AN INTRODUCTION TO THE DIFFERENTIAL-INTEGRAL FORMULATION OF THE LABOUR-THEORY OF VALUE

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1. FOREWORD

In this study, we analyse the foundations of the labour-theory of value. After discussing and assigning certain dimensions to the fundamental concepts, we obtain the basic dimensional relations in section 3. Furthermore, on the basis of a particular macro-economic model, we discuss the consistency and coherence of these dimensional relations in section 4.

In section 5, by using the techniques of differential-integral calculus, we obtain the equations of commodity M, labour E, and value D from the dimensional relations. These differential-integral equations yield a dynamical description of the economic model that we have considered.

In section 6, we briefly summarize our results. Furthermore, we discuss a justification of the (theoretical) methodology that we have used in the present study. In addition, we briefly consider the problem of measurement in relation to our theoretical framework of the labour-theory of value.

This study in an introduction to the differential-integral formulation of the labour-theory of value. We believe that we have presented a new perspective that would be fruitful for further developments.

I would like to give my special thanks to Dr. Yalçın Koç, Associate Professor in Philosophy, for his contribution to my understanding of Science in general. His conceptual contribution to this study has a major importance and has enabled a new scientific approach to the labour-theory of value. This scientific approach consists of the simultaneous consideration of the theoretical methods of natural sciences, foundations of the labour-theory of value, and certain philosophical concerns.

I would also like to thank my thesis advisor Dr. Deniz Gökçe without whose help this study would not have been completed. His extremely attractive lectures and the discussions we have made on the methodology of economics have been the major motivation of this study.

Finally, I would like to thank my family for their invaluable support, my sister Ayşe İnhan for her patience in our discussions and for her exquisite coffees, and Bakiye Takmaz, for her cooperation in typing the manuscript.

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CONTENTS

LIST OF FIGURES

2. INTRODUCTION

In any study on the foundations of Economics, the primary concept that requires a thorough understanding is the concept of value. Furthermore, analysis of the description of value formation is essential. A particular theory's understanding of economic reality (i.e., totality of economic facts) certainly depends on how the theory concieves the concept of value and the process of value formation. In this study, we are concerned with the foundations of the labour-theory of value.

The labour-theory of value has been first discussed by Adam Smith. A more systematic account, however, was developed by David Ricardo in the first decades of the nineteenth century and by the second half of the same century, the explicit foundations of the labour-theory of value were laid.

Recently, the labour-theory of value has become a subject of rigorous analysis in view of its fundamental postulates and in opposition to the neo-classical utility-value theory.(1)

The labour-theory of value aims at a formulation of explicit relations between the concepts of labour-power, commodity and value so that one can obtain the ratios of exchange and objective measures of exchange between commodities in an economy where commodities are produced by labour-power and other commodities.(2) The ratio of exchange is determined by an independent variable common to both commodities in the exchange. This objective factor is defined as the necessary

(1) See Meek (1956), Seton (1957), Morishima (1961,1973), Akyüz (1980a)
(2) For the basic conceptual considerations of the labour-theory of value, see: Schumpeter (1972), Seligman (1962), Divitçioğlu (1982) and Akyüz (1980b).

amount of labour used in the production of that specific commodity. Thus, it is accepted that the exchange value of a commodity (or value, in general, under the defined conditions of commodity production) is equal to the time spent by the necessary amount of labour in its production. In other words, in the process of the exchange of commodities in the market, actually, according to the labour-theory of value, the quantities of labour (which are necessary to produce the exchanged commodities) are exchanged.

By means of the above considerations and the assumptions that commodities are exchanged on the basis of the value criteria which we defined above, the labour-theory of value constructs a general model where the inputs are commodities and labour-power. This model, furthermore, is considered as the recycling of the chain of production, where all inputs and outputs are given in terms of commodities whose values are expressed in terms of the labour time spent on them. Thus, in the production of each commodity, there is a direct contribution of labour through human labour-power, and an indirect contribution of labour through the means of production used.

In every production cycle, the value of the output must be greater than the value of the input so that the concepts of surplus and growth of the economy are non-trivial. The input commodities (i.e., means of production and labour-power) are termed as the initial Capital, and the output as the final Capital. For Capital to grow, the labour-theory of value claims that there must be a mechanism within the production cycle which increases the initial capital. The labour-theory of value consequently argues that the initial capital increases not through the unequal exchange in the market, but on the contrary, through a certain economical mechanism within the production cycle, assuming equal exchange on the basis of the objective value.

The difference between the value of the final products and the value of the total input is termed as the surplus value. According to the labour-theory of value, surplus value is obtained by the difference between the value which the labour-power creates during the production period and the value of the labour-power itself. In this statement, there is the implicit assertion that the labour-power is such an entity that it actually creates more value than the value potentially attributed to itself.

The value of labour-power is defined as the necessary amount of labour required to produce the goods with which the labour-power can reproduce itself under the given social conditions. The concept of the value of labour-power corresponds to the concept of wage. Although the concept of wage corresponds to the value of potential labour-power, it does not correspond to the value this labour-power creates during the period of production. The latter is generally greater than the former; the difference between them is termed as the surplus value. It is through this distinction between labour-power and labour that one can start understanding the conceptual framework of the labour-theory of value.

The consistency of the labour-theory of value under specific assumptions and the possibility of the transformation of values to prices has been a subject of constant debate for nearly a century. Analyses have been made as to what happens to the theory when prices deviate from values, when wages deviate from the value of labour-power, and when profits deviate from individual surplus values. Furthermore, it is still an issue of debate wether the theory is still consistent when the above deviations occur under the conditions of different technologies in different industries and when the economy resides on an equilibrium profit rate (1).

(1) There have been different solutions to such problems; See Bortkiewicz (1952), Winternitz (1948), Meek (1956), Seton (1957) Morishima and Seton (1961), Morishima (1973).

In order to bring forth a solution to the above mentioned problems and to develop a consistent model with which practical applicability is possible, an extensive use of Mathematics is required. Mathematical abstraction not only enables a description of the mechanism under study, but also leads the way to results which could not be previously foreseen. Mathematics allows the selection of the crucial parameters which determine the behaviour of the model and thus enables the testing of the model.

The models describing the labour-theory of value have, almost exclusively been expressed with the methods of linear algebra. The most recent ones being matrix algebra analyses using Leontiev matrices and production coefficients.(1) These static models are considered as adequate abstractions to prove the consistency of the main premises and the transformability of values and prices. (2) By means of such static models however, one cannot obtain a dynamic description; it is not possible to calculate the rate of change of the variables with respect to time.

A dynamical description of the economical situation seems, in general, to be closer to reality than the static description. Thus, in order to construct a dynamical description and to overcome the above mentioned difficulties and inadequacies, we found it absolutely necessary to proceed in the following manner:

First, a dimensional analysis of the fundamental concepts of the labour-theory of value is absolutely necessary. A clear understanding of the fundamental concepts, consistency

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(1) See Morishima and Seton (1961), Morishima (1973), Akyüz (1980)

(2) In our approach to the labour-theory of value, the latter does not constitute a conceptual problem; see section 3.

of the conceptual framework and the relations of these concepts with each other as well as with the facts of reality is the very first essential step in the construction of a sound theoretical analysis. Without the proper dimensional definitions of the basic concepts and their basic functions, it is impossible to develop the proper mathematical tools to reconstruct the mathematical framework of the labour-theory of value. Dimensional analysis gains further importance when we consider the fact that in Economics, every theory has developed its own concepts and their dimensional relations and interpretations.(1) Thus, the first step in our study consists of the construction of the dimensional framework of concepts such as Labour (E), Labour-power (E_g), Product (U), etc., and the basic dimensional equations.

We, then, construct a model to which the labour theory of value is applied. Although this model is simple, it has a generalised form from a global point of view. On the basis of this model and dimensional analysis, we obtain other complex concepts such as surplus value. Furthermore, the parameters of price and wage in distinction to the value of commodity and the value of labour-power are introduced in order to consider deviations of these parameters from values in our model.

We, then, give new definitions of the basic concepts in view of the dynamical description of our model, by using the methods of differential-integral calculus. We discuss the application of these dynamical parameters to our model by analysing the meaning of these abstractions we have made. Our analysis, however, is by no means complete. This study is just a brief introduction to a new conceptual analysis of the labour-theory of value.

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(1) See De Jong (1967 pp.21-23)

3. DIMENSIONAL ANALYSIS

In a foundational study, it is absolutely essential to analyse the main concepts of a theory with respect to well defined dimensions, and to specify the relations among these concepts in a dimensional context. Dimensional analysis provides a method to check the inner consistency of a theory.

Different theories, however, assign different meanings to certain concepts. For example, the concept of "Capital" has a different definition in the labour theory of value than it has in the neo-classical approach. A discussion of how these concepts are related to reality and to what degree they reveal it belongs to the foundations of Economics and depends on an analysis of how different theories see the world differently. Such a discussion, obviously, requires dimensional analysis as well.

A dimension can be defined as a set of additive quantities (De Jong, 1967, p.7). Any quantity is considerd as a number (representing the result) of a measurement multiplied by a certain unit.

The choice of dimensions is not arbitrary but depends totally on the foundations of the theory, that is, on how the theory describes the world.

We classify the concepts (i.e., the parameters) into two groups: (a) primitive concepts (or, irreducible concepts) which correspond to the primary dimensions, and (b) complex concepts (or, molecular concepts) which correspond to a coherent agglomeration of the primary dimensions.

In classical mechanics, for instance, mass M, length L and time T are the primary dimensions. The complex concepts (or, parameters) can be defined in terms of M, L and T. The dimension of velocity v (i.e., $[v]$) (1) is a compound of length L and time T such that:

$$[v] = [LT^{-1}] \qquad \ldots (3.1)$$

Thus, velocity v is a complex (or, molecular) parameter, whereas length L and time T are primitive (or, irreducible) concepts.

There are also dimensionless entities which are obtained when quantities of the same dimension are divided by each other. These are ratios which can be expressed with numbers since they have no units. Furthermore, constants pertaining to a certain theory can be considered as dimensionless entities of that theory. Dimensionless quantities by means of which one writes proper algebraic equalites, however, play no important role in dimensional analysis.

Every theory in Economics chooses a different set of primitive parameters. As a simple example, let us consider Irving Fisher's equation of exchange:

$$MV = PT \qquad \ldots (3.2)$$

In this equation, M symbolises the net stock of money, V, its velocity (or rather, its frequency t^{-1}, where t denotes proper time) of circulation, P, the general price level, and T (trade volume), the flow of goods. The dimension of T is assigned to be $[R]$, that is the set of all goods including labour. Thus, the theoretical concepts are based on three primary dimensions: M, R and t.

J. M. Keynes, on the other hand, draws a distinction between investment goods, consumption goods and labour. The first two can be classified as produced goods to which the

(1) $[A]$ indicates the dimension of A.

dimension of $[R_p]$ can be assigned. Labour, on the other hand, has the dimension of $[R_a]$. These constitute the two primary dimensions in the Keynesian theory. (1) We will now consider the dimensional analysis of the labour theory of value.

The theoretical framework of the labour-theory of value is constructed on the basis of the following concepts: Product U, exchange Ex, commodity M, labour E, labour-power E_g, time t, and value D.

Among the above, labour-power E_g and time t are the primitive concepts to which we assign the dimensions of $[E_g]$ and $[t]$ respectively. We will show that all other complex concepts of the labour-theory of value can be expressed in terms of these two dimensions.

A dimensional analysis of the conceptual framework of the labour-theory of value yields the following definitions:

Df.1 $[M] = [U\ (Ex)]$

Df.2 $[E] = [Ut]$

Df.3 $[E_g] = [Et^{-1}]$

Df.4 $[D] = [EU^{-1}]$

The first definition is a direct outcome of the definition of a <u>commodity</u> in the labour-theory of value. According to this definition, a <u>commodity</u> is a <u>product</u> which satisfies the following conditions:

(1) For further details, see: De Jong (1967).

1) A product which is produced to be exchanged
2) A product which is produced to satisfy a socially determined human need.(1)

From the first necessary condition in the definition of a commodity, it can be seen that without the process of exchange, the concept of commodity cannot be defined. For notational simplicity, however, we will assume hereafter in our analysis that the dimension of Ex is an identity element such that:

$$[M] = [U] \qquad \ldots (3.3)$$

Equation (3.3) means that the concept of commodity has the dimension of the concept of product [U] . From now on, we will use [U] as the dimension corresponding to the concept of commodity.

The concept of commodity is the starting point of abstraction in the labour-theory of value. The frame of analysis within which the labour-theory of value takes shape is described as commodity production.

The concept of commodity production has a historical content. That is, it has changed its character with certain stages in history. Without considering the details of this historical development, we can assert that, at present, what marks the characteristics of commodity production is the fact that labourpower has become a commodity in the market, deprived from the ownership of the means of production. This means that, in the present frame of analysis, labour-power is an input commodity taking part in the process of production, and that it will have the same properties as any other commodity. By substituting Df.2 into Df.3, one obtains that the dimension of labour-power E_g is identical to the dimension of product U, which can be exchanged as a commodity in the market.

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(1) See Morishima (1973), Akyüz (1980)

From the two main properties defining the concept commodity, the labour-theory of value derives two distinct notions of value related to every commodity. These notions are the exchange value, which is created in the context of exchange relations, and the use-value, which related to the commodity's property of satisfying a socially determined human need. The use-value is a general property of a commodity and is independent of the mode of production. Human needs are rich in variety and quality depending on many factors such as income, culture, production and distribution relations etc. Thus, the use-value of a commodity is said to be socially determined. A commodity can have different use-values depending on whether it is to be used during consumption or whether as an input to a production process. The labour-theory of value thus concludes that the use-value by itself is inadequate in reflecting the social relations and the mechanism behind the phenomenon of exchange. Hence, the labour-theory of value analyses the formation of value in the context of exchange. According to the labour-theory of value, exchange has to occur on equal terms. That is, assuming that the simplest form of exchange consists of a binary relation, equal quantities have to contribute from both sides in the process of exchange. But since the use-values taking part in exchange are incomparable, there must be something general, common to both commodities, something which can constitute an objective measure for all commodities regardless of their use-values, so that a quantitative comparison can be made. This comparable objective measure which determines the ratio of exchange is defined to be the socially necessary human labour used in a certain period of time to produce a given commodity.

Human labour as such, is directly proportional to the quantity of commodity produced and also to the time spent in the transformation of labour-power to labour during the production of the given commodity. This proportionality is expressed in Df.2.

An analogy can be drawn between the dimensional equation of labour and work in classical mechanics. In classical mechanics, work W is equal to the product of force F and distance:

$$W = Fd \qquad \ldots (3.4)$$

Although the square of time t takes part inversely in acceleration a which is implicit in force F, the concept of time in Physics and the concept of labour-time in the labour-theory of value should not be confused. The time mentioned in the labour-theory of value is the time spent in the transformation of labour-power to labour during production. In a sense, such a time plays the role of distance d in the formula for work W in eqn. (3.4). This is the reason why there is a direct proportionality between labour and time. Had there been an inverse proportionality, we should be in a position to say that labour increases as the time spent in the transformation of labour-power to labour decreases; such a statement is obviously wrong. Consequently, the proportionality is direct.

The socially necessary labour which generate the exchange-value (or, value under the defined conditions of commodity production) is an abstract entity.

Value, as in Df.4, is defined as the necessary average simple labour in the production of one unit of a commodity in an industry under average social production conditions (i.e., technical level of the industry, labour intensity, labour productivity etc.). This abstract labour can be viewed as the consummation of general average human labour-power under the given technological conditions. Thus, the socially necessary labour is not a standart constant like the speed of light c in physics. There exists no standart measure of value in Economics much to the disappointment of Ricardo who felt the need to specify a measure of value which would itself be invariant. Such a perfect measure of value does not exist. The measure of value that we have discussed so far is a dynamic one by its very nature; it changes with the changing conditions of technology and production.

Df.3 makes a very important distinction between labour-power and labour. This distinction is crucial for the labour-theory of value. Labour-power can be described as the ability to generate labour and consequently commodities, analogous to the concept of power in mechanics. Labour, on the other hand, analogously corresponds to the work done in actuality. Labour-power is a potential while labour is actual. Labour is the realisation of the labour-power during the labour-time (or, production time) t.

Labour-power is inversely proportional to time. From Df.3, one obtains that

$$[E] = [E_g \, t] \qquad \ldots (3.5)$$

This crucial distinction between labour E and labour-power E_g is not explicitly considered in the literature on the foundations of the labour-theory of value. The Neo-Classical theory, on the other hand, defines the concept of wage as the return to labour for its contribution to production. When the distinction between labour-power and labour is made, this assertion of the Neo-classical theory becomes paradoxical, since that which is not actual (i.e., labour-power) cannot have an actual value.

In the labour theory of value, however, value is attributed not to labour, but to the labour-power. Since the labour-power E_g and time t are the primitive concepts of the labour theory of value, by substituting Df.3 into Df.4, we clearly see that value D is proportional to $(E_g/U)t$. Consequently, what is essential to the description of economic reality is not the value of labour, but the value of labour-power. It is in this sense that labour-power is a commodity whereas labour itself is not. Thus, wages should be thought of as corresponding not to the value of labour (which is paradoxical as in the Neo-classical theory), but to the value of labour-power.

Without the above distinction between labour and labour-power, it is not possible to understand the concept of surplus in the labour theory of value. The value of labour-power, and the value created when this labour-power is realised in time t are totally distinct entities and their difference constitutes the concept of surplus. The Neo-classical school, however, by ignoring such a fundamental distinction, excludes the concept of surplus from its vocabulary.

The last dimensional equation in Df.4 is closely related to the above discussions, value is expressed as the amount of labour used in the production of a unit of commodity. value can also be expressed by substituting the equivalent of $[E]$ in Df. 3 into Df.4:

$$[D] = \left[\frac{E_g t}{U}\right] \qquad \text{... (3.6)}$$

Let us further investigate the dimentional equations. By substituting Df.2 into Df.3, we obtain:

$$[E_g] = \left[\frac{U.t}{t}\right] = [U] \qquad \text{... (3.7)}$$

The result in egn. (3.7) corresponds with the previous assertion that, under the given conditions of commodity production, labour-power is a commodity; therefore, it has the dimension of $[U]$

By substituting egn. (3.7) into egn. (3.6), we find that:

$$[D] = \left[\frac{Ut}{U}\right] = [t] \qquad \ldots (3.8)$$

This means that value has the dimension of time; and therefore should be expressed with the unit of time. This result also corresponds with the definition of value in the literature of the labour-theory of value (1).

It should be emphasized that the exchange-value of a commodity cannot be measured in isolation according to the abstract labour used in its production, but only through the means of another commodity (i.e., the product U) which is quantitatively (or rather, dimensionally) identical to labour -power E_g.

In general, value of a commodity is measured in terms of money. In the labour theory of value, money is defined as a particular commodity whose use-value specifies the exchange-value of other commodities. Let $[P]$ denote the dimension of money; then :

Df.5 $[P] = [U]$

(1) See Schumpeter (1972), Seligman (1963), Akyüz (1980)

The following model is a simple representation of the production process as the labour-theory of value describes it:

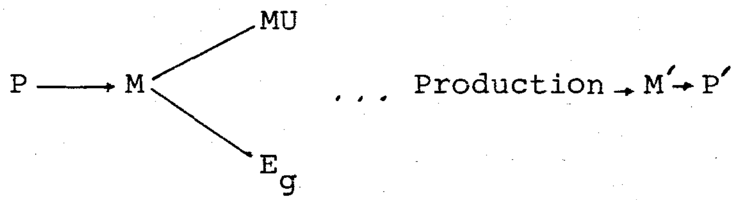

Figure 1

Let us consider the above model as an abstraction of the cycle of production. In the beginning, a quantity P of money is transformed into a quantity M of commodities through exchange. In fact, M is constituted by labour-power E_g and the means of production MU. During the process of production, the labour-power E_g is transformed into labour E. By means of the means of production MU and the transformation of E_g into E, the output commodity bundle M' is created during time t. M' is in turn converted to money P' and the cycle is completed.

In view of the dimensional analysis we have made, it is possible to express P, M, MU, M' and P' in terms of the dimension of labour-power E_g. In other words, money, the means of production, the output and obviously the labour-power in the process of production all have the dimension $[E_g]$. Consequently, we observe that the entities in the above model are conceptually (or, dimensionally) coherent and consistent.

The labour-theory of value asserts that if the value of M' (i.e., $D_{M'}$) is greater than the value of M (i.e., D_M), then there would be economic growth in the above model in direct proportion to $D_{M'} - D_M$. The difference $D_{M'} - D_M$ is termed as the surplus-value and is directly proportional to the difference between the value created by labour-power E_g (i.e., $D_{M'}$) and the value of labour-power itself (i.e., D_{E_g}). The implicit proposition in this assertion is that surplus value is generated within the production cycle while value is conserved during exchange.

It is clear that the dimension of surplus-value AD is that of value D:

$$\text{Df.6} \qquad [AD] = [D]$$

We should define, however, another important concept which is linked with the concept of surplus value; this is the concept of capital. The quantity of commodities with which production starts is termed as the initial capital, while the quantity obtained in the end is the final capital. As we observed in the above model expressed by figure 1, the initial capital is increased in terms of value. This property marks the definition of capital. That is, the labour-theory of value defines capital as value which creates surplus-value.

The concept of capital is dynamic in the following sense. In the beginning of the production process, it is a quantity of money; then, it is transformed into the means of production MU and the labour-power E_g. After the production process has ended, it is a new bundle of output commodities M' , which is in turn transformed to money P' , being increased in direct proportion to the difference $D_{M'} - D_M$. Thus, the dimension of capital is also that of $[E_g]$, in consistence with other entries in the model in figure 1.

(It should be remembered that, $[U] = [E_g]$; see eqn. (3.7).)

Furthermore, a distinction between price and the value of a commodity , and also between wage and the value of labour-power is necessary if one expects the above model to describe the reality. According to the labour-theory of value, prices and wages can deviate from the values of the corresponding commodities. Consequently, one must consider the application of different technologies among industries. There is a similarity in the case of the deviations of the profit from individual surpluses.

One of the main areas of research on the foundations of the labour-theory of value is on the mechanism of the conservation of value under the above described conditions. On the literature on the labour-theory of value, this area of research is called the Transformation Problem. For a solution of this problem, a dimentional compatibility between the system of values and that of deviated prices (or, prices of production) is necessary.

In view of the dimensional analysis in the present study, prices pri, wage wag and profit pro have the dimension of value D:

Df.7 $[pri] = [D]$

Df.8 $[wag] = [D]$

Df.9 $[pro] = [D]$

Thus, from a conceptual point of view, the transformation problem is not a genuine problem, since prices have the same dimension as values. It seems that this problem is an indirect outcome of the particular model and the mathematical methods used in its analysis.(1)

A different solution to the transformation problem, however, is beyond the scope of the present study. As it has been already mentioned, our purpose in the present section is to develop a dimensional analysis of the fundamental conceptsof the labour-theory of value. On the basis of this dimensional analysis, we will analyse a simple macro-model in the following section.

(1) For the solutions of the transformation problem, see Bortkiewicz (1952); Winternitz (1948); Meek (1956), Seton (1957), Morishima (1973); Morishima and Seton (1961), Akyüz (1980).

4. A SIMPLE MACRO-MODEL

In this section, we analyse the process of production and derive the mathematical expressions for complex concepts such as surplus-value in the context of a macro-economic model. The model we will construct is simple. However, it is a very general macro-model with consistent initial assumptions. To this model, we apply the technique of dimensional analysis developed in the previous section.

Our model is a two-sector model. The first sector produces means of production whereas the second sector produces consumer goods. In each sector, the inputs are means of production and labour-power.

The model represents a cycle of production with an aggregate production time t. The means of production taking part and the labour-power used is totally consumed within the production process. The labour-power in both sectors is homogeneous; that is, the time necessary to reproduce the total labour-power used in the process of production for both sectors is t'

Let us now describe the production cycle with the following picture:

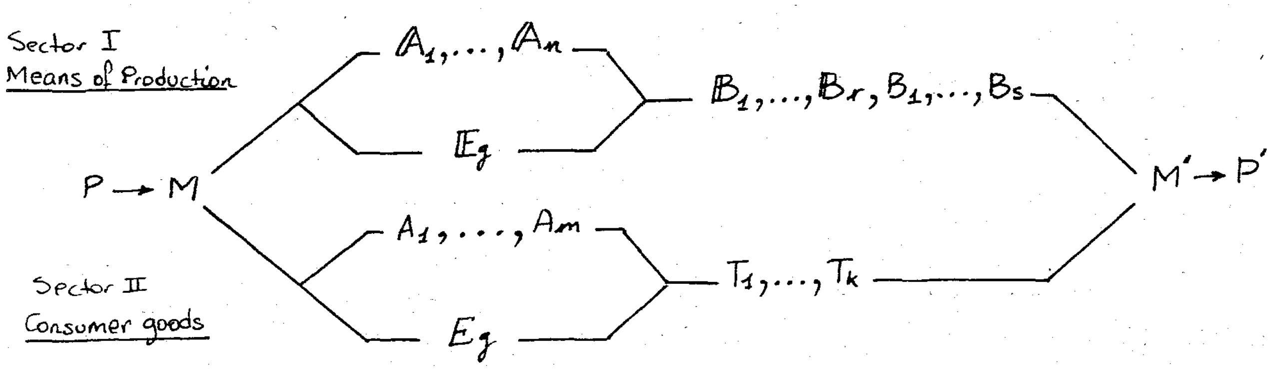

Figure 2

This is an aggregative model where $\mathbb{A}_1, \ldots, \mathbb{A}_n$ are the means of production used in the production of new means of production $\mathbb{B}_1, \ldots, \mathbb{B}_r$ for sector I and $B_1, \ldots, B_s$ for sector II. $A_1, \ldots, A_m$ are the means of production to be used in the production of consumer goods $T_1, \ldots, T_k$. $\mathbb{E}_g$ is the total labour-power used in sector I and E_g is the total labour-power used in sector II.

We are interested in obtaining the total surplus value in the model in figure 2. In view of our discussion in section 3, total surplus is the difference between the total value of the output and the total value of the input.

To express value D in general, we use the relation in Df.4 in section 3:

$$[D] = \left[\frac{E}{U}\right] \qquad \ldots (4.1)$$

To calculate the value of the total input, we shall first express all inputs in terms of labour and then divide this expression by the total input in terms of commodity (or, product). From Df.2 in section 3, we know that:

$$[E_I] = [U_I t] \qquad \ldots (4.2)$$

Thus :

$$E_I = \sum_{i=1}^{n} (\mathbb{A}_i)t + \sum_{j=1}^{m} (A_j)t + \mathbb{E}_g t' + E_g t' \qquad \ldots (4.3)$$

$$U_I = \sum_{i=1}^{n} \mathbb{A}_i + \sum_{j=1}^{m} A_j + \mathbb{E}_g + E_g \qquad \ldots (4.5)$$

In these expressions, t denotes the total production time (or, labour-time) spent in which the means of production are totally consumed, and t' denotes the total labour-time necessary to reproduce the total labour-power input to the production process. In other words, t' expresses the value of the total labour-power, or total wage bill in value terms.

The value of total input D_I will then be:

$$D_I = \frac{\sum_{i=1}^{n}(\mathbb{A}_i)t + \sum_{j=1}^{m}(A_j)t + \mathbb{E}_g t' + E_g t'}{\sum_{i=1}^{n}\mathbb{A}_i + \sum_{j=1}^{m}A_j + \mathbb{E}_g + E_g} \quad \ldots (4.5)$$

If we make the simplifying assumption that n=m we have :

$$D_I = \frac{\sum_{i=1}^{n}(\mathbb{A}_i + A_i)t + (\mathbb{E}_g + E_g)t'}{\sum_{i=1}^{n}(\mathbb{A}_i + A_i) + (\mathbb{E}_g + E_g)} \quad \ldots (4.6)$$

The calculate the value of the total output Do, we proceed in a similar manner:

$$D_o = \frac{\sum_{i=1}^{r}(\mathbb{B}_i)t + \sum_{i=1}^{s}(B_i)t + \sum_{i=1}^{k}(T_i)t}{\sum_{i=1}^{n}\mathbb{B}_i + \sum_{i=1}^{s}B_i + \sum_{i=1}^{k}T_i} = t \quad \ldots (4.7)$$

Surplus-value AD is the difference between

D_I and Do:

$$AD = Do - D_I \quad \ldots (4.8)$$

Therefore, from eqns. (4.6) and (4.7), we obtain:

$$AD = t - \frac{\sum_{i=1}^{n}(\mathbb{A}_i + A_i)t + (\mathbb{E}_g + E_g)t'}{\sum_{i=1}^{n}(\mathbb{A}_i + A_i) + (\mathbb{E}_g + E_g)} \quad \ldots (4.9)$$

Simplifying eqn. (4.9), we obtain:

$$AD = t\left[1 - \frac{\sum_{i=1}^{n}(\mathbb{A}_i + A_i) + \left(\frac{t'}{t}\right)(\mathbb{E}_g + E_g)}{\sum_{i=1}^{n}(\mathbb{A}_i + A_i) + (\mathbb{E}_g + E_g)}\right] \quad \ldots (4.10)$$

In the above general expression, the ratio $(\frac{t'}{t})$ plays a crucial role in determining the surplus-value AD:

If $(\frac{t'}{t}) = 1$, then $AD = 0$

If $(\frac{t'}{t}) < 1$, then $AD > 0$, and

If $(\frac{t'}{t}) > 1$, then $AD < 0$.

Therefore, in order that surplus-value is positive, t' should be smaller than t.

The ratio $(\frac{t'}{t})$ gains further importance if t' is interpreted as the value of labour-power or the wage bill, ant t as the value created in the process of production. If $t' = t$, that is, if the value created is totally payed to the labour-power, than the surplus is zero.

We can simplify the expression for the total surplus AD in eqn. (4.10) by means of the following definitions:

<u>Df.1</u> $\xi = (\frac{t'}{t})$

<u>Df.2</u> $$\tau = \frac{(\mathbb{E}_g + E_g)}{\sum_{i=1}^{n}(\mathbb{A}_i + A_i) + (\mathbb{E}_g + E_g)}$$

ξ in Df.1 denotes the ratio of return to the labour-power from the value created. τ in Df.2 is a <u>dimensionless quantity</u> that reflects the measure of labour intensity in production. Using these two definitions, we obtain:

$$AD = t\left[1-\left(\left(1-\tau\right)+\xi\tau\right)\right] \qquad \ldots (4.11)$$

and, thus:

$$AD = t\tau\left(1-\xi\right) \qquad \ldots (4.12)$$

From the above, it can be observed that surplus value AD increases in direct proportion with τ (i.e., labour intensity), Furthermore, AD increases with decreasing ξ; that is, AD increases with decreasing returns to labour-power.

After having defined surplus-value, we can also express the other parameters of the labour-theory of value in terms of τ and ξ. The profit rate r, for example, is defined as:

$$r = \frac{AD}{c + V} \qquad \ldots (4.13)$$

in the labour-theory of value. AD is the surplus, C, the value of the means of production, and V, the value of labour-power.

We obtain V in terms of ξ as follows:

$$V = t' = t\xi \qquad \ldots (4.14)$$

We can also find an expression forC. Since:

$$C = t - AD - V \qquad \ldots (4.15)$$

By substituting AD in eqn. (4.12) into eqn. (4.15), we obtain:

$$c = t - \tau t(1-\xi) - t\xi \qquad \text{... (4.16)}$$

$$c = t(1-\xi)(1-\tau) \qquad \text{... (4.17)}$$

Thus:

$$r = \frac{t\tau(1-\xi)}{t(1-\tau)(1-\xi) + t\xi} \qquad \text{... (4.18)}$$

and:

$$r = \frac{\tau(1-\xi)}{1-\tau(1-\xi)} \qquad \text{... (4.19)}$$

In eqn. (4.19) the profit rate r increases with increasing τ (i.e., labour-power intensity) and decreasing ξ (i.e., returns to labour-power).

It is possible to develop the model in figure 2 so that the deviations of prices from values, and wages from the value of labour-power can be considered. To do this, it is necessary to define <u>price</u> p and <u>wage</u> w as follows:

<u>Df.2</u> $p = \alpha_i t_i$

<u>Df.3</u> $w = \gamma_i t_i'$

Where α_i and γ_i are dimensionless quantities indicating the deviations from values of commodities, and labour-power respectively. More specifically, α_i and γ_i can be defined as follows:

<u>Df.4</u> $$\alpha_i = \left(\frac{P_i}{M_i}\right)$$

<u>Df.5</u> $$\gamma_i = \left(\frac{P_i}{(E_g)_i}\right)$$

Where P_i is the money payed to the commodity M_i, and to the labour-power $(E_g)_i$, respectively. P_i, M_i and $(E_g)_i$ have the dimensions of commodity U. Consequently, if P_i is different from M_i and $(E_g)_i$, the ratios α_i and γ_i are different from 1. Thus, p and w, in <u>Df.4</u> and <u>Df.5.</u>, deviate.

Nevertheless, in order to expand the model via the considerations in definitions 2 to 5, our simple model must be reconstructed. The present model is an aggregative one where the values of all the commodities and values of the labour-power are assumed to be the same for descriptive purposes and to check the consistency of the specified dimensions. It is obviously a model with a high level of abstraction (of the economical reality). Such a reconstruction of the present model, however, is beyond the scope of the present study. In the following section, we apply the techniques of differential-integral calculus to the dimensional equations we obtained in the previous section.

5. BASIC DIFFERENTIAL - INTEGRAL EQUATIONS OF COMMODITY, LABOUR AND VALUE

Differential-integral calculus provides us with the mathematical tools by means of which one can analyse functional relations. Thus, an application of the techniques of differential-integral calculus to the functional relations between the concepts of labour-power E_g, labour E, commodity U, and value t yields dynamical descriptions of these concepts throughout the process of change. Consequently, one obtains a dynamical interpretation of the model we have developed in the previous section. Before considering the basic dimensional equations, however, we will very briefly introduce the mathematical tools which will be used.(1)

If R is a region in which the independent variables $(x_1,...,x_n)$ may vary, and if a unique value u is assigned to each point $(x_1,...x_n)$ of this region according to some definite relation, then $u = f(x_1,...,x_n)$ is said to be a function of the independent variables $x_1,...,x_n$.

Let us consider a function of 2 variables $u = f(x,y)$. If we keep $y=y_0$ as constant, then we obtain a function of one variable which can be geometrically represented by the intersection of the plane $y = y_0$ and the surface $u(x,y)$

(1) See R,Courant (1936)

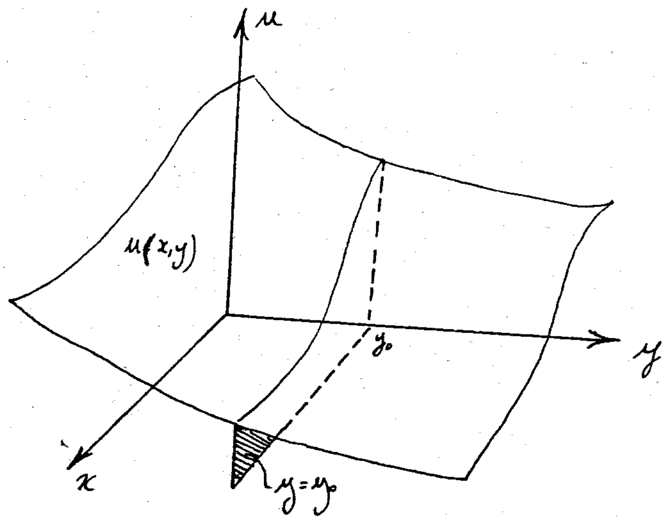

Figure 3

The curve of the above intersection can be represented by $u = f(x, y_o)$. If we differentiate this function in the usual way at the point $x=x_o$, we obtain the partial derivative of $f(x,y)$ with respect to x at (x_o, y_o); that is, $\frac{\partial f(x,y)}{\partial x}$

Geometrically, the above partial derivative denotes the slope of the surface $u = f(x,y)$ at (x_o, y_o) in the direction of the x-axis.

A differentiable function $f(x,y)$ can be approximated, in an increment, to a linear function called the differential of the function which can be expressed as:

$$du = \frac{\partial f(x,y)}{\partial x} dx + \frac{\partial f(x,y)}{\partial y} dy \qquad \ldots (5.1)$$

The above linear approximation in eqn. (5.1) becomes more accurate as the increments dx and dy become smaller. We make use of this concept (i.e., the concept of total differential) in dealing with the line and surface integrals of Labour E and value D.

In analysing a continuous function f(x,y) in the rectangular region $\alpha \leqslant x \leqslant \beta$ and $a \leqslant y \leqslant b$, one can keep the quantity x fixed and integrate f(x,y) which is now a function of y alone, over the interval $a \leqslant y \leqslant b$. This is expressed as:

$$F(x) = \int_a^b f(x,y)\, dy \qquad \ldots (5.2)$$

The quantity kept fixed is called a parameter. Thus, in eqn. (5.2), F is a function of the parameter x.

Another method of analysis is to integrate the function along a path and obtain a line integral. If the variables x and y can be expressed as functions of a single parameter, say t, such that

$$x = x(t) \quad \text{and} \quad y = y(t) \qquad \ldots (5.3)$$

Thus, f(x,y) becomes F(x(t), y(t)). The line integral then is expressed as an ordinary integral with one parameter such that:

$$\int_C dF = \int_{t=a}^{t=b} \left(\frac{\partial F(x(t),y(t))}{\partial x}\frac{dx}{dt} + \frac{\partial F(x(t),y(t))}{\partial y}\frac{dy}{dt} \right) \qquad \ldots (5.4)$$

Having assigned the primitive concepts as labour-power E_g and time t in section 3, we can express the Commodity U, labour E, and value D as functions of E_g and t, in view of the dimensional equations:

$$U = f(E_g) \qquad \dots (5.5)$$

$$E = f(E_g, t) \qquad \dots (5.6)$$

$$D = f(E_g, U^{-1}, t) \qquad \dots (5.7)$$

Differentiating eqn. (5.5) with respect to labour-power E_g, we obtain

$$dU = f'(E_g)\, dE_g \qquad \dots (5.8)$$

Thus the slope of the curve $f(E_g)$ gives the change in the commodity U with respect to labour-power E_g. Integrating eqn. (5.8), we have:

$$U = \int_a^b f'(E_g)\, dE_g \qquad \dots (5.9)$$

$f(E_g)$, intuitively. describes labour-power productivity and can be empirically obtained in view of the economic model at issue.

Thus, eqn. (5.9) yields the first differential-integral equation expressing commodity U. To express labour E in a similar manner, we obtain the total differential of eq. (5.6):

$$dE = \frac{\partial E}{\partial E_g} \cdot dE_g + \frac{\partial E}{\partial t} \cdot dt \qquad \dots (5.10)$$

If we express E_g and t as functions of the parameter τ such that:

$$E_g = x(\tau) \quad \text{and} \quad t = y(\tau) \qquad \ldots (5.11)$$

Then:

$$dF(\tau) = \left[\frac{\partial F(x(\tau),y(\tau))}{\partial E_g} \frac{dx(\tau)}{d\tau} + \frac{\partial F(x(\tau),y(\tau))}{\partial t} \frac{dy(\tau)}{d\tau} \right] d\tau \qquad \ldots (5.12)$$

And the line integral becomes:

$$E = \int_C dE = \int_{\tau=a}^{\tau=b} F(\tau)\, d\tau \qquad \ldots (5.13)$$

Equation (5.13) is the second differential-integral equation of the labour-theory of value. One can geometrically interpret this line integral as follows:

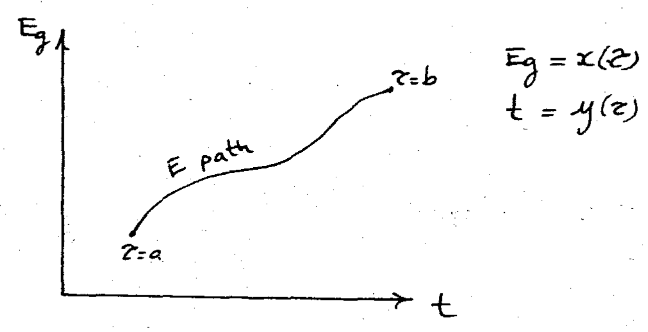

Figure 4

That is, the line integral in eqn. (5.13) yields the length of the path on $F(x(\tau), y(\tau))$ in the economical space of the dimension of labour-power E_g and time t. The shape of the curve F obviously depends on the particular model at issue.

An alternative method of analysis is to assign E_g (1) as a fixed parameter and obtain :

$$E = f(E_g) = \int_a^b f(E_g, t)\,dt \qquad \ldots (5.14)$$

The differential-integral equation of value D can be obtained by taking the total differential of eqn. (5.7):

$$dD = \frac{\partial f}{\partial E_g} dE_g + \left(-\frac{1}{U^2}\right) \frac{\partial f}{\partial U} dU + \frac{\partial f}{\partial t} dt \qquad \ldots (5.15)$$

Taking E_g, U and t as functions of the parameter τ :

$$E_g = x(\tau)$$

$$U = y(\tau) \qquad \ldots (5.16)$$

$$t = z(\tau)$$

we obtain the $F(\tau)$:

(1) Eg can be interpreted as the same thing as the quantity of employment in its Keynesian definition. See De Jong (1967) p. 31

$$F(\tau) = \frac{\partial f}{\partial E_q}\frac{dx(\tau)}{d\tau} - \frac{1}{U^2}\frac{\partial f}{\partial u}\frac{dy(\tau)}{d\tau} + \frac{\partial f}{\partial t}\frac{dz(\tau)}{d\tau} \quad \ldots (5.17)$$

Thus:

$$D = \int_C dD = \int_{\tau=a}^{\tau=b} F(\tau)\, d\tau \quad \ldots (5.18)$$

Consequently, in eqn.'s (5.9), (5.13) and (5.18), we obtained the differential integral formulation of commodity U, labour E, and value D, respectively.

6. CONCLUSION

In this study, we obtained the fundamental differential-integral equations of the labour-theory of value. To achieve this aim we found it essential first to make a dimensional analysis of the main concepts.

The conceptual framework of the labour-theory of value is built on two primitive (i.e., irreducible) concepts. These are labour-power E_g and time t.

The complex concepts (i.e., concepts which are reducible to mathematical relations between E_g and t) such as commodity, labour, and value are explicated in the form of dimensional equations in section 3. These dimensional equations provide the functional relations that form the basis of differential-integral equations in section 5.

In section 4, we developed a simple, but a very generalised macro-economic model. In view of this particular model, we discussed the consistency and the coherence of the dimensional relations which are obtained in section 3. Our results show that, in deriving a complex concept such as surplus value AD, we arrive at a mathematical expression that behaves in accordance with the foundations of the labour-theory of value. On the basis of similar considerations in section 3, one can claim that our dimensional analysis and the implications we have derived from it are consistent and coherent.

In the last section, by applying the techniques of the differential-integral calculus to the basic dimensional relations, we obtain the differential-integral equations of commodity U, labour E, and value D. These equations describe the economical reality dynamically. Our results, however, should be considered sa a first step in developing the differential-integral approach to the labour theory of value.

Particularly, the mathematical economic-spaces we have obtained in relation to the eqn.'s (5.5), (5.6) and (5.7) need further interpretation. Furthermore, to obtain a complete dynamical description of the economic reality in view of the labour-theory of value,one should apply the techniques of differential-integral calculus, to other parameters such as surplus-value AD, rate of profit r, etc..

By reconsidering and modifying our simple macro-model, one can include micro descriptions in the analysis. The present model is aggregative and too general to enable the explicit consideration of problems such as deviations of prices and wages from values and of profits from individual surpluses in certain industries, and the average profit rate which balances such deviations.(1) Methods which allow more detailed analysis on the basis of goods and industries have been recently developed.(2) Therefore for further developments a micro consideration of the issues we have discussed in the present study is necessary.

In this study, we developed a method of analysis that seems to be in accordance with the methods of natural sciences. Starting from certain conceptual abstractions, we obtained mathematical relations and differential-integral equations which dynamically describe a macro-economic reality. On the basis of such dynamical descriptions, one can obtain certain <u>predictions</u> and <u>explanations</u> of the <u>economic phenomena</u> in view of the model we have considered. It should be empha-

(1) These deviations constitute the <u>transformation problem</u>. However, we showed in section 3 that, from a conceptual (or, dimensional) point of view, the transformation problem is not a genuine problem.
(2) The recent methods are matrix algebra applications in conjunction with Leontiev type input-output analysis,see Morishima and Seton (1961)

sized that, in Physics for example, such predictions and explanations pertain to the underlying abstract model and are approximations of the (actual) physical reality. Similarly, this is the case in the present study. The predictions and explanations of economic phenomena that one obtaines from the differential-integral equations in section 5, pertain to our abstract macro-economic model; they can be considered as approximations of actual economic situations. It should be noted that, being an approximation cannot be considered as an essential shortcoming of the theoretical enterprise in the present study. Otherwise, one should be in a position to consider exact mathematical sciences such as Physics as inexact.

It seems likely that, on the basis of the problem of measurement in social scienses, certain objections can be raised against our results.

The problem of measurement in Economics is beyond the scope of the present study. Let us briefly argue,however, that objections which are raised on the basis of the measurement problem are not defeating; although they are relevant.

In obtaining the curves in the economic-spaces in section 5, one would be confronted with the problem of measurement. It is by means of such curves that one can apply the differential-integral equation and obtain predictions and explanations of actual economic phenomena. To obtain these curves, one should measure certain parameters such as labour-power E_g and (production) time t. Certain difficulties that inhere in empirical matters in the measurement process do not consititute an impossibility of the theoretical (dynamical) description of the economical reality. By means of certain measurement, approximation , and error calculation techniques,one can in principle provide the mathematical theory with curves that have empirical content (i.e., that reveal a particular economic situation empirically in view of certain parameters).

The issue of measurement and its relation to theoretical descriptions, however, should be made the subject of another study.

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