OPERATING ROOM SCHEDULING WITH SEQUENCE-DEPENDENT & STOCHASTIC SURGERY DURATIONS

by

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ABSTRACT

OPERATING ROOM SCHEDULING WITH SEQUENCE-DEPENDENT & STOCHASTIC SURGERY DURATIONS

Operating rooms are the most costly part of hospitals. In order to increase their efficiencies, hospitals should first increase the efficiency of their operating rooms. In this thesis, we consider the next-day operating room scheduling problem both for single operating room and multiple operating rooms. It is assumed that surgeries have uncertain durations and distributions of surgery durations are dependent on the sequence of the surgeries within an operating room. In this problem, sequence-dependency comes from the location of surgeries within an operating room instead of the relationship between two successive surgeries. It is aimed to sequence and schedule sequence-dependent surgeries by minimizing the weighted sum of expected waiting time of patients, idle time of operating rooms, and overtime of the hospital staff. In order to find solutions to the problem, Sample Average Approximation(SAA) method is used. Then, the effect of different parameters on penalty of ignoring sequence-dependent surgery durations is analyzed. Furthermore, L-Shaped method and five different heuristics are introduced to decrease the computation time further. We test the performance of these heuristics by comparing them with the solutions found by using SAA. According to results, Modified Myopic Heuristic performs better than other heuristics for single operating room problem. For multiple operating rooms, giving more weight to overtime increases the cost occurred by ignoring the sequence-dependent surgeries. It is also observed that SVF Heuristic is an important way to decrease the computation time of complex scheduling problems.

ÖZET

SIRALAMAYA BAĞLI VE STOKASTİK AMELİYAT SÜRELERİ İLE AMELİYATHANE ÇİZELGELEMESİ

Hastanelerin en maliyetli kısımlarını ameliyathaneler oluşturur. Dolayısıyla hastaneler verimliliklerini arttırmak için işe ameliyathanelerden başlamalıdır. Bu tez çalışmasında, ertesi-gün ameliyathane çizelgelenmesi problemi hem tek ameliyathaneli hem de çok ameliyathaneli durumlar için ele alındı. Ameliyatların sürelerinin rastsal değşkenler olduğu ve ameliyat sürelerinin ameliyatların ameliyathanedeki sırasına göre değiştiği varsayıldı. Bu problemde, ameliyat sürelerinin sıralamaya bağlı olması ardışık iki ameliyat arasındaki ilişki yerine ameliyatın yer aldığı sıradan kaynaklandı. Hastaların bekleme sürelerinin, ameliyathanelerin boşta kalan sürelerinin, ve personelin fazla mesai sürelerinin ağırlıklı toplamlarını en aza indirgeyecek şekilde ameliyatların sıralanması ve onlara yeterli süre atanması amaçlandı. Problemi çözmek için Örnek Ortalama Yaklaşım(SAA) yöntemi kullanıldı. Daha sonra değişik parametrelerin sıralamaya bağlı ameliyatların sıralamaya bağlı olmadığını varsayma maliyeti üzerindeki etkileri analiz edildi. Ayrıca probleme daha kısa sürede sonuç bulabilmek amacıyla L-Shaped yöntemi ve beş farklı sezgisel yöntem kullanıldı. Bu sezgisel yöntemlerin performansları sezgisel yöntemlerden elde edilen sonuçlarla Örnek Ortalama Yaklaşım yöntemiyle elde edilen sonuçlar karşılaştırılarak test edildi. Değiştirilmiş Miyopik yöntemin tek ameliyathane olan durumlarda diğer sezgisel yöntemlerden daha iyi performans gösterdiği gözlemlendi. Fazla sayıda ameliyathane olması durumunda, personelin fazla mesai sürelerinin ağırlığı arttırıldığında, sıralamaya bağlı ameliyatların sıralamaya bağlı olmadığını varsayma maliyetinin arttığı görüldü. Ayrıca ameliyatları artan varyans değerine göre sıralamanın(SVF), problemin çözüm süresini ciddi anlamda kısalttığı gözlemlenmiştir.

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$D_{i,k}$	Assigned duration to i^{th} sequence in k^{th}operating room
$F_{i,j}(x)$	Cumulative distribution function of $T_{i,j,k}$
$f_{i,j}(x)$	Density function of $T_{i,j,k}$
m	Number of operating rooms
N	Number of surgeries
n_k	Number of surgeries assigned to k^{th} operating room
o_k	Overtime of k^{th} operating room
P	Number of scenarios
S	Set of surgeries that need to be scheduled for the next day
s_i	Standard deviation of the surgery duration of the surgery j
$s_{i,j,k}$	Idle time of j^{th} surgery when it is assigned to i^{th} sequence in k^{th} operating room
$T_{i,j,k}$	Surgery duration of j^{th} surgery when it is assigned to i^{th} sequence in k^{th} operating room
$w_{i,j,k}$	Waiting time of j^{th} surgery when it is assigned to i^{th} sequence in k^{th} operating room
Y_i	Random variable of the convolution of the distributions of surgery durations of the first i surgeries
α_1	Unit cost of idle time
α_2	Unit cost of waiting time
α_3	Unit cost of overtime
Θ	Expected recourse function
$\mu_{i,j}$	Mean of the surgery duration of j^{th} surgery when it is assigned to i^{th} sequence

LIST OF ACRONYMS/ABBREVIATIONS

$C(Myp)$	Cost of Using Myopic Heuristic
$C(Mod)$	Cost of Using Veteran' Heuristic
$C(V)$	Cost of Using Modified Myopic Heuristic
CV	Coefficient of Variation
PI	Penalty of Using Sequence-Dependent Surgery Durations
SAA	Sample Average Approximation Method
SMIP	Stochastic Mixed Integer Program
SVF	Shortest-Variance-First Heuristic

1. INTRODUCTION

In today's world, there exists an increasing growth in healthcare expenditures [1]. According to [2], see Figure 1.1, proportion of healthcare expenditures in total GDP in US increased from 5.0% in 1960 to 17.8% in 2015. A third of these healthcare expenditures is accounted for hospital expenditures [2]. In order to reduce their expenditures, many hospitals have been looking for ways to increase the efficient use of their resources. According to the Health Care Financial Management Association, operating rooms(ORs) account for more than 40 % of a hospital's revenues and costs [3]. Therefore, it is crucial to increase the efficiency of operating rooms in order to increase the efficiency of the whole system in hospitals. In return, increasing the efficiency of hospitals will lead to a decrease in healthcare expenditures.

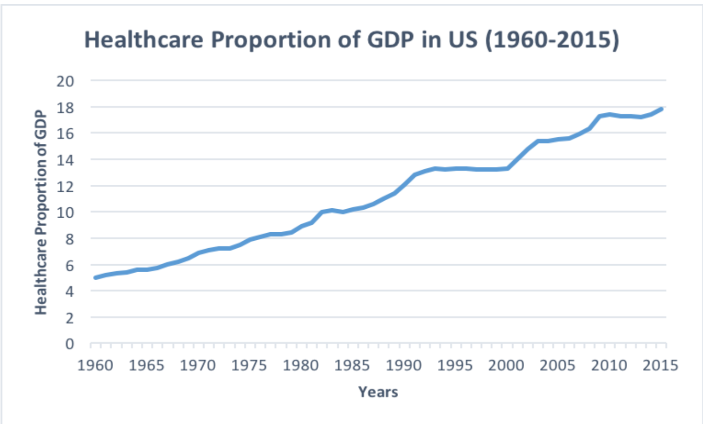


Figure 1.1. Healthcare Proportion of GDP in US Between 1960 and 2015.

In literature, there are several studies on operating room planning and scheduling. However, most studies deal with a single operating room since additional decisions are required when there are more than one operating room. Furthermore, surgery durations are assumed to be deterministic in a large portion of studies due to the computational burden of stochastic scheduling problems. In this study, surgeries have uncertain durations and both single operating room and multiple operating room scheduling problems are considered.

Performance of hospital staff (surgeons, nurses, anesthesiologists, technicians) changes according to time of the day [4, 5]. In the morning, most of the hospital staff are energetic, and this may result in a decrease in surgery durations of early surgeries. Moreover, hospital staff don't want to stay beyond planned working time of the day, and this may also reduce surgery durations of late surgeries. These situations have motivated us to study on sequence-dependent surgery durations and to measure the effect of ignoring sequence-dependent surgery durations.

Majority of studies on sequence-dependent procedures focus on sequence-dependent setup times. Therefore, sequence-dependency comes from the relationship between two successive procedures. In our problem, sequence-dependency is related to the location of surgeries within an operating room. According to our model, distributions of surgery durations change if they are scheduled first or last. When the literature is further investigated, sequence-dependent surgery durations are not considered in any of studies on operating room scheduling. Only one study [6] used sequence-dependent setup times between the surgeries, but surgery durations and setup times between surgeries are taken as deterministic in this article.

Operating room planning and scheduling problems have some constraints that make the problem difficult. First, many factors including ability of hospital staff, type of surgeries, and possible complications encountered during surgeries affect the uncertainty of surgery durations and it is important to predict surgery durations as accurate as possible. If surgeries last shorter than predicted surgery durations, operating rooms remain idle and utilization of operating rooms decreases. If surgeries last longer than predicted surgery durations, patients will wait for their surgeries and this will result in dissatisfaction of patients. Moreover, computational burden of the problem increases significantly as the number of the operating rooms and the number of the surgeries increase, since there are too many possible assignments of surgeries to operating rooms and sequences of surgeries within operating rooms.

According to their hierarchical decision levels, operating room scheduling problems are divided into three categories as strategic, tactical, and operational [7, 8].

Strategic level problems have long-term planning horizon. Long-term capacity of operating rooms and allocation of operating rooms over different specialties are determined in strategic level. Temporary capacity of operating rooms and allocation of surgeries to different days in a week are determined in tactical level. In operational level, daily schedules of operating rooms are determined. Therefore, our problem belongs to operational level category of operating room scheduling problems.

In this thesis, we consider a next-day operating room scheduling problem for both single operating room and multiple operating rooms. Emergent surgeries are ignored, and only elective surgeries are included. It is assumed that surgeries have uncertain durations and distributions of surgery durations are dependent on the sequence within operating rooms. Furthermore, it is assumed that a surgery cannot start before its planned time even if the previous surgery is completed earlier than the planned time. On the other hand, it may start later than planned time if the previous surgery is completed after the planned time. The objective is to minimize the expected weighted sum of waiting time of patients, idle time of operating rooms, and overtime of the hospital staff. During the thesis, the mathematical model of the problem is formulated as a stochastic mixed integer program as a first step. Then, Sample Average Approximation(SAA) method is used to find solutions to the problem, and five different heuristics and L-Shaped method are introduced to decrease the computation time of the problem.

To our knowledge, operating room scheduling with sequence-dependent and uncertain surgery durations has not been considered in the literature. Therefore, our primary contribution is to consider sequence-dependent surgery durations in a stochastic environment and analyze the effect of ignoring sequence-dependent surgery durations. Furthermore, a pattern for the extension of Shortest-Variance-First heuristic to multiple operating rooms is developed. Its optimality is proven for small cases, and it is expected that this pattern is optimal in general.

This thesis is organized as follows. Chapter 2 reviews the relevant literature on operating room scheduling. Chapter 3 defines the next-day operating room scheduling problem with sequence dependent surgery durations, and provides the mathematical formulation of the problem. In Chapter 4, solution methods used to find solutions to the problem are provided. Experimental settings and numerical results obtained for these settings are presented in Chapter 5. Furthermore, Chapter 5 provides some managerial insights obtained from the analysis of the results. Finally, Chapter 6 summarizes the results of experiment and significant implications of these results.

2. LITERATURE REVIEW

Hospitals are getting highly interested in reducing their costs and increasing satisfaction of their patients. A large portion of hospital costs incur in operating rooms. Therefore, operating room planning and scheduling problem draws great attention in recent years. In the first section, review articles on operating room scheduling will be summarized. Then, articles on operating room scheduling will be divided into two categories. In the first part, articles include only scheduling of the surgeries will be discussed. In the second part, articles deal with both sequencing and scheduling of surgeries will be discussed. Finally, last section summarizes several articles on scheduling with sequence-dependent setup times. Although studies on operating room scheduling have not considered sequence-dependent and uncertain surgery durations so far, there exists studies on machine scheduling with sequence-dependent setup times between successive jobs.

2.1. Review Articles

There are a number of review articles on operating room scheduling. Blake and Carter [9] classified studies according to performance measures and level of decisions(strategic, administrative, tactical). They suggested that operating room scheduling should be done together with other operations in hospitals. Cardoen *et al.* [10] reviewed the research on operating room and scheduling. They classified articles according to 7 criteria: patient characteristics(emergent or elective surgeries), performance measure(waiting time, utilization, and so forth), decision level(sequencing or scheduling), type of analysis, solution method, uncertainty(deterministic vs. stochastic), and applicability of research. They stated that most of the researchers use deterministic surgery durations due to computational complexity of stochastic problems. There are a large number of studies tested with real data, but it is difficult to apply these improvements on hospitals since high-degree of data availability and system integration are required for their implementation. Erdogan and Denton [11] listed the factors, which make operating room scheduling complicated. First, uncertainty in surgery durations, uncertainty in the number of patients to be scheduled, and uncertainty in the portion of the type of the surgeries make the problem difficult. Furthermore, high fixed costs of operating rooms and variable costs of resources increase the complexity of the problem. Studies are divided into four categories according to solution methods: queuing, simulation, optimization, and heuristics. They emphasized deficiencies in operating room scheduling literature, which are real time scheduling on the day of the surgery, uncertain demand of patients, and focusing on the whole system instead of only considering operating rooms. Throughout this chapter, our focus will be on stochastic operating room scheduling problem.

2.2. Literature on Scheduling Surgeries

In this section, articles that only deal with assignment of surgery durations are considered. All articles assume that sequence of surgeries is known at the beginning.

Weiss [12] studied initially on scheduling of surgeries when the sequence of the surgeries is given at the beginning. The objective function is the weighted sum of waiting time and idle time. Both simulation and analytical methods are used for finding solutions. It is found that the problem can be reduced to "Newsvendor" problem when there are two surgeries. If the number of surgeries is more than 2, a myopic heuristic is used. Surgery durations are found by the convolution of the distributions of surgery durations.

Wang [13] developed methods in order to minimize the weighted delay customer delay time and the system completion time. In the article, it is assumed that surgery durations are distributed exponentially, and they are independent. It is found that the objective function is convex. Therefore, first-order-condition provides the optimal surgery durations. Although surgery durations are independent and identically distributed in the analysis, it is observed that the assigned surgery durations increase first, and then decrease. They call this pattern as a dome-shaped pattern. According to numerical results, the problem converges faster and solution time decreases, as the weight of customer delay time decreases. In his following study [14], he assumed that surgery durations are independent and identically distributed with phase-type distribution, and he found the flow-time(waiting time + surgery duration) distributions of the surgeries. He developed different methods to calculate the expected values of these distributions.

Robinson and Chen [15] developed a robust heuristic in order to minimize the weighted sum of the waiting time of the patients and the idle time of surgeons. The sequence of the patients is specified at the beginning. The distributions of surgery durations are derived from an empirical distribution of surgery durations. Sample Average Approximation method is used for finding solutions. According to numerical results, it is observed that the heuristic method provides solutions on the average within 0.5% of the optimal solution. In the worst case, the solution is within 20% of the optimal solution for more than 3 surgeries.

Denton and Gupta [16] expressed the problem as two-stage stochastic linear program. The objective function is to minimize the weighted cost of the customer waiting, server idling, and tardiness with respect to a chosen session length. They found the lower and upper bounds of the objective function, and then they used a version of L-Shaped method with sequential bounding. Results showed that the weighted cost increases linearly as the standard deviation of surgery durations increases or the number of surgeries to be scheduled increases.

Begen and Queyranne [17] assumed that the surgery durations are distributed with a joint discrete probability function. The objective function is the expected total underage and overage costs. They found that the optimal surgery durations can be calculated in polynomial time.

2.3. Literature on Both Sequencing and Scheduling Surgeries

In this section, studies on sequencing of surgeries and assignment of surgery durations to these surgeries are discussed. Some articles sequence and schedule surgeries simultaneously, and others use heuristics for sequencing or scheduling of surgeries. This section will be divided into two categories according to the number of operating rooms used.

2.3.1. Single Operating Room Problem

Weiss [12] also studied on jointly determining optimal sequence of surgeries and surgery durations. The objective function is the weighted sum of waiting time and idle time. According to the results, surgeries which have distribution of surgery durations with fatter tails should be ordered first in order to obtain optimal sequences. However, this rule may not be optimal when the number of surgeries is more than 2.

Wang [18] studied on sequencing and scheduling a finite number of N customers optimally, and used exponential distribution for service times. The objective function is the weighted sum of the expected flow times and system completion time. He used a two-phase problem solution. In the first phase, the optimal service order is determined. The optimal service times are found in the second phase. It is found that sequencing jobs in order of decreasing mean rates gives the optimal sequence. Furthermore, optimal service times are calculated by solving N nonlinear equations. Single operating room scheduling problem is a type of problem that Wang considered in this article.

Vander Bosch and Dietz [19] scheduled surgeries at regularly-spaced times. A given set of patients are scheduled into several appointment days, and the objective function is the weighted sum of the expected waiting time and expected overtime. They used Lattice Algorithm, in which they interchanged a pair of surgeries at each step. When there is no longer improvement in the objective value, the algorithm stops. Although our problem is next-day operating room scheduling problem, this article considers a multi-day operating room scheduling problem.

Dexter and Traub [20] considered elective case scheduling in order to maximize the utilization of the operating rooms. They used two heuristics for scheduling of surgeries: Earliest Start Time Heuristic and Latest Start Time Heuristic. The first heuristic is good at predicting start time, and the second heuristic can eliminate overtime.

According to the results, three conditions should be satisfied in order to maximize the utilization of operating rooms. The first condition is that if the service has filled its regular operating time, then new surgery should be scheduled into another service's regular operating time. The second condition is that a service should not schedule a surgery into another service as long as it can complete the surgery in its regular operating time. The last condition is that a surgery should not be scheduled to be completed in an overutilized operating room, if it can start earlier in another service's operating room.

Lebowitz [21] tried to answer that how should operating rooms be scheduled in order to maximize the throughput and to minimize waiting time and overtime. He stated that short procedures contain less variability than long procedures, and they have smaller deviations. Therefore, scheduling the short procedure first maximizes on-time performance. A Monte Carlo Simulation applied to the problem, and random numbers are generated from normal distribution. According to the results, it is seen that scheduling short procedures first increased the efficiency of operating rooms.

Denton *et al.* [22] used a two-stage stochastic programming model in order to sequence surgeries and schedule surgery durations simultaneously. The objective function is the weighted sum of waiting time, idle time, and overtime. Sample Average Approximation method is used for finding solutions. Furthermore, four different heuristics are used for sequencing the surgeries: (1) sequencing the surgeries in order of increasing mean of durations, (2) sequencing the surgeries in order of increasing variance of durations, (3) sequencing the surgeries in order of increasing coefficient of variation of durations, and (4) interchange heuristic. The last heuristic resembles L-Shaped method, since it first initializes the master problem and then add optimality cut to the master problem. According to the results, it is observed that sequencing surgeries in order of increasing variance of durations outperforms other heuristics in terms of solution quality and ease of implementation.

Hans *et al.* [23] assigned a planned slack overtime variable to operating rooms. By minimizing this planned slack variable, it is aimed to make the surgery schedule robust against overtime. The aim is to maximize capacity utilization and to minimize the overtime. According to the results, the best heuristic among the heuristics used in the article was regret-based random sampling. Moreover, they observed that surgeries with similar duration variability are generally clustered on the same time.

Mancilla and Storer [24] proved that finite scenario Sample Average Approximation method is NP-Complete. They developed a heuristic method based on Bender's Decomposition, and the results are compared with the solutions found by SAA method. When the other heuristics and the heuristic based on Bender's Decomposition are compared, it is found that heuristic based on Bender's gives significantly better results than sort-by-variance heuristic when the unit costs are unequal. On the other hand, the computation time of the proposed method is larger than the computation time of sort-by-variance heuristic.

Baker [25] studied on Single Machine Stochastic Scheduling Problem. The objective function is the weighted sum of the total expected tardiness and earliness cost. He found that when the job processing times are distributed normally and earliness and tardiness costs are the same for each job, the optimal sequence is obtained by sequencing the surgeries in order of increasing variance. The results can be used in operating room scheduling problem.

Guda *et al.* [26] found that Shortest-Variance-First rule is optimal under the assumption of dilation ordering of processing durations. The objective function is the weighted sum of earliness and tardiness cost, and it is assumed that the tardiness and earliness costs are same for each job.

2.3.2. Multiple Operating Room Problem

As mentioned earlier, majority of studies on operating room scheduling considers single operating room problem. This section summarizes a list of articles that consider multiple operating room scheduling problem.

Denton *et al.* [27] developed a simulation model for a multiple operating room problem. In this article, the uncertainty related to the intake process, surgical procedure, and recovery process is concerned. The objective function is the weighted sum of waiting time and overtime. Results are presented based on real data. They used a simple simulated annealing method in order to improve patient arrival schedule. According to the results, they found that even a simple scheduling heuristic may result in improvements in the waiting time and overtime.

Lamiri *et al.* [28] developed a stochastic model for operating room scheduling with elective and emergent surgeries. It is assumed that surgery durations of elective surgeries are known and deterministic. They proposed a Monte Carlo Optimization Method that combines Monte Carlo Simulation and Mıxed Integer Program. In this model, elective surgeries are assigned into different periods over planning horizon in order to minimize the sum of elective patient related cost and overtime cost of operating rooms. This article differs from our problem in that the planning horizon consists more than one day.

Lamiri *et al.* [29] continued to study on the same problem. In this article, they assumed that operating rooms are identical, and SAA method is used for finding solutions. They compared the results of different optimization models. According to the results, Monte Carlo optimization is the best method, and Taboo Search method is the best heuristic method. Furthermore, they proved that Monte Carlo optimization method converges exponentially to a real optimal solution.

Denton *et al.* [30] developed two methods in order to minimize the total sum of the fixed cost of opening operating rooms and the variable cost of overtime relative to a fixed length of day. The first method is two-stage stochastic linear program with binary decisions in the first stage, and simple recourse in the second stage. The binary decisions in the first stage are the number of operations to be opened, and the assignment of surgeries to operating rooms. The second method tries to minimize the maximum cost associated with uncertainty of surgery durations. According to the results, it is found that the second method is faster than the first method. Furthermore, it benefits from limiting the worst-case outcome of the recourse problem.

Batun *et al.* [31] used L-Shaped method for multiple operating room and multiple surgeries problem. In the first stage, the number of operating rooms to be opened, the assignment of surgeries into operating rooms, and the sequence of the surgeries within operating rooms are determined. Therefore, this stage only includes binary variables. In the second stage, the weighted sum of the waiting time, idle time, and overtime is calculated by using the values of binary variables obtained in the first stage. This phase consists only continuous variables. In order to fasten the algorithm, a lower bound is added to the master problem, and it is observed that the computation time decreases significantly.

Zhao and Li [6] developed a method for sequence-dependent setup times of surgeries. It is aimed to decide on the number of operating rooms to open, the assignment of surgeries to operating rooms, and the sequence of surgeries within an operating room. It is assumed that surgeries belong to different types, and each operating room allow only a set of surgery types. Furthermore, it is assumed that setup time and surgery durations are deterministic. Mixed Integer Nonlinear Programming model and Constraint Programming model are used to solve the problem. According to results, it is observed that Constraint Programming model gives more efficient results in terms of computation time and solution quality.

2.3.3. Scheduling with Sequence-Dependent Setup Times

Although sequence-dependent surgery durations have not been considered sufficiently in operating room scheduling problems, there are several studies on machine scheduling with sequence-dependent setup times. As in [6], majority of studies on machine scheduling problems with sequence-dependent setup times focus on deterministic setup times due to the computational burden of stochastic problems. In this section, a number of studies on machine scheduling with sequence-dependent setup times are summarized.

Guinet [32] proposed an assignment algorithm for scheduling of jobs when there exists a number of identical parallel machines. The objective is to minimize the maximum completion time of the jobs or the mean completion time of jobs. The algorithm proposed is an extension of Hungarian algorithm.

Soroush and Fredendall [33] studied on single machine scheduling problem. It is assumed that processing times are uncertain and normally distributed, but due dates are deterministic. The objective is to minimize the total expected earliness and tardiness cost. Three heuristic methods are proposed for finding solutions to the problem. According to the computational experiments, two of these heuristics are good at finding optimal sequences.

Lee *et al.* [34] proposed a three phase heuristic for scheduling of jobs when there exists only one machine. It is assumed that setup times are sequence-dependent and all jobs are available for processing at time zero. The objective is to minimize the total weighted tardiness. In the first phase, a number of parameters that characterize the problem are calculated. By using these parameters, schedule of jobs is found based on a priority rule in the second phase. In the last phase, the schedule obtained in the second phase is improved by using a local improvement procedure. Then, Lee *et al.* [35] extended the same problem to a number of parallel and identical machines. The same heuristic is used for the solution of the problem, and simulated annealing method is applied in the third phase.

Anglani *et al.* [36] developed a robust approach for the scheduling of jobs when there exists a number of parallel machines. It is assumed that processing times are uncertain. A fuzzy mathematical programming model is formulated in order to balance the trade-off between total setup costs and robustness in demand satisfaction. A heuristic method is used in the solution of the problem since the problem is a Nonlinear Mixed Integer Program. According to the result, average deviation from the optimal solution is less than 1.5%.

Table 2.1. Comparison of articles.

property	Our Problem	6	12	13	14	15	16	17	18	19	22	24	25	27	28	29	30	31
elective	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
emergent															✓	✓		
next-day	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓			✓	✓
multi-day										✓					✓	✓		
deterministic		✓								✓								
stochastic	✓		✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓
independent	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
correlated								✓										
waiting time	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓				✓
idle time	✓		✓			✓	✓	✓			✓	✓	✓					✓
overtime	✓	✓					✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓

Table 2.1 gives a summary table for comparison of properties of some articles in this literature review and our thesis. The first two rows specify the patient characteristics. The next two rows show the planning horizon. The next two rows specify whether surgery durations are deterministic or stochastic. The next two rows show whether surgery durations are independently distributed or correlated. The last three rows are the performance measures used in the models. According to this table, it is seen that the most closely related problem to our problem is [31]. The difference is the distribution of the surgery durations. They estimated the distributions of surgery durations from historical data. Furthermore, surgery durations are not sequence-dependent.

In conclusion, there are several studies on the scheduling and sequencing of the surgeries. It is nearly impossible to solve these problems analytically, and different heuristics are developed in order to solve these problems. However, there is no study on the scheduling and sequencing of sequence-dependent stochastic surgeries. Only one study ([6]) considers the sequence-dependent setup times between two surgeries, but the nature of this problem and our problem is completely different. The problem uses deterministic setup and surgery durations. Therefore, the problem only deals with the sequencing of the surgeries. On the other hand, our problem uses stochastic surgery durations, and its computational complexity increases significantly as the number of surgeries and the number of operating rooms increase. When studies on machine scheduling with sequence-dependent setup times are investigated, it is observed that majority of these studies also use deterministic setup times as in [6]. All in all, our problem becomes unique in its nature.

3. MATHEMATICAL MODEL

3.1. Motivation

In operating room scheduling problems, high uncertainty of surgery durations makes the problem challenging. There are lots of factors affecting surgery durations, such as time of the day, ability of hospital staff, surgery type, and so on. In order to deal with this uncertainty, the problem is modeled as Stochastic Mixed Integer Programming (SMIP).

3.2. Properties & Assumptions

This problem is a next-day operating room scheduling problem. The list of the surgeries is known one day before the day of the surgeries. Therefore, all of the surgeries are assumed to be elective. It is assumed that surgery durations of different surgeries have independent distributions. Furthermore, distributions of surgery durations may change according to sequence of surgeries.

In this problem, there are 3 objectives:

- to minimize the total expected waiting time of the patients between the assigned starting time and actual starting time of their surgeries
- to minimize the total expected idle time of the operating rooms between two successive surgeries
- to minimize the total expected overtime of the operating rooms

By giving weights to each of these objectives, a multi-objective optimization problem is developed.

In this model, it is assumed that constraints of the problem will be satisfied by all possible values that existing random variables can take.

There exists three surgery operations completed within an operating room, and these are setup time before the surgery, surgery duration, and clean-up time after the surgery, respectively. During setup time, both operating room and patient are prepared for the surgery. During clean-up time, operating room will be cleaned. Furthermore, patient will become awake and he/she will be transported to post-anesthesia-care-unit during clean-up time. In this problem, it is assumed that assigned surgery durations also include the setup time and the clean-up time, and they are considered as a whole. Therefore, when a surgery is completed, the following surgery starts immediately in the planned schedule. There is no need to leave an empty space for setup time and clean-up time.

It is assumed that surgeries, which are ranked as the first in each operating room, start without any waiting time. Therefore, waiting time variables for these surgeries take zero value. It is also assumed that a surgery cannot start earlier than its planned time, even if the operating room becomes idle. On the other hand, a surgery may start later than its planned time, if previous surgery is not completed on time.

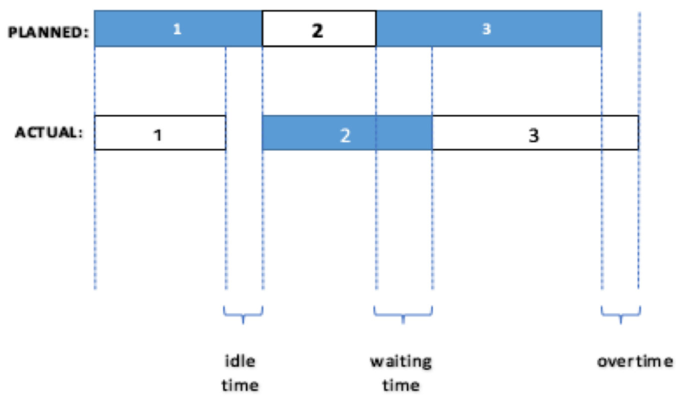


Figure 3.1. A Sample Realization.

Figure 3.1 provides a sample realization of surgeries within an operating room. As mentioned earlier, the first surgery starts without any waiting time. It is seen that the first surgery lasts shorter than its planned time. According to the previous assumption, the second surgery cannot start until its planned starting time and the operating room remains idle until that time. It is seen that the second surgery lasts longer than its planned time. This means that the next surgery will be delayed until the completion of the second surgery. This delay is equal to the waiting time of the patient for the third surgery. In this problem, it is assumed that the end of the day is taken as the planned completion time of the last surgery. Since the last surgery lasts longer than its planned time in Figure 3.1, this time interval is the overtime.

3.3. Notation

In this section, the notation of the model will be defined. Summary of the parameter set can be found in Table 3.1, and summary of the decision variable set can be found in Table 3.2.

Let $S = \{1, ..., N\}$ be the set of surgeries that need to be scheduled for the next day. There are m operating rooms, and each operating room deals with n_k surgeries for $k = \{1, ..., m\}$. It is known that $\sum_{k=1}^{m} n_k = N$. In this problem, it is assumed that the number of surgeries assigned to each operating room is determined in advance by hospital management. Therefore, n_k values are given as parameters to the model. As mentioned in the previous section, this model has three objectives, and the objective function is defined as the weighted sum of these objectives. These weights are the unit costs of these three objectives. α_1 is defined as the unit cost of the idle time, α_2 is defined as the unit cost of the waiting time, and α_3 is defined as the unit cost of the overtime.

$T_{i,j,k}$ is the random duration of surgery j, which has cumulative distribution function $F_{i,j}(x)$ and density function $f_{i,j}(x)$, when it is assigned to i^{th} sequence in k^{th} operating room. It is assumed that surgery durations are not dependent on operating room since they are assumed to be identical. Therefore, $F_{i,j}(x)$ and $f_{i,j}(x)$ do not have an index for operating room.

There are two types of decision variables in this model. Continuous variables are used for assignment of surgery durations, and binary variables are used for sequence of surgeries. $D_{i,k}$ is the assigned duration of i^{th} scheduled surgery in k^{th} operating room. $w_{i,j,k}$ is the waiting time of the patient who is waiting for surgery j, which is assigned

Table 3.1. Parameter set.

S	set of surgeries that need to be scheduled for the next day
m	number of operating rooms
N	number of surgeries
n_k	number of surgeries assigned to k^{th} operating room
α_1	the unit cost of idle time
α_2	the unit cost of waiting time
α_3	the unit cost of overtime
$T_{i,j,k}$	random duration of surgery j when it is assigned to i^{th} sequence in k^{th} operating room
$F_{i,j}(x)$	cumulative distribution function of $T_{i,j,k}$
$f_{i,j}(x)$	density function of $T_{i,j,k}$

to i^{th} sequence in k^{th} operating room. It takes positive value only if $(i-1)^{th}$ scheduled surgery in k^{th} operating room lasts longer than its planned time. $s_{i,j,k}$ is the idle time of the operating room right after i^{th} scheduled surgery in k^{th} operating room. It is the time interval between the completion time of i^{th} scheduled surgery and the planned starting time of $(i+1)^{th}$ scheduled surgery in k^{th} operating room. o_k is the excessive time over the total planned time of all surgeries assigned to k^{th} operating room. It takes positive value if the last scheduled surgery in k^{th} operating room lasts longer than its planned time.

$$x_{i,j,k} = \begin{cases} 1, & \text{if surgery j is assigned to } i^{th} \text{ sequence in } k^{th} \text{ operating room} \\ 0, & \text{otherwise} \end{cases}$$

Table 3.2. Decision variable set.

$w_{i,j,k}$	waiting time for surgery j when it is assigned to i^{th} sequence in k^{th} operating room
$s_{i,j,k}$	idle time of operating room k before $(i+1)^{th}$ scheduled surgery when surgery j is assigned to i^{th} sequence in k^{th} operating room
o_k	overtime in k^{th} operating room
$D_{i,k}$	duration assigned to i^{th} sequence in k^{th} operating room
$x_{i,j,k}$	binary variable which is 1 if surgery j is assigned to i^{th} sequence in k^{th} operating room

3.4. The Mathematical Model

In this section, objective function and constraints of the mathematical model will be defined in detail.

$$\text{minimize} \quad E\left[\sum_{k=1}^{m}\left(\sum_{j=1}^{N}\left(\sum_{i=1}^{n_k}\alpha_1 * s_{i,j,k} + \sum_{i=2}^{n_k}\alpha_2 * w_{i,j,k}\right) + \alpha_3 * o_k\right)\right] \tag{3.1}$$

subject to:

$$-\sum_{j=1}^{N} w_{i,j,k} + \sum_{j=1}^{N} w_{i+1,j,k} - \sum_{j=1}^{N} s_{i,j,k} + D_{i,k} = \sum_{j=1}^{N} T_{i,j,k} * x_{i,j,k} \tag{3.2}$$

$i = 1, ..., n_k - 1 \quad \& \quad k = 1, ..., m$

$$-\sum_{j=1}^{N} w_{n_k,j,k} - \sum_{j=1}^{N} s_{n_k,j,k} + D_{n_k,k} + o_k = \sum_{j=1}^{N} T_{n_k,j,k} * x_{n_k,j,k} \tag{3.3}$$

$k = 1, ..., m$

$$\sum_{k=1}^{m}\sum_{i=1}^{n_k} x_{i,j,k} = 1 \qquad j = 1, ..., N \tag{3.4}$$

$$\sum_{j=1}^{N} x_{i,j,k} = 1 \qquad i = 1, ..., n_k \quad \& \quad k = 1, ..., m \tag{3.5}$$

$$w_{i,j,k} \leq M * x_{i,j,k} \qquad i = 1, ..., n_k, \quad j = 1, ..., N \quad \& \quad k = 1, ..., m \tag{3.6}$$

$$s_{i,j,k} \leq M * x_{i,j,k} \qquad i = 1, ..., n_k, \quad j = 1, ..., N \quad \& \quad k = 1, ..., m \tag{3.7}$$

$$w_{i,j,k} \geq 0 \qquad i = 1, ..., n_k, \quad j = 1, ..., N \quad \& \quad k = 1, ..., m \tag{3.8}$$

$$s_{i,j,k} \geq 0 \qquad i = 1, ..., n_k, \quad j = 1, ..., N \quad \& \quad k = 1, ..., m \tag{3.9}$$

$$o_k \geq 0 \qquad k = 1, ..., m \tag{3.10}$$

$$D_{i,k} \geq 0 \qquad i = 1, ..., n_k \quad \& \quad k = 1, ..., m \tag{3.11}$$

The objective function (3.1) is the expected weighted cost of the waiting time of the patients, the idle time of the operating rooms, and the overtime of the hospital staff.

Constraints (3.2) and (3.3) provide the relation between the waiting time of the patients, the idle time of the operating rooms, and the overtime of the hospital staff. The actual completion time of a surgery is the sum of planned starting time of the surgery, waiting time of the patient for that surgery and the actual duration of that surgery. According to Constraint (3.2), if the actual completion time of a surgery is later than the planned completion time of the surgery, the next patient will wait for his/her surgery. On the other hand, if it is earlier than the planned completion time of the surgery, operating room will remain idle until the planned starting time of the next surgery. This constraint is defined for all surgeries except the last scheduled surgery within each operating room. According to Constraint (3.3), if the actual completion time of the last scheduled surgery is later than the planned completion time of that surgery, an overtime will occur. Constraints (3.4) and (3.5) ensure that a surgery is assigned to exactly one sequence in exactly one OR, and each sequence in each OR is assigned to exactly one surgery. Constraints (3.6) and (3.7) ensure that the waiting time and the idle time variables take 0 value unless j^{th} surgery is assigned to i^{th} sequence in k^{th} operating room. In these constraints, M indicates a large number. Constraints (3.8), (3.9), (3.10), and (3.11) define the non-negativity restrictions of the waiting time, the idle time, the overtime, and the assigned duration variables, respectively.

Although the nature of the problem is Stochastic Mixed Integer Problem(SMIP), it is nearly impossible to find an exact solution. Therefore, different approaches are developed in order to come up with approximate solutions to the problem. In the next chapter, these approaches are defined in detail.

4. SOLUTION METHODOLOGY

It is difficult to solve a Stochastic Mixed Integer Program optimally. Since the aim is to extract some managerial insights instead of finding the exact solutions, different approaches are applied to find solutions to the problem. As a first step, Sample Average Approximation method will be defined in this section. By generating a number of scenarios for random variables and estimating the objective function with these scenarios, Sample Average Approximation method provides solutions to difficult problems. Then, L-Shaped method will be defined, which will help solving problems when Sample Average Approximation method becomes inadequate. In the last section, different heuristics, which are used to sequence and schedule surgeries, will be explained with our reasons to use them.

4.1. Sample Average Approximation

4.1.1. Motivation

It is difficult to find an exact solution to a Stochastic Mixed Integer Program due to its computational complexity. Since the nature of the problem is stochastic, it is nearly impossible to consider every single possible value that random variables can take. In these kind of problems, it is found that Sample Average Approximation(SAA) method provides reasonable results, and it is commonly used in operating room scheduling problems as mentioned in Literature Review ([22, 24, 37, 38]).

In this method, a number of scenarios for surgery durations are generated based on the distributions of these surgery durations. By using these scenarios, it is aimed to obtain a general solution to the problem. The objective value is calculated by taking the average of objective values of these scenarios. As the number of scenarios increases, more possible values of random variables are considered, and the result become more reliable.

All constraints and parameters used in this method are the same with those used in the mathematical model. The only difference in this method is the decision variables, since they keep the scenario number as an index.

4.1.2. Notation

In this section, the notation of Sample Average Approximation Method will be defined. Summary of the parameter set can be found in Table 4.1, and summary of the decision variable set can be found in Table 4.2.

Let $S = \{1, ..., N\}$ be the set of surgeries that need to be scheduled for the next day. There are m operating rooms, and each operating room deals with n_k surgeries for $k = \{1, ..., m\}$. It is known that $\sum_{k=1}^{m} n_k = N$. Furthermore, there are P scenarios. As in the previous section, α_1 is defined as the unit cost of the idle time, α_2 is defined as the unit cost of the waiting time, and α_3 is defined as the unit cost of the overtime.

$T_{i,j,k,p}$ is the random duration of surgery j, which has cumulative distribution function $F_{i,j}(x)$ and density function $f_{i,j}(x)$, in p^{th} scenario when it is assigned to i^{th} sequence in k^{th} operating room.

$D_{i,k}$ is the assigned duration of i^{th} scheduled surgery in k^{th} operating room. $w_{i,j,k,p}$ is the waiting time of the patient who is waiting for surgery j, which is assigned to i^{th} sequence in k^{th} operating room in p^{th} scenario. $s_{i,j,k,p}$ is the idle time of the operating room right after i^{th} scheduled surgery in k^{th} operating room in p^{th} scenario. $o_{k,p}$ is the excessive time over the total planned time of all surgeries assigned to k^{th} operating room in p^{th} scenario. Essentially, all variables other than $D_{i,k}$ and $x_{i,j,k}$ get an extra index for scenario number in this method.

$$x_{i,j,k} = \begin{cases} 1, & \text{if surgery j is assigned to } i^{th} \text{ sequence in } k^{th} \text{ operating room} \\ 0, & \text{otherwise} \end{cases}$$

Table 4.1. Parameter set of SAA.

S	set of surgeries that need to be scheduled for the next day
m	number of operating rooms
N	number of surgeries
P	number of scenarios
n_k	number of surgeries assigned to k^{th} operating room
α_1	the unit cost of idle time
α_2	the unit cost of waiting time
α_3	the unit cost of overtime
$T_{i,j,k,p}$	random duration of surgery j in p^{th} scenario when it is assigned to i^{th} sequence in k^{th} operating room
$F_{i,j}(x)$	cumulative distribution function of $T_{i,j,k}$
$f_{i,j}(x)$	density function of $T_{i,j,k}$

Table 4.2. Decision variable set of SAA.

$w_{i,j,k,p}$	waiting time for surgery j when it is assigned to i^{th} sequence in k^{th} operating room in p^{th} scenario
$s_{i,j,k,p}$	idle time of operating room k before $(i+1)^{th}$ scheduled surgery when surgery j is assigned to i^{th} sequence in k^{th} operating room in p^{th} scenario
$o_{k,p}$	overtime in k^{th} operating room in p^{th} scenario
$D_{i,k}$	duration assigned to i^{th} sequence in k^{th} operating room
$x_{i,j,k}$	binary variable which is 1 if surgery j is assigned to i^{th} sequence in k^{th} operating room

4.1.3. The Model of SAA

In this section, the objective function and constraints of the Sample Average Approximation method will be defined in detail.

$$\text{minimize} \quad \frac{1}{P} * \sum_{p=1}^{P} \left(\sum_{k=1}^{m} \left(\sum_{j=1}^{N} \left(\sum_{i=1}^{n_k} \alpha_1 * s_{i,j,k,p} + \sum_{i=2}^{n_k} \alpha_2 * w_{i,j,k,p} \right) + \alpha_3 * o_{k,p} \right) \right) \tag{4.1}$$

subject to:

$$-\sum_{j=1}^{N} w_{i,j,k,p} + \sum_{j=1}^{N} w_{i+1,j,k,p} - \sum_{j=1}^{N} s_{i,j,k,p} + D_{i,k} = \sum_{j=1}^{N} T_{i,j,k,p} * x_{i,j,k} \tag{4.2}$$

$i = 1, ..., n_k - 1, \quad k = 1, ..., m \quad \& \quad p = 1, ..., P$

$$-\sum_{j=1}^{N} w_{n_k,j,k,p} - \sum_{j=1}^{N} s_{n_k,j,k,p} + D_{n_k,k} + o_{k,p} = \sum_{j=1}^{N} T_{n_k,j,k,p} * x_{n_k,j,k} \tag{4.3}$$

$k = 1, ..., m \quad \& \quad p = 1, ..., P$

$$\sum_{k=1}^{m} \sum_{i=1}^{n_k} x_{i,j,k} = 1 \qquad j = 1, ..., N \tag{4.4}$$

$$\sum_{j=1}^{N} x_{i,j,k} = 1 \qquad i = 1, ..., n_k \quad \& \quad k = 1, ..., m \tag{4.5}$$

$$w_{i,j,k,p} \leq M * x_{i,j,k} \qquad i = 1, ..., n_k, \quad j = 1, ..., N, \quad k = 1, ..., m \quad \& \quad p = 1, ..., P \tag{4.6}$$

$$s_{i,j,k,p} \leq M * x_{i,j,k} \qquad i = 1, ..., n_k, \quad j = 1, ..., N, \quad k = 1, ..., m \quad \& \quad p = 1, ..., P \tag{4.7}$$

$$w_{i,j,k,p} \geq 0 \qquad i = 1, ..., n_k, \quad j = 1, ..., N, \quad k = 1, ..., m \quad \& \quad p = 1, ..., P \tag{4.8}$$

$$s_{i,j,k,p} \geq 0 \qquad i = 1, ..., n_k, \quad j = 1, ..., N, \quad k = 1, ..., m \quad \& \quad p = 1, ..., P \tag{4.9}$$

$$o_{k,p} \geq 0 \qquad k = 1, ..., m \quad \& \quad p = 1, ..., P \tag{4.10}$$

$$D_{i,k} \geq 0 \qquad i = 1, ..., n_k \quad \& \quad k = 1, ..., m \tag{4.11}$$

The objective function (4.1) is the average weighted cost of the waiting time of the patients, the idle time of the operating rooms, and the overtime of the surgeons.

Constraints (4.2) and (4.3) provide the relation between the waiting time of the patients, the idle time of the operating rooms, and the overtime of the surgeons. Constraints (4.4) and (4.5) ensure that a surgery is assigned to exactly one sequence in exactly one OR, and each sequence in each OR is assigned to exactly one surgery. Constraints (4.6) and (4.7) ensure that the waiting time and the idle time variables take 0 value unless j^{th} surgery is assigned to i^{th} sequence in k^{th} operating room. In these constraints, M indicates a large number. Constraints (4.8), (4.9), (4.10), and (4.11) define the non-negativity restrictions of the waiting time, the idle time, the overtime, and the assigned duration variables, respectively.

4.1.3.1. Symmetry Breaking Constraints. In terms of sequencing of surgeries, there are N! possible allocation of surgeries. If operating rooms are not identical, every allocation will be different from another allocation. As the number of surgeries and the number of operating rooms increase, the number of possible allocation of surgeries will increase significantly, and in turn the computation time will increase. However, it is possible to decrease the number of allocations that should be considered, if operating rooms are identical. In this case, same allocation will be encountered more than once. In order to eliminate this possibility, two constraints are added to the existing model. These constraints prevent obtaining the same solution again. They are called Symmetry Breaking Constraints.

These constraints are below:

$$\sum_{k=1}^{j} \sum_{i=1}^{n_k} x_{i,j,k} = 1 \qquad j = 1, ..., m \qquad (4.12)$$

$$\sum_{k=r}^{min(j,m)} \sum_{i=1}^{n_k} x_{i,j,k} - \sum_{a=r-1}^{j-1} \sum_{i=1}^{n_k} x_{i,a,r-1} \leq 0 \qquad j = 2, ..., N \quad \& \quad r = 2, ..., min(j, m) \tag{4.13}$$

Constraint (4.12) helps assigning the surgeries to operating rooms in the lexicographical order, since assigning surgeries lexicographically will give feasible solution to the problem [39]. According to this constraint, surgery 1 should be assigned to the first operating room. Surgery 2 can be assigned to the first or the second operating room. By following this pattern, surgery (m-1) can be assigned to the first (m-1) operating rooms and surgery m can be assigned to all operating rooms. For this constraint, it is assumed that $N \geq m$.

Constraint (4.13) tries to prevent assigning surgeries to operating rooms that have a larger OR number index than the smallest OR number index of empty operating rooms [30]. For example, surgery 4 cannot be assigned to the third operating room, if there exists no surgery in the second operating room.

4.2. L-Shaped Method

As the number of operating rooms and the number of surgeries increase, the problem size increases significantly. In turn, the computation time of the problem increases significantly, as well. When SAA method cannot provide solution in a reasonable computation time by itself, L-Shaped method ([40–42]) is applied to the problem. It is applied in [31], and they obtained good results for their problem.

In this method, there are two stages as the solution of the master problem and the solution of the sub problem. In the first stage of the problem, surgeries are assigned to operating rooms and their sequences are determined according to the solution of a Mixed Integer Program. The objective of this problem is to minimize the value of variable Θ, which is the expected recourse function. Θ is the expected value of the objective function of the sub problem given the sequences of surgeries in their assigned operating rooms. In the second stage, the total weighted sum of the waiting time of the patients for the surgeries, the idle time of the operating rooms, and the overtime of the surgeons is minimized. As in the previous section, Sample Average Approximation method is used to solve the sub problem. Since the values of the binary variables are assigned in the first stage, the second stage of L-Shaped method is a Linear Programming problem. The master problem and the sub problem will be defined throughout this section. In Figure 4.1, the algorithm of the L-Shaped method can be found.

Require α_1, α_2, α_3, m & N;
Require Parameters of distributions of surgery durations for $i = 1, 2, \ldots, N/m$ and $j = 1, 2, \ldots, N$;
$w \Leftarrow 0$;
$\Theta \Leftarrow -\infty$;
while $w > \Theta$ **do**
  solve Master Problem and obtain the value of Θ;
  x values in Sub Problem $\Leftarrow$ values of x decision variables in Master Problem;
  Generate scenarios and obtain T matrix
  solve Sub Problem;
  $w \Leftarrow \pi^T * (h - T * x)$;
  add $\Theta \geq \pi^T * (h - T * x)$ to the Master Problem;
end while
w is the optimal value of the problem.

Figure 4.1. L-Shaped Method Algorithm.

As an input, the algorithm requires $\alpha_1, \alpha_2, \alpha_3$ weights, the number of operating rooms (m), and the total number of surgeries (N). It is assumed that each operating room will have the equal number of the surgeries. Furthermore, parameters of distributions of surgery durations for each surgery and each sequence should be provided.

According to the algorithm, w and Θ variables are initialized as a first step. w is equal to 0, and Θ is equal to $-\infty$ at the beginning. w is the objective value of the sub problem, and Θ is the objective value of the master problem. At each step of this algorithm, Θ and w values are obtained respectively. The algorithm continues to iterate as long as the value of w is larger than the value of Θ. At each iteration, an optimality cut is added to the master problem, and these cuts help Θ to converge to the optimal value of the problem.

As it can be seen in the algorithm, w is calculated as $\pi^T * (h - T * x)$ in the algorithm. This value is also equal to the objective value. π is the matrix of optimal simplex multipliers. h is the vector of constants of each constraint. h will be ignored in this problem, since there exists no constants in all constraints. T is the coefficient matrix of the values of the $x_{i,j,k}$ binary variables. This matrix consists the values generated in different scenarios. As mentioned earlier, if the value of w is larger than Θ, an optimality cut is generated and this cut is $\Theta \geq \pi^T * (-T * x)$. As opposed to the use of x in the calculation of the value of w, x defines the matrix of the $x_{i,j,k}$ binary variables in this constraint.

4.2.1. Master Problem Model

The objective function and constraints of the master problem will be defined in this part. The objective value of the master problem is the expected recourse function, which is the expected value of the objective function of the sub problem given the sequences of surgeries in their assigned operating rooms. The model for the master problem is below:

$$\text{minimize} \quad \Theta \tag{4.14}$$

subject to:

$$\sum_{k=1}^{m}\sum_{i=1}^{n_k} x_{i,j,k} = 1 \qquad j = 1, ..., N \tag{4.15}$$

$$\sum_{j=1}^{N} x_{i,j,k} = 1 \qquad i = 1, ..., n_k \quad \& \quad k = 1, ..., m \tag{4.16}$$

$$\sum_{k=1}^{j}\sum_{i=1}^{n_k} x_{i,j,k} = 1 \qquad j = 1, ..., m \tag{4.17}$$

$$\sum_{k=r}^{min(j,m)}\sum_{i=1}^{n_k} x_{i,j,k} - \sum_{a=r-1}^{j-1}\sum_{i=1}^{n_k} x_{i,a,r-1} \leq 0 \qquad j = r, ..., N \quad \& \quad r = 2, ..., min(j, m) \tag{4.18}$$

Constraints (4.15) and (4.16) ensure that a surgery is assigned to exactly one sequence of exactly one OR, and each sequence of each OR is assigned to exactly one surgery. Constraints (4.17) and (4.18) are defined as symmetry breaking constraints in Sample Average Approximation method section. They prevent solving the problem more than once for the same assignment and schedule of the surgeries, if the operating rooms are identical.

4.2.2. Subproblem Model

The objective function and constraints of the sub problem will be defined in this part. The assignment of the surgeries to the operating rooms and the sequencing of the surgeries within these operating rooms are done in the first stage, and these values are given to the sub problem as a parameter. The model is the following:

$$\text{minimize} \quad \frac{1}{P} * \sum_{p=1}^{P}\left(\sum_{k=1}^{m}\left(\sum_{j=1}^{N}\left(\sum_{i=1}^{n_k}\alpha_1 * s_{i,j,k,p} + \sum_{i=2}^{n_k}\alpha_2 * w_{i,j,k,p}\right) + \alpha_3 * o_{k,p}\right)\right) \tag{4.19}$$

subject to:

$$-\sum_{j=1}^{N} w_{i,j,k,p} + \sum_{j=1}^{N} w_{i+1,j,k,p} - \sum_{j=1}^{N} s_{i,j,k,p} + D_{i,k} = \sum_{j=1}^{N} T_{i,j,k,p} * x_{i,j,k} \tag{4.20}$$

$i = 1, ..., n_k - 1, \quad k = 1, ..., m \quad \& \quad p = 1, ..., P$

$$-\sum_{j=1}^{N} w_{n_k,j,k,p} - \sum_{j=1}^{N} s_{n_k,j,k,p} + D_{n_k,k} + o_{k,p} = \sum_{j=1}^{N} T_{n_k,j,k,p} * x_{n_k,j,k} \tag{4.21}$$

$k = 1, ..., m \quad \& \quad p = 1, ..., P$

$$w_{i,j,k,p} \leq M * x_{i,j,k} \qquad i = 1, ..., n_k, \quad j = 1, ..., N, \quad k = 1, ..., m \quad \& \quad p = 1, ..., P \tag{4.22}$$

$$s_{i,j,k,p} \leq M * x_{i,j,k} \qquad i = 1, ..., n_k, \quad j = 1, ..., N, \quad k = 1, ..., m \quad \& \quad p = 1, ..., P \tag{4.23}$$

$$w_{i,j,k,p} \geq 0 \qquad i = 1, ..., n_k, \quad j = 1, ..., N, \quad k = 1, ..., m \quad \& \quad p = 1, ..., P \tag{4.24}$$

$$s_{i,j,k,p} \geq 0 \qquad i = 1, ..., n_k, \quad j = 1, ..., N, \quad k = 1, ..., m \quad \& \quad p = 1, ..., P \tag{4.25}$$

$$o_{k,p} \geq 0 \qquad k = 1, ..., m \quad \& \quad p = 1, ..., P \tag{4.26}$$

$$D_{i,k} \geq 0 \qquad i = 1, ..., n_k \quad \& \quad k = 1, ..., m \tag{4.27}$$

The objective function (4.19) is the average weighted cost of the waiting time of the patients, the idle time of the operating rooms, and the overtime of the surgeons.

Constraints (4.20) and (4.21) provide the relation between the waiting time of the patients, the idle time of the operating rooms, and the overtime of the surgeons.

Constraints (4.22) and (4.23) ensure that the waiting time and the idle time variables take 0 value unless j^{th} surgery is assigned to i^{th} sequence in k^{th} operating room. In these constraints, M indicates a large number. Constraints (4.24), (4.25), (4.26), and (4.27) define the non-negativity restrictions of the waiting time, the idle time, the overtime, and the assigned duration variables, respectively.

4.3. Heuristics

In this section, a number of heuristics are defined for allocation of surgery durations, which are Myopic Heuristic, Veteran's Heuristics, Modified Myopic Heuristic, and Expectation Heuristic. Furthermore, a pattern for applying Shortest-Variance-First(SVF) Heuristic to multiple operating rooms is discovered, and it is proven for 3 small cases.

Each problem consists N! possible allocation and sequence of surgeries. Therefore, the number of possible allocations increases significantly as the number of surgeries and the number of operating rooms increase. This increase also leads to an important increase in computation time. As mentioned earlier, this thesis aims to provide some managerial insights on operating room scheduling instead of finding the exact solution. For this reason, even approximate solutions that can be obtained in a reasonable time enable us to obtain some insights. In the next chapter, performances of heuristics will be compared.

4.3.1. Heuristics For Assignment of Surgery Durations

In this section, four different heuristics are used to determine surgery durations before solving the problem. By eliminating the variable $D_{i,k}$ from the model, it is aimed to decrease the computation time.

4.3.1.1. Myopic Heuristic. As indicated in its name, this heuristic assigns a surgery duration for a surgery, as if it is the only surgery waiting to be scheduled. It does not consider the relationship between two successive surgeries, and regard a surgery as isolated from other surgeries. According to [12], the problem is considered as Newsvendor problem if there is only one operating room and two surgeries. In that article, it is also found that the optimal surgery duration is on a quantile of a distribution that is dependent on α_1 and α_2 values.

In this problem, it is assumed that this rule is also valid when there are multiple surgeries and multiple operating rooms. It is assumed that each surgery is independent, and each surgery duration is calculated by using only the distribution of the corresponding surgery duration.

In order to clarify the method, the model will be defined as a single operating room problem, and there will be no index for operating room. At the second part of Myopic Heuristic section, the part of the code added for Myopic heuristic will be defined.

As in the previous sections, α_1,α_2,α_3, and N are used as parameters. $T_{i,j}$ is the random duration of surgery j if it is assigned to i^{th} sequence. $F_{i,j}$ is cumulative distribution function of $T_{i,j}$, and $f_{i,j}$ is density function of $T_{i,j}$. $D_{i,j}$ is the assigned duration of surgery j if it is assigned to i^{th} sequence.

In this method, each surgery will have a separate objective function, and each surgery duration will be calculated by using its own objective function. These objective functions are below:

$$\min \quad \alpha_1 * E\left[(D_{i,j} - T_{i,j})^+\right] + \alpha_2 * E\left[(T_{i,j} - D_{i,j})^+\right] \qquad i = 1, ..., N-1 \quad \& \quad j = 1, ..., N$$

$$\min \quad \alpha_1 * E\left[(D_{i,j} - T_{i,j})^+\right] + \alpha_3 * E\left[(T_{i,j} - D_{i,j})^+\right] \qquad i = N \quad \& \quad j = 1, ..., N$$

Require α_1, α_2, α_3, & N;
Require $F_{i,j},\ i = 1, 2, \ldots, N \quad \& \quad j = 1, 2, \ldots, N$;
for $i = 1$ to N **do**
  for $j = 1$ to N **do**
    if $i < N$ **then**
      $D_{i,j} \Leftarrow F_{i,j}^{-1}\left(\frac{\alpha_2}{\alpha_1+\alpha_2}\right)$;
    else
      $D_{i,j} \Leftarrow F_{i,j}^{-1}\left(\frac{\alpha_3}{\alpha_1+\alpha_3}\right)$;
    end if
  end for
end for

Figure 4.2. Myopic Heuristic Algorithm.

Figure 4.2 gives the algorithm for the assignment of surgery durations when there is a single operating room and multiple surgeries. As in the algorithm, if the surgery is not the last surgery of the day, surgery duration is the $\left(\frac{\alpha_2}{\alpha_1+\alpha_2}\right)^{th}$ quantile of the corresponding distribution. In this situation, α_1 is the unit overage cost, and α_2 is the unit underage cost as in Newsvendor problem. If the surgery is the last surgery of the day, surgery duration is the $\left(\frac{\alpha_3}{\alpha_1+\alpha_3}\right)^{th}$ quantile of the corresponding distribution. In this situation, α_3 will be the unit underage cost as in Newsvendor problem.

In case of multiple operating rooms and multiple surgeries, all constraints used in Sample Average Approximation (SAA) method will remain the same in the model. In addition to those constraints, constraint (4.28) will be added to the model in order to assign surgery durations according to Myopic Heuristic method. $F_{i,j}^{-1}$ is the inverse of cumulative distribution function of $T_{i,j}$.

$$D_{i,k} = \sum_{j=1}^{N} x_{i,j,k} * F_{i,j}^{-1} \qquad i = 1, ..., n_k \quad \& \quad k = 1, ..., m \tag{4.28}$$

4.3.1.2. Veteran's Heuristic. In Veterans Hospital in US, each patient is supposed to be present in hospital in the morning and wait for their sequences. Therefore, when a surgery is completed, the next surgery immediately starts. There is no idle time between two consecutive surgeries. In this case, convolution of distributions of surgery durations of the first r surgeries is required for $r = 1, ..., N$ in order to estimate the completion time of surgeries and N is the number of surgeries. Then, the total assigned duration for this list of surgeries can be calculated by using Newsvendor formula. This method will start from the surgery assigned to the first sequence, and consecutively total assigned durations will be calculated by using convolutions. Each surgery duration will be the difference between two successive total assigned durations.

This method is only used for single operating room problem, therefore there will be no index for the operating room. Moreover, it is assumed that the order of surgeries is known at the beginning in this method. There will be only one index that will indicate the order of the surgery.

$D(i)$ is defined as the total assigned duration for the first i surgeries. The assigned duration of the surgery that is assigned to i^{th} sequence is calculated as below:

$$D_i = D(i) - D(i-1) \tag{4.29}$$

Y_i is the random variable of the convolution of the distributions of surgery durations of the first i surgeries. Therefore, it can be indicated as:

$$Y_i = \sum_{r=1}^{i} T_r \tag{4.30}$$

In this method, each Y_i will have a separate objective function for $i = 1, ..., N$ and $D(i)$ values will be calculated by using these objective functions. Furthermore, $G(i)$ is cumulative distribution function of Y_i, and g_i is density function of Y_i. These objective functions are below:

$$\min \quad \alpha_1 * E\left[(D(i) - Y_i)^+\right] + \alpha_2 * E\left[(Y_i - D(i))^+\right] \qquad i = 1, ..., N-1$$

$$\min \quad \alpha_1 * E\left[(D(i) - Y_i)^+\right] + \alpha_3 * E\left[(Y_i - D(i))^+\right] \qquad i = N$$

Require α_1, α_2, α_3, & N;
Require $G_i,\ i = 1, 2, \ldots, N$;
for $i = 1$ to N **do**
  if $i < N$ **then**
    $D(i) \Leftarrow G_i^{-1}\left(\frac{\alpha_2}{\alpha_1+\alpha_2}\right)$;
  else
    $D(i) \Leftarrow G_i^{-1}\left(\frac{\alpha_3}{\alpha_1+\alpha_3}\right)$;
  end if
end for
$D_1 \Leftarrow D(1)$;
for $i = 2$ to N **do**
  $D_i \Leftarrow D(i) - D(i-1)$;
end for

Figure 4.3. Veteran's Heuristic Algorithm.

Figure 4.3 gives the algorithm for the assignment of surgery durations when Veteran's Heuristic is used. As in the algorithm, if the surgery is not the last surgery of the day, the sum of surgery durations is the $\left(\frac{\alpha_2}{\alpha_1+\alpha_2}\right)^{th}$ quantile of the corresponding convolution. In this situation, α_1 is the unit overage cost, and α_2 is the unit underage cost as in Newsvendor problem. If the surgery is the last surgery of the day, the sum of surgery durations is the $\left(\frac{\alpha_3}{\alpha_1+\alpha_3}\right)^{th}$ quantile of the corresponding convolution. In this situation, α_3 will be the unit underage cost as in Newsvendor problem.

In this method, if $\alpha_2 \leq \alpha_3$, then it is certain that the assigned duration for the last surgery will have a positive value. If $\alpha_2 > \alpha_3$, then the assigned duration for the last surgery may take a negative value. If this is the case, D_N will take 0 value.

There is a drawback of this method. Veteran's Heuristic assumes that there will be no idle time between two successive surgeries. When a surgery is completed, the next surgery will start immediately. However, there will be assigned durations for each surgery in the problem of concern. If a surgery is completed and there exists time until the next surgery, the operating room will remain empty until the assigned duration finishes.

4.3.1.3. Modified Myopic Heuristic. In Myopic Heuristic method, surgery durations are calculated independently. Therefore, the waiting time of patients for surgeries is not considered. Therefore, Modified Myopic Heuristic is developed in order to consider the waiting time.

As in Veteran's Heuristic method, this method is only used for single operating room problem, therefore there will be no index for the operating room. Moreover, it is assumed that the order of the surgeries is known at the beginning in this method. There will be only one index that will indicate the order of the surgery.

D_i^{myp} is the assigned duration for the surgery assigned to i^{th} sequence when Modified Myopic Heuristic method is used.

As it is seen in Figure 4.4, F_i is cumulative distribution function of T_i. The assigned duration of the surgery assigned to the first sequence has the same value of the assigned duration of the surgery found with Myopic Heuristic method, since there is no waiting time for the first surgery. For other surgeries, the assigned duration of a surgery found with Myopic Heuristic will be added to the waiting time of the patient for that surgery in order to obtain the assigned duration of that surgery with Modified Myopic Heuristic method.

Require α_1, α_2, α_3, & N;

Require $F_i,\ i = 1, 2, \ldots, N$;

for $i = 1$ to N **do**

  if $i = 1$ **then**

    $D_i^{myp} \Leftarrow F_i^{-1}\left(\frac{\alpha_2}{\alpha_1+\alpha_2}\right)$;

  else if $i = N$ **then**

    $D_i^{myp} \Leftarrow F_i^{-1}\left(\frac{\alpha_3}{\alpha_1+\alpha_3}\right) + E\left[(T_{i-1} - D_{i-1})^+\right]$;

  else

    $D_i^{myp} \Leftarrow F_i^{-1}\left(\frac{\alpha_2}{\alpha_1+\alpha_2}\right) + E\left[(T_{i-1} - D_{i-1})^+\right]$;

  end if

end for

Figure 4.4. Modified Myopic Heuristic Algorithm.

4.3.1.4. Expectation Heuristic. Many hospitals in the world assigns surgery durations by simply taking the average of similar surgeries. In this method, the assigned surgery duration of surgery j when it is assigned to i^{th} sequence is just the mean of the distribution of the surgery duration of the corresponding surgery as it is used in hospitals.

As in Myopic Heuristic method, all constraints used in Sample Average Approximation (SAA) method will remain the same in the model. In addition to those constraints, constraint (4.31) will be added to the model in order to assign surgery durations according to Expectation Heuristic method. $\mu_{i,j}$ is the mean of the distribution of the surgery duration of surgery j when it is assigned to i^{th} sequence.

$$D_{i,k} = \sum_{j=1}^{N} x_{i,j,k} * \mu_{i,j} \qquad i = 1, ..., n_k \quad \& \quad k = 1, ..., m \qquad (4.31)$$

4.3.2. Heuristics For Sequencing

Next-day operating room scheduling problem is a type of Stochastic Mixed Integer Program. There are lots of binary variables to be determined, and this makes the computation time a significant problem for finding solutions. In order to decrease the computation time, the following heuristic method is defined.

4.3.2.1. Shortest-Variance-First Heuristic. In the article [25], Baker proved that if there is a single operating room and surgery durations are distributed normally, sequencing the surgeries in increasing order of variance gives the optimal solution. Furthermore, it is assumed that the unit costs of earliness and tardiness are equal for all surgeries. Assume that standard deviation of surgery durations is denoted as s_i, which means the standard deviation of the surgery duration of the surgery i. Since it is assumed that surgery durations are distributed normally, the standard deviation of the completion time of surgery j is equal to $\sqrt{\sum_{a=1}^{j} s_a^2}$. Baker reduces the objective function of the problem to the sum of the standard deviation of completion times of surgeries.

In this method, it is aimed to apply the same rule for multiple operating rooms. In order to apply the rule, the optimal assignment of surgeries to operating rooms should be found as a first step. A pattern for optimal assignment of surgeries to operating rooms is discovered, and this pattern is proven for 2 operating rooms-4 surgeries, 3 operating rooms-6 surgeries, and 2 operating rooms-6 surgeries.

It is assumed that operating rooms are identical in each of the cases below. It is also assumed that same number of surgeries are assigned to each operating room. The surgeries within an operating room are sorted in increasing order of standard deviation of surgery durations based on Baker's proof.

Theorem 4.1. *If there are 2 operating rooms and 4 surgeries, the optimal sequence of the surgeries is* $1 \rightarrow 3$ *for OR-1 and* $2 \rightarrow 4$ *for OR-2 when* $s_1 < s_2 < s_3 < s_4$ *and the assumptions above are valid.*

Proof. Surgeries are sorted in increasing order of standard deviation of surgery durations ($s_1 < s_2 < s_3 < s_4$).

If operating rooms are not assumed to be identical, there would be $\binom{4}{2} = 6$ cases to be tested. However, it is assumed that the operating rooms are identical. By assigning surgery 1 to OR-1 in each case, there will be only $\binom{3}{1} = 3$ cases to be tested.

Table 4.3. Cases for Theorem 4.1.

Case Number	**OR-1**	**OR-2**	**Objective Function**
1	1→2	3→4	$s_1 + \sqrt{s_1^2 + s_2^2} + s_3 + \sqrt{s_3^2 + s_4^2}$
2	1→3	2→4	$s_1 + \sqrt{s_1^2 + s_3^2} + s_2 + \sqrt{s_2^2 + s_4^2}$
3	1→4	2→3	$s_1 + \sqrt{s_1^2 + s_4^2} + s_2 + \sqrt{s_2^2 + s_3^2}$

Let's start with comparing Case-2 and Case-3. Since s_1 and s_2 are common terms in both cases, they are ignored. The square of the remaining terms are taken for both cases.

Table 4.4. Comparison of Case-2 & Case-3 - Step 1 (Theorem 4.1).

Case Number	**Compared Value**
2	$s_1^2 + s_2^2 + s_3^2 + s_4^2 + 2 * \sqrt{(s_1^2 + s_3^2) * (s_2^2 + s_4^2)}$
3	$s_1^2 + s_2^2 + s_3^2 + s_4^2 + 2 * \sqrt{(s_1^2 + s_4^2) * (s_2^2 + s_3^2)}$

Common terms are eliminated, and the square of the remaining terms are taken for both cases.

Table 4.5. Comparison of Case-2 & Case-3 - Step 2 (Theorem 4.1).

Case Number	**Compared Value**
2	$(s_1 * s_2)^2 + (s_1 * s_4)^2 + (s_2 * s_3)^2 + (s_3 * s_4)^2$
3	$(s_1 * s_2)^2 + (s_1 * s_3)^2 + (s_2 * s_4)^2 + (s_3 * s_4)^2$

$(s_1 * s_2)^2$ and $(s_3 * s_4)^2$ are the common terms in both cases, and they are eliminated. $(s_1 * s_3)^2 + (s_2 * s_4)^2$ is subtracted from both cases.

Table 4.6. Comparison of Case-2 & Case-3 - Step 3 (Theorem 4.1).

Case Number	**Compared Value**
2	$(s_1 * s_4)^2 - (s_1 * s_3)^2 + (s_2 * s_3)^2 - (s_2 * s_4)^2$
3	0

When the remaining terms in Case-2 are arranged, it becomes $(s_1^2 - s_2^2)*(s_4^2 - s_3^2)$. According to the initial assumption, $s_1 < s_2$ and $s_3 < s_4$. Therefore, Case-2 gives a negative value, whereas Case-3 is equal to 0. Since Case-3 gives larger objective value, it is eliminated.

The next step is to compare the objective values of Case-1 and Case-2. Since s_1 is the common terms in both cases, it is ignored.

Table 4.7. Comparison of Case-1 & Case-2 - Step 1 (Theorem 4.1).

Case Number	**Compared Value**
1	$s_3 + \sqrt{s_1^2 + s_2^2} + \sqrt{s_3^2 + s_4^2}$
2	$s_2 + \sqrt{s_1^2 + s_3^2} + \sqrt{s_2^2 + s_4^2}$

Since $s_3 > s_2$, $\sqrt{s_3^2 + s_4^2}$ is larger than $\sqrt{s_2^2 + s_4^2}$. If $s_3 + \sqrt{s_1^2 + s_2^2}$ is also larger than $s_2 + \sqrt{s_1^2 + s_3^2}$, the comparision will be done. Then, let's take the square of these terms and compare them.

Table 4.8. Comparison of Case-1 & Case-2 - Step 2 (Theorem 4.1).

Case Number	**Compared Value**
1	$s_1^2 + s_2^2 + s_3^2 + 2 * \sqrt{(s_1 * s_3)^2 + (s_2 * s_3)^2}$
2	$s_1^2 + s_2^2 + s_3^2 + 2 * \sqrt{(s_1 * s_2)^2 + (s_2 * s_3)^2}$

There is only one different term between Case-1 and Case-2, and these values are $(s_1 * s_3)^2$ and $(s_1 * s_2)^2$. Since $s_3 > s_2$, $(s_1 * s_3)^2$ is larger than $(s_1 * s_2)^2$.

Since the objective value of Case-1 is larger than the objective value of Case-2, Case-1 is also eliminated. Since Case-2 gives the minimum objective value, the sequence of surgeries in Case-2 is optimal. In the optimal sequence, surgery 1 precedes surgery 3, and surgery 2 precedes surgery 4. □

Theorem 4.2. *If there are 3 operating rooms and 6 surgeries, the optimal sequence of the surgeries is* $1 \rightarrow 4$ *for OR-1,* $2 \rightarrow 5$ *for OR-2, and* $3 \rightarrow 6$ *for OR-3 when* $s_1 < s_2 < s_3 < s_4 < s_5 < s_6$ *and the assumptions above are valid.*

Proof. Surgeries are sorted in increasing order of standard deviation of surgery duration $(s_1 < s_2 < s_3 < s_4 < s_5 < s_6)$.

If operating rooms are not assumed to be identical, there would be $\binom{6}{2} * \binom{4}{2} = 90$ cases to be tested. When the operating rooms are identical, there will be $\binom{5}{1} * \binom{4}{2} = 30$ cases to be tested by assigning surgery 1 to OR-1 in each case. The number of cases are decreased further by using Theorem 4.1. There are 5 possible surgeries that can be assigned to OR-1. When a surgery is assigned to OR-1, the remaining problem will be 2 operating rooms and 4 surgeries problem and the sequence of the surgeries of this problem is known in advance. Therefore, the proof will continue with 5 possible cases.

Table 4.9. Cases for Theorem 4.2

Case Number	OR-1	OR-2	OR-3	Objective Function
1	1→2	3→5	4→6	$s_1 + \sqrt{s_1^2 + s_2^2} + s_3 + \sqrt{s_3^2 + s_5^2} + s_4 + \sqrt{s_4^2 + s_6^2}$
2	1→3	2→5	4→6	$s_1 + \sqrt{s_1^2 + s_3^2} + s_2 + \sqrt{s_2^2 + s_5^2} + s_4 + \sqrt{s_4^2 + s_6^2}$
3	1→4	2→5	3→6	$s_1 + \sqrt{s_1^2 + s_4^2} + s_2 + \sqrt{s_2^2 + s_5^2} + s_3 + \sqrt{s_3^2 + s_6^2}$
4	1→5	2→4	3→6	$s_1 + \sqrt{s_1^2 + s_5^2} + s_2 + \sqrt{s_2^2 + s_4^2} + s_3 + \sqrt{s_3^2 + s_6^2}$
5	1→6	2→4	3→5	$s_1 + \sqrt{s_1^2 + s_6^2} + s_2 + \sqrt{s_2^2 + s_4^2} + s_3 + \sqrt{s_3^2 + s_5^2}$

Let's start with comparing Case-4 and Case-5. Since s_1, s_2, s_3, and $\sqrt{s_2^2 + s_4^2}$ are common terms in both cases, they are ignored. The square of the remaining terms are taken for both cases.

Table 4.10. Comparison of Case-4 & Case-5 - Step 1 (Theorem 4.2).

Case Number	Compared Value
4	$s_1^2 + s_3^2 + s_5^2 + s_6^2 + 2 * \sqrt{(s_1^2 + s_5^2) * (s_3^2 + s_6^2)}$
5	$s_1^2 + s_3^2 + s_5^2 + s_6^2 + 2 * \sqrt{(s_1^2 + s_6^2) * (s_3^2 + s_5^2)}$

Common terms are eliminated, and the square of the remaining terms are taken for both cases.

Table 4.11. Comparison of Case-4 & Case-5 - Step 2 (Theorem 4.2).

Case Number	Compared Value
4	$(s_1 * s_3)^2 + (s_1 * s_6)^2 + (s_3 * s_5)^2 + (s_5 * s_6)^2$
5	$(s_1 * s_3)^2 + (s_1 * s_5)^2 + (s_3 * s_6)^2 + (s_5 * s_6)^2$

$(s_1 * s_3)^2$ and $(s_5 * s_6)^2$ are the common terms in both cases, and they are eliminated. $(s_1 * s_5)^2 + (s_3 * s_6)^2$ is subtracted from both cases.

Table 4.12. Comparison of Case-4 & Case-5 - Step 3 (Theorem 4.2).

Case Number	**Compared Value**
4	$(s_1 * s_6)^2 - (s_1 * s_5)^2 + (s_3 * s_5)^2 - (s_3 * s_6)^2$
5	0

When the remaining terms in Case-4 are arranged, it becomes $(s_1^2 - s_3^2)*(s_6^2 - s_5^2)$. According to the initial assumption, $s_1 < s_3$ and $s_5 < s_6$. Therefore, Case-4 gives a negative value, whereas Case-5 is equal to 0. Since Case-5 gives larger objective value, it is eliminated.

The next step is to compare the objective values of Case-3 and Case-4. Since s_1, s_2, s_3, and $\sqrt{s_3^2 + s_6^2}$ are common terms in both cases, they are ignored. The square of the remaining terms are taken for both cases.

Table 4.13. Comparison of Case-3 & Case-4 - Step 1 (Theorem 4.2).

Case Number	**Compared Value**
3	$s_1^2 + s_2^2 + s_4^2 + s_5^2 + 2 * \sqrt{(s_1^2 + s_4^2) * (s_2^2 + s_5^2)}$
4	$s_1^2 + s_2^2 + s_4^2 + s_5^2 + 2 * \sqrt{(s_1^2 + s_5^2) * (s_2^2 + s_4^2)}$

Common terms are eliminated, and the square of the remaining terms are taken for both cases.

Table 4.14. Comparison of Case-3 & Case-4 - Step 2 (Theorem 4.2).

Case Number	**Compared Value**
3	$(s_1 * s_2)^2 + (s_1 * s_5)^2 + (s_2 * s_4)^2 + (s_4 * s_5)^2$
4	$(s_1 * s_2)^2 + (s_1 * s_4)^2 + (s_2 * s_5)^2 + (s_4 * s_5)^2$

$(s_1 * s_2)^2$ and $(s_4 * s_5)^2$ are the common terms in both cases, and they are eliminated. $(s_1 * s_4)^2 + (s_2 * s_5)^2$ is subtracted from both cases.

Table 4.15. Comparison of Case-3 & Case-4 - Step 3 (Theorem 4.2).

Case Number	**Compared Value**
3	$(s_1 * s_5)^2 - (s_1 * s_4)^2 + (s_2 * s_4)^2 - (s_2 * s_5)^2$
4	0

When the remaining terms in Case-3 are arranged, it becomes $(s_1^2 - s_2^2)*(s_5^2 - s_4^2)$. According to the initial assumption, $s_1 < s_2$ and $s_4 < s_5$. Therefore, Case-3 gives a negative value, whereas Case-4 is equal to 0. Since Case-4 gives larger objective value, it is eliminated.

The next step is to compare the objective values of Case-2 and Case-3. Since s_1, s_2, and $\sqrt{s_2^2 + s_5^2}$ are common terms in both cases, they are ignored.

Table 4.16. Comparison of Case-2 & Case-3 - Step 1 (Theorem 4.2).

Case Number	**Compared Value**
2	$\sqrt{s_1^2 + s_3^2} + s_4 + \sqrt{s_4^2 + s_6^2}$
3	$\sqrt{s_1^2 + s_4^2} + s_3 + \sqrt{s_3^2 + s_6^2}$

Since $s_4 > s_3$, $\sqrt{s_4^2 + s_6^2}$ is larger than $\sqrt{s_3^2 + s_6^2}$. If $\sqrt{s_1^2 + s_3^2} + s_4$ is also larger than $\sqrt{s_1^2 + s_4^2} + s_3$, the comparision will be done. Then, let's take the square of these terms and compare them.

Table 4.17. Comparison of Case-2 & Case-3 - Step 2 (Theorem 4.2).

Case Number	**Compared Value**
2	$s_1^2 + s_3^2 + s_4^2 + 2 * \sqrt{(s_1 * s_4)^2 + (s_3 * s_4)^2}$
3	$s_1^2 + s_3^2 + s_4^2 + 2 * \sqrt{(s_1 * s_3)^2 + (s_3 * s_4)^2}$

There is only one different term between Case-2 and Case-3, and these values are $(s_1 * s_4)^2$ and $(s_1 * s_3)^2$. Since $s_4 > s_3$, $(s_1 * s_4)^2$ is larger than $(s_1 * s_3)^2$. Therefore, the objective value of Case-2 is larger than the objective value of Case-3, and Case-2 is also eliminated.

The last comparision is between Case-1 and Case-3. Since s_1 and s_3 are common terms in both cases, they are ignored.

Table 4.18. Comparison of Case-1 & Case-3 - Step 1 (Theorem 4.2).

Case Number	**Compared Value**
1	$\sqrt{s_1^2 + s_2^2} + \sqrt{s_3^2 + s_5^2} + s_4 + \sqrt{s_4^2 + s_6^2}$
3	$\sqrt{s_1^2 + s_4^2} + s_2 + \sqrt{s_2^2 + s_5^2} + \sqrt{s_3^2 + s_6^2}$

It is seen that $\sqrt{s_3^2 + s_5^2}$ is larger than $\sqrt{s_2^2 + s_5^2}$, and $\sqrt{s_4^2 + s_6^2}$ is larger than $\sqrt{s_3^2 + s_6^2}$. If the sum of remaining terms in Case-1 are also larger than the sum of remaining terms in Case-3, the comparision will be done. Let's take the square of the remaining terms.

Table 4.19. Comparison of Case-1 & Case-3 - Step 2 (Theorem 4.2).

Case Number	**Compared Value**
1	$s_1^2 + s_2^2 + s_4^2 + 2 * \sqrt{(s_1 * s_4)^2 + (s_2 * s_4)^2}$
3	$s_1^2 + s_2^2 + s_4^2 + 2 * \sqrt{(s_1 * s_2)^2 + (s_2 * s_4)^2}$

There is only one different term between Case-1 and Case-3, and these values are $(s_1 * s_4)^2$ and $(s_1 * s_2)^2$. Since $s_4 > s_2$, $(s_1 * s_4)^2$ is larger than $(s_1 * s_2)^2$. Therefore, the objective value of Case-1 is larger than the objective value of Case-3, and Case-1 is also eliminated. Case-3 provides the minimum objective value, and the sequence of surgeries in Case-3 gives the optimal sequence of surgeries. In the optimal sequence, surgery 1 precedes surgery 4 in OR-1, surgery 2 precedes surgery 5 in OR-2, and surgery 3 precedes surgery 6 in OR-3. □

Theorem 4.3. *If there are 2 operating rooms and 6 surgeries, the optimal sequence of the surgeries is* $1 \to 3 \to 5$ *for OR-1 and* $2 \to 4 \to 6$ *for OR-2 when* $s_1 < s_2 < s_3 < s_4 < s_5 < s_6$ *and the assumptions above are valid.*

Appendix A provides the proof of Theorem 4.3. In the rest of the thesis, this pattern is assumed to be true for all cases due to the significant sign of its correctness. Furthermore, the assumptions of normally distributed surgeries and equal unit costs are disregarded in this method.

As in Myopic Heuristic method and Expectation Heuristic Method, all constraints used in Sample Average Approximation (SAA) method will remain the same in the model. In addition to those constraints, constraint (4.32) will be added to the model in order to sequence the surgeries according to Shortest-Variance-First Rule. $V_{i,j}$ is the variance of the distribution of the surgery duration of surgery j when it is assigned to

i^{th} sequence.

$$\sum_{j=1}^{N} x_{i-1,j,k} * V_{i-1,j} - \sum_{j=1}^{N} x_{i,j,k} * V_{i,j} \leq 0 \qquad i = 1, ..., n_k \quad \& \quad k = 1, ..., m \tag{4.32}$$

5. NUMERICAL RESULTS

In order to understand the possible results of the algorithms defined in previous sections, a set of experiments are defined, and the results of these experiments are analyzed. By using the results obtained in this section, some useful insights will be provided throughout the section. These experiments are divided into 2 categories. The first part of the experiments assumes that sequence of the surgeries is known in advance. Therefore, only surgery durations are assigned by using algorithms. These experiments are designed for Single OR-2 Surgeries and Single OR-Multiple Surgeries, respectively. The second part of the experiments is used to sequence and schedule surgeries simultaneously and it is designed for Multiple ORs-Multiple Surgeries.

In Single OR-2 Surgeries experiment, the effect of multiplier, mean of surgery durations, coefficient of variation of surgery durations, and unit costs on penalty of ignoring sequence-dependent surgery durations are analyzed, respectively. Furthermore, relationship between these parameters and cost of using Myopic Heuristic, Veteran's Heuristic, and Modified Myopic Heuristic are evaluated.

In Single OR-Multiple Surgeries experiment, the effect of number of surgeries, first-order multiplier, last-order multiplier, mean of surgery durations, coefficient of variation of surgery durations, and unit costs on cost of using same heuristics are analyzed, respectively.

In Multiple ORs-Multiple Surgeries experiment, the effect of number of operating rooms, first-order multiplier, last-order multiplier, mean of surgery durations, coefficient of variation of surgery durations, and unit costs on penalty of ignoring sequence-dependent surgery durations are analyzed, respectively.

5.1. Scheduling Results

In these experiments, it is assumed that the sequences of the surgeries are predetermined, and these lists are available at the beginning of the day. Therefore, there are no binary variables in the model. The problem becomes a Linear Program. These experiments are prepared for two problems. The first problem is the scheduling of two surgeries within a single operating room. The second problem is the scheduling of multiple surgeries within a single operating room.

5.1.1. Single OR - 2 Surgeries

As a first step, an experiment is prepared for two surgeries within a single operating room. In this experiment, it is aimed to investigate whether or not ignoring sequence-dependent surgery durations affects the total weighted cost of the waiting time of the patients for the surgeries, the idle time of the operating rooms, and the overtime of the surgeons. Furthermore, Myopic Heuristic, Veteran's Heuristic, and Modified Myopic Heuristic are also used for the assignment of the surgery durations in order to measure their efficiency. The relationship between different parameter settings and penalty of ignoring sequence-dependent surgery durations with these settings is analyzed in detail throughout the section.

5.1.1.1. Experimental Setting. In this experimental setting and in other experimental settings used throughout the thesis, the log-normal distribution is used for distributions of the surgery durations, since it is recommended mostly in the literature for distributions of the surgery durations. Table 5.1 provides the parameter set of this experiment. It is assumed that the mean of the surgery durations is chosen from the set $\{0.5, 1, 2\}$, and the coefficient of variation of the surgery durations is chosen from the set $\{0.4, 0.8, 1.2\}$. By using these parameters, the mean of the log of the log-normal distribution(μ) and the standard deviation of the log of the log-normal distribution(σ)

are calculated as follows:

$$\sigma = \sqrt{ln\left(cv\left[X\right]^2 + 1\right)} \tag{5.1}$$

$$\mu = ln\left(E\left[X\right]\right) - \frac{\sigma^2}{2} \tag{5.2}$$

In these equations, $E[X]$ is the mean of the surgery durations, and $cv[X]$ is the coefficient of the variation of the surgery durations. These parameters are calculated, since μ and σ are required for the log-normal data generation in R platform.

In this experiment, time unit is taken as an hour, and the expected surgery durations are between half an hour and two hours. As mentioned earlier, this experiment is designed for single operating room and two surgeries within this operating room. It is assumed that distributions of surgery durations for these two surgeries are independent. Therefore, μ and σ values for these two surgeries are determined independently.

Table 5.1. Parameter set for Single OR-2 Surgeries.

Parameter	**Description**	**Range**
E[X]	Mean(Log-normal)	$\{0.5, 1, 2\}$
cv[X]	Coefficient of variation(Log-normal)	$\{0.4, 0.8, 1.2\}$
a	Multiplier	$\{0.8, 0.95, 0.983, 0.99, 1.1, 1.3\}$
α_1	Unit cost of idle time	$\{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8\}$
α_2	Unit cost of waiting time	$\{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8\}$

In order to measure the magnitude of the effect of sequence-dependent surgery durations on the objective value, 6 different cases are defined. These cases are determined according to the value chosen from the set $a = \{0.8, 0.95, 0.983, 0.99, 1.1, 1.3\}$. If a is equal to 0.8 in the parameter set, the mean and the standard deviation of the surgery duration of the surgery assigned to first sequence are reduced by 20 percent.

If a is equal to 1.3 in the parameter set, the mean and the standard deviation of the surgery duration of the surgery assigned to the first sequence are increased by 30 percent. By doing this, the coefficient of the variation of the surgery duration of the surgery assigned to the first sequence is preserved. According to the new values of the mean and the standard deviation, μ and σ values are updated.

Sample Average Approximation(SAA) method is used for obtaining solutions in this experiment. Therefore, a predetermined number of scenarios are generated for each member of the parameter set. This parameter set provides $3x3x3x3x36x6 = 17496$ different test instances. There are 3 options for the mean of the surgery duration of each surgery, 3 options for the standard deviation of the surgery duration of each surgery, 6 members of a, and 36 different combinations for $\{\alpha_1, \alpha_2, \alpha_3\}$. For each member of the parameter set, random data is generated for two possible sequences. R platform is used for data generation. A separate file with .dat extension is formed for each member of the parameter set and each sequence. Then, a script for the implementation of the model is written on Cplex 12.5. It is observed that the computation time for the problem is 0.5 seconds on the average if the number of scenarios is taken as 1000. If the number of scenarios is 20000, the computation time for the problem increases to 5 seconds on the average.

At the beginning, it is expected that objective values of two possible sequences should be the same, if the mean and the standard deviation of surgery durations of surgeries are the same. Since the data is generated randomly, it is observed that there exists a small difference between the objective values of two possible sequences. In order to obtain consistent results, the difference between these two objective values should be below a specified value. According to the results, the maximum percentage difference between the objective values is 10% for 1000 scenarios. If the number of scenarios is 10000, the maximum percentage difference is 3%. If the number of scenarios is 20000, the maximum percentage difference is 2%. It is observed that results are more consistent, if the number of scenarios is 20000. Therefore, 20000 scenarios can be used for SAA method.

Before solving the problem with SAA method by using 20000 scenarios, it is aimed to compare the computation time and the objective value of the solution of the problem with 20000 scenarios and those of the solution of the problem with 400 scenarios and 50 replications. A small parameter set is defined for this analysis, and it is formed by choosing 10 percent of the parameter set randomly. Analysis is done by using the results obtained for this small parameter set.

The average computation time of the problem with 20000 scenarios is 4.95 seconds. On the other hand, the average computation time of the problem with 400 scenarios and 50 replications is 23.08 seconds. As the number of scenarios decreases, the computation time of the problem decreases. However, it starts to increase, if the number of replications increases. When replications are used for the solution of the problem, there will be an extra loop for these replications. This loop increases the computation time of the problem, as well. This creates a disadvantage for using replications in the solution of the problem.

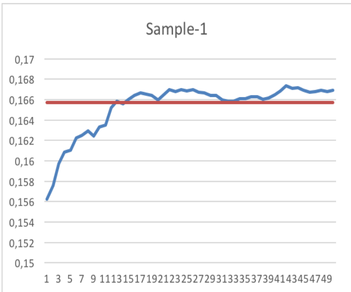
When the minimum objective value of the problem with 20000 scenarios is compared with the average of the minimum objective values of the replications for the problem with 400 scenarios and 50 replications, it is observed that the average cost of the problem with 400 scenarios and 50 replications is less than the cost of the problem with 20000 scenarios in 57.6% of all solutions.

Table 5.2. Percentage difference between the average cost of the problem with 400 scenarios and 50 replications and the cost of the problem with 20000 scenarios.

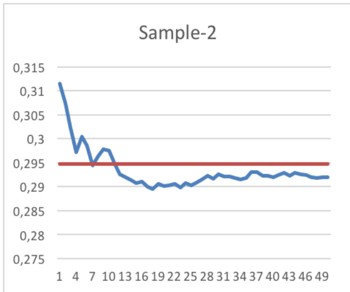
Min	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
-27.46%	-4.38%	-0.86%	-0.92%	2.61%	19.92%

According to 5.2, the average cost of the problem with 400 scenarios and 50 replications is at most 27.46% less than the cost of the problem with 20000 scenarios.

The average cost of the problem with 400 scenarios and 50 replications is at most 19.92% larger than the cost of the problem with 20000 scenarios. Between the lower quartile and the upper quartile, it is observed that costs of the problem with 20000 scenarios and the problem with 400 scenarios and 50 replications are proximate.



(a)



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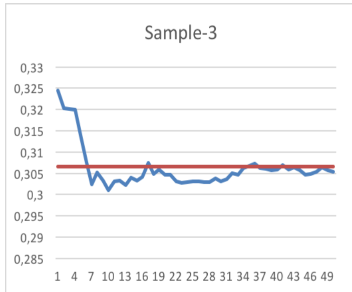

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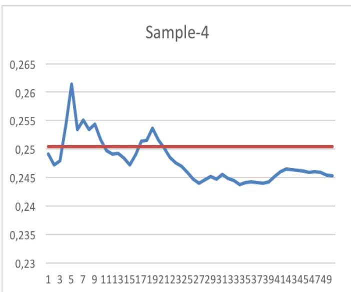

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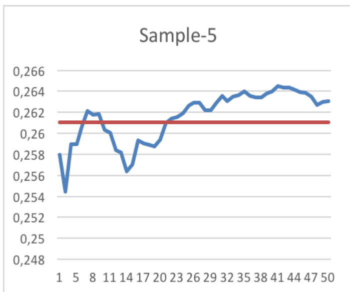

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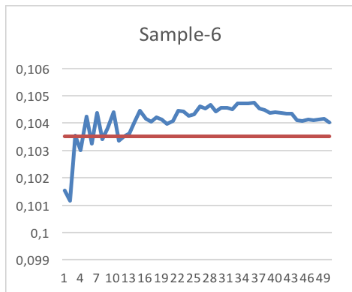

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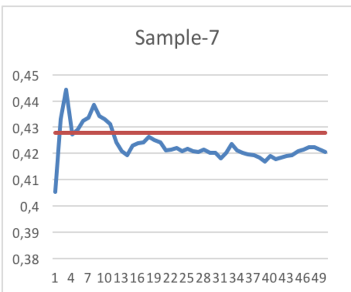

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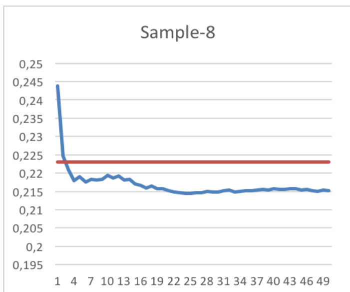

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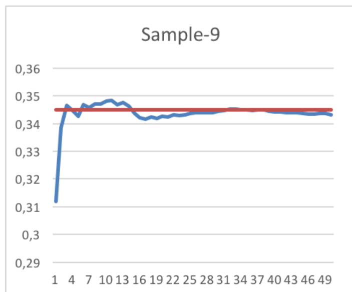


(i)

Figure 5.1. Number of Replications vs. Average Cost of Replications.

Figure 5.1 is prepared in order to visualize whether solutions with 400 scenarios and a number of replications converge to a value. In these graphics, x-axis represents the number of replications, and y-axis represents the average cost of these replications. When the number of replications is 1, the average cost is the cost of the first replications. When the number of replications is 2, the average cost is the average of the costs of first two replications. This pattern continues until the number of replications is equal to 50. The blue line shows the trend of the average cost of the replications. The red line is the cost of the problem with 20000 scenarios under the same parameter setting. Each graphic in Figure 5.1 shows that the average cost converges to a value, as the number of replications increases. It is important to note that the solutions used in Figure 5.1 are randomly chosen from the parameter set. In 6 of these 9 graphics, the cost of the problem with 20000 scenarios is higher than the average cost of the problem with 400 scenarios and 50 replications.

The advantage of using a number of replications with small number of scenarios is the computation time. However, since the size of the problem is small in this experiment, replications do not reduce the computation time. Furthermore, it is difficult to assign values to decision variables when solutions are obtained by using replications. Therefore, solutions are obtained by using 20000 scenarios without any replications in order to analyze results.

By changing only one parameter at a time and keeping others the same, changes in the objective value are analyzed. These parameters are the multiplier of the mean and the standard deviation of the surgery assigned to the first sequence, the mean and the coefficient of the variation of the surgery duration of both surgeries, α_1, and α_2 values, respectively. As a first step, the effect of the sequence on the objective value as a result of the change in the parameter is calculated. This effect is the percentage difference between the minimum objective value of two possible sequences and the objective value obtained when sequence-dependent surgery durations are ignored. It is also called as penalty of ignoring sequence-dependent surgery durations and it can be calculated as below:

$$PI = \frac{obj^i - obj}{obj} \times 100\% \tag{5.3}$$

In this equation, PI is penalty of ignoring sequence-dependent surgery durations. Furthermore, obj^i is the objective value obtained when sequence-dependent surgery durations are ignored and obj is the objective value obtained when sequence-dependent surgery durations are considered. If PI is high, it means that the change in the parameter has a significant effect on the sequence of the surgeries. From now on, PI will be used as an abbreviation of penalty of ignoring sequence-dependent surgery durations. The second factor calculated to evaluate the effect of the sequence is the percentage difference between the makespan of surgeries obtained when sequence-dependent surgery durations are ignored and the makespan of surgeries obtained when sequence-dependent surgery durations are considered. It can be represented with the following equation:

$$\Delta M = \frac{makespan^i - makespan}{makespan} \times 100\% \tag{5.4}$$

In this equation, ΔM is the percentage difference between two makespan values. $makespan^i$ is the makespan of surgeries obtained when sequence-dependent surgery durations are ignored and $makespan$ is the makespan of surgeries obtained when sequence-dependent surgery durations are considered.

5.1.1.2. Results for SAA. Results of the first experiment will be analyzed step by step according to the changes in the parameter set. Only important findings will be presented in this section, remaining results can be found in Appendix B.

(i) Effect of The Multiplier of The Mean and The Standard Deviation of The Surgery Duration of The Surgery Assigned to The First Sequence:

Table 5.3 gives the summary table of the effect of the multiplier of the mean and the standard deviation of the surgery assigned to the first sequence on penalty of ignoring sequence-dependent surgery durations when other parameters are kept the same.

Table 5.3. Multiplier vs. PI.

Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.8	0.233%	1.977%	3.651%	5.103%	7.789%	22.030%
0.95	0.001%	0.087%	0.175%	0.322%	0.396%	4.213%
0.983	0.000%	0.011%	0.026%	0.110%	0.056%	4.977%
0.99	0.000%	0.005%	0.013%	0.085%	0.030%	4.675%
1.1	0.026%	0.294%	0.563%	0.864%	1.269%	9.150%
1.3	0.297%	2.183%	4.039%	6.163%	9.043%	43.270%

According to Table 5.3, it is observed that as the change in the mean and the standard deviation of the surgery assigned to the first sequence increases, penalty of ignoring sequence-dependent surgery durations increases. This result makes sense. If a surgery is highly affected by the time of the day, ignoring this situation will create inefficiencies in the system.

Table 5.4. Multiplier vs. percentage difference between makespan of surgeries.

Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.8	2.15%	7.70%	10.06%	9.96%	12.20%	17.87%
0.95	-2.03%	1.60%	2.36%	2.35%	3.09%	8.67%
0.983	-5.69%	0.30%	0.78%	0.77%	1.25%	7.26%
0.99	-6.07%	0.00%	0.48%	0.47%	0.91%	6.90%
1.1	-10.18%	-6.03%	-4.67%	-4.70%	-3.36%	2.45%
1.3	-26.50%	-17.54%	-13.93%	-14.01%	-10.41%	-2.27%

According to 5.4, it is observed that as the multiplier of the mean and the standard deviation of the surgery assigned to the first sequence increases, the sum of the surgery durations increases. On the other hand, as the multiplier of the mean and the standard deviation of the surgery assigned to the first sequence decreases, the sum of the surgery durations decreases. Furthermore, it is observed that as the change in the mean and the standard deviation of the surgery assigned to the first sequence increases, the percentage change of the sum of surgery durations increases. When these results are evaluated, it can be concluded that if a surgery lasts shorter when it is scheduled first, the sum of surgery durations will decrease. On the other hand, if a surgery lasts longer when it is scheduled first, the sum of surgery durations will increase to consider this change.

(ii) Effect of The Coefficient of Variation of Surgery Durations:

Table 5.5 gives the summary table of the effect of the coefficient of variation of the surgery duration of surgery 1 on penalty of ignoring sequence-dependent surgery durations when other parameters are kept the same.

Table 5.5. CV of the surgery duration of surgery 1 vs. PI.

CV of Surgery Duration of Surgery 1	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.4	0.00%	0.05%	0.63%	3.09%	4.39%	43.27%
0.8	0.00%	0.03%	0.33%	1.70%	2.19%	31.83%
1.2	0.00%	0.03%	0.26%	1.53%	1.66%	28.38%

According to Table 5.5, it is observed that as the coefficient of variation of surgery duration of surgery 1 decreases, penalty of ignoring sequence-dependent surgery durations increases.

Table 5.6 gives the summary table of the effect of the coefficient of variation of the surgery duration of surgery 2 on penalty of ignoring sequence-dependent surgery durations when other parameters are kept the same.

According to Table 5.6, it is observed that as the coefficient of variation of surgery duration of surgery 2 decreases, penalty of ignoring sequence-dependent surgery durations increases.

Table 5.6. CV of the surgery duration of surgery 2 vs. PI.

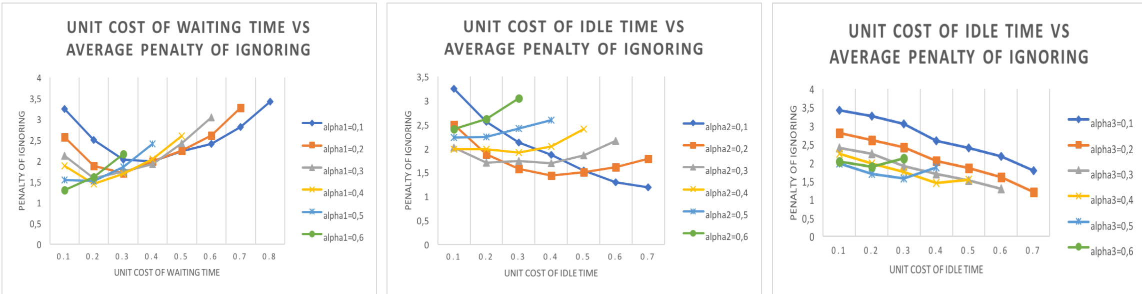
CV of Surgery Duration of Surgery 2	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.4	0.00%	0.05%	0.64%	3.11%	4.40%	43.27%
0.8	0.00%	0.03%	0.32%	1.69%	2.20%	24.71%
1.2	0.00%	0.03%	0.27%	1.53%	1.67%	26.22%

As it is known coefficient of variation provides relative variability. Therefore, it can be said that if the coefficient of variation is small, the variability will be small, as well. When coefficient of variation is small, surgery durations will be assigned more precisely. On the other hand, if it takes a high value, the cost will increase due to imprecise surgery durations. That's why, penalty of ignoring sequence-dependent surgery durations will decrease as coefficient of variation increases.

(iii) Effect of Unit Costs:

Figure 5.2 provides a summary of the relationship between unit costs and average penalty of ignoring sequence-dependent surgery durations. Furthermore, tables of the results can be found in Appendix B. According to Figure 5.2, there is a negative correlation between the unit cost of idle time and average penalty of ignoring sequence-dependent surgery durations when unit cost of waiting time or unit cost of overtime is kept constant at a small value. When one of these costs is kept stable at a high value, then the correlation becomes positive. Furthermore, it is found that the largest value of average penalty of ignoring sequence-dependent surgery durations is observed when the unit cost of the waiting time takes its maximum possible value and the unit cost of the overtime takes its minimum possible value.

In this section, results are obtained by using SAA method. Although results for single operating room and two surgeries are obtained in a reasonable computation time, the computation time increases as the number of operating rooms and the number of surgeries increase. In the following sections, heuristics will be used for finding solutions



(a) (b) (c)

Figure 5.2. Unit Costs vs. Average PI.

for single operating room and two surgeries problem. Then, these heuristics will also be adapted for finding solutions for single operating room and multiple surgeries problem and multiple operating rooms and multiple surgeries problem.

Three heuristics will be used for single operating room and two surgeries problem, and these heuristics are Myopic Heuristic, Veteran's Heuristic, and Modified Myopic Heuristic, respectively. The parameter set will be the same with the parameter set used in the previous experiment. A separate file with .dat extension is formed for each member of this parameter set and each sequence. Number of scenarios generated for SAA method is taken as 20000 for each member of the parameter set.

5.1.1.3. Results for Myopic Heuristic. In this section, results of single operating room and two surgeries problem obtained with Myopic Heuristic will be analyzed. The algorithm for the assignment of surgery durations in Myopic Heuristic can be found in Section 4.3.1.1.

After the assignment of the surgery durations is completed, these values are added to the model as constraints, and the objective value of the problem is obtained by solving the code in Cplex.

In order to measure the efficiency of Myopic Heuristic for single operating room and two surgeries problem, the percentage difference between the minimum objective value of two possible sequences and the objective value obtained when the surgery durations are assigned by using Myopic Heuristic is calculated. This value is also called as cost of using Myopic Heuristic and it can be calculated as below:

$$C(Myp) = \frac{obj^{myp} - obj}{obj} \times 100\% \tag{5.5}$$

In this equation, $C(Myp)$ is cost of using Myopic Heuristic. Furthermore, obj^{myp} is the objective value obtained with Myopic Heuristic. As in the previous section, only important findings will be listed in this section. Remaining results can be found in Appendix C.

(i) Effect of The Multiplier of The Mean and The Standard Deviation of The Surgery Duration of The Surgery Assigned to The First Sequence:

Table 5.7 gives the summary table of the effect of the multiplier of the mean and the standard deviation of the surgery assigned to the first sequence on the cost of using Myopic Heuristic when other parameters are kept the same.

Table 5.7. Multiplier vs. $C(Myp)$, N=2.

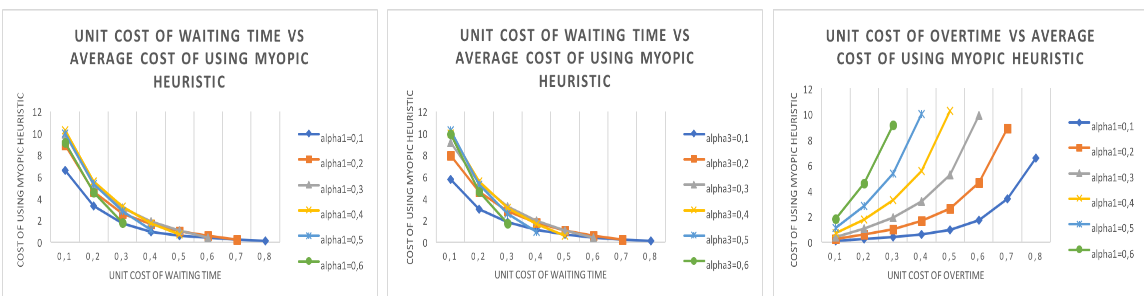
Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.8	0.00%	0.58%	1.98%	3.72%	5.54%	21.53%
0.95	0.00%	0.56%	1.79%	3.16%	4.51%	20.24%
0.983	0.00%	0.58%	1.85%	3.28%	4.65%	21.62%
0.99	0.00%	0.59%	1.85%	3.27%	4.68%	21.27%
1.1	0.00%	0.70%	2.05%	3.67%	5.19%	23.73%
1.3	0.00%	0.80%	2.45%	4.29%	6.09%	27.54%

According to Table 5.7, the cost of using Myopic Heuristic takes its minimum value, when the multiplier is equal to 0.95. On the other hand, the cost of using Myopic Heuristic takes its maximum value, when the multiplier is equal to 1.3. It is expected that as the multiplier gets closer to 1, the cost of using Myopic Heuristic should decrease. However, it is observed that as the multiplier increases from 0.95 to 0.99, the cost of using Myopic Heuristic increases. Therefore, the cost of using Myopic Heuristic is not dependent on the multiplier, it is robust to the changes in the multiplier.

(ii) Effect of Unit Costs:

Tables of the results can be found in Appendix C. As the results in this section are evaluated together, it is found that as the unit cost of the idle time increases and the unit cost of the overtime decreases, the cost of using Myopic Heuristic increases first, and then it starts to decrease. Furthermore, the difference between the values of upper quartile and lower quartile decreases as the unit cost of the idle time increases.

Figure 5.3 provides the summary of the relationship between unit costs and cost of using Myopic Heuristic. It is seen that as the unit cost of waiting time increases, cost of Using Myopic Heuristic decreases when the unit cost of idle time or unit cost of overtime is kept the same. Since Myopic Heuristic assigns durations independently, it disregards the effect of waiting time. Therefore, it gives more weight to the cost of waiting time to eliminate waiting time. It is also seen that as the unit cost of overtime increases, the cost of using Myopic Heuristic increases when the unit cost of idle time is kept the same. This is also reasonable, since as the unit cost of overtime increases, the unit cost of waiting time decreases when the unit cost of the idle time remains the same.



(a) (b) (c)

Figure 5.3. Unit Costs vs Average $C(Myp)$.

5.1.1.4. Results for Veteran's Heuristic. In this section, results of single operating room and two surgeries problem obtained with Veteran's Heuristic will be analyzed. The algorithm for the assignment of surgery durations in Veteran's Heuristic can be found in Section 4.3.1.2.

After the assignment of the surgery durations is completed, these values are added to the model as constraints as in the previous section, and the objective value of the problem is obtained by solving the code in Cplex.

In order to measure the efficiency of Veteran's Heuristic for single operating room and two surgeries problem, the percentage difference between the minimum objective value of two possible sequences and the objective value obtained when the surgery durations are assigned by using Veteran's Heuristic is calculated. This value is also called as cost of using Veteran's Heuristic and it can be calculated as below:

$$C(V) = \frac{obj^v - obj}{obj} \times 100\% \tag{5.6}$$

In this equation, $C(V)$ is cost of using Veteran's Heuristic. Furthermore, obj^v is the objective value obtained with Veteran's Heuristic. As in the previous section, only important findings will be listed in this section. Remaining results can be found in Appendix D.

(i) Effect of The Multiplier of The Mean and The Standard Deviation of The Surgery Duration of The Surgery Assigned to The First Sequence:

Table 5.8 gives the summary table of the effect of the multiplier of the mean and the standard deviation of the surgery assigned to the first sequence on cost of using Veteran's Heuristic when other parameters are kept the same.

According to Table 5.8, cost of using Veteran's Heuristic takes its minimum value, when the multiplier is equal to 0.95. On the other hand, cost of using Veteran's

Table 5.8. Multiplier vs. $C(V)$, N=2.

Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.8	0.00%	0.29%	0.89%	1.95%	2.44%	26.09%
0.95	0.00%	0.22%	0.64%	1.72%	1.67%	24.32%
0.983	0.00%	0.23%	0.66%	1.78%	1.76%	24.34%
0.99	0.00%	0.23%	0.65%	1.78%	1.71%	24.90%
1.1	0.00%	0.30%	0.77%	2.03%	1.97%	28.01%
1.3	0.00%	0.39%	1.00%	2.44%	2.49%	32.22%

Heuristic takes its maximum value, when the multiplier is equal to 1.3. It is expected that as the multiplier gets closer to 1, cost of using Veteran's Heuristic should decrease. However, it is observed that as the multiplier increases from 0.95 to 0.99, cost of using Veteran's Heuristic increases. Therefore, cost of using Veteran's Heuristic is not dependent on the multiplier, it is robust to the changes in the multiplier.

(ii) Effect of The Coefficient of Variation of Surgery Durations:

Table 5.9 gives the summary table of the effect of the coefficient of variation of the surgery duration of surgery 1 on cost of using Veteran's Heuristic when other parameters are kept the same.

Table 5.9. CV of the surgery duration of surgery 1 vs. $C(V)$, N=2.

CV of Surgery Duration of Surgery 1	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.4	0.00%	0.30%	0.87%	2.15%	2.36%	32.22%
0.8	0.00%	0.27%	0.77%	1.97%	1.98%	31.86%
1.2	0.00%	0.23%	0.67%	1.73%	1.78%	24.00%

According to Table 5.9, it is observed that cost of using Veteran's Heuristic decreases as the coefficient of variation of surgery duration of surgery 1 increases.

Table 5.10 gives the summary table of the effect of the coefficient of variation of the surgery duration of surgery 2 on cost of using Veteran's Heuristic when other parameters are kept the same.

Table 5.10. CV of the surgery duration of surgery 2 vs. $C(V)$, N=2.

CV of Surgery Duration of Surgery 2	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.4	0.00%	0.30%	0.87%	2.15%	2.37%	32.22%
0.8	0.00%	0.28%	0.67%	1.97%	1.99%	31.31%
1.2	0.00%	0.23%	0.67%	1.73%	1.78%	24.00%

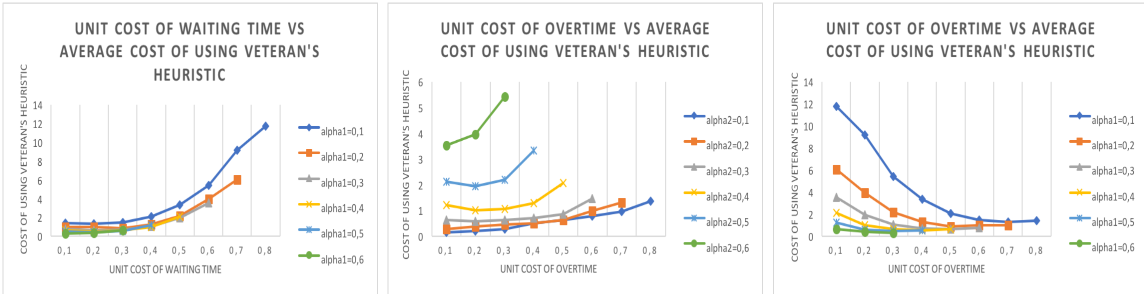
According to Table 5.10, it is observed that cost of using Veteran's Heuristic decreases as the coefficient of variation of surgery duration of surgery 2 increases.

When the results are analyzed, it is observed that there is a negative correlation between the coefficient of variation of the surgery durations and the cost of using Veteran's Heuristic.

(iii) Effect of Unit Costs:

Tables of results can be found in Appendix D. As the results in this section are evaluated together, it is found that as the unit cost of the idle time increases and the unit cost of overtime decreases, cost of using Veteran's Heuristic decreases generally.

Figure 5.4 shows that cost of using Veteran's Heuristic is positively correlated with the unit cost of waiting time, and negatively correlated with the unit cost of idle time. In Veteran's Heuristic, it is assumed that when a surgery is completed, the next surgery will start immediately. There should not be any idle time between two successive surgeries. Therefore, Veteran's Heuristic gives better results as the unit cost of idle time increases. Finally, cost of using Veteran's Heuristic increases, as the unit cost of overtime increases when the unit cost of waiting time is kept the same. On the



(a) (b) (c)

Figure 5.4. Unit Costs vs Average $C(V)$.

other hand, cost of using Veteran's Heuristic decreases, as the unit cost of overtime increases when idle time is kept constant.

5.1.1.5. Results for Modified Myopic Heuristic. In this section, results of single operating room and two surgeries problem obtained with Modified Myopic Heuristic will be analyzed. The algorithm for the assignment of surgery durations in Modified Myopic Heuristic can be found in Section 4.3.1.3. Since log-normal distribution is used for distributions of surgery durations, the value of $E\left[(T_{i-1}-D_{i-1})^{+}\right]$ in the algorithm can be calculated as below:

$$e^{\mu_{i-1}+\frac{\sigma_{i-1}^2}{2}} * \Phi\left(\frac{\mu_{i-1}+\sigma_{i-1}^2-ln\left(D_{i-1}^{myp}\right)}{\sigma_{i-1}}\right) - D_{i-1} * \left(1-\Phi\left(\frac{ln\left(D_{i-1}^{myp}\right)-\mu_{i-1}}{\sigma_{i-1}}\right)\right)$$

$\Phi\,(.)$ is cumulative distribution function of the standard normal distribution. The surgery duration of the surgery assigned to i^{th} sequence is the sum of the value calculated above and the surgery duration of the surgery assigned to i^{th} sequence by using Myopic Heuristic.

After the assignment of the surgery durations is completed, these values are added to the model as constraints as in the previous sections, and the objective value of the problem is obtained by solving the code in Cplex.

In order to measure the efficiency of Modified Myopic Heuristic for single operating room and two surgeries problem, the percentage difference between the minimum objective value of two possible sequences and the objective value obtained when the surgery durations are assigned by using Modified Myopic Heuristic is calculated. This value is also called as cost of using Modified Myopic Heuristic and it can be calculated as below:

$$C(Mod) = \frac{obj^{mod} - obj}{obj} \times 100\% \tag{5.7}$$

In this equation, $C(Mod)$ is cost of using Modified Myopic Heuristic. Furthermore, obj^{mod} is the objective value obtained with Modified Myopic Heuristic. As in the previous sections, only important findings will be listed in this section. Remaining results can be found in Appendix E.

(i) Effect of The Multiplier of The Mean and The Standard Deviation of The Surgery Duration of The Surgery Assigned to The First Sequence:

Table 5.11 gives the summary table of the effect of the multiplier of the mean and the standard deviation of the surgery assigned to the first sequence on cost of using Modified Myopic Heuristic when other parameters are kept the same.

Table 5.11. Multiplier vs. $C(Mod)$, N=2.

Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.8	0.00%	0.08%	0.40%	1.40%	1.52%	21.47%
0.95	0.00%	0.08%	0.30%	0.92%	0.93%	25.81%
0.983	0.00%	0.09%	0.33%	0.97%	0.96%	26.27%
0.99	0.00%	0.08%	0.33%	0.97%	0.97%	27.01%
1.1	0.00%	0.12%	0.42%	1.18%	1.19%	30.99%
1.3	0.00%	0.17%	0.61%	1.59%	1.64%	37.18%

According to Table 5.11, cost of using Modified Myopic Heuristic takes its minimum value, when the multiplier is equal to 0.95. On the other hand, cost of using Modified Myopic Heuristic takes its maximum value, when the multiplier is equal to 1.3. It is expected that as the multiplier gets closer to 1, cost of using Modified Myopic Heuristic should decrease. However, it is observed that as the multiplier increases from 0.95 to 0.99, cost of using Modified Myopic Heuristic increases. Therefore, cost of using Modified Myopic Heuristic is not dependent on the multiplier, it is robust to the changes in the multiplier.

(ii) Effect of The Coefficient of Variation of Surgery Durations:

Table 5.12 gives the summary table of the effect of the coefficient of variation of the surgery duration of surgery 1 on cost of using Modified Myopic Heuristic when other parameters are kept the same.

Table 5.12. CV of the surgery duration of surgery 1 vs. $C(Mod)$, N=2.

CV of Surgery Duration of Surgery 1	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.4	0.00%	0.06%	0.29%	0.92%	0.97%	22.95%
0.8	0.00%	0.12%	0.42%	1.18%	1.19%	25.57%
1.2	0.00%	0.13%	0.46%	1.42%	1.36%	37.18%

According to Table 5.12, it is observed that cost of using Modified Myopic Heuristic increases as the coefficient of variation of surgery duration of surgery 1 increases.

Table 5.13 gives the summary table of the effect of the coefficient of variation of the surgery duration of surgery 2 on cost of using Modified Myopic Heuristic when other parameters are kept the same.

According to Table 5.13, it is observed that cost of using Modified Myopic Heuristic increases as the coefficient of variation of surgery duration of surgery 2 increases.

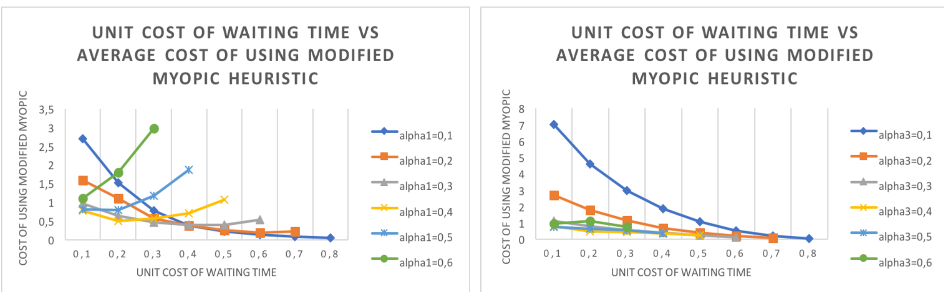
Table 5.13. CV of the surgery duration of surgery 2 vs. $C(Mod)$, N=2.

CV of Surgery Duration of Surgery 2	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.4	0.00%	0.06%	0.29%	0.92%	0.97%	23.07%
0.8	0.00%	0.12%	0.42%	1.17%	1.17%	25.57%
1.2	0.00%	0.13%	0.46%	1.42%	1.36%	37.18%

When the results are analyzed, it is observed that there is a positive correlation between the coefficient of variation of the surgery durations and cost of using Modified Myopic Heuristic.

(iii) Effect of Unit Costs:

Tables of results can be found in Appendix E. According to Figure 5.5, it is observed that when the unit cost of the idle is kept constant at a small value, cost of using Modified Myopic Heuristic decreases as the unit cost of waiting time increases. Furthermore, cost of using Modified Myopic Heuristic increases as the unit cost of the waiting time increases when the unit cost of the idle time is kept constant at a high value.



(a) (b)

Figure 5.5. Unit Costs vs Average $C(Mod)$.

Results also show that cost of using Modified Myopic Heuristic decreases, as the unit cost of the overtime remains the same and the unit cost of the waiting time increases.

5.1.1.6. Comparative Analysis of Heuristics. In this section, results of Myopic Heuristic, Veteran's Heuristic, and Modified Myopic Heuristic are compared. According to the results, cost of using Myopic Heuristic is less than cost of using Veteran's Heuristic and cost of using Modified Myopic Heuristic in 4.68% of all solutions. In 58.29% of all solutions, cost of using Modified Myopic Heuristic is less than cost of using Myopic Heuristic and cost of using Veteran's Heuristic.

Table 5.14. Heuristics vs. costs of using these heuristics.

Heuristic	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
Myopic	0.00%	0.64%	1.99%	3.57%	5.11%	27.54%
Veteran's	0.00%	0.27%	0.76%	1.95%	2.03%	32.22%
Modified Myopic	0.00%	0.10%	0.39%	1.17%	1.16%	37.18%

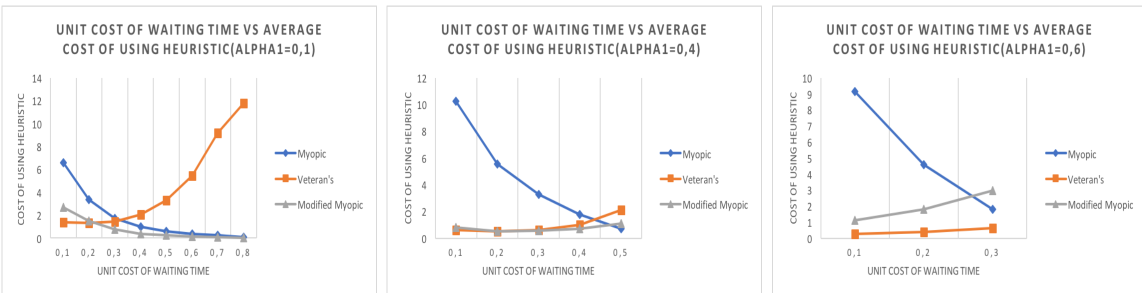
The results between the upper quantile and the lower quantile show that cost of using Myopic Heuristic is more than cost of using Veteran's Heuristic and cost of using Modified Myopic Heuristic. On the other hand, cost of using Modified Myopic Heuristic is less than other costs between lower quantile and upper quantile. In conclusion, Modified Myopic Heuristic can be preferred instead of other heuristics used in this experiment.

In this experiment, the sequence of the surgeries obtained without using heuristic can be different from the sequence of the surgeries obtained by using heuristics. In 5.76% of all solutions, the sequence of the surgeries obtained by using Myopic Heuristic is different from the sequence obtained without using heuristics. This value is 2.94% for Veteran's Heuristic, and 3.21% for Modified Myopic Heuristic. Table 5.15 gives the summary table of heuristics and costs of using these heuristics when the sequences of the surgeries are the same.

Table 5.15. Heuristics vs. costs of using these heuristics(same sequence).

Heuristic	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
Myopic	0.00%	0.58%	1.83%	3.25%	4.52%	27.24%
Veteran's	0.00%	0.26%	0.73%	1.89%	1.96%	32.22%
Modified Myopic	0.00%	0.09%	0.36%	1.10%	1.08%	37.18%

According to Table 5.15, it is observed that eliminating the results with different sequences does not affect the values of the statistics too much. That is because only a small portion of all solutions provides different sequences.

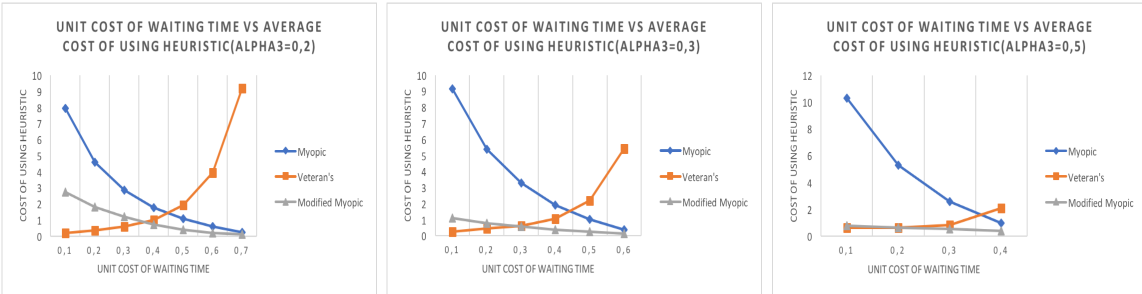


(a) (b) (c)

Figure 5.6. α_2 vs Average Cost of Using Heuristic (α_1 is constant.).

Figure 5.6 gives the relationship between the unit cost of waiting time and cost of using heuristics. When the unit cost of idle time is small, Veteran's Heuristic provides better results for a small unit cost of waiting time. As the unit cost of waiting time increases, Modified Myopic Heuristic starts to provide better results. As the unit cost of idle time increases, Veteran's Heuristic becomes dominant.

Figure 5.7 gives the relationship between the unit cost of waiting time and cost of using heuristic when the unit cost of overtime is kept constant. When the unit cost of overtime is small, Veteran's Heuristic provides better results for a small unit cost of waiting time. As the unit cost of waiting time increases, Modified Myopic Heuristic



(a) (b) (c)

Figure 5.7. α_2 vs Average Cost of Using Heuristic (α_3 is constant.).

starts to give better results. As the unit cost of overtime increases, Veteran's Heuristic provides better results for small unit cost of waiting time again. However, Modified Myopic Heuristic becomes dominant.

As Figure 5.6 and Figure 5.7 are evaluated together, it is found that it is better to use Veteran's Heuristic when unit cost of idle time is high, unit cost of overtime is small, or unit cost of waiting time is small. When the unit cost of idle time is small or unit cost of overtime is high, it is better to use Modified Myopic Heuristic. These are important findings from the evaluation of experiment.

5.1.2. Single OR - Multiple Surgeries

In this experiment, there will be only one operating room, and there will be more than 2 surgeries. It is aimed to investigate whether or not ignoring sequence-dependent surgery durations affects the total weighted cost of the waiting time of the patients for the surgeries, the idle time of the operating rooms, and the overtime of the surgeons. Furthermore, Myopic Heuristic, Veteran's Heuristic, and Modified Myopic Heuristic are also used for the assignment of the surgery durations in order to measure their efficiency as the number of the surgeries increases. The relationship between different parameter settings and cost of using heuristics with these settings is analyzed in detail throughout the section.

5.1.2.1. Experimental Setting. In this experiment, the parameter set is different from the parameter set used in the previous section. Table 5.16 provides the parameter set of this experiment. Number of surgeries within an operating room takes values between 3 and 7. In order to get rid of the effect of the order of the surgeries on the solution, it is assumed that all surgeries are log-normally distributed with the same parameters. Mean of the surgery durations is chosen from the set $\{0.5, 1, 2\}$, and the coefficient of variation of the surgery durations is chosen from the set $\{0.4, 0.8, 1.2\}$.

Table 5.16. Parameter set for Single OR-Multiple Surgeries.

Parameter	**Description**	**Range**
N	Number of surgeries	$\{3, 4, 5, 6, 7\}$
E[X]	Mean(Log-normal)	$\{0.5, 1, 2\}$
cv[X]	Coefficient of variation(Log-normal)	$\{0.4, 0.8, 1.2\}$
a_{first}	First-order multiplier	$\{0.9, 0.95, 0.983\}$
a_{last}	Last-order multiplier	$\{0.8, 0.9\}$
α_1	Unit cost of idle time	$\{0.2, 0.4, 0.6\}$
α_2	Unit cost of waiting time	$\{0.2, 0.4, 0.6\}$

Mean and standard deviation of the surgery durations of the surgery assigned to the first sequence and the surgery assigned to the last sequence are multiplied by multipliers. As in the previous experiment, the value of the coefficient of variation will remain the same, since both mean and standard deviation will be multiplied by the same value. Multipliers are used to observe whether or not distributions of the surgery durations are dependent on the order of the surgeries within an operating room. Multiplier of the surgery assigned to the first sequence is chosen from the set $a_{first} = \{0.9, 0.95, 0.983\}$, and it is called first-order multiplier. Multiplier of the surgery assigned to the last sequence is chosen from the set $a_{last} = \{0.8, 0.9\}$, and it is called last-order multiplier.

Furthermore, the unit cost of the idle time(α_1), the unit cost of the waiting time(α_2), and the unit cost of the overtime(α_3) take values as the multiples of 0.2. Therefore, there will be 6 different combinations of $\{\alpha_1, \alpha_2, \alpha_3\}$. This parameter set has $3x3x3x2x6x5 = 1620$ members. There are 3 options for the mean of the surgery durations, 3 options for the standard deviation of the surgery durations, 3 options for first-order-multiplier, 3 options for last-order-multiplier, 6 different combinations for $\{\alpha_1, \alpha_2, \alpha_3\}$, and 5 options for the number of surgeries. For each member of the parameter set, randam data is generated by using R platform. A separate file with .dat extension is formed for each member of the parameter set. As in the previous experiment, SAA method is used for the solution of the problem. Number of scenarios generated for each member of the parameter set is taken as 10000.

5.1.2.2. Results for Myopic Heuristic. In this section, results of single operating room and multiple surgeries problem obtained with Myopic Heuristic will be analyzed. In order to measure the efficiency of Myopic Heuristic for single operating room and multiple surgeries problem, the percentage difference between the objective value obtained without using heuristic and the objective value obtained when the surgery durations are assigned by using Myopic Heuristic is calculated. This value is also represented as $C(Myp)$ as in the previous experiment.

(i) Effect of The Number of Surgeries:

Table 5.17 gives the summary table of the effect of the number of surgeries on cost of using Myopic Heuristic for Multiple Surgeries when other parameters are kept the same.

According to Table 5.17, the uncertainty increases as the number of surgeries increases, and this leads to an increase in the cost of using Myopic Heuristic for Multiple Surgeries. When the number of surgeries is equal to 3, the cost of using Myopic Heuristic for Multiple Surgeries takes values between 2.6 and 30.71. On the other hand, the cost of using Myopic Heuristic for Multiple Surgeries takes values between

Table 5.17. Number of surgeries vs. $C(Myp)$.

Number of Surgeries	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
3	2.60%	9.08%	13.33%	15.53%	24.16%	30.71%
4	5.65%	17.55%	22.98%	26.38%	39.51%	46.13%
5	8.63%	25.55%	34.26%	37.23%	52.23%	63.33%
6	11.18%	33.76%	46.33%	48.16%	66.13%	79.99%
7	13.58%	40.85%	58.55%	59.01%	81.49%	98.06%

13.58 and 98.06, when the number of surgeries is equal to 7.

(ii) Effect of First-Order Multiplier of The Mean and The Standard Deviation of The Surgery Duration of The Surgery Assigned to The First Sequence:

Both the mean and the standard deviation of the surgery duration of the surgery assigned to the first sequence is multiplied by the value chosen from the set $\{0.9, 0.95, 0.983\}$.

Results are analyzed separately for each different value of the number of surgeries. According to the results, the cost of using Myopic Heuristic for Multiple Surgeries increases, as the first-order multiplier increases. Since this pattern is observed for each different value of the number of surgeries, summary tables of the effect of the first-order multiplier of the mean and the standard deviation of the surgery assigned to the first sequence on the cost of using Myopic Heuristic for Multiple Surgeries for 3 surgeries and 7 surgeries are provided in Table 5.18.

According to Table 5.18, it is observed that cost of using Myopic Heuristic for Multiple Surgeries increases as the first-order multiplier increases and gets closer to 1.

Table 5.18. First-order multiplier vs. $C(Myp)$, N=3 and N=7.

Number of surgeries = 3						
First-order Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.9	2.60%	8.45%	13.03%	15.24%	23.85%	28.30%
0.95	2.62%	9.16%	13.54%	15.51%	24.35%	30.71%
0.983	2.89%	9.32%	13.25%	15.85%	24.80%	29.54%
Number of surgeries = 7						
First-order Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.9	13.58%	39.77%	57.40%	58.65%	80.60%	94.65%
0.95	13.87%	40.50%	58.72%	59.00%	81.05%	94.86%
0.983	13.91%	41.21%	58.86%	59.40%	82.42%	98.06%

(iii) Effect of Last-Order Multiplier of The Mean and The Standard Deviation of The Surgery Duration of The Surgery Assigned to The Last Sequence:

Both the mean and the standard deviation of the surgery duration of the surgery assigned to the last sequence is multiplied by the value chosen from the set $\{0.8, 0.9\}$.

Results are analyzed separately for each different value of the number of surgeries. According to the results, the cost of using Myopic Heuristic for Multiple Surgeries decreases, as the last-order multiplier increases. Since this pattern is observed for each different value of the number of surgeries, summary tables of the effect of the last-order multiplier of the mean and the standard deviation of the surgery assigned to the last sequence on cost of using Myopic Heuristic for Multiple Surgeries for 3 surgeries and 7 surgeries are provided in Table 5.19.

According to Table 5.19, it is observed that cost of using Myopic Heuristic for Multiple Surgeries decreases as the last-order multiplier increases and gets closer to 1.

Table 5.19. Last-order multiplier vs. $C(Myp)$, N=3 and N=7.

Number of surgeries = 3						
Last-order Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.8	2.74%	9.27%	13.54%	15.97%	25.40%	30.71%
0.9	2.60%	8.91%	13.05%	15.09%	23.51%	27.84%
Number of surgeries = 7						
Last-order Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.8	13.91%	41.97%	58.86%	59.46%	82.49%	98.06%
0.9	13.58%	39.97%	58.16%	58.16%	80.46%	95.54%

(iv) Effect of Unit Costs:

In this section, the effect of the unit cost of the idle time on cost of using Myopic Heuristic for Multiple Surgeries when the unit cost of the waiting time is kept constant at a value is analyzed. As in the previous section, the results are analyzed separately for each different value of the number of surgeries.

When there exists a small number of surgeries, it is observed that there is a negative correlation between the unit cost of the idle time and the cost of using Myopic Heuristic for Multiple Surgeries. Table 5.20 gives the summary table of the effect of the unit cost of the idle time on the cost of using Myopic Heuristic for Multiple Surgeries when there are 3 surgeries. Furthermore, it is seen that cost of using Myopic Heuristic decreases as the unit cost of waiting time increases, or unit cost of overtime increases.

When the number of surgeries is more than or equal to 5, the relationship changes. Table 5.21 gives the summary table of the effect of the unit cost of the idle time on cost of using Myopic Heuristic for Multiple Surgeries when there are 7 surgeries.

Table 5.20. α_1 vs. $C(Myp)$, N=3.

The unit cost of the waiting time = 0.2						
Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.2	22.24%	24.76%	25.95%	26.10%	27.80%	30.71%
0.4	21.70%	23.74%	25.39%	25.29%	26.73%	29.15%
0.6	14.25%	15.41%	17.83%	17.55%	19.51%	21.03%
The unit cost of the waiting time = 0.4						
Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.2	6.40%	8.65%	11.01%	10.22%	11.52%	12.40%
0.4	7.93%	9.23%	9.66%	9.58%	10.07%	10.51%
The unit cost of the waiting time = 0.6						
Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.2	2.60%	3.70%	4.78%	4.46%	5.19%	5.85%

According to Table 5.21, cost of using Myopic Heuristic for Multiple Surgeries increases first, and it starts to decrease, as the unit cost of the idle time increases when the unit cost of the waiting time is kept constant at 0.2. When the unit cost of the waiting time is equal to 0.4, there exists a positive correlation between the unit cost of the idle time and cost of using Myopic Heuristic for Multiple Surgeries.

When the results are analyzed together, it is observed that cost of using Myopic Heuristic for Multiple Surgeries decreases, as the unit cost of the waiting time increases when the unit cost of the idle time is kept constant at a value. Since Myopic Heuristic gives more importance to the cost of the waiting time, the results are consistent with what is expected.

Table 5.21. α_1 vs. $C(Myp)$, N=7.

The unit cost of the waiting time = 0.2						
Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.2	71.71%	78.09%	84.16%	82.38%	85.80%	88.57%
0.4	75.01%	77.93%	88.19%	85.94%	90.96%	98.06%
0.6	59.86%	61.34%	72.10%	71.59%	80.31%	85.62%
The unit cost of the waiting time = 0.4						
Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.2	7.85%	9.88%	14.30%	13.30%	15.98%	17.61%
0.4	10.36%	11.82%	12.39%	12.44%	13.10%	14.59%
The unit cost of the waiting time = 0.6						
Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.2	13.58%	15.79%	24.55%	23.49%	29.86%	32.05%

5.1.2.3. Results for Veteran's Heuristic. In this section, results of single operating room and multiple surgeries problem obtained with Veteran's Heuristic will be analyzed. In order to measure the efficiency of Veteran's Heuristic for single operating room and multiple surgeries problem, the percentage difference between the objective value obtained without using heuristic and the objective value obtained when the surgery durations are assigned by using Veteran's Heuristic is calculated. This value is also represented as $C(V)$ as in the previous experiment.

(i) Effect of The Number of Surgeries:

Table 5.22 gives the summary table of the effect of the number of surgeries on cost of using Veteran's Heuristic for Multiple Surgeries when other parameters are kept the same.

Table 5.22. Number of surgeries vs. $C(V)$.

Number of Surgeries	Min	Lower Quartile	Median	Mean	Upper Quartile	Max
3	1.76%	3.82%	5.76%	7.39%	8.67%	22.86%
4	3.48%	7.13%	10.37%	12.10%	15.14%	32.81%
5	5.83%	10.70%	15.41%	17.05%	21.00%	42.29%
6	8.45%	14.20%	20.44%	22.28%	27.76%	51.92%
7	11.07%	18.66%	25.16%	27.51%	33.65%	61.65%

According to Table 5.22, the uncertainty increases as the number of surgeries increases, and this leads to an increase in cost of using Veteran's Heuristic for Multiple Surgeries. When the number of surgeries is equal to 3, cost of using Veteran's Heuristic for Multiple Surgeries takes values between 1.76 and 22.86. On the other hand, cost of using Veteran's Heuristic for Multiple Surgeries takes values between 11.07 and 61.65, when the number of surgeries is equal to 7.

(ii) Effect of First-Order Multiplier of The Mean and The Standard Deviation of The Surgery Duration of The Surgery Assigned to The First Sequence:

Results are analyzed separately for each different value of the number of surgeries. According to the results, cost of using Veteran's Heuristic for Multiple Surgeries increases, as the first-order multiplier increases. Since this pattern is observed for each different value of the number of surgeries, summary tables of the effect of the first-order multiplier of the mean and the standard deviation of the surgery assigned to the first sequence on cost of using Veteran's Heuristic for Multiple Surgeries for 3 surgeries and 7 surgeries are provided in Table 5.23.

According to Table 5.23, it is observed that cost of using Veteran's Heuristic for Multiple Surgeries increases as the first-order multiplier increases and gets closer to 1.

Table 5.23. First-order multiplier vs. $C(V)$, N=3 and N=7.

Number of surgeries = 3						
First-order Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.9	1.85%	3.74%	5.62%	7.17%	8.03%	21.34%
0.95	1.90%	3.89%	5.63%	7.39%	8.66%	22.28%
0.983	1.76%	4.03%	5.98%	7.62%	8.81%	22.86%
Number of surgeries = 7						
First-order Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.9	11.07%	18.60%	24.86%	27.22%	33.21%	61.15%
0.95	11.20%	18.66%	25.27%	27.51%	33.79%	61.65%
0.983	11.21%	18.62%	25.45%	27.80%	34.09%	61.28%

(iii) Effect of Last-Order Multiplier of The Mean and The Standard Deviation of The Surgery Duration of The Surgery Assigned to The Last Sequence:

Results are analyzed separately for each different value of the number of surgeries. According to the results, cost of using Veteran's Heuristic for Multiple Surgeries decreases, as the last-order multiplier increases. Since this pattern is observed for each different value of the number of surgeries, summary tables of the effect of the last-order multiplier of the mean and the standard deviation of the surgery assigned to the last sequence on cost of using Veteran's Heuristic for Multiple Surgeries for 3 surgeries and 7 surgeries are provided in Table 5.24.

According to Table 5.24, it is observed that the cost of using Veteran's Heuristic for Multiple Surgeries decreases as the last-order multiplier increases and gets closer to 1.

Table 5.24. Last-order multiplier vs. $C(V)$, N=3 and N=7.

Number of surgeries = 3						
Last-order Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.8	1.86%	4.29%	6.22%	7.68%	9.00%	22.86%
0.9	1.76%	3.71%	5.49%	7.11%	8.00%	20.72%
Number of surgeries = 7						
Last-order Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.8	11.20%	19.11%	25.10%	27.74%	33.54%	61.65%
0.9	11.07%	18.14%	25.27%	27.28%	33.84%	60.51%

(iv) Effect of Unit Costs:

In this section, the effect of the unit cost of the idle time on cost of using Veteran's Heuristic for Multiple Surgeries when the unit cost of the waiting time is kept constant at a value is analyzed. There exists a negative correlation between the unit cost of the idle time and cost of using Veteran's Heuristic for Multiple Surgeries. Table 5.25 gives the summary table of the effect of the unit cost of the idle time on cost of using Veteran's Heuristic for Multiple Surgeries when there are 3 surgeries. As mentioned in the previous experiment, Veteran's Heuristic gives more weight to the cost of idle time. Therefore, the results are consistent with our expectation. Furthermore, there is a positive correlation between unit cost of waiting time and cost of using Veteran's Heuristic.

As the number of surgeries increases, the relationship remains the same. Table 5.26 gives the summary table of the effect of the unit cost of the idle time on cost of using Veteran's Heuristic for Multiple Surgeries when there are 7 surgeries.

Table 5.25. α_1 vs. $C(V)$, N=3.

The unit cost of the waiting time = 0.2						
Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.2	3.69%	4.58%	5.43%	5.52%	6.12%	8.22%
0.4	2.69%	3.19%	3.77%	3.88%	4.40%	5.56%
0.6	1.76%	2.16%	2.66%	2.67%	3.13%	3.99%
The unit cost of the waiting time = 0.4						
Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.2	6.19%	6.74%	8.58%	8.67%	10.22%	12.51%
0.4	4.50%	5.13%	6.31%	6.33%	7.18%	9.04%
The unit cost of the waiting time = 0.6						
Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.2	12.74%	14.09%	18.00%	17.29%	19.70%	22.86%

According to Table 5.26, cost of using Veteran's Heuristic for Multiple Surgeries decreases, as the unit cost of the idle time increases.

When the results are analyzed together, it is observed that the cost of using Veteran's Heuristic for Multiple Surgeries increases, as the unit cost of the waiting time increases and the unit cost of the overtime decreases when the unit cost of the idle time is kept constant at a value. Since Veteran's Heuristic gives more importance to the cost of the idle time, the results are consistent with what is expected.

(v) Effect of The Coefficient of Variation of Surgery Durations:

Summary tables of the effect of the coefficient of variation of surgery durations on cost of using Veteran's Heuristic for Multiple Surgeries for 3 surgeries and 7 surgeries

Table 5.26. α_1 vs. $C(V)$, N=7.

The unit cost of the waiting time = 0.2						
Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.2	19.11%	21.27%	24.83%	24.90%	28.35%	32.34%
0.4	14.36%	15.49%	18.29%	18.30%	20.59%	23.98%
0.6	11.07%	11.94%	14.04%	14.09%	15.75%	19.45%
The unit cost of the waiting time = 0.4						
Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.2	27.99%	30.31%	35.44%	36.19%	41.51%	47.01%
0.4	20.04%	21.81%	26.28%	26.07%	29.38%	33.62%
The unit cost of the waiting time = 0.6						
Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.2	32.20%	34.87%	43.46%	45.51%	57.63%	61.65%

are provided in Table 5.27.

According to Table 5.27, there exists a negative correlation between the coefficient of variation of the surgery durations and the cost of using Veteran's Heuristic for Multiple Surgeries.

5.1.2.4. Results for Modified Myopic Heuristic. In this section, results of single operating room and multiple surgeries problem obtained with Modified Myopic Heuristic will be analyzed. In order to measure the efficiency of Modified Myopic Heuristic for single operating room and multiple surgeries problem, the percentage difference between the objective value obtained without using heuristic and the objective value obtained when the surgery durations are assigned by using Modified Myopic Heuristic

Table 5.27. CV of surgery durations vs. $C(V)$, N=3 and N=7.

Number of surgeries = 3						
CV of Surgery Durations	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.4	2.69%	4.73%	7.29%	8.97%	10.98%	22.86%
0.8	2.04%	3.78%	5.76%	7.43%	8.55%	19.91%
1.2	1.76%	3.06%	4.79%	5.79%	6.53%	14.46%
Number of surgeries = 7						
CV of Surgery Durations	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.4	15.43%	21.18%	29.98%	33.46%	43.10%	61.65%
0.8	12.75%	18.33%	25.16%	27.00%	35.34%	44.72%
1.2	11.07%	15.31%	21.03%	22.08%	29.40%	35.51%

is calculated. This value is also represented as $C(Mod)$ as in the previous experiment.

(i) Effect of The Number of Surgeries:

Table 5.28 gives the summary table of the effect of the number of surgeries on cost of using Modified Myopic Heuristic for Multiple Surgeries when other parameters are kept the same.

According to Table 5.28, the uncertainty increases as the number of surgeries increases, and this leads to an increase in cost of using Modified Myopic Heuristic for Multiple Surgeries. When the number of surgeries is equal to 3, cost of using Modified Myopic Heuristic for Multiple Surgeries takes values between 0.53 and 7.57. On the other hand, cost of using Modified Myopic Heuristic for Multiple Surgeries takes values between 4.78 and 25.79, when the number of surgeries is equal to 7.

Table 5.28. Number of surgeries vs. $C(Mod)$.

Number of Surgeries	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
3	0.53%	1.08%	1.63%	2.29%	2.80%	7.57%
4	0.77%	2.74%	3.73%	4.67%	6.01%	11.84%
5	1.24%	5.29%	6.34%	7.56%	9.51%	17.01%
6	3.04%	7.99%	9.86%	10.81%	13.26%	21.87%
7	4.78%	10.44%	13.36%	14.14%	17.68%	25.79%

(ii) Effect of First-Order Multiplier of The Mean and The Standard Deviation of The Surgery Duration of The Surgery Assigned to The First Sequence:

Results are analyzed separately for each different value of the number of surgeries. According to the results, cost of using Modified Myopic Heuristic for Multiple Surgeries increases, as the first-order multiplier increases. Since this pattern is observed for each different value of the number of surgeries, summary tables of the effect of the first-order multiplier of the mean and the standard deviation of the surgery assigned to the first sequence on cost of using Modified Myopic Heuristic for Multiple Surgeries for 3 surgeries and 7 surgeries are provided in Table 5.29.

According to Table 5.29, it is observed that cost of using Modified Myopic Heuristic for Multiple Surgeries increases as the first-order multiplier increases and gets closer to 1.

(iii) Effect of Last-Order Multiplier of The Mean and The Standard Deviation of The Surgery Duration of The Surgery Assigned to The Last Sequence:

Results are analyzed separately for each different value of the number of surgeries. According to the results, cost of using Modified Myopic Heuristic for Multiple Surgeries decreases, as the last-order multiplier increases. Since this pattern is observed for each

Table 5.29. First-order multiplier vs. $C(Mod)$, N=3 and N=7.

Number of surgeries = 3						
First-order Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.9	0.53%	1.08%	1.57%	2.19%	2.76%	6.27%
0.95	0.67%	1.08%	1.66%	2.29%	2.88%	7.57%
0.983	0.77%	1.11%	1.69%	2.40%	2.86%	6.87%
Number of surgeries = 7						
First-order Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.9	4.78%	10.04%	13.12%	13.99%	17.66%	25.28%
0.95	5.02%	10.67%	13.39%	14.13%	17.68%	25.68%
0.983	4.87%	10.46%	13.52%	14.30%	17.72%	25.79%

different value of the number of surgeries, summary tables of the effect of the last-order multiplier of the mean and the standard deviation of the surgery assigned to the last sequence on cost of using Modified Myopic Heuristic for Multiple Surgeries for 3 surgeries and 7 surgeries are provided in Table 5.30.

According to Table 5.30, it is observed that cost of using Modified Myopic Heuristic for Multiple Surgeries decreases as the last-order multiplier increases and gets closer to 1.

(iv) Effect of Unit Costs:

In this section, the effect of the unit cost of the idle time on cost of using Modified Myopic Heuristic for Multiple Surgeries when the unit cost of the waiting time is kept constant at a value is analyzed. There exists a negative correlation between the unit cost of the idle time and cost of using Modified Myopic Heuristic for Multiple Surgeries. Table 5.31 gives the summary table of the effect of the unit cost of the idle time on cost

Table 5.30. Last-order multiplier vs. $C(Mod)$, N=3 and N=7.

Number of surgeries = 3						
Last-order Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.8	0.53%	1.15%	1.65%	2.40%	2.95%	7.57%
0.9	0.62%	1.07%	1.60%	2.19%	2.73%	6.40%
Number of surgeries = 7						
Last-order Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.8	4.87%	10.51%	13.49%	14.28%	18.09%	25.64%
0.9	4.78%	10.22%	13.24%	14.01%	17.62%	25.79%

of using Modified Myopic Heuristic for Multiple Surgeries when there are 3 surgeries.

As the number of surgeries increases, the relationship remains the same. Table 5.32 gives the summary table of the effect of the unit cost of the idle time on cost of using Modified Myopic Heuristic for Multiple Surgeries when there are 7 surgeries.

According to Table 5.32, cost of using Modified Myopic Heuristic for Multiple Surgeries decreases, as the unit cost of the idle time increases and the unit cost of the overtime decreases.

When the results are analyzed together, it is observed that the cost of using Modified Myopic Heuristic for Multiple Surgeries decreases, as the unit cost of the waiting time increases and the unit cost of the overtime decreases when the unit cost of the idle time is kept constant at a value.

5.1.2.5. Comparative Analysis of Heuristics. In this section, results of Myopic Heuristic, Veteran's Heuristic, and Modified Myopic Heuristic for multiple surgeries are com-

Table 5.31. α_1 vs. $C(Mod)$, N=3.

The unit cost of the waiting time = 0.2						
Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.2	4.67%	5.10%	5.67%	5.66%	6.22%	7.57%
0.4	0.85%	1.24%	2.13%	2.00%	2.57%	3.57%
0.6	0.53%	0.77%	0.91%	1.08%	1.51%	1.87%
The unit cost of the waiting time = 0.4						
Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.2	1.25%	2.24%	2.69%	2.57%	2.90%	3.52%
0.4	0.67%	1.02%	1.10%	1.10%	1.20%	1.40%
The unit cost of the waiting time = 0.6						
Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.2	0.62%	1.15%	1.50%	1.36%	1.63%	1.94%

pared. When the number of surgeries is equal to 3, cost of using Modified Myopic Heuristic is less than other costs in 88.58% of all solutions. When the number of surgeries is equal to 4 or 5, cost of using Modified Myopic Heuristic is less than other costs in 88.889% of all solutions. This value is 89.815% when the number of surgeries is equal to 6, and 93.593% when the number of surgeries is equal to 7.

According to Table 5.33, the minimum average cost is obtained by using Modified Myopic Heuristic. On the other hand, the maximum average cost is obtained by using Myopic Heuristic. This pattern is the same for each different value of the number of surgeries. As in the single operating room and two surgeries experiment, Modified Myopic Heuristic can be preferred over other heuristics in general.

Table 5.32. α_1 vs. $C(Mod)$, N=7.

The unit cost of the waiting time = 0.2						
Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.2	18.45%	21.16%	23.34%	22.76%	24.19%	25.79%
0.4	12.58%	13.63%	17.70%	16.93%	18.90%	22.28%
0.6	5.24%	5.86%	9.80%	9.76%	13.14%	16.21%
The unit cost of the waiting time = 0.4						
Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.2	7.85%	9.88%	14.30%	13.30%	15.98%	17.61%
0.4	10.36%	11.82%	12.39%	12.44%	13.10%	14.59%
The unit cost of the waiting time = 0.6						
Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.2	4.78%	6.03%	10.13%	9.65%	12.60%	14.16%

When 6 possible $\{\alpha_1, \alpha_2, \alpha_3\}$ investigated, there is only one combination $\{0.2, 0.2, 0.6\}$ gives Veteran's Heuristic as the best heuristic in n=3 case. In n=7 case, all combinations give Modified Myopic Heuristic as the best heuristic. Therefore, as the number of surgeries within an operating room increases, Modified Myopic Heuristic becomes dominant among all heuristics.

When the computation time of the heuristics is investigated, it is observed that the single operating room and two surgeries problem with 20000 scenarios can be solved in approximately 5 seconds. When Myopic Heuristic is used, the computation time reduces to 1 seconds. This value is 4 seconds for Veteran's Heuristic, and 3 seconds for Modified Myopic Heuristic. The computation time is approximately the same in the single operating room and 3 surgeries problem with 10000 scenarios. When the number of surgeries is equal to 4 and the number of scenarios is taken as 10000, the

Table 5.33. Heuristics vs. costs of using these heuristics for multiple surgeries.

Number of surgeries = 3						
Heuristic	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
Myopic	2.60%	9.08%	13.33%	15.53%	24.16%	30.71%
Veteran's	1.76%	3.83%	5.76%	7.39%	8.67%	22.86%
Modified Myopic	0.53%	1.08%	1.63%	2.29%	2.80%	7.57%
Number of surgeries = 7						
Heuristic	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
Myopic	13.58%	40.85%	58.55%	59.01%	81.49%	98.06%
Veteran's	11.07%	18.66%	25.16%	27.51%	33.65%	61.65%
Modified Myopic	4.78%	10.44%	13.36%	14.14%	17.68%	25.79%

computation time is approximately 15 seconds. The computation time reduces to 2 seconds with Myopic Heuristic, 10 seconds with Veteran's Heuristic, and 5 seconds with Modified Myopic Heuristic. When the number of surgeries is equal to 5 and the number of scenarios is taken as 10000, the computation time is approximately 62 seconds. The computation time reduces to 4 seconds with Myopic Heuristic, 25 seconds with Veteran's Heuristic, and 10 seconds with Modified Myopic Heuristic. When the number of surgeries is equal to 6 and the number of scenarios is taken as 10000, the computation time is approximately 155 seconds. The computation time reduces to 7 seconds with Myopic Heuristic, 40 seconds with Veteran's Heuristic, and 15 seconds with Modified Myopic Heuristic. When the number of surgeries is equal to 7 and the number of scenarios is taken as 10000, the computation time is approximately 306 seconds. The computation time reduces to 12 seconds with Myopic Heuristic, 65 seconds with Veteran's Heuristic, and 21 seconds with Modified Myopic Heuristic. Therefore, it is more advantageous to use heuristics, as the number of surgeries increases as in Figure 5.8.

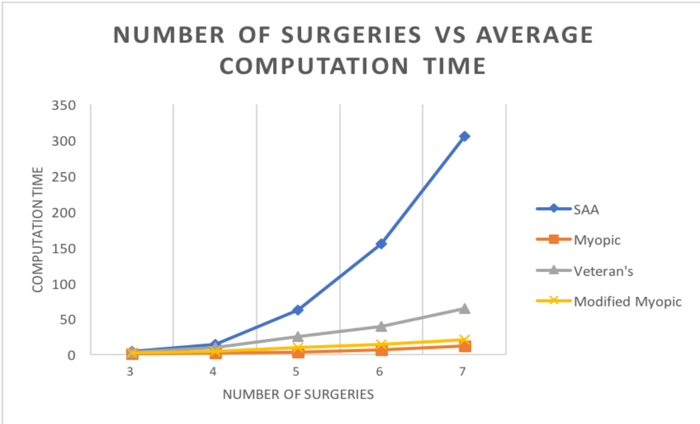


Figure 5.8. Number of Surgeries vs. Average Computation Time.

5.1.3. Managerial Insights for Single Operating Room

According to the analysis of numerical results, we obtained some useful suggestions for hospitals. These suggestions are listed below.

- In this thesis, results are obtained for the next day operating room scheduling problem with sequence-dependent surgery durations. We calculated the effect of disregarding sequence-dependent surgery durations on the objective value. When unit costs and penalty of ignoring sequence-dependent surgery durations are compared, it is found that the largest average penalty is obtained where the unit cost of waiting time takes its maximum possible value, and the unit cost of overtime takes its minimum possible value. Therefore, hospital management should consider sequence-dependent surgery durations especially when they give highest weight to the waiting time of their patients.
- It is found that penalty of ignoring sequence-dependent surgery durations increases as the unit cost of idle time decreases when the unit cost of waiting time is kept at a small value. On the other hand, penalty of ignoring sequence-dependent surgery durations increases as the unit cost of idle time increases when the unit cost of waiting time is kept at a high value.
- Three different heuristics are used for assigning surgery durations in single operating room problem. These heuristics are Myopic Heuristic, Veteran's Heuristic,

and Modified Myopic Heuristic. When there are only two surgeries, it is found that Veteran's Heuristic performs better than other heuristics when the hospital gives more weight to idle time of their operating rooms or less weights to waiting time of their patients and overtime of their hospital staff. When the hospital gives less weight to the idle time of their operating rooms or more weight to the overtime of their hospital staff, using Modified Myopic Heuristic provides better results than other heuristics. As the number of surgeries increases, Modified Myopic Heuristic becomes dominant.

5.2. Scheduling & Sequencing Results

In these experiments, schedule of the surgeries is not known at the beginning. Both the scheduling and sequencing of the surgeries are done simultaneously during the experiments. This experiment is used for the scheduling and the sequencing of multiple surgeries within multiple operating rooms.

5.2.1. Multiple ORs & Multiple Surgeries

In this experiment, there will be multiple operating rooms, and there will be more than 2 surgeries for each operating room. As in the previous experiments, it is aimed to investigate whether or not ignoring sequence-dependent surgery durations affects the total weighted cost of the waiting time of the patients for the surgeries, the idle time of the operating rooms, and the overtime of the surgeons. Furthermore, L-Shaped method is used to find solutions to the problem. In order to decrease the computation time further, Shortest-Variance-First Heuristic is used to sequence the surgeries. Finally, Expectation Heuristic is used to give an initial lower bound to the objective function of master problem in L-Shaped method. The weighted cost obtained from Expectation Heuristic is used as a lower bound for the objective function of master problem. Then, the relationship between different parameter settings and penalty of ignoring sequence-dependent surgery durations with these settings is analyzed in detail throughout the section.

5.2.1.1. Experimental Setting. Table 5.34 provides the parameter set of the experiment.Number of operating rooms is chosen from the set $\{2, 3, 4\}$, and the number of surgeries for each operating room is taken as 3. As mentioned earlier, surgery durations are distributed log-normally. For each surgery, two different values for mean and coefficient of variation of surgeries durations are assigned. Mean of surgery durations is generated randomly from $U[0.5, 1]$ for small case, and from $U[1, 4]$ for large case. Coefficient of variation of surgery durations is generated randomly from $U[0.4, 0.8]$ for small case, and from $U[0.8, 1.2]$ for large case. Table 5.35 gives the list of mean and coefficient of variation of surgery durations for each surgeries. It is assumed that all surgeries will take their small mean values, if the status for mean is small in parameter set. On the other hand, they will take their large mean values, if status for mean is large in parameter set. The same situation is also valid for coefficient of variation of surgery durations. If there exists two operating rooms, first 6 surgeries are used in the experiment. If there exists three operating rooms, first 9 surgeries are used. If there exists four operating rooms, all surgeries listed are used.

Table 5.34. Parameter set for Multiple ORs-Multiple Surgeries.

Parameter	**Description**	**Range**
m	Number of operating rooms	$\{2, 3, 4\}$
E[X]	Mean(Log-normal)	$U[0.5, 1]$ or $U[1, 4]$
cv[X]	Coefficient of variation(Log-normal)	$U[0.4, 0.8]$ or $U[0.8, 1.2]$
a_{first}	First-order multiplier	$\{0.9, 0.95, 0.983\}$
a_{last}	Last-order multiplier	$\{0.8, 0.9\}$
$\{\alpha_1, \alpha_2, \alpha_3\}$	Combination of unit costs	$\{\{0.2, 0.2, 0.6\},\{0.2, 0.6, 0.2\},\{0.6, 0.2, 0.2\},\{0.33, 0.33, 0.33\}\}$

As in the previous experiment, mean and standard deviation of the surgery durations of surgeries assigned to the first sequence in each operating room and surgeries assigned to the last sequence in each operating room are multiplied by multipliers. The value of the coefficient of variation will remain the same, since both mean and standard deviation will be multiplied by the same value. Multipliers are used to observe whether or not distributions of the surgery durations are dependent on the order of

Table 5.35. List of surgeries.

	Mean of Surgery Durations		**CV of Surgery Durations**	
Surgery No	**Small**	**Large**	**Small**	**Large**
1	0.826002	1.512699	0.464173	1.111993
2	0.836677	3.451817	0.629322	1.160489
3	0.675973	2.648434	0.578598	1.060725
4	0.52426233	3.67677446	0.525743	1.114683
5	0.7713	2.19291	0.658008	0.992704
6	0.697872	1.020716	0.614661	1.072327
7	0.918719	1.830141	0.634059	0.928344
8	0.643209	3.248767	0.5222	1.184154
9	0.68731	3.149823	0.736488	0.960846
10	0.520074	2.201534	0.41216	1.14916
11	0.852874	2.498977	0.430257	0.954548
12	0.621697	1.564187	0.685311	0.86679

the surgeries within an operating room. Multiplier of surgeries assigned to the first sequence in each operating room is chosen from the set $a_{first} = \{0.9, 0.95, 0.983\}$, and it is called first-order multiplier. Multiplier of surgeries assigned to the last sequence in each operating room is chosen from the set $a_{last} = \{0.8, 0.9\}$, and it is called last-order multiplier.

Furthermore, there will be 4 different combinations of $\{\alpha_1, \alpha_2, \alpha_3\}$. $\{0.2, 0.2, 0.6\}$ case is used in order to observe the effect of overtime. $\{0.2, 0.6, 0.2\}$ case is used in order to observe the effect of waiting time. $\{0.6, 0.2, 0.2\}$ case is used in order to observe the effect of idle time. Finally, $\{0.33, 0.33, 0.33\}$ case is used to understand equal case situation.

This parameter set has $3x3x2x4x2x2 = 288$ members. There are 3 options for the number of operating rooms, 3 options for first-order-multiplier, 3 options for last-order-multiplier, 4 different combinations for $\{\alpha_1, \alpha_2, \alpha_3\}$, 2 options for mean of surgery durations, and 2 options for coefficient of variation of surgery durations. Since L-Shaped method works iteratively, new data is generated on Cplex OPL for each iteration.

In this section, the key measure is the effect of the sequence on the objective value as a result of the change in the parameter. This effect is percentage difference between the objective value obtained when sequence-dependent surgeries considered and the objective value obtained when sequence-dependent surgeries disregarded. It is called as penalty of ignoring sequence-dependent surgery durations(PI) as in the single operating room and two surgeries experiment.

5.2.1.2. Results for L-Shaped Method.

(i) Effect of The Number of Operating Rooms:

Table 5.36 gives the summary table of the effect of the number of operating rooms on penalty of ignoring sequence-dependent surgery durations when other parameters are kept the same.

Table 5.36. Number of operating rooms vs. PI.

Number of ORs	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
2	0.00%	0.59%	1.03%	1.56%	2.20%	6.78%
3	0.05%	0.68%	1.17%	1.58%	2.25%	5.29%
4	0.09%	0.84%	1.96%	2.01%	2.64%	6.52%

According to Table 5.36, the uncertainty increases as the number of operating rooms increases, and this leads to an increase in penalty of ignoring sequence-dependent surgeries. However, these values are close to each other.

(ii) Effect of First-Order Multiplier of The Mean and The Standard Deviation of The Surgery Duration of Surgeries Assigned to The First Sequence:

Results are analyzed separately for each different value of the number of operating rooms. According to the results, there exists no pattern for 2 and 3 operating rooms. Penalty of ignoring sequence-dependent surgery durations is minimum when the first-order multiplier is equal to 0.95 in these number of operating rooms. When the number of operating rooms is equal to 4, however, penalty of ignoring sequence-dependent surgery durations increases as the first-order multiplier increases. Table 5.37 provides the summary of the relationship between the first-order multiplier and penalty of ignoring sequence-dependent surgery durations.

Table 5.37. First-order multiplier vs. PI, m=2-4.

Number of ORs = 2						
First-order Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.9	0.29%	0.79%	1.21%	1.92%	2.27%	5.03%
0.95	0.04%	0.39%	0.76%	1.31%	1.45%	6.78%
0.983	0.00%	0.47%	1.13%	1.39%	2.28%	3.518%
Number of ORs = 3						
First-order Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.9	0.06%	0.80%	1.11%	1.58%	1.94%	5.29%
0.95	0.10%	0.59%	1.04%	1.49%	2.24%	4.37%
0.983	0.05%	0.67%	1.36%	1.68%	2.70%	3.82%
Number of ORs = 4						
First-order Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.9	0.09%	1.80%	2.21%	2.37%	3.03%	6.52%
0.95	0.12%	1.20%	1.87%	1.89%	2.53%	3.88%
0.983	0.18%	0.60%	1.86%	1.70%	2.50%	4.88%

According to Table 5.37, it is observed that penalty of ignoring sequence-dependent surgery durations increases as the first-order multiplier increases and gets closer to 1 only for 4 operating rooms.

(iii) Effect of Last-Order Multiplier of The Mean and The Standard Deviation of The Surgery Duration of Surgeries Assigned to The Last Sequence:

Results are analyzed separately for each different value of the number of operating rooms. According to the results, penalty of ignoring sequence-dependent surgery durations decreases, as the last-order multiplier increases. Table 5.38 gives the summary of the relationship between the last-order multiplier and penalty of ignoring sequence-dependent surgery durations.

Table 5.38. Last-order multiplier vs. PI, m=2-4.

Number of ORs = 2						
Last-order Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.8	0.14%	0.71%	1.86%	1.90%	2.62%	5.03%
0.9	0.00%	0.45%	0.79%	1.15%	1.20%	6.78%
Number of ORs = 3						
Last-order Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.8	0.06%	0.69%	1.54%	1.89%	3.04%	5.29%
0.9	0.05%	0.63%	1.04%	1.28%	1.94%	3.82%
Number of ORs = 4						
Last-order Multiplier	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.8	0.18%	1.96%	2.03%	2.33%	3.14%	6.52%
0.9	0.09%	0.70%	1.78%	1.66%	2.32%	5.70%

According to Table 5.38, it is observed that penalty of ignoring sequence-dependent surgery durations decreases as the last-order multiplier increases and gets closer to 1.

(iv) Effect of Mean of Surgery Durations:

As mentioned earlier, surgeries have two possible mean values. If mean status is small in the parameter set, all surgeries take their small mean values. If the status is large, then all surgeries take their large mean values. Table 5.39 gives the summary of the relationship between the mean status and penalty of ignoring sequence-dependent surgery durations.

Table 5.39. Mean status vs. PI, m=2-4.

Number of ORs = 2						
Mean Status	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
small	0.00%	0.35%	0.62%	0.95%	1.14%	3.81%
large	0.29%	0.81%	1.88%	2.10%	2.64%	6.78%
Number of ORs = 3						
Mean Status	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
small	0.31%	0.74%	1.00%	1.24%	1.62%	2.46%
large	0.05%	0.56%	1.36%	1.74%	2.81%	5.29%
Number of ORs = 4						
Mean Status	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
small	0.12%	0.71%	1.87%	1.89%	2.64%	5.70%
large	0.09%	0.98%	2.10%	2.13%	2.67%	6.52%

According to Table 5.39, penalty of ignoring sequence-dependent surgery durations increases as the mean of surgery durations increases.

(v) Effect of CV of Surgery Durations:

As mentioned earlier, surgeries have two possible coefficient of variation values. If CV status is small in the parameter set, all surgeries take their small coefficient of variation values. If the status is large, then all surgeries take their large coefficient of variation values. Table 5.40 gives the summary of the relationship between the CV status and penalty of ignoring sequence-dependent surgery durations.

Table 5.40. CV status vs. PI, m=2-4.

Number of ORs = 2						
CV Status	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
small	0.12%	0.86%	1.94%	2.16%	3.03%	6.78%
large	0.00%	0.32%	0.66%	0.88%	1.06%	2.52%
Number of ORs = 3						
CV Status	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
small	0.10%	0.93%	2.02%	2.15%	3.14%	5.29%
large	0.05%	0.54%	0.83%	1.08%	1.55%	3.04%
Number of ORs = 4						
CV Status	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
small	0.09%	1.76%	2.39%	2.51%	3.12%	6.52%
large	0.12%	0.67%	1.34%	1.41%	2.19%	3.27%

According to Table 5.40, penalty of ignoring sequence-dependent surgery durations decreases as the coefficient of variation of surgery durations increases. This will be true, since when the coefficient of variation is small, the standard deviation will be large. As the standard deviation increases, penalty of ignoring increases.

(vi) Effect of Unit Costs:

In this section, four possible combinations of unit costs are tested in terms of penalty of ignoring sequence-dependent surgery durations. As in the previous parts, the results are analyzed separately for each different value of the number of operating rooms. Table 5.41 gives the summary table of the effect of the unit cost combination on penalty of ignoring sequence-dependent surgery durations.

Table 5.41. Unit cost combination vs. PI, m=2-4.

Number of ORs = 2						
Unit Cost Combination	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
$\{0.2, 0.2, 0.6\}$	0.30%	0.79%	1.18%	1.78%	2.18%	5.03%
$\{0.2, 0.6, 0.2\}$	0.14%	0.42%	1.04%	1.36%	1.87%	6.78%
$\{0.33, 0.33, 0.33\}$	0.12%	0.64%	0.76%	1.54%	2.56%	4.86%
$\{0.6, 0.2, 0.2\}$	0.00%	0.53%	1.09%	1.49%	2.30%	4.74%
Number of ORs = 3						
Unit Cost Combination	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
$\{0.2, 0.2, 0.6\}$	0.69%	1.05%	1.54%	1.92%	2.24%	5.29%
$\{0.2, 0.6, 0.2\}$	0.12%	0.56%	0.72%	1.32%	1.76%	4.16%
$\{0.33, 0.33, 0.33\}$	0.06%	0.89%	1.85%	1.72%	2.60%	4.36%
$\{0.6, 0.2, 0.2\}$	0.05%	0.44%	0.77%	1.26%	1.90%	3.78%
Number of ORs = 6						
Unit Cost Combination	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
$\{0.2, 0.2, 0.6\}$	0.12%	1.54%	2.23%	2.34%	2.60%	6.52%
$\{0.2, 0.6, 0.2\}$	0.09%	0.67%	1.90%	1.88%	2.69%	5.70%
$\{0.33, 0.33, 0.33\}$	0.32%	0.97%	1.44%	1.84%	2.53%	5.44%
$\{0.6, 0.2, 0.2\}$	0.21%	0.76%	1.97%	1.88%	2.70%	3.86%

According to Table 5.41, there exists no certain pattern between combinations. However, when the unit cost of idle time is kept as 0.2, it is observed that penalty of ignoring sequence-dependent surgery durations decreases, when the unit cost of waiting time increases. This situation is observed in all operating room cases. Furthermore, when the unit cost of waiting time is kept as 0.2, it is observed that penalty of ignoring sequence-dependent surgery durations decreases as the unit cost of idle time increases. Again, this situation is observed in all situations. When overall results are evaluated, it is observed that penalty of ignoring sequence-dependent surgery durations is high, when overtime has significant weight or all unit costs are equal.

(vii) Performance Comparison:

In L-Shaped method, 36 binary variables are used for 2 operating room problem. This number is 81 for 3 operating rooms and 144 for 4 operating rooms. According to Table 5.42, the number of iterations is approximately proportional to number of operating rooms on the average.

Table 5.42. Number of operating rooms vs. number of iterations

Number of ORs	**Min**	**Mean**	**Max**
2	2	2.61364	14
3	2	4.9	104
4	2	8.77273	315

According to Table 5.43, the solution time increases significantly as the number of operating rooms increases. Other than these computations, L-Shaped method is also used without SVF Heuristic for only 2 operating rooms due to the computational time constraints. According to results, the objective values are so close, and this difference can be ignored. On the other hand, the solution time is 167.57 seconds on the average, and 2650.69 seconds at maximum. Number of iterations is 26.02 on the average, and 373 at maximum. This values are significantly larger than those of solutions with SVF Heuristic. Therefore, it is observed that using SVF Heuristic helps finding solutions in

Table 5.43. Number of operating rooms vs. number of solution time.

Number of ORs	**Min**	**Mean**	**Max**
2	12.265	25.1645	99.52
3	16.625	174.704	1471.56
4	24.517	1315.37	8853.2

reasonable time interval.

5.2.2. Managerial Insights for Multiple Operating Rooms

According to the analysis of numerical results, we obtained some useful suggestions for hospitals if there are multiple operating rooms. These suggestions are listed below.

- In order to consider multiple operating rooms, the results are obtained for 2,3, and 4 operating rooms. When penalty of ignoring sequence-dependent surgery durations is calculated, it is found that penalty of ignoring sequence-dependent surgery durations is high when overtime has significant weight or all unit costs are equal. This situation is reverse of the single operating room case, where the largest average penalty of ignoring sequence-dependent surgery durations is obtained for minimum overtime value. However, there are only 4 unit cost combinations for multiple operating rooms case due to the computational complexity. The minimum possible overtime value was 0.1 on single operating room case. It should be tested for multiple operating rooms case, as well.
- It is found that penalty of ignoring sequence-dependent surgery durations increases as the unit cost of idle time decreases when the unit cost of waiting time is kept at 0.2. Furthermore, penalty of ignoring sequence-dependent surgery durations decreases as the unit cost of waiting time increases when the unit cost of idle time is kept at 0.2. Therefore, hospital management should especially care on the cost occurred by ignoring sequence-dependent surgeries when they give

less weights to idle time and waiting time and high weight to overtime.

6. CONCLUSION

Operating rooms are the major part of hospital revenues, and also the most costly part of healthcare services. Therefore, increasing the efficiency of operating rooms is crucial for increasing the utilization of operating rooms, increasing satisfaction of patients from the services, and decreasing the cost of operating rooms. Throughout this thesis, we consider the next day operating room scheduling problem for both single operating room and multiple operating rooms cases. It is assumed that all surgeries are elective and surgery durations are uncertain. It is aimed to investigate whether surgery durations are dependent on the order of surgeries within operating room. In Chapter 2, we review the relevant literature on operating room scheduling. Most of the studies on operating room scheduling problem deal with deterministic surgery durations due to the computational complexity of stochastic problems. To our knowledge, we are first to study on the operating room scheduling problem where surgery durations are uncertain and dependent on the sequence within operating room.

In Chapter 3, we formulate the problem as Stochastic Mixed Integer Program. Since this problem has a stochastic nature, all possible values of random variables should be considered to obtain the optimum solution. However, it is nearly impossible. Therefore, Sample Average Approximation method is defined to come up solutions with the problem in Chapter 4. By generating a number of scenarios and taking the average of the objective values of these scenarios, an approximate solution is found for the problem. As the number of scenarios increases, this approximation converges to the optimal solution. Although SAA method provides good results for small cases, it becomes insufficient as the number of surgeries and the number of operating rooms increase. At this point, L-Shaped method is used to find solutions for multiple operating rooms problem. Since scheduling surgeries and assigning durations to these surgeries simultaneously make the problem even harder, we developed five heuristics to decrease the computation time.

In Chapter 5, a number of experiments are developed to analyze the possible outcomes of the problem. Then, numerical results are obtained for these experiments. For single operating room problem, Myopic Heuristic, Veteran's Heuristic, and Modified Myopic Heuristic are used, respectively. Multiple operating room problem is solved by using L-Shaped method. However, since it converges slowly for small cases, we use also SVF Heuristic for sequencing surgeries. Moreover, we use Expectation Heuristic to provide a lower bound for the master problem.

According to the results of single operating room problem, Veteran's Heuristic is better than other heuristics when the unit cost of idle time is high, the unit cost of waiting time of patients for surgeries is small, or the unit cost of overtime of hospital staff is small. Modified Myopic Heuristic performs better than other heuristics if the unit cost of idle time is small or the unit cost of overtime is high. When the number of surgeries increases, Modified Myopic Heuristic becomes dominant. Results for multiple operating rooms show that penalty of ignoring sequence-dependent surgery durations increases as the unit cost of idle time or the unit cost of waiting time decreases. Furthermore, penalty of ignoring sequence-dependent surgery durations is directly proportional to mean status, whereas inversely proportional to coefficient of variation status.

In conclusion, next-day operating room scheduling is a challenging problem especially when the surgery durations are stochastic. Therefore, since our aim is to provide some managerial insights to the hospital instead of finding optimum solutions, we develop different heuristics, and evaluate their performance. Accordingly, we provide some useful insights to hospitals on Chapter 5, and we hope that these insights will help to increase the efficiency of hospitals. Furthermore, a pattern for optimal allocation of surgeries to operating rooms under SVF rule is discovered during the thesis. It is proven for three cases, which are 2 OR-4 surgeries, 2 OR- 6 operations, and 3 OR-6 operations. For now, this pattern is not proven for general m OR-n surgeries case. As a future research, we are interested to prove this pattern analytically. Moreover, pre-operative process and pro-operative process of surgeries should be integrated into the current problem, since considering the process as a whole provides more realistic results. Finally, additional heuristics should be developed and performance of existing heuristics should be improved in order to be able to come up with solutions to the problems with large number of operating rooms and large number of surgeries.

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APPENDIX A: PROOF OF THEOREM 4.3

Proof. Surgeries are sorted in the increasing order of standard deviation of surgery duration ($s_1 < s_2 < s_3 < s_4 < s_5 < s_6$).

If operating rooms are not assumed to be identical, there would be $\binom{6}{3} = 20$ cases to be tested. When the operating rooms are identical, there will be $\binom{5}{2} = 10$ cases to be tested by assigning surgery 1 to OR-1 in each case.

Table A.1. Cases for Theorem 4.3.

Case Number	OR-1	OR-2	Objective Function
1	1→2→3	4→5→6	$s_1 + \sqrt{s_1^2+s_2^2} + \sqrt{s_1^2+s_2^2+s_3^2} + s_4 + \sqrt{s_4^2+s_5^2} + \sqrt{s_4^2+s_5^2+s_6^2}$
2	1→2→4	3→5→6	$s_1 + \sqrt{s_1^2+s_2^2} + \sqrt{s_1^2+s_2^2+s_4^2} + s_3 + \sqrt{s_3^2+s_5^2} + \sqrt{s_3^2+s_5^2+s_6^2}$
3	1→2→5	3→4→6	$s_1 + \sqrt{s_1^2+s_2^2} + \sqrt{s_1^2+s_2^2+s_5^2} + s_3 + \sqrt{s_3^2+s_4^2} + \sqrt{s_3^2+s_4^2+s_6^2}$
4	1→2→6	3→4→5	$s_1 + \sqrt{s_1^2+s_2^2} + \sqrt{s_1^2+s_2^2+s_6^2} + s_3 + \sqrt{s_3^2+s_4^2} + \sqrt{s_3^2+s_4^2+s_6^2}$
5	1→3→4	2→5→6	$s_1 + \sqrt{s_1^2+s_3^2} + \sqrt{s_1^2+s_3^2+s_4^2} + s_2 + \sqrt{s_2^2+s_5^2} + \sqrt{s_2^2+s_5^2+s_6^2}$
6	1→3→5	2→4→6	$s_1 + \sqrt{s_1^2+s_3^2} + \sqrt{s_1^2+s_3^2+s_5^2} + s_2 + \sqrt{s_2^2+s_4^2} + \sqrt{s_2^2+s_4^2+s_6^2}$
7	1→3→6	2→4→5	$s_1 + \sqrt{s_1^2+s_3^2} + \sqrt{s_1^2+s_3^2+s_6^2} + s_2 + \sqrt{s_2^2+s_4^2} + \sqrt{s_2^2+s_4^2+s_5^2}$
8	1→4→5	2→3→6	$s_1 + \sqrt{s_1^2+s_4^2} + \sqrt{s_1^2+s_4^2+s_5^2} + s_2 + \sqrt{s_2^2+s_3^2} + \sqrt{s_2^2+s_3^2+s_6^2}$
9	1→4→6	2→3→5	$s_1 + \sqrt{s_1^2+s_4^2} + \sqrt{s_1^2+s_4^2+s_6^2} + s_2 + \sqrt{s_2^2+s_3^2} + \sqrt{s_2^2+s_3^2+s_5^2}$
10	1→5→6	2→3→4	$s_1 + \sqrt{s_1^2+s_5^2} + \sqrt{s_1^2+s_5^2+s_6^2} + s_2 + \sqrt{s_2^2+s_3^2} + \sqrt{s_2^2+s_3^2+s_4^2}$

Let's start with comparing Case-1 and Case-2. s_1 and $\sqrt{s_1^2+s_2^2}$ are common terms in both cases, and they are ignored. Since s_4 is larger than s_3, $\sqrt{s_4^2+s_5^2} + \sqrt{s_4^2+s_5^2+s_6^2}$ is larger than $\sqrt{s_3^2+s_5^2} + \sqrt{s_3^2+s_5^2+s_6^2}$. Therefore, if the sum of remaining terms in Case-1 is larger than the sum of remaining terms in Case-2, Case-1 will be eliminated. Let's take the square of the remaining terms.

Table A.2. Comparison of Case-1 & Case-2 - Step 1 (Theorem 4.3).

Case Number	**Compared Value**
1	$s_1^2 + s_2^2 + s_3^2 + s_4^2 + 2 * \sqrt{(s_1 * s_4)^2 + (s_2 * s_4)^2 + (s_3 * s_4)^2}$
2	$s_1^2 + s_2^2 + s_3^2 + s_4^2 + 2 * \sqrt{(s_1 * s_3)^2 + (s_2 * s_3)^2 + (s_3 * s_4)^2}$

Common terms are eliminated, and the square of the remaining terms are taken for both cases.

Table A.3. Comparison of Case-1 & Case-2 - Step 2 (Theorem 4.3).

Case Number	**Compared Value**
1	$(s_1 * s_4)^2 + (s_2 * s_4)^2 + (s_3 * s_4)^2$
2	$(s_1 * s_3)^2 + (s_2 * s_3)^2 + (s_3 * s_4)^2$

$(s_3 * s_4)^2$ is the common term in both cases, and it is eliminated. $(s_1 * s_3)^2 + (s_2 * s_3)^2$ is subtracted from both cases.

Table A.4. Comparison of Case-1 & Case-2 - Step 3 (Theorem 4.3).

Case Number	**Compared Value**
1	$(s_1 * s_4)^2 - (s_1 * s_3)^2 + (s_2 * s_4)^2 - (s_2 * s_3)^2$
2	0

When the remaining terms in Case-1 are arranged, it becomes $(s_1^2 + s_2^2)*(s_4^2 - s_3^2)$. According to the initial assumption, $s_3 < s_4$. Therefore, Case-1 gives a positive value, whereas Case-2 is equal to 0. Since Case-1 gives larger objective value, it is eliminated.

The next step is to compare the objective values of Case-2 and Case-6. Since s_1 is the common term in both cases, it is ignored. Since s_3 is larger than s_2, and s_5 is larger than s_4, $\sqrt{s_3^2+s_5^2+s_6^2}$ is larger than $\sqrt{s_2^2+s_5^2+s_4^2}$. If $\sqrt{s_1^2+s_2^2}+s_3$ is larger than $\sqrt{s_1^2+s_3^2}+s_2$, and $\sqrt{s_1^2+s_2^2+s_4^2}+\sqrt{s_3^2+s_5^2}$ is larger than $\sqrt{s_1^2+s_3^2+s_5^2}+\sqrt{s_2^2+s_4^2}$, the objective value of Case-2 will be larger than the objective value of Case-6.

Let's first compare $\sqrt{s_1^2+s_2^2}+s_3$ and $\sqrt{s_1^2+s_3^2}+s_2$. Take the square of these terms.

Table A.5. Comparison of Case-2 & Case-6 - Step 1 (Theorem 4.3).

Case Number	Compared Value
2	$s_1^2+s_2^2+s_3^2+2*\sqrt{(s_1*s_3)^2+(s_2*s_3)^2}$
6	$s_1^2+s_2^2+s_3^2+2*\sqrt{(s_1*s_2)^2+(s_2*s_3)^2}$

Common terms are eliminated, and the square of the remaining terms are taken for both cases.

Table A.6. Comparison of Case-2 & Case-6 - Step 2 (Theorem 4.3).

Case Number	Compared Value
2	$(s_1*s_3)^2+(s_2*s_3)^2$
6	$(s_1*s_2)^2+(s_2*s_3)^2$

$(s_2*s_3)^2$ is the common term in both cases, and it is eliminated. Since s_3 is larger than s_2, $(s_1*s_3)^2$ is also larger than $(s_1*s_2)^2$. As a second step, $\sqrt{s_1^2+s_2^2+s_4^2}+\sqrt{s_3^2+s_5^2}$ and $\sqrt{s_1^2+s_3^2+s_5^2}+\sqrt{s_2^2+s_4^2}$ are compared. The square of these terms are taken.

Table A.7. Comparison of Case-2 & Case-6 - Step 3 (Theorem 4.3).

Case Number	Compared Value
2	$s_1^2 + s_2^2 + s_3^2 + s_4^2 + s_5^2 + 2 * \sqrt{(s_1^2 + s_2^2 + s_4^2) * (s_3^2 + s_5^2)}$
6	$s_1^2 + s_2^2 + s_3^2 + s_4^2 + s_5^2 + 2 * \sqrt{(s_1^2 + s_3^2 + s_5^2) * (s_2^2 + s_4^2)}$

Common terms are eliminated, and the square of the remaining terms are taken for both cases.

Table A.8. Comparison of Case-2 & Case-6 - Step 4 (Theorem 4.3).

Case Number	Compared Value
2	$(s_1 * s_3)^2 + (s_1 * s_5)^2 + (s_2 * s_3)^2 + (s_2 * s_5)^2 + (s_3 * s_4)^2 + (s_4 * s_5)^2$
6	$(s_1 * s_2)^2 + (s_1 * s_4)^2 + (s_2 * s_3)^2 + (s_3 * s_4)^2 + (s_2 * s_5)^2 + (s_4 * s_5)^2$

$(s_2 * s_3)^2$, $(s_2 * s_5)^2$, $(s_3 * s_4)^2$, and $(s_4 * s_5)^2$ are the common terms and they are eliminated. $(s_1 * s_2)^2 + (s_1 * s_4)^2$ is subtracted from both cases.

Table A.9. Comparison of Case-2 & Case-6 - Step 5 (Theorem 4.3).

Case Number	Compared Value
2	$(s_1 * s_3)^2 - (s_1 * s_2)^2 + (s_1 * s_5)^2 - (s_1 * s_4)^2$
6	0

When the remaining terms in Case-2 are arranged, it becomes $s_1^2 * (s_3^2 - s_2^2 + s_5^2 - s_4^2)$. According to the initial assumption, $s_2 < s_3$ and $s_4 < s_5$. After all steps, it is proven that Case-2 gives larger objective value, and it is eliminated.

In this step, Case-3 and Case-4 are compared. Since s_1, $\sqrt{s_1^2 + s_2^2}$, s_3, and $\sqrt{s_3^2 + s_4^2}$ are common terms in both cases, they are ignored. The square of the remaining terms are taken for both cases.

Table A.10. Comparison of Case-3 & Case-4 - Step 1 (Theorem 4.3).

Case Number	Compared Value
3	$(s_1 * s_3)^2 + (s_1 * s_4)^2 + (s_1 * s_6)^2 + (s_2 * s_3)^2 + (s_2 * s_4)^2 + (s_2 * s_6)^2 + (s_3 * s_5)^2 + (s_4 * s_5)^2 + (s_5 * s_6)^2$
4	$(s_1 * s_3)^2 + (s_1 * s_4)^2 + (s_1 * s_5)^2 + (s_2 * s_3)^2 + (s_2 * s_4)^2 + (s_2 * s_5)^2 + (s_3 * s_6)^2 + (s_4 * s_6)^2 + (s_5 * s_6)^2$

Common terms are eliminated. $(s_1 * s_5)^2 + (s_2 * s_5)^2 + (s_3 * s_6)^2 + (s_4 * s_6)^2$ is subtracted from both cases.

Table A.11. Comparison of Case-3 & Case-4 - Step 2 (Theorem 4.3).

Case Number	Compared Value
3	$(s_6^2 - s_5^2) * (s_1^2 + s_2^2 - s_3^2 - s_4^2)$
4	0

$s_1 + s_2$ is less than $s_3 + s_4$. Therefore, Case-3 gives negative value, whereas Case-4 is equal to 0. Therefore, Case-4 is eliminated.

In this step, Case-3 and Case-6 will be compared. s_1^2 is common in both cases, and this term is ignored. Since s_3 is larger than s_2, $\sqrt{s_3^2 + s_4^2 + s_6^2}$ is also larger than $\sqrt{s_2^2 + s_4^2 + s_6^2}$. Furthermore, it is proven that $\sqrt{s_1^2 + s_2^2} + s_3$ is larger than $\sqrt{s_1^2 + s_3^2} + s_2$ while comparing Case-2 and Case-6. If $\sqrt{s_1^2 + s_2^2 + s_5^2} + \sqrt{s_3^2 + s_4^2}$ is larger than $\sqrt{s_1^2 + s_3^2 + s_5^2} + \sqrt{s_2^2 + s_4^2}$, the objective value of Case-3 will be larger than the objective value of Case-6.

$\sqrt{s_1^2 + s_2^2 + s_4^2} + \sqrt{s_3^2 + s_5^2}$ and $\sqrt{s_1^2 + s_3^2 + s_5^2} + \sqrt{s_2^2 + s_4^2}$ are compared below. The square of these terms are taken.

Table A.12. Comparison of Case-3 & Case-6 - Step 1 (Theorem 4.3).

Case Number	Compared Value
3	$s_1^2+s_2^2+s_3^2+s_4^2+s_5^2+2*\sqrt{(s_1^2+s_2^2+s_5^2)*(s_3^2+s_4^2)}$
6	$s_1^2+s_2^2+s_3^2+s_4^2+s_5^2+2*\sqrt{(s_1^2+s_3^2+s_5^2)*(s_2^2+s_4^2)}$

Common terms are eliminated, and the square of the remaining terms are taken for both cases.

Table A.13. Comparison of Case-3 & Case-6 - Step 2 (Theorem 4.3).

Case Number	Compared Value
3	$(s_1*s_3)^2+(s_1*s_4)^2+(s_2*s_3)^2+(s_2*s_4)^2+(s_3*s_5)^2+(s_4*s_5)^2$
6	$(s_1*s_2)^2+(s_1*s_4)^2+(s_2*s_3)^2+(s_3*s_4)^2+(s_2*s_5)^2+(s_4*s_5)^2$

$(s_1*s_4)^2$, $(s_2*s_3)^2$, and $(s_4*s_5)^2$ are the common terms and they are eliminated. $(s_1*s_2)^2+(s_3*s_4)^2+(s_2*s_5)^2$ is subtracted from both cases.

Table A.14. Comparison of Case-3 & Case-6 - Step 3 (Theorem 4.3).

Case Number	Compared Value
3	$(s_3^2-s_2^2)*(s_1^2+s_5^2-s_4^2)$
6	0

Since s_2 is less than s_3, and s_4 is less than s_5, the remaining value in Case-3 is positive. Therefore, the objective value of Case-3 is larger than the objective value of Case-6. Case-3 is eliminated.

In this step, Case-5 and Case-10 are compared. Since s_1 and s_2 are the common terms, they are eliminated. If $\sqrt{s_1^2+s_5^2}+\sqrt{s_2^2+s_3^2}$ is larger than $\sqrt{s_1^2+s_3^2}+\sqrt{s_1^2+s_5^2}$, and $\sqrt{s_1^2 + s_5^2 + s_6^2} + \sqrt{s_2^2 + s_3^2 + _4^2}$ is larger than $\sqrt{s_1^2 + s_3^2 + s_4^2} + \sqrt{s_2^2 + s_5^2 + s_6^2}$, the objective value of Case-10 will be larger than the objective value of Case-6.

Let's first compare $\sqrt{s_1^2 + s_5^2} + \sqrt{s_2^2 + s_3^2}$ and $\sqrt{s_1^2 + s_3^2} + \sqrt{s_1^2 + s_5^2}$. Take the square of these terms.

Table A.15. Comparison of Case-5 & Case-10 - Step 1 (Theorem 4.3).

Case Number	**Compared Value**
5	$s_1^2 + s_2^2 + s_3^2 + s_5^2 + 2 * \sqrt{(s_1 * s_2)^2 + (s_1 * s_5)^2 + (s_2 * s_3)^2 + (s_3 * s_5)^2}$
10	$s_1^2 + s_2^2 + s_3^2 + s_5^2 + 2 * \sqrt{(s_1 * s_2)^2 + (s_1 * s_3)^2 + (s_2 * s_5)^2 + (s_3 * s_5)^2}$

Common terms are eliminated, and the square of the remaining terms are taken for both cases.

Table A.16. Comparison of Case-5 & Case-10 - Step 2 (Theorem 4.3).

Case Number	**Compared Value**
5	$(s_1 * s_2)^2 + (s_1 * s_5)^2 + (s_2 * s_3)^2 + (s_3 * s_5)^2$
10	$(s_1 * s_2)^2 + (s_1 * s_3)^2 + (s_2 * s_5)^2 + (s_3 * s_5)^2$

$(s_1 * s_2)^2$ and $(s_3 * s_5)^2$ are the common terms, and they are eliminated. Then, $(s_1 * s_5)^2 + (s_2 * s_3)^2$ is subtracted from both cases.

Table A.17. Comparison of Case-5 & Case-10 - Step 3 (Theorem 4.3).

Case Number	**Compared Value**
5	0
10	$(s_2^2 - s_1^2) * (s_5^2 - s_3^2)$

Since s_1 is less than s_2 and s_3 is less than s_5, the value in Case-10 becomes positive. As a second step, $\sqrt{s_1^2 + s_5^2 + s_6^2} + \sqrt{s_2^2 + s_3^2 +_4^2}$ and $\sqrt{s_1^2 + s_3^2 + s_4^2} + \sqrt{s_2^2 + s_5^2 + s_6^2}$ are compared. The square of these terms are taken, and common terms are eliminated.

Table A.18. Comparison of Case-5 & Case-10 - Step 4 (Theorem 4.3).

Case Number	**Compared Value**
5	$(s_1 * s_2)^2 + (s_1 * s_5)^2 + (s_1 * s_6)^2 + (s_2 * s_3)^2 + (s_3 * s_5)^2 + (s_3 * s_6)^2 + (s_2 * s_4)^2 + (s_4 * s_5)^2 + (s_4 * s_6)^2$
10	$(s_1 * s_2)^2 + (s_1 * s_3)^2 + (s_1 * s_4)^2 + (s_2 * s_5)^2 + (s_3 * s_5)^2 + (s_4 * s_5)^2 + (s_2 * s_6)^2 + (s_3 * s_6)^2 + (s_4 * s_6)^2$

$(s_1 * s_2)^2, (s_3 * s_5)^2, (s_3 * s_6)^2, (s_4 * s_5)^2$, and $(s_4 * s_6)^2$ are common terms, and they are eliminated. Then, $(s_1 * s_2)^2 + (s_1 * s_6)^2 + (s_2 * s_3)^2 + (s_2 * s_4)^2$ is subtracted from both cases.

Table A.19. Comparison of Case-5 & Case-10 - Step 5 (Theorem 4.3).

Case Number	**Compared Value**
5	0
10	$(s_2^2 - s_1^2) * (s_5^2 + s_6^2 - s_3^2 - s_4^2)$

Therefore, the remaining value in Case-10 is again positive, whereas remaining value in Case-5 is equal to 0. Case-10 gives larger objective value, and it is eliminated.

In this step, Case-5 and Case-6 will be compared. s_1, $\sqrt{s_1^2 + s_3^2}$, and s_2 are common terms, and they are eliminated. Since s_5 is larger than s_4, $\sqrt{s_2^2 + s_5^2 + s_6^2}$ is also larger than $\sqrt{s_2^2 + s_4^2 + s_6^2}$. If the sum of remaining terms are larger in Case-5, the objective value of Case-5 will be larger than the objective value of Case-6. The square of the remaining terms are taken, and compared below.

Table A.20. Comparison of Case-5 & Case-6 - Step 1 (Theorem 4.3).

Case Number	Compared Value
5	$s_1^2 + s_2^2 + s_3^2 + s_4^2 + s_5^2 + 2 * \sqrt{(s_1^2 + s_3^2 + s_5^2) * (s_2^2 + s_5^2)}$
6	$s_1^2 + s_2^2 + s_3^2 + s_4^2 + s_5^2 + 2 * \sqrt{(s_1^2 + s_3^2 + s_5^2) * (s_2^2 + s_4^2)}$

Common terms are eliminated, and the square of the remaining terms are taken for both cases.

Table A.21. Comparison of Case-5 & Case-6 - Step 2 (Theorem 4.3).

Case Number	Compared Value
5	$(s_1 * s_2)^2 + (s_1 * s_5)^2 + (s_2 * s_3)^2 + (s_3 * s_5)^2 + (s_2 * s_4)^2 + (s_4 * s_5)^2$
6	$(s_1 * s_2)^2 + (s_1 * s_4)^2 + (s_2 * s_3)^2 + (s_3 * s_4)^2 + (s_2 * s_5)^2 + (s_4 * s_5)^2$

$(s_1 * s_2)^2$, $(s_2 * s_3)^2$, and $(s_4 * s_5)^2$ are the common terms and they are eliminated. $(s_1 * s_4)^2 + (s_3 * s_4)^2 + (s_2 * s_5)^2$ is subtracted from both cases.

Table A.22. Comparison of Case-5 & Case-6 - Step 3 (Theorem 4.3).

Case Number	Compared Value
5	$(s_5^2 - s_4^2) * (s_1^2 + s_3^2 - s_2^2)$
6	0

Since s_4 is less than s_5, and s_2 is less than s_3, the remaining value in Case-5 is positive. Therefore, the objective value of Case-5 is larger than the objective value of Case-6. Case-5 is eliminated.

In this step, Case-6 and Case-7 will be compared. s_1, $\sqrt{s_1^2+s_3^2}$, s_2, and $\sqrt{s_2^2+s_4^2}$ are common terms, and they are eliminated. The square of the remaining terms are taken.

Table A.23. Comparison of Case-6 & Case-7 - Step 1 (Theorem 4.3).

Case Number	Compared Value
6	$(s_1*s_2)^2+(s_1*s_4)^2+(s_1*s_6)^2+(s_2*s_3)^2+(s_3*s_4)^2+(s_3*s_6)^2+(s_2*s_5)^2+(s_4*s_5)^2+(s_5*s_6)^2$
7	$(s_1*s_2)^2+(s_1*s_4)^2+(s_1*s_5)^2+(s_2*s_3)^2+(s_3*s_4)^2+(s_3*s_5)^2+(s_2*s_6)^2+(s_4*s_6)^2+(s_5*s_6)^2$

Common terms are eliminated, and $(s_1*s_6)^2+(s_3*s_6)^2+(s_2*s_5)^2+(s_4*s_5)^2$ is subtracted from both cases.

Table A.24. Comparison of Case-6 & Case-7 - Step 2 (Theorem 4.3).

Case Number	Compared Value
6	$(s_6^2-s_5^2)*(s_1^2+s_3^2-s_2^2-s_4^2)$
7	0

The sum of remaining values in Case-6 takes a negative value, whereas the remaining value is equal to 0 in Case-7. Since the objective value of Case-7 is larger, it is eliminated.

The next step is to compare the objective values of Case-6 and Case-8. Since s_1 and s_2 are the common terms in both cases, they are ignored. If $\sqrt{s_1^2+s_3^2}+\sqrt{s_2^2+s_4^2}$ is less than $\sqrt{s_1^2+s_4^2}+\sqrt{s_2^2+s_3^2}$, and $\sqrt{s_1^2+s_3^2+s_5^2}+\sqrt{s_2^2+s_4^2+s_6^2}$ is less than $\sqrt{s_1^2+s_4^2+s_5^2}+\sqrt{s_2^2+s_3^2+s_6^2}$, the objective value of Case-6 will be less than the objective value of Case-8.

Let's first compare $\sqrt{s_1^2+s_3^2}+\sqrt{s_2^2+s_4^2}$ and $\sqrt{s_1^2+s_4^2}+\sqrt{s_2^2+s_3^2}$. Take the square of these terms.

Table A.25. Comparison of Case-6 & Case-8 - Step 1 (Theorem 4.3).

Case Number	Compared Value
6	$s_1^2 + s_2^2 + s_3^2 + s_4^2 + 2 * \sqrt{(s_1 * s_2)^2 + (s_1 * s_4)^2 + (s_2 * s_3)^2 + (s_3 * s_4)^2}$
8	$s_1^2 + s_2^2 + s_3^2 + s_4^2 + 2 * \sqrt{(s_1 * s_2)^2 + (s_1 * s_3)^2 + (s_2 * s_4)^2 + (s_3 * s_4)^2}$

Common terms are eliminated, and the square of the remaining terms are taken for both cases.

Table A.26. Comparison of Case-6 & Case-8 - Step 2 (Theorem 4.3).

Case Number	Compared Value
6	$(s_1 * s_2)^2 + (s_1 * s_4)^2 + (s_2 * s_3)^2 + (s_3 * s_4)^2$
8	$(s_1 * s_2)^2 + (s_1 * s_3)^2 + (s_2 * s_4)^2 + (s_3 * s_4)^2$

$(s_1 * s_2)^2$ and $(s_3 * s_4)^2$ are common terms in both cases, and they are eliminated. $(s_1 * s_4)^2 + (s_2 * s_3)^2$ is subtracted from both cases.

Table A.27. Comparison of Case-6 & Case-8 - Step 3 (Theorem 4.3).

Case Number	Compared Value
6	0
8	$(s_2^2 - s_1^2) * (s_4^2 - s_3^2)$

Since s_1 is less than s_2, and s_3 is less than s_4, the remaining value in Case-8 takes a positive value.

As a second step, $\sqrt{s_1^2 + s_3^2 + s_5^2} + \sqrt{s_2^2 + s_4^2 + s_6^2}$ and $\sqrt{s_1^2 + s_4^2 + s_5^2} + \sqrt{s_2^2 + s_3^2 + s_6^2}$ are compared. The square of these terms are taken.

Table A.28. Comparison of Case-6 & Case-8 - Step 4 (Theorem 4.3).

Case Number	**Compared Value**
6	$s_1^2 + s_2^2 + s_3^2 + s_4^2 + s_5^2 + s_6^2 + 2 * \sqrt{(s_1^2 + s_3^2 + s_5^2) * (s_2^2 + s_4^2 + s_6^2)}$
8	$s_1^2 + s_2^2 + s_3^2 + s_4^2 + s_5^2 + s_6^2 + 2 * \sqrt{(s_1^2 + s_4^2 + s_5^2) * (s_2^2 + s_3^2 + s_6^2)}$

Common terms are eliminated, and the square of the remaining terms are taken for both cases.

Table A.29. Comparison of Case-6 & Case-8 - Step 5 (Theorem 4.3).

Case Number	**Compared Value**
6	$(s_1 * s_2)^2 + (s_1 * s_4)^2 + (s_1 * s_6)^2 + (s_2 * s_3)^2 + (s_3 * s_4)^2 + (s_3 * s_6)^2 + (s_2 * s_5)^2 + (s_4 * s_5)^2 + (s_5 * s_6)^2$
8	$(s_1 * s_2)^2 + (s_1 * s_3)^2 + (s_1 * s_6)^2 + (s_2 * s_4)^2 + (s_3 * s_4)^2 + (s_4 * s_6)^2 + (s_2 * s_5)^2 + (s_3 * s_5)^2 + (s_5 * s_6)^2$

Common terms are eliminated, and $(s_1 * s_4)^2 + (s_2 * s_3)^2 + (s_3 * s_6)^2 + (s_4 * s_5)^2$ is subtracted from both cases.

Table A.30. Comparison of Case-6 & Case-8 - Step 6 (Theorem 4.3).

Case Number	**Compared Value**
6	0
8	$(s_4^2 - s_3^2) * (-s_1^2 + s_2^2 + s_6^2 - s_5^2)$

According to the sum of remaining values, Case-8 takes a positive value. Since the objective value of Case-8 is larger, it is eliminated.

The last step is to compare the objective values of Case-6 and Case-9. Since s_1 and s_2 are the common terms in both cases, they are ignored. Furthermore, it is proven that $\sqrt{s_1^2 + s_3^2} + \sqrt{s_2^2 + s_4^2}$ is less than $\sqrt{s_1^2 + s_4^2} + \sqrt{s_2^2 + s_3^2}$ while comparing Case-6 and

Case-8. If $\sqrt{s_1^2+s_3^2+s_5^2}+\sqrt{s_2^2+s_4^2+s_6^2}$ is less than $\sqrt{s_1^2+s_4^2+s_6^2}+\sqrt{s_2^2+s_3^2+s_5^2}$, the objective value of Case-6 will be less than the objective value of Case-9.

The square of the remaining terms are taken.

Table A.31. Comparison of Case-6 & Case-9 - Step 1 (Theorem 4.3).

Case Number	**Compared Value**
6	$s_1^2+s_2^2+s_3^2+s_4^2+s_5^2+s_6^2+2*\sqrt{(s_1^2+s_3^2+s_5^2)*(s_2^2+s_4^2+s_6^2)}$
9	$s_1^2+s_2^2+s_3^2+s_4^2+s_5^2+s_6^2+2*\sqrt{(s_1^2+s_4^2+s_6^2)*(s_2^2+s_3^2+s_5^2)}$

Common terms are eliminated, and the square of the remaining terms are taken for both cases.

Table A.32. Comparison of Case-6 & Case-9 - Step 2 (Theorem 4.3).

Case Number	**Compared Value**
6	$(s_1*s_2)^2+(s_1*s_4)^2+(s_1*s_6)^2+(s_2*s_3)^2+(s_3*s_4)^2+(s_3*s_6)^2+(s_2*s_5)^2+(s_4*s_5)^2+(s_5*s_6)^2$
9	$(s_1*s_2)^2+(s_1*s_3)^2+(s_1*s_5)^2+(s_2*s_4)^2+(s_3*s_4)^2+(s_4*s_5)^2+(s_2*s_6)^2+(s_3*s_6)^2+(s_5*s_6)^2$

Common terms are eliminated, and $(s_1*s_4)^2+(s_1*s_6)^2+(s_2*s_3)^2+(s_2*s_5)^2$ is subtracted from both cases.

Table A.33. Comparison of Case-6 & Case-9 - Step 3 (Theorem 4.3).

Case Number	**Compared Value**
6	0
9	$(s_2^2-s_1^2)*(-s_3^2+s_4^2+s_6^2-s_5^2)$

According to the sum of remaining values, Case-9 takes a positive value. Since the objective value of Case-9 is larger, it is eliminated. Case-6 provides the minimum objective value, and the sequence of surgeries in Case-6 gives the optimal sequence of surgeries. In the optimal sequence, surgery 1 precedes surgery 3 and surgery 3 precedes surgery 5 in OR-1, surgery 2 precedes surgery 4 and surgery 4 precedes surgery 6 in OR-2. □

APPENDIX B: RESULTS OF SAA METHOD FOR m=1 & N=2

B.1. Effect of The Mean of Surgery Durations

Table B.1. Mean of the surgery duration of surgery 1 vs. PI.

Mean of Surgery Duration of Surgery 1	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.5	0.00%	0.03%	0.36%	2.00%	2.16%	43.27%
1	0.00%	0.04%	0.45%	2.34%	2.61%	31.83%
2	0.00%	0.03%	0.36%	1.98%	2.16%	24.31%

Table B.2. Mean of the surgery duration of surgery 2 vs. PI.

Mean of Surgery Duration of Surgery 2	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.5	0.00%	0.03%	0.36%	2.01%	2.20%	26.22%
1	0.00%	0.04%	0.45%	2.32%	2.57%	43.27%
2	0.00%	0.04%	0.36%	2.00%	2.20%	28.38%

B.2. Effect of Unit Costs

Table B.3. α_2 vs. PI, α_1=0.1.

Unit Cost of Waiting Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.06%	0.73%	3.24%	2.97%	43.27%
0.2	0.00%	0.04%	0.29%	2.50%	2.92%	16.90%
0.3	0.00%	0.04%	0.35%	2.02%	2.53%	19.36%
0.4	0.00%	0.04%	0.38%	1.98%	2.22%	20.91%
0.5	0.00%	0.05%	0.45%	2.23%	2.50%	28.38%
0.6	0.00%	0.05%	0.49%	2.40%	2.78%	21.48%
0.7	0.00%	0.05%	0.57%	2.80%	3.31%	25.64%
0.8	0.00%	0.06%	0.65%	3.42%	3.84%	26.22%

Table B.4. α_2 vs. PI, α_1=0.2.

Unit Cost of Waiting Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.03%	0.34%	2.56%	2.41%	23.01%
0.2	0.00%	0.03%	0.31%	1.88%	2.30%	16.91%
0.3	0.00%	0.03%	0.36%	1.70%	1.93%	17.73%
0.4	0.00%	0.04%	0.39%	1.98%	2.08%	19.48%
0.5	0.00%	0.04%	0.41%	2.24%	2.39%	18.66%
0.6	0.00%	0.04%	0.54%	2.61%	2.88%	20.66%
0.7	0.00%	0.06%	0.64%	3.26%	3.51%	22.92%

Table B.5. α_2 vs. PI, α_1=0.3.

Unit Cost of Waiting Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.03%	0.25%	2.12%	2.50%	16.82%
0.2	0.00%	0.03%	0.32%	1.57%	1.71%	19.16%
0.3	0.00%	0.03%	0.37%	1.74%	1.82%	19.51%
0.4	0.00%	0.04%	0.43%	1.91%	2.09%	16.11%
0.5	0.00%	0.04%	0.51%	2.41%	2.57%	20.77%
0.6	0.00%	0.06%	0.68%	3.04%	3.42%	24.31%

Table B.6. α_2 vs. PI, α_1=0.4.

Unit Cost of Waiting Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.03%	0.29%	1.87%	2.17%	12.86%
0.2	0.00%	0.03%	0.26%	1.44%	1.57%	13.06%
0.3	0.00%	0.03%	0.34%	1.69%	1.86%	14.54%
0.4	0.00%	0.04%	0.42%	2.04%	2.31%	15.29%
0.5	0.00%	0.05%	0.51%	2.59%	3.04%	18.38%

Table B.7. α_2 vs. PI, α_1=0.5.

Unit Cost of Waiting Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.02%	0.28%	1.54%	1.81%	11.64%
0.2	0.00%	0.03%	0.27%	1.51%	1.67%	12.84%
0.3	0.00%	0.03%	0.39%	1.85%	1.94%	14.00%
0.4	0.00%	0.05%	0.52%	2.40%	2.71%	17.05%

Table B.8. α_2 vs. PI, α_1=0.6.

Unit Cost of Waiting Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.03%	0.27%	1.29%	1.41%	12.17%
0.2	0.00%	0.03%	0.33%	1.61%	1.66%	12.07%
0.3	0.00%	0.05%	0.45%	2.16%	2.25%	14.11%

Table B.9. α_2 vs. PI, α_1=0.7.

Unit Cost of Waiting Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.02%	0.24%	1.19%	1.25%	10.59%
0.2	0.00%	0.04%	0.35%	1.78%	1.78%	16.07%

APPENDIX C: RESULTS OF MYOPIC HEURISTIC FOR m=1 & N=2

C.1. Effect of The Mean of Surgery Durations

Table C.1. Mean of the surgery duration of surgery 1 vs. $C(Myp)$, N=2.

Mean of Surgery Duration of Surgery 1	Min	Lower Quartile	Median	Mean	Upper Quartile	Max
0.5	0.00%	0.54%	1.80%	3.32%	4.56%	26.60%
1	0.00%	0.78%	2.40%	3.92%	5.72%	27.24%
2	0.00%	0.60%	1.88%	3.46%	4.74%	27.54%

Table C.2. Mean of the surgery duration of surgery 2 vs. $C(Myp)$, N=2.

Mean of Surgery Duration of Surgery 2	Min	Lower Quartile	Median	Mean	Upper Quartile	Max
0.5	0.00%	0.55%	1.78%	3.33%	4.54%	26.60%
1	0.00%	0.78%	2.40%	3.91%	5.76%	27.24%
2	0.00%	0.59%	1.87%	3.46%	4.77%	27.54%

C.2. Effect of The Coefficient of Variation of Surgery Durations

Table C.3. CV of the surgery duration of surgery 1 vs. $C(Myp)$, N=2.

CV of Surgery Duration of Surgery 1	Min	Lower Quartile	Median	Mean	Upper Quartile	Max
0.4	0.00%	0.50%	1.80%	3.55%	5.06%	27.54%
0.8	0.00%	0.71%	2.08%	3.64%	5.27%	24.63%
1.2	0.00%	0.71%	2.07%	3.51%	5.04%	25.50%

Table C.4. CV of the surgery duration of surgery 2 vs. $C(Myp)$, N=2.

CV of Surgery Duration of Surgery 2	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.4	0.00%	0.50%	1.80%	3.55%	5.03%	27.54%
0.8	0.00%	0.71%	2.08%	3.64%	5.27%	24.63%
1.2	0.00%	0.71%	2.09%	3.51%	5.02%	25.81%

C.3. Effect of Unit Costs

Table C.5. α_1 vs. $C(Myp)$, α_2=0.1.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.01%	1.60%	5.50%	6.59%	10.88%	21.70%
0.2	0.14%	3.99%	7.76%	8.89%	13.66%	26.69%
0.3	0.29%	5.61%	9.42%	9.92%	14.35%	27.54%
0.4	0.56%	6.55%	10.24%	10.27%	14.08%	25.85%
0.5	1.09%	6.20%	9.90%	9.98%	13.43%	23.43%
0.6	1.11%	6.13%	9.04%	9.14%	12.17%	20.26%
0.7	1.57%	5.89%	8.03%	7.94%	10.29%	14.89%
0.8	2.01%	4.82%	5.66%	5.73%	6.78%	12.25%

Table C.6. α_1 vs. $C(Myp)$, α_2=0.2.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.72%	2.41%	3.36%	5.40%	14.60%
0.2	0.05%	2.10%	3.96%	4.63%	7.30%	12.84%
0.3	0.11%	2.60%	4.77%	5.27%	7.74%	14.60%
0.4	0.36%	3.12%	5.37%	5.56%	7.78%	13.40%
0.5	0.51%	3.42%	5.28%	5.37%	7.31%	11.21%
0.6	0.86%	3.44%	4.71%	4.60%	5.88%	7.95%
0.7	1.04%	2.65%	3.05%	3.02%	3.46%	4.50%

Table C.7. α_1 vs. $C(Myp)$, α_2=0.3.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.35%	1.21%	1.72%	2.73%	7.48%
0.2	0.02%	0.99%	2.21%	2.60%	4.05%	7.33%
0.3	0.09%	1.55%	2.96%	3.16%	4.72%	7.96%
0.4	0.28%	1.97%	3.33%	3.27%	4.50%	6.97%
0.5	0.41%	2.10%	2.95%	2.84%	3.61%	4.98%
0.6	0.61%	1.61%	1.85%	1.80%	2.06%	2.73%

Table C.8. α_1 vs. $C(Myp)$, α_2=0.4.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.20%	0.66%	0.96%	1.54%	3.85%
0.2	0.00%	0.69%	1.43%	1.65%	2.60%	4.67%
0.3	0.07%	1.06%	1.90%	1.92%	2.72%	4.39%
0.4	0.24%	1.26%	1.84%	1.75%	2.24%	3.08%
0.5	0.36%	1.01%	1.15%	1.12%	1.26%	1.66%

Table C.9. α_1 vs. $C(Myp)$, α_2=0.5.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.15%	0.41%	0.61%	0.95%	2.52%
0.2	0.01%	0.50%	0.94%	1.03%	1.56%	2.53%
0.3	0.09%	0.69%	1.10%	1.06%	1.43%	2.00%
0.4	0.17%	0.62%	0.72%	0.69%	0.81%	1.06%

Table C.10. α_1 vs. $C(Myp)$, α_2=0.6.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.11%	0.30%	0.39%	0.60%	1.60%
0.2	0.02%	0.34%	0.56%	0.58%	0.82%	1.34%
0.3	0.07%	0.35%	0.43%	0.42%	0.50%	0.68%

Table C.11. α_1 vs. $C(Myp)$, α_2=0.7.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.09%	0.19%	0.23%	0.36%	0.92%
0.2	0.02%	0.17%	0.24%	0.23%	0.29%	0.49%

Table C.12. α_1 vs. $C(Myp)$, α_2=0.8.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.05%	0.09%	0.10%	0.13%	0.33%

APPENDIX D: RESULTS OF VETERAN'S HEURISTIC FOR m=1 & N=2

D.1. Effect of The Mean of Surgery Durations

Table D.1. Mean of the surgery duration of surgery 1 vs. $C(V)$, N=2.

Mean of Surgery Duration of Surgery 1	Min	Lower Quartile	Median	Mean	Upper Quartile	Max
0.5	0.00%	0.22%	0.67%	1.80%	1.86%	31.60%
1	0.01%	0.36%	0.91%	2.17%	2.31%	31.86%
2	0.00%	0.24%	0.73%	1.88%	1.95%	32.22%

Table D.2. Mean of the surgery duration of surgery 2 vs. $C(V)$, N=2.

Mean of Surgery Duration of Surgery 2	Min	Lower Quartile	Median	Mean	Upper Quartile	Max
0.5	0.00%	0.22%	0.66%	1.80%	1.85%	31.60%
1	0.01%	0.35%	0.91%	2.17%	2.31%	31.41%
2	0.00%	0.23%	0.72%	1.88%	1.94%	32.22%

D.2. Effect of Unit Costs

Table D.3. α_1 vs. $C(V)$, α_2=0.1.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.20%	0.67%	1.37%	1.71%	12.00%
0.2	0.00%	0.14%	0.42%	0.96%	1.13%	9.37%
0.3	0.00%	0.13%	0.38%	0.77%	0.93%	7.48%
0.4	0.00%	0.12%	0.33%	0.63%	0.75%	6.26%
0.5	0.00%	0.10%	0.29%	0.50%	0.61%	5.07%
0.6	0.00%	0.07%	0.18%	0.27%	0.39%	1.54%
0.7	0.00%	0.06%	0.14%	0.20%	0.30%	1.21%
0.8	0.00%	0.04%	0.09%	0.14%	0.22%	0.75%

Table D.4. α_1 vs. $C(V)$, α_2=0.2.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.01%	0.35%	0.76%	1.30%	1.57%	12.18%
0.2	0.00%	0.26%	0.56%	0.97%	1.23%	9.06%
0.3	0.01%	0.18%	0.42%	0.63%	0.92%	5.11%
0.4	0.00%	0.15%	0.34%	0.49%	0.73%	2.26%
0.5	0.01%	0.13%	0.30%	0.44%	0.63%	2.13%
0.6	0.00%	0.11%	0.26%	0.37%	0.55%	1.66%
0.7	0.00%	0.10%	0.20%	0.28%	0.42%	1.29%

Table D.5. α_1 vs. $C(V)$, α_2=0.3.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.03%	0.65%	1.24%	1.45%	1.91%	6.80%
0.2	0.02%	0.32%	0.67%	0.85%	1.18%	3.27%
0.3	0.02%	0.25%	0.52%	0.70%	1.05%	2.52%
0.4	0.02%	0.23%	0.47%	0.63%	0.93%	2.43%
0.5	0.02%	0.21%	0.45%	0.58%	0.84%	2.47%
0.6	0.03%	0.21%	0.44%	0.62%	0.88%	3.39%

Table D.6. α_1 vs. $C(V)$, α_2=0.4.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.07%	0.99%	1.89%	2.07%	2.78%	7.31%
0.2	0.02%	0.57%	1.09%	1.28%	1.72%	4.58%
0.3	0.04%	0.45%	0.86%	1.06%	1.52%	3.90%
0.4	0.06%	0.42%	0.77%	1.01%	1.40%	4.01%
0.5	0.08%	0.45%	0.85%	1.20%	1.66%	5.86%

Table D.7. α_1 vs. $C(V)$, α_2=0.5.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.13%	1.52%	2.97%	3.33%	4.45%	12.40%
0.2	0.10%	0.96%	1.80%	2.19%	2.94%	7.90%
0.3	0.14%	0.84%	1.51%	1.94%	2.77%	7.13%
0.4	0.15%	0.85%	1.56%	2.11%	3.09%	6.97%

Table D.8. α_1 vs. $C(V)$, α_2=0.6.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.28%	2.44%	4.72%	5.43%	7.45%	19.56%
0.2	0.22%	1.66%	3.19%	3.95%	5.75%	13.83%
0.3	0.32%	1.58%	2.94%	3.53%	5.17%	9.90%

Table D.9. α_1 vs. $C(V)$, α_2=0.7.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.51%	3.74%	7.32%	9.18%	13.12%	31.86%
0.2	0.48%	2.98%	5.53%	6.04%	8.73%	16.41%

Table D.10. α_1 vs. $C(V)$, α_2=0.8.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.81%	6.23%	11.44%	11.76%	15.66%	32.22%

APPENDIX E: RESULTS OF MODIFIED MYOPIC FOR m=1 & N=2

E.1. Effect of The Mean of Surgery Durations

Table E.1. Mean of the surgery duration of surgery 1 vs. $C(Mod)$, N=2.

Mean of Surgery Duration of Surgery 1	Min	Lower Quartile	Median	Mean	Upper Quartile	Max
0.5	0.00%	0.07%	0.31%	1.10%	1.06%	36.01%
1	0.00%	0.18%	0.50%	1.31%	1.36%	37.18%
2	0.00%	0.08%	0.34%	1.11%	1.07%	36.54%

Table E.2. Mean of the surgery duration of surgery 2 vs. $C(Mod)$, N=2.

Mean of Surgery Duration of Surgery 2	Min	Lower Quartile	Median	Mean	Upper Quartile	Max
0.5	0.00%	0.07%	0.31%	1.11%	1.06%	36.01%
1	0.00%	0.17%	0.51%	1.31%	1.35%	37.18%
2	0.00%	0.08%	0.34%	1.10%	1.08%	36.54%

E.2. Effect of Unit Costs

Table E.3. α_1 vs. $C(Mod)$, α_2=0.1.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.34%	1.52%	2.70%	3.50%	21.47%
0.2	0.00%	0.35%	0.88%	1.59%	2.10%	12.75%
0.3	0.00%	0.24%	0.58%	0.97%	1.19%	7.79%
0.4	0.00%	0.15%	0.42%	0.78%	0.83%	8.64%
0.5	0.00%	0.13%	0.41%	0.81%	1.15%	7.71%
0.6	0.00%	0.17%	0.58%	1.11%	1.73%	6.76%
0.7	0.00%	0.47%	1.51%	2.72%	3.93%	15.52%
0.8	0.02%	1.63%	4.96%	7.02%	9.35%	37.18%

Table E.4. α_1 vs. $C(Mod)$, α_2=0.2.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.15%	0.74%	1.52%	1.98%	14.57%
0.2	0.00%	0.27%	0.62%	1.11%	1.60%	9.04%
0.3	0.00%	0.18%	0.43%	0.63%	0.90%	5.05%
0.4	0.00%	0.12%	0.33%	0.50%	0.77%	2.25%
0.5	0.00%	0.15%	0.45%	0.79%	1.25%	4.48%
0.6	0.01%	0.31%	1.03%	1.80%	2.64%	10.62%
0.7	0.02%	1.12%	3.15%	4.61%	6.39%	24.50%

Table E.5. α_1 vs. $C(Mod)$, α_2=0.3.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.08%	0.39%	0.77%	1.13%	7.04%
0.2	0.00%	0.12%	0.37%	0.56%	0.91%	3.02%
0.3	0.00%	0.12%	0.31%	0.46%	0.72%	1.92%
0.4	0.00%	0.12%	0.36%	0.57%	0.89%	3.21%
0.5	0.00%	0.21%	0.69%	1.18%	1.71%	7.12%
0.6	0.01%	0.70%	1.95%	2.98%	4.21%	16.26%

Table E.6. α_1 vs. $C(Mod)$, α_2=0.4.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.04%	0.21%	0.38%	0.58%	2.80%
0.2	0.00%	0.09%	0.25%	0.38%	0.59%	1.69%
0.3	0.00%	0.10%	0.25%	0.39%	0.60%	2.11%
0.4	0.00%	0.14%	0.40%	0.70%	1.03%	4.50%
0.5	0.00%	0.39%	1.16%	1.86%	2.66%	10.31%

Table E.7. α_1 vs. $C(Mod)$, α_2=0.5.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.03%	0.13%	0.23%	0.36%	1.26%
0.2	0.00%	0.07%	0.19%	0.26%	0.40%	1.07%
0.3	0.00%	0.07%	0.23%	0.40%	0.60%	2.56%
0.4	0.00%	0.22%	0.65%	1.07%	1.58%	6.54%

Table E.8. α_1 vs. $C(Mod)$, α_2=0.6.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.02%	0.09%	0.14%	0.21%	0.87%
0.2	0.00%	0.04%	0.13%	0.19%	0.29%	1.14%
0.3	0.00%	0.10%	0.30%	0.54%	0.82%	3.65%

Table E.9. α_1 vs. $C(Mod)$, α_2=0.7.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.02%	0.05%	0.09%	0.13%	0.64%
0.2	0.00%	0.04%	0.12%	0.22%	0.33%	1.51%

Table E.10. α_1 vs. $C(Mod)$, α_2=0.8.

Unit Cost of Idle Time	**Min**	**Lower Quartile**	**Median**	**Mean**	**Upper Quartile**	**Max**
0.1	0.00%	0.01%	0.03%	0.05%	0.08%	0.40%