THE ANALYSIS OF CHARACTERISTIC DYNAMICS OF STOCK MANAGEMENT STRUCTURES

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ABSTRACT

THE ANALYSIS OF CHARACTERISTIC DYNAMICS OF STOCK MANAGEMENT STRUCTURES

Stock management is a dynamic task which is often found in managerial, physical, and biological systems. The aim in stock management is to bring a stock at a desired level and maintain it at that level by taking corrective actions. Stock management task imposes difficulties to the decision maker, which results in unwanted oscillations. In this thesis, different stock management structures are modeled and analyzed. We carry out complete parametric analysis of stock management problems with continuous delays of first, second, and third orders aiming to obtain the range of values for different characteristic dynamics of stock. For parametric analysis, we use control theoretic approaches. We first provide different stock management structures modeled using stock-flow diagrams of system dynamics methodology. Secondly, we obtain the corresponding simplified differential equations of the system dynamics model and, based on the differential equations, we obtain block diagrams. Thirdly, we convert simplified differential equations of the model from time domain to s-domain using Laplace transformation technique and obtain the transfer function. Fourthly, the characteristic equation of the transfer function is determined. Finally, we determine the critical values of the decision parameters at which a qualitative change in dynamics is observed by analyzing the roots of the characteristic equation. The critical values that are reported in this thesis are valid for all durations of the delay between the corrective actions and their eventual results on the stock. We also obtained a few counterintuitive results such as increasing the level of aggressiveness in stock corrections can completely eliminate oscillations in one of the cases. Aiming to build a bridge between system dynamics and control theory, the corresponding block diagrams of many basic system dynamics models are provided in the thesis and also in one of its appendices.

ÖZET

STOK YÖNETİMİ YAPILARININ KARAKTERİSTİK DİNAMİKLERİNİN ANALİZİ

Stok yönetimi; yönetimsel, fiziksel ve biyolojik sistemlerde sık karşılaşılan dinamik bir problemdir. Stock yönetiminin amacı, düzeltici faaliyetlerde bulunarak bir stoğu istenilen seviyeye getirmek ve stoğu o seviyede tutmaktır. Stok yönetimi problemi, karar vericiyi istenmeyen kararsız dalgalanmalarla da sonuçlanabilecek zorluklarla karşı karşıya bırakmaktadır. Bu tezde, farklı stok yönetimi yapıları modellenmiş ve analiz edilmiştir. Birinci, ikinci ve üçüncü dereceden sürekli gecikmeli stok yönetimi problemlerinin çeşitli karakteristik dinamiklerini elde edebilmek için parametrik değer aralıkları belirlemeyi amaçlayarak eksiksiz bir analiz yapılmıştır. Parametrik analiz için kontrol teorisi yöntemlerini kullandık. İlk olarak, sistem dinamiği yöntemine ait stok-akış diyagramları kullanılarak modellenmiş olan farklı stok yönetimi yapıları sunuldu. İkinci olarak, sistem dinamiği modellerinin diferansiyel denklemleri ve bu denklemlere dayanarak sistem dinamiği modellerine karşılık gelen blok diyagramları elde ettik. Üçüncü olarak, Laplace dönüşümü tekniğini kullanarak modellerin diferansiyel denklemlerini zaman tanım kümesinden frekans tanım kümesine çevirerek transfer fonksiyonları elde ettik. Dördüncü olarak, transfer fonksiyonlarının karakteristik denklemlerini belirledik. Son olarak, karakteristik denklemlerin köklerini analiz ederek dinamiklerde niteliksel değişimin gözlemlendiği kritik karar parametreleri değerlerini belirledik. Bu tezde raporlanan kritik değerler, düzeltici faaliyetler ile bu faaliyetlerin stok üzerindeki nihai sonuçları arasındaki gecikme süresinin bütün değerleri için geçerlidir. Ayrıca, beklenmeyen bazı sonuçlar da elde ettik. Örnek durumların birinde, stok seviyesini istenilen düzeye getirmek için yapılan düzeltme miktarlarının şiddetini arttırmak dalgalanmaları tamamen yok etmektedir. Sistem dinamiği ve kontrol teorisi arasında bir köprü kurulması amaçlandığından, birçok temel sistem dinamiği modeline karşılık gelen blok diyagramlar tezde ve tezin eklerinden birinde verilmiştir.

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θ	Duration of a delay; a nonnegative constant value
$\delta(t)$	Unit impulse function

LIST OF ACRONYMS/ABBREVIATIONS

a	A constant value
a_1	A constant value
a_2	A constant value
adt	Acquisition Delay Time
AF	Acquisition Flow
bf	Birth fraction; a nonnegative constant value
cf	Contact fraction; a nonnegative constant value
CF	Control Flow
df	Death fraction; a nonnegative constant value
DT	Simulation time step; a positive constant value between zero and one
$f(t)$	A function of t, which stands for time
$f'(t)$	First derivative of $f(t)$
$f''(t)$	Second derivative of $f(t)$
$f^{(n)}$	n^{th} derivative of $f(t)$
$F(s)$	Laplace transform of $f(t)$
$g(s)$	Transfer function in the Laplace domain
I	Infected; a stock variable
if	Infection fraction; a nonnegative constant value
$L[f(t)]$	Laplace transform of $f(t)$
$L^{-1}[f(t)]$	Inverse Laplace transform of $f(t)$
LF	Loss Flow
p	Population; a stock variable
ra	Relative Aggressiveness
rf	Removal fraction; a nonnegative constant value
s	A complex variable used in frequency domain
S	Stock
S^*	Desired Stock
SA	Stock Adjustment
sat	Stock Adjustment Time

SL	Supply Line
SL^*	Desired Supply Line
SLA	Supply Line Adjustment
$Step(t)$	Unit step function
Sus	Susceptible; a stock variable
t	Time
$u(s)$	Input variable in the Laplace domain
wsl	Weight of Supply Line
$x(t)$	A function of t
$X(s)$	Laplace transform of $x(t)$
$x_1(t)$	A function of t
$X_1(s)$	Laplace transform of $x_1(t)$
$x_2(t)$	A function of t
$X_2(s)$	Laplace transform of $x_2(t)$
$y(s)$	Output variable in the Laplace domain

1. INTRODUCTION

In this thesis, different stock management structures are modeled and analyzed. Stock management is a dynamic task that requires continuous effort. The aim in stock management is to bring a stock at a desired level and maintain it at that level by taking corrective actions (Mutallip, 2013; Sterman, 2000). Stock management structures are often found in managerial, physical, and biological systems. For example, inventory control appears in managerial systems (Barlas, 2002; Barlas and Ozevin, 2004; Riddalls and Bennett, 2002a, Sipahi and Delice, 2010), the control of the level of production capacity also appears in managerial systems (Paich and Sterman, 1993; Chapter 20 in Sterman, 2000; Vlachos *et al.*, 2007), the temperature of a room that is regulated by an air-conditioning device constitutes a physical system, the control of the angular position of a helicopter is a result of human-physical system interactions (Şeker and Yasarcan, 2010), and the regulation of blood glucose in a healthy person is a part of a biological system (Herdem and Yasarcan, 2010).

1.1. Delays in Stock Management

Stock management is subject to many studies (Angerhofer at al., 2000; Akkermans and Vos, 2003; Edali and Yasarcan, 2014; Chaharsooghi *et al.*, 2008; Größler *et al.*, 2008; Sterman, 1989; Yasarcan and Barlas, 2005a; Yasarcan, 2010 and 2011). According to the experimental studies carried out with human participants, stock management task imposes difficulties to the decision maker (or to the controller), which results in unwanted oscillations and costs associated with these dynamics (Barlas and Ozevin, 2004; Diehl and Sterman, 1995; Moxnes, 2000; Sengupta *et al.*, 1999; Sterman, 1989; Yasarcan, 2010). The intrinsic cause of difficulty in stock management is the existence of delays because time delays separate causes and their effects in time and, perhaps, also in space. Regardless of the increase in the level of complexity caused by the existence of a delay, it must be represented in the model of a dynamic system because models containing time delays are more realistic compared to the models that do not represent delay causing structures, which is also valid for the models of stock management tasks (Barlas, 2002; Chapter 11 in Sterman, 2000; Yasarcan, 2011).

The existence of a delay causing structure brings difficulty to the stock management task that may result in excessive stock, stock deficit, or intolerable oscillations around the desired level. These undesirable dynamics bring about additional costs (Barlas and Gündüz, 2011; Li and Liu, 2013; Mutallip and Yasarcan, 2014; Riddalls and Bennett, 2002a; Rydzak and Sawicka, 2008; Sengupta, *et al.*, 1999; Sipahi and Delice, 2010; Sterman, 1989; Yasarcan, 2011). For example, in inventory control, overshoot of the inventory level leads to an increase in holding costs while undershoot of the inventory level results in an increase in backlog costs or lost sales costs (Edali and Yasarcan, 2015; IE 413, Unpublished Lecture Notes; Sterman, 1989). As the aim in stock management is to bring a stock at a desired level and maintain it at that level and as it is costly to have oscillatory dynamics, eliminating oscillations is suggested as a solution (Edali and Yasarcan, 2015; Riddalls and Bennett, 2002a and 2002b; Sipahi and Delice, 2010; Sterman, 1987, 1989; Yasarcan and Barlas, 2005a; Yasarcan, 2011).

1.2. Physical and Decision Parameters

A stock management model usually represents two aspects of the stock management task: (i) the physical process, (ii) the decision making process. Sterman (1989) suggests using anchor-and-adjust heuristic to represent the decision making processes of human participants and he claims that the suggested heuristic is a good representation. Therefore, we represent the decision making process using the anchor-and-adjust heuristic. In this thesis, the parameters related to the physical aspect of the stock management task are called *physical parameters* and the parameters related to the decision making processes, the parameters of the anchor-and-adjust heuristic, are called *decision parameters*.

1.3. Stock Management Studies and Motivation for this Thesis

Sterman (1989) suggests optimum values for two decision parameters for each individual stock management task in The Beer Game; there exists an inventory management problem for each echelon on the supply-chain in the game. Each one of these cascading stock management problems involves a discrete supply line delay (i.e., pure supply line delay; infinite order supply line delay). Therefore, the suggested optimum values are only valid for such problems. Yasarcan and Barlas (2005a) use one of those optimum values to obtain non-oscillatory dynamics in stock management with different structural types of delays (material delay, information delay, control via a secondary stock, and composite delay). Mutallip and Yasarcan (2014) optimizes one of the parameter values for stock management problems with continuous delays (first, second, third, fourth, and eight order delays) using a simple penalty function that accumulates the differences between the stock and its desired level. They also suggest ways to select meaningful values for the other decision parameter. In addition to the results given for continuous delay cases, they validate the suggested values by Sterman (1989) for a discrete delay case. Riddalls and Bennett (2002a) analyze a stock management problem with a discrete delay from a control theoretic perspective, where the results obtained by them are also in accordance with the results reported by Sterman (1989). Barlas and Ozevin (2004) uses two stock management tasks (one with a first order delay and the other one with a discrete delay) in their experiments where they use human participants as decision makers. They fit different ordering rules to the data obtained from the participants aiming to evaluate the adequacy of literature suggested ordering polices. According to their results, the widely used anchor-and-adjust heuristic can represent the orders placed by participants who make relative smoother adjustments, but the same heuristic is weak in representing the orders placed by participants who have sudden jumps in their orders. Mikati (2010) studies the problem from a different angle and he caries out simulation experiments in determining the effect of batch sizes on the average duration of delay under different operational conditions. Yasarcan and Barlas (2005b) diverts from the rest literature because they assume that the loss from the stock depends proportionally on the stock itself. However, none of these studies makes a through stability analysis for a stock management problem with a continuous delay. Note that discrete delay (pure delay, infinite order delay) has a simple mathematical expression. Thus, it is widely used in representing delays. However, continuous delays can more realistically represent a delay causing structure compared to discrete delay (Mutallip and Yasarcan, 2014). Mikati (2010) used a first order material delay structure in representing delays in a production-inventory system and argued that, for their case, it is a better representation compared to discrete delay. Venkateswaren and Son (2004) suggested using a higher order continuous delay structure rather than using a discrete delay to obtain a more correct behavioral representation of delay.

The motivation for this thesis is to close this gap in the literature by carrying out a complete parametric analysis of stock management problems with continuous delays of first, second, and third orders aiming to obtain the range of values for different characteristic dynamics of stock such as goal-seeking behavior (i.e., asymptotic approach to the desired level of stock), stable oscillations, and unstable oscillations. Higher orders are excluded from this thesis because the similarity between the results of second order and third order supply line delay cases indicate that carrying out analysis for higher orders would not add much to the study. Furthermore, it gets more and more difficult to carry out the analysis as the order increases.

1.4. Methodology

In the thesis, the decision parameters of the stock management structures, which are modeled using system dynamics (SD) methodology, are analyzed using control theoretic approaches. We first provide different stock management structures modeled using stock-flow diagrams of SD methodology. Secondly, we obtain the corresponding simplified differential equations of the SD model and, based on the differential equations, we obtain block diagrams. Note that obtaining block diagrams is not a necessary step for the parametric analysis. However, we present block diagrams as there is a side purpose of this thesis, which is explained in the next paragraph. Thirdly, we convert simplified differential equations of the model from time domain to s-domain using Laplace transformation technique and obtain the transfer function. Fourthly, the characteristic equation of the transfer function is determined. Finally, we determine the critical values of the decision parameters at which a qualitative change in dynamics is observed by analyzing the roots of the characteristic equation.

In this thesis, we modeled different stock management structures using SD methodology because (i) SD is often used in modeling stock management structures, and (ii) SD has a strong focus on the validity of the constructed models, which is partially achieved by explicitly representing all problem related elements of the system in the model. Control theoretic analytical analysis is carried out in the thesis because, although SD has a strong focus on constructing valid models, it’s a simulation based approach; analytical analysis is either hard or impossible because some basic and most non-basic SD models involve complexities such as nonlinear relations. As a side purpose, this thesis aims to build a bridge between SD and control theory (CT). Accordingly, (i) SD methodology is introduced in the next chapter (Chapter 2), (ii) in chapter 2, we also provided block diagrams of two example SD models, (iii) in Appendix A, a summary of Laplace transforms, transfer functions, and block diagrams are given, and (iv) in Appendix B, 13 additional basic SD models and their corresponding block diagrams are constructed. The differences between the two systems approach, SD and CT, are caused by the focus of the studies carried out in the two fields. The main focus of the studies using system dynamics is to either make structural changes to the system and/or develop control policies in managing it (see, for example, Sterman, 2000; Yasarcan, 2011). However, the studies using control theory focus on either optimizing parameter values to obtain the optimal response from the system and/or come up with values that produce stable dynamics (see, for example, Riddalls and Bennett, 2002a; Sipahi and Delice, 2010; Zhou and Disney, 2005).

1.5. Descriptions of the Chapters of the Thesis

The system dynamics model of a stock management structure without delay, its different mathematical representations, and its corresponding block diagram are given in Chapter 3. In chapters 4, 5, 6, and 7, system dynamics models of stock management structures with a first, second, third, and infinite order supply line delays are presented, respectively. In these chapters, sets of corresponding differential equations and block diagrams are also provided. We carry out parametric analyses and provide tables representing the range of parameter values for different characteristic dynamics in chapters 3, 4, 5, and 6. The results provided in Section 7.5 are already present in other studies and given in this thesis as a quick reference. In addition to the results that are present in the literature, we have shown that the characteristic behaviors discussed in Section 7.5 are independent of the delay duration that exists in the physical process. In Chapter 8, we explain how to obtain a set of values for the decision parameters that will produce the desired dynamics.

2. STOCK-FLOW DIAGRAMS, BLOCK DIAGRAMS, AND MATHEMATICAL REPRESENTATIONS FOR DYNAMIC SYSTEMS

In system dynamics (SD) methodology, a formal mathematical model of a dynamic system consists of a stock-flow diagram and a set of equations, which together correspond to a set of approximate integral equations. It is also possible to express these models as a set of differential equations (Barlas, 2002; Forrester, 1961 and 1971; IE 533, Unpublished Lecture Notes; IE 550, Unpublished Lecture Notes; Sterman, 2000). One of the aims of this chapter is to introduce the main modeling tool (i.e., stock-flow diagrams) of SD methodology and to show different mathematical representations of SD models. Therefore, after giving the stock-flow diagram and equations of two example SD models, we also provide their approximate integral equations, difference equations, differential equations, and integral equations.

Jay Wright Forrester, the founder of SD, developed the field adapting servomechanistic ideas (Forrester, 2007; Lane, 2007). Today, servomechanism theory is known as classical control theory. The side purpose of this thesis is to build a bridge between SD and its roots (i.e. control theory). For this purpose, we constructed block diagrams of well known generic SD models providing details about SD modeling concepts. Note that a block diagram represents a set of differential equations in frequency domain. Therefore, it is natural that a block diagram of an SD model can be obtained. The link that we aim to establish between the two systems fields will help control theorists to understand SD models and will assist system dynamicists in representing their models using block diagrams, which will hopefully enable both the control theorist and the system dynamicist to use the analysis methods of control theory in analyzing SD models.

The first example given in this chapter is a basic population model and the second example is an epidemic model. In Appendix B, we give SD models, corresponding differential equations, and block diagrams of 15 additional commonly used structures: compounding, draining, first-order linear, production, goal seeking (stock adjustment), capacitated growth, growth with overshoot, a first order and a third order continuous material delay, a first order and a third order continuous information delay, discrete material delay, discrete information delay, oscillating, and simple goal setting. Block diagrams that we present are not only exact replicas of their corresponding SD models, but they also include all the details present in the SD models. We carefully formulated SD models and their corresponding block diagrams and verified their behavior by simulating them and by observing the same exact behavior from the SD model and its block diagram. In Appendix A, a summary of Laplace transforms, transfer functions, and block diagrams are also provided as a quick reference.

2.1. Basic Population Model

The basic population model explains the growth and decay processes for a population of any kind such as a bacteria population, animal population, or human population. Stock-flow diagram of a basic population model is given in Figure 2.1.

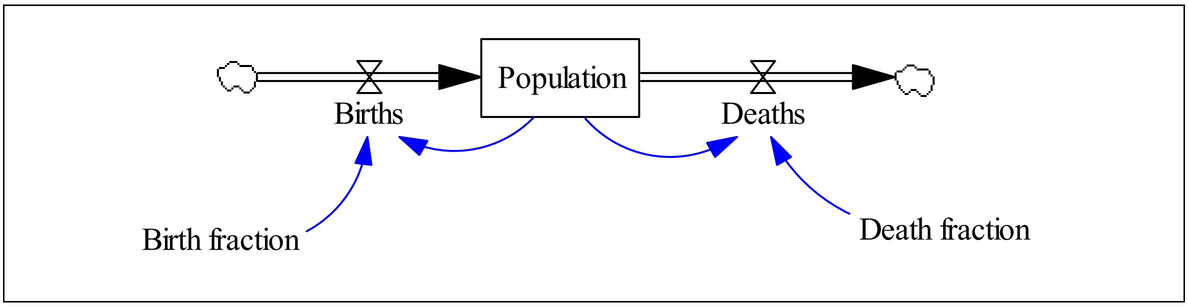

Figure 2.1. Stock-flow diagram of the basic population model (Barlas, 2002).

In the stock-flow diagram given in Figure 2.1, “Population” is a stock variable, which is an accumulation formed over time. “Population” (p) can only change via “Births” and “Deaths”, which are flow variables. There can be one, two, or more than two flows attached to a stock variable. In this simple example, there are only two flows attached to p, where “Births” is the inflow and “Deaths” is the outflow. Therefore, “Births” fills in and “Deaths” drains out p. “Birth fraction” (bf) and “Death fraction” (df) are the parameters of the population model, which consists of the stock-flow diagram given in Figure 2.1 and the equations 2.1 and 2.2.

$$Births = bf \times p \tag{2.1}$$

$$Deaths = df \times p \tag{2.2}$$

To be able to simulate the model, numerical values must be assigned to *bf*, *df*, and simulation-time-step (*DT*). *bf* and *df* can assume non-negative values and *DT* can assume a value between zero and one. If the value of *DT* is strictly between zero and one, the model corresponds to an approximate integral equation. If the value of *DT* is one, the model corresponds to a difference equation. *DT* cannot be equal to zero (Barlas, 2002; Forrester, 1961 and 1971; IE 533, Unpublished Lecture Notes; IE 550, Unpublished Lecture Notes; Sterman, 2000). If *bf* is bigger than *df*, populations grows exponentially. If *bf* is less than *df*, population decays exponentially.

2.2. Different Representations of the Basic Population Model

The relationship between the stock variable, which is *p*, and the flow variables attached to it, which are "Births" and "Deaths", imply Equation 2.3 (see Figure 2.1).

$$p_{t+DT} = p_t + (Births - Deaths) \times DT \tag{2.3}$$

Inserting equations 2.1 and 2.2 into Equation 2.3 and simplifying the equation result in Equation 2.4, which is the corresponding approximate integral equation of the model (IE 533, Unpublished Lecture Notes).

$$p_{t+DT} = p_t + (bf - df) \times p_t \times DT \tag{2.4}$$

In continuous time simulation, an approximate integral equation or a set of approximate integral equations are used; the value assigned to *DT* must strictly be less than one and greater than zero (IE 533, Unpublished Lecture Notes; IE 550, Unpublished Lecture Notes).

In discrete time simulation, a difference equation or a set of difference equations are used. Assigning one to *DT* in Equation 2.4 and simplifying the equation result in Equation

2.5, which is the corresponding difference equation of the model (IE 533, Unpublished Lecture Notes).

$$p_{t+DT} = (1 + bf - df) \times p_t \tag{2.5}$$

Equation 2.4 can be re-written as Equation 2.6.

$$\frac{p_{t+DT} - p_t}{DT} = (bf - df) \times p \tag{2.6}$$

Equation 2.7, which is the corresponding differential equation of the model, is obtained from Equation 2.6 by taking the limit of *DT* to zero (IE 533, Unpublished Lecture Notes).

$$\frac{dp}{dt} = \lim_{DT \to 0^+} \left(\frac{p_{t+DT} - p_t}{DT} \right) = (bf - df) \times p \tag{2.7}$$

Equation 2.7 can be re-written as Equation 2.8.

$$\int_{p_0}^{p_t} dp = \int_0^t (bf - df) \times p \times dt \tag{2.8}$$

Equation 2.9, which is the corresponding integral equation of the model, is obtained from Equation 2.8 (IE 533, Unpublished Lecture Notes).

$$p_t = p_0 + \int_0^t (bf - df) \times p \times dt \tag{2.9}$$

Block diagram of the basic population model is given in Figure 2.2.

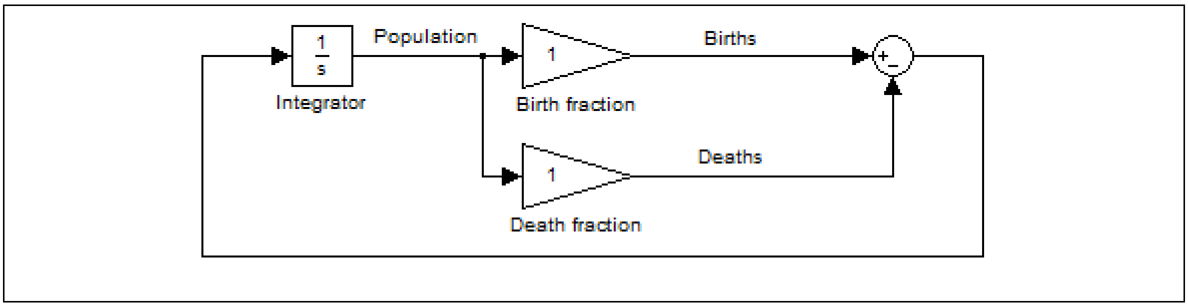

Figure 2.2. Block diagram of a basic population model.

The diagram given in Figure 2.2 represents all the details present in the SD model. "Birth fraction" and "Death fraction" are represented by gain blocks for multiplication. "Population" is the input signal of "Birth fraction" and "Death fraction". "Births", which is the output signal of "Birth fraction", is obtained by multiplying "Population" with "Birth fraction". "Deaths", which is the output signal of "Death fraction", is obtained by multiplying "Population" with "Death fraction". "Births" and "Deaths" are the input signals of a summation block. After the summation process, net rate of births (or deaths) is obtained. Net rate is the input signal of the integrator. As a consequence of the integration process, "Population", which is a stock variable, is obtained.

In Appendix A, a summary of Laplace transforms, transfer functions, and block diagrams are given. In Appendix B, 13 additional basic SD models and their corresponding block diagrams are also provided as a future reference.

2.3. Epidemic Model

The epidemic model explains how infections spread over time. Initially, a few people are infected and they slowly spread the infection. As the infected population grows, the spread of infections gets faster as the number of contacts per unit time increases. However, as the susceptible population declines, the spread of the infections starts to slow down following a very fast infection rate. This behavior is known as boom-then-bust and it is not only observed in the spread of infections, but also in the spread of information. Stock-flow diagram of the epidemic model is given in Figure 2.3.

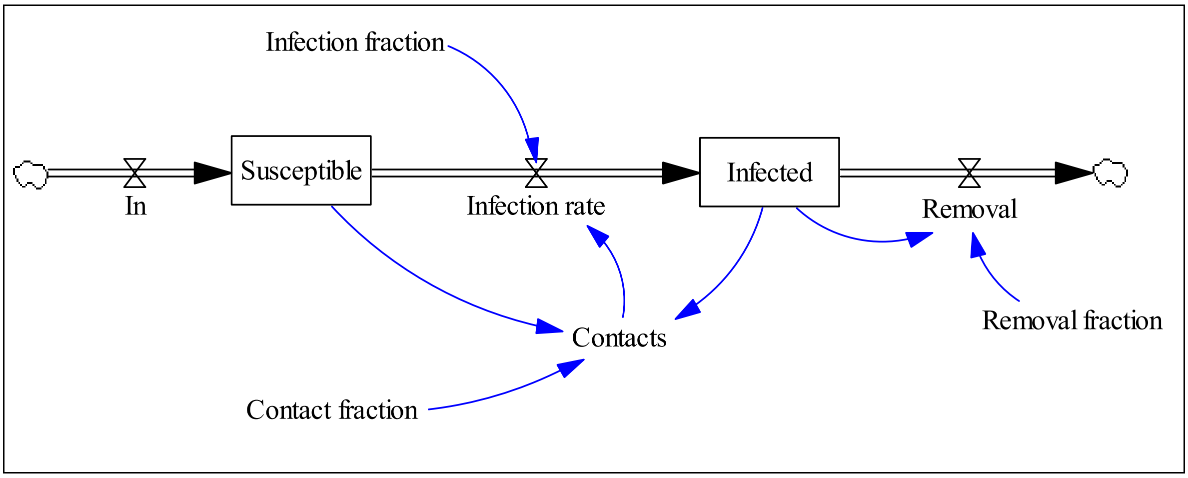

Figure 2.3. Stock-flow diagram of the epidemic model (Barlas, 2002).

In the stock-flow diagram given in Figure 2.3, “Susceptible” and “Infected” are stock variables, which are accumulations formed over time. “Susceptible” (*Sus*) can only change via “In” and “Infection rate” and “Infected” (*I*) can only change via “Infection rate” and “Removal”. “In”, “Infection rate”, and “Removal” are flow variables. “In” is the inflow of *Sus*, “Infection rate” is the outflow of *Sus* and, simultaneously, it is the inflow of *I*, and “Removal” is the outflow of *I*. Therefore, “In” fills in and “Infection rate” drains out *Sus* and “Infection rate” fill in and “Removal” drain out *I*. “Contacts” is the one and only intermediate calculation variable (i.e. auxiliary variable) of the model. In more complex models, usually there are many auxiliary variables. “Infection fraction” (*if*), “Contact fraction” (*cf*), and “Removal fraction” (*rf*) are the parameters of the epidemic model, which consists of the stock-flow diagram given in Figure Figure 2.3 and the equations 2.10, 2.11 and 2.12. *if*, *cf*, and *rf* can only assume nonnegative values.

$$Infection\ rate = if \times Contacts \tag{2.10}$$

$$Contacts = cf \times Sus \times I \tag{2.11}$$

$$Removal = rf \times I \tag{2.12}$$

To be able to simulate the model, numerical values must be assigned to *if*, *cf*, *rf,* and simulation-time-step (*DT*).

2.4. Different Representations of the Epidemic Model

The relationship between the stock variables, which are *Sus* and *I*, and the flow variables attached to them, which are “In”, “Infection rate”, and “Removal”, imply equations 2.13 and 2.14 (see Figure Figure 2.3).

$$Sus_{t+DT} = Sus_t + (In - Infection\ rate) \times DT \tag{2.13}$$

$$I_{t+DT} = I_t + (Infection\ rate - Removal) \times DT \tag{2.14}$$

Inserting equations 2.10, 2.11, and 2.12 into equations 2.13 and 2.14 and simplifying the equations result in equations 2.15 and 2.16, which are the corresponding set of approximate integral equations of the model.

$$Sus_{t+DT} = Sus_t + (In - if \times cf \times Sus \times I) \times DT \tag{2.15}$$

$$I_{t+DT} = I_t + (if \times cf \times Sus \times I - rf \times I) \times DT \tag{2.16}$$

Assigning one to *DT* in equations 2.15 and 2.16 and simplifying the equations result in equations 2.17 and 2.18, which are the corresponding set of difference equations of the model.

$$Sus_{t+DT} = Sus_t + (In - if \times cf \times Sus \times I) \tag{2.17}$$

$$I_{t+DT} = I_t + (if \times cf \times Sus \times I - rf \times I) \tag{2.18}$$

Equations 2.15 and 2.16 can be re-written as equations 2.19 and 2.20.

$$\frac{Sus_{t+DT} - Sus_t}{DT} = (In - if \times cf \times Sus \times I) \tag{2.19}$$

$$\frac{I_{t+DT} - I_t}{DT} = \left(if \times cf \times Sus \times I - rf \times I\right) \tag{2.20}$$

Equations 2.21 and 2.22, which are the corresponding set of differential equations of the model, are obtained from equations 2.19 and 2.20 by taking the limit of *DT* to zero.

$$\frac{dS}{dt} = \lim_{DT \to 0^+} \left(\frac{Sus_{t+DT} - Sus_t}{DT} \right) = \left(In - if \times cf \times Sus \times I\right) \tag{2.21}$$

$$\frac{dI}{dt} = \lim_{DT \to 0^+} \left(\frac{I_{t+DT} - I_t}{DT} \right) = \left(if \times cf \times Sus \times I - rf \times I\right) \tag{2.22}$$

Equations 2.21 and 2.22 can be re-written as equations 2.23 and 2.24.

$$\int_{Sus_0}^{Sus_t} dSus = \int_0^t \left(In - if \times cf \times Sus \times I\right) \times dt \tag{2.23}$$

$$\int_{I_0}^{I_t} dI = \int_0^t \left(if \times cf \times Sus \times I - rf \times I\right) \times dt \tag{2.24}$$

Equations 2.25 and 2.26, which are the corresponding set of integral equations of the model, is obtained from equations 2.23 and 2.24.

$$Sus_t = Sus_0 + \int_0^t \left(In - if \times cf \times Sus \times I\right) \times dt \tag{2.25}$$

$$I_t = I_0 + \int_0^t \left(if \times cf \times Sus \times I - rf \times I\right) \times dt \tag{2.26}$$

Block diagram of the epidemic model structure is given in Figure 2.4.

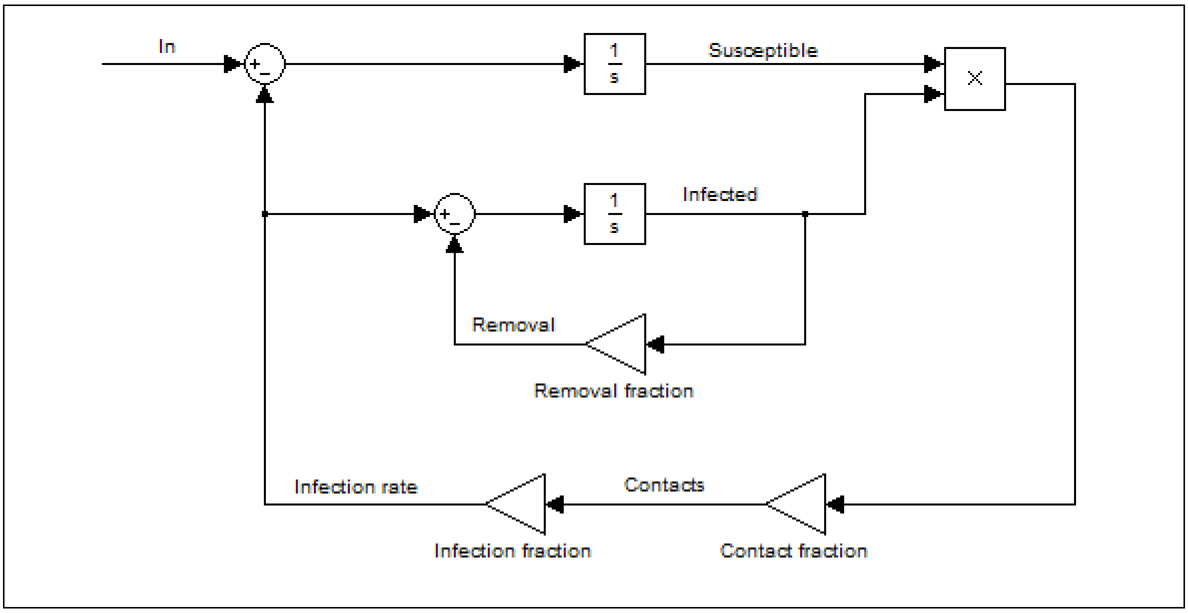

Figure 2.4. Block diagram of an epidemic model.

The diagram given in Figure 2.4 represents all the details present in the SD model. “Removal fraction”, “Contact fraction” and “Infection fraction” are represented by gain blocks. “Infected” is the input signal of “Removal fraction”. “Removal”, which is the output signal of “Removal fraction”, is obtained by multiplying “Infected” with “Removal fraction”. “Contacts” is obtained by multiplying “Susceptible” times “Infected” with “Contact fraction”. “Contacts” is the input signal of “Infection fraction”. “Infection rate”, which is the output signal of “Infection fraction”, is obtained by multiplying “Contacts” by “Infection fraction”. Stock variables, “Susceptible” and “Infected”, are obtained as a consequence of two integration processes. “Infected” is obtained by integrating a net flow that is obtained by subtracting “Removal” from “Infection rate”. “Susceptible” is obtained by integrating a net flow that is obtained by subtracting “Infection rate” from “In”.

In Appendix A, a summary of Laplace transforms, transfer functions, and block diagrams are given. In Appendix B, 13 additional basic SD models and their corresponding block diagrams are also provided as a future reference.

3. STOCK MANAGEMENT STRUCTURE WITH A FIRST ORDER SUPPLY LINE DELAY

In this chapter, a stock management structure that involves a first order supply line delay will be analyzed. In the following chapters, stock management structures with a second order delay, third order delay, and discrete delay (i.e., infinite order delay, pure delay) will be analyzed. The physical process depicted by the model that will be introduced in this chapter is almost the same for the models presented in the next three chapters (4, 5, and 6) except for the order of the delay structure. Moreover, all the parameters and the decision making process are exactly the same in all of the models given in the chapters 3, 4, 5, and 6. Therefore, the decision making process, the parameters, and most of the variables will only be discussed in detail in this chapter. The results for a stock management structure without a delay are presented in Appendix C for completeness and as a preliminary part of the study. Note that a stock management structure without a delay cannot show rich dynamics as it is way too simple to represent a real stock management system. More realistic stock management structures involve delays and they can show richer dynamics as presented in this and following chapters.

3.1. System Dynamics Model of the Stock Management Structure with a First Order Supply Line Delay

Stock-flow diagram of a stock management structure with a first order supply line delay is given in Figure 3.1.

The model equations are 3.1-3.5.

$$CF = LF + SA + SLA \tag{3.1}$$

$$SA = \frac{S^* - S}{sat} \tag{3.2}$$

$$SLA = wsl \times \frac{SL^* - SL}{sat} \tag{3.3}$$

$$SL^* = adt \times LF \tag{3.4}$$

$$AF = \frac{SL}{adt} \tag{3.5}$$

where *adt* stands for "Acquisition Delay Time", *AF* stands for "Acquisition Flow", *CF* stands for "Control Flow", *LF* stands for "Loss Flow", *S* stands for "Stock", *S** stands for "Desired Stock", *SA* stands for "Stock Adjustment", *sat* stands for "Stock Adjustment Time", *SL* stands for "Supply Line", *SL** stands for "Desired Supply Line", *SLA* stands for "Supply Line Adjustment", and *wsl* stands for "Weight of Supply Line".

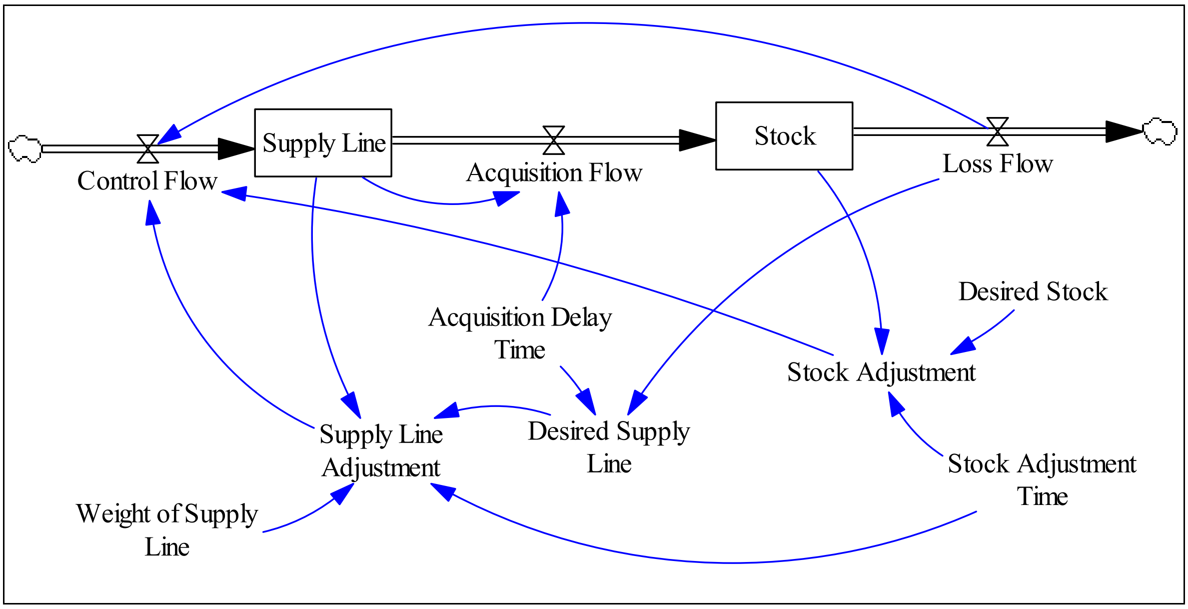

Figure 3.1. Stock-flow diagram of the stock management structure with a first order supply line delay.

The diagram in Figure 3.1 and equations 3.1-3.5 define a stock management structure with a first order supply line delay.

3.2. The Physical Process Depicted by the Stock Management Model

The physical process of the stock management task depicted by the model diagram given in Figure 3.1 consists of *adt*, *AF*, *LF*, *S*, and *SL*. *S* and *SL* are the state variables (i.e., stock variables) of this model, where *S* is the main variable of concern that is subject to control and *SL* represents the past decisions that have not yet reached the *S*. *LF* is the outflow of *S* and it drains S. In realistic systems, *LF* cannot be instantaneously measured and reported. Therefore, it is usually estimated with a smoothing method. However, in this thesis, we are not concerned with the difficulties in the estimation of *LF*. Therefore, estimation formation is not included in the models. *LF* is still an important aspect of the model because it serves as the input to the model.

adt is the duration of the delay between the corrective actions and their eventual results on the stock and its value is strictly greater than zero. In this study, we assume that *adt* cannot be controlled by the decision maker; it is inherent to the physical process. *AF* is a delayed version of *CF* and it represents the decisions that arrive to and have an effect on *S* (Equation 3.5). The physical process is defined by the stock-flow relations that are present in Figure 3.1 and Equation 3.5.

3.3. The Decision Making Process Depicted by the Stock Management Model

The decision making process depicted by the model given in Figure 3.1 consists of *CF*, *SA*, *sat*, *SLA*, *S**, *SL**, and *wsl*. *CF* represents the decisions; it is the output of the decision making process and it is input to the physical process of the stock management task (Equation 3.1). *SA* and *SLA* are the adjustment terms (equations 3.2 and 3.3). *SL** is the desired level of *SL* that is necessary to maintain an outflow (i.e., *AF*) equal to *LF* so as to be able to maintain *S* at its desired level. If *SL** is not correctly calculated, a steady-state error will be obtained (Equation 3.4). *S** is an input to the decision making process, which is assumed to be determined by processes that are excluded from this study; *S** is a predetermined constant value.

LF, *S*, and *SL* are also the inputs to the decision making process. The decision making process represented in the model is known as the anchor-and-adjust heuristic

(Sterman, 1989). The decision making process is defined by equations 3.1-3.4. Note that these equations are valid also for chapters 4, 5, and 6.

The two important decision parameters that can be controlled by the decision maker are *sat* and *wsl*; *sat* is the intended time to close the gap between the stock and its desired level and *wsl* is the relative importance given to the supply line compared to the stock. The order of the delay, *adt*, *sat*, and *wsl* determine the nature of the dynamics observed in stock. As the order of the delay and *adt* are assumed to be inherent to the physical process, the decision maker needs to decide on the values of *sat* and *wsl* to obtain the desired dynamics in *S*, which is the main concern of this thesis.

3.4. Simplified Set of Differential Equations of the Stock Management Structure with a First Order Supply Line Delay

As mentioned in Section 4.1, the diagram in Figure 3.1 and equations 3.1-3.5 define a stock management structure with a first order supply line delay. The simplified set of differential equations that corresponds to this structure is given in equations 3.6 and 3.7.

$$\frac{dS}{dt} = AF - LF = \frac{SL}{adt} - LF \tag{3.6}$$

$$\frac{dSL}{dt} = CF - AF = LF + \frac{S^* - S}{sat} + wsl \times \frac{adt \times LF - SL}{sat} - \frac{SL}{adt} \tag{3.7}$$

3.5. Block Diagram of the Stock Management Structure with a First Order Supply Line Delay

Block diagram of the stock management structure with a first order supply line delay is given in Figure 3.2.

The diagram given in Figure 3.2 represents all the details present in the SD model which is given in Figure 3.1 and equations 3.1-3.5.

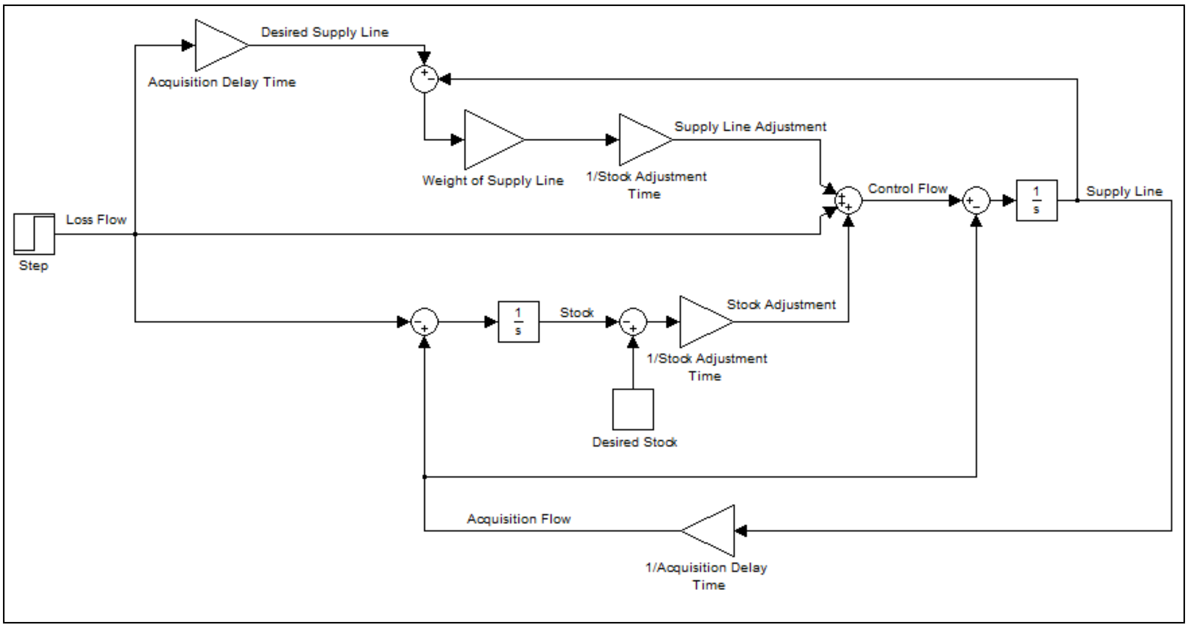

Figure 3.2. Block diagram of the stock management structure with a first order supply line delay.

3.6. Analysis of the Dynamics of Stock Management Structure with a First Order Supply Line Delay

Laplace transform of the simplified set of differential equation of the stock management structure with a first order supply line delay (equations 3.6 and 3.7), by assuming *S** equals to zero, is given in equations 3.8 and 3.9.

$$s \times S(s) - S(0) = \frac{SL(s)}{adt} - LF(s) \tag{3.8}$$

$$s \times SL(s) - SL(0) = LF(s) - \frac{S(s)}{sat} + \frac{adt \times LF(s) - SL(s)}{sat} \times wsl - \frac{SL(s)}{adt} \tag{3.9}$$

Assuming $S(0) = 0$ and $SL(0) = 0$, and arranging equations 3.8 and 3.9, equations 3.10 and 3.11 are obtained.

$$SL(s) = adt \times s \times S(s) + adt \times LF(s) \tag{3.10}$$

$$SL(s)\times\left(s+\frac{wsl}{sat}+\frac{1}{adt}\right)=LF(s)-\frac{S(s)}{sat}+\frac{wsl\times adt}{sat}\times LF(s) \tag{3.11}$$

Inserting Equation 3.10 into Equation 3.11 and arranging, Equation 3.12 is obtained.

$$\left[adt\times s^2+\left(\frac{adt\times wsl}{sat}+1\right)\times s+\frac{1}{sat}\right]\times S(s)=-adt\times s\times LF(s) \tag{3.12}$$

Transfer function of this system is the ratio of output variable ($S(s)$) to the input variable ($LF(s)$) in the Laplace domain, which is given in Equation 3.13.

$$\frac{S(s)}{LF(s)}=-\frac{adt\times s}{adt\times s^2+\left(\frac{adt\times wsl}{sat}+1\right)\times s+\frac{1}{sat}} \tag{3.13}$$

The denominator of the transfer function gives characteristic equation of the system (Equation 3.14).

$$adt\times s^2+\left(\frac{adt\times wsl}{sat}+1\right)\times s+\frac{1}{sat}=0 \tag{3.14}$$

Equation 3.14 is solved in MATLAB and characteristic roots of the characteristic equation are given in equations 3.15 and 3.16.

$$s_1=-\frac{sat+adt\times wsl+\sqrt{adt^2\times wsl^2+2\times adt\times sat\times wsl-4\times adt\times sat+sat^2}}{2\times adt\times sat} \tag{3.15}$$

$$s_2=-\frac{sat+adt\times wsl-\sqrt{adt^2\times wsl^2+2\times adt\times sat\times wsl-4\times adt\times sat+sat^2}}{2\times adt\times sat} \tag{3.16}$$

For stability property, real parts of the characteristic roots must be analyzed. If real parts all of the roots are negative (all of the roots are located in the left-half of the s-plane), the system is stable. If real part of the at least one of the roots is positive (at least one of the root is located in the right-half of the s-plane), the system is unstable.

The characteristic roots of the characteristic equation are complex conjugate (equations 3.15 and 3.16), therefore, have the same real parts. Since *adt*, *sat* and *wsl* are always positive parameters, the real parts of the characteristic roots are always negative, which is given in Equation 3.17.

$$-\frac{sat + adt \times wsl}{2 \times adt \times sat} < 0 \tag{3.17}$$

Therefore, the system is always stable which is also consistent with Yasarcan (2003).

To determine the parameter values that make stock show an oscillatory behavior, the term in the square root of the characteristic roots must be analyzed. If the term in the square root is negative, the value of characteristic root becomes complex and the stock starts to oscillate. An oscillatory behavior is obtained if the condition given in Equation 3.18 is satisfied.

$$adt^2 \times wsl^2 + 2 \times adt \times sat \times wsl - 4 \times adt \times sat + sat^2 < 0 \tag{3.18}$$

3.7. Relative Aggressiveness

A new parameter that is first introduced by (Mutallip and Yasarcan, 2014), namely, "Relative Aggressiveness" (*ra*), is defined as the ratio of *adt* to *sat* (Equation 3.19). Both *sat* and *adt* are strictly positive parameters, thus, ra is strictly greater than zero too. *ra* can be used together with *wsl* to determine the nature of the stock behavior (i.e., the characteristic dynamics). In this way, the analysis becomes simpler as the three parameter space (*wsl*, *sat*, *adt*) is reduced to two (*wsl*, *ra*). Accordingly, Equation 3.18 can be rewritten in terms of *ra* and is given in Equation 3.20.

$$ra = \frac{adt}{sat} \tag{3.19}$$

$$ra^2 \times wsl^2 + 2 \times ra \times (wsl - 2) + 1 < 0 \tag{3.20}$$

3.8. Results

For different *wsl* values between 0 and 1, critical values of *ra* are obtained by equating the right hand side of the inequality given in Equation 3.20 to zero and solving it. The results are reported in Table 3.1.

Table 3.1. Critical *ra* values obtained for different *wsl* values.

wsl	ra_1	ra_2
0.00	0.250	-
0.01	0.251	39,800
0.02	0.253	9,900
0.05	0.256	1,560
0.10	0.263	379.7
0.15	0.271	164.2
0.20	0.279	89.72
0.30	0.296	37.48
0.40	0.318	19.68
0.50	0.343	11.66
0.60	0.375	7.403
0.70	0.417	4.889
0.80	0.477	3.273
0.90	0.577	2.139
0.95	0.668	1.659
0.97	0.727	1.463
0.99	0.826	1.235
1.00	-	-

For *ra* values below or equal to ra_1 or above or equal to ra_2, stock shows no oscillations; it shows a goal seeking behavior (i.e., asymptotic approach to the desired level of stock). If a selected *ra* value is between ra_1 and ra_2, stock shows oscillatory behavior in a stable manner. For wsl equal to 0, stable oscillations can be observed for all ra values greater than 0.25. For *wsl* above or equal to 1, there are no real nonnegative values of *ra* that can make the stock oscillate. Therefore, there are no oscillations for *wsl* above or equal to 1. For *wsl* values less than 1, as *wsl* increases, the range of interval of *ra* in which stock shows oscillatory behavior decreases. In Chapter 7, it is explained how *ra* values can be used to obtain *sat* values as *sat* is the decision parameter and *ra* is only used to simplify the analysis.

3.9. Validity of Results for All Durations of Delay

The results (i.e., the characteristic behaviors) presented in Section 3.8 only depend on *wsl* (the relative importance given to the supply line compared to the stock) and *ra* (the ratio of *adt* to *sat*) and they are valid for all *adt* values because Equation 3.20 does not explicitly contain *adt* (i.e., the duration of delay between the corrective actions and their eventual results on the stock).

A stock management structure with a first order supply line delay is capable of producing only non-oscillatory behavior (i.e., goal seeking) and stable oscillations. However, a stock management structure with higher order delays is capable to produce richer dynamics including unstable oscillations as will be discussed in the following chapters (4, 5, and 6).

4. STOCK MANAGEMENT STRUCTURE WITH A SECOND ORDER SUPPLY LINE DELAY

In this chapter, a stock management structure that involves a second order supply line delay will be analyzed. The physical process depicted by the model that will be introduced in this chapter is almost the same for the model presented in Chapter 3 except for the order of the delay structure. Moreover, all the parameters and the decision making process are exactly the same in all of the models given in the chapters 3, 4, 5, and 6. Therefore, the decision making process, the parameters, and most of the variables are only discussed in detail in Chapter 3 when they are introduced for the first time.

4.1. System Dynamics Model of the Stock Management Structure with a Second Order Supply Line Delay

Stock-flow diagram of a stock management structure with a second order supply line delay is given in Figure 4.1.

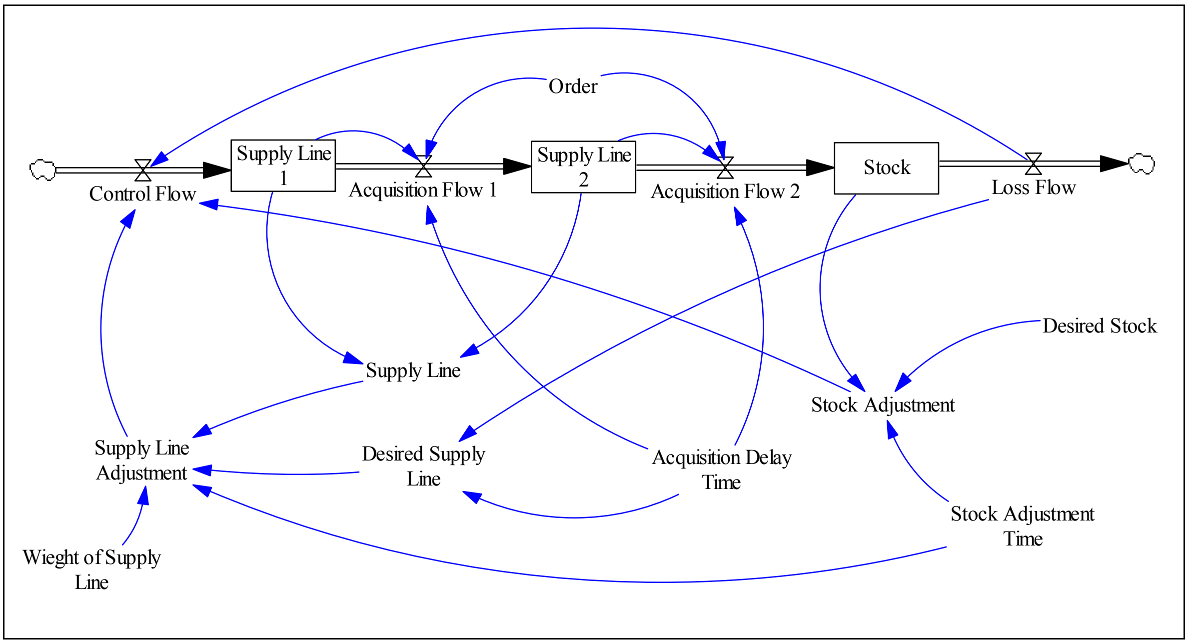

Figure 4.1. Stock-flow diagram of the stock management structure with a second order supply line delay.

The model equations are 3.1-3.4 and 4.1-4.3.

$$SL = SL1 + SL2 \tag{4.1}$$

$$AF1 = \frac{SL1}{adt/Order} \tag{4.2}$$

$$AF2 = \frac{SL2}{adt/Order} \tag{4.3}$$

where *AF1* stands for "Acquisition Flow 1", *AF2* stands for "Acquisition Flow 2", *SL1* stands for "Supply Line 1", and *SL2* stands for "Supply Line 2".

4.2. Simplified Set of Differential Equations of the Stock Management Structure with a Second Order Supply Line Delay

The diagram in Figure 4.1 and equations 3.1-3.4 and 4.1-4.3 define a stock management structure with a second order supply line delay. The simplified set of differential equations that corresponds to this structure is given in equations 4.4, 4.5 and 4.6.

$$\frac{dS}{dt} = AF2 - LF = \frac{SL2}{adt/Order} - LF \tag{4.4}$$

$$\frac{dSL1}{dt} = CF - AF1 = LF + \frac{S^* - S}{sat} + wsl \times \frac{adt \times LF - SL}{sat} - \frac{SL1}{adt/Order} \tag{4.5}$$

$$\frac{dSL2}{dt} = AF1 - AF2 = \frac{SL1}{adt/Order} - \frac{SL2}{adt/Order} \tag{4.6}$$

4.3. Block Diagram of the Stock Management Structure with a Second Order Supply Line Delay

Block diagram of the stock management structure with a second order supply line delay is given in Figure 4.2.

The diagram given in Figure 4.2 represents all the details present in the SD model which is given in Figure 4.1 and equations 3.1-3.4 and 4.1-4.3.

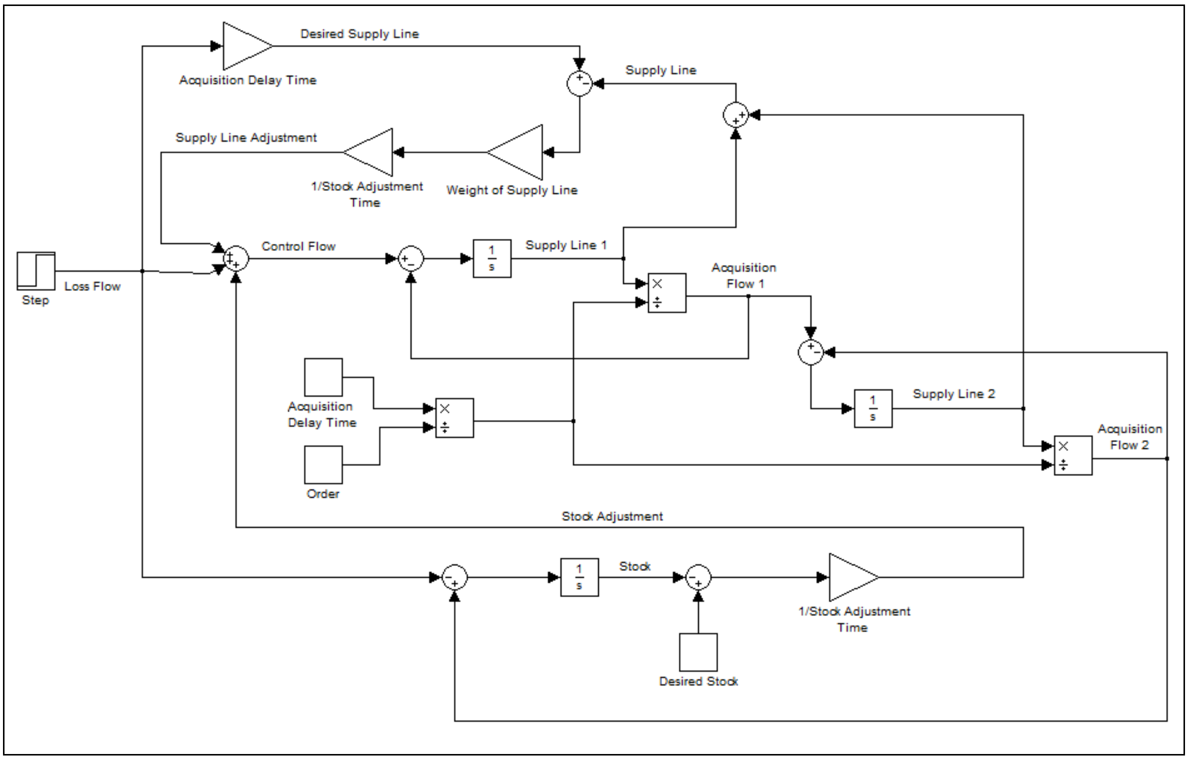

Figure 4.2. Block diagram of the stock management structure with a second order supply line delay.

4.4. Analysis of the Stock Management Structure with a Second Order Supply Line Delay

Laplace transform of the simplified set of differential equation of the stock management structure with a second order supply line delay (equations 4.4, 4.5 and 4.6), by assuming S^* equals to zero, is given in equations 4.7, 4.8, and 4.9.

$$s \times S(s) - S(0) = \frac{SL2(s)}{adt/Order} - LF(s) \tag{4.7}$$

$$s \times SL1(s) - SL1(0) = LF(s) - \frac{S(s)}{sat} + \frac{adt \times LF(s) - SL(s)}{sat} \times wsl - \frac{SL1(s)}{adt/Order} \tag{4.8}$$

$$s \times SL2(s) - SL2(0) = \frac{SL1(s)}{adt/Order} - \frac{SL2(s)}{adt/Order} \tag{4.9}$$

Assuming *S*(0) equals to zero and arranging Equation 4.7, Equation 4.10 is obtained.

$$SL2(s) = (adt/Order) \times s \times S(s) + (adt/Order) \times LF(s) \tag{4.10}$$

Assuming *SL2*(0) equals to zero, Equation 4.9 can be re-written as Equation 4.11.

$$SL1(s) = SL2(s) \times \left[(adt/Order) \times s + 1 \right] \tag{4.11}$$

Inserting Equation 4.10 into Equation 4.11 and arranging, Equation 4.12 is obtained.

$$\begin{aligned} SL1(s) = {} & \left[(adt/Order)^2 \times s^2 + (adt/Order) \times s \right] \times S(s) \\ & + \left[(adt/Order)^2 \times s + (adt/Order) \right] \times LF(s) \end{aligned} \tag{4.12}$$

SL(s) can be written as Equation 4.13.

$$SL(s) = SL1(s) + SL2(s) \tag{4.13}$$

Assuming *SL1*(0) equals to zero and inserting Equation 4.10, Equation 4.12, and Equation 4.13 into Equation 4.8, and arranging, Equation 4.14 is obtained.

$$\begin{bmatrix} \left(adt/Order\right)^2 \times s^3 + \left(2\times\left(adt/Order\right)+\dfrac{wsl}{sat}\times\left(adt/Order\right)^2\right)\times s^2 \\ +\left(2\times\dfrac{wsl}{sat}\times\left(adt/Order\right)+1\right)\times s+\dfrac{1}{sat} \end{bmatrix}\times S(s)$$
$$=\begin{bmatrix} -\left(adt/Order\right)^2 \times s^2 + \left(-2\times\left(adt/Order\right)-\dfrac{wsl}{sat}\times\left(adt/Order\right)^2\right)\times s \\ +\left(wsl\times\dfrac{adt}{sat}-2\times\dfrac{wsl}{sat}\times\left(adt/Order\right)\right) \end{bmatrix}\times LF(s) \tag{4.14}$$

Transfer function of the system is given in Equation 4.15.

$$\frac{S(s)}{LF(s)}=\frac{\begin{bmatrix} -\left(adt/Order\right)^2 \times s^2 + \left(-2\times\left(adt/Order\right)-\dfrac{wsl}{sat}\times\left(adt/Order\right)^2\right)\times s \\ +\left(wsl\times\dfrac{adt}{sat}-2\times\dfrac{wsl}{sat}\times\left(adt/Order\right)\right) \end{bmatrix}}{\begin{bmatrix} \left(adt/Order\right)^2 \times s^3 + \left(2\times\left(adt/Order\right)+\dfrac{wsl}{sat}\times\left(adt/Order\right)^2\right)\times s^2 \\ +\left(2\times\dfrac{wsl}{sat}\times\left(adt/Order\right)+1\right)\times s+\dfrac{1}{sat} \end{bmatrix}} \tag{4.15}$$

Characteristic equation of the system is given in Equation 4.16.

$$\begin{gathered} \left(adt/Order\right)^2 \times s^3 + \left(2\times\left(adt/Order\right)+\frac{wsl}{sat}\times\left(adt/Order\right)^2\right)\times s^2 \\ +\left(2\times\frac{wsl}{sat}\times\left(adt/Order\right)+1\right)\times s+\frac{1}{sat}=0 \end{gathered} \tag{4.16}$$

Equation 4.16 can be rewritten in terms of *ra* using Equation 3.19 and is given in Equation 4.17.

$$\begin{gathered} \left(adt/Order\right)^3 \times s^3 + \left(2\times\left(adt/Order\right)^2 + wsl\times\left(ra/Order\right)\times\left(adt/Order\right)^2\right)\times s^2 \\ +\left(2\times wsl\times\left(ra/Order\right)\times\left(adt/Order\right)+\left(adt/Order\right)\right)\times s + ra/Order = 0 \end{gathered} \tag{4.17}$$

4.5. Results

Equation 4.17 is used in the analysis. If the real parts of all s-roots are less than zero, the resulting behavior is stable and, if at least one is greater than zero, the resulting behavior is unstable. If all s-roots have no imaginary parts, the resulting behavior is non-oscillatory and, if at least two have imaginary parts, the resulting behavior is oscillatory. Note that it is not possible to have odd number of roots having imaginary parts.

We assigned 2 to *Order* as the stock management structure presented in this chapter has a second order supply line delay. Equation 4.17 explicitly contains *adt* too. Therefore, we arbitrarily assigned 6 to *adt* to be able to carry out the analysis. This arbitrary value selection does not make the analysis invalid as will be explained in Section 5.6. The analysis is repeated for all values of *wsl* from 0 to 1 with an increment of 0.01. For each value of *wsl*, all possible values of *ra* (i.e., *ra* > 0) are considered in determining the changes in the signs of the real and imaginary parts of s-roots (i.e. characteristic roots) that are obtained from Equation 4.17. Table 4.1 gives examples of real and imaginary parts of the s-roots for some selected values of *wsl* and *ra* for different characteristic behaviors. For example, if *wsl* is 0 and *ra* is 0.2, the stock shows goal seeking behavior (i.e., no oscillations) as all roots are real (i.e., they have no imaginary parts) and less than zero; if *wsl* is 0.60 and *ra* is 8, the stock shows stable oscillations as all real parts of the roots are negative and two roots that are complex conjugates have imaginary parts; if *wsl* is 0.05 and *ra* is 50, the stock shows unstable oscillations as two of the roots have positive real parts and the same two roots also have imaginary parts.

As a result of the analysis, four different intervals of *wsl* is determined. The characteristic behaviors that can be obtained in each interval are visualized in Figure 4.3. For *wsl* equals to zero, as *ra* increases, the behavior of stock first changes from goal seeking to stable oscillations, and, later, to unstable oscillations. For *wsl* values between 0.01 and 0.11, as *ra* increases, the behavior of stock first changes from goal seeking to stable oscillations, later, from stable oscillations to unstable oscillations, and, finally, from unstable oscillations back to stable oscillations. The change from unstable to stable oscillations was not expected because as aggressiveness in corrections grows, we usually expect to see the unstable oscillations get even stronger. For *wsl* values between 0.12 and

0.99, as *ra* increases, the behavior of stock changes from goal seeking to stable oscillations and no further change occurs. For *wsl* equals to 1, stock shows only goal seeking behavior.

Table 4.1. s-roots for different sets of *wsl* and *ra* values.

wsl	*ra*	s_1	s_2	s_3
0.00	0.2	-0.04435	-0.19580	-0.42652
0.00	3	-0.01822+0.29634i	-0.01822-0.29634i	-0.63023
0.00	10	0.07224+0.47232i	0.07224-0.47232i	-0.81114
0.05	0.25	-0.06168	-0.17285	-0.43422
0.05	4	-0.01667+0.33292i	-0.01667-0.33292i	-0.66667
0.05	50	0.11837+0.82910i	0.11837-0.82910i	-1.32008
0.05	300	-0.00315+1.32585i	-0.00315-1.32585i	-3.16037
0.60	0.4	-0.09007	-0.19513	-0.42147
0.60	8	-0.25639+0.29929i	-0.25639-0.29929i	-0.95389
1.00	2	-0.33334	-0.33334	-0.33334

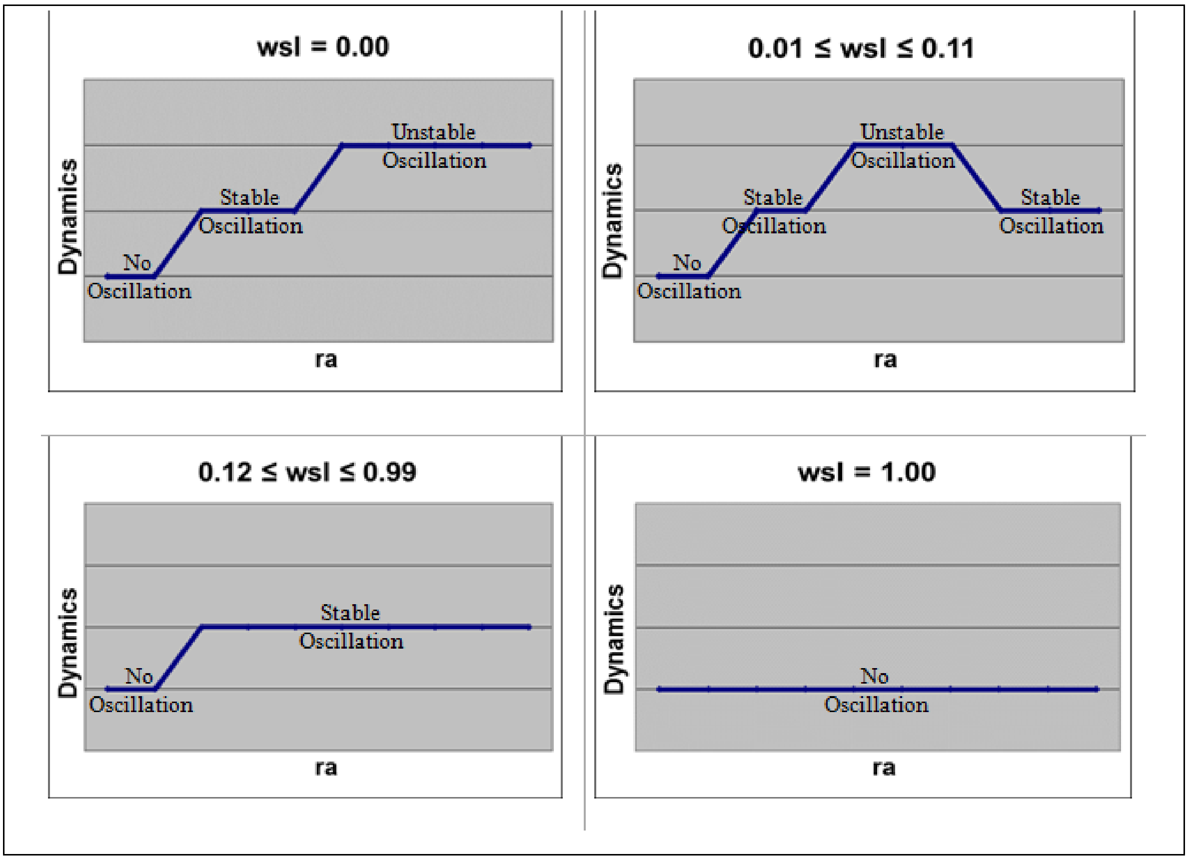

Figure 4.3. Different stock dynamics that can be obtained in the four different intervals of *wsl* values.

After these four intervals of *wsl* are obtained, we determine critical *ra* values for selected representative values of *wsl*. As described in the first paragraph of this section, all possible *ra* values are considered for each selected value of *wsl* to determine the critical *ra* values at which the dynamics change (Table 4.2). The behavior of stock first changes from goal seeking to stable oscillations at *ra* equal to ra_1, it changes from stable oscillations to unstable oscillations at *ra* equal to ra_2, and it changes from unstable oscillations back to stable oscillations at *ra* equal to ra_3. Therefore, for $0 < ra < ra_1$, goal seeking behavior is obtained; for $ra_1 \leq ra < ra_2$ and $ra_3 \leq ra$, stable oscillations is obtained; for $ra_2 \leq ra < ra_3$, unstable oscillations is obtained. If a value for ra_1, ra_2, or ra_3 is not reported in Table 4.2, it implies that there is no such value. Hence, for example, if *wsl* is selected as 0.60, stock shows goal seeking behavior for *ra* values less than 0.4696 and it shows stable oscillations for all *ra* values greater than or equal to this value. The reported values of ra_1, ra_2, or ra_3 are not exact; they are reported with only four meaningful digits. Therefore, if *ra* is selected as 0.4695, the stock will show goal seeking behavior, but one cannot know how exactly it will behave between the range $0.4695 < ra < 0.4696$ by just looking at Table 4.2.

Table 4.2. Critical *ra* values obtained for different *wsl* values.

wsl	ra_1	ra_2	ra_3
0.00	0.2962	4.000	-
0.01	0.2978	4.213	9,496
0.03	0.3012	4.730	939.7
0.05	0.3048	5.432	294.6
0.08	0.3102	7.224	86.53
0.10	0.3138	10.00	40.00
0.11	0.3158	14.70	22.50
0.12	0.3178	-	-
0.30	0.3584	-	-
0.60	0.4696	-	-
0.90	0.7912	-	-
0.95	0.9582	-	-
0.99	1.307	-	-
1.00	-	-	-

In Chapter 7, it is explained how *ra* values can be used to obtain *sat* values as *sat* is the decision parameter and *ra* is only used to simplify the analysis.

4.6. Validity of Results for All Durations of Delay

Different than the equation analyzed in the previous chapter (Equation 3.20), the equation analyzed in this chapter (Equation 4.17) explicitly contains *adt*. Therefore, a value must be assigned to *adt* to be able to obtain the intervals for *wsl* and *ra* values that produce different dynamics. Accordingly, in the second paragraph of Section 4.5, it is stated that 6 is assigned to *adt*. However, assigning an arbitrary value to *adt* does not change the dynamic behavior because *adt* does not change the signs of the real and imaginary parts of the s-roots, but it only affects the numerical values in a proportional way as presented in Table 4.3. Thus the results (i.e., the characteristic behaviors) only depend on *wsl* (the relative importance given to the supply line compared to the stock) and *ra* (the ratio of *adt* to *sat*) and they are valid for all *adt* values. The s-roots given in Table 4.3 are obtained as an example for five arbitrarily selected values of *adt*; *wsl* and ra are arbitrarily selected as 0.2 and 10, respectively. The proportional change of the s-roots can be observed in Table 4.3.

Table 4.3. s-roots obtained for different *adt* values (*wsl* = 0.2, *ra* = 10).

adt	s_1	s_2	s_3
3	-0.13753+0.91649i	-0.13753-0.91649i	-1.72493
6	-0.06877+0.45824i	-0.06877-0.45824i	-0.86247
12	-0.03438+0.22912i	-0.03438-0.22912i	-0.43123
18	-0.02292+0.15275i	-0.02292-0.15275i	-0.28749
24	-0.01719+0.11456i	-0.01719-0.11456i	-0.21562

If s-roots are obtained for a set of values of *wsl*, *ra*, and *adt*, the new values of s-roots for the same *wsl* and *ra* values, but for a new value of *adt* can be obtained using Equation 4.18.

$$(s_i)_{new} = \frac{(adt)_{old}}{(adt)_{new}} \times (s_i)_{old} \qquad \textit{for all } i \tag{4.18}$$

5. STOCK MANAGEMENT STRUCTURE WITH A THIRD ORDER SUPPLY LINE DELAY

In this chapter, a stock management structure that involves a third order supply line delay will be analyzed. The physical process depicted by the model that will be introduced in this chapter is almost the same for the model presented in Chapter 3 except for the order of the delay structure. Moreover, all the parameters and the decision making process are exactly the same in all of the models given in the chapters 3, 4, 5, and 6. Therefore, the decision making process, the parameters, and most of the variables are only discussed in detail in Chapter 3 when they are introduced for the first time.

5.1. System Dynamics Model of the Stock Management Structure with a Third Order Supply Line Delay

Stock-flow diagram of a stock management structure with a third order supply line delay is given in Figure 5.1.

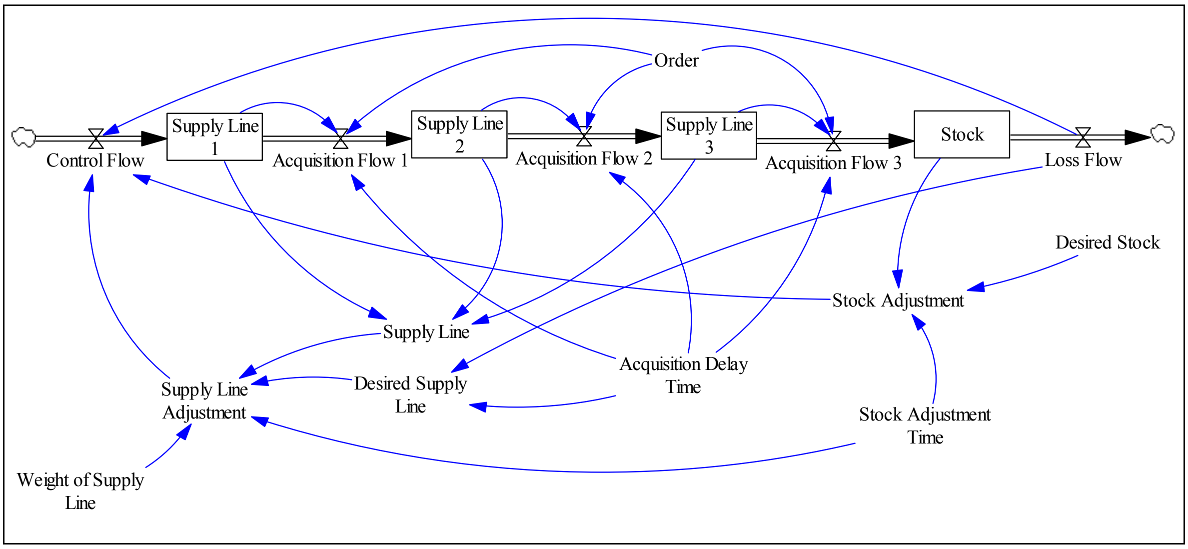

Figure 5.1. Stock-flow diagram of the stock management structure with a third order supply line delay.

The model equations are 3.1-3.4 and 5.1-5.4.

$$SL = SL1 + SL2 + SL3 \tag{5.1}$$

$$AF1 = \frac{SL1}{adt/Order} \tag{5.2}$$

$$AF2 = \frac{SL2}{adt/Order} \tag{5.3}$$

$$AF3 = \frac{SL3}{adt/Order} \tag{5.4}$$

where *AF1* stands for “Acquisition Flow 1”, *AF2* stands for “Acquisition Flow 2”, *AF3* stands for “Acquisition Flow 3”, *SL1* stands for “Supply Line 1”, *SL2* stands for “Supply Line 2”, and *SL3* stands for “Supply Line 3”.

5.2.Simplified Set of Differential Equation of the Stock Management Structure with a Third Order Supply Line Delay

The diagram in Figure 5.1 and equations 3.1-3.4 and 5.1-5.4 define a stock management structure with a third order supply line delay. The simplified set of differential equation that corresponds to this structure is given in equations 5.5, 5.6, 5.7, and 5.8.

$$\frac{dS}{dt} = AF3 - LF = \frac{SL3}{adt/Order} - LF \tag{5.5}$$

$$\frac{dSL1}{dt} = CF - AF1 = LF + \frac{S^* - S}{sat} + wsl \times \frac{adt \times LF - SL}{sat} - \frac{SL1}{adt/Order} \tag{5.6}$$

$$\frac{dSL2}{dt} = AF1 - AF2 = \frac{SL1}{adt/Order} - \frac{SL2}{adt/Order} \tag{5.7}$$

$$\frac{dSL3}{dt} = AF2 - AF3 = \frac{SL2}{adt/Order} - \frac{SL3}{adt/Order} \tag{5.8}$$

5.3.Block Diagram of the Stock Management Structure with a Third Order Supply Line Delay

Block diagram of the stock management structure with a third order supply line delay is given in Figure 5.2.

The diagram given in Figure 5.2 represents all the details present in the SD model which is given in Figure 5.1 and equations 3.1-3.4 and 5.1-5.4.

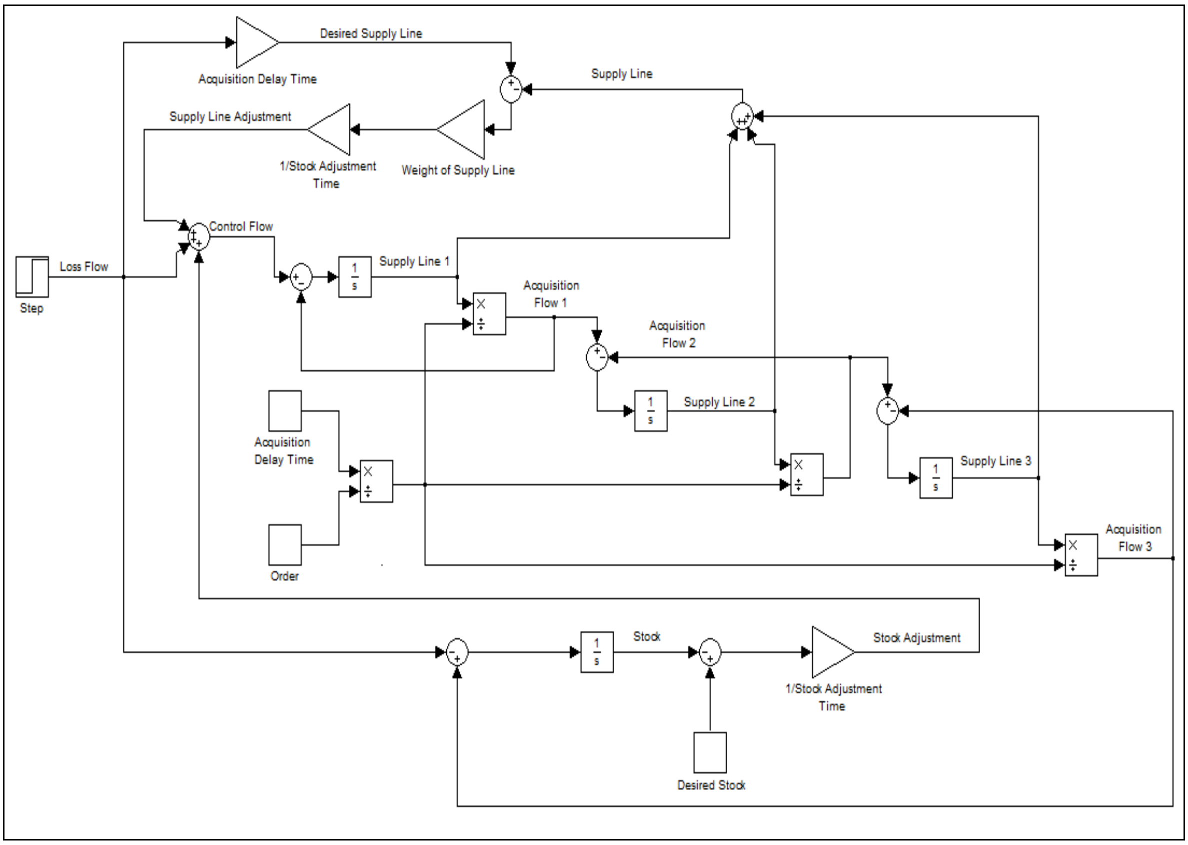

Figure 5.2. Block diagram of the stock management structure with a third order supply line delay.

5.4. Analysis of the Stock Management Structure with a Third Order Supply Line Delay

Laplace transform of the simplified set of differential equation of the stock management structure with a third order supply line delay (equations 5.5, 5.6, 5.7, and 5.8), by assuming *S** equals to zero, is given in equations 5.9, 5.10, 5.11, and 5.12.

$$s \times S(s) - S(0) = \frac{SL3(s)}{adt/Order} - LF(s) \tag{5.9}$$

$$s \times SL1(s) - SL1(0) = LF(s) - \frac{S(s)}{sat} + \frac{adt \times LF(s) - SL(s)}{sat} \times wsl - \frac{SL1(s)}{adt/Order} \tag{5.10}$$

$$s \times SL2(s) - SL2(0) = \frac{SL1(s)}{adt/Order} - \frac{SL2(s)}{adt/Order} \tag{5.11}$$

$$s \times SL3(s) - SL3(0) = \frac{SL2}{adt/Order} - \frac{SL3}{adt/Order} \tag{5.12}$$

Assuming *S*(0) equals to zero and arranging Equation 5.9, Equation 5.13 is obtained.

$$SL3(s) = (adt / Order) \times s \times S(s) + (adt / Order) \times LF(s) \tag{5.13}$$

Assuming *SL3*(0) equals to zero, Equation 5.12 can be re-written as Equation 5.14.

$$SL2(s) = SL3(s) \times [(adt / Order) \times s + 1] \tag{5.14}$$

Inserting Equation 5.13 into Equation 5.14 and arranging, Equation 5.15 is obtained.

$$\begin{aligned} SL2(s) = {} & [(adt / Order)^2 \times s^2 + (adt / Order) \times s] \times S(s) \\ & + [(adt / Order)^2 \times s + (adt / Order)] \times LF(s) \end{aligned} \tag{5.15}$$

Assuming *SL2*(0) equals to zero, Equation 5.11 can be re-written as Equation 5.16.

$$SL2(s)\left[\left(adt/Order\right)\times s+1\right]=SL1(s) \tag{5.16}$$

Inserting Equation 5.15 into Equation 5.16, Equation 5.17 is obtained.

$$\begin{aligned} SL1(s) = & \left[\left(adt/Order\right)^3\times s^3+2\times\left(adt/Order\right)^2\times s^2+\left(adt/Order\right)\times s\right]\times S(s) \\ & +\left[\left(adt/Order\right)^3\times s^2+2\times\left(adt/Order\right)^2\times s+\left(adt/Order\right)\right]\times LF(s) \end{aligned} \tag{5.17}$$

SL(s) can be written as Equation 5.18.

$$SL(s)=SL1(s)+SL2(s)+SL3(s) \tag{5.18}$$

Assuming *SL1*(0) equals to zero and inserting Equation 5.13, Equation 5.15, Equation 5.17 and Equation 5.18 into Equation 5.10, and arranging, Equation 5.19 is obtained.

$$\begin{aligned} & \begin{bmatrix} \left(adt/Order\right)^3\times s^4+\left(3\times\left(adt/Order\right)^2+\dfrac{wsl}{sat}\times\left(adt/Order\right)^3\right)\times s^3 \\ +\left(3\times\left(adt/Order\right)+3\times\dfrac{wsl}{sat}\times\left(adt/Order\right)^2\right)\times s^2 \\ +\left(3\times\dfrac{wsl}{sat}\times\left(adt/Order\right)+1\right)\times s+\dfrac{1}{sat} \end{bmatrix}\times S(s) \\ = & \begin{bmatrix} -\left(adt/Order\right)^3\times s^3+\left(-3\times\left(adt/Order\right)^2-\dfrac{wsl}{sat}\times\left(adt/Order\right)^3\right)\times s^2 \\ +\left(-3\times\left(adt/Order\right)-3\times\dfrac{wsl}{sat}\times\left(adt/Order\right)^2\right)\times s \\ +\left(wsl\times\dfrac{adt}{sat}-3\times\dfrac{wsl}{sat}\times\left(adt/Order\right)\right) \end{bmatrix}\times LF(s) \end{aligned} \tag{5.19}$$

Transfer function of the system is given in Equation 5.20.

$$\frac{S(s)}{LF(s)}=\frac{\left[\begin{array}{l}-(adt/Order)^3\times s^3+\left(-3\times(adt/Order)^2-\frac{wsl}{sat}\times(adt/Order)^3\right)\times s^2\\ \quad+\left(-3\times(adt/Order)-3\times\frac{wsl}{sat}\times(adt/Order)^2\right)\times s\\ \quad+\left(wsl\times\frac{adt}{sat}-3\times\frac{wsl}{sat}\times(adt/Order)\right)\end{array}\right]}{\left[\begin{array}{l}(adt/Order)^3\times s^4+\left(3\times(adt/Order)^2+\frac{wsl}{sat}\times(adt/Order)^3\right)\times s^3\\ \quad+\left(3\times(adt/Order)+3\times\frac{wsl}{sat}\times(adt/Order)^2\right)\times s^2\\ \quad+\left(3\times\frac{wsl}{sat}\times(adt/Order)+1\right)\times s+\frac{1}{sat}\end{array}\right]} \tag{5.20}$$

Characteristic equation of the system is given in Equation 5.21.

$$\begin{array}{l}(adt/Order)^3\times s^4+\left(3\times(adt/Order)^2+\frac{wsl}{sat}\times(adt/Order)^3\right)\times s^3\\ \quad+\left(3\times(adt/Order)+3\times\frac{wsl}{sat}\times(adt/Order)^2\right)\times s^2\\ \quad+\left(3\times\frac{wsl}{sat}\times(adt/Order)+1\right)\times s+\frac{1}{sat}=0\end{array} \tag{5.21}$$

Equation 5.21 can be rewritten in terms of *ra* using Equation 3.19 and is given in Equation 5.22.

$$\begin{array}{c}(adt/Order)^4\times s^4+\left(3\times(adt/Order)^3+wsl\times(ra/Order)\times(adt/Order)^3\right)\times s^3\\ +\left(3\times(adt/Order)^2+3\times wsl\times(ra/Order)\times(adt/Order)^2\right)\times s^2\\ +\left(3\times wsl\times(ra/Order)\times(adt/Order)+(adt/Order)\right)\times s+(ra/Order)=0\end{array} \tag{5.22}$$

5.5. Results

Equation 5.22 is used in the analysis. If the real parts of all s-roots are less than zero, the resulting behavior is stable and, if at least one is greater than zero, the resulting behavior is unstable. If all s-roots have no imaginary parts, the resulting behavior is non-oscillatory and, if at least two have imaginary parts, the resulting behavior is oscillatory. Note that it is not possible to have odd number of roots having imaginary parts.

We assigned 3 to *Order* as the stock management structure presented in this chapter has a third order supply line delay. Equation 5.22 explicitly contains *adt* too. Therefore, we arbitrarily assigned 6 to *adt* to be able to carry out the analysis. This arbitrary value selection does not make the analysis invalid as will be explained in Section 6.6. The analysis is repeated for all values of *wsl* from 0 to 1 with an increment of 0.01. For each value of *wsl*, all possible values of *ra* (i.e., *ra* > 0) are considered in determining the changes in the signs of the real and imaginary parts of s-roots (i.e. characteristic roots) that are obtained from Equation 5.22. In the previous chapter, real and imaginary parts of the s-roots for some selected values of *wsl* and *ra* for different characteristic behaviors were provided in Table 4.1. Although, the numerical values will be different for a third order delay, the interpretation of the results is essentially the same. Thus, a table similar to Table 4.1 is not given in this chapter.

As a result of the analysis, four different intervals of *wsl* is determined. The characteristic behaviors that can be obtained in each interval are visualized in Figure 5.3. For *wsl* values between 0.00 and 0.11, as *ra* increases, the behavior of stock first changes from goal seeking to stable oscillations, and, later, to unstable oscillations. For *wsl* values between 0.12 and 0.19, as *ra* increases, the behavior of stock first changes from goal seeking to stable oscillations, later, from stable oscillations to unstable oscillations, and, finally, from unstable oscillations back to stable oscillations. The change from unstable to stable oscillations was not expected because as aggressiveness in corrections grows, we usually expect to see the unstable oscillations get even stronger. For *wsl* values between 0.20 and 0.99, as *ra* increases, the behavior of stock changes from goal seeking to stable oscillations and no further change occurs. For *wsl* equals to 1, stock shows only goal seeking behavior.

After these four intervals of *wsl* are obtained, we determine critical *ra* values for selected representative values of *wsl*. As described in the first paragraph of this section, all possible *ra* values are considered for each selected value of *wsl* to determine the critical *ra* values at which the dynamics change (Table 5.1).

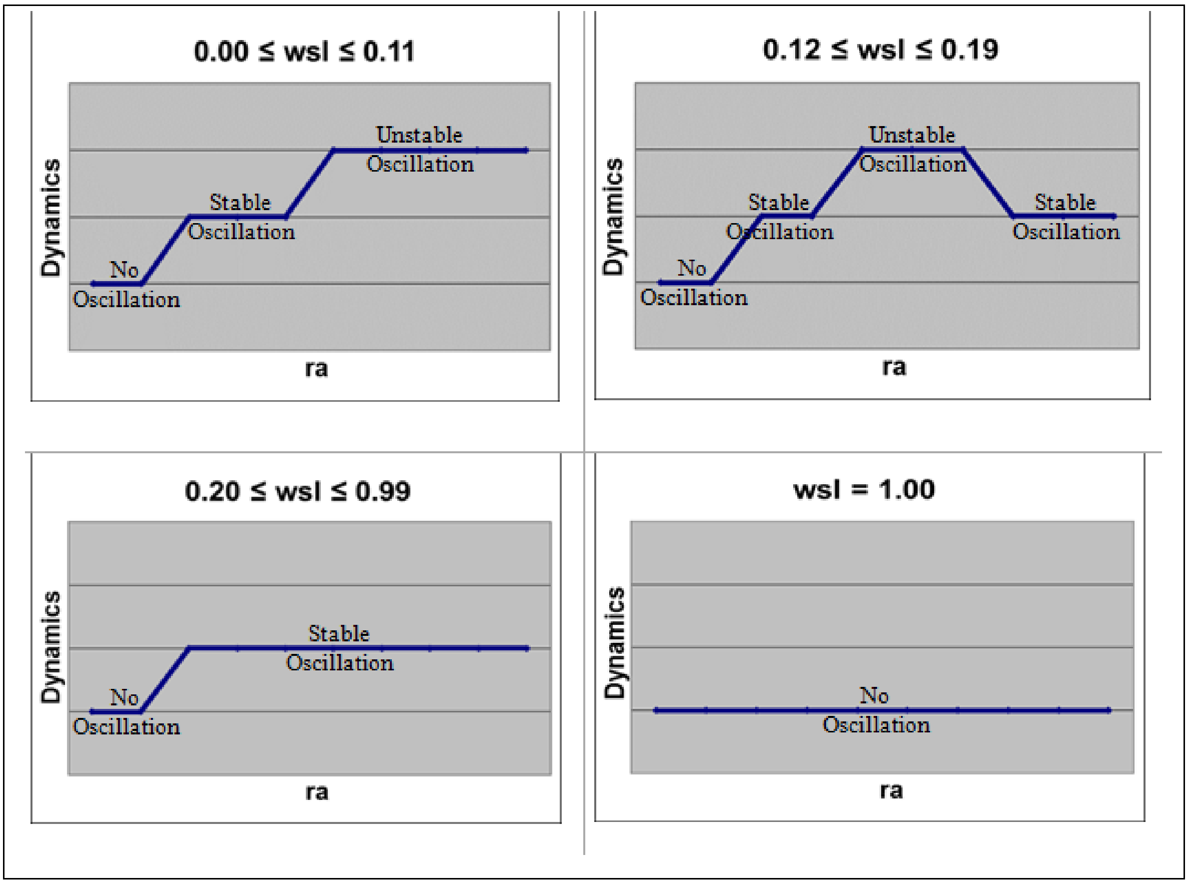

Figure 5.3. Different stock dynamics that can be obtained in the four different intervals of *wsl* values.

The behavior of stock first changes from goal seeking to stable oscillations at *ra* equal to ra_1, it changes from stable oscillations to unstable oscillations at *ra* equal to ra_2, and it changes from unstable oscillations back to stable oscillations at *ra* equal to ra_3. Therefore, for $0 < ra < ra_1$, goal seeking behavior is obtained; for $ra_1 \leq ra < ra_2$ and $ra_3 \leq ra$, stable oscillations is obtained; for $ra_2 \leq ra < ra_3$, unstable oscillations is obtained. If a value for ra_1, ra_2, or ra_3 is not reported in Table 5.1, it implies that there is no such value. Hence, for example, if *wsl* is selected as 0.15; stock shows goal seeking behavior for *ra* values less than $2.619\text{x}10^{-13}$; it shows stable oscillations for all *ra* values greater than or equal to this value, but less than 5.418; it shows unstable oscillations for all *ra* values greater than or equal to 5.418, but less than 121.9; it shows stable oscillations for all *ra* values greater than or equal to 121.9.

Table 5.1. Critical *ra* values obtained for different *wsl* values.

wsl	ra_1	ra_2	ra_3
0.00	2.230×10^{-13}	2.667	-
0.03	2.297×10^{-13}	2.942	-
0.08	2.421×10^{-13}	3.579	-
0.11	2.519×10^{-13}	4.152	-
0.12	2.537×10^{-13}	4.397	799.0
0.15	2.619×10^{-13}	5.418	121.9
0.17	2.679×10^{-13}	6.573	59.48
0.19	2.747×10^{-13}	8.987	29.49
0.20	2.783×10^{-13}	-	-
0.30	3.185×10^{-13}	-	-
0.50	4.446×10^{-13}	-	-
0.80	1.114×10^{-12}	-	-
0.90	2.233×10^{-12}	-	-
0.99	2.228×10^{-11}	-	-
1.00	-	-	-

In Chapter 8, it is explained how *ra* values can be used to obtain *sat* values as *sat* is the decision parameter and *ra* is only used to simplify the analysis.

5.6. Validity of Results for All Durations of Delay

Different than the equation analyzed in Chapter 3 (Equation 3.20), but similar to the equation analyzed in Chapter 4 (Equation 4.17), the equation analyzed in this chapter (Equation 5.22) explicitly contains *adt*. Therefore, a value must be assigned to *adt* to be able to obtain the intervals for *wsl* and *ra* values that produce different dynamics. Accordingly, in the second paragraph of Section 5.5, it is stated that 6 is assigned to *adt*. However, assigning an arbitrary value to *adt* does not change the dynamic behavior because *adt* does not change the signs of the real and imaginary parts of the s-roots, but it only affects the numerical values in a proportional way as presented in Table 5.2. Thus the results (i.e., the characteristic behaviors) only depend on *wsl* (the relative importance given to the supply line compared to the stock) and *ra* (the ratio of *adt* to *sat*) and they are valid for all *adt* values. The s-roots given in Table 5.2 are obtained as an example for four arbitrarily selected values of *adt*; *wsl* and ra are arbitrarily selected as 0.2 and 6, respectively. The proportional change of the s-roots can be observed in Table 5.2.

Table 5.2. s-roots obtained for different *adt* values (*wsl* = 0.2, *ra* = 6).

adt	s_1	s_2	s_3	s_4
3	-0.03396+0.76821i	-0.03396-0.76821i	-1.66604-0.77890i	-1.66604+0.77890i
6	-0.01698+0.38411i	-0.01698-0.38411i	-0.83302-0.38945i	-0.83302+0.38945i
12	-0.00849+0.19205i	-0.00849-0.19205i	-0.41651-0.19473i	-0.41651+0.19473i
18	-0.00566+0.12804i	-0.00566-0.12804i	-0.27767-0.12982i	-0.27767+0.12982i

If s-roots are obtained for a set of values of *wsl*, *ra*, and *adt*, the new values of s-roots for the same *wsl* and *ra* values, but for a new value of *adt* can be obtained using Equation 4.18 given in the previous chapter.

6. STOCK MANAGEMENT STRUCTURE WITH A DISCRETE DELAY

In this chapter, a stock management structure that involves an infinite order supply line delay (i.e., discrete delay, pure delay) will be analyzed. The physical process depicted by the model that will be introduced in this chapter is almost the same for the model presented in Chapter 3 except for the order of the delay structure. Moreover, all the parameters and the decision making process are exactly the same in all of the models given in the chapters 3, 4, 5 and 6. Therefore, the decision making process, the parameters, and most of the variables are only discussed in detail in Chapter 3 when they are introduced for the first time.

6.1. System Dynamics Model Stock Management Structure with a Discrete Delay

Stock-flow diagram of a stock management structure with a discrete delay is given in Figure 6.1.

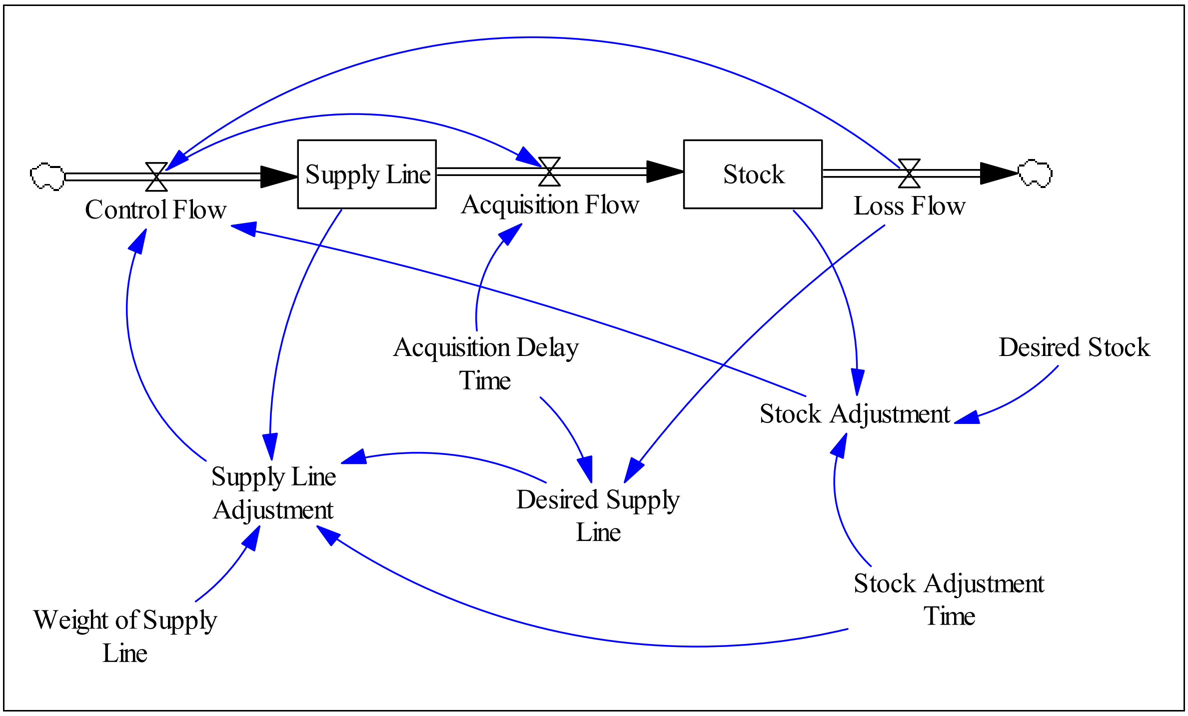

Figure 6.1. Stock-flow diagram of the stock management structure with a discrete delay.

The model equations are 3.1-3.4 and 6.1.

$$AF(t) = \begin{cases} \dfrac{SL(0)}{adt} & \text{for} \quad 0 \le t < adt \\ CF(t - adt) & \text{for} \quad t \ge adt \end{cases} \tag{6.1}$$

6.2. Simplified Set of Differential Equation of the Stock Management Structure with a Discrete Delay

The diagram in Figure 6.1 and equations 3.1-3.4 and 6.1 define a stock management structure with a discrete delay. The simplified set of differential equation that corresponds to this structure is given in equations 6.2 and 6.3.

$$\frac{dS}{dt} = AF - LF = CF(t - adt) - LF \tag{6.2}$$

$$\frac{dSL}{dt} = CF - AF = LF + \frac{S^* - S}{sat} + wsl \times \frac{adt \times LF - SL}{sat} - CF(t - adt) \tag{6.3}$$

6.3. Block Diagram of the Stock Management Structure with a Discrete Delay

Block diagram of the stock management structure with a discrete delay is given in Figure 6.2.

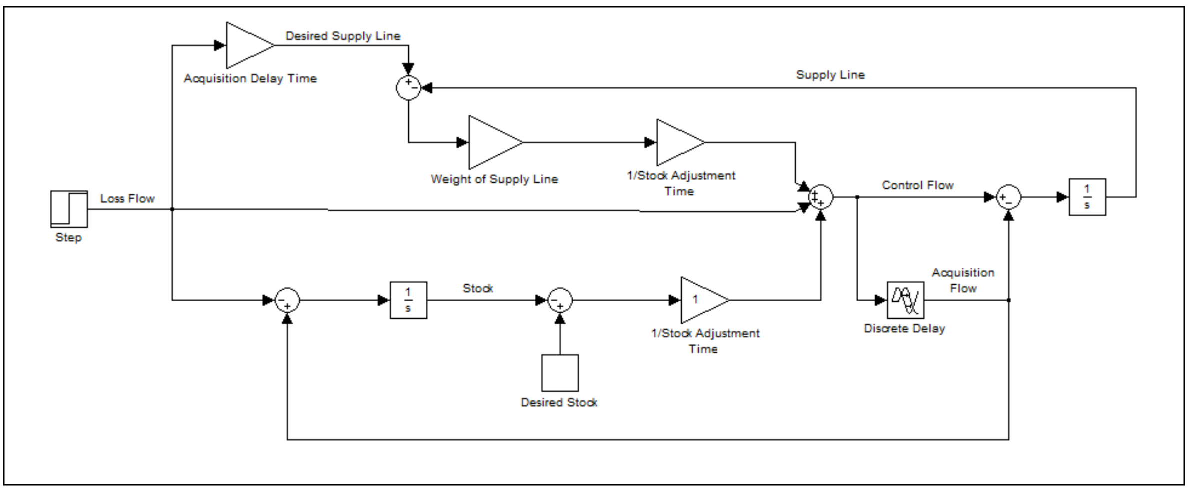

Figure 6.2. Block diagram of the stock management structure with a discrete delay.

6.4. Analysis of the Stock Management Structure with a Discrete Delay

Laplace transform of the simplified set of differential equation of the stock management structure with a discrete delay (equations 6.2 and 6.3), by assuming *S** equals to zero, is given equations 6.4 and 6.5.

$$s \times S(s) - S(0) = e^{-adt \times s} \times CF(s) - LF(s) \tag{6.4}$$

$$s \times SL(s) - SL(0) = LF(s) - \frac{S(s)}{sat} + \frac{adt \times LF(s) - SL(s)}{sat} \times wsl - e^{-adt \times s} \times CF(s) \tag{6.5}$$

Laplace transform of the "Control Flow" (*CF*(*s*)), is given in Equation 6.6.

$$CF(s) = LF(s) - \frac{S(s)}{sat} + \frac{adt \times LF(s) - SL(s)}{sat} \times wsl \tag{6.6}$$

Assuming *S*(0)=0 and *SL*(0)=0, and inserting Equation 6.6 into equations 6.4 and 6.5, equations 6.7 and 6.8 are obtained.

$$s \times S(s) = e^{-adt \times s} \times \left(LF(s) - \frac{S(s)}{sat} + \frac{adt \times LF(s) - SL(s)}{sat} \times wsl \right) - LF(s) \tag{6.7}$$

$$\begin{aligned} s \times SL(s) = LF(s) - \frac{S(s)}{sat} + \frac{adt \times LF(s) - SL(s)}{sat} \times wsl \\ - e^{-adt \times s} \times \left(LF(s) - \frac{S(s)}{sat} + \frac{adt \times LF(s) - SL(s)}{sat} \times wsl \right) \end{aligned} \tag{6.8}$$

Equation 6.8 can be re-written as Equation 6.9.

$$SL(s) = \frac{\left[\begin{aligned} & LF(s) - \frac{S(s)}{sat} + \frac{adt \times LF(s)}{sat} \times wsl \\ & - e^{-adt \times s} \times \left(LF(s) - \frac{S(s)}{sat} + \frac{adt \times LF(s)}{sat} \times wsl \right) \end{aligned} \right]}{\left(s + \frac{wsl}{sat} - e^{-adt \times s} \times \frac{wsl}{sat} \right)} \tag{6.9}$$

Inserting Equation 6.9 into Equation 6.7 and arranging, Equation 6.9 is obtained.

$$S(s)\times\left[s+\frac{e^{-adt\times s}}{sat}-\frac{e^{-adt\times s}\times\frac{wsl}{sat^{2}}-\left(e^{-adt\times s}\right)^{2}\times\frac{wsl}{sat^{2}}}{s+\frac{wsl}{sat}-e^{-adt\times s}\times\frac{wsl}{sat}}\right]$$
$$=LF(s)\times\left[\frac{\left(\begin{array}{c}e^{-adt\times s}+e^{-adt\times s}\times wsl\times\frac{adt}{sat}-e^{-adt\times s}\times\frac{wsl}{sat}-e^{-adt\times s}\times\frac{wsl^{2}\times adt}{sat^{2}}\\+\left(e^{-adt\times s}\right)^{2}\times\frac{wsl}{sat}+\left(e^{-adt\times s}\right)^{2}\times\frac{wsl^{2}\times adt}{sat^{2}}\end{array}\right)}{s+\frac{wsl}{sat}-e^{-adt\times s}\times\frac{wsl}{sat}}-1\right] \tag{6.10}$$

Characteristic equation of the system is given in Equation 6.11.

$$s+\frac{e^{-adt\times s}}{sat}-\frac{e^{-adt\times s}\times\frac{wsl}{sat^{2}}-\left(e^{-adt\times s}\right)^{2}\times\frac{wsl}{sat^{2}}}{s+\frac{wsl}{sat}-e^{-adt\times s}\times\frac{wsl}{sat}}=0 \tag{6.11}$$

6.5. Results

Since Equation 6.11 has infinitely many s-roots due to transcendental term ($e^{-adt\times s}$), it is impossible to find all of the s-roots. Therefore, the characteristic dynamics of a discrete delay structure cannot be determined by analyzing s-roots of the characteristic equation (Sipahi and Delice, 2010).

If the forced part (*LF*) of Equation 6.2 is removed, Equation 6.12 is obtained.

$$\frac{dS}{dt}=AF(t)=CF(t-adt) \tag{6.12}$$

If the forced part (*LF*) is removed and *S** is assumed to zero (without loss of generality, since the system is linear (Riddalls and Bennett, 2002b, 2003), Equation 6.13 is obtained.

$$CF(t) = -\frac{S}{sat} - wsl \times \frac{S\,L}{sat} \tag{6.13}$$

Taking the derivative of Equation 6.13 and inserting Equation 6.12 and Equation 6.3, Equation 6.14 is obtained.

$$\frac{dCF(t)}{dt} = -\frac{CF(t-adt)}{sat} - \frac{wsl}{sat} \times \left[CF(t) - CF(t-adt)\right] \tag{6.14}$$

Arranging Equation 6.14, Equation 6.15 is obtained.

$$\frac{dCF(t)}{dt} = -\frac{wsl}{sat} \times CF(t) - \left(\frac{1-wsl}{sat}\right) \times CF(t-adt) \tag{6.15}$$

The stability properties of Equation 6.15 were derived in Bellman and Cooke (1963) as reported by Riddalls and Bennett (2002b, 2003). Note that the variables names that we use in this thesis and the variable names used in Riddalls and Bennett (2002b, 2003) are not the same, but they exactly correspond to each other. The system defined by Equation 6.15 is asymptotically stable independent of delay magnitude if and only if $wsl \geq 0.5$. If, however, $wsl < 0.5$, the maximum delay (*adt**) for which the system is asymptotically stable is given in Equation 6.16.

$$adt^* \leq \frac{\arccos\left(wsl/(wsl-1)\right)}{\frac{1}{sat} \times \sqrt{1 - 2 \times wsl^2}} \tag{6.16}$$

Equation 6.16 can be rewritten in terms of *ra* using Equation 3.19 and is given in Equation 6.17.

$$ra \le \frac{\arccos\left(wsl/(wsl-1)\right)}{\sqrt{1-2\times wsl^2}} \tag{6.17}$$

Therefore for *wsl* < 0.5, for stability, *ra* must satisfy the condition defined by Equation 6.17. For a few selected *wsl* values, critical *ra* values are given in Table 6.1. *ra* values higher than ra_{cr} values produce unstable oscillations.

Table 6.1. Critical *ra* values.

wsl	ra_{cr}
0.0	1.5708
0.1	1.6992
0.2	1.9011
0.3	2.2238
0.4	2.7898

6.6. Validity of Results for All Durations of Delay

The results (i.e., the characteristic behaviors) discussed in Section 7.5 only depend on *wsl* (the relative importance given to the supply line compared to the stock) and *ra* (the ratio of *adt* to *sat*) and they are valid for all *adt* values because Equation 6.17 does not explicitly contain *adt* (i.e., the duration of delay between the corrective actions and their eventual results on the stock).

7. THE VALUES OF THE DECISION PARAMETERS THAT WILL PRODUCE THE DESIRED DYNAMICS

In this chapter, we explain how the tables provided in this thesis are used to obtain values for the decision parameters (*wsl*, *sat*) that will produce the desired dynamics. The results provided in the thesis are valid for all *adt* values and the value of the decision parameter *sat* can be obtained for a given *adt* using Equation 7.1.

$$sat = \frac{adt}{ra} \tag{7.1}$$

The duration of delay between the corrective actions and their eventual results on the stock, *adt*, is a physical parameter and belongs to the physical process. Thus, we assume that the decision maker does not have a control over it. After a decision maker determines the desired dynamics, if he wants to choose a set value for the two decision parameters, *wsl* and *sat*, he must first use a table provided in the thesis (i.e., Table 3.1, Table 4.2, Table 5.1, or Table 6.1 depending on the order of the delay present in the physical process). Later, he must use the *adt* value present in the physical process, the *wsl* and *ra* values obtained from the corresponding table, and Equation 7.1 to obtain a value for *sat*. Thus, as a result, a set of *wsl* and *sat* values is obtained.

7.1. Example Decision Parameter Value Selection for a Stock Management Model with a First Order Supply Line Delay

We assume a stock management model with a first order supply line delay and with duration of 5 units of time. We arbitrarily select *wsl* as 0.2. According to Table 3.1, if *ra* is selected below 0.279, stock shows a goal seeking behavior and the approach of the stock to its desired level gets faster as *ra* increases; see stock dynamics in Figure 7.1 for *ra* values of 0.15 (Line 2), 0.2 (Line 3), and 0.25 (Line 4).

Line 1 in Figure 7.1, Figure 7.2, Figure 7.3, Figure 7.4, Figure 7.7, Figure 7.8, Figure 7.11, Figure 7.12, and Figure 7.13 represents “Desired Stock” for all experiments in those figures. If *ra* is equal to or bigger than 0.279 and smaller than 89.72, stock shows oscillatory behavior in a stable manner; see stock dynamics in Figure 7.1 for *ra* values of 0.5 (Line 5) and 1 (Line 6).

After oscillatory behavior is observed, the highest value that stock attains starts to increase as *ra* increases, but the period of oscillations shortens; see stock dynamics in Figure 7.2 for *ra* values of 0.75 (Line 2), 1 (Line 3), 2 (Line 4), and 3 (Line 5).

The highest value that stock attains increases until *ra* becomes equal to 5, after that value, it starts to decrease as *ra* increases (Figure 7.3). However, the period of oscillations continues to shorten as *ra* increases.

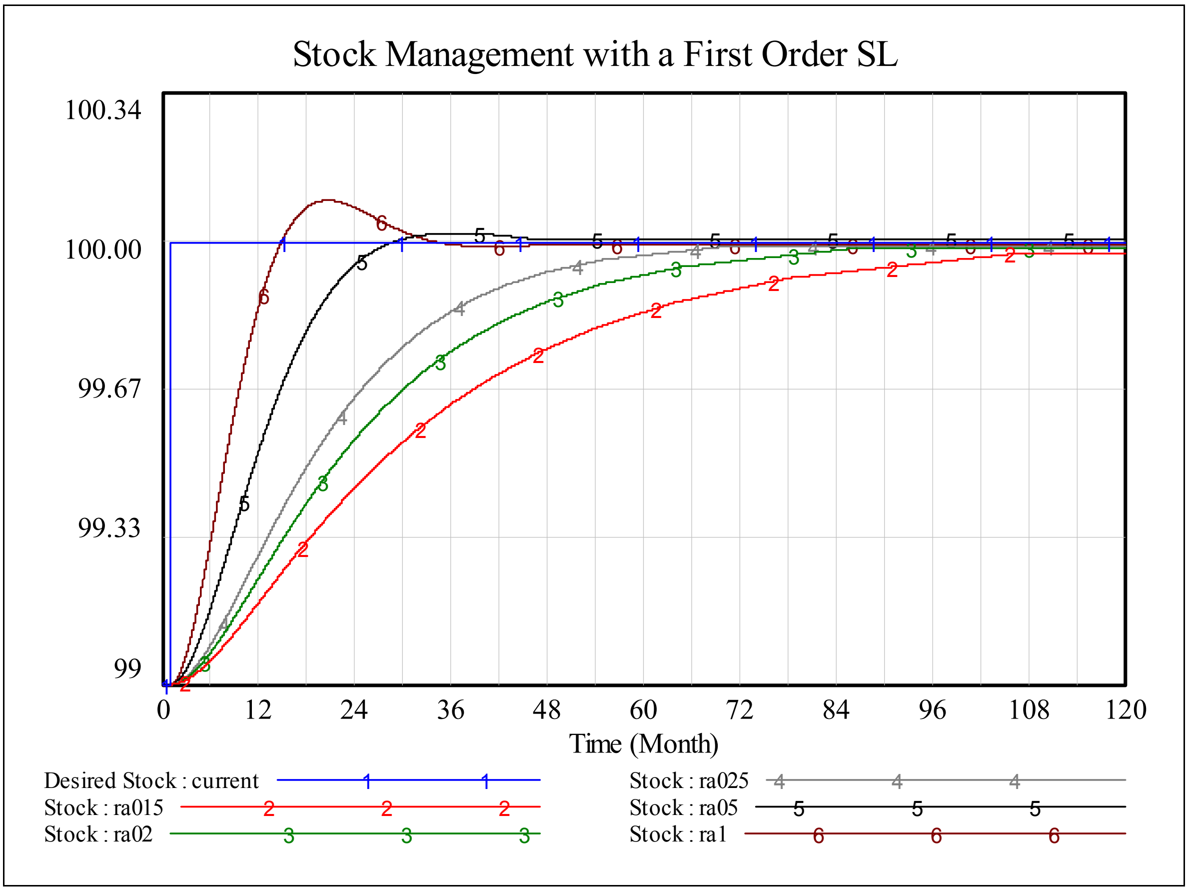

Figure 7.1. Different stock dynamics for *ra* values of 0.15, 0.2, 0.25, 0.5, and 1.

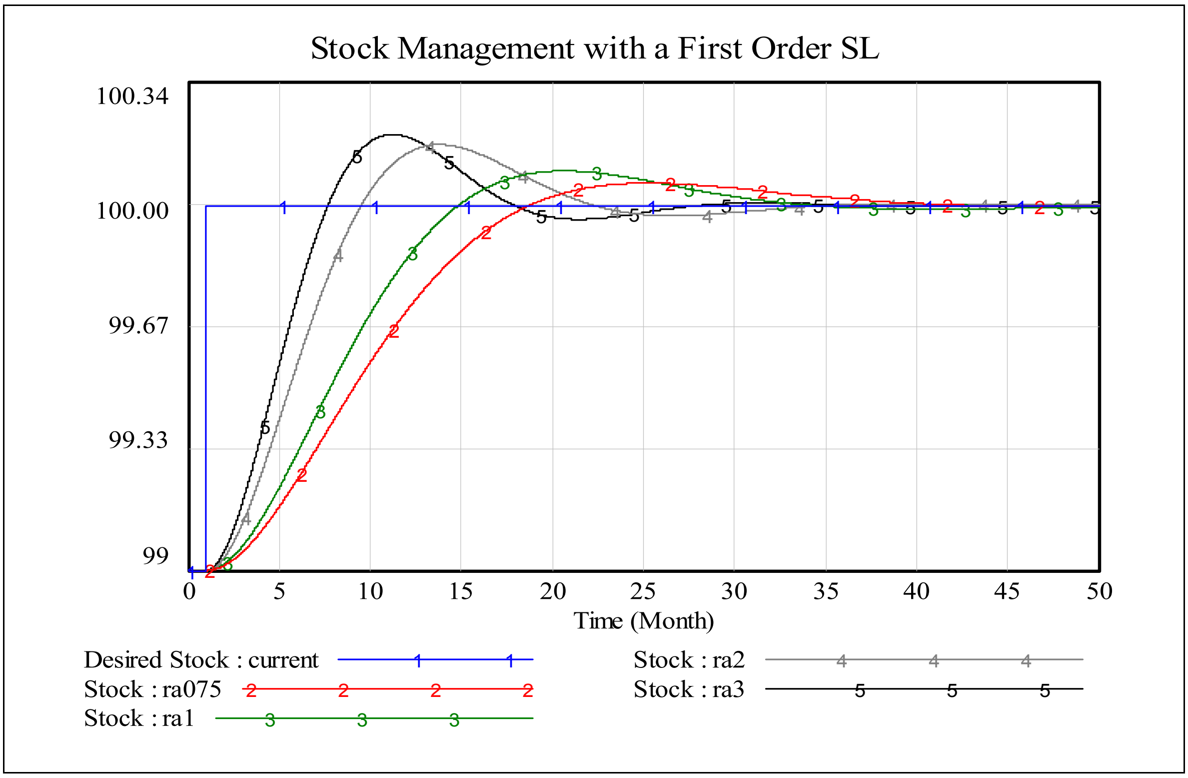

Figure 7.2. Different stock dynamics for *ra* values of 0.75, 1, 2, and 3.

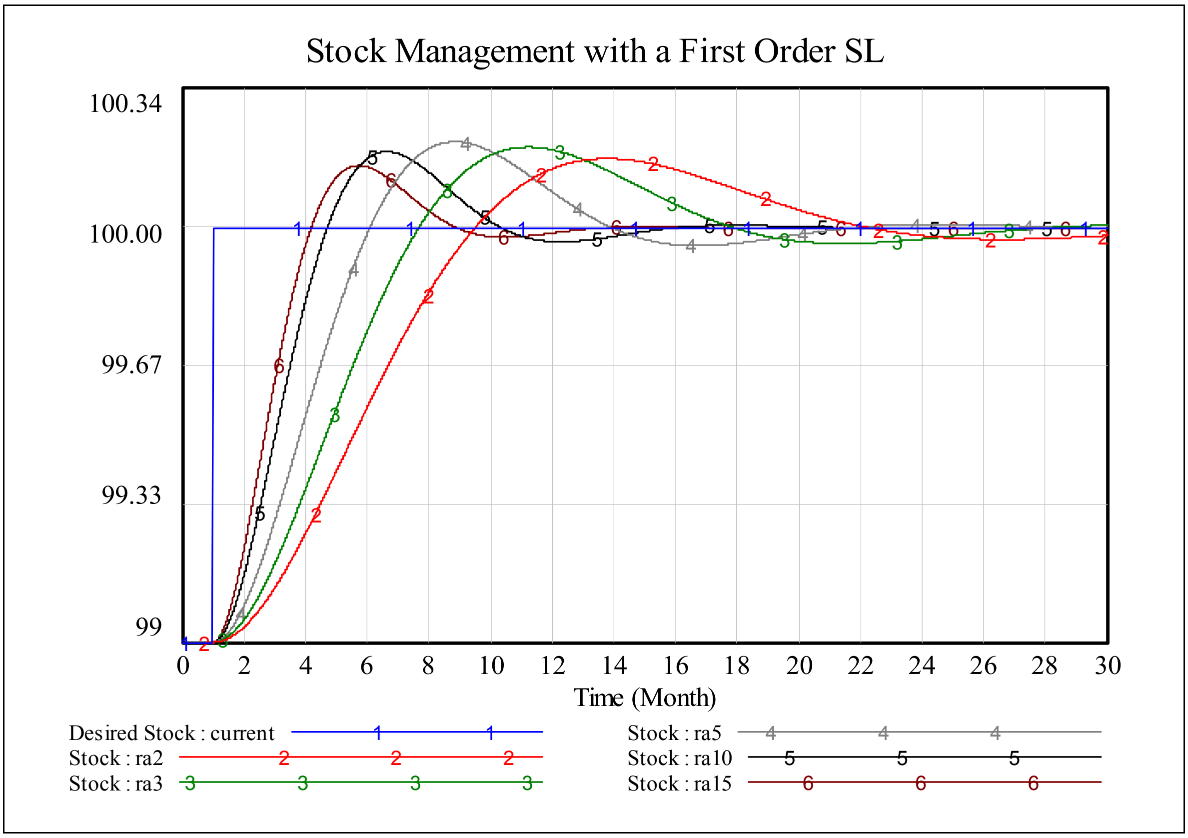

Figure 7.3. Different stock dynamics for *ra* values of 2, 3, 5, 10, and 15.

The highest value that stock attains decreases as *ra* further increases (Figure 7.4). If *ra* is selected equal to or greater than 89.72, stock shows a goal seeking behavior again. Period of the oscillations always shortens as *ra* increases, which can be seen in Figure 7.1, Figure 7.2, Figure 7.3, and Figure 7.4.

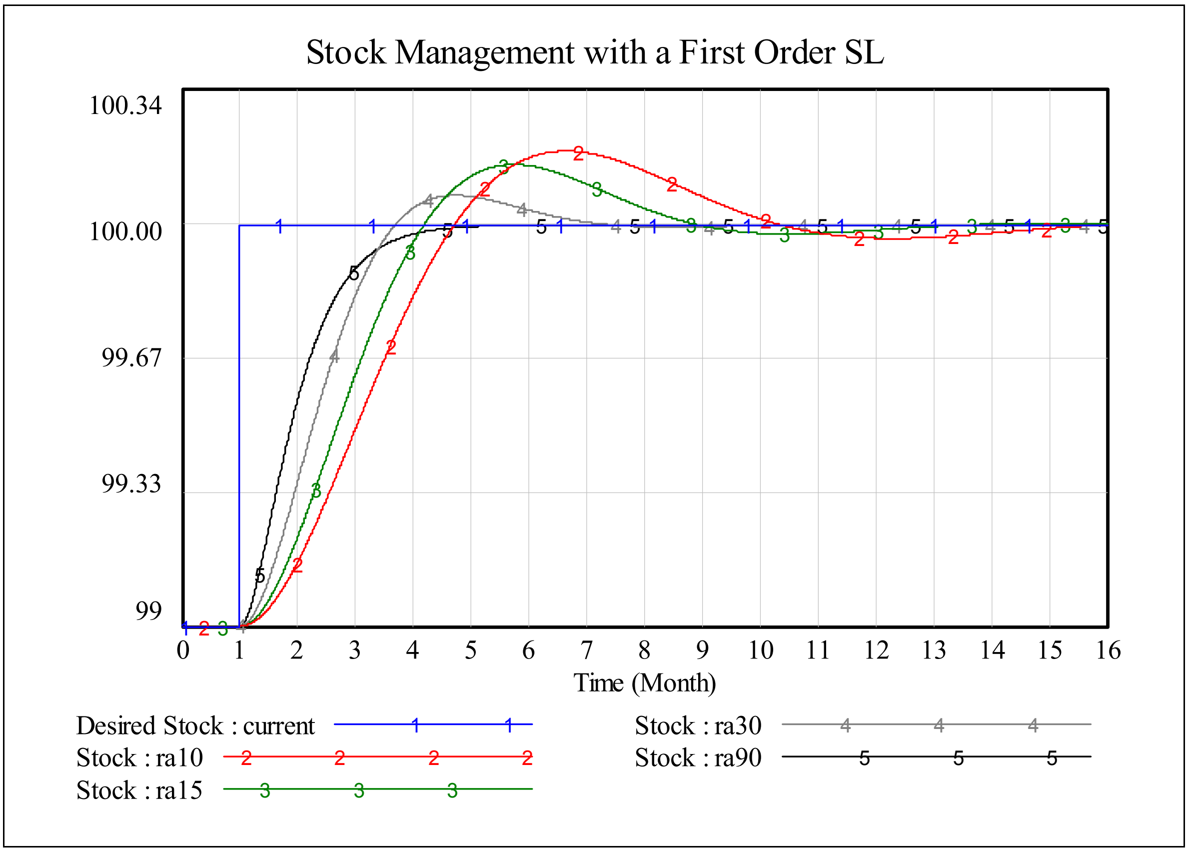

Figure 7.4. Different stock dynamics for *ra* values of 10, 15, 30, and 90.

If the desired dynamics is stable oscillations, according to Table 3.1, there are infinitely many alternatives of set of *wsl* and *ra* values for stable oscillations. For *wsl* equals to 0.2, if the desired dynamics is stable oscillations with comparatively higher amplitude values, *ra* can be chosen equal to 5. Since *adt* is 5, using Equation 7.1, *sat* is obtained as 1. Thus, the set of values for the decision parameters, *wsl* and *sat*, are 0.2 and 1, respectively. If the desired dynamics is a goal seeking behavior with a fast approach to the desired level of stock, *ra* can be chosen equal to 90. Since *adt* is 5, using Equation 7.1, *sat* is obtained as $0.0\overline{5}$. Therefore, the set of values for the decision parameters, *wsl* and *sat*, are 0.2 and $0.0\overline{5}$, respectively.

The initial value of *Control flow* increases as *ra* increases (i.e., as *sat* decreases), which can be seen in Figure 7.5 and Figure 7.6.

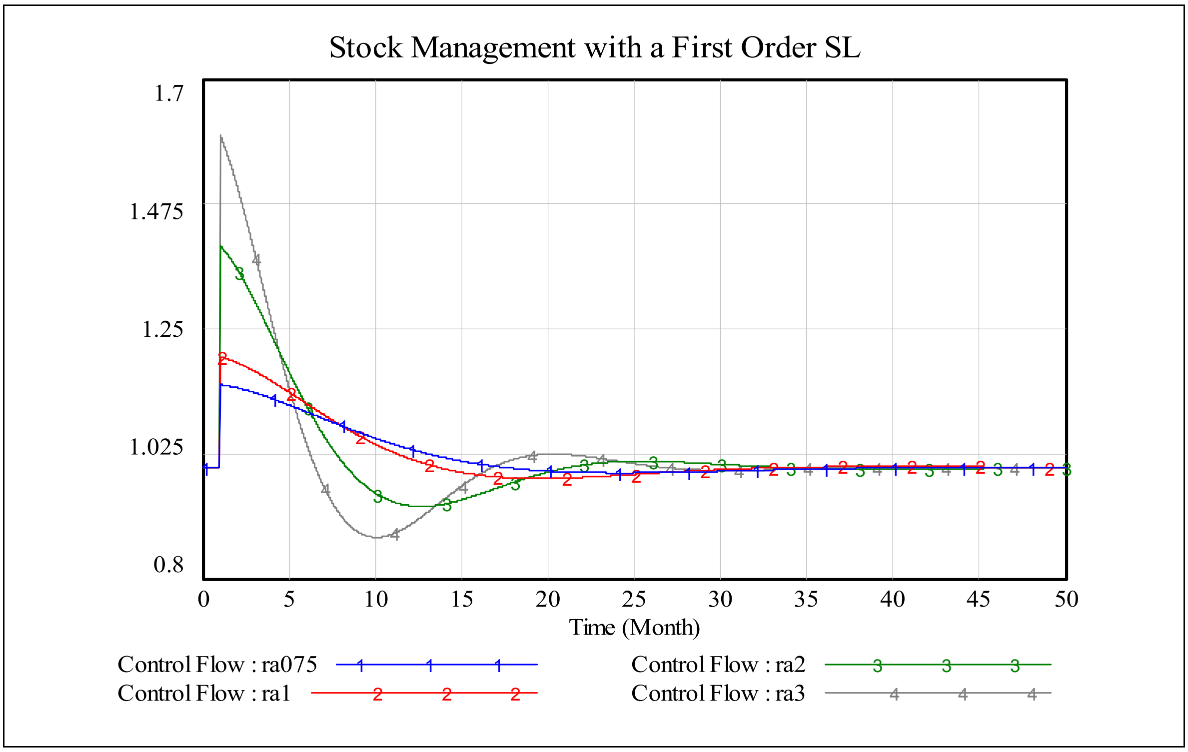

Figure 7.5. Different control flow dynamics for *ra* values of 0.75, 1, 2, and 3.

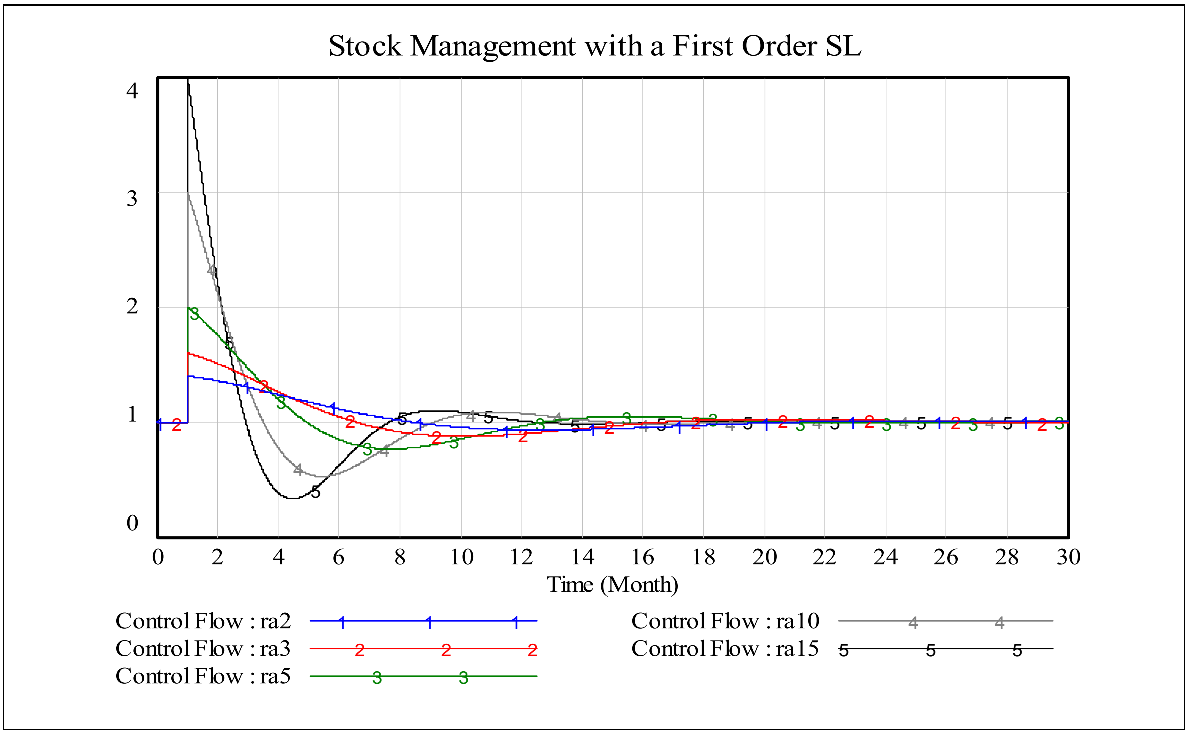

Figure 7.6. Different control flow dynamics for *ra* values of 2, 3, 5, 10, and 15.

7.2. Comparison of the Dynamics with the Literature Suggested Values

For *wsl* equals to 1, stock always shows a goal seeking behavior regardless of the level of aggressiveness in corrections, which is the main reason behind selecting *wsl* as 1 in many studies (Sterman, 1989; Yasarcan and Barlas, 2005a; Yasarcan, 2011). If *wsl* is 1, the approach of the stock to its desired level gets faster as *ra* increases; see stock dynamics in Figure 7.7 for *ra* values of 1 (Line 2), 5 (Line 3), 90 (Line 4), and 150 (Line 5). However, this increase in the rate of approach becomes negligible after a point; for example, compare the dynamics for *ra* values of 90 (Line 4) and 150 (Line 5) in Figure 7.7.

wsl equals to 1 and *sat* equals to 1 (*ra* equals to 5 in this case) are the decision parameter values suggested by the literature for optimum stock behavior (Sterman, 1989). However, for *wsl* equals to 0.2 and *sat* equals to $0.0\overline{5}$ (*ra* equals to 90), stock approaches to its desired level faster than the literature suggested *wsl* and *sat* values, which can be seen in Figure 7.8. This result is surprising because ignoring most of the past decisions (i.e., supply line) and making extremely aggressive corrections produce a better behavior than the literature suggested decision parameter values that suggest that supply line must fully be considered and aggressive corrections must be avoided. Surprisingly a fast and stable approach of stock to its desired level is obtained by using wsl = 0.2 and sat = $0.0\overline{5}$.

For *wsl* equals to 0.2 and *sat* equals to $0.0\overline{5}$ (*ra* equals to 90), first value of the control flow is higher than for *wsl* equals to 1 and *sat* equals to 1 (*ra* equals to 5) which is shown in Figure 7.9. Although the approach of stock to its desired level is faster for wsl = 0.2 and sat = $0.0\overline{5}$, control flow is more smooth for *wsl* equals to 1 and *sat* equals to 1.

For continuous delays, it is known that if *wsl* equals to 1, stock cannot show oscillatory behavior and *sat* can be chosen any value greater than 0 including very low *sat* values, which corresponds to very aggressive corrections. However, *wsl* equals to 1 cannot give as fast approach as *wsl* equals to 0.2 even when *sat* is chosen very small (even if a higher *ra* value for *wsl* = 1 is used). For example, initial sizes of the control flow are the same for *wsl* equals to 1 and *ra* equals to 90 and *wsl* equals to 0.2 and *ra* equals to 90 (Figure 7.10).

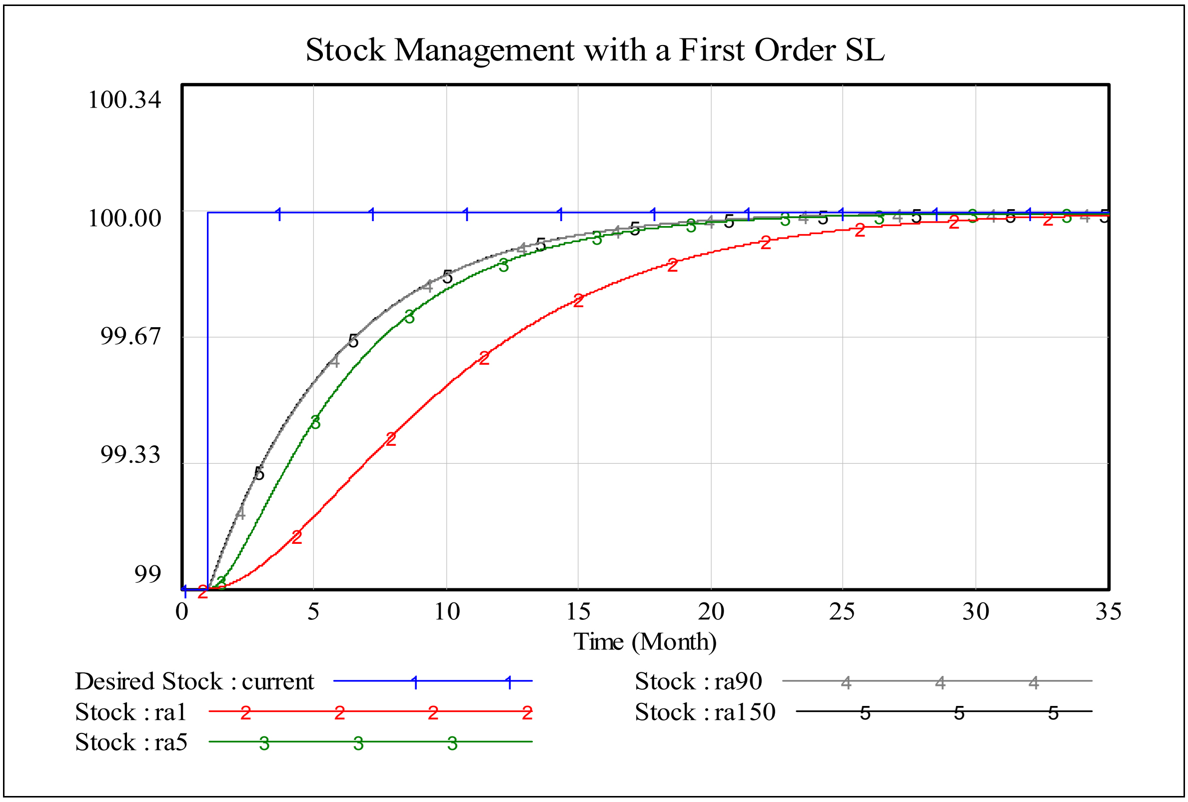

Figure 7.7. Different stock dynamics for *wsl* equals to 1 and *ra* values of 1, 5, 90, and 150.

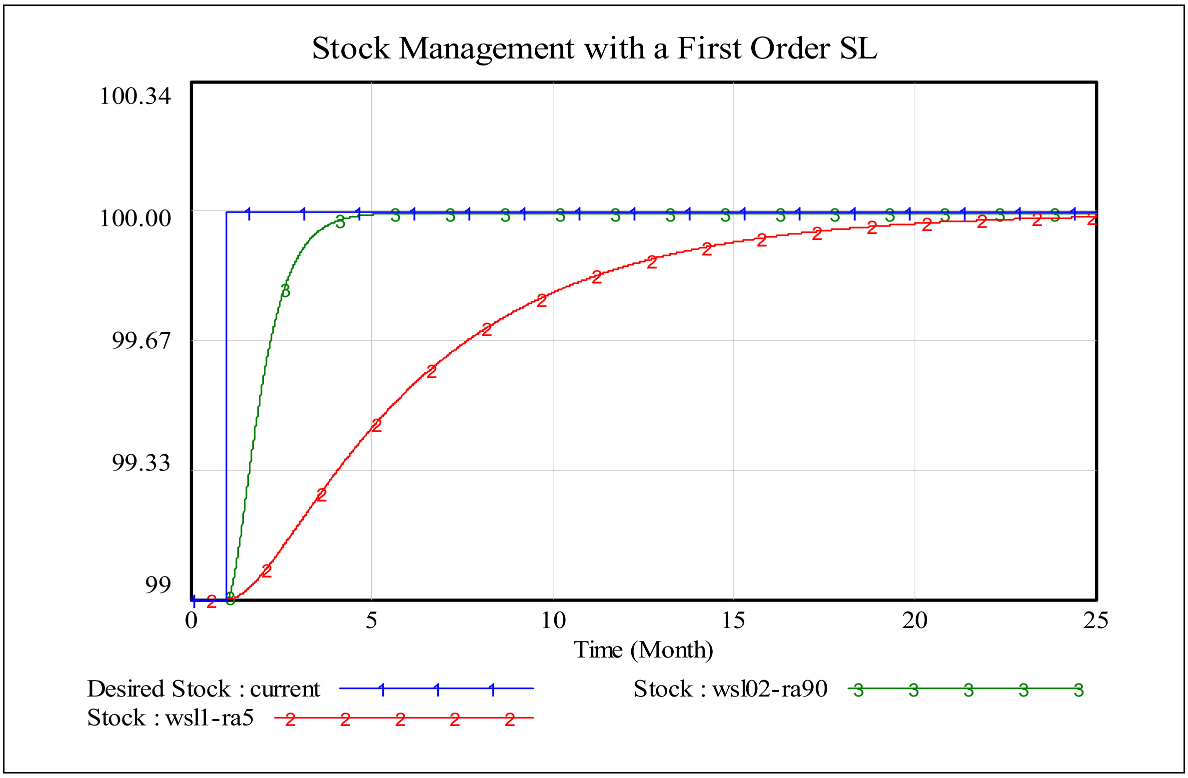

Figure 7.8. Comparison of different stock dynamics generated by literature suggested values (wsl = 1 and sat = 1) and (wsl = 0.2 and sat = $0.0\overline{5}$).

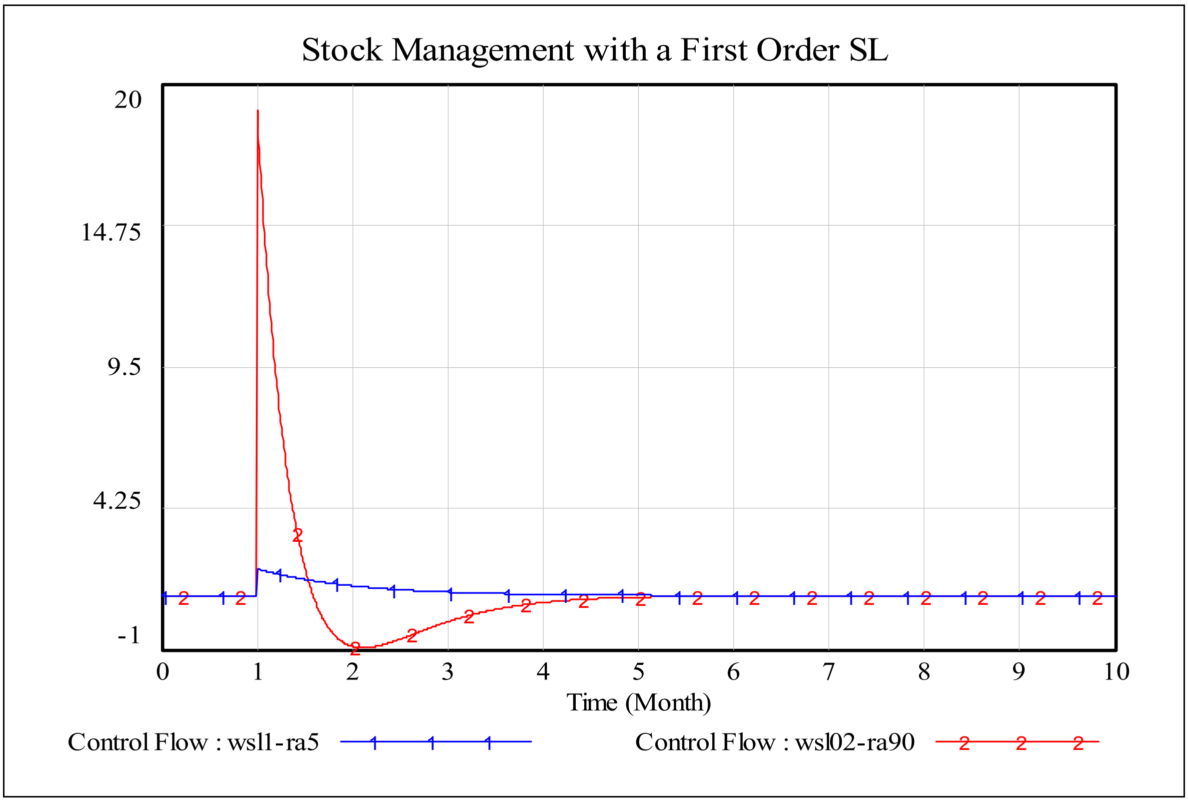

Figure 7.9. Comparison of different control flow dynamics generated by literature suggested values (wsl = 1 and sat = 1) and (wsl = 0.2 and sat = $0.0\overline{5}$).

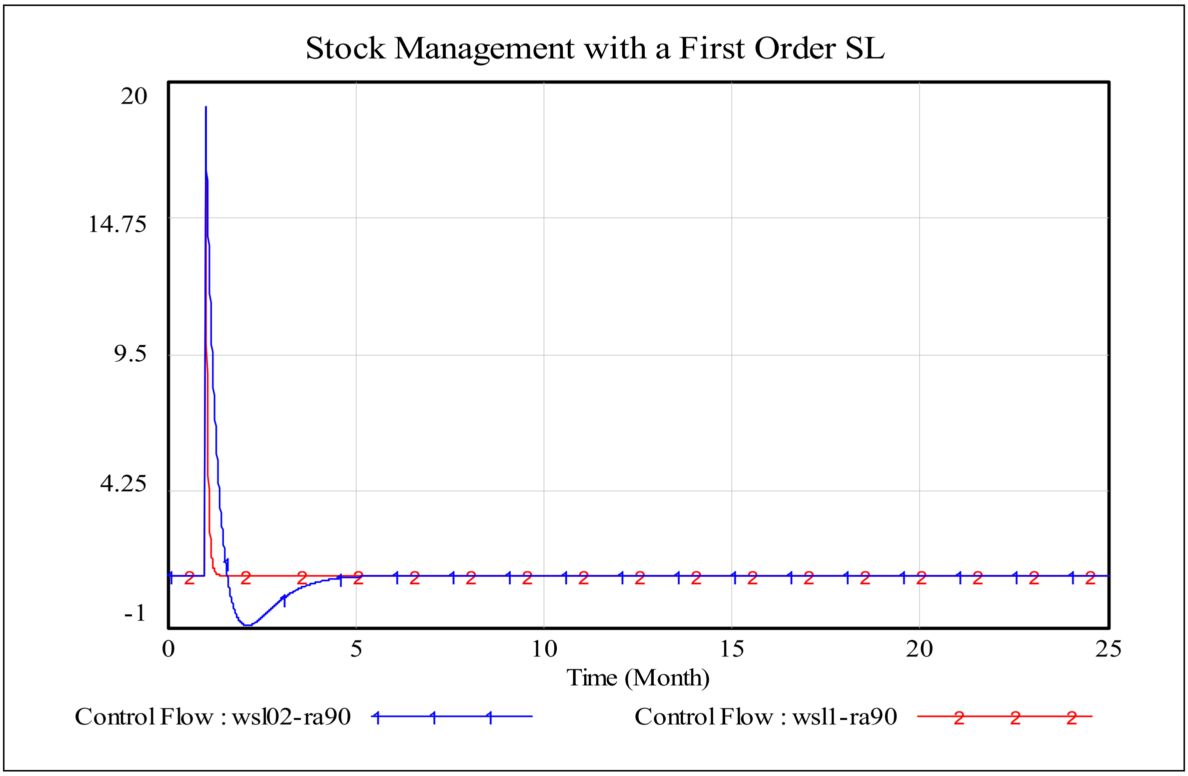

Figure 7.10. Same initial sizes of control flow for *wsl* equals to 1 and *wsl* equals to 0.2.

For the same initial size of the control flow, *wsl* equals to 0.2 can give faster approach to the desired level of the stock (Figure 7.11).

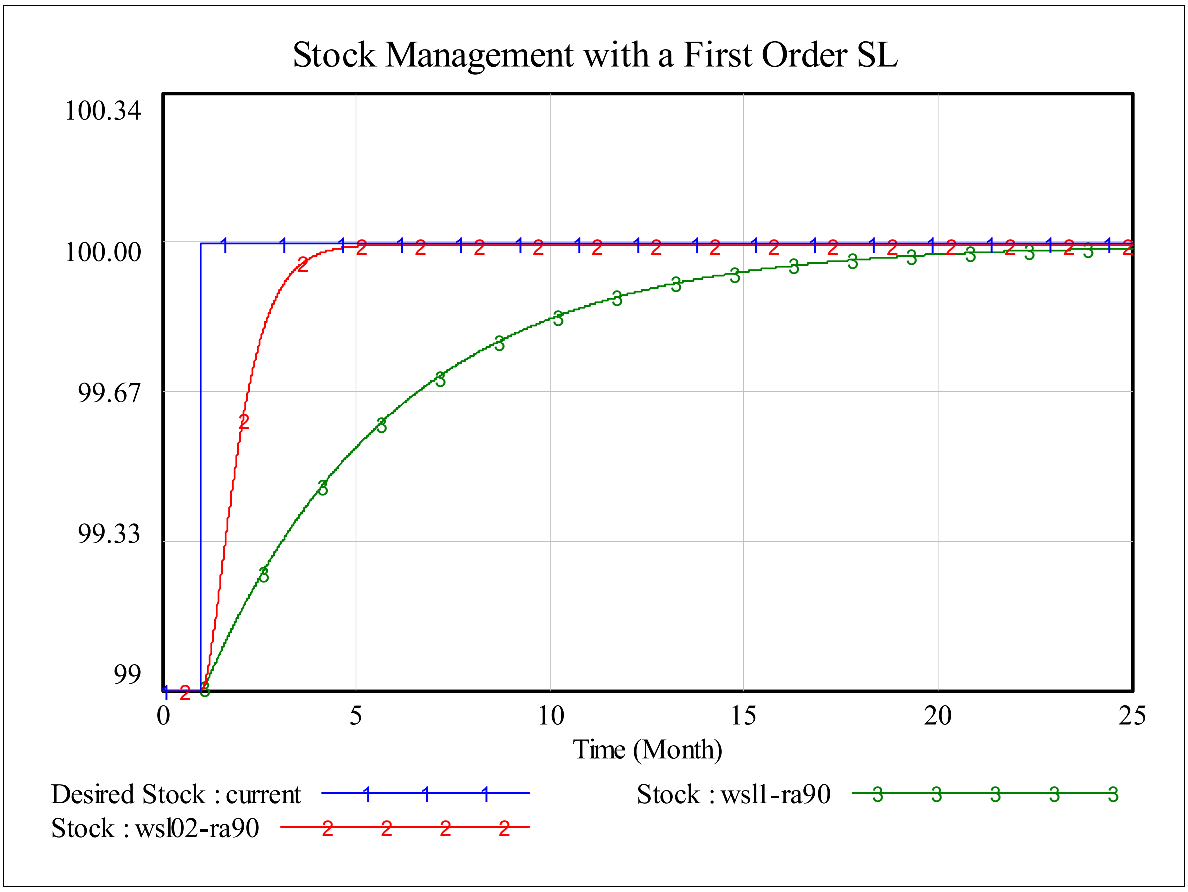

Figure 7.11. Stock dynamics for the same initial sizes of control flow for *wsl* equals to 1 and *wsl* equals to 0.2.

7.3. Example Decision Parameter Value Selection for a Stock Management Model with a Third Order Supply Line Delay

We assume a stock management model with a third order supply line delay and with duration of 5 units of time. We arbitrarily select *wsl* as 0.17. According to Table 5.1, if *ra* is selected below 2.679×10^{-13}, stock shows a goal seeking behavior. If *ra* is between 2.679×10^{-13} and 6.573, stock shows stable oscillatory behavior. If *ra* is between 6.573 and 59.48, stock shows unstable oscillatory behavior. If *ra* is above or equal to 59.48, stock shows stable oscillatory behavior again. Different dynamics for *ra* above 59.48 are given in Figure 7.12.

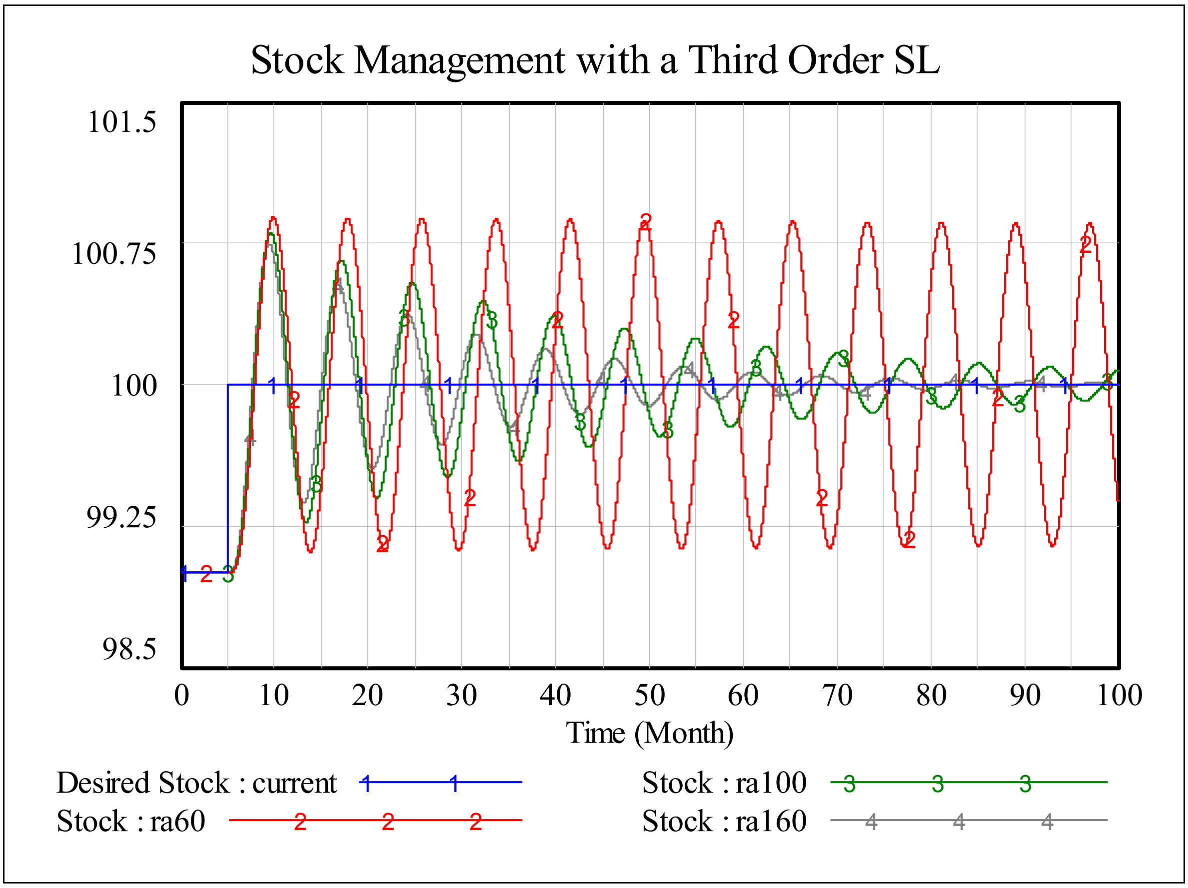

Figure 7.12. Different stock dynamics for *ra* values of 60, 100, and 160.

The amplitude of the oscillations diminishes faster as *ra* increases. For example, if the desired dynamics is stable oscillations with comparatively smaller amplitude values, *ra* can be chosen equal to 160. Since *adt* equals to 5, using Equation 7.1, *sat* is obtained as 0.03125. Thus, the set of values for the decision parameters, *wsl* and *sat*, are 0.17 and 0.03125, respectively.

ra can be chosen equal to 10, if the desired dynamics is unstable oscillations with comparatively smaller amplitude values (Figure 7.13). Since *adt* equals to 5, using Equation 7.1, *sat* is obtained as 0.5. Therefore, the set of values for the decision parameters, *wsl* and *sat*, are 0.17 and 0.5, respectively.

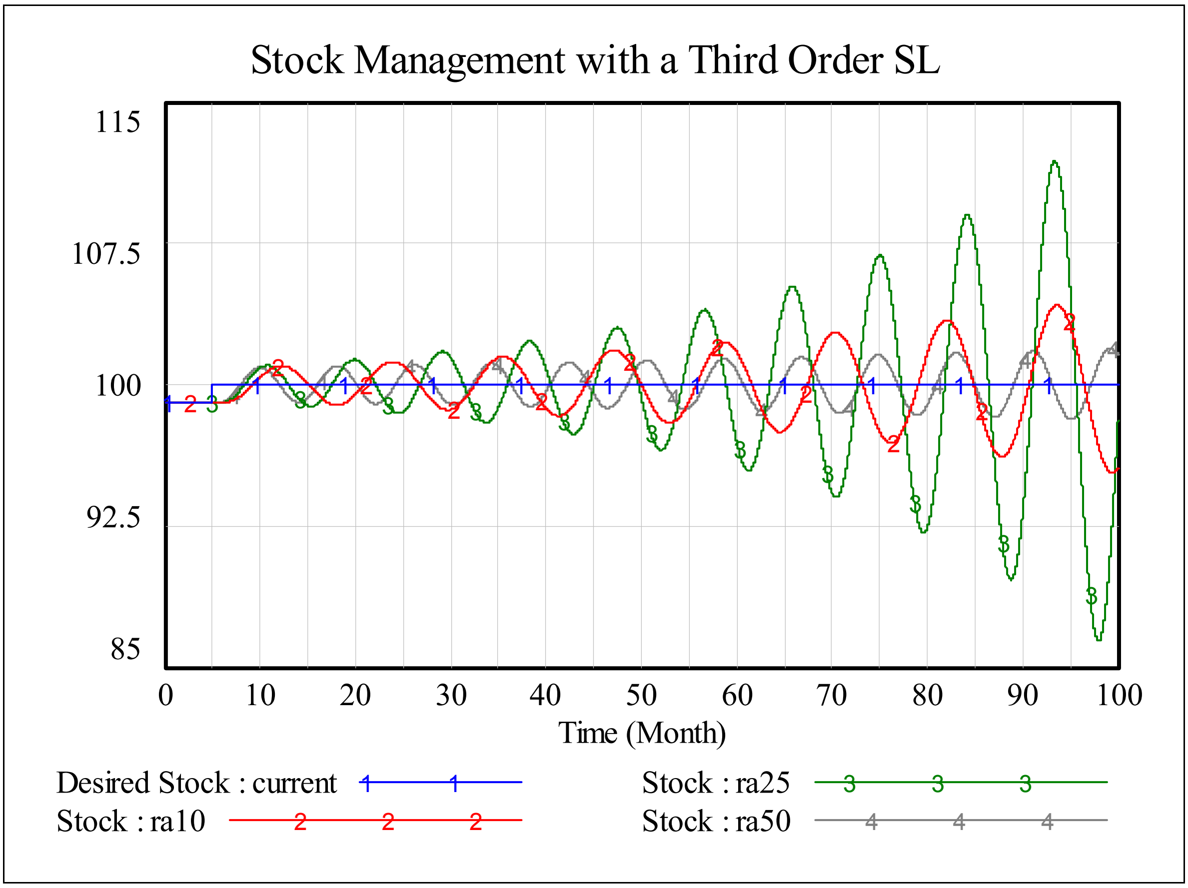

Figure 7.13. Different stock dynamics for *ra* values of 10, 25, and 50.

8. CONCLUSION

In this thesis, different stock management structures were modeled and analyzed. We carried out complete parametric analysis of stock management problems with continuous delays of first, second, and third orders and obtained the range of values for different characteristic dynamics of stock. We avoided black-box approach and block diagrams of different structures were provided in full detail inspired from system dynamics (SD) methodology, which has a strong focus on the validity of the constructed models by explicitly representing all problem related elements in the model.

The stock of *the basic stock management structure without delay* always exhibits goal seeking behavior and never oscillates, which confirms the results reported in the literature. The stock of *the stock management structure with a first order supply line delay* can only show either goal seeking behavior or stable oscillations, which confirms the results reported in the literature too. However, we also obtained a counterintuitive result for this structure. Assume that the stock is producing stable oscillations for a given set of acquisition delay time (*adt*; the duration of delay between the corrective actions and their eventual results on the stock), weight of supply line (*wsl*; the relative importance given to the supply line compared to the stock), and stock adjustment time (*sat*; the intended time to close the gap between the stock and its desired level) values. It is expected that decreasing *sat* (i.e., increasing aggressiveness in making corrections) would strengthen the existing oscillations. In spite of this intuitive expectation, our results show that this relationship between the strength of oscillations and *sat* exists up to a point and decreasing *sat* below that point, makes oscillations disappear and, surprisingly, stock starts to show a goal seeking behavior; very aggressive corrections can completely eliminate oscillations.

The stock of *the stock management structure with a second order supply line* delay is capable of producing goal seeking behavior, stable oscillations, and unstable oscillations, which is true for stock management structures with higher order supply line delays. This also confirms to the results reported in the literature. However, similar to the first order delay case, we obtained unexpected results for the second order delay and the third order delay cases. Although the critical values that give different characteristic dynamics for these two cases are different, there is a different interval of *wsl* values for each of these cases that produce the unanticipated results. Accordingly, if a *wsl* value is selected from the aforementioned interval, as *sat* decreases, first, stock starts to oscillate in a stable manner, later, stock starts to oscillate in an unstable manner, and as aggressiveness further increases (i.e., *sat* decreases), although one intuitively expects the unstable oscillations get wilder, stock unexpectedly starts to oscillate again in a stable manner. Another surprising result is obtained by ignoring most of the past decisions and making extremely aggressive corrections, which contradicts the literature that suggests that supply line must fully be considered and aggressive corrections must be avoided. We show that a selected set of decision parameters values gives a faster approach of stock to its desired level compared to the literature suggested values without sacrificing the stability in stock dynamics.

The results reported in this thesis can be used to obtain goal seeking behavior, stable oscillations, or unstable oscillations for stock management structures with a continuous supply line delay of first, second, and third orders. The results that we obtained are valid for all durations of delay between the corrective actions and their eventual results on the stock.

Aiming to build a bridge between SD and control theory (CT), the corresponding block diagrams of many basic SD models are provided in the thesis and also in one of its appendices. Block diagrams that we present are the exact replicas of their corresponding SD models. Thus, they include all the details present in the SD models. Each SD model and its corresponding block diagram are simulated and the same exact behavior is obtained. We hope that the bridge that we tried to establish will help control theorists to understand and analyze SD models and will assist system dynamicists in representing their models using block diagrams.

The similarity between the results of second order and third order supply line delay cases indicate that carrying out analysis for higher orders would not add much to the study and also it gets more and more difficult to carry out the analysis as the order increases. Regardless of these facts, we are planning to carry out parametric analysis of stock management with fourth and higher order continuous supply line delays as a continuation of this study to make sure that there is no further interesting results. Analyzing supply chains involving a series of cascading stock management structures with continuous delays is a possible path for future studies.

APPENDIX A: LAPLACE TRANSFORMS, TRANSFER FUNCTIONS, AND BLOCK DIAGRAMS

Laplace transform is widely used in control theory, which is a method of converting a set of ordinary differential equations to a set of algebraic equations that can be easily solved. A transfer function is the ratio of a system's output to its input in the Laplace domain, which is also known as the frequency domain. Block diagrams are often used to represent dynamic systems in control theory. Each block in a block diagram has at least two Laplace domain signals connected to it, one input signal and an output signal, and an associated transfer function that transforms the input signal into the output signal. Blocks are connected via their signals (i.e. the output signal generated by a block can be the input to another block). Thus, a complete block diagram represents the dynamic relationship between one input or many inputs to a system and one output or many outputs of that same system (Bequette, 2007; Seborg *et al.*, 2004).

A.1. Laplace Transform Method

Laplace transform of a time domain function, *f(t)*, is given in Equation A.1 (Bequette, 2007).

$$F(s) = L[f(t)] = \int_0^{\infty} f(t) \times e^{-st} \times dt \tag{A.1}$$

where s is a complex variable.

A.1.1. An Example of Laplace Transformation: Exponential Function

In this sub-section, the Laplace transform of an exponential function is obtained as an example (Equation A.6).

$$L\left[e^{-at}\right]=\int_{0}^{\infty} e^{-at}\times e^{-st}\times dt \tag{A.2}$$

where a is a constant. Equation A.2 can be re-written as Equation A.3.

$$L\left[e^{-at}\right]=\lim_{b\to\infty}\int_{0}^{b} e^{-(s+a)\times t}\times dt \tag{A.3}$$

$$L\left[e^{-at}\right]=\lim_{b\to\infty}\left[\frac{e^{-(s+a)\times t}}{-(s+a)}\right]_{0}^{b} \tag{A.4}$$

$$L\left[e^{-at}\right]=\lim_{b\to\infty}\left[\frac{e^{-(s+a)\times b}}{-(s+a)}+\frac{1}{s+a}\right] \tag{A.5}$$

Finally, Equation A.6, which is the Laplace transform of an exponential function, is obtained from Equation A.5.

$$L\left[e^{-at}\right]=\frac{1}{s+a} \tag{A.6}$$

A.1.2. The Generic Form of the Laplace Transform of a Time Delayed Function

If $f(t)$ represents the value of a particular function at time t, its time delayed version, $f(t-\theta)$, represents the value of that function at time $t-\theta$, where θ, which is a positive constant, is the duration of the delay. In this sub-section, the generic Laplace transform of a delayed function is derived (Equation A.12).

$$L\left[f(t-\theta)\right]=\int_{0}^{\infty} f(t-\theta)\times e^{-st}\times dt \tag{A.7}$$

Equation A.7 can be re-written as Equation A.8.

$$L[f(t-\theta)] = \int_0^\infty f(t-\theta) \times e^{-s(t-\theta+\theta)} \times dt \tag{A.8}$$

$$L[f(t-\theta)] = \int_0^\infty f(t-\theta) \times e^{-s(t-\theta)} \times e^{-s\theta} \times dt \tag{A.9}$$

$$L[f(t-\theta)] = e^{-s\theta} \times \int_0^\infty f(t-\theta) \times e^{-s(t-\theta)} \times d(t-\theta) \tag{A.10}$$

If a change of variables, $t^* = t - \theta$ is used to integrate the function (Equation A.10), Equation A.11 is obtained.

$$L[f(t-\theta)] = e^{-s\theta} \times \int_0^\infty f(t^*) \times e^{-st^*} \times dt^* \tag{A.11}$$

Finally, Equation A.12, which is the generic form of the Laplace transform of a time-delayed function, is obtained from Equation A.11.

$$L[f(t-\theta)] = e^{-s\theta} \times F(s) \tag{A.12}$$

According to Equation A.12, Laplace transform of the delayed version of a function equals to $e^{-s\theta}$ times the Laplace transform of that function (Bequette, 2007; Seborg *et al.*, 2004).

A.1.3. The Generic Form of the Laplace Transform of a First Order Derivative

The generic form of the Laplace transform of a first order derivative of a function is obtained by using integration by parts technique (Equation A.17).

$$L\left[\frac{df(t)}{dt}\right] = \int_0^\infty \frac{df(t)}{dt} \times e^{-st} \times dt \tag{A.13}$$

Equation A.13 can be re-written as Equation A.14.

$$L\left[\frac{df(t)}{dt}\right] = \lim_{b\to\infty} \int_0^b e^{-st} \times \frac{df(t)}{dt} \times dt \tag{A.14}$$

$$L\left[\frac{df(t)}{dt}\right] = \lim_{b\to\infty} \left[f(t) \times e^{-st}\Big|_0^b + s \times \int_0^b f(t) \times e^{-st} \times dt \right] \tag{A.15}$$

$$L\left[\frac{df(t)}{dt}\right] = \lim_{b\to\infty} \left[f(b) \times e^{-sb} - f(0) + s \times \int_0^b f(t) \times e^{-st} \times dt \right] \tag{A.16}$$

Finally, Equation A.17, which is the generic form of the Laplace transform of a first order derivative of a function, is obtained from Equation A.16.

$$L\left[\frac{df(t)}{dt}\right] = s \times L[f(t)] - f(0) \tag{A.17}$$

A.1.4. Laplace Transforms of Commonly Used Time Domain Functions

Laplace transforms are used for solving most dynamic problems and, in solving such a problem, Laplace transform tables are usually used to save time. Accordingly, a Laplace transform table for some of the common functions is also provided in this Appendix (Table A.1). For a more comprehensive Laplace transform table, see, for example, Bequette (2007) or Seborg *et al.* (2004).

Unit impulse ($\delta(t)$) is defined by Equation A.18 and its integral from negative infinity to positive infinity, which is equal to one, is given in Equation A.19.

$$\delta(t) = \lim_{\varepsilon\to 0^+} \begin{Bmatrix} \dfrac{1}{\varepsilon} & \text{for} \quad 0 \le t \le \varepsilon \\ 0 & \text{for} \quad t > \varepsilon \end{Bmatrix} \tag{A.18}$$

$$\int_{-\infty}^{+\infty} \delta(t) \times dt = \int_{0}^{\varepsilon} \delta(t) \times dt = \lim_{\varepsilon \to 0^+} \int_{0}^{\varepsilon} \frac{1}{\varepsilon} \times dt = 1 \tag{A.19}$$

Table A.1. Laplace transforms of common time domain functions.

Time domain function	**Laplace domain function**
$f(t)$	$F(s)$
$\delta(t)$ (Equation A.18)	1
$Step(t)$ (Equation A.20)	$\frac{1}{s}$
a	$\frac{a}{s}$
$f(t-\theta)$	$e^{-\theta s} \times F(s)$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{s+a}$
$t \times e^{-at}$	$\frac{1}{(s+a)^2}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$\frac{df(t)}{dt}$	$s \times L[f(t)] - f(0)$
$\frac{d^n f(t)}{dt}$	$s^n \times F(s) - s^{n-1} \times f(0) - s^{n-2} \times f'(0) - s^{n-3} \times f''(0) - \cdots - f^{(n-1)}(0)$

Unit step ($Step(t)$) is defined by Equation A.20.

$$Step(t) = \begin{Bmatrix} 0 & \text{for} & t < 0 \\ 1 & \text{for} & t \geq 0 \end{Bmatrix} \tag{A.20}$$

It is also possible to use a Laplace transform table (Table A.1) to obtain the inverse Laplace transform of a Laplace domain function, which is defined by Equation A.21.

$$L^{-1}[F(s)] = f(t) \tag{A.21}$$

Note that the inverse Laplace transform of the Laplace transform of a time domain function is itself (Equation A.22).

$$f(t) = L^{-1}[L[f(t)]] \tag{A.22}$$

A.1.5. Solving Linear Differential Equations Using Laplace Transforms: An Example First Order Equation

To solve a differential equation with Laplace transform, Laplace transform of both sides of the differential equation must be taken. Then, the resulting algebraic equation must be solved for $L[f(t)]$. Finally, the inverse transform must be taken by using Laplace transform table.

An example first order differential equation is given below (Equation A.23):

$$\frac{dx(t)}{dt} - x(t) = e^{t} \tag{A.23}$$

Initial condition is $x(0) = a$, where a is a constant value.

Taking the Laplace of Equation A.23 gives Equation A.24.

$$L\left[\frac{dx(t)}{dt} - x(t)\right] = L\left[e^{t}\right] \tag{A.24}$$

$$L\left[\frac{dx(t)}{dt}\right] - L[x(t)] = L\left[e^{t}\right] \tag{A.25}$$

Equation A.26 is obtained from Equation A.25 using Table A.1.

$$\left(s \times L[x(t)] - x(0)\right) - L[x(t)] = \frac{1}{s-1} \tag{A.26}$$

Equation A.26 can be re-written as Equation A.27.

$$(s-1) \times L[x(t)] - a = \frac{1}{s-1} \tag{A.27}$$

Solving for $L[x(t)]$ gives Equation A.28.

$$L[x(t)] = \frac{1}{(s-1)^2} + \frac{a}{(s-1)} \tag{A.28}$$

Equation A.29 is obtained by inverting Equation A.28 to the time domain using Laplace transform table (Table A.1).

$$x(t) = t \times e^t + a \times e^t \tag{A.29}$$

A.1.6. Solving Linear Differential Equations Using Laplace Transforms: An Example Set of First Order Equations

An example of a set of first order differential equations is given below (equations A.30 and A.31):

$$\frac{dx_1(t)}{dt} + x_2(t) = 1 \tag{A.30}$$

$$x_1(t) - \frac{dx_2(t)}{dt} = 0 \tag{A.31}$$

Initial conditions are $x_1(0) = a_1$ and $x_2(0) = a_2$, where a_1 and a_2 are constants.

Taking the Laplace of equations A.30 and A.31 gives equations A.32 and A.33.

$$L\left[\frac{dx_1(t)}{dt}\right]+L[x_2(t)]=L[1] \tag{A.32}$$

$$L[x_1(t)]-L\left[\frac{dx_2(t)}{dt}\right]=L[0] \tag{A.33}$$

Equations A.34 and A.35 are obtained from equations A.32 and A.33 using Table A.1.

$$s\times X_1(s)-x_1(0)+X_2(s)=\frac{1}{s} \tag{A.34}$$

$$X_1(s)-s\times X_2(s)+x_2(0)=0 \tag{A.35}$$

Equations A.34 and A.35 can be re-written as equations A.36 and A.37.

$$s\times X_1(s)+X_2(s)=\frac{1}{s}+a_1 \tag{A.36}$$

$$X_1(s)-s\times X_2(s)=-a_2 \tag{A.37}$$

Equation A.38 is obtained from Equation A.37.

$$X_1(s)=-a_2+s\times X_2(s) \tag{A.38}$$

Equation A.39 is obtained by inserting Equation A.38 into Equation A.36.

$$-a_2\times s+s^2\times X_2(s)+X_2(s)=\frac{1}{s}+a_1 \tag{A.39}$$

Solving for $X_2(s)$ gives Equation A.40.

$$X_2(s) = \frac{1}{s \times (1 + s^2)} + \frac{a_1}{s^2 + 1} + \frac{a_2 \times s}{s^2 + 1} \tag{A.40}$$

Equation A.41 is obtained from Equation A.40.

$$X_2(s) = \frac{1}{s} - \frac{(1 - a_2) \times s}{s^2 + 1} + \frac{a_1}{s^2 + 1} \tag{A.41}$$

Equation A.42 is obtained by inserting Equation A.41 into Equation A.38.

$$X_1(s) = -a_2 + s \times \left[\frac{1}{s} - \frac{(1 - a_2) \times s}{s^2 + 1} + \frac{a_1}{s^2 + 1} \right] \tag{A.42}$$

Equation A.42 is simplified to Equation A.43.

$$X_1(s) = -a_2 + 1 - (1 - a_2) \times \left[1 - \frac{1}{s^2 + 1} \right] + \frac{a_1 \times s}{s^2 + 1} \tag{A.43}$$

$$X_1(s) = \frac{(1 - a_2)}{s^2 + 1} + \frac{a_1 \times s}{s^2 + 1} \tag{A.44}$$

Using Table A.1, the two Laplace domain functions (equations A.41 and A.44) is inverted to the time domain, which are given below:

$$x_1(t) = (1 - a_2) \times \sin(t) + a_1 \times \cos(t) \tag{A.45}$$

$$x_2(t) = 1 - (1 - a_2) \times \cos(t) + a_1 \times \sin(t) \tag{A.46}$$

A.1.7. Solving Linear Differential Equations Using Laplace Transforms: An Example Second Order Equation

An example second order linear differential equation is given below (Equation A.47):

$$\frac{d^2x(t)}{dt^2} - 4\times\frac{dx(t)}{dt} + 4\times x(t) = e^{-2\times t} \tag{A.47}$$

Initial conditions are $x(0) = a_1$ and $\dot{x}(0) = a_2$, where a_1 and a_2 are constants.

Taking the Laplace of Equation A.47 gives Equation A.48.

$$L\left[\frac{d^2x(t)}{dt^2}\right] - 4\times L\left[\frac{dx(t)}{dt}\right] + 4\times L[x(t)] = L\left[e^{-2\times t}\right] \tag{A.48}$$

Equation A.49 is obtained from Equation A.48 using Table A.1.

$$\left(s^2\times X(s) - s\times x(0) - \dot{x}(0)\right) - 4\times\left(s\times X(s) - x(0)\right) + 4\times X(s) = \frac{1}{s+2} \tag{A.49}$$

Equation A.49 can be re-written as Equation A.50.

$$\left(s^2 - 4\times s + 4\right)\times X(s) - a_1\times s - a_2 + 4\times a_1 = \frac{1}{s+2} \tag{A.50}$$

Solving for $X(s)$ gives Equation A.51.

$$X(s) = \frac{1}{\left(s^2 - 4\times s + 4\right)\times\left(s+2\right)} + \frac{a_1\times s}{s^2 - 4\times s + 4} + \frac{\left(a_2 - 4\times a_1\right)}{s^2 - 4\times s + 4} \tag{A.51}$$

Equation A.52 is obtained from Equation A.51.

$$X(s) = \frac{1}{16\times(s+2)} + \left(a_1 - \frac{1}{16}\right)\times\frac{1}{s-2} + \left(\frac{1}{4} - 2\times a_1 + a_2\right)\times\frac{1}{(s-2)^2} \tag{A.52}$$

Equation A.53 is obtained by inverting Equation A.52 to the time domain using Laplace transform table (Table A.1).

$$x(t) = \frac{1}{16} \times e^{-2\times t} + \left(a_1 - \frac{1}{16}\right) \times e^{2\times t} + \left(\frac{1}{4} - 2 \times a_1 + a_2\right) \times t \times e^{2\times t} \qquad \text{(A.53)}$$

A.2. Transfer Functions and Block Diagrams

Transfer function of a dynamic system is the ratio of the output variable to the input variable in the Laplace domain. In general, $g(s)$ is used to represent a transfer function that is defined in Equation A.54 (Bequette, 2007; Seborg *et al.*, 2004).

$$g(s) = \frac{y(s)}{u(s)} \qquad \text{(A.54)}$$

where $u(s)$ is the input variable and $y(s)$ is the output variable in the Laplace domain.

A.2.1. Block Diagrams

Transfer functions are often used in block diagrams. The relationship between input and output defined by Equation A.54 is depicted in Figure A.1.

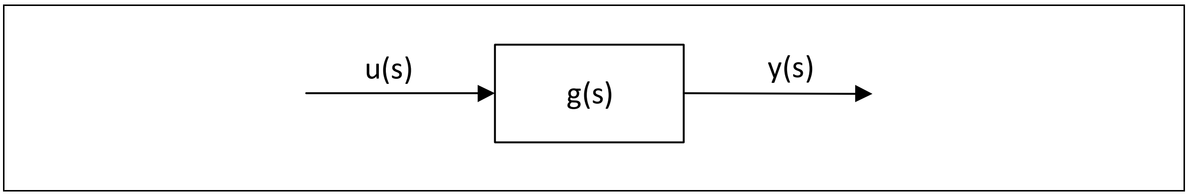

Figure A.1. The most basic block diagram representing an input-output relationship in the Laplace domain.

Block diagrams have three main types of elements which are signals, unary operator blocks, and m-ary (i.e., many-ary) operator blocks given in Figure A.2.

In the block diagram representation given in Figure A.2, $r(s)$, $u(s)$, $y_1(s)$, and $y_2(s)$ are signals. $u(s)$ is the input signal and $y_1(s)$ is the output signal of $g_1(s)$. $y_1(s)$ is the input signal and $y_2(s)$ is the output signal of $g_2(s)$. $g_1(s)$ and $g_2(s)$ are unary operator blocks which operates on the input signals connected to them with the transfer functions to form the output signals. $r(s)$ and $y_2(s)$ are the input variables and $u(s)$ is the output variable of the summation block which is an m-ary operator block. An m-ary operator block is shown as a circle and has two or more input variables and a single output variable (Wescott, 2006).

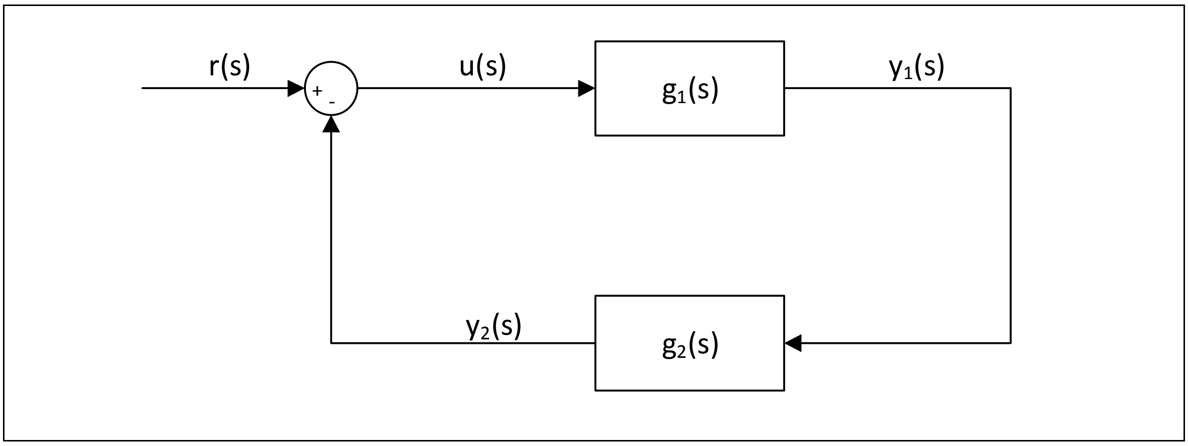

Figure A.2. A block diagram with two blocks.

A.2.2. Reduced Block Diagrams

A generic example is given in Figure A.3.

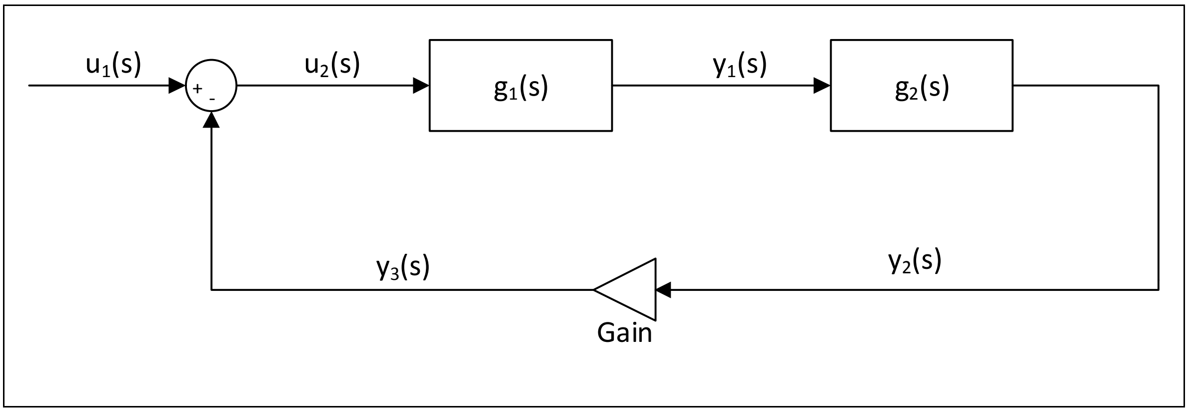

Figure A.3. A generic example of block diagram.

The relationship between inputs and outputs of the system are given in equations A.55 and A.56, A.57 and A.58.

$$g_1(s) = \frac{y_1(s)}{u_2(s)} \tag{A.55}$$

$$g_2(s) = \frac{y_2(s)}{y_1(s)} \tag{A.56}$$

$$y_3(s) = y_2(s) \times Gain \tag{A.57}$$

$$u_1(s) - y_s(s) = u_2(s) \tag{A.58}$$

Combining $g_1(s)$ and $g_2(s)$ into a single transfer function, $g_3(s)$ is obtained which is given in Equation A.59.

$$g_3(s) = g_1(s) \times g_2(s) = \frac{y_1(s)}{u_2(s)} \times \frac{y_2(s)}{y_1(s)} = \frac{y_2(s)}{u_2(s)} \tag{A.59}$$

Equation A.58 can be rewritten as Equation A.60.

$$u_1(s) - y_2(s) \times Gain = \frac{y_2(s)}{g_1(s) \times g_2(s)} \tag{A.60}$$

$$u_1(s) = \frac{y_2(s)}{g_1(s) \times g_2(s)} + y_2(s) \times Gain \tag{A.61}$$

$$u_1(s) = y_2(s) \times \left[\frac{1 + Gain \times g_1(s) \times g_2(s)}{g_1(s) \times g_2(s)} \right] \tag{A.62}$$

Overall transfer function of the process, $g_4(s)$ is given in Equation A.63.

$$g_4(s) = \frac{y_2(s)}{u_1(s)} = \frac{g_1(s) \times g_2(s)}{1 + Gain \times g_1(s) \times g_2(s)} \tag{A.63}$$

The reduced form of the block diagram given in Figure A.3 is depicted in Figure A.4, which is also a block diagram and an equivalent of the diagram given in Figure A.3.

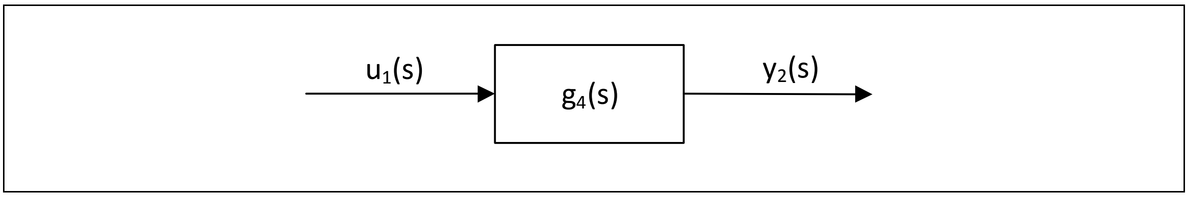

Figure A.4. Reduced form of the example block diagram.

APPENDIX B: THE CORRESPONDING BLOCK DIAGRAMS OF BASIC SYSTEM DYNAMICS MODELS

We first convert generic system dynamics model structures to differential equations and, later, we obtain corresponding block diagrams of these structures from the differential equations. To save space, the derivation process is not provided, but only the resulting differential equations and block diagrams are given. If the reader is interested in the derivation process, he/she can read the appendices A and B and carry out derivations himself/herself. Note that a comprehensive model usually contains one or many of these generic structures. Moreover, a constant of a simple structure may turn into a variable, even into a state variable, in a more complex model.

B.1. Compounding Structure

Stock-flow diagram of a compounding structure is given in Figure B.1.

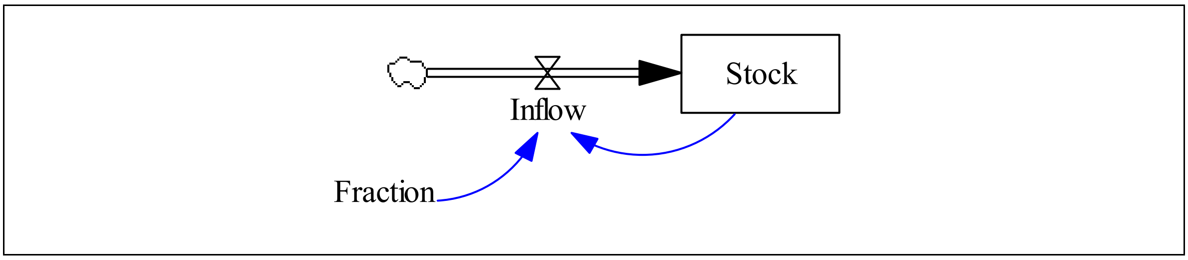

Figure B.1. Stock-flow diagram of the compounding structure.

The inflow equation of the model is given in Equation B.1.

$$Inflow = Fraction \times Stock \tag{B.1}$$

where “Fraction” is a nonnegative constant value.

The diagram in Figure B.1 and Equation B.1 define a compounding structure. The simplified differential equation that corresponds to this structure is given in Equation B.2.

$$\frac{dStock}{dt} = Inflow = Fraction \times Stock \tag{B.2}$$

Block diagram of the compounding structure is given in Figure B.2.

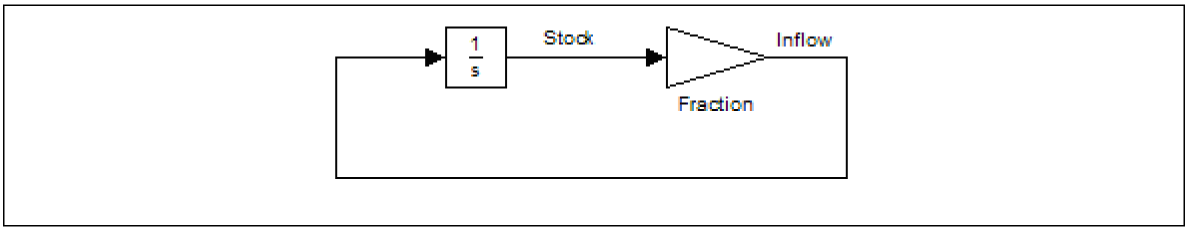

Figure B.2. Block diagram of the compounding structure.

B.2. Draining Structure

Stock-flow diagram of a draining structure is given in Figure B.3.

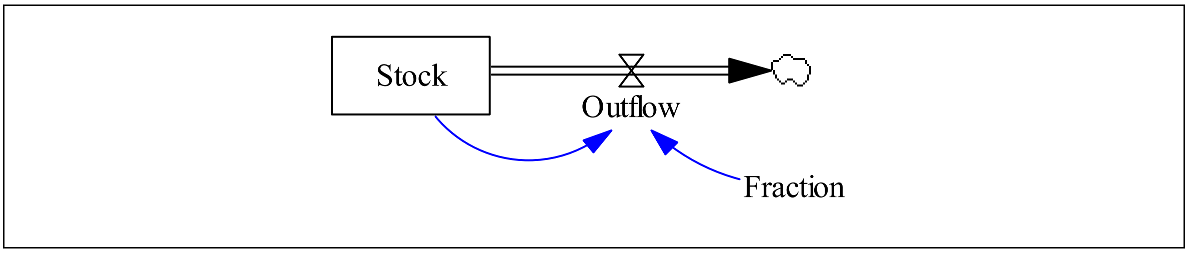

Figure B.3. Stock-flow diagram of the draining structure.

The outflow equation of the model is given in Equation B.3.

$$Outflow = Fraction \times Stock \tag{B.3}$$

where “Fraction” is a nonnegative constant value.

The diagram in Figure B.3 and Equation B.3 define a draining structure. The simplified differential equation that corresponds to this structure is given in Equation B.4.

$$\frac{dStock}{dt} = -Outflow = -Fraction \times Stock \tag{B.4}$$

Block diagram of the draining structure is given in Figure B.4.

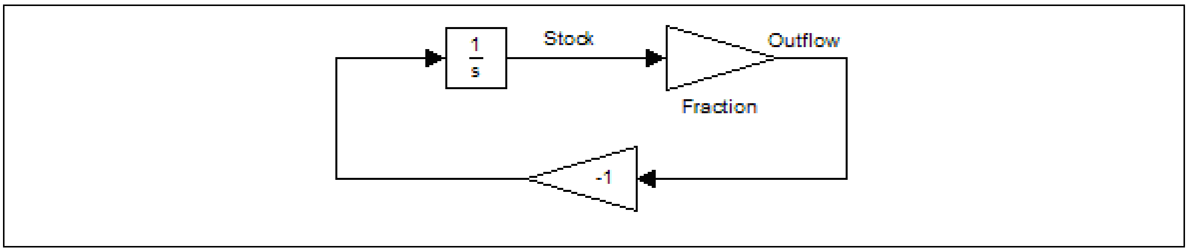

Figure B.4. Block diagram of the draining structure.

B.3. First Order Linear Structure

Stock-flow diagram of a first order linear structure is given in Figure B.5.

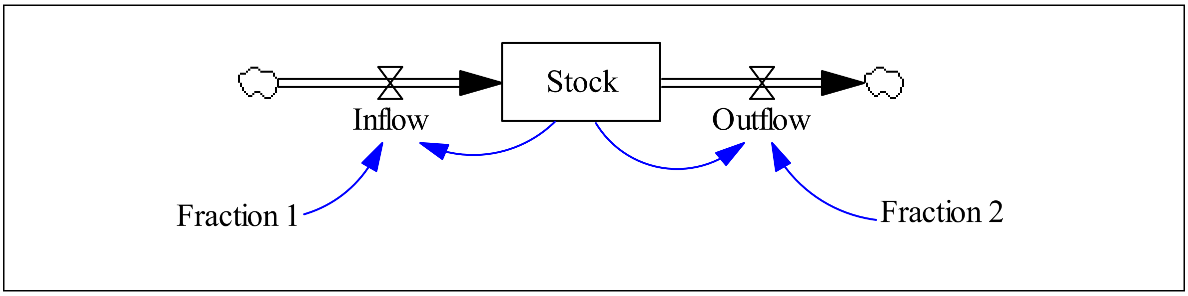

Figure B.5. Stock-flow diagram of the first order linear structure.

The inflow and outflow equations of the model are given in equations B.5 and B.6.

$$Inflow = Fraction\ 1 \times Stock \tag{B.5}$$

$$Outflow = Fraction\ 2 \times Stock \tag{B.6}$$

where “Fraction 1” and “Fraction 2” are nonnegative constant values.

The diagram in Figure B.5 and equations B.5 and B.6 define a first order linear structure. The simplified differential equation that corresponds to this structure is given in Equation B.7.

$$\frac{dStock}{dt} = Inflow - Outflow = (Fraction\ 1 - Fraction\ 2) \times Stock \qquad \text{(B.7)}$$

Block diagram of the first order linear structure is given in Figure B.6.

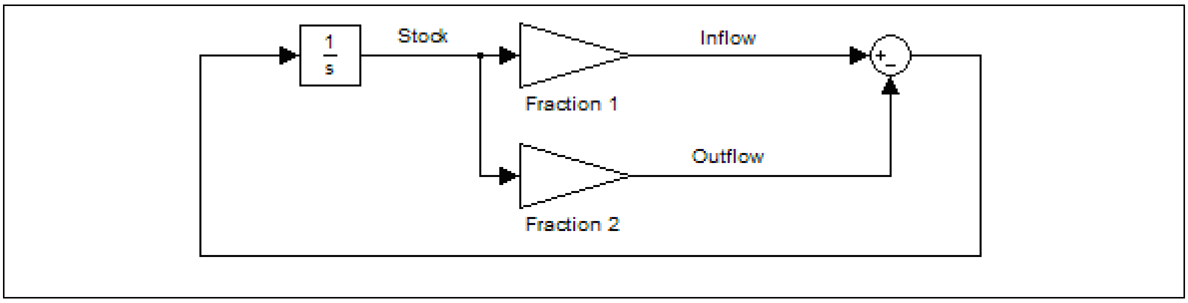

Figure B.6. Block diagram of the first order linear structure.

B.4. Production Structure

Stock-flow diagram of a production structure is given in Figure B.7.

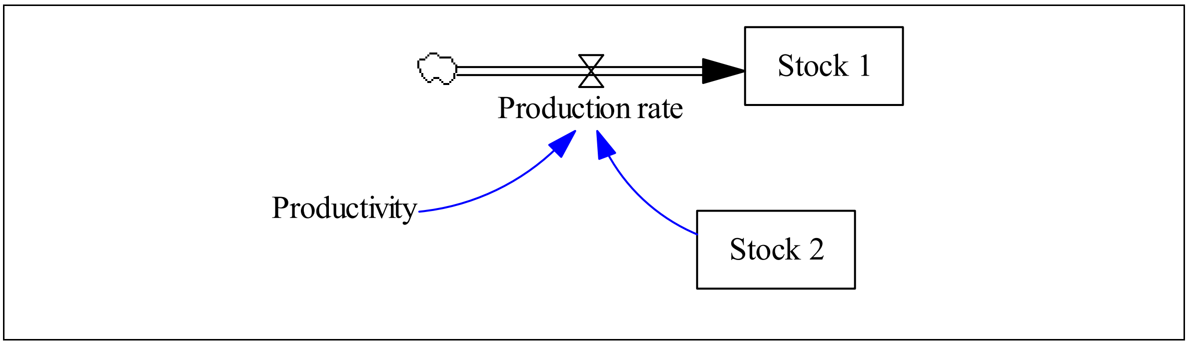

Figure B.7. Stock-flow diagram of the production structure.

The inflow equation of the model is given in Equation B.8.

$$Production\ \ rate = Productivity \times Stock\ 2 \qquad \text{(B.8)}$$

where "Productivity" is a nonnegative constant value.

The diagram in Figure B.7 and Equation B.8 define a production structure. The simplified differential equation that corresponds to this structure is given in Equation B.9.

$$\frac{dStock\ 1}{dt} = Production\ \ rate = Productivity \times Stock\ 2 \tag{B.9}$$

Flows are not connected to the state variable named “Stock 2” because the focus of this structure is on representing “Production rate” flow. Similar to other simple model structures, the production structure usually is a part of a more comprehensive model.

Block diagram of the production structure is given in Figure B.8.

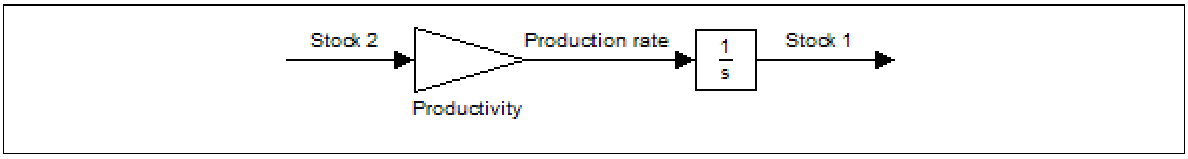

Figure B.8. Block diagram of the production structure.

B.5. Goal Seeking Structure

Stock-flow diagram of a goal seeking structure, which is also known as stock adjustment structure, is given in Figure B.9.

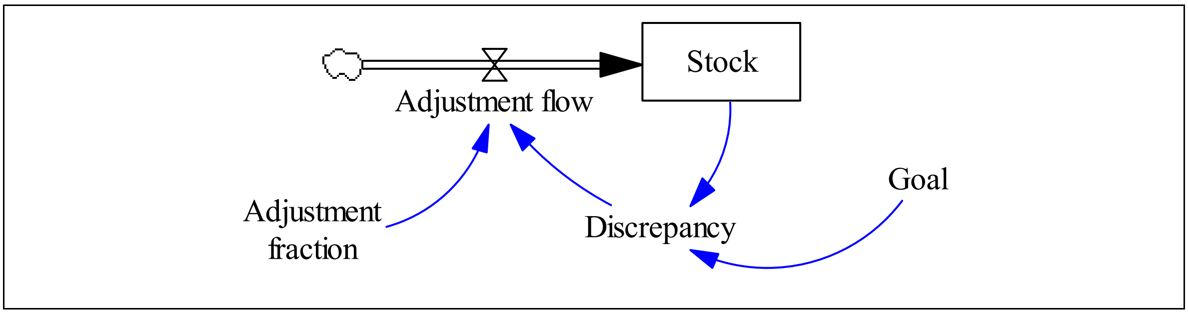

Figure B.9. Stock-flow diagram of the goal seeking structure.

The model equations are B.10 and B.11.

$$Adjustment\ \ flow = Adjustment\ \ fraction \times Discrepancy \tag{B.10}$$

$$Discrepancy = Goal - Stock \tag{B.11}$$

where “Goal” is a constant value and “Adjustment fraction” is a nonnegative constant value.

The diagram in Figure B.9 and equations B.10 and B.11 define a goal seeking structure. The simplified differential equation that corresponds to this structure is given in Equation B.12.

$$\frac{dStock}{dt} = Adjustment\ flow = Adjustment\ fraction \times (Goal - Stock) \tag{B.12}$$

Block diagram of the goal seeking structure is given in Figure B.10.

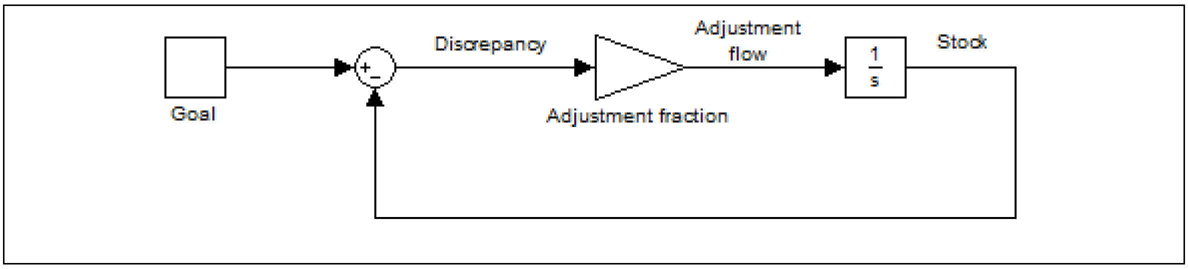

Figure B.10. Block diagram of the goal seeking structure.

B.6. Capacitated Growth Structure (S Shaped Growth Caused by a Capacity Limit)

Stock-flow diagram of a capacitated growth structure is given in Figure B.11.

The model equations are B.13-B.17.

$$Inflow = Fraction\ 1 \times Stock \tag{B.13}$$

$$Fraction\ 1 = Effect\ of\ Ratio\ on\ Fraction\ 1 \times Standard\ Fraction\ 1 \tag{B.14}$$

$$Effect\ of\ Ratio\ on\ Fraction\ 1 = f(Ratio) \tag{B.15}$$

$$Ratio = \frac{Stock}{Capacity} \tag{B.16}$$

$$Outflow = Fraction\ 2 \times Stock \tag{B.17}$$

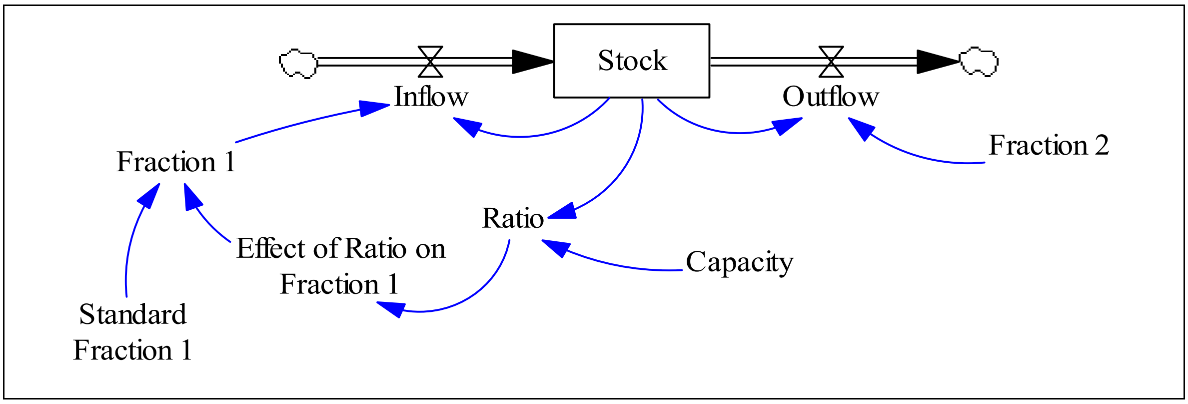

Figure B.11. Stock-flow diagram of the capacitated growth structure.

The diagram in Figure B.11 and equations B.13-B.17 define a capacitated growth structure. The simplified differential equation that corresponds to this structure is given in Equation B.18.

$$\begin{aligned}\frac{dStock}{dt} &= Inflow - Outflow \\ &= f\left(\frac{Stock}{Capacity}\right) \times Standard\ Fraction\ 1 \times Stock - Fraction\ 2 \times Stock\end{aligned} \tag{B.18}$$

Block diagram of the capacitated growth structure is given in Figure B.12.

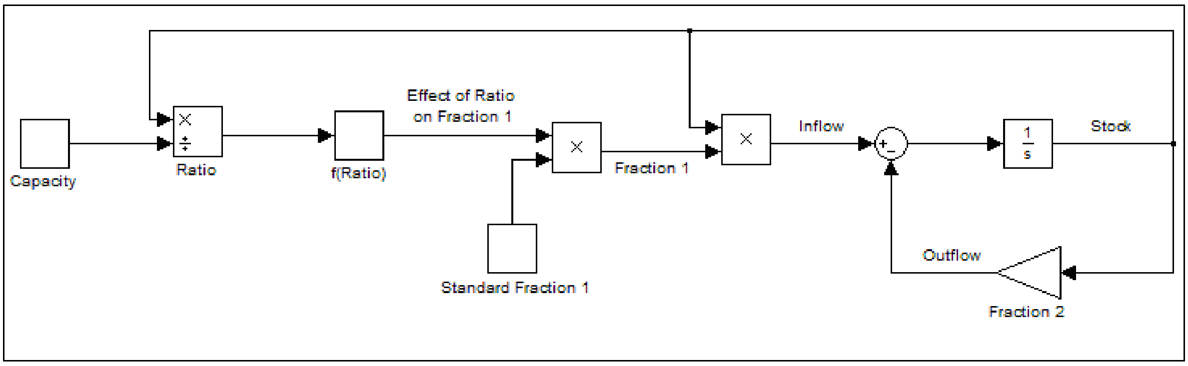

Figure B.12. Block diagram of the capacitated growth structure.

As an example, assume that *f*(*Ratio*) is given by Equation B.19.

$$Effect\ of\ \ Ratio\ on\ Fraction\ 1 = f\left(Ratio\right) = 1 - 0.75 \times Ratio \qquad \text{(B.19)}$$

The corresponding part of the block diagram representing *f*(*Ratio*), which is obtained under this assumption (Equation B.19), is given in Figure B.13.

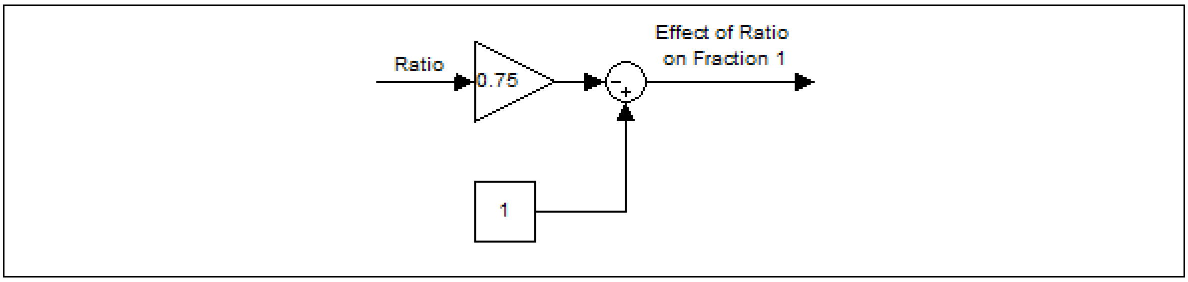

Figure B.13. Block diagram of an example *f*(*Ratio*).

B.7. Growth with Overshoot Structure (Caused by a Delayed Effect of Capacity Limit)

Stock-flow diagram of a growth with overshoot structure is given in Figure B.14.

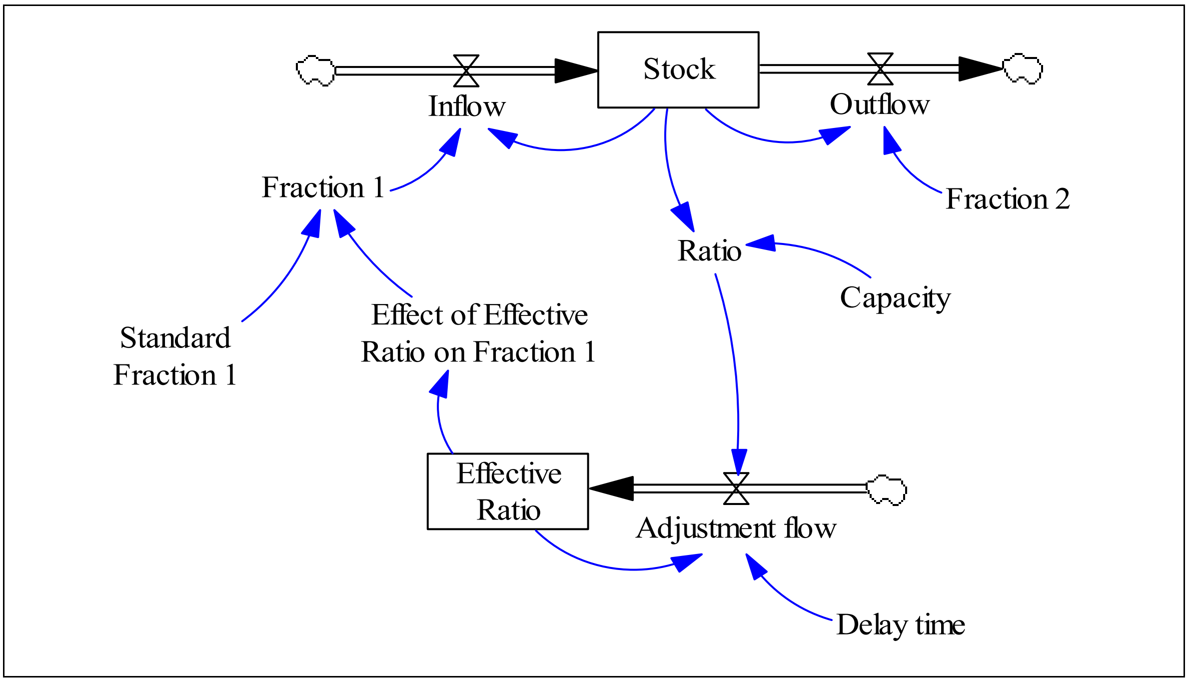

Figure B.14. Stock-flow diagram of the growth with overshoot structure.

The model equations are B.20-B.25.

$$Inflow = Fraction\ 1 \times Stock \tag{B.20}$$

$$Fraction\ 1 = Standard\ Fraction\ 1 \times Effect\ of\ \ Effective\ Ratio\ on\ Fraction\ 1 \tag{B.21}$$

$$Effect\ of\ \ Effective\ Ratio\ on\ Fraction\ 1 = f\left(Effective\ Ratio\right) \tag{B.22}$$

$$Adjustment\ \ flow = \frac{Ratio - Effective\ Ratio}{Delay\ time} \tag{B.23}$$

$$Ratio = \frac{Stock}{Capacity} \tag{B.24}$$

$$Outflow = Fraction\ \ 2 \times Stock \tag{B.25}$$

The diagram in Figure B.14 and equations B.20-B.25 define a growth with overshoot structure. The simplified set of differential equations that corresponds to this structure is given in equation B.26 and B.27.

$$\begin{aligned} \frac{dStock}{dt} &= Inflow - Outflow \\ &= Standard\ \ Fraction\ 1 \times f\left(Effective\ Ratio\right) \times Stock - Fraction\ \ 2 \times Stock \end{aligned} \tag{B.26}$$

$$\frac{dEffective\ Ratio}{dt} = Adjustment\ \ flow = \frac{\dfrac{Stock}{Capacity} - Effective\ Ratio}{Delay\ time} \tag{B.27}$$

Block diagram of the growth with overshoot structure is given in Figure B.15.

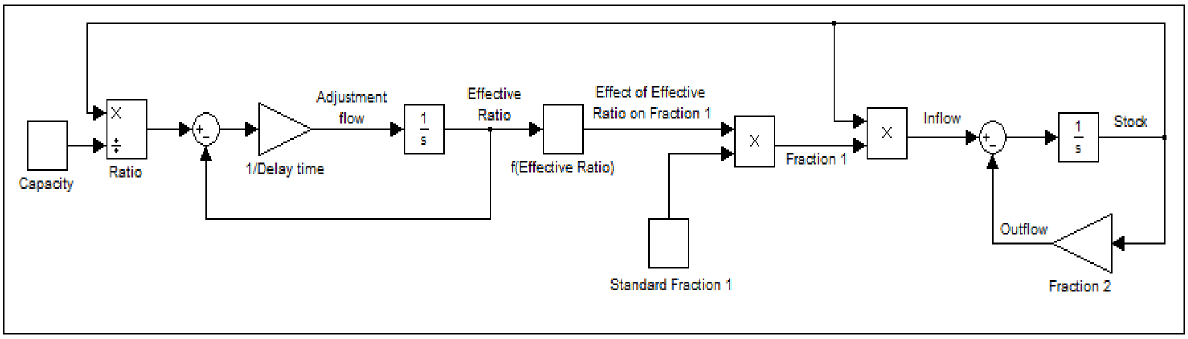

Figure B.15. Block diagram of the growth with overshoot structure.

B.8. Continuous Material Delay Structures

Stock-flow diagram of a first order continuous material delay structure is given in Figure B.16.

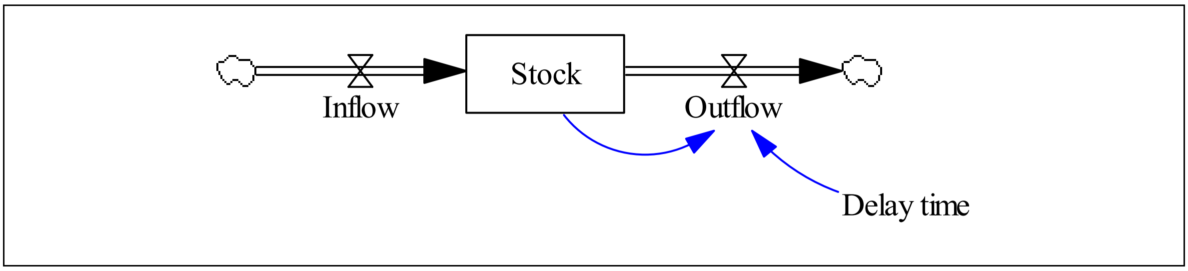

Figure B.16. Stock-flow diagram of the first order continuous material delay structure.

Outflow equation of the model is given in Equation B.28.

$$Outflow = \frac{Stock}{Delay\ time} \tag{B.28}$$

The diagram in Figure B.16 and Equation B.28 define a first order continuous material delay structure. The simplified differential equation that corresponds to this structure is given in Equation B.29.

$$\frac{dStock}{dt} = Inflow - Outflow = Inflow - \frac{Stock}{Delay\ time} \tag{B.29}$$

where “Delay time” is a nonnegative constant value.

Block diagram of the first order continuous material delay structure is given in Figure B.17.

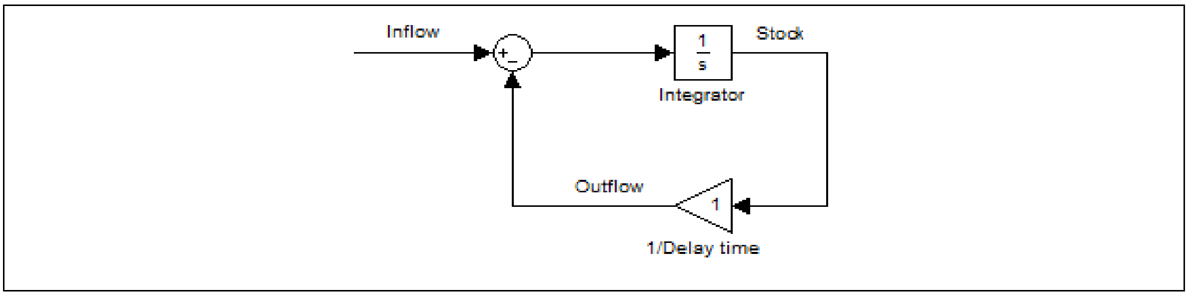

Figure B.17. Block diagram of the first order continuous material delay structure.

Stock-flow diagram of a third order continuous material delay structure is given in Figure B.18.

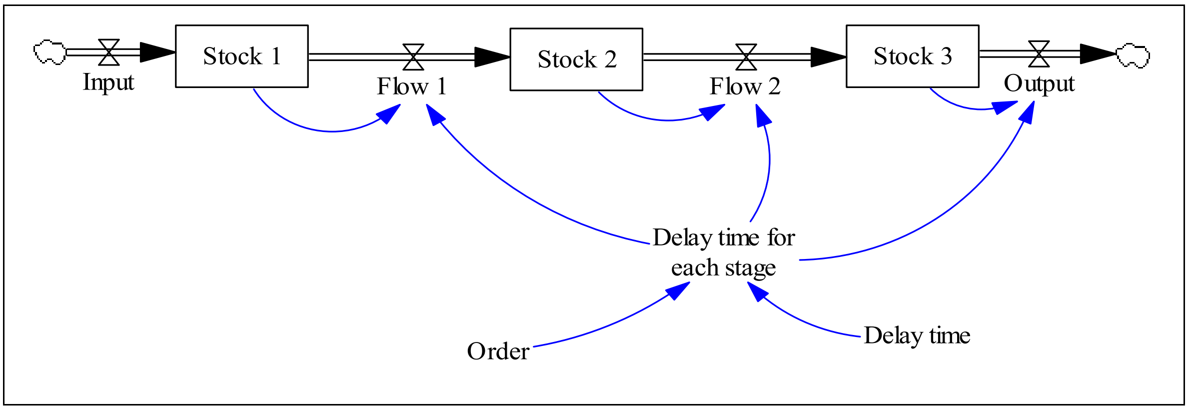

Figure B.18. Stock-flow diagram of the third order continuous material delay structure.

The model equations are B.30-B.33.

$$Flow\,1 = \frac{Stock\ 1}{Delay\ time\ for\ each\ stage} \tag{B.30}$$

$$Flow\,2 = \frac{Stock\ 2}{Delay\ time\ for\ each\ stage} \tag{B.31}$$

$$Output = \frac{Stock\,3}{Delay\ time\ for\,each\,stage} \tag{B.32}$$

$$Delay\ time\ for\,each\,stage = \frac{Delay\ time}{Order} \tag{B.33}$$

The diagram in Figure B.18 and equations B.30-B.33 define a third order continuous material delay structure. The simplified set of differential equations that corresponds to this structure is given in equations B.34, B.35, and B.36.

$$\frac{dStock\,1}{dt} = Input - Flow\,1 = Input - \frac{Stock\,1}{Delay\ time/Order} \tag{B.34}$$

$$\frac{dStock\,2}{dt} = Flow\,1 - Flow\,2 = \frac{Stock\,1}{Delay\ time/Order} - \frac{Stock\,2}{Delay\ time/Order} \tag{B.35}$$

$$\frac{dStock\,3}{dt} = Flow\,2 - Output = \frac{Stock\,2}{Delay\ time/Order} - \frac{Stock\,3}{Delay\ time/Order} \tag{B.36}$$

where “Delay time” and “Order” are nonnegative constant values and “Order” corresponds to the number of state variables (i.e., stocks) in the material structure.

Block diagram of the third order continuous material delay structure is given in Figure B.19.

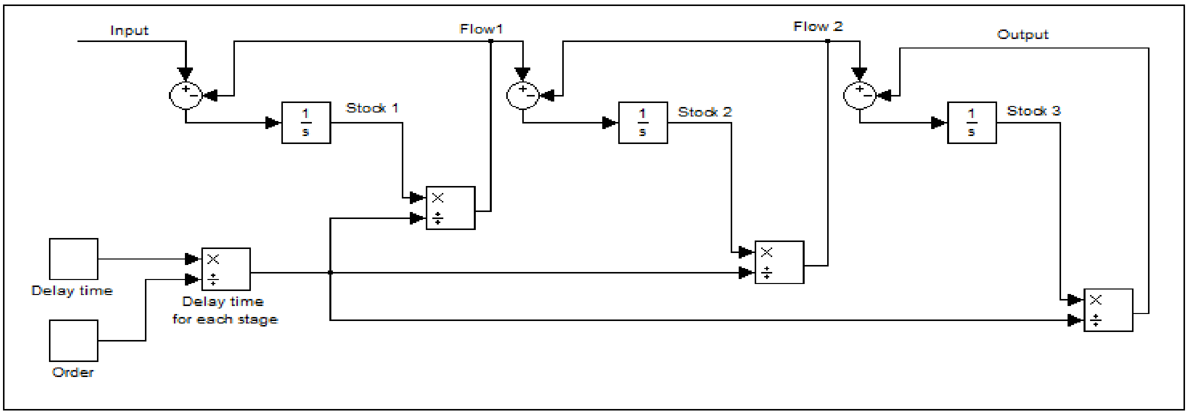

Figure B.19. Block diagram of the third order continuous material delay structure.

Note that every material delay structure is an application of aforementioned draining structure. Hence, a material delay structure contains one or many draining structures.

B.9. Continuous Information Delay Structures

Stock-flow diagram of a first order continuous information delay structure is given in Figure B.20.

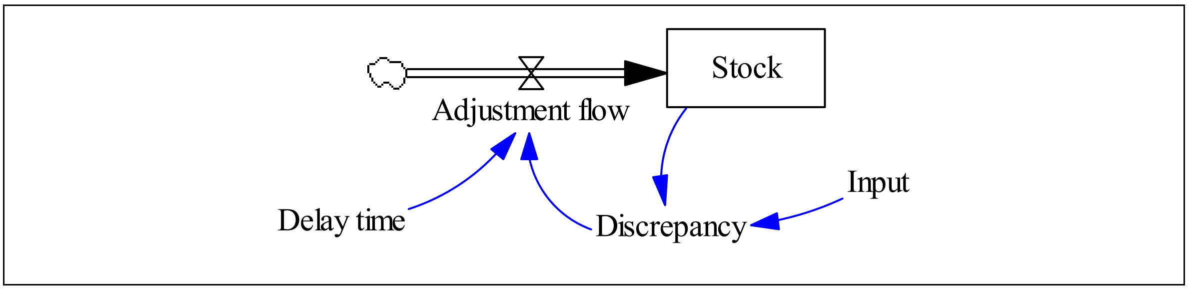

Figure B.20. Stock-flow diagram of the first order continuous information delay structure.

Equations of the model are given in equations B.37 and B.38.

$$Adjustment\ flow = \frac{Discrepancy}{Delay\ time} \tag{B.37}$$

$$Discrepancy = Input - Stock \tag{B.38}$$

The diagram in Figure B.20 and equations B.37 and B.38 define a first order continuous information delay structure. The simplified differential equation that corresponds to this structure is given in Equation B.39.

$$\frac{dStock}{dt} = Adjustment\ flow = \frac{Input - Stock}{Delay\ time} \tag{B.39}$$

where “Delay time” is a nonnegative constant value.

Block diagram of the first order continuous information delay structure is given in Figure B.21.

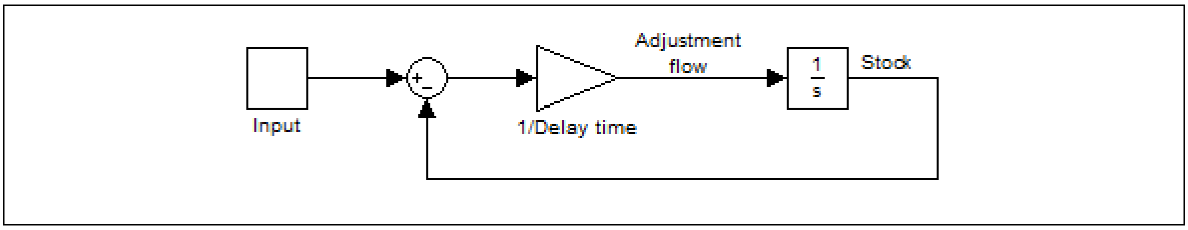

Figure B.21. Block diagram of the first order continuous information delay structure.

Stock-flow diagram of a third order continuous information delay structure is given in Figure B.22.

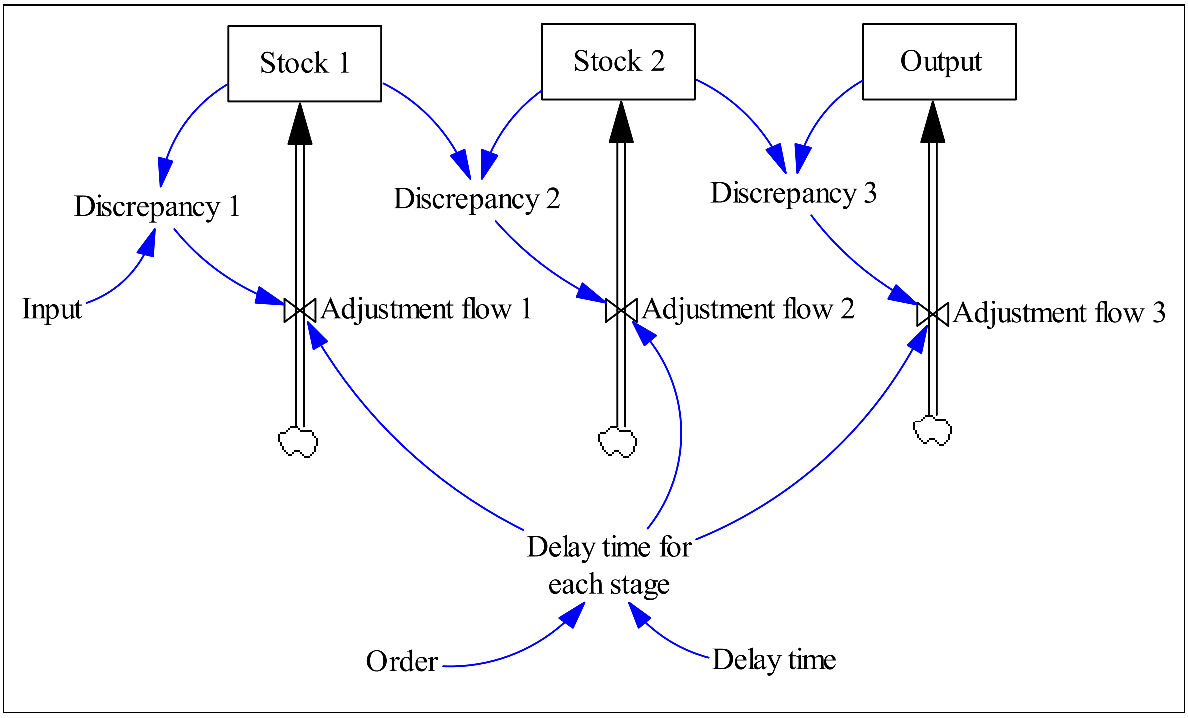

Figure B.22. Stock-flow diagram of the third order continuous information delay structure.

The model equations are B.40-B.46.

$$Adjustment\ flow1 = \frac{Discrepancy\,1}{Delay\ time\ for\ each\ stage} \tag{B.40}$$

$$Adjustment\ flow2 = \frac{Discrepancy\,2}{Delay\ time\ for\, each\, stage} \tag{B.41}$$

$$Adjustment\ flow3 = \frac{Discrepancy\,3}{Delay\ time\ for\, each\, stage} \tag{B.42}$$

$$Discrepancy\,1 = Input - Stock\ 1 \tag{B.43}$$

$$Discrepancy\,2 = Stock\ 1 - Stock\ 2 \tag{B.44}$$

$$Discrepancy\,3 = Stock\ 2 - Output \tag{B.45}$$

$$Delay\ time\ for\, each\, stage = \frac{Delay\ time}{Order} \tag{B.46}$$

The diagram in Figure B.22 and equations B.40-B.46 define a third order continuous information delay structure. The simplified differential equations that correspond to this structure are given in equations B.47, B.48, and B.49.

$$\frac{dStock\ 1}{dt} = Adjustment\ flow1 = \frac{Input - Stock\ 1}{Delay\ time/Order} \tag{B.47}$$

$$\frac{dStock\ 2}{dt} = Adjustment\ flow2 = \frac{Stock\ 1 - Stock\ 2}{Delay\ time/Order} \tag{B.48}$$

$$\frac{dOutput}{dt} = Adjustment\ flow3 = \frac{Stock\ 2 - Output}{Delay\ time/Order} \tag{B.49}$$

where “Delay time” and “Order” are nonnegative constant values.

Block diagram of the third order continuous information delay structure is given in Figure B.23.

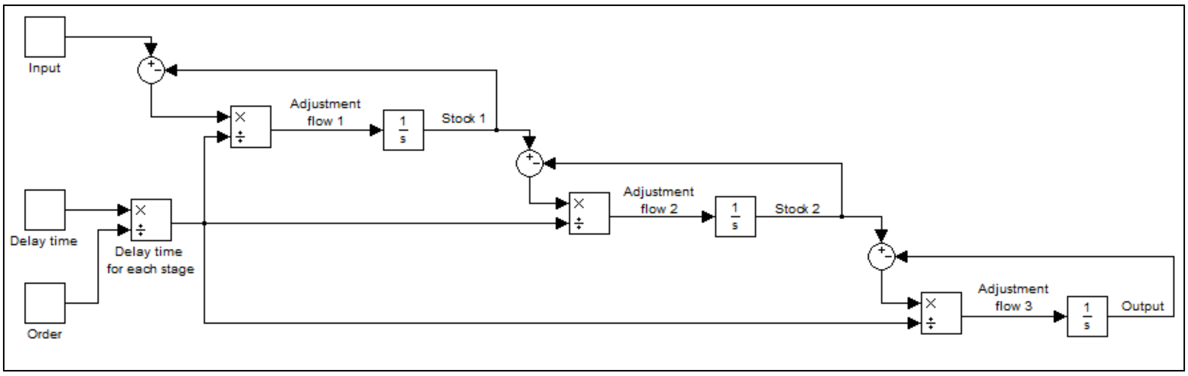

Figure B.23. Block diagram of the third order continuous information delay structure.

Note that every information delay structure is an application of aforementioned goal seeking structure. Hence, an information delay structure contains one or many goal seeking structures.

B.10. Discrete Material Delay Structure (Pure Delay)

Stock-flow diagram of a discrete material delay structure is given in Figure B.24.

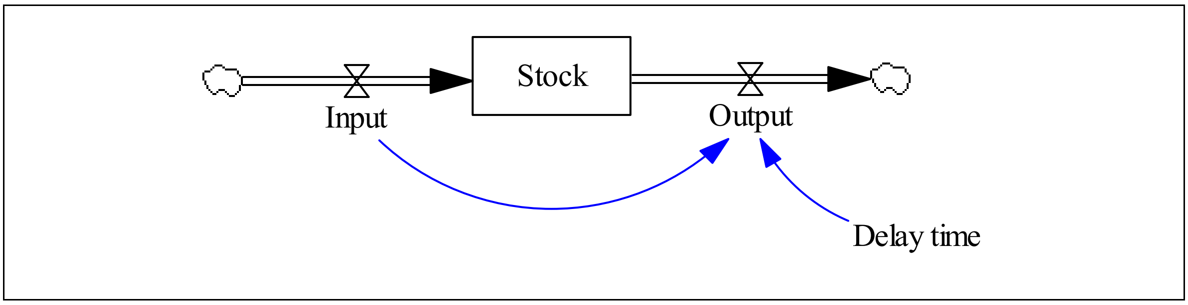

Figure B.24. Stock-flow diagram of the discrete material delay structure.

Output equation of the model is given in Equation B.50.

$$Output(t) = \begin{cases} \dfrac{Stock(0)}{Delay\ time} & \text{for} \quad 0 \le t < Delay\ time \\ Input(t - Delay\ time) & \text{for} \quad t \ge Delay\ time \end{cases} \tag{B.50}$$

where “Delay time” is a nonnegative constant value.

The diagram in Figure B.24 and Equation B.50 define an infinite order (i.e., discrete) material delay structure. The simplified differential equation that corresponds to this structure is given in Equation B.51.

$$\frac{dStock}{dt} = Input - Output$$
$$= \begin{cases} Input(t) - \dfrac{Stock(0)}{Delay\ time} & \text{for} \quad 0 \le t < Delay\ time \\ Input(t) - Input(t - Delay\ time) & \text{for} \quad t \ge Delay\ time \end{cases} \tag{B.51}$$

Block diagram of the discrete material delay structure is given in Figure B.25.

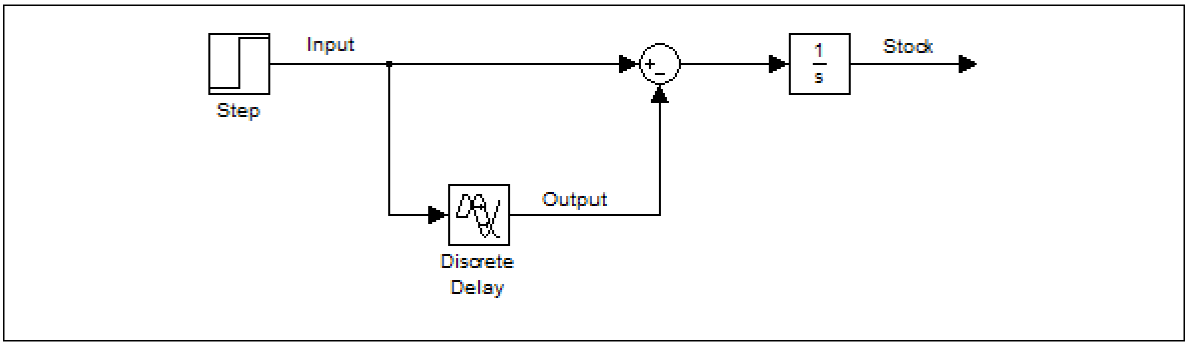

Figure B.25. Block diagram of the discrete material delay structure.

B.11. Discrete Information Delay Structure (Pure Delay)

Stock-flow diagram of a discrete information delay structure is given in Figure B.26.

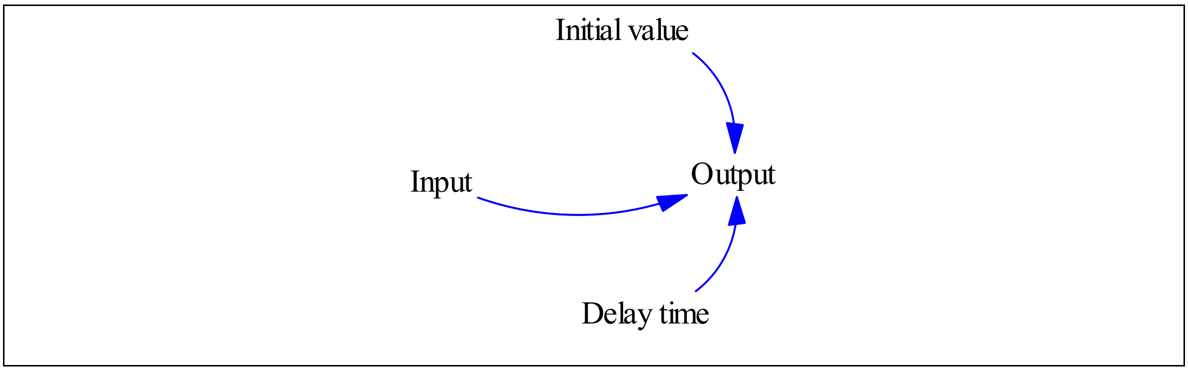

Figure B.26. Stock-flow diagram of the discrete information delay structure.

Output equation of the model is given in Equation B.52.

$$Output(t) = \begin{cases} Initial\ value & \text{for} \quad 0 \le t < Delay\ time \\ Input(t - Delay\ time) & \text{for} \quad t \ge Delay\ time \end{cases} \tag{B.52}$$

where "Delay time" is a nonnegative constant value.

The diagram in Figure B.26 and Equation B.50 define an infinite order (i.e., discrete) information delay structure. Note that there is no simplified differential equation that corresponds to this structure as there is no stock in Figure B.26.

Block diagram of the discrete information delay structure is given in Figure B.27.

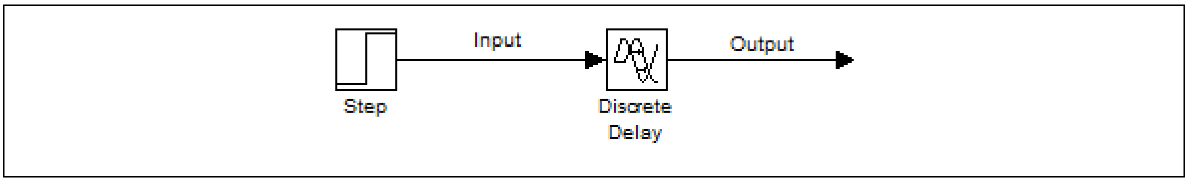

Figure B.27. Block diagram of the discrete information delay structure.

B.12. Oscillating Structure

Stock-flow diagram of an oscillating structure is given in Figure B.28.

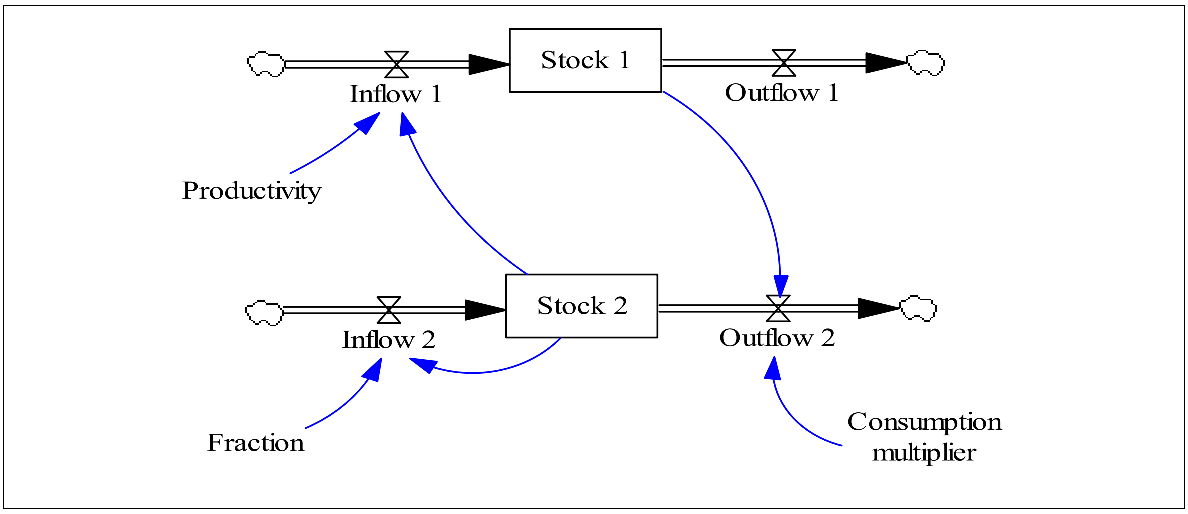

Figure B.28. Stock-flow diagram of the oscillating structure.

The model equations are B.53, B.54, and B.55.

$$Inflow\ 1 = Productivity \times Stock\ 2 \tag{B.53}$$

$$Inflow\ 2 = Fraction \times Stock\ 2 \tag{B.54}$$

$$Outflow\ 2 = Consumption\ multiplier \times Stock\ 1 \tag{B.55}$$

The diagram in Figure B.28 and equations B.53, B.54, and B.55 define an oscillating structure. The simplified differential equations that correspond to this structure are given in equations B.56 and B.57.

$$\frac{dStock\ 1}{dt} = Inflow\ 1 - Outflow\ 1 = Productivity \times Stock\ 2 - Outflow\ 1 \tag{B.56}$$

$$\begin{aligned}\frac{dStock\ 2}{dt} &= Inflow\ 2 - Outflow\ 2 \\ &= Fraction \times Stock\ 2 - Consumption\ multiplier \times Stock\ 1\end{aligned} \tag{B.57}$$

where "Productivity" and "Consumption multiplier" are nonnegative constant values and "Fraction" is a constant value.

Block diagram of the oscillating structure is given in Figure B.29.

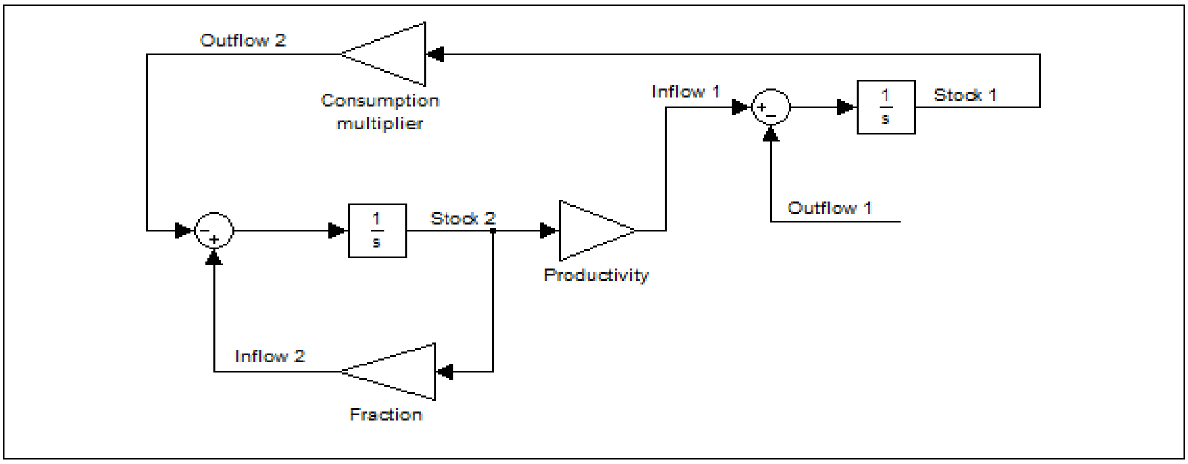

Figure B.29. Block diagram of the oscillating structure.

B.13. Simple Goal Setting Structure

Stock-flow diagram of the simple goal setting structure is given in Figure B.30.

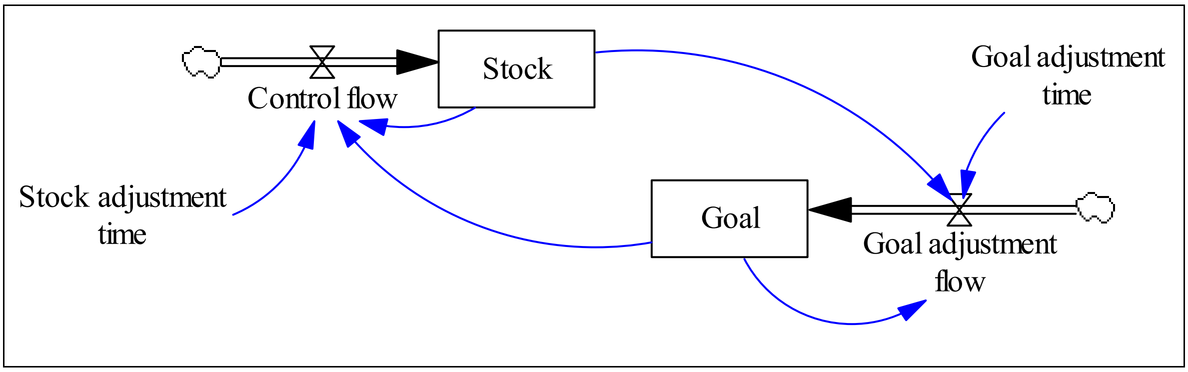

Figure B.30. Stock-flow diagram of the simple goal setting structure.

The model equations are B.58 and B.59.

$$Control\ flow = \frac{Goal - Stock}{Stock\ adjustment\ time} \tag{B.58}$$

$$Goal\ adjustment\ flow = \frac{Stock - Goal}{Goal\ adjustment\ time} \tag{B.59}$$

The diagram in Figure B.30 and equations B.58 and B.59 define the simple goal setting structure. The simplified set of differential equations that correspond to this structure are given in equations B.60 and B.61.

$$\frac{dStock}{dt} = Control\ flow = \frac{Goal - Stock}{Stock\ adjustment\ time} \tag{B.60}$$

$$\frac{dGoal}{dt} = Goal\ adjustment\ flow = \frac{Stock - Goal}{Goal\ adjustment\ time} \tag{B.61}$$

where "Stock adjustment time" and "Goal adjustment time" are nonnegative constant values.

Block diagram of the simple goal setting structure is given in Figure B.31.

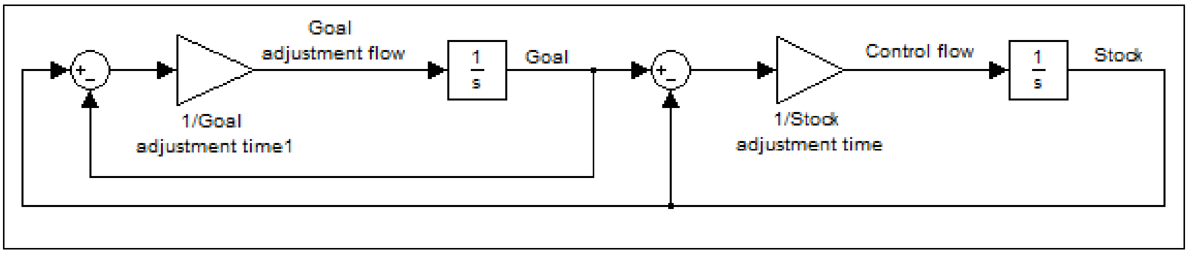

Figure B.31. Block diagram of the simple goal setting structure.

APPENDIX C: BASIC STOCK MANAGEMENT STRUCTURE WITHOUT DELAY

This appendix aims to summarize the results for a stock management structure without a delay as a preliminary part of the study. The results reported in this appendix have already been reported in the literature.

C.1. System Dynamics Model of the Basic Stock Management Structure without Delay

Stock-flow diagram of a basic stock management structure without delay is given in Figure C.1.

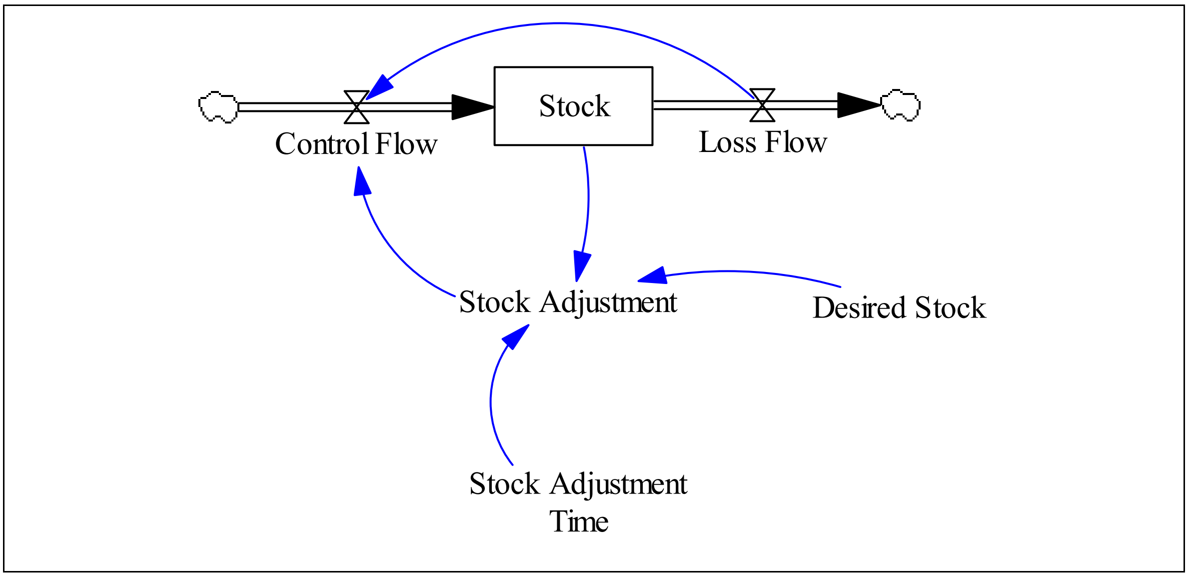

Figure C.1. Stock-flow diagram of the basic stock management structure without delay.

In the stock-flow diagram given in Figure C.1, "Stock" (S) is an accumulation formed over time. "Control Flow" (CF) is the inflow and "Loss Flow" (LF) is the outflow of S. "Desired Stock" (S^*) and "Stock Adjustment Time" (sat) are the parameters of this simple stock management model, which consists of the stock-flow diagram given in Figure C.1 and the equations C.1 and C.2.

$$CF = LF + SA \tag{C.1}$$

$$SA = \frac{S^* - S}{sat} \tag{C.2}$$

To be able to simulate the model, numerical values must be assigned to *S**, *sat*, and simulation-time-step (DT); see the previous chapter and the following section. Note that *sat* must strictly be greater than zero.

C.2. The Approximate Integral Equation of the Basic Stock Management Structure without Delay

The relationship between the stock variable, which is *S*, and the flow variables attached to it, which are *CF* and *LF*, imply Equation C.3 (see Figure C.1).

$$S_{t+DT} = S_t + (CF - LF) \times DT \tag{C.3}$$

Inserting equations C.1 and C.2 into Equation C.3 and simplifying the equation result in Equation C.4, which is the corresponding approximate integral equation of the model.

$$S_{t+DT} = S_t + \frac{S^* - S_t}{sat} \times DT \tag{C.4}$$

C.3. The Difference Equation of the Basic Stock Management Structure without Delay

Assigning one to DT in Equation C.4 and simplifying the equation result in Equation C.5, which is the corresponding difference equation of the model.

$$S_{t+DT} = S_t + \frac{S^* - S_t}{sat} \tag{C.5}$$

C.4. The Differential Equation of the Basic Stock Management Structure without Delay

Equation C.4 can be re-written as Equation C.6.

$$\frac{S_{t+DT} - S_t}{DT} = \frac{S^* - S}{sat} \tag{C.6}$$

Equation C.7, which is the corresponding differential equation of the model, is obtained from Equation C.6 by taking the limit of DT to zero.

$$\frac{dS}{dt} = \lim_{DT \to 0^+} \left(\frac{S_{t+DT} - S_t}{DT} \right) = \frac{S^* - S}{sat} \tag{C.7}$$

C.5. The Integral Equation of the Basic Stock Management Structure without Delay

Equation C.7 can be re-written as Equation C.8.

$$\int_{S_0}^{S_t} dS = \int_0^t \frac{S^* - S}{sat} \times dt \tag{C.8}$$

Equation C.9, which is the corresponding integral equation of the model, is obtained from Equation C.8.

$$S_t = S_0 + \int_0^t \frac{S^* - S}{sat} \times dt \tag{C.9}$$

C.6. Block Diagram of the Basic Stock Management Structure without Delay

Block diagram of the basic stock management structure without delay is given in Figure C.2.

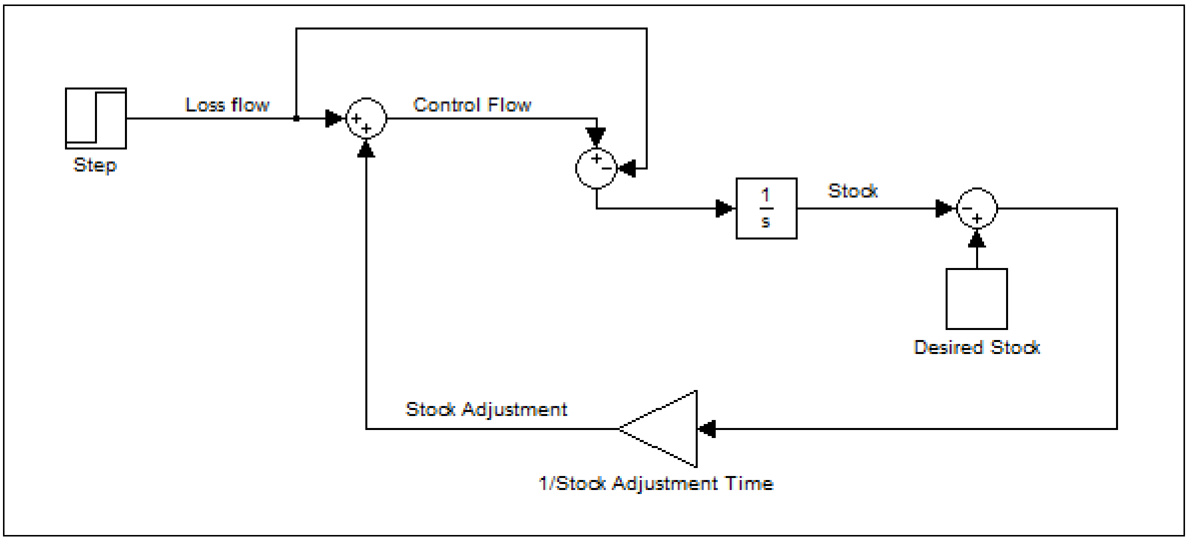

Figure C.2. Block diagram of the basic stock management structure without delay.

C.7. Stability Analysis of the Basic Stock Management Structure without Delay

To determine the stability property of a system, characteristic roots of the characteristic equation must be analyzed. The characteristic equation originates from the Laplace transform of the differential equation. Initial conditions do not play role on the stability of the state of the system, thus, they are assumed to be zero (Bequette, 2007; Nise, 2004; Ogata, 2002; Seborg *et al.*, 2004; Sipahi and Delice, 2010).

Laplace transform of the simplified differential equation of the basic stock management structure (Equation C.7) is given in Equation C.10 by assuming "Desired Stock" equals to zero.

$$s \times S(s) - S(0) = -\frac{S(s)}{sat} \tag{C.10}$$

Characteristic equation is obtained by arranging Equation C.10 and assuming $S(0)$ = 0 and given in Equation C.11.

$$S(s) \times (sat \times s + 1) = 0 \tag{C.11}$$

The characteristic root of the characteristic equation is given in Equation C.12.

$$s = -\frac{1}{sat} \tag{C.12}$$

As mentioned before, *sat* is a parameter that is strictly greater than zero. Therefore, the characteristic root given in Equation C.12 is always negative and has no complex part, which implies that *S* is always stable and never oscillates; it always shows a goal seeking behavior (i.e., asymptotic approach to the desired level of stock). This result is consistent with the results reported by Sterman (2000) and Yasarcan (2003, 2011).

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