ANALYSIS OF A SUPPLY CHAIN WITH FORECAST EVOLUTION AND SUPPLY DISRUPTION

by

Hilmi Erbilgin

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ABSTRACT

ANALYSIS OF A SUPPLY CHAIN WITH FORECAST EVOLUTION AND SUPPLY DISRUPTION

In this thesis, we analyze two supply chain models. In the first model, we develop a centralized supply chain consisting one warehouse and two retailers. The aim of this model is to find the total optimal order quantity for the warehouse. We assume that the warehouse faces yield uncertainty. At the beginning of the period, both retailers place orders and the warehouse starts supplying them. However, if there is not enough on hand inventory at the warehouse at the time of order, then it should allocate it to the retailers. After receiving units by the retailers, stochastic customer demand is observed and inventory costs are occurred. We develop and present a formula to calculate the total optimal order quantity that minimizes the total system cost and the formula can be solved in numerically; therefore we examine results in the numerical analysis section. In the second model, we consider a decentralized supply chain with one manufacturer and one retailer under a wholesale price contract. The goals of this model are to find the total order quantity for the retailer, the total advance production amount for manufacturer and the wholesale price. The manufacturer is subject to yield uncertainty. In addition, retailer can update its forecast after obtaining better information. At the beginning of the period, both of them agree on wholesale price and the manufacturer starts producing before receiving the retailer's order. When the retailer obtains forecast update, it gives total order quantity to the manufacturer. If advance production quantity is not sufficient to meet the retailer's total amount, then the manufacturer makes outsourcing for the remaining quantity. After receiving total units by the retailer, customer demand is satisfied with a retailer price and the market uncertainty is realized. We develop and present an explicit formula for the retailer's problem. The manufacturer's problem can only be solved in numerically and accordingly the wholesale price can. We also examine a numerical analysis for the model.

ÖZET

TAHMİN EVRİMİ VE TEDARİK KESİNTİSİ ALTINDA BİR TEDARİK ZİNCİRİNİN ANALİZİ

Bu tez çalışmasında, iki farklı tedarik zinciri modelini inceledik. İlk modelde, bir depo ve iki perakendeciden oluşan merkezi bir tedarik zinciri modeli geliştirdik. Bu modelimizde, depo ürün belirsizliği ile karşılaşmaktadır. Periyodun başında, perakendeciler ilk tahmin değerlerine göre depodan ürünleri sipariş ederler, depo tedarik için ürünleri hazırlamaya başlar. Fakat, elindeki envanter toplam siparişi karşılamaya yetmiyorsa, o miktarı perakendecilere paylaştıracaktır. Perakendeciler ise ürünleri aldıktan sonra rassal olan müşteri talepleri karşılamaya başlarlar ve periyot sonunda envanter maliyetleri ortaya çıkar. Bu modeldeki amacımız, toplam beklenen sistem maliyetini en aza çekecek toplam optimum sipariş miktarını bulmaktır. Bu doğrultuda, öncelikle toplam sistem maliyeti ve optimum sipariş miktarını hesaplamak için formüller sunduk. Optimum sipariş miktarı formülü ancak nümerik olarak çözülebildiğinden, sonuçlar ve verimliliğini nümerik analiz kısmında tartıştık. İkinci modelimizde ise merkezi olmayan, birbirlerine toptan satış kontratı ile bağlı olan bir üretici ve bir perakendeciden oluşan bir tedarik zinciri düşündük. Üreticimiz yine ürün belirsizliğine ve perakendecimiz ise talep güncelleme hakkına sahiptir. Bunlara göre, dönemin başında üretici perakendecinin sipariş miktarını almadan ön üretime başlar. Perakendeci tahmin güncellemesini yaptıktan sonra toplam siparişini üreticiye bildirir. Ön üretim miktarı siparişi karşılıyorsa üretici bu miktarı gönderir fakat bu yeterli değilse kalan miktarı, toptan satış kontratlarına göre, dış kaynaklı tedarik etmek ve perakendeciye teslim etmek zorundadır. Bu model için amaçlarımız, perakendeci için optimum sipariş miktarını bulmak, üretici için ön üretim miktarını bulmak ve toptan satış fiyatını bulmaktır. Bunları dikkate alarak, öncelikle perakendeci için açık bir formül bulduk. Üretici için bulduğumuz formül ise sadece nümerik olarak incelenebilir bir formüldü ve bu toptan satış fiyatının açık şekilde bulunmasını da etkiledi. Modelimiz için sayısal bir analiz sunarak çeşitli sonuçlar elde ettik.

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b_i	unit backordering cost for retailer *i*, charged for the units which are short at the end of the demand period, where $i = 1, 2$
c_i	unit purchasing cost for retailer *i*, where $i = 1, 2$
c_1	per unit production cost before the forecast update is realized
c_2	per unit outsourcing cost after the forecast update is realized
D_i	demand of retailer *i* during demand period, where $i = 1, 2$
F	random variable representing the forecast update
$F(.), f(.)$	cdf and pdf of forecast update
$F_{i,0}$	forecast of demand for each retailer *i* at the beginning of the lead time period, where $i = 1, 2$
$F_{i,1}$	forecast of demand for retailer *i* at the beginning of the demand period, where $i = 1, 2$
$G(.), g(.)$	cdf and pdf of residual market uncertainty
h_i	holding cost per unit keeping at location *i* at the end the demand period, where $i = 1, 2$
L	lead time period for the centralized supply chain
r	per unit retail price
u	total order quantity for the warehouse
u^*	optimum total order quantity for the warehouse
u_i	stock amount that is sent from warehouse to retailer *i*, where $i = 1, 2$
u_a	manufacturer's advance production quantity
u_a^*	manufacturer's optimal advance production quantity
v	random variable representing the market uncertainty
$v_{i,0}$	demand uncertainty for retailer *i*, where $i = 1, 2$
$F_{v_{i,0}}(.)$ & $f_{v_{i,0}}(.)$	cdf and pdf of demand uncertainty
$v_{i,1}$	market uncertainty for retailer *i*, where $i = 1, 2$
$F_{v_{i,1}}(.)$ & $f_{v_{i,1}}(.)$	cdf and pdf of residual market uncertainty
w	per unit wholesale price
w^*	manufacturer's optimal wholesale price

y	retailer's total order quantity
$y^*(\mu)$	retailer's optimal total order quantity given the forecast update
z	random variable denoting the yield uncertainty ratio
$Z(.)$, $z(.)$	cdf and pdf of yield uncertainty
μ	realization of the forecast update F

1. INTRODUCTION

Supply chain management is an extensive area and includes strategic, tactical, and operational management decisions. It consists of the movement and storage of raw materials, work-in process inventory, and finished products from point of origin to final customers. The aim of a supply chain is to provide products of the right quality, at the right time and in the right amount. However, supply chains are subject to different types of uncertainties. Early studies in inventory theory emphasize mostly demand uncertainty, but in recent researches supply uncertainty is also taken into consideration. In literature, supply uncertainty is divided into three main categories: random disruptions, yield uncertainty and stochastic lead times. Disruption occurs when the supply process of a company is interrupted, so it cannot receive and send any items until the disruption is over. Yield uncertainty occurs when the amount provided by the supplier as a random variable which either depends on the quantity ordered or is independent of it. The last type of supply uncertainty is stochastic lead-times which is a random amount of time spent until the exact amount of order received by the retailer. In this thesis, we generate two different models and both of them have yield uncertainty which caused by the manufacturer or warehouse.

Moreover, since supply chains grow globally, sharing of supply risk becomes more important against supply uncertainty in supply chain management. In this view, supply risk is shared centralized or decentralized. Decision of centralized or decentralized can mitigate the supply risk. In centralized environment, i.e. centralized systems, can be controlled centrally, or information about distribution centers can be monitored centrally. In decentralized systems, all players such as retailer, supplier, manufacturer or warehouse etc. control themselves and either little or no information about other players in the system. In supply systems, two opposite effects have been detected, risk *pooling* effect, which occurs when inventory held at one echelon at centralized systems, and *risk diversification* effect, which occurs when inventory is held at decentralized different locations. In our study, in the first model, we set up a centralized supply chain with one warehouse and two retailers. Unlike the first model, we have a decentralized system with a supplier and a retailer in the second model.

In addition to supply uncertainty and centralized or decentralized decision making, in many supply chains, the upstream company (supplier) must make an adjustment to the demand of the downstream company (retailer) so that retailers wait until the last possible minute to place their order after receiving a better forecast information. The last minute ordering is used in various industries such as automotive, apparel and electronics. Supply network starts to work after receiving advance demand information for related retailers, and if it is necessary, additional batch is produced or outsourced in order to satisfy the demand. Through this process, in comparison with respect to revenue management, retailers are in more advantageous situation rather than suppliers. Since suppliers encounter time constraints, their production cost increases. In our second model, we have a similar system that works in the same wholesale price before and after the forecast evolution between a supplier and a retailer.

In our first model, we analyze a centralized system that consists of one warehouse and two retailer model with supply uncertainty. In order to improve customer service levels and decrease lead times, global companies that have several distribution centers are usually operated centrally. Despite the fact that supply chain problems in One-Warehouse Multiple Retailer (OWMR) are investigated for many years, studies focusing on the effect of supply uncertainty are still needed due to unsolved different problems. One of our aims in this thesis is to analyze a supply chain which is provided by an unreliable warehouse, i.e. a warehouse may not satisfy the demand. Our model is a specialized version of OWMR, which consists of one warehouse and two identical retailers. Retailers are assumed to serve in a close location to each other and similar customer segments. Therefore, their unit backorder and holding costs are not different from each other. Furthermore, warehouse does not intend to keep stock since related holding cost of warehouse charges much more than retailers. At the beginning of each ordering period, the retailers place their orders according to their forecast information to the warehouse and warehouse starts preparing to supply their orders. We assume that there is no fixed cost associated with any order. On the other hand, at the end of the lead time warehouse observes yield ratio which is a random variable distributed as uniform or beta, etc. As a result, due to unreliable warehouse, retailers may not be able to receive the quantity that they order. The core of this research is to investigate the total order amount allocation and their inventory costs for the retailers. At the end of the lead time, warehouse provides and allocates total quantity ordered by the retailers so that their inventory costs are minimum. After receipt of the shipment by the retailers, customer demand occurs at the retailer level and consequently holding and backorder costs are charged. Demand values for the retailers over the planning horizon are assumed to be stochastic because of market uncertainty which is a realized after the demand period. This system is operated by a central decision maker process in the warehouse and as it has the full information on stock levels, cost, and forecast parameters of the retailers. The warehouse decides how much to satisfy the total demand and allocates these amounts to the retailers taking into account a total system cost consideration. In this model, there is only one decision maker which is the warehouse. Under these circumstances, in any period, when placing an ordering and making an allocation decision, the warehouse aims to minimize the total system wide cost associated with supply uncertainty structure. In this model, R simulation software and Matlab software are used to get numerical analysis.

In the second model, we develop a decentralized supply chain with a single manufacturer with a supply uncertainty, i.e. yield uncertainty and a single retailer. Moreover, the retailer has an opportunity to update her order after receiving a better demand information that is also known as last minute ordering and not new in some industries such as apparel industry. In recent studies and surveys claim that most of the industries increase their just-in-time purchases and reduce buying associated with long term forecasts. One aim in this part of investigation is to analyze a decentralized supply chain with an unreliable manufacturer and advance demand information. In the supply system, after retailer setting its own retailer price, both manufacturer and retailer agree on a wholesale price and sign the formal procurement contract. Secondly, the manufacturer makes a decision on how much to produce in advance with a unit price cost before the receiving the order amount from the retailer and starts producing this advance amount. Based on the contract and after the realizing forecast uncertainty, retailer buys a product from the manufacturer before the actual demand takes place. We assume that there is not any fixed ordering cost. Since the manufacturer has to satisfy all order quantities after the forecast update, if it is necessary, the manufacturer outsources the remaining batch. This is reasonable given the time constraint that manufacturer is faced. At that point, outsource production is more costly than own production and the manufacturer should observe the difference without reflecting its current prices. The main feature of this division is to decide how much the manufacturer should produce to maximize its profit as well as retailer profit. Demand values for the retailer is obtained from forecast and market uncertainty. Unlike the previous model, both forecast and market uncertainty are assumed to random variables. Before the demand period, the retailer obtains forecast update after a market research. Finally, market uncertainty is realized and retailer satisfies demand at a fixed retailer price. Additionally, we make an assumption that the wholesale price is determined between own production cost and retailer price to allow both the manufacturer and the retailer make profit. This model is operated under the decentralized decisions, so the manufacturer and retailer do not have detailed information of each other. In this setting, the wholesale price, the manufacturer's advance production quantity and the retailer's order quantity are determined subsequently. We formalized the all decisions made by using backward induction algorithms. Therefore, we firstly sort out the retailer's problem since it is only affected by the wholesale price. And then, the manufacturer's advance production problem is resolved due to the fact that it is highly dependent on the retailer's order decision and the wholesale price. Lastly, we show the wholesale price affects all other decisions. The retailer and the manufacturer target to maximize their profits after their sales season. Like the first model, in order to get numerical solution we use R simulation software and Matlab software.

To the best of our knowledge, there is a growing tendency to develop inventory models with supply uncertainty in centralized and decentralized supply chains in the literature. Also, in recent years, models with advance demand information is another focused area in inventory management. In our study, in the first model, based on results and analysis we make three major contributions. Firstly, we model a centralized supply chain under supply uncertainty. In this model, we have one warehouse and two identical retailers. Secondly, we make the system characterization which provides computational procedure for the total production quantity and total system-wide cost for the whole system. Lastly, we present a numerical study to comprehend better understanding of the cost minimization.

In the second model, we investigate the decentralized supply chain with supplier uncertainty and advance demand information. Our contributions in this part are as follows. Firstly, we model the decentralized supply chain under supply disruptions and with advance demand information and forecast update. Secondly, we provide formulations for the retailer's order amount and expected profit as well as the manufacturer's advance production quantity and expected profit. At that point, we make a numerical analysis for the wholesale price. Finally, we present a numerical study for all studies of this part.

The remainder of this thesis is organized as follows. In Chapter 2, we examine the literature about supply uncertainties, centralized and decentralized supply chains, and advance demand information and forecast update. In Chapter 3, we analyze a centralized one warehouse and two-retailer model with supply uncertainty. In Chapter 4, we study a decentralized supply chain with one manufacturer and one retailer model with supply uncertainty, advance demand information and forecast update. Finally, Chapter 5 concludes the thesis, summarizes our research and discusses future research directions.

2. LITERATURE SURVEY

In this thesis, we develop one centralized (one warehouse and two-retailer supply chain with supply uncertainty) and one decentralized (one warehouse and one retailer) supply chain with supply uncertainty and advance demand information models. Hence, our focus is mainly based on supply uncertainty, centralized or decentralized decision making and advance demand information or forecast update.

2.1. Supply Uncertainty

Supply uncertainty is widely discussed topic in the literature and divided into three main categories, which are random disruptions, stochastic lead times, and yield uncertainty.

2.1.1. Random Disruptions

Random disruptions in the supply chain models refer to the models with supply uncertainty in two different, randomly occurring available or unavailable states. Random disruptions may occur due to a wide range of reasons like natural disasters, labor strikes and so on. In the literature, modeling different random disruption and ordering policies are commonly studied. Parlar and Berkin (1991) analyze the inventory problem where the supply availability and unavailability last for a random duration. Under the main assumption that supply and no supply periods are randomly distributed, they build an objective function to solve optimal order quantities by using the concepts of renewal theory. Furthermore, Parlar and Perry (1995) consider the availability of the supplier as a two state continuous time Markov Chain where one state corresponds to availability and the other state corresponds to unavailability. Again using the renewal theory, they find out long term average cost for learning the state of the supplier. They also examine that waiting time for the next ordering. Following studies by Parlar and Perry (1996) extend the findings by Parlar and Berkin (1991) and incorporate multiple-supplier problems under one all suppliers with similar characterizations assumption, and they propose a model and observe that as the number of suppliers get larger, the problem get close to classical EOQ models. Mohebbi and Hao (2006) extend Parlar and Berkin's (1991) contributions by adding a non-zero random lead time under random demand. Mohebbi (2004) model the availability of suppliers as a renewal process under the random demand. Likewise, Özekici and Parlar (1999) analyze the optimality of base stock policies under the fixed ordering cost is zero and (s,S) policy when fixed ordering cost is non-zero. Gullu *et al.* (1997) examine Bernoulli distributed supplier availability and indicate that the order-up-to level policy is optimal and find out a newsboy-like formula in order to find optimal inventory levels. Further studies Gullu *et al.* (1999) extend their Gullu *et al*. (1997) paper and study partial availability under a multi-echelon system. They provide a newsboy-like formula for optimal order-up-to levels.

On the other hand, some studies take random disruptions both at supplier and retailer consideration. For example, Qi *et al*. (2009) focus on a problem with both retailer and supplier face random disruptions and they conclude the cost function is quasi-convex. In 2010, Qi *et al*. extend their earlier study to determinate the location of the retailers with respect to supplier location, and show that it is important to consider supply disruptions at the supply chain design phase. Sargut and Qi (2012) analyze two echelon supply chain problem which considers random disruptions and stochastic lead times at supplier, depending on both availabilities of retailer and supplier. They indicate that if the retailer becomes more vulnerable, the effect of supplier vulnerability on the optimal order level decreases.

Comparison of the suppliers that is in the different availability position (i.e. available or unavailable) is another research area. Chopra *et al*. (2007) design a single period model in which one supplier is cheaper but is subject to random disruptions while the other supplier is more expensive but much more reliable. They offer that retailer should order from the cheaper supplier, and remaining from the expensive supplier. Schmitt and Snyder (2012) extend this work over multiple periods. They compare two cases with respect to optimal order quantities and find that using single-period approximation causes increase in cost and distorts order quantities. Likewise, Qi (2013) considers the same problem and incorporate retailer wait time and fixed ordering cost into the model.

As a result, in our both models only warehouse or manufacturer is subject to supply disruption and we assume that there is no adverse environmental effect to cut off the supply during the analysis period. Therefore, we are informed about random disruptions but not use in our models.

2.1.2. Stochastic Lead Time

Stochastic lead time is also studied extensively in the inventory management literature. When the lead time is stochastic, exact amount is received by the companies but waits in a random amount of time. Kaplan (1970) analyzes a dynamic inventory model with stochastic lead time and shows that multidimensional optimization problem with a stochastic lead time converges to one-dimensional optimization problems. Anupindi *et al*. (1996)'s is an extensive research of Kaplan (1970), taking non-stationary stochastic lead time in consideration and provide several heuristics for different lead time distributions.

Some researchers consider optimal order size and order policy with stochastic lead times. For this perspective, Liberatore (1978) conducts a study of continuous deterministic demand and develop a stochastic lead time inventory model. This study is shown to be a generalization of the EOQ model with backordering. Alp *et al*. (2003) analyze optimal lot size quantity problem considering stochastic lead times and they present a model with charging fixed cost every order time regardless of the batch size. They further devise a dynamic programming for order quantities and time. In 2005, Tang and Grubbström study manufacturing and remanufacturing systems with random lead times. The study compares the cycle ordering policy and dual sourcing ordering policy for both manufacturing and remanufacturing systems with respect to economic inferences related to lead time length.

Moreover, lead time distribution and its relationships with other parameters like demand; cost etc. is another research area. Hayya *et al*. (2005) consider lead time distributed with exponentially. They demonstrate the cost of variability according to lead time distribution using different parameters. They also point out that if the system reaches to zero inventory, then lead time variability will be reduced. In a recent paper, Hoque (2013) models a vendor-buyer integrated production-inventory system and examines whether when lead time is normal distributed as it is more appropriate distribution type associated with cost examination rather than exponential. In 2013, Hoque extends his previous work by adding equal and unequal batches of a lot. In this research, a manufacturer-buyer integrated inventory model with a normal distribution of lead times is developed. According to this model, there is an important cost reduction compared to lead times of exponential distribution.

Finally, some researchers examine economic inferences in inventory systems with random lead time. Janakiraman and Roundy (2004) model an inventory system with periodic review, stochastic demand and lead times. They assume that crossing order not occurs during the lead time and they mainly focus on lost sales. Their main important result is the lost sales cost as a function of inventory on hand plus inventory in transit. This is commonly used to compute optimal base stock level for inventory systems with lost sales. Banerjee and Meitei (2010) analyze single period inventory model with stochastic demand and lead time and investigate profit analysis. After numerical examples, they explore that optimal inventory policy is different depending on whether stochastic lead time and selling price decline are taken into consideration. Xiao *et al*. (2010) analyze a game theoretic three-stage supply chain model with demand uncertainty by considering ordering, wholesale price and random lead time. They conclude that when the lead time increases, the retailer wants more unit of product and the manufacturer declines the unit wholesale price. They also show that higher unit holding cost has an impact to decrease optimal lead time. Nasri *et al*. (2012) study researching quality adjusted EOQ model with stochastic lead times by taking flexibility and quality improvement into consideration and provide two main significant results: One is that flexibility and quality can be improved by using setup reduction, the second outcome is strategic investment for flexibility and quality improvement in view of stochastic lead time.

To sum up, in our both models we assume that lead time is negligible since warehouse or manufacturer and retailers are in a close location and their lead time only includes the production time not the transportation time. Also, we assume that transshipment is not available between the retailers. Hence, we are not interested in lead times whether stochastic or not.

2.1.3. Yield Uncertainty

The third group of supply uncertainty is yield uncertainty which occurs when the amount provided by the supplier is a random variable. For this perspective, Yano and Lee (1995) give a comprehensive review about determining lot sizes when production or procurement yields are random. They focus on system costs, yield uncertainty and performance with respect to random yields. Gerchak *et al*. (1988) analyze a finite horizon problem with stochastic demand and random yield. They prove that order point is not related to yield uncertainty for a single period problem. However, if the number of period increases the problem converges to infinite problem and the order point of infinite problem is not smaller than the order point of the certain problem. Konak *et al*. (2011) develop a dynamic approach for inventory model to study batch sizes in a single or multi-stage problem with random yields in each stage. Gupta and Cooper (2005) examine different stochastic comparison techniques in relations to production yield management and find that a yield rate is smaller in the convex order it provides higher expected profit. The study also identifies the properties of the yield rate distributions. Recently, Zhang *et al*. (2014) provide a detailed periodic review lot sizing problems with yield uncertainty, disruptions, and capacity restrictions in a model. As a result, in our both models, we consider yield uncertainty caused by the manufacturer or warehouse. In centralized model, warehouse has an allocation problem due to yield uncertainty and in the decentralized model, according to their contracts, manufacturer deals with full supply since retailer wants to receive total ordered amount without regarding yield uncertainty.

Yield uncertainty with capacity restriction for manufacturer or warehouse is an important subject in inventory management. Capacity restriction prevents manufacturer to produce more as well as hold extra unit in inventory. In this subject, Wang and Gerchak (1996) analyze a periodic review inventory problem with both random yield and variable capacity. They prove that minimum expected cost function is quasi-convex and then find a single critical point for the initial stock level for the finite horizon problem for each period. Therefore, if the initial stock is greater than this critical point, the optimal production is zero; else it is greater than zero. Their research also shows that finite problem is converged to infinite problem. Erdem an Özekici (2002) conduct a study of a system where uncertain yield because of randomness in capacity of supplier. Their main finding is that a base stock level policy is optimal for single, multiple, or infinite problems and the order-up to level is related to state of the environment. Grasman (2009) works on dynamic and linear programming transformations to develop a method to optimize policy decision process, given capacity restrictions and yield uncertainty. They suggest that their order policy results improve decision making capabilities of complex system environments. In 2008, Abdel-Malek *et al*. extend a previous work on a capacitated multi-product model with adding yield uncertainty for general distribution functions and show that variance of yield distribution plays a significant role in determining optimum order quantities. Yan (2012) studies a decentralized model where the unreliable suppliers decide their own capacity levels and the retailers make their orders based on suppliers' capacity levels. What Yan (2012) finds that supplier capacity levels and retailers' order quantity from each supplier remain same when expected shortage for each supplier. Consequently, in our models, we consider yield uncertainty but not with capacity restriction for manufacturer or warehouse as we are mainly interested in how yield uncertainty affects the ordering and producing policies.

In order to prevent and mitigate of the effect of yield randomness, there are several techniques are offered. Some of them are adding extra cost and preventing extra cost. Grosfeld-Nir *et al*. (2000) model a system where retailer receives full order to satisfy its demand. If it does not (i.e. there may be defective units), then further manufacturing and inspection are required, so consequently the optimal production cost depends on inspection cost. Similarly, Agnihothri *et al*. (2000) examine the effects of yield uncertainty making decision on order quantity when there is tardiness cost associated with lateness. They demonstrate that the optimal order policy is defined by some specific yield distributions and service level constraints. Furthermore, Sloan (2004) considers a Markov decision process model with random demand and binomial yield. The study is mainly focus on yield uncertainty caused by equipment condition and aims to choose simultaneously equipment maintenance schedule to optimize total system cost. After they compare simultaneously and sequentially choosing, the former is beneficial more than the latter with respect to total cost of the system. Also, Lin and Hou (2005) study an inventory system with random yield variable and consider yield variability reduction through capital investment. They utilize the optimal capital investment and ordering policies to obtain minimum expected total system cost. Cheong and Song (2013) consider improving performance of the uncertain supply with respect to supply risk. They analyze the newsvendor purchasing decision under yield uncertainty and find that partial supply risk information is needed to evaluate the order quantity to improve the overall profit, but there is need more information to get better results for stochastic conditions. In our first model, there is not any penalty related with yield uncertainty for the warehouse. However, the second model is similar to Grosfeld-Nir *et al*. (2000) model, but we include outsourcing cost for unfilled order quantity not the inspection cost, which causes a decrease in total expected profit of the manufacturer due to high outsourcing cost.

Supplier comparison and supplier selection is another area in this topic. Yang *et al*. (2007) model a system with a buyer that facing random demand decides order quantities from its several suppliers with different yield rates and prices. They provide a mathematical formulation for the buyer's profit maximization problem and suggest a method similar to newsvendor approach to select appropriate suppliers using Newton search method. Likewise, Yan *et al*. (2012) approach a supply chain problem incorporating a retailer and several unreliable suppliers. They investigate how yield uncertainty affects the supplier selection and retailer's expected profit in both independent random yields and correlated random yields. They show that retailer might do better under unreliable suppliers rather than reliable suppliers with respect to profit maximization. Yan and Wang (2013) extend Yan *et al*. (2012) by adding proof of the optimality of sourcing in EOQ model with general random yield. As a result, in both models in this thesis, we have only one manufacturer or warehouse and retailer is supplied from them. Thus, there is not any selection decision for the warehouse and manufacturer.

Lastly, cost analysis and price decision are extensively studied in inventory supply networks with yield uncertainty. Grosfeld-Nir *et al*. (2005) analyze a two-echelon inventory system with deterministic demand and yield uncertainty. Their objective is to minimize the total setup and variable production costs. They prove that the expected cost of any production policy can be determined by solving a finite set of linear equations for any yield distributions. Li and Zheng (2006) analyze the joint replenishment and pricing control mechanism under the uncertain demand and yield. They derive a threshold value to determine the optimal replenishment policy, i.e. it is optimal to produce if and only if the starting inventory in a period below the threshold value. Their main conclusion is that the threshold of replenishment does not depend on the yield uncertainty in the single period problem. Moreover, Li *et al*. (2012) conduct an inventory model with deterministic demand and random yield under wholesale price contract. They examine ordering time and the profit losses of the supply chain members due to random yield supply. Their results are provided under more generalized yield distribution so can be applicable in many industries. In a recent paper, Li *et al*. (2014) examine the remanufacturing and pricing decision under random demand and yield. They work on two strategies, first remanufacturing then pricing and first pricing then remanufacturing. They determine optimal remanufacturing quantity and price for both situations. They also show that first remanufacturing is more desirable for low manufacturing cost and market price sensitivity than first pricing. Consequently, in centralized model, we consider reducing total system-wide cost and are not interested in determining purchasing, holding and backordering costs. Moreover, in decentralized model, we would like to increase expected profits and make a decision for wholesale price.

To sum up, we examine the all supply uncertainty types and develop our models with yield uncertainty. In the first model, there is an allocation problem caused by yield uncertainty between the retailers; however, central warehouse solves this problem so that total system cost will be minimum. In the second model, retailer places its order without knowing anything about yield ratio of manufacturing. At the same time, manufacturer starts producing in advance to satisfy the retailer’s order. However, at the end of production lead time it faces the yield ratio and should satisfy the full order of retailer according to their contract. In this setting, both the retailer and the manufacturer aim to maximize their profit.

2.2. Centralized or Decentralized Decision

The other discussed subject in this thesis is conditions considered in centralized or decentralized decision making-process. In the literature, centralized and decentralized systems are either separately or jointly analyzed. We firstly discuss the centralized and decentralized supply chain systems in this section.

There has been a growing body of research focusing on centralized and decentralized settings as well as their comparisons. For example, Chen and Chen (2005) conduct a study of the effects of centralization and decentralization on multi-item replenishment environment. They model centralized and decentralized systems and show the optimal properties of the both models to minimize the cost. Saharidis *et al*. (2009) compare the centralized and decentralized replenishment policies controlling base stock, echelon base stock, and partial backordering with regards to maximize the total profit. They conclude that the performance of each strategy depends on the environment and variations in the certain system parameters on the optimal system parameters which are found by using Markov chains. Likewise, Duan and Liao (2013) provide the optimal replenishment policies of capacitated centralized and decentralized supply chains strategies using a simulation and optimization techniques. They conclude that it is valuable to implement centralized system and coordinate the decentralized system so that each component can be benefited.

In addition, Leung (2010) generates a model to optimize the inventory decision in multi-stage models in a multi firm supply chain. The study derives optimal solutions to the three and four stage models using perfect squares method. In addition, sharing the coordination is beneficial between the firms. In 2011, Leung extends the earlier works by adding backorders linear and as well as fixed cost to some/all retailers. Optimal solutions for three and four stage models are also again obtained. Schmitt *et al*. (2012) examine the optimal system design in multi-location system and analyze the expected costs and cost variances associated with centralized or decentralized settings, which are subjected to supply disruptions. They show that if the demand is deterministic and supply is disrupted, decentralized system is optimal because of cost variance reduction. However, when the demand is random and supply has no disruptions, centralized system is optimal due to risk pooling effect. They also express that if supply and demand are random, decentralized is the best option for the risk-averse firm. In a recent paper, Jafarian *et al*. (2014) analyze the causes of bullwhip effects in centralized and decentralized systems in a three stage supply chain with a manufacturer, a wholesaler, and a retailer using response surface method. They conclude that wholesaler’s order batching is the main cause for the decentralized system and chain’s order batching is the main cause for the centralized system.

Similarly, Anderson and Bao (2010) compare the price competition in both centralized and decentralized systems. They indicate that market shares play a critical role in price competition and the coefficient variation of market shares determines the best supply chain design strategy. Jeong (2012) considers the full return contract in a supply chain with a risk-free manufacturer and a risk neutral retailer under a partial information sharing policy. The order quantity is decided by the retailer and rebate price is determined by the manufacturer. The study provides an optimal solution for both centralized and decentralized supply chain networks. Egri (2013) discusses Jeong (2012)'s finding in making a correction in an equation in the model and concludes the condition of information privacy can be violated in partial information sharing. In our models, we would like to investigate centralized and decentralized supply systems and their effects separately not together.

In our first model, we have a centralized supply chain system, i.e. there is a central authority to make decisions for the whole system so that minimize the total system operating cost. Seo *et al*. (2002) examine an inventory system with a centralized warehouse and multiple retailers, which each facility uses continuous-review batch ordering policy. They develop the order risk policy to calculate effective stock policies under the assumption that warehouse send all orders to the retailers for a fixed lead time. Later work by Rezapour and Farahani (2010) model a centralized supply chain system under deterministic price-dependent demands and rival chain. They analyze the both system to obtain an equilibrium. They also provide multiple choices for facility location with respect to inventory and shipment decisions. More recently, Ivanov *et al*. (2014) consider a centralized multi-stage supply network which is defined as a non-stationary dynamic system. They demonstrate that multi-objective problem formulation is not only cost oriented optimization, but also advantageous to find a feasible solution for even unbalanced supply and demand cases that do not consider capacity situations. Such formulation also leads retailer to improving service level. As a result, in our first model, we analyze a centralized system with random demand and yield uncertainty and central warehouse decide on allocation between two independent retailers so that minimize the system wide cost.

The second model is decentralized supply chain system, i.e. each facility can make its own decisions individually. Under certain conditions, such as lack of material and information flow, decentralized supply chain networks are considered to be more effective for each facility than centralized supply chain systems. Lee and Billington (1993) construct a decentralized supply chain model for a company to control their material flow and examine the new system with regard to cost-benefit analysis. Chen (1999) analyzes a system where both material and information flows in the supply chain are subject to temporary delay. In order to prevent the system from this postponement, the optimal order or production decisions are made individually for each division in the same firm. Additionally, the same study also provides the impact of accurate customer demand information on supply chain performance. Furthermore, Waller *et al.* (2006) examine the effects of cross docking on the changes in the inventory level in a decentralized supply chain and find that if the number of stores increases the benefit of cross docking decreases because of low safety stock level and high service level constraints. Similarly, Sheu and Son (2008) characterize the replenishment policy deviations on decentralized supply chain performance. They firstly model a benchmark and examine how different settings deviate from this benchmark using simulation model techniques. They show that performance of decentralized supply chain is connected with several types of replenishment policies and policy deviations.

Game theory becomes an important tool to analyze decentralized supply chain with conflicting objectives. Wang *et al*. (2004) approach to decentralized supply chain systems by using game theory methods to derive Nash equilibrium. They consider a model with a supplier and n retailers. If the supplies are sufficient, non-cooperative behavior is obtained; in contrast, if they are insufficient, there is non-cooperative behavior and competition between retailers as well as supplier. Likewise, Bernstein and Federgruen (2005) examine condition equilibrium between among competing retailers under stochastic demand, depending on their own retail price for both competing and noncompeting retailers. Morales and Vermeulen (2009) study a game in which an action taken bounds for other players' potential choices and indicate that this system has equilibrium for inventory cost minimization.

Moreover, Liu *et al*. (2007) construct a Stackelberg game to analyze the effects of pricing and lead time decisions in a decentralized supply network composed of a retailer and a supplier. By the assumption, supplier is the leader and retailer is the follower. In comparison, decentralized system is generally less efficient than centralized system according to double marginalization effect. They also provide a detailed study related to market and operational factors and show that both retailers and suppliers should primarily focus on their internal factors rather than choosing a coordination strategy. Subramanian *et al*. (2006) study contracts in decentralized settings, and add quantity flexibilities to mitigate the bullwhip effects using simulation and optimization techniques. In a recent paper, Ruiz-Benitez and Muriel (2014) investigate a wholesale-price contract and a buy-back contract between a manufacturer and a retailer that faces random demand. It is shown that higher profits are obtained when manufacturer and retailer act as an individual player without considering any information about consumer returns and supply chain profits increase since retailer has an extra logistic cost related to consumer return. In our second model, we have a similar model as Liu *et al*. (2007) to determine the wholesale price. We develop a Stackelberg game to analyze wholesale price and determine how yield uncertainty affects the expected profit and wholesale price.

Information sharing is one of the most important subjects in decentralized supply networks. Generally, players in the system want to know the other's information but do not want to share their own information with each other. In this view, Li (2010) analyze the inventory behavior in decentralized supply chains. Optimal inventory policies are constructed for each firm with or without information sharing and are investigated for n-stage systems, different batch ordering policies, fixed set up cost and Markovian customer demand. Nativi and Lee (2012) analyze the information sharing strategies in a decentralized setting with an application of radio frequency identification technique (RFID). They compare the systems with or without RFID. They demonstrate that profits are increased with more returns. In our second model, information sharing between the manufacturer and the retailer is not available, since retailer has no information about the manufacturer's production yield ratio. Retailer places its order and wants to receive full amount.

2.3. Advance Demand Information and Forecast Update

The last subject that we mention in this thesis is advance demand information and forecast update. In many supply networks, adjustments between the upstream firm and the downstream firm are appreciated. A well-known example is quick response, which defined as a process that provides faster response to retailer orders. Fisher and Raman (1996) discuss quick response and a demand estimation method in a response-based production model and find an increase in profits. Similarly, Iyer and Bergen (1997) examine the effects of quick response in apparel industry between a manufacturer and a retailer channel and show that quick response may not be better off for the manufacturer.

Furthermore, Wheng and Parlar (1999) study the effect of advance ordering. They investigate joint-stocking and prior sale discount decisions for multiple effects of coordination. Likewise, Gallego and Özer (2001) develop a model with advance demand information and show that optimal results for the a state-dependent (s,S) policy where set-up cost is zero and base stock policy is set-up for the zero cost. Donselaar *et al*. (2001) examine inventory reduction using advance demand information in different settings in a project with based supply chains. They conclude that advance demand information is valuable for large projects with high profit expectations. As a result, in the second model in this thesis, advance demand information is used by the manufacturer. At the beginning of the period, manufacturer starts producing advance units according to its forecast. At the end of the production lead time, retailer places order quantity which is known the forecast update and the manufacturer faces yield ratio, outsourcing may be required so that satisfy the full order of retailer.

The impact of supply chain contracts in decentralized supply systems are also analyzed in the literature. Donohue (2000) analyzes supply contracts with forecast update for two different production types; one is cheap and requires a long lead time and the other is expensive and quick turnaround. The study focuses on return option to achieve channel coordination in this paper. Pareto optimal solution for two different production types is also examined. Lariviere and Porteus (2001) study supply chain contracts for only wholesale price, not consider administrative cost in wholesale price. They point out coefficient of variation as the main factor to check changes in wholesale prices and profits in decentralized supply network. Cachon (2003) examine supply chain contracts such as buy-back contracts, flexibility contracts, and revenue sharing contracts. Kleindorfer and Wu (2003) analyze the integrating long term and short term contracts with respect to transaction costs. Consequently, in our decentralized model, we focus on a price-only contract, i.e. there is no change in price before and after the forecast updates and determines the wholesale price in different scenarios.

Tang *et al*. (2003) develop a system called advance booking discount that enables customers to promise to their orders at a discount price before the selling season. They determine the benefits of that program and evaluate the discount price so that retailer profit is to be maximum level. Cachon (2004) investigates the inventory risk in a supply chain with a newsvendor model. The study shows that a retailer who orders total quantity before the selling season faces inventory risk in a push contract setting, nevertheless, retailer who orders totally after the demand realization then manufacturer faces inventory risk. Advance purchase discounts with two wholesale prices present intermediate allocation risk for both retailer and manufacturer. Özer *et al*. (2006) analyze dual purchase contract with a forecast update. The earlier study conducts a wholesale price contract where a retailer orders after forecast update and a dual purchase contract where a retailer orders before and after forecast update. They also provided that market uncertainty is a key factor to decide a wholesale price contract. The second model in this thesis can be considered as an extension of this paper but it differs as we also incorporated yield uncertainty in our models. In a recent study, Bakal and Karakaya (2013) analyze a decentralized supply network with a retailer that has an opportunity to update demand after observing perfect demand information and a manufacturer. They investigate not only ordering policies but also benefits of flexibilities for retailer and manufacturer.

Sharing demand forecast information is one of the significant topics between manufacturers and retailers. Lariviere and Cachon (2001) investigate ways of the sharing demand forecast between a manufacturer and a retailer and show that albeit costly, the manufacturer with a high demand forecast is interested in sharing forecast information with the supplier. Chen (2003) provides a review for asymmetric information models discussion in supply chain. Özer and Wei (2006) study credible forecast information sharing between a supplier and a manufacturer and develop two different contracts. One is nonlinear capacity reservation where manufacturer may fee to reserve capacity and the second is with advance purchase contract. They conclude that the capacity reservation contract provides the supplier to determine the forecast information of the manufacturer as well as advance purchase contract signals the forecast information for the manufacturer. Tan *et al*.

(2007) develop a model with imperfect advance demand information and demonstrate that the optimal ordering policy for state-dependent order-up-to type and optimal order level is an increasing function of the advance demand information size. They also provide an upper bound for the order-up-to level for different periods. In 2009, Tan *et al*. examine a system that includes one ordering and one rationing decision under two demand classes with imperfect advance demand information. They solve the rationing problem and suggest solution methods for ordering problem using Monte Carlo simulation. In the centralized model, information sharing is not available since warehouse is the only decision maker for the whole system. On the contrary, in the decentralized model, the manufacturer needs to observe the forecast update of retailer for his decision. Therefore, information sharing between the manufacturer and the retailer is required.

Finally, order timing is analyzed in inventory systems with advance demand information. Ferguson *et al*. (2005) consider the impact of forecast updating on the order timing when partial information updating is presented. They investigate supply chain members' timing choices and optimal decisions under partial demand information updating. Taylor (2006) studies order timing without a forecast update and retailer sets the selling price. He also examines the effect of retailer sales and information asymmetry. He explores that if information is symmetric, the manufacturer prefers to sell late since demand is not affected by the sales effort. On the other hand, retailer has superior information about market demand, and then manufacturer prefers to sell early.

3. A CENTRALIZED ONE WAREHOUSE-TWO RETAILER MODEL WITH SUPPLY UNCERTAINTY AND FORECASTS EVOLUTION

In the first part of Chapter 3, we briefly describe the forecast/demand relationship and the cost model of the centralized supply chain. In the second part, we characterize the optimum order quantity and the cost functions of the system. Finally, in the third part, we have a numerical analysis section including method, results and managerial insights.

3.1. Model Description

3.1.1. Development of the Demand and the Forecast Model

In our supply chain model, there are two similar but independent retailers. These retailers buy products from the warehouse associated with their forecasts and the warehouse decides how much to send to each retailer if there is not enough product on hand. We assume that warehouse has no capacity restriction.

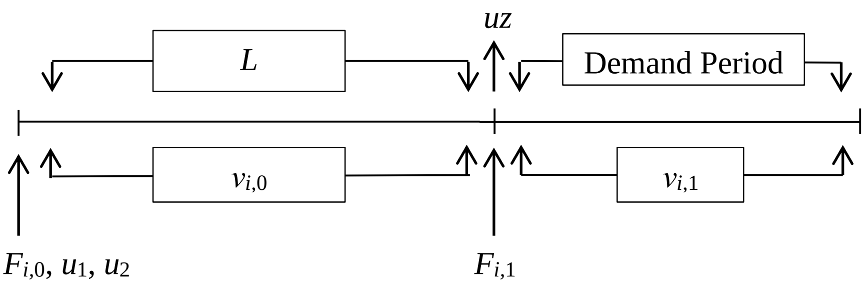

Figure 3.1. Time and Order of Events of the Centralized Model.

At the beginning of the lead time period (L), retailers place their own orders (u_i). Let $F_{i,0}$ be the forecast of demand for each retailer i at the beginning of the lead time period After L, retailers face demand uncertainty ($v_{i,0}$), however after getting better information on demand, they determine their new forecast variables. Let $F_{i,1}$ be the forecast of demand for retailer i at the beginning of the demand period . The variable ($v_{i,1}$) represents the residual market uncertainty and is realized after the demand period. It is critical to calculate demand variable for each retailer. D_i is the demand of retailer i during demand period, where i = 1, 2 for the retailers. In this model, D_i is correlated to $F_{i,1}$ with $v_{i,1}$ and $F_{i,1}$ is related to $F_{i,0}$ and $v_{i,0}$. The forecasts and demand relations are as follows:

$$F_{i,1} = F_{i,0} v_{i,0}$$

$$D_i = F_{i,1} v_{i,1} = F_{i,0} v_{i,0} v_{i,1}$$

We assume that the multiplication of the two independent log-normal distributions is a log-normal distribution. Therefore, according to properties of log-normal distribution demand is also distributed log-normal due to multiplication of log-normal random variable.

Forecast variables ($F_{i,0}$ & $F_{i,1}$) are values that determined by the retailers with different techniques such as Holt's method, Winter's method etc. We model demand uncertainty ($v_{i,0}$) as a continuous random variable with a cdf and pdf of $F_{v_{i,0}}(.)$ & $f_{v_{i,0}}(.)$ respectively and similarly residual market uncertainty ($v_{i,1}$) as a continuous random variable with a cdf and pdf of $F_{v_{i,1}}(.)$ & $f_{v_{i,1}}(.)$ respectively. We assume that both of them are distributed log-normal and $E[V_0] = E[V_1] = 1$ with a standard deviations 0.03, 0.1 and 0.5.

3.1.2. Development of the Cost Model

Our model is a single period centralized supply chain model consisting of one warehouse and two retailers. In this model, warehouse observes inventory level of retailers, then retailers place their orders with respect to their forecast variables and warehouse sends their orders if the total order is available. If warehouse does not have enough inventory to satisfy the total order amount of the retailers, then it allocates on hand inventory to the retailers in order to minimize the total system-wide cost.

First of all, as it is seen in the Figure 3.2., at the beginning of the lead time period (L), retailer-1 and retailer-2 place their orders of size u_1 and u_2 from the warehouse. We assume that their initial inventory is zero. Let u denote the total order quantities for each retailer (u_i). Warehouse starts to satisfy total order of retailers (u). At the end of L, warehouse observes yield uncertainty ratio and faces on hand inventory size as uz. Let z be

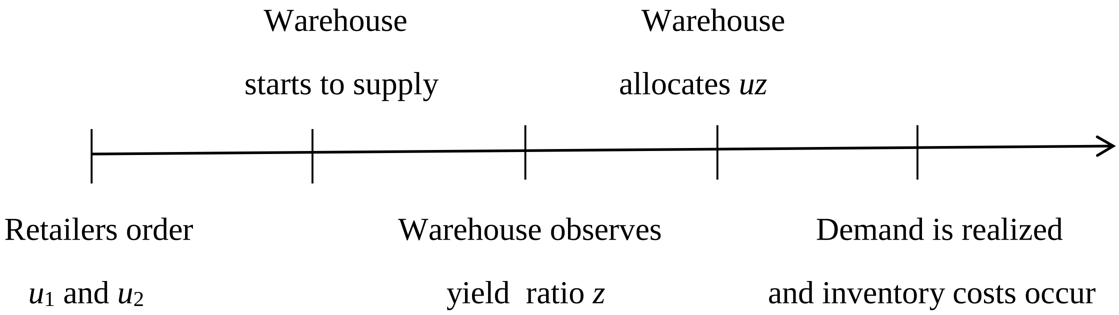

Figure 3.2. Sequence of Events.

the random variable denoting the yield uncertainty ratio. We assume that z has a cdf and a pdf of $Z(.)$ and $z(.)$ respectively and distributed between 0 and 1. Moreover, at that point, warehouse allocates u_1 and u_2 such that $u_1 + u_2 = uz$. Retailers pay c per each unit received. After receipt of the allocation quantities by the retailers, demands are observed, and relevant end-of-period inventory holding and backordering costs are charged at each location. At that point, we also assume that there is no transshipment between the retailers. Let h_i be the holding cost per unit keeping at location i at the end the demand period, where i = 1, 2 for the retailers. Additionally, b_i is the unit backordering cost for retailer i, charged for the units which are short at the end of the demand period.

Warehouse supplies at most amount of u, when z=1; therefore, backordering cost is not charged at warehouse location. Furthermore, holding cost is only charged at the retailer level. Since holding cost at the warehouse is much more than at retailer level, warehouse does not intend to keep inventory at its location. We also assume that lead-time from the warehouse to the retailers is negligible as they are in a close location and there is no transshipment between the retailers. Accordingly, $L_i\ (u_i \mid F_{i,1})$ be the single period holding and backorder cost in the demand period for retailer i given the forecast state of the beginning of demand period.

$$L_i\left(u_i \mid F_{i,1}\right) = \sum_i b_i\left(D_i - u_i\right)^+ + h_i\left(u_i - D_i\right)^+ \qquad (3.1)$$

where $D_i = F_{i,1}\nu_{i,1}$

For notational convenience, J_1 ($F_{i,1}$,u,z) be the minimum expected cost of operating the whole system.

$$J_1\left(F_{i,1},u,z\right) \ = \ min \ \{\sum_i u_i c_i + L_i\left(u_i \mid F_{i,1}\right)\} \tag{3.2}$$

Because of yield, demand and residual market uncertainty are random variables, we define the J_0 ($F_{i,0}$, u) be the expected value of J_1 ($F_{i,1}$,u,z). We solve this equation so that minimize the total system-wide cost over $u \geq 0$.

$$J_0\left(F_{i,0},u\right) \ = E_{z,v \ i,0}\left[J_1\left(F_{i,1},u,z\right) \ \right] \tag{3.3}$$

To solve this problem, we consider a dynamic programming (DP) method, which for breaks major problems down into simpler smaller problems.

$$\begin{aligned} &Min \quad \sum_i u_i c_i + L_i\left(u_i \mid F_{i,1}\right) \\ &\text{s.t.} \quad u_1 + u_2 = uz \\ &\qquad u_1,\ u_2 \ \geq 0. \end{aligned} \tag{3.4}$$

In order to obtain u_i^* for i =1, 2, we check Karush Kuhn Tucker (KKT) conditions to solve J_1 ($F_{i,1}$,u,z).

3.2. Derivation of the Optimum Order Quantity and the Cost Functions

Total cost equation of the system is as follows:

$$\begin{aligned} &\text{Min} \quad c_1u_1 + c_2u_2 + L_1(u_1 \mid F_{1,1}) + L_2(u_2 \mid F_{2,1}) \\ &\text{s.t.} \quad u_1 + u_2 = uz \\ &\qquad u_1,\ u_2 \ \geq 0. \end{aligned}$$

If we write the KKT conditions of this equation:

$$\text{Min} \quad \sum_i u_i c_i + L_i\left(u_i \mid F_{i,1}\right) + \lambda \left(u_1 + u_2 - uz\right)$$

$$\text{s.t.} \quad u_1 + u_2 = uz$$

$$u_{1,}\ u_2 \geq 0. \tag{3.5}$$

The KKT conditions can be stated as follows. The vector *u*, total of u_1 and u_2, is the optimal solution of the equation if the following conditions hold.

$$u_1 + u_2 = uz \qquad u_{1,}\ u_2 \geq 0 \tag{1}$$

$$c_i + \frac{\delta}{\delta u_i} L_i\left(u_i \mid F_{i,1}\right) + \lambda = 0 \qquad \lambda \text{ unrestricted} \tag{2}$$

$$vu_1 = 0$$

$$vu_2 = 0 \qquad \text{v} \geq 0 \tag{3}$$

Before proceeding any further, we should discuss the optimality conditions. Firstly, the first condition states that candidate solution point must be feasible; constraints of the problem must satisfy it. This is usually known as the *primal feasibility*. Second condition is referred as the *dual feasibility*, corresponds to feasibility of a problem related to the original problem. Finally, the last condition is known as *complementary slackness*. Here λ and v are called Lagrangian multipliers corresponding to constraints $u_1 + u_2 = uz$ and $u_{1,}$ $u_2 \geq 0$ respectively.

Let us take the derivative of the Equation 3.5 with respect to u_1 and u_2 respectively.

$$\sum_i u_i c_i + L_i\left(u_i \mid F_{i,1}\right) + \lambda \left(u_1 + u_2 - uz\right)$$

$$c_1 + \frac{\delta}{\delta u_1} L_1\left(u_1 \mid F_{1,1}\right) + \lambda = 0$$

$$c_2 + \frac{\delta}{\delta u_2} L_2\left(u_2 \mid F_{2,1}\right) + \lambda = 0$$

$$u_1 + u_2 = uz$$

where $L_i\left(u_i \mid F_{i,1}\right) = h_i E\left[\left(u_i - F_{i,1}v_{i,1}\right)^+\right] + b_i E\left[\left(F_{i,1}v_{i,1} - u_i\right)^+\right]$

$$= h_i \int_0^{u_i/F_{i,1}} (u_i - F_{i,1}v_{i,1})\mathrm{d}F_{v_{i,1}}(v) + b_i \int_{u_i/F_{i,1}}^{\infty} (F_{i,1}v_{i,1} - u_i)\mathrm{d}F_{v_{i,1}}(v)$$

Putting this variable into the derivative Equation 3.5, we obtain the followings:

$$\frac{\delta}{\delta u_i} L_i(u_i \mid F_{i,1}) = h_i \int_0^{u_i/F_{i,1}} \mathrm{d}F_{v_{i,1}}(v) - b_i \int_{u_i/F_{i,1}}^{\infty} \mathrm{d}F_{v_{i,1}}(v)$$

$$= h_i \int_0^{u_i/F_{i,1}} \mathrm{d}F_{v_{i,1}}(v) - b_i(1 - \int_0^{u_i/F_{i,1}} \mathrm{d}F_{v_{i,1}}(v))$$

$$= - b_i + (h_i + b_i) \int_0^{u_i/F_{i,1}} \mathrm{d}F_{v_{i,1}}(v)$$

Hence, we complete the derivation of the equation and it is as follows:

$$c_i + \frac{\delta}{\delta u_i} L_i\left(u_i \mid F_{i,1}\right) + \lambda = 0$$

$$c_i - b_i + (h_i + b_i) \int_0^{u_i/F_{i,1}} \mathrm{d}F_{v_{i,1}}(v) + \lambda = 0$$

At that point, it is easily seen $\int_0^{u_i/F_{i,1}} \mathrm{d}F_{v_{i,1}}(v)$ is the cdf of the $v_{i,1}$ and this leads us to obtaining a new and familiar equation. Let $F_{vi,1}$ be the cdf of the $v_{i,1}$.

$c_i - b_i + \lambda + (h_i + b_i)\ F_{v_{i,1}}(\frac{u_i}{F_{i,1}})$ =0 and then we get ;

$$F_{v_{i,1}}(\frac{u_i}{F_{i,1}}) = \frac{b_i - c_i - \lambda}{h_i + b_i}$$

Let us take the inverse of both side of the equation and we get:

$$u_i = F^{-1}_{v_{i,1}}\left(\frac{b_i - c_i - \lambda}{h_i + b_i}\right)F_{i,1}$$

$$u_1 + u_2 = uz \qquad (3.6)$$

Accordingly, Equation 3.6 is a Newsboy-like formula. In the Equation 3.6 u_i is a function of λ and in this function $F_{i,1}$ is the forecast variable of the retailers. This equation provides warehouse allocating the on hand inventory to the retailers under the $u_1 + u_2 = uz$ constraint.

At the beginning of the period, if $F_{i,1}$, u and z are given then λ can be solved mathematically. Since we have not enough information to sort out this problem at the beginning of the period, we reconsider that retailers have similar properties and are located in close areas; and assume that their purchasing, holding and backordering costs are identical. Also, $v_{1,1}$ and $v_{2,1}$ have identical and independent distribution for two retailers. These assumptions are important to calculate easily optimal order quantity and the total system-wide cost. Therefore,

- $c_1 = c_2 = c$, $h_1 = h_2 = h$, $b_1 = b_2 = b$ and
- $v_{1,1} = v_{2,1} = v_1$.

If we rewrite the u_i equation according to above assumption:

$$u_i = F^{-1}_{v_1}\left(\frac{b - c - \lambda}{h + b}\right)F_{i,1}$$

is a kind of newsboy equation again. Let us write this into the constraint equation, i.e. sum of individual order quantities is total order quantity, of the original problem.

$$u_1 + u_2 = uz$$

$$F^{-1}_{v_1}\left(\frac{b - c - \lambda}{h + b}\right)F_{1,1} + F^{-1}_{v_1}\left(\frac{b - c - \lambda}{h + b}\right)F_{2,1} = uz$$

$$F^{-1}_{v_1}\left(\frac{b - c - \lambda}{h + b}\right) = \frac{uz}{F_{1,1} + F_{2,1}}$$

Finally, we find that u_1 and u_2 values:

$$u_1 = \frac{uz}{F_{1,1}+F_{2,1}} F_{1,1} \tag{3.7}$$

$$u_2 = \frac{uz}{F_{1,1}+F_{2,1}} F_{2,1} \tag{3.8}$$

It is observed if the parameter values of the retailers are equal, warehouse allocates the orders as a proportion of the total inventory according to their forecast variables after getting demand uncertainties. It also leads us to help to solve the optimal order quantity and the total cost problems of the system. Let us put u_1 and u_2 values to the original equation.

$$\begin{aligned} \text{Min} \quad & \sum_i \frac{uz}{F_{1,1}+F_{2,1}} F_{i,1}c_i + L_i\left(\frac{uz}{F_{1,1}+F_{2,1}} F_{i,1} \mid F_{i,1}\right) \\ \text{s.t.} \quad & \frac{uz}{F_{1,1}+F_{2,1}} F_{1,1} + \frac{uz}{F_{1,1}+F_{2,1}} F_{2,1} = uz \\ & \frac{uz}{F_{1,1}+F_{2,1}} F_{1,1} \ , \quad \frac{uz}{F_{1,1}+F_{2,1}} F_{2,1} \geq 0 \end{aligned} \tag{3.9}$$

where $F_{i,1} = F_{i,0}\, v_{i,0}$

We will calculate the $E_{z,v_{i,0}} \left[\sum_i \frac{uz}{F_{1,1}+F_{2,1}} F_{i,1}c_i + L_i\left(\frac{uz}{F_{1,1}+F_{2,1}} F_{i,1} \mid F_{i,1}\right)\right]$ over $u \geq 0$.

In order to make it easier, we can separate the equation in two parts:

$$\begin{aligned} J_0(F_{i,0}, u) = {} & E_{z,v_{i,0}} \left[\sum_i \frac{uz}{F_{1,1}+F_{2,1}} F_{i,1}c_i \right] \\ & + E_{z,v_{i,0}} \left[\sum_i L_i\left(\frac{uz}{F_{1,1}+F_{2,1}} F_{i,1} \mid F_{i,1}\right) \right] \end{aligned} \tag{3.10}$$

We start to characterize for the first part of the equation. This part is the total purchasing cost part of the retailers.

$$E_{z,v_{i,0}}\left[\sum_i \frac{uz}{F_{1,1}+F_{2,1}} F_{i,1}c_i\right] = u\,c\,z\,\underbrace{\left(\frac{F_{1,0}v_{1,0}}{F_{1,0}v_{1,0}+F_{2,0}v_{2,0}}+\frac{F_{2,0}v_{2,0}}{F_{1,0}v_{1,0}+F_{2,0}v_{2,0}}\right)}_{=1}$$

$$= E\,[u\,c\,z\,] = u\,c\,E\,[z]$$

The second part of the equation, which is the part of the total inventory cost of both retailers, is as follows:

In particular, we define α_i to get simpler equation:

$$\alpha_i = \left(\frac{z}{F_{1,0}v_{1,0}+F_{2,0}v_{2,0}} F_{i,0}v_{i,0}\right)$$

$$E_{z,v_{i,0}}\left[\sum_i L_i(u\alpha_i \mid F_{i,1})\right] = h_i \int_0^{u/\alpha_i} (u\alpha_i - F_{i,0}v_{i,0}v_{i,1})\mathrm{d}F(D_i)$$

$$+b_i \int_{u/\alpha_i}^{\infty} (F_{i,0}v_{i,0}v_{i,1} - u\alpha_i)\mathrm{d}F(D_i)$$

where $D_i = F_{i,0}\, v_{i,0}\, v_{i,1}$.

After getting these simpler equations, we need to check the first order conditions on with respect to "*u*" of the total inventory cost equation. In this equation, "*u*" is our decision variable and in order to calculate "*u*" we need to take derivative of this.

$$= c\,E\,[z] + \sum_i E[h\alpha_i \int_0^{u_i/\alpha_i} \mathrm{d}F(D_i) - b\alpha_i \int_{u_i/\alpha_i}^{\infty} \mathrm{d}F(D_i)\,]$$

$$= c\,E\,[z] + \sum_i E[-b\alpha_i + (h+b)\alpha_i \int_0^{u_i/\alpha_i} \mathrm{d}F(D_i)]$$

$$= c\,E\,[z] + \sum_i E[\alpha_i(-b+(h+b)P\{F_{i,0}v_{i,0}v_{i,1} \le u\alpha_i\})]$$

where P {.} is the probability function.

In order to check convexity of the function: we take the second derivative of the equation J_0 ($F_{i,0}$, u) with respect to "u" and we obtain:

$$\sum_i E_{z,v_{i,0}}[\frac{z}{F_{1,0}v_{1,0}+F_{2,0}v_{2,0}}F_{i,0}v_{i,0}((h+b)f_{v_{i,1}}(\frac{uz}{F_{1,0}v_{1,0}+F_{2,0}v_{2,0}}))(\frac{z}{F_{1,0}v_{1,0}+F_{2,0}v_{2,0}})]>0$$

then the equation has a local minimum point at u and we can say that the equation is convex.

Due to convexity of the problem, we can equal to this function to 0 so that we find the total optimal order quantity.

$$0 = c\,\mathrm{E}\,[z] + \sum_i E_{z,v_{i,0}}[\frac{z}{F_{1,0}v_{1,0}+F_{2,0}v_{2,0}}F_{i,0}v_{i,0}(-b+(h+b)$$

$$P\{v_{i,1} \le \frac{uz}{F_{1,0}v_{1,0}+F_{2,0}v_{2,0}}\})] \tag{3.11}$$

Consequently, Equation 3.11 provides us to find the optimal order quantity for the centralized supply chain system consists of one warehouse and two identical retailers. We also calculate the expected total system-wide cost after obtaining optimum total order quantity. In the next section, we make a numerical analysis related to these equations and analyze the results under different scenarios. Due to complication of the problem, we use R and Matlab software to get numerical results and examine them.

3.3. Numerical Analysis

In this part, we carry out a numerical analysis on the behavior of the order up to levels and expected total system-wide cost with respect to different parameters such as yield uncertainty ratio, purchasing, and backordering costs.

We analyze the optimal order up to levels and expected total system-wide cost in different parameter combinations in a single period. The different patterns for forecast values, backordering costs, purchasing cost, yield uncertainty ratio, demand and market uncertainty parameters are summarized in Table 3.1. The total forecast of the retailers at the beginning of the period $F_{1,1} + F_{2,1} = 100$ and we assume that forecast of the Retailer-1 is slightly more than forecast of the Retailer-2. Therefore, $F_{1,1}$ =60 for Retailer-1 and $F_{2,1}$ =40 for Retailer-2 are determined. Since it is a centralized supply chain system, the warehouse observes the inventory levels of the retailers and supplies them according to their forecast values. In the literature, yield uncertainty is seen generally beta or uniform distributed since it can be taken values between 0 and 1. We make an assumption that yield uncertainty is beta distributed and has different expected values (0.5, 0.75, 0.9 and 1) and variances 1/12. At that point, we need to obtain α and β as the shape parameters according to following expected value and variance equations and are shown in the Table B.1. in Appendix B:

$$E[z] = \frac{\alpha}{\alpha + \beta}$$

$$Var[z] = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

According to equations; calculation is as follows:

$$0.5 = \frac{\alpha}{\alpha + \beta} \quad \text{and} \quad \frac{1}{12} = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

From that, we obtain $\alpha = 1$ and $\beta = 1$. They are also applied for different expected values of yield uncertainty ratios.

Furthermore, demand and market uncertainty values are distributed log-normal. Their means are 1 and variances are (0.03, 0.1 0.5). For these values, we find the μ and σ values, which are shown in Table B.1. in Appendix B, for each parameters according to following equations:

$$E[v] = exp(\mu + 1/2\ \sigma^2)$$

$$Var[v] = exp(2\mu + \sigma^2)(exp(\sigma^2) - 1)$$

The ways of calculation of demand and market uncertainty values are like:

$$1 = exp(\mu + 1/2\ \sigma^2)$$

$$exp(1) = \mu + 1/2\ \sigma^2$$

$$0 = \mu + 1/2\,\sigma^2$$

$$-\mu = 1/2\,\sigma^2$$

and after getting this equation we put μ the variance equation into:

$$0.03 = exp\,(2\mu + (-2\,\mu))(exp(-2\,\mu) - 1)$$

We take the results as μ = -0.014779 and σ = 0.171927. Similarly, we present other parameters as in this way.

Finally, the holding cost of the retailers is fixed to h = 1 and unit purchasing cost c = (2, 4) for both of the retailers.

In order to obtain optimal order up to levels and expected total system-wide cost, we use Monte Carlo simulation technique on R and Matlab softwares, because getting exact solutions takes much more time to calculate and therefore we develop two codes for both equations using both softwares are found in C.1. and C.2. in Appendix C; however we only analyze the R results in this section. When we calculate optimum order quantity "*u*", we use trial-error method since our equation cannot be solved exactly. Our main aim is to find *u* that is closest to 0.

3.3.1. Algorithm of Finding the Optimum Order Quantity

1. Step: We enter an order quantity *u* to the system with constant variables.

```
u<-71
h<-1
b<-10
c<-4
F1<-60
F2<-40
```

2. Step: At that point, we generate 10^6 random variables for yield (beta, α, β), demand and market uncertainty (lognormal, mu, sigma) variables.

```
n<-1000000
alfa<-0.9375
beta<-0.3125
```

mu<-(-0.047655)

sigma<-0.308723

z <-rbeta (n,alfa,beta)

v_{10}*<-rlnorm(n,mu,sigma)*

v_{20}*<-rlnorm(n,mu,sigma)*

v_{11}*<-rlnorm(n,mu,sigma)*

v_{21}*<-rlnorm(n,mu,sigma)*

<u>3. Step:</u> Code works for inner calculations for probability and inner cost functions as the following:

w_1<-(v_{11}-$uz/(F_1v_{10}+F_2v_{20})$<=0)

w_2<-(v_{21}-$uz/(F_1v_{10}+F_2v_{20})$<=0)

h_0<-cz

h_1<-$F_1v_{10}z/(F_1v_{10}+F_2v_{20})(-b+(h+b)w_1)$

h_2<-$F_2v_{20}z/(F_1v_{10}+F_2v_{20})(-b+(h+b)w_2)$

F<-$h_0+h_1+h_2$

<u>4. Step:</u> Finally, we find mean and standard error of the total equation in order to calculate optimum order quantity.

OptU<-mean(F)

StError<-sd(F)/sqrt(n)

print(OptU)

print(StError)

<u>5. Step:</u> If *OptU* has the closest value to 0 while it has also sign change, it is the optimum point, and else we try these steps for different *u* value.

Moreover, total system cost code works in a similar fashion. When we obtained *u* from the above algorithm we find the total system cost.

3.3.2. Algorithm of Finding the Total System Cost

<u>1. Step:</u> We enter the optimum order quantity and constant variables to the system.

u<-71

h<-1

b<-5

c<-4

F_1*<-60*

F_2*<-40*

<u>2.Step:</u> 10^6 random variables for yield (beta α, β), demand and market uncertainty(lognormal, mu, sigma) variables are generated.

n<-1000000

alfa<-0.9375

beta<-0.3125

mu<-(-0.047655)

sigma<-0.308723

z <-rbeta (n,alfa,beta)

v_{10}*<-rlnorm(n,mu,sigma)*

v_{20}*<-rlnorm(n,mu,sigma)*

v_{11}*<-rlnorm(n,mu,sigma)*

v_{21}*<-rlnorm(n,mu,sigma)*

<u>3.Step:</u> Code facilities the inner calculations; $Cost_0$ is the purchasing cost function, $Cost_{11}$&$Cost_{12}$ are the holding and backordering cost for the retailer 1 and $Cost_{21}$&$Cost_{22}$ are the holding and backordering cost for the retailer 2.

$Cost_0$*<-ucz*

$Cost_{11}$*<-hpmax(*$uzF_1v_{10}/(F_1v_{10}+F_2v_{20})-F_1v_{10}v_{11}$*,0)*

$Cost_{12}$*<-bpmax(*$F_1v_{10}v_{11}-uzF_1v_{10}/(F_1v_{10}+F_2v_{20})$*,0)*

$Cost_{21}$*<-hpmax(*$uzF_2v_{20}/(F_1v_{10}+F_2v_{20})-F_2v_{20}v_{21}$*,0)*

$Cost_{22}$*<-bpmax(*$F_2v_{20}v_{21}-uzF_2v_{20}/(F_1v_{10}+F_2v_{20})$*,0)*

TotalCost<-$Cost_0+Cost_{11}+Cost_{12}+Cost_{21}+Cost_{22}$

<u>4. Step:</u> Lastly, total system cost is calculated with standard errors.

ExpCost<-mean(TotalCost)

StError<-sd(TotalCost)/sqrt(n)

print(ExpCost)

print(StError)

Since we do not get the exact solution of the problem and using Monte Carlo simulation technique, we take standard error parameters and examine it in the numerical analysis section. If we work codes for *u* =71, we get the results;

```
> print(OptU)
[1] -0.0172054
> print(StError)
[1] 0.001479337
```

and *u* = 72;

```
> print(OptU)
[1] 0.02200059
> print(StError)
[1] 0.001513927
```

As we can see that optimum order quantity is between 71 and 72, we choose the *u*=71 since it is more close the 0 value. The total system cost is for *u* = 71;

```
> print(ExpCost)
[1] 456.1771
> print(StError)
[1] 0.04962756
```

The scenarios which are generated by the combinations of forecast values, yield uncertainty ratio, backorder and unit purchasing cost, demand and market uncertainty parameters are listed in Table D.1. in Appendix D.

Table 3.1. Parameter Patterns.

	c=2		c=4	
	var (v)	b	var (v)	b
E[z]=0.5	0.03	5	0.03	5
	0.03	10	0.03	10
	0.1	5	0.1	5
	0.1	10	0.1	10
	0.5	5	0.5	5
	0.5	10	0.5	10
E[z]=0.75	0.03	5	0.03	5
	0.03	10	0.03	10
	0.1	5	0.1	5
	0.1	10	0.1	10
	0.5	5	0.5	5
	0.5	10	0.5	10
E[z]=0.9	0.03	5	0.03	5
	0.03	10	0.03	10
	0.1	5	0.1	5
	0.1	10	0.1	10
	0.5	5	0.5	5
	0.5	10	0.5	10
E[z]=1	0.03	5	0.03	5
	0.03	10	0.03	10
	0.1	5	0.1	5
	0.1	10	0.1	10
	0.5	5	0.5	5
	0.5	10	0.5	10

We analyze 48 different scenarios for each of order up to levels and expected total system-wide cost. We calculate the optimum order up to level, total system cost, standard error of the total system cost and % penalty cost of ignoring supply uncertainty for each scenario. Penalty cost parameter helps us investigating yield uncertainty effects on total system-wide cost. After acquiring optimum cost for any variables, then we recalculate them with the situation without any yield uncertainty. As it is expected that optimum costs with z=1 values cost are bigger than original optimum cost, we subtract the original optimum cost from it, then divide by original optimum cost with z=1 values and lastly multiply with 100 in order to get % penalty cost. Penalty in cost is calculated as the following in mathematically:

$$\%\ Penalty\ Cost = \frac{\text{Optimum Cost (E[z]=1)}values - \text{Optimum Cost}}{\text{Optimum Cost (E[z]=1)}values}\ 100$$

Results that we obtained from the different scenarios are listed as the following:

As unit purchasing cost of the products increases with similar yield uncertainty ratio and other constant parameters such as holding and backorder cost, optimum order quantity decrease.

Table 3.2. Total System Cost when Constant Yield Uncertainty Ratio.

		c=2	c=4
z=0.5	Var(v)	0.03	
	Backorder Cost	b=5	b=5
	u*	143	103
	Total Cost	333.6631	455.6919

Also, while yield uncertainty approaches to certain values, i.e. when *E[z]* is close to 1, optimum order quantity again decreases.

Table 3.3. Optimum Order Quantity when Increasing Yield Uncertainty Ratio.

		c=2		c=4	
	Var (v)	0.1		0.1	
	Backorder Cost	b=5	b=10	b=5	b=10
z=0.5	u*	141	203	87	151
z=0.75	u*	108	142	71	113
z=0.9	u*	94	118	65	98
z=1	u*	93	117	65	98

Interestingly, when c=2, increasing in variance of the demand and market uncertainties causes slight reduction in optimum order quantities for b=5, but increase in optimum order quantities for b=10. One example of this is as follows:

Table 3.4. Optimum Order Quantity with Different Backorder Cost and c=2.

		c=2					
	Var (v)	0.03		0.1		0.5	
	Backorder Cost	b=5	b=10	b=5	b=10	b=5	b=10
z=0.5	u*	143	195	141	203	115	203
z=0.75	u*	110	133	108	142	86	144
z=0.9	u*	98	111	94	118	73	118
z=1	u*	97	111	93	117	72	117

This is because if the backordering cost (b=10) is much more than the unit purchasing cost it stimulates to increase in total cost when there is a short of stock. Hence, warehouse accepts the risk of out of stock under small backordering costs.

In contrast, when c=4, for both b=5 and b=10 there is a decrease in optimum order quantities when increase in variance of demand and market uncertainties.

Table 3.5. Optimum Order Quantity with Different Backorder Cost and c=4.

		c=4					
	Var (v)	0.03		0.1		0.5	
	Backorder Cost	b=5	b=10	b=5	b=10	b=5	b=10
z=0.5	u*	103	151	87	151	50	128
z=0.75	u*	87	114	71	113	38	95
z=0.9	u*	80	101	65	98	35	80
z=1	u*	80	101	65	98	33	79

Moreover, we observe that total system cost continues to increase when variance goes up since uncertainty in demand increases. In addition, confidence of total system cost is measured with standard error parameters. If the variances increase, standard error of the total cost also increases and confidence of measurement decreases in spite of generating 10^7 variables.

Table 3.6. Total System Cost with Different Variance Parameters when c=2 and c=4.

		c=2			
z=0.5	Var(v)	0.03		0.1	
	u*	143	195	141	203
	Total Cost	333.6631	487.5113	351.3156	517.0054
	St.ErrorTC	0.033060	0.075952	0.047509	0.093568

		c=4			
z=0.5	Var(v)	0.1	0.1	0.5	0.5
	u*	87	151	50	128
	Total Cost	464.9764	691.1113	483.2769	791.8041
	St.ErrorTC	0.050289	0.093749	0.125499	0.231257

Finally, penalty cost value is an important point in this analysis. It compares the actual total cost and total cost with z=1 values. Therefore, penalty cost value provides us interpreting yield uncertainty ratios over total system-wide cost. As it is seen from the results, penalty cost decreases when yield has more certain values and under increasing demand and market uncertainties.

Table 3.7. Penalty Cost (%) in Increasing Yield Uncertainty Ratio.

		c=2			
	Var(v)	0.03		0.1	
	Backorder Cost	b=5	b=10	b=5	b=10
z=0.5	% Penalty Cost	7.0569	15.7018	5.6366	12.4458
z=0.75	% Penalty Cost	1.8825	4.4168	1.3253	3.2569
z=0.9	% Penalty Cost	0.0208	0.0134	0.0286	0.0233

If we compare the penalty cost values among different unit purchasing costs, we observe that they have lower values in bigger purchasing costs because of lower ordering units.

Table 3.8. Penalty Cost (%) when c=2 and c=4.

		c=2		c=4	
	Var(v)	0.03		0.03	
	Backorder Cost	b=5	b=10	b=5	b=10
z=0.5	% Penalty Cost	7.0569	15.7018	1.0943	7.0504
z=0.75	% Penalty Cost	1.8825	4.4168	0.2247	1.5686
z=0.9	% Penalty Cost	0.0208	0.0134	0.0004	0.0027

In conclusion, we recommend some managerial insights. In such a supply chain environment, backordering cost is one of the key parameter affecting optimum order quantity as well as total system cost. Therefore, it should be estimated carefully by the central control mechanism. In particular, relationship between the backordering cost and unit purchasing cost should be identified extensively and should not be ignored totally. As it is seen from the results, if the backorder cost and purchasing cost differ much more, the warehouse accepts being stock out. The other critical parameter is variance of demand and market uncertainties. When variances have large values, although the optimum order quantity generally decreases, the total system cost increases, indicating that fluctuations in demand and market variances have a negative impact on total system costs despite the fact that warehouse decrease optimum order quantity. Finally, yield uncertainty ratio is the main factor affecting optimum order quantity and total system cost. Expectedly, more certain values present low optimum order quantities and total system cost. At that point, in order to prevent from uncertainty in yield there are some actions that can be taken, including providing supply much more than required, investing for the equipment and improving quality and maintenance techniques.

4. A DECENTRALIZED MANUFACTURER / RETAILER SUPPLY CHAIN SYSTEM WITH SUPPLY UNCERTAINTY AND ADVANCE DEMAND INFORMATION

In the first section of Chapter 4, we briefly describe the demand model and the wholesale price contract of the decentralized supply chain consisting of one warehouse and one retailer. In the second part, we characterize the retailer's optimum order decision, manufacturer's optimum advance production decision and the pricing decision. Finally, in the third part, we have a numerical analysis section including method, results and managerial insights.

4.1. Model Description

4.1.1. The Demand Model

In this decentralized environment, there is one manufacturer and one retailer. The retailer places order amount from the manufacturer before the demand period. The manufacturer produces and satisfies all orders with respect to their wholesale contract basis. At that point the manufacturer faces yield uncertainty after the production lead time, however, retailer is not interested in yield uncertainty of manufacturer.

After receiving quantities by the retailer, the demand D is realized and retailer satisfies customer demands if inventory on hand is enough. Let $r \geq 0$ be the retailer's fixed unit sales price in the market. Demand can be stated as a form of:

$D=Fv$

where F and v are random variables.

The retailer learns F, which is forecast update, before the demand period starts. Forecast update can be obtained after a market research, which indicates that F is continuous random variable with a cdf and pdf of F (.) and f (.), respectively. We use μ for the realization of F. In addition, v is a continuous random variable and represents the residual market uncertainty. It is obtained after realizing demand period. We assume that $E[v]=1$ and $v \geq 0$, hence, the mean demand before obtaining the forecast update is $\bar{\mu} = E[F]$. Also, G (.) and g (.) are the cdf and pdf of random variable v respectively.

The contract arranges the behavior of the companies and the resulting profits. In the next section, we analyze the wholesale price contract between the manufacturer and the retailer.

4.1.2. Wholesale Price Contract

The supply chain system consists of one manufacturer and one retailer in a decentralized model, i.e. each company makes its decision individually independent of the other. The order of events under the wholesale price contract is summarized in Figure 4.1.

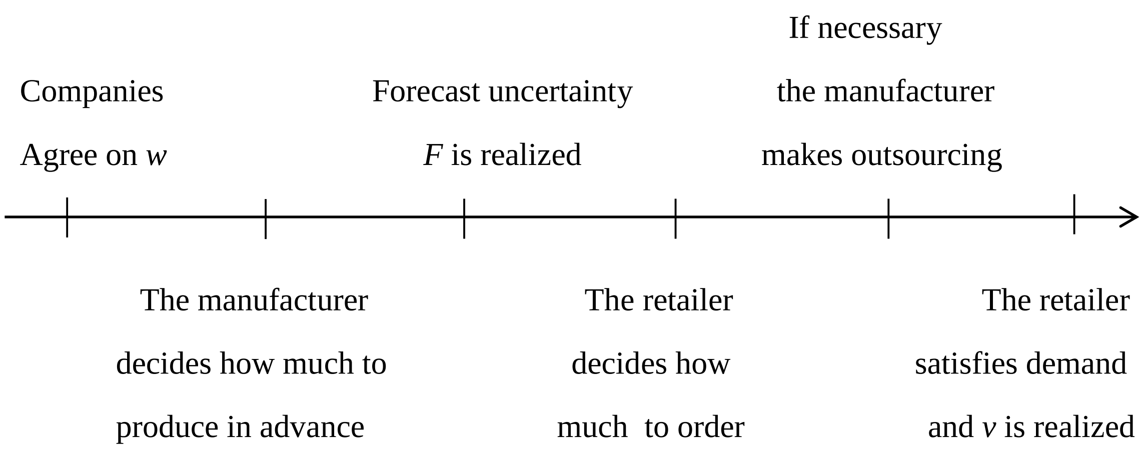

Figure 4.1. Time and order of events under the wholesale price.

First of all, the manufacturer and the retailer both agree on wholesale price w and sign the contract. We assume that there is no process related to setting wholesale price at this point. Secondly, the manufacturer decides how much to produce in advance (u_a) with a per unit production cost c_1 before the retailer places its order. However, at the end of production lead time, the manufacturer faces yield uncertainty ratio (z), with an inventory $u_a z$ on hand. We assume that yield uncertainty ratio is a random variable and lies between 0 and 1. Let Z (.) and z (.) be the cdf and the pdf of z. Then, the retailer obtains the forecast update μ, which is the realization of F, and makes a decision how much to order from the manufacturer with the determined wholesale price. Accordingly, if on hand inventory at the manufacturer is not sufficient to satisfy the retailer's total order quantity ($u_a z \leq y$), then the manufacturer should make outsourcing for the remaining batch $(y - u_a z)^+$ with an outsourcing production cost of c_2. Due to short production lead time, orders should be ready in a quick way. Therefore, outsourcing is required at this stage, and we assume that unit production cost is smaller than outsourcing cost per unit, i.e. $c_1 \leq c_2$. Lastly, the retailer satisfies the demand from on hand inventory with a fixed price r and the residual market uncertainty v is obtained after demand period. Furthermore, we assume that $c_1 \leq w \leq r$; otherwise it is never profitable for the manufacturer to produce or retailer to place any order.

In order to solve this problem, we make three decisions. The first is for the wholesale price, the second is for the manufacturer's advance production quantity, and the last one is for the retailer's order quantity. We use the backward induction method in order to get optimal solutions, which mean that we solve first the last decision point. The manufacturer's production decision has no influence on the retailer's ordering policy since total amount ordered by the retailer should be supplied according to their contract agreement. Therefore, we first solve the retailer's ordering decision for a given wholesale price as it only dependent on the wholesale price. Accordingly, the manufacturer's advance production decision is related to the retailer's order decision and the wholesale price. Hence, next we solve the manufacturer's production problem. The wholesale pricing decision has an impact on all other decisions; therefore, we solve the optimal wholesale price last.

4.2. Optimal Order Decisions

4.2.1. The Retailer's Decision

If the given order quantity y and the forecast update μ, then the retailer' expected profit function is as follows:

$$\Pi^r (y, \mu) = rE_v [\min (y, \mu v)] - wy \qquad (4.1)$$

In order to obtain optimum order, we need to take derivative of both sides of the equation. Firstly, we make it simpler.

$$\prod^r(y,\mu) = rE_y[\min(y,\mu v)] - wy$$

$$\prod^r(y,\mu) = rE_y[y - \max(y - \mu v, 0)] - wy$$

$$= ry - rE_y[\max(y - \mu v, 0)] - wy$$

$$= ry - r\int_0^{y/\mu}(y - \mu v)g(v)dv - wy$$

Now, we take the derivative of this function with respect to *y*.

$$\frac{\delta \prod^r(y,\mu)}{\delta y} = r - r\int_0^{y/\mu} dG(v) - w$$

$$0 = r - rG_v(y/\mu) - w$$

$$y^*(\mu) = G_{v^{-1}}\left(\frac{r-w}{r}\right)\mu \quad (4.2)$$

Equation 4.2 provides the retailer to determine optimal order quantity to maximize its profit.

4.2.2. The Manufacturer's Decision

It is critical to initiate advance production before the forecast update for the manufacturer, since it is an important opportunity to reduce the production costs. On the other hand, that the retailer's order quantity is unknown is a serious obstacle at that point. The manufacturer also encounters excess inventory problem.

Let $u_a \geq 0$ units be the manufacturer's advance production quantity. After deciding how much to produce in advance, manufacturer starts producing and finally stock them due to fact that the retailer places orders only after the forecast update. Since the manufacturer faces yield uncertainty ratio *z*, on hand inventory may not be sufficient to supply the retailer's full order. Therefore, the manufacturer makes outsourcing the remaining units for a higher unit cost than its own production cost.

If the advance production quantity and the retailer's optimal order quantity are given, then the expected profit of manufacturer under the wholesale price is as follows:

$$\prod^{m}(w,u_a) = wE_F[y(F)] - c_1u_a - c_2E_{F,z}[y(F) - u_az]^+ \quad (4.3)$$

To get optimal advance production quantity for the manufacturer, we should make the Equation 4.3 simpler. Firstly, we define $\rho = G_{v^{-1}}(\frac{r-w}{r})$ for our calculations.

$$\prod^{m}(w,u_a) = wE_F[y(F)] - c_1u_a - c_2E_{F,z}[y(F) - u_az]^+$$

$$\prod^{m}(w,u_a) = w(\bar{\mu}\rho) - c_1u_a - c_2\int_0^1 \int_{u_az/\rho}^{\infty} (\mu\rho - u_az) f_\mu(\mu) d\mu f_z(z) dz$$

Then take the derivative of this function with respect to u_a:

$$\frac{\delta \prod^{m}(w,u_a)}{\delta u_a} = -c_1 + c_2\int_0^1 \int_{u_az/\rho}^{\infty} (z) f_\mu(\mu) d\mu f_z(z) dz$$

$$0 = -c_1 + c_2\int_0^1 z(1 - F(\frac{u_az}{\rho}) f_z(z) dz \quad (4.4)$$

Since F (.) is increasing on μ, we can conclude that Π^m (w, u_a) is concave in u_a and u_a^* is the solution of the first order condition. Unfortunately, we cannot obtain the exact optimal advance production quantity $u_a{}^*$, since the Equation 4.4 can only be solved in numerically.

Theorem 4.1. The manufacturer optimally produces $u_a{}^*$ with respect to Equation 4.4 units in advance. Moreover, $u_a{}^*$ is decreasing in c_1.

This theorem states the manufacturer's advance production decision. Part 1 shows the optimal advance production quantity. In part 2, as it is expected, the optimal production quantity increases with respect to the cost view from advance production. If the advance production cost is lower than outsourcing cost c_2, then the manufacturer intends to produce more. Moreover, it causes excess inventory.

4.2.3. Wholesale Pricing Decision

When characterizing the retailer's optimal order quantity and the manufacturer's advance production quantity, we have assumed that the wholesale price w is given. It is acceptable for retailers to take the wholesale price as given. In this part, we analyze the wholesale price setting scenario where the manufacturer decides it as a Stackelberg leader.

The Stackelberg leadership model is described as a strategic game in economics, where the leader firm moves first and the follower firms move sequentially. In our model, the manufacturer is the leader and the retailer is the follower. After the manufacturer sets the wholesale price, the retailer gives order amount to the manufacturer.

In order to obtain the manufacturer's optimal wholesale price w^*, we replace $u_a{}^*$ instead of u_a in Equation 4.3 and then we solve for:

$$w^* = \arg\max_w \prod^m (w, u_a{}^*)$$

$$\prod^m (w, u_a{}^*) = wE_F[y(F)] - c_1 u_a{}^* - c_2 E_{F,z}[y(F) - u_a{}^* z]^+$$

$$\prod^m (w, u_a{}^*) = w(\bar{\mu}\rho) - c_1 u_a{}^* - c_2 \int_0^1 \int_{u_a{}^* z/\rho}^{\infty} (\mu\rho - u_a{}^* z) f_\mu(\mu) d\mu f_z(z) dz$$

However, since we cannot find a simple equation for $u_a{}^*$, we cannot take the derivative of this equation with respect to w. Because, $u_a{}^*$ is dependent on $\rho = G_{v^{-1}}(\frac{r-w}{r})$ and we cannot find a relationship between $u_a{}^*$ and w. Therefore, only numerical computations can be used to interpret the wholesale price. In the numerical analysis of this section, after computations we comment on deciding the manufacturer's optimal wholesale price.

4.3. Numerical Analysis

In this part of the Chapter 4, we conduct a numerical analysis on the behavior of the manufacturer's optimal wholesale price, the retailer's optimal order quantity and the expected profit as well as the manufacturer's advance production quantity decision and the expected profit with respect to different parameters such as uncertainty random variables and various outsourcing cost. As we have stated earlier, the manufacturer's advance production quantity and the wholesale pricing decision can only be solved numerically. While conducting numerical analysis, we use Matlab and R simulation software.

We analyze this decentralized supply chain model in both view of the manufacturer and the retailer. The different patterns for random variable representing the forecast update, outsourcing cost, yield uncertainty ratio and residual market uncertainty ratio parameters are summarized in Table 4.1.

We consider that the forecast update F is distributed lognormal with a mean of 100 and with a mean of 200 and so that we can compare both situations with respect to the wholesale price and the expected profit and determine coefficient of variation (CV) of F as (0.05, 0.1, 0.2). Using the following equation, variance of F can be found (25, 100, 400) respectively for mean of 100 and (100, 400, 1600) respectively for mean of 200.

$$CV = \frac{\sigma}{\mu}$$

After determining variance of variables, we find the μ and σ parameters for forecast update for the means of 100 and 200 separately.

$$E[v] = exp(\mu + 1/2\ \sigma^2)$$
$$Var[v] = exp(2\mu + \sigma^2)(exp(\sigma^2) - 1)$$
$$100 = exp(\mu + 1/2\ \sigma^2)$$
$$25 = exp(2\mu + \sigma^2)(exp(\sigma^2) - 1)$$

Solving both equations mutually, then we get μ = 4.603922 and σ = 0.0024969.

Moreover, we make an assumption for yield uncertainty z which has beta distribution with a means of (0.5, 0.75, 0.9, 1) and variance of 1/12. For this distribution, we need shape parameters of beta distribution α and β and obtain them with the following formulas shown in Table B.2. in Appendix B:

$$E[z] = \frac{\alpha}{\alpha+\beta}$$

$$Var[z] = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

According to equations; calculation of shape parameters are as follows:

$$0.75 = \frac{\alpha}{\alpha+\beta} \quad \text{and } 1/12 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

From that, we obtain $\alpha = 0.9375$ and $\beta = 0.3125$. They are also applied for different expected values of yield uncertainty ratios.

Furthermore, residual market uncertainty value is log-normal distributed with mean equal to 1 and variances to (0.03, 0.1 0.5). For these values, we report the μ and σ values in Table B.2. in Appendix B for each parameter according to following equations:

$$E[v] = exp(\mu + 1/2\ \sigma^2)$$

$$Var[v] = exp(2\mu + \sigma^2)(exp(\sigma^2) - 1)$$

The calculation of demand and market uncertainty values are:

$$1 = exp(\mu + 1/2\ \sigma^2)$$

$$exp(1) = \mu + 1/2\ \sigma^2$$

$$0 = \mu + 1/2\ \sigma^2$$

$$-\mu = 1/2\ \sigma^2$$

and after getting this equation we put μ the variance equation into:

$$0.1 = exp\ (2\mu + (-2\mu))*(exp(-2*\mu) - 1)$$

We take the results as $\mu = -0.047655$ and $\sigma = 0.09531$. Similarly, we present other parameters as in this way.

The manufacturer's advance production cost is fixed to $c_1 = 2$ and retailer price $r = 10$. Also, we assume that $c_1 \leq w \leq r$.

Table 4.1. Parameter Patterns for the Decentralized Model.

E[F]=100,200			
E[z]	c2	var(v)	CvF
0.5, 0.75, 0.9, 1	5	0.03	0.05
	5		0.1
	5		0.2
	5	0.1	0.05
	5		0.1
	5		0.2
	5	0.5	0.05
	5		0.1
	5		0.2
	8	0.03	0.05
	8		0.1
	8		0.2
	8	0.1	0.05
	8		0.1
	8		0.2
	8	0.5	0.05
	8		0.1
	8		0.2
	20	0.03	0.05
	20		0.1
	20		0.2
	20	0.1	0.05
	20		0.1
	20		0.2
	20	0.5	0.05
	20		0.1
	20		0.2

Assumption 4.1. At that point, while determining c_2 we assume that $E[z]*c_2 > c_1$. We defer the explanation of this assumption in Appendix A. Hence, c_2 is equal to 2.5* c_1, 4* c_1 and 10* c_1.

To get numerical results associated with our model, we use Monte Carlo simulation technique on R and Matlab software due to the fact that getting exact solutions take much more time for a solution. We develop three different codes reported in C.3., C.4. and C.5. in Appendix C. One is set in Matlab for the retailer's optimum order quantity and the expected profit with a given wholesale price, the second one is formed in R and calculates the advance production decision for the manufacturer, and the last one is for the manufacturer's expected profit.

We first should determine the wholesale price, however, for this we need the manufacturer's optimal advance production quantity $u_a{}^*$, therefore we calculate all possible $u_a{}^*$ with different wholesale price parameters. To calculate $u_a{}^*$, we use trial-error method to get an optimal solution. Optimum $u_a{}^*$ is the value that closest to 0.

4.3.1. Algorithm of Determining the Optimum Advance Production Quantity and the Wholesale Price

1. Step: We enter an order quantity *u* and whole price *w* to the system with constant variables.

u<-82

w<-9.6

r<-10

c_1*<-2*

c_2*<-5*

2. Step: For random variables, we define the yield, demand and market uncertainty variables for the system.

alfa<-1

beta<-1

muF<-4.603922

sigmaF<-0.049969

muv<-(-0.014779)

sigmav<-0.171927

<u>3. Step:</u> This process continues until we get u_a^* with given all *w* values where the point the sign of the equation changes.

<u>4. Step:</u> After getting u_a^* with given all *w* values, now we calculate the total expected profit of the manufacturer to determine the optimum wholesale price *w*. We enter the u_a^* with the related *w* value.

```
u<-82
w<-9.6
r<-10
c1<-2
c2<-5
```

<u>5. Step:</u> 10^6 random variables for yield (beta α, β), demand (lognormal, muF, sigmaF) and market uncertainty (lognormal, muv, sigmav) variables are generated.

```
n<-1000000
alfa<-1
beta<-1
muv<-(-0.014779)
sigmav<-0.171927
muF<-4.603922
sigmaF<-0.049969
z<-rbeta (n,alfa,beta)
m<-rlnorm(n,muF,sigmaF)
```

<u>6. Step:</u> Code works for the inner calculations including revenue for the manufacturer, the cost for the advance production and the cost for the outsourcing if it is needed. Then we have,

```
ExpProfit<-mean(TotalProfit)
StError<-sd(TotalProfit)/sqrt(n)
print(ExpCost)
print(StError)
```

7. Step: For all $u_a{}^*$ and w, we find the all expected profits of the manufacturer. If this values remains to increase, then we try another bigger $u_a{}^*$ and w value until obtaining a low profit value.

For example, $w = 9.5$ and $u_a{}^* = 84$ with the expected profit = 372.8893

$w = 9.6$ and $u_a{}^* = 82$ with the expected profit = 373.5273

$w = 9.7$ and $u_a{}^* = 80$ with the expected profit = 372.1721

Then, $w^* = 9.6$ and $u_a{}^* = 82$ are determined the optimum wholesale price and advance production quantity by this way.

Now, we have the optimum wholesale price value that is the need for a next step to calculate the optimum order quantity for the retailer for a given w. In order to calculate the retailer's optimum order quantity y^*, we take certain results as we find an exact equation for it. At the same time, we solve the expected profit equation of the retailer using Monte Carlo simulation technique.

4.3.2. Algorithm of Determining the Expected Profit of the Retailer

1. Step: We enter the optimum wholesale price to the system and retailer sale price.

w = 9.60;

r = 10;

2. Step: The code firstly works for determining the optimum order quantity (delta).

p=(r-w)/r;

delta =logninv(p,muv,sigmav);

3. Step: 10^6 random variables for demand (lognormal, muF, sigmaF) and market uncertainty (lognormal, muv, sigmav) variables are generated.

n=1000000;

muv=(-0.014779);

sigmav=0.171927;

muF=4.603922;

sigmaF=0.049969;

4. Step: For all *i=1:n* values, the inner functions are calculated and finally we have;

meanOpty=mean(Opty);

StErrory =std(Opty)/sqrt(n); and

meanProfit = meanProf1-meanCost1;

StErrorp = std(Profit1-Cost1)/sqrt(n);

If we calculate the optimum order quantity and the expected profit for the retailer for $w^* = 9.6$; we have:

E {Opt(y)} = 0.72923,

E {Profit} = 27.2729,

StError {Profit} = 0.012436.

The scenarios generated by the combinations of forecast update values; yield uncertainty ratio, outsourcing cost, and residual market uncertainty parameters are listed in Table D.2. in Appendix D. We analyze 216 different scenarios for each of the optimal wholesale prices, the manufacturer's optimal advance production quantity and the expected profit and the retailer's optimal order quantity and the expected profit. We also calculate the standard errors of the expected profits of the manufacturer and the retailer. In addition, % loss in profit due to ignoring supply uncertainty parameter for the manufacturer is computed for each scenario. This parameter provides us to comprehend how yield uncertainty impacts on the wholesale price as well as the manufacturer's and the retailer's decisions.

Loss in profit parameter is obtained by the following method: Firstly, we have the optimum expected profits for each scenario. Secondly, we recalculate the expected profits without any yield uncertainty parameters (z=1). As it is expected, original optimum expected values are bigger than expected profits without yield uncertainty parameters. Therefore, we subtract it from the original optimum expected profit, then divide again original optimum expected profit and multiply by 100 to get % loss in profit. Loss in cost is calculated as the following in mathematically:

$$\% \, Loss \, in \, Profit = \frac{\text{Optimum Expected Profit} - \text{Expected Profit (E[z]=1)} \, values}{\text{Optimum Expected Profit}} * 100$$

Results that we have obtained are listed as the following:

Based on our results, in both *E [F]* =100 and *E [F]* =200 situations, an increase in yield certainty causes a decrease in the wholesale price. It never changes for all parameters. It also has a positive impact on the expected profits of the manufacturer and the retailer, i.e. total system profit increases.

Table 4.2. Wholesale Price when Increasing Yield Uncertainty Ratio for E [F] =100 and E [F] =200.

		c2	c2=2.5c1			
		Var(v)	0.03		0.1	
		Cvf	0.05	0.1	0.05	0.1
E[F] = 100	E[z]=0.5	w*	9.60	9.60	9.20	9.22
		Man Profit	373.5273	372.4766	291.8145	290.8818
		Retailer Prof	27.2729	27.2437	43.2657	42.0140
	E[z]=0.75	w*	9.49	9.49	9.01	9.01
		Man Profit	464.0228	460.7695	368.9748	366.1685
		Retailer Prof	35.3462	35.3462	55.2727	55.2727
		c2	c2=2.5c1			
		Var(v)	0.03		0.1	
		Cvf	0.05	0.1	0.05	0.1
E[F] = 200	E[z]=0.5	w*	9.60	9.60	9.20	9.21
		Man Profit	747.7232	747.0172	584.1794	583.6082
		Retailer Prof	54.5528	54.5528	86.5665	85.2609
	E[z]=0.75	w*	9.48	9.48	9.00	9.01
		Man Profit	930.1213	927.9239	739.7235	737.8867
		Retailer Prof	72.1785	72.1785	111.7668	110.5138

If the yield certainty ratio and the variance of the forecast update *F* increase and the variance of the residual market uncertainty *v* is fixed, then the wholesale price *w* decrease.

Moreover, there are some increments in the wholesale prices and advance order quantity along the variances while increase in variance of forecast update.

Table 4.3. Wholesale Price with Constant Market Uncertainty, Different Variance of Forecast Update and Yield Uncertainty Ratio.

			c2=4c1		
		Var(v)	0.03		
		Cvf	0.05	0.1	0.2
E[F] = 100	E[z]=0.5	w*	9.71	9.72	9.72
		u*	101	101	102
	E[z]=0.75	w*	9.54	9.54	9.57
		u*	86	88	92
	E[z]=0.9	w*	9.45	9.46	9.50
		u*	77.3	79.3	82.4
	E[z]=1	w*	9.37	9.38	9.43
		u*	78.24	80.50	84.15

On the other hand, advance production quantity generally decreases as the variance of forecast update increases in *E[z]* = 0.5, since uncertainty in demand causes less advance production for the manufacturer and the manufacturer takes risk to make them outsource for the extra units. In addition, in increasing variance of forecast update, the manufacturer increases the wholesale price as well as increase advance production quantity so that avoids large outsourcing cost.

Table 4.4. Advance Order Quantity with Constant Market Uncertainty, Different Variance of Forecast Update and Yield Uncertainty Ratio.

		c2	c2=2.5c1		
		Var(v)	0.1		
		Cvf	0.05	0.1	0.2
E[F] = 200	E[z] = 0.5	w*	9.20	9.21	9.23
		u*	139	139	137
	E[z] = 0.75	w*	9.00	9.01	9.02
		u*	135	136	138
	E[z] = 0.9	w*	8.87	8.88	8.90
		u*	131.80	132.10	132.30
	E[z] = 1	w*	8.74	8.76	8.78
		u*	134.70	135.02	135.81

If the variance of the residual market uncertainty increases and the variance of the forecast update is fixed, then the wholesale price and advance production quantity decrease as well as the expected profit of the manufacturer. Interestingly, although retailer places order less, her expected profit increases. We also conclude that when retailer wants to place order more due to uncertainties in demand, the manufacturer sets a high wholesale price as it expected and thus the retailer have to decrease her order quantity with a given price. Lastly, total system profit has lower values under the large variances.

Table 4.5. Profit of the Manufacturer and the Retailer with Different Variance of the Residual Market Uncertainty.

		c2	c2=2.5c1		
		Var(v)	0.3	0.1	0.5
		Cvf	0.1		
E[F] = 100	E[z] = 0.5	w*	9.60	9.22	8.37
		y*	0.7292	0.6153	0.4369
		u*	82	69	49
		Man Profit	372.4766	290.8818	169.3940
		Retailer Prof	27.2437	42.0140	52.7488
	E[z] = 0.75	w*	9.49	9.01	7.87
		y*	0.7439	0.6408	0.4918
		u*	80.1	69.0	53
		Man Profit	460.7695	366.1685	225.0003
		Retailer Prof	35.3462	55.2727	75.9601
	E[z] = 0.9	w*	9.42	8.88	7.56
		y*	0.7520	0.6551	0.5250
		u*	76.20	66.40	53.20
		Man Profit	507.3926	406.6219	256.6048
		Retailer Prof	40.5832	63.6128	91.7515
	E[z] = 1	w*	9.37	8.77	7.34
		y*	0.7574	0.6664	0.5484
		u*	77.29	68.01	55.97
		Man Profit	543.4616	438.2269	282.1962
		Retailer Prof	44.3747	70.9656	103.5979

Change in outsourcing cost also affects the system parameters. Since the manufacturer faces yield uncertainty after the advance production time, outsourcing may be required for the remaining batches associated with the forecast update from the retailer. As it is expected, an increase in outsourcing cost increments in the wholesale price with similar variance parameters. An increase in advance production quantity is also observed.

However, this increases production costs and decreases the expected profit of the manufacturer. Due to high wholesale prices, the retailer decreases the order amount and causing decrease in the expected profit. At that point, for *E[z]* = 0.5 and c_2 =10*c_1, the manufacturer determines the wholesale price values very closely to retailer price, so the expected profit of the retailer is very small.

Table 4.6. Wholesale Price with Different Outsourcing Cost.

		c2	c2=4c1			c2=10c1		
		Var(v)	0.03			0.03		
		Cvf	0.05	0.1	0.2	0.05	0.1	0.2
E[F] = 100	E[z] = 0.5	w*	9.71	9.72	9.72	9.93	9.94	9.96
		u*	101	101	102	145	145	143
		Man Profit	287.82	286.34	280.50	63.02	60.90	52.31
	E[z] = 0.75	w*	9.54	9.54	9.57	9.71	9.72	9.75
		u*	86	88	92	112	113	115
		Man Profit	417.30	414.48	404.25	279.86	277.64	268.25
	E[z] = 0.9	w*	9.45	9.46	9.50	9.56	9.57	9.63
		u*	77.3	79.3	82.4	78.5	83.2	91.6
		Man Profit	489.34	480.31	461.59	398.08	385.89	358.79
	E[z] = 1	w*	9.37	9.38	9.43	9.37	9.40	9.45
		u*	78.24	80.50	84.15	80.65	85.28	94.61
		Man Profit	548.43	538.38	517.77	544.50	530.02	498.88

Furthermore, if we compare the system with *E [F]* =100 and *E [F]* =200, the wholesale price does not change reasonably. Since an increase in the mean of the forecast update, both the advance production quantity and the expected profit of the manufacturer increase.

Table 4.7. Wholesale Price with *E [F]* =100 and *E [F]* =200.

	c2	c2=2.5c1					
	Var(v)	0.1					
		E[F]=100			E[F]=200		
	Cvf	0.05	0.1	0.2	0.05	0.1	0.2
E[z] = 0.5	w*	9.20	9.22	9.23	9.20	9.21	9.23
	u*	70	69	66	139	139	137
	Man Profit	291.81	290.88	288.17	584.17	583.60	581.00
	Retailer Prof	43.27	42.01	41.41	86.56	85.26	82.81
E[z] = 0.75	w*	9.01	9.01	9.03	9.00	9.01	9.02
	u*	67.9	69.0	69.1	135	136	138
	Man Profit	368.97	366.16	359.01	739.72	737.88	732.40
	Retailer Prof	55.27	55.27	53.94	111.76	110.51	109.23

Finally, % loss of profit due ignoring supply uncertainty parameter is a significant value in this analysis. It provides us to investigate the actual total profit and total profit with z=1 values. The results obtained are similar for $E[F] = 100$ and $E[F] = 200$. Basically, an increase in outsourcing cost and the variance of the residual market uncertainty increase in loss of profit, in contrast, an increase in the variance of the forecast update decreases loss in profit. Interestingly, for $E[z] = 0.5$ and $c_2 = 10*c_1$ scenarios, we obtain that the loss of profit values are negative. We have stated earlier that the manufacturer set very high wholesale price in this environment. Because of that, certain values ($z = 1$) parameters does not sufficient to make profit for the manufacturer since outsourcing cost is very high. This is not a profitable system for the manufacturer.

We summarize our results as the following. In a decentralized supply chain consists of a manufacturer and a retailer, variances of the forecast update and the market uncertainty are important factors. Small variance values do not make critical impacts on the decision; however large variances are dangerous for both the manufacturer and the retailer. The outsourcing cost for the manufacturer is also critical. As the manufacturer encounters yield uncertainty, outsourcing is inevitable for it in order to comply with the order demand. The manufacturer also avoids producing more because of high production cost and excess inventory in its stocks. Finally, yield uncertainty ratio is one of the key elements for the manufacturer. It affects the manufacturer's advance production quantity, the wholesale pricing decisions and as well as the retailer's decision. Also, it forces to make outsourcing for the manufacturer. As a result, the manufacturer should review its production system and improve their quality.

5. CONCLUSION

In this thesis, we consider a centralized supply chain with a single item and stochastic demand under yield uncertainty in a single period review where the system consists of one warehouse and two retailers in the first part. Retailers are assumed to be identical with respect to their purchasing, holding and backordering costs since they are in a close proximity. The warehouse does not intend to keep excess inventory since holding cost of the warehouse is much more than holding cost of the retailers. Yield uncertainty caused by the warehouse is distributed between 0 and 1 as it is commonly used in a similar way in the literature. We also assume that yield uncertainty has uniform or beta distribution. After lead time, the warehouse sends order to the retailer associated with their forecast values if the on hand inventory is enough to supply in full. If it is not sufficient to meet the total order size of the retailers, then the warehouse makes a decision to allocate on hand inventory to the retailers. The objective of this section is to find the total optimal order quantity for the warehouse and the total optimal expected system-wide cost at the end of the demand period.

First of all, we develop the forecast-demand and the total expected system-wide cost of the model. Total system cost equation includes purchasing cost and inventory cost after the demands are realized at the retailer level. Moreover, in order to solve total optimum order quantity, we analyze the total system cost equation. After checking Karush Kuhn Tucker conditions, we obtain a similar newsboy equation; however, it is complicate to solve. At this point, we assume that retailer's parameters are identical. We show that the warehouse allocates the on hand inventory with respect to proportion of the related retailer's forecast size over total forecast amount, and that total optimal order quantity amount equation can only be solved in numerically. Secondly, for this purpose, we set codes of equations on Matlab and R simulation softwares. We use Monte Carlo simulation techniques on softwares to get quick results. In order to reduce standard errors of the analysis, we generate large number of sample repetitions. Finally, we make a numerical analysis for the model. We find that as backordering cost of the retailer increases, the total optimal order quantity decreases. Furthermore, if yield certainty increases, the optimal order quantity decreases. Lastly, we describe a new parameter called penalty cost.

Associated with this parameter, if yield uncertainty has low values, and then penalty cost values increase.

The second model in this thesis is a decentralized supply chain consists of a manufacturer and the retailer in single period. In this problem, the retailer buys the product from the manufacturer. The manufacturer produces and satisfies all orders given by the retailer with respect to their contract. At that point, the manufacturer starts to produce an advance quantity before obtaining the retailer's order amount. At the end of the production lead time, the manufacturer faces yield uncertainty. However, the manufacturer waits until the retailer gives her order after obtaining forecast update. If necessary (i.e. on hand inventory of the manufacturer is not enough to meet all order of the retailer), the manufacturer makes outsourcing for the remaining batches. We assume that the outsourcing production cost is always much more than advance production cost of the manufacturer due to time and set-up constraints. After receiving total order quantity by the retailer, the market demand is realized and the retailer satisfies customer demand with a retailer price. We make two assumptions that advance production cost is smaller than the wholesale price and the wholesale price is smaller than the retailer price. Finally, the residual market uncertainty is realized after demand period and the period is ended. The aim of this part is to solve total optimal order quantity of the retailer, the optimal advance production amount of the manufacturer, their expected profits and the optimal wholesale price of the manufacturer.

In order to solve this problem, three decisions are made sequentially. One is the wholesale price decision, the other is the manufacturer's advance production problem and the last one is the retailer's order quantity problem. We use backward induction method to solve these problems; the last problem is solved in the first and so on. Furthermore, we analyze the retailer' optimal order quantity and find an exact equation for it. After that, we characterize the manufacturer's advance production quantity equation which is solved using numerical analysis and we examine the wholesale price since it affects all other decisions. Finally, we conduct a numerical analysis for this supply chain model. We find that while increasing yield certainty, there is a decrease in the wholesale prices. Also, variances of the forecast update and market uncertainty have a positive or negative impacts on the wholesale price depending on yield uncertainty. Outsourcing cost also affects the wholesale prices, when an increase in outsourcing cost causes increase in the wholesale price. Lastly, we describe a loss in profit parameter to investigate yield uncertainty effects on the expected profits. We conclude that an increase in the outsourcing cost and the variance of the residual market uncertainty increase in the loss of profit. On the other hand, an increase in the variance of the forecast update causes a decrease in the loss profit.

The first model of this thesis can be further extended for dissimilar retailers and in multi-periods. Therefore, allocation policy can be a significant problem. In addition to these, the warehouse keep excess inventory and back orders with different yield uncertainty patterns. Moreover, priority of one retailer is considered to be extended. The second model of this thesis can be extended to dual purchase contract design with yield uncertainty parameters. Additionally, contract with discounts for the unmeet products can be analyzed in the future.

APPENDIX A: ASSUMPTION 4.1.

We derive the total expected profit of the manufacturer as the following:

$$\prod^{m}(w,u_a) = wE_F[y(F)] - c_1 u_a - c_2 E_{F,z}[y(F) - u_a z]^{+}$$

After taking derivation of this equation with respect to u_a:

$$\frac{\delta \prod^{m}(w,u_a)}{\delta u_a} = -c_1 + c_2 \int_0^1 \int_{u_a z/\rho}^{\infty} (z) f_\mu(\mu) d\mu f_z(z) dz \qquad [\rho = G_{v^{-1}}(\frac{r-w}{r})]$$

$$= -c_1 + c_2 \int_0^1 z(1 - F(\frac{u_a z}{\rho}) f_z(z) dz$$

$$= -c_1 + c_2 E[z] - c_2 \int_0^1 zF(\frac{u_a z}{\rho}) f_z(z) dz > 0 \qquad \text{if} \quad E[z] * c_2 > c_1$$

$$\int_0^1 zF(\frac{u_a z}{\rho}) f_z(z) dz = (c_2 E[z] - c_1) / c_2 \qquad \text{if} \quad u^* = 0 \text{ is optimal.}$$

APPENDIX B: PARAMETERS OF SCENARIOS FOR THE MODELS

Table B.1. Parameters of Scenarios for the Centralized Supply Chain Model.

z~Beta	α	β
E [z] = 0.5	1	1
E [z] = 0.75	0.9375	0.3125
E [z] = 0.9	0.072	0.008

Var (v)	μ	σ
0.03	-0.014779	0.171927
0.1	-0.047655	0.308723
0.5	-0.202733	0.636761

Table B.2. Parameters of Scenarios for the Decentralized Supply Chain Model.

z~Beta	α	β
E [z] = 0.5	1	1
E [z] = 0.75	0.9375	0.3125
E [z] = 0.9	0.072	0.008

Var (v)	μ	σ
0.03	-0.014779	0.171927
0.1	-0.047655	0.308723
0.5	-0.202733	0.636761

E [F]	Cv(F)	Var (F)	μ	σ
100	0.05	25	4.603922	0.049969
	0.1	100	4.600195	0.099751
	0.2	400	4.585560	0.198042
200	0.05	100	5.297069	0.049969
	0.1	400	5.293342	0.099751
	0.2	1600	5.278707	0.198042

APPENDIX C: SOFTWARE CODES FOR THE MODELS

C.1. R-Code for the Centralized Supply Chain for the Total Optimum Order Quantity

```
n<-1000000
u<-72
h<-1
b<-5
c<-4
F1<-60
F2<-40
alfa<-0.9375
beta<-0.3125
mu<-(-0.047655)
sigma<-0.308723

z  <-rbeta (n,alfa,beta)
v10<-rlnorm(n,mu,sigma)
v20<-rlnorm(n,mu,sigma)
v11<-rlnorm(n,mu,sigma)
v21<-rlnorm(n,mu,sigma)
w1<-(v11-uz/(F1*v10+F2*v20)<=0)
w2<-(v21-uz/(F1*v10+F2*v20)<=0)

h0<-c*z
h1<-F1*v10*z/(F1*v10+F2*v20)*(-b+(h+b)*w1)
h2<-F2*v20*z/(F1*v10+F2*v20)*(-b+(h+b)*w2)
F<-h0+h1+h2

OptU<-mean(F)
StError<-sd(F)/sqrt(n)
print(OptU)
print(StError)
```

C.2. R-Code for the Centralized Supply Chain for the Total Expected System-Wide Cost

```
n<-1000000
u<-71
h<-1
b<-5
c<-4
F1<-60
F2<-40
alfa<-0.9375
beta<-0.3125
mu<-(-0.047655)
sigma<-0.308723
z  <-rbeta (n,alfa,beta)
v10<-rlnorm(n,mu,sigma)
v20<-rlnorm(n,mu,sigma)
v11<-rlnorm(n,mu,sigma)
v21<-rlnorm(n,mu,sigma)

Cost0<-u*c*z
Cost11<-h*pmax(uz*F1*v10/(F1*v10+F2*v20)-F1*v10*v11,0)
Cost12<-b*pmax(F1*v10*v11-uz*F1*v10/(F1*v10+F2*v20),0)
Cost21<-h*pmax(uz*F2*v20/(F1*v10+F2*v20)-F2*v20*v21,0)
Cost22<-b*pmax(F2*v20*v21-uz*F2*v20/(F1*v10+F2*v20),0)
TotalCost<-Cost0+Cost11+Cost12+Cost21+Cost22
ExpCost<-mean(TotalCost)
StError<-sd(TotalCost)/sqrt(n)
print(ExpCost)
print(StError)
```

C.3. R-Codes for the Decentralized Supply Chain for the Manufacturer's Optimal Advance Production Quantity

```
MPu<-function (u,w,r=10,c1=2,c2=5){
H<-numeric ()
for (i in 1:length(u)) {
u<-u
w<-w
r<-r
c1<-c1
c2<-c2

alfa<-1
beta<-1
muF<-4.603922
sigmaF<-0.049969
muv<-(-0.014779)
sigmav<-0.171927


p<-(r-w)/r
delta<-qlnorm(p,muv,sigmav)

# calculate c1
h0<-(-c1)

# calculate int
int1<-function(z){(1-
plnorm(uz/delta,muF,sigmaF))*z*dbeta(z,alfa,beta)}
h2<-integrate(int1,0,1)[[1]]

# The whole equation
H[i]=h0+c2*h2
print(H[i])
}
}
MPu(u=82,w=9.6)
```

C.4. R-Codes for the Decentralized Supply Chain for the Manufacturer's Total Expected Profit

```
n<-1000000
u<-82
w<-9.6
c1<-2
c2<-5
r<-10

alfa<-1
beta<-1
muv<-(-0.014779)
sigmav<-0.171927
muF<-4.603922
sigmaF<-0.049969

z<-rbeta (n,alfa,beta)
m<-rlnorm(n,muF,sigmaF)
p<-(r-w)/r
delta<-qlnorm(p,muv,sigmav)

h0<-w*m*delta
Cost1<-c1*u
Cost2<-c2*pmax(m*delta-uz,0)

TotalProfit<-h0-Cost1-Cost2
ExpProfit<-mean(TotalProfit)
StError<-sd(TotalProfit)/sqrt(n)
print(ExpCost)
print(StError)
```

C.5. Matlab Code for the Decentralized Supply Chain for the Retailer's Optimal Order Quantity and Total Expected Profit

```
clear all
n=1000000;
w=9.60;
r=10;
muv=(-0.014779);
sigmav=0.171927;
muF=4.603922;
sigmaF=0.049969;
p=(r-w)/r;
delta=logninv(p,muv,sigmav);
vvector = zeros(n,1);
mvector = zeros(n,1);
Opty =zeros(n,1);
Profit1 =zeros(n,1);
Cost1=zeros(n,1);
for i=1:n
    v=lognrnd(muv,sigmav);
    m=lognrnd(muF,sigmaF);
    vvector(i) =v;
    mvector(i) =m;
    Opty(i)    = delta;
    Profit1(i) = r*min(m*delta,m*v);
    Cost1   (i) = w*m*delta;
end
meanOpty=mean(Opty);
StErrory =std(Opty)/sqrt(n);
meanProf1=mean(Profit1);
meanCost1=mean(Cost1);
meanProfit = meanProf1-meanCost1;
StErrorp = std(Profit1-Cost1)/sqrt(n);
disp(['E{Opt(y)} = ' num2str(meanOpty)])
disp(['E{Profit} = ' num2str(meanProfit)])
disp(['StError{Profit} = ' num2str(StErrorp)])
```

APPENDIX D: RESULT OF SCENARIOS FOR THE MODELS

Table D.1. Result of Scenarios for the Centralized Supply Chain Model.

		c=2					
	Var(v)	0.03		0.1		0.5	
	Backorder Cost	b=5	b=10	b=5	b=10	b=5	b=10
z=0.5	u*	143	195	141	203	115	203
	Total Cost	333.6631	487.5113	351.3156	517.0054	403.1317	630.4340
	St.Error of TC	0.033060	0.075952	0.047509	0.093568	0.116922	0.211058
	% Penalty Cost	7.0569	15.7018	5.6366	12.4458	3.0391	6.8456
z=0.75	u*	110	133	108	142	86	144
	Total Cost	294.6412	396.9548	321.9585	442.2923	388.3369	585.1085
	St.Error of TC	0.029699	0.068066	0.043804	0.082935	0.114764	0.203169
	% Penalty Cost	1.8825	4.4168	1.3253	3.2569	0.7233	1.6112
z=0.9	u*	98	111	94	118	73	118
	Total Cost	274.6058	351.3471	309.4028	414.2872	383.8694	576.1381
	St.Error of TC	0.029625	0.071523	0.043286	0.084528	0.114457	0.202869
	% Penalty Cost	0.0208	0.0134	0.0286	0.0233	0.0310	0.0106
z=1	u*	97	111	93	117	72	117
	Total Cost	250.4167	282.1479	289.2579	353.1194	371.7850	533.0347
	St.Error of TC	0.017637	0.024840	0.037413	0.055748	0.112498	0.191979

Table D.1. Result of Scenarios for the Centralized Supply Chain Model Continue.

		c=4					
	Var(v)	0.03		0.1		0.5	
	Backorder Cost	b=5	b=10	b=5	b=10	b=5	b=10
z=0.5	u*	103	151	87	151	50	128
	Total Cost	455.6919	658.3695	464.9764	691.1113	483.2769	791.8041
	St.Error of TC	0.027314	0.065487	0.050289	0.093749	0.125499	0.231257
	% Penalty Cost	1.0943	7.0504	0.9030	5.7788	0.4933	3.2401
z=0.75	u*	87	114	71	113	38	95
	Total Cost	442.4826	581.4135	456.1496	632.3192	479.8959	761.5039
	St.Error of TC	0.026468	0.058451	0.049454	0.086233	0.125310	0.226569
	% Penalty Cost	0.2247	1.5686	0.1588	1.2451	0.1800	0.7381
z=0.9	u*	80	101	65	98	35	80
	Total Cost	435.5069	542.0584	451.9276	607.3663	478.7269	752.1697
	St.Error of TC	0.026262	0.058956	0.049153	0.085109	0.125142	0.224872
	% Penalty Cost	0.0004	0.0027	0.0182	0.0090	0.0099	0.0644
z=1	u*	80	101	65	98	33	79
	Total Cost	428.5287	492.9323	446.7658	566.0806	476.5313	726.6881
	St.Error of TC	0.024751	0.033649	0.048474	0.085109	0.124976	0.221925

Table D.2. Result of Scenarios for the Decentralized Supply Chain Model.

		c2	c2=2.5c1					
		Var(v)	0.03			0.1		
		Cvf	0.05	0.1	0.2	0.05	0.1	0.2
E[F]=100	E[z]=0.5	w*	9.60	9.60	9.60	9.20	9.22	9.23
		y*	0.7292	0.7292	0.7292	0.6179	0.6153	0.6140
		u_a*	82	82	79	70	69	66
		Man Profit	373.527	372.476	369.227	291.814	290.881	288.176
		St.Error of M	0.0365	0.0375	0.0401	0.0310	0.0314	0.0336
		Retailer Prof	27.272	27.243	27.236	43.265	42.014	41.417
		St.Error of R	0.0124	0.0127	0.0137	0.0272	0.0270	0.0280
		% Loss of Profit	1.223	1.074	0.769	2.537	2.300	2.049
	E[z]=0.75	w*	9.49	9.49	9.50	9.01	9.01	9.03
		y*	0.7439	0.7439	0.7426	0.6408	0.6408	0.6385
		u_a*	79.0	80.1	80.4	67.9	69.0	69.1
		Man Profit	464.022	460.769	452.482	368.974	366.168	359.011
		St.Error of M	0.0350	0.0365	0.0432	0.0297	0.0309	0.0356
		Retailer Prof	35.346	35.346	34.635	55.272	55.272	53.944
		St.Error of R	0.0147	0.0151	0.0162	0.0322	0.0322	0.0340
		% Loss of Profit	0.434	0.388	0.260	0.700	0.645	0.539
	E[z]=0.9	w*	9.41	9.42	9.42	8.88	8.88	8.91
		y*	0.7531	0.7520	0.7520	0.6551	0.6551	0.6518
		u_a*	75.90	76.20	76.20	66.10	66.40	66.10
		Man Profit	513.938	507.392	494.351	412.315	406.621	395.258
		St.Error of M	0.0346	0.0367	0.0445	0.0300	0.0315	0.0371
		Retailer Prof	41.358	40.583	40.612	63.612	63.612	61.704
		St.Error of R	0.0164	0.0167	0.0182	0.0358	0.0358	0.0374
		% Loss of Profit	0.028	0.038	0.063	0.048	0.096	0.133
	E[z]=1	w*	9.37	9.37	9.39	8.76	8.77	8.78
		y*	0.7574	0.7574	0.7553	0.6675	0.6664	0.6654
		u_a*	76.61	77.29	77.87	67.51	68.01	68.61
		Man Profit	550.871	543.461	528.757	444.721	438.226	425.257
		St.Error of M	0.0088	0.0173	0.0332	0.0071	0.0140	0.0268
		Retailer Prof	44.375	44.374	42.891	71.662	70.965	70.256
		St.Error of R	0.0171	0.0177	0.0190	0.0387	0.0390	0.0412

Table D.2. Result of Scenarios for the Decentralized Supply Chain Model Continue.

		c2	c2=2.5c1			c2=4c1		
		Var(v)	0.5			0.03		
		Cvf	0.05	0.1	0.2	0.05	0.1	0.2
E[F]=100	E[z]=0.5	w*	8.37	8.37	8.39	9.71	9.72	9.72
		y*	0.4369	0.4369	0.4346	0.7113	0.7094	0.7094
		u_a*	49	49	47	101	101	102
		Man Profit	170.048	169.394	167.469	287.823	286.346	280.504
		St.Error of M	0.02170	0.02203	0.02275	0.05988	0.05962	0.06112
		Retailer Prof	52.748	52.748	51.965	19.346	18.638	18.638
		St.Error of R	0.05267	0.05267	0.05373	0.00986	0.00981	0.00981
		% Loss of Profit	9.239	8.223	7.128	9.931	8.500	6.026
	E[z]=0.75	w*	7.86	7.87	7.89	9.54	9.54	9.57
		y*	0.4929	0.4918	0.4897	0.7375	0.7375	0.7335
		u_a*	52.4	53	52.9	86	88	92
		Man Profit	227.126	225.000	219.528	417.307	414.482	404.254
		St.Error of M	0.02281	0.02327	0.02531	0.05364	0.05449	0.05793
		Retailer Prof	76.482	75.960	75.012	31.643	31.643	29.462
		St.Error of R	0.07010	0.07040	0.07176	0.01312	0.01312	0.01447
		% Loss of Profit	2.057	1.784	1.497	2.032	1.774	1.243
	E[z]=0.9	w*	7.50	7.56	7.62	9.45	9.46	9.50
		y*	0.5314	0.5250	0.5186	0.7486	0.7474	0.7426
		u_a*	53.60	53.20	52.60	77.3	79.3	82.4
		Man Profit	264.643	256.604	247.558	489.342	480.317	461.592
		St.Error of M	0.02423	0.02442	0.02735	0.05443	0.05571	0.05996
		Retailer Prof	94.764	91.751	88.598	38.372	37.624	34.588
		St.Error of R	0.08330	0.08157	0.08199	0.01541	0.01576	0.01634
		% Loss of Profit	1.593	0.266	0.294	0.065	0.098	0.052
	E[z]=1	w*	7.28	7.34	7.42	9.37	9.38	9.43
		y*	0.5548	0.5484	0.5399	0.7574	0.7564	0.7509
		u_a*	56.12	55.97	55.67	78.24	80.50	84.15
		Man Profit	287.561	282.196	271.619	548.435	538.384	517.778
		St.Error of M	0.00469	0.00913	0.01729	0.00895	0.01749	0.03322
		Retailer Prof	106.553	103.597	99.176	44.379	43.617	39.851
		St.Error of R	0.09158	0.08975	0.08977	0.01724	0.01749	0.01806

Table D.2. Result of Scenarios for the Decentralized Supply Chain Model Continue.

		c2	c2=4c1					
		Var(v)	0.1			0.5		
		Cvf	0.05	0.1	0.2	0.05	0.1	0.2
E[F]=100	E[z]=0.5	w*	9.42	9.44	9.45	8.79	8.81	8.84
		y*	0.5869	0.5837	0.5821	0.3876	0.3852	0.3814
		u_a*	84	83	84	55	55	55
		Man Profit	220.46	219.19	214.43	121.17	120.38	117.27
		St.Error of M	0.048	0.049	0.049	0.031	0.031	0.032
		Retailer Prof	30.024	28.865	28.265	35.398	34.596	33.398
		St.Error of R	0.021	0.020	0.021	0.038	0.038	0.038
		% Loss of Profit	14.815	12.933	10.663	38.316	34.546	28.704
	E[z]=0.75	w*	9.12	9.14	9.14	8.12	8.16	8.20
		y*	0.6279	0.6254	0.6254	0.4647	0.4603	0.4559
		u_a*	73	75	79	54	55	57
		Man Profit	328.85	326.48	317.72	196.92	195.144	188.72
		St.Error of M	0.045	0.046	0.048	0.033	0.033	0.034
		Retailer Prof	48.285	46.989	46.989	63.985	62.091	60.315
		St.Error of R	0.029	0.029	0.029	0.061	0.060	0.060
		% Loss of Profit	2.880	2.477	2.086	6.609	5.871	4.769
	E[z]=0.9	w*	8.91	8.95	8.98	7.67	7.72	7.83
		y*	0.6518	0.6475	0.6442	0.5133	0.5079	0.4961
		u_a*	67.3	68.7	71.5	53.0	54.0	55.0
		Man Profit	390.84	383.01	366.89	244.12	238.01	225.502
		St.Error of M	0.047	0.047	0.050	0.0371	0.0368	0.0371
		Retailer Prof	61.727	59.082	57.201	85.940	83.344	77.703
		St.Error of R	0.034	0.034	0.035	0.0771	0.0759	0.07431
		% Loss of Profit	0.171	0.172	0.259	0.717	0.740	0.745
	E[z]=1	w*	8.76	8.79	8.81	7.30	7.36	7.49
		y*	0.6675	0.6644	0.6624	0.5527	0.5463	0.5325
		u_a*	68.95	70.71	74.23	57.09	58.14	59.68
		Man Profit	442.57	433.76	415.65	285.80	278.52	263.862
		St.Error of M	0.0072	0.0141	0.0268	0.00482	0.00935	0.01751
		Retailer Prof	71.656	69.579	68.282	105.729	102.509	95.404
		St.Error of R	0.03873	0.03865	0.04045	0.09063	0.08897	0.08706

Table D.2. Result of Scenarios for the Decentralized Supply Chain Model Continue.

		c2	c2=10c1					
		Var(v)	0.03			0.1		
		Cvf	0.05	0.1	0.2	0.05	0.1	0.2
E[F]=100	E[z]=0.5	w*	9.93	9.94	9.96	9.91	9.91	9.93
		y*	0.6458	0.6398	0.6245	0.4593	0.4593	0.4465
		u_a*	145	145	143	103	104	105
		Man Profit	63.023	60.904	52.311	43.939	42.319	36.280
		St.Error of M	0.12240	0.12496	0.12501	0.08901	0.08932	0.09048
		Retailer Prof	4.280	3.633	2.372	3.749	3.749	2.841
		St.Error of R	0.00383	0.00355	0.00276	0.00531	0.00531	0.00460
		% Loss of Profit	361.342	321.878	283.564	521.083	472.184	432.039
	E[z]=0.75	w*	9.71	9.72	9.75	9.43	9.46	9.50
		y*	0.7113	0.7094	0.7035	0.5853	0.5805	0.5738
		u_a*	112	113	115	92	92	95
		Man Profit	279.861	277.641	268.259	213.929	212.135	204.450
		St.Error of M	0.11590	0.11650	0.11828	0.09462	0.09522	0.09713
		Retailer Prof	19.341	18.612	16.507	29.454	27.698	25.391
		St.Error of R	0.00994	0.00995	0.00972	0.02068	0.02020	0.01970
		% Loss of Profit	25.613	19.673	13.752	33.058	26.139	19.080
	E[z]=0.9	w*	9.56	9.57	9.63	9.16	9.18	9.25
		y*	0.7349	0.7335	0.7247	0.6230	0.6205	0.6114
		u_a*	78.5	83.2	91.6	66.6	70.4	77.3
		Man Profit	398.081	385.894	358.791	312.468	302.233	279.469
		St.Error of M	0.13329	0.13422	0.13432	0.11302	0.11308	0.11329
		Retailer Prof	30.190	29.439	25.088	45.728	44.475	40.192
		St.Error of R	0.01333	0.01343	0.01294	0.02834	0.02827	0.02749
		% Loss of Profit	0.599	0.459	0.493	1.373	1.400	1.505
	E[z]=1	w*	9.37	9.40	9.45	8.77	8.81	8.89
		y*	0.7574	0.7542	0.7486	0.6664	0.6624	0.6540
		u_a*	80.65	85.28	94.61	70.96	74.89	82.66
		Man Profit	544.506	530.028	498.884	439.112	426.382	399.230
		St.Error of M	0.00949	0.01862	0.03571	0.00779	0.01526	0.02932
		Retailer Prof	44.379	42.088	38.373	70.932	68.289	63.009
		St.Error of R	0.01719	0.01715	0.01755	0.03858	0.03800	0.03903

Table D.2. Result of Scenarios for the Decentralized Supply Chain Model Continue.

		c2	c2=10c1		
		Var(v)	0.5		
		Cvf	0.05	0.1	0.2
E[F]=100	E[z]=0.5	w*	9.80	9.81	9.83
		y*	0.2208	0.2179	0.2117
		u_a*	50	49	49
		Man Profit	18.7342	17.9316	15.0320
		St.Error of M	0.040782	0.041710	0.039396
		Retailer Prof	3.5505	3.3577	2.9177
		St.Error of R	0.007662	0.007242	0.006741
		% Loss of Profit	1331.133	1230.109	1167.181
	E[z]=0.75	w*	8.87	8.90	8.93
		y*	0.3777	0.3739	0.3701
		u_a*	60	60	62
		Man Profit	116.8623	115.6703	119.6821
		St.Error of M	0.062544	0.061265	0.062627
		Retailer Prof	32.3413	31.2222	30.0957
		St.Error of R	0.036449	0.035736	0.035652
		% Loss of Profit	66.347	54.310	45.012
	E[z]=0.9	w*	8.13	8.24	8.45
		y*	0.4636	0.4514	0.4278
		u_a*	49.5	51.2	54.1
		Man Profit	184.9246	177.3311	161.2384
		St.Error of M	0.083627	0.080073	0.079225
		Retailer Prof	63.5459	58.4993	49.2624
		St.Error of R	0.060752	0.057505	0.051878
		% Loss of Profit	5.205	5.232	5.369
	E[z]=1	w*	7.34	7.43	7.63
		y*	0.5484	0.5389	0.5176
		u_a*	58.40	60.93	65.41
		Man Profit	282.9472	272.5074	250.7415
		St.Error of M	0.005354	0.010486	0.020165
		Retailer Prof	103.6028	98.7266	87.9529
		St.Error of R	0.088978	0.086453	0.081842

Table D.2. Result of Scenarios for the Decentralized Supply Chain Model Continue.

		c2	c2=2.5c1					
		Var(v)	0.03			0.1		
		Cvf	0.05	0.1	0.2	0.05	0.1	0.2
E[F]=200	E[z]=0.5	w*	9.60	9.60	9.60	9.20	9.21	9.23
		y*	0.7292	0.7292	0.7292	0.6179	0.6166	0.6140
		u_a*	164	164	163	139	139	137
		Man Profit	747.72	747.017	744.955	584.17	583.60	581.00
		St.Error of M	0.0723	0.07315	0.07450	0.0607	0.0614	0.0624
		Retailer Prof	54.552	54.552	54.552	86.566	85.260	82.813
		St.Error of R	0.0246	0.02467	0.02467	0.0542	0.0541	0.0537
		% Loss of Profit	1.420	1.269	0.962	2.798	2.537	2.080
	E[z]=0.75	w*	9.48	9.48	9.48	9.00	9.01	9.02
		y*	0.7451	0.7451	0.7451	0.6419	0.6408	0.6396
		u_a*	157	159	161	135	136	138
		Man Profit	930.12	927.923	921.533	739.72	737.88	732.40
		St.Error of M	0.0686	0.06987	0.07335	0.0593	0.0597	0.0619
		Retailer Prof	72.178	72.178	72.178	111.76	110.51	109.23
		St.Error of R	0.0299	0.02997	0.02997	0.0648	0.0646	0.0649
		% Loss of Profit	0.479	0.446	0.333	0.739	0.677	0.618
	E[z]=0.9	w*	9.40	9.40	9.41	8.87	8.88	8.90
		y*	0.7542	0.7542	0.7531	0.6561	0.6551	0.6529
		u_a*	151.60	152.10	152.50	131.80	132.10	132.30
		Man Profit	1034.3	1027.9	1014.77	830.39	824.63	813.19
		St.Error of M	0.0684	0.0695	0.07372	0.0595	0.0602	0.0628
		Retailer Prof	84.223	84.223	82.756	128.58	127.40	124.78
		St.Error of R	0.0330	0.0330	0.03363	0.0718	0.0711	0.0713
		% Loss of Profit	0.035	0.035	0.037	0.070	0.061	0.074
	E[z]=1	w*	9.35	9.36	9.39	8.74	8.76	8.78
		y*	0.7595	0.7585	0.7553	0.6695	0.6675	0.6654
		u_a*	152.82	153.43	154.16	134.70	135.02	135.81
		Man Profit	1109.1	1101.7	1086.94	895.934	889.442	876.405
		St.Error of M	0.00891	0.01767	0.03463	0.00722	0.01430	0.02802
		Retailer Prof	91.836	90.313	85.723	145.817	143.314	140.604
		St.Error of R	0.03473	0.03467	0.03456	0.07848	0.07738	0.07774

Table D.2. Result of Scenarios for the Decentralized Supply Chain Model Continue.

		c2	c2=2.5c1			c2=4c1		
		Var(v)	0.5			0.03		
		Cvf	0.05	0.1	0.2	0.05	0.1	0.2
E[F]=200	E[z]=0.5	w*	8.36	8.37	8.39	9.71	9.71	9.72
		y*	0.4380	0.4369	0.4346	0.7113	0.7113	0.7094
		u_a*	98	98	97	202	202	202
		Man Profit	340.503	340.100	338.755	576.480	575.614	572.679
		St.Error of M	0.04299	0.04347	0.04324	0.11984	0.11974	0.11962
		Retailer Prof	106.481	105.484	103.615	38.692	38.692	37.248
		St.Error of R	0.10568	0.10536	0.10502	0.01975	0.01975	0.01982
		% Loss of Profit	9.502	9.114	8.235	10.790	9.741	8.411
	E[z]=0.75	w*	7.84	7.86	7.88	9.54	9.54	9.56
		y*	0.4951	0.4929	0.4908	0.7375	0.7375	0.7349
		u_a*	104	105	106	170	172	175
		Man Profit	455.617	454.213	450.968	835.814	834.681	828.995
		St.Error of M	0.04562	0.05666	0.04654	0.10715	0.10764	0.10919
		Retailer Prof	155.152	152.868	150.934	63.285	63.285	60.386
		St.Error of R	0.14122	0.14038	0.14011	0.02749	0.02749	0.02722
		% Loss of Profit	2.041	1.961	1.985	2.317	2.008	1.750
	E[z]=0.9	w*	7.50	7.55	7.60	9.45	9.45	9.47
		y*	0.5314	0.5261	0.5208	0.7486	0.7486	0.7463
		u_a*	106.80	106.10	105.40	152.20	154.50	158.30
		Man Profit	526.953	522.394	513.153	987.330	978.540	960.583
		St.Error of M	0.04787	0.04768	0.04879	0.10881	0.10939	0.11122
		Retailer Prof	189.560	184.507	179.331	76.705	76.705	73.707
		St.Error of R	0.16611	0.16257	0.16013	0.03110	0.03110	0.03116
		% Loss of Profit	0.262	0.287	0.258	0.083	0.052	0.083
	E[z]=1	w*	7.27	7.29	7.34	9.36	9.38	9.39
		y*	0.5559	0.5538	0.5484	0.7585	0.7564	0.7553
		u_a*	111.85	112.02	111.94	154.22	156.26	160.77
		Man Profit	580.520	575.130	564.362	1106.74	1096.88	1076.78
		St.Error of M	0.00474	0.00937	0.02825	0.00906	0.01791	0.03498
		Retailer Prof	214.607	212.618	207.141	90.318	87.282	85.677
		St.Error of R	0.18317	0.18202	0.17965	0.03435	0.03394	0.03470

Table D.2. Result of Scenarios for the Decentralized Supply Chain Model Continue.

		c2	c2=4c1					
		Var(v)	0.1			0.5		
		Cvf	0.05	0.1	0.2	0.05	0.1	0.2
E[F]=200	E[z]=0.5	w*	9.42	9.42	9.45	8.78	8.81	8.82
		y*	0.5869	0.5869	0.5821	0.3888	0.3852	0.3839
		u_a*	167	167	166	111	110	110
		Man Profit	441.586	441.006	438.437	242.771	242.421	240.760
		St.Error of M	0.09804	0.09822	0.09827	0.06466	0.06535	0.06414
		Retailer Prof	59.975	59.975	56.562	71.546	69.133	68.515
		St.Error of R	0.04238	0.04238	0.04115	0.07850	0.07685	0.07642
		% Loss of Profit	15.875	14.637	13.103	39.798	37.295	33.886
	E[z]=0.75	w*	9.12	9.12	9.15	8.12	8.14	8.17
		y*	0.6279	0.6279	0.6242	0.4647	0.4625	0.4592
		u_a*	145	146	149	108	108	110
		Man Profit	658.953	657.865	652.945	394.637	393.811	390.363
		St.Error of M	0.09105	0.09185	0.09237	0.06700	0.06660	0.06685
		Retailer Prof	96.483	96.483	92.730	128.052	126.202	123.195
		St.Error of R	0.05874	0.05874	0.05801	0.12188	0.12101	0.12001
		% Loss of Profit	3.119	2.783	2.535	6.990	6.343	5.662
	E[z]=0.9	w*	8.90	8.94	8.98	7.67	7.71	7.77
		y*	0.6529	0.6486	0.6442	0.5133	0.5090	0.5026
		u_a*	132.80	133.90	136.60	104.40	105.10	106.60
		Man Profit	789.434	781.666	765.859	494.152	488.220	475.896
		St.Error of M	0.09509	0.09483	0.09579	0.07476	0.07415	0.07389
		Retailer Prof	124.745	119.548	114.291	172.073	167.980	161.918
		St.Error of R	0.06997	0.06822	0.06730	0.15357	0.15087	0.14776
		% Loss of Profit	0.197	0.157	0.147	0.629	0.602	0.636
	E[z]=1	w*	8.75	8.77	8.78	7.29	7.33	7.38
		y*	0.6685	0.6664	0.6654	0.5538	0.5495	0.5442
		u_a*	135.92	137.68	141.64	112.60	113.53	115.83
		Man Profit	893.841	885.147	867.502	578.785	571.590	557.048
		St.Error of M	0.00736	0.01455	0.02837	0.00489	0.00964	0.01869
		Retailer Prof	144.617	141.979	140.487	212.773	207.882	202.548
		St.Error of R	0.07768	0.07695	0.07789	0.18121	0.17921	0.01767

Table D.2. Result of Scenarios for the Decentralized Supply Chain Model Continue.

		c2	c2=10c1					
		Var(v)	0.03			0.1		
		Cvf	0.05	0.1	0.2	0.05	0.1	0.2
E[F]=200	E[z]=0.5	w*	9.93	9.94	9.95	9.91	9.91	9.93
		y*	0.645	0.639	0.632	0.459	0.459	0.446
		u_a*	289	287	285	206	206	201
		Man Profit	127.1	126.2	121.7	88.50	87.69	84.14
		St.Error of M	0.261	0.259	0.245	0.182	0.183	0.178
		Retailer Prof	8.561	7.273	5.993	7.491	7.475	5.685
		St.Error of R	0.007	0.006	0.006	0.010	0.010	0.008
		% Loss of Profit	386.09	359.14	323.20	554.39	517.02	469.8
	E[z]=0.75	w*	9.71	9.71	9.74	9.43	9.45	9.48
		y*	0.711	0.711	0.705	0.585	0.582	0.577
		u_a*	223	224	224	184	183	183
		Man Profit	560.68	559.68	554.91	428.54	427.70	424.1
		St.Error of M	0.232	0.233	0.234	0.192	0.192	0.191
		Retailer Prof	38.64	38.64	34.39	58.88	56.50	53.09
		St.Error of R	0.019	0.019	0.018	0.041	0.041	0.039
		% Loss of Profit	29.93	25.432	19.851	38.225	32.720	25.70
	E[z]=0.9	w*	9.56	9.57	9.62	9.16	9.18	9.23
		y*	0.734	0.733	0.726	0.623	0.620	0.614
		u_a*	152.10	156.70	164.70	128.90	132.50	139.2
		Man Profit	807.3	796.4	771.3	634.5	625.0	604.5
		St.Error of M	0.267	0.266	0.264	0.227	0.225	0.223
		Retailer Prof	60.37	58.91	51.58	91.54	88.99	82.86
		St.Error of R	0.026	0.026	0.024	0.056	0.055	0.053
		% Loss of Profit	0.513	0.558	0.578	1.314	1.170	1.242
	E[z]=1	w*	9.36	9.38	9.39	8.75	8.79	8.83
		y*	0.758	0.756	0.755	0.668	0.664	0.660
		u_a*	156.58	161.08	170.81	138.00	141.49	149.3
		Man Profit	1102.9	1089.0	1059.9	890.48	878.23	852.7
		St.Error of M	0.009	0.018	0.037	0.007	0.015	0.030
		Retailer Prof	90.318	87.245	85.686	144.64	139.16	133.9
		St.Error of R	0.034	0.033	0.034	0.076	0.076	0.075

Table D.2. Result of Scenarios for the Decentralized Supply Chain Model Continue.

		c2	c2=10c1		
		Var(v)	0.5		
		Cvf	0.05	0.1	0.2
E[F]=200	E[z]=0.5	w*	9.80	9.81	9.83
		y*	0.2208	0.2179	0.2117
		u_a*	99	98	96
		Man Profit	37.7506	37.1925	35.7373
		St.Error of M	0.087876	0.086785	0.084610
		Retailer Prof	7.1321	6.7098	5.8457
		St.Error of R	0.015124	0.014401	0.013176
		% Loss of Profit	1399.784	1345.400	1238.317
	E[z]=0.75	w*	8.87	8.90	8.93
		y*	0.3777	0.3739	0.3701
		u_a*	119	118	118
		Man Profit	234.2215	233.6062	231.2557
		St.Error of M	0.124147	0.123046	0.122530
		Retailer Prof	64.6662	62.3220	60.2191
		St.Error of R	0.072804	0.071124	0.069432
		% Loss of Profit	0.745	0.666	0.546
	E[z]=0.9	w*	8.13	8.18	8.25
		y*	0.4636	0.4581	0.4503
		u_a*	95.90	97.80	102.10
		Man Profit	376.8287	370.1127	354.8186
		St.Error of M	0.168977	0.166607	0.164034
		Retailer Prof	127.1366	122.5178	116.2504
		St.Error of R	0.121220	0.118020	0.113920
		% Loss of Profit	5.096	5.320	5.335
	E[z]=1	w*	7.31	7.33	7.42
		y*	0.5516	0.5491	0.5399
		u_a*	113.88	117.02	122.10
		Man Profit	576.0040	565.8847	545.0284
		St.Error of M	0.005404	0.010707	0.020982
		Retailer Prof	210.0916	208.0413	198.5380
		St.Error of R	0.180310	0.178950	0.173400

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