A FORMAL MODEL AND A MATHEMATICAL PROGRAMMING BASED ANALYSIS FRAMEWORK FOR TACTICAL-LEVEL PLANNING OF HOME CARE SYSTEMS

by

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Submitted to the Institute for Graduate Studies in
Science and Engineering in partial fulfillment of
the requirements for the degree of
Doctor of Philosophy

Graduate Program in Department of Industrial Engineering

Boğaziçi University

2015

To my family…

ACKNOWLEDGEMENTS

It has been a great opportunity to work with Assoc. Prof. Ali Tamer Ünal. His way of handling obstacles that we have faced throughout the study was exceptional. It is his endless tolerance, sensibility and talent in managing advisor student relations that provide the most suitable working conditions and realize this study. This dissertation would not have been possible without his supervision and contribution. Working with him has been a great experience for me.

I would like to thank my thesis committee members who have been extremely helpful and indulgent all the time. I want to express my gratitude to Assoc. Prof. Zeki Caner Taşkın for his moral support, encouragement and support throughout this research. I am very grateful to Prof. Orhan Feyzioğlu for his interest and suggestions that provided insightful discussions and comments throughout this research.

I am also grateful to Assist. Prof. Gönenç Yücel for his invaluable contributions and suggestions throughout this study.

I would also like to thank Prof. Ahmet Gündüz Ulusoy for taking part in my defense committee, his comprehensive comments and suggestions.

This thesis has been supported by Boğaziçi University Research Fund Grant No: 6366.

ABSTRACT

A FORMAL MODEL AND A MATHEMATICAL PROGRAMMING BASED ANALYSIS FRAMEWORK FOR TACTICAL-LEVEL PLANNING OF HOME CARE SYSTEMS

Home care is a viable alternative to delivering health care services, especially for the elderly. Due to various operational characteristics, home care organizations are significantly different from the traditional institutionalized health care delivery systems, such as hospitals. In this study, we first propose a formal object model in which we define the entities involved in a home care system and their relations. Later we demonstrate that, the proposed formal model can be used to develop mathematical programming models that include generic system-level constraints and represent tactical-level aggregate planning problems in perspectives of various stakeholders, such as regulators, patients, health care institutions, funding bodies and system resources. We develop an agent-based simulation algorithm to provide a general understanding on how the actors interact and the system evolves based on actors' simplistic decision making. We also propose an iterative optimization algorithm that involves mathematical programming based decision making for actor groups in a systematic manner so as to come up with an agreed-upon solution that reflects the collective wellbeing of all system actors. The total of formal model, generic and system level constraints, actors' perspectives as well as two different modeling approaches constitutes an analysis framework which can be employed in analyzing the evolutionary progress of the overall system. Our numerical study shows that the proposed analysis framework and the algorithms representing two different and complementing approaches are valuable tools in investigating the evolution of a health care system including home care as an alternative means of service.

ÖZET

EVDE BAKIM SİSTEMLERİNİN TAKTİKSEL DÜZEYDE PLANLAMASI İÇİN BİÇİMSEL BİR MODEL VE MATEMATİKSEL PROGRAMLAMA TABANLI ANALİZ PLATFORMU

Evde bakım özellikle yaşlılar için, sağlık hizmetleri sunumunun uygun bir alternatifidir. Çeşitli operasyonel özellikler nedeniyle, evde bakım kuruluşları hastane gibi geleneksel kurumsal sağlık sunum sistemlerinden önemli farklılıklar gösterir. Bu çalışmada, öncelikle evde bakım sisteminde yer alan unsurları ve ilişkilerini tanımladığımız biçimsel bir nesne modeli öneriyoruz. Sonrasında önerilen bu biçimsel modelin, sistemin genel kısıtlamaları ile düzenleyiciler, hastalar, sağlık kurumları, finansman sağlayıcılar ve sistem kaynakları gibi çeşitli paydaşların bakış açılarından taktik seviye bütünleşik planlama problemlerini temsil edecek matematiksel programlama modelleri geliştirmek için kullanılabileceğini gösteriyoruz. Aktörlerin nasıl etkileşime girdiğini ve sistemin bu aktörlerin basit kararlar almalarına dayalı evrimsel olarak nasıl ilerlediğine dair genel bir anlayış sağlamak için etmen-tabanlı bir benzetim algoritması geliştirilmiştir. Ayrıca aktörlerin, tüm sistem aktörlerinin müşterek refahını yansıtan üzerinde anlaşılmış bir çözüme erişebilecek şekilde matematiksel programlama tabanlı karar vermelerini öngören tekrarlamalı bir eniyileme algoritması öneriyoruz. Biçimsel model, genel ve sistem düzeyindeki kısıtlamalar ve aktörlerin bakış açılarının yanı sıra iki farklı modelleme yaklaşımı, tüm sistemin evrimsel ilerlemesini analiz etmede kullanılabilecek bir analiz çerçevesi oluşturmaktadır. Sayısal çalışmamız, önerdiğimiz analiz çerçevesinin ve iki farklı ve birbirini tamamlayıcı yaklaşımları temsil eden algoritmaların evde bakımın alternatif bir hizmet sunum türü olduğu sağlık sisteminin evrimsel işleyişini araştırmada değerli araçlar olduğunu göstermektedir.

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LIST OF ACRONYMS / ABBREVIATIONS

ABMS	Agent-Based Modeling and Simulation
CT	Computed Tomography
MIP	Mixed Integer Programming
MRI	Magnetic Resonance Imaging
MS	Management Science
OR	Operations Research
SGK	Social Security Institution
UML	Unified Modeling Language
VRPTW	Vehicle Routing Problem with Time Windows

1. INTRODUCTION

Healthcare systems have complicated built-in dynamics. Various actors and related components take place in these systems. Main actors are patients, institutions, healthcare professionals, funding bodies and regulators, all of which have their own unique needs and expectations in the system.

Patients are the people that demand healthcare services due to their specific medical conditions. They strive to receive the best possible service at reasonable levels of spending. Patients choose among alternative services provided by healthcare institutions. Whether they are private or public organizations, these institutions have commercial perspectives. They hire healthcare professionals and acquire various equipment and supplies to deliver services. They mainly focus on margins and efficiency. Healthcare professionals constitute a major and critical type of system resources. They try to achieve job satisfaction. Funding bodies such as insurers or sponsors cover for expenses of patients' they are associated with. Regulators in healthcare systems issue policies or legislations and usually have large and long-term impacts on the system.

All of these system actors are in interaction. Actions of an actor usually have profound effects over the others. Every actor's goals are different from each other and almost always, they are in conflict. The relations among system actors and components are complicated and nonlinearity is a common phenomenon in the system.

The presence of different actors, their interactions and conflicting goals, as well as nonlinear relations complicate the ongoing behavior of a healthcare system. The evolution of such a system through time is an interesting research topic for us, in this respect.

We think that in order to observe actors' responses against others' actions, policy changes and various dynamics, one should have a framework that can be employed in analyzing the evolutionary progress of the healthcare system through time. The outcomes of factors and impacts are interesting for us at an aggregate level. Therefore we focus on tactical level concerns in this study, rather than operational details.

To identify and describe system components and their relations, we develop and propose a formal system model using object-oriented programming paradigm. The model is represented by Unified Modeling Language (UML).

Our formal model provides a complete formalization of system actors, their tactical level actions and other related system components. Every system component has its specific attributes that represent its characteristics and each actor has its own perspective of a system component. The perspectives of actors over system components are represented by utility functions.

Healthcare systems have generic constraints that limit actors' actions. There are limitations on receiving a certain level of healthcare service, system's total available resource supply or a particular institution's available resources. All these generic constraints within the system can be formulated via mathematical expressions that use our formal model's relations, components and their attributes. Using the utility functions we can formulate generic objective functions for actors. As a result, our proposed formal model enables one to formulate mathematical programming models over a healthcare system and analyze its actors' decision making activities.

Over the last decades, home care has shown a considerable growth within the healthcare system, especially in developed countries. Larger portions of healthcare workers are employed in this field every year. Being an alternative to institution based healthcare, home care provides more beneficial results in certain situations over its counterpart. Home based care delivery has several specific characteristics that differentiate it from institution based care. Hence, we consider home care to be a significant part of the overall healthcare system and develop our formal model so as to accommodate these characteristics.

Costs and revenues associated with healthcare system components are important concerns from actors' perspectives. However, we observe that actors' goals and actions are not solely based on these aspects. Attributes such as availability, reliability, conformance and convenience related to the delivery of a healthcare service all have significant importance in the system. Therefore, our utility functions encapsulate and represent a collective view of all monetary and non-monetary factors related to delivery of services.

Introducing such a utility concept that inherently express cost related matters enables us to drop funding related actors and attributes, as well as costs or revenues related components from our analysis without loss of generality.

Being uniquely based on actors' goals and expectations, utility functions are different for each actor and determining a single set of actions that is most favorable for every actor at any setting is very hard.

For this purpose, we develop an agent-based simulation algorithm to simulate different actors' actions and responses against system-level impacts over time using our formal model. This simulation algorithm employs simple but rational decision rules for system's individual actors (called "agents") and iterates through simulation periods. The results of this algorithm provide a general understanding on how the system evolves based on actors' simplistic and mostly "myopic" decision making processes.

Based on our formal model, we also develop mathematical programming formulations for every system actor so as to optimize its actions in a given system setting. Based on these formulations we develop an iterative optimization algorithm that incorporates utility considerations of every system actor in a systematic manner and come up with an agreed-upon solution that reflects the collective utility of the overall system.

With the help of these two different approaches, one can analyze the impact of several factors over actors' utilities and actions, moreover, the evolution of the overall system.

We generate numerical scenarios where the system is subjected to specific system-level, major policy changes. We implement our algorithms to investigate the effects of these impacts over the evolution of the system. We observe that the proposed analysis framework and algorithms are valuable tools in investigating the evolution of a health care system including home care as an alternative means of service.

Section 2 presents our survey over the related literature. Section 3 presents the details of the proposed analysis framework. Section 4 introduces our simulation and optimization algorithms defined using the analysis framework. Section 5 shows the details of our numerical study, where we implement our proposed approaches within the analysis framework over alternative scenarios. Section 6 summarizes our conclusions and remarks.

2. LITERATURE SURVEY

Healthcare systems have been drawing increased attention of Operations Research and Management Science (OR/MS) researchers and practitioners over the years. Simulation, queueing theory, heuristics and mathematical programming are the most common techniques applied to aid decision making in healthcare, in this respect. Papageorgiou (1978), Boldy and O'Kane (1982), Pierskalla et al. (1989), Pierskalla and Brailer (1994), Royston (1998), Brailsford and Vissers (2011), Rais and Viana (2011), Hulshof et al. (2012), Dobrzykowski et al. (2013) and Xing et al. (2013) review the progress of research in the field.

Brandeau et al. (2004) classify the healthcare OR/MS literature into three broad areas. *Clinical applications* correspond to optimization efforts in design and planning of medical tests, treatments and procedures. *Public policy and economic analysis* involves risk estimation, cost and outcome modeling, as well as screening and disease prevention in healthcare. *Healthcare operations management* refers to decision making activities in planning and control of operations and resources in healthcare.

The extensive research in healthcare operations management resulted in review papers that focus on particular healthcare resources. Facilities, operating theatres, departments (outpatient, inpatient, emergency, etc.), workforce (nurses, residents, doctors, therapists, etc.) as well as equipment (beds, intensive care units, MRIs, CTs, etc.) are among critical healthcare resources. Daskin and Dean (2004) review several location models for healthcare facilities. Cardoen et al. (2010), May et al. (2011) and Guerriero and Guido (2011) review various approaches applied in planning and scheduling of *operating theatres*; Cayirli and Veral (2003) review scheduling of *outpatient services*. Aboueljinane et al. (2013) review simulation literature in *emergency service* operations. Gupta and Denton (2008) review *appointment scheduling* in general critical care services. Cheang et al. (2003), Burke et al. (2004), Ernst et al. (2004) and Van den Bergh et al. (2013) review the literature related to scheduling of healthcare workforce.

Literature includes several reviews that focus on specific modeling approaches and techniques. Lakshmi and Iyer (2013) review *queueing theory* applications in healthcare management. Jun et al (1999), Eldabi et al. (2007), Thorwarth and Arisha (2009), Günal and Pidd (2010), Katsaliaki and Mustafee (2011) and Mielczarek and Uzialko-Mydlikowska (2012) review various studies in healthcare that are based on *simulation*.

A widely accepted framework, which is proposed by Antony (1965), involves decision making activities in healthcare operations management to be studied at strategic, tactical and operational planning levels. Boldy and O'Kane (1982), Guerriero and Guido (2011), Hulshof et al. (2012), Dobrzykowski et al. (2013) adopt this categorization in their reviews. We provide a brief description of these planning levels from a healthcare operations management perspective.

Strategic decisions represent the highest level of decision making. It involves design and development decisions that will be realized and remain effective in long term. The decisions at this level usually require substantial amount of investment. Location selection for facilities, layout design and high level resource capacity planning decisions can be considered at this decision level.

Tactical decisions involve aggregate-level planning and allocation of resources to meet service demand. Being shaped by strategic decisions, tactical decisions determine resource levels in medium term. Shift or patient assignments to workforce as well as hiring and firing decisions mostly reside in tactical decision levels.

Operational decisions relate directly to the execution of healthcare activities. Employee-to-patient assignments, workforce scheduling and routing of as well as detailed planning of critical resources and determining logistic decisions of materials are at this level.

Home care is a rapidly growing type of service delivery within healthcare systems, especially in developed countries. Tarricone and Tsouros (2008) locate home care to be at the intersection between healthcare and social systems, including medical and paramedical services as well as help services such as personal care, housekeeping and social support provided in people’s home. They list several factors that highlight the importance of growing home care need in Europe. According to their report, changes in demography, social life, epidemiology, science, expectations, and policies drive the need for home care. We elaborate more on planning of home care systems in the rest of this survey.

Benzarti et al. (2010) review operations management issues in home health care and define five main problems studied in the literature as resource dimensioning, partitioning a territory into districts, allocation of resources into districts, assigning workforce to patients or visits and routing of the workforce.

Adopting from Antony (1965), we classify home care planning efforts based on decision making horizons, effective time buckets and impacts of decisions.

- high-level policy making and resource capacity dimensioning are *strategic level* decisions
- districting territories, assignment of workforce among and within these districts and to patients at aggregate levels are *tactical level* decisions
- detailed assignment of individual employees to patients, visits or shifts and routing decisions are *operational level* decisions

Table 2.1 presents the categorization of the home care operations management literature based on this classification.

During our literature survey, we have not encountered a study that we can classify within the strategic level planning efforts in home care systems.

De Angelis (1998) develops a stochastic linear programming model to optimize admission schedule of home care patients within a district subject to workforce capacity constraints.

Several researchers studied the home care districting problem, where a territory is partitioned into districts. Blais et al. (2003) propose a tabu search procedure to make this partitioning while balancing workload and considering mobility concerns of nurses' assigned to each partition. Lahrichi et al. (2006) analyzes their solution for upcoming years and propose to allow sharing of nurses among different districts based on their current workloads. Hertz and Lahrichi (2009) propose a mixed integer mathematical programming formulation that consider sharing of nurse capacities and solve it by exact algorithm. They also propose a tabu search algorithm for a modified version of this model containing additional objectives and associated nonlinear constraints. Benzarti et al. (2013) propose two mathematical models for nurses' allocation among districts with objectives that balance workload and minimize travel distance.

Table 2.1. Classification of reviewed papers on home care operations management

Tactical Level Planning:	Operational Level Planning:
De Angelis (1998) Blais et al. (2003) Lahrichi et al. (2006) Hertz and Lahrichi (2009) Benzarti et al. (2013)	Begur et al. (1997) Bertels and Fahle (2006) Eveborn et al. (2006) Akjiratikarl et al. (2007) Bredström and Rönnqvist (2008) Chahed et al. (2009) Trautsamwieser and Hirsch (2011) Nickel et al. (2012) An et al. (2012) Rendl et al. (2012) Rasmussen et al. (2012) Liu et al. (2013) Hiermann et al. (2013) Cappanera and Scutellà (2013) Allaoua et al. (2013) Duque et al. (2014) Carello and Lanzarone (2014) Liu et al. (2014) Mankowska et al. (2014)

Operational level planning of home care systems involve staff rostering and routing components so as to meet clients' home care service demand. These components result in problems that are very hard to solve. Sahin et al. (2013) proposes a framework that identifies and categorizes the factors that increase the operational complexity in home care settings. Heuristic or metaheuristic based techniques, exact algorithms and hybrid algorithms have been proposed to solve variations of problems at this level of home care planning.

Many authors study routing of home care workers based on the well-known vehicle routing perspective. Introduction of temporal constraints over timing of visits, results in a Vehicle Routing Problem with Time Windows (VRPTW) with many depots, where nurses correspond to depots housing single vehicles. Temporal constraints may be the patients' or workers' timing preferences for visits, the synchronization of the workforce at visits, etc.

Begur et al. (1997) develop a decision support tool that employs heuristics-based solution procedures for daily patient assignments and corresponding routes of home care nurses. Bertels and Fahle (2006) compare the performances of several search algorithms to determine nurses-to-patient assignments and routing of nurse visits. Eveborn et al. (2006) assign workers to visits heuristically using a repeated matching algorithm. Akjiratikarl et al. (2007, 2008) propose metaheuristic-based solution procedures for the problem to minimize the total distance travelled by the workers. Trautsamwieser and Hirsch (2011) proposes a metaheuristic solution approach based on variable neighborhood search for routing of nurses. Nickel et al. (2012) define two levels of planning, where weekly schedules are generated by master planning and last minute changes are incorporated by operational level planning, where both levels are solved using heuristic-based procedures. An et al. (2012) develop a heuristic algorithm for developing visiting schedules for home care nurses to minimize their total travel time.

Bredström and Rönnqvist (2008) propose a VRPTW model focusing on the effects of temporal constraints that incorporate time-based synchronization workers at locations and solve it by an optimization based heuristic procedure. Rasmussen et al. (2012) decompose the problem and use dynamic column generation in a branch-and-price algorithm. Liu et al. (2013) and Liu et al. (2014) addresses versions of this problem in home care logistics and test their proposed metaheuristic methods on several instances. Mankowska et al. (2014) propose a heuristic-based solution methodology for the problem with several temporal dependencies.

Rendl et al. (2012) applies various metaheuristics to solve the home care scheduling problem where nurses can select among alternative modes of transportation, where initial solutions are generated via a constraint programming-based clustering heuristic. Hiermann et al. (2013) evaluate the performances of different heuristics and metaheuristics for the multimodal home health care scheduling.

Chahed et al. (2009) integrates the production sequence of drugs together with the routing of nurses administering them at patients' home, while respecting drugs’ shelf lives and solves the problem with an exact method. Cappanera and Scutellà (2013) propose integer linear programming models that employ a priori scheduling profiles in order to combine different decision levels. Allaoua et al. (2013) propose a matheuristic approach for the combined vehicle routing and staff rostering problems. Duque et al. (2014) propose a mathematical programming based solution methodology to optimize service levels and total travel distance for workforce assignment problem faced by a home care organization currently operating in Belgian regions. Carello and Lanzarone (2014) develop a cardinality-constrained robust model that considers continuity of care and uncertainty as main aspects in nurse-to-patient assignment.

This study is focused on developing a formal model that describes the tactical level components and associations in healthcare systems. Based on this formal model, we develop an analysis framework to study actors’ tactical level planning decisions in presence of system-level policy changes. These policy changes may be considered as strategic level decisions of regulating or funding bodies within the system.

3. A MATHEMATICAL PROGRAMMING BASED ANALYSIS FRAMEWORK FOR HEALTH CARE SYSTEMS INCLUDING HOME CARE

The wellbeing of actors in a healthcare system depend on the balancing demand and supply. Patients are the sources of demand and all physical entities that take place in delivering healthcare services constitute to system supply.

There are many factors that may cause imbalance between demand and supply in a healthcare system. Internal factors such as actors' unique and conflicting objectives, as well as external factors related to economy or politics influence the system occasionally or continuously. All these factors result in variations in actors' decision making.

In order to observe each actor's response to these factors, one should have a tool that can be employed in analyzing the evolutionary progress through reaching a good balance between demand and supply.

In this section we describe the details of such a tool that we name as an "analysis framework" for healthcare systems including home care. This proposed framework is based on a formal system model. Using the elements of this formal model, we define a set of generic system constraints that are valid for all system actors. We describe the perspectives of different system actors that represent their goals and expectations. We also define exemplary system-level functions that formalize certain operational dynamics within the system.

3.1. Proposed Formal Model

Our formal model formalizes the tactical level components, their characteristics and associations of a health care system that includes home care. This model is developed by using object-oriented programming paradigm and represented by UML as described by Rumbaugh et al. (2005).

Class diagram of the overall system including system components and their relations is presented in Figure 3.1. This diagram denotes the classes of our formal model and the associations in between as well as general attributes and methods of each class. An object within the system is an instance of one of these classes. Objects indices are denoted by lower case letters in little boxes in upper left corners of these classes. Top sections of classes denote class names. The lower section includes main attributes of classes and existing functions of the class are in the lowest section. The associations between classes in class diagrams follow the standard UML notation and definitions.

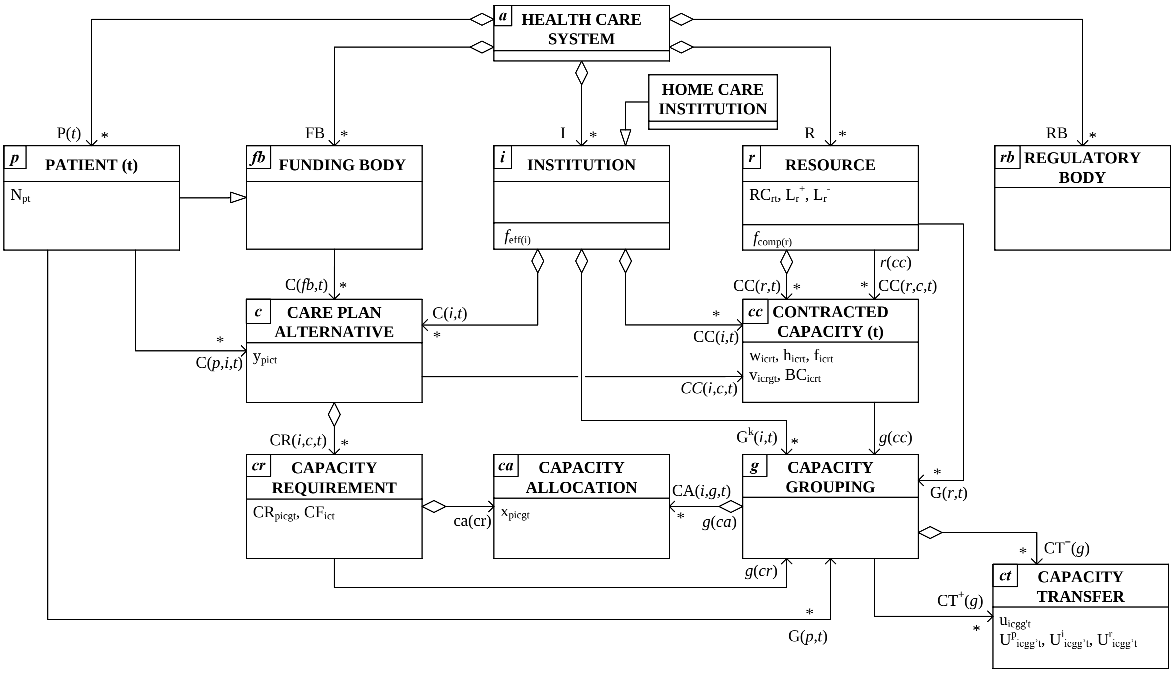

Figure 3.1. Class diagram of the overall system

Class descriptions, notations of indices, object-related notations including relevant attributes and functions of our formal model are presented in Tables 3.1 through 3.7. In the rest of this section, we will provide a brief description of the elements and the structure of this formal model.

In formalizing the system components at tactical level, we do not focus on actors or system components individually. Instead, we make use of a *grouping* concept. Every class in our formal model corresponds to a group that is based on the common and significant characteristics of a system component (actors, capacity requirements, capacity allocations, capacity transfers, capacity groupings and contracted capacities). This enables us to make aggregations over individual system components and study the system from a tactical perspective.

Time ($\boldsymbol{t}$), being a critical issue in formalizing the system, is also aggregated into time buckets. Therefore time is discrete and each time period can be a week, a month, etc. Most objects are associated with a particular period or a set of periods within a predetermined planning horizon ($\boldsymbol{T}$).

A health care system ($\boldsymbol{a}$) has several actor groups. These are patients, health care institutions, resources, funding bodies and regulatory bodies. All actor groups are denoted with objects in our formal model.

A health care institution is a service provider that makes use of various types of resources to deliver health care services. Hospitals, medical centers or offices are such institutions. Our formal model uses institution groups to differentiate institutions based on their different characteristics. Similarities related to types of provided services, service standards and target patient groups may form different institution groups in our formal model. Institutions offering a set of services at similar standards and prices, or those located within the same area, are of same type and therefore belong to the same **institution group** object ($\boldsymbol{i}$). Institutions providing home care services as an alternative service, as well as stand-alone home care providers, form up a special subset called home care institutions, among the set of all institution groups ($\boldsymbol{I}$).

The health care service provided by institutions in tactical planning levels is denoted by a care plan ($\boldsymbol{c}$). Institutions may provide alternative care plans in a time period ($\boldsymbol{C}(\boldsymbol{i}, \boldsymbol{t})$) among which the patients choose from, such as home care, hospital-based care, or a combination of both.

Every system resource has different qualifications, preferences and characteristics. Resources are grouped based on these attributes. Institutions utilize resources to provide services. Each system resource can be included within a **resource group** ($\boldsymbol{r}$) that makes an aggregation based on various characteristics. Different attributes of resources such as resource types (physicians, nurses, medical staff, etc.) area of profession (e.g. physiotherapist, pulmonologist, oncologist, etc. for physicians; intensive care, elderly care, etc. for nurses), title (academic doctors or medical practitioners for physicians), years of experience (<5 years, ≥10 years, etc.) and location of residence (based on districts, territories, etc.) may be defined as different resource groups. The system includes a collection of different resource groups ($\boldsymbol{R}$), where each group has a timely available capacity ($\boldsymbol{RC_{rt}}$).

Table 3.1. Formalization of system's actors their attributes and notation

t	Index for time periods
T	Set of time periods (t = 1, 2, ...) within the planning horizon
a	Index for health care system object
i	Index for institution group object denoting its type
I	Set of all *i*'s in the system
c	Index for care plan alternative group object denoting its type
$C(i,t)$	Set of all *c*'s provided by *i* at *t*
r	Index for resource group object denoting its type
R	Set of all *r*'s in the system
RC_{rt}	Standard capacity of *r* at time *t*
L_r^+	Standard lead time for increasing the contracted capacity (hiring) of *r*
L_r^-	Standard lead time for decreasing the contracted capacity (firing) of *r*
fb	Index for funding body group object denoting its type
FB	Set of all *fb*'s in the system
rb	Index for regulatory body object denoting its type
RB	Set of all *rb*'s in the system

In this study, we refer to health care workforce as the main system resource. However, our analysis framework is designed to accommodate other types of resources, such as medical equipment or consumables, as well.

One of the unique properties of home care environments is that resources may not be completely owned by home care institutions. Institutions acquire portions of resources' capacities by signing full-time, part-time or flexible contracts. In this way, total capacity of a resource can be distributed among multiple institutions. If resource is an equipment or a consumable, contracting corresponds to lease or sales agreement between the institution and supplier. Hiring and firing decisions made over a resource contracting occurs only after certain lead times ($\boldsymbol{L_r^+}$ and $\boldsymbol{L_r^-}$, respectively).

Funding-related components of our formal model are very abstract and may require further analysis to represent the actual dynamics of a health care funding system. Kutzin (2001) and Mossialos and Dixon (2002) provide a conceptual framework on the insurance function in health care to analyze health care financing systems. There are alternative mechanisms for raising funds for health care. These are classified as revenue collection, fund pooling and purchasing. For the time being, we generalize these funding related issues with funding body groups. A funding body is any kind of a sponsor that provides funds for patients' care plan expenditures. A **funding body group** ($\boldsymbol{fb}$) can be the patient group itself, their families, public insurance, private insurance companies, etc. Funding bodies related with a patient group provide coverage for portions of its care plan expenses. System contains a set of alternative funding bodies ($\boldsymbol{FB}$).

A **regulatory body group** ($\boldsymbol{rb}$) is a decision maker that affect the system at strategic, tactical and operational levels. Set of all regulatory body groups ($\boldsymbol{RB}$) contain authorities that regulate, legislate or audit the operations of health care institutions based on laws, rules or standards.

Table 3.2. Formalization of patients and care plan selections

p	Index for patient group object denoting its patient type
$P(t)$	Set of all *p*'s entering the system at *t*
N_{pt}	Group size of *p* entering the system at *t*
y_{pict}	Percentage of *p* selecting *i*'s *c* at *t*
$C(p,i,t)$	Set of all *c*'s provided by *i* for *p* at time *t*

Patients in the system are people who require medical care. Delivery of both institution-based and home-based healthcare services are affected by certain essential factors based on patient demographics. Disease rates, income distributions, certain social and cultural aspects of the patient population such as priorities, preferences and traditional tendencies are influential in patients' decision making. Selection of home-based care plans may depend on aspects such as proximity to institutional care, location of residence, type of disease or service related expenses, etc. Our analysis framework incorporates these influential factors by classifying the population into patient groups ($\boldsymbol{p}$), where each group denotes a certain subset of the population having similar attributes and constitutes a demand for health care services.

At each time period, a set of patient groups ($\boldsymbol{P}(\boldsymbol{t})$) enter the system. Each group has a size that denotes the number of patients in the patient group ($\boldsymbol{N_{pt}}$). A certain percentage of each patient group entering the system ($\boldsymbol{y_{pict}}$) selects from set of all care plan alternatives offered by an institution group ($\boldsymbol{C}(\boldsymbol{p}, \boldsymbol{i}, \boldsymbol{t})$).

A **contracted capacity** object ($\boldsymbol{cc}$) denotes the amount of a resource group's capacity acquired by an institution for a particular period ($\boldsymbol{w_{icrt}}$). Institutions make hiring ($\boldsymbol{h_{icrt}}$) and firing ($\boldsymbol{f_{icrt}}$) decisions to increase and decrease their contracted amounts of resource capacities. Institutions may leave certain amounts of contracted capacities idle to act as buffer capacities ($\boldsymbol{BC_{icrt}}$) against demand or supply fluctuations in the system, due to institutional policies, regulations or system standards. Institution groups contract capacities from resource groups for utilizing in their alternative care plans offerings ($\boldsymbol{CC}(\boldsymbol{i}, \boldsymbol{c}, \boldsymbol{t})$).

Table 3.3. Formalization of institutions' capacity contracting schemes

cc	Index for contracted capacity objects
w_{icrt}	Capacity of *r* that is contracted by *i* for *c* at *t*
h_{icrt}	Capacity of *r* that is hired by *i* for *c* at *t*
f_{icrt}	Capacity of *r* that is fired by *i* for *c* at *t*
BC_{icrt}	Buffer capacity of *r* contracted by *i* for *c* at *t*
$CC(i, c, t)$	Set of all *cc*'s of *i* to be utilized in *c* at *t*

In practice, institutions may not prefer hiring or firing workforce due to many concerns like associated costs, corporate values, employee satisfaction, and so forth. Therefore, these decisions are usually penalized or constrained in decision making to avoid hiring and firing decisions so as to keep workforce at certain levels, which in turn leads to unsatisfied demand or idle capacities.

Total workforce within a health care system including home care can be thought as a pool of employees having certain specialties and skills. Hiring and firing means contracting or releasing portions of this workforce pool as required, in this respect. Most of the health care workforce is affiliated with several institutions and they have the ability to deliver services on call. Since fulfilling patients' demand is a main priority, mechanisms that limit hiring or firing decisions through fixed costs or budget limitations may lead to unsatisfied demand or underutilization of system resources.

Our study focus on investigating operational dynamics of the system so as to achieve best resource utilization levels while fulfilling patients' demand. Besides, budget-related limitations at this level of planning can be easily surpassed through taking out loans. Based on this perspective, we have decided not to include hiring or firing costs. However, these components can be included in the proposed formal model as additional attributes or functions. All these concerns related to the workforce are valid for other types of system resources, as well.

Table 3.4. Formalization of institutions' capacity grouping schemes

g	Index for capacity grouping objects
ct	Index for capacity transfer objects
ca	Index for capacity allocation objects
$G(i,t)$	Set of all g’s in i’s grouping scheme
v_{icrgt}	Capacity of r contracted by i for c and transferred to g at t
$G^k(i,t)$	Set of all kth level g's within grouping scheme of i at t where k=1,2
$CT^+(g)$	Set of all ct's from which g transfers capacity
$CT^-(g)$	Set of all ct's to which g transfers capacity
$u_{icgg't}$	Capacity transferred within i's c from g to g' at t

Our formal model defines **capacity grouping** ($\boldsymbol{g}$), **capacity transfer** ($\boldsymbol{ct}$) and **capacity allocation** ($\boldsymbol{ca}$) objects to represent an institution group's tactical-level capacity planning decisions. Contracted capacities, capacity groupings, together with capacity transfers build up a capacity planning framework that we call a *grouping scheme* for institution groups. The grouping scheme is used to make various aggregations over institutions' contracted capacities and enable aggregate level capacity transfers to meet service demand within the tactical level planning horizon.

We introduce the formal structure of resource contracting and grouping schemes with an illustrative example. In this example, a home care institution group’s tactical level capacity plan is described, where home care nursing resources are planned within the grouping scheme.

The goal of planning is to meet service requirements of different patient groups in an efficient manner. Nurse capacity is classified so as to match demand characteristics and efficiency concerns. Some patients demand nurse with high experience (≥5 years). Moreover, patients to be served live in two different districts (A or B), and therefore nurse capacity is further classified based on nurses’ location of residence to enable better planning in matching demand with supply.

Based on demanded nursing experience and location of residence, four patient groups are defined. The first patient group lives in A and demands a total of 260 hours of nursing capacity. This group does not require high experience, so we denote them as requiring <5 years of experience. The second group lives in B and demands a total of 110 hours of capacity with <5 years of experience. Third and fourth groups live in A and B, respectively. These two groups demand capacity having ≥5 years of experience with amounts of 90 and 220 hours, respectively.

The grouping scheme of this institution which contracts and utilizes nurses to meet patient demand includes the objects and associations presented using the hierarchical network structure of Figure 3.2. Nodes correspond to resource groups, contracted capacities and capacity groupings, whereas arcs correspond to certain object attributes and capacity transfers.

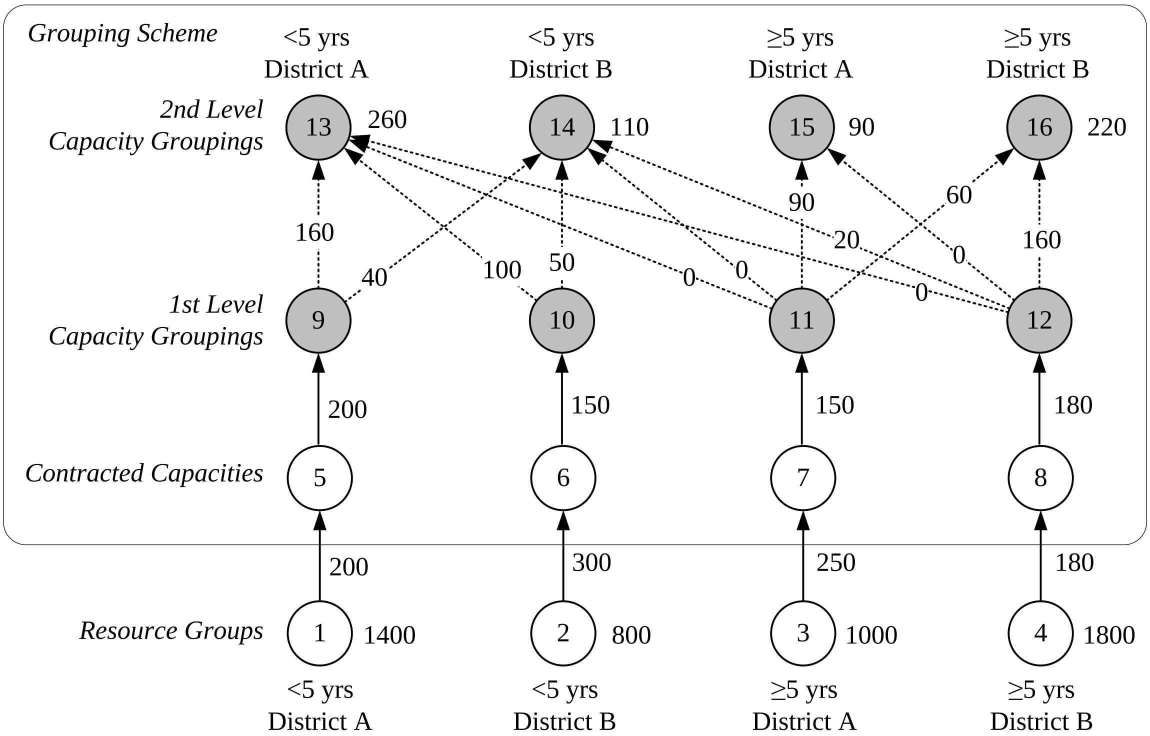

Figure 3.2. Illustration of institutions' grouping schemes

The total amount of available nursing capacity in the illustration is 5000 hours, 2800 of which has ≥5 years of experience and the rest having less experience. 1000 of those with ≥5 years of experience lives in area A, whereas the rest lives in area B. On the other hand, 1400 hours of nurses with <5 years of experience live in area A and the rest lives in area B. These available capacities are denoted in numbers next to nodes 1 to 4 at the bottom of Figure 3.2.

Resource contracting transactions of an institution results in certain portions of the system's total available resource capacities be transferred into its grouping scheme (as indicated with numbers 200, 300, 250 and 180 next to the arcs outgoing from bottom nodes and ingoing into contracted capacity nodes 5 to 8). Portions of these capacities will be transferred among capacity groupings within the institution's grouping scheme ($\boldsymbol{G(i,t)}$) to satisfy care plans' capacity requirements selected by patient groups. Note that outgoing capacity flows from contracted capacities indicate their utilization ($\boldsymbol{v_{icrgt}}$). Capacities that are not transferred out of contracted capacities remain idle.

There are two levels of capacity grouping nodes within a grouping scheme. Capacity requirements of care plans are defined in terms of the characteristics depicted at 2nd level capacity groupings ($\boldsymbol{G^2(i,t)}$). Capacity allocations will be done from capacities aggregated at this level (illustrated with nodes 13 to 16). The 1st level capacity groupings ($\boldsymbol{G^1(i,t)}$) aggregate over contracted capacities (illustrated with nodes 9 to 12).

Capacity aggregated in 1st level groupings are transferred to 2nd level and then allocated to meet care plan requirements. The dotted directed lines denote possible capacity transfers between these two grouping levels. Amounts of aggregate capacities transferred are denoted next to these lines. Capacity transfers through vertical dotted lines indicate an aggregated capacity at 1st level being utilized and allocated efficiently (e.g. nursing capacity with <5 years of experience, residing at A is allocated to meet the patient of patient living in A requiring <5 years of experience) whereas others indicate certain types of inefficiencies (nursing capacity living in B is allocated to meet the demand in A, etc.). Inefficiencies that are presented in this illustration are due to extra transportation requirement and/or allocating experienced nursing capacity for a task that can be accomplished by a lower profile.

Any type of tactical-level capacity planning decisions and possible inefficiencies can be modeled and investigated using our formal model and the associated grouping scheme. The exemplary capacity plan covers 260 hours of <5 years, District A demand by allocating 100 hours of capacity, inefficiently. Another inefficient allocation depicted in the illustration is allocating 20 hours of nursing capacity living in B having ≥5 years of experience to meet the demand of patients in the same area, but not requiring such a level of experience.

Capacity transfer objects denote capacity planning decisions and are represented by capacity flows among capacity groupings. The associations among an institution's capacity groupings, capacity transfers and contracted capacities form up the grouping scheme of that institution that provides the framework of tactical-level capacity planning concerns. Each capacity grouping has a list of ingoing and outgoing capacity transfers (denoted by $\boldsymbol{CT^+(g)}$ and $\boldsymbol{CT^-(g)}$, respectively). Each capacity transfer object is associated with two capacity groupings that correspond to capacity transfer's source and destination objects. Within the grouping scheme, our formal model allows to keep tracks of both efficient and inefficient capacity transfers that occur to fulfill each patient group's demand. Since efficiency of capacity allocations is the main measure of planning performance, such a mechanism enables us to investigate system's evolution in greater detail. Amount of capacity transfer between 1st and 2nd level capacity grouping nodes ($\boldsymbol{u_{icgg't}}$) indicates the target patient group (i.e. the 2nd level capacity grouping) and the origin of capacity (i.e. the resource group) in order to track the efficiency (or equivalently, the utility) of the corresponding capacity transfer and resulting allocation. This efficiency is a measure of utility and may be interpreted differently from different system actors' perspectives.

Care plans provided by institutions may have various capacity requirements ($\boldsymbol{cr}$) belonging to different resource groups along the care plan's duration. Amount of capacity requirement ($\boldsymbol{CR_{picgt}}$) is usually set by management or standards. The degree of satisfying a capacity requirement is a performance indicator for an institution's planner. All capacity requirements of a care plan ($\boldsymbol{CR(i,c,t)}$) should be fulfilled. Due to temporal issues, a planner may decide to satisfy a capacity requirement by inefficient means. Moreover, the amount of capacity allocated for a service can differ from its capacity requirement. If capacity is scarce, the planner may choose to assign less capacity than the required amount or even not to make a capacity allocation at all. On the other hand, if capacity is abundant, excess capacity can be allocated for services (in case such an allocation produce benefits in terms of service levels or profitability). In order to handle such issues, capacity requirements and amounts of capacity allocations ($\boldsymbol{x_{picgt}}$) are handled separately in our formal model.

Table 3.5. Formalization of institutions' capacity allocations

cr	Index for capacity requirement objects
CR_{picgt}	Standard amount of capacity requirement from g for i's c for p at t
$CR(i,c,t)$	Set of all cr for i's c at t
x_{picgt}	Capacity allocated by i's c from g for p at t

Grouping scheme introduced in this study allows one to model various tactical-level planning concerns within the system, such as total travel distance and workload balancing of workforce as well as compatibility of patient-to-nurse assignments. All these concerns may be represented by appropriate utility functions from different perspectives.

Illustrated example of Figure 3.2 involves only a single care plan alternative for the institution under consideration (i.e. home care). Our formal model uses additional sets of nodes and associations for every care plan alternative provided by institutions. The structure of nodes and associations for each care plan alternative are unique based on its dynamics and requirements. Figure 3.3 illustrates the case of multiple care plan alternatives.

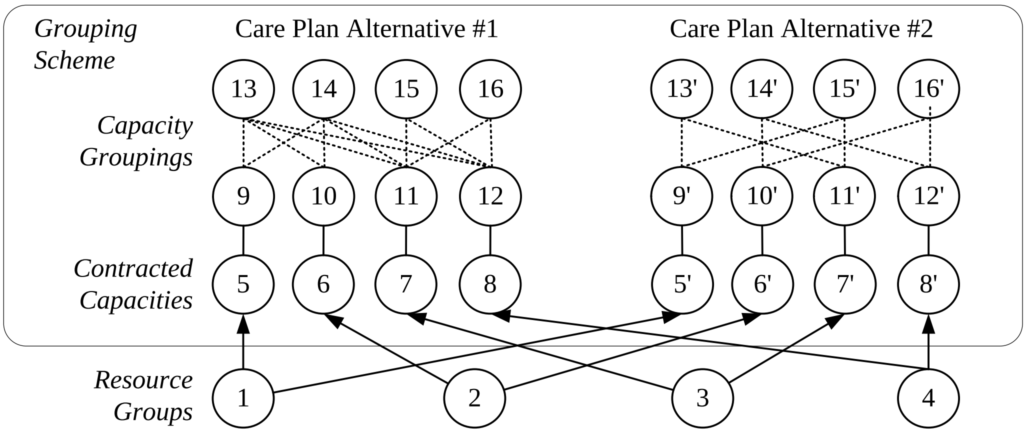

Figure 3.3. Grouping scheme for multiple care plan alternatives case

3.2. Generic System Constraints

Based on the formal model described in Section 3.1, we can define and formulate a number of generic constraints that are relevant to tactical level decision making processes in a healthcare environment including home care. Additional definitions of our formal model that are used in the upcoming sections are presented in Tables 3.6 through 3.11.

Table 3.6. Additional definitions related to formalization of system components

$ca(cr)$	*ca* intended to fulfill *cr*
$CA(i, g, t)$	Set of all *ca*'s that are allocated via *g* at *t*

Every expression within our formal model and generic constraints may turn out to be a decision variable or a given system parameter under different perspectives at various levels of planning. By using relevant subsets of the generic constraints, introducing specific objectives in various actors' perspectives and making assumptions (if necessary), we can formulate various optimization problems of a home care environment within a healthcare system.

Satisfying patients' healthcare demand is a main priority and therefore, all patients in a patient group select a care plan.

$$\sum_{\forall i \in I, \forall c \in C(p,i,t)} y_{pict} = 1, \qquad \forall t \leq T, \forall p \in P(t) \tag{3.1}$$

$$y_{pict} \geq 0, \qquad \forall t \leq T, \forall p \in P(t), \forall i \in I, \forall c \in C(p,i,t) \tag{3.2}$$

Every capacity requirement of a selected care plan should be fulfilled by capacity allocations. Whenever a portion of a patient group selects a care plan alternative at a given time period, all capacity requirements of that plan should be covered by the capacity allocations. Note also that we assume capacity allocations to cover for the whole capacity requirements. Defining capacity requirements and allocations separately, our formal model allows variations on this generic constraint.

$$CR_{picgt}\, N_{pt}\, y_{pict} \leq x_{picgt},$$
$$\forall t \leq T, \forall p \in P(t), \forall i \in I, \forall c \in C(p,i,t), \forall cr \in CR(i,c,t) \tag{3.3}$$

$$x_{picgt} \geq 0,$$
$$\forall t \leq T, \forall p \in P(t), \forall i \in I, \forall c \in C(p,i,t), \forall cr \in CR(i,c,t), \forall ca(cr) \tag{3.4}$$

Capacity allocations are done via capacities transferred within institutions' grouping schemes. Capacity transfers within a grouping scheme conveys capacity from an aggregation of contracted capacities (i.e. a 1st level grouping) to the appropriate aggregation, from which allocations are made (i.e. a 2nd level capacity grouping). Note that selection of a care plan alternative results in capacity allocations for all resources involved in its delivery.

$$\sum_{\forall ca \in CA(i,g,t)} x_{picgt} \leq \sum_{\forall ct \in CT^{+}(g)} u_{icg'gt}, \qquad \forall t \leq T, \forall i \in I, \forall g \in G^{2}(i,t) \tag{3.5}$$

$$u_{icgg't} \geq 0, \qquad \forall t \leq T, \forall i \in I, \forall g' \in G^{2}(i,t), \forall ct \in CT^{+}(g') \tag{3.6}$$

Total of capacity flowing out of a 1st level capacity grouping node indicate utilized portions of the associated aggregation of contracted resource capacities.

$$\sum_{\forall ct \in CT^{-}(g)} u_{icgg't} \leq v_{icrgt}, \qquad \forall t \leq T, \forall i \in I, \forall c \in C(i,t), \forall cc \in CC(i,c,t) \quad (3.7)$$

$$v_{icrgt} \geq 0, \qquad \forall t \leq T, \forall i \in I, \forall c \in C(i,t), \forall cc \in CC(i,c,t) \quad (3.8)$$

The capacity transferred into the grouping scheme cannot exceed the total contracted amount.

$$v_{icrgt} \leq w_{icrt}, \qquad \forall t \leq T, \forall i \in I, \forall c \in C(i,t), \forall cc \in CC(i,c,t) \quad (3.9)$$

$$w_{icrt} \geq 0, \qquad \forall t \leq T, \forall i \in I, \forall c \in C(i,t), \forall cc \in CC(i,c,t) \quad (3.10)$$

Amount of resource increase and decrease are subject to lead times and form a balance between consecutive time periods (for applicable periods).

$$w_{icrt} = w_{icr(t-1)} + h_{icr(t-L_r^+)} - f_{icr(t-L_r^-)},$$
$$\forall t \leq T, \forall i \in I, \forall c \in C(i,t), \forall cc \in CC(i,c,t) \quad (3.11)$$

$$h_{icrt} \geq 0, \qquad \forall t \leq T, \forall i \in I, \forall c \in C(i,t), \forall cc \in CC(i,c,t) \quad (3.12)$$

$$f_{icrt} \geq 0, \qquad \forall t \leq T, \forall i \in I, \forall c \in C(i,t), \forall cc \in CC(i,c,t) \quad (3.13)$$

Total of contracted capacities of a resource cannot exceed its total available capacity.

$$\sum_{\forall i \in I, \forall c \in C(i,t)} w_{icrt} \leq RC_{rt}, \qquad \forall t \leq T, \forall r \in R \quad (3.14)$$

3.3. Actors' Perspectives and Objectives

Each of the five groups of actors (patients, resources, institutions, funding bodies and regulatory bodies) has its own perspective of the healthcare system including home care.

Patients demand the best possible services within their reach, wealth and available financial coverage. Institutions aim for sustainable growth and profits. Resources, whether they are employees themselves or equipment and consumables from a supplier, try to achieve the highest levels of revenues as well as good working conditions. Insurers or sponsors try to achieve lowest possible service expenses without jeopardizing service standards and patients' satisfaction. Regulators try to maximize the overall utility and efficiency of the system to maintain sustainability.

Every healthcare system includes financing mechanisms. Service deliveries have an associated price that corresponds to revenue for the institution and earnings for the resource whose capacity is utilized. The price, in this respect is a cost for the financing mechanism whether be it the patient himself, his family, the insurer, etc. So, all types of costs are important concerns with the system. However, we observe that the patient's decision making is not solely based on costs.

Various types of attributes related to delivery of services effect patients' decision making in a healthcare system. Availability, reliability, conformance and convenience of the service, and resources delivering the service are influential factors. All these factors and associated costs collectively define the choices of patients' in a healthcare system.

For this reason, we consider healthcare to be more of a utility based system. Instead of cost minimization, our analysis framework focuses on utility maximization. However involved costs are included in actor's utility functions through negative impacts on utility levels for patients and positive impacts on resources' utilities.

Regulatory bodies do strategic planning and shape the long term behavior of the system through legislations and policies. Their strategic decisions such as policy changes affect the operation of the system as well as all planning efforts. Tactical and operational plans use regulators' strategic decisions as inputs.

As a result, actors and components related to financing and regulating the delivery of services are kept apart from the focus of this study. Though, our formal model and analysis framework is designed so as to allow investigating the effects of financial or regulatory factors.

Table 3.7. Formalization of actors' utility gains via capacity transfers

$U^{p}_{icgg't}$	*p*'s utility gain upon selecting *i*'s *c* and being transferred unit capacity from *g* to *g'* at *t*
$U^{i}_{icgg't}$	*i*'s utility gain upon transferring unit capacity from *g* to *g'* within *c* at *t*
$U^{r}_{icgg't}$	*r*'s utility gain upon transferring its unit capacity from *g* to *g'* in *i*'s *c* at *t*
$U^{r(idle)}_{icrt}$	*r*'s utility gain for its unit capacity remaining idle in *i*'s *c* at *t*
$U^{i(idle)}_{icrt}$	*i*'s utility gain for its unit contracted capacity of *r* remaining idle within *c* at *t*
$G^{k}(p,t)$	Set of all k^{th} level *g*'s that are of the same type with *p* at *t*
$G^{k}(r,t)$	Set of all k^{th} level *g*'s that are of the same type with *r* at *t*
CF_{ict}	Correction factor of *i*'s *c* based on its standard capacity requirements at *t*

Our formal model uses utility functions to represent the perspectives of actor groups. Main objectives are maximizing utilities which might correspond to satisfaction maximization, cost minimization, or a combination of both. Each capacity allocation made as a result of capacity planning efforts results in a utility gain in the system for each actor. Whether be the utility function linear or nonlinear, each capacity allocation corresponds to a certain value or a level of utility gain. Our formal model uses the grouping scheme as a tool to model the utility gains of actors. Each capacity flow within an institution's grouping scheme indicates a contracted resource's capacity being allocated to satisfy a patient group's demand for a care plan alternative. Table 3.7 presents actors' utility gains per unit flows in our formal model. Note that each 1st level grouping node $\boldsymbol{g}$, is associated with a contracted resource group $\boldsymbol{r}$, and each 2nd level grouping node $\boldsymbol{g'}$, is associated with a patient group $\boldsymbol{p}$.

Utility gains of a *patients' perspective* may be influenced by several factors related to institutions' capacity allocations. Different combinations of a patient's income level, his specific expectations, institution's type, allocated resource's type (or qualification), type of care plan (home-based, hospital, etc.), coverage provided by funding bodies, as well as ease or necessity of transportation (in case of hospitalization) may result in different gains for that patient, in this respect. In such an environment, patients try to choose an optimal mix from available care plan alternatives and maximize their utilities. Patients' perspective can be formulated as the total utility gained by selecting care plans, and the resulting total utility gained by capacity transfers within grouping schemes. Each capacity transferred to a patient has a unit utility value ($\boldsymbol{U^{p}_{icgg't}}$). If an institution chooses to allocate capacities from different sources for a care plan's delivery, the resulting utility level will be a weighted average of utility gains from those sources, where weights are determined based on the amount of flows from different contracted capacity nodes.

$$max\ \ Z(P,T) = \sum\nolimits_{\forall t \le T, \forall p \in P(t), \forall g \in G^2(p,t), \forall ct \in CT^+(g)} U^{p}_{icg'gt}\, CF_{ict}\, u_{icg'gt} \qquad (3.15)$$

Note that a correction factor ($\boldsymbol{CF_{ict}}$) is employed to avoid unjust utility gains from care plans that require more capacity flows than others due to their standard capacity requirements.

Institutions' perspective focuses on contracting and allocating resources' capacities efficiently to deliver services, so as to fulfill patient demand. The institutions' utilities are mostly determined by their costs. These costs incur due to resource contracting in general and other extra costs such as resources' transportation needs for home care delivery. Efficient and inefficient capacity transfers within an institution's grouping scheme are measures of its total utility. For this reason, institutions' perspective can also be formulated as the total utility gained by determining the capacity flows within their grouping scheme in allocating capacities for selected care plans.

$$max\ \ Z(I,T) = \sum\nolimits_{\forall t \le T, \forall i \in I, \forall g \in G^1(i,t), \forall ct \in CT^-(g)} U^{i}_{icgg't}\, CF_{ict}\, u_{icgg't} \qquad (3.16)$$

Resources' perspective mainly corresponds to the perspective of the workforce. The healthcare workforce contracts portions of its capacity to institution groups which are then assigned to meet the capacity requirements of patients' care plans through grouping schemes. Resources' utility gains are based on this contracting procedure and allocations. Type of the institution group the resource works for, as well as the degree of match between the resource qualification level and the patient expectations are influential factors in terms of resources' total utility gains, which can be formulated as the following expression.

$$max \;\; Z(R,T) = \sum_{\forall t \le T, \forall r \in R, \forall g \in G^1(r,t), \forall ct \in CT^-(g)} U^r_{icgg't} \, CF_{ict} \, u_{icgg't} \qquad (3.17)$$

Note that contracted capacities that remain idle are assumed not to affect the utility of resources. This can also be included by defining particular utility gains for unit idle capacities, multiplying them with total idle capacity and adding the product to the above expression. Institution utilities involving utility gains with idle contracted capacities can be expressed similarly.

$$max \; Z(R,T) = \sum_{\forall t \le T, \forall r \in R, \forall g \in G^1(r,t), \forall ct \in CT^-(g)} U^r_{icgg't} \, CF_{ict} \, u_{icgg't}$$
$$+ \sum_{\forall t \le T, \forall r \in R, \forall g \in G^1(r,t), \forall ct \in CT^-(g)} U^{r(idle)}_{icrt} \, CF_{ict} \left(w_{icrt} - \sum_{\forall ct \in CT^-(g)} u_{icgg't} \right) \quad (3.18)$$

These expressions denoting actors' perspectives can be used as objective functions in various optimization models. Therefore compositions and levels of actors' utility functions has profound effects over the system. Even though our formal model is designed to model the specific dynamics of home care, it holds for all other care services as classified by Hulshof et al. (2012), as well.

3.4. System-Level Functions

Using our proposed formal model, one can define functions over system components to model various system-level relations and mechanisms. In this section, we define two such functions that involve competition among institutions for contracting resources and institutions' efficiency gains due to economies of scale. Table 3.8 presents the formalization of these system-level functions.

Table 3.8. Formalization of system-level functions

$f^{r}_{comp(r)}$	*r*'s utility gain coefficient from *r*'s competition function
$f^{i}_{comp(r)}$	*i*'s utility gain coefficient from *r*'s competition function
$f^{p}_{comp(r)}$	*p*'s utility gain coefficient from *r*'s competition function
$f_{eff(i)}$	*i*'s efficiency gain function

In an environment where institutions compete to acquire resource capacities, unit costs of resource capacities may not remain fixed and tend to increase. This increase will affect utility gains of system actors. We introduce competition effect functions for resources ($\boldsymbol{f_{comp(r)}}$) that describe the relation of increasing unit capacity costs with scarce system resources. Given the overall contracted percentage of a resource group's total available capacity, these functions return utility gain coefficients. Increased percentage values, decrease patient and institution utilities due to increased resource costs, but favor resource utilities due to increased earnings. The general structures of competition functions from actors' perspectives are usually nonlinear and are illustrated in Figure 3.4.

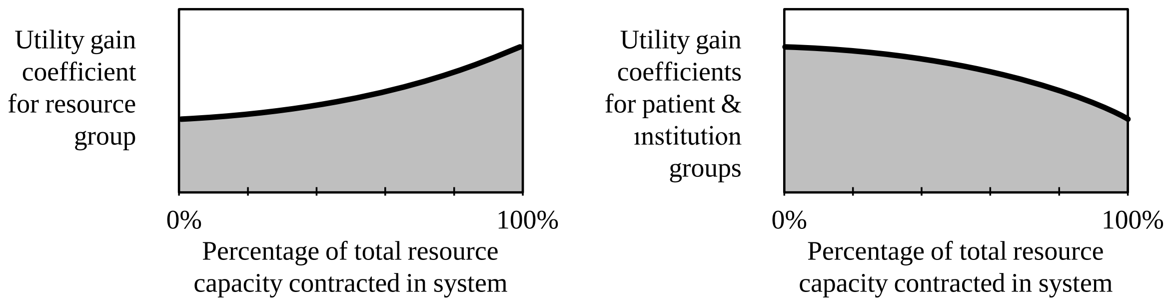

Figure 3.4. Illustration of actors' utility coefficients based on competition levels

Resources' perspectives can then be expressed as follows with this additional system-level function. Other actor perspectives can be expressed similarly.

$$max\ \ Z(R,T) = \textstyle\sum_{\forall t\leq T, \forall r\in R, \forall g\in G^1(r,t), \forall ct\in CT^-(g)} f^{r}_{comp(r)}\ U^{r}_{icgg't}\ CF_{ict}\ u_{icgg't} \quad (3.19)$$

The second system-level function we define involve institutions' efficiency gains due to economies of scale. Both in hospitalization and home care, increasing demand towards care plans of an institution group decreases capacity requirements of that group and thereby increase its efficiency. For instance, as the number of contracted patients by a home care organization increases, resources will require less and less transportation times due to shorter distances between locations. Likewise, number of resources required to look after patients in a hospital ward is not directly proportional with the number of patients. These relations are usually nonlinear.

To incorporate such a mechanism, we introduce an efficiency gain function ($\boldsymbol{f_{eff(i)}}$) having a structure as illustrated in Figure 3.5. Based on the percentage of patients selecting an institution group's care plans, this function returns a corresponding capacity requirement coefficient value. Capacity fulfillment constraints can then be modified as follows so as to incorporate this system-level function.

$$f_{eff(i)}\, CR_{picgt}\, N_{pt}\, y_{pict} \leq x_{picgt} \tag{3.20}$$

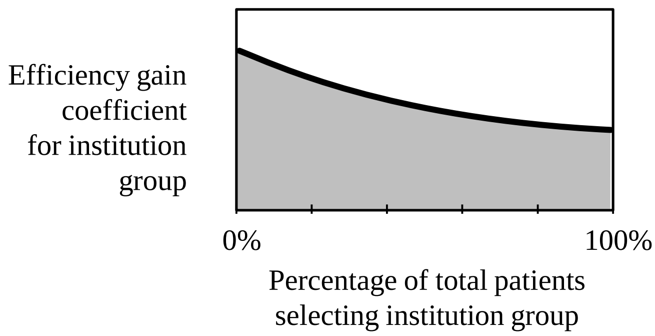

Figure 3.5. Illustration of efficiency gain functions

3.5. Linearization of System-Level Functions

The functions introduced in Equations (3.19) and (3.20) include nonlinearity. In order to avoid the difficulties of nonlinear optimization, one can make certain assumptions and apply linearization techniques to end up with Mixed Integer Programming (MIP) formulations. In this section, we introduce such linearization procedures adopted from the literature to such nonlinear functions.

3.5.1. Linearization of Competition-Based Utility Functions

Table 3.9 presents additional indices, variables and parameters of our formal model that are used in the linearization of utility functions based on different levels of competition over resources.

Incorporating institutions' competition over contracting resources requires formulating the percentage of a resource group's total capacity contracted in the system at a time period. The following expression represents this percentage.

$$\frac{\sum_{\forall cc \in CC(r,t)} w_{icrt}}{RC_{rt}} \tag{3.21}$$

In this study, we assume the nonlinear utility coefficient functions of patients, institutions and resources to have a step-function structure as illustrated in Figure 3.6. This assumption allows these functions to return discrete utility gain coefficient values for different resource contracting percentages in the system.

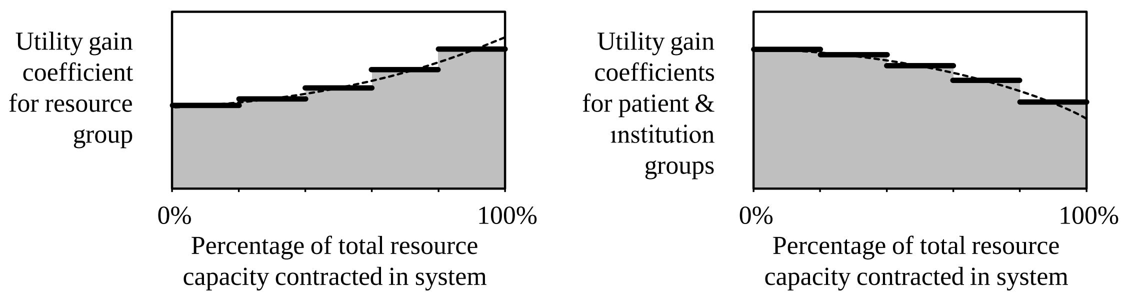

Figure 3.6. Illustrations of actors’ linearized utility coefficients

Table 3.9. Additional definitions for linearization of utility coefficient functions

$G^k(p,t)$	Set of all *k*th level *g*'s from which *p*'s capacity requirement can be allocated at *t* where k=1,2
$G^k(r,t)$	Set of all *k*th level *g*'s to which *r* can allocate capacity at *t* where k=1,2
$CC(r,t)$	Set of all *cc*'s of *r* at *t*
l	Index for percentage intervals of $\sum_{\forall cc\in CC(r,t)} w_{icrt}/RC_{rt}$
L	Set of all *l* indices
s_{rtl}	Nonnegative number that equals to the percentage value $\sum_{\forall cc\in CC(r,t)} w_{icrt}/RC_{rt}$when it is inside *l* at *t*
z_{rtl}	Equal to 1 if s_{rtl} is positive, 0 otherwise
LB_l	Lower bound of *l*
UB_l	Upper bound of *l*
$F^{p,l}_{comp(r),t}$	Utility gain coefficient of linearized $f^p_{comp(r)}$ corresponding to *l* at *t*
$F^{i,l}_{comp(r),t}$	Utility gain coefficient of linearized $f^i_{comp(r)}$ corresponding to *l* at *t*
$F^{r,l}_{comp(r),t}$	Utility gain coefficient of linearized $f^r_{comp(r)}$ corresponding to *l* at *t*
$d_{icgg't}$	Free variable indicating the total utility of capacity flow from *g* to *g'* in *i*'s *c* at *t* for patients
$e_{icgg't}$	Free variable indicating the total utility of capacity flow from *g* to *g'* in *i*'s *c* at *t* for institutions
$f_{icgg't}$	Free variable indicating the total utility of capacity flow from *g* to *g'* in *i*'s *c* at *t* for resources
M^i_{icgt}	A sufficiently large number $\left(= \max_{\forall l}\left(F^{i,l}_{comp(r),t}\right) U^r_{icgg't}\, CF_{ict}\, RC_{rt}\right)$

The objective functions of actor groups incorporating competition over resources can then be linearized. This procedure is adopted from Bradley et al. (1977) and requires introduction of additional model components.

The original objective function from institutions' perspective depicted in Equation (3.16) can be linearized by replacing the objective with Equation (3.22) and introducing Equations (3.23) - (3.30).

$$max\ \ Z(I,T) = \sum_{\forall t\le T,\forall i\in I,\forall g\in G^1(i,t),\forall ct\in CT^-(g)} e_{icgg't} \tag{3.22}$$

$$\frac{\sum_{\forall cc\in CC(r,t)} w_{icrt}}{RC_{rt}} = \sum_{\forall l\in L} s_{rtl}\ , \qquad \forall t \le T, \forall r \in R \tag{3.23}$$

$$\sum_{\forall l \in L} z_{rtl} = 1, \qquad \forall t \leq T, \forall r \in R \tag{3.24}$$

$$LB_l \, z_{rtl} \leq s_{rtl}, \qquad \forall t \leq T, \forall r \in R, \forall l \in L \tag{3.25}$$

$$s_{rtl} \leq UB_l \, z_{rtl}, \qquad \forall t \leq T, \forall r \in R, \forall l \in L \tag{3.26}$$

$$s_{rtl} \geq 0, \qquad \forall t \leq T, \forall r \in R, \forall l \in L \tag{3.27}$$

$$z_{rtl} \in \{0, 1\}, \qquad \forall t \leq T, \forall r \in R, \forall l \in L \tag{3.28}$$

$$F^{i,l}_{comp(r),t} \, U^{i}_{icgg't} \, CF_{ict} \, u_{icgg't} + \, M^{i}_{icgt} \, (1 - z_{rtl}) \geq e_{icgg't}$$
$$\forall t \leq T, \forall i \in I, \forall g \in G^1(i,t), \forall ct \in CT^-(g), \forall l \in L \tag{3.29}$$

$$e_{icgg't} \geq 0, \qquad \forall t \leq T, \forall i \in I, \forall g \in G^1(i,t), \forall ct \in CT^-(g) \tag{3.30}$$

Note that, patients' and resources' perspectives can also be linearized similarly based on their own utility coefficient functions and introduction of related variables.

As a result, patients' perspective will involve the objective function presented in Equation (3.31) subject to Equations (3.23) – (3.28) together with Equations (3.32) and (3.33).

$$max \;\; Z(P,T) = \sum_{\forall t \leq T, \forall p \in P(t), \forall g \in G^2(p,t), \forall ct \in CT^+(g)} d_{icgg't} \tag{3.31}$$

$$F^{p,l}_{comp(r),t} \, U^{p}_{icgg't} \, CF_{ict} \, u_{icgg't} + \, M^{p}_{icgt} \;\, (1 - z_{rtl}) \geq d_{icgg't}$$
$$\forall t \leq T, \forall p \in P(t), \forall g \in G^2(p,t), \forall ct \in CT^+(g), \forall l \in L \tag{3.32}$$

$$d_{icgg't} \geq 0, \qquad \forall t \leq T, \forall p \in P(t), \forall g \in G^2(p,t), \forall ct \in CT^-(g) \tag{3.33}$$

Resources' perspective will involve the objective function presented in Equation (3.34) subject to Equations (3.23) – (3.28) together with Equations (3.35) and (3.36).

$$max\ \ Z(R,T) = \textstyle\sum_{\forall t \le T, \forall r \in R, \forall g \in G^1(r,t), \forall ct \in CT^-(g)} f_{icgg't} \tag{3.34}$$

$$F^{r,l}_{comp(r),t}\, U^r_{icgg't}\, CF_{ict}\, u_{icgg't} + \ M^r_{icgt}\ \ (1 - z_{rtl}) \ge f_{icgg't},$$
$$\forall t \le T, \forall r \in R, \forall g \in G^1(r,t), \forall ct \in CT^-(g), \forall l \in L \tag{3.35}$$

$$f_{icgg't} \ge 0, \qquad \forall t \le T, \forall r \in R, \forall g \in G^1(r,t), \forall ct \in CT^-(g) \tag{3.36}$$

3.5.2. Linearization of Efficiency Gains Functions

Efficiency gains due to economies of scale by an institution group (***i***) require calculating the percentage of total patients selecting its care plans. This percentage can be formulated with the following expression.

$$\frac{\sum_{\forall p \in P(t), \forall c \in C(p,i,t)} NP_{pt}\, y_{pict}}{\sum_{\forall p \in P(t)} NP_{pt}} \tag{3.37}$$

Nonlinear capacity requirement coefficient functions are also assumed to have a step-function structure as illustrated in Figure 3.7. These step functions return discrete coefficient values for different timely selection percentages.

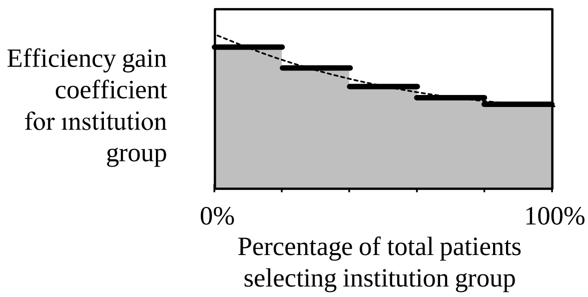

Figure 3.7. Illustration of linearized efficiency gain functions

Linearization of the capacity fulfillment constraints is done similar to the previous procedure. Additional indices, variables and parameters that allow this procedure are presented in Table 3.10. The original capacity fulfillment constraint depicted in Equation (3.20) can be linearized by replacing it with Equations (3.38) - (3.44).

Table 3.10. Additional definitions for linearization of efficiency gains

j	Index for percentage intervals of $\sum_{\forall p\in P(t),\forall c\in C(p,i,t)} NP_{pt}\, y_{pict} / \sum_{\forall p\in P(t)} NP_{pt}$
J	Set of all *j* indices
n_{itj}	Nonnegative number that equals to the percentage value $\sum_{\forall p\in P(t),\forall c\in C(p,i,t)} NP_{pt}\, y_{pict} / \sum_{\forall p\in P(t)} NP_{pt}$ when it is inside *j* at *t*
m_{itj}	Equal to 1 if n_{itj} is positive, 0 otherwise
LB_j	Lower bound of *j*
UB_j	Upper bound of *j*
$F^{j}_{eff(i),t}$	Function value of piece-wise linear $f_{eff(i)}$ corresponding to *j* for *i* at *t*
M_{picgt}	A sufficiently large number ($= \max_{\forall j} F^{j}_{eff(i),t}\; CR_{picgt}\, N_{pt}$)

$$\frac{\sum_{\forall p\in P(t),\forall c\in C(p,i,t)} NP_{pt}\, y_{pict}}{\sum_{\forall p\in P(t)} NP_{pt}} = \sum_{\forall j\in J} n_{itj}, \qquad \forall t \le T, \forall i \in I \tag{3.38}$$

$$\sum_{\forall j\in J} m_{itj} = 1, \qquad \forall t \le T, \forall i \in I \tag{3.39}$$

$$LB_j\, m_{itj} \le n_{itj}, \qquad \forall t \le T, \forall i \in I, \forall j \in J \tag{3.40}$$

$$n_{itj} \le UB_j\, m_{itj}, \qquad \forall t \le T, \forall i \in I, \forall j \in J \tag{3.41}$$

$$F^{j}_{eff(i),t}\, CR_{picgt}\, N_{pt}\, y_{pict} \le \sum_{\forall CA(cr)} x_{picgt} + M_{picgt}\left(1 - m_{itj}\right),$$
$$\forall t \le T, \forall p \in P(t), \forall i \in I, \forall c \in C(p,i,t), \forall cr \in CR(i,c,t), \forall j \in J \tag{3.42}$$

$$n_{itj} \ge 0, \qquad \forall t \le T, \forall i \in I, \forall j \in J \tag{3.43}$$

$$m_{itj} \in \{0, 1\}, \qquad \forall t \le T, \forall i \in I, \forall j \in J \tag{3.44}$$

Note that the constraint sets (3.29) and (3.42) together cause reductions in total utility gains of actors by gains in efficiency. In order to avoid it, one should divide the utility gain expression with the attained efficiency gain level to correct the total utility gained.

As a result, constraint sets (3.29) are replaced by those in (3.45). Additional parameters that allow this procedure are presented in Table 3.11. Constraint sets (3.32) and (3.35) should also be modified accordingly, resulting in Equations (3.46) and (3.47), respectively.

Table 3.11. Additional definitions for linearization of system level functions

$M_{icgt}^{p(r)}$	A sufficiently large number $\left(= \max\limits_{\forall j,\forall l}\left(\frac{F_{comp(r),t}^{p,l}}{F_{eff(i),t}^{j}}\right) U_{icgg't}^{p}\, CF_{ict}\, RC_{rt}\right)$
$M_{icgt}^{i(r)}$	A sufficiently large number $\left(= \max\limits_{\forall j,\forall l}\left(\frac{F_{comp(r),t}^{i,l}}{F_{eff(i),t}^{j}}\right) U_{icgg't}^{i}\, CF_{ict}\, RC_{rt}\right)$
$M_{icgt}^{r(r)}$	A sufficiently large number $\left(= \max\limits_{\forall j,\forall l}\left(\frac{F_{comp(r),t}^{r,l}}{F_{eff(i),t}^{j}}\right) U_{icgg't}^{r}\, CF_{ict}\, RC_{rt}\right)$

$$\left(\frac{F_{comp(r),t}^{i,l}}{F_{eff(i),t}^{j}}\right) U_{icgg't}^{i}\, CF_{ict}\, u_{icgg't} + M_{icgt}^{i(r)}(1 - z_{rtl}) + M_{icgt}^{i(r)}(1 - m_{itj}) \geq e_{icgg't}$$

$$\forall t \leq T, \forall i \in I, \forall g \in G^{1}(i,t), \forall ct \in CT^{-}(g), \forall l \in L, \forall j \in J \qquad (3.45)$$

$$\left(\frac{F_{comp(r),t}^{p,l}}{F_{eff(i),t}^{j}}\right) U_{icgg't}^{p}\, CF_{ict}\, u_{icgg't} + M_{icgt}^{p(r)}(1 - z_{rtl}) + M_{icgt}^{p(r)}(1 - m_{itj}) \geq d_{icgg't}$$

$$\forall t \leq T, \forall i \in I, \forall g \in G^{1}(i,t), \forall ct \in CT^{-}(g), \forall l \in L, \forall j \in J \qquad (3.46)$$

$$\left(\frac{F_{comp(r),t}^{r,l}}{F_{eff(i),t}^{j}}\right) U_{icgg't}^{r}\, CF_{ict}\, u_{icgg't} + M_{icgt}^{r(r)}(1 - z_{rtl}) + M_{icgt}^{r(r)}(1 - m_{itj}) \geq f_{icgg't}$$

$$\forall t \leq T, \forall i \in I, \forall g \in G^{1}(i,t), \forall ct \in CT^{-}(g), \forall l \in L, \forall j \in J \qquad (3.47)$$

4. PROPOSED ALGORITHMS OVER THE ANALYSIS FRAMEWORK

We are interested in how the system and its actors respond to changes in demand, supply, system-level functions, and actors' utilities. In order to study the evolution of the system in presence of such changes, we develop and implement two different approaches.

- agent-based modeling and simulation
- mathematical programming based optimization

Based on our analysis framework, we propose an **agent-based modeling and simulation** approach. This agent-based model is a modified version of our formal model. As opposed to group-based decision making of our formal model, the agent-based model involves actors making their decisions individually. Using this modified model, we develop a simulation algorithm, where each actor applies simple rules to come up with better decisions in its own perspective. Actors making decisions one at a time, lead to the evolution of the overall system through time in presence of system constraints and functions. The results of this simulation study provide us reference cases, where agents make local decisions based on their myopic views.

We also propose a **mathematical programming based optimization algorithm**, which is designed to consider all the aspects introduced in our formal model. This algorithm enables actors to act in light of others' perspectives, generic constraints and system-level functions. It represents an efficient way of decision making where each actor group and acquire the maximum utility within the system based on their global views.

With the help of these algorithms representing two different approaches (myopic and global), one can model the effects of a variety of changes, trends or impacts over the actors and the overall system. These algorithms may be used to aid system level policy making, investment and pricing decisions and to simulate actor behaviors.

4.1. Agent-Based Modeling and Simulation

Based on the components and associations described in our proposed formal model and analysis framework, an agent-based simulation algorithm is developed to simulate actors' decision making and system's evolution over time. This algorithm employs simple and rational decision rules for system's individual actors (called agents) and iterates through the simulation periods. The results of this simulation provide us valuable benchmarks on how the system evolves based on actors' simplistic decision making.

Agent-based modeling and simulation (ABMS) is a widely used technique to model the behavior of complex systems as a whole, in presence of autonomous agents having their own attributes, making decisions individually, and interacting with other agents in the system. Epstein (1996) and Railsback and Grimm (2011) introduce the general concepts of ABMS.

Healthcare systems including home care are such complex systems. Therefore, we implemented ABMS methodology to investigate tactical-level decision making behavior of patients, resources and institutions.

We make minor modifications on the object definitions of our formal model to implement ABMS. Every component in our formal model introduced in section 3 represents a grouping. For our agent-based model, we disaggregate all these groupings into individual objects, to enable agent-based decision making. As a result, patient agents select institution agents' care plan alternatives. Resource agents' capacities are fully or partially contracted by institution agents. These capacities are then aggregated and transferred within institution's capacity grouping scheme and allocated to meet the specific capacity requirements of patients' selected care plans. All associations, attributes and functions of the formal model apart from the grouping concept remain the same.

Based on this agent-based model, we develop an agent-based simulation algorithm where tactical level actions of individual actors are taken based on simple decision rules. Figure 4.1 and Figure 4.2 introduce the pseudo-code of our proposed agent-based simulation algorithm.

(1) Create all system agents with associated components, initialize resource capacity contracts and corresponding competition levels for first period (t = 1)

Resources are contracted such that each resource agent of a resource group are assigned one at a time to every institutions' healthcare delivery channels (home care and hospital) so that every resource is equally distributed among institutions and their alternative care plan offerings

(2) For all the patients entering the system at t and chosen at random,

(2.1) Based on institutions' current resource capacities, currently efficiency gains, and the number of patients that have not yet selected a care plan alternative, decide whether there is a need to impose the patient to select a specific type of care plan alternative so that all remaining patients are able to select a care plan

(2.2) Determine utility gains for the patient's care plan alternatives (whether it is imposed, the provider has available capacity or not) and create a list of alternatives in non-increasing order of their corresponding utility gains

(2.3) Select the first alternative in this list which is in conformance with impositions (if any) and whose provider has enough available capacity, and update provider's current capacity transfers based on newly achieved efficiency gains

(2.4) Allocate capacity via most efficient capacity transfers, where provider's utilities are tie breakers and create hiring decisions for future periods to eliminate any inefficiency that occurs during the allocation

(2.5) If the selected care plan alternative were NOT the very first alternative in the list, create hiring decisions for the first care plan alternative's provider such that it may fulfill the patient's unsatisfied demand in the future

(continued in Figure 4.2)

Figure 4.1. Pseudo-code of the agent-based simulation algorithm

The algorithm initially creates all system agents and contracts among institution and resource agents, such that every resource is equally distributed among institutions. Then, for each period in the simulation horizon, entering patient agents select a care plan alternative. The selection is based on the patient's utility gain associated with the care plan as well as resource availabilities to perform the service. In the later stages of this selection process, the algorithm constrains patients to select care plan alternatives such that the total available capacity can serve every entering patient. This means that some patients may not be able to select its most preferred care plan due to system's capacity constraints. Selections change the capacity requirements of institutions based on efficiency gains.

Once all the patient demand is covered within the system. Institutions make capacity hiring decisions for their fully utilized capacities (referred as buffer capacity) to meet unfulfilled patient demand in the future. Firing decisions are created for capacities that remain idle. The algorithm corrects all capacity related decisions to avoid excessive hiring or firing within lead times. Resource contracts result in changes in resource competition levels and alter utility gains of system actors.

(3) For all the capacity groupings of all institutions' and chosen at random,

- *(3.1a) If the capacity grouping is fully utilized, correct hiring decisions (if any) to avoid excessive hiring or firing within the resource contracting lead time such that at least a predetermined buffer capacity will be hired after resource contracting lead time and the system's total available capacity can sustain the patient demand after the resource contracting lead time*
- *(3.1b) If the capacity grouping is NOT fully utilized, create firing decisions to fire the amount equal to the idle capacity and correct this decision based on previous period's decisions to avoid excessive hiring or firing within the resource contracting lead time, and the system's total available capacity can sustain the patient demand after the resource contracting lead time*
- *(3.2a) If the corrected decision at the capacity grouping is a firing decision, fire resource capacities selected at random, until there will be no excess capacity at the grouping after resource contracting lead time*
- *(3.2b) If the corrected decision at the capacity grouping is a hiring decision, hire required capacity from available system resources selected at random that have idle capacities after resource contracting lead time such that those having portions of their capacity already contracted by the institution have priority over others*

(4) Determine competition levels for next period based on existing resource contracts

(5) If $t = T$ stop, else set $t = t + 1$, and go to (2)

Figure 4.2. Pseudo-code of the agent-based simulation algorithm (continued)

The algorithm is designed so as to comply with the modified formal model, generic system constraints, actors' perspectives and system level functions. All capacity constraints within the system are preserved throughout the algorithm. System level functions are updated over the iterations to enable better decision making for the actors.

An efficient allocation means making a perfect match among actors (in terms of utility gains) in allocating capacity. Allocations that do not comply with this definition are considered as inefficient. Institutions try to compensate for a period's unfulfilled demand in future periods via resource hiring. They also acquire buffer capacities for their fully utilized resources to enable fulfillment for unsatisfied demand in later periods. Idle capacities are fired. Actors' decision making is determined by their utilities as well as system constraints as imposed in our analysis framework.

4.2. Mathematical Programming Based Optimization

Based on our formal model and the overall analysis framework, we develop and propose an optimization algorithm. This mathematical programming based algorithm is designed to determine actors' best actions in presence of each other's conflicting objectives and system-level impacts.

Using our analysis framework, we can define complete tactical-level mathematical programming models for every actor group (patients, institutions and resources) based on their perspectives, generic constraints and system-level functions. Each of these models are utility maximization models for the corresponding actor.

Patients' model, $Model(k, P, T)$ has its objective function expressed in Equation (3.31) and is subject to constraints expressed in Equations (3.1), (3.2), (3.4) – (3.14), (3.23) – (3.28), (3.33), (3.38) – (3.44), (3.46).

Institutions' model, $Model(k, I, T)$ has its objective expressed in Equation (3.22) and is subject to constraints expressed in Equations (3.1), (3.2), (3.4) – (3.14), (3.23) – (3.28), (3.30), (3.38) – (3.45).

Resources' model, $Model(k, R, T)$ has its objective expressed in Equation (3.34) and is subject to constraints expressed in Equations (3.1), (3.2), (3.4) – (3.14), (3.23) – (3.28), (3.36), (3.38) – (3.44), (3.47).

The objective functions of all these models are different and mostly conflicting with each other. For this reason, problem solutions for different actors will be divergent from each other and finding a solution which is best for all actors is not possible.

As a remedy to this, we design an iterative optimization algorithm that enables us to find an overall solution represents the collective utilities of actors having different perspectives. This proposed algorithm involves solving these models in a manner that enables different perspectives to come to an agreed-upon solution.

The algorithm solves mathematical programming models corresponding to all three actors' perspectives at each iteration, such that patients' model is solved first, institutions' model second, and resources' last. This sequence is chosen so as to create a demand-driven procedure. Solution of each model is called a "phase" in the algorithm. Figure 4.3 illustrates these three phases solved consecutively over the iterations.

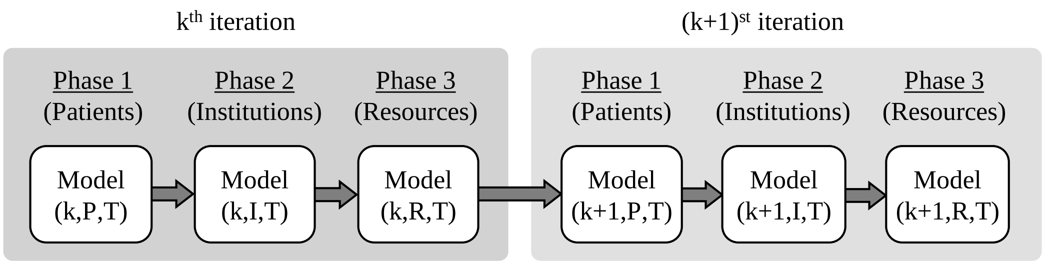

Figure 4.3. Illustration of phases solved consecutively over iterations

Generic system constraints and other system-level functions remain the same for all these models. In addition to these constraint sets, our algorithm uses the solutions of every model to generate constraint sets and append them to upcoming phases as additional constraints in a way which we describe next.

The objective function expression of patients' model at k^{th} iteration is as follows

$$Z_k(P,T) = \sum_{\forall t \leq T, \forall p \in P} Z_k(p,t) \tag{4.1}$$

where

$$Z_k(p,t) = \sum_{\forall g \in G^2(p,t), \forall ct \in CT^+(g), \forall l \in L} d_{icgg't} \tag{4.2}$$

$Z_k(p,t)$ denotes the total utility gained by a patient group *p* at period *t* at kth iteration.

Let $z_k^*(p,t)$ be the total utility value gained which is calculated by Equation (4.3), where $d^*_{icgg't}$ is the optimal (or best found) value for variable $d_{icgg't}$ based on the solution of $Model(k,P,T)$.

$$z_k^*(p,t) = \sum_{\forall g \in G^2(p,t), \forall ct \in CT^+(g), \forall l \in L} d^*_{icgg't} \tag{4.3}$$

Let $C(k, \Delta_k, P, T)$ be the set of constraints defined in Equation (4.4).

$$Z_k(p,t) \geq (1 - \Delta_k)\, z_k^*(p,t)\,, \qquad \forall t \leq T, \forall p \in P \tag{4.4}$$

$(1 - \Delta_k)$ is a multiplier based on the iteration number of the algorithm, where $0 \leq \Delta_k \leq 1$ and $\Delta_k \in \mathbb{R}$.

The left hand side of Equation (4.4) represents each patient groups' utility gains for each period and the right hand side is a percentage of its best value in $Model(k,P,T)$. So, $C(k, \Delta_k, P, T)$ imposes a lower bound on patient groups' utility gains based on the solution of $Model(k,P,T)$, which represents each patient groups' perspective .

Our algorithm adds the constraint set $C(k, \Delta_k, P, T)$ to the forthcoming three models (i.e. $Model(k,I,T)$, $Model(k,R,T)$ and $Model(k+1,P,T)$). We apply the same procedure for institutions' and resources' models and determine the corresponding constraint sets $C(k, \Delta_k, I, T)$ and $C(k, \Delta_k, R, T)$, respectively. Note that these constraint sets may reduce the feasible region of the associated models, but they do not cause infeasibility.

Figure 4.4 illustrates this procedure, where outgoing arrows from 1st, 2nd and 3rd phases of the kth iteration indicate constraint sets being generated by the solutions of the corresponding models. Arrows' destinations indicate the three forthcoming models to which these constraint sets will be added. For the sake of simplicity, the illustration presents the addition of constraint sets generated at kth iteration only.

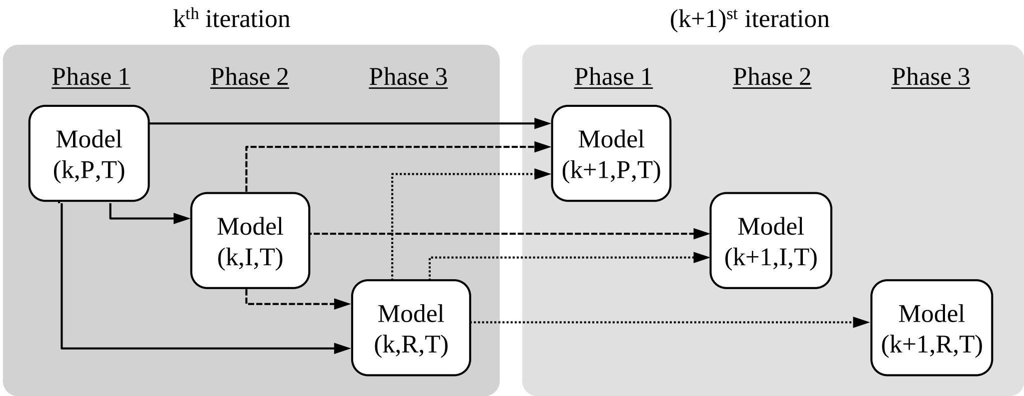

Figure 4.4. Illustration of constraint sets being added to forthcoming models

The optimization algorithm decreases Δ_k in a systematic manner over the iterations so as to introduce tighter lower bounds for actors' utility gains and enable convergence to a certain and what we call an "agreed-upon" utility level for each actor group within the system. In the last iteration of the algorithm (i.e. the Kth iteration), Δ_k is set to zero. Figure 4.5 presents the pseudo-code of the proposed optimization algorithm.

(1) *Initialize* Δ_k *for* $k = 1, 2, \ldots, K$
Initialize capacity contracts w_{icrt} *for* $t = 1, \forall i \in I, \forall c \in C(i,t), \forall cc \in CC(i,c,t)$
Set k = 1

(2) *Solve* $Model(k,P,T)$ *subject to existing generated constraint sets* $C(k-1,\Delta_{k-1},P,T)$, $C(k-1,\Delta_{k-1},I,T)$ *and* $C(k-1,\Delta_{k-1},R,T)$ *and generate constraint set* $C(k,\Delta_k,P,T)$ *based on the solution*

(3) *Solve* $Model(k,I,T)$ *subject to existing generated constraint sets* $C(k-1,\Delta_{k-1},I,T)$, $C(k-1,\Delta_{k-1},R,T)$ *and* $C(k,\Delta_k,P,T)$, *and generate constraint set* $C(k,\Delta_k,I,T)$ *based on the solution*

(4) *Solve* $Model(k,R,T)$ *subject to existing generated constraint sets* $C(k-1,\Delta_{k-1},R,T)$, $C(k,\Delta_k,P,T)$ *and* $C(k,\Delta_k,I,T)$, *and generate constraint set* $C(k,\Delta_k,R,T)$ *based on the solution*

(5) *If* $k = K$, *STOP, else set* $k = k + 1$ *and go to (2)*

Figure 4.5. Pseudo-code of the proposed optimization algorithm

5. NUMERICAL STUDY

Based on our analysis framework, we implement our proposed approaches on a healthcare system setting. Our aim is to investigate the effects of a number of policy changes over the actors' decision making processes and over the system as a whole.

The healthcare setting introduced in this section is theoretical and it is designed based on our interviews with health care professionals. We introduce the details of this setting under six main groups.

- demand and supply settings
- system-level functions settings
- utility functions settings
- policy changes settings
- algorithm settings
- key metrics for output analysis

In the upcoming subsections, we describe the details of these settings. After that, we present the analysis of the obtained outputs.

5.1. Demand and Supply Settings

A constant service demand of 3.000 patients entering the system is to be satisfied over the planning horizon of 18 periods. These patients are classified in three patient groups based on their income levels. The number of high, moderate and low income patients at each group are 250, 750 and 2000, respectively.

There are two types of care plan alternatives for these patients.

- hospital based care
- home based care

These care plan alternatives are performed by nurses. The monthly standard nurse capacity requirements for these care plan alternatives are assumed to be 4 hours and 8 hours, respectively. Duration of every care plan alternative is a one month.

The system is assumed to have just enough resources to cover total healthcare demand when 60% of the population selects hospitals and 40% selects home care. Based on this assumption, there are 105 nurses each having a monthly capacity of 160 hours. Nurses are grouped into three equally sized groups (i.e. 35 nurses each) with respect to their level of experience (high, moderate and low). Standard lead time for hiring or firing a nurse is assumed to be two months.

Institutions are grouped based on their cost levels. There are high, moderate and low cost institutions in the system. Each group has two institutions in operation.

Our optimization algorithm uses actor groups, whereas every single patient, resource and institution correspond to an agent for our agent-based simulation algorithm.

5.2. System-Level Function Settings

The system level function that determines competition over resource groups is assumed to have three distinct levels, namely low, moderate and high. The level of competition over a resource group is based on the percentage of the contracted portion of its total available capacity. The structure of this system-level function is presented in Table 5.1 and is illustrated in Figure 5.1. Utilities of all actors are affected by the current competition level in the system.

Table 5.1. Competition levels for a resource group’s contracted capacity percentages

Percentage of a resource group’s capacity contracted	Competition level for the resource group
below 50%	Low
between 50% and 80%	Moderate
over 80%	High

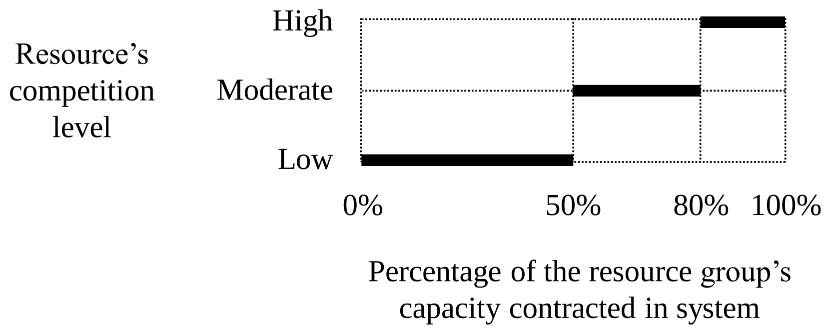

Figure 5.1. Competition levels for a resource group's contracted capacity percentages

System level function that determines institutions' efficiency gains due to economies of scale is also assumed to have three levels. Institutions may have low, moderate or high efficiency gain levels based on the percentage of the patient population they serve.

The structure of this system-level function is presented in Table 5.2 and is illustrated in Figure 5.2. Realized capacity requirements are affected by institutions' current efficiency gain coefficients.

Table 5.2. Efficiency gain levels for an institution group's patient selection percentages

Percentage of total patients selecting the institution group	Efficiency gain level for the institution group	Efficiency gain coefficient for the institution group
Below 25%	Low	1.00
Between 25% and 50%	Moderate	0.88
Over 50%	High	0.82

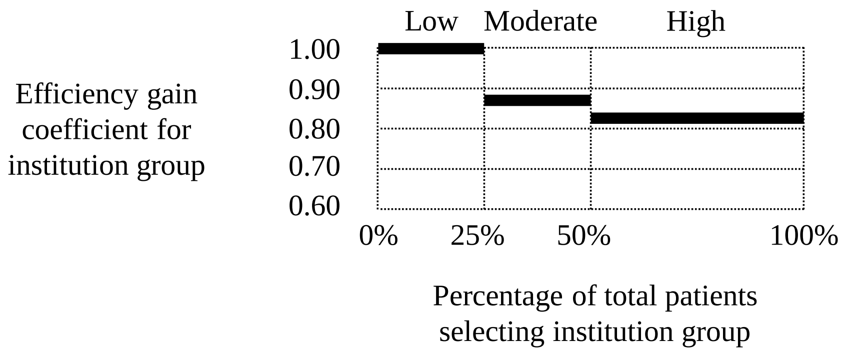

Figure 5.2. Efficiency gain levels for an institution group's patient selection percentages

5.3. Utility Function Settings

Utility gains from different factors and sources are considered to be additive and the total utility gained from capacity transfers within institutions' grouping schemes is assumed to have two types of components.

- adequacy of matching between actor groups
- coverage/incentive level provided by funding bodies

Health care services are delivered to patients by institutions through utilization of resources. Patient gains utility based on the institution and the resource delivering the service. Institution gains utility from the patient and the resource delivering the service. Resource gains utility based on the institution she works for and the patient that she delivers service to. It is assumed that the mismatch among the actor groups degrades actors' utilities.

Degradation in utilities is due to a number of factors such as patient and employee expectations, associated prices and costs of services, as well as level of luxury and other value-adding service features. For instance, patients with high income prefer high cost institutions where they are served by nurses with high levels of experience.

Unit utilities gained by two interacting actor groups (A and B) during service delivery is presented in Table 5.3. Delivery of a healthcare service involve three actor groups interacting with each other. For instance, patient with high income serviced by a highly experienced nurse contracted by high cost institution would gain a utility of 0.30 (0.15 from patient-institution and 0.15 from patient-resource interactions). If only the institution were in a moderate cost group in this service delivery, then the total utility gain for the patient would be 0.25 (0.10 from patient-institution and 0.15 from patient-resource interactions). We assume that adequacy of matching is not affected by resource competition levels.

The second component of an actor's utility gain is due to the level of coverage or incentive it receives from a service delivery.

Funders, insurers or policy-makers, whether they be public or private organizations, provide coverage for patients. They may choose to provide full or partial coverage for healthcare service expenditures. The level of coverage effects patients' utility gains. They may also grant incentives for institutions or resources for taking part in delivery of these services. The incentive might be related to the payment made in exchange of services, tax incentives rewarded, bonuses received, etc. The level of such incentives have effects on institutions' or resources' utility gains.

Table 5.3. Utility gains from actor groups in interaction

		Actor group B		
		High	Moderate	Low
Actor group A	High	0.15	0.10	0.00
	Moderate	0.10	0.15	0.10
	Low	0.00	0.10	0.15

The level of coverage or incentive provided in the system is assumed to be affected by the level of competition over resources that are utilized in services. For instance, as the competition over a resource increase, its cost will increase for the institution, which will be reflected to the patient in terms of increased payments or premiums. Assumed values of competition-based utility functions of system actors for hospital based care and home based care are presented in Table 5.4, Table 5.5 and Table 5.6.

It is assumed that competition over resources results in increased utility levels for resources, but decreased levels for patients and institutions for all care types. Resources are assumed to be indifferent for type of care they provide.

Home care provides higher utility for patients and lower utility for institutions in low and moderate competition levels. On the other hand home care provides lower utility for patients and higher utility for institutions at high competition levels.

Including the competition-based utility gains, a patient with high income, gains a total utility of 0.75 when receiving home based care service from a moderate cost institution by a highly experienced nurse, whose competition level is low and home care is moderately covered by his insurer. Components of the total utility gained in this example are 0.10 from patient-institution interaction, 0.15 from patient-resource interaction and 0.50 from moderately covered home based care service delivered by a resource that has low level of competition in the system. Note that, such a calculation of total utility gain is made possible by our utilities' additivity assumption.

Table 5.4. Patients' competition-based utility gains for hospital and home based care

		Level of competition		
Type of care	Level of coverage	Low	Moderate	High
Hospital based care	High	0.50	0.45	0.35
	Moderate	0.40	0.35	0.25
	Low	0.30	0.25	0.15
Home based care	High	0.60	0.50	0.30
	Moderate	0.50	0.40	0.20
	Low	0.40	0.30	0.10

Table 5.5. Institutions' competition-based utility gains for hospital and home based care

		Level of competition		
Type of care	Level of incentive	Low	Moderate	High
Hospital based care	High	0.60	0.45	0.20
	Moderate	0.55	0.40	0.15
	Low	0.50	0.35	0.10
Home based care	High	0.50	0.45	0.35
	Moderate	0.45	0.40	0.30
	Low	0.40	0.35	0.25

Table 5.6. Resources' competition-based utility gains for hospital and home based care

		Level of competition		
Type of care	Level of incentive	Low	Moderate	High
Hospital based care	High	0.35	0.40	0.50
	Moderate	0.25	0.30	0.40
	Low	0.15	0.20	0.30
Home based care	High	0.35	0.40	0.50
	Moderate	0.25	0.30	0.40
	Low	0.15	0.20	0.30

5.4. Policy Change Settings

There are a number of possible policy changes over a healthcare system that include home care as an alternative means of service. These policy changes can be classified as strategic planning decisions made by funding or regulatory bodies and usually have profound and long-term effects over actors' decision making processes and the overall system. Since such policy changes are influential in tactical planning levels, they are interesting for us in investigating the evolution of the system through time.

There are four types of policy changes considered in our numerical study.

- changes in coverage levels of care plan alternatives for patients
- changes in incentive levels of care plan alternatives for resources
- changes in incentive levels of care plan alternatives for institutions
- changes in system's resource supply

Changes in coverage and incentive levels of care plan alternatives are depicted with timely unit utility gain levels attributed to capacity transfers within institutions' grouping schemes. Changing the resource supply means addition or removal of system resources.

Timing of the policy change and the way actors react to this change is to be analyzed in this numerical study.

For this purpose we define two different scenarios (scenario no.1 and scenario no. 2) based on different policy change settings. Alternative levels of coverage or incentives are determined as low, moderate and high, whereas changes in resource supply are determined as user defined percentages that may denote possible increases or decreases in system's resource supply. Table 5.7 and Table 5.8 describe the initial policies and the changes applied in these policies under these two scenarios.

Scenario no.1 is intended to represent the situation in Turkey as of 2015, where hospital based care is almost fully covered in public hospitals but home based care services are not covered at all by the Social Security Institution (SGK). With this scenario we would like to investigate the effects of introducing coverage for home based care of patients as well as providing incentives for both institutions and resources for performing these types of services. The resource pool within the territory is assumed to be constant in this scenario.

Table 5.7. Policy change settings for scenario no. 1

Scenario No. 1		
Policy Type	Initial policy	Changed policy
Patients' coverage for hospital care	High	-
Patients' coverage for home care	Low	High
Institutions' incentive for hospital care	High	-
Institutions' incentive for home care	Low	High
Resources' incentive for hospital care	High	-
Resources' incentive for home care	Low	High
Resource supply	105 nurses	-

Scenario no.2 is chosen to represent a developed country where home care is a major part of the healthcare system and coverages/incentives for actor groups are set to certain constant levels. The major policy change in this scenario is determined as increasing the resource supply within the territory that the healthcare system is located in. This increase can be achieved through designating the healthcare workforce among territories.

Table 5.8. Policy change settings for scenario no. 2

Scenario No. 2		
Policy Type	Initial policy	Changed policy
Patients' coverage for hospital care	Moderate	-
Patients' coverage for home care	Moderate	-
Institutions' incentive for hospital care	Moderate	-
Institutions' incentive for home care	Moderate	-
Resources' incentive for hospital care	Moderate	-
Resources' incentive for home care	Moderate	-
Resource supply	105 nurses	+ 20%

5.5. Algorithm Settings

We implement our proposed simulation and optimization algorithms over our healthcare system setting introduced so far.

The policy changes under different scenarios of this numerical study are expected to have profound effects over the system actors and their actions. Being able to simulate these affects through our proposed agent-based simulation algorithm is expected to give us an idea of how the system would evolve based on very simplistic and individual decision making processes. Our optimization algorithm, on the other hand, is expected to denote a set of optimized actions that represent a collective decision making process. The simulation algorithm represents local (myopic) perspectives, whereas the optimization algorithm has a global view of the system over the whole planning horizon.

The number of periods in simulation runs is chosen to be 108 (=18×6) to have a clear view of the system's evolution through time. The first 54 (18×3) periods are considered as warm-up periods and policy change is activated at 55th period.

Resource contracts for the first periods of both algorithms are set by assigning resource agents of resource groups one at a time to every institutions' healthcare delivery channels (home care and hospital) so that every resource is equally distributed among institutions and their care plan alternatives.

We use 10% buffer ratio in the simulation algorithm and make five replications for every run.

In the mathematical programming based optimization algorithm, the planning horizon is chosen to be 22 months. Since the standard lead time for nurse hiring and firing is two months, we will be discarding the initial and final two months from analysis, which leaves us an 18-period planning horizon.

Our optimization algorithm decreases Δ_k in a systematic manner over the iterations. Δ_1 is chosen to be 0.30 and it is decremented 0.02 at every iteration, which results in a total of 16 iterations for the optimization algorithm. Table 5.9 illustrates the values of Δ_k values used over the iterations of the algorithm.

Table 5.9. Values of Δ_k's over the iterations

Iteration (k)	1	2	3	…	15	16
Δ_k	0.30	0.28	0.26	…	0.02	0.00

Both the agent-based simulation and mathematical programming based optimization algorithms are coded in C# programming language. Models over the optimization algorithm iterations are solved using CPLEX Optimization Studio 12.5.1 over a computer with an Intel® Core™ i7-4510U CPU @ 2.60 GHz processor, where solution times for optimization models are limited to 3,600 seconds and relative MIP gap tolerance is set to 0.01.

5.6. Key Metrics for Output Analysis

We define the following metrics as our measures for analyzing the outputs of proposed simulation and optimization algorithms.

- patient, institution and resource utilities and system's total utility
- institution groups' total resource contracts to be utilized at each care plan alternatives

- patient groups’ total selection quantities for home and hospital based care alternatives
- competition levels of system resources
- efficiency gain levels achieved by institutions

Actors’ utilities represent their well-being based on utility gains through delivery of healthcare services. Their total represent system’s overall well-being. Capacity contracts between institutions and resources denote the utilization of system’s resource capacity and its distribution among care plan alternatives (i.e. hospital and home care). The timely distribution of patients among these alternatives represents the evolution of healthcare service demand. Competition over resources and efficiency gains achieved by institutions are the two main system-level functions we have introduced in this study. Therefore they are considered to have influence over system’s evolution.

Note that these metrics are chosen as a sample set from the pool of all possible metrics associated with objects of our proposed analysis framework. These metrics, can be defined and tracked at various further levels of details. Those we define and report in this section are aggregate measures and do not include details about distinct patient, institution, resource or capacity-related groups. However, the impact of a policy change on each and every actor group (e.g. patients with high, moderate and low income) can be observed separately in more detail.

5.7. Output Analysis for Scenario No. 1

As indicated in Table 5.7, the policy changes for scenario no. 1 are set as follows.

- utilities related to hospital care remain “High”
- utilities related to all actors for home care are increased from “Low” to “High”
- resource supply remains the same

5.7.1. Simulation Algorithm Results for Scenario No. 1

Figures 5.3‒5.6 represent the changes in system total and actor groups’ utility levels through simulation runs for five replications, where vertical lines at each figure at period 55 indicate the timing of policy changes.

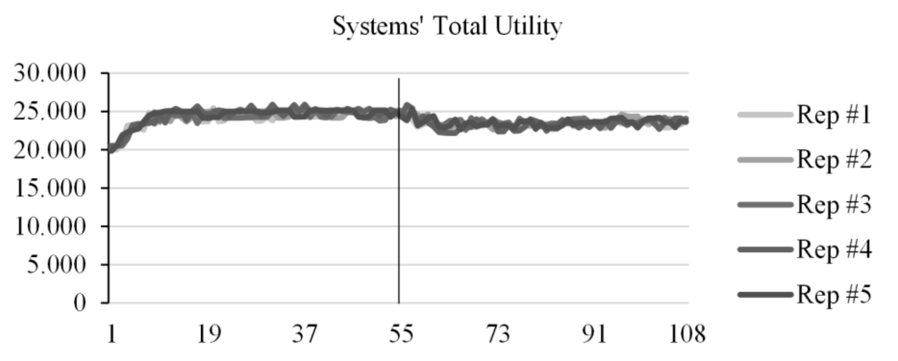

Figure 5.3. System’s total utility (scenario no. 1)

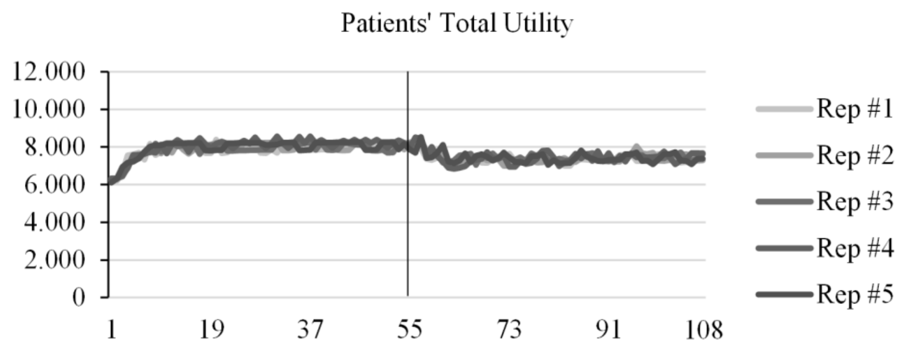

Figure 5.4. Patient groups’ total utility (scenario no. 1)

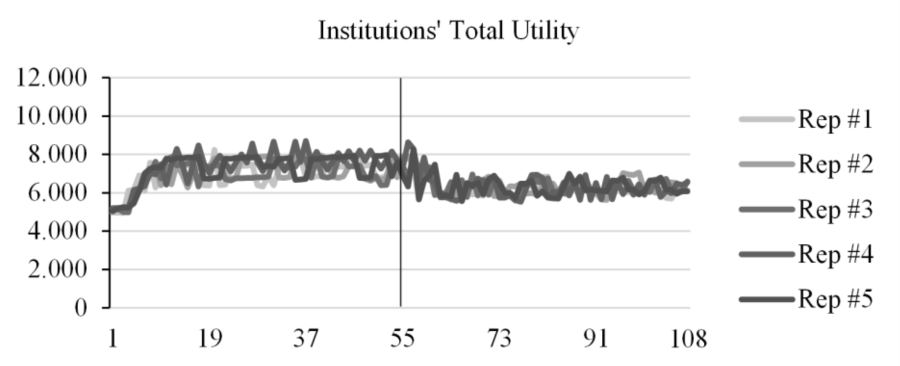

Figure 5.5. Institution groups’ total utility (scenario no. 1)

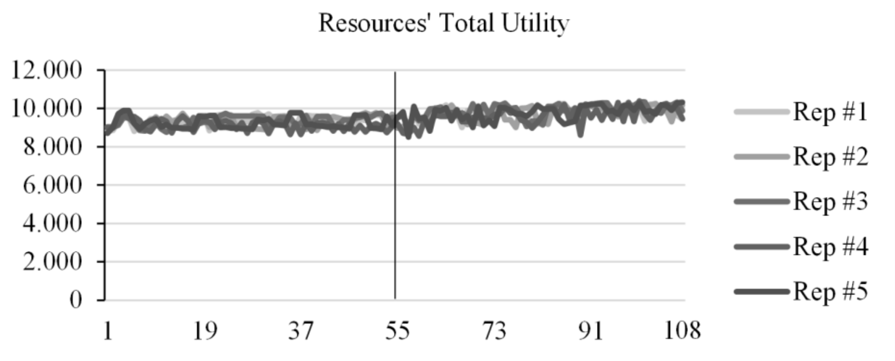

Figure 5.6. Resource groups' total utility (scenario no. 1)

Attained utility gains indicate that the policy changes under consideration result in lower utility levels for patients and institutions, and higher utilities for resources. One would expect that increasing the utility gains for a care plan alternative (i.e. home care) would result in increased, or at least constant levels of patient utilities.

Figures 5.7 and 5.8 reveal institutions' contracted capacities from resources for alternative care types through simulation runs for replications. Increased utility levels offered by home care services result in some patients choosing home based care services. Since home care requires more capacity than its counterpart, the amount of resources contracted in the system increases.

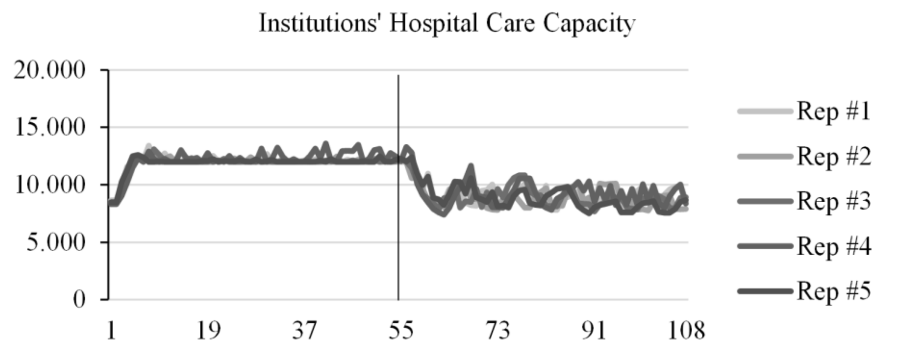

Figure 5.7. Institutions' hospital care capacities (scenario no. 1)

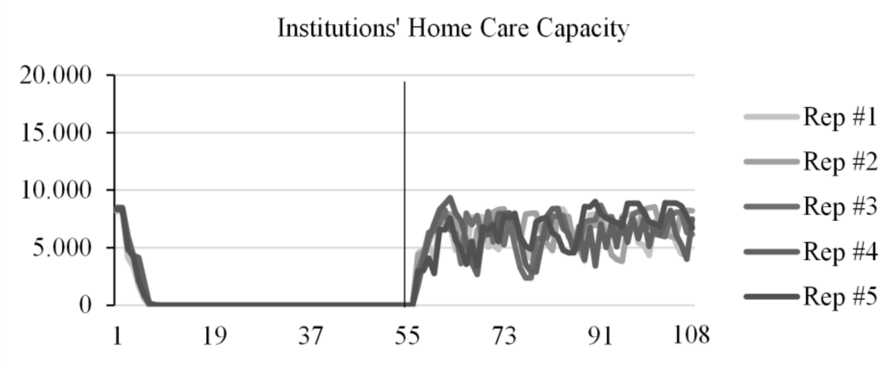

Figure 5.8. Institutions' home care capacities (scenario no. 1)

Figure 5.9 represents the distribution of system's contracted capacities among care plan alternatives calculated by averaging over replications. The dotted lines in Figure 5.9 indicates the total available resource capacity in the system.

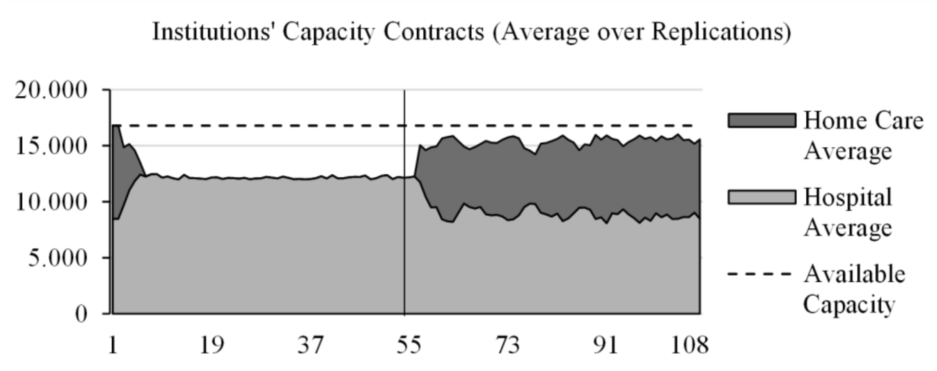

Figure 5.9. Distribution of capacities among care plan alternatives (scenario no. 1)

Figure 5.10 shows patients' care plan choices over the simulation runs. When we consider Figures 5.9 and 5.10 simultaneously, we see that patients' selection of home based care plans occurs only up to certain levels and therefore system resources are not fully utilized by institutions.

The factor that withholds all patients from selecting home care turns out to be the increased competition levels over resources. Figures 5.11, 5.12 and 5.13 show the changes of resource competition levels for resource groups #1, #2 and #3 through simulation runs where 0, 1 and 2 in vertical axes denote low, moderate and high competition levels, respectively. One would observe that even though there are up and down fluctuations among levels, all resources' competition levels tend to increase right after the occurrence of policy changes.

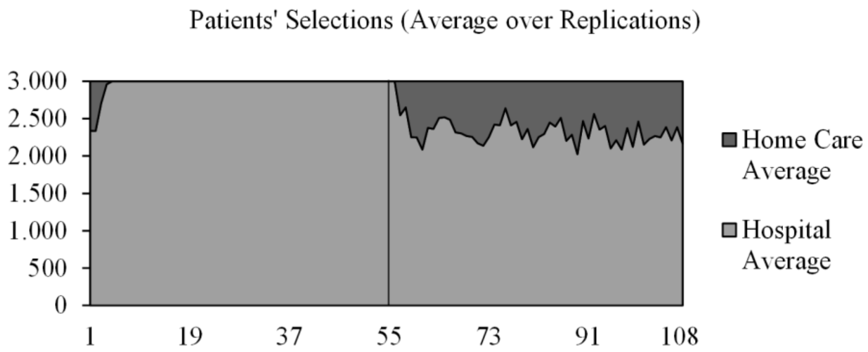

Figure 5.10. Distribution of care plan selections for simulation runs (scenario no. 1)

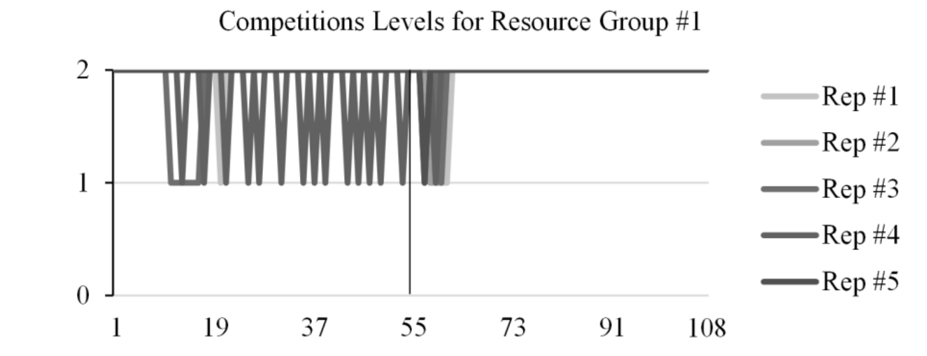

Figure 5.11. Competition levels of resource group #1 (scenario no. 1)

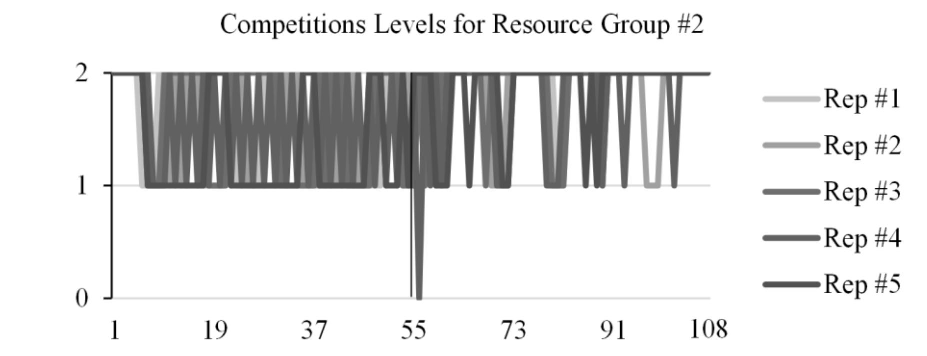

Figure 5.12. Competition levels of resource group #2 (scenario no. 1)

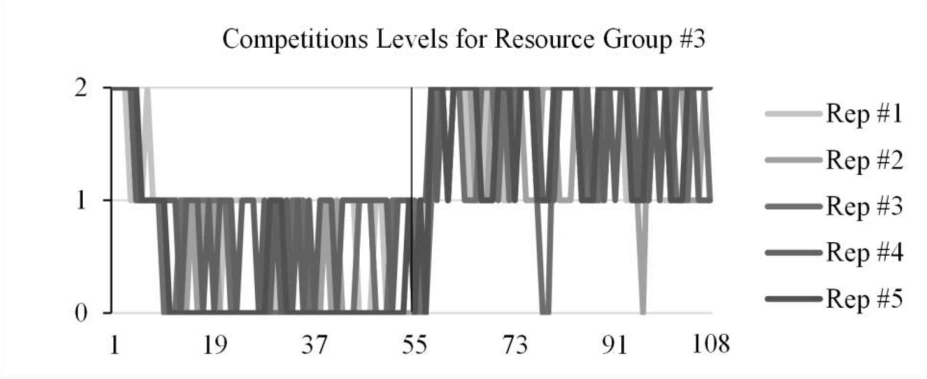

Figure 5.13. Competition levels of resource group #3 (scenario no. 1)

So, when home care becomes beneficial for patients, they select this alternative up to the extend where required additional resource levels lead to increased resource competition and thereby undesirably low patient utilities (which are denoted in Table 5.4).

Figures 5.14, 5.15 and 5.16 show the changes of efficiency gain levels for institution groups #1, #2 and #3 through simulation runs where 0, 1 and 2 in vertical axes denote low, moderate and high gains, respectively.

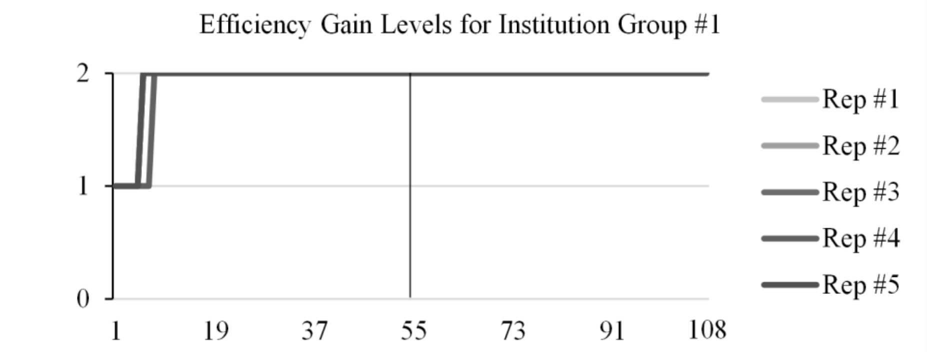

Figure 5.14. Efficiency gain levels of institution group #1 (scenario no. 1)

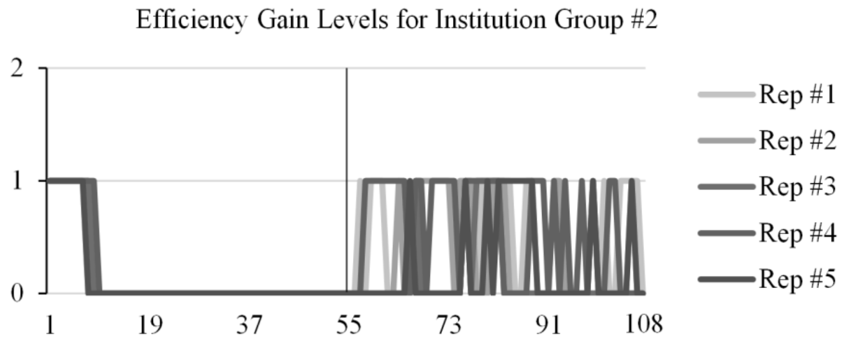

Figure 5.15. Efficiency gain levels of institution group #2 (scenario no. 1)

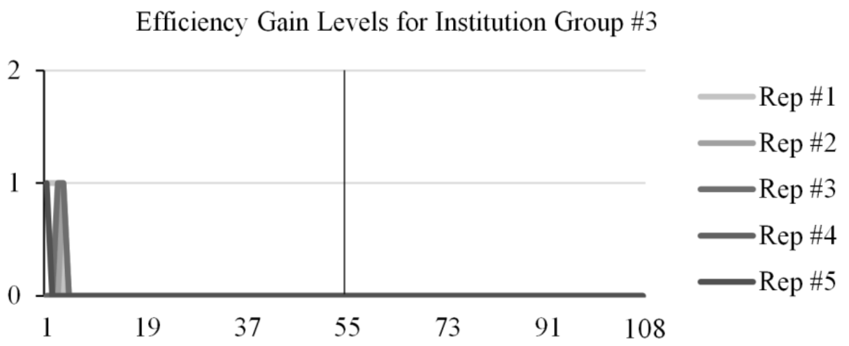

Figure 5.16. Efficiency gain levels of institution group #3 (scenario no. 1)

5.7.2. Optimization Algorithm Results for Scenario No. 1

The total utilities of the overall system and each actor group summed over all periods through the iterations of our optimization algorithm are presented in Figures 5.17 and 5.18.

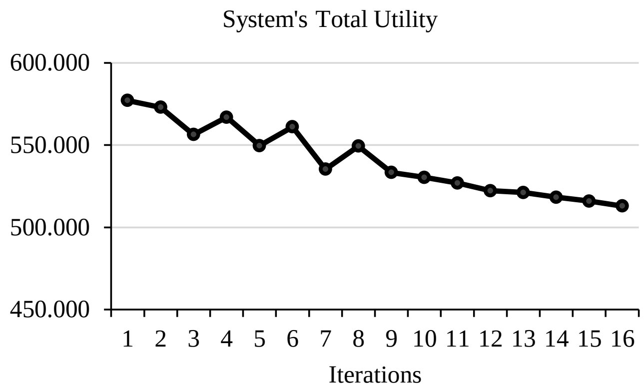

Figure 5.17. System’s total utilities through optimization iterations (scenario no. 1)

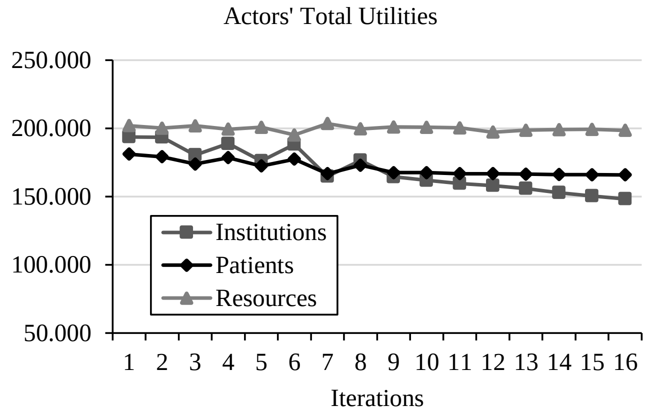

Figure 5.18. Actors’ total utilities through optimization iterations (scenario no. 1)

Further investigation of these results show our optimization algorithm’s process of arriving at an agreed-upon solution. Figures 5.19, 5.20 and 5.21 present the details of these utilities for each period within the planning horizon throughout algorithm’s iterations.

One observes that patients’ and institutions’ overall utility levels drop, whereas resource utilities converge at an in-between level through the optimization algorithm’s iterations.

Timely utilities of the 16th iteration are the main outputs of our optimization algorithm and are denoted with hollow circles in figures. This output can be considered as an agreed-upon solution that consider all actor groups’ well-being in presence of policy changes.

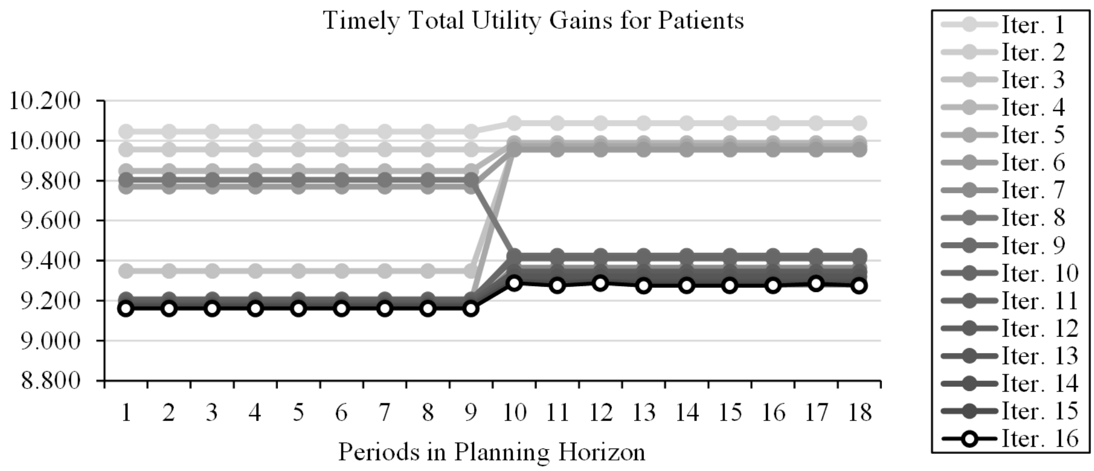

Figure 5.19. Patients’ timely utilities through optimization iterations (scenario no. 1)

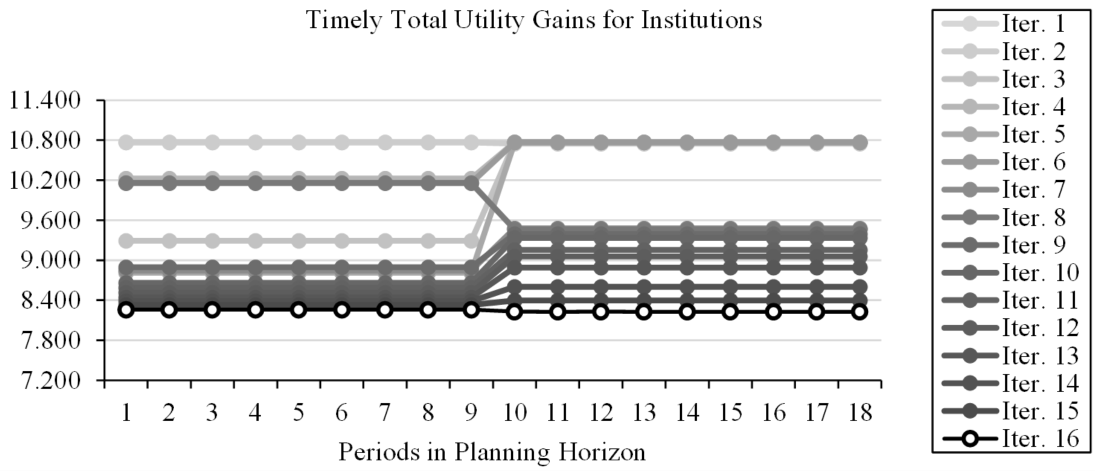

Figure 5.20. Institutions' timely utilities through optimization iterations (scenario no. 1)

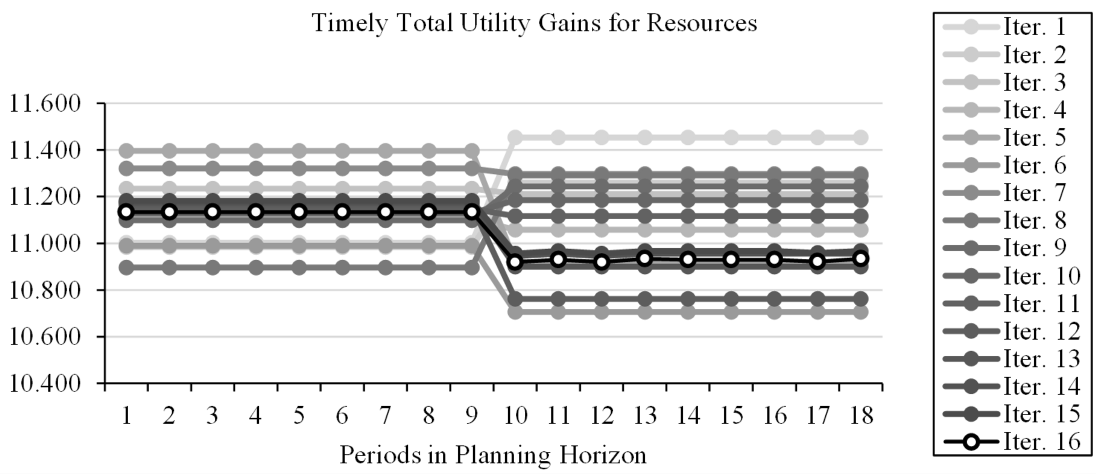

Figure 5.21. Resources' timely utilities through optimization iterations (scenario no. 1)

Figure 5.22 represents capacities contracted by institutions to be utilized in alternative care plans, namely hospital and home care, as well as the system's total available capacity as a result of the optimization algorithm. Figure 5.23 shows the distribution of patients among care plan alternatives.

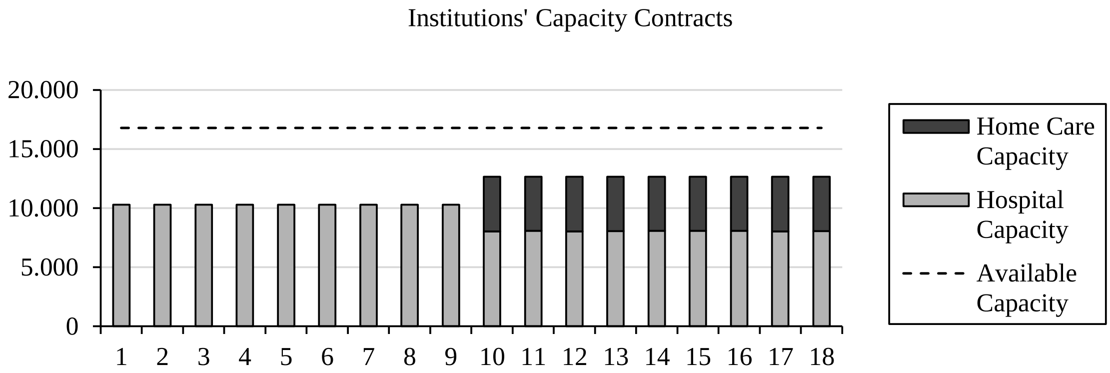

Figure 5.22. Institutions’ timely capacity contracts (scenario no. 1)

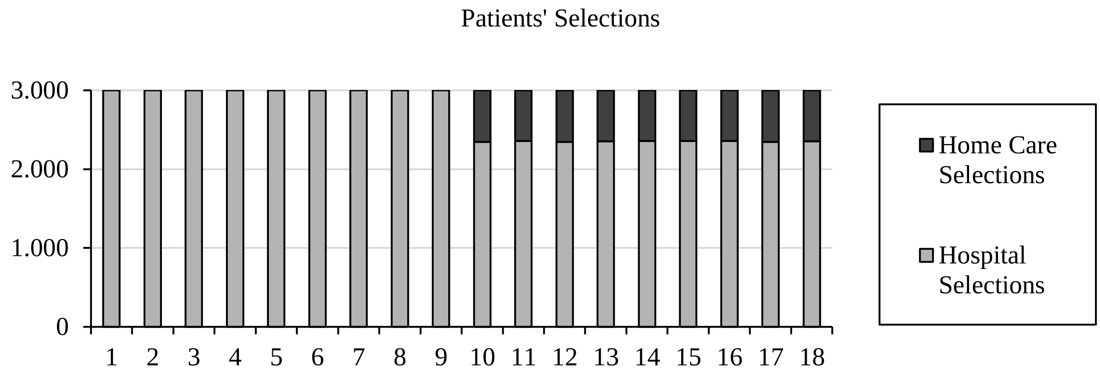

Figure 5.23. Patients’ timely care plan selections (scenario no. 1)

Figure 5.24 and Figure 5.25 present the timely change in resources’ competition levels and institutions’ efficiency gain levels through the planning horizon. 0, 1 and 2 in vertical axes denote low, moderate and high levels of these system-level functions, respectively.

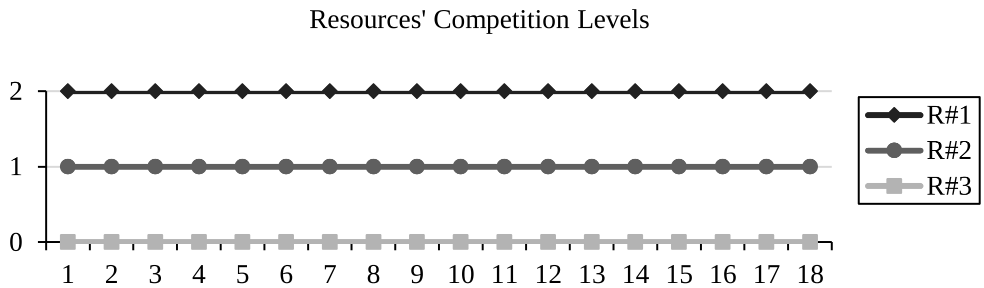

Figure 5.24. Resources’ timely competition levels (scenario no. 1)

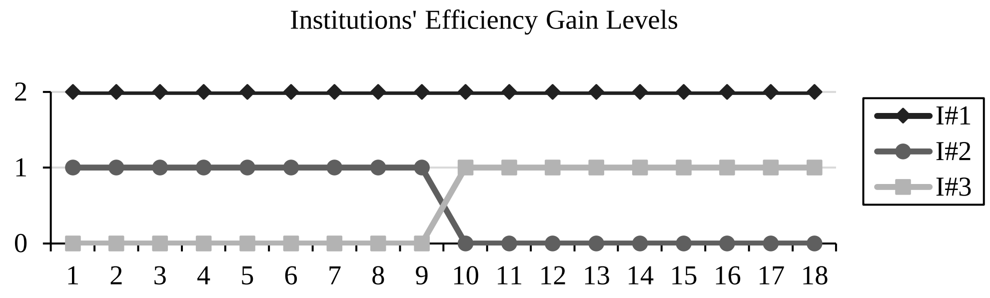

Figure 5.25. Institutions' timely efficiency gain levels (scenario no. 1)

5.8. Output Analysis for Scenario No. 2

As indicated in Table 5.7, the policy changes for scenario no. 2 are set as follows.

- utilities related to all actors for all care types remain "Moderate"
- system's resource supply is increased by 20 percent

5.8.1. Simulation Algorithm Results for Scenario No. 2

Figures 5.26−5.29 represent the changes in system total and actor groups' utility levels through simulation runs for five replications, where vertical lines at each figure at period 55 indicate the timing of policy changes.

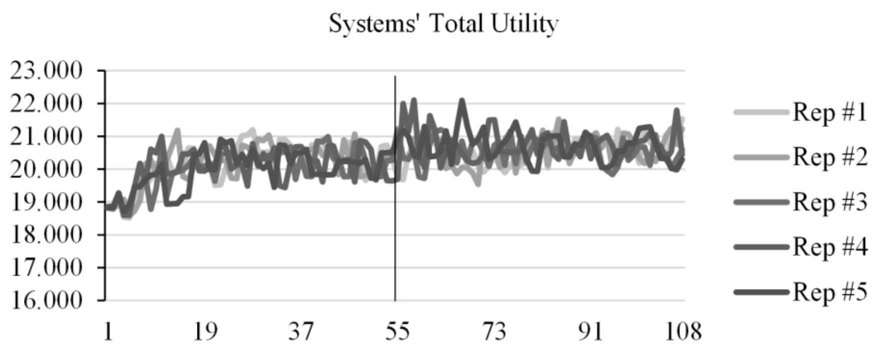

Figure 5.26. System's total utility (scenario no. 2)

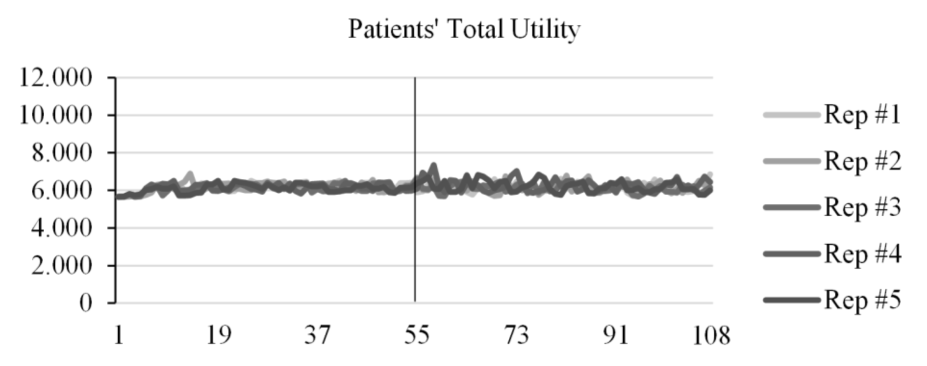

Figure 5.27. Patient groups’ total utility (scenario no. 2)

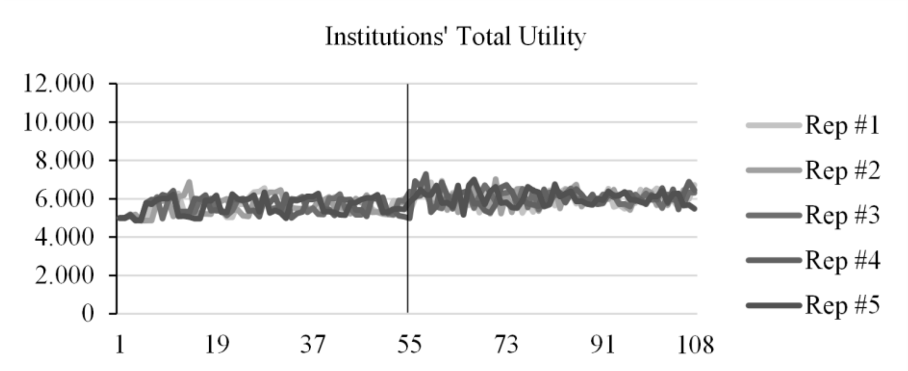

Figure 5.28. Institution groups’ total utility (scenario no. 2)

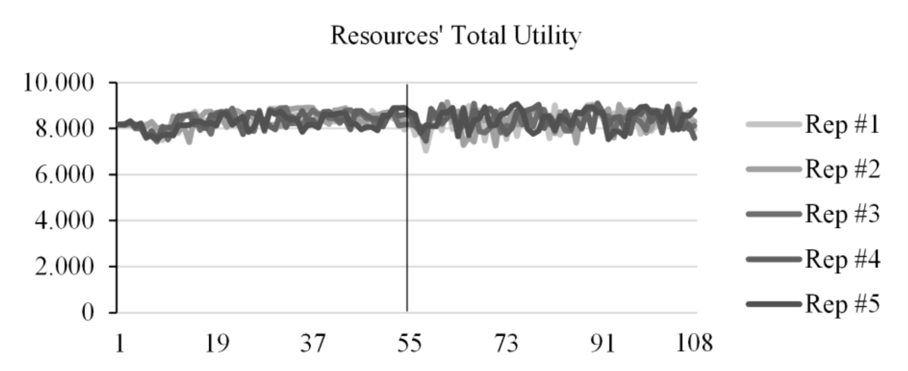

Figure 5.29. Resource groups’ total utility (scenario no. 2)

One may expect that increasing resource availability would result in higher utilities for patients and lower utilities for resources. Attained utility gains indicate that the policy change do not result in an obvious shift for neither patients nor resources. Institutions achieve higher utilities upon the increase in resource supply, though.

Figures 5.30 and 5.31 show institutions’ contracted capacities for alternative care types through simulation runs for replications.

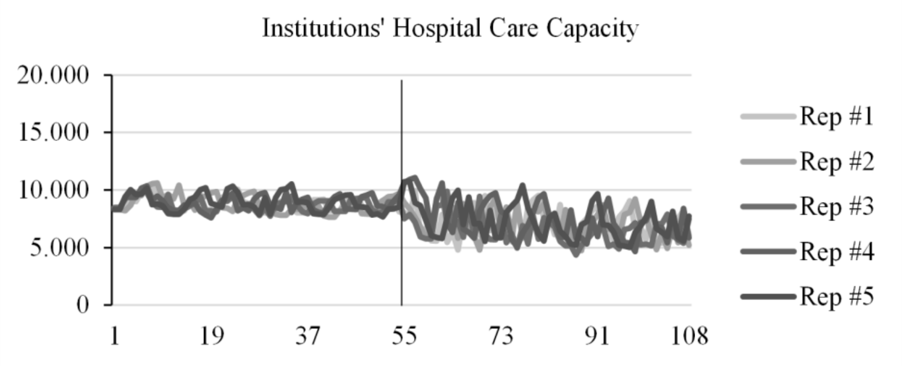

Figure 5.30. Institutions’ hospital care capacities (scenario no. 2)

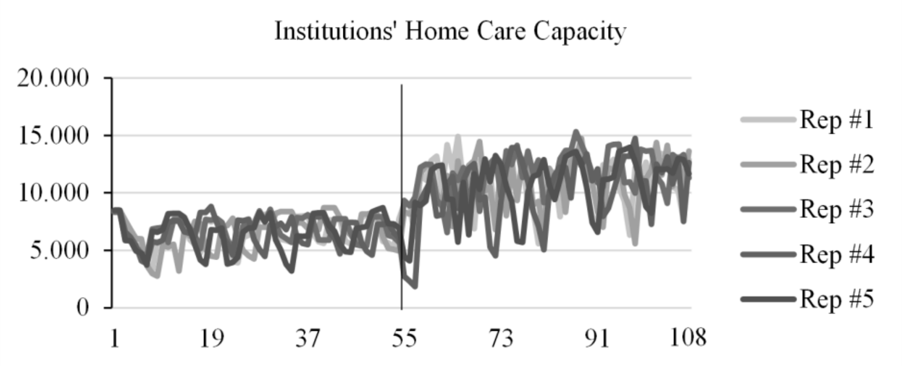

Figure 5.31. Institutions’ home care capacities (scenario no. 2)

Figure 5.32 represents the amount of capacities contracted for care plan alternatives calculated by averaging over replications. Figures 5.30 – 5.32 reveal that increased resource supply favors home based care which requires more capacity than its counterpart.

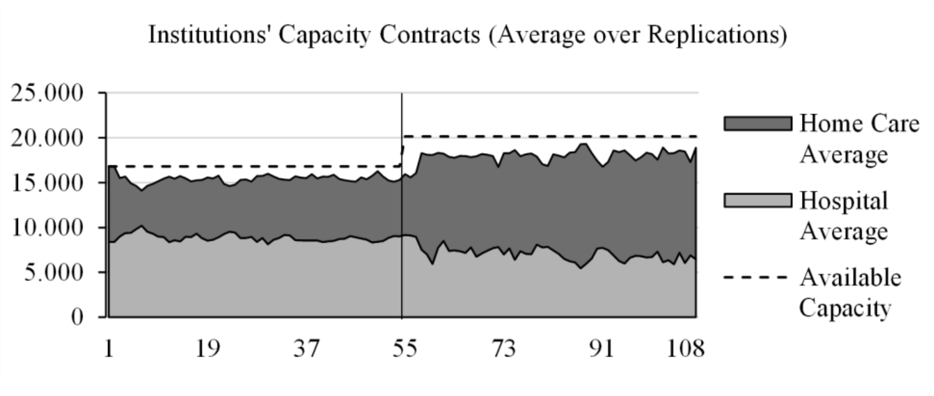

Figure 5.32. Distribution of capacities among care plan alternatives (scenario no. 2)

Figure 5.33 shows patients' care plan choices over the simulation runs. We observe that almost half of the patient population chooses home based care as a result of the policy change.

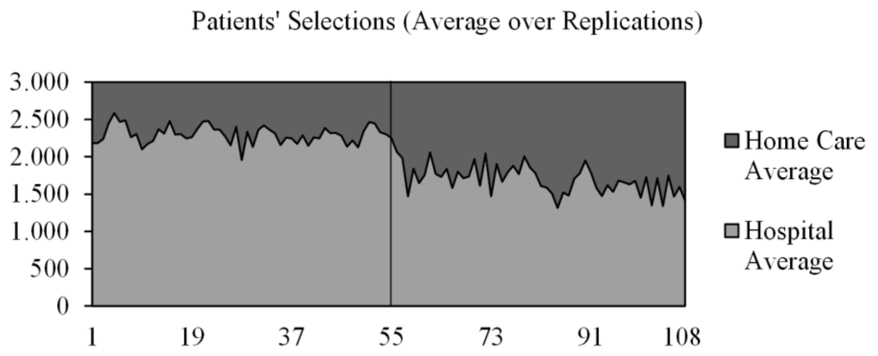

Figure 5.33. Distribution of care plan selections for simulation runs (scenario no. 2)

As in scenario no. 1, increased competition over resources withholds patients selecting home care and institutions contracting all available resources.

Figures 5.34, 5.35 and 5.36 show the resource competition for resource groups #1, #2 and #3 through simulation runs. Policy change results in instantaneous shifts to lower competition levels for resource groups. However these rare shifts are not permanent and patients choosing the more beneficial but resource consuming home care option shifts the competition level back up.

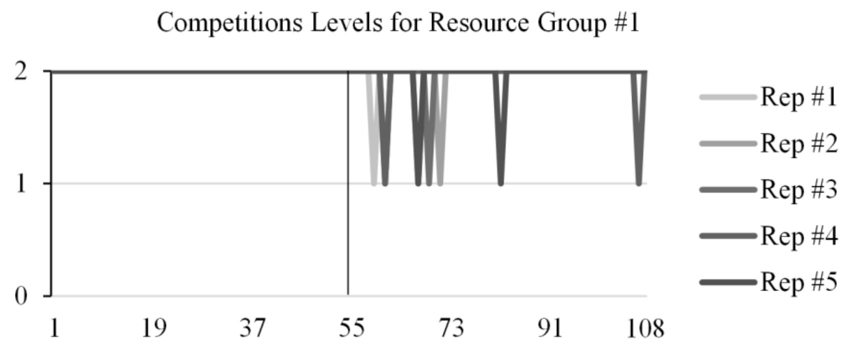

Figure 5.34. Competition levels of resource group #1 (scenario no. 2)

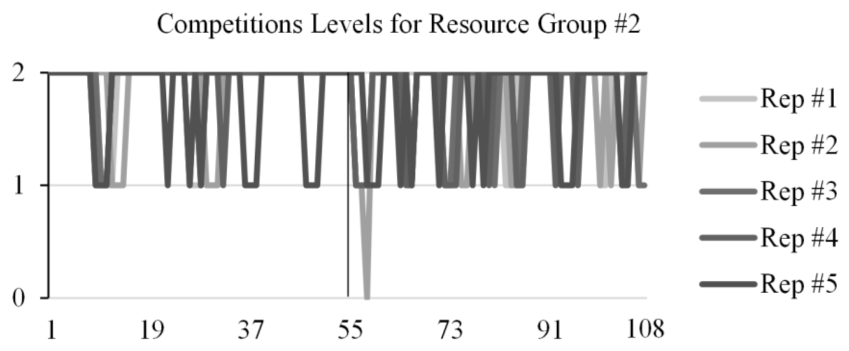

Figure 5.35. Competition levels of resource group #2 (scenario no. 2)

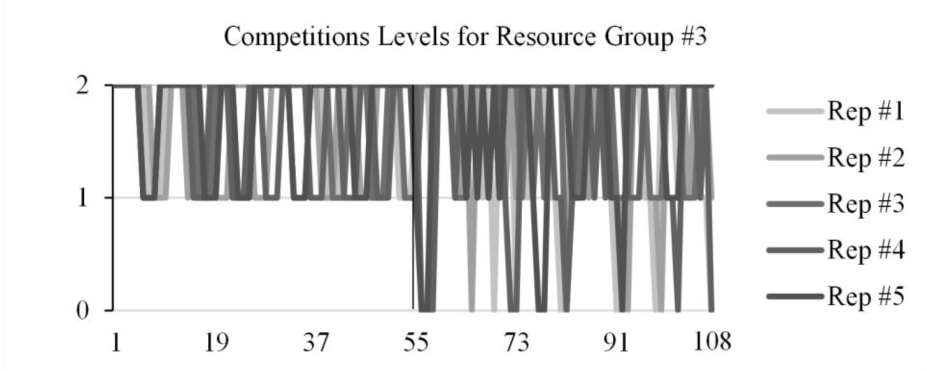

Figure 5.36. Competition levels of resource group #3 (scenario no. 2)

Figures 5.37, 5.38 and 5.39 show the changes of efficiency gain levels for institution groups #1, #2 and #3 through simulation runs.

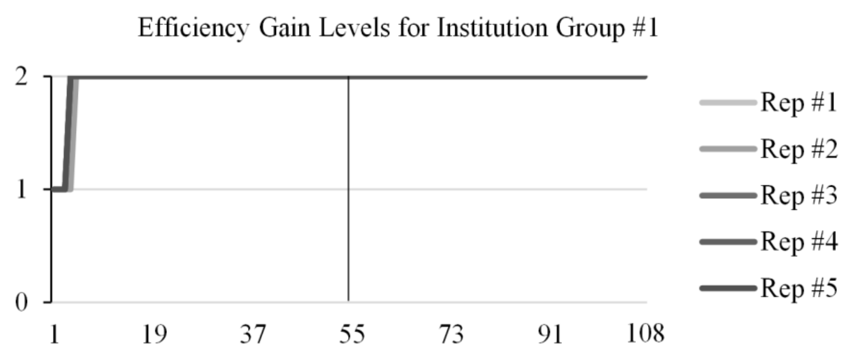

Figure 5.37. Efficiency gain levels of institution group #1 (scenario no. 2)

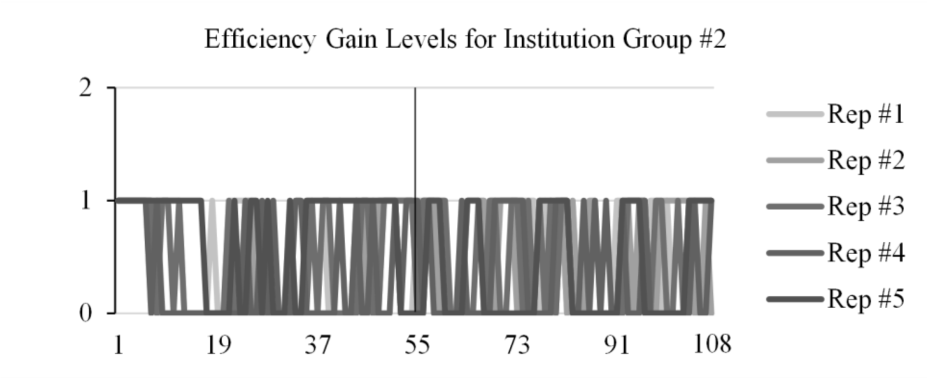

Figure 5.38. Efficiency gain levels of institution group #2 (scenario no. 2)

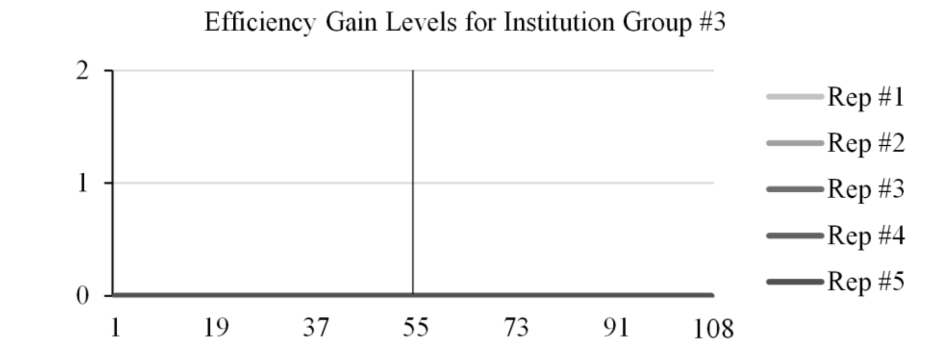

Figure 5.39. Efficiency gain levels of institution group #3 (scenario no. 2)

5.8.2. Optimization Algorithm Results for Scenario No. 2

The total utilities of the overall system and each actor group summed over all periods through the iterations of our optimization algorithm are presented in Figures 5.40 and 5.41.

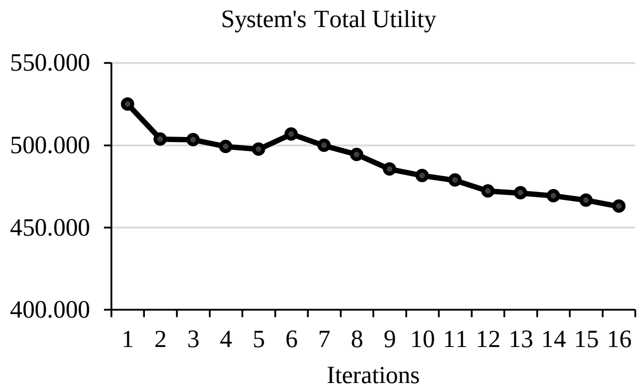

Figure 5.40. System’s total utilities through optimization iterations (scenario no. 2)

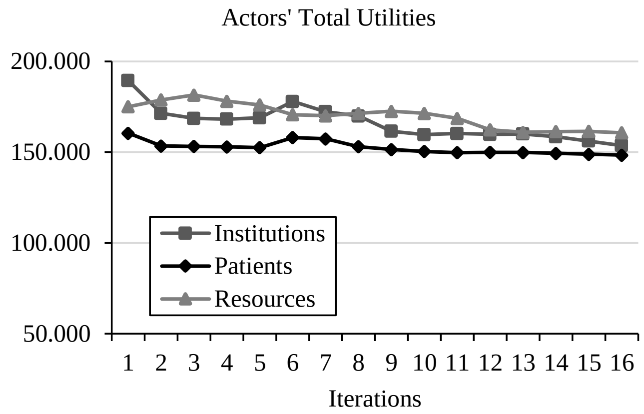

Figure 5.41. Actors' total utilities through optimization iterations (scenario no. 2)

Details of these results show the timely utilities of these actors. Figures 5.42, 5.43 and 5.44 present the progress of our optimization algorithm through iterations until it reaches a solution as a result of the 16th iteration (denoted by hollow circles). We observe that all actors' overall utility levels drop as the algorithm proceeds in this scenario.

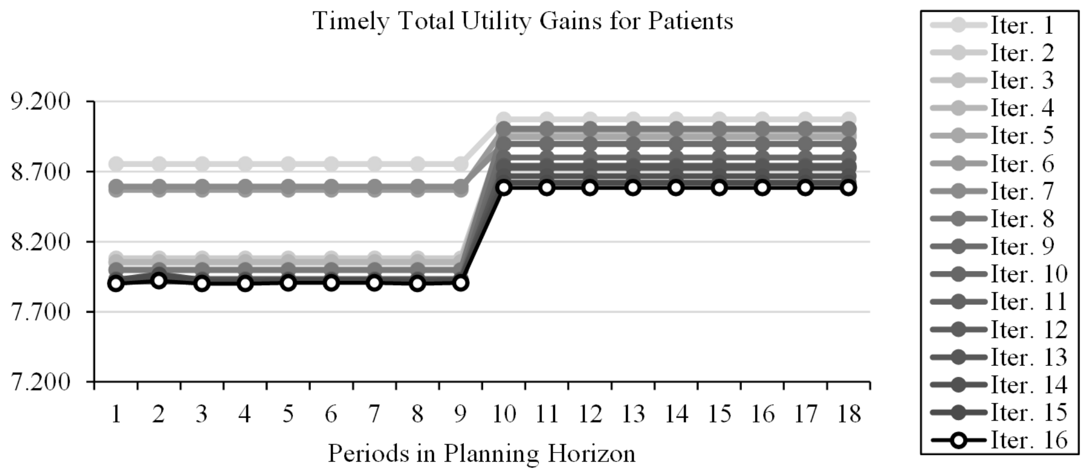

Figure 5.42. Patients' timely utilities through optimization iterations (scenario no. 2)

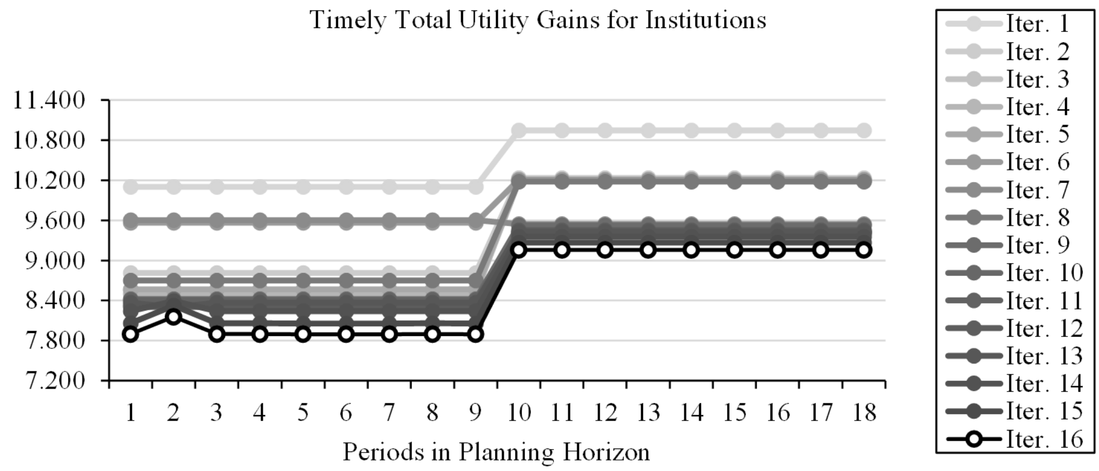

Figure 5.43. Institutions’ timely utilities through optimization iterations (scenario no. 2)

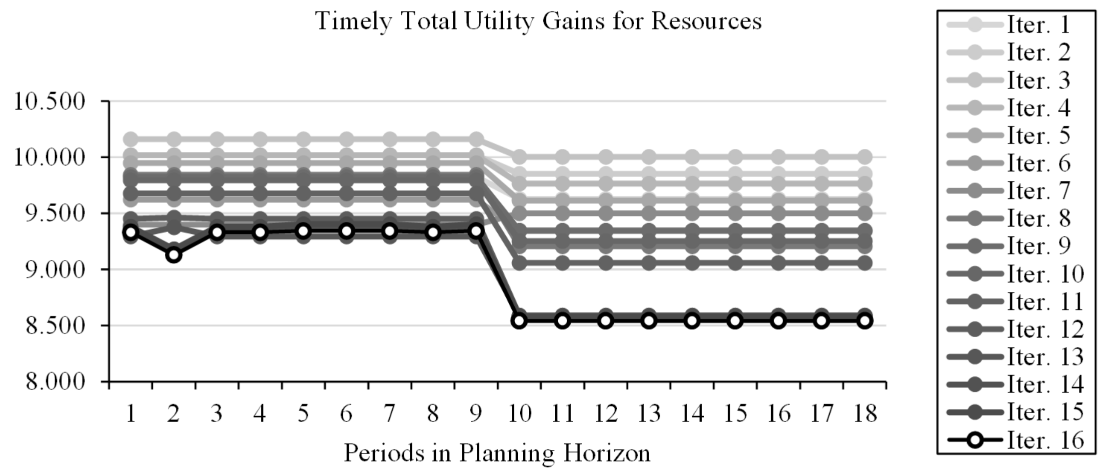

Figure 5.44. Resources’ timely utilities through optimization iterations (scenario no. 2)

Figure 5.45 represents contracted capacities by institutions to be utilized in alternative care plans and the system’s total available capacity determined by optimization algorithm. Figure 5.23 shows the distribution of patients among care plan alternatives.

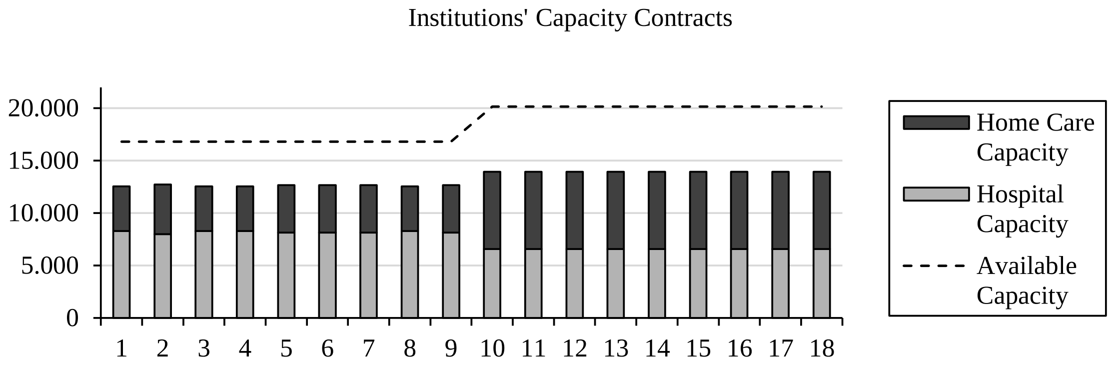

Figure 5.45. Institutions’ timely capacity contracts (scenario no. 2)

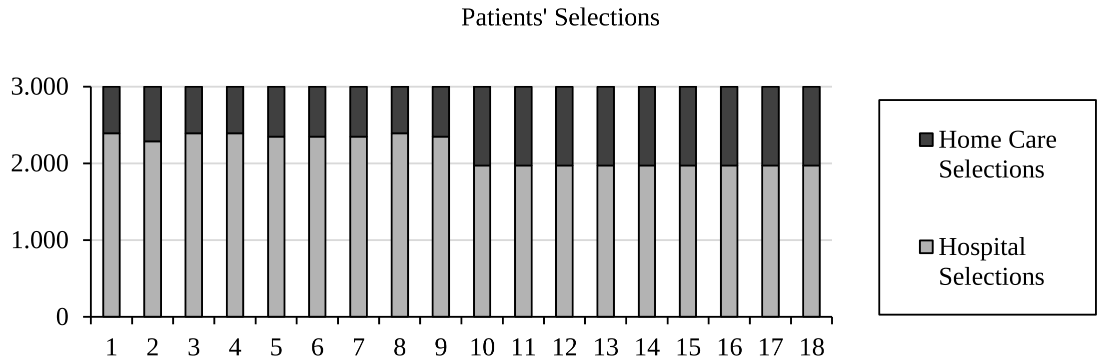

Figure 5.46. Patients’ timely care plan selections (scenario no. 2)

Figure 5.47 and Figure 5.48 present the timely change in resources’ competition levels and institutions’ efficiency gain levels through the planning horizon.

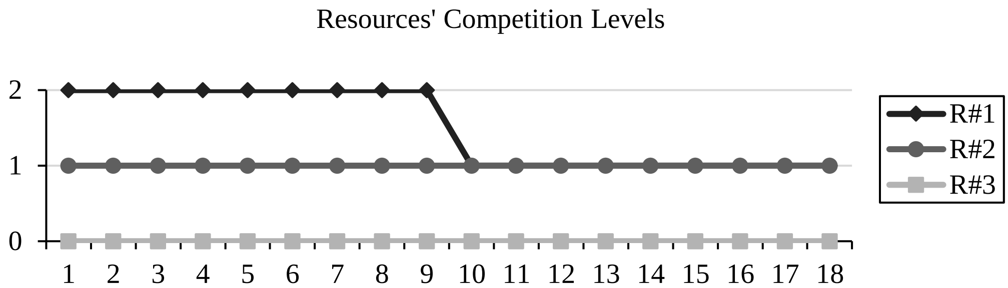

Figure 5.47. Resources’ timely competition levels (scenario no. 2)

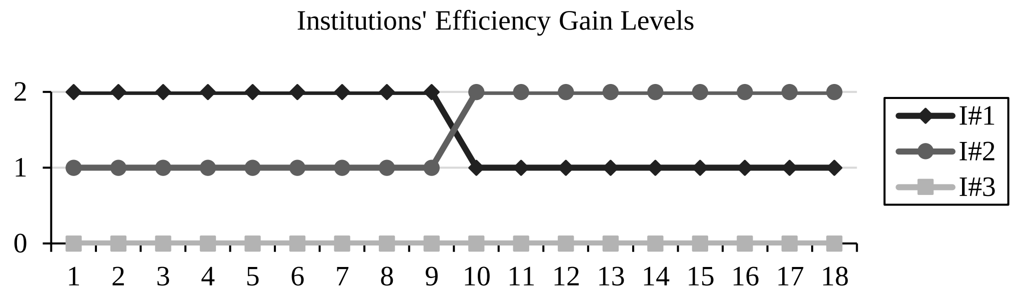

Figure 5.48. Institutions’ timely efficiency gain levels (scenario no. 2)

5.9. A Remark on Utility Function Settings

The utility levels that we associate with capacity flows are critical in actors’ decision making and the wellbeing of the overall system. As presented in section 5.3, utility functions have two types of components, namely adequacy of matching and coverage/incentive provided. In section 5.7 the effects of varying coverage/incentive levels via policy changes are investigated. Now we focus on the effects of changes in the other utility component, the adequacy of matching.

For this purpose, we change the utility levels associated with adequacy of matching between actor groups and compare the results with the original values introduced in Table. 5.3. Table 5.10 present the modified utility levels in this respect. By doing so, we compare the outputs of settings having different utility function structures.

Table 5.10. Utility gains from actor groups in interaction (modified version)

		Actor group B		
		High	Moderate	Low
Actor group A	High	0.50	0.30	0.00
	Moderate	0.30	0.50	0.30
	Low	0.00	0.30	0.50

Values presented in Table 5.10 are higher than those of Table 5.3. In order to analyze the effect of such an increase in the weights of a utility component, we implement our algorithms in two additional scenarios, namely scenario no. 3 and scenario no. 4. Policy settings for these scenarios are presented in Table 5.11. Note that the policies are kept constant throughout these two scenarios. The only difference between them is that scenario no. 3 uses the adequacy of matching utilities of Table. 5.3., whereas Scenario no. 4 uses those of Table. 5.10.

Table 5.11. Policy change settings for scenarios no. 3 and no. 4

Scenario No. 3 and Scenario No. 4		
Policy Type	Initial policy	Changed policy
Patients' coverage for hospital care	Moderate	-
Patients' coverage for home care	Moderate	-
Institutions' incentive for hospital care	Moderate	-
Institutions' incentive for home care	Moderate	-
Resources' incentive for hospital care	Moderate	-
Resources' incentive for home care	Moderate	-
Resource supply	105 nurses	-

For demonstration purposes, we present only a selection of the results obtained from simulation runs over scenarios 3 and 4.

Figure 5.49 and Figure 5.50 present the amount of capacities contracted for care plan alternatives calculated by averaging over replications for scenarios no. 3 and no. 4.

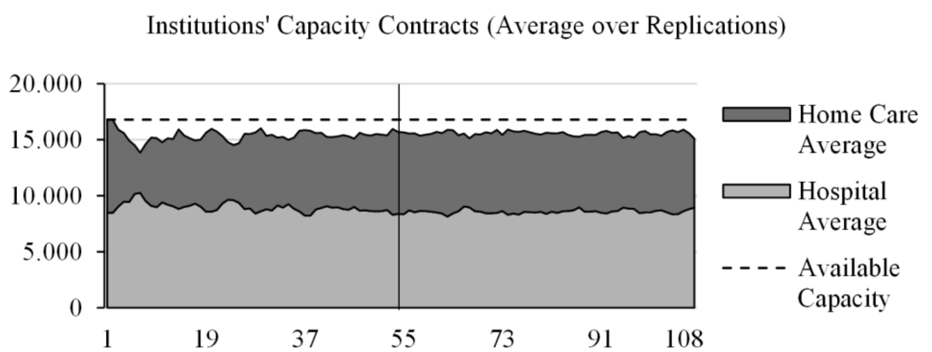

Figure 5.49. Distribution of capacities among care plan alternatives (scenario no. 3)

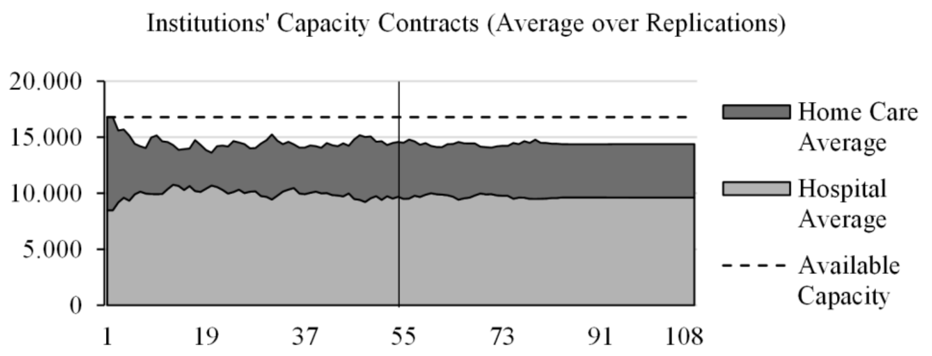

Figure 5.50. Distribution of capacities among care plan alternatives (scenario no. 4)

Figure 5.51 and Figure 5.52 show patients' care plan choices over the simulation runs over replications for scenarios no. 3 and no. 4.

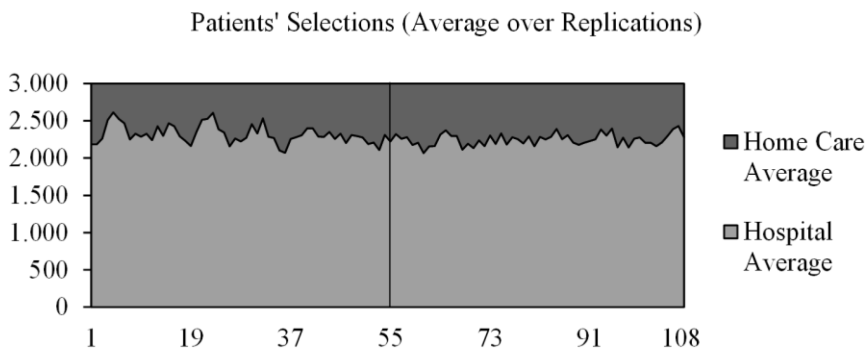

Figure 5.51. Distribution of care plan selections for simulation runs (scenario no. 3)

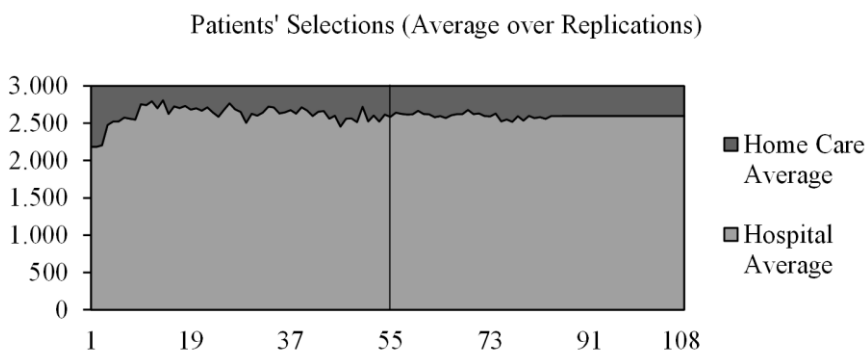

Figure 5.52. Distribution of care plan selections for simulation runs (scenario no. 4)

These results show that increasing the weight of adequacy of matching among actor groups results in reduced selection of home care services. This is due to the fact that the additional utility from selecting a matching actor group becomes more favorable than the utility from selecting home based care. Increased resource competition resulting from home care selection also favors efficient use of resource capacities and therefore, hospital based care alternative.

This analysis demonstrates an illustration of the impact of changing utility function structure. Here we reemphasize the importance of developing utility functions that represent actors' perspectives correctly for better analysis of system's evolution.

6. CONCLUSIONS & REMARKS

There are many factors and issues that create imbalance and affect the evolution of a health care system including home care as an alternative means of service. The analysis framework that we develop in this study explains and formalizes the main concerns related to tactical-level decision making processes of actors within such a system.

The system has certain limitations that every actor has to comply with. Using our formal model, we develop a generic mathematical programming based constraints set that express these limitations related to patients' service selections, resources' capacity contracting and institutions' capacity planning.

Responses of actors, to others' actions or system-level impacts are significant issues in terms of planning activities. Each actor has a perspective and these perspectives are expressed by utility functions in our analysis framework. Decision alternatives are evaluated in terms of their associated utility gains. Therefore, our analysis framework is based on utility maximization which we believe to be a correct approach when we consider the priorities in a healthcare system including home care. This mechanism allows us to generalize various aspects that have influence over system components simultaneously, such as costs, preferences, convenience, and so forth.

In addition to generic constraints and actors' perspectives, the system involves several mechanisms and that we call system-level functions. These functions affect the system and the actors within the system. We introduce two such functions that describe changes in utilities based on competition over resources and efficiency gains due to economies of scale. We also show how we integrate these functions into our analysis framework.

The system has inherent nonlinear relations. We show that by applying specific linearization techniques under certain assumptions, our analysis framework can incorporate such nonlinearity.

Various analysis can be made using our analysis framework. We introduce two different approaches and techniques in this respect which enables one to analyze the system's evolution through time.

Our proposed agent-based simulation algorithm describe how the system evolves based on simple decision making of system agents which represent system actors, where the choices and corresponding utilities for actors are determined by the algorithm. It reflects actors' individualistic and myopic behavior based solely on their simplistic but rational decisions. This approach is a representation of what basically happens when the system lacks the presence of an authority or a decision maker that considers the system as a whole.

We also propose an optimization algorithm that iteratively solves mathematical programming models from different perspectives to optimality within a special systematic manner, so as to decide the best set of actions for every system actor while maintaining the utilities of others at certain levels. It has a global and system-wide perspective and tries to improve the collective utilities of all actor groups. This algorithm represents some kind of an idealistic behavior and a corresponding set of actions for the system actors, which may not be easily achieved in practice.

We think that both these algorithms generate valuable results, and therefore must be used as complementing approaches. If there is a major gap in actors' utility levels, patient selections or capacity contracts between the results of the two algorithms, then regulating authorities may decide to take additional measures to direct the system for its own behalf. Defining quotes or limitations for system's or institutions' resource contracts as well as delivered services and associated pricing may be considered in this respect.

We define two scenarios in our numerical study, where effects and outcomes of different policy changes and alternative utility function structures are analyzed using the proposed framework and algorithms. We observe that the tools proposed in this study generates valuable results and can be used to investigate the system's evolutionary progress through time.

A possible extension to this research may be to include the system's regulatory body as a fourth actor group and allow tuning of policy changes through simulation runs and optimization iterations. Note that from a regulator's perspective care plan selections and capacity transfers might be considered as given parameters and possible changes in utility levels or system resources might be the main decision concerns. In such settings, regulator's objective shall be changing policies in a way such that every actor's wellbeing is considered.

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