix by assigning related instantaneous rates.
- Change the diagonal of state transition matrix with -v_is.
- Transpose the state transition matrix.
- To remove linear dependency, replace the last row with a row vector composed of 1s which stand for normalization equation of the steady state probabilities
- Form a column vector b composed of 0 which has 1 only at the last row.

- Solve Ax = b where A is the matrix composed at the previous step and x is the column vector of steady state probabilities of states.

- Calculate P_T^N , P_E^N , $\bar{n}$, p_b and p_s

5.1.1. Steps of the Developed Solution Algorithm

The state of the system at any time is denoted by (α_1, α_2, n) where α_1 and α_2 denote the machine states of M_1 and M_2, and n denotes the buffer state. In order to determine the instantaneous transition rates from system state (α_1, α_2, n) to system state (α_1', α_2', n'), first the state space for the given buffer capacity should be extracted. State spaces for some values of N are presented in Appendix B.

5.1.1.1. Definitions of Machine and Buffer states. In the model M_1 has 9 possible machine states and M_2 has 7 possible machine states to keep part type and quality state information of the machines. Table 5.1 shows the symbols and definition of states. To be able to generally comment on machine states, it is beneficial to classify machine states as W, B, F and S (general machine states) where $W = \{0, 1, \bar{0}, \bar{1}\}$, $B = \{b0, b1, b\bar{0}, b\bar{1}\}$, $S = \{s0, s1\}$, F = $\{F\}$.. This classification will also be useful while calculating the performance measures.

Table 5.1. Definition of machine states for 2 machine 1 finite buffer model

M_1	**State definition**
0	The machine is operating and producing good manufacturing parts
1	The machine is operating and producing bad manufacturing parts, but the operator does not know this yet.
$\overline{0}$	The machine is operating and producing good remanufacturing parts
$\overline{1}$	The machine is operating and producing bad remanufacturing parts, but the operator does not know this yet
$b0$	The machine is blocked and the blocked part is a good manufacturing part.
$b1$	The machine is blocked and the blocked part is a bad manufacturing part
$b\overline{0}$	The machine is blocked and the blocked part is a good remanufacturing part
$b\overline{1}$	The machine is blocked and the blocked part is a bad remanufacturing part
F	The machine is being repaired
M_2	**State definition**
0	The machine is operating and producing good manufacturing parts
1	The machine is operating and producing bad manufacturing parts, but the operator does not know this yet.
$\overline{0}$	The machine is operating and producing good remanufacturing parts
$\overline{1}$	The machine is operating and producing bad remanufacturing parts, but the operator does not know this yet
$s0$	The machine is starving and the last part it produced was a good part (so machine's quality state is good)
$s1$	The machine is starving and the last part it produced was a bad part (so machine's quality state is bad)
F	The machine is being repaired

Since both the number of parts (buffer level) and the arrangement of part types in the buffer should be kept in the buffer states, they are coded as shown in Table 5.2. The empty state is represented with 1. First a manufacturing part (M) is added to buffer by coding the state with 2 and a remanufacturing part (R) is added by coding the state with 3 when number of parts in the buffer is 1. Recursively for n parts in the buffer, the buffer states for n-1 is taken and first a (M) is added to the end of all states with n-1 parts, then a (R) is added to the end of all states with n-1 parts sequentially. The order of the parts in the representation below shows the arrangement of manufacturing and remanufacturing parts in the buffer. The part at the beginning of the buffer (at the right) is the part which will be taken by M_2 next, and the part at the end (at the left) is the part which was produced by M_1 last.

Table 5.2. Buffer states for 2 machine 1 finite buffer model

Buffer state	Parts in the queue					
1	E					
2	M					
3	R					
4	M	M				
5	R	M				
6	M	R				
7	R	R				
8	M	M	M			
9	R	M	M			
10	M	R	M			
11	R	R	M			
12	M	M	R			
13	R	M	R			
14	M	R	R			
15	R	R	R			
...	...	...	...	...	...	
58	M	R	M	R	R	
59	R	R	M	R	R	
60	M	M	R	R	R	
61	R	M	R	R	R	
62	M	R	R	R	R	
...	...	...	...	...	...	...

So there will be a total of $2^{N+1}-1$ different buffer states each characterizing an arrangement of manufactured and remanufactured parts where N denotes the buffer capacity. Also it should be noted that the last 2^N states refer the buffer states at which buffer is full and the number of parts in the buffer (buffer level) is $\lfloor \log_2(n) \rfloor$ where is n is the coded buffer state.

5.1.1.2. Forming the State Space. The state space of the model is generated by producing the appropriate Cartesian product of machine and buffer states as shown in Table 5.3.

Table 5.3. Forming the state space of 2 machine 1 finite buffer model

General Machine states M_1	M_2	Appropriate Buffer States	Number of states M_1	M_2	Buffer	Total number of states
W	W	All *n*	4	4	2^{N+1}-1	$16(2^{N+1}-1)$
B	W	*n*=Full	4	4	2^N	$16(2^N)$
F	W	All *n*	1	4	2^{N+1}-1	$4(2^{N+1}-1)$
W	S	*n*=Zero	4	2	1	8
W	F	*n* =Zero	4	1	1	$4(2^{N+1}-1)$
B	F	*n* =Full	4	1	2^N	$4(2^N)$
F	S	*n* =Zero	1	2	1	2
F	F	All *n*	1	1	2^{N+1}-1	$(2^{N+1}-1)$
B	S	Not possible	4	2	0	0

As seen in the table above total number of system states is $25(2^{N+1}-1) + 20(2^N) + 10$ which increases exponentially with the increase of buffer capacity *N*. Figure 5.1. shows the exponentially increasing CPU time of the algorithm because of the number of states.

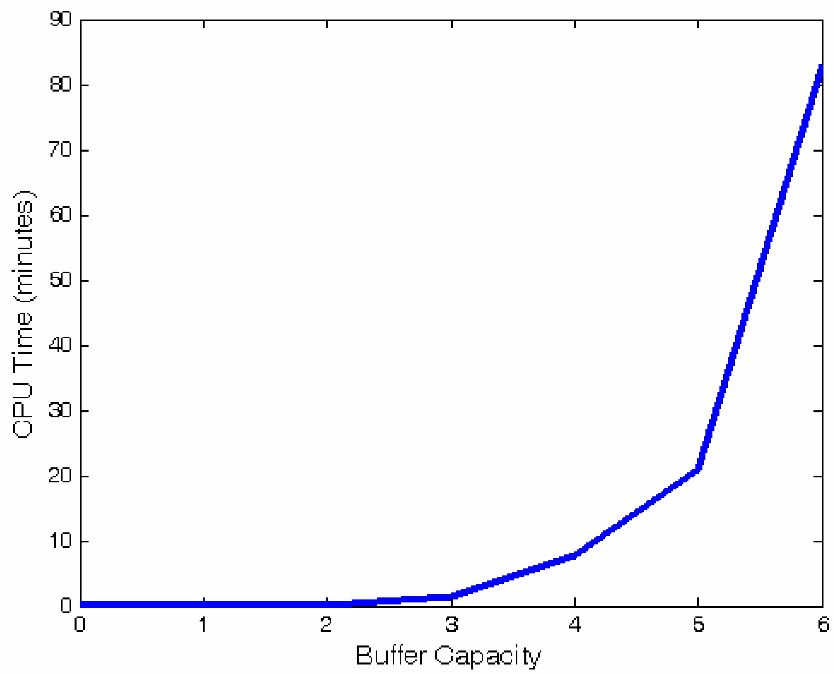

Figure 5.1 CPU time versus buffer capacity

5.1.1.3. Determining the v_is. The process changes it states when an event occurs. There are six possible events which can cause system to make a transition and change its state. These are

- M_1 finishes a part and tries to add that part to the end of the queue (if it is not blocked or failed).
- M_2 finishes a part and tries to pull the first part from the queue (if it is not starved or failed).
- M_1 fails (if it is not blocked or failed).
- M_2 fails (if it is not starved or failed).
- M_1 gets fixed (if it is failed).
- M_2 gets fixed (if it is failed).

The event which occurs first among all possible events for a state changes the state of the system. So v_i is the sum of the rates of all possible events that can occur at that state. v_i s are obtained by summing the appropriate rates from Table 5.4.

With respect to the quality state of the machine failure rates are different. At states 0 and $\overline{0}$ machines can experience only operational failures with rate λ_{fi}, besides at states 1 and $\overline{1}$ machines can either fail operationally or can be detected by the inspector with rate $\lambda_{fi} + \lambda_{di}$.

Table 5.4. Rates for v_i s

	M₁	***M₂***
0	$\mu_{m1}+\lambda_{f1}$	$\mu_{m2}+\lambda_{f2}$
1	$\mu_{m1}+\lambda_{f1}+\lambda_{d1}$	$\mu_{m2}+\lambda_{f2}+\lambda_{d2}$
$\bar{0}$	$\mu_{r1}+\lambda_{f1}$	$\mu_{r2}+\lambda_{f2}$
$\bar{1}$	$\mu_{r1}+\lambda_{f1}+\lambda_{d1}$	$\mu_{r2}+\lambda_{f2}+\lambda_{d2}$
b0	0	-
b1	0	-
$b\bar{0}$	0	-
$b\bar{1}$	0	-
s0	-	0
s1	-	0
F	μ_{f1}	μ_{f2}

For instance the v_i rate of the ($\bar{1}$, 0, n) states is $\mu_{r1}+\lambda_{f1}+\lambda_{d1}+\ \mu_{m2}+\lambda_{f2}$ for all n (n = 1, 2, ...2^{N+1}-1)

5.1.1.4. Determining the Transitions. At the next step the algorithm constructs the transition matrix by checking whether or not there is a transition between each pair of states and if there is it assigns the appropriate instantaneous transition rate to that transition. In order to obtain the transition matrix, the algorithm examines all six possible events which can cause system to make a transition and change its state.

Transitions caused by event 'M_1 finishes producing a part' change the state of M_1 when M_2 is not starving. When M_2 is starving, this event causes a change in the states of both machines. These changes are driven by the logic of the process which also takes the buffer transitions into account.

For all buffer levels other than the full, when M_1 finishes a part and M_2 is not starving at that time, M_1 adds a manufacturing or remanufacturing part to the end of the queue. The new state of M_1 is determined according to the part it starts to process and the quality state of the machine. Buffer state transition will be

- from n to $2n$, if the part added to the queue is a *manufacturing* part

- from n to $2n+1$, if the part added to the queue is a *remanufacturing* part

When M_2 is starving which means buffer is empty, the production of M_1 changes both of the machine states. The part finished by M_1 is taken by M_2 instantly and the buffer remains empty as it was before. The state of M_2 is determined according to the part it takes and its quality state before starvation.

When the buffer is full, M_1 can not add the part to the queue when it finishes one and it changes its state to the corresponding blocking state which contains the information of blocked part type and quality state of the machine. The buffer state of the system does not change.

Transitions caused by event 'M_2 finishes producing a part' change the state of M_2 when M_1 is not blocked. When M_1 is blocked, this event causes a change in the states of both machines. These changes are driven by the logic of the process which also takes the buffer transitions into account.

For all buffer levels other than the empty, when M_2 finishes a part and M_1 is not blocked at that time, M_2 takes the first part -a manufacturing or a remanufacturing part- from the queue. The new state of M_2 is determined according to the part it starts to process and the quality state of the machine. Buffer state transition will be

- from n to $n-2^{\lfloor \log_2(n) \rfloor -1}$, if the first part in the queue is a *manufacturing* part

- from n to $n-2^{\lfloor \log_2(n) \rfloor}$, if the first part in the queue is a *remanufacturing* part

When M_1 is blocked which means buffer is full, the production of M_2 changes both of the machine states. As soon as a part is taken from the queue by M_2, the part blocked in M_1 is added to the queue and M_1 takes a new part to the system. The buffer level remains full, however the buffer state chances as a result of added and removed parts to and from the queue. Buffer state transition will be

- from n to $2n - 2^{\lfloor \log_2(n) \rfloor}$, if both added and removed parts are *manufacturing* parts

- from n to $2n - 2^{\lfloor \log_2(n) \rfloor} + 1$, if the added part is a *remanufacturing* and the removed one is a *manufacturing* part.

- from n to $2n - 2^{\lfloor \log_2(n) \rfloor + 1}$, if the added part is a *manufacturing* and the removed one is a *remanufacturing* part.

- from n to $2n - 2^{\lfloor \log_2(n) \rfloor + 1} + 1$, if both added and removed parts are *remanufacturing* parts

When the buffer is empty, M_2 can not take a part from the queue and it changes its state to the corresponding starving state which contains the information of quality state of the machine. The buffer state of the system does not change.

Transitions caused by event 'M_1 fails' change the state of M_1 when M_1 is working and the rest of the system states remain same.

Similarly, transitions caused by event 'M_2 fails' changes the state of M_2 when M_2 is working and the rest of system states remain same.

Similarly, transitions caused by event 'M_2 fails' changes the state of M_2 when M_2 is working and the rest of system states remain same.

Transitions caused by event 'M_2 gets fixed' change the state of M_2 and buffer when M_1 is not blocked or the buffer is not empty in the same way as it changes for the event 'M_2 finishes producing a part'. When M_1 is blocked which means the buffer is full, the repair of M_2 triggers M_2 to take the first part from the queue and M_1 to send its blocked part to the queue and take a new part. At this situation buffer state changes as it changes for the event 'M_2 finishes producing a part' and M_1 is blocked. When the buffer is empty, M_2 eventually starves with a good quality state, and buffer remains empty.

5.1.1.5. Constructing the system of equations. Since all states communicate and are positive recurrent (starting from any state mean time to return that state is finite), sufficient condition for existence of limiting probabilities is satisfied. At the steady state

$$\begin{aligned} v_j P_j &= \sum_{k \neq j} q_{kj} P_k \\ \sum_j P_j &= 1 \end{aligned} \tag{5.1}$$

Similarly, we can write

$$\begin{aligned} \sum_{k \neq j} q_{kj} P_k - v_j P_j &= 0 \\ \sum_j P_j &= 1 \end{aligned} \tag{5.2}$$

In the matrix form we can rewrite the same equations as shown below where Q_{jk} is the state transition matrix, P_j is the column vector of steady state probabilities, I is an identity matrix with the dimensions of total number of states, v_j is a column vector, and 1 is a column vector composed of 1s with the dimension of total number of states x 1.

$$\left((Q_{jk})' - I\, v_j\right) P_j = 0 \tag{5.3}$$

$$P_j{}'1 = 1 \tag{5.3}$$

To remove linear dependency in equation (5.3) we replace the last equation at the last row (or one of the rows which can be selected arbitrarily) with equation (5.4) and denote this new matrix with *A*. Then the system of equations to obtain the steady state probabilities is $A\ P_j = b$ where *b* is the column vector of the changed right hand side.

5.1.1.6. Performance Measures. Algebraically solving $A\ P_j = b$ gives the steady state probabilities of all states which will be used calculating performance measures of the system P_T^N , P_E^N , $\bar{n}$, p_b and p_s. To obtain performance measures, steady state probabilities must be aggregated through the appropriate states and multiplied by relevant quantities if necessary. Calculations of P_T^N , P_E^N , $\bar{n}$, p_b and p_s are done as shown in the equations below.

$$P_T^N = \sum_{n=1}^{2^{N+1}-1} \sum_{\alpha_1 \in M1} \sum_{\alpha_2 = 0,1} P(\alpha_1, \alpha_2, n)\mu_{m2} + \sum_{n=1}^{2^{N+1}-1} \sum_{\alpha_1 \in M1} \sum_{\alpha_2 = \bar{0}, \bar{1}} P(\alpha_1, \alpha_2, n)\mu_{r2} \tag{5.5}$$

$$P_E^N = P_T^N\left(p_R \frac{P_{\bar{0}}^1}{P_{\bar{0}}^1 + P_{\bar{1}}^1}\frac{P_{\bar{0}}^2}{P_{\bar{0}}^2 + P_{\bar{1}}^2} + (1-p_R)\frac{P_0^1}{P_0^1 + P_1^1}\frac{P_0^2}{P_0^2 + P_1^2}\right) \tag{5.6}$$

$$\bar{n} = \sum_{n=1}^{2^{N+1}-1}\sum_{\alpha_1 \in M1}\sum_{\alpha_2 \in M2} P(\alpha_1,\alpha_2,n)\lfloor \log_2(n) \rfloor \tag{5.7}$$

$$p_b = \sum_{n=1}^{2^{N+1}-1}\sum_{\alpha_1 \in B}\sum_{\alpha_2 \in M2} P(\alpha_1,\alpha_2,n) \tag{5.8}$$

$$p_s = \sum_{n=1}^{2^{N+1}-1}\sum_{\alpha_1 \in M1}\sum_{\alpha_2 \in S} P(\alpha_1,\alpha_2,n) \tag{5.9}$$

where $W=\{0,\ 1, \bar{0},\ \bar{1}\}$, $B = \{\ b0,\ b1,\ b\bar{0},\ b\bar{1}\}$, $S = \{s0,\ s1\}$, $\mathrm{F} = \{F\}$, $M1 = W \cup B \cup F$, $M2 = W \cup S \cup F$ are the predefined sets of states.

5.2. Validation of Two Machine One Finite Buffer Model

The total and effective production rates have been compared with a discrete-event, discrete-part simulation for different cases and buffer sizes. Table 5.5 shows the absolute per cent differences of simulation and the algebraic solution algorithm. The parameters for these cases are shown in the Appendix A.

Table 5.5. Validation of two machine one finite buffer model

	N	**0**	**1**	**2**	**3**	**4**	**5**
Case 7	**PT**	7.67%	8.79%	9.35%	10.90%	10.44%	10.51%
	PE	8.28%	9.10%	4.91%	2.80%	2.26%	0.65%
Case 9	**PT**	11.78%	3.67%	6.27%	8.36%	7.91%	8.15%
	PE	16.16%	14.40%	7.21%	3.98%	1.86%	1.31%
Case 10	**PT**	12.17%	6.44%	7.43%	7.65%	8.38%	7.46%
	PE	13.85%	7.42%	1.96%	0.75%	2.11%	2.14%
Case 18	**PT**	6.98%	5.33%	8.31%	8.46%	8.87%	10.01%
	PE	10.85%	11.33%	3.74%	3.73%	1.92%	0.46%

Figures 5.2-5.5. show the per cent absolute differences for P^T and P^E. As it is seen from the figures error on P^E decreases when buffer size increases. It can be inferred that the variance of the zero buffer case is higher than the others and as buffer size increases the variability of the system decreases.

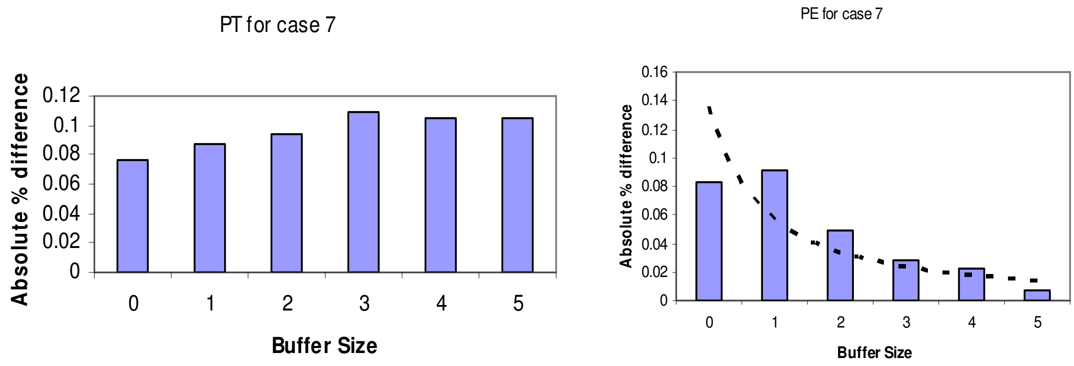

Figure 5.2. Absolute per cent differences of algebraic algorithm and simulation (Case7)

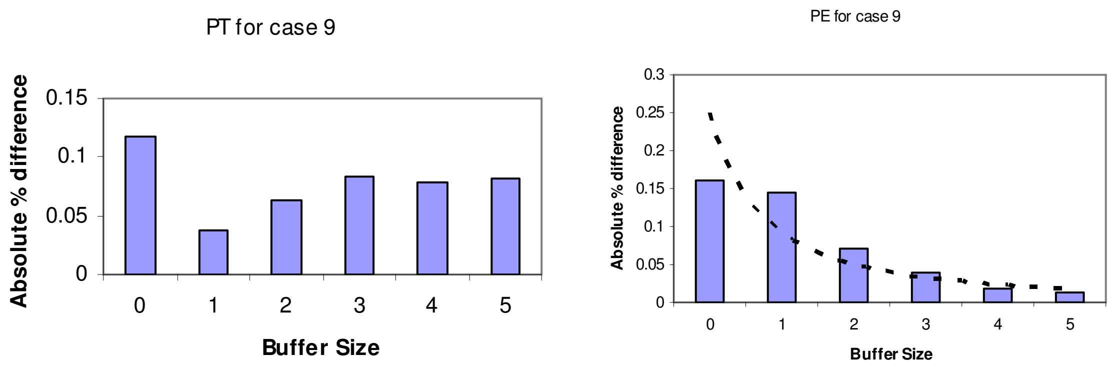

Figure 5.3. Absolute per cent differences of algebraic algorithm and simulation (Case9)

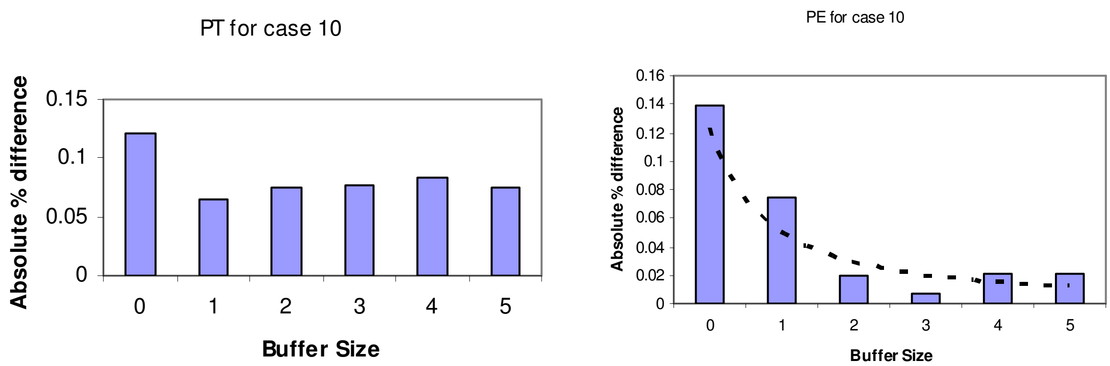

Figure 5.4. Absolute per cent differences of algebraic algorithm and simulation (Case10)

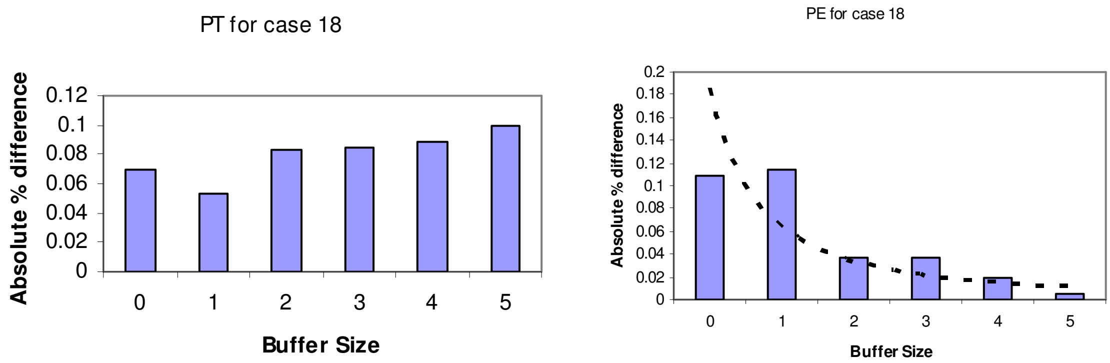

Figure 5.5. Absolute per cent differences of algebraic algorithm and simulation (Case18)

6. NUMERICAL RESULTS AND SENSITIVITY ANALYSIS

6.1. Single Machine Case

In this section some sensitivity analysis are presented to extract inference about the behaviors of the single machine model which will also be used in qualitative interpretations of 2M1B models. Moreover, sensitivity analysis validates the single machine model again by illustrating intuitively expected patterns. Rates of the single machine model obtained from system parameters were given in equation (4.6). Sensitivity analysis is done via observing the response of performance measures to changes in these parameters which are given in Appendix C for each case.

An increase in p_R means that the proportion of remanufactured parts in the system increases. As seen at Figure 6.1., this will increase P^T and decrease P^E because of the assumptions based on hybrid manufacturing systems' characteristics: Manufacturing a new part takes longer ($\mu_m < \mu_r$) and remanufactured parts cause more wear and tear (*g* < *t)*

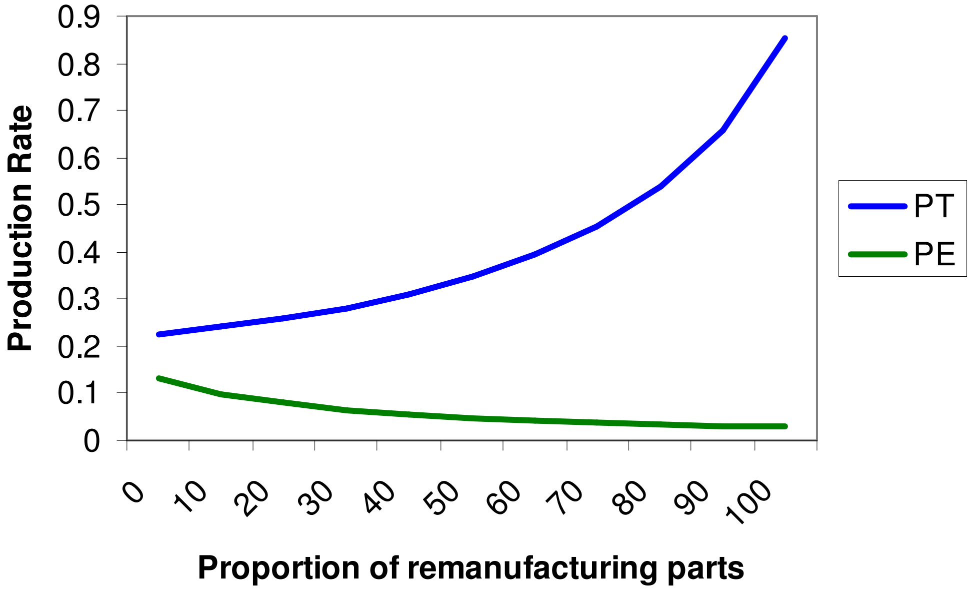

Figure 6.1. Sensitivity analysis for single machine model: Effect of p_R on P^T and P^E

As a consequence system yield decreases as illustrated in Figure 6.2.

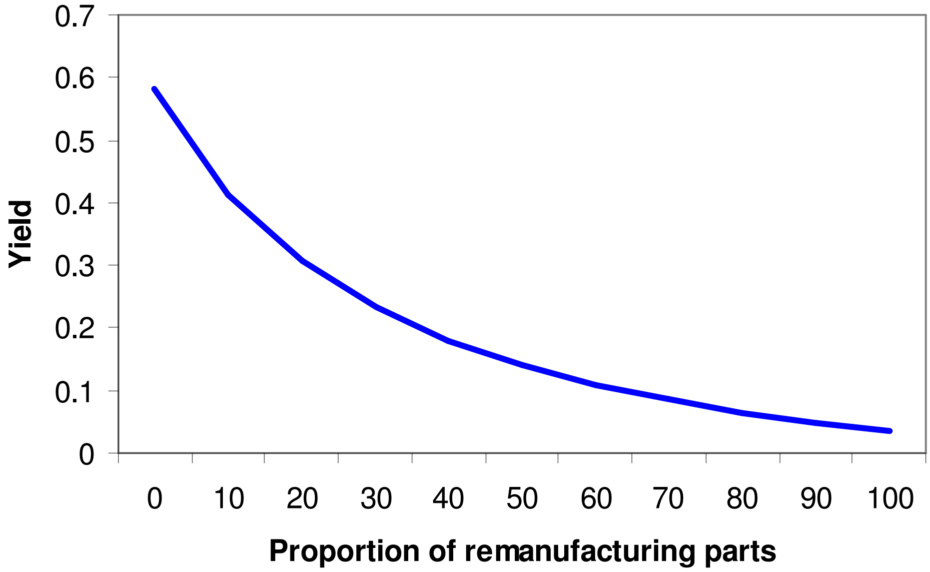

Figure 6.2. Sensitivity analysis for single machine model: Effect of p_R on system yield.

MTQF for remanufacturing operations decreases when an increase at $p^r{}_f$ occurs. That leads the system to have more quality failures and less stable operations. The system will spend more time at $\overline{1}$ state. There will be inspections which will stop machine for a repair (other than operational failures) at state $\overline{1}$. So machine will be at the repair state more frequently and the total time spent there will be longer. So both P^T and P^E will decrease with a decreasing slope as shown in Figure 6.3.

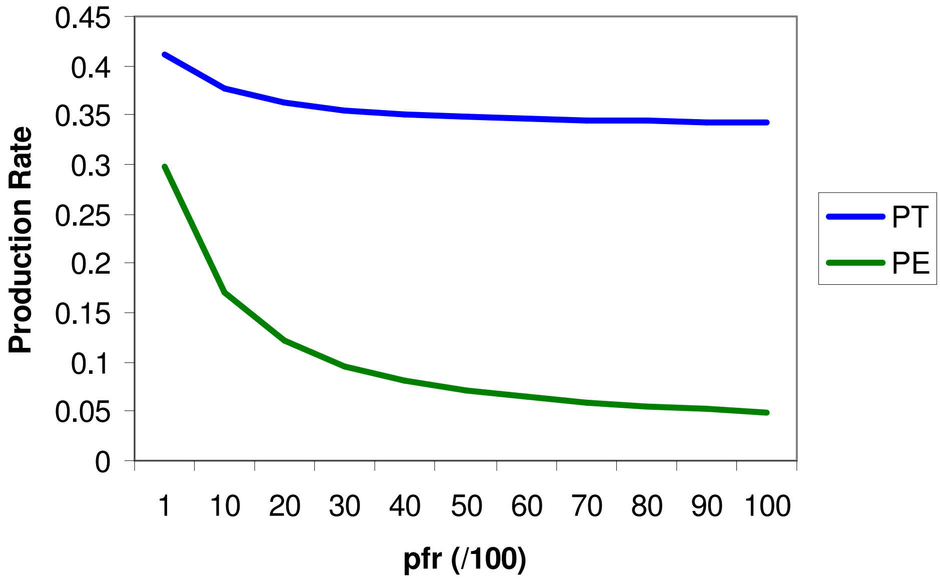

Figure 6.3. Sensitivity analysis for single machine model: Effect of p^r_f on P^T and P^E

The yield drastically decreases also when the system experiences more failures which is illustrated in Figure 6.4.

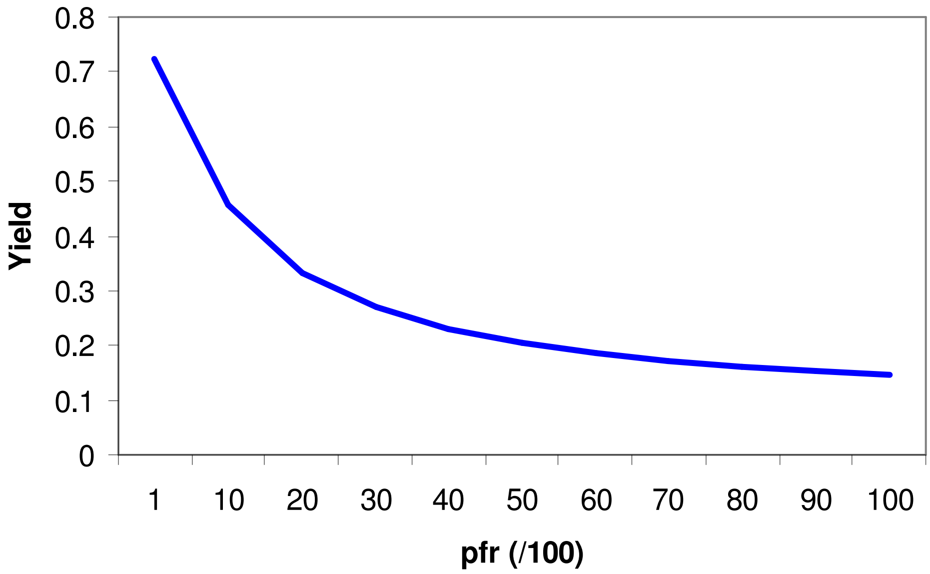

Figure 6.4. Sensitivity analysis for single machine model: Effect of p^m_f on system yield.

When *MTTD* increases, the system has less strict inspection which leads system to stay in bad production states longer and move to repair states less frequently. The effect of this situation on P^T and P^E turns out to be as seen is Figure 6.5. The system will spend more time on production states which increase P^T. But the source of this increase is the longer time spent at bad production states, so P^E worsens. The system yield diminishes with a decreasing slope as shown in Figure 6.6.

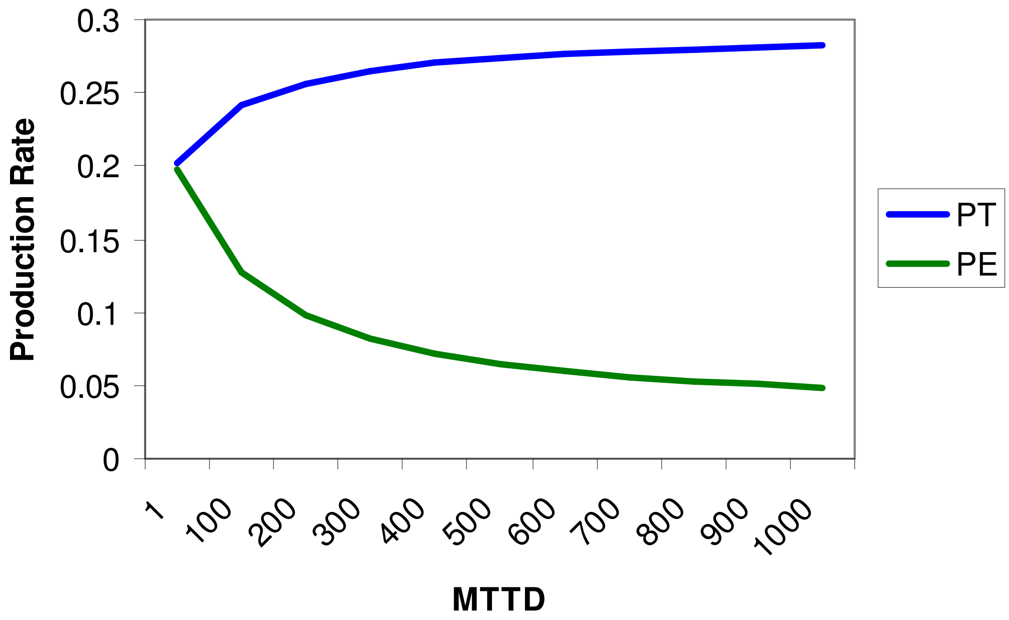

Figure 6.5. Sensitivity analysis for single machine model: Effect of *MTTD* on P^T and P^E

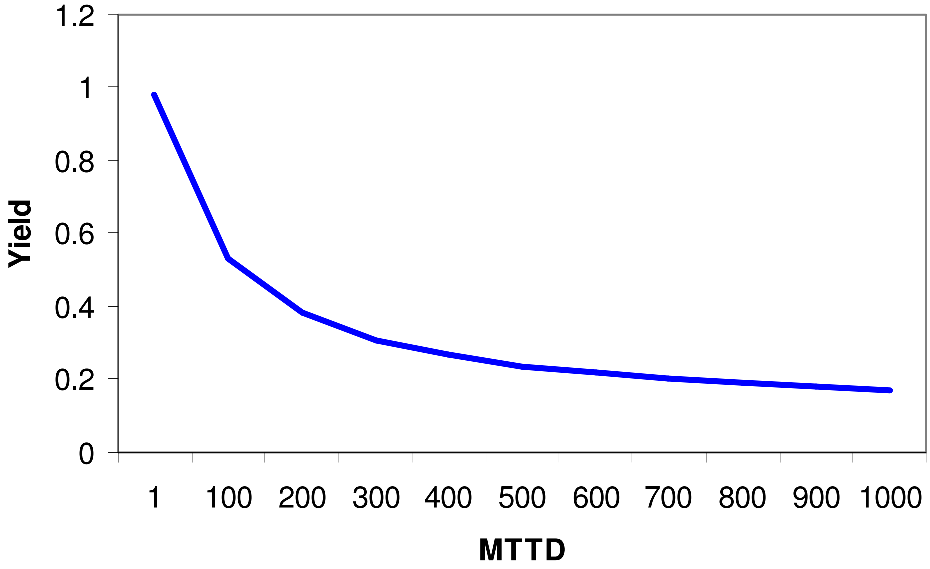

Figure 6.6. Sensitivity analysis for single machine model: Effect of *MTTD* on yield

An activity which will reduce the rates of operational failures (such as periodic maintenance) improves both P^T and P^E as illustrated in Figure 6.7.

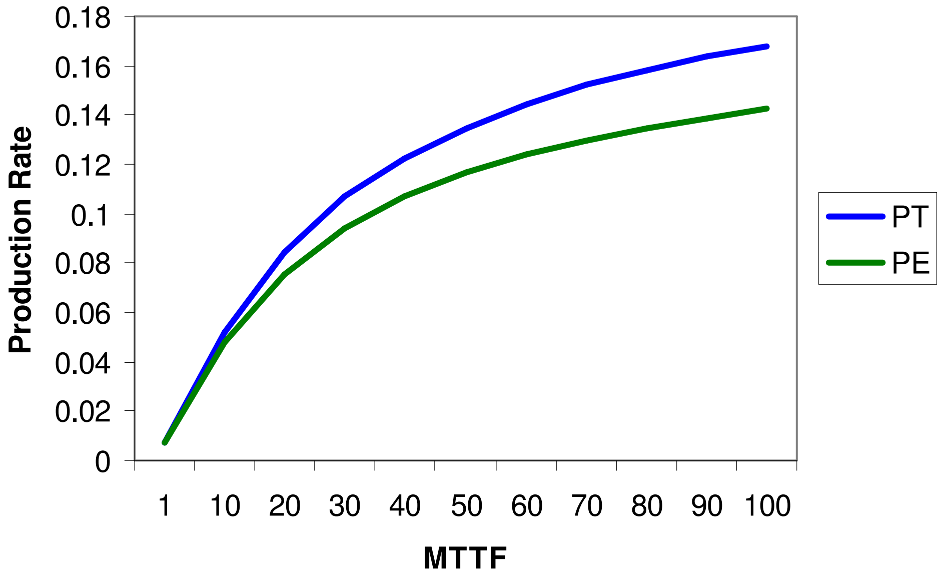

Figure 6.7. Sensitivity analysis for single machine model: Effect of *MTTF* on P^T and P^E

On the other hand, system yield will be affected negatively from this situation as seen in Figure 6.8, because operational failures sometimes catch the system on bad producing states and fix the quality state. But it should be noted that the decrease on the yield is not as much as in the increase of $p^r{}_f$ or the increase of *MTTD* instances in Figures 6.4 and 6.6.

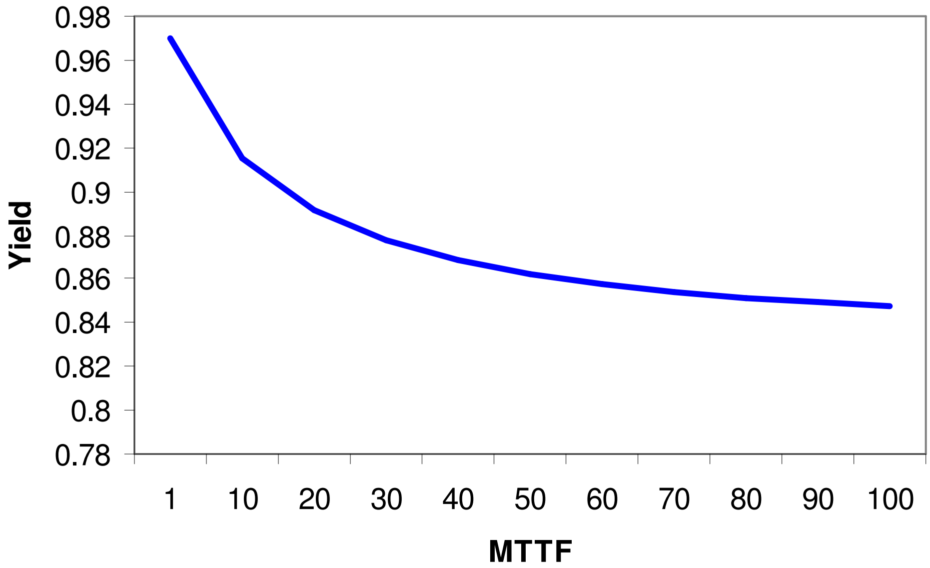

Figure 6.8. Sensitivity analysis for single machine model: Effect of *MTTF* on yield.

The time spent for a repair activity would be used for production if there were not any failures which will improve P^T and P^E. In Figure 6.9 the opposite situation is presented.

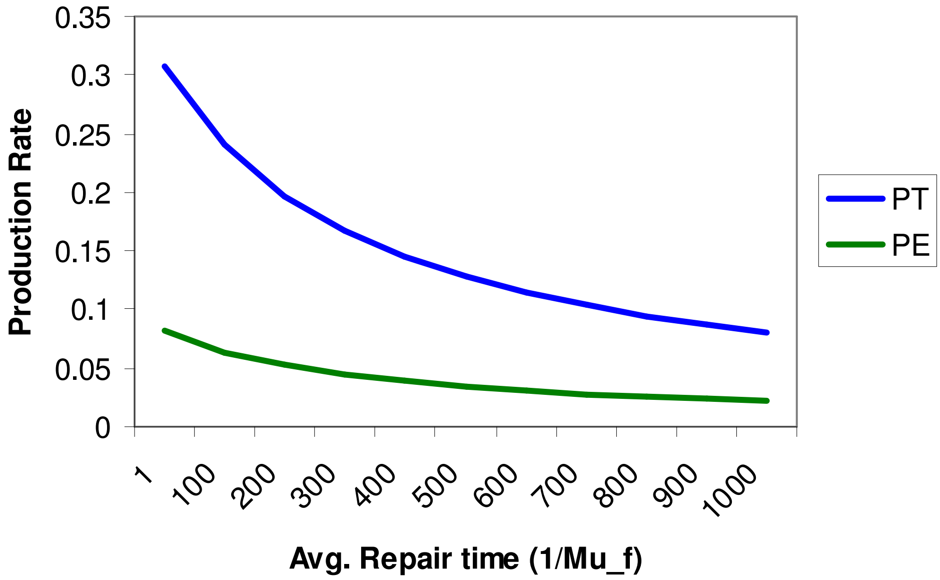

Figure 6.9. Sensitivity analysis for single machine model: Effect of average repair time on P^T and P^E

However the yield of the system is not affected for the fraction between reducing P^T and P^E stays constant. Since yield is a performance measure of the quality which is not related with time, but the number of parts produced, the time spent without production does not influence the yield.

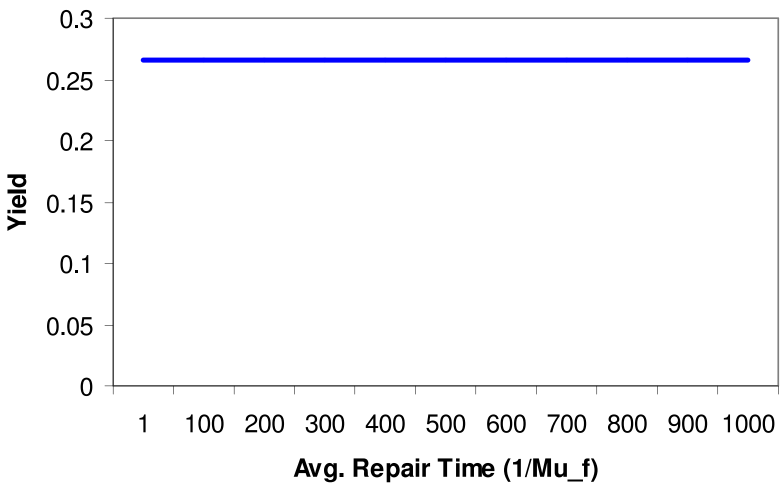

Figure 6.10. Sensitivity analysis for single machine model: Effect of μ_f on yield.

6.2. Special 2M1B Cases

This section includes sensitivity analysis of 2M1B hybrid transfer lines at extreme cases: when buffer size is zero and infinity. Finite buffer 2M1B systems will perform in the interval determined by these two extreme cases. Parameters used in sensitivity analysis are given in Appendix C.

As stated earlier, the first machine never suffers from blockage at the infinite buffer case whereas at the zero buffer case system gets blocked and starves most frequently. Thus according to the individual production rates of M_1 and M_2 system either tends to block or starve more often. There are cases where M_1 is faster or M_2 is faster. Each of the sensitivity analysis carried out in this section are done for these alternative two cases which are obtained by changing the places of the machines (or may be perceived as shifting the flow to opposite direction). Sensitivity figures of the alternative cases are illustrated in Appendix D.

As in the single machine case, when proportion of remanufacturing parts in the system increases, P^Ts increase for remanufacturing parts take less time to produce and P^Es decrease they cause more wear and tear.

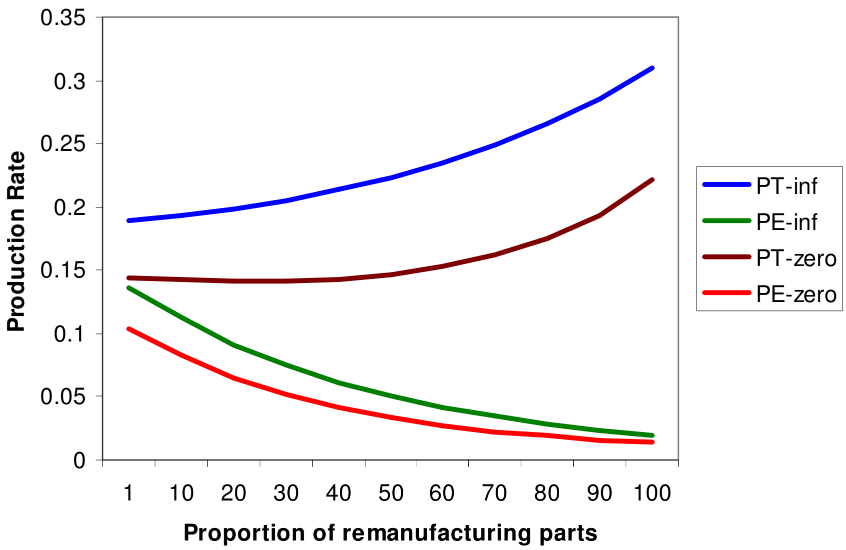

Figure 6.11. Sensitivity analysis for extreme cases: Effect of p_r on P^T and P^E

The system performance measures respond to an increase at *MTTD* of M_1 similarly for the two cases where M_1 has a greater individual production rate than M_2 or it has not - except the total production rate at infinite buffer-. When M_1 is faster, P_∞^T do not increase since M_2 determines the total production rate of the system. This situation is illustrated in Figure 6.12 and its alternative case is supplied in Appendix D.

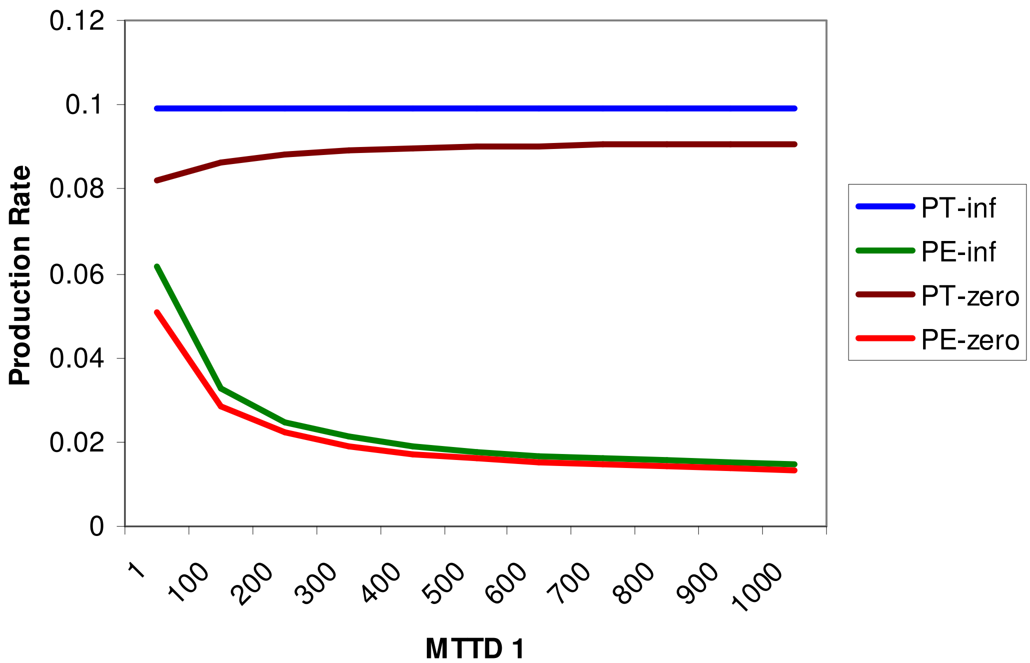

Figure 6.12. Sensitivity analysis for extreme cases: (1) Effect of *MTTD1* on P^T and P^E

When M_2 is faster, P_∞^T increases, because in that case *MTTD* increment means less starvation for M_2 as illustrated in Figure 6.13.

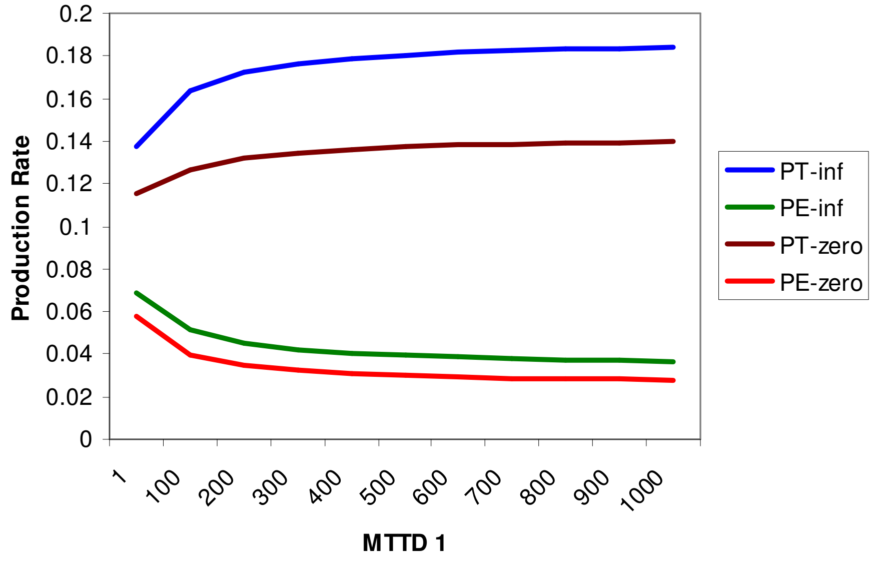

Figure 6.13. Sensitivity analysis for extreme cases: (2) Effect of *MTTD1* on P^T and P^E

An increase of *MTQF* for remanufacturing causes reduction at all performance measures as shown in Figure 6.14. A similar pattern is observed for *MTQF* increase for manufacturing. Alternative cases which are shown in the appendix D perform alike.

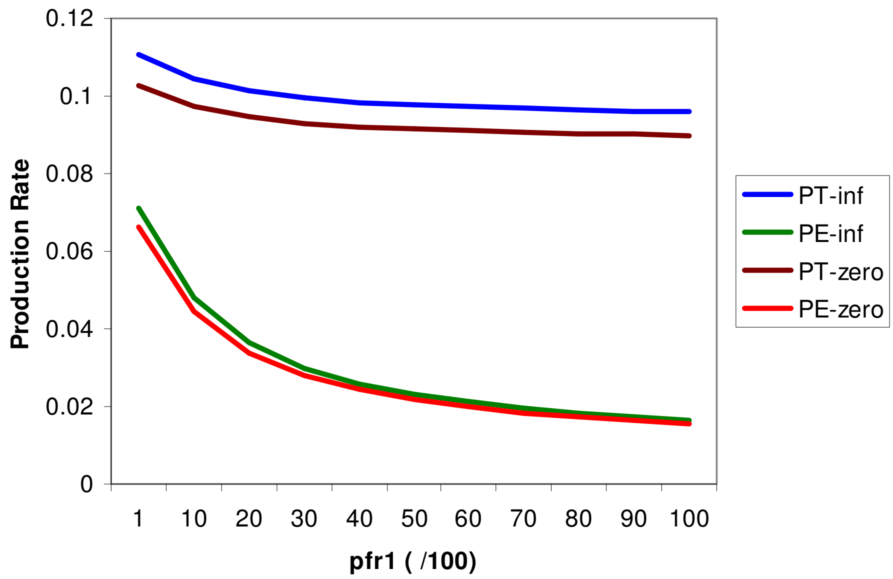

Figure 6.14. Sensitivity analysis for extreme cases: Effect of p^r_{f1} on P^T and P^E

In Figure 6.15 the dashed line represents the point at which the individual total production rate of M_1 is equal to of its M_2. The increase of μ_{m1} starts to cause starvation at 30 for the infinite buffer case which means at that point M_2 become faster. Also it should be noticed, both P^Es (zero and infinite) increase first, then they decrease. The increase occurs because manufacturing causes less quality failures. On the other hand, the reason of the decrease is that manufacturing parts start to take too much time even they have good quality.

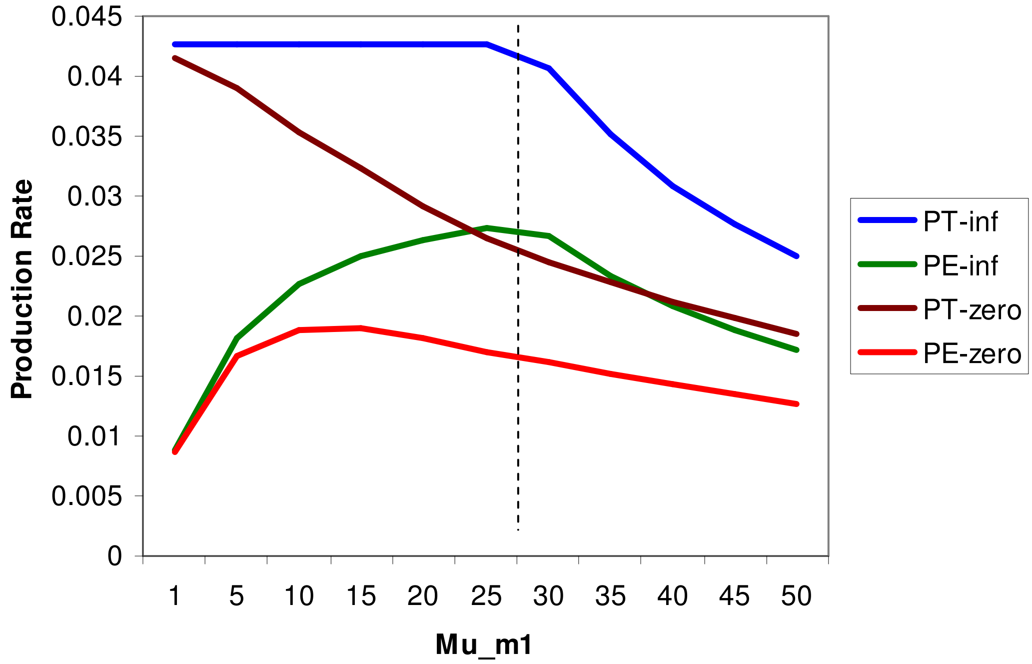

Figure 6.15. Sensitivity analysis for extreme cases: Effect of μ_{m1} on P^T and P^E

6.3. Two Machine One Finite Buffer Case

Sensitivity analysis of the finite buffer two machine line especially provides information about the effect of buffer size. Figure 6.16 shows trends of total and effective production rates changing with buffer size for case #10. Both of the production rates are increasing as buffer size increases and also it should be noted that they are converging to values of the infinite buffer size which are obtained by using the analytical solution.

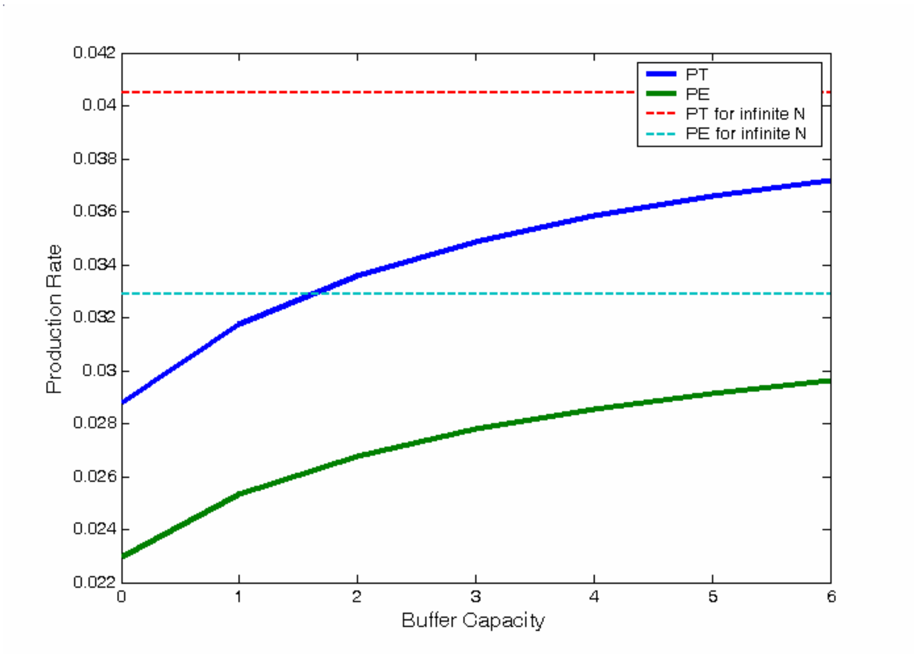

Figure 6.16. Production rates versus buffer capacity

Since the buffer offers more flexibility to the line, when there is more buffer capacity the system suffers from blockage and starvation less. Figure 6.17 illustrates blockage and starvation probabilities versus buffer size for case #10.

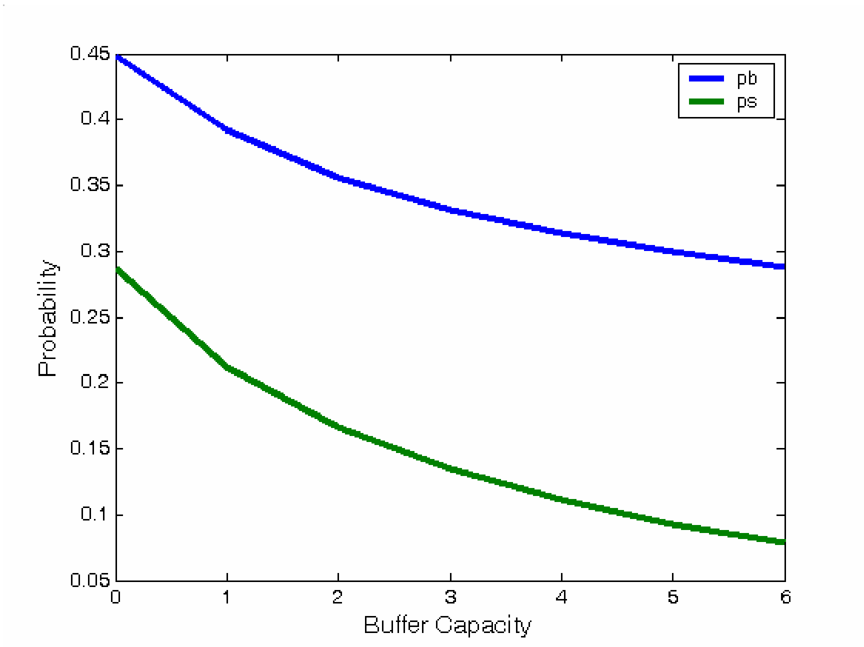

Figure 6.17. Blockage and starvation probabilities versus buffer capacity

Also the average inventory level increases when it is allowed to take more parts to buffer. This is demonstrated in Figure 6.18

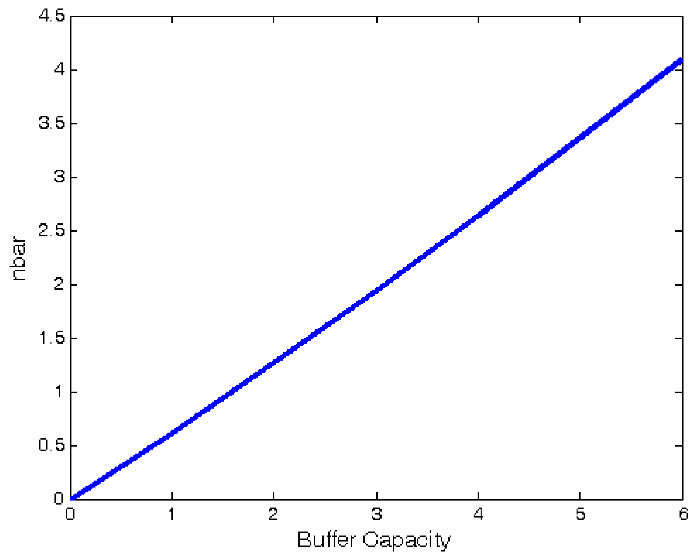

Figure 6.18. Average inventory level versus buffer capacity

The sensitivity analyses which are done for the previous cases are repeated in this section to comprehend the effect of buffer size on performance measures.

When *MTTD* of M_1 increases, there exists less stoppage reasoning from inspections. Since M_1 will be operational more, total production rate will improve whereas effective production decrease and average buffer level increases. The blockage probability also increases for average buffer gets closer to the buffer capacity, thus, the starvation probability decreases. Figure 6.19 shows the average buffer levels for different buffer capacities under the increase of *MTTD1*. The effects of p^r_{f1} on the other performance measures are illustrated in Appendix D.

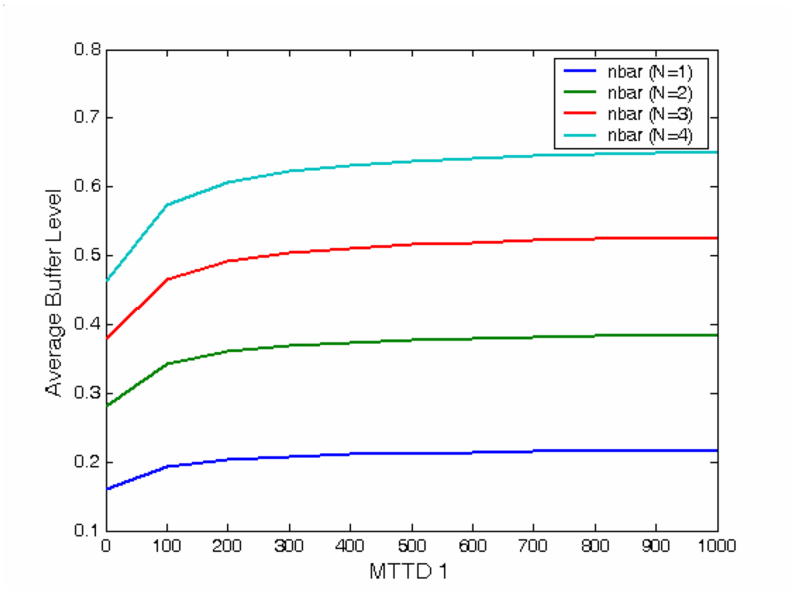

Figure 6.19. Sensitivity analysis for finite buffer case: Effect of *MTTD1* on average buffer level.

The figures of performance measure responses to a *MTQF1* incremental are given in appendix D which are as they are in previous sensitivity analysis. We can see from those figures that the first few increases in the buffer capacity affects more, then the curves start to get closer to each other.

The last analysis of section 5.2 where individual production rates of the system shift are reproduced here for intermediate buffer capacities. The increase of μ_{m1} starts to cause starvation after it becomes the slower machine. Small buffer capacity systems are more sensitive to such changes which are illustrated through Figure 6.20-6.22

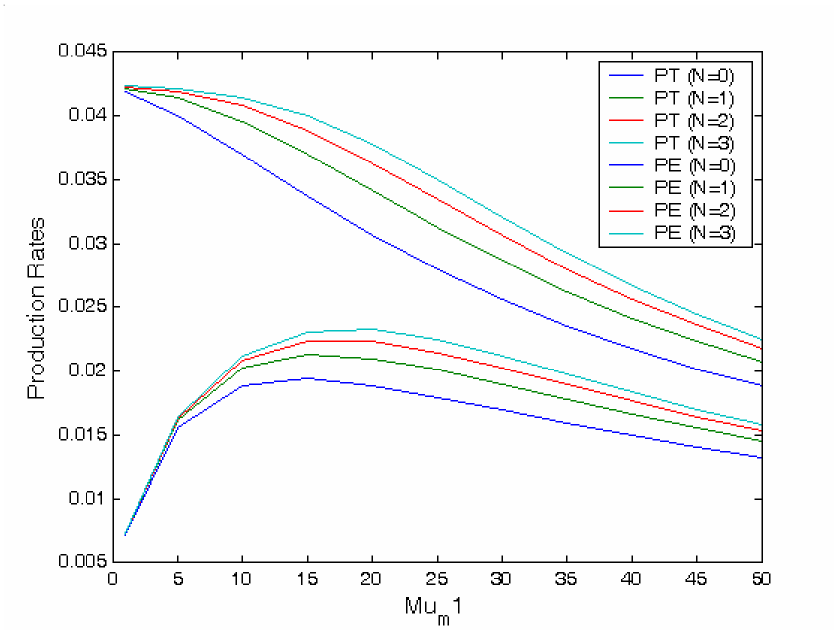

Figure 6.20. Sensitivity analysis for finite buffer case: Effect of μ_{m1} on average total and effective production rates

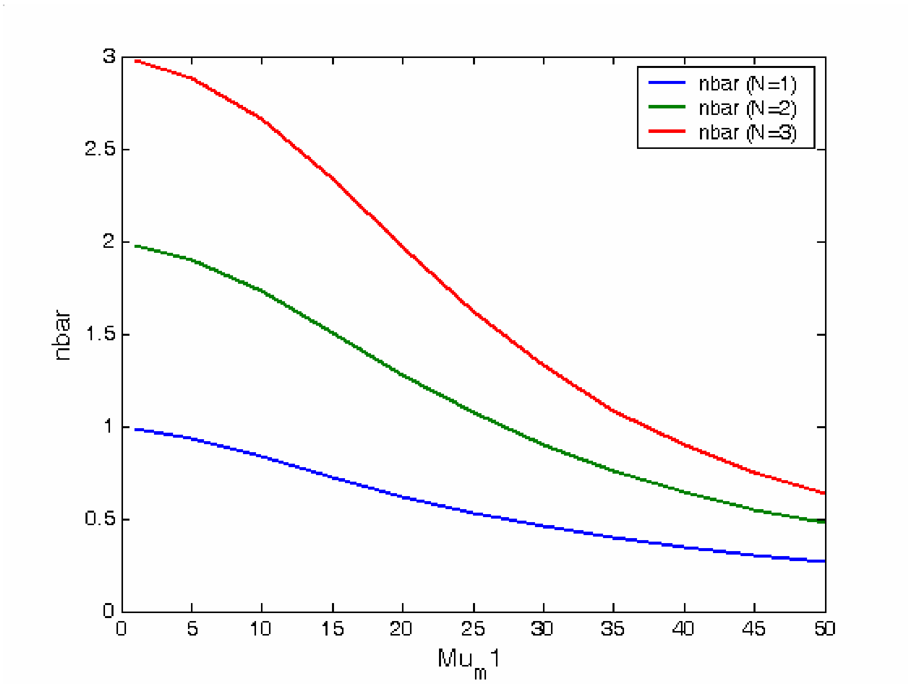

Figure 6.21. Sensitivity analysis for finite buffer case: Effect of μ_{m1} on nbar

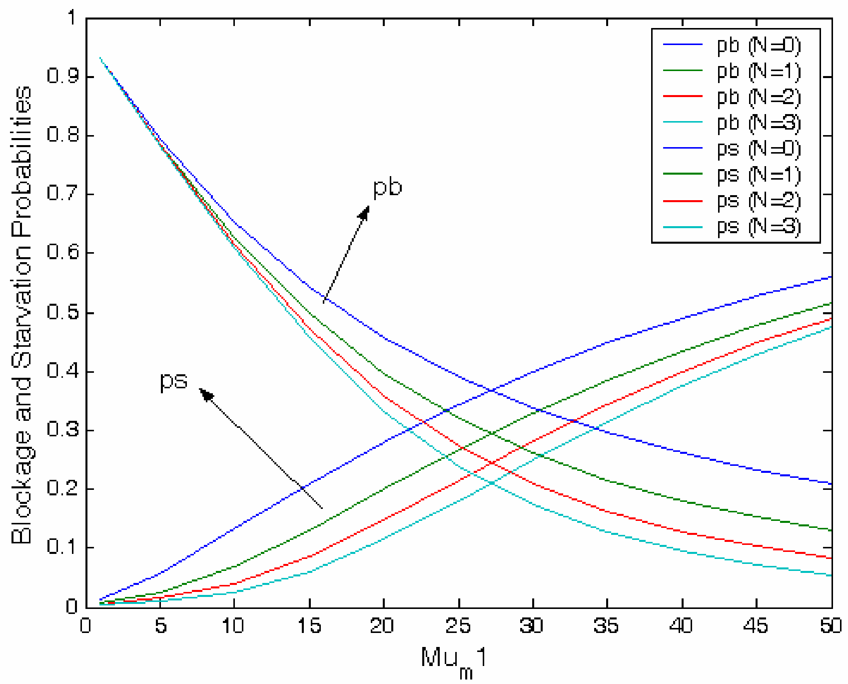

Figure 6.22. Sensitivity analysis for finite buffer case: Effect of μ_{m1} on blockage and starvation probabilities.

7. RESULTS

There are two major objectives in manufacturing shop floors which are valid also for hybrid manufacturing shop floors: To increase effective productivity and to increase quality. So, the obtained results from the analysis based on analytic models of the problem can be gathered according to these two main objectives. This section includes interpretations of sensitivity analysis and results.

7.1. To Increase Effective Productivity

Improving the individual throughput of each operation and increasing the buffer space are typical ways to increase the production rate of manufacturing systems (Kim and Gershwin, 2005). But if machines can have quality failures, there are some other ways to increase the effective production rate such as increasing the yield of each operation and making more strict inspections. Stabilizing operations (decreasing *MTTF, MTQF* for new and returned parts), thus improving the yield of individual operations, will increase effective throughput of the system as it is seen in Figures 6.3, 6.4, 6.7, 6.8 for single machine and 6.14 for two machines..

On the other hand, reducing the mean time to detect (*MTTD*) will increase the effective production rate only if the quality failure is persistent as it is assumed in our models. This situation is seen in Figures 6.5, 6.6, 6.12, 6.13 for different cases. In contrast, if the quality failure is Bernoulli which means the quality of each part is independent of the others, it will decrease the effective production rate. Therefore, stopping the line does not reduce the number of bad parts in the future. In a situation in which machines produce defective parts frequently and inspection is poor, increasing inspection reliability would be more effective than increasing buffer size to improve the effective production rate. Contrarily if inspection is reliable, and the machine still produces bad parts frequently, increasing machine stability would be more effective than increasing buffer size.

Manufacturing takes less time compared to remanufacturing; however remanufacturing causes more quality failures. Thus, from the effective productivity point of view the proportion of remanufacturing parts is important. Even though if the inspection operation is very strict, increasing the proportion of remanufacturing parts can result in less effective production rate depending on the repair times.

7.2. To Increase Quality

There are two major ways to improve quality. One is to increase the yield of individual operations and the other is to perform more strict inspection. Having preventive maintenance on machines and using robust engineering techniques to stabilize operations have been suggested as tools to increase yield of individual operations (Kim, 2005). Both approaches increase the Mean Time to Quality Failure (*MTQF*) (i.e. decrease g and t). On the other hand, the inspection policy aims to detect bad parts as soon as possible and prevent their flow to downstream operations. More strict inspection decreases the mean time to detect (*MTTD*) (increases h and therefore increases $f = h + p$). It is optimal to use a combination of both methods to improve quality.

Since returned parts cause more quality failures, the comparison of *MTQF* of returned items and new products is very important. Further action that can be done to increase quality is to reject remanufacturing by abandoning not only its benefits to system performance measures but also its economic attraction which is not considered in this study. Alternatively, using dedicated resources for different part types will give flexibility of adjusting different quality policies for different lines.

8. CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

Different characteristics of manufacturing and remanufacturing operations (raw materials and returned parts) add new aspects to quality and productivity dimensions of the hybrid production shop floor.

In this study, at first, a model has been developed and analyzed to evaluate the steady-state probabilities of a single machine of the hybrid production shop with quality and operational failures. Then by using the single machine model, extreme cases for 2M1B system are examined. A model for finite buffer systems is constructed and an algebraic solution algorithm is developed for the finite buffer 2M1B model. Finally, results derived from numerical experimentation and sensitivity analysis are obtained.

It is seen that in hybrid shop floor there are many different situations difficult to make appropriate decisions to increase productivity and quality for a production engineer who is not aware of behaviors of the system. This study contributes to practice by summarizing its analytical findings about the system. In addition developed models for zero, infinite and finite buffer provide background information to the researchers interested.

Then a decomposition technique, that divides a long transfer line into multiple 2-machine-1-buffer models, could be developed. Currently, we are working on 2M1B models. The results of our analysis can be used to decompose larger hybrid production systems. Furthermore, multiple failure modes can be considered in single machine and two machine lines for the hybrid shop floor.

For further work, estimation of performance measures for large buffer capacities and long line analysis with decomposition can be studied. Also to reveal the economic attraction of remanufacturing, models which take a cost function into account can make contribution. Finally, systems having both persistent type and Bernoulli type failures should be investigated.

APPENDIX A: PARAMETERS AND RATES FOR SIMULATIONS STUDIES

Table A. 1. Parameters for the single machine simulations.

Case #	1	2	3	4	5	6	7	8	9	10
p_R	0.25	0.25	0.333	0.333	0.5	0.25	0.25	0.333	0.5	0.5
$1/\mu_m$	10	10	10	10	9	10	10	10	10	9
$1/\mu_r$	2	2	5	2	2	2	2	10	2	2
p_f^m	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.05	0.01	0.1
p_f^r	0.05	0.1	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.3
MTTF	200	200	300	300	100	200	200	200	300	100
MTTD	100	100	300	300	50	200	200	200	300	50
$1/\mu_f$	20	10	20	10	10	20	20	20	10	10

Table A. 2. Rates for the single machine simulations.

Case #	1	2	3	4	5	6	7	8	9	10
r	0.9	0.9	0.9	0.9	1	0.9	0.9	1	0.9	1
m	13.5	13.5	3.6	9	4.5	13.5	13.5	2	4.5	4.5
g	0.001	0.001	0.001	0.001	0.0011	0.001	0.001	0.005	0.001	0.0111
t	0.025	0.05	0.01	0.025	0.025	0.025	0.025	0.005	0.025	0.15
p	0.005	0.005	0.0033	0.0033	0.01	0.005	0.005	0.005	0.0033	0.01
f	0.015	0.015	0.0067	0.0067	0.03	0.01	0.01	0.01	0.0067	0.03
k	0.0556	0.1111	0.0556	0.1111	0.1	0.0556	0.0556	0.05	0.1111	0.1
l	0.0037	0.0074	0.0139	0.0111	0.0222	0.0037	0.0037	0.025	0.0222	0.0222

Table A. 3. Parameters for the infinite buffer simulations

Case #	1	2	3	4	5	6	7	8	9	10
p_R	0.10	0.30	0.20	0.30	0.20	0.30	0.40	0.25	0.40	0.30
p_{f1}^{m}	0.01	0.02	0.01	0.10	0.01	0.02	0.05	0.01	0.10	0.01
p_{f1}^{r}	0.05	0.10	0.05	0.20	0.05	0.10	0.15	0.05	0.20	0.01
μ_{m1}	10	10	10	10	25	20	25	25	20	10
μ_{r1}	5	5	5	5	15	10	10	10	15	10
λ_{f1}	400	400	400	300	400	400	200	500	500	400
λ_{d1}	100	200	200	100	200	200	100	50	50	100
μ_{f1}	30	60	60	60	60	60	60	120	120	30
μ_{m2}	10	10	20	25	10	15	20	20	25	20
μ_{r2}	5	5	10	15	5	5	15	15	5	10
p_{f2}^{m}	0.01	0.02	0.01	0.02	0.01	0.02	0.02	0.01	0.10	0.02
p_{f2}^{r}	0.05	0.10	0.05	0.10	0.05	0.10	0.08	0.05	0.20	0.08
μ_{f2}	30	60	60	60	60	60	60	60	120	60
λ_{d2}	100	200	200	100	200	200	100	100	50	100
λ_{f2}	400	400	400	300	400	400	300	300	500	400
N	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞

Table A. 4. Rates for the infinite buffer simulations

Case #	1	2	3	4	5	6	7	8	9	10
r_1	0.01	0.03	0.02	0.03	0.008	0.015	0.016	0.01	0.02	0.03
m_1	0.18	0.14	0.16	0.14	0.0533	0.07	0.06	0.075	0.04	0.07
g_1	0.001	0.002	0.001	0.01	0.0004	0.001	0.002	0.0004	0.005	0.001
t_1	0.01	0.02	0.01	0.04	0.0033	0.01	0.015	0.005	0.01333	0.001
p_1	0.0025	0.0025	0.0025	0.0033	0.0025	0.0025	0.005	0.002	0.002	0.0025
$f_1=h_1+p_1$	0.0125	0.0075	0.0075	0.0133	0.0075	0.0075	0.015	0.022	0.022	0.0125
k_1	0.0315	0.0137	0.0148	0.0137	0.0144	0.0137	0.0132	0.0073	0.0055	0.0233
l_1	0.0017	0.0029	0.0018	0.0029	0.0021	0.0029	0.0035	0.0009	0.0027	0.01
r_2	0.01	0.03	0.01	0.012	0.02	0.02	0.02	0.0125	0.016	0.015
m_2	0.18	0.14	0.08	0.0466	0.16	0.14	0.04	0.05	0.12	0.07
g_2	0.001	0.002	0.0005	0.0008	0.001	0.0013	0.001	0.0005	0.004	0.001
t_2	0.01	0.02	0.005	0.0066	0.01	0.02	0.005	0.0033	0.04	0.008
p_2	0.0025	0.0025	0.0025	0.0033	0.0025	0.0025	0.003	0.0033	0.002	0.0025
$f_2=h_2+p_2$	0.0125	0.0075	0.0075	0.0133	0.0075	0.0075	0.013	0.0133	0.022	0.0125
k_2	0.0315	0.0137	0.0148	0.0132	0.0148	0.0145	0.011	0.0133	0.0073	0.0137
l_2	0.0017	0.0029	0.0018	0.0034	0.0018	0.002	0.006	0.0033	0.0009	0.0029

Table A. 5. Parameters for the zero and finite buffer simulations

Case #	1	2	3	4	5	6	7	8	9
p_R	0.1	0.3	0.2	0.3	0.2	0.3	0.4	0.25	0.3
p_{f1}^{m}	0.01	0.02	0.01	0.1	0.01	0.02	0.05	0.01	0.01
p_{f1}^{r}	0.05	0.1	0.05	0.2	0.05	0.1	0.15	0.05	0.01
μ_{m1}	0.1	0.1	0.1	0.1	0.04	0.05	0.04	0.04	0.1
μ_{r1}	0.2	0.2	0.2	0.2	0.0666	0.1	0.1	0.1	0.1
λ_{f1}	0.0025	0.0025	0.0025	0.00333	0.0025	0.0025	0.005	0.002	0.0025
λ_{d1}	0.01	0.005	0.005	0.01	0.005	0.005	0.01	0.02	0.01
μ_{f1}	0.0333	0.0166	0.0166	0.0166	0.0166	0.0166	0.0166	0.0083	0.0333
μ_{m2}	0.1	0.1	0.05	0.04	0.1	0.0666	0.05	0.05	0.05
μ_{r2}	0.2	0.2	0.1	0.0666	0.2	0.2	0.0666	0.0666	0.1
p_{f2}^{m}	0.01	0.02	0.01	0.02	0.01	0.02	0.02	0.01	0.02
p_{f2}^{r}	0.05	0.1	0.05	0.1	0.05	0.1	0.08	0.05	0.08
μ_{f2}	0.0333	0.0166	0.0166	0.0166	0.0166	0.0166	0.0166	0.0166	0.0166
λ_{d2}	0.01	0.005	0.005	0.01	0.005	0.005	0.01	0.01	0.01
λ_{f2}	0.0025	0.0025	0.0025	0.0033	0.0025	0.0025	0.0033	0.0033	0.0025
N	0	0	0	0	0	0	0	0	0

Table A. 6. Parameters for the zero and finite buffer simulations-continued

Case #	10	11	12	13	14	15	16	17	18
p_R	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.0001
p_{f1}^{m}	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
p_{f1}^{r}	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
μ_{m1}	0.0667	0.05	0.125	0.0625	0.0666	0.05	0.0666	0.05	0.05
μ_{r1}	0.125	0.0625	0.25	0.0833	0.125	0.1	0.1	0.125	0.125
λ_{f1}	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
λ_{d1}	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
μ_{f1}	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
μ_{m2}	0.05	0.0666	0.0625	0.125	0.05	0.0666	0.05	0.0666	0.0666
μ_{r2}	0.0625	0.125	0.0833	0.25	0.1	0.125	0.125	0.1	0.1
p_{f2}^{m}	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
p_{f2}^{r}	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
μ_{f2}	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
λ_{d2}	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
λ_{f2}	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
N	0	0	0	0	0	0	0	0	0

Table A. 7. Rates for the zero and finite buffer simulations

Case #	1	2	3	4	5	6	7	8	9
r_1	0.0100	0.0300	0.0200	0.0300	0.0080	0.0150	0.0160	0.0100	0.0300
m_1	0.1800	0.1400	0.1600	0.1400	0.0533	0.0700	0.0600	0.0750	0.0700
g_1	0.0010	0.0020	0.0010	0.0100	0.0004	0.0010	0.0020	0.0004	0.0010
t_1	0.0100	0.0200	0.0100	0.0400	0.0033	0.0100	0.0150	0.0050	0.0010
p_1	0.0025	0.0025	0.0025	0.0033	0.0025	0.0025	0.0050	0.0020	0.0025
$f_1=h_1+p_1$	0.0125	0.0075	0.0075	0.0133	0.0075	0.0075	0.0150	0.0220	0.0125
k_1	0.0300	0.0117	0.0133	0.0117	0.0133	0.0117	0.0100	0.0063	0.0233
l_1	0.0033	0.0050	0.0033	0.0050	0.0033	0.0050	0.0067	0.0021	0.0100
r_2	0.0100	0.0300	0.0100	0.0120	0.0200	0.0200	0.0200	0.0125	0.0150
m_2	0.1800	0.1400	0.0800	0.0467	0.1600	0.1400	0.0400	0.0500	0.0700
g_2	0.0010	0.0020	0.0005	0.0008	0.0010	0.0013	0.0010	0.0005	0.0010
t_2	0.0100	0.0200	0.0050	0.0067	0.0100	0.0200	0.0053	0.0033	0.0080
p_2	0.0025	0.0025	0.0025	0.0033	0.0025	0.0025	0.0033	0.0033	0.0025
$f_2=h_2+p_2$	0.0125	0.0075	0.0075	0.0133	0.0075	0.0075	0.0133	0.0133	0.0125
k_2	0.0300	0.0117	0.0133	0.0117	0.0133	0.0117	0.0100	0.0125	0.0117
l_2	0.0033	0.0050	0.0033	0.0050	0.0033	0.0050	0.0067	0.0042	0.0050

Table A. 8. Rates for the zero and finite buffer simulations - continued

Case #	10	11	12	13	14	15	16	17	18
r_1	0.0167	0.0125	0.0313	0.0156	0.0167	0.0125	0.0167	0.0125	0.0000
m_1	0.0938	0.0469	0.1875	0.0625	0.0938	0.0750	0.0750	0.0938	0.1250
g_1	0.0007	0.0005	0.0013	0.0006	0.0007	0.0005	0.0007	0.0005	0.0005
t_1	0.0063	0.0031	0.0125	0.0042	0.0063	0.0050	0.0050	0.0063	0.0063
p_1	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050
$f_1=h_1+p_1$	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150
k_1	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0200
l_1	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0000
r_2	0.0125	0.0167	0.0156	0.0313	0.0125	0.0167	0.0125	0.0167	0.0000
m_2	0.0469	0.0938	0.0625	0.1875	0.0750	0.0938	0.0938	0.0750	0.1000
g_2	0.0010	0.0013	0.0013	0.0025	0.0010	0.0013	0.0010	0.0013	0.0013
t_2	0.0044	0.0088	0.0058	0.0175	0.0070	0.0088	0.0088	0.0070	0.0070
p_2	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050
$f_2=h_2+p_2$	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150
k_2	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0200
l_2	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.000002

APPENDIX B: SYSTEM STATES FOR DIFFERENT BUFFER SIZES

Table B. 1 System states for N = 0.

α_1	α_2	n	α_1	α_2	n	α_1	α_2	n	α_1	α_2	n
0	0	[1]	0bar	1	[1]	b0	0bar	[1]	b1bar	0bar	[1]
0	1	[1]	0bar	0bar	[1]	b0	1bar	[1]	b1bar	1bar	[1]
0	0bar	[1]	0bar	1bar	[1]	b0	F	[1]	b1bar	F	[1]
0	1bar	[1]	0bar	s0	[1]	b1	0	[1]	F	0	[1]
0	s0	[1]	0bar	s1	[1]	b1	1	[1]	F	1	[1]
0	s1	[1]	0bar	F	[1]	b1	0bar	[1]	F	0bar	[1]
0	F	[1]	1bar	0	[1]	b1	1bar	[1]	F	1bar	[1]
1	0	[1]	1bar	1	[1]	b1	F	[1]	F	s0	[1]
1	1	[1]	1bar	0bar	[1]	b0bar	0	[1]	F	s1	[1]
1	0bar	[1]	1bar	1bar	[1]	b0bar	1	[1]	F	F	[1]
1	1bar	[1]	1bar	s0	[1]	b0bar	0bar	[1]			
1	s0	[1]	1bar	s1	[1]	b0bar	1bar	[1]			
1	s1	[1]	1bar	F	[1]	b0bar	F	[1]			
1	F	[1]	b0	0	[1]	b1bar	0	[1]			
0bar	0	[1]	b0	1	[1]	b1bar	1	[1]			

Table B. 2. System states for N =1

α_1	α_2	n	α_1	α_2	n	α_1	α_2	n	α_1	α_2	n	α_1	α_2	n
0	0	[1]	1bar	s0	[1]	1bar	0	[2]	F	0	[2]	b0	0	[3]
0	1	[1]	1bar	s1	[1]	1bar	1	[2]	F	1	[2]	b0	1	[3]
0	0bar	[1]	1bar	F	[1]	1bar	0bar	[2]	F	0bar	[2]	b0	0bar	[3]
0	1bar	[1]	F	0	[1]	1bar	1bar	[2]	F	1bar	[2]	b0	1bar	[3]
0	s0	[1]	F	1	[1]	1bar	F	[2]	F	F	[2]	b0	F	[3]
0	s1	[1]	F	0bar	[1]	b0	0	[2]	0	0	[3]	b1	0	[3]
0	F	[1]	F	1bar	[1]	b0	1	[2]	0	1	[3]	b1	1	[3]
1	0	[1]	F	s0	[1]	b0	0bar	[2]	0	0bar	[3]	b1	0bar	[3]
1	1	[1]	F	s1	[1]	b0	1bar	[2]	0	1bar	[3]	b1	1bar	[3]
1	0bar	[1]	F	F	[1]	b0	F	[2]	0	F	[3]	b1	F	[3]
1	1bar	[1]	0	0	[2]	b1	0	[2]	1	0	[3]	b0bar	0	[3]
1	s0	[1]	0	1	[2]	b1	1	[2]	1	1	[3]	b0bar	1	[3]
1	s1	[1]	0	0bar	[2]	b1	0bar	[2]	1	0bar	[3]	b0bar	0bar	[3]
1	F	[1]	0	1bar	[2]	b1	1bar	[2]	1	1bar	[3]	b0bar	1bar	[3]
0bar	0	[1]	0	F	[2]	b1	F	[2]	1	F	[3]	b0bar	F	[3]
0bar	1	[1]	1	0	[2]	b0bar	0	[2]	0bar	0	[3]	b1bar	0	[3]
0bar	0bar	[1]	1	1	[2]	b0bar	1	[2]	0bar	1	[3]	b1bar	1	[3]
0bar	1bar	[1]	1	0bar	[2]	b0bar	0bar	[2]	0bar	0bar	[3]	b1bar	0bar	[3]
0bar	s0	[1]	1	1bar	[2]	b0bar	1bar	[2]	0bar	1bar	[3]	b1bar	1bar	[3]
0bar	s1	[1]	1	F	[2]	b0bar	F	[2]	0bar	F	[3]	b1bar	F	[3]
0bar	F	[1]	0bar	0	[2]	b1bar	0	[2]	1bar	0	[3]	F	0	[3]
1bar	0	[1]	0bar	1	[2]	b1bar	1	[2]	1bar	1	[3]	F	1	[3]
1bar	1	[1]	0bar	0bar	[2]	b1bar	0bar	[2]	1bar	0bar	[3]	F	0bar	[3]
1bar	0bar	[1]	0bar	1bar	[2]	b1bar	1bar	[2]	1bar	1bar	[3]	F	1bar	[3]
1bar	1bar	[1]	0bar	F	[2]	b1bar	F	[2]	1bar	F	[3]	F	F	[3]

APPENDIX C: PARAMETERS FOR SENSITIVITY ANALYSIS

Table C. 1. Parameters for single machine sensitivity analysis: p_R

Case #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
p_R	1	10	20	30	40	50	60	70	80	90	100
$1/\mu_m$	4	4	4	4	4	4	4	4	4	4	4
$1/\mu_r$	1	1	1	1	1	1	1	1	1	1	1
p_f^m	1	1	1	1	1	1	1	1	1	1	1
p_f^r	10	10	10	10	10	10	10	10	10	10	10
$1/\mu_f$	50	50	50	50	50	50	50	50	50	50	50
MTTD	400	400	400	400	400	400	400	400	400	400	400
MTTF	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

Table C. 2. Parameters for single machine sensitivity analysis: p^r_f

Case #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
p_R	25	25	25	25	25	25	25	25	25	25	25
$1/\mu_m$	2	2	2	2	2	2	2	2	2	2	2
$1/\mu_r$	1	1	1	1	1	1	1	1	1	1	1
p_f^m	1	1	1	1	1	1	1	1	1	1	1
p_f^r	1	10	20	30	40	50	60	70	80	90	100
$1/\mu_f$	50	50	50	50	50	50	50	50	50	50	50
MTTD	100	100	100	100	100	100	100	100	100	100	100
MTTF	200	200	200	200	200	200	200	200	200	200	200

Table C. 3. Parameters for single machine sensitivity analysis: *MTTD*

Case #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
p_R	25	25	25	25	25	25	25	25	25	25	25
$1/\mu_m$	4	4	4	4	4	4	4	4	4	4	4
$1/\mu_r$	1	1	1	1	1	1	1	1	1	1	1
p_f^m	1	1	1	1	1	1	1	1	1	1	1
p_f^r	10	10	10	10	10	10	10	10	10	10	10
$1/\mu_f$	50	50	50	50	50	50	50	50	50	50	50
MTTD	1	100	200	300	400	500	600	700	800	900	1000
MTTF	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

Table C. 4. Parameters for single machine sensitivity analysis: *MTTF*

Case #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
p_R	25	25	25	25	25	25	25	25	25	25	25
$1/\mu_m$	4	4	4	4	4	4	4	4	4	4	4
$1/\mu_r$	1	1	1	1	1	1	1	1	1	1	1
p_f^m	1	1	1	1	1	1	1	1	1	1	1
p_f^r	10	10	10	10	10	10	10	10	10	10	10
$1/\mu_f$	50	50	50	50	50	50	50	50	50	50	50
MTTD	20	20	20	20	20	20	20	20	20	20	20
MTTF	1	10	20	30	40	50	60	70	80	90	100

Table C. 5. Parameters for single machine sensitivity analysis: μ_f

Case #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
p_R	25	25	25	25	25	25	25	25	25	25	25
$1/\mu_m$	4	4	4	4	4	4	4	4	4	4	4
$1/\mu_r$	1	1	1	1	1	1	1	1	1	1	1
p_f^m	1	1	1	1	1	1	1	1	1	1	1
p_f^r	10	10	10	10	10	10	10	10	10	10	10
$1/\mu_f$	1	100	200	300	400	500	600	700	800	900	1000
MTTD	400	400	400	400	400	400	400	400	400	400	400
MTTF	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

Table C. 6. Parameters of sensitivity analysis for extreme cases: p_r

Case #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
p_R	1	10	20	30	40	50	60	70	80	90	99.9
p_{f1}^m	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
p_{f1}^r	5	5	5	5	5	5	5	5	5	5	5
μ_{m1}	2	2	2	2	2	2	2	2	2	2	2
μ_{r1}	1	1	1	1	1	1	1	1	1	1	1
λ_{f1}	200	200	200	200	200	200	200	200	200	200	200
λ_{d1}	100	100	100	100	100	100	100	100	100	100	100
μ_{f1}	50	50	50	50	50	50	50	50	50	50	50
μ_{m2}	4	4	4	4	4	4	4	4	4	4	4
μ_{r2}	2	2	2	2	2	2	2	2	2	2	2
p_{f2}^m	1	1	1	1	1	1	1	1	1	1	1
p_{f2}^r	8	8	8	8	8	8	8	8	8	8	8
μ_{f2}	50	50	50	50	50	50	50	50	50	50	50
λ_{d2}	100	100	100	100	100	100	100	100	100	100	100
λ_{f2}	200	200	200	200	200	200	200	200	200	200	200

Table C. 7. Parameters of sensitivity analysis for extreme cases: (1) *MTTD1*

Case #	1	2	3	4	5	6	7	8	9	10	11
p_R	25	25	25	25	25	25	25	25	25	25	25
p_{f1}^m	1	1	1	1	1	1	1	1	1	1	1
p_{f1}^r	5	5	5	5	5	5	5	5	5	5	5
μ_{m1}	2	2	2	2	2	2	2	2	2	2	2
μ_{r1}	1	1	1	1	1	1	1	1	1	1	1
λ_{f1}	400	400	400	400	400	400	400	400	400	400	400
λ_{d1}	1	100	200	300	400	500	600	700	800	900	1000
μ_{f1}	50	50	50	50	50	50	50	50	50	50	50
μ_{m2}	10	10	10	10	10	10	10	10	10	10	10
μ_{r2}	4	4	4	4	4	4	4	4	4	4	4
p_{f2}^m	2	2	2	2	2	2	2	2	2	2	2
p_{f2}^r	8	8	8	8	8	8	8	8	8	8	8
μ_{f2}	50	50	50	50	50	50	50	50	50	50	50
λ_{d2}	200	200	200	200	200	200	200	200	200	200	200
λ_{f2}	500	500	500	500	500	500	500	500	500	500	500

Table C. 8. Parameters of sensitivity analysis for extreme cases: (2) *MTTD1*

Case #	1	2	3	4	5	6	7	8	9	10	11
p_R	25	25	25	25	25	25	25	25	25	25	25
p_{f1}^m	2	2	2	2	2	2	2	2	2	2	2
p_{f1}^r	10	10	10	10	10	10	10	10	10	10	10
μ_{m1}	5	5	5	5	5	5	5	5	5	5	5
μ_{r1}	2	2	2	2	2	2	2	2	2	2	2
λ_{f1}	200	200	200	200	200	200	200	200	200	200	200
λ_{d1}	1	100	200	300	400	500	600	700	800	900	1000
μ_{f1}	50	50	50	50	50	50	50	50	50	50	50
μ_{m2}	2	2	2	2	2	2	2	2	2	2	2
μ_{r2}	1	1	1	1	1	1	1	1	1	1	1
p_{f2}^m	1	1	1	1	1	1	1	1	1	1	1
p_{f2}^r	5	5	5	5	5	5	5	5	5	5	5
μ_{f2}	50	50	50	50	50	50	50	50	50	50	50
λ_{d2}	150	150	150	150	150	150	150	150	150	150	150
λ_{f2}	200	200	200	200	200	200	200	200	200	200	200

Table C. 9. Parameters of sensitivity analysis for extreme cases: (1) p^r_f

Case #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
p_R	25	25	25	25	25	25	25	25	25	25	25
p^m_{f1}	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
p^r_{f1}	1	1	1	1	1	1	1	1	1	1	1
μ_{m1}	2	2	2	2	2	2	2	2	2	2	2
μ_{r1}	1	1	1	1	1	1	1	1	1	1	1
λ_{f1}	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
λ_{d1}	200	200	200	200	200	200	200	200	200	200	200
μ_{f1}	50	50	50	50	50	50	50	50	50	50	50
μ_{m2}	10	10	10	10	10	10	10	10	10	10	10
μ_{r2}	4	4	4	4	4	4	4	4	4	4	4
p^m_{f2}	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
p^r_{f2}	2	2	2	2	2	2	2	2	2	2	2
μ_{f2}	50	50	50	50	50	50	50	50	50	50	50
λ_{d2}	200	200	200	200	200	200	200	200	200	200	200
λ_{f2}	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

Table C. 10.. Parameters of sensitivity analysis for extreme cases: μ_{m1}

Case #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
p_R	25	25	25	25	25	25	25	25	25	25	25
p^m_{f1}	1	1	1	1	1	1	1	1	1	1	1
p^r_{f1}	5	5	5	5	5	5	5	5	5	5	5
μ_{m1}	1	5	10	15	20	25	30	35	40	45	50
μ_{r1}	2	2	2	2	2	2	2	2	2	2	2
λ_{f1}	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
λ_{d1}	200	200	200	200	200	200	200	200	200	200	200
μ_{f1}	50	50	50	50	50	50	50	50	50	50	50
μ_{m2}	25	25	25	25	25	25	25	25	25	25	25
μ_{r2}	10	10	10	10	10	10	10	10	10	10	10
p^m_{f2}	2	2	2	2	2	2	2	2	2	2	2
p^r_{f2}	10	10	10	10	10	10	10	10	10	10	10
μ_{f2}	50	50	50	50	50	50	50	50	50	50	50
λ_{d2}	200	200	200	200	200	200	200	200	200	200	200
λ_{f2}	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

APPENDIX D: PLOTS OF SENSITIVITY ANALYSIS

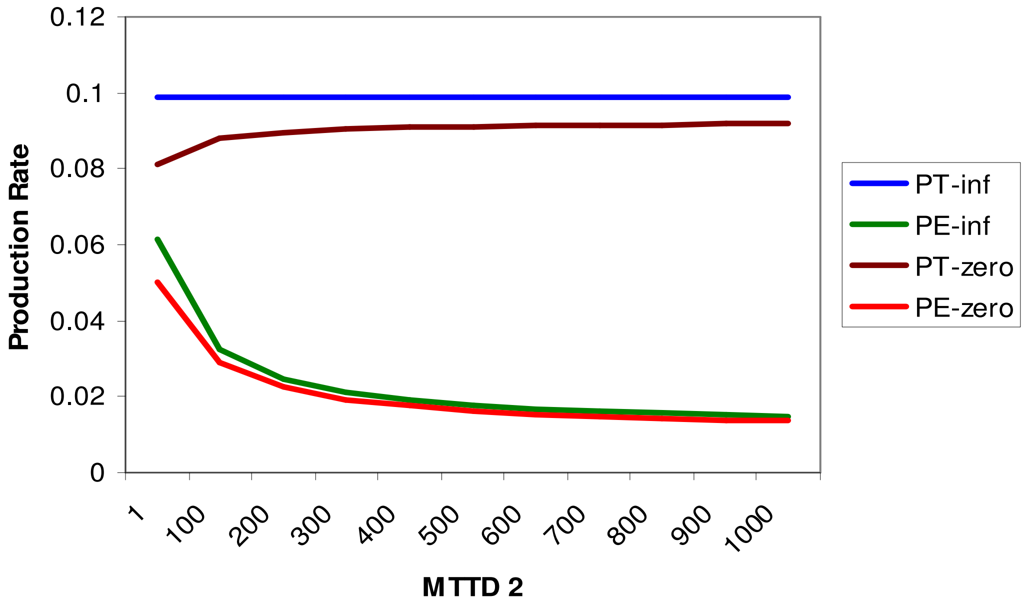

Figure D. 1 Alternative case for: Effect of *MTTD2* on P^T and P^E

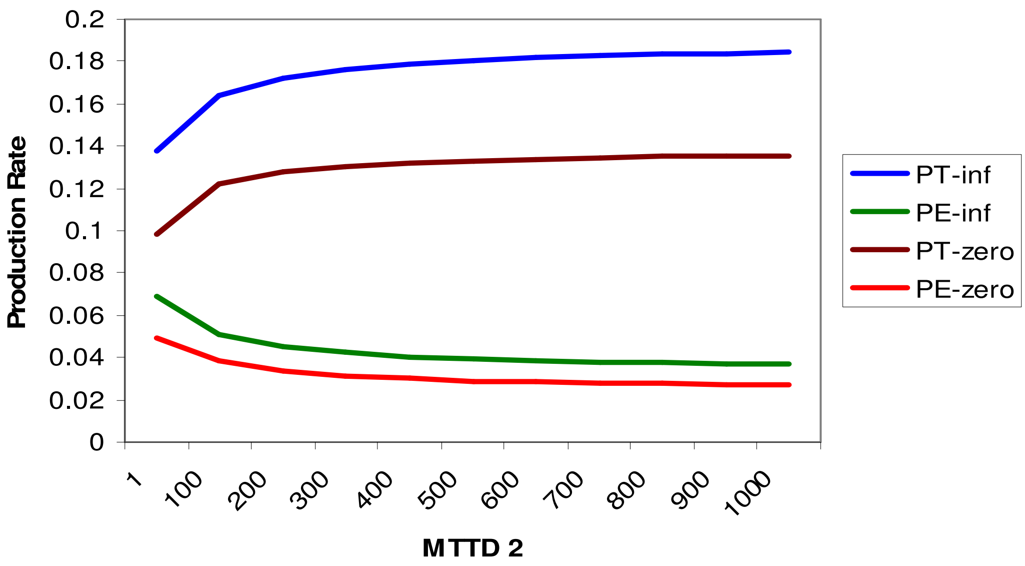

Figure D. 2. Sensitivity analysis for extreme cases: (1) Effect of *MTTD2* on P^T and P^E

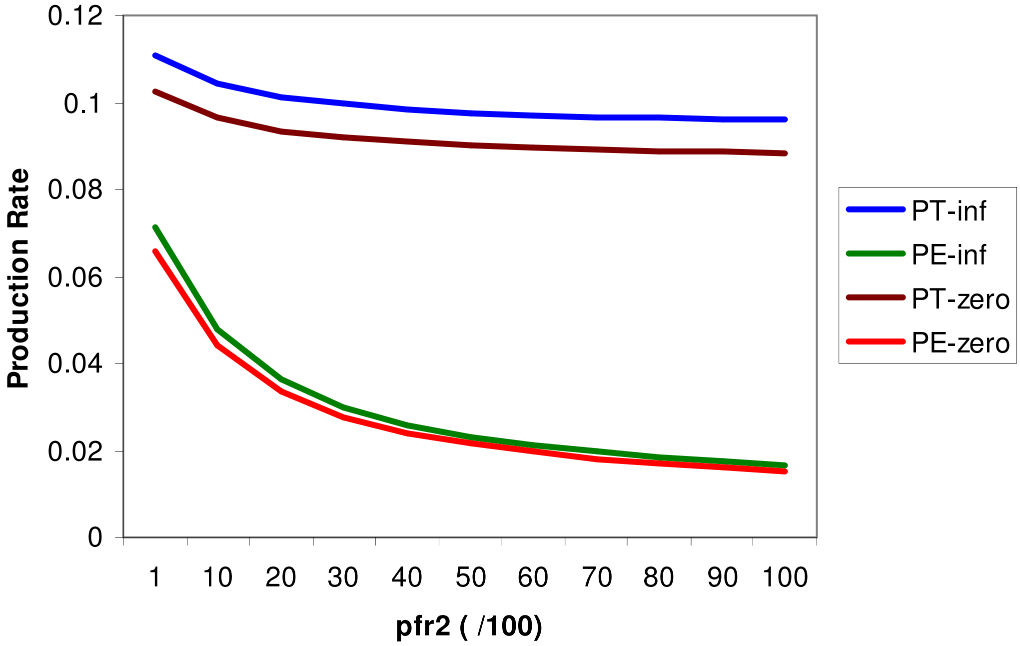

Figure D. 3. Sensitivity analysis for extreme cases: Effect of $p^r{}_f2$ on P^T and P^E

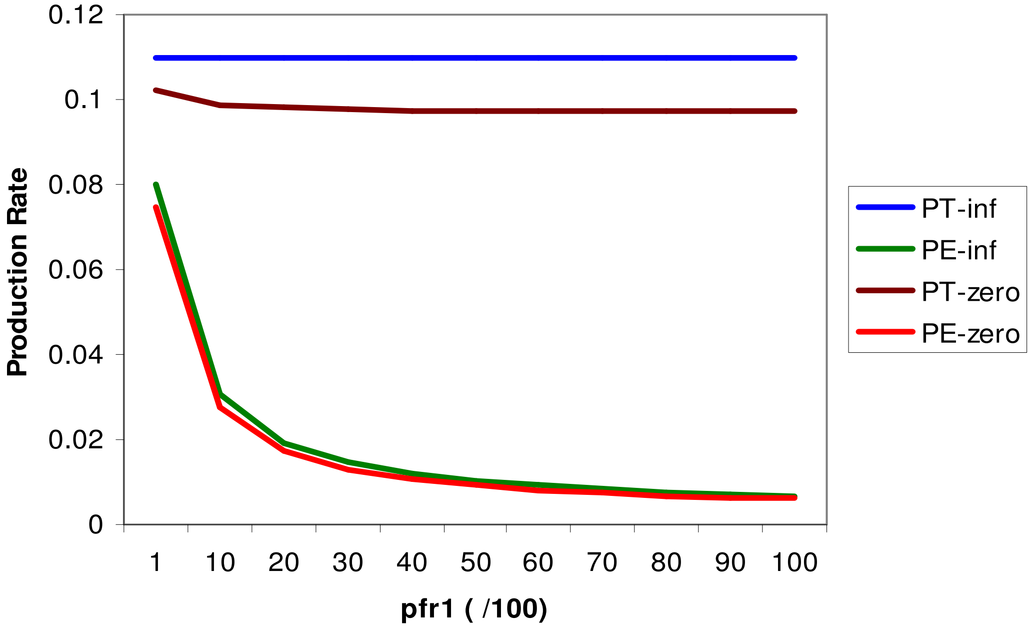

Figure D. 4. Sensitivity analysis for extreme cases :(1) Effect of $p^r{}_f1$ on P^T and P^E

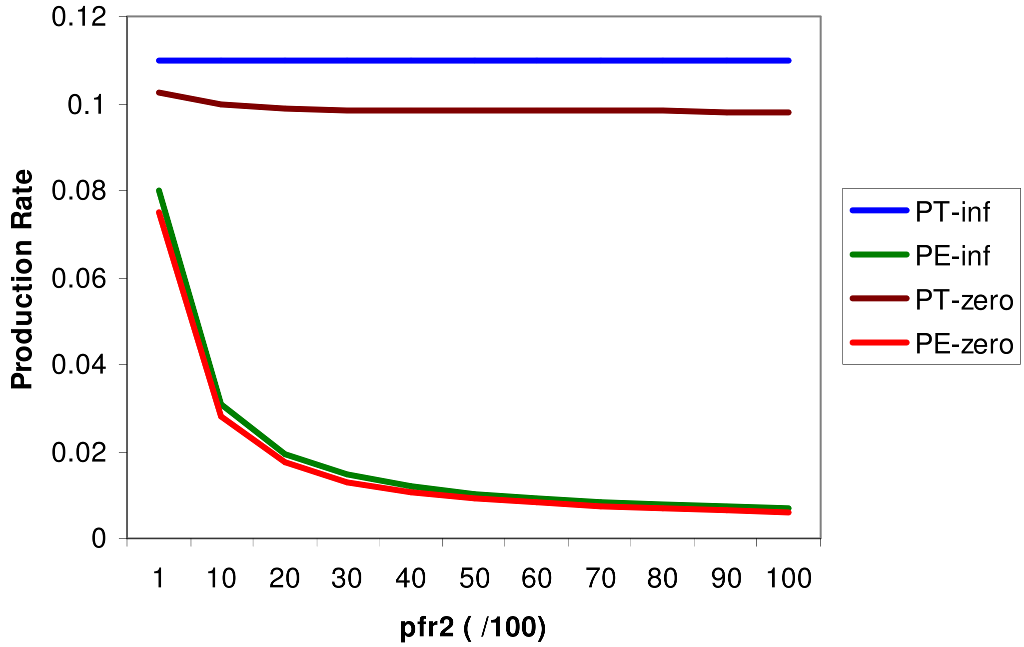

Figure D. 5. Sensitivity analysis for extreme cases: (2) Effect of p^r_f2 on P^T and P^E

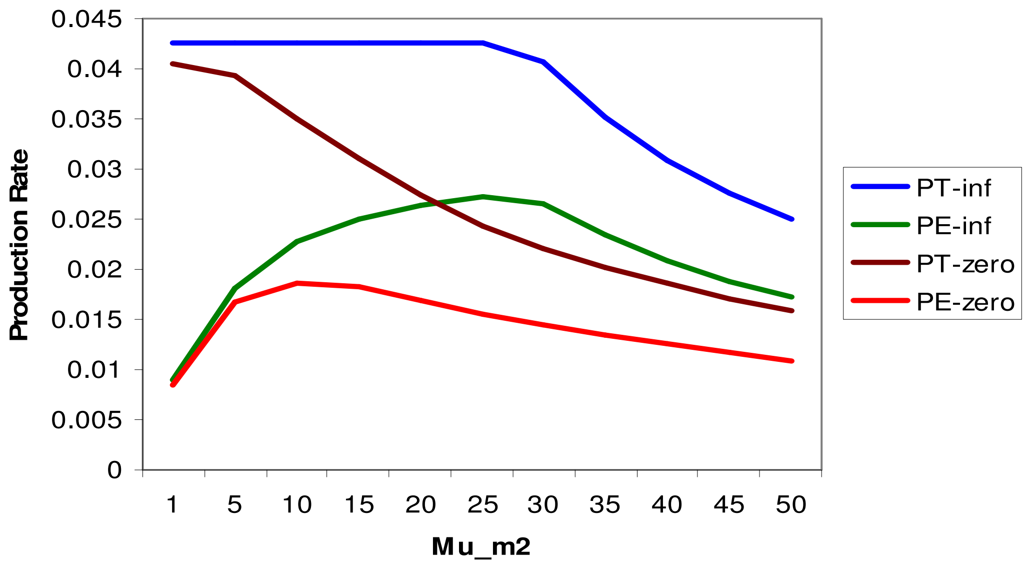

Figure D. 6. Sensitivity analysis for extreme cases: Effect of μ_{m2} on P^T and P^E

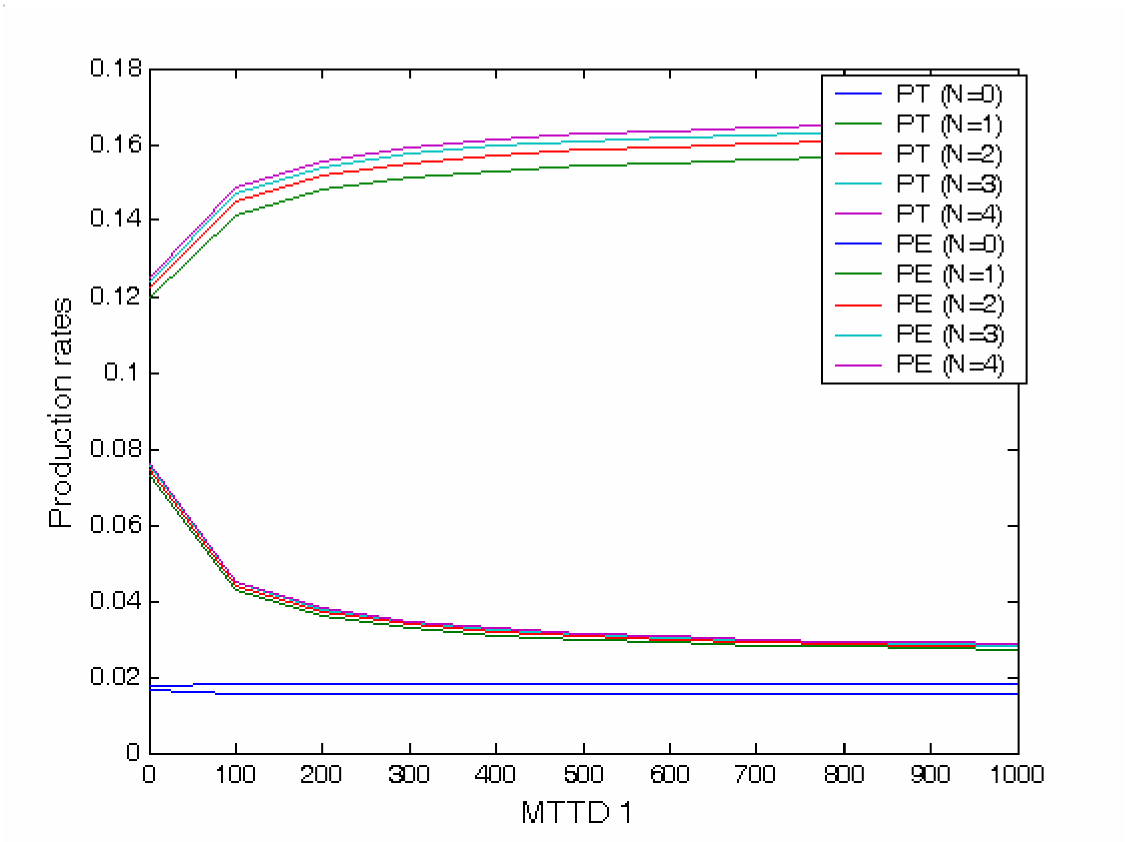

Figure D. 7. Sensitivity analysis for finite buffer case: Effect of *MTTD1* on P^T and P^E

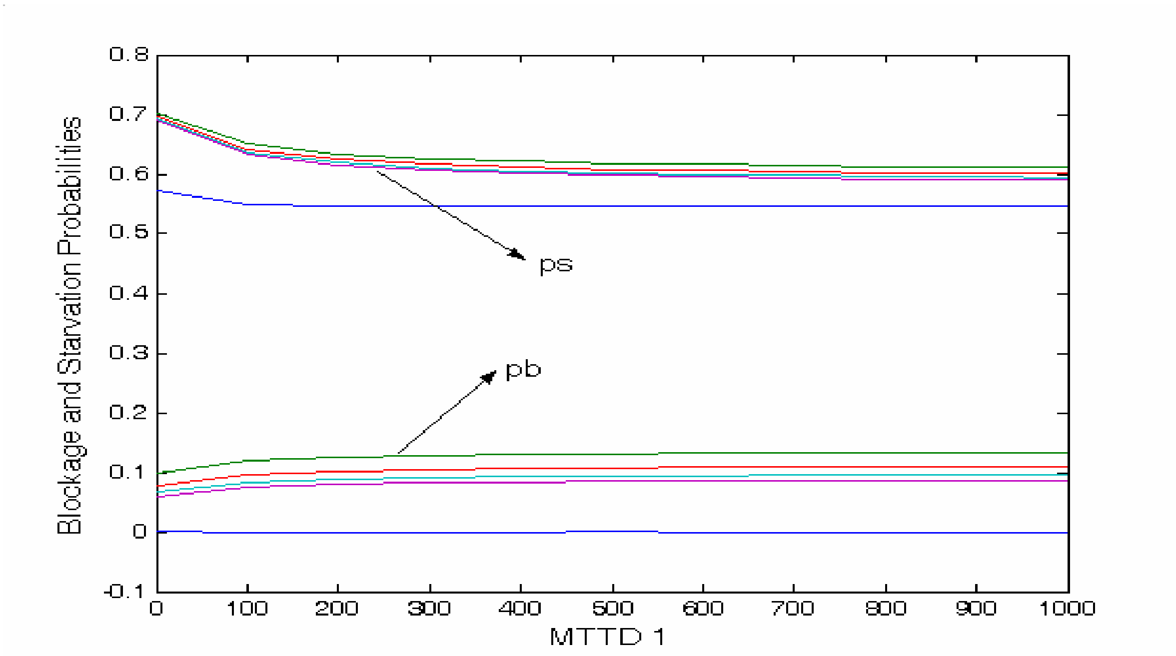

Figure D. 8. Sensitivity analysis for finite buffer case: Effect of *MTTD1* on p_b and p_s

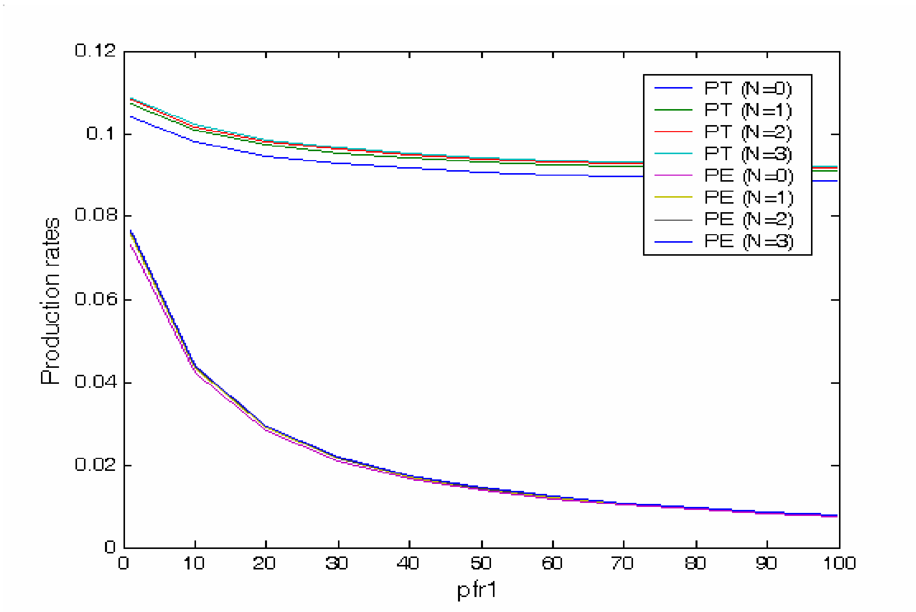

Figure D. 9. Sensitivity analysis for finite buffer case: Effect of *MTQF1* on P^T and P^E

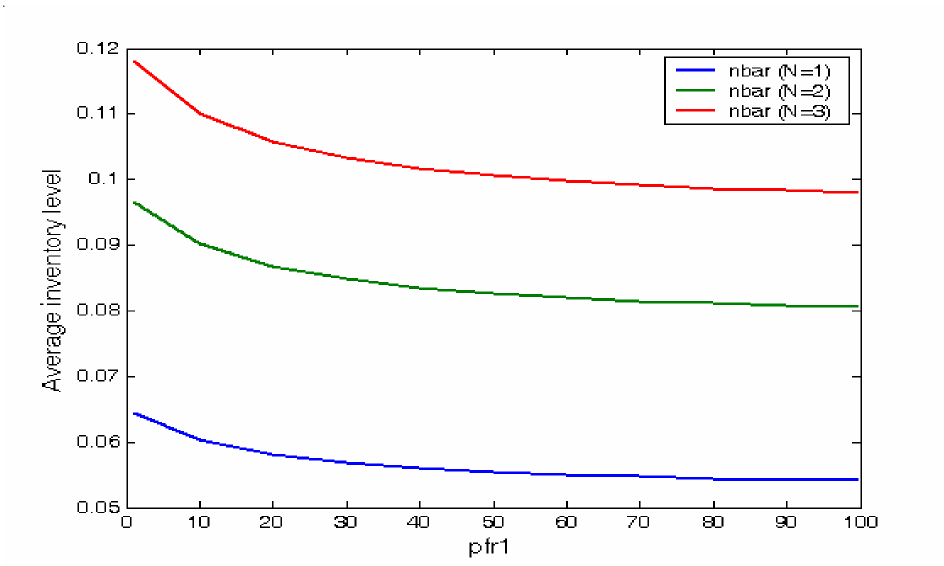

Figure D. 10. Sensitivity analysis for finite buffer case: Effect of *MTQF1* on average buffer level

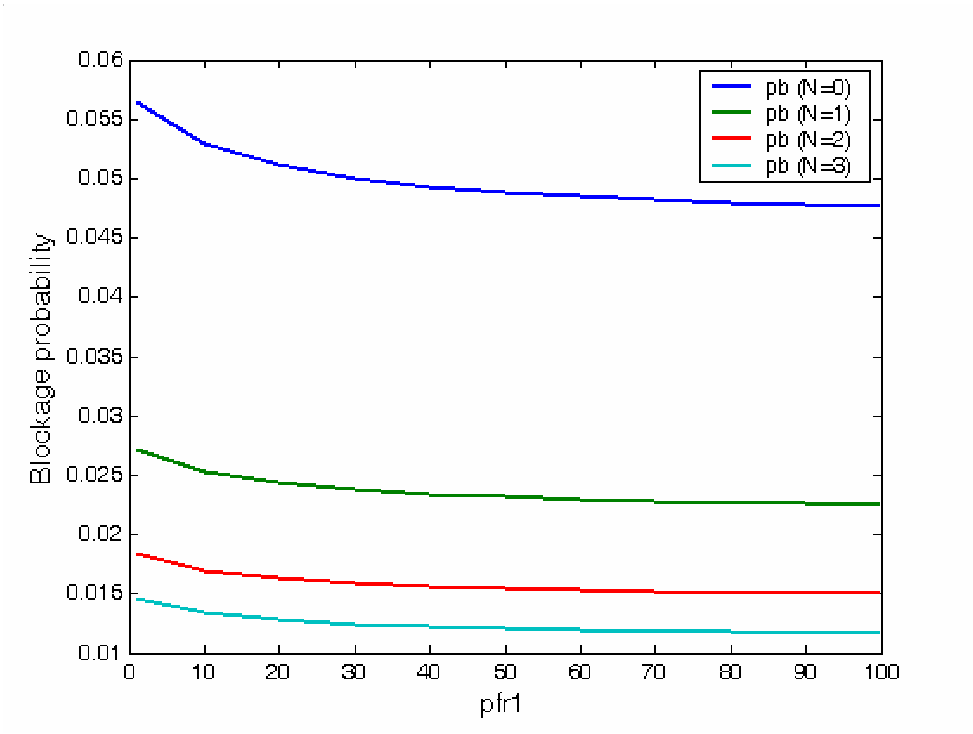

Figure D. 11 Sensitivity analysis for finite buffer case: Effect of *MTQF1* on p_b

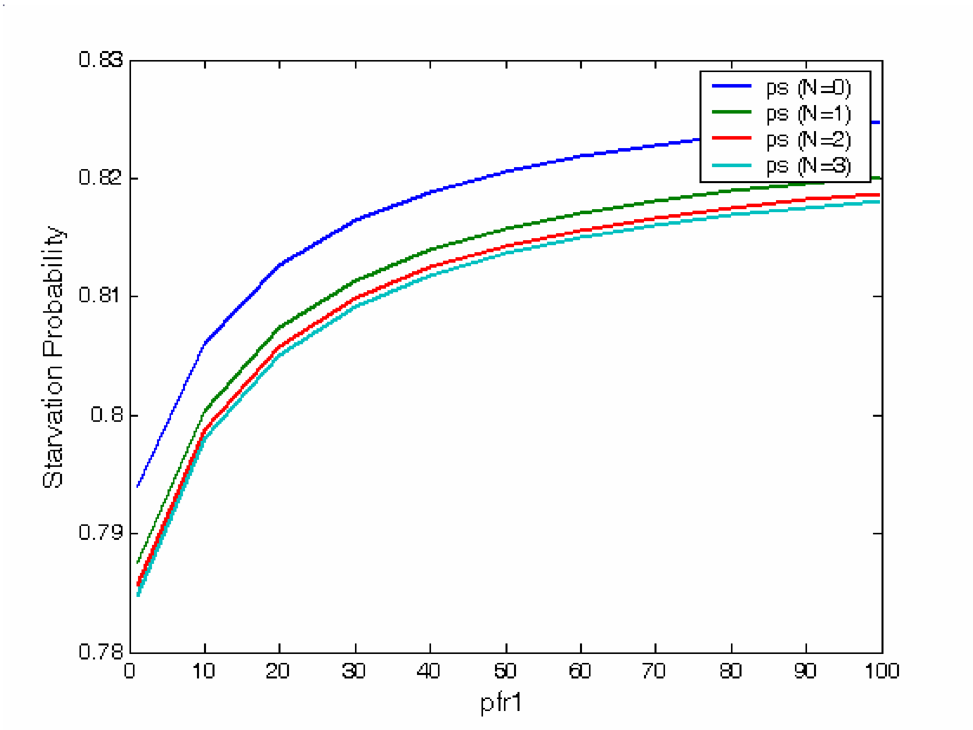

Figure D. 12 Sensitivity analysis for finite buffer case: Effect of *MTQF1* on p_s

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