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IN-SITU PARTIALLY PENETRATING VARIABLE HEAD TESTS

by

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B.S. in C.E., Boğaziçi University, 1993

Submitted to the Institute for Graduate Studies in Science and Engineering in partial fullfilment of the requirements for the degree of

Master of Science

in

Civil Engineering

Boğaziçi University

1996

ACKNOWLEDGEMENTS

When I first started to work on this thesis, no one would have imagined that I would finish it at Imperial College in Silwood Park, in the United Kingdom. Life is in deed worth to live but such unknowns that emerge throughout the linear pace of the life renders it more and more attractive for us.

There are a lot of people who contributed throughout this study. I am deeply grateful to all of them. But first of all I want to thank God gave me health, strength and reasoning. During my most troublesome times of trial and suffering, He was the only one having footprints in the sand.

My Thesis supervisor Dr. Cem B. Avcı encouraged, supported and guided me at all stages of this dissertation. I have gained a lot from his experience and knowledge. The high quality standards he was expecting throughout this study taught me how to search and search invariably for "the better". I am thankful to him for lightening the path I was and is pursuing. I want to thank Dr. Osman Börekçi for constantly being interested in my work and raising precious suggestions. He contributed a lot to my background on fluid mechanics and hydraulics. His presence in the Examining Committee forced me to pay great attention to every little detail of this study. Dr. Orhan Yenigün was also a member of the Examining Committee and made valuable suggestions on the final shape of the thesis. I am thankful to him.

My family, namely my mother Nilüfer Kılanç and my aunt Nazmiye Ergeneman were always with me when I needed them. I do not know what I would have done without their assistance. They always tried to understand and support me in whatever I did. When struggling with problems throughout this study I realised once more their important role in my life. Thank you Mum, thank you my sweet Aunt... I also want to thank my Uncle, Nuh Ergeneman, for having a precious personality and a brilliant career which I adored and considered as my ultimate goal to reach since I was borne.

I want to raise my sincere thanks to the academic staff of the Civil Engineering Department of Boğaziçi University who built up my Civil Engineering background and constantly interested in the progress of this study, particularly Dr. Erol Güler, Dr. Gülay (Aşkar) Altay, Dr. Atilla Akkoyunlu, Dr. Gökhan Baykal, Dr. Turan Durgunoğlu and Dr. Erol İnelmen. I also want to thank Dr. John Mather from Royal Holloway Bedford New College, University of London, who contributed a lot to my background on Groundwater Hydraulics.

My friends both in Turkey and in England were always by my side when I needed them. It is unfortunately not possible to name all of them here. My special thanks goes to Harald Struwe for letting me use his own computer, Feyza Türkeş for never giving up encouraging me, Arzu Çamlıdere for being 'my older sister' by my side, Gisele De Franca Oliveria for constantly trying to understand me (You are a star!), Song Soo Kim ('Are you OK?'), Susan Dickinson ('Go away Burak, you're working me very hard!'), Cheryl Case, Nilay Tatlısöz, Pınar Özkan (thanks to Internet), Kemal Burak Hanoğlu, Duygu Onat ('Hello,...') and members of the Geoenvironment Team Osman Pekarun, Emir Günim and Levent Veryeri. I also want to thank Güzide Meltem Erer for always being perfect to me.

I want to dedicate this thesis to my family and who care for and believe in me and what I do.

IN-SITU PARTIALLY PENETRATING VARIABLE HEAD TESTS

ABSTRACT

Variable head tests (slug tests) represent one of the in-situ permeability tests that can be performed to characterise the hydraulic properties of porous media. Groundwater quality remediation projects and the associated risk assessment studies have increased the need for improved methods of analysis of data collected from in-situ tests. Analysis procedures were developed in the present study for slug tests performed in monitoring wells partially penetrating confined or unconfined, anisotropic, finite depth aquifers with the ultimate goal of estimating the hydraulic parameters as accurately as possible. To this end, a new set of governing equation and boundary conditions were analysed which were free from some inherent assumptions utilised by previously established methods of analysis. The effect of skin friction, wellbore and aquifer storage characteristics were incorporated. The applicability of the present model for confined and unconfined aquifers was investigated by analysing slug test data obtained from the literature. The results indicated close matches with actual field values. The effect of specific yield of the aquifer on the response of well-aquifer systems under unconfined conditions was studied. The results suggested that the well-aquifer response is influenced by the specific yield towards the later times of the test.

DEĞİŞKEN SEVİYELİ KISMİ DELEN SAHA DENEYLERİ

ÖZET

Değişken seviyeli saha deneyleri (slug deneyleri) geçirgen ortamların hidrolik özelliklerini belirlemek için kullanılan metodlardan biridir. Yeraltı suyu kalitesini iyileştirme projeleri ve buna bağlı risk değerlendirme çalışmaları saha deneylerinden elde edilen verilerin analiz metodlarının iyileştirilmesi ihtiyacını doğurmuştur. Bu çalışmada serbest yüzeyli ve basınç altındaki akiferleri kısmi delen gözlem kuyularında yapılan slug deneyleri ile ilgili yeni analiz metodları geliştirilmiştir; güdülen temel amaç hidrolik parametrelerin en iyi şekilde tahmin edilmesi olmuştur. Bu amaçla, şu ana kadar kabul görmüş analiz metodlarının yapmış olduğu basitleştirici kabullerden arınmış denklem ve sınır koşulları kullanılmıştır. Kuyu etrafındaki örselenmiş bölgeden kaynaklanan basınç kayıpları, kuyu ve akiferin (storage) depolama özellikleri de analize dahil edilmiştir. Geliştirilen çözümlerin geçerliliği literatürden alınan deney verileri kullanılarak incelenmiştir. Elde edilen sonuçlar gerçek saha değerleri ile yakın uyum göstermiştir. Serbest yüzeyli koşullarda akiferin özgül verim (specific yield) değerinin akifer-kuyu sistemini nasıl etkilediği de bu çalışmada incelenmiştir. Sonuçlar, testin ilerleyen safhalarında özgül verimin etkili olduğunu ortaya koymuştur.

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Section 2

A	Parameter used in Bouwer and Rice Method (1976)
B	Parameter used in Bouwer and Rice Method (1976)
C	Parameter used in Bouwer and Rice Method (1976)
D	Thickness of the study formation
F	Shape factor
h	Head in the well bore at time t
H	Original head in the well bore
H_0	Initial change in the well head
ΔH_{skin}	Pressure drop across the 'skin'
k	Intrinsic permeability
k_h	Horizontal hydraulic conductivity
k_v	Vertical hydraulic conductivity
K	Hydraulic conductivity
K_x	Hydraulic conductivity in x direction
K_y	Hydraulic conductivity in y direction
K_z	Hydraulic conductivity in z direction
K_θ	Angular hydraulic conductivity
L	Length of the screen
q(t)	Rate of inflow at the piezometer tip
Q	Volumetric rate across the skin (Section 2.3.2.)
Q	Inflow into the well bore (2.3.4.3.)
r_c	Casing radius
r_s	Screen radius
r_{seff}	Effective screen radius
r_w	Radial distance of undisturbed portion of aquifer from centreline of the well bore
R_e	Effective radius over which y dissipates

s_{b1}	Surface on which Neuman conditions prevail
s_{b2}	Surface on which Dirichlet conditions prevail
$s_{boundary}$	Union of s_{b1} and s_{b2}
s_{open}	Open surface (screen) of the well bore
S	Storativity
S_c	Specific storage coefficient
t	Time
T_0	Basic time lag
W	Dimensionless time-scale used by Cooper et. al. (1967)
y	Piezometric head difference between inside and outside of the well
α	Storage parameter used by Cooper et. al. (1967)
μ	Viscosity
ν	Region affected by the slug test

Section 3, 4, 5, and 6

Throughout this sections the symbols overlain by a ~ have dimension; the symbols with an overbar are transformed by Laplace transformation.

A	Arbitrary coefficient
A_0	A subsidiary group
A_n	A subsidiary group
B	Arbitrary coefficient
C_D	Dimensionless coefficient describing the storage characteristics
D	Thickness of the confined aquifer or the height of water column for unconfined aquifer
E	Arbitrary coefficient
F	Arbitrary coefficient
f(t)	A function

f(x,t)	A function
h	Hydraulic head within the aquifer
h^*	Average head along the screen
h_0	Instantaneous water level change in the well bore
h_w	Water head in the well bore
ΔH	Pressure drop across the 'skin' (Footnote 8)
I0, K0	Modified Bessel Functions of second kind of order zero
I1, K1	Modified Bessel Functions of second kind of order one
k	Arbitrary constant
K_r	Radial hydraulic conductivity
K_z	Hydraulic conductivity in z direction
L	Screen length
m	An integer
n	An integer
N_{steh}	An even number representing the number of Stehfest points;
p	Independent variable of Laplace transformation
q_0	A subsidiary group used with Modified Bessel Function
q_n	A subsidiary group used with Modified Bessel Function
q_{scr}	Darcy flux across the screen
Q	Influx / outflux to of from the well bore
r	Radial distance from the centre of the well bore
S_c	Specific storage coefficient
S_k	Skin factor
S_w	Dimensional skin factor given by Carslaw and Jaeger (1949)
S_y	Specific yield
t	Time
W_i	A weight coefficient depending on N_{steh},
z_a	Distance from the bottom of the screen to the confining lower boundary
z_b	Distance from the top of the screen to the confining lower boundary
$\mathcal{L}$	Laplace transformation

1. INTRODUCTION

As viewed from a spacecraft, the earth appears to have a blue-green cast due to the vast amount of water covering the globe. Contradicting this fact, only a very small portion of earth's total water supply is available to humans as fresh water. The great majority of water on earth, namely 97.2 percent of the total, is saline and held by oceans. The remaining 2.8 percent is kept by land areas and mainly retained by ice caps and glaciers. This renders surface and groundwater the only available freshwater reserves for human consumption. (Figure 1.1)

Groundwater, which can be described as the subsurface water that occurs beneath the water table in fully saturated soils and geologic formations, constitutes 98 percent of the total freshwater reserve of earth. It is an important water supply for municipalities, agriculture and industry with an increasing trend in its use over the past decades (Figure 1.2.). Twenty six percent of all Canadians and over half the population of the United States (US henceforth) depends on groundwater as drinking water supply (Kruus et. al., 1991). In England and Wales some 32 percent of public water supplies are derived from groundwater with rising this figure up to 75 percent in parts of southern England (Mather, 1994). In the arid and semi-arid parts of Africa and the Middle East groundwater is the only source of supply. Besides its significance as a natural resource, groundwater also constitutes the baseflow component of streams, rivers and lakes. Therefore all studies related to groundwater such as its utilisation, protection, contamination, remediation are directly associated with one of the basic needs of humans, i.e. **water**.

The primary motivation for the study of groundwater has traditionally been its importance as a resource. During late 1970s, however, much of the emphasis in groundwater investigations in industrialised countries has shifted from problems of groundwater supply to considerations of groundwater quality simply because of the increase in public awareness and concern about the state of the

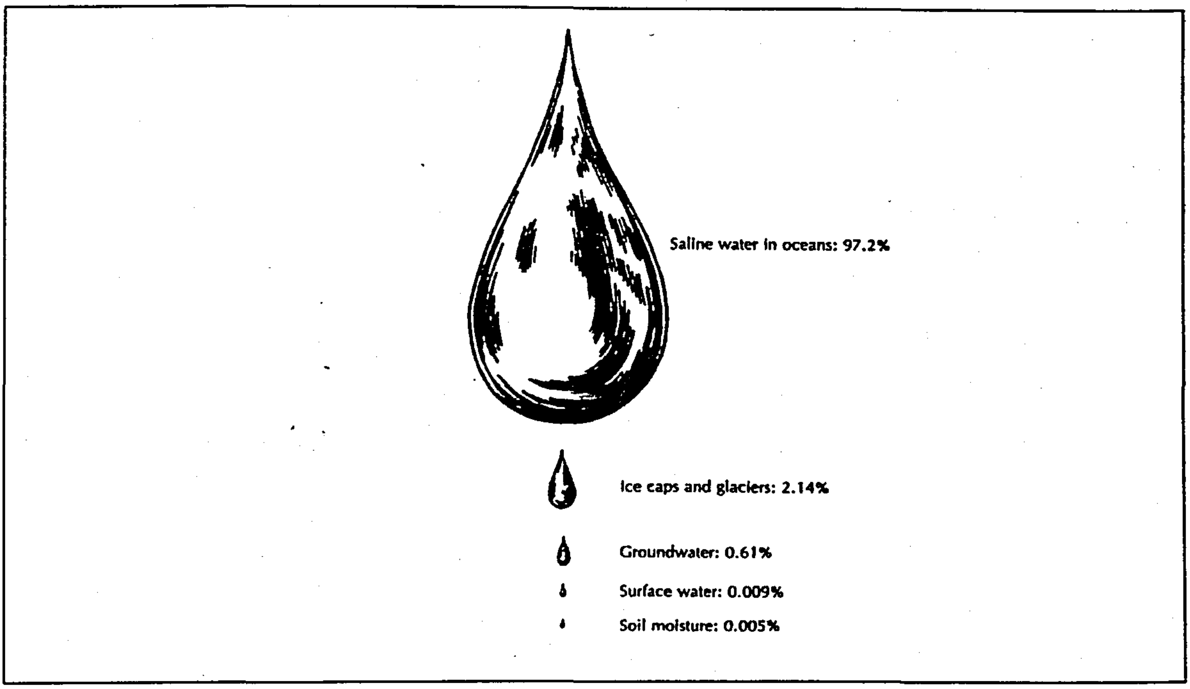

FIGURE 1.1. Worlds water supply (Source: Fetter, 1988)

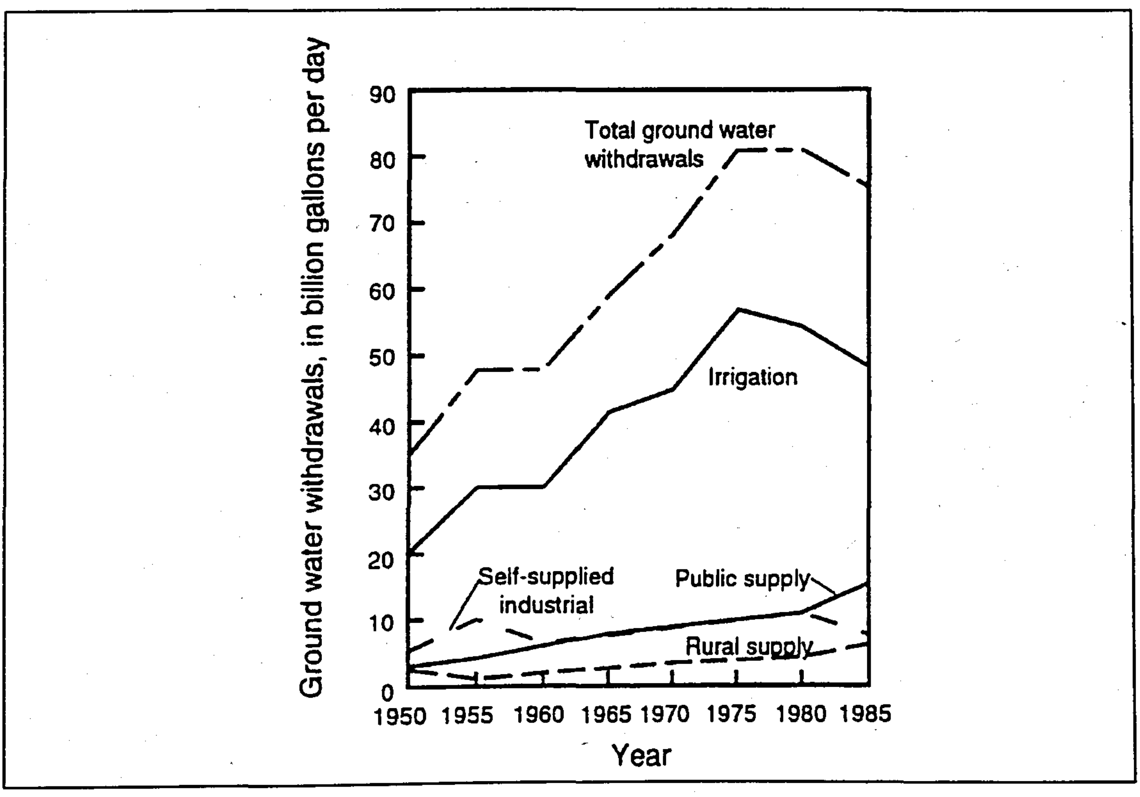

FIGURE 1.2. Trends in groundwater usage in the US (Source: Bedient et. al., 1994)

environment (Freeze and Cherry, 1979). Particularly, the discovery of hazardous wastes at Love Canal in Niagara, New York (1979) and at the Denver Arsenal in Colorado, in the US, and Lekkerkirk near Rotterdam, in the Netherlands (1980), attracted public and scientific attention towards the potential problems of groundwater pollution and subsequent health risks, and prompted the so called **hazardous waste decade** of 1980s. The response of the governments to these discoveries has been revising their environmental policies and implementing tight legislative frameworks on the protection and improvement of the environment, natural resources and public health. Enforcing bodies, such as Environmental Protection Agency, EPA in US and National Rivers Authority, NRA (recently Environment Agency), in UK were commissioned to administer the pertinent legislation and to search for, rank and remediate, if necessary, the polluted sites. With all these changes hydrologists, hydrogeologists, civil and environmental engineers and other scientists were involved characterising, evaluating and remediating hazardous waste sites with respect to groundwater contamination. The field of groundwater has seen an explosion in the number of risk assessment and remedial investigation studies related to thousands of abandoned and hazardous waste disposal sites and leaking tanks across the world (Bedient, et. al., 1994). Figure 1.3 demonstrates the characteristic pathways of contaminants from waste disposal facilities to groundwater systems, which are frequently encountered in such studies.

The evaluation of the consequences of actual or potential contaminant releases into the environment and associated regulatory liabilities of such occurrences usually necessitates the utilisation of well established hazard, performance and/or risk assessment methods. The Hazard Ranking System (HRS), the Site Ranking System (SRS), the Remedial Action Priority System (RAPS) and the Multimedia Environmental Pollutant Assessment System (MEPAS) which are used in the US; the Hazard Assessment of Landfill Operations (HALO) and the Safety Assessment Comparison (SACO) methodologies employed in the United Kingdom (UK henceforth); and PRIAF

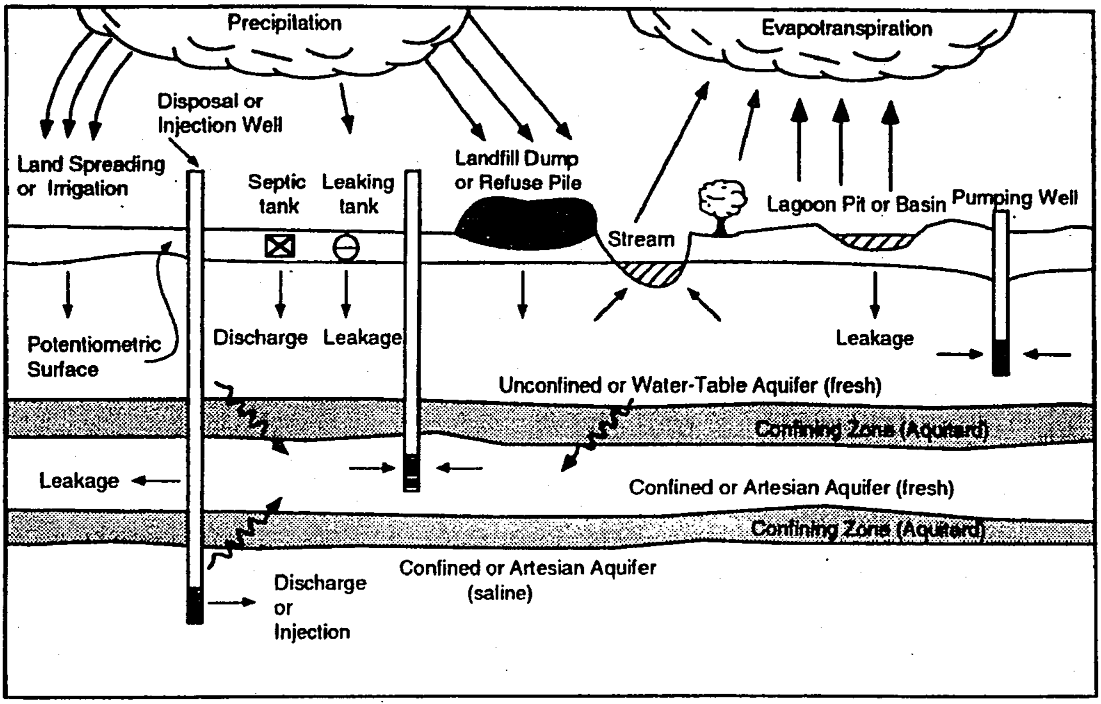

FIGURE 1.3. Pathways of contaminants from waste disposal activities to groundwater systems (Source: Bedient et. al., 1994)

system exercised in the Netherlands are typical examples of such methods (Little, 1996). The conceptual representations of two of these methods, namely SACO and RASP methodologies, are given in Figures 1.4 and 1.5., respectively. As reflected by these figures groundwater constitutes one of the major pathways of contaminant transport within the environment and therefore should be included in any hazard, performance and risk assessment methods as the geosphere transport media.

All the above mentioned widely accepted methods are founded on physics-based modelling of the environment using empirical, analytical, semi-analytical and computational algorithms; with the ultimate goal of describing the investigated physical system as accurately as possible. An increase in the level of sophistication of the model might improve its analysis accuracy **but the analysis can never be more accurate than the information on which it is based** (Whelan et. al., 1992). The quality of the parameters characterising physical

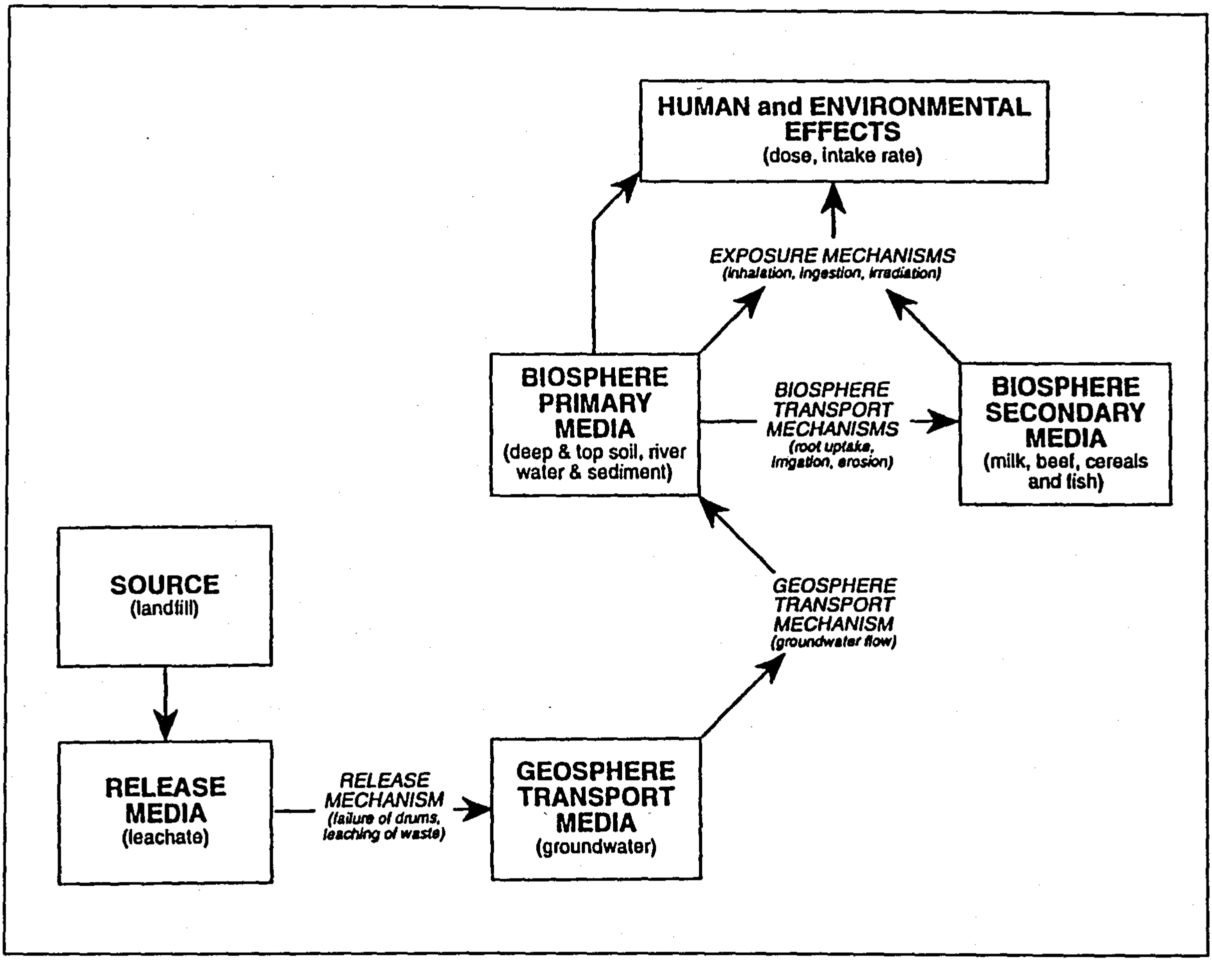

FIGURE 1.4. Conceptual Model of SACO Methodology (Source: Little, 1996)

phenomena embraced within a model is of vital importance for the overall reliability of the model. Consequently during last decades a significant amount of effort in this field focused mainly on determination of such parameters as accurately as possible. Adsorption coefficient of chemicals describing their environmental fate behaviour within soils, dispersion coefficient of contaminants in the atmosphere used by plume models, **hydraulic conductivity and storativity of geologic formations governing groundwater movement,** can be cited as typical examples of such parameters.

The present study focuses on mathematical modelling of one of the field tests that is widely used in practice for determining hydraulic properties (hydraulic conductivity and storativity) of geologic formations, namely **in-situ variable head**

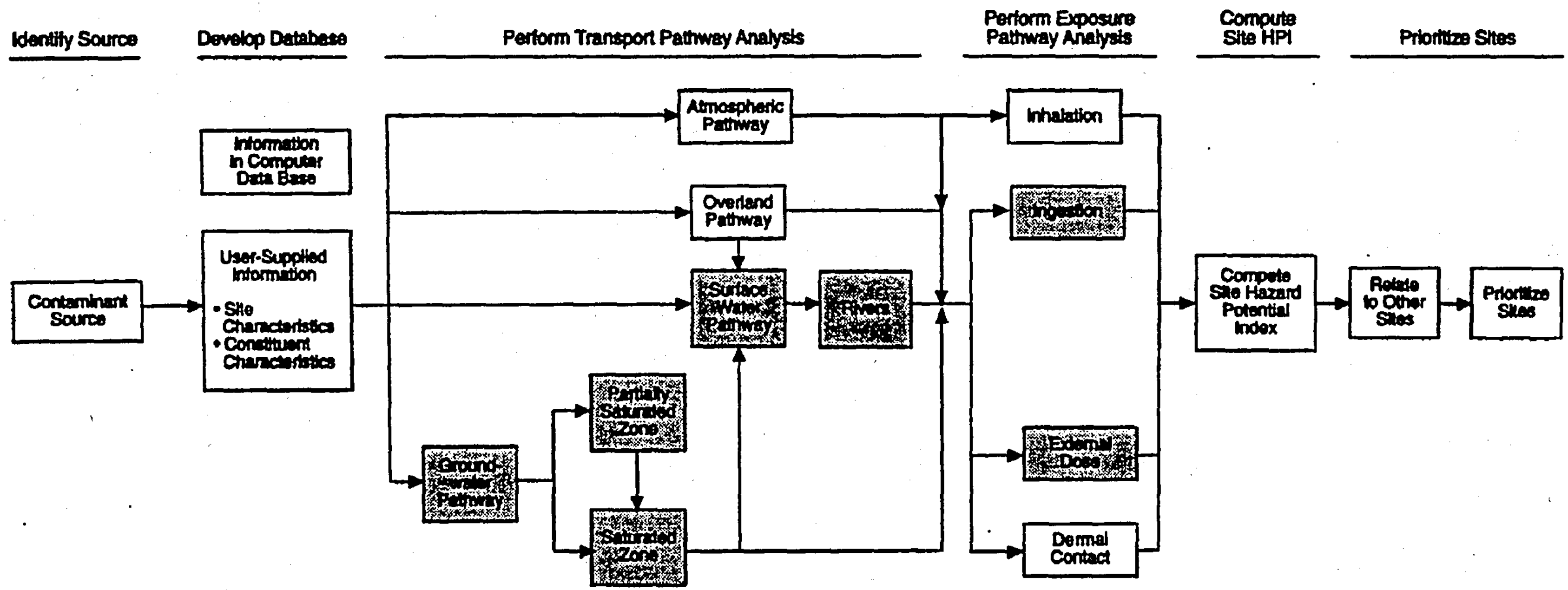

FIGURE 1.5. Conceptual model of RASP methodology (Source: Little, 1996)

tests or simply **slug tests**. The test procedure consists of creating a relatively fast initial change in the original water level present in the well casing and subsequently measuring the water level recovery. The hydraulic properties of the porous media are calculated using the water level changes observed in the casing together with the information near the well screen, well geometry and volume of initial water level displacement. Analysis procedures were developed for slug tests performed in monitoring wells partially penetrating confined or unconfined, anisotropic, finite depth aquifers with the ultimate goal of estimating hydraulic parameters as accurately as possible. To this end, a new set of governing equation and boundary conditions were analysed which were free from some inherent assumptions utilised by previously established methods of analysis. The solution procedures used by Dougherty and Babu (1984) for confined conditions were adapted to solve the governing equations in Laplace domain both for confined and unconfined aquifers. Real time solutions were obtained by inverting them numerically using the Stehfest algorithm; the reliability of this algorithm for similar problems is widely accepted in research community. The applicability of the developed procedures for confined and unconfined conditions were investigated by analysing slug test data obtained from the literature. One of the boundary conditions used in this study for unconfined settings, which describes the water table as a transient material boundary (Type 2.b.), was never investigated before during the evolution of the slug test theory, to the best of authors knowledge.

2. GENERAL CONSIDERATIONS AND LITERATURE REVIEW ON IN-SITU VARIABLE HEAD (SLUG) TESTS

The variable head test is one of the most commonly used techniques by the hydrogeologists for estimating hydraulic properties in the field (Kruseman and de Ridder, 1989). This technique, which is quite simple in practice, consist of creating a relatively fast initial change in the original water level present in the well casing and subsequently measuring the water level recovery. The procedure can be conducted in monitoring wells, piezometers or standpipes (Avci, et. al., 1995) and besides its major utilisation for hydraulic conductivity determination, can also be used **(i)** to measure the observation well response time in leaky aquifer tests (Black and Kipp, 1977), **(ii)** to assess the degree of well development, **(iii)** to specify response criteria for observation wells in aquifer test designs (Black, 1978), and **(iv)** as a tool for monitoring the effects of clogging in small diameter observation wells (Dax, 1987).

In-situ variable head tests can be conducted in two different basic forms, namely the rising head test and the falling head test. Historically, an initial increase in water level was termed the 'slug method' (Ferris and Knowles, 1954) and the initial decrease the 'bailer method' (Skibitze, 1958). However this distinction is not maintained today (Chirlin, 1989) and both methods are called 'the slug test method'. This term is invariably used in this study to refer in-situ variable head tests.

Although slug test analysis methods were introduced into the hydrogeology and petroleum engineering four decades ago, its utilisation was initially hampered by several technical and practical difficulties until mid 1970s[1] (Black, 1978). The development of pressure transducers and simultaneous use of electronical measuring devices (millivolt chart recorder, strip chart recorder) enabled accurate

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[1] In hydraulic settings with high transmissivity values, the initial slug dissipates very quickly and manual monitoring of water level is practically not possible.

and fast water level readings to be obtained during slug tests and caused this method to become one of the classical techniques of hydrogeology during the last two decades. The advantages of slug test method against other field methods are summarised below:

(a) the test is cost effective because the required amount of equipment and man-power for conduction of the test is small:

- no pumping apparatus and observation wells are required;
- only a small amount of water (if any) is required to initiate the test;
- small-diameter boreholes normally used in geologic exploration may serve for slug testing (double duty) in a hydraulic testing program (Domenico and Schwartz, 1990) ;

(b) this testing technique requires relatively short duration for completion;

(c) slug tests can be conducted in formations of low hydraulic conductivity where pumping tests would not be practical to proceed[2];

(d) particularly on the sites of suspected and actual contamination, use of slug tests does not require the withdrawal and subsequent disposal of contaminated groundwater;

(e) widely used slug test data analysis methods[3] are easily perceivable for field practitioners.

2.1. Typical Slug Test Procedures

The simplest way of creating the initial change in the water table is adding or removing a known volume of water to or from the well bore. However a number of draw-backs of these procedures, particularly the technical problems associated

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[2] this renders the application of slug tests particularly popular in the investigations of hazardous waste sites where the low hydraulic conductivity of the geologic formations is the major concern.

[3] Hvorslev (1951), Cooper et. al. (1967), and Bouwer and Rice (1976) Methods

with pouring water down a well (Black, 1978) are reported in literature and limiting their practical use. The initial water level change in the well may also be achieved by suddenly introducing or removing a solid cylinder of known volume. As schematically shown in Figure 2.1.1. such a slug test is conducted by first measuring the static level in the well and then lowering a rod of known displacement inside the well below the static water level. After the equilibrium is attained in the well, the rod is pulled rapidly from the well, which will cause an instantaneous drop in the water level. The recovery of the water level is than observed. Monitoring the water level can be done manually in low permeability systems but is not possible in higher yield systems because of very quick dissipation of initial perturbation. Figures 2.1.2. and 2.1.3. demonstrates typical (overdamped[4]) slug test data plotted on semi-logarithmic paper with linear and logarithmic timescales, respectively.

Over the last decades, in order to overcome encountered technical problems, to increase the level of control on test parameters, to improve the testing accuracy and to satisfy specific site conditions several slug test practices and related test configurations were developed. Two examples of such commonly used configurations, one used for multilevel slug tests and the other employing a pressure supply for creating the initial head, are presented in Figures 2.1.4. and 2.1.5., respectively.

[4] Adjective, describing the aquifer-well response to the slug test and discussed in detail in the next section.

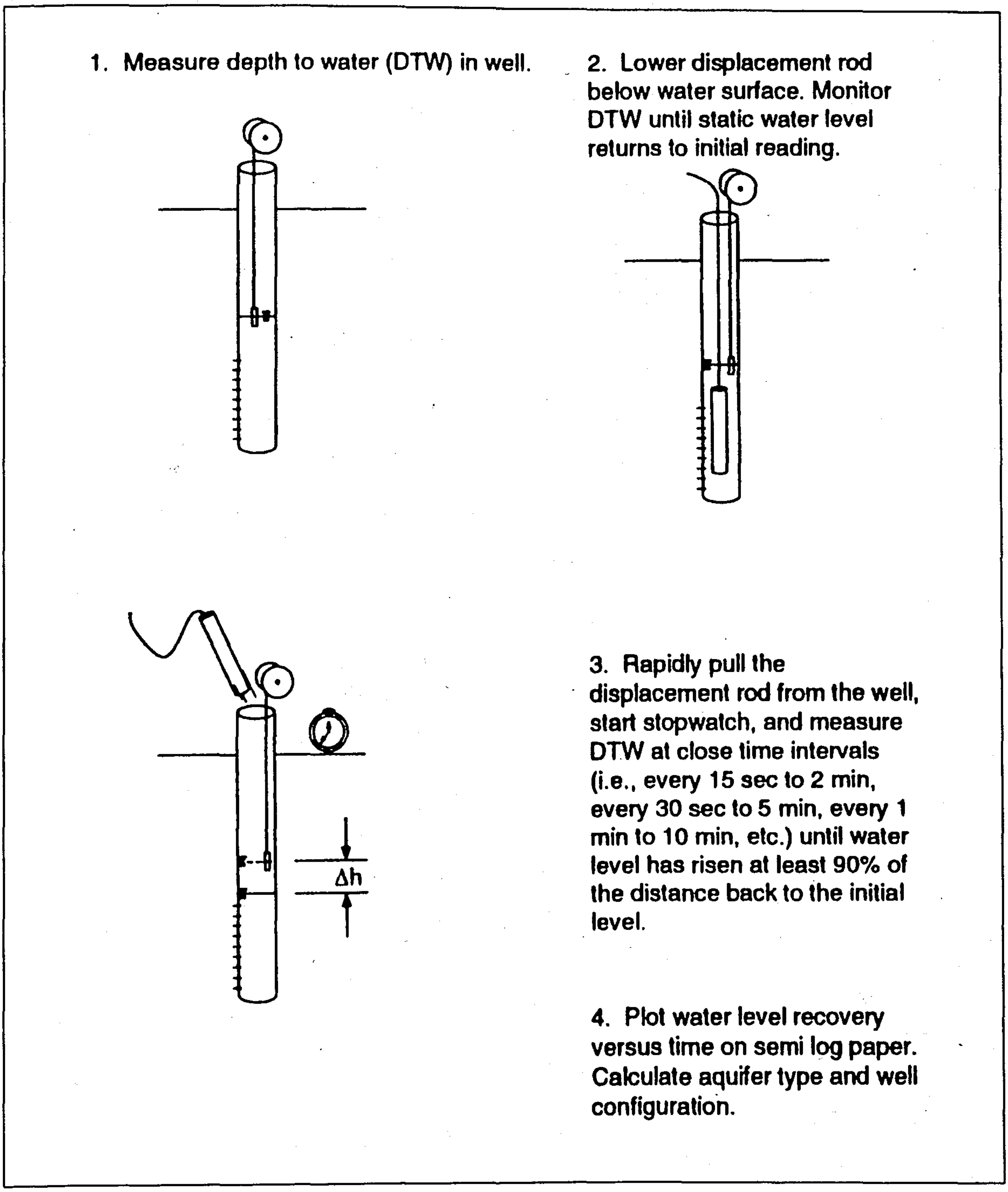


FIGURE 2.1.1 Typical slug test procedure (Source: Bedient et. al., 1994)

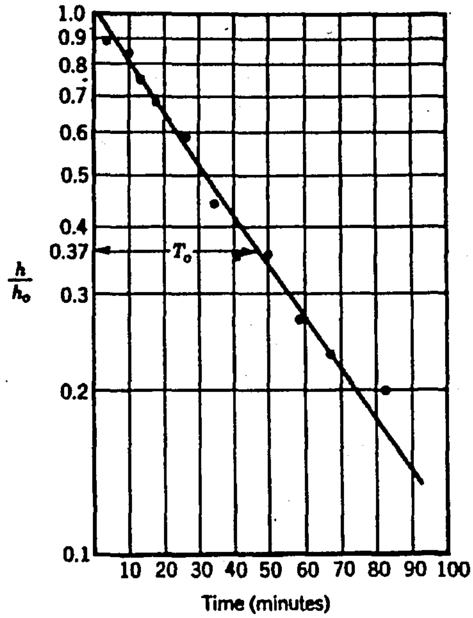


FIGURE 2.1.2. Field response of a slug test, semi-logarithmic plot with logarithmic well head scale (Source: Bedient et. al. 1994)

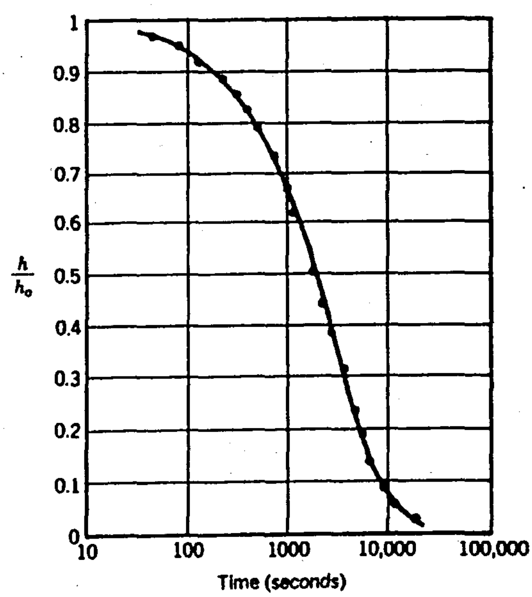


FIGURE 2.1.3. Field response of a slug test, semi-logarithmic plot with logarithmic time scale (Source: Bedient et. al. 1994

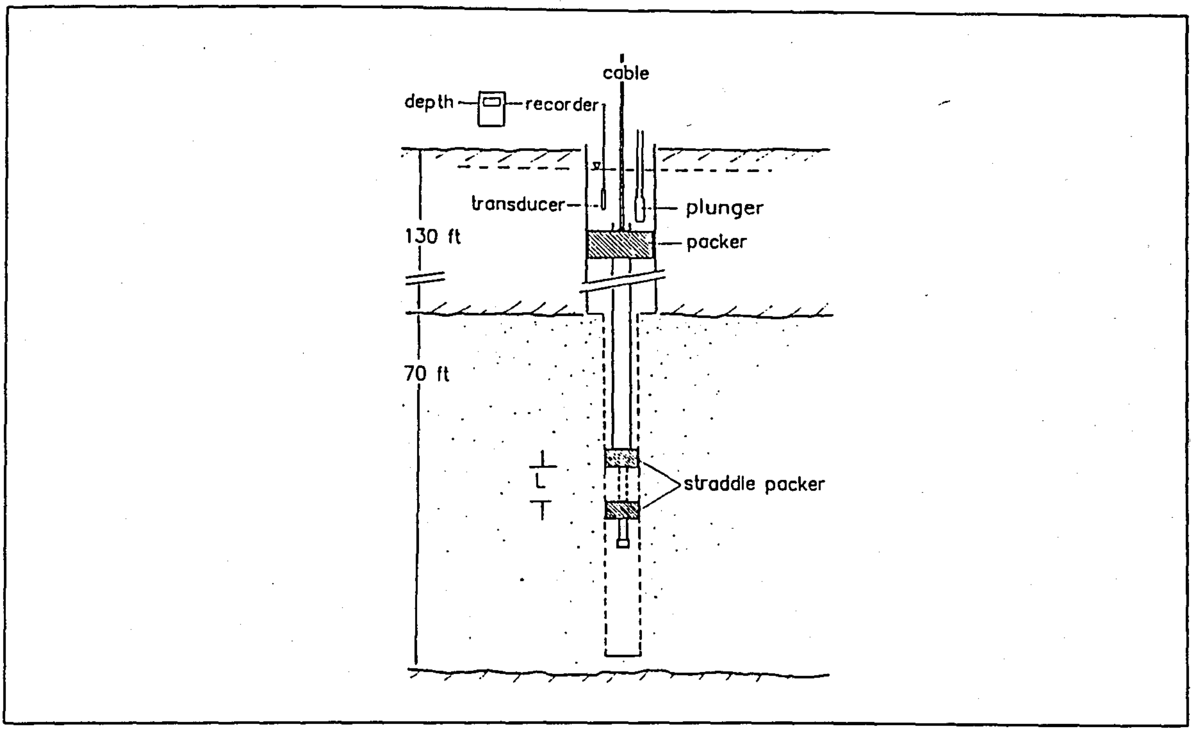


FIGURE 2.1.4 Multilevel slug test apparatus (Source: Widdowson et. al., 1990)

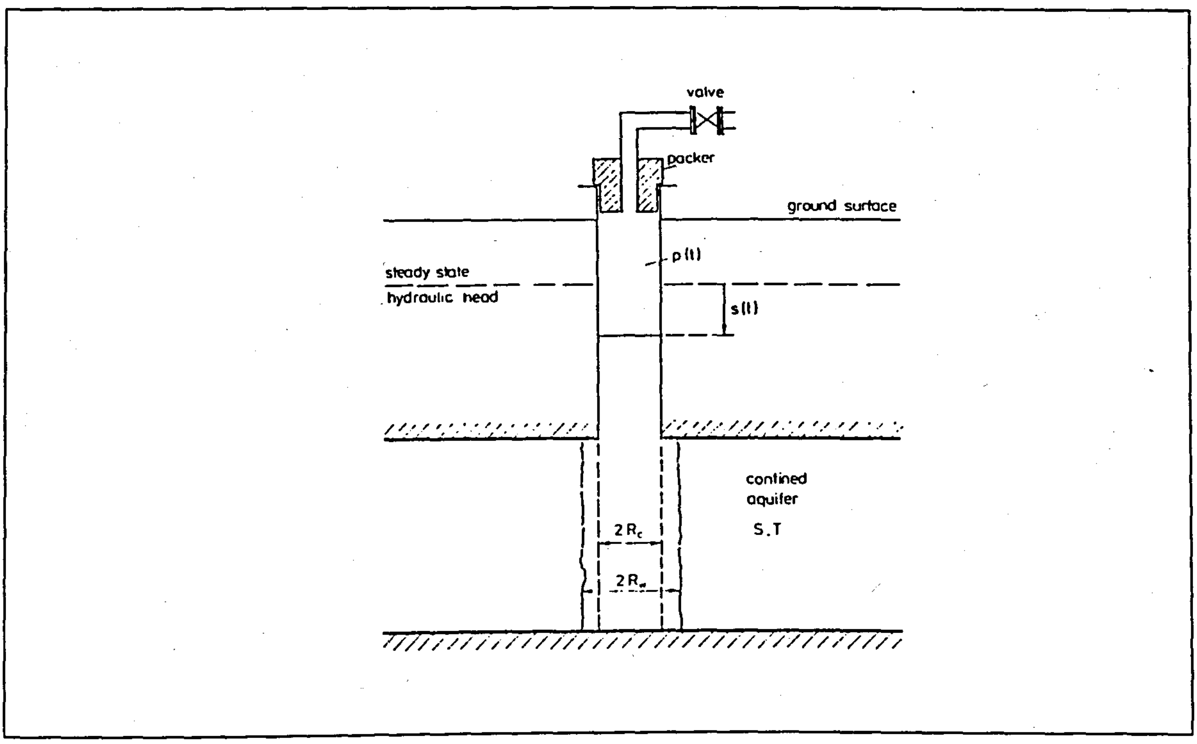


FIGURE 2.1.5. Arrangement of a slug test which employs a pressure supply (Source: Marchall and Barczewski, 1989)

2.2. The Response of a Well-Aquifer System to a Slug Test

The force-free response of a well-aquifer system to an abruptly induced change of water level in the well is analogous to the classical mechanical system of a mass on a spring in a viscous medium. The water in the well corresponds to the mass, and the aquifer corresponds to the spring. Depending on the mass of water in the well and the hydraulic characteristics of the aquifer, especially its hydraulic conductivity, the response of the water level in the well may be either as an overdamped, critically damped, or underdamped oscillator. According to van der Kamp (1976) when the system is overdamped, the water level returns to the equilibrium level in an approximately exponential manner. In the underdamped case the water level oscillates about the equilibrium level. The critical damping case is the transition between the two types of response. Three actual cases of well responses observed in three different wells in Cap Pele Aquifer are given in Figure 2.2.1.: well 2C reflects underdamped response, well 2A experiences critical damping and well 9A demonstrates overdamped repines. In practice, the most frequently encountered aquifer-well response is the overdamped case. Critically and underdamped cases are observed occasionally, especially if the hydraulic conductivity of the aquifer is high, the storage characteristics are insignificant and the mass of water in the well is large (deep wells).

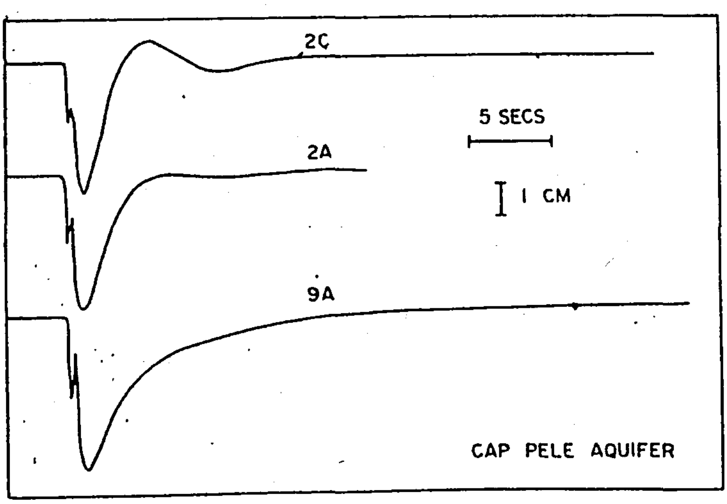


FIGURE 2.2.1. Well responses in Cap Pele Aquifer
(Source: van der Kamp, 1976)

2.3. The Evolution of the Slug Test Theory

2.3.1 General Considerations

In the process of any well testing, an input impulse (perturbation) is provided to the aquifer and the resulting response is measured. The aquifer response is governed by parameters such as the hydraulic conductivity, storage coefficient, specific yield, skin effect, distance to boundaries, fracture properties, double porosity coefficient, etc. (Horne, 1990). Based on an understanding of the aquifer physics, a mathematical model of the dependence of the response on these reservoir parameters is developed. Then by matching the model response (the type curves) to the measured aquifer response it is inferred that the model parameters take the same values of the aquifer parameters (Figure 2.3.1.1.). Well test interpretation is therefore an **inverse problem** in that model parameters are inferred by analysing model response to a given input, however, the quality of the parameter estimations depends highly on the quality of the mathematical model, i.e. how accurate it formulates the physics of the investigated system.

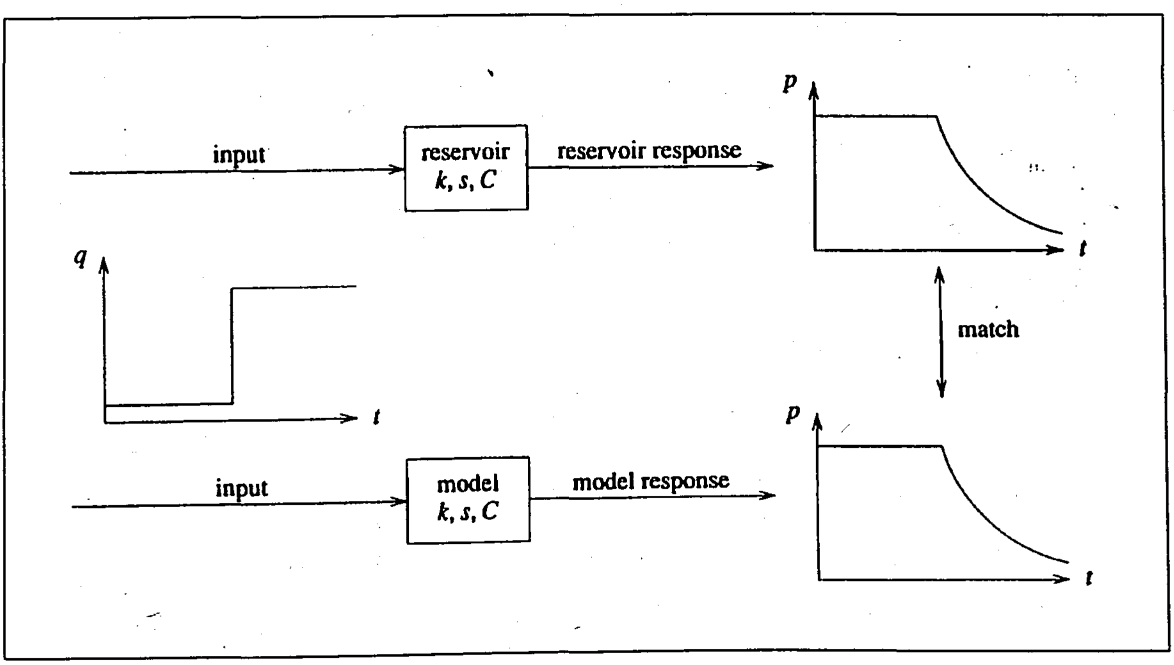


FIGURE 2.3.1.1. Schematical of inverse problem (Source: Horne, 1990)

During the last 40 years many mathematical models describing slug test response of aquifers in varying complexity have been developed with the ultimate goal of estimating unknown hydraulic parameters as accurately as possible. These parameters include invariably the hydraulic conductivity (or transmissivity), and in some models the specific storage or storage coefficient and a skin factor (Chirlin, 1989). The next section explains the development of slug test models in greater detail.

2.3.2. Model Development for Slug Tests

The induced groundwater flow within an aquifer experiencing a slug test can be formulated as a transient boundary-value problem. The following six points should be known for describing such a problem completely (Freeze and Cherry, 1979):

- the size and shape of the region of flow;
- the governing equation of flow within the region;
- the boundary conditions around the boundaries of the region;
- the initial conditions in the region;
- the spatial distribution of the hydrogeologic parameters that control the flow; and
- a mathematical method of solution.

During the last four decades, the evolution of the slug test theory progressed by attempts of solving this transient boundary value problem with the use of several simplifying assumptions on the above six points. The introduction of such assumptions were aimed at **(i)** casting the problem into a mathematically solvable form[5]; **(ii)** neglecting various physical phenomena which were presumed

[5] Typical examples: (i) semi-infinite aquifer conditions assumed by Hvorslev (1951)
(ii) storage characteristics of the aquifer
(iii) methods focusing on only one kind of aquifer types

to be insignificant for the system; or **(iii)** restricting the analysis into a preferred field setting. However such assumptions compromise the generality and the accuracy of the obtained solution and consequently the quality of estimated parameters. Therefore, since the introduction of the first analysis methods on slug tests into the hydrogeology literature, much work has been directed at removing one or more of previously accepted simplifying assumptions.

The most general setting for a slug test is depicted in Figure 2.3.2.1. A well penetrating the porous formation is open over a surface S_{open} and impermeable elsewhere. The radius of the open portion (screen) is r_s whereas the casing radius is denoted by r_c; the water level within the well is assumed to change within the casing only. The region that is affected by the slug test within the formation is delineated by ν and consists of surfaces s_{open} and $s_{boundary}$. The latter can further be subdivided into two types, namely:

s_{b1}: on which Neuman conditions prevail, frequently describing surfaces with no-flow across them;

s_{b2}: where Dirichlet conditions hold and the head is specified along the surface which may or may not change with time.

The shape of ν is a model assumption (finite of infinite medium) and the conditions s_{b1} and s_{b2} along with s_{open} determine the boundary conditions of the problem (e.g. confined or unconfined aquifer, partial penetration, etc.).

The most general form the governing equation describing groundwater flow within the aquifer is given in Cartesian coordinates by

$$S_c \frac{\partial h}{\partial t} = K_x \frac{\partial^2 h}{\partial x^2} + K_y \frac{\partial^2 h}{\partial y^2} + K_z \frac{\partial^2 h}{\partial z^2} \qquad (2.1)$$

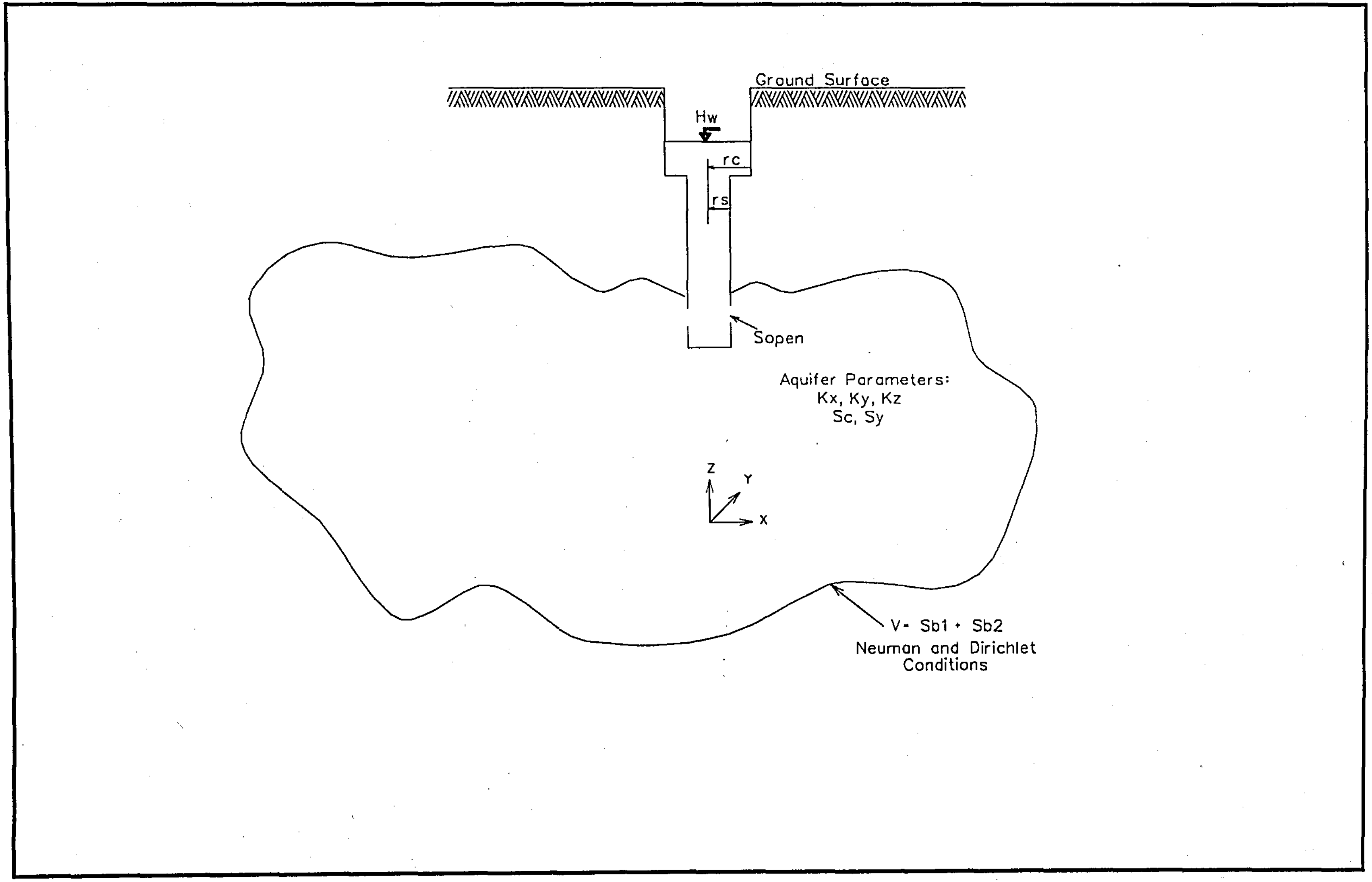


FIGURE 2.3.2.1. The most general setting of a slug test

using a coördinate system which coincide with principal directions of anisotropy. In this equation the specific storage coefficient S_c and hydraulic conductivity terms K_x, K_y, K_z are functions of x, y, z and t; i.e. $S_c(x,y,z,t)$, $K_x(x,y,z,t)$, $K_y(x,y,z,t)$, $K_z(x,y,z,t)$. These terms reduce to $S_c(t)$, $K_x(t)$, $K_y(t)$ and $K_z(t)$ if the formation under investigation is homogeneous along the coordinate axes and can further be represented by constant values S_c, K_x, K_y and K_z if the aquifer properties does not change in time.

In polar coordinates equation (2.1) reads[6]

$$S_c \frac{\partial h}{\partial t} = K_r \frac{1}{r} \frac{\partial}{\partial r}\left[r \frac{\partial h}{\partial r}\right] + K_\theta \frac{1}{r^2} \frac{\partial^2 h}{\partial \theta^2} + K_z \frac{\partial^2 h}{\partial z^2} \tag{2.2}$$

For 'no angular flow' case equation (2.2) simplifies to

$$S_c \frac{\partial h}{\partial t} = K_r \frac{\partial^2 h}{\partial r^2} + K_r \frac{1}{r} \frac{\partial h}{\partial r} + K_z \frac{\partial^2 h}{\partial z^2} \tag{2.3}$$

representing an axisymmetric flow field around the well bore. Ignoring the vertical flow component within the formation yields the following equation which describes radial flow only:

$$\frac{S_c}{K_r} \frac{\partial h}{\partial t} = \frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} \tag{2.4}$$

The left hand side terms in equations (2.1) - (2.4) drop out if the storage characteristics of the formation are postulated to be negligible. If the formation under investigation is isotropic K_r and K_z become equal to each other and can be represented by a common parameter K; the governing equation (2.1) takes the form

[6] The conversion of equation (2.1) from Cartesian to polar coordinates can be found in O'Neil (1992) in greater detail.

$$\frac{S_c}{K}\frac{\partial h}{\partial t} = \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} \tag{2.5}$$

for the transient case and

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \tag{2.6}$$

for the quasi steady-state conditions.

Both equations (2.5) and (2.6) are very well known partial differential equations, **Poisson** and **Laplace** equations, respectively. Equation (2.5) is termed as the **Diffusion Equation** in hydrogeology literature and has been extensively used to describe transient flow problems over the past years.

The inner boundary conditions regarding the macroscopic mass balance of the well bore and the nature of pressure transmission between well and aquifer along s_{open} (well screen) has attracted a considerable amount of research during the evolution of slug test theory. These points are discussed in subsequent sections.

- **The Skin Effect**: In real world there is often a zone surrounding the well which is invaded by mud filtrate or cement during the drilling or completion of the well causing higher pressure drops than expected. This zone may have lower permeability than the reservoir and thereby acts as a 'skin' around the well bore casing. In Figure 2.3.2.2, ΔH represents this extra pressure drop caused by the skin effect. Carslaw and Jaeger (1949) introduced the concept of **dimensionless skin factor** in order to quantify the pressure drop across the skin.

$$S_k = \frac{k\,L}{Q\,\mu}\Delta h_{skin} \tag{2.7}$$

where k is the intrinsic permeability, m is viscosity, L is the length of screen and Q is the volumetric rate across the skin. The pressure drop in this equation is assumed to vary linearly with volumetric rate. Hurst (1953) and van Everdingen (1953) later defined a dimensionless skin factor by relating the Darcy flux (volumetric rate / area) to the pressure drop across the screen. Their factor for circular wells varies by a factor of 2π from Carslaw and Jaeger one[7].

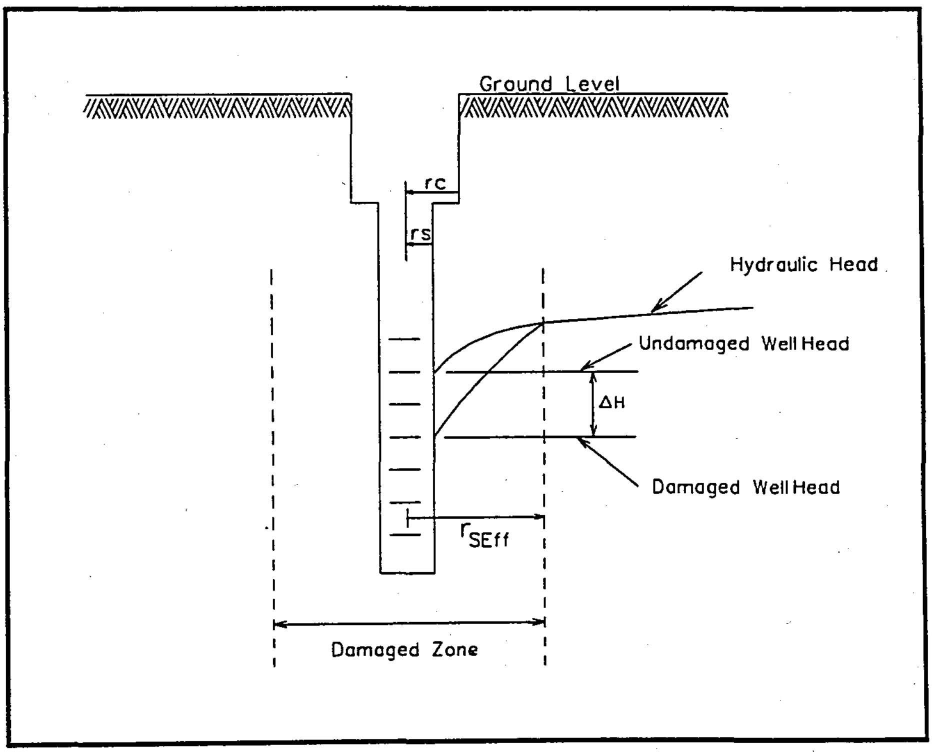


FIGURE 2.3.2.2. Schematical representation of skin effect

[7] The derivation of these factors are further discussed in Section 3.2.

The quantification of skin effect can also be done in terms of an **effective well bore radius** which can be defined as the radius that the well appears to have due to the reduction in flow caused by skin effect (Horne, 1990). The relation correlating r_{eff} and S_k is given by Earlougher (1977) as

$$r_{seff} = r_s \, e^{-S_k} \tag{2.8}$$

By the help of this equation, if one of these parameters is known or can be estimated, the other one can be calculated.

Although encountered occasionally , if the disturbed zone around the well has a higher hydraulic conductivity than the formation itself, the dimensionless skin factor takes on negative values. The usual range of dimensionless skin factors (Darcy flux approach) is reported to be between -5 and 20 (Horne, 1990).

2.3.3. Literature Review

The development of the slug test theory in the groundwater and petroleum engineering fields progressed in two major groups :

(i) models and analysis techniques that consider the inertial effects of the water column within the well bore and as such concentrating on underdamped and critically damped slug test responses, or aiming at obtaining general solutions that cover all three possible cases including overdamped case. These solution procedures are beyond the scope of this study and not discussed further.

(ii) models and analysis techniques which ignore the inertial effects of the water in the well and as such developing solutions only for overdamped slug tests.

During the first two decades of the so called "four decades of slug test" (Chirlin, 1990) much effort was put on the development of new slug test analysis techniques which made use of some significant simplifying assumptions. Although the likely negative effects of such assumptions on the generality of the obtained solutions were well realised in hydrogeology community, these analysis techniques received a wide acceptance in practice. Starting mid 1970s and with the advent of the hazardous waste decade of 1980s, research in slug test theory inclined on method refinement and verification, with the ultimate goal of estimating hydraulic properties of study formations as accurately as possible. The principal motivation behind this change of track was the introduction of tight environmental legislations which required field test methods to comply with high quality assurance standards. Much work focused on the parameter prediction capabilities and overall validity of widely accepted and applied methods. In particular existing methods were examined on whether or not

- they include the effect of well bore and aquifer storage;
- they respect the vertical flow component within the aquifer tested; two immediate relevant inferences are the effects of partial penetration and anisotropy on parameter estimations;
- they consider the skin effects.

Since late 1980s, along with model verification studies, research on slug tests inclined on sensitivity analyses, automation of analysis methods, development of new data analysis techniques in order to overcome curve matching problems (due to the nonuniqueness of the type curves, chapter 57) and very recently determination of the radius of influence of slug tests using observation wells. A chronological list of progresses in overdamped slug test theory is given below.

1951 :Although not using the term "slug test" directly, **Hvorslev** pointed out for the first time that variable head tests can be used to estimate in-situ hydraulic conductivity. He presented solutions for homogeneous, isotropic or anisotropic, infinite or semi-infinite aquifer settings, neglecting the storage properties of the aquifer. His analysis method is considered the simplest interpretation of piezometer recovery data (Freeze and Cherry, 1979) and very widely accepted in practice. The details of this method is given in section 2.3.4.1.

1954 :Ferris and Knowles introduced the term "slug test" into the hydrogeology literature by extending Theis' line source solution for pumping wells. They employed an instantaneous line source approach to describe slug tests in an infinite medium; storage effects of the well bore were not accommodated by their approach. They showed that aquifer transmissivity can be estimated from the slope of a plot of head versus reciprocal of time. The validity of this approach was scrutinised by Bredehoeft et. al. (1966) and Cooper et. al. (1967).

1958 :**Skibitze** donated the term "bail test" to the hydrogeology terminology corresponding to in-situ rising head tests.

1966 :**Bredehoeft et. al.** showed the significance of the effect of well bore storage on the response of well-aquifer systems through an electric analogue investigation. They pointed out that Ferris and Knowles (1954) line source approach which neglects the effects of well bore storage can only be a good approximation to the finite well bore solution for sufficiently large values of time.

1967 :**Cooper et. al.** presented solution procedures for slug tests conducted under fully penetrating confined aquifer settings. Considering both the well bore and aquifer storage characteristics, they provided a set of semilog type curves which serve for estimating hydraulic properties of the tested aquifer. Their approach assume that purely radial flow occurs within the study formation. This method is very widely accepted in hydrogeology field practice for confined conditions. The application principles of this technique is explained in section 2.3.4.2. in greater detail.

1970 :**van Pollen and Weber**
and
1972 :**Kohlhaas** introduced Cooper et. al. (1967) slug test type curves to the petroleum engineering community.

1973 :**Papadopoulos et. al.** generated and published a new set of type curves using Cooper et. al. method. Their principal motivation for this study was the need of new type curves for slug tests conducted in formations with very low hydraulic conductivity.

1975 :**Ramey et. al.**, combining both the skin effect and the well bore storage into a single correlation group, developed three different kinds of type curves for slug tests, namely for early, intermediate and late times. They further introduced the concept of radius of investigation of slug tests, which would later attract attention within the research community with the increasing concern of how distant slug test estimations could be representative on the field. Ramey et. al. further derived an expression for estimating dimensionless skin factor by effective radius of the well. They have concluded that transmissivity estimates by Cooper et. al. (1967) method are not sensitive to skin effects.

1976 :**Bouwer and Rice** presented an analysis method for slug tests conducted in unconfined finite thickness formations; the method accommodates the well bore storage properties but not aquifer storativity. Employing empirical relationships developed from electrical-analogue simulations, the authors presented expressions for determining the hydraulic conductivity of the study formation. This method received very wide acceptance in practice and is currently the most commonly used method for determining hydraulic conductivity under unconfined conditions (Bedient et. al., 1994), particularly by field application of slug tests in shallow wells that partially penetrate the formation (Hyder and Butler, 1995). Further details on this method is given in section 2.3.4.3.

1978 :Dagan proposed a mathematical-numerical model for solving the Laplace equation expressed in cylindrical coordinates for direct application to the double packer and partially penetrating slug test problems. Assuming constant head upper boundary the method includes the effects of location of screen with respect to free water surface. The use of this approach is limited by the numerical problems encountered for low aspect ratios. Dagan pointed out that the screen length should be at least 50 times greater than the screen radius as a applicability criteria of this method.

1978 :Black adapted the method of Bouwer and Rice (1976) for applications with the procedure used by Cooper et. al. (1967). One type curve was obtained by plotting exp $(10)^t$ versus t on a semi-logarithmic plot of type curves of Cooper et. al. This type curve may be used to obtain a match point in the same manner as the other type curves. The author suggested to use of this unified set of type curves (Figure 2.3.3.1.) without any preconceptions about the aquifer conditions. The unified approach of Black is very convenient to use and very widely applied in practice. In 1986, Pandit and Miner developed a computer program on slug test data analysis making use of this unified method.

1980 :Bredehoeft and Papadopoulos, pointing out very long test durations in tight formations, suggested a pressurised slug test procedure, where the well is filled with fluid causing an initial change in the head, to be followed by an instantaneous compression of the fluid in the well. The decay of the pressure yields the same time response to be analysed by the slug test theory. The change in the well bore storage due to the excess pressure applied on the water in the well, decreases test durations considerable (Figure 2.3.3.2.). In 1982, Neuzil suggested several modifications in the test configurations after questioning Bredehoeft and Papadopoulos' assumptions on the nature of selected initial conditions and well bore and water compressibilities.

1984 :Faust and Mercer considered the effects of a finite thickness skin on the response of slug tests by using numerical and analytical models. They concluded that under certain conditions the early time response is related to the skin zone properties rather than the actual aquifer hydraulic parameters, whereas the late time response is controlled by the aquifer properties. The authors pointed out that Cooper et. al. (1967) type curves should be used with caution if there is a possibility of the presence of a well damage. They further recommended the utilisation of Ramey et. al. (1975) method if the skin is of infinitesimal thickness. Their approach was then scrutinised by Moench and Hsieh (1985); these authors showed that both Ramey et. al. and Faust and Mercer methods would yield the same result if the corresponding parameters were selected properly.

1984 :Dougherty and Babu, primarily focusing on the solution procedures in fractured porous reservoirs, developed dimensionless type curves for variable head tests under confined conditions. In the present study their solution procedures were adapted to extend the solutions to unconfined aquifer settings.

1984 :Nguyen and Pinder proposed a solution procedure for slug tests by solving axisymmetric form of governing equation for partially penetrating confined settings. The method accommodates the well bore and aquifer storage properties but not the skin effect. This method received a fair amount of use. However, in 1993 Hyder and Butler showed that the parameter estimates obtained by this approach must be viewed cautiously due to an error in the analytical solution procedure upon which the model is based (Hyder et. al., 1994)

1986 :Sagaev presented solution procedures for slug tests under fully penetrating confined conditions including well bore storage and skin effects. Solving purely radial groundwater equation, the author discussed the aquifer response with three kinds of type curves, namely early, intermediate and late times, in relation with different degrees of skin effect and well bore storage.

1987 :Dax introduced a new analysis method including the partial penetration effects through the use of Cooper et. al. method and as such incorporated the effect of storage properties of the aquifer in his analysis. The author showed that the model was applicable to thick unconfined aquifers and therefore could be used to determine the storativity of unconfined formations along with the hydraulic conductivity whereas the widely used Bouwer and Rice method can only estimate the hydraulic conductivity of the formation.

1987 :Hayashi et. al. focused on determining the range of screen lengths for which the conventional type curves are valid for conventional slug test data analysis techniques. They concluded that the effect of the presence of axial flow become more pronounced when the ratio of the test interval length to the well bore diameter is small or the specific storage of the study formation is low. The authors suggested that the use of conventional type curves should be restricted to test intervals that are more than 20 times longer than the well bore diameter. They further presented a new set of type curves which are valid for smaller aspect ratios.

1988 :Karasaki et. al. presented solutions for various models of slug tests under different test geometries and conditions encountered in field practice for which existing solutions were inadequate. They pointed out that when the screen of the well is much smaller than the thickness of the formation and when the formation is isotropic, the flow may better be approximated by spherical flow. They stressed the fact that analyses of slug test results suffer problems on nonuniqueness more than other well tests. The type curves for most cases have similar shapes in semilog plots rendering curve matching difficult. The authors stated that log-log type plots, however, may reflect some features that may not be evident in semilog plots . Karasaki et. al. concluded that because of the nonuniqueness problem, slug test data should not be used solely to predict hydraulic properties of the study formation; all other information such as geological and geophysical data should be considered.

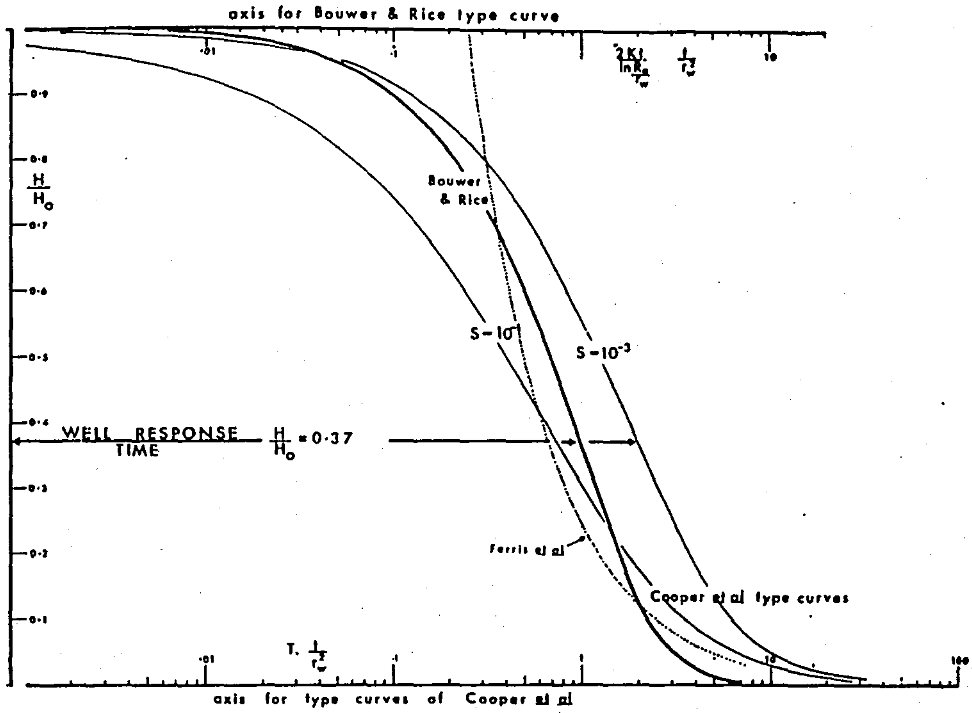


FIGURE 2.3.3.1. Type curves given by Black (1978) (Source: Black, 1978)

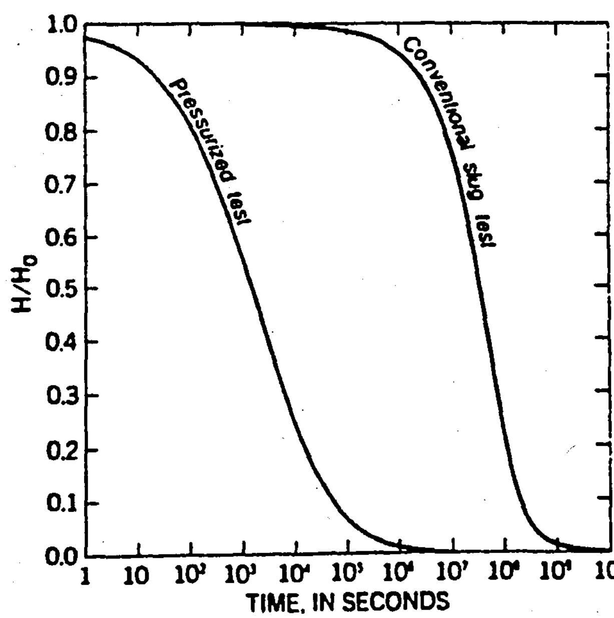


FIGURE 2.3.3.2. Field responses of conventional and pressurised slug tests (Source: Bredehoeft and Papadopoulos, 1980)

1989 :**Chirlin** compared Hvorslev (1951) and Cooper et. al. (1967) methods aiming at determining the effects of incompressibility assumption inherent to Hvorslev method. The author concluded that under fully penetrating confined conditions, the head in the aquifer of the Hvorslev model can substantially differ from that of Cooper et. al. model.

1990 :**Widdowson et. al.** presented a multilevel slug test technique which provides data for determining the local value of hydraulic conductivity at a specific elevation, under confined and unconfined, fully or partially penetrating, homogeneous and anisotropic conditions. The authors developed a numerical model for data analysis, which was patterned after Dagan's (1978) mathematical numerical model but less susceptible to numerical problems. The well bore and aquifer storage properties were accommodated within the model whereas no skin effect was considered. The findings of their paper along with the results presented by Melville et. al. (1991), were used extensively in the present study. Further details are given in Section 4.1.2.

1992 :**Hinsby et. al.** introduced a mini slug test method for the determination of local hydraulic conductivity values in unconfined sandy aquifers. The authors pointed out that deviations of the field data from a straight line at later times when plotted on semi-log paper can be explained with the presence of phenomena similar to the delayed yield response observed in pumping tests. They further determined that the use of specific storage coefficient yields better results with Dax (1987) method than of using specific yield. Consequently, they concluded that for early and intermediate times the water levels measured within the well bore will be influenced by the specific storage coefficient rather than the specific yield of the aquifer.

1994 :**Hyder et. al.** presented solutions to a mathematical model describing the groundwater flow induced by a slug test under confined or unconfined conditions considering partially penetration, anisotropy and finite thickness skin. For unconfined case they assumed constant head water table boundary, i.e. that water table is not effected by the head change within the well bore during the test. The solution was obtained by employing a series of integral transforms (Laplace and Fourier transformations) and then inverting from the transform-space by fast Fourier Transform techniques and Stehfest Algorithm. The primary purpose of this study was stated by the authors as assessing the viability of conventional methods of slug test data analysis.

Present Study:Analysis procedures were developed for slug tests performed in monitoring wells under confined or unconfined conditions. The mathematical model under investigation incorporates the effects of partial penetration, anisotropy and infinitesimal well skin. For confined aquifer solutions no-flow boundary conditions were assumed at the top and the bottom of the aquifer. For unconfined formations two water table elevation conditions were considered: **(i)** constant head boundary similar to the Bouwer and Rice (1976) method; and **(ii)** transient water table boundary based on Neuman (1972) conditions developed for constant discharge analysis in unconfined conditions. To the best of the authors knowledge the latter condition was never investigated throughout the evolution of the slug test theory. Subsequent sections are devoted to the detailed explanation and evaluation of the present approach.

2.3.4. Principles of Widely Accepted Methods of Analysis

2.3.4.1. Hvorslev Method (1951) In his original work "Time Lag and Soil Permeability in Groundwater Observations", Hvorslev proposed theoretical and experimental methods for determination of the 'time lag' and its influence on the results of groundwater observations. Reviewing the irregularities in groundwater conditions and principal sources of error in groundwater observations, he derived formulas for determination of the flow of water through various types of intakes or well points. He also expanded the solutions to include the anisotropy of the study formation and finally briefly discussed in the closing section of his paper the potential in situ use of the proposed analysis method.

Hvorslev's basic time-lag method, which can be applied to various well piezometer and standpipe geometries, is principally based on the assumption that the rate of inflow q, at the piezometer tip at any time t is proportional to the hydraulic conductivity, K, of the soil and to the unrecovered head difference (H - h) (Figure 2.3.4.1.1.). The method was developed for a wide variety of formation settings : full or partially penetration in an infinite or semi-infinite below, or confined, homogeneous and anisotropic aquifers. The storage properties of the aquifer was not embraced within the model, i.e. the aquifer compressibility was ignored. For the unconfined case the phreatic surface was assumed to remain constant throughout the test. The following equation determines the head in the well bore:

$$q(t) = \pi r^2 \frac{dh}{dt} = F K (H - h) \tag{2.9}$$

where F is a factor (called shape factor) that depends on the shape and dimensions of the intake area. Hvorslev also defined the basic time lag:

$$T_0 = \frac{\pi r^2}{F K} \tag{2.10}$$

and described it literally ".....the time required time required for equalisation of the this (initial) pressure difference when the original rate of flow .....is maintained". Substituting the basic time lag expression into equation (2.9) yields

$$\frac{H - h}{H - H_0} = e^{-t/T_0} \tag{2.11}$$

with H_0 is the initial head difference. This relation indicates an exponential decline in recovery rate with time and as such implies that field data should follow the same behaviour.

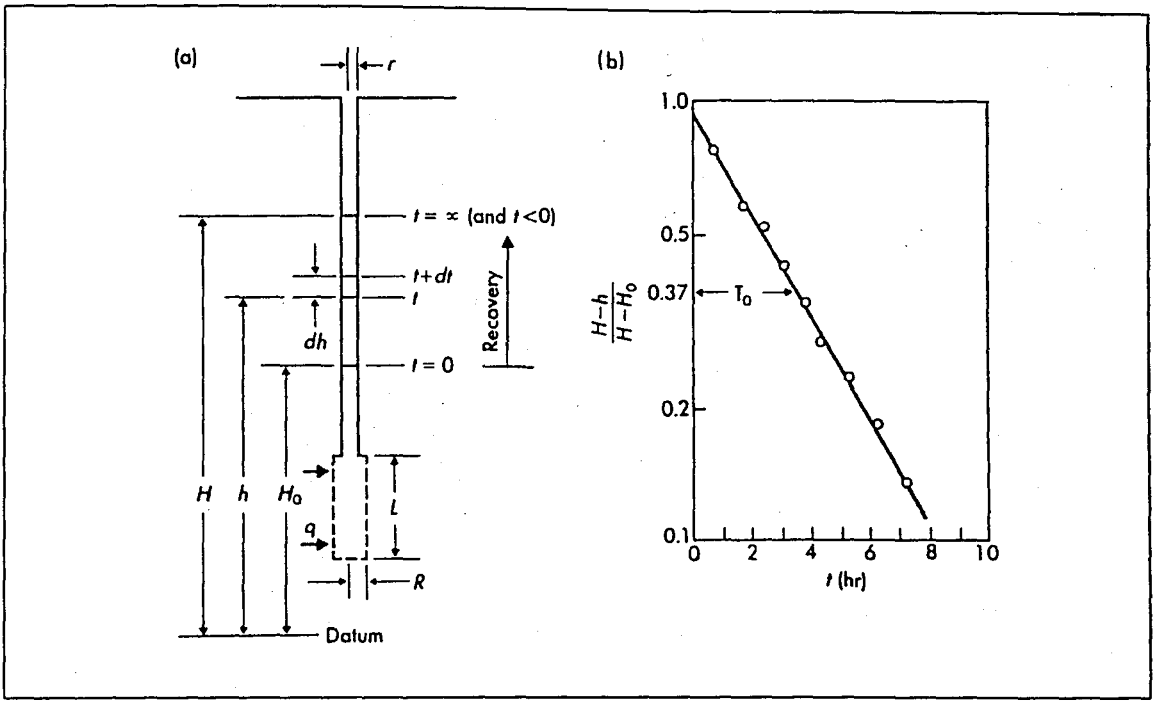


FIGURE 2.3.4.1.1. Hvorslev Method (1951), (a) the test geometry (b) determination of the basic time lag (Source: Freeze and Cherry, 1979)

In order to determine the hydraulic conductivity of the study formation equation (2.11) is employed. The unknown T_0 is determined by first plotting the field data in a semi-logarithmic scale as shown in Figure (2.3.4.1.1.). Noting that for $\frac{(H-h)}{(H-H_0)} = 0.37$, t / T_0 assumes the value 1, the basic time lag is read directly as the time intercept on the field curve where normalised head equals 0.37. The unknown value of F can be determined using derivations given in Hvorslev's original study. Hvorslev developed explicit formulas for determination of hydraulic conductivity values by constant head, variable head and time lag tests for permeameters and various types of borings and piezometers and listed them in his paper. All available formulas are given in Figure 2.3.4.1.2., as adapted from Hvorslev's original work.

Hvorslev method is very widely used in practice. In the US, Environmental Protection Agency, EPA, cited this method for use in unconfined aquifers and

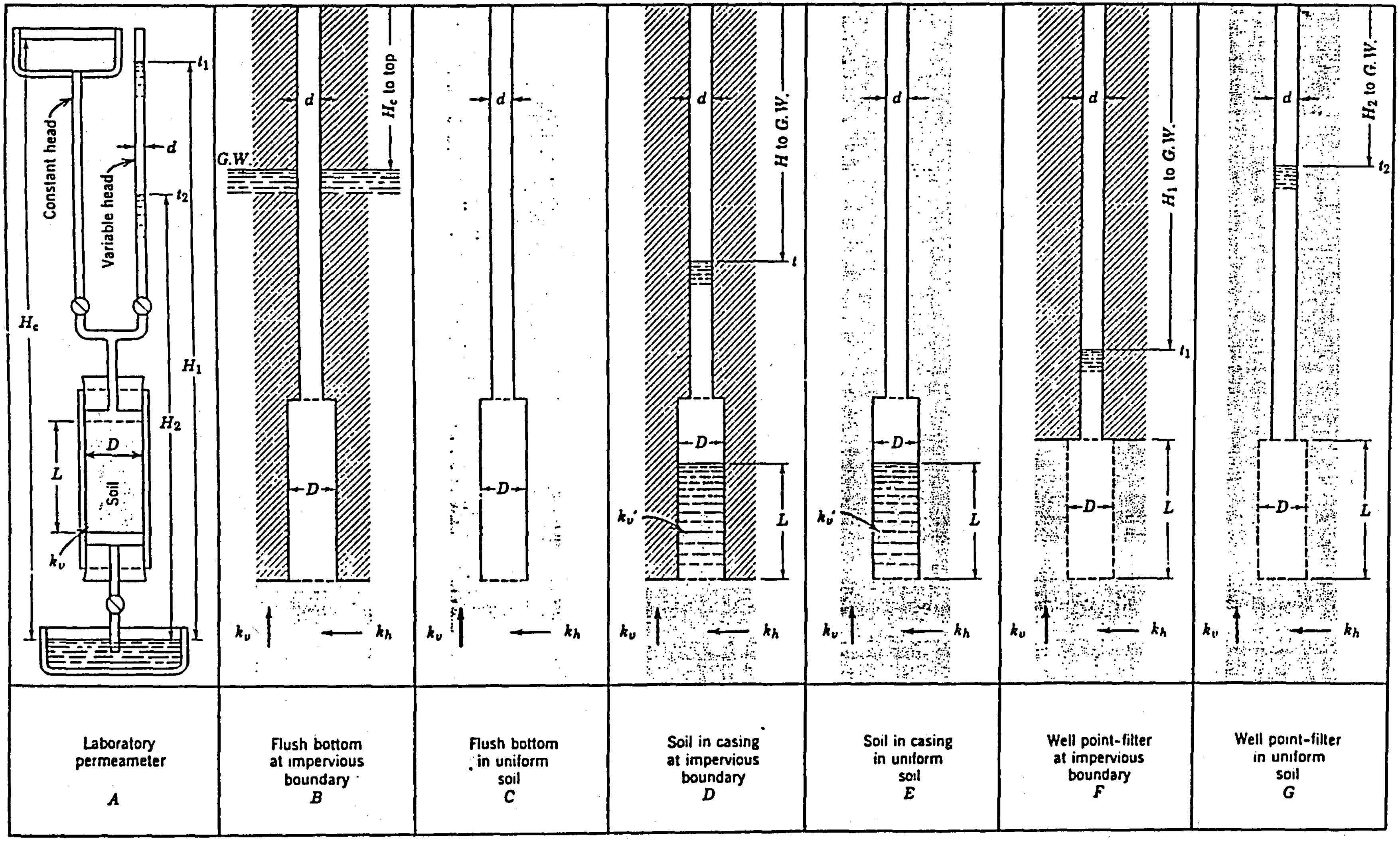
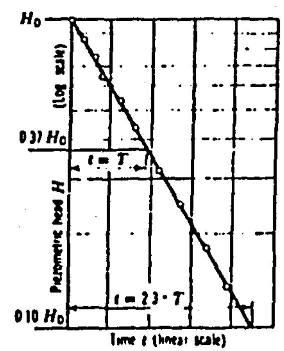


FIGURE 2.3.4.1.2. Formulas for determination of hydraulic conductivity by Hvorslev Method (1951) (Source: Akgüner, 1995)

Case	Constant head	Variable Head	Basic Time Lag	Notation
A	$k_v = \frac{4 \cdot q \cdot L}{\pi \cdot D^2 \cdot H_c}$	$k_v = \frac{d^2 \cdot L}{D^2 \cdot (t_2 - t_1)} \cdot \ln\frac{H_1}{H_2}$ $k_v = \frac{L}{t_2 - t_1} \cdot \ln\frac{H_1}{H_2}$ for $d=D$	$k_v = \frac{d^2 \cdot L}{D^2 \cdot T}$ $k_v = \frac{L}{T}$ for $d=D$	D = Diameter, intake, sample (cm) d = Diameter, standpipe (cm) L = Length, intake, sample (cm) H_c = Constant piez. head (cm) H_1 = Piez. head for $t=t_1$ (cm) H_2 = Piez. head for $t=t_2$ (cm) q = Flow of water (cm^3/sec) t = Time (sec) T = Basic time lag (sec) k_v' = Vert. perm. casing (cm/sec)
B	$k_m = \frac{q}{2 \cdot D \cdot H_c}$	$k_m = \frac{\pi \cdot d^2}{8 \cdot D \cdot (t_2 - t_1)} \cdot \ln\frac{H_1}{H_2}$ $k_m = \frac{\pi \cdot D}{8 \cdot (t_2 - t_1)} \cdot \ln\frac{H_1}{H_2}$ for $d=D$	$k_m = \frac{\pi d^2}{8 \cdot D \cdot T}$ $k_m = \frac{\pi \cdot D}{8 \cdot T}$ for $d=D$	
C	$k_m = \frac{q}{2.75 \cdot D \cdot H_c}$	$k_m = \frac{\pi \cdot d^2}{11 \cdot D \cdot (t_2 - t_1)} \cdot \ln\frac{H_1}{H_2}$ $k_m = \frac{\pi \cdot D}{11 \cdot (t_2 - t_1)} \cdot \ln\frac{H_1}{H_2}$ for $d=D$	$k_m = \frac{\pi \cdot d^2}{11 \cdot D \cdot T}$ $k_m = \frac{\pi \cdot D}{11 \cdot T}$ for $d=D$	
D	$k_v' = \frac{4 \cdot q\left(\frac{\pi}{8} \cdot \frac{k_v'}{k_v} \cdot \frac{D}{m} + L\right)}{\pi \cdot D^2 \cdot H_c}$	$k_v' = \frac{d^2 \cdot \left(\frac{\pi}{8} \cdot \frac{k_v'}{k_v} \cdot \frac{D}{m} + L\right)}{D^2 \cdot (t_2 - t_1)} \cdot \ln\frac{H_1}{H_2}$ $k_v = \frac{\frac{\pi}{8} \cdot \frac{D}{m} + L}{t_2 - t_1} \cdot \ln\frac{H_1}{H_2}$ for $k_v = k_v'; d=D$	$k_v' = \frac{d^2 \cdot \left(\frac{\pi}{8} \cdot \frac{k_v'}{k_v} \cdot \frac{D}{m}\right) + L}{D^2 \cdot T}$ $k_v = \frac{\frac{\pi}{8} \cdot \frac{D}{m} + L}{T}$ for $k_v' = k_v; d=D$	k_v = Vert. perm. ground (cm/sec) k_h = Horz. perm. ground (cm/sec) k_m = Mean coeff. perm. (cm/sec) m = Transformation ratio
E	$k_v' = \frac{4 \cdot q\left(\frac{\pi}{11} \cdot \frac{k_v'}{k_v} \cdot \frac{D}{m} + L\right)}{\pi \cdot D^2 \cdot H_c}$	$k_v' = \frac{d^2 \cdot \left(\frac{\pi}{11} \cdot \frac{k_v'}{k_v} \cdot \frac{D}{m} + L\right)}{D^2 \cdot (t_2 - t_1)} \cdot \ln\frac{H_1}{H_2}$ $k_v = \frac{\frac{\pi}{11} \cdot \frac{D}{m} + L}{t_2 - t_1} \cdot \ln\frac{H_1}{H_2}$ for $k_v = k_v'; d=D$	$k_v' = \frac{d^2 \cdot \left(\frac{\pi}{11} \cdot \frac{k_v'}{k_v} \cdot \frac{D}{m} + L\right)}{D^2 \cdot T}$ $k_v = \frac{\frac{\pi}{11} \cdot \frac{D}{m} + L}{T}$ for $k_v' = k_v; d=D$	$k_m = \sqrt{k_h \cdot k_v}$ $m = \sqrt{k_h / k_v}$ $\ln = \log_e = 2.3 \log_{10}$
F	$k_h = \frac{q \cdot \ln\left[\frac{2mL}{D} + \sqrt{1 + \left(\frac{2mL}{D}\right)^2}\right]}{2 \cdot \pi \cdot L \cdot H_c}$	$k_h = \frac{d^2 \cdot \ln\left[\frac{2mL}{D} + \sqrt{1 + \left(\frac{2mL}{D}\right)^2}\right]}{8 \cdot L \cdot (t_2 - t_1)} \cdot \ln\frac{H_1}{H_2}$ $k_h = \frac{d^2 \cdot \ln\left(\frac{4mL}{D}\right)}{8 \cdot L \cdot (t_2 - t_1)} \cdot \ln\frac{H_1}{H_2}$ for $\frac{2mL}{D} > 4$	$k_h = \frac{d^2 \cdot \ln\left[\frac{2mL}{D} + \sqrt{1 + \left(\frac{2mL}{D}\right)^2}\right]}{8 \cdot L \cdot T}$ $k_h = \frac{d^2 \cdot \ln\left(\frac{4mL}{D}\right)}{8 \cdot L \cdot T}$ for $\frac{2mL}{D} > 4$	
G	$k_h = \frac{q \cdot \ln\left[\frac{mL}{D} + \sqrt{1 + \left(\frac{mL}{D}\right)^2}\right]}{2 \cdot \pi \cdot L \cdot H_c}$	$k_h = \frac{d^2 \cdot \ln\left[\frac{mL}{D} + \sqrt{1 + \left(\frac{mL}{D}\right)^2}\right]}{8 \cdot L \cdot (t_2 - t_1)} \cdot \ln\frac{H_1}{H_2}$ $k_h = \frac{d^2 \cdot \ln\left(\frac{2mL}{D}\right)}{8 \cdot L \cdot (t_2 - t_1)} \cdot \ln\frac{H_1}{H_2}$ for $\frac{mL}{D} > 4$	$k_h = \frac{d^2 \cdot \ln\left[\frac{mL}{D} + \sqrt{1 + \left(\frac{mL}{D}\right)^2}\right]}{8 \cdot L \cdot T}$ $k_h = \frac{d^2 \cdot \ln\left(\frac{2mL}{D}\right)}{8 \cdot L \cdot T}$ for $\frac{mL}{D} > 4$	



Determination basic time lag T

FIGURE 2.3.4.1.2. (continued) Formulas for determination of hydraulic conductivity by Hvorslev Method (1951)

according to Chirlin (1989) at least one RI/FS for a Superfund top-10 NPL site relies heavily on this method.

2.3.4.2. Cooper et. al. Method (1967) For slug tests run in well bores that are fully penetrating confined aquifers, Cooper et. al. developed a test interpretation procedure by assuming that the aquifer is homogeneous, isotropic, and infinite in radial extent (Figure 2.3.4.2.1.). Their formulation includes the effects of storage characteristics of both the well bore and the test formation. The authors assumed that purely radial flow conditions prevail within the study formation, which can be justified with the fully penetrating nature of the test setting. By applying Laplace transformation to the nonsteady radial groundwater movement equation and employing relevant boundary conditions, authors ended up with Laplace domain solution of head distribution within the aquifer. The time domain solution was obtained from solutions of analogous heat flow problems (Carslaw and Jaeger, 1949). Cooper et. al. gave the following expression describing the change of water level within the casing after an initial change in water level of H_0 is applied:

$$H = \left(\frac{8\,H_0\,\alpha}{\pi^2}\right) \int_0^\infty e^{-\frac{Wu^2}{\alpha}} \frac{du}{\left(u\,\Delta(u)\right)} \qquad (2.12)$$

Here, S and T are storativity and transmissivity of the aquifer, respectively, and α and W are defined as:

$$\alpha = \frac{r_s^2}{r_c^2} S \quad \text{and} \quad W = \frac{T\,t}{r_c^2} \qquad (2.13)$$

By numerically evaluating the integral in the equation (2.12), the authors generated a family of type curves for different α values (Figure 2.3.4.2.2.) and drawn them on semi-logarithmic paper. To determine the hydraulic parameters of the study formation a curve matching procedure is applied which is quite similar to Theis graphical method (Wenzel, 1942). The field data, which are normalised

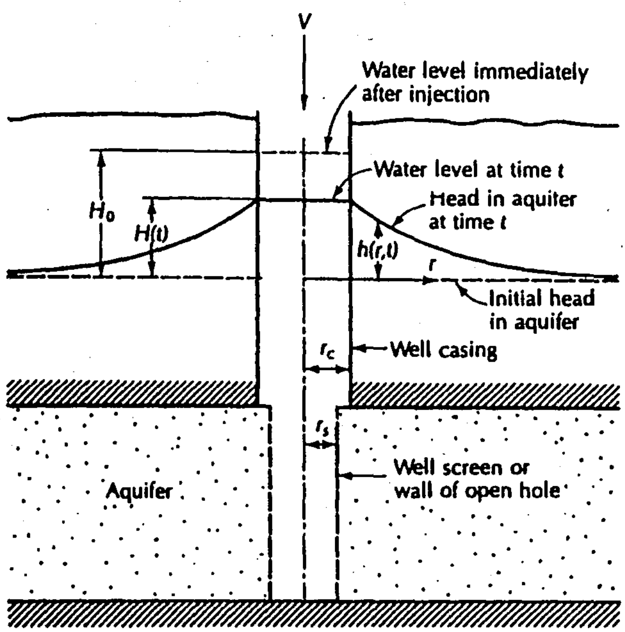


FIGURE 2.3.4.2.1. Cooper et. al. (1967) slug test setup
(Source: Bedient et. al., 1994)

with the initial excess water height H_0 within the well bore, is plotted on semi-logarithmic paper on the same scale as the type curves. Keeping the arithmetic axes coincident, a best fit position between the field data and type curves is determined. For an arbitrary point on coinciding linear axes, both W and t values are read and substituted in equations (2.13) to end up with the unknown value of transmissivity T. The value of storativity S can be determined from the definition of α. However, because the curve matching procedure depends upon the shapes of the type curves which are very similar to each other when α differs one order of magnitude, determination of S by this method is considered unreliable (Freeze and Cherry, 1979). Cooper et. al., on the contrary, pointed out that by a knowledge of geologic conditions and other considerations about the site it is possible to estimate S within an order of magnitude and thereby eliminate some of the doubt as to what value of α should be used for matching the data plot.

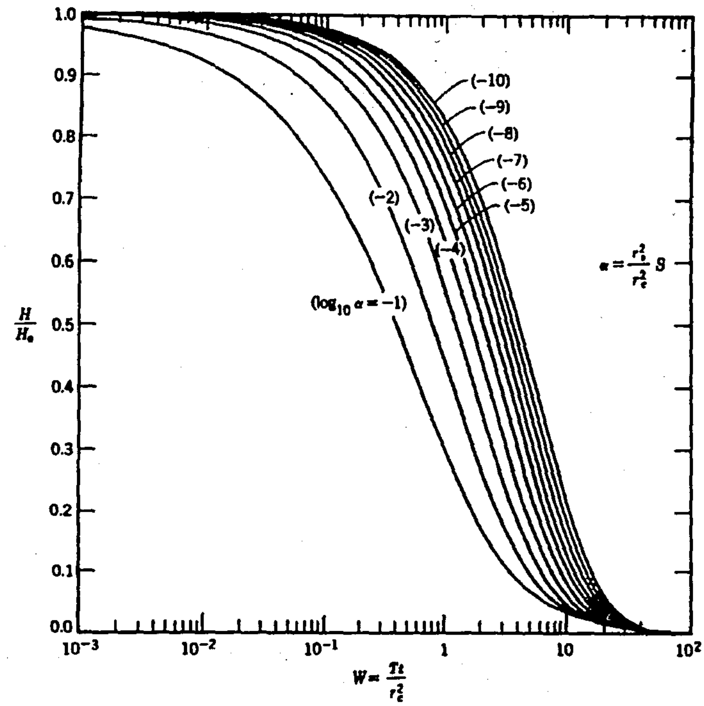


FIGURE 2.3.4.2.2. Cooper et. al. (1967) type curves
(Source: Bedient et. al. 1994)

Cooper et. al. method is very widely used in practice for fully and partially penetrating settings under confined conditions, although originally it was developed only for fully penetrating wells. When analysing partially penetrating wells with Cooper et. al. method, one automatically assumes that the formation thickness D is equal to the screen length of the well; this is an immediate inference of purely radial flow assumption and inherent to the development of this method of analysis.

Chirlin (1989) states, that Cooper et. al. method can be used as a standard for fully penetrating confined conditions, particularly against which to judge the effect of incompressibility assumption. However, the findings of the present study along with the results Melville et. al. (1991) reflect potential discrepancies in the parameter estimations of this method.

2.3.4.3. Bouwer and Rice Method (1976) The most commonly used method for determining hydraulic conductivity under unconfined conditions is the Bouwer and Rice method (Bedient et. al., 1994), particularly by field application of slug tests in shallow wells that partially penetrate the formation (Hyder and Butler, 1995). This approach employs empirical relationships developed from electrical-analogue simulations and based on several simplifying assumptions such as **(i)** the study formation is homogeneous and isotropic, **(ii)** the specific storage of the formation is negligible, **(iii)** flow above the water table can be ignored[8], **(iv)** the upper boundary of the problem is constant throughout the test duration, and **(v)** no skin effect is present. The test model used by Bouwer and Rice is shown in Figure 2.3.4.3.1.

Bouwer and Rice Method is principally based the Thiem equation of steady flow which describes the relationship between the inflow into the borehole and the drawdown around it, this equation in its modified form reads:

$$Q = \frac{2\pi K_r L y}{\ln(R_e / r_w)} \tag{3.14}$$

where y : piezometric head difference between inside and outside of the well;

R_e : effective radius over which y is dissipated;

r_w : radial distance of undisturbed portion of aquifer from centreline;

K_r : hydraulic conductivity of the study formation;

Q : inflow into the well bore; and

L : length of the open portion of the well.

Combining equation (3.14) with the macroscopic mass balance equation yields the following expression:

[8] This assumption renders both falling and rising head test being mathematically identical.

$$K_r = \frac{r_c^2 \ln(R_e/r_w)}{2L} \frac{1}{t} \ln \frac{y_0}{y_t} \tag{3.15}$$

for the hydraulic conductivity value of the study formation, where y_0 = y at time 0 and y_t = y at time t.

Bouwer and Rice determined the unknown value of R_e using an electrical resistance network analog for different values of r_w, L, H, and D. The following equations were given by the authors, along with Figure 2.3.4.3.2., for determining ln R_e / r_w:

Partially penetrating wells:

$$\ln \frac{R_e}{r_w} = \left[\frac{1.1}{L_w/r_w} + \frac{A + B\ln(H - L_w)/r_w}{L_e/r_w} \right]^{-1} \tag{3.16}$$

Fully penetrating wells:

$$\ln \frac{R_e}{r_w} = \left[\frac{1.1}{L_w/r_w} + \frac{C}{L_e/r_w} \right]^{-1} \tag{3.17}$$

The validity of this widely used method was scrutinised over the last years and recently Hyder and Butler (1995) pointed out that for slug tests performed in homogeneous, isotropic formations that would be classified as aquifers, Bouwer and Rice method provides estimates within 30 per cent of actual field values. They further stated that in less permeable, clay rich formations estimates may overpredict formation conductivity by more than 100 per cent.

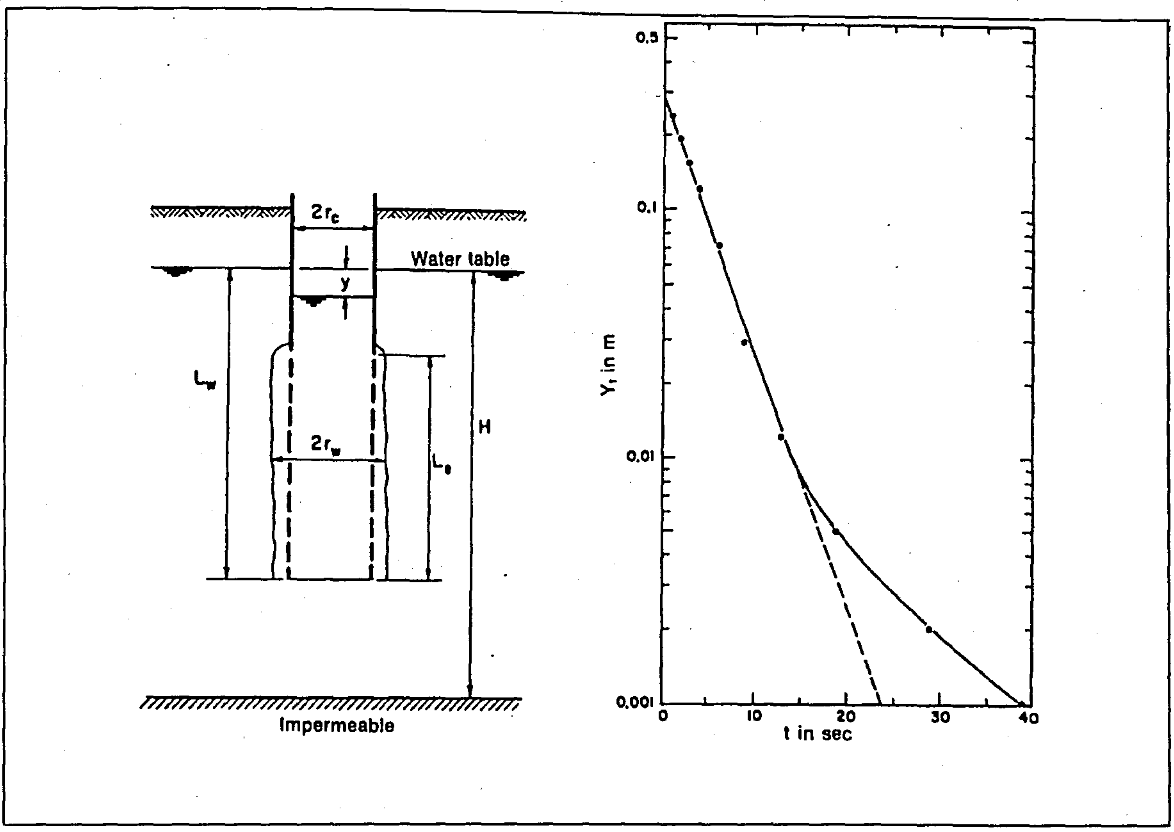


FIGURE 2.3.4.3.1. The test model used by Bouwer and Rice (1976)
(Source: Bedient et. al., 1994)

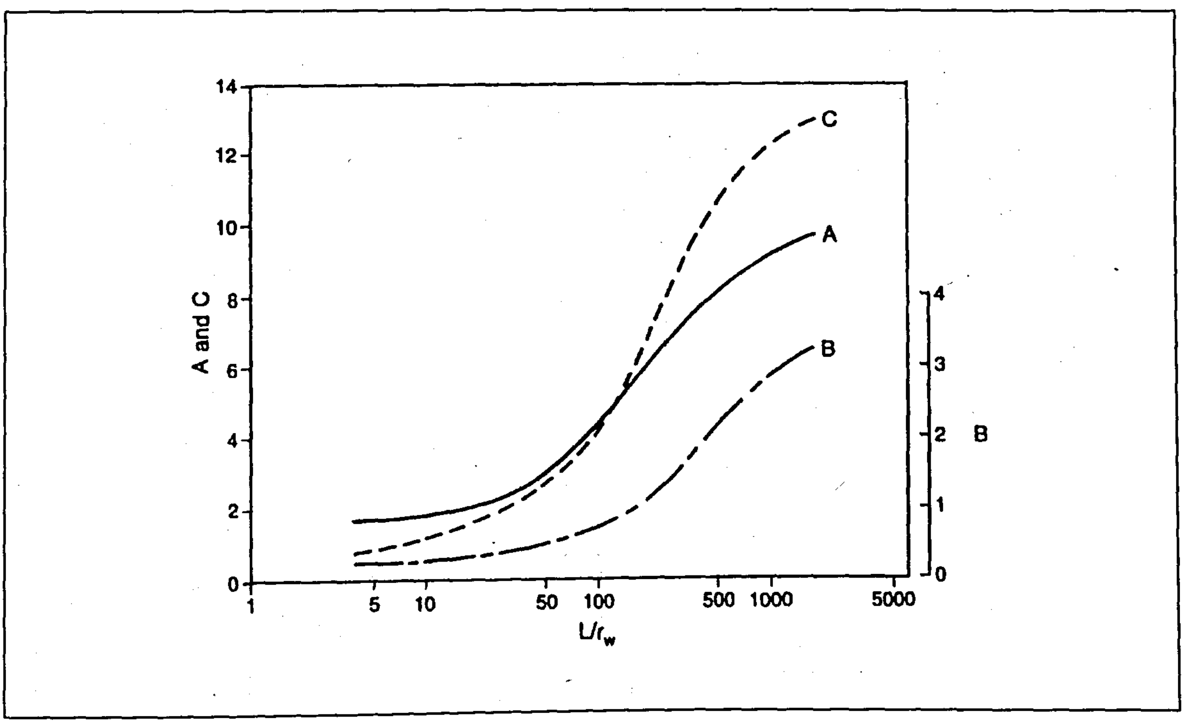


FIGURE 2.3.4.3.2. Curves relating A, B, and C to L / r_w
(Source: Bouwer and Rice, 1976)

3. THEORETICAL SOLUTION OF THE PROBLEM

3.1 Description of the Problem

The slug test model under analysis for both confined and unconfined cases is shown in Figure 3.1.1. The cylindrical coordinate system used to define the governing groundwater flow equation is shown on these figures. The problem domain is assumed to be homogeneous, anisotropic[1], infinite in radial extent and finite in depth. The thickness of the confined aquifer or the height of water column for the unconfined aquifer is represented by the variable **D**. The radial and vertical hydraulic conductivities, specific storage coefficient and specific yield are defined as $\mathbf{K_r}$, $\mathbf{K_z}$, $\mathbf{S_c}$, $\mathbf{S_y}$, respectively. The well casing and screen radii are given by $\mathbf{r_c}$ and $\mathbf{r_s}$ whereas the screen length **L** is defined by the variables $\mathbf{z_a}$ and $\mathbf{z_b}$ representing the distance from the bottom and the top of screen, respectively to the confining lower no-flow boundary. An instantaneous water level change $\mathbf{h_0}$ is imposed above the initial water level present in the casing to create a pressure disturbance in the system and the water levels $\mathbf{h_w(t)}$ are subsequently measured in the casing as they return to their original levels. The hydraulic head distribution in the aquifer following the instantaneous water level change in the casing is given by **h(r,z,t)**.

Throughout the derivations, variables overlain by a ~ have dimension, whereas the ones with an overbar indicate that the variable is transformed by Laplace transformation.

3.2 Mathematical Formulation of the Problem

The differential equation governing the nonsteady and axisymmetric hydraulic head distribution within the aquifer is given by

[1] The principle axes of anisotropy are assumed to be parallel to the axes of the cylindrical coordinate system used.

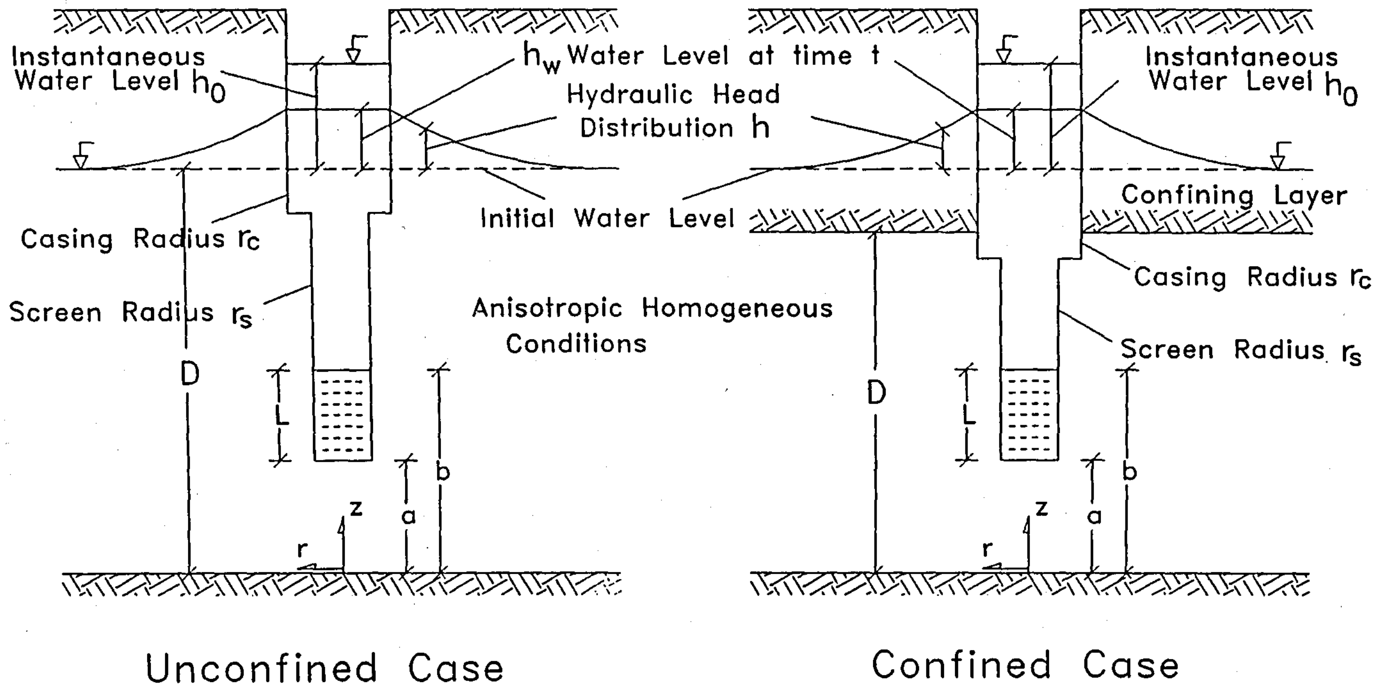


FIGURE 3.1.1. The slug test model under analysis

$$\tilde{K}_r\left(\frac{\partial^2\tilde{h}}{\partial\tilde{r}^2}+\frac{1}{\tilde{r}}\frac{\partial\tilde{h}}{\partial\tilde{r}}\right)+\tilde{K}_z\frac{\partial^2\tilde{h}}{\partial\tilde{z}^2}=\tilde{S}_c\frac{\partial h'}{\partial\tilde{t}} \tag{3.1}$$

which considers both the radial as well as the axial flow in the vicinity of the screened zone[2].

There are several boundary conditions which must be determined to complete the formulation of the problem; one of these conditions is obtained from the macroscopic mass balance of the well bore and relates the rate of change in the water level measured in the casing to the flux occurrence across the screened zone. The macroscopic mass balance of the well bore requires that the rate of mass change be equal to the total flux through the screen, plus/minus the influx/outflux **Q**, if any, from the top of the well bore[3]. If the vertical pressure gradients in the well bore are assumed to be hydrostatic (i.e. the head in the well $\mathbf{h_w}$ is independent of the depth and hence varies only with time) (Van Everdingen and Hurst, 1949; Dougherty and Babu, 1984) and the effect of compressibility on volume change is ignored[4], the following equation is obtained:

$$\tilde{Q}-\pi\tilde{r}_c^2\frac{\partial\tilde{h}_w}{\partial\tilde{t}}+2\pi\tilde{r}_s\tilde{L}\tilde{K}_r\left.\frac{\partial\tilde{h}}{\partial\tilde{r}}\right|_{\tilde{r}=\tilde{r}_s}=0 \tag{3.2}$$

---

[2] The derivation of this equation utilises Darcy's law; throughout the derivations presented in this study the groundwater flow in the aquifer induced by the slug injection is assumed to be laminar and the linear nature of the Darcy's law is conserved. Further details can be found in Freeze and Cherry (1979, pp. 69-76)

[3] For a typical slug test procedure no additional water is added to or removed from the borehole after the initial perturbation, i.e. Q is zero, but throughout the derivations presented in this study the term Q is kept for not loosing the generality of the solution. This enabled the final solution of the problem to be given in three different ways, namely variable head, constant head and the general form.

[4] This assumption is not valid if the well bore is pressurised to reduce the test duration. For such cases the compressibility should be introduced into the macroscopic mass balance equation (Bredehoeft et. al., 1980)

The third expression on the left hand side of this equation simply gives the radial flowrate into the well bore by virtue of Darcy's law[5].

One of the critical assumptions of the present formulation is that the head along the screen is constant and varies only with time. This is a common postulation used in the analysis of similar type of problems (Dougherty and Babu, 1984; Hyder et. al. 1994; Hyder and Butler, 1995) and is principally based on the assumption that the head within the well bore varies only with time. An immediate deduction of this assumption is that the flux along the screen is constant, independent of depth and is a function of time only[6]. The constant value of the head is chosen as the average head along the screen and obtained by using the Mean Value Theorem[7] as:

$$\tilde{h}^*\Big|_{\tilde{r}=\tilde{r}_s} = \frac{1}{\tilde{L}}\int_{\tilde{z}_a}^{\tilde{z}_b} \tilde{h}\Big|_{\tilde{r}=\tilde{r}s} d\tilde{z} \tag{3.3}$$

The relationship between h_w, the average head h^* in the aquifer just outside the well bore and the hydraulic head losses which may occur from the possible presence of a skin zone near the screened zone is given by:

$$\tilde{h}_w = \tilde{h}^*\Big|_{\tilde{r}=\tilde{r}_s} - \tilde{S}_k \tilde{K}_r \frac{\partial \tilde{h}^*}{\partial \tilde{r}}\Bigg|_{\tilde{r}=\tilde{r}_s} \tag{3.4}$$

---

5 The use of Darcy's law implies the assumption of laminar flow across the screen.

6 This assumption is consistent with the homogeneous nature of the present problem domain, but should be revised for cases where radial hydraulic conductivity is a function on z.

7 **The First Mean Value Theorem of Integrals** (Stephenson, 1991)

if f(x) is continuous in $a \le x \le b$, then

$$\int_a^b f(x)\,dx = (b-a)\,f(\xi)$$

where $a \le \xi \le b$.

This relation was originally introduced by van Everdingen (van Everdingen, 1949) and Hurst (Hurst, 1953) where S stands for the skin factor[8].

The flux conditions along the cylindrical area describing the well borehole are given by

$$K_r \left.\frac{\partial \tilde{h}}{\partial \tilde{r}}\right|_{\tilde{r}=\tilde{r}_s} = \begin{cases} 0 & \text{for} \quad z < z_a \\ -\tilde{q}_{scr} & \text{for} \quad z_a \le z \le z_b \\ 0 & \text{for} \quad z > z_b \end{cases} \tag{3.5}$$

---

[8] **The relationship between Hurst (1953) and Carslaw&Jaeger (1949) dimensionless skin factors**

Carslaw and Jaeger (1949) introduced the dimensionless skin factor as

$$S_w = \frac{L\,\Delta H}{Q} K_r \tag{8.1}$$

where L is the screen length, K_r the radial hydraulic conductivity, Q the volumetric rate and ΔH the head loss across the screen. For a circular well with an infinitesimal skin, Q can be expressed as

$$Q = 2\pi r_s \int_0^L K_r \left.\frac{\partial h}{\partial r}\right|_{r=r_s} dz \tag{8.2}$$

Substituting the above equation into equation (8.1) yields

$$\Delta H = 2\pi r_s S_w K_r \int_0^L \left.\frac{\partial h}{\partial r}\right|_{r=r_s} dz \quad \text{or simply} \quad \Delta H = 2\pi r_s S_w K I \tag{8.3}$$

with I representing the integral term. Equation (3.4) of the present method takes on the following form for ΔH

$$\Delta\tilde{H} = \tilde{S}\tilde{K}_r \left.\frac{\partial \tilde{h}^*}{\partial \tilde{r}}\right|_{r=r_s} d\tilde{z} \tag{8.4}$$

and using the relation (3.3), equation (8.4) can be expressed as

$$\Delta\tilde{H} = \frac{\tilde{S}\tilde{K}_r}{\tilde{L}} I \tag{8.5}$$

Introducing the dimensionless form of S (p.49) yields

$$\Delta\tilde{H} = \frac{S\,\tilde{r}_s}{\tilde{L}} I \tag{8.6}$$

The relation between S_w and S is obtained by equalling (8.3) and (8.6) to each other to end up:

$$S_w = 2\pi S$$

where $-q_{scr}$[9] describes the average flux along the screen. These conditions imply that no flow occurs above and below the screened zone along $r = r_s$, and hence suggests that the well bore is cased above and below the open interval through the aquifer thickness D. The unknown magnitude of q_{scr} will be obtained at an intermediate part of the solution and be used as a subsidiary parameter.

The change in the head is assumed to approach zero as r approaches infinity, i.e.:

$$\lim_{\tilde{r} \to \infty} \tilde{h}(\tilde{r}, \tilde{z}, \tilde{t}) = 0 \qquad (3.6)$$

The upper boundary condition of the problem domain depends on whether the aquifer is confined or unconfined, whereas the lower no-flow boundary condition is the same for both cases; the following conditions are formulated in the present analysis:

Type 1 Confined aquifer

$$\left.\frac{\partial \tilde{h}}{\partial \tilde{z}}\right|_{\tilde{z}=0} = 0 \quad \text{and} \quad \left.\frac{\partial \tilde{h}}{\partial \tilde{z}}\right|_{\tilde{z}=\tilde{D}} = 0 \qquad (3.7)$$

stating impermeable boundary conditions both at the top and the bottom of the aquifer;

Type 2.a. Unconfined aquifer

$$\left.\frac{\partial \tilde{h}}{\partial \tilde{z}}\right|_{\tilde{z}=0} = 0 \quad \text{and} \quad \left.\tilde{h}\right|_{\tilde{z}=\tilde{D}} = 0 \qquad (3.8)$$

[9] The minus sign is chosen arbitrarily for easing further derivations involving q_{scr}. Omitting it would yield the same solution.

stating no-flow boundary condition at the bottom and that the initial water level remain constant during the variable head test. This formulation has been widely used (Hvorlev (1951), Bouwer and Rice (1976), Dagan (1978), Hyder et. al. (1994), Hyder and Butler (1995)) and assumes that the fluctuations in the water table caused by the instantaneous water level change in the casing is negligible.

Type 2.b. Unconfined aquifer

$$\left.\frac{\partial \tilde{h}}{\partial \tilde{z}}\right|_{z=0} = 0 \quad \text{and} \quad -\tilde{K}_z \left.\frac{\partial \tilde{h}}{\partial \tilde{z}}\right|_{\tilde{z}=\tilde{D}} = \tilde{S}_y \left.\frac{\partial \tilde{h}}{\partial \tilde{t}}\right|_{\tilde{z}=\tilde{D}} \tag{3.9}$$

with no-flow boundary condition at the bottom of the aquifer. The upper water table boundary condition is considered being transient as derived by Neuman (1972) ; this upper boundary condition was originally obtained by using a perturbation technique based on the assumption that the water table fluctuations are much smaller than the depth of the water column in the aquifer.

To compete the formulation of the problem, the following initial conditions are assumed:

$$\tilde{h}_w(0) = \tilde{h}_0 \quad \text{and} \quad \tilde{h}(\tilde{r}, \tilde{z}, 0) \quad \text{for} \quad \tilde{r}_s < \tilde{r} < \infty \tag{3.10}$$

stating that the initial head in the aquifer is constant and equals zero and the head in the well bore differs from that of the aquifer by an amount of h_0.

3.3. Solution of the Problem

The solution procedures are aimed at obtaining an expression for the water level variation in the casing $\mathbf{h_w(t)}$ in the Laplace domain using the dimensionless governing equation and corresponding boundary conditions. The following steps are carried out to this end.

3.3.1. Dimensionless Analysis

Casting the investigated problem into dimensionless form by making use of dimensionless variables is a common method employed in well test analyses (Horne, 1990). The importance of dimensionless parameters is that they simplify the aquifer models by embodying the aquifer parameters, such as $\mathbf{S_y}$ or $\mathbf{K_r}$, thereby reducing the total number of unknowns. They have the additional advantage of providing model solutions that are independent of any particular unit system. The present problem is converted into dimensionless form using the following variables as the basic dimensionless variables:

$$h = \frac{\tilde{h}}{\tilde{h}_0} \quad ; \quad h_w = \frac{\tilde{h}_w}{\tilde{h}_0} \quad ; \quad r = \frac{\tilde{r}}{\tilde{r}_s} \quad ; \quad L = \frac{\tilde{L}}{\tilde{r}_s} \quad ; \quad D = \frac{\tilde{D}}{\tilde{r}_s}$$

$$z = \frac{\tilde{z}}{\tilde{r}_s} \quad ; \quad z_a = \frac{\tilde{z}_a}{\tilde{r}_s} \quad ; \quad z_b = \frac{\tilde{z}_b}{\tilde{r}_s} \quad ; \quad \zeta = \frac{\tilde{K}_z}{\tilde{K}_r} \quad ; \quad \delta = \frac{\tilde{L}}{\tilde{D}} \tag{3.11}$$

Inserting these variables into the governing and boundary condition equations[10] yield the following subsidiary dimensionless groups.

$$t = \frac{\tilde{K}_r}{\tilde{S}_c\, \tilde{r}_s^2}\,\tilde{t} \quad ; \quad C_D = \frac{\tilde{r}_c^2}{2\tilde{L}\,\tilde{r}_s^2\,\tilde{S}_c} \quad ; \quad Q = \frac{\tilde{Q}}{2\pi \tilde{L}\,\tilde{h}_0\,\tilde{K}_r}$$

$$S_k = \frac{\tilde{K}_r}{\tilde{r}_s}\,\tilde{S}_k \quad ; \quad S_y = \frac{\tilde{S}_y}{\tilde{r}_s\,\tilde{S}_c\,\zeta} \tag{3.12}$$

---

[10] Note that $\quad \dfrac{\partial \tilde{h}}{\partial \tilde{r}} = \dfrac{\tilde{h}_0}{\tilde{r}_s}\dfrac{\partial h}{\partial r} \quad ; \quad \dfrac{\partial^2 \tilde{h}}{\partial \tilde{z}^2} = \dfrac{\tilde{h}_0}{\tilde{r}_s^2}\dfrac{\partial h}{\partial z} \quad$ and $\quad \dfrac{\partial^2 \tilde{h}}{\partial \tilde{r}^2} = \dfrac{\tilde{h}_0}{\tilde{r}_s^2}\dfrac{\partial^2 h}{\partial r^2}$

Consequently, the equations describing the problem in dimensionless form read:

The governing equation:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} + \zeta\frac{\partial^2 h}{\partial z^2} = \frac{\partial h}{\partial \tilde{t}} \tag{3.13}$$

Boundary conditions:

$$Q - C_D\frac{\partial h_w}{\partial t} + \left.\frac{\partial h}{\partial r}\right|_{r=1} = 0 \tag{3.14}$$

$$\left.h^*\right|_{r=1} = \frac{1}{L}\int_{z_a}^{z_b} \left.h\right|_{r=1} dz \tag{3.15}$$

$$h_w = \left.h^*\right|_{r=1} - S_k\left.\frac{\partial h^*}{\partial r}\right|_{r=1} \tag{3.16}$$

$$\left.\frac{\partial h}{\partial r}\right|_{r=1} = \begin{cases} 0 & \text{for} \quad z < z_a \\ -q_{scr} & \text{for} \quad z_a \le z \le z_b \\ 0 & \text{for} \quad z > z_b \end{cases} \tag{3.17}$$

$$\lim_{\tilde{r}\to\infty} \tilde{h}(\tilde{r}, \tilde{z}, \tilde{t}) = 0 \tag{3.18}$$

Type 1. Confined Aquifer:

$$\left.\frac{\partial h}{\partial z}\right|_{z=0} = 0 \quad \text{and} \quad \left.\frac{\partial h}{\partial z}\right|_{z=D} = 0 \tag{3.19}$$

Type 2.a. Unconfined Aquifer with constant water table boundary:

$$\left.\frac{\partial h}{\partial z}\right|_{z=0} = 0 \quad \text{and} \quad h\big|_{z=D} = 0 \tag{3.20}$$

Type 2.b. Unconfined Aquifer with transient water table boundary:

$$\left.\frac{\partial h}{\partial z}\right|_{z=0} = 0 \quad \text{and} \quad -\left.\frac{\partial h}{\partial z}\right|_{z=D} = S_y \left.\frac{\partial h}{\partial t}\right|_{z=D} \tag{3.21}$$

Initial conditions

$$\tilde{h}_w(0) = \tilde{h}_0 \quad \text{and} \quad \tilde{h}(\tilde{r}, \tilde{z}, 0) \quad \text{for} \quad \tilde{r}_s < \tilde{r} < \infty \tag{3.22}$$

3.3.2. Laplace Transformation

Laplace transforms have been used extensively in development of solutions to the problems in diffusion and heat transport systems as well as in many aquifer problems of diverse settings. The power of the Laplace transformation stems from the fact that one of the independent variables of a partial differential equation is eliminated and thus in many cases simplifying the solution of the transformed equation and the boundary conditions, just by making use of relevant initial conditions. The obtained solution in the Laplace domain can then be inverted to the real domain either by analytical or numerical inversion methods (Dougherty and Babu, 1984, Hyder et. al. 1994, Dougherty, 1989 and Avci, 1994)

The Laplace Transform, with the presumption that f(t) is defined for $t \geq 0$, is defined as

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-pt}\, f(t)\, dt \tag{3.23}$$

for all p such that the improper integral converges.

Applying the Laplace Transformation to the dimensionless governing and boundary condition equations, and employing corresponding initial conditions[11,12] yield:

The governing equation:

$$\frac{\partial^2 \bar{h}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{h}}{\partial r} + \zeta \frac{\partial^2 \bar{h}}{\partial z^2} = p\,\bar{h} \tag{3.24}$$

Boundary Conditions:

$$\bar{Q} + \left.\frac{\partial \bar{h}}{\partial r}\right|_{r=1} = C_D\left(p\,\bar{h}_w - 1\right) \tag{3.25}$$

$$\left.\bar{h}^*\right|_{r=1} = \frac{1}{L}\int_{z_a}^{z_b} \left.\bar{h}\right|_{r=1} dz \tag{3.26}$$

$$\bar{h}_w = \left.\bar{h}^*\right|_{r=1} - S_k \left.\frac{\partial \bar{h}^*}{\partial r}\right|_{r=1} \tag{3.27}$$

---

[11] Note that for f(x, t) $\mathcal{L}\left(\frac{\partial f}{\partial x}\right) = \int_0^{\infty} e^{-pt}\frac{\partial f}{\partial x}dt = \frac{\partial}{\partial x}\int_0^{\infty} e^{-pt}dt = \frac{\partial}{\partial x}\left[\mathcal{L}(f)\right]$
because the partial derivative is with respect to x and the integral is to t.

[12] Note that $\mathcal{L}\left\{\frac{\partial f(t)}{\partial t}\right\} = p\,\mathcal{L}\{f(t)\} - f(0)$

$$\left.\frac{\partial \bar{h}}{\partial r}\right|_{r=1} = \begin{cases} 0 & \text{for} \quad z < z_a \\ -\bar{q}_{scr} & \text{for} \quad z_a \le z \le z_b \\ 0 & \text{for} \quad z > z_b \end{cases} \tag{3.28}$$

$$\lim_{\tilde{r}\to\infty} \tilde{h}\left(\tilde{r}, \tilde{z}, \tilde{t}\right) = 0 \tag{3.29}$$

Type 1. Confined Aquifer

$$\left.\frac{\partial \bar{h}}{\partial z}\right|_{z=0} = 0 \quad \text{and} \quad \left.\frac{\partial \bar{h}}{\partial z}\right|_{z=D} = 0 \tag{3.30}$$

Type 2.a. Unconfined aquifer with constant water table boundary

$$\left.\frac{\partial \bar{h}}{\partial z}\right|_{z=0} = 0 \quad \text{and} \quad \left.\bar{h}\right|_{z=D} = 0 \tag{3.31}$$

Type 2.b. Unconfined aquifer with transient water table boundary

$$\left.\frac{\partial \bar{h}}{\partial z}\right|_{z=0} = 0 \quad \text{and} \quad -\left.\frac{\partial \bar{h}}{\partial z}\right|_{z=D} = S_y\, p\, \left.\bar{h}\right|_{z=D} \tag{3.32}$$

3.3.3. Fourier Series Solution of the Problem

The boundary value problem described by the equations (3.24)-(3.32) are solved using **separation of variables method**[13] by attempting a solution of the form

[13] Different nomenclature is used in literature for this method, such as **Fourier Method** (O'Neil, 1992) and **Product Method** (Kreyzsig, 1993)

$$\bar{h}(r, z, p) = \mathcal{R}(r)\,\mathcal{Z}(z)\,\mathcal{P}(p) \tag{3.33}$$

and then solving for $\mathcal{R}$, $\mathcal{Z}$ and $\mathcal{P}$. This solution procedure was adapted by Dougherty and Babu (1984) for the solution of hydraulics problems in fractured porous reservoirs which included the derivation of dimensionless type curves for variable head tests under confined and isotropic conditions.

Substituting the expression (3.33) into the governing differential equation (3.24) yields

$$\frac{\mathcal{R}''}{\mathcal{R}} + \frac{1}{r}\frac{\mathcal{R}'}{\mathcal{R}} - p = \zeta\frac{\mathcal{Z}''}{\mathcal{Z}} = \lambda \tag{3.34}$$

with λ being the separation variable, and produces the following two ordinary differential equations

$$\mathcal{Z}'' + \frac{\lambda}{\zeta}\mathcal{Z} = 0 \quad \text{and} \quad \mathcal{R}'' + \frac{1}{r}\mathcal{R}' - (p + \lambda)\mathcal{R} = 0 \tag{3.35}$$

Both these differential equations, with relevant boundary conditions, are **Sturm-Liouville** type problems[14] having λ as corresponding eigenvalues. The general solutions for $\mathcal{Z}$ and $\mathcal{R}$ are given then

$$\mathcal{Z}(z) = A\sin(kz) + B\cos(kz) \quad \text{with} \quad k^2 = \sqrt{\frac{\lambda}{\zeta}} \tag{3.36}$$

and

[13] A typical **Sturm-Liouville** problem involves a differential equation defined on an interval together with conditions the solution and/or its derivative is to satisfy at the endpoints of the interval; the form of the endpoint conditions determines the type of the problem: *regular*, *periodic* or *singular* (O'Neil, 1992). The boundary conditions of the present problem renders $\mathcal{Z}$ and $\mathcal{R}$ of being regular type.

$$\mathcal{R}(r) = E\,I_0\left[(p+\lambda)^{\frac{1}{2}}\,r\right] + F\,K_0\left[(p+\lambda)^{\frac{1}{2}}\,r\right] \tag{3.37}$$

where A, B, E and F are arbitrary constants. I_0 and K_0 in equation (3.37) stands for **Modified Bessel Functions** [15]of second kind of order zero and their graphical representations are given in Figure 3.3.3.1.

The first derivatives of the $\mathcal{Z}$ and $\mathcal{R}$ with respect to the corresponding independent variables are given as[16]

$$\mathcal{Z}' = \frac{1}{k}\left[A\cos(kz) - B\sin(kz)\right] \tag{3.38}$$

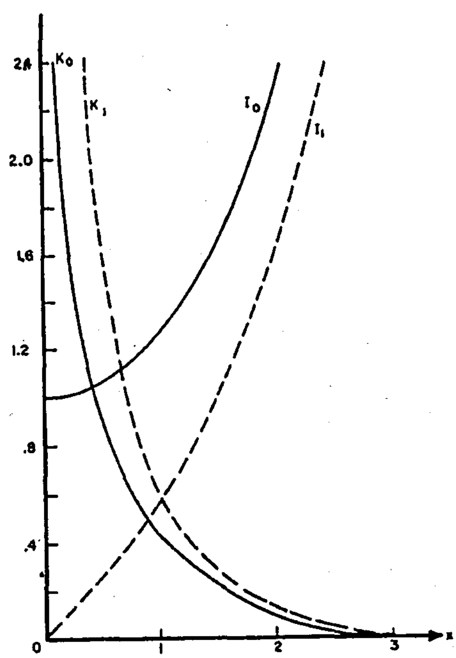


FIGURE 3.3.3.1. Graphical representation of Modified Bessel Functions (Souce: Abramowitz and Stegun, 1972)

[15] Detailed information on this subject can be found in O'Neil (1992) (pp 371-399), Abramowitz and Stegun (1972) (pp. 374-385)

[16] Note that $\frac{\partial K_0[r]}{\partial r} = -K_1[r]$ and $\frac{\partial I_0[r]}{\partial r} = I_1[r]$

and

$$\mathcal{R}' = (p+\lambda)^{\frac{1}{2}}\left\{E\, I_1\left[(p+\lambda)^{\frac{1}{2}} r\right] - F\, K_1\left[(p+\lambda)^{\frac{1}{2}} r\right]\right\} \tag{3.39}$$

These expressions will be used subsequently.

The next step in developing the particular solution of the governing differential equation (3.24) is the determination of the arbitrary constants A, B, E, F and the eigenvalues λ in the general solutions of $\mathcal{Z}$ and $\mathcal{R}$, which satisfy the boundary conditions of the investigated aquifer

3.3.3.1. Solution for Confined Aquifer Case (Type 1) Type 1 boundary conditions require that

$$A = 0 \quad \text{and} \quad \lambda_n = \zeta \frac{n^2 \pi^2}{D^2} \quad \text{for} \quad n = 0, 1, 2, 3\ldots \tag{3.40}$$

and therefore leaving the solution of $\mathcal{Z}$ as:

$$\mathcal{Z}(z) = B_n \cos\left(\frac{n\pi}{D} z\right) \tag{3.41}$$

As $r \to \infty$, $I_0[(p+\lambda)^{1/2} r] \to \infty$ (Figure 3.3.3.1.), leading to an unbounded solution and violating the boundary condition given in (3.29). Thus E must be 0 leaving the solution of $\mathcal{R}$ as:

$$\mathcal{R}(r) = F_n K_0\left[\left(p + \frac{n^2\pi^2}{D^2}\zeta\right)^{\frac{1}{2}} r\right] \tag{3.42}$$

Inserting (3.41) and (3.42) into (3.33) yield

$$\bar{h}(r,z,p) = \mathcal{R}(r)\,\mathcal{Z}(z)\,\mathcal{P}(p) = B_n\,F_n\,\mathcal{P}(p)\,K_0\left[\left(p + \frac{n^2\pi^2}{D^2}\zeta\right)^{\frac{1}{2}} r\right]\cos\left(\frac{n\pi}{D}z\right) \qquad (3.43)$$

Incorporating the arbitrary constants B_n and F_n into $\mathcal{P}(p)$ and naming the new term as α_n converts equation (3.43) into:

$$\bar{h}(r,z,p) = \alpha_n\,K_0\left[\left(p + \frac{n^2\,\pi^2}{D^2}\zeta\right)^{\frac{1}{2}} r\right]\cos\left(\frac{n\,\pi}{D}z\right) \qquad (3.44)$$

For each n = 1, 2, 3, 4, ... there is a function, that satisfies the governing equation (3.24) and the boundary conditions given in (3.30), where α_n is yet to be determined. The solution is then attempted to be developed by an infinite superposition of h_ns, employing the Fundamental Theory of Partial Differential Equations[17]. The infinite series expansion of h takes the form

$$\bar{h} = \sum_{n=0}^{\infty}\alpha_n\,K_0[q_n r]\cos\left(\frac{n\,\pi}{D}z\right) \quad \text{with} \quad q_n = \left(p + \frac{n^2\,\pi^2}{D^2}\zeta\right)^{\frac{1}{2}} \qquad (3.45)$$

and

$$\bar{h} = \alpha_0 K_0[q_0 r] + \sum_{n=1}^{\infty}\alpha_n\,K_0[q_n r]\cos\left(\frac{n\,\pi}{D}z\right) \qquad (3.46)$$

when the first term of the expansion (n = 0) is expressed separately. The boundary conditions (3.28) representing the flux condition along the cylindrical

[17] **The Fundamental Theory of Partial Differential Equations**

If u_1 and u_2 are any solutions of a linear homogeneous partial differential equation in some region R, then $u=c_1u_1+c_2u_2$, where c_1 and c_2 are any constants, is also a solution of that equation in R.

This theorem is also named as the superposition or linearity principle. (Kreyszig, 1993) region describing the well bore are used next for determining the unknown α_ns. Substituting the infinite series expansion of h into (3.28) with

$$\frac{\partial \bar{h}}{\partial r} = -\left\{\alpha_0 q_0 K_1[q_0 r] + \sum_{n=1}^{\infty} \alpha_n q_n K_1[q_n r] \cos\left(\frac{n\pi}{D} z\right)\right\} \tag{3.47}$$

yields

$$-\left[\alpha_0 q_0 K_1[q_0] + \sum_{n=1}^{\infty} \alpha_n q_n K_1[q_n] \cos\left(\frac{n\pi}{D} z\right)\right] = \begin{cases} 0 & \text{for} \quad z < z_a \\ -\bar{q}_{scr} & \text{for} \quad z_a \le z \le z_b \\ 0 & \text{for} \quad z > z_b \end{cases} \tag{3.48}$$

Recognising the left hand side of the above equation as the **Fourier Cosine Expansion**[18] of the function given on the right hand side leads to equations (3.49) and (3.50) for α_0 and (3.51) and (3.52) for α_n:

$$-\alpha_0 q_0 K_1[q_0] = \frac{1}{D}\int_0^L \left.\frac{\partial \bar{h}}{\partial r}\right|_{r=1} dz = \frac{1}{D}\int_{z_a}^{z_b} (-\bar{q}_{scr})\, dz = -\frac{L\,\bar{q}_{scr}}{D} \tag{3.49}$$

$$\alpha_0 = \frac{L\,\bar{q}_{scr}}{q_0 K_1[q_0] D} \tag{3.50}$$

$$-\alpha_n q_n K_1[q_n] = \frac{2}{D}\int_0^L \left.\frac{\partial \bar{h}}{\partial r}\right|_{r=1} \cos\left(\frac{n\pi}{D} z\right) dz = \frac{1}{D}\int_{z_a}^{z_b} (-\bar{q}_{scr}) \cos\left(\frac{n\pi}{D} z\right) dz \tag{3.51}$$

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[18] **The Definition of Fourier Cosine Series on [0,L]**

if f is integrable on [0,L], the Fourier cosine series of f on [0,L] is

$$a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{\Delta} x\right)$$

with

$$a_0 = \frac{1}{\Delta}\int_0^{\Delta} f(x)\,dx \quad \text{and} \quad a_n = \frac{1}{\Delta}\int_0^{\Delta} f(x) \cos\left(\frac{n\pi}{\Delta} x\right) dx$$

$$\alpha_n = \frac{2\,\overline{q}_{scr}}{n\,\pi\,q_n\,K_1[q_n]}\left[\sin\left(\frac{n\,\pi}{D}z_b\right) - \sin\left(\frac{n\,\pi}{D}z_a\right)\right] \tag{3.52}$$

Substitution of the determined α_ns into the equation (3.44) produces the following expression which describes the hydraulic head distribution within the aquifer in the Laplace domain:

$$\begin{aligned}\overline{h}(r,z,p) = {} & \frac{L\,\overline{q}_{scr}}{D}\frac{K_0[q_0 r]}{q_0 K_1[q_0]} + \\ & \sum_{n=1}^{\infty}\frac{2\,\overline{q}_{scr}}{n\,\pi}\frac{K_0[q_n r]}{q_n\,K_1[q_n]}\left[\sin\left(\frac{n\,\pi}{D}z_b\right) - \sin\left(\frac{n\,\pi}{D}z_a\right)\right]\cos\left(\frac{n\,\pi}{D}z\right)\end{aligned} \tag{3.53}$$

All the parameters in the above equation, except q_{scr}, have known values. The next step in the solution procedure is using the equations (3.25) and (3.27) in order to eliminate q_{scr} and eventually derive the expression for h_w in closed form.

The insertion of equation (3.53) into (3.26) yields the average head along the screen. It reads:

$$\begin{aligned}\overline{h}^{*}\Big|_{r=1} = -\frac{1}{L}\Bigg\{ & \int_{z_a}^{z_b}\frac{L\,\overline{q}_{scr}}{D}\frac{K_0[q_0]}{q_0 K_1[q_0]}\,dz + \\ & \sum_{n=1}^{\infty}\int_{z_a}^{z_b}\frac{2\,\overline{q}_{scr}}{n\,\pi}\frac{K_0[q_n]}{q_n\,K_1[q_n]}\left[\sin\left(\frac{n\,\pi}{D}z_b\right) - \sin\left(\frac{n\,\pi}{D}z_a\right)\right]\cos\left(\frac{n\,\pi}{D}z\right)dz\Bigg\}\end{aligned} \tag{3.54}$$

or simply

$$\overline{h}^{*}\Big|_{r=1} = \overline{q}_{scr}(A_0 + A) \tag{3.55}$$

by defining A_0 and A as

$$A_0 = \frac{K_0[q_0]\,\delta}{q_0 K_1[q_0]} \quad \text{and} \quad A = \sum_{n=1}^{\infty}\frac{2\,\overline{q}_{scr}}{\delta\,n^2\,\pi^2}\frac{K_0[q_n]}{q_n\,K_1[q_n]}\left[\sin\left(\frac{n\,\pi}{D}z_b\right) - \sin\left(\frac{n\,\pi}{D}z_a\right)\right]^2 \tag{3.56}$$

The equations (3.25) and (3.27) in their new forms given below

$$\overline{Q} - \overline{q}_{scr} = C_D\left(p\,\overline{h}_w - 1\right) \tag{3.57}$$

and

$$\overline{h}_w = \overline{q}_{scr}\left[\left(A_0 + A\right) + S_k\right] \tag{3.58}$$

comprises a system with two equations and two unknowns, h_w and q_{scr}. Solving these equations by eliminating q_{scr} results

$$\overline{h}_w = \frac{\left(C_D + \overline{Q}\right)\left[\left(A_0 + A\right) + S_k\right]}{C_D p\left[\left(A_0 + A\right) + S_k\right] + 1} \tag{3.59}$$

which describes the head within the well bore in Laplace domain. For a conventional slug test procedure, i.e. Q = 0, the solution takes the form

$$\overline{h}_w = \frac{C_D\left[\left(A_0 + A\right) + S_k\right]}{C_D p\left[\left(A_0 + A\right) + S_k\right] + 1} \tag{3.60}$$

3.3.3.2. Solution for Unconfined Aquifer Case (Type 2.a.) The solution procedures conducted for this case are very similar to the ones completed for Type 1 boundary conditions. The only, but crucial, difference is that the flux condition along the screen can not be approximated by a Fourier series expansion for this case. Thus, another solution procedure is introduced for the determination of α_ns and discussed widely in the text.

Type 2.a boundary conditions along with boundary condition (3.29) require that

$$A = 0 \quad \text{and} \quad \lambda_n = \zeta\frac{\left(n + \frac{1}{2}\right)^2 \pi^2}{D^2} \quad \text{for} \quad n = 0, 1, 2, 3\ldots \tag{3.61}$$

consequently leading to

$$\mathcal{Z}(z) = B_n \cos\left(\frac{\left(n+\frac{1}{2}\right)\pi}{D} z\right) \tag{3.62;}$$

$$\mathcal{R}(r) = F_n K_0\left[\left(p + \frac{\left(n+\frac{1}{2}\right)^2 \pi^2}{D^2}\zeta\right)^{\frac{1}{2}} r\right] \tag{3.63}$$

and then

$$\bar{h}(r, z, p) = \alpha_n K_0\left[\left(p + \frac{\left(n+\frac{1}{2}\right)^2 \pi^2}{D^2}\zeta\right)^{\frac{1}{2}} r\right] \cos\left(\frac{\left(n+\frac{1}{2}\right)\pi}{D} z\right). \tag{3.64}$$

The solution of h is attempted to be expressed as an infinite superposition of h_ns with the same reasoning discussed in Section 3.3.3.1. This ends up with:

$$\bar{h} = \sum_{n=0}^{\infty} \alpha_n K_0[q_n r] \cos\left(\frac{\left(n+\frac{1}{2}\right)\pi}{D} z\right) \quad \text{with} \quad q_n = \left(p + \frac{\left(n+\frac{1}{2}\right)^2 \pi^2}{D^2}\zeta\right)^{\frac{1}{2}} \tag{3.65}$$

The boundary conditions (3.28) representing the flux condition along the cylindrical region describing the well bore are used next for determining the unknown α_ns. Substituting the infinite series expansion of h into (3.28) with

$$\frac{\partial \bar{h}}{\partial r} = -\sum_{n=0}^{\infty} \alpha_n q_n K_1[q_n r] \cos\left(\frac{\left(n+\frac{1}{2}\right)\pi}{D} z\right) \tag{3.66}$$

yields

$$-\sum_{n=0}^{\infty} \alpha_n q_n K_1[q_n] \cos\left(\frac{(n+\frac{1}{2})\pi}{D} z\right) = \begin{cases} 0 & \text{for} \quad z < z_a \\ -\overline{q}_{scr} & \text{for} \quad z_a \le z \le z_b \\ 0 & \text{for} \quad z > z_b \end{cases} \qquad (3.67)$$

In contrast to the solution procedure conducted in Section 3.3.3.1., the summation on the left hand side of (67) can not be recognised as the Fourier Cosine Series expansion of the function on the right hand side because of the (n+ ½) term within the cosine parameter. However, multiplying this equation by

$$\cos\left(\frac{m\pi}{D} z\right) \qquad (3.68)$$

and recalling **the orthogonality relationship** of cosine function, which, is in its modified form[19], states

$$\int_0^{\Delta} \cos\left(\frac{m\pi}{\Delta} x\right) \cos\left(\frac{n\pi}{\Delta} x\right) dx = 0 \quad \text{if} \quad m \neq n \qquad (3.69)$$

leads to

$$\int_0^D \left.\frac{\partial \overline{h}}{\partial r}\right|_{r=1} \cos\left(\frac{(m+\frac{1}{2})\pi}{D} z\right) dz = -\sum_{n=0}^{\infty} \int_0^D \alpha_n q_n K_1[q_n] \cos\left(\frac{(n+\frac{1}{2})\pi}{D} z\right) \cos\left(\frac{(m+\frac{1}{2})\pi}{D} z\right) dz \qquad (3.70)$$

and

[19] The general form of the orthogonality relationship reads

$$\int_{-\pi}^{\pi} \cos(mx)\cos(nx)\,dx = 0 \quad \text{if} \quad m \neq n.$$

Mathematical proof of both the general form and (3.69) can be found in O'Neil, 1992 (p.1012 and p. 1176, respectively)

$$-\int_{z_b}^{z_a} q_{scr} \cos\left(\frac{\left(n+\frac{1}{2}\right)\pi}{D} z\right) dz = -\alpha_n q_n K_1[q_n] \int_0^D \cos^2\left(\frac{\left(n+\frac{1}{2}\right)\pi}{D} z\right) dz \qquad (3.71)$$

Employing the trigonometric relation

$$\cos^2 x = \frac{1+\cos 2x}{2} \qquad (3.72)$$

when solving the integrations in (3.71) yields

$$-\frac{q_{scr} D}{\left(n+\frac{1}{2}\right)\pi}\left[\sin\left(\frac{\left(n+\frac{1}{2}\right)\pi}{D} z_a\right) - \sin\left(\frac{\left(n+\frac{1}{2}\right)\pi}{D} z_b\right)\right] = -\alpha_n q_n K_1[q_n]\left[\int_0^D dz + \int_0^D \cos\left(\frac{(2n+1)\pi}{D} z\right) dz\right] \qquad (3.73)$$

and

$$\alpha_n = \frac{2\,\overline{q}_{scr}}{\left(n+\frac{1}{2}\right)\pi\, q_n\, K_1[q_n]}\left[\sin\left(\frac{\left(n+\frac{1}{2}\right)\pi}{D} z_b\right) - \sin\left(\frac{\left(n+\frac{1}{2}\right)\pi}{D} z_a\right)\right] \qquad (3.74)$$

and eventually ends up with

$$\overline{h}(r,z,p) = \sum_{n=0}^{\infty} \frac{2\,\overline{q}_{scr}}{\left(n+\frac{1}{2}\right)\pi} \frac{K_0[q_n r]}{q_n K_1[q_n]}\left[\sin\left(\frac{\left(n+\frac{1}{2}\right)\pi}{D} z_b\right) - \sin\left(\frac{\left(n+\frac{1}{2}\right)\pi}{D} z_a\right)\right]\cos\left(\frac{\left(n+\frac{1}{2}\right)\pi}{D} z\right) \qquad (3.75)$$

The corresponding average head along the screen is obtained as:

$$\left.\overline{h}^*\right|_{r=1} = -\frac{1}{L}\sum_{n=0}^{\infty}\int_{z_a}^{z_b} \frac{2\,\overline{q}_{scr}}{\left(n+\frac{1}{2}\right)\pi} \frac{K_0[q_n]}{q_n K_1[q_n]}\left[\sin\left(\frac{\left(n+\frac{1}{2}\right)\pi}{D} z_b\right) - \sin\left(\frac{\left(n+\frac{1}{2}\right)\pi}{D} z_a\right)\right]\cos\left(\frac{\left(n+\frac{1}{2}\right)\pi}{D} z\right) dz \qquad (3.76)$$

or simply

$$\left.\overline{h}^*\right|_{r=1} = A\,\overline{q}_{scr} \qquad (3.77)$$

where A is given by:

$$A = \sum_{n=0}^{\infty} \frac{2}{\delta\,(n+\frac{1}{2})^2\,\pi^2}\,\frac{K_0[q_n]}{q_n\,K_1[q_n]}\left[\sin\!\left(\frac{(n+\frac{1}{2})\,\pi}{D}\,z_b\right) - \sin\!\left(\frac{(n+\frac{1}{2})\,\pi}{D}\,z_a\right)\right]^2 \tag{3.78}$$

Solving equations (3.25) and (3.27) simultaneously after inserting (3.78) yields

$$\overline{h}_w = \frac{\left(C_D + \overline{Q}\right)\left[A + S_k\right]}{C_D\,p\left[A + S_k\right] + 1} \tag{3.79}$$

in its most general form. For conventional slug test case (3.79) assumes the following form:

$$\overline{h}_w = \frac{C_D\left[A + S_k\right]}{C_D\,p\left[A + S_k\right] + 1} \tag{3.80}$$

<u>3.3.3.3. Solution for Unconfined Aquifer Case (Type 2.b.)</u> The solution for this case proceeds same as the solution of Type 2.a. The only difference lies in the determination procedure of eigenvalues λ of $\mathcal{J}$.

The no flow boundary condition given in (3.32) requires A = 0 and leaves (3.36) as:

$$\mathcal{J}(z) = B\cos(kz) \quad \text{with} \quad k^2 = \sqrt{\frac{\lambda}{\zeta}} \tag{3.81}$$

Substituting (3.81) into the transient water table condition presented in (3.32) yields:

$$S_y\,p\,\mathcal{R}(r)\,\mathcal{P}(p)\,B\cos(kD) = \mathcal{R}(r)\,\mathcal{P}(p)\,B\,k\sin(kD) \tag{3.82}$$

and consequently

$$\frac{S_y\, p\, D}{\varphi} = \tan(\varphi) \quad \text{with} \quad \varphi = k\, D \tag{3.83}$$

Although equation (3.83) can not be solved algebraically, there are infinitely many positive solutions; these solutions are demonstrated in Figure 3.3.3.3.1. Calling these solutions φ_n infers the following eigenvalues :

$$\lambda_n = \frac{\varphi_n^2\, \zeta}{D^2} \quad \text{for} \quad n = 1, 2, 3, 4 \ldots \tag{3.84}$$

With these eigenvalues $\mathcal{Z}$ gets the form

$$\mathcal{Z}(z) = B_n \cos\left(\frac{\varphi_n}{D}\, z\right) \tag{3.85}$$

whereas, not to violate (3.29), $\mathcal{R}$ becomes

$$\mathcal{R}(r) = F_n\, K_0\left[\left(p + \frac{\varphi_n^2\, \zeta}{D^2}\right)^{\frac{1}{2}} r\right] \tag{3.86}$$

The infinite series expansion for h is obtained then as:

$$\bar{h} = \sum_{n=0}^{\infty} \alpha_n\, K_0[q_n r] \cos\left(\frac{\varphi_n}{D}\, z\right) \quad \text{with} \quad q_n = \left(p + \frac{\varphi^2\, \zeta}{D^2}\right)^{\frac{1}{2}} \tag{3.87}$$

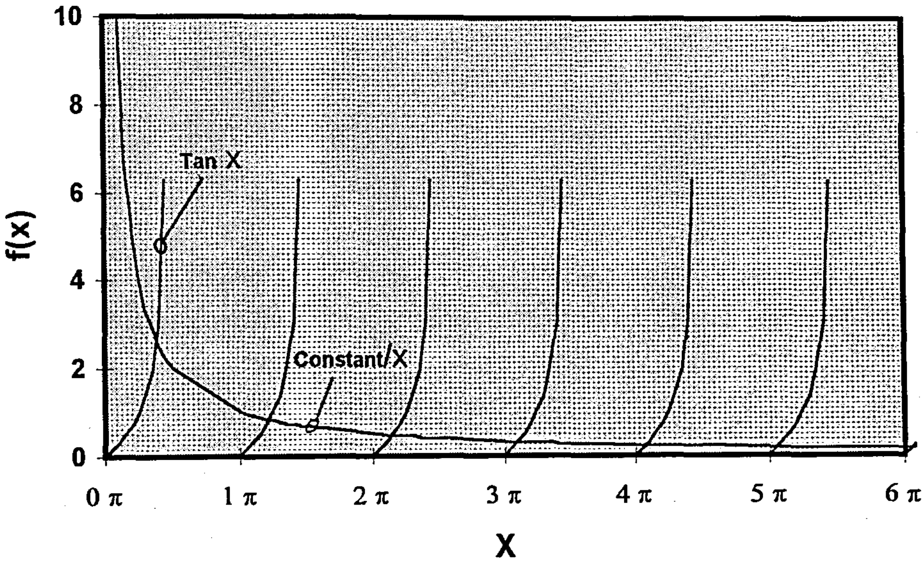

FIGURE 3.3.3.3.1. Graphical representation of equation (3.83)

Proceeding the same solution procedure presented in Section 3.3.3.2 between equations (3.66) and (3.73) yields

$$\alpha_n = \frac{2\,\overline{q}_{scr}\left[\sin\left(\frac{\varphi_n}{D}z_b\right) - \sin\left(\frac{\varphi_n}{D}z_a\right)\right]}{q_n\,K_1\left[q_n\left(\varphi_n + \frac{\sin(2\varphi_n)}{2}\right)\right]} \tag{3.88}$$

and

$$\overline{h} = \sum_{n=1}^{\infty} \frac{2\,\overline{q}_{scr}K_0[q_n\,r]\left[\sin\left(\frac{\varphi_n}{D}z_b\right) - \sin\left(\frac{\varphi_n}{D}z_a\right)\right]}{q_n\,K_1[q_n]\left(\varphi_n + \frac{\sin(2\varphi_n)}{2}\right)}\cos\left(\frac{\varphi_n}{D}z\right). \tag{3.89}$$

Eventually, the solution for h_w is obtained as

$$\bar{h}_w = \frac{\left(C_D + \bar{Q}\right)\left[A + S_k\right]}{C_D\, p\left[A + S_k\right] + 1} \tag{3.90}$$

where A is given by

$$A = \sum_{n=1}^{\infty} \frac{2\, K_0\left[q_n\right]\left[\sin\left(\frac{\varphi_n}{D} z_b\right) - \sin\left(\frac{\varphi_n}{D} z_a\right)\right]^2}{\delta\, q_n\, K_1\left[q_n\right]\varphi_n\left(\varphi_n + \frac{\sin(2\varphi_n)}{2}\right)} \tag{3.91}$$

For the conventional slug test procedure the solution for h_w assumes the form:

$$\bar{h}_w = \frac{C_D\left[A + S_k\right]}{C_D\, p\left[A + S_k\right] + 1} \tag{3.92}$$

3.4. Numerical Inversion of the Transformed Solution

The transform solutions developed in the previous sections need to be inverted into time domain for having practical importance. This can be achieved in two alternative methods; analytical inversion or numerical approximate inversion. The analytical inversion of presently developed expressions is too complicated due to the necessity of integration in the complex plain. Consequently an approximate numerical solution is attempted to this end.

There are numerous methods for numerical inversion in the literature. Most are in the form of a polynomial approximation of which the Fourier series is most commonly used because of calculation ease (Durbin, 1973). Alternative methods were developed using Gaussian formula for integration of the inversion integral (Piessens, 1971). Practical difficulties are likely to be encountered when applying aforementioned methods because the series expansion coefficients are functions of the transform solution and the number of terms in the approximating polynomial which renders calculations cumbersome.

In the present study the numerical method originally developed by Gaver (1966) and revised and developed in the form of an algorithm by Stehfest (1970) is used. Over the past years this algorithm has been successfully employed in the field of petroleum engineering and hydrogeology, particularly to calculate transient pressure response in porous materials (Moench and Ogata, 1981; Da Prat, 1981). The algorithm is known to be suitable when the solution in the time domain is smooth, monotonic and without rapid oscillations and therefore ideal for overdamped slug tests. However, its utilisation for underdamped cases is plaughed due to the oscillatory nature of the well-aquifer system response. This condition was also observed by Kipp (1985) who concluded to use Crump (1976) algorithm for the numerical inversion.

In Stehfest algorithm, the following formulation is used for obtaining the inverse of a transform function:

$$f(t) = \mathcal{L}^{-1}\left[\bar{f}(p)\right] \approx \frac{\ln 2}{t}\sum_{i=1}^{N_{steh}} W_i \bar{f}\left(\frac{i \ln 2}{t}\right) \tag{3.93}$$

where

	$\mathcal{L}^{-1}$	:	inverse Laplace transformation;
	p	:	the transform variable;
	f	:	the solution in the Laplace domain;
	t	:	time;
	N_{steh}	:	an even number representing the number of Stehfest points;
	W_i	:	a weight coefficient depending only on N_{steh}, and given by:

$$Wi = (-1)^{(N_{steh}/2)+i} \sum_{k=(i+1)/2}^{\min(i, N_{steh}/2)} \frac{k^{(N_{steh}/2)}(2k)!}{\left(N_{steh}/2 - k\right)!\; k!\; (k-1)!\,(i-k)!\,(2k-i)!} \tag{3.94}$$

where k is calculated by integer arithmetics.

The main advantages of Stehfest algorithm over other numerical inversion techniques can be listed as:

- no complex number calculations are required, all involved parameters are real;
- the value of W_i depends only on the selected value of N_{steh}. Thus they are evaluated once and stored for all subsequent points in time;
- a small number of Stehfest Numbers (N_{steh}) are required for each point in time.

Because of these features, Stehfest algorithm can conveniently be employed by field practitioners using hand-held programmable calculators (Moench and Ogata 1981, based on personal communication with Ramey, 1978).

The accuracy of Stehfest approximation is sensitive to the selection of the number of Stehfest points (N_{steh}). Theoretically the accuracy can be increased by increasing N_{steh}. However, in computer applications after a point rounding errors worsen the result (Moench and Ogata, 1981) and the value of N_{steh} should be determined for each problem and computer (Dougherty and Babu, 1984). In the present study eight Stehfest points were used (N_{steh} = 8) and the underlying reasons are discussed in Appendix A.

4. APPLICATIONS OF THE PROPOSED METHOD

The validity and applicability of the proposed approach were investigated by analysing field data previously published in literature. Two FORTRAN programs were developed to this end:

(i) **Type_Curve_Generator** Code (called TCG henceforth) which generates dimensionless type curves for each investigated aquifer setting using a timescale given by $\frac{t}{2\gamma C_d} = \frac{\tilde{t} K_r D}{r_s^2}$ [1];

(ii) **Curve_Match_Program** (CMP) which estimates the hydraulic properties of investigated formation by matching TCG generated type curves and field data following the scheme explained in section 2.3.1.

Both programs were structured suitable for large scale parametric studies and simple sensitivity analyses, and extensively used throughout this study. Further information on these programs along with their listings are given in Appendix A.

4.1. Confined Aquifers

4.1.1. Fully Penetrating Case

Cooper et. al. (1967) demonstrated the validity of their method by analysing slug test data obtained in a well near Dawsonville, Georgia. This well cased to **24 m** with **15.2 cm** (6 inches) casing and drilled as a **15.2 cm** open hole to a depth of **122 m**. The test was conducted by sudden withdrawal of a long weighted float from the well, causing an initial change in the water level of **0.560 m**. Observed

[1] This scale is independent of the screen length.

recovery values are given in Table 4.1.1.1. and graphically shown in Figure 4.1.1.1.

By analysing the recovery data with curve matching procedure, the authors ended up with the following values for aquifer transmissivity and storativity

$$\text{Transmissivity} = 5.3 \text{ cm}^2/\text{sec}$$
$$\text{Storativity} = 10^{-3}$$

corresponding to **4.35 x 10^{-4} cm/sec** and **8.2×10^{-6} m^{-1}** for radial hydraulic conductivity and specific storage values, respectively. The Cooper et. al. method assumes purely radial flow by neglecting the axial flow component within the aquifer. This assumption is often considered to be a valid approach in practice if the well fully penetrates the formation being tested and/or there is evidence of presence of anisotropy.

For quantifying the hydraulic conductivity of the formation by the present method, dimensionless type curves were generated for the above aquifer setting using the TCG program; the obtained curves are depicted in Figure 4.1.1.2. The skin coefficient S_k was assumed to be zero during calculations because Cooper et. al. did not include the skin effect in their original solution development. By executing CMP, the type curve for **C_D= 5 x 10^1** was determined to fit the test data best. A typical curve match procedure for the present aquifer setting is demostrated through 4 consecutive Figures, 4.1.1.3. and 4.1.1.6. The following values were obtained for formation transmissivity and storativity:

$$\text{Transmissivity} = 3.6 \text{ cm}^2/\text{sec}$$
$$\text{Storativity} = 10^{-2}$$

showing differences between parameter estimations of the present and Cooper et al methods. The transmissivity of the formation estimated by the present method is **68** percent of the one obtained by Cooper et. al. method. The storativity , on the other hand, is one order of magnitude higher than the Cooper et. al. result.

The sensitivity of the dimensionless type curves of the above site setting to the effects of skin friction and anisotropy was investigated by the present method and depicted in figures 4.1.1.7. and 4.1.1.8. respectively. It is apparent that the skin friction shifts the type curves to the right on the dimensionless time scale indicating that the calculated hydraulic conductivity values would be smaller than the actual hydraulic conductivity of the formation. Figure 4.1.1.9. depicts type curves that were obtained for different skin frictions (solid curves), with C_d fixed to 5×10^1, along with type curves without a skin but for different C_D values. The similar shape of both solid and dashed lines (especially for high C_Ds), however, renders the estimation of hydraulic conductivity using curve matching scheme difficult. This condition was also observed by Ramey and Agarwal (1972), who concluded that transmissivity estimates by the Cooper et. al. method are not sensitive to skin effects. However problems stemming from the so called "non-uniqueness of solutions in slug test theory" is not uncommon (Karasaki et. al., 1988) in practice and was discussed in section 2.3.3.

Figure 4.1.1.7. depicts that the degree of anisotropy does not have an effect on well response in case of full penetration; all type curves of different ζ values fall on each other ($C_D = 5 \times 10^1$). This phenomena can be explained by the prevailing radial flow conditions within the aquifer.

TABLE 4.1.1.1. Slug test data given by Cooper et. al. (1967)

t (sec)	Dimensionless Head
3	0.816
6	0.700
9	0.616
12	0.550
15	0.500
18	0.450
21	0.400
24	0.366
27	0.334
30	0.300
33	0.266
36	0.250
39	0.234
42	0.200
45	0.193
48	0.166
51	0.159
54	0.146
57	0.134
60	0.127
63	0.116

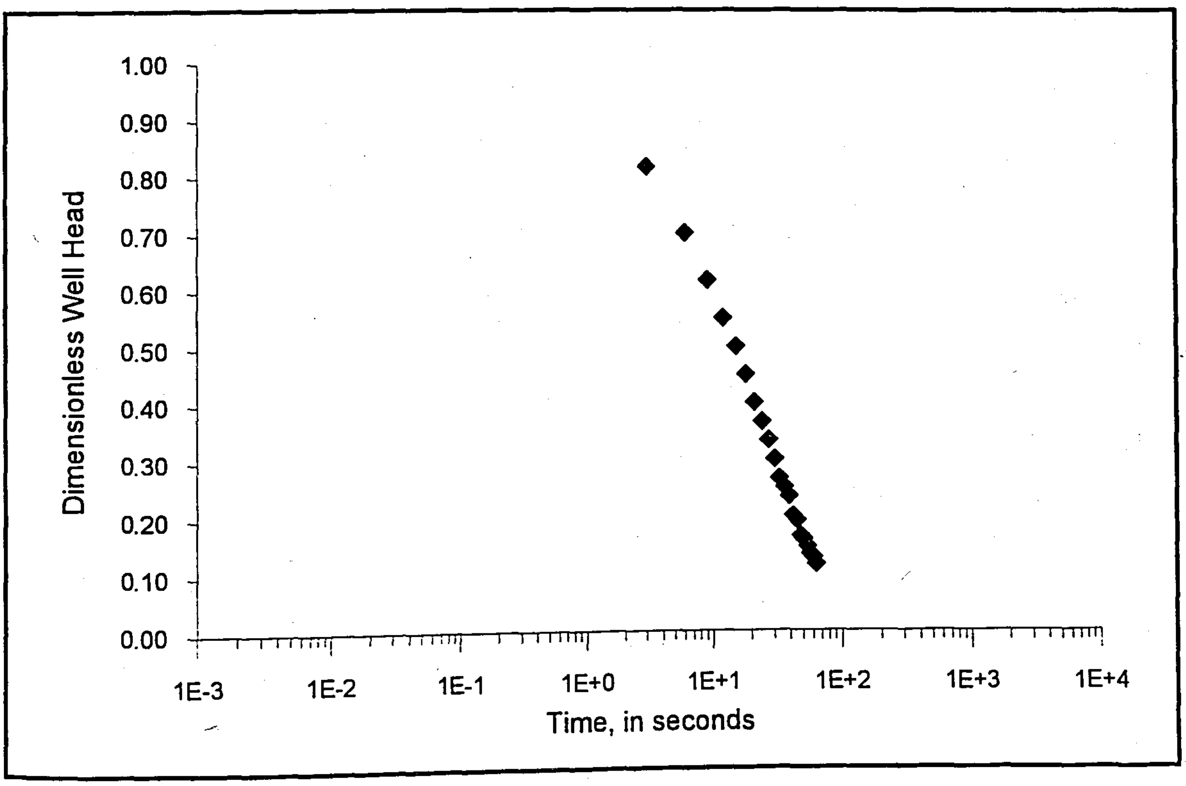

FIGURE 4.1.1.1. Slug test data given by Cooper et. al. (1967)

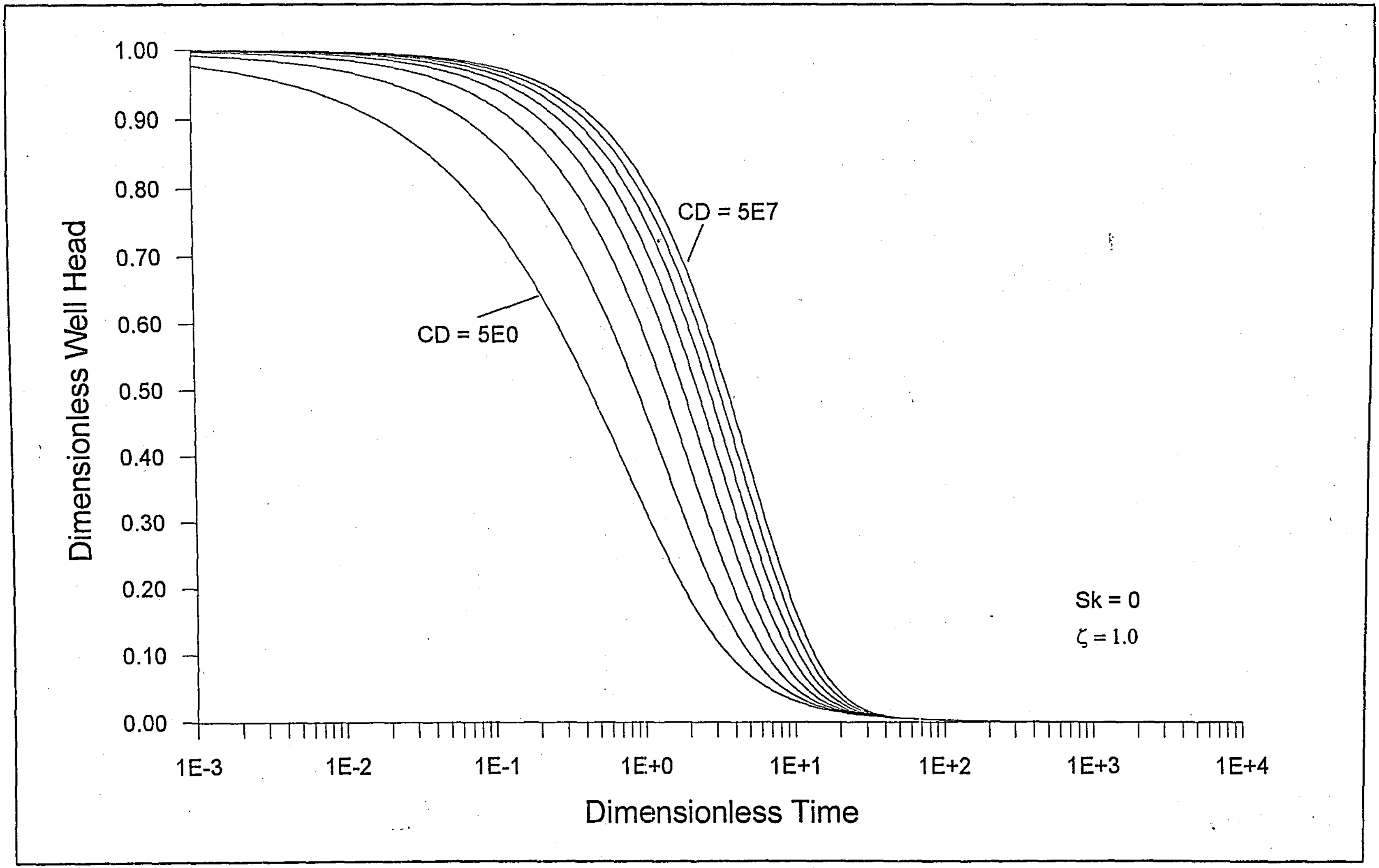

FIGURE 4.1.1.2. Dimensionless type curves for Cooper et. al. aquifer setting (1967)

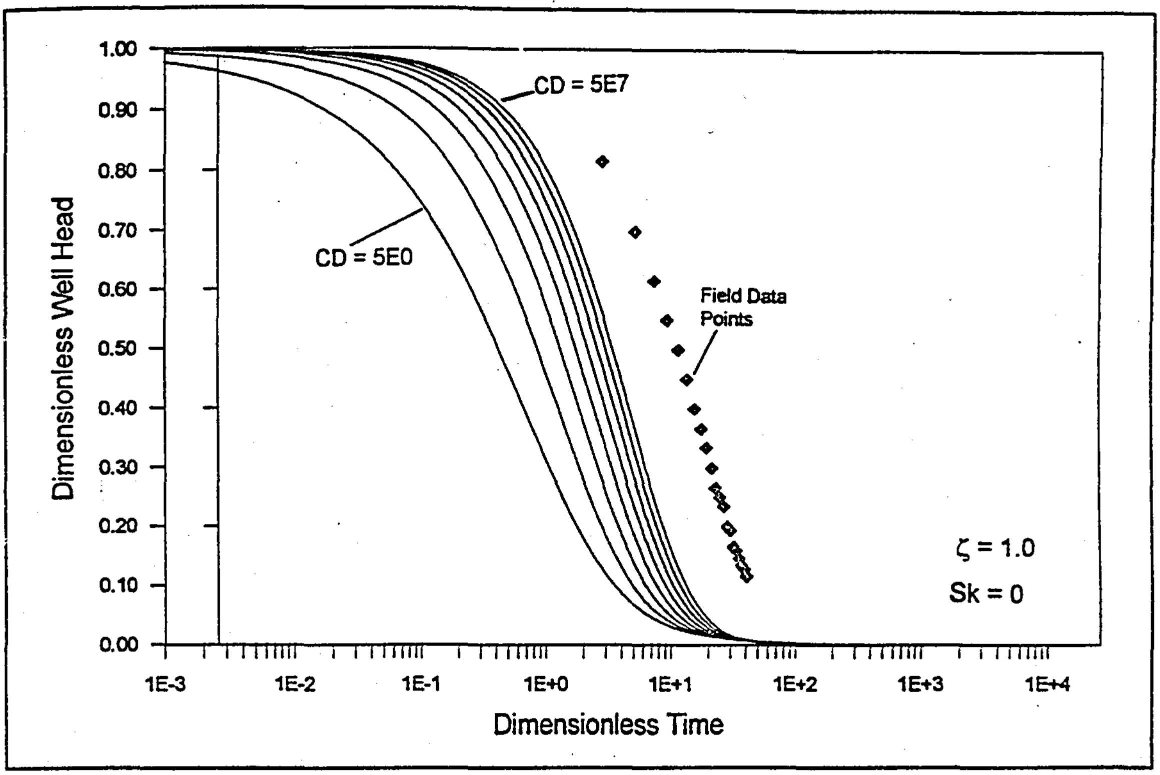

FIGURE 4.1.1.3. Curve matching procedure for Cooper et. al. (1967) aquifer setting, Step 1

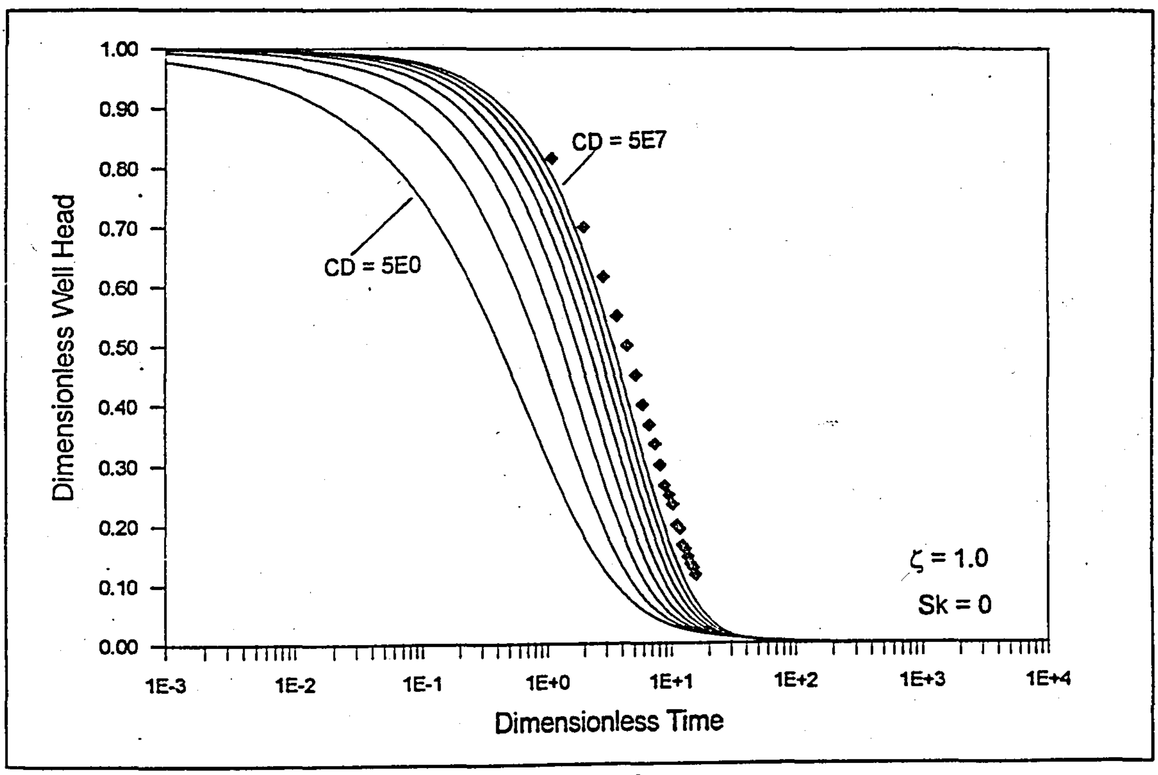

FIGURE 4.1.1.4. Curve matching procedure for Cooper et. al. (1967) aquifer setting, Step 2

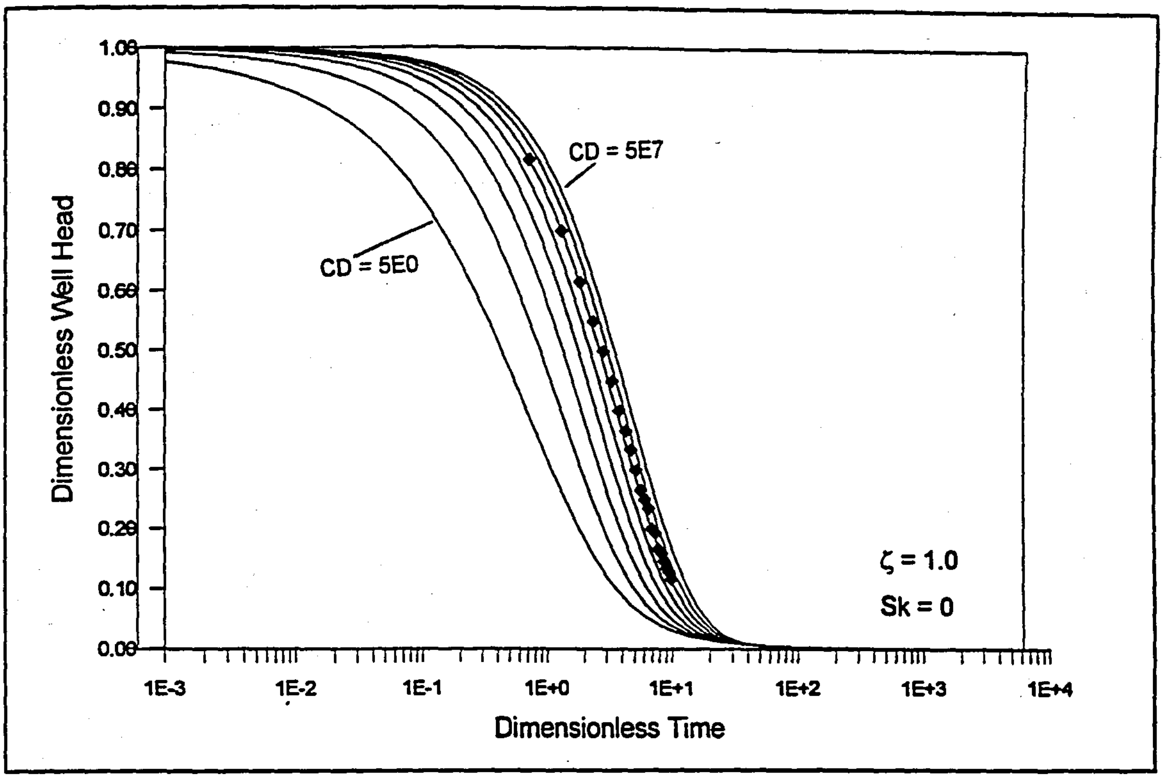

FIGURE 4.1.1.5. Curve matching procedure for Cooper et. al. (1967) aquifer setting, Step 3

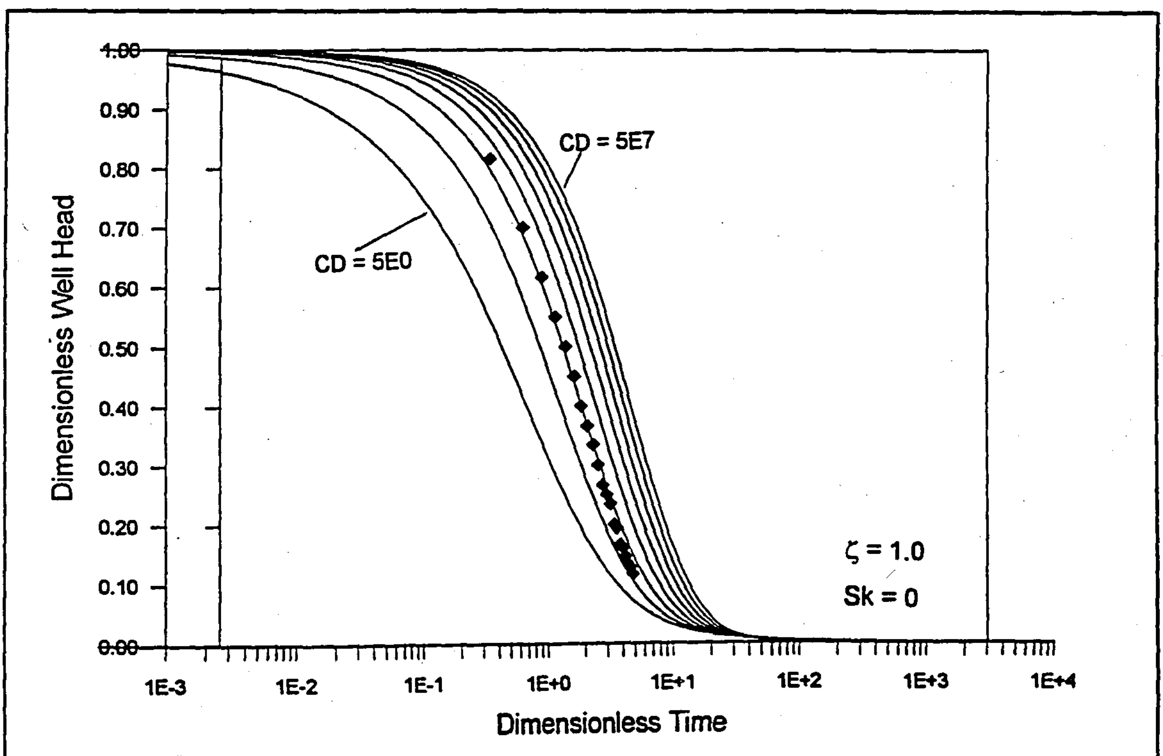

FIGURE 4.1.1.6. Curve matching procedure for Cooper et. al. (1967) aquifer setting, Step 4

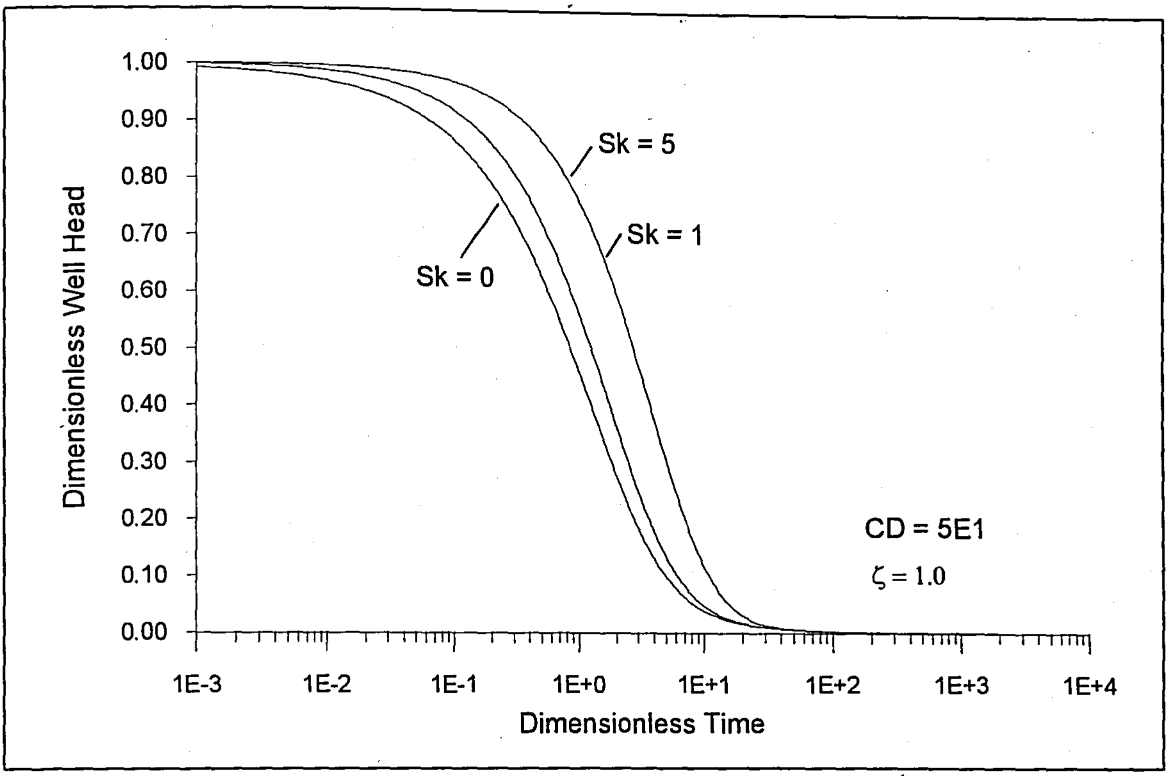

FIGURE 4.1.1.7. The effect of skin friction on dimensionless type curves Cooper et. al. aquifer settting (1967)

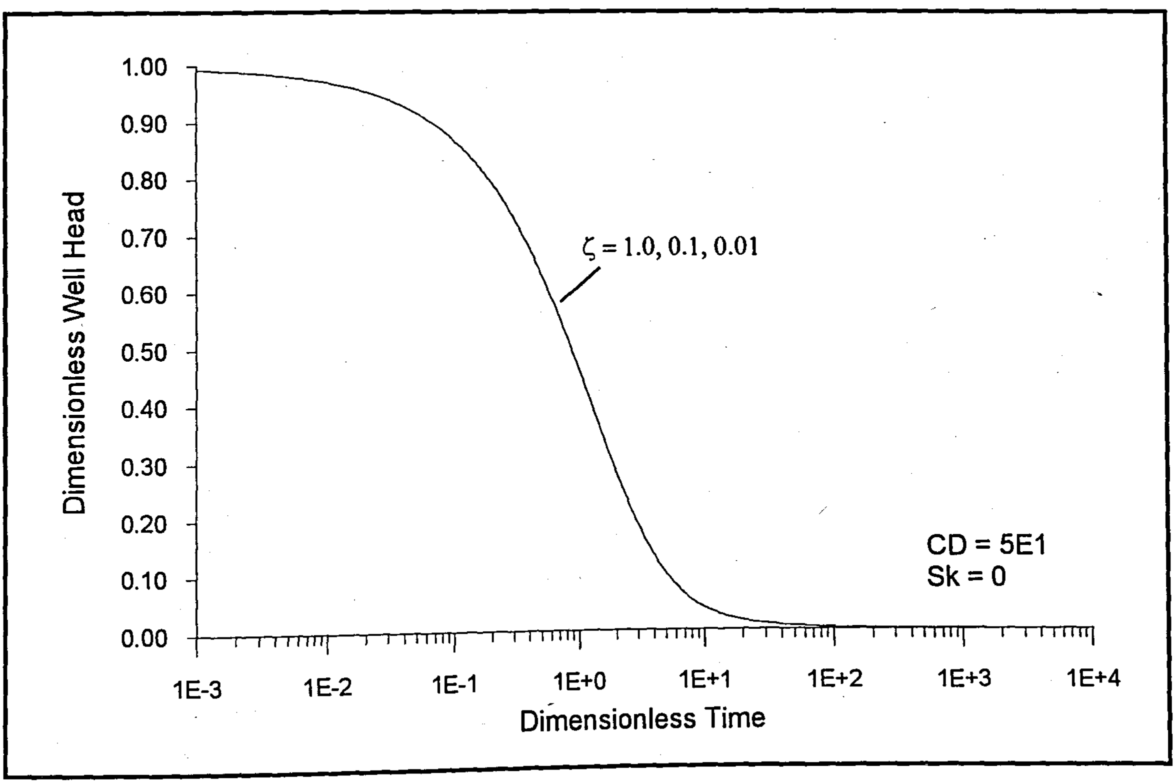

FIGURE 4.1.1.8. The effect of anisotropy on dimensionless type curves Cooper et. al. aquifer setting (1967)

4.1.2. Partial Penetrating Case

The prediction capabilities of the presently derived equations depicting axial as well as radial flow conditions under partially penetrating, anisotropic and confined conditions were evaluated by analysing multilevel slug test data collected by Melville et. al.. (1991) using a double packer system in three wells in a confined, granular aquifer near Mobile, Alabama. Soil sampling and geophysical data indicated homogeneous material throughout the **21.2 m** (70 ft) thickness of the aquifer which is located 39.6 m (130 ft) below the land surface. The geology and the well placement at the site, and the multilevel slug test unit designed and used for these tests are shown in Figures 4.1.2.1. and 4.1.2.2., respectively. The authors stated that in the general utilisation of multilevel slug testing, vertical flow in the annular region outside the well casing or screen can invalidate the test. Therefore special care was given during well installation and completion for not developing skin effects; skin friction of the screen was assumed to be zero by the authors during data analysis. The screened zone length **L** isolated by the straddle packer was **1.11 m** with the well casing $\mathbf{r_c}$ and screen $\mathbf{r_s}$ radii given as **0.051 m** (4 inches PVC) and **0.0762 m** (6 inches PVC), respectively. Before proceeding to data analysis, Melville et. al. tested the reproducibility of the behaviour of wells; tests repeated at a given elevation were generally found reproducible with the maximum difference in slopes of the straight line fits of the data among all the repeated tests to be 11 per cent. Figure 4.1.2.3 demostrates the reproducibility of the data at well E6; The authors used the slug test data collected in this well for data analysis in their study. The data collected at various depths in Well E6 in the aquifer is graphically presented in Figure 4.1.2.4.

Two methods of analysis were used by Melville et. al. (1991) to estimate the hydraulic properties of the aquifer:

(i) Cooper et. al. (1967) method assuming that purely radial flow occurs during the slug test; the aquifer thickness under these conditions becomes equivalent to the screened zone length; and

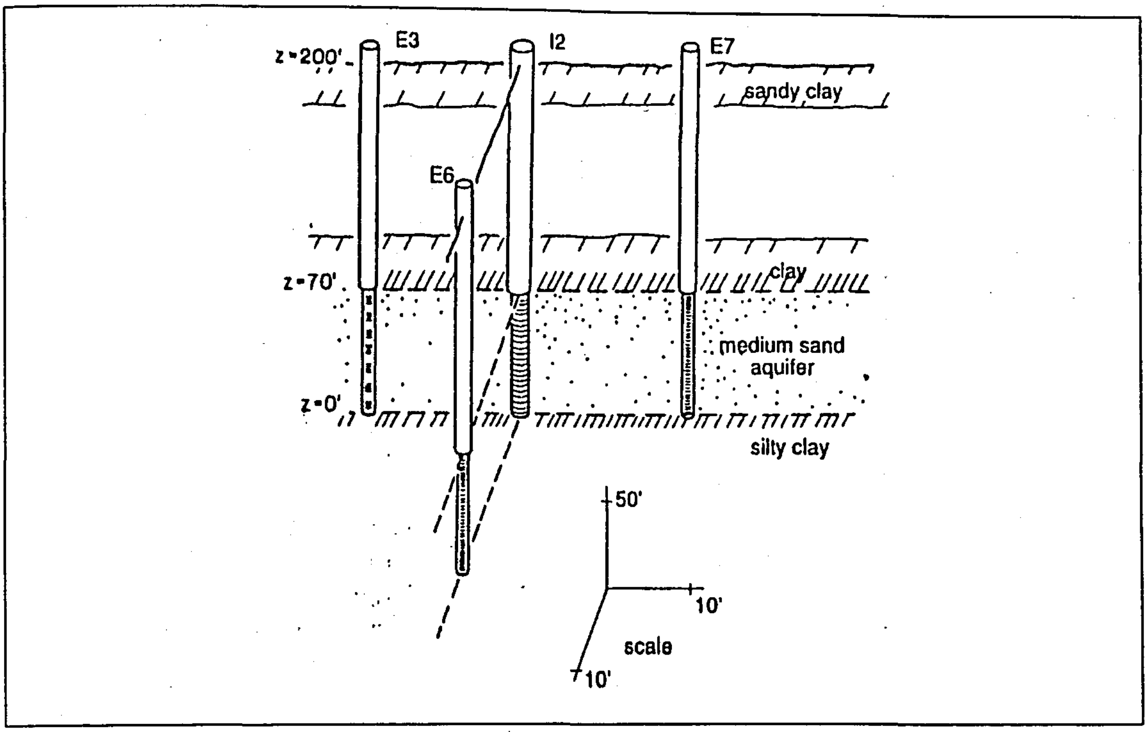

FIGURE 4.1.2.1. Geology and well placement at the Mobile site
(Source: Melville et. al., 1991)

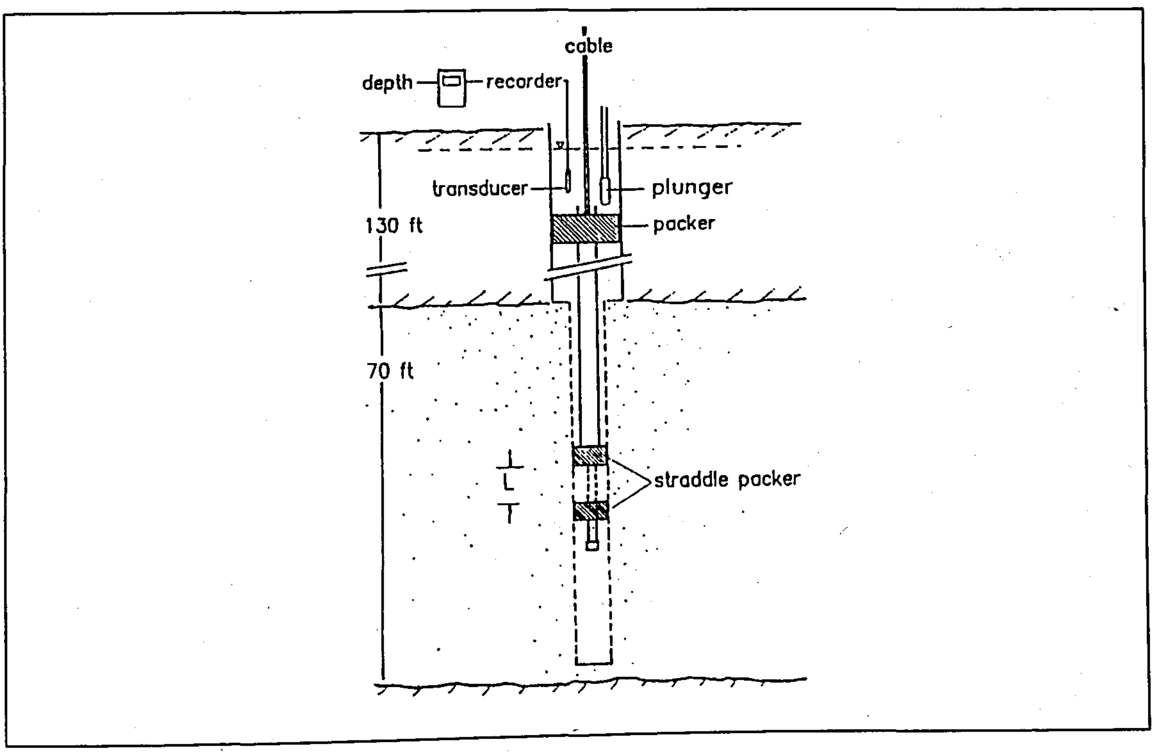

FIGURE 4.1.2.2. Multilevel slug test apparatus at the Mobile test site
(Source: Melville et. al., 1991)

(ii) Widdowson et. al. (1990) numerical method of analysis where radial as well as axial flow under anisotropic conditions could be analysed. The model was based on a finite element model (EFLOW) with modifications for multilevel slug tests. The authors stated that this technique follows the approach of Dagan (1978) in that curve matching and determination of an effective radius of influence are not required. The latter statement here indicates that the method was developed for no skin cases and valid only naturally developed wells, as was deliberately expressed in the original paper. The main advantage of the method over other commonly used models was stated being applicable to a complete range of multilevel slug test geometries and not suffering from numerical difficulties that plague other data analysis techniques (Bouwer and Rice, 1976; Dagan, 1978) when values of L are small.

The authors used the numerical model with $\zeta = K_z / K_r$ ratio of 0.1 in the analysis of the slug test data in order to determine the validity of the radial flow assumption[2]. The results of both methods are listed in Table 4.1.2.1. and demonstrate that the hydraulic conductivities calculated based on method two are about **two thirds** of the values based on method one, the radial flow model. At this point, the findings of Melville et. al. combined with the discussions in section 4.1.1. of this study end up with an important conclusion about the validity of the Cooper et. al. method : The difference between Cooper et. al. method and Method 2 estimations (**67** per cent), shown in Table 4.1.2.1. is very close to the difference observed between the present and Cooper et. al. methods (**68** percent) (Section 4.1.1.). This fact strongly suggests the presence of a systematic bias (c.a. **30** per cent in magnitude) in parameter estimations of Cooper et. al. method and raises doubts about its validity.

The prediction capabilities of the presently derived equations were further investigated by analysing multilevel slug test data collected in Well E6. The data

[2] Considering $\zeta = 0.1$ limits the significance of the axial flow component within the formation and therefore represents an induced radial flow condition.

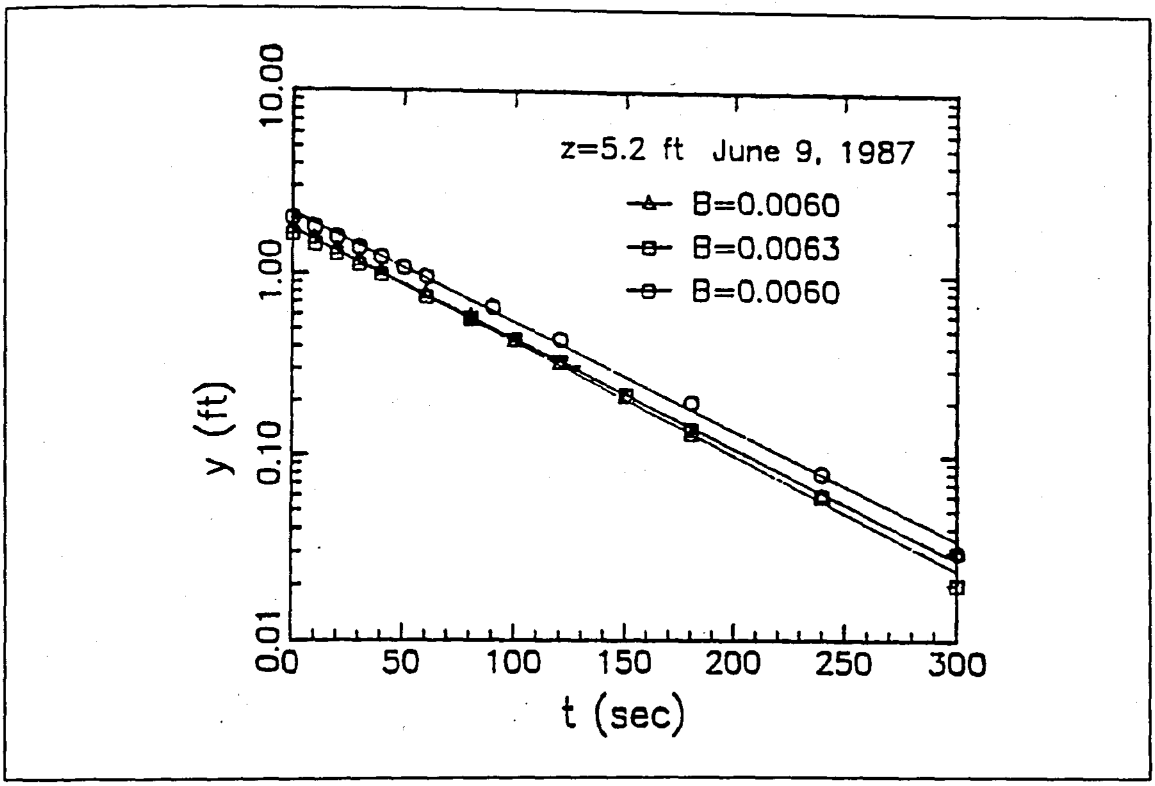

FIGURE 4.1.2.3. Reproducibility of data at well E6
(Source: Melville et. al., 1991)

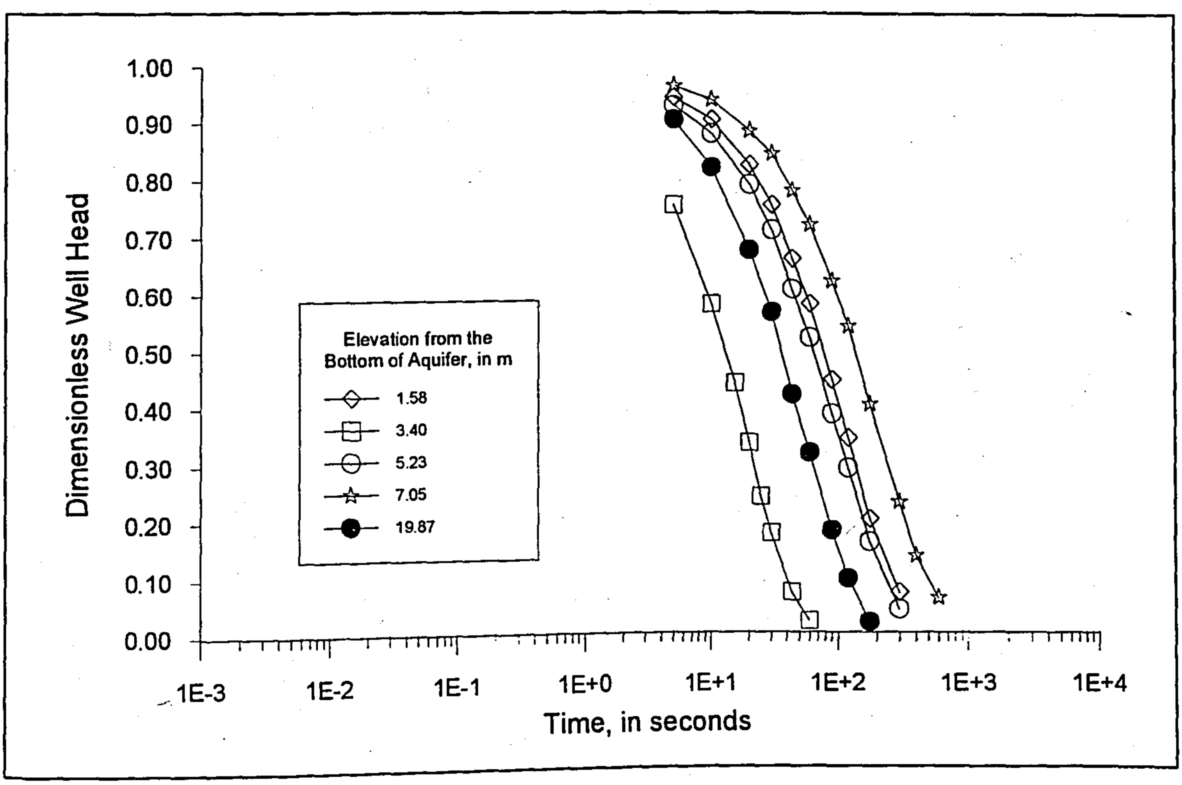

FIGURE 4.1.2.4. Slug test data of Melville et. al.(1996)

were analysed by generating dimensionless type curves for various C_D values using TCG program under isotropic as well as anisotropic conditions for each of the depths where the double packer was located.

The obtained type curves are shown through Figures 4.1.2.5. and 4.1.2.14 for both isotropic and anisotropic conditions. Figures 4.1.2.15. to 4.1.2.19., on the other hand, depict type curves for two different C_D values under isotropic and anisotropic conditions. It is evident that the effect of anisotropy within the aquifer increases with increasing C_D, i.e. decreasing specific storage coefficient of the formation. The hydraulic properties of the aquifer were quantified by matching the dimensionless type curves with the data for a best fit by the help of CMP. The results of the analysis for both isotropic and anisotropic conditions at various depths are listed in Table 4.1.2.2.

TABLE 4.1.2.1 Comparison of Method 1 and Method 2 results

	Hydraulic Conductivity Values, in m/day		
Elevation from Aquifer Bottom	Method 1 Cooper et al. (1967)	Method 2 Melville et al., (1990)	Ratio
in m.	(Radial flow model)	Induced radial flow with ζ=0.1	Method 2 / Method 1
1.58	10.21	6.78	66%
3.40	68.00	49.37	73%
5.23	12.25	7.87	64%
7.05	5.84	3.57	61%
19.82	22.61	16.57	73%

TABLE 4.1.2.2. Hydraulic conductivity estimates of the present method

	Hydraulic Parameters for Isotropic Conditions	
Elevation from Aquifer Bottom	Hydraulic Conductivity, m/s	Specific Storage Coefficient
in m.	in m/s	1/m
1.58	5.98	2.01×10^{-4}
3.40	40.77	2.01×10^{-8}
5.23	6.43	2.01×10^{-3}
7.05	2.95	2.01×10^{-3}
19.82	14.48	2.01×10^{-8}

Hydraulic Parameters for Anisotropic Conditions		
Hydraulic Conductivity, m/s	Specific Storage Coefficient	Elevation from Aquifer Bottom
in m/s	1/m	in m.
8.38	2.01×10^{-4}	1.58
54.29	2.01×10^{-8}	3.40
9.39	2.01×10^{-3}	5.23
4.38	2.01×10^{-3}	7.05
18.56	2.01×10^{-8}	19.82

The present method yielded higher hydraulic conductivity values for anisotropic case than for isotropic case at all elevations. This is consistent with concepts introduced so far and the difference between the results indicate the importance of vertical flow within the aquifer during a slug test.

The following three tables 4.1.2.3, 4.1.2.4. and 4.1.2.5. collate all hydraulic conductivity values estimated by different methods discussed in this section and reflect relative differences between them.

TABLE 4.1.2.3. Comparison of the results of Method 1, Method 2 and the present method

	Hydraulic Conductivity Values, in m/day			
Elevation from Aquifer Bottom	Method 1 Cooper et al, (1967)	Method 2 Melville et al., (1990)	Present Method	
in m.	(Radial flow model)	ζ=0.1	ζ=0.1	ζ=1.0, considering axial flow in the aquifer
1.58	10.21	6.78	8.38	5.98
3.40	68.00	49.37	54.29	40.77
5.23	12.25	7.87	9.39	6.43
7.05	5.84	3.57	4.38	2.95
19.82	22.61	16.57	18.56	14.48

TABLE 4.1.2.4. Comparison of the results of Method 1 and the present method

	Hydraulic Conductivity Values, in m/day		
Elevation from Aquifer Bottom	Method 1 Cooper et al, (1967)	Present Method	Ratio
in m.	(Radial flow model)	ζ=1.0, i.e. isotropic	Present Method / Method 1
1.58	10.21	5.98	59%
3.40	68.00	40.77	60%
5.23	12.25	6.43	52%
7.05	5.84	2.95	51%
19.82	22.61	14.48	64%

TABLE 4.1.2.5. Comparison of the results of Method 2 and the present method

	Hydraulic Conductivity Values, in m/day		
Elevation from Aquifer Bottom	Method 2 Melville et al., (1990)	Present Method	Ratio
in m.	ζ=0.1	ζ=0.1	Method 2 / Present Method
1.58	6.78	8.38	81%
3.40	49.37	54.29	91%
5.23	7.87	9.39	84%
7.05	3.57	4.38	82%
19.82	16.57	18.56	89%

The following points can be inferred by considering all result:

•The results obtained under anisotropic conditions by the present approach and the numerical model yielded quite similar hydraulic conductivity values,

•The present method ended up with the lowest hydraulic conductivity values under isotropic conditions whereas Cooper et. al. method yielded the highest figures. The discrepancy between two methods is 57 percent, higher than the systematic bias determined previously . This increase can be explained by the presence of axial flow[3].

• Overall, the above results indicate that the use of the Cooper et. al. (1967) method is not suitable for the analysis of the slug test data collected in the present setting.

[3] The bias was determined to be about 30 percent under radial flow conditions in section 4.1.1. and at the beginning of this section.

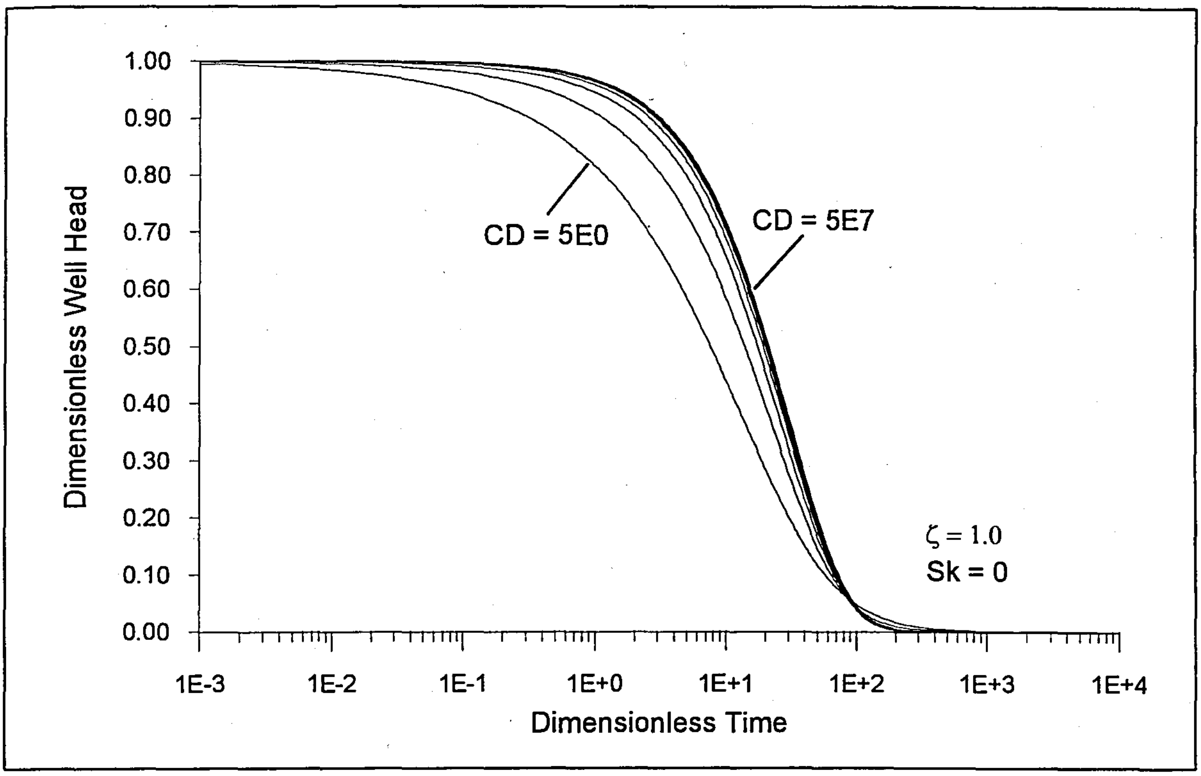

FIGURE 4.1.2.5. Dimensionless type curves for isotropic conditions, z = 1.58 m (Melville et. al. aquifer setting (1991))

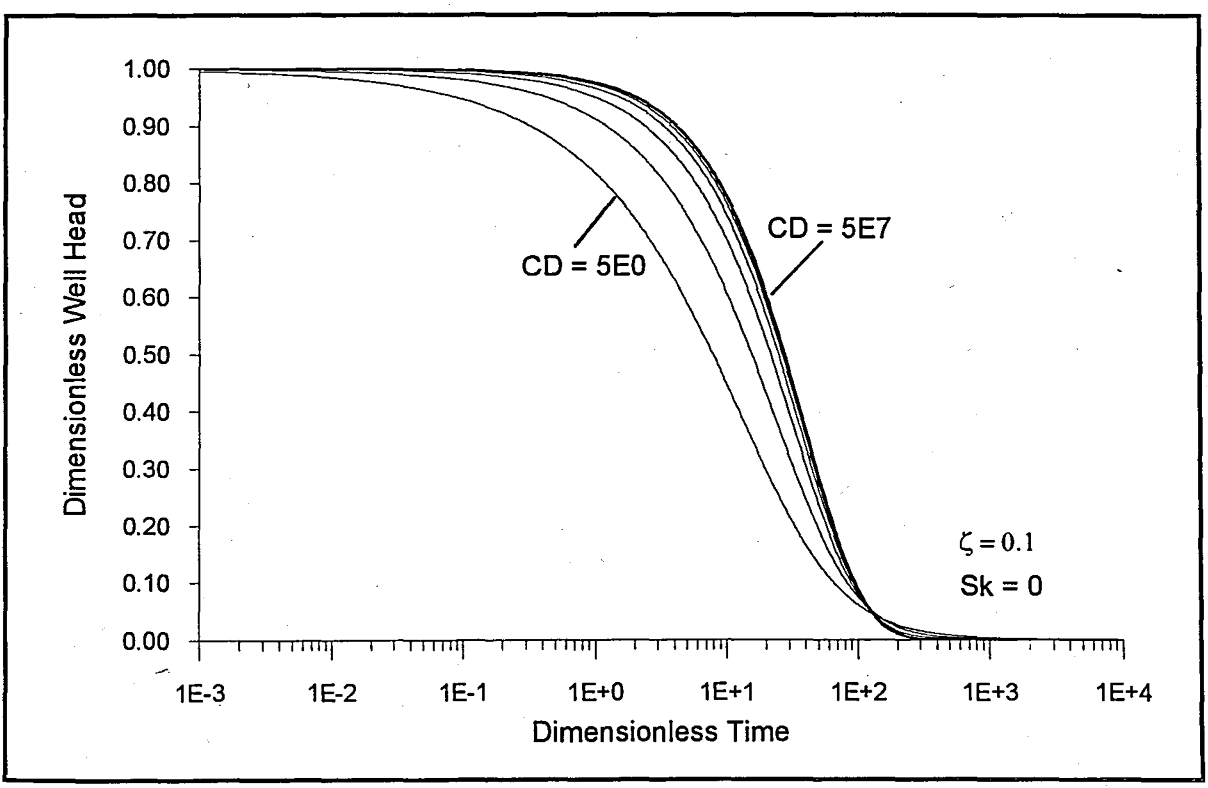

FIGURE 4.1.2.6. Dimensionless type curves for anisotropic conditions, z = 1.58 m (Melville et. al. aquifer setting (1991))

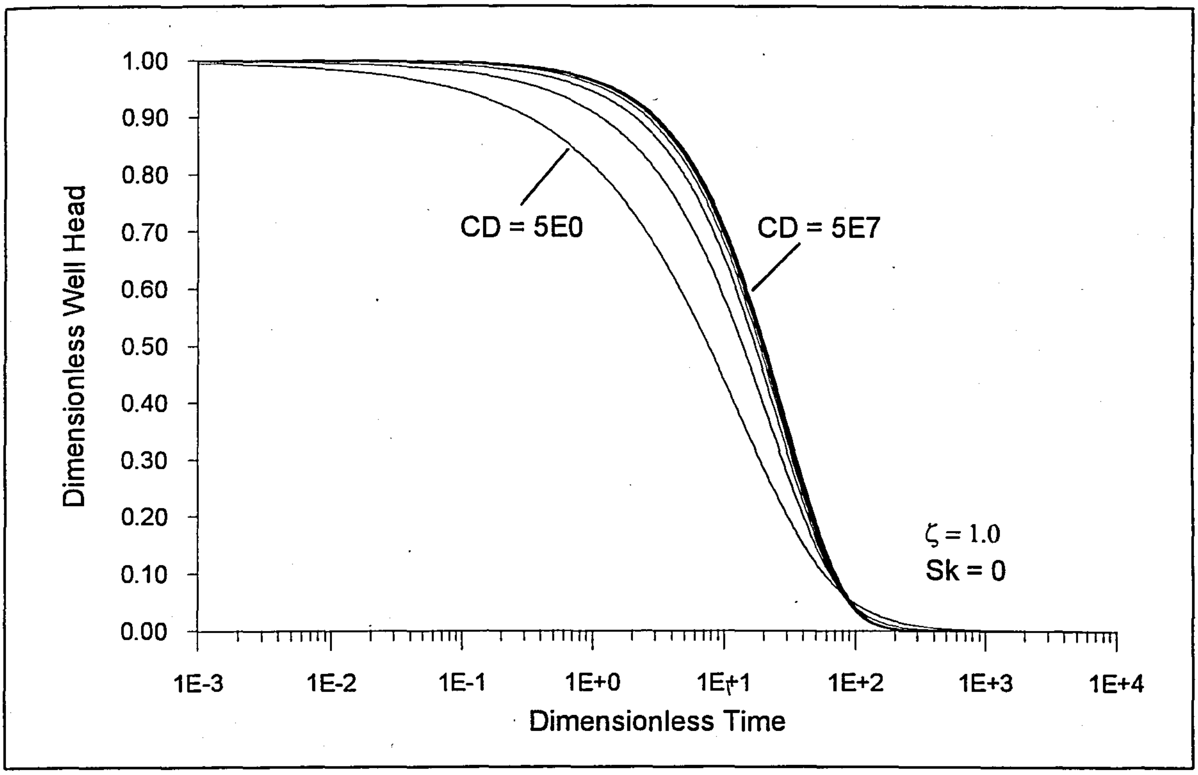

FIGURE 4.1.2.7. Dimensionless type curves for isotropic conditions, z = 3.40 m (Melville et. al. aquifer setting (1991))

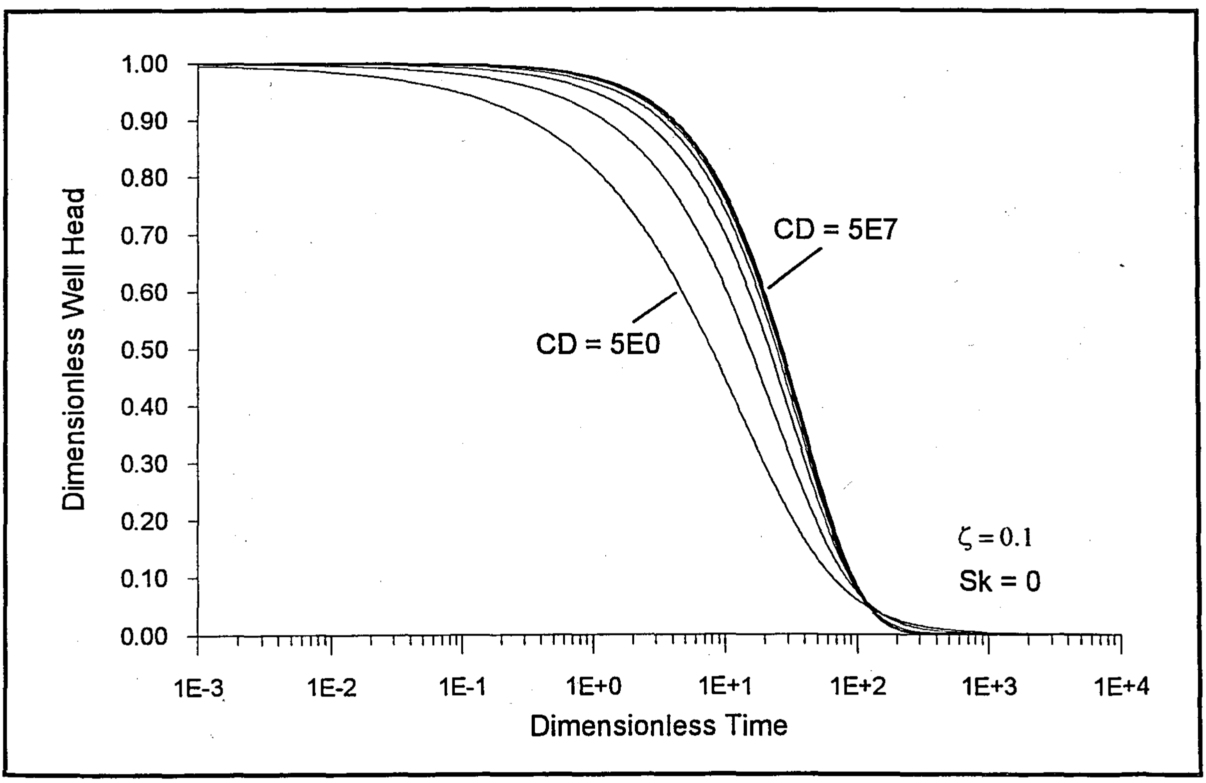

FIGURE 4.1.2.8. Dimensionless type curves for anisotropic conditions, z = 3.40 m (Melville et. al. aquifer setting (1991))

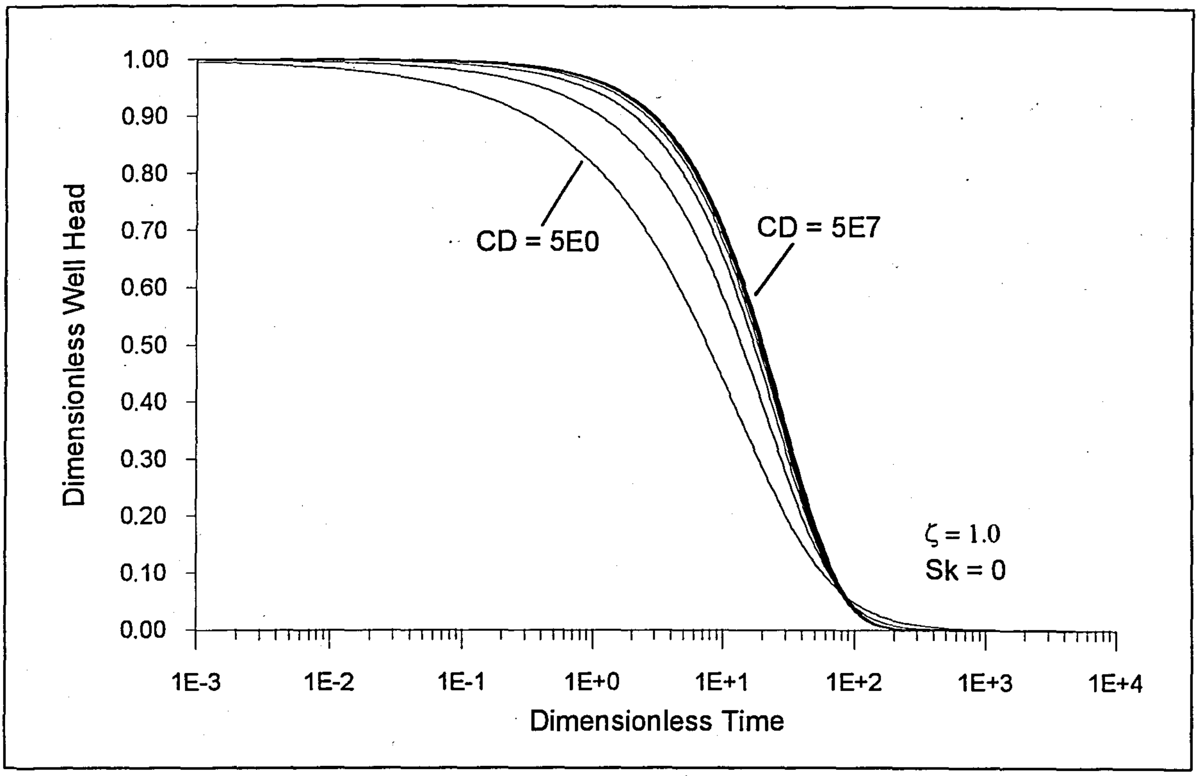

FIGURE 4.1.2.9. Dimensionless type curves for isotropic conditions, z = 5.23 m (Melville et. al. aquifer setting (1991))

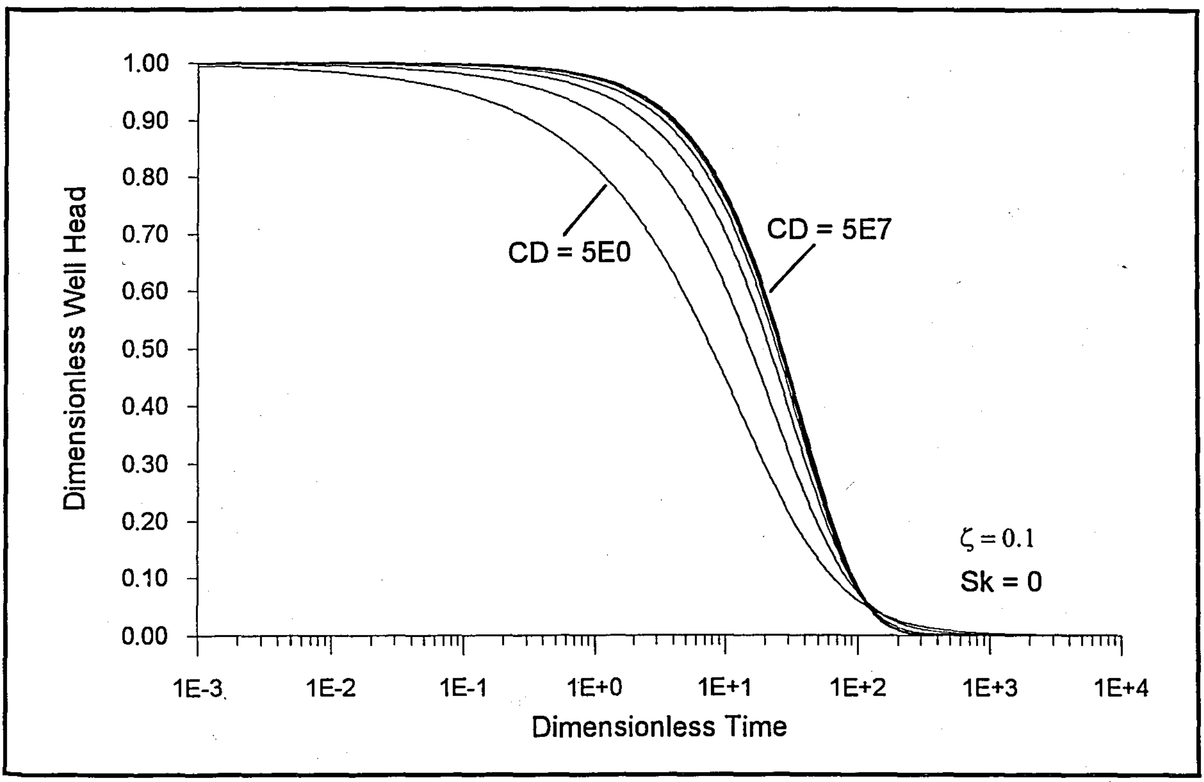

FIGURE 4.1.2.10. Dimensionless typecurves for anisotropic conditions, z = 5.23 m (Melville et. al. aquifer setting (1991))

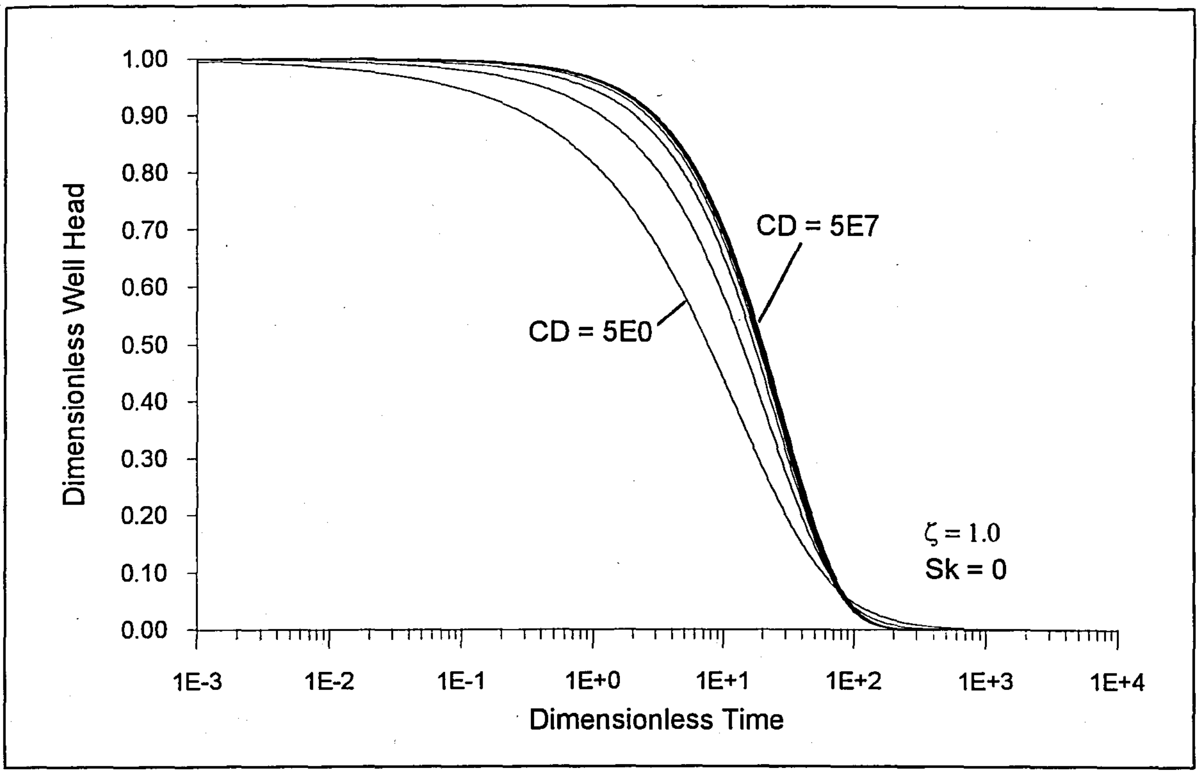

FIGURE 4.1.2.11. Dimensionless type curves for isotropic conditions, z = 7.05 m (Melville et. al. aquifer setting (1991))

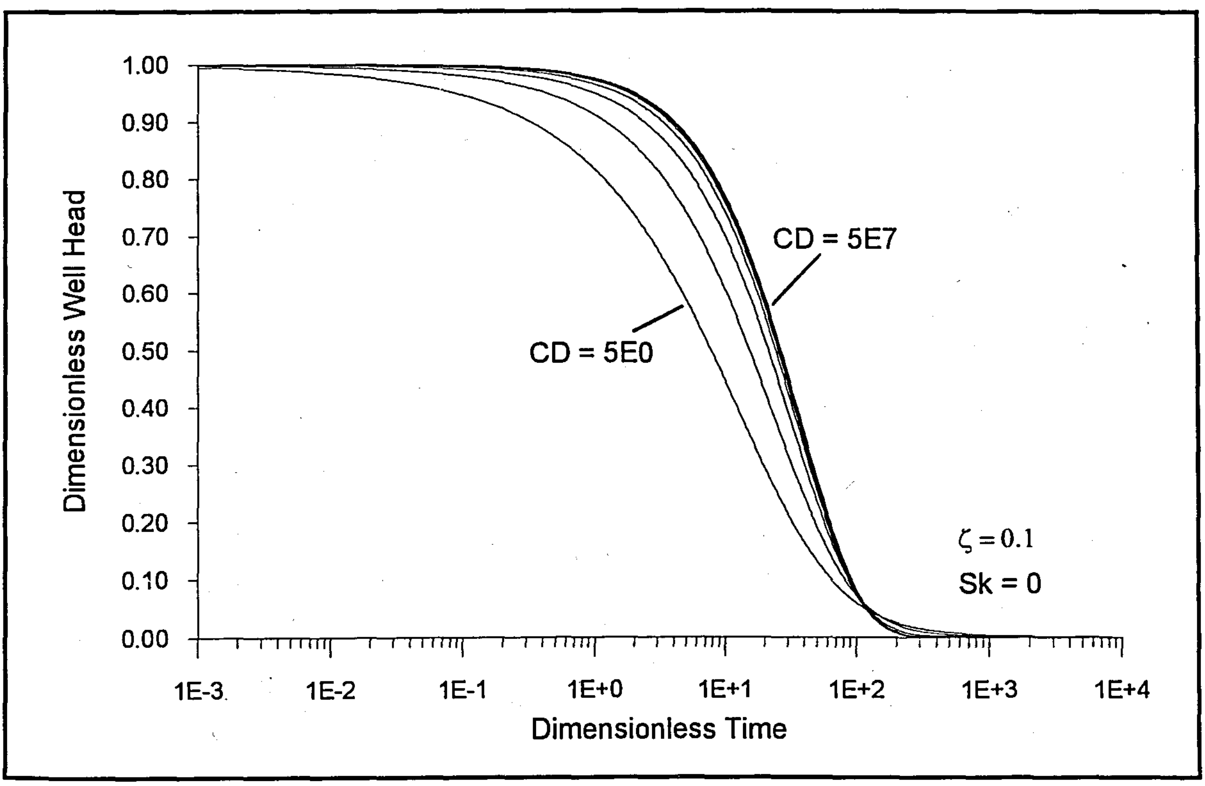

FIGURE 4.1.2.12. Dimensionless type curves for anisotropic conditions, z = 7.05 m (Melville et. al. aquifer setting (1991))

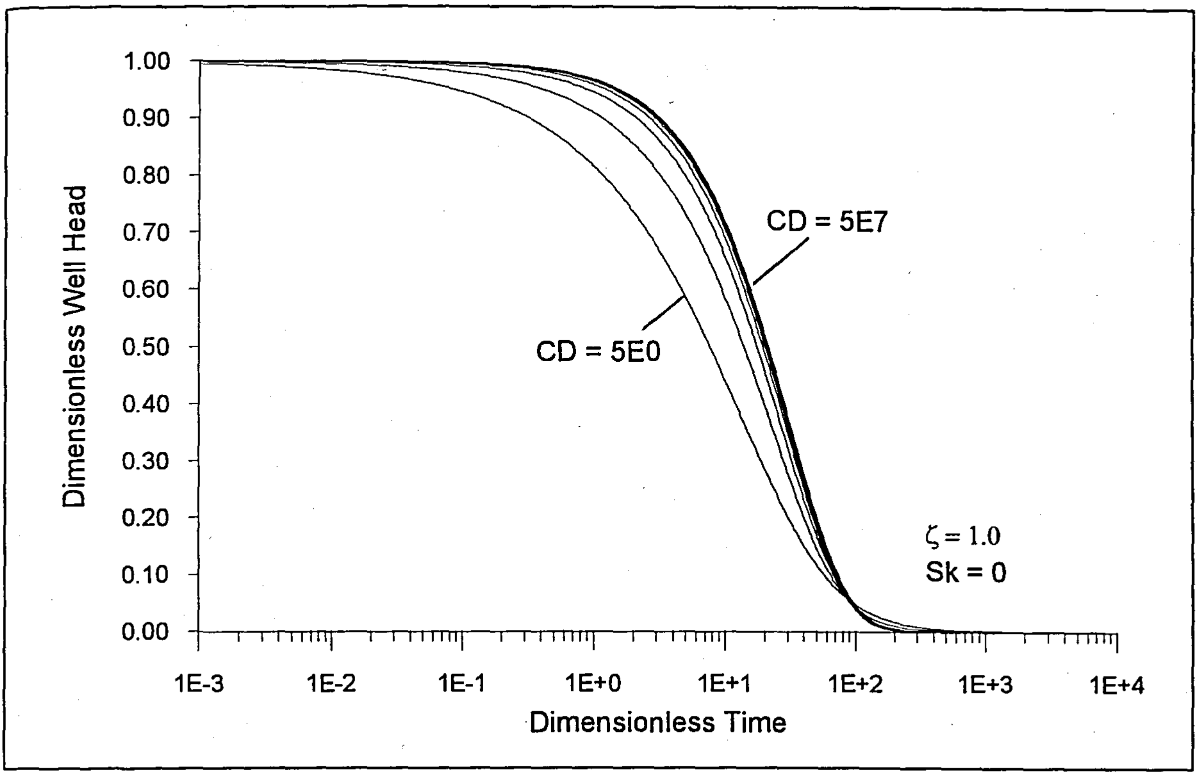

FIGURE 4.1.2.13. Dimensionless type curves for isotropic conditions, z = 19.82 m (Melville et. al. aquifer setting (1991))

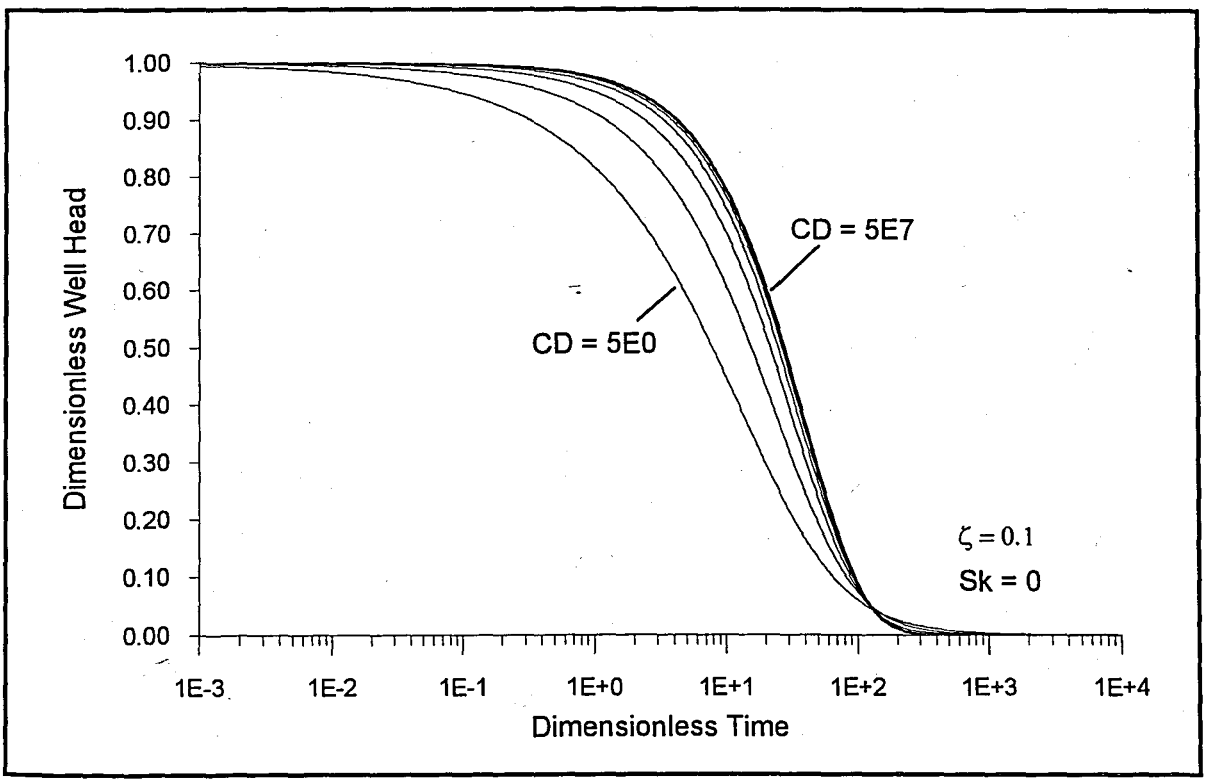

FIGURE 4.1.2.14. Dimensionless type curves for anisotropic conditions, z = 19.82 m (Melville et. al. aquifer settings (1991))

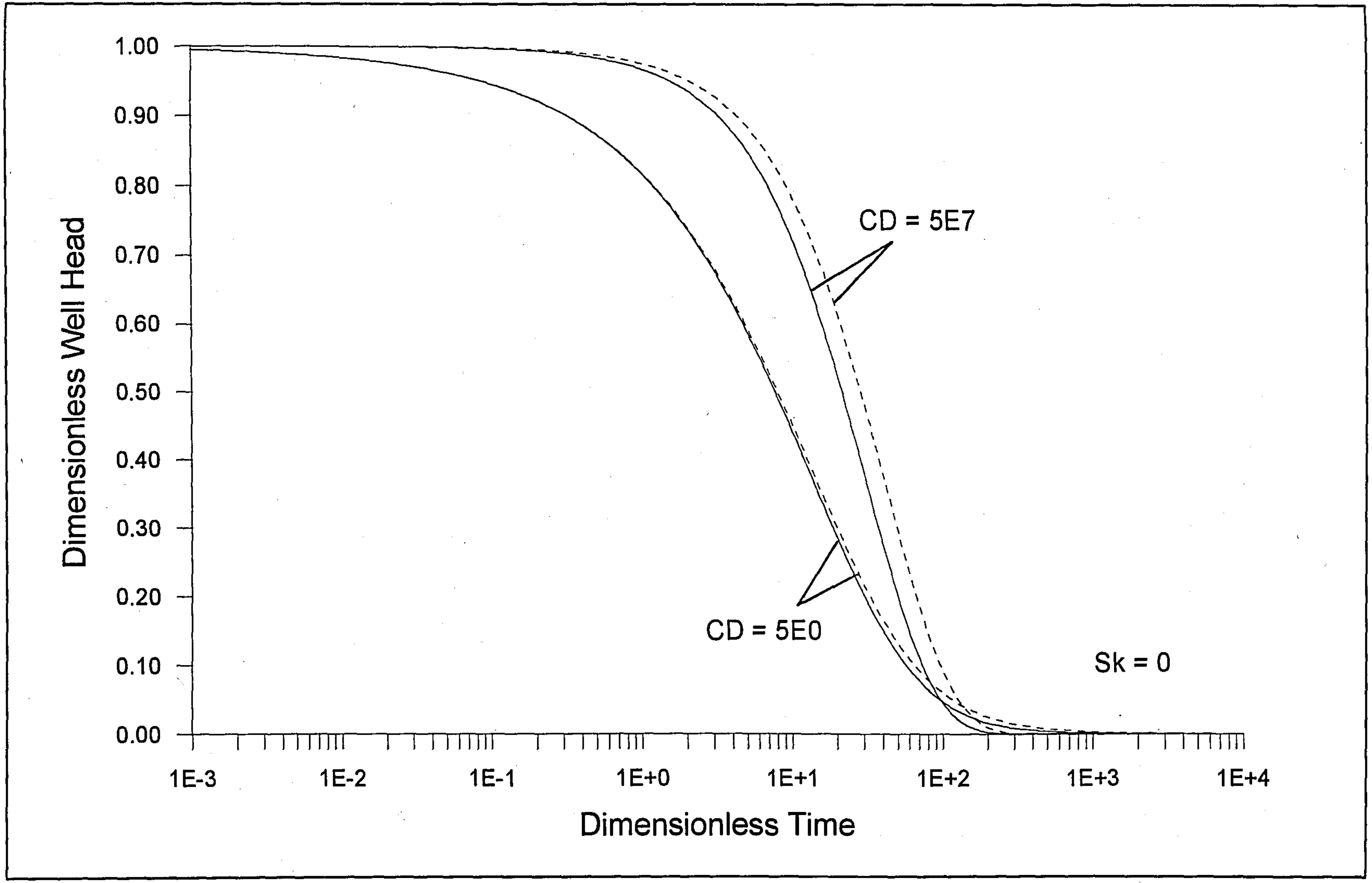

FIGURE 4.1.2.15. Comparison of dimensionless type curves for isotropic and anisotropic conditions, z = 1.58 m (solid curve: isotropic, dashed curve: anisotropic, Melville et. al. aquifer setting (1991))

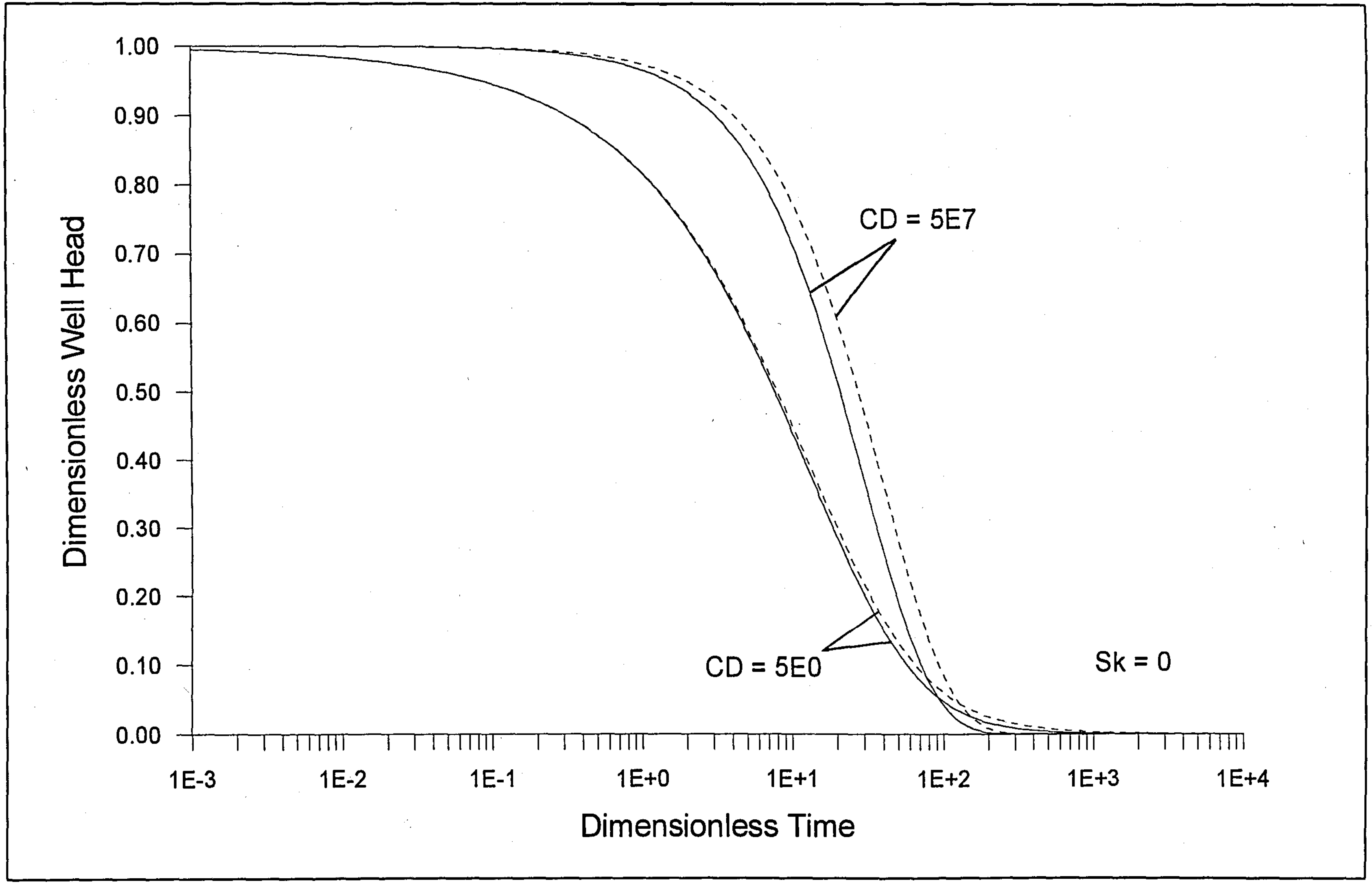

FIGURE 4.1.2.16. Comparison of dimensionless type curves for isotropic and anisotropic conditions, z = 3.40 m (solid curve: isotropic, dashed curve: anisotropic, Melville et. al. aquifer setting (1991))

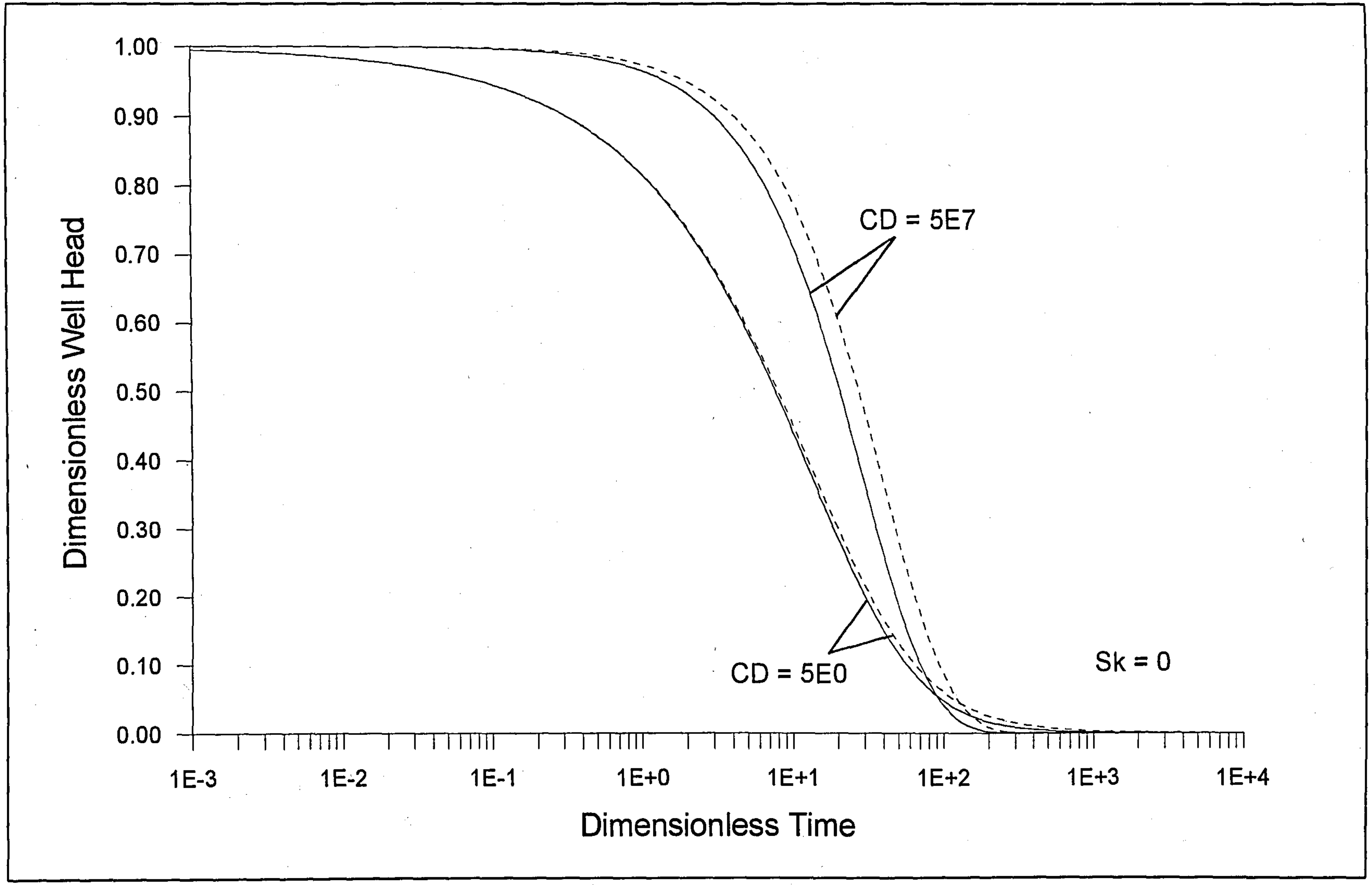

FIGURE 4.1.2.17. Comparison of dimensionless type curves for isotropic and anisotropic conditions, z = 5.23 m
(solid curve: isotropic, dashed curve: anisotropic, Melville et. al. aquifer setting (1991))

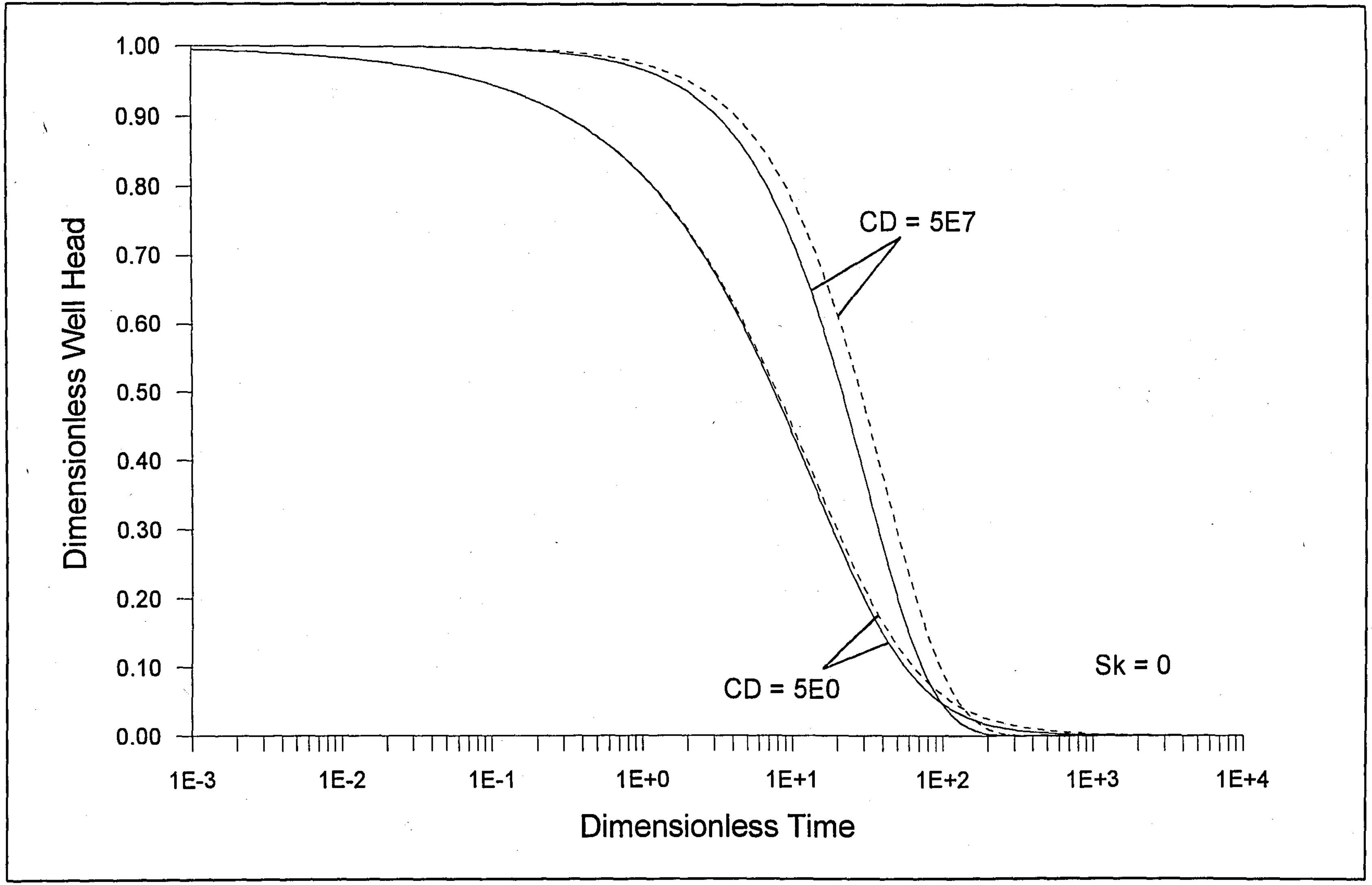

FIGURE 4.1.2.19. Comparison of dimensionless type curves for isotropic and anisotropic conditions, z = 19.82 m (solid curve: isotropic, dashed curve: anisotropic, Melville et. al. aquifer setting (1991))

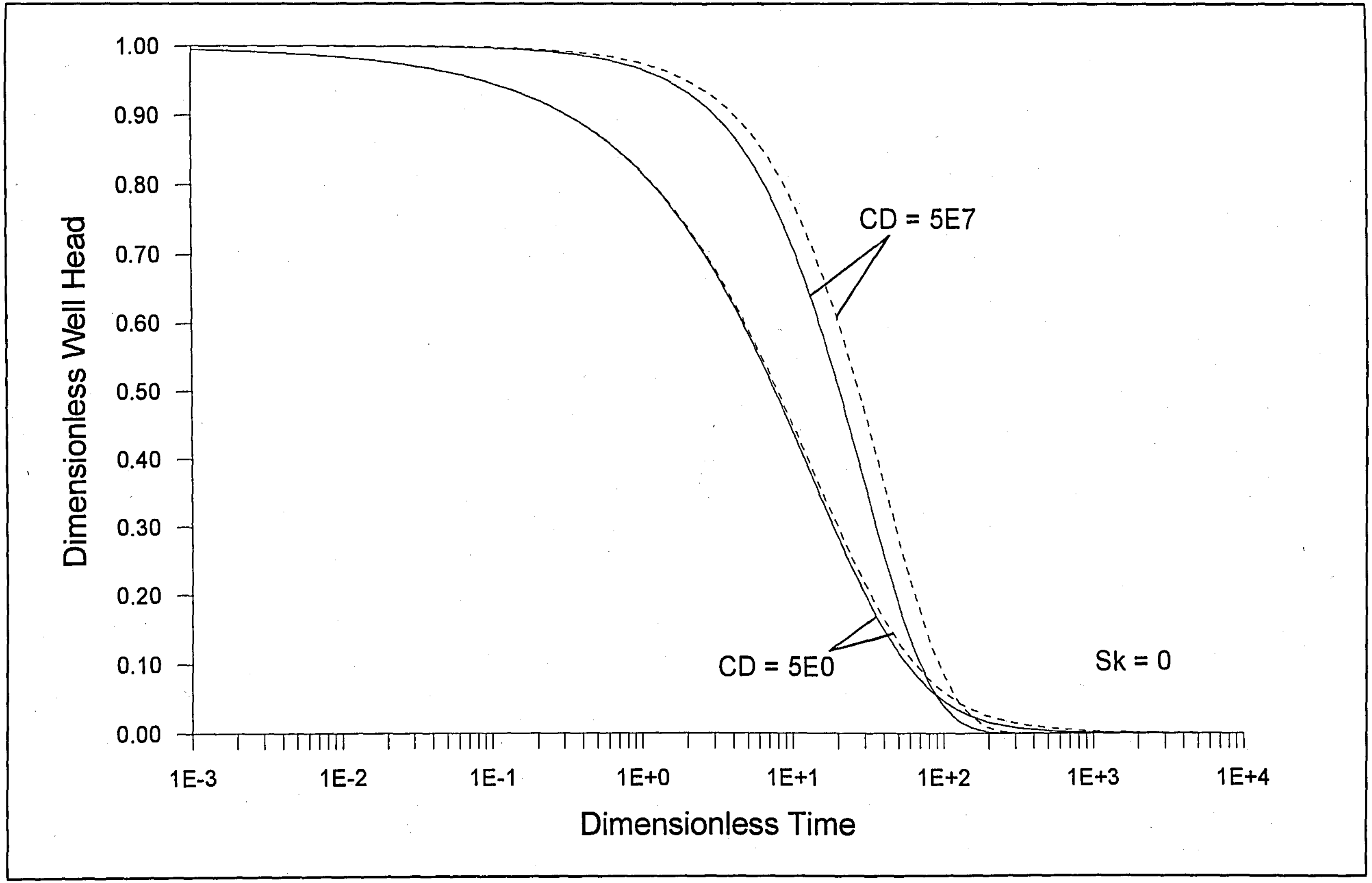

FIGURE 4.1.2.18. Comparison of dimensionless type curves for isotropic and anisotropic conditions, z = 7.05 m (solid curve: isotropic, dashed curve: anisotropic, Melville et. al. aquifer setting (1991))

The sensitivity of the dimensionless type curves to the effects of several factors were investigated and the findings are summarised below.

Skin Friction: The effect of skin friction on water level values measured in the casing can be seen from the dimensionless type curves shown in Figure 4.1.2.20. which were generated for the 1.58 m packer setup. The presence of skin friction shifts the type curves to the right on the dimensionless time scale, indicating that the calculated hydraulic conductivities would be smaller than the actual hydraulic conductivity of the formation.

Anisotropy: Figure 4.1.2.21. illustrates the effects of different anisotropy coefficients on the dimensionless type curves for the 1.58 m packer setup. The results indicate that the presence of anisotropy in the formation will shift the curves to the right and cause an increase in the calculated horizontal hydraulic conductivity values. This behaviour can be explained by the diminishing weight of the vertical flow component in the overall response of the formation to the slug test

Partial Penetration: The effect of partial penetration on the dimensionless type curves were investigated by analysing the Mobile aquifer setting for a set of hypothetical packer setups. The centre point of the open portion for each packer setup was assumed to coincide with the midpoint of the aquifer, i.e. z = 10.6 m. The generated type curves are shown in figures 4.1.2.22. and 4.1.2.23. The results point out that the type curves are shifted to the left with increasing ratio of screen length to aquifer thickness.

Distance to Nearest Aquifer Boundary: The dimensionless type curves generated for z = 1.58 m and z = 7.05 m are given in Figures 4.1.2.24. and

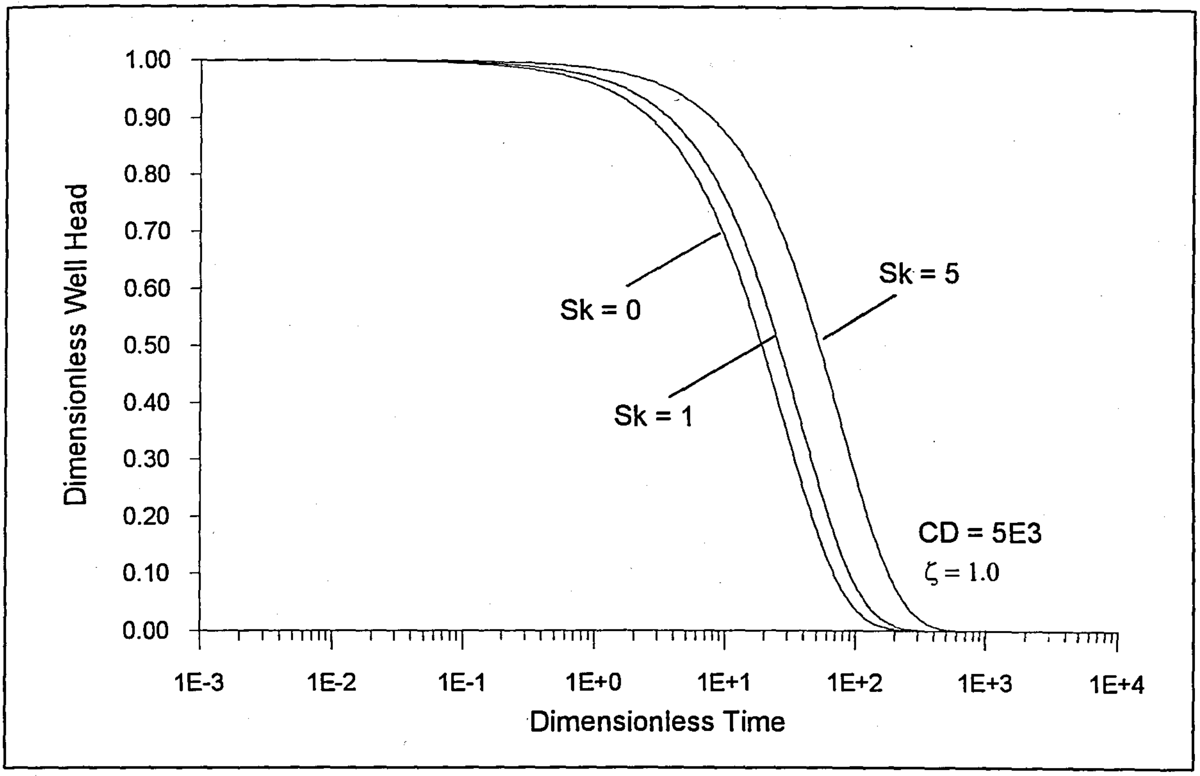

FIGURE 4.1.2.20. The effect of skin friction on dimensionless type curves, z = 1.58 m (Melville et. al. aquifer setting (1991))

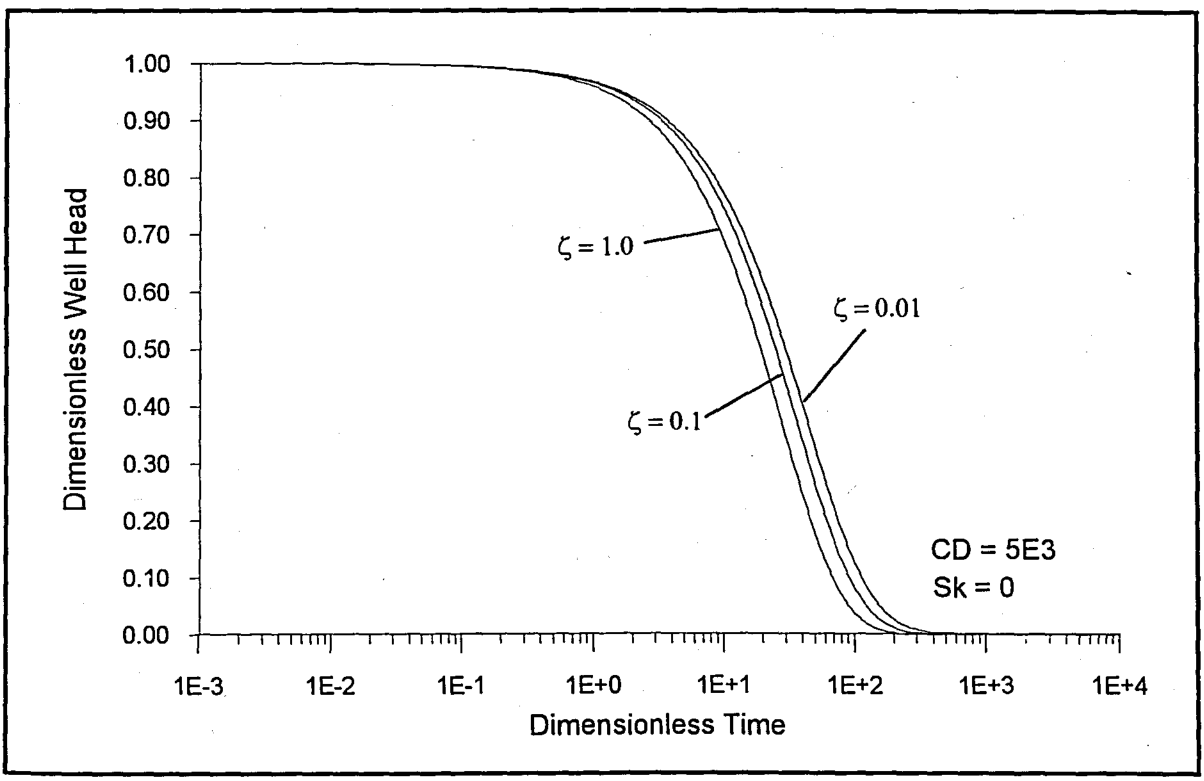

FIGURE 4.1.2.21. The effect of anisotropy on dimensionless type curves, z = 1.58 m (Melville et. al. aquifer setting (1991))

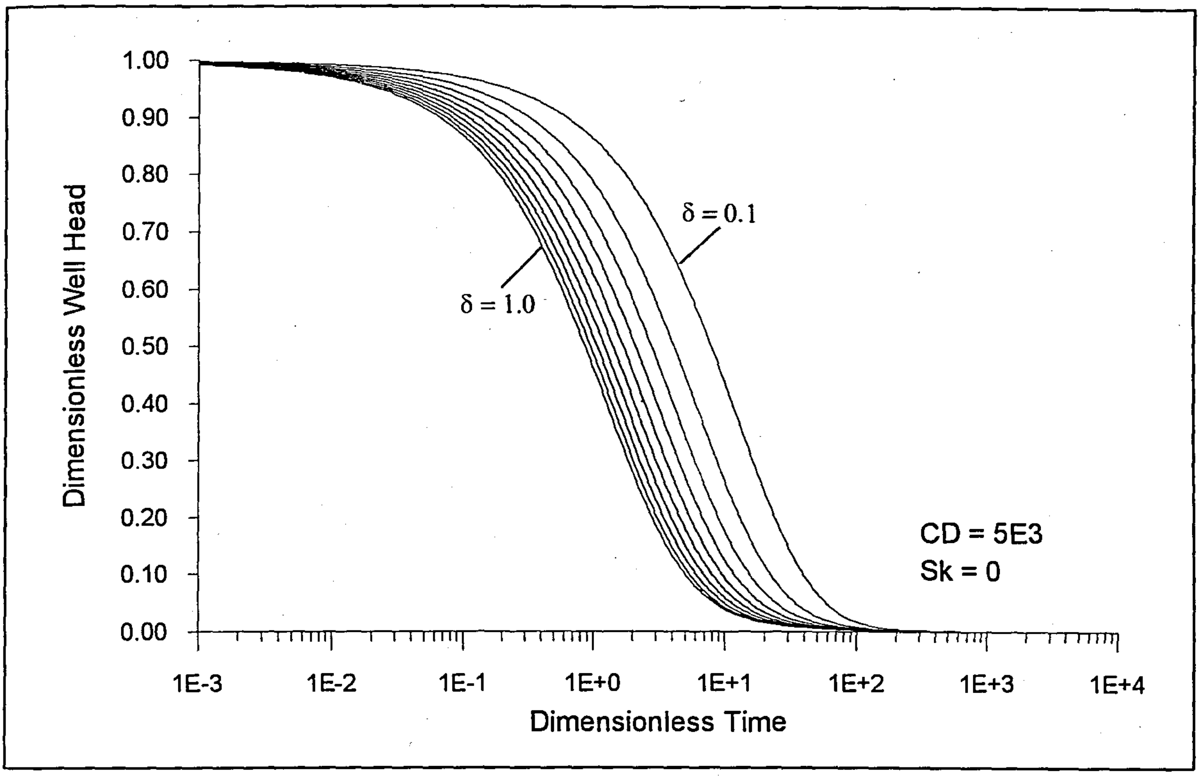

FIGURE 4.1.2.22. The effect of partial penetration on dimensionless type curves under isotropic conditions (Melville et. al. aquifer setting (1991))

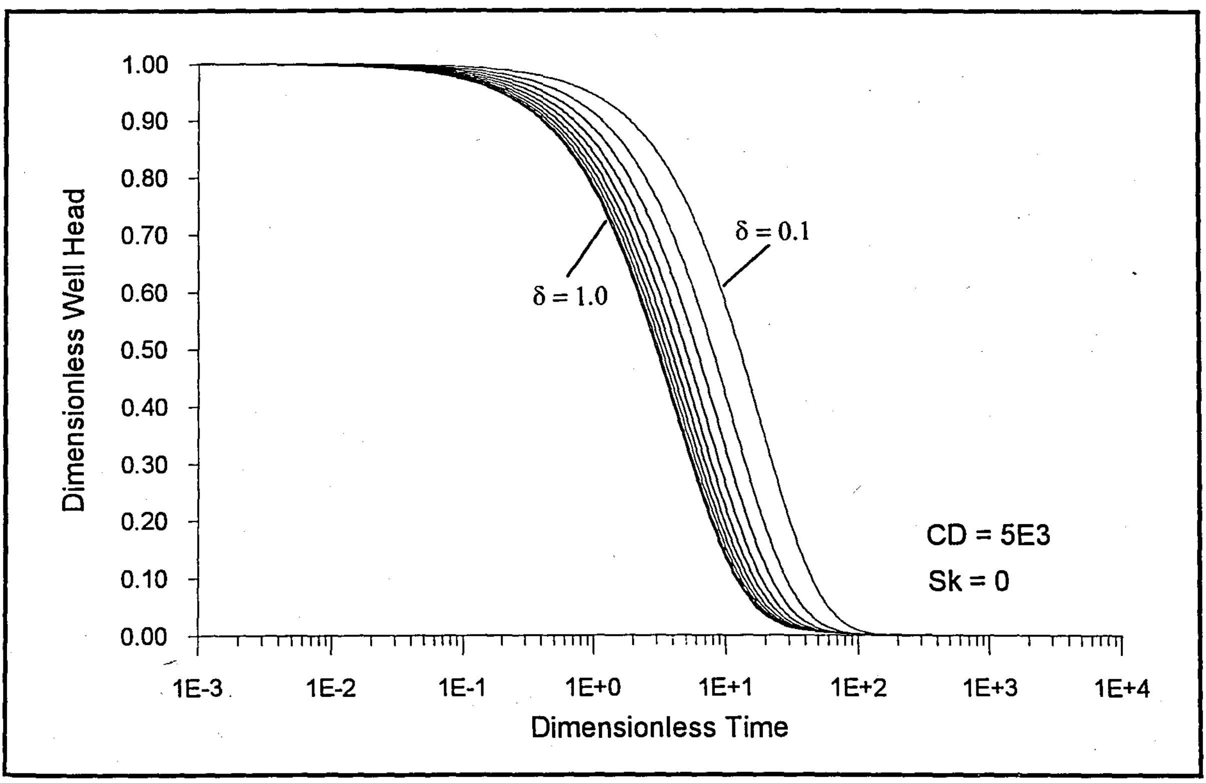

FIGURE 4.1.2.23. The effect partial penetration on dimensionless type curves under anisotropic conditions (Melville et. al. aquifer setting (1991))

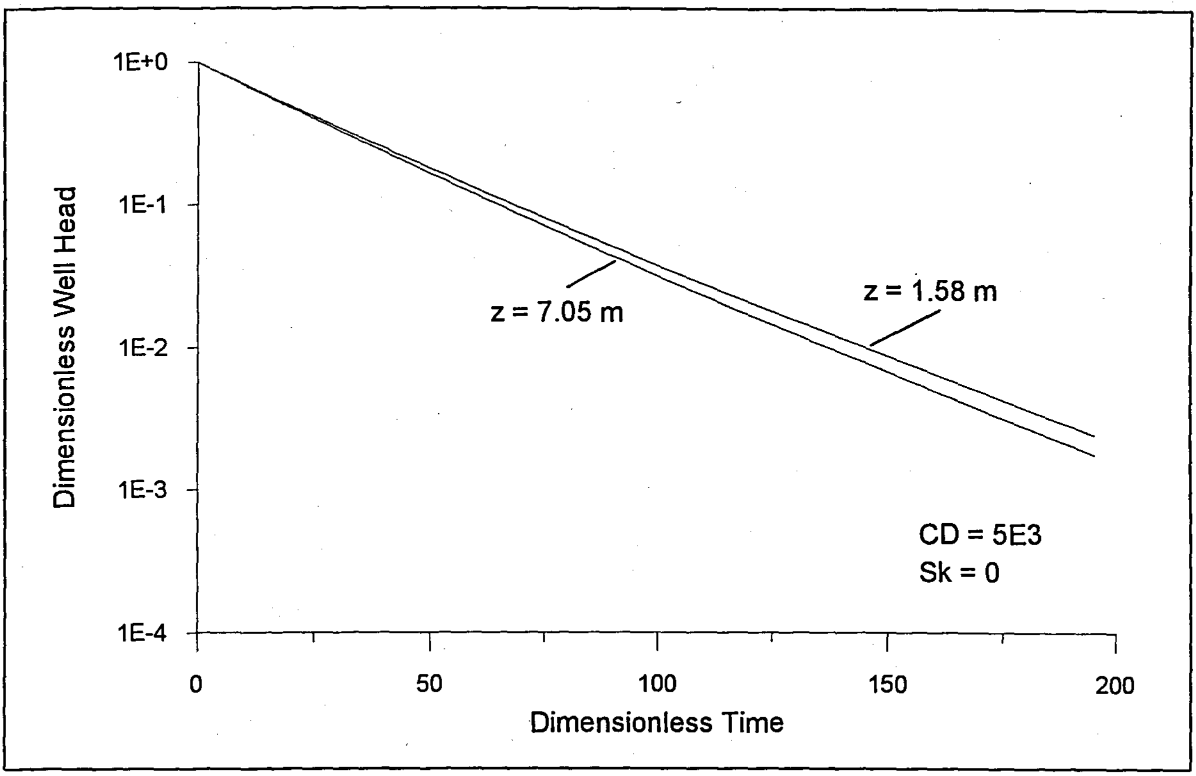

FIGURE 4.1.2.24. The effect of distance to the nearest boundary on dimensionless type curves under isotropic conditions (Melville et. al. aquifer setting (1991))

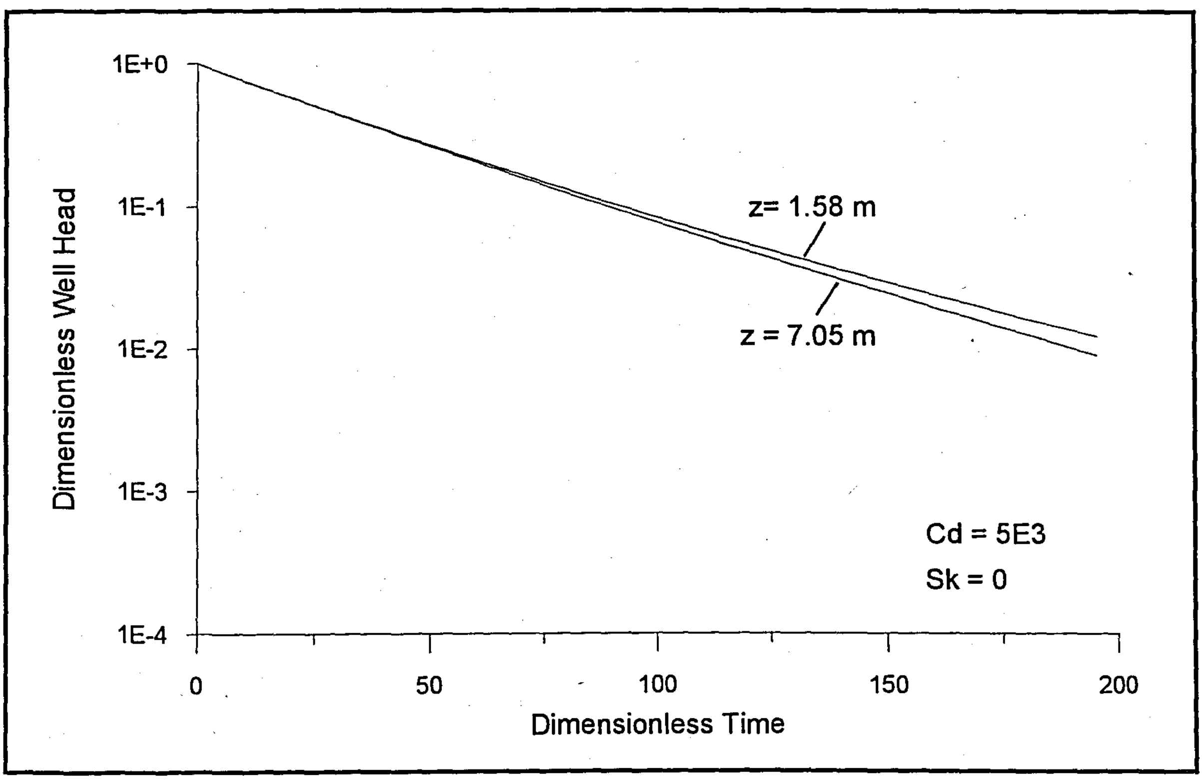

FIGURE 4.1.2.25. The effect of distance to the nearest boundary on dimensionless type curves under anisotropic conditions (Melville et. al. aquifer setting (1991))

4.1.2.25. in semilogarithmic scale for isotropic and anisotropic conditions, respectively The drawdown curves reflect a clear difference between the two elevations indicating the effect of no flow boundary on the well response; the response becomes slower as the screen approaches to the boundary.

4.2. Unconfined Aquifers

The predictive ability of the presently derived formulations for the unconfined aquifer conditions was investigated by analysing the slug test data presented by Bouwer and Rice (1976). The data were obtained in a cased well with screen and casing radii 0.0762 m in the alluvial deposits of the Salt River bed west of Phoenix, Arizona. The static water table was at a depth of 3 m whereas the overall thickness of the water column within the aquifer was 80 m. The bottom of the 4.56 m long screen was at 5.5 m depth from the water table surface. The field data obtained is shown in Figure 4.2.1. and listed in Table 4.2.1.

4.2.1. Evaluation of Type 2.a. Solutions

The slug test data were analysed by generating dimensionless type curves for various C_D values using TCG program under isotropic conditions; these curves are shown in Figure 4.2.1.1. The hydraulic properties of the aquifer were quantified then by matching the dimensionless type curves with the data for a best fit by the help of CMP. The best match with the field data was obtained for $C_D = 5 \times 10^1$ which yielded the following values:

Hydraulic Conductivity = 4.8×10^{-4} m/sec

Storage Coefficient = 0.022

Bouwer and Rice (1976) indicated that the values of the hydraulic conductivity were expected to be in the range of 1.16×10^{-4} m/sec and 6×10^{-4} m/sec based on

TABLE 4.2.1. Slug test data given by Bouwer and Rice (1976)

t (sec)	Dimensionless Head
1	0.793
2	0.655
3	0.517
4	0.448
6	0.241
9	0.100
13	0.041
19	0.017
63	0.116

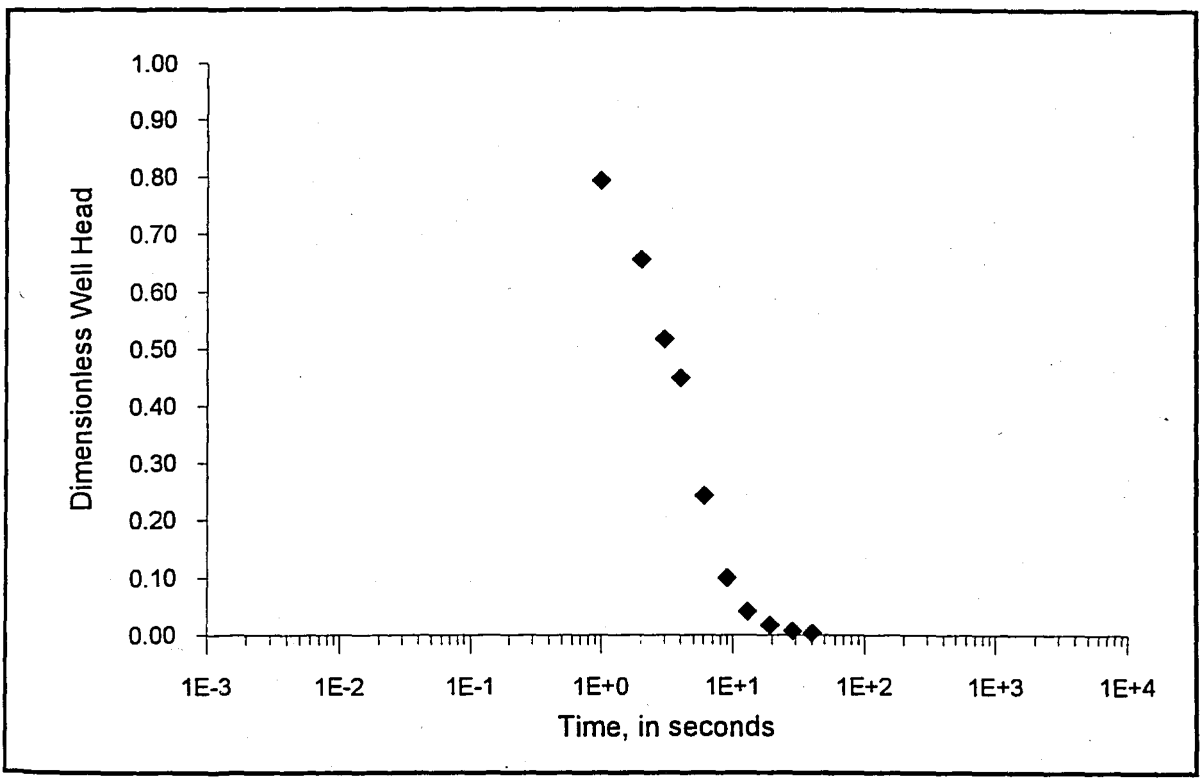

FIGURE 4.2.1. Slug test data given by Bouwer and Rice (1976)

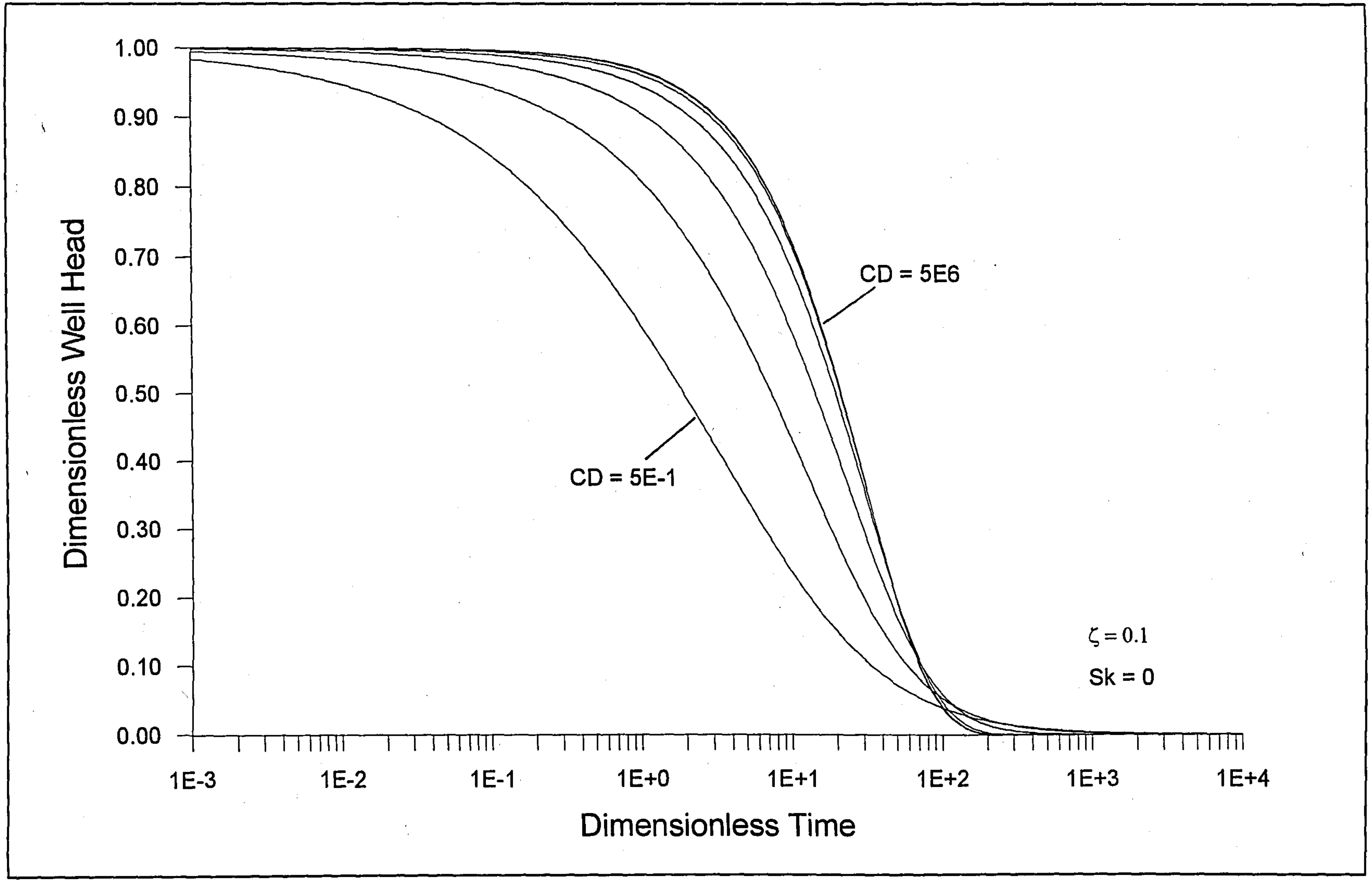

FIGURE 4.2.1.1. Dimensionless type curves for Bouwer and Rice aquifer setting (1976)

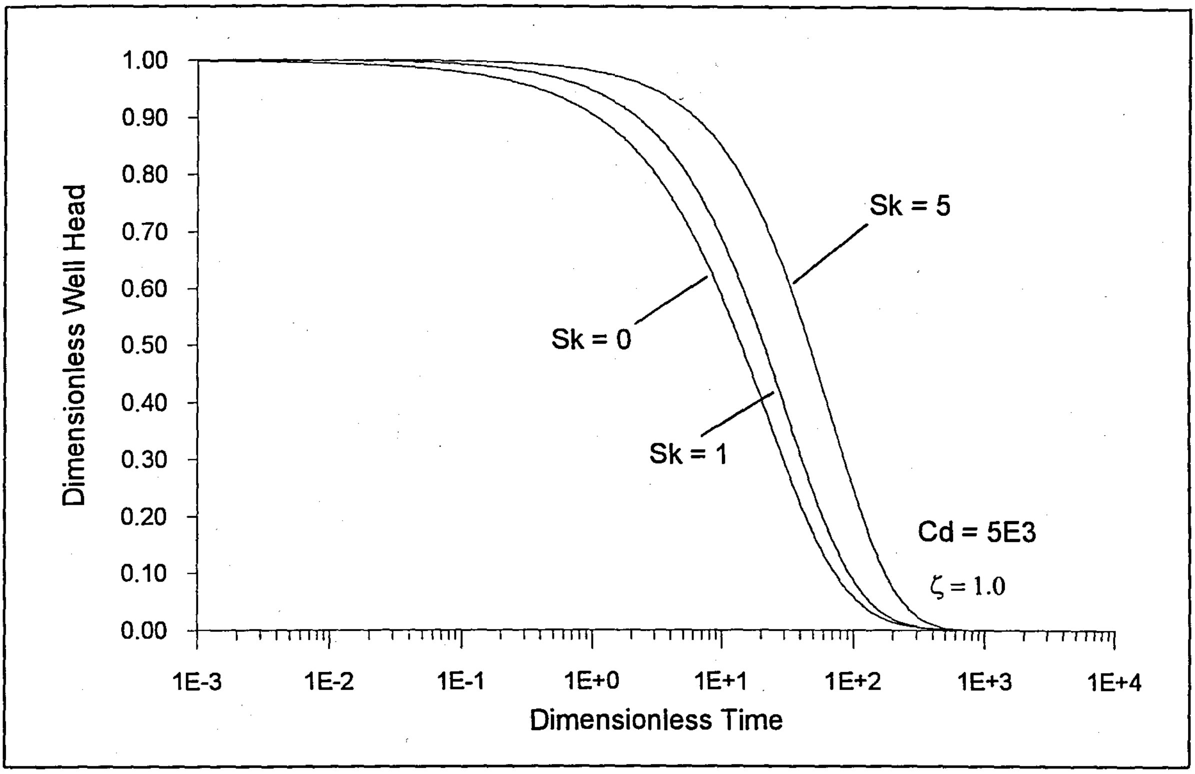

FIGURE 4.2.1.2. The effect of skin friction on dimensionless type curves (Bouwer and Rice aquifer setting (1976))

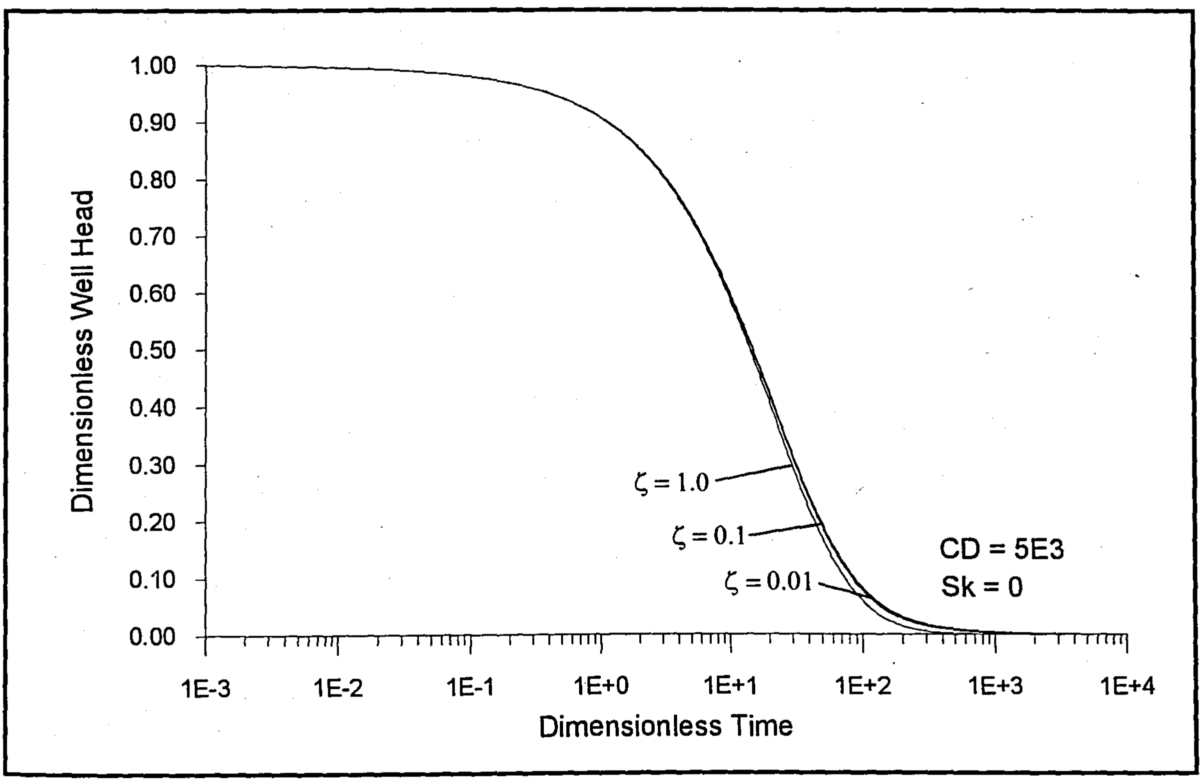

FIGURE 4.2.1.3. The effect of anisotropy on dimensionless type curves (Bouwer and Rice aquifer setting (1976))

additional hydrogeologic information. The estimated value of hydraulic conductivity by the present formulation falls within this range.

The sensitivity of the dimensionless type curves to the effects of skin friction and anisotropy was investigated for the above aquifer setting and relevant type curves are plotted in Figures 4.2.1.2. and 4.2.1.3. The figures indicate that the skin friction, like in the confined case, shifts the type curves to the right. The presence of anisotropy has the same effect, however is less pronounced than under confined conditions.

4.2.2. Evaluation of Type 2.b. Solutions

The expressions obtained using Type 2.b. boundary conditions incorporate the possible effects of the specific yield and fluctuations of the water table into the analysis by considering the upper boundary as a moving material surface. The effects of this boundary condition might be expected to emerge at later times by making an analogy to the delayed yield phenomena observed in unconfined aquifers; these phenomena were successfully modelled by Neuman (1972). Figure 4.2.2.1. shows the dimensionless type curves generated by TCG program for different S_y values using Bouwer and Rice aquifer setting. These curves were obtained for isotropic conditions with C_D fixed to 5×10^1; this C_D value was determined previously using Type 2a analysis. Figure 4.2.2.1. indicates that the dimensionless type curves seem to be very close to each other at all early, intermediate and late times; the curves are hardly to distinguish from each other. However, if the curves are plotted on a semi-log graph with linear time scale as shown in Figure 4.2.2.2., the effect of different Sy values on the shape of dimensionless type curves can be seen clearly. Figure 4.2.2.3. depicts the same curves on log-log scale reflecting **a unique S shape** whereas the type curve obtained by Type 2a analysis doesn't exhibit any inflection points. This S shaped deviation from non-yield (Type 2.a) case resembles the deviation of the response of unconfined aquifers from ideal Theis solution type curves under pumping conditions and infers the capability of the presently developed method of simulating the effects of water table level changes during slug tests.

The effect of different anisotropy values on Type 2.b type curves are depicted in Figures 4.2.2.4., 4.2.2.5 and 4.2.2.6.. Type curves presented in Figure 4.2.2.5. are generated with a vertical hydraulic conductivity value twice as much as the radial component (ζ = 2.0). This condition is unlikely to be encountered in practice, however, artificially magnifies the effect of the moving water table; in fact different type curves can easily be identified in this figure (Comparing with Figure 4.2.2.1. (where ζ = 1.0.). Figures 4.2.2.5 and 4.2.2.6. which present the type curves on semi-log scale at early times and on log-log scale at late times, respectively, indicate the effect of anisotropy on well response clearly; the effect of the moving boundary become more pronounced with an increase in vertical hydraulic conductivity.

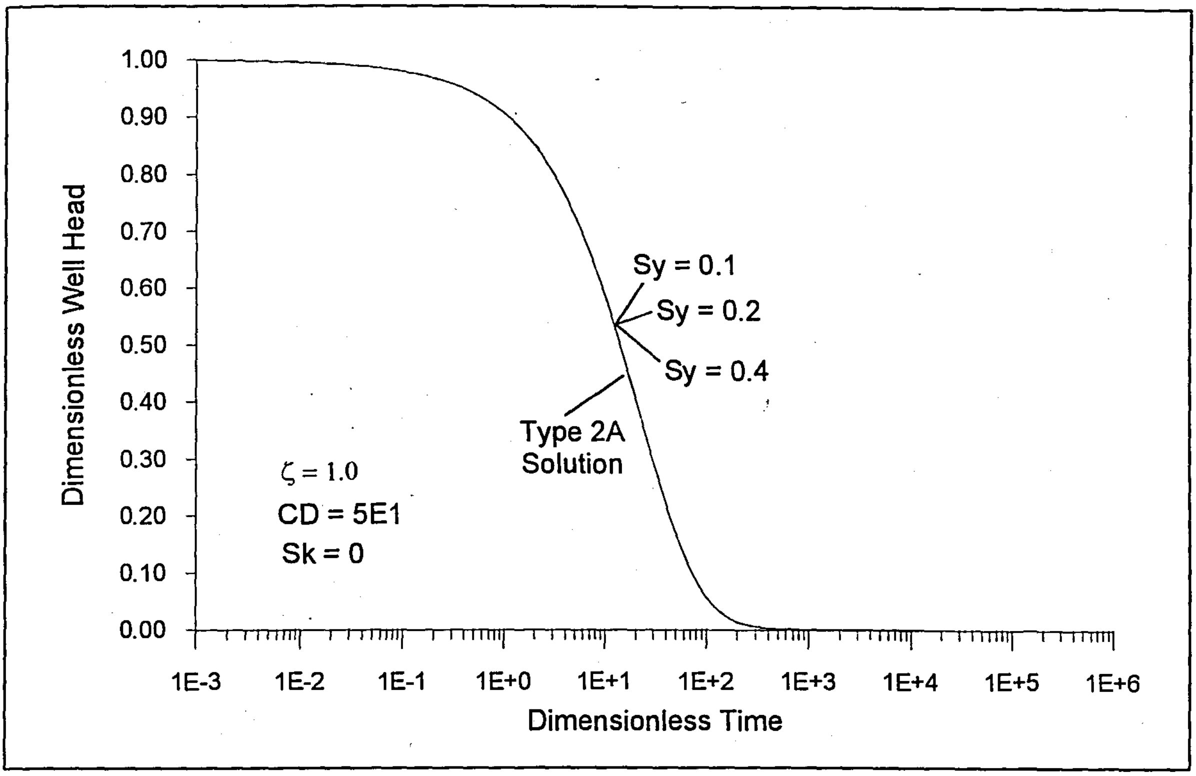

FIGURE 4.2.2.1. Dimensionless type curves for different Sy values, logarithmic time-scale (Bouwer and Rice aquifer setting (1976))

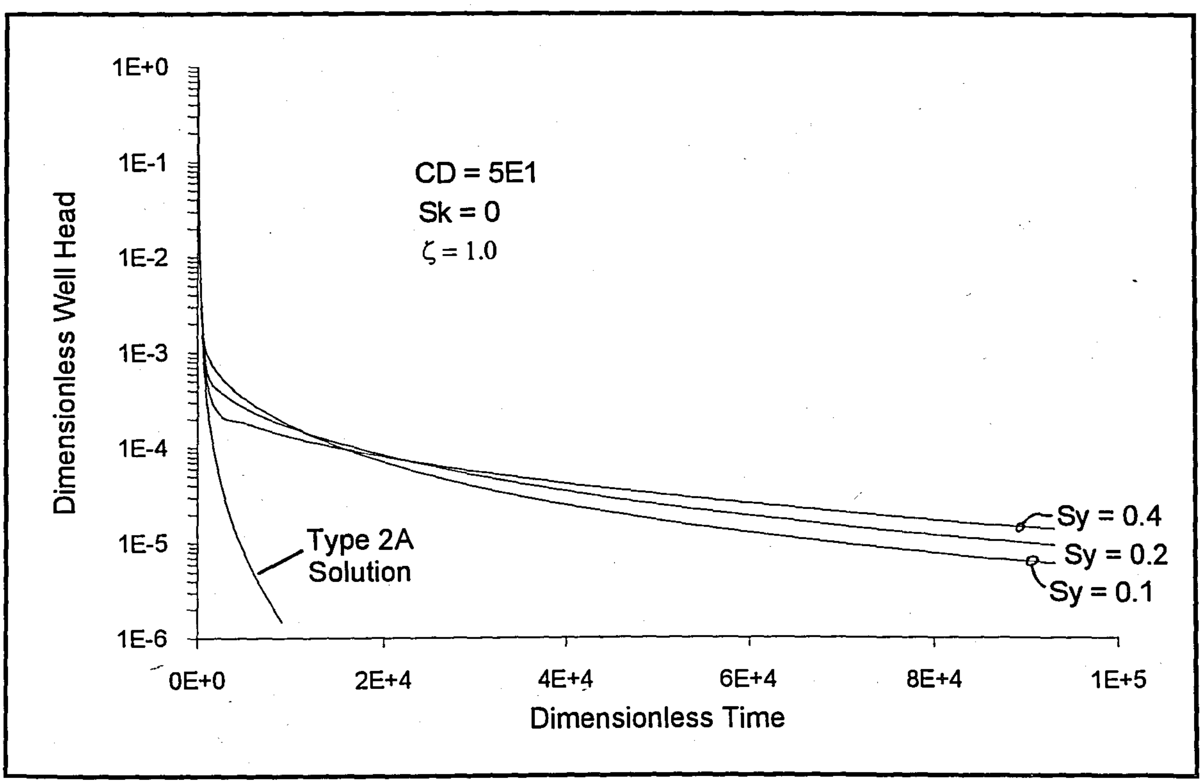

FIGURE 4.2.2.2. Dimensionless type curves for different Sy values, logarithmic well-head scale (Bouwer and Rice aquifer setting (1976))

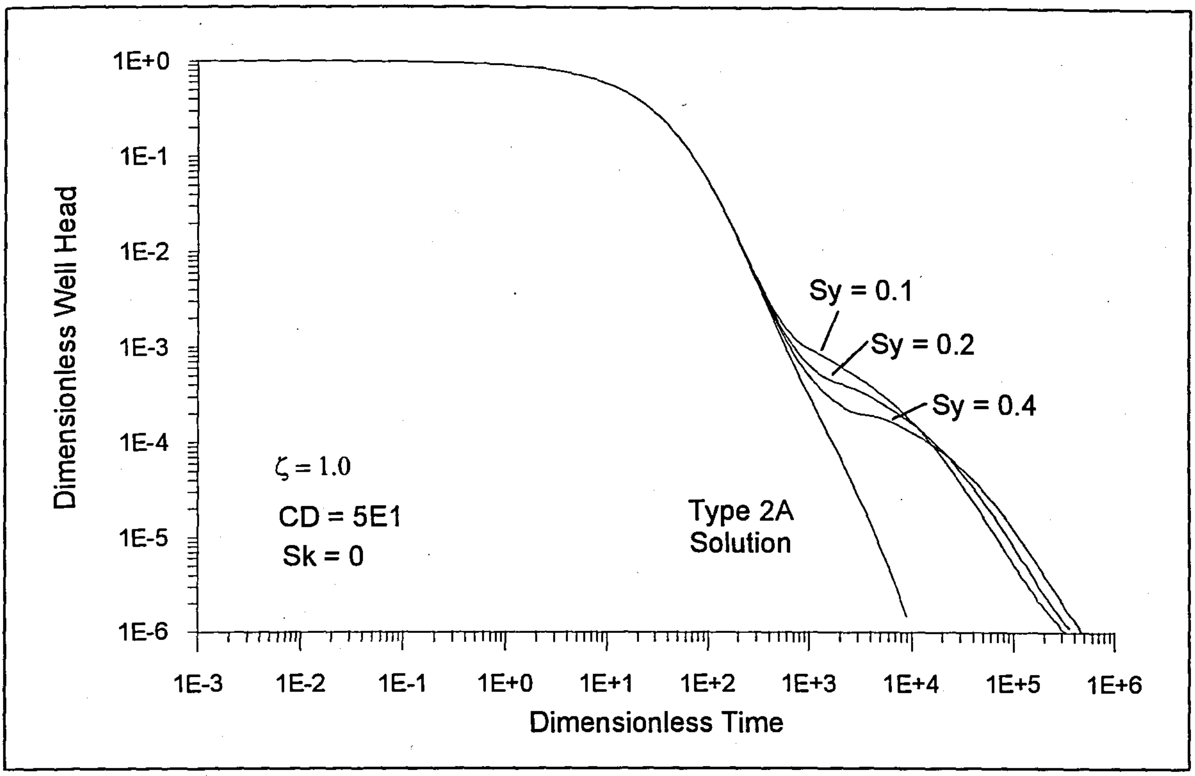

FIGURE 4.2.2.3. Dimensionless type curves for different Sy values, log-log scale (Bouwer and Rice aquifer setting (1976))

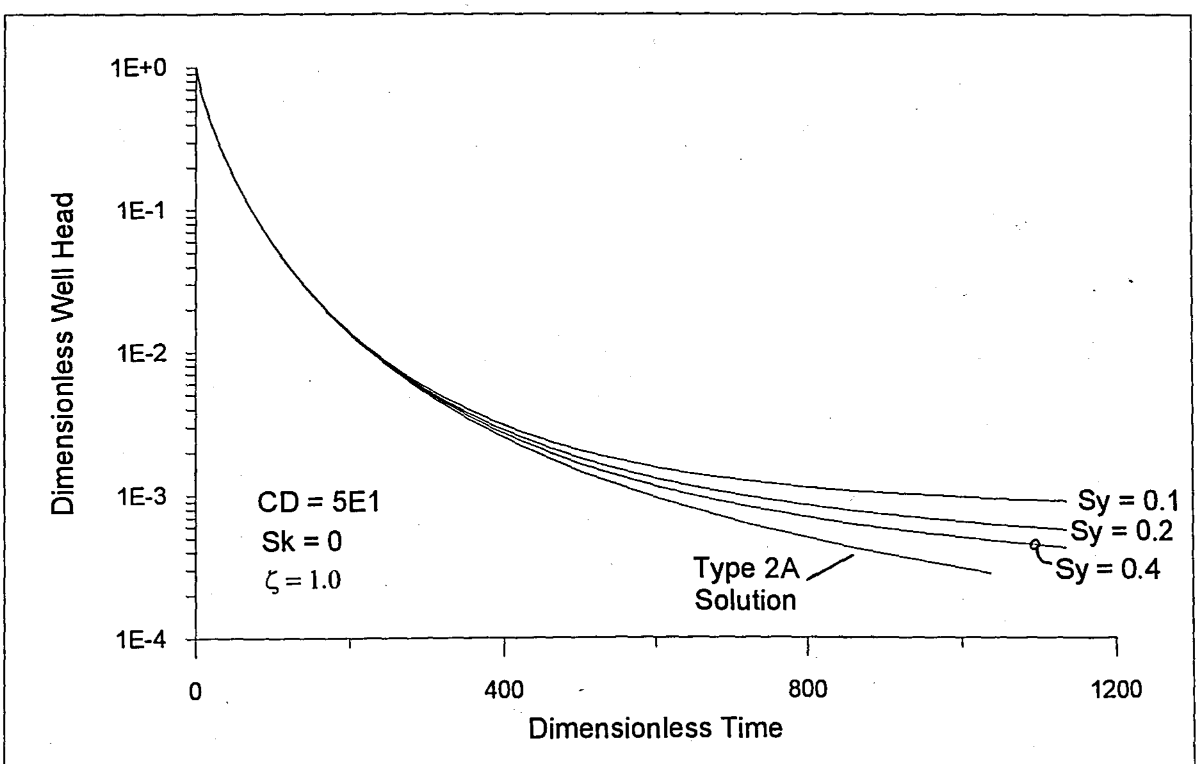

FIGURE 4.2.2.4. Dimensionless type curves for different Sy values, logarithmic well-head scale, intermediate test times, (Bouwer and Rice aquifer setting (1976))

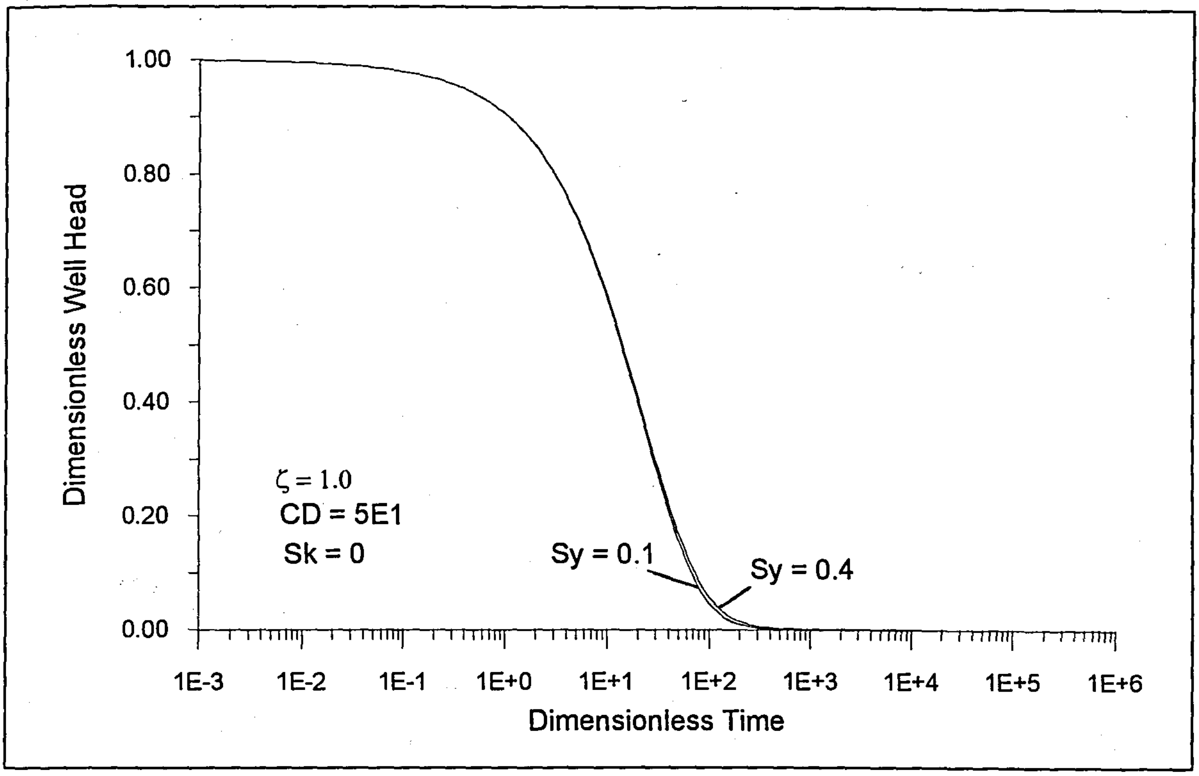

FIGURE 4.2.2.5. Dimensionless type curves for different Sy values, logarithmic time-scale, artificially magnified vertical flow, (Bouwer and Rice aquifer setting (1976))

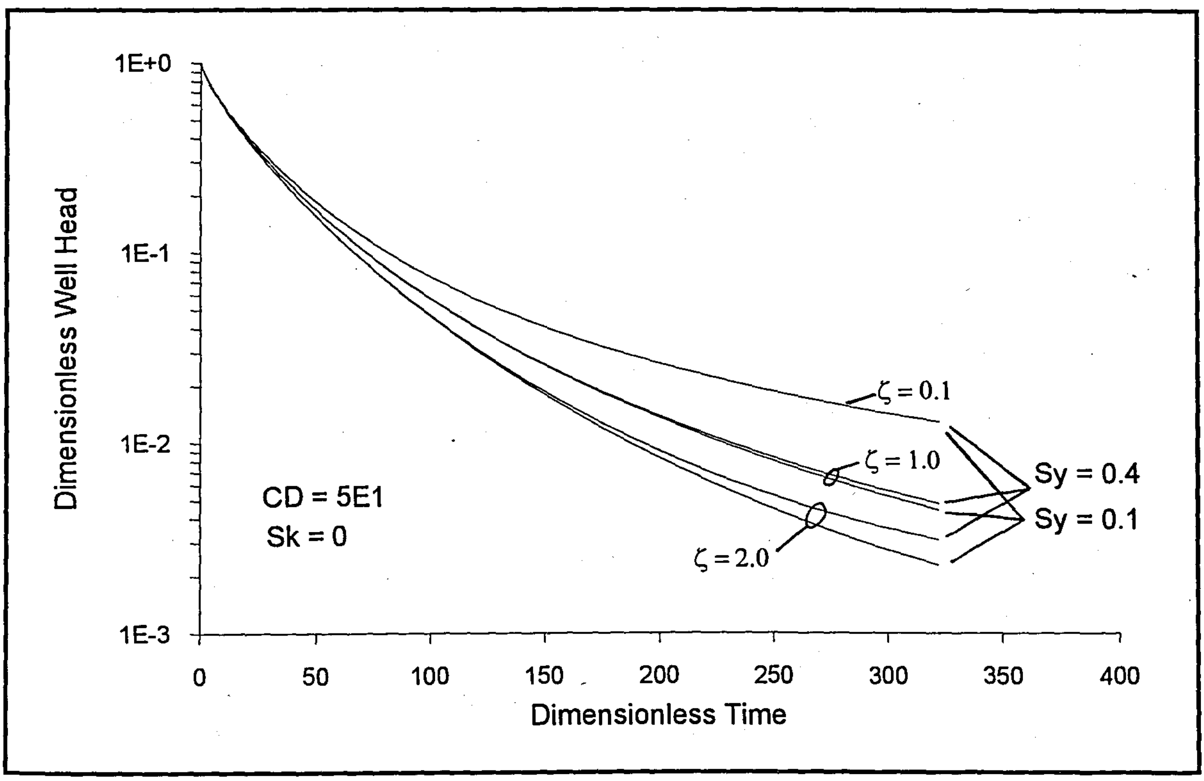

FIGURE 4.2.2.6. The effect of anisotropy (vertical flow) on dimensionless type curves for different Sy values, logarithmic well-head scale, eary test times, (Bouwer and Rice aquifer setting (1976)

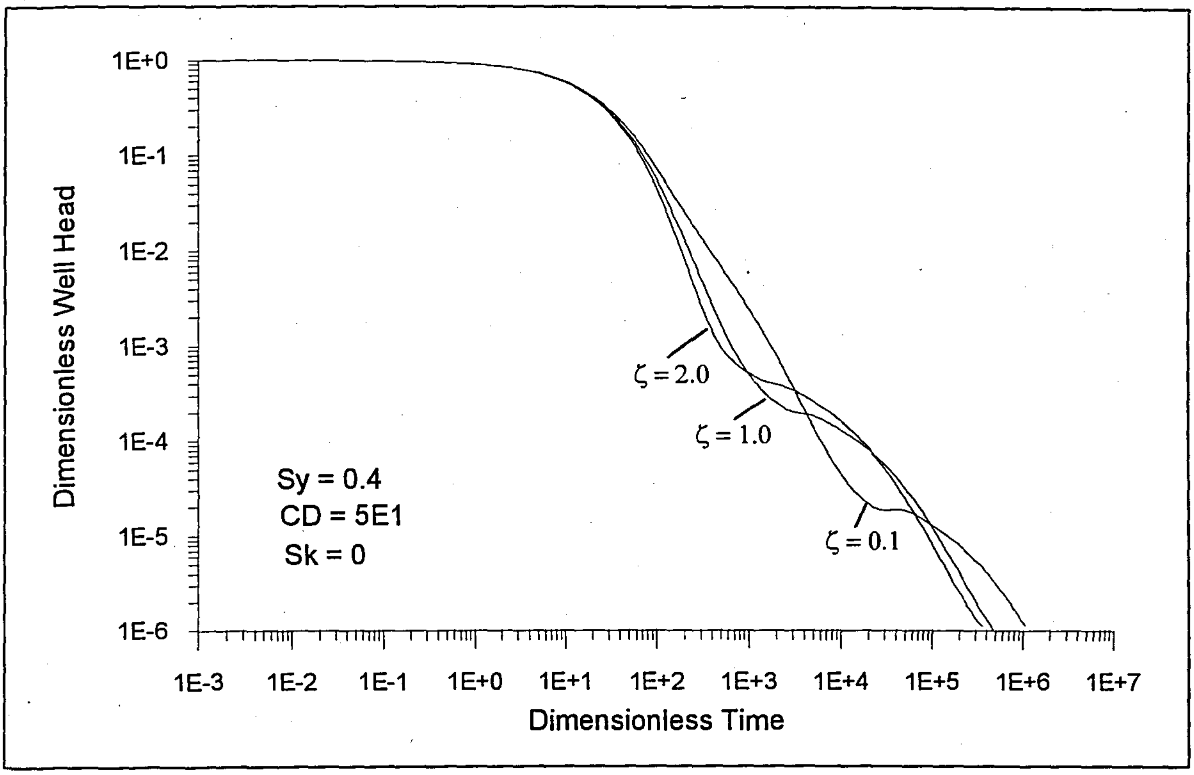

FIGURE 4.2.2.7. The effect of anisotropy (vertical flow) on dimensionless type curves for different Sy values, log-log scale (Bouwer and Rice aquifer setting (1976))

5. FURTHER RESEARCH

The following points were not covered in the present study but bear further investigation:

- Development of the present solutions for **Derivative Plot Method:** Derivative plots have been gaining much attention in the advent of very sensitive downhole instruments (Bourdet 1983 quoted by Karasaki et. al. 1988). The basic idea behind this method is to magnify the rate of head change rate so that the inflection points appear as maxima and minima. The use of type curves which depict the derivative of the head in the well can sometimes be very useful while curve matching procedures to overcome the "nonuniqueness problem" of slug test type. Figure 5.1 and Figure 5.2 show the conventional and derivative plots for Cooper et. al. (1967) type curves given by Karasaki et. al. (1988), respectively. The expressions obtained in the present study can be developed for such an analysis, as well.

- Development of the present solution for determination of **Radius of Influence (observation wells):** The capabilities and limitations of slug tests in characterising the hydraulic properties of study formations have been receiving considerable amount of attention during the last years. An investigation of the radial dependence of Cooper et. al. analytical solution (1967) for a slug test in a confined aquifer showed that the use of one or more observation wells can vastly improve the parameter estimates (McElwee et. al., 1995). The presently developed solutions can similarly be manipulated to end up with formulae expressing the radial hydraulic head distribution within the study formation.

- Scrutinising the validity of the assumption that the head in the well is independent of depth: Dougherty and Babu (1984) states that this assumption is critical posing a solvable problem and it is not easy to relax it and obtaining an analytical solution, however, any solution obtained from such a model would be very powerful. The present model can be reconsidered to serve to this end.

- Evaluating the validity of the assumption that the flux through the screen is independent of depth: Any simple distribution of flux along the screen would be included in the present formulations. This would yield only changes in the Fourier coefficients α_n (Section 3.3.3.)

- Extending the present expressions and computer codes for large scale sensitivity analyses and parametric studies: A similar study was completed recently by Hyder as a Ph.D. dissertation at University of Kansas, US (1994).

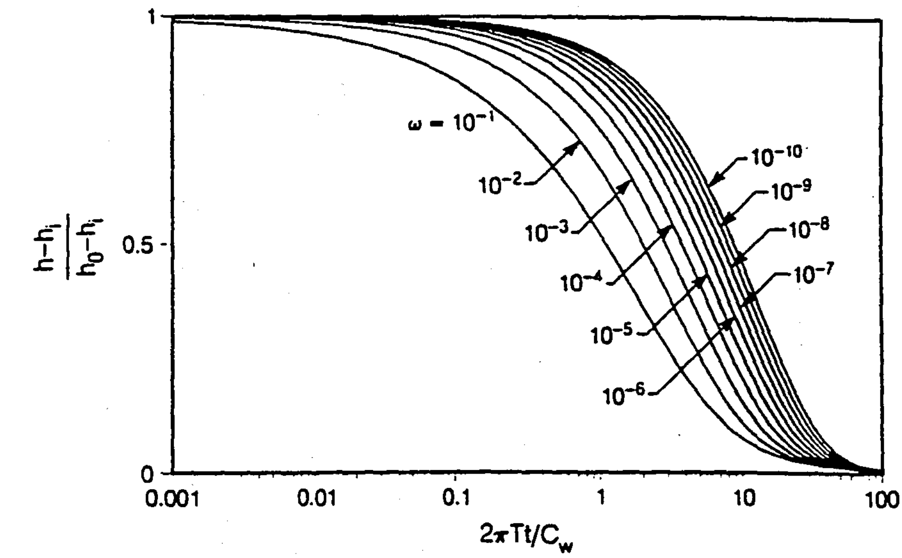

FIGURE 5.1 Type Curves after Cooper et. al. (1967)
(Source: Karasaki et. al., 1988)

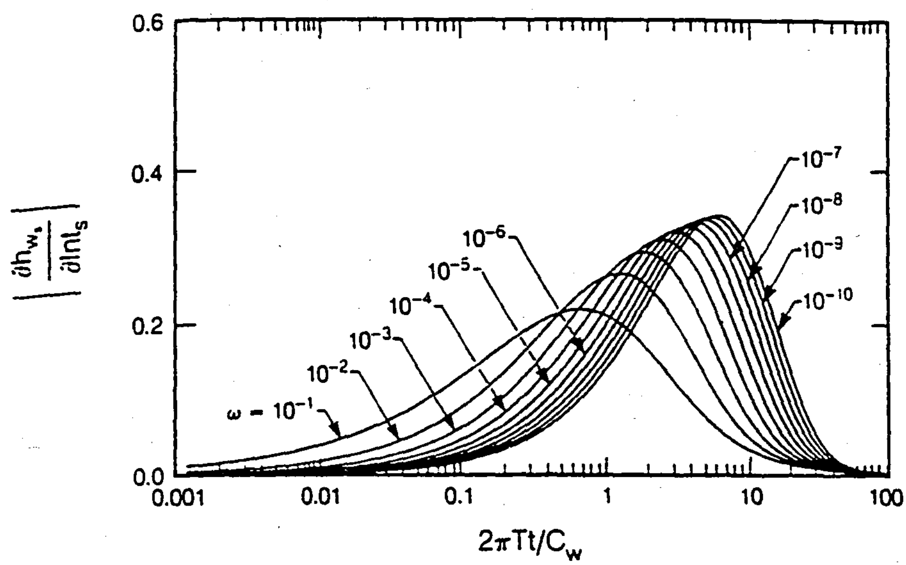

FIGURE 5.2 Derivative plot for Cooper et. al. model (1967)
(Source: Karasaki et. al., 1988)

6 SUMMARY AND CONCLUSIONS

The following points can be listed describing the steps paced and the results obtained in the present study:

- Analysis procedures were developed for slug tests performed in monitoring wells partially penetrating confined or unconfined, anisotropic, finite depth aquifers with the ultimate goal of estimating hydraulic parameters of the study formation as accurately as possible. To this end, a new set of governing equation and boundary conditions were analysed which were free from some inherent assumptions utilised by previously established methods of analysis. The expressions for the water level variation in the casing and the hydraulic head distributions were obtained in the Laplace Domain and inverted back by a numerical method namely the Stehfest algorithm. Two computer programs were developed for generating dimensionless type curves and determining unknown hydraulic properties of study formation.

- The applicability of the presently developed procedures for confined and unconfined aquifers were investigated by analysing slug test data obtained from the literature. Both fully and partially penetrating conditions were analysed for confined aquifers; the first by using Cooper et. al. aquifer setting (1967) and the latter by analysing Melville et. al. (1991) multilevel slug test data. The results obtained by the present method were compared with previous analysis results based on analytical and numerical methods. The results indicated that the present method provided close results to those obtained by the numerical method. The results also indicated that axial flow should be taken into account in the analysis of data collected from partially penetrating wells even if the aquifer might exhibit anisotropic properties. Present findings together with the observations of Melville et. al. (1991) reflected a potential bias in parameter estimations of Cooper et. al. (1967) method.

- The analysis procedures for unconfined aquifer conditions were developed for two water table boundary conditions; **Type 2.a.** : constant head boundary condition; and **Type 2.b.**: transient boundary condition. The data collected from an unconfined aquifer (Bouwer and Rice aquifer setting, 1976) were analysed using the presently developed procedures for Type 2.a. conditions; the estimated aquifer parameters were within the range of expected values. Dimesionless type curves that were generated using Type 2.b. solution yielded unique S shaped log-log curves resembling response curves observed during pumping tests in unconfined aquifers (delayed yield phenomena). These curves reflected the fact that the water levels measured in the casing were influenced by the specific yield of the aquifer towards the later times of the test.

APPENDIX A

The details of TCG and CMP Codes along with their listings are given herein.

The computer programs developed in the present study were designed suitable for large scale parametric studies and small scale sensitivity analyses. The schematical representation of data flow between and within the TCG and CMP programs is given in Figure A.1. The listings of the programs along with explanations about relevant input and output files are provided in the subsequent pages.

Two program parameters were considered to be of immense importance for the type curves generated by the TCG code, namely the number of summation terms used in Fourier series solution and the Stehfest number used when inverting the Laplace domain solutions back to the real time:

<u>Number of summation terms</u>: Dougherty and Babu (1984) stated in their original work, which was the main inspiration of the present study, that 25 terms were enough for obtaining reliable type curves. Several runs were carried out in the present study aiming at selecting the best value which optimises the accuracy versus the length of run time. The Figures A.2. to A.9. demonstrate the effect of N on dimensionless type curves clearly; type curves for N = 10 and N = 500 differ considerably, however, N = 250 type curves coincide perfectly with the latter one. In the present study N was selected equal to 250.

<u>Stehfest Number</u>: The accuracy of Stehfest approximation is sensitive to the selection of the number of Stehfest points (N_{steh}). Theoretically the accuracy can be increased by increasing N_{steh}. However, in computer applications after a point rounding errors worsen the result (Moench and Ogata, 1981) and the value of N_{steh} should be determined for each problem and computer (Dougherty and Babu, 1984). Moench and Ogata stated that the value of N_{steh} to use for maximum accuracy is approximately proportional to the number of significant figures that the computer in use holds. Dougherty and Babu stated that a value of 18 had been found suitable for most reservoir problems on IBM 3081 with 64-bit word length. Although TGC and CMP codes were developed in double precision, N_{steh} was selected to be 8 for the present study. The underlying logic bases on the fact that

the coefficients given by Press et. al. (1990) used in the sub-functions of Modified Bessel Functions are in single precision.

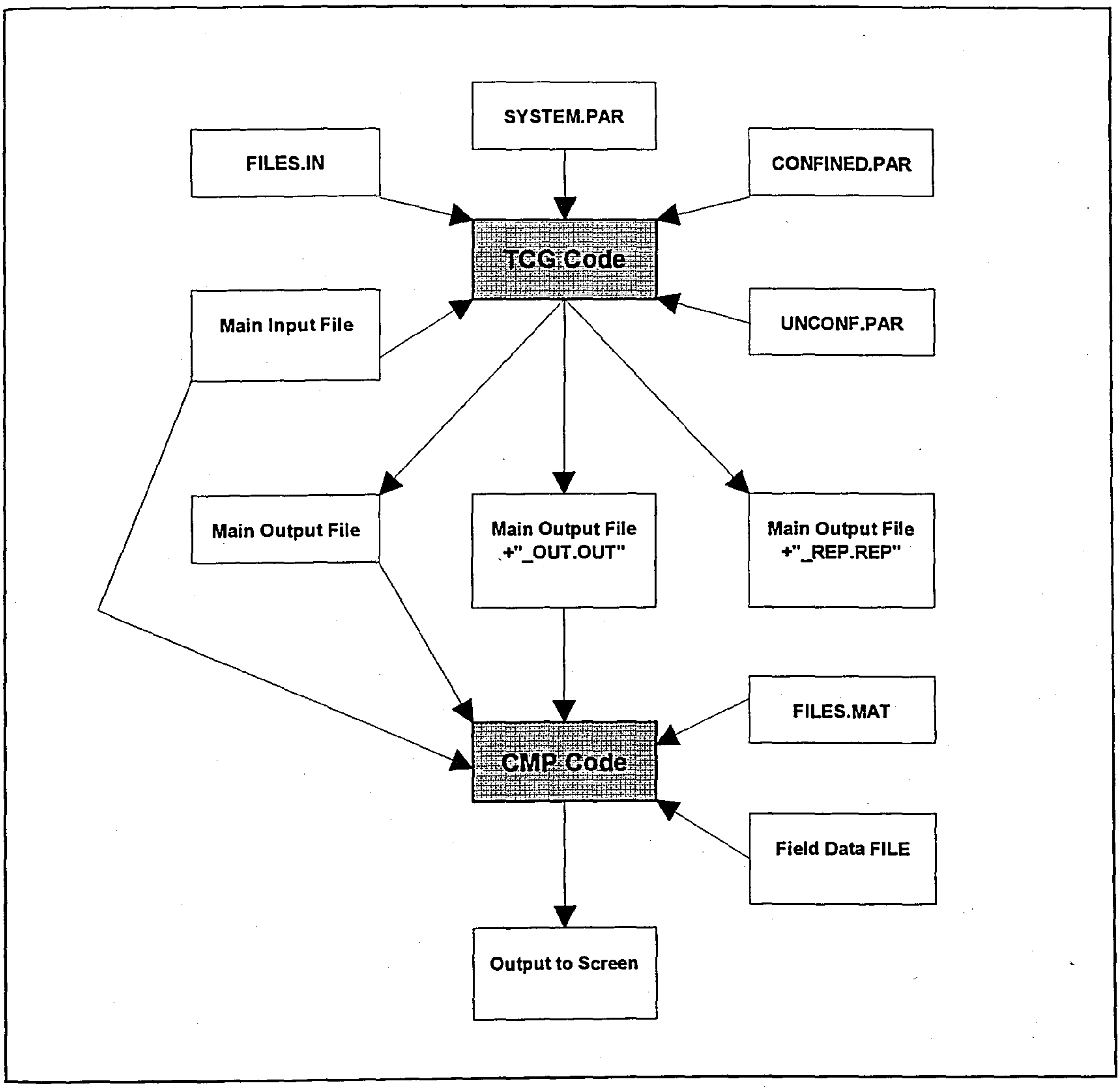

FIGURE A. 1. Schematical representation of data transfer between and within the programs

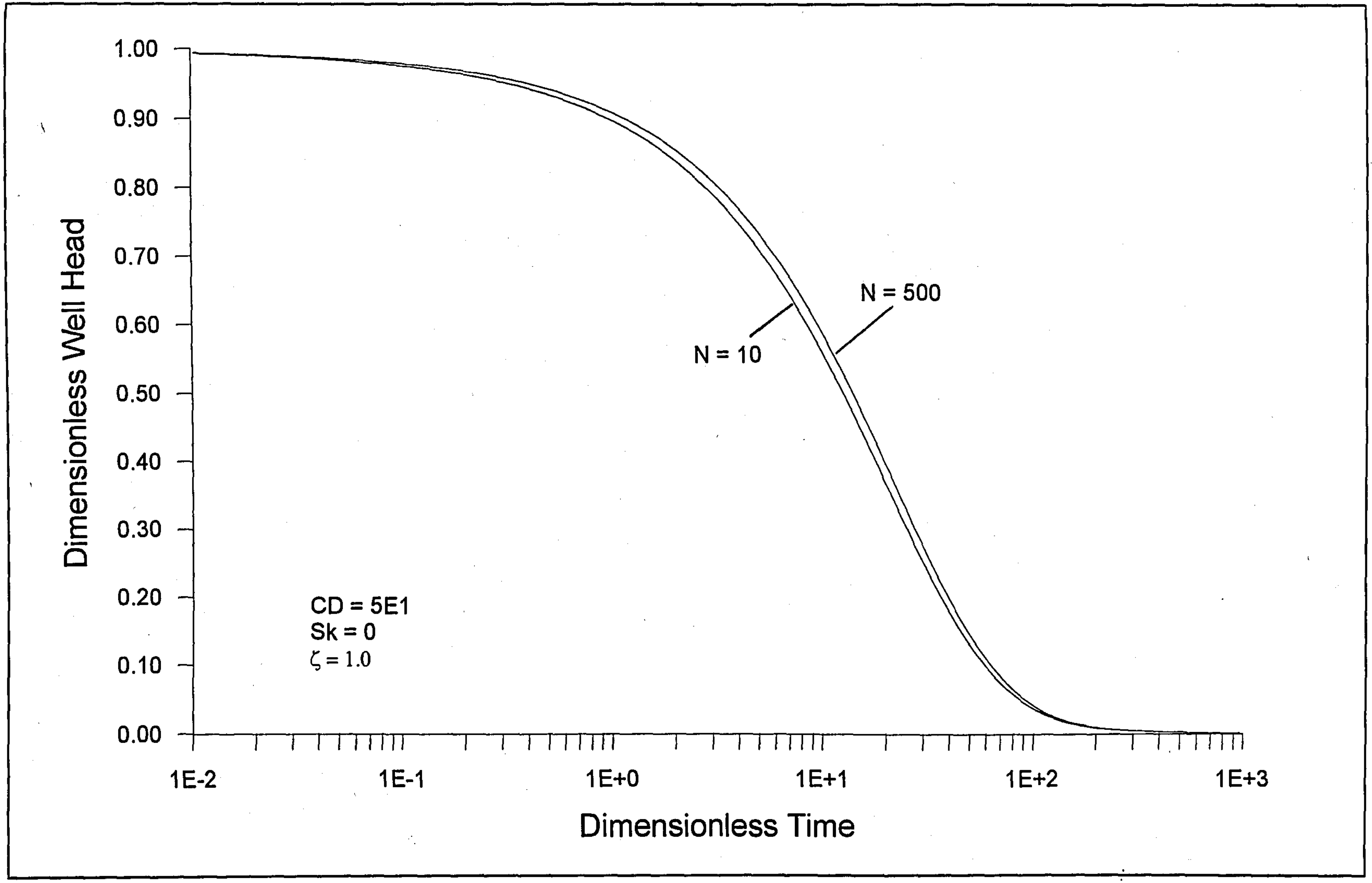

FIGURE A.2. The effect of the number of infinite series expansion terms, N, on the shape of dimensionless type curves, CD = 5E1, N = 10 and 500

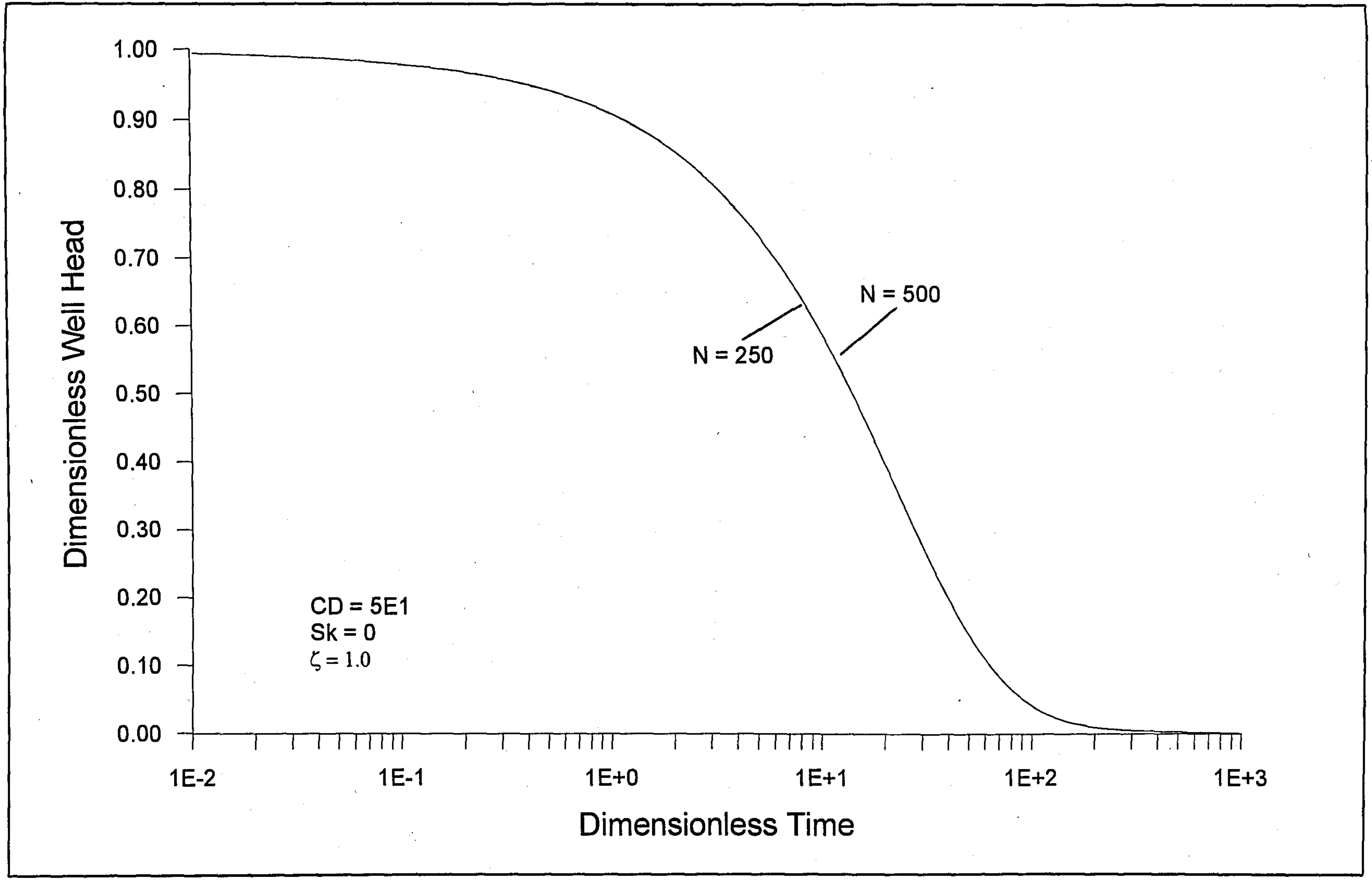

FIGURE A.3. The effect of the number of infinite series expansion terms, N, on the shape of dimensionless type curves, CD = 5E1, N =250 and 500

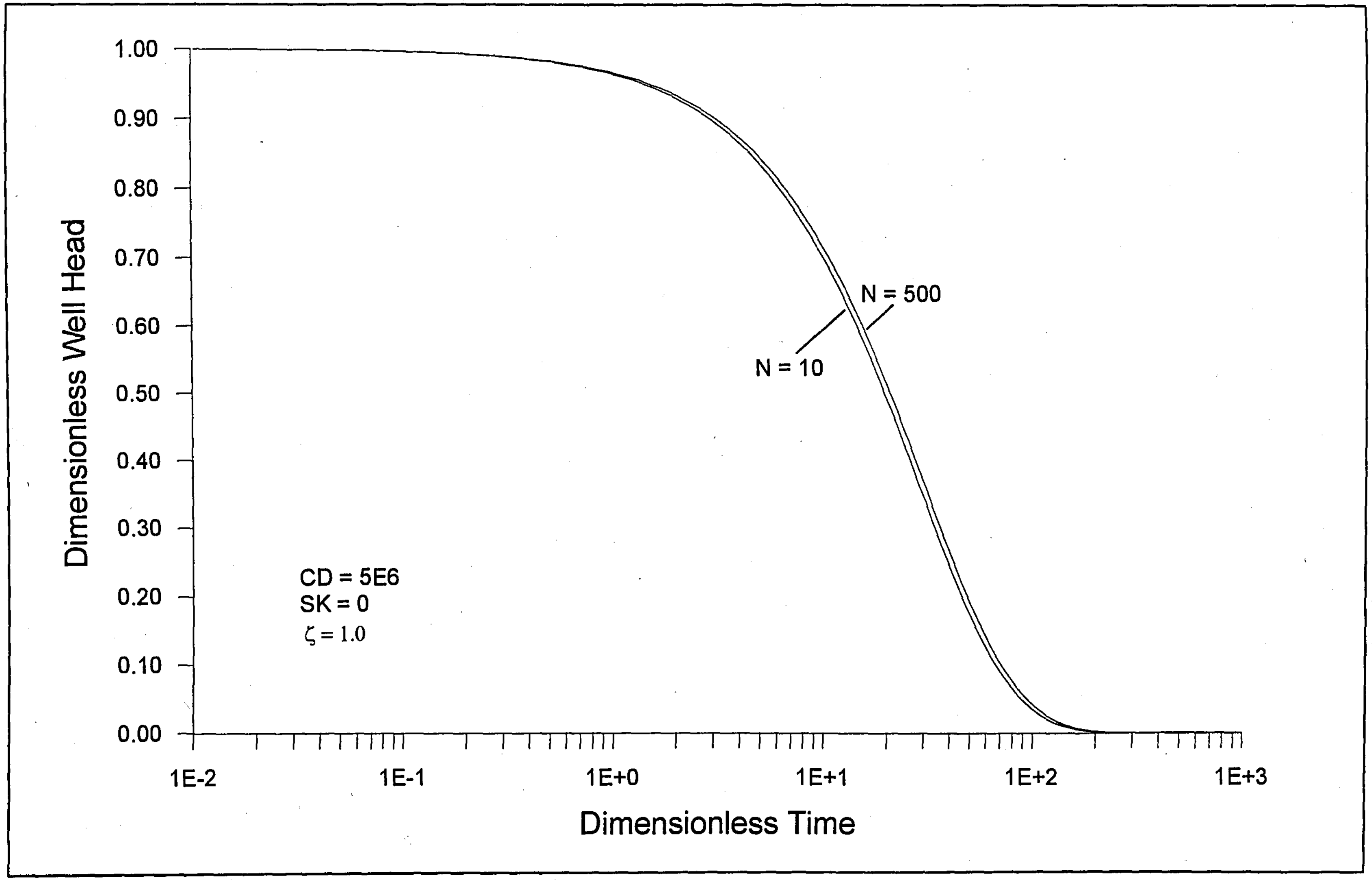

FIGURE A.4. The effect of the number of infinite series expansion terms, N, on the shape of dimensionless type curves, CD = 5E6, N = 10 and 500

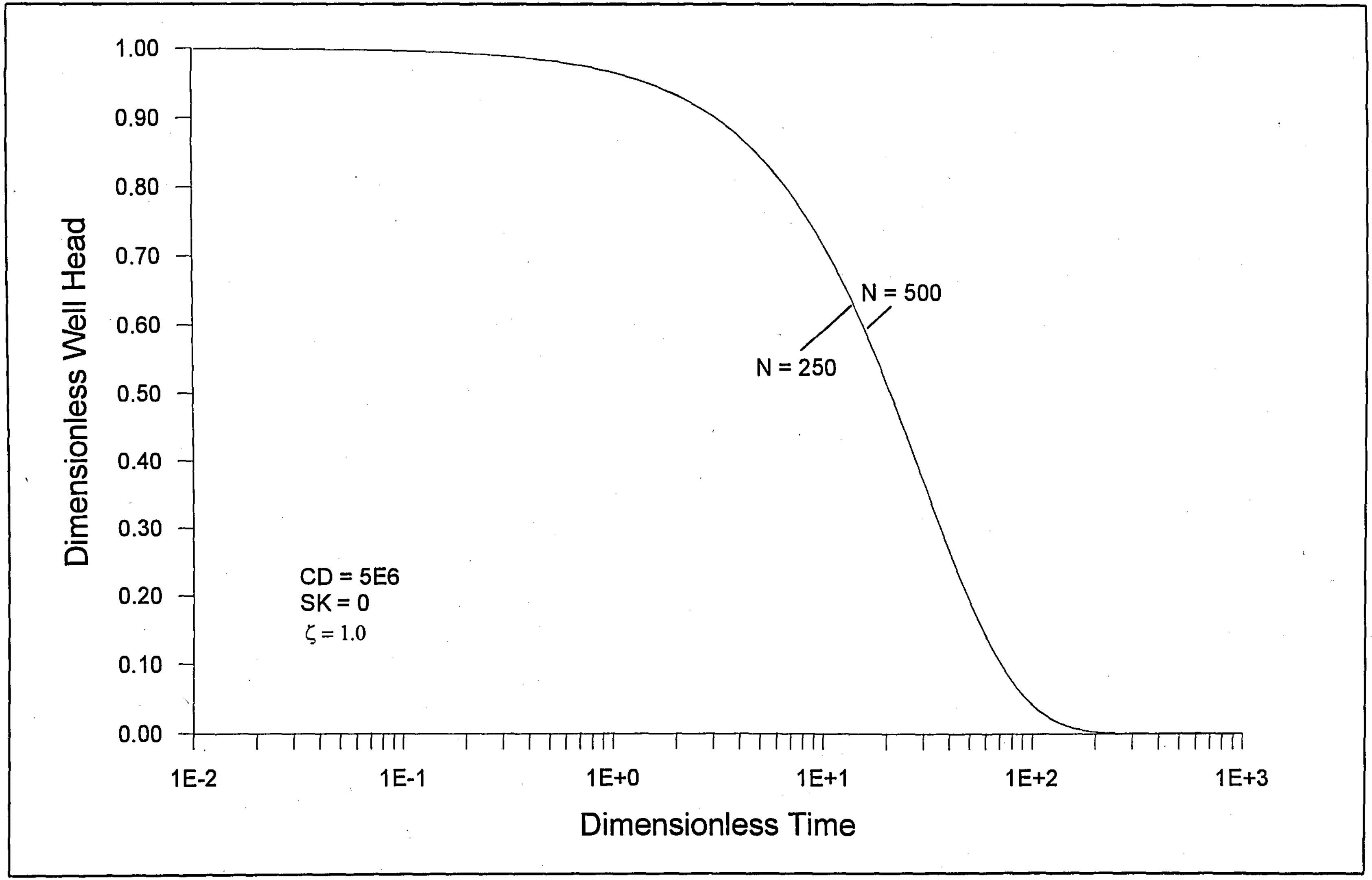

FIGURE A.5. The effect of the number of infinite series expansion terms, N, on the shape of dimensionless type curves, CD = 5E6, N = 250 and 500

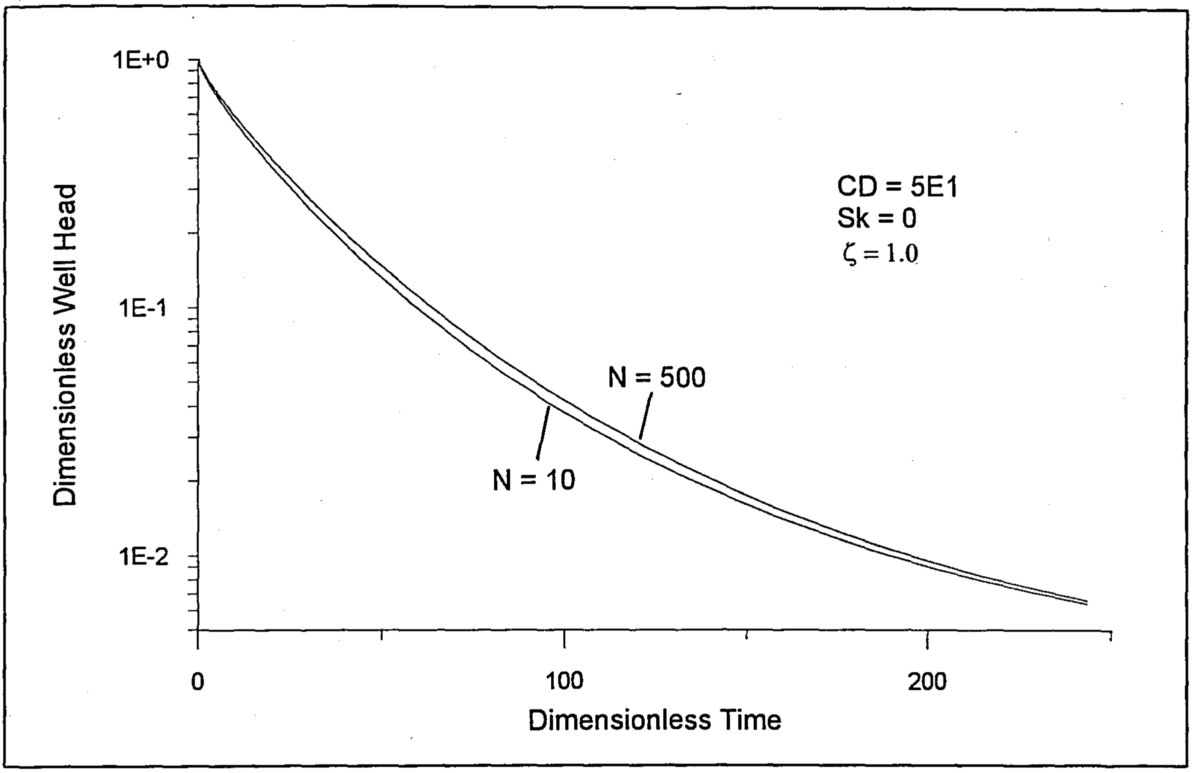

FIGURE A.6. The effect of the number of infinite series expansion terms, N, on the shape of dimensionless type curves, CD = 5E1, early times, N = 10 and 500

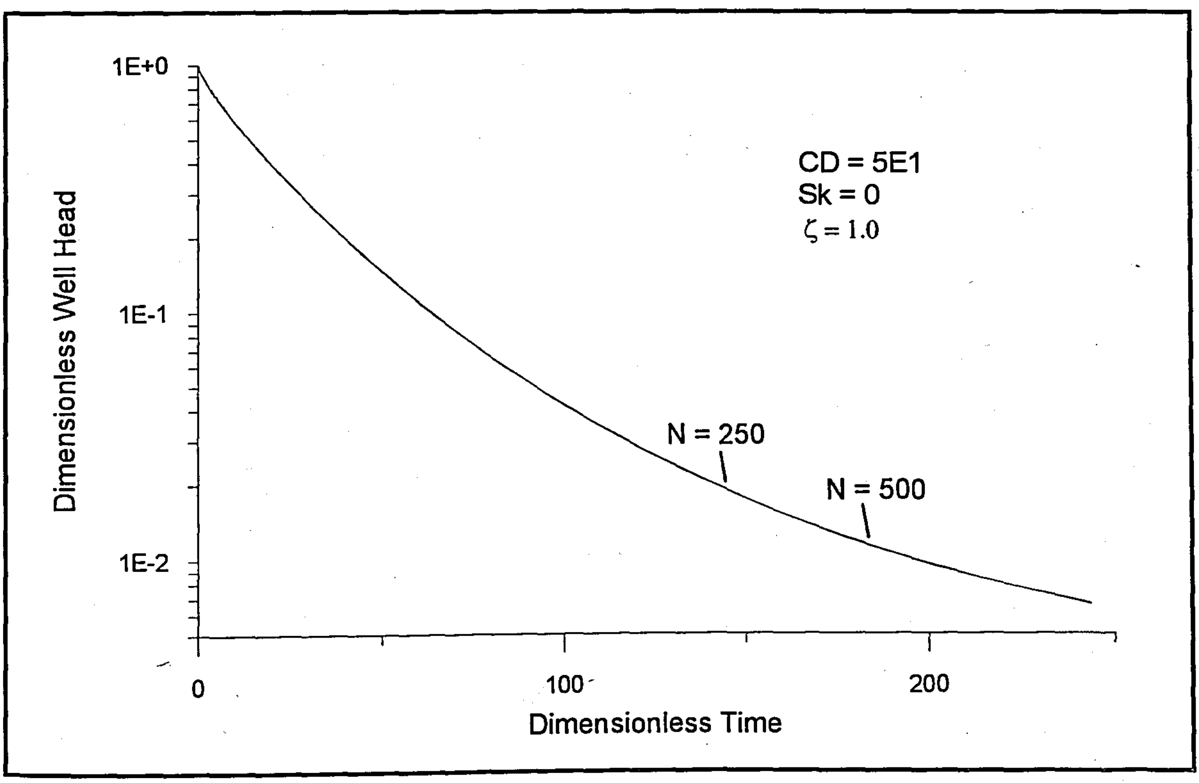

FIGURE A.7. The effect of the number of infinite series expansion terms, N, on the shape of dimensionless type curves, CD = 5E1, early times, N = 250 and 500

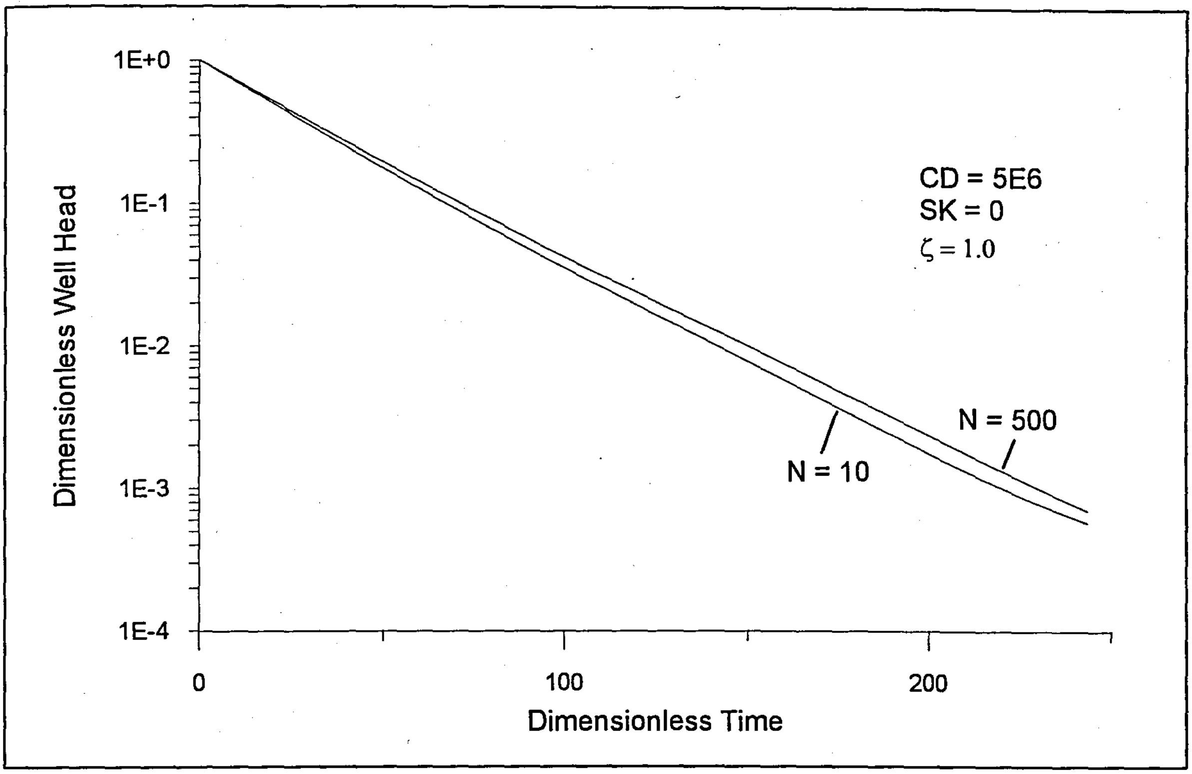

FIGURE A.8. The effect of the number of infinite series expansion terms, N, on the shape of dimensionless type curves, CD = 5E6, early times, N = 10 and 500

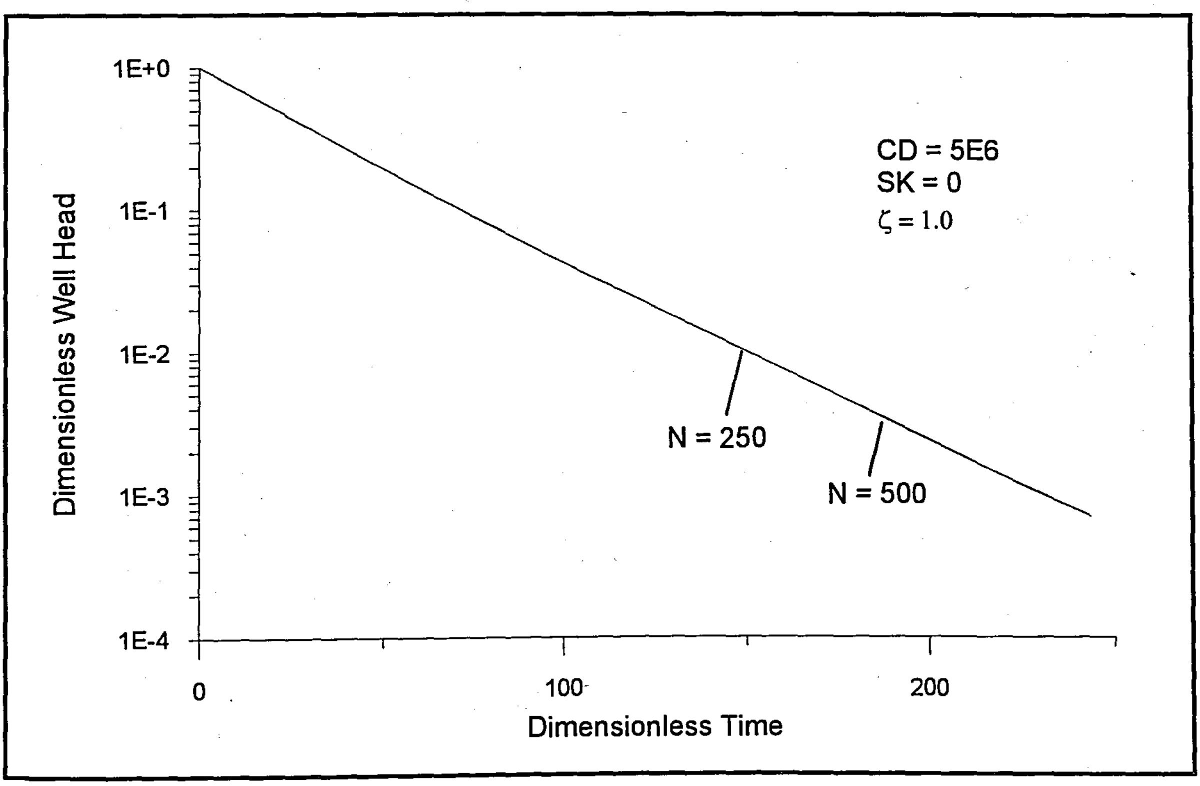

FIGURE A.9. The effect of the number of infinite series expansion terms, N, on the shape of dimensionless type curves, CD = 5E6, early times, N = 250 and 500

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**********************************************************************
*                        TYPE_CURVE_GENERATOR                        *
*                                                                    *
*                   developed by Burak KILANÇ                        *
*                                                                    *
*                   Bogaziçi University, 1996                        *
**********************************************************************

**********************************************************************
*                        MAIN PROGRAM STARTS                         *
*                                                                    *

      PROGRAM TYPE_CURVE_GENERATOR_(TCG)

**********************************************************************
*        Parameter Declaration for Large Scale Sensitivity Analyses  *
**********************************************************************

      PARAMETER(MAXT=1025,MAXCD=20,MAXZETA=5,MAXSKD=5,MAXSY=9)

**********************************************************************
*                     Declarations of Variables used                 *
**********************************************************************

      DOUBLE PRECISION G,H,P,V,S,SN,A,FA
      DOUBLE PRECISION D,ZETA,SY,RS,RC,ZA,ZB
      DOUBLE PRECISION DD,SYD,SKD,ZAD,ZBD,CD,TD,TDC
      DOUBLE PRECISION HWD
      DOUBLE PRECISION SLUG1,SLUG2,SLUG3
      DOUBLE PRECISION EPSILON,TLOWER,TUPPER,DELTA
      DOUBLE PRECISION PI

      REAL AJ

      INTEGER NTERM,TSTEP,FLNUM,AQUIFTYP,NEGATIVE
      INTEGER CDNUM,ZETANUM,SKDNUM,SYNUM
      INTEGER IK,I1,I2,I3,I4,I5,I6,ITER
      INTEGER I,M,K,NH

      CHARACTER*12 FLIN,FLDAT,FLOUT,FLREP
      CHARACTER*5  FL

      DIMENSION G(20),H(20),P(20),V(20)
      DIMENSION TD(MAXT),TDC(MAXT),HWD(MAXT)
      DIMENSION CD(MAXCD),ZETA(MAXZETA),SKD(MAXSKD)
      DIMENSION SY(MAXSY),SYD(MAXSY)

      PI=4.D0*DATAN(1.D0)
      ITER=0
```

```
***********************************************************************
*                 1. Arrangement of input and output files            *
*             2.  Inputting and outputting necessary parameters       *
*            3. Casting relevant variables into dimensionless form    *
***********************************************************************

      OPEN (81,FILE='SYSTEM.PAR')
      OPEN (82,FILE='FILES.IN')

      READ(81,*) NTERM,N
      READ(81,*) EPSILON
      READ(81,*) TLOWER,TUPPER,TSTEP
      CLOSE(81)

      READ(82,*) FLNUM

      DO 1111 I1=1,FLNUM

      READ(82,101) FLIN
      READ(82,103) FL

      FLOUT=(FL(1:(INDEX(FL,' ')-1)))//'_OUT.OUT'
      FLREP=(FL(1:(INDEX(FL,' ')-1)))//'_REP.REP'

      OPEN (91,FILE=FLOUT)
      OPEN (92,FILE=FLREP)

      OPEN(83,FILE=FLIN)

      READ(83,*) AQUIFTYP
      READ(83,*) D,ZA,ZB,RS,RC

      CLOSE(83)

      DD=D/RS
      ZAD=ZA/RS
      ZBD=ZB/RS

      IF (AQUIFTYP.EQ.1) THEN
         OPEN(84,FILE='CONFINED.PAR')
      ELSE
         OPEN(84,FILE='UNCONF.PAR')
      ENDIF

      READ(84,*) CDNUM
      READ(84,*) (CD(IK),IK=1,CDNUM)
      READ(84,*) ZETANUM
      READ(84,*) (ZETA(IK),IK=1,ZETANUM)
      READ(84,*) SKDNUM
      READ(84,*) (SKD(IK),IK=1,SKDNUM)

      IF(AQUIFTYP.NE.3) THEN
         SYNUM=1
         SY(1)=0.
         GOTO 1201
      ENDIF

      READ(84,*) SYNUM
      READ(84,*) (SY(IK),IK=1,SYNUM)

1201  CLOSE(84)

      WRITE(91,*) (CDNUM*ZETANUM*SKDNUM*SYNUM)
```

```
      WRITE(92,200)
      WRITE(92,*)
      WRITE(92,201)
      WRITE(92,210) AQUIFTYP
      WRITE(92,202) D,ZA,ZB
      WRITE(92,203) RS,RC
      WRITE(92,*)
      WRITE(92,204)
      WRITE(92,205) N,NTERM
      WRITE(92,206) EPSILON
      WRITE(92,207) TUPPER,TLOWER
      WRITE(92,208) ((2**TSTEP)+1)
      WRITE(92,209) CDNUM,ZETANUM,SKDNUM,SYNUM
      WRITE(92,*)
      WRITE(92,200)
```

```
***********************************************************************
*          Main Body of the Code Including  the Stehfest Algorithm    *
***********************************************************************
```

```
      DO 2222 I2=1,CDNUM
      DELTA=(DLOG(TUPPER)-DLOG(TLOWER))/(2**TSTEP)

      DO 2301 IK=1,(2**TSTEP)+1
         TDC(IK)=DEXP(DLOG(TLOWER)+DELTA*(IK-1))
         TD(IK)=2*((ZB-ZA)/D)*CD(I2)*TDC(IK)
2301  CONTINUE

      DO 3333 I3=1,ZETANUM
      DO 4444 I4=1,SKDNUM
      DO 5555 I5=1,SYNUM

      SYD(I5)=2*(ZB-ZA)*RS/(RC**2)*CD(I2)*SY(I5)/ZETA(I3)

      NEGATIVE=0

      NH=N/2
      G(1)=1
      DO 6666 I6=1,(2**TSTEP)+1
        ITER=ITER+1
        WRITE(*,110) I1,FLNUM,I2,CDNUM,I3,ZETANUM,I4,SKDNUM,
     &              I5,SYNUM,I6,((2**TSTEP)+1),
     &              ITER,(CDNUM*ZETANUM*SKDNUM*SYNUM*((2**TSTEP)+1))*FLNUM
        DO 6710 I=2,N
          G(I)=G(I-1)*I
6710    CONTINUE
        H(1)=2/G(NH-1)
        DO 6720 I=2,NH-1
          H(I)=I**(NH)*G(2*I)/G(NH-I)/G(I)/G(I-1)
6720    CONTINUE
        H(NH)=NH**(NH)*G(N)/G(NH)/G(NH-1)
        SN=(-1.0D0)**(NH+1)
        DO 6730 I=1,N
          V(I)=0.0D0
          M=I
          IF (I.GE.NH) M=NH
          AJ=(I+1.)/2.
          DO 6740 K=AJ,M
            IF (I.EQ.K) GO TO 6725
            IF (I.EQ.2*K) GO TO 6727
            V(I)=V(I)+H(K)/G(I-K)/G(2*K-I)
```

```
            GO TO 6740
6725        V(I)=V(I)+H(K)/G(2*K-I)
            GO TO 6740
6727        V(I)=V(I)+H(K)/G(I-K)
6740      CONTINUE
          V(I)=SN*V(I)
          SN=-SN
6730    CONTINUE
        A=DLOG(2.0D0)/TD(I6)
        DO 6750 I=1,N
          S=I*A

          IF (AQUIFTYP.EQ.1) THEN
             P(I)=SLUG1(NTERM,S,DD,ZAD,ZBD,CD(I2),ZETA(I3),SKD(I4),PI)
          ELSE IF (AQUIFTYP.EQ.2) THEN
             P(I)=SLUG2(NTERM,S,DD,ZAD,ZBD,CD(I2),ZETA(I3),SKD(I4),PI)
          ELSE
             P(I)=SLUG3(NTERM,S,DD,ZAD,ZBD,CD(I2),ZETA(I3),SKD(I4)
     &                       ,SYD(I5),EPSILON,PI)
          ENDIF

6750    CONTINUE
        FA=0.0D0
        DO 6760 I=1,N
          FA=FA+V(I)*P(I)
6760    CONTINUE
        FA=A*FA
        HWD(I6)=FA
6666  CONTINUE

6999  CONTINUE


      FLDAT=(FL(1:(INDEX(FL,' ')-1)))//
     &(CHAR(ICHAR('0')+I2))//(CHAR(ICHAR('0')+I3))//
     &(CHAR(ICHAR('0')+I4))//(CHAR(ICHAR('0')+I5))//'.DAT'

      WRITE(91,101) FLDAT

      OPEN(93,FILE=FLDAT)

      DO 5603 IK=1,(2**TSTEP)+1
         IF(HWD(IK).LE.0.) THEN
           NEGATIVE=NEGATIVE+1
           GOTO 5603
         ENDIF
         WRITE(93,105) TDC(IK),HWD(IK)
5603  CONTINUE

      WRITE(92,*)
      WRITE(92,301) FLIN, FLDAT
      WRITE(92,302) CD(I2),ZETA(I3)
      WRITE(92,303) SKD(I4),SY(I5),SYD(I5)
      WRITE(92,*)

      CLOSE(93)

5555  CONTINUE
4444  CONTINUE
3333  CONTINUE
2222  CONTINUE

      WRITE(92,200)
```

```
1111  CONTINUE

************************************************************************
*                        Format Declarations                           *
************************************************************************

101    FORMAT(A12)
103    FORMAT(A5)
105    FORMAT(2(E15.8))
110    FORMAT(' Case: ',I1,'/',I1,2X,'Step: ',4(I2,'/'I2,2X),
      &I3,'/',I3,2X,I7,'/',I7)
200    FORMAT(' ==================================================
      &=========')
201    FORMAT(' AQUIFER & TEST PARAMETERS:')
202    FORMAT(' D = ',E10.4,5X,'Za= ',E10.4,5X,'Zb=',E10.4)
203    FORMAT(' Rs= ',E10.4,5X,'Rc= ',E10.4,5X)
204    FORMAT(' SYSTEM PARAMETERS:')
205    FORMAT(' N = ',I3,12X,'NTERM= ',I5)
206    FORMAT(' Epsilon = ',E10.4)
207    FORMAT(' Axis Limits: TUpper  = ',E10.4,5X,'TLower = ',E10.4,5X)
208    FORMAT(' Time period divided into ',I5,' points')
209    FORMAT(' ',I2,' Cds, ',I2,' Zetas, ',I2, ' Skds, ',I2,' Sys were
      & investigated')
210    FORMAT(' Aquifer Type : ',I2)
301    FORMAT(' Input File: ',A12,5X,'Output File: ',A12)
302    FORMAT(' Cd  = ',E10.4,5X,'Zeta= ',E10.4)
303    FORMAT(' Skd = ',E10.4,5X,'Sy  = ',E10.4,5X,'Syd= 'E10.4)

************************************************************************
*                          Final Operations                            *
************************************************************************

       CLOSE(82)
       CLOSE(91)
       CLOSE(92)

       END

*                                                                      *
*                       END of the MAIN PROGRAM                        *
************************************************************************

************************************************************************
*                     DECLARATION of SUB-FUNCTIONS                     *
*                                                                      *

************************************************************************
*                   Sub-Function for Type 1 Solutions                  *
************************************************************************

       DOUBLE PRECISION FUNCTION
      &        SLUG1(NTERM,S,DD,ZAD,ZBD,CD,ZETA,SKD,PI)

       INTEGER NTERM
       DOUBLE PRECISION S,DD,ZAD,ZBD,CD,ZETA,SKD,PI
       DOUBLE PRECISION RHO,Q0,QN,A0,A,C
       DOUBLE PRECISION BESSK0,BESSK1
```

```
      INTEGER I

      RHO=(ZBD-ZAD)/DD
      Q0=DSQRT(S)
      A0=(RHO*BESSK0(Q0))/(Q0*BESSK1(Q0))
      A=0.D0

      DO 10 I=1,NTERM
         QN=DSQRT(S+ZETA*((I*PI/DD)**2))
         C=BESSK0(QN)/BESSK1(QN)
         A=A+(((DSIN(I*PI*ZBD/DD)-DSIN(I*PI*ZAD/DD))**2)*C)/
     &         ((I**2)*QN)
10    CONTINUE

      A=A0+A*2.D0/(RHO*(PI**2))

      SLUG1=(CD*(A+SKD))/(CD*(A+SKD)*S+1)

      RETURN
      END
```

```
**********************************************************************
*                  Sub-Function for Type 2.a Solutions               *
**********************************************************************
```

```
      DOUBLE PRECISION FUNCTION
     &        SLUG2(NTERM,S,DD,ZAD,ZBD,CD,ZETA,SKD,PI)

      INTEGER NTERM
      DOUBLE PRECISION S,DD,ZAD,ZBD,CD,ZETA,SKD,PI
      DOUBLE PRECISION RHO,QN,A,C
      DOUBLE PRECISION BESSK0,BESSK1
      INTEGER I

      RHO=(ZBD-ZAD)/DD
      A=0.D0

      DO 10 I=1,NTERM
         QN=DSQRT(S+ZETA*(((I+5.D-1)*PI/DD)**2))
         C=BESSK0(QN)/BESSK1(QN)
         A=A+(((DSIN((I+5.D-1)*PI*ZBD/DD)-DSIN((I+5.D-1)*PI*ZAD/DD))**2)
     &        *C)/(((I+5.D-1)**2)*QN)
10    CONTINUE

      A=A*2.D0/(RHO*(PI**2))

      SLUG2=(CD*(A+SKD))/(CD*(A+SKD)*S+1)

      RETURN
      END
```

```
**********************************************************************
*                  Sub-Function for Type 2.b Solutions               *
**********************************************************************
```

```
      DOUBLE PRECISION FUNCTION
     &        SLUG3(NTERM,S,DD,ZAD,ZBD,CD,ZETA,SKD,SYD,EPSILON,PI)

      INTEGER NTERM
      DOUBLE PRECISION S,DD,ZAD,ZBD,CD,ZETA,SKD,SYD,EPSILON,PI
      DOUBLE PRECISION RHO,QN,A,C
```

```
      DOUBLE PRECISION XLOW,XMID,XUP,XMIDPREV,F,X,ROOT
      DOUBLE PRECISION BESSK0,BESSK1
      INTEGER I

      F(X)=DTAN(X)-((S*SYD*DD)/X)

      RHO=(ZBD-ZAD)/DD
      A=0.D0

      ROOT=-(1.D0/2.D0)*PI

      DO 10 I=1,NTERM

         XLOW=(I-1.D0)*PI
         XUP=ROOT+PI
         XMIDPREV=0.D0

20       XMID=(XLOW+XUP)/2.D0

         IF ((XMIDPREV-XMID).EQ.0.D0) THEN
           GOTO 30
         ENDIF

         IF (F(XMID).GT.EPSILON) THEN
            XUP=XMID
            XMIDPREV=XMID
            GOTO 20
         ENDIF

         IF (F(XMID).LT.(-1.D0*EPSILON)) THEN
            XLOW=XMID
            XMIDPREV=XMID
            GOTO 20
         ENDIF

30       ROOT=XMID

         QN=DSQRT(S+ZETA*((ROOT/DD)**2))
         C=BESSK0(QN)/(BESSK1(QN)*QN*ROOT*(ROOT+DSIN(2*ROOT)/2.D0))
         A=A+(((DSIN(ROOT*ZBD/DD)-DSIN(ROOT*ZAD/DD))**2)*C)

10    CONTINUE

      A=A*2.D0/RHO

      SLUG3=(CD*(A+SKD))/(CD*(A+SKD)*S+1)

      RETURN
      END

*************************************************************************
*                Sub-Function for Modified Bessel Function K0           *
*        adapted from Numerical Recipes in Pascal (Press et. al., 1990) *
*************************************************************************

      DOUBLE PRECISION FUNCTION BESSK0(X)
      DOUBLE PRECISION X,BESSI0
      DOUBLE PRECISION Y,P1,P2,P3,P4,P5,P6,P7,
     *    Q1,Q2,Q3,Q4,Q5,Q6,Q7
      DATA P1,P2,P3,P4,P5,P6,P7/-0.57721566D0,0.42278420D0,0.23069756D0,
     *    0.3488590D-1,0.262698D-2,0.10750D-3,0.74D-5/
      DATA Q1,Q2,Q3,Q4,Q5,Q6,Q7/1.25331414D0,-0.7832358D-1,0.2189568D-1,
```

```
*     -0.1062446D-1,0.587872D-2,-0.251540D-2,0.53208D-3/
      IF (X.LE.2.0D0) THEN
        Y=X*X/4.0
        BESSK0=(-DLOG(X/2.0D0)*BESSI0(X))+(P1+Y*(P2+Y*(P3+
*            Y*(P4+Y*(P5+Y*(P6+Y*P7))))))
      ELSE
        Y=(2.0D0/X)
        BESSK0=(DEXP(-X)/DSQRT(X))*(Q1+Y*(Q2+Y*(Q3+
*            Y*(Q4+Y*(Q5+Y*(Q6+Y*Q7))))))
      ENDIF
      RETURN
      END
```

```
************************************************************************
*              Sub-Function for Modified Bessel Function K1             *
*   adapted from Numerical Recipes in Pascal (Press et. al., 1990)      *
************************************************************************
```

```
      DOUBLE PRECISION FUNCTION BESSK1(X)
      DOUBLE PRECISION X,BESSI1
      DOUBLE PRECISION Y,P1,P2,P3,P4,P5,P6,P7,
*     Q1,Q2,Q3,Q4,Q5,Q6,Q7
      DATA P1,P2,P3,P4,P5,P6,P7/1.0D0,0.15443144D0,-0.67278579D0,
*     -0.18156897D0,-0.1919402D-1,-0.110404D-2,-0.4686D-4/
      DATA Q1,Q2,Q3,Q4,Q5,Q6,Q7/1.25331414D0,0.23498619D0,-0.3655620D-1,
*     0.1504268D-1,-0.780353D-2,0.325614D-2,-0.68245D-3/
      IF (X.LE.2.0D0) THEN
        Y=X*X/4.0D0
        BESSK1=(DLOG(X/2.0D0)*BESSI1(X))+(1.0D0/X)*(P1+Y*(P2+
*          Y*(P3+Y*(P4+Y*(P5+Y*(P6+Y*P7))))))
      ELSE
        Y=2.0D0/X
        BESSK1=(DEXP(-X)/DSQRT(X))*(Q1+Y*(Q2+Y*(Q3+
*          Y*(Q4+Y*(Q5+Y*(Q6+Y*Q7))))))
      ENDIF
      RETURN
      END
```

```
************************************************************************
*              Sub-Function for Modified Bessel Function I0             *
*   adapted from Numerical Recipes in Pascal (Press et. al., 1990)      *
************************************************************************
```

```
      DOUBLE PRECISION FUNCTION BESSI0(X)
      DOUBLE PRECISION X,AX
      DOUBLE PRECISION Y,P1,P2,P3,P4,P5,P6,P7,
*     Q1,Q2,Q3,Q4,Q5,Q6,Q7,Q8,Q9
      DATA P1,P2,P3,P4,P5,P6,P7/1.0D0,3.5156229D0,3.0899424D0,1.2067492D
*0,
*     0.2659732D0,0.360768D-1,0.45813D-2/
      DATA Q1,Q2,Q3,Q4,Q5,Q6,Q7,Q8,Q9/0.39894228D0,0.1328592D-1,
*     0.225319D-2,-0.157565D-2,0.916281D-2,-0.2057706D-1,
*     0.2635537D-1,-0.1647633D-1,0.392377D-2/
      IF (DABS(X).LT.3.75D0) THEN
        Y=(X/3.75D0)**2
        BESSI0=P1+Y*(P2+Y*(P3+Y*(P4+Y*(P5+Y*(P6+Y*P7)))))
      ELSE
        AX=DABS(X)
        Y=3.75D0/AX
        BESSI0=(DEXP(AX)/DSQRT(AX))*(Q1+Y*(Q2+Y*(Q3+Y*(Q4
*          +Y*(Q5+Y*(Q6+Y*(Q7+Y*(Q8+Y*Q9))))))))
```

```
      ENDIF
      RETURN
      END

***********************************************************************
*              Sub-Function for Modified Bessel Function I1           *
*     adapted from Numerical Recipes in Pascal (Prees et. al., 1990)  *
***********************************************************************

      DOUBLE PRECISION FUNCTION BESSI1(X)
      DOUBLE PRECISION X,AX
      DOUBLE PRECISION Y,P1,P2,P3,P4,P5,P6,P7,
     *     Q1,Q2,Q3,Q4,Q5,Q6,Q7,Q8,Q9
      DATA P1,P2,P3,P4,P5,P6,P7/0.5D0,0.87890594D0,0.51498869D0,
     *    0.15084934D0,0.2658733D-1,0.301532D-2,0.32411D-3/
      DATA Q1,Q2,Q3,Q4,Q5,Q6,Q7,Q8,Q9/0.39894228D0,-0.3988024D-1,
     *    -0.362018D-2,0.163801D-2,-0.1031555D-1,0.2282967D-1,
     *    -0.2895312D-1,0.1787654D-1,-0.420059D-2/
      IF (DABS(X).LT.3.75D0) THEN
        Y=(X/3.75D0)**2
        BESSI1=X*(P1+Y*(P2+Y*(P3+Y*(P4+Y*(P5+Y*(P6+Y*P7))))))
      ELSE
        AX=DABS(X)
        Y=3.75D0/AX
        BESSI1=(DEXP(AX)/DSQRT(AX))*(Q1+Y*(Q2+Y*(Q3+Y*(Q4+
     *      Y*(Q5+Y*(Q6+Y*(Q7+Y*(Q8+Y*Q9))))))))
      ENDIF
      RETURN
      END

*                                                                     *
*                   END of SUB-FUNCTION DECLARATIONS                  *
***********************************************************************
```

File: SYSTEM.PAR

Status: Input file

Content: Parameters used for Stehfest algorithm and other mathematical operations
Parameters describing the timescale

General structure: (all parameters in general format)

1st Line

Number of Fourier series expansion terms
Number of Stehfest numbers

2nd Line

Error criterion (epsilon) for the bisection method used in Slug3 Sub-function (Type 2.b. solution)

3rd Line

Lower limit of the time scale (initial dimensionless time)
Upper limit of the time scale (final dimensionless time)
Parameter determining the interval number the timescale will be divided to (to the powers of 2, namely : 5 -> 32 intervals, 6 -> 64 intervals .....)

****

Example File:

500 8
1.D-4
1.D-3 1.D3 8

****

File: CONFINED.PAR

Status: Input file

Content: Parameters used for confined aquifer case

General structure:

1st and 2nd Lines

Number of C_D values to be analyzed

C_D values

3rd and 4th Lines

Number of ζ values (anisotropy coefficient) to be analyzed

ζ values

5th and 6th Lines

Number of S_k values (skin factor) to be analyzed

S_k values (dimensionless)

****

Example File:

```
6
1.D2 1.D3 1.D4 1.D5 1.D6 1.D8
3
1.0 0.1 0.01
3
0 1 5
```

****

File: UNCONF.PAR

Status: Input file

Content: Parameters used for unconfined aquifer case

General structure:

1st and 2nd Lines

Number of C_D values to be analyzed

C_D values

3rd and 4th Lines

Number of ζ values (anisotropy coefficient) to be analyzed

ζ values

5th and 6th Lines

Number of S_k values (skin factor) to be analyzed

S_k values (dimensionless)

7th and 8th Lines

Number of S_y values (specific yield) to be analyzed

S_y values (dimensional)

****

Example File:

4
5.D1 5.D2 5.D4 5.D5
2
0.1 1.0

2

0 1 5

2

0.05 0.1 0.15 0.20 0.25

****

File: FILES.IN

Status: Input file

Content: Number of cases to be processed

Input and relevant output file names for cases to be investigated

General structure:

1st Line

Number of cases to be processed

2nd Line

Name of the input file containing the aquifer parameters of Case 1

3rd Line[1]

Name of the output file relevant to Case 1. The type curve values will be written in this file

4th Line

Name of the input file containing the aquifer parameters of Case 2

5th Line[1]

Name of the output file relevant to Case 2. The type curve values will be written in this file

.

.

.

.

[1] The output name must not be longer than 5 characters

****

Example File:

3

OKTAYG

OG

B&RA

OUT1

B&RB

OUT2

****

File: Main Input File (name user defined)
The input file name is read from FILES.IN

Status: Input file

Content: Aquifer parameters and test geometry

General structure:

1st Line

Aquifer (Analysis) type

1. Confined condition (Type 1)
2. Unconfined (Type 2.a.)
3. Unconfined (Type 2.b.)

2nd Line

Depth of the aquifer (confined case) or the height of the water column (unconfined case)
The distance from the bottom of the screen to the lower boundary
The distance from the top of screen to the lower boundary
Screen Radius
Casing Radius

(All parameters in dimensional values)

****

Example File: (for Melville et. al. Aquifer setting (1991))

1
21.336 1.031748 2.138172 0.0508 0.07239

****

File: Main Output File (name user defined)

The output file name is read from FILES.IN and modified by the code adding a 4 digit code. The code indicates the set of parameters used in the analysis and was given in CONFINED.PAR or UNCONF.PAR

(Example: FNAME1231.DAT : 1st C_D value
2nd ζ value
3rd S_k value
1st S_y value
given in CONFİNED.PAR or UNCONF.PAR

Status: Output file

Content: The dimensionless time and well head values obtained by the presently developed solution procedures

Future Use: As an input file for graphical production

****

Example File: (time - head)

1.00E-03	9.95E-01
1.03E-03	9.95E-01
1.06E-03	9.95E-01
1.10E-03	9.94E-01
1.13E-03	9.94E-01
1.17E-03	9.94E-01

.
.
.

8.82E+03	9.07E-05
9.10E+03	8.67E-05
9.39E+03	8.29E-05
9.69E+03	7.92E-05
1.00E+04	7.58E-05

****

File: Output file name + "_OUT.OUT"
Example filename : CASE1_OUT.OUT
The output file name is read from FILES.IN

Status: Output file

Content: Output file names processed for each case investigated

Future Use: As an input file of CMP Program

****

File: Output file name + "_REP_REP"
Example filename : CASE1_REP.REP
The output file name is read from FILES.IN

Status: Output file

Content: Descriptive parameters of each case investigated by the program

Future Use: Primarily, for the convenience of the program user for post-checking parameters investigated in each case

```
***********************************************************************
*                         CURVE_MATCH_PROGRAM                         *
*                                                                     *
*                    developed by Burak KILANÇ                        *
*                                                                     *
*                     Bogaziçi  University, 1996                      *
***********************************************************************

***********************************************************************
*                         MAIN PROGRAM STARTS                         *
*                                                                     *

     PROGRAM CURVE_MATCH_PROGRAM_(CMP)

***********************************************************************
*         Parameter Declaration for Large Scale Sensitivity Analyses  *
***********************************************************************

      PARAMETER(MAXDP=20,MAXFILE=4500)

***********************************************************************
*                     Declarations of Variables used                  *
***********************************************************************

      INTEGER I1,I2,I3,I4,INT,SEP
      INTEGER FIELDNUM,FLNUM

      DOUBLE PRECISION FT,FHWD,TDC,HWD,T,H,TPREV,HPREV,TSLP,FSLP
      DOUBLE PRECISION ERROR,TERROR,AUX,D,ZA,ZB,RS,RC,KR,KRAVG

      CHARACTER*12 FFILE,TFILE,TFL,AUXFL,AQUIFL

      DIMENSION TDC(MAXDP),HWD(MAXDP)
      DIMENSION FT(MAXDP),FHWD(MAXDP)
      DIMENSION FSLP(MAXDP),TSLP(MAXDP)
      DIMENSION KR(MAXDP)
      DIMENSION TERROR(MAXFILE),TFL(MAXFILE)

***********************************************************************
*               1. Arrangement of input and output files              *
*            2. Inputting and outputting necessary parameters         *
***********************************************************************

      OPEN(80,FILE='FILES.MAT')
      READ(80,105) FFILE
      READ(80,105) TFILE
      READ(80,105) AQUIFL
      CLOSE(80)

      OPEN(81,FILE=FFILE)
      READ(81,*) FIELDNUM
      DO 20 I2=1,FIELDNUM
           READ(81,*) FT(I2),FHWD(I2)
20    CONTINUE
      CLOSE(81)

      OPEN(85,FILE=AQUIFL)
      READ(85,*)
```

```
      READ(85,*) D,ZA,ZB,RS,RC
      CLOSE(85)

      OPEN(82,FILE=TFILE)

      READ(82,*) FLNUM

************************************************************************
*                 Determination of the Degree of Best-Match with      *
*                          Forward Differences Method                 *
************************************************************************

      DO 10 I1=1,FLNUM

         TERROR(I1)=0

         READ(82,*) TFL(I1)
         OPEN(83,FILE=TFL(I1))

         DO 30 I3=1,FIELDNUM
33            READ(83,*) T,H
              IF (H.GT.FHWD(I3)) THEN
                TPREV=T
                HPREV=H
                GOTO 33
              ENDIF
              HWD(I3)=FHWD(I3)
              TDC(I3)=TPREV*((T/TPREV)**((FHWD(I3)-HPREV)/(H-HPREV)))
30       CONTINUE

         CLOSE(83)

         DO 35 I5=1,FIELDNUM-1
           IF (I5.EQ.1) GOTO 35
           FSLP(I5)=ABS((FHWD(I5+1)-FHWD(I5))/LOG10(FT(I5+1)/FT(I5)))
           TSLP(I5)=ABS((HWD(I5+1)-HWD(I5))/LOG10(TDC(I5+1)/TDC(I5)))
           ERROR=ABS((FSLP(I5)-TSLP(I5))/FSLP(I5)*100)
           TERROR(I1)=TERROR(I1)+ERROR

35       CONTINUE

10       CONTINUE

         CLOSE(82)

************************************************************************
*              Insertion Sorting of the Degree of Best-Match of       *
*                  the Set of Type Curves under Investigation         *
************************************************************************

          SEP=FLNUM/2

39        INT=0

          DO 40 I4=1,FLNUM-SEP
             IF(TERROR(I4).GT.TERROR(I4+SEP)) THEN
               INT=1
               AUX=TERROR(I4)
               AUXFL=TFL(I4)
               TERROR(I4)=TERROR(I4+SEP)
```

```
              TFL(I4)=TFL(I4+SEP)
              TERROR(I4+SEP)=AUX
              TFL(I4+SEP)=AUXFL
            ENDIF
40        CONTINUE

          IF(INT.EQ.1) GOTO 39
          SEP=SEP/2
          IF(SEP.GE.1) GOTO 39
```

```
*************************************************************************
*                Determination of the Hydraulic Conductivity            *
*************************************************************************
```

```
          DO 3333 IK=1,FLNUM

          OPEN(88,FILE=TFL(IK))

          KRAVG=0.

          DO 90 I3=1,FIELDNUM
99             READ(88,*) T,H
               IF (H.GT.FHWD(I3)) THEN
                 TPREV=T
                 HPREV=H
                 GOTO 99
               ENDIF
               HWD(I3)=FHWD(I3)
               TDC(I3)=TPREV*((T/TPREV)**((FHWD(I3)-HPREV)/(H-HPREV)))
90        CONTINUE

          DO 50 I4=1,FIELDNUM
               KR(I4)=(RC**2)*(TDC(I4))/(D*(FT(I4)))
               KRAVG=KRAVG+KR(I4)
50        CONTINUE

          KRAVG=KRAVG/FIELDNUM

          WRITE(*,*) TFL(IK),(KRAVG*86400)

          CLOSE(88)
3333      CONTINUE
```

```
*************************************************************************
*                        Format Declarations                            *
*************************************************************************
```

```
101       FORMAT (4(E10.5,2X))
102       FORMAT (5(F10.5,2X))
105       FORMAT (A12)
```

```
*************************************************************************
*                          Final Operations                             *
*************************************************************************
```

```
          CLOSE(82)
          CLOSE(83)
C         CLOSE(66)
```

```
      END
*                                                                              *
*                        END of the MAIN PROGRAM                               *
********************************************************************************
```

File: FILES.MAT

Status: Input file

Content: The names of the files containing field and type curve data to be matched
The name of the file containing the aquifer setting under investigation

General structure:

<u>1st Line</u>

Name of the file containing the field data

<u>2nd Line</u>

Name of the file containing the type curves

<u>3rd Line</u>

The name of the file containing the aquifer setting under investigation

****

Example File:

OKTAYG.FLD
OG_OUT.OUT
OKTAYG

****

File: Output file name + "_OUT.OUT"

Status: Input file (created by TCG program automatically)

Content: The number and names of the files containing type curve data

General structure:

1st Line

Number of the files to be processed

2nd Line

Name of the first file to be processed

3rd Line

Name of the second file to be processed

.
.
.
.

File: Main Input File of TCG program

Status: Shared input file

Content: Aquifer parameters and test geometry

General structure:

As described on page 139

****

File: Main Output File of TCG program

Status: Shared input file

Content: Dimensionless time and well head generated by TCG Code

General structure:

As described on page 140

File: File name given by the user

Status: Input file

Content: Field slug test data

General structure:

1st Line

Number of the data points to be read

Subsequent Lines

Time in seconds, dimensionless well head

****

Example File:

```
10
5 9.450000E-001
10 9.050000E-001
20 8.250000E-001
30 7.550000E-001
43.94 6.600000E-001
60 5.800000E-001
90 4.450000E-001
120.82 3.430000E-001
177.83 2.000000E-001
300 7.000000E-002
```

****

APPENDIX B

Two papers of different contents were extracted from the present study and one of them was already published, the other one is under revision. Herein, the published one is presented.

GEOENVIRONMENT 2000

VOLUME 1

Characterization, Containment, Remediation, and Performance in Environmental Geotechnics

Proceedings of a Specialty Conference sponsored by the Geotechnical Engineering Division and the Environmental Engineering Division of the American Society of Civil Engineers

Co-Sponsored by the
American Institute of Professional Geologists
Association of Engineering Geologists
Association of Ground Water Scientists & Engineers
Association of Engineering Firms Practicing in Geosciences (ASFE)
National Science Foundation - Geomechanical, Geotechnical & Geo-Environmental Systems

New Orleans, Louisiana
February 24-26, 1995

Edited by Yalcin B. Acar and David E. Daniel

Geotechnical Special Publication No. 46

Published by the
American Society of Civil Engineers
345 East 47th Street
New York, New York 10017-2398

CONTENTS

1. Characterization/Speciation

VARIABLE HEAD TEST ANALYSIS FOR PARTIALLY PENETRATING WELLS

Cem B. Avcı[1], Erol Güler[2], Member, ASCE, Burak Kılanç[3]

ABSTRACT

The search for improved analysis procedures of in situ permeability tests continues to be an area of interest for researchers due to the need for accurate data in ground water remediation projects. The present study focuses on the analysis of variable head test (slug test) data obtained from partially penetrating wells screened in unconfined and confined aquifers below the water table. Expressions describing the water level variation in the casing are obtained in the Laplace domain and are subsequently inverted numerically to obtain dimensionless type curves. The applicability and the limitations of the existing methods of analysis of the variable head tests performed in confined and unconfined aquifers are examined in view of the presently developed procedures.

INTRODUCTION

The variable head permeability test is one of the in-situ permeability tests which can be performed in monitoring wells to obtain hydrogeological information during the course of groundwater studies. The test presently investigated is performed by creating a relatively fast initial change in the original water level present in the well

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casing and recording the subsequent water level changes (Figure 1); this test is also known as a 'slug' test. The test procedures can be conducted in monitoring wells, piezometers, or standpipes. The hydraulic properties of the porous media are calculated using the water level changes observed in the casing together with the hydrogeologic information near the well screen, well geometry and volume of initial water level displacement. Accurate analysis of the collected data is required especially when the hydraulic property values are planned to be used in ground water contamination transport predictions and risk assessment studies.

Several methods have been developed over the years for the analysis of variable head tests. Hvorslev (1951) developed solutions for wells, piezometers or standpipes with various geometry and penetration (full or partial penetration) for cases of infinite and semi-infinite, confined and unconfined homogeneous anisotropic media. The analysis procedures developed by Cooper et al. (1967) are applicable for variable head tests conducted in confined, radially infinite, homogeneous and isotropic aquifers. The Bouwer and Rice (1976) method can be used to analyze tests performed in fully or partial penetrating wells in unconfined, homogeneous isotropic aquifers. Karasaki et al. (1988) provided solutions for tests occurring under the following conditions: dominant linear flow, radial flow with boundaries, and two layer and concentric composite models with different flow geometries between the inner and outer regions. Continued interest in the development of new techniques for analysis of variable head tests are evident in the work of Peres et al. (1989) and Marschall and Barczewski (1989).

Recent articles by Chirlin (1989), Chapuis (1989) and Avcı (1994) have shown that the use of some of the well known methods of analysis may lead to inaccurate characterization of the hydraulic property values. The assumption of negligible storage coefficient inherent in the development of the Hvorslev (1951) solutions for the fully penetrating well and the nonpenetrating well cases were demonstrated to limit the use of these solutions by Chirlin (1989) and Avcı (1994), respectively. Chapuis (1989) used numerical methods to demonstrate that the shape factors derived by Hvorslev (1951) may cause inaccuracies in the prediction of the hydraulic conductivity. The derivation of the Bouwer and Rice (1976) method was based on neglecting the storage coefficient and the use of curves derived from the results of a resistance network analog; the authors pointed out that the use of these curves could differ up to 25% with the analog results in certain cases. The validity of a purely radial flow occurrence and the screened zone required for the use of Cooper et al (1967) analysis procedure has not been thoroughly investigated.

The present investigation focuses on the development of analysis techniques for the variable head tests performed in partially penetrating wells under confined or unconfined conditions. The method of solution is based on the approach developed by Dougherty and Babu (1984) for the flow from a partially penetrating well in a double porosity reservoir. The analysis is conducted by determining the Laplace domain expressions of the water level changes in the casing and subsequently inverting the expressions using numerical methods to generate dimensionless type curves in the real time domain. The applicability and the limitations of the available methods of analysis for the variable head tests performed in confined and unconfined aquifers are also examined using the proposed approach.

GOVERNING EQUATIONS

The variable head permeability test model that is under analysis is shown in Figure 1. The aquifer is taken to be homogeneous, isotropic and infinite in radial extent; variable head tests are conducted under confined as well as unconfined conditions. The variable D represents either the thickness of the confined aquifer or the water column for the unconfined aquifer. The hydraulic conductivity and specific storage coefficient are defined as K and S_o respectively. The well is taken to have a casing radius r_c and a screen radius r_s; the well screen length L is defined by the variables a and b representing the distance from the top and bottom of screen, respectively to the confining lower boundary. The test is performed by imposing an instantaneous water level change h_o above the initial water level present in the casing and subsequently measuring water levels h_w in the casing as they return to their original levels. The hydraulic head distribution in the aquifer following the instantaneous water level change in the casing is given by h. The hydraulic head losses created by the presence of a 'skin' zone surrounding the well borehole are characterized by a skin factor coefficient S (Sageev, 1986). The cylindrical coordinate system used to define the governing ground water flow equation is shown in Figure 1.

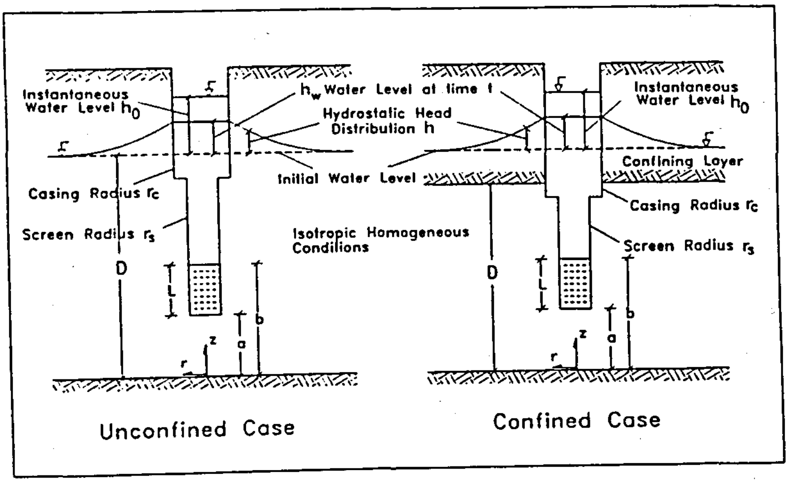

Figure 1. Schematic of the Variable Head Test

The equation governing the hydraulic head distribution is given by:

$$\frac{\partial^2 h}{\partial r^2}+\frac{1}{r}\frac{\partial h}{\partial r}+\frac{\partial^2 h}{\partial z^2}=\frac{S_c}{K}\frac{\partial h}{\partial t} \qquad (1)$$

The following variables are used to nondimensionalize the above governing equation

$$h_d=\frac{h}{h_0}\ ,\ h_{wd}=\frac{h_w}{h_0}\ ,\ r_d=\frac{r}{r_s}\ ,\ z_d=\frac{z}{r_s}\ ,\ L_d=\frac{L}{r_s}\ ,\ D_d=\frac{D}{r_s} \qquad (2)$$

$$a_d=\frac{a}{r_s}\ ,\ b_d=\frac{b}{r_s}\ ,\ \gamma=\frac{L}{D}\ ,\ t_d=\frac{tK}{r_s^2 S_C}\ ,\ S_d=\frac{SK}{r_s}\ ,\ C_d=\frac{r_c^2}{2r_s^2 S_c L}$$

to obtain:

$$\frac{\partial^2 h_d}{\partial r_d^2}+\frac{1}{r_d}\frac{\partial h_d}{\partial r_d}+\frac{\partial^2 h_d}{\partial z_d^2}=\frac{\partial h_d}{\partial t_d} \qquad (3)$$

The dimensionless form of the equation relating the rate of volume change in the casing to the flux through the screened zone is given by:

$$\frac{dh_{wd}}{dt_d}=\frac{1}{C_d}\frac{\partial h_d}{\partial r_d}\Big|_{r_d=1} \qquad (4)$$

The relationship between h_{wd}, the average head in the screen h^*_d and the hydraulic head losses which may occur from the possible presence of a skin zone near the screened zone is given by:

$$h_{wd}=h_d^* - S_d\frac{\partial h_d}{\partial r_d}\Big|_{r_d=1} \qquad (5)$$

where S_d is the dimensionless Skin factor (Sageev, 1986). The expression for the average head in the screen is given by:

$$h_d^* = \frac{1}{L_d}\int_{a_d}^{b_d} h_d\Big|_{r_d=1} dz_d \qquad (6)$$

The flux conditions along the cylindrical area above and below the screen locations are given by:

$$\frac{\partial h_d}{\partial r_d}\Big|_{r_d=1}=0 \quad z_d<a_d \quad and \quad z_d>b_d \tag{7}$$

These conditions imply that no radial flow occurs above and below the screened zone along $r_d = 1$. The conditions for the upper and lower boundaries of the aquifer are:

$$\frac{\partial h_d}{\partial z_d}=0 \quad z_d=0,D_d \tag{8}$$

for a confined aquifer with impermeable boundaries at the top and bottom of the aquifer and

$$h_d=0 \quad z_d=D_d \quad ; \quad \frac{\partial h_d}{\partial z_d}=0 \quad z_d=0 \tag{9}$$

for an unconfined aquifer with an impermeable boundary at the bottom and an initial water table level at $z_d = D_d$ which remains constant during the variable head test. This formulation was used by Bouwer and Rice (1976) and assumes that the fluctuations in the water table caused by the instantaneous water level change in the casing are negligible. The initial conditions are given by:

$$h_d=0 \quad h_{wd}=1 \quad t_d=0 \tag{10}$$

SOLUTION DEVELOPMENT

The analysis procedure for the variable head test was developed by obtaining an expression for the water level variation in the casing h_w in the Laplace domain using the dimensionless governing equation and corresponding boundary conditions; these expressions are subsequently inverted into the real time domain using numerical methods to obtain a set of dimensionless type curves. This method is a common method used to develop analysis procedures for in situ permeability tests (Moench and Ogata, 1984; Karasaki et al., 1988; Dougherty, 1989 and Avcı, 1994). The Laplace transform defined as:

$$\bar{h}_d=\int_0^{\infty} e^{-pt_d}\, h_d\, dt_d \tag{11}$$

is applied to the dimensionless equation and boundary conditions yielding:

$$\frac{\partial^2 \bar{h}_d}{\partial r^2_d}+\frac{1}{r_d}\frac{\partial^2 \bar{h}_d}{\partial r_d}+\frac{\partial^2 \bar{h}_d}{\partial z^2}=p\,\bar{h}_d \tag{12}$$

$$\bar{h}_{wd}=\bar{h}^*_d-S_d\frac{\partial \bar{h}_d}{\partial r_d}\Big|_{r_d=1} \tag{13}$$

$$\frac{\partial \bar{h}_d}{\partial r_d}\Big|_{r_d=1}=0 \quad z_d<a_d \text{ and } z_d>b_d \tag{14}$$

The Laplace domain expressions for the conditions along the upper and lower extent of the aquifer become:

$$\frac{\partial \bar{h}_d}{\partial z_d}=0 \qquad z_d=0,D_d \tag{15}$$

for a confined aquifer and

$$\bar{h}_d=0 \quad z_d=D_d\,; \quad \frac{\partial \bar{h}_d}{\partial z_d}=0 \quad z_d=0 \tag{16}$$

for an unconfined aquifer. The solution to $\bar{h}_{wd}$ was developed for the confined condition by Dougherty and Babu (1984); this research focused primarily on the solution of hydraulics problems in fractured porous reservoirs which included the derivation of dimensionless type curves for variable head tests under confined conditions. The solution procedure for the variable head test performed under unconfined conditions are developed in the following sections. The expression for $\bar{h}_d$ is taken to be an infinite series expansion as follows:

$$\bar{h}_d=\sum_{n=0}^{\infty}\bar{\alpha}_n(r_d,p)\,Cos\left(\frac{(n+\frac{1}{2})\pi z_d}{D_d}\right) \tag{17}$$

which satisfies the unconfined aquifer boundary conditions. Inserting this expression into equation (12) yields :

$$\sum_{n=0}^{\infty}\left[\frac{d\bar{\alpha}_n}{dr_d^2}+\frac{1}{r_d}\frac{d\bar{\alpha}_n}{dr_d}-\left(p+\frac{(n+\frac{1}{2})^2\pi^2}{D_d^2}\right)\bar{\alpha}_n\right]Cos\left(\frac{(n+\frac{1}{2})\pi z_d}{D_d}\right)=0 \tag{18}$$

This equation is satisfied when:

$$\frac{d^2\bar{\alpha}_n}{dr_d^2}+\frac{1}{r_d}\frac{d(\bar{\alpha}_n)}{dr_d}-\left(p+\frac{(n+\frac{1}{2})^2\pi^2}{D_d^2}\right)\bar{\alpha}_n=0 \tag{19}$$

The solution to this ordinary differential equation is given by:

$$\bar{\alpha}_n=\beta_n(p)K_0(g_n r_d) \qquad n=0,1,2,... \tag{20}$$

where

$$g_n=\left(p+\frac{(n+\frac{1}{2})^2\pi^2}{D_d^2}\right)^{1/2} \tag{21}$$

and K_0 is the Modified Bessel function of the second kind of order zero. The expression for the hydraulic head distribution thus becomes:

$$\bar{h}_d=\beta_0 K_0(g_o r_d)+\sum_{n=1}^{\infty}\beta_n K_0(g_n r_d)\,Cos\left(\frac{(n+\frac{1}{2})\pi z_d}{D_d}\right) \tag{22}$$

The coefficients β_n are determined using the flux boundary along the screen and casing conditions to yield the following expression:

$$\bar{h}_d=\gamma c\left[\frac{K_0(g_0 r_d)}{g_0 K_1(g_0)}+\frac{2}{\gamma}\sum_{n=1}^{\infty}\frac{K_0(g_n r_d)}{(n+\frac{1}{2})\pi g_n K_1(g_n)}\left[Sin\left(\frac{(n+\frac{1}{2})\pi b_d}{D_d}\right)-\right.\right.$$

$$\left.\left.\left(-Sin\left(\frac{(n+\frac{1}{2})\pi a_d}{D_d}\right)Cos\left(\frac{z_d\,(n+\frac{1}{2})\pi}{D_d}\right)\right)\right]\right] \tag{23}$$

where c is the expression for the time dependent flux through the screen and K_1 is the Modified Bessel function of the second kind of order 1. The expression for the average hydraulic head along the screened zone, the relationship between the water level measured in the casing and the flux conditions along the screen and the average hydraulic head at the screened zone are used to determine the following expression for the water level measured in the casing:

$$\bar{h}_{wd} = \frac{C_d(A + S_d)}{1 + pC_d(A + S_d)} \tag{24}$$

where

$$A = \frac{2}{\gamma}\sum_{n=0}^{\infty} \frac{K_0(g_n)}{(n+\frac{1}{2})^2\pi^2 g_n K_1(g_n)}\left[Sin\left(\frac{(n+\frac{1}{2})\pi b_d}{D_d}\right) - Sin\left(\frac{(n+\frac{1}{2})\pi a_d}{D_d}\right)\right]^2 \tag{25}$$

The expression obtained by Dougherty and Babu (1984) for the confined conditions is similar to equation (24) and is given by:

$$\bar{h}_{wd} = \frac{C_d\,[(A_0 + A)\gamma + S_d]}{1 + pC_d[(A_0 + A)\gamma + S_d]} \tag{26}$$

where

$$A_0 = \frac{K_0(g_0)}{g_0 K_1(g_0)} \tag{27}$$

$$A = \frac{2}{\gamma^2}\sum_{n=1}^{\infty} \frac{K_0(g_n)}{n^2\pi^2 g_n K_1(g_n)}\left[Sin\left(\frac{n\pi b_d}{D_d}\right) - Sin\left(\frac{n\pi a_d}{D_d}\right)\right]^2 \tag{28}$$

These expressions are inverted back into the real time domain to obtain dimensionless type curves using the numerical inversion method developed by Stehfest (1970). The variable head test data are often analyzed by matching the measured water levels in the casing with the set of dimensionless type curves to determine the hydraulic conductivity and specific storage coefficient of the aquifer.

APPLICATIONS

Confined Aquifers

The validity of the derivations and the numerical inversion scheme was tested by obtaining the values of the dimensionless type curves for various storage coefficient values for a fully penetrating well and comparing those values with the results predicted by Cooper et al. (1967). Figure 2 indicates the dimensionless type curves for the fully penetrating case obtained by the present method; the predicted values were found to be the same as the Cooper et al. (1967) results.

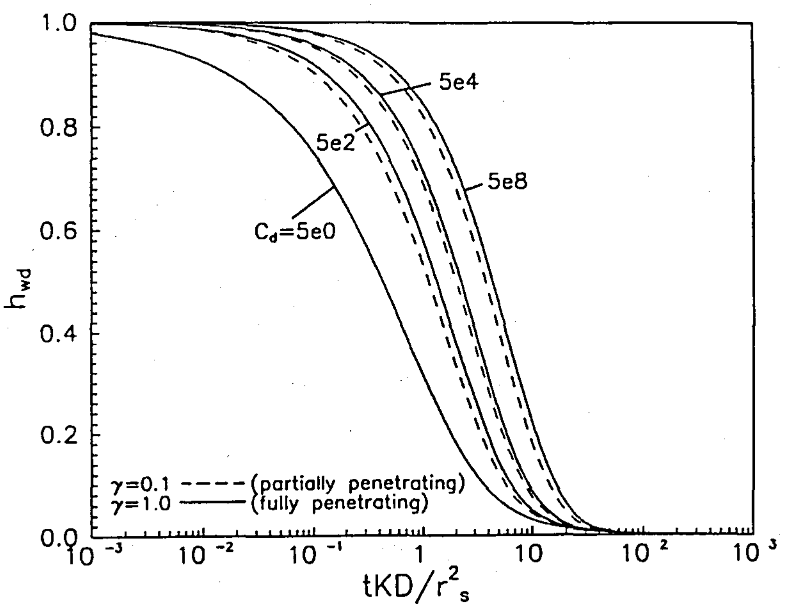

Figure 2. Dimensionless Type Curves and Partial Penetration Effects

The partial penetration effects on the dimensionless type curves were examined by plotting the results on a new dimensionless time scale given by $t_d/2\gamma C_d = tKD/r^2_s$. This allows the new time scale to be independent of the screen length; the results are shown in Figure 2. The shape of the curves are shifted slightly to the left with decreasing ratio of screen length to aquifer thickness. It was also noted during the simulations that the type curves for partially penetrating wells were almost independent of the location of the screen relative to the upper and lower boundaries of the aquifer.

Multilevel variable head test data was collected by Melville et al. (1991) in a 21.2m thick confined, granular aquifer near Mobile, Alabama using a double packer system in a borehole with a 5cm radius; the separation length of the packers was 1.11m. Some of the water level data obtained from Melville et al. (1991) which were collected at various elevations from the bottom of the aquifer along the borehole are shown in Figure 3. The data was analyzed using the present approach by generating a set of dimensionless curves based on the storage coefficient of the aquifer (setting C_d) and the particular location of the packer system (specifying a_d, b_d, S_d and γ).

The hydraulic properties of the aquifer was determined by matching the dimensionless type curves with the data by finding a best fit. The best fit position is obtained between the field data and the type curves by keeping the arithmetic axes coincident. A time value for the field data is equated at a match point with a value of the dimensionless variable tKD / r^2_s. The hydraulic conductivity value is obtained knowing the values of the time variable t, the dimensionless variable, the depth of the aquifer D and screen radius r_s. The data fit for the first test performed at z=1.58m is shown in Figure 4. The results of the analysis are listed in Table 1.

The present method of analysis provides lower values than the calculations obtained by Melville et al. (1991) for Method 1 which was based on the Cooper et al. (1967) method assuming radial flow occurrence. This is expected since the inclusion of the vertical flow dimension in the governing equations will allow flow occurrence in the vertical direction resulting in the reduction of the hydraulic conductivity value. The results obtained in Method 2 was based on the use of a numerical method which took into account the vertical flow occurrence; the results from the numerical method were obtained using an aquifer anisotropy K_r/K_z of 10:1 which forced a greater horizontal flow occurrence in the vicinity of the screen. The authors indicated that despite the conditions which favored horizontal flow occurrence, the vertical components of the seepage influenced the values of the hydraulic conductivity such that the values calculated by Method 2 were about two-thirds of the values calculated by Method 1. The present analysis method which assumes isotropic conditions and vertical flow occurrence provided hydraulic conductivity values that were almost half of those calculated assuming radial flow occurrence during the permeability test.

Table 1. Analysis of the Variable Head Test Data for Confined Conditions

Elevation (m)	Hydraulic Conductivity (m/day) Calculated by		
	Present Method	Melville et al. (1991)	
		Method 1	Method 2
1.58	5.71	10.21	6.78
3.40	35.98	68.00	49.37
5.23	6.35	12.25	7.87
7.05	3.17	5.84	3.57
19.82	11.64	22.61	16.57

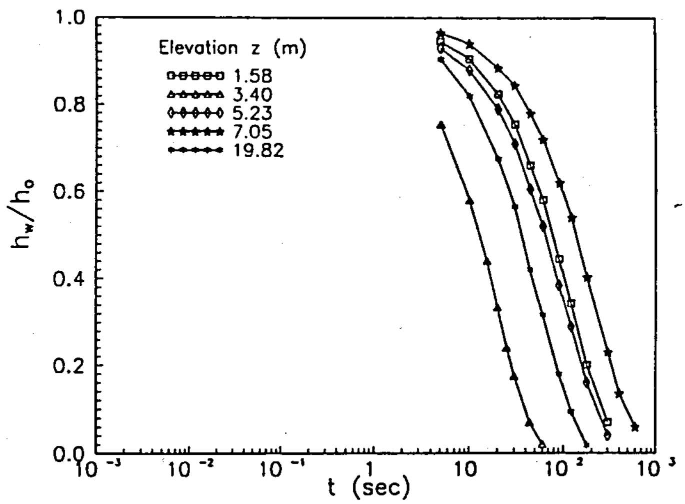

Figure 3. Variable Head Test Data from Melville et al. (1991)

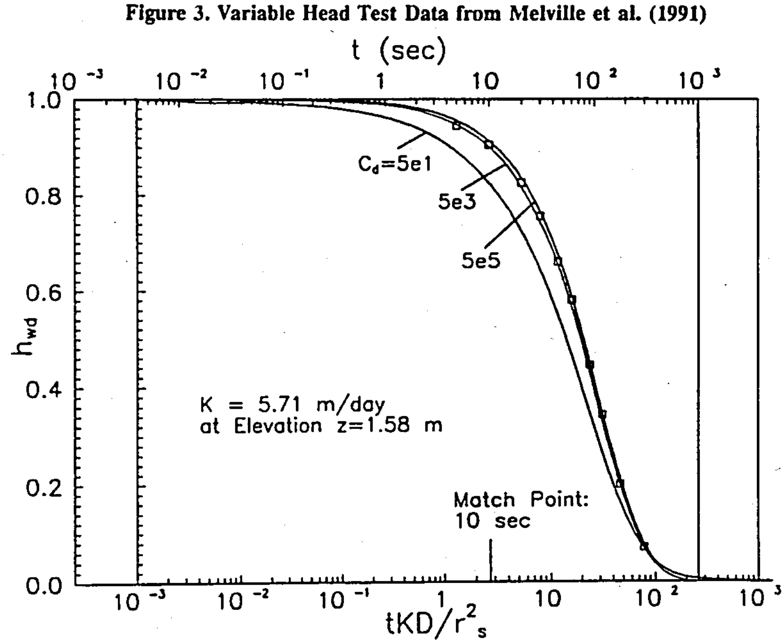

Figure 4. Curve Matching Procedures

The effect of the skin friction on the water level values measured in the casing was investigated for the above case setting for the variable head test conducted at elevation 1.58m. The dimensionless type curves shown in Figure 5 indicate that the skin friction shifts the type curves to the right on the dimensionless time scale. The presence of the skin friction would therefore cause an underestimation of the actual hydraulic conductivity of the formation.

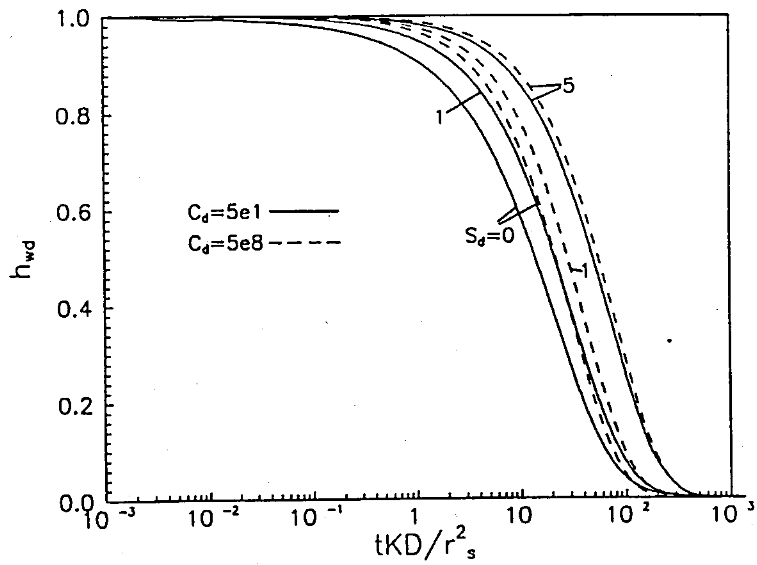

Figure 5. Effects of the Skin Friction for Two Specific Storage Coefficient Conditions

Unconfined Conditions

The variable head tests performed under unconfined conditions were examined assuming that the water table surface did not change during the variable head test performance. Figure 6 shows the effects of the specific storage coefficient on the dimensionless type curves for a fully penetrating well under confined conditions and the effects of partial penetration for the unconfined conditions. The specific storage coefficient did not have any effect on the water level variations for a fully penetrating case due to the the constant water level boundary condition at the phreatic surface.

The variable head test data presented in Bouwer and Rice (1976) was used to compare the predictive ability of the present method. The test was performed in a monitoring well with a 4.56m screen, 0.076m radius located in a 80m thick aquifer; the bottom of the casing was located at 5.5m depth from the water table surface. The analysis of the data is shown in Figure 7.

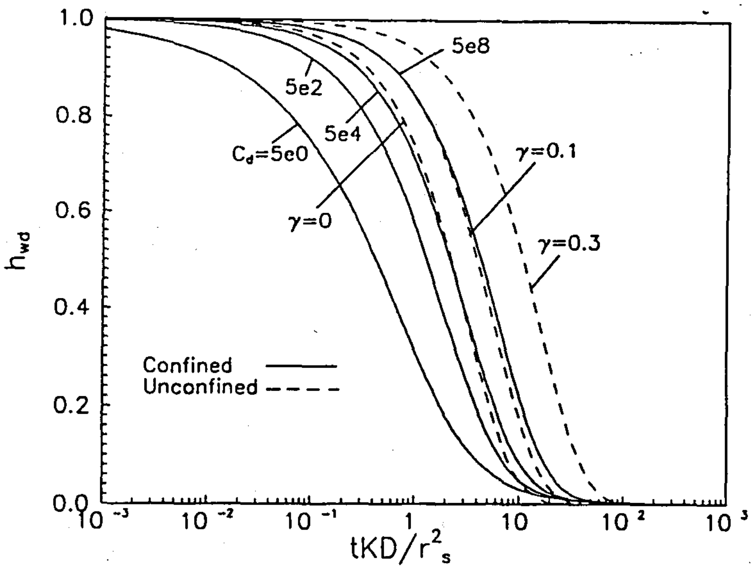

Figure 6. Dimensionless Type Curves for Confined and Unconfined Conditions

A hydraulic conductivity value of $5x10^{-4}$ m/sec was calculated with the present method whereas Bouwer and Rice (1976) predicted a value of $3.6x10^{-4}$ m/sec. Both values should be acceptable since the authors indicated that the values of the hydraulic conductivity were expected to be in the range of $1.16x10^{-4}$ m/sec and $6x10^{-4}$ m/sec based on additional hydrogeologic information.

One advantage of the present method over the Bouwer and Rice (1976) method is the elimination of the uncertainties that are associated with the determination of the coefficients in the Bouwer and Rice (1976) method that were based on the results of a resistance network analog. These coefficients are determined through the use of graphs which could lead up to 25% difference between the results of the analog and the estimated values. The methods developed by Hvorslev (1951) are often used to analyze data obtained under unconfined conditions. The advantage of the present method over the Hvorslev type solutions is a better representation of the flux conditions occurring near the screened zone rather than the use of shape factors that are based on ellipsoid approximations.

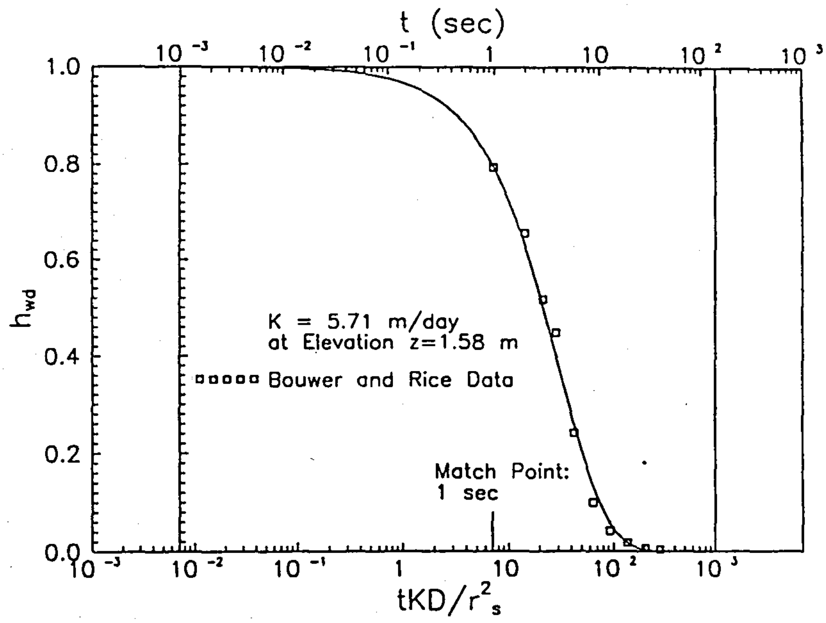

Figure 7. Curve Matching for Variable Head Test Data in an Unconfined Aquifer

CONCLUSIONS

An analytical procedure was developed and reviewed for the variable head tests performed in monitoring wells under unconfined and confined conditions. The solution for the water level variation in the casing was obtained in the Laplace domain and was subsequently inverted into the real domain using numerical methods. Dimensionless type curves were obtained for the analysis of field data; two case studies were examined using the proposed approach.

The approach presented in this paper was noted to have certain features which allow a more accurate analysis of the field data compared to other analytical techniques. The improved accuracy is due to a realistic portrayal of the flow field around the screened zone including the vertical component of the flow.

ACKNOWLEDGEMENTS

This project was supported in part by the Boğaziçi University Research Funding Institution under Project no. 94A00428.

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