ANALYSIS & DESIGN OF REINFORCED CONCRETE CIRCULAR WATER TANKS

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A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE AT THE SCHOOL OF ENGINEERING OF ROBERT COLLEGE-ISTANBUL.

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CONTENTS .

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Introduction

THE problem of designing reinforced concrete tanks is faced in many industrial and water supply fields, as tanks are needed for sludge digestion, storage, distribution, or filtration. Each case has its problems. The digestion tank, for example, undergoes the effect of its chemical contents; gas holders may suffer from corrosion, and so on. But, as all these tanks are of reinforced concrete, the general method of design is basically the same for all.

This thesis has been intended to expound the theory and practice pertaining to the design of reinforced concrete circular water tanks, with an eye to the exposition of the method of superposition in analysis and design, and considering the tanks as structures obeying the general theory of shells with bending.

From the method to be explained it should become easy to handle the problem of superposing any other effect, that does not exist in the case of water tanks, on those effects already existing in these tanks. To design containers of hot liquids, for instance, the extra stresses due to the temperature gradient are superposed on the stresses already assumed to exist in the structure due to pre-existing conditions.

As earthquakes have caused a lot of damage to water tanks, especially elevated ones, the method of calculating stresses due to earthquakes has been explained, and the way to superimpose them on other stresses has been clarified. Formulae for calculation of earthquake stresses have been derived and expressed in a closed form. Other formulae that are needed have been also derived for different boundary conditions at the roof/wall and the wall/floor connections.

Where soil comes into the picture, the classical theory of Winkler-Zimmerman has been applied, with the assumption that the foundation is elastic.

This present work of analysis and design is given here in three sections:

I. Analysis & Design of an Elevated Reinforced Concrete Circular Water Tank,

II. Analysis & Design of a Ground-supported Reinforced Concrete Circular Water Tank.

III. Analysis & Design of an Underground Reinforced Concrete Circular Water Tank.

A special Appendix has been prepared for reference, giving the derivation of some of the formulae and equations that have been used in the text.

(I)

ANALYSIS & DESIGN OF AN ELEVATED REINFORCED CONCRETE CIRCULAR WATER TANK.

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(A)

(i) Economical Analysis:-

It is not easy to get at the optimum solution for the problem of determination of the economical dimensions of the tank because of the many interconnecting factors affecting this solution. However, the following approach can be used to choose dimensions within satisfactorily economical limits.

Let :

V = volume of the tank, cu.ft. $= \pi D^2 H/4$
C_w = unit cost of the wall, T.L./sq.ft.
C_r = unit cost of the roof, T.L./sq.ft.
C_f = unit cost of the floor, T.L./sq.ft. - excluding the cost of the circular girder.
H = height of the tank, ft.
D = diameter of the tank, ft.
A_w = area of the wall, sq.ft. $= \pi DH$
A_r = area of the roof, sq.ft. $= \pi D^2/4$
A_f = area of the floor, sq.ft. $= \pi D^2/4$

$\therefore$ Total cost is

$$C = A_w.C_w + A_r.C_r + A_f.C_f$$

$$= (4V/D).C_w + (\pi D^2/4)(C_r + C_f).$$

Differentiating with respect to D gives:

$$\frac{dC}{dD} = -(4V/D^2).C_w + (\pi D/2)(C_r + C_f)$$

If this is set equal to zero, it will give the optimum diameter D as:

$$D = 2\sqrt[3]{\frac{V.C_w}{(C_r + C_f)}}$$

(ii) Design:

Choosing the case in which the relation between C_w, C_r, and C_f is given by the equality

$$C_w = C_r + C_f$$

the optimum diameter will be:

$$D = \sqrt[3]{\frac{8V}{\pi}}.$$

Let the required capacity of the tank be equal to 50,000 gallons; or V = 6700 cu.ft.

$\therefore \quad D = \sqrt[3]{\frac{8 \times 6700}{\pi}} = 25.8$ ft. -- say 26 ft.

$\therefore \quad H = 12.6$ ft.

Add 1.4 ft. for clearance thus making $H = 14$ ft.

(B)
The Roof Slab

(i) Analysis:

The solution for the problem of a uniformly loaded plate(circular plate) with clamped edges is given* as:

$$M_r = \frac{q}{16}\left[a^2(1 + v) - r^2(3 + v)\right]$$

$$M_t = (q/16)\left[a^2(1 + v) - r^2(1 + 3v)\right]$$

where:

a = radius of the plate, (r = distance from centre),
v = Poisson's ratio,
q = intensity of the uniformly distributed load,
M_r = bending moment per unit length, acting in the radial direction, and
M_t = bending moment per unit length, acting in the circumferential direction.

(ii)Design:

Let the slab thickness be	= 6"	= 0.5 ft.
Hence, weight of roof slab	= (0.5) x 150	= 75 lb./sq.ft.
Assume snow load is		= 5 lb./sq.ft.
Total load on slab is		= 80 lb./sq.ft.

At the edge of the slab the moment is:

$(M_r)_{r=a} = -qa^2/8 = -1690$ ft-lb.per ft. of periphery.

Let the uniform thickness of the wall be = 9" = 0.75 ft.

For the wall ; $H^2/Dh = (14)^2/26 \times 0.75 = 10$

$\therefore$ stiffness of the wall = $1.010\ Eh^3/H$ **

where the coefficient k = 1.010 corresponds to $H^2/Dh = 10$.

The stiffness of the circular slab is***:

$= 0.104\ Eh^3/a$

Relative stiffness of the wall $= 1.010 \times (9)^3/14 = 52.5$

Relative stiffness of the slab $= 0.104 \times (6)^3/13 = 1.72$

These stiffnesses determine the distributing factors as:
0.03 for the slab; 0.97 for the wall.

(*) "Theory Of Plates & Shells", S.Timoshenko; McGraw-Hill Book Company, Inc., 1959, p.55.

(**) From Table XVIII of the pamphlet "Circular Concrete Tanks Without Prestressing"-issued by the Portland Cement Association. (***)Table XIX, same reference.

Distribution of moments:

	Wall	Roof slab
Distribution factors	0.97	0.03
Fixed End Moments	0	-1690
Distributed moments	+1640	+ 50
Final Moments	+1640	-1640 ft.lb.

Shear $V = \pi q a^2 / 2\pi a = qa/2$
$= 80x13/2 = 520$ lb.per ft.of periphery.

∴ unit shear is:

$v = V/jbd = 520/0.875x12x4.5 = 11$ psi

Choosing $f'_c = 3000$ psi, we will have:

$11 < 0.03 f'_c = 90$ psi. .... O.K.

On distribution of moments a moment of 50 ft.lb./ft. was induced at the edge of the slab; hence, to the coefficients* multiplied by(qa^2) in the expressions for the moments M_r,M_t at any point between the centre and edge of the slab we should add;

$$+ 50/qa^2 = + 50/80x(13)^2 = + 0.0038$$

The result of this operation is given in Schedule I.

Schedule I

Point	0.0a	0.1a	0.2a	0.3a	0.4a	0.5a	0.6a	0.7a	0.8a	0.9a	1.0a
M_v Coeff.	+0.075	+0.073	+0.067	+0.057	+0.043	+0.025	+0.003	-0.023	-0.053	-0.087	-0.125
Add	+0.0038 ←										→ +0.0038
Final Coeff. of M_v	+.0788	+.0768	+.0708	+.0608	+.0468	+.0288	+.0068	-.0192	-.0492	-.0832	-.1212
M_r/ft	+1070	+1040	+960	+820	+620	+390	+ 92	-260	-660	-1125	-1640
M_r/Segment	0	+107	+192	+246	+248	+195	+55.2	-182	-528	-1012	-1640
M_t Coeff.	+.075	+.074	+.071	+.066	+.059	+.050	+.039	+.026	+.011	-.006	-.025
Add	+.0038 ←										→ +0.0038
Final Coeff. of M_t	+.0788	+.0778	+.0748	+.0698	+.0628	+.0538	+.0428	+.0298	+.0148	-.0022	-.0212
M_t/ft	+1070	+1050	+1010	+950	+850	+730	+580	+405	+200	- 30	-290

The final values of M_r,M_t are plotted in Fig.1,p.5.

(*) op.cit. ,Table XII.

Fig. 1 - Variation of radial and tangential moments in the roof slab.of the elevated tank.

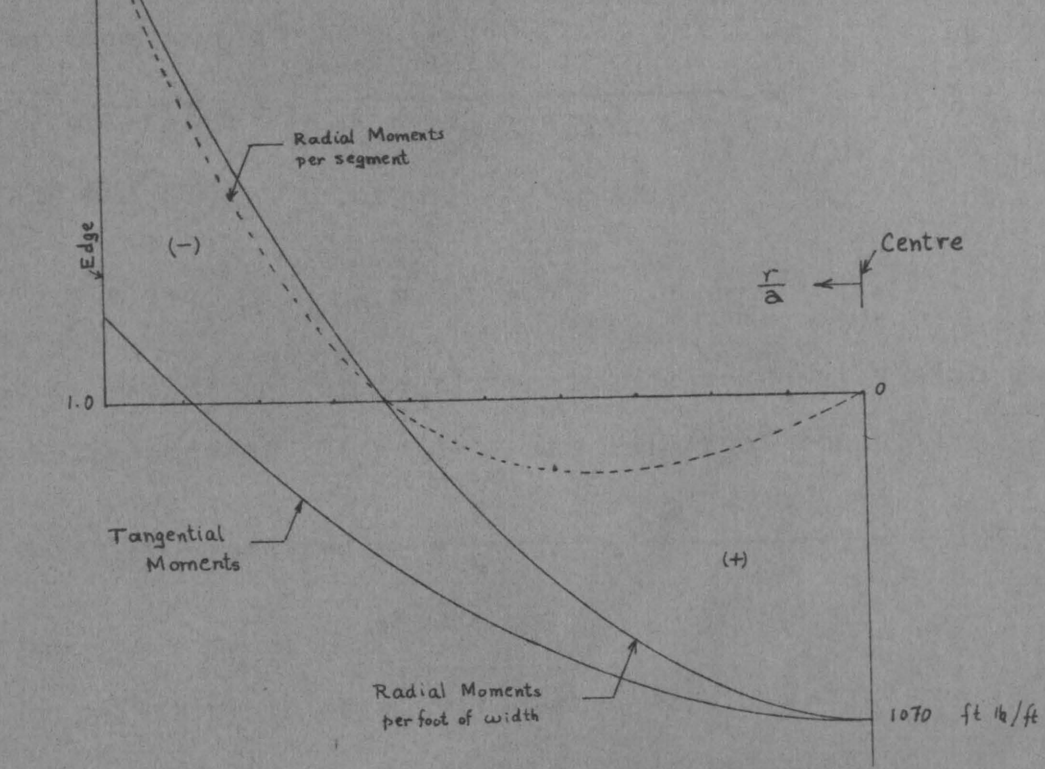
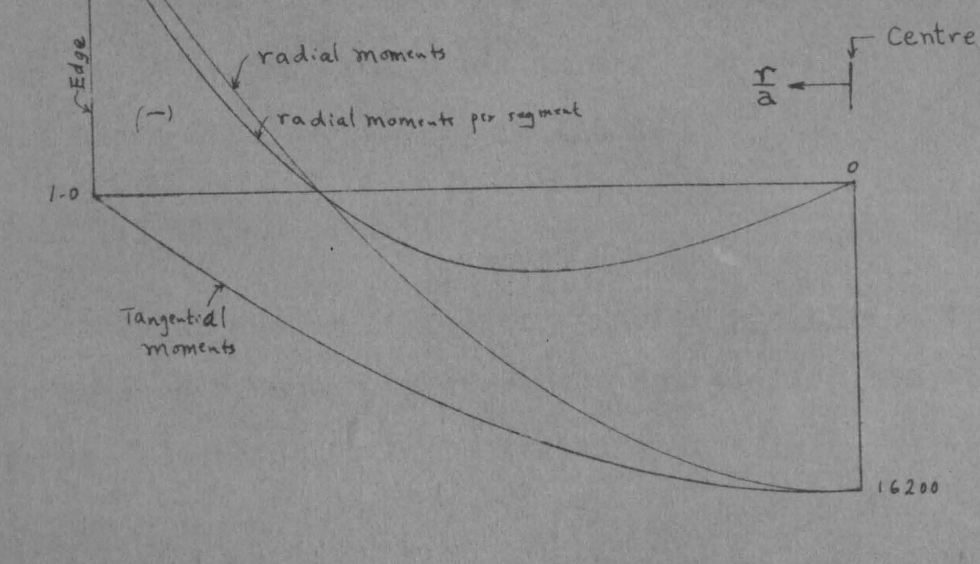


Fig. 2 - Variation of radial and tangential moments in the floor slab of the elevated tank.



NEGATIVE REINFORCEMENT:

Maximum negative M = -1640 ft.lb per ft.

$$A_s = \frac{M}{f_s jd} = \frac{1640 \times 12}{20,000 \times 0.875 \times 4.5} = 0.250 \text{ sq. in.}$$

USE # 4 ϕ @ 9.5'' o.c.

This gives $A_s = 0.2 \times \frac{12}{9.5} = 0.252$ sq. in.

This reinforcement is placed in the top of the slab and the outside at the wall of the wall/roof connection.

Total no. required $= \frac{2 a}{9.5} = \frac{2 \times 13 \times 12}{9.5} = 103$ bars.

Say 104 bars of # 4 ϕ.

By referring to **Fig. 1** (dashed line) :

$\frac{1}{2} \times 104 = 52$ bars, are discontinued at:

0.16a + 12 diameters = 0.16 x 13 + 12 x 0.5/12 = 2.58'

Say at 2'7" from the inner side of the wall.

The other 52 bars are discontinued at a distance of:

0.38a + 12 diameters = 0.38 x 13 + 12 x 0.5/12 = 5.44'.

Say at 5' 6" from the inner side of the wall.

All above bars are placed <u>radially</u>.

POSITIVE REINFORCEMENT:

The largest number of radial bars for positive moments is between 0.3a ~ 0.4a where the dashed line has its maximum value. At point 0.4a the radial moment/ft = 620 ft-lb/ft.

and the length of the concentric circle through 0.4a is:

$$2\pi(0.4a) = 32.7'$$

At point 0.4a

$$A_s = \frac{32.7M}{f_s\, jd} = \frac{32.7 \times 620 \times 12}{20{,}000 \times 0.875 \times 5} = 2.78 \text{ sq. in.}$$

Here d = 5"

USE 14 $\#4\phi$ giving $A_s = 2.8$.

These bars are arranged as shown below. (Fig. 3)

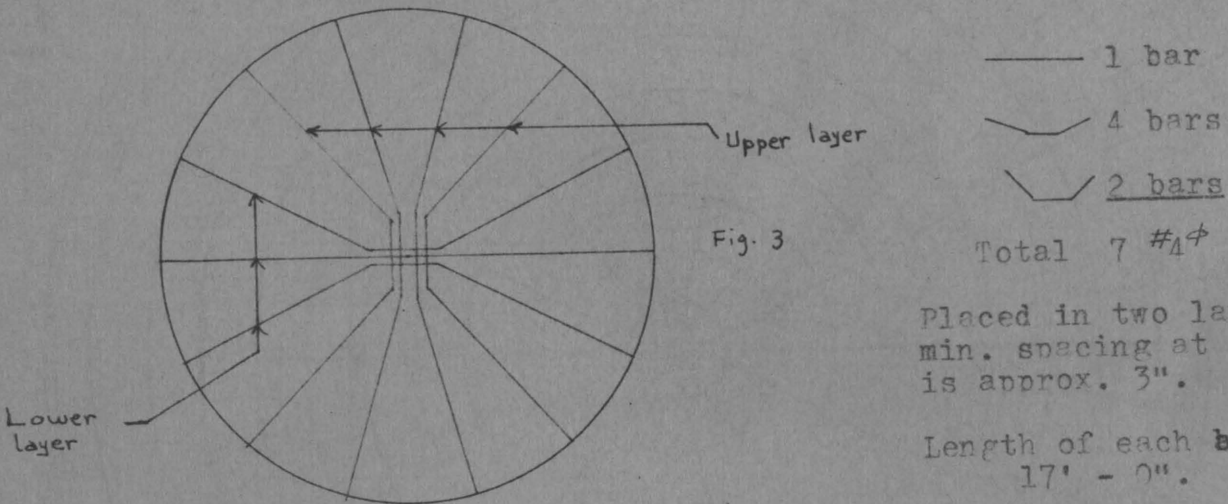


Fig. 3

Total 7 $\#4\phi$

Placed in two layers min. spacing at center is approx. 3".

Length of each bar is 17' - 0".

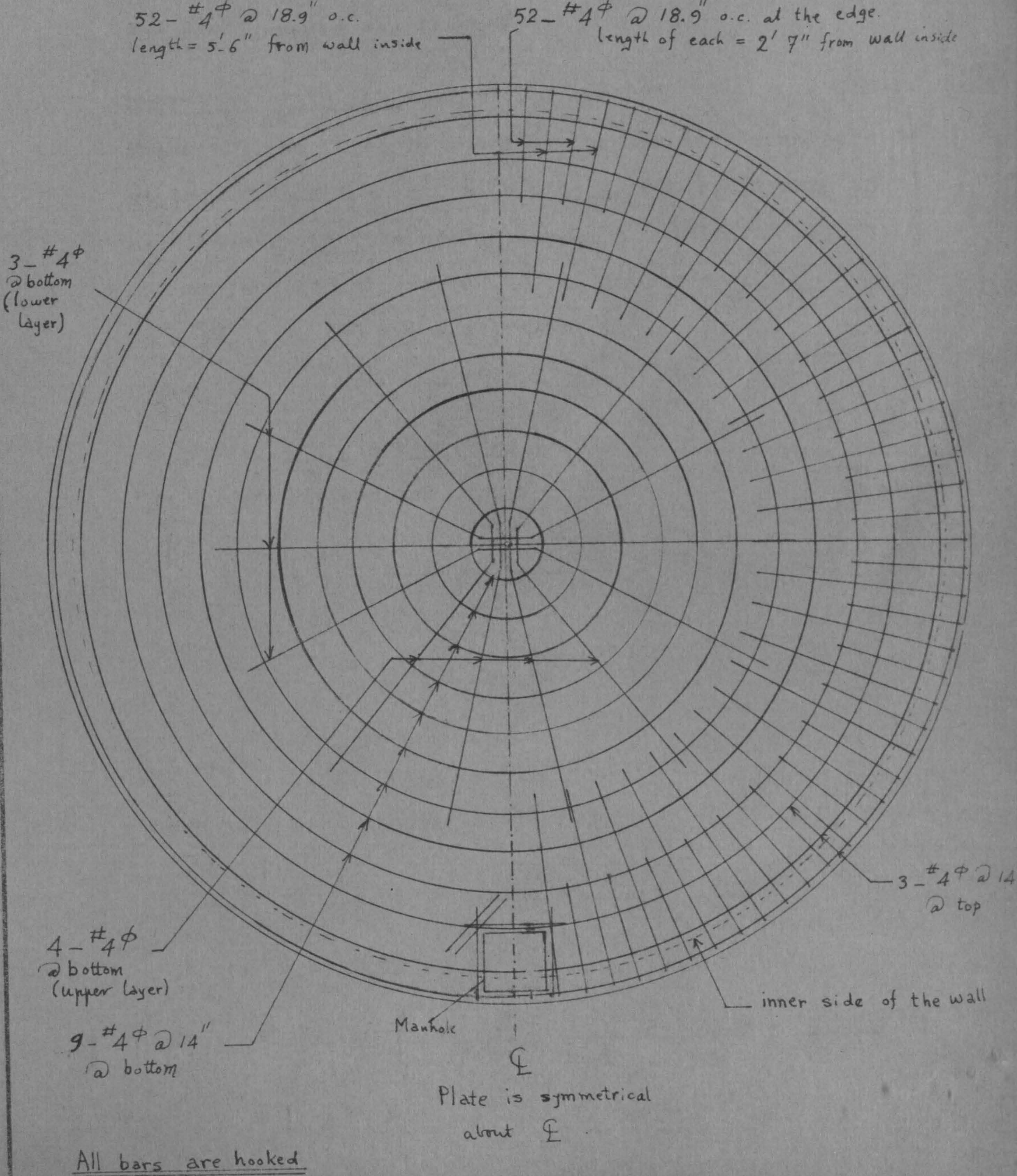
Ring bars should be placed in accordance with the tangential moment curve in Fig. 1. Maximum area of steel is required near the centre and is:

$$A_s = \frac{M}{f_s\, jd} = \frac{1070 \times 12}{20{,}000 \times 0.785 \times 4.5} = 0.166 \text{ Sq. in.}$$

USE $\#4\phi$ @ 14" o.c. (giving $A_s = 0.17$ sq. in.)

Areas of ring bars decrease gradually from the centre toward point (0.9a). Inside this point the bars are all in the bottom, outside it bars are in the top. Laps are

PLATE 1 : Reinforcing steel of the roof slab of the elevated tank.



spliced according to ACI (1956) Code requirements.

Smallest ring bar will have a radius of 14".

For requirements see PLATE I.

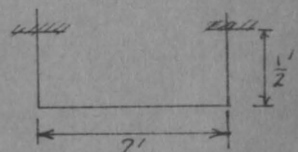
DESIGN OF STEEL LADDER

These steps will be fixed in the wall for the inside and outside stairs. Taking the load on a step = 160 lb. it acts as a cantilever. Let its projection = ½' and length = 2 ft.

One projecting bar carries 80 lb. at its end.

$$M_{max.} = 80 \times \tfrac{1}{2} = 40 \text{ ft-lb.}$$

$$= 40 \times 12 = 480 \text{ lb-in.}$$



dimensions of one step
Fig. 4

$$Z = \frac{M}{\sigma} = \frac{0.480}{20.000} = 0.024 \text{ in}^3.$$

for a circular section.

$$Z = \frac{I}{C} = \frac{r^4}{4(r)} = \frac{r^3}{4} = 0.024$$

$$r^3 = 0.096 = 0.0305$$

$$r = 0.31"$$

USE #5ϕ giving area $A_s = 0.31$ sq. in.

There will be a spacing of 1½ ft.

$$\tau_{max.} = \frac{VQ}{It} \; ; \quad Q = \frac{c^2}{2} \cdot \frac{4c}{3} = \frac{2c^3}{3}$$

$$t = 2c, \quad I = \frac{\pi c^4}{4}.$$

$$\tau_{max.} = \frac{VQ}{It} = \frac{V}{2c} \cdot \frac{2c^3}{3} \cdot \frac{4}{\pi c^4} = \frac{4V}{3\pi c^2} = \frac{4}{3}\,\frac{V}{A}$$

$$\tau_{max} = \frac{4}{3} \cdot \frac{80}{0.31} = 344 \text{ psi} \quad 13000 \text{ psi} \quad \text{O.K.}$$

For inside and outside ladders USE two vertical rods (welded at connection with projecting portions) of #5ϕ having a length of 14' each. The ladder will extend along a column(with steps embedded in it)up to the surface of the ground.

DESIGN OF THE MANHOLE:

Dimensions are 2' x 2'

Approximate design: Take strip AB whose length is about 14' and it is 1 ft. wide.

Load on top is q=80 lb/sq. ft.

M = 1960 ft-lb.

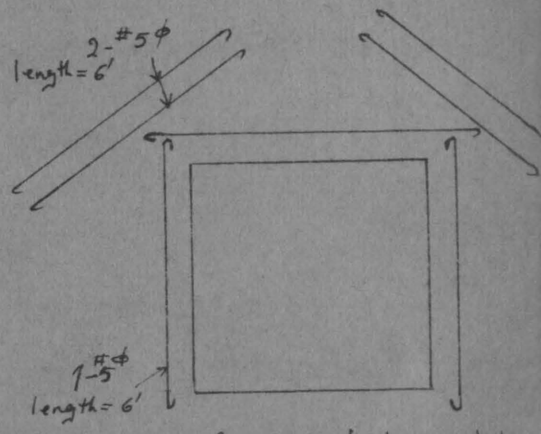


Fig. 5 - Reinforcement of the manhole

$$A_s = \frac{M}{f_s\, jd} = \frac{1960 \times 12}{20{,}000 \times 0.875 \times 5} = 0.269 \text{ in}^2.$$

USE 1 #5ϕ giving A_s = 0.31 in^2, its length is 2 x 3 = 6'

By the same approximation use 1 #5ϕ in the two other parallel sides and 2 #5ϕ bars at the corner of the manhole @ 3" o.c. @ the bottom.

DESIGN OF THE COVER OF THE MANHOLE:

This could be designed according to the exact theory of plates; but for such a small plate an approximate design is preferable.

From exact theory:

$$M_x = M_y = 0.0479\, qa^2.$$

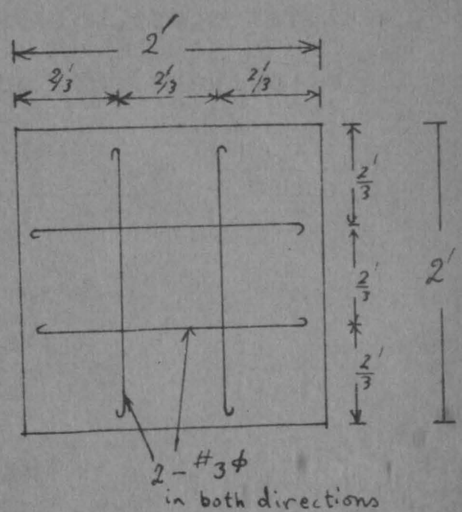


Fig. 6 - Reinforcement of the cover of the manhole.

(this is for $v = 0.3$) if it is changed to $v = 0.2$ we get:

$$M_x = M_y = 0.0442\ qa^2$$
$$= 0.0442\ (80)(2)^2$$
$$= 7.07\ \text{ft-lb.}$$

which is very small.

Hence USE 2 - #3ϕ in each direction at the bottom of the cover.

(C) ANALYSIS OF THE WALL OF THE TANK:

(i) Analysis of the fixed base/free top condition:

Given a circular tank of constant wall thickness h, the water(or liquid) pressure at any depth will be:(see Fig. 7)

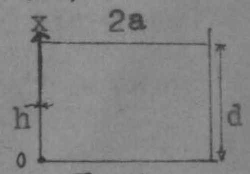


Fig. 7

$$Z = -\gamma(d - x) \ ....(1)$$

This pressure will be carried by horizontal ring action, and by vertical bending action.

Taking a one-unit wide vertical strip of the wall, we have:

$$D. \frac{d^4w}{dx^4} = Z_V = Z - Z_H \ .... (2)$$

where Z_V, Z_H are the portions of the liquid pressure which are supported by vertical action, and ring action, respectively.

For a horizontal strip of unit depth, the ring tension is constant at a given depth and is equal to:

$$N_\varphi = Z_H \cdot a$$

∴ Horizontal tensile stress is:

$$\sigma_t = N\varphi/h = Z_H \cdot a/h$$

and radial strains will be:

$$\varepsilon_a = \sigma_t/E = Z_H . a/Eh$$

Hence, the outward radial deflection will be :

$$w = \varepsilon_a \cdot a = Z_H . a^2/Eh$$

$$\therefore \quad Z_H = Eh/(a^2.w)$$

From equation (2):

$$D.\frac{d^4w}{dx^4} + Eh/(a^2.w) = Z = -\gamma(d-x) \quad \ldots\ldots(3)$$

This equation is similar to that of a beam on elastic foundation(*)

$$EI.\frac{d^4w}{dx^4} + kw = Z$$

whose solution includes the term $= \sqrt[4]{k/4EI}$

where k is the modulus of the foundation.

In the case at hand, the case of the wall of the circular tank, the term corresponding to(k) is:

$$\beta = \sqrt[4]{3(1-\nu^2)/a^2h^2} = \sqrt[4]{Eh/4a^2D}$$

Then, from eq.(3), we have:

$$\frac{d^4w}{dx^4} + 4\beta^4.w = Z/D = -\gamma(d-x)/D$$

The final solution of this equation is given(**) for the case of a free top and rigidly fixed bottom (assuming that $h \ll a$, and $h \ll d$), as:

$$w = -\frac{\gamma a^2 d}{Eh}\left[1 - \frac{x}{d} - \theta(\beta x) - \left(1 - \frac{1}{\beta d}\right)\zeta(\beta x)\right]$$

Then, ring tension will be:

$$N_\varphi = -\frac{Eh}{a}w = \gamma a d\left[1 - \frac{x}{d} - \theta(\beta x) - \left(1 - \frac{1}{\beta d}\right)\zeta(\beta x)\right]$$

and the moment in the wall will be:

$$M_x = -D\frac{d^2w}{dx^2} = \frac{\gamma a d h}{\sqrt{12(1-\nu^2)}}\left[-\zeta(\beta x) + \left(1 - \frac{1}{\beta d}\right)\theta(\beta x)\right]$$

In the above expressions the following notation was used:

$$\theta(\beta x) = e^{-\beta x}.\cos\beta x$$

$$\zeta(\beta x) = e^{-\beta x}.\sin\beta x.$$

(The values of these functions are tabulated, for different values of (βx), in Timoshenko's "Theory of Plates & Shells", and in Heteneyi's "Beams of Elastic Foundation").

The shear at any point on the wall is obtained by differentia-

$$Q_x = \frac{d}{dx}(M_x) = \frac{\gamma a d h}{\sqrt{12(1-v^2)}}\Big[\beta e^{-\beta x}\sin\beta x - \beta e^{-\beta x}\cos\beta x + (1-\frac{1}{\beta d})(-\beta e^{-\beta x}\cos\beta x - \beta e^{-\beta x}\sin\beta x)\Big]$$

$$\therefore \quad Q_x = \frac{\gamma a.h}{\sqrt{12(1-v^2)}}\Big[\varphi(\beta x) - 2\beta d.\theta(\beta x)\Big]$$

where :

$$\varphi(\beta x) = e^{-\beta x}.(\sin\beta x + \cos\beta x)$$

(ii) Analysis of the hinged base/free top condition:

The actual condition of the bottom of the wall is, in fact, not absolutely rigid; it may lie somewhere between the rigid end and the hinged end conditions. Therefore expressions for the wall with hinged end are developed below.

The solution of the free top/fixed bottom case results in the expression:

$$w = e^{-\beta x}.(C_3.\cos\beta x + C_4.\sin\beta x) - \gamma(d-x)a^2/Eh$$

The constants C_3 and C_4 where determined by applying the boundary conditions:

$$w = 0 \quad \text{at the top}$$

$$\frac{dw}{dx} = 0 \quad \text{at the base}$$

The result is that given on page(12).

But in the hinged base/free top case the two boundary conditions are:

$$(1) \quad (w)_{x=0} = 0 = C_3 - \gamma d.a^2/Eh$$

$$\therefore \quad C_3 = \gamma d.a^2/Eh$$

$$(2) \quad (M_x)_{x=0} = 0 \;; \text{ or } \frac{d^2w}{dx^2} = 0$$

$$\frac{d^2w}{dx^2} = \Big[\beta^2 C_3 e^{-\beta x}(\cos\beta x + \sin\beta x) - \beta C_3 e^{-\beta x}(-\beta\sin\beta x + \beta\cos\beta x) - \beta^2 C_4 e^{-\beta x}(\cos\beta x - \sin\beta x) + \beta C_4 e^{-\beta x}(-\beta\sin\beta x - \beta\cos$$

At $x = 0$ we have :

$$\frac{d^2w}{dx^2} = -2\beta^2.C_4 = 0$$

$$\therefore \quad C_4 = 0$$

$$\therefore \quad w = C_3 . e^{-\beta x} . \cos \beta x - \gamma (d - x) a^2 / Eh$$

$$= \frac{\gamma d a^2}{Eh} . e^{-\beta x} . \cos \beta x - \gamma (d - x) a^2 / Eh$$

$$\therefore \quad w = - \frac{\gamma d . a^2}{Eh} \left\{ 1 - \frac{x}{d} - \theta (\beta x) \right\}$$

Therefore:

$$N_\varphi = - \frac{E.h}{a} . w = + \gamma a . d \left\{ 1 - \frac{x}{d} - \theta (\beta x) \right\}$$

And the moment will be:

$$M_x = - \frac{\gamma a.d.h}{\sqrt{12(1 - v^2)}} . \zeta (\beta x)$$

and:

$$Q_x = - \frac{\gamma a.d.h}{\sqrt{12(1 - v^2)}} . \psi (\beta x)$$

where

$$\psi (\beta x) = e^{-\beta x} . (\cos \beta x - \sin \beta x).$$

(iii) Analysis of the effect of moment at the top of the wall:

Continuity at the wall/roof connection causes a momen whose effect should be superposed on other effects.

As the actual condition of the tank is such that the bott is hinged rather than fixed, only the effect of a moment at the top with the farther end hinged will be superposed on the previously discussed effects on the wall.

The expressions for deflection, moment, and shear on a bear on elastic foundation is given for the case of a concentrated moment (M_o) at one end with the other end hinged as(*):

$$w = \frac{2 M_o \lambda^2}{k} . \frac{\cosh \lambda l \sinh \lambda x' \sin \lambda x - \cos \lambda l \sinh \lambda x \sin \lambda x'}{\cosh^2 \lambda l - \cos^2 \lambda l}$$

$$M = \frac{M_o}{\cosh^2 \lambda l - \cos^2 \lambda l} \left[\cosh \lambda l \cos \lambda x \cosh \lambda x' - \cos \lambda l \cosh \lambda x \cos \lambda x' \right]$$

$$Q = \frac{- M_o \lambda}{\cosh^2 \lambda l - \cos^2 \lambda l} \left[\cosh \lambda l (\cos \lambda x \sinh \lambda x' + \sin \lambda x \cosh \lambda x') + \cos \lambda l (\sinh \lambda x \cos \lambda x' + \cosh \lambda x \sin \lambda x') \right]$$

(*) "Beams on Elastic Foundation", by Hetenyi, pp. 61-62

In these equations

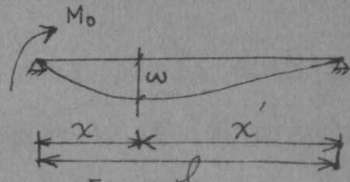


Fig. 8

$$= \sqrt[4]{k/4EI}$$

where (k) is the foundation modulus.

For the case of a plate this will be

$$\lambda = \beta = \sqrt[4]{3(1-v^2)/a^2.h^2} \quad \text{(*)} \quad ....(4)$$

Hence, the value of (k) to be used in the case of the plate is:

$$k = E.h(1-v^2)/a^2 \quad ....(5)$$

In applying the above equations note that (x) is measured from the top of the wall, (x') from the bottom, (see Fig. 8).

The hoop tension can be developed from the expression for deflection, where

$$N_\varphi = - \frac{E.h.w}{a}$$
$$= - \frac{Eh}{a} \frac{2M_0\lambda^2}{k} \left[\frac{\cosh\lambda l \sinh\lambda x' \sin\lambda x - \cos\lambda l \sinh\lambda x \sin\lambda x'}{\cosh^2\lambda l - \cos^2\lambda l} \right]$$

Using the above equations, (4), and (5), it can be shown that:

$$N_\varphi = - \frac{2M_0\lambda^2 a}{(1-v^2)} \frac{\cosh\lambda l \sinh\lambda x' \sin\lambda x - \cos\lambda l \sinh\lambda x \sin\lambda x'}{\cosh^2\lambda l - \cos^2\lambda l}$$

Attention should be drawn here to the fact that these equations are based on the assumption that the <u>outward</u> deflection is <u>positive</u>.

(iv) Analysis of the effect of water surging due to earthquake:

As an earthquake takes place water inside the tank will move, and additional stresses will develop in the wall because of surging which causes oscillatory increases and decreases in the hydrostatic pressure. The manner in which the pressure varies under these circumstances is rather complicated. Therefore an assumption is made here, that the resulting water pressure will

(*) "Theory of Plates & Shells", by S.Timoshenko & S. Woinowsky
... 468

be uniform along the height of the tank.

Considering only the case of the tank as a shell with a fixed base and hinged top, and having an inner uniform pressure

$$Z = -p \quad ,$$

and referring to the equation

$$\frac{d^2w}{dx^2} + 4\beta^2 . w = Z/D \quad ....(6)^{(*)}$$

the particular solution for this case will be:

$$-p/(4\beta^4 D) = -p.a^2/Eh = f(x)$$

The general solution of eq.(6) is:

$$w = e^{\beta x}.(C_1.\cos\beta x + C_2.\sin\beta x) + e^{-\beta x}.(C_3.\cos\beta x + C_4.\sin\beta x) + f(x).$$

Here f(x) is as shown above. By changing the exponential functions in the expression for (w) according to the identity:

$$\cosh x \pm \sinh x = e^{\pm x}$$

and letting the constants be as follows:

$$C_1 + C_2 \equiv C_4 \ , \ C_1 - C_2 \equiv C_3 \ , \ C_2 + C_4 \equiv C_2 , \ C_2 - C_4 \equiv C_1$$

we finally get:

$$w = -(pa^2/Eh) + C_1.\sin\beta x.\sinh\beta x + C_2.\sin\beta x.\cosh\beta x + C_3.\cos\beta x.\sinh\beta x + C_4.\cos\beta x.\cosh\beta x. \quad .....(7)$$

If the origin of coordinates is chosen at the midpoint of the height of the wall, (w) will be an even function of (x); hence

$$C_2 = C_3 = 0.$$

The remaining constants can be determined by satisfying the edge conditions.

For the fixity at the ends in earthquake analysis two cases are considered here:

(*) op. cit. p. 468, eq.(276).

(a) Top & Bottom Hinged:

This condition is solved in the literature(*); the final equations representing the deflection, moment, and hoop tension in a shell, hinged at both ends and subjected to an inner uniform pressure, are (after some modification):

$$w = -\frac{p.a^2}{Eh}\left\{1 - \frac{2\sin\alpha.\sinh\alpha}{\cos 2\alpha + \cosh 2\alpha}.(\sin\beta x.\sinh\beta x) - \frac{2\cos\alpha.\cosh\alpha}{\cos 2\alpha + \cosh 2\alpha}.(\cos\beta x.\cosh\beta x)\right\}$$

$$M_x = -\frac{p}{\beta^2}\left[\frac{\sin\alpha.\sinh\alpha}{\cos 2\alpha + \cosh 2\alpha}.(\cosh\beta x.\cos\beta x) - \frac{\cos\alpha.\cosh\alpha}{\cos 2\alpha + \cosh 2\alpha}.(\sin\beta x.\sinh\beta x)\right]$$

$$N_\varphi = p.a\left\{1 - \frac{2\sin\alpha.\sinh\alpha}{\cos 2\alpha + \cosh 2\alpha}.(\sin\beta x.\sinh\beta x) - \frac{2\cos\alpha.\cosh\alpha}{\cos 2\alpha + \cosh 2\alpha}.(\cos\beta x.\cosh\beta x)\right\}$$

$$Q_x = -\frac{p}{\beta}\left\{\frac{\sin\alpha.\sinh\alpha}{\cos 2\alpha + \cosh 2\alpha}.(\sinh\beta x.\cos\beta x - \cosh\beta x.\sin\beta x) - \frac{\cos\alpha.\cosh\alpha}{\cos 2\alpha + \cosh 2\alpha}.(\sin\beta x.\cosh\beta x + \cos\beta x.\sinh\beta x)\right\}$$

where $\alpha = \beta d/2$

(b) Top & Bottom Fixed:

Referring to eq.(7)page(16), we see that the constants C_1 and C_4 should satisfy the two conditions:

(i) $(w)_{x=d/2} = 0$

(ii) $\left(\frac{dw}{dx}\right)_{x=d/2} = 0$

Now: (since $C_2 = C_3 = 0$),

$$w = -p.a^2/Eh + C_1.\sin x.\sinh x + C_4.\cos x.\cosh x \quad \text{.....(8)}$$

From the first boundary condition one obtains:

$$C_1(\sin\alpha.\sinh\alpha) + C_4(\cos\alpha.\cosh\alpha) = + p.a^2/Eh \quad \text{....(i)}$$

From the second boundary condition one obtains:

$$C_1(\cos\alpha\sinh\alpha + \sin\alpha\cosh\alpha) + C_4(\cos\alpha\sinh\alpha - \sin\alpha\cosh\alpha) = 0.\text{(i}$$

(*) op.cit. pp. 476 - 477

Solving equations (i) and (ii) we get:

$$C_1 = - \frac{(p.a^2/Eh)(\cos\alpha.\sinh\alpha - \sin\alpha.\cosh\alpha)}{\sinh\alpha.\cosh\alpha + \sin\alpha.\cos\alpha}$$

$$C_4 = + \frac{(p.a^2/Eh)(\cos\alpha.\sinh\alpha + \sin\alpha.\cosh\alpha)}{\sinh\alpha.\cosh\alpha + \sin\alpha.\cos\alpha}$$

Substituting the values of C_1 and C_4 into eq.(8),we get:

$$w = -\frac{p}{4D\beta^4}\left\{1 + \left(\frac{\cos\alpha\sinh\alpha - \sin\alpha\cosh\alpha}{\sinh\alpha\cosh\alpha + \sin\alpha\cos\alpha}\right)\sin\beta x \sinh\beta x - \left(\frac{\cos\alpha\sinh\alpha + \sin\alpha\cosh\alpha}{\sinh\alpha\cosh\alpha + \sin\alpha\cos\alpha}\right)\cos\beta x \cosh\beta x\right\}$$

$$M_x = +\frac{p}{2\beta^2}\left\{\left(\frac{\cos\alpha\sinh\alpha - \sin\alpha\cosh\alpha}{\sinh\alpha\cosh\alpha + \sin\alpha\cos\alpha}\right)\cos\beta x \cosh\beta x + \left(\frac{\cos\alpha\sinh\alpha + \sin\alpha\cosh\alpha}{\sinh\alpha\cosh\alpha + \sin\alpha\cos\alpha}\right)\sin\beta x \sinh\beta x\right\}$$

$$N_\varphi = +p a\left\{1 + \left(\frac{\cos\alpha\sinh\alpha - \sin\alpha\cosh\alpha}{\sinh\alpha\cosh\alpha + \sin\alpha\cos\alpha}\right)\sin\beta x \sinh\beta x - \left(\frac{\cos\alpha\sinh\alpha + \sin\alpha\cosh\alpha}{\sinh\alpha\cosh\alpha + \sin\alpha\cos\alpha}\right)\cos\beta x \cosh\beta x\right\}$$

$$Q = +\frac{p}{\beta}\left\{\left(\frac{\sin\alpha\cosh\alpha}{\sinh\alpha\cosh\alpha + \sin\alpha\cos\alpha}\right)\sin\beta x \cosh\beta x + \left(\frac{\cos\alpha\sinh\alpha}{\sinh\alpha\cosh\alpha + \sin\alpha\cos\alpha}\right)\cos\beta x \sinh\beta x\right\}$$

It remains now to develop a method to determine the value of the pressure (p) approximately for use in design.

Supposing that the earthquake moves the water in the tank in a direction perpendicular to AB in Fig.(9) and to the right, the left portion will undergo stresses of opposite sign to those developed in the right half.

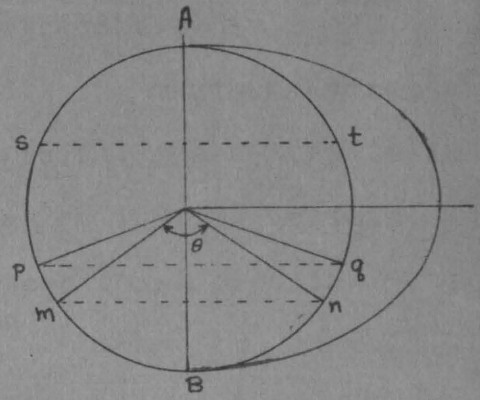


Fig.9-Pressure on the tank wall due to water surging.

Divide the arc AB into portions whose projections on the diameter AB are equal. Calculation is presented here for five equal parts. The mass in each portion (taking a unit depth of water in the tank) can be determined by calculating the area to which it corresponds. For the elevated tank the following calculations are presented:

Area of segment mnB $= \frac{1}{2}R^2.(\theta - \sin\theta)$

$\sin\theta = 0.787$, $\theta = 1.81$

$\therefore$ mass of water in this segment $= 1 \times \frac{62.4}{g}\left\{\frac{1}{2}(13)^2(1.81-0.787)\right\}$

$= 5400/g$

$\therefore$ Pressure on arc $\widehat{Bn}$ is:

$$p = \frac{F}{A} = \frac{(5400/g).(0.1g)}{(1)(13)(1.81/2)} = 46 \text{ lb/ft}^2.$$

Similar calculations give the following results:

Pressure on arc $\widehat{qn}$ = 131 lb/sq.ft.

Pressure on arc $\widehat{qt}$ = 172 lb/sq.ft.

These values are approximate; in fact, if we take a narrow strip at the middle with width = 1 unit and depth = 1 unit, the mass of water will be

$$m = 1 \times 1 \times 26 \times 62.4/g \text{ lb.}$$

$\therefore$ Pressure $= m.a/A = \frac{1 \times 1 \times 26 \times 62.4 \times 0.1}{1 \times 1} = 162$ #/sq.ft.

The value 162#/sq.ft. is the maximum value of the surge pressure, and it may occur at any point on this circumference, depending on the orientation of the shake of the earthquake.

Comparing the value 162#/sq.ft. with the maximum hydrostatic pressure $p = \gamma H = 62.4 \times 14 = 875$#/sq.ft, we see that it is about 1/5 of it, so one may think of neglecting it; In this work this value of surge pressure has been considered in calculations.

This completes the analysis of the wall of the elevated tank. For the dimensions of the chosen tank, the following table and figures have been prepared for use in design. Tables I VI, together with the graphs plotted from them, show clearly the superposition procedure and how it affects the final design of the wall of the tank.

Table I : Moments, shears, and hoop tensions in the wall of the elevated tank, due to hydrostatic pressure.

Point on the wall	M (ft-lb/ft)		Q (lb/ft)		N_φ (lb/ft)	
	Fixed	Hinged	Fixed	Hinged	Fixed	Hinged
1.0 d	- 9	- 3	- 3	- 4	- 19	- 30
0.9 d	- 17	- 11	- 6	- 7	+ 1140	+ 1115
0.8 d	- 23	- 23	- 2	- 9	+ 2460	+ 2280
0.7 d	- 14	- 34	+ 14	- 4	+ 3650	+ 3520
0.6 d	+ 32	- 27	+ 47	+ 19	+ 4980	+ 4880
0.5 d	+ 145	+ 33	+ 95	+ 70	+ 6200	+ 6300
0.4 d	+ 308	+ 170	+ 127	+ 142	+ 6960	+ 7600
0.3 d	+ 487	+ 427	+ 84	+ 211	+ 6700	+ 8300
0.2 d	+ 450	+ 715	- 160	+ 172	+ 5030	+ 7700
0.1 d	- 185	+ 770	- 745	- 171	+ 2090	+ 5000
0.0 d	- 2080	0	- 1740	- 1050	0	0

In designing the wall of the elevated tank, a careful analysis of wall/floor connection should be made. In the present work the wall base and the floor of the tank are chosen to be supported by a circular girder which rests on eight columns. Analysis of the circular girder is given on page (32) .

Table II

Point on the wall	M ft-lb/ft	Q lb/ft	N_φ lb/ft
1.0 d	+1640	+685	0
0.9d	+608	+523	+2440
0.8d	+202.5	+283	+2270
0.7d	+3.6	+ 2.5	+970
0.6d	-110	+ 0.7	+ 530
0.5d	- 86	-28.5	+88
0.4d	-41	-26.5	-84
0.3d	-27.5	-28.5	-194
0.2d	-1.3	- 5.5	-76.5
0.1d	+1.8	0	-36.8
0d	0	+1.9	0

Table II : Effect of top moment on M, Q, N_φ (elevated tank)

Table VII

Point on the wall	M ft-lb/ft	N_φ lb/ft	Q lb/ft
1.0d	0	+10100	0
0.9d	0	+10100	0
0.8d	0	+10100	0
0.7d	-12.6	+10500	+13.7
0.6d	+17.8	+10550	+23.8
0.5d	+65	+10600	+44.7
0.4d	+138	+10100	+59.1
0.3d	+206	+8750	+28
0.2d	+169	+6010	-103
0.1d	-158	+2330	-395
0d	-1010	0	-850

Table VII : Effect of water surge on M, Q, N_φ. (Ground-supported tank).

Table III

Combined effects of hydrostatic pressure and moment @ top of elevated tank, on M, Q, N_φ.
(Fixed Bottom)

Point on the wall	M (ft-lb)/ft			Q (lb.)/ft			N_φ (lb.)/ft		
	Table I Fixed	Table II	Total	Table I Fixed	Table II	Total	Table I Fixed	Table II	Total
1.0 d	− 9	+1640	+1631	− 3	+685	+682	− 19	0	− 19
0.9 d	− 17	+608	+591	− 6	+523	+517	+1140	+2440	+3580
0.8 d	− 23	+202.5	+179.5	− 2	+283	+281	+2460	+2270	+4730
0.7 d	− 14	+3.6	−10.4	+14	+2.5	+16.5	+3650	+970	+4620
0.6 d	+32	−110	−78	+47	+0.7	+47.7	+4980	+530	+5510
0.5 d	+145	−86	+59	+95	−28.5	+66.5	+6200	+88	+6288
0.4 d	+308	−41	+267	+127	−26.5	+100.5	+6960	−84	+6876
0.3 d	+487	−27.5	+495.5	+84	−28.5	+55.5	+6700	−194	+6506
0.2 d	+450	−1.3	+448.7	−160	−5.5	−165.5	+5030	−76.5	+4953.5
0.1 d	−185	+1.8	−183.2	−745	0	−745	+2090	−36.8	+2053.2
0 d	−2080	0	−2080	−1740	+1.9	−1738	0	0	0

Table IV

Combined effects of hydrostatic pressure and moment @ top of elevated tank, on M, Q, N_φ
(Hinged Bottom)

Point on the wall	M ft-lb/ft			Q lb/ft			N_φ lb/ft		
	Table I Hinged	Table II	Total	Table I Hinged	Table II	Total	Table I Hinged	Table II	Total
1.0 d	−3	+1640	+1637	−4	+685	+681	−30	0	−30
0.9 d	−11	+608	+597	−7	+523	+516	+1115	+2440	+3555
0.8 d	−23	+202.5	+179.5	−9	+283	+274	+2280	+2270	+4550
0.7 d	−34	+3.6	−30.4	−4	+2.5	−1.5	+3520	+970	+4490
0.6 d	−27	−110	−137	+19	+0.7	+19.7	+4880	+530	+5410
0.5 d	+33	−86	−53	+70	−28.5	+41.5	+6300	+88	+6388
0.4 d	+170	−41	+129	+142	−26.5	+115.5	+7600	−84	+7516
0.3 d	+427	−27.5	+399.5	+211	−28.5	+182.5	+8300	−194	+8106
0.2 d	+715	−1.3	+713.7	+172	−5.5	+166.5	+7700	−76.5	+7623.5
0.1 d	+770	+1.8	+768.2	−171	0	−171	+5000	−36.8	+4963.2
0	0	0	0	−1050	+1.9	−1048	0	0	0

Table V

Effect of water surge on the walls of the elevated Tank.

Point on the wall	M ft-lb/ft		Q lb/ft		N_φ lb/ft	
	Fixed	Hinged	Fixed	Hinged	Fixed	Hinged
1.0 d	−490	0	+400	+210	+14.7	+4200
0.9 d	−74.6	+144	+178	+28.7	+495	+2750
0.8 d	+71.2	+131	+47	−35	+1270	+2100
0.7 d	+92.2	+73	−9.2	−40.8	+1880	+1880
0.6 d	+69.8	+27.5	−15.2	−24.3	+2180	+1870
0.5 d	+58.4	+10.6	0	0	+2280	+1880
0.4 d	+69.8	+27.5	+15.2	+24.3	+2180	+1870
0.3 d	+92.2	+73	+9.2	+40.8	+1880	+1880
0.2 d	+71.2	+131	−47	+35	+1270	+2100
0.1 d	−74.6	+144	−178	−28.7	+495	+2750
0	−490	0	−400	−210	+14.7	+4200

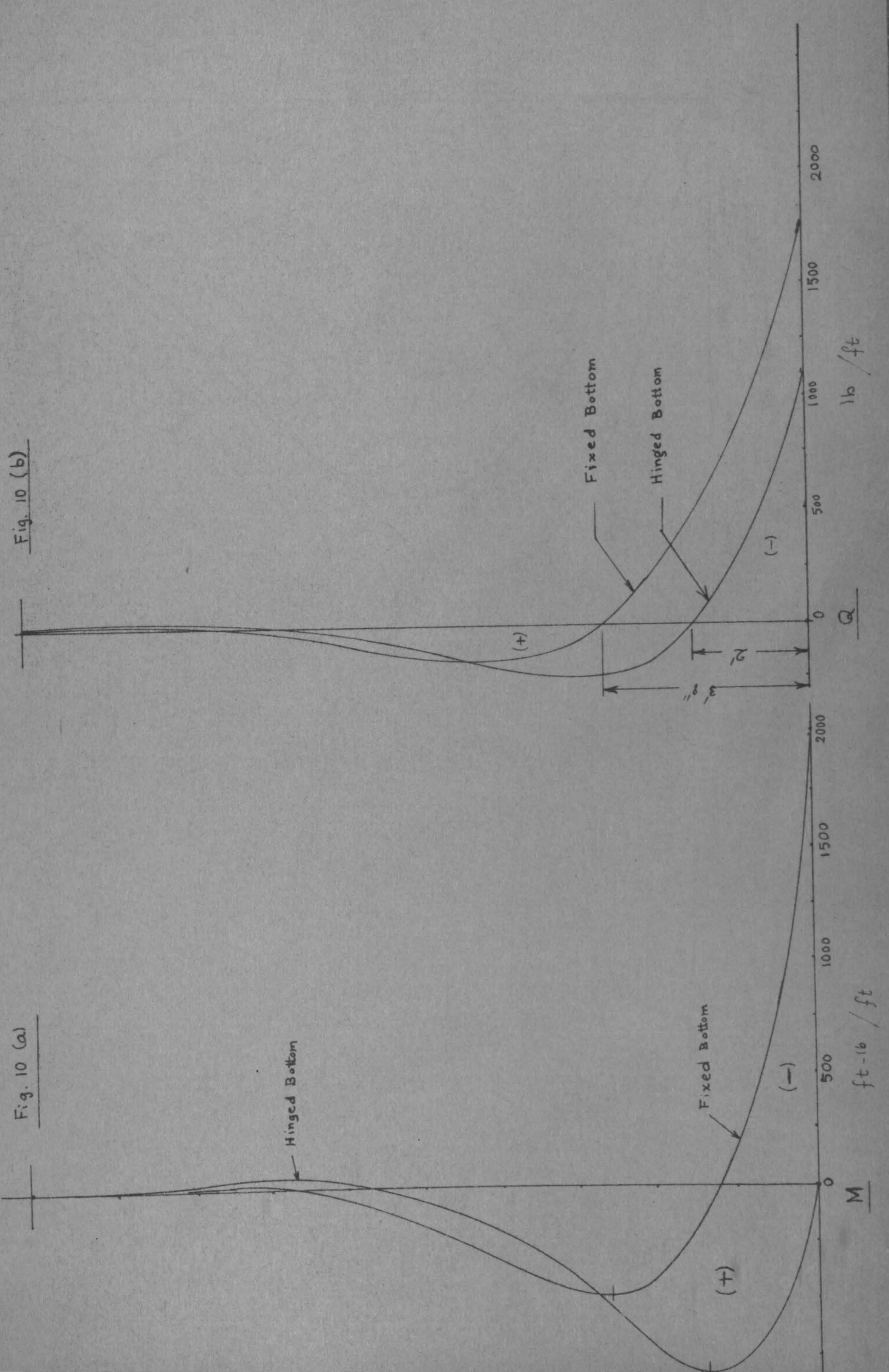
Table VI

Combined effect of hydrostatic pressure, top moment, and water surge, on M, Q, N_φ in wall of the elevated tank.

Point on the wall	M ft-lb/ft		Q lb/ft.		N_φ lb/ft.	
	Fixed	Hinged	Fixed	Hinged	Fixed	Hinged
1.0 d	+1141	+1637	+1082	+891	−4	+4170
0.9 d	+516	+741	+695	+544.7	+4075	+6305
0.8 d	+251	+310	+328	+239	+6000	+6650
0.7 d	+82	+43	+6	−42	+6500	+6370
0.6 d	−8	−110	+33	−5	+7690	+7280
0.5 d	+117	−42	+67	+42	+8568	+8268
0.4 d	+337	+157	+116	+140	+9056	+9386
0.3 d	+552	+473	+65	+223	+8386	+9986
0.2 d	+520	+845	−213	+202	+6224	+9724
0.1 d	−258	+912	−923	−200	+2548	+7713
0	−2570	0	−2138	−1258	+15	+4200

Fig. 10

Variation of moments and shears along the wall
Due to water pressure (Table I)



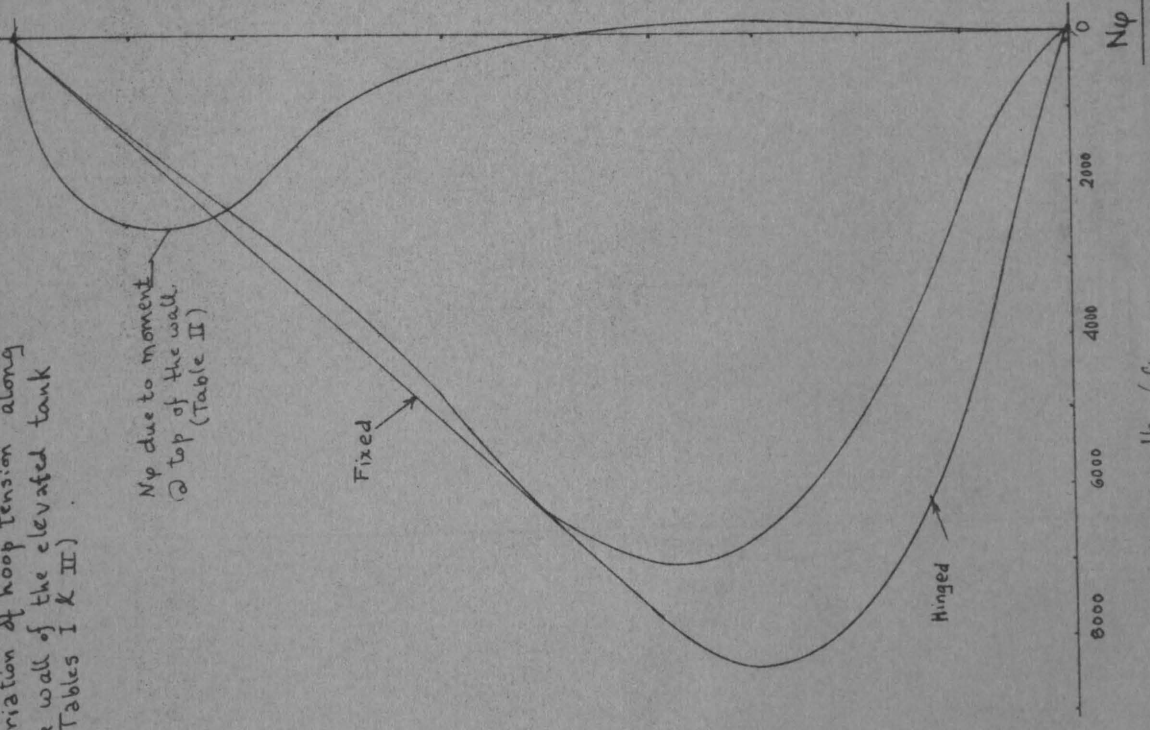
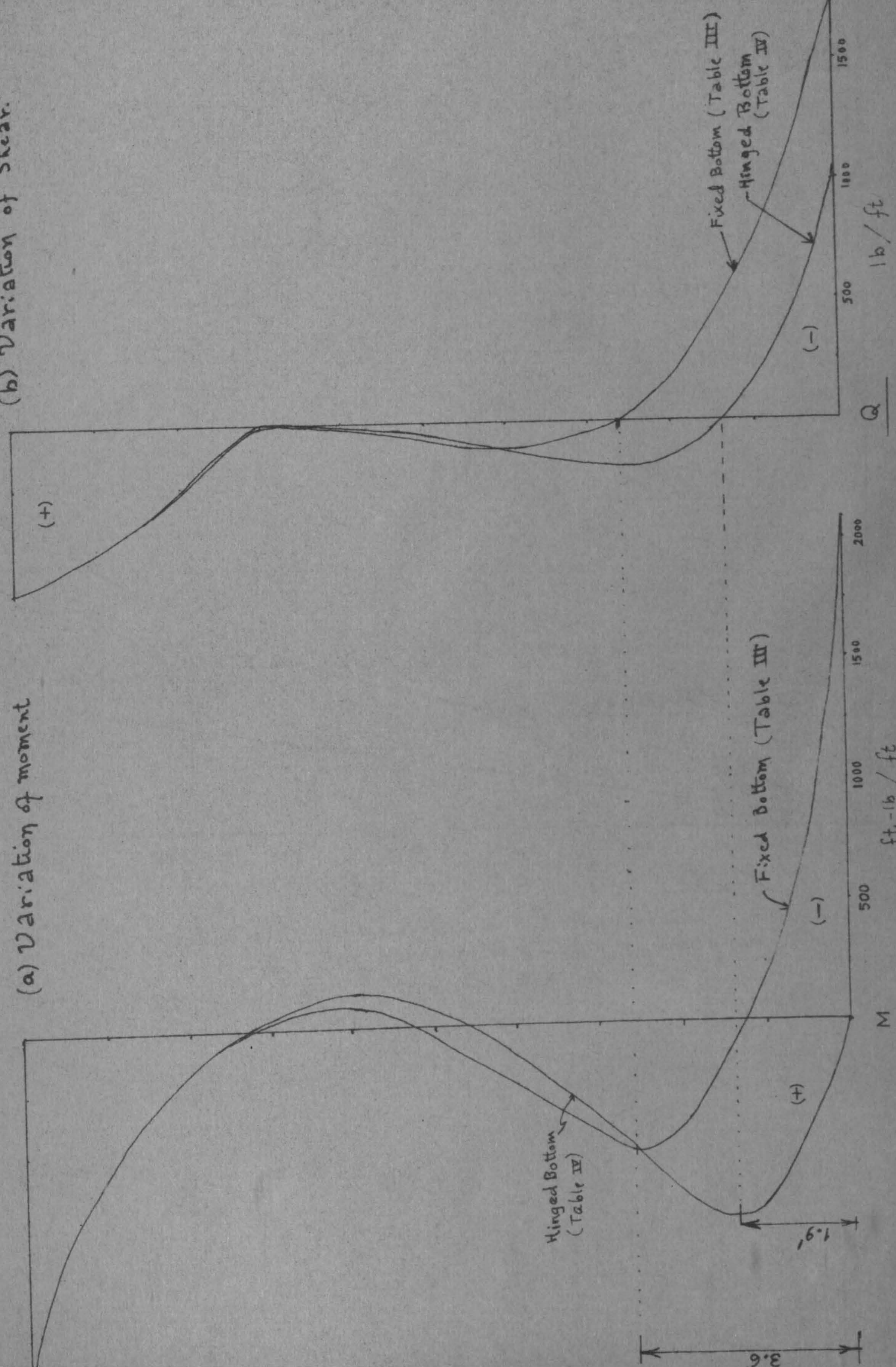


Fig. 10 (C)

Variation of hoop tension along the wall of the elevated tank (Tables I & II)

Fig. 11 - a, b

Combined effects of hydrostatic pressure and the moment @ top of the wall. (Tables III & IV).



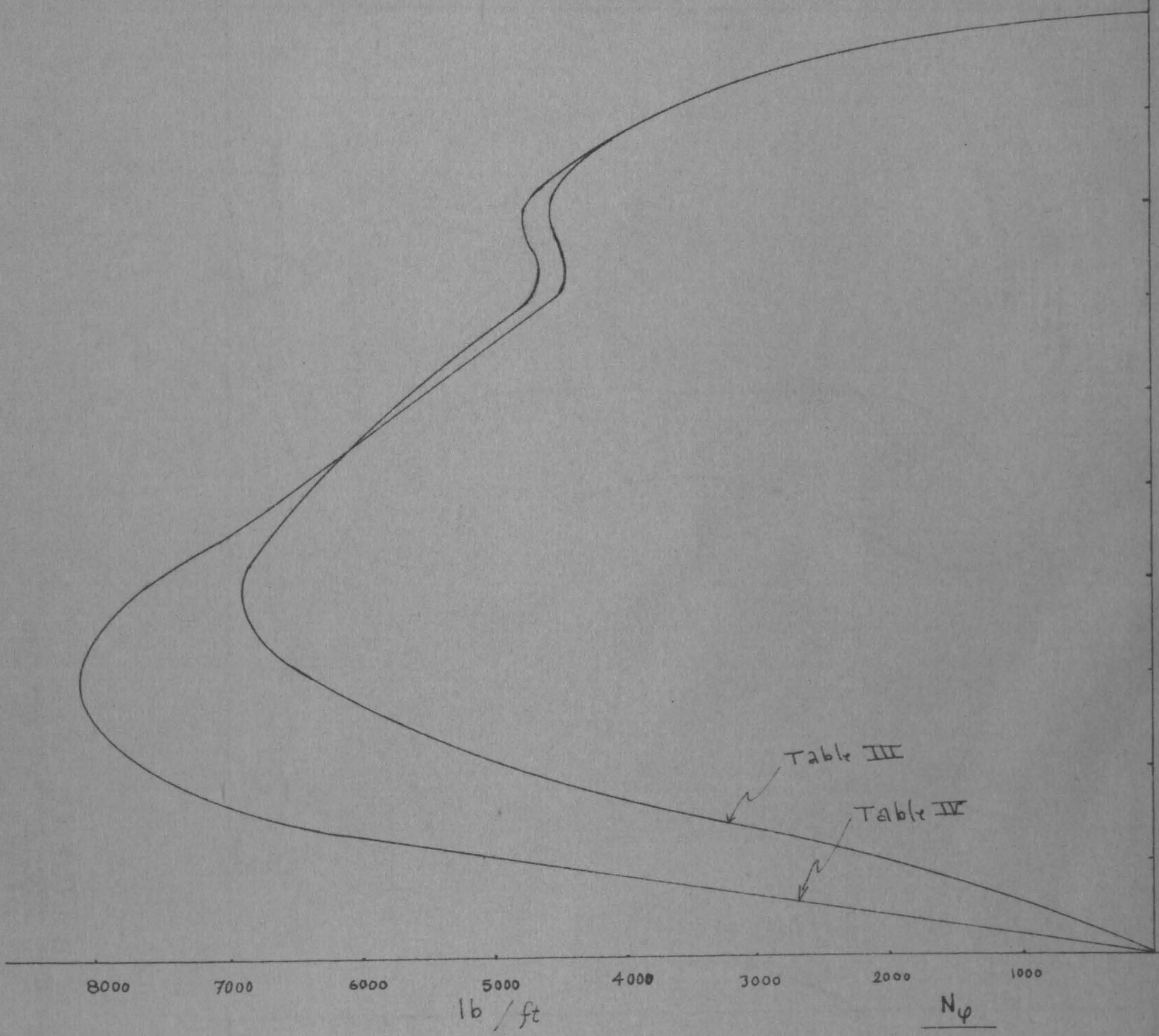
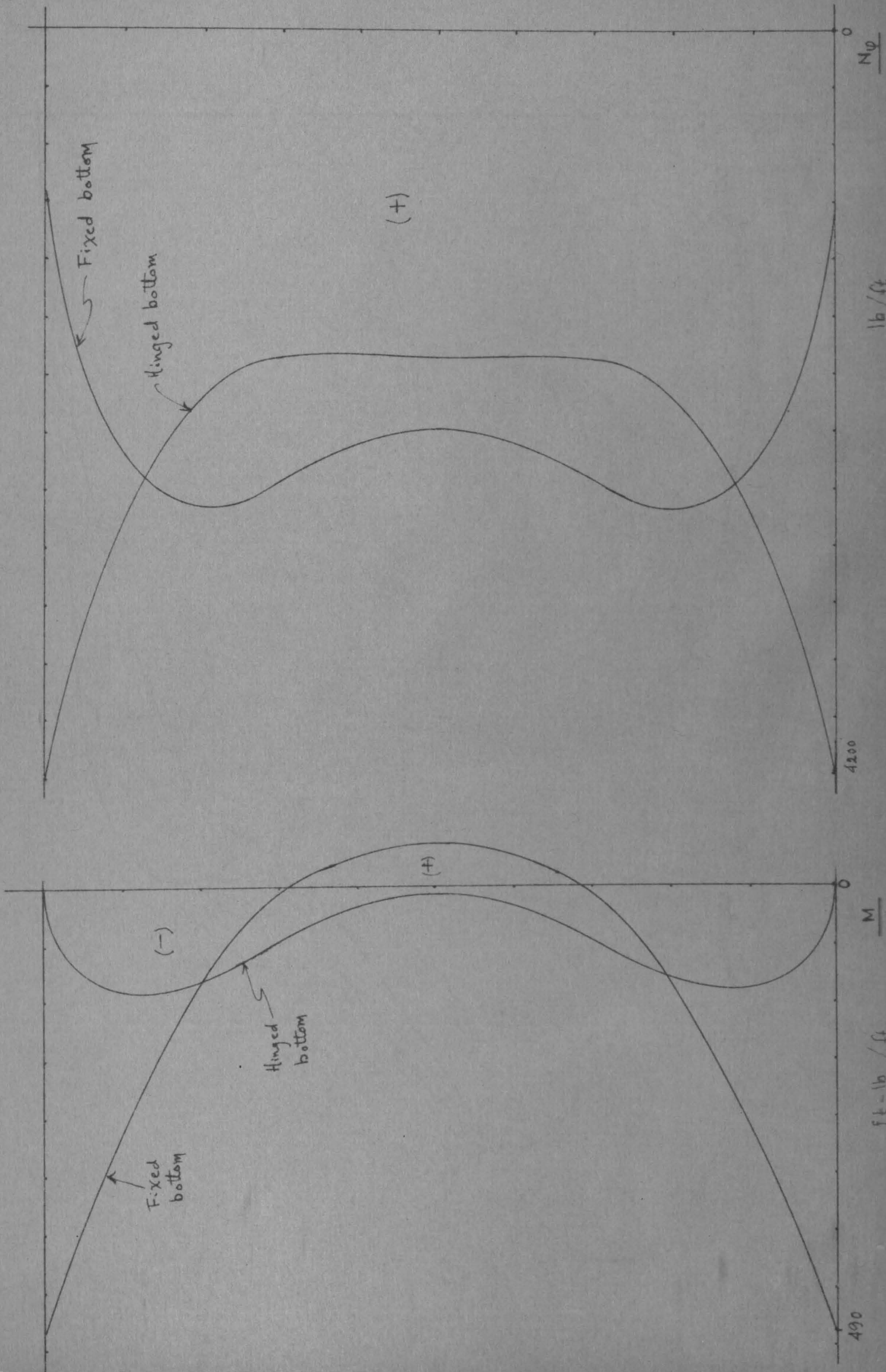


Fig. 11 (c) : combined effects of hydrostatic pressure and moment @ top of wall, on hoop tension. (Tables III, IV).

Fig. 12
Effect of water surge on the wall (Table V)



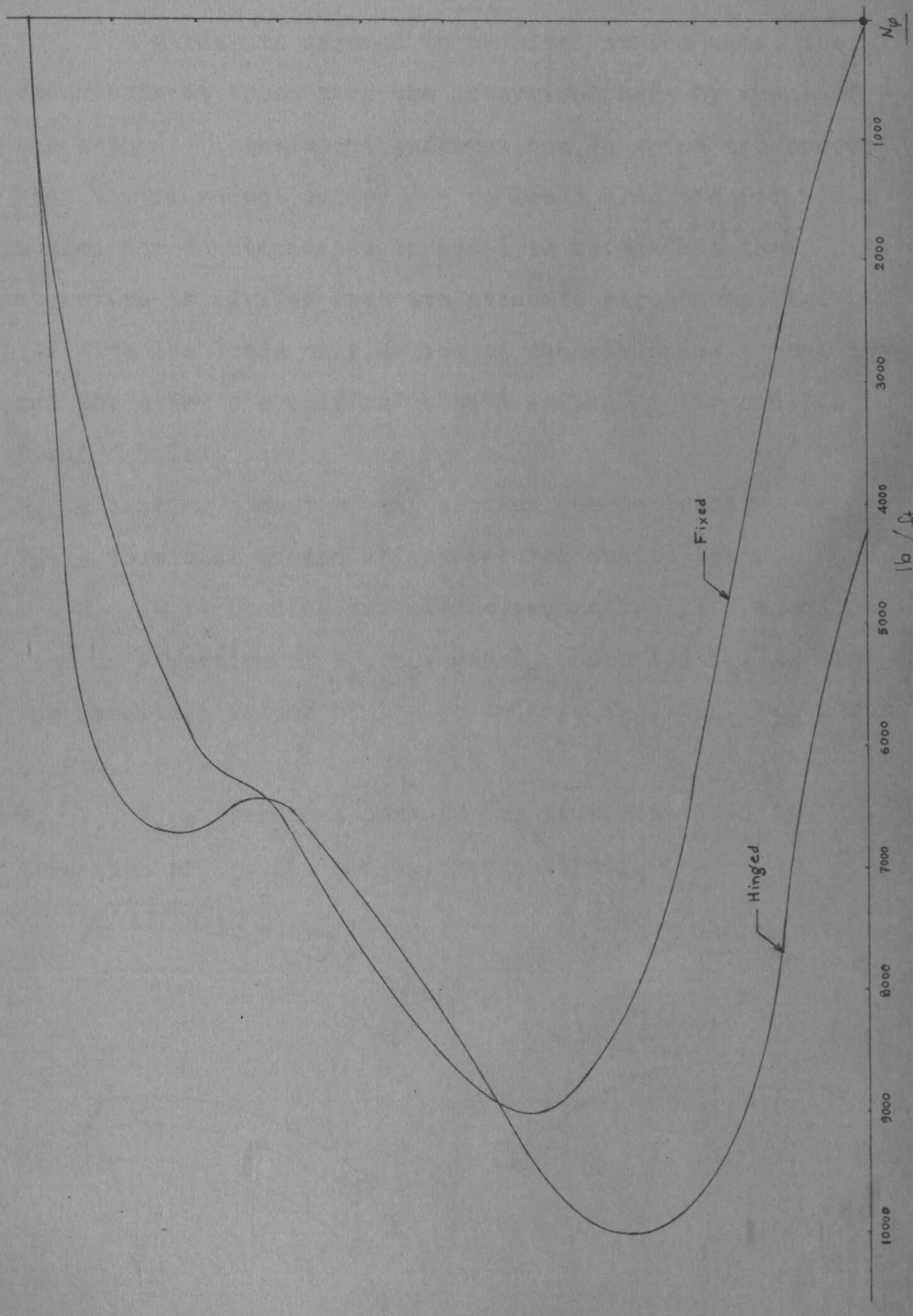


Fig. 13
Combined effects of hydrostatic pressure, water surge, and moment @ top of the wall of the elevated tank
(Tables VII)

The Circular Girder

(i) Analysis:

The girder is assumed to be fixed at the ends; the redundants at these ends are determined here by means of the method of consistent deformations, in which the concept that "the redundant motion due to loads plus the redundant motion due to redundants is equal to zero" is applied. Thus the structure is divided into two separate structures, Fig. (14), (a) with the loads, only, acting on the girder (as a cantilever) and the other (b) has only redundants acting at its end. In Fig. (15) let:

M_o = bending moment at any section due to loads,

T_o = torsional moment at any section due to loads,

M_a, M_b, M_c = bending moments due, respectively, to a unit load in direction of X_a, X_b, and X_c. Here X_a, X_b, X_c are the numerical values of the redundants M_{ba}, T_{ba}, F_{ba} shown in Fig. (15)

T_a, T_b, T_c = torsional moments due to a unit load in direction of X_a, X_b, and X_c, respectively.

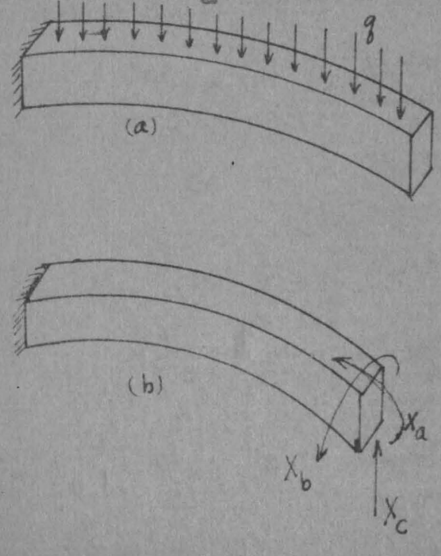
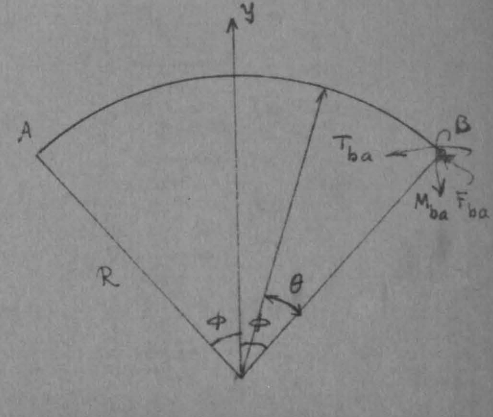


Fig. 14

Fig. 15



Let, also:

δ_{aa} = displacement in direction of X_a due to $X_a = 1$ unit,

δ_{ab} = displacement in direction of X_a due to $X_b = 1$ unit,

δ_{ba}, δ_{bc}, etc. have similar definitions.

δ_{ao}, δ_{bo}, δ_{co} = displacements in directions of X_a, X_b, and X_c, respectively, due to loads.

Then, neglecting the effect of temperature, we have:

Total displacement in direction of X_a due to redundants =

$$X_a \cdot \delta_{aa} + X_b \cdot \delta_{ab} + X_c \cdot \delta_{ac}$$

Total displacement in direction of X_b due to redundants =

$$X_a \cdot \delta_{ba} + X_b \cdot \delta_{bb} + X_c \cdot \delta_{bc}$$

Total displacement in direction of X_c due to redundants =

$$X_a \cdot \delta_{ca} + X_b \cdot \delta_{cb} + X_c \cdot \delta_{cc}$$

Since the summation of redundant motions due to loads and to redundants is equal to zero, we have:

$$X_a \cdot \delta_{aa} + X_b \cdot \delta_{ab} + X_c \cdot \delta_{ac} + \delta_{ao} = 0$$

$$X_a \cdot \delta_{ba} + X_b \cdot \delta_{bb} + X_c \cdot \delta_{bc} + \delta_{bo} = 0$$

$$X_a \cdot \delta_{ca} + X_b \cdot \delta_{cb} + X_c \cdot \delta_{cc} + \delta_{co} = 0$$

Since the main effect on motion will be due the bending and torsional moments, the values of the displacements will be:

$$\delta_{ao} = \int \frac{M_a \cdot M_o}{EI} ds + \int \frac{T_a \cdot T_o}{GJ} ds$$

$$\delta_{bo} = \int \frac{M_b \cdot M_o}{EI} ds + \int \frac{T_b \cdot T_o}{GJ} ds$$

$$\delta_{co} = \int \frac{M_c \cdot M_o}{EI} ds + \int \frac{T_c \cdot T_o}{GJ} ds$$

$$\delta_{aa} = \int \frac{M_a^2}{EI} ds + \int \frac{T_a^2}{GJ} ds$$

$$\delta_{ab} = \int \frac{M_a \cdot M_b}{EI} ds + \int \frac{T_a \cdot T_b}{GJ} ds$$

$$\delta_{ac} = \int \frac{M_a . M_c}{EI} ds + \int \frac{T_a . T_c}{GJ} ds$$

$$\delta_{bb} = \int \frac{(M_b)^2}{EI} ds + \int \frac{(T_b)^2}{GJ} \qquad \text{Eq.s(A)}$$

$$\delta_{bc} = \int \frac{M_b . M_c}{EI} ds + \int \frac{T_b . T_c}{GJ} ds$$

$$\delta_{cc} = \int \frac{M_c^2}{EI} ds + \int \frac{T_c^2}{GJ} ds$$

If $X_a = 1$, then

$M_a = \cos(\phi - \theta)$, $T_a = -\sin(\phi - \theta)$.

If $X_b = 1$, then

$M_b = \sin(\phi - \theta)$, $T_b = \cos(\phi - \theta)$. Eq.s(B)

If $X_c = 1$, then

$M_c = R\sin\theta$, $T_c = R(1-\cos\theta)$.

From equations A, B the following results are obtained(*) after integrating from (0) to (2ϕ):

Eq.s(C)

$$\delta_{aa} = \frac{R}{2EI}(2\phi + \sin 2\phi) + \frac{R}{2GJ}(2\phi - \sin 2\phi).$$

$$\delta_{bb} = \frac{R}{2EI}(2\phi - \sin 2\phi) + \frac{R}{2GJ}(2\phi + \sin 2\phi).$$

$$\delta_{cc} = \frac{R^3}{EI}\left(\phi - \frac{\sin 4\phi}{4}\right) + \frac{R^3}{GJ}\left(3\phi - 2\sin 2\phi + \frac{\sin 4\phi}{4}\right).$$

$$\delta_{ab} = 0$$

$$\delta_{ac} = \frac{R^2}{2EI}.\sin\phi(2\phi + \sin 2\phi) + \frac{R^2}{2GJ}.\sin\phi(2\phi - \sin 2\phi).$$

$$\delta_{bc} = \frac{R^2}{2EI}.\cos\phi.(\sin 2\phi - 2\phi) + \frac{R^2}{2GJ}.(3\sin\phi - 2\phi\cos\phi - \sin\phi\cos 2\phi$$

$$\delta_{ao} = \frac{-R^3 q}{EI}(\sin\phi - \sin^3\phi - \cos\phi) - \frac{R^3}{GJ}(3\sin\phi - \sin^3\phi - 3\cos\phi).$$

$$\delta_{bo} = \frac{R^3 q}{2}\left(\frac{1}{EI} - \frac{1}{GJ}\right)(2\phi - \sin 2\phi)\sin\phi$$

$$\delta_{co} = \frac{R^4 q}{2EI}(\sin^2 2\phi - 2\cos 2\phi - 2\phi) - \frac{R^4 q}{2GJ}(\sin^2 2\phi - 2^2 - 4\sin 2\phi).$$

According to Maxwell's law of reciprocal relationship:

$$\delta_{ba} = \delta_{ab}$$

$$\delta_{ca} = \delta_{ac}$$

$$\delta_{cb} = \delta_{bc}$$

(*) See Appendix.

(ii) Design

Weight of roof and floor slabs $= 2(13)^2\pi(\frac{1}{2})(150) = 79500$ lb.

Weight of wall $= 2\pi(13)(0.75)(14)(150)$

$= 129,000$ lb.

Weight of girder itself $= 2\pi(13)(1.5)(1.5)(150)$

$= 27,500$ lb.

Total $= 236,000$ lb.

$q = \frac{236,000}{2\ (13)} = 2900$ lb. per ft. of periphery.

Let section of the girder be 18" x 18"

$$I = (1/12)\ bh^3 = (1/12)(18)^4 = 8,748\ in^4.$$

$$J = \frac{bh}{12}(b^2 + h^2) = \frac{1}{12} \cdot (18 \times 18)(324 + 324)$$

$$= 17496\ in^4.$$

$v = 0.2$

$$G = \frac{E}{2(1 + v)} = 0.417E$$

$$GJ = 0.834EI$$

$= 22^\circ\ 30' = 0.3927$ radians.

Substituting values of function of ϕ

into equations(C) on page (34) we get:

$$\delta_{aa} = + \frac{10.31}{EI} \qquad \delta_{bb} = + \frac{12.14}{EI}$$

$$\delta_{cc} = + \frac{350}{EI} \qquad \delta_{ab} = 0$$

$$\delta_{ac} = + \frac{51.22}{EI} \qquad \delta_{bc} = + \frac{9.82}{EI}$$

$$\delta_{ao} = - \frac{176.26}{EI} q \qquad \delta_{bo} = - \frac{6.58}{EI} q$$

$$\delta_{co} = - \frac{1330.5}{EI} q .$$

Hence:

$$X_a \frac{(10.31)}{EI} + 0 + X_c \frac{51.22}{EI} = \frac{176.26}{EI} q$$

$$0 + X_b \frac{12.14}{EI} + X_c \frac{9.82}{EI} = \frac{6.58}{EI} q$$

$$X_a \frac{51.22}{EI} + X_b \frac{9.82}{EI} + X_c \frac{350}{EI} = \frac{1330.5}{EI} q$$

$$X_a = - 22.35q$$

$$X_b = - 3.52q$$

$$X_c = + 5.13q$$

X_c could have been determined by the eq:

$$X_c = qR \phi = q(13)(0.3927) = + 5.1q$$

X_b can also be determined from statics by taking moments about the line AB. Thus only the first equation obtained above is necessary to solve for X_a.

Note that $X_a (= M_{ba})$ is not perpendicular to the cross-section at A and B; X_b $(= T_{ba})$ is also not acting in plane of the cross setins at A and B. X_a has been chosen to be parallel to line AB, X_b normal to it.

∴ The moment at A and B acting normal to the section is:

$$X_a \cdot \cos(22^\circ 30') = - 22.35q\ (0.92388) = - 20.65q$$

The torsion at A or B acting in plane of the section is:

$X_b \cdot \sin(22^{\circ}30') = -3.52q(0.38268) = -1.35q$

$\therefore M_a = -20.65 \times 2900 = -60,000$ ft-lb.

$T_a = -1.35 \times 2900 = -3,900$ ft-lb.

This torsion is relieved by that which is produced by the weight of the floor slab resting on the circular girder. The torsion effect of the D.L. of the floor is determined mathematically in what follows:

This approximate method is based on consideration of a simply supported slab which is formed by joining the points ABCD shown in Fig.(16) The sides AD, BC are considered simply supported, the curves AB, DC are approximated by the lines AB and DC.

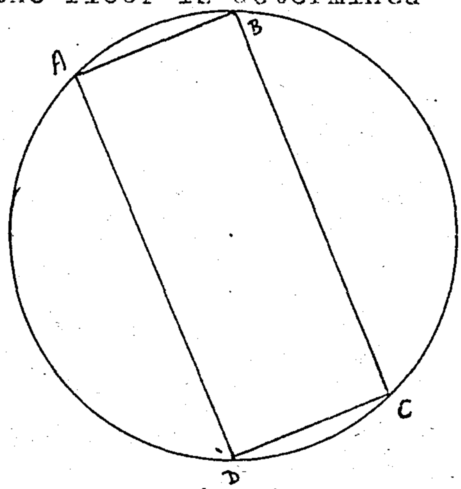


Fig. (16)

The floor slab resting on the cirlcular girder and columns.

(a) Taking ABCD with coordinates and boundary conditions as shown, (Fig. 17) we can determine the slope at the edge AB as follows:

Let moment applied at edge AB be

$M_y = f(E_m)$

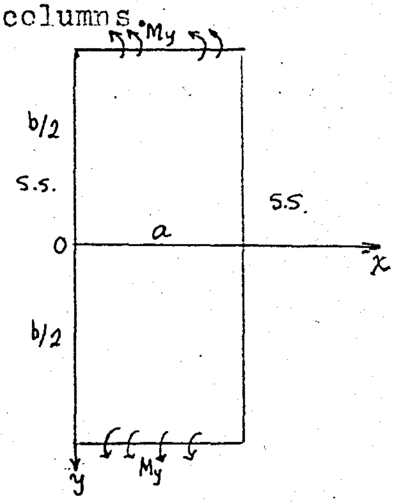


Fig. (17)
Simply supported slab, with moment M_y.

which causes a deflection surface w*:

$$W_m = \frac{a^2}{2\pi^2 D} \sum_{m=1}^{\infty} \frac{\sin\frac{m\pi x}{a}}{m^2 \cosh\alpha_m} E_m\left(\alpha_m \tanh\alpha_m \cosh\frac{m\pi y}{a} - \frac{m\pi y}{a}\sinh\frac{m\pi y}{a}\right) \qquad \ldots\ldots (9)$$

Here a $\equiv$ AB, b $\equiv$ BC

where E_m is to be determined later.

$$\alpha_m = \frac{m\pi b}{2a}$$

$$\frac{\partial w_m}{\partial y} = \frac{a^2}{2\pi^2 D} \sum_{m=1}^{\infty} \frac{\sin\frac{m\pi x}{a}}{m^2 \cosh\alpha_m} E_m \left\{ \frac{m\pi}{a}.\alpha_m \tanh\alpha_m \sinh\frac{m\pi y}{a} - \frac{m\pi y}{a}.\frac{m\pi}{a}\cosh\frac{my\pi}{a} - \frac{m\pi}{a}.\sinh\frac{m\pi y}{a} \right\}$$

The slope at y = b/2 will be:

$$\left(\frac{\partial w_m}{\partial y}\right)_{y=b/2} = \frac{a^2}{2\pi^2 D} \sum_{m=1}^{\infty} \frac{\sin\frac{m\pi x}{a}}{m^2 \cosh\alpha_m} E_m \cdot \frac{m\pi}{a}\left\{\alpha_m \tanh\alpha_m \sinh\frac{m\pi b}{2a} - \frac{m\pi b}{2a}\cosh\frac{m\pi b}{2a} - \sinh\frac{m\pi b}{2a}\right\}$$

$$= \frac{a}{2\pi D} \sum_{m=1}^{\infty} \frac{\sin\frac{m\pi x}{a}}{m \cosh\alpha_m} E_m \left\{\alpha_m \tanh\alpha_m \sinh\alpha_m - (\cosh\alpha_m)\alpha_m - \sinh\alpha_m \right\}$$

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* op. cit. p. 182.

$$\left(\frac{\partial w_m}{\partial y}\right)_{y=b/2} = \frac{a}{2\pi D}\sum_{m=1}^{\infty} \sin\frac{m\pi x}{a}\cdot\frac{1}{m}\cdot E_m\left\{\alpha_m\cdot\tanh^2\alpha_m - \alpha_m - \tanh\alpha_m\right\}$$

(b) Taking a uniformly loaded rectangular plate simply supported at the four edges, and supporting a load q'(= dead weight of the floor slab)(*), the deflection surface is :

$$w_s = \frac{4q'a^4}{\pi^5 D}\sum_{m=1,3,5}^{\infty}\frac{1}{m^5}\left(1-\frac{\alpha_m\tanh\alpha_m+2}{2\cosh\alpha_m}\cdot\cosh 2\alpha_m\cdot y/b + \frac{\alpha_m}{2\cosh\alpha_m}\cdot\frac{2y}{b}\cdot\sinh\frac{2\alpha_m y}{b}\right)\sin\frac{m\pi x}{a}$$

$$\frac{\partial w_s}{\partial y} = \frac{4q'a^4}{\pi^5 D}\sum_{m=1,3,5}^{\infty}\frac{1}{m^5}\cdot\sin\frac{m\pi x}{a}\cdot\left\{-\frac{2\alpha_m}{b}\cdot\frac{\alpha_m\tanh\alpha_m+2}{2\cosh\alpha_m}\cdot\sinh\frac{2\alpha_m y}{b} + \frac{2\alpha_m\cdot\alpha_m}{b\;2\cosh\alpha_m}\cdot\frac{2y}{b}\cdot\cosh\frac{2\alpha_m y}{b} + \left(\frac{2}{b}\right)\frac{\alpha_m}{2\cosh\alpha_m}\cdot\sinh\frac{2\alpha_m y}{b}\right.$$

$$\left(\frac{\partial w_s}{\partial y}\right)_{y=b/2} = -\frac{2q'a^3}{\pi^5 D}\sum_{m=1,3,5}^{\infty}\frac{1}{m^4}\sin\frac{m\pi x}{a}\left[\alpha_m\tanh^2\alpha_m - \alpha_m - \tanh\alpha_m\right]$$

(c) Let ϕ= the final slope at the edges of the plate and the circular girder. From St. Venant's analysis the angle of twist per unit length is given(*) as:

$$= (1/\beta b h^3)(T/G)$$

where b,h are the width and height of the beam, respectively; β is a coefficient depending on the ratio (b/h); T is the torsional moment, and G is the modulus of rigidity of the beam.

is due to the accumulation of torsional effect from the end of the beam in the direction of the centre; the torsion in the middle is zero. Torsion at any point A, (Fig. 18) is given by the equation:

(*) See the reference : "Advanced Mechanics Of Materials by Fred B.Seely & James O.Smith; second edition, 1952, New York John Wiley & Sons, Inc. - p. 271

$$T_A = \int_0^A M_y . dx$$

where M_y is to be taken as that corresponding to (w_m) (eq. 9)

$$M_y = - D . \left(\frac{\partial^2 w_m}{\partial y^2} + v . \frac{\partial^2 w_m}{\partial x^2} \right)$$

It can be shown that:

$$\left[\frac{\partial w_m}{\partial y^2} \right]_{y=b/2} = - \frac{1}{D} . \sum_{m=1}^{\infty} E_m . \sin \frac{m\pi x}{a}$$

$$\left[\frac{\partial \omega_m}{\partial x^2} \right]_{y=b/2} = 0$$

$$\therefore \quad M_y = - D . \frac{\partial^2 \omega_m}{\partial y^2} = \sum_{m=1}^{\infty} E_m . \sin \frac{m\pi x}{a}$$

Hence:

$$T_x = \int M_y . dx = \int \left(\sum_{m=1}^{\infty} E_m . \sin \frac{m\pi x}{a} \right) dx$$

$$= - \left[\sum_{m=1}^{\infty} E_m . \frac{a}{m\pi} . \cos \frac{m\pi x}{a} \right]_{\delta}^{a/2}$$

$$= + \sum_{m=1}^{\infty} E_m . \frac{a}{m\pi} . \left(\cos \frac{m\pi\delta}{a} - \cos \frac{m\pi}{2} \right)$$

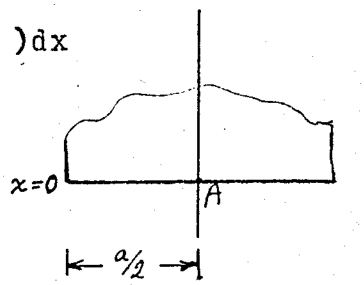


Fig. 18

where (δ) is the abscissa of any point between $x = 0$ and $x = a/2$.

Knowing the torsion, the twisting angle can be determined:

$$\phi = \int_0^{\delta} T_x . dx . \ 1/\beta b h^3 G$$

$$\therefore \quad = \int_0^{\delta} \sum_{m=1}^{\infty} E_m . \frac{a}{m\pi} . \left(\cos \frac{m\pi x}{a} - \cos \frac{m\pi}{2} \right) dx . 1/\beta b h^3 G$$

$$= \left[\sum_{m=1}^{\infty} E_m \frac{a}{m\pi} \left(\frac{a}{\pi m} \sin \frac{m\pi x}{a} - x \cos \frac{m\pi}{2} \right) \right]_0^{\delta} 1/\beta b h^3 G$$

$$\phi = \sum_{m=1}^{\infty} E_m \frac{a}{m\pi} \left[\frac{a}{m\pi} \sin \frac{m\pi\delta}{a} - \delta \cos \frac{m\pi}{2} \right] . \ 1/\beta b h^3 G$$

Taking only odd values of (m), we have

$$\phi = \sum_{m=1,3,5}^{\infty} E_m \frac{a^2}{m^2 \pi^2} \left[\sin \frac{m\pi}{a} x \right] . \ 1/\beta b h^3 G \quad \cdots (10)$$

Equating $\phi = \left(\frac{\partial \omega_s}{\partial y} \right)_{y=b/2} + \left(\frac{\partial \omega_m}{\partial y} \right)_{y=b/2}$ and taking odd values of(m):

$$\therefore \quad E_m = \frac{2 q' a^2}{\pi^4 D} \frac{\sum \frac{1}{m^3} \left(\alpha_m \tanh^2 \alpha_m - \alpha_m + \tanh \alpha_m \right)}{\sum \left\{ \left(\frac{a}{2D} \right) \left(\alpha_m \tanh^2 \alpha_m - \alpha_m - \tanh \alpha_m \right) - a/m\pi \beta b h^3 G \right\}}$$

The above is summed up in the fact that the final shape of the edge of the plate on top of the circular girder is given by the angle of twist (ϕ) in eq. (10) .

For numerical evaluation of the final approximate torsion the following steps are taken:

(i) At the edge of the circular plate the moment is

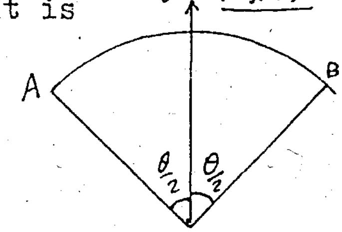


Fig. 19

$$M_{r\ r=a} = -q'.a^2/8$$

∴ Torsion at point A shown (Fig. 19) is:

$$T_1 = \int_0^{\theta} M dx = \int_0^{\pi\theta/180} (-q'a^2/8).a.d\theta$$

$$= -(q'.a^3/8).(\theta\pi/180) \quad \ldots\ldots\ldots\ldots\ldots (11)$$

(ii) The moment at the built in edge of a rectangular plate with two opposite edges simply supported and the others fixed, is(*):

$$\left(M_y\right)_{y=b/2} = \frac{4q'a^2}{\pi^3} \sum_{m=1,3,5,\ldots}^{\infty} \frac{\sin\frac{m\pi x}{a}}{m^3} \cdot \frac{\alpha_m - \tanh\alpha_m(1+\alpha_m\tanh\alpha_m)}{\alpha_m - \tanh\alpha_m(\alpha_m\tanh\alpha_m - 1)}$$

The torsion at point A (Fig. 20) will be:

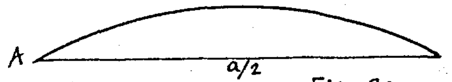


Fig. 20

$$T_2 = \int_0^{a/2} M_y dx =$$

$$= \frac{4q'a^2}{\pi^3}\int_0^{a/2} \sum_{m=1,3,5,\ldots}^{\infty} \frac{\sin\frac{m\pi x}{a}}{m^3} \frac{\alpha_m - \tanh\alpha_m(1+\alpha_m\tanh\alpha_m)}{\alpha_m - \tanh\alpha_m(\alpha_m\tanh\alpha_m - 1)} dx$$

$$= + \frac{4q'a^3}{\pi^4} \sum_{m=1,3,5,\ldots}^{\infty} \frac{1}{m^4} \frac{\alpha_m - \tanh\alpha_m(1+\alpha_m\tanh\alpha_m)}{\alpha_m - \tanh\alpha_m(\alpha_m\tanh\alpha_m - 1)} \quad \ldots (12)$$

(iii) The moment at the end of a rectangular flexible plate is:

$$M_y = \sum_{m=1,3,5}^{\infty} E_m \cdot \sin\frac{m\pi x}{a}$$ (only odd values are taken as we will finally have to equate terms with odd values of m).

$$M_y = \sum_{m=1,3,5,\ldots}^{\infty} \sin\frac{m\pi x}{a} \frac{2q'a^2}{m^3\pi^4 D} \left\{ \frac{\alpha_m\tanh^2\alpha_m - \alpha_m + \tanh\alpha_m}{\frac{a}{2D}(\alpha_m\tanh^2\alpha_m - \alpha_m - \tanh\alpha_m) - \frac{a}{m\pi\beta b h^3 G}} \right\}$$

∴ Torsion at point A is (Fig. 21):

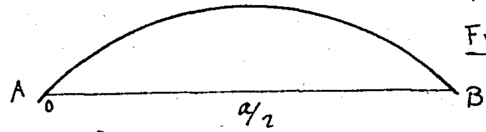


Fig. 21

$$T_3 = \int_0^{a/2} M_y dx$$

$$= \frac{2q'a}{\pi^4 D}\int_0^{a/2} \sum_{m=1,3,5,\ldots}^{\infty} \left(\sin\frac{m\pi x}{a}\right) \frac{1}{m^3} \left\{ \frac{\alpha_m\tanh^2\alpha_m - \alpha_m + \tanh\alpha_m}{\frac{1}{2D}(\alpha_m\tanh^2\alpha_m - \alpha_m - \tanh\alpha_m) - \frac{1}{m\pi\beta b h^3 G}} \right\}$$

$$= \frac{2q'a^2}{\pi^5 D} \sum_{m=1,3,5,\ldots}^{\infty} \frac{1}{m^4} \left\{ \frac{\alpha_m\tanh^2\alpha_m - \alpha_m + \tanh\alpha_m}{\frac{1}{2D}(\alpha_m\tanh^2\alpha_m - \alpha_m - \tanh\alpha_m) - \frac{1}{m\pi\beta b h^3 G}} \right\} \quad \ldots (13)$$

(*) op.cit. p. 186

(iv) The final inward torsion (T) will be:

$$T = T_1 \cdot (T_3/T_2)$$

Using the expressions for T_1, T_2, and T_3 the final form of (T) is:

$$T = -\frac{q' \cdot a^2}{16D} \cdot \left(\frac{\theta}{180}\right) \sum_{m=1,3,5..}^{\infty} \frac{\alpha_m - \alpha_m \tanh^2 \alpha_m + \tanh \alpha_m}{\frac{1}{m\pi\beta bh^3 G} - \frac{1}{2D}\left(\alpha_m \tanh^2 \alpha_m - \alpha_m - \tanh \alpha_m\right)} \quad \text{----} (14)$$

This torsion Torsion T is a relieving one as it acts inwards, therefore it has to be subtracted from the torsion obtained on page (37).

The variation of the relieving torsion on the girder is seen to be almost linear; this fact can be checked by noting that the expression for (T) eq. 14 is almost insensitive to the variation of (m); its denominator in particular exhibits this fact.

By taking the first three terms of the Fourier series for (T), it can be shown that the torsion can be expressed as:

$$T = -(0.1248) \cdot q' \cdot a^2 \cdot (\theta/180)$$

where (q') is the uniform load consisting of the weight of the floor slab plus the weight of the water; thus q' = 950#/sq.ft. From the dimensions of the floor slab and the positions of the column tops, the length (a), which is the line AB, can be found to be equal to (9.949 ft.)

$$\therefore T = -11735\ (\theta/180)$$

At $\theta = \pi/8$ we have

$$T_{max} = 1466 \text{ ft.lb.}$$

This value came out to be rather small due to the fact that the shell wall and the girder below the plate act both as deep beams where shearing effects are directed to the tops of the columns; besides, some torsion is taken off by this effect. Knowing the value of (T) the girder can now be designed.

Steel for bending moment:

Fig. (22) gives the bending moment along the girder

The value 34.7 q at the center can be obtained from statics.

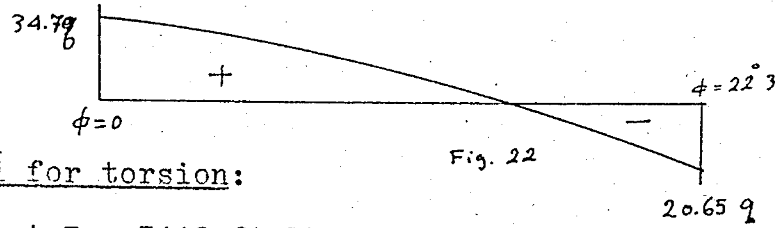


Fig. 22

At the center:

1- Longitudinal steel for torsion:

Torsional moment = - 3442 ft.lb.
(at the fixed end)

Relieving Torsion= 1466 -do-

Final torsion at the fixed end = 1976 ft.lb.

According to the German code (*) longitudinal steel for torsion is: (Fig. 23)

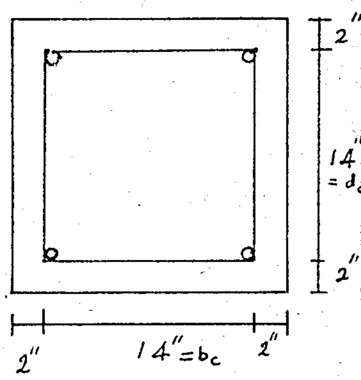


Fig. 23

$A_s = (A_T \, . \, S) / (2 \, b_c \, d_c \, f_s)$

$S = (14 + 14) / 2 = 14"$

$A_s = \dfrac{(1976 \times 12) \times 14}{2 \times 14 \times 14 \times 20,00} = 0.0423$ sq.in (one bar)

$A_s = 4 \times 0.0423 = 0.1692$ sq.in.

2- Steel for bending moment:

$A_s = M/f_s jd = \dfrac{100,000 \times 12}{20,000 \times 0.875 \times 16} = 4.29$ sq,in.

$4.29 + 0.1692 = 4.4592$ sq.in.

Use 4 - 1.5" Ø bars giving $A_s = 4.9$ sq.in.

Two of these bars are discontinued at a distance of 3'-5" on both sides of the center.

At the fixed edge:

$A_s = M/f_s jd = \dfrac{60,000 \times 12}{20,000 \times 0.875 \times 16} = 2.54$ sq.in.

Use 4- (15/16") Ø giving $A_s = 2.76$ sq.in.

(*) Journal of the American Concrete Institute, January 1964 - an article by Paul Zia, p. 1

Two of these are discontinued at 3" from each end.

Steel for torsional moment:

According to the German code (*), shearing stress is:

$$T = M_T/A.D = \frac{4.07 \times (1976\times12)}{(18\times18)\times 18} = 16.5 \angle 264 \text{ psi.} \quad \text{(OK)}$$

$$A_V = M_T.t/2b_c d_c f_V = \frac{(1976\times12)\ t}{2\times14\times14\times20{,}000} = 0.00302\ t$$

For loops of #4 bars : t = 0.196/0.00302 = 65"

But use a maximum spacing of 18" = 1.5' (The section actually needs to be changed; but it is left here as it is to show the general procedure) 10.2/1.5 = 6.8 .. use 7 loops. See Fig. 24.

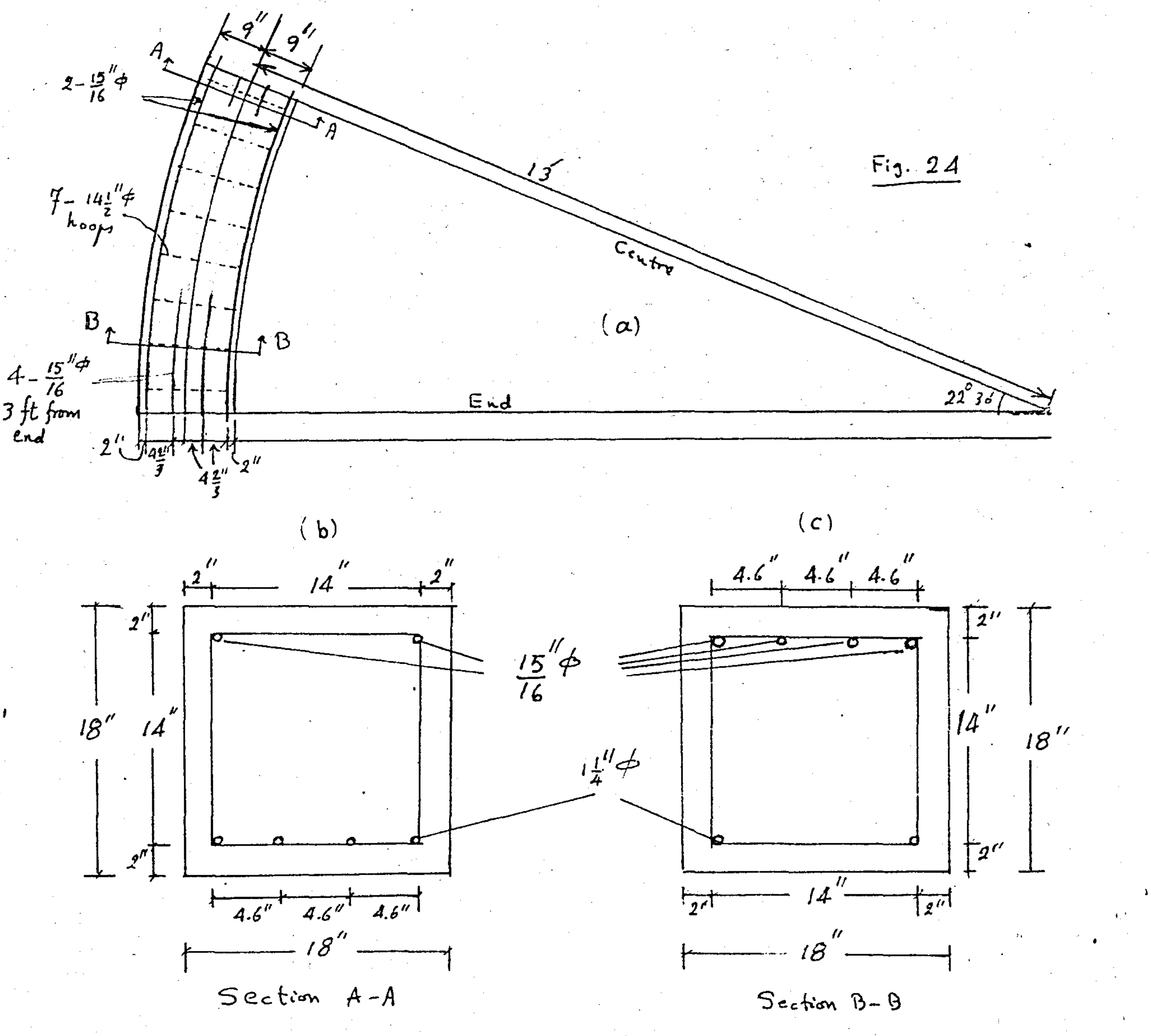


Fig. 24

In torsional and moment calculations the direct effect of water was not considered since the shell wall and the circular girder, both act as a deep beam, where shears distribute themselves such that the effect goes to the top of the eight columns.

C) Design of walls: (See pp. 20-31)

(i) For hoop tension in the fixed edge condition, the max. hoop tension is + 9056 lb., and it occurs at 0.4 x 14' = 5.6' from the top.

In the hinged case the max. hoop tension = + 9986 lb. and it occurs at 0.3 x 14 = 4.2' from the top.

$$A_s = N_{max.}/fs = \frac{9986}{14{,}000} = 0.71 \text{ sq. in. per ft.}$$

Use #5 Ø @ 10.5" o.c. in each of two curtains giving a total area of 0.71 sq.in.

For top portion:

$$A_s = \frac{6650}{14000} = 0.475 \text{ sq.in.}$$

.. Use #5 Ø @ 15" o.c. in each of two curtains giving $A_s = 0.49$ sq.in.

For bottom portion use same as for top portion.

These bars are lapped for a length of 40 diameters =40x0.625=25" (The laps are tied with 16 gauge soft iron wire to keep the bars from being disturbed on placing concrete.)

Max. tensile stress in the concrete including the effect of shrinkage is: $f_c = \frac{C\ E_s\ As + T_{max.}}{A_c + nA_s}$ (*)

$$= \frac{0.0003 \times 30 \times 10^6 \times 0.71 + 9986}{9 \times 12 + 10 \times 0.71} = 142 \text{ psi.} \angle 300 \text{ psi.} \quad (O.K.)$$

(*) "Circular Concrete Tanks Without Prestressing"-Portland Cement Association.

(ii) <u>For Moments:</u>

The max. positive moment occurs at the top where $M = + 1637$ ft-lb (in the hinged case) - 1640 ft.-lb.

∴ continue the reinforcement at the top of the roof slab @ the wall / roof connection; i.e. Use #4 ∅ @ 9.5" o.c. up to 0.7 x 14 + 33 diameters. = 10' 9" from the top.

Use same reinforcement in the inside curtain, with connection extending into the <u>bottom</u> of the <u>roof</u> slab a distance of 2'.

At the bottom of the wall $M_{max.} = 2570$ ft.-lb.

This moment will be opposed by that resulting from the dead weight of the floor slab, and the weight of water weight.

Weight of floor slab $= \frac{6.5}{12} \times 1 \times 1 \times 150 = 81$ lb./sq.ft.

Weight of water $= 1 \times 1 \times 14 \times 62.5 = 875$ " " "

Total......= 956 " " "

Moment @ edge of floor slab due to its weight + wt. of water is:

$M = -q.a^2/8 = \frac{956\ (13)^2}{8} = -20,200$ ft.-lb./ft.

20,200 - 2570 = 17630 ft.-lb./ft.

This moment will be distributed between the wall and the floor slab according to their stiffnesses.

Let thickness of the floor slab = 6.5"

Relative stiffness of the wall $= 1.010 \times \frac{(9)^3}{14} = 52.5$

" " " " slab $= 0.104 \times \frac{(6.5)^3}{13} = 3.37$

∴ distributing factors are:

Wall = 52.5/(52.5 + 3.37) = 0.94

Slab = ...0.06

Moment distribution:

	Wall	Floor
Distribution factor	0.94	0.06
FEM.	+ 0	- 17630
Distributed M.	+ 16,570	+ 1,060
Final Moment.	+ 16,570	-16,570

∴ at the bottom of the wall induced net moment = + 4023 ft.lb/ft.

$A_s = M/f_s jd$ =(16570 x 12)/(20,000 x 0.875 x 7.5) = 1.51 sq.in.

But by inserting a haunch of 6" x 6" at the bottom, the amount of steel can be reduced to:

A_s = (16,570) x 12/20,000 x 0.875(9 +6.5 - 2) = 0.835 sq.in.

∴ Use #8 ∅ @ 11 1/4"o.c.

This reinforcement will extend also in the vertical direction (in the wall) a distance of:

0.3 x 14 + 33 diameters = 6' from the <u>bottom</u>.

Moment induced at the bottom of the wall due to earthquake is: 51,300 lb.-ft./ft. (see column design.)

The ACI allows an increase of 33 % of allowable stresses in case of earthquakes.

The moment of ± 51,300 ft.lb/ft. will be again distributed between the wall and floor slab according to their stiffnesses:

	Wall	Floor
Distribution facor:	0.94	0.06
FEM	-51,300	0.00
	+48,300	+3000
	- 300	+3000

Considering the net induced moment at the wall we have:

$A_s = M/f_s jd$ = 48,300/(20,000 x 1.33 x 0.875 x 13.5) = 1.85 sq.in.

Use #9 ∅ @ 6 ½ "

This reinforcement will extend vertically in the wall a distance of 3', and also in the circular girder a distance of 15". Whem it happens to be within the column area let it extend a distance of 2' inside the column.

D) Design Of The Floor Slab:

Due to the weight of the water and the dead weight of the floor slab, the moment at the edge will be :

$M = -q\ a^2/8 = -$ 20,200 ft.-lb./ft. (see before)

It has been already shown that a moment of + 1,060 ft.lb is been induced at the edge of the floor on distributing the 20,200 ft.-lb./ft. moment. From the moment of 51,300 ft.-lb/ft. resulting from earthquake, the moment induced at the floor is + 3000 ft.-lb./ft.

∴ 3000 + 1,060 = 4,060 ft.-lb./ft.

∴ $4{,}060/qa^2 = +\ 4{,}060/(956 \times 13^2) = +\ 0.025$.

This value will be added to the coefficients (*) of M_r, M_t

The result is given in schedule II.

Final values of M_r , M_t are obtained as usual by multiplying their final coefficient by $qa^2 = 956 \times 13^2 = 162{,}000$

Values of M_r,M_t are plotted in Fig. (2) page 5.

Negative reinforcement:

Max. negative moment is $-$ 162,000 ft.-lb./ft.

$A_s = M/f_s jd = 162{,}000 \times 12/20{,}000 \times 0.875 \times 5 = 2.22$ sq.in.

Use 1-3/16" Ø @ 6" (giving $A_s = 2.22$ sq.in.)

(*) Portland Cement Association Pamphlet, Table XII.

SCHEDULE II:

Point / Moment	0.0a	0.1a	0.2a	0.3a	0.4a	0.5a	0.6a	0.7a	0.8a	0.9a	1.0a	
M_v Coeff.	+.075	+.073	+.067	+.057	+.043	+.025	+.003	-.023	-.053	-.087	-.125	
Add	+.025	←								→	+0.025	
Final M_r Coeff.	+.1	+.098	+.092	+.082	+.068	+.05	+.028	+.002	-.028	-.062	-.100	
M_v/ft	+16200	+15900	+14900	+13300	+11000	+8100	+4530	+3250	-4530	-10000	-16200	
M_v/Seg.	0	+1600	+2300	+4000	+4400	+4000	+2700	+2300	-3600	-9000	-16200	
M_t Coeff.	+0.075	+0.074	+0.071	+0.066	+0.059	+0.05	+0.039	+0.026	+0.011	-0.006	-0.025	
Add	+0.025	←								→	+0.025	
Final M_t Coeff.	+.1	+.099	+.096	+.091	+.084	+.075	+.064	+.051	+.036	+.019	0	
M_t/ft	+16200	+16000	+15500	+14700	+13600	+12100	+10400	+8200	+5800	+3100	0	

This reinforcement is placed at the top of the slab and into the wall. Total number of bars required is:

2 (13) x 12/6 = 163.28 say 164 bars.

By referring to Fig. (2), 82 of these bars are discontinued at a distance of :

$0.14\ a + 12$ diameters $= 0.14 \times 13 + 12 \times \frac{1.1075}{12} = 1.82 + 1.1075$
$= 2.9275$, say at 2' 11" from the inside of the wall.

The other 82 top bars are discontinued at a distance of $0.3\ a + 12$ diameters $= 0.13 \times 13 + 12 \times \frac{1.1075}{12}$
$= 3.9 + 1.1075$, say at 5' from the inside of the wall.

Positive Reinforcement:

Reinforcement for positive moments is needed mostly between points 0.3a, 0.5a where the dashed line has its max. value. At point 0.4a the radial moment per segment is + 4400 ft.-lb./ft; and the length of the concentric circle through

$0.4a = 2 \ (0.4 \times 13) = 32.7$ ft.

.. At point 0.4a:

$$A_s = 32.7 \times M/f_s jd = 32.7 \times 11{,}000 \times 12/20{,}000 \times 0.875 \times 5$$

$$= 49.3 \text{ sq.in.}$$

Use 32 - 1 7/16" Ø (giving $A_s = 51.9$ sq.in.)

These bars will be arranged as follows:

There will be eight radial bars in each quadrant; the whole being in two layers crossing at the center. According to the dashed line in Fig. (2) page 5, length of each bar, which makes two bars in the slab, will be 19-6". The bars are placed as shown in Fig. (25) with minimum spacing of 3" where they cross at the center.

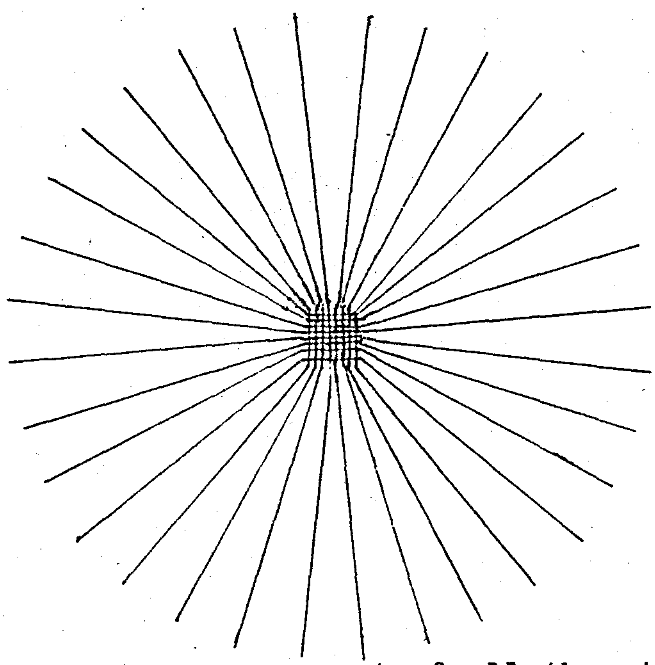


Fig. 25

4 bars

4 bars

4 bars

4 bars

Total 16 - 17/16" Ø

each is 19' 6" long

Minimum spacing @ centre, where bars cross, is 3".

The arrangement of all the steel is shown in plate II.

E- <u>Design Of Columns:</u>

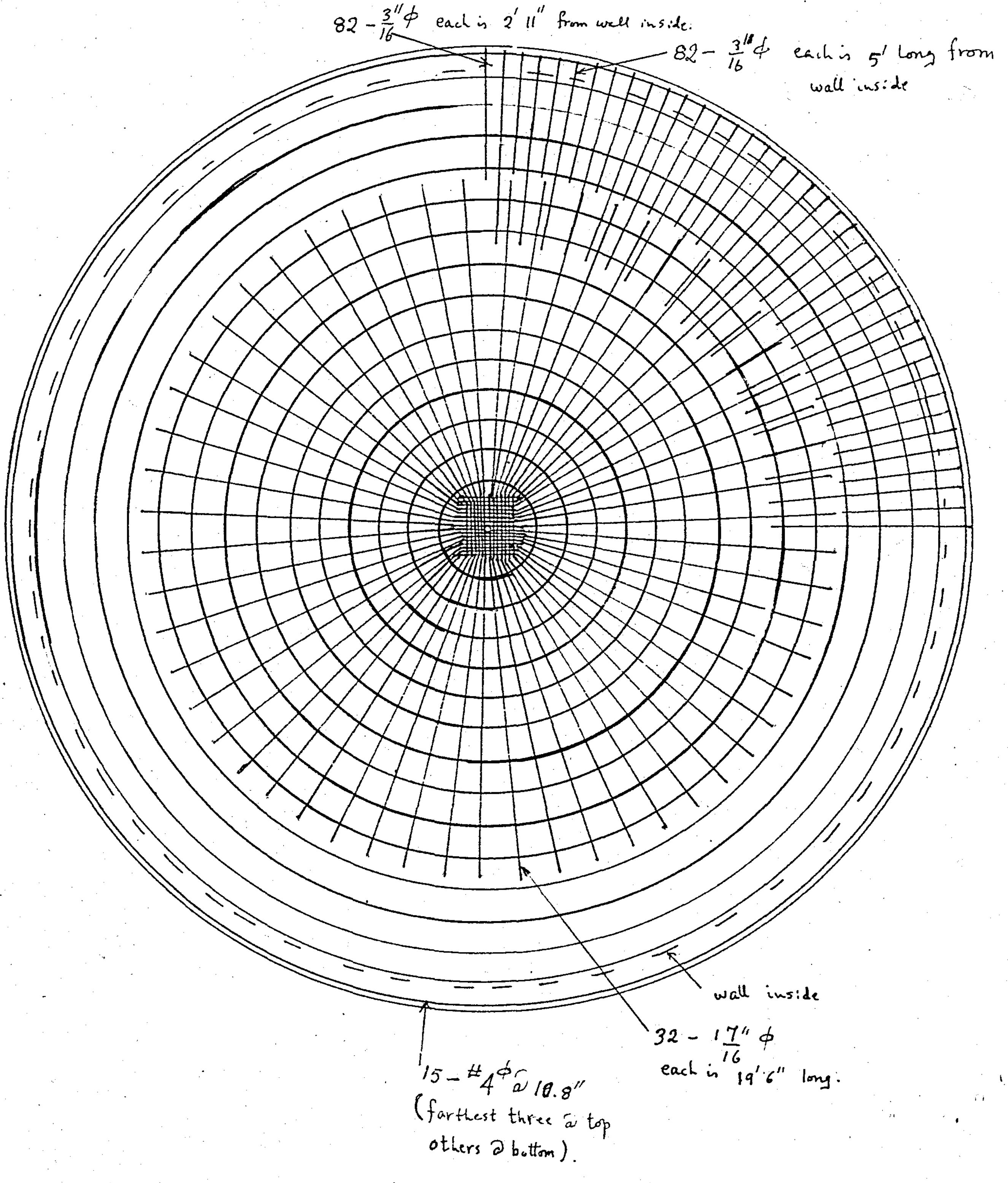
The columns are to be designed for the following:

1- Dead load of the tank, and water load.

2- Direct load on the columns resulting from the earthquake (or wind); this will be either compression or tension.

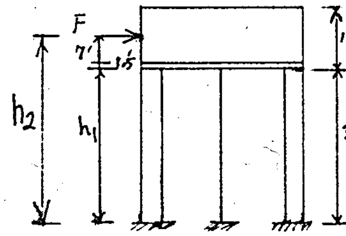
Plate II

Reinforcing steel for the floor slab of the elevated tank .



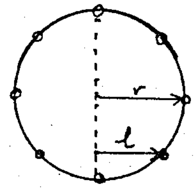
3- Flexural stresses resulting from earthquake (or wind) moments on the column.

For the case of fixed footings, if n = the number of columns (which are supposed to be identical), the resisting moment will be equal to the sum of the moments at the bases of the separate columns. The value of the latter is(*)



$$n.\frac{F}{n}\ .\ \frac{h_1}{2} = F\ h_1/2$$

$$\therefore F\ h_2 = F\ h_1/2 + \frac{V_r}{r}\sum l^2 \qquad \ldots\ldots\ldots \quad (15)$$



where V_r is the axial load due to earthquake (or wind) in the outermost column. See Fig. 26

Fig. 26

Weight of the wall	$= 2\pi(13)\ (0.75) \times 14 \times 150$	= 128,000 lb.
Wt. of roof slab	$= \pi(13^2 \times \frac{1}{2} \times 150)$	= 40,000 lb.
" " floor "	$= \pi\ 13^2 \times 6.5/12 \times 150$	= 43,500 lb.
Wt. of Circular girder	$= 2\pi(13)(1.5)91.5) \times 150$	= 27,500 lb.
Total weight of empty tank (without columns and foundation)		= 239,000 lb.
Weight of the water	$= \pi(13)^2(14) \times 62.5$	= 465,000 lb.
Total weight without columns & foundation		= 704,000 lb.

∴ Load on one column = 88,000 lb.

When an earthquake takes place, the heavy top-tank will resist the motion at the foundation; this entails a horizontal force (F) at the middle of the tank, where:

$F = ma = 704,000/32.2\ (0.1 \times 32.2) = 70,400$ lb.

Total moment @ the level of the circular beam is:

$70,400 \times 7 = 492,800$ ft.-lb. ; $\frac{492,800}{2\pi(13)} = 51,300$ lb.-ft./ft.

This is the value used for designing the end of the wall.

(*)R.Concrete, Water Towers,Bunkers,Silos,& Gantries, W.S.Gray, Concrete Publications Ltd., London 1953, page 159.

Applying eq.(15) we get:

$$70400 \times 38.5 = 70400 \times \frac{30}{2} + \frac{V_r}{13} \cdot 2(13)^2 + 4(9.19)^2$$

where $l = r.\cos\theta = 13 \times 0.7071 = 9.19$ ft.
$r = 13$ ft.

$\therefore V_r = 31825$ lb.
= maximum direct load on the column due to earthquake.

Shear per column = 70400/8 = 8800 lb.

Moment on column = 8800 x 7 = 61600 ft.lb.

Choosing a section of column = 20" x 20" :

Weight of one column $= (20/12)^2 \times 30 \times 150$
= 12500 lb.

Total direct force on the bottom of each column is

= 88000 + 12500 = 100500 lb.

Assuming that the column is reinforced by 4- 1 1/4"Ø vertical bars,(i.e. $p_g = 4.9/400 = 0.01225$) the moment of inertia of the transformed section is determined as follows:

$$(1/12)bh^3 = (1/12)(20)^4 = 13333 \text{ in}^4$$

$$(n-1)A_s.y^2 = 9 \times 4.9 \times (8)^2 = 2820 \text{ in}^4$$

$$\therefore I = 16153 \text{ in}^4$$

The safe axial load will be:

$$P = 0.18 f'_c . A_g + 0.8 A_s . f_s$$

$$= 0.18 \times 3000 \times 400 + 0.8 \times 4.9 \times 20000$$
$$= 294400 \text{ lb} > 100500 \text{ lb}$$

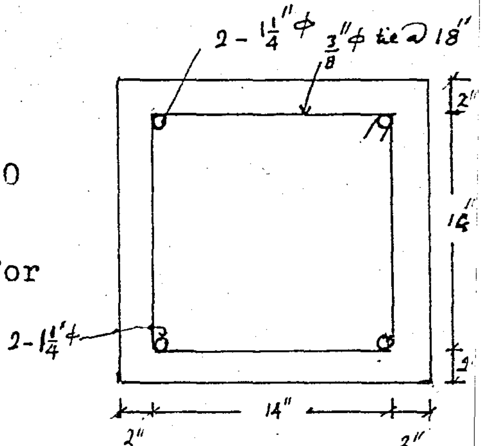


Fig. 27

The nominal allowable axial unit stress for tied columns is given as:

$$F_a = 0.8(0.225 f'_c + f_s . p_g)$$

$$= 0.8(0.225 \times 3000 + 20000 \times 0.01225$$
$$= 736 \text{ psi.}$$

For 3000 psi concrete the allowable bending unit stress that would be permitted if bending stress only existed is: $F_b = 1350$ps

The nominal axial stress f_a = axial load/area of member
= (100500 + 31825)/400 = 330.8 psi.

The bending unit stress f_b = bending moment/section modulus of transformed section(uncracked)

$$\therefore \quad f_b = \frac{61600 \times 12 \times 10}{16153} = 458 \text{ psi.}$$

For a safe design:

$$f_a/F_a + f_b/F_b \leq 1$$

$$\frac{330.8}{736} + \frac{458}{1350} = 0.449 + 0.339 = 0.788 \leq 1 \qquad \text{(O.K)}$$

Under the above conditions the combined stresses will be:

$$\text{(i)} \quad \frac{100500 + 31825}{444} + 458 = \begin{matrix} +756 \text{ psi.} \\ -160 \text{ psi.} \end{matrix}$$

$$\text{(ii)} \quad \frac{100500 - 31825}{444} + 458 = \begin{matrix} +613 \text{ psi.} \\ -303 \text{ psi.} \end{matrix}$$

where 444 sq.in. is the equivalent area of the section of the column $= 400 + (10-1) \times 4.9 = 444$ sq.in.

The above values of stresses are seen to be below allowables.

Shear on column is ;

$$v = \frac{V}{0.875\,bd} = 8800/(0.875 \times 18 \times 18) = 31 \text{ psi.}$$

As ties USE 3/8" Ø @ 18". See Fig. 27.

(F) Design Of The Foundation

(i) Analysis:

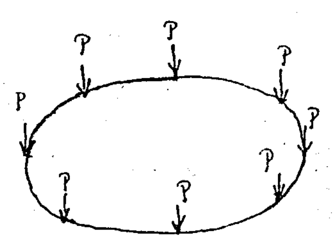


Fig. 28
Symmetric loading of ring foundation.

The condition of stress resulting in the case where the tank is not subjected to wind or earthquake load, is that of a ring under a symmetric case of loading as shown in Fig.(28).

If $P(\xi)$ is the intensity of a load at the section $\varphi = \xi$ (Fig. 29) then, for the case of symmetric loading, $P(\xi)$ will be an even function of (ξ) and the equations for the deflection $w(\varphi)$, angle of twist $\beta(\varphi)$, bending moment $M_x(\varphi)$, and twisting moment $M_z(\varphi)$ will be given

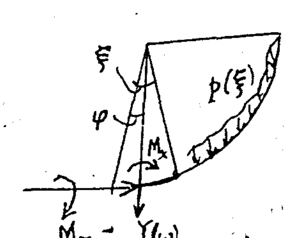


Fig. 29

by the equations (*) :

$$w(\varphi) = \frac{2}{\pi kR}\int_0^{\pi/2} P(\xi)d\xi + \frac{4R^2}{\alpha\mu\pi}\sum_{n=2,4,..}^{\infty} Y_n.\cos n\varphi.\int_0^{\pi/2} P(\xi)\cos n\xi\, d\xi$$

$$\beta(\varphi) = \frac{4R}{\alpha\mu\pi}\sum_{n=2,4,6,..}^{\infty} B_n.\cos n\varphi.\int_0^{\pi/2} P(\xi).\cos n\xi\, d\xi$$

$$M_x(\varphi) = \frac{4R}{\mu\pi}\sum_{n=2,4,6...}^{\infty} M_{xn}.\cos n\varphi.\int_0^{\pi/2} P(\xi).\cos n\xi\, d\xi$$

$$M_z(\varphi) = \frac{4R}{\mu\pi}\sum_{n=2,4,6..}^{\infty} M_{zn}.\sin n\varphi.\int_0^{\pi/2} P(\xi).\cos n\xi\, d\xi$$

If the load is antisymmetrical, $P(\xi)$ will be an <u>odd</u> function of (ξ). In this case the expressions for deflections, angle of twist, and moments will be:

$$w(\varphi) = \frac{4R^2}{\alpha\mu\pi}\sum_{n=1,3,5,..}^{\infty} Y_n.\sin n\varphi.\int_0^{\pi/2} P(\xi).\sin n\xi\, d\xi$$

$$\beta(\varphi) = \frac{4R}{\alpha\mu\pi}\sum_{n=1,3,5}^{\infty} B_n.\sin n\varphi.\int_0^{\pi/2} P(\xi).\sin n\xi\, d\xi$$

$$M_x(\varphi) = \frac{4R}{\mu\pi}\sum_{n=1,3,5..}^{\infty} M_{xn}.\sin n\varphi.\int_0^{\pi/2} P(\xi).\sin n\xi\, d\xi$$

$$M_z(\varphi) = -\frac{4R}{\mu\pi}\sum_{n=1,3,5..}^{\infty} M_{zn}.\cos n\varphi.\int_0^{\pi/2} P(\xi).\sin n\xi\, d\xi$$

The following nomenclature applies to the above formulae:

$$Y_n = \frac{\mu n^2 - 1}{n^6 - 2n^4 + n^2(1 \pm \lambda) \pm \lambda/\mu}$$

$$B_n = -\frac{(1 \pm \mu)\, n^2}{1 + \mu n^2}\,.\,Y_n$$

$M_{xn} = B_n + n^2.Y_n$; $M_{zn} = -\dfrac{M_{xn}}{n}$

μ = C/EI_x ($C = GJ$ = torsional rigidity of the beam)

$P(\xi)$ = force(distributed or concentrated.

R = radius of the circular foundation(beam)

I_x = moment of inertia of beam cross-section @ the x-axis.

α = EI_x/R

λ = kR^4/EI_x

k = elastic modulus of the foundation

E = Youngś modulus of the beam.

It should be noted that the value of(k) which is usually given in tables is the subgrade modulus measured for the case of 1 sq.ft

(*) "Deflection of Circular Beams Resting on Elastic Foundations Obtained be Methods of Harmonic Analysis"-by Enrico Volterra; Journal of Applied Mechanics, June 1953, pp. 227 - 232.

bearing plate. This value should be changed if the width of the foundation is different from 1 ft. If the beam is (b) ft. wide, the corresponding modulus of foundation will be (*) :

$$k' = k.\left\{\frac{b+1}{2b}\right\}^2$$

(ii) Design:

Radius of the ring foundation is

$$R = (26 + 2x0.75)/2 = 13.75 \text{ ft.}$$

Under a symmetric loading the period of the harmonic expressions is(8) since we have 8 symmetrically placed loads. Hence the expressions for deflection, angle of twist, and moments will be:

$$w(\varphi) = \frac{4P}{\pi R k} + \frac{8PR^2}{\alpha\mu\pi}\sum_{n=8,16,24,..}^{\infty} Y_n \cos n\varphi \cos n(22^\circ 30')$$

$$\beta(\varphi) = \frac{8RP}{\alpha\mu\pi}\sum_{n=8,16,24}^{\infty} B_n \cos n\varphi \cos n(22^\circ 30')$$

$$M_x(\varphi) = \frac{8RP}{\mu\pi}\sum_{n=8,16,..}^{\infty} M_{xn} \cos n\varphi \cos n(22^\circ 30')$$

$$M_z(\varphi) = \frac{8RP}{\mu\pi}\sum_{n=8,16,..}^{\infty} M_{zn} \sin n\varphi \cos n(22^\circ 30')$$

Only the first three terms in the above series will be considered in calculations.

Choosing a section of 30"x30" for the ring, the following values are obtained:

$I_x = 6.75 \times 10^4$ in^4 ; J = polar moment of inertia $= 95.625 \times 10^3$ in^4

$C = GJ = 11.95 \times 10^{10}$ lb.in^2 = torsional rigidity,

$\mu = 0.59$, α (**) $= 12.273 \times 10^8$ lb.in.

$k = 200$ lb/in^2/in.

$$\therefore\ k' = k\left(\frac{b+1}{2b}\right)^2 = 200\left(\frac{30+1}{2x30}\right)^2 = 53.38 \text{ lb./in}^2$$

The final results of the calculations are shown in terms of P, the

(*) See Terzaghi and Peck's " Soil Mechanics In Engineering Practice", p.

(**) See "Theory Of Plates & Shells", by S. Timoshenko and S. Woinowsky-Krieger, McGraw-Hill Book Company, Inc. Second Edition 1959, p. 259.

load of one column, in Table (IX).

Table(IX)
Symmetric case of loading .

φ	w (inches)	β (radians)	M_x (in-lb)	M_z (in-lb)
0^0	0.000,012,054P	$+140.952\times10^{-12}$P	-3.394,572 P	0
5^0	0.000,012,056P	$+112.751\times10^{-12}$P	-2.698,513 P	+0.276,882 P
10^0	0.000,012,061 P	$+37.169\times10^{-12}$P	-1.657,137 P	+0.487,575 P
15^0	0.000,012,067 P	-61.187×10^{-12}P	+0.047,698 P	+0.523,423 P
20^0	0.000,012,069 P	-155.124×10^{-12}P	+5.711,748 P	+0.278,198 P
22.5^0	0.000,012,074 P	-168.967×10^{-12}P	+6.718,914 P	0

The load P is determined as follows:(for symmetric loading):

Load on a column = ... 100500 lb.

weight of the foundation = $\frac{2\pi(13.75)(30/12)^2}{8/150}$ = 10120 lb.

∴ P = ... 110620 lb.

Using this value of P, Table (X) is obtained:

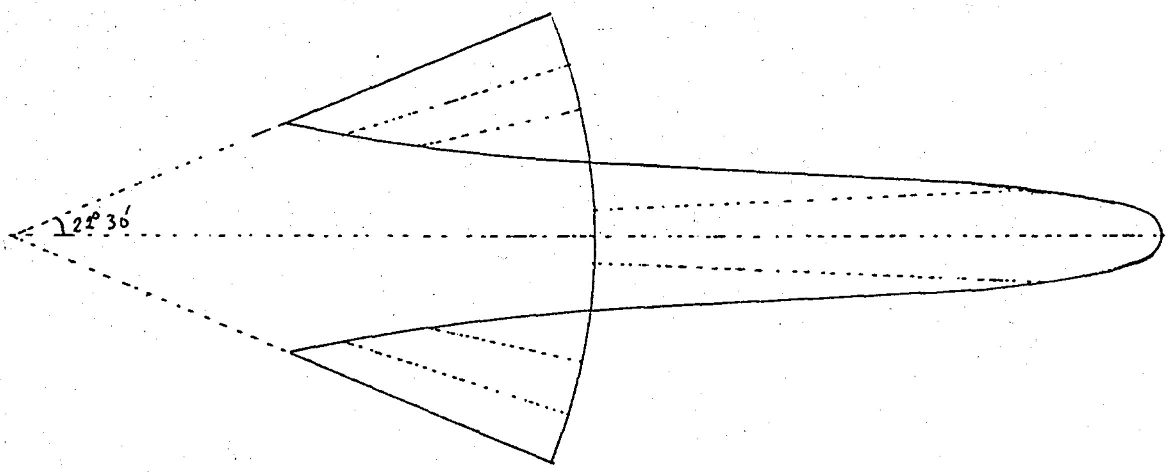
Table (X)
Symmetric Case of Loading

φ	w (inches)	β (radians)	M_x (in-lb)	M_z (in-lb)
0^0	1.33341	$+155.92\times10^{-7}$	-375,550	0
5^0	1.33363	$+124.72\times10^{-7}$	-298,500	+30,630
10^0	1.33418	$+42.23\times10^{-7}$	-183,300	+53,940
15^0	1.33485	-67.68×10^{-7}	+ 5,280	+57,900
20^0	1.33507	-171.59×10^{-7}	+613,830	+30,775
22.5^0	1.33562	-186.91×10^{-7}	+743,250	0

The variation of M_x, M_z along the portion between any two columns is plotted in Fig.(30).

In a similar manner the antisymmetric case of loading is analysed;however ,in this case,the Fourier expressions have to be taken for all odd values of n. These expressions will be :

(a)



(b)

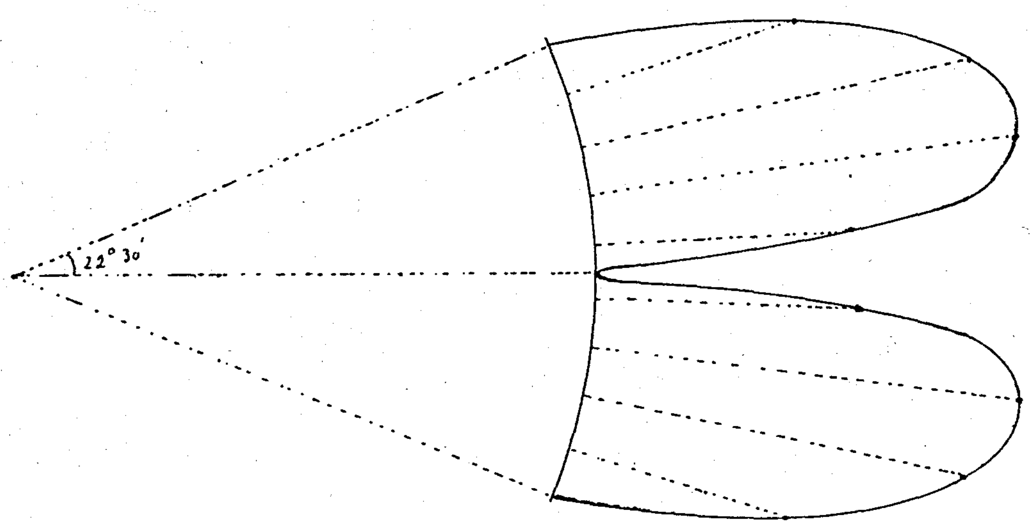


Fig. 30 : (a) Variation of bending moment along the ring foundation in the symmetric case.

(b) Variation of twisting moment along the ring foundation in the symmetric case.

$$w(\varphi) = \frac{8RP}{\alpha\mu\pi}\sum_{n=1,3,5,..}^{\infty} Y_n \sin n\varphi \sin n(22^\circ 30')$$

$$\beta(\varphi) = \frac{8RP}{\alpha\mu\pi}\sum_{n=1,3,5}^{\infty} B_n \sin n\varphi \sin n(22^\circ 30')$$

$$M_x(\varphi) = \frac{8RP}{\mu\pi}\sum_{n=1,3,5}^{\infty} M_{xn} \sin n\varphi \sin n(22^\circ 30')$$

$$M_z(\varphi) = -\frac{8RP}{\mu\pi}\sum_{n=1,3,5}^{\infty} M_{zn} \cos n\varphi \sin n(22^\circ 30')$$

The value of P which has to be used in the above expressions is

$$P = 100500 + 31825 = 132325 \text{ lb.}$$

Using this value Table (XI) was obtained.

Table (XI)
Antisymmetric Case of Loading

φ	M_x (in.lb.)	M_z (in.lb.)
0°	0	+ 2188390
22.5°	+ 2386000	+ 29900
45°	+ 1656580	- 1429900
67.5°	- 4596840	+ 36880
90°	- 2534950	0

The values of the bending and twisting moments are plotted in Fig. (31).

By referring to Tables (X),(XI) and Fig.s (30) &(31) it is seen that the maximum positive moments and max. negative moments occur in the case of antisymmetric loading. These values will be used in designing the ring foundation.

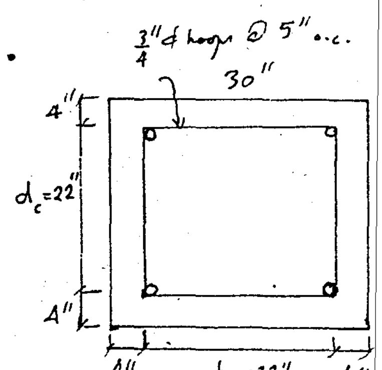


Fig. 32

STEEL FOR TWISTING MOMENT

Max. twisting moment = 2188390 in.lb.

$$A_V = \frac{M_T \cdot t}{2 \cdot b_c \cdot d_c \cdot f_v} = \frac{2188390\ t}{2x22x22x1.33x20000} = 0.085\ t$$

For 3/4" ø hoops , t = 0.4418/0.085 = 5.19"

∴ USE 3/4" ø hoops @ 5" o.c. (Fig. 32)

The area of the longitudinal reinforcement at each corner will be:

$$A_s = \frac{M_T \cdot S}{2 \cdot b_c \cdot d_c \cdot f_s} = \frac{2188390x\ (22+22)/2}{2x22x22x20000x1.33} = 1.869 \text{ in.}^2$$

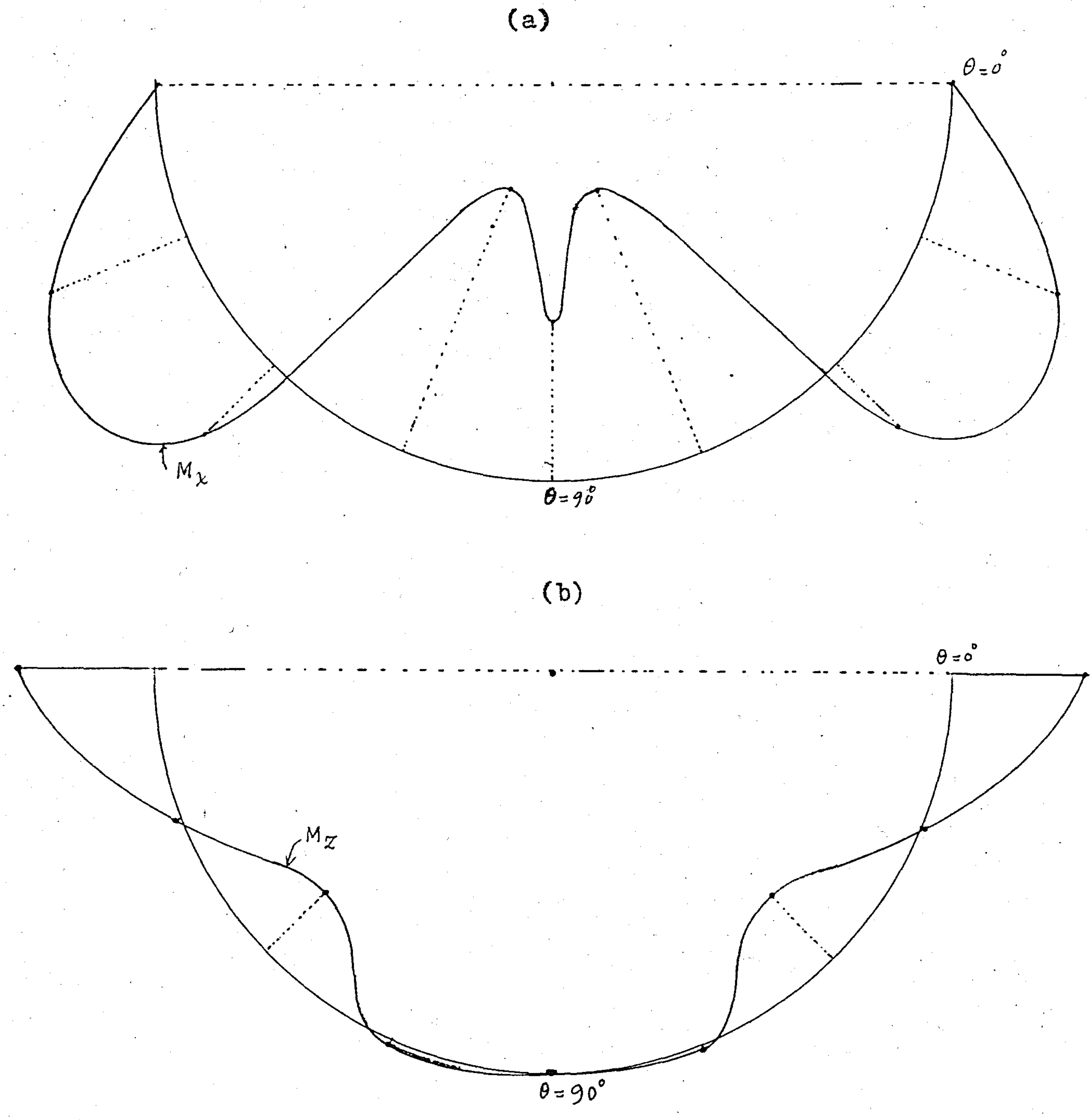


Fig. 31

(a) Variation of bending moment along the ring foundation in the antisymmetric case.

(b) Variation of twisting moment along the ring foundation in the antisymmetric case.

STEEL FOR BENDING MOMENTS

1. Positive moment: its max. occurs @ $\varphi = 22^{o}30'$, or the middle point between any two column bases.

   $A_s = M/f_s.j.d = 2386000/(20000x1.33x0.875x26) = 0.396$ in.^{2}

   At the base of a column area of steel is

   $A_s = 1656580/(20000 \times 1.33 \times 0.875 \times 26) = 0.274$ in.^{2}

2. Negative moment:

   At a column base area of steel is

   $A_s = 2534950/(20000 \times 1.33 \times 0.875 \times 26) = 0.419$ in.^{2}

   At midpoint:

   $A_s = 4596840/(20000 \times 1.33 \times 0.875 \times 26) = 1.34$ in.^{2}

Combining the effects of torsional and bending moments, the reinforcement will be as follows:

At the base of a column:

Positive steel = 0.274 in.^{2} + 2(1.869) bars,

∴ USE 2- 1 $\frac{9}{16}$" Ø at corners, and 2- 7/16" Ø in between.

Negative steel = 0.419 in.^{2} + 2(1.869) bars,

∴ USE 2- 1 $\frac{9}{16}$" Ø at corners, and 2- ½" Ø in between.

At the midpoint between the bases of two columns:

positive steel = 0.396 + 2(1.869) bars   USE 2-1 $\frac{9}{16}$"φ @ corners, & 2-½"φ in between
negative steel = 1.34 + 2(1.869) bars

∴ USE 2- 1 $\frac{9}{16}$" Ø at corners, and 3- 3/4" Ø in between.

Length of positive and negative reinforcement at the base of a column will be 7 ft., 3.5ft. on each side of the centre.

Details of the reinforcement are shown in Fig. (33)

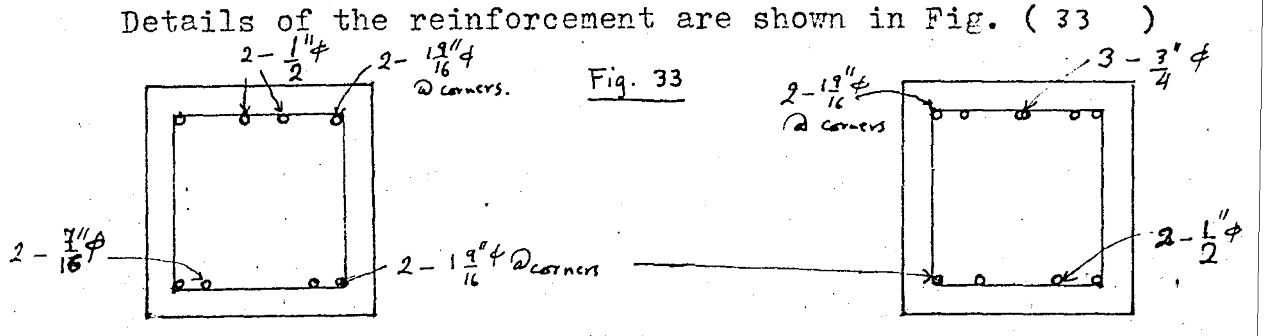


Fig. 33

Section under a column   Section @ midpoint between columns

(II)

ANALYSIS & DESIGN OF A GROUND-SUPPORTED REINFORCED CONCRETE CIRCULAR WATER TANK

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(A)

(i) Economical Analysis:-

Using the same notation on page(2), we have:

Total cost $C = A_w . C_w + A_f . C_f$

$$= \pi D . \frac{4V}{\pi D^2} . C_w + \frac{\pi D^2}{4} . C_f$$

$$\frac{dC}{dD} = 0 = -\frac{4V}{D^2} . C_w + \frac{\pi D}{2} . C_f$$

∴ The optimum diameter will be :

$$D = 2 . \sqrt[3]{V . C_w / \pi C_f}$$

(ii) Let the required capacity of the tank be 250000 gallons, or 33500 cu.ft. Assuming the relation between C_w, C_f is given by the equality: $C_w = 2C_f$

then $D = 2 . \sqrt[3]{C_f . 2V / \pi C_f}$

∴ $D = 2 . \sqrt[3]{2 \times 33500/\pi} = 55.4$ ft.

Let it be 57 ft. Then $H = 33500/(\pi)(57/2)^2 = 13.1$ ft.

Adding 0.9 ft for clearance makes $H = 14$ ft.

(B)

DESIGN OF WALL

Since the height of this tank is the same as that of the elevated one, the stresses developed in the tank will be exactly as those determined for the case of free top/fixed bottom, and for free top/hinged bottom since we do not know exactly how the subgrade is going to behave. The results for hinged and fixed bottom cases are given in Table I and plotted on Fig. (10).

The effect of water surging during an earthquake is to be superposed on the hydrostatic effect of water; but the mass of water in this case is much larger than that in the elevated one. Furthermore, here we have to consider the case of a free top/fixed bottom wall, subjected to an inner pressure which is assumed to be uniform; this pressure is that resulting from water surging. Following is the development of the solution to the effect of water surging during an earthquake.

Solution:

In the general solution for the shell problem, viz.

$$w = C_1 \sin\beta x.\sinh\beta x + C_2 \sin\beta x.\cosh\beta x + C_3 \cos\beta x.\sinh\beta x + C_4 \cos\beta x.\cosh\beta x + f(x).$$

the particular solution f(x) results from solving the equation:

$$\frac{d^4 w}{dx^4} + 4\beta^4 w = \frac{Z}{D} = -p/D$$

This is the case because a uniform outward pressure (-p) is assumed to exist in the case of earth-shaking. The particular solution will be:

$$w_1 = -p/4\beta^4 D = -pa^2/Eh$$

The four conditions to solve for the four unknowns C_1, C_2, C_3, C_4 are

(1) $(w)_{x=0} = 0 = -pa^2/Eh + C_4$

$$\therefore C_4 = pa^2/Eh \quad \ldots\ldots\ldots\ldots \text{(i)}$$

(2) $\left(\frac{dw}{dx}\right)_{x=0} = 0 = \{C_1(\beta\cos\beta x \sinh\beta x + \beta\sin\beta x\cosh\beta x) + C_2(\beta\cos\beta x.\cosh\beta x + \beta\sin\beta x.\sinh\beta x) + C_3(-\beta\sin\beta x.\sinh\beta x + \beta\cos\beta x.\cosh\beta x) + C_4(-\beta\sin\beta x.\cosh\beta x + \cos\beta x.\sinh\beta x)\}_{x=0}$

$$\therefore C_2 + C_3 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

(3) $[M_x]_{x=d} = -D\left[\frac{d^2w}{dx^2}\right]_{x=d} = 0$

$\therefore \left[\frac{d^2w}{dx^2}\right]_{x=d} = 0 = \{ C_1\beta(-\beta\sin\beta x.\sinh\beta x + \beta\cos\beta x.\cosh\beta x + \beta\cos\beta x.\cosh\beta x + \beta\sin\beta x.\sinh\beta x) + C_2\beta(-\beta\sin\beta x.\cosh\beta x + \beta\cos\beta x.\sinh\beta x + \beta\cos\beta x.\sinh\beta x + \beta\sin\beta x.\cosh\beta x) + C_3\beta(-\beta\cos\beta x.\sinh\beta x - \beta\sin\beta x.\cosh\beta x - \beta\sin\beta x.\cosh\beta x - \beta\sin\beta x\cos\beta x + \beta\cos\beta x\sin$

$+ C_4\beta(-\beta\cos\beta x.\cosh\beta x - \beta\sin\beta x.\sinh\beta x - \beta\sin\beta x.\sinh\beta x + \cos\beta x\cosh\beta x)$

$\therefore \quad C_1\cos\beta d.\cosh\beta d + C_2\cos\beta d.\sinh\beta d - C_3\sin\beta d.\cosh\beta d$

$- C_4\sin\beta d.\sinh\beta d = 0$ .... (iii)

(4) $[Q]_{x=0} = \frac{d}{dx}(M_x)_{x=0} = 0$

$\therefore \{C_1(-\beta\sin\beta x.\cosh\beta x + \beta\cos\beta x.\sinh\beta x) + C_2(-\beta\sin\beta x.\sinh\beta x +$
$\beta\cos\beta x.\cosh\beta x) - C_3(\beta\cos\beta x.\cosh\beta x + \sin\beta x.\sinh\beta x)$
$- C_4(\beta\cos\beta x.\sinh\beta x + \beta\sin\beta x.\cosh\beta x)\}_{x=0} = 0$

$\therefore C_1(-\sin\beta d.\cosh\beta d + \cos\beta d.\sinh\beta d) + C_2(-\sin\beta d\sinh\beta d +$
$\cos\beta d.\cosh\beta d) - C_3(\cos\beta d.\cosh\beta d + \sin\beta d.\sinh\beta d) -$
$C_4(\cos\beta d.\sinh\beta d + \sin\beta d.\cosh\beta d) = 0$ .... (iv)

From equations i,ii,iii we get:

$C_1(\cos\beta d.\cosh\beta d) + C_2(\cos\beta d.\sinh\beta d + \sin\beta d.\cosh\beta d)$
$- \frac{pa^2}{Eh}(\sin\beta d.\sinh\beta d) = 0$ .... (v)

From equations i, ii,iv we get:

$C_1(-\sin\beta d.\cosh\beta d + \cos\beta d.\sinh\beta d) + C_2(2\cos\beta d.\cosh\beta d)$
$- \frac{pa^2}{Eh}(\cos\beta d.\sinh\beta d + \sin\beta d.\cosh\beta d) = 0$ .... (vi)

From the above equations the constants are determined:

$$C_1 = -\left(\frac{pa^2}{Eh}\right).\frac{\cos^2\beta d.\sinh^2\beta d + \sin^2\beta d.\cosh^2\beta d}{\cos^2\beta d + \cosh^2\beta d}$$

$$C_2 = -C_3$$

$$= \left(\frac{pa^2}{Eh}\right).\frac{\sinh\beta d.\cosh\beta d + \sin\beta d.\cos\beta d}{\cos^2\beta d + \cosh^2\beta d}$$

Using the values of these constants ,the closed expressions for the deflection,hoop tension,shear,and bending moment are obtained It is to be noted here that the resulting expressions are based on the following ones,given in terms of (w):

$$N = -\frac{Eh}{a}.w$$

$$M_x = -D.\frac{d^2w}{dx^2}$$

$$Qx = \frac{d}{dx}(M_x) = -D.\frac{d^3w}{dx^3}$$

Following are the final closed form expressions of the deflection, hoop tension, bending moment, and shear in the wall of the tank due to water surging during an earthquake. It should be remembered here that the variable (x) is zero at the bottom of the tank, and is equal to d, the depth of the tank, at the top of the wall.

$$W = -\frac{pa^2}{Eh}.\Big\{1 + \frac{\cos^2\beta d.\sinh^2\beta d + \sin^2\beta d.\cosh^2\beta d}{\cos^2\beta d + \cosh^2\beta d}.\sin\beta x\sinh\beta x - \frac{\sinh\beta d.\cosh\beta d + \sin\beta d.\cos\beta d}{\cos^2\beta d + \cosh^2\beta d}.(\sin\beta x.\cosh\beta x - \cos\beta x.\sinh\beta x) - \cos\beta x.\cosh\beta x\Big\}$$

$$N_\varphi = pa\Big\{1 + \frac{\cos^2\beta d.\sinh^2\beta d + \sin^2\beta d.\cosh^2\beta d}{\cos^2\beta d + \cosh^2\beta d}.\sin\beta x.\sinh\beta x - \frac{\sinh\beta d.\cosh\beta d + \sin\beta d.\cos\beta d}{\cos^2\beta d + \cosh^2\beta d}.(\sin\beta x\cosh\beta x - \cos\beta x\sinh\beta x) - \cos\beta x.\cosh\beta x\Big\}$$

$$M_x = \frac{p}{2\beta^2}.\Big\{\frac{\cos^2\beta d.\sinh^2\beta d + \sin^2\beta d.\cosh^2\beta d}{\cos^2\beta d + \cosh^2\beta d}.(\cos\beta x.\cosh\beta x) - \frac{\sinh\beta d.\cosh\beta d + \sin\beta d.\cos\beta d}{\cos^2\beta d + \cosh^2\beta d}.(\cos\beta x\sinh\beta x + \sin\beta x\cosh\beta x) + \sin\beta x.\sinh\beta x\Big\}$$

$$Q_x = \frac{p}{2\beta}\Big\{\frac{\cos^2\beta d.\sinh^2\beta d + \sin^2\beta d.\cosh^2\beta d}{\cos^2\beta d + \cosh^2\beta d}.(\cos\beta x.\sinh\beta x - \sin\beta x.\cosh\beta x) - \frac{\sinh\beta d\cosh\beta d + \sin\beta d.\cos\beta d}{\cos^2\beta d + \cosh^2\beta d}.(2\cos\beta x.\cosh\beta x) + \cos\beta x.\sinh\beta x + \sin\beta x.\cosh\beta x\Big\}$$

Values of M, N, and Q are calculated at the tenth points of the height of the wall, and are recorded in Table VII (p. 21)

Procedure of the wall design is the same as that of the elevated tank; hence, the actual design of the wall is omitted to avoid repetition.

The combined effects of water surge and hydrostatic pressure are given in Table VIII (p. 66) for both cases: hinged, and fixed bottom.

Table VIII.

Combined effects of hydrostatic pressure and water surge on the wall of the ground-supported tank.

Point on the wall	M ft-lb/ft		Q lb./ft.		N_φ lb/ft	
	Bottom Fixed	Bottom Hinged	Fixed	Hinged	Fixed	Hinged
1.0 d	- 9	-3	- 3	- 4	+10081	+10070
0.9 d	-17	- 11	- 6	- 7	+11240	+11215
0.8 d	-23	-23	- 2	- 9	+12560	+12380
0.7 d	-27	-47	+28	+10	+14150	+14020
0.6 d	+50	-9	+71	+43	+15530	+15430
0.5 d	+910	+98	+140	+115	+16800	+16900
0.4 d	+446	+308	+186	+201	+17060	+17700
0.3 d	+693	+633	+112	+239	+15450	+17050
0.2 d	+619	+884	-263	+ 69	+11040	+13710
0.1 d	-343	+612	-1140	-566	+4420	+7330
0	-3090	-1010	-2590	-1900	0	0

Design Of The Floor Slab

(A) Analysis:

The floor of the tank is subjected at its circumference to a uniformly distributed load (P_o) per unit run, due to the weight of the wall, and to a moment (M_o) per unit run, due to hydrostatic pressure. In analysis two analogous cases are discussed here:

(i) The case of a circular slab with a uniformly distributed load P_o per unit run:

The equation of the deflection curve of the plate in this case is(*)

$$w_1 = C_1 . Z_1(\eta r) + C_2 . Z_2(\eta r)$$

where $\eta = \sqrt[4]{k_o/D}$

$$C_1 = - \frac{P_o \eta}{k_o} . \frac{Z_1(\eta a) + \frac{1-\nu}{\eta a} Z_2'(\eta a)}{Z_1(\eta a) Z_2'(\eta a) - Z_1'(\eta a) Z_2(\eta a) + \frac{1-\nu}{\eta a}[Z_1'^2(\eta a) + Z_2'^2(\eta a)]}$$

$$C_2 = - \frac{P_o \eta}{k_o} . \frac{Z_2(\eta a) + \frac{1-\nu}{\eta a} Z_1'(\eta a)}{Z_1(\eta a) Z_2'(\eta a) - Z_1'(\eta a) Z_2(\eta a) + \frac{1-\nu}{\eta a}[Z_1'^2(\eta a) + Z_2'^2(\eta a)]}$$

and Z_1, Z_2, etc. are the Bessel functions whose values can be obtained from tables(**) if $\eta a < 6$; If $a > 6$ the following asymptotic values of these functions can be used to evaluate them:

$$Z_1(x) = + \alpha \cos\sigma \quad , \quad Z_2 = - \alpha \sin\sigma$$

$$Z_1'(x) = + \alpha(\cos\sigma - \sin\sigma)/\sqrt{2}$$

$$Z_2'(x) = - \alpha(\cos\sigma + \sin\sigma)/\sqrt{2}$$

where (x) is the argument of the functions, and

$$\sigma = x/\sqrt{2} - \pi/8 \quad ; \quad \alpha = (1/\sqrt{2\pi x}).e^{x/\sqrt{2}}$$

(ii) The case of a circular slab, of radius a, which is loaded by a moment M_o per unit length of the periphery.

In this case the deflection is:

$$w_2 = D_1 . Z_1(\eta r) + D_2 . Z_2(\eta r)$$

(*),(**) Hetenevi's "Beams..." pp. 106-107.

where
$$D_1 = \frac{M_o\eta^2}{k_o}\cdot\frac{Z_1'(\eta a)}{Z_1(\eta a)Z_2'(\eta a) - Z_1'(\eta a)Z_2(\eta a) + \frac{1-v}{\eta a}\left[Z_1'^2(\eta a) + Z_2'^2(\eta a)\right]}$$
$$D_2 = \frac{M_o\eta^2}{k_o}\cdot\frac{Z_2'(\eta a)}{Z_1(\eta a)Z_2'(\eta a) - Z_1'(\eta a)Z_2(\eta a) + \frac{1-v}{\eta a}\left[Z_1'^2(\eta a) + Z_2'^2(\eta a)\right]}$$

Knowing that:
$$M_r = -D.\left(\frac{\partial^2\omega}{\partial r^2} + \frac{v}{r}.\frac{\partial w}{\partial r}\right)$$
$$M_t = -D.\left(\frac{1}{r}.\frac{\partial\omega}{\partial r} + v.\frac{\partial^2\omega}{\partial r^2}\right)$$

and utilizing the properties of Bessel functions, it can be shown that:
$$M_r = -\alpha D\left\{\left[(C_1-C_2)\cos\sigma - (C_1 + C_2)\sin\sigma\right].\left(\frac{v}{r} - \frac{1}{\eta r}\right)/\sqrt{2} - (C_1.\sin\sigma + C_2.\cos\sigma)\right\}$$
$$M_t = -\alpha D\left\{\left[(C_1-C_2)\cos\sigma - (C_1+C_2)\sin\sigma\right]\left(\frac{1}{r} - \frac{v}{\eta r}\right)/\sqrt{2} - v.(C_1.\sin\sigma + C_2.\cos\sigma)\right\}$$

The same results apply for (w_2) after replacing C_1 and C_2 by D_1 and D_2, respectively.

As $r \rightarrow 0$ it becomes more convenient to evaluate M_r, M_t by using the asymptotic values of the Z-functions:
$$Z_1(\eta r) = +1 \quad , \quad Z_2(\eta r) = -\eta^2.r^2/4$$
$$Z_1'(\eta r) = -\eta^3.r^3/16 \quad , \quad Z_2'(\eta r) = -\eta r/2$$

Thus
$$\frac{\partial^2 w_1}{\partial r^2} = -\frac{3C_1}{16}.\eta^2.r^2 - C_2/2$$
$$\frac{\partial w_1}{\partial r} = -\frac{C_1}{16}.\eta^3.r^3 - C_2.\eta r/2$$
$$\therefore (M_r)_{r\to 0} = + D.C_2.(1 + v\eta)/2 \quad , \text{ and}$$
$$(M_t)_{r\to 0} = + D.C_2.(\eta + v)/2$$

Similar expressions hold for (w_2) by using D_1 and D_2 for C_1, C_2.

(B) Design:

Assuming that the slab thickness = 12 in., the following values will be used in calculations:

P_o = 0.75 x 14 x 150 = 1580 lb/ft of periphery

$$k_o' = k_o\left\{\frac{b+1}{2b}\right\}^2 = 200\left\{\frac{58+1}{2\text{x}58}\right\}^2 = 50 \text{ lb/in}^3.$$

η^a = 8.85

C_1 = 0.000025 C_2 = 0.000017

D_1 = 0.000239 D_2 = - 0.000091

Calculations of the moments at the tenth points of the radius of floor slab are recorded in Table (XII) and plotted in figures (34) and (35). The effects of M_o and P_o are superposed and plotted in Fig. (35).

It is seen from these graphs that most of the moments are effective near the edge of the slab, and that the maximum value of the bending moment is not at the edge, but at some short distance from the edge. This will draw the attention to the idea of thickening the floor slab near the end so as to reduce the steel needed there.

The moments given in Table (XII) can be used to determine the required area of steel in the same manner that the roof and floor slabs of the elevated tank were designed. Again, to avoid repetition, this detail will not be included here.

TABLE XII

Point	M_r			M_t		
	Due to P_o	Due to M_o	Σ	Due to P_o	Due to M_o	Σ
0	- 640	- 500	- 1140	- 130	- 95	- 225
0.1a	- 475	- 450	- 925	- 200	- 110	- 310
0.2a	- 800	- 10	- 810	- 300	0	- 300
0.3a	- 1050	- 300	- 1350	- 400	- 50	- 450
0.4a	+ 3000	- 1600	+ 2400	+ 1000	- 1600	- 600
0.5a	+ 1250	+ 24000	+ 25250	+ 90	+ 8000	+ 8090
0.6a	+ 60	+ 2800	+ 2860	+ 1200	+ 3440	+ 4640
0.7a	+ 13250	- 72000	+ 58750	+ 3150	- 20000	- 16850
0.8a	+ 19140	- 200420	- 181280	+ 3200	- 32000	- 26800
0.9a	- 45100	- 160000	- 205100	- 60	- 40000	- 40060
1.0a	- 18800	- 406575	- 425375	- 2240	- 103700	- 105940

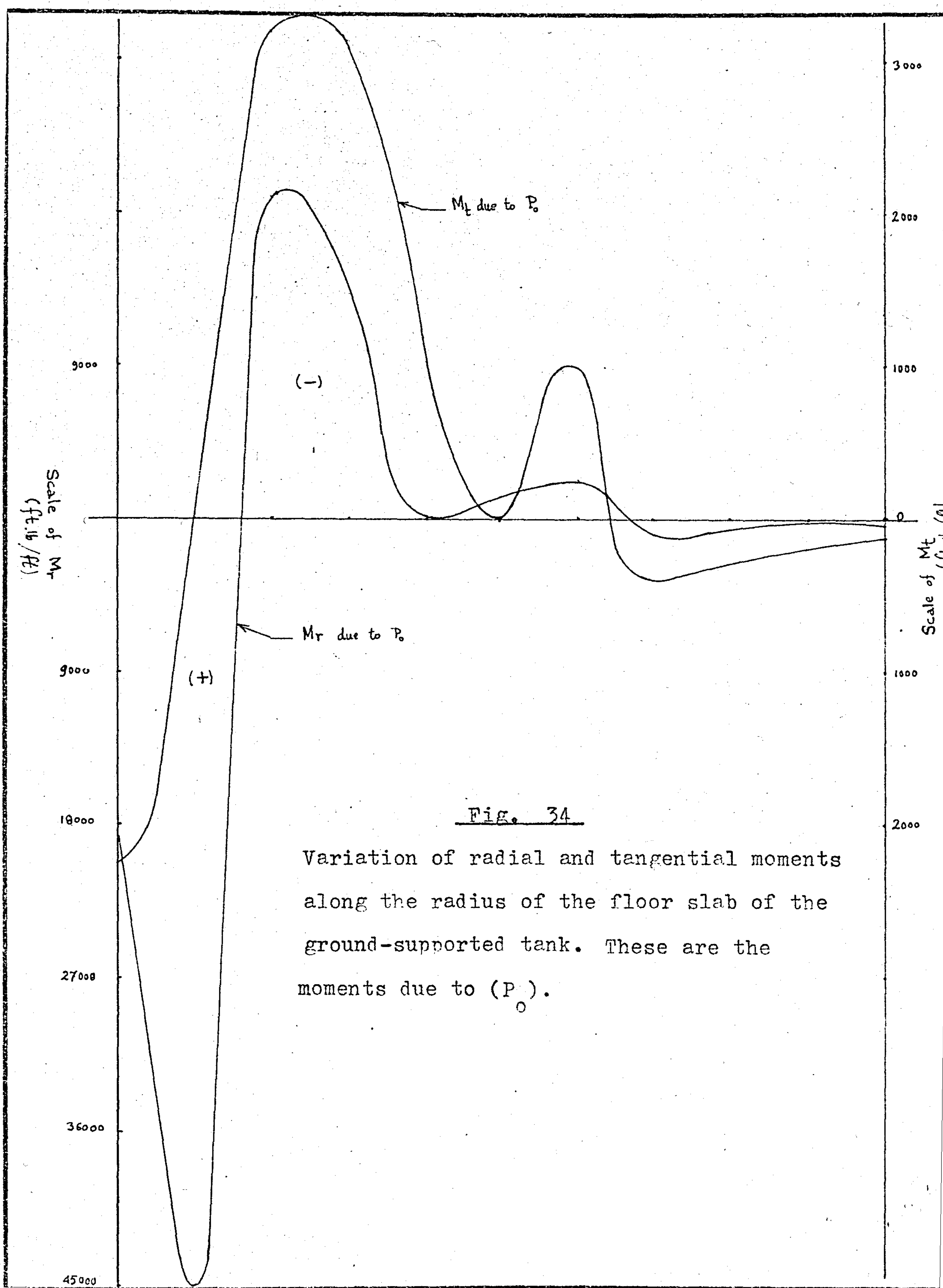

Fig. 34

Variation of radial and tangential moments along the radius of the floor slab of the ground-supported tank. These are the moments due to (P_o).

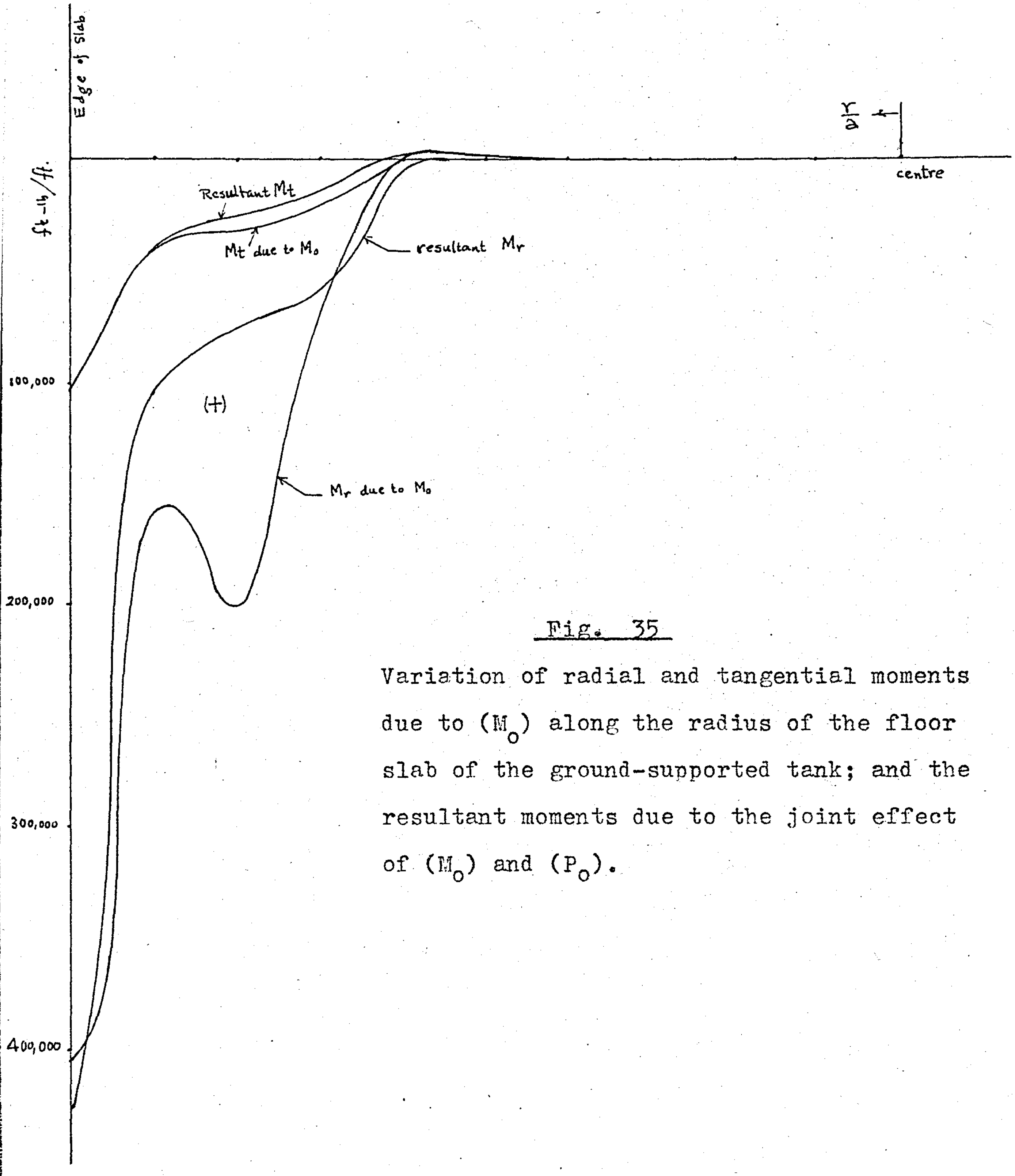

Fig. 35

Variation of radial and tangential moments due to (M_o) along the radius of the floor slab of the ground-supported tank; and the resultant moments due to the joint effect of (M_o) and (P_o).

(III)

ANALYSIS & DESIGN OF AN UNDERGROUND REINFORCED CONCRETE CIRCULAR WATER TANK.

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The analysis of this type of tanks is based to a great extent on the same line of analysis followed in the cases of the elevated and ground-supported tanks. Similarities and differences between the underground tanks and the other two types are cited below as a guide in analysis and design.

(1) The economical analysis is equally applicable to all of the three types.

(2) The analysis of a moment at the top of the wall of the elevated tank is applicable to the underground one. In the former, the moment at the wall/roof connection resulted from the weight of the roof plus snow load. In the latter, the same situation arises if the wall/roof connection is continuous, but, this time, the superposed load on top of the floor should be considered; it may be the surcharge of a highway or a soil fill, for example.

(3) The analysis for earthquake stresses applied in the first two cases is also applicable to the third. The analysis for a fixed bottom/fixed top, which may be needed here, is already dealt with in the elevated tank design.

(4) Stresses resulting from hydrostatic pressure are the same in the elevated and the underground tanks if the latter has a continuous wall/roof connection in particular-(as a matter of fact, this boundary condition will not have a serious effect on the resulting stresses, if the wall is deep enough.

(5) The analysis of the floor of the underground tank resembles that of the ground-supported one;and the design of both resembles that of the roof and floor slabs of the elevated tank. However, if the underground tank is too wide columns will be needed to the support the roof. In case one column is used in the middle of the slab,the moments can be determined from the deflection formula:

$$w = (P_0/4\,\eta^2.D).Z_3(\eta r)$$

where P_o is the load of the column, $Z_3(\eta r)$ is a Bessel function whose asymptotic value is:

1) $+\frac{1}{2}$ as $x \rightarrow 0$

2) $+\sqrt{2/\pi\eta r}\,.\left(e^{-\eta r/\sqrt{2}}\right).\sin\left(\frac{\eta r}{\sqrt{2}} + \frac{\pi}{8}\right)$ as $x \rightarrow \infty$

These asymptotic values are often applicable since the middle load P_o causes a dishing effect in such a manner that the deflection is usually confined to the near vicinity of the applied load.

(6) The underground tank exhibits a new feature that was not encountered on analysing the elevated and ground-supported tanks. It is that a special care is needed in analysis of the wall which has to support the surrounding soil pressure,especially when the tank is empty. This active soil pressure is given by the formula:

$$P_R = \tfrac{1}{2}\gamma H^2.\ \frac{1 - \sin\phi}{1 + \sin\phi}.\ \cos i$$

where: P_R = the resultant pressure on the wall,

γ = unit weight of the soil,

H = height of the wall,

i = angle of slope of the back fill, and

Ø = angle of internal friction for the soil.

Stresses resulting from active soil pressure should be taken by properly placed steel depending on the intensity of the pressure.

(7) Another feature of the underground tanks is yet to be considered, viz. consideration of the stability of the tank in wet ground. The upward pressure of the ground water may be high enough to lift or tilt the tank, especially when it is empty. In stability analysis the upward pressure is assumed to be uniform, and it has to be less than the total weight of the tank. If it happens that the upward pressure exceeds the total downward force (weight), the tank has to be made heavier by building an additional toe at the edges, or by constructing batters on the sides of the wall.

CONCLUSION.

It should become clear from such a work of analysis that the mathematical approach to problems of design is rather lengthy and is often complicated, and not superior to approximate methods of design. This was clear in the case of the relieving torsion whose value was rather small and could have been neglected after making some justified assumptions as to the behaviour of the circular girder with respect to the floor slab. However, this approach is often useful, and sometimes necessary, in explaining the way forces behave in the tank structure, and in applying the methods of elasticity to engineering structures. The method of superposition has been sufficiently clarified by the formulae derived for the different factors that bring about stresses in the tank, especially those due to earthquake and wind effects.

The elevated tank design shows that it is much better to build a steel one rather than a concrete tank if the size of the tank is considerably huge, hence more rigid than others. Such huge concrete tanks bring about problems regarding the size of the columns and the supporting foundation. Whereas concrete tanks are not allowed to deflect, steel tanks are allowed to have about 15 inches of deflection at the top of the tank. This flexibility in structure is sometimes more recommended than rigidity, although many support the opposite view. The steel tank tower may have ties with springs connecting opposite columns together, so that one tie is stretched while the other is relaxed during an earthquake. This device helps in getting at a considerably flexible and slender structure in which aesthetic considerations are satisfied, unlike the case with a very rigid and massive structure.

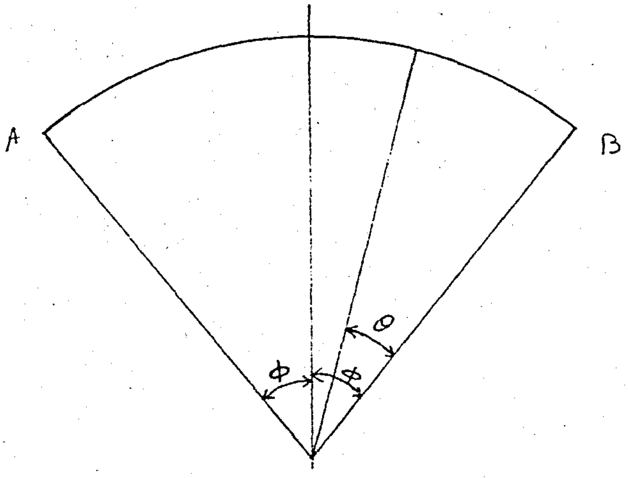
Thus it is recommended that concrete tanks be built if they are ground-supported or below ground. In case a tank is ground-supported, the smaller the diameter, the more economical it is as compared to a similar rectangular tank. As the ground-supported tank gets larger, it will be better to have it rectangular rather than circular, and design will then be of vertical slabs subjected to hydrostatic, or any, pressure with sides fixed or hinged.

The floor of the ground-supported tank exhibits the fact that its edges are subjected to huge moments which have their maximum value at some distance from the edge. These moments can be partly offset by building a toe at the edge, thus helping the floor/wall connection in supporting the applied moments.

The variation of the moments in this floor slab shows that, as the tank gets wider, the momental effect on the centre decreases; this leads to the idea of designing the floor, especially in the middle, by using one or two steel meshes instead of placing the bars radially as in floors with small diameters. Furthermore, this momental variation suggests that the vicinity of the edges of the slab be thickened more than the middle so as to provide a higher effective depth, and, consequently, a lower steel percentage.

APPENDIX

Derivation of the formulae for determination of the redundants in the circular girder of the elevated tank:



1. δ_{aa} :-

$M_a = \cos(\phi - \theta)$

$T_a = \sin(\phi - \theta)$

$ds = R\, d\theta$

$$\delta_{aa} = \int_0^{2\phi} \frac{M_a^2}{EI} ds + \int_0^{2\phi} \frac{T_a^2}{GJ} ds$$

$$= \int_0^{2\phi} \frac{R}{EI} \cos^2(\phi-\theta)\, d\theta + \frac{R}{GJ}\int_0^{2\phi} \sin^2(\phi-\theta) d\theta$$

$$= \frac{R}{EI}\int_0^{2\phi} \frac{1}{2}\left[1 + \cos 2(\phi-\theta)\right] d\theta + \frac{R}{GJ}\int_0^{2\phi} \frac{1}{2}\left[1 - \cos 2(\phi-\theta)\right] d\theta$$

$$= \frac{R}{2EI}\int_0^{2\phi} d\theta + \frac{R}{2EI}\int_0^{2\phi}\cos 2(\phi-\theta)\, d\theta + \frac{R}{2GJ}\int_0^{2\phi} d\theta - \frac{R}{2GJ}\int_0^{2\phi}\cos 2(\phi-\theta)\, d\theta$$

$$= \frac{2R\phi}{2EI} + \frac{2R\phi}{2GJ} + \left(\frac{R}{2EI} - \frac{R}{2GJ}\right)\int_0^{2\phi} \cos 2(\phi-\theta)\, d\theta$$

Now: $$\int_0^{2\phi} \cos 2(\phi-\theta)\, d\theta = \left[\frac{1}{2}\cos 2\phi \sin 2\theta - \frac{1}{2}\sin 2\phi \cos 2\theta\right]_0^{2\phi}$$

$$= \frac{1}{2}\left[\cos 2\phi\,(2\sin 2\phi \cos 2\phi) - \sin 2\phi\,(2\cos^2 2\phi - 1) + \sin 2\phi\right] = \frac{1}{2}\left[2\sin 2\phi\right]$$

$$= \sin 2\phi$$

$$\therefore\ \delta_{aa} = \frac{R}{2EI}(2\phi + \sin 2\phi) + \frac{R}{2GJ}(2\phi - \sin 2\phi).$$

2. δ_{bb} :- $M_b = \sin(\phi-\theta)$

$T_b = \cos(\phi-\theta)$

$ds = R\,d\theta$

$$\delta_{bb} = \int_0^{2\phi} \frac{M_b^2}{EI}\,ds + \int_0^{2\phi} \frac{T_b^2}{GJ}\,ds$$

$$= \frac{R}{EI}\int_0^{2\phi} \sin^2(\phi-\theta)\,d\theta + \frac{R}{GJ}\int_0^{2\phi}\cos^2(\phi-\theta)\,d\theta$$

From the similarity between the expressions,

$$\delta_{bb} = \frac{R}{2EI}(2\phi - \sin 2\phi) + \frac{R}{2GJ}(2\phi + \sin 2\phi).$$

3. δ_{cc} :- $M_c = R\sin\theta$,

$T_c = R(1-\cos\theta)$

$ds = R\,d\theta$

$$\delta_{cc} = \int_0^{2\phi} \frac{M_c^2}{EI}\,ds + \int_0^{2\phi} \frac{T_c^2}{GJ}\,ds$$

$$= \frac{R^3}{EI}\int_0^{2\phi}\sin^2\theta\,d\theta + \frac{R^3}{GJ}\int_0^{2\phi}(1-\cos\theta)^2\,d\theta$$

$$\therefore \delta_{cc} = \frac{R^3}{EI}\left[\phi - \frac{\sin 4\phi}{4}\right] + \frac{R^3}{GJ}\left[3\phi - 2\sin 2\phi + \frac{\sin 4\phi}{4}\right]$$

4. $$\delta_{ab} = \int_0^{2\phi}\frac{M_a M_b}{EI}\,ds + \int_0^{2\phi}\frac{T_a T_c}{GJ}\,ds$$

$$= \frac{R}{EI}\int_0^{2\phi}\cos(\phi-\theta)\sin(\phi-\theta)\,d\theta - \frac{R}{GJ}\int_0^{2\phi}\cos(\phi-\theta)\sin(\phi-\theta)\,d\theta$$

This results in:

$$\delta_{ab} = 2\left(\frac{R}{EI} - \frac{R}{GJ}\right)\sin^2\phi\cos^2\phi\left[\cos^2\phi - 1 + \sin^2\phi\right] = \underline{\underline{0}}$$

5. $$\delta_{ac} = \int_0^{2\phi} \frac{M_a M_c}{EI} ds + \int_0^{2\phi} \frac{T_a T_c}{GJ} ds$$

$$= \frac{R^2}{EI} \int_0^{2\phi} \sin\theta \cos(\phi-\theta) d\theta - \frac{R^2}{GJ} \int_0^{2\phi} \sin(\phi-\theta)(1-\cos\theta) d\theta$$

Now $$\int_0^{2\phi} \sin\theta \cos(\phi-\theta) d\theta = \frac{1}{2} \sin\phi (2\phi + \sin 2\phi)$$

and $$\int_0^{2\phi} \sin(\phi-\theta)(1-\cos\theta) d\theta = \frac{1}{2} \sin\phi (\sin 2\phi - 2\phi)$$

$$\therefore \delta_{ac} = \frac{R^2}{2EI} \sin\phi (2\phi + \sin 2\phi) + \frac{R^2}{2GJ} \sin\phi (2\phi - \sin 2\phi)$$

6. $$\delta_{bc} = \int_0^{2\phi} \frac{M_b M_c}{EI} ds + \int_0^{2\phi} \frac{T_b T_c}{GJ} ds$$

$$\int_0^{2\phi} \frac{M_b M_c}{EI} ds = \frac{R^2}{EI} \int_0^{2\phi} \sin(\phi-\theta) \sin\theta d\theta$$

$$= \frac{R^2}{2EI} \cos\phi (\sin 2\phi - 2\phi)$$

$$\int_0^{2\phi} \frac{T_b T_c}{GJ} ds = \frac{R^2}{GJ} \int_0^{2\phi} \cos(\phi-\theta)(1-\cos\theta) d\theta$$

$$= \frac{R^2}{GJ} \left\{ \int_0^{2\phi} \cos(\phi-\theta) d\theta - \int_0^{2\phi} \cos\theta \cos(\phi-\theta) d\theta \right.$$

$$\int_0^{2\phi} \cos(\phi-\theta) d\theta = 2\sin\phi \;; \int_0^{2\phi} \cos\theta \cos(\phi-\theta) d\theta = \phi\cos\phi + \sin\phi \cos^2\phi$$

Hence $$\int_0^{2\phi} \frac{T_b T_c}{GJ} = \frac{R^2}{2GJ} \left[3\sin\phi - 2\phi\cos\phi - \sin\phi \cos 2\phi \right]$$

$$\therefore \delta_{bc} = \frac{R^2}{2EI} \cos\phi \left[\sin 2\phi - 2\phi \right] + \frac{R^2}{2GJ} \left[3\sin\phi - 2\phi\cos\phi - \sin\phi \cos 2\phi \right]$$

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