ASSESSING BENEFITS OF DIRECT SHIPMENT IN A DETERMINISTIC SUPPLY CHAIN

by

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ABSTRACT

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Today's manufacturing establishments have to improve their operations such as logistics, distribution and inventory. Integration of production-distribution systems and scheduling are widely used in manufacturing applications to perform production in an efficient way. This study analyses the interaction and integration between the manufacturing facility and the warehouse. We are interested in an evaluation of two approaches: Direct shipment from the plant to customers vs. warehouse replenishment in an environment where Economic Lot Scheduling Problem arises. We consider a single machine in the plant which produces two seperate items.

An MIP model is built up and a wide range of situations are generated with parameters having different values in order to carry out the experiments on a variety of different problems, and to make a good comparison between the two approaches. The models are run for all the problem settings on GAMS where CPLEX is invoked. During the study, the integration of GAMS and MS Excel is achieved through which GAMS directly reads the input and then writes the output data. Besides, the development of a computer programme using Visual Basic is made which creates case-based demand profiles as Excel input-data spreadsheets. This provides an effective and fast analysis tool which is especially important for people in practice. After investigations, the results are reported and analysis is presented.

This study evaluates the benefits of direct shipment and consolidated warehouse shipment, in a deterministic, single-machine multi-item environment, and presents the results of the comparison and contributes to practical use and applicability of the model by the help of computer technology.

ÖZET

DETERMİNİSTİK TEDARİK ZİNCİRİNDE FABRİKADAN MÜŞTERİYE DOĞRUDAN SATIŞIN DEĞERLENDİRİLMESİ

Günümüzün üretim kuruluşlarının, aralarında lojistik, dağıtım ve envanterin de bulunduğu operasyonlarını iyileştirmeleri gereklidir. Üretim-dağıtım sistemlerinin entegrasyonu ve çizelgeleme, daha verimli üretim yapabilmek için üretim uygulamalarında en çok kullanılan yöntemlerdir. Bu çalışma, üretim yeri ile depo arasındaki etkileşimi ve entegrasyonu analiz etmektedir. Bu konuda iki yaklaşıma odaklanılmıştır: ELSP probleminin yeraldığı bir üretim yerinden doğrudan müşteriye satış ile depodan satış. Üretim yerinde, iki ayrı ürünün tek bir makinada üretilmesi sözkonusudur.

Bir MIP modeli kurulmuş ve deneyleri pekçok değişik problem seti üzerinde gerçekleştirmek ve iki yaklaşımın karşılaştırmasını iyi yapabilmek için farklı parametre değerleri ile geniş bir durum yelpazesi oluşturulmuştur. Tüm problem setleri için model CPLEX çözücüsünün işletildiği GAMS üzerinde koşturulmuştur. Çalışma sırasında, MS Excel'den doğrudan veri okumak ve sonuçları doğrudan Excel'e yazmak için GAMS ve MS Excel entegre edilmiştir. Ayrıca, Visual Basic ile her bir senaryo için girdi talep verilerini oluşturan ve bu kayıtları tutacak ayrı Excel dosyaları oluşturan bir bilgisayar programı geliştirilmiştir.. Bu uygulama, özellikle pratikte büyük faydası olacak efektif ve hızlı bir analiz aracıdır. Koşumların ardından sonuçlar rapor edilmiş ve analizler sunulmuştur.

Bu çalışma, tek makina – çok ürünün olduğu deterministik bir çevrede fabrikadan doğrudan müşteriye satış ile depodan satışı değerlendirmekte, karşılaştırma sonuçlarını ortaya koymakta ve bilgisayar teknolojisinin yardımı ile analizin pratik kullanımına ve uygulanabilirliğine katkıda bulunmaktadır.

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LIST OF ABBREVIATIONS

APICS	The Association for Operations Management
BP	Basic Period
CC	Common Cycle
DP	Dynamic Programming
EBP	Extended Basic Period
ELSP	Economic Lot Scheduling Problem
EOQ	Economic Order Quantity
GAMS	General Algebraic Modeling System
GDX	GAMS Data Exchange
JIT	Just-In-Time
MIP	Mixed Integer Programming
NP	Non-deterministic Polynomial
WH	Warehouse

1. INTRODUCTION

In today's competitive, fast-moving environment where there is a need for higher efficiency, improved operational performance due to fast product design, manufacturing and distribution, companies continuously search for ways to improve their operations among which logistics, distribution and inventory management play a main role. In such a challenging business environment, a company must use all of its available resources efficiently. In order to meet these requirements, they should continuously improve their production system performances, maintain competitiveness by facilitating flexibility, reduced response time to changes in their production systems, disturbances in their production plants and reduced response time to customer orders. To achieve that, entities of the production system should be well organized and cooperate with each other while performing their own tasks. Integration of production-distribution systems and scheduling are widely used in manufacturing applications to perform production in an efficient way.

A study done by the Global Logistics Research Team at Michigan State University in 1995 on the logistics aspects of several world class companies shows that the management of these enterprises recognize the fact that the ability to visualize and develop cooperative relationships with other firms throughout the supply chain is critical to world class logistics performance (Sarmiento and Nagi, 1999). The study also shows that leading organizations are developing increased unification among supply chain partners. One of the main motivations of this thesis arose at that point, namely to analyse the interaction and integration between the manufacturing facility and the warehouse. We are interested in an evaluation of two approaches: Direct shipment from the plant to customers vs. warehouse replenishment. We analyze the cases of direct shipping, shipping through a consolidation terminal/warehouse and a combination of both options.

Many companies process more than one item on their facility. In general, having specialized machines and equipment for each type of item that a company processes would be expensive. In such cases, it is typically more economical to purchase one high-speed machine that is capable of processing many products than to purchase many dedicated machines (Dobson, 1987; Gallego and Roundy, 1992). Such high-speed facilities typically have limited capacity and are only capable of processing one item at a time, requiring a setup each time a different item is scheduled for processing (Gallego and Roundy, 1992). Hence, the dominant characteristics of a single-machine multi-item system are the following:

- A single machine
- Multiple items
- One item processed at a time
- Limited capacity but sufficient to satisfy total demand
- A setup time between the processing of different items
- Items may differ in cost structure and machine utilization

The system described above refers to the Economic Lot Scheduling Problem (ELSP). In this study, we analyze the trade-offs between direct-plant shipment, consolidated warehouse replenishment and a combination of both options in an environment where ELSP arises. We consider a single machine in the plant which produces two several items.

We are motivated by the following facts: First, in spite of the awareness and willingness for cooperative efforts, there seems to be a lack of tools and methods that can help managers in the analysis of the integrated systems. To undertake the optimization of integrated problems, we can make use of the increasing capabilities of computer technology. Second, as companies move towards the reduction in inventory levels, models for the production-distribution problem will become more important. Due to the fact that to this date very little work exists in this direction, the analysis of production-distribution systems still is a wide open area for research (Sarmiento and Nagi, 1999). Further, Chen and Vairaktarakis (2005) state that recent reductions in inventory levels have created a closer linkage between production and distribution in practice. Therefore, more academic research is needed to model direct production-distribution problem-solving techniques that can be used in practice. In addition, the problem of inventory control in a single-machine multi-item system exists in a variety of industries. Obviously, although there is an extensive research on the ELSP, the models are not widely used in practice. The reason for this should be investigated, but one reason for the low practical use may be due to the mathematical skill that is needed for implementation and use of these models. Increasing the practical usefulness and making practitioners aware of the theoretical models in order to increase the practical use may be the most important challenge for researchers and practitioners together.

A single-machine multi-item system has many practical applications and can be found in many industries. For instance, it can be found in process industries, especially in consumer goods industries. The Association for Operations Management defines process industries as the group of manufacturers that produce items by mixing, separating, forming and/or performing chemical reactions (Brander *et al.*, 2005). Paint manufacturers, refineries, and breweries are examples of process industries. According to Silver et al. (1998), single-machine multi-item systems apply most commonly in continuous flow processes, although they are also found in batch flow processes. Brander *et al.* (2005) state that continuous flow process industries are characterized by one (or very few) process steps and a low added value. Examples of flow process industries include bulk chemicals, glass manufacturing, paper production and steel & aluminum production. They also argue that batch process industries are characterized, in general, by several processing steps, a mainly convergent material flow ("assembly") and a high added value. Examples of batch process industries include pharmaceutical, other fine chemical industries and refineries. Textile, plastic extrusion, printing, packing, and tire industries are also examples of applications where single-machine multi-item systems can be found. In continuous manufacturing, main processing operations have very high startup and shutdown costs. The machine also incurs a high changeover cost when it has to switch over from one product to another.

Last, a project done by a cross-functional team from Monsanto Co., a large multi-national with headquarters in the USA within Crop Protection business, and from Massachusetts Institute of Technology (MIT), under the auspices of the Integrated Supply Chain Management consortium at MIT caught our attention. Their mission was to develop and apply a model of the production, inventory and distribution network to guide strategic level decisions about Monsanto's Crop Protection business. The solution to the level production problem was to ship directly from the plant to retailers. Direct shipments and vendor managed inventory also offers a process enhancement with a single point of contact for warranty, returns, technical support and customer service.

This study aims at developing a model for evaluating the benefits of direct shipment and consolidated warehouse shipment, and of the combination of two like a hybrid system in a deterministic, single-machine multi-items environment and presenting the results of the comparison. The goal is at the same time to contribute to practical use and applicability of the model by the help of computer technology.

We aim to handle the following aspects:

- When should each item be produced? (Scheduling)
- How many units should be shipped to warehouse when? (Transshipment)
- When should the items be sold directly from plant, and when from warehouse? (Integration of production-inventory-distribution)

The rest of the study is organized as follows. In section 2, the results of the literature survey conducted are provided. Section 3 defines the problem, introduces the notation, assumptions are stated and the mathematical model is presented. In section 4, results of the computational analysis are given. Last section presents the conclusions, discussion, and identifies future research directions.

2. LITERATURE REVIEW

In the scope of this literature survey, we have focused on three categories. The first category presents an overview of the previous studies within inventory control. Secondly, the studies related with integrated analysis of production-distribution systems are reviewed. In the third category, the studies about Economic Lot Scheduling Problem (ELSP) are presented.

2.1. Inventory Control

Many of us deal with inventories daily, normally without taking any bigger notice of them. For instance, we store food and many other items at home since it simplifies our daily life. However, we try to keep the inventories low since we otherwise would run out of space or money. Food may also be out of date if stored too long time, which presumes some kind of strategy behind the purchase of perishable items. Therefore, we are all familiar with the need to manage inventories intuitively.

Inventories can be seen as stockpiles of raw materials, components, work in process, spare parts, and finished goods and play an important role in many activities. For instance, hospitals rely on supplies of medicines, blood for transfusions, surgical equipment, and other things. We would also be grounded or delayed without airlines' inventories of jet fuel. Furthermore, restaurants could not operate accurately without inventories of food. These are only some examples were lack of inventories could have negative effects. Most of the items that we use at home or at work have been objects for a long chain of production and distribution activities that are fed by diverse inventories.

Inventories can be categorised from how they are created. For instance, Krajewski and Ritzman (2004) identify four different types of inventories in this context: cycle, safety, anticipation, and pipeline. Cycle inventory results from attempts to order or produce in batches instead of one unit at a time and is the portion of the total inventory that varies directly with the lot size. Safety stock is held by a company to protect against uncertainties in demand, lead time, and supply. Anticipation stock is used to build up inventory when the production capacity might be lower than the expected future demand, e.g. in cases when demand vary due to seasons. Finally, the pipeline inventory consists of items moving from point to point in the material flow system. Additionally, Silver et al. (1998) define congestion stocks, which occur when multiple items share the same production equipment, particularly when there are significant setup times. These items compete for limited capacity and inventories are built up in order for the equipment to become available.

As mentioned above, lack of inventories could have serious negative effects in many situations. Axsäter (2000) argues that the two main reasons for inventories are economies of scale and uncertainties. Lambert and Stock (1993) mean that inventories serve five purposes within a firm; they enable the firm to achieve economies of scale, they balance supply and demand, they enable specialization in manufacturing, they provide protection from uncertainties in demand and order cycle, and they act as a buffer between critical interfaces within the channel of distribution.

As seen, there are many reasons for inventories. Of course, there are also reasons against inventories. First, inventories represent a monetary investment in goods on which a firm must pay, rather than receive, interest. Inventory holding cost is the cost of keeping items on hand, including interest, storage and handling, taxes, insurance, and shrinkage (Krajewski and Ritzman, 2004). The annual cost to maintain one unit in inventory typically ranges from 20 to 40 percent of its value (Ballou, 1999; Krajewski and Ritzman, 2004). Interest or opportunity cost arises since a company may obtain a loan or forgo the opportunity of an investment that promises an attractive return to finance the inventory. This is usually the largest component of the holding cost, often as high as 15 percent (Krajewski and Ritzman, 2004). Inventory also needs space and must be moved into and out of storage, which implies storage and handling cost. High inventories may imply higher taxes in the end of the year as well as higher insurance costs. High inventories may also imply costs for pilferage, obsolescence, and deterioration. Ballou (1999) argues that inventories also can mask quality problems. Due to all these positive and negative effects, managing inventories carefully is important. Zipkin (2000) means that managing inventories well is the difference between corporate success and failure.

For the last two decades, inventories have become one of the dimensions upon which companies compete on a global scale and many of the successful companies keep inventories lean (Zipkin, 2000). Axsäter (2000) states that the objective of inventory control is to balance conflicting goals. One goal is to keep inventory levels down to make cash available for other purposes. On the other hand, the purchasing manager may wish to order large batches to get volume discounts. The production manager normally wants high inventories to support long production runs and avoid time-consuming setups. Additionally, lack of inventory may shut down the production line due to missing materials. The marketing manager would like to have high inventories over a broad range of finished goods to allow quick response to customer demands. Segerstedt (1999) means that, through production and inventory control, profitability is achieved by the difficult balancing of resource utilization (high), capital and inventory investments (low) and market services (high). These objectives may be contradictory in some aspects but cooperate in others depending on the situation. For instance, high utilization of resources may imply long delivery times resulting in low market service. Therefore, a balance between these objectives must be found as illustrated in Figure 2.1. (Olhager, 2000).

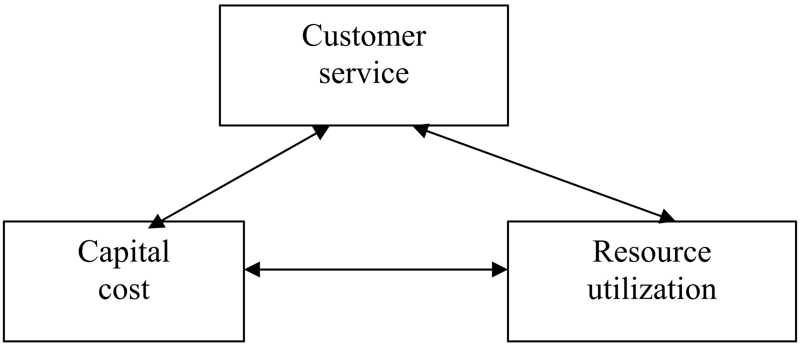

Figure 2.1. Contradictory objectives.

A company must find a balance between these objectives that coincides with other aggregate objectives of the company. If this is done in a good way, we make one step towards achieving the mission of logistics, which is getting the right goods or services to the right place, at the right time, and in the desired condition, while making the greatest contribution to the firm (Ballou, 1999).

An inventory control system can be designed so that the inventory position is monitored continuously or at certain time intervals. The continuous review system triggers an order, which will be delivered after a certain lead time, as soon as the inventory is below a certain level. The other alternative is denoted periodic review, since it considers the inventory position only at certain time intervals, which in general are constant. The two most common ordering policies connected with inventory control are denoted (R,Q) and (s,S) policy. When using an (R,Q) policy, the two parameters R (Reorder point) and Q (Order quantity) need to be determined. The reorder point determines when it is time to place an order and the order quantity determines the amount to be ordered. The (s,S) policy is similar to the (R,Q) policy. When the inventory position is below s, the quantity up to a maximum level, S, is ordered. This policy differs from the (R,Q) policy in the way that we do not need to order a specific order quantity or multiples thereof.

Axsäter (2000) classifies inventory control from the problem that is studied and argues that there are mainly three branches of problems. First, control problems for a single inventory with items that can be handled independently. For this, one of the most well-known results in the inventory control area may be used; the economic order quantity (EOQ). EOQ originates from Harris (1913), but it is also associated with Wilson (1934) and known as the Wilson lot size. The EOQ is modified to other settings as well, e.g. finite production rate, quantity discounts, allowed backorders etc. The EOQ assumes that each item can be considered independently and pays no attention to scheduling.

The second branch of inventory control problems in the classification leaves the assumption of independent items and considers coordinated replenishments of two types. The first reason for coordinated replenishments is to get a smooth production load, for instance when a group of items is produced in the same production line. The other reason is where orders for a group of items should be triggered at the same time, so called joint replenishments. For instance, we may want joint replenishments to get discounts or to reduce transportation costs. The third branch of problems in inventory control considers multi-level systems, where several inventories are connected to each other, for instance in a production system with several stocks or in a distribution system with a central warehouse and retailers (Axsäter, 2000).

2.2. Integrated Analysis of Production-Distribution Systems

Every production company's desire is to produce at a competitive cost while capturing a significant share of market demand, so that reasonable profits are obtained. In today's competitive, fast-moving environment where there is a need for higher efficiency, improved operational performance due to fast product design, manufacturing and distribution, companies continuously search for ways to improve their operations among which logistics, distribution and inventory management play a main role. In this area, the flow of physical goods from manufacturers to customers is a major focus. Moreover, product delivery at a reasonable cost has recently become a critical factor in the survival of emerging e-businesses (Lee and Whang, 2001). Recently, a new approach to the analysis of production and distribution operations has been identified, which has proven to be of significant relevance to companies that have adopted it. This approach is based on the integration of decisions of different functions (e.g., supply process, distribution, inventory management, production planning, facilities location, etc.) into a single optimization model. Sarmiento and Nagi (1999) states that the basic idea behind these model is to simultaneously optimize decision variables of different functions that have traditionally been optimized sequentially, in the sense that the optimized output of one stage becomes the input to the other (first setting inventory levels and then scheduling distribution, for instance). A recent study done by the Global Logistics Research Team at Michigan State University in 1995 on the logistics aspects of several world class companies shows that the management of these enterprises recognize the fact that the ability to visualize and develop cooperative relationships with other firms throughout the supply chain is critical to world class logistics performance (Sarmiento and Nagi, 1999).

Due to their applicability in real-world situations, the multi-echelon systems have caught the researchers' attention. The multi-echelon inventory/distribution systems represent a special category of inventories encountered in practice where several installations are involved. A special type of such inventory systems deals with one-warehouse and N-retailers (e.g., a chain of stores supplied by a single regional warehouse). Models that consider warehousing/distribution as the first echelon and retailer or the end customer as the second echelon covers the "single supply and multiple demand locations" problem presented by a system with one depot (warehouse) that supplies multiple customers (retailers). In this problem, the warehouse is the sole supplier of N retailers. Customer demand occurs at each retailer at a constant rate. This demand must be met as it occurs over an infinite horizon without either shortages or backlogging. Orders placed by retailers generate demands at the warehouse. There is a holding cost rate per unit stored per unit time and a fixed charge for each order placed at the warehouse and at each retailer. The demand rates, holding unit costs, and setup costs are stationary and facility dependent. Delivery of orders is assumed to be instantaneous, that is, lead times are assumed to be zero. The goal is to find a policy with minimum or near-minimum long-run average cost.

This one-warehouse and N-retailers system was examined by Schwarz (1973) and he showed that the form of the optimal policy can be very complex; in particular, it requires that the order quantity at one or more of the locations varies with time even though all relevant demand and cost factors are time invariant. Thus, he considered the possibility of restricting the class of strategies, where the order quantity at each location does not change with time and he determined the necessary conditions of an optimal policy. Moreover, he provided a solution method for the one-warehouse and N identical retailers problem and suggested heuristic solutions for the general case. Schwarz (1973) proved that an optimal policy can be found in the set of ‘‘basic’’ policies. A basic policy is any feasible policy where deliveries are made to the warehouse only when the warehouse has zero inventory and, at least one retailer has zero inventory. Moreover, deliveries are made to any given retailer only when that retailer has zero inventory. In addition, all deliveries made to any given retailer between successive deliveries to the warehouse are of equal size.

In the context of integrated analysis, one-warehouse and N-retailers problem has been analyzed by Federgruen and Zipkin (1984), Federgruen *et al.* (1986), Burns *et al.* (1985), Anily and Federgruen (1990), Viswanathan and Mathur (1997) and Chandra (1993). The problem is quite recent and not much literature is available on it. This problem is sometimes confused by researchers with the Inventory/Routing Problem. The “one depot, multiple retailers” problem is a tactical problem that considers a depot that allocates a product (or products) to several retailers in such a way that overall costs are minimized (Sarmiento and Nagi, 1999). These costs generally include holding and shortage at retailers, and transportation costs. The decision variables of interest are shipment sizes and delivery routes. Given that the formulation of the problem is NP-hard [1], heuristic solutions have been developed. The general approach taken to the solution is to analyze the case of direct shipments between depot and retailers and use this analysis as a base of for the development of algorithms for the case when delivery routes are also to be determined. With the exception of Anily and Federgruen and Viswanathan and Mathur which only present the routing case, all other authors present both cases, direct shipments and shipments through routes.

Burns *et al.* (1985) consider an infinite horizon problem and develop an analytical method to minimize overall costs. They are interested in comparing the cost performance of the two distribution strategies: direct shipping and peddling. Inventory costs are included in the objective function in an aggregate form, i.e., the cost of holding inventory at the depot, on transit and at the retailers is obtained by approximating the time that the product spends in the system and multiplying it by an interest factor. For direct shipping they obtain an EOQ type of solution by trading off inventory and transportation costs. If the shipment size obtained from the solution greater than the capacity of the truck (which is given) then a full-truck load is scheduled. For the peddling strategy a full-truck load is optimal shipment size.

Just-In-Time (JIT) environment where frequent, small shipments are usually required between suppliers and manufacturers many times transportation contracts for specific volumes are made to assure proper supply. In such situations emergency shipments are contracted by suppliers whenever customer's demand presents a sudden increase and a higher amount of product is required at the customer's location. The problem faced by the logistic manager is to decide the number of vehicles required to make the shipments in such a way that a balance between spare capacity in contracted vehicles and use of emergency shipments are achieved. Yano and Gerchak (1989) analyze this problem. According to Sarmiento and Nagi (1999), two important contributions of this work are the following. First, it deals with the transportation system at a strategic level, i.e., it determines the optimal size for the regular fleet and establishes the time between shipments. Second, it shows that full-truck loads and one vehicle per shipment are not

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[1] The term NP-hard is used to distinguish difficult problems with respect to the likelihood of finding an efficient method for solving the problem.

always optimal, this is due to the fact that depending on demand variability it can be more profitable to have spare space in the regular vehicles, even if more than one truck is needed per trip.

In "production-inventory-distribution-inventory" problems, Blumenfeld *et al.* (1985) and Benjamin (1989) consider several locations on each echelon for the integrated analysis and present formulations of deterministic models. Blumenfeld *et al.* (1985) are interested in analyzing the existed trade-offs between transportation, inventory holding and production setup costs in the network. They analyze the cases of direct shipping, shipping through a consolidation terminal and a combination of both options, and obtain shipment sizes that trade-off these costs. Benjamin (1989) considers the simultaneous optimization of the production lot size problem, the transportation problem and the EOQ problem. He accounts for supply constraints and explicitly considers inventory costs; his interest is to find optimal production sizes for supply points and order quantities for demand points. His model assumes an unconstrained transportation system and direct shipments between each node, therefore, no routing decisions have to be taken. Although this work considers multiple products, no product to truck allocation decisions are made. He presents a comparison between the simultaneous and the separate optimization approaches and finds that the magnitude of the advantage of optimizing the problem simultaneously, depends on the relative size of setup and holding costs at each of the supply and demand points. This is an interesting remark since it highlights the fact that simultaneous optimization is not always better.

Chien (1993) analyzes the case of direct shipments between a single supply location and a single demand point. Demand for the product follows a known probability distribution. The transportation cost is fixed per shipment and the truck has limited capacity. The objective is to maximize the expected profit of the operation by determining a joint optimal production rate and shipment size. The problem is solved by an iterative procedure which yields solutions that are within 0.2 - 3.8% of optimality in terms of expected profits.

Barnes-Schuster and Bassok (1997) study a single-depot / multi-retailer system with independent stochastic stationary demands, linear inventory costs, and backlogging at the retailers over an infinite horizon. They also involve the transportation cost between the depot and the retailers. They consider a specific policy of direct shipments. That is, a lower bound on the long run average cost per period for the system over all order/delivery strategies is developed. They examine the strategy of direct shipping with fully loaded trucks via comparison to the delivered lower bound. They show that, from a practical point of view, in situations where normal demand distributions are known or can be estimated, and truck sizes are close to the mean of demand, very good results can be expected from a direct shipping policy.

Gallego and Simchi-Levi (1990) compare the total cost of implementing a full truck direct shipping strategy with a lower bound on all possible replenishment strategies, obtaining a surprisingly simple result. In direct shipping, each retailer sends goods via one or more trucks which leave the depot with the goods, delivers them to the retailer, and then returns to the depot. Full truck direct shipping permits only full-truck loads of goods to be delivered to the retailer. The authors develop a lower bound and show that the effectiveness of full truck direct shipping over all inventory-routing strategies is at least 94 per cent whenever the Economic Lot Size of each of the retailers is at least 71 per cent of vehicle capacity.

Abdul-Jalbar *et al.* (2003) introduce near-optimal policies for inventory/distribution systems with one-warehouse and N- retailers considering an instantaneous demand pattern at the warehouse. They study two cases: if the warehouse and the retailers belong to the same firm (centralization), or if the warehouse and retailers belong to different firms (decentralization). The results show that as the number of retailers increases so does the number of instances where the decentralized policy is better when the parameters are generated using the same uniform distribution. In addition, given a number of retailers, under specific conditions of the unit replenishment and holding costs at the warehouse, they argue that the centralized policy can provide better solutions.

Motivated by applications in the computer and food catering service industries, Chen and Vairaktarakis (2005) study an integrated scheduling model of production and distribution operations. In this model, a set of jobs (i.e., customer orders) are first processed in a processing facility (e.g., manufacturing plant or service center) and then delivered to the customer directly without intermediate inventory. Their objective is to find a joint schedule of production and distribution such that both customer service level and total distribution cost is optimized. Customer service level is measured by a function of the times when jobs are delivered to the customer. Chen (2004) points out that most existing production-inventory-distribution models study strategic or tactic levels of decisions, and a few addresses integrated decisions at the detailed job-scheduling level. In all these models, production and distribution are indirectly linked through inventory, and inventory cost is a significant portion of the total cost. None of these models is applicable to the scheduling problems encountered in direct order systems.

Liu *et al.* (2003) study a mixed truck delivery system that allows both hub-and-spoke and direct shipment delivery modes. In a direct shipment system, each supplier operates independently with its own fleet delivering goods to customers. Each vehicle visits only one customer in a trip. When there are multiple suppliers in the delivery region, especially when customers have common suppliers, another type of delivery system can be utilized. In such a system, goods from all suppliers are collected and consolidated in a central facility, called the hub, and then redistributed to the customers. In reality, suppliers and customers are located quite randomly and delivery quantities vary from order to order. The advantage of one of the systems over the other is neither obvious nor unchanged from day to day. In this situation, a mixed delivery system can be beneficial and better than either of the two pure delivery systems. Such a mixed system can be viewed as a hub-and-spoke system allowing some orders to be directly shipped whenever beneficial. Therefore, in the mixed system, different delivery modes may be used for different shipments depending on the quantity to be shipped and geographical locations of the supplier and the customer. The authors develop a heuristic algorithm to determine the mode of delivery for each demand and to perform vehicle routing in both modes of deliveries. Computational experiments are carried out to compare the mixed system with the pure hub-and-spoke system and the pure direct shipment system. They conclude that the mixed system is more effective than both pure systems. The delivery plan produced using the heuristic for the mixed system saves about four per cent in total distance on average compared with the best of the pure systems. The savings is about 10 per cent on average if compared with any one of the pure systems.

Liu *et al.* (2003) states that direct shipment should be utilized when the lead-time requirement is tight, the goods need to be isolated, or the shipment is large. According to them, the hub-and-spoke system takes advantage of the economies of scale in vehicle utilization. It can also improve customer service in terms of delivery frequency. When direct shipment is used, smaller suppliers need to wait until a sufficient amount of goods are ordered to maintain cost effectiveness in transportation. With the hub-and-spoke system, suppliers can provide a higher frequency of delivery (improved service quality) by combining the demands or orders of others. Intuitively, when the customers of each supplier are located very close to the supplier and the delivery quantity is large enough to justify the shipping of goods with full truckloads, the direct shipment system is better. Otherwise, the hub-and-spoke system is more appropriate.

Started in 1995, a project was conducted by a cross-functional team from Monsanto Co., a large multi-national firm with headquarters in the USA within Crop Protection business, and from Massachusetts Institute of Technology (MIT), under the auspices of the Integrated Supply Chain Management consortium at MIT. The mission was to develop and apply a model of the production, inventory and distribution network to guide strategic level decisions about Monsanto's Crop Protection business. A solution to the level production problem was to ship directly from the plant to retailers. Retailers participating in this program were compensated for their efforts. Thus, a portion of the overall cost savings was passed into the channel in the hope that this would help to make the products more price competitive. In distribution, one of the ways that the company and the channel could improve is via direct shipments to Trade Partners and retailers. Increases in direct flow (avoiding excessive use of public storage) resulted in cost savings of $0.03-0.06 per pound of shipped product. Some of these cost savings will be passed along to the end user, so the improvement in supply chain management will result in a more competitive market position (Graves *et al.,* 1996).

A study done by Oracle (2005) states that local distributors quote prices that are somewhat higher than wholesalers because they are closer to the customer’s location. Purchasing directly from a wholesaler or a manufacturer offers correspondingly lower prices but higher shipping costs. After analysis, buyers often find that they continually overpay for shipping. It also says that direct shipments and vendor managed inventory offers a process enhancement with a single point of contact for warranty, returns, technical support and customer service.

2.3. Economic Lot Scheduling Problem (ELSP)

The ELSP is a time-honored problem that has been around since 1915. Elmaghraby (1978) defines it as the problem of accommodating cyclical production patterns when several products are made on a single machine or facility. At any point of time the machine is either idle or producing a particular item. However, when two or more items compete for the machine, the phenomenon "interference" will occur, that is, it will be required to produce two items at the same time, which is physically impossible (Elmaghraby, 1978). Dobson (1987) considers the ELSP as the problem of scheduling of several products on a single facility so as to minimize holding and setup costs. Silver et al. (1998) define the ELSP as:

"*finding a cycle length, a production sequence, production times, and idle times, so that the production sequence can be completed in the chosen cycle, the cycle can be repeated over time, demand can be fully met, and annual inventory and setup costs can be minimized*". (Silver et al., 1998)

The assumptions of a restricted version of the problem normally define the traditional ELSP. These assumptions are (Bomberger, 1966; Doll and Whybark, 1973):

- Only one item can be produced at a time
- Production capacity is constrained but sufficient to meet total demand
- Production rates are deterministic and constant
- There is a setup cost and a setup time associated with producing each item
- Setup costs and times are independent of production order
- All demand must be met in the periods in which they occur
- Demand rates are deterministic and constant
- Inventory costs are directly proportional to inventory levels

Zipkin (1991) describes ELSP as follows:

- There are several items, all of which must be produced on a single machine. The machine can make only one product at a time.
- The supply and demand characteristics of each product are like those in the single-product economic lot size problem: There is a constant demand rate, and all demand must be filled immediately. Production also occurs at a constant rate. There may be a setup time required before production can begin.
- There is a constant holding cost rate for carrying inventory of each product. There may also be a setup cost.
- The problem is to determine a production schedule, a complete specification of which items are to be produced, when, and in what quantities. The criterion is long-run average cost.

The problem has attracted the attention of many researchers, partly because it captures many important features of frequently encountered real scheduling problems, and partly because, though simple to state, it seems to be quite difficult to solve. Indeed, very little is known about truly optimal policies (Zipkin, 1991). ELSP is difficult problem to solve due to the need to satisfy the production capacity constraint and the constraint to produce only one item at a time. Hsu (1983) and Gallego and Shaw (1997) have shown that it is NP-hard. Hsu (1983) has shown that even the question of finding a feasible schedule is computationally hard (NP complete).

Elmaghraby (1978) presents an overview of the ELSP and a review of early contributions to the problem. He divides the contributions into two categories:

I. Analytical approaches that achieve *the optimum of a restricted version of the original problem.*
II. Heuristic approaches that achieve *"good" solutions of the original problem.*

Most research has focused on relatively simple classes of policies, especially cyclic schedules, where the schedule is designed so that the entire system is periodic. There are three broad categories of solutions that have been found useful, classified as the common cycle (CC) approach, the basic period (BP) approach, and the extended basic period (EBP) approach. In the CC approach, the cycle time for each item is an integer multiple of that basic period where the cycle time of each item are equal to each other. The basic period approach extends the common cycle approach and allows the items to have different cycle times as long as they are multiples of a basic period. The extended basic period approach relaxes the condition on the value of the basic period to be large enough to accommodate the production of all items. Instead, the basic period only has to be long enough to cover the average setup and production times for all products (Lopez and Kingsman, 1991).

According to Davis (1990), two or more items may compete for the same facility when:

- The load on the facility exceeds capacity resulting in overlaps in the schedule to satisfy output requirement.
- A startup of an item may appear prior to the completion of the production run of the economic lot size for another product in order to avoid a stockout of the product.

According to Rogers (1958), it may be possible to find feasible schedules when there are few items sharing the same facility and when there is considerable free schedule space because of high production rates relative to the demand rates. In case of numerous items, the number of potential interferences increases.

Elmaghraby (1978) stated criticism against the heuristic models that there is no systematic way to test the feasibility of a given set of parameters and also that there are no definite procedures for the escape from infeasibility once it is established. Haessler and Hogue (1976) present a procedure for finding a feasible solution when the frequencies are determined by systematically increasing the basic period and halving the frequencies.

Dobson (1987) presents a formulation that provides feasible schedules by allowing the lot sizes and thus the cycle times for each product to vary over time. Zipkin (1991) presents a heuristic for constructing a feasible schedule from power-of-two frequencies. He schedules the items in equal subintervals starting with the most frequently produced items, then the next most frequent items and so on. When the items are assigned, the schedule may not be feasible since operations may overlap if the load of any subinterval exceeds its capacity. Therefore, Zipkin (1991) also presents a procedure for an evaluation of this. Also

Yao et al. (2003) present a heuristic for testing the feasibility of a given solution to the ELSP. They propose integer programming models and a heuristic to solve them. Levén and Segerstedt (2005) present a method for adjusting order quantities derived from heuristic procedures for the ELSP. The method considers the runout times (current inventory levels divided with expected demand rates) of the items and if the present situation indicates a future stockout situation the order quantities are reduced in order to prevent shortages.

Nilsson and Segerstedt (2005) argue that heuristic techniques such as Doll and Whybark (1973) and Goyal (1975) can find solutions where the production can be scheduled during a time interval, the initial inventory level is the same as the final, and the schedule presents no shortages. However, they mean that traditional approximations for the inventory holding costs disagree with actual costs since it is difficult to schedule and sequence the items such that the inventory level reaches zero at each start of production.

Most research on the ELSP assume setup times that are independent on the order in which the items are processed, which, according to Lopez and Kingsman (1991), is not a valid assumption for many real world problems. Instead, in many cases, the sequence of the setups affects the times and costs. For instance, this could appear when setups involve cleaning of equipment or wastes of material. Maxwell (1964) states that producing in wrong sequence entails too much productive time lost due to setups. Therefore, in a problem considering sequence-dependent setups, the sequence in which the products should be processed is important in order to minimize the inventory holding cost and setup cost. Lopez and Kingsman (1991) argue that a common cycle approach can be used. The common cycle approach results in low costs and cycle times, since the products are sequenced in an optimal manner to reduce the total setup time, permitting smaller lot sizes and lower costs. Some researchers have presented heuristics for the ELSP with sequence-dependent setups. Numerous papers addresses heuristic procedure for the problem with sequence-dependent setups, including Galvin (1987), Maxwell (1964), Delporte and Thomas (1977), Dobson (1992), and Wagner and Davis (2002).

3. PROBLEM STATEMENT AND MATHEMATICAL MODEL

This chapter states the problem, introduces the notation and assumptions, and presents the mathematical model.

In a manufacturing facility, goods can be delivered to the customers in various ways to fullfil their orders. The solution which the mathematical model brings in this thesis considers two different approaches to achieve order satisfaction. First one is to use a warehouse in which orders are transferred to. The second approach is the possibility of direct shipment where the manufacturer gets in contact directly with the customer and satisfies the requested order directly from the plant. Thus, there are two common types of delivery systems: the direct shipment system and consolidated warehouse shipment system.

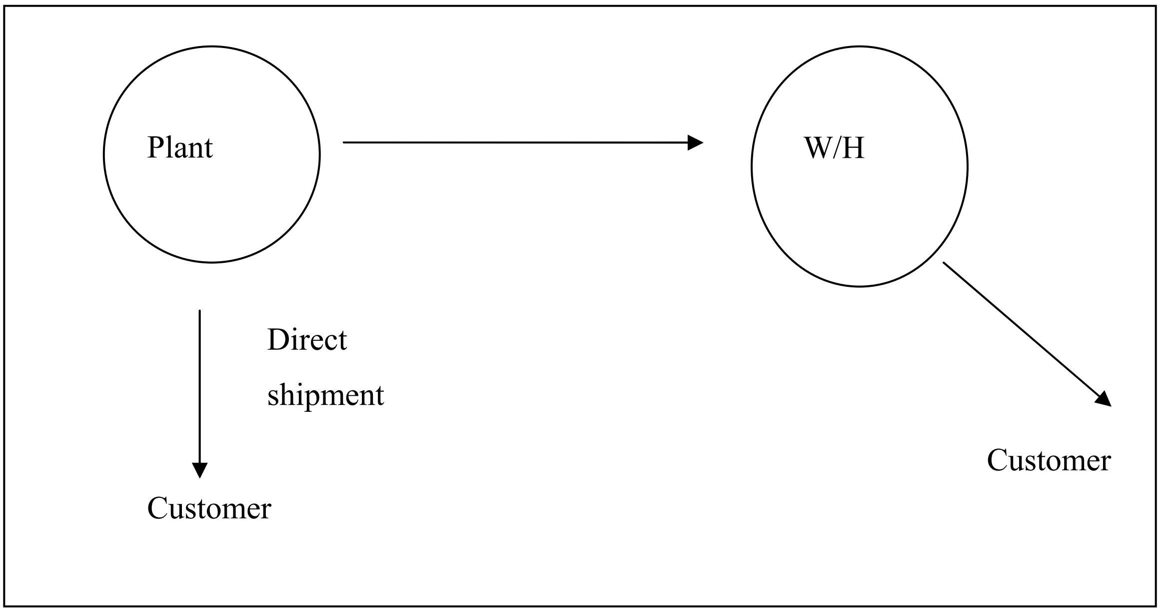

Figure 3.1. Direct shipment & consolidated warehouse shipment

We address the following situation:

The time is broken up into periods, the present period is called one, the next period is called two, and the final period is called T. The assumption is that we have a manufacturing facility producing two different items over T periods of time. At each time period, the plant faces demands for these two items from the customer. Depending on the result of the optimization problem, orders placed at the manufacturing facility may also generate demands at the warehouse of the plant which is a part of the whole demand. If the decision is to meet a portion of the demand from the warehouse then this part of the demand will be satisfied via consolidated warehouse replenishment, and the rest will be served by direct shipment from the manufacturing facility. Each period's demand to be satisfied directly from the plant must be met on time from inventory or current production. The demands must be met as it occurs without shortages which lead to a "Just-In-Time" strategy where minimum stock level must be determined to reduce its cost and maximize the gain with always satisfied customers. We assume that no backorder is permitted. We do not restrict the demand rate to be constant. Four different scenarios are considered for the demand rates:

1. Demand equal over time: In this scenario, in each period, the customer places same size order for the products.
2. Increasing demand over time: Here, as times goes on over T time periods the demand size increases.
3. Decreasing demand over time: In this scenario, as times goes on over the ten time periods the demand size increases.
4. Seasonal demand: The distribution of the demand over time is like a Normal distribution function, e.g. demand at the tails of the periods are low whereas it makes peaks when we are in the middle of the time horizon.

In the manufacturing facility, there is a single machine which processes the two items. Therefore, only one item can be produced at a time and the model will decide which item to produce in each time period to achieve optimum results. The machinary capacity is an important constraint that affects the production model. The production rates for each item are equal and constant over time.

In our problem, we have the following scenario:

- Demands for the two products arrive at the manufacturing facility.

- The plant decides on the portion of the demand to be satisfied both by the plant itself and by the depot.
- The manufacturing facility decides on the production schedule, e.g. when to produce which item.
- To satisfy the demand portion of the depot, goods are transported to the warehouse by homogeneous vehicle with a limited capacity. We assume that there is an infinite supply of vehicles.

We do not consider the distance and the location of the customer, and do not therefore include them in our problem. The only assumption we make here is that the customer is closer to the depot than to the manufacturing facility which directs the customer supplying from the warehouse. But, since the goods are transported from plant to the warehouse, this causes a price difference, like the product selling price at the plant is lower than the price at the depot and that creates another reason for the customer to decide to go for the manufacturing plant itself. Here in this production model, we are not interested in the routing problem and assume that the customer itself comes either to the plant or warehouse to get his/her products.

The plant has a limited storage capacity. There is a holding cost per each unit stored per time period charged at the end of each time period both in manufacturing facility and in the warehouse. That stock cost is also another factor to fit in “Just-In-Time” policies. We assume that the holding cost in the warehouse is more than the inventory holding cost in the plant. During any period in which production takes place a production cost is incurred in the plant which is proportional to the units produced. Depending on the sequence of the production, a fixed setup cost for each production change incurs. For the items shipped from the manufacturing facility to the warehouse, there is a transshipment cost consisting of two partial costs. One is the fixed shipment cost per truck and the other is shipment cost per unit transported to the depot. This is why one must also consider the best usage of trucks by adjusting the loading and transfer strategies to create most efficient actions. In this model, the inventory position is reviewed at the end of each period, and then the production decision is made.

In our model, there is no inventory in the warehouse at the beginning of the first time period whereas in the plant we start with an initial inventory for each item. The initial inventory level is set to a multiple of the maximum demand faced over T periods of time.

Lead times and setup times are assumed to be zero. In the plant, there is a space constraint whereas in the warehouse we do not apply such a restriction. Thus, you can not store a big amount of goods in the plant but can keep an infinite amount in the warehouse.

Mixed Integer Programming (MIP) is a widely used technique of mathematical programming and several applications to real-world problems are quoted in the literature. Our first aim is to develop a mixed integer model. Hereafter we use the notation shown below:

N	number of items
T	number of time periods
i	product type index ($i = 1,2,\ldots, N$);
t	time period index ($t = 1,2,\ldots.., T$);
$d_{i,t}$	demand for product i at time t;
$\overline{d}_i$	average demand for product i over the whole time horizon;
S	space capacity at manufacturing facility;
C	truck capacity;
c_i	unit production cost of item i;
h^M_i	production inventory holding cost of item i;
h^W_i	warehouse inventory holding cost of item i;
k_i	unit shipment cost from the production plant to the warehouse;
V	fixed shipment cost per truck;
μ_i	production rate for item i;
ρ	utilization rate;
K_i	fixed setup cost at the production plant for item i;
w_i	unit space of item i;
P^M_i	price of product i at plant;
P^W_i	price of product i at warehouse;
$I^M_{i,t}$	inventory of item i at the end of period t at the manufacturing facility;

$I^M_{i,0}$	starting inventory of item i at the manufacturing facility;
$I^W_{i,t}$	inventory of item i in period t at warehouse;
$I^W_{i,0}$	starting inventory of item i at warehouse;
P_i	average price of product i;
$\overline{\cos t}$	average cost per item;
$\overline{\mu}$	average production rate;
$d^M_{i,t}$	demand for item i at time t at plant;
$d^W_{i,t}$	demand for item i at time t at warehouse;
$Y_{i,t}$	setup change, shows whether production starts from start-up or production changes from one product to another;
$y_{i,t}$	number of units shipped of i^{th} item from the production plant to warehouse at time t;
$X_{i,t}$	$= \begin{cases} 1 \text{ if } i^{th} \text{ item is produced in time period } t, \\ 0 \ \text{ otherwise;} \end{cases}$
n_t	number of trucks going from plant to warehouse at time t;
α_i	ratio of demand for item i at plant;
R	revenue;
f_1	total inventory holding cost at plant;
f_2	total production cost ;
f_3	total setup cost;
f_4	total inventory holding cost at warehouse;
f_5	total transshipment cost;
f	profit.

The model

Objective function:

$$\text{MAX} \quad f = R - (f_1 + f_2 + f_3 + f_4 + f_5) \text{ where}$$

$$R = \sum_{i=1}^{N} \sum_{t=1}^{T} (\alpha_i * d_{i,t} * P^M_i + (1-\alpha_i) * d_{i,t} * P^W_i)$$

$$f_1 = \sum_{i=1}^{N} \sum_{t=1}^{T} h^M{}_i * I^M{}_{i,t}$$

$$f_2 = \sum_{i=1}^{N} \sum_{t=1}^{T} X_{i,t} * c_i * \mu_i$$

$$f_3 = \sum_{i=1}^{N} \sum_{t=1}^{T} Y_{i,t} * K_i$$

$$f_4 = \sum_{i=1}^{N} \sum_{t=1}^{T} h^W{}_i * I^W{}_{i,t}$$

$$f_5 = \sum_{i=1}^{N} \sum_{t=1}^{T} y_{i,t} * k_i + \sum_{t=1}^{T} n_t * V$$

$\Rightarrow$ Objective function:

$$\text{MAX} \quad \sum_{i=1}^{N} \sum_{t=1}^{T} (\alpha_i * d_{i,t} * P^M{}_i + (1-\alpha_i) * d_{i,t} * P^W{}_i)$$
$$- \Big[\sum_{i=1}^{N} \sum_{t=1}^{T} h^M{}_i * I^M{}_{i,t} + \sum_{i=1}^{N} \sum_{t=1}^{T} X_{i,t} * c_i * \mu_i + \sum_{i=1}^{N} \sum_{t=1}^{T} Y_{i,t} * K_i$$
$$+ \sum_{i=1}^{N} \sum_{t=1}^{T} h^W{}_i * I^W{}_{i,t} + \Big(\sum_{i=1}^{N} \sum_{t=1}^{T} y_{i,t} * k_i + \sum_{t=1}^{T} n_t * V \Big) \Big]$$

Constraints:

$$I^M{}_{i,0} + X_{i,1} * \mu_i - \alpha_i * d_{i,1} - y_{i,1} = I^M{}_{i,1} \text{ for } \forall i = 1,..,N; \quad (1a)$$

$$I^M{}_{i,t} + X_{i,t+1} * \mu_i - \alpha_i * d_{i,t+1} - y_{i,t+1} = I^M{}_{i,t+1} \text{ for } \forall i = 1,...,N; \ \forall t = 1,2,...,T-1; \quad (1b)$$

$$I^W{}_{i,0} + y_{i,1} - (1- \alpha_i) * d_{i,1} = I^W{}_{i,1} \text{ for } \forall i = 1,...,N; \quad (2a)$$

$$I^{W}_{i,t} + y_{i,t+1} - (1-\alpha_i) * d_{i,t+1} = I^{W}_{i,t+1} \text{ for } \forall i = 1,...,N, \; \forall t = 1,2,...,T-1; \quad (2b)$$

$$I^{M}_{i,t} \geq 0 \quad \forall i = 1,...,N \; ; \; \forall t = 1,2,...,T \quad (3)$$

$$I^{W}_{i,t} \geq 0 \quad \forall i = 1,...,N \; ; \; \forall t = 1,2,...,T \quad (4)$$

$$\sum_{i=1}^{N} X_{i,t} \leq 1 \; ; \; X_{i,t} \in \{0,1\} \; ; \; \forall t = 1,2,...,T \quad (5)$$

$$Y_{i,t} \geq 0 \quad ; \forall i = 1,...,N \; ; \; \forall t = 1,2,...,T \quad (6)$$

$$Y_{i,t} \geq X_{i,t} - X_{i,t-1} \quad ; \; \forall i = 1,...,N \; ; \; \forall t = 1,2,...,T \quad (7)$$

$$\sum_{i=1}^{N} w_i * I^{M}_{i,t} \leq S \; ; \; \forall t = 1,2,...,T \quad (8)$$

$$n_t \geq \frac{\sum_i yi,t}{C} \; ; \; n_t \text{ integer} \quad \forall t = 1,2,...,T \quad (9a)$$

$$n_t \leq \frac{\sum_i yi,t}{C} + 1; \; \forall t = 1,2,...,T \quad (9b)$$

$$0 \leq \alpha_i \leq 1; \; \forall i = 1,...,N \quad (10)$$

The objective function aims to maximize the profit gained, which is calculated by the subtraction of all the related costs from the total revenue. Since all demand will be satisfied we can obtain the revenue by multiplying the related prices (prices at the plant P^{M}_{i} and prices at the warehouse P^{W}_{i}) with the regarding demand portions $\alpha_i * d_{i,t}$ and $(1-\alpha_i) * d_{i,t}$.

Constraint set (1a), (1b) ensures that each next period's ending inventory $I^{M}_{i,t+1}$ is equal to the subtraction of the demand portion satisfied directly by plant $\alpha_i * d_{i,t}$, units shipped to the warehouse during the period $y_{i,t+1}$ from the sum of beginning inventory on-hand $I^{M}_{i,t}$ and units produced during that period $X_{i,t+1} * \mu_i$. Constraint set (2a), (2b) presents that the ending inventory of the next period in the depot $I^W_{i,t+1}$ is obtained when we add units $y_{i,t+1}$ transported from the plant to the warehouse to the beginning inventory of that period $I^W_{i,t}$ and subtract the demand portion satisfied by the warehouse $(1- \alpha_i) * d_{i,t+1}$. Constraint (3) and (4) ensure that all demand are met on time by restricting each period's ending inventory to be nonnegative. That only single product can be produced at a time is taken account by constraint (5) where $X_{i,t}$ can only be zero or 1. Having $X_{i,t}$ equal to zero means that in period t no item is produced of product i whereas $X_{i,t}$ with value of 1 indicates that item i is produced in period t. By constraints (6) and (7), the setup change is controlled. A setup change occurs when the production starts from a start-up or a breakdown, or when there is change in production from one product to another. Therefore, for example, if item i is produced in the first, third and fourth period during the whole 5 time periods we would have $X_{i,1} = X_{i,3} = X_{i,4} = 1$ and $X_{i,2} = X_{i,5} = 0$. This would imply that there two setup changes during the 5 periods of time. The reason for having $X_{i,2} = 0$ might be a breakdown, no production at all or production of another product in period 2. The manufacturing facility has a limited storage capacity which is reflected by a limit on end-of-period inventory with constraint (8). The trucks transporting goods from the plant to the warehouse have a limited capacity. Therefore, constraints (9a) and (9b) calculate the number of trucks and impose a limit of the number: The number of trucks must be integer. Finally, constraint (10) guarantees that this portion can only be within the range [0-1]. The special case of having $\alpha_i = 1$ shows that all demand for item i is directly satisfied by plant. In opposite, $\alpha_i = 0$ means that demand is wholly met by the warehouse. In addition, having the same model with $\alpha_i = 0$ and a single item would turn the problem into a sort of Wagner-Within algorithm.

4. RESULTS OF THE MODEL AND COMPUTATIONAL ANALYSIS

This section explains the purpose of computations, states the parameter values and problem instances, the approach of implementation and presents the results and observations.

4.1. The Purpose of the Computations

The aim of computations is to assess direct shipment versus consolidated warehouse shipment with the use of many different problem cases using different parameter values. The main concern of the assessment is to analyze α_i which gives the ratio of the demand that is to be satisfied directly from the plant. In other words, the goal of the observations is to find out when or under which conditions it is more beneficial to sell goods directly from plant and/or from the warehouse, and which parameters influence the result more. Before starting analysis between direct and consolidated shipments and explore the impact of parameters, we should decide on the experiments, problem instances.

4.2. Parameters and Problem Instances

We generated cases and problem instances in the following manner:

1. We specify the overall size of the model through the parameters N and T, the number products and the number of time periods where N is set to 2 and T is set to 10.
2. Average demand of each product $\overline{d}_i$ take the values of 50, 100 and 150. We use the relationship of $\overline{d}_i$ s such that $\overline{d}_1 - \overline{d}_2 \geq 0$ which indicates two possibilities which are $\overline{d}_1 = \overline{d}_2$ and $\overline{d}_1 - \overline{d}_2 > 0$. The described assumption addresses that the average demand for item 2 can not be more than the average demand for item 1 which results in six demand levels. Since we have four different demad rates over time (equal over time, increasing over time, decreasing over time and seasonal over

time) as mentioned in section 3, we have overall 24 demand cases as combinations of six demand levels and four demand structures as summarized in Table 4.1.

3. Given the freedom to choose other parameters, unit space of each item i is supposed to be equal and fixed to 1, and starting inventory of all items at warehouse is fixed to zero whereas starting inventory of each item i at plant is set to 1.78 times of the maximum demand of that item among 10 time periods. That is, $I^{W}_{i,0} = 0$ and $I^{M}_{i,0} = 1.78 * \max \{ d_{i,t} \}$ for $\forall i = 1,2$ as given in Table 4.2. (This helps the comparability and ensures the feasibility.)

Table 4.1. Demand cases as combinations of six demand levels and four demand structures

Demand level	Demand structure	i	Time period t									
			1	2	3	4	5	6	7	8	9	10
SC1	equal	1	50	50	50	50	50	50	50	50	50	50
		2	50	50	50	50	50	50	50	50	50	50
	increasing	1	5	15	25	35	45	55	65	75	85	95
		2	5	15	25	35	45	55	65	75	85	95
	decreasing	1	95	85	75	65	55	45	35	25	15	5
		2	95	85	75	65	55	45	35	25	15	5
	seasonal	1	6	28	50	72	94	94	72	50	28	6
		2	6	28	50	72	94	94	72	50	28	6
SC2	equal	1	100	100	100	100	100	100	100	100	100	100
		2	100	100	100	100	100	100	100	100	100	100
	increasing	1	10	30	50	70	90	110	130	150	170	190
		2	10	30	50	70	90	110	130	150	170	190
	decreasing	1	190	170	150	130	110	90	70	50	30	10
		2	190	170	150	130	110	90	70	50	30	10
	seasonal	1	10	55	100	145	190	190	145	100	55	10
		2	10	55	100	145	190	190	145	100	55	10

SC3	equal	1	150	150	150	150	150	150	150	150	150	150
		2	150	150	150	150	150	150	150	150	150	150
	increasing	1	60	80	100	120	140	160	180	200	220	240
		2	60	80	100	120	140	160	180	200	220	240
	decreasing	1	240	220	200	180	160	140	120	100	80	60
		2	240	220	200	180	160	140	120	100	80	60
	seasonal	1	16	83	150	217	284	284	217	150	83	16
		2	16	83	150	217	284	284	217	150	83	16

SC4	equal	1	100	100	100	100	100	100	100	100	100	100
		2	50	50	50	50	50	50	50	50	50	50
	increasing	1	10	30	50	70	90	110	130	150	170	190
		2	5	15	25	35	45	55	65	75	85	95
	decreasing	1	190	170	150	130	110	90	70	50	30	10
		2	95	85	75	65	55	45	35	25	15	5
	seasonal	1	10	55	100	145	190	190	145	100	55	10
		2	6	28	50	72	94	94	72	50	28	6

Demand level	Demand structure	i	Time period t									
			1	2	3	4	5	6	7	8	9	10
SC5	equal	1	150	150	150	150	150	150	150	150	150	150
		2	50	50	50	50	50	50	50	50	50	50
	increasing	1	60	80	100	120	140	160	180	200	220	240
		2	5	15	25	35	45	55	65	75	85	95
	decreasing	1	240	220	200	180	160	140	120	100	80	60
		2	95	85	75	65	55	45	35	25	15	5
	seasonal	1	16	83	150	217	284	284	217	150	83	16
		2	6	28	50	72	94	94	72	50	28	6
SC6	equal	1	150	150	150	150	150	150	150	150	150	150
		2	100	100	100	100	100	100	100	100	100	100
	increasing	1	60	80	100	120	140	160	180	200	220	240
		2	10	30	50	70	90	110	130	150	170	190
	decreasing	1	240	220	200	180	160	140	120	100	80	60
		2	190	170	150	130	110	90	70	50	30	10
	seasonal	1	16	83	150	217	284	284	217	150	83	16
		2	10	55	100	145	190	190	145	100	55	10

The abbreviations used in Table 4.1 indicate the following demand levels:

SC1: $\overline{d}_1 = \overline{d}_2 = 50$;

SC2: $\overline{d}_1 = \overline{d}_2 = 100$;

SC3: $\overline{d}_1 = \overline{d}_2 = 150$;

SC4: $\overline{d}_1 = 100$, $\overline{d}_2 = 50$;

SC5: $\overline{d}_1 = 150$, $\overline{d}_2 = 50$;

SC6: $\overline{d}_1 = 150$, $\overline{d}_2 = 100$;

Table 4.2. Starting inventory of each item i at plant

Demand structure	i	$I^M_{i,0}$ of each demand level					
		SC1	SC2	SC3	SC4	SC5	SC6
Equal	1	89	178	267	178	267	267
	2	89	178	267	89	89	178
Increasing	1	169.10	338.2	427.2	338.2	427.2	427.2
	2	169.10	338.2	427.2	169.1	169.1	338.2
Decreasing	1	169.10	338.2	427.2	338.2	427.2	427.2
	2	169.10	338.2	427.2	169.1	169.1	338.2
Seasonal	1	167.32	338.2	505.52	338.2	505.52	505.52
	2	167.32	338.2	505.52	167.32	167.32	338.2

4. We make no difference between the production rates of each item. The production rates are generated by using the following formula where five different values are chosen for the utilization rate ρ. Therefore, we use five different production rates for each set of 24 scenarios which results in 120 combinations having the other parameters not changed. Since it's a batch-production, the production rate is actually the batch size.

$$\rho = \frac{\overline{d_1} + \overline{d_2}}{\mu}; \text{ where } \mu = \mu_1 = \mu_2$$

The values assigned to the utilization rate ρ and corresponding batch sizes for each demand level are shown in Table 4.3.

Table 4.3. Batch sizes corresponding to each utilization rate and demand level

Demand level	Utilization rate ρ				
	0.5	0.75	0.85	0.95	0.97
	Production rate / Batch size μ				
	μ (1)	μ (2)	μ (3)	μ (4)	μ (5)
SC1	200	133.33	117.65	105.26	103.09
SC2	400	266.67	235.30	210.53	206.19
SC3	600	400	352.94	315.80	309.30
SC4	300	200	176.47	157.89	154.64
SC5	400	266.67	235.29	210.53	206.20
SC6	500	333.33	294.12	263.16	257.73

5. Space capacity of the manufacturing facility S and the truck capacity C are scaled to fit to two parameters, unit space w_i and average demand level $\bar{d}_i$ and a given percentage, S % and C % respectively. S and C are obtained as follows:

$$S = S\,\% * \sum_{i=1}^{2} w_i * \bar{d}_i$$

$$C = C\,\% * \sum_{i=1}^{2} w_i * \bar{d}_i$$

   Three different scenarios of S are analyzed, in one of which there is no space constraint actually where S % is set to 10; in the second and third S % is set to 0.75 and 0.25 respectively, so that each period, the plant can only hold 75% or 25% of the whole items ordered in a period. For the truck capacity C, two cases are investigated where C % is set to 0.8 and 0.5. That is, each period, the truck can transport either 80% or 50% of the demand to be met.

6. In our experiments, we analyze both identical and unidentical products, where the difference maker is the unit production cost. In investigating identical products, the unit production cost c_i is the same for two products and take the value of 3 whereas in case of unidentical products c_1 becomes two times greater than c_2. ($c_1 = 6$; $c_2 = 3$).

7. Plant inventory holding cost h^M_i and warehouse inventory holding cost h^W_i are generated from unit production cost in such a way that $h^M_i = c_i * r_1$ and $h^W_i = h^M_i * r_2$ where r_1 takes the value 0.1 and r_2 0.15. The holding costs of each

product is equal to each other. Thus, we have the following inventory holding cost cases for the two possibilities of unit production cost as listed in Table 4.4.

Table 4.4. Unit inventory holding costs h^M_i and h^W_i

Unit production cost structure	i	h^M_i	h^W_i
$c_1 = 3 = c_2 = 3$	1	0.3	0.45
	2	0.3	0.45
$c_1 = 6 > c_2 = 3$	1	0.6	0.9
	2	0.3	0.45

8. Unit shipment cost k_i is calculated in a similar way to inventory holding costs with the following formula: $k_i = b * c_i$ where $b = 0.2$. So, unit shipment costs take the values shown in Table 4.5. Similarly, fixed setup cost at the production plant K_i is generated as a function of c_i which is $K_i = j * c_i$ where $j = 10$.

Table 4.5. Unit shipment costs k_i and fixed setup cost K_i

Unit production cost structure	i	k_i	K_i
$c_1 = 3 = c_2 = 3$	1	0.6	30
	2	0.6	30
$c_1 = 6 > c_2 = 3$	1	1.2	60
	2	0.6	30

9. Fixed shipment cost per truck V is dependent on the truck capacity C and is generated from the formula $V = a * C$ where $a = 1$. In Table 4.6. the values taken by the space capacity of the manufacturing facility S, the truck capacity C and fixed shipment cost per truck V are given as per demand level.

Table 4.6. S, C and V values for each demand level

Demand level	S			C		V	
	Uncapacitated	75%	25%	80%	50%	80%	50%
SC1	1000	75	25	80	50	80	50
SC2	2000	150	50	160	100	160	100
SC3	3000	225	75	240	150	240	150
SC4	1500	112.5	37.5	120	75	120	75
SC5	2000	150	50	160	100	160	100
SC6	2500	187.5	62.5	200	125	200	125

10. For generating the price of each product at plant, an average cost per item $\overline{\cos t}$ is calculated and defined with the following formula:

$$\overline{\cos t} = \Big\{ \sum_{i=1}^{2} c_i * \overline{d}_i + \sum_{i=1}^{2} \frac{\overline{d}_i}{2} * \frac{h_i^M + h_i^W}{2} + \sum_{i=1}^{2} \frac{\overline{d}_i * k_i}{2}$$

$$+ \frac{V}{T*C} \left\lceil \frac{T*\overline{d}_1 + T*\overline{d}_2}{2} \right\rceil + \left\lceil \frac{\overline{d}_1 + \overline{d}_2}{\overline{\mu}} \right\rceil * \frac{K_1 + K_2}{2} \Big\} * \frac{1}{\overline{d}_1 + \overline{d}_2}$$

By multiplying the average cost obtained as above with a constant e = 1.7 we find the price of each product at plant P^M_i. The price of product i at warehouse is classified as low, medium and high price. By low P^W_i, we set it equal to the P^M_i whereas by high warehouse price we assign the value of $2* P^M_i$ where two is the factor g. In generating the medium warehouse price, we analyzed all 120 basic cases where there is no space capacity constraint in plant, 80 per cent capacity availibility of truck and identical products are produced, and we observed the behaviour of α_i s by changing the values of g. The results are given in Figure 4.1.

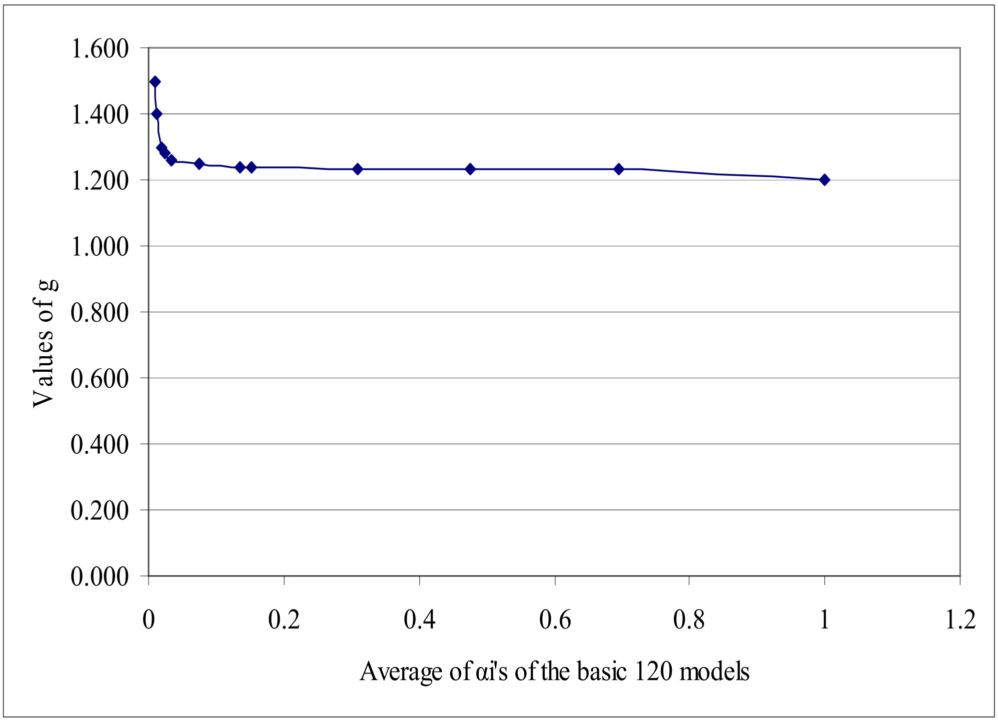

Figure 4.1. Distribution of α_i according to the values of factor g

Based on the results shown, g value is chosen to be 1.23 where the average of α_i 's becomes 0.5 which indicates that half of demand is met by the warehouse.

Thus, the generation of the models is governed by demand level, demand structure and production rate / batch size which make the basic 120 models. We run these 120 models each time by changing the values of parameters as explained above. Beside the parameters, these factors influence the results, and we intent to observe the behavior of the solutions with respect to them. To ensure variety in models, we anticipate almost every parameter of which we have 36 combinations. Therefore, in our study we intent to run and analyze 4320 models and get the following outputs:

α_i	ratio of demand for item i at plant;
f	profit;
R	revenue;
f_1	total inventory holding cost at plant;
f_2	total production cost ;
f_3	total setup cost;

f_4	total inventory holding cost at warehouse;
f_5	total transshipment cost;
$X_{i,t}$	production per time period
h^M_i	production inventory holding cost of item i;
h^W_i	warehouse inventory holding cost of item i;
K_i	fixed setup cost at the production plant for item i;
$y_{i,t}$:	number of units shipped of ith item from the production plant to warehouse at time t;
n_t	number of trucks going from plant to warehouse at time t;

GAMS Implementation

In the implementation phase, CPLEX MIP optimiser is used which we accessed through the GAMS (General Algebraic Modeling System) modeling language described by Brooke, Kendrick, Meeraus and Raman (1998). In using GAMS to invoke CPLEX, the default options are used in all cases. To understand the behaviour of the model more clearly and to save computational running time, 120 models are run once at a time, and GAMS is integrated with Microsoft Excel for reading and writing data. It should be emphasized that the more parameters and constraints the solver should consider the longer it takes to get the results. Therefore, the running time gets longer as the problem becomes more difficult to solve. To be able to run 120 models once at a time, GAMS GDX (GAMS Data Exchange) facilities are used which are very new tools developed and explained by GAMS Development Corporation in 2006.

A GDX file is a file that stores values of one or more GAMS symbols such as sets, parameters, variables and equations. GDX file can be used to prepare data for a GAMS model, store results of the same model using different parameters etc. A GDX file has some utilities enabling integration with Excel spreadsheets and running many different models at the same time.

GDXXRW utility is a utility to read and write Excel spreadsheet data. This utility requires the presence of Microsoft Excel and therefore can only be used on a PC running the Windows operating system with Microsoft Excel installed. GDXXRW can read multiple ranges in a spreadsheet, in a sheets of a file or multiple files, and write data to a 'GDX' file, or read from a 'GDX' file, and write the data to different ranges in a spreadsheet or different sheets and files. This utility is used in our implementation of running models with different parameter values in that possible values parameters can take are stored in Excel spreadsheets such as $d_{i,t}$ of each demand profile or production rates for each demand level, and read from the specified areas of the specified sheets. In that way, almost every input data organized in Excel spreadsheets are directly read by GAMS during the execution phase of the model which allows high speed in running models with changing input data values. In Figure 4.2., an example for reading spreadsheet is given. In this example, the values of $d_{i,t}$ are read from the file named cases.xls, in which the data is stored in sheet Scenario1.

To make data reading process faster and more efficient, the development of a computer programme using Visual Basic is realized which creates case-based demand profiles as Excel input-data spreadsheets. In this programme, the user sets the number of periods and also the two average demand levels for each item. The result is the demand in each period of time $d_{i,t}$ concerning the four different demand structures like equal over time, increasing over time, decreasing over time, and seasonal over time.

```
 $CALL GDXXRW.EXE cases.xls  par=d rng=Scenario1!A3:K5
Parameter d(i,t);
$GDXIN cases.gdx
$LOAD d
$GDXIN
```

Figure 4.2. Example for reading Excel spreadsheet data with GDXXRW utility

GDXXRW utility does not only performed for reading data but also writing data to the Excel spreadsheets. By using that facility, we are able to write outputs of the model such as profit, α_i, total inventory holding cost at plant, total production cost, total setup cost, total inventory holding cost at warehouse, total transshipment cost, revenue into an Excel sheet and generate designed charts directly after the exeqution. This ability provides a fast and very good analysis method of the results. Figure 4.3. shows an example for writing data to an Excel file. In this example, the output total production cost f_2 is written into an Excel spreadsheet named “ProductionCost” of the file result.xls.

```
execute_unload "result.gdx"    f2;
execute_unload "result.gdx";
execute 'gdxxrw.exe result.gdx var=f2 rng=ProductionCost!';
```

Figure 4.3. Example for writing data into an Excel spreadsheet with GDXXRW utility

Another GDX utility is GDXMERGE which combines multiple GDX files into one file. Symbols with the same name, dimension and type are combined into a single symbol of higher dimension. The added dimension has the file name of the combined file as its unique element. The result of the merge is written to a file called merged.gdx. With the help of GDXMERGE, we are able to run a model as many times as it’s required with different values of parameters, save GDX files for each run and combine them into a single one so that we can read the results of all the models run from the merged.gdx file and write them into an Excel sheet by using the GDXXRW utility. In Figure 4.4., an example for how to merge different GDX file is given. In this example, the model is run twice and the outputs f and α_i are saved into files r001.gdx and r002.gdx respectively. Then, using GDXMERGE utility, they are combined into the file merged.gdx. The results for α_i and f are then saved under the names AllAlfa and Final.

```
solve test using mip maximizing f ;
execute_unload "r001.gdx" f, alfa.l;
-----------
solve test using mip maximizing f ;
execute_unload "r002.gdx" f, alfa.l;

$call gdxmerge  r00*.gdx
$gdxin merged.gdx
$load  AllAlfa = alfa
$load Final = f
$gdxin
option AllAlfa:8;
option Final:8;
```

Figure 4.4. Example for running multiple models at once and combining them

Results and Observations

In order to be able to understand the behavior and the results of the problem in every case and to be able too see the whole picture, first a preliminary analysis is made. The preliminary analysis is made very deeply by taking account each parameter's effect to the solution. At that stage, not only the profit, α_i and revenues, but also all outputs like setup changes, production, inventory are analyzed. This analysis becomes the basis for the whole assessment and helps even guess the behaviour of the model and the output results of the remaining cases.

For that purpose, the basic 120 models with identical products are run for which the unit production costs are the same, i.e. $c_1 = c_2 = 3$. The other changing parameter values in the preliminary analysis are as follows:

S: Uncapacitated, and 75%
C: 80%
$h^M_1 = h^M_2 = 0,3$

$h^W_1 = h^W_2 = 0{,}45$

$k_1 = k_2 = 0{,}6$

$K_1 = K_2 = 30$

V is determined by C as explained above.

P^M_i is determined by the $\overline{\cos t}$.

P^W_i is once set to low (equal) price and once to medium price.

Thus, 120 basic models are run 4 times, once with no capacity limits of S with equal P^M_i to P^W_i, once with no capacity limits of S with equal $P^W_i = 1.23 * P^M_i$, and the two situations again with 75 per cent S capacity limit, and a total of 480 models are investigated. The output results of each run is reported in Appendix A. We give the four situations the listed names:

Situation1: No Space Constraint – Low WH Price
Situation2: No Space Constraint – Medium WH Price
Situation3: 75% Space Constraint – Low WH Price
Situation4: 75% Space Constraint – Medium WH Price

These four situations are instances from the big class of cases where the unit production cost of each item is equal to each other ($c_1 = c_2 = 3$) and we imply 80 per cent truck capacity limit. Thus, considering all 4320 cases, we classify the cases into four main groups:

Group1: Identical unit production cost – 80 per cent truck capacity
Group2: Identical unit production cost – 50 per cent truck capacity
Group3: Unidentical unit production cost – 80 per cent truck capacity
Group4: Unidentical unit production cost – 50 per cent truck capacity

As it is seen, our basic 4 situations for the preliminary analysis fall into Group1.

Observation 1: In all of 480 models, the average cost $\overline{\cos t}$ has the same value within each scenario like SC1, SC2, etc. $\overline{\cos t}$ changes from scenario like scenario, but remains the same in a scenario among the 4 different situtations as it is reported in Table 4.7. This is beacuse $\overline{\cos t}$ is mainly influenced by the total amount of demand and very little affected by the other parameters included in the function of $\overline{\cos t}$, since they mostly appear as ratios which is not desperately different from situation to situation. So, the other parameters in the formula can be seen as unavoidable. As expected, we observe a monotonic trend of $\overline{\cos t}$ subject to the total amount of demand, i.e. the lower the demand the the higher the average cost as shown in Figure 4.5.

Table 4.7. $\overline{\cos t}$ according to each scenario in all 4 basic situations in Group1

Scenario	$\overline{\cos t}$
SC1(50-50)	4.27
SC2 (100-100)	4.12
SC3 (150 – 150)	4.07
SC4 (100-50)	4.17
SC5 (150 -50)	4.12
SC6 (150 – 100)	4.09

The observation about the average cost is also valid for the scenarios in the remaining situations of the other three groups. Table 4.8 shows the value of $\overline{\cos t}$ in each different group.

Table 4.8. $\overline{\cos t}$ according to each scenario in Group2, Group3, Group4

Scenario	Group2	Group3	Group4
SC1(50-50)	4.29	6.16	6.18
SC2 (100-100)	4.14	5.94	5.96
SC3 (150 – 150)	4.09	5.86	5.88
SC4 (100-50)	4.19	6.59	6.61
SC5 (150 -50)	4.14	6.81	6.83
SC6 (150 – 100)	4.11	6.24	6.26

Although the average cost of SC2 and SC5 are the same in Group1 and Group2 it becomes different in the other groups. This is due to the unidentical unit production cost in the last two groups despite of having the same total amount for both items. Since the demand for item 1 in SC5 higher than the demand for item 2 the average cost becomes also higher in Group3 and Group4. Having different average cost in each group within each scenario shows also how it is affected by the truck capacity.

Observation 2: Revenue R which is a function of α_i, $d_{i,t}$, P^M_i and P^W_i becomes the same value in each scenario of Situation1 and Situation3 since in this situation $P^M_i = P^W_i$. This statement is true for overall problem: In cases where the price of the plant does not differ from the price of the warehouse the revenue becomes always the same value within each scenario. The statement above can be extended like that: For special cases, where $\alpha_i = 1$ (all demand is satisfied by the plant) and $\alpha_i = 0$ (all demand is satisfied by the warehouse), the revenue states again the same within each scenario. This is for example is observed in cases where high price is implied in the warehouse where $2*P^M_i = P^W_i$. In high price cases the model sends almost all items produced to the warehouse and sells from there. So, α_i becomes 0, and despite having different prices the revenue remains the same in these cases.

In other remaining two situations of the preliminary analysis, R values changes in every model since every model has different values of α_i and P^M_i is not equal to P^W_i. As it is shown in Figure 4.5., in Situation1 and Situation3, revenue increases when the demand increases, independent of α_i. The behavior of revenue in other situtions will explain itself when the behaviour of profit and α_i, is reported.

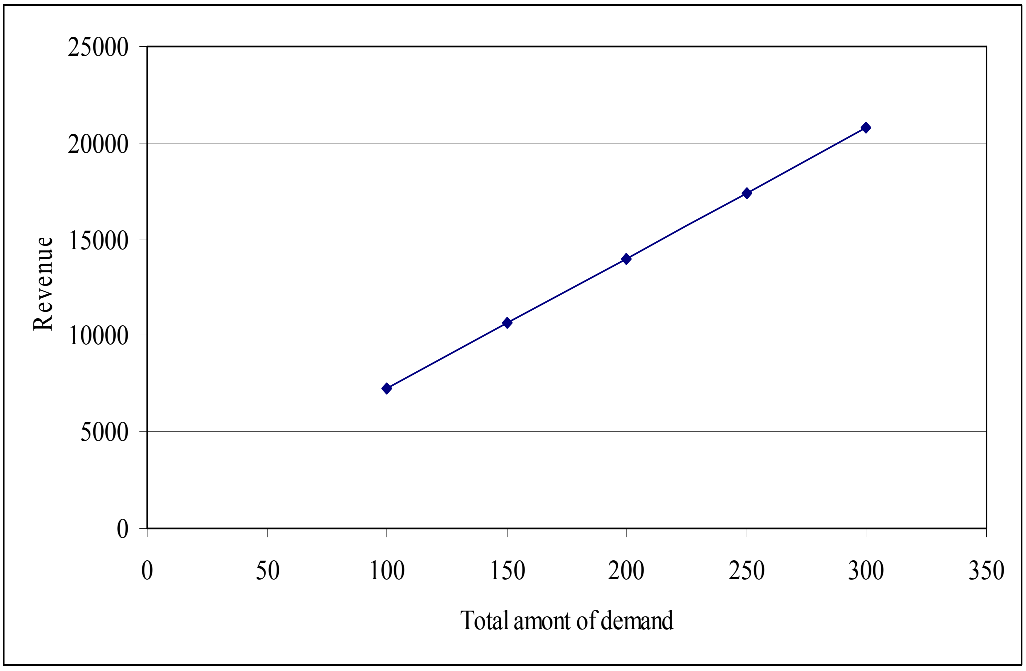

Figure 4.5. Revenue with varying amount of total demand in Low WH Price cases

Observation 3: In the preliminary analysis, at first, the profit trend is investigated while varying the demand level, demand structure and batch size for each of the 4 situations (Table A.1., A.2., A.3. and A.4. at Appendix A)

In order to analyze the cases better and visualize the outputs, the profit results of each scenario with respect to batch size, in other words production rate, are shown graphically (Figure 4.6., 4.7., 4.8., 4.9., 4.10., 4.11.).

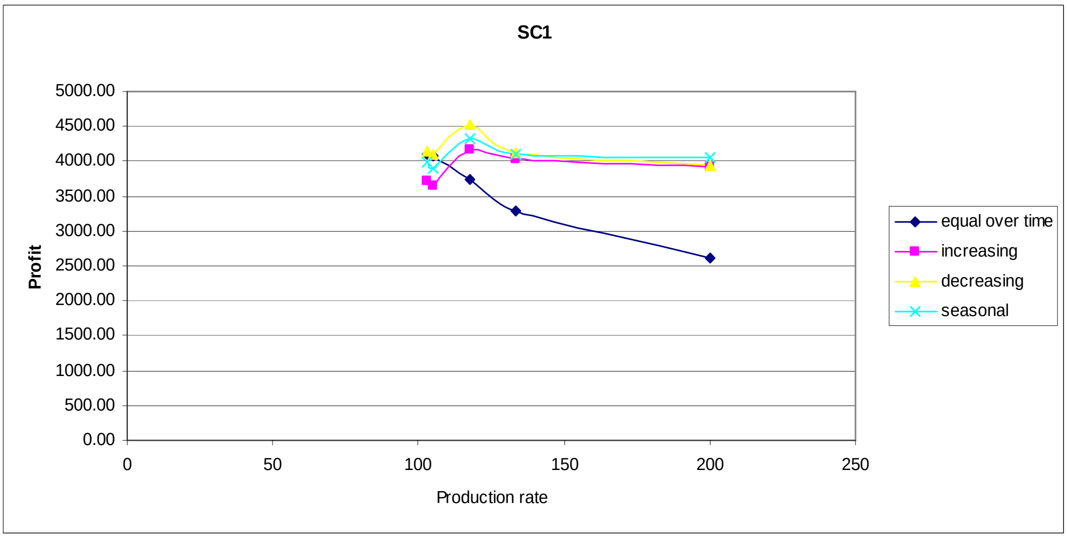

Figure 4.6. Profit with respect to batch size in Situation1, SC1

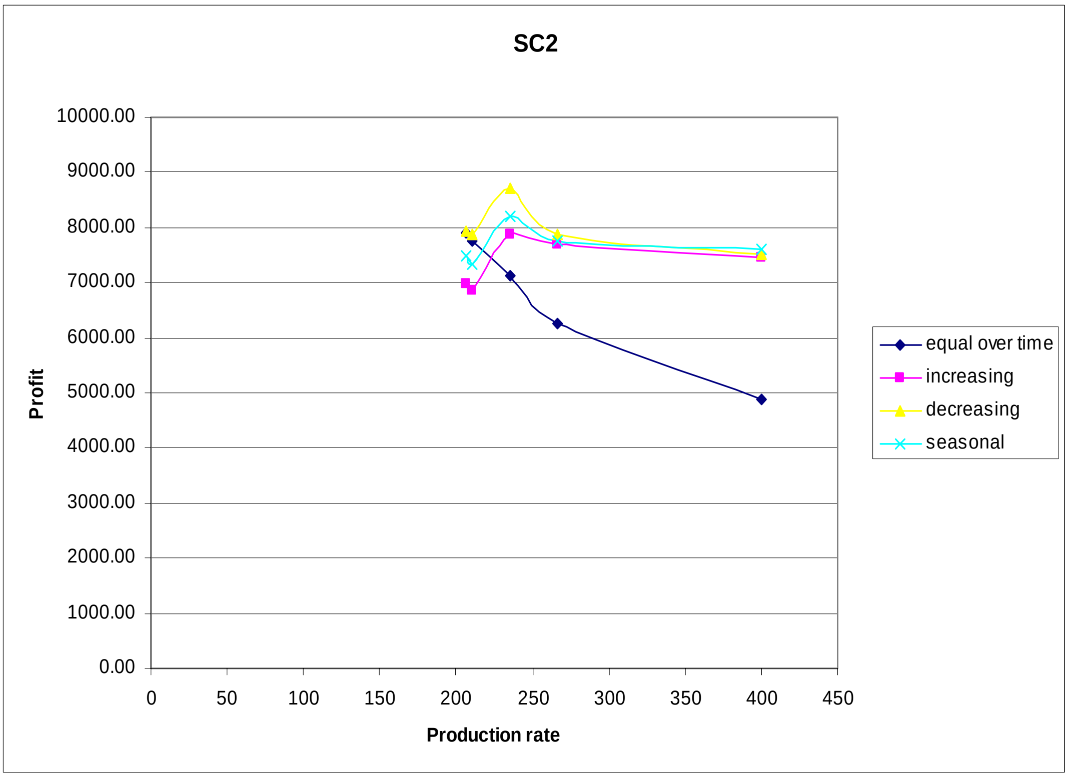

Figure 4.7. Profit with respect to batch size in Situation1, SC2

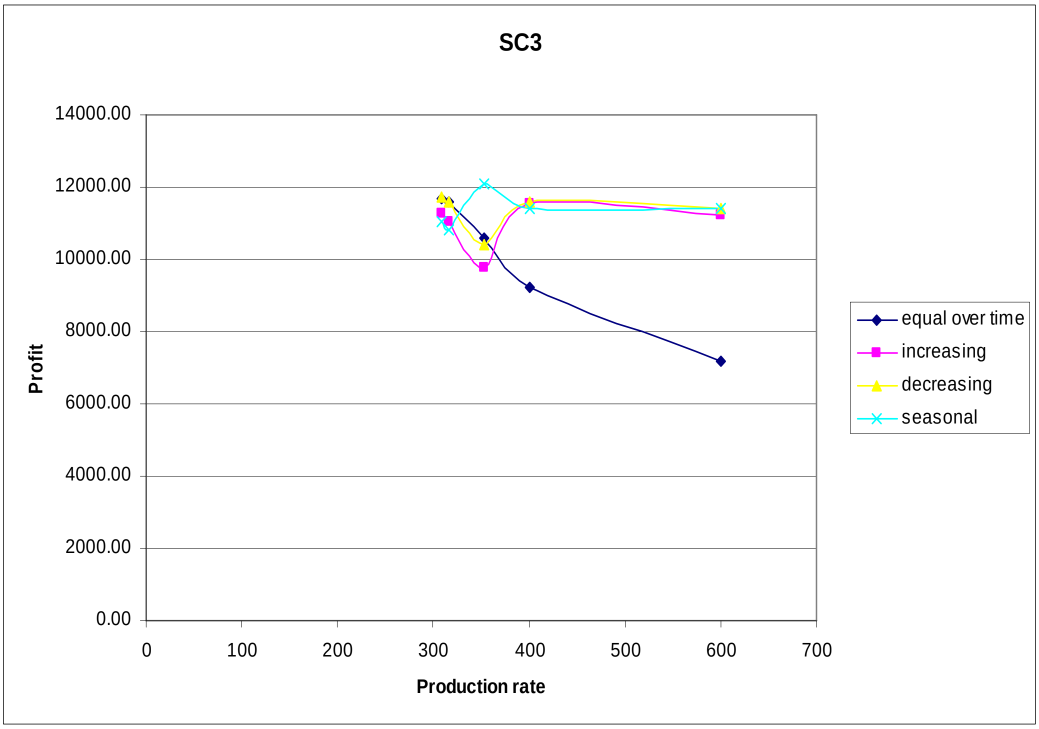

Figure 4.8. Profit with respect to batch size in Situation1, SC3

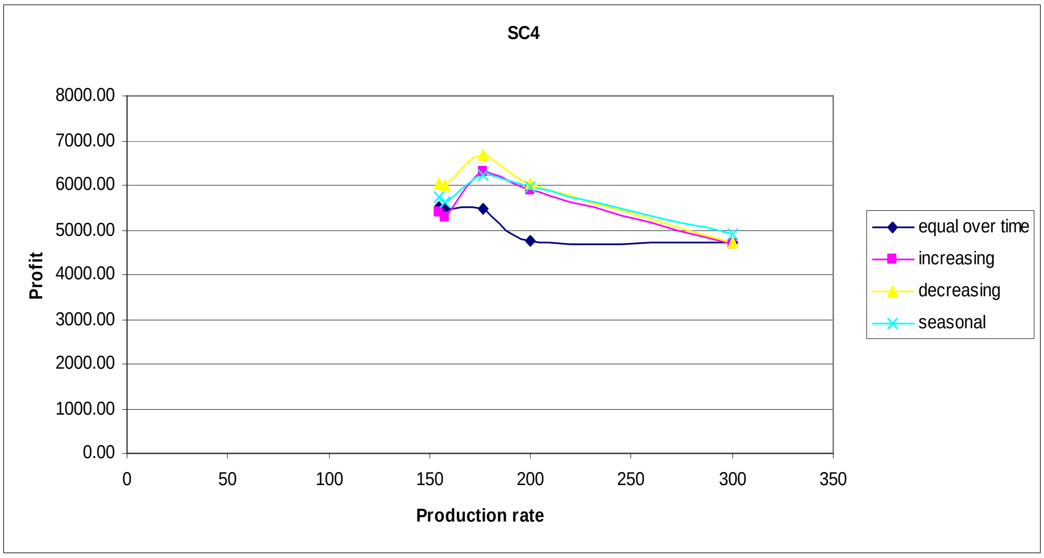

Figure 4.9. Profit with respect to batch size in Situation1, SC4

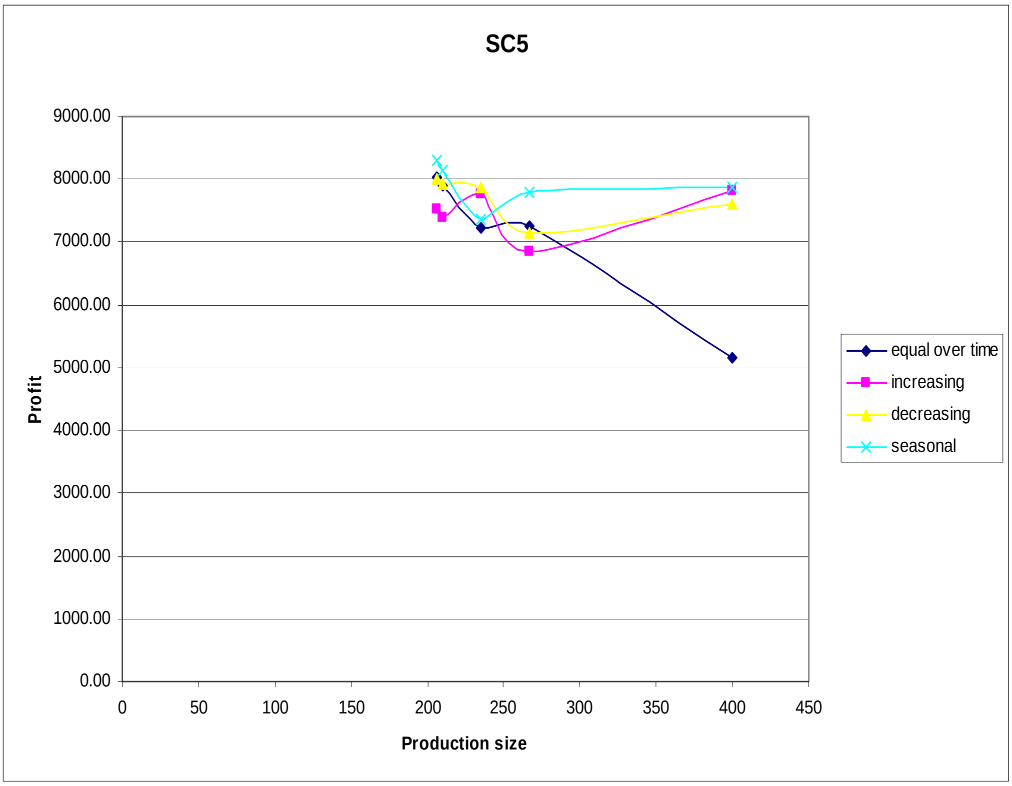

Figure 4.10. Profit with respect to batch size in Situation1, SC5

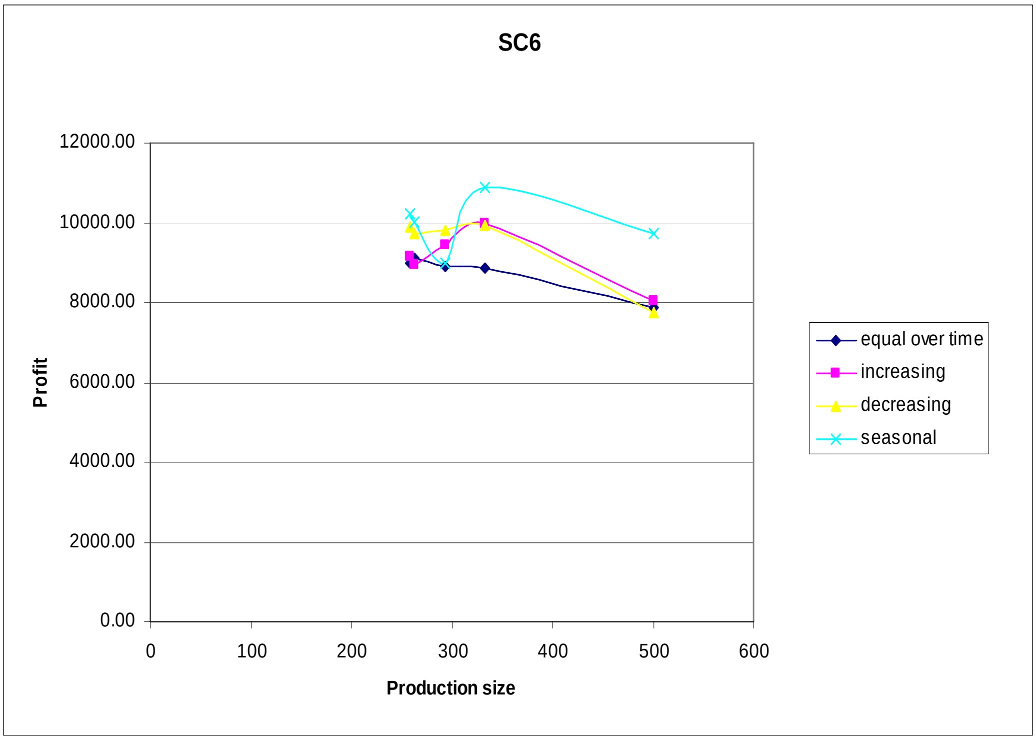

Figure 4.11. Profit with respect to batch size in Situation1, SC6

Observation 3.1.: The behavior of profit can mainly be explained by the production cost f_2, because production cost is more dominant among other costs. That's why it is very important to understand the production cost trend. Another important point is that production cost f_2 takes the same value among all four situations of the preliminary analysis and is only affected by demand level, demand structure and production rate, keeping unit production cost unchanged. Obviously, this fact can be explained by the concern of meeting the demand, i.e. in every situation's demand profile and batch size, the same amount of demand must be satisfied which is independent of price and space capacity. Since production is the same among the situations, setup cost is also the same in Situation1 and Situation2, and also in Situation3 and Situation4 since although the warehouse price does not affect the production schedule, the space capacity constraint may influence in order to optimize the space and truck utilization, production cost f_2 and transportation cost.

Observation 3.2.: The observation above about the production cost f_2 can easily be extended to the whole problem. Production cost remains always the same and takes the same value within each scenario. This implies that the production cost is independent of space capacity, truck capacity and price. We also observe a change if we go from one group to another, i.e. if we change the unit production cost. For example, in the case of unidentical unit production cost, where the unit production cost of item 1 is two times greater than the second, the production cost becomes high as expected. In short, production cost is only affected by demand level, demand structure, production rate and unit production cost. Since the statement influences all cases and production cost affects the result more than the other cost items, a detailed analysis of production cost f_2 is presented.

Observation 4: First, the production cost in SC1(demand level: 50, 50) with varying batch size is shown in Figure 4.12. The trend observed is the same as in SC2 (demand level: 100, 100), SC4 (demand level: 100, 50) and SC3 (demand level: 150, 150) with only difference in the demand structures "increasing" and "decreasing" (Figure 4.13.). In general, production cost decreases as production rate μ_i decreases since the production is in batches. It means that although the production frequency may be fast the cost decreases because in each production lower amount of item is produced. For example, in SC1, production frequency, $\sum_{t} X_{i,t}$, for the demand structure “equal over time” is given in Table 4.9. When the production rate is at its maximum the production frequency becomes its minimum. However, since the batch size is very big the prodution cost is at its maximum as well. This is very good observed in demand structure “equal over time” since the starting inventory of “equal over time” much different than the others and thus depends more on production to meet the demand. In other demand structures of SC1, SC3 and SC5, where the production rate is faster, the production frequency starts slower since in these cases the starting inventory is high. But, at production rates μ (4) and μ (5) where production rate is slow (i.e. batch size is small) the production frequency reaches its maximum and becomes the same among μ (4) and μ (5) since the rates are very close to each other. That is one of the reasons for the production cost behaviour in demand structures other than “equal over time” . In μ (3), production cost becomes minimum in demand structures other than “equal over time” because it is still enough to produce with the same frequency like in μ (2) to satisfy the demand. But, since the batch size here is smaller f_2 makes its minimum peak, and becomes more in the slower rates because of faster production frequency. Unlike SC1, SC2 and SC4, in SC3, we observe a different trend of f_2 in the demand structures “decreasing” and “increasing”, i.e. production cost reaches its maximum at μ (3) as the production frequency here increases from μ (2) to μ (3) unlike from μ (3) to μ (4) in SC1, SC2 and SC4.

Table 4.9. $\sum_t X_{i,t}$ with varying production rate in SC1, $\bar{d}_1 = \bar{d}_2 = 50$

Demand structure	Production rate	$\sum_t X_{1,t}$	$\sum_t X_{2,t}$
equal over time	μ (1)	3	3
	μ (2)	4	4
	μ (3)	4	4
	μ (4)	4	4
	μ (5)	4	4
increasing	μ (1)	2	2
	μ (2)	3	3
	μ (3)	3	3
	μ (4)	4	4
	μ (5)	4	4
decreasing	μ (1)	2	2
	μ (2)	3	3
	μ (3)	3	3
	μ (4)	4	4
	μ (5)	4	4
seasonal	μ (1)	2	2
	μ (2)	3	3
	μ (3)	3	3
	μ (4)	4	4
	μ (5)	4	4

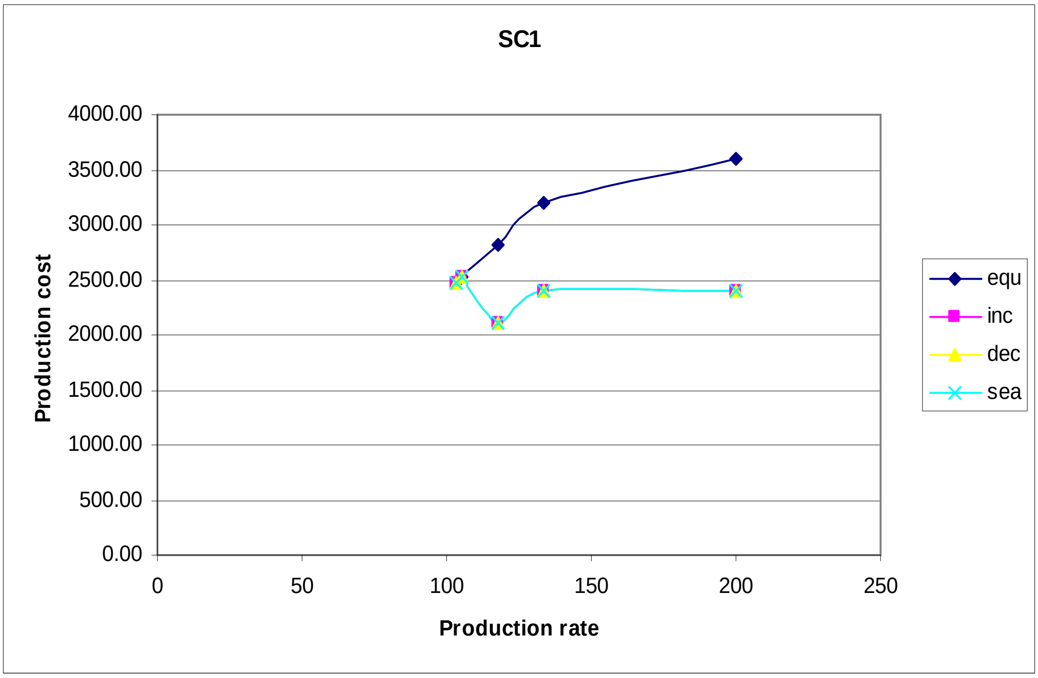

Figure 4.12. Production cost with varying production rate of SC1

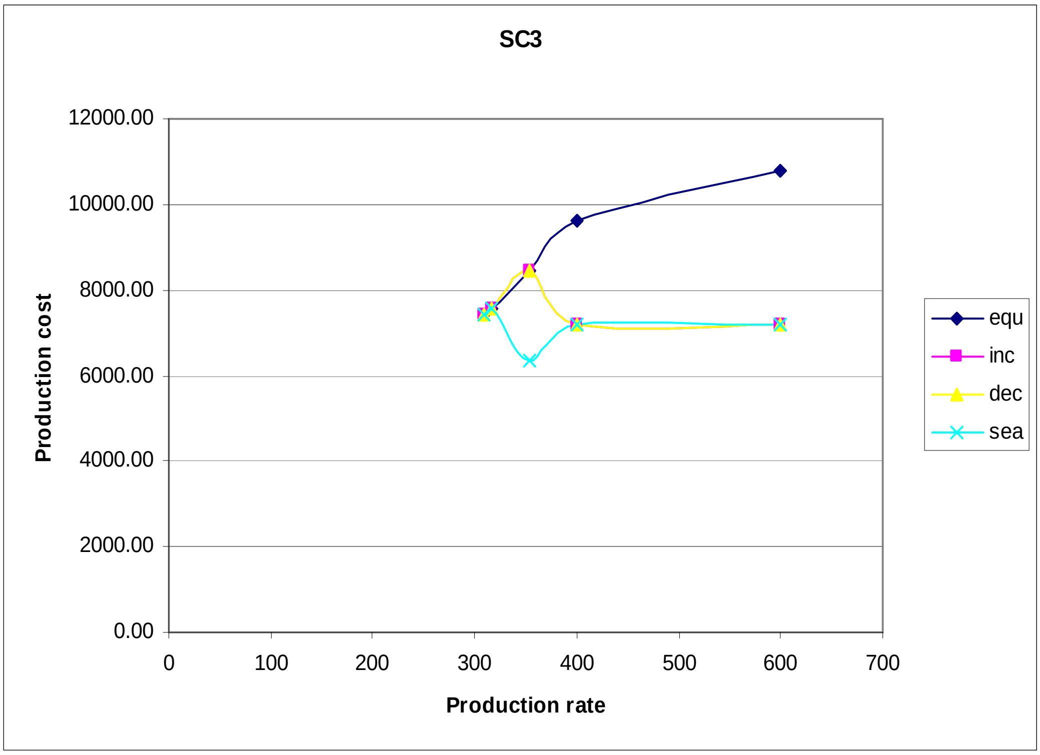

Figure 4.13. Production cost with varying production rate of SC3

To analyze the production cost trend in other scenarios SC5 (demand level: 150, 50) and SC6 (demand level: 150, 100), and to compare the results with resprect to scenario, batch size and demand profile, we present the production frequencies in Table 4.10.

Table 4.10. $\sum_{t} X_{i,t}$ with varying production rate in all scenarios

Demand structure	Production rate	$\sum_{t} X_{i,t}$ in Scenarios											
		SC1		SC2		SC3		SC4		SC5		SC6	
equal over time	μ (1)	3	3	3	3	3	3	3	2	4	2	3	2
	μ (2)	4	4	4	4	4	4	5	2	5	2	4	3
	μ (3)	4	4	4	4	4	4	5	3	6	2	5	3
	μ (4)	4	4	4	4	4	4	6	3	6	2	5	4
	μ (5)	4	4	4	4	4	4	6	3	6	2	5	4
increasing & decreasing	μ (1)	2	2	2	2	2	2	3	2	3	1	3	2
	μ (2)	3	3	3	3	3	3	4	2	5	2	4	2
	μ (3)	3	3	3	3	4	4	4	2	5	2	4	3
	μ (4)	4	4	4	4	4	4	5	3	6	2	5	3
	μ (5)	4	4	4	4	4	4	5	3	6	2	5	3
seasonal	μ (1)	2	2	2	2	2	2	3	2	3	1	3	2
	μ (2)	3	3	3	3	3	3	4	2	4	2	3	2
	μ (3)	3	3	3	3	3	3	4	2	5	2	4	3
	μ (4)	4	4	4	4	4	4	5	3	5	2	4	3
	μ (5)	4	4	4	4	4	4	5	3	5	2	4	3

Due to the same reasons mentioned and the production frequency results given in Table 4.14 , we have the following trend for SC5 and SC6.

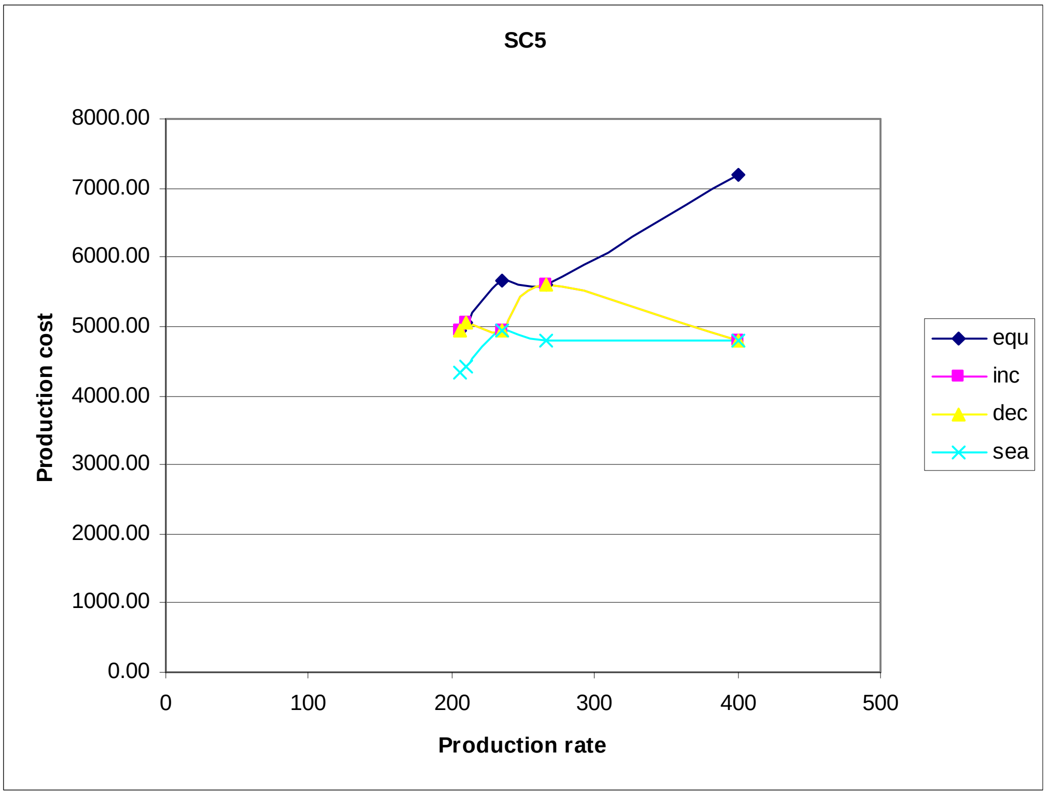

Figure 4.14. Production cost with varying production rate of SC5

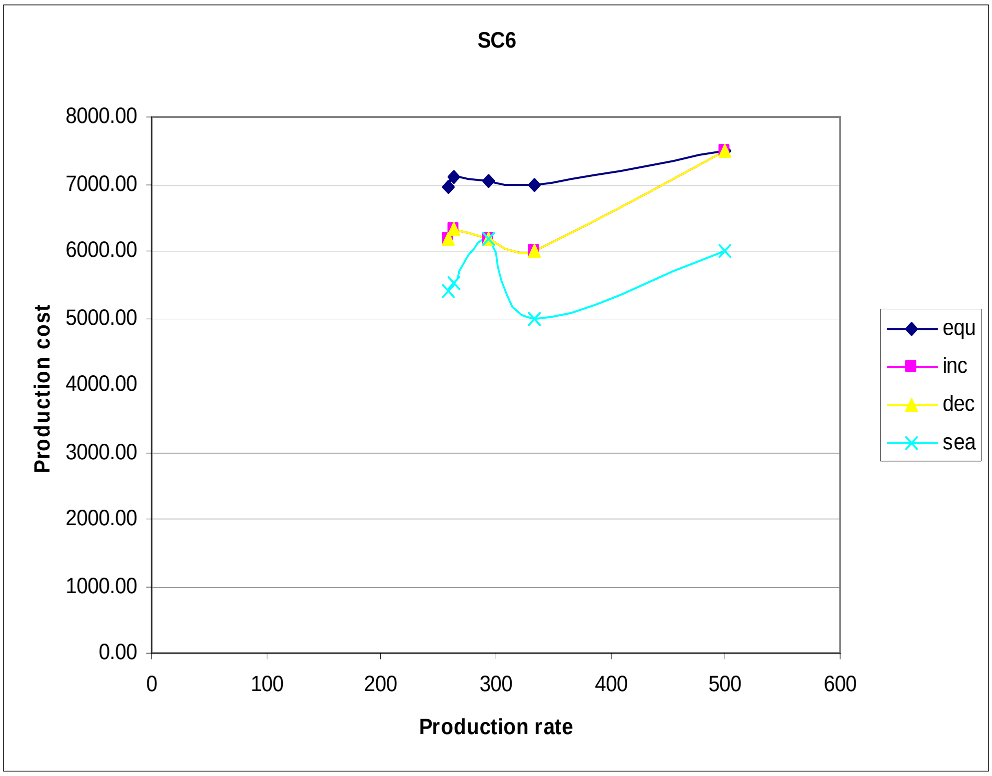

Figure 4.15. Production cost with varying production rate of SC6

Since the production cost is the main cost over all costs such as inventory holding cost, it explains the behaviour of profit. Lastly, the higher is demand level the more is the production cost which increases proportional to the demand level. That is why the highest production cost is made in SC3.

Observation 5: In Situation1 of the preliminary analysis, where warehouse price is equal to the plant price and there is no space constraint in the plant, it makes no sense to send products to warehouse and satisfy some amount of demand from the warehouse. Otherwise, that would cause only more cost while revenue does not change. Thus, in all 120 models of Situation1, α_i s are equal to one which means that all demand is satisfied directly from the plant. That also results in no inventory holding cost at warehouse f_4 and no transshipment cost f_5.

Observation 6: Where production cost takes the same value in Situation1, inventory holding cost at plant f_1 makes the difference in profit. In Situation1 and Situation2, where there is no space constraint, the trend of inventory holding cost at plant f_l shows close similarity. In Figure 4.16., the behavior of f_l of Situation1 SC4 (demand level: 100, 50) is presented. For SC1 and SC2, a similar trend is observed. Obviously, plant inventory holding cost in "increasing" demand structure is higher than the other demand structures. This is the result of having huge starting inventory on-hand which is consumed more in the last periods since the demand in the first periods is not high. Therefore, the inventory is kept for a long time. Unlike "increasing" demand structure, the inventory holding cost at plant f_l is small than the others due to the fact that the starting inventory is early consumed. So, not much inventory is held over the whole time horizon. In "equal over time", we have an increasing function as a result of having little starting inventory, increasing production cost and having the production schedule distributed over the whole time horizon like the demand distribution. In "decreasing" demand structure, the model starts with the production first in the first periods and consumes for meeting the demand. It then increases as production rate increases since this time batch size is big when production occurs but demand is low, so inventory at plant remains.

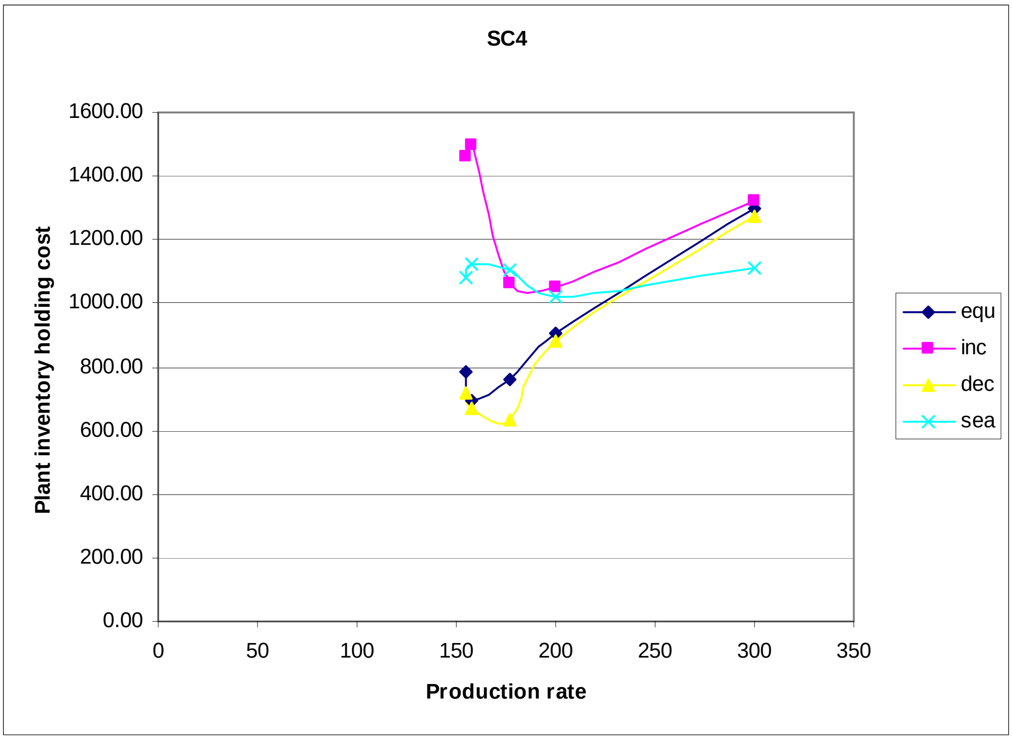

Figure 4.16. Inventory holding cost at plant with varying production rate of SC4 in Situation1

In “increasing” demand structure, the model first tries to consume the starting inventory and produce afterwards in the last periods. Since we already have inventory on-hand, inventory holding cost becomes maximum and decreases up to μ (3) because the production decreases as well. But, after μ (3), it makes a change and increases, no more inventory on-hand is to be used and production starts. It produces high amount of items which results in inventory. In “seasonal” demand structure, the production follows the demand distribution where production frequency increases as the production rate decreases. Since we have inventory on-hand at the beginning, production occurs in mittle of time horizon, and it produces in almost every mittle period as μ decreases. That is why the graph of f_I has a very low slope. The trend only makes difference in SC5 and SC6 in that the trend makes a maximum pick point at μ (3) due to the production as explained before. For Situation1 and Situation2, a similar trend is observed due to the fact that there is no space constraint in both cases, and it does not hold much inventory in the warehouse even when it sends items to the warehouse in Situation2 because it prefers to send approximately the amount of $d^W_{i,t}$. That’s why inventory holding cost does not very much change among these two situations. However, it is different in Situation3 and Situation4.

Observation 7: In Table B.1. at Appendix B, the α_i values are given for Situation2, where the space at plant has no restriction but we have a medium price at warehouse. As we see, α_i in SC1 for “equal over time” demand structure takes the values 0.40 and 0.00 respectively and is not affected by the production rate. The behaviour of μ among the scenarios in “equal over time” is also little influenced by the demand level. This is because the model lets each time-period one truck go from plant to the warehouse, sends as much as it should be satisfied by the warehouse and keeps no inventory holding cost at warehouse. Since the production is the same among all situations total amount of inventory held in both plant and warehouse does not change. It only makes difference in the total amount cost since unit holding cost at warehouse more than at plant. The higher the demand level the less product is sent to warehouse because in high levels of demand, the transportation cost and inventory holding cost increases which can not be balanced by the revenue. So, the model sends products to warehouse only if the revenue win is greater than the extra cost. For example, although it makes sense to satisfy a certain amount of demand from the warehouse in “ equal over time”, α_i takes the value 1 in other demand structures and high demand levels, because in these demand structures we have already inventory on-hand causing holding cost. In cases where some demand is met by warehouse, like in SC1, both the product shipped and the number of truck follow the demand distribution.

Observation 8: Analyzing Situation3 where the prices are equal to each other but there is a restriction on the space capacity at plant shows that the revenues gained in Situation1 are more than in Situation3. Obviously, the fact can be explained by the unwillingness of the model of sending products to the warehouse. Since the prices are equal meeting some demand from warehouse only causes more cost such as transshipment and warehouse inventory holding cost. That is why all demand is satisfied directly by the plant in Situation1 where extra transshipment and holding cost is decreased to zero. Unlike Situation1, in Situation3, the model has to send products to warehouse because there is not enough space to hold all the inventory at plant. So, meeting some demand from the warehouse is an obligation in this situation. In order to minimize transshipment & inventory holding cost at warehouse, the model tries to keep the most possible amont of products in the plant and the transshipment only occurs when the inventory at plant reaches the maximum space capacity available. It also takes into account the periods where production occures and sends the products in these periods to the warehouse. In Table B.2. at Appendix B, α_i values in Situation3 are presented. The average of α_i values here is 0.53. Although the average is close to the average in Situation2 it is better distributed since almost every model makes transshipment. We observe an increasing trend of transshipment cost with varying production rate in every scenario due to the fact that the more the batch size the more difficult to hold them at plant as shown in Figure 4.17. So, shipment becomes unavoidable. The transshipment cost f_5 in “increasing” is higher than in other demand structures since the starting inventory is consumed more in the last periods and inventory holding cost at plant would be very high as explained in Situation1. So, the model has to make more transshipment. However, the costs are not very different from each other since transshipment is due to the space limit. They only get higher values as demand level changes, i.e. the higher demand level the more transshipment cost.

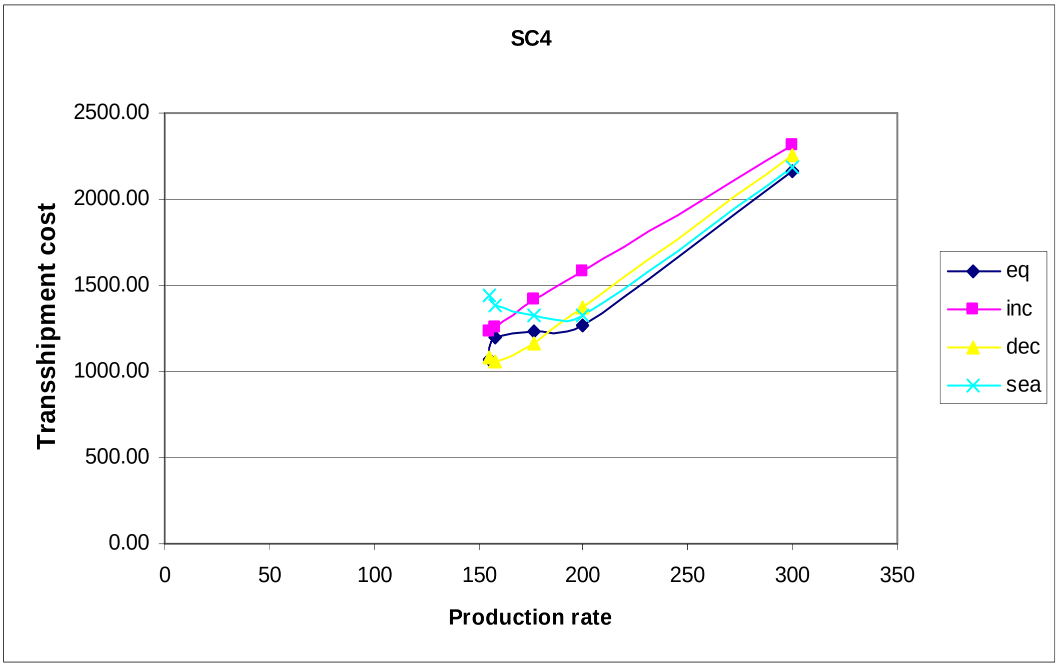

Figure 4.17. Transshipment cost with varying production rate of SC4

The explanations above also are the reason for the behavior of inventory holding cost at warehouse f_4. The trend is like the trend of f_1, inventory holding cost at plant, in Situation1 where there is no capacity constraint. It means that the behavior of f_1 in Situation1 is transferred to f_4 in Situation3. Another reason for that is total amount of inventory remains the same in every situation due to same production, and that only distribution of total inventory makes the difference in behavior of f_1 and f_4.

Observation 9: In Table B.3. at Appendix B, α_i values in Situation4 are given. It shows that in almost every scenario α_i becomes maximum in the "equal over time" demand structure. That means that, in Situation4, where warehouse price is medium, the demand is satisfied more by the plant in "equal over time". This is because transshipment follows the demand distribution and thus the model sends product to warehouse each period which causes high transshipment cost. The minimum of α_i values are observed in "decreasing" demand structure since it is better to send just in the early periods of time horizon in order to minimize the inventory holding cost at plant and sell the products to a higher price. The average of α_i values in Situation4 is 0.09 which shows that the demand mostly satisfied by warehouse. This is logical and the lowest value compared to other situations because the model already has to make transshipment due to the space capacity constraint. It increases the number of products shipped in order to make more profit by utilizing the transshipment. The transshipment costs and inventory holding costs in plant are close to each other within each scenario and do not show a certain trend. On of the analysis in transshipment cost in Situation3 and Situation4 is that within each demand structure of a demand level the transshipment is maximum where there is a huge difference between the production cost of that demand structure and of the others. For example, in Situation4 SC1, the production cost in ”equal over time” with the productin rate μ (1) is much more than in the other demand structures. Here, the transshipment cost also becomes maximum. However, if the difference of f_2 is not very high then starting inventory is taken into account as a second factor like in seasonal demand structure of SC5 and SC6. Lastly, when starting inventories also close to each other, usually when the production rate is fast then f_5 becomes high in the “increasing” demand structure in order to get rid of inventory holding cost at plant.

Observation 10: In all cases where high price is applied α_i becomes zero meaning that all demand is satisfied from the warehouse. The output is affected nor by demand profile, costs neither by space & truck capacity. This shows how great effect the price has on deciding where to sell. We can state that the model sends all items to warehouse, and a warehouse-shipment occurs when there is a noticeable price difference between warehouse and plant. The observation also addresses that price is more important than the other parameters influencing the result.

Observation 11: In Table 4.11., Table 4.12., Table 4.13., and Table 4.14., average α_i values are given for each demand level, each truck capacity, each space capacity at plant and price policy with only exception: Among this combinations where α_i gets 1 or 0 in all cases the results are not given since they are out of the observation. Here, we see that, in low price policy, in other words where there is no price difference between warehouse and plant, α_i is not affected by truck capacity. The outcome for α_i does also not change if we have products with unidentical unit production cost. This can be explained by the fact that the plant actually is not willing to send products to the warehouse because of equal selling prices at both side. In such a case, sending items to the warehouse does only cause more cost like transshipment and inventory holding cost at warehouse. Therefore, selling some amount of product not from the plant is due to the obligation. It is an obligation since there is a space restriction at plant. As only space is causing transshipment to warehouse we do not observe a change in α_i values if we change truck capacity or produce unidentical items. In such a case, a change in α_i values can be observed if we change the space limit of the plant. If we increase the space capacity of the plant the model keeps more items in the plant.

Observation 12: Apart from the low-price policy, in the medium-price policy, comparing the α_i values within each demand level by keeping the products the same type but changing the truck capacity we see that the model is then affected by the truck capacity. Increasing the truck capacity results in an increase in the α_i values as well. This means that the model keeps items more in the plant. Increasing the size of the truck causes extra cost, i.e. the fixed transshipment cost per truck gets higher. On the other hand, the model should use more truck if it would send the same amount of product to the warehouse with smaller size of trucks. Because it results in high cost the model decides on increasing the α_i values so that lower amount of items are transshipped to the warehouse, and more is sent directly from the plant.

Table 4.11. Average α values for the given cases of identical products with 80 per cent truck capacity

Demand level	Plant Space – Price Policy Cases			
	75% Space Capacity – Medium Price	75% Space Capacity – Low Price	25% Space Capacity – Medium Price	25% Space Capacity – Low Price
SC1	0.07	0.51	0.07	0.18
SC2	0.12	0.52	0.07	0.19
SC3	0.09	0.57	0.10	0.19
SC4	0.07	0.52	0.08	0.18
SC5	0.08	0.54	0.09	0.18
SC6	0.12	0.53	0.11	0.19

Table 4.12. Average α values for the given cases of identical products with 50 per cent truck capacity

Demand level	Plant Space – Price Policy Cases			
	75% Space Capacity – Medium Price	75% Space Capacity – Low Price	25% Space Capacity – Medium Price	25% Space Capacity – Low Price
SC1	0.03	0.52	0.04	0.17
SC2	0.05	0.53	0.02	0.18
SC3	0.05	0.59	0.05	0.19
SC4	0.04	0.55	0.03	0.17
SC5	0.04	0.55	0.05	0.15
SC6	0.03	0.55	0.05	0.17

Observation 13: In Table 4.20. and Table 4.21., α_i values for both items are given since we present here products with unidentical unit production cost and want to be able to

see the difference compared to the identical products if there is any. It is observed that the α_i values of item 1 are higher than the α_i values of item 2. So, the model is more willing to send products with lower unit production cost to the warehouse and to keep the items with high unit production cost in the plant. The reason for that is that the items with high unit production cost cause higher unit shipment cost, and inventory holding cost. Therefore, the model tries to keep more amount of item 1 in the plant compared to item 2.

Table 4.13. Average α_i values for the given cases of unidentical products with 80 per cent truck capacity

Demand level	Plant Space – Price Policy Cases			
	75% Space Capacity – Medium Price	75% Space Capacity – Low Price	25% Space Capacity – Medium Price	25% Space Capacity – Low Price
SC1	0.04 (0.06 – 0.01)	0.49 (0.71 – 0.26)	0.04 (0.08 – 0.01)	0.16 (0.24 – 0.08)
SC2	0.04 (0.08 – 0.00)	0.51 (0.71 – 0.32)	0.04 (0.07 – 0.01)	0.16 (0.24 – 0.08)
SC3	0.05 (0.10 – 0.00)	0.54 (0.75 – 0.32)	0.03 (0.05 – 0.01)	0.17 (0.24 – 0.10)
SC4	0.03 (0.05 – 0.00)	0.50 (0.76 – 0.23)	0.06 (0.10 – 0.01)	0.15 (0.25 – 0.06)
SC5	0.02 (0.05 – 0.00)	0.48 (0.76 – 0.21)	0.05 (0.09 – 0.00)	0.16 (0.25 – 0.06)
SC6	0.05 (0.10 – 0.00)	0.49 (0.75 – 0.22)	0.07 (0.12 – 0.02)	0.16 (0.27 – 0.06)

Table 4.14. Average α_i values for the given cases of unidentical products with 80 per cent truck capacity

Demand level	Plant Space – Price Policy Cases			
	75% Space Capacity – Medium Price	75% Space Capacity – Low Price	25% Space Capacity – Medium Price	25% Space Capacity – Low Price
SC1	0.01 (0.02 – 0.00)	0.50 (0.72 – 0.28)	0.03 (0.06 – 0.00)	0.15 (0.21 – 0.09)
SC2	0.02 (0.04 – 0.00)	0.51 (0.69 – 0.32)	0.04 (0.08 – 0.00)	0.15 (0.21 – 0.09)
SC3	0.05 (0.09 – 0.00)	0.58 (0.71 – 0.45)	0.03 (0.05 – 0.00)	0.15 (0.21 – 0.08)
SC4	0.01 (0.01 – 0.00)	0.49 (0.74 – 0.25)	0.02 (0.04 – 0.00)	0.15 (0.25 – 0.05)
SC5	0.01 (0.02 – 0.00)	0.49 (0.76 – 0.22)	0.02 (0.05 – 0.00)	0.14 (0.24 – 0.03)
SC6	0.02 (0.03 – 0.00)	0.49 (0.76 – 0.23)	0.03 (0.06 – 0.00)	0.16 (0.26 – 0.05)

5. CONCLUSIONS AND FURTHER RESEARCH

This study addresses the interaction and integration between the manufacturing facility and the warehouse, and analyzes the trade-offs between direct-plant shipment, consolidated warehouse replenishment and a combination of both options in a single-machine two-item system environment where demands are deterministic but have different profiles.

We developed a mathematical model for evaluating the benefits of direct shipment and consolidated warehouse shipment, and of the combination of two like a hybrid system, and found the answers for when and in which order each item is produced, how many units are shipped to warehouse when, and when the items should be sold directly from plant, and when from warehouse. To fulfill that, the study is completed by a set of computational tests to validate the model itself, taking into account the possible scenarios and possible variations of the relevant parameters. We also focused our attention on undertaking the optimization of the problem by making use of the increasing capabilities of computer technology. To reach this goal and also to make our computations more systematic and fast, we developed a tool which creates each possible demand scenario in an Excel spreadsheet. The integration of MS Excel and GAMS is achieved so that GAMS connects to different areas in different files to reads/writes from/to speadsheets. Each time GAMS run 120 models at the same time for each of which it reads the input data and then writes the results. This functionality provides fast and an easy way of customizing modifying scenarios, allows optimizing the model and analyzing the cases automatically. Once all needed graphics, functions are specified in Excel all is needed to be done is to run GAMS. As result, it gives all required reports. Therefore, the proposed tool developed during the study will fill the lack of tools and methods that can help managers in the analysis of the integrated systems. We believe that the study increases the practical value of the model for practitioners, and the practical aspect of the study is of considerable industrial significance.

The results obtained show that how domain the production cost is among other costs such as setup, transshipment, inventory holding cost, and how significant it influences the profit. The profit trends are almost the same; the behaviour of profit can mainly be explained by the production cost. In general, production cost decreases as production rate μ decreases since the production is in batches. The higher demand level the more is the production cost which increases proportional to the demand level. Where warehouse price is equal to the plant price, it makes no sense to send products to warehouse and satisfy some amount of demand from the warehouse. Plant inventory holding cost in "increasing" demand structure is higher than in the other demand structures. In "decreasing" demand structure, the model starts with the production in the first periods and consumes items for meeting the demand. It then increases as production rate increases since this time batch size is big when production occurs but demand is low, so inventory at plant remains. With analysis, we observed how capacity constraint introduced by the plant and by the truck change the results. Another conclusion is that demand structure and production rate (batch size) are important factors that should always be taken into account. Besides, it is concluded how deeply the result is affected by the price policy. In the tests done during the thesis work, it is seen that the price difference between the plant and the warehouse is the most important parameter. That items with unidentical unit production cost do not have much influence on the decision variables of the model is another interesting outcome of our analysis. For managerial decision making, it is very important to know about the demand structure since the profit result depends on it. In addition, the tool generated during the thesis study would help the managers to make scenario analysis and decide on the strategy they would follow in different cases.

In this study, we made our investigations under some assumptions. Of course, changing these can always lead to areas of further research. Further research can preferably consider stochastic environments. The problem studied here may be extended from two products to n products. Besides, traveling costs due to distances may be another aspect to go. Times such as setup times and traveling times may be included in further research. Another area would be to add the customer aspect into the model and integrate the behavior of the customer into the model as well.

APPENDIX A. PROFITS OF FOUR BASIC SITUATIONS

Table A.1. Profits of Situation1, No Space Constraint – Low WH Price

Demand level	Demand structure	Batch size				
		μ (1)	μ (2)	μ (3)	μ (4)	μ (5)
SC1	Equal	2610.75	3280.75	3738.40	4085.49	4108.28
	Increasing	3915.15	4035.15	4169.27	3641.99	3717.52
	Decreasing	3945.15	4115.15	4529.27	4099.89	4152.68
	Seasonal	4060.83	4090.83	4316.71	3895.04	3975.78
SC2	Equal	4891.50	6261.50	7127.38	7747.82	7916.55
	Increasing	7440.30	7680.30	7877.95	6863.98	6985.04
	Decreasing	7500.30	7870.30	8698.54	7869.77	7945.35
	Seasonal	7590.30	7740.30	8192.06	7348.72	7480.20
SC3	Equal	7172.25	9242.25	10571.07	11596.46	11664.83
	Increasing	11221.05	11551.05	9768.70	11031.58	11258.16
	Decreasing	11401.05	11611.05	10388.11	11595.26	11723.63
	Seasonal	11411.13	11411.13	12088.78	10823.76	11035.98
SC4	Equal	4711.13	4741.13	5480.54	5489.55	5517.10
	Increasing	4687.73	5887.73	6300.67	5282.99	5396.28
	Decreasing	4702.73	6022.73	6666.84	5984.83	6049.01
	Seasonal	4895.57	5945.57	6224.39	5621.88	5742.99
SC5	Equal	5161.50	7251.50	7227.97	7900.97	8040.26
	Increasing	7808.10	6838.10	7760.45	7381.78	7532.84
	Decreasing	7613.10	7143.10	7874.28	7925.73	8000.01
	Seasonal	7870.98	7780.98	7355.69	8147.30	8293.14
SC6	Equal	7861.88	8881.88	8922.46	9106.09	9002.49
	Increasing	8040.68	9960.68	9443.62	8932.78	9151.60
	Decreasing	7740.68	9940.68	9805.38	9747.52	9881.81
	Seasonal	9725.72	10905.72	8993.36	10033.61	10215.92

Table A.2. Profits of Situation2, No Space Constraint – Medium WH Price

Demand level	Demand structure	Batch size				
		μ (1)	μ (2)	μ (3)	μ (4)	μ (5)
SC1	Equal	2677.23	3347.23	3804.88	4121.97	4114.76
	Increasing	3893.91	3949.68	4103.21	3676.17	3752.05
	Decreasing	3961.45	4144.68	4545.56	4099.91	4181.01
	Seasonal	4126.93	4091.21	4347.09	3895.42	3989.48
SC2	Equal	4929.81	6299.81	7165.69	7722.97	7924.86
	Increasing	7350.30	7680.30	7807.36	6863.98	7015.04
	Decreasing	7445.60	7870.30	8627.42	7813.42	7945.35
	Seasonal	7626.44	7740.30	8162.06	7318.72	7480.20
SC3	Equal	7182.38	9252.38	10581.21	11387.12	11614.96
	Increasing	11221.05	11551.05	9768.70	11061.58	11228.16
	Decreasing	11401.05	11611.05	10388.11	11625.26	11723.63
	Seasonal	11411.13	11381.13	12088.78	10793.76	11065.98
SC4	Equal	4733.52	4793.52	5532.93	5541.94	5569.50
	Increasing	4661.66	5825.32	6345.04	5283.01	5396.31
	Decreasing	4660.97	5979.54	6639.53	5927.03	5977.26
	Seasonal	4820.66	5922.91	6446.30	5621.88	5742.99
SC5	Equal	5199.81	7079.81	7366.87	7939.28	8108.57
	Increasing	7758.53	6808.10	7710.93	7261.88	7502.84
	Decreasing	7458.53	7060.20	7692.53	7872.83	7918.72
	Seasonal	7840.98	7780.98	7325.69	8036.11	8211.96
SC6	Equal	7886.09	8876.09	8946.68	9130.31	9026.71
	Increasing	7811.95	9757.99	9413.62	8902.78	9121.60
	Decreasing	7590.68	10110.68	9628.91	9747.52	9929.13
	Seasonal	9545.72	10875.72	9023.36	9894.66	10185.92

Table A.3. Profits of Situation3, 75% Space Constraint – Low WH Price

Demand level	Demand structure	Batch size				
		μ (1)	μ (2)	μ (3)	μ (4)	μ (5)
SC1	Equal	551.83	1930.08	2462.80	3069.94	2851.29
	Increasing	2171.34	2593.46	2733.10	2295.99	2389.26
	Decreasing	2149.38	2828.35	3414.60	3124.37	3210.19
	Seasonal	2457.04	2885.47	2921.42	2549.31	2702.59
SC2	Equal	773.65	3560.15	4595.59	5869.88	5372.59
	Increasing	3952.67	4796.93	5046.21	4261.98	4452.78
	Decreasing	3908.76	5331.58	6439.20	5978.75	6120.38
	Seasonal	4491.92	5405.48	5428.10	4812.34	5004.01
SC3	Equal	995.48	5190.23	6728.39	8639.81	7893.88
	Increasing	6201.61	7650.44	6470.10	8449.43	8719.99
	Decreasing	6494.94	7685.31	6823.58	8870.77	9395.92
	Seasonal	6558.96	7960.94	7960.25	7089.39	7375.75
SC4	Equal	2113.99	3111.05	3920.87	4040.86	4115.82
	Increasing	1818.91	3845.52	4660.54	3260.39	3439.56
	Decreasing	1925.15	4209.41	5299.29	4718.13	4748.56
	Seasonal	2247.44	4163.93	4928.34	3722.69	3905.38
SC5	Equal	1218.40	5450.57	5550.11	6388.90	6599.21
	Increasing	4717.04	4260.32	5637.97	5368.44	5554.01
	Decreasing	4586.62	4673.62	6035.94	6548.64	6757.03
	Seasonal	4993.13	5438.85	4726.99	5907.07	6081.56
SC6	Equal	3634.88	5838.63	6515.22	6841.01	6733.96
	Increasing	3503.62	6767.81	6570.90	6514.16	6741.32
	Decreasing	2917.44	7220.84	7197.12	7499.90	7894.61
	Seasonal	5504.46	7716.53	6004.48	7295.41	7520.41

Table A.4. Profits of Situation4, 75% Space Constraint – Medium WH Price

Demand level	Demand structure	Batch size				
		μ (1)	μ (2)	μ (3)	μ (4)	μ (5)
SC1	Equal	1835.87	2819.47	3528.12	3907.71	3867.72
	Increasing	3531.03	3698.07	3830.67	3121.74	3238.63
	Decreasing	3560.41	3876.50	4390.06	3853.93	3912.34
	Seasonal	3752.27	3802.08	4050.13	3492.22	3622.82
SC2	Equal	3245.07	5310.78	6630.06	7519.28	7464.32
	Increasing	6541.08	6925.04	7283.66	5722.70	5954.25
	Decreasing	6686.36	7277.70	8272.37	7254.50	7346.36
	Seasonal	7014.10	7001.42	7487.57	6409.73	6687.10
SC3	Equal	4654.27	7764.89	9723.70	10851.99	11026.09
	Increasing	10295.98	10694.79	8041.84	10012.38	10422.24
	Decreasing	10439.81	10716.45	9016.50	10889.56	11032.70
	Seasonal	10241.50	10265.03	11134.07	9412.19	9787.49
SC4	Equal	4035.41	3997.67	5099.40	5189.67	5212.76
	Increasing	3542.39	5290.17	5683.14	4447.42	4626.44
	Decreasing	3615.89	5564.51	6454.29	5548.58	5618.99
	Seasonal	3873.22	5449.21	6136.93	4969.47	5125.86
SC5	Equal	3528.78	6718.16	6926.33	7550.23	7655.30
	Increasing	7052.68	5764.72	7157.16	6632.34	6845.97
	Decreasing	7017.00	6157.00	7298.66	7544.37	7594.85
	Seasonal	7193.24	6974.90	6469.70	7611.01	7703.43
SC6	Equal	6896.58	8481.31	8363.91	8539.32	8496.26
	Increasing	6477.88	9438.92	8638.43	7904.01	8168.57
	Decreasing	6029.51	9558.14	9160.62	9171.37	9494.25
	Seasonal	8615.31	10282.38	7756.04	9229.47	9537.56

APPENDIX B. α_i VALUES IN BASIC SITUATIONS

Table B.1. α_i values in Situation2

Production rate	Demand structure	α_i values for each demand level											
		SC1		SC2		SC3		SC4		SC5		SC6	
		α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2
μ (1)	equ	0.40	0.00	0.40	0.00	0.40	0.00	0.30	0.00	0.27	0.00	0.33	0.00
	inc	0.00	0.22	1.00	1.00	1.00	1.00	0.18	0.00	0.08	0.00	0.00	0.00
	dec	0.00	0.32	0.00	0.12	1.00	1.00	0.00	0.00	0.00	0.17	1.00	1.00
	sea	0.10	0.00	0.24	0.00	1.00	1.00	0.09	0.00	1.00	1.00	1.00	1.00
μ (2)	equ	0.40	0.00	0.40	0.00	0.40	0.00	0.30	0.00	0.27	0.00	0.33	0.00
	inc	0.25	0.00	1.00	1.00	1.00	1.00	0.31	0.00	1.00	1.00	0.54	0.00
	dec	0.10	0.00	1.00	1.00	1.00	1.00	0.09	0.00	0.48	0.00	1.00	1.00
	sea	0.11	0.00	1.00	1.00	1.00	1.00	0.06	0.00	1.00	1.00	1.00	1.00
μ (3)	equ	0.40	0.00	0.40	0.00	0.40	0.00	0.30	0.00	0.27	0.00	0.00	0.50
	inc	0.00	0.25	1.00	1.00	1.00	1.00	0.17	0.00	0.25	0.36	1.00	1.00
	dec	0.00	0.32	0.00	0.00	1.00	1.00	0.22	0.00	0.00	0.17	1.00	1.00
	sea	0.00	0.11	1.00	1.00	1.00	1.00	0.30	0.00	1.00	1.00	1.00	1.00
μ (4)	equ	0.40	0.00	0.40	0.00	0.40	0.00	0.30	0.00	0.27	0.00	0.33	0.00
	inc	0.08	0.00	1.00	1.00	1.00	1.00	0.19	0.00	0.06	0.00	1.00	1.00
	dec	0.77	0.00	1.00	0.37	1.00	1.00	0.25	0.00	0.48	0.00	1.00	1.00
	sea	0.00	0.11	1.00	1.00	1.00	1.00	1.00	1.00	0.59	0.00	1.00	1.00
μ (5)	equ	0.40	0.00	0.00	0.40	0.00	0.40	0.30	0.00	0.27	0.00	0.33	0.00
	inc	0.00	0.25	1.00	1.00	1.00	1.00	0.19	0.00	1.00	1.00	1.00	1.00
	dec	0.12	0.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.89	1.00	1.00
	sea	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.59	0.00	1.00	1.00

Table B.2. α_i values in Situation3

Production rate	Demand structure	α_i values for each demand level											
		SC1		SC2		SC3		SC4		SC5		SC6	
		α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2
μ (1)	equ	0.20	0.45	0.20	0.45	0.20	0.45	0.56	0.00	0.50	0.00	0.42	0.00
	inc	0.34	0.42	0.34	0.42	0.36	0.13	0.47	0.35	0.48	0.16	0.55	0.10
	dec	0.37	0.23	0.38	0.22	0.36	0.17	0.56	0.00	0.50	0.00	0.43	0.07
	sea	0.48	0.25	0.48	0.25	0.25	0.48	0.59	0.14	0.53	0.18	0.40	0.01
μ (2)	equ	0.38	0.75	0.75	0.38	0.38	0.75	1.00	0.08	0.93	0.00	0.50	0.38
	inc	0.56	0.27	0.56	0.27	0.45	0.49	0.75	0.00	0.68	0.80	0.50	0.40
	dec	0.63	0.41	0.23	0.94	0.47	0.47	0.87	0.28	0.83	0.00	0.94	0.00
	sea	0.69	0.76	0.74	0.70	0.70	0.75	0.76	0.60	0.68	0.80	0.38	0.56
μ (3)	equ	0.38	0.75	0.38	0.75	0.75	0.38	0.88	0.24	1.00	0.00	0.98	0.20
	inc	0.37	0.42	0.42	0.37	0.84	0.87	0.74	0.02	0.78	0.12	0.66	0.56
	dec	0.31	0.63	0.31	0.63	0.52	1.00	0.87	0.28	0.92	0.50	0.89	0.22
	sea	0.29	0.48	0.73	0.17	0.17	0.73	0.55	0.72	0.69	0.44	0.55	0.88
μ (4)	equ	0.40	0.70	0.70	0.40	0.70	0.40	0.90	0.15	0.95	0.00	0.88	0.28
	inc	0.42	0.89	0.89	0.42	0.86	0.75	0.69	0.63	0.83	0.59	0.70	0.82
	dec	0.73	0.75	0.73	0.75	0.99	0.48	0.85	0.50	0.94	0.54	0.94	0.41
	sea	0.40	0.81	0.58	0.67	0.58	0.67	0.75	0.43	0.63	0.41	0.53	0.79
μ (5)	equ	0.06	0.69	0.69	0.06	0.06	0.69	0.89	0.23	0.94	0.00	0.86	0.29
	inc	0.42	0.87	0.73	0.64	0.85	0.74	0.75	0.61	0.83	0.59	0.70	0.81
	dec	0.58	0.80	0.80	0.58	0.83	0.78	0.83	0.50	0.92	0.54	0.54	1.00
	sea	0.93	0.21	0.92	0.20	0.57	0.66	0.73	0.20	0.53	0.80	0.66	0.49

Table B.3. α_i values in Situation4

Production rate	Demand structure	α_i values for each demand level											
		SC1		SC2		SC3		SC4		SC5		SC6	
		α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2
µ (1)	equ	0.31	0.06	0.27	0.10	0.17	0.20	0.04	0.00	0.00	0.00	0.18	0.13
	inc	0.00	0.00	0.06	0.00	0.00	0.00	0.00	0.00	0.12	0.00	0.00	0.08
	dec	0.13	0.00	0.00	0.06	0.05	0.00	0.00	0.00	0.00	0.00	0.04	0.44
	sea	0.00	0.03	0.01	0.05	0.09	0.00	0.00	0.00	0.06	0.00	0.00	0.11
µ (2)	equ	0.21	0.00	0.00	0.00	0.11	0.00	0.00	0.00	0.02	0.00	0.08	0.00
	inc	0.09	0.00	0.04	0.00	0.19	0.00	0.03	0.00	0.01	0.11	0.00	0.18
	dec	0.05	0.00	0.00	0.05	0.00	0.09	0.04	0.00	0.00	0.00	0.00	0.28
	sea	0.00	0.04	0.00	0.09	0.05	0.00	0.07	0.00	0.00	0.00	0.00	0.00
µ (3)	equ	0.18	0.19	0.19	0.18	0.05	0.00	0.04	0.00	0.18	0.10	0.00	0.23
	inc	0.01	0.09	0.21	0.13	0.34	0.25	0.00	0.00	0.12	0.00	0.41	0.03
	dec	0.04	0.00	0.10	0.00	0.00	0.09	0.19	0.00	0.19	0.27	0.00	0.06
	sea	0.00	0.18	0.04	0.37	0.00	0.00	0.29	0.00	0.22	0.21	0.18	0.30
µ (4)	equ	0.17	0.09	0.34	0.06	0.15	0.10	0.20	0.12	0.08	0.00	0.15	0.00
	inc	0.15	0.15	0.15	0.15	0.26	0.09	0.11	0.24	0.26	0.00	0.35	0.00
	dec	0.00	0.00	0.00	0.51	0.05	0.00	0.48	0.00	0.25	0.00	0.19	0.2
	sea	0.07	0.09	0.16	0.14	0.16	0.14	0.15	0.14	0.17	0.00	0.10	0.1
µ (5)	equ	0.00	0.08	0.00	0.69	0.31	0.01	0.24	0.00	0.14	0.00	0.25	0.0
	inc	0.11	0.15	0.04	0.22	0.24	0.08	0.09	0.20	0.23	0.00	0.32	0.00
	dec	0.00	0.00	0.00	0.00	0.18	0.00	0.08	0.00	0.23	0.00	0.21	0.13
	sea	0.15	0.11	0.11	0.15	0.11	0.15	0.19	0.00	0.16	0.00	0.20	0.00

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