Analysis of the Competition Under Yield and Demand Uncertainty with Yield-Dependent Price

by

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ABSTRACT

Analysis of the Competition Under Yield and Demand Uncertainty with Yield-Dependent Price

In this study, the competition of two manufacturers in a two stage game is analyzed. At the beginning of the first stage, the manufacturers decide on the amount of the area to be leased in the presence of random yield and demand. At the end of this period, the yield is realized and the manufacturers try to optimize their optimum production quantity with respect to the random demand and realized yield.

Both the centralized and decentralized game analysis are included in the study. The centralized analysis acts as a benchmark for the decentralized game results. In the centralized analysis, the optimality of the decision variables are discussed and the sensitivity analysis of the decision variables is performed. As far as decentralized analysis is concerned, the unique equilibrium is characterized. Comparative statics results are presented to study the effect of different parameters on the equilibrium.

For both centralized and decentralized settings, a computational study is presented. First stage results show that the centralized game expected profit is higher than the sum of the decentralized game expected profits which implies that coordination is necessary. Moreover, if there is a low yield expectation, competitors should consider coordination options more. Another noteworthy result is that as the expected profit is concerned, as maximum capacity of the market increases, inefficiency due to the competition decreases.

ÖZET

Verime Bağlı Fiyatlandırmayla Birlikte Verim ve Talep Belirsizliği Altında Rekabet Analizi

Bu çalışmada, iki aşamalı bir oyunda iki üreticinin rekabeti analiz edilmiştir. Birinci aşamanın sonunda üreticiler, belirsiz verim ve talep durumunda kiralanması gereken optimum alan miktarına karar verirler. Bu dönemin sonunda verim gerçekleşir ve üreticiler gerçekleşen verim ve belirsiz talep ışığında üretim miktarlarını, beklenen karlarını en yüksek noktaya ulaştıracak şekilde belirlemeye çalışırlar.

Hem merkezi hem de dağıtılmış oyun analizlerini incelenmiştir. Merkezi analiz, dağıtılmış oyun sonuçları için bir karşılaştırma kriteri olarak kullanılmıştır. Merkezi oyun analizinde, karar değişkenlerinin idealliği tartışılmış ve farklı parametrelere göre sensitivite analizleri yapılmıştır. Dağıtılmış oyun analizinde ise her iki aşamadaki eşsiz denge noktaları karakterize edilmiştir. Ayrıca değişik parametrelerin denge noktalarına etkilerine dair karşılaştırmalı statik sonuçları verilmiştir.

Hem merkezi hem de dağıtılmış analiz için sayısal çalışmalar sunulmuştur. Ilk aşamada sonuçları merkezi oyun sonuçlarının dağıtılmış oyun sonuçları toplamından büyük olduğu gözlenmiştir. Bu da dağıtılmış oyunda da merkezi oyun sonuçlarına ulaşabilmek için koordinasyonun gerekliliğini göstermektedir. Bunun yanısıra, düşük verim beklentisi durumunda, üreticiler koordinasyon opsiyonlarını düşünmelidirler. Bir diğer önemli sonuç ise, her iki aşamayı kapsayan beklenen kara bakıldığında, pazar kapasitesi arttıkça rekabet yüzünden oluşan kayıpların azalmakta olmasıdır.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS/ABBREVIATIONS

α_j	Fraction of the unsatisfied demand of the manufacturer i which can be satisfied by the manufacturer j,$\alpha \in (0,1]$ (random variable)
ϕ, Φ	Density function and CDF of first manufacturers demand in the decentralized case
ψ, Ψ	Density function and CDF of the second manufacturers demand in the decentralized case
b_j	Cost of the lost sales per unit after demand realization for the manufacturer j
c_j	Unit leasing cost of the olive trees for the manufacturer j
$c_{p,j}$	Unit processing cost of the production for the manufacturer j
$D_j(p(u))$	Demand for the end product of the manufacturer j(random variable)
$E_j[\Pi_i]$	The expected profit function of the manufacturer j at the stage i
f, F	Density function and CDF of the manufacturer's demand in the centralized case
h_j	Salvage value of the olives after the first stage for the manufacturer j
q_j	The amount of the olives that will be used for the olive oil production by the manufacturer j
Q_j	The area to be leased by the manufacturer j
$p_j(u)$	Price of the end product for the manufacturer j
s_j	Salvage value of the end product after demand realization for the manufacturer j
u	Realized yield(random variable)
y, Y	Density function and CDF of the yield, u, on a support of [0,M]

1. INTRODUCTION

Due to the globalization, competition among firms have increased significantly. In most of the sectors, oligopoly is the governing model. In order to survive, firms should minimize their costs while adopting to the changes in the costs, in the prices and in the conditions of the competition. In this study, the aim is to analyze the competition of two manufacturers and characterize their behavior when demand and yield of production is random. The motivation is olive oil production process, where the yield and the demand has randomness.

Crops of the olive oil trees can be harvested every two years. After the olives are collected, they should be processed in 48 hours for the olive oil production. Because of economical issues, the manufacturers prefer to lease the olive trees before olive production occurs rather than to buy olives from the farmer at the time of production. Another advantage of leasing is obtaining a pure end product. Olives growing in different areas may not be at the same quality. With leasing option, such a case is prevented. Yet, the manufacturers bear all the risks related to the yield and they also overtake all the cost of the leased area during the growing phase. Therefore, to meet the demand of any year, the quantity to be leased should be determined before two years. Another important issue to consider is that the newly obtained olive oil can not be blended with the older product because of quality and taste issues.Moreover, bottled olive oil has a shelf life of one year on the average. Thus, meeting demand with the new product has significant importance. Therefore, the model consists of two stages. At the end of the first stage, the yield is realized and at the end of the second stage, the demand is realized. The decision variables of the manufacturers are the number of trees to be leased and the quantity to be produced, in two consecutive stages, respectively.

In the setting of this study, two manufacturers compete for the excess demand of olive oil in the same market which can not be met by the other player. Because many manufacturers process the olives coming from the same region, the taste and properties of their end products resemble each other, such that the end product of one manufacturer can be substituted with the product of the other manufacturer. Hence, such a competition is a realistic case. Yet, the optimal leasing areas and the optimal production quantities of the manufacturers will be effected by this competition. In order to determine the optimal points for both players and characterize the game, the game theoretical analysis of this setting is studied.

Price is determined by the market, therefore both the manufacturers sell their end products at the same market price. Yet, the market demand is influenced by the market price,i.e. if end product price is high less consumers will buy the end product such that demand is going to decrease. Thus, a price-dependent demand function is employed. In the years with low yield, a rise in the olive oil prices has been observed. This is mainly because the olive oil demand of the market is mostly satisfied with the olives growing in a specific area. In the years with low yield, the supply of olive oil will decrease. A fall in the supply will lead to a rise in the olive oil prices. Because of this inverse relationship, the price function, used in this model, depends on the yield. Specifically, random demand is contingent on an exogenous parameter, price which is itself contingent on random yield.

The objective of this thesis is to investigate the possible outcomes of a two-stage game in the presence of two random variables. The olive oil production process definitely contains all the necessary model characteristics. At the beginning of the first stage, the player(or players) decide on the optimum leasing area by taking the random demand, random yield and the expected profit functions into consideration. After the yield is realized at the end of the first period, the production quantity which maximizes the expected profit with respect to the random demand and realized yield, is determined by the player(or players). Both the demand and the price functions are assumed to be yield-dependent. In the decentralized game, information is known to the players. Moreover, all the players make their decisions simultaneously.

We first analyze the centralized model. In this setting, there is only one manufacturer who makes all the decisions in order to maximize its own profit. The results imply that in both stages, the expected profit functions are concave and there is unique optimum leasing and production values. Moreover, according to the sensitivity analysis, the change in the cost parameters lead to contrary movement of the optimality whereas the impact of the salvage parameters changes with respect to the stage.

Then we continue to analyze the decentralized model. In this model, two manufacturers are competing for the excess of olive oil demand and all their information is known to every player. The manufacturers try to maximize their expected profits in every stage. The game played in both stages is investigated and the results point that there exists a unique equilibrium point in every stage. In addition to this finding, the equilibrium point belonging to the first stage definitely effects the second stage equilibrium. We also provide the comparative static analysis for the optimal decision variables. Generally, the results obtained in the centralized model are valid for this game, too, i.e. the manufacturer whose parameters do change, replies to these changes as if it is the only manufacturer whereas its competitor responses are just the opposite. To give an example, if some parameter change leads to an increase of the optimum value of the decision variable belonging to one manufacturer, the optimality belonging to the other manufacturer decreases because of the same parameter change.

Then we provide the numerical analysis regarding both the centralized and decentralized cases. An additive type of demand function which reflects the actual demand pattern of olive oil is employed. Not only the results obtained are compared to the other ones belonging to the same case and same stage, but also the results for the centralized and decentralized cases are compared with each other. For the first stage, two scenarios, called as expected and best yield case are studied. The main outcome is that in every stage, the centralized case expected profit and optimum point is higher than the sum of the ones belonging to the two players in the decentralized case. Another significant result is that the best yield case scenarios lead to less expected profit values.

The main contribution of this thesis is the two-stage Nash game characterization in the presence of two random variables which effect both stages. In some of the studies that appeared in the literature, the game played between consecutive agents configuring the supply chain and the contractual issues are investigated. There are also papers analyzing the centralized games with two stages. The second contribution is characterization of the behaviour of the equilibrium with respect to various model parameter changes.

The rest of the thesis is organized as follows. Chapter 2 includes a literature survey about the studies which deal with similar model characteristics, as well as Nash games. In Chapter 3, the centralized model and its details are shown explicitly. The analytical results regarding the centralized model and the sensitivity analysis are also given in this chapter. In Chapter 4, the decentralized game played by two manufacturers is characterized. Again, the analytical and comparative statics results are displayed. In Chapter 5, the numerical analysis for both games are included. Eventually, the study is concluded in Chapter 6.

2. LITERATURE REVIEW

Game theory is one of the widely employed tools in order to characterize the behavior under vertical and horizontal competition. Cachon and Netessine study the game theory applications, both cooperative and non-cooperative games in static as well as dynamic settings, relevant to supply chain in detail [1]. Parlar and Leng review the game theoretic applications in supply chain management which contains information about the inventory games, production and pricing competition, games with other attributes and games with joint decisions [2].

Parlar employs game theory for the analysis of the inventory decisions of a substitutable product which has a random demand [3]. In this study, the Nash game played by two manufacturers who produce substitutable products is analyzed. The substitution rate determines the amount of products that can be sold to the customers of the competitor. The findings imply that each player has a unique response function given other player's production amount. Moreover, it is proven that there exists only one Nash equilibrium at which both manufacturers maximize their expected profits. It is also shown that if one player behaves in order to damage the other player's profit, the problem for the damaged player becomes the classical newsboy problem. Another outcome is that cooperation between players lead to higher expected profits for both manufacturers.

In the literature, vast amount of studies have been conducted about the horizontal competition in oligopolies. They can be divided mainly into two categories. The studies in the first category is about the competition in the sales price, called as Bertrand competition. The ones belonging to the second group, including this study, deals with the competition in quantity, also named as Cournot competition. Bernstein and Federgruen analyze the first- and second-order monotonicity properties of the equilibrium points with respect to quantity-dependent cost and price-dependent demand functions for the Bertrand and Cournot competition cases in oligopolies and duopolies [4]. Both functions also depend on one other parameter and different combinations of these dependencies are analyzed. Bernstein and Federgruen also deal with the decentralized supply chain analysis under random demand [5]. In this paper, the demand is stated to depend only on the retail price of the retailer if there is no competition and states the optimal order quantities regarding this case. Then, the case where retailers are competing and their demands depend on the retail prices of all the players, is investigated. In both cases demand function is in the multiplicative form.

The settings with two stages deal mostly with options where the manufacturers can buy options at the beginning of the first stage. These options can be exercised at the beginning of the second stage in order to avoid lost sales. [6],[7]. Barnes-Schuster et al. study a model which consists of two periods with correlated demand [6]. In both periods, there is demand realization. Products can be ordered for both periods at the beginning of the first period, but the amount of order for the second period can be changed by the buyer at the beginning of the second period. The flexibility offered by the options and different coordination contracts between the buyer and supplier are investigated under such a model. Li and Liu also analyze such a setting, but in their model the manufacturer has a limited capacity to offer [7]. Keskinocak et al., study a setting that also consists of two stages at which the supplier announces her wholesale prices and the manufacturer decides on the quantity to buy in each stage, as well as the sales price of the end product [8]. The decentralized analysis of this setting is studied where the supplier has a capacity limitation.

There are many studies including a deterministically or stochastically modelled price dependent demand function. Petruzzi and Dada discuss the effects of the additive and multiplicative types of error terms on the optimality in a single period model [9]. In this model, stocking quantity and selling price decisions are made simultaneously. The paper also contains various sales scenarios for multi period models and price characterization. Arcelus, Kumar and Srinivasan also study the impact of different demand functions, i.e. deterministic and stochastic cases with price functions having additive or multiplicative error terms, on a setting where a retailer faces price-dependent demand [10]. In this paper, also the effect of trade incentives on the price and ordering quantity determination is investigated. Liu, Fry and Raturi analyze the single period, one manufacturer-one supplier setting with deterministic and stochastic price-dependent demand functions [11]. In this study, the demand function is stochastic with an additive type of error term.

A distinguishing part of this study is that it contains two random variables included in the demand and yield. Parlar and Wang also discuss the impact of random yield on the inventory models [12]. Shen and Pang analyze the supply chain coordination issue with capacity options in the presence of random demand, random supply and a spot market [13]. The importance of the options contract for all the supply chain agents, as well as for the whole supply chain is studied. Abdel-Malek et al. investigate a setting where one gardener decides on the production quantities for different kinds of products in the presence of random yield, random demand and capacity constraints [14]. The studied scenarios are unconstraint case, active constraint case and tight constraint case. The results obtained by using different distribution functions, as well as different methods, for random demand and random yield are displayed. Wang also studies a setting with random yield and uncertain demand which consists of a manufacturer and a distributer [15]. This paper is merely concentrated on the vertical cooperation. Two scenarios, traditional and Vendor Managed Inventory, i.e. VMI, arrangements, are analyzed. The results imply that both arrangements lead to less profit than the one obtained in the integrated supply chain analysis. Jones, Lowe and Traub analyze a model that consists of two stages at which production can occur [16]. Like in our model, both the yield and the demand are assumed to be random.

Kazaz studies a model which shows similarity to our model [17]. In this paper, to meet the random demand one manufacturer is going to produce olive oil in the presence of random yield. The model consists of two stages, in the first stage, the manufacturer decides on the optimum leasing area whereas in the second stage, the optimum olive oil production quantity is determined. Both the demand and the price of the olive oil are assumed to be yield-dependent. The manufacturer has two options for olive acquisition. He can lease an area and produce olive oil from the olives grown there. If the yield is low, he can also buy olives from a spot market. The change in the optimality of the decision variables with respect to the differing olive acquisition choices are investigated.

Our setting resembles the setting of Kazaz's work, in the randomness of demand and yield [17]. Moreover, in both studies, there are two stages in which the yield and the demand are realized, respectively. The main difference between these studies is that we consider two competing firms for the excess demand of the olive oil. In our study horizontal competition is analyzed whereas in Kazaz's study only the centralized solutions are characterized. In addition to this difference, there is a spot market in Kazaz's paper from where the manufacturer can buy extra quantities in the second stage.

3. THE CENTRALIZED ANALYSIS

In this study, competition of two manufacturers are analyzed in the presence of random demand and random yield of production. In the centralized game, both manufacturers are assumed to belong to the same company as if there is only one olive oil manufacturer present. The model consists of two stages. The order of events is:

- At the beginning of the first stage, the area to be leased is determined.
- At the end of this stage, yield is realized and it is observed.
- At the beginning of the second stage, the manufacturer determines the quantity of olives to be used for olive oil production.
- At the end of the second stage, demand is realized.

Figure 3.1 visualizes the model.

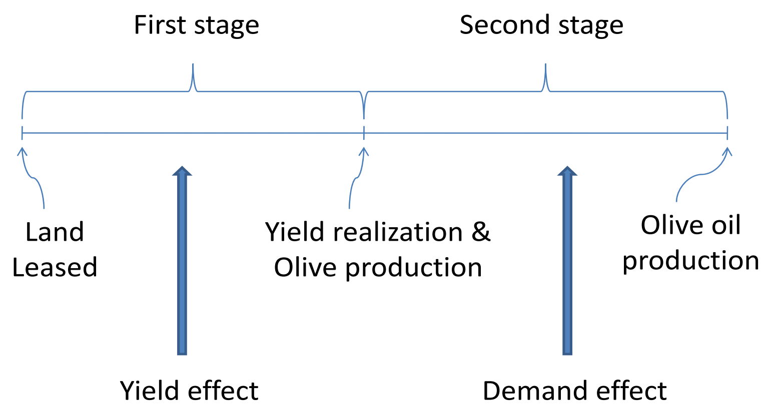

Figure 3.1. Model description

At the beginning of the first stage, the manufacturer decides on the area to be leased by considering the expected profit function with respect to the random yield and the random demand. In this model, the farmer is assumed to have unlimited area to lease such that the manufacturer/manufacturers can always lease all the demanded area. At the end of the first stage, the yield is realized. The only decision variable for the manufacturer will be the quantity to produce in order to meet the random demand. Therefore, in the second stage, the manufacturer tries to determine the optimal amount of the olive oil to be produced according to the demand expectation.

The main costs for the manufacturers are the leasing, manufacturing and shortage costs. The following notation is used in the centralized analysis:

c_p:	Unit processing cost of the production for the manufacturer
c:	Unit leasing cost of the land for the manufacturer
h:	Salvage value of the olives after the first stage for the manufacturer
b:	Cost of the lost sales per unit olive oil after demand realization for the manufacturer
s:	Salvage value of the olive oil after demand realization
u:	Olive yield(random variable) on a support of [0,M], with $0 < M < \infty$
y, Y:	Density function and CDF of the yield, u,
$D(p(u))$:	Demand for the olive oil contingent on price (random variable), $D \in [0, \infty]$
f, F:	Density function and CDF of $D(p(u))$
$p(u)$:	Exogenous price of the olive oil contingent on yield

The notation of the decision variables are as follows:

Q:	The area to be leased by the manufacturer
q:	The amount of the olives that will be used for the olive oil production by the manufacturer

In this model, the price of the end product is assumed to depend on the yield realized. As yield increases, the total quantity of olives produced also increases which leads to overproduction of olive oil. Because of excess supply of olive oil, the olive oil price decreases in order to increase the olive oil demand such that the levels of supply and demand are balanced. Hence, the relationship between the price and the yield can be stated as inverse.

As seen in the notation section, the demand depends on the price of the end product. Because the end product price depends on the yield from the leasing area, the demand for the end product is also yield-dependent. As yield increases, the price of the end product decreases. The fall in the end product price leads to a rise of the end product demand. Thus, the relationship between the end product demand, $D(p(u))$, and the yield, u, is an indirect relationship.

There are many types of demand functions which include the randomness in various forms, such as additive or multiplicative. One example for the additive type of price-dependent demand function is:

$$D(p(u)) = K - \beta p(u) + \varepsilon \tag{3.1}$$

In this equation, ε is the random error term, β is the rate term representing the influence of the end product price on the demand, $p(u)$ is a non-increasing deterministic function of u and K stands for the maximum deterministic market demand. The distribution of the random demand, $D(p(u))$ necessarily depends on the realized yield, u and it is denoted as $F(x; u)$. Its density is denoted as $f(x; u)$. The analysis of the first and second stages are stated in the subsequent sections.

3.1. Model Assumptions

There are some restrictions on the costs. The constraints regarding the costs can be stated as follows:

1. $h < c$
2. $s > h$
3. $s < h + c_p$

4. $p(0) \geq p(M) \geq c + c_p > s$

The salvage value of the olives after yield realization should be less than the leasing cost of the olive trees such that the manufacturer is forced to consider the demand of the olive oil by deciding on the amount of the area to be leased. This condition is satisfied by the first assumption. The second and third assumptions ensure that the salvage value of the olive oil should be greater than the salvage value of the olives while being less than the sum of the salvage value of the olives and the processing cost in order to avoid salvaging all the end product. Such a condition prevents the case of producing olive oil from all the olives present without determining the necessary amount of end product to meet the demand. Finally, the last assumption states that the lowest price of the end product should be greater than the sum of the production and leasing costs, which is greater than the salvage value of the olive oil. This condition enables the manufacturer to prefer selling all its product to the market rather than salvaging.

3.2. Analysis of the Second Stage

At the beginning of the second stage, the yield is realized and the manufacturer tries to decide on the amount of the olive oil production with respect to the random demand. The profit function of the manufacturer can be written as:

$$\begin{aligned} \Pi_2(q; Q, u) = &- c_p q + p(u)\min\{q, D(p(u))\} - b(D(p(u)) - q)^+ \\ &s(q - D(p(u)))^+ + h(Qu - q) \end{aligned} \tag{3.2}$$

where $(x)^+$ denotes $\max\{0, x\}$. The first term is the processing cost of the olive oil. The second term is the gain obtained by meeting the demand or selling the produced olive oil. The third term represents the cost of lost sales because of the unmet demand. The fourth term is the salvage gain obtained by the olive oil which is left after all the demand is met. The last term is the revenue obtained from the olives which are not being used for olive oil production.

In order to determine the production quantity, the manufacturer looks at its expected profit function. When expectation is taken over the demand, the function becomes:

$$\begin{aligned} E[\Pi_2(q;Q,u)] = &- c_p q + h(Qu - q) + [p(u) - s + b]\int_0^q x f(x;u)dx - b\mu(u) \\ &+ qF(q)[s - p(u) - b] + [p(u) + b]q \end{aligned} \tag{3.3}$$

Because production can not be larger than the yield obtained from the leased land, the manufacturer decides its production quantity as follows:

$$\begin{aligned} max \quad E[\Pi_2(q;Q,u)] = &- c_p q + h(Qu - q) \\ &+ \int_q^\infty (p(u)q - b(x - q))f(x;u)dx \\ &+ \int_0^q (p(u)x + s(q - x))f(x;u)dx \\ s.t. \quad q \leq Qu & \end{aligned} \tag{3.4}$$

The first term in the expected profit function represents the processing cost of the olive oil. The second term stands for the salvage value of the olives that are not being used for the olive oil production. The third term represents the case where the production is not high enough to cover the demand. The last term stands for the high production case, where the demand is satisfied and the remaining part of the olive oil is salvaged.

Proposition 3.2.1. *The expected profit of the manufacturer, $E[\Pi_2(q;Q,u)]$ (c.f.(3.4)) is strictly concave in q and the optimal amount of the olive oil can be found from:*

$$q^* = \begin{cases} Qu & if\, q' > Qu \\ q' & if\, q' \leq Qu \end{cases}$$

where

$$q' = F^{-1}\left[\frac{p(u) + b - c_p - h}{p(u) + b - s}\right] \tag{3.5}$$

which is definitely positive by Assumption 3*.*

Proof: See Appendix A.1.

3.3. Analysis of the First Stage

At the beginning of the first stage, the main issue is to determine the leasing area by taking the randomness in the demand and in the yield into consideration. The manufacturer's problem is:

$$\max_{Q} -cQ + E_u \left[\max_{q \leq Qu} E_D \left[\Pi_2(q; Q, u) \right] \right] \tag{3.6}$$

Therefore, the expected profit function of the company in the first stage is stated as follows:

$$E[\Pi_1(q, Q, u)] = -cQ + \int_0^{u^*} Z(Q, u) y(u) du + \int_{u^*}^{M} W(Q, q^*, u) y(u) du \tag{3.7}$$

where

$$\begin{aligned} W(Q, q^*, u) = & - c_p q^* + h(Qu - q^*) \\ & + \int_0^{q^*} (p(u)x) + s(q^* - x) f(x; u) dx \\ & + \int_{q^*}^{\infty} (p(u)q^* - b(x - q^*)) f(x; u) dx \end{aligned}$$

$$\begin{aligned} Z(Q, u) = & - c_p Qu + \int_{Qu^*}^{\infty} (p(u)Qu - b(x - Qu)) f(x; u) dx \\ & + \int_0^{Qu^*} (p(u)x + s(Qu - x)) f(x; u) dx \end{aligned}$$

In the rest of this section, u^* is defined as the yield at which the optimal amount of olives to be processed, namely q^*, is equal to the amount of the olives obtained from the leased area, i.e. Qu^* [17]. If the realized yield is lower than u^*, all the olives produced are going to be used in the olive oil production in order to meet as much demand as possible whereas if the realized yield is higher than u^*, the rest of the olives which are not being used in olive oil production, are salvaged.

The first term in equation (3.7) represents the leasing cost of the area. The second term stands for the case, where the yield is higher than the u^*, which is the yield at which the leased area can only cover the expected optimum olive oil amount. The last term represents the case at which the yield is lower than the u^* in which case all yield is converted to olive oil.

Proposition 3.3.1. *The expected profit of the manufacturer as given in (3.7) is strictly concave in Q and the amount of the leasing area satisfying the first order condition (Equation 3.8) maximizes the total expected profit in the first stage.*

$$\begin{aligned}\frac{dE[\Pi_1(q,Q,u)]}{dQ} &= -c + h\int_{u^*}^{M} uy(u)du \\ &+ \int_0^{u^*}\left[-c_p u + su + \int_{Qu}^{\infty}((p(u)+b-s)uf(x;u)dx)\right]y(u)du = 0 \qquad (3.8)\end{aligned}$$

Proof: See Appendix A.2.

The centralized analysis we propose in this thesis is a specific case of Kazaz [17]. In his study, the manufacturer leases the area at the beginning of the first stage. After observing the demand, he can also buy olives from the spot market in order to meet the demand which is not allowed in our model.

3.4. Sensitivity Analysis

In this model, there are various parameters such as salvage values, backorder costs, etc., that affect the amount of the expected profit, as well as the amount of the olive oil and leasing area which maximize the expected profits, in both stages. Their influence on the optimal olive oil production and leasing area can be shown analytically. The results for the second stage can be stated as follows:

Proposition 3.4.1. *In the second stage, the amount of the olive oil produced by the manufacturer, q^*, increases when*

- *the lost sales cost, b, decreases*
- *the processing cost of the olive oil, c_p, decreases*
- *the salvage value of the olive, h, decreases*
- *the salvage value of the olive oil, s, increases*

Proof: See Section A.3 in Appendix A.

These results are in line with the expected behaviour of the manufacturer. As lost sales costs rise, the manufacturer will prefer to produce more in order to increase its possibility to meet the demand. As the processing cost of the olive oil increases, it will be more reasonable to decrease the olive oil production because it will not be as profitable as before. The decrease in the olive salvage value will lead to a fall in the production since the manufacturer will not prefer to have unnecessary olive stock at the end of the period. Eventually, as the olive oil salvage value increases, it will be more profitable to produce more.

In the first stage, the main issue is to determine the amount of the leasing area with respect to the expected profit function which depends on the random yield and random demand. Again, the effects of the cost parameters on the decision parameter are obtained analytically as follows:

Proposition 3.4.2. *In the first stage, the amount of the area leased by the manufacturer, Q^*, increases when*

- *the cost of leasing a unit area, c, decreases*
- *the backorder cost, b, increases*
- *the processing cost of the olive oil, c_p, decreases*
- *the salvage value of the olive, h, increases*
- *the salvage value of the olive oil, s, increases*

Proof: See Appendix A.4.

The results in Proposition 3.4.2 are also in line with the expectations. As leasing cost increases, the manufacturer will prefer to lease less area. The rise in the backorder cost will lead to more stocking, thus more area is leased. When the cost of the olive oil process falls, the manufacturer will decide on more production because of profit maximization issues. Increasing the salvage values of olive or olive oil will lead to a larger leased area because the manufacturer will be less afraid to keep extra stock on hand.

4. THE DECENTRALIZED ANALYSIS

The model employed in the decentralized analysis is the same as the one used in the centralized analysis except the number of manufacturers and demand substitution, i.e. in the decentralized analysis, the competition of two manufacturers are investigated under random yield and demand. We assume that there are two olive oil producers that buy their olives from the same region. Hence, both producers are subject to the same yield. The notation used in this chapter is stated below:

$c_{p,j}$:	Unit processing cost of the production for the manufacturer j
c_j:	Unit leasing cost of the olive trees for the manufacturer j
h_j:	Salvage value of the olives after the first stage for the manufacturer j
b_j:	Cost of the lost sales per unit after demand realization for the manufacturer j
s_j:	Salvage value of the end product after demand realization for the manufacturer j
$p_j(u)$:	Exogenous price of the end product for both manufacturers
$D_j(p(u))$:	Demand for the end product of the manufacturer j(random variable), $D_j \geqslant 0$
ϕ, Φ:	Density function and CDF of $D_1(p(u))$
ψ, Ψ:	Density function and CDF of $D_2(p(u))$
α_j:	Fraction of the unsatisfied demand of the manufacturer i which can be satisfied by the manufacturer j,$\alpha \in (0, 1]$
u:	Realized yield(random variable), common for all players
y, Y:	Density function and CDF of the yield, u, on a support of [0,M] with $0 < M < \infty$

The notation of the decision variables is as follows:

Q_j: The area to be leased by the manufacturer j

q_j: The amount of the olives that will be used for the olive oil production by the manufacturer j

In this setting, all the cost and demand information is common to all players. All the assumptions stated in the centralized analysis are also valid. Furthermore, if a retailer is short of olive oil in the market, a proportion of its customers go and buy olive oil from the other retailer and vice versa. Hence, there is demand substitution. The manufacturers play a Nash game in stage one over the leased land area. Then they play another Nash game in stage two over the quantities to be produced. The analysis of the game begins from the second stage.

4.1. Analysis of the Second Stage

In the second stage, i.e. after yield is realized, the manufacturers are going to decide on optimal production quantities of the end products by considering their expected profit functions. This setting resembles the one studied by Parlar [3]. The only difference is the yield dependency of the demand function. Since yield has already been realized in this stage, the yield can be considered as one of the input parameters. The profit function regarding the first manufacturer is written as follows:

$$\begin{aligned}\Pi_2^1(q_1;q_2,Q_1,u) = &- c_{p1}q_1 + h_1(Q_1u - q_1) - b_1(D_1(p(u)) - q_1) \\ &+ p(u)\min\{q_1, D_1(p(u)) + \alpha_1(D_2(p(u)) - q_2)^+\} \\ &+ s_1(q_1 - D_1(p(u)) - \alpha_1(D_2(p(u)) - q_2)^+)^+\end{aligned} \tag{4.1}$$

In this equation, the first term stands for the processing cost of the olive oil. The second term represents the gain obtained by salvaging the rest of the olives which are not used in olive oil production whereas the third term expresses the lost sales cost.The turnover obtained from olive oil sale is reflected in the fifth term. The last term stands for the salvage gain obtained from the excess olive oil production. After taking expectations over D_1 and D_2, the expected profit function obtained for the first manufacturer can be stated as follows:

$$
\begin{aligned}
max \quad E_1[\Pi_2(q_1; q_2, Q_1, u)] = & -c_{p1}q_1 + h_1(Q_1u - q_1) \\
& + \int_{q_1}^{\infty} (p(u)q_1 - b_1(x_1 - q_1))\phi(x_1)dx_1 \\
& + \int_{-\infty}^{q_1} \int_{-\infty}^{q_2} p(u)x_1\phi(x_1)\psi(x_2)dx_2dx_1 \\
& + \int_{-\infty}^{q_1} \int_{-\infty}^{q_2} s_1(q_1 - x_1)\phi(x_1)\psi(x_2)dx_2dx_1 \\
& + \int_{q_2}^{\infty} \int_{-\infty}^{q_1-\alpha_1(x_2-q_2)} p(u)(x_1 + \alpha_1(x_2 - q_2))\phi(x_1)\psi(x_2)dx_1dx_2 \\
& + \int_{q_2}^{\infty} \int_{-\infty}^{q_1-\alpha_1(x_2-q_2)} s_1(q_1 - x_1 - \alpha_1(x_2 - q_2))\phi(x_1)\psi(x_2)dx_1dx_2 \\
& + \int_{q_2}^{\infty} \int_{q_1-\alpha_1(x_2-q_2)}^{q_1} p(u)q_1\phi(x_1)\psi(x_2)dx_1dx_2 \\
s.t. \quad & q_1 \leq Q_1u
\end{aligned}
\tag{4.2}
$$

where both x_1 and x_2 depend on the olive oil price, $p(u)$. In the expected profit function for the second stage, the first term represents the production cost of olive oil, the second term states the salvage value of the unprocessed olives. The third term is stating the expected profit function when the first manufacturer can not meet its own demand. The fourth term represents the case where both the manufacturers are satisfying their own demands. The case at which the first manufacturer can meet all the demand whereas the second manufacturer can not satisfy its own demand is stated in the fifth term. Eventually, the last term includes the case of the first manufacturer satisfying only its own demand and the second manufacturer not meeting its own. The expected profit function of the second manufacturer can be written in the same manner.

In order to find out whether there is an equilibrium point in olive oil production amount for both of the manufacturers, their expected profit functions should be characterized.

Proposition 4.1.1. *The expected profits of both manufacturers in the second stage, as given in (4.2) is strictly concave in q_i and the amount of the olive oil obtained from the* *first order condition maximizes the total expected profit in the second stage.*

Proof: See Section A.5 in Appendix A.

Clearly an analogous result holds for the second manufacturer.

Because in the second stage, the only decision variable for the manufacturer j is q_j, the production quantity, the production amount maximizing the expected profit is calculated with respect to the first derivative of the expected profit function and is stated as q_1^*. However, like in the centralized case, because of the constraint, the optimal production quantity, i.e. $q_1^{'}$ is obtained as follows:

$$q_1^*(q_2) = \begin{cases} Q_1 u & \text{if } q_1^{'}(q_2) > Q_1 u \\ q_1^{'}(q_2) & \text{if } q_1^{'}(q_2) \leq Q_1 u \end{cases}$$

where the term $q_1^{'}(q_2)$ is the optimum olive oil production amount (given q_2) satisfying the FOC as shown below:

$$\begin{aligned} \frac{dE_1[\Pi_2(q_1; q_2, Q_1, u)]}{dq_1} = & - c_{p1} - h_1 + p(u) + b_1 - b_1\Phi(q_1) + (s_1 - p(u))\Phi(q_1)\Psi(q_2) \\ & + \int_{q_2}^{\infty} (s_1 - p(u))\Phi(q_1 - \alpha_1(x_2 - q_2))\psi(x_2)dx_2 = 0 \end{aligned} \tag{4.3}$$

$q_1^*(q_2)$ is the best response function of the first manufacturer. Due to the Proposition 4.1.1, the response function is unique.

In the rest of the study, u_j^* is defined as the yield at which the optimal amount of olives to be processed, namely $q_j^{'}(q_i)$, is equal to the amount of the olives obtained from the leased area, i.e. $Q_j u_j^*$.

The expected profit function of the second manufacturer can be written in the same manner. Thus, it is also concave leading to the existence of one optimal quantity for each yield, q_1 and Q_2. Because for every yield, leasing area and competitors production quantity, there is only one optimum production quantity for both players, the response functions can be stated as unique.

The response functions of both manufacturers are displayed in the Figure 4.1.

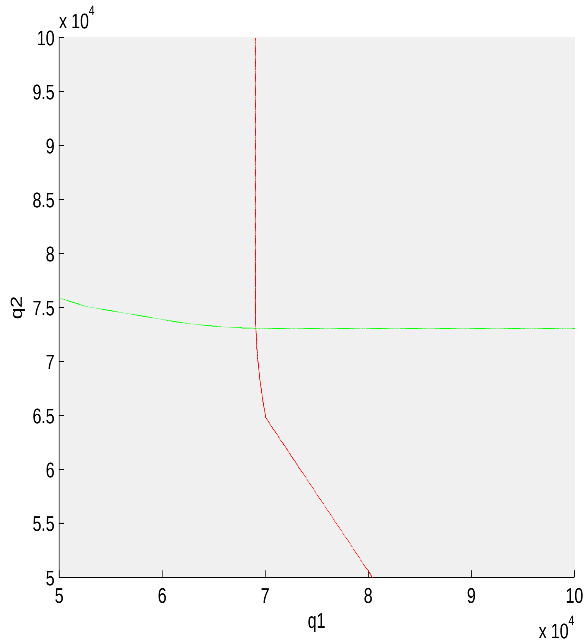

Figure 4.1. Graphical Illustration of the Response Functions

The optimal production quantities of both manufacturers depend on the yield, leasing area and the optimal quantity of the other manufacturer. Thus, competition between these manufacturers will effect their optimal quantities in the second stage. Therefore, it is a question whether an equilibrium point exists and whether it is unique. In order to characterize the equilibrium point(or points), the Hessian matrices of the expected profit functions of both manufacturers are investigated.

Theorem 4.1.1. *In the second stage of the decentralized game played by two manufacturers:*

i)There exists at least one Nash equilibrium.

ii)The equilibrium is unique.

Proof: See Appendix A.6.

This result is already available in Parlar [3]. The only difference is the capacity restriction of Qu on production quantity q.

Depending on the realized yield and the constraint, the equilibrium point can be one of the four pairs of $(q_1^{'},q_2^{'})$, $(Q_1u,q_2^{'})$, $(q_1^{'},Q_2u)$ and $(Q_1u,\ Q_2u)$. The first pair is the equilibrium point of the case when both $q_1^{'}$ and $q_2^{'}$ are satisfying the constraint. The second pair represents the case when the the first manufacturer has less olives than he needs for the maximization of the expected profit function whereas in the third pair case, the reverse is true. Eventually, the last pair represents the case where both manufacturers can only process the olives obtained from the leasing area. The equilibrium can be any point in the shaded area in Figure 4.2.

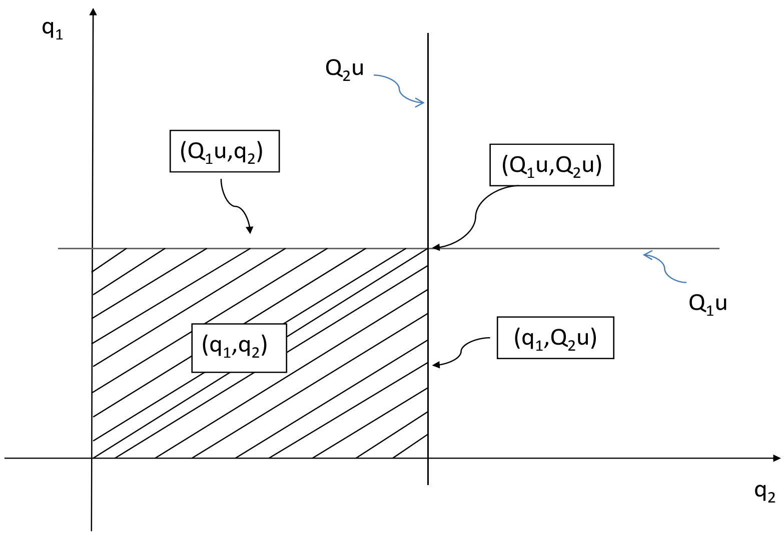

Figure 4.2. Graphical Illustration of the Equilibrium Pairs

4.2. Analysis of the First Stage

In this stage, the expected profit function of the first manufacturer depends on the area the second manufacturer is going to lease, as well as the optimal olive oil production quantities of both manufacturers obtained from the expected profit functions of the second stage. Like in the centralized case, in the first stage, expectation over the yield and demand is taken in order to obtain the optimal leasing area. Yet, in this case, there are two different u^* terms, i.e. u_1^* and u_2^*, and their relationship to each other affects the expected profit function. u_1^* is found from the equality $Q_1 u_1^* = q_1^{'}$ and u_2^* is obtained from the equality $Q_2 u_2^* = q_2^{'}$.

In the first case, where u_1^* is less than u_2^*, the expected profit function of the first manufacturer in the first stage is:

$$\begin{aligned} E_1[\Pi_1(Q_1; q_1^*, q_2^*, Q_2, u)] = &- cQ_1 + \int_{u_2^*}^{M} V(Q_1; q_1^*, q_2^*, u) y(u) du \\ &+ \int_{u_1^*}^{u_2^*} W_1(Q_1; q_1^*, Q_2, u) y(u) du \\ &+ \int_{0}^{u_1^*} Z(Q_1; Q_2, u) y(u) du \end{aligned} \tag{4.4}$$

where

$$\begin{aligned} V(Q_1; q_1^*, q_2^*, u) = &-c_{p1} q_1^* + h_1(Q_1 u - q_1^*) \\ &+ \int_{q_1^*}^{\infty} (p(u) q_1^* - b_1(x_1) - q_1^*)) \phi(x_1, u) dx_1 \\ &+ \int_{-\infty}^{q_1^*} \int_{-\infty}^{q_2^*} p(u) x_1 \phi(x_1, u) \psi(x_2, u) dx_2 dx_1 \\ &+ \int_{-\infty}^{q_1^*} \int_{-\infty}^{q_2^*} s_1(q_1^* - x_1) \psi(x_2, u) dx_2 dx_1 \\ &+ \int_{q_2^*}^{\infty} \int_{-\infty}^{q_1^* - \alpha_1(x_2) - q_2^*)} p(u)(x_1 + \alpha_1(x_2 - q_2^*)) \phi(x_1, u) \psi(x_2, u) dx_1 dx_2 \\ &+ \int_{q_2^*}^{\infty} \int_{-\infty}^{q_1^* - \alpha_1(x_2 - q_2^*)} s_1(q_1^* - x_1 - \alpha_1(x_2 - q_2^*)) \phi(x_1, u) \psi(x_2, u) dx_1 dx_2 \\ &+ \int_{q_2^*}^{\infty} \int_{q_1^* - \alpha_1(x_2 - q_2^*)}^{q_1^*} p(u) q_1^* \phi(x_1, u) \psi(x_2, u) dx_1 dx_2 \end{aligned} \tag{4.5}$$

$$
\begin{aligned}
W_1(Q_1; q_1^*, Q_2, u) = & -c_{p1} q_1^* + h_1(Q_1 u - q_1^*) \\
& + \int_{q_1^*}^{\infty} (p(u) q_1^* - b_1(x_1 - q_1^*)) \phi(x_1, u) dx_1 \\
& + \int_{-\infty}^{q_1^*} \int_{-\infty}^{Q_2 u} p(u) x_1 \phi(x_1, u) \psi(x_2, u) dx_2 dx_1 \\
& + \int_{-\infty}^{q_1^*} \int_{-\infty}^{Q_2 u} s_1(q_1^* - x_1) \psi(x_2, u) dx_2 dx_1 \\
& + \int_{Q_2 u}^{\infty} \int_{-\infty}^{q_1^* - \alpha_1(x_2 - Q_2 u)} p(u)(x_1 + \alpha_1(x_2 - Q_2 u)) \phi(x_1, u) \psi(x_2, u) dx_1 dx_2 \\
& + \int_{Q_2 u}^{\infty} \int_{-\infty}^{q_1^* - \alpha_1(x_2 - Q_2 u)} s_1(q_1^* - x_1 - \alpha_1(x_2 - Q_2 u)) \phi(x_1, u) \psi(x_2, u) dx_1 dx_2 \\
& + \int_{Q_2 u}^{\infty} \int_{q_1^* - \alpha_1(x_2 - Q_2 u)}^{q_1^*} p(u) q_1^* \phi(x_1, u) \psi(x_2, u) dx_1 dx_2
\end{aligned}
$$

$$
\begin{aligned}
Z(Q_1; Q_2, u) = & -c_{p1} Q_1 u + \int_{Q_1 u}^{\infty} (p(u) Q_1 u - b_1(x_1 - Q_1 u)) \phi(x_1, u) dx_1 \\
& + \int_{-\infty}^{Q_1 u} \int_{-\infty}^{Q_2 u} p(u)(x_1) \phi(x_1, u) \psi(x_2, u) dx_2 dx_1 \\
& + \int_{-\infty}^{Q_1 u} \int_{-\infty}^{Q_2 u} s_1(Q_1 u - x_1) \phi(x_1, u) \psi(x_2, u) dx_2 dx_1 \\
& + \int_{Q_2 u}^{\infty} \int_{-\infty}^{Q_1 - \alpha_1(x_2 - Q_2 u)} p(u)(x_1 + \alpha_1(x_2 - Q_2 u)) \phi(x_1, u) \psi(x_2, u) dx_1 dx_2 \\
& + \int_{Q_2 u}^{\infty} \int_{-\infty}^{Q_1 - \alpha_1(x_2 - Q_2 u)} s_1(Q_1 u - x_1 - \alpha_1(x_2 - Q_2 u)) \phi(x_1, u) \psi(x_2, u) dx_1 dx_2 \\
& + \int_{Q_2 u}^{\infty} \int_{Q_1 u - \alpha_1(x_2 - Q_2 u)}^{Q_1 u} p(u) Q_1 u \phi(x_1, u) \psi(x_2, u) dx_1 dx_2
\end{aligned}
\tag{4.6}
$$

In the expected profit equation, the first term represents the cost of the leasing area. The terms $V(Q_1; q_1^*, q_2^*, u)$, $W_1(Q_1; q_1^*, Q_2, u)$ and $Z(Q_1; Q_2, u)$ represent the expected profit function of the first manufacturer in the case where the equilibrium of the second stage is at (q_1^*, q_2^*), $(q_1^*, Q_2 u)$ and $(Q_1 u, Q_2 u)$, respectively.

On the other hand, if u_1^* is greater than u_2^*, the expected profit function of the first manufacturer becomes:

$$
\begin{aligned}
E_1[\Pi_1(Q_1; q_1^*, q_2^*, Q_2, u)] = & - cQ_1 + \int_{u_1^*}^{M} V(Q_1, q_1^*, q_2^*, u) y(u) du \\
& + \int_{u_2^*}^{u_1^*} W_2(Q_1; q_2^*, u) y(u) du \\
& + \int_{0}^{u_2^*} Z(Q_1; Q_2, u) y(u) du
\end{aligned} \tag{4.7}
$$

where the functions $V(Q_1, q_1^*, q_2^*, u)$ and $Z(Q_1; Q_2, u)$ are the same functions written for the previous case, i.e. the equations (4.5) and (4.6) respectively, and $W_2(Q_1; q_2^*, u)$ is stated below:

$$
\begin{aligned}
W_2(Q_1; q_2^*, u) = & -c_{p1} Q_1 u + \int_{Q_1 u}^{\infty} (p(u) Q_1 u - b_1(x_1 - Q_1 u)) \phi(x_1, u) dx_1 \\
& + \int_{-\infty}^{Q_1 u} \int_{-\infty}^{q_2^*} p(u) x_1 \phi(x_1, u) \psi(x_2, u) dx_2 dx_1 \\
& + \int_{-\infty}^{Q_1 u} \int_{-\infty}^{q_2^*} s_1 (Q_1 u - x_1) \phi(x_1, u) \psi(x_2, u) dx_2 dx_1 \\
& + \int_{q_2^*}^{\infty} \int_{-\infty}^{Q_1 - \alpha_1 (x_2 - q_2^*)} p(u)(x_1 + \alpha_1 (x_2 - q_2^*)) \phi(x_1, u) \psi(x_2, u) dx_1 dx_2 \\
& + \int_{q_2^*}^{\infty} \int_{-\infty}^{Q_1 - \alpha_1 (x_2 - q_2^*)} s_1 (Q_1 u - x_1 - \alpha_1 (x_2 - q_2^*)) \phi(x_1) \psi(x_2) dx_1 dx_2 \\
& + \int_{q_2^*}^{\infty} \int_{Q_1 u - \alpha_1 (x_2 - q_2^*)}^{Q_1 u} p(u) Q_1 u \phi(x_1, u) \psi(x_2, u) dx_1 dx_2
\end{aligned}
$$

In the expected profit equation, the first term represents the cost of the leasing area. The terms $V(Q_1; q_1^*, q_2^*, u)$, $W(Q_1; q_2^*, u)$ and $Z(Q_1; Q_2, u)$ represent the expected profit function of the first manufacturer in the case where the equilibrium of the second stage is at (q_1^*, q_2^*), $(Q_1 u, q_2^*)$ and $(Q_1 u, Q_2 u)$, respectively.

Proposition 4.2.1. *The expected profits of both manufacturers in the first stage, as given in the equations (4.4) and (4.7) are strictly concave in Q_j and the size of the leasing area from the first order condition maximizes the total expected profit in the first stage.*

Proof: See Appendix A.7.

$Q_1^*(Q_2)$ is a response function of the first manufacturer. Because of Proposition 4.2.1, this response function is unique. The same is also true for the second manufacturer's response function, $Q_2^*(Q_1)$.

This concavity implies the existence of one optimal leasing area with respect to the expected yield and to the leased area of the competitor. The size of the leasing area satisfying the first order conditions can be defined as the optimal one. Again, there are two cases regarding the relationship of u_j^*s to each other. If u_1^* is less than u_2^*, the FOC is:

$$\begin{aligned}\frac{dE_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1} &= -c_1 + \int_{u_1^*}^{M} h_1 u y(u) du \\ &+ \int_0^{u_1^*} (-c_{p1} + p(u) + b_1 - b_1\Phi(Q_1 u)) u y(u) du \\ &+ \int_0^{u_1^*} (s_1 - p(u))\Phi(Q_1 u)\Psi(Q_2 u) u y(u) du \\ + \int_0^{u_1^*}\int_{Q_2 u}^{\infty} (s_1 - p(u))\Phi(Q_1 u - \alpha_1(x_2 - Q_2 u))&\psi(x_2,u) u y(u) dx_2 du = 0\end{aligned} \tag{4.8}$$

This is the case where Q_1^* is found from when $u_1^* \leq u_2^*$.

Yet, if $u_1^* > u_2^*$, the FOC becomes:

$$\begin{aligned}\frac{dE_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1} &= -c_1 + \int_{u_1^*}^{M} h_1 uy(u)du \\ &+ \int_{u_2^*}^{u_1^*} (-c_{p1} + p(u) + b_1 - b_1\Phi(Q_1u))uy(u)du \\ &+ \int_{u_2^*}^{u_1^*} (s_1 - p(u))\Phi(Q_1u)\Psi(q_2)uy(u)du \\ &+ \int_{u_2^*}^{u_1^*}\int_{q_2}^{A_2} (s_1 - p(u))\Phi(Q_1u - \alpha_1(x_2 - q_2))\psi(x_2,u)uy(u)dx_2du \\ &+ \int_{0}^{u_2^*} (-c_{p1} + p(u) + b_1 - b_1\Phi(Q_1u))uy(u)du \\ &+ \int_{0}^{u_2^*} (s_1 - p(u))\Phi(Q_1u)\Psi(Q_2u)uy(u)du \\ &+ \int_{0}^{u_2^*}\int_{Q_2u}^{\infty} (s_1 - p(u))\Phi(Q_1u - \alpha_1(x_2 - Q_2u))\psi(x_2,u)uy(u)dx_2du = 0\end{aligned} \tag{4.9}$$

The manufacturers here again play a Nash game, where they both decide on the leasing area at the same time without knowing each others decisions. The response functions of the first and second manufacturer help to analyze the nature of the game and to find out the equilibrium point or points if they exist. So, the response functions are investigated accordingly.

Theorem 4.2.1. *In the first stage of the decentralized game played by two manufacturers:*
i)There exists at least one Nash equilibrium.
ii)The equilibrium is unique.

Proof: See Appendix A.8.

The unique equilibrium point is located at Q_1^*, Q_2^* obtained from the FOC of the expected profit functions stated for the manufacturers.

4.3. Comparative Statics

Like in the centralized analysis, in the decentralized case, the parameters play an important role on the optimization of the decision variables, as well as their equilibrium points, in both stages. The effect they cause on the optimal olive oil production and leasing area can be shown analytically. The results for the second stage are:

Proposition 4.3.1. *In the second stage, the amount of the olive oil produced by the manufacturer j increases when*

- *its lost sales cost, b_j, decreases*
- *its processing cost of the olive oil, c_{pj}, decreases*
- *the salvage value of the olive, h_j, decreases*
- *the salvage value of the olive oil, s_j increases*

The other manufacturers' optimal production amount decreases in this case.

Proof: See Section A.9 in Appendix A.

The expectation was the manufacturers to behave like in the centralized case when their cost parameters change. The competitor manufacturer is going to respond to these changes by making the opposite move. This is again the expected behavior. If the optimal value of the first manufacturer's decision parameter rises, it most probably will meet its own demand such that the second manufacturer can not gain any extra customers leading to a fall in the optimal production decision.

In the first stage, the main issue is to determine the amount of the leasing area with respect to the expected profit function which depends on the random yield and random demand. Again, the effects of the cost parameters on the decision parameter are obtained analytically as follows:

Proposition 4.3.2. *In the first stage, the amount of the area leased by the manufacturer j increases when*

- *the cost of leasing a unit area,* c_j*, decreases*
- *its lost sales cost,* b_j*, increases*
- *its processing cost of the olive oil,* c_{pj}*, decreases*
- *the salvage value of the olive,* h_j*, increases*
- *the salvage value of the olive oil,* s_j*, increases*

The other manufacturers optimal leasing area size decreases in this case.

Proof. See Appendix A.10. □

The same reasoning explained for the behaviour of the manufacturers to the changes of the second stage can be used in order to define the changes in the equilibrium in this stage, too. Therefore, the manufacturers change their strategies as if they were the monopoly when their cost parameters change. This will effect the response of their competitors. In this stage, again the results are in line with the expected ones, i.e. the manufacturer whose parameters change behaves like in the centralized case while the other manufacturer responds with the opposite move.

5. NUMERICAL ANALYSIS

In this chapter, the aim is to analyze numerically the behaviour of the manufacturers with respect to changing parameters. The optimum and equilibrium values of the decision variables and the expected profit function for the first stage are calculated in the centralized and decentralized cases. Then, the impact of the changing parameters on the optimality, as well as Nash equilibrium is investigated and the results are reported.

In order to characterize and analyze the competition, the form of the demand function is significantly important. At the beginning of the study, the demand is stated to be price-dependent and stochastic. The type of demand function studied further is a demand function consisting of a deterministic part as well as an additive type of error term. It can be written as:

$$D(p(u)) = K - \beta p(u) + \varepsilon \tag{5.1}$$

for the centralized analysis. Here, K is the maximum olive oil market size, β is the rate of the decrease in olive oil demand per increase of the price for the product, $p(u)$ is the non increasing deterministic price function depending on u and ε is the random error term. ε is distributed within the range $[-B_1,B_2]$ with $B_1, B_2 > 0$. For the decentralized analysis, the demand function becomes:

$$D_j(p(u)) = K_j - \beta_j p(u) + \varepsilon_j \quad j = 1, 2 \tag{5.2}$$

where K_j is the maximum deterministic capacity of the olive oil demand for the manufacturer j, β_j is the rate of the decrease in olive oil demand per increase of the price for the product of the manufacturer j and ε_j is the random error term. ε_1 and ε_2 are distributed within the range $[-B_{11},B_{12}]$ and $[-B_{21},B_{22}]$ where all B_{11}, B_{12},B_{21} and B_{22} are nonnegative constants. The probability distribution functions and the cumulative probability distribution functions of ε_1, ε_2 are g,G and f,F, respectively. The probability functions of the random error terms in the demand functions, present in the centralized and decentralized case, are assumed to be uniform for the computational study.

The exogenous price function, $p(u)$, present in both functions 5.2 and 5.1 is assumed to be yield dependent due to the nature of this model. There is an inverse relationship between the exogenous price and the yield. This is mainly because in most of the industries, as well as in olive oil sector, scarce supply leads to increase in the price. The price function used in the numerical analysis is:

$$p(u) = 19.86 - 9.93 * u \tag{5.3}$$

which is obtained from sales data analysis included in Kazaz's paper [17]. The cost parameters whose effect on the optimality and the Nash equilibrium as reported in Sections 3.4 and 4.3 are set to be constant. For the value determination of these cost parameters, Kazaz [17] is taken as the reference, too. The unit leasing area cost is set to be 2.64, while the processing cost of olive oil is taken as 3.13. The salvage values of olive and olive oil are determined as 1.97 and 4. Eventually, the lost sales cost is set to 5 as in [17]. The probability distribution function of the yield is assumed to be uniform between 0 and 1 for the computational study.

In centralized analysis, the changing parameters are the maximum olive oil market capacity, K, and the boundaries of the random error term in demand function. In decentralized analysis, the fraction of unsatisfied customers who can be satisfied by the competitor, α_j is also added to the parameters K_j, B_{11}, B_{12},B_{21} and B_{22}. In all cases, the parameters are set to two levels, defined as high and low. For centralized analysis, a 2×2 factorial design is used whereas in the decentralized analysis, the experimental design employed is $2 \times 3 \times 3$ type. In order to define the significance of the effects of the parameters on the optimality, ANOVA is applied and the results are tabulated in Appendix B.

All computations are carried out using MATLAB v. 7.0. "fsolve" function from

Matlab Optimization Toolbox(v. 7.0) is also used in order to find out the values of the optimal decision variables.

5.1. Analysis of the Second Stage

In this stage, the decision variable is the production amount of the olive oil. Hence, the investigation is concentrated on how much the parameters do influence the optimum production size in this stage. The analysis for the centralized and decentralized cases are displayed in the following sections.

5.1.1. Centralized Model

In the centralized case, the changing parameters are K, ranges for ε and u. The maximum olive oil market capacity is set to be 125000 and 150000, whereas the boundaries for the error term changes from $(-10000, 10000)$ to $(-15000, 15000)$. All these four cases are investigated in four different yield values, 0.25, 0.5, 0.75 and 1, respectively. The results are displayed in Table 5.1.

As it can be easily observed, the optimum olive oil production amount decreases and the expected profit of the manufacturer rises as yield of the olive falls. This mainly displays the effect of the yield on the olive oil price, i.e. as yield decreases, the price for the olive oil increases and thus, demand for the olive oil decreases. However, the price of the end-product depends on the yield, too. Therefore, decreasing yield leads to an increase in the olive oil price such that although there is less production at low yields, the expected profit rises.

The maximum capacity of the olive oil market also has a major influence on the decision variable, q. Increase in K results in the rise of q, as well as the expected profit.

The variability of the random error term in the demand function also influences the optimal value of the decision variable q^* and $E[\Pi_2(q; Q, u)]$. As the error term boundaries get larger in absolute value, i.e. the variance increases, the optimal pro-

Table 5.1. The Numerical Illustration of Responses for the Centralized Model

K	$[-B_1, B_2]$	u	q^*	$E[\Pi_2]$
125000	(-10000,10000)	0.25	61395	643590
125000	(-15000,15000)	0.25	61286	641480
150000	(-10000,10000)	0.25	75473	789550
150000	(-15000,15000)	0.25	75351	787210
125000	(-10000,10000)	0.5	118720	1310100
125000	(-15000,15000)	0.5	123030	1305500
150000	(-10000,10000)	0.5	143720	1610500
150000	(-15000,15000)	0.5	148030	1604900
125000	(-10000,10000)	0.75	120950	1176000
125000	(-15000,15000)	0.75	125130	1170300
150000	(-10000,10000)	0.75	145950	1442100
150000	(-15000,15000)	0.75	150130	1436300
125000	(-10000,10000)	1	123060	1029700
125000	(-15000,15000)	1	127050	1023900
150000	(-10000,10000)	1	148060	1261400
150000	(-15000,15000)	1	152050	1255500

duction size definitely rises. As $E[\Pi_2(q; Q, u)]$ is concerned, the difference between the cases is small compared to the difference of q^*s. Moreover, at lower yield, the expected profit slightly increases as variability rises. Yet, as yield gets higher, the reverse is true.

According to ANOVA results, as q is concerned, all the parameters have significant influence on its value. In addition to this, only maximum capacity of the olive oil market and the yield has major impact on the expected profit of the manufacturer.

5.1.2. Decentralized Model

In the decentralized case, the changing parameters are K_j, α_j, boundaries of the probability function belonging to the ε_j, i.e. $[B_{11}, B_{12}]$, $[B_{21}, B_{22}]$, and u. Two levels of each parameter is investigated. For K_j, the levels are 50000 and 75000, whereas the boundaries for ε_j change from $(-5000, 5000)$ and $(-10000, 10000)$. The last parameter

Table 5.2. The Numerical Illustration of Responses for the Decentralized Model (u=0.25)

K_1	α_1	$[B_{11}, B_{12}]$	K_2	α_2	$[B_{21}, B_{22}]$	$q_1^{'}$	$q_2^{'}$
50000	0.3	(-5000,5000)	75000	0.3	(-5000,5000)	37028	62028
50000	0.3	(-5000,5000)	75000	0.7	(-5000,5000)	37028	62033
50000	0.7	(-5000,5000)	75000	0.3	(-5000,5000)	37033	62028
75000	0.3	(-5000,5000)	75000	0.3	(-5000,5000)	62028	62028
75000	0.3	(-5000,5000)	75000	0.7	(-5000,5000)	62028	62033
75000	0.7	(-5000,5000)	75000	0.3	(-5000,5000)	62033	62028
50000	0.3	(-10000,10000)	75000	0.3	(-5000,5000)	41429	62032
50000	0.3	(-10000,10000)	75000	0.7	(-5000,5000)	41429	62042
50000	0.7	(-10000,10000)	75000	0.3	(-5000,5000)	41434	62032
75000	0.3	(-10000,10000)	75000	0.3	(-5000,5000)	66429	62032
75000	0.3	(-10000,10000)	75000	0.7	(-5000,5000)	66429	62042
75000	0.7	(-10000,10000)	75000	0.3	(-5000,5000)	66434	62032
50000	0.3	(-5000,5000)	75000	0.3	(-10000,10000)	37032	66429
50000	0.3	(-5000,5000)	75000	0.7	(-10000,10000)	37032	66434
50000	0.7	(-5000,5000)	75000	0.3	(-10000,10000)	37042	66429
75000	0.3	(-5000,5000)	75000	0.3	(-10000,10000)	62032	66429
75000	0.3	(-5000,5000)	75000	0.7	(-10000,10000)	62032	66434
75000	0.7	(-5000,5000)	75000	0.3	(-10000,10000)	62042	66429

constituting the design of the experiment is α_j which takes 0.3 or 0.7. The optimal production amounts constituting the Nash equilibrium are investigated.

The results for the yield values 0.25, 0.5, 0.75 and 1 are displayed in Tables 5.2, 5.3, 5.4 and 5.5, respectively.

Like in the centralized case, as yield increases, the production quantities at equilibrium of both manufacturers rise. K_j being low or high also significantly effects the Nash equilibrium. As expected, the increase in α_j leads to the increase in the optimal production amount of the manufacturer j. However, the Nash equilibrium changes only dramatically when K of the player is at low level and the variability of the random

Table 5.3. The Numerical Illustration of Responses for the Decentralized Model (u=0.5)

K_1	α_1	$[B_{11}, B_{12}]$	K_2	α_2	$[B_{21}, B_{22}]$	$q_1^{'}$	$q_2^{'}$
50000	0.3	(-5000,5000)	75000	0.3	(-5000,5000)	39418	64418
50000	0.3	(-5000,5000)	75000	0.7	(-5000,5000)	39418	64424
50000	0.7	(-5000,5000)	75000	0.3	(-5000,5000)	39424	64418
75000	0.3	(-5000,5000)	75000	0.3	(-5000,5000)	64418	64418
75000	0.3	(-5000,5000)	75000	0.7	(-5000,5000)	64418	64424
75000	0.7	(-5000,5000)	75000	0.3	(-5000,5000)	64424	64418
50000	0.3	(-10000,10000)	75000	0.3	(-5000,5000)	43726	64423
50000	0.3	(-10000,10000)	75000	0.7	(-5000,5000)	43726	64436
50000	0.7	(-10000,10000)	75000	0.3	(-5000,5000)	43732	64423
75000	0.3	(-10000,10000)	75000	0.3	(-5000,5000)	68726	64423
75000	0.3	(-10000,10000)	75000	0.7	(-5000,5000)	68726	64436
75000	0.7	(-10000,10000)	75000	0.3	(-5000,5000)	68732	64423
50000	0.3	(-5000,5000)	75000	0.3	(-10000,10000)	39423	68726
50000	0.3	(-5000,5000)	75000	0.7	(-10000,10000)	39423	68732
50000	0.7	(-5000,5000)	75000	0.3	(-10000,10000)	39436	68726
75000	0.3	(-5000,5000)	75000	0.3	(-10000,10000)	64423	68726
75000	0.3	(-5000,5000)	75000	0.7	(-10000,10000)	64423	68732
75000	0.7	(-5000,5000)	75000	0.3	(-10000,10000)	64436	68726

error term belonging to his demand function is larger than the one belonging to the competitors demand function.

The ANOVA results point that K_1, $[B_{11}, B_{12}]$ and u affect the optimum olive oil production size of the first manufacturer, whereas $[B_{11}, B_{12}]$, $[B_{21}, B_{22}]$ and u influences the second manufacturers' decision.

5.1.3. Comparison

As the results of the centralized and decentralized cases are analyzed, the optimal production amount in the centralized case exceeds the sum of the ones belonging to

Table 5.4. The Numerical Illustration of Responses for the Decentralized Model (u=0.75)

K_1	α_1	$[B_{11}, B_{12}]$	K_2	α_2	$[B_{21}, B_{22}]$	q_1'	q_2'
50000	0.3	(-5000,5000)	75000	0.3	(-5000,5000)	41774	66774
50000	0.3	(-5000,5000)	75000	0.7	(-5000,5000)	41774	66782
50000	0.7	(-5000,5000)	75000	0.3	(-5000,5000)	41782	66774
75000	0.3	(-5000,5000)	75000	0.3	(-5000,5000)	66774	66774
75000	0.3	(-5000,5000)	75000	0.7	(-5000,5000)	66774	66782
75000	0.7	(-5000,5000)	75000	0.3	(-5000,5000)	66782	66774
50000	0.3	(-10000,10000)	75000	0.3	(-5000,5000)	45953	66780
50000	0.3	(-10000,10000)	75000	0.7	(-5000,5000)	45953	66797
50000	0.7	(-10000,10000)	75000	0.3	(-5000,5000)	45962	66780
75000	0.3	(-10000,10000)	75000	0.3	(-5000,5000)	70953	66780
75000	0.3	(-10000,10000)	75000	0.7	(-5000,5000)	70953	66797
75000	0.7	(-10000,10000)	75000	0.3	(-5000,5000)	70962	66780
50000	0.3	(-5000,5000)	75000	0.3	(-10000,10000)	41780	70953
50000	0.3	(-5000,5000)	75000	0.7	(-10000,10000)	41780	70962
50000	0.7	(-5000,5000)	75000	0.3	(-10000,10000)	41797	70953
75000	0.3	(-5000,5000)	75000	0.3	(-10000,10000)	66780	70953
75000	0.3	(-5000,5000)	75000	0.7	(-10000,10000)	66780	70962
75000	0.7	(-5000,5000)	75000	0.3	(-10000,10000)	66797	70953

both manufacturers in the decentralized case. This is mainly because of the α_j term used in the decentralized case. Only a part of the demand which is not met by the first manufacturer can be met by the second manufacturer, and vice versa. Thus, there is a chance that some customers do not add to the total demand, therefore the total production size that maximizes the expected profit functions of both manufacturers will be smaller than the one obtained from the FOC in the centralized case. However, the difference can be defined as minor with respect to the results obtained.

As the ANOVA results for both cases are investigated, it is observed that the capacity of the olive oil market and the yield very much contributes to the determination of olive oil production size of the manufacturers in the centralized and decentralized

Table 5.5. The Numerical Illustration of Responses for the Decentralized Model (u=1)

K_1	α_1	$[B_{11}, B_{12}]$	K_2	α_2	$[B_{21}, B_{22}]$	$q_1^{'}$	$q_2^{'}$
50000	0.3	(-5000,5000)	75000	0.3	(-5000,5000)	44072	69072
50000	0.3	(-5000,5000)	75000	0.7	(-5000,5000)	44072	69083
50000	0.7	(-5000,5000)	75000	0.3	(-5000,5000)	44083	69072
75000	0.3	(-5000,5000)	75000	0.3	(-5000,5000)	69072	69072
75000	0.3	(-5000,5000)	75000	0.7	(-5000,5000)	69072	69083
75000	0.7	(-5000,5000)	75000	0.3	(-5000,5000)	69083	69072
50000	0.3	(-10000,10000)	75000	0.3	(-5000,5000)	48065	69080
50000	0.3	(-10000,10000)	75000	0.7	(-5000,5000)	48065	69102
50000	0.7	(-10000,10000)	75000	0.3	(-5000,5000)	48076	69080
75000	0.3	(-10000,10000)	75000	0.3	(-5000,5000)	73065	69080
75000	0.3	(-10000,10000)	75000	0.7	(-5000,5000)	73065	69102
75000	0.7	(-10000,10000)	75000	0.3	(-5000,5000)	73076	69080
50000	0.3	(-5000,5000)	75000	0.3	(-10000,10000)	44080	73065
50000	0.3	(-5000,5000)	75000	0.7	(-10000,10000)	44080	73076
50000	0.7	(-5000,5000)	75000	0.3	(-10000,10000)	44102	73065
75000	0.3	(-5000,5000)	75000	0.3	(-10000,10000)	69080	73065
75000	0.3	(-5000,5000)	75000	0.7	(-10000,10000)	69080	73076
75000	0.7	(-5000,5000)	75000	0.3	(-10000,10000)	69102	73065

cases. Moreover, the change in the variability of the random error term mainly influences the optimality of the decision variables.

5.2. Analysis of the First Stage

In the first stage, the optimum leasing area is determined by taking the randomness of demand and yield into consideration. In this stage, the effects of the same parameters, except the yield, analyzed in the previous section on the expected profit as well as on the optimality and u^* are investigated. In order to decide on the optimal production size with respect to the demand and yield expectations, the second stage results are used. Two scenarios are investigated. The first one, called Expected Scenario, uses the expectation of the production with respect to the random demand

and yield. The second scenario, Best Yield Case Scenario, is based on realizing the maximum amount of production in order to cover as much demand as possible. The results of both cases are studied separately. In the following sections, the results for these cases are displayed and discussed.

5.2.1. Centralized Analysis

In the centralized analysis of the first stage, K and $[B_1, B_2]$ are the parameters influencing the decision variable and the expected profit of the manufacturer. The levels of K and the range of the distribution function are the same as in the second stage centralized analysis. The results obtained are displayed in Tables 5.6 and 5.7.

Table 5.6. The Numerical Illustration of Responses for the Centralized Model-Expected Scenario

K	$[-B_1, B_2]$	u^*	Q^*	$E[\Pi_1]$
125000	(-10000,10000)	0.49	245580	228000
125000	(-15000,15000)	0.51	245142	224190
150000	(-10000,10000)	0.48	301891	278350
150000	(-15000,15000)	0.49	301404	274770

In the scenario which is based on the expectation of the production size with respect to the yield and demand, both parameters have a larger influence on the expected profit than on the optimal size of leasing area. As u^* values are concerned, at the low level of K, u^* values seem to be higher compared to the high level results. The reverse is true for the variability of the error term, at low levels of variability, u^* values are also low. Another observation is that as variance increases, u^* also rises, but the expected profit decreases. This is mainly because of the yield-dependent price setting. As u^* gets higher, the expected price of the end product decreases leading to a decrease in the expected profit. According to the ANOVA results, the maximum capacity of the olive oil market has a greater influence on Q and $E[\Pi_2]$ than the boundaries of the probability distribution function belonging to the random error term. By the determination of u^* no one of the parameters seem to be important.

Table 5.7. The Numerical Illustration of Responses for the Centralized Model-Best Yield Case Scenario

K	$[-B_1, B_2]$	u^*	Q^*	$E[\Pi_2]$
125000	(-10000,10000)	0.50	245017	227620
125000	(-15000,15000)	0.52	244656	223910
150000	(-10000,10000)	0.49	301333	277940
150000	(-15000,15000)	0.51	300915	274460

In the best yield case scenario, the expected profit of the manufacturer seems to be more affected by the change of the parameters than the optimal leasing area. At higher capacity of the olive oil market, both the expected profit and optimal leasing area are higher. This observation is also true for low level of $[B_1, B_2]$. u^* values are higher at low level of K and high level of error term variability. The ANOVA results show that both the parameters have the same impact on u^* while a change in K affects Q and $E[\Pi(Q, q)]$ more than a change in the boundaries of the probability density function belonging to the random error term.

When both cases are compared, the main observation is that in both cases, the impact of the parameters on Q and $E[\Pi(Q; q, u)]$ is the same. The only difference between the expected and best yield case scenarios is the weight of the parameters in the determination of u^*. This is mainly because of the method used in the determination of q^*. In the best yield case, q^* used is the maximum production amount observed in the second stage which significantly depends on the both parameters investigated. However, in the expected scenario, q^* is the expectation of the production amounts over the yield. Thus, the effect of K and $[B_1, B_2]$ on q^* has been reduced.

5.2.2. Decentralized Analysis

In the decentralized analysis of the first stage, the parameters as well as their levels constituting the experimental design are the same as in the second stage decentralized analysis. The influenced variables are u_1^*, u_2^*, Q_1^*, Q_2^*, $E_1[\Pi_1]$ and $E_2[\Pi_1]$. The results obtained are displayed in Tables 5.8 and 5.9.

Table 5.8. The Numerical Illustration of Responses for the Decentralized Model-Expected Scenario

K_1	α_1	$[B_{11}, B_{12}]$	K_2	α_2	$[B_{21}, B_{22}]$	Q_1^*	Q_2^*	$E_1[\Pi_1]$	$E_2[\Pi_1]$
50000	0.3	(-5000,5000)	75000	0.3	(-5000,5000)	78102	133991	79927	130980
50000	0.3	(-5000,5000)	75000	0.7	(-5000,5000)	77964	134617	79886	131380
50000	0.7	(-5000,5000)	75000	0.3	(-5000,5000)	78634	133934	80383	130940
75000	0.3	(-5000,5000)	75000	0.3	(-5000,5000)	134062	134062	130780	130780
75000	0.3	(-5000,5000)	75000	0.7	(-5000,5000)	134001	134740	130770	131120
75000	0.7	(-5000,5000)	75000	0.3	(-5000,5000)	134740	134001	131120	130770
50000	0.3	(-10000,10000)	75000	0.3	(-5000,5000)	78576	133720	74235	131710
50000	0.3	(-10000,10000)	75000	0.7	(-5000,5000)	78533	134327	74283	132280
50000	0.7	(-10000,10000)	75000	0.3	(-5000,5000)	79679	133885	74283	132280
75000	0.3	(-10000,10000)	75000	0.3	(-5000,5000)	134054	134520	126700	131240
75000	0.3	(-10000,10000)	75000	0.7	(-5000,5000)	133989	135327	126630	131640
75000	0.7	(-10000,10000)	75000	0.3	(-5000,5000)	134922	134557	127220	131160
50000	0.3	(-5000,5000)	75000	0.3	(-10000,10000)	78297	133750	80407	131150
50000	0.3	(-5000,5000)	75000	0.7	(-10000,10000)	78236	134471	80365	131790
50000	0.7	(-5000,5000)	75000	0.3	(-10000,10000)	79180	133676	80830	130750
75000	0.3	(-5000,5000)	75000	0.3	(-10000,10000)	134520	134054	131240	126700
75000	0.3	(-5000,5000)	75000	0.7	(-10000,10000)	134557	134922	131160	127220
75000	0.7	(-5000,5000)	75000	0.3	(-10000,10000)	135327	133989	131640	126630

In the expected scenario, the difference in the maximum capacities of the olive oil market strongly influences the equilibrium point as well as the expected profit value. When both manufacturers have the same maximum capacity values, the equilibrium point shifts in favor of the manufacturer with the error term having larger variability, so does the expected profit value, too. The parameter α_j also effects the Nash equilibrium and expected profits of the manufacturers. The manufacturer having a larger α_j, decides on leasing more area and the competitor does the opposite.

According to the ANOVA results, K_1 and $[B_{11}, B_{12}]$ has importance by the determination of u_1^* whereas u_2^* depends only on the variabilities of the random error terms belonging to the manufacturers. As Nash equilibrium is concerned, the maximum capacity of the olive oil market plays an important role. Moreover, the levels of α_1 and $[B_{21}, B_{22}]$ influences the optimal value of Q_1 whereas Q_2^* is effected by α_1 and α_2. K_1 and $[B_{11}, B_{12}]$ are the parameters having a major impact on the expected profits of both manufacturer while α_1 and $[B_{21}, B_{22}]$ also influence $E_1[\Pi_1]$ and $E_2[\Pi_1]$, respectively.

Table 5.9. The Numerical Illustration of Responses for the Decentralized Model-Best Yield Case Scenario

K_1	α_1	$[B_{11}, B_{12}]$	K_2	α_2	$[B_{21}, B_{22}]$	Q_1^*	Q_2^*	$E_1[\Pi_1]$	$E_2[\Pi_1]$
50000	0.3	(-5000,5000)	75000	0.3	(-5000,5000)	77369	133431	82747	134360
50000	0.3	(-5000,5000)	75000	0.7	(-5000,5000)	77319	134090	82680	134580
50000	0.7	(-5000,5000)	75000	0.3	(-5000,5000)	77956	133373	83045	134340
75000	0.3	(-5000,5000)	75000	0.3	(-5000,5000)	133470	133470	134330	134330
75000	0.3	(-5000,5000)	75000	0.7	(-5000,5000)	133413	134108	134310	134470
75000	0.7	(-5000,5000)	75000	0.3	(-5000,5000)	134108	133413	134470	134310
50000	0.3	(-10000,10000)	75000	0.3	(-5000,5000)	77925	134277	73913	137000
50000	0.3	(-10000,10000)	75000	0.7	(-5000,5000)	77881	134930	73833	137400
50000	0.7	(-10000,10000)	75000	0.3	(-5000,5000)	78913	134493	74657	136740
75000	0.3	(-10000,10000)	75000	0.3	(-5000,5000)	133395	134897	127910	135410
75000	0.3	(-10000,10000)	75000	0.7	(-5000,5000)	133320	135905	127760	135810
75000	0.7	(-10000,10000)	75000	0.3	(-5000,5000)	134213	134823	128260	135400
50000	0.3	(-5000,5000)	75000	0.3	(-10000,10000)	77653	133209	83341	128420
50000	0.3	(-5000,5000)	75000	0.7	(-10000,10000)	77586	133967	83283	128660
50000	0.7	(-5000,5000)	75000	0.3	(-10000,10000)	78497	133131	83725	128410
75000	0.3	(-5000,5000)	75000	0.3	(-10000,10000)	134897	133395	135410	127910
75000	0.3	(-5000,5000)	75000	0.7	(-10000,10000)	134823	134213	135400	128260
75000	0.7	(-5000,5000)	75000	0.3	(-10000,10000)	135905	133320	135810	127760

When the results of the best yield case scenario are analyzed, it is seen that the maximum capacity of the olive oil market is significantly important in the determination of the equilibrium point, as well as the expected profit functions of both manufacturers. The impact of α_j can also be observed in the results, as α_j increases, both Q_j and $E_j[\Pi_1]$ increases whereas the competitor decides on a lower optimal leasing area and its expected profit falls. The level of the variability of the random error term influences the equilibrium, too.

The ANOVA results show a strong relationship between K_1 and u_1^*. The level of first manufacturers random error term variability also effects u_1^* while u_2^* depends on the level of boundaries belonging to the probability density function of the second manufacturers random error term. The equilibrium point is influenced by K_1, $[B_{11}, B_{12}]$, α_1 and α_2. The levels of K_1 and $[B_{11}, B_{12}]$ impact the expected profit of the first manufacturer while the level of $[B_{21}, B_{22}]$ has a significant importance in the determination of the second manufacturers expected profit.

Table 5.10. Comparison of Expected Profits in Centralized and Decentralized Settings in the First Stage

K_1	α_1	$[B_{11}, B_{12}]$	K_2	α_2	$[B_{21}, B_{22}]$	Δ_e	Δ_w
50000	0.3	(-5000,5000)	75000	0.3	(-5000,5000)	92	95
50000	0.3	(-5000,5000)	75000	0.7	(-5000,5000)	93	95
50000	0.7	(-5000,5000)	75000	0.3	(-5000,5000)	93	96
75000	0.3	(-5000,5000)	75000	0.3	(-5000,5000)	94	97
75000	0.3	(-5000,5000)	75000	0.7	(-5000,5000)	94	97
75000	0.7	(-5000,5000)	75000	0.3	(-5000,5000)	94	97
50000	0.3	(-10000,10000)	75000	0.3	(-5000,5000)	91	93
50000	0.3	(-10000,10000)	75000	0.7	(-5000,5000)	92	93
50000	0.7	(-10000,10000)	75000	0.3	(-5000,5000)	92	93
75000	0.3	(-10000,10000)	75000	0.3	(-5000,5000)	94	96
75000	0.3	(-10000,10000)	75000	0.7	(-5000,5000)	94	96
75000	0.7	(-10000,10000)	75000	0.3	(-5000,5000)	94	96
50000	0.3	(-5000,5000)	75000	0.3	(-10000,10000)	94	95
50000	0.3	(-5000,5000)	75000	0.7	(-10000,10000)	95	95
50000	0.7	(-5000,5000)	75000	0.3	(-10000,10000)	94	95
75000	0.3	(-5000,5000)	75000	0.3	(-10000,10000)	94	96
75000	0.3	(-5000,5000)	75000	0.7	(-10000,10000)	94	96
75000	0.7	(-5000,5000)	75000	0.3	(-10000,10000)	94	96

5.2.3. Comparison

When the centralized and decentralized cases are compared to each other, the total profit obtained in the decentralized case is smaller than the profit in the centralized case. Again, the reason for this outcome is the presence of the α_j term. When the results for the expected and best yield case scenarios in both cases are investigated, the difference in the total leased area between the centralized and decentralized cases in both scenarios are nearly the same, whereas the expected profit difference between the centralized and decentralized cases in the best yield case scenario is smaller compared to the one of the expected scenario. Yet, in both scenarios, the differences in total expected profits are minor compared to ones in the total leasing area.

Table 5.10 shows the ratio of the total expected profits calculated in the decentralized and centralized game for both scenarios. The terms Δ_e and Δ_w are representing the ratio of the total expected profit of all players in the decentralized game to the centralized game expected profit in percentage where the subscripts e and w denote the expected and best yield case scenario, respectively. As the sum of the expected profits belonging to the manufacturers' in the decentralized and centralized game are analyzed, it is observed that the maximum ratio is 97 percent while the minimum ratio is 91 percent. In the first stage, the most significant factor effecting the ratio is the maximum market capacity. Increase in the maximum market capacity, K yields in closer total expected profit values in the centralized and decentralized games without any further coordination attempts. When the expected and best yield case scenarios are compared to eachother, the best yield case scenario gives closer gains than the expected scenario. This is mainly because the best yield case scenario assumes the yield is going to be high. Hence, it can be stated that producing more with high yield assumption leads to more profit share. Again, the variability of the error term and the α term are likely less significant factors in the determination of the expected profit ratios.

ANOVA results imply in all cases, the level of the maximum capacity of the olive oil market plays the most significant role on the optimality and therefore on the profit. In the centralized case, both parameters are proven to be significant. However, in the decentralized case, the importance of the parameters changes with respect to the scenario and variable investigated.

6. CONCLUSION

Today, supply chain management has become the most effective tool for increasing profits. Most of the companies try to find the optimal production quantity in order to increase their market shares and to compete with other manufacturers without damaging their profit margin. Therefore, there are lots of studies analyzing vertical and horizontal competition in different kinds of supply chain models by using game theory. Horizontal competition is characterized by Nash games whereas in vertical competition cases, Stackelberg analysis is more natural. Price dependent, random demand models are widely employed because it generally reflects the behaviour of the end customers. In most of the studies, there is only one random variable, generally demand. A large proportion of the studies with random demand and random yield deal with assembly systems.

In this study, two manufacturers are competing for the price-dependent and random demand in the presence of random production yield. The model consists of two stages at which the manufacturers try to determine the optimal decision variables in order to maximize their expected profits. Both manufacturers decide at the same time, thus the game they play in this setting is a Nash game. Besides, all the information is common to every competitor. The objective of this study is to analyze the behaviour, as well as to find out if there is a Nash equilibrium in terms of the decision variables.

First of all, the centralized case with only one governing manufacturer is investigated in Chapter 3. The findings show that in both stages, there is an optimum point at which the manufacturer's expected profit is maximized. This result is actually in line with the general newsvendor solution. In addition, sensitivity analysis is performed with respect to the cost and salvage parameters. All the results for the analyzed parameters, except the olive salvage value, are actually expected results as the newsvendor solution suggest, i.e. decreasing cost and salvage parameters yield in increasing and decreasing of the optimal decision variables, respectively. However, as the olive salvage value decreases, the responses of the manufacturer in two stages are different. To be more specific, the fall in the olive salvage value leads to an increase in the leased amount but to a decrease in the olive oil production quantity.

In Chapter 4, the decentralized case is studied. In this case, two manufacturers are assumed to meet the overall demand. The results for the second stage imply that there is a unique optimal point for every manufacturer at any yield which maximizes its individual expected profits. However, the optimality depends on the leased area and the realized yield. Therefore, like in the centralized case, each player's optimal production quantity can be the one optimizing its expected profit or its maximum olive oil amount obtained from the olives grown in the leased area. Moreover, the analysis shows that the game played has only one equilibrium point in every stage. In the second stage, this unique equilibrium point can be of the four optimality pairs depending on the actualized yield and leased areas. Besides, comparative statics is used in order to analyze the behaviour of the manufacturers in terms of changing parameters. The comparative statics results imply that the manufacturer whose parameters change, acts like he is a monopol in the market. However, its competitor makes the opposite moves as first manufacturers parameters change.

Chapter 5 contains the numerical analysis. In this chapter, the demand function is assumed to have an additive type of error term. For the first stage, two different scenarios are studied, one of which takes the expected olive oil production amount in the second stage as input whereas the other one uses the maximum olive oil production quantity. Results of the centralized and decentralized cases are compared with each other where centralized analysis outcomes act as a benchmark. As expected, the optimal decision variables, as well as the expected profits of the centralized case manufacturer in both stages is greater than the sum of the same variables belonging to the two manufacturers in the decentralized case. This is caused mainly by the rate term α_j, fraction of the unsatisfied demand of the manufacturer i which can be satisfied by the manufacturer j. In the decentralized setting, the manufacturer can not meet the whole demand which is lost by its competitor, because the customer may not be willing to buy another product. However, in the centralized setting, it is assumed that there does not exist any other olive oil supplier and the customer is definitely going to buy the on-shelf olive oil.

As the second stage results are analyzed, we can say that areas with high yield expectation are most probably leading to a less profitable business because of the yield-dependent price setting. Therefore, a high yield case may enable the manufacturers to gain more profit without any further coordination attempts. Another significant outcome is that in a market with high maximum capacity, manufacturers will be more willing to enter the game because the optimum production amounts are larger than the case with a smaller maximum market capacity, leading to more profit. To sum up, as the second stage is concerned we can say that small producers(as captured by K) should not engage in a production game with the large manufacturers, especially if they are facing a high demand uncertainty. If the yield is high, probability to gain more profit share increases for small manufacturers with low demand risk.

The first stage results imply that the sum of the expected profits in both scenarios of the decentralized game are definitely closer to the expected profit obtained in the centralized setting. We can state that manufacturers will prefer to begin the game with high yield expectation as the worst-case scenario suggests, since it enables the decentralized game players to get higher expected profits as compared to the ones obtained in the expected scenario. Besides, it can be stated that the expected profits of manufacturers with low maximum market capacity and high demand variance are lower than the ones belonging to the manufacturers with high maximum market capacity and high demand variance. Thus, manufacturers which are more prone to the demand risk should enter the game only if their maximum market capacity is large enough. To wrap up, the first stage results for both scenarios imply that like in the second stage, small manufacturers, especially the ones with high demand variances, should not enter the leasing game either. Moreover, all the results for both stages point that under low yield expectation, it will be better to find a coordination mechanism for all the players in the decentralized setting.

To summarize, in this thesis, the aim is to find out some managerial insights about the behaviour of two players competing in a two-stage setting. Since the olive oil production sector contains most of the model assumptions used, the focus is on this sector. Possible further research topics are the analysis of the vertical competition between the farmer and the manufacturers in a decentralized setting and the horizontal competition analysis of the decentralized setting when there is limited area to lease. In this thesis, the centralized setting constitute from one manufacturer. However, it may also be reasonable to define the centralized setting as there are two manufacturers belonging to the same company. With such a benchmark, both the centralized and decentralized analysis can provide more realistic insights.

APPENDIX A: PROOFS

A.1. Proof of Proposition 3.2.1

In order to define the behavior of the profit function, it is necessary to look at the second derivative with respect to the olive oil production quantity. The first derivative with respect to q is:

$$\begin{aligned} \frac{dE[\Pi_2(q; , Q, u)}{dq}] &= -c_p - h + s + \int_q^{A_2} (p(u) + b - s) f(x, u) dx \\ &= c_p - h + s + (p(u) + b - s)(1 - F(q)) \end{aligned} \tag{A.1}$$

The second derivative of the expected profit function with respect to q is taken as follows:

$$\frac{d^2 E[\Pi_2(q; Q, u)]}{dq^2} = -(p(u) + b - s) f(q) \tag{A.2}$$

The first term is positive due to the last assumption defined in Section 3.1. Since the probability function is also positive, the second order derivative with respect to q is negative because of the negativity sign. Therefore, it can be stated that the expected profit function is concave with respect to the olive oil production amount. Moreover, to determine the optimal amount of the olive oil to be produced, the first order condition is sufficient. However, because of the yield constraint, the optimum olive oil production quantity, i.e. q^* is stated as follows:

$$q^* = \begin{cases} Qu & \text{if } q^{'} > Qu \\ q^{'} & \text{if } q^{'} \leq Qu \end{cases}$$

where the $q^{'}$ is obtained from the first order condition as follows:

$$q^{'} = F^{-1} \left(\frac{p(u) + b - c_p - h}{p(u) + b - s} \right) \tag{A.3}$$

The optimum production quantity is either Qu or $q^{'}$. In the case it is Qu, it will be strictly positive because both the leased area and the yield are positive terms. In the other case, the term $q^{'}$ should be analyzed. First, we need to check the existence of $q^{'}$. In order $q^{'}$ to exist, the term inside the inverse probability function, $\frac{p(u)+b-c_p-h}{p(u)+b-s}$, should be bounded by zero and one. Due to the third model assumption in Section 3.1, the sum of the processing cost and the salvage value of the olives is greater than the salvage value of the olive oil. Therefore, the denominator is greater than the numerator. This ensures the existence of the optimum production quantity. Because the optimum olive oil production quantity is obtained by the inverse probability distribution and the bounds of the probability distribution are positive, the result is strictly positive, too. $\square$

A.2. Proof of Proposition 3.3.1

In order to define the behavior of the profit function, the second derivative with respect to the area to be leased, i.e. Q should be investigated. The first order derivative of $\frac{dE[\Pi_1(Q;q,u)]}{dQ}$ with respect to Q is:

$$\begin{aligned}\frac{dE[\Pi_1(q,Q,u)]}{dQ} =& -c_1 + \int_{u^*}^{M} huy(u)du + \int_0^{u^*}\int_0^{Qu}(-c_p+s)uf(x;u)dxy(u)du \\ &+ \int_0^{u^*}\int_{Qu}^{\infty}(p(u)+b-s)uf(x;u)dxy(u)du = 0\end{aligned} \tag{A.4}$$

The second derivative is taken as follows:

$$\begin{aligned}\frac{d^2E[\Pi_1(Q;q,u)]}{dQ^2} =& -\frac{du^*}{dQ}hu^*y(u^*) - \int_0^{u^*}u^2(p(u)+b-s)f(Qu)y(u)du \\ &+ \frac{du^*}{dQ}(-c_pu^* + su^* + \int_{Qu^*}^{\infty}(p(u)+b-s)u^*f(x;u)dx)y(u^*)\end{aligned} \tag{A.5}$$

The term $\frac{du^*}{dQ}$ is obtained by implicit differentiation of the function $Qu^* = q^{'}$ as follows:

$$Q\frac{du^*}{dQ} + u^* = \frac{dq^{'}}{dQ} = 0 \tag{A.6}$$

Therefore, the term $\frac{du^*}{dQ}$ equals to $-\frac{u^*}{Q}$. By replacing and rearranging the equation A.5, we obtain:

$$\begin{aligned}\frac{d^2E[\Pi_1(Q;q,u)]}{dQ^2} = &- \int_0^{u^*} u^2(p(u)+b-s)f(Qu)y(u)du \\ &+ \frac{u^{*2}}{Q}((p(u^*)+b-h-s)-(p(u^*)+b-s)F(Qu^*))y(u)\end{aligned} \tag{A.7}$$

At u^*, Qu^* is equal to $q^{'}$. It is also known that $q^{'} = F^{-1}(\frac{p(u)+b-c_p-h}{p(u)+b-s})$. By making the replacements according to these equalities, the equation A.7 becomes:

$$\frac{d^2E[\Pi_1(Q;q,u)]}{dQ^2} = - \int_0^{u^*} u^2(p(u)+b-s)f(Qu)y(u)du \tag{A.8}$$

u^2 is always positive. $(p(u)+b-s)$ is positive because the lost sales cost of the end product is always greater than the salvage value by the last assumption stated in Section 3.1. The probability functions are also positive due to their nature. Thus, all the terms inside the integral are positive. Due to the negative sign at the beginning, the second derivative of the expected profit function with respect to the leasing area is negative which proves the concavity of the profit function with respect to Q.

In order to define whether the optimum leasing area exists, it is necessary to look at the first order conditions. After some rearrangements, the first order derivative of the expected profit function with respect to Q becomes:

$$\begin{aligned}\frac{dE[\Pi_1(q,Q,u)]}{dQ} = &- c + h\int_{u^*}^{M} uy(u)du \\ &+ \int_0^{u^*}((p(u)+b-c_p)-(p(u)+b-s)F(Qu))uy(u)du = 0\end{aligned} \tag{A.9}$$

In the equation A.9, the first term is strictly negative whereas the second and third terms are positive. In order to check whether an optimum leasing area exists, the limiting cases, 0 and ∞ are investigated.

Case i) As Q approaches zero, the expected yield, u^* goes to infinity which causes the second term in the equation A.9 to drop. The cumulative probability function within the third term, i.e. $F(Qu)$, also goes to zero such that the equality can be redefined as follows:

$$\lim_{Q \to 0} \frac{dE[\Pi_1(q, Q, u)]}{dQ} = -c + \int_0^M (p(u) + b - c_p) u y(u) du \tag{A.10}$$

Case ii) As Q approaches to infinity, the expected yield, u^* goes to zero which causes the third term in the equation A.9 to drop. Thus, the equality can be redefined as follows:

$$\lim_{Q \to \infty} \frac{dE[\Pi_1(q, Q, u)]}{dQ} = -c + \int_0^M h u y(u) du \tag{A.11}$$

As it can be observed, the integral boundaries in both cases are $[0, M]$ which enables the comparison between the results of these two cases. Because of the first and fourth model assumptions, the salvage value of the olive oil is less than the sum of the processing cost of the olive oil and the salvage value of the olives. Thus, the following is true:

$$\lim_{Q \to \infty} \frac{dE[\Pi_1(q, Q, u)]}{dQ} < \lim_{Q \to 0} \frac{dE[\Pi_1(q, Q, u)]}{dQ} \tag{A.12}$$

The equation A.12 implies a decrease in the slope of the profit function. Since the expected profit function is concave in Q, we need to show that the expected profit value obtained at the first extreme case, i.e. $Q \to 0$, is larger than the one obtained at the second extreme case, i.e. $Q \to \infty$ in order to prove the existence of the Q value which satisfies the FOC. As Q gets closer to zero, the leasing cost drops, u^* gets closer to ∞ and the expected profit becomes:

$$E[\Pi_1(q,Q,u)] = \int_0^M Z(Q,u)y(u)du \tag{A.13}$$

where

$$\begin{aligned} Z(Q,u) = &- c_pQu - \int_{Qu^*}^{\infty} bxf(x;u)dx \\ &+ \int_0^{Qu^*} (p(u)-s)xf(x;u)dx \end{aligned}$$

In this case, the expected profit goes to an infinite value as Q gets closer to zero. In the opposite case, i.e. when Q gets closer to infinity, u^* gets closer to zero and the expected profit function becomes:

$$E[\Pi_1(q,Q,u)] = -cQ + \int_0^M W(Q,q^*,u)y(u)du \tag{A.14}$$

where

$$\begin{aligned} W(Q,q^*,u) = &- c_pq^* + h(Qu-q^*) \\ &+ \int_0^{q^*} (p(u)x) + s(q^*-x)f(x;u)dx \\ &+ \int_{q^*}^{\infty} (p(u)q^* - b(x-q^*))f(x;u)dx \end{aligned}$$

In this case, terms with q^* contribute very little as compared to the terms Q. Because the first assumption suggests that the olive salvage value is less than the unit leasing cost, we can say that the term $-cQ$ definitely is larger than the rest of the expected profit equation, i.e. $\int_0^M W(Q,q^*,u)y(u)du$ in absolute value. Thus, the expected profit of the manufacturer goes to $-\infty$ as Q goes infinity. When both cases are compared, the statement

$$\lim_{Q\to\infty} E[\Pi_1(q,Q,u)] < \lim_{Q\to 0} dE[\Pi_1(q,Q,u)] \tag{A.15}$$

is true. Since the expected profit function is proven to be concave, the equation A.15

ensures that the there exists a Q within the valid boundaries, i.e. 0 and $-\infty$, which maximizes the expected profit. □

A.3. Proof of Proposition 3.4.1

The reaction of the olive oil production amount to the changing parameters can be obtained by checking its first derivatives with respect to those parameters. The optimum production quantity of the olive oil, denoted by q^*, should satisfy the following equation.

$$F(q^*) = \frac{(p(u) + b - c_p - h)}{(p(u) + b - s)} \tag{A.16}$$

The influence of the parameters are obtained by implicit differentiation as follows:

*i)*Effect of changing backorder cost on the optimum olive oil production amount:

$$\frac{dq^*}{db} = \frac{-c_p - h - s}{(p(u) + b - s)^2 f(q^*)} \tag{A.17}$$

The numerator is strictly negative whereas the denominator is definitely positive. Thus, $\frac{dq^*}{db}$ is negative, leading to an inverse relationship between q^* and b.

*ii)*Effect of changing processing cost on the optimum olive oil production amount:

$$\frac{dq^*}{dc_p} = \frac{-1}{(p(u) + b - s) f(q^*)} \tag{A.18}$$

In this case, again, the numerator is strictly negative. The term $p(u) + b - s$ is positive because of the last model assumption present in Section 3.1. Moreover, the probability density function in the denominator is strictly positive due to its nature. Therefore, $\frac{dq^*}{dc_p}$ is negative, too, leading to an inverse relationship between q^* and c_p.

*iii)*Effect of changing salvage value of olive on the optimum olive oil production amount:

$$\frac{dq^*}{dh} = \frac{-1}{(p(u) + b - h)} f(q^*) \tag{A.19}$$

This case very much resembles the case of the processing cost change, where the numerator is negative and the denominator is strictly positive due to the second and fourth model assumptions. This implies a negative relationship between q^* and h.

*iv)*Effect of changing salvage value of olive oil on the optimum olive oil production amount:

$$\frac{dq^*}{dh} = \frac{p(u) + b - c_p - h}{(p(u) + b - s)^2} f(q^*) \tag{A.20}$$

The term in the numerator, $p(u) + b - c_p - h$, is negative due to the first and fourth model assumptions made in Section 3.1. The terms in the denominator are both positive. Therefore, the relationship between q^* and s can be defined as positive. □

A.4. Proof of Proposition 3.4.2

To analyze the effects of the cost parameters on the decision variable, $\frac{d^2E[\Pi_1(q,Q,u)]}{dQda}$ is analyzed where a represent the investigated cost parameter.

*i)*Effect of changing the leasing cost on the size of the leasing area:

$$\begin{aligned} \frac{d^2E[\Pi_1(q,Q,u)]}{dQdc} =& -1 - \frac{du^*}{dc}(h + c_p - p(u^*) - b)u^*y(u^*) \\ & - \frac{du^*}{dc}(p(u^*) + b - s)F(Qu^*)u^*y(u^*) \end{aligned} \tag{A.21}$$

It is stated that Qu^* is equal to $q^{'}$ by definition. Therefore, the term $F(Qu^*)$ can be replaced by $F(q^{'})$, which is equal to $\frac{p(u)+b-c_p-h}{p(u)+b-s}$. By rearranging, the second and third terms in the equation A.21 drop and it becomes:

$$\frac{d^2E[\Pi_1(q,Q,u)]}{dQdc} = -1 \tag{A.22}$$

which is strictly negative implying an inverse relationship between Q^* and c. In the rest of the proofs in this section, the same methodology is used.

*ii)*Effect of changing backorder cost on the size of the leasing area:

$$\begin{aligned}\frac{d^2E[\Pi_1(q,Q,u)]}{dQdb} &= \int_0^{u^*}(1-F(Qu))uy(u)du - \frac{du^*}{db}(h+c_p-p(u^*)-b)u^*y(u^*)\\ &- \frac{du^*}{db}(p(u^*)+b-s)F(Qu^*)u^*y(u^*)\end{aligned} \tag{A.23}$$

Due to the FOC, the second and third terms are equal to zero and the equality becomes:

$$\frac{d^2E[\Pi_1(q,Q,u)]}{dQdb} = \int_0^{u^*}(1-F(Qu))uy(u)du \tag{A.24}$$

Because the cumulative probability function can not be greater than 1, all the terms within the integral are all positive. Thus,$\frac{d^2E[\Pi_1(q,Q,u)]}{dQdb}$ is positive and the change in the backorder cost leads to a positive change in the optimum size of the leasing area.

*iii)*Effect of changing processing cost on the size of the leasing area:

$$\begin{aligned}\frac{d^2E[\Pi_1(q,Q,u)]}{dQdc_p} &= -\int_0^{u^*}uy(u)du - \frac{du^*}{dc_p}(h+c_p-p(u^*)-b)u^*y(u^*)\\ &- \frac{du^*}{dc_p}(p(u^*)+b-s)F(Qu^*)u^*y(u^*)\end{aligned} \tag{A.25}$$

Again, because of the FOC of the second stage, $\frac{d^2E[\Pi_1(q,Q,u)]}{dQdc_p}$ is equal to $-\int_0^{u^*}uy(u)du$ which is strictly negative. Therefore, the influence of the processing cost changes on the optimum size of the leasing area is negative.

*iv)*Effect of changing salvage value of olive on the size of the leasing area:

$$\begin{aligned}\frac{d^2E[\Pi_1(q,Q,u)]}{dQdh} &= \int_0^{u^*} uy(u)du - \frac{du^*}{dh}(h+c_p-p(u^*)-b)u^*y(u^*)\\ &- \frac{du^*}{dh}(p(u^*)+b-s)F(Qu^*)u^*y(u^*)\\ &= \int_0^{u^*} uy(u)du\end{aligned} \tag{A.26}$$

As seen, $\frac{d^2E[\Pi_1(q,Q,u)]}{dQdh}$ is positive. This results in a positive relationship.

*v)*Effect of changing salvage value of olive oil on the size of the leasing area:

$$\begin{aligned}\frac{d^2E[\Pi_1(q,Q,u)]}{dQds} &= \int_0^{u^*} F(Qu)uy(u)du - \frac{du^*}{ds}(h+c_p-p(u^*)-b)u^*y(u^*)\\ &- \frac{du^*}{ds}(p(u^*)+b-s)F(Qu^*)u^*y(u^*)\\ &= \int_0^{u^*} F(Qu)uy(u)du\end{aligned} \tag{A.27}$$

Again, the equation A.27 is positive implying a positive influence of the olive oil salvage value on the size of the leasing area. □

A.5. Proof of Proposition 4.1.1

The characterization of the expected profit function belonging to the manufacturer 1 in the second stage with respect to the decision variable q_1 can be done by looking at its second derivative as follows:

$$\begin{aligned}\frac{d^2E_1[\Pi_2(q_1;q_2,Q_1,u)]}{dq_1^2} =& -b\phi(q_1)+(s-p(u))\phi(q_1)\Psi(q_2)\\ &+\int_{q_2}^{A_2}(s-p(u))\phi(q_1-\alpha_1(x_2-q_2))\psi(x_2,u)dx_2\end{aligned} \tag{A.28}$$

The first term in the equation A.28 is negative due to the sign. The second and the third terms are also negative because the salvage value is always less than the price of the end product, i.e. $s-p(u)$ is negative. So, $\frac{d^2E_1[\Pi_2(q_1;q_2,Q_1,u)]}{dq_1^2}$ is negative implying the concavity of the expected profit function with respect to the decision variable q_1. The same is also valid for the second manufacturer. □

A.6. Proof of Theorem 4.1.1

*i)*To state whether an equilibrium exists, the response functions should be characterized. Cachon and Netessine [1] states that for a two-player non-zero sum game, there exists at least one Nash equilibrium if the game is supermodular, i.e. $\frac{d^2E_j}{dq_j dq_i} \geq 0$.

$$\frac{d^2E_1[\Pi_2(q_1;q_2,Q_1,u)]}{dq_1dq_2} = \int_{q_2}^{\infty}(s_1 - p(u))\alpha_1\Phi(q_1 - \alpha_1(x_2 - q_2))\psi(x_2,u)dx_2 \qquad (A.29)$$

$$\frac{d^2E_2[\Pi_2(q_2;q_1,Q_2,u)]}{dq_2dq_1} = \int_{q_1}^{\infty}(s_2 - p(u))\alpha_2\Psi(q_2 - \alpha_2(x_1 - q_1))\phi(x_1,u)dx_1 \qquad (A.30)$$

The terms $(s_j - p(u))$ are negative due to the fourth model assumption. Thus, the equations A.29 and A.30 are negative and the game is submodular. Hence, by defining a new variable x_2 which is equal to $(-q_2)$ and taking the first and second derivatives with respect to this variable, the game becomes supermodular implying that there exists an equilibrium.

*ii)*In order to find out if there is one equilibrium point or many, the Hessian matrix is checked. The Implicit Function theorem and contraction mapping argument imply that a game has only one equilibrium point if the expression:

$$\left|\frac{d^2E_i[\Pi_2]}{dq_idq_j}\right| < \left|\frac{d^2E_i[\Pi_2]}{dq_i^2}\right| \qquad (A.31)$$

is true for all players of the game[1].

$$\begin{aligned}\frac{d^2E_1[\Pi_2(q_1;q_2,Q_1,u)]}{dq_1^2} =& - b_1\phi(q_1) + (s_1 - p(u))\phi(q_1)\Psi(q_2) \\ &+ \int_{q_2}^{\infty}(s_1 - p(u))\phi(q_1 - \alpha_1(x_2 - q_2))\psi(x_2,u)dx_2\end{aligned} \qquad (A.32)$$

$$\frac{d^2E_1[\Pi_2(q_1;q_2,Q_1,u)]}{dq_1dq_2} = \alpha_1 \int_{q_2}^{\infty} (s_1 - p(u))\phi(q_1 - \alpha_1(x_2 - q_2))\psi(x_2,u)dx_2 \quad \text{(A.33)}$$

As explained before, both equations, A.32 and A.33, are negative. The term multiplied by α_1, which is strictly between 0 and 1, in the equation A.29 and the third term in the equation A.28 are the same terms. Hence, the equation A.29 is definitely smaller than the third term of the equation A.28. Moreover, the first and second terms of the equation A.28 are also negative. Therefore, when the two equations are compared to each other, it can be easily seen that the equation A.28 is always less than A.29, ensuring that the inequality A.31 holds for all q_1, q_2 pairs. Because the second manufacturer's profit function displays the same characteristics, it can be stated that this inequality is true also for the second manufacturer. Since this condition holds for the expected profit functions of both manufacturers, there exists a unique equilibrium point. □

A.7. Proof of Proposition 4.2.1

The characterization of the expected profit function belonging to the manufacturer j in the first stage with respect to the decision variable Q_j can be done by looking at its second derivative. First, the case when $u_1^* < u_2^*$ is analyzed:

$$\begin{aligned}\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1^2} = &\frac{du_1^*}{dQ_1}(-c_{p1} - h_1 + p(u_1^*) + b_1 - b_1\Phi(Q_1u_1^*)\\ &+(s_1 - p(u_1^*))\Phi(Q_1u_1^*)\Psi(Q_2u_1^*)\\ &+\int_{Q_2u}^{\infty}(s_1 - p(u_1^*))\Phi(Q_1u - \alpha_1(x_2 - Q_2u))\psi(x_2,u)u_1^*y(u_1^*)dx_2)\\ &+\int_0^{u_1^*}(-b_1\phi(Q_1u) + (s_1 - p(u))\phi(Q_1u)\Psi(Q_2u))u^2y(u)du\\ &+\int_0^{u_1^*}\int_{Q_2u}^{\infty}(s_1 - p(u))\phi(Q_1u - \alpha_1(x_2 - Q_2u))\psi(x_2,u)u^2y(u)dx_2du\end{aligned}$$

(A.34)

It is known that $Q_1u_1^* = q_1^{'}$ and $q_1^{'}$ is the olive oil production quantity that satisfies the first order conditions of the second stage. When the equation A.34 is investigated, it can be easily noticed that the multiplier of the term $\frac{du_1^*}{dQ_1}$ equals to the first order condition of the second stage, i.e. equation 4.3, because of the equality $Q_1u_1^* = q_1^*$. Because at q_1^*, FOC is equal to zero by definition, the equationA.34 becomes:

$$\begin{aligned}\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1^2} &= \int_0^{u_1^*}(-b_1\phi(Q_1u)+(s_1-p(u))\phi(Q_1u)\Psi(Q_2u))u^2y(u)du \\ &+\int_0^{u_1^*}\int_{Q_2u}^{\infty}(s_1-p(u))\phi(Q_1u-\alpha_1(x_2-Q_2u))\psi(x_2,u)u^2y(u)dx_2du\end{aligned} \tag{A.35}$$

The first term within the integral is negative due to the sign. The second and the third terms are also negative because the salvage value is always less than the price of the end product, i.e. $(s_1 - p(u))$ is negative due to the fourth model assumption. Thus, $\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1^2}$ is negative in this case.

When $u_1^* \geq u_2^*$, the $\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1^2}$ becomes:

$$\begin{aligned}\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1^2} &= \frac{du_1^*}{dQ_1}(-c_{p1}-h_1+p(u_1^*)+b_1-b_1\Phi(Q_1u_1^*) \\ &+(s_1-p(u_1^*))\Phi(Q_1u_1^*)\Psi(q_2^*) \\ &+\int_{q_2^*}^{\infty}(s_1-p(u_1^*))\Phi(Q_1u-\alpha_1(x_2-q_2^*))\psi(x_2,u))u_1^*y(u_1^*)dx_2 \\ &+\int_0^{u_2^*}(-b_1\phi(Q_1u)+(s_1-p(u))\phi(Q_1u)\Psi(Q_2u))u^2y(u)du \\ &+\int_0^{u_2^*}\int_{Q_2u}^{\infty}(s_1-p(u))\phi(Q_1u-\alpha_1(x_2-Q_2u))\psi(x_2,u)u^2y(u)dx_2du \\ &+\int_{u_2^*}^{u_1^*}(-b_1\phi(Q_1u)+(s_1-p(u))\phi(Q_1u)\Psi(q_2^*))u^2y(u)du \\ &+\int_{u_2^*}^{u_1^*}\int_{q_2^*}^{\infty}(s_1-p(u))\phi(Q_1u-\alpha_1(x_2-q_2^*))\psi(x_2,u)u^2y(u)dx_2du\end{aligned} \tag{A.36}$$

Because of the same reasoning, explained in the previous case, $\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1^2}$ becomes:

$$\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1^2} = +\int_0^{u_2^*}(-b_1\phi(Q_1u)+(s_1-p(u))\phi(Q_1u)\Psi(Q_2u))u^2y(u)du$$
$$+\int_0^{u_2^*}\int_{Q_2u}^{\infty}(s_1-p(u))\phi(Q_1u-\alpha_1(x_2-Q_2u))\psi(x_2,u)u^2y(u)dx_2du$$
$$+\int_{u_2^*}^{u_1^*}(-b_1\phi(Q_1u)+(s_1-p(u))\phi(Q_1u)\Psi(q_2^*))u^2y(u)du$$
$$+\int_{u_2^*}^{u_1^*}\int_{q_2^*}^{\infty}(s_1-p(u))\phi(Q_1u-\alpha_1(x_2-q_2^*))\psi(x_2,u)u^2y(u)dx_2du \tag{A.37}$$

Because all the terms within the integrals are negative, $\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1^2}$ is again negative, which ensures the concavity of the expected profit function with respect to the decision variable Q_1. The same is also valid for the second manufacturer. □

A.8. Proof of Theorem 4.2.1

The methodology used in the proof of 4.1.1 is employed in this proof, too.

i) The supermodularity of the game should be checked in order to prove the existence of the Nash equilibrium point. Like the expected profit function, this analysis also consists of two cases with respect to the relationship of u_1^* and u_2^*. When u_1^* is less than u_2^*, the $\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1dQ_2}$ equals to zero. In the other case, $\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1dQ_2}$ is:

$$\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1dQ_2} = \int_0^{u_2^*}\int_{Q_2u}^{\infty}(s_1-p(u))u^2\alpha_1\phi(Q_1u-\alpha_1(x_2-Q_2u))\psi(x_2,u)y(u)dx_2du \tag{A.38}$$

The equation A.38 is also negative due to the $s_1-p(u)$ term which is strictly less than zero because of the model assumptions. Therefore, in general:

$$\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1dQ_2} \le 0 \tag{A.39}$$

which implies that the game is submodular with respect to the optimal leasing areas. Again, a new variable X_2 is defined and the derivatives are taken with respect to it such that the game becomes supermodular meaning that equilibrium exists.

ii) For the determination of the equilibrium, the Hessian matrix is analyzed. As explained in Theorem 4.1.1, the Implicit Function theorem and contraction mapping argument state that a game has only one equilibrium point if the expression:

$$\left|\frac{d^2E_1[\Pi_1]}{dQ_1dQ_2}\right| < \left|\frac{d^2E_1[\Pi_1]}{dQ_1^2}\right| \tag{A.40}$$

is true for all players of the game[1].

When u_1^* is less than u_2^*:

$$\begin{aligned}\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1^2} &= \int_0^{u_1^*}(-b_1\phi(Q_1u)+(s_1-p(u))\phi(Q_1u)\Psi(Q_2u))u^2y(u)du \\ &+\int_0^{u_1^*}\int_{Q_2u}^{\infty}(s_1-p(u))\phi(Q_1u-\alpha_1(x_2-Q_2u))\psi(x_2,u)u^2y(u)dx_2du\end{aligned} \tag{A.41}$$

$$\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1dQ_2} = 0 \tag{A.42}$$

In this case, the inequality A.40 strictly holds.

When u_1^* is greater than u_2^*:

$$\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1^2} = +\int_0^{u_2^*}(-b_1\phi(Q_1u)+(s_1-p(u))\phi(Q_1u)\Psi(Q_2u))u^2y(u)du$$
$$+\int_0^{u_2^*}\int_{Q_2u}^{\infty}(s_1-p(u))\phi(Q_1u-\alpha_1(x_2-Q_2u))\psi(x_2,u)u^2y(u)dx_2du$$
$$+\int_{u_2^*}^{u_1^*}(-b_1\phi(Q_1u)+(s_1-p(u))\phi(Q_1u)\Psi(q_2^*))u^2y(u)du$$
$$+\int_{u_2^*}^{u_1^*}\int_{q_2^*}^{\infty}(s_1-p(u))\phi(Q_1u-\alpha_1(x_2-q_2^*))\psi(x_2,u)u^2y(u)dx_2du \tag{A.43}$$

$$\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1dQ_2} = \int_0^{u_2^*}\int_{Q_2u}^{\infty}(s_1-p(u))u^2\alpha_1\phi(Q_1u-\alpha_1(x_2-Q_2u))\psi(x_2,u)y(u)dx_2du \tag{A.44}$$

As seen, equation $\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1dQ_2}$ is the last term of the equation $\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1^2}$ multiplied with α_1, which is positive and is between 0 and 1. Because both terms are negative, it can be stated that the last term of $\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1^2}$ is smaller than $\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1dQ_2}$. Because the other terms in the equation $\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1^2}$ are also negative, in absolute values, $\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1^2}$ is strictly greater than $\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1dQ_2}$ which proves the uniqueness of the Nash equilibrium. □

A.9. Proof of Proposition 4.3.1

To analyze the effects of the parameters on the optimality, the argumentation stated by Cachon and Netessine [1] is used. This study suggests that the changes in the optimum, i.e. equilibrium, values with respect to any parameter can be defined as follows:

$$\frac{dq_1^*}{da} = -\frac{\frac{d^2E_1[\Pi_2(q_1;q_2,Q_1,u)]}{dq_1da}\frac{d^2E_2[\Pi_2(q_2;q_1,Q_2,u)]}{dq_2^2} - \frac{d^2E_1[\Pi_2(q_1;q_2,Q_1,u)]}{dq_1dq_2}\frac{d^2E_2[\Pi_2(q_2;q_1,Q_2,u)]}{dq_2da}}{\mid H \mid} \tag{A.45}$$

$$\frac{dq_2^*}{da} = -\frac{\frac{d^2E_1[\Pi_2(q_1;q_2,Q_1,u)]}{dq_1^2}\frac{d^2E_2[\Pi_2(q_2;q_1,Q_2,u)]}{dq_2da} - \frac{d^2E_1[\Pi_2(q_1;q_2,Q_1,u)]}{dq_1da}\frac{d^2E_2[\Pi_2(q_2;q_1,Q_2,u)]}{dq_2dq_1}}{\mid H \mid} \quad (A.46)$$

In these equations, the term in the denominator, $\mid H \mid$, is the determinant of the Hessian matrix. The Hessian matrix of the profit functions in this stage is :

$$H = \begin{vmatrix} \frac{\partial^2 E_1[\Pi_2]}{\partial q_1^2} & \frac{\partial^2 E_1[\Pi_2]}{\partial q_1 \partial q_2} \\ \frac{\partial^2 E_2[\Pi_2]}{\partial q_2 \partial q_1} & \frac{\partial^2 E_2[\Pi_2]}{\partial q_2^2} \end{vmatrix}$$

where

$$\begin{aligned} \frac{d^2E_1[\Pi_2(q_1;q_2,Q_1,u)]}{dq_1^2} =& - b_1\phi(q_1) + (s_1 - p(u))\phi(q_1)\Psi(q_2) \\ &+ \int_{q_2}^{\infty}(s_1 - p(u))\phi(q_1 - \alpha_1(x_2 - q_2))\psi(x_2,u)dx_2 \end{aligned} \quad (A.47)$$

$$\frac{d^2E_1[\Pi_2(q_1;q_2,Q_1,u)]}{dq_1dq_2} = \int_{q_2}^{\infty}(s_1 - p(u))\alpha_1\phi(q_1 - \alpha_1(x_2 - q_2))\psi(x_2,u)dx_2 \quad (A.48)$$

$$\begin{aligned} \frac{d^2E_2[\Pi_2(q_2;q_1,Q_2,u)]}{dq_2^2} =& - b_2\psi(q_2) + (s_2 - p(u))\psi(q_2)\Phi(q_1) \\ &+ \int_{q_1}^{\infty}(s_2 - p(u))\psi(q_2 - \alpha_2(x_1 - q_1))\phi(x_1,u)dx_1 \end{aligned} \quad (A.49)$$

$$\frac{d^2E_2[\Pi_2(q_2;q_1,Q_2,u)]}{dq_2dq_1} = \int_{q_1}^{\infty}(s_2 - p(u))\alpha_2\psi(q_2 - \alpha_2(x_1 - q_1))\phi(x_1,u)dx_1 \quad (A.50)$$

Because both expected functions are submodular, the term $\frac{d^2E_1[\Pi_2(q_1;q_2,Q_1,u)]}{dq_1^2}\frac{d^2E_2[\Pi_2(q_2;q_1,Q_2,u)]}{dq_2^2}$ will be always greater than the term $\frac{d^2E_1[\Pi_2(q_1;q_2,Q_1,u)]}{dq_1dq_2}\frac{d^2E_2[\Pi_2(q_2;q_1,Q_2,u)]}{dq_2dq_1}$. Thus, the determinant of the Hessian matrix is definitely positive.

The cost parameters of the first manufacturer only effects the first manufacturer's profit function. Therefore, $\frac{d^2E_2[\Pi_2(q_2;q_1,Q_2,u)]}{dq_2da}$ equals to zero if a is one of the first manufacturers' cost parameters ,leading the second term in the numerator of the function A.45 to drop. The same is also true for the optimality of the second manufacturer. So, the equations A.45 and A.46 become:

$$\frac{dq_1^*}{da} = -\frac{\frac{d^2E_1[\Pi_2(q_1;q_2,Q_1,u)]}{dq_1 da}\frac{d^2E_2[\Pi_2(q_2;q_1,Q_2,u)]}{dq_2^2}}{\mid H \mid} \tag{A.51}$$

$$\frac{dq_2^*}{da} = -\frac{\frac{d^2E_1[\Pi_2(q_1;q_2,Q_1,u)]}{dq_1 da}\frac{d^2E_2[\Pi_2(q_2;q_1,Q_2,u)]}{dq_2 dq_1}}{\mid H \mid} \tag{A.52}$$

The denominator is proven to be positive, while $\frac{d^2E_2[\Pi_2(q_2;q_1,Q_2,u)]}{dq_2^2}$ is negative. In this equality, the only term that changes sign, thus the relationship, with respect to the cost parameter investigated is $\frac{d^2E_1[\Pi_2(q_1;q_2,Q_1,u)]}{dq_1 da}$. Its analysis is done as follows:

*i)*The effect of the lost sales cost:

To determine the responses of the first and second manufacturer to a change in the lost sales cost of the first manufacturer, the sign of $\frac{d^2E_1[\Pi_2(q_1;q_2,Q_1,u)]}{dq_1 db_1}$ should be investigated as follows:

$$\frac{d^2E_1[\Pi_2(q_1;q_2,Q_1,u)]}{dq_1 db_1} = 1 - \Phi(q_1) \geq 0 \tag{A.53}$$

This term being positive makes the equation A.45 also positive, pointing to a direct effect on the optimality. Then, its influence on the second manufacturer is investigated. The terms in the equation A.46 are all negative except $\frac{d^2E_1[\Pi_2(q_1;q_2,Q_1,u)]}{dq_1 db_1}$. With this result, the second manufacturers reaction to lost sales cost changes of the first manufacturer is proven to be negative. Moreover, these findings state a change in the equilibrium, i.e. q_1^* of the Nash equilibrium pair increases and q_2^* decreases as the lost sales cost of the first manufacturer rises.

*ii)*The effect of the processing cost:

The same analysis as done in *i)* is applied in this case , too. It begins with the first manufacturer:

$$\frac{d^2E_1[\Pi_2(q_1; q_2, Q_1, u)]}{dq_1 dc_{p1}} = -1 \leq 0 \tag{A.54}$$

Because of this negativity, the equation A.45 is going to be also negative whereas the equation A.46 is positive. Thus, the first manufacturer will respond to the fall in its processing cost by rising its production while the second mmanufacturer will produce less olive oil. All these result in a change of equilibrium in favour of the second manufacturer as the processing cost of the first manufacturer rises.

*iii)*The effect of the olive salvage value:

In order to define the influence of the olive salvage value on the optimal results, the following analysis is done:

$$\frac{d^2E_1[\Pi_2(q_1; q_2, Q_1, u)]}{dq_1 dh_1} = -1 \leq 0 \tag{A.55}$$

This outcome ensures the equation A.45 to be negative and A.46 to be positive, resulting in a rise of the q_1^* and in a fall of the q_2^* when the olive salvage value of the first manufacturer increases.

*iv)*The effect of the olive oil salvage value:

As the salvage value for the first manufacturer is concerned:

$$\frac{d^2E_1[\Pi_2(q_1; q_2, Q_1, u)]}{dq_1 ds_1} = \Phi(q_1)\Psi(q_2) + \int_{q_2}^{\infty} \Phi(q_1 - \alpha_1(x_2 - q_2))\psi(x_2, u)dx_2 \geq 0 \tag{A.56}$$

Because this equality is positive, the equation A.45 is strictly positive, implying a positive relationship between the first manufacturers lost sales cost and olive oil production size. The second manufacturer is also influenced by this change. Because $\frac{d^2E_1[\Pi_2(q_1;q_2,Q_1,u)]}{dq_1 ds_1}$, the equation A.46 is strictly less than zero, leading to a negative effect on the optimality.

With these information on hand, it can be stated that an increase in the olive oil salvage value of the first manufacturer results in an equilibrium higher in q_1 and lower in q_2. □

A.10. Proof of Proposition 4.3.2

The methodology used in A.9 is applied in order to prove the effects of the cost parameters on the optimal results and equilibrium points.

$$\frac{dQ_1^*}{da} = -\frac{\frac{d^2E_1[\Pi_1(Q_1;Q_2,q_1^*,q_2^*,u)]}{dQ_1da}\frac{d^2E_2[\Pi_1(Q_2;Q_1,q_2^*,q_1^*,u)]}{dQ_2^2} - \frac{d^2E_1[\Pi_1(Q_1;Q_2,q_1^*,q_2^*,u)]}{dQ_1dQ_2}\frac{d^2E_2[\Pi_1(Q_2;Q_1,q_1^*,q_2^*,u)]}{dQ_2da}}{\mid H \mid} \tag{A.57}$$

$$\frac{dQ_2^*}{da} = -\frac{\frac{d^2E_1[\Pi_1(Q_1;Q_2,q_1^*,q_2^*,u)]}{dQ_1^2}\frac{d^2E_2[\Pi_1(Q_2;Q_1,q_2^*,q_1^*,u)]}{dQ_2da} - \frac{d^2E_1[\Pi_1(Q_1;Q_2,q_1^*,q_2^*,u)]}{dQ_1da}\frac{d^2E_2[\Pi_1(Q_2;Q_1,q_2^*,q_1^*,u)]}{dQ_2dQ_1}}{\mid H \mid} \tag{A.58}$$

The Hessian matrix of the profit functions in this stage is:

$$H = \begin{vmatrix} \frac{\partial^2 E_1[\Pi_1]}{\partial Q_1^2} & \frac{\partial^2 E_1[\Pi_1]}{\partial Q_1 \partial Q_2} \\ \frac{\partial^2 E_2[\Pi_1]}{\partial Q_2 \partial Q_1} & \frac{\partial^2 E_2[\Pi_1]}{\partial Q_2^2} \end{vmatrix}$$

All the terms constituting the Hessian matrix are different for the two cases mentioned in the section 4.2. For the case where u_1^* is less than u_2^*, the terms are written as follows:

$$\begin{aligned} \frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1^2} &= \int_0^{u_1^*}(-b_1\phi(Q_1u) + (s_1 - p(u))\phi(Q_1u)\Psi(Q_2u))u^2y(u)du \\ &+ \int_0^{u_1^*}\int_{Q_2u}^{\infty}(s_1 - p(u))\phi(Q_1u - \alpha_1(x_2 - Q_2u))\psi(x_2,u)u^2y(u)dx_2du \end{aligned} \tag{A.59}$$

$$\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1dQ_2} = 0 \tag{A.60}$$

$$\begin{aligned}\frac{d^2E_2[\Pi_1(Q_2;q_2^*,q_1^*,Q_1,u)]}{dQ_2^2} &= + \int_0^{u_1^*} (-b_2\phi(Q_2u) + (s_2 - p(u))\psi(Q_2u)\Phi(Q_1u))u^2y(u)du \\ &+ \int_0^{u_1^*}\int_{Q_1u}^{\infty} (s_2 - p(u))\psi(Q_2u - \alpha_2(_1 - Q_1u))\phi(x_1,u)u^2y(u)dx_1du \\ &+ \int_{u_1^*}^{u_2^*} (-b_2\psi(Q_2u) + (s_2 - p(u))\psi(Q_2u)\Phi(q_1^*))u^2y(u)du \\ &+ \int_{u_1^*}^{u_2^*}\int_{q_1^*}^{\infty} (s_2 - p(u))\psi(Q_2u - \alpha_2(x_1 - q_1^*))\phi(x_1,u)u^2y(u)dx_1du\end{aligned} \tag{A.61}$$

$$\frac{d^2E_2[\Pi_1(Q_2;q_2^*,q_1^*,Q_1,u)]}{dQ_2dQ_1} = \int_0^{u_1^*}\int_{Q_1u}^{\infty} (s_2 - p(u))u^2\alpha_2\psi(Q_2u - \alpha_2(x_1 - Q_1u))\phi(x_1,u)y(u)dx_1du \tag{A.62}$$

In this case, the determinant of the Hessian is greater than zero because all the terms are negative and the game is submodular implying that the first term in the determinant calculation is strictly greater than the second term.

When u_2^* is less than u_1^*, the terms become:

$$\begin{aligned}\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1^2} &= + \int_0^{u_2^*} (-b_1\phi(Q_1u) + (s_1 - p(u))\phi(Q_1u)\Psi(Q_2u))u^2y(u)du \\ &+ \int_0^{u_2^*}\int_{Q_2u}^{\infty} (s_1 - p(u))\phi(Q_1u - \alpha_1(x_2 - Q_2u))\psi(x_2,u)u^2y(u)dx_2du \\ &+ \int_{u_2^*}^{u_1^*} (-b_1\phi(Q_1u) + (s_1 - p(u))\phi(Q_1u)\Psi(q_2^*))u^2y(u)du \\ &+ \int_{u_2^*}^{u_1^*}\int_{q_2^*}^{\infty} (s_1 - p(u))\phi(Q_1u - \alpha_1(x_2 - q_2^*))\psi(x_2,u)u^2y(u)dx_2du\end{aligned} \tag{A.63}$$

$$\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1dQ_2} = \int_0^{u_2^*}\int_{Q_2u}^{\infty}(s_1-p(u))u^2\alpha_1\phi(Q_1u-\alpha_1(x_2-Q_2u))\psi(x_2,u)y(u)dx_2du \tag{A.64}$$

$$\frac{d^2E_2[\Pi_1(Q_2;q_2^*,q_1^*,Q_1,u)]}{dQ_2^2} = \int_0^{u_2^*}(-b_2\psi(Q_2u)+(s_2-p(u))\psi(Q_2u)\Phi(Q_1u))u^2y(u)du$$
$$+\int_0^{u_2^*}\int_{Q_1u}^{\infty}(s_2-p(u))\psi(Q_2u-\alpha_2(x_1-Q_1u))\phi(x_1,u)u^2y(u)dx_1du \tag{A.65}$$

$$\frac{d^2E_2[\Pi_1(Q_2;q_2^*,q_1^*,Q_1,u)]}{dQ_2dQ_1} = 0 \tag{A.66}$$

Again, in this case the determinant of the Hessian matrix is proven to be positive due to the submodularity. Therefore, we can conclude that $\mid H \mid$ is always greater than zero. So, the only term needed to define the relationship between the parameter and optimality is $\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1da}$ where a is the cost parameter investigated.

*i)*The effect of the unit leasing cost:

Again, the two cases should be analyzed separately. In the first case, i.e. $u_1^* \leq u_2^*$, the term $\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1dc_1}$ is:

$$\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1dc_1} = -1+\frac{du_2^*}{dc_1}(-h_1-c_{p1}+p(u_2^*)+b_1-b_1\Phi(Q_1u_2^*))u_2^*y(u_2^*)$$
$$+\frac{du_2^*}{dc_1}(s_1-p(u_2^*))\Phi(Q_1u_2^*)\Psi(Q_2u_2^*)u_2^*y(u_2^*)$$
$$+\frac{du_2^*}{dc_1}(\int_{Q_2u}^{\infty}(s_1-p(u))\Phi(Q_1u-\alpha_1(x_2-Q_2u))\psi(x_2,u)uy(u)dx_2) \tag{A.67}$$

Because $\frac{du_2^*}{dc_1}$ equals to zero, the equation becomes:

$$\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1dc_1} = -1 \leq 0 \tag{A.68}$$

In the second case, the analysis is done as follows:

$$\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1dc_1} = -1 + \frac{du_1^*}{dc_1}(-h_1 - c_{p1} + p(u_1^*) + b_1 - b_1\Phi(Q_1u_1^*))u_1^*y(u_1^*)$$
$$+\frac{du_1^*}{dc_1}(s_1 - p(u_1^*))\Phi(Q_1u_1^*)\Psi(Q_2u_1^*)u_1^*y(u_1^*)$$
$$+\frac{du_1^*}{dc_1}(\int_{Q_2u}^{\infty}(s_1 - p(u))\Phi(Q_1u - \alpha_1(x_2 - Q_2u))\psi(x_2,u)uy(u)dx_2) \tag{A.69}$$

The multiplier of the term $\frac{du_1^*}{dc_1}$ equals to zero due to the first stage first order conditions. Therefore, the equation again becomes:

$$\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1dc_1} = -1 \leq 0 \tag{A.70}$$

All these point out that the equation A.57 is negative. Moreover, the equation A.58 is positive. They both imply that the equilibrium changes in favour of the second manufacturer.

In the following cases, the same analysis is done.

*ii)*The effect of the lost sales cost:

$$\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1db_1} = \int_0^{u_1^*}(1 - \Phi(Q_1u))uy(u)du \geq 0 \tag{A.71}$$

The positiveness of this term leads to the equations A.57 and A.58 being positive and negative respectively. Thus, between q_1^* and b_1, there is a positive relationship whereas between q_2^* and b_1, the relationship is negative.

*iii)*The effect of the processing cost:

$$\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1dc_{p1}} = -\int_0^{u_1^*}\Phi(Q_1u)uy(u)du \leq 0 \tag{A.72}$$

Like in the case of the unit leasing cost, $\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1dc_{p1}}$ is negative, resulting in the equation A.57 being negative and A.58 being positive. So, the effect of the processing cost is negative on the first manufacturer's optimality and positive on the second manufacturer's optimal leasing area size.

*iv)*The effect of the olive salvage value:

$$\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1dh_1} = \int_0^{u_1^*} \Phi(Q_1u)uy(u)du \geq 0 \tag{A.73}$$

As observed, the equality A.73 is definitely positive. Hence, the equation A.57 is positive which points to a positive relationship. In addition to this, the equation A.58 is negative such that the second manufacturer is affected negatively from the change of the olive oil salvage value.

*v)*The effect of the olive oil salvage value:

$$\begin{aligned}\frac{d^2E_1[\Pi_1(Q_1;q_1^*,q_2^*,Q_2,u)]}{dQ_1ds_1} &= \int_0^{u_1^*} \Phi(Q_1u)\Psi(Q_2u)uy(u)du \\ &+ \int_0^{u_1^*}\int_{Q_2u}^{\infty} \Phi(Q_1u-\alpha_1(x_2-Q_2u))\psi(x_2,u)uy(u)dx_2du \geq 0\end{aligned} \tag{A.74}$$

This case very much resembles to the one of olive salvage value, i.e. the equations A.57 and A.58 are positive and negative, respectively. This results in a change of optimal leasing size areas favorable to the first manufacturer. □

APPENDIX B: TABLES

B.1. Anova Tables for the Second Stage of the Centralized Analysis

Table B.1. ANOVA Table for q

Analysis of variance table (ANOVA) Response q					
Source	df	Sum of Squares	Mean Square	F Value	p value Prob > F
K	1	2500000000	2500000000	201504.6	< 2.2e-16
Boundaries for ε	1	71233600	71233600	5741.6	< 2.2e-16
Yield	1	91934720	91934720	7410.1	< 2.2e-16
Residual	12	148880	12407		

Table B.2. ANOVA Table for $E[q; Q, u]$

Analysis of variance table (ANOVA) Response $E[q; Q, u]$					
Source	df	Sum of Squares	Mean Square	F Value	p value Prob > F
K	1	3.3276e+11	3.3276e+11	187.4902	1.097e-08
Boundaries for ε	1	4.3560e+07	4.3560e+07	0.0245	0.8781
Yield	1	7.1012e+11	7.1012e+11	400.1131	1.392e-10
Residual	12	2.1297e+10	1.7748e+09		

B.2. Anova Tables for the Second Stage of the Decentralized Analysis

Table B.3. ANOVA Table for q_1

Analysis of variance table (ANOVA) Response q_1					
Source	df	Sum of Squares	Mean Square	F Value	p value Prob > F
K_1	1	1.1250e+10	1.1250e+10	1.6308e+06	¡2e-16
Boundaries for ε_1	1	2.8442e+08	2.8442e+08	4.1229e+04	¡2e-16
α_1	1	1.6950e+03	1.6950e+03	2.4570e-01	0.6218
Boundaries for ε_2	1	8.6700e+02	8.6700e+02	1.2570e-01	0.7241
α_2	1	8.333e-02	8.333e-02	1.208e-05	0.9972
Yield	1	4.7818e+08	4.7818e+08	6.9317e+04	¡2e-16
Residual	65	4.4840e+05	6.8980e+03		

Table B.4. ANOVA Table for q_2

Analysis of variance table (ANOVA) Response q_2					
Source	df	Sum of Squares	Mean Square	F Value	p value Prob > F
K_1	1	8.104e-22	8.104e-22	1.175e-25	1.0000
Boundaries for ε_1	1	70674847	70674847	10244.9659	¡2e-16
α_1	1	434	434	0.0629	0.8027
Boundaries for ε_2	1	213743002	213743002	30984.0046	¡2e-16
α_2	1	1261	1261	0.1828	0.6704
Yield	1	478181470	478181470	69316.7810	¡2e-16
Residual	65	448402	6898		

B.3. Anova Tables for the First Stage of the Centralized Analysis

Table B.5. ANOVA Table for u^*

Analysis of variance table (ANOVA) Response u^*-Expected Scenario					
Source	df	Sum of Squares	Mean Square	F Value	p value Prob > F
K	1	0.000225	0.000225	9	0.2048
Boundaries for ε	1	0.000225	0.000225	9	0.2048
Residual	1	0.000025	0.000025		

Table B.6. ANOVA Table for Q

Analysis of variance table (ANOVA) Response Q-Expected Scenario					
Source	df	Sum of Squares	Mean Square	F Value	p value Prob > F
K	1	3168170082	3168170082	5278084.27	0.0002771
Boundaries for ε	1	213906	213906	356.36	0.0336922
Residual	1	600	600		

Table B.7. ANOVA Table for $E[Q;q]$

Analysis of variance table (ANOVA) Response $E[Q;q]$-Expected Scenario					
Source	df	Sum of Squares	Mean Square	F Value	p value Prob > F
K	1	2546716225	2546716225	192568.3	0.001451
Boundaries for ε	1	13653025	13653025	1032.4	0.019807
Residual	1	13225	13225		

Table B.8. ANOVA Table for u^*

Analysis of variance table (ANOVA) Response u^*-Best Yield Case Scenario					
Source	df	Sum of Squares	Mean Square	F Value	p value Prob > F
K	1	1e-04	1e-04	6.6042e+28	2.477e-15
Boundaries for ε	1	4e-04	4e-04	2.6417e+29	1.239e-15
Residual	1	1.514e-33	1.514e-33		

Table B.9. ANOVA Table for Q

Analysis of variance table (ANOVA) Response Q-Best Yield Case Scenario					
Source	df	Sum of Squares	Mean Square	F Value	p value Prob > F
K	1	3168282656	3168282656	3900625.00	0.0003223
Boundaries for ε	1	151710	151710	186.78	0.0464991
Residual	1	812	812		

Table B.10. ANOVA Table for $E[Q;q]$

Analysis of variance table (ANOVA) Response $E[Q;q]$-Best Yield Case Scenario					
Source	df	Sum of Squares	Mean Square	F Value	p value Prob > F
K	1	2543689225	2543689225	192339.45	0.001452
Boundaries for ε	1	12924025	12924025	977.24	0.020358
Residual	1	13225	13225		

B.4. Anova Tables for the First Stage of the Decentralized Analysis

Table B.11. ANOVA Table for u_1^*

Analysis of variance table (ANOVA) Response u_1^*-Expected Scenario					
Source	df	Sum of Squares	Mean Square	F Value	p value Prob > F
K_1	1	0.0056889	0.0056889	180.7059	1.352e-08
Boundaries for ε_1	1	0.0064000	0.0064000	203.2941	6.932e-09
α_1	1	0.0001000	0.0001000	3.1765	0.1000
Boundaries for ε_2	1	0.0000333	0.0000333	1.0588	0.3238
α_2	1	1.521e-32	1.521e-32	4.831e-28	1.0000
Residual	12	0.0003778	0.0000315		

Table B.12. ANOVA Table for u_2^*

Analysis of variance table (ANOVA) Response u_2^*-Expected Scenario					
Source	df	Sum of Squares	Mean Square	F Value	p value Prob > F
K_1	1	0.00000556	0.00000556	1.0	0.3370
Boundaries for ε_1	1	0.00111111	0.00111111	200.0	7.607e-09
α_1	1	0.00000278	0.00000278	0.5	0.4930
Boundaries for ε_2	1	0.00270000	0.00270000	486.0	4.462e-11
α_2	1	0.00000833	0.00000833	1.5	0.2442
Residual	12	0.00006667	0.00000556		

Table B.13. ANOVA Table for q_1

Analysis of variance table (ANOVA) Response Q_1-Expected Scenario					
Source	df	Sum of Squares	Mean Square	F Value	p value Prob > F
K_1	1	1.4054e+10	1.4054e+10	2.4245e+05	¡2e-16
Boundaries for ε_1	1	9.8805e+04	9.8805e+04	1.7045e+00	0.216181
α_1	1	2.8185e+06	2.8185e+06	4.8621e+01	1.49e-05
Boundaries for ε_2	1	5.6942e+05	5.6942e+05	9.8228e+00	0.008625
α_2	1	9.1300e+03	9.1300e+03	1.5750e-01	0.698436
Residual	12	6.9562e+05	5.7969e+04		

Table B.14. ANOVA Table for q_2

Analysis of variance table (ANOVA) Response Q_2-Expected Scenario					
Source	df	Sum of Squares	Mean Square	F Value	p value Prob > F
K_1	1	802644	802644	19.5024	0.0008412
Boundaries for ε_1	1	168784	168784	4.1011	0.0656917
α_1	1	541941	541941	13.1679	0.0034585
Boundaries for ε_2	1	19441	19441	0.4724	0.5049675
α_2	1	1545854	1545854	37.5607	5.109e-05
Residual	12	493874	41156		

Table B.15. ANOVA Table for $E_1[\Pi_1(Q_1; q_1^*, q_2^*, Q_2, u]$

Analysis of variance table (ANOVA) Response $E_1[\Pi_1(Q_1; q_1^*, q_2^*, Q_2, u]$-Expected Scenario					
Source	df	Sum of Squares	Mean Square	F Value	p value Prob > F
K_1	1	1.1847e+10	1.1847e+10	61823.6292	¡2e-16
Boundaries for ε_1	1	1.0017e+08	1.0017e+08	522.7592	2.908e-11
α_1	1	1.1106e+06	1.1106e+06	5.7955	0.03307
Boundaries for ε_2	1	6.4218e+05	6.4218e+05	3.3513	0.09209
α_2	1	3.1690e+03	3.1690e+03	0.0165	0.89981
Residual	12	2.2995e+06	1.9162e+05		

Table B.16. ANOVA Table for $E_2[\Pi_1(Q_2; q_2^*, q_1^*, Q_1, u]$

Analysis of variance table (ANOVA) Response $E_2[\Pi_1(Q_2; q_2^*, q_1^*, Q_1, u]$-Expected Scenario					
Source	df	Sum of Squares	Mean Square	F Value	p value Prob > F
K_1	1	13226939	13226939	9.7776	0.008742
Boundaries for ε_1	1	10314803	10314803	7.6249	0.017237
α_1	1	460136	460136	0.3401	0.570547
Boundaries for ε_2	1	11466075	11466075	8.4759	0.013048
α_2	1	686408	686408	0.5074	0.489883
Residual	12	16233400	1352783		

Table B.17. ANOVA Table for u_1^*

Analysis of variance table (ANOVA) Response u_1^*-Best Yield Case Scenario					
Source	df	Sum of Squares	Mean Square	F Value	p value Prob > F
K_1	1	0.0156056	0.0156056	55.4669	7.76e-06
Boundaries for ε_1	1	0.0032111	0.0032111	11.4133	0.005485
α_1	1	0.0001000	0.0001000	0.3554	0.562133
Boundaries for ε_2	1	0.0003571	0.0003571	1.2694	0.281907
α_2	1	1.065e-32	1.065e-32	3.786e-29	1.00000
Residual	12	0.0033762	0.0002813		

Table B.18. ANOVA Table for u_2^*

Analysis of variance table (ANOVA) Response u_2^*-Best Yield Case Scenario					
Source	df	Sum of Squares	Mean Square	F Value	p value Prob > F
K_1	1	0.00000556	0.00000556	0.0282	0.869334
Boundaries for ε_1	1	0.00027778	0.00027778	1.4122	0.257672
α_1	1	0.00000278	0.00000278	0.0141	0.907370
Boundaries for ε_2	1	0.00217302	0.00217302	11.0477	0.006067
α_2	1	0.00000833	0.00000833	0.0424	0.840372
Residual	12	0.00236032	0.00019669		

Table B.19. ANOVA Table for q_1

Analysis of variance table (ANOVA) Response Q_1-Best Yield Case Scenario					
Source	df	Sum of Squares	Mean Square	F Value	p value Prob > F
K_1	1	1.4249e+10	1.4249e+10	45180.3960	¡2e-16
Boundaries for ε_1	1	1.6167e+06	1.6167e+06	5.1261	0.04289
α_1	1	2.8522e+06	2.8522e+06	9.0434	0.01092
Boundaries for ε_2	1	2.5811e+05	2.5811e+05	0.8184	0.38345
α_2	1	1.1224e+04	1.1224e+04	0.0356	0.85352
Residual	12	3.7846e+06	3.1539e+05		

Table B.20. ANOVA Table for q_2

Analysis of variance table (ANOVA) Response Q_2-Expected Scenario					
Source	df	Sum of Squares	Mean Square	F Value	p value Prob > F
K_1	1	388080	388080	1.2135	0.292253
Boundaries for ε_1	1	3222025	3222025	10.0747	0.008009
α_1	1	636272	636272	1.9895	0.183785
Boundaries for ε_2	1	105248	105248	0.3291	0.576788
α_2	1	1713096	1713096	5.3565	0.039161
Residual	12	3837766	319814		

Table B.21. ANOVA Table for $E_1[\Pi_1(Q_1; q_1^*, q_2^*, Q_2, u]$

Analysis of variance table (ANOVA) Response $E_1[\Pi_1(Q_1; q_1^*, q_2^*, Q_2, u]$-Best Yield Case Scenario					
Source	df	Sum of Squares	Mean Square	F Value	p value Prob > F
K_1	1	1.2400e+10	1.2400e+10	1264.9945	1.560e-13
Boundaries for ε_1	1	1.3491e+08	1.3491e+08	13.7632	0.002981
α_1	1	6.9750e+05	6.9750e+05	0.0712	0.794189
Boundaries for ε_2	1	9.7677e+06	9.7677e+06	0.9965	0.337872
α_2	1	1.2352e+04	1.2352e+04	0.0013	0.972266
Residual	12	1.1763e+08	9.8024e+06		

Table B.22. ANOVA Table for $E_2[\Pi_1(Q_2; q_2^*, q_1^*, Q_1, u]$

Analysis of variance table (ANOVA) Response $E_2[\Pi_1(Q_2; q_2^*, q_1^*, Q_1, u]$-Best Yield Case Scenario					
Source	df	Sum of Squares	Mean Square	F Value	p value Prob > F
K_1	1	2170139	2170139	0.2381	0.634361
Boundaries for ε_1	1	8840711	8840711	0.9701	0.344108
α_1	1	201003	201003	0.0221	0.884406
Boundaries for ε_2	1	96472406	96472406	10.5856	0.006911
α_2	1	255208	255208	0.0280	0.869888
Residual	12	109362960	9113580		

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