OPTIMAL AMOUNT OF RESOURCE EXTRACTION UNDER DUOPOLY

ESRA DERELİ

BOĞAZİÇİ UNIVERSITY

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OPTIMAL AMOUNT OF RESOURCE EXTRACTION UNDER DUOPOLY

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Esra Dereli

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Thesis Abstract

Esra Dereli, “Optimal Amount of Resource Extraction under Duopoly”

The purpose of this thesis is to analyze the optimal amount of depletable natural resource extraction under duopoly through a dynamic model. The starting point comes from the world boron market in which there are two big players (U.S.A. and Turkey). It is assumed that both of these countries extract resources from their own stocks and sell in the same market. The model is solved in two periods. Specifically, it aims to find equilibrium that at least one country does not extract in the first period, but in the second. The conditions for the existence of such an equilibrium are analyzed. Furthermore, the effect of expected increase in demand in the second period is examined. Excel - Solver is used for simulation analysis to see the resource extraction amounts graphically under different conditions. Simulation analysis allows analyzing the effect of stocks, discount factors, marginal costs and expected increasing demand in the next periods and consumer surplus.

Tez Özeti

Esra Dereli, "Düopol Durumunda Optimum Doğal Kaynak Çıkarma Miktarı"

Bu tezin amacı, tükenebilen doğal kaynakların optimum çıkarılma miktarının düopol durumunda dinamik bir model yolu ile analiz edilmesidir. Tezin başlangıç noktası dünya bor pazarında iki büyük oyuncu (Türkiye ve A.B.D) olmasından gelmektedir. Bu iki ülkenin kendi stoklarını çıkarttıkları ve aynı pazarda sattıkları varsayılmaktadır. Model iki period için çözülmüştür. Özellikle, ülkelerde birinin veya her ikisinin ilk period üretmediği ikinci periyod ürettiği dengelerin bulunması amaçlanmıştır. Bu dengelerin varlığı analiz edilmiştir ve ikinci periyodda talebin artmasının beklentisinin etkisi araştırılmıştır. Farklı durumlardaki kaynak çıkarma miktarlarını grafiksel olarak görmek için Excel-Solver'ı kullanarak simulasyon analizleri yapıldı. Simulasyon analizleri stokların, iskonto faktörlerinin, marjinal maliyetlerin, ileriki periyodlarda talep artışı beklentisinin ve tüketici rantının etkisinin analiz edilmesine olanak sağlar.

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CHAPTER 1

INTRODUCTION

We have solved a two period dynamic model for depletable natural resources under the assumption of duopoly. We have searched for the equilibriums that at least one country does not extract in the first period but in the second and we have analyzed the effect of increasing demand.

Since the starting point of the thesis comes from boron market, first of all we search the basic properties of world boron market and then we have made literature review on depletable natural resource economics.

Overview on Boron Market

Boron is the non metal element number five on the periodic chart with the symbol B. It does not occur alone in nature. Borates are minerals which contain boric oxide or boron-oxygen molecules. Most products and government statistical data are shown in terms of B_2O_3, rather than B.

Glass and ceramics, soaps and detergents, agriculture, timber preservation, fire retardants, cosmetics and medicine are the main usage areas of today’s market, although more than 25% of the world’s borates are used in numerous other consumer products.

According to the official web site of Eti Maden, Turkey has the largest boron reserves that 72,20% of the world reserves. The U.S.A and Russia are the other countries that have a significant amount of boron reserves. The table below shows the amount of boron reserves of all countries.

Table 1. World Boron Reserves (Thousand Tons – $B_2 O_3$)

Country	Proven Reserve	Probable Possible Reserve	Total Reserve	Percent in Total (%)
Turkey	227.000	624.000	851.000	72.20
U.S.A.	40.000	40.000	80.000	6.80
Russia	40.000	60.000	100.000	8.50
China	27.000	9.000	36.000	3.10
Chile	8.000	33.000	41.000	3.50
Bolivia	4.000	15.000	19.000	1.60
Peru	4.000	18.000	22.000	1.90
Argentina	2.000	7.000	9.000	0.80
Kazakhstan	14.000	1.000	15.000	1.30
Serbia	3	0	3	0.30
Total	369.000	807.000	1,176.000	100

Source: http://www.etimaden.gov.tr

Borate production is centered in the U.S.A, Turkey, Argentina, Chile, Bolivia, Peru, Russia, and China. The United States is the leader in the production of refined borates and boric acid whereas Turkey is the world's major supplier of mineral concentrates and also produces large amount of boric acid and refined borates. Almost 90% of Turkey's production is sold on the export market. Fig. 1 shows the production levels of each country.

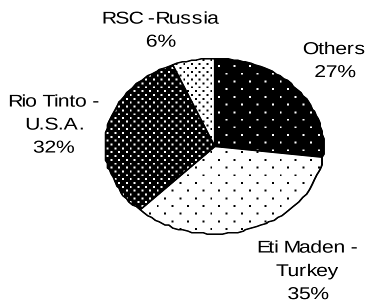

Fig. 1 Production of Boron by Major Countries, 2005 (%)

Source: http://www.etimaden.gov.tr

The table below gives the consumption levels of boron by regions for the years 2006 and 2007. Demand for boron products is growing over time because of the economic improvement in the world.

Table 2. World's Boron Consumption

	2006		**2007**		
Region	**Ton**	**Ratio**	**Ton**	**Ratio**	**Variation**
Western and Southern Europe	675.645	18,70%	632.085	16,8%	-6,4%
North Europe	40.820	1,10%	43.920	1,2%	7,6%
Eastern Europe	141.040	3,9%	180.050	4,8%	27,7%
North America	1.013.250	28,0%	899.250	23,9%	-11,3%
Central and South America	393.886	10,9%	416.200	11,1%	5,7%
Middle East	30.412	0,8%	50.278	1,3%	65,3%
Asia	1.286.339	35,6%	1.498.836	39,8%	16,5%
Ocenia	17.132	0,5%	18.700	0,5%	9,2%
Africa	14.450	0,4%	23.150	0,6%	60,2%
Total	**3.612.974**		**3.712.469**		**4,0%**

Source: http://www.etimaden.gov.tr

Literature Review

The natural resource literature goes back to Harold Hotelling (1931). He is accepted as the origin of resource economics. He had two purposes in writing the 1931 paper: (1) to assess the policy debates arising out of the conservation movement and (2) to develop a theory of natural resources. His main questions are, what the optimal production rate is, what the effect of uncertainty and estimate is, whether it is more profitable to complete the extraction within a finite time, what the optimal exploitation rate is if the mine is publicly owned, what the effect of regulation or taxation is on private owners, what the difference is under monopoly, duopoly and free competition, what the effect of extraction cost is. He deals with nonrenewable natural resources. He has created the Hotelling Rule which states that for efficient extraction path and competitive resource industry equilibrium, the price of an exhaustible resource must grow at a rate equal to the rate of interest. Hence, he has shown that competitive equilibrium equals the socially optimum level. Hotelling stated that the present value of a unit extracted must be the same for all periods. Furthermore, he showed that under the condition that demand is stable and the demand curve does not intersect the price axis at a finite price output declines monotonically and reaches zero. If demand intersects the price axis at a finite time, then the time of exhaustion will be finite. When he examined the rate of extraction of the monopolist he found that marginal revenue will grow at the rate of interest, not price as in the case of free competition and he reached the conclusion that the monopolist will deplete more slowly. He argued that extraction costs increases as the mine goes deeper. Hence, the one who has more resources has cost advantage according to Hotelling's view. All the questions raised in his paper are not answered in his paper.

Devarajan and Fischer (1981) briefly summarize Hotelling's paper, pointing out its importance. But nothing has been done for forty years in natural economics after Hotelling's paper. According to them, this is because of the difficulty of mathematics in resource economics and more serious problems of that time. With the oil crisis, natural resource economics has become a more attractive issue in their view.

The aim of Stiglitz' paper (1976) is to compare the rate of extraction of an exhaustible natural resource in competitive markets with that of a monopolist. He does his analysis in a two period model and infinite time horizon. In the infinite time model, he solves continuous time dynamic equilibrium. He assumes constant elasticity of demand in all models and he solves the problem starting from zero extraction costs, then adds extraction costs and then solves the problem with increased elasticity of demand. The basic results of Stiglitz are; 1. Monopoly prices and competitive equilibrium prices will be the same under constant elasticity of demand, and zero extraction costs are assumed. 2. In the case of existence of extraction costs and increased elasticity of demand monopolists are more "conservation minded" than competitive markets (Stiglitz, 1976). Stiglitz also examines the existence of speculators. He finds that if the elasticity of demand is decreasing and the price of the natural resource increasing more than the rate of interest and moreover if zero storage costs are assumed, these would create an incentive for speculators to buy the natural resource and store it and he finds out that the competitive and monopolist equilibrium will be the same under those assumptions.

Fischer and Laxminarayan (2004) have written about extraction of non-renewable natural resources under monopoly. However, their contribution is that they analyzed the effect of monopoly if the monopolist is able to sell to two markets with different prices. They have solved the problem by using continuous dynamic equilibrium tools. They assumed no extraction costs, constant elasticity of demand in which elasticity is greater than 1 and no arbitrage opportunities. Contradicting Hotelling's and Stiglitz view that the monopolist is conservationist, they found that a monopolist selling to two markets with different demand curves depletes the resource more rapidly than the social planner. This result suggests that when a monopolist is able to price discriminate between two sets of customers, then the group with relatively elastic demand is better off, and the group with relatively inelastic demand is worse off with monopoly provision than under the social planner. (Fischer and Laxminarayan, 2004). Moreover, there may be welfare losses because of unregulated monopoly.

Koulovatianos and Mirman (2004) examine the relationship between dynamic duopoly and the relative size of firms. They assume duopoly with different sized firms and they solve the optimal intertemporal amount of the capital use by using Bellman equations in the infinite time horizon. They find out that if larger firms have cost advantages, then the supply of all firms declines leading to higher industry growth. Moreover, they find that if cost advantage does not exist then this symmetry results in aggressive competition.

Koulovatianos and Mirman (2007) analyze the effect of market structure on industry growth. In their analysis, market structure depends on dynamic externality which arises due to the non-excludability of the capital stock and market externality which leads to Cournot type supply competition. They examine four market structures which are dynamic monopoly; two monopolists selling the product in their own separate markets but utilizing capital from the same provider, a duopoly with firms selling in the same markets and utilizing capital from the same provider, and a duopoly with firms utilizing capital from separate providers. They use the Bellman equation for solving those problems. They find out that more firms result in higher aggregate supply and lower growth. Comparing dynamic monopoly with two monopolists and dynamic monopoly with duopoly (which utilizes capital from the same provider) enables them to analyze the impact of market externality in addition to dynamic externality. They find that depending on the demand elasticity, the aggregate supply of two monopolists can be higher than the aggregate supply of duopolists.

Petra Huck (2005) tries to solve the dynamic equilibrium of a three player non-renewable natural resource model. However, because of the difficulty in mathematics she prefers to run a simulation and she examines the effect of parameters by running simulations.

CHAPTER 2

THE MODEL

Two Periods Model

Assumptions

We assume that there are two countries in the market denoted by *A* and *T* selling the same natural resource in the same market but exploiting the resource from their own stocks. Two periods are assumed in the model.

The stock of country *i* at time *t* is denoted by $s_{i.t}$ and the amount of natural resource extracted at time *t* by country *i* is denoted by $k_{i,t}$, , i=*A,T*. s_T and s_A are initial stocks of Country A and Country T, respectively. And it is assumed that s_T is greater than s_A i.e. $s_T = ms_A, m > 1$.

It is assumed that the U.S.A and Turkey have different discount factors denoted by β_T and β_A. Furthermore, β_T is assumed to be smaller than β_A . The logic behind this is the fact that Turkey is a developing country whereas the U.S.A is a developed country. Therefore, the U.S.A gives more importance to the future than Turkey.

Demand is assumed to be linear and is of the form, $P_t(k_t) = a - b(k_{T,t} + k_{A,t})$, where a and b are positive constants.

Cost is proportional to the amount of extraction at time *t* and a constant i.e. $C_{i,t} = c_i k_{i,t}$. Since Hotelling (1931) states that the exploitation cost of a country with less resources is more than the country with greater resource and this is the case in the real world, it is assumed that $c_T < c_A$.

The problem faced by each country

The maximization problem of Turkey is;

$$\max(a - bk_{T,1} - bk_{A,1})k_{T,1} - c_T k_{T,1} + \beta_T \left[(a - bk_{T,2} - bk_{A,2})k_{T,2} - c_T k_{T,2}\right]$$

subject to

$k_{T,1} + k_{T,2} \leq s_T$ and $k_{T,t} \geq 0, t = 1,2.$

Then the Lagrange Multiplier of Turkey is,

$$L_T = (a - bk_{T,1} - bk_{A,1} - c_T)k_{T,1} + \beta_T \left[(a - bk_{T,2} - bk_{A,2} - c_T)k_{T,2}\right] - \lambda_{T,0}(k_{T,1} + k_{T,2} - s_T)$$
$$+ \lambda_{T,1}k_{T,1} + \lambda_{T,2}k_{T,2}$$

The maximization problem of the U.S.A is;

$$\max(a - bk_{T,1} - bk_{A,1})k_{A,1} - c_A k_{A,1} + \beta_A \left[(a - bk_{T,2} - bk_{A,2})k_{A,2} - c_A k_{A,2}\right]$$

subject to

$k_{A,1} + k_{A,2} \leq s_A$ and $k_{A,t} \geq 0, t = 1,2.$

Then the Lagrange Multiplier of the U.S.A is

$$L_A = (a - bk_{T,1} - bk_{A,1} - c_A)k_{A,1} + \beta_A \left[(a - bk_{T,2} - bk_{A,2} - c_A)k_{A,2}\right] - \lambda_{A,0}(k_{A,1} + k_{A,2} - s_A)$$
$$+ \lambda_{A,1}k_{A,1} + \lambda_{A,2}k_{A,2}$$

The necessary Kuhn-Tucker conditions for country *i* are;

(1) $\dfrac{\partial L_i}{\partial k_{i,1}} = 0$

(2) $\dfrac{\partial L_i}{\partial k_{i,2}} = 0$

(3) $k_{i,1} + k_{i,2} \leq s_i$

(4) $k_{i,t} \geq 0, t = 1, 2$

(5) $\lambda_{i,o}(k_{i,1} + k_{i,2} - s_i) = 0$

(6) $\lambda_{i,j}k_{i,1} = 0, \text{ j} = 1,2$

(7) $\lambda_j \geq 0, \text{ j} = 0, 1, 2$

Then, (1) and (2) and the maximization problem of each country give us the following equations,

(8) $a - 2bk_{T,1} - bk_{A,1} - c_T = \lambda_{T,0} - \lambda_{T,1}$

(9) $\beta_T (a - 2bk_{T,2} - bk_{A,2} - c_T) = \lambda_{T,0} - \lambda_{T,2}$

(10) $a - 2bk_{A,1} - bk_{T,1} - c_A = \lambda_{A,0} - \lambda_{A,1}$

(11) $\beta_A (a - 2bk_{A,2} - bk_{T,2} - c_A) = \lambda_{A,0} - \lambda_{A,2}$

Since there are 6 parameters in the problem which are $\lambda_{T,0}, \lambda_{T,1}, \lambda_{T,2}$, $\lambda_{A,0}, \lambda_{A,1}, \lambda_{A,2}$ and these parameters can take positive values or they can be 0, there are 2^6 cases in the problem.

Since it is not meaningful to analyze all the cases, in the next pages we examine the interesting cases where at least one country does not extract any resources in the first period but extracts them in the second period.

The following four cases search the equilibrium for the first period in which none of the countries extract resources and for the second period when both of the countries extract a positive amount of resource. We show that there is no such equilibrium. These four cases are;

1. Both countries do not extract in the first period and finish their resources in the second,

2. Both countries do not extract in the first period, extract resources in the second period but they do not finish their resources,

3. Both countries do not extract in the first period, extract in the second but Turkey finishes its resources whereas the U.S.A does not,

4. Both countries do not extract in the first period, extract in the second but the U.S.A finishes its resources whereas Turkey does not.

CASE 1.

Both countries do not extract in the first period and finish their resources in the second i.e.

$\lambda_{T,0} > 0, \lambda_{T,1} > 0, \lambda_{T,2} = 0, \lambda_{A,0} > 0, \lambda_{A,1} > 0, \lambda_{A,2} = 0.$

Since $k_{T,1} = 0$ & $k_{A,1} = 0$,

By (8) $a - c_T = \lambda_{T,0} - \lambda_{T,1}$

By (9) $\beta_T(a - 2bk_{T,2} - bk_{A,2} - c_T) = \lambda_{T,0}$

Then $\lambda_{T,1}$ is found to be,

$\lambda_{T,1} = (\beta_T - 1)(a - c_T) - \beta_T(2bk_{T,2} + bk_{A,2})$

Since $(a - c_T)$>0 (Otherwise, $\lambda_{T,0} < 0$), $0 < \beta_T < 1$ and $k_{T,2}$ and $k_{A,2}$ are positive numbers, then the above equation is negative which contradicts with the necessary Kuhn-Tucker conditions. Therefore, there is no such equilibrium.

CASE 2.

Both countries do not extract in the first period and extract resources in the second period but they do not finish their resources i.e.

$\lambda_{T,0} = 0, \lambda_{T,1} > 0, \lambda_{T,2} = 0, \lambda_{A,0} = 0, \lambda_{A,1} > 0, \lambda_{A,2} = 0.$

Since $k_{T,1} = 0$ & $k_{A,1} = 0$,

By (8) $a - c_T = -\lambda_{T,1}$

By (9) $\beta_T(a - 2bk_{T,2} - bk_{A,2} - c_T) = 0$

Since $\lambda_{T,1}$ should be a positive number, then $(a - c_T)$ should be negative. And from the above equation we get $(a - c_T) = 2bk_{T,2} + bk_{A,2}$. But since $(a-c_T)$ is negative then at least one of $k_{T,2}$ and $k_{A,2}$ should be negative which is a contradiction to the necessary Kuhn-Tucker conditions. Hence, we conclude that there is no equilibrium as stated in the Case 2.

CASE 3.

Both countries do not extract in the first period and extract in the second but Turkey finishes its resources whereas the U.S.A does not i.e.

$\lambda_{T,0} > 0, \lambda_{T,1} > 0, \lambda_{T,2} = 0, \lambda_{A,0} = 0, \lambda_{A,1} > 0, \lambda_{A,2} = 0.$

Since $k_{T,1} = 0$ & $k_{A,1} = 0$,

By (10) $a - c_A = -\lambda_{A,1}$

By (11) $\beta_A(a - 2bk_{A,2} - bk_{T,2} - c_A) = 0$

Since $\lambda_{A,1}$ should be a positive number, then $(a-c_A)$ should be negative. And from the above equation we get $(a-c_A) = 2bk_{A,2} + bk_{T,2}$. But since $(a-c_A)$ is negative then at least one of $k_{T,2}$ and $k_{A,2}$ should be negative which is a contradiction to the necessary Kuhn-Tucker conditions. Hence, we conclude that there is no equilibrium as stated in the Case 3.

CASE 4.

Both countries do not extract in the first period, extract in the second but the U.S.A finishes its resources whereas Turkey does not i.e.

$\lambda_{T,0} = 0, \lambda_{T,1} > 0, \lambda_{T,2} = 0, \lambda_{A,0} > 0, \lambda_{A,1} > 0, \lambda_{A,2} = 0.$

This is the symmetric case of Case 3. Therefore, the solution is similar;

Since $k_{T,1} = 0$ & $k_{A,1} = 0$,

By (8) $a - c_T = -\lambda_{T,1}$

By (9) $\beta_T (a - 2bk_{T,2} - bk_{A,2} - c_T) = 0$

Since $\lambda_{T,1}$ should be a positive number, then $(a - c_T)$ should be negative. And from the above equation we get $(a - c_T) = 2bk_{T,2} + bk_{A,2}$. But since $(a - c_T)$ is negative then at least one of $k_{T,2}$ and $k_{A,2}$ should be negative. Hence, this is a contradiction to the necessary Kuhn-Tucker conditions. Then, we conclude that there is no equilibrium as stated in the Case 4.

The above four cases show that there is no equilibrium that neither of the countries extract any resource in the first period but both extract in the second. The logic behind this is the fact that the case that both countries do not extract in the first period but extract in the second period is less profitable than they extract that amount of resource in the first period. Because, future is less valuable than today, they discount it. The only reason not to extract today but to extract tomorrow is to have an advantage tomorrow, such as prices should be higher, demand is expected to increase. However, there is not such opportunity in the second period. Therefore, we did not find such equilibrium.

Here we have analyzed other interesting cases that one of the countries does not extract in the first period but the other does and both countries extract in the second period.

CASE 5.

The U.S.A does not extract any resources in the first period but extracts in the second whereas Turkey extracts in both periods and both of the countries finish their resources i.e.

$\lambda_{T,0} > 0, \lambda_{T,1} = 0, \lambda_{T,2} = 0, \lambda_{A,0} > 0, \lambda_{A,1} > 0, \lambda_{A,2} = 0.$

By the above parameters we get

$k_{A,1} = 0, k_{A,2} = s_A, k_{T,2} = s_T - k_{T,1}$.

By (8) and (9), $a - 2bk_{T,1} - c_T = \beta_T(a - 2bk_{T,2} - bs_A - c_T) = \lambda_{T,0}$

Letting $s_A = s$ and $s_T = ms$, we find $k_{T,1}$ and $k_{T,2}$ as;

$$k_{T,1} = \frac{b\beta_T s(2m+1) + (1-\beta_T)(a-c_T)}{2b(1+\beta_T)}$$

$$k_{T,2} = \frac{(\beta_T - 1)(a-c_T) + bs(2m-\beta_T)}{2b(1+\beta_T)}$$

The necessary Kuhn-Tucker conditions to have such equilibrium are:

1. $\lambda_{T,0} > 0$ if $2(a-c_T) > bs(2m+1)$ and $(a-c_T) > 0$.

2. $k_{T,1} > 0$ if $\frac{b\beta_T s(2m+1) + (1-\beta_T)(a-c_T)}{2b(1+\beta_T)} > 0$

This condition satisfies since $0 < \beta_T < 1$ and $(a-c_T) > 0$.

3. $k_{T,2} > 0$ if $bs(2m-\beta_T) > (1-\beta_T)(a-c_T)$

Since the right hand side is positive, $2m$ should be greater than β_T.

4. By (11) we find $\lambda_{A,0}$ as;

$\lambda_{A,0} = \beta_A(a - 2bs_A - bk_{T,2} - c_A)$. When we insert $k_{T,2}$ into this equation we find that $\lambda_{A,0} > 0$ if $(a-c_A) > 2bs + \frac{(\beta_T - 1)(a-c_T) + bs(2m-\beta_T)}{2(1+\beta_T)}$

When we arrange this we find that;

$\lambda_{A,0} > 0$ if $2(1+\beta_T)(a-c_A)+(1-\beta_T)(a-c_T) > bs(4+2m+3\beta_T)$

5. By (10) and (11) we find $\lambda_{A,1}$ as;

$$\lambda_{A,1} = (\beta_A - 1)(a-c_A) - b\beta_A s(2+m) + \frac{b\beta_T s(2m+1)+(1-\beta_T)(a-c_T)}{2(1+\beta_T)}(1+\beta_A)$$

(12) $\lambda_{A,1} > 0$ if $\dfrac{b\beta_T s(2m+1)+(1-\beta_T)(a-c_T)}{2(1+\beta_T)}(1+\beta_A) > (1-\beta_A)(a-c_A)+b\beta_A s(2+m)$

In order to simplify the above equation (12) let $\beta_A = \beta_T = \beta$. Then,

$$\lambda_{A,1} = -3b\beta s(1+\beta)+(1-\beta^2)(2c_A - c_T - a)$$

If $c_A \leq c_T$ then $\lambda_{A,1} < 0$ and then we conclude that if the countries have identical discount factors and identical cost functions (or the one that does not extract in the first period has lower cost), then there is no equilibrium.

If $c_A > c_T$, then $\lambda_{A,1} > 0$ if $(1-\beta)(2c_A - c_T - a) > 3b\beta s$

In the light of the above discussions when we examine the necessary condition for $\lambda_{A,1} > 0$, we see that this condition holds depending on the discount factors between countries and the cost functions i.e. β_T should be smaller than β_A and c_T should be smaller than c_A and the difference between discount factors should be big enough for the equation (12) to hold.

6. The other necessary conditions (i.e. $\lambda_{T,1} = 0, \lambda_{T,2} = 0, \lambda_{A,2} = 0$) are satisfied during the calculations.

Hence, we can conclude that there is equilibrium that U.S.A does not extract any resources in the first period but extracts in the second whereas

Turkey extracts in both periods and both of the countries finish their resources if necessary Kuhn-Tucker conditions that are stated above are satisfied. In order for these conditions to be satisfied, U.S.A should give much more importance to the future than Turkey and Turkey's cost extraction should be smaller than U.S.A.

CASE 6.

Turkey does not extract any resources in the first period but extracts in the second whereas the U.S.A extracts in both periods and both of the countries finish their resources i.e.

$\lambda_{T,0} > 0, \lambda_{T,1} > 0, \lambda_{T,2} = 0, \lambda_{A,0} > 0, \lambda_{A,1} = 0, \lambda_{A,2} = 0.$

By the above parameters we get $k_{T,1} = 0, k_{T,2} = s_T, k_{A,2} = s_A - k_{A,1}$.

By (10) and (11) $a - 2bk_{A,1} - c_A = \beta_A(a - 2bk_{A,2} - bs_T - c_A)$

Then we find $k_{A,2}$ and $k_{A,1}$ as;

$$k_{A,1} = \frac{b\beta_A s(2+m) + (1-\beta_A)(a-c_A)}{2b(1+\beta_A)}$$

$$k_{A,2} = \frac{bs(2-\beta_A m) - (1-\beta_A)(a-c_A)}{2b(1+\beta_A)}$$

Necessary Kuhn-Tucker conditions are;

1. $\lambda_{A,0} > 0$ if $2(a-c_A) > bs(2+m)$ and $(a-c_A) > 0$.

2. $k_{A,1} > 0$ if $\dfrac{b\beta_A s(2+m) + (1-\beta_A)(a-c_A)}{2b(1+\beta_A)} > 0$ and this condition satisfies since $0 < \beta_A < 1$ and $(a-c_A) > 0$

$\lambda_{T,0} > 0$ if $2(a-c_T) > bs(2m+1)$ and $(a-c_T) > 0$.

3. $k_{A,2} > 0$ if $bs(2-\beta_A m) > (1-\beta_A)(a-c_A)$

In order for this equation to be satisfied, $\beta_A m$ should be smaller than 2. We can emphasize that if the difference between the stocks of the countries is too big (i.e. bigger than $\beta_A m$) then there is no equilibrium where the bigger country does not extract any resources but the smaller country extracts in the first period and both countries extracts in the second period and they finish their resources.

4. By (9) $\lambda_{T,0}$ is found to be, $\lambda_{T,0} = \beta_T(a - 2bms - bk_{A,2} - c_T)$. Inserting $k_{A,2}$ into this equation we get,

$\lambda_{T,0} > 0$ if $2(1+\beta_A)(a-c_T) + (1-\beta_A)(a-c_A) > bs(2+4m+3m\beta_A)$

5. By (8), (9) and $k_{A,1}$ we find $\lambda_{T,1}$ as,

$$\lambda_{T,1} = (\beta_T - 1)(a-c_T) - b\beta_T s(2m+1) + \frac{b\beta_A s(2+m) + (1-\beta_A)(a-c_A)}{2(1+\beta_A)}(1+\beta_T)$$

$$\lambda_{T,1} > 0 \text{ if } \frac{b\beta_A s(2+m) + (1-\beta_A)(a-c_A)}{2(1+\beta_A)}(1+\beta_T) > b\beta_T s(2m+1) + (1-\beta_T)(a-c_T)$$

If $\beta_A = \beta_T = \beta$, then the above equation becomes;

$$(1-\beta)(a-c_A) > \frac{3m}{2}b\beta s + (1-\beta)(a-c_T)$$

Since $c_A > c_T$, then $\lambda_{T,1} < 0$ which is a contradiction to the necessary Kuhn Tucker conditions. Hence, we can conclude that if the discount factors of the countries are the same then there is no equilibrium that Turkey does not extract any resources in the first period but extracts in the second whereas the U.S.A extracts in both periods and both of the countries finish their resources. Otherwise, if discount factors are not the same then the above condition which is very tight condition should hold.

CASE 7.

Turkey extracts in both periods and finishes its resources whereas the U.S.A does not extract in the first period, extracts in the second and does not finish its resources i.e.

$\lambda_{T,0} > 0, \lambda_{T,1} = 0, \lambda_{T,2} = 0, \lambda_{A,0} = 0, \lambda_{A,1} > 0, \lambda_{A,2} = 0.$

The above parameters imply $k_{T,1} > 0, k_{T,2} > 0, k_{A,1} = 0, k_{A,2} < s_A$.

By (10) $-\lambda_{A,1} = a - c_A - bk_{T,1}$

Since $\lambda_{A,1} > 0$, then $bk_{T,1} > a - c_A$

By (8) $\lambda_{T,0} = a - 2bk_{T,1} - c_T$

Since $\lambda_{T,0} > 0$, then $a - c_T > 2bk_{T,1}$

Then, when we combine the above equations we get;

$$a - c_T > 2(a - c_A)$$

Although c_T is smaller than c_A, the difference among them should be too much in order for this equation to hold. In the real world the difference between marginal costs are not so much. Therefore, in the real world, there is no such equilibrium.

CASE 8.

U.S.A extracts in both periods and finishes its resources whereas Turkey does not extract in the first period, extracts in the second and does not finish its resources i.e.

$\lambda_{T,0} = 0, \lambda_{T,1} > 0, \lambda_{T,2} = 0, \lambda_{A,0} > 0, \lambda_{A,1} = 0, \lambda_{A,2} = 0.$

The above parameters imply $k_{T,1} = 0, k_{T,2} < s_T, k_{A,1} > 0, k_{A,2} > 0$.

By (8) $-\lambda_{T,1} = a - c_T - bk_{A,1}$

Since $\lambda_{T,1} > 0$, then $bk_{A,1} > a - c_T$

By (10) $\lambda_{A,0} = a - 2bk_{A,1} - c_A$

Since $\lambda_{A,0} > 0$, then $a - c_A > 2bk_{A,1}$

Then, when we combine the above equations we get;

$$a - c_A > 2(a - c_T)$$

But since $c_A > c_T$, then the above equation does not hold and we can conclude that there is not an equilibrium that U.S.A extracts in both periods and finishes its resources whereas Turkey does not extract in the first period and extracts in the second and does not finish its resources

Expected Increase in Demand

Assumptions

All of the assumptions that we make in the standard two period model hold, except demand. Now, demand is assumed to increase in the second period i.e. demand in the first period is the same as before;

$P_1(k_1) = a - b(k_{T,1} + k_{A,1})$

And the demand in the second period is assumed to be

$P_2(k_2) = A - b(k_{T,2} + k_{A,2})$ in which $A > a$.

The problem faced by each country

The maximization problem of Turkey is;

$\max(a - bk_{T,1} - bk_{A,1})k_{T,1} - c_T k_{T,1} + \beta_T \lfloor (A - bk_{T,2} - bk_{A,2})k_{T,2} - c_T k_{T,2} \rfloor$

subject to

$k_{T,1} + k_{T,2} \leq s_T$ and $k_{T,t} \geq 0, t = 1,2.$

Then the Lagrange Multiplier of Turkey is,

$L_T = (a - bk_{T,1} - bk_{A,1} - c_T)k_{T,1} + \beta_T \lfloor (A - bk_{T,2} - bk_{A,2} - c_T)k_{T,2} \rfloor - \lambda_{T,0}(k_{T,1} + k_{T,2} - s_T)$
$+ \lambda_{T,1}k_{T,1} + \lambda_{T,2}k_{T,2}$

The maximization problem of the U.S.A is;

$\max(a - bk_{T,1} - bk_{A,1})k_{A,1} - c_A k_{A,1} + \beta_A \lfloor (A - bk_{T,2} - bk_{A,2})k_{A,2} - c_A k_{A,2} \rfloor$

subject to

$k_{A,1} + k_{A,2} \leq s_A$ and $k_{A,t} \geq 0, t = 1,2.$

Then the Lagrange Multiplier of the U.S.A is

$$L_A = (a - bk_{T,1} - bk_{A,1} - c_A)k_{A,1} + \beta_A \lfloor (A - bk_{T,2} - bk_{A,2} - c_A)k_{A,2} \rfloor - \lambda_{A,0}(k_{A,1} + k_{A,2} - s_A)$$
$$+ \lambda_{A,1}k_{A,1} + \lambda_{A,2}k_{A,2}$$

The necessary Kuhn-Tucker conditions for country i are the same as shown in the second period model.

Then, first and second necessary Kuhn-Tucker conditions give the following equations;

(13) $a - 2bk_{T,1} - bk_{A,1} - c_T = \lambda_{T,0} - \lambda_{T,1}$

(14) $\beta_T(A - 2bk_{T,2} - bk_{A,2} - c_T) = \lambda_{T,0} - \lambda_{T,2}$

(15) $a - 2bk_{A,1} - bk_{T,1} - c_A = \lambda_{A,0} - \lambda_{A,1}$

(16) $\beta_A(A - 2bk_{A,2} - bk_{T,2} - c_A) = \lambda_{A,0} - \lambda_{A,2}$

In the following cases, we have shown that if demand is assumed to increase in the second period, it is possible to have equilibrium that none of the countries extracts in the first period but both of the countries extract in the second.

CASE 9.

Both countries do not extract in the first period but finishes all the resources in the second period i.e.

$\lambda_{T,0} > 0, \lambda_{T,1} > 0, \lambda_{T,2} = 0, \lambda_{A,0} > 0, \lambda_{A,1} > 0, \lambda_{A,2} = 0.$

Then, $k_{T,1} = 0, k_{T,2} = s_T, k_{A,1} = 0, k_{A,2} = s_A$

The necessary Kuhn-Tucker conditions in order to have such equilibrium are;

1. $\lambda_{T,0}$ is found to be; by (14) $\beta_T(A-2bk_{T,2}-bk_{A,2}-c_T)=\lambda_{T,0}$

$\lambda_{T,0}>0$ if $A-c_T>2bms+bs$

2. By (13) and (14) $\lambda_{T,1}$ is found to be,

$\lambda_{T,1}=\beta_T(A-c_T)-(a-c_T)-\beta_T(2bms+bs)$

$\lambda_{T,1}>0$ if $\beta_T(A-c_T)-(a-c_T)>\beta_T(2bms+bs)$

3. By (16) $\lambda_{A,0}=\beta_A(A-2bk_{A,2}-bk_{T,2}-c_A)$

Then, $\lambda_{A,0}>0$ if $A-c_A>2bs+bms$

4. By (15) and (16) $\lambda_{A,1}=\beta_A(A-c_A)-(a-c_A)-\beta_A(2bs+bms)$

$\lambda_{A,1}>0$ if $\beta_A(A-c_A)-(a-c_A)>\beta_A(2bs+bms)$

In CASE 1, we have shown that there is no equilibrium, that "both of the countries do not extract in the first period but finish all the resources in the second". However, when we add the assumption that demand increases in the second period we can find such an equilibrium. Extraction of both countries only in the second period can become more profitable under the above conditions.

CASE 10.

Both of the countries do not extract in the first period but extract in the second and they do not finish their resources i.e.

$\lambda_{T,0}=0,\lambda_{T,1}>0,\lambda_{T,2}=0,\lambda_{A,0}=0,\lambda_{A,1}>0,\lambda_{A,2}=0.$

Then, $k_{T,1} = 0, k_{T,2} < s_T, k_{A,1} = 0, k_{A,2} < s_A$

By (14) $A - c_T = 2bk_{T,2} + bk_{A,2}$

By (16) $A - c_A = 2bk_{A,2} + bk_{T,2}$

Then from the above equations we find $k_{T,2}$ and $k_{A,2}$ to be;

$$k_{T,2} = \frac{A - 2c_T + c_A}{3b}$$

$$k_{A,2} = \frac{A - 2c_A + c_T}{3b}$$

Since $\frac{\partial k_{T,2}}{\partial A} > 0$ and $\frac{\partial k_{A,2}}{\partial A} > 0$, as A increases the extraction amount of both countries increases until the amount of their stocks.

The necessary Kuhn-Tucker conditions in order to have such equilibrium are;

1. By (13) $a - c_T = -\lambda_{T,1}$

$\lambda_{T,1} > 0$ if $(a - c_T) < 0$

2. By (15) $a - c_A = -\lambda_{A,1}$

$\lambda_{A,1} > 0$ if $(a - c_A) < 0$

3. $\lambda_{T,0} = 0$ (i.e. $k_{T,2} < \mathrm{s_T}$) if $\frac{A - 2c_T + c_A}{3b} < ms$

4. $\lambda_{A,0} = 0$ (i.e. $k_{A,2} < \mathrm{s_A}$) if $\frac{A - 2c_A + c_T}{3b} < s$

5. $\lambda_{T,2} = 0$ (i.e. $k_{T,2} > 0$) if $\frac{A - 2c_T + c_A}{3b} > 0$

6. $\lambda_{A,2} = 0$ (i.e. $k_{A,2} > 0$) if $\frac{A - 2c_A + c_T}{3b} > 0$

CASE 11.

Both countries do not extract in the first period but extracts in the second period and Turkey finishes its resources but the U.S.A does not i.e.

$\lambda_{T,0} > 0, \lambda_{T,1} > 0, \lambda_{T,2} = 0, \lambda_{A,0} = 0, \lambda_{A,1} > 0, \lambda_{A,2} = 0.$

Then $k_{A,1} = 0, k_{T,1} = 0, k_{A,2} < s_A, k_{T,2} = s_T$.

By (16) $\beta_A (A - 2bk_{A,2} - bk_{T,2} - c_A) = 0$

Then $k_{A,2}$ is found to be,

$$k_{A,2} = \frac{A - bs_T - c_A}{2b}$$

Here, the amount of $k_{A,2}$ is positively related to *A*. Therefore, we can say that the extraction amount of the U.S.A increases if *A* increases.

The necessary Kuhn-Tucker conditions are;

1. By (15) $a - c_A = -\lambda_{A,1}$

Then $\lambda_{A,1} > 0$ if $a - c_A < 0$

2. $\lambda_{A,2} = 0$ if $\frac{A - bs_T - c_A}{2b} > 0$

3. $\lambda_{A,0} = 0$ if $\dfrac{A - bs_T - c_A}{2b} < s$

4. By (14), $\lambda_{T,0} = \beta_T (A - 2bk_{T,2} - bk_{A,2} - c_T)$

Then $\lambda_{T,0} > 0$ if $\lambda_{T,0} = \beta_T (A - 2bs_T - bk_{A,2} - c_T)$

5. By (13) and (14) $\lambda_{T,1}$ is found as

$\lambda_{T,1} = \beta_T (A - c_T) - (a - c_T) - \beta_T (2bs_T + bk_{A,2})$. Then $\lambda_{T,1} > 0$ if

$\beta_T (A - c_T) - (a - c_T) - \beta_T (2bs_T + bk_{A,2}) > 0$

CASE 12.

Both countries do not extract in the first period but extract in the second period and Turkey does not finish its resources whereas the U.S.A does i.e. $\lambda_{T,0} = 0, \lambda_{T,1} > 0, \lambda_{T,2} = 0, \lambda_{A,0} > 0, \lambda_{A,1} > 0, \lambda_{A,2} = 0.$

Above parameters give; $k_{A,1} = 0, k_{T,1} = 0, k_{A,2} = s_A, k_{T,2} < s_T$.

By (14) $k_{T,2}$ is found as; $k_{T,2} = \dfrac{A - bs_A - c_T}{2b}$

The necessary Kuhn-Tucker conditions in order to have such equilibrium are;

1. By (13) $a - c_T = -\lambda_{T,1}$. Then $\lambda_{T,1} > 0$ if $a - c_T < 0$

2. $\lambda_{T,2} = 0$ if $\dfrac{A - bs_A - c_T}{2b} > 0$

3. $\lambda_{T,0} = 0$ if $\frac{A - bs_A - c_T}{2b} < s_T$

4. By (16) $\lambda_{A,0} = \beta_A(A - 2bk_{A,2} - bk_{T,2} - c_A)$. Then $\lambda_{A,0} > 0$ if $(A - 2bs_A - bk_{T,2} - c_A) > 0$

5. By (15) and (16) $\lambda_{A,1} = \beta_A(A - c_A) - (a - c_A) - \beta_A(2bs_A + bk_{T,2})$

Then $\lambda_{A,1} > 0$ if $\beta_A(A - c_A) - (a - c_A) - \beta_A(2bs_A + bk_{T,2}) > 0$

In the first part of the model, we showed that if demand is stable in both periods there is no case where none of the countries extract in the first period but both of the countries extracts in the first period. However, with the assumption that demand increases in the second period, we showed that there can be such equilibrium if the necessary Kuhn Tucker conditions are satisfied.

Hence, we can conclude that the assumption that demand increases in the second period raise the possibility of having an equilibrium that at least one country does not extract in the first period but extracts in the second.

Infinite Time Model

Assumptions

Time is assumed to be continuous and the horizon is infinite. There are two countries denoted by *A* and *T* selling the same natural resource in the same market but exploiting the resource from their own stocks. Consumer demand is assumed to be linear.

Stock of country i at time t is denoted by $s_{i.t}$ and the amount of natural resource extracted at time t by country i, $i=A,T$ is denoted by $k_{i,t}$.

Since we assume depletable natural resource, change in stock is expressed as

$$\dot{s}_{T,t} = -k_{T,t}$$

$$\dot{s}_{A,t} = -k_{A,t}$$

And the cost function is assumed to depend on the amount of the resource extracted and denoted as

$$C_A(k_{A,t}) = c_A k_{A,t}$$

$$C_T(k_{T,t}) = c_T k_{T,t}$$

The problem faced by each country

Each country is directly affected by the strategy of the other country. Therefore, the maximization problem of each country contains the extraction amount of the other.

The maximization problem of Turkey is

$$\max \int_0^{\infty} \left[(a - bk_{T,t} - bk_{A,t})k_{T,t} - c_T k_{T,t} \right] e^{-rt} dt$$

subject to $\dot{s}_{T,t} = -k_{T,t}$

$$\int_0^{\infty} k_{T,t} dt \le s_{T,0},\ s_{T,0} \text{ is given}$$

$$k_{T,t} > 0.$$

And the maximization problem of the U.S.A is,

$$\max \int_0^\infty \left[(a - bk_{T,t} - bk_{A,t})k_{A,t} - c_A k_{A,t} \right] e^{-rt} dt$$

subject to $\dot{s}_{A,t} = -k_{A,t}$

$$\int_0^\infty k_{A,t} dt \le s_{A,0},\ s_{A,0} \text{ is given}$$

$$k_{A,t} > 0.$$

The Solution Method for the Problem

We use Hamiltonian method in order to solve for the problem. Hamiltonian functions of both countries are,

$$\mathrm{H}_T = (a - bk_T - bk_A - c_T)k_T e^{-rt} + \lambda_T(-k_T)$$
$$\mathrm{H}_A = (a - bk_T - bk_A - c_A)k_A e^{-rt} + \lambda_A(-k_A)$$

$\lambda_i, i = A, T$ is the shadow price which shows the marginal value of an additional unit of *s* at time *t*.

Necessary conditions are

$$\frac{\partial \mathrm{H}_T}{\partial k_T} = 0 \text{ and } \frac{\partial \mathrm{H}_T}{\partial s_T} = -\dot{\lambda}_T$$
$$\frac{\partial \mathrm{H}_A}{\partial k_A} = 0 \text{ and } \frac{\partial \mathrm{H}_A}{\partial s_A} = -\dot{\lambda}_A$$

And the transversality conditions are

$$\lim_{t\to\infty} \lambda_T s_T = 0,$$
$$\lim_{t\to\infty} \lambda_A s_A = 0.$$

$$\frac{\partial H_T}{\partial k_T} = e^{-rt}(a - 2bk_T - bk_A - c_T) - \lambda_T = 0$$

$$\frac{\partial H_T}{\partial s_T} = 0 = -\dot{\lambda}_T, \text{ then } \lambda_T \text{ is constant.}$$

$$\frac{\partial H_A}{\partial k_A} = e^{-rt}(a - 2bk_A - bk_T - c_A) - \lambda_A = 0,$$

$$\frac{\partial H_A}{\partial s_A} = 0 = -\dot{\lambda}_A, \text{ then } \lambda_A \text{ is constant.}$$

Since λ_T and λ_A are constants, by the transversality condition we can say that both of the countries finish their resources i.e.

$$\lim_{t\to\infty} s_T = 0,$$

$$\lim_{t\to\infty} s_A = 0.$$

We did not complete the solution of the infinite time model. For the further research, the infinite time solution can be examined.

In order to see the extraction amounts for infinite time, we used simulation techniques for different cases.

CHAPTER 3

SIMULATION MODELS

In the simulation part, we have applied the same simulation method with Petra Huck (2005).

We used excel solver to find the extraction path. We analyzed the following cases;

- Monopoly
- Two identical countries
- Two countries with different initial stocks
- Two countries with different initial stocks and different discount factors
- Two countries with different initial stocks, different discount factors and different marginal costs
- Two countries with different initial stocks, different discount factors, different marginal costs and increase in demand in the twenty first period.

Assumptions

As in the theoretical part demand is assumed to be linear and is of the form:

$P(k) = a - bk$

r is the discount factor

s is the amount of stock in the initial period *c* is the marginal cost

For the simulation part a, b, c, s and r are chosen parallel to the values used in Huck's (2005) paper.

Both countries maximize their profit and discount the future. The maximization problem of Turkey is;

$$\max \sum_{t=1}^{100} (\frac{1}{1+r})^t (a - bk_{T,t} - bk_{A,t} - c_T) k_{T,t}$$

The maximization problem of the U.S.A is;

$$\max \sum_{t=1}^{100} (\frac{1}{1+r})^t (a - bk_{T,t} - bk_{A,t} - c_A) k_{A,t}$$

Optimal Extraction Path of Monopoly

The parameters are chosen as; *a = 80; b = 5; c = 2; r = 0,06; s = 600.* The optimal extraction path found is shown in the graph below;

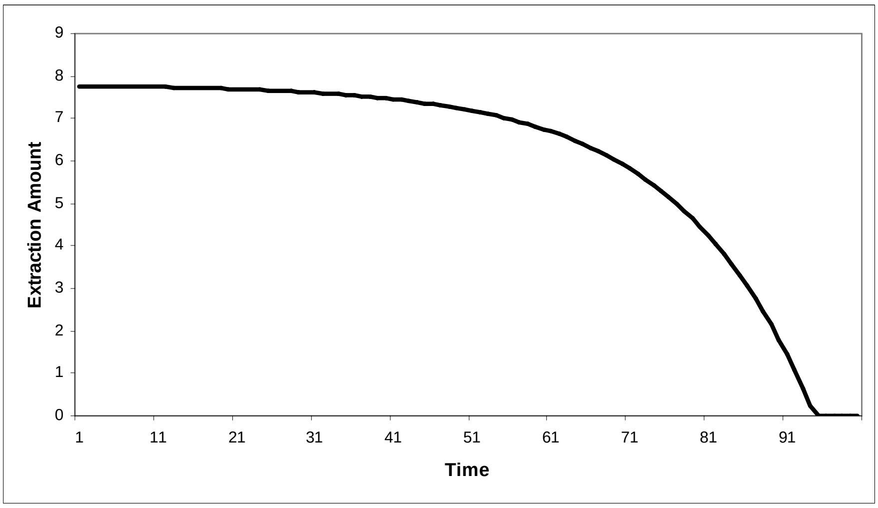

Fig2. The Optimal Extraction Path of Monopoly

A monopolist depletes its resources at a decreasing rate and depletes all the stock in ninety forth period.

Optimal Extraction Path of Duopoly – Identical Two Countries

In this part we assume two identical countries; Turkey and U.S.A denoted as T and A, respectively. The parameters are chosen as a = 80; b = 5; $c_T = 2$; $c_A = 2$; $r_T = 0{,}06$; $r_A = 0{,}06$; $s_T = 300$; $s_A = 300$.

Fig3. shows the optimal extraction path for two identical countries;

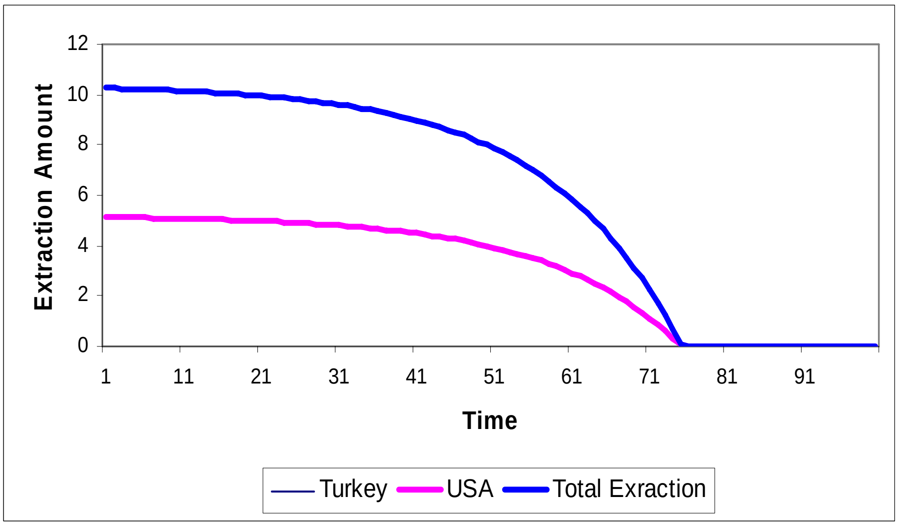

Fig3. The Optimal Extraction Path of Duopoly – Identical Two Firms

Figure 3 shows that two identical duopolists deplete the natural resource at seventy fifth period. When optimal extraction path for two identical countries is compared with that of monopoly, it can be seen that in the case of duopoly the resource is depleted earlier than in monopoly as Hotelling and Stiglitz proved. Stiglitz (1976) showed that “a monopolist is more conservation minded than a competitive market”.

Optimal Extraction Path of Duopoly – Different Initial Stocks

In this part, we examined the optimal extraction path of two countries with identical discount factors, marginal costs but different initial stocks.

The parameters are chosen as $a = 80; b = 5; c_T = 2; c_A = 2; r_T = 0{,}06; r_A = 0{,}06; s_T = 450; s_A = 150$. Then, the optimal extraction path is found to be

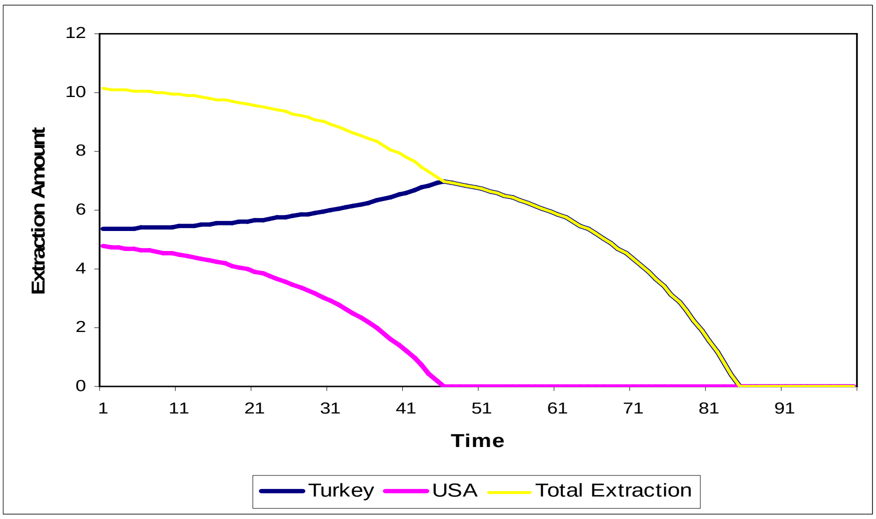

Fig4. The Optimal Extraction Path of Two Countries with Different Initial Stocks

Figure 4 shows that the country with greater amount of stock increases its extraction amount until the country with smaller stock depletes its resource, then it behaves as a monopoly and the whole stock is finished in period eighty four which is earlier than a monopolist but later than two identical countries.

Optimal Extraction Path of Duopoly – Different Initial Stocks and Different Discount Factors

In this part, parameters are chosen as $a = 80; b = 5; c_T = 2; c_A = 2; r_T = 0{,}10; r_A = 0{,}06; s_T = 450; s_A = 150$, and the optimal extraction path of two countries with different initial stocks and different discount factors is found as

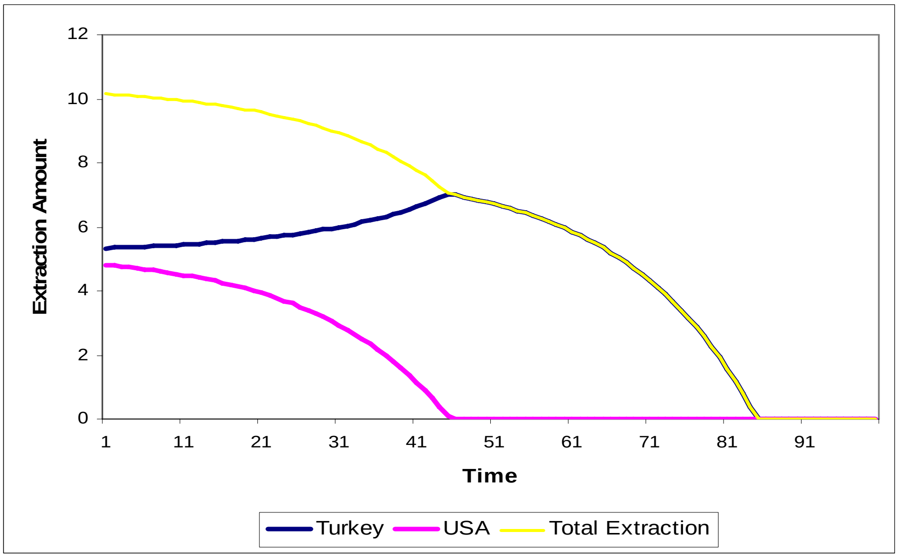

Fig5. The Optimal Extraction Path of Two Countries with Different Initial Stocks and Different Discount Factors

We added different discount rates to the previous analysis. We have increased the discount factor of Turkey, since Turkey gives less importance to the future than the U.S.A. However, we have seen that there is not much difference with Fig4.

Optimal Extraction Path of Duopoly – Different Initial Stocks, Different Discount Factors, Different Costs

In this part, we aim at finding the effect of cost. Therefore, we have chosen parameters as $a = 80; b = 5; c_T = 2; c_A = 4; r_T = 0,10; r_A = 0,06; s_T = 450; s_A = 150$. Then, the optimal path graph for these parameters is found as

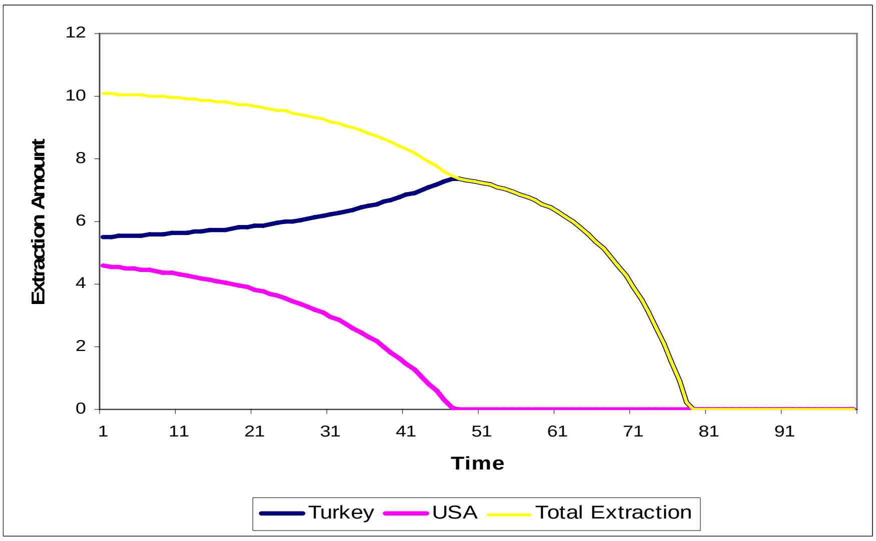

Fig6. The Optimal Extraction Path of Two Countries with Different Initial Stocks, Different Cost Factors and Different Stocks

In order to understand the effect of different cost we compare this graph with Fig5. Then, it can be seen that if one country's cost is greater than the other's, then the stock is depleted earlier.

Optimal Extraction Path of Duopoly –Increase in Demand

In this part of the simulation, we have assumed that demand increases in the twenty first period. In the first 20 periods, demand is assumed to be of the form $P(k) = a - bk$ and in the rest of the periods demand is of the form $P(k) = A - bk$, where $A > a$.

We made the simulation for different values of A.

First parameters are *a = 80 (until 21st period); A = 160 (between 21st and 100th period); b = 5;* $c_T = 2$; $c_A = 4$; $r_T = 0{,}10$; $r_A = 0{,}06$; $s_T = 450$; $s_A = 150$. And the optimal extraction path is found to be;

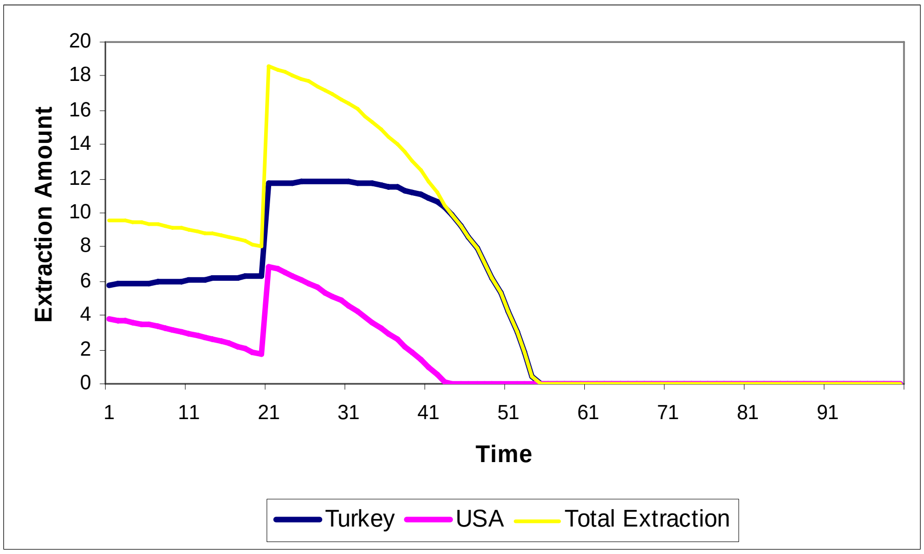

Fig7. The Optimal Extraction Path of Duopoly – With the Assumption of Increasing Demand (A = 160)

From Fig7, it can be easily seen that there is a jump in the exploitation rate in the period of increased demand. The total extraction period became shorter than other cases.

Secondly, we have increased A much more and searched for the result. Holding all other parameters equal A is set to be 450 in this case and then the graph is found to be

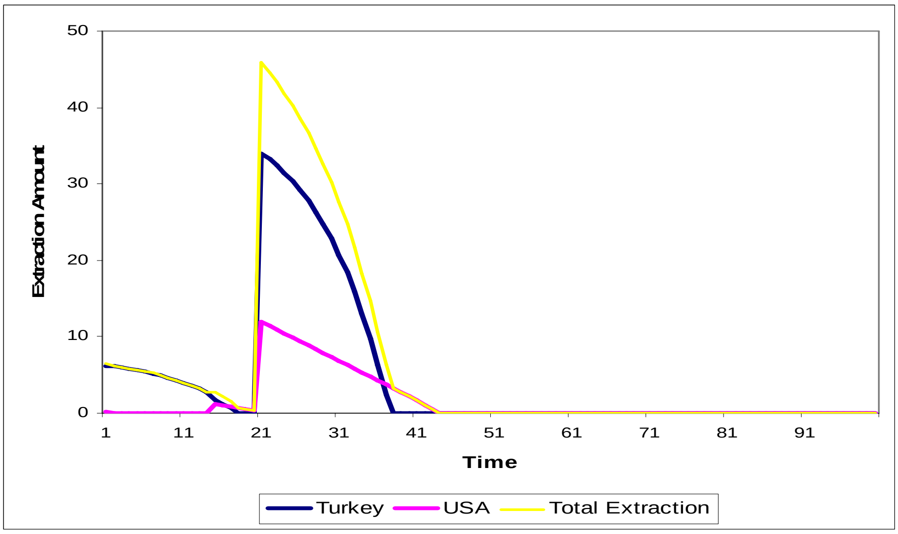

Fig8. The Optimal Extraction Path of Duopoly – With the Assumption of Increasing Demand (A = 450)

In this case, Turkey starts with a positive amount of extraction and then decreases extraction amount to zero for a couple of periods, but then in the period of increasing demand, extraction amount rises. On the other hand, the U.S.A does not extract for 15 periods, then increases its extraction amount, then decreases to zero again and with the increase in demand its amount of exploitation also jumps. However, the interesting case here is that the U.S.A, the country with lower stock depletes its resource later than Turkey. In all other cases, the U.S.A depleted earlier than Turkey.

Optimal Extraction Path of Duopoly - Consumer Surplus is added

In this case, we assume that the U.S.A maximizes its profit and consumer surplus i.e. the maximization problem of the U.S.A becomes

$$\max \sum_{t=0}^{100} (\frac{1}{1+r})^t (a - bk_{T,t} - bk_{A,t} - c_A)k_{A,t} + \frac{b(k_{A,t} + k_{T,t})^2}{2}$$

Turkey maximizes its profit as in the earlier cases. The parameters are chosen as; $a = 80; b = 5; c_T = 2; c_A = 2; r_T = 0{,}06; r_A = 0{,}06; s_T = 450; s_A = 150$. Then, the optimal extraction path given by Solver is

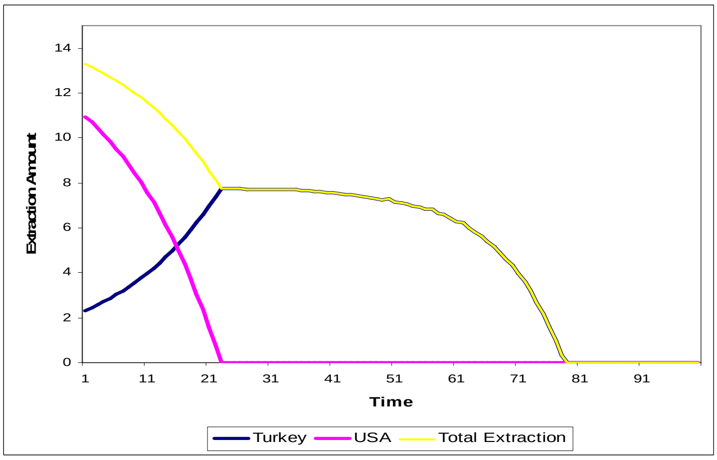

Fig9. The Optimal Extraction Path of Duopoly – Consumer surplus is added

In this case, U.S.A starts with extracting 10.95 units of stock, however in the case before we add consumer surplus U.S.A extracts 4.59 units of stock. Although it has depleted its resource at 47^{th} period at the case without consumer surplus, when we add consumer surplus it has finished its resource at the period 22. Hence, we can conclude that if one country takes into account the consumer surplus, then it depletes its resource earlier.

CHAPTER 4

CONCLUSION

This research analyzed the optimal extraction amount of a depletable natural resource and tried to find a solution for the existence of an equilibrium, where countries do not extract in the first and extract in the second period.

We showed that there is no equilibrium that for the first period none of the countries extract resources and for the second period both of the countries extract a positive amount of resource. However, if demand is expected to increase in the second period, there can be an equilibrium, that any extraction occurs in the first period but extract in the second period depending on the amount of increase in the demand.

Moreover, if the U.S.A gives much more importance to the future than Turkey and Turkey's cost is less than the U.S.A and the necessary conditions described in Case 5 are satisfied then there is an equilibrium, that the U.S.A does not extract any resources in the first period but extracts in the second whereas Turkey extracts in both periods and both of the countries finish their resources. And if the necessary conditions in Case 6 are satisfied then there is an equilibrium that Turkey does not extract any resources in the first period but extracts in the second whereas the U.S.A extracts in both periods and both of the countries finish their resources. However, these two conditions are too tight. In the real world these conditions cannot be satisfied.

In the simulation part, we see that in the monopoly, resource is depleted slower than that of duopoly. If one of the countries in the duopoly has greater stock the resource finishes later than in both countries of identical stocks.

Furthermore, if the discount factor is found not to have much effect on the extraction amounts and the effect of marginal cost is analyzed it is seen that if one country's cost is greater than the other, than the stock is depleted earlier. Expected increase in demand in the future results in slow depletion in the early periods and rises in the period of increase in demand. Depending on the amount of increase in demand, countries can choose not to extract any amount in the early periods until increase in demand occurs.

Solving the problem for infinite time is the subject of further research.

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