CURVED FINITE ELEMENTS FOR CYLINDRICALLY AND AXI-SYMMETRICALLY CURVED SHELLS

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SYNOPSIS

Assuming displacement functions and using energy method of derivation,

1-a 20x20 stiffness matrix of a curved, rectangular, finite element of a cylindrical shell, without rigid body displacement modes.

2-a 24x24 stiffness matrix of a curved, rectangular, finite element of a cylindrical shell with displacement functions modified to include rigid body modes, and a slope continuity term added as a deflection parameter.

3-a 24x24 stiffness matrix of a curved, rectangular, finite element of a cylindrical shell with displacement functions modified to include rigid body modes and rotation about the axis perpendicular to the element surface.

4-a 15x15 stiffness matrix of a doubly curved, triangular finite element of a spherical shell without rigid body modes.

are developed.

All four elements possess the actual curvatures of the shells that they are taken from.

Thin shell assumptions are used in all the derivations of the element stiffness matrices.

Static equilibrium test is conducted on elements 3 and 4. Also using elements 3 and 4, an example problem is solved.

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INTRODUCTION

In the finite element analysis, the structure is divided into finite number of two or three dimensional elements connecte at nodal points. Finite element method of analysis is normally carried out by the stiffness method of analysis, description of which is abundantly available in the literature(1)[1],(2). The only part of the analysis considered herein is the formulation of the element stiffness matrices, which define the behavior of the element itself.

There are different aproaches to obtain the element stiffness matrix. Energy method of derivation makes use of displacemen functions and minimum potential energy principal. In the statics method of derivation element stiffness matrices are constructed by obtaining corner forces due to each displacement mode. Energy method will be used herein.

The types of elements taken into account here are cylindrically and axisymmetrically curved finite elements.

In the literature several different types of elements are constructed and compared for a cylindrical shell. R.H. Gallag (3), in this Ph.D. thesis, constructed a 24x24 stiffness matrix, where the displacement functions did not include rigid body modes. Bogner, Fox and Schmit(4), used Hermite interpolation formu to get the 48x48 stiffness matrix of a cylindrical shell elemen in their paper. Their stiffness matrix is very sensitive, with rigid body modes. J.J. Connor and C.Brebbia's(5) rectangular shell element is derived following shallow shell theory.

1 Numbers in parentheses refer to corresponding items in the references.

They did not take into consideration the rigid body modes also. G.Cantin and R.W.Clough (6) in their paper use the same method of construction for their cylindrical shell,24x24 element stiffness matrix as done herein.They include rigid body displacement modes in their displaçement functions,which are different from those used herein.

As for the doubly curved,triangular,revolutionary shell element the only known stiffness matrix is the one constructed by Ş.Utku (7),but he followed shallow shell theory.

The first stiffness matrix derived herein for a rectangular finite element of a cylindrical shell is for an element with five deflection parameters at each corner $(u,v,w,\theta_x,\theta_y)$, yielding a total of twenty degrees of freedom for the element. The element is assumed to have infinite stiffness for the twisting moments.The membrane displacement functions do not satisfy compatability and equilibrium at the edges but within the element there is equilibrium.The stresses vary linearly with the distance along the axis they are perpendicular and shear is constant along both directions of axes.The displacement functions do not account for all the rigid body displacement modes of the element.

The second cylindrical shell rectangular finite element has six deflection parameters at each node,with a total of 24 degrees of freedom for the element. As a deflection parameter a term which accounts for slope continuity is added The displacement functions are the same as those of the first element,but they are modified to include the rigid body displacement modes of the element.

The third element stiffness matrix derived for a cylindrical shell finite element has also 24 degrees of freedom,6 at each node.Rotation about the axis perpendicular to the element surface is added as the 6^{th} deflection parameter.The displacement functions assumed are different from those used in the above mentioned two elements.The displacement functions used are modified to include rigid body displacements.

The triangular finite element of a sphericalshell has five deflection parameters at each node,rotation about the vertical axis,Θ_z,is neglected assuming element infinitely stiff against twisting moments,in the plane of the element.The element has a total of 15 degrees of freedom.The membrane displacement functions do not have equilibrium and compatability at the edges,but there is equilibrium within the element.In the bending displacement function the higher order twisting terms are combined. the displacement functions do not contain all the rigid body displacements.

For all four elements the member axes are chosen such that they coincide with the principal radii of curvatures.

In appendix II two computer programs are presented. Program STIFF calculates the element stiffness matrix in numerical form,given the dimensions and the radius of the element.Program SHELL can solve problems up to 36 unknowns which is a rather coarse mesh idealization when shell problems are solved. The transformation matrices between nodal coordinates and polynomial coefficients along with stiffness matrices in terms of polynomial co-efficients for the cylindrical shell elements are also given in appendix II.

CHAPTER 1

ENERGY METHOD OF DERIVATION OF STIFFNESS MATRICES

Using assumed finite element deformation patterns the stiffness matrix of an element of any shape,form or material properties can be constructed by a standart analysis procedure called the energy method.The energy method is carried out as follows:

1-The internal displacements u,v,w should be expressed in terms of displacement functions F.

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{bmatrix} F \end{bmatrix} \begin{Bmatrix} a \end{Bmatrix} \tag{1.1}$$

The amplitutes of the displacement functions are represented by the generalized coordinates, $\{a\}$, which should be equal to the number of degrees of freedom of the element.

2- The nodal displacement components, $\{d\}$, in terms of generalized coordinates, $\{a\}$, are obtained by mere substitution of the nodal coordinates into displacement function matrix $[F]$.

$$\{d\} = [A] \{a\} \tag{1.2}$$

3-For further use, the generalized coordinates,

$\{a\}$,in terms of nodal displacements, $\{d\}$, are obtained by a simple inversion process.

$$\{a\}=[A^{-1}]\{d\} \tag{1.2a}$$

For simplicity let,

$$[B]=[A^{-1}] \tag{1.2b}$$

Substituting (1.2b) into (1.2a) ,

$$\{a\}=[B]\{d\} \tag{1.2c}$$

4- The element strains, ε ,can be related to displacements by a matrix of differential operators.

$$\{\varepsilon\}=[Q]\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \tag{1.3}$$

Matrix $[Q]$ contains the differential operators. When bending is also present,curvature terms should also be included in the $\{\varepsilon\}$.The contents of $[Q]$ is both strain and curvature relations for a shell element.

Substituting (1.1) into (1.3),

$$\{\varepsilon\}=[Q][F]\{a\} \tag{1.3a}$$

Performing the apropriate differentiation processes,

$$\{\varepsilon\}=[G_o]\{a\} \tag{1.3b}$$

Substituting (1.2c) into (1.3b) yields,

$$\{\varepsilon\}=[G_0][B]\{d\} \quad (1.3c)$$

5- After obtaining strain-displacement relations, the stress-strain equations can be constructed.

$$\{\sigma\}=[D]\{\varepsilon\} \quad (1.4)$$

$[D]$ is a matrix containing elastic constants related to the specific elastic properties of the finite element material, which can be isotropic, orthotropic, elasto-plastic-etc. Substituting (1.3c) in (1.4), the element stresses become,

$$\{\sigma\}=[D][G_0][B]\{d\} \quad (1.4a)$$

6- The strain energy given strains and stresses is,

$$U=\frac{1}{2}\int_V(\varepsilon_x\sigma_x+\varepsilon_y\sigma_y+\varepsilon_z\sigma_z+\gamma_{xy}\tau_{xy}+\gamma_{xz}\tau_{xz}+\gamma_{zy}\tau_{zy})\,dV \quad (1.5)$$

Or in a more compact form, the strain energy is,

$$U=\frac{1}{2}\int_V\{\varepsilon\}^T\{\sigma\}\,dV \quad (1.5a)$$

Substituting in (1.5a), (1.4), strain energy becomes,

$$U=\frac{1}{2}\int_V\{\varepsilon\}^T[D]\{\varepsilon\}\,dV \quad (1.5b)$$

Another form of strain energy can be obtained by substituting in (1.5b), the strain displacement relation (1.3c).

$$U=\frac{1}{2}\int_V \{d\}^T [B]^T [G_o]^T [D][G_o][B]\{d\}\, dV \quad (1.5c)$$

The matrices d and B contain constants so they can be left out of integration. If we define,

$$[H_o]=\int_V [G_o]^T [D][G_o]\, dV \quad (1.5d)$$

Equation (1.5c) becomes:

$$U= \{d\}^T [B]^T [H_o][B]\{d\} \quad (1.5e)$$

Castigliano's Theorem states that:

$$\frac{\partial U}{\partial d_i} = p_i \quad (1.6)$$

Where p_i are the external forces and d_i are the actual displacements at the point of application of p_i, in the direction of p_i. Also the relation between the element forces and corner displacements are given by,

$$p_i = K_{ij}\, d_j \quad (1.6a)$$

Substituting (1.6a) into (1.6) gives:

$$\frac{\partial^2 U}{\partial d_i \partial d_j} = K_{ij} \qquad (1.6b)$$

Following the same steps from equation(1.6) on,using equation (1.5e), the stiffness matrix of an element can be obtained.Therefore

$$[K]=[B]^T [H_o] [B] \qquad (1.7)$$

$[K]$ is the nodal point stiffness matrix.

CHAPTER 2

DERIVATION OF THE STIFFNESS MATRIX OF A 20x20, CURVED, CYLINDRICAL SHELL FINITE ELEMENT.

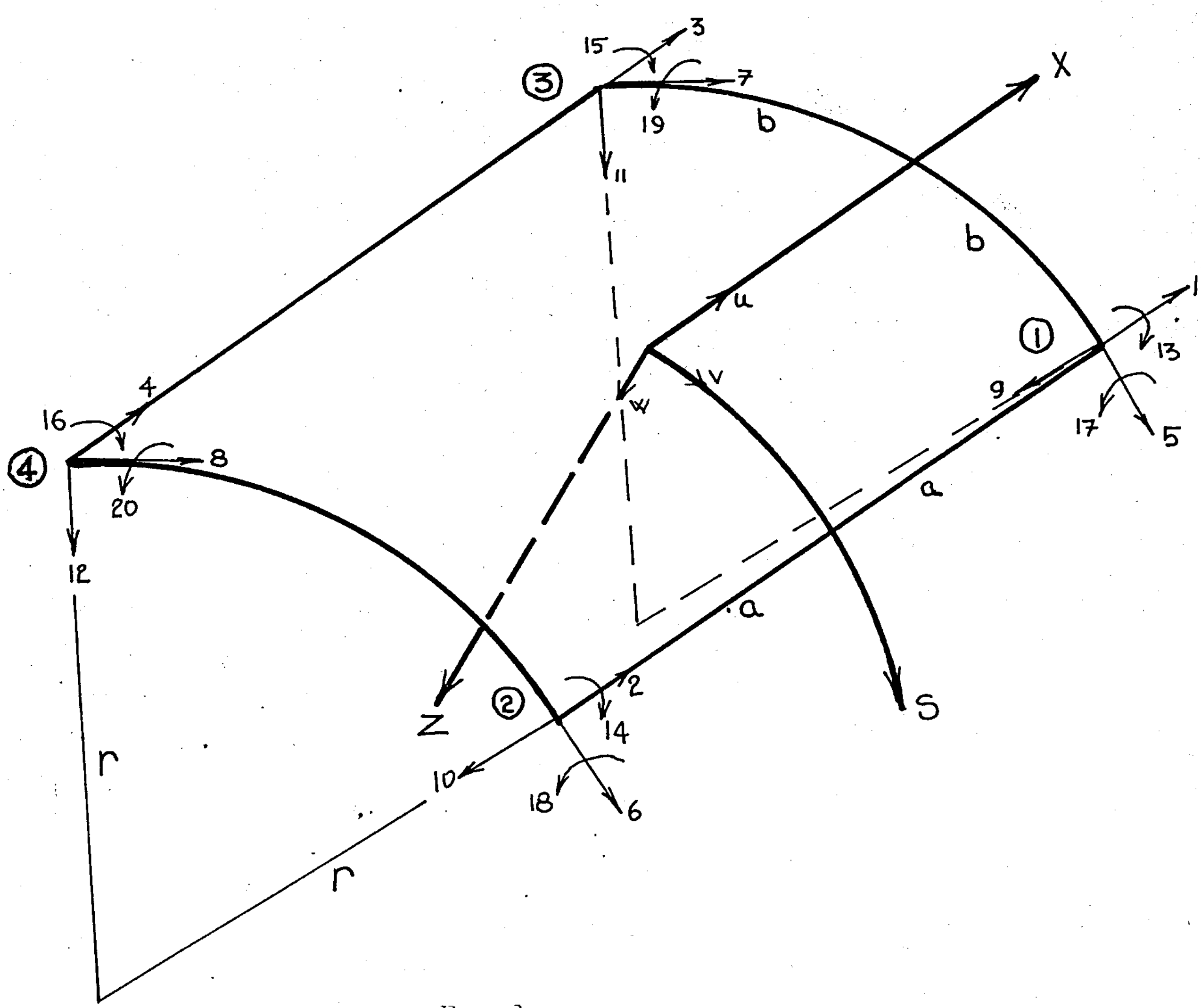


Fig.1

The displacement functions chosen are:

$$u = a_1 + a_2X + a_3S - a_4(\mu X^2 + S^2) + 2a_5XS \qquad (2.1a)$$

$$v = 2a_4XS - a_5(X^2 + \mu S^2) + a_6 + a_7X + a_8S \qquad (2.1b)$$

$$w = a_9 + a_{10}X + a_{11}S + a_{12}X^2 + a_{13}XS + a_{14}S^2 + a_{15}X^3 + $$
$$+ a_{16}X^2S + a_{17}S^2X + a_{18}S^3 + a_{19}X^3S + a_{20}S^3X \qquad (2.1c)$$

To simplify the inversion process corner displacement parameters are chosen in the following order:

$$\{d\}^T = [u_1, u_2, u_3, u_4, v_1, v_2, v_3, v_4, w_1, w_2, w_3, w_4,$$
$$\theta_{x1}, \theta_{x2}, \theta_{x3}, \theta_{x4}, \theta_{s1}, \theta_{s2}, \theta_{s3}, \theta_{s4}] \qquad (2.2a)$$

Where,

$$\theta_{xi} = \frac{\partial w}{\partial s} + \frac{v}{r} \qquad (2.2b)$$

and,

$$\theta_{xi} = -\frac{\partial w}{\partial x} \qquad (2.2c)$$

Using 2.1a,b,c and 2.2a,b,c ,the transformation mattrix, $[A]$,between nodal displacements, $\{d\}$, and polynomial coefficients, $\{a\}$,can be found.

$$\{d\} = [A]\{a\} \qquad (2.2d)$$

Matrix A is given explicitly in appendix I, part 1, table 1.

The strain-displacement matrix, taken from Novozhilov (8), can be expressed as:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xs} \\ \varkappa_x \\ \varkappa_s \\ \varkappa_{xs} \end{Bmatrix} = \begin{bmatrix} \partial/\partial x & 0 & 0 \\ 0 & \partial/\partial s & -1/r \\ \partial/\partial s & \partial/\partial x & 0 \\ 0 & 0 & \partial^2/\partial x^2 \\ 0 & \partial/r\partial s & \partial^2/\partial s^2 \\ 0 & \partial/r\partial x & \partial^2/\partial x\partial s \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \qquad (2.3)$$

The matrix $[D]$ of elastic constants, relating element stresses and strains,

$$[\sigma] = [D]\{\varepsilon\} \qquad (2.4)$$

is now:

$$\begin{bmatrix} D_1 & D_2 & & & \\ D_2 & D_1 & & 0 & \\ & & D_3 & & \\ & 0 & & D_4 & D_5 \\ & & & D_5 & D_6 \end{bmatrix} \qquad (2.4a)$$

Where,

$D_1 = E/(1-\mu^2)$ $\qquad$ $D_4 = E\, t^2/(12(1-\mu^2))$

$D_2 = \mu D_1$ $\qquad$ $D_5 = \mu D_4$

$D_3 = (1-\mu)D_1/2$ $\qquad$ $D_6 = 2(1-\mu)D_4$ $\qquad$ (2.4b)

D_6 is modified in order to be consistent with strain-displacement relations.

Using the above given relations one can easily obtain,

$$[G_o]^T [D][G_o] \qquad (2.5)$$

After integrating over the volume the stiffness matrix in terms of polynomial coefficients, $[H_o]$ is derived. $[H_o]$ is listed in appendix I, part 1, table 2.

To obtain the stiffness matrix K , program STIFF is used, in which the operation,

$$[K] = [B]^T [H_o][B] \qquad (2.6)$$

is done.

CHAPTER 3.

DERIVATION OF THE STIFFNESS MATRIX OF A 24x24, CURVED, CYLINDRICAL SHELL FINITE ELEMENT WITH DISPLACEMENT FUNCTIONS MODIFIED TO INCLUDE RIGID BODY DISPLACEMENTS. SLOPE CONTINUITY TERM IS USED AS A DEFLECTION PARAMETER.

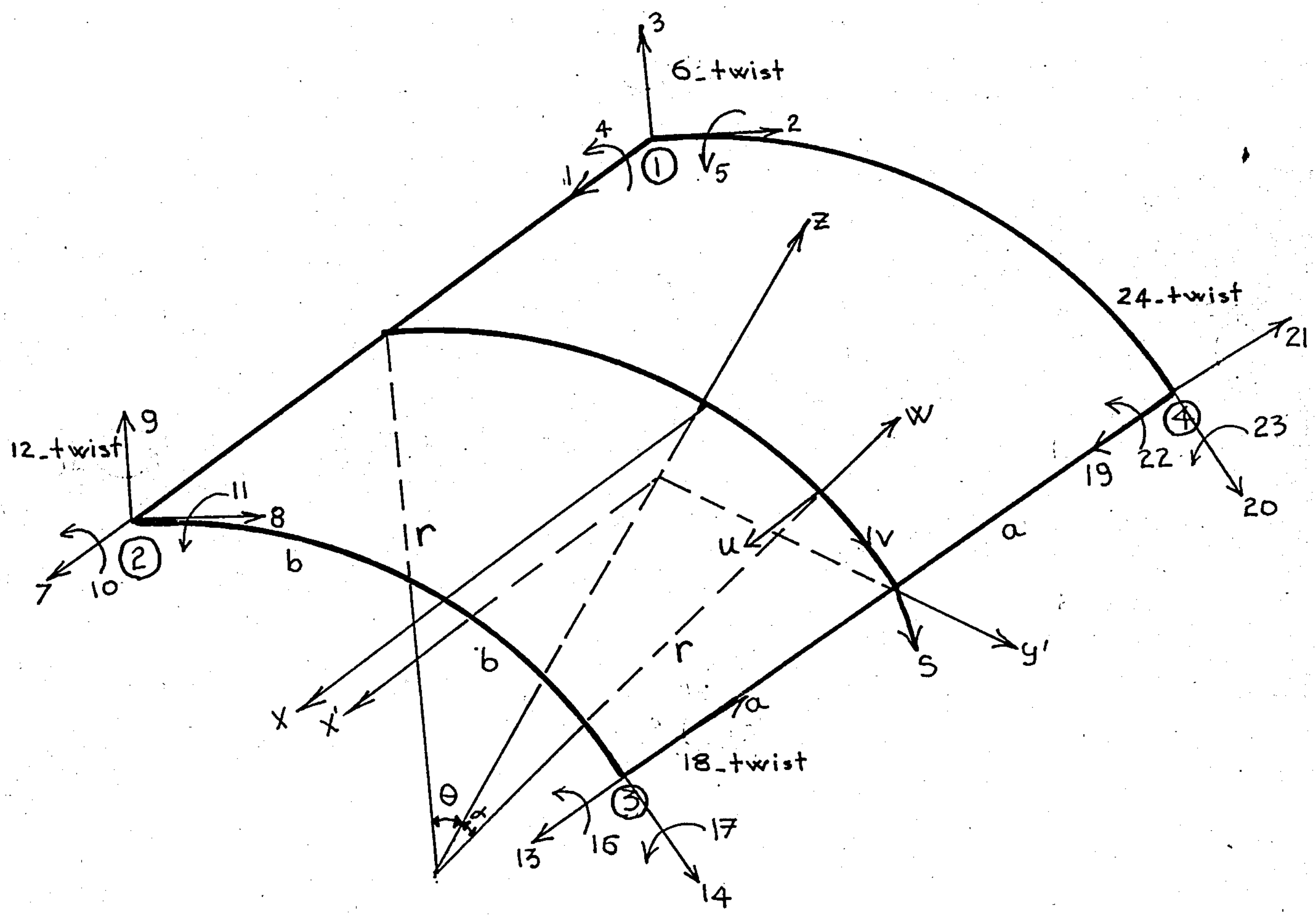


Fig. 2

The displacement functions assumed are:

$$u = -a_1(\mu X^2 + S^2) + 2a_2XS + a_3X + a_4S + a_5 \quad (3.1)$$

$$v = -a_2(X^2 + \mu S^2) + 2a_1XS + a_6X + a_7S + a_8 \quad (3.2)$$

$$\begin{aligned} w = {} & a_9X^3S^3 + a_{10}X^3S^2 + a_{11}X^3S + a_{12}X^3 + \\ & + a_{13}X^2S^3 + a_{14}X^2S^2 + a_{15}X^2S + a_{16}X^2 + \\ & + a_{17}XS^3 + a_{18}XS^2 + a_{19}XS + a_{20}X + a_{21}S^3 + \\ & + a_{22}S^2 + a_{23}S + a_{24} \end{aligned} \quad (3.3)$$

RIGID BODY DISPLACEMENT MODES[1]

One can easily verify the following relation between therigid body translation of components $\delta_{x'}, \delta_{y'}, \delta_{z'}$ in the x',y',z',system and the assumed displacement field (6). Fig.2.

$$\begin{Bmatrix} u(x,s) \\ v(x,s) \\ w(x,s) \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \begin{Bmatrix} \delta_{x'} \\ \delta_{y'} \\ \delta_{z'} \end{Bmatrix} \quad (3.4)$$

A similar relation exists between the small amplitute rigid body rotation components $\theta_{x'}, \theta_{y'}, \theta_{z'}$, in x',y',z' system and the chosen displacement function.This relation is:

1- Relations (3.4) and (3.5) are derived in appendix I.part 2.

$$\begin{Bmatrix} u(x,s) \\ v(x,s) \\ w(x,s) \end{Bmatrix} = \begin{bmatrix} 0 & r(\cos\alpha-\cos\theta) & -r\sin\alpha \\ -r(1-\cos\alpha\cos\theta) & x\sin\alpha & x\cos\alpha \\ r\sin\alpha\cos\theta & -x\cos\alpha & x\sin\alpha \end{bmatrix} \begin{Bmatrix} \theta_{x'} \\ \theta_{y'} \\ \theta_{z'} \end{Bmatrix} \quad (3.5)$$

The terms corresponding to rigid body displacement functions, u, v, and w are respectively:

$$R_u = a_3X + a_4S + a_5 \quad (3.6a)$$

$$R_v = a_6X + a_7S + a_8 \quad (3.6b)$$

$$R_w = a_{20}X + a_{23}S + a_{24} \quad (3.6c)$$

Therefore,

$$\delta_{x'} = a_5 \quad (3.7a)$$

$$\delta_{y'} = a_8 \quad (3.7b)$$

$$\delta_{z'} = a_{24} \quad (3.7c)$$

and,

$$\theta_{x'} = \frac{\partial R}{\partial y}w = a_{23} \quad (3.8a)$$

$$\theta_{y'} = -\frac{\partial R}{\partial x}w = -a_{20} \quad (3.8b)$$

$$\theta_{z'} = \frac{\partial R}{\partial x}v = a_6 \quad (3.8c)$$

Substituting the above values into relations (3.4) and (3.5), we see that the polynomial terms in the displacement functions cannot satisfy the rigid body displacements alone. Therefore we must change the assumed displacement functions to include the terms obtained from equations (3.5) and (3.4). After moifying the new expressions are obtained:

$$u = -a_1(\mu X^2 + S^2) + 2a_2XS + a_3X + a_4S + a_5 - a_6R\sin\alpha - a_{20}R(\cos\alpha - \cos\theta) \quad (3.9a)$$

$$v = 2a_1XS - a_2(X^2 + \mu S^2) + a_6X\cos\alpha + a_7S + a_8\cos\alpha - a_{20}X\sin\alpha - a_{23}R(1 - \cos\alpha\cos\theta) - a_{24}\sin\alpha \quad (3.9b)$$

$$\begin{aligned} w = {} & a_9X^3S^3 + a_{10}X^3S^2 + a_{11}X^3S + a_{12}X^3 + \\ & + a_{13}X^2S^3 + a_{14}X^2S^2 + a_{15}X^2S + a_{16}X^2 + \\ & + a_{17}XS^3 + a_{18}XS^2 + a_{19}XS + a_{20}X\cos\alpha + \\ & + a_{21}S^3 + a_{22}S^2 + a_{23}R\sin\alpha\cos\theta + a_{24}\cos\alpha + \\ & + a_6X\sin\alpha + a_8\sin\alpha \end{aligned} \quad (3.9c)$$

The deflection parameters are:

$$\{d\}^T = [u_1, v_1, w_1, \theta_{x1}, \theta_{s1}, \theta_{z1}, u_2, v_2, w_2, \theta_{x2}, \theta_{s2}, \theta_{z2}, u_3, v_3, w_3, \theta_{x3}, \theta_{s3}, \theta_{z3}, u_4, v_4, w_4, \theta_{x4}, \theta_{s4}, \theta_{z4}] \quad (3.10)$$

Where,

$$\theta_{xi} = \frac{\partial w}{\partial s} - \frac{v}{R} \qquad (3.10a)$$

$$\theta_{si} = -\frac{\partial w}{\partial x} \qquad (3.10b)$$

$$\theta_{zi} = \frac{\partial^2 w}{\partial x \partial s} \qquad (3.10c)$$

By apropriate differentiation and substitution transformation matrix, A ,is obtained,

$$\left\{ d \right\} = \left[A \right] \left\{ a \right\} \qquad (3.10d)$$

Matrix $[A]$ is tabulated in appendix I, part 2 table 3.

The strain-displacement matrix is:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_s \\ \varepsilon_{xs} \\ \chi_x \\ \chi_s \\ \chi_{xs} \end{Bmatrix} = \begin{bmatrix} \partial/\partial x & 0 & 0 \\ 0 & \partial/\partial s & 1/r \\ \partial/\partial s & \partial/\partial x & 0 \\ 0 & 0 & -\partial^2/\partial x^2 \\ 0 & \partial/r\partial s & -\partial^2/\partial s^2 \\ 0 & 2^{*}\partial/r\partial x & -2^{*}\partial^2/\partial x\partial s \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \qquad (3.11)$$

* Factor, 2, is needed as work done by the twisting moment, numerically equal on both faces. They are not written separately.

Matrix [D] ,for elastic constants for this case is,

$$[D] = \begin{bmatrix} D_1 & D_2 & & & & \\ D_2 & D_1 & & & 0 & \\ & & D_3 & & & \\ & & & D_4 & D_5 & \\ & 0 & & D_5 & D_4 & \\ & & & & & D_6 \end{bmatrix} \qquad (3.12)$$

Where,

$D_1 = E/(1-\mu^2)$ $\qquad$ $D_4 = Et^2/12(1-\mu^2)$

$D_2 = \mu D_1$ $\qquad$ $D_5 = \mu D_4$ $\qquad$ (3.12a)

$D_3 = (1-\mu)D_1/2$ $\qquad$ $D_6 = (1-\mu)D_4/2$

Then by appropriate differentiation of the displacement functions,and triple matrix multiplication, we can obtain,

$$[G_o]^T[D][G_o] \qquad (3.13)$$

After the integration of (3.13)over the volume $\left[H_o\right]$ is obtained.$\left[H_o\right]$ is given in appendix I,part 2,table 4,explicitly.

To get the member stiffness matrix program STIFF should be used,in which the operation,

$$\left[B\right]^T \left[H_o\right]\left[B\right] . \qquad (3.14)$$

is done.

NUMERICAL EXAPLE

Making use of three way symmetry,one octant of a circular cylinder,with built in edges,is analysed.The cylinder is assumed to be loaded with a uniform internal pressure.(Fig.2a).A 2x3 mesh is used.

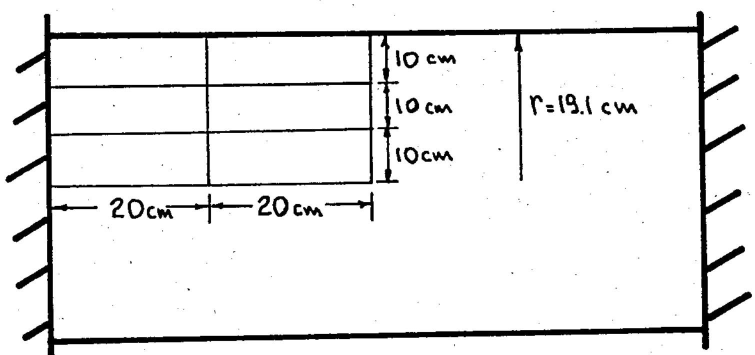


Fig.2a

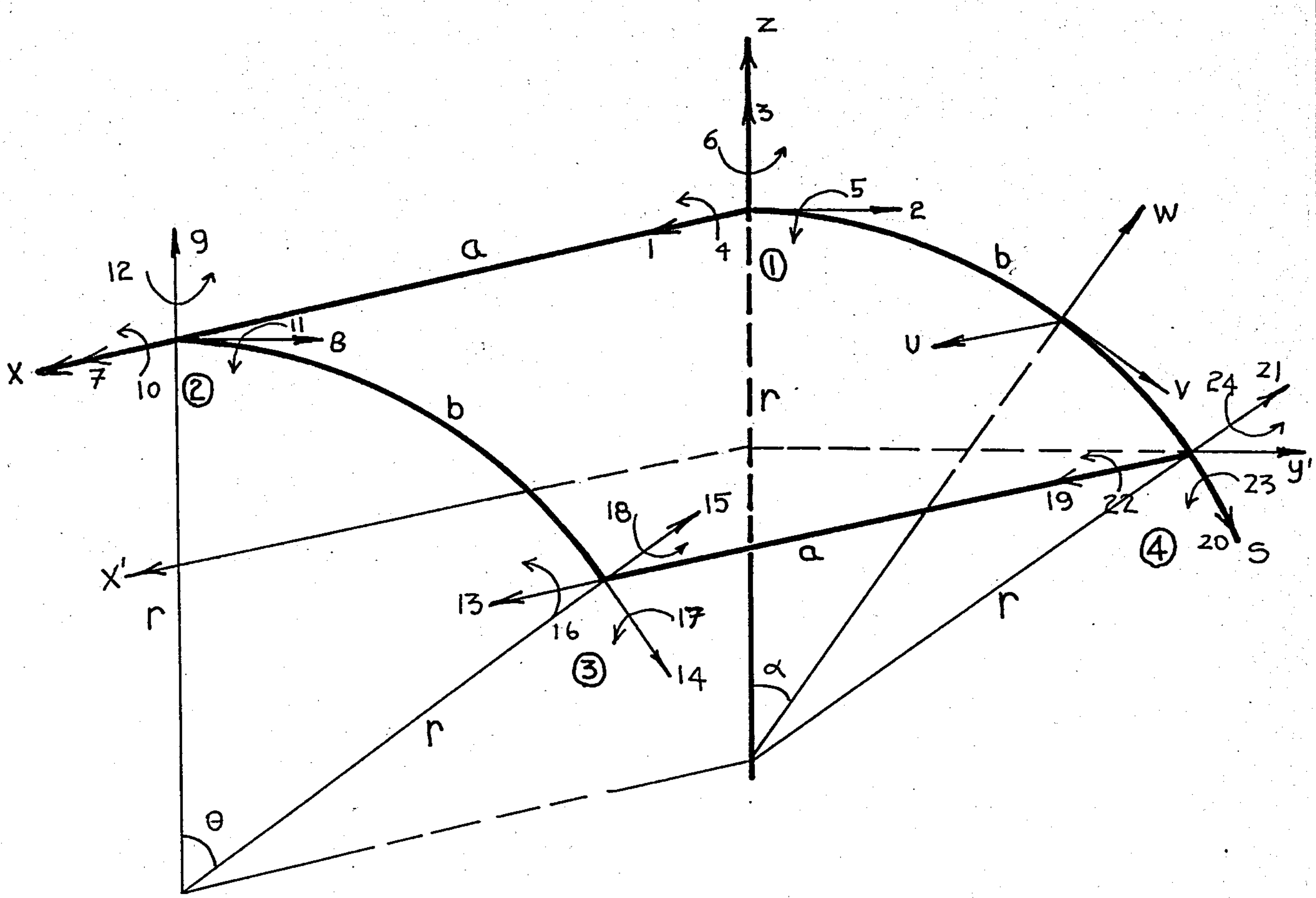
DATA:

Modulus of elasticity (E)	:	210 t/cm^2
Thickness (H)	:	0.3 cm
Poisson's ratio (VU)	:	0.3
Load	:	0.02 t/cm^2

In Timoshenko(11),the deflection at the center of the cylinder is given to be 1.16×10^{-3} cms.The result of our analysis yielded 1.0617×10^{-4} cms.for the deflection at the center. The complete set of results is given in appendix I,part 2,along with the stiffness matrix used,in numerical form.

CHAPTER 4

DERIVATION OF THE STIFFNESS MATRIX OF A 24x24, CURVED, CYLINDRICAL SHELL FINITE ELEMENT WITH DISPLACEMENT FUNCTIONS MODIFIED TO INCLUDE RIGID BODY DISPLACEMENTS AND ROTATION ABOUT VERTICAL AXIS.



The displacement functions chosen are:

$$u = a_1 + a_2X + a_3S + a_4XS + a_5S^2 + a_6S^2X \quad (4.1)$$

$$v = a_7 + a_8X + a_9S + a_{10}XS + a_{11}X^2 + a_{12}X^2S \quad (4.2)$$

$$w = a_{13} + a_{14}X + a_{15}S + a_{16}X^2 + a_{17}XS + a_{18}S^2 + a_{19}X^3 + a_{20}X^2S + a_{21}XS^2 + a_{22}S^3 + a_{23}X^3S + a_{24}XS^3 \quad (4.3)$$

To be able to invert the transformation matrix between the nodal coordinates and polynomial coefficients, A , two changes are done. The first one is the addition of the new terms S^2, S^2X in u, and X^2, X^2S in v displacement functions. These terms are found by a trial and error process (12) . The second change is done in the location of the member axis, it is moved to corner no.1 (Fig.3).

RIGID BODY DISPLACEMENT MODES

The relations (3.4) and (3.5), derived in Chapter 3, between the rigid body translations, rotations and the assumed displacement field are also valid for the element considered here. The only change is, now the angle θ is the whole angle between the two horizantal sides of the element (Fig.3).

The terms corresponding to rigid body displacement functions in u, v, and w are respectively:

$$R_u = a_1 + a_2X + a_3S \tag{4.4a}$$

$$R_v = a_7 + a_8X + a_9S \tag{4.4b}$$

$$R_w = a_{13} + a_{14}X + a_{15}S \tag{4.4c}$$

Therefore,

$$\delta_{x'} = a_1 \tag{4.5a}$$

$$\delta_{y'} = a_7 \tag{4.5b}$$

$$\delta_{z'} = a_{13} \tag{4.5c}$$

and,

$$\theta_{x'} = \frac{\partial R_w}{\partial \bar{y}} = a_{15} \tag{4.6a}$$

$$\theta_{y'} = -\frac{\partial R_w}{\partial \bar{x}} = -a_{14} \tag{4.6b}$$

$$\theta_{z'} = \frac{\partial R_v}{\partial \bar{x}} = a_8 \tag{4.6c}$$

Substituting the above obtained values into relations (3.4) and (3.5), and introducing the resulting terms into the displacement functions (4.1),(4.2) and (4.3),the displacement field becomes to be:

$$u = +a_1 + a_2X + a_3S + a_4XS + a_5S^2 + a_6S^2X - a_8R\sin\alpha - $$

$$- a_{14}R(\cos\alpha - \cos\theta) \tag{4.7a}$$

$$v = a_7\cos\alpha + a_8X\cos\alpha + a_9S + a_{10}SX + a_{11}X^2 + a_{12}X^2S$$

$$-a_{13}\sin\alpha - a_{14}X\sin\alpha - a_{15}R(1-\cos\theta\cos\alpha) \tag{4.7b}$$

$$w = a_7\sin\alpha + a_8X\sin\alpha + a_{13}\cos\alpha + a_{14}X\cos\alpha +$$

$$+ a_{15}R\sin\alpha\cos\theta + a_{16}X^2 + a_{17}SX + a_{18}S^2 + a_{19}X^3 +$$

$$+ a_{20}SX^2 + a_{21}XS^2 + a_{22}S^3 + a_{23}X^3S + a_{24}S^3X \tag{4.7c}$$

The deflection parameters used are:

$$[d]^T = [u_i, v_i, w_i, \theta_{xi}, \theta_{yi}, \theta_{zi}] \tag{4.8}$$

Where,

$$\theta_{xi} = \frac{\partial W}{\partial S} - \frac{V}{R} \tag{4.9a}$$

$$\theta_{yi} = \frac{\partial W}{\partial X} \tag{4.9b}$$

$$\theta_{zi} = \frac{1}{2}\left(\frac{\partial V}{\partial X} - \frac{\partial U}{\partial S}\right) \tag{4.9c}$$

By apropriate differentiation of displacement functions and substituting the nodal coordinates, the transformation matrix A is obtained.

$$[d] = [A]\{a\} \tag{4.10}$$

The matrix $[A]$ is tabulated in appendix I, part 3, table 5.

The matrices for strain-displacement relations and for elastic constants are the same as those given in Chapter 3, (3.11) and (3.12) respectively.

Then by apropriate differentiation of the displacement functions according to (3.11).and after triple matrix multiplication .we can obtain:

$$[G_o]^T [D][G_o] \tag{4.11}$$

After integrating (4.11) over the volume $[H_o]$ is obtained. $[H_o]$ is given in appendix I, part 3, table 6.

To get the member stiffness matrix, program STIFF should be used, in which the operation,

$$[B]^T [H_o][B] \tag{4.12}$$

is done.

NUMERICAL EXAPLE

The same problem stated in Chapter 3.Fig.2a, is solved for comparison.Using the same data and 2x3 meshbut the third cylindrical shell element the deflection at tne center of the cylinder is found to be 4.7323×10^{-4} cms.The complete set of results is given in appendix I,part 3.

CHAPTER 5.

DERIVATION OF THE STIFFNESS MATRIX OF A 15x15, TRIANGULAR, DOUBLY CURVED, SPHERICAL SHELL FINITE ELEMENT.

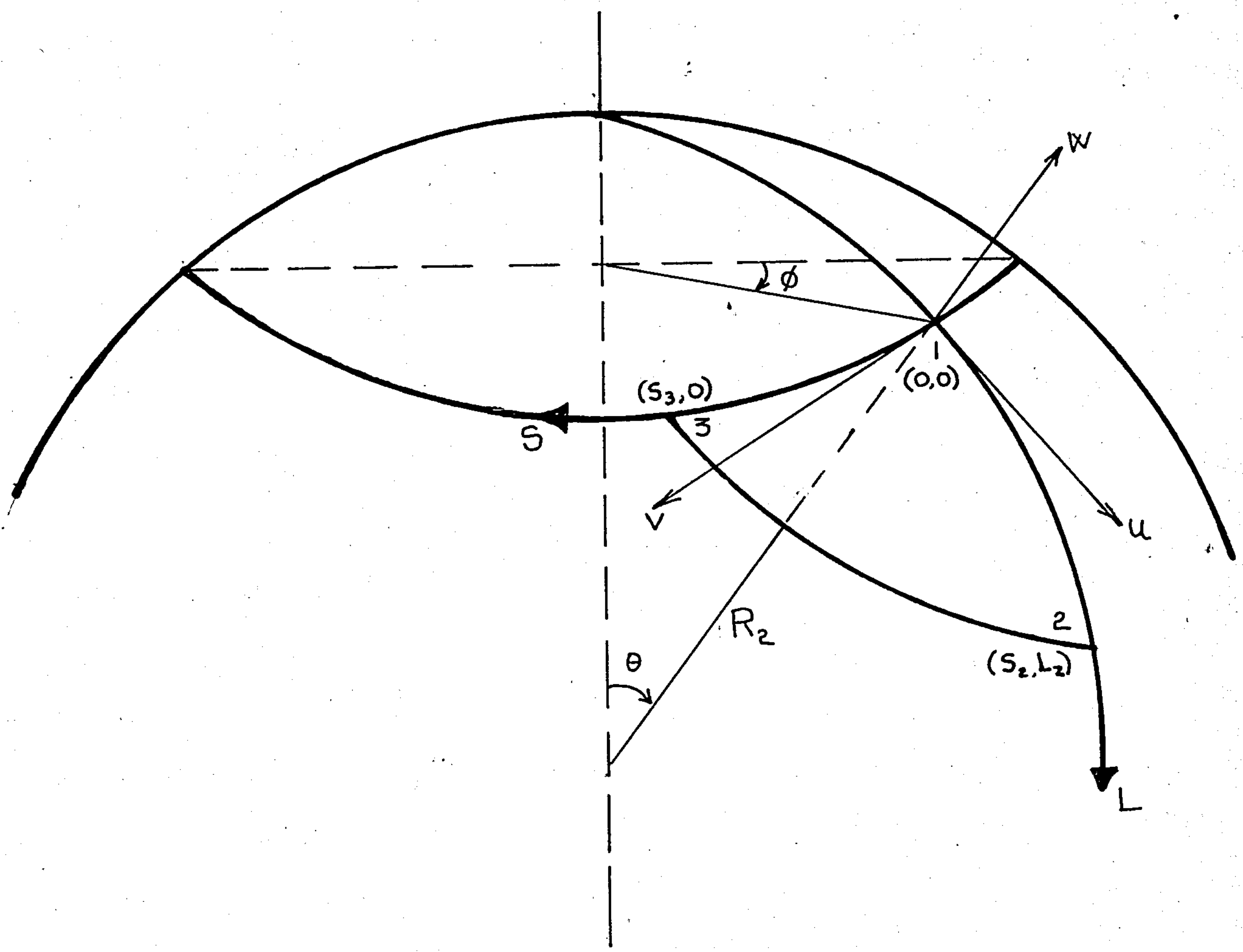


Fig. 4

Displacement functions of the element are taken from Zienkiewicz and Holister (9). They are:

$$u = a_1 + a_2S + a_3L \tag{5.1a}$$

$$v = a_4 + a_5S + a_6L \tag{5.1b}$$

$$w = a_7 + a_8S + a_9L + a_{10}S^2 + a_{11}SL + a_{12}L^2 + a_{13}S^3 + a_{14}(S^2L + SL^2) + a_{15}L^3 \tag{5.1c}$$

The deflection parameters are chosen in the following order:

$$\{d\}^T = [u_1, u_2, u_3, u_4, v_1, v_2, v_3, v_4, w_1, w_2, w_3, w_4, \theta_{L1}, \theta_{L2}, \theta_{L3}, \theta_{L4}, \theta_{s1}, \theta_{s2}, \theta_{s3}, \theta_{s4}] \tag{5.2}$$

Where,

$$\theta_{Li} = -\frac{\partial w}{\partial s} \tag{5.2a}$$

$$\theta_{si} = \frac{\partial w}{\partial L} \tag{5.2b}$$

Using (4.1a,b,c) and (4.2a,b), A matrix is constructed.

$$\{d\} = [A]\{a\} \tag{5.3}$$

$[A]$ is tabulated in appendix I, part 4, table 7.

The strain-displacement relations are taken from Novozhilov (8). They are,

$$\varepsilon_L = \partial u/\partial L + w/R_1 \tag{5.4a}$$

$$\varepsilon_S = \partial v/\partial s + u\cot\theta/R_2 + w/R_2 \tag{5.4b}$$

$$\varepsilon_{LS} = \partial u/\partial s + \partial v/\partial L - v\cot\theta/R_2 \tag{5.4c}$$

$$\mathcal{X}_L = \partial u/R_1\partial L - \partial^2 w/\partial L^2 \tag{5.4d}$$

$$\mathcal{X}_S = \frac{\partial}{\partial s}\left(\frac{\partial w}{\partial s} - \frac{v}{R_2}\right) - \frac{\cot\theta}{R_2}\left(\frac{\partial w}{\partial L} - \frac{u}{R_1}\right) \tag{5.4e}$$

$$\mathcal{X}_{LS} = -\frac{\partial}{\partial L}\left(\frac{\partial w}{\partial s} - \frac{v}{R_2}\right) + \frac{1}{R_1}\left(\frac{\partial u}{\partial s} - \frac{v\cot\theta}{R_2}\right) \tag{5.4f}$$

Matrix D for elastic constants is:

$$[D] = \begin{bmatrix} D_1 & D_{12} & & & & \\ D_{12} & D_1 & & & 0 & \\ & & D_3 & & & \\ & & & D_{44} & D_{45} & \\ & 0 & & & & \\ & & & D_{45} & D_{44} & \\ & & & & & D_{66} \end{bmatrix} \tag{5.5}$$

Where,

$$D_{11} = E/(1-\mu^2) \qquad D_{44} = Et^2/12(1-\mu^2)$$

$$D_{12} = D_{11}\mu \qquad D_{45} = \mu D_{44} \qquad (5.5a)$$

$$D_{33} = (1-\mu)D_{11}/2 \qquad D_{66} = 2(1-\mu)D_{44}$$

By simple differentiation and triple matrix multiplication,

$$\left[G_o\right]^T\left[D\right]\left[G_o\right] \qquad (5.6)$$

is obtained.The results of (4.6) is listed in appendix I, part 4, column by column.

INTEGRATION OVER THE VOLUME OF THE TRIANGLE

Due to the general shape of the triangle, to reduce the effort of integration triangular coordinates (ε , η) are used.The origin of the member axes is taken at node 1.(Fig.3),so that S_1.L_1 becomes zero.Also node 3 is assumed to lie on the S-axis,as a result L_3 is zero.It is also assumed θ is constant for an element.It should be taken as the average of the top and bottom corner θ's.

Because the results of the integration processes were long expressions, in order not to make the stiffness matrix in terms of polynomial coefficients, $[H_o]$, clumsy, back substitution is not done. In appendix I, part 4, the results of the integration processes along with the method of integration is given.

To obtain the member stiffness matrix, $[K]$, program STIFF should be used, in which,

$$[B]^T [H_o] [B] \qquad (5.7)$$

is done.

DISCUSSIONS AND CONCLUSIONS

It is obvious that analysis of cylindrical shells by curvedylindrical shell elements is more advantageous than flat-plateelement idealization.A curved finite element will represent the actual physical behavior of the shell where a flat-plate element will not,unless a very fine mesh is used in the analysis,which will increase the effort of calculation.

The 20x20 cylindrical shell finite element stiffness matrix developed herein is a rather primitive one. The element,in the first place,has 20 degrees of freedom because the effects of twisting moments are ignored This will lead to discrepancies in problems where twisting effect is present.In the second place the element will not respond to all the rigid body displacements.As may be seen from the derivation of the 24x24 cylindrical shell element stiffness matrix,the polynomial displacement functions cannot even approximate rigid body displacements.Therefore when analysing a cylindrical shell by the 20x20 cylindrical shell finite element a fine mesh should be used to get the exact elastic solutions.When a fine mesh is used the elements will almost be flat,but in any case they will yield better results than flat-plate elements.The 20x20 cylindrical shell finite element can be used when a coarse idea of the results are needed,since the computational effort will be less than that of flat-plate idealization.

The 24x24 cylindrical shell finite element derived in Chapter 3,is advantageus to use as can be seen from the result of the example . Even a coarse mesh system gives a fair idea of the exact solution.The addition of the slope continuity term into the corner displacement matrix has no physical meaning but it increases the sensitivity of the element.A static equilibrium test is conducted on the element used in solving the problem stated in Chapter 3. The results of the equilibrium test are satisfactory.The computer program for the equilibrium test along with the results are given in appendix I.part 2.It is shown in the literature,(6),that solving problems using an element which contains rigid body modes give significantly better results in coarse mesh systems than using elements not modified to include rigid body modes.

The convergence to the exact solution could not be tested because of the limited capacity of the computer used.As stated before,using program SHELL problems only up to 36 unknows can be solved.Also tests to show the advantages of inclusion of curvature in the element is not performed,but it is obvious.A curved finite element will represent the system analysed exactly.

The third 24x24 cylindrical shell finite element derived in Chapter 4,is better behaved than both of the above discussed elements.This is concluded from the comparison of the result of the problem solved using the second and the third elements.Using the second element. the deflection in the middle of the cylinder is found to be off by 90.5%,whereas using the third element the error is 59.5%, which is quite good in coarse mesh systems.

The same equilibrium test is also conducted using the third element stiffness matrix.The twisting moments around the vertical axis at each corner were included in the static equilibrium test.Relative to the magnitute of the numbers in the stiffness matrix,the result of the equilibrium test is exceptionally good.The results are given in appendix I,part 3.

The convergence to the exact solution could not be tested because of the capacity of the computer used.

Up to now the analysis of axi-symmetrically curved shells were done by idealizing the shells as a series of conical frustra joined at nodal circles or flat-plate elements (10).This idealization will not represent the actual behavior of the system.The elements used have single or no curvature at all.The triangular, spherical shell finite element whose stiffness matrix is constructed here,represents the actual physical shape of the system to be analysed.In the coarse mesh system it would give better results.Unfortunately no testing was conducted using spherical shell element stiffness matrix.The stiffness matrix contains long expressions. Actually the element derived herein is not the ideal element to be used in the analysis of axi-symmetric shells. It lacks the twisting moments.Also the displacement functions are not modified to include all the rigid body displacement modes of the element.

As a result it is concluded that the third cylindrical shell element is far better behaved than both the second and the first elements.The third 24x24 cylindrical shell element is an effective element to be used in the analysis of arbitrary cylindrical shells.

The 15x15 spherical shell triangular element is not the ideal elment to analize an axi-symmetric shell but it can be used to get a coarse idea of the exact results.

APPENDIX I

PART 1

TABLE 1. TRANSFORMATION MATRIX BETWEEN POLYNOMIAL COEFFICIENTS AND NODAL COORDINATES. (2.2d)

(Left hand side)

$$
[A]=\begin{bmatrix}
1 & a & b & c^* & 2ab & 0 & 0 & 0 & 0 & 0 \\
1 & -a & b & c & -2ab & 0 & 0 & 0 & 0 & 0 \\
1 & a & -b & c & -2ab & 0 & 0 & 0 & 0 & 0 \\
1 & -a & -b & c & 2ab & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2ab & d^* & 1 & a & b & 0 & 0 \\
0 & 0 & 0 & -2ab & d & 1 & -a & b & 0 & 0 \\
0 & 0 & 0 & -2ab & d & 1 & a & -b & 0 & 0 \\
0 & 0 & 0 & 2ab & d & 1 & -a & -b & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & a \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -a \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & a \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -a \\
0 & 0 & 0 & 2ab/r & d/r & 1/r & a/r & b/r & 0 & 0 \\
0 & 0 & 0 & -2ab/r & d/r & 1/r & -a/r & b/r & 0 & 0 \\
0 & 0 & 0 & -2ab/r & d/r & 1/r & a/r & -b/r & 0 & 0 \\
0 & 0 & 0 & 2ab/r & d/r & 1/r & -a/r & -b/r & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1
\end{bmatrix}
$$

* $c = -(\mu a^2 + b^2)$ $\qquad$ $d = -(a^2 + \mu b^2)$

TABLE 1. CONTINUED.
(Right hand side)

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
b	a^2	ab	b^2	a^3	a^2b	ab^2	b^3	a^3b	ab^3
b	a^2	$-ab$	b^2	$-a^3$	a^2b	$-ab^2$	b^3	$-a^3b$	$-ab^3$
$-b$	a^2	$-ab$	b^2	a^3	$-a^2b$	ab^2	$-b^3$	$-a^3b$	$-ab^3$
$-b$	a^2	ab	b^2	$-a^3$	$-a^2b$	$-ab^2$	$-b^3$	a^3b	ab^3
1	0	a	$2b$	0	a^2	$2ab$	$3b^2$	a^3	$3ab^2$
1	0	$-a$	$2b$	0	a^2	$-2ab$	$3b^2$	$-a^3$	$-3ab^2$
1	0	a	$-2b$	0	a^2	$2ab$	$3b^2$	a^3	$3ab^2$
1	0	$-a$	$-2b$	0	a^2	$-2ab$	$3b^2$	$-a^3$	$-3ab^2$
0	$-2a$	$-b$	0	$-3a^2$	$-2ab$	$-b^2$	0	$-3a^2b$	$-b^3$
0	$2a$	$-b$	0	$-3a^2$	$2ab$	$-b^2$	0	$-3a^2b$	$-b^3$
0	$-2a$	b	0	$-3a^2$	$2ab$	$-b^2$	0	$3a^2b$	b^3
0	$2a$	b	0	$-3a^2$	$-2ab$	$-b^2$	0	$3a^2b$	b^3

PART 1.

TABLE 2. STIFFNESS MATRIX IN TERMS OF POLYNOMIAL COEFFICIENTS.

0																			
0	2,2																		
0	0	3,3																	
0	0	0	4.4																
0	0	0	0	5,5															
0	0	0	0	0	0									Symmetrical					
0	0	7,3	0	0	0	7,7													
0	8,2	0	0	0	0	0	8,8												
0	9,2	0	0	0	0	0	9,8	9,9											
0	0	0	10.4	0	0	0	0	0	10,10										
0	0	0	0	0	0	0	0	0	0	11,11									
0	12,2	0	0	0	0	0	12,8	12,9	0	0	12,12								
0	0	0	0	0	0	13,7	0	0	0	0	0	13,13							
0	14,2	0	0	0	0	0	14,8	14,9	0	0	14.12	0	14.14						
0	0	0	15,4	0	0	0	0	0	15,10	0	0	0	0	15,15					
0	0	0	0	16,5	0	0	0	0	0	16,11	0	0	0	0	16,16				
0	0	0	17,4	0	0	0	0	0	17,10	0	0	0	0	17,15	0	17,17			
0	0	0	0	18.5	0	0	0	0	0	18,11	0	0	0	0	18,16	0	18,18		
0	0	0	0	0	0	19,7	0	0	0	0	0	19,13	0	0	0	0	0	19,19	
0	0	0	0	0	0	20,7	0	0	0	0	0	20,13	0	0	0	0	0	20,19	20,20

Note: Each term should be multiplied by the thickness of the element.

TABLE 2. CONTINUED

$k(2,2) = 4abD_1$

$k(8,2) = 4abD_2$

$k(9,2) = -4abD_2$

$k(12,2) = -4a^3bD_2/3r$

$k(14,2) = -4b^3aD_2/3r$

$k(3,3) = 4abD_3$

$k(7,3) = 4abD_3$

$k(4,4) = 16ba^3(1-\mu^2)D_1/3 + 16ba^3D_4/3r^2 + 16b^3aD_6/3r^2$

$k(10,4) = -8a^3b(1-\mu^2)D_1/3r$

$k(15,4) = -8a^5b(1-\mu^2)D_1/3r + 16ba^3D_5/r$

$k(17,4) = -8a^3b^3(1-\mu^2)D_1/9r + 16ba^3D_4/3r + 16b^3aD_6/3r$

$k(5,5) = 16b^3a(1-\mu^2)D_1/3 + 16b^3a\mu^2D_4/3r^2 + 16b^3aD_6/3r^2$

$k(16,5) = -16b^3a\mu D_5/3r - 16a^3bD_6/3r$

$k(18,5) = -16b^3a\mu D_4/r$

$k(7,7) = 4abD_3 + 4abD_6/r^2$

$k(13,7) = 4abD_6/r$

$k(19,7) = 4ba^3D_6/r$

$k(20,7) = 4b^3aD_6/r$

$k(8,8) = 4abD_1 + 4abD_4/r^2$

$k(9,8) = -4abD_1/r$

$k(12,8) = -4ba^3D_1/3r + 8abD_5/r$

$k(14,8) = -4b^3aD_1/3r + 8abD_4/r$

$k(9,9) = 4abD_1/r^2$

$k(12,9) = 4a^3bD_1/3r^2$

$k(14,9) = 4ab^3D_1/3r^2$

$k(10,10) = 4a^3bD_1/3r^2$

$k(15,10) = 4a^5bD_1/5r^2$

$k(17,10) = 4a^3b^3D_1/9r^2$

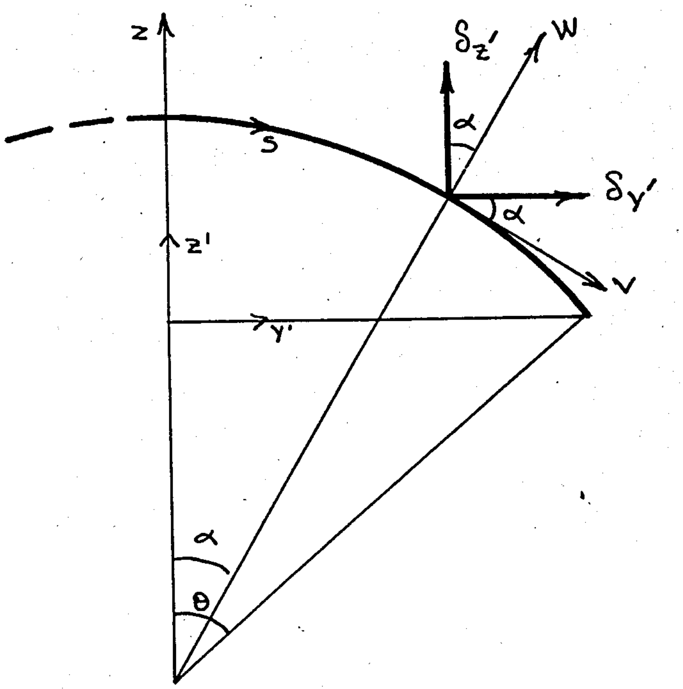
TABLE 2. CONTINUED

$k(11,11) = 4b^3aD_1/3r^2$
$k(16,11) = 4a^3b^3D_1/5r^2$
$k(12,12) = 4a^5bD_1/5r^2 + 16abD_4$
$k(14,12) = 4a^3b^3D_1/9r^2 + 16abD_5$
$k(13,13) = 4a^3b^3D_1/9r^2 + 4abD_6$
$k(19,13) = 4a^5b^3D_1/15r^2 + 4a^3bD_6$
$k(20,13) = 4b^5a^3D_1/15r^2 + 4b^3aD_6$
$k(14,14) = 4b^5aD_1/5r^2 + 16abD_6$
$k(15,15) = 4a^7bD_1/7r^2 + 48a^3bD_4$
$k(17,15) = 4a^5b^3D_1/15r^2 + 16a^3bD_5$
$k(16,16) = 4a^5b^3D_1/15r^2 + 16b^3aD_4/3 + 16a^3bD_6/3$
$k(18,16) = 4a^3b^5D_1/15r^2 + 16ab^3D_5$
$k(17,17) = 4b^5a^3D_1/15r^2 + 16a^3bD_4/3 + 16b^3aD_6/3$
$k(18,18) = 4b^7aD_1/7r^2 + 48b^3aD_4$
$k(19,19) = 4a^7b^3D_1/21r^2 + 16a^3b^3D_4 + 36a^5bD_6/5$
$k(20,19) = 4a^5b^5D_1/25r^2 + 16b^3a^3D_5 + 4a^3b^3D_6$
$k(20,20) = 4b^7a^3D_1/21r^2 + 16a^3b^3D_4 + 36b^7aD_6/5$

PART 2.

DERIVATION OF THE EFFECT OF RIGID BODY TRANSLATIONS AND ROTATIONS ON THE DISPLACEMENT FIELD.

$\delta_{y'}, \delta_{z'}$



$$u = \delta_{x'}$$

$$V = \cos\alpha\, \delta_{y'} - \sin\alpha\, \delta_{z'}$$

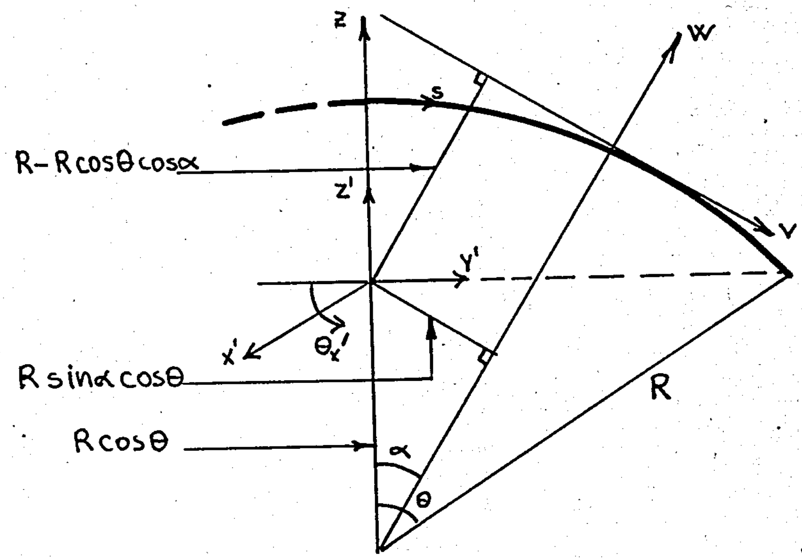
$$W = \sin\alpha\, \delta_{y'} + \cos\alpha\, \delta_{z'}$$

$\theta_{x'}$

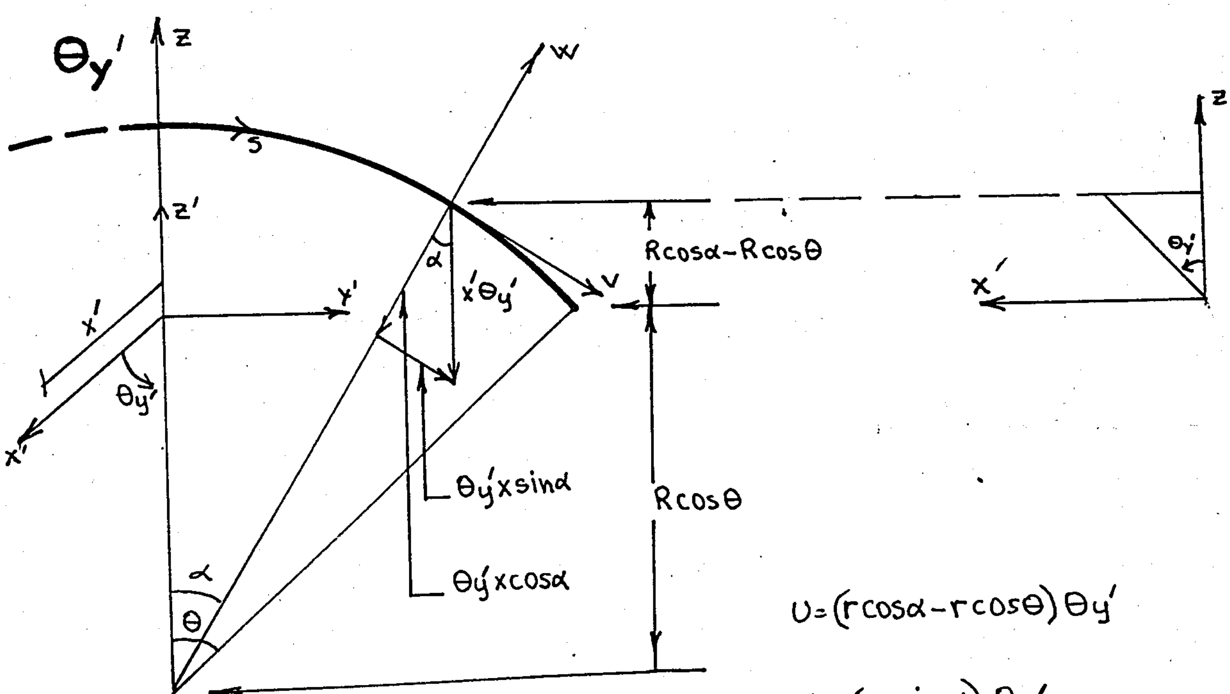
$u = 0$

$v = -r(1-\cos\alpha\cos\theta)\cdot\theta_{x'}$

$w = (r\sin\alpha\cos\theta)\theta_{x'}$



$\theta_{y'}$

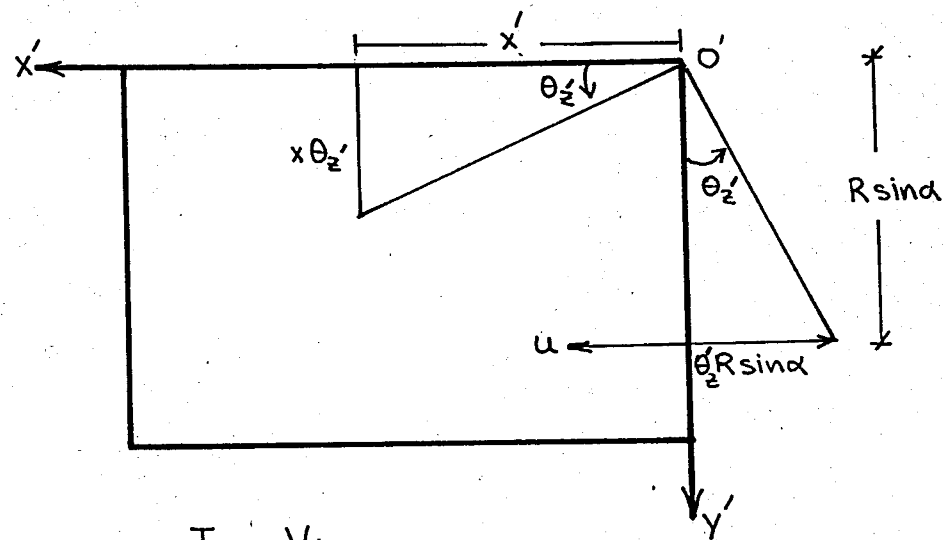


$u = (r\cos\alpha - r\cos\theta)\theta_{y'}$

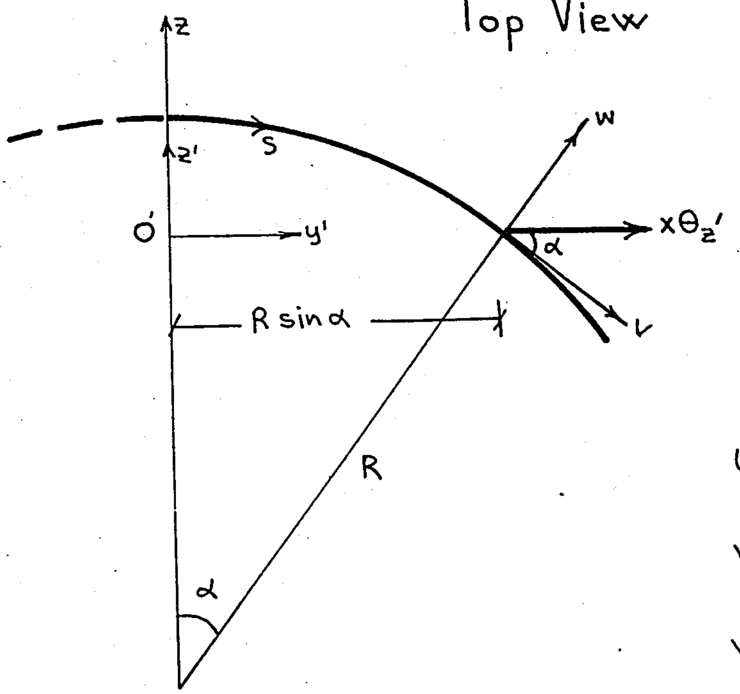
$v = (x\sin\alpha)\theta_{y'}$

$w = -(x\cos\alpha)\theta_{y'}$

θ_z'



Top View



$$u = (-r\sin\alpha)\,\theta_z'$$

$$v = (x\cos\alpha)\,\theta_z'$$

$$w = (x\sin\alpha)\,\theta_z'$$

PART 2.

TABLE 3. TRANSFORMATION MATRIX BETWEEN POLYNOMIAL COEFFICIENTS AND NODAL COORDINATES. (3.10d)
(Left hand side)

$$
[A]=\begin{bmatrix}
e^* & 2ab & -a & -b & 1 & rS^* & 0 & 0 & 0 & 0 & 0 & 0 \\
2ab & f^* & 0 & 0 & 0 & -aC^* & -b & C & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & aS & 0 & -S & a^3b^3 & -a^3b^2 & a^3b & -a^3 \\
-2ab/r & -f/r & 0 & 0 & 0 & 0 & b/r & 0 & -3a^3b^2 & 2a^3b & -a^3 & 0 \\
0 & 0 & 0 & 0 & 0 & S & 0 & 0 & 3a^2b^3 & -3a^2b^2 & 3a^2b & -3a^2 \\
0 & 0 & 0 & 0 & 0 & C/r & 0 & 0 & 9a^2b^2 & -6a^2b & 3a^2 & 0 \\
e & -2ab & a & -b & 1 & -rS & 0 & 0 & 0 & 0 & 0 & 0 \\
-2ab & f & 0 & 0 & 0 & -aC & b & C & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -aS & 0 & S & -a^3b^3 & -a^3b^2 & -a^3b & -a^3 \\
2ab/r & -f/r & 0 & 0 & 0 & 0 & -b/r & 0 & -3a^3b^2 & -2a^3b & -a^3 & 0 \\
0 & 0 & 0 & 0 & 0 & -S & 0 & 0 & -3a^2b^3 & -3a^2b^2 & -3a^2b & -3a^2 \\
0 & 0 & 0 & 0 & 0 & C/r & 0 & 0 & 9a^2b^2 & 6a^2b & 3a^2 & 0 \\
e & 2ab & -a & b & 1 & -rS & 0 & 0 & 0 & 0 & 0 & 0 \\
2ab & f & 0 & 0 & 0 & aC & b & C & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & aS & 0 & S & a^3b^3 & a^3b^2 & a^3b & a^3 \\
-2ab/r & -f/r & 0 & 0 & 0 & 0 & -b/r & 0 & 3a^3b^2 & 2a^3b & a^3 & 0 \\
0 & 0 & 0 & 0 & 0 & -S & 0 & 0 & -3a^2b^3 & -3a^2b^2 & -3a^2b & -3a^2 \\
0 & 0 & 0 & 0 & 0 & C/r & 0 & 0 & 9a^2b^2 & 6a^2b & 3a^2 & 0 \\
e & -2ab & a & b & 1 & -rS & 0 & 0 & 0 & 0 & 0 & 0 \\
-2ab & f & 0 & 0 & 0 & -aC & b & C & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -aS & 0 & S & -a^3b^3 & -a^3b^2 & -a^3b & -a^3 \\
2ab/r & f/r & 0 & 0 & 0 & 0 & -b/r & 0 & -3a^3b^2 & -2a^3b & -a^3 & 0 \\
0 & 0 & 0 & 0 & 0 & -S & 0 & 0 & -3a^2b^3 & -3a^2b^2 & -3a^2b & -3a^2 \\
0 & 0 & 0 & 0 & 0 & C/r & 0 & 0 & 9ab^2 & 6a^2b & 3a^2 & 0
\end{bmatrix}
$$

$e = -(\mu a^2 + b^2)$ $\qquad S = \sin\theta$

$f = -(a^2 + \mu b^2)$ $\qquad C = \cos\theta$

TABLE 3. CONTINUED.
(Right hand side)

0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	$-as$	0	0	$-rs^2$	s
$-a^2b^3$	a^2b^2	$-a^2b$	a^2	ab^3	$-ab^2$	ab	$-ac$	$-b^3$	b^2	$-rcs$	c
$3a^2b^2$	$-2a^2b$	a^2	0	$-3ab^2$	$2ab$	$-a$	0	$3b^2$	$-2b$	1	0
$-2ab^3$	$2ab^2$	$-2ab$	$2a$	b^3	$-b^2$	b	$-c$	0	0	0	0
$-6ab^2$	$4ab$	$-2a$	0	$3b^2$	$-2b$	1	s/r	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	as	0	0	$-rs^2$	$-s$
a^2b^3	a^2b^2	a^2b	a^2	$-ab^3$	$-ab^2$	$-ab$	$-ac$	b^3	b^2	rsc	c
$3a^2b^2$	$2a^2b$	a^2	0	$-3ab^2$	$-2ab$	$-a$	0	$3b^2$	$2b$	1	0
$2ab^3$	$2ab^2$	$2ab$	$2a$	$-b^3$	$-b^2$	$-b$	$-c$	0	0	0	0
$-6ab^2$	$-4ab$	$-2a$	0	$3b^2$	$2b$	1	s/r	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	$-as$	0	0	$-rs^2$	$-s$
a^2b^3	a^2b^2	a^2b	a^2	ab^3	ab^2	ab	ac	b^3	b^2	rsc	c
$3a^2b^2$	$2a^2b$	a^2	0	$3ab^2$	$2ab$	a	0	$3b^2$	$2b$	1	0
$-2ab^3$	$-2ab^2$	$-2ab$	$-2a$	$-b^3$	$-b^2$	$-b$	$-c$	0	0	0	0
$6ab^2$	$4ab$	$2a$	0	$3b^2$	$2b$	1	$-s/r$	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	as	0	0	$-rs^2$	$-s$
a^2b^3	a^2b^2	a^2b	a^2	$-ab^3$	$-ab^2$	$-ab$	$-ac$	b^3	b^2	rsc	c
$3a^2b^2$	$2a^2b$	a^2	0	$-3ab^2$	$-2ab$	$-a$	0	$3b^2$	$2b$	1	0
$2ab^3$	$2ab^2$	$2ab$	$2a$	$-b^3$	$-b^2$	$-b$	$-c$	0	0	0	0
$-6ab^2$	$-4ab$	$-2a$	0	$3b^2$	$2b$	1	$-s/r$	0	0	0	0

PART 2.

TABLE 4. STIFFNESS MATRIX IN TERMS OF POLYNOMIAL COEFFICIENTS. (3.14)

$[H_0]$ = (symmetrical)

1,1																							
0	2,2																						
0	0	3,3																					
0	0	0	4,4																				
0	0	0	0	0																			
0	0	0	0	0	0																		
0	0	7,3	0	0	0	7,7																	
0	0	0	0	0	0	0	0																
0	0	0	0	0	0	0	0	9,9															
10,1	0	0	0	0	0	0	0	0	10,10														
0	0	0	0	0	0	0	0	11,9	0	11,11													
12,1	0	0	0	0	0	0	0	0	12,10	0	12,12												
0	13,2	0	0	0	0	0	0	0	0	0	0	13,13											
0	0	14,3	0	0	0	14,7	0	0	0	0	0	0	14,14										
0	15,2	0	0	0	0	0	0	0	0	0	0	15,13	0	15,15									
0	0	16,3	0	0	0	16,7	0	0	0	0	0	0	16,14	0	16,16								
0	0	0	0	0	0	0	0	17,9	0	17,11	0	0	0	0	0	17,17							
18,1	0	0	0	0	0	0	0	0	18,10	0	18,12	0	0	0	0	0	18,18						
0	0	0	0	0	0	0	0	19,9	0	19,11	0	0	0	0	0	19,17	0	19,19					
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
0	21,2	0	0	0	0	0	0	0	0	0	0	21,13	0	21,15	0	0	0	0	0	21,21			
0	0	22,3	0	0	0	22,7	0	0	0	0	0	0	22,14	0	22,16	0	0	0	0	0	22,22		
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Note: Each term should be multiplied by the thickness of the element.

TABLE 4. CONTINUED.

$k(1,1) = 16ba^3(1-\mu^2)D_1/3 + 16ba^3D_4/3r^2 + 64b^3aD_6/3r^2$

$k(10,1) = 8b^3a^5(1-\mu^2)D_1/15r - 16a^3b^3D_5/3r - 16a^5bD_4/5r - 64a^3b^3D_6/3r$

$k(12,1) = 8a^5b(1-\mu^2)D_1/5r - 16a^3bD_5/3r$

$k(18,1) = 8a^3b^3(1-\mu^2)D_1/9r - 16a^3bD_4/3r - 64b^3aD_6/3r$

$k(2,2) = 16b^3a(1-\mu^2)D_1/3 + 16\nu^2ab^3D_4/3r^2 + 64ba^3D_6/3r^2$

$k(13,2) = 16\nu b^5a(D_5/5r + 16\mu a^3b^3D_4/3r + 64a^3b^3D_6/3r$

$k(15,2) = 16b^3a\mu D_5/3r + 64ba^3D_6/3$

$k(21,2) = 16\mu b^3aD_4/r$

$k(3,3) = 4abD_1$

$k(7,3) = 4abD_2$

$k(14,3) = 4a^3b^3D_2/9r$

$k(16,3) = 4ba^3D_2/3r$

$k(22,3) = 4b^3aD_2/3r$

$k(4,4) = 4abD_3$

$k(7,7) = 4abD_1 + 4abD_4/r^2$

$k(14,7) = 4a^3b^3D_1/9r - 8b^3aD_5/3r - 8a^3bD_4/3r$

$k(16,7) = 4ba^3D_1/3r - 8abD_5/r$

$k(22,7) = 4b^3aD_1/3r - 8abD_4/r$

$k(9,9) = 4a^7b^7D_1/49r^2 + 48a^3b^7D_4/7 + 288a^5b^5D_5/25 + 144a^7b^3D_4/21$
$\quad + 1296a^5b^5D_6/25$

$k(11,9) = 4a^7b^5D_1/35r^2 + 48a^3b^5D_4/5 + 48a^5b^3D_5/5 + 432a^5b^3D_6/15$

$k(17,9) = 4a^5b^7D_1/35r^2 + 48a^5b^3D_4/5 + 48b^5a^3D_5/5 + 432a^3b^5D_6/15$

$k(19,9) = 4a^5b^5D_1/25r^2 + 16a^3b^3D_6$

$k(10,10) = 4a^7b^5D_1/35r^2 + 48b^5a^3D_4/5 + 16a^7bD_4/7 + 96a^5b^3D_5/15 + 576b^3a^5D_6/15$

$k(12,10) = 4a^7b^3D_1/21r^2 + 16a^3b^3D_4 + 48a^5bD_5/5$

$k(18,10) = 4a^5b^5D_1/25r^2 + 16a^3b^3D_5/3 + 16a^5bD_4/5 + 192a^3b^3D_6/9$

$k(11,11) = 4a^7b^3D_1/21r^2 + 16a^3b^3D_4 + 144a^5bD_6/5$

$k(17,11) = 4a^5b^5D_1/25r^2 + 16a^3b^3D_5 + 16a^3b^3D_6$

TABLE 4. CONTINUED.

$k(19,11) = 4b^3a^5D_1/15r^2 + 16ba^3D_6$

$k(12,12) = 4a^7bD_1/7r^2 + 48a^3bD_4$

$k(18,12) = 4a^5b^3D_1/15r^2 + 16a^3bD_5$

$k(13,13) = 4a^5b^7D_1/35r^2 + 16b^7aD_4/7 + 48b^3a^5D_4/5 + 96b^5a^3D_5/15$
$\quad + 576a^3b^5D_6/15$

$k(15,13) = 4a^5b^5D_1/25r^2 + 16b^5aD_4/5 + 16a^3b^3D_5/3 + 64a^3b^3D_6/3$

$k(21,13) = 4a^3b^7D_1/21r^2 + 48b^5aD_5/5 + 16a^3b^3D_4$

$k(14,14) = 4a^5b^5D_1/25r^2 + 16b^5aD_4/5 + 16a^5bD_4/5 + 32a^3b^3D_5/9 + 256a^3b^3D_6/9$

$k(16,14) = 4a^5b^3D_1/15r^2 + 16b^3aD_4/3 + 16a^3bD_5/3$

$k(22,14) = 4a^3b^5D_1/15r^2 + 16b^3aD_5/3 + 16a^3bD_4/3$

$k(15,15) = 4a^5b^3D_1/15r^2 + 16ab^3D_4/3 + 64a^3bD_6/3$

$k(21,15) = 4a^3b^5D_1/15r^2 + 16b^3aD_5$

$k(16,16) = 4a^5bD_1/5r^2 + 16abD_4$

$k(22,16) = 4a^3b^3D_1/9r^2 + 16abD_5$

$k(17,17) = 4b^7a^3D_1/21r^2 + 16a^3b^3D_4 + 144ab^5D_6/5$

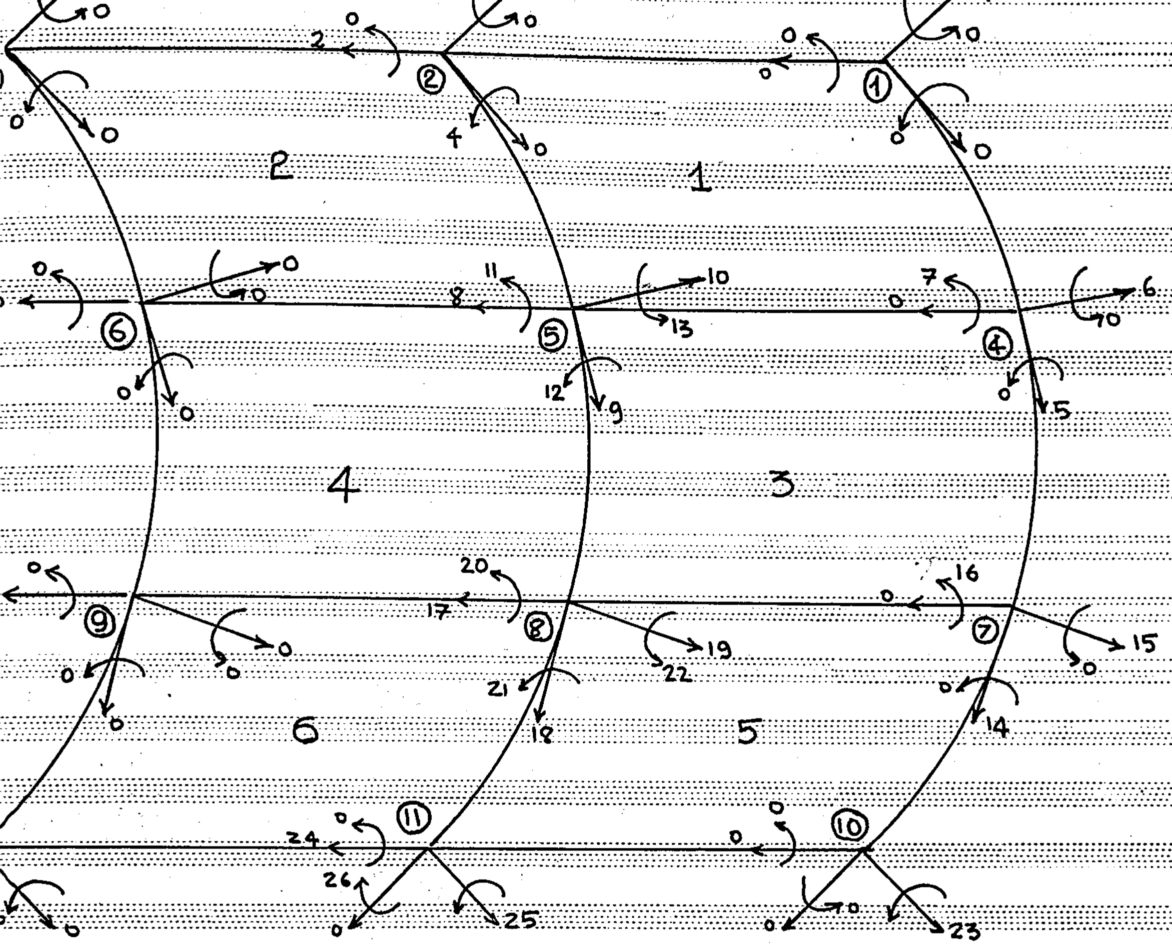
$k(19,17) = 4a^3b^5D_1/15r^2 + 16b^3aD_6$

$k(18,18) = 4a^3b^5D_1/15r^2 + 16a^3bD_4/3 + 64b^3aD_6/3$

$k(19,19) = 4a^3b^3D_1/9r^2 + 16abD_6$

$k(21,21) = 4b^7aD_1/7r^2 + 48b^3aD_4$

$k(22,22) = 4ab^5D_1/5r^2 + 16abD_4$



NDRICAL SHELL ANALYSIS BY FINITE ELEMENTS
210.

EET= 2.

SSON,S RATIO= .30
ESS= 0.3
5.00
NTAL SIDE= 10.00
19.1
=0. (ALL UNITS KIP-FOOT,EXCEPT FEED A AND I IN INCHES)
=1. (ALL UNITS KIP-FOOT)
=2. (METRIC UNITS HOMOGENEOUS THROUGHOUT)

NUMBER OF MEMBERS = 6
NUMBER OF JOINTS = 12
NUMBER OF UNKNOWNS= 26

MEMBER STIFFNES; MATRIX

234.27	112.84	-13.19	-20.64	37.55	-68.74
112.84	465.54	-105.49	-133.71	303.00	-384.18
-13.19	-105.49	95.24	310.17	-298.18	904.24
-20.64	-133.71	310.17	1757.29	-1074.74	5158.27
37.55	303.00	-298.18	-1074.74	1663.04	-4399.65
-68.74	-384.18	904.24	5158.27	-4399.65	22182.11
8.02	11.15	13.19	20.64	-37.55	68.74

11.15	199.84	-55.63	-78.30	227.17	-315.13
-13.19	-55.63	11.93	74.18	-85.09	405.27
-20.64	-78.30	74.18	482.30	-416.84	2580.27
-37.55	-227.17	85.09	416.84	-575.59	2254.45
68.74	315.13	-405.27	-2580.27	2254.45	-13534.53
-181.10	-112.63	13.98	24.67	-53.06	82.31
-112.63	-262.46	57.79	114.11	-261.13	510.03
-13.98	-57.79	10.51	-46.75	4.28	-288.64
24.67	114.11	46.75	249.91	-303.97	1222.87
-53.06	-261.13	-4.28	-303.97	299.43	-1712.24
-82.31	-510.03	-288.64	-1222.87	1712.24	-5627.56
-61.20	-11.36	-13.98	-24.67	53.06	-82.31
11.36	-394.35	135.32	261.67	-389.64	750.31
-13.98	-135.32	-22.77	-180.10	62.57	-495.82
24.67	261.67	180.10	638.37	-507.99	1754.48
53.06	389.64	62.57	507.99	-413.89	1892.03
82.31	750.31	495.82	1754.48	-1892.02	5447.90

8.02	-11.15	-13.19	-20.64	-37.55	68.74
11.15	199.84	-55.63	-78.30	-227.17	315.13
13.19	-55.63	11.93	74.18	85.09	-405.27
20.64	-78.30	74.18	482.30	416.84	-2580.27
-37.55	227.17	-85.09	-416.84	-575.59	2254.45
68.74	-315.13	405.27	2580.27	2254.45	-13534.53
234.27	-112.84	13.19	20.64	37.55	-68.74
-112.84	465.54	-105.49	-133.71	-303.00	384.18
13.19	-105.49	95.24	310.17	298.18	-904.24
20.64	-133.71	310.17	1757.29	1074.74	-5158.27
37.55	-303.00	298.18	1074.74	1663.04	-4399.65
-68.74	384.18	-904.24	-5158.27	-4399.65	22182.12
-61.20	11.36	13.98	24.67	53.06	-82.31
-11.36	-394.35	135.32	261.67	389.64	-750.31
13.98	-135.32	-22.77	-180.10	-62.57	495.82
-24.67	261.67	180.10	638.37	507.99	-1754.48
53.06	-389.64	-62.57	-507.99	-413.89	1892.02
82.31	-750.31	-495.82	-1754.48	-1892.02	5447.90
-181.10	112.63	-13.98	-24.67	-53.06	82.31
112.63	-262.46	57.79	114.11	261.13	-510.03
13.98	-57.79	10.51	-46.75	-4.28	288.64
-24.67	114.11	46.75	249.91	303.97	-1222.87
-53.06	261.13	4.28	303.97	299.43	-1712.24
-82.31	510.03	288.64	1222.87	1712.24	-5627.56

-181.10	-112.63	-13.98	24.67	-53.06	-82.31
112.63	-262.46	-57.79	114.11	-261.13	-510.03
13.98	57.79	10.51	46.75	-4.28	-288.64
24.67	114.11	-46.75	249.91	-303.97	-1222.87
-53.06	-261.13	4.28	-303.97	299.43	1712.24
82.31	510.03	-288.64	1222.87	-1712.24	-5627.56
-61.20	-11.36	13.98	-24.67	53.06	82.31
11.36	-394.35	-135.32	261.67	-389.64	-750.31
13.98	135.32	-22.77	180.10	-62.57	-495.82
24.67	261.67	-180.10	638.37	-507.99	-1754.48
53.06	389.64	-62.57	507.99	-413.89	-1892.02
-82.31	-750.31	495.82	-1754.48	1892.03	5447.90
234.27	112.84	13.19	-20.64	37.55	68.74
112.84	465.54	105.49	-133.71	303.00	384.18

-20.64	-133.71	95.24	-310.17	298.18	904.24
37.55	303.00	-310.17	1757.29	-1074.74	-5158.27
68.74	384.18	298.18	-1074.74	1663.04	4399.65
8.02	11.15	904.24	-5158.27	4399.65	22182.12
-11.15	199.84	-13.19	20.64	-37.55	-68.74
13.19	55.63	55.63	-78.30	227.17	315.13
-20.64	-78.30	11.93	-74.18	85.09	405.27
-37.55	-227.17	-74.18	482.30	-416.84	-2580.27
-68.74	-315.13	-85.09	416.84	-575.59	-2254.45
		-405.27	2580.27	-2254.45	-13534.53

-61.20	11.36	-13.98	24.67	53.06	82.31
-11.36	-394.35	-135.32	261.67	389.64	750.31
-13.98	135.32	-22.77	180.10	62.57	495.82
-24.67	261.67	-180.10	638.37	507.99	1754.48
53.06	-389.64	62.57	-507.99	-413.89	-1892.02
-82.31	750.31	-495.82	1754.48	1892.03	5447.90
-181.10	112.63	13.98	-24.67	-53.06	-82.31
112.63	-262.46	-57.79	114.11	261.13	510.03
-13.98	57.79	10.51	46.75	4.28	288.64
-24.67	114.11	-46.75	249.91	303.97	1222.87
-53.06	261.13	-4.28	303.97	299.43	1712.24
82.31	-510.03	288.64	-1222.87	-1712.25	-5627.56
8.02	-11.15	13.19	-20.64	-37.55	-68.74
11.15	199.84	55.63	-78.30	-227.17	-315.13
-13.19	55.63	11.93	-74.18	-85.09	-405.27
20.64	-78.30	-74.18	482.30	416.84	2580.27
-37.55	227.17	85.09	-416.84	-575.59	-2254.45
-68.74	315.13	405.27	-2580.27	-2254.45	-13534.53
234.27	-112.84	-13.19	20.64	37.55	68.74
-112.84	465.54	105.49	-133.71	-303.00	-384.18
-13.19	105.49	95.24	-310.17	-298.18	-904.24
20.64	-133.71	-310.17	1757.29	1074.74	5158.27
37.55	-303.00	-298.18	1074.74	1663.04	4399.65
68.74	-384.18	-904.24	5158.27	4399.65	22182.12

```
                CODE NUMBERS
0.              FIRST JO.SECOND JO.THIRD JO.FOURTH JO.
                1           2           5           4
      1    0    0    0    2    0    3    0    4    0    8    9   10   11   12   13    0    5
  7
                2           3           6           5
  0    3    0    4                   0    0    0    0    0                                  8    9
 11   12   13
                4           5           8           7
  5    6    7    0    0    8    9   10   11   12   13   17   18   19   20   21   22    0   14
 16
                5           6           9           8
  9   10   11   12   13    0    0    0    0    0    0                                      17   18
 20   21   22
                7           8          11          10
 14   15   16    0    0   17   18   19   20   21   22   24    0   25    0   26    0    0    0
  0
                8           9          12          11
 18   19   20   21   22    0    0    0    0    0                                          24    0
  0   26    0
```

DIRECT JOINT LOADS

.01	0.00	.02	0.00	0.00	.02	0.00	0.00
0.00	.04	0.00	0.00	0.00	0.00	.02	0.00
0.00	0.00	.04	0.00	0.00	0.00	.01	0.00
.02	0.00						

JOINT DEFORMATIONS

.00010617	2	-.00000679	3	.00012154
-.00000294	5	0.00000000	6	.00010617
0.00000000	8	-.00000679	9	0.00000000
.00012154	11	0.00000000	12	-.00000294
0.00000000	14	0.00000000	15	.00010617
0.00000000	17	-.00000679	18	0.00000000
.00012154	20	0.00000000	21	-.00000294
0.00000000	23	.00010617	24	-.00000679
.00012154	26	-.00000294		

MEMBER END FORCES IN COMMON AXES

NO.	1	2	3	4	5	6
	-.004	-.037	.009	.016	-.033	
	.054	.004	-.038	.010	.016	
	.032	-.053	.004	.038	.010	
	-.016	.032	.053	-.004	.037	
	.009	-.016	-.033	-.054		
	-.004	-.031	.009	.017	-.032	
	.058	.004	-.015	.003	.003	
	.011	-.016	.004	.015	.003	
	-.003	.011	.016	-.004	.031	
	.009	-.017	-.032	-.058		
	-.004	-.037	.010	.016	-.033	
	.054	.004	-.038	.010	.016	
	.032	-.053	.004	.038	.010	
	-.016	.032	.053	-.004	.037	
	.009	-.016	-.033	-.054		
	-.004	-.031	.009	.017	-.032	
	.058	.004	-.015	.003	.003	
	.011	-.016	.004	.015	.003	
	-.003	.011	.016	-.004	.031	
	.009	-.017	-.032	-.058		
	-.004	-.037	.009	.016	-.033	
	.054	.004	-.038	.010	.016	
	.032	-.053	.004	.038	.010	
	-.016	.032	.053	-.004	.037	
	.010	-.016	-.033	-.054		
	-.004	-.031	.009	.017	-.032	
	.058	.004	-.015	.003	.003	
	.011	-.016	.004	.015	.003	
	-.003	.011	.016	-.004	.031	
	.009	-.017	-.032	-.058		

```
ZZJOB
ZZFORX 4
      DIMENSION C(24,24),E(6)
C     EQUILIBRIUM TEST OF CYLINDRICAL SHELL ELEMENT
      READ 2,R,A,AN
      READ 1,C
 1    FORMAT(5E16.8)
 2    FORMAT(8F10.3)
      PUNCH 2,R,A,AN
      DO 10 J=1,24
C     TEST 1
      E(1)=C(1,J)+C(7,J)+C(13,J)+C(19,J)
C     TEST 2
      E(2)=(C(2,J)+C(8,J)+C(14,J)+C(20,J))*COS(AN)
     1+(C(15,J)+C(21,J)-C(3,J)-C(9,J))*SIN(AN)
C     TEST 3
      E(3)=(C(2,J)+C(8,J)-C(14,J)-C(20,J))*SIN(AN)
     1+(C(3,J)+C(9,J)+C(15,J)+C(21,J))*COS(AN)
C     TEST 4
      E(4)=C(4,J)+C(10,J)+C(16,J)+C(22,J)
     1-(C(2,J)+C(8,J)+C(14,J)+C(20,J))*R*SIN(AN)*SIN(AN)
     2+(C(15,J)+C(21,J)-C(3,J)-C(9,J))*R*COS(AN)*SIN(AN)
C     TEST 5
      E(5)=(C(5,J)+C(11,J)+C(17,J)+C(23,J))*COS(AN)
     1+(C(2,J)-C(8,J)+C(14,J)-C(20,J))*A*SIN(AN)
     2+(C(3,J)+C(21,J)-C(9,J)-C(15,J))*A*COS(AN)
     3+(-C(6,J)-C(12,J)+C(18,J)+C(24,J))*SIN(AN)
C     TEST 6
      E(6)=(C(3,J)-C(9,J)+C(15,J)-C(21,J))*A*SIN(AN)
     1+(C(5,J)+C(11,J)-C(17,J)-C(23,J))*SIN(AN)
     2+(C(1,J)+C(7,J)-C(13,J)-C(19,J))*R*SIN(AN)
     3+(C(8,J)-C(2,J)+C(14,J)-C(20,J))*A*COS(AN)
      PUNCH 4,J,E
 4    FORMAT(/35H EQUILIBRIUM TEST RESULTS OF COLUMN,4X,I4/(6F13.2))
 10   CONTINUE
      END


 EQUILIBRIUM TEST RESULTS OF COLUMN      1
          .00         0.00         0.00         0.00         0.00         -

 EQUILIBRIUM TEST RESULTS OF COLUMN      2
          .00         0.00         -.03          .01        75.73         -8

 EQUILIBRIUM TEST RESULTS OF COLUMN      3
          .00         0.00          .01          .07       -71.51        -35

 EQUILIBRIUM TEST RESULTS OF COLUMN      4
          .00          .01          .01          .37      -501.75       -157

 EQUILIBRIUM TEST RESULTS OF COLUMN      5
          .00         -.01         -.03         -.27       481.90        117

 EQUILIBRIUM TEST RESULTS OF COLUMN      6
          .00          .06          .05         1.28     -2164.54       -428

 EQUILIBRIUM TEST RESULTS OF COLUMN      7
          .00         0.00         0.00         0.00         0.00         -

 EQUILIBRIUM TEST RESULTS OF COLUMN      8
          .00         0.00         -.03          .01       -75.73          8
```

```
EQUILIBRIUM TEST RESULTS OF COLUMN        9
        .00          0.00          .01          .07         71.51        35
EQUILIBRIUM TEST RESULTS OF COLUMN       10
        .00           .01          .01          .37        501.75       157
EQUILIBRIUM TEST RESULTS OF COLUMN       11
        .00           .01          .03          .27        481.90       117
EQUILIBRIUM TEST RESULTS OF COLUMN       12
        .00          -.06         -.05        -1.28      -2164.53      -428
EQUILIBRIUM TEST RESULTS OF COLUMN       13
        .00          0.00         0.00         0.00          0.00
EQUILIBRIUM TEST RESULTS OF COLUMN       14
        .00          0.00          .03          .01         75.73         8
EQUILIBRIUM TEST RESULTS OF COLUMN       15
        .00          0.00          .01         -.07         71.51       -35
EQUILIBRIUM TEST RESULTS OF COLUMN       16
        .00           .01         -.01          .37       -501.75       157
EQUILIBRIUM TEST RESULTS OF COLUMN       17
        .00          -.01          .03         -.27        481.90      -117
EQUILIBRIUM TEST RESULTS OF COLUMN       18
        .00          -.06          .05        -1.28       2164.53      -428
EQUILIBRIUM TEST RESULTS OF COLUMN       19
        .00          0.00         0.00         0.00          0.00
EQUILIBRIUM TEST RESULTS OF COLUMN       20
        .00          0.00          .03          .01        -75.73        -8
EQUILIBRIUM TEST RESULTS OF COLUMN       21
        .00          0.00          .01         -.07        -71.51        35
EQUILIBRIUM TEST RESULTS OF COLUMN       22
        .00           .01         -.01          .37        501.75      -157
EQUILIBRIUM TEST RESULTS OF COLUMN       23
        .00           .01         -.03          .27        481.90      -117
EQUILIBRIUM TEST RESULTS OF COLUMN       24
        .00           .06         -.05         1.28       2164.53      -428
```

PART 3.

TABLE 5. TRANSFORMATION MATRIX BETWEEN NODAL COORDINATES AND POLYNOMIAL COEFFICIENTS (4.10)

$[A] =$

1	0	0	0	0	0	0	0	0	0	0	0	0	$-r(1-c\theta)$	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	$-r(1-c\theta)$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0
0	0	$-1/2$	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	a	0	0	0	0	0	0	0	0	0	0	0	$-r(1-c\theta)$	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	a	0	0	a^2	0	0	0	$-r(1-c\theta)$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	a	0	a^2	0	0	a^3	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	$-a^2/r$	0	0	0	1	0	a	0	0	a^2	0	0	a^3	0
0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	$-2a$	0	0	$-3a^2$	0	0	0	0	0
0	0	$-1/2$	$-a/2$	0	0	0	1	0	0	a	0	0	0	0	0	0	0	0	0	0	0	0	0
1	a	b	ab	b^2	b^2a	0	$-rs\theta$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	$c\theta$	$ac\theta$	b	ab	a^2	a^2b	$-s\theta$	$-as\theta$	$-rs^2\theta$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	$s\theta$	$as\theta$	0	0	0	0	$c\theta$	$ac\theta$	$rc\theta s\theta$	a^2	ab	b^2	a^3	ba^2	ab^2	b^3	a^3b	b^3a
0	0	0	0	0	0	0	0	$-b/r$	$-ab/r$	$-a^2/r$	$-a^2b/r$	0	0	1	0	a	$2b$	0	a^2	$2ab$	$3b^2$	a^3	$3b^2a$
0	0	0	0	0	0	0	$-s\theta$	0	0	0	0	0	$-c\theta$	0	$-2a$	$-b$	0	$-3a^2$	$-2ab$	$-b^2$	0	$-3a^2b$	$-b^3$
0	0	$-1/2$	$-a/2$	$-b$	$-ab$	0	$c\theta$	0	$b/2$	a	ab	0	$-s\theta$	0	0	0	0	0	0	0	0	0	0
1	0	b	0	b^2	0	0	$-rs\theta$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	$c\theta$	0	b	0	0	0	$-s\theta$	0	$-rs^2\theta$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	$s\theta$	0	0	0	0	0	$c\theta$	0	$rs\theta c\theta$	0	0	b^2	0	0	0	b^3	0	0
0	0	0	0	0	0	0	0	$-b/r$	0	0	0	0	0	1	0	0	$2b$	0	0	0	$3b^2$	0	0
0	0	0	0	0	0	0	$-s\theta$	0	0	0	0	0	$-c\theta$	0	0	$-b$	0	0	0	$-b^2$	0	0	$-b^3$
0	0	$-1/2$	0	$-b$	0	0	$c\theta$	0	$b/2$	0	0	0	$-s\theta$	0	0	0	0	0	0	0	0	0	0

$c\theta = \cos\theta$ $s\theta = \sin\theta$

PART 3.

TABLE 6. STIFFNESS MATRIX IN TENMS OF POLYNOMIAL COEFFICIENTS.

$[H_Q] =$ (symmetrical)

0																							
0	2,2																						
0	0	3,3																					
0	4,2	4,3	4,4																				
0	0	5,3	5,4	5,5																			
0	6,2	6,3	6,4	6,5	6,6																		
0	0	0	0	0	0	0																	
0	0	0	0	0	0	0	0																
0	9,2	0	9,4	0	9,6	0	0	9,9															
0	10,2	10,3	10,4	10,5	10,6	0	0	10,9	10,10														
0	0	11,3	11,4	11,5	11,6	0	0	0	11,10	11,11													
0	12,2	12,3	12,4	12,5	12,6	0	0	12,9	12,10	12,11	12,12												
0	0	0	0	0	0	0	0	0	0	0	0	0											
0	0	0	0	0	0	0	0	0	0	0	0	0	0										
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0									
0	16,2	0	16,4	0	16,6	0	0	16,9	16,10	0	16,12	0	0	0	16,16								
0	17,2	0	17,4	0	17,6	0	0	17,9	17,10	17,11	17,12	0	0	0	17,16	17,17							
0	18,2	0	18,4	0	18,6	0	0	18,9	18,10	0	18,12	0	0	0	18,16	18,17	19,18						
0	19,2	0	19,4	0	19,6	0	0	19,9	19,10	0	19,12	0	0	0	19,16	19,17	19,18	19,19					
0	20,2	0	20,4	0	20,6	0	0	20,9	20,10	20,11	20,12	0	0	0	20,16	20,17	20,18	20,19	20,20				
0	21,2	0	21,4	0	21,6	0	0	21,9	21,10	21,11	21,12	0	0	0	21,16	21,17	21,18	21,19	21,20	21,21			
0	22,2	0	22,4	0	22,6	0	0	22,9	22,10	0	22,12	0	0	0	22,16	22,17	22,18	22,19	22,20	22,21	22,22		
0	23,2	0	23,4	0	23,6	0	0	23,9	23,10	23,11	23,12	0	0	0	23,16	23,14	23,18	23,19	23,20	23,21	23,22	23,23	
0	24,2	0	24,4	0	24,6	0	0	24,9	24,10	24,11	24,12	0	0	0	24,16	24,17	24,18	24,19	24,20	24,21	24,22	24,23	24,24

Note: Each term should be multiplied by the element thickness.

TABLE 6. CONTINUED.

$k(2,2) = abD_1$
$k(4,2) = ab^2D_1/2$
$k(6,2) = ab^3D_1/3$
$k(9,2) = ab\mu D_1$
$k(10,2) = a^2b\mu D_1/2$
$k(12,2) = a^3b\mu D_1/3$
$k(16,2) = a^3b\mu D_1/3r$
$k(17,2) = a^2b^2\mu D_1/4r$
$k(18,2) = ab^3\mu D_1/3r$
$k(19,2) = a^4b\mu D_1/4r$
$k(20,2) = a^3b^2\mu D_1/6r$
$k(21,2) = a^2b^3\mu D_1/6r$
$k(22,2) = ab^4\mu D_1/4r$
$k(23,2) = a^4b^2\mu D_1/8r$
$k(24,2) = a^2b^4\mu D_1/8r$
$k(3,3) = abD_3$
$k(4,3) = a^2bD_3/2$
$k(5,3) = ab^2D_3$
$k(6,3) = a^2b^2D_3/2$
$k(10,3) = ab^2D_3/2$
$k(11,3) = a^2bD_3$
$k(12,3) = a^2b^2D_3/2$
$k(4,4) = ab^3D_1/3 + a^3bD_3/3$
$k(5,4) = a^2b^2D_3/2$
$k(6,4) = ab^4D_1/4 + a^3b^2D_3/2$
$k(9,4) = b^2a\mu D_1/2$
$k(10,4) = a^2b^2\mu D_1/4 + a^2b^2D_3/4$
$k(11,4) = 2a^3bD_3/3$

TABLE 6. CONTINUED.

$k(12,4)= a^3 b^2 \mu D_1/6 + a^3 b^2 D_3/2$
$k(16,4)= a^3 b^2 \mu D_1/6r$
$k(17,4)= a^2 b^3 \mu D_1/6r$
$k(18,4)= a b^4 \mu D_1/4r$
$k(19,4)= a^4 b^2 \mu D_1/8r$
$k(20,4)= a^3 b^3 \mu D_1/9r$
$k(21,4)= a^2 b^4 \mu D_1/8r$
$k(22,4)= a b^5 \mu D_1/5r$
$k(23,4)= a^4 b^3 \mu D_1/12r$
$k(24,4)= a^2 b^5 \mu D_1/10r$
$k(5,5)= 4 a b^3 D_3/3$
$k(6,5)= 4 a^2 b^3 D_3/6$
$k(10,5)= 2 a b^3 D_3/3$
$k(11,5)= a^2 b^2 D_3$
$k(12,5)= 4 a^2 b^3 D_3/6$
$k(6,6)= a b^5 D_1/5 + 4 a^3 b^3 D_3/9$
$k(9,6)= a b^3 \mu D_1/3$
$k(10,6)= a^2 b^3 \mu D_1/6 + a^2 b^3 D_3/3$
$k(11,6)= 4 a^3 b^2 D_3/6$
$k(12,6)= a^3 b^3 \mu D_1/9 + 4 a^3 b^3 D_3/9$
$k(16,6)= a^3 b^3 \mu D_1/9r$
$k(17,6)= a^2 b^4 \mu D_1/8r$
$k(18,6)= a b^5 \mu D_1/5r$
$k(19,6)= a^4 b^3 \mu D_1/12r$
$k(20,6)= a^3 b^4 \mu D_1/12r$
$k(21,6)= a^2 b^5 \mu D_1/10r$
$k(22,6)= a b^6 \mu D_1/6r$
$k(23,6)= a^4 b^4 \mu D_1/16r$

TABLE 6. CONTINUED.

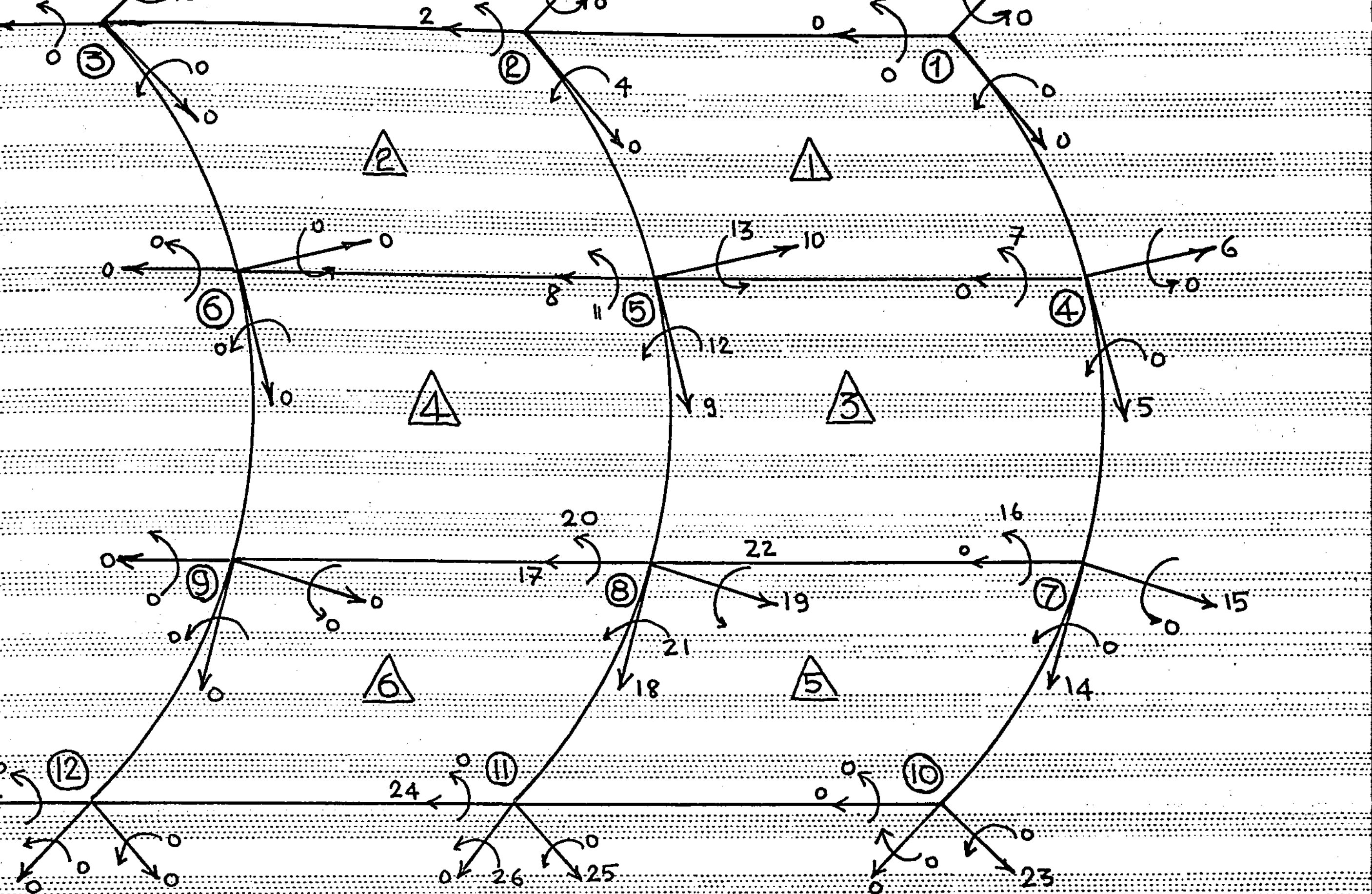
$k(24,6) = a^2b^4\mu D_1/12r$
$k(9,9) = abD_1 + abD_4/r^2$
$k(10,9) = a^2bD_1/2 + a^2bD_4/2r^2$
$k(12,9) = a^3bD_1/3 + a^3bD_4/3r^2$
$k(16,9) = a^3bD_1/3r - 2abD_5/r$
$k(17,9) = a^2b^2D_1/4r$
$k(18,9) = ab^3D_1/3r - 2abD_4/r$
$k(19,9) = a^4bD_1/4r - 6a^2bD_5/2r$
$k(20,9) = a^3b^2D_1/6r - b^2aD_5/r$
$k(21,9) = a^2b^3D_1/6r - a^2bD_4/r$
$k(22,9) = ab^4D_1/4r - 3b^2aD_4/r$
$k(23,9) = a^4b^2D_1/8r - 6a^2b^2D_5/4r$
$k(24,9) = a^2b^4D_1/8r - 6a^2b^2D_4/4r$
$k(10,10) = a^3bD_1/3 + ab^3D_3/3 + a^3bD_4/3r^2 + 4ab^3D_6/3r^2$
$k(11,10) = a^2b^2D_3/2 + 2a^2b^2D_6/r^2$
$k(12,10) = a^4bD_1/4 + a^2b^3D_3/3 + a_4bD_4/4r^2 + 8a^2b^3D_6/6r^2$
$k(16,10) = a^4bD_1/4r - a^2bD_5/r$
$k(17,10) = a^3b^2D_1/6r - 2ab^2D_6/r$
$k(18,10) = a^2b^3D_1/6r - a^2bD_4/r$
$k(19,10) = a^5bD_1/5r - 2a^3bD_5/r$
$k(20,10) = a^4b^2D_1/8r - a^2b^2D_5/2r - 2a^2b^2D_6/r$
$k(21,10) = a^3b^3D_1/9r - 2a^3bD_4/3r - 8ab^3D_6/3r$
$k(22,10) = a^2b^4D_1/8r - 6a^2b^2D_4/4r$
~~$k(23,10) = a^5b^2D_1/10r - a^3b^2D_5/R - 2a^3b^2D_6/r$~~
$k(24,10) = a^3b^4D_1/12r - a^3b^2D_4/r - 3ab^4D_6/r$
$k(11,11) = 4a^3bD_3/3 + 16a^3bD_6/3r^2$
$k(12,11) = 4a^3b^2D_3/6 + 16a^3b^2D_6/6r^2$
$k(17,11) = -4a^2bD_6/r$

TABLE 6. CONTINUED.

$k(20,11) = -16a^3bD_6/3r$
$k(21,11) = -16a^2b^2D_6/4r$
$k(23,11) = -6a^4bD_6/r$
$k(24,11) = -4a^2b^3D_6/r$
$k(12,12) = a^5bD_1/5 + 4a^3b^3D_3/9 + a^5bD_4/5r^2 + 16a^3b^3D_6/9r^2$
$k(16,12) = a^5bD_1/5r - 2a^3bD_5/3r$
$k(17,12) = a^4b^2D_1/8r - 2a^2b^2D_6/r$
$k(18,12) = a^3b^3D_1/9r - 2a^3bD_4/3r$
$k(19,12) = a^6bD_1/6r - 6a^4bD_5/4r$
$k(20,12) = a^5b^2D_1/10r - a^3b^2D_5/3r - 16a^3b^2D_6/6r$
$k(21,12) = a^4b^3D_1/12r - a^4bD_4/2r - 16a^2b^3D_6/6r$
$k(22,12) = a^3b^4D_1/12r - a^3b^2D_4/r$
$k(23,12) = a^6b^2D_1/12r - 6a^4b^2D_5/8r - 3a^4b^2D_6/8r$
$k(24,12) = a^4b^4D_1/16r - 6a^4b^2D_4/8r - 3a^2b^4D_6/8r$
$k(16,16) = a^5bD_1/5r^2 + 4abD_4$
$k(17,16) = a^4b^2D_1/8r^2$
$k(18,16) = a^3b^3D_1/9r^2 + 4abD_5$
$k(19,16) = a^6bD_1/6r^2 + 6a^2bD_4$
$k(20,16) = a^5b^2D_1/10r^2 + 2ab^2D_4$
$k(21,16) = a^4b^3D_1/12r^2 + 2a^2bD_5$
$k(22,16) = a^3b^4D_1/12r^2 + 6ab^2D_5$
$k(23,16) = a^6b^2D_1/12r^2 + 3a^2b^2D_4$
$k(24,16) = a^4b^4D_1/16r^2 + 3a^2b^2D_5$
$k(17,17) = a^3b^3D_1/9r^2 + 4abD_6$
$k(18,17) = a^2b^4D_1/8r^2$
$k(19,17) = a^5b^2D_1/10r^2$
$k(20,17) = a^4b^3D_1/12r^2 + 4a^2bD_6$
$k(21,17) = a^3b^4D_1/12r^2 + 4ab^2D_6$

TABLE 6. CONTINUED.

$k(22,17) = a^2 b^5 D_1/10r^2$
$k(23,17) = a^5 b^3 D_1/15r^2 + 4a^3 b D_6$
$k(24,17) = a^3 b^5 D_1/15r^2 + 4b^3 a D_6$
$k(18,18) = a b^5 D_1/5r^2 + 4ab D_4$
$k(19,18) = a^4 b^3 D_1/12r^2 + 6a^2 b D_5$
$k(20,18) = a^3 b^4 D_1/12r^2 + 2ab^2 D_5$
$k(21,18) = a^2 b^5 D_1/10r^2 + 2a^2 b D_4$
$k(22,18) = a b^6 D_1/6r^2 + 6ab^2 D_4$
$k(23,18) = a^4 b^4 D_1/16r^2 + 3a^2 b^2 D_5$
$k(24,18) = a^2 b^6 D_1/12r^2 + 3a^2 b^2 D_4$
$k(19,19) = a^7 b D_1/7r^2 + 12a^3 b D_4$
$k(20,19) = A^6 b^2 D_1/12r^2 + 3a^2 b^2 D_4$
$k(21,19) = a^5 b^3 D_1/15r^2 + 4a^3 b D_5$
$k(22,19) = a^4 b^4 D_1/16r^2 + 9a^2 b^2 D_5$
$k(23,19) = a^7 b^2 D_1/14r^2 + 6a^3 b^2 D_4$
$k(24,19) = a^5 b^4 D_1/20r^2 + 6a^3 b^2 D_5$
$k(20,20) = a^5 b^3 D_1/15r^2 + 4ab^3 D_4/3 + 16a^3 b D_6/3$
$k(21,20) = a^4 b^4 D_1/16r^2 + a^2 b^2 D_5 + 4a^2 b^2 D_6$
$k(22,20) = a^3 b^5 D_1/15r^2 + 4ab^3 D_5$
$k(23,20) = a^6 b^3 D_1/18r^2 + 2a^2 b^3 D_4 + 6a^4 b D_6$
$k(24,20) = a^4 b^5 D_1/20r^2 + 2a^2 b^3 D_5 + 4a^2 b^3 D_6$
$k(21,21) = a^3 b^5 D_1/15r^2 + 4a^3 b D_4/3 + 16b^3 a D_6/3$
$k(22,21) = a^2 b^6 D_1/12r^2 + 3a^2 b^2 D_4$
$k(23,21) = a^5 b^4 D_1/20r^2 + 2a^3 b^2 D_5 + 4a^3 b^2 D_6$
$k(24,21) = b^6 a^3 D_1/18r^2 + 2a^3 b^2 D_4 + 6b^4 a D_6$
$k(22,22) = a b^7 D_1/7r^2 + 12ab^3 D_4$
$k(23,22) = a^4 b^5 D_1/20r^2 + 6a^2 b^3 D_5$
$k(24,22) = b^7 a^2 D_1/14r^2 + 6a^2 b^3 D_4$
$k(23,23) = a^7 b^3 D_1/21r^2 + 4a^3 b^3 D_4 + 36a^5 b D_6/5$
$k(24,23) = a^5 b^5 D_1/25r^2 + 4a^3 b^3 D_5 + 4a^3 b^3 D_6$
$k(24,24) = a^3 b^7 D_1/21r^2 + 4a^3 b^3 D_4 + 36ab^5 D_6/5$



CYLINDRICAL SHELL ANALYSIS BY FINITE ELEMENTS

210.

EET= 2.

SSON,S RATIO= .30

ESS= .30

10.000

NTAL SIDE= 20.000

19.100

=0. (ALL UNITS KIP-FOOT,EXCEPT FEED A AND I IN INCHES)

=1. (ALL UNITS KIP-FOOT)

=2. (METRIC UNITS HOMOG:NEOUS THROUGHOUT)

NUMBER OF MEMBERS = 6
NUMBER OF JOINTS = 12
NUMBER OF UNKNOWNS= 26

MEMBER STIFFNESS MATRIX

540.29	-27039.17	-292.02	-143.10	-5706.14	-604712.22
039.16	12119.88	121.86	37.03	2560.80	270340.75
292.02	121.86	-2.61	-16.06	21.49	2906.92
143.10	37.04	-16.06	148.96	-10.73	1325.59
706.13	2560.79	21.49	-10.73	558.92	57035.43
712.37	270340.88	2906.92	1325.60	57035.52	6043062.70
354.07	-25638.72	-275.48	-98.37	-5408.80	-573211.56
329.04	-13120.26	-138.74	-48.66	-2764.27	-293176.79
278.40	126.12	9.02	58.42	26.56	2769.50

1130.87	503.04	7.51	[illegible]	[illegible]	[illegible]
4231.52	256761.25	2768.25	63.23	101.52	11263.41
7963.27	25905.48	278.20	1089.18	54114.01	5739408.20
3338.73	-12670.50	-134.59	102.90	5467.74	579188.20
-329.99	146.60	-6.58	-39.96	-2672.70	-283247.27
519.44	-221.95	35.31	-53.68	31.29	3296.28
1272.88	570.04	4.20	196.99	-51.38	-5185.38
3953.76	258828.98	2777.15	-42.12	119.61	12713.70
9931.09	26772.41	289.30	1106.51	54621.34	5785745.60
0607.07	13660.86	152.41	138.57	5647.20	598735.69
-320.86	138.78	9.47	71.20	2872.95	305862.41
545.92	-244.75	-41.17	22.80	28.81	3204.98
1248.63	-546.06	-6.69	-178.28	-46.69	-5450.17
9990.43	268111.69	2898.02	37.90	-113.47	-12471.21
			1308.27	56528.19	5995112.30

7354.10	29329.06	-278.40	-510.62	-1130.87	-574231.72
5638.72	-13120.26	126.12	238.24	503.04	256761.19
-275.48	-138.74	9.02	39.91	7.51	2768.25
-98.36	-48.66	58.42	207.13	63.23	1089.17
5408.80	-2764.27	26.56	47.17	101.52	54113.95
5211.87	-293177.02	2769.51	5088.09	11263.44	5739410.70
4438.64	27812.39	-261.60	-483.21	-1067.42	-544715.66
7812.38	14253.84	-142.76	-266.63	-559.14	-278560.52
-261.60	-142.76	-2.16	-29.94	13.93	2630.55
-483.21	-266.63	-29.94	-172.06	19.13	4850.26
1067.42	-559.14	13.93	19.13	48.73	10719.57
4715.82	-278560.66	2630.55	4850.26	10719.59	5453296.20
4987.85	-28099.44	264.32	487.74	1076.49	550323.00
6871.98	13720.33	-124.46	-238.45	-513.98	-269021.47
-315.37	-165.38	9.22	41.02	5.98	3157.35
493.85	252.34	-41.02	-193.90	-12.65	-4947.60
1215.49	-632.00	5.98	12.65	24.07	12170.09
9122.46	-280695.77	2639.61	4868.39	10749.79	5496631.60
5804.89	-29042.01	275.68	506.09	1121.81	568624.39
9023.00	-14842.80	142.29	268.03	573.29	290599.97
-306.24	-157.57	-6.84	-35.46	3.48	3066.03
520.33	275.15	35.46	181.38	-8.43	-5212.36
1191.25	608.01	-3.48	-8.43	-17.93	-11927.64
3805.33	-290880.45	2760.44	5069.96	11233.23	5694460.30

7963.28	28338.74	-329.99	519.44	-1272.88	-578953.72
5905.47	-12670.50	146.60	-221.95	570.04	258828.86
278.20	-134.59	-6.58	35.31	4.20	2777.15
102.90	-39.96	-53.68	196.99	-42.12	1106.49
5467.74	-2672.70	31.29	-51.38	119.61	54621.26
9188.37	-283247.37	3296.28	-5185.38	12713.70	5785745.70
4987.83	26871.98	-315.37	493.85	-1215.49	-549122.26
3099.42	13720.32	-165.38	252.34	-632.00	-280695.55
264.32	-124.46	9.22	-41.02	5.98	2639.61
487.74	-238.45	41.02	-193.90	12.65	4868.39
1076.48	-513.98	5.98	-12.65	24.07	10749.77
0323.02	-269021.52	3157.35	-4947.60	12170.09	5496631.20
5571.65	-27151.43	318.09	-498.38	1224.56	554845.00
7151.43	13309.64	-147.08	224.15	-586.84	-271350.72
318.09	-147.08	-2.16	29.94	13.93	3165.41
-498.38	224.15	29.94	-172.06	-19.13	-4965.71
1224.56	-586.84	13.93	-19.13	48.73	12200.30

7379.46	-28059.29	327.27	-514.91	1263.81	573230.99
9322.74	-14348.36	164.67	-253.33	645.59	292996.54
308.95	-153.44	9.27	-40.19	8.22	3075.09
-524.86	266.54	-40.19	192.21	-12.65	-5230.48
1200.31	586.47	-8.21	12.65	-36.02	-11957.84
4666.43	-281079.68	3287.22	-5167.26	12683.50	5741411.30

9931.10	-30607.07	-320.86	545.92	1248.63	-599990.22
6772.40	13660.85	138.78	-244.75	-546.06	268111.55
289.30	152.41	9.47	-41.17	-6.69	2898.02
138.57	71.20	22.80	-178.28	37.90	1308.26
5647.19	2872.95	28.81	-46.69	-113.47	56528.12
8735.87	305862.48	3204.98	-5450.18	-12471.23	5995111.70
6804.88	-29023.00	-306.24	520.33	1191.25	-568804.96
9041.99	-14842.79	-157.57	275.15	608.01	-290880.21
275.68	142.29	-6.84	35.46	-3.48	2760.44
506.08	268.03	-35.46	181.38	-8.43	5069.95
1121.81	573.29	3.48	-8.43	-17.93	11233.21
8624.32	290599.97	3066.02	-5212.35	-11927.66	5694458.20
7379.47	29322.74	308.95	-524.86	-1200.31	574666.20
8059.28	-14348.36	-153.44	266.54	586.47	-281079.56
327.27	164.67	9.27	-40.19	-8.21	3287.22
-514.91	-253.33	-40.19	192.21	12.65	-5167.26
1263.81	645.59	8.22	-12.65	-36.02	12683.49
3231.06	292996.60	3075.09	-5230.48	-11957.86	5741410.60
9356.52	30307.33	318.14	-541.39	-1239.57	594129.09
0307.33	15519.32	171.02	-295.72	-645.22	303627.37
-318.14	171.02	-2.16	29.94	-13.93	3195.92
-541.39	-295.72	29.94	-172.06	19.13	-5432.05
1239.57	-645.21	-13.93	19.13	48.73	-12441.01
4129.33	303627.45	3195.92	-5432.05	-12441.02	5949777.30

```
                   CODE NUMBERS
.                  FIRST JO.SECOND JO.THIRD JO.FOURTH JO.
                   1                 2                  5                  4
      1    0    0    0    2    0    3    0    4    0    8    9   10   11   12   13    0    5
 7
                   2                 3                  6                  5
 0    3    0    4              0    0    0    0    0                                      8    9
11   12   13
                   4                 5                  8                  7
 5    6    7    0    0    8    9   10   11   12   13   17   18   19   20   21   22    0   14
16
                   5                 6                  9                  8
 9   10   11   12   13    0    0    0    0    0    0                                     17   18
20   21   22
                   7                 8                 11                 10
14   15   16    0    0   17   18   19   20   21   22   24    0   25    0   26    0    0    0
 0
                   8                 9                 12                 11
18   19   20   21   22    0    0    0    0    0                                          24    0
 0   26    0

                   DIRECT JOINT LOADS

 .01        0.00        .02        0.00        0.00        .02        0.00        0.00
```

0.00	0.00	.04	0.00	0.00	0.00	.02	0.0
.02	0.00		0.00	0.00	0.00	.01	0.0

JOINT DEFORMATIONS

1

1	.00047323	2	-.00011810	3	.00165656
4	-.00001174	5	-.00084655	6	.00158987
7	.00003585	8	-.00004342	9	.00045416
0	.00204415	11	-.00002701	12	-.00005456
3	-.00003274	14	.00095757	15	.00397033
6	-.00004243	17	.00013354	18	.00051780
9	.00317725	20	-.00004187	21	.00021717
2	-.00003330	23	.00607141	24	.00017792
5	.00453641	26	.00007982		

MEMBER END FORCES IN COMMON AXES

. NO.	1	2	3	4	5	6
1	-.037	-.013	.009	.018	-.012	
	.327	-.039	-.055	.007	.002	
	.021	.458	.033	.014	.011	
	-.027	.021	.299	.043	.053	
	.007	-.002	-.022	.497		
2	.039	-.050	.012	.023	-.021	
	-.422	.036	.019	-.004	-.017	
	-.002	-.319	-.035	-.013	.001	
	-.013	-.00..	-.380	-.041	.017	
	.008	0.000	-.020	-.340		
3	.042	-.053	.012	.002	-.021	
	-.477	.037	-.019	.002	-.015	
	.014	-.286	-.041	.050	.016	
	-.044	.025	-.467	-.039	.020	
	.007	.008	-.029	-.290		
4	-.030	-.012	.018	.043	-.015	
	.328	-.037	-.020	-.011	-.047	
	-.003	.418	.034	-.025	.008	
	-.044	0.000	.350	.032	.057	
	.002	.028	-.022	.405		
5	-.040	-.020	.012	-.008	-.014	
	.329	-.038	-.059	.002	-.011	
	.015	.487	.039	.014	.016	
	-.042	.027	.302	.039	.061	
	.010	.003	-.032	.497		
6	.046	-.048	.018	.027	-.019	
	-.424	.030	.015	-.014	-.057	
	-.008	-.281	-.037	.010	.009	
	-.053	.002	-.388	-.039	.020	
	.003	.035	-.027	-.347		

```
ZZOBB
ZZFORX 4
      DIMENSION C(24,24),E(6)
C     EQUILIBRIUM TEST OF CYLINDIRICAL SHELL ELEMENT
      READ 2,R,A,AN
      READ 1,C
 1    FORMAT(5E16.8)
 2    FORMAT(8F10.3)
      PUNCH 2,R,A,AN
      DO 10 J=1,24
C     TEST 1
      E(1)=C(1,J)+C(7,J)+C(13,J)+C(19,J)
C     TEST 2
      E(2)=(C(2,J)+C(8,J)+C(14,J)+C(20,J))*COS(AN)
     1+(C(15,J)+C(21,J)-C(3,J)-C(9,J))*SIN(AN)
C     TEST 3
      E(3)=(C(2,J)+C(8,J)-C(14,J)-C(20,J))*SIN(AN)
     1+(C(3,J)+C(9,J)+C(15,J)+C(21,J))*COS(AN)
C     TEST 4
      E(4)=C(4,J)+C(10,J)+C(16,J)+C(22,J)
     1-(C(2,J)+C(8,J)+C(14,J)+C(20,J))*R*SIN(AN)*SIN(AN)
     2+(C(15,J)+C(21,J)-C(3,J)-C(9,J))*R*COS(AN)*SIN(AN)
C     TEST 5
      E(5)=(C(5,J)+C(11,J)+C(17,J)+C(23,J))*COS(AN)
     1+(C(2,J)-C(8,J)+C(14,J)-C(20,J))*A*SIN(AN)
     2+(C(3,J)+C(21,J)-C(9,J)-C(15,J))*A*COS(AN)
     3+(-C(6,J)-C(12,J)+C(18,J)+C(24,J))*SIN(AN)
C     TEST 6
      E(6)=(C(6,J)+C(12,J)+C(18,J)+C(24,J))*COS(AN)
     1+(C(5,J)+C(11,J)-C(17,J)-C(23,J))*SIN(AN)
     2+(C(1,J)+C(7,J)-C(13,J)-C(19,J))*R*SIN(AN)
     3+(C(8,J)-C(2,J)+C(14,J)-C(20,J))*A*COS(AN)
     4+(C(3,J)-C(9,J)+C(15,J)-C(21,J))*A*SIN(AN)
      PUNCH 4,J,E
 4    FORMAT(/35H EQUILIBRIUM TEST RESULTS OF COLUMN,4X,I4/(6F13.2))
 10   CONTINUE
      END
ZZZ


EQUILIBRIUM TEST RESULTS OF COLUMN       1
         .00         0.00         -.14          .14         -.99      -140.

EQUILIBRIUM TEST RESULTS OF COLUMN       2
         .00         0.00          .06         -.06          .41        62.

EQUILIBRIUM TEST RESULTS OF COLUMN       3
         .00         0.00         0.00         0.00         0.00          .

EQUILIBRIUM TEST RESULTS OF COLUMN       4
         .00         0.00         0.00          .04         0.00          .

EQUILIBRIUM TEST RESULTS OF COLUMN       5
         .00         0.00          .01         -.01          .08        13.

EQUILIBRIUM TEST RESULTS OF COLUMN       6
         .19          .06         1.45        -1.43         9.78      1406.

EQUILIBRIUM TEST RESULTS OF COLUMN       7
         .00         0.00         -.13          .14         -.89      -133.
```

EQUILIBRIUM TEST RESULTS OF COLUMN 8

.00	0.00	-.07	.07	-.44	-68.

EQUILIBRIUM TEST RESULTS OF COLUMN 9

.00	0.00	0.00	0.00	0.00	.

EQUILIBRIUM TEST RESULTS OF COLUMN 10

.00	0.00	0.00	0.00	0.00	1.

EQUILIBRIUM TEST RESULTS OF COLUMN 11

.00	0.00	0.00	0.00	.01	2.

EQUILIBRIUM TEST RESULTS OF COLUMN 12

.09	.06	1.38	-1.39	8.80	1335.

EQUILIBRIUM TEST RESULTS OF COLUMN 13

.00	0.00	.13	-.14	.92	134.

EQUILIBRIUM TEST RESULTS OF COLUMN 14

.00	0.00	-.06	.07	-.45	-65.

EQUILIBRIUM TEST RESULTS OF COLUMN 15

.00	0.00	0.00	0.00	0.00	.

EQUILIBRIUM TEST RESULTS OF COLUMN 16

.00	0.00	0.00	0.00	0.00	-1.

EQUILIBRIUM TEST RESULTS OF COLUMN 17

.00	0.00	0.00	0.00	.01	2.

EQUILIBRIUM TEST RESULTS OF COLUMN 18

.09	.07	1.39	-1.40	9.18	1347.

EQUILIBRIUM TEST RESULTS OF COLUMN 19

.01	0.00	.14	-.14	.96	139.

EQUILIBRIUM TEST RESULTS OF COLUMN 20

.00	0.00	.07	-.07	.48	71.

EQUILIBRIUM TEST RESULTS OF COLUMN 21

.00	0.00	0.00	0.00	0.00	.

EQUILIBRIUM TEST RESULTS OF COLUMN 22

.00	0.00	0.00	0.00	-.01	-1.

EQUILIBRIUM TEST RESULTS OF COLUMN 23

.00	0.00	0.00	.01	-.02	-2.

EQUILIBRIUM TEST RESULTS OF COLUMN 24

.19	.07	1.45	-1.43	9.76	1396.

PART 4.

TABLE 7. TRANSFORMATION MATRIX BETWEEN NODAL COORDINATES AND POLYNOMIAL COEFFICIENTS. (5.3)

$$
[A] = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & S_2 & L_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & S_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & S_2 & L_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & S_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & S_2 & L_2 & S_2^2 & S_2L_2 & L_2^2 & S_2^3 & (S_2^2L_2+L_2^2S_2) & L_2^3 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & S_3 & 0 & S_3^2 & 0 & 0 & S_3^3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -2S_2 & -L_2 & 0 & -3S_2^2 & -(2S_2L_2+L_2) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -2S_3 & 0 & 0 & -3S_3^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & S_2 & 2L_2 & 0 & (S_2^2+2S_2L_2) & 3L_2^2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & S_3 & 0 & 0 & S_3^2 & 0
\end{bmatrix}
$$

PART 4.

THE RESULTS OF THE TRIPLE MATRIX MULTIPLICATION,

$$[G_o]^T[D][G_o]$$

FOR THE 15x15, SPHERICAL SHELL FINITE ELEMENT LISTED COLUMN BY COLUMN.

IN WRITING THE COLUMNS SYMMETRY IS MADE USE OF.

COLUMN 1

(1,1) $D_1 \cot^2\theta / R_2^2 + D_{44} \cot^2\theta / R_2^4$

(2,1) $S D_1 \cot^2\theta / R_2^2 + S D_{44} \cot^2\theta / R_2^4$

(3,1) $D_{12} \cot\theta / R_2 + L D_1 \cot^2\theta / R_2^2 + D_{45} \cot\theta / R_2^3 + L D_{44} \cot^2\theta / R_2^4$

(4,1) 0

(5,1) $D_1 \cot\theta / R_2 - D_{44} \cot\theta / R_2^3$

(6,1) 0

(7,1) $(D_{12} + D_1) \cot\theta / R_2^2$

(8,1) $S(D_{12} + D_1) \cot\theta / R_2^2$

(9,1) $L(D_{12} + D_1) \cot\theta / R_2^2 - D_{44} \cot^2\theta / R_2^3$

(10,1) $S^2(D_{12} + D_1) \cot\theta / R_2^2 + 2 D_{44} \cot\theta / R_2^2$

(11,1) $SL(D_{12} + D_1) \cot\theta / R_2^2 - S D_{44} \cot^2\theta / R_2^3$

(12,1) $L^2(D_{12} + D_1) \cot\theta / R_2^2 - 2 D_{45} \cot\theta / R_2^2 - 2 L D_{44} \cot^2\theta / R_2^3$

(13,1) $S^3(D_{12} + D_1) \cot\theta / R_2^2 + 6 S D_{44} \cot\theta / R_2^2$

(14,1) $(S^2L + L^2S)(D_{12} + D_1) \cot\theta / R_2^2 - 2 S D_{45} \cot\theta / R_2^2 + D_{44} \left[2L - (S^2 + 2SL) \cot\theta / R_2 \right] \cot\theta / R_2^2$

(15,1) $L^3 (D_{12} + D_1) \cot\theta / R_2^2 - 6 L D_{45} \cot\theta / R_2^2 - 3 L^2 D_{44} \cot^2\theta / R_2^3$

COLUMN 2

(1,2) $SD_1 \cot^2\theta / R_2^2 + SD_{44} \cot^2\theta / R_2^4$

(2,2) $S^2 D_1 \cot^2\theta / R_2^2 + S^2 D_{44} \cot^2\theta / R_2^4 + D_{66} / R_2^2 + D_3$

(3,2) $SD_{12} \cot\theta / R_2 + SLD_1 \cot^2\theta / R_2^2 + S \cot\theta D_{45} / R_2^3 + SL \cot^2\theta D_{44} / R_2^4$

(4,2) $-D_{66} \cot\theta / R_2^3 - \cot\theta D_3 / R_2$

(5,2) $SD_1 \cot\theta / R_2 - S \cot\theta D_{44} / R_2^3 - SD_6 \cot\theta / R_2^3 - S \cot\theta D_3 / R_2$

(6,2) $D_6 (1/R_2^2 - L \cot\theta / R_2^3) + D_3 (1 - L \cot\theta / R_2)$

(7,2) $S(1+\mu) D_1 \cot\theta / R_2^2$

(8,2) $S^2 (1+\mu) D_1 \cot\theta / R_2^2$

(9,2) $SL(1+\mu) D_1 \cot\theta / R_2^2 - S \cot^2\theta D_{44} / R_2^3$

(10,2) $S^3 (1+\mu) D_1 \cot\theta / R_2^2 + 2S \cot\theta D_{44} / R_2^2$

(11,2) $S^2 L (1+\mu) D_1 \cot\theta / R_2^2 - S^2 \cot^2\theta D_{44} / R_2^3 - D_6 / R_2$

(12,2) $SL^2 (1+\mu) D_1 \cot\theta / R_2^2 - 2S \cot\theta D_{45} / R_2^2 - 2SL \cot^2\theta D_{44} / R_2^3$

(13,2) $S^4 (1+\mu) D_1 \cot\theta / R_2^2 + 6S^2 \cot\theta D_{44} / R_2^2$

(14,2) $(S^3 L + L^2 S^2)(1+\mu) D_1 \cot\theta / R_2^2 - 2S^2 \cot\theta D_{45} / R_2^2 + D_{44} [2L - (S^2 + 2SL) \cot\theta / R_2] S \cot\theta / R_2^2$
$- 2D_6 (S+L) / R_2$

(15,2) $L^3 S (1+\mu) D_1 \cot\theta / R_2^2 - 6SL \cot\theta D_{45} / R_2^2 - 3L^2 S \cot^2\theta D_{44} / R_2^3$

COLUMN 3

(3,3) $D_1 + L\cot\theta D_{12}(2 + L\cot\theta D_1/D_{12}R_2)/R_2 + (D_{44}/R_2 + L\cot\theta D_{45}/R_2^2)/R_2 + [D_{45}/R_2 + \frac{L\cot\theta}{R_2^2}$

$\times L\cot\theta/$

(4,3) 0

(5,3) $D_{12} + L\cot\theta D_1/R_2 - D_{45}/R_2^2 - L\cot\theta D_{44}/R_2^3$

(6,3) 0

(7,3) $(1+\mu)D_1(L\cot\theta/R_2 + 1)/R_2$

(8,3) $S(1+\mu)D_1(L\cot\theta/R_2+1)/R_2$

(9,3) $L(1+\mu)D_1(L\cot\theta/R_2+1)/R_2 - D_{45}\cot\theta/R_2^2 - L\cot^2\theta D_{44}/R_2^3$

(10,3) $S^2(1+\mu)D_1(L\cot\theta/R_2+1)/R_2 + 2D_{45}/R_2 + 2L\cot\theta D_{44}/R_2^2$

(11,3) $SL(1+\mu)D_1(L\cot\theta/R_2+1)/R_2 - S\cot\theta D_{45}/R_2 - SL\cot^2\theta D_{44}/R_2^3$

(12,3) $L^2(1+\mu)D_1(L\cot\theta/R_2+1)/R_2 - [2D_{44} + 2L\cot\theta D_{45}/R_2]/R_2 - [2D_{45} + 2L\frac{\cot\theta}{R_2}D_{44}]\frac{L\cot}{R_2^2}$

(13,3) $S^3(1+\mu)D_1(L\cot\theta/R_2+1)/R_2 + 6SD_{45}/R_2 + 6SL\cot\theta D_{44}/R_2^2$

(14,3) $(S^2L+L^2S)(1+\mu)D_1(L\cot\theta/R_2+1)/R_2 + [-2SD_{44} + D_{45}[2L-(S^2+2SL)\cot\theta/R_2]]/R_2 +$

$+ [-2SD_{45} + D_{44}(2L-(S^2+2SL)\cot\theta/R_2]L\cot\theta/R_2^2$

(15,3) $L(1+\mu)D_1(L\cot\theta/R_2+1)/R_2 - [6LD_{44} + 3L^2\cot\theta D_{45}/R_2]/R_2$

COLUMN 4

(4,4) $\cot^2\theta D_3/R_2^2 + \cot^2\theta D_{66}/R_2^4$

(5,4) $s\cot^2\theta D_3/R_2^2 + s\cot^2\theta D_6/R_2^4$

(6,4) $-\cot\theta(1-L\cot\theta/R_2)D_3/R_2 - \cot\theta(1/R_2 - L\cot\theta/R_2^2)D_6/R_2^2$

(7,4) 0

(8,4) 0

(9,4) 0

(10,4) 0

(11,4) $\cot\theta D_6/R_2^2$

(12,4) 0

(13,4) 0

(14,4) $2\cot\theta(s+L)D_6/R_2^2$

(15,4) 0

COLUMN 5

(5,5) $D_1 + S^2\cot^2\theta\, D_3/R_2^2 + D_{44}/R_2^2 + S^2\cot^2\theta\, D_6/R_2^4$

(6,5) $-S\cot\theta(1 - L\cot\theta/R_2)D_3/R_2 - S\cot\theta\, D_6(1/R_2 - L\cot\theta/R_2^2)/R_2^2$

(7,5) $(1+\mu)D_1/R_2$

(8,5) $S(1+\mu)D_1/R_2$

(9,5) $L(1+\mu)D_1/R_2 + \cot\theta\, D_{44}/R_2^2$

(10,5) $S^2(1+\mu)D_1/R_2 - 2D_{44}/R_2$

(11,5) $SL(1+\mu)D_1/R_2 + S\cot\theta\, D_{44}/R_2^2 + S\cot\theta\, D_6/R_2^2$

(12,5) $L^2(1+\mu)D_1/R_2 + 2D_{45}/R_2 + 2L\cot\theta\, D_{44}/R_2^2$

(13,5) $S^3(1+\mu)D_1/R_2 - 6S\,D_{44}/R_2$

(14,5) $(S^2L+L^2S)(1+\mu)D_1/R_2 + 2S\,D_{45}/R_2 - D_{44}\left[2L - (S^2+2SL)\frac{\cot\theta}{R_2}\right]/R_2 + 2S\cot\theta(S+L)D_6/R_2^2$

(15,5) $L^3(1+\mu)D_1/R_2 + 6L\,D_{45}/R_2 + 3L^2\cot\theta\, D_{44}/R_2^2$

COLUMN 6

(6,6) $(1-L\cot\theta/R_2)^2(D_3+D_6/R_2^2)$

(7,6) 0

(8,6) 0

(9,6) 0

(10,6) 0

(11,6) $(L\cot\theta/R_2-1)D_6/R_2$

(12,6) 0

(13,6) 0

(14,6) $-2(1-L\cot\theta/R_2)(S+L)D_6/R_2$

(15,6) 0

COLUMN 7

(7,7) $(1+\mu)D_1/R_2^2$

(8,7) $S(1+\mu)D_1/R_2^2$

(9,7) $L(1+\mu)D_1/R_2^2$

(10,7) $S^2(1+\mu)D_1/R_2^2$

(11,7) $SL(1+\mu)D_1/R_2^2$

(12,7) $L^2(1+\mu)D_1/R_2^2$

(13,7) $S^3(1+\mu)D_1/R_2^2$

(14,7) $(S^2L+L^2S)(1+\mu)D_1/R_2^2$

(15,7) $L^3(1+\mu)D_1/R_2^2$

COLUMN 8

(8,8) $S^2(1+\mu)D_1/R_2^2$

(9,8) $SL(1+\mu)D_1/R_2^2$

(10,8) $S^3(1+\mu)D_1/R_2^2$

(11,8) $S^2L(1+\mu)D_1/R_2^2$

(12,8) $L^2S(1+\mu)D_1/R_2^2$

(13,8) $S^4(1+\mu)D_1/R_2^2$

(14,8) $(S^3L+L^2S^2)D_1(1+\mu)/R_2^2$

(15,8) $L^3S(1+\mu)D_1/R_2^2$

COLUMN 9

(9,9) $L^2(1+\mu)D_1/R_2^2 + \cot^2\theta D_{44}/R_2^2$

(10,9) $S^2L(1+\mu)D_1/R_2^2 - 2\cot\theta D_{44}/R_2^2$

(11,9) $SL^2(1+\mu)D_1/R_2^2 + S\cot^2\theta D_{44}/R_2^2$

(12,9) $L^3(1+\mu)D_1/R_2^2 + (2D_{45} + 2L\cot\theta D_{44}/R_2)\cot\theta/R_2$

(13,9) $S^3L(1+\mu)D_1/R_2^2 - 6S\cot\theta D_{44}/R_2$

(14,9) $(S^2L^2 + L^3S)(1+\mu)D_1/R_2^2 - \{-2SD_{45} + D_{44}[2L - (S^2+2SL)\cot\theta/R_2]\}\cot\theta/R_2$

(15,9) $L^4(1+\mu)D_1/R_2^2 + (6LD_{45} + 3L^2\cot\theta D_{44}/R_2)\cot\theta/R_2$

COLUMN 10

(10,10) $S^4(1+\mu)D_1/R_2^2 + 4D_{44}$

(11,10) $S^3L(D_{12}+D_1)/R_2^2 - 2S\cot\theta D_{44}/R_2$

(12,10) $L^2S^2(1+\mu)D_1/R_2^2 - 4D_{45} - 4L\cot\theta D_{44}/R_2$

(13,10) $S^5(1+\mu)D_1/R_2^2 + 12SD_{44}$

(14,10) $(S^4L+L^2S^3)(1+\mu)D_1/R_2^2 - 4SD_{45} + 2D_{44}[2L - ((S^2+2SL)\cot\theta/R_2)\cot\theta/R_2]$

(15,10) $L^3S^2(1+\mu)D_1/R_2^2 - 12LD_{45} - 6L^2\cot\theta D_{44}/R_2$

COLUMN 11

(11,11) $S^2L^2(1+\mu)D_1/R_2^2 + S^2\cot^2\theta D_{44}/R_2^2 + D_6$

(12,11) $SL^3(1+\mu)D_1/R_2^2 + 2S\cot\theta D_{45}/R_2 + 2LS\cot^2\theta D_{44}/R_2^2$

(13,11) $S^4L(1+\mu)D_1/R_2^2 - 6S^2\cot\theta D_{44}/R_2$

(14,11) $(S^3L^2+L^3S^2)(1+\mu)D_1/R_2^2 - \left\{-2SD_{45} + D_{44}\left[2L-(S^2+2SL)\cot\theta/R_2\right]\right\}S\cot\theta/R_2 + 2D_6(S+L)$

(15,11) $L^4S(1+\mu)D_1/R_2^2 + 6SL\cot\theta D_{45}/R_2 + 3L^2S\cot^2\theta D_{44}/R_2^2$

COLUMN 12

(12,12) $L^4(1+\mu)D_1/R_2^2 + 4D_{44}(1+L^2\cot^2\theta/R_2^2) + 8L\cot\theta D_{45}/R_2$

(13,12) $S^3L^2(1+\mu)D_1/R_2^2 - 12sD_{45} - 12SL\cot\theta D_{44}/R_2$

(14,12) $(S^2L^3+L^4S)(1+\mu)D_1/R_2^2 + \left[2L-(S^2+2SL)\cot\theta/R_2\right]\left[-2D_{45}-2L\cot\theta D_{44}/R_2\right] + 4SD_{44} + 4SL\cot\theta D_{45}/R_2$

(15,12) $L^5(1+\mu)D_1/R_2^2 + 12LD_{44} + 6L^2\cot\theta D_{45}/R_2 + 12L^2\cot^2\theta D_{45} + 6L^3\cot^2\theta D_{44}/R_2$

COLUMN 13

(13,13) $S^6(1+\mu)D_1/R_2^2 + 36S^2D_{44}$

(14,13) $(S^5L+L^2S^4)(1+\mu)D_1/R_2^2 - 12S^2D_{45} + 6SD_{44}[2L-(S^2+2SL)\cot\theta/R_2]$

(15,13) $L^3S^3(1+\mu)D_1/R_2^2 - 36SLD_{45} - 18L^2S\cot\theta D_{44}/R_2$

COLUMN 14

$$[*] = [2L - (S^2 + 2SL)\cot\theta / R_2]$$

(14,14) $(S^2L + L^2S)^2(1+\mu)D_1/R_2^2 + 4S^2D_{44} - 2SD_{45}[2L - (S^2+2SL)\cot\theta/R_2] - 2[*]SD_{45} +$

$$+ D_{44}[*]^2 + 4(S+L)^2 D_6$$

(15,14) $(S^2L^4 + SL^5)(1+\mu)D_1/R_2^2 + 12SLD_{44} + 6L^2S\cot\theta D_{45}/R_2 - 6[*]LD_{45} -$

$$-3[*]L^2\cot\theta D_{44}/R_2$$

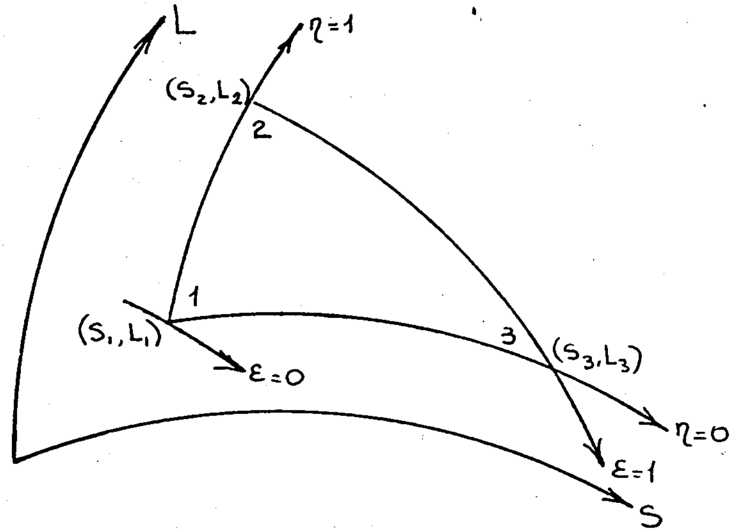
COLUMN 15

(15,15) $L^6(1+\mu)D_1/R_2^2 + 36L^2D_{44} + 18L^3\cot\theta D_{45}/R_2 + 18L^3\cot\theta D_{45}/R_2 + 9L^4\cot^2\theta D_{44}/R_2^2$

PART 4.

INTEGRATION OF INDIVIDUAL TERMS TO OBTAIN THE STIFFNESS MATRIX IN TERMS OF POLYNOMIAL COEFFICIENTS, FOR THE 15x15, TRIANGULAR, SPHERICAL SHELL ELEMENT USING TRIANGULAR COORDINATES.

TRIANGULAR COORDINATES AND JACOBIAN TRANSFORMATION.



$S = C_1 + C_2\varepsilon + C_3\eta + C_4\varepsilon\eta$ $\quad L = d_1 + d_2\varepsilon + d_3\eta + d_4\varepsilon\eta$

1- $\varepsilon=0 \quad \eta=0$

a) $S=S_1 \qquad C_1=S_1$

b) $L=L_1 \qquad d_1=L_1$

2- $\varepsilon=1 \quad \eta=0$

a) $S=S_3 \qquad C_2=S_3-S_1$

b) $L=L_3 \qquad d_2=L_3-L_1$

3- $\varepsilon=0 \quad \eta=1$

a) $S=S_1 \qquad C_3=0$

b) $L=L_1 \qquad d_3=0$

4- $\varepsilon=1 \quad \eta=1$

a) $S=S_2 \qquad C_4=S_2-S_3$

b) $L=L_2 \qquad d_4=L_2-L_3$

$$S = S_1 + (S_3 - S_1)\varepsilon + (S_2 - S_3)\varepsilon\eta$$

$$S = S_1 + \varepsilon(S_{31} - S_{32}\eta)$$

$$L = L_1 + (L_3 - L_1)\varepsilon + (L_2 - L_3)\varepsilon\eta$$

$$L = L_1 + \varepsilon(L_{31} - L_{32}\eta)$$

$$J = \begin{vmatrix} S_{31} - S_{32}\eta & -S_{32}\varepsilon \\ L_{31} - L_{32}\eta & -L_{32}\varepsilon \end{vmatrix} = \varepsilon(S_{32}L_{31} - L_{32}S_{31}) = A\varepsilon$$

SAMPLE INTEGRATION:

$$\int_V S\,dV = \int_{\varepsilon=0}^{1}\int_{\eta=0}^{1} A\varepsilon\left[S_1+\varepsilon(S_{31}-S_{32}\eta)\right]d\varepsilon\, d\eta\, t$$

$$= At\int_{\varepsilon=0}^{1}\left[\varepsilon S_1+\varepsilon^2\left(S_{31}-\frac{S_{32}}{2}\right)\right]$$

$$= At\left[\frac{S_1}{2}+\frac{1}{3}\left(S_{31}-\frac{S_{32}}{2}\right)\right]$$

$$= \frac{At}{6}\left(S_1+S_2+S_3\right) \quad \text{but} \quad S_1=0$$

$$= \frac{At}{6}\left(S_2+S_3\right)$$

INTEGRATION OF:

$S^2 \quad = -At\left(S_2^3-S_3^3\right)/12S_{32}$

$S^3 \quad = -At\left(S_2^4-S_3^4\right)/20S_{32}$

$S^4 \quad = -At\left(S_2^5-S_3^5\right)/30S_{32}$

$S^5 \quad = -At\left(S_2^6-S_3^6\right)/42S_{32}$

$S^6 \quad = -At\left(S_2^7-S_3^7\right)/56S_{32}$

INTEGRATION OF:

$$L \qquad = AtL_2/6$$

$$L^2 \qquad = AtL_2^2/12$$

$$L^3 \qquad = AtL_2^3/20$$

$$L^4 \qquad = AtL_2^4/30$$

$$L^5 \qquad = AtL_2^5/42$$

$$L^6 \qquad = AtL_2^6/56$$

$$SL \qquad = At(S_3+S_2)L_2/4\times 6$$

$$S^2L \qquad = \frac{AtL_2}{5S_{32}^2}\left[\frac{1}{4}\left(S_2^4-S_3^4\right)+\frac{S_3}{2}\left(S_3^3-S_2^3\right)\right]$$

$$S^3L \qquad = \frac{AtL_2}{6S_{32}^2}\left[\frac{1}{5}\left(S_2^5-S_3^5\right)+\frac{S_3}{4}\left(S_3^4-S_2^4\right)\right].$$

$$S^4L \qquad = \frac{AtL_2}{7S_{32}^2}\left[\frac{1}{6}\left(S_2^6-S_3^6\right)+\frac{S_3}{5}\left(S_3^5-S_2^5\right)\right]$$

$$S^5L \qquad = \frac{AtL_2}{8S_{32}^2}\left[\frac{1}{7}\left(S_2^7-S_3^7\right)+\frac{S_3}{6}\left(S_3^6-S_2^6\right)\right]$$

INTEGRATION OF:

$L^2S \quad = A + L_2^2(3S_2 + S_3)/60$

$L^3S \quad = A + L_2^3(4S_2 + S_3)/120$

$L^4S \quad = A + L_2^4(5S_2 + S_3)/210$

$L^5S \quad = A + L_2^5(6S_2 + S_3)/336$

$L^2S^2 \quad = A + L_2^2(6S_2^2 + 3S_2S_3 + S_3^2)/30x6$

$L^3S^2 \quad = A + L_2^3(10S_2^2 + 4S_3S_2 + S_3^2)/42x10$

$L^4S^2 \quad = A + L_2^4(15S_2^2 + 5S_3S_2 + S_3^2)/56x15$

$L^2S^3 \quad = A + L_2^2(10S_2^3 + 6S_3S_2^2 + 3S_3^2S_2 + S_3^3)/42x10$

$L^2S^4 \quad = A + L_2^2(15S_2^4 + 10S_3S_2^3 + 6S_3^2S_2^2 + 3S_3^3S_2 + S_3^4)/56x15$

$L^3S^3 \quad = A + L_2^3(20S_2^3 + 10S_2^2S_3 + 4S_3^2S_2 + S_3^3)/56x20$

APPENDIX II

```
ZZJOB
ZZFOR
*LDISKSHELL
C     CYLINDRICAL SHELL ANA,YSIS
      DIMENSION NCODE(24,10),SM(36,37),P(26),V(26),JNO(4,10),B(24,24)
      DIMENSION MDUM(24)
 999  CONTINUE
      READ 1
 1    FORMAT(1HX,79H
     1                                        )
       READ 5555,MS
 5555 FORMAT(2I4)
      PUNCH 1
      READ 2,E,AIFEET,VU,H,S,A,R
      READ 555,((B(I,J),I=1,MS),J=1,MS)
 555  FORMAT(5E16.8)
      PUNCH 101,E,AIFEET,VU,H,S,A,R
 101  FORMAT(4X,2HE=,F14.0//4X,7HAIFEET=,F6.0//
     14X,16HPOISSON,S RATIO=,F10.2/1X,10HTHICKNESS=,F10.3/
     21X,5HSIDE=,F10.3/1X,16HHORIZANTAL SIDE=,F10.3/7HRADIUS=,F10.3)
      PUNCH 1011
 1011 FORMAT(1X,64HAIFEET=0. (ALL UNITS KIP-FOOT,EXCEPT FEED A AND I
     1 IN INCHES)/1X,31HAIFEET=1.  (ALL UNITS KIP-FOOT)/1X,
     247HAIFEET=2. (METRIC UNITS HOMOGENEOUS THROUGHOUT)/)
C     CONVERT PSI TO KSF IF ENGLISH SYSTEM
      IF(AIFEET-1.)102,102,103
 102  E=E*(144./1000.)
 103  CONTINUE
      READ 4,ME,NJ,N
      PUNCH 40,ME,NJ,N
 40   FORMAT(/10X,20HNUMBER OF MEMBERS = ,I4/10X,19HNUMBER OF JOINTS  =I
     14/10X,20HNUMBER OF UNKNOWNS= ,I4/)
 4    FORMAT(9I4,4X,5F8.3)
 2    FORMAT(7F10.3)
      PUNCH 12
 12   FORMAT(///20X,12HCODE NUMBERS/7HMEM NO.,12X,9HFIRST JO.,
     110HSECOND JO.,9HTHIRD JO.,10HFOURTH JO.)
      FAC=1.
      IF(AIFEET-0.01)861,862,862
 861  FAC=144.
 862  CONTINUE
      DO 16 I=1,ME
      L=I
      READ 44,MNO,J1,J2,J3,J4
      PUNCH 444,MNO,J1,J2,J3,J4
 444  FORMAT(I4,12X,4(I4,10X))
      JNO(1,I)=J1
      JNO(2,I)=J2
      JNO(3,I)=J3
      JNO(4,I)=J4
      READ 44,(MDUM(J),J=1,MS)
      PUNCH 44,(MDUM(J),J=1,MS)
      DO 18 J=1,MS
 18   NCODE(J,L)=MDUM(J)
 44   FORMAT(20I4)
 13   FORMAT(I3,4I4,1X,20I3)
 16   CONTINUE
      PUNCH 4,MS
      CALL SGEN(N,SM,ME,MS,NCODE,B)
      PUNCH 4,MS
C     READ LOAD DATA
      DO 54 I=1,N
 54   P(I)=0.
```

```
      PUNCH 50
 50   FORMAT(///20X,18HDIRECT JOINT LOADS/)
      READ 51,(P(J),J=1,N)
 51   FORMAT(8F10.2)
      PUNCH 51,(P(J),J=1,N)
      JP=N+1
      DO 681 I=1,N
 681  SM(I,JP)=P(I)
C     SOLVE BY GAUSS ELIMINATION -
      PUNCH 57
 57   FORMAT (//20X,18HJOINT DEFORMATIONS/)
      PUNCH 4,MS
      C^LL GAUSS(SM,N,1)
      P JNCH 4,MS
      DO 682 I=1,N
 682  V(I)=SM(I,JP)
      CALL BACK(ME,MS,NCODE,V,SM,B)
      GO TO 999
 8581 DUM=LOGF(DUM)
      DUM=ABS(1.)
      DUM=1.E-20
      DUM=EXPF(DUM)
      END
ZZZZ
ZZJOB
ZZFOR
*LDISKSGEN
      SUBROUTINE SGEN(N,SM,ME,MS,NCODE,B)
      DIMENSION SM(36,37),B(24,24),NCODE(24,10)
C     GENERATE MASTER STIFFNESS MATRIX
      DO 17 K=1,N
      DO 17 L=1,N
 17   SM(K,L)=0.
      DO 9 M=1,ME
      DO 8 I=1,MS
      SAYN1=1.
      K=NCODE(I,M)
      IF(K)20,8,22
 20   SAYN1=-1.
      K=-K
 22   DO 70 J=1,MS
      SAYN2=1.
      L=NCODE(J,M)
      IF(L)30,70,32
 30   SAYN2=-1.
      L=-L
 32   SM(K,L)=SM(K,L)+B(I,J)*SAYN1*SAYN2
 70   CONTINUE
 8    CONTINUE
 9    CONTINUE
      RETURN
      E^ID
ZZZZ
ZZJOB
ZZFOR
*LDISKGAUSS
      SUBROUTINE GAUSS(A,N,NLOAD)
      DIMENSION A(36,37)
      NP=N+NLOAD
      DO 13 K=1,N
      KP=K+1
      R=1./A(K,K)
      DO 11 J=KP,NP
```

```
 11    A(K,J)=R*A(K,J)
       DO 13 I=1,N
       IF(I-K) 14,13,14
 14    S=A(I,K)
       DO 12 J=KP,NP
 12    A(I,J)=A(I,J)-S*A(K,J)
 13    CONTINUE
 101   FORMAT(18I4)
       NPL=N+1
       DO 777 J=NPL,NP
       INDEX=J-N
       PUNCH 101  ,INDEX
       PUNCH 47,(L,A(L,J),L=1,N)
 777   PUNCH 471
 47    FORMAT(3(I4,F20.10))
 471   FORMAT(////)
       RETURN
       END
ZZZZ
ZZJOB
ZZFOR
*LDISKBACK
       SUBROUTINE BACK(ME,MS,NCODE,V,SM,B)
       DIMENSION NCODE(24,10),V(26),SM(36,37),B(24,24)
C      MULTIPLY MEMBER DEFORMATIONS BY MEMBER STIFFNESSES
       P JNCH 85
 85    FORMAT (///,20X,32HMEMBER END FORCES IN COMMON AXES//9H MEM. NO.,
      18X,1H1,10X,1H2,10X,1H3,13X,1H4,10X,1H5,10X,1H6/)
       DO 86 I=1,ME
       DO 87 K=1,MS
       KN=NCODE(K,I)
       SAYN=1.
       IF(KN) 88,89,9
 88    KN=-KN
       SAYN=-1.
 90    SM(K,1)=V(KN)*SAYN
       GO TO 87
 89    SM(K,1)=0.
 87    CONTINUE
C      PRODUCT OF STIFFNESS WITH DEFORMATIONS WHICH ARE STORED INTO
C      THE FIRST COLOMN OF SMMATRIX
       DO 91 J=1,MS
       SUM=0.
       DO 92 K=1,MS
 92    SUM=SUM+B(J,K)*SM(K,1)
 91    SM(J,2)=SUM
       PUNCH 93,I,(SM(J,2),J=1,MS)
 93    FORMAT(I5,6X,5F12.3/(11X,5F12.3))
 86    CONTINUE
       RETURN
       END
ZZZZ
```

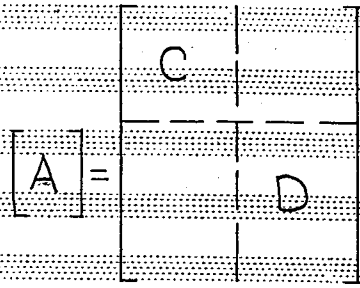
```
ZZJOB
ZZFOR
*LDISKSTIFF1
C     FORMATION OF THE MEMBER STIFFNESS MATRIX
      DIMENSION T(24,24),C(24,24),V(24),DUM(1),DAM(1,1)
      COMMON T,C,V,DUM,DAM
      READ 2,E,VU,H,B,A,R,AN
 2    FORMAT(7F10.3)
      DO 16 J=1,1
 9    DUM(J)=0.
 16   DAM(J,J)=0.
      CALL TEBIR(E,H,VU,B,A,R,T)
      CALL TEIKI(E,H,VU,B,A,R,T)
      CALL TEUC(E,H,VU,B,A,R,T)
      CALL CECE(A,B,VU,R,AN,C)
      DEM=SIN(DEM)
      DEM=COS(DEM)
      CALL EXIT
      END
ZZZZ
ZZJOB
ZZFOR
*LDISKSTIFF2
      DIMENSION T(24,24),C(24,24),V(24),DUM(1),DAM(1,1)
      COMMON T,C,V,DUM,DAM
      CALL INVERT(C,24,24,DET,COND)
      PRINT 2,((C(I,J),J=1,6),I=1,24)
      PRINT 2,((C(I,J),J=7,12),I=1,24)
      PRINT 2,((C(I,J),J=13,18),I=1,24)
      PRINT 2,((C(I,J),J=19,24),I=1,24)
 2    FORMAT(2X,6F13.3)
      CALL TRIPLE(C,T,C,24,24,V)
      PUNCH 6
      PUNCH 5,((C(I,J),J=1,6),I=1,24)
      PUNCH 6
      PUNCH 5,((C(I,J),J=7,12),I=1,24)
      PUNCH 6
      PUNCH 5,((C(I,J),J=13,18),I=1,24)
      PUNCH 6
      PUNCH 5,((C(I,J),J=19,24),I=1,24)
      PUNCH 6
 5    FORMAT(6F13.2)
 6    FORMAT(///)
      PUNCH 555,C
 555  FORMAT(5E16.8)
 1181 DEM=LOGF(DEM)
      DEM=ABS(11.)
      DEM=1.E-20
      DEM=EXPF(DEM)
      CALL EXIT
      END
ZZZZ
ZZJOB
ZZFOR
*LDISKINVERT
      SUBROUTINE INVERT(A,N,M,DET,COND)
      DIMENSION A(2),IP(24)
C     DIMENSION A(2),IP(24)
C     N=ACTUAL SIZE TO BE INVERTED
C     M=DIMENSION SIZE OF THE MATRIX TO BE INVERTED
      MN1=M*(N-1)
      EPS=1.E-20
      DO 41 I=1,N
```

```
 41     IP(I)=I
C       IP(K) WILL KEEP TRACK OF WHICH ROW ENDS UP AS K*TH PIVOT ROW
        DET=1.
C
C       INVERSION STARTS
        DO 198 K=1,N
        DO 199 K=1,N
        MK=M*K-M
        KK=K+MK
C       FIND MAXIMUM ELEMENT IN K*TH COLUMN
        AMAX=ABS(A(KK))
        IMAX=K
        IF(K-N) 4,65,65
  4     KP=K+1
        DO 60 I=KP,N
        IK=I+MK
        AIK=ABS(A(IK))
        IF(AIK-AMAX) 60,60,5
  5     AMAX=AIK
        IMAX=I
 60     CONTINUE
C       TEST FOR SINGULARITY
 65     IF(AMAX-EPS) 300,6,6
  6     LAST=MN1+K
C       INTERCHANGE ROWS K AND IMAX
        IF(K-IMAX) 3,100,3
  3     IMAXJ=IMAX
        DO 75 KJ=K,LAST,M
        T=A(KJ)
        A(KJ)=A(IMAXJ)
        A(IMAXJ)=T
 75     IMAXJ=IMAXJ+M
        J=IP(K)
        IP(K)=IP(IMAX)
        IP(IMAX)=J
C       COMPUTE DETERMINANT
 100    CONTINUE
C       DIVIDE K*TH ROW BY A(K,K)
        T=1./A(KK)
        A(KK)=1.0
        DO 140 KJ=K,LAST,M
 140    A(KJ)=A(KJ)*T
C       SUBTRACT A(I,K) TIMES THE K*TH ROW FROM THE OTHER ROWS
        DO 199 I=1,N
        IF(I-K) 1,199,1
  1     IK=I+MK
        T=A(IK)
        A(IK)=0.0
        IJ=I
        DO 190 KJ=K,LAST,M
        A(IJ)=A(IJ)-T*A(KJ)
 190    IJ=IJ+M
 199    CONTINUE
 198    CONTINUE
C
C       RESTORE PROPER COLUMN ORDER IN THEINVERSE
        DO 250 K=1,N
        MK=M*K-M
C       COLUMN NOW OCCUPYING K*TH POSITION IS ACTUALLY COLUMN ...
 210    J=IP(K)
C       ...OF THE INVERSE. HENCE ...
        IF(J-K) 2,250,2
C       RELOCATE COLUMN K TO ITS FINAL POSITION
```

```
 2      MJ=M*J-M
        DO 225 I=1,N
        IJ=I+MJ
        IK=I+MK
        T=A(IJ)
        A(IJ)=A(IK)
 225    A(IK)=T
C       ADJUST IP RECORD
        IP(K)=IP(J)
        IP(J)=J
C       AND CHECK NEW K*TH COLUMN
        GO TO 210
 250    CONTINUE
        RETURN
C
C       PROCEDURE FOR SINGULAR OR NEARLY SINGULAR MATRIX.

 300    PUNCH 310,K,AMAX
 310    FORMAT(6H  STEPI3,7H PIVOT=1PE15.8,24H, INVERSION DISCONTINUED/
       1)
        DET=0.
        COND=1.E38
        RETURN
        END
ZZZZ
ZZJOB
ZZFOR
*LDISKTRIPLE
        SUBROUTINE TRIPLE(A,C,D,N,NR,V)
        DIMENSION A(24,24),C(24,24),D(24,24),V(24)
C       A*=(A TRANSPOSE)*C*D
C       A,C,D ARE AVAILABLE THRU ARGUMENT OR COMMON STATEMENT
C       N=ROWS OF C,NR=COLUMNS OF C
C       MULTIPLY(A TRANSPOSE)*C,STORE IN C
        DO 10 J=1,NR
        DO 11 I=1,N
        SUM=0.
        DO 12 K=1,N
 12     SUM=SUM+A(K,I)*C(K,J)
 11     V(I)=SUM
        DO 13 I=1,N
 13     C(I,J)=V(I)
 10     CONTINUE
C       MULTIPLY(NEW C)*D,STORE IN A
        DO 20 J=1,NR
        DO 21 I=1,N
        SUM=0.
        DO 22 K=1,NR
 22     SUM=SUM+C(I,K)*D(K,J)
 21     V(I)=SUM
        DO 23 I=1,N
 23     A(I,J)=V(I)
 20     CONTINUE
        RETURN
        END
ZZZZ
```

```
ZZJOB    Ho and A Matrices for the 20x20 Cylindrical Shell Element.
ZZFOR
*LDISKCC
      SUBROUTINE CC(A,S,VU,R,C)
      DIMENSION    C(20,20)
      DO 220 I=1,20
      DO 220 J=1,20
      C(I,J)=0.
 220  CONTINUE
      C(1,1)=1.
      C(2,1)=1.
      C(3,1)=1.
      C(4,1)=1.
      C(5,6)=1.
      C(6,6)=1.
      C(7,6)=1.
      C(8,6)=1.
      C(1,2)=A
      C(3,2)=A
      C(5,7)=A
      C(7,7)=A
      C(2,2)=-A
      C(4,2)=-A
      C(6,7)=-A
      C(8,7)=-A
      C(1,3)=S
      C(2,3)=S
      C(5,8)=S
      C(6,8)=S
      C(3,3)=-S
      C(4,3)=-S
      C(7,8)=-S
      C(8,8)=-S
      C(1,4)=-(VU*A*A+S*S)
      C(2,4)=C(1,4)
      C(3,4)=C(1,4)
      C(4,4)=C(1,4)
      C(1,5)=2.*A*S
      C(2,5)=-C(1,5)
      C(3,5)=-C(1,5)
      C(4,5)=C(1,5)
      C(5,4)=C(1,5)
      C(6,4)=-C(1,5)
      C(7,4)=-C(1,5)
      C(8,4)=C(1,5)
      C(5,5)=-(A*A+VU*S*S)
      C(6,5)=C(5,5)
      C(7,5)=C(5,5)
      C(8,5)=C(5,5)
      C(16,8)=-S/R
      C(15,8)=-S/R
      C(14,8)=S/R
      C(13,8)=S/R
      C(16,7)=-A/R
      C(15,7)=A/R
      C(14,7)=-A/R
      C(13,7)=A/R
      C(16,6)=1./R
      C(15,6)=1./R
      C(14,6)=1./R
      C(13,6)=1./R
      C(16,5)=-(A*A+VU*S*S)/R
      C(15,5)=-(A*A+VU*S*S)/R
```



```
      C(14,5)=-(A*A+VU*S*S)/R
      C(13,5)=-(A*A+VU*S*S)/R
      C(16,4)=2.*S*A/R
      C(15,4)=-2.*A*S/R
      C(14,4)=-2.*A*S/R
      C(13,4)=2.*S*A/R
      RETURN
      END
ZZZZ
ZZJOB
ZZFOR
*LDISKDD
      SUBROUTINE DD(A,S,D)
      DIMENSION D(12,12)
      DO 11 L=1,3
      DO 10 N=1,4
      GO TO(201,202,203,204),N
 201  X=A
      Y=S
      GO TO 60
 202  X=-A
      Y=S
      GO TO 60
 203  X=A
      Y=-S
      GO TO 60
 204  X=-A
      Y=-S
 60   I=(L-1)*4+N
      GO TO(101,102,103),L
 101  D(I,1)=1.
      D(I,2)=X
      D(I,3)=Y
      D(I,4)=X*X
      D(I,5)=X*Y
      D(I,6)=Y*Y
      D(I,7)=X**3
      D(I,8)=X*X*Y
      D(I,9)=Y*Y*X
      D(I,10)=Y**3
      D(I,11)=(X**3)*Y
      D(I,12)=X*(Y**3)
      GO TO 10
 102  D(I,1)=0.
      D(I,2)=0.
      D(I,3)=1.
      D(I,4)=0.
      D(I,5)=X
      D(I,6)=2.*Y
      D(I,7)=0.
      D(I,8)=X*X
      D(I,9)=2.*Y*X
      D(I,10)=3.*Y*Y
      D(I,11)=X**3
      D(I,12)=3.*X*Y**2
      GO TO 10
 103  D(I,1)=0.
      D(I,2)=-1.
      D(I,3)=0.
      D(I,4)=-2.*X
      D(I,5)=-Y
      D(I,6)=0.
      D(I,7)=-3.*X*X
```

```
      D(I,8)=-2.*Y*X
      D(I,9)=-Y*Y
      D(I,10)=0.
      D(I,11)=-3.*X*X*Y
      D(I,12)=-Y**3
 10   CONTINUE
 11   CONTINUE
      RETURN
      END
ZZZZ
ZZJOB
ZZFOR
*LDISKTT
      SUBROUTINE TT(E,H,VU,S,A,R,T)
      DIMENSION T(20,20)
      A1=4.*A*S
      D11=E*H/(1.-VU*VU)
      D12=VU*D11
      D44=E*(H**3)/(12.*(1.-VU*VU))
      D66=D33*(H**2)/3.
      D45=VU*D44
      D33=E*H/(2.+2.*VU)
      DO 224 I=1,20
      DO 224 J=1,20
      T(I,J)=0.
 224  CONTINUE
      T(5,5)=16.*A*(S**3.)*D11*(1.-VU*VU)/3.
     1+16.*(S**3.)*A*VU*VU*D44/(3.*R*R)+(16.*S*(A**3.)*D66)/(3.*R*R)
      T(4,4)=16.*S*(A**3.)*D11*(1.-VU**2.)/3.
     1+16.*S*(A**3.)*D44/(3.*R*R)+16.*(S**3.)*A*D66/(3.*R*R)
      T(2,2)=A1*D11
      T(3,3)=A1*D33
      T(7,7)=A1*D33+A1*D66/(R*R)
      T(8,8)=A1*D11+A1*D44/R**2.
      T(8,2)=A1*D12
      T(7,3)=A1*D33
      T(2,8)=T(8,2)
      T(3,7)=T(7,3)
      RETURN
      END
ZZZZ
ZZJOB
ZZFOR
*LDISKUU
      SUBROUTINE UU(E,H,VU,S,A,R,U)
      DIMENSION U(8,12)
      A1=4.*A*S
      D11=E*H/(1.-VU*VU)
      D12=VU*D11
      D33=E*H/(2.+2.*VU)
      D44=E*(H**3)/(12.*(1.-VU*VU))
      D45=VU*D44
      D66=D33*(H**2)/3.
      DO 221 I=1,20
      DO 221 J=1,20
      U(I,J)=0.
 221  CONTINUE
      U(2,1)=-A1*D12/R
      U(2,4)=-4.*S*(A**3.)*D12/(3.*R)
      U(2,6)=(U(2,4))*S*S/(A*A)
      U(4,2)=(-8.*S*(A**3.)*(D11-/U*D12))/(3.*R)
      U(4,7)=-8.*(A**5.)*S*D11*(1.-VU*VU)/(5.*R)+16.*S*(A**3.)*D45/R
      U(5,8)=(-16.*(S**3
```

$$[H_0] = \begin{bmatrix} T & U \\ U^T & W \end{bmatrix}$$

```
      U(5,10)=-16.*(S**3.)*A*VU*D44/R
      U(7,5)=A1*D66/R
      U(7,11)=4.*S*(A**3.)*D66/R
      U(7,12)=4.*(S**3.)*A*D66/R
      U(8,4)=-4.*S*(A**3.)*D11/(3.*R)+2.*A1*D45/R
      U(8,1)=-A1*D11/R
      U(8,6)=-4.*(S**3.)*A*D11/(3.*R)+2.*A1*D44/R
      U(4,9)=-8.*(A**3.)*(S**3.)*D11*(1.-VU*VU)/(9.*R)
     1+16.*S*(A**3.)*D44/(3.*R)+16.*(S**3.)*A*D66/(3.*R)
      RETURN
      END
ZZZZ
ZZJOB
ZZFOR
*LDISKWW
      SUBROUTINE WW(E,H,VU,;,A,R,W)
      DIMENSION W(12,12)
      D11=E*H/(1.-VU*VU)
      D12=VU*D11
      D44=E*(H**3)/(12.*(1.-VU*VU))
      D45=VU*D44
      D66=D33*(H**2)/3.
      D33=E*H/(2.+2.*VU)
      A1=4.*A*S
      ET=4.*(A**3.)*S/(R*R)
      ES=4.*A*(S**3.)/(R*R)
      DO 223 I=1,20
      DO 223 J=1,20
  223 W(I,J)=0.
      W(1,1)=A1*D11/R**2.
      W(1,4)=ET*D11/3.
      W(1,6)=ES*D11/3.
      W(2,2)=ET*D11/3.
      W(2,7)=ET*A*A*D11/5.
      W(2,9)=ET*S*S*D11/9.
      W(3,3)=ES*D11/3.
      W(3,8)=W(2,9)
      W(3,10)=ES*S*S*D11/5.
      W(4,4)=W(2,7)+4.*A1*D44
      W(4,6)=W(2,9)+4.*A1*D45
      W(5,5)=ET*S*S*D11/9.+A1*D66
      W(5,11)=ET*A*A*S*S*D11/15.+ET*D66*R*R
      W(5,12)=ES*A*A*S*S*D11/15.+ES*D66*R*R
      W(6,6)=W(3,10)+4.*A1*D44
      W(7,7)=ET*(A**4.)*D11/7.+12.*ET*D44*R*R
      W(7,9)=ET*A*A*S*S*D11/15.+4.*ET*D45*R*R
      W(8,8)=ET*A*A*S*S*D11/15.+4.*ES*D44*R*R/3.
     1+4.*ET*D66*R*R/3.
      W(8,10)=(W(7,9))*S*S/(A**2.)
      W(9,9)=ES*S*S*A*A*D11/15.+4.*ET*D44*R*R/3.+4.*ES*D66*R*R/3.
      W(10,10)=ES*(S**4.)*D11/7.+12.*ES*D44*R*R
      W(11,11)=ET*(A**4.)*S*S*D11/21.+4.*ET*S*S*D44*R*R
     1+).*ET*A*A*D66*R*R/5.
      W(11,12)=ET*A*A*(S**4.)*D11/25.+4.*ES*A*A*R*R*D45+ES*A*A*R*R*D66
      W(12,12)=ES*(S**4.)*A*A*D11/21.+4.*ET*S*S*D44*R*R
     I+9.*ES*S*S*D66*R*R/5.
      DO 17 L=2,12
      LM=L-1
      DO 17 J=1,LM
   17 W(L,J)=W(J,L)
      RETURN
      END
ZZZZ
```

H_o and A Matrices for the 24x24 Cylindrical Shell Element with Slope Continuity Term.

```
ZZJOB
ZZFOR
*LDISKTEBIR
      SUBROUTINE TEBIR(E,H,VU,B,A,R,T)
      DIMENSION T(24,24)
      D1=E*H/(1.-VU*VU)
      D2=VU*D1
      D3=E*H/(2.+2.*VU)
      D4=(H**2)*D1/12.
      D5=VU*D4
      D6=E*(H**3)/(24.*(1.+VU))
      DO 16 I=1,24
      DO 16 J=1,24
 16   T(I,J)=0.
      T(1,1)=16.*B*(A**3)*(1.-VU*VU)*D1/3.+16.*B*(A**3)*D4/(3.*R**2)
     1+64.*A*(B**3)*D6/(3.*R*R)
      T(1,10)=(8.*(B**3)*(A**5)*(1.-VU*VU)*D1/(15.*R))
     1-(48.*(A**3)*(B**3)*D5/(9.*R))-(16.*(A**5)*B*D4/(5.*R))
     2-(192.*(A**3)*(B**3)*D6/(9.*R))
      T(1,12)=8.*(A**5)*B*(1.-VU*VU)*D1/(5.*R)-48.*(A**3)*B*D5/(3.*R)
      T(1,18)=8.*(A**3)*(B**3)*(1.-VU*VU)*D1/(9.*R)
     1-16.*(A**3)*B*D4/(3.*R)-64.*(B**3)*A*D6/(3.*R)
      T(2,2)=16.*(B**3)*A*(1.-VU*VU)*D1/(3.)
     1+16.*VU*VU*A*(B**3)*D4/(3.*R*R)+64.*B*(A**3)*D6/(3.*R*R)
      T(2,13)=16.*VU*A*(B**5)*D5/(5.*R)
     1+48.*VU*(A**3)*(B**3)*D4/(9.*R)+192.*(A**3)*(B**3)*D6/(9.*R)
      T(2,15)=16.*(B**3)*A*VU*D5/(3.*R)+64.*B*(A**3)*D6/(3.*R)
      T(2,21)=48.*VU*A*(B**3)*D4/(3.*R)
      T(3,3)=4.*A*B*D1
      T(3,7)=4.*A*B*D2
      T(3,14)=4.*(A**3)*(B**3)*D2/(9.*R)
      T(3,16)=4.*B*(A**3)*D2/(3.*R)
      T(3,22)=4.*(B**3)*A*D2/(3.*R)
      T(4,4)=4.*A*B*D3
      T(7,7)=4.*A*B*D1+(4.*A*B*D4/(R*R))
      T(7,14)=4.*(A**3)*(B**3)*D1/(9.*R)-8.*(B**3)*A*D5/(3.*R)
     1-8.*(A**3)*B*D4/(3.*R)
      T(7,16)=4.*B*(A**3)*D1/(3.*R)-8.*A*B*D5/R
      T(7,22)=4.*(B**3)*A*D1/(3.*R)-8.*A*B*D4/R
      T(9,9)=4.*(A**7)*(B**7)*D1/(49.*R*R)+144.*(A**3)*(B**7)*D4/21.
     1+288.*(A**5)*(B**5)*D5/25.+144.*(B**3)*(A**7)*D4/21.
     2+1296.*(A**5)*(B**5)*D6/25.
      T(9,11)=4.*(A**7)*(B**5)*D1/(35.*R*R)+144.*(A**3)*(B**5)*D4/15.
     1+144.*(A**5)*(B**3)*D5/15.+432.*(A**5)*(B**3)*D6/15.
      T(9,17)=(4.*(A**5)*(B**7)*D1/(35.*R*R))
     1+(144.*(B**5)*(A**3)*D5/15.)
     2+(144.*(A**5)*(B**3)*D4/15.)+(432.*(B**5)*(A**3)*D6/15.)
      T(9,19)=4.*(A**5)*(B**5)*D1/(25.*R*R)+144.*(A**3)*(B**3)*D6/9.
      T(10,10)=4.*(A**7)*(B**5)*D1/(35.*R*R)+144.*(B**5)*(A**3)*D4/15.
     1+16.*(A**7)*B*D4/7.+96.*(A**5)*(B**3)*D5/15.
     2+576.*(B**3)*(A**5)*D6/15.
      T(10,12)=4.*(A**7)*(B**3)*D1/(21.*R*R)+144.*(A**3)*(B**3)*D4/9.
     1+48.*(A**5)*B*D5/5.
      T(10,18)=4.*(A**5)*(B**5)*D1/(25.*R*R)+48.*(A**3)*(B**3)*D5/9.
     1+16.*(A**5)*(B)*D4/5.+192.*(A**3)*(B**3)*D6/9.
      T(11,11)=4.*(A**7)*(B**3)*D1/(21.*R*R)+144.*(A**3)*(B**3)*D4/9.
```

```
     1+144.*(A**5)*B*D6/5.
      T(11,17)=4.*(A**5)*(B**5)*D1/(25.*R*R)+144.*(A**3)*(B**3)*D5/9.
     1+144.*(A**3)*(B**3)*D6/9.
      T(11,19)=4.*(B**3)*(A**5)*D1/(15.*R*R)+16.*B*(A**3)*D6
      T(12,12)=4.*(A**7)*B*D1/(7.*R*R)+144.*(A**3)*B*D4/3.
      T(12,18)=4.*(A**5)*(B**3)*D1/(15.*R*R)+16.*(A**3)*(B)*D5
      T(13,13)=4.*(A**5)*(B**7)*D1/(35.*R*R)+16.*(B**7)*A*D4/7.
     1+144.*(B**3)*(A**5)*D4/15.+96.*(B**5)*(A**3)*D5/15.
     2+576.*(A**3)*(B**5)*D6/15.
      T(13,15)=4.*(A**5)*(B**5)*D1/(25.*R*R)+16.*(B**5)*A*D4/5.
     1+48.*(A**3)*(B**3)*D5/9.+192.*(A**3)*(B**3)*D6/9.
      T(13,21)=4.*(A**3)*(B**7)*D1/(21.*R*R)+48.*(B**5)*A*D5/5.
     1+144.*(A**3)*(B**3)*D4/9.
      T(14,14)=4.*(A**5)*(B**5)*D1/(25.*R*R)+16.*(B**5)*A*D4/5.
     1+16.*(A**5)*B*D4/5.+32.*(A**3)*(B**3)*D5/9.
     2+256.*(A**3)*(B**3)*D6/9.
      T(14,16)=4.*(A**5)*(B**3)*D1/(15.*R*R)+16.*(B**3)*A*D4/3.
     1+16.*(A**3)*B*D5/3.
      T(14,22)=(4.*(A**3)*(B**5)* )1/(15.*R*R))+(16.*(B**3)*A*D5/3.)
     1+(16.*(A**3)*B*D4/3.)
      T(15,15)=(4.*(A**5)*(B**3)*D1/(15.*R*R))+(16.*A*(B**3)*D4/3.)
     1+(64.*(A**3)*B*D6/3.)
      T(15,21)=(4.*(A**3)*(B**5)*D1/(15.*R*R))+(16.*(B**3)*A*D5)
      T(16,16)=4.*(A**5)*B*D1/(5.*R*R)+16.*A*B*D4
      T(16,22)=4.*(A**3)*(B**3)*D1/(9.*R*R)+16.*A*B*D5
      T(17,17)=4.*(B**7)*(A**3)*D1/(21.*R*R)+16.*(A**3)*(B**3)*D4
     1+144.*A*(B**5)*D6/5.
      T(17,19)=4.*(A**3)*(B**5)*D1/(15.*R*R)+16.*(B**3)*A*D6
      T(18,18)=(4.*(A**3)*(B**5)*D1/(15.*R*R))+(16.*(A**3)*B*D4/3.)
     1+(64.*(B**3)*A*D6/3.)
      T(19,19)=4.*(A**3)*(B**3)*D1/(9.*R*R)+16.*A*B*D6
      T(21,21)=4.*(B**7)*A*D1/(7.*R*R)+144.*(B**3)*A*D4/3.
      T(22,22)=4.*A*(B**5)*D1/(5.*R*R)+16.*A*B*D4
      DO 17 L=2,24
      LM=L-1
      DO 17 J=1,LM
 17   T(L,J)==T(J,L)
      R:TURN
      END
ZZZZ
ZZJOB
ZZFOR
*LDISKCECE
      SUBROUTINE CECE(A,B,VU,R,AN,C)
      DIMENSION C(24,24)
      DO 9 J=1,24
      DO 9 I=1,24
 9    C(I,J)=0.
      DO 11 N=1,4
      DO 10 L=1,6
      GO TO(201,202,203,204),N
 201  X=-A
      S=-B
      GO TO 60
 202  X=A
      S=-B
      GO TO 60
 203  X=A
      S=B
      GO TO 60
 204  X=-A
      S=B
 60   I=(N-1)*6+L
```

```
      GO TO(101,102,103,104,105,106),L
  101 C(I,1)=-(VU*X*X+S*S)
      C(I,2)=2.*S*X
      C(I,3)=X
      C(I,4)=S
      C(I,5)=1.
      C(I,6)=-R*SIN(S/R)
      GO TO 10
  102 C(I,1)=2.*S*X
      C(I,2)=-(X*X+VU*S*S)
      C(I,6)=X*COS(AN)
      C(I,7)=S
      C(I,8)=COS(AN)
      C(I,20)=-X*SIN(S/R)
      C(I,23)=-R*SIN(S/R)*(SIN(S/R))
      C(I,24)=-SIN(S/R)
      GO TO 10
  103 C(I,9)=(X**3)*(S**3)
      C(I,10)=(X**3)*(S**2)
      C(I,11)=(X**3)*S
      C(I,12)=X**3
      C(I,13)=(X**2)*(S**3)
      C(I,14)=(X**2)*(S**2)
      C(I,15)=(X**2)*S
      C(I,16)=X**2
      C(I,17)=X*(S**3)
      C(I,18)=X*(S**2)
      C(I,19)=X*S
      C(I,20)=X*COS(AN)
      C(I,21)=S**3
      C(I,22)=S**2
      C(I,23)=R*(SIN(S/R))*(COS(AN))
      C(I,24)=COS(AN)
      C(I,6)=X*SIN(S/R)
      C(I,8)=SIN(S/R)
      GO TO 10
  104 C(I,1)=-2.*S*X/R
      C(I,2)=(X*X+VU*S*S)/R
      C(I,7)=-S/R
      C(I,9)=3.*(X**3)*(S**2)
      C(I,10)=2.*(X**3)*S
      C(I,11)=X**3
      C(I,13)=3.*(X**2)*(S**2)
      C(I,14)=2.*S*X**2
      C(I,15)=X**2
      C(I,17)=3.*X*(S**2)
      C(I,18)=2.*S*X
      C(I,19)=X
      C(I,21)=3.*(S**2)
      C(I,22)=2.*S
      C(I,23)=1.
      GO TO 10
  105 C(I,6)=-SIN(S/R)
      C(I,9)=-3.*(X**2)*(S**3)
      C(I,10)=-3.*(X**2)*(S**2)
      C(I,11)=-3.*(X**2)*S
      C(I,12)=-3.*X**2
      C(I,13)=-2.*X*(S**2)*S
      C(I,14)=-2.*X*(S**2)
      C(I,15)=-2.*S*X
      C(I,16)=-2.*X
      C(I,17)=-(S**3)
      C(I,18)=-(S**2)
```

```
      C(I,19)=-S
      C(I,20)=-COS(AN)
      GO TO 10
  106 C(I,6)=(COS(AN))/R
      C(I,9)=9.*(X**2)*(S**2)
      C(I,10)=6.*(X**2)*S
      C(I,11)=3.*(X**2)
      C(I,13)=6.*X*(S**2)
      C(I,14)=4.*S*X
      C(I,15)=2.*X
      C(I,17)=3.*(S**2)
      C(I,18)=2.*S
      C(I,19)=1.
      C(I,20)=-(SIN(S/R))/R
   10 CONTINUE
   11 CONTINUE
      RETURN
      END
ZZZZ
```

H_o and A Matrices for the 24x24 Cylindrical Shell Element with Rotation About Vertical Axis.

```
ZZJOB
ZZFOR
*LDISKTEBIR
      SUBROUTINE TEBIR(E,H,VU,B,A,R,T)
      DIMENSION T(24,24)
      DO 888 I=1,24
      DO 888 J=1,24
 888  T(I,J)=0.
      A2=A**2
      A3=A**3
      A4=A**4
      A5=A**5
      A6=A**6
      A7=A**7
      B2=B**2
      B3=B**3
      B4=B**4
      B5=B**5
      B6=B**6
      B7=B**7
      D1=E*H/(1.-VU*VU)
      D2=VU*D1
      D3=E*H/(2.+2.*VU)
      D4=(H**2)*D1/12.
      D5=VU*D4
      D6=E*(H**3)/(24.*(1.+VU))
      T(2,2)=A*B*D1
      T(4,2)=B2*A*D1/2.
      T(6,2)=B3*A*D1/3.
      T(9,2)=A*B*VU*D1
      T(10,2)=A2*B*VU*D1/2.
      T(12,2)=A3*B*VU*D1/3.
      T(16,2)=A3*B*VU*D1/(3.*R)
      T(17,2)=A2*B2*VU*D1/(4.*R)
      T(18,2)=A*B3*VU*D1/(3.*R)
      T(19,2)=A4*B*VU*D1/(4.*R)
      T(20,2)=A3*B2*VU*D1/(6.*R)
      T(21,2)=A2*B3*VU*D1/(6.*R)
      T(22,2)=B4*A*VU*D1/(4.*R)
      T(23,2)=A4*B2*VU*D1/(8.*R)
      T(24,2)=A2*B4*VU*D1/(8.*R)
      T(3,3)=A*B*D3
      T(4,3)=A2*B*D3/2.
      T(5,3)=B2*A*D3
      T(6,3)=A2*B2*D3/2.
      T(10,3)=B2*A*D3/2.
      T(11,3)=A2*B*D3
      T(12,3)=A2*B2*D3/2.
      T(4,4)=A*B3*D1/3.+A3*B*D3/3.
      T(5,4)=A2*B2*D3/2.
      T(6,4)=A*B4*D1/4.+A3*B2*D3/3.
      T(9,4)=B2*A*VU*D1/2.
      T(10,4)=A2*B2*VU*D1/4.+A2*B2*D3/4.
      T(11,4)=2.*A3*B*D3/3.
      T(12,4)=A3*B2*VU*D1/6.+A3*B2*D3/3.
      T(16,4)=A3*B2*VU*D1/(6.*R)
```

```
      T(17,4)=A2*B3*VU*D1/(6.*R)
      T(18,4)=A*B4*VU*D1/(4.*R)
      T(19,4)=A4*B2*VU*D1/(8.*R)
      T(20,4)=A3*B3*VU*D1/(9.*R)
      T(21,4)=A2*B4*VU*D1/(8.*R)
      T(22,4)=A*B5*VU*D1/(5.*R)
      T(23,4)=A4*B3*VU*D1/(12.*R)
      T(24,4)=A2*B5*VU*D1/(10.*R)
      T(5,5)=4.*A*B3*D3/3.
      T(6,5)=4.*A2*B3*D3/6.
      T(10,5)=2.*A*B3*D3/3.
      T(11,5)=A2*B2*D3
      T(12,5)=4.*A2*B3*D3/6.
      T(6,6)=A*B5*D1/5.+4.*A3*B3*D3/9.
      T(9,6)=A*B3*VU*D1/3.
      T(10,6)=A2*B3*VU*D1/6.+A2*B3*D3/3.
      T(11,6)=4.*A3*B2*D3/6.
      T(12,6)=A3*B3*VU*D1/9.+4.*A3*B3*D3/9.
      T(16,6)=A3*B3*D1*VU/(9.*R)
      T(17,6)=A2*B4*VU*D1/(8.*R)
      T(18,6)=A*B5*VU*D1/(5.*R)
      T(19,6)=A4*B3*VU*D1/(12.*R)
      T(20,6)=A3*B4*VU*D1/(12.*R)
      T(21,6)=A2*B5*D1*VU/(10.*R)
      T(22,6)=A*B6*VU*D1/(6.*R)
      T(23,6)=A4*B4*VU*D1/(16.*R)
      T(24,6)=A2*B6*VU*D1/(12.*R)
      T(9,9)=A*B*D1+A*B*D4/(R*R)
      T(10,9)=A2*B*D1/2.+A2*B*D4/(2.*R*R)
      T(12,9)=A3*B*D1/3.+A3*B*D4/(3.*R*R)
      T(16,9)=A3*B*D1/(3.*R)-2.*A*B*D5/R
      T(17,9)=A2*B2*D1/(4.*R)
      T(18,9)=A*B3*D1/(3.*R)-2.*A*B*D4/R
      T(19,9)=A4*B*D1/(4.*R)-6.*A2*B*D5/(2.*R)
      T(20,9)=A3*B2*D1/(6.*R)-B2*A*D5/R
      T(21,9)=A2*B3*D1/(6.*R)-A2*B*D4/R
      T(22,9)=A*B4*D1/(4.*R)-3.*B2*A*D4/R
      T(23,9)=A4*B2*D1/(8.*R)-6.*A2*B2*D5/(4.*R)
      T(24,9)=A2*B4*D1/(8.*R)-6.*A2*B2*D4/(4.*R)
      RETURN
      END
ZZZZ
ZZJOB
ZZFOR
*LDISKTEIKI
      SUBROUTINE TEIKI(E,H,VU,B,A,R,T)
      DIMENSION T(24,24)
      A2=A**2
      A3=A**3
      A4=A**4
      A5=A**5
      A6=A**6
      A7=A**7
      B2=B**2
      B3=B**3
      B4=B**4
      B5=B**5
      B6=B**6
      B7=B**7
      D1=E*H/(1.-VU*VU)
      D2=VU*D1
      D3=E*H/(2.+2.*VU)
      D4=(H**2)*D1/12.
```

```
      D5=VU*D4
      D6=E*(H**3)/(24.*(1.+VU))
      T(10,10)=A3*B*D1/3.+A*B3*D3/3.+A3*B*D4/(3.*R*R)
     1+4.*A*B3*D6/(3.*R*R)
      T(11,10)=A2*B2*D3/2.+2.*A2*B2*D6/(R*R)
      T(12,10)=A4*B*D1/4.+A2*B3*D3/3.+A4*B*D4/(4.*R*R)
     1+8.*A2*B3*D6/(6.*R*R)
      T(16,10)=A4*B*D1/(4.*R)-A2*B*D5/R
      T(17,10)=A3*B2*D1/(6.*R)-2.*A*B2*D6/R
      T(18,10)=A2*B3*D1/(6.*R)-A2*B*D4/R
      T(19,10)=A5*B*D1/(5.*R)-2.*A3*B*D5/R
      T(20,10)=A4*B2*D1/(8.*R)-A2*B2*D5/(2.*R)-2.*A2*B2*D6/R
      T(21,10)=A3*B3*D1/(9.*R)-2.*A3*B*D4/(3.*R)-8.*A*B3*D6/(3.*R)
      T(22,10)=A2*B4*D1/(8.*R)-6.*A2*B2*D4/(4.*R)
      T(23,10)=A5*B2*D1/(10.*R)-A3*B2*D5/R-2.*A3*B2*D6/R
      T(24,10)=A3*B4*D1/(12.*R)-A3*B2*D4/R-3.*B4*A*D6/R
      T(11,11)=4.*A3*B*D3/3.+16.*A3*B*D6/(3.*R*R)
      T(12,11)=4.*A3*B2*D3/6.+16.*A3*B2*D6/(6.*R*R)
      T(17,11)=-4.*A2*B*D6/R
      T(20,11)=-16.*A3*B*D6/(3.*R)
      T(21,11)=-16.*A2*B2*D6/(4.*R)
      T(23,11)=-6.*A4*B*D6/R
      T(24,11)=-4.*A2*B3*D6/R
      T(12,12)=A5*B*D1/5.+4.*A3*B3*D3/9.+A5*B*D4/(5.*R*R)
     1+16.*A3*B3*D6/(9.*R*R)
      T(16,12)=A5*B*D1/(5.*R)-2.*A3*B*D5/(3.*R)
      T(17,12)=A4*D1*B2/(8.*R)-2.*A2*B2*D6/R
      T(18,12)=A3*B3*D1/(9.*R)-2.*A3*B*D4/(3.*R)
      T(19,12)=A6*B*D1/(6.*R)-6.*A4*B*D5/(4.*R)
      T(20,12)=A5*B2*D1/(10.*R)-A3*B2*D5/(3.*R)-16.*A3*B2*D6/(6.*R)
      T(21,12)=A4*B3*D1/(12.*R)-A4*B*D4/(2.*R)-16.*A2*B3*D6/(6.*R)
      T(22,12)=A3*B4*D1/(12.*R)-B2*A3*D4/R
      T(23,12)=A6*B2*D1/(12.*R)-6.*A4*B2*D5/(8.*R)-3.*A4*B2*D6/R
      T(24,12)=A4*B4*D1/(16.*R)-6.*A4*B2*D4/(8.*R)-3.*B4*A2*D6/R
      T(16,16)=A5*B*D1/(5.*R*R)+4.*A*B*D4
      T(17,16)=A4*B2*D1/(8.*R*R)
      T(18,16)=A3*B3*D1/(9.*R*R)+4.*A*B*D5
      T(19,16)=A6*B*D1/(6.*R*R)+6.*A2*B*D4
      T(20,16)=A5*B2*D1/(10.*R*R)+2.*A*B2*D4
      T(21,16)=A4*B3*D1/(12.*R*R)+2.*A2*B*D5
      T(22,16)=A3*B4*D1/(12.*R*R)+6.*B2*A*D5
      T(23,16)=A6*B2*D1/(12.*R*R)+3.*A2*B2*D4
      T(24,16)=A4*B4*D1/(16.*R*R)+3.*A2*B2*D5
      RETURN
      END
ZZZZ
ZZJOB
ZZFOR
*LDISKTEUC
      SUBROUTINE TEUC(E,H,VU,B,A,R,T)
      DIMENSION T(24,24)
      A2=A**2
      A3=A**3
      A4=A**4
      A5=A**5
      A6=A**6
      A7=A**7
      B2=B**2
      B3=B**3
      B4=B**4
      B5=B**5
      B6=B**6
      B7=B**7
```

```
      D1=E*H/(1.-VU*VU)
      D2=VU*D1
      D3=E*H/(2.+2.*VU)
      D4=(H**2)*D1/12.
      D5=VU*D4
      D6=E*(H**3)/(24.*(1.+VU))
      T(17,17)=A3*B3*D1/(9.*R*R)+4.*A*B*D6
      T(18,17)=A2*B4*D1/(8.*R*R)
      T(19,17)=A5*B2*D1/(10.*R*R)
      T(20,17)=A4*B3*D1/(12.*R*R)+4.*A2*B*D6
      T(21,17)=A3*B4*D1/(12.*R*R)+4.*A*B2*D6
      T(22,17)=A2*B5*D1/(10.*R*R)
      T(23,17)=A5*B3*D1/(15.*R*R)+4.*A3*B*D6
      T(24,17)=A3*B5*D1/(15.*R*R)+4.*B3*A*D6
      T(18,18)=A*B5*D1/(5.*R*R)+4.*A*B*D4
      T(19,18)=A4*B3*D1/(12.*R*R)+6.*A2*B*D5
      T(20,18)=A3*B4*D1/(12.*R*R)+2.*A*B2*D5
      T(21,18)=A2*B5*D1/(10.*R*R)+2.*A2*B*D4
      T(22,18)=A*B6*D1/(6.*R*R)+6.*A*B2*D4
      T(23,18)=A4*B4*D1/(16.*R*R)+3.*A2*B2*D5
      T(24,18)=A2*B6*D1/(12.*R*R)+3.*A2*B2*D4
      T(19,19)=A7*B*D1/(7.*R*R)+12.*A3*B*D4
      T(20,19)=A6*B2*D1/(12.*R*R)+3.*A2*B2*D4
      T(21,19)=A5*B3*D1/(15.*R*R)+4.*A3*B*D5
      T(22,19)=A4*B4*D1/(16.*R*R)+9.*A2*B2*D5
      T(23,19)=A7*B2*D1/(14.*R*R)+6.*A3*B2*D4
      T(24,19)=A5*B4*D1/(20.*R*R)+6.*A3*B2*D5
      T(20,20)=A5*B3*D1/(15.*R*R)+4.*A*B3*D4/3.+16.*A3*B*D6/3.
      T(21,20)=A4*B4*D1/(16.*R*R)+A2*B2*D5+4.*A2*B2*D6
      T(22,20)=A3*B5*D1/(15.*R*R)+4.*A*B3*D5
      T(23,20)=A6*B3*D1/(18.*R*R)+2.*B3*A2*D4+6.*A4*B*D6
      T(24,20)=A4*B5*D1/(20.*R*R)+2.*A2*B3*D5+4.*A2*B3*D6
      T(21,21)=A3*B5*D1/(15.*R*R)+4.*A3*B*D4/3.+16.*B3*A*D6/3.
      T(22,21)=A2*B6*D1/(12.*R*R)+3.*A2*B2*D4
      T(23,21)=A5*B4*D1/(20.*R*R)+2.*A3*B2*D5+4.*A3*B2*D6
      T(24,21)=B6*A3*D1/(18.*R*R)+2.*A3*B2*D4+6.*B4*A*D6
      T(22,22)=A*B7*D1/(7.*R*R)+12.*A*B3*D4
      T(23,22)=A4*B5*D1/(20.*R*R)+6.*A2*B3*D5
      T(24,22)=B7*A2*D1/(14.*R*R)+6.*A2*B3*D4
      T(23,23)=A7*B3*D1/(21.*R*R)+4.*A3*B3*D4+36.*A5*B*D6/5.
      T(24,23)=A5*B5*D1/(25.*R*R)+4.*A3*B3*D5+4.*A3*B3*D6
      T(24,24)=A3*B7*D1/(21.*R*R)+4.*A3*B3*D4+36.*A*B5*D6/5.
      DO 17 J=1,24
      DO 17 I=J,24
 17   T(J,I)=T(I,J)
      RETURN
      END
ZZZZ
ZZJOB
ZZFOR
*LDISKCECE
*LIST PRINTER
      SUBROUTINE CECE(A,B,VU,R,AN,C)
      DIMENSION C(24,24)
      DO 9 J=1,24
      DO 9 I=1,24
 9    C(I,J)=0.
      DO 11 N=1,4
      DO 10 L=1,6
      GO TO(201,202,203,204),N
 201  X=0.
      S=0.
      GO TO 60
```

```
202   X=A
      S=0.
      GO TO 60
203   X=A
      S=B
      GO TO 60
204   X=0.
      S=B
60    I=(N-1)*6+L
      GO TO(101,102,103,104,105,106),L
101   C(I,1)=1.
      C(I,2)=X
      C(I,3)=S
      C(I,4)=X*S
      C(I,5)=S*S
      C(I,6)=S*S*X
      C(I,8)=-R*SIN(S/R)
      C(I,14)=-R*(COS(S/R)-COS(AN))
      GO TO 10
102   C(I,7)=COS(S/R)
      C(I,8)=X*COS(S/R)
      C(I,9)=S
      C(I,10)=S*X
      C(I,11)=X*X
      C(I,12)=X*X*S
      C(I,13)=-SIN(S/R)
      C(I,14)=-X*SIN(S/R)
      C(I,15)=-R*(1.-COS(AN)*COS(S/R))
      GO TO 10
103   C(I,7)=SIN(S/R)
      C(I,8)=X*SIN(S/R)
      C(I,13)=COS(S/R)
      C(I,14)=X*COS(S/R)
      C(I,15)=R*SIN(S/R)*COS(AN)
      C(I,16)=X*X
      C(I,17)=S*X
      C(I,18)=S*S
      C(I,19)=X**3
      C(I,20)=S*X*X
      C(I,21)=X*S*S
      C(I,22)=S**3
      C(I,23)=(X**3)*S
      C(I,24)=(S**3)*X
      GO TO 10
104   C(I,9)=-S/R
      C(I,10)=-S*X/R
      C(I,11)=-X*X/R
      C(I,12)=-X*X*S/R
      C(I,15)=1.
      C(I,17)=X
      C(I,18)=2.*S
      C(I,20)=X*X
      C(I,21)=2.*S*X
      C(I,22)=3.*S*S
      C(I,23)=X**3
      C(I,24)=3.*S*S*X
      GO TO 10
105   C(I,8)=-SIN(S/R)
      C(I,14)=-COS(S/R)
      C(I,16)=-2.*X
      C(I,17)=-S
      C(I,19)=-3.*X*X
      C(I,20)=-2.*S*X
```

```
      C(I,21)=-S*S
      C(I,23)=-3.*X*X*S
      C(I,24)=-S**3
      GO TO 10
  106 C(I,3)=-1./2.
      C(I,4)=-X/2.
      C(I,5)=-S
      C(I,6)=-S*X
      C(I,8)=COS(S/R)
      C(I,10)=S/2.
      C(I,11)=X
      C(I,12)=S*X
      C(I,14)=-SIN(S/R)
   10 CONTINUE
   11 CONTINUE
      RETURN
      END
ZZZZ
```

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