PRICING IN SERVICE SYSTEMS WITH STRATEGIC CUSTOMERS

by

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ABSTRACT

PRICING IN SERVICE SYSTEMS WITH STRATEGIC CUSTOMERS

In this thesis, we examine the pricing decision process in service systems with strategic customers. We present three systems throughout the study. In the first system, priority classes are offered to the customers and price level for every priority class is determined to maximize profit. Two models are created and comparisons are done in order to determine how parameter values affect pricing decision and priority class choices. The second system consists of a single server and homogeneous customers but different delay information levels exist. Three information levels are examined and effects of delay information level on pricing decision and customers' behavior are investigated. In the third system, there exists randomness in service valuation and waiting cost. We discuss the cost of uncertainty in service systems by regarding probabilistic parameter values.

ÖZET

STRATEJİK MÜŞTERİLERİN BULUNDUĞU SERVİS SİSTEMLERİNDE FİYATLANDIRMA

Bu tezde,stratejik müşterilere sahip servis sistemlerindeki fiyatlandırma karar süreci incelenmiştir. Çalışma boyunca üç sistem sunulmuştur. Birinci sistemde,müşterilere öncelikli sınıflar sunulmuş ve bu öncelikli sınıflar için karı maksimize eden fiyat seviyelere belirlenmiştir. İki model yaratılmış ve parametre değerlerinin karar fiyatına ve öncelikli sınıf seçimlerine etkisini belirlemek amaçlı karşılaştırmalar yapılmıştır. İkinci sistem tek bir sunucu ve aynı özellikli müşterilerden oluşmuştur. Fakat, farklı bekleme süreleri bilgisi mevcuttur. Üç seviyede bekleme süresi bilgisi değerlendirilmiştir ve bekleme süresi bilgisinin karar fiyata ve müşteri davranışlarına etkisi araştırılmıştır. Üçüncü sistemde, servise verilen değer ve bekleme süresi maliyeti üzerinde belirsizlik vardır. Servis sistemlerindeki bu belirsizliğin maliyeti olasılıksal parametre değerleri düşünülerek işlenmiştir.

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c	Unit time waiting cost of customers
G	Profit of service providers
k	Revenue charged from all customers
m	Number of servers
p	Price charged from customers for the service they receive
p_i	Price charged from customers with Type i priority class
q	Probability of choosing Type 1 priority class
R	Service valuation of customers
R_i	Service valuation for Type i priority class
S	Willingness to wait of customers
S_i	Time spent in the server by the customers choosing Type i priority class
U	Utility of customers
U_i	Utility of customers choosing Type 1 priority class
v	Additional cost for Type 1 priority class
W	Customer waiting time in the service system
W_i	Customer waiting time with Type i priority class in the service system
Λ	Customer arriving rate to the service system
λ	Effective customer arriving rate to the service system
λ_1	Customer effective arrival rate to Type 1 priority class
λ_2	Customer effective arrival rate to Type 2 priority class
μ	Service rate in the service system
μ_c	Expectation of unit time waiting cost
μ_R	Expectation of service valuation of customers
μ_u	Expectation of R-cE[W
Π	Probability that all servers are busy

ρ	Utilization of servers
ρ_1	Utilization of servers by customers with Type 1 priority class
ρ_2	Utilization of servers by customers with Type 2 priority class
σ_c^2	Variance of unit time waiting cost
σ_R^2	Variance of service valuation of customers
σ_u^2	Variance of R-cE[W]

1. INTRODUCTION

In recent years, the importance of pricing decision process in various service systems has become so apparent that more research on service systems has been done in the literature. The intention of customers and service providers in service systems are different. While a customer seeks to maximize his individual utility, a server provider operates to maximize his profit. Therefore, there is a need for an equilibrium behavior pattern for these different behavior types.

Service systems are generally queuing type systems because there is limited number of service providers but there are many customers. Therefore, many service systems include queues that brings the displeasure of waiting in the queue for customers. Because waiting sensitivity of customers are different from each other, service providers can charge higher price from the customers who are less tolerable to wait in exchange for shorter waiting time. In most of the priority queue literature, the greatest problem is faced by the service provider that is how to manage service differentiation problem and how to create an optimal price and priority menu in order to maximize his own profit.

In another area of research, the effect of delay information on pricing policy is studied. Information sharing between customers and service providers has always had considerable effects on pricing processes. Since recent developments in technology make information sharing between two parts more reliable and easier, a question has emerged that is how information sharing level between customers and service providers affect their behaviors and strategies.

Research on pricing strategies in service systems has always been widespread. However, most studies regard the models in which service value received by customers are known. A new research area has been quite popular recently that cover the problems with unknown service valuations and waiting sensitivity.

These research areas are the motivation for our study. In this thesis, we study the pricing decision process in service systems and the factors affecting the process. In the first part, we consider an M/M/m service system where there are priority classes. Our first models are different from the literature in that the market is homogeneous and the customers are not differentiated out of the service system. Therefore, there is no difference among their waiting sensitivities and waiting cost for them are same. However, they have to choose their priority classes within the service system that affects their service rewards.

We offer a base model in order to maximize the service provider's profit. However, we need the expected waiting time for different priority classes in order to generate profit function and find the optimal price. Kella and Yechiali (1985) have previously calculated the waiting time for different priority classes so we benefit from the study done by them. In addition, we present another model by changing our profit function that is alternative for our base model. The alternative profit function has different properties for different parameter values. Therefore, we examine all possible cases and determine corresponding optimal price decisions. Furthermore, we generate numerical results demonstrating which factors affect priority decisions of customers and how they are correlated.

Another area of interest in this thesis is how delay information level between the service provider and customers affect the service provider's strategy and customers' behaviors. We have offered three delay information levels and generated their respective profit functions. Furthermore, we observe that two of them are quite similar to each other so we only determine the optimal price decisions for our two different information levels. Because determination of optimal pricing policy is not possible through mathematical calculations, we compute our numerical results by using matlab programs.

Our numerical results indicate that different delay information creates different price decisions and yields different profit for service provide. Moreover, we also calculate customers' expected utility in order to observe how customers are affected from sharing delay information. Comparison of the results shows us that delay information affect the service provider and customers in opposite directions.

We also investigate service systems where each customer has own service valuation and a unique waiting cost. In order to generate our model, we suppose both parameter have log normal distribution. This assumption enable us to determine optimal price level and compute maximum profit through matlab programs.

We compute numerical results for different mean and variance values that indicate us how optimal price is affected by uncertainty and expected values. Then, we determine optimal price levels by ignoring variances and compute corresponding profit. The comparison of this profit level with maximum profit level gives us the cost of ignoring uncertainty in service systems.

This thesis is structured as follows. In Chapter 2, we review the literature on the price and demand interaction in the service systems with multiple servers and multiple customer classes. We also review the literature on delay information effects on pricing decision policies and customer strategies. There are also some paper in this chapter regarding uncertain service valuation and delay time cost. In Chapter 3, we present our M/M/m service system models that offer priority classes and regards how pricing strategy and priority classes are correlated. In Chapter 4, we propose three different models with different information levels and compare their corresponding pricing decisions and customers behaviors. Therefore, we calculate the effective arrival rate, maximum profit levels and expected customer utility for different information levels. In Chapter 5, a service system is modeled for which revenue and penalty cost of customers are random parameters. Moreover, we present how uncertainty affect pricing decisions and optimal policies. Finally, conclusions are drawn and directions for future work are offered in Chapter 6.

2. LITERATURE REVIEW

The focus of this study is on the interaction between price and demand in service systems. In this chapter a brief literature survey is provided on this topic. The papers are examined in three groups. First group includes the papers that study the price and demand interaction in the service systems with multiple customer classes. In the second group, the papers examine the effects of different delay information levels on pricing decision. Finally, the third group of papers regard the systems in which customer service utility information and delay time cost is unknown (Brill and Posner , 1977)

2.1. Service Systems with Multiple Customer Classes

In this thesis, we firstly examine pricing decision of service systems where different customer classes exist with different priority service level. Queuing models with multiple priority classes have always been an attractive research area in the literature. Therefore, in this section we only present some papers that are substantially relevant to our study.

Adiri and Yechiali (1974) studies a basic service system modeled as M/M/1 system with separate queues and queues have priorities on one another. They assume that customers entering to the system have full information about the system and may choose to leave the system without receiving service. Different cases are considered including monopoly and non monopoly systems with both preemptive and non preemptive priority policies. They calculate optimal control limits of priority-pricing policies and leaving policies for non monopoly systems. They determine these control limits first for two priorities and for more priorities. Then, the control limits for non monopoly systems are properly transformed for monopoly systems with basic changes necessitated by steady state condition.

Ha (1998) models another service system in which customers are able to choose their service rates based on their waiting sensitivity. He states that congestion externalities prevent the service system from reaching its optimality. Therefore, he offers a incentive compatible pricing menu for customers which guarantees customers to choose their actual service rate. He also considers the optimal arrival rate of customers while determining the optimal pricing menu which is similar to our model we dealt with in 3.

Maglaras and Zeevi (2005) also study a model of service systems delivering different types of services to heterogeneous customers. Their objective is to design such service systems so that the service providers have best possible revenue. In their model, customers have to make their own choices by considering both price and crowding effect. The crowding effect is especially important for customers because the customers choosing to have best-effort type service can only be served by remaining capacity of the system. Because there are a lot of decision variables in the model, they follow approximations and make relaxations on objective function rather than exact solutions. Then, they show that the results of deterministic relaxation yield optimal policy for the system with high congestion and high congestion leads to increase in revenues.

Guler*et al.* (2014) state a price menu for multiple customers classes which have different sensitivities for waiting and service value. Because the system in this study is modeled as an inventory system with M/M/1, different stock policies are considered. Therefore, they make decisions on optimal priority rule, optimal inventory policy and optimal pricing. They derive simple formulations for single customer class case through continuous approximation. They also make conclusions on two customer classes case through numerical examples.

Zhou *et al.* (2014) analyze pricing decision of service companies in order to maximize their revenues in presence of customer heterogeneity and in absence of price discrimination. They discover that both properties and potential of each customer group are determining factors in pricing decision. Therefore, the service companies may prefer to serve either one class of customers or multiple classes according to potential in each customer groups. Different scenarios are regarded for two customer classes and optimal pricing policy is determined for different parameters. They also result that this study may be extended for more customer classes in case of dominating customer case.

Niels (1972) studies multi server queuing systems in which it is assumed that the net benefit of an arriving customer is inversely proportional to the number of customers exist in the system. Therefore, there is a threshold value of the number of customers in the system above which coming customers do not enter to the system if balking is possible. If there is a capacity limit on the system less than threshold value, then it is proven that expected net benefit of customers increases. He also offers to charge a fixed price from customers entering to the system in order to satisfy both social and economic optimality.

Printezis and Burnetas (2008) analyze an M/M/m service system which offer identical service to their customers. In their study, there are no differences among customers but they are allowed to observe the system state before entering. An option is offered at a fixed price which allow customers to by pass the waiting. Their aim is to determine the optimal price for an option in order to maximize service provider's revenue. They solve their problem by modeling a two stage problem between service provider and customers.

Gilland and Warsing (2009) state the effects of priority pricing on customer delay costs. They discover that priority price of each customer class manufacturer charges in order to maximize its revenue also minimizes the total delay time of each customer class. Therefore, they conclude that both the manufacturer and the customers reach their best if they behave independently. Another result they attain is that an increase in the number of customer classes leads to an increase in manufacturer revenue at a decreasing rate.

Doroudi *et al.* (2013) also study service systems with multi class customer types. They assume that delay sensitivity of each customer is different and their delay sensitivity is proportional to their service valuation. Then, they also offer price menus for different customer classes by assuming that customer service valuations coming from exponential, uniform and pareto distributions. They improve previous studies by extending two customer classes to multiple classes and computing closed form price and priority allocations. Comparison of their offered prices with ideal prices customers reveal illustrates the effects of service valuation variability on prices and service providers' profit.

Afeche and Pavlin (2013) consider service systems in heterogeneous customer market. In order to extract more from impatient customers, they offer a price and lead time menu. In their model, they rank customer preferences according to lead times and determine how much they pay. They also investigate under which circumstances pooling of different customer types gives better results and whether implementation of artificial delay is a good choice for service providers. Finally, they study the correlation between price lead time menu and capacity of service system with market characteristics.

Chen and Wan (2003) studies price competition between two different make to order firms which can be considered as multi server queuing systems. The service capacities of these two firms are different as well as their service valuations and their waiting costs. Therefore, their model is similar to service systems with multiple customer classes in many aspects. Their findings prove that the firm with faster service, greater service valuations and less waiting cost can mostly charge additional price similar to prioritized customer classes in service systems. Their contribution to literature is their results on how an increase or decrease in service capacity affect previous optimal price and how much.

Armony and Haviv (2003) also study competition between two service providers both of which offer identical service. The customers in their model make their choices according to their service price and delay time costs. There are two decision levels one of which is the decision of customers whether to enter the system or not and which service provider they choose. The second decision is the decision of service providers on what price they should charge. In this study, correlation between these two decision processes are analyzed. The difference of this study among literature is different waiting cost parameter for each customer classes.

Maglaras *et al.* (2013) model a service system with heterogeneous customers as a multi server queuing system. They made a deterministic relaxation on profit maximization problem which leads to an optimal priority rule and optimal price option for two customer classes case. One contribution of this study is on whether differentiated service offering is a reasonable choice based on customer demand elasticity. It offers artificial delay time for some customer classes in order to extract more from impatient customers. Then, these studies are extended for more than two customers classes. Finally, this study also makes contribution on topic of traffic analysis in queuing systems by indicating how operating regimes are emerged.

Hanbali *et al.* (2013) analyze the waiting time distributions of multiple customer classes in a non preemptive M/G/c through some approximations. The key point of this study is its aim to make an accurate estimate for the waiting times of multiple customer classes. Therefore, it requires the waiting time distribution of each customer classes as well as its expected waiting time. In order to find an appropriate waiting time distribution per customer class, the first two moments of conditional waiting time for every customer class are estimated. Finally, the study is extended for service time distributions in order make faster calculations.

The papers we cover in this section study mostly pricing decisions in service systems. They analyze the interactions between different decision parameters and variables in order to determine optimal pricing policy. Most of them offer optimal price level for two types of customer classes and some extend their studies for more customer classes types.

2.2. Service Systems with Different Delay Information Level

While modeling different service systems, share of delay information with customers is substantial decision for pricing process. Service providers may choose to inform customers about their delay time at different levels. Moreover, the papers presented in this section analyze pricing process in service systems with different delay information levels.

Naor (1969) is one of the first studying the regulation of queue size by an entering fee. He models an M/M/1 queuing system and assumes that an arriving customer can see the queue length which is determined by the strategy followed by customers. Therefore, the distribution of delay time in queue length is affected by customers strategy while it also affects customers' choice on joining the queue. He determines an entering price aiming revenue maximization and calculate the threshold value of queue length above which customers do not join to the queue. It is also possible to determine another entering price which is Pareto optimal for M/M/1 queuing models. When these two prices are compared with each others it is seen that they are equal to each other for most parameter values (Edelson and Hilderbrand , 1975).

Brill and Posner (1977) offer a new method in order to determine the stationary waiting time distribution in M/M/1 queues. Because delay time information is important for customers' decisions on joining a queue, determination of waiting time distribution is critical for service providers. Therefore, they introduce new methods in order generate closed form mathematical equations for waiting time distributions. They use Volterra integral equations as their starting points. They conclude that their results can also be extended for multi server queuing systems.

Stidham (1985) also studies a single server queuing systems. He present the difference between open loop control system and closed loop control system. In his study on open loop control system, he states that knowledge of customers on current state of queuing system creates a natural control mechanism for queuing systems.

Then he compares average benefits of queuing systems which are controlled either dynamically or statically.

Bell and Stidham (1983) analyze the joining decision of customers to a queuing system with K different servers. The difference of their study is their assumption on customers' knowledge about expected waiting time. In their study, customers do not observe the queue length instead they only know the service distribution of each server and per unit time waiting cost at every server. Under these conditions, they discover that arrival rate to each server is different from the optimal arrival rate under full information case. It is seen that customers overload the servers with greater service rate and smaller unit time waiting cost. Therefore, congestion level in the whole system increase and system optimality cannot be satisfied.

Deacon and Sonstelie (1985) interpret data obtained from a natural experiment in order to determine the cost of waiting time for customers. In the experiment, customers have two options which are either purchase their gasoline at a high price without waiting or buy it at a low price with waiting in the queue. The data is used to model the customer's choice of station which is also transformed into an economic model in order to estimate the value of waiting time in the queue. They conclude that the cost of waiting time cannot be determined accurately with current data but it is possible to calculate lower and upper bound on the cost of waiting time.

Hassin (1986) analyzes a queuing model in which customers' waiting time is regarded as a service quality measurement. In this model, service providers are able to hide the actual queue length information from customers but reveal the known distribution of waiting times. He searches in this study whether the hidden queue length information prevents social optimality. His results indicate that hidden waiting time information generally prevents society's benefit to reach optimality and it is never socially optimal in case that service providers' choose to share the hidden information with customers but it is not allowed.

Hui and Tse (1996) examine two different waiting information types on customers' choices which are waiting duration information and queuing information. An experiment is conducted as a 3 factorial design. They find that both information types have considerable effect on customers' decisions for intermediate and long waiting times but not short. Furthermore, waiting time information do not have an important impact on the acceptability of waiting but queuing do. Finally, they agree on that queuing information is preferable compared to waiting duration information.

Kumar *et al.* (1997) address the issue that how waiting time guarantees affect customers' waiting time experiences. In the model they develop, it is assumed that customers can observe the queue length and have estimations for their waiting times based on the observed queue length and estimated service rate. They determine customers' satisfaction by measuring their estimation for waiting time before and during their service. They conclude that whenever service times are less than customers' expectation customers' satisfaction increases and waiting time guarantee also increases satisfaction. However, in case that service times are greater than expectation customers with guaranteed waiting time have less satisfaction than the others.

Chen and Murray (2004) model a make to order monopoly firm in order to determine the price level it sets and the capacity level it chooses. Their study is different from other pricing studies due to the importance of delay. They state that the queue length is unobservable for customers in most cases. It is assumed that a customer cannot know the existing state of the system but knows the average of system. Therefore, all customers without difference play the same game simultaneously. Customers' choice depends on the price level, the reward received from firm and expected delay time in the queue. They also investigate the correlation between price and delay time because they both have direct relationship with customer demand.

Dobson and Pinker (2006) investigate the value of delay time information for customers. Since development in technologies makes information sharing with customers possible, it becomes a big question whether to share information with customers is beneficial for firms. Thus, they start their study by analyzing the factors affecting such a benefit and they derive the conditions under which sharing state dependent delay time information with customers is beneficial. Because the information level can be changed, they develop many scenarios. Their results show that sharing information is not always for the good of firms but it is beneficial whenever customers' waiting time sensitivity is more heterogeneous.

Whitt (1999) models two M/M/s/r queuing systems in which balking and reneging are possible. Then, he measures the effect of anticipated delay information on service system. In the first model, he assumes that customers have no information on current system state and he calculate the joining probability of coming customers to the queuing system. In the second model, customers know the number of people in the queue at their arrival time. They also know the remaining service time of the customers who are currently served. Then, their joining probabilities are also calculated. The results show us that reneging is not observed in model 2 since customers have waiting time information instead more balking are observed. There are more customers in the system for model 1 and customers are more likely to experience less waiting time in model 2.

Guo and Zipkin (2007) analyze single server queuing systems with different levels of delay information. In order to observe the effects of different delay information on customers' decisions and so the whole system, they introduce three levels of delay information which are no information, partial information and full information. Arriving customers use the given information for their decision of joining to queue or not. Moreover, they generate closed form joining probabilities for most cases and show that more information generally decrease throughput level. They also determine the sufficient conditions under which both the customers and the service provider have more benefits with more information. In other cases, it is seen that more information makes the system worse. Davidson (1988) addresses the question that how the information level of customers on prices affect their service providers' choice. He observes the system equilibrium state with different information levels. He states that if customers have no information about prices then all service providers set same prices. Moreover, he concludes that market segmentation exists effectively whenever customers have full information .

Papers stated in this section comprise service system models in which different levels of delay information exist. Because knowledge of delay information for customers are sometimes impossible or it is required in some other cases, researchers try to determine the effects of information level on service systems by generating different models and make analyses on them.

2.3. Uncertain Service Utility and Delay Cost

There are two important decision parameters determining price in service systems. These are service reward for customers and per unit time delay cost for customers. Moreover, there is also a recent approach regarding them as uncertain variables and studying their correlations.

Carmon *et al.* (1995) analyze the cost function of waiting time and its effect on service reward in their study because their study requires information about both cost of waiting and service reward. Their main aim is investigating the psychological cost of waiting time for service systems and they conclude that service reward and waiting cost are proportional to each other.

Veeraraghavan *et al.* (2011) study how customers make a selection between different service providers if their service value is unknown. Because service value is a variable in their study, they give defective information to customers about service providers' service value. Then, it is seen that more customers choose the service provider with greater service value. However, it causes its delay time to increase due to congestion. Therefore, they take the correlation between service value and delay cost into account in order to determine the system equilibrium.

Debo and Veeraraghavan (2014) study a single server queuing system where both service rate of the system and service value of customers are also unknown. They aim to determine joining equilibrium under such an uncertainty. They show that it is not possible to determine single threshold value for queue length under existence of uncertainty. There is still a threshold queue length if low and high service values are close to each other. Therefore, customers do not join the queue above this threshold level. Moreover, if there is a large gap between low and high service values, they offer a sputtering equilibrium. Under sputtering equilibrium, a lower threshold queue length exists and joining or balking decision are indifferent at this level. However, customers are more inclined to joining the queue as the queue length is move away from threshold level.

Larsen (1998) models an M/M/1 queuing system in which service reward has a specific distribution. He investigates the sensitivity of pricing decision on the variance of the reward distribution. In his study he assume that the reward received by customers is a random variable with know distribution which is uniform distribution in his case. Then he tests the sensitivity for two different models. In the first model customers only know the steady state queue information and in the second model they have more information about the queue length. His results indicate that smaller variance causes total welfare to decrease while it leads to both decrease and increase in service provider's profit depending on parameter values.

Cui *et al.* (2009) analyze queuing systems under information asymmetry. They offer a probabilistic admission control through which they charge greater price from customers with greater service reward. In their model, the service provider does not know service valuation of each customer so it is important to ensure customers to reveal their actual service valuations. They offer a service price menu which induce customers to make true choices. They also introduce different delay time sensitivity and service valuation of each customer in order to make price menu for different variable ranges.

Doroudi *et al.* (2013) make a recent study on service systems facing heterogeneous customers with unknown service valuations and delay costs. It is assumed that each customer has different service valuations coming from a continuous distribution and customer delay cost is proportional to their service valuations. In their study, they assume customer service valuations coming from exponential, uniform and Pareto distributions. They generate closed form formulations for revenues of M/M/1 queues and find numerical solutions for generalized cases.

In this section, we cover the papers that regard service utility and delay cost as variables. The papers of this section examine the interaction between service valuations and waiting costs and most of them derive conclusions on how these variables affect optimal pricing policies in service systems.

3. PRICING IN MULTI SERVER QUEUES WITH PRIORITIES - TWO CUSTOMER CLASSES

In this section, we present M/M/m service systems in which customers are not differentiated out of the service system but they have to choose one of the different priority classes offered by the system whenever they enter to the service system. In order to keep our system simple, we suppose that there are two types of priority classes called Type 1 and Type 2. Customers who choose Type 1 priority class has non preemptive priority over customers who choose Type 2 priority class. Here non preemptive priority means that when a customer choosing Type 2 class is in service and another customer choosing Type 1 class arrives to the system, the Type 2 customer is not interrupted and the server provider proceeds with Type 2 customer. In our model, the customers who prefer to be Type 1 rather than Type 2 have to pay greater price for their service. However, they gain more benefit from the service they received and are exposed to smaller waiting time. Therefore, choice of priority class is a decision parameter for customers. In our model, it is also assumed that there is no service failure in the system and balking is not possible so every customer coming to the service system have to choose either Type 1 class or Type 2 class.

In our model, we assume that customers arrive to the service system with Poisson distribution rate Λ. Moreover, the service time for all customers is identical without considering which priority classes they choose and it has an exponential distribution with rate μ. There are m servers identical to each other. In order to guarantee that the queue length for both priority classes do not explode, we suppose the following condition is satisfied

$$\frac{\Lambda}{m\mu} < 1. \tag{3.1}$$

Because balking is not possible and all customers arriving to the service system with rate Λ choose either Type 1 priority class or Type 2 priority class, the arrival rate of

Type 1 customers λ_1 and Type 2 customers λ_2 satisfies

$$\lambda_1 + \lambda_2 = \Lambda. \tag{3.2}$$

In order to determine the net utility function for both customer types, we need to calculate the expected waiting times for both customer types. Customer waiting times in M/M/c service systems has been previously calculated by using probabilistic equivalence between M/G/c queues with multiple servers' vacations and M/M/c queues for a single customer type (Kella and Yechiali , 1985). In our M/M/m service system model, we also calculate the expected customer waiting times but for different customer types with different priority levels which are Type 1 and Type 2.

Denoting the probability that all servers are busy by Π, it is computed as

$$\Pi = \frac{(\Lambda/\mu)^m}{m!(1-\rho)}[\sum_{i=0}^{m-1} \frac{(\Lambda/\mu)^i}{i!} + \frac{(\Lambda/\mu)^m}{m!(1-\rho)}]^{-1} \tag{3.3}$$

where

$$\rho = \frac{\lambda}{m\mu} \tag{3.4}$$

$$\rho_1 = \frac{\lambda_1}{m\mu}. \tag{3.5}$$

Then, the expected customer waiting times for Type 1 customers and Type 2 customers are calculated as

$$E[W_1] = \frac{\Pi}{m\mu(1-\rho_1)} \tag{3.6}$$

$$E[W_2] = \frac{\Pi}{m\mu(1-\rho_1)(1-\rho)}. \tag{3.7}$$

3.1. Base Model

In this model, a service system is considered which charges a service fee p_i whenever entering customer choose Type i priority class. It is previously stated that $p_1 > p_2$ because customers selecting Type 1 priority class have higher priority over customers choosing Type 2 priority class. Moreover, an arriving customer chooses Type 1 priority class with some probability which is equal to q. Therefore, arrival rate of customers with Type 1 priority class is Λq while arrival rate of customers with Type 2 priority class equals to Λ(1 - q).

The customer's choice of their priority class is based on his economic consideration which is non negative utility of the customer. Here, utility of a customer who chooses Type i priority class is calculated as

$$U_i(q) = R_i - p_i - cE[W_i]. \tag{3.8}$$

Because we assume that customers are not differentiated outside of the service system, the waiting cost for all customers are unique which is equal to c. However, the benefit customers received from the service system is different for different priority classes like the service price charged for different priority classes.

Then, we solve the equations stated below supposing that 0 <q<1 in order to determine the maximum service prices for which customers keep to choose both priority classes

$$U_1(q) = 0 \tag{3.9}$$

$$U_2(q) = 0. \tag{3.10}$$

When we make the utility for Type 1 priority class equal to zero, q becomes

$$q = (m\mu - \frac{c\Pi}{R_1 - p_1})\frac{1}{\Lambda}. \tag{3.11}$$

Because we assume that q is between 0 and 1, the following conditions can be generated

$$\frac{R_1 - p_1}{c} > \frac{\Pi}{m\mu} \tag{3.12}$$

$$\frac{R_1 - p_1}{c} < \frac{\Pi}{m\mu(1-\rho)}. \tag{3.13}$$

In order q making U_1 equal to 0 above also satisfies U_2(q)=0, we need

$$R_1 - p_1 > (1-\rho)(R_2 - p_2) \tag{3.14}$$

or

$$p_1 - (1-\rho)p_2 = R_1 - R_2(1-\rho). \tag{3.15}$$

The equations generated above show us that service prices of both priority classes are required to be linearly related to each other in order the entering probability for both priority classes to be positive.

Our fundamental aim is to determine the price levels maximizing the service provider's profit. Various profit functions can be generated for M/M/m service systems. In this section, we generate a profit function in which a unit cost v for each additional server unit is incurred and it is subtracted from the service provider's total revenue charged from customers choosing both Type 1 and Type 2 priority classes. Therefore, our profit function is computed as

$$G(p_1, p_2) = p_1\Lambda q + p_2\Lambda(1 - q) - vm. \tag{3.16}$$

The mathematical model, objective function of which is our profit function, is required to be solved in order to determine the service prices maximizing service provider's profit.

$$\begin{aligned}
&\max G(p_1, p_2) \\
&\text{s.t.} \\
&\qquad p_1 - (1-\rho)p_2 = R_1 - (1-\rho)R_2 \\
&\qquad \frac{\Pi}{m\mu} < \frac{R_1 - p_1}{c} < \frac{\Pi}{m\mu(1-\rho)} \\
&\qquad q = (m\mu - \frac{c\Pi}{R_1 - p_1})\frac{1}{\Lambda}
\end{aligned} \tag{3.17}$$

If we obtain G(p_1,p_2) as G(p_1) using 3.17, we can take the derivative of profit function with respect to p_1.

$$G(p_1) = p_1\Lambda(m\mu - \frac{c\Pi}{R_1 - p_1})\frac{1}{\Lambda} + (\frac{R_1 - p_1}{1-\rho} + R_2)\Lambda(1 - (m\mu + \frac{c\Pi}{R_1 - p_1})\frac{1}{\Lambda}) - vm \tag{3.18}$$

The first derivative of G(P_1) with respect to P_1 is

$$\frac{dG(p_1)}{dp_1} = m\mu + \frac{\Lambda}{1-\rho} - \frac{m\mu}{1-\rho} + \frac{1}{R_1 - p_1}(-c\Pi + \frac{c\Pi}{1-\rho}) \tag{3.19}$$

$$+\frac{1}{(R_1 - p_1)^2}(-p_1 c\Pi + \frac{p_1 c\Pi}{1-\rho} - \frac{R_1 c\Pi}{1-\rho} + R_2 c\Pi). \tag{3.20}$$

which is equal to

$$\frac{dG}{dp_1} = -\frac{c\Pi(R_1 - R_2)}{(R_1 - p_1)^2}. \tag{3.21}$$

Then, the second derivative is

$$\frac{\partial^2 G(p_1)}{\partial {p_1}^2} = \frac{2\Pi c}{(R_1 - p_1)^2}(\frac{R_2 - R_1}{R_1 - p_1}). \tag{3.22}$$

Because the second derivative is negative as long as $(R_2\text{-}R_1)/(R_1\text{-}p_1) < 0$, p_1 value that satisfies this condition and makes our profit function concave. There are two possible conditions satisfying the concavity condition. The first one requires R_2-R_1 to be negative while R_1-p_1 is positive and the second one requires R_2-R_1 to be positive while R_1-p_1 is negative. However, realization of the the second condition is not possible because it is previously assumed that customers choosing Type 2 priority class has gained less benefit from the service they received than Type 1 priority class so positive R_2-R_1 is not possible. Therefore, the only condition required to be satisfied for concavity is $p_1 < R_1$. If this condition is satisfied, p_1 value that makes the first derivative of our profit function zero gives the global maximum of our profit function.

However, the first derivative of G(p_1) with respect to p_1 is always negative because customers selecting Type 1 priority class has always gained greater benefit than customers with Type 2 priority class so R_1-R_2 is always positive. This indicates that our profit function is always decreasing so the minimum p_1 value satisfying the constraints in 3.17 becomes our optimal price for the service provider that maximizes his profit. The constraints in 3.17 set bounds to p_1 that is

$$R_1 - \frac{c\Pi}{m\mu(1-\rho)} < p_1 < R_1 - \frac{c\Pi}{m\mu}. \tag{3.23}$$

Therefore, profit maximizing price level for Type 1 priority class is equal to minimum possible value of p_1 that is approximately

$$p_1 = R_1 - \frac{c\Pi}{m\mu(1-\rho)}. \tag{3.24}$$

Therefore, p_2 is equal to

$$p_2 = R_2 - \frac{c\Pi}{m\mu(1-\rho)^2}. \tag{3.25}$$

These price levels make the selection probability of Type 1 priority class that is q equal to 1 so all customers entering to the service system choose Type 1 priority class with this profit function.

Therefore, we generate alternative profit functions for which existence of different priority classes improves the service provider's profit.

3.2. Alternative Models

In this subsection, an alternative model is offered in which every customer who chooses Type 1 priority class creates an additional cost v for the service system compared to the customers selecting Type 2 priority class. Because Type 1 priority class requires better service, it costs more for the service provider. Therefore, additional cost v is defined and it is assumed that it increases linearly with the expected number of customers choosing Type 1 priority class in the whole service system. Therefore, we are interested in the expected number of customers with Type 1 priority class in the queue and in the servers. It is natural that customers who choose Type 1 priority class have to pay greater price for the service they received than the other customers. Therefore, we assume that an additional service price p is charged from only those who choose Type 1 priority class. Here, p corresponds to p_1-p_2 in 3.1. Their benefit is also greater than the benefit with Type 2 priority class that is $R_1 > R_2$. Thus, we define the difference between them as R that is

$$R = R_1 - R_2. \tag{3.26}$$

The customers here choose their priority classes based on their economic regards again that is non negative utility difference between priority class 1 and priority class 2

$$\Delta U = R - p + c(E[W_2] - E[W_1]). \tag{3.27}$$

Because customers who enter to service systems choose Type 1 priority class as long as their utility is greater than or equal to 0, the best additional price p from the service provider's perspective can be found when the utility difference function is set to zero. Therefore, the relationship between p and λ_1 can be stated as

$$p = R + c(E[W_2] - E[W_1]) \tag{3.28}$$

where the difference between expected waiting time of customers with Type 1 and Type 2 priority classes is computed as

$$E[W_2] - E[W_1] = \frac{\rho}{(1-\rho)} \frac{\Pi}{m\mu(1-\rho_1)}. \tag{3.29}$$

Therefore, the additional price function is equal to

$$p = R + c\frac{\rho}{(1-\rho)} \frac{\Pi}{m\mu(1-\rho_1)}. \tag{3.30}$$

Because all customers entering to the service system pay a fixed price but customers selecting Type 1 priority class also pay an additional service price p and create an additional cost v, we can generate our profit function for this model as

$$G(\lambda_1) = k + P(\lambda_1)\lambda_1 - vE(N_1). \tag{3.31}$$

Here k is a fixed revenue obtained from all customers so it corresponds to Λp_2 in 3.1. Therefore, customers' priority class choices do not affect k in this model. Moreover, E[N_1] is the number of customers choosing Type 1 priority class in the whole service system so it is equal to the multiplication of arrival rate of customers with Type 1 priority and the total time spent by them in the whole system that is summation of time spent in the queue and time spent in the server. Thus, E[N_1] is computed as

$$E[N_1] = \lambda_1(E[W_1] + E[S_1]) = \lambda_1(\frac{\Pi}{m\mu(1-\rho_1)} + \frac{1}{\mu}). \tag{3.32}$$

The λ_1 in the above equation stands for the arrival rate of type 1 customers to the service system.

Moreover, our profit function can be expanded as

$$G(\lambda_1) = k + (R + c\frac{\rho}{(1-\rho)}\frac{\Pi}{m\mu(1-\rho_1)})\lambda_1 - v\frac{\Pi\rho_1}{(1-\rho_1)} + v\rho_1 m. \tag{3.33}$$

In order to determine the additional price level and type 1 customer arrival rate to the service system (λ_1) that maximizes our profit, the first derivative of G(λ_1) is taken with respect to λ_1

$$\frac{dG(\lambda_1)}{d\lambda_1} = R - \frac{v}{\mu} + \frac{1}{(1-\rho_1)^2}[c\frac{\rho}{(1-\rho)}\frac{\Pi}{m\mu} - v\frac{\Pi}{m\mu}]. \tag{3.34}$$

When the first derivative of G(λ_1) is made equal to zero, calculated λ_1 value gives us a critical point of our profit that is

$$\lambda_1 = m\mu(1 - \sqrt{\frac{v\frac{\Pi}{m\mu} - c\frac{\rho}{(1-\rho)}\frac{\Pi}{m\mu}}{R - \frac{1}{\mu}}}). \tag{3.35}$$

It also becomes our optimal λ_1 that maximizes profit if it satisfies two boundary conditions. The first of them guarantees that the queue length does not explode and it is stated below

$$0 < \rho_1 < 1. \tag{3.36}$$

It can be also shown as

$$0 < \sqrt{\frac{v\frac{\Pi}{m\mu} - c\frac{\rho}{(1-\rho)}\frac{\Pi}{m\mu}}{R - \frac{1}{\mu}}} < 1 \tag{3.37}$$

if we calculate ρ_1 by using the critical λ_1 value stated above. The second condition to check is whether critical λ_1 value is within the possible range of λ_1 that is $[0,\Lambda]$. It is also given below

$$0 < \lambda_1 < \Lambda. \tag{3.38}$$

The second condition can also be written as

$$1 - \frac{\Lambda}{m\mu} < \sqrt{\frac{v\frac{\Pi}{m\mu} - c\frac{\rho}{(1-\rho)}\frac{\Pi}{m\mu}}{R - \frac{1}{\mu}}} < 1 \tag{3.39}$$

by using the critical λ_1 value. Because the second condition guarantees the first condition in 3.37, we can proceed only with the second interval that says

$$(1 - \frac{\Lambda}{m\mu})^2(R - \frac{v}{\mu}) < v\frac{\Pi}{m\mu} - c\frac{\rho}{(1-\rho)}\frac{\Pi}{m\mu} < (R - \frac{v}{\mu}). \tag{3.40}$$

In order to determine whether our critical λ_1 value makes our profit function maximum, the second derivative of our profit function must be calculated. The λ_1 value that makes first derivative zero becomes our profit maximizing arrival rate as long as second derivative is negative and our profit function is concave. Checking out the second derivative,

$$\frac{\partial^2 G(\lambda_1)}{\partial {\lambda_1}^2} = (c\frac{\rho}{(1-\rho)}\frac{\Pi}{m\mu} - v\frac{\Pi}{m\mu})(-2)(-\frac{1}{m\mu})(\frac{1}{(1-\rho_1)^3}). \tag{3.41}$$

In order to check concavity of G(λ_1) we should check the sign of

$$[c\frac{\rho}{(1-\rho)}\frac{\Pi}{m\mu} - v\frac{\Pi}{m\mu}]. \tag{3.42}$$

Therefore, there are two possible cases for concavity of profit function that are analyzed below.

3.2.1. Case I: Positive Second Derivative of G(λ_1)

Our profit function becomes convex function and the following condition is satisfied

$$[c\frac{\rho}{(1-\rho)}\frac{\Pi}{m\mu} - v\frac{\Pi}{m\mu}] > 0. \tag{3.43}$$

Here, we do not have to make further analyses based on whether boundary conditions are satisfied or not because our critical λ_1 value does gives minimum profit. Therefore, we only consider the end points of our possible λ_1 range that is 0 and Λ. Either G(0) or G(Λ) gives us maximum achievable profit.

$$G(0) = k \tag{3.44}$$

$$G(\Lambda) = k + (R + c\frac{\rho}{(1-\rho)}\frac{\Pi}{m\mu(1-\rho)})\Lambda - v\frac{\Pi\rho}{(1-\rho)} + v\rho m \tag{3.45}$$

If G(Λ) is greater than G(0), our optimal λ_1 value becomes Λ otherwise it becomes 0. Therefore, it can be concluded that customers always choose same priority class as long as our profit function is convex.

3.2.2. Case II: Negative Second Derivative of G(λ_1)

Our profit function becomes concave and the following condition is satisfied

$$[c\frac{\rho}{(1-\rho)}\frac{\Pi}{m\mu} - v\frac{\Pi}{m\mu}] \leq 0. \tag{3.46}$$

We again make further analyses in order to determine whether boundary conditions are satisfied or not.

Case (a): Critical λ_1 value with a negative value

$$v\frac{\Pi}{m\mu} - c\frac{\rho}{(1-\rho)}\frac{\Pi}{m\mu} > (R - \frac{v}{\mu}) \tag{3.47}$$

If the boundary condition above is hold, λ_1 and ρ_1 have negative values that is not possible. Moreover, we need to check first order derivative again considering this boundary condition

$$\frac{dG(\lambda_1)}{d\lambda_1} = R - \frac{v}{\mu} + \frac{1}{(1-\rho_1)^2}[c\frac{\rho}{(1-\rho)}\frac{\Pi}{m\mu} - v\frac{\Pi}{m\mu}] < 0. \tag{3.48}$$

Therefore, it is concluded that G(λ_1) is always concave decreasing function so the

λ_1 value maximizing our profit function equals to possible minimum value of λ_1 that is equal to zero.

$$\lambda_1{}^* = 0 \tag{3.49}$$

Case (b): Critical λ_1 value within possible range

$$(1 - \frac{\Lambda}{m\mu})^2(R - \frac{v}{\mu}) < v\frac{\Pi}{m\mu} - c\frac{\rho}{(1-\rho)}\frac{\Pi}{m\mu} < (R - \frac{v}{\mu}) \tag{3.50}$$

is satisfied. Therefore, we make the first order derivative of G(λ_1) equal to zero and compute the corresponding λ_1 value.

$$\frac{dG(\lambda_1)}{d\lambda_1} = R - \frac{v}{\mu} + \frac{1}{(1-\rho_1)^2}[c\frac{\rho}{(1-\rho)}\frac{\Pi}{m\mu} - v\frac{\Pi}{m\mu}] = 0 \tag{3.51}$$

Because G(λ_1) is a concave function, λ_1 computed above maximizes our profit function and becomes our optimal λ_1.

$$\lambda_1{}^* = m\mu(1 - \sqrt{\frac{v\frac{\Pi}{m\mu} - c\frac{\rho}{(1-\rho)}\frac{\Pi}{m\mu}}{R - \frac{1}{\mu}}}) \tag{3.52}$$

Case (c): Critical λ_1 value beyond Λ

The following condition exists

$$(1 - \frac{\Lambda}{m\mu})^2(R - \frac{v}{\mu}) > v\frac{\Pi}{m\mu} - c\frac{\rho}{(1-\rho)}\frac{\Pi}{m\mu}. \tag{3.53}$$

λ_1 value greater than Λ is not possible. Therefore, we need to check first order derivative with this condition.

$$\frac{dG(\lambda_1)}{d\lambda_1} = R - \frac{v}{\mu} + \frac{1}{(1-\rho_1)^2}[c\frac{\rho}{(1-\rho)}\frac{\Pi}{m\mu} - v\frac{\Pi}{m\mu}] = 0 \tag{3.54}$$

Within this boundary, there is a real λ_1 value that makes $G(\lambda_1)$ equal to zero. It is computed as

$$\lambda_1 = m\mu(1 - \sqrt{\frac{v\frac{\Pi}{m\mu} - c\frac{\rho}{(1-\rho)}\frac{\Pi}{m\mu}}{R - \frac{1}{\mu}}}). \tag{3.55}$$

Because P is a concave function and its maximum value is obtained at a λ_1 value greater than Λ, the optimal λ_1 maximizing our profit function is equal to closest λ_1 value to calculated above that is equal to Λ.

$$\lambda_1{}^* = \Lambda \tag{3.56}$$

Moreover, our optimal λ_1 values for different cases are summarized in Table 3.1.

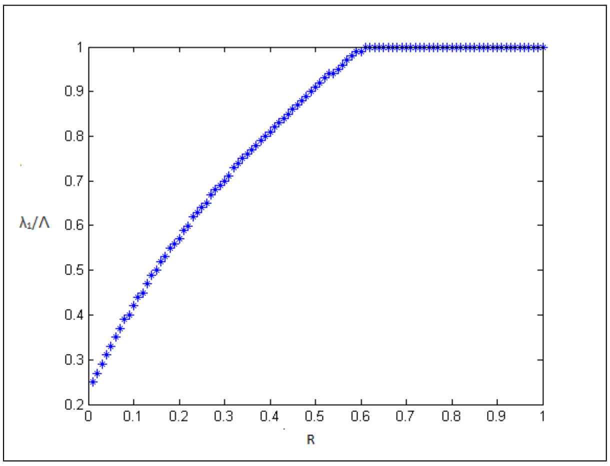
Table 3.1. Optimal λ_1 values under different scenarios.

	$critical\lambda_1 < 0$	$critical\lambda_1$ between $(0,\Lambda)$	$critical\lambda_1 > \Lambda$
Convex $G(\lambda_1)$	$\lambda_1^* = \Lambda$	$\lambda_1^* = 0$ or Λ	$\lambda_1^* = 0$
Concave $G(\lambda_1)$	$\lambda_1^* = 0$	$\lambda_1^* =$ critical λ_1	$\lambda_1^* = \Lambda$

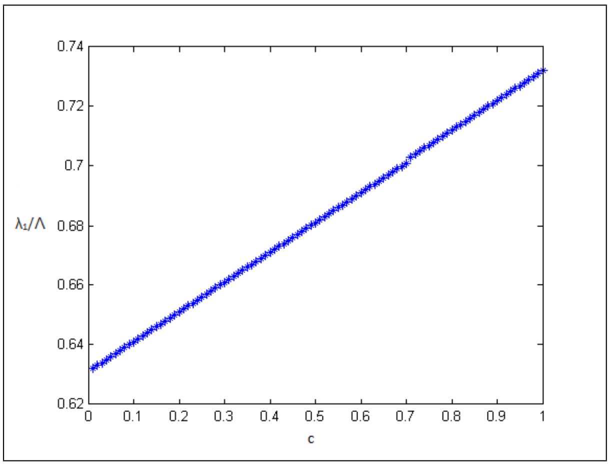
3.3. Numerical Results and Interpretation

In this section, we investigate how parameter values affect optimal choice rate of Type 1 priority class and corresponding profit values. We have explained the necessary conditions to have an optimal λ_1 value between 0 and Λ so that there are both types of customers exist in service systems. Because we are only interested in service systems where there are customers with both Types of priority classes, we guarantee that concavity condition is satisfied in the analyses stated below.

Consider the graphs in Figure 3.1a and Figure 3.1b.



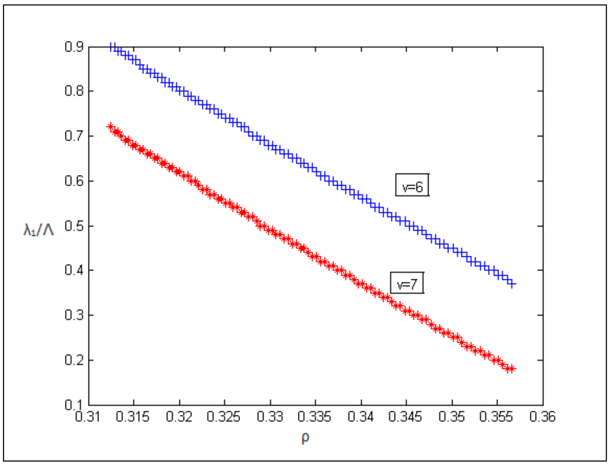
(a) Relationship between additional service value for priority class 1 and priority class 1 choice rate.



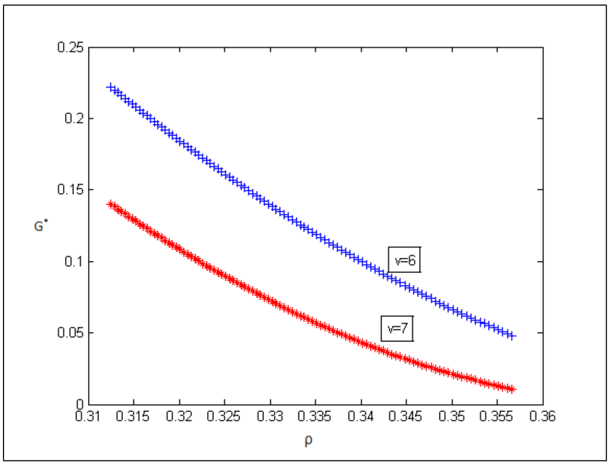
(b) Relationship between unit time waiting cost for priority class 1 and priority class 1 choice rate.

Figure 3.1. Priority class 1 choice rate relationship versus R and c.

The first graph 3.2(a) indicates the relationship between additional service value R customers receive in case that they choose Type 1 priority class and choice rate of Type 1 priority class. While R value increases in the graph, the choice rate of Type 1 priority class also increases as one expects. There is also a similar relationship between unit time waiting cost c and choice rate of Type 1 priority class since increasing unit waiting cost causes more customers to choose higher priority class to decrease their waiting time and cost. When two graphs are compared, it is observed that additional service value R has greater effect on customers priority class choices. A unit increase in R value leads bigger increase in choice rate of Type 1 priority class compared to unit waiting cost.



(a) Relationship between rho and priority class 1 choice rate.



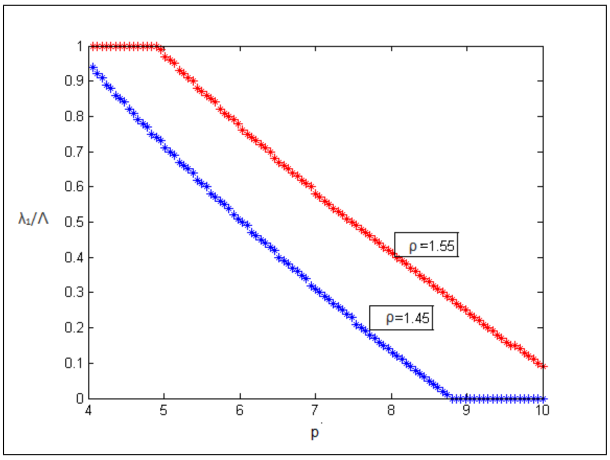
(b) Relationship between rho and profit.

Figure 3.2. Priority class 1 choice rate and profit relationship with ρ.

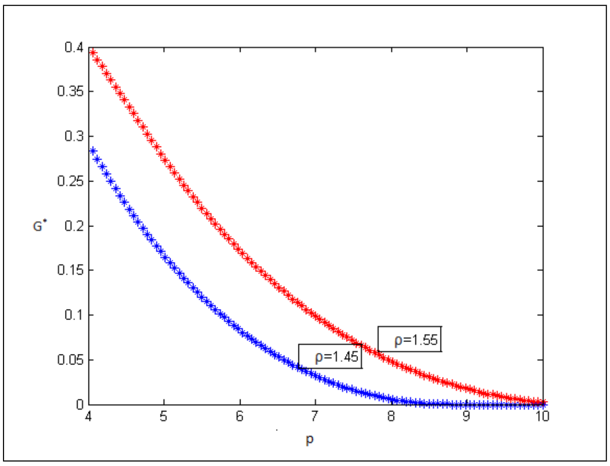
The graphs in Figure 3.2a and Figure 3.2b present the relationship between Type 1 priority class choice rate-ρ and profit-ρ respectively for two different values of additional cost v. In Figure 3.2a, we observe that there is a negative relationship between Type 1 priority class choice rate and ρ. As the utilization of service system increases, choice rate of Type 1 priority class also decreases. The reason for this kind of customer behavior is greater additional price for Type 1 priority class. However, the more striking result is seen in Figure 3.2b. The profit decreases while the utilization of service system increases. Service utilization affect profit function via its effects on Type 1 priority class choices and additional service prices. The graph shows us that the combination of these effects create a negative relationship between profit and ρ.

We can also observe the effects of additional cost v for Type 1 priority class on its choice rate and profit. The graph in Figure 3.2a presents that greater v value (v=7) causes less customers to select Type 1 priority class so choice rate is smaller for greater v value. The cause of such a relationship is the service provider's pricing policy. When additional cost of Type 1 priority class increases, the service provider desires to have less customers with Type 1 so he decreases additional price p for higher priority. Furthermore, smaller profit can be obtained for greater v values as expected since it directly decreases our profit value.

We also examine the additional cost v for Type 1 priority class with Type 1 class choice rate and profit value in order to prove the conclusions we state above. Consider the graphs in Figure 3.3a and 3.3b. The results derived from these graphs are consistent with our results observed in Figure 3.2a and Figure 3.2b. While the cost for Type 1 priority class increases, its choice rate decreases. Moreover, increasing cost v causes profit to decrease in the Figure 3.2b. These graphs also state additional cost v relationship with Type 1 priority class choice rate and profit for two different ρ values confirming our previous results.



(a) Relationship between additional cost 'v' and priority class 1 choice rate.



(b) Relationship between additional cost 'v' and profit.

Figure 3.3. Priority class 1 choice rate and profit relationship with v.

4. PRICING IN M/M/1 SERVICE SYSTEM WITH WILLINGNESS TO WAIT DISTRIBUTION

In this chapter, we present M/M/1 service systems where there is no service service differentiation but various delay sensitivity among customers exists. The idea behind this system is "willingness to wait" of customers. Therefore, a random variable S is introduced to the model that denotes an arriving customer's willingness to wait time in the system until receiving service. In order to keep our model simple, it is assumed that the distribution of S is known as exponential distribution.

In this model, it is assumed that arriving customers enter to the system as long as their utility is greater than or equal to zero so balking is possible.Moreover, customers come to the service system with Poisson distribution whose rate is equal to Λ. The service duration for all customers is identical and has an exponential distribution with rate μ. In order to prevent the service queue from exploding, we suppose the following condition is satisfied

$$\rho = \frac{\Lambda}{\mu} < 1. \tag{4.1}$$

Since balking is possible in this model, every customer who comes to the service system does not have to enter to the system. Therefore, we define a new variable λ that denotes the effective arrival rate of customers to the service system. It is still impossible for service queues to explode because the effective arrival rate cannot be greater than Λ.

It is also assumed that every customer's willingness to wait level denoted previously by S is unique and exponentially distributed with parameter θ. Moreover, there is no service failure in the system.

Guo and Zipkin (2007) have also analyzed M/M/1 queuing systems where every customer has different importance level for time. They compare various models that are differentiated with customers' waiting time information level as no information, partial information and full information. In this section, we extend their models with partial information by generating different profit functions for possible waiting cost computations.

Our fundamental aim in this study is to find the optimal service price that maximizes service provider's profit. We basically generate three different utility functions for our three models with different waiting time information levels. In our first and second models, we assume that the distribution of waiting time is known, but their waiting time cost computations are different in corresponding utility functions. In the first model, the waiting time cost increases linearly as waiting time increases whereas in the second model a fixed cost c is implemented whenever a customer's waiting time exceeds his willingness to wait level. However, they give similar results so we analyze them in the same section. In our third model, it assumed that the waiting time distribution is not known but the expected value of waiting time is known and waiting time cost again increases linearly with waiting time.

4.1. Models

4.1.1. The Distribution of Waiting Time is Known

In this model, it is assumed that customers' waiting time distribution is known that is waiting time distribution of an M/M/1 queuing model. And we offer two possible utility functions that can be generated with known waiting time. In the first case, we assume that waiting time cost of a customer is lineally increasing with the waiting time whenever his waiting time exceeds threshold value s. If waiting time does not exceed S, then no waiting cost is implemented. Therefore, the utility function for customers can be calculated as stated below.

$$U = R - p - cE[(W - S)^+] \tag{4.2}$$

The expectation in the formula above can be computed as

$$E[(W - S)^+] = \int_0^\infty P\{(W - S)^+ > t\}dt. \tag{4.3}$$

which can be expanded as

$$\int_0^\infty \rho e^{-\mu(1-\rho)(t+S)}dt = e^{-\mu(1-\rho)S}E[W]. \tag{4.4}$$

Our utility function for this model becomes equal to

$$U = R - p - c\rho e^{-\mu(1-\rho)S}. \tag{4.5}$$

Thus, our effective arrival rate λ can be computed from the equation below.

$$\Lambda P(U \geqslant 0) = \Lambda P(R - p - ce^{-\mu(1-\rho)S}E[W] \geqslant 0) \tag{4.6}$$

Because we know that S is exponentially distributed with rate θ our effective arrival rate λ becomes

$$\lambda = \Lambda e^{-\theta[-\ln(R-p)/(\mu(1-\rho))+\ln c/(\mu(1-\rho))+\ln\rho/(\mu(1-\rho))-\ln\mu/(\mu(1-\rho))-\ln(1-\rho)/(\mu(1-\rho))]}. \tag{4.7}$$

Therefore, service price p becomes equal to

$$p = R - e^{\frac{\mu(1-\rho)}{\theta}\ln\lambda - \frac{\mu(1-\rho)}{\theta}\ln\Lambda + \ln c + n\rho - \ln\mu - \ln(1-\rho)}. \tag{4.8}$$

Then, our profit G(λ) can be calculated as stated below

$$G(\lambda) = (R - e^{\frac{\mu(1-\rho)}{\theta}\ln\lambda - \frac{\mu(1-\rho)}{\theta}\ln\Lambda + \ln c + n\rho - \ln\mu - \ln(1-\rho)})\lambda. \tag{4.9}$$

If we define a new variable K as

$$K = \frac{\mu(1-\rho)}{\theta}\ln\lambda - \frac{\mu(1-\rho)}{\theta}\ln\Lambda + \ln c + n\rho - \ln\mu - \ln(1-\rho). \tag{4.10}$$

The first derivative of G(λ) with respect to λ becomes

$$\frac{dG(\lambda)}{d\lambda} = -K'e^{K}\lambda + (R - e^{K}). \tag{4.11}$$

Checking out the second derivative

$$\frac{\partial^2 G(\lambda)}{\partial\lambda^2} = -e^{K}[K''\lambda + (K')^2\lambda + 2K'] \tag{4.12}$$

where

$$K' = -\frac{\ln\lambda}{\theta} + \frac{(1-\rho)}{\theta\rho} + \frac{\ln\Lambda}{\theta} + \frac{1}{\lambda} + \frac{1}{\mu(1-\rho)} \tag{4.13}$$

and

$$K'' = -\frac{1}{\lambda\theta} - \frac{1}{\theta\mu\rho^2} - \frac{1}{\lambda^2} + \frac{1}{\mu^2(1-\rho)^2} \tag{4.14}$$

Moreover, the λ value that makes the first derivative of G(λ) equal to zero becomes our optimal arrival rate as long as G(λ) is a concave function. By using Equation 4.8, we can also calculate the optimal price p that maximizes the service provider's profit. However, we cannot determine the optimal λ value through mathematical calculations due to complexity of the equations. Therefore, we use matlab in order to determine optimal λ in Equation 4.2

In our second case, we assume that the distribution of waiting time is still known but the waiting cost does not increase as the waiting time increases instead a fixed cost c is implemented whenever a customer's waiting time exceeds his willingness to wait level. Moreover, waiting time distribution of customers is also known and the utility function of customers can be calculated as stated below.

$$U = R - p - cE[1_{\{W-S\}}] \tag{4.15}$$

The expectation in the utility function above can be calculated as

$$E[1_{\{W-S\}}] = P\{W > S\}. \tag{4.16}$$

Therefore, our utility function for this model is equal to

$$U = R - p - c\rho e^{-\mu(1-\rho)S}. \tag{4.17}$$

From the equations above, it is seen that there is no difference between the utility functions of both the first and the second cases. Because both models are similar to each other, we do not make further calculations for our second case.

4.1.2. The Expectation of Waiting Time is Known

In this model, it is supposed that customers' waiting time distribution is not known but their expected waiting time is known. The waiting time cost of a customer is linearly increasing with expected waiting time if expected waiting time is greater than a customer's willingness to wait level. The utility function for this model is generated as

$$U = R - p - c(E[W] - S)^+. \tag{4.18}$$

Our effective arrival rate λ is calculated below.

$$\Lambda P(U \geqslant 0) = \Lambda P(R - p - c(E[W] - S)^{+} \geqslant 0) \tag{4.19}$$

The probability of non negative utility values becomes equal to

$$P((E[W] - S)^{+} \leqslant \frac{R-p}{c}) = E[1_{\{(E[W]-S)^{+} \leqslant \frac{R-p}{c}\}}]. \tag{4.20}$$

The probability above is equal to zero if p is greater than R value meaning that non negative utility is not possible without greater R than p. The probability can be calculated in two parts

$$E[1_{\{(E[W]-S)^{+} \leqslant \frac{R-p}{c}\}}] = \int_{0}^{E[W]} 1_{\{(E[W]-S)^{+} \leqslant \frac{R-p}{c}\}} f_s(x)dx + \int_{E[W]}^{\infty} 1_{\{(E[W]-S)^{+} \leqslant \frac{R-p}{c}\}} f_s(x)dx \tag{4.21}$$

. That is equal to

$$E[1_{\{(E[W]-S)^{+} \leqslant \frac{R-p}{c}\}}] = \int_{E[W]-\frac{R-p}{c}}^{E[W]} f_s(x)dx + \int_{E[W]}^{\infty} 1 f_s(x)dx. \tag{4.22}$$

Therefore, the probability of on negative utility is equal to

$$P(U > 0) = \begin{cases} 1 & E[W] - \frac{R-p}{c} < 0, p < R \\ P(S \geq E[W] - \frac{R-p}{c}) & E[W] - \frac{R-P}{c} \geq 0, p < R \\ 0 & otherwise \end{cases} \tag{4.23}$$

Case (a): E[W] - $\frac{R-p}{c} \geq 0$ *and* $p<R$

The effective arrival rate λ to the service system under this condition is calculated as

$$\lambda = \Lambda P(S \geq E[W] - \frac{R-p}{c}). \tag{4.24}$$

Thus, λ becomes

$$\lambda = \Lambda e^{-\theta(\frac{\rho}{\mu(1-\rho)} - \frac{R-p}{c})}. \tag{4.25}$$

and effective price becomes equal to

$$p = R - \frac{c}{\theta}\ln\lambda + \frac{c}{\theta}\ln\Lambda - \frac{c\rho}{\mu(1-\rho)}. \tag{4.26}$$

Our profit G(λ) can be calculated as stated below.

$$G(\lambda) = \left(R - \frac{c}{\theta}\ln\lambda + \frac{c}{\theta}\ln\Lambda - \frac{c\rho}{\mu(1-\rho)}\right)\lambda \tag{4.27}$$

In order to find the optimal value of price and effective arrival rate that maximizes our profit function, the first and the second derivative of profit function need to be computed. The first derivative of G(λ) with respect to λ becomes

$$\frac{dG(\lambda)}{d\lambda} = R - \frac{c}{\theta}\ln\lambda - \frac{c}{\theta} + \frac{c}{\theta}\ln\Lambda - \frac{c}{\theta}\frac{2\rho-\rho^2}{(1-\rho)^2}. \tag{4.28}$$

Checking out the second derivative

$$\frac{\partial^2 G(\lambda)}{\partial\lambda^2} = -\frac{c}{\theta\lambda} - \frac{c}{\mu^2}\frac{2}{(1-\rho)^3} \tag{4.29}$$

The second derivative of G(λ) is always smaller than zero so our profit function is concave. Furthermore, the λ value that makes the first derivative of G(λ) equal to zero becomes our effective arrival rate maximizing our profit function. The optimal price level can also be calculated by using the equation in 4.26. However, we cannot compute the optimal λ value through mathematical calculations because of complexity of the equations. Therefore, we use matlab in order to determine the optimal lambda in 4.25.

Case (b): E[W] - $\frac{R-p}{c}$<0 and p<R

Because the probability of non negative utility is equal to 1 under this condition, the effective arrival rate λ becomes

$$\lambda = \Lambda \tag{4.30}$$

and the price level satisfying this condition is

$$p < R - \frac{c\Lambda}{\mu(\mu - \Lambda)}. \tag{4.31}$$

Therefore, the optimal price level becomes its possible maximum value that is

$$p = R - \frac{c\Lambda}{\mu(\mu - \Lambda)} \tag{4.32}$$

and the maximum profit for this case is equal to

$$G(\lambda) = R - \frac{c\Lambda}{\mu(\mu - \Lambda)}\Lambda. \tag{4.33}$$

4.2. Numerical Results and Interpretation

The optimal price level maximizing service provider's profit is given in B.1 for the case where the distribution of waiting time is known and in B.2 for the case where the expectation of waiting time is known.

The willingness to wait level of customers is an important factor affecting the system equilibrium level. It directly affects the customers' utility so their effective rate of arrival λ to the system. The graph in Figure 4.1 present the relationship between θ and customers effective arrival rate λ for our two cases.

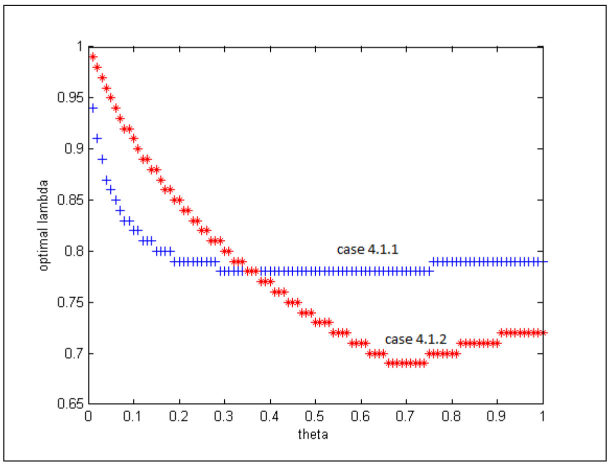


Figure 4.1. The relationship between θ and effective arrival rate.

The graph indicates that optimal λ value decreases for our both cases while θ value increases. Customers' willingness to wait level is exponentially distributed with rate θ. Therefore, their willingness to wait level decreases as θ increases. It is a natural result that customers are less willing to enter to the service system if their waiting tolerance is smaller.

We also study the effect of θ on maximum possible profit level that can be obtained by service providers. Thus, we state the graph in Figure 4.2 indicating the relationship between θ and profit.

The graph shows us that the possible maximum profit generally decreases while θ value increases. However, there is an exception for case 4.1.2 where θ has considerably small values. The reason for such a graphical trend is the piecewise structure of the profit function generated in 4.1.2. Furthermore, it is not surprising that the maximum achievable profit decreases for our both models because an increase in θ causes the effective arrival rate λ to decrease. It has a positive relationship with maximum profit directly.

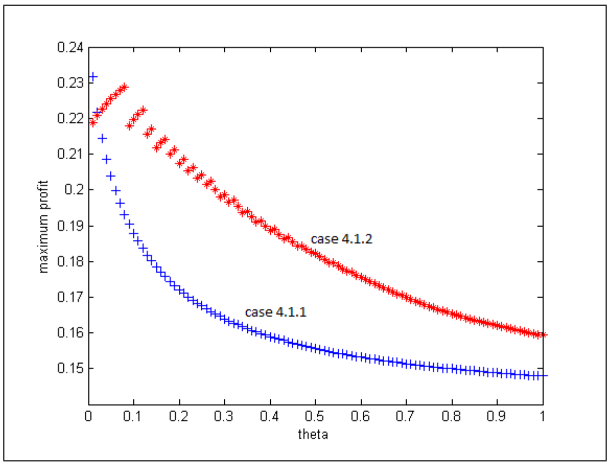


Figure 4.2. The relationship between θ and possible maximum profit.

If we compare the maximum profit values for two different models with the same θ value, we observe that the maximum achievable profit is greater in case 4.1.2 than in case 4.1.1 except for considerably small values of θ where the profit function of 4.1.2 has break point. Because it is assumed that only the expectation of waiting time is known for case 4.1.2 and it is less information sharing with customers compared to the case where distribution of waiting time is known, the service provider can charge greater price from the customers in case 4.1.2. Therefore, it is consistent with greater achievable profit amount.

From above, we conclude that the service provider in our first model is better off than in our second model. Furthermore, we also analyze the customers situation for our both model by comparing their expected utility in Figure 4.3.

Figure 4.3 displays that the expected utility of customers is greater in case 4.1.1 than in case 4.1.2 except for the smaller θ values. The exception interval here is consistent with the exception interval in Figure 4.2 indicating the maximum possible profit. Furthermore, it is anticipated that the expected utility of customers is greater in case 4.1.1 because the distribution of waiting time is known for this case and information sharing is greater that enables customers to make their decisions more rational.

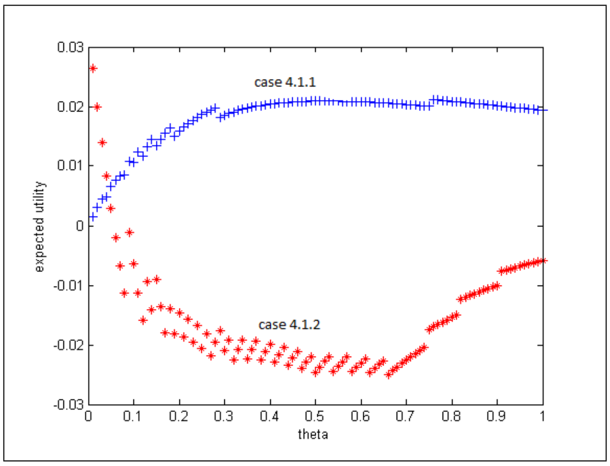


Figure 4.3. The relationship between θ and expected utility of customers.

5. PRICING IN M/M/1 SERVICE SYSTEM WITH RANDOM REVENUE AND PENALTY COST PARAMETERS

In this chapter, we analyze M/M/1 service systems where each customer has a unique valuation for the service he receives and a unique waiting cost per unit time. Moreover, customers' properties are unobservable for the service provider. Therefore, both revenue and penalty cost of customers are random parameters.

It is assumed that customers enter to the service system as long as their utility is greater than or equal to zero. Arriving process of customers is a Poisson process with rate Λ. Furthermore, service duration for every customer is exponentially distributed with rate μ. We guarantee that the condition below is always satisfied.

$$\frac{\Lambda}{\mu} < 1 \tag{5.1}$$

Our specific approach in this study is to determine the optimal price level and maximum profit for the service systems with random revenue and penalty cost parameters. We also analyze how different mean, variance and correlation values affect pricing policy and maximum profit in these systems. Therefore, we determine the optimal price level ad maximum possible profit for specific parameter values in 5.2 and then compare our results. Moreover, we also investigate the change in maximum profit level if the variability in revenue and penalty cost parameters is ignored.

5.1. Derivations

Customers' utility function as presented in previous sections equals to

$$U(\lambda) = R - p - cE[W] \tag{5.2}$$

The R and c values in the above equation are random parameters. We assume that the value computed below has approximately normal distribution with corresponding mean and variance parameters.

$$R - cE[W] \sim N(\mu_u, {\sigma_u}^2) \tag{5.3}$$

where

$$\mu_u = \mu_R - E[W]\mu_c \tag{5.4}$$

and

$${\sigma_u}^2 = {\sigma_R}^2 + {\sigma_c}^2 E[W]^2 - 2E[W]\mathrm{cov}(R, c). \tag{5.5}$$

If effective arrival rate is λ, then utilization becomes

$$\rho = \frac{\lambda}{\mu}. \tag{5.6}$$

Therefore, expected waiting time above is calculated as

$$E[W] = \frac{\rho}{\mu(1-\rho)}. \tag{5.7}$$

Furthermore, the covariance between R and c is calculated as

$$\mathrm{cov}(R, c) = correlation(R, c)\sigma_R \sigma_c \tag{5.8}$$

The effective arrival rate to the service system is

$$\lambda = \Lambda P(U(\lambda) \geq 0) = \Lambda P(N(\mu_u, {\sigma_u}^2) \geq p). \tag{5.9}$$

Therefore, price in the above equation can be computed as

$$P(\lambda) = \mu_u + \sigma_u \Phi^{-1}\left(1 - \frac{\lambda}{\Lambda}\right). \tag{5.10}$$

Then, the profit function G(λ) is calculated as

$$G(\lambda) = \lambda P(\lambda) = \lambda\left(\mu_u + \sigma_u \Phi^{-1}\left(1 - \frac{\lambda}{\Lambda}\right)\right). \tag{5.11}$$

We write matlab programs in order to determine the price level maximizing service provider's profit and corresponding effective arrival rate by an exhaustive search as G(λ) is highly non linear.

5.2. Numerical Results

In this section, we determine the maximum possible profit and optimal price level that gives this profit. The code that determine the optimal price level and maximum profit is given in C.1.

We also study how parameter uncertainty affects maximum possible profit if it is ignored. Therefore, we firstly compute the optimal price level P(λ) when parameter values are regarded as constant.

$$P(\lambda) = R - c\frac{\rho}{\mu(1-\rho)} \tag{5.12}$$

Then, we compute our profit G(P) if we set price P(λ) in existence of uncertainty. Furthermore, we also calculate the optimal price level $P^*(\lambda^*)$ by considering uncertainty in parameter values.

$$P^*(\lambda^*) = \mu_u + \sigma_u \Phi^{-1}\left(1 - \frac{\lambda^*}{\Lambda}\right) \tag{5.13}$$

We also calculate our maximum profit G(P^*) for P^*. Finally, we compare our profit values G(P) and G(P^*) in order to determine the cost of uncertainty. If we define value of information as

$$ValueofInformation = 100\left(\frac{G(P^*) - G(P)}{G(P)}\right), \tag{5.14}$$

the cost of ignoring uncertainty can be determined by the value of information.

In this section, we compute expected waiting time, optimal price, maximum profit and value of information for specific parameter values. The parameter values considered in this section is given in Table 5.1.

Table 5.1. Parameter values.

μ_R	1.5	5.0	
μ_c	0.1	0.5	1.0
CorrelationofVariation	0.0	0.1	0.3

We consider all combinations of these parameter values. In order to determine both negative and positive correlation between c and R values, the correlation coefficient is taken as -0.8 and 0.8. Moreover, we also assume that c and R values are independent by considering their correlation coefficient as zero.

The results obtained from above parameter combinations are also given in this section. Because the data obtained is extensive, we summarize this data by the R code as stated below

```
>data<-read.delim(file.choose(), header=TRUE)
```

$>attach(data)$

$>aggdata <\text{-}aggregate(data,\ by=list(c_{mean},\ c_{coefficientofvariation}),\ FUN=mean,$
$na.rm=TRUE)$

5.2.1. Effects of Uncertainty in c

The data obtained using the code above is given in Table 5.2. It indicates that the expected waiting time decreases as c_{mean} increases. Because greater waiting cost increases negative utility, less customers enter to the service system. Thus, the expected waiting time also decreases. Moreover, expected waiting time decreases while the coefficient of variation increases. Greater coefficient of variation means greater variance in c. Therefore, greater uncertainty exists whenever the coefficient of variation increases and less customers choose entering to the service system due to uncertainty.

Table 5.2. Average results for different parameter values of unit cost c.

c_{mean}	$c_{coeffofvar}$	Expected Waiting Time	Price	Profit	Value of Inf.
0.10	0.00	3.377250000	2.660766667	2.064266667	0.984251440
0.10	0.10	3.266572222	2.659227778	2.053033333	0.985263200
0.10	0.30	3.103366667	2.658355556	2.033938889	0.989902511
0.50	0.00	1.396633333	2.475933333	1.508383333	0.672695767
0.50	0.10	1.369372222	2.481338889	1.501838889	0.691659392
0.50	0.30	1.319888889	2.496400000	1.490711111	0.823224982
1.00	0.00	0.899116667	2.375566667	1.206483333	1.522879723
1.00	0.10	1.181594444	2.409744444	1.280361111	1.554471598
1.00	0.30	1.141177778	2.432333333	1.276038889	1.837750559

Similar results are also obtained for possible maximum profit. While the coefficient of variation increases, maximum profit decreases. Because more uncertainty decreases customer entering rate to the system, the maximum profit service providers can achieve also decreases. Furthermore, the effect of coefficient of variation on optimal price is complex. The table indicates that optimal price increases with increasing coefficient of variation for bigger c values while it decreases with increasing coefficient of variation for smaller c values. The reason for this behavior is our normal distribution assumption. Coefficient of variation and optimal price relationship changes according to the interval where the effective customer arrival rate λ is within it. Our price equation presents that optimal price decreases with increasing variance if λ is greater than 0.5 and it increases with decreasing variance if λ is less than 0.5. Therefore, there is a positive relationship between them for greater c and negative relationship for smaller c. We also analyze the cost of ignoring uncertainty by observing Value of Information. The table shows that Value of Information increases while variance of c increases as it is expected.

5.2.2. Effects of Uncertainty in R

Table 5.3 is created for different R_{mean} and $R_{coefficientofvariation}$ values. It shows us that expected waiting time increases while R_{mean} increases. It is a natural result because greater R_{mean} leads to more positive utility and more customers choose entering to the service system. Furthermore, the expected waiting time decreases while the coefficient of variation increases. This result is also similar to the relationship between $c_{variance}$ and expected waiting time. It can be concluded that less customers choose entering to the system whenever uncertainty in a system increases. Therefore, increasing variance decreases the expected waiting time.

Table 5.3. Average results for different parameter values of service valuation R.

R_{mean}	$R_{coeffofvar}$	Expected Waiting Time	Price	Profit	Value of Inf.
1.50	0.00	1.565385185	1.073437037	0.611325926	0.142769706
1.50	0.10	1.434670370	1.091403704	0.599507407	0.622373492
1.50	0.30	1.105544444	1.185925926	0.582522222	3.335531336
5.00	0.00	3.038300000	3.949770370	2.805500000	0.062271818
5.00	0.10	2.638265385	3.863634615	2.660280769	1.063965404
5.00	0.30	1.645384615	3.957357692	2.384957692	1.561083030

We also expect similar results for possible maximum profit. Because variance increase in R decreases customer entering rate to the service system, maximum profit also decreases. Thus, maximum profit decreases with increasing coefficient of variation. The effect of coefficient of variation on optimal price is again complicated due to the reason stated in 5.2.2. Moreover, the table indicates that value of information increases while coefficient of variation increases. Therefore, it can be said that cost of ignoring uncertainty increases with increasing uncertainty in a service system.

5.2.3. Effects of Different Correlation Coefficients

We also compute same numerical values for different correlation coefficients between unit waiting cost c and service valuation R. These results are summarized in Table 5.4.

Table 5.4. Average results for different correlation coefficients.

Correlation Coefficient	Expected Waiting Time	Price	Profit	Value of Inf.
0	2.012616667	2.528114815	1.633785185	1.115620902
-0.8	1.792761111	2.524048148	1.580709259	1.248201604
0.8	1.879612963	2.497725926	1.590524074	0.987543885

When we compare our expected waiting times for different correlation coefficients, we observe that whenever c and R values are independent, the expected waiting time becomes minimum. Therefore, existence of relationship between c and R whether it is negative or positive decreases expected waiting time. It is also seen that negative correlation coefficient decreases expected waiting time more than positive correlation coefficient. Moreover, we expect similar results for both profit and value of information. If profit values are compared, it is seen that the greatest profit is obtained whenever there is no correlation between c and R. This result is estimated because greatest waiting time means greatest effective arrival rate. Therefore, greater profit is achievable. In addition, negative correlation coefficient leads to smaller profit than positive correlation coefficient. When value of informations are compared, it is seen that the greatest value of information takes place whenever a negative correlation coefficient exists. Because R and c values affect profit in different directions, negative relationship between them increases their effect on the profit even more. Therefore, ignorance of uncertainty in this case causes greatest cost. On the other hand, the positive correlation between them balance their individual effect on the profit so value of information is smaller for this case.

6. CONCLUSION

In the scope of this thesis, we investigate the pricing policy in service systems with strategic customers. We study three different service system models throughout the thesis. Our first model has an M/M/c queuing system with priority classes. The customers in this model choose their customer classes within the system. The second system includes an M/M/1 queuing model with single type customer. In this system, we create three sub models for which information levels are different. In our third model, we consider another M/M/1 queuing model but we assume that both service valuation and waiting cost of customers are uncertain.

For each of our system models, we assume that customers are willing to receive service as long as their utility is non negative. Moreover, the service price affecting customers' utility is set by the service provider. Therefore, customers' demand for service are determined by this price level. Our fundamental aim for every model is to maximize the service provider's profit.

In order to find the utility of customers, we calculate the expected waiting time for different priority classes in our first service system model. Then, we generate a profit function for which all customers are forced to choose first priority class in order to maximize service provider's profit. Therefore, we modify our profit function such that both priority class Types are chosen by the customers with different rates depending on additional service price for the Type 1 priority class. Moreover, we generate numerical results. They indicate which factors affect customers' priority class choices for the parameter values satisfying concavity condition.

We observe that an increase in both unit time waiting cost and service valuation drives the customers to choose Type 1 priority class more in order to satisfy their utility requirements. It is also seen that as system utilization level increases, choice rate of Type 1 priority class decreases. On the other hand, an increase in service utilization increases service provider's profit. Another factor affecting Type 1 priority class choice rate is the additional price charged from the customers who are willing to have faster service. We observe that an increase in additional service price creates a decrease in both Type 1 priority class choice rate and profit.

For our service system models with different delay information, we regard two cases where distribution of waiting time is known and expectation of waiting time is known respectively. Then, we develop three different profit functions in order to determine the price levels maximizing profit. It is assumed that customers have a willingness to wait level in these sub models so there is no implemented waiting cost below this threshold value in their utility functions. Therefore, the price levels that maximizes profit functions are directly affected from customers' willingness to wait level. Moreover, we also compute the expected utility of customers for every sub model. Then, we compare our results in order to observe the effects of different delay information levels on service providers' profit and customers' expected utility.

In result comparisons, we conclude that customers effective arrival rate decreases as their willingness to wait level decreases for both cases. We also find that maximum possible profit generally decreases as customers' willingness to wait level decreases. However, there is an exception for the case that expectation of waiting time is known due to its piecewise structure. When we compare the maximum possible profit for two different cases, we observe that maximum profit is greater for the case that expectation of waiting time is known than the case that the distribution of waiting time is known. We also conclude that because there is less information sharing where expectation of waiting time is known, it is natural to achieve greater profit for the service provider. When we compare customers' expected utility for our two cases, a consistent result is seen that is customers who have less information also have less expected utility.

In our last service system model, we are dealing with uncertain service valuation and unit time waiting cost problem. In order to analyze such systems, we make a normality assumption. Then, we generate the function yielding optimal price level. For different parameter values, we determine the optimal price level and maximum achievable profit. By comparing these results, we show the effect of parameter uncertainties on the optimal price and maximum profit.

We observe that expected waiting time and maximum profit decrease as variance of unit time waiting cost increases. In order to determine the cost of ignoring uncertainty in service systems, we calculate the value of information. When we compare the value of information for different unit time waiting cost variances, it is seen that cost of ignoring uncertainty increases while unit time waiting cost variance increases. Same results are also obtained for different variance values of service valuation. When we analyze the effects of correlation coefficient between service valuation and unit time waiting cost on the value of information, we observe that if there is a negative correlation between them, the cost of ignoring uncertainty is greatest. And it is smallest if they are positively correlated with each other.

Future research based on our study may be conducted in several areas. First, our first study can be expanded with more priority classes and different profit functions can be generated for various service systems. It is possible for various delay information models to analyze service systems where information sharing is required by the government or it is prohibited. Moreover, more distribution assumptions can be hold for uncertain waiting cost and service valuations of customers.

APPENDIX A: MATLAB CODE FOR MULTI SERVER QUEUES WITH PRIORITIES

A.1. Relationship Between R and Priority Class 1 Choice Rate

```
clear all,close all;
lambda=1;
v=6;
c=0.1;
mu=1.5;
m=2;
rho=lambda/(m*mu);
sum = 0;
for i=0:(m-1)
sum = sum + (m*rho)^i/ factorial(i);
end
sum = sum + ((m*rho)^m) /(factorial(m)*(1-m*rho));
Pay=((m*rho)^m) /(factorial(m)*(1-m*rho))/ sum;
for j=1:100
R = j/100;
maxprofit=0;
maxprice=0;
maxlambda=0;
for i=1:100
lambda1 =(i/100)*lambda;
rho1=lambda1/(m*mu);
profit(i)=lambda1*(R+c*(rho/(1-rho))*(Pay/(m*mu*(1-rho1))))
-v*(Pay*rho1/(1-rho1))+rho1*m;
price(i)= R+c*(rho/(1-rho))*(Pay/(m*mu*(1-rho1)));
lambda2(i)=lambda1;
```

```
if profit(i)>maxprofit
maxprofit=profit(i);
maxprice=price(i);
maxlambda=lambda2(i);
else
maxprofit=maxprofit;
maxprice=maxprice;
maxlambda=maxlambda;
end
end
R2(j)= R;
lambda3(j)= maxlambda;
disp(R);
disp(maxlambda);
end
plot(R2,lambda3,'b*');
xlabel('R');
ylabel('lambda1/lambda');
```

A.2. Relationship Between c and Priority Class 1 Choice Rate

```
clear all,close all;
lambda=1;
R=0.25;
v=6;
mu=1.5;
m=2;
rho=lambda/(m*mu);
sum = 0;
for i=0:(m-1)
```

```
sum = sum + (m*rho)^i/factorial(i);
end
sum = sum + ((m*rho)^m) /(factorial(m)*(1-m*rho));
Pay=((m*rho)^m) /(factorial(m)*(1-m*rho))/ sum;
for j=1:100
c = j/100;
maxprofit=0;
maxprice=0;
maxlambda=0;
for i=1:100
lambda1 =(i/100)*lambda;
rho1=lambda1/(m*mu);
profit(i)=lambda1*(R+c*(rho/(1-rho))*(Pay/(m*mu*(1-rho1))))
-v*(Pay*rho1/(1-rho1))+rho1*m;
price(i)= R+c*(rho/(1-rho))*(Pay/(m*mu*(1-rho1)));
lambda2(i)=lambda1;
if profit(i)>maxprofit
maxprofit=profit(i);
maxprice=price(i);
maxlambda=lambda2(i);
else
maxprofit=maxprofit;
maxprice=maxprice;
maxlambda=maxlambda;
end
end
c2(j)= c;
lambda3(j)= maxlambda;
end
plot(c2,lambda3,'b*');
xlabel('c');
```

```
ylabel('lambda1/lambda');
```

A.3. Relationship Between ρ and Priority Class 1 Choice Rate

```
clear all,close all;
lambda=1;
R=0.25;
c=0.1;
m=2;
v1=6;
for j=1:100
mu= 1.4+0.2*(j/100);
rho=lambda/(m*mu);
sum = 0;
for i=0:(m-1)
sum = sum + (m*rho)^i/factorial(i);
end
sum = sum + ((m*rho)^m) /(factorial(m)*(1-m*rho));
Pay=((m*rho)^m) /(factorial(m)*(1-m*rho))/ sum;
maxprofit=0;
maxprice=0;
maxlambda=0;
for i=1:100
lambda1 =(i/100)*lambda;
rho1=lambda1/(m*mu);
profit(i)=lambda1*(R+c*(rho/(1-rho))*(Pay/(m*mu*(1-rho1))))
-v1*(Pay*rho1/(1-rho1))+rho1*m;
price(i)= R+c*(rho/(1-rho))*(Pay/(m*mu*(1-rho1)));
lambda2(i)=lambda1;
if profit(i)>maxprofit
```

```
maxprofit=profit(i);
maxprice=price(i);
maxlambda=lambda2(i);
else
maxprofit=maxprofit;
maxprice=maxprice;
maxlambda=maxlambda;
end
end
rho2(j)= lambda/(m*mu);
lambda3(j)= maxlambda;
end
v1=7;
for j=1:100
mu= 1.4+0.2*(j/100);
rho=lambda/(m*mu);
sum = 0;
for i=0:(m-1)
sum = sum +(m*rho)^i/factorial(i);
end
sum = sum + ((m*rho)^m) /(factorial(m)*(1-m*rho));
Pay=((m*rho)^m) /(factorial(m)*(1-m*rho))/ sum;
maxprofit=0;
maxprice=0;
maxlambda=0;
for i=1:100
lambda1 =(i/100)*lambda;
rho1=lambda1/(m*mu);
profit(i)=lambda1*(R+c*(rho/(1-rho))*(Pay/(m*mu*(1-rho1))))
-v1*(Pay*rho1/(1-rho1))+rho1*m;
price(i)= R+c*(rho/(1-rho))*(Pay/(m*mu*(1-rho1)));
```

```
lambda2(i)=lambda1;
if profit(i)>maxprofit
maxprofit=profit(i);
maxprice=price(i);
maxlambda=lambda2(i);
else
maxprofit=maxprofit;
maxprice=maxprice;
maxlambda=maxlambda;
end
end
rho3(j)= lambda/(m*mu);
lambda4(j)= maxlambda;
end
plot(rho2,lambda3,'b+',rho3,lambda4,'r*');
xlabel('rho');
ylabel('lambda1/lambda');
```

A.4. Relationship Between ρ and Profit

```
clear all,close all;
lambda=1;
R=0.25;
c=0.1;
m=2;
v1=6;
for j=1:100
mu= 1+0.6*(j/100);
rho=lambda/(m*mu);
sum = 0;
```

```
for i=0:(m-1)
sum = sum + (m*rho)^i/factorial(i);
end
sum = sum + ((m*rho)^m) /(factorial(m)*(1-m*rho));
Pay=((m*rho)^m) /(factorial(m)*(1-m*rho))/ sum;
maxprofit=0;
maxprice=0;
maxlambda=0;
for i=1:100
lambda1 =(i/100)*lambda;
rho1=lambda1/(m*mu);
profit(i)=lambda1*(R+c*(rho/(1-rho))*(Pay/(m*mu*(1-rho1))))
-v1*(Pay*rho1/(1-rho1))+rho1*m;
price(i)= R+c*(rho/(1-rho))*(Pay/(m*mu*(1-rho1)));
lambda2(i)=lambda1;
if profit(i)>maxprofit
maxprofit=profit(i);
maxprice=price(i);
maxlambda=lambda2(i);
else
maxprofit=maxprofit;
maxprice=maxprice;
maxlambda=maxlambda;
end
end
rho2(j)= lambda/(m*mu);
profit2(j)= maxprofit;
end
v1=7;
for j=1:100
mu= 1+0.6*(j/100);
```

```
rho=lambda/(m*mu);
sum = 0;
for i=0:(m-1)
sum = sum + (m*rho)^i/factorial(i);
end
sum = sum + ((m*rho)^m) /(factorial(m)*(1-m*rho));
Pay=((m*rho)^m) /(factorial(m)*(1-m*rho))/ sum;
maxprofit=0;
maxprice=0;
maxlambda=0;
for i=1:100
lambda1 =(i/100)*lambda;
rho1=lambda1/(m*mu);
profit(i)=lambda1*(R+c*(rho/(1-rho))*(Pay/(m*mu*(1-rho1))))
-v1*(Pay*rho1/(1-rho1))+rho1*m;
price(i)= R+c*(rho/(1-rho))*(Pay/(m*mu*(1-rho1)));
lambda2(i)=lambda1;
if profit(i)>maxprofit
maxprofit=profit(i);
maxprice=price(i);
maxlambda=lambda2(i);
else
maxprofit=maxprofit;
maxprice=maxprice;
maxlambda=maxlambda;
end
end
rho3(j)= lambda/(m*mu);
profit3(j)= maxprofit;
end
plot(rho2,profit2,'b+',rho3,profit3,'r*');
```

```
xlabel('rho');
ylabel('G');
```

A.5. Relationship Between v and Priority Class 1 Choice Rate

```
clear all,close all;
lambda=1;
R=0.25;
c=0.1;
m=2;
mu=1.45;
rho=lambda/(m*mu);
sum = 0;
for i=0:(m-1)
sum = sum + (m*rho)^i/factorial(i);
end
sum = sum + ((m*rho)^m) /(factorial(m)*(1-m*rho));
Pay=((m*rho)^m) /(factorial(m)*(1-m*rho))/ sum;
for j=1:100
v = 4+6*(j/100);
maxprofit=0;
maxprice=0;
maxlambda=0;
for i=1:100
lambda1 =(i/100)*lambda;
rho1=lambda1/(m*mu);
profit(i)=lambda1*(R+c*(rho/(1-rho))*(Pay/(m*mu*(1-rho1))))
-v*(Pay*rho1/(1-rho1))+rho1*m;
price(i)= R+c*(rho/(1-rho))*(Pay/(m*mu*(1-rho1)));
lambda2(i)=lambda1;
```

```
if profit(i)>maxprofit
maxprofit=profit(i);
maxprice=price(i);
maxlambda=lambda2(i);
else
maxprofit=maxprofit;
maxprice=maxprice;
maxlambda=maxlambda;
end
end
v2(j)= v;
lambda3(j)= maxlambda;
end
mu=1.55;
rho=lambda/(m*mu);
sum = 0;
for i=0:(m-1)
sum = sum + (m*rho)^i/factorial(i);
end
sum = sum + ((m*rho)^m) /(factorial(m)*(1-m*rho));
Pay=((m*rho)^m) /(factorial(m)*(1-m*rho))/ sum;
for j=1:100
v = 4+6*(j/100);
maxprofit=0;
maxprice=0;
maxlambda=0;
for i=1:100
lambda1 =(i/100)*lambda;
rho1=lambda1/(m*mu);
profit(i)=lambda1*(R+c*(rho/(1-rho))*(Pay/(m*mu*(1-rho1))))
-v*(Pay*rho1/(1-rho1))+rho1*m;
```

```
price(i)= R+c*(rho/(1-rho))*(Pay/(m*mu*(1-rho1)));
lambda2(i)=lambda1;
if profit(i)>maxprofit
maxprofit=profit(i);
maxprice=price(i);
maxlambda=lambda2(i);
else
maxprofit=maxprofit;
maxprice=maxprice;
maxlambda=maxlambda;
end
end
v3(j)= v;
lambda4(j)= maxlambda;
end
plot(v2,lambda3,'b*',v3,lambda4,'r*');
xlabel('v');
ylabel('lambda1/lambda');
```

A.6. Relationship Between v and Profit

```
clear all,close all;
lambda=1;
R=0.25;
c=0.1;
m=2;
mu=1.45;
rho=lambda/(m*mu);
sum = 0;
for i=0:(m-1)
```

```
sum = sum + (m*rho)^i/factorial(i);
end
sum = sum + ((m*rho)^m) /(factorial(m)*(1-m*rho));
Pay=((m*rho)^m) /(factorial(m)*(1-m*rho))/ sum;
for j=1:100
v = 4+6*(j/100);
maxprofit=0;
maxprice=0;
maxlambda=0;
for i=1:100
lambda1 =(i/100)*lambda;
rho1=lambda1/(m*mu);
profit(i)=lambda1*(R+c*(rho/(1-rho))*(Pay/(m*mu*(1-rho1))))
-v*(Pay*rho1/(1-rho1))+rho1*m;
price(i)= R+c*(rho/(1-rho))*(Pay/(m*mu*(1-rho1)));
lambda2(i)=lambda1;
if profit(i)>maxprofit
maxprofit=profit(i);
maxprice=price(i);
maxlambda=lambda2(i);
else
maxprofit=maxprofit;
maxprice=maxprice;
maxlambda=maxlambda;
end
end
v2(j)= v;
profit3(j)= maxprofit;
end
mu=1.55;
rho=lambda/(m*mu);
```

```
sum = 0;
for i=0:(m-1)
sum = sum + (m*rho)^i/factorial(i);
end
sum = sum + ((m*rho)^m) /(factorial(m)*(1-m*rho));
Pay=((m*rho)^m) /(factorial(m)*(1-m*rho))/ sum;
for j=1:100
v = 4+6*(j/100);
maxprofit=0;
maxprice=0;
maxlambda=0;
for i=1:100
lambda1 =(i/100)*lambda;
rho1=lambda1/(m*mu);
profit(i)=lambda1*(R+c*(rho/(1-rho))*(Pay/(m*mu*(1-rho1))))
-v*(Pay*rho1/(1-rho1))+rho1*m;
price(i)= R+c*(rho/(1-rho))*(Pay/(m*mu*(1-rho1)));
lambda2(i)=lambda1;
if profit(i)>maxprofit
maxprofit=profit(i);
maxprice=price(i);
maxlambda=lambda2(i);
else
maxprofit=maxprofit;
maxprice=maxprice;
maxlambda=maxlambda;
end
end
v3(j)= v;
profit4(j)= maxprofit;
end
```

```
plot(v2,profit3,'b*',v3,profit4,'r*');
xlabel('v');
ylabel('G');
```

APPENDIX B: MATLAB CODE FOR M/M/1 SERVICE SYSTEMS WITH WILLINGNESS TO WAIT DISTRIBUTION

B.1. Optimal Price Calculation with Known Distribution of Waiting Time

```
clear all,close all;
lambda=1;
R=0.25;c=0.1;mu=1.5;teta=1;maxprofit=0;maxprice=0;maxlambda=0;
for j=1:100
clambda1 =(j/100)*lambda;
crho=clambda1/mu;
cprofit(j)= clambda1*(R-c*log(clambda1)/teta+c*log(lambda)/teta
-((c*crho)/(mu*(1-crho))));
cprice(j)=(R-c*log(clambda1)/teta+c*log(lambda)/teta
-((c*crho)/(mu*(1-crho))));
clambda2(j)=clambda1;
if cprice(j)< R
criticalprofit = cprofit(j);
criticalprice = cprice(j);
criticallambda = clambda2(j);
criticalrho = clambda2(j)/mu;
break
end
end
for i=1:100
lambda1 =(i/100)*lambda;
rho=lambda1/mu;
profit(i)= lambda*(R-exp(mu*(1-rho)*log(lambda)/teta-mu*(1-rho)
log(lambda)/teta+log(c)+log(rho)-log(mu)-log(1-rho)));
```

```
price(i)=(R-exp(mu*(1-rho)*log(lambda1)/teta-mu*(1-rho)*log(lambda)/teta
+log(c)+log(rho)-log(mu)-log(1-rho)));
lambda2(i)=lambda1;
if profit(i)>maxprofit
maxprofit=profit(i);
maxprice=price(i);
maxlambda=lambda2(i);
maxrho=maxlambda/mu;
else
maxprofit=maxprofit;
maxprice=maxprice;
maxlambda=maxlambda;
maxrho=maxlambda/mu;
end
end
if maxprice > R
maxprofit=criticalprofit;
maxprice=criticalprice;
maxlambda=criticallambda;
maxrho=criticalrho;
end
disp(maxprice);
disp(maxprofit);
disp(maxlambda);
Eutility = R-maxprice-c*(maxrho/((1-maxrho)*mu))*teta/(mu*(1-maxrho)+teta)
```

B.2. Optimal Price Calculation with Known Expected Waiting Time

```
clear all,close all;
lambda=1;
```

```
R=0.25;
c=0.1;
mu=1.5;
teta=1;
maxprofit=0;
maxprice=0;
maxlambda=0;
for j=1:100
clambda1 =(j/100)*lambda;
crho=clambda1/mu;
cprofit(j)= clambda1*(R-c*log(clambda1)/teta+c*log(lambda)/teta
-((c*crho)/(mu*(1-crho))));
cprice(j)=(R-c*log(clambda1)/teta+c*log(lambda)/teta
-((c*crho)/(mu*(1-crho))));
clambda2(j)=clambda1;
if cprice(j)< R
criticalprofit = cprofit(j);
criticalprice = cprice(j);
criticallambda = clambda2(j);
criticalrho = clambda2(j)/mu;
break
end
end
for i=1:100
lambda1 =(i/100)*lambda;
rho=lambda1/mu;
profit(i)= lambda1*(R-c*log(lambda1)/teta+c*log(lambda)/teta
-(c*rho)/(mu*(1-rho)))
price(i)=(R-c*log(lambda1)/teta+c*log(lambda)/teta
-((c*rho)/(mu*(1-rho))))
lambda2(i)=lambda1;
```

```
if price(i)> R
price(i)=0;
profit(i)=0;
lambda2(i)=0;
end
if profit(i)>maxprofit
maxprofit=profit(i);
maxprice=price(i);
maxlambda=lambda2(i);
maxrho=maxlambda/mu;
else
maxprofit=maxprofit;
maxprice=maxprice;
maxlambda=maxlambda;
maxrho=maxlambda/mu;
end
end
disp(maxprice);
disp(maxprofit);
disp(maxlambda);
Eutility = R- maxprice - c (maxrho/((1-maxrho)*mu)
+exp((-teta)*maxrho/((1-maxrho)*mu))/teta - 1/teta)
```

B.3. Relationship Between θ and Effective Arrival Rate

```
clear all,close all;
lambda=1;
R=0.25;
c=0.1;
mu=1.5;
```

```
for k=1:100
teta = k/100
maxprofit=0;
maxprice=0;
maxlambda=0;
for j=1:100
clambda1 =(j/100)*lambda;
crho=clambda1/mu;
cprofit(j)= clambda1*(R-c*log(clambda1)/teta+c*log(lambda)/teta
-((c*crho)/(mu*(1-crho))));
cprice(j)=(R-c*log(clambda1)/teta+c*log(lambda)/teta
-((c*crho)/(mu*(1-crho))));
clambda2(j)=clambda1;
if cprice(j)< R
criticalprofit = cprofit(j);
criticalprice = cprice(j);
criticallambda = clambda2(j);
criticalrho = clambda2(j)/mu;
break
end
end
for i=1:100
lambda1 =(i/100)*lambda;
rho=lambda1/mu;
profit(i)= lambda1*(R-exp(mu*(1-rho)*log(lambda1)/teta-mu*(1-rho)
log(lambda)/teta+log(c)+log(rho)-log(mu)-log(1-rho)));
price(i)=(R-exp(mu*(1-rho)*log(lambda1)/teta-mu*(1-rho)*log(lambda)/teta
+log(c)+log(rho)-log(mu)-log(1-rho)));
lambda2(i)=lambda1;
if profit(i)>maxprofit
maxprofit=profit(i);
```

```
maxprice=price(i);
maxlambda=lambda2(i);
maxrho=maxlambda/mu;
else
maxprofit=maxprofit;
maxprice=maxprice;
maxlambda=maxlambda;
maxrho=maxlambda/mu;
end
end
if maxprice > R
maxprofit=criticalprofit;
maxprice=criticalprice;
maxlambda=criticallambda;
maxrho=criticalrho;
end
lambda3(k) = maxlambda;
teta1(k)= teta;
end
for k=1:100
teta = k/100
maxprofit=0;
maxprice=0;
maxlambda=0;
for j=1:100
clambda1 =(j/100)*lambda;
crho=clambda1/mu;
cprofit(j)= clambda1*(R-c*log(clambda1)/teta+c*log(lambda)/teta
-((c*crho)/(mu*(1-crho))));
cprice(j)=(R-c*log(clambda1)/teta+c*log(lambda)/teta
-((c*crho)/(mu*(1-crho))));
```

```
clambda2(j)=clambda1;
if cprice(j)< R
criticalprofit = cprofit(j);
criticalprice = cprice(j);
criticallambda = clambda2(j);
criticalrho = clambda2(j)/mu;
break
end
end
for i=1:100
lambda1 =(i/100)*lambda;
rho=lambda1/mu;
profit(i)= lambda1*(R-c*log(lambda1)/teta+c*log(lambda)/teta-
((c*rho)/(mu*(1-rho))));
price(i)=(R-c*log(lambda1)/teta+c*log(lambda)/teta
-((c*rho)/(mu*(1-rho))));
lambda2(i)=lambda1;
if profit(i)>maxprofit
maxprofit=profit(i);
maxprice=price(i);
maxlambda=lambda2(i);
maxrho=maxlambda/mu;
else
maxprofit=maxprofit;
maxprice=maxprice;
maxlambda=maxlambda;
maxrho=maxlambda/mu;
end
end
if maxprice > R
maxprofit=criticalprofit;
```

```
maxprice=criticalprice;
maxlambda=criticallambda;
maxrho=criticalrho;
end
lambda4(k) = maxlambda;
teta2(k)= teta;
end
plot(teta1, lambda3,'b+',teta2, lambda4,'r*');
xlabel('theta');
ylabel('optimal lambda');
```

B.4. Relationship Between θ and Possible Maximum Profit

```
clear all,close all;
lambda=1;
R=0.25;
c=0.1;
mu=1.5;
for k=1:100
teta = k/100
maxprofit=0;
maxprice=0;
maxlambda=0;
for j=1:100
clambda1 =(j/100)*lambda;
crho=clambda1/mu;
cprofit(j)= clambda1*(R-c*log(clambda1)/teta+c*log(lambda)/teta
-((c*crho)/(mu*(1-crho))));
cprice(j)=(R-c*log(clambda1)/teta+c*log(lambda)/teta
-((c*crho)/(mu*(1-crho))));
```

```
clambda2(j)=clambda1;
if cprice(j)< R
criticalprofit = cprofit(j);
criticalprice = cprice(j);
criticallambda = clambda2(j);
criticalrho = clambda2(j)/mu;
break
end
end
for i=1:100
lambda1 =(i/100)*lambda;
rho=lambda1/mu;
profit(i)= lambda1*(R-exp(mu*(1-rho)*log(lambda1)/teta-
mu*(1-rho)*log(lambda)/teta+log(c)+log(rho)-log(mu)-log(1-rho)));
price(i)=(R-exp(mu*(1-rho)*log(lambda1)/teta-mu*(1-rho)*log(lambda)/teta
+log(c)+log(rho)-log(mu)-log(1-rho)));
lambda2(i)=lambda1;
if profit(i)>maxprofit
maxprofit=profit(i);
maxprice=price(i);
maxlambda=lambda2(i);
maxrho=maxlambda/mu;
else
maxprofit=maxprofit;
maxprice=maxprice;
maxlambda=maxlambda;
maxrho=maxlambda/mu;
end
end
if maxprice> R
maxprofit=criticalprofit;
```

```
maxprice=criticalprice;
maxlambda=criticallambda;
maxrho=criticalrho;
end
profit1(k) = maxprofit;
teta1(k)= teta;
end
for k=1:100
teta = k/100
maxprofit=0;
maxprice=0;
maxlambda=0;
for j=1:100
clambda1 =(j/100)*lambda;
crho=clambda1/mu;
cprofit(j)= clambda1*(R-c*log(clambda1)/teta+c*log(lambda)/teta
-((c*crho)/(mu*(1-crho))));
cprice(j)=(R-c*log(clambda1)/teta+c*log(lambda)/teta
-((c*crho)/(mu*(1-crho))));
clambda2(j)=clambda1;
if cprice(j)< R
criticalprofit = cprofit(j);
criticalprice = cprice(j);
criticallambda = clambda2(j);
criticalrho = clambda2(j)/mu;
break
end
end
for i=1:100
lambda1 =(i/100)*lambda;
rho=lambda1/mu;
```

```
profit(i)= lambda1*(R-c*log(lambda1)/teta+c*log(lambda)/teta
-(c*rho)/(mu*(1-rho));
price(i)=(R-c*log(lambda1)/teta+c*log(lambda)/teta
-((c*rho)/(mu*(1-rho))));
lambda2(i)=lambda1;
if profit(i)>maxprofit
maxprofit=profit(i);
maxprice=price(i);
maxlambda=lambda2(i);
maxrho=maxlambda/mu;
else
maxprofit=maxprofit;
maxprice=maxprice;
maxlambda=maxlambda;
maxrho=maxlambda/mu;
end
end
if maxprice > R
maxprofit=criticalprofit;
maxprice=criticalprice;
maxlambda=criticallambda;
maxrho=criticalrho;
end
profit2(k) = maxprofit;
teta2(k)= teta;
end
plot(teta1, profit1,'b+',teta2, profit2,'r*');
xlabel('theta');
ylabel('maximum profit');
```

B.5. Relationship Between θ and Expected Utility of Customers

```
clear all,close all;
lambda=1;
R=0.25;
c=0.1;
mu=1.5;
for k=1:100
teta = k/100
maxprofit=0;
maxprice=0;
maxlambda=0;
for j=1:100
clambda1 =(j/100)*lambda;
crho=clambda1/mu;
cprofit(j)= clambda1*(R-c*log(clambda1)/teta+c*log(lambda)/teta
-((c*crho)/(mu*(1-crho))));
cprice(j)=(R-c*log(clambda1)/teta+c*log(lambda)/teta
-((c*crho)/(mu*(1-crho))));
clambda2(j)=clambda1;
if cprice(j)< R
criticalprofit = cprofit(j);
criticalprice = cprice(j);
criticallambda = clambda2(j);
criticalrho = clambda2(j)/mu;
break
end
end
for i=1:100
lambda1 =(i/100)*lambda;
rho=lambda1/mu;
```

```
profit(i)= lambda1*(R-exp(mu*(1-rho)*log(lambda1)/teta-mu*(1-rho)
log(lambda)/teta+log(c)+log(rho)-log(mu)-log(1-rho)));
price(i)=(R-exp(mu*(1-rho)*log(lambda1)/teta-mu*(1-rho)*log(lambda)/teta
+log(c)+log(rho)-log(mu)-log(1-rho)));
lambda2(i)=lambda1;
if profit(i)>maxprofit
maxprofit=profit(i);
maxprice=price(i);
maxlambda=lambda2(i);
maxrho=maxlambda/mu;
else
maxprofit=maxprofit;
maxprice=maxprice;
maxlambda=maxlambda;
maxrho=maxlambda/mu;
end
end
if maxprice > R
maxprofit=criticalprofit;
maxprice=criticalprice;
maxlambda=criticallambda;
maxrho=criticalrho;
end
utility1(k)=R-maxprice-c*(maxrho/((1-maxrho)*mu))
teta/(mu*(1-maxrho)+teta);
teta1(k)= teta;
end
for k=1:100
teta = k/100
maxprofit=0;
maxprice=0;
```

```
maxlambda=0;
for j=1:100
clambda1 =(j/100)*lambda;
crho=clambda1/mu;
cprofit(j)= clambda1*(R-c*log(clambda1)/teta+c*log(lambda)/teta
-((c*crho)/(mu*(1-crho))));
cprice(j)=(R-c*log(clambda1)/teta+c*log(lambda)/teta
-((c*crho)/(mu*(1-crho))));
clambda2(j)=clambda1;
if cprice(j)< R
criticalprofit = cprofit(j);
criticalprice = cprice(j);
criticallambda = clambda2(j);
criticalrho = clambda2(j)/mu;
break
end
end
for i=1:100
lambda1 =(i/100)*lambda;
rho=lambda1/mu;
profit(i)= lambda1*(R-c*log(lambda1)/teta+c*log(lambda)/teta
-((c*rho)/(mu*(1-rho))));
price(i)=(R-c*log(lambda1)/teta+c*log(lambda)/teta
-((c*rho)/(mu*(1-rho))));
lambda2(i)=lambda1;
if profit(i)>maxprofit
maxprofit=profit(i);
maxprice=price(i);
maxlambda=lambda2(i);
maxrho=maxlambda/mu;
else
```

```
maxprofit=maxprofit;
maxprice=maxprice;
maxlambda=maxlambda;
maxrho=maxlambda/mu;
end
end
if maxprice > R
maxprofit=criticalprofit;
maxprice=criticalprice;
maxlambda=criticallambda;
maxrho=criticalrho;
end
utility2(k) = R - maxprice - c * (maxrho/((1-maxrho)*mu))
teta / (mu*(1-maxrho)+teta);
teta2(k)= teta;
end
plot(teta1, utility1,'b+',teta2, utility2,'r*');
xlabel('theta');
ylabel('expected utility');
```

APPENDIX C: MATLAB CODE FOR M/M/1 SERVICE SYSTEMS WITH RANDOM REVENUE AND PENALTY COST PARAMETERS

C.1. Optimal Price Calculation with Random Revenue and Penalty Cost Parameters

```
clear all,close all;
lambda=1;
mu=1;
mu_c=0.1;
cv_c=0;
var_c=(mu_c * cv_c)^2;
mu_R=1.5;
cv_R=0.1;
var_R=(mu_R * cv_R)^2;
corr=0;
cov=corr*sqrt(var_R*var_c);
givenprice=1.2;
maxprofit=0;
maxprice=0;
maxlambda=0;
for i=1:10000
lambda1 =(i/10000)*lambda;
rho=lambda1/mu;
price(i)= norminv((1-lambda1/lambda),mu_R-(rho/((1-rho)*mu))*mu_c,
sqrt((var_R + var_c * (rho/((1 - rho) * mu))^2)-2*cov*(rho/((1-rho)*mu))));
profit(i)= lambda1* norminv((1-lambda1/lambda),mu_R-(rho/((1-rho)*mu))
mu_c,(sqrt(var_R + var_c * (rho/((1 - rho) * mu))^2)-2*cov*(rho/((1-rho)*mu))));
lambda2(i)=lambda1;
```

```
if price(i)<givenprice
ourlambda=lambda2(i-1);
ourprice=givenprice;
ourprofit=ourlambda*ourprice;
break;
end
end
disp(ourlambda);
disp(ourprice);
disp(ourprofit);
```

C.2. Effective Arrival Rate with Given Price

```
clear all,close all;
lambda=1;
mu=1;
mu_c=1;
cv_c=0.3;
var_c=(mu_c*cv_c)^2;
mu_R=5;
cv_R=0.3;
var_R=(mu_R*cv_R)^2;
corr=0.8;
cov=corr*sqrt(var_R*var_c);
maxprofit=0;
maxprice=0;
maxlambda=0;
for i=1:10000
lambda1 =(i/10000)*lambda;
rho=lambda1/mu;
```

```
if sqrt(var_R + var_c * (rho/((1 - rho) * mu))^2 - 2 * cov * (rho/((1 - rho) * mu)))==0
if mu_R - mu_c*(rho/((1-rho)*mu))<0
profit(i)=0;
lambda2(i)=0;
else
price(i)=mu_R - (rho/((1 - rho) * mu)) * mu_c;
profit(i)=lambda1 * (mu_R - (rho/((1 - rho) * mu)) * mu_c);
lambda2(i)=lambda1;
if profit(i)>maxprofit
maxprofit=profit(i);
maxprice=price(i);
maxlambda=lambda2(i);
maxrho=maxlambda/mu;
else
maxprofit=maxprofit;
maxprice=maxprice;
maxlambda=maxlambda;
maxrho=maxlambda/mu;
end
end
else
if mu_R-mu_c*(rho/((1-rho)*mu))<0
profit(i)=0;
lambda2(i)=0;
else
price(i)= norminv((1-lambda1/lambda),mu_R-(rho/((1-rho)*mu))*mu_c,
sqrt((var_R + var_c * (rho/((1 - rho) * mu))^2)-2*cov*(rho/((1-rho)*mu))));
profit(i)= lambda1* norminv((1-lambda1/lambda),mu_R-(rho/((1-rho)*mu))*
mu_c,sqrt(var_R + var_c * (rho/((1 - rho) * mu))^2-2*cov*(rho/((1-rho)*mu))));
lambda2(i)=lambda1;
if profit(i)>maxprofit
```

```
maxprofit=profit(i);
maxprice=price(i);
maxlambda=lambda2(i);
maxrho=maxlambda/mu;
else
maxprofit=maxprofit;
maxprice=maxprice;
maxlambda=maxlambda;
maxrho=maxlambda/mu;
end
end
end
end
disp(maxrho/((1-maxrho)*mu));
disp(mu_R-(maxrho/((1-maxrho)*mu))*mu_c);
disp(sqrt(var_R + var_c * (maxrho/((1 - maxrho) * mu))^2
-2*cov*(maxrho/((1-maxrho)*mu))));
if sqrt(var_R + var_c * (maxrho/((1 - maxrho) * mu))^2-
2*cov*(maxrho/((1-maxrho)*mu)))==0
a=0;
disp(a);
else
disp(norminv((1-maxlambda/lambda),0,1));
end
disp(maxlambda);
disp(maxprice);
disp(maxprofit);
```

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