R O B E R T   C O L L E G E

S C H O O L   O F   E N G I N E E R I N G

DEPARTMENT OF MECHANICAL ENGINEERING

THESIS FOR M.S. DEGREE

" THEORETICAL ANALYSIS OF SOLID PROPELLANT ROCKETS, PREPARATION OF A SOLİD PROPELLANT, STATIC TESTING OF THE SOLID PROPELLANT IN A TEST ROCKET ON A TEST STAND, DETERMINATION OF DESIGN PARAMETERS, AND DESIGN OF AN 80 POUND THRUST ROCKET "

I N S T R U C T O R

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P R E P A R E D   B Y

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	Lab. Design Thesis		

Contents :

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Nov. I4 , 1962

Professor Necdet Eraslan
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Dear Sır ,

Upon your recommendation , I wish to take as my theisis subject ; " The Theoretical Analysis of'Solid Propellant Rockets' and the design of an 80 lbs. Thrust Rocket."

The theisis will consist of a theoretical survey of the working principle of the solid propellant rockets , the development of a design procedure , the building of an 80 lbs. thrust rocket and its static testing on a test-stand. In this rocket CORDİTE type propellant or another readily castable propellant will be used. The different burning types will be discussed, the basic principle of the grain design will be explained.

The function of the test stand will be ; the determination of the thrust , the specific impulse and the recording of the pressure variation in the rocket. For this purpose strain gages , the BRUSH recorder , and engine indicator will be used. The drum of the latter will be rotated by a record playing electric motor.

The stability of the rocket to be launched will be checked both theoretically and experimentally.

Approved November 15, 1962
N. Eraslan

Respectfully Yours ,

Nurhan Harput

Nurhan Harput

ROCKETS IN GENERAL

Momentum Principle :

The thrust force of a rocket is the reaction experienced by its structure due to the ejection of high velocity matter.

The motion of a vehicle through a fluid medium relies on the forces imparted to it by change of momentum.

In rocket propulsion small gas masses are thrown back , or carried within the vehicle and ejected at very high velocities. The force acting on the rocket can be determined by the momentum principle of fluid mechanics.

$$\text{Thrust on Rocket} = \text{External Force} = \dot{m}\, v_2 + (P_2 - p_3)\, A_2 \qquad \text{(I)}$$
$$= (\dot{w}/g)\, v_2 + (p_2 - p_3)\, A_2$$

The first part of the equation represents Momentum Thrust , and the second part represents Pressure Thrust.

Where ;

$\dot{m}$ = mass flow rate. $\dot{w}$ = weight flow rate.

v_2 = exhaust velocity of gases. A_2 = exhaust jet area.

p_2 = exhaust pressure. p_3 = air pressure.

Proof of equation (I) :

Resultant of External Forces = Rate of change of Momentum

$$\sum F = dm/dt\ (v_2 - v_I)$$

$dm/dt = \dot{m} = -ve$, and $v_I = 0$, therefore $F_{thrust} = \dot{m} v_2$ and

$F_{pressure} = (p_2 - p_3)\, A_2$,

therefore;

$$F_{thrust} + F_{pressure} = F = \dot{m}\, v_2 + (p_2 - p_3)\, A_2$$

Now , if the exhaust pressure is less than the surrounding air pressure , the pressure thrust is negative. Since this condition gives low thrust and is undesirable, the rocket exhaust nozzle is so designed that the exhaust pressure is equal to or slightly higher than the air pressure.

When the air pressure is equal to exhaust pressure, the pressure thrust is zero, and thrust is expressed as :

$$F = (\dot{W}/g)\ v_2 \qquad (2)$$

This condition gives a maximum thrust for a given propellant and a chamber pressure.

The rocket nozzle design , which permits the expansion of the propellant products to the pressure that is exactly equal to the pressure of the surrounding air, is the rocket nozzle with <u>Optimum Expansion Ratio</u>.

Thrust varies with altitude , since atmospheric decreases with altitude. Therefore thrust increases at higher altitudes.

$$\text{Effective Exhaust Velocity} = c = F/(\dot{W}/g) = v_2 + (p_2 - p_3)\, A_2 g/\dot{W} \qquad (3)$$

This equation is derived from equation (I) and when $p_2 = p_3$ then the effective exhaust velocity is equal to the exhaust velocity of the propellant gases , v_2.

E F F I C I E N C I E S

Combustion Efficiency :

It is defined as ; the Ratio of the actual and the ideal heat of reactionper unit of propellant

Internal Efficiency :

It is defined as ; the Ratio of kinetic energy of the exhaust jet and the chemical energy of the propellants.

$$E_{int.} = \frac{I/2\ (\dot{W}/g)\ c^2}{\dot{W}\ Q_R\ J} = \frac{c^2}{2\ g\ J\ Q_R} \qquad (4)$$

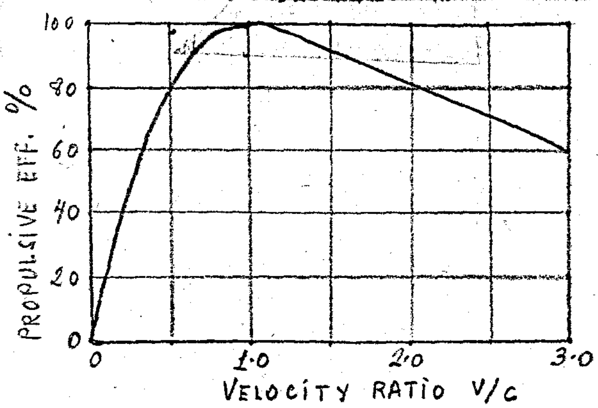
where ;

Q_R = Heat of reaction per unit of propellant at chamber conditions.

Propulsive Efficiency :

It determines how much of the kinetic energy of the exhaust jet is useful for propulsion.

$$E_{prop.} = \frac{\text{Vehicle Energy}}{\text{Vehicle Energy + Residual Kinetic jet Energy}}$$



PROPULSIVE EFFICIENCY AT VARYING VELOCITIES.

$$E_{prop.} = \frac{F\ v}{F\ v + I/2\ (\dot{W}/g)(c-v)^2} = \frac{2\ v/c}{I + (v/c)^2} \quad (5)$$

where ;

v = absolute vehicle velocity.

For maximum propulsive efficiency , the forward rocket velocity should be equal to exhaust velocity.

Total Efficiency :

It is defined as , the Ratio of vehicle energy to the chemical energy augmented by kinetic energy of the propellants.

$$(6) \quad E_{Total} = \frac{F\ v}{\dot{W}Q_R J + I/2\ \dot{W}/g\ v^2} = \frac{c\ v}{gQ_R J + v^2/2} = \frac{2\ v/c}{I/E_{in} + (v/c)^2}$$

DEFINITIONS

Specific Impulse : (specific thrust)

It is , the thrust that can be obtained from an equivalent rocket which has a propellant weight flow rate of unity.

$$I_s = \frac{F}{\dot{W}} = \frac{c}{g} \quad (7)$$

where ;

I_s = Specific impulse in pounds of thrust per pound per second of propellant flow.

F = Thrust in pounds.

$\dot{W}$ = weight flow rate in pounds per second.

Impulse : (Total Impulse)

$$I_t = \int_0^t F\ dt = Is\ \dot{W}\ dt \quad (8)$$

where ;

I_t = Total Impulse in pounds per second.

t = Duration in seconds.

For constant Thrust : $I_t = F\ t = I_s\ W$ (9)

where ;,

W = Total weight of effective propellant, in pounds.

Specific Propellant Consumption :

It is defined as ; the required propellant flow to produce one pound of thrust in an equivalent rocket.

$$w_s = I/I_s = \dot{W}/F \text{ , Pounds/pounds.second} \qquad (I0)$$

Specific fuel or Propellant consumption :

It is defined as ; pounds of fuel or propellant per unit time per pound of thrust.

NOZZLE THEORY

Ideal Rocket :

The following assumptions have been made in addition to the preliminary assumption of ; one dimensional flow through ideal rockets.

I - The working substance (propellant products) is homogeneous in composition throughout the rocket chamber.

2 - Working substance obeys perfect gas laws.

3 - No friction.

4 - No heat transfer across rocket wall, therefore flow is adiabatic . Flow is assumed to be isentropic through the nozzle which is politropic actually.

5 - Propellant flow is steady and constant.

6 - All exhaust gases leaving rocket nozzle have an axially directed velocity.

7 - Gas velocity is uniform across any section normal to nozzle axis.

8 - Chemical equilibrium is established within rocket chamber and does not shift in the nozzle.

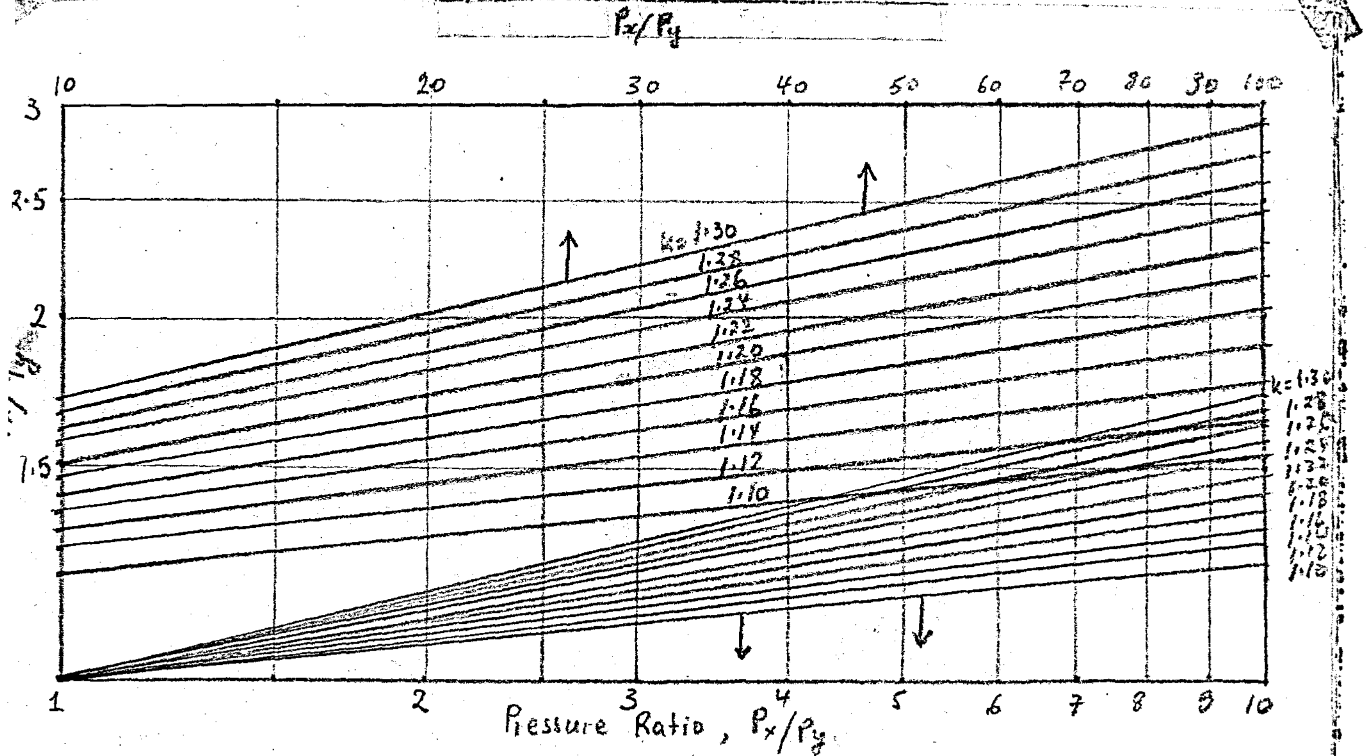
Isentropic Flow through nozzles : (Ideal Nozzles)

The derivation of the equations that are valid for this kind of flow will not be shown here , since they are derived by Proffesor N. Eraslan during his Gas Dynamics lectures ,already.

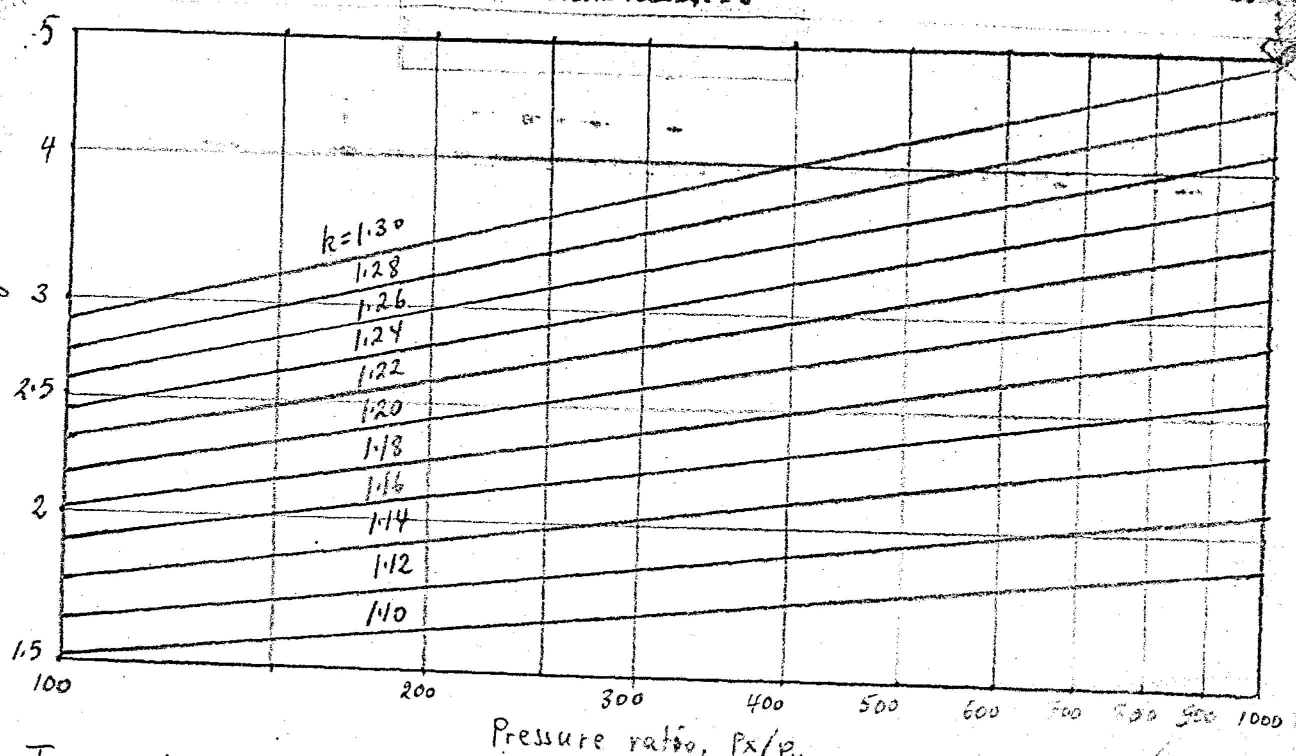
The subscripts I and 2 denote nozzle inlet and exit conditions.

$$\text{Nozzle Exhaust Velocity} = v_2 = \sqrt{\frac{2\,g\,k}{k-I} R\,T_I \left(\left(I - (p_2/p_I)^{\frac{k-I}{k}}\right)\right) + v_I^2} \qquad (II)$$

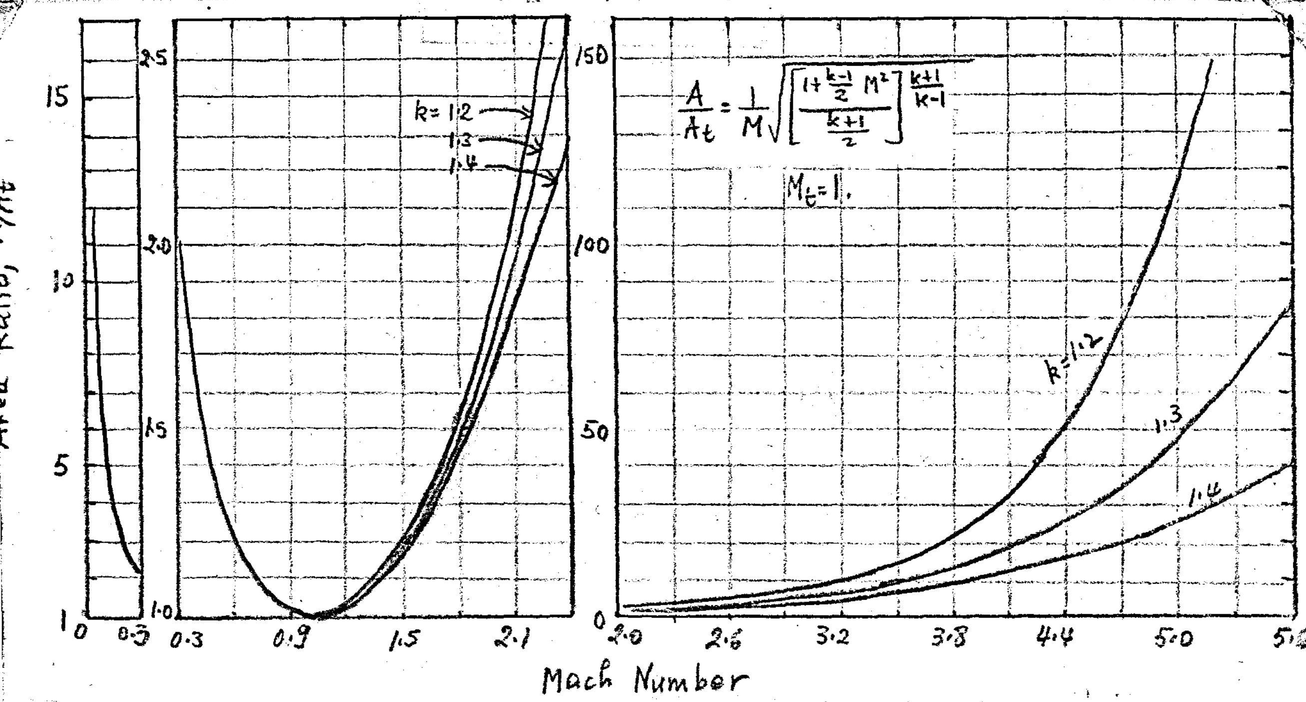
This equation also holds for any two points within the nozzle.



Temperature Ratio versus pressure-ratio for isentropic process.



Temperature ratio versus pressure ratio for isentropic process.



Relation of Area Ratio to Mach Number for Isentropic flow in De Laval Nozzle.

If chamber cross section is large compared to nozzle section then v_I is negligible. Chamber temperature T_I is equal to nozzle inlet temperature, and for an isentropic nozzle flow process, it also is equal to stagnation temperature. Then ;

$$v_2 = \sqrt{\frac{2\,g\,k}{k - I}\,R\,T_o\left(\left(I - (p_2/p_0)^{\frac{k-I}{k}}\right)\right)} = \sqrt{\frac{2\,g\,k\,R'\,T_o\,E}{(k-I)\,M}} \qquad (I2)$$

where ;

$E = I - (p_2/p_I)^{\frac{k-I}{k}}$ (I3) which is Ideal Cycle Efficiency of constant pressure engine cycle operating between p_2 and p_I.

When p_I/p_2 approaches infinity then v_2 becomes maximum, i.e.

$$v_2(\text{maximum}) = \sqrt{\frac{2\,g\,k}{k - I}\,R\,T_o} \qquad (I4)$$

Very high gas velocities can be obtained in rocket nozzles. There is appreciable temperature drop of combustion gases through a rocket nozzle. Increase of kinetic energy of gases is derived from a decrease in enthalpy which is roughly proportional to decrease in temperature.

Nozzle Area Expansion Ratio is defined as :

$$\varepsilon = \frac{A_2}{A_t} = \frac{\text{Nozzle exit area}}{\text{Throat area}} \qquad (I5)$$

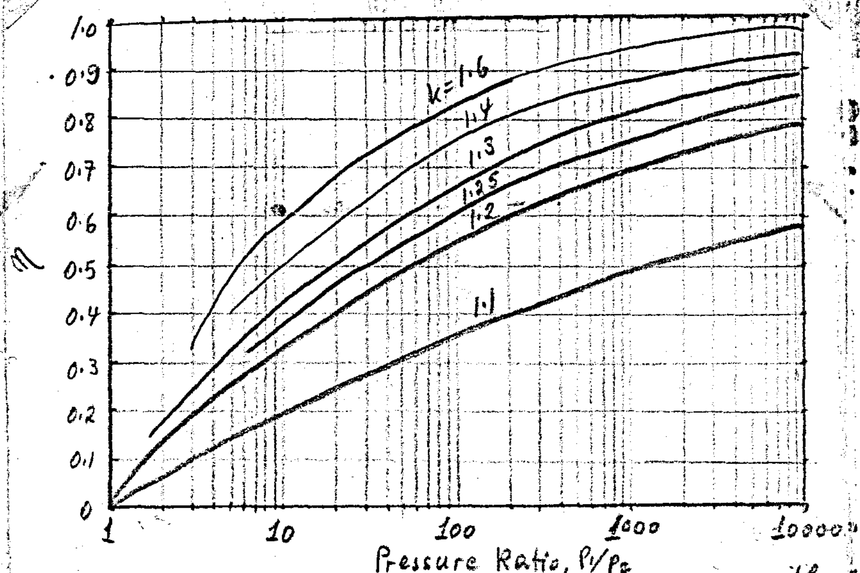
For any Isentropic flow process , such as that occurs in Rocket nozzles , weight flow is given by the following equation , which is computed from equations of continuity , isentropic flow relations and equation (I2) ; where equation (I2) is derived from conservation of Energy principle :

$$\text{Weight Flow} = \dot{W} = \frac{A_x\,p_I}{R}\,2\,g\,J\left(\left(\left(\frac{C_p}{T_I}\left(\left((p_x/p_I)^{\frac{2}{k}} - (p_x/p_I)^{\frac{k+I}{k}}\right)\right)\right)\right)\right)^{\frac{I}{2}} \qquad (I6)$$

between any section x and the nozzle inlet section I.

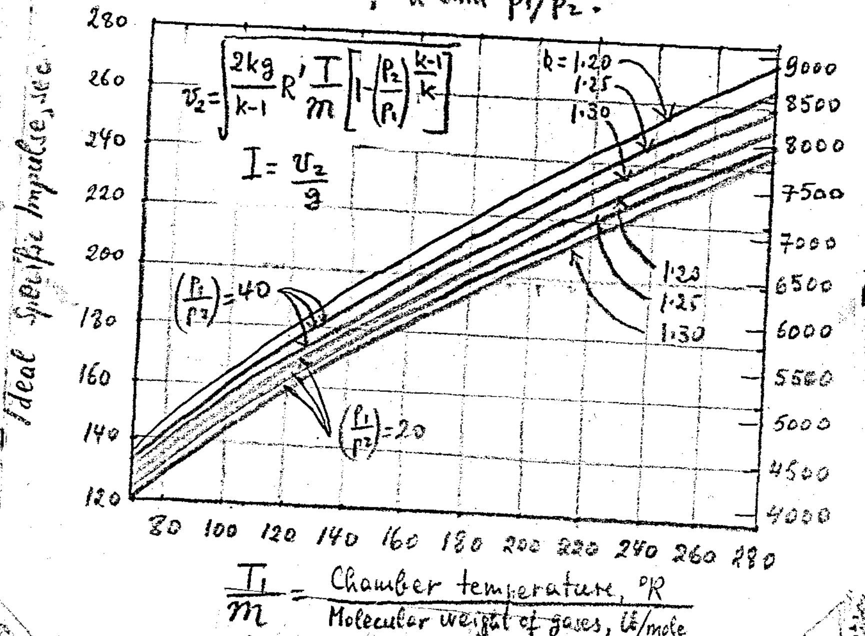
Maximum gas flow per unit area occurs at throat and a unique gas pressure corresponds to this maximum flow.

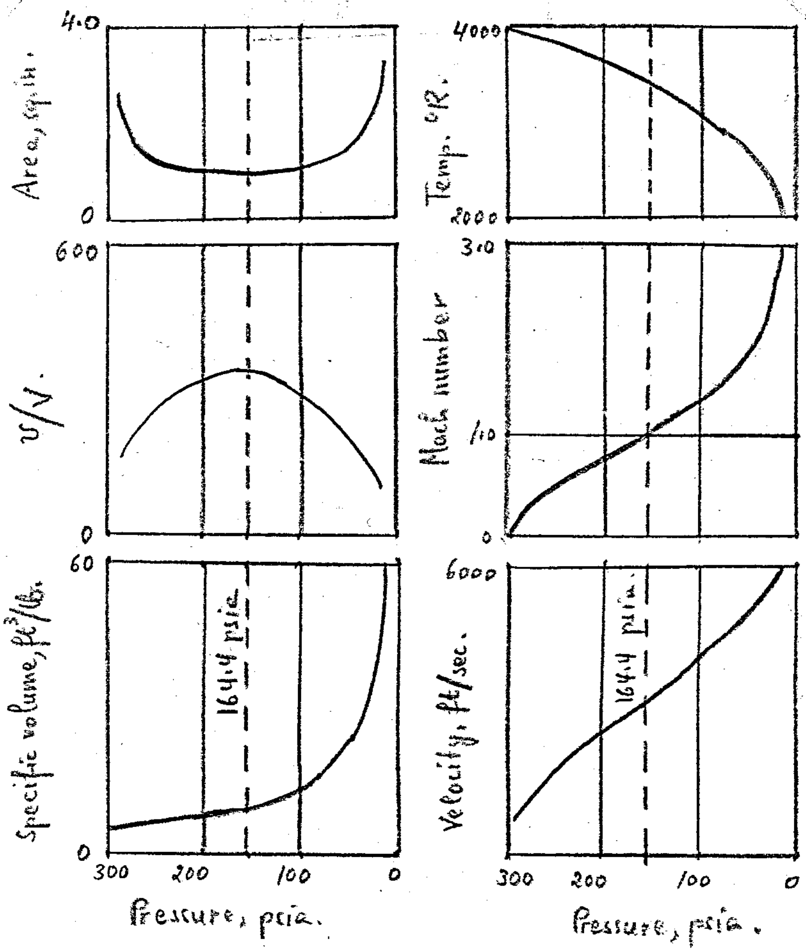
Throat pressure for which isentropic weight flow is maximum is called critical pressure.



Change of ideal constant pressure cycle efficiency η with pressure ratio P_1/P_2.

Specific Impulse and exhaust velocity as functions of the chamber temperature and the molecular weight for various values of k and P_1/P_2.





Typical variation of velocity, area, temperature, specific volume, and Mach Number with pressure in a rocket nozzle.

$$p_t/p_I = (2/k+I)^{k/k-I} \qquad (I7)$$

All rocket thrust chambers have sufficient combustion chamber pressure to attain critical pressure at throat.

At the critical point :

$$V_t = V_I \, (k+I/2)^{I/k-I} \qquad (I8)$$

$$T_t = T_I \, (2/k+I) \qquad (I9)$$

$$v_t = (g \; k \; R \; T_t)^{I/2} = \left(\frac{2\,g\,k}{k+I} \, R \; T_I \right)^{I/2} \qquad (20)$$

In nozzles in which critical conditions prevail :

I - Throat velocity is local acoustic velocity.

2 - M=I at the throat.

Divergent portion of a nozzle permits a further decrease in pressure and increase in velocity above acoustic velocity.

Sonic or supersonic conditions can be attained , if critical pressure prevails at throat, i,e. if p_2/p_I =or smaller than the value given by equation (I7).

In rocket design we have supersonic nozzles , where :

$v_t = a_t$, v_2 greater than v_t , p_2/p_I greater than $(2/k+I)^{k/k-I}$

Using these relationships and the continuity equation we can find the following relation for a flow through critical section of a supersonic nozzle :

$$W = \frac{A_t \; v_t}{V_t} = A_t \; p_I \; g \; \frac{\sqrt{k\,((2/k+I))^{k+I/k-I}}}{\sqrt{g \; k \; R \; T_I}} \qquad (2I)$$

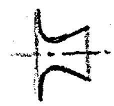
and from Perfect gas law;

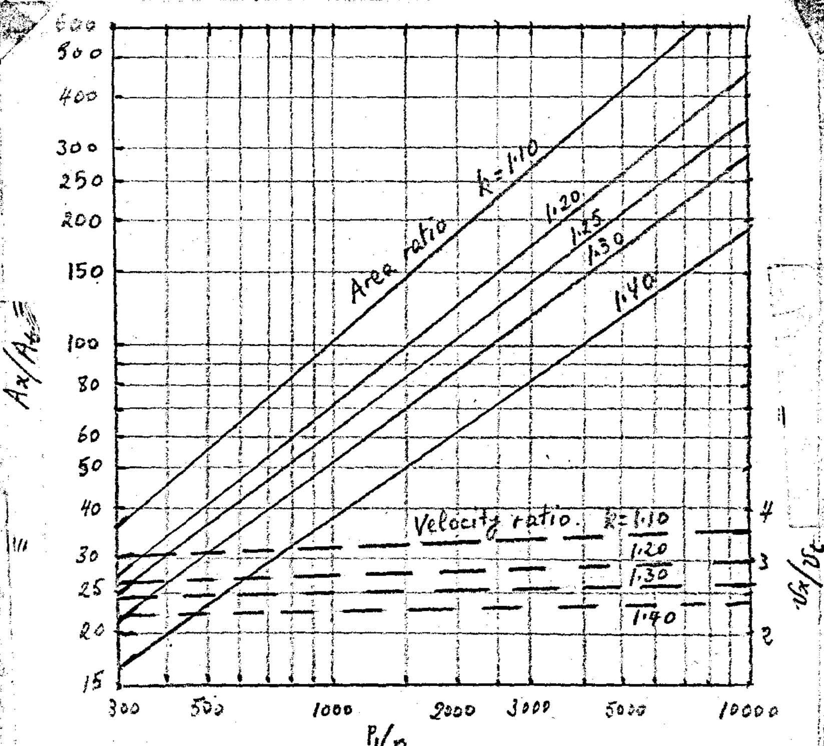
$$\frac{A_t}{A_x} = V_t/V_x \times v_x/v_t = \left(\frac{k+I}{2}\right)^{\frac{I}{k-I}} (p_x/p_I)^{\frac{I}{k}} \sqrt{\frac{k+I}{k-I}\left(\left(I-(p_x/p_I)^{k-I/k}\right)\right)}$$

where x is downstream subscript. (22)

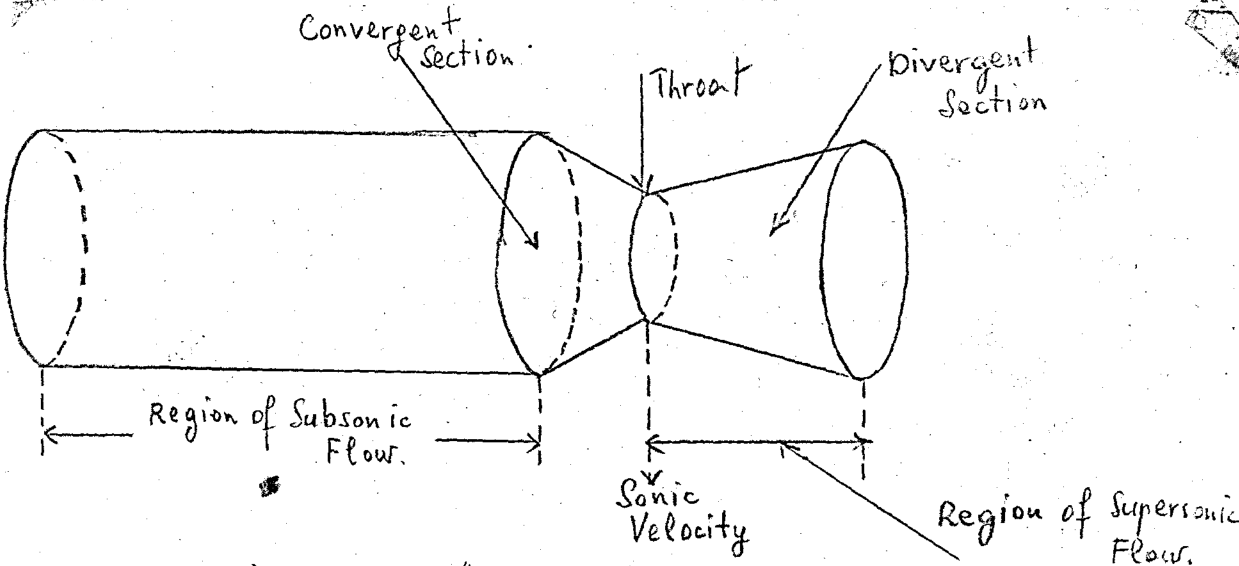
when $p_x = p_2$ then $A_x/A_t = A_2/A_t = \varepsilon$ from equation (I5).

Nozzle Types

Throat Velocity:	$v_t < a_t$	$v_t = a_t$	$v_t = a_t$
Exit Velocity:	$v_2 < a_2$	$v_2 = v_t$	$v_2 > v_t$
Shape.			
Pressure Ratio.:	$\frac{p_1}{p_2} < \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}}$	$\frac{p_1}{p_2} = \frac{p_1}{p_t} = \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}}$	$\frac{p_1}{p_2} > \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}}$
	SUBSONIC	SONIC	SUPERSONIC



Area and velocity ratios as functions of pressure ratio for the diverging section of a supersonic nozzle.



CONVERGENT-DIVERGENT "DELAVAL" NOZZLE.

Similarly, an expression for the ratio of the velocity at any point downstream of the throat with pressure p_x and the throat velocity may be written from equations (I2) and (20) :

$$\frac{v_x}{v_t} = \sqrt{\frac{k+I}{k-I}\left(\left(I-(p_x/p_1)^{\frac{k-I}{k}}\right)\right)} = \sqrt{\frac{k+I}{k-I}E} \qquad (23)$$

When exit pressure coincides with atmospheric pressure, ($p_x=p_2$), these equations apply for optimum expansion conditions.

The thrust on the rocket unit structure is caused by the action of the pressure of the combustion gases against the rocket chamber, injector and nozzle surfaces.

The axial thrust F can be determined by summing up all the pressures acting on all area elements dA, which are projected into a plane normal to the nozzle axis,

$$F = \int p \, dA \qquad (24)$$

Both external and internal pressures should be considered. Internal pressure in the chamber is high and decreases steadily in the nozzle, while the external pressure is equal to atmospheric pressure and is constant throughout. The force obtained by this summation equals that obtained by momentum principle :

$$F = \frac{v_2 \, W}{g} + (p_2 - p_3)\, A_2 \qquad (I)$$

This equation can be expanded by modifying it and substituting v_2 v_t, and V_t from equations (I2), (I8) and (20) and we obtain the IDEAL THRUST EQUATION.

$$F = \frac{A_t \, v_t \, v_2}{g \, V_t} + (p_2-p_3)A_2 = A_t \, p_I \sqrt{\frac{2k^2}{k-I}\left(\frac{2}{k+I}\right)^{\frac{k+I}{k-I}}\left(\left(I-\left(\frac{p_2}{p_I}\right)^{\frac{k-I}{k}}\right)\right)} + \frac{p_2-p_3}{p_I}\frac{A_2}{A_t} \qquad (25)$$

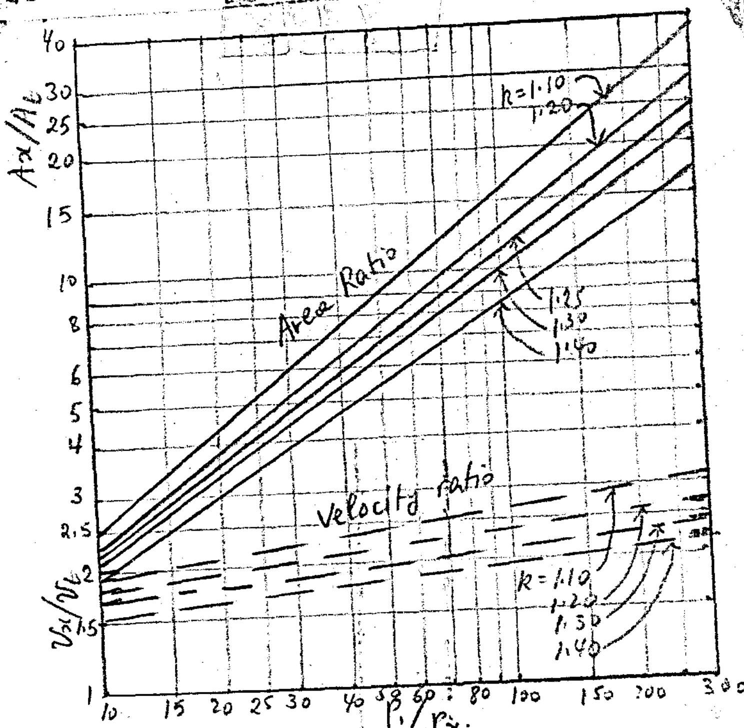
If Thrust coefficient is defined as :

$$C_F = \sqrt{\frac{2k^2}{k-I}\left(\frac{2}{k+I}\right)^{\frac{k+I}{k-I}}\left(\left(I-(p_2/p_I)^{\frac{k-I}{k}}\right)\right)} + \frac{p_2-p_3}{p_I}\frac{A_2}{A_t} \qquad (26)$$

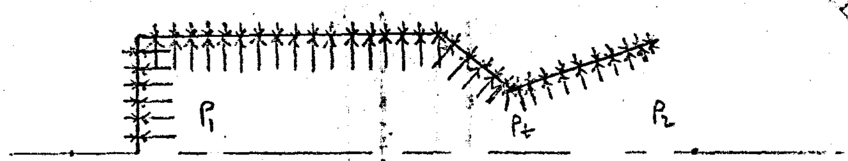
Then :

$$F = C_F \, A_t \, p_I \qquad (27)$$

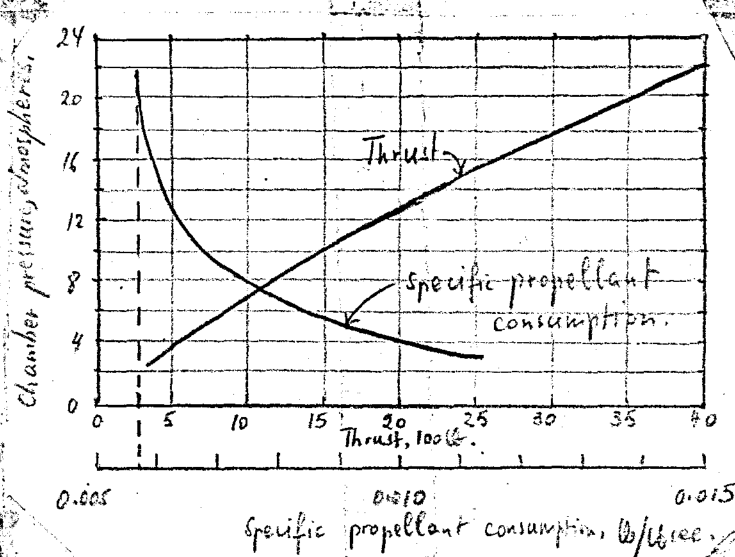
For optimum expansion, when $p_2 = p_3$, last term of C_F is zero.

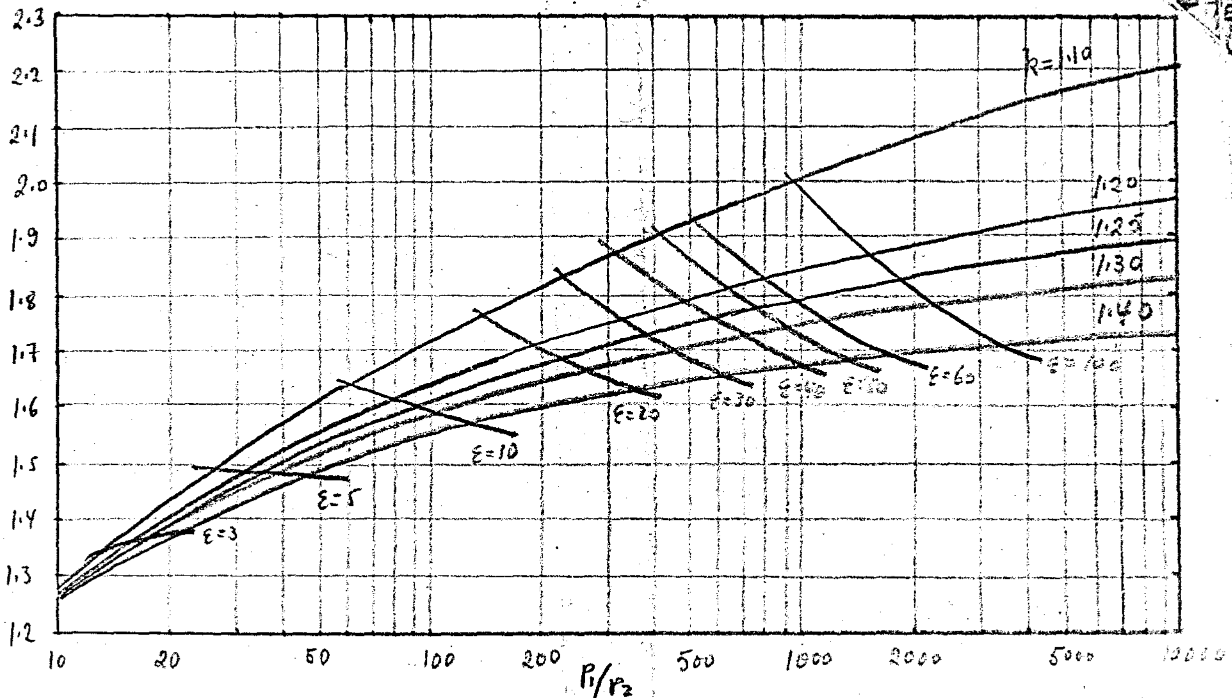


Area and Velocity ratios as functions of pressure ratio for the diverging section of a supersonic nozzle.



Free body diagram of pressure forces acting on rocket chamber and nozzle walls.





Thrust coefficient C_F as a function of pressure ratio, area ratio, and specific heat ratio for optimum expansion conditions.

Underexpanding Nozzle :

It discharges the fluid at a pressure greater than the external pressure , because exit area is too small. Therefore expansion is incomplete and continues outside.

Overexpanding Nozzle :

It is a nozzle in which the fluid is expanded to a lower pressure than the external pressure. It has a too large exit area.

In an overexpanding nozzle , for high external pressures , a separation of jet takes place in the divergent section of the nozzle and oblique shock waves will accompany. Point of separation travels , upstream with increasing external pressure. Flow at nozzle exit is supersonic.

Separation does not alter the direction of axial thrust, since the flow separates uniformly over one cross section of the divergent nozzle cone of conventional rocket designs.

Loss of thrust due to over or under expansion can be determined from C_F. Pressure thrust is positive for underexpansion and negative for overexpansion.

Effect of over or underexpansion is reduction of exhaust velocity and so loss of energy. In overexpansion, since the jet separates from nozzle wall, and nozzle does not flow full, a large portion of the nozzle is not utilized , so the nozzle is longer than necessary , which increases weight and size and decreases the flight performance.

The tendency in rocket design is either an optimum expansion ratio nozzle or slightly underexpanding nozzle.

The Characteristic Exhaust Velocity ; is defined as :

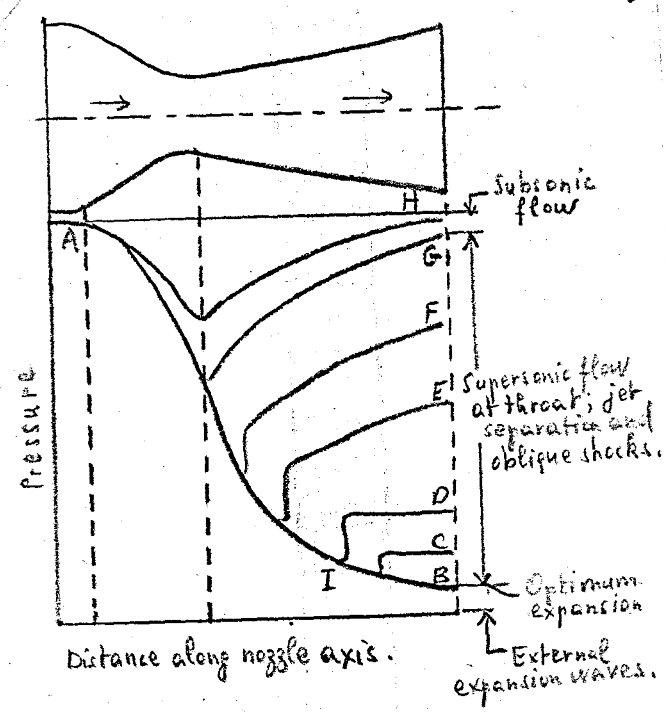
$$c^{*} = \frac{c}{C_F} = \frac{p_I A_t g}{\dot{w}} = \frac{g I_s}{C_F} = \frac{\sqrt{g k R T_I}}{k \sqrt{(2/k+I)^{(k+I)/(k-I)}}} \qquad (28)$$

using equations ; (I) , (25) and (2I).

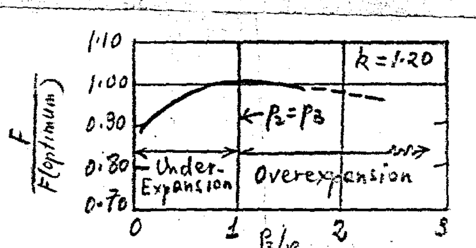
Real Nozzles

Shapes: Almost any symmetrical and well rounded nozzle shape will have very low losses.

A divergent cone half angle between I2 to I8 degrees is optimum Since nozzles with discontinuities in wall contour cause shock waves,



Distribution of pressure in a De Laval nozzle for different flow conditions.



Theoretical reduction of thrust due to under- or overexpansion.

all nozzle sections are well rounded. Nozzle exit section has sharp edge because a rounded edge permits overexpansion and flow seperation.

The Nozzle angle correction factor ; which is used to correct the non-axial component of gas velocity is defined as :

$$\lambda = I/2\ (\ I + \cos\alpha) \qquad (29)$$

where ,

α = Nozzle cone divergence half angle.

The flow in a real nozzle differs from that of ideal nozzle because of friction effects,heat transfer,imperfect gases,non-axial flow and non-uniformity of working substance and flow distribution. The degree of departure is indicated by the Energy Conversion Efficiency of a nozzle, which is the ratio of the Kinetic Energy per unit of flow of the jet leaving the nozzle to the Kinetic Energy per unit of flow of an ideal leaving an ideal nozzle, that is supplied with same working substance at same initial state and velocity and expands to the same exit pressure as real nozzle. This relationship is expressed as :

$$e = \frac{(v_2)_a^2}{(v_2)_i^2} = \frac{(v_2)_a^2}{(v_I)_a^2 + c_p(T_I - T_2)} \qquad (30)$$

where ,

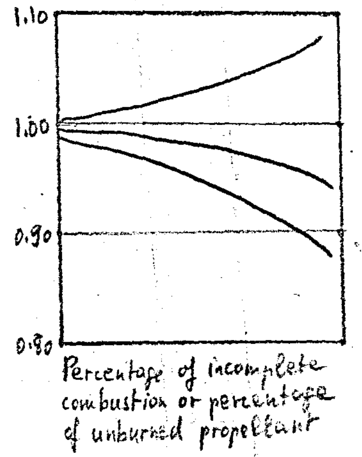
e denotes the energy conversion efficiency , v_I and v_2 the velocities at the nozzle inlet and exit, and c_pT_I and c_pT_2 the respective enthalpies for an ideal isentropic expansion.

The velocity correction factor ; is defined as the square root of the energy conversion efficiency. It is also approximately equal to the ratio of the actual specific impulse to the ideal specific impulse.

The discharge correction factor ; is defined as the ratio o f the mass flow rate in a real nozzle to that of an ideal nozzle which expands an ideal working fluid from the same initial conditions to the same exit pressure.

$$\zeta_d = \dot{W}_a/\dot{W}_i = \frac{\dot{W}_a\, c}{F\, g} = \frac{\dot{W}_a\sqrt{g\, k\, R\, T_I}}{A_t p_1 g\, k\sqrt{(2/k+I)^{(k+I)/(k-I)}}} \qquad (3I)$$

using equations (7) and (2I) .



Thrust, discharge, and velocity correction factors as functions of the completeness of combustion

Usually actual flow rate is larger than the ideal flow rate , since heat transfer to nozzle walls lowers temperature and increases density and flow rate, also specific heat ratio changes so as to increase discharge correction factor in actual nozzles.

The actual thrust is lower than the ideal thrust and can be calculated by an empirical thrust correction factor :

$$F_a = \zeta_F F_i = \zeta_F C_F p_1 A_t = \zeta_F c \dot{w}/g \qquad (32)$$

where ,

$$\zeta_F = \zeta_v \zeta_d = F_a / F_i \qquad (33)$$

In actual rocket nozzles the gas velocity is lower near the walls than in the middle and is a function of the nozzle shape , injection method in liquid propellants ,

ROCKET PROPELLANT PERFORMANCE CALCULATIONS

Rockets utilize the heat liberated in the combustion of chemical propellants as a source of energy.

A high specific impulse means a saving in weight of propellant carried and a reduction of the necessary tank size, or rocket body size.

The ideal specific impulse and the exhaust velocity of a rocket propellant system can be calculated for optimum expansion from equations (I2) and (7).

The calculation of these equations require the evaluation of flame temperature T_I , mean molecular weight of product gases M , and specific heat ratio of hot gases k.

Since the above mentioned equations are for ideal conditions the values calculated will be 5-I2 % more than the actual values , since only a portion of correction done in previous page are due to combustion inefficiencies.

Quantity of energy available from a given rocket propellant is determined by the chemical nature of the oxidizer and fuel molecules as well as by the chemical nature of the reaction gas products.

The method explained below is for calculation of v and I_s

for any given propellant combination at any given chamber pressure and mixture ratio.

Assumptions and Definitions

Chemical reaction is assumed to take place in an ideal rocket. The product gases are assumed to be in chemical equilibrium at the chamber pressure and no heat is transferred to the walls.There is no friction , and the gas mixture is assumed to be homogeneous in composition. In an ideal rocket , the grain design of solid propellant has no effect on ideal chemical equilibrium of combustion gases,

Heat of Formation : It is the change of enthalpy which results when a compound is formed at standard conditions from its elements isothermally and at constant pressure.

Heat of Reaction : It is the change in enthalpy which occurs when products are formed from reactants at standard conditions,namely at constant reference temperature and pressure.

Combustion reactions are exothermic. Heat of reaction in general terms is :

$$Q_R = \sum((n\ (Q_F)_{products})) - \sum((n\ (Q_F)_{reactants})) \quad (34)$$

Mixture of Gases

In rocket combustion devices the working substance consists of a mixture of several gases, which can analytically be treated as a perfect gas. The specific heat , the molecular weight, and the specific heat ratio of a mixture can be determined from gas analysis and the properties of the individual gases. A propellant product gas mixture is usually defined by its chemical analysis which i s a volumetric or molar analysis.

Volumetric analysis expresses the amount of a particular gas as a percentage of the total gas mixture volume.

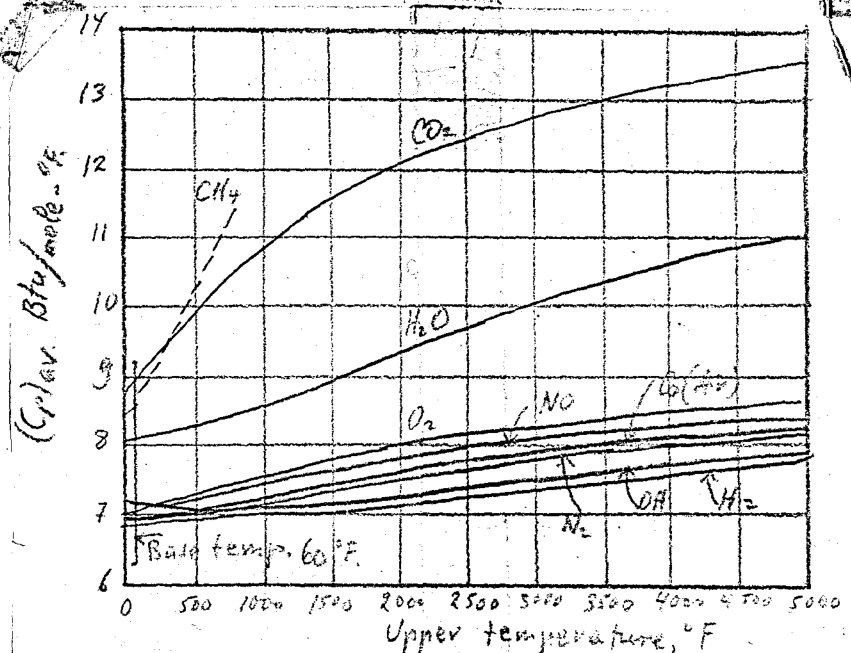
Molar analysis gives the fraction of moles of each gas for one mole of mixture.

$$\text{Molecular weight of a gas mixture} = M = \sum(X\ m_x) \quad (35)$$

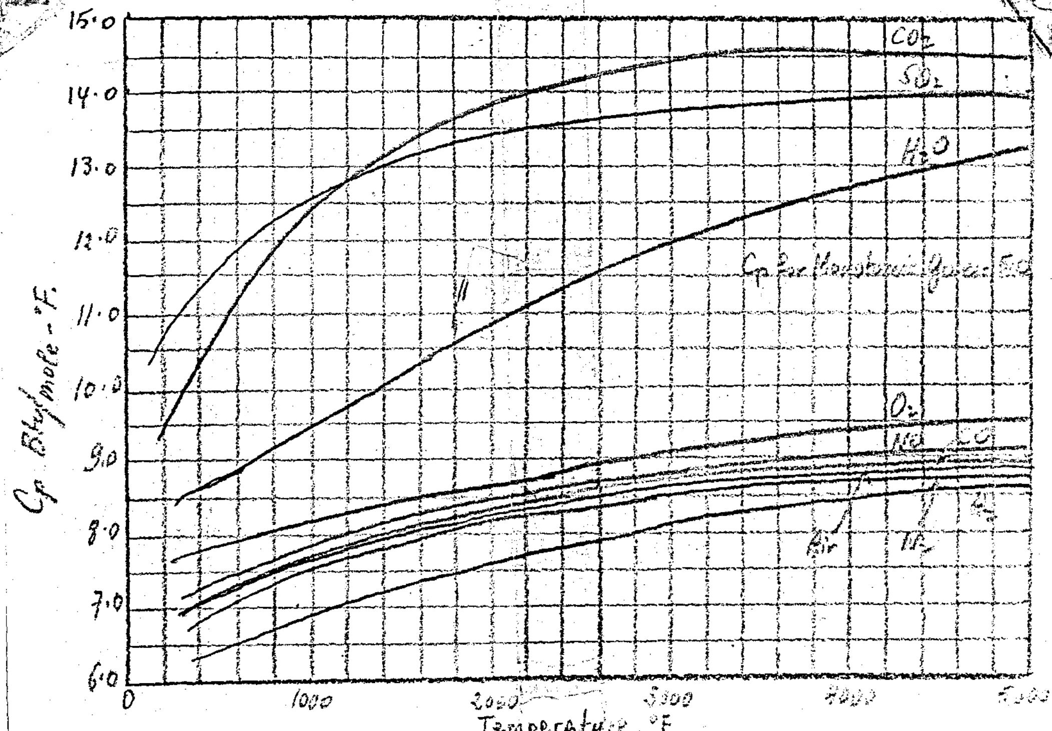
$$\text{Gas constant} = R = R'/M \quad (36)$$

$$\text{Specific Heat per Mole} = C = \sum(X\ C_x) \quad (37)$$

$$\text{Specific Heat Ratio} = k = C_p/C_v = C_p/\ C_p - 1.99 \quad (38)$$



Average molar specific heats of gases at constant pressure between 60°F and abscissa temperature.



Molar specific heats of gases at constant pressure versus temp.

Chemical Equilibrium

A chemical reaction is said to be in equilibrium when the reactants are being transformed into products at the same rate that the products are being reverted into the reactants. This condition of equilibrium is concerned with reversible reactions.

The equilibrium constant for perfect gases depends only on temperature and is defined as :

$$K_p = \frac{p\,A^a\; p\,B^b\; p\,C^c}{p\,Z^z\; p\,Y^y} \qquad (39)$$ when concentration is expressed in pressure units.

$$K_n = \frac{X\,A^a\; X\,B^b}{X\,Z^z\; X\,Y^y} \qquad (40)$$ when concentrations expressed in volume or molar %.

where ,

a,b,c, are number of moles of substances A,B,C, and p_A, p_B, partial pressures, and X_A, X_B, are mole fractions.

The equilibrium of rocket combustion gases is very sensitive to temperature. When the overall temperature is raised, the chemical reaction will tend to shift in such a manner as to absorb heat. If the temperature is very suddenly decreased ,the reaction velocities are so low that very little change in concentration and composition occurs.

The effect of pressure on equilibrium depends on molar proportions. In most rocket reactions combustion temperature slightly increases with increased chamber pressure.

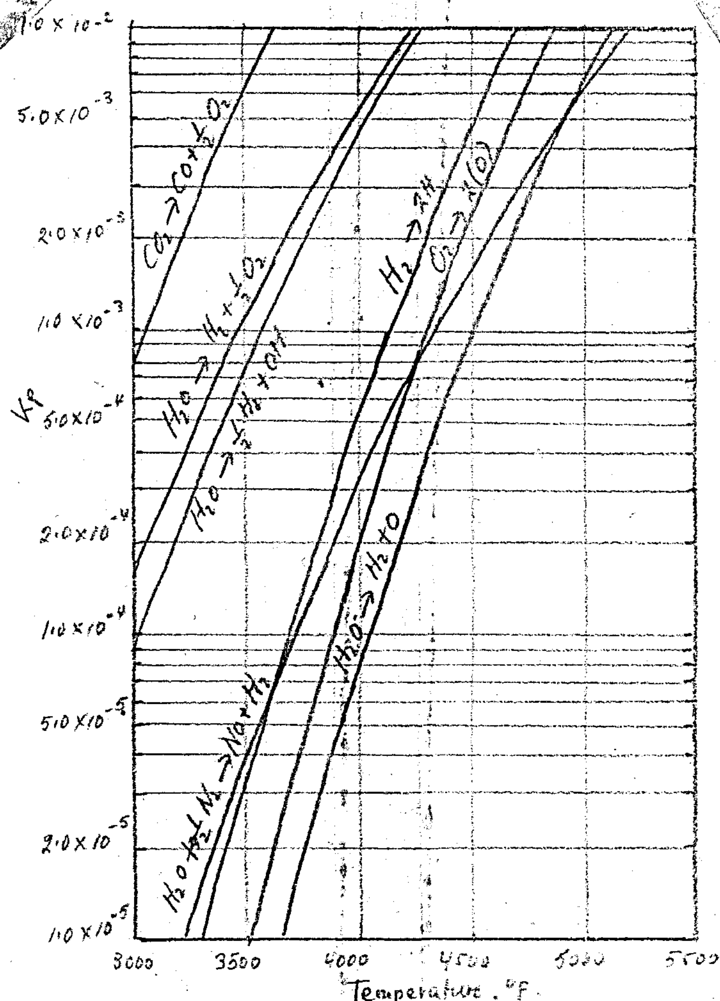
Dissociation

High combustion temperature in rocket assists the dissociation of some of reaction products by permitting them to break up into smaller molecules and monatomic constituents. This dissociation takes place if combustion temperature exceeds 4800 degree F.

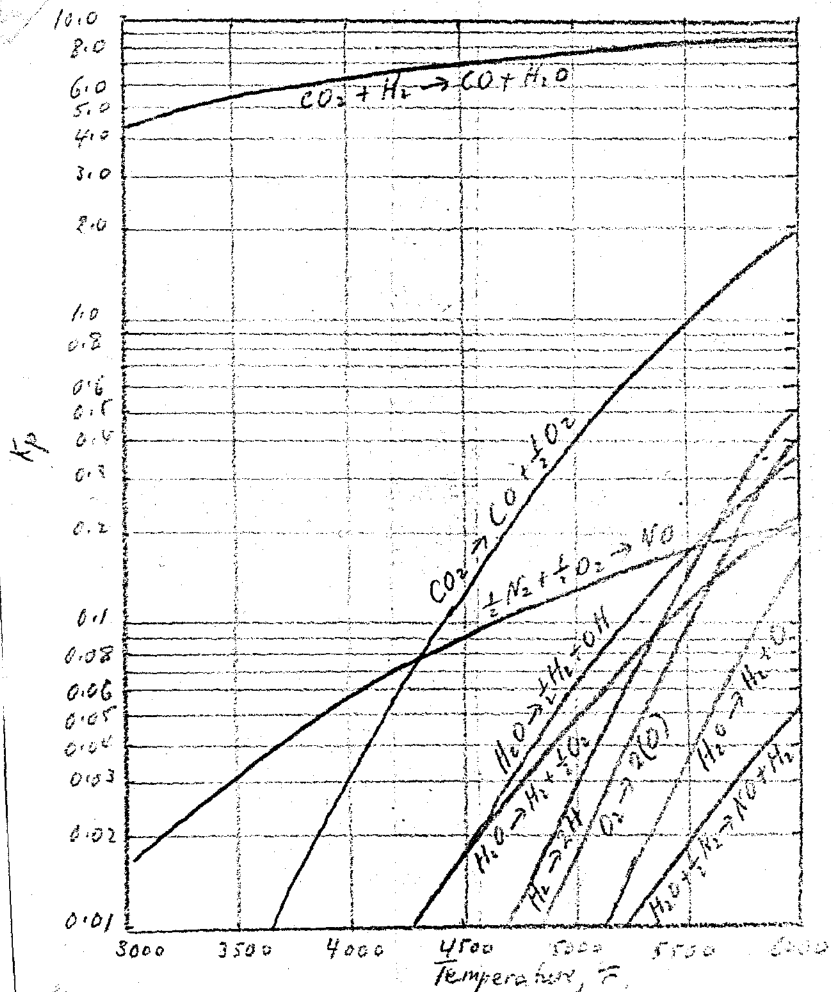
Effects of dissociation for rocket propulsion units :

I - Dissociation consumes energy and therefore decreases the amount of energy available for conversion into kinetic energy, limiting maximum attainable exhaust velocity and flame temperature.

2 - Dissociation usually increases average specific volume of combustion gases and lowers average molecular weight of products. This slightly raises exhaust velocity.



Equilibrium constant Kp versus temperature.



Equilibrium constant K_p versus temperature.

3 - When gas stream temperature is lowered reassociation occurs. In an isentropic gas expansion nozzle temperature drops towards nozzle exit,here equilibrium composition changes and additional heat is liberated at exit.
4 - Boundary layer near the wall is cooler than main body of gases because of conduction through the wall. This may cause reassociation and additional heat release in the boundary layer.So presence of dissociated reaction products can increase heat transfer to the wall.

In general dissociation limits Specific Impulse.

Determination of Combustion temperature and Composition of Reaction Products

Rocket performance calculations may be solved by the principle of conservation of energy , provided chemical equilibrium is established at constant pressure within the combustion chamber and assumptions for ideal rockets are valid.

Calculation of combustion temperature and the product gas composition are based on equating heat of reaction of propellant combination and rise in enthalpy of product gases.

$$((Q_R))_{T_o} = \sum ((n_p \int_{T_o}^{T} C_p \, dT)) = \Delta h \qquad (41)$$

where ,

$(Q_R)_{T_o}$ = Heat of reaction of propellant combustion at reference temperature T_o.

$C_p \, dT$ = enthalpy change necessary to heat one mole of each product gas from reference temp. T_o to flame temperature T.

n_p = number of moles of each product gas.

C_p = average molar specific heat at constant pres. between T, T_o.

Δh = Total enthalpy gain of reaction products.

The above equation solved for T will give combustion temperature.

Combustion Efficiency

The ratio of the actual change in enthalpy per unit propellant mixture to the calculated change in enthalpy necessary to transform the propellants from the initial conditions to the products at the calculated chamber temperature. In solid propellants combustion efficiency is a function of grain design and degree of mixing between several solid constituents and increases with increased combustion temperature.

Chemical Reaction in Nozzle

In ideal nozzles expansion is assumed to be isentropic and chemical composition of gases do not change. In a real rocket nozzle equilibrium will shift since temperature and pressure drop continuously in expansion. For most propellants burning and recombination in the nozzle will permit further release of chemical energy into heat and will cause a higher exit temperature.At nozzle exit also average molecular weight of most products will increase. This will decrease exhaust velocity but the effect of heat release is greater so exhaust velocity will increase also rocket performance increases.

Another argument is that chemical reaction is frozen at nozzle entrance because individual gas particles travel at supersonic speeds through nozzle in such a short time that it may not be possible to maintain instantaneous chamical equilibrium as the pressure decreases. Calculation of I_s on this basis is referred to as Specific Impulse at frozen equilibrium.

Specific Impulse values calculated to amount for the reaction occuring in the nozzle are referred to as shifting equilibrium values, because equilibrium shifts as gases pass from high temperature and pressure to low pressure and temperature regions.

Actual nozzle exit condition lies in between changing equilibrium and frozen reaction conditions.

Results of Thermochemical Computations

A high exhaust velocity can be obtained by lowering the molecular weight of combustion products or by increasing the chemical energy per unit of propellant weight, which in turn increases combustion temperature.

The dissociation of combustion gases into monatomic constituents and radicals lowers the combustion temperature because energy is consumed which otherwise would be available for raising the temperature of the gases.

Optimum rocket performance is obtained at optimum mixture ratio which occurs usually at a value which is richer in fuel than the stoichiometric mixture ratio, at which all the fuel is theoretically completely oxidized, and flame temperature is a maximum.

Molecular weight tends to decrease as fuel concentration is increased over stoichiometric value.Changes in chamber pressure changes exact value of optimum mixture ratio.In solid propellant rockets optimum mixture ratio determines the proportions oxidizer and fuel to be used during propellant preparation.

Propellant mixtures with low combustion temperature but relatively high exhaust velocities are desirable since design of rocket units is thereby simplified.

Increased chamber pressure increases chamber temperature and I_s.

Heat Transfer

Some of the significant applications of heat transfer theory in the field of rocketry :

I - Heat transfer from the hot reaction gases to the walls of the combustion device, such as solid propellant combustion chambers and nozzles.

In almost all solid propellant rockets uncooled chambers are used. In an uncooled unit the operating time is short and heat transfer is transient phenomenon. In this chambers, reduction of heat transfer to the wall by means of insulation and clever design permits a lowering of chamber weights.

2 - Selection of propellants with favourable heat transfer characteristics.

Mechanism of heat transmission to solid propellant grains is not well understood.

3 - Jet flame heating

The heat transfer from jet flames ,or from hot exhaust streams to the surrounding equipment presents design problems.This includes radiation and convection effects of the jet on equipment of rocket test stands.

4 - Heat transfer to flying vehicles at high speed.

Aerodynamic heating of vehicles can result ina substantial rise in the skin temperature. This requires the rocket hardware to operate at elevated temperature conditions, and it often causes the heating of propellants particularly in flying vehicles which use integral skin wall designs of solid propellant chambers.

Ordinary heat transfer equations cannot be used to determine heat transfer rates in rockets since rocket conditions differ from ordinary heat transfer problems. The differences are :

I - The energy release per unit volume is very large.

2 - Heat transfer rates are very large.

3 - Combustion temperatures in rockets (4000 to 6000 F^o) is higher than in standard furnaces.

4 - The surface velocities of the gases along the chamber wall are very high, increasing heat transferred by convection.

5 - Some properties of hot propellant gases are not very well known.

6 -The boundary layer effects are difficult to evaluate analytically.

7 - The equations developed for gas film coefficients assume an equilibrium velocity profile, which implies also a length of several diameters of chamber ahead of the section to which the heat transfer computations are applied. Such an equilibrium condition cannot normally be established in a rocket combustion device where length to diameter ratio is small and where turbulence tends to diminish insulating effect of the boundary layer.

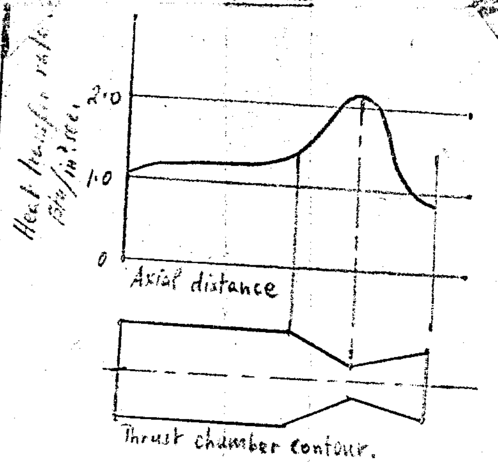
8 - Grain design in solid propellant rockets has a profound influence on the gas film. By a change in grain design it is possible to change heat flux and wall temperature. In solid propellant rockets heat transfer to the wall is affected by chamber liner design , grain configuration.

9 - Pressure effects which become significant above 4500 F^o are usually neglected in rocket heat transfer calculations.

Rocket Thrust Chamber Heat Transfer

Heat transfer rate varies within the rocket and is usually highest at, and immediately upstream of the nozzle throat, with local wall temperatures having highest values in the region. Rocket thrust chambers with local heat transfer rates of 0.I to I0 Btu/sec.in^2. have been successful. The largest part of heat is transferred by convection a small part by radiation.

Amount of heat transfer increases as exposed wall surface of combustion chamber increases. For constant chamber pressure , surface



Typical heat transfer rate distribution along thrust chamber wall.

area of combustion chamber generally increases with thrust.

Variation in chamber pressure seriously effect heat transfer. Increase of heat transfer with chamber pressure imposes design limits on the maximum practical chamber pressure.

Heat Transfer Failures

The wall temperature on the gas side exceeds the value at which the material is readily melted or oxidized. The local loss of material and local heating weakens the wall so remaining material is inadequate to take the imposed load. This failure is characterised by melting, erosion or severe oxidation on the gas side surface of the wall. It occurs primarily in uncooled chambers and uncooled nozzles.

Uncooled Combustion Devices (Most Solid Propellant Rockets)

Uncooled walls act essentially as heat sponges and absorb heat from the hot gases. Heat is transferred from the hot gases to the wall, and during operation a changing temperature gradient exists across the wall. During the combustion process the gas side of the wall is always hotter than the outside wall surface. At the completion of rocket's operation the wall temperatures tend to equalize.

Heat transferred across the hot surface of the wall must be less than the heat absorbing capacity of the wall material below the critical temperature. If heat transfer to atmosphere is neglected :

$$Q\Delta t = - k A (dT/dL)\Delta t = W \tilde{c} \Delta T \qquad (42)$$

where ,

ΔT = average wall temperature increment , F^{o}. W=Weight of wall,lbs.

Q = Heat,Btu/sec. transferred across area A, sq.in.

dT/dL = Temperature gradient of heat flow near hot wall surface.

$\tilde{c}$ = Specific heat of wall material.

Δt = Time increment , secs.

Above equation indicates the following trends:

I - Short operating duration (t) should favor uncooled chamber design, since it minimizes amount of heat to be absorbed by the wall.

2 - Low conductivity is desirable by using an insulating layer on the hot gas side, such as ceramic linings and ceramic nozzle inserts used in solid propellant rockets.

3 - High value of $\bar{c}$ of the wall promotes high heat absorbing capacity.

4 - High conductivity of wall material reduce temperature gradients within the wall so thermal stresses and tend to increase heat absorbing capacity of the wall material.

5 - High value of maximum permissible stress at the highest possible elevated temperatures will permit a large increase in wall temperature without weakening of the wall.

Heat transfer to Flying Vehicles

At high flight speeds the boundary layer around the vehicle is heated aerodynamically to relatively high temperatures. This behaviour of the boundary layer governs the heating of vehicle's skin, aerodynamic drag, structural requirement, and heating and possible boil-off of propellants. A laminar boundary layer transfers less heat than a turbulent one.

Rockets carried externally by high speed aircraft can be subject to intense skin heating. In some of these applications solid propellant rockets are subjected to excessive temperatures on the integral skin wall, so that their physical properties deteriorate or danger of involuntary ignition exists.

Jet Flames

The jet from a rocket will heat its surroundings by means of :

I - Conduction and convection to objects directly in the jet.

2 - Radiation to surrounding environment.

Jet vanes, test stand flame deflectors, other equipment fall into I) and test stand equipment, etc. fall into 2).

The flame shape or region of hot gases is a function of nozzle geometry, degree of underexpansion, the atmospheric pressure, chemical composition and vehicle velocity. Approximate formula is :

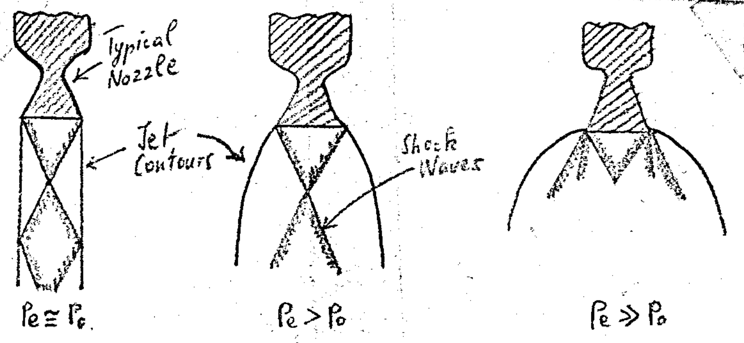
$$l = \sqrt{F/f} \qquad (43)$$

where ; l = length of visible flame, ft.

f = empirical factor with a value of I0 for the given dimensions.

F = Thrust, lbs.

If gas pressure at exit equals atmospheric pressure, the jet has a cylindrical boundary. At altitude, or when nozzle exit pressure is



Typical jet contours for different ratios of the nozzle exit gas pressure p_e and the freestream atmospheric pressure p_0.

arger than atmospheric pressure , the jet has a tendency to spread out.

In conical nozzles shock waves are visible in the exhaust jet. In certain solid propellants , the exhaust jet is incandescently luminous or cloudy, and this phenomenon obscures the shock wave pattern. The shock waves are a series of expansion and compression waves starting at exit of nozzle.

SOLİD PROPELLANT ROCKET FUNDAMENTALS

I - Principle Components of Solid Propellant Rockets :

Principle components are : propellant , hardware such as chamber , nozzle or mounting pads , and igniter.

(i) Propellant - Solid propellants have a plastic like appearance and burn on their exposed surfaces to form hot exhaust gases which produce a reaction force. A physical mass of the propellant is the grain Some rockets have more than one grain in the same chamber.

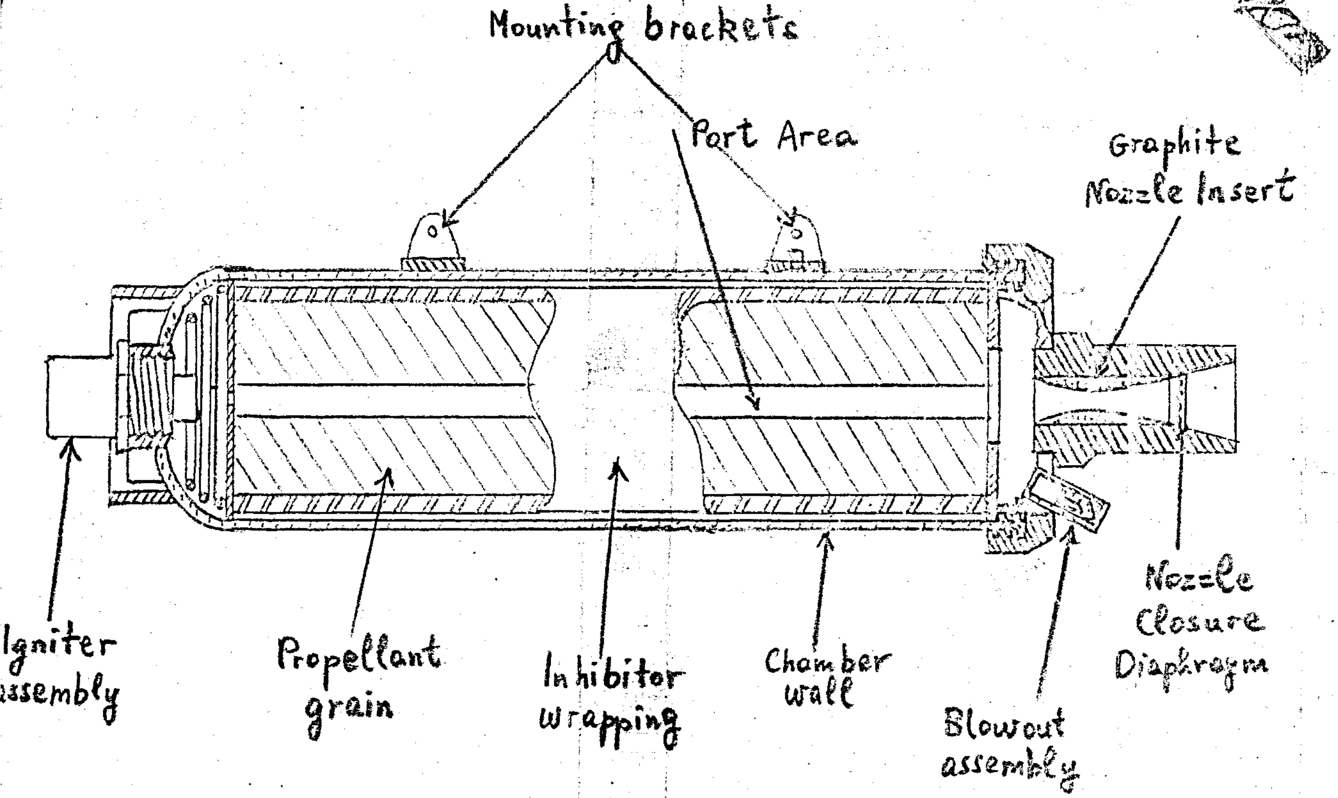
A solid propellant is either composed of heterogenous mixture of several chemicals or a homogenous charge of special chemicals. When propellant charge is ignited,a well designed grain burns smoothly without surges or detonation. The combustion consumes the grain in a direction normal to burning surface.

Shape,size,exposed burning surface,and geometrical form of the grain influence burning characteristics of rocket and determine the operating pressure, thrust and duration.

(ii) Hardware- Solid propellant is confined in a combustion chamber and reaction gases are exhausted through an exhaust nozzle.

Hardware of solid propellant rockets also include ; provisions for assembly or disassembly of the unit, mounting pads, burst diaphragm or other safety provisions to prevent overpressurization of the chamber and means for holding propellant grain in place.These hardware components are usually uncooled and have to withstand severe heating.

(iii) Igniter - Combustion of propellant grain starts by means of an igniter which is started by electric current or percussion action.



CTIONAL VIEW OF TYPICAL SOLID PROPELLANT ROCKET.

2 - Grain Configuration :

Thrust is equal to the product of exhaust velocity and mass flow rate. With a large flow rate , large thrust is achieved.Thi is obtained by a large burning surface or fast burning rate or both.

A low thrust of longer duration can be obtained if exposed burnin area is small. Therefore variation of thrust is obtained by varying geometric form and therefore exposed surface of propellant charge.

Exposed burning surface is also limited by inhibitors, which are inert or slow burning chemicals. They are applied to grain surfaces , where burning is to be prevented. Inhibitors are either applied by bo nding sheets of inhibitor,by wrapping by a tape,or by dipping the gra

When inhibitor is applied to inner surface of chamber ,it acts to reduce heat transfer to the wall and is referred to as a liner.

Since inhibitor restricts burning , solid propellant units with inhibitors are called restricted burning rockets, and ones without in hibitors unrestricted burning rockets.

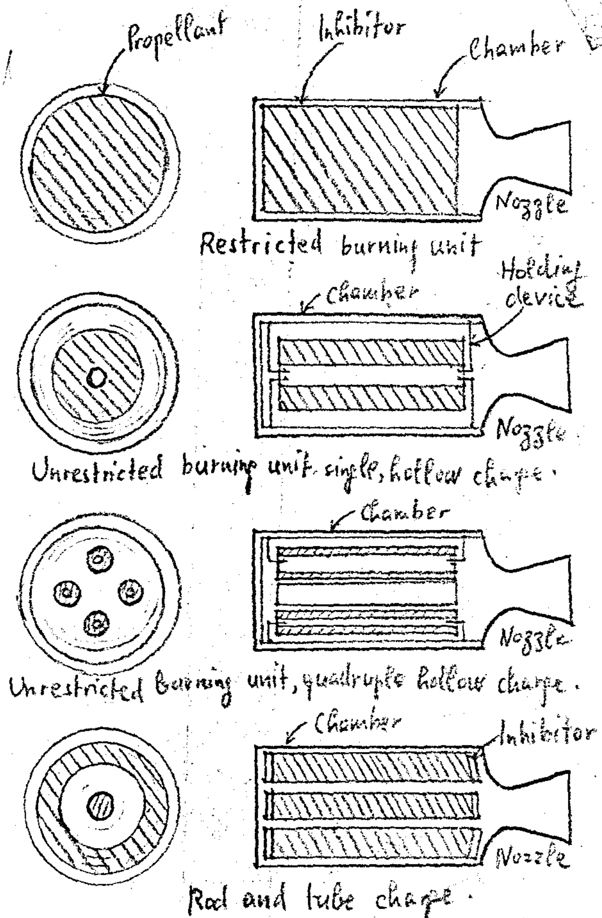
Geometrical arrangement of grain charges depend on desired variat ion of burning area during operation,characteristics of propellant an duration. In most configuration burning area varies with time.

If burning area and therefore gas evolution rate , chamber pressu re, and thrust increase with burning time, then rocket is said to hav progressive burning characteristics.If they decrease with time, then rocket is said to have regressive burning characteristics.

A grain which maintains approximately constant burning area and constant thrust has neutral burning characteristics.

Many grains have perforations , that is , holes in the propellant charge to increase the burning surface. In perforated grains, propell ant burns on all exposed and unrestricted surfaces. The gases created in the forward end have to flow past a portion of the propellant grai to the nozzle,and if their velocities are high they may cause erosion over the burning propellant surfaces which they have passed.

Many grain configuration,especially unrestricted types have larg void space which is necessary for the escape of combustion products nozzle exit, and for mounting provisions and to provide for thermal expansion. This void space if small, space factor is large which per



Typical solid propellant grain configuration.

for compact and lighter design. In computation of space factor , inhibitors , liners, etc. are not included in the volume of propellant.

3 -Combustion of Solid Propellants

In most solid propellant rockets,no arrangement is made for t control of the combustion process during the burning of the propellan If there are no mechanical devices installed in the unit , control of the burning is only possible by the use of a suitable grain with inhibitor coatings before the unit is used.

During the burning the solid propellant regulates itself. Fundamental property involved in this self-control burning is the rate o burning which is the rate at which a burning surface receeds in a direction normal to itself , inches/second. Most ordinary propellants have 0.03 to 2.5 in./sec. burning rate at 2000 lbs./sq.in. chamber pressure. For some restricted units an approximate formula is :

$$r = a\, p_c^n \qquad (44)$$

where ,

r = burning rate,in./sec. p_c = chamber pressure ,lbs/sq.in.

a = constant , 0.05-0.002 for restricted propellants.

n = constant , 0.4-0.85 for restricted propellants.

Actually burning rates are determined in static firing tests of rocket units. Burning rates actually depend on : the gas pressure on the propellant,p;temperature of the propellant charge before ignition,T_p;velocity of gas flow past the charge surface,V;time since the start of burning,t;the position in the charge,x;the direction in whic the normal to the burning surface point,w,

$$r = r(p,T_p,V,t,x,w) \qquad (45)$$

Variation in burning rate with time after start of combustion is due to gradual heating of the propellant and motor tube by radiant heat transfer from the hot propellant gas. It is a more important effect for hot propellants than for cool propellants.The propellant grai must be slightly transparent to radiation to permit internal heating.

Both variation with position and direction are due to inhomogeneities and anisotropies introduced into structure of propellant during its manufacture.

It will simplify the general theory if if only the variables of chamber pressure, propellant temperature and gas flow velocity are dependent. The effect of temperature is seperable from pressure and velocity dependence:

$$r = f_1 (T_p) \quad f_2 (p,V) \qquad (46)$$

Temperature dependence is expressed as :

$$f_1 (T_p) = \frac{A}{T_1 - T_p} \qquad (47)$$

where ; A and T_1 are empirical constants.

Pressure and velocity dependence is seperable in some cases but not in others :

$$f_2 (p_c,V) = a' (p/I000)^n (I+kqV) \quad \text{or} \quad a' (p/I000)^n f_3 (V) \qquad (48)$$

where;

q = density of propellant gas flowing past the grain.

a' and k = Empirical constants.

f_3 (V) has a peculiar dependence :

$$f_3 (V) = \begin{cases} I & \text{if } V \text{ less than } V_o \\ I + k_v (V-V_o) & \text{if } V_o \text{ less than } V \end{cases} \qquad (49)$$

where ; k_v , V_o are empirical constants , and V_o= threshold velocit

Frequently an expression without threshold velocity is used :

$$f_3 (V) = I + k V \qquad (50)$$

Erosive constant k (I+kqV) ranges from 0.05 to 0.7 sq.in. sec./lb.

Final burning rate expressions are then as follows :

$$r = \frac{a'}{T_1 - T_p} (p/I000)^n \quad (I + kqV) \qquad (5I)$$

For negligible velocity this reduce to :

$$r = a (p/I000)^n \qquad \text{with } a = a'/ T_1 - T_p \qquad (52)$$

where ;p = psia.

a = 0.I-I.0 in/sec. n = 0.2 - 0.8

Stability of the Burning Surface

Combustion Geometry : It is important that the regular behaviour of the burning surface should be stable so that burning surface area does not change in an unpredictable manner. If a disturbance is represented by an irregularity of the surface simple geometrical considerations, assuming burning in a normal direction, indicate that the surface will smooth out and continue its regular recession as long as the desired surface is flat. A constant area for the burning surface is the important requirement since a small change in the burning surface area will produce a large change in chamber pressure.

Charge Deformation Problems : Propellant charge is subject to pressure and to axial and radial acceleration forces. These forces produce deformation of the charge.

In case of tubular and internal burning propellants where flows through ducts along the grain, there is a serious situation. General pressure level does not lead to a deformation of the charge, but the chamber pressure at forward end of rocket is larger than that at the exhaust nozzle end of the charge. The unbalance of pressure lead to longitudinal compressive stress, bulging the charge into the gas duct area. Acceleration forces also add onto this compressive stress. The bulging leads to a restriction which increases the pressure drop. At high propellant temperature burning rate is higher, propellant is softer, and normal duct area is reduced by thermal expansion of the propellant. The higher pressure produce large accelerations. All these effects result in increased pressure drop and bulging, which may cause failure of rocket.

Let us consider only pressure drop due to the necessary gas flow velocity. If cross sectional area of propellant is normally A_{pn}, effect of pressure drop increase it to A_p, where

$$A_p = A_{pn} (I + 2\mu/E \ \sigma) \qquad (53)$$

$$\sigma = p_F - p_N \qquad (54)$$

p_F= Front pressure.

p_N= Rear pressure. μ = Poisson's ratio. E=Elastic Modulus

If ; A_q= Internal cross sectional area of combustion chamber tube

Then duct area ; $A_d = A_q - A_p$, $A_{dn} = A_q - A_{pn}$ (55)

Assume ; $A_q = 2\ A_{pn}$

If propellant were perfectly rigid , pressure drop would be less , $_n$, since no bulge would obstruct duct area.

Combining equations (53) and (55) :

$$A_q - A_d = A_q - A_{dn} (I + 2\mu / E \ \sigma)$$

$$A_d = A_{dn} (I + 2\mu / E \ \sigma)$$

$$A_d - A_{dn} = A_{dn}\ 2\mu / E \ \sigma$$

$$A_d - A_{dn} / A_{dn} = 2\mu / E \ \sigma$$

$$\sigma = E/2\mu (A_d/A_{dn} - I)$$

$$\sigma = E/2\mu (I/ A_{dn/} A_d - I)$$

$$\sigma/\sigma_n = I/2\mu \ E/\sigma_n (I/ A_{dn/A_d} - I) \qquad (56)$$

There is a critical elastic modulus and if the actual modulus of propellant is less than that, the rocket will blow up. Note that reduction in actual modulus with temperature is more important effect th- an increase of critical elastic modulus with temperature.

Steady State Dynamics of Radial - Burning Grains

In internal burning and radial burning grains erosive burning if it is mass flow dependent , can introduce erosive instability into combustion process.

If the burning rate remains the same at every point on the surface , a hollow cylindrical charge , restricted from burning on the ends , maintains a constant burning surface throughout the burning period.So it is assumed that burning surface, S_b , is constant. But, as the solid propellant burns away the duct area A_d , for parallel gas flow increases. In usual design , velocity of gas flow at exhaust nozzle is very much larger than the velocity of recession of the burning surface. During the time necessary for combustion of I % of solid propellant , gas evolution corresponds to about twice the duct volume within motor, so assumption of steady state will be a satisfactory approximation.

Properties of combustion gas are assumed the same across any given cross section. Propellant gas is assumed ideal. Skin drag and frictional forces of internal gas flow on propellant grain are neglected.

Between front of combustion chamber and exhaust nozzle end of the charge, thepressure of gas stream decreases and velocity of gas flow increases. Between exhaust nozzle end of the charge and throat of nozzle no further combustion mass is added to flow. In this portion there is a sudden expansion of gas at entrance which lead to loss of stagnation pressure which is neglected.

Mass flow through an exhaust nozzle is :

$$\dot{m}_o = p_o \, A_t \, / \, \bar{c}^* \qquad (57)$$

where ;

p_o = stagnation pressure related to velocity and pressure at the end of the charge.

Rate of mass combustion is :

$$\dot{m}_b = \bar{r} \, S_b \, q_b \qquad (58)$$

Mass flow out of exhaust nozzle equals rate of mass combustion. Equating equations (57) and (58) and writing in terms of $K = S_b/A_t$ (59)

$$K = p_o \, / \, q_p \, \bar{r} \, \bar{c}^* \qquad (60)$$

Introducing parameters :

$$\phi^+ = p_F/p_o \quad (61) \qquad \phi_r = \bar{r}/r_F \quad (62) \qquad \phi = \phi^+ \, \phi_r \quad (63)$$

where ; r_F , p_F = burning rate and gas pressure at the front of the grain , we get :

$$K = p_o/q_p \, \bar{r} \, c^* = p_F/ \, q_p \, c^* \, \phi^+ \, r_F \, \phi_r = p_F \, / \, q_p \, c^* \, r_F \, \phi \quad (64$$

where;

K = Ratio of burning surface to throat area.

ϕ_r and ϕ^+ may be determined in terms of ratio of gas duct area to exhaust nozzle throat area , A_d/A_t , and the front chamber pressure.

<u>Internal Gas Flow</u>

Analytical technique used is to find variation in conditions along the grain,relative to front conditions,in terms of local Mach Numbe

Mach number at the nozzle end of the grain is determined by the ratio of exhaust nozzle throat area to gas duct area.

Conservation of mass , momentum , and energy at any section x , together with equation of state are :

$$A_d \ d(qV)/dx = q_p \ r \ dS_b/dx = q_p \ r \ \emptyset \qquad (64)$$

$$OL = S_b \qquad (65)$$

$$p_F - p = qV^2 \qquad (66)$$

$$I/2 \ V^2 = c_p \ (T_o - T) \qquad (67)$$

$$p = q \ R \ T \qquad (68)$$

where ;

q,V,p,T = gas density, velocity , pressure, temperature at any section x.

r = Burning rate at x. dS_b/dx = constant.

A_d = gas duct area, $R = \bar{R}/m$

O = perimeter of burning surface.

T_o (= T_F) = adiabatic constant pressure flame temperature for combustion of solid propellant.

Local Mach Number = M ---- $M^2 = V^2/a^2 = V^2/k \ R \ T$ (69)

where ;

a = local velocity of sound. k = ratio of specific heats.

Mass Flow Density is dimensionless and equal to :

$$g = q \ V \ / \ q_F \ a_F \qquad (70)$$

where ;

a_F = Sonic velocity at front section.

q_F = gas density at front section.

Solving for local conditions in terms of front conditions and local Mach Number we get : (proofs will not be given since proved already in gas Dynamics by Professor N. Eraslan)

$$p/p_F = I \ / \ I + k \ M^2 \qquad (7I)$$

$$T/T_F = T/T_o = I \ / \ I + k-I/2 \ M^2 \qquad (72)$$

$$q/q_F = \frac{I + \frac{k-I}{2} M^2}{I + k M^2} \qquad (73)$$

$$V/a_F = \frac{M}{(I + \frac{k-I}{2} M^2)^{I/2}} \qquad (74)$$

$$g = \frac{M}{I + k M^2} (I + \frac{k-I}{2} M^2)^{I/2} \qquad (75)$$

$$M^2 = \frac{((I - 2(k+I)g^2))^{I/2} - (I - 2 k g^2)}{66 (k-I) - 2 k^2 g^2))} \qquad (76)$$

Local stagnation pressure is p_o :

$$p_o/p = (T_F/T)^{k/k-I} \qquad (77)$$

$$p_o/p_F = (I + \frac{k-I}{2} M^2)^{k/k-I} \;/\; I + k M^2 \qquad (78)$$

Internal sonic flow :

Physically , supersonic flow is not attained adjacent to a burning surface where mass is being added.

Differentiating equation (75) :

$$\frac{dM}{M} = \frac{(I + k M^2) (I + k-I/2 M^2)}{I - M^2} \frac{dg}{g} \qquad (79)$$

We see that ; if M is less than I Mach number increases with g, and if M is greater than I Mach number decreases as mass flow increase

The flow cannot become supersonic. Variation of duct area , due to variation in burning rate , may cause the sonic position to jump from place to place along the grain. Uniform flow is established if A_d is larger than A_t and if pressure level rises so that actual mass flow can increase with increasing pressure.

If M=I , we have equation (64) for sonic flow , which will unable one to solve for steady state front chamber pressure.

- -

Recent Developments in Combustion of Solid Propellant Rocket Engines

Research was made on the following areas :

I - Influence of composition on ballistic performance.

2 - Erosive burning characteristics of propellants.(polyurethane propellants)

3 - Combustion instability phenomena.

I - In early stages of propellant development , a simple propellant based on ammonium perchlorate and an improved polyurethane binder was investigated. To get good physical properties 20 to 22.5 % by weight binder was used. Increase in energy was achieved by using aluminium alloy additives and high energy plasticizer. In initial work , only anticaking agent and catalyst were added to promote curing. But this propellant was unstably burned. Then by addition of alluminium alloy 5 % by weight , combustion was stabilized. With this as reference propellant, present aim is to produce :

(a) Propellant with low burning rates.

(b) Propellants with high burning rates.

(c) High energy propellants.

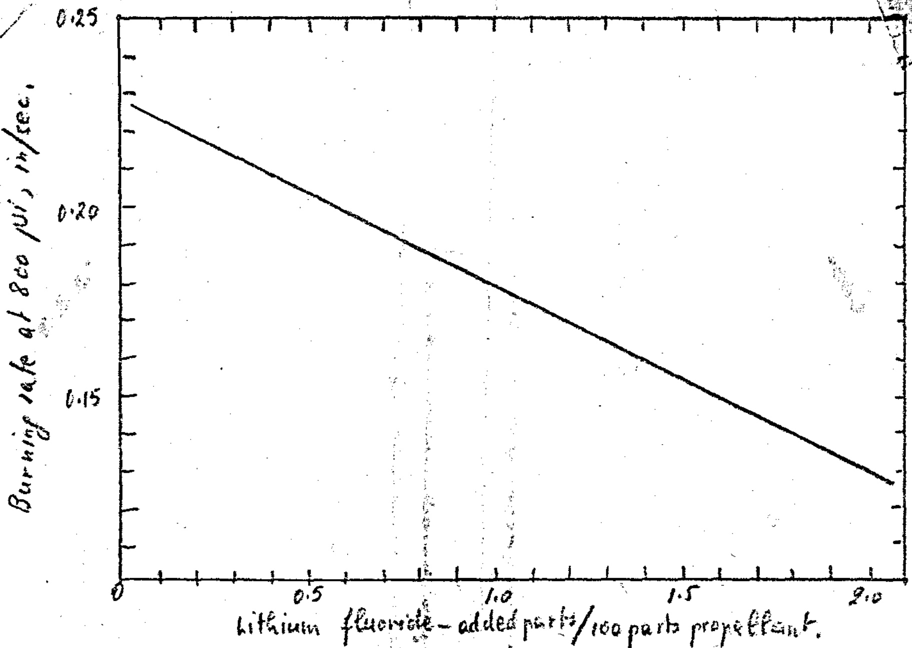
(a) Low burning rate propellants :

Its production is attempted by including in the composition several parts by weight of an endothermic coolant. Since the propellant binder was cured by chemical means , additives are selected very carefully. Alkali metal halides were considered chemically inert. It was investigated that the effect was quantitatively related to latent heat of volatization per unit weight and <u>Lithium Fluoride</u> was the most effective additive. It was found that for up to 2 % , the depression in burning rate was proportional to concentration of lithium fluoride.

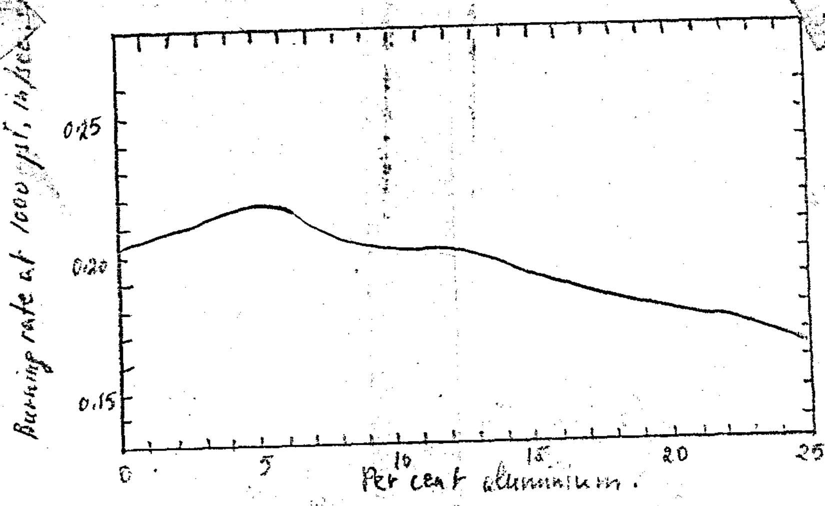
Next approach to develope slow burning rate propellants was the use of formulations using large proportions of metal additives . e.g. Aluminium content. For rocket engine development it is decided to concentrate on a slow burning propellant with a rate of 0.I8 in./sec.

(b) Propellants with high burning rates :

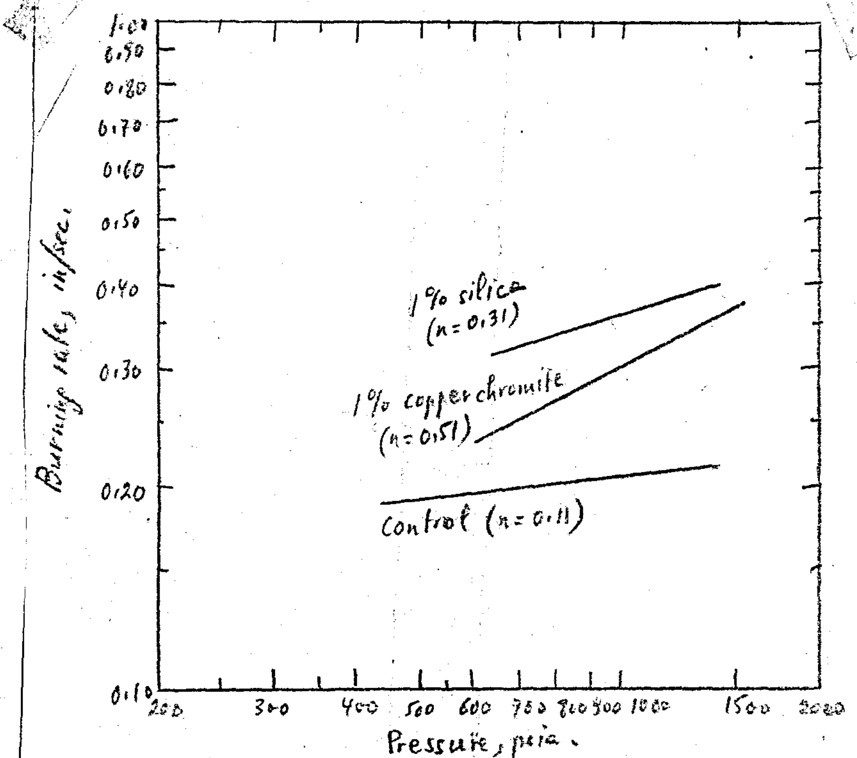
First approach is to use hotter propellants , and this is



Effect of lithium fluoride on burning rate.



Influence of aluminium content on burning rate - 25 % binder.



Burning rates of propellants incorporating copper chromite and silica combustion catalysts.

achieved by reducing binder ratio, to fuel. This procedure increases performance and burning rate up to a level.

Other approaches :

1) Addition of potassium perchlorate - objections are : high value of pressure is associated.
2) Addition of heterogeneous gas phase catalyst - objections are Loss of performance.
3) Control of crystal deflegration by oxidant additives.

Copper chromite and finally divided silica accelerate burning rate of propellant without changing other parameters, and this is used for item 2).

Control of rate of crystal deflegration is investigated by agglomeration of additives with ammonium perchlorate, and this is used for item 3).

(c) High Energy Propellants :

For developing more energetic propellants the path of sacrifice of polymeric binder for alluminium is not followed. Rather, the binder phase is retained at a high level and an explosive plasticizer is added to improve overall stoichiometry of propellant.

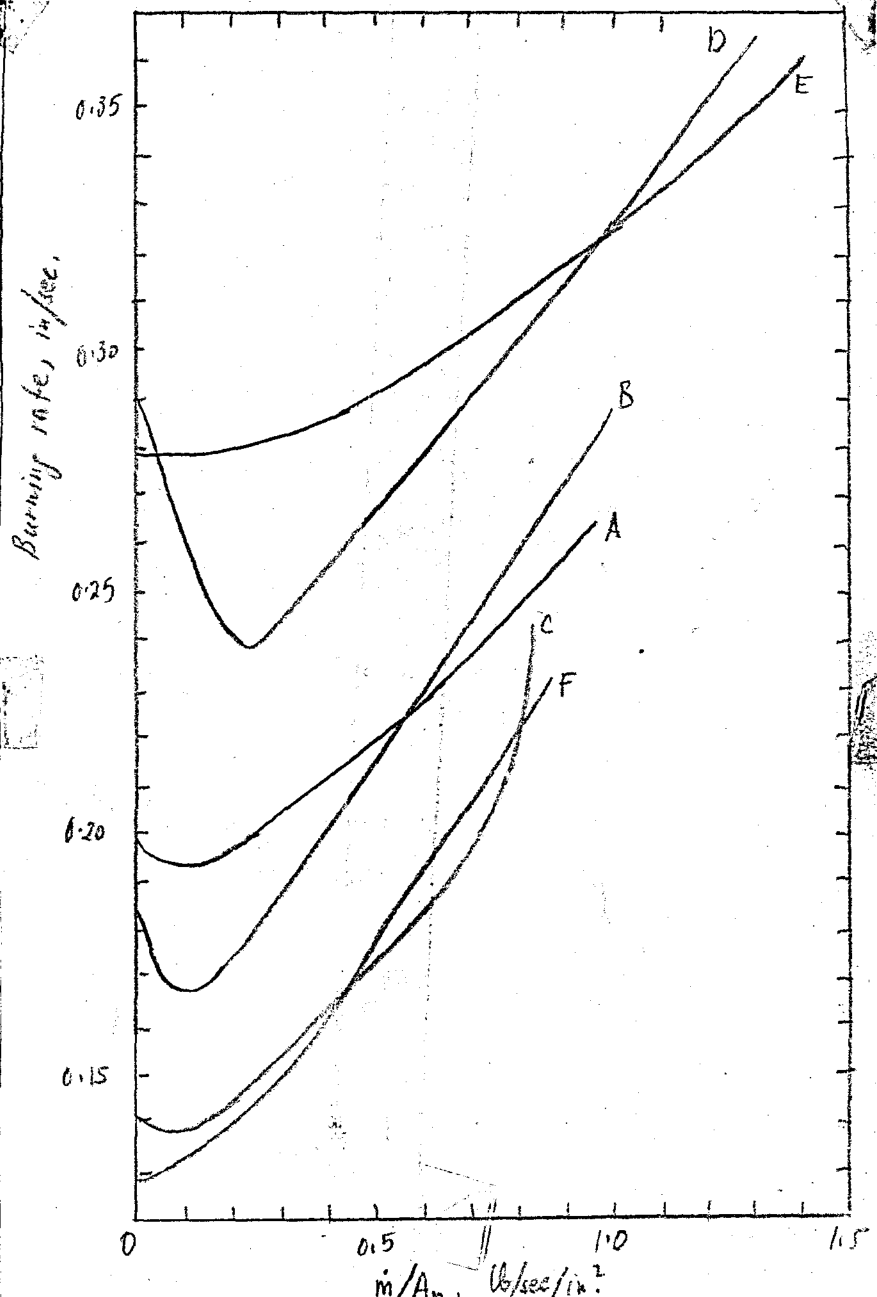
2 - Erosive Burning of Propellants :

Canadian Armament Research and Development Establishment has determined erosive burning by inserting conductivity probes in test grains and comparing the station burning rates at various times during burning.

Taking medium energy propellant as reference, it is seen that,; negative erosion is quite apparent in the hotter and slightly faster burning propellants and appears to be less severe in cooler,slow burning compositions.

Also, there appears to be little major change in erosion characteristics as the weight fraction of the solids in combustion products increase from 0 to 30 %.

Another point is that , erosive burning effect is still quite noticable at high values of port to throat area ratio.



Variation of burning rate with local mass flowrate per unit conduit area.

3-Unstable Combustion

Instability arises because the rocket engine contains an acoustical cavity whose walls are reacting surfaces which for certain geometrical configurations are capable of amplifying small pressure disturbances. These pressure disturbances lead to undesirable and dangerous over-pressures , severe mechanical vibration , excessive heat transfer , grain rupture and abnormal thrust-time curves.

Two Main instability :

I - The transverse mode where the gas flow sloshes across the section of the cavity.

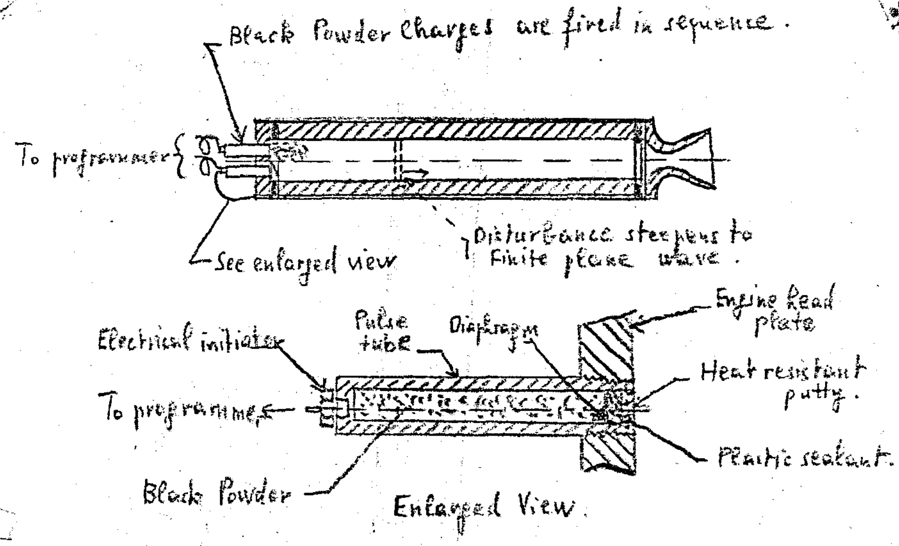
2 - The longitudinal mode where macrosonic disturbance travels backwards and forwards along the longitudinal axis of the cavity at about local sonic velocity.

A combination of these two leads to a highly undesirable increas in burning rate of propellants , and under appropriate conditions , amplification of the initial disturbance occurs up to a limiting value fixed by system losses.

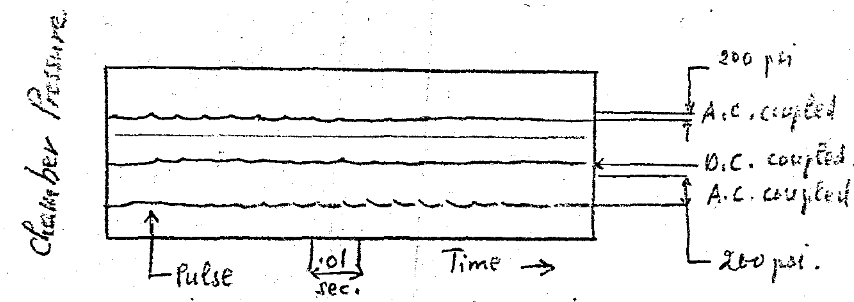
One source of damping is the poor reflection characteristics of the ends, in particular , the nozzle end naturally has higher losses. Other sources of damping arise from normal irreversible fluid dynamic viscous effects and mechanical exitation of the rubbery propellant grain.

The type of instability is determined by both the engine design and the type of propellant. Certain slow burning propellants, in suitable grain geometries, been found on occasion to be sensitive to unforced initiation of longitudinal mode of unstable combustion. This instability cab be overcomed by a pulsing technique developed at CARDE.

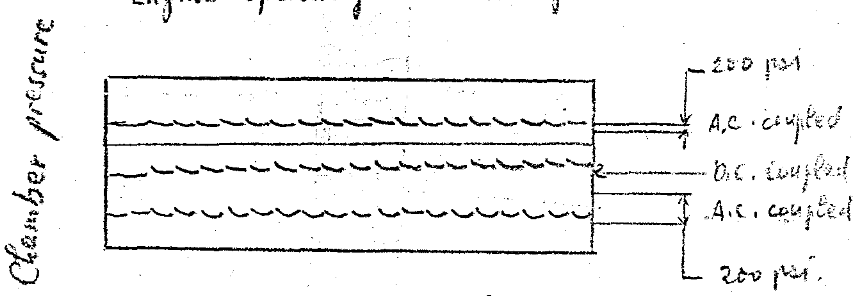
This thechnique consists of firing a small gunpowder charge which initiates a pressure disturbance at the head end of the rocket engine. The pulse propogating up and down the cavity may be either amplified or reduced, depending on the system losses and energy input. High frequency recording equipment show the change in wave form of the pulse. It is found that when engine apperars unstable, the pulses quickly build up to a limiting amplitude and increase the mean engine working pressure due to increased burning rate. After initiation the naturally



Carde Pulse Technique.



Engine Operating in Stable Regime.



Engine Operating in Unstable Regime.

High Frequency pressure recordings of Pulse Technique applied to a 10 in. dia. × 12.5 in. long engine.

occuring instability is identical with artifically induced one. It is guessed that naturally occuring phenomena may be caused by relatively small burning defects which occur in large grains.

By the use of this technique the aim is to conduct a fundamental investigation on two distinct lines. First, to gain an understanding of the effect of engine operating parameters, such as pressure and cavity shape on instability process associated with a given propellant. The step is to compare the behaviour of different propellants in engines of fixed geometry.

For example : Propellant B ; burned stably in several engine designs. Propellant C ; used when slower burning propellant needed. It is more erosive , but contrary to the fact that more erodible propellants are more stable , it is unstable.

Propellant Data

Parameters	Propellants A	B	C
Oxidizer, % by weight	75	70	70
Polyurethane fuel,% by weight	25	25	25
Aluminium , % by weight	-	5	5
Lithium Fluoride, added parts	-	-	I.8
Flame temperature, ^{o}K	2I00	2340	2220
Mean molecular weight	2I.54	22.I6	22.I4
Rocket burning rate at I000 psi in./sec.	0.2I	0.24	0.I5
Characteristic velocity,c^{+},ft/sec.	4650	4820	4560
Specific Impulse at I000 psi lbf-sec/lbm	2I6	22I	2I0
Propellant density, lb/in^3.	0.0580	0.0584	0.0585

Aim of CARDE :

Selecting stable propellants, seeking out a phenomenological understanding of the problem.

By multiple pulsing techniques the effects of grain geometry and initial chamber pressure on stability has been investigated for propellants A,B,C.

C is the most unstable and B is the most stable . For A and C stable operation occurs only in low pressur regions.

The stability limit dividing the stable and unstable regions is a function of the instantaneous grain geometry.

With slow burning propellants the lower stability limit occurs at too low a pressure for use in booster rockets operating at about 1000 psi. But for high performance space rockets operating at low pressures, the very slow burning lithium fluoride based propellants may not be unattractive.

The promising slow burning propellants are highly aluminized ones which are highly erosive to the rocket hardware.

Combustion Stability acceptance of Rocket Engines :

This new technique (pulse technique) means that the response of the propulsion systems to a disturbance capable of trigerring combustion instability in the finite wave axial mode can be verified in the first prototype firings.

Transverse instability is not a problem in aluminized propellants but axial instability might pose some difficulties.

This testing technique gives firm guarantee that an engine will not transition from stable to unstable operation in the axial mode over the required temperature range. After a few firin tests design configuration is qualified.

Finally let us make a definition of instability : Instability is assesed as the ratio of the unstable burning pressure increase over the stable operating pressure.

Weight of Propellant Burned :

Propellant flow rate is given by the following expression :

$$\dot{W} = \frac{d\,W_u}{dt} = A_b\, q_b\, r \qquad (80)$$

where ;

A_b = burning area. r = Burning rate. q_b = propellant density.

Total weight of effective propellant is defined as :

$$W_u = q_b \int A_b\, r\, dt \qquad (81)$$

It is difficult to obtain instantaneous propellant flow rate, $\dot{w}$ or effective exhaust velocity experimentally. But total impulse and total propellant weight consumed during test can be measured. Propellant weight is determined by weighing rocket before and after a test. Effective propellant weight is less than total propellant weight because some grain design permit small portions of propellant to remain unburned during combustion.

Total Impulse :

Total impulse can be accurately determined in testing by integrating area under a thrust-time curve. For this reason average specific impulse is usually calculated from total measured impulse and effective propellant weight.

Total impulse is defined as : the integration of thrust over operating duration , t_b :

$$I_t = \int_0^{t_b} F \, dt = \bar{F} \, t_b \, , \qquad (82)$$

where ; $\bar{F}$ = average thrust during burning duration , t_b.

Effective Burning time :

It does not include the thrust build up and thrust decay period. During these periods total impulse is neglected when calculating average performance values.

Other Performance Parameters :

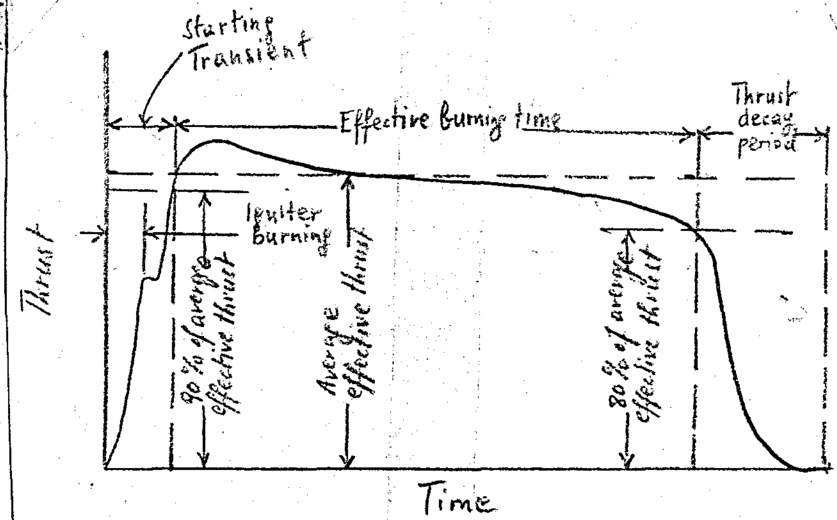
Thrust , Exhaust velocity and Specific Impulse are defined in the first pages of the report so they will not be repeated here.

We have the following performance parameters :

$$\frac{\text{Total Impulse}}{\text{Loaded weight ratio}} = \frac{I_t}{W_g} \qquad (83)$$

$$\frac{\text{Thrust}}{\text{Weight}} = \frac{F}{W_g} \qquad (84)$$

When weight of inhibitors , hardware, etc. become small in relation to propellant weight ; $I_t/W_g \longrightarrow I_t/W_u = I_s$ so higher values of I_t/W_g give better design of rocket.



Typical solid propellant thrust-time diagram.

Volume Impulse : Total impulse per unit volume of propellant grain ;

$$I_v = I_t \;/\; V_b \qquad (85)$$

Finally we have the basic relation of parameters affecting performance which is derived from principle of conservation of matter.

Propellant mass burned per unit time equals increase in gas mass per unit time in combustion chambér plus mass flowing through exhaust nozzle per unit time.

$$A_b \; r \; p_b = d/dt \; (q_c \; V_c) + A_t \; p_c \; ((\; g \; k \; / \; R \; T_c \; (2/k+I)^{\frac{k+I}{k-I}} \;))^{I/2} \qquad (86)$$

This equation is useful in numerical solutions of transient condition. The change of gas mass may be neglected and from there a relation for the ratio of burning area to throat area is obtained and chamber pressure may be expressed as a function of this ratio.

Performance Limitations :

Characteristics of individual propellants limit; thrust , burning, duration and shape of rocket.

Temperature Sensitivity :

Initial temperature of grain affect the performance. On a hot day a solid propellant rocket operates at a higher chamber pressure and thrust than on a cool day. Initial temperature of grain has a decided effect on burning rate, but total impulse is unaffected.

Methods of accurately controlling thrust of a given propellant grain with varying ambient grain temperature are :

I - Variation of burning surface ar the length and shape of propellant grain.

2 - Replacement of the nozzle with one having corrected throat area and expansion ratio.

Temperature changes affect the equilibrium pressure and burning rate.

Temperature sensitivity is defined as the % change of thrust pe unit temperature change.

Temperature sensitivity coefficient of equilibrium pressure, at particular value of K (Area ratios) is defined as :

$$\Pi_K = (\partial \ln p / \partial T)_K = I/p_c \, (\partial p / \partial t)_K \quad (87)$$

Temperature sensitivity coefficient at ~~equilibrium~~ pressure p_c, of burning rate is defined as :

$$\sigma_p = (\partial \ln r / \partial T)_p = I/r \, (\partial r/\partial T)_p \quad (88)$$

Temperature Limits :

In geographical locations where nights are cold and days are very warm , solid propellant charges are subject to temperature effects. During cold, propellant charges become brittle and subject to cracking and also rapid change of temperature cause cracking, which increases burning area , therefore mass flow rate and chamber pressure which cause overloading of chamber walls and result in chamber failure.

Overheating of propellant charge prior to firing cause propellants to become weak, so they are unable to withstand high chamber pressure or acceleration of vehicle. This failure is again due to increased burning area . Therefore propellants have temperature limits which may be widened by use of additives sometimes.

Pressure Limits :

Below a certain pressure the combustion becomes unstable, so operating chamber pressure must be always above Lower Pressure Limit. (well above atmospheric pressure) The lower combustion pressure may provide a safety since propellants will not ignite inadvertantly or even explosion of chamber takes place , they will not sustain combustion at the ambient pressure.

Because many grains have regressive burning characteristics, during thrust decay period , there is always a propellant residue which cease to burn when chamber pressure falls below the lower pressure limit, which is ineffective in giving impulse to rocket.

Pressure is determined by throat area. When throat area exceeds a certain value, combustion pressure will approach lower limit and burning will tend to become unstable. This throat area is called the critical throat area , the actual throat area must be always smaller.

The chamber pressure should be safely above the lower combustion pressure.

Above upper pressure limit, the burning rate increases rapidly and ata certain point burning becomes abnormal, detonation occurs which usually shutters the chamber.(above 6000 lbs/sq.in.)

Deterioration :

Many propellants deteriorate with storage. Some propellants experience a low order chemical reaction during storage, which change properties of grain. This is inhibited by adding certain chemicals. Other propellants absorb moisture which softens and weakens the charge.

Handling Precautions :

It is necessary to handle solid propellant rockets carefully and avoid impact or shock loads on rocket during storage and handling to prevent undesirable cracks , which cause uncontrolled increases of effective burning rate.

Size and Duration Limitations :

Duration is limited because of practicle problems of constructing very large and heavy chambers , and because of excessive heating of hardware parts in prolonged operation.

For any propellant charge size there is an upper limit of maximum thrust by disproportionately large nozzles in relation to port area and propellant volume.

Advantages and Disadvantages of Solid Propellants compared with Liquid Propellants :

I - Simpler in construction and design.
2 - Usually the lighter unit for low total impulse application
3 - Few servicing problems
4 - Believed to be more reliable .
5 - Sometimes difficult to handle .

SOLID PROPELLANTS : I - Classification :

Any one solid propellant includes usually two or more of the following :

I - Oxidizer (nitrates or perchlorates)

2 - Fuel (Organic resins or plastics)

3 - Chemical compound combining fuel and oxidizer qualities.

4 - Additives (to control fabrication process,burning rate,etc.)

5 - Inhibitors (bounded,taped or dip-dried onto propellant) to restrict burning surface.

Two types of Propellants :

I - Composite propellant - A fuel and an oxidizer , neither of which burn satisfactorily without the other. It consists of crystalline , finally ground oxidizers dispersed in a matrix of a fuel compound.

2 - Double base propellants - Contains unstable chemical compounds, such as nitrocellulose, which are capable of combustion in the absence of all other material. There name is given so because many are based largely on a colloid of nitroglycerin and nitrocellulose.

Most propellants contain from 4 to 8 chemicals. In addition additives are used to control physical and chemical properties of propellant. Purposes of additives are :

I) To accelerate or deccelerate burning rate (catalyst)

2) To increase chemical stability or avoid deterioration during storage.

3) To control processing properties during fabrication.

4) To control radiation absorption properties of burning propellant.

5) To increas physical strength and decrease elastic deformation.

6) To minimize temperature sensitivity.

2 - Desired Properties :

a) Ahigh release of chemical energy , promotes a high performance and therefore a high value of combustion temperature and specific impulse.

b) A low molecular weight of combustion products is desirable to increase specific impulse.

c) Solid propellant should be stable for a long period and should not deterdorate physically or chemically during storage.

d) High density of solid propellant permits the use of a small chamber volume and therefore small chamber weight.

e) Solid propellant should be unaffected by atmospheric conditions, e.g. should not be hygroscopic.

f) Propellant should not be subject to accidental ignition . That is , auto-ignition temperature shoul be high , lower combustion limit should be higher than atmospheric pressure, and it should be insensitive to impact.

g) Propellant should have high physical strength properties , e.g. tensile , compressive , shear strength should be high.

h) Small coefficient of thermal expansion will minimize relative motion within chamber and thermal stresses within stored grain.

i) Propellant composition should be chemically inert during storage and operation and not require special materials for chamber or nozzle construction.

j) Propellant should have desirable fabrication properties . e.g. fluidity during casting , easy control of chemical processes.

k) Propellant's performance properties and fabrication technic shouldbe insensitive to impurities or small processing variations to simplify its production , inspection , and reduce cost.

l) Low temperature sensitivity.

m) Exhaust gas should be smokless to avoid their deposition on operational location and to avoid detection in military use.

n) Propellant should lend itself to bonding to metal parts , to application of inhibitors,to production techniques, and should be amenable to the use of a simple igniter.

o) Exhaust should be non-luminous and non-toxic.

p) Method of propellant preparation should be simple.

q) Solid propellant conductivity and specific heat should be such as to control heat transfer to grain.

r) Propellant grain should be opaque to radiation, to prevent ignition at locations other than burning surface.

s) Propellant should resist erosion.

3 - Basic Chemicals :

Oxidizers:

a) The Perchlorates :

(i) Sodium perchlorate $Na\,Cl\,O_4$ 52 % available oxygen.

(ii) Potassium perchlorate $K\,Cl\,O_4$ 46 % available oxygen.

(iii) Magnesium perchlorate $Mg\,(Cl\,O_4)_2$ 34 % available oxygen.

(iv) Ammonium perchlorate $N\,H_4\,Cl\,O_4$ 25.2 % available oxygen.

All produce H Cl and chlorine compounds upon reaction with fuels. Their exhaust is toxic and corrosive. H Cl forms a dangerous fog. All form a dense smoky exhaust with exception of $N\,H_4\,Cl\,O_4$. Ammonium and Potassium perchlorate are only slightly soluble in water so can be used for propellants exposed to moisture.

Perchlorates are produced by electrolysis of chlorides. Their oxidizing potential is high so they are found in propellants of high specific impulse. They are available in white crystal form,and crystal size influences fabrication and burning rate.

b) Inorganic Nitrates :

(i) Potassium Nitrate $K\,N\,O_3$ 39.5 % available oxygen.

(ii) Sodium Nitrate $Na\,N\,O_3$ 47 % available oxygen.

(iii) Ammonium Nitrate $N\,H_4\,N\,O_3$ 20 % available oxygen.

First two have undesirable smoke in exhaust due to solid material formed in combustion products. Ammonium nitrate is smokless , has non-toxic exhaust gas , but oxidizing potential is low, so it is used for low performance, and low burning rate. Ammonium nitrate is produced from nitric acid and ammonia. They are cheap.

c) Organic Nitrates :

(i) Nitroglycerin $C_3\,H_5\,(O\,N\,O_2\,)_3$

(ii) Diethyleneglycol dinitrate , DEGN $(\,C\,H_2 - C\,H_2 - O\,N\,O_2)_2\,O$

(iii) Nitrocellulose $C_6\,H_7\,O_2\,(O\,N\,O_2\,)_3$

All are unstable and capable of oxidizing their organic material.

Nitroglycerin is slightly soluble in water , it causes violent detonation on slight shock. It is less sensitive in solid form. Gelatinization of nitroglycerin with nitrocellulose results in a stable material which is the basis of double base propellants. With double base propellants additives are used to control physical properties , stability during storage , etc. Increase in nitrocellulose increases physical strength. Increase in nitroglycerine increases performance and burning rate.

d) Aromatic Nitro Compounds :

(i) Ammonium Picrate $C_6 H_2 (N O_2)_3 O N H_4$

(ii) Trinitrotoluene $C H_3 C_6 H_2 (N O_2)_3$; TNT.

(iii) Dinitrotoluene $C H_3 C_6 H_3 (N O_2)_2$; DNT.

4 - Fuels :

Fuels are selected for their ability to be oxidized,for adding desirable physical properties to mixtures, and desirable fabrication characteristics. In fabrication many fuels are mixed with crystalline oxidizer, while fuel is in liquid state and at elevated temperatures. After that due to temperature drop it undergoes a physical or chemical change and hardens the grain.

a) Asphalt-Oil type fuels :

Asphalt is heated , liquified and mixed with oxidizers, and then mixture is cast into rocket chambers and allowed to cool to a semi-solid state. Because it tends to crack, special oils are mixed with it to improve its characteristics at low temperatures. Their maximum operating temperature is limited because oil softens it , and becomes unable to withstand high operating temperatures.

b) Plastic Fuels :

Various components of plastic are mixed, the oxidizer is added, and the mixture is cast before plastic sets.(Thermosetting - plastics)

c) Rubber type Fuels :

Several synthetic rubber are used for propellant bases. Elastic properties permit simplerdesign provisions for thermal expansion and contraction of propellant during storage, and better transfer of pressure loads.

5 - Manufacture of some Solid Propellants

A) Fuel-Oxidizer type Propellants (Composite Propellants)

Processing Steps :

a) Grinding of Oxidizer Crystals :

Particle size of crystals effect burning rate, processing properties, and physical properties of propellant. Decrease of particle size results in increase of burning rate. A grinder is used for grinding of crystals to proper sizes under controlled atmosphere conditions , and it is a mechanical process difficult to control.

b) Propellant Mixing :

Proper proportions of crystalline oxidizer , fuel and additives are thoroughly mixed in a suitable equipment. Some fuels require elevated temperatures to attain sufficient fluidity to permit mixing. Other fuels undergo a chemical change and release heat to form the grain. With this last type the grains must be cast, or extruded.

c) Casting of grain :

Many propellants which have a crystalline oxidizing agent are cast into proper grain shape. Either cast directly into combustion chamber, or into special molds. To form required port area a metal core is inserted into the mold. To prevent air bubble formation in grain during casting, a vacuum is sometimes applied.

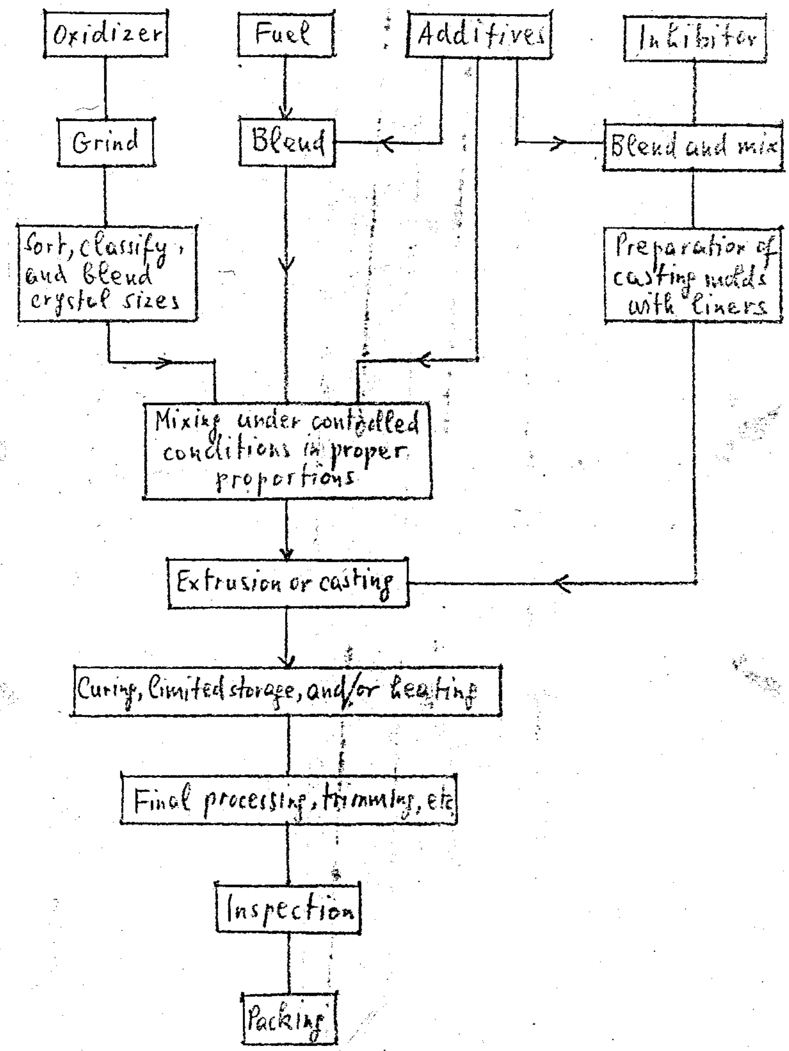
d) Curing :

After casting, propellant is cured. With some fuels, this involves a slow and controlled cooling process, with others a chemical reaction in propellant. The higher temperature,the faster the cooling.

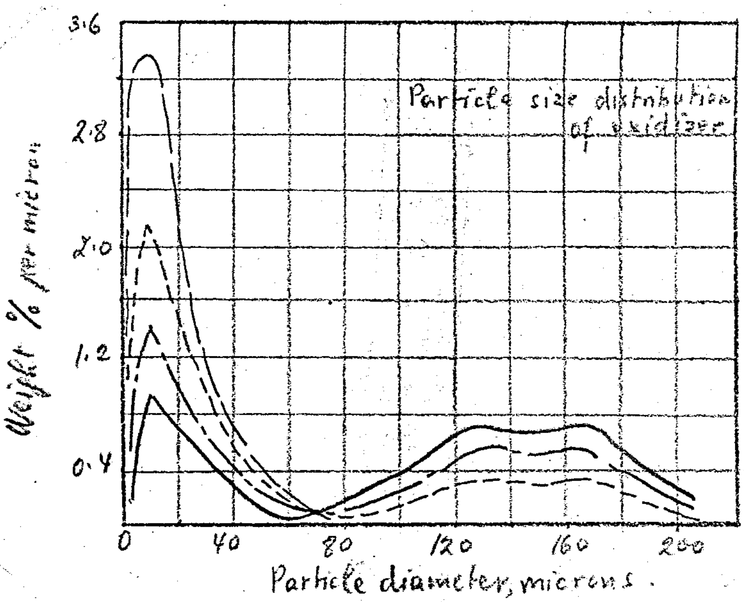
e) Final Processing :

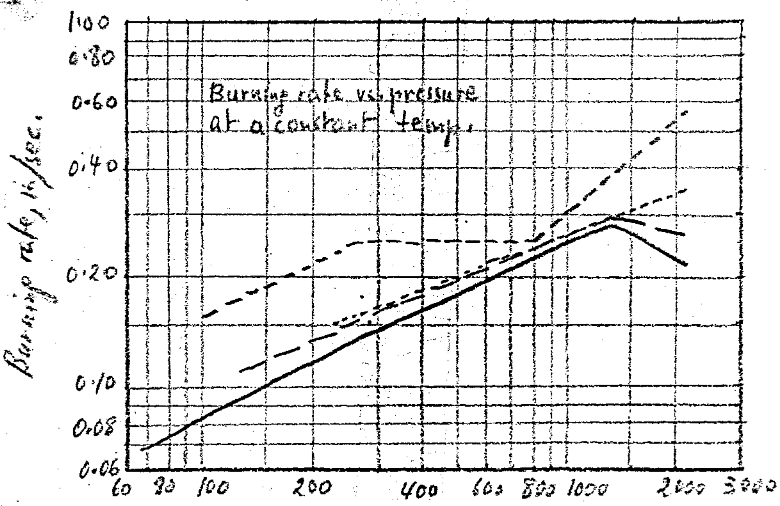
Withdrawal of metal core, cutting of excess propellant in the risers and sealing of the chamber is included in final processing. If a mold is used , the grain is removed from it, cut to proper size, inhibitors applied and inserted into the chamber.

Pressure molding is used for very viscous mixtures.



General processing sequences in the preparation of a fuel-oxidizer composite type propellant.





Effect of oxidizer particle size on the burning rate of a heterogeneous propellant using ammonium perchlorate.

B) Double Base type Propellants (Colloid of Nitrocellulose and Nitroglycerine) :

They are prepared by a solvent or solventless extrusion process, or sometimes by casting process.

In solvent process, a volitile solvent is mixed with the ingradients to improve mixing, casting and extrusion qualities. Resulting grains are dried to remove solvent. However, solvent may cause cracks by evaporation. This process is desirable for thin sectioned propellant grain configuration.

In solventless process blending and mixing is accomplised by running the propellant through heated differential rollers. The material is extruded through a suitable die at very high pressures. Extruded grain is heated to relieve internal stresses. Extrusion process permits exact control of grain size.

In casting process , nitroglycerine liquid with additives is cast into an evacuated mold which is preloaded with small pellets of nitrocellulose. Then mold is heated and nitrocellulose forms a homogeneous solid with the liquid mixture. This process permits manufacture of very large grains.

C) Pressed Powder Charges :

Several propellants are made by pressing or forming mixtures of loose, small particles into grain shape.(Black Powder)

Black Powder manufacture requires mixing ingredients in proper proportions and forming them, sometimes under pressure, into desired grain shape. A binder such as oil is added.

Mixture of Ammonium Nitrate and Guanadine Nitrate with appropriate catalysts for decomposition are used for small rockets. Loose powder compressed under high pressure (7000 lbs/ sq.in.) into suitable molds, forms a charge which is hard and rock-like in appearance.

- -

DESİGN OF SOLİD PROPELLANT ROCKETS

I - Chamber Pressure Selection :

Considerations affecting choice of chamber pressure are :

a) The pressure must be above the lower combustion pressure limit to insure steady smooth burning at all initial grain temperatures.

b) Sometimes high chamber pressures is desired, which increases specific impulse and also for a given thrust it permits the use of a smaller throat.

c) In general overall weight increases with chamber pressure, which justifies heavier supports, more difficult handling, higher heat transfer to metal parts and increased costs. Therefore it is best to chose minimum possible chamber pressure.

d) Desired burning rate also affects selection of chamber pressure , since burning rate is an exponential function of chamber pressure.

There is no general rule for the selection of optimum chamber pressure, the choice must be made on the merits of the specific application.

2 - Grain Selection :

Some of the factors affecting the selection of the geometry of solid propellant charge are :

a) Many applications present a fundamental geometrical limitation , e.g. maximum missile diameter, maximum overall length.

b) In high performance applications it is important to obtain high loading density in chamber and minimize void space and port area, and ineffective propellant remaining after operation.

c) Grain geometry influence heating of the walls and to some extent, of the nozzle. E.g. bonding of internally burning grain to chamber wall, reduce heat transfer and permit the use of a thinner wall.

d) Proper grain design prevents or controls erosive burning.Erosion is the result of increased and erratic burning rate at grain areas where high local gas velocities cause wearing of exposed surfaces.

e) Desired thrust program determines desired burning area program. Burning surface relation with time can be altered to give progressive regressive or neutral burning. e.g. simple rod grain in regressive , internally burning tube grain in progressive.

By partial inhibiting of exposed surfaces by varying grain design by tapering the ends, etc. various thrust programs can be achieved.

f) Physical strength characteristics of a given propellant limits grain geometry.

3 - Burning Rate and Erosion :

Burning rate progresses in a direction normal to exposed burning surface and at a rate which is a function of combustion pressure , ambient grain temperature and propellant properties.

Approximate formulas are :

$$r = a\, p_c^n \qquad (89)$$

$$r = a + b\, p_c \qquad (90)$$

where ;

r = burning rate. p_c = Chamber pressure.

a,b = constants for a given initial grain temperature.

These formulas are insufficient for design purposes, and graphical presentation of propellant data must be used.

Burning is a function of chemical composition, method of propellant preparation, initial grain temperature, burning time, gas velocity adjacent to grain, radiation pattern and geometrical shape of grain. Burning process is very complex and involves interacting reactions in solid,liquid, gaseous phases at high pressures and temperatures.

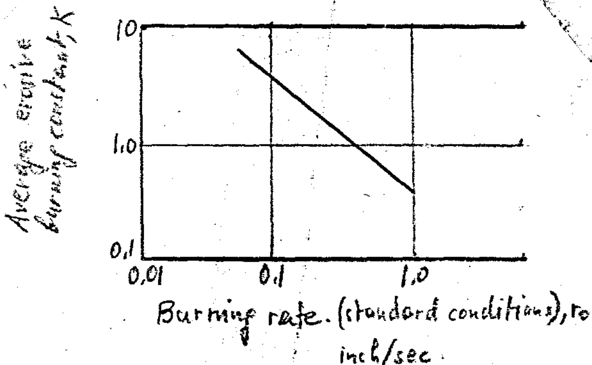
There is a very high temperature gradient between solid propellant and hot gases at burning surface.

Burning rate is determined empirically and presented in plots.

Erosive Burning ; indicates the burning rate of a solid propellant affected by the flow of high velocity gases parallel to burning surface. Erosion usually occurs inside the flow channel near the nozzle and where gas velocity is high.

Erosion is characterized by an increase in burning rate, and expressed by an erosion coefficient ; e.

$$e = r/r_o \qquad (91)$$



Typical variation of erosive constant with burning rate.

where ;

r = burning rate with erosion.

r_o = burning rate without erosion.(without gas flow parallel to surface)

$$e = I + K \; \dot{w}/w^{+} \qquad (92)$$

where ;

K = Erosion burning constant.

$\dot{w}$ = Weight flow rate through channel, lbs/sec.

w^{+}= Weight flow rate which produces M=I in constant area channel.

By permitting erosion under controlled conditions it is possible to decrease channel volume and increase relative amount of propellant in chamber. Erosion is most pronounced at beginning and decreases as flow channel becomes larger.

4 - Forces acting on Grain :

A - Dynamic Loads :

Reasons of dynamic forces acting on propellant grain are :

a) Grain is pushed towards the nozzle due to gas pressure difference between aft and forward end of those grains where hot gases flow in channels parallel to chamber axis.

b) Axial accelerations of vehicle push grain toward the nozzle.

c) Friction of gas flowing through port channels cause a force towards the nozzle.

d) Forces are created also by impact of gases against protrusions or cross perforations.

e) Side acceleration caused by maneuvres put unusual loads on grain.

f) Shock loads or sudden pressurization of chamber by igniter gases.

The grain and grain supports must withstand these forces if design is to be successful.

Support of grain is accomplished by different methods :

a) Grain is inherently strong enough to withstand imposed loads and forces are resisted by the converging section of nozzle or closure around nozzle.

b) Grain is reinforced by bonding structural members into or onto the grain.

c) Grain is mechanically supported by special fixtures at one or more stations in rocket.

In a) and c) grain is supported near the nozzle. So the forces acting on it usually load it in compression as a column. So the column strength is the most significant physical property.

In b) strength of grain and support depends on shear transfer, to keep it from failing mechanically.

Allowable shear and compression forces vary with propellant formulation, rate of loading, manufacturing method, and ambient temperature of grain.

B - Static Loads :

Materials used in solid propellant preparation change in linear dimension and volume with changes in temperature. Thus uneven changes in ambient temperature during storage or shipment induce stresses in grain. e.g. Simple mechanically supported and unrestrained grains may change their length, then stresses introduced by restraining the grain inside the chamber while temperature changes must be kept low to prevent cracking and deformation. Rubber-like propellant formulations may absorb this stress internally. But, double-base propellants do not permit case bonding and must be supported in a flexible manner in the chamber. In these cases of design , propellant is allowed to expand and contract. Sometimes a large spring is used to keep propellant properly positioned with respect to nozzle. In addition to thermal expansion stresses, grain must withstand, to normal handling shocks and loads imposed on a typical rocket.

5 - Pressure and Velocity Distribution along length of Rockets with Burning Surfaces Parallrl to Rocket Axis :

An analysis is made on the following assumptions :

a) The pressure is constant at any one cross-section.

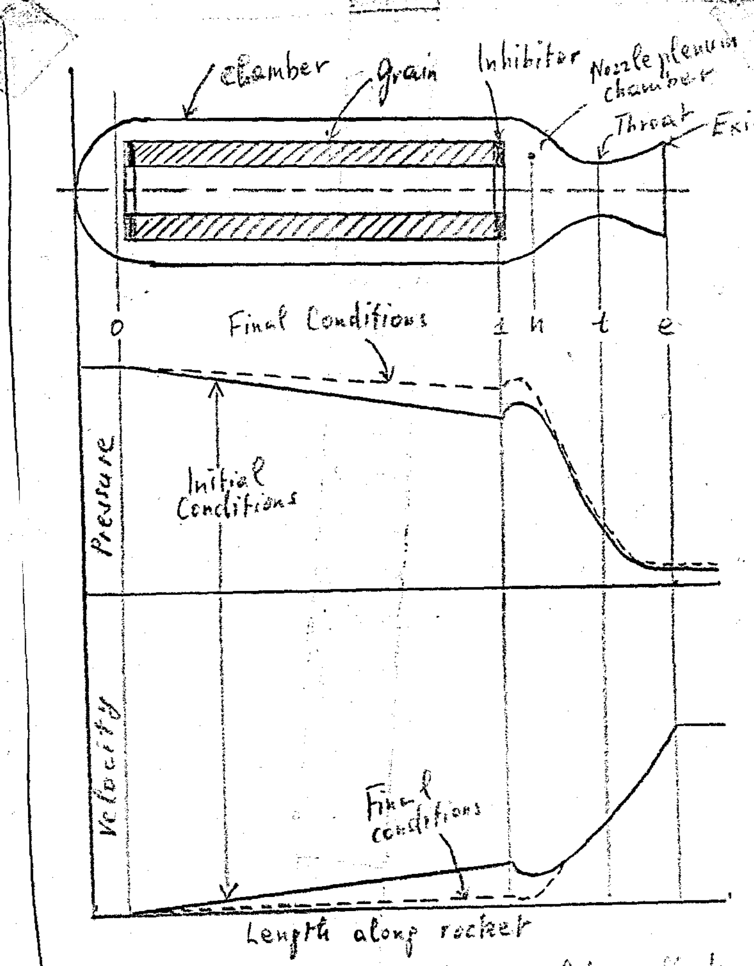
b) The perfect gas laws apply.

c) There is no friction loss.

d) The thermodynamic processes are adiabatic.

e) The port area, A_p , is constant throughout the length of the grain at any instant. There is no erosive burning.

f) Gas is generated on the exposed surfaces of the port areas only.



Pressure and Velocity history in solid propellant rocket.

Pressure is a maximum at the forward end of the rocket chamber and diminishes along length of the grain. Mass is added at all burning surfaces and this mass is accelerated to a velocity corresponding to the available flow area.

Force due to pressure difference between any two cross section acting on the area, A_p equals to the time rate of change of flow momentum.

$$(p_o - p) A_p = \dot{m} v \qquad (93)$$

where ;

p_o = static or stagnation pressure at 0 section.

p = static pressure at any section.

v = gas velocity. $\dot{m}$ = mass flow rate. A_p = port area.

As port area increases with burning time, pressure drop becomes less pronounced. It will be negligible if flow velocity is small.

After passing the grain section , the gases enter the plenum chamber before the nozzle, where they have a velocity decrease, static pressure increase and a loss in total pressure.

Relations for gas expansion depend on geometry of grain and can be approximated by analysis.

Conclusions for grain design with burning occuring on exposed port areas are :

I - Combustion pressure is not uniform along length of chamber. Therefore burning rate also varies, and is fastest near front end.

2 - Because of pressure losses in channels , effective chamber pressure is lower than front end stagnation pressure. Losses of flow should be known for exact computation.

3 - Pressure and burning rate at any section vary with burning time as port area increases.

4 - If pressure losses in plenum chamber are small compared to loss in available energy incurred in accelerating gasses in flow channels, then specific impulse ratio and total pressure ratio across grain may be given by the following equations :

$$(p_o/p_I)_{st.} = I + k M_I^2 \qquad (94)$$

$$(p_o/p_I)_{tot.} = I + k M_I^2 \; / \; (I + k-I/2 \, M_I^2)^{k/k-I} \qquad (95)$$

$$((I_s/(I_s)_{id.})) = \frac{I - \dfrac{((\,(I + k\,M_I^2)\,p_e/p_o\,))^{k-I/k}}{I + k-I/2\,M_I^2}}{I - (\,p_e/p_o\,)^{k-I/k}}$$

where ; (96)

I_s = Specific impulse for optimum expansion.

$(I_s)_{id.}$ = Specific impulse of equivalent rocket which has no internal pressure losses.

6 - Grain Volume and Weight Relations :

Volume occupied by propellant is :

$$V_b = W_b/q_b = \bar{F}\,t_b\,g\,/\,q_b\,c = V_c\,/\,w \qquad (97)$$

where ;

V_c = chamber volume. $\bar{F}$ = average thrust.

w = space factor depending on geometrical grain arrangement.

t_b = effective burning duration of propellant.

c = effective exhaust velocity.

Space factor w, is edfined as :

$$w = \frac{V_b + \text{Void Space} + \text{Liner Volume}}{V_b} = V_c\,/\,V_b \qquad (98)$$

For restricted burning units ; w = I.03 to I.25.

For unrestricted burning units ; w = I.2 to 6.

Length of a restricted end-burning propellant charge is defined as :

$$L_b = r\,t_b \qquad (99)$$

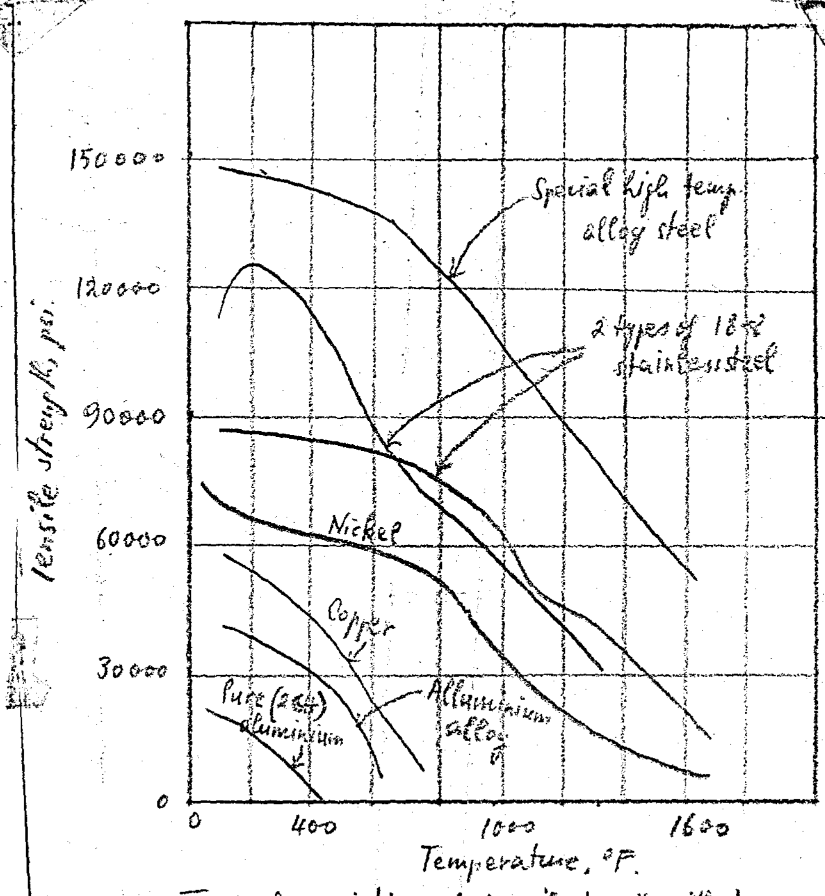
Cross sectional area A_b of a restricted charge is found from volume, weight and density relation as follows :

$$L_b\,A_b\,q_b = W_b \qquad (I00)$$

7 - Hardware Design for Solid Propellant Rockets :

Chamber :

It contains all the grain and must withstand pressure, statrting surges and often severe heating. Exact pressure to which chamber is subjected is difficult to determine.



Typical variation of tensile strength with temp.

Typical set of pressure design factors :

Nominal steady state operating pressure ----------- 100 %.
Pressure at maximum specified grain operating temperature 120%
Design surge pressure ------------------------------ 150 %.
Hydrostatic proof pressure ------------------------- 165 %.
Design yield pressure ------------------------------ 185 %.
Design burst pressure ------------------------------ 205 %.

Temperature of the wall , which carries pressure load determines physical properties of stress-carrying members, therefore largely design and weight of chamber. Since load carrying ability of wall diminishes as it becomes hot, heating of chamber wall must be limited by insulations which will reduce required wall thickness.

High strength heat treated steels are used for light weight units. Fast heating of inner wall surface produces a temperature gradient, therefore thermal stresses across the wall.

Chamber design must also provide for attachment of nozzle , support points for mounting, safety plugs, handling hooks , and provisions for loading the grain.

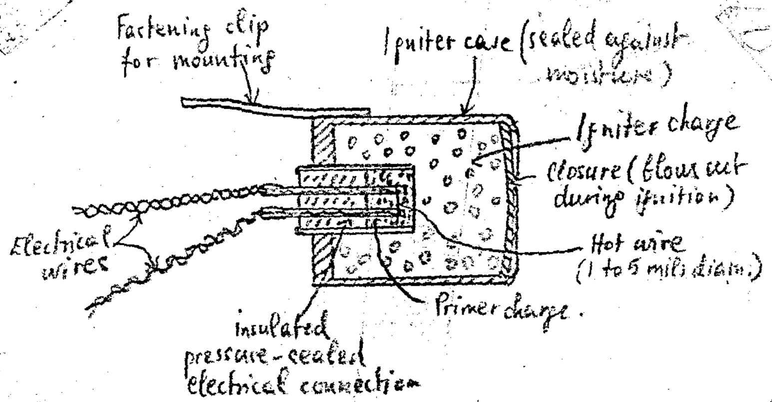
Nozzles :

Nozzles are uncooled for solid propellant units. Nozzle sections are subjected to severe conditions because of relatively high heat transfer rates and mechanical erosion effects of hot, high velocity gases.

For short periods metal nozzles with protective coatings are used. Extra metal is used to act as a heat sink. For larger periods , ceramic nozzles or graphite nozzle inserts are used. Erosion of nozzle throat should be negligibly small to control variation in performance over burning time. Multiple nozzlesare used in solid propellant rockets sometimes. Accurate alignment of nozzle axis with center of gravity of rocket is important, to minimize flight errors. To prevent entry of moisture into chamber and to aid ignition , a closure across the nozzle is provided.

Igniters :

Pyrotechnic type of igniters are used for solid propellant rockets. Sometimes they are put at the forward end , so that ignition



Pyrotechic Igniter.

gases will sweep past the complete propellant charge before reaching the nozzle. Often they are held in propellant charge before and electric wires are connected through nozzle.

Pyrothecnic igniter contains electrically heated wire surrounded by a small amount of primer which is sensitive to temperature and ignite readily and burn when heated. Main igniter charge is adjacent to primer,it produces a hot flame which ignites rocket grain. Igniter cases are sealed usually to prevent abrorption of moisture,and there is a closure which blows out during ignition.

Igniter pressure is the most important design consideration. If it is too high , then nozzle closure will blow out or the shock will fracture the grain. If it is too low then ignition of rocket's charge will not take place. A good ignition theory is not accepted and a empirical approach is generally valid.

An example of igniter is a steel body containing an ignition pellet and a flashcap (electrical wire immersed in black powder). The pellet may consist of ammonium perchlorate or phenal-formaldehyde plastic-potassium perchlorate. The pellet should have a length/diameter ratio of less than 3. Typical units are : I5 grams per pellet,with I.5 in. burning surface,in order to ignite I8 in. of main propellant burning surface.

To achieve smooth combustion,the propellant grain mixture must be homogeneous,ignition should be widely distributed at the same moment.

Another example of electric ignition is the following : a U-bend ina resistance wire is formed and each end is connected to a long piece of lead wire; then one free lead is connected directly to one terminal of I2-volt battery.The other lead is connected to one pole of a switch which has its other pole connected to the other terminal of the battery. The U-bend of the resistance wire is coated with candy propellant and is introduced through the nozzle of rocket into motor and placed against the propellant grain. When the switch is closed,resistance wire heats up and ignites the propellant cast around it. As this propellant lead burns,it ignites the propellant grain.2 switches are used for safety.This igniter is used for sugar-potassium nitrate propellant.

Accessories :

Safety Diaphragm :

It permits the gases to escape when the chamber pressure becomes excessively high. It must be insulated from the chamber so that predetermined bursting pressure will not change with heating by hot gases.

Deflector Cap :

It is located at the downstream of the diaphragm. It directs the gases in several directions to give a very small thrust when gases pass through ruptured diaphragm.

Bursting Diaphragm with an electrically operated pyrothechnic blow-off charge :

It is used to stop operation of solid propellant rocket. Either extra nozzle area ~~is~~ opened by diaphragm may lower chamber pressure to prevent continuous active high thrust combustion, or it may be located so as to nullify thrust produced by exhaust nozzle .

Handling Hooks , Flanges , Brackets , Moisture Seals , Protective Dust Caps , Inspection Ports are the other accesories.

Flanges are used for loading and replacing purposes of propellants.

Brackets are for upright storage and for mounting rockets in vehicles.

- -

EXPERIMENTS , RESULTS AND DESIGN CALCULATIONS

Static firing test were done on a test stand designed by me , and in a I.25 in.diameter I4 in. long test rocket.

Two types of propellants were used in these tests. The propellant mixtures were prepared in the laboratory by me with the help of rocket club members.

The two type of propellants used were the following :

I - Zinc and Sulphur Propellant.

2 - Sugar and Potassium Nitrate Propellant.

Methods of Preparation :

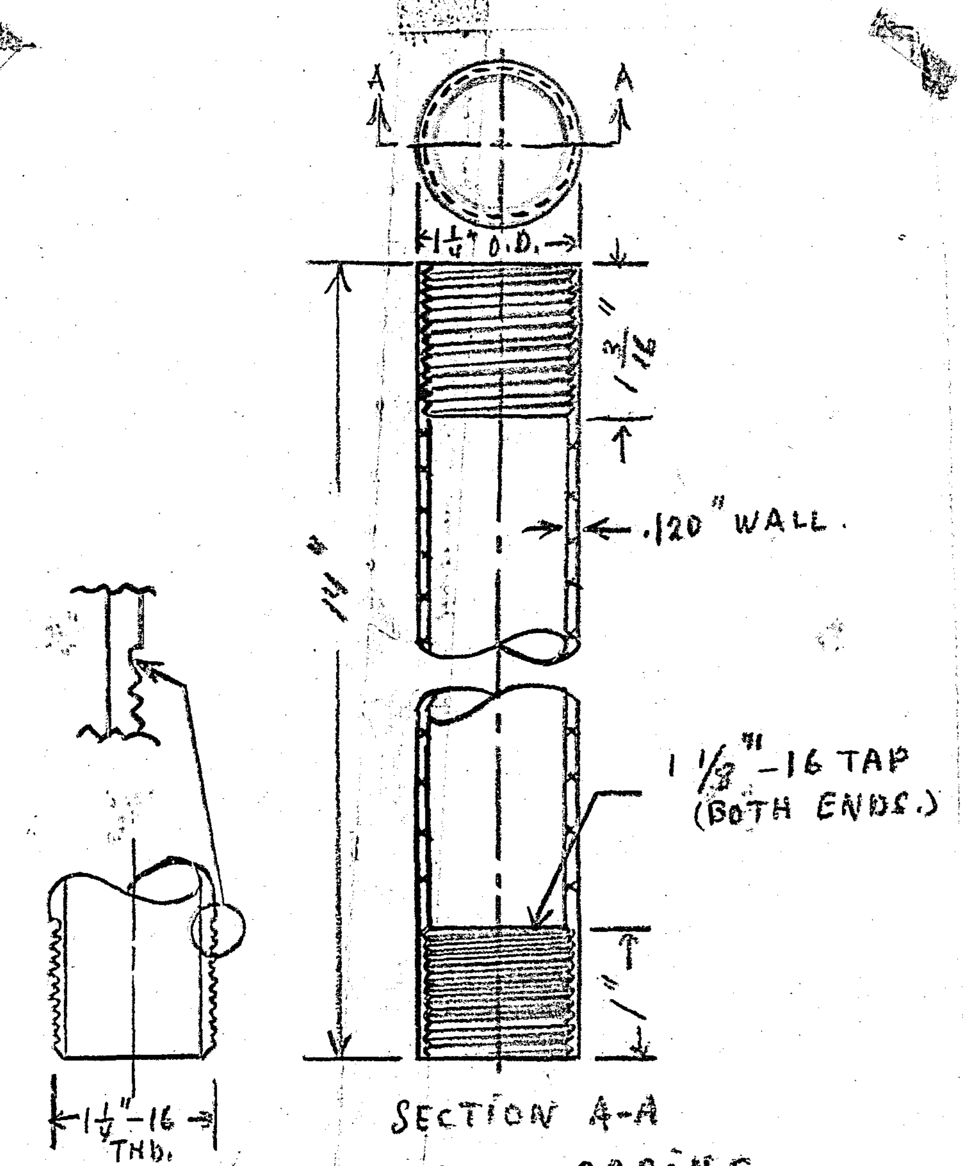
I - Zinc and Sulphur Propellant :

Finely powdered and dry four parts by weight of ~~sulphur~~ Zinc and one part by weight of sulphur are mixed thoroughly, moisture being kept out during mixing. Several proportions of zinc and sulphur were tried and the best burning characteristics were observed during the use of the above mentioned proportions.

If the materials are lumpy or the particles vary greatly in size, or if they are uniformly too large, grind each part in a mortar and sift before mixing. Do not grind any two chemicals together. They may explode. Never work near a flame and never try to burn a bit of propellant just to see how it burns. When zinc and sulphur are thoroughly mixed , the propellant can be tamped gently and carefully into the rocket motor.

2 - Sugar and Potassium Nitrate :

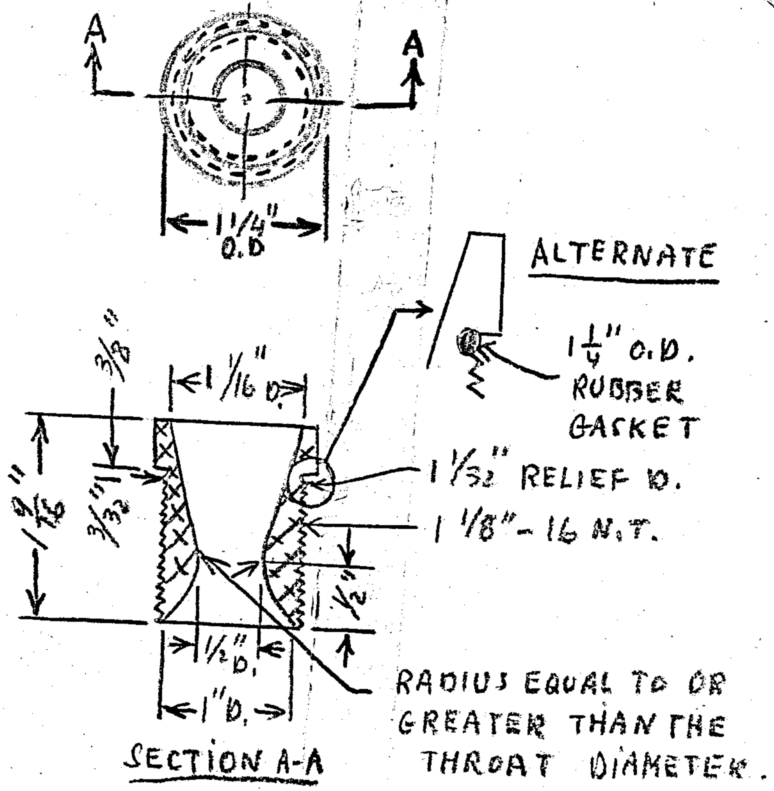
A mixture consisting of two parts sugar and three parts potassium nitrate by weight is taken. One trick is to melt the sugar slowly in a water bath or in a double boiler. The use of caramel candy works well. Keep candy dry and melt it by heating in a water bath in a double boiler up to 350 oF. Use a good and thoroughly tested thermometer for the measurement of temperature. After heating the candy to 350 ^{o}F,potassium nitrate is slowly stirred into the molten candy. If the potassium nitrate is lumpy or coarse, grind and sift it separately before using. Since Potassium Nitrate absorbs moisture from air, it should be stored



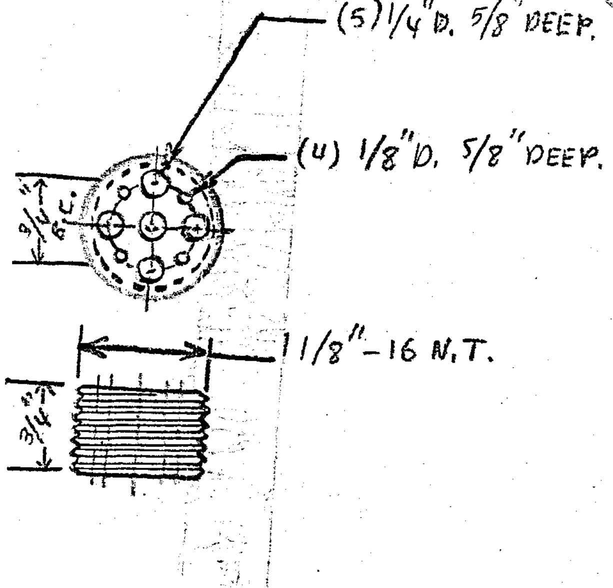
ALTERNATE.

SECTION A-A
ROCKET CASING

TEST ROCKET



ROCKET NOZZLE.



NOSE PLUG

TEST ROCKET

in a dry place and not handled in the open on humid days. Stirring of potassium nitrate into molten candy is done in small portions at a time. If temperature rises above 350 ^{o}F, remove it from water or oil bath, continue to stir until mixture cools down well below 350 ^{o}F, and then continue heating and adding remaining portions of Potassium Nitreate. When all is added, the hot melted substance is poured into the motor. Upon cooling it solidifies and forms a nice grain. This propellant approximately burns at 0.8 in./sec. rate, and at 700 lbs/sq.in. pressure.

Observations : The preliminary static firing tests were performed to see how the propellants burn and it was observed that ; the Zinc and Sulphur mixture burned with a fast burning rate, and gave a quite high thrust but it had non-homogeheous burning characteristics. The Sugar and Potassium Nitrate mixture burned with a slower burning rate , developed a smaller thrust but it had homogeneous burning characteristics.

Then it was decided to use Sugar and Potassium nitrate mixture in the final tests with some modifications such as the addition of Alumininium Powder and the use of plastic material instead of sugar.

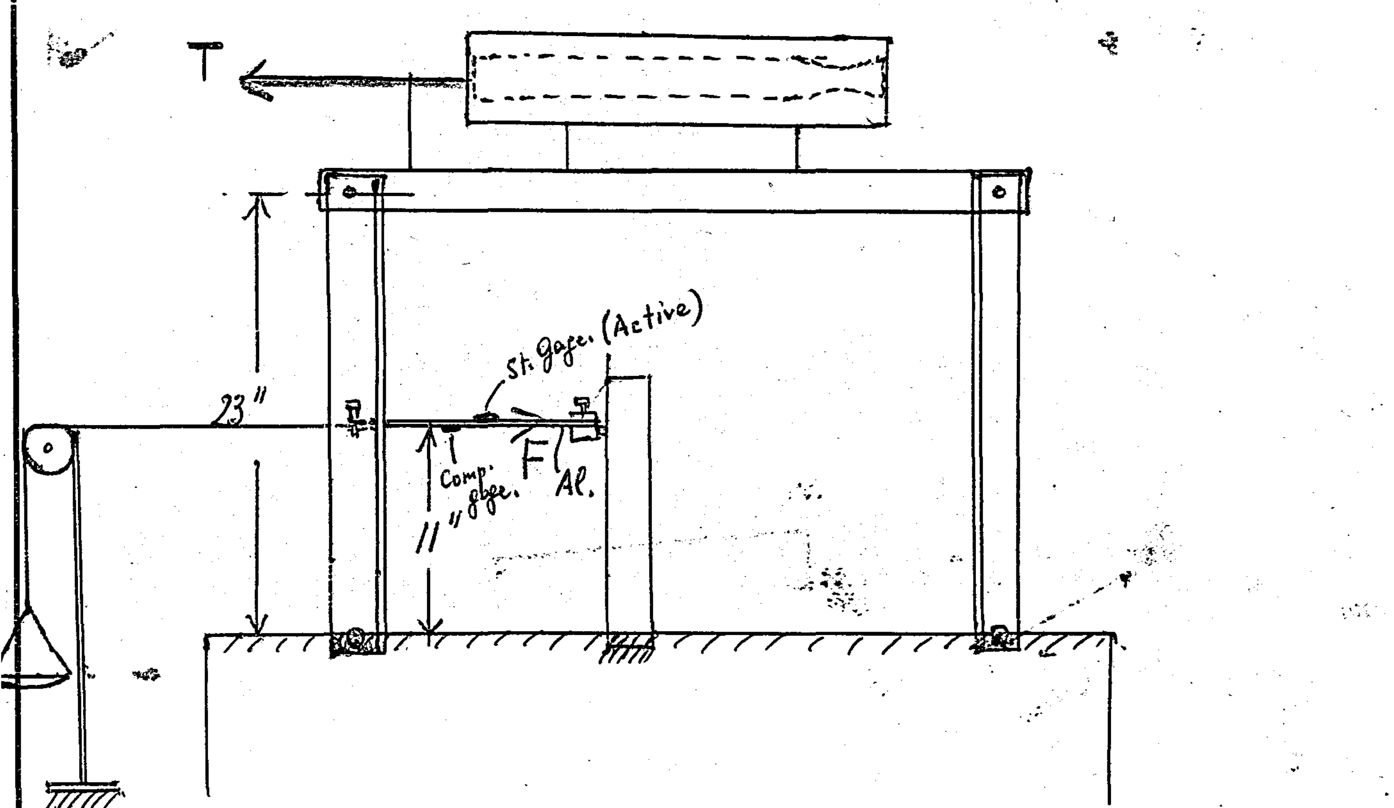
- <u>The Test Stand :</u>

The stand which supports the rocket unit is designed so that it permits a slight motion in the direction of the thrust. The magnitude of the thrust is then measured by a thrust-sensing element, such as by means of a strain gage set up on the aluminium piece as shown on the representative figure and the pictures, and the calibration is done on a strain bridge which is very sensitive.

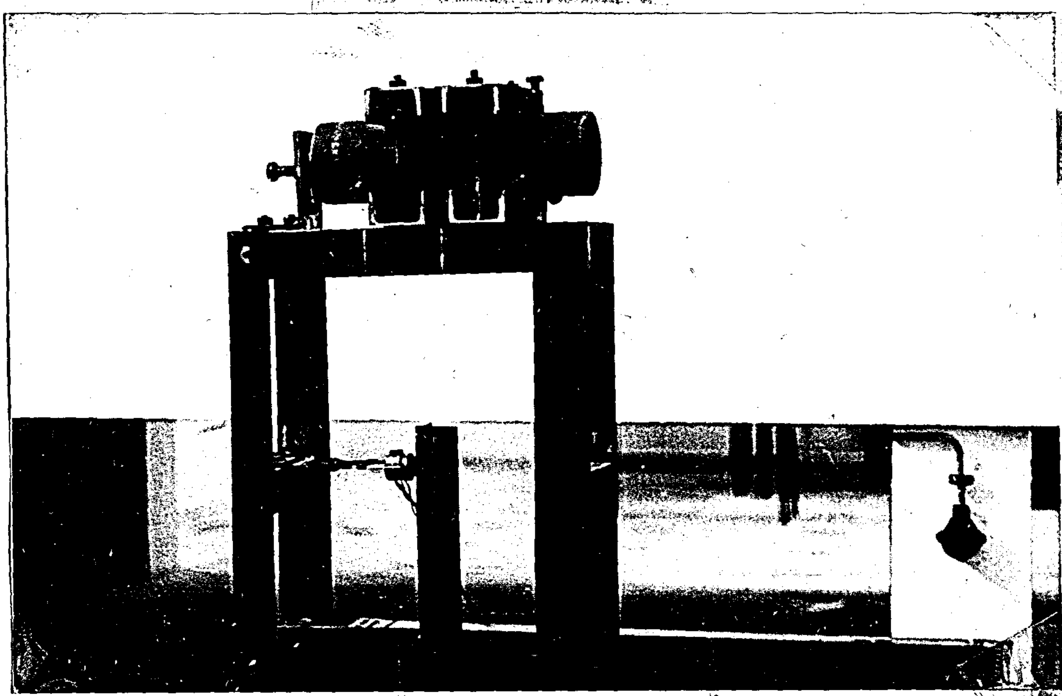
TThe most important point in the design of the test stand is to minimize thrust errors, such as errors due to friction. Frictionn may be reduced by the use of well lubricated journal bearings or ball bearings.

We use two strain gages. Since the deformation is also sensitive to temperature , we use a compensating gage which is not glued to the aluminium piece and is only sensitive to temperature. The other gage which is glued to the aluminium piece is the active gage which is sensitive to strain.

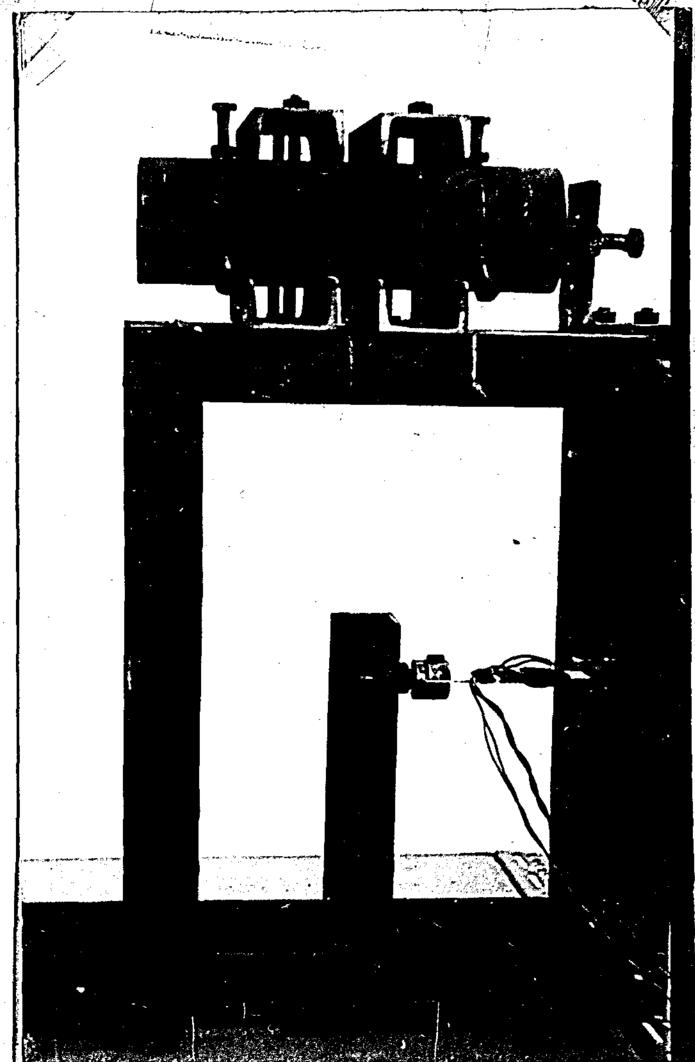
A wire is attached to one side of the test stand , which passes over a pulley and has a weight pan attached to its other end. This arrangement is for the purpose of drawing strain vs. thrust curve for the test stand. By putting different weights in the pan we can measure the strain for each weight and draw a curve and then when we fire the rocket and measure the strain, with that strain we can get the corresponding thrust from the curve.



Rocket Thrust = Force on Aluminium Piece x II / 23.



TEST STAND.



TEST STAND

Working Principle of Strain Gage :

The resistance of a wire is given by the following equation :

$$R = q\ l/A$$

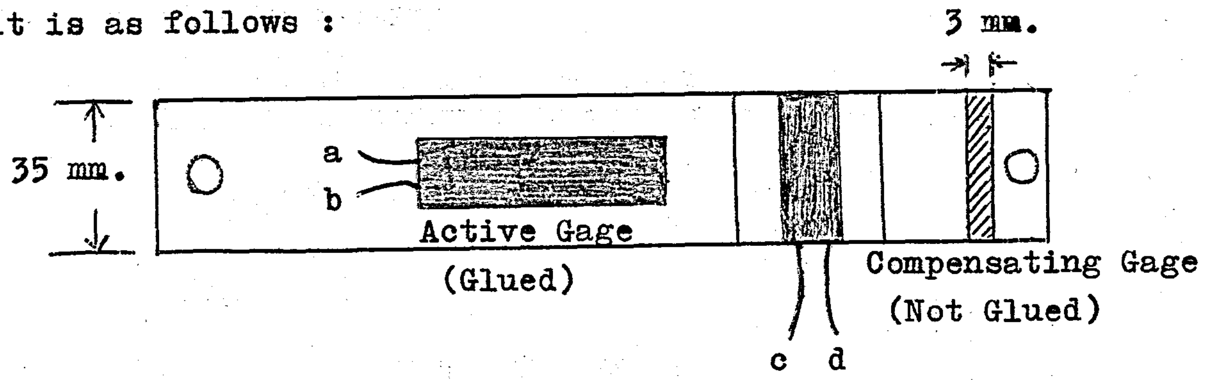
where ;

R : Resistance , q : constant , l : length of wire

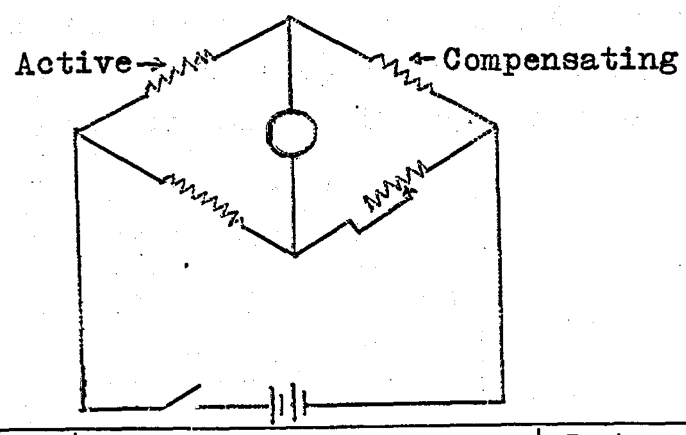
A : cross sectional area of the wire.

Then if a force F is applied on the wire the length of the wire increases to l' and the crosssection decreases to A'. Since the increase in l is not equal to the decrease in A , the resistance R changes. This change in R is detected by the strain gage and the strain bridge. Since the resistance also changes with temperature we use one active gage and one compensating gage . The active gage is sensitive to both the strain and temperature while the compensating gage is only sensitive to temperature.

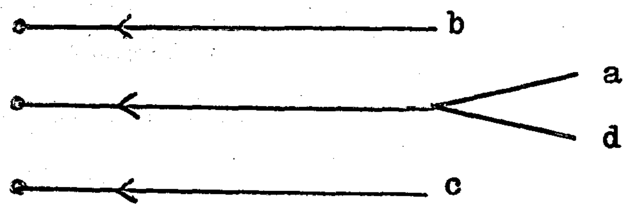
The diagram of the aluminium bar with the strain gages on it is as follows :



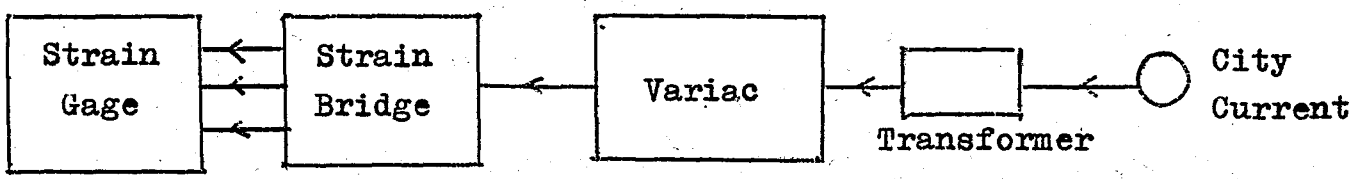
The strain bridge operates like a common Windstone bridge.



The wires (b) and (c) are soldered and connected to one of the 11 chanels of the strain bridge. Also the wires (a) and (d) are connected to the same channel. Then we have the following wiring diagram :



The instrument set up is as follows :



The instruments have the following specifications :

Strain Bridge : (Manufactured according to specifications set up by Cambridge University)

Strain Bridge Name Plate :

Tecquipment
Research and Development Engineers
limited
Nottingham
Unit No: I8726I

MADE IN ENGLAND

Variac : 220 volts voltage is supplied to it through a transformer . The function of variac is to supply voltage in the range 0-220 V

Transformer : It is used to step up the voltage of II0 volts to 220 volts and feed it to the variac. Its specifications are as follows

AROS MILA NO
Autotransformer
No:56I VI000
Volt II0/220 Hz 50
Nod 40/90

- -

Calibration :

The calibration and determination of different values of strain for different values of thrust , and then drawing of graphs of measured strain versus calculated strain and measured strain versus thrust are done as follows :

I - Calculation of Strain :

$$\text{Strain} = \text{Stress} / E$$

$E = 10 \times 10^6$ psi.

$$\text{Stress} = \text{Force/Area} = \text{Weight/Crossectional area of AL.}$$

Area $= 35 \times 3 / (25.4)^2 = 0.163$ sq.in.

Weight $= W \times 2.2$ lbs.

$\underline{\text{STRAIN}} = 2.2 \times W / 0.163 \times 10^7 = \underline{1.35 \times W \times 10^{-6}}$ Micro in./in.

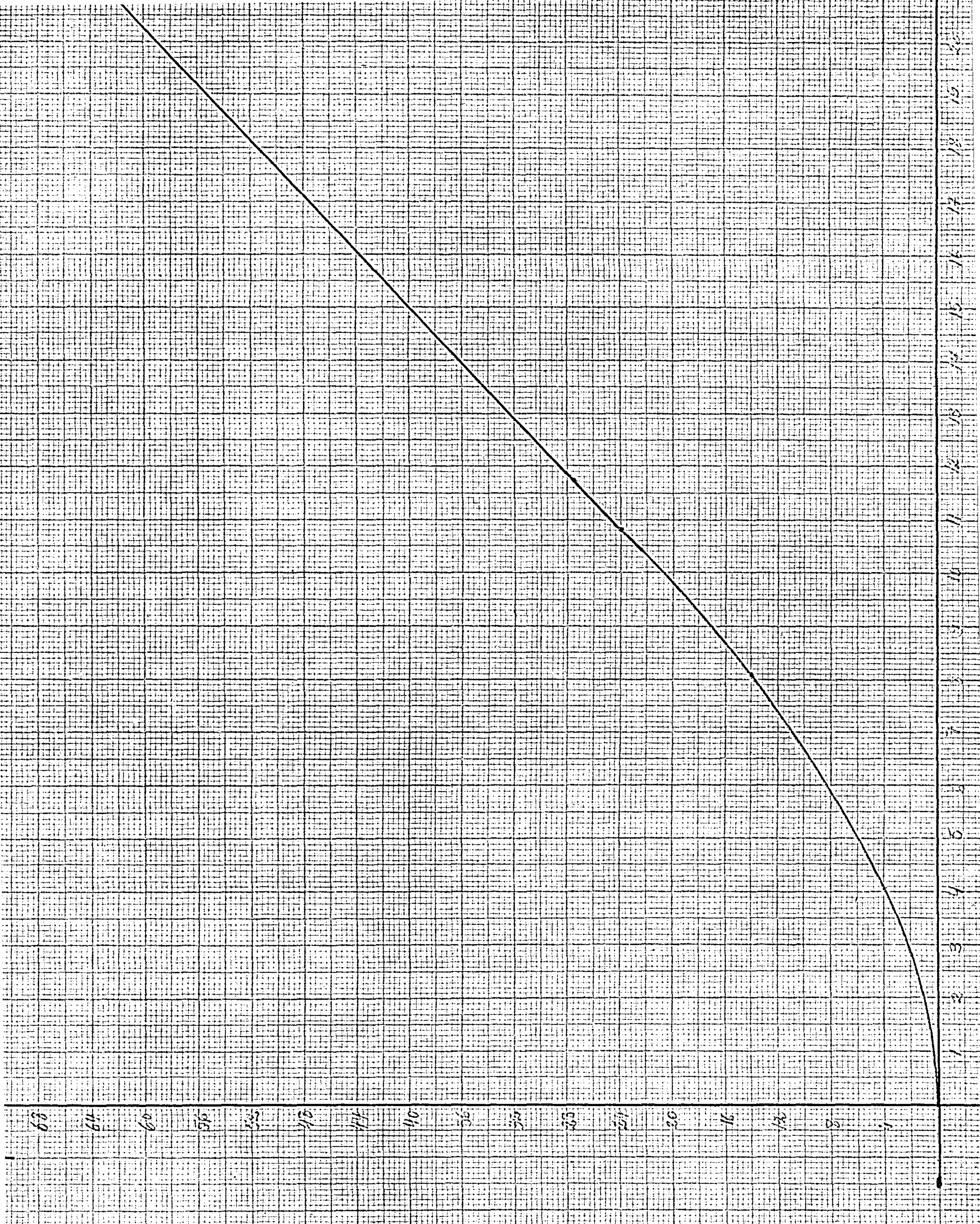
The Strain Bridge measures $1.35 \times W$.

By putting the different weight values in the above equation we get the calculated experiment values of the strain . At the same time we measure values of strain recorded by the instrument.

Then we have the following table of results :

W , Kg.	$Strain_{exper.}$	$Strain_{measured.}$
3	4.05	4
6	8.1	14
8	10.8	24
11	14.8	40
16	21.6	65

Now we can draw two curves. one , as calculated strain versus measured strain, and the other $11/23 \times$ Weight = Thrust versus measured strain . From the second curve we can determine thrust directly after firing with the measured strain value. From the first curve we can obtain the calculated strain value with the measured strain value, and then from the following equation we can get the thrust :



Strain$_{exp.}$ vs. Strain$_{mes.}$

DATE 23 rd. May . I963.

Hor. e x IO^{-6} Vert. mi.in./in. NAME Nurhan Harput.

Thrust = Strain / I.35 x II / 23

- After this calibration we had a firing test . The propella used was Potassium Nitrate , Sugar mixture formed into a radial bu ning grain , and had Zinc,Sulfur mixture throughout the hole to ac elerate the burning rate, and to be able to develope a larger thru But unfortunately this propellant mixture did not give any thrust. it was decided to make another test with a narrower nozzle and bla powder as propellant.

The photographs taken during this final firing test are at ached to the next page. In the first picture we can see the white dense smoke rising from the jet. In the second picture the bright flame of the exhaust jet can be clearly visualized.

Design Procedure :

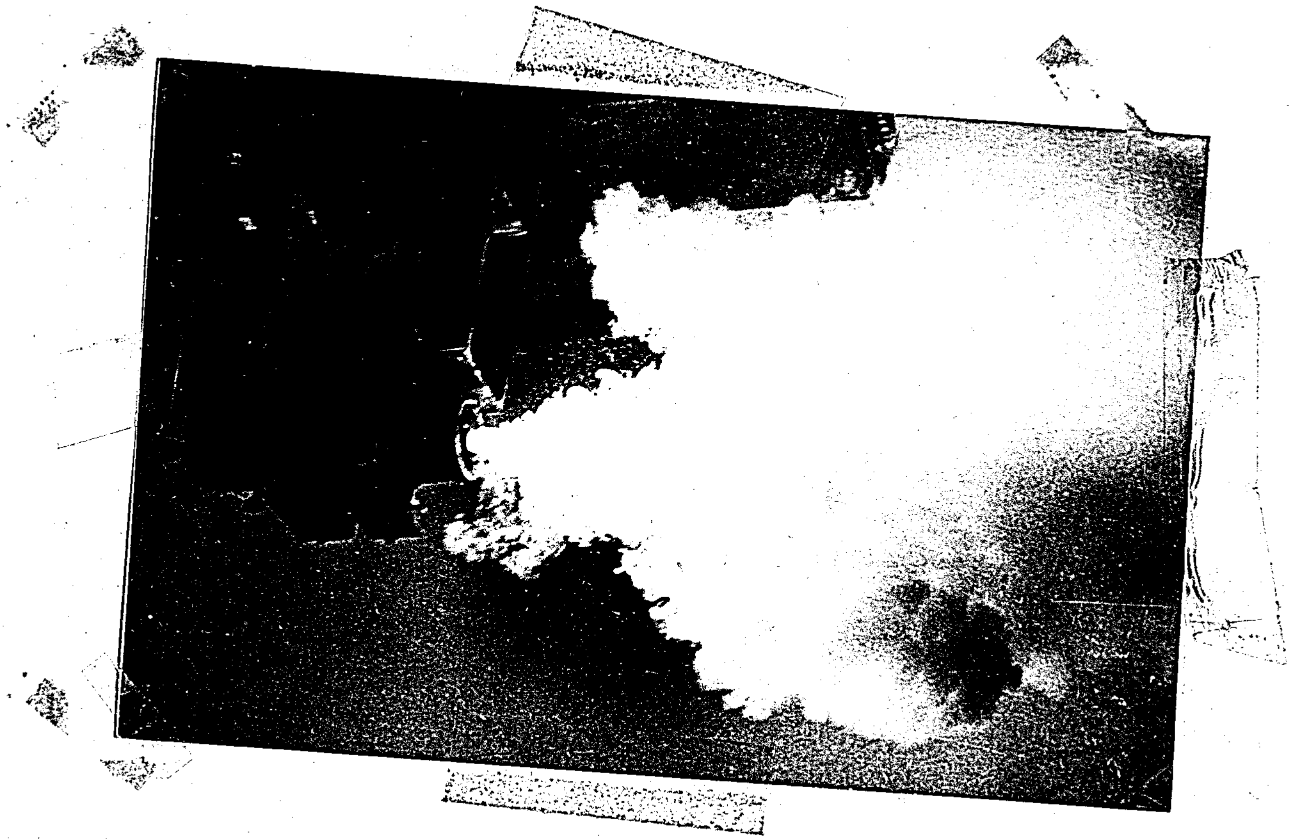
It is necessary first to determine the burning c racteristics of the propellant used in order to be able to design rocket. The propellant which we are going to use when launching th Aluminium rocket is not determined yet. Therefore assuming that we are going to use propellant "X" , we design the rocket as follows:

I - First we weigh our test rocket empty and then we fil the propellant in and weigh again , the difference in weights give the weight of propellant used. This will be used in the determina of burning rate.

2 - Then we fire the rocket and measure the strain from th strain bridge and from the curve we determine the thrust. At the s time we measure the time from the start of burning to the end, th gives the burning time.

3 - Flow rate is obtained by dividing the weight of prope nt with the burning time , we get "Y" lbs./sec.

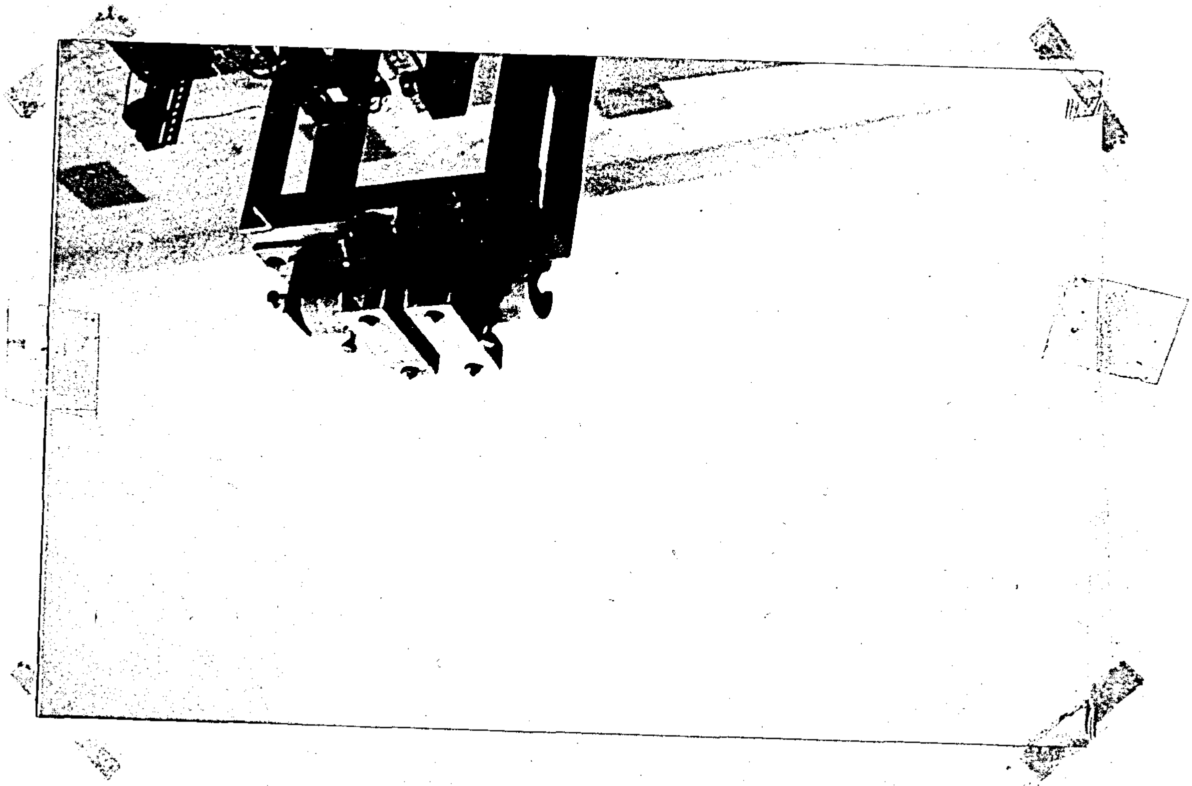
4 - The burning rate is the velocity at which a solid pro llant is consumed during operation. It is measured in a direction rmal to the propellant surface.



Static Firing Test of Potassium Nitrate , Sugar propellant , of a radial burning charge containing Zinc , Sulhfur mixture throughout the hole.

A white dense smoke is rising from the jet.

23 rd. May . 1963 .



Static Firing Test of Potassium Nitrate , Sugar Propellant , of a radial burning charge containing Zinc , Sulfur mixture throughout the hole.

The bright flame of the exhaust jet is clearly visualized.

23 rd. May . 1963 .

Knowing the length of burning surface and the burning time can determine the burning rate by dividing the first with the se in in./sec.

We can determine the burning rate for a desired chamber pr re , for I000 psi. for example , from the formula :

$$r = a\, p_c^n$$

where a and n are empirical constants and given.

5 - Knowing the flow rate and the thrust we can determine exhaust velocity from the equation :

$$c = T / Q$$

where c is the effective exit velocity.

The design is done according to optimum conditions therefo the pressure thrust term is neglected in the thrust equation.

6 - Knowing the exit velocity , we can determine the Speci impulse from the following equation :

$$c = I_s \times g$$

7 - We can measure the density of our propellant mixture b dividing the weight with the volume occupied.

8 - The combustion chamber temperature can be calculated f the isentropic flow equations , knowing the chamber pressure.

9 - The thrust coefficient is taken from the graphs assumi a value for k and for optimum expansion and pressure ratio of $\frac{I00}{I4}$

After determination of these values we come to the design o our rocket. The desired grain shape is a radial burning charge.

I - The total impulse is obtained from the following equat

$$I_t = T \times t_b = I_s \times W_u$$

Knowing the thrust desired and the burning time desired we can ge the total impulse. Then by dividing the total impulse with the Sp fic impulse we get the effective propellant weight , W_u.

2 - The required burning surface for producing the desired rust is found as follows :

$$T = Q \times I_s = q_b \times A_b \times r \times I_s$$

Then ;

$$A_b = T / q_b \times r \times I_s$$

Knowing the density of the propellant , the burning rate and the Specific Impulse , we can calculate the required burning surface for the desired thrust from the above equation.

3 - The web thickness will be :

$$b = 2 \times r \times t_b$$

which can be calculated readily from the known burning rate and the desired burning time.

4 - The maximum diameter of the grain is determined by a given maximum outside diameter , by the thickness of the chamber wall, and by allowance for an annular passage space or port area sufficiently large to avoid erosion. The wall thickness of the cylindrical shell is determined by the conventional relation :

$$t_w = \frac{p_c \times \text{radius}}{\text{stress}}$$

Using a safety factor of about 200 % with the desired chamber pressure , and the maximum tensile stress of Aluminium, the wall thicness can be found from the above equation for a given diameter.

5 - The port area is determined from the following relations:

$$Q = q_b \times A_b \times I_s \quad \text{and} \quad V = R \times T / p$$

then ;

$$A_p = V \times Q / c = Q \times R \times T / p \times \& = \frac{q_b \times r \times A_b \times R \times T}{p \times c}$$

Knowing the propellant density , burning rate , burning area , chamber temperature and pressure and exit velocity , then the port area is determined from the above equation.

6 - Then thos port area is subtracted from total crossectioal area and the total crossectional area of propellant is found. Then the length of propellant charge is found from the following relation:

$$L_b = V / A = (W_u / q_b) / A$$

Since we know , effective propellant weight , density and area of propellant , we can determine length of propellant from the above equation.

7 - The nozzle throat area can be found from the following equation :

$$A_t = T / C_F \times p_c.$$

Since we know the thrust coefficient and chamber pressure , we can determine the nozzle throat area for desired thrust.

From this nozzle throat area , the nozzle throat diameter is calculated, and from the isentropic flow relations and the continuity equation we can get the nozzle exit area, and nozzle exit diameter.

The nozzle convergence and divergence half angles will be 30^{o} , and 15^{o} respectively. According to these angles and the radius of nozzle , the nozzle length is determined readily.

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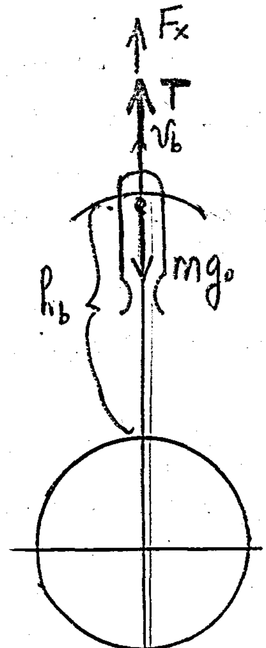

FLIGHT OF ROCKETS

T = Thrust of rocket.

F_x = Resultant of external forces

F_x and T are in the same direction.

Then ; $T + F_x = m \times dv/dt$ (I)

since ; $T = Q \times c$ and $Q = - dm/dt$

then ; $F_x = m \times dv/dt + c \times dm/dt$ (2)

Case I : $F_x = 0$. Therefore we have from equation (2) $\frac{dm}{m} = - \frac{dv}{c}$

Integrating between limits mo and m , and o and v , respectively we get :

$$m = m_o \times e^{-v/c} \qquad (3)$$

where m = total mass at a moment, and

$$m_o = m_{empty} + m_{pay\ load} + m_{propellant}$$

When the rocket reaches the burnout altitude , it will move with no acceleration, with an initial velocity. At burnout altitude so the equation becomes :

$$e^{v_b/c} = I + m_{oprop.} / m_{empty} + m_{pay.} \qquad (4)$$

Since ; $c = I_s \times g$, we see that increasing I_s increases payload.

Case 2 : Only attractive gravity force is present and no aerodynamic drag, therefore

$$-m \times g = m \times dv / dt + c \times dm / dt \qquad (5)$$

Following exactly the same procedure as in first case , we get :

$$m = m_o \times e^{-(gt+v / c)} \qquad (6)$$

Now we calculate the velocity at burnout altitude assuming no aerodynamic drag , no gravity and constant thrust.

$$T = m \times dv/dt = (m_o - Qt) \times dv/dt \qquad (7)$$

Integrating this equation between the limits 0 and v_b , and 0 and t_b , and replacing m_o/m_{empty} = R(mass ratio) , and $T/Q = c = I_s\, g$, we get the following equation for velocity at burnout altitude :

$$v_b = I_s \times g \times \ln R \qquad (8)$$

This represents the characteristics of a rocket and is called , Rocket Index.

Finally we can calculate the burnout altitude , assuming gravitational force present. Therefore the equation becomes :

$$T - m \times g_o = m \times dv/dt \qquad (9)$$

From here we get ;

$$dv/dt = d^2h/dt^2 = T / (m_o - Qt) - g_o$$

Integrating this equation between the limits 0 and h_b , and 0 and t_b , respectively , and integrating twice, and denoting :

$$\frac{Q \times t_b}{m_o} = s = \frac{\text{Initial mass of propellant}}{\text{Take off mass of rocket}}$$

The burnout altitude is given with the following equation :

$$h_b = c \times t_b \; ((\; (I + (I-s/s) \ln (I-s)) \;)) - I/2\, g\, t_b^2 \qquad (IO$$

B I B L I O G R A P H Y

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3 - Rocket Handbook for Amateurs

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