MATHEMATICAL MODELING AND OPTIMIZATION OF VESSEL ROUTE SELECTION AND REFUELING DECISIONS

by

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To my family,

for their love and endless support

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ABSTRACT

MATHEMATICAL MODELING AND OPTIMIZATION OF VESSEL ROUTE SELECTION AND REFUELING DECISIONS

Maritime transportation is a volatile and growing sector. In order to survive in this competitive sector, maritime companies have to continuously strive for lower costs in performing vessel operations. Today, fuel (bunker) comprises about 50-60 percent of a vessel's voyage expenses. This study aims the mathematical modeling and optimization of refueling policy and routing decisions. Refueling policy consists of the refueling ports and amounts of a vessel. The selection of refueling port affects the route of the vessel.

Two mathematical models are developed in this study. One of them is the Single Voyage Bunkering Model, which is a mixed integer programming model and defines the refueling ports and amounts of a vessel whose a few number of loading/discharging ports are certain in a short term. The second model, the Long Term Bunkering Model, is developed as an infinite stage stochastic dynamic programming model. This model seeks to determine the refueling amount of a vessel at a port according to the arrival fuel amount of the vessel and the expected future destination from that port.

Single Voyage Bunkering Model is applied to a real problem by the CPLEX solver within GAMS v23.3.3 software. The Long Term Bunkering Model is also solved by using real data and by different dynamic programming algorithms that are programmed in Visual Studio 2005 with C#. Apart from classic dynamic programming solution procedures, two new heuristic algorithms are also developed. At the end of the study, the results obtained are compared and discussed.

ÖZET

GEMİ ROTA SEÇİMİ VE YAKIT İKMALİ KARARLARININ MATEMATİKSEL MODELLEME VE OPTİMİZASYONU

Deniz taşımacılığı, değişken ve büyüyen bir sektördür. Bu rekabetçi sektörde ayakta kalabilmek için, denizcilik şirketleri, gemi operasyonlarını daha düşük maliyetlerle yürütmeye çabalamak zorundadırlar. Günümüzde bir geminin sefer maliyetlerinin yaklaşık yüzde 50-60'ını yakıt (bunker) oluşturabilmektedir. Bu çalışma, gemilerin yakıt ikmal politika ve rota kararlarının matematiksel modellenmesi ve optimizasyonunu amaçlamaktadır. Yakıt alma politikası, geminin yakıt alacağı limanlar ve yakıt ikmal miktarlarından oluşmaktadır. Yakıt ikmal limanının seçilmesi, geminin rotasını da etkilemektedir.

Bu çalışmada iki matematiksel model geliştirilmiştir. Bunlardan biri, Tek Seferlik Yakıt İkmali modeli bir karışık tamsayı programlama modeli olup, kısa vadede birkaç yükleme/tahliye limanı kesinleşmiş olan bir geminin, yakıt ikmal limanlarını ve miktarlarını belirlemektedir. İkinci model olan Uzun Vadeli Yakıt İkmali modeli, bir sonsuz aşamalı rassal dinamik programlama modeli olarak geliştirilmiştir. Bu model, geminin bir limana varış yakıt miktarına göre bu limandaki yakıt ikmal miktarının ve bu limandan sonra gelecek beklenen varış yerinin belirlenmesini araştırmaktadır .

Tek Seferlik Yakıt İkmali modeli, gerçek bir problem için GAMS v23.3.3 yazılımındaki CPLEX çözücüsü tarafından çözülmüştür. Uzun Vadeli Yakıt İkmali modeli de gerçek veriler kullanılarak ve farklı dinamik programlama algoritmalarının Visual Studio 2005'de C# ile programlanmasıyla denenmiştir. Klasik dinamik programlama çözüm prosedürlerinin yanısıra iki yeni heuristik algoritma da geliştirilmiştir. Çalışmanın sonunda elde edilen sonuçlar karşılaştırılmış ve tartışılmıştır.

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LIST OF SYMBOLS/ABBREVIATIONS

ALT I	Alternative Heuristic I
ALT II	Alternative Heuristic II
DP	Dynamic Programming
DWT	Death Weight Tonnage
HS	High Sulfur
HVPI	Hybrid Value-Policy Iteration
IFO	Intermediate Fuel Oil
IP	Integer Programming
L/D	Loading/Discharging
LS	Low Sulfur
LTB	Long-Term Bunkering
MDP	Markov Decision Process
MDO	Marine Diesel Oil
MGO	Marine Gas Oil
MIP	Mixed-Integer Programming
MT	Metric Ton
NM	Nautical Miles
PI	Policy Iteration
SDP	Stochastic Dynamic Programming
SP	Set Partitioning
SVB	Single Voyage Bunkering
TSP	Travelling Salesman Problem
VI	Value Iteration

1. INTRODUCTION

The existence of maritime transportation goes back to 3200 B.C. In this era, Egyptians had maritime trade routes going as far as Indonesia. After the invention of steam engine in 15th century, larger and faster ships were built to sail the longer trade routes. This is regarded as the beginning of the rise in maritime transportation [1]. In 20th century, diesel engines stimulated the development of the sector and today maritime transportation dominates the international trade with a share of 90 per cent.

Since 1970, globalization in trade and industrialization has increased the international seaborne trade by 68 per cent. As a result, world maritime fleet has grown in a similar fashion as demonstrated in Figure 1.1. Especially, containerships and bulk carriers have shown a significant increase in world seas [2].

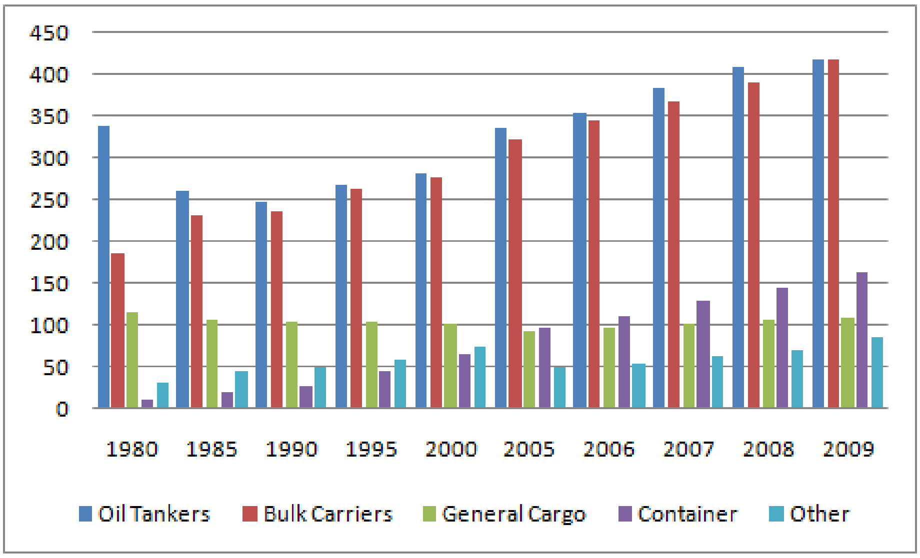

Figure 1.1. World fleet by vessel type (millions of dwt)

The growth of world maritime fleet indicates a tough competition among shipping companies. In order to survive in this competitive sector, companies have to cope with uncertainty which is the main challenge in most maritime planning problems [3]. For instance, weather conditions may affect the sailing time of a ship. An unpredictable delay may change all loading and unloading plans, adversely. A voyage order to an unexpected geographical area may force the ship to refuel from a port having higher fuel prices.

Hence, it is essential to predict uncertain occurrences based on past experiences. When unknown factors are so estimated, these data can be used to devise robust plans that provide the flexibility to respond to sudden changes with less deviation from expected costs.

1.1. Maritime Planning and Bunkering

Bunker planning is one of the major issues that require a systematic approach. Bunker is the general term used for marine fuels, which dates back to the days when ships were powered by coal, since it was the name of depots where coal was stored [4]. Today bunker is the name of any type of fuel used aboard ships and refueling is known as bunkering.

In the maritime world, there is quite a diversity in bunker fuels. They are classified as Marine Gas Oil (MGO), Marine Diesel Oil (MDO), Intermediate Fuel Oil (IFO), Medium Fuel Oil (MFO) and Heavy Fuel Oil (HFO). IFO has also two types as IFO 180 and IFO 380. These bunkers also differ in terms of sulfur content as Low Sulfur (LS) and High Sulfur (HS) [5]. Vessels generally use both diesel oil and fuel oil. Types of bunkers that a vessel uses depend on technical characteristics of that vessel. Hence, this diversity makes the refueling more complicated, even when ignoring the chemical quality properties of bunkers.

The cost of bunker fuel is a central component of vessels' operating expenses. It is estimated that bunker costs are 50-60 per cent of vessels' voyage costs [6]. As being petroleum products, marine fuels have changing market conditions. Bunker prices not only differ between ports, but also may differ between suppliers in the same port. Additionally, since bunker is a crude oil product, its prices change daily and it generally follows an increasing trend. Therefore, refueling time and amount of a vessel is a critical decision. In addition, refueling port decision is related with the route of vessel. Since route selection is a medium-term decision, refueling of vessel can be regarded as tactical planning problem [3].

Selection of refueling port and amount are not simple decisions from many aspects. First of all, refueling cost does not only cover fuel purchasing cost. It is a combination of direct and associated costs of intake. Indirect refueling cost includes the cost of time that the vessel spends for refueling and port charges. When the vessel waits long hours for refueling at a port which has lower fuel price, it may not be a cost saving action due to loss from hourly revenue or even costs of late delivered cargos.

Other factors affecting refueling are vessel and port characteristics and vessel's shipping service. Fuel tanker capacity and type of the ship directly influence amount of bunkering. For instance, if the vessel is a chemical tanker, for some cases, liquid cargos that the vessel carries may require heat. Therefore, extra fuel is needed to keep the cargo at required temperature. Ports are also influential due to their congestions and other operational conditions. Apart from these factors, shipping service type of vessel can be regarded as the main determinant of its refueling strategy.

Depending on operational and commercial differences, shipping services are classified as liner, tramper and industrial shipping. Liner ships operate within a schedule similar to a bus line and have a fixed port rotation with published dates of calls. On the other hand, tramper ship does not sail in a fixed routing or schedule and is available at short notice to load any cargo between any two ports. In industrial shipping, the shipping company is the owner of the cargo. The shipper can be the owner or hire the vessel [7].

In case of a liner, number of port calls, route and required fuel consumptions are known well in advance. Hence, it is easy to plan a refueling schedule by just considering possible fuel price changes. On the other hand, it is more complicated to perform a refueling strategy for a tramper vessel, since port calls of the vessel are not known precisely. Accurate information about port calls of a tramper is only available on short notice (for instance just three port calls away). Therefore, tramper refueling plans require continuous revision.

Today, most of the maritime planning problems requires a decision making process for which the decision maker has to cope with many known and foreseen factors. As the fleets of maritime companies grow, problems get larger and complicated to solve by only experience and gut feeling. Although all companies have managers and experts who are captains or generally have sea experience and knowledge, planners from other disciplines and approaches are also needed since most problems, like refueling decisions, highly benefit from new solution methods based on optimization tools. These new maritime planning crews are capable of applying both experience and up-to-date optimization methods.

1.2. General Information

In order to gain a general overview on maritime logistics, a brief knowledge on maritime terminology is necessary. Instead of detailed technical terms, this section presents simple definitions which are helpful to comprehend the issues and previous studies that are covered. Additionally, cost structure of a voyage is described.

1.2.1. Terminology

- Port call is the port that vessel visits for discharging or loading cargo.
- Voyage includes a sequence of port calls. A vessel that has no cargo starts from an origin port and visits multiple loading/discharging (L/D) ports until completing all cargo transport jobs and having no remaining cargo.
- Routing is assignment of a sequence of port calls to a vessel whereas scheduling is the planning of port call times and other tasks on specified route.
- Spot (voyage) charter is a hiring contract for a particular vessel to move a single cargo between specified origin and L/D ports in the immediate future. This type of contract is common for tramp vessels.
- Time charter is a contract for the hire of a vessel for a specified period of time. The charterer is a person or firm who enters into a contract with the vessel owner for the transportation of (specified or unspecified) cargo and pays for the bunker fuel, fresh water, port charges, etc. in addition to charter hire.

- Laydays or laycan is range of dates. The hire contract must start within laycan. If the vessel does not arrive at the last day of laycan, charterer can cancel the contract.
- Laytime is the time allowed by the vessel owner to the voyage charterer to carry out the cargo L/D operations. Laytime may be expressed as a certain number of days or number of tons of cargo loaded/unloaded per day. [8,9]
- Barging is the term used for cargo handling operation that is performed by small boats from shore to vessel or vice versa. The small boats are called barge that are used for carrying of goods on rivers, canals and shallow waters. Bunker barge is used to supply bunker to vessel.

1.2.2. Voyage Costs

Voyage costs consist of fixed costs and variable costs. Voyage fixed costs do not depend on volume of freight, whereas variable costs mostly include handling costs directly related to the volume of cargo.

Vessel costs and port charges are main voyage fixed cost components. Vessel costs consist of crew, maintenance, insurance, vessel depreciation and management fees. Port charges include fees for all received services in port. [10]

Bunker costs cover the consumption of all fuel types and constitute the highest portion of a voyage cost, while depending on both duration of voyage, speed and type of vessel. A vessel use diesel oil, fuel oil and lubricants for its engine system.

Apart from voyage cost, unexpected costs due to operational delays or idleness of vessel can occur. When the arrival time of a vessel exceeds a specific time, a penalty cost associated with late arrival is realized. In addition, each extra hour that vessel uses can be regarded a loss from hourly vessel profit. Another cost item associated with the delays at port is bunker consumption for auxiliary systems. Generally, vessels consume IFO only during sailing and consume MDO or MGO mostly at ports. MDO and MGO prices are higher than IFO prices and their consumption at ports can be regarded as a cost due to unexpected delays.

1.3. Objectives of the Study

The objective of this study is to model and investigate the vessel refueling problem with the aim of guiding maritime refueling decisions by optimization-based methods. Two optimization models are developed for the refueling problems in case of tramper shipping. Unlike other models presented in maritime literature, opportunity cost of vessel due to time spent at a port is included in the models.

The reason for the study and the modeling approach adopted is related with the general trends in maritime sector. As companies are more willing to perform only their core business and outsource their transportation needs, tramp shipping is becoming more popular in the world seas. Although world tramper fleet is growing, there are few studies on tramper operations. This study also aims to contribute to this area of maritime literature.

The first model is formulated as a mixed-integer programming (MIP) model. It provides a simple solution for a specific application in tramper vessel routing and operations. In this special case, a set of scheduled ports of the tramper vessel is known and the vessel may refuel either from this set of ports or from specific refueling ports on the route.

Next, the main model is formulated as an infinite stage stochastic dynamic programming (SDP) model. In this case, it is assumed that voyages between ports follow a Markov Process, while fuel prices and port charges differ from port to port. Refueling tonnage of a vessel in a port depends on the amount of fuel in its tanks when the vessel arrives at the port. Therefore, state variables are associated with both the identity of current port and the on-board fuel level.

Several approaches and related computational algorithms are applied to solve this dynamic programming (DP) model. The first approach is a policy iteration (PI) method, which is based on updating refueling results for each state, as the total cost improves. The second algorithm is a value iteration (VI) method, which is based on updating cost values recursively starting from an initial value function set. The third approach is a hybrid value-policy iteration (HVPI) method. In addition to these "optimal seeking" approaches, alternative heuristic approaches are also applied, which are based on comparing different policies that are generated from combinations of various thumb-rules applied at states. These combinations are used in Bellman's equations (which is based on the bunker prices and value functions) in a sequence.

Although fuel prices are non-uniform among ports, their stochastic nature is not included in this study. Stochastic prices can be a topic in a further study.

The following chapter overviews maritime literature and other studies of airline literature related to refueling management.

In Chapter 3, mathematical models and computational algorithms are presented. The assumptions related to mathematical models are explained in detail. This section also covers data compilation and analysis.

Chapter 4 includes the optimization cases of both the DP and the MIP models. Verification, validation and output analysis are presented. In addition, computational algorithms are compared in this section.

Chapter 5 contains the conclusions and further study opportunities on maritime bunkering. Tables and figures are presented in Appendices.

2. LITERATURE SURVEY

In this section, previous studies on some maritime planning problems are presented. Although various planning issues are examined in maritime literature, bunkering is not a well-emphasized subject. Only a recent study on bunkering is available. Two airline fuel management studies are also examined.

In the literature, there is a lack of attention to maritime planning problems. The main reasons are uncertainty and variety involved in planning problems. Uncertainty requires continuous revision, while diversity in operations make the customization difficult to achieve. Another reason is the long tradition of maritime industry, to build barriers to interaction with other disciplines [11].

Maritime literature can be classified into strategic, tactical and operational planning problems. Strategic problems include selection of fleet size, mix and market, maritime supply chain, transportation network and ship design. Tactical planning consists of routing and scheduling decisions. Bunker planning can be regarded as a tactical planning problem since it affects the route selection of vessel. Operational planning covers short-term decisions on voyage leg, cruising speed and ship loading [3].

In the strategic area, two studies consider maritime transportation as a ring of supply chain. Magala and Sammons [12] present two models considering a port as an element of supply chain and they use the system theory and bundling in order to increase efficiency of the whole system rather than individual transportation modes. Panayides [13] reviews studies that are related to the convergence of maritime transport and logistics. This review highlights the changing roles of ports in intermodal transportation systems.

In tactical planning, Fagerholt [7] present an IP model which generates weekly routes for a fleet of liner vessels. Their model optimizes transportation costs including fuel and port expenses, while the solution algorithm suggested consists of two phases. The route generation phase solves a Travelling Salesman Problem (TSP) and the generated routes are used as input in an IP model.

Ting and Tzeng [10] examine costs of liner services and give a DP model on scheduling decisions. Decisions at the stages of this DP model constitute cruising speed, quay crane dispatching and buffer time, which is a tolerance time between estimated and actual berthing. The objective is to minimize the total expected variation between available berth time-windows to estimated berth time-windows of the planned voyage. In the cost analysis, voyage fixed and freight variable costs are well-defined. Bunker costs are included in voyage fixed costs.

Brown et al. [14] present an elastic Set Partitioning (SP) model for scheduling crude oil tankers; the costs covered include daily opportunity cost of ship time, port and canal charges, demurrage cost and bunker fuel. The elastic term indicates that the penalties in objective function can be either positive or negative values. The model assigns cargos to owned vessels or spot charter. Elastic SP enables adding demurrage costs having elastic penalties to the objective function. The solution procedure suggested consists of three phases, which are schedule generation, cost calculation and model optimization.

Sherali et al. [15] formulate a MIP model for the routing and scheduling problem of oil tankers. The fleet consists of time-chartered, spot-chartered and company-owned vessels and the model assigns vessels to demands and to voyage legs. It also selects routes considering time and canal dues. The objective is to minimize operational cost of selected legs, penalty costs of violating delivery dates and the chartering costs. Unlike other models, not only late deliveries but also early deliveries are penalized. The model seeks to better utilize company-owned vessels and to use chartered vessels only when needed.

Although scheduling and routing models are generally discussed for liner shipping, Christiansen et al. [16] also review a tramper shipping model. Instead of minimizing the costs, as done in industrial shipping, the model maximize profit which is gained from operating the vessel fleet and servicing the cargoes by spot charters. This model also features a SP formulation, which is a favored tool in most vessel scheduling problems.

Huang and Karimi [17] present a study on scheduling trans-shipment operations for chemical cargo vessels. The transfer operation from one vessel to another is called trans-shipment. Trans-shipment is a necessary operation since large deep-sea vessels cannot enter some ports or canals. In trans-shipment operations, accurate scheduling is crucial since both donor and recipient ships must be on-time at the meeting location. Three models are formulated, which give optimal sequences, timings and positions of cargos by considering some operational constraints, arrival times and time-charter costs.

Apart from scheduling and operational models, there is a study on the vessel bunkering problem. Besbes and Savin [18] present stochastic DP models for both liner and tramper vessels. Liner vessels follow a cyclical route, which only requires refuelling cost minimization. However, trampers have a choice of next destination port from available jobs. Hence, the problem combines route selection with refueling decisions and the objective becomes profit maximization. The models are formulated as average cost per stage stochastic DP in infinite horizon. State variables are defined as current port, arrival bunker level, price and reward vectors for the tramper case. For the liner case, rewards are not considered since route is known. The optimal policy is found for three cases. In the first case, bunker prices are deterministic and uniform in a specified network of ports. In the second case, bunker prices are deterministic but non-uniform. The third case is based on stochastic uniform bunker prices. Bunker price dynamics are modeled as a two-state Markov process. Bunker price of a port depends on both crude oil prices and local factors. Crude oil prices are computed through a mean reverting process which is found to give better description than geometric Brownian motion. Since DP models are not easy to solve optimally even for small problem sizes, two heuristics are developed. Optimal location cycle policy heuristic (OLC) separates routing and bunkering decisions to handle the large combinatorial nature of the tramper problem. Optimal location-inventory cycle heuristic (OLIC) retains both decisions and non-uniformity of prices, but ignores stochastic variations of prices and rewards. Performances of two heuristics are compared in a numerical study.

Fuel management models are also discussed in airline literature. Stroup and Wollmer [19] present a refueling model for the airline industry. This LP model determines the refueling amounts for an aircraft. Flight schedules of aircrafts are known (which are similar to liner vessels). There are supplier and station constraints. Supplier constraints are upper and lower bounds on the amounts of fuel that can be purchased from a supplier. Station constraints are allowable landing and take-off amounts of aircrafts and also lower bounds on fuel reserve of stations. Fuel prices are non-uniform and constant. Unlike vessels, aircraft fuel consumption is assumed to be a linear function of on-board fuel at take-off.

Abdelghany et al. [20] also suggest a model which determines the optimal amount of ferry fuel to be loaded at each airport for a given aircraft route. Fuel prices are non-uniform among airports and refueling cost consists of purchasing cost, supply costs and direct operating costs. An aircraft consumes more fuel as the weight of fuel on-board increases. Excess fuel also increases maintenance costs. Direct operating costs include cost of extra burned fuel and maintenance cost due to excess weight. Hence, the model seeks a trade-off between refueling from airports having low prices and the extra fuel consumption cost due to flying the aircraft with heavier fuel weight. Unit price of fuel in the aircraft before taking off is evaluated as a weighted average. In order to update this value at each airport, fuel price at the current airport and unit fuel price in aircraft at the previous airport is weighted by refueling amounts. The model is applied to single-stage and multi-stage fuel strategies. In the single-stage strategy, fuel loaded for a flight is fully consumed at that flight. But in the multi-stage case, after one flight is completed, the aircraft is allowed to consume the remaining excess fuel on tanks in the next flights, which may be more than one in number.

3. MATHEMATICAL MODELS AND COMPUTATIONAL METHODS

In this section, compilation and analysis of the required data for the developed models is discussed. Then the Single Voyage Bunkering (SVB) and the Long-Term Bunkering (LTB) models are presented. In order to have a clear understanding of the LTB model, the related DP methodology is briefly reviewed before the model formulation. Assumptions for both models are given and finally four computational algorithms to solve the LTB model are discussed.

3.1. Data Compilation and Analysis

This part of the study consists of data compilation and analysis required for the bunkering problems. Required data are obtained from a maritime fleet management company of Turkey. The company manages both company-owned and chartered chemical tramp vessels of various sizes in a wide geographical area, including South Asia, Mediterranean Sea, North-West Europe, North America, US Gulf and Caribbean. Historical data on voyages of these vessels is examined and these data are utilized in both the SVB and the LTB model developments, as well as in implementation of these models.

Main factors considered in bunkering decisions are information on voyages, bunker (fuel) prices and characteristics of ports that are within the sailing network of vessel. Characteristics of ports include port charges and time duration for refueling operations. Sailing network of ports include ports that are frequently visited for L/D and ports that are close to the L/D ports and that are competitive in terms of bunker purchasing.

Experienced decision makers can estimate some factors and takes voyage information into account, then manually choose the next refueling amount and port from a set of alternatives. However, as the fleets grow and voyages of vessels get more complex, it becomes difficult to pinpoint the best alternative. Therefore, all data on hand must be analyzed and used in order to come up with a decision that fits expected and worst case scenarios. In addition, since tramp vessels do not have a long term fixed voyage course, data must be revised continuously.

3.1.1. Bunker Prices

As mentioned previously, unit prices of bunkers differ among ports. As crude oil products, their prices experience daily fluctuations. Figure 3.1 indicates this volatile structure of bunker prices in four major ports. [21]

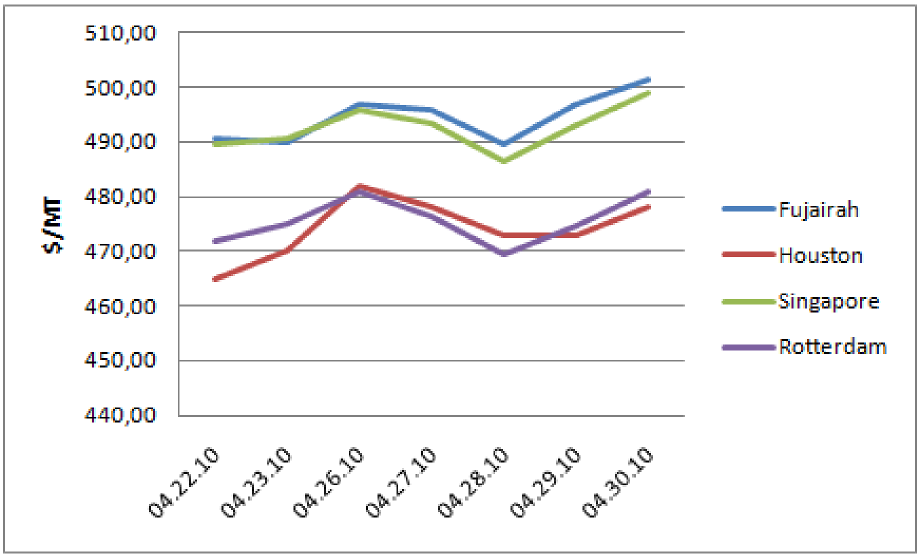

Figure 3.1. HS IFO 380 Bunker Price Trends (dollars per MT)

Bunker prices are considered as a data set that cannot be known precisely due to this volatility. Another reason is lack of price information. Bunker prices are not reported for all ports and are not accessible for all shipping companies. There are firms that shipping companies may consult about prices. (Some internet sites on bunkers report prices in return for a subscription fee.) Bunker prices used in the implementation phase of this study are obtained from data provided by this kind of sites, as well as from data obtained from the previously mentioned fleet management company (Unfortunately, this company did not have any continuous agreement with a bunker price provider). Refueling processes are planned by the operations staff, considering the current known operational conditions and price information, which is obtained via port agents or directly from suppliers, just before bunkering decision.

In this study, bunker prices are collected for a network of ports that are frequently visited by the vessels of the company for L/D and/or bunkering purposes. Due to lack of data availability, prices at some ports are approximated based on previous data. For instance, suppose current bunker price at a specific port is unknown but the historical one or two spot price data at this port are available. Then, the current price at this port can be estimated by using this scarce data in tandem with the full (past and present) bunker price data set of a second “base” port. Simply, it is assumed that the relative price between the bunker price at the port in question and at the base port remains unchanged. Accordingly, the past prices at first port are calculated as the percentage value of the past prices at the base port. Then, the average percentage is obtained and the current unknown/unavailable price at the first port is estimated with respect to this average percentage. This method is applied in price estimation of Sines, Le Havre and Tallinn ports. In this analysis, bunker price at Rotterdam is used as the base price, since Rotterdam is a major port whose past and current price data are readily available in public, as well as in company records. Sines, Le Havre and Tallinn are the ports whose current price values are not available. Although it is also difficult to find historical price data for these ports, two or three values of past records for these ports are collected. The bunker price estimations for Sines, Le Havre and Tallinn are indicated in Table 3.1, Table 3.2 and Table 3.3, respectively.

Table 3.1. Sines port bunker price analysis

Date	Bunker Prices		Per cent of Rotterdam
	Rotterdam	Sines	
23.01.2006	305	317	103.93
10.07.2006	305	324	106.23
22.10.2007	423	451	106.62
Average percentage:			105.59
22.04.2010	474	**501**	**105.59**

Table 3.2. Le Havre port bunker price analysis

Date	Bunker Prices		Per cent of
	Rotterdam	Le Havre	Rotterdam
23.01.2006	305	305	100
10.07.2006	305	314	102.95
22.10.2007	423	438	103.54
Average percentage:			102.16
22.04.2010	474	**484**	**102.16**

Table 3.3. Tallinn port bunker price analysis

Date	Bunker Prices		Per cent of
	Rotterdam	Tallinn	Rotterdam
23.01.2006	305	307	100.65
10.07.2006	305	321	105.24
22.10.2007	423	451	106.62
12.12.2008	208	175	84.13
Average percentage:			104.17
22.04.2010	474	**494**	**104.17**

As can be seen in Table 3.1, there are three historical price values for two ports. Current price of Rotterdam is also known (which is 474 $/MT). If the current price at Sines as a percentage of Rotterdam price can be estimated, current unknown price at port Sines can also be projected, since the current price of Rotterdam is available. Therefore, the three past data values are used to obtain average percentage of bunker price of Sines and this value is applied to the current bunker price at Rotterdam. On average, the price at Sines is 105,59 per cent of price at Rotterdam. As a result, the current price at Sines is estimated as 500.5 $/MT, which is 105.59 per cent of 474 $/MT.

Although this simple approach of estimation may lead to erroneous values, prices are also subjectively adjusted based on the current prices over all ports. For instance, the estimated bunker price at Tallinn port (493 $/MT) is regarded as very high compared to other ports' prices at that time. Hence, Tallinn price is adjusted to 480 $/MT, considering other bunker prices at the Great Belt, since Tallinn port is close to that region.

In some cases, only one past price at a port within the network is available and in some case, no bunker price data may be available for a certain port. An unknown price of this kind is estimated based on the bunker prices at other ports in the same region. For example, bunker price at Constantza is estimated in this way: It is estimated with respect to the price at Odessa, since both ports are in the Black Sea region. This kind of rough estimation can also be applied for ports that belong to same country. Generally, ports belonging to same country and/or region have similar price profiles. For example, bunker prices at UK ports are assumed to have the same price at Port Falmouth, a UK port where bunker prices are readily available in public sources. However, in this kind of estimation, current prices of other ports of the region are also considered, since two ports of the same country may have different prices. For instance, there are significant price differences between French ports on the Mediterranean Sea and the Atlantic. As a result of all these estimations and adjustments, the price data of ports considered in this study are obtained as deployed in Table A.1.

3.1.2. Voyage Information

As explained previously, a voyage consists of multiple port calls. Although port calls of tramp vessels regarding future voyages may be unknown, since they are similar, they can be predicted from historical data. A tramp vessel sails between two ports according to available and appropriate cargo to be carried. Fortunately, cargo transport destinations for a port shows a similar behavior in the medium-term. The next port call also depends on the region of current port, time constraints and characteristics of the vessel. Company records on voyages confirm this behavior. According to the data, when a vessel sails from North Europe to North America, generally she stays there for a while for voyages between ports in the same region. Frequent voyage legs between Houston and New Orleans is a prominent example. In addition to frequent regional transports, some ports can be regarded as major ports that have various links to ports in other regions. Rotterdam and Gibraltar are such two major ports.

In this study, two types of voyage data is used. The first type is real data of one voyage that is used in the implementation of the SVB model. The second type is historical data on vessel movements between ports and it is used to obtain the transition probabilities of the LTB model implementation. Based on historical voyage records, frequently visited ports are determined and they are chosen as a network of ports. Since historical records of a vessel cover only the recent past periods, the number of past voyages of one vessel are not sufficient to obtain port-to-port transitions. Therefore, all vessel records are used to compile the sailing frequencies between ports. It is a reasonable approximation, since all vessels are chemical tankers and similar in terms of characteristics.

In order to obtain transition probabilities between port pairs, the number of port-to-port movements are counted, while some geographically close ports are aggregated into a single port. Algeciras-Ceuta and Eastham-Runcorn port pairs are two examples of such mergings. Sailing time between the Algeciras-Ceuta port pair or the Eastham-Runcorn port pair is less than two hours at 13 NM sailing speed. Under this practice, some ports that are frequently visited but do not have sufficient data about bunker prices and port charges, can be merged with a nearby port having more abundant cost data. Resulting port-to-port transition probabilities are deployed in Table A.2.

Another required data set is the port-to-port distances. This data set is obtained in terms of days from the Netpas Distance 2.5 and other distance calculator sources in the internet. (An average speed of 13NM is used in distance calculations). Then, the distance data is deployed in the computation of port-to-port bunker consumptions of vessels since daily average bunker consumption of a vessel is available as a technical property of the vessel. The resulting travel times and average fuel consumptions between ports are presented in Table A.3 and Table A.4.

On the other hand, bunker consumptions between ports are necessarily continuous parameters. These data are approximated into discrete values to simplify the developed DP model and its solution procedures (Otherwise, the LTB model would have become an infinite state problem). In order to discretize values, on board bunker amounts, port-to-port consumptions and alternative bunkering amounts are approximated into the closest multiple of a predefined interval value. Table 3.4 and Table 3.5 indicate the approximation for a five ports network. Since in real life, bunkers are also acquired in discrete amounts, discretization of these values are in line with the real life bunkering operations.

Table 3.4. Continuous port-to-port consumption values

(in hours)	Lavera	Barcelona	Ceuta	Sines	Rotterdam
Lavera	0.0	10.8	39.6	57.6	118.8
Barcelona		0.0	28.8	46.8	108.0
Ceuta			0.0	18.0	79.2
Sines				0.0	64.8
Rotterdam					0.0

Table 3.5. Discrete port-to-port consumptions in multiples of 10 hours

(in hours)	Lavera	Barcelona	Ceuta	Sines	Rotterdam
Lavera	0	10	40	60	120
Barcelona		0	30	50	110
Ceuta			0	20	80
Sines				0	60
Rotterdam					0

3.1.3. Port Characteristics

Bunker price is the major factor that determines bunkering amount and port. However, a decision based on only bunker prices may not lead to lower overall bunkering costs. The choice of bunkering port also depends on port characteristics, such as charges and operation/waiting times in ports.

Port charges include operational and service costs of a vessel. During the time spent in port, a vessel receives services from its shipping agent company that prepares the documents required for cargoes to be transported and deals with insurance and customs matters on behalf of the vessel. Agent simply plays a role of managing vessel requirements at port in return for fee. Other cost items included in port charges can be barging and calling costs.

In order to compile data on port charges of the ports in the considered network, company records are deployed. As performed in estimation of bunker prices, when records are not sufficient for precise estimation of port charges, this cost item of a port is approximated by the port charges of other ports in the same region (or in the same country). In some cases, either barging or calling cost are available and the available one is used as the port charge. For some ports, both calling cost and barging cost are zero in past records. Hence, the overall port charges of these ports are approximated to the average port charges of the region. The resulting approximate port charges in dollars for some ports are presented in Table 3.6.

Table 3.6. Port charges in some ports

Ceuta	Gibraltar	Syros	Kiel Canal	Tallinn
2,461	5,192	1,000	0	2,363
Skaw	Kalilimenes	Piraeus	Malta	Kumkale
3,596	1,200	3,326	5,349	0

Another important factor is the operation time spent in port. Operation time refers to both the bunkering time elapsed and the time spent for port operations such as L/D. Time spent in bunkering can be regarded as a loss from revenue generation of the vessel. Ports have different traffic congestions or other operational properties. In addition to bunker price and port charges, expected average bunkering time in port is also important in overall cost considerations. Therefore, the time elapsed for only bunkering purposes is referred as waiting time throughout this study. Average waiting times are approximated from company records and experience of operation stuff.

Unfortunately, the waiting time data available in the target company's records are scarce (less than three instances for many ports). Therefore, a rough approximation method is deployed in the estimation of mean and variance of waiting times. For sample sizes greater than or equal to three, mean and variance of the sample are used. For samples having two observations, mean of the sample is used but variance is approximated as an equally weighted average of sample's own variance and overall variance of all ports. Overall variance of all data available is used as the variance of ports having one observation. This one observation is used as the mean waiting time of that port. If no observation is on hand, then the average waiting time and variance is estimated as the overall mean and variance of waiting times of all ports.

Additionally, each port has a lower bound on bunker supply such that a vessel must receive at least this amount of bunker. Although lower limits differ between ports, they are 100 MT on average.

3.2. The Single Voyage Bunkering Model

As mentioned previously, voyage information of tramp vessels are known just a short time before the beginning of a voyage. Therefore, bunkering activities are planned only considering port calls of a voyage that are certain at the bunker planning time. The SVB model plans bunkering activities in short terms. It deploys bunkering ports and bunker amounts of a vessel for only a voyage plan, which consists of scheduled three or four L/D port calls.

In the SVB model, a voyage consists of a set of scheduled port calls for L/D activities. Vessel may either refuel from these scheduled ports or may refuel from other ports on the route. Other ports that the vessel can stop by for only bunkering should be located near or on the route of the vessel to avoid longer additional travel efforts. The developed model minimizes the sum of five types of incurred costs:

- Cost of bunker purchased from scheduled ports, which is directly dependent on bunkering amount.
- Cost of bunker purchased from other ports on route, which is directly dependent on bunkering amount.
- Cost of visiting a port, which is independent from purchased bunker amount.
- Penalty charges associated with late arrival damages to a scheduled port, which is dependent on the time overrun.
- An additional cost to represent the waiting time uncertainty and the associated potential damages.

Cost of visiting a port consists of port charges, opportunity cost due to waiting times at ports and cost of additional efforts. Opportunity cost refers to hourly loss from vessel's revenue when the vessel is idle, (i.e. neither sailing nor involved in L/D activities at a port) or refueling. As mentioned in the previous section, port charges are not considered for scheduled L/D ports, since the vessel must visit these ports and spend time for operations L/D in any case.

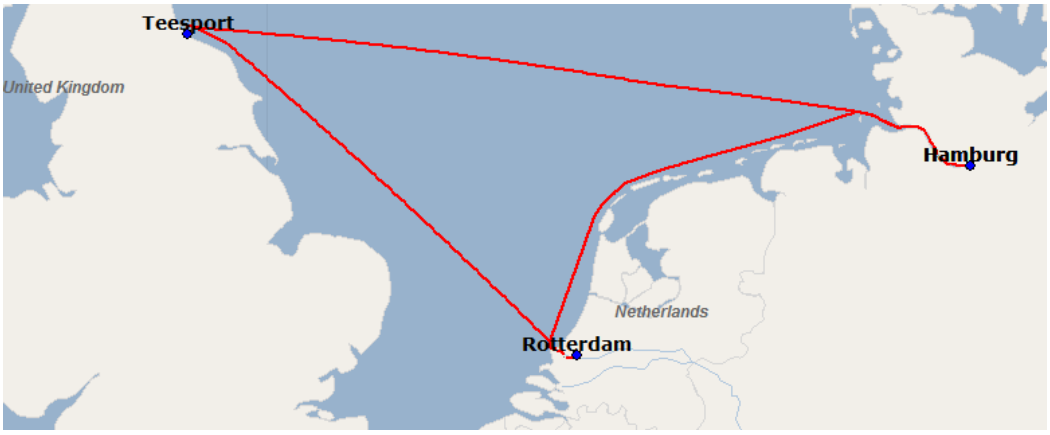

Figure 3.2. Example for cost of additional effort

Additional travel efforts consist of cost of additional travel both in terms of bunker consumption and time. In order to visit a port which is not on the established route, extra travel time and bunker is required, since the vessel stray from the schedule course. An example can be illustrated in Figure 3.2. If Hamburg and Teesport are scheduled ports, additional cost of visiting Rotterdam for only bunkering is approximately 13 hours and 9.5 MT bunker consumption. On the other hand, bunker prices and other costs at Rotterdam may be lower than any other L/D port in the voyage plan. Therefore, this additional costs may lead to lower overall cost, considering the bunkering costs associated with other legs of the voyage.

As indicated, possible bunkering ports are predetermined for each leg of a voyage. In order to determine the empty bunker space of a vessel arriving at an “bunkering only” port, the bunker consumed between this port and the starting port of the leg must be known. For instance, the distance between Hamburg and Rotterdam in Figure 3.2 is that kind of distance item.

Variance of waiting time at a port is considered as a measure of waiting time uncertainty of that port. Two ports may have the same average waiting times, but variances of waiting times may be different. The port having higher waiting time variance carries the risk of requiring longer hours in bunkering. Therefore, in order to reflect the impact of this risk, waiting time variance is also included in the model by assigning an appropriate importance weight to the risk associated with waiting time variance. This weight is expected to reflect the decision maker's risk attitude towards uncertain waiting times.

As mentioned previously, the total time that a vessel spends for only bunkering at a port is referred as waiting time. Since a reasonable amount of such waiting time at each port and some variation in the travel time between ports is expected, a "slack" time is defined and allowed for each leg of the voyage (a scheduled port pair). Accordingly, delays in excess of this slack time represent a late arrival at the next scheduled port. In other words, a vessel will not experience laycan violations as long as slack times are not violated (as explained in Section 1.2.1, laycans indicate the time windows in which a vessel must arrive at the scheduled ports). If the waiting time at bunkering ports (between a pair of L/D ports) exceeds this slack time, then the penalty charges associated with late arrival is applied due to overrunning of the slack time.

A MIP optimization model is constructed to reflect the above discussed issues and considerations. The sets and parameters used in the model are defined as follows:

S : the set of scheduled ports for L/D operations;

R : the set of ports for only refueling ;

$P = |S \cup R|$: the number of ports under consideration;

$P(i)$:the set of ports associated with voyage leg ending with port $i \in S$;

p_i : unit bunker price at port $i \in S \cup R$;

c_i : fixed cost of bunkering at port $i \in S \cup R$;

w_i : waiting time at port $i \in S \cup R$;

v_i : variance of waiting time at port $i \in S \cup R$;

C : bunker capacity of a vessel;

d_i : travel distance of voyage leg which ends at port $i \in S$, in terms of days of travel time;

t_i : additional travel distance of visiting $i \in R$ in terms of days of travel time;

f_i : lower limit on the amount of bunker that can be purchased at port $i \in S \cup R$;

D : penalty charges associated with late arrival (per time unit day);

b : average bunker consumption per unit time (day);

β : importance weight of waiting time variance;

I : on board bunker amount at the beginning of voyage;

L : lower bound on bunker amount on board on arrival at a port;

T : slack time allowed in a leg of the voyage;

q_i : travel distance of port $i \in S \cup R$ to the starting port of voyage leg associated with i;

The decision variables used in the model are as follows:

x_i : the amount of bunkering at port $i \in S \cup R$;

u_i : the time in excess of allowed slack time in leg ending with port $i \in S$;

y_i : binary variable indicating whether the vessel refuels port $i \in S \cup R$ or not;

The model is formulated as follows:

$$z = \min \sum_{i \in S \cup R} p_i x_i + \sum_{i \in \mathcal{S} \cup R} c_i y_i + D \sum_{i \in \mathcal{S}} u_i + \beta D \sum_{i \in \mathcal{S} \cup R} v_i \tag{3.1}$$

s.t.

$$x_i \le C - I - \sum_{\substack{k<i \\ k \in S \cup R}} x_k + b \left(\sum_{\substack{k \le i \\ k \in S}} d_k + \sum_{\substack{k<i \\ k \in R}} t_k y_k + y_i q_i \right) \qquad \forall\, i \in S \cup R \tag{3.2}$$

$$I + \sum_{\substack{k<i \\ k \in S \cup R}} x_k - b \left(\sum_{\substack{k \le i \\ k \in S}} d_k + \sum_{\substack{k<i \\ k \in R}} t_k y_k + y_i q_i \right) \ge L \qquad \forall\, i \in S \cup R \tag{3.3}$$

$$u_i \ge \sum_{k \in P(i)} y_k \left(t_k + w_k \right) - T \qquad \forall\, i \in S \tag{3.4}$$

$$x_i \leq My_i \qquad \forall i \in S \cup R \tag{3.5}$$

$$x_i \geq f_i y_i \qquad \forall i \in S \cup R \tag{3.6}$$

$$u_i \geq 0 \qquad \forall i \in S \tag{3.7}$$

$$x_i \geq 0 \qquad \forall i \in S \cup R \tag{3.8}$$

$$y_i = 0,1 \qquad \forall i \in S \cup R \tag{3.9}$$

The objective function (3.1) minimizes the sum of five sets of costs. The first term is the bunker purchasing costs from scheduled and "refueling only" ports. The second term indicates the fixed cost of visiting ports for refueling. Generally, this cost item is lower at scheduled ports. The third and fourth terms are extra charges associated with late arrival (due to slack time violation) and uncertainty cost due to variance of waiting times. Although the extra charge due to violation of slack time and the waiting variances do not refer the same cost item, the parameter D is used in both costs, and it is left to the "weight adjuster" coefficient "β" to adjust the relative importance of these cost items. Constraint set (3.2) includes capacity constraints and assures that the amount of bunker received at a port cannot exceed the empty space in bunker tanks. If the associated port is a scheduled port, the last term of the constraint ($y_i q_i$) is zero, since the scheduled port is already the starting port of a voyage leg. ($q_i = 0$, $\forall i \in S$). This principal is also valid for constraint set (3.3), which ensure that on board bunker is at least as large as the required safety amount when a vessel arrives at a port. Constraint sets (3.4) and (3.7) indicate that if the additional travel time and waiting time at a bunkering port exceed the slack time (actually the vessel refuels only once in a leg, as explained below), exceeded time is a positive value and zero otherwise. Constraint sets (3.8) and (3.9) are non-negativity and integrality constraints, respectively.

The Constraint set (3.4) does not restrict the vessel from visiting multiple bunkering only ports. In case of multiple bunkering port calls in a voyage leg, the port-to-port distances of bunkering ports must be also considered, in order to determine the bunker consumptions between them. For instance, in Figure 3.3, Kalilimenes and Piraeus are two bunkering ports of the voyage leg between Mersin and Istanbul (L/D ports). If the vessel first refuels at Kalilimenes and then at Piraeus, the bunker consumption between Piraeus and Kalilimenes is important data. However, as indicated, the travel distance of a bunkering port to the starting port of its voyage (q_i) is sufficient to discourage multiple bunkering calls in a leg since the penalty cost associated with slack time violation and the costs of visiting bunkering ports do not allow the vessel to stop for bunkering more than once in a leg anyway. Hence, this property of the problem simplifies the model formulation.

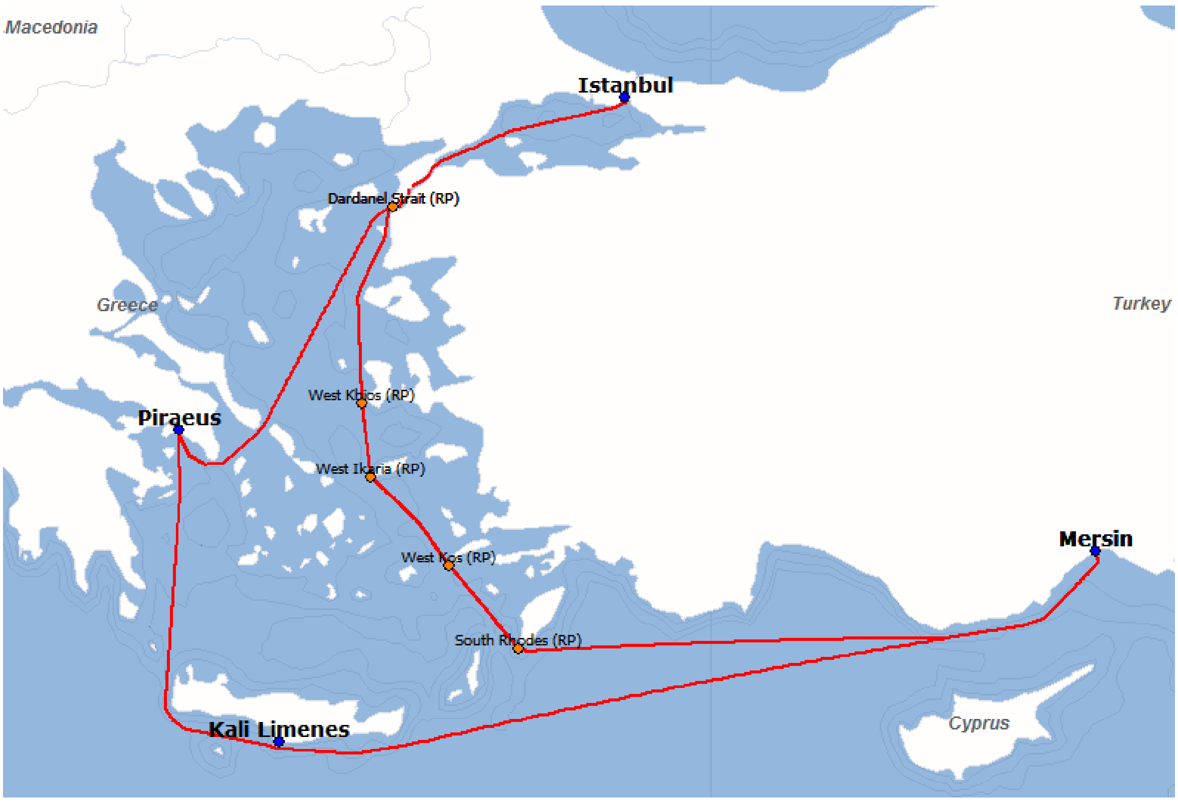

Figure 3.3. Example of more than one bunkering ports in a voyage leg

3.3. The Long Term Bunkering Model

In this section, the LTB model is formulated as a discounted DP problem with finite state space. Before detailed formulation of the LTB model is given, a brief overview on the DP methodology used in such models is presented. In order to clarify these DP topics, different problem types in related areas are explained with examples.

3.3.1. The Dynamic Programming Methodology

DP is a modelling approach to sequential decision problems that usually require optimization over time and facilitates the analysis of the structural properties of these problems, as well as aiming for the optimal solutions of these problems. DP is most suitable in many real life applications such as budgeting, inventory control and network problems [22]. DP solution approach is based on the idea of making an optimal decision at each stage, while considering best possible sum of present and expected future costs or revenues. The general structure of a DP problem can be best illustrated with an example.

Inventory control is a well-known dynamic decision problem. In this case, a problem in finite horizon is considered. The decision periods are discrete time intervals. The decision or control variable of this problem is the quantity to be ordered for an item at the beginning of each period. A policy, prescribes the way a decision is to be made at each point in time; hence, it is the sequence of all control variables of the problem. The stock available at the beginning of a period can be the state variable, which always needs to contain a sufficient and efficient summary of available information affecting the future of the inventory process. The state variable includes the available information at that point in time. Control variable (action) is determined based on the stochastic demand of the item during each period. The costs incurred at a period are holding or shortage costs of inventory and ordering cost. Hence, the first cost item is a function of the state variable and the second cost item depends on the control variable. The problem is explained with a simple formulation as follows:

s_t: inventory at the beginning of period t;

x_t: ordered amount at the beginning of period t;

d_t: demand in period t;

$p_{i,j}(x,t)$: probability at time t that the next state is j, given that the current state is i and action is x.

$$s_{t+1} = x_t + s_t - d_t \tag{3.10}$$

$$p_{i,j}\left(x,t\right) = P\left\{s_{t+1} = j \mid s_t = i,\ x_t = x\right\} \tag{3.11}$$

As indicated in Equation (3.10), the stock at the beginning of the next period depends on the current stock, amount ordered at the beginning of the current period and demand during the period. Therefore, the next state depends on the probability distribution of a random factor (demand). Accordingly, the probability indicated in (3.11) is also the probability of demand at period *t* to be a value dependent on order amount. Hence, action to be taken is a determinant of the transition probability between states.

In DP, transition between states can be deterministic or stochastic. As in this example, in the basic models, transitions usually have the memoryless property of Markov Chains, which indicates that the next state only depends on the current state, but not other previous states. For the inventory example, in each state there are a number of possible events that can result with a transition, which indicate a Markov Decision Process (MDP). Stochastic DP problems with discrete states, decisions and times are called MDP [22]. In this study, the MDP of interest is actually a continuous time irreducible Markov Chain. It does not have any absorbing state or subset of states and all states are aperiodic. In other words, there is a single recurrent class of states that any state in the class is reachable from any other state in the class. Hence, the Markov Chain is ergodic.

MDPs are in general solved by recursive equations called "Bellman's equations". Each equation of the problem has a value function which is cost (or reward) due to action on a particular state. Bellman's equations of the inventory control problem is given as:

$$X\left(s_t\right) = \begin{cases} x_t = I - s_t, & s_t < I \\ 0, & otherwise \end{cases} \tag{3.12}$$

I : ordering point of the inventory level;

π : a policy that is a sequence of functions $\left\{\mu_0, \mu_1, \ldots, \mu_{N-1}\right\}$, where $\mu_i\left(s_i\right) = x_i$;

c : cost per unit order;

$h(s_t)$: one period holding (or shortage) cost of s_t;

$$V^{\pi}(s_t) = h(s_t) + c\mu_t(s_t) + \beta \mathrm{E}^{\pi}\left[\sum_{i=t}^{T} V(s_i)\right] \tag{3.13}$$

$$V(s_t) = h(s_t) + \min_{x_t \in X(s_t)}\left[cx_t + \sum_{d_t} P(s_{t+1} = x_t + s_t - d_t) V(s_{t+1})\right] \tag{3.14}$$

The Equation (3.13) defines a value function for a particular policy π. As indicated in optimality Equation (3.14), recursive part is the expected value, V, of the future periods. Since all value functions include such recursions, finding optimal decision for one period at each stage leads us to an optimal policy. Hence, Bellman's optimality principal indicates that:

> "Regardless of the decisions taken to enter a particular state in a particular stage, the remaining decisions made for leaving that stage must constitute an optimal policy."[24]

Therefore, in finite horizon problems, whatever the intial state and action are, remaining actions give an optimal policy based on the first state. An optimal policy can be constructed by backward induction, which is based on finding an optimal policy for the last period, then extending this operation for the last two period and continuing in this manner until achieving an overall optimal policy.

Bellman's optimality condition is also valid for infinite horizon MDPs. Infinite horizon models are deployed in the following situations:

- The number of stages is really infinite or a large number which is not precisely known and end of planning horizon is far distant in the future.
- The length of the planning horizon can be random or subject to control with no specific upper bound.
- The infinite horizon asuumption may simplify the solution, even if the planning horizon is finite and known [25].

The main difference between finite and infinite stage problems is that the system is regarded stationary in infinite stage problems. “Stationary” term indicates time homogeneity; that is, the system equation, cost per stage, distribution of random parameters, state and control spaces do not change in time [26]. In contrast with finite stage problems, optimal policies of these problems are also stationary (time-homogeneous). Therefore, these policies can be computed and implemented relatively easily. The solution methods of two types are also different. Another difference is that unlike finite horizon case, infinite horizon problems does not have a starting state.

There are four main classes of infinite horizon problems. The first class is stochastic shortest path problems. The number of stages is finite but stage length is random and depends on the policy. The second problem type is discounted problems with bounded cost per stage. The absolute value of cost per stage is bounded, which is the condition of limit existence, since the problem becomes infinite stage problem as the number of stage goes to infinity (see Equation (3.15)).The third type is discounted and undiscounted problems with unbounded cost per stage. The fourth type gives consideration to average cost per stage. The sum of the total cost or reward does not need to be finite since the average cost is used [25].

Since the infinite stage problem is independent from time, the Bellman’s equation for the inventory problem can be converted into a steady-state form, as indicated in Equation (3.15). Consequently, the discounted problem Equation (3.16) of a particular stationary policy π does not have any time dimension. The discount factor is denoted by $0 < \beta < 1$. The role of the discount factor is to deemphasize rewards and cost that might be obtained in a more distant or unknown future.

$$V(s) = \lim_{t \to \infty} V(s_t) \tag{3.15}$$

$$V^{\pi}(s) = h(s) + c\mu(s) + \beta \sum_{s' \in S} P(s' \mid s, \mu(s)) V^{\pi}(s') \tag{3.16}$$

$$V(s) = h(s) + \min_{x \in X(s)} \left(cx + \beta \sum_{s' \in S} P(s' \mid s, x) V(s') \right) \tag{3.17}$$

The policy $\pi = \{\mu, \mu, \ldots, \mu\}$ is a stationary optimal policy if and only if $\mu(s)$ gives the minimum value V for each s. In other words, whatever the state at a point in time, the stationary policy that gives the minimum is optimal. The optimality condition is stated in Equation (3.17). The solution of the set of linear equations that consists of the Equation (3.16) of each state is unique for a stationary policy.

Individual DP equations does not seem hard to solve but, computational complexity of the DP algorithm increases exponentially with dimensionality of the state set, which makes it impractical in large-scale applications. Many practical problems suffer one of the three curses of dimensionality. These are state, action and outcome or random parameter spaces. As an example, computational complexity of inventory control problem is affected from the number of product types. If there are M types of product with N different inventory levels, the DP problem will have N^M number of states.

Regarding the computational aspects, there are several algorithms to solve infinite horizon problems. Unlike backward induction of the finite stage case that start from last period, infinite horizon algorithms does not have any last period to start iteratively. For a conjecture of optimal policy, the set of linear equations can be solved and compared with other results to come up with a point where no further improvement in value functions is possible. Computational methods of infinite horizon problems are explained in the following sections.

3.3.2. Model Structure

The aim of the LTB model is to plan the bunkering decisions of tramp vessels in the long term. Short term voyage information of a tramp vessel is known just before the beginning of the voyage period. Although bunkering decisions for a voyage can be arranged after the voyage details become available, as performed in the SVB model, these decisions do not consider the unknown future voyages. Hence, cost optimization does not cover other port calls in the unknown future. If the optimization is performed regarding the predicted future port calls, the resulting overall bunkering costs of the associated multiple voyages have a potential to significantly improve the short term planning results.

The idea behind the long term planning can be explained with a scenario. For instance, a tramp vessel has a known voyage that includes port calls at Rotterdam, Ceuta and Augusta. Short term plan suggests refueling at Ceuta for a certain amount of bunker. However, when the next voyage becomes certain (as from Augusta to Istanbul then to Odessa), the vessel would need more bunkering at Augusta to sail onwards. If Ceuta is better than Augusta in terms of bunkering cost, more bunker could be acquired from Ceuta to decrease overall costs, if future voyages of the vessel were considered. Therefore, long term planning have a good potential to lower overall cost.

The LTB model is presented in the form of an infinite stage discounted DP problem. The reason of applying this class of DP is to have a model that fits tramp vessels operations' structure, as well as to plan bunkering in the long term. As mentioned previously, infinite stage modeling is applied when the end of planning horizons are random and/or cannot be known precisely. Sailing behaviors of tramp vessels constitute port-to-port transitions that are uncertain and varying. Therefore, infinite stage model is well-suited for the tramp vessel problem. Discounted problem type with bounded cost per stage is applied due to computational ease and nature of the cost structure. Since the aim is to optimize long term total cost instead of average cost per stage, the discounted type of modeling is appropriate. Additionally, since the cost at every stage is bounded, as the number of stages goes to infinity, value functions of the distant future converges to zero [26]. Hence, infinite stage modeling in applicable.

In the LTB problem, there is a network of ports that tramp vessels frequently visits for L/D operations and/or bunkering. Port-to-port transition probabilities can be estimated from previous records of vessels. Since bunkering amount at a port depends on the vessel's arrival bunker level, the state of the system has two components, which are, current port and on board arrival bunker level of the vessel. Therefore, the next state only depends on the current state, independent from other previous states, and this form indicates that the state transitions are MDPs. The transition matrix has no periodicity and it consists of a single recurrent class of states, which means that the Markov chain is irreducible. Therefore, the limiting distribution exists.

A stationary policy of the model consists of amounts that the vessel should refuel at each state. In other words, decision variable of each state is the amount of bunker that is to be purchased at a port with a known on board amount. The sum of arrival and received bunker at port should enable the vessel to sail onward to other ports that have a positive transition probability from the current port. Even when this probability is small, the vessel must be able to sail if this voyage leg is arranged. Therefore, the departure bunker on board should be sufficient to allow sailing to the most distant port having a positive transition probability. It means that the model should not let the tramp vessel lose a job due to lack of bunker.

However, a penalty cost is applied to "unplanned bunkering" accomplished to avoid being short of bunker for small transition probability, long duration next destinations. According to the established procedures, when a vessel arrive at a port, it refuels a certain amount, which is predetermined and ordered before the vessel's arrival and before the declaration of the next port. The model determines this amount before arrival to the port based on the levels of the state variables. However, this amount may not be sufficient to sail to a distant port having a small transition probability. If an order is placed to sail to that distant port, the vessel must receive additional bunker to fulfill that voyage leg. The model applies a penalty cost to that "unplanned bunkering" and the "predetermined bunkering" amounts for each state (port and on board bunker level pair) constitutes a stationary policy. The penalty cost can be interpreted as a type of spot pricing or surcharge of rushed deliveries. As the penalty cost decreases, it is expected that unplanned bunkering (after next destination becomes certain) would increase, but this means that the importance of applying a predetermined plan decreases. The cost of unplanned bunkering decision is evaluated based on new bunker prices of the port. The predetermined amount is evaluated and arranged at the previously set bunker price, which is generally less than the spot price.

As mentioned previously, the bunker consumptions between ports are continuous values. In order to have a finite set of states, bunker consumption, on board (arrival) bunker amounts and refueling amounts are discretized. Therefore, the number of states become the product of the number of ports and the number of on board bunker levels.

The parameters and control variables of the LTB model are as follows:

$N = \{1, 2, \ldots, n\}$ is the network of ports;

$A = \{(i, j) : i, j \in N\}$ is the set of connecting nodes in N ;

K is the number of on board (arrival) bunker levels;

$\mathcal{I} = \{I_k : LB \le I_k \le C, k = 0, \ldots, K-1\}$ is the set of discretized arrival bunker levels;

$B = \{b_{ij} \in \mathbb{Z}_+ : (i, j) \in A\}$ is amount of bunker consumption in the arcs of defined network;

C is the bunker capacity of a vessel;

$U(i, I_k) = \{u_{iI_k} : 0 \le u_{iI_k} \le C - I_k\}$ is the amount of bunkering at state (i, I_k);

$J(i, u_{iI_k}) = \{j : j \in N, b_{ij} \le I_k + u_{iI_k}\}$ is the set of ports that are reachable from $i \in N$ when arrival bunker amount is I_k and u_{iI_k} is the bunkering amount at state (i, I_k);

$\mathcal{V} = \{v_{ij} : (i, j) \in A\}$ is the set of transition probabilities between ports in the network;

$\mathcal{V}^0 = \{v_i^0 : i \in N\}$ is the set of steady state probabilities of the ports in the network;

$P = \{p_i : i \in N\}$ is the set of fuel prices at each port in the network

p^0 is the average cost of bunker on board;

LB : is the lower bound on the amount of bunker on board, which is equal to I_0 ;

w_i is the fixed cost of visiting port i ;

$D = \{d_{ij} : d_{ij} = v_{ij} b_{ij}, i \in N, j \in N\}$ is the set of expected bunker consumptions between ports;

pm is the multiplier to obtain penalty cost of unplanned bunkering;

f_i is the average bunkering amount at port i for a stationary policy;

$\pi:\left\{u_{1I_1},u_{1I_2},\ldots,u_{nI_K}\right\}$ is the stationary policy;

The Bellman's equation is formulated as follows:

$$H\left(i,I_k\right)=\min_{L_i\le u_{iI_k}\le C-I_k}\left\{\begin{array}{l} h\left(u_{iI_k}\right)+\alpha\sum\limits_{j\in J\left(i,u_{iI_k}\right)} v_{ij}\left(p^0 b_{ij}+H\left(j,I_k+u_{iI_k}-b_{ij}\right)\right)+ \\ \alpha\sum\limits_{j\notin J\left(i,u_{iI_k}\right)} v_{ij}\,g\left(b_{ij}+LB-I_k-u_{iI_k}\right)+\alpha\sum\limits_{j\notin J\left(i,u_{iI_k}\right)} v_{ij}\,p^0 b_{ij}+ \\ \alpha\sum\limits_{j\notin J\left(i,u_{iI_k}\right)} v_{ij}H\left(j,LB\right) \end{array}\right\} \tag{3.18}$$

where

$$\mathcal{V}^0=\mathcal{V}^0\mathcal{V} \tag{3.19}$$

$$\sum_{i\in\mathcal{N}} v_i^0=1 \tag{3.20}$$

$$h\left(i,u_{iI_k}\right)=w_i+p_i u_{iI_k} \tag{3.21}$$

$$g\left(i,b_{ij}+LB-I_k-u_{iI_k}\right)=\left(pm*w_i+w_i\right)+\left(pm*p_i+p_i\right)(b_{ij}+LB-I_k-u_{iI_k}) \tag{3.22}$$

$$f_i=\frac{\sum\limits_k u_{iI_k}+\sum\limits_k\sum\limits_{j\notin J\left(i,u_{iI_k}\right)} v_{ij}\left(b_{ij}+I_0-I_k-u_{iI_k}\right)}{K} \tag{3.23}$$

If we denote $\mathcal{M}$ as "min" operator, then the Equation (3.18) can be expressed in matrix form as follows:

$$\mathcal{M}^{\pi}\left(H\right)=h^{\pi}+\alpha p^0 D+\alpha V^{\pi} g^{\pi}+\alpha VH^{\pi} \tag{3.24}$$

The equation (3.18) consists of two main recursive parts. The first recursive part includes value functions of the states that are reachable from the current state without extra bunkering. The second recursion consists of states that are not reachable with bunkering policy π and require extra bunker. In other words, the predetermined policy π is not sufficient to sail to some of the ports. Therefore, bunkering amount is increased to a level that is sufficient to reach that distant ports. As indicated, unplanned bunker is purchased at a penalty cost.

The first term $h\left(u_{iI_k}\right)$ of Equation (3.18) is the purchasing cost of bunker at port i including fixed and variable costs. The second term includes total discounted expected cost of bunker consumption to sail to reachable ports with bunker amount $u_{iI_k} + I_k$. The recursion term in this part, $H\left(j, I_k + u_{iI_k} - b_{ij}\right)$, is the future expected cost associated with transitions to other states. The state variable $I_k + u_{iI_k} - b_{ij}$ is simply the amount of bunker on board on arrival at port j, and the total of these value functions is also an expected value. The second discounted part of the equation covers the cost to sail to distant ports that are not reachable with the predetermined u_{iI_k} bunkering amount. In order to reach these ports, the vessel receives extra bunker (at penalty cost). This amount is denoted by $b_{ij} + LB - I_k - u_{iI_k}$ and penalty cost of receiving that much bunker is $g\left(b_{ij} + LB - I_k - u_{iI_k}\right)$. The final two terms are the total expected consumption and recursion costs associated with transition to distant ports. Since a voyage leg to a distant port becomes certain after the first bunkering decision, total purchasing cost of extra bunkers is also expected value. Bunker level of the state variable of a distant port is LB (or equivalently I_0), since, because of penalty costs, only minimum required level of bunker to sail to that port should be acquired as extra (unplanned) amount.

The average bunkering amount at each port is evaluated in Equation (3.23) for a predetermined bunkering policy. The first part of this equation is the equally weighted average of the predetermined amounts and the second part is the expected unplanned bunkering. As explained, a vessel performs additional (unplanned) bunkering when it is required to sail to a distant port, (which is unreachable with the predetermined amount of bunkering). Therefore, in order to estimate the total expected unplanned bunkering, unplanned bunkering amounts associated with states are multiplied with the probability of sailing to the respective distant ports and aggregated.

$$p^0 = P\mathcal{V}^0 F \Big/ \sum_{i\in\mathcal{N}} f_i v_i^0 = \left[[p_1, p_2, \ldots, p_n] \begin{bmatrix} v_1^0 \\ v_2^0 \\ \vdots \\ v_n^0 \end{bmatrix} [f_1, f_2, \ldots, f_n] \right] \Big/ \sum_{i\in\mathcal{N}} f_i v_i^0 \tag{3.25}$$

In order to calculate the consumption costs between ports, the cost of a unit bunker on board, p^0, needs to be estimated based on Equation (3.25). The model is solved iteratively, for each time the p^0 value is improved. Thereby, iteratively improving the estimate for the cost of fuel consumed between two ports. Initially, refuelling amounts are assumed to be uniform among ports (which is denoted by $f_1 = f_2 = \ldots = f_n$). Steady state transition probabilities are evaluated based on the Equations (3.19) and (3.20). Then, the average cost per unit of onboard fuel is calculated and used to evaluate the fuel consumption cost of a transition. Once an optimal policy is found, it is used to revise the unit cost of fuel on board. Then, an improved optimal policy is found by using the new p^0 value and this procedure is continued until achieving no further improvement in value functions or change in optimal stationary policy.

The optimal long run cost is evaluated from the average bunkering amounts at ports as deployed in Equation (3.26). The sum of all value functions is used to observe the policy improvements based on the computation algorithms. However, this sum is not a meaningful measure of overall long term cost. For instance, as the number of bunker levels increases, this sum also increases due to the increase in the number of states (The number of value functions is equal to the number of states). Solution procedure deployed is detailed in the following sections.

$$\text{Optimal long run cost} = \sum_i v_i^0 \left(f_i p_i + w_i \right) \tag{3.26}$$

3.4. Assumptions of Models

The SVB and the LTB models are based on some assumptions. The assumptions that are used in both models are as follows:

- Speed and route changes due to sea conditions such as weather, canal traffic etc. are neglected.
- Bunker prices, port and canal fees are exempt from time related changes.
- There is a lower limit on the bunker supply amount of each port, such that a vessel must receive at least this amount if the decision to bunker at that port is made. In the current implementation, these limits are assumed to be equal for each port of the network.
- Models are formulated for one type of bunker. Simultaneous planning for more than one type of bunkers is not considered.
- Waiting times are not proportional with bunkering amounts.
- Speed of vessel is not proportional with the weight of cargo. The average speed of vessel is used.
- Predetermined maritime routes are used between ports.
- Ship-to-ship bunkering is not considered.

Another assumption of the SVB model is that the port charges at scheduled L/D ports are zero. Since the vessel visits these ports in any case, the port charges due to L/D operations do not affect the overall bunkering costs of the problem.

Additionally, in the LTB model it is assumed that the occurrence frequency of various arrival bunker levels at different ports are the same. In reality, this may not be the case; for instance, the vessel may arrive at a specific port with 100 MT bunker on board more frequently than 120 MT. Accordingly, in the LTB model, the average bunkering from a port is evaluated as an equally weighted average of optimal policies in that port for each level of on board bunker. (i.e. $\sum_k u_{iI_k} \big/ K$ part of the Equation (3.23))

3.5. Computation Algorithms for The Long Term Model

In this section, the computational methodologies used to solve the infinite stage DP model are explained. There are several algorithmic strategies for solving infinite horizon problems. Three of these methods, the Policy Iteration (PI), the Value Iteration (VI) and the Hybrid Value-Policy Iteration (HVPI) approaches are explained in the following subsections. Additionally, alternative heuristic algorithms are presented.

As indicated before, in infinite stage problems, since there is no last period to start with, it is not possible to apply backward induction. Of course, all possible stationary policies could be tried until achieving the lowest sum of value functions. However, this approach is not efficient due to curse of dimensionality. Hence, the algorithms to be presented depend on some sort of convergence in value functions, which is faster than evaluation of all policies.

The linear programming (LP) approach is also a widely used method in solving DP models. In the LP approach to infinite stage DP models, the Bellman's equations are the constraints and the model is of the maximin type if the objective is reward maximization (and of the minimax type if the objective is cost minimization). As the number of states increases, the LP model becomes hard to handle due to the increase in the number of constraints. Therefore, the algorithms to be presented can be used instead of the LP approach, since they can easily be implemented to problems with large state spaces.

3.5.1. The Policy Iteration Algorithm

The PI algorithm is based on generating an improving sequence of policies to the MDP problem. First, a policy is chosen and used to find initial value functions. Then, the new values are used to find an improved policy. This continues until no further improvement is possible. The general algorithm is described using the notation of the LTB model as follows:

Step 0: Initialization:

i. Compute steady state probabilities by $\mathcal{V}^0 = \mathcal{V}^0\mathcal{V}$ and $\sum_{i\in\mathcal{N}} v_i^0 = 1$.

ii. Set $p^0{}_0 = \sum_{i\in\mathcal{N}} p_i v_i^0$.

iii. Select a policy $\pi_0^0 = \left\{u_{iI_k}, u_{iI_k} = 0 \;\; \forall (i, I_k)\right\}$.

iv. Set outer iteration $t_1 = 1$ and inner iteration $t_2 = 1$.

Step 1: Policy Evaluation:

Given a policy $\pi_{t_1}^{t_2-1} = \left\{\left(u_{1I_1}\right)_{t_1}^{t_2-1}, \left(u_{1I_2}\right)_{t_1}^{t_2-1}, \ldots, \left(u_{nI_K}\right)_{t_1}^{t_2-1}\right\}$;

i. Compute the $h^{\pi_{t_1}^{t_2-1}}$, $g^{\pi_{t_1}^{t_2-1}}$ and $V^{\pi_{t_1}^{t_2-1}}$ matrix.

ii. Compute $H^{\pi_{t_1}^{t_2}}$ matrix by solving the system of linear equations

$$H^{\pi_{t_1}^{t_2}} = \left(I - \alpha V\right)^{-1}\left(h^{\pi_{t_1}^{t_2-1}} + \alpha p^0 D + \alpha V^{\pi_{t_1}^{t_2-1}} g^{\pi_{t_1}^{t_2-1}}\right)$$

Step 2: Policy Improvement:

Obtain a new policy π'_{t_1} based on Equation (3.28)

If $\left(u_{iI_k}\right)'_{t_1} = \left(u_{iI_k}\right)_{t_1}^{t_2-1}$ ($\pi'_{t_1} = \pi_{t_1}^{t_2-1}$) for all states $\left(i, I_k\right)$

Go to Step 3.

Else

Set $t_2 = t_2 + 1$, $\pi_{t_1}^{t_2} = \pi'_{t_1}$ and go to Step 1.

Step 3: Update Cost of Bunker On Board:

i. Compute average bunkering amounts by Equation 3.23.

ii. Compute cost of bunker on board by

$$p^0{}_{t_1} = P\mathcal{V}^0 F \Big/ \sum_{i\in\mathcal{N}} f_i v_i^0 = \left[\left[p_1, p_2, \ldots, p_n\right]\begin{bmatrix} v_1^0 \\ v_2^0 \\ \vdots \\ v_n^0 \end{bmatrix}\left[f_1, f_2, \ldots, f_n\right]\right] \Big/ \sum_{i\in\mathcal{N}} f_i v_i^0$$

If $p^0{}_{t_1} = p^0{}_{t_1-1}$

Stop, optimal policy is $\pi^* = \pi_{t_1}^{t_2-1} = \pi'_{t_1}$.

Else

Set $t_1 = t_1 + 1$, $t_2 = 1$ and go to Step 1.

Figure 3.4. The policy iteration algorithm

(i) Step 1 (Initialization): Select an initial arbitrary stationary policy π^0 and set the iteration number $t=0$.

(ii) Step 2 (Policy Evaluation): Given the stationary policy π^t, compute the corresponding unknown $H(i,I_k)$ values by solving the following system of linear Equations (3.27) (where I denotes an identity matrix).

$$H^{\pi^t}=(I-\alpha V)^{-1}\left(h^{\pi^t}+\alpha p^0 D+\alpha V^{\pi^t}g^{\pi^t}\right) \tag{3.27}$$

(iii) Step 3 (Policy Improvement): Find a new stationary policy π^{t+1} that satisfies the following condition for each state:

$$u_{iI_k}^{t+1}=\underset{u_{iI_k}\in U(i,I_k)}{\arg\min}\left\{\begin{array}{l} h\left(u_{iI_k}\right)+\alpha\displaystyle\sum_{j\in J\left(i,u_{iI_k}\right)} d_{ij}p^0+\alpha\sum_{j\notin J\left(i,u_{iI_k}\right)} v_{ij}g\left(b_{ij}+I_0-I_k-u_{iI_k}\right) \\ +\alpha\displaystyle\sum_{j\notin J\left(i,u_{iI_k}\right)} v_{ij}\left(p^0 b_{ij}+H^t\left(j,I_k+u_{iI_k}-b_{ij}\right)\right) \\ +\alpha\displaystyle\sum_{j\notin J\left(i,u_{iI_k}\right)} v_{ij}H^t\left(j,I_0\right) \end{array}\right\} \tag{3.28}$$

(iv) Step 4: If $u_{iI_k}^{t+1}=u_{iI_k}^t$ for all states (i,I_k), then set optimal policy $\pi^*=\pi^t=\left\{u_{1I_1}^t,u_{1I_2}^t,\ldots,u_{nI_K}^t\right\}$; otherwise set $t=t+1$, return to Step 2 and repeat the process.

In general, the PI algorithm converges to an optimal solution in a remarkably small number of iterations. Typically fewer than 10 to 20 policy iterations are required to find an optimal solution [27]. However, solving the system of linear equations indicated becomes computationally burdensome as the number of states and decision variables increases.

The solution procedure of the LTB model consists of two types of iterations. At the end of each main iteration, the cost of bunker on board, namely p^0, is updated. In the sub iteration, the main policy iteration idea is applied to find an optimal policy associated with the current p^0 value.

The overall algorithm is presented in Figure 3.4. The inner iteration refers to policy iteration. After a number of inner iterations, a tentative optimal policy is reached. Then, p^0 value is updated (in the outer iteration). The initial policy π_0^0 can be any set of refueling policies; in this study, it is selected as either $\{0,0,...,0\}$ or $\{C-I_0, C-I_1,...,C-I_{K-1}\}$.In other words, the algorithm starts with minimum or maximum bunkering possible at each state.

3.5.2. The Value Iteration Algorithm

The VI algorithm is the most common algorithm used in solving DP problems, due to its ease of implementation. It is based on iterative estimation of value functions. At each iteration, the estimated value function of a state determines the decision variable for that state. Hence, the VI algorithm is also called as the successive approximation algorithm.

Unlike the PI algorithm, solving a set of linear equations is not necessary in the VI algorithm. Instead, the VI algorithm starts with arbitrary values of value functions and sequentially updates them by applying these values to recursive parts of the Bellman's equations. This process continues until convergence of value functions is obtained. Another difference from the PI approach is that, in the VI algorithm, at each iteration, an optimal policy is found for the values obtained from the previous iteration. At each iteration, a policy that gives the lowest state values is chosen. Then, the values of this policy is used to devise a new policy. The convergence of the algorithm refers to the small difference between error bounds. In other words, the algorithm continues until the largest value function change becomes a negligible value, almost zero. The general VI algorithm can be described as follows:

(i) Step 1 (Initialization): Select an initial set of value functions and set iteration number $t=0$. Generally, $H^0(i,I_k)=0$ is set as initial state values, while this selection affects the convergence rate of algorithm.

(ii) Step 2 (Value Improvement): For each state (i,I_k), compute the $H^t(i,I_k)$ values and obtain the optimal policy π^t satisfying the following equation for all states.

$$H^t(i,I_k)=\min_{L_i\le u_{iI_k}\le C-I_k}\left\{\begin{array}{l} h\left(u_{iI_k}\right)+\alpha\sum\limits_{j\in J\left(i,u_{iI_k}\right)} v_{ij}\left(p^0 b_{ij}+H^{t-1}\left(j,I_k+u_{iI_k}-b_{ij}\right)\right)+ \\ \alpha\sum\limits_{j\notin J\left(i,u_{iI_k}\right)} v_{ij}\, g\left(b_{ij}+I_0-I_k-u_{iI_k}\right)+\alpha\sum\limits_{j\notin J\left(i,u_{iI_k}\right)} v_{ij}\, p^0 b_{ij}+ \\ \alpha\sum\limits_{j\notin J\left(i,u_{iI_k}\right)} v_{ij} H^{t-1}\left(j,I_0\right) \end{array}\right\}$$

(iii) Step 3 (Error Evaluation): Compute the error values $\underline{\boldsymbol{e}}_{\boldsymbol{t}}$ and $\overline{\boldsymbol{e}}_{\boldsymbol{t}}$**.** If $\overline{\boldsymbol{e}}_{\boldsymbol{t}}-\underline{\boldsymbol{e}}_{\boldsymbol{t}}\le\varepsilon$, mark π^t as optimal policy, otherwise set $t=t+1$ and go to Step 2. ε is chosen as a value sufficiently close to zero.

$$\underline{e}_t=\frac{\alpha}{1-\alpha}\min_{i,I_k}\left[H^t(i,I_k)-H^{t-1}(i,I_k)\right]$$

$$\bar{e}_t=\frac{\alpha}{1-\alpha}\max_{i,I_k}\left[H^t(i,I_k)-H^{t-1}(i,I_k)\right]$$

Although the VI is a less computationally complex algorithm, it converges more slowly compared to the PI algorithm. Referring to the number of states and the number of actions as N and M, respectively, the running time for each iteration in the VI is $O(MN^2)$. Therefore, it is a polynomial time algorithm, (as long as the total number of iterations required for convergence is polynomial). The PI algorithm is also a polynomial time algorithm, where policy improvements can be performed in $O(MN^2)$ arithmetic operations and linear system of equations can be solved in $O(N^3)$ operations. Computational performances of these methods is compared in the following sections. The VI algorithm of the LTB model is presented in Figure 3.5.

Step 0: Initialization:

i. Compute steady state probabilities by $V^0 = V^0 V$ and $\sum_{i\in\mathcal{N}} v_i^0 = 1$.

ii. Set $p^0{}_0 = \sum_{i\in\mathcal{N}} p_i v_i^0$.

iii. Set $H_0^0(i, I_k) = 0$ for each state (i, I_k)

iv. Set outer iteration $t_1 = 1$ and inner iteration $t_2 = 1$.

Step 1: Value Improvement:

Obtain an optimal policy $\pi_{t_1}^{t_2} = \left\{ \left(u_{1I_1}\right)_{t_1}^{t_2}, \left(u_{1I_2}\right)_{t_1}^{t_2}, \ldots, \left(u_{nI_K}\right)_{t_1}^{t_2} \right\}$ and compute the $H_{t_1}^{t_2}(i, I_k)$ values that satisfy the following system of equations:

$$H_{t_1}^{t_2} = \min_{\pi_{t_1}^{t_2}} \left(h^{\pi_{t_1}^{t_2}} + \alpha p^0 D + \alpha V^{\pi_{t_1}^{t_2}} g^{\pi_{t_1}^{t_2}} + \alpha V H_{t_1}^{t_2 - 1} \right)$$

Step 2: Error Evaluation

Compute the error values $\underline{e}_t$ and $\bar{e}_t$ from the Equations (3.30) and (3.31).

If $\bar{e}_t - \underline{e}_t \le \varepsilon$; Optimal policy is $\pi_{t_1}^{t_2}$. Set $t_2 = 1$. Go to Step 3.

Else; Set $t_2 = t_2 + 1$. Go to Step 1

Step 3: Update cost of bunker on board:

i. Compute average bunkering amounts by Equation 3.23.

ii. Compute cost of bunker on board by

$$p^0{}_{t_1} = PV^0F \Big/ \sum_{i\in\mathcal{N}} f_i v_i^0 = \left[\left[p_1, p_2, \ldots, p_n \right] \begin{bmatrix} v_1^0 \\ v_2^0 \\ \vdots \\ v_n^0 \end{bmatrix} \left[f_1, f_2, \ldots, f_n \right] \right] \Big/ \sum_{i\in\mathcal{N}} f_i v_i^0$$

If $p^0{}_{t_1} = p^0{}_{t_1 - 1}$; Stop, optimal policy is $\pi_{t_1}^{t_2}$.

Else; Set $t_1 = t_1 + 1$. Set $H_0^0(i, I_k) = 0$ for each state (i, I_k) .
Go to Step 1.

Figure 3.5. The value iteration algorithm

3.5.3. The Hybrid Value-Policy Iteration Algorithm

In the VI approach, state values are not the steady state values of the policy. In contrast, the PI approach determines the true, steady state values for a policy. Therefore, the PI requires more arithmetic operations, but it converges faster than the VI. The HVPI algorithm uses advantageous properties of the PI and the VI methods. The algorithm updates values at a policy for a specific number of times, then (if the error values are sufficiently small) policy evaluation is performed to proceed to a new policy. Therefore, the hybrid algorithm converges faster than the VI algorithm and it is easier to implement in terms of computational complexity, compared to the PI algorithm.

The algorithm consists of two main steps that reflect policy generation and partial policy evaluation. The HVPI implementation on the LTB model is described in Figure 3.6.

3.5.4. Alternative Heuristic Algorithms

Apart from previously discussed three algorithms, two alternative heuristic algorithms are developed. The heuristic algorithms highly depend on a policy evaluation step like that of the PI. Therefore, the overall computation time is also long in these algorithms. Although these two heuristic algorithms converge slowly and generate near optimal solutions, frequently they do generate optimal solutions and they can be regarded as alternative methods.

In the first heuristic algorithm (ALT I), for a given policy, a group of states having values that makes the highest or lowest contribution to the sum of value functions is selected and all decision variable combinations, (namely, the set of all possible refueling amounts of these states) are generated. These combinations are used to derive different policies from the previously evaluated policy and the derived policy that gives the best total of value functions is chosen as an improved policy. Since policy evaluation is a process that consists of solving a set of linear equations as many as the number of states, the choice of group size is critical. When the group size increases, the number of policies to be evaluated and the time of convergence increases as well. The algorithm is described in Figure 3.7. In consistent with the previously described algorithms, the outer iteration indicates the number of p^0 updates and the inner iteration indicates the total number of policy improvements.

Step 0: Initialization:

i. Compute steady state probabilities by $\mathcal{V}^0 = \mathcal{V}^0\mathcal{V}$ and $\sum_{i\in\mathcal{N}} v_i^0 = 1$.

ii. Set $p^0{}_0 = \sum_{i\in\mathcal{N}} p_i v_i^0$.

iii. Set $H_0^0(i, I_k) = 0$ for each state (i, I_k).

iv. Select value iteration limit M .

v. Set outer iteration $t_1 = 1$ and inner iteration $t_2 = 1$.

Step 1: Policy Generation:

Obtain a policy $\pi_{t_1}^{t_2} = \left\{ \left(u_{1I_1}\right)_{t_1}^{t_2}, \left(u_{1I_2}\right)_{t_1}^{t_2}, \ldots, \left(u_{nI_K}\right)_{t_1}^{t_2} \right\}$ from Equation (3.28) of each state (i, I_k)

Step 2: Partial Policy Improvement

Set $m = 0$ and let $H_{t_1}^{t_2}(0) = \min_{\pi_{t_1}^{t_2}} \left(h^{\pi_{t_1}^{t_2}} + \alpha p^0 D + \alpha V^{\pi_{t_1}^{t_2}} g^{\pi_{t_1}^{t_2}} + \alpha V H_{t_1}^{t_2 - 1} \right)$

If $H_{t_1}^{t_2}(0) - H_{t_1}^{t_2-1} \le \varepsilon$, go to Step 3.

Else

While $m \le M$

i. Compute $H_{t_1}^{t_2}(m) = \min_{\pi_{t_1}^{t_2}} \left(h^{\pi_{t_1}^{t_2}} + \alpha p^0 D + \alpha V^{\pi_{t_1}^{t_2}} g^{\pi_{t_1}^{t_2}} + \alpha V H_{t_1}^{t_2}(m-1) \right)$.

ii. Set $m = m + 1$.

Set $H_{t_1}^{t_2}(M) = H_{t_1}^{t_2}$, $t_2 = t_2 + 1$ and go to Step 1.

Step 3: Update cost of bunker on board:

i. Compute average bunkering amounts by Equation 3.23.

ii. Compute cost of bunker on board by

$$p^0{}_{t_1} = P\mathcal{V}^0 F \Big/ \sum_{i\in\mathcal{N}} f_i v_i^0 = \left[\left[p_1, p_2, \ldots, p_n \right] \begin{bmatrix} v_1^0 \\ v_2^0 \\ \vdots \\ v_n^0 \end{bmatrix} \left[f_1, f_2, \ldots, f_n \right] \right] \Big/ \sum_{i\in\mathcal{N}} f_i v_i^0$$

If $p^0{}_{t_1} = p^0{}_{t_1 - 1}$

Stop, optimal policy is $\pi_{t_1}^{t_2}$.

Else

Set $t_1 = t_1 + 1$ and go to Step 1.

Figure 3.6. The hybrid value-policy iteration algorithm

Step 0: Initialization:

i. Compute steady state probabilities by $\mathcal{V}^0 = \mathcal{V}^0\mathcal{V}$ and $\sum_{i\in\mathcal{N}} v_i^0 = 1$.

ii. Set $p^0{}_0 = \sum_{i\in\mathcal{N}} p_i v_i^0$.

iii. Select group size G and a policy $\pi_0^0 = \left\{u_{iI_k}, u_{iI_k} = 0 \;\; \forall (i, I_k)\right\}$.

iv. Set outer iteration $t_1 = 1$ and inner iteration $t_2 = 1$.

Step 1: Policy Evaluation:

Given a policy $\pi_{t_1}^{t_2-1} = \left\{\left(u_{1I_1}\right)_{t_1}^{t_2-1}, \left(u_{1I_2}\right)_{t_1}^{t_2-1}, \ldots, \left(u_{nI_K}\right)_{t_1}^{t_2-1}\right\}$;

i. Compute $H^{\pi_{t_1}^{t_2-1}}$ matrix by solving system of linear equation

$$H^{\pi_{t_1}^{t_2-1}} = \left(I - \alpha V\right)^{-1}\left(h^{\pi_{t_1}^{t_2-1}} + \alpha p^0 D + \alpha V^{\pi_{t_1}^{t_2-1}} g^{\pi_{t_1}^{t_2-1}}\right)$$

ii. Compute contributions of states as $Cont\left(H^{\pi_{t_1}^{t_2-1}}\right) = \alpha V^{\pi_{t_1}^{t_2-1}} H^{\pi_{t_1}^{t_2-1}}$

Step 2: Policy Improvement:

i. Select a new group of states (G number of states) having the highest (or lowest) sum of contributions.

ii. Devise alternative temporary π'_{t_1} policies from $\pi_{t_1}^{t_2-1}$ by only changing u_{iI_k} values of selected G number of states and select the policy that gives lowest $\sum_{i,I_k} H^{\pi'_{t_1}}\left(i, I_k\right)$.

If $\sum_{i,I_k} H^{\pi'_{t_1}}\left(i, I_k\right) < \sum_{i,I_k} H^{\pi_{t_1}^{t_2-1}}\left(i, I_k\right)$,

Set $t_2 = t_2 + 1$ and $\pi_{t_1}^{t_2} = \pi'_{t_1}$, go to Step 1.

Else

If still unused group of states exist, go to (i)

Else; go to Step 3.

Step 3: Update Cost of Bunker On Board: Obtain $p^0{}_{t_1}$ value.

If $p^0{}_{t_1} = p^0{}_{t_1-1}$; stop, optimal policy is $\pi^* = \pi_{t_1}^{t_2}$.

Else; set $t_1 = t_1 + 1$ and go to Step 1.

Figure 3.7. The heuristic algorithm I (ALT I)

The second alternative algorithm (ALT II) improves the policy by only changing the refueling amount of selected one port at each iteration. The ports are sorted according to the sum of bunker price and port charges. The algorithm starts from the port having the highest (or lowest) sum of costs and all possible refueling amount options of each state (port and on board bunker level pair) are used to devise the policies. The devised policy that gives a lower sum of value functions is an improved policy. The ALT II algorithm is described in Figure 3.8.

Initialization step provides a sorted list of ports based on their $p_i + w_i$ values. Each port on the sorted list is tested sequentially. Then, in the current policy, the u_{iI_k} value of a selected port is changed for each I_k value and the devised policy is evaluated. If the devised policy gives a lower sum of value functions, then this policy is set to be the improved new policy.

The starting refueling amount of a state (i, I_k) can be either zero or $C - I_k$ which are lower and upper limits on the refueling amount, respectively. Additionally, the ports can be sorted in ascending or descending order of the sum of bunker prices and port charges. Therefore, the ALT II algorithm is classified into four types as indicated in Table 3.7. In the case considered in this study, Type 2 and Type 3 generate the actual optimal policy as the PI, the VI and the HVPI algorithms. The detailed optimality and convergence properties of algorithms are discussed in the Section 4.2.3.

Table 3.7: Types of the ALT II

	Sorting Method of Ports	
Starting Amount	Ascending	Descending
Lower Bound	Type 1	Type 2
Upper Bound	Type 3	Type 4

Step 0: Initialization:

i. Compute steady state probabilities by $\mathcal{V}^0 = \mathcal{V}^0\mathcal{V}$ and $\sum_{i\in\mathcal{N}} v_i^0 = 1$.

ii. Set $p^0{}_0 = \sum_{i\in\mathcal{N}} p_i v_i^0$.

iii. Generate the sorted set of ports as $S = \{i' : w_{i'} + p_{i'} < w_{i'+1} + p_{i'+1}, i' \in \mathcal{N}\}$ in ascending (or descending) order of $p_i + w_i$ values.

iv. Select a policy $\pi_0^0 = \{u_{iI_k}, u_{iI_k} = 0 \;\; \forall (i, I_k)\}$ or $\pi_0^0 = \{u_{iI_k}, u_{iI_k} = C - I_k \;\; \forall (i, I_k)\}$

v. Set outer iteration $t_1 = 1$ and outer iteration $t_2 = 1$.

Step 1: Policy Improvement:

Given a policy $\pi_{t_1}^{t_2-1} = \left\{\left(u_{1I_1}\right)_{t_1}^{t_2-1}, \left(u_{1I_2}\right)_{t_1}^{t_2-1}, \ldots, \left(u_{nI_K}\right)_{t_1}^{t_2-1}\right\}$;

i. Select a port $i' \in S$.

ii. Devise an alternative policy π'_{t_1} from $\pi_{t_1}^{t_2-1}$ by changing the $u_{i'I_k}$ of selected $i' \in S$.

iii. Evaluate the devised policy π'_{t_1} by solving the system of linear equations:

$$H^{\pi'_{t_1}} = \left(I - \alpha V\right)^{-1}\left(h^{\pi'_{t_1}} + \alpha p^0 D + \alpha V^{\pi'_{t_1}} g^{\pi'_{t_1}}\right)$$

If $\sum_{i,I_k} H^{\pi'_{t_1}}\left(i, I_k\right) < \sum_{i,I_k} H^{\pi_{t_1}^{t_2-1}}\left(i, I_k\right)$

Set improved policy $\pi_{t_1}^{t_2} = \pi'_{t_1}$, $t_2 = t_2 + 1$ and go to (i).

Else

Go to Step 2.

Step 3: Update Cost of Bunker On Board:

i. Compute cost of bunker on board by

$$p^0{}_{t_1} = P\mathcal{V}^0 F \Big/ \sum_{i\in\mathcal{N}} f_i v_i^0 = \left[\left[p_1, p_2, \ldots, p_n\right]\begin{bmatrix} v_1^0 \\ v_2^0 \\ \vdots \\ v_n^0 \end{bmatrix}\left[f_1, f_2, \ldots, f_n\right]\right] \Big/ \sum_{i\in\mathcal{N}} f_i v_i^0$$

If $p^0{}_{t_1} = p^0{}_{t_1-1}$

Stop, optimal policy is $\pi^* = \pi_{t_1}^{t_2}$.

Else

Set $t_1 = t_1 + 1$ and go to Step 1.

Figure 3.8. The heuristic algorithm II (ALT II)

4. VERIFICATION, VALIDATION AND OUTPUT COMPARISONS

In this chapter, the applications of the SVB and the LTB models to a pilot vessel refueling case described below is discussed. As mentioned previously, the pilot problems are associated with a specific Turkish maritime company (all port cost, distance data, and voyage and vessel characteristics are based on this company's records and experiences). The company vessels operate in several continents, but only the North European and Mediterranean regions are used in the models, since more historical voyage data are available for these regions. The results of different data sets are interpreted and compared with the past real refueling applications. Additionally, for validation purposes, the models are run by different (and extreme) data values regarding bunker prices and transition probabilities.

For the LTB model, four different data sets are used. The data sets are generated for different number of ports and discretization of bunker levels. The previously presented algorithms are applied to solve the LTB model and outputs of these algorithms are compared in terms of optimality and speed of convergence.

4.1. Implementation of The Single Voyage Bunkering Problem

In order to verify and validate the SVB model, it is applied to a real life problem. The data of the problem is obtained from the previously mentioned maritime company. In this case, a vessel of the company has a scheduled voyage, which includes four scheduled L/D port calls. The voyage consists of three legs and the voyage duration is approximately 22 days without visiting any other port for bunkering. This time interval includes the L/D operation times at ports. The vessel is to depart from Huelva (in Spain) and sail to Thamesport (in UK), Vyborg (in Russia) and Tekirdağ (in Turkey), in the order given. The voyage is illustrated in Figure 4.1.

The vessel has predetermined routing points between these scheduled ports. For instance, from Thamesport to Vyborg, the vessel has to sail thorough the Kiel Canal, which is a predetermined routing point for this leg of the voyage. However, from Vyborg to

Gibraltar, the vessel sails thorough the other routing point option, Skaw Area of Denmark. Therefore, the bunkering port alternatives are chosen from ports that are located as close as possible to this predetermined route.

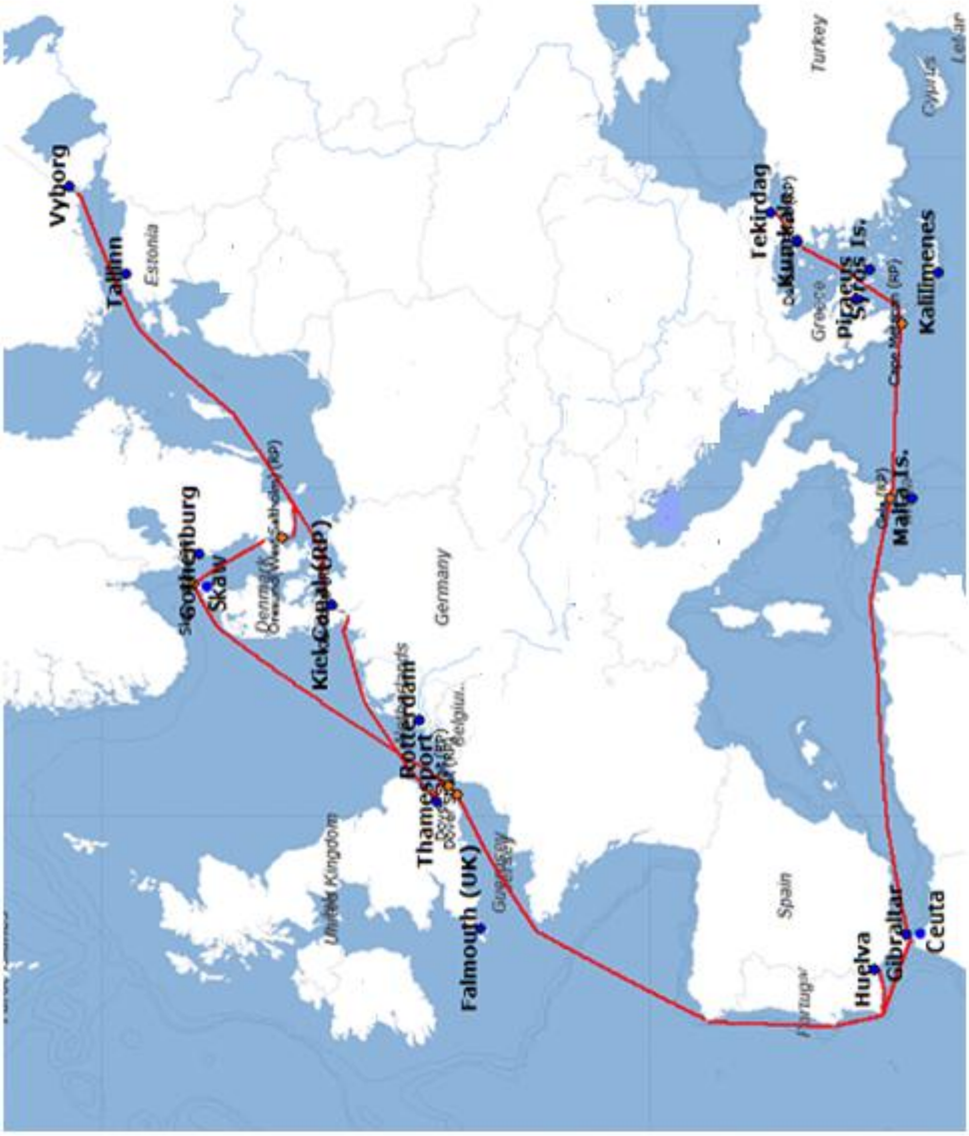

Figure 4.1. Voyage map of the SVB model application

The SVB problem is solved by the CPLEX solver within GAMS v23.3.3 software (see Appendix C). The results are compared and discussed for different scenarios.

4.1.1. Parameters of The Single Voyage Bunkering Model

As indicated, the SVB problem consists of four L/D port calls. The vessel may refuel at these ports but also may visit other ports located close to this predetermined route. Apart from the L/D ports, there are 13 possible bunkering ports on the route. For instance, the first leg of the voyage is from Huelva to Thamesport. The vessel may also refuel at Ceuta, Gibraltar or Falmouth in this leg of the voyage. These three ports are the alternative bunker ports selected between Huelva and Thamesport. Due to slack time restriction, the vessel is expected to visit at most one bunkering port in a voyage leg. The ports are in the order of the voyage legs based on the SVB model formulation. A port of a voyage leg may appear in another voyage leg with a different port index.

Table 4.1. The time parameters of the SVB problem

	Port	i	t(i) (days)	w(i) (hrs)	v(i) (hrs)	q(i) (days)
Leg 1	**Huelva**	0	0.00	11.52	112.34	0.00
	Ceuta	1	0.60	5.01	2.95	0.40
	Gibraltar	2	0.60	9.23	23.14	0.40
	Falmouth	3	0.10	4.43	1.96	3.00
Leg 2	**Thamesport**	4	0.00	11.52	112.34	0.00
	Rotterdam	5	0.75	26.95	65.47	0.50
	Kiel canal	6	0.00	10.20	79.23	1.30
	Tallinn	7	0.10	8.10	61.01	3.90
Leg 3	**Vyborg**	8	0.00	11.52	112.34	0.00
	Tallinn	9	0.05	8.10	61,01	0.50
	Skaw	10	0.00	12.33	77.29	2.60
	Gothenburg	11	0.23	3.50	112.34	2.60
	Rotterdam	12	0.33	26.95	65.47	4.10
	Falmouth	13	0.10	4.43	1.96	5.10
	Ceuta	14	0.00	5.01	2.95	8.30
	Gibraltar	15	0.00	9.23	23,14	8.30
	Malta	16	0.11	18.34	186,7	11.40
	Kalilimenes	17	0.82	22.15	323,5	13.00
	Pireaus	18	0.16	8.66	48,61	13.00
	Syros	19	0.08	11.50	112,3	13.00
	Kumkale	20	0.04	3.80	112,3	13.60
	Tekirdag	21	0.00	11.52	112,3	0.00

The time and cost parameters of the pilot problem are displayed in Table 4.1 and Table 4.2.

Table 4.2. The cost parameters of the SVB problem

	Port	i	p(i)	c(i)	Port Charges	Waiting Cost	Travel Cost (bunker)	Travel Cost (time)
Leg 1	**Huelva**	0	505	3,360	0	3,360	0	0
	Ceuta	1	505	13,576	2,461	1,461	5,454	4,200
	Gibraltar	2	492	17,397	5,192	2,692	5,313	4,200
	Falmouth	3	508	5,998	3,092	1,292	914	700
Leg 2	**Thamesport**	4	508	3,360	0	3,360	0	0
	Rotterdam	5	474	24,109	4,600	7,860	6,399	5,250
	Kiel canal	6	485	2,975	0	2,975	0	0
	Tallinn	7	480	6,289	2,363	2,362	864	700
Leg 3	**Vyborg**	8	506	3,360	0	3,360	0	0
	Tallinn	9	480	5,507	2,363	2,362	432	350
	Skaw	10	479	7,192	3,596	3,596	0	0
	Gothenburg	11	479	7,613	3,000	1,020	1,983	1,610
	Rotterdam	12	474	17,585	4,600	7,860	2,816	2,310
	Falmouth	13	508	5,998	3,092	1,292	914	700
	Ceuta	14	505	3,922	2,461	1,461	0	0
	Gibraltar	15	492	78,843	5,192	2,692	0	0
	Malta	16	490	7,089	0	5,349	970	770
	Kalilimenes	17	484	20,544	1,200	6,460	7,144	5,740
	Pireaus	18	486	8,371	3,326	2,526	1,400	1,120
	Syros	19	486	5,614	1,000	3,354	700	560
	Kumkale	20	480	1,733	0	1,108	346	280
	Tekirdag	21	502	3,360	0	3,360	0	0

In Table 4.1, the values in the t_i column indicate the additional travel time required to visit ports for bunkering. This value is zero for some ports; for instance, in Leg 2, the vessel needs to sail through Kiel Canal anyway, therefore, the vessel may refuel at Kiel port without any additional travel costs (neither due to extra bunker consumption nor extra time spent), hence these costs are zero at Kiel. The other columns indicate the mean and variance of waiting times and the distance of ports to the starting port of their legs (see Section 3.2 for notation). The waiting time data of the scheduled L/D ports are not available in records. Therefore, the mean waiting time (w_i) and the variances of waiting time (v_i) parameters of the L/D ports are estimated as from the overall mean and variances of other ports (see Section 3.1.3). The distance of a L/D port to the starting port of the voyage leg (q_i) is zero by definition. The direct travel times between L/D ports are displayed in Table 4.3.

In Table 4.2, the cost of visiting a port (c_i) is the sum of port charges, opportunity cost due to waiting times and additional travel costs (due to bunker consumption and extra time spent). The t_i values are used to obtain additional travel efforts, which are indicated in the travel cost columns. The travel cost time column is obtained by multiplying t_i with daily opportunity cost of the vessel. The travel cost bunker column is obtained by multiplying t_i with p_i and daily average bunker consumption. The waiting cost column is obtained by multiplying the average waiting time with hourly opportunity cost of the vessel.

Table 4.3. Direct distances between the schedule L/D ports

	Departure	Arrival	Direct d_i Distance (days)	Required Bunker (MT)
Leg 1	Huelva	Thamesport	3.90	70.20
Leg 2	Thamesport	Vyborg	3.70	66.60
Leg 3	Vyborg	Tekirdag	13.76	247.68

In this problem, the allowed slack times in voyage legs are assumed to be the same and 12 hours. The bunker capacity of the vessel is 341 MT and daily opportunity cost of the vessel is assumed to be 7,000 \$. Waiting time costs are computed by using this value. The penalty cost associated with potential damages due to overrunning the slack time is assumed to be 5,000 \$ per day. The importance weight of waiting time variance is selected as 0.1. The lower limit on the amount of bunker that can be purchased at a port is 60 MT and it is assumed to be the same for all ports. The daily average bunker consumption of the vessel is 18 MT (at 13 NM speed). The cost of additional bunker consumption to visit a bunkering port is evaluated at the bunker price of that port. For instance, this cost at Ceuta is evaluated by multiplying 505 $ with 18 MT and 0,6 days. These parameters are displayed in Table 4.4.

Table 4.4. Other parameters of the SVB model

Vessel Bunker Capacity	Daily Average Bunker Consumption	Lower Limit of Bunkering	Slack Time	Daily Opportunity Cost	Penalty of Exceeding Slack Time	Importance Weight of Variance
341 MT	18 MT	60 MT	0.5 days	7000 $	5000 $	0.1

4.1.2. Verification of The Single Voyage Bunkering Model

In order to verify the SVB model, the problem is solved by deploying some extreme data. The considered cases are as follows:

- Case 1: The on board bunker amount at the beginning of the voyage is changed to 170.8 MT which is the required amount to complete the Leg 1 and Leg 2 with safety bunker level (34 MT) remaining on board at the end of Leg 2 (see Table 4.3). In other words, the vessel should be able to sail to Vyborg without bunkering at any other bunkering port on the course of the route. The associated results indicate that the vessel should refuel 187.68 MT at Kiel (Port 6) in Leg 2 and then refuel 60 MT at Vyborg (Port 8). Although the vessel could refuel in Leg 3 (ports after Vyborg), the model prefers to refuel at Vyborg, since bunkering at scheduled ports are advantageous due to their zero port charges and additional travel time of visiting. Therefore, although many ports of the Leg 3 has lower bunker prices, the model do not prefer to refuel at these ports.

Table 4.5. The voyage result of the Case 1

Starting Bunker	Departure		Arrival		Bunkering On Arrival Port
170.80	Port	On board	Port	On board	
1	Huelva	170.80	Thamesport	100.60	0
2	Thamesport	100.60	**Kiel Canal**	77.20	187.68
3	Kiel Canal	264.88	**Vyborg**	221.68	60.00
4	Vyborg	281.68	Tekirdag	34.00	0

In order to check whether the vessel arrives Tekirdag (last port of the voyage) with the on board bunker at the lowest allowable level (34 MT), the sailing and bunkering actions are monitored. The related bunkering plan of the voyage is displayed in Table 4.5. The total bunkering amount in this case is 247.68 MT. The final bunker amount on board is 34 MT, as expected.

- Case 2: The on board bunker amount at Huelva (starting port of the voyage) is assumed to be 34, which is the lowest possible amount. Then, the vessel is forced to refuel 93.6 MT at Huelva. Although the bunker price and port charges at Huelva are high, the vessel must refuel at that port, in order to sail to the nearest port. As expected, the vessel completes the Leg 1 and then receives 230.88 MT at Kiel Canal, since Kiel is cheaper than Thamesport. Then, the vessel completes Leg 2 and receives 60 MT at Vyborg, which is the sufficient amount to complete Leg 3 without visiting any other port on the route for bunkering. The results of this case are indicated in Table 4.6.

Table 4.6. The results of the Case 2

Starting Bunker	Departure		Arrival		Bunkering On Arrival Port
34	Port	On board	Port	On board	
1	**Huelva**	127.6	Thamesport	57.4	0
2	Thamesport	57.4	**Kiel Canal**	34.0	230.8
3	Kiel Canal	264.8	**Vyborg**	221.6	60.0
4	Vyborg	281.6	Tekirdag	34.0	0

- Case 3: In order to observe the behavior of the model in extreme cases, prices and port charges are changed to zero or increased to extreme values. First, the bunker price at Tallinn port is set to zero. The reason of selecting the Tallinn port is that it is a bunkering port for both Leg 2 and Leg 3 of the voyage. Then, the vessel receives 60 MT at Huelva, then refuels 179.38 MT at Kiel Canal (Port 6) in Leg 2 and then visits Tallinn (Port 9) to receive 60 MT bunker in Leg 3. Although the vessel could visit Tallinn in Leg 2, it visits in Leg 3, since Kiel is still cheaper than Tallinn in Leg 2. The model does not select Vyborg, since Tallinn is also cheaper than Vyborg now.

- Case 4: In order to observe that the behavior of the model when bunkering is discouraged at Vyborg, the bunker price and port charges at that port (third scheduled port) is increased to 1,000 \$ and 100,000 \$, respectively. ($p_8 = 1,000$, $c_8 = 100,000$). The outcome indicates that the vessel should receive 60 MT at Huelva, then refuel 178.48 MT at Kiel Canal and then refuel 60 MT at Ceuta (Port 14) in Leg 3 instead of Vyborg. The bunker price and port charges at Ceuta are not the lowest values among the ports of Leg 3. Kumkale would be a cheaper choice in terms of bunker price and port charges. However, the departure bunker amount at Kiel Canal is not sufficient to sail to Kumkale.

In the second run, the bunker price and port charges at Ceuta are assumed to be 1,000 \$ and 100,000 \$, respectively. Then, the optimal solution indicates 60 MT bunkering at Tallinn (Port 9) in Leg 3 instead of Ceuta (Port 14).

4.1.3. Scenario Analysis of The Single Voyage Bunkering Model

This problem is solved for different scenarios. The scenarios are generated for different values of on board bunker amount at the beginning of the voyage and safety bunker level. The parameters of the scenarios are displayed in Table 4.7.

Table 4.7. The scenarios of the SVB problem

Scenario	Safety Level (MT)	Beginning Bunker Amount
1	34	120
2	34	140
3	50	120
4	34	200

- Scenario 1: The optimal solution is

$x_0 = 60$, $x_6 = 178.48$, $x_8 = 60$, $y_0 = 1$, $y_6 = 1$, $y_8 = 1$ and $u_2 = 0.16$.

According to this policy, the vessel should refuel at Huelva at the beginning of Leg 1 (Port 0) to receive 60 MT bunker, then is should refuel at Kiel Canal in Leg 2 (Port 6) to receive 178.48 MT bunker and 60 MT at Vyborg (Port 8) at the end of Leg 2. The slack time tolerance is violated in Leg 2 due to the time spent in Kiel Canal. The optimal cost of bunkering for that voyage is 942,804 $.

- Scenario 2: The optimal solution is

$x_6 = 218.48$, $x_{14} = 60$, $y_6 = 1$, $y_{14} = 1$ and $u_2 = 0.16$.

According to this policy, the vessel should refuel at Kiel Canal in Leg 2 (Port 6) to receive 218.48 MT bunker and then is should refuel at Ceuta in Leg 3 (Port 14) to receive 60 MT bunker. The slack time tolerance is violated in Leg 2 due to the time spent in Kiel Canal. The optimal cost of bunkering for that voyage is 888,798 $.

- Scenario 3: The optimal solution is

$x_0 = 60$, $x_6 = 194.48$, $x_8 = 60$, $y_0 = 1$, $y_6 = 1$, $y_8 = 1$ and $u_2 = 0.16$.

According to this policy, the vessel should refuel at Huelva at the beginning of Leg 1 (Port 0) to receive 60 MT bunker, then is should refuel at Kiel Canal in Leg 2 (Port 6) to receive 194.48 MT bunker and 60 MT at Vyborg (Port 8) at the end of Leg 2. The slack time tolerance is violated in Leg 2 due to the time spent in Kiel Canal. The optimal cost of bunkering for that voyage is 990,404 $.

- Scenario 4: The optimal solution is

$x_6 = 158.48$, $x_8 = 60$, $y_6 = 1$, $y_8 = 1$ and $u_2 = 0.16$.

In other words, according to this policy, the vessel should refuel at Kiel Canal in Leg 2 (Port 6) to receive 158.48 MT bunker and then is should refuel at Vyborg (Port 8) at the end of Leg 2 to receive 60 MT bunker. The slack time tolerance is violated in Leg 2 due to the time spent in Kiel Canal. The optimal cost of bunkering for that voyage is 678,859 $.

The GAMS inputs and outputs of the above discussed SVB model scenarios are given in Appendix C.

4.2. Implementation of The Long Term Bunkering Model

In this section, the LTB model is applied to real life data. The model is deployed on four different data sets. The first two data sets include five ports. In the first five port data set, the intervals of discretized bunker levels is set as 34 MT, which is 10 per cent of the bunker capacity of the employed vessel (340 MT). The bunker level intervals of the second five ports data set is set as 10 MT. The outputs are compared in order to observe any possible effects of this discretization on the optimal policy.

The LTB model is also applied to two data sets of 23 ports. The map indicating the location of the related ports are displayed in Figure 4.2. The bunker level intervals are determined in a similar fashion (i.e. discretized bunker level intervals being 34 MT in one data set and 10 MT in the other one.). As indicated in Section 3.3.2, the number of states is very much effected by the size of these intervals, since the state space consists of (port-on board bunker level) pairs. The properties of all data sets are displayed in Table 4.8.

Table 4.8. Structure of the Data Sets considered in the LTB model

	Number of ports	Bunker level interval	Number of bunker levels	Number of states
Data Set I	5	34	10	50
Data Set II	5	10	31	155
Data Set III	23	34	10	230
Data Set IV	23	10	31	713

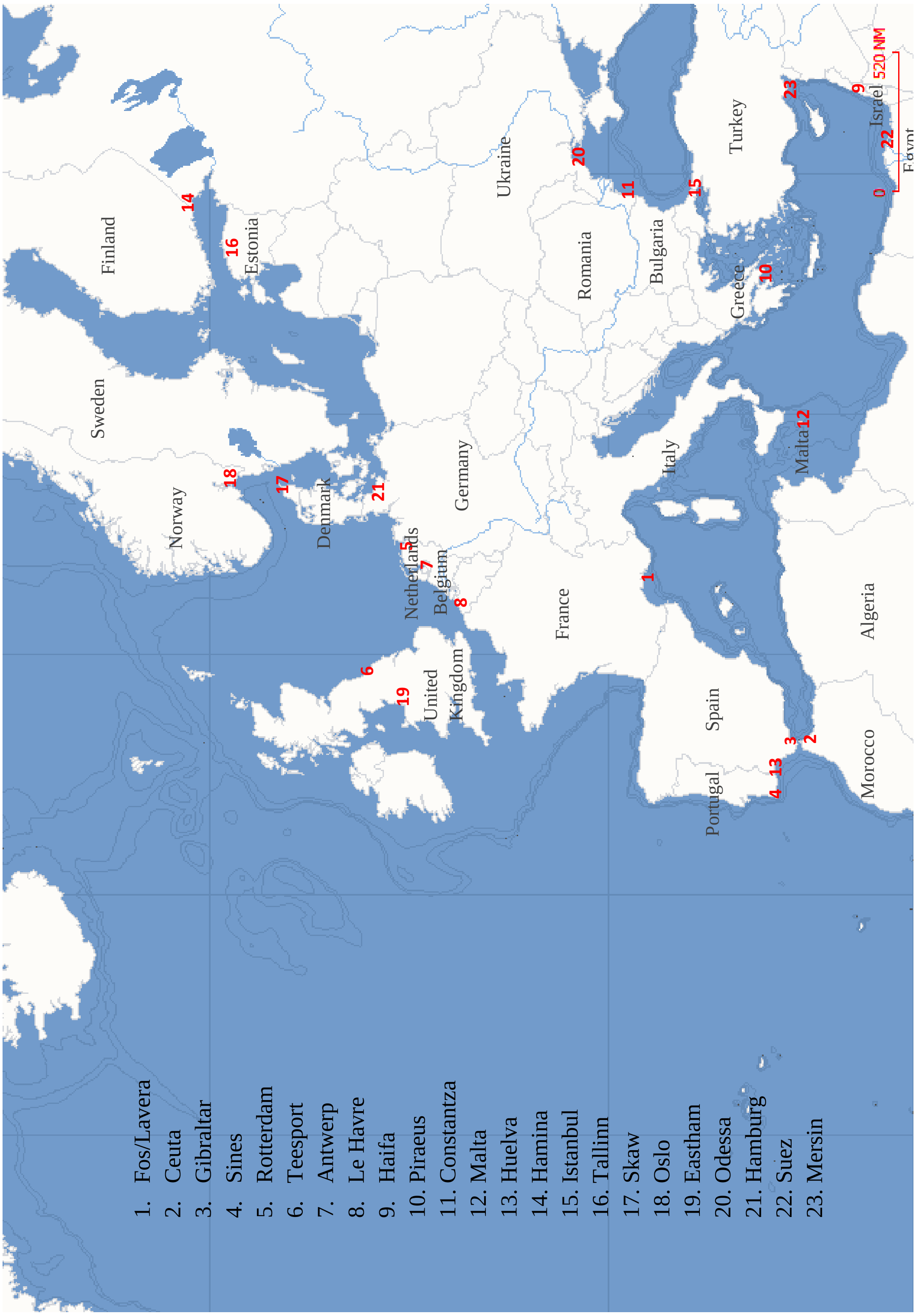

Figure 4.2. Map showing the locations of 23 ports considered in the cases investigated

All data sets are solved by the PI, VI and HVPI algorithms optimally and the rate of convergence properties of these algorithms are compared. Then, the developed ALT I and ALT II heuristic algorithms are applied to these data sets. The outputs of the heuristic algorithms are compared with optimal results obtained through the PI algorithm.

4.2.1. The Data of The Long Term Bunkering Model

Problem sets are first solved by the PI, the VI and the HVPI algorithms. These three algorithms generate an optimal policy. Then, the ALT I and the ALT II algorithms are applied to the same problem sets. All algorithms are encoded in Microsoft Visual Studio 2005 with C Sharp (C#) programming language and they are implemented on a PC with 2.13 GHz Intel(R) Core(TM)2 Duo processor and 4 GBytes of RAM, under the Windows 7 operating system. The codes are provided in the CD accompanying this thesis.

The remaining parameters of the LTB model in the five ports problem (i.e. Data Set I and Data Set II including ports Fos/Lavera, Ceuta, Gibraltar, Sines and Rotterdam) are displayed in Table 4.9 and Table 4.10. The bunker prices and port charges at these five ports are presented in Table A.1. As indicated, port-to-port bunker consumptions are evaluated at 13 NM vessel speed.

Table 4.9. Bunker consumptions between port pairs in the five port problems

	Fos/Lavera	Ceuta	Gibraltar	Sines	Rotterdam
Fos/Lavera	0.0	39.6	39.6	57.6	118.8
Ceuta	39.6	0.0	1.8	18.0	79.2
Gibraltar	39.6	1.8	0.0	18.0	79.2
Sines	57.6	18.0	18.0	0.0	64.8
Rotterdam	118.8	79.2	79.2	64.8	0.0

Table 4.10. Transition probability matrix between port pairs in the five port problems

Port No		Fos/Lavera	Ceuta	Gibraltar	Sines	Rotterdam
0	Fos/Lavera	0.000	0.666	0.333	0.000	0.000
1	Ceuta	0.200	0.000	0.000	0.800	0.000
2	Gibraltar	0.333	0.333	0.000	0.333	0.000
3	Sines	0.000	0.100	0.000	0.000	0.900
4	Rotterdam	0.498	0.251	0.251	0.000	0.000

The discretized approximate port-to-port bunker consumptions between port pairs are displayed in Table 4.11 and Table 4.12.

Table 4.11. Discretized bunker consumptions between port pairs in the five port problem at 34 MT bunker level intervals

	Fos/Lavera	Ceuta	Gibraltar	Sines	Rotterdam
Fos/Lavera	0	34	34	68	102
Ceuta	34	0	0	34	68
Gibraltar	34	0	0	34	68
Sines	68	34	34	0	68
Rotterdam	102	68	68	68	0

Table 4.12. Discretized bunker consumptions between port pairs in the five port problem at 10 MT bunker level intervals

	Fos/Lavera	Ceuta	Gibraltar	Sines	Rotterdam
Fos/Lavera	0	40	40	60	120
Ceuta	40	0	0	20	80
Gibraltar	40	0	0	20	80
Sines	60	20	20	0	60
Rotterdam	120	80	80	60	0

As mentioned previously, in order to have a finite state space, the on board (arrival) bunker levels are also discretized. The lower bounds on the arrival bunker amounts are assumed to be 34 MT and 40 MT for Data Set I and Data Set II, respectively. As indicated, if a vessel is to refuel at a port, there is lower limit on the bunker amount it receives. These limits are set at 102 MT and 60 MT for Data Set I and Data Set II, respectively. According to these limits and the lower bounds on the arrival bunker levels, the discretized bunkering amounts are determined in the intervals 102-306 MT and 60-300 MT for Data Set I and Data Set II, respectively. Accordingly, the number of refueling levels is determined as 10 in Data Set I and Data III, and as 26 in Data Set II and Data Set IV. The arrival bunker and refueling levels for Data Set I and Data Set III are displayed in Table 4.13 and Table 4.14. The arrival and refueling bunker levels of Data Set II and Data Set IV are displayed in Table 4.15 and Table 4.16, respectively.

Table 4.13. Arrival bunker levels (Data Set I and Data Set III)

Level no	0	1	2	3	4	5	6	7	8	9
Arrival l level	34	68	102	136	170	204	238	272	306	340

Table 4.14. Refueling levels (Data Set I and Data Set III)

Level no	0	1	2	3	4	5	6	7
Refuel level	0	102	136	170	204	238	272	306

Table 4.15. Arrival bunker levels (Data Set II and Data Set IV)

Level no	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Arrival level	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190
Level no	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
Arrival level	200	210	220	230	240	250	260	270	280	290	300	310	320	330	340	

Table 4.16. Refueling levels (Data Set II and Data Set IV)

Level no	0	1	2	3	4	5	6	7	8	9	10	11	12
Refuel level	0	60	70	80	90	100	110	120	130	140	150	160	170
Level no	13	14	15	16	17	18	19	20	21	22	23	24	25
Refuel level	180	190	200	210	220	230	240	250	260	270	280	290	300

Next, the LTB model with Data Set I is solved by the PI, VI and HVPI algorithms, respectively. The penalty cost multiplier (pm in the LTB model formulation) is first selected as 0.5. In other words, the cost of unplanned bunkering is set as 50 per cent more that of the predetermined bunkering. The results for different pm values are analyzed in the following sections.

4.2.2. Verification of The Long Term Bunkering Model

In order to test whether the LTB model accurately reflects the behavior of the bunkering processes and correctly respond to changes in the parameters, it is solved at different parameter settings, each named a “trial”. The related experimentation is done on the five port problem mostly with 31 on board bunker level intervals (Data Set II). The reasons being that with only five ports, the effects of specific parameter changes are crispier (more differentiated) and easier to trace and with 31 bunker level intervals. Only the first trial (Trial 0) is performed with the Data Set I.

The effects of parameter changes on average bunkering are displayed in Table 4.17. Base case values are associated with the original parameter setting regarding Data Set II. The trials and their results are explained below.

Table 4.17. Average bunkering levels in the base case and in the trials

	Lavera	Ceuta	Gibraltar	Sines	Rotterdam
	f[0]	**f[1]**	**f[2]**	**f[3]**	**f[4]**
Base Case	26.14	16.45	16.45	11.61	101.93
Trial 0	3.39	3.40	2.26	9.52	15.28
Trial 1	145.16	4.38	4.19	11.61	26.61
Trial 2	7.74	73.55	128.06	60.97	23.86
Trial 3	7.74	93.87	101.93	78.71	28.39
Trial 4	8.06	8.71	8.71	11.61	30.96
Trial 5	8.71	16.45	8.71	11.61	31.93
Trial 6	7.74	1.42	1.39	145.16	27.58
Trial 7	16.45	1.42	1.39	60.00	25.16

- Trial 0: ($pm = -1$) This trial is pursued to test the general sensibility of the PI, the VI and the HVPI algorithms in generating feasible and optimal solutions. (As explained, these three algorithms generate the same optimal solutions) The penalty multiplier is changed to -1 in Data Set I, which reduces the cost of unplanned bunkering to zero. Accordingly, all predetermined bunkering amounts become zero and the total unplanned bunkering increases to 748 MT, which is the sum of unplanned bunker amounts of all states. As expected, this results implies that the vessel should only refuel when needed and at an amount sufficient to sail to the declared ports. The unplanned bunkering amounts for each current port-destination port pair are displayed in Table 4.18. For instance, the unplanned bunkering at Port 0 (Fos) is 34 MT when the arrival on board bunker level is 34 MT and the destination port is Port 1 (Ceuta). This result is reasonable since there is a positive probability to sail from Fos to Ceuta (see Table 4.10) and the bunker consumption between these two ports is 34 MT (see Table 4.11). Therefore, in order to arrive at Ceuta with just safety on board bunker level (34 MT), due to nature of the LTB model (the second recursive part of the Bellman's equation), the vessel should receive exactly 34 MT of unplanned bunkering. (Since all other unplanned amounts between remaining port pairs and bunker levels are zero, they are not displayed in Table 4.18)

Table 4.18. The unplanned bunkering amounts of Trial 0

Current Port	Next Port	Arrival Amount	Unplanned Amount
0	1	34	34
0	2	34	34
1	0	34	34
1	3	34	34
2	0	34	34
2	3	34	34
3	1	34	34
3	4	34	68
3	4	68	34
4	0	34	102
4	0	68	68
4	0	102	34
4	1	34	68
4	1	68	34
4	2	34	68
4	2	68	34

Trial 1: ($w_0 = 0, \ \ p_0 = 0$) This trial is pursued to observe the effect of bunker prices and port charges on bunkering decisions and the possible effects of the transition probabilities. In this experimentation, the port charges and bunker prices at Port 0 (Fos/Lavera) are reduced to zero. Then, the average bunkering at Port 0 increases significantly, while it decreases at Port 4 (see Table 4.17). The cheapest port of base case is Port 4. In Trial 1, the cheapest port is Port 0, at which the vessel has a positive transition probability to sail to Port 4. The response implies that the vessel should more bunker at the cheapest port (Port 1) instead of Port 4 and then proceed on its voyage.

- Trial 2: ($w_4 = 10,000, \ \ p_4 = 1,000$) This trial is pursued to observe the relations between average bunkering amounts at different ports. In this experimentation, the bunker price and port charges at Port 4 (Rotterdam) are increased. Accordingly, in the resulting optimal solution, the bunkering amounts at Port 1 (Ceuta) and Port 2 (Gibraltar) increase significantly, since both ports have significant transitions from Port 4. The bunkering amount at Port 3 (Sines) also increases, since the transition probability from Port 3 to Port 4 is high ($v_{34} = 0,9$).

- Trial 3: ($w_4 = 10,000$, $p_4 = 1,000$, $v_{40} = 1$) This trial is pursued to observe the effects of the transition probabilities on bunkering decisions. In this experimentation, in addition to changes in Trial 2, the transition probability from Port 4 to Port 1 is increased to unity. Hence, the next port to Port 4 becomes Port 1. In the resulting optimal solution, the bunkering amounts at Port 1 and Port 3 increase, since the probability of a voyage from Port 4 to Port 1, then to Port 3 is high.

- Trial 4: (All port charges and prices are set the same, but not zero; $v_{40} = 1$) This trial is pursued to observe the effects of the network structure and connections on bunkering decisions. In the resulting optimal solution, since Port 4 is more "centrally located" (in the sense that three ports are accessible from Port 4), the amount bunkered at this port increases with respect to other ports.

- Trial 5: (All port charges and prices are set the same but not zero. $v_{40} = 1$, $v_{14} = 0.4$, $v_{13} = 0.4$) This trial is also pursued to observe the effects of the network structure and connections on bunkering decisions. In this experiment, the Port 4 now can be accessed from Port 1. As expected, the bunkering amount at Port 1 increases.

- Trial 6: ($p_3 = 200$, $pm = 0.1$) This trial is pursued to observe the combined effects of bunker prices and the unplanned bunkering price mark-up (the pm parameter). The price at the most expensive port is decreased. Hence, the bunkering at this port increases, while the same value decreases in Port 1 and Port 4. The total unplanned bunkering becomes 290 MT, since its price mark-up is decreased.

- Trial 7: ($p_3 = 200$, $p_0 = 200$, $pm = 0.1$) This trial is pursued to observe the effects of port charges on bunkering decisions. Total unplanned bunkering becomes 320 MT, since its price mark-up is decreased. The bunkering amount at Port 0 increases while the same amount at Port 3 decreases, since the port charge at Port 3 is higher. (see Table A.1)

- Trial 8: In addition to these trials, different pm values are also experimented with. As pm decreases (i.e. the penalty cost of unplanned bunkering decreases), unplanned bunkering becomes more attractive with respect to predetermined bunkering. The results for different pm values are displayed in Table 4.19.

Table 4.19. Optimal long term costs of data sets for different pm values

pm	Data Set I		Data Set II		Data Set III		Data Set IV	
	Cost	Unplanned amount	Cost	Unplanned amount	Cost	Unplanned amount	Cost	Unplanned amount
1.5	25,950	0	20,962	0	23,180	0	24,378	0
0.5	25,950	0	20,962	0	22,743	68	23,643	70
0.0	25,950	0	20,962	0	17,930	952	20,905	2390
-0.1	25,950	0	20,962	0	11,683	3128	16,071	8890
-0.5	9,079	748	8,676	2200	6,746	5338	7,362	20830
-1.0	9,079	748	8,676	2200	6,747	5338	7,362	20830

Additionally, as the number of states increases, unplanned bunkering becomes more attractive in higher pm values. The total unplanned bunkering of all data sets is zero at when $pm = 1.5$. In other words, when the cost of unplanned bunkering is higher than the predetermined bunkering, the policy indicates that the vessel should not receive unplanned bunker at a port. The pm values higher than 1.5 does not change the optimal solution, since the unplanned bunkering at these values increases the overall cost. If $pm = -1$, then the cost of unplanned bunkering becomes zero. In this case, the vessel should only receive the required bunker to sail the next destination ports (see Trial 0).

4.2.3. The Scenario Analysis of The Long Term Bunkering Model

The optimal solution obtained for the five port problem (under Data Set I) is displayed in Table 4.20 (of course this same solution is generated by the PI, the VI and the HVPI algorithms). It is noted that extra bunkering (unplanned bunkering) amounts are all zero. As indicated, the predetermined bunkering amount depends on the arrival bunker level and the port. For instance, 136 MT bunkering at Lavera port means that the vessel should refuel that much amount of bunker at Lavera when it arrives at that port with 34 MT bunker. It can be observed from the policy associated with Rotterdam that the bunkering amount generally increases as the available space in bunker tanks increases. The average bunkering at each port is evaluated as equally weighted average of arrival bunkering suggested for different states as displayed in Table 4.21. It is noted that the average bunkering amount at Rotterdam is highest, which is possibly due to lowest bunker price at Rotterdam (474 $/MT) and / or the number of positive transition probabilities from other ports into Rotterdam. The Lavera and Sines ports have the same bunker prices (514 $/ MT), while Sines has the highest port charges. Therefore, refueling at these two ports at lower amounts, as the optimal policy suggest, is quite reasonable.

Table 4.20: The optimal bunkering policy of the five port problem (under Data Set I)

u(i,I)		**34**	**68**	**102**	**136**	**170**	**204**	**238**
Fos/Lavera	**0**	136	0	0	0	0	0	0
Ceuta	**1**	102	102	102	0	0	0	0
Gibraltar	**2**	102	136	102	0	0	0	0
Sines	**3**	102	102	0	0	0	0	0
Rotterdam	**4**	238	204	170	136	102	136	102

Table 4.21. The average bunkering amounts of the five port problem (under Data Set I)

Port Name	**Bunkering At Each Port**	
Fos/Lavera	**f[0]**	13.6
Ceuta	**f[1]**	30.6
Gibraltar	**f[2]**	34.0
Sines	**f[3]**	20.4
Rotterdam	**f[4]**	108.8

The optimal long run cost is found as 25,950 $, while the overall average cost of bunker on board $\left(p^0\right)$ is 488.33 $/MT. The PI algorithm converges in 12 iterations and 2 updates of p^0 value. The computational performances of other algorithms are presented in the following sections.

The optimal solution obtained for the five port problem (under Data Set II) is displayed in Table 4.22 and Table 4.23. As in the Data Set I case, unplanned bunkering amounts are all zero. However, the optimal solution and the optimal value are somewhat different (than that of the results under Data Set I). The optimization process takes advantage of the additional bunkering choices made available by the redefined discretization and fine-tunes the bunkering decisions leading to decreases in bunkering amounts and average bunkering. The results is a lower optimal long run cost at 20,962 \$. The final updated cost of bunker on board is found to be 484.5 \$/MT , which is also less than the p^0 value of the Data Set I.

Table 4.22. The bunkering policy of the five port problem (under Data Set II)

u(i,I)	Fos/Lavera	Ceuta	Gibraltar	Sines	Rotterdam
	0	**1**	**2**	**3**	**4**
40	60	80	80	60	240
50	60	70	70	60	230
60	60	60	60	60	220
70	90	60	60	60	210
80	0	60	60	60	200
90	0	60	60	60	190
100	0	60	60	0	180
110	0	60	60	0	170
120	0	0	0	0	160
130	0	0	0	0	150
140	0	0	0	0	140
150	0	0	0	0	130
160	0	0	0	0	120
170	0	0	0	0	110
180	0	0	0	0	100
190	0	0	0	0	90
200	0	0	0	0	80
210	0	0	0	0	70
220	0	0	0	0	60
230	0	0	0	0	60
240	0	0	0	0	60
250	0	0	0	0	70
260	0	0	0	0	60
270	0	0	0	0	60
280	0	0	0	0	0

Table 4.23. The average bunkering amounts of the five port problem (under Data Set II)

Port Name	Bunkering At Each Port	
Fos/Lavera	**f[0]**	8.709
Ceuta	**f[1]**	16.451
Gibraltar	**f[2]**	16.451
Sines	**f[3]**	11.612
Rotterdam	**f[4]**	101.935

The optimal solutions obtained for the 23 port problem (under Data Set III and the Data Set IV) are displayed in Table 4.24 and Table 4.25.

Table 4.24. The optimal solution for 23 port problem (under Data Set III)

u(i,I)		34	68	102	136	170	204	238	Average Bunkering f(i)
Fos/Lavera	**0**	102	102	0	0	0	0	0	20.40
Ceuta	**1**	102	102	0	0	0	0	0	20.71
Gibraltar	**2**	102	102	0	0	0	0	0	20.40
Sines	**3**	102	0	0	0	0	0	0	10.20
Rotterdam	**4**	272	238	204	170	136	102	102	122.40
Teesport	**5**	102	102	0	0	0	0	0	20.40
Antwerp	**6**	102	102	0	0	0	0	0	20.40
Le Havre	**7**	0	0	0	0	0	0	0	0.49
Haifa	**8**	102	102	136	0	0	0	0	34.00
Piraeus	**9**	204	170	136	102	102	0	0	71.40
Constantza	**10**	170	136	102	0	0	0	0	40.80
Malta	**11**	136	0	0	0	0	0	0	13.60
Huelva	**12**	102	102	0	0	0	0	0	20.40
Hamina	**13**	136	102	102	102	0	0	0	44.20
Istanbul	**14**	102	136	102	0	0	0	0	34.00
Tallinn	**15**	0	0	0	0	0	0	0	0.00
Skaw	**16**	0	0	0	0	0	0	0	0.00
Oslo	**17**	102	0	0	0	0	0	0	10.20
Eastham	**18**	102	0	0	0	0	0	0	10.20
Odessa	**19**	170	136	102	102	102	0	0	61.20
Hamburg	**20**	0	0	0	0	0	0	0	0.00
Suez	**21**	102	102	0	0	0	0	0	20.40
Mersin	**22**	102	0	0	0	0	0	0	10.20

Table 4.25. The optimal solution for 23 port problem (under Data Set IV)

u(i,I)	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250	260	270	280	Average Bunkering f(i)
0	90	80	70	60	60	60	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	19.355
1	100	90	80	70	60	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	18.798
2	130	120	110	100	90	80	70	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	24.516
3	60	60	60	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	11.613
4	280	270	260	250	240	230	220	210	200	190	180	170	160	150	140	130	120	110	100	90	80	70	60	60	60	130.000
5	70	60	60	60	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	13.871
6	110	100	90	80	70	60	60	70	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	22.581
7	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4.009
8	120	110	100	90	80	70	60	60	70	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	30.322
9	260	250	240	230	220	210	200	190	180	170	160	150	140	130	120	110	100	90	80	70	60	60	60	60	0	114.190
10	120	110	100	90	80	70	60	60	60	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	30.000
11	60	60	60	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	11.612
12	70	60	60	70	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	14.193
13	150	140	130	120	110	100	90	80	70	60	60	60	60	60	0	0	0	0	0	0	0	0	0	0	0	41.630
14	100	90	80	70	60	60	70	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	22.903
15	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.935
16	60	60	70	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6.129
17	60	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7.742
18	60	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7.742
19	200	190	180	170	160	150	140	130	120	110	100	90	80	70	60	60	60	60	60	60	0	0	0	0	0	72.580
20	70	60	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10.000
21	70	60	60	60	60	80	70	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	14.838
22	60	60	60	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	11.612

4.2.4. Comparison of The Solution Algorithms

The five port and the 23 port pilot problems (each under 10 interval and 31 interval discretizations) are solved by the presented solution algorithms. Then, the results and the performances of the algorithms are compared. As mentioned, the PI, the VI and the HVPI algorithms generate optimal solutions. However, the computational complexity and the rate of convergence properties of these algorithms are different. The alternative heuristic algorithms also generate optimal solutions in most cases. The optimality, rate of convergence and computational complexity properties of these algorithms are also analyzed and compared.

First, the rate of convergence properties of the PI, the VI and the HVPI algorithms are analyzed. The comparison of the PI and the VI algorithms are presented in Table 4.26. In the VI method, each iteration is simple and quickly executed, however, since the number of iterations are much higher than that of the PI method, total execution time of VI is higher.

Table 4.26. The comparison of the PI and the VI algorithms

Data Sets	Policy Iteration (PI)			Value Iteration (VI)		
	p_0 updates	total iterations	time (sec.)	p_0 updates	total iterations	time (sec.)
Data Set I	2	12	0.36	2	5256	78
Data Set II	2	12	1.28	2	5251	278
Data Set III	2	10	3	2	6503	439
Data Set IV	2	12	74	2	6664	1904

The HVPI algorithm is a combination of the PI and the VI algorithms. As illustrated in Figure 3.5, the HVPI algorithm updates values of a policy for a specific number of times (number of value iterations is denoted by M) , then if the error values (differences) are sufficiently small, policy evaluation is performed to proceed to a new policy. Therefore, the number of value iterations affects the rate of convergence of the algorithm. The convergence times of the HVPI algorithm for different data sets are displayed in Table 4.27. Like the other algorithms, the cost of bunker on board is updated two times in the HVPI. As the number of value iterations (M) in each main iteration of the algorithm increases, the time of convergence decreases. In addition, the number of policy evaluation step decreases. Therefore, the total number of operations to solve linear equations decreases.

Table 4.27. The time of convergence of the HVPI algorithms at data sets

M	Data Sets	p_0 updates	Total Iterations	Time (sec.)
5	5 ports 34 MT intervals	2	878	7
	5 ports 20 MT intervals	2	878	21
	5 ports 10 MT intervals	2	878	43
	23 ports 34 MT intervals	2	1086	60
	23 ports 20 MT intervals	2	1092	238
	23 ports 10 MT intervals	2	1113	281
20	5 ports 34 MT intervals	2	252	3
	5 ports 20 MT intervals	2	252	6
	5 ports 10 MT intervals	2	253	12
	23 ports 34 MT intervals	2	312	20
	23 ports 20 MT intervals	2	314	96
	23 ports 10 MT intervals	2	320	116
50	5 ports 34 MT intervals	2	106	2
	5 ports 20 MT intervals	2	106	2
	5 ports 10 MT intervals	2	106	6
	23 ports 34 MT intervals	2	130	15
	23 ports 20 MT intervals	2	130	64
	23 ports 10 MT intervals	2	134	90
100	5 ports 34 MT intervals	2	54	1
	5 ports 20 MT intervals	2	54	1
	5 ports 10 MT intervals	2	54	5
	23 ports 34 MT intervals	2	68	15
	23 ports 20 MT intervals	2	63	46
	23 ports 10 MT intervals	2	68	60

Another interesting observation on the implementation of the HVPI algorithm is the relationship between the number of value iterations in each main iteration, size of the states space and the change in total iterations. As the number of value iterations in each state (M) increases, the number of total iterations not only decrease but also get closer for different sizes of states. In other words, it can be interpreted that the total number iterations becomes less dependent on the number of states in higher M values. For instance, the total number of iterations for the 23 ports data get closer when the value iteration number is increased to 20 and in higher M values, they become equal. It can be observed from the difference between the total number of iterations of the five port and 23 port data sets when M=100. The total number of iterations for the five ports are almost equal at all M values, since their number of states are closer. In general, it can be claimed that the number of on board bunker levels has an effect on the number of total iterations. The M value does not affect the number of p_0 updates.

Table 4.28. The results of Data Set I obtained through the ALT I heuristic algorithm

pm	Group Size	Long Term Cost	Iterations	Total Unplanned Amount	Final p_0
0.5	2	26,504	44	68	489.43
	3	26,505	29	68	489.43
	4	28,075	25	34	492.63
1	2	26,504	47	68	489.43
	3	26,504	26	68	489.43
	4	28,075	27	34	492.63
1.5	2	26,504	46	68	489.43
	3	28,075	30	34	492.62
	4	28,075	25	34	492.63

Apart from these optimizing algorithms, the heuristic algorithms are also compared. As explained previously, in the ALT I heuristic algorithm, a group of states having values that makes the highest (or lowest) contribution to sum of value functions is selected and all refueling amount combinations of these ports are evaluated to improve the current policy (see Figure 3.6). The group size of this selected states affects the quality of the results. As the group size decreases, the best solution obtained gets closer to the optimal solution. This situation is exemplified in Table 4.28. The Data Set I is solved by the ALT I heuristic for different pm values. Generally, as the group size increases, the number of groups decreases and the long run cost gets closer to the optimal cost (25,950 \$ with $pm = 0.5$). The comparison of two states (namely, a group having any two states) provides a larger search for improved policies. If an improved policy is obtained from a group having two states, then new groups are generated according to this new improved policy. Hence, the chance of any state to form a group with another state different than the previous one increases. In other words, the diversity of groups is higher in groups having two states. Additionally, as the group size increases, the time of convergence increases due to the increase in the number of policy evaluation steps. Therefore, it is claimed that as the group size decreases, the computational complexity decreases as well. The policy results of the ALT I algorithm for the Data Set I is displayed in Table 4.29.

Table 4.29. The policy results of Data Set I obtained through the ALT I heuristic algorithm ($pm = 0.5$)

Group Size	u(i,I)		34	68	102	136	170	204	238	f(i)
4	Fos/Lavera	**0**	136	102	0	0	0	102	0	23,800
	Ceuta	**1**	102	102	136	0	170	0	102	34,000
	Gibraltar	**2**	102	102	136	0	0	102	0	44,200
	Sines	**3**	102	136	0	0	102	0	0	34,000
	Rotterdam	**4**	272	238	0	170	136	102	0	93,490
3	Fos/Lavera	**0**	136	102	0	0	0	0	0	23,800
	Ceuta	**1**	102	102	102	0	0	0	0	30,600
	Gibraltar	**2**	136	102	0	0	0	0	0	26,064
	Sines	**3**	102	102	0	0	0	0	0	20,400
	Rotterdam	**4**	238	204	170	136	102	136	102	108,800
2	Fos/Lavera	**0**	136	102	0	0	0	0	0	23,800
	Ceuta	**1**	102	102	102	0	0	0	0	30,600
	Gibraltar	**2**	136	102	0	0	0	0	0	26,064
	Sines	**3**	102	102	0	0	0	0	0	20,400
	Rotterdam	**4**	238	204	170	136	102	136	102	108,800

As displayed in Table 4.28, the pm value affects the long run cost. As the pm value increases, the predetermined bunkering becomes more attractive and the long run cost decreases when group size is two. In general, it can be claimed that the solution of ALT I heuristic algorithm gets closer to the optimal solution as the pm value increases. In other words, the ALT I algorithm generates its best solution at higher pm value compared to the PI, the VI and the HVPI algorithms (which generate optimal solutions). The optimal solution of the PI, the VI and the HVPI algorithms for the Data Set I (at $pm = 0.5$) is also compared with the best solution of the ALT I algorithm as displayed in Table 4.30. Although the ALT I algorithm does not generate optimal policy, it converges faster than the VI algorithm.

Table 4.30. Comparison of the ALT I heuristic algorithm with optimizing algorithms

pm	Algorithm	Long Term Cost	Number of Iterations	Time (sec.)	Total Unplanned Amount
0.5	PI	25,950	12	0.57	0
	VI	25,950	5256	51	0
	HVPI (M=100)	25,950	54	1	0
1.5	ALT I	26,504	44	72	68

The ALT II heuristic algorithm is also applied to the data sets. As mentioned, this algorithm consists of different versions (see Table 3.7). The ALT II heuristic may generate optimal solutions. The best solutions obtained by the ALT II heuristic for the four data sets are displayed in Table 4.31. It can be observed that the Type 3 version of the heuristic algorithm seems to generate better long term cost (and optimal policy). (The heuristic algorithm generates optimal policy for Data Set I in all versions due to the smaller state space.) The better performance of Type 3 is somewhat intuitive, since this version of the ALT II starts with zero bunkering at the ports having higher costs. Then, other cheaper ports are assigned large bunkering amounts. Therefore, bunkering at ports having higher costs is minimized. (see Figure 3.7)

Table 4.31. The best long term costs generated by the ALT II heuristic algorithm for the LTB problem defined by Data Set I, II and III

	Data Set I	Data Set II	Data Set III
Type 1	25,950	23,180	22,743
Type 2	25,950	22,544	22,743
Type 3	25,950	23,180	22,743
Type 4	25,950	23,180	24,052

Table 4.32. Computational performance of the ALT II algorithm on the LTB problem defined by Data Set III

	Type 1	Type 2	Type 3	Type 4
Long Term Cost	22,743	22,743	22,743	24,052
Unplanned Bunker	68	68	68	68
Number of p_0 Updates	5	4	4	3
Final p_0	485.750	485.751	485752	484.950
Number of Iterations	679	321	247	216
Computation Time (min.)	29	31	27	22

Although the ALT II algorithm generates good (and in many cases optimal) policies, it converges in a long a time. As indicated in Table 4.32, the time of convergence in Type 3 is 25 minutes. But still, the total number of iterations is lower than the other versions due to the initial policy. All problems are solved with $pm = 0.5$. As mentioned previously, when the unplanned bunkering amounts are zero at some pm value, there is no need to get results for higher pm values than that one , since the higher pm values would also necessarily generate zero unplanned bunkering amounts and the same predetermined bunkering policy. This principle is also valid for the ALT II heuristic algorithm. The problems are solved for higher pm values by the ALT II algorithm and the best solutions are not affected by this change.

In Table 4.33, the best solutions obtained by the ALT I and the ALT II algorithms are compared with the optimal policy obtained by the PI, the VI and the HVPI algorithms for the LTB problem defined by Data Set I. As indicated above, the ALT II algorithm generates the optimal policy at all types, since the number of states is small.

Table 4.33: Comparison of best solutions of the heuristic algorithms for LTB problem defined by Data Set I

Algorithm	u(i,I)		34	68	102	136	170	204	238	f(i)
PI (VI and HVPI)	Fos/Lavera	**0**	136	0	0	0	0	0	0	13.60
	Ceuta	**1**	102	102	102	0	0	0	0	30.60
	Gibraltar	**2**	102	136	102	0	0	0	0	34.00
	Sines	**3**	102	102	0	0	0	0	0	20.40
	Rotterdam	**4**	238	204	170	136	102	136	102	108.80
ALT I	Fos/Lavera	**0**	136	102	0	0	0	0	0	23.80
	Ceuta	**1**	102	102	102	0	136	0	0	30.60
	Gibraltar	**2**	136	102	0	0	0	0	0	26.06
	Sines	**3**	102	0	0	0	0	0	0	20.40
	Rotterdam	**4**	238	204	170	136	102	136	102	108.80
ALT II	Fos/Lavera	**0**	136	0	0	0	0	0	0	13.60
	Ceuta	**1**	102	102	102	0	0	0	0	30.60
	Gibraltar	**2**	102	136	102	0	0	0	0	34.00
	Sines	**3**	102	102	0	0	0	0	0	20.40
	Rotterdam	**4**	238	204	170	136	102	136	102	108.80

5. CONCLUSIONS AND FURTHER STUDIES

Throughout this study, the vessel refueling planning problem is investigated. The bunkering (refueling) issue is an important cost item to maritime companies and in the highly competitive maritime sector, it is vital to manage the bunkering activities in a most cost effective manner. In liner vessels, it is easier to plan the bunkering activities in the long term, but in tramp vessels, the planning is more difficult, since a typical voyage includes a number of scheduled port calls, which are mostly uncertain until the starting time of the voyage.

The bunkering cost of a vessel does not only cover the purchasing cost of the bunker. There are some other indirect cost associated with visiting a port for only refueling (such as waiting costs and additional travel efforts). The developed SVB model is applied to the tramper vessel problem considering these cost items. The model is formulated as a MIP problem, which consists of the scheduled L/D ports and other alternative ports in the neighborhood of the predetermined route for only bunkering purposes. The model is applied to a real life problem and the outcomes are examined and interpreted.

In order to plan the bunkering activities in long term, the LTB model is developed, as an infinite stage stochastic DP problem. In this model, the transition probabilities between ports are assumed to be known and the objective is set to minimize the long term refueling cost of a vessel. In the model, the unplanned bunkering at a port has a higher cost (penalty cost) than the predetermined bunkering. The model is constructed based on data sets, partially containing real data and partially containing fictitious data generated based on interviews with experts; then some well known infinite stage DP algorithms are deployed for its solution. Additionally, two new heuristic algorithms are developed and applied to the case problems. The results of all algorithms are discussed and compared.

The main findings can be summarized as the following:

- Apart from bunker prices, the port charges and waiting times at port do affect the bunkering port decision and the amount. The bunkering port decisions influence the route of the vessel.
- Level of penalty charges due to unplanned bunkering affects the long term bunkering policy of a vessel.
- As the bunker level intervals increases, the optimal long term bunkering cost decreases. However, due to the increase in the number of states in the LTB model, the time of convergence of the solution algorithms increases.
- Although the LTB model can be solved optimally with the PI, the VI and the HVPI algorithms, heuristic algorithms can be also applied to generate near optimal solutions.

As a further work, the SVB and the LTB models can be expanded in terms of cost structures. Since the bunkers are crude oil products, their prices change in time. The port charges are also subjected to changes. Therefore, the models can be revised considering the affects of cost changes of the ports.

Additionally, in reality, vessels use more than one type of bunkers. In this study, this fact is neglected and the models are developed for only one type of bunker. In a further work, the model can be refined to include multiple bunker types. Then, the port visits of the vessel may change substantially, since visiting a bunkering port for only one type of bunker may not be cost effective and refueling for all bunker needs at the same time can be more efficient. In this case, all bunker availabilities, prices and consumptions may affect the refueling policy.

The laytime and laycan constraints of the vessel can also be detailed in future models. In this case, the cost due to violation of time at ports (laytime violation) and the cost of losing a contract (laycan violation) are included in the cost structure in a rather simplistic manner. The trade-off between the cost of losing a contract due to laycan violation and cost of bunkering could be included in the LTB model. In case of laytime violation, the demurrage cost could be also a cost item in the model.

Additionally, the selection of L/D contracts can also be the subject of the models. In this case, the models could become profit maximization instead of cost minimization, since the L/D ports are to be selected based expected revenues, as well as related transportation and operational costs.

The subject of this study is not a well-discussed issue in the maritime literature. The works on maritime transportation are generally on vessel scheduling and routing issues. Especially for liner ships, there are many works in the literature but not on the bunkering problems. It is hoped that this study will trigger further research and from different perspectives on the subject in future.

APPENDIX A: DATA SETS OF MODELS

Table A.1. Port charges and bunker prices of the 23 port problem

Ports	Port Charges (dollars)	Bunker Prices ($/MT IFO 380)
Fos/Lavera	4,000	514
Ceuta	2,461	505
Gibraltar	5,192	492
Sines	8,000	514
Rotterdam	7,860	474
Teesport	3,000	508
Antwerp	7,000	476
Le Havre	4,000	503
Haifa	3,000	498
Piraeus	3,326	486
Constantza	3,000	495
Malta	5,349	490
Huelva	2,400	495
Hamina	2,500	480
İstanbul	2,817	502
Tallinn	2,362	480
Skaw	3,596	479
Oslo	2,500	485
Eastham	3,092	508
Odessa	3,000	500
Hamburg	4,602	476
Suez	8,263	501
Mersin	2,700	500

Table A.2. Port-to-port transition probability matrix of the 23 port problem

Transition probabilities		Fos	Ceuta	Gibraltar	Sines	Rotterdam	Teesport	Antwerp	Le Havre	Haifa	Piraeus	Constantza	Malta	Huelva	Hamina	Istanbul	Tallinn	Skaw	Oslo	Eastham	Odessa	Hamburg	Suez	Mersin
Fos/Lavera	1	0.000	0.400	0.200	0.000	0.000	0.000	0.000	0.000	0.200	0.000	0.000	0.000	0.200	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Ceuta	2	0.091	0.000	0.000	0.363	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.091	0.363	0.000	0.091	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Gibraltar	3	0.250	0.250	0.000	0.250	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.250	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Sines	4	0.000	0.077	0.000	0.000	0.692	0.000	0.154	0.077	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Rotterdam	5	0.095	0.047	0.047	0.000	0.000	0.143	0.143	0.047	0.143	0.000	0.000	0.000	0.047	0.143	0.000	0.000	0.000	0.047	0.000	0.000	0.047	0.047	0.000
Teesport	6	0.000	0.000	0.000	0.000	0.111	0.000	0.111	0.000	0.000	0.000	0.000	0.000	0.000	0.333	0.000	0.000	0.111	0.222	0.000	0.000	0.111	0.000	0.000
Antwerp	7	0.000	0.000	0.000	0.000	0.400	0.100	0.000	0.100	0.000	0.000	0.000	0.000	0.100	0.000	0.000	0.000	0.000	0.000	0.300	0.000	0.000	0.000	0.000
Le Havre	8	0.000	0.000	0.000	0.000	0.428	0.000	0.428	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.143	0.000	0.000	0.000	0.000	0.000	0.000
Haifa	9	0.000	0.000	0.200	0.000	0.000	0.000	0.000	0.000	0.000	0.100	0.000	0.100	0.000	0.000	0.300	0.000	0.000	0.000	0.000	0.000	0.000	0.100	0.200
Piraeus	10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.143	0.000	0.428	0.143	0.000	0.000	0.143	0.000	0.000	0.000	0.000	0.143	0.000	0.000	0.000
Constantza	11	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.111	0.000	0.000	0.222	0.000	0.333	0.000	0.000	0.000	0.000	0.333	0.000	0.000	0.000
Malta	12	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.333	0.666	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Huelva	13	0.200	0.000	0.200	0.000	0.400	0.000	0.000	0.200	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Hamina	14	0.000	0.000	0.055	0.000	0.166	0.278	0.055	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.111	0.111	0.222	0.000	0.000	0.000	0.000	0.000
İstanbul	15	0.000	0.077	0.000	0.000	0.000	0.000	0.000	0.000	0.077	0.000	0.461	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.385	0.000	0.000	0.000
Tallinn	16	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Skaw	17	0.000	0.000	0.000	0.000	0.000	0.000	0.333	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.333	0.000	0.000	0.333	0.000	0.000
Oslo	18	0.000	0.000	0.000	0.000	0.143	0.000	0.143	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.571	0.000	0.000	0.000	0.143	0.000	0.000
Eastham	19	0.000	0.000	0.000	0.000	0.500	0.000	0.250	0.250	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Odessa	20	0.083	0.000	0.000	0.000	0.166	0.000	0.000	0.000	0.000	0.166	0.166	0.000	0.000	0.000	0.166	0.000	0.000	0.000	0.000	0.083	0.000	0.083	0.083
Hamburg	21	0.000	0.000	0.000	0.000	0.333	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.333	0.333	0.000	0.000	0.000	0.000	0.000
Suez	22	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.500	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.500	0.000	0.000	0.000
Mersin	23	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.428	0.143	0.000	0.143	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.143	0.000	0.143	0.000

Table A.3. Port-to-port travel times in the 23 port problem

Travel times (days)		Fos	Ceuta	Gibraltar	Sines	Rotterdam	Teesport	Antwerp	Le Havre	Haifa	Piraeus	Constantza	Malta	Huelva	Hamina	Istanbul	Tallinn	Skaw	Oslo	Eastham	Odessa	Hamburg	Suez	Mersin
Fos/Lavera	1	0.0	2.2	2.2	3.2	6.6	7.0	6.6	5.9	5.1	3.4	5	2.0	2.6	10.3	4.4	10.0	7.9	8.2	6.3	5.5	7.4	5.1	5.0
Ceuta	2	2.2	0.0	0.1	1.0	4.4	4.8	4.4	3.7	6.4	4.7	6.4	3.1	0.4	8.1	5.8	7.8	5.7	6.0	4.1	6.9	5.2	6.4	6.3
Gibraltar	3	2.2	0.1	0.0	1.0	4.4	4.8	4.4	3.7	6.4	4.7	6.4	3.2	0.4	8.1	5.8	7.8	5.7	6.0	4.1	6.9	5.2	6.4	6.3
Sines	4	3.2	1.0	1.0	0.0	3.6	4.02	3.6	2.95	7.2	5.7	7.37	4.12	0.58	7.33	6.7	7.08	4.87	5.25	3.33	7.83	4.42	7.37	7.25
Rotterdam	5	6.6	4.4	4.4	3.6	0.0	0.87	0.46	0.79	10.8	9.12	10.08	7.5	4.12	3.92	10.12	3.62	1.46	1.79	2.04	11.21	1.0	10.8	10.6
Teesport	6	7.0	4.8	4.8	4.02	0.87	0.0	1.1	1.0	11.25	9.6	11.2	8.0	4.6	3.9	10.0	3.6	1.4	1.7	2.3	11.7	1.3	11.2	11.1
Antwerp	7	6.6	4.4	4.4	3.6	0.46	1.1	0.0	0.8	10.8	9.12	10.8	7.5	4.12	4.2	10.16	4.0	1.8	2.1	2.3	11.2	1.3	10.8	10.7
Le Havre	8	5.9	3.7	3.7	2.95	0.79	1.0	0.8	0.0	10.12	8.46	10.1	6.9	3.46	4.5	9.46	4.3	2.1	2.4	1.6	10.6	1.6	10.1	10.0
Haifa	9	5.1	6.4	6.4	7.2	10.8	11.25	10.8	10.12	0.0	2.1	3.2	3.3	6.8	14.5	2.6	14.2	12.0	12.4	10.5	3.7	11.6	0.8	0.8
Piraeus	10	3.4	4.7	4.7	5.7	9.12	9.6	9.12	8.46	2.1	0.0	1.8	1.7	5.1	12.8	1.1	12.6	10.4	10.7	8.8	2.2	9.9	2.2	1.9
Constantza	11	5.0	6.4	6.4	7.37	10.08	11.2	10.8	10.1	3.2	1.8	0.0	3.3	6.8	14.5	0.6	14.2	12.0	12.4	10.5	0.6	11.6	3.4	3.0
Malta	12	2.0	3.1	3.2	4.12	7.5	8.0	7.5	6.9	3.3	1.7	3.3	0.0	3.5	11.2	2.7	11.0	8.8	9.1	7.2	3.8	8.3	3.3	3.2
Huelva	13	2.6	0.4	0.4	0.58	4.12	4.6	4.12	3.46	6.8	5.1	6.8	3.5	0.0	7.9	6.1	7.6	5.4	5.7	3.8	7.2	4.9	6.8	6.7
Hamina	14	10.3	8.1	8.1	7.33	3.92	3.9	4.2	4.5	14.5	12.8	14.5	11.2	7.9	0.0	13.9	0.3	2.4	2.8	5.3	15	2.5	14.5	14.4
İstanbul	15	4.4	5.8	5.8	6.7	10.12	10.0	10.16	9.46	2.6	1.1	0.6	2.7	6.1	13.9	0.0	13.6	11.4	11.8	9.8	1.1	10.9	2.8	2.4
Tallinn	16	10.0	7.8	7.8	7.08	3.62	3.6	4	4.3	14.2	12.6	14.2	11.0	7.6	0.3	13.6	0.0	2.2	2.6	5.1	14.7	2.2	14.2	14.1
Skaw	17	7.9	5.7	5.7	4.87	1.46	1.4	1.8	2.1	12.0	10.4	12.0	8.8	5.4	2.4	11.4	2.2	0.0	0.4	2.9	12.5	1.1	12.0	11.9
Oslo	18	8.2	6.0	6.0	5.25	1.79	1.7	2.1	2.4	12.4	10.7	12.4	9.1	5.7	2.8	11.8	2.6	0.4	0.0	3.1	12.8	1.4	12.4	12.3
Eastham	19	6.3	4.1	4.1	3.33	2.04	2.3	2.3	1.6	10.5	8.8	10.5	7.2	3.8	5.3	9.8	5.1	2.9	3.1	0.0	10.9	3.1	10.5	10.4
Odessa	20	5.5	6.9	6.9	7.83	11.21	11.7	11.2	10.6	3.7	2.2	0.6	3.8	7.2	15.0	1.1	14.7	12.5	12.8	10.9	0.0	12.0	3.9	3.5
Hamburg	21	7.4	5.2	5.2	4.42	1.0	1.3	1.3	1.6	11.6	9.9	11.6	8.3	4.9	2.5	10.9	2.2	1.1	1.4	3.1	12.0	0.0	11.6	11.5
Suez	22	5.1	6.4	6.4	7.37	10.8	11.2	10.8	10.1	0.8	2.2	3.4	3.3	6.8	14.5	2.8	14.2	12.0	12.4	10.5	3.9	11.6	0.0	1.4
Mersin	23	5.0	6.3	6.3	7.25	10.6	11.1	10.7	10.0	0.8	1.9	3.0	3.2	6.7	14.4	2.4	14.1	11.9	12.3	10.4	3.5	11.5	1.4	0.0

Table A.4. Port-to-port average bunker consumptions in the 23 port problem (13 NM speed)

Average Bunker Consumptions (MT)		Fos	Ceuta	Gibraltar	Sines	Rotterdam	Teesport	Antwerp	Le Havre	Haifa	Piraeus	Constantza	Malta	Huelva	Hamina	Istanbul	Tallinn	Skaw	Oslo	Eastham	Odessa	Hamburg	Suez	Mersin
Fos/Lavera	1	0.00	39.60	39.60	57.60	118.80	126.00	118.80	106.20	91.80	61.20	90.00	36.00	46.80	185.40	79.20	180.00	142.20	147.60	113.40	99.00	133.20	91.80	90.00
Ceuta	2	39.60	0.00	1.80	18.00	79.20	86.40	79.20	66.60	115.20	84.60	115.20	55.80	7.20	145.80	104.40	140.40	102.60	108.00	73.80	124.20	93.60	115.20	113.40
Gibraltar	3	39.60	1.80	0.00	18.00	79.20	86.40	79.20	66.60	115.20	84.60	115.20	57.60	7.20	145.80	104.40	140.40	102.60	108.00	73.80	124.20	93.60	115.20	113.40
Sines	4	57.60	18.00	18.00	0.00	64.80	72.36	64.80	53.10	129.60	102.60	132.66	74.16	10.44	131.94	120.60	127.44	87.66	94.50	59.94	140.94	79.56	132.66	130.50
Rotterdam	5	118.80	79.20	79.20	64.80	0.00	15.66	8.28	14.22	194.40	164.16	181.44	135.00	74.16	70.56	182.16	65.16	26.28	32.22	36.72	201.78	18.00	194.40	190.80
Teesport	6	126.00	86.40	86.40	72.36	15.66	0.00	19.80	18.00	202.50	172.80	201.60	144.00	82.80	70.20	180.00	64.80	25.20	30.60	41.40	210.60	23.40	201.60	199.80
Antwerp	7	118.80	79.20	79.20	64.80	8.28	19.80	0.00	14.40	194.40	164.16	194.40	135.00	74.16	75.60	182.88	72.00	32.40	37.80	41.40	201.60	23.40	194.40	192.60
Le Havre	8	106.20	66.60	66.60	53.10	14.22	18.00	14.40	0.00	182.16	152.28	181.80	124.20	62.28	81.00	170.28	77.40	37.80	43.20	28.80	190.80	28.80	181.80	180.00
Haifa	9	91.80	115.20	115.20	129.60	194.40	202.50	194.40	182.16	0.00	37.80	57.60	59.40	122.40	261.00	46.80	255.60	216.00	223.20	189.00	66.60	208.80	14.40	14.40
Piraeus	10	61.20	84.60	84.60	102.60	164.16	172.80	164.16	152.28	37.80	0.00	32.40	30.60	91.80	230.40	19.80	226.80	187.20	192.60	158.40	39.60	178.20	39.60	34.20
Constantza	11	90.00	115.20	115.20	132.66	181.44	201.60	194.40	181.80	57.60	32.40	0.00	59.40	122.40	261.00	10.80	255.60	216.00	223.20	189.00	10.80	208.80	61.20	54.00
Malta	12	36.00	55.80	57.60	74.16	135.00	144.00	135.00	124.20	59.40	30.60	59.40	0.00	63.00	201.60	48.60	198.00	158.40	163.80	129.60	68.40	149.40	59.40	57.60
Huelva	13	46.80	7.20	7.20	10.44	74.16	82.80	74.16	62.28	122.40	91.80	122.40	63.00	0.00	142.20	109.80	136.80	97.20	102.60	68.40	129.60	88.20	122.40	120.60
Hamina	14	185.40	145.80	145.80	131.94	70.56	70.20	75.60	81.00	261.00	230.40	261.00	201.60	142.20	0.00	250.20	5.40	43.20	50.40	95.40	270.00	45.00	261.00	259.20
Istanbul	15	79.20	104.40	104.40	120.60	182.16	180.00	182.88	170.28	46.80	19.80	10.80	48.60	109.80	250.20	0.00	244.80	205.20	212.40	176.40	19.80	196.20	50.40	43.20
Tallinn	16	180.00	140.40	140.40	127.44	65.16	64.80	72.00	77.40	255.60	226.80	255.60	198.00	136.80	5.40	244.80	0.00	39.60	46.80	91.80	264.60	39.60	255.60	253.80
Skaw	17	142.20	102.60	102.60	87.66	26.28	25.20	32.40	37.80	216.00	187.20	216.00	158.40	97.20	43.20	205.20	39.60	0.00	7.20	52.20	225.00	19.80	216.00	214.20
Oslo	18	147.60	108.00	108.00	94.50	32.22	30.60	37.80	43.20	223.20	192.60	223.20	163.80	102.60	50.40	212.40	46.80	7.20	0.00	55.80	230.40	25.20	223.20	221.40
Eastham	19	113.40	73.80	73.80	59.94	36.72	41.40	41.40	28.80	189.00	158.40	189.00	129.60	68.40	95.40	176.40	91.80	52.20	55.80	0.00	196.20	55.80	189.00	187.20
Odessa	20	99.00	124.20	124.20	140.94	201.78	210.60	201.60	190.80	66.60	39.60	10.80	68.40	129.60	270.00	19.80	264.60	225.00	230.40	196.20	0.00	216.00	70.20	63.00
Hamburg	21	133.20	93.60	93.60	79.56	18.00	23.40	23.40	28.80	208.80	178.20	208.80	149.40	88.20	45.00	196.20	39.60	19.80	25.20	55.80	216.00	0.00	208.80	207.00
Suez	22	91.80	115.20	115.20	132.66	194.40	201.60	194.40	181.80	14.40	39.60	61.20	59.40	122.40	261.00	50.40	255.60	216.00	223.20	189.00	70.20	208.80	0.00	25.20
Mersin	23	90.00	113.40	113.40	130.50	190.80	199.80	192.60	180.00	14.40	34.20	54.00	57.60	120.60	259.20	43.20	253.80	214.20	221.40	187.20	63.00	207.00	25.20	0.00

APPENDIX B: RESULTS OF THE MODELS

Table B.1. The best near optimal solution of the ALT II-Type 2 for Data Set II

Port Name	Bunkering At Each Port	
Fos/Lavera	**f[0]**	8.709
Ceuta	**f[1]**	16.451
Gibraltar	**f[2]**	16.451
Sines	**f[3]**	11.612
Rotterdam	**f[4]**	117.741

Table B.2. The near optimal policy of the Data Set III solved by ALT II-Type 4

u(i,I)		34	68	102	136	170	204	238	f(i)
Fos/Lavera	**0**	136	102	0	0	0	0	0	20.4
Ceuta	**1**	102	102	102	0	0	0	0	20.7
Gibraltar	**2**	102	102	0	0	0	0	0	20.4
Sines	**3**	102	0	0	0	0	0	0	10.2
Rotterdam	**4**	306	272	238	204	170	136	102	142.8
Teesport	**5**	102	102	0	0	0	0	0	20.4
Antwerp	**6**	102	102	0	0	0	0	0	20.4
Le Havre	**7**	102	0	0	0	0	0	0	0.5
Haifa	**8**	102	102	136	0	0	0	0	34.0
Piraeus	**9**	204	170	136	102	102	0	0	71.4
Constantza	**10**	170	136	102	0	0	0	0	40.8
Malta	**11**	136	0	0	0	0	0	0	13.6
Huelva	**12**	102	102	0	0	0	0	0	20.4
Hamina	**13**	136	102	102	102	0	0	0	44.2
Istanbul	**14**	102	136	102	0	0	0	0	34.0
Tallinn	**15**	0	0	0	0	0	0	0	0.0
Skaw	**16**	0	0	0	0	0	0	0	0.0
Oslo	**17**	102	0	0	0	0	0	0	10.2
Eastham	**18**	102	0	0	0	0	0	0	10.2
Odessa	**19**	170	136	102	102	102	0	0	61.2
Hamburg	**20**	0	0	0	0	0	0	0	0.0
Suez	**21**	102	102	0	0	0	0	0	20.4
Mersin	**22**	102	0	0	0	0	0	0	10.2

Table B.3. The near optimal policy of the Data Set IV solved by ALT II-Type I

u(i,I)	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250	260	270	280	f(i)
0	90	80	70	60	60	60	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9.35
1	100	90	80	70	60	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	18.79
2	130	120	110	100	90	80	70	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	24.52
3	60	60	60	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	11.61
4	280	270	260	250	240	230	220	210	200	190	180	170	160	150	140	130	120	110	100	90	80	70	60	60	60	130.00
5	70	60	60	60	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	13.87
6	110	100	90	80	70	60	60	70	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	22.58
7	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4.00
8	120	110	100	90	80	70	60	60	70	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	30.32
9	260	250	240	230	200	190	180	170	180	150	140	130	120	110	100	90	80	70	60	60	60	60	0	0	0	114.19
10	120	110	100	90	80	70	60	70	60	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	30.32
11	60	60	60	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	11.61
12	70	60	60	70	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	14.19
13	150	140	130	120	110	100	90	80	70	60	60	60	60	60	0	0	0	0	0	0	0	0	0	0	0	41.63
14	100	90	80	70	60	60	70	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	22.90
15	260	0	0	0	0	210	200	190	180	170	160	150	140	130	120	110	100	90	80	70	60	60	60	60	0	83.87
16	60	60	70	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6.13
17	60	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7.74
18	60	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7.74
19	200	190	180	170	160	150	140	130	120	110	100	90	80	70	60	60	60	60	60	60	0	0	0	0	0	72.58
20	70	60	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10.00
21	70	60	60	60	60	80	70	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	14.84
22	60	60	60	60	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	11.61

APPENDIX C: GAMS CODES OF THE SVB MODEL

```
sets
i/0*21/,
k/1*3/;

alias
(i,j);

scalar
h/18/,
G/341/,
onboard/120/,
DM/5000/,
beta/0.1/,
f/60/,
S/0.5/,
LB/34/;

parameters

index(k)/1 4,2 8,3 21/,

p(i)/0 505,1 505,2 492,3 508,4 508,5 474,6 485,7 480,8 506,9 480,10 479,
11 479,12 474,13 508,14 505,15 492,16 490,17 484,18 486,19 486,20 480,21
502/,

c(i)/0 3360,1 13576.25,2 17397.68,3 5998,4 3360.50,5 24109.42,6 2975,
7 6289,8 3360,9 5508,10 7192.25,11 7613.89,12 17586,13 5998,14 3922,
15 7884,16 7089,17 20544,18 8372,19 5614,20 1750,21 3360/,

w(i)/0 0.46,1 0.21,2 0.38,3 0.18,4 0.46,5 1.12,6 0.66,7 0.34,8 0.46,9
0.34,10 0.51,
11 0.15,12 1.12,13 0.18,14 0.21,15 0.38,16 0.76,17 0.92,18 0.36,19
0.48,20 0.16,
21 0.46/,

d(i)/0 0,1 0,2 0,3 0,4 3.9,5 0,6 0,7 0,8 3.7,9 0,10 0,11 0,12 0,13 0,14
0,15 0,
16 0,17 0,18 0,19 0,20 0,21 13.76/,

t(i)/0 0,1 0.6,2 0.6,3 0.1,4 0,5 0.75,6 0,7 0.1,8 0,9 0.05,10 0,11
0.23,12 0.33,
13 0.1,14 0,15 0,16 0.11,17 0.82,18 0.16,19 0.08,20 0.04,21 0/,

v(i)/0 4.68,1 0.12,2 0.96,3 0.08,4 4.68,5 2.73,6 3.30,7 2.54,8 4.68,9
2.54,
10 3.22,11 4.68,12 2.73,13 0.08,14 0.12,15 0.96,16 7.78,17 13.47,18 2.02,
19 4.68,20 4.68,21 4.68/,
```

```
q(i)/0 0,1 0.4,2 0.4,3 3,4 0,5 0.5,6 1.3,7 3.9,8 0,9 0.5,10 2.6,11 2.6,
12 4.1,13 5.1,14 0,15 0,16 11.4,17 13,18 13,19 13,20 13.6,21 0/;

variables
z;

positive variables
x(i),u(k);

binary variables
y(i);

equations

objective,  leg1, leg2, leg3, const2, const1, coupling1, coupling2;

objective..z=e=sum(i, c(i)*x(i))+sum(i,p(i)*y(i))
+DM*sum(k,u(k))+beta*sum(i,y(i)*v(i)*DM);

const1(i)..x(i)=l=G-sum((j)$(ord(j)<ord(i)), x(j))
+h*sum((j)$(ord(j)<=ord(i)), d(j))+h*sum((j)$(ord(j)<ord(i)), t(j)*y(j))
+h*y(i)*q(i)-onboard;


const2(i)..onboard+sum(j$(ord(j)<ord(i)), x(j))-h*sum(j$(ord(j)<=ord(i)),
d(j))
-h*sum(j$(ord(j)<ord(i)), t(j)*y(j))-h*y(i)*q(i)=g=LB;

leg1..u('1')=g=sum(j$(ord(j)<index('1')),y(j)*(t(j)+w(j)))-S;

leg2..u('2')=g=sum(j$((ord(j)<index('2')) and
(ord(j)>=index('1'))),y(j)*(t(j)+w(j)))-S;

leg3..u('3')=g=sum(j$((ord(j)<index('3')) and
(ord(j)>=index('2'))),y(j)*(t(j)+w(j)))-S;

coupling1(i)..x(i)=l=1000*y(i);

coupling2(i)..x(i)=g=f*y(i);

model submodel /all/;

solve submodel using MIP minimizing z;

display x.l, y.l, u.l, z.l;
```

```
               S O L V E      S U M M A R Y

     MODEL   submodel            OBJECTIVE  z
     TYPE    MIP                 DIRECTION  MINIMIZE
     SOLVER  CPLEX               FROM LINE  87

**** SOLVER STATUS     1 Normal Completion
**** MODEL STATUS      8 Integer Solution
**** OBJECTIVE VALUE           942804.0000

 RESOURCE USAGE, LIMIT          0.129      1000.000
 ITERATION COUNT, LIMIT        27    2000000000

ILOG CPLEX       Nov  1, 2009 23.3.3 WIN 13908.15043 VIS x86/MS Windows
Cplex 12.1.0, GAMS Link 34

Cplex MIP uses 1 of 2 parallel threads. Change default with option
THREADS.
MIP status(102): integer optimal, tolerance
Fixed MIP status(1): optimal
Solution satisfies tolerances.

MIP Solution:       942804.000000    (26 iterations, 0 nodes)
Final Solve:        942804.000000    (1 iterations)

Best possible:      931670.205617
Absolute gap:        11133.794383
Relative gap:            0.011809


                       LOWER     LEVEL     UPPER    MARGINAL

---- VAR z              -INF  9.4280E+5     +INF       .

---- VAR x

      LOWER     LEVEL     UPPER    MARGINAL

0       .       60.000     +INF        .
1       .         .        +INF  10601.250
```

2	.	.	+INF	14422.680
3	.	.	+INF	3023.000
4	.	.	+INF	385.500
5	.	.	+INF	21134.420
6	.	178.480	+INF	.
7	.	.	+INF	3314.000
8	.	60.000	+INF	.
9	.	.	+INF	2533.000
10	.	.	+INF	4217.250
11	.	.	+INF	4638.890
12	.	.	+INF	14611.000
13	.	.	+INF	3023.000
14	.	.	+INF	947.000
15	.	.	+INF	4909.000
16	.	.	+INF	4114.000
17	.	.	+INF	17569.000
18	.	.	+INF	5397.000
19	.	.	+INF	2639.000
20	.	.	+INF	.
21	.	.	+INF	3360.000

---- VAR u

	LOWER	LEVEL	UPPER	MARGINAL
1	.	.	+INF	5000.000
2	.	0.160	+INF	.
3	.	.	+INF	5000.000

---- VAR y

	LOWER	LEVEL	UPPER	MARGINAL
0	.	1.000	1.000	25945.000
1	.	.	1.000	32695.000
2	.	.	1.000	33102.000
3	.	.	1.000	7303.000
4	.	.	1.000	5148.000
5	.	.	1.000	51351.500
6	.	1.000	1.000	5435.000

7	.	.	1.000	7105.000
8	.	1.000	1.000	25946.000
9	.	.	1.000	4427.500
10	.	.	1.000	2089.000
11	.	.	1.000	15135.500
12	.	.	1.000	19510.500
13	.	.	1.000	5903.000
14	.	.	1.000	565.000
15	.	.	1.000	972.000
16	.	.	1.000	10270.500
17	.	.	1.000	51130.000
18	.	.	1.000	10064.000
19	.	.	1.000	7110.000
20	.	.	1.000	-1.220E+6
21	.	.	1.000	2842.000

```
**** REPORT SUMMARY :        0     NONOPT
                             0 INFEASIBLE
                             0  UNBOUNDED
```

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