TACTICAL COMPETITION OF TWO AIRLINES IN FLEET ASSIGNMENT

by

Ceyda Yaba

B.S., Industrial Engineering, Boğaziçi University, 2013

Submitted to the Institute for Graduate Studies in
Science and Engineering in partial fulfillment of
the requirements for the degree of
Master of Science

Graduate Program in Industrial Engineering
Boğaziçi University
2015

ACKNOWLEDGEMENTS

First of all, I would like to thank Prof. Taner Bilgiç for his valuable advices and comments. I have learned and gained a lot of experience during my graduate studies, thanks to him. Also, I would like to thank Assoc. Prof. Caner Taşkın and Assist. Prof. Murat Kaya for being in my thesis jury.

I would like to thank my mom, my dad and my brother for their endless support for my studies. With the encouragement I get from them, I completed my Master's Degree. Also, I would like to thank my friends Merve, Merve and Serpil for the support on the study nights. In addition, my assistant friends Beste and Betül provided me such a great motivation during my studies. I can thank many more of my friends for their encouragement.

ABSTRACT

TACTICAL COMPETITION OF TWO AIRLINES IN FLEET ASSIGNMENT

We analyze two airlines competing in the same market at a tactical level using an itinerary-based fleet assignment model. The market demand is contingent on the fares chosen by the airlines. The airlines solve their own fleet assignment models and can choose to spill customers on some itineraries. The recapture rates of these customers are also contingent on the fares chosen by airlines. Therefore, the dimensions of competition are both the demand and the recapture rates in the same market. Using a logit model for allocating the demand and the recapture rates between the airlines, we seek for equilibrium behavior of the airlines with exogenous fares in a computational setting. We argue that in this tactical level competition, variables like fleet type, aircraft capacity is as important as the fares.

ÖZET

İKİ HAVAYOLU ŞİRKETİNİN FİLO ATAMA PROBLEMİNDE TAKTİKSEL DÜZEYDEKİ REKABETİ

Aynı piyasadaki iki havayolunun taktiksel düzeydeki rekabeti güzergah tabanlı filo atama modeliyle incelenmektedir. Piyasadaki talepler havayolları tarafından belirlenen fiyatlara bağlıdır. Her havayolu kendi filo atama problemini çözdükten sonra müşterileri diğer güzergahlarda uçurmaya karar verebilir. Bu müşterilerin diğer güzergahları hangi oranda tercih edeceği de havayolları tarafından belirlenen fiyatlara bağlıdır. Dolayısıyla havayolları arasındaki rekabet aynı piyasadaki talepte ve diğer güzergahları tercih etme oranında görülebilir. Havayolları arasındaki piyasa talebi paylaştırılırken ve diğer güzergahları tercih etme oranı hesaplanırken logit fonksiyonu kullanılmıştır ve rekabet ortamında verilen fiyatlarla havayollarının denge davranışı aranmaktadır. Ayrıca taktiksel seviyedeki rekabette filo tipi, uçak kapasitesi gibi değişkenlerin de fiyatlar kadar önemli olduğu tartışılmaktadır.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS iii
ABSTRACT iv
ÖZET v
LIST OF FIGURES viii
LIST OF TABLES x
LIST OF SYMBOLS xii
LIST OF ACRONYMS/ABBREVIATIONS xiv
1. INTRODUCTION 1
2. LITERATURE SURVEY 3
  2.1. Operations Research in Airline Operations 3
  2.2. Fleet Assignment Models 6
  2.3. Itinerary Based Fleet Assignment 8
  2.4. Discrete Choice Modeling 10
  2.5. Competition among Airlines 10
3. PROBLEM DEFINITION 15
  3.1. Discrete Choice Model 18
  3.2. Itinerary Based Fleet Assignment 20
    3.2.1. Basic Fleet Assignment Model 20
    3.2.2. Itinerary-Based Fleet Assignment Model 23
4. ANALYSIS AND RESULTS 27
  4.1. Duopoly with Small Time-Space Networks 27
  4.2. Duopoly with Realistic Time-Space Networks 29
    4.2.1. Similar Airlines in the Competition 31
      4.2.1.1. Identical Fleet Types 31
      4.2.1.2. Different Fleet Types 33
    4.2.2. Duopoly with a Market Leader Airline 36
      4.2.2.1. Identical Fleet Types 36
      4.2.2.2. Different Fleet Types 39
5. CONCLUSION 43

APPENDIX A: PROFITS OF THE FLEET ASSIGNMENT 45
REFERENCES 69

LIST OF FIGURES

Figure 3.1. Order of Events. 16

Figure 3.2. Level of Competition between Airlines. 17

Figure 3.3. Time-Space Network Example. 20

Figure 3.4. A Node with Incoming and Outgoing Arcs. 21

Figure 4.1. Time-Space Network for the First Airline. 27

Figure 4.2. Time-Space Network for the Second Airline. 27

Figure 4.3. Response Functions of the Airlines. 29

Figure 4.4. Response Functions of Similar Airlines with Identical Fleets. . . . 31

Figure 4.5. Average Number of Empty Seats for the First Airline. 32

Figure 4.6. Average Number of Empty Seats for the Second Airline. 33

Figure 4.7. Response Functions of Similar Airlines with Different Fleets. . . . 34

Figure 4.8. Average Number of Empty Seats for the First Airline. 35

Figure 4.9. Average Number of Empty Seats for the Second Airline. 36

Figure 4.10. Response Functions of with the Market Leader Airline with Identical Fleets. 37

Figure 4.11. Average Number of Empty Seats for the First Airline. 38

Figure 4.12. Average Number of Empty Seats for the Second Airline. 39

Figure 4.13. Response Functions of with the Market Leader Airline with Different Fleets. 40

Figure 4.14. Average Number of Empty Seats for the First Airline. 41

Figure 4.15. Average Number of Empty Seats for the Second Airline. 42

LIST OF TABLES

Table 3.1. Independent variables of discrete choice model. 18

Table 3.2. Sets, parameters, and decision variables of basic FAM. 22

Table 3.3. Sets, parameters, and decision variables of IFAM. 24

Table 4.1. Profits of airlines at the equilibrium points. 30

Table 4.2. Operating costs of the aircraft types. 31

Table 4.3. Profits of similar airlines at the equilibrium points with identical fleets. 32

Table 4.4. Profits of similar airlines at the equilibrium points with different fleets. 34

Table 4.5. Profits of with the market leader airline at the equilibrium points with identical fleets. 37

Table 4.6. Profits of with the market leader airline at the equilibrium points with different fleets. 41

Table A.1. Profit of the first airline when the airlines are similar and fleet types are identical. 45

Table A.2. Profit of the second airline when the airlines are similar and fleet types are identical. 48

Table A.3. Profit of the first airline when the airlines are similar and fleet types are different. 51

Table A.4. Profit of the second airline when the airlines are similar and fleet types are different. 54

Table A.5. Profit of the first airline when it is the market leader, with identical fleets. 57

Table A.6. Profit of the second airline when the first airline is the market leader, with identical fleets. 60

Table A.7. Profit of the first airline when it is the market leader, with different fleets. 63

Table A.8. Profit of the second airline when the first airline is the market leader, with different fleets. 66

LIST OF SYMBOLS

A	Set of airports
AIR	Set of airlines
b_p^r	Recapture rate from itinerary p to itinerary r
$C_{k,i}$	Cost of assigning fleet type k to flight i
$CL(k)$	Set of flights that pass the count time when it is flown by fleet type k
D_p	Demand of itinerary p
$f_{k,i}$	Binary fleet assignment variable
$fare_p$	Fare of itinerary p
$\overline{fare}_{air}$	Fare vector of itinereries for airline air
$I(k, o, t)$	Set of incoming flights to node k,o,t
K	Set of fleet types
L	Set of flight legs
M	Set of markets
MD_m	Market demand of market m
$morning_p$	Dummy variable for itineraries departing in the morning
N	Set of nodes in the time-space network
N_k	Number of aircrafts of fleet type k
$nonstop_p$	Dummy variable for nonstop itineraries
$O(k, o, t)$	Set of outgoing flights from node k,o,t
P	Set of itineraries
P_m	Set of itineraries in market m
$SEATS_k$	Number of seats in fleet type k
$stop_p$	Dummy variable for one-stop itineraries
T	Sorted set of all event times
t_p^r	Number of passengers redirected from itinerary p to itinerary r
$time_p$	travel time of itinerary p
V_p	Utility function value for itinerary p

X	Independent variable vector of utility function
$y_{k,o,t,t+1}$	Number of aircrafts on the ground of fleet type k, at airport o, after time t
β	Coefficient vector of utility function
δ_i^p	Binary parameter for defining if itinerary p includes flight leg i
Π_{air}	Profit of airline *air*
Π_{air}^*	Maximum profit of airline *air*

LIST OF ACRONYMS/ABBREVIATIONS

CAB	Civil Aeronautics Board
FAA	Federal Aviation Administration
FAM	Fleet Assignment Model
FAP	Fleet Assignment Problem
IFAM	Itinerary Based Fleet Assignment Model
LP	Linear Programming
MILP	Mixed Integer Linear Programming
O-D	Origin Destination
PMM	Passenger Mix Model
QSI	Quality Service Index

1. INTRODUCTION

The deregulations in the airline markets provide free entry, however an airline survives depending on its efficiency while managing the operations. Therefore the use of operations research techniques gain importance while the airlines are making their decisions in every stage [1].

The growing literature in airline operations generates new concepts and terminology. Some of the terminology adopted from the literature is given next.

The first thing to come to mind when airlines are mentioned is flight legs. A “flight leg” connects two airports with a nonstop trip. However, there might be more than one flight that connects an origin-destination (O-D) pair, which is defined as an “itinerary”. A “market” consists of one O-D pair. Passenger can choose between different itineraries in a market [2].

The flight legs are operated by aircrafts. An aircraft is a member of a “fleet family” which can be operated by same crew and has same cockpit design. More specifically, a “fleet type” defines every aircraft. For example, Boeing B767-300 is a fleet type; every aircraft with same type has the same capacity [3].

This thesis focuses on two airlines competing in the same markets. Most of the literature about the competition focus on either the network formation or the scheduling competition between the airlines. However, we focus on the tactical competition. As the airlines are free to choose their fares for the itineraries, we aim to find the equilibrium fares with given flight networks. The schedule is given for the flights, and the airlines choose their fares in order to determine the demands. Also, we argue the effects of the fleet mix.

Another discussion for this thesis is the similarity of the airlines. We can say that an airline would have better brand utility therefore it is more preferred than its competitive airline. Therefore, we analyze two cases, when the airlines are indifferent from the viewpoint of passengers and when the first airline has a competitive advantage because of some historical reasons.

Our main contribution is to pose the competition problem at a tactical level that includes fare competition along with capacity of fleets. We compute the equilibrium fares under different fleet types and argue that the fleet type has a determining effect on the fare competition.

Our motivation was from the fact that Turkish Airlines was the monopoly for a very long time. After deregulations in Turkish domestic market new airlines are emerged. The effect of the competition should be observed for the airlines, and this study tries to explain these effects with different settings. Rather than looking at the competition at higher levels of operation, such as network construction or schedule decisions, we look for the effects of tactical competition with different fleet types.

The following chapters are organized as follows: Chapter 2 provides the literature review necessary for this research. Chapter 3 explains the models used in this thesis. Then, in Chapter 4 the results of different scenarios are discussed. The detailed results are given in Appendix A.

2. LITERATURE SURVEY

The literature for airline operations are growing rapidly. There are numerous researches for every phase of airline operations. This chapter summarizes some of the relevant literature used in this thesis. Section 2.1 provides the literature about the topics that use the operations research techniques used in airlines. Then, Section 2.2 focuses on the fleet assignment models developed in the literature, while the concept of itinerary based fleet assignment models are described in Section 2.3. Section 2.4 summarizes the research done for discrete choice modeling techniques. Finally Section 2.5 looks through the literature about competition between the airlines.

2.1. Operations Research in Airline Operations

As Barnhart *et al.* [4] state airlines have a major impact on contributing to the global economy and it is expected to grow more in future decades. Therefore, as airline companies grow, they need to give more importance in operations research techniques, in order to increase their shares on the market. The problems that arise in this sector are closely related to operations research techniques and with the help of information technology; it can be easily implemented. These techniques have been applied to many different problems in this area. Schedule design, fleet assignment, aircraft routing, crew scheduling can be counted as examples [1].

The problems are solved individually and the output of one problem is used as an input to the other stage. It would be better to solve the whole problem at once, but even these individual problems are too large. It is possible to integrate some of the problems together and come up with new algorithms to deal with this complexity [4].

According to Bazaargan [1], schedule design is the first step to in airline planning stages where the aim is to decide on origins, destinations and departure times of the flights. This stage takes into account the demand forecasts, the physical capacity of the airline, and the behavior of its competitor airlines.

Belobaba *et al.* [5] mention that airlines mostly operates with hub-and-spoke networks while determining the O-D pairs as opposed to point-to-point route structure because hubs allow less flights to cover more markets. Point-to-point routes need to cover all of the O-D pairs with one flight, which is very costly. However, hub-and-spoke network structure is more efficient, as passengers can be connected to a low demand O-D by the hub airport and airline is more profitable and less aircraft is used. Decision regarding the departure times is a very complicated task. It affects many other decisions for the airline including its fleet and crew. Because of the complexity, manual schedule designs are used in practice. However, new optimization models are emerging [4]. As an example Lohatepanont and Barnhart [6] suggest using an existing timetable and adding or removing flights to the schedule to obtain a better schedule.

After obtaining a feasible and profitable schedule, the next step is to determine the assignment of available fleets to planned flights with a minimum cost [4]. This problem is known as the fleet assignment model (FAM). As this problem is the base model for this thesis, it will be discussed in Section 2.2 in more detail.

The solution of FAM gives which flight should be flown with which type of aircraft. However, individual aircrafts with specific tail numbers need to be assigned to flights, which is the routing problem. Routing should also be feasible with respect to maintenance constraints [1]. Airlines must satisfy the maintenance requirements that are imposed by authorities like FAA (Federal Aviation Administration). However, the airline companies apply stricter rules for their maintenance schedule. Clarke *et al.* [7] extends the fleet assignment model to capture maintenance requirements and crew considerations. They discuss the basic fleet assignment model and give some techniques to decrease the size of the model. Then, they explain how to take into account maintenance constraints and crew requirements without making the model too big to be solved. In the paper by Gopalan and Talluri [8], given the flight assignment, they provide a maintenance routing that every aircraft with specific tail number have to visit the maintenance station in at most three days.

The next problem to solve is crew scheduling. This involves assigning crews to flights. After fuel cost, crew cost is the second largest cost for airlines. This cost can be manageable if better schedules are found, therefore this problem attracts many researchers. The crew scheduling problem consists of two parts: crew pairing and crew assignment [1]. Klabjan *et al.* [9] defines the crew pairing problem as a set partitioning problem. They try to find feasible pairings, which are defined as "crew itineraries", by applying heuristic algorithms to deal with the complexity of the problem. The crews are subject to some rules defined by FAA and the airline company. They are assigned to flights in a working day, called duties. The pairings should include flight sequences that start and end at the crew base. The objective is to minimize cost by partitioning all flights. Crew assignment problem, known as crew rostering, follows crew pairing phase. The solution of crew rostering problem assigns pairings and off-days to crews along with another must do activities. Gamache *et al.* [10] suggests that the problem takes into account the qualifications and the number of crews needed to operate that pairing, as each aircraft type cannot be operated by any member of the crew. They used column generation approach to deal with the large scale of the problem.

All of these problems use the techniques from operations research and researchers try to come up with better solutions all the time as the sector grows rapidly. According to Barnhart *et al.* [4] the papers in air transportation industry with operation research methodologies exceeds 1000 in the past 50 years.

Another important part of operations for a profitable airline is the revenue management step. There is an extensive literature for airline revenue management techniques as it provides the sales revenue for the airlines which is an essential part for profitability.

Mcgill and Van Ryzin [11] provide a review for the revenue management problem in the past forty years. The revenue management problem has an objective of maximizing revenues by searching the most profitable "booking policies". Booking policies decide if the "booking classes" should be open or not. The booking classes are defined for the bookings that have common characteristics. The decision is to find the optimal reservations for the number of seats for the specific booking classes. The dilemma is if too many seats are reserved for low fare classes then the high fare demand is spilled due to capacity. On the other hand, fewer seats are assigned to low fare classes, and then the plane can depart empty. Therefore, this problem does not have a straightforward solution.

Talluri and Van Ryzin [12] propose that the “bid-price control” method as a revenue management is not optimal. Bid-price control allows a passenger to book a seat in an itinerary if the price of that itinerary exceeds the sum of the threshold prices of the legs that constitute it. They suggest a dynamic programming approach, rather than a deterministic control to find optimal bid-price controls.

According to Belobaba and Farkas [13], the spill estimation is really crucial for the fleet assignment. They combine the revenue management techniques to the spill estimation process, they do not consider average price but the different prices for every fare class.

Another research that uses the revenue management decision is by Bish *et al.* [14]. They consider that airline do not have use a fixed capacity while deciding on fare class capacity, and can change aircrafts with different capacities. That way they are more flexible when they observe the real demands.

2.2. Fleet Assignment Models

The aim of the fleet assignment problem (FAP) is to assign different aircraft types to the flight legs with minimum operating cost given a schedule. There are many flights scheduled for a day and FAP is an essential part of the airline operations since it affects other stages of the operation. Therefore, this problem captures a lot of interest from the researchers [3].

As mentioned in Sherali *et al.* [3], basic fleet assignment models assume that airlines operate with same schedule every day. However, in practice, this is not true; demand varies from day to day in a week. In addition, these basic models use flight-based assignments. Most of the models are subject to same type of constraints. These are covering constraints that assure every flight leg is assigned to one type of aircraft, flow conservation constraints, and availability constraints in which number of used aircrafts cannot exceed number of available aircrafts.

Abara as one of the earliest researchers that works on this problem used “connection based network structure”. Connection based networks are one of the conventions that is used in fleet assignment problems. Nodes represent times of arrival or departures of flights, additionally there are source and sink nodes. There are different types of arcs, “flight connection arcs” connect arrival nodes to departure nodes, “terminating arcs” connect arrival nodes to sink node, and “originating arcs” connect source node to departure nodes [3]. According to Abara [15], every aircraft that is assigned to a flight can connect another flight that has the departure time exceeds the first one’s arrival time by minimum permitted time. This “flight-to-flight connections” are called turns. The constraints are the usual constraints mentioned above. Decision variables define feasible turns for aircraft types.

The other network structure that is more widely used in the literature is time-space network. In this network, arcs correspond to flights or time spent on the ground, and nodes represent an airport at a specific time. Berge and Hopperstad [16] define this FAP as a multi commodity network flow problem. Hane *et al.* [17] also worked on this problem, with the same types of constraints and solved the problem with several preprocessing steps in order to reduce its size.

According to Rushmeier and Kontogiorgis [18], the time space network cannot identify which aircraft is on the ground, in contrast to connection based networks. Connection based networks consider connection possibilities, which results in aircraft utilization. From the complexity point of view, time space network generates less number of variables as opposed to connection based networks since connection networks take into account all possible connections [3].

More advanced researches based on FAP are emerging in the literature, since technology improvements and competiveness of the sector allows these new areas. Integration FAP with other problems in the airline operations, mentioned in the previous section, is one of the improvements [3].

Clarke *et al.* [7] add additional constraints to cover different maintenance checks to the basic FAP. Also, they consider crews that needs to stay overnight away from the base station, which incurs as a cost to airlines. They provide a model integrated with fleet assignment to overcome this situation.

Sandhu and Klabjan [19] propose an integrated fleeting and crew pairing model. The suggested model realizes simultaneous assignment of fleets and pairings. New constraints are introduced in order to prevent aircraft-routing infeasibility. Then, they compare two solution methodologies: Lagrangian relaxation with column generation and Benders decomposition.

In the FAPs, demands are forecasted as flight-based and a passenger that cannot be accommodated due to capacity restrictions is assumed to be lost. However, another extension to the FAM is using itinerary based demands and considering spill and recapture effects [3]. These models in the literature will be discussed in Section 2.3.

2.3. Itinerary Based Fleet Assignment

Most of the fleet assignment models assume that objective is maximizing the “unconstrained revenue” minus the “assignment cost”. However, in reality the capacity of the aircraft should be considered so that, the revenue generated from this assignment would be more realistic. If a passenger is rejected from a flight because of capacity restrictions, he either chooses not to fly or chooses other flights offered by other airlines, so that airline “spills” that customer’s demand, the customer is lost. Another possibility is that the customer chooses the other itineraries offered by that airline, this time airline have the opportunity of “recapturing” the demand, which results in revenue generation [2].

The passenger mix model (PMM) is integrated with basic fleet assignment models. PMM is described by Kniker [20]. He suggests that the PMM provides the solution as the "assignment of passengers to the flight legs" given the schedule with assigned fleets.

Itineraries that have more than one flight for an OD pair naturally affect the demand of the individual flights. On the contrary, the supply is available seats in a flight leg. This interaction is captured in the itinerary-based fleet assignment models. Farkas [21] proposes an itinerary based model with this "network effect", but he does not consider recapture.

Barnhart *et al.* [2] improve the model proposed by Farkas, with adding the recapture effect. They integrate the basic FAM and PMM, with an objective of minimizing assignment cost. The capacities of the aircrafts are important part of the model, and with the itinerary-based assignments the dependency between supply and demand is considered. They provide a "recapture rate" definition based on QSIs.

Wang *et al.* [22] also suggested an itinerary based fleet assignment model, with spill and recapture based on "attractiveness of the itineraries" in the whole market not only the individual airline. The recapture ratios provide the probability of choosing that itinerary if a passenger is spilled based on the attractiveness of the itinerary.

Lohatepanont and Barnhart [6] enhance the mentioned itinerary based models by integrating schedule planning to the itinerary based fleet assignment. They suggest that given a planned schedule, there are optional flights that can be cancelled. They use a "base schedule" instead of creating an entirely new schedule.

Atasoy *et al.* [23] add the pricing decision to the itinerary based fleeting model with schedule planning. In the model demand and price are also decision variables. The demand is shared among the itineraries in the market according to a choice model.

2.4. Discrete Choice Modeling

In the early 2000's airlines began to use the discrete choice methodology while forecasting the demands for their itineraries. Discrete choice modeling allows to observe that an individual can choose between itineraries based on the characteristics of the itinerary, such as price, travelling time, departure time, and others [24].

The first choice models introduced are based on Quality Service Indexes (QSI). QSI methodology is proposed by U.S government in 1957. The choice of a passenger is based on the “quality of the itinerary”, and the attributes to explain this quality has some weights. These weights are calculated via some statistical methods or experts decide on what should that weights be [24].

Coldren *et al.* [25] provides a multinomial logit model for deciding the market shares. They provide a utility function for each itinerary, which is a linear model with independent variables that represent different characteristics of the itinerary. The share of the itinerary is the exponential utility of that itinerary divided by the sum of all of the exponential utilities of the itineraries in the same market.

Atasoy and Bierlaire [26] use the logit model to describe the choice strategy of the passengers. They used “revealed preference data” based on a real booking data and “stated preference data” which are collected by surveys. The dependent variables include price, travel time, number of stops, and the departure time. They also study the price elasticity of the demand model.

2.5. Competition among Airlines

All the models discussed above were solved for monopoly markets. However, in reality, airlines compete with each other, and they try to capture the largest market share in order to increase their revenues.

The U.S. market was highly regulated by the Civil Aeronautics Board (CAB) until

1970s and there are restrictions on the free entry and exit of airlines, in addition to the restrictions on the fares of flights and the markets to be operated on of the existing airlines. In 1976, CAB allowed the price discounts on different customers, "Airline Deregulation Act" two years later allowing new airlines to enter the domestic market and they dropped the restrictions on the fares. Deregulation is the start of competition among airlines in the domestic market. It has many advantages and disadvantages which are addressed by the researchers [27].

Even though academics argue that there will be perfect competition with deregulation, there were some abnormalities that were not predicted before the observation of the deregulated markets. Airlines prefer to merge, although the free entry and exit were not limited anymore, especially between the airlines serving in crowded airports. In addition to merged airlines, code sharing agreements were done between the regional and major airlines for the small markets. Other unpredicted results of deregulation include the frequent flyer programs and tickets sold by travel agents. Other than these, the hub-and-spoke network structures influenced the competition too much [28].

After deregulation, airlines started to choose their own network structures, which lead to the hub-and-spoke network structure. With the ease of hub airports, that can support more traffic flow than the spoke airports, the passengers in a low demand market can be connected to their destination via the hub airport with a one-stop flight [28]. Competition with selection of hubs and networks are introduced in the literature. Some of the examples can be found in the following paragraphs.

Passengers benefit from hubs with high level of frequencies while airlines use the advantage of "scale economies" by using larger aircrafts with higher utilizations. Kanafani and Ghobrial [29] suggested a model that seeks for equilibrium in the network of an airline with a single hub. They introduce new routes to the network based on a multinomial logit choice modeling and calculate the overall profit of the network with added routes. They compare the networks for a single airline until they reach equilibrium.

Hansen [30] proposed an “n-player non-cooperative game” with an application from U.S. domestic market. In order to simplify the strategy sets for airlines in competition, he considered only hub competition rather than the general competition. There are two types of airlines in the game hub airlines and point-to-point airlines. Hub airlines can have a direct service between their own hubs, and connect the other passengers between spokes via the hubs. On the other hand, point-to-point airlines serve their passengers with direct connections between every O-D pairs they operate in. Point-to-point airlines and hub airlines decide on the service frequency in every O-D pair that they serve by maximizing their own profits. The model was applied to U.S. domestic market; the result gave “quasi-equilibrium” rather than real equilibrium points.

Adler [31] proposes that airlines need to find most profitable Hub-and-Spoke networks in order to survive in the market. Two stage Nash game is discussed, where in the first stage a network is generated with an integer linear program. In the second stage a nonlinear optimization model is solved to maximize profit based on the networks chosen. The frequency, sizes of the aircrafts and prices are taken into account as variables.

Competition on networks is also discussed by Takebayashi and Kanafani [32]. They introduced one-to-one competition between a hub-and-spoke airline and a point-to-point airline. They assumed that the hub-and-spoke airline was already in the market and point-to-point airline maximizes its profit after seeing other airline’s actions. The second level of the optimization is then solved by maximizing the passengers’ utilities based on the optimal solutions of the airlines.

Adler [33] introduces a nonlinear mathematical model such that fare, frequency and aircraft size are decision variables and the objective is to maximize profit. The competition can be defined as “non-cooperative, two-stage game” for more than one airline and airlines decide on their hub-and-spoke networks in addition to the decision variables. Airlines choose their networks at the same time in the first phase, and then the mathematical model is solved. The aim is to find Nash equilibrium. The proposed model is applied to a Western Europe market.

Another competition level among airlines is deciding on the schedules of their operation. Hong and Harker [34] provide two models for schedules of the airlines. First model has two stages. The first stage is modeled for assigning slots at airports to the airlines. In the second stage an equilibrium model is solved "slot prices and flight schedules and fares." Second model takes the allocation of slots endogenously and provides the solution for schedules, fares and routes along with passenger choices of carriers and priorities of airlines in landing.

Vaze and Barnhart [35] design a model for competition of airlines in congested airports. Airlines decide on their flight frequencies with given slot constraints. The market share model is a S-curve model and the objective is to find a Nash equilibrium by solving individual optimization models successively.

Wei and Hansen [36] develop a nested logit model in order to decide on whether aircraft size, service frequency, seat availability and fare have an impact on the market share of the airline. They discuss this model in a duopoly market, where a customer chooses either one of the firms or he does not travel at all.

Wei and Hansen [37] also discuss how airlines' make their choices regarding aircraft size and service frequency. Different game theoretic models are proposed in the paper. The decisions are based on costs and demands of the airlines. A Nash game, a Stackelberg game, and a two-level hierarchical game are introduced.

Hansen and Liu [38] consider the frequency competition among airlines and they compare S-curve model with a model based on "schedule delay". Schedule delay is the difference between the available flight time and when the passenger wants to travel. They assume that objective of every airline is to maximize their own profits.

The deregulation started in 1983 with the Civil Aviation Law ("Türk Sivil Havacılık Kanunu"). The air transport sector would be more competitive and safe with this law.

With the new entrants to the sector, their shares increased in the market, so that Turkish Airlines, which was the only carrier before the deregulation, have been modernized and their fleets are improved to achieve higher standards. After the economic crisis in 2001 in Turkey, Turkish Airlines and other domestic carriers are allowed to choose their fares. Also, they are choosing their schedules in the different times of the day so that the available capacity is utilized efficiently. The deregulation in the domestic markets have a significant mpact on the passengers carried in total, since the new entrants provide lower fares for the itineraries, as opposed to the monopolistic market [39].

This thesis provides a competition model between two airlines, in which they compete on fares with their given schedules. They compete in same markets and they want to capture the most of the market share by adjusting the fares of their itineraries. The airlines then solve their itinerary based fleet assignment models and they can spill and recapture demand based on the capacity of their fleets. The details will be discussed in Chapter 3.

3. PROBLEM DEFINITION

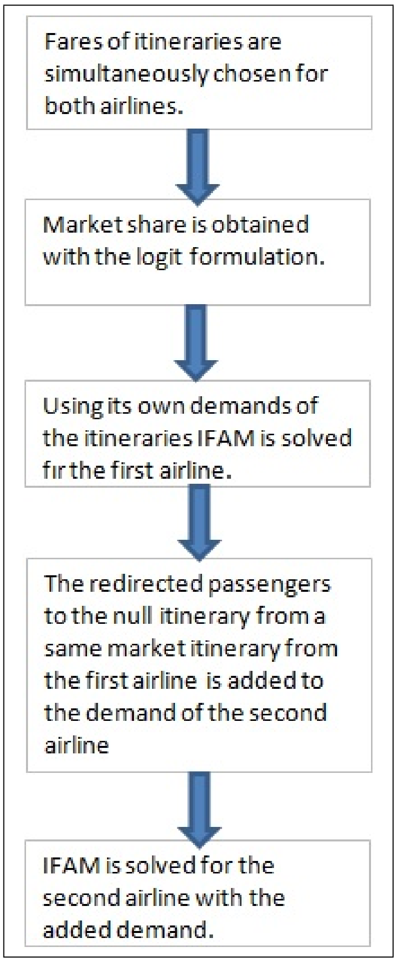
The main purpose of this thesis is to find equilibrium scenarios for the fares of two competing airlines under different fleet compositions. They have their own fleet assignment decisions, but they have dependency while deciding on their fares since they compete in the same markets.

The first step in the analysis is to determine the market demands of the itineraries distributed by the discrete choice model defined in Section 3.1. The market demand and the fares of the itineraries are exogenous to the model. The market consists of itineraries of both airlines in addition to the null itinerary (which indicates that the customer choose not to fly). Most of the demand is captured by the itinerary with the highest utility value among the itineraries in the same market with the given fares and networks of the airlines.

After determining on the demand of the itineraries, IFAM for the first airline is solved. It is assumed that the first airline is a larger airline than the second airline, hence solves IFAM first. The solution of IFAM gives the fleet assignment decisions, with the number of aircrafts on the ground at given times. Another decision of IFAM is the number of passengers that are redirected to other itineraries, either to the itineraries of the first airline or to the null itinerary. The redirected passengers to the null itinerary are lost to the first airline. We assume that these customers now consider to travel with the second airline and we add them to the demands of the itineraries of the second airline.

The next process is to find the solution of IFAM for the second airline, with the extra demand from the null spill of the first airline in the same market. The extra demand is shared to the itineraries of the second airline with the discrete choice model. Then IFAM is solved for the second airline. The details of IFAM is given in Section 3.2.2.

The order of events described above is summarized in Figure 3.1.



Fares of itineraries are simultaneously chosen for both airlines.

Market share is obtained with the logit formulation.

Using its own demands of the itineraries IFAM is solved fir the first airline.

The redirected passengers to the null itinerary from a same market itinerary from the first airline is added to the demand of the second airline

IFAM is solved for the second airline with the added demand.

Figure 3.1. Order of Events.

The competition between the airlines are based on the fare structure of their itineraries. We expect that the demand share of an itinerary increases as its fare decreases. However, the airlines can lose customers if they cannot accommodate all of their passengers. Hence, the fleet mix of the airline has an effect on the profit too. The level of competition is outlined in Figure 3.2.

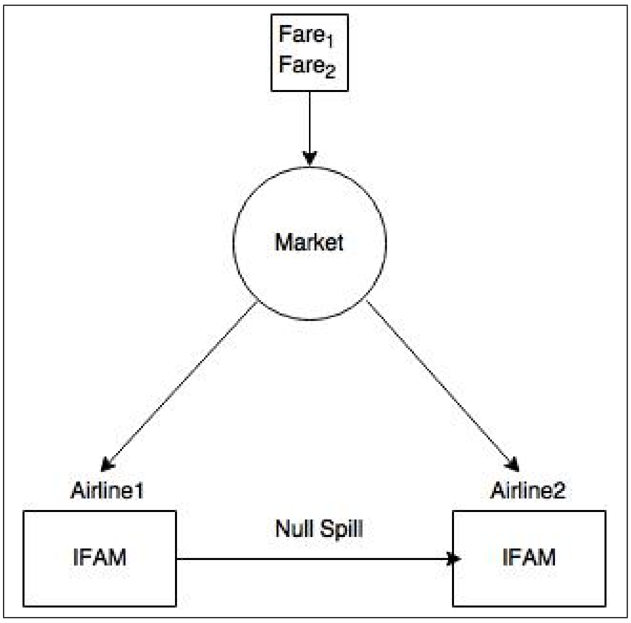


Figure 3.2. Level of Competition between Airlines.

The airlines are risk neutral profit maximizers subject to the fleet assignment constraints in the game defined in this thesis. The airlines decide on their prices as a first step to capture the demand from the market. Then they solve the fleet assignment problems individually. This process is repeated by increasing the fares by a fixed step size. The equilibrium solutions occur at the fares where both of the airlines maximize their profits at the same fare vectors.

Airlines maximize their profits as given in Equations 3.1 and 3.2. The profit of an airline is dependent on the competitive airline's fare vector. The equilibrium is searched for the fare vectors of the airlines where maximum profits occur.

$$\Pi_1^* = \max \Pi_1(\overline{fare_1} | \overline{fare_2}) \tag{3.1}$$

$$\Pi_2^* = \max \Pi_1(\overline{fare_2} | \overline{fare_1}) \tag{3.2}$$

The next sections provide the details of the individual models that are used for the competitive environment. They are originally defined for the monopolies but we modified them for the duopoly case for this thesis as needed.

3.1. Discrete Choice Model

Discrete choice modeling strategy allows airlines to forecast demands more accurately while increasing the revenues in return. These models estimate the behavior of individual passengers. There are different methodologies to construct a discrete choice models. In this thesis we refer to the model introduced by Atasoy and Bierlaire [26].

A chosen itinerary have a utility function value that effects the choice of the customer. The utility function for itinerary p can be written as follows:

$$V_p = \beta X \tag{3.3}$$

In Equation 3.3 the X vector represents the independent variables, whereas β vector represents the coefficients of the independent variables. The independent variables can be chosen by the modeler such as price of the itinerary, travel time of the itinerary, level of service of the itinerary (nonstop or more stops), the carrier and so on [24].

The utility function described by Atasoy and Bierlaire chooses the following independent variables summarized in the Table 3.1.

Table 3.1. Independent variables of discrete choice model.

Variables	
$fare_p$	fare of itinerary p.
$time_p$	total travel time of itinerary p.
$nonstop_p$	dummy variable, it takes value 1 if itinerary p is a non-stop itinerary, 0 otherwise
$stop_p$	dummy variable, it takes value 1 if itinerary p is a one-stop itinerary, 0 otherwise
$morning_p$	dummy variable, it takes value 1 if first flight of itinerary p departs in the morning, between 07:00 and 11:00

The utility function used in this thesis use the same model parameters and coefficients described by Atasoy and Bierlaire. They differentiate between cabin classes (economy or business), but we did not use different cabin classes, all of the passengers are traveling in the same cabin class.

$$\begin{aligned} V_p = \quad & 0.0851 log(fare_p/100) nonstop_p + (-1.47) log(fare_p/100) stop_p \\ & + (-0.0204) time_p nonstop_p + (-0.0705) time_p stop_p \\ & + 0.282 morning_p \end{aligned} \tag{3.4}$$

Atasoy and Bierlaire used the logarithm of the price since the argue that an increase in the price does not have a linear effect on the utility of a customer. An increase of 1 dollar does not have the same effect as an increase of 100 dollars [26].

The calculated utility function in Equation 3.4 is used for the logit model, that gives the choice probability of an itinerary.

$$Prob(p) = \frac{exp(V_p)}{\sum_{r \in P_m} exp(V_r)} \quad \forall m \in M. \tag{3.5}$$

The Equation 3.5 calculates the probability of choosing itinerary p. The numerator is the exponential of the utility value of itinerary p. The denominator is the sum of the exponential utility values of all itineraries that are in the same market with p, including the null itinerary and the itineraries of the competitive airline.

Discrete choice model with a logit formulation is really helpful for determining the parameters for IFAM. It provides the information of individual choices of passengers, rather than a quality indication of an airline.

3.2. Itinerary Based Fleet Assignment

Itinerary based fleet assignment model (IFAM) is introduced by Barnhart et. al. by integrating basic fleet assignment model (BFAM) with passenger mix model (PMM) to capture the effect of demand, spill and recapture [2]. The following sections provide the definition of the BFAM, followed by the definition of IFAM.

3.2.1. Basic Fleet Assignment Model

Most of the basic fleet assignment models used in the industry is very similar to the model proposed by Hane *et al.* [17]. FAM can be defined as minimizing assignment cost subject to assignment, aircraft balance and available aircraft constraints.

Time-space network is defined for keeping track of the aircraft flow balance. An example of a time space network is given in the following Figure 3.3.

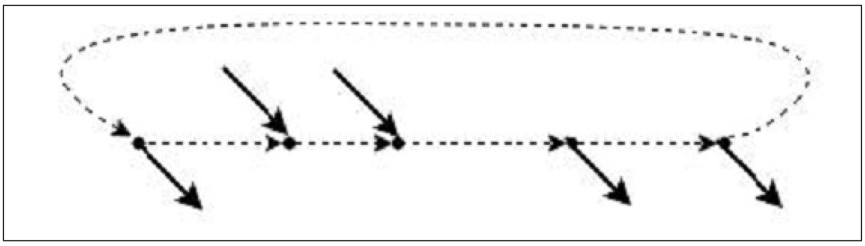


Figure 3.3. Time-Space Network Example.

As seen in the Figure 3.3, nodes represent an airport at a given time, and emanating arcs from a node represent flights departing from an airport at a given departure time, incoming arcs to a node represent arrival at a given arrival time. There are also ground arcs between the adjacent nodes of an airport, shown with dashed arrows, that represent the number of aircrafts on the ground between these times. With the ease of time-space network, the flow balance of a node can be represented easily. The wrap around arcs represent the end of the day, the arc that connects the first node and the last node. Most of the FAMs use daily schedule, if a plane stays at an airport at night, it should satisfy the flow balance of the first node in the morning [1]. Figure 3.4 shows a node with incoming and outgoing arcs that satisfy the flow balance.

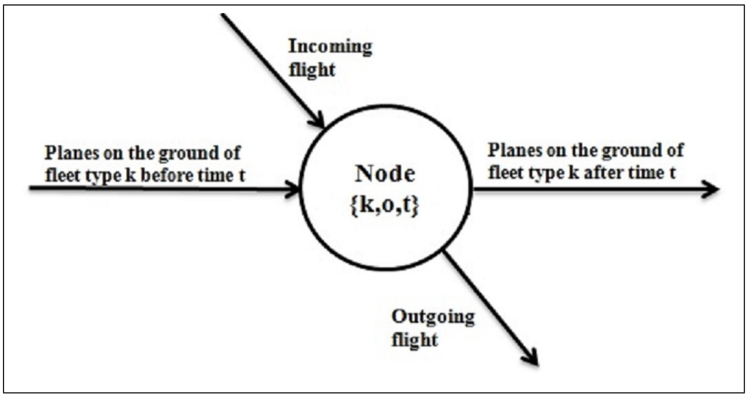


Figure 3.4. A Node with Incoming and Outgoing Arcs.

The flow balance of a node is defined as number of aircrafts on the incoming arcs should be equal to the number of aircrafts on the outgoing arcs. In this context, the incoming arcs are either the flights incoming to that airport at that time or the aircrafts on the ground before that time. Also, the outgoing arcs are either the flights that are outgoing from that airport at that time or the aircrafts on the ground right after that time.

When a time-space network is used as the network structure, FAM uses sets, parameters and variables as shown in Table 3.2. This notation is taken from Barnhart *et al.* [2].

Table 3.2. Sets, parameters, and decision variables of basic FAM.

Sets	
A	set of airports indexed by o.
L	set of flights indexed by i.
K	set of fleet types indexed by k.
T	sorted set of all event times (arrival or departure) at all airports, indexed by t. The event at time t occurs before the event at time $t+1$. If number of event times is m, $t = t_m$ is the last event before the count time, and $t = t_1$ is the first event after the count time.
N	set of nodes in the time space network indexed by k, o, t.
$CL(k)$	set of flights that pass the count time when it is flown by fleet type k.
$I(k, o, t)$	set of incoming flights to node k, o, t.
$O(k, o, t)$	set of outgoing flights from node k, o, t.
Parameters	
$C_{k,i}$	cost of assigning flight leg i is to fleet type k.
N_k	number of aircrafts of fleet type k.
Decision Variables	
$f_{k,i}$	binary variable that takes the value 1 if flight leg i is assigned to fleet type k.
$y_{k,o,t,t+1}$	number of aircrafts of fleet type k on the ground at airport o between the times t and $t+1$.

The mathematical model for the basic FAM is given below, using the sets, decision variables, and parameters defined in Table 3.2.

Basic FAM:

$$\min \sum_{i \in L} \sum_{k \in K} C_{k,i} f_{k,i} \tag{3.6}$$

s.t.

$$\sum_{k \in K} f_{k,i} = 1, \quad \forall i \in L, \tag{3.7}$$

$$y_{k,o,t-1,t} + \sum_{i \in I(k,o,t)} f_{k,i} - y_{k,o,t,t+1} - \sum_{i \in O(k,o,t)} f_{k,i} = 0, \quad \forall k, o, t, \tag{3.8}$$

$$\sum_{o \in A} y_{k,o,t_m,t_1} + \sum_{i \in CL(k)} f_{k,i} \leq N_k, \quad \forall k \in K, \tag{3.9}$$

$$f_{k,i} \in \{0, 1\} \quad \forall k \in K, \forall i \in L, \tag{3.10}$$

$$y_{k,o,t,t+1} \geq 0 \quad \forall k, o, t. \tag{3.11}$$

Equation 3.6 is the objective function which minimizes the assignment cost of aircrafts to the flights. Constraints 3.7 guarantee that every flight is assigned to exactly one fleet type. Constraints 3.8 satisfy the flow balance of all nodes in the network as defined in the Figure 3.4. The count constraints 3.9 make sure that more than the number of aircrafts cannot be assigned to the flights. The count time is defined for these constraints so that the number of aircrafts assigned can be easily tracked even if they are flying or on the ground.

3.2.2. Itinerary-Based Fleet Assignment Model

Barnhart *et al.* developed an itinerary based fleet assignment model (IFAM) in order to include the spill and recapture effects, and the capacity of the network. This model is built upon the basic FAM described in the previous section [2].

There are additional sets, parameters and decision variables introduced for IFAM given in the Table 3.3. Others that are used in the model is provided with the basic FAM formulation.

Table 3.3. Sets, parameters, and decision variables of IFAM.

Sets	
P	set of itineraries indexed by p or r.
M	set of all markets indexed by m.
P_m	set of itineraries in market m
Parameters	
$SEATS_k$	number of seats available on fleet type k.
D_p	demand of itinerary p.
MD_m	market demand of market m.
$fare_p$	fare of itinerary p.
b_p^r	recapture rate from p to r.
δ_i^p	binary parameter that takes the value 1 if flight i is included in itinerary p, 0 otherwise.
Decision variables	
t_p^r	number of passengers requesting itinerary p but are redirected to itinerary r.

The demand of an itinerary is is calculated by the discrete choice model described in Section 3.1. Therefore demand for an itinerary is obtained by multiplying the total demand of the market including that itinerary and probability of choosing that itinerary as seen in Equation 3.12.

$$D_p = MD_m Prob(p) \quad \forall p \in P_m \tag{3.12}$$

Recapture rate effects the number of passengers that are spilled to another itinerary. Not 100% of the passenger that are spilled from itinerary p to r chooses to itinerary r. Recapture rates are needed for IFAM in order to decide on the fraction of passengers that will be redirected to r from p. They are also calculated by the discrete choice model defined in Section 3.1 as shown in Equation 3.13.

$$b_p^r = \begin{cases} 1 & \text{if } p = r \\ \frac{exp(V_r)}{\sum_{l \in P_m \backslash p} exp(V_l)} & \text{if } p \neq r \end{cases} \tag{3.13}$$

The recapture rate takes value of 1 if p and r are the same itinerary. However, if p is not equal to r, the recapture rate is calculated as the exponential of the utility value of itinerary r, divided by the sum of the exponential values of utilities of itineraries in the same market as r, except itinerary p, including null itinerary.

The mathematical model for IFAM is given below with the sets, parameters, and decision variables defined in Table 3.3 in addition to definitions provided in Seciton 3.2.1.

IFAM:

$$\max \sum_{m \in M} \sum_{\substack{p \in P_m \\ p \neq 0}} fare_p D_p - \sum_{i \in L} \sum_{k \in K} C_{k,i} f_{k,i} - \sum_{m \in M} \sum_{\substack{p \in P_m \\ p \neq 0}} \sum_{r \in P_m} fare_p t_p^r$$

$$+ \sum_{m \in M} \sum_{\substack{p \in P_m \\ p \neq 0}} \sum_{\substack{r \in P_m \\ r \neq 0}} b_p^r fare_r t_p^r \tag{3.14}$$

s.t.

$$\sum_{k \in K} f_{k,i} = 1, \quad \forall i \in L, \tag{3.15}$$

$$y_{k,o,t-1,t} + \sum_{i \in I(k,o,t)} f_{k,i} - y_{k,o,t,t+1} - \sum_{i \in O(k,o,t)} f_{k,i} = 0, \quad \forall k, o, t, \tag{3.16}$$

$$\sum_{o \in A} y_{k,o,t_m,t_1} + \sum_{i \in CL(k)} f_{k,i} \leq N_k, \quad \forall k \in K, \tag{3.17}$$

$$\sum_{k \in K} SEATS_k f_{k,i} + \sum_{m \in M} \sum_{r \in P_m} \sum_{p \in P_m} \delta_i^p t_p^r - \sum_{m \in M} \sum_{\substack{r \in P_m \\ r \neq 0}} \sum_{p \in P_m} \delta_i^p b_r^p t_r^p \geq \sum_{m \in M} \sum_{\substack{p \in P_m \\ p \neq 0}} \delta_i^p D_p, \quad \forall i \in L, \tag{3.18}$$

$$\sum_{r \in P_m} t_p^r \leq D_p, \quad \forall m \in M, \forall p \in P_m, p \neq 0, \tag{3.19}$$

$$f_{k,i} \in \{0,1\}, \quad \forall k \in K, \forall i \in L, \tag{3.20}$$

$$y_{k,o,t,t+1} \geq 0, \quad \forall k, o, t, \tag{3.21}$$

$$t_p^r \geq 0, \quad \forall m \in M, \forall p \in P_m, \forall r \in P_m. \tag{3.22}$$

The objective function (3.14) is maximizing revenue. It consists of the unconstrained revenue and the revenue from recapture subtracted by spill cost and operating cost. The constraints (3.15), (3.16), and (3.17) are the same constraints of the basic FAM. IFAM expands the basic FAM with extra constraints. Constraints (3.18) ensure that the demand of a flight with spilled and recaptured passengers cannot exceed the capacity of the aircraft after assignment. Constraints (3.19) satisfy that number of passengers who are redirected from itinerary p to itinerary r cannot be more than the unconstrained demand of itinerary p. Lastly, constraints (3.20), (3.21), and (3.22) are about the types of decision variables.

There are small modifications in our model that have somewhat different assumptions from the IFAM proposed by Barnhart *et al.* [2]. They assume that all of the itineraries are operated in the same market, however in our case there are several markets with several itineraries to choose from. Therefore the set M is introduced which represents the all markets. Also, it is assumed that an itinerary can spill its passengers to the other itineraries that are in the same market or to the null itinerary. Null itinerary is defined as the passenger is lost, either to the competitive airline or the passenger chooses not to fly. This is a no revenue option for the airline [23].

4. ANALYSIS AND RESULTS

All of the models are implemented by Gurobi Optimizer with Python interface [40]. Gurobi uses branch and bound based on LP while solving MILPs. We used the default solution technique of Gurobi while solving IFAM for airlines with default tolerences for integer optimality.

4.1. Duopoly with Small Time-Space Networks

This section provides a small example for solving the problem defined in Chapter 3 regarding the competitive environment. The results are given for the sake of explaining how the equilibrium fares are searched.

Initially, we tested our model for a duopoly market with small time-space networks. The airlines are competing in one market with several itineraries. The time-space networks can be seen in Figures 4.1 and 4.2.

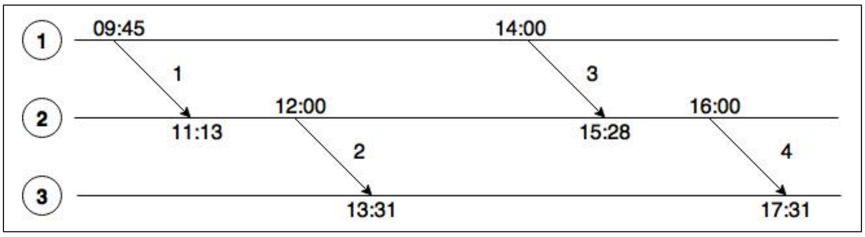


Figure 4.1. Time-Space Network for the First Airline.

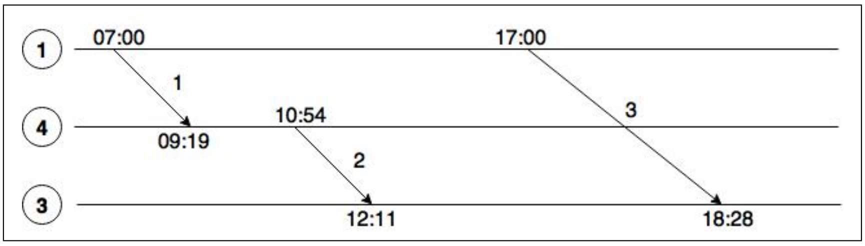


Figure 4.2. Time-Space Network for the Second Airline.

As seen in Figure 4.1, the first airline has four flights numbered from 1 to 4. These flights are the nonstop itineraries for the first airline. Also, the Flight 1 and Flight 2, and Flight 3 and Flight 4 constitute the one-stop itineraries of the first airline. These one-stop itineraries link Airport 1 to Airport 3 via Airport 2. Also, Flight 1 is a morning itinerary.

Figure 4.2 shows the time-space network of the second airline. It has three flights, as nonstop itineraries, and Flight 1 and Flight 2 form a one-stop itinerary. This one-stop itinerary and Flight 3 are the itineraries of the market of Airport 1 to Airport 3. Flight 1 and Flight 2 are morning itineraries.

The airlines compete in the O-D pair of Airport 1 and Airport 3 with the aforementioned itineraries. Other itineraries are not part of the competition, however they are essential for the operation. The problem is solved for 50 instances, with the competitive environment explained in Chapter 3. The fares of the competitive itineraries are increased by 10% in every loop, and every fare vector is numbered from 1 to 50, with 1 representing the base fares. The base fares are the lowest level of the itinerary fares, on the average of 315.33 TL for the first airline, and on the average of 247.75 TL for the second airline.

The response function for the airlines is given in Figure 4.3. The diamonds show the fare vector of the first airline given the fare vector of the second airline, whereas the squares show the fare vector of the second airline given the fare vector of the first airline at where the maximum profits occur given the competitive airline fares vectors. As it is seen in the figure there are multiple equilibria where the diamonds and squares are overlapped.

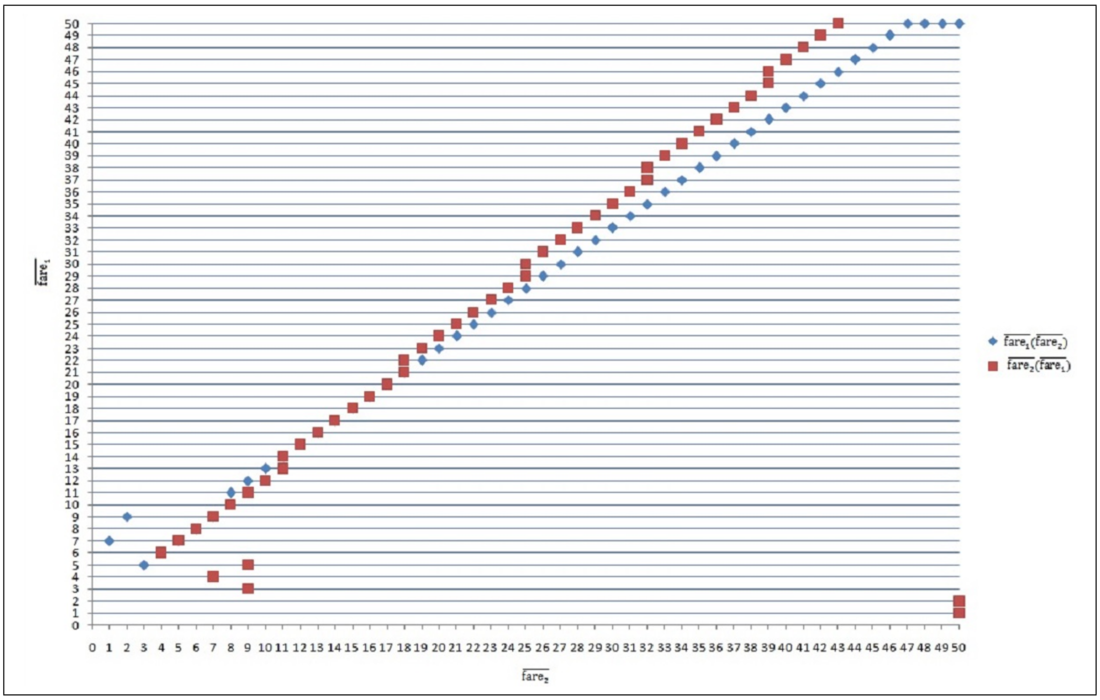


Figure 4.3. Response Functions of the Airlines.

The profits for the airlines at the equilibrium points are given in Table 4.1. The last row written in bold shows the non-dominated (Pareto efficient) equilibrium.

4.2. Duopoly with Realistic Time-Space Networks

This section provides solutions for a duopoly with real sized time-space networks for a given daily schedule. These airlines are competing in the domestic Turkish market. There are 104 O-D pairs in total of which 95 of them are served by the both airlines. The first airline has167 flights and 378 nonstop and one-stop itineraries operating in 19 airports; whereas the second airline has 170 flights and 235 nonstop and one-stop itineraries operating in 20 airports.

The size of a typical IFAM is 11841 variables, 9318 constraints for the first airline and 13457 variables, 11256 constraints for the second airline. It is solved on the average of 106 seconds to optimality. We solve 225 such problems to search for the equilibrium.

Table 4.1. Profits of airlines at the equilibrium points.

$\overline{fare_1^*}$	$\overline{fare_2^*}$	Π_1^*	Π_2^*
6	4	84679.56	69299.79
7	5	90737.51	75102.34
8	6	96830.12	80890.68
9	7	102951.98	86666.87
14	11	127927.45	115425.74
15	12	132127.06	121198.21
16	13	140337.81	126961.32
17	14	146564.13	132716.02
18	15	152802.71	138463.11
19	16	159052.38	144203.27
20	17	165312.17	149937.11
21	**18**	**171581.24**	**155665.13**

With these given time-space networks for the airlines, we search for the equilibrium for different scenarios. First we will assume that the airlines are similar in the competition, that is the passengers do not have a brand preference when choosing an itinerary to travel. Then, we will add a brand utility for the first airline, where it is assumed that first airline has an advantage from the viewpoint of the passengers, they will have a tendency to choose the first airline's itineraries over the second airline's itineraries. Both of the assumptions will be solved for two different scenarios: the airlines have the same fleet mix and the airlines have different fleet mix. All of these scenarios are solved for 15 instances. The fares of the competitive itineraries are increased by 25% in every loop, and every fare vector is numbered from 1 to 15, with 1 representing the base fares. The base fares are the lowest fares for the itineraries, the average base fare for the first airline is 126.47 TL and for the second airline is 100.67 TL.

The operating costs of the aircrafts are also the part of the objective function. Depending on the capacity of the aircrafts the costs per hour change. The costs of the aircraft types used in the analysis are given in Table 4.2.

Table 4.2. Operating costs of the aircraft types.

Fleet type capacity	210	120	110	30
Operating Cost (TL/h)	2300	1800	1700	1400

4.2.1. Similar Airlines in the Competition

It is assumed that both airlines are indifferent for a passenger. Utility of an itinerary just depends on the fare, travel time, if the itinerary is nonstop or one-stop, and if the itinerary departs in the morning or not. We have two different scenarios tried for this case depending on the fleet mix of the airlines.

4.2.1.1. Identical Fleet Types. Both of the airlines have the identical fleet types. We assumed that there are enough number of aircrafts for all types of the fleets to avoid infeasibility. There are three types of fleets with the capacities of 110, 210, and 120 passengers, respectively.

The response functions for the fares of the airlines are given in Figure 4.4.

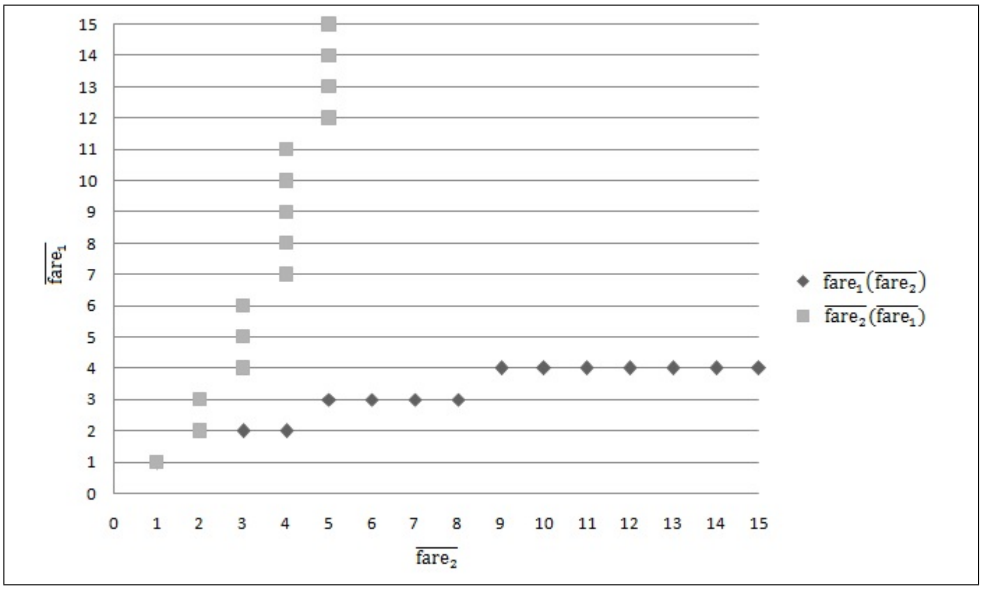


Figure 4.4. Response Functions of Similar Airlines with Identical Fleets.

The diamonds show the responses of the first airline response as a function of the fare vector of the second airline. The squares show the responses of the second airline response as a function of the fare vector of the first airline. As it is seen in Figure 4.4 there are two equilibrium points at (1, 1) and (2, 2). The profits at the equilibrium points are given in Table 4.3. The row written in bold shows the non-dominated (Pareto efficient) equilibrium. The first airline has more profit than the second airline in both of the equilibrium points. The reason for this may be that the first airline offers more itineraries than the second airline, hence captures more passengers. Since the aircrafts in the fleet mix are fairly large, they can accommodate all of the passengers demanding the itineraries when the fares are small and demand is large.

Table 4.3. Profits of similar airlines at the equilibrium points with identical fleets.

$\overline{fare_1^*}$	$\overline{fare_2^*}$	Π_1^*	Π_2^*
1	1	800399.2	763696.3
2	**2**	**959099.6**	**923498.5**

The average number of empty seats on the fleeted schedule is calculated, for every fare vector of an airline given the competitive airline's fare vector. Figures 4.5 and 4.6 shows the graphs of the average number of empty seats for a given fare vector of a competitive airline.

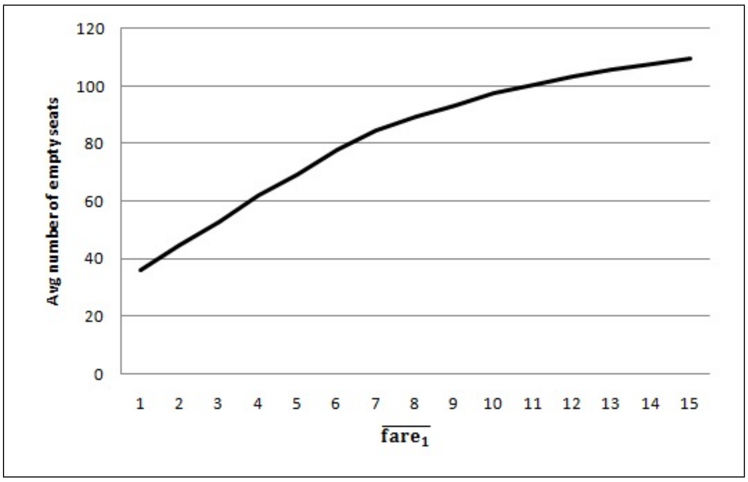


Figure 4.5. Average Number of Empty Seats for the First Airline.

The number of empty seats are increasing with the increasing fares for the first airline because the demands are decreasing and the airline cannot decrease the capacity as it has relatively large fleets as seen on Figure 4.5. As all of the fleet types have large number of seats, the airline cannot assign a smaller aircraft for a empty itinerary.

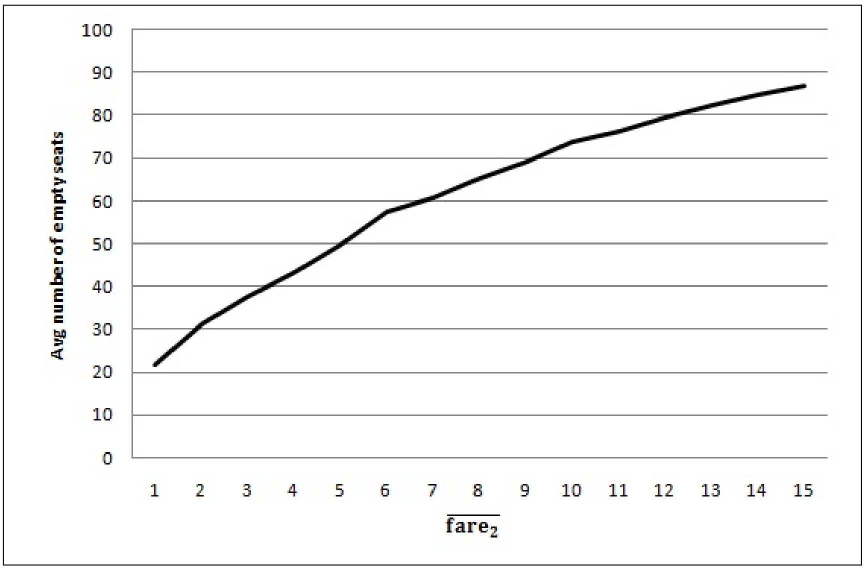


Figure 4.6. Average Number of Empty Seats for the Second Airline.

The second airline has the similar fleet types as the first airline, therefore the average number of empty seats are increasing with increasing fares, since the demands are lost shown in Figure 4.6. Hence, the same reason applies here too, assigned fleets are too large for the demand, and itineraries are flying with large number of empty seats.

4.2.1.2. Different Fleet Types. We now allow the airlines to have different fleet types. We assumed that there are enough number of aircrafts for all types of the fleets to avoid infeasibility. There are three types of fleets with the capacities of 110, 210, and 120 for the first airline; whereas the second airline has three types of fleets with capacities 120, 210, and 30 passengers. The response functions for the fares of the airlines are given in Figure 4.7.

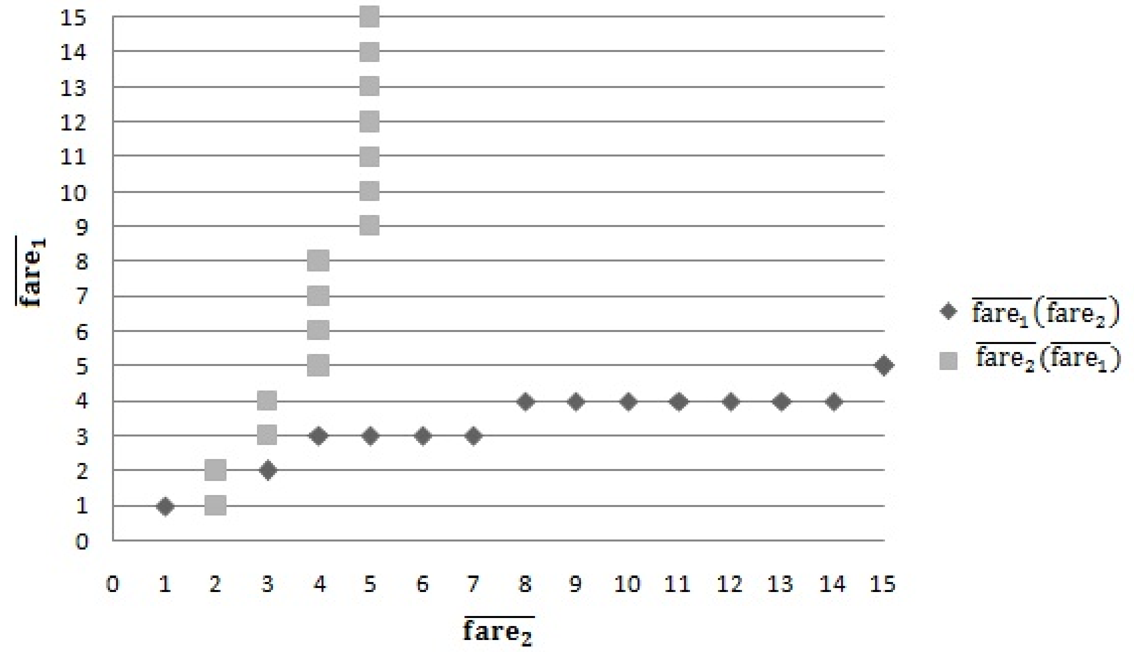


Figure 4.7. Response Functions of Similar Airlines with Different Fleets.

The diamonds show the responses of the first airline response as a function of the fare vector of the second airline. The squares show the responses of the second airline response as a function of the fare vector of the first airline. As it is seen in Figure 4.7 there is only one equilibrium point at fare level (2, 2). The profit at the equilibrium point is given in Table 4.4.

Table 4.4. Profits of similar airlines at the equilibrium points with different fleets.

$\overline{fare_1^*}$	$\overline{fare_2^*}$	Π_1^*	Π_2^*
2	**2**	**959099.6**	**860104.4**

The first airline has more profit than the second airline in the equilibrium. Since the first airline offers more itineraries than the second airline, there is more demand for the first airline. Also, the capacities of the aircrafts are larger for the first airline, that is more passengers are utilized, without spilling them.

When we compare these profits of the airlines with the scenario involving same fleet types, we observe that the first airline has the same profit as before since it has the same fleet mix and its problem is not affected since the equilibrium fares for both airlines are at the same level too. However, the second airline has lower profit than the previous case. The second airline chooses to spill the passengers rather than accommodating its demand. Therefore, while reducing its operating cost, it also loses customers by assigning the small aircrafts.

The average number of empty seats on the fleeted schedule is calculated, for every fare vector of an airline given the competitive airline's fare vector shown in figures 4.11 and 4.12 shows the graphs of the average number of empty seats for a given fare vector of a competitive airline.

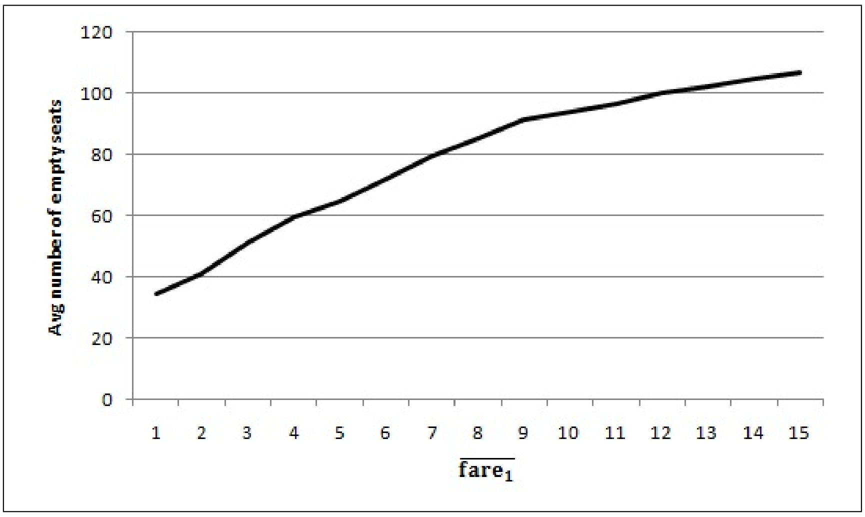


Figure 4.8. Average Number of Empty Seats for the First Airline.

The number of empty seats are increasing with the increasing fares for the first airline because the demands are decreasing as seen on Figure 4.11. As all of the fleet types are larger with respect to the number of seats, the airline cannot assign a smaller aircraft for an itinerary with smaller demand.

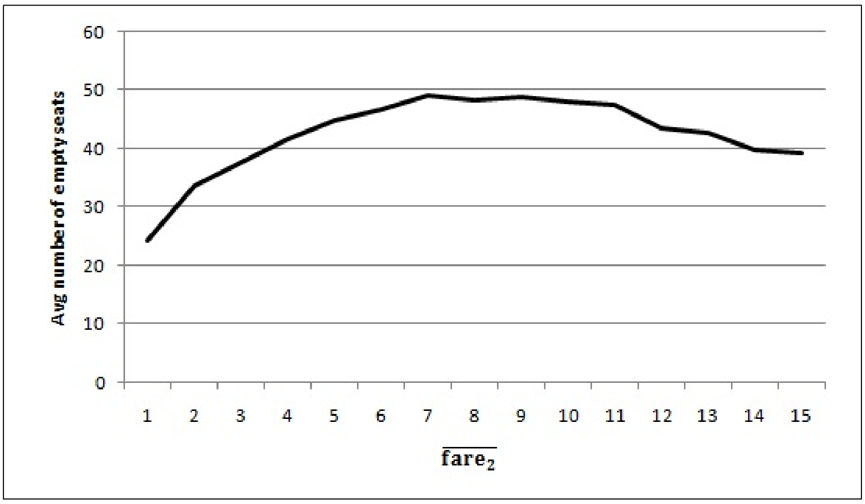


Figure 4.9. Average Number of Empty Seats for the Second Airline.

The second airline has a concave behavior for the average number of empty seats. The second airline has the opportunity to assign smaller aircrafts when the demands of its itineraries are low, then the average number of empty seats are first increasing then decreasing with increasing fares.

4.2.2. Duopoly with a Market Leader Airline

We assume that the first airline has more advantage than the second airline, because it is the market leader. Therefore, the passengers determine their itineraries, also according to the brand, where first airline itineraries have larger utility. Then we added a brand utility to the first airline itineraries, which is 10% of the absolute value of the original utility. The same scenarios are tested for this case as well, where the airlines have the same fleet and they have different fleets.

4.2.2.1. Identical Fleet Types. Both of the airlines have the identical fleet types. We assumed that there are enough number of aircrafts for all types of the fleets to avoid infeasibility. There are three types of fleets with the capacities of 110, 210, and 120 passengers, respectively. The response functions for the fares of the airlines are given in Figure 4.10.

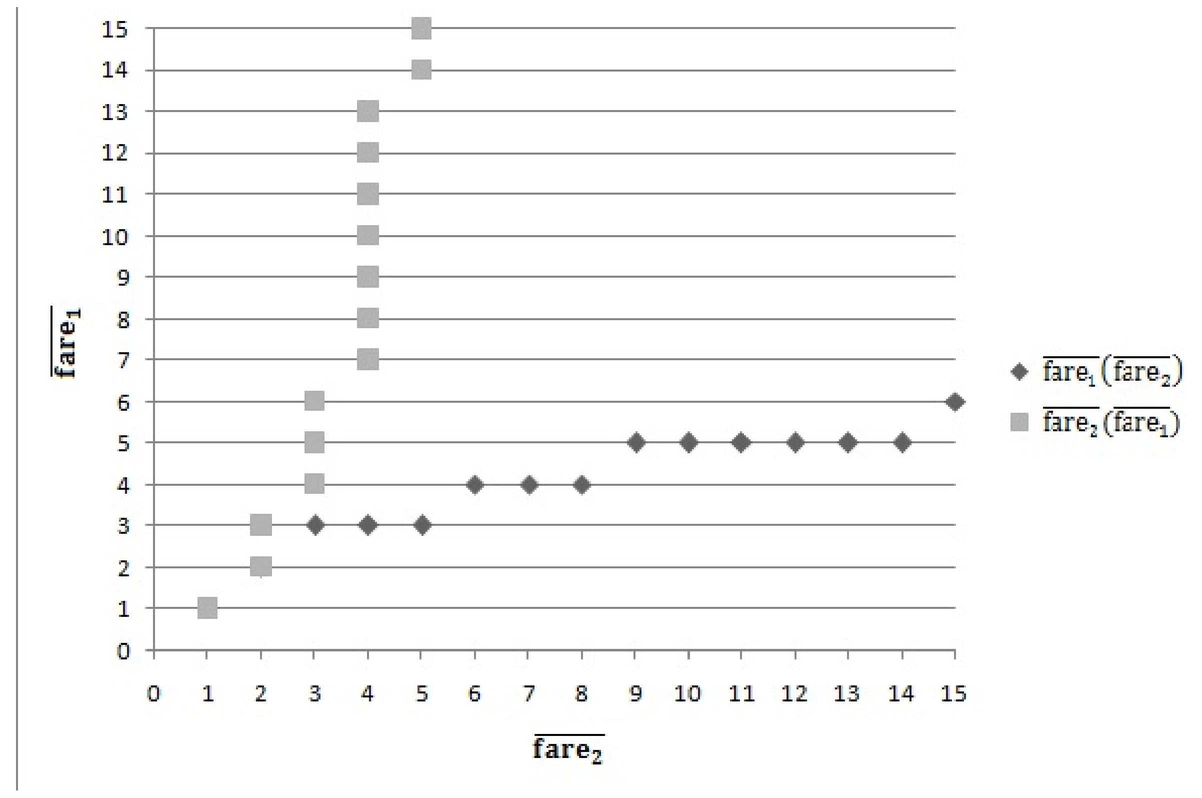


Figure 4.10. Response Functions of with the Market Leader Airline with Identical Fleets.

The diamonds show the responses of the first airline response as a function of the fare vector of the second airline. The squares show the responses of the second airline response as a function of the fare vector of the first airline. As it is seen in Figure 4.13 there are two equilibrium points at fare levels (1, 1) and (2, 2). The profits at the equilibrium points are given in Table 4.5. The row written in bold shows the non-dominated (Pareto efficient) equilibrium. The first airline has obviously more

Table 4.5. Profits of with the market leader airline at the equilibrium points with identical fleets.

$\overline{fare_1^*}$	$\overline{fare_2^*}$	Π_1^*	Π_2^*
1	1	847570.4	740048.6
2	**2**	**1043940**	**893240.7**

profit than the second airline at the equilibrium points, since attracts more passengers because of its advantage. Also, when we compare the profits with the case when the airlines are similar, we can say that being advantageous results in higher profit in the first airline,however the second airline has less profit than the previous case with identical fleets. Since the market demand does not change, it is natural that higher utilities for the first airline results in higher profits.

The average number of empty seats on the fleeted schedule is calculated, for every fare vector of an airline given the competitive airline's fare vector. Figures 4.11 and 4.12 shows the graphs of the average number of empty seats for a given fare vector of a competitive airline.

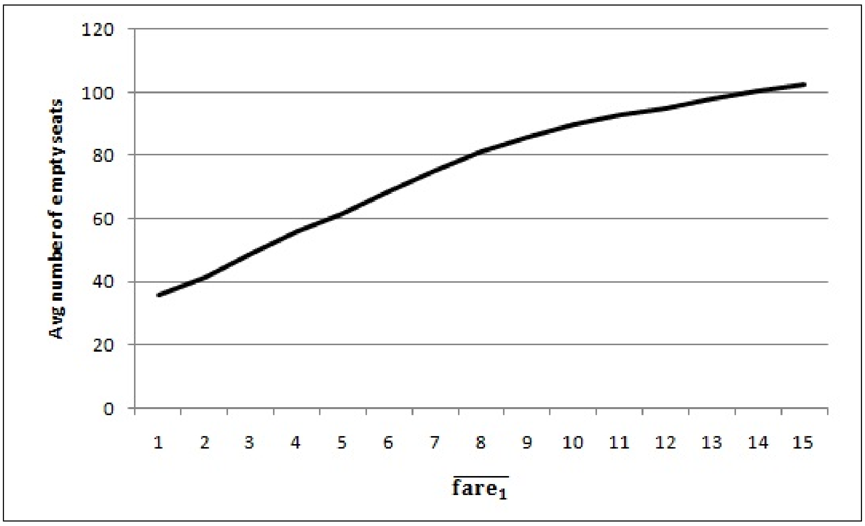


Figure 4.11. Average Number of Empty Seats for the First Airline.

The number of empty seats are increasing with the increasing fares for the first airline because the demands are decreasing and the airline cannot decrease the capacity as it has relatively large fleets as seen on Figure 4.11. As all of the fleet types have large number of seats, the airline cannot assign a smaller aircraft for a empty itinerary.

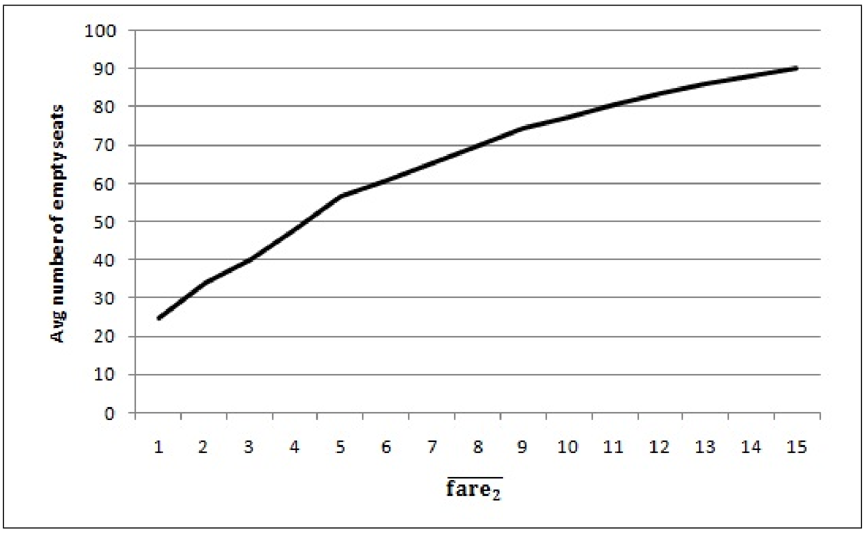


Figure 4.12. Average Number of Empty Seats for the Second Airline.

The second airline has the similar fleet types as the first airline, therefore the average number of empty seats are increasing with increasing fares, since the demands are lost as seen on Figure 4.12. Hence, the same reason applies here too, assigned fleets are too large for the demand, and itineraries are flying with large number of empty seats.

4.2.2.2. Different Fleet Types. Now the airlines have different fleet types. We assumed that there are enough number of aircrafts for all types of the fleets to avoid infeasibility. There are three types of fleets with the capacities of 110, 210, and 120 passengers for the first airline, and 120, 210, and 30 passengers for the second airline. The response functions for the fares of the airlines are given in Figure 4.13.

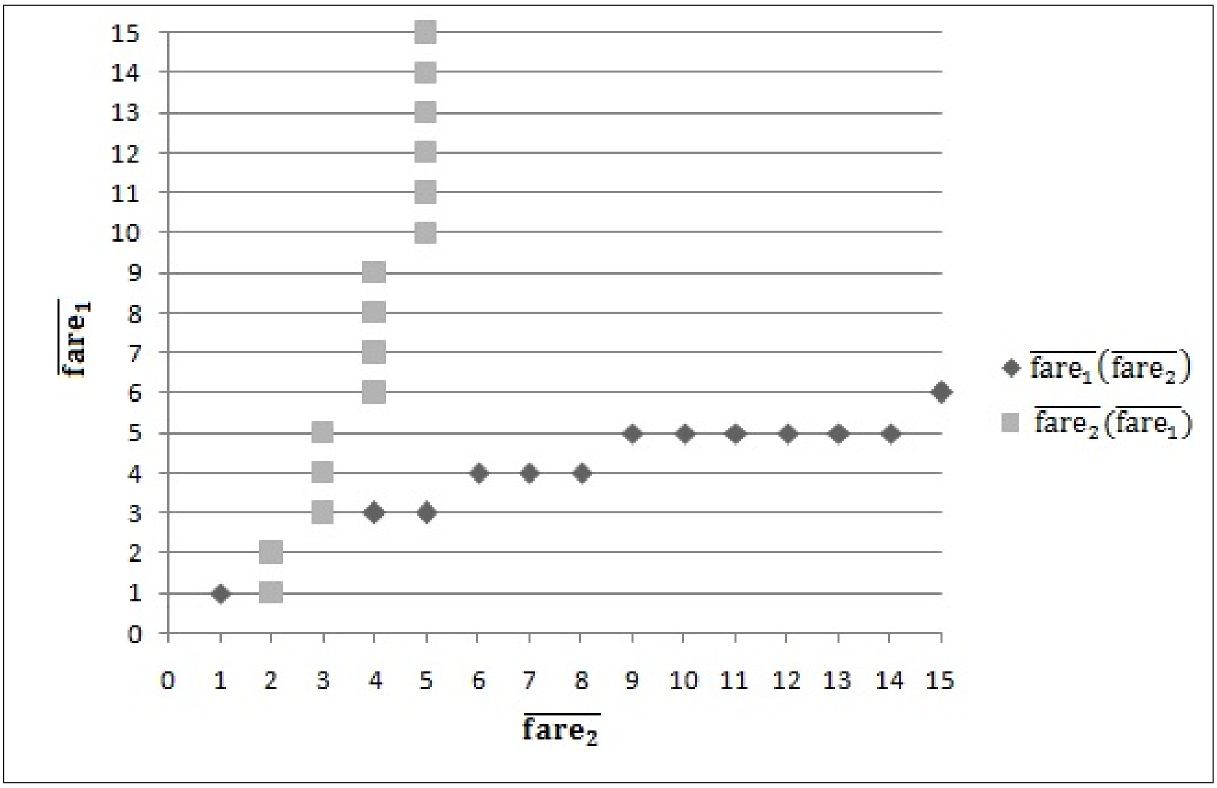


Figure 4.13. Response Functions of with the Market Leader Airline with Different Fleets.

The diamonds show the responses of the first airline response as a function of the fare vector of the second airline. The squares show the responses of the second airline response as a function of the fare vector of the first airline. As it is seen in Figure 4.13 there are two equilibrium points at fare levels (2, 2) and (3, 3). The profits at the equilibrium points are given in Table 4.6. The row written in bold shows the non-dominated (Pareto efficient) equilibrium. This case results in higher fares levels for both airlines. The first airline can increase its fares since it captures more demand, then the second airline gain the advantage of increasing its fares.

The first airline has obviously more profit than the second airline at the equilibrium points, since attracts more passengers because of its advantage. Also, when we compare the profits with the case when the airlines are similar, we can say that being advantageous results in higher profit in the first airline,however the second airline has

Table 4.6. Profits of with the market leader airline at the equilibrium points with different fleets.

$\overline{fare_1^*}$	$\overline{fare_2^*}$	Π_1^*	Π_2^*
2	2	1043936	832336.7
3	**3**	**1191255**	**950933.2**

less profit than the previous case with different fleets. Since the market demand does not change, it is natural that higher utilities for the first airline results in higher profits.

The average number of empty seats on the fleeted schedule is calculated, for every fare vector of an airline given the competitive airline's fare vector. Figures 4.14 and 4.15 shows the graphs of the average number of empty seats for a given fare vector of a competitive airline.

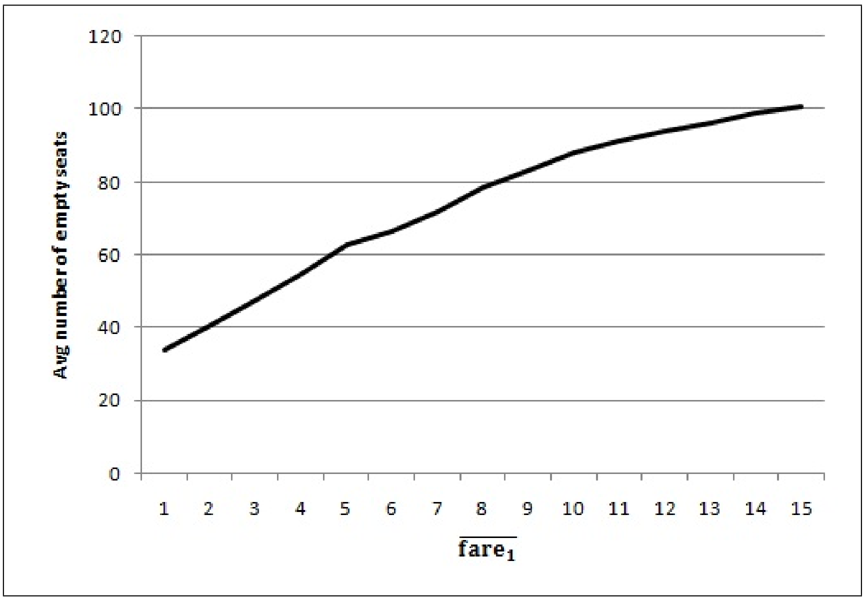


Figure 4.14. Average Number of Empty Seats for the First Airline.

The number of empty seats are increasing with the increasing fares for the first airline because the demands are decreasing and the airline cannot decrease the capacity as it has relatively large fleets as seen on Figure 4.14. As all of the fleet types have large number of seats, the airline cannot assign a smaller aircraft for a empty itinerary.

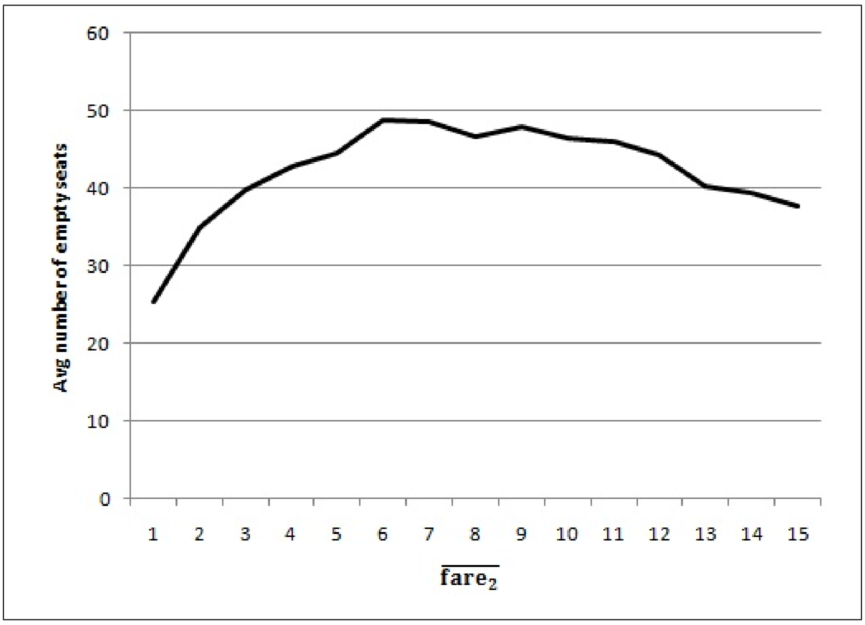


Figure 4.15. Average Number of Empty Seats for the Second Airline.

The second airline has the different fleet types as the first airline, therefore the average number of empty seats are concave according to increasing fares, since the second airline can choose to assign smaller aircrafts as seen on Figure 4.15. Therefore, with the opportunity to assign small aircrafts the second airline benefits from the higher utilizations for its aircrafts.

5. CONCLUSION

We solved itinerary based fleet assignment problems for two airlines that are competing in the same markets. They are dependent on each other, because passengers are choosing the itineraries that they wish to travel from either one of the airlines, or they do not travel at all. Therefore, the airlines determine their fares as to maximize their own profits based on the shared demand. The main result from this environment is the computation of fare vectors of the itineraries.

First of all, we assumed that the passengers are indifferent of the airlines while choosing itineraries that they want to travel. Then, we solved the problem for the identical fleet mix for the both airlines. There exist two equilibrium points and one non-dominated equilibrium. Also, we can say that when aircrafts are large, the airlines cannot assign a small aircraft as a response to decreasing demands. However, having the same types of fleets have advantages for the airlines, because the crew and maintenance requirements are satisfied without having too many people specialized on different equipment.

The problem is also solved for the airlines owning different fleets. There exist a unique equilibrium point regarding the fare vectors. In this case, the first airline has larger aircrafts and cannot assign smaller aircrafts to respond the decreasing demand. On the contrary, the second airline has the opportunity to assign smaller aircrafts when the demands are decreasing, hence the aircrafts are used with high utilizations. Although this seems as an advantage, different fleet types require different crew to operate. The costs associated with crew and maintenance are not considered in this thesis.

Secondly, we add a brand utility to the first airline so that its demands are higher than usual even if its fares are high. We also test this setting with identical fleet mix for both of the airlines. The average number of empty seats are increasing with increasing fares for both airlines because of the reason explained before. Also, we observe that the profit of the first airline is larger, when compared to the case with indifferent airlines, and the profit of the second airline is lower. This is an obvious result since the utilities of the passengers are higher when traveling with the itineraries of the first airline. Therefore, they choose to travel with the first airline even with the itineraries with high fares. For the market leader airline, the competition helps as the second airline drives fares up and this increases the profits.

When we solve the problem for the case where the first airline has more utility for its itineraries than the second airline with different fleet types, we see that the first airline again has more profit than the second airline. Also, the average number of empty seats are increasing for the first airline since it has fleet types with larger capacities. On the other hand, the second airline manages to increase the aircraft utilization by responding with smaller aircrafts to decreasing demands.

For a future work, we can add the fleet type requirements to the model in order to find a complete operational solutions from fleet assignment to crew pairing with maintenance requirements. All of these problems are too large individually, however obtaining optimal solutions without the dependency of the individual stages would improve the solutions remarkably.

Another research area for the future may be adding different cabin classes for the airlines, and adding passenger demand requesting business or economy classes. Also, as a more realistic case different fare classes can be added to the model, so that revenue management is also included. In reality, airlines try to attract customers by applying campaigns and allocating very cheap seats for its passengers so that the demands are increased. Although it captures the reality, our model gives an average behavior for the airlines without having complexity on the fare vectors.

APPENDIX A: PROFITS OF THE FLEET ASSIGNMENT

Table A.1. Profit of the first airline when the airlines are similar and fleet types are identical.

		$\overline{fare_2}$				
		1	2	3	4	5
$\overline{fare_1}$	1	800399.2339	953954.2452	1051353.256	1119186.565	1167099.226
	2	748722.4926	959099.5842	1105713.884	1209293.984	1280534.082
	3	661263.8567	886448.1807	1064047.161	1195033.043	1295162.509
	4	568858.9141	794162.7433	979641.3595	1129689.864	1243808.165
	5	480674.969	698864.1372	884756.8193	1039068.317	1166429.298
	6	401029.2266	607573.1296	788342.2135	942251.0406	1070733.471
	7	330777.3311	525512.126	697805.9697	847278.5361	975514.8509
	8	269265.9358	451610.1073	615070.3648	758775.9499	882745.9009
	9	215427.1467	385955.4316	540511.7074	676740.7971	797101.8332
	10	168206.5086	327768.4199	473368.6933	603642.5923	717501.5781
	11	126650.7261	276137.5843	413298.9027	536727.1994	646068.1421
	12	89932.42507	230213.1725	359512.2188	476419.9166	580671.9894
	13	57347.87746	189237.7212	311260.4058	422028.3703	521205.5707
	14	28304.32109	152551.6603	267865.2222	372893.5911	467255.1439
	15	2304.420996	119587.7371	228726.3514	328412.6735	418237.5324

Table A.1. Profit of the first airline when the airlines are similar and fleet types are identical (cont.).

		$\overline{fare_2}$				
		6	7	8	9	10
$\overline{fare_1}$	1	1200257.278	1224412.057	1242548.331	1257040.371	1268519.748
	2	1333348.342	1374519.389	1405307.993	1428906.821	1448144.084
	3	1370391.393	1428168.907	1470589.177	1503339.969	1529321.415
	4	1333207.439	1406503.28	1461772.197	1507353.717	1544477.522
	5	1266127.999	1343743.501	1408732.554	1461929.436	1505069.317
	6	1178783.615	1267294.438	1337484.919	1393708.407	1441409.986
	7	1082357.732	1173246.252	1249451.007	1312774.59	1364524.877
	8	989242.1868	1079425.818	1155550.416	1220796.367	1276418.076
	9	900371.3121	988669.0389	1066083.042	1130434.269	1184983.979
	10	817975.8751	904103.6695	978102.0289	1042849.792	1099024.514
	11	741430.8245	825466.7607	897879.442	960176.0332	1013818.324
	12	672513.1399	752314.9678	822766.4116	884125.2082	936380.5833
	13	609113.8658	686282.6808	753160.8342	812379.8393	864063.3804
	14	551179.393	625282.705	690198.458	746975.7863	796271.4856
	15	498360.5923	569307.6487	631816.4634	686720.8241	734670.2085

Table A.1. Profit of the first airline when the airlines are similar and fleet types are identical (cont.).

		$\overline{fare_2}$				
		11	12	13	14	15
$\overline{fare_1}$	1	1277835.159	1285396.609	1291602.774	1296762.472	1301069.96
	2	1463147.267	1474910.649	1484440.96	1492407.042	1499016.314
	3	1550720.94	1567961.969	1582167.155	1594093.35	1604192.525
	4	1574797.373	1599513.427	1619717.635	1636055.121	1649855.39
	5	1541207.019	1571486.468	1596635.211	1618001.537	1636160.716
	6	1481559.059	1515605.832	1543284.82	1567350.259	1587668.884
	7	1407627.295	1443484.323	1473774.55	1499632.894	1522703.739
	8	1322356.148	1361575.791	1394451.881	1422132.946	1446196.619
	9	1232749.142	1273707.106	1309361.891	1339055.15	1364673.945
	10	1146010.717	1186151.996	1221224.213	1251859.924	1278782.025
	11	1061487.144	1102902.11	1138511.102	1168686.26	1194856.138
	12	981917.7801	1021527.524	1056708.368	1087682.545	1114765.417
	13	908973.746	947442.7316	981229.3611	1010897.88	1036686.606
	14	839943.128	878175.8277	911493.3994	940193.8306	965904.7093
	15	775979.3461	813000.2115	845509.3978	874105.2573	899310.7718

Table A.2. Profit of the second airline when the airlines are similar and fleet types are identical.

		$\overline{fare_1}$				
		1	2	3	4	5
$\overline{fare_2}$	1	763696.3023	904889.1126	997734.8098	1060408.135	1102016.42
	2	748199.3918	923498.5197	1051640.61	1143359.883	1207661.299
	3	707986.7335	896931.7907	1044888.93	1157358.801	1241350.874
	4	666848.6432	841125.1748	1003477.612	1130235.13	1226575.753
	5	652418.9328	778937.4091	943694.7794	1077117.593	1183177.373
	6	643587.4134	739358.475	880304.9359	1016608.937	1125187.694
	7	619708.7815	680034.7007	816796.8421	951553.3088	1063094.527
	8	616888.8364	645301.4074	752037.9515	886763.3787	998186.1677
	9	552810.7571	617524.8538	712872.0767	823366.2166	933435.3064
	10	542166.4787	598615.3684	646899.3829	763788.1993	870815.7898
	11	507528.3287	553668.2662	606757.8684	709387.5003	812114.4648
	12	515885.4149	585917.5614	561596.2834	662536.3417	756109.684
	13	615113.0634	516154.9099	550628.2588	616930.1951	703658.7842
	14	535401.5836	570884.5274	535117.8885	592210.8621	654288.6359
	15	689122.4677	505807.9499	497411.8143	555026.9904	608265.1592

Table A.2. Profit of the second airline when the airlines are similar and fleet types are identical (cont.).

		$\overline{fare_1}$				
		6	7	8	9	10
$\overline{fare_2}$	1	1131944.008	1152663.785	1167560.296	1179242.227	1188709.452
	2	1254581.344	1290535.809	1317875.601	1339168.639	1355685.179
	3	1304387.142	1351657.214	1386999.145	1414815.852	1437474.754
	4	1302285.273	1361407.019	1408030.169	1446190.133	1475468.398
	5	1267358.332	1334625.393	1387920.911	1430940.543	1466048.032
	6	1213900.495	1285753.667	1343755.301	1391215.154	1430331.641
	7	1154709.061	1229385.736	1289924.344	1339541.589	1380692.927
	8	1089841.263	1165873.05	1229093.302	1280664.895	1323970.937
	9	1025027.789	1101122.056	1164907.737	1218146.711	1262939.639
	10	961021.4757	1036676.684	1099900.943	1153386.626	1198548.567
	11	900041.6333	974141.4364	1036630.517	1089334.289	1134391.487
	12	842173.8473	914787.5648	976228.162	1028386.874	1072789.712
	13	787040.8399	857809.9122	918055.5642	969412.9765	1013175.141
	14	735194.1948	804116.1337	862821.8234	912886.0721	955786.5088
	15	686498.5271	753317.6541	810427.8656	859320.272	901273.2362

Table A.2. Profit of the second airline when the airlines are similar and fleet types are identical (cont.).

		$\overline{fare_1}$				
		11	12	13	14	15
$\overline{fare_2}$	1	1196220.509	1201990.706	1206687.568	1210556.276	1213620.5
	2	1368712.874	1379407.493	1387989.29	1394917.566	1400705.038
	3	1456263.008	1471403.105	1483917.922	1494457.606	1503286.981
	4	1498299.758	1517423.401	1533069.048	1545809.285	1556560.306
	5	1494938.724	1518778.657	1538388.736	1555222.441	1569549.301
	6	1462721.754	1489879.63	1512368.229	1531669.765	1548182.441
	7	1414803.204	1443869.95	1468431.909	1489311.999	1507211.66
	8	1360388.001	1391429.815	1417739.739	1439589.512	1458402.951
	9	1300225.276	1331639.35	1358423.02	1381709.337	1401392.292
	10	1236666.201	1269160.015	1296955.581	1320320.808	1340643.794
	11	1172742.961	1205501.119	1233585.356	1257592.285	1278533.375
	12	1110219.261	1142729.787	1170702.588	1194867.053	1215800.728
	13	1050638.226	1082839.182	1110370.318	1134129.201	1155001.921
	14	992617.8378	1024342.435	1051762.534	1075539.48	1096247.171
	15	937370.6308	968525.2966	995470.195	1018870.724	1039277.034

Table A.3. Profit of the first airline when the airlines are similar and fleet types are different.

		$\overline{fare_2}$				
		1	2	3	4	5
$\overline{fare_1}$	1	800399.2339	953954.2452	1051353.256	1119186.565	1167099.226
	2	748722.4926	959099.5842	1105713.884	1209293.984	1280534.082
	3	661263.8567	886448.1807	1064047.161	1195033.043	1295162.509
	4	568858.9141	794162.7433	979641.3595	1129689.864	1243808.165
	5	480674.969	698864.1372	884756.8193	1039068.317	1166429.298
	6	401029.2266	607573.1296	788342.2135	942251.0406	1070733.471
	7	330777.3311	525512.126	697805.9697	847278.5361	975514.8509
	8	269265.9358	451610.1073	615070.3648	758775.9499	882745.9009
	9	215427.1467	385955.4316	540511.7074	676740.7971	797101.8332
	10	168206.5086	327768.4199	473368.6933	603642.5923	717501.5781
	11	126650.7261	276137.5843	413298.9027	536727.1994	646068.1421
	12	89932.42507	230213.1725	359512.2188	476419.9166	580671.9894
	13	57347.87746	189237.7212	311260.4058	422028.3703	521205.5707
	14	28304.32109	152551.6603	267865.2222	372893.5911	467255.1439
	15	2304.420996	119587.7371	228726.3514	328412.6735	418237.5324

Table A.3. Profit of the first airline when the airlines are similar and fleet types are different (cont.).

		$\overline{fare_2}$				
		6	7	8	9	10
$\overline{fare_1}$	1	1200257.278	1224412.057	1242548.331	1257040.371	1268519.748
	2	1333348.342	1374519.389	1405307.993	1428906.821	1448144.084
	3	1370391.393	1428168.907	1470589.177	1503339.969	1529321.415
	4	1333207.439	1406503.28	1461772.197	1507353.717	1544477.522
	5	1266127.999	1343743.501	1408732.554	1461929.436	1505069.317
	6	1178783.615	1267294.438	1337484.919	1393708.407	1441409.986
	7	1082357.732	1173246.252	1249451.007	1312774.59	1364524.877
	8	989242.1868	1079425.818	1155550.416	1220796.367	1276418.076
	9	900371.3121	988669.0389	1066083.042	1130434.269	1184983.979
	10	817975.8751	904103.6695	978102.0289	1042849.792	1099024.514
	11	741430.8245	825466.7607	897879.442	960176.0332	1013818.324
	12	672513.1399	752314.9678	822766.4116	884125.2082	936380.5833
	13	609113.8658	686282.6808	753160.8342	812379.8393	864063.3804
	14	551179.393	625282.705	690198.458	746975.7863	796271.4856
	15	498360.5923	569307.6487	631816.4634	686720.8241	734670.2085

Table A.3. Profit of the first airline when the airlines are similar and fleet types are different (cont.).

		$\overline{fare_2}$				
		11	12	13	14	15
$\overline{fare_1}$	1	1277835.159	1285396.609	1291602.774	1296762.472	1301069.96
	2	1463147.267	1474910.649	1484440.96	1492407.042	1499016.314
	3	1550720.94	1567961.969	1582167.155	1594093.35	1604192.525
	4	1574797.373	1599513.427	1619717.635	1636055.121	1649855.39
	5	1541207.019	1571486.468	1596635.211	1618001.537	1636160.716
	6	1481559.059	1515605.832	1543284.82	1567350.259	1587668.884
	7	1407627.295	1443484.323	1473774.55	1499632.894	1522703.739
	8	1322356.148	1361575.791	1394451.881	1422132.946	1446196.619
	9	1232749.142	1273707.106	1309361.891	1339055.15	1364673.945
	10	1146010.717	1186151.996	1221224.213	1251859.924	1278782.025
	11	1061487.144	1102902.11	1138511.102	1168686.26	1194856.138
	12	981917.7801	1021527.524	1056708.368	1087682.545	1114765.417
	13	908973.746	947442.7316	981229.3611	1010897.88	1036686.606
	14	839943.128	878175.8277	911493.3994	940193.8306	965904.7093
	15	775979.3461	813000.2115	845509.3978	874105.2573	899310.7718

Table A.4. Profit of the second airline when the airlines are similar and fleet types are different.

		$\overline{fare_1}$				
		1	2	3	4	5
$\overline{fare_2}$	1	705124.5751	831673.9634	914653.519	967977.5127	1004581.054
	2	710617.2651	860104.3848	975397.9227	1058842.401	1117843.896
	3	691097.8482	856034.0664	984297.8582	1081685.455	1157333.061
	4	664235.612	820158.2934	964619.6215	1077708.132	1161146.554
	5	661431.5993	773570.2033	923749.8847	1043457.418	1139630.458
	6	662145.0267	746328.3177	875576.3722	1000010.582	1099428.944
	7	645498.6978	698266.2682	824506.5376	950019.4978	1052167.367
	8	650266.1639	671890.6287	768139.0619	895416.6896	1000411.865
	9	589687.8547	651616.2152	738948.5415	840192.1787	943534.0602
	10	579073.5357	636325.7118	680987.3549	788303.7719	890085.0685
	11	547271.4145	592995.5829	645447.1648	742287.2355	837412.806
	12	555804.2328	625178.1207	602029.1194	699443.0738	788540.5675
	13	651614.4947	557363.2428	592236.7163	656753.5137	740616.8953
	14	575597.123	610216.9376	577437.0242	633714.2324	693509.7864
	15	726202.2017	548002.2061	539483.8586	597438.3071	649595.7563

Table A.4.Profit of the second airline when the airlines are similar and fleet types are different (cont.).

		$\overline{fare_1}$				
		6	7	8	9	10
$\overline{fare_2}$	1	1030574.153	1049029.586	1063340.524	1073790.135	1081679.928
	2	1161227.61	1193862.333	1218394.029	1236837.087	1250949.028
	3	1215673.075	1259420.709	1292800.496	1319516.297	1341180.257
	4	1228453.659	1281036.966	1323145.21	1357855.877	1384648.244
	5	1214367.488	1274510.28	1322546.396	1361428.921	1393051.923
	6	1179990.417	1245224.146	1296844.503	1339749.843	1375347.689
	7	1136161.195	1205232.428	1260146.396	1306043.661	1344029.553
	8	1084599.672	1154507.754	1212657.895	1260952.123	1301154.111
	9	1030581.742	1100862.533	1160307.858	1209932.152	1251212.794
	10	974932.9864	1046177.895	1104737.142	1154482.267	1196201.553
	11	921119.7496	991833.2614	1051322.989	1100690.58	1141594.237
	12	869052.1187	938761.1707	997419.8005	1048035.136	1090519.397
	13	819314.5693	886038.3479	943732.2827	993167.9774	1035611.117
	14	772646.8297	837816.1274	893219.9105	940936.5913	981740.473
	15	726498.745	791863.6577	846565.6062	892812.6284	932609.1794

Table A.4. Profit of the second airline when the airlines are similar and fleet types are different (cont.).

		$\overline{fare_1}$				
		11	12	13	14	15
$\overline{fare_2}$	1	1088020.713	1093256.998	1097592.415	1101148.856	1103964.059
	2	1262306.619	1271519.225	1279001.398	1285218.094	1290444.112
	3	1358594.184	1372686.854	1384348.812	1394418.456	1402825.035
	4	1405927.549	1423904.562	1438499.022	1450637.228	1460947.352
	5	1419086.238	1441086.535	1459083.041	1474299.457	1487399.508
	6	1404904.636	1429686.785	1450759.135	1468511.615	1483741.887
	7	1375060.504	1401579.754	1424066.737	1443156.466	1459466.401
	8	1334409.735	1363022.693	1387375.152	1407807.536	1425452.544
	9	1285988.094	1315551.903	1340778.897	1362356.735	1380502.767
	10	1231689.536	1261950.669	1288053.799	1309903.807	1329021.72
	11	1177091.185	1207336.176	1233875.836	1256988.792	1276584.037
	12	1125972.756	1155937.368	1181925.004	1204201.169	1223506.644
	13	1072237.43	1103275.234	1129610.304	1152644.025	1172546.097
	14	1017366.141	1048451.269	1075530.655	1098736.305	1118712.301
	15	966690.7129	996672.4442	1022848.907	1045824.281	1065755.347

Table A.5. Profit of the first airline when it is the market leader, with identical fleets.

		$\overline{fare_2}$				
		1	2	3	4	5
$\overline{fare_1}$	1	847570.4245	993546.1076	1087884.35	1154934.837	1200482.871
	2	833062.9186	1043940.266	1188128.051	1282586.226	1352425.743
	3	769120.308	1011110.719	1191261.316	1322510.232	1420584.039
	4	700618.3392	944789.8347	1146550.503	1301212.474	1418259.224
	5	630206.6409	874764.6429	1079963.389	1248964.622	1380063.677
	6	563038.8308	802623.6633	1007870.808	1179465.884	1321770.06
	7	500720.1779	731212.6223	933217.1285	1105569.67	1249403.332
	8	443916.6762	665546.9906	861396.5478	1030592.214	1175868.089
	9	392549.7678	604576.9356	793105.3506	959618.5543	1101482.41
	10	346164.0665	548643.4278	730777.302	891093.9318	1031196.243
	11	304254.4066	497601.4228	672703.795	827946.0446	964075.9668
	12	266326.3176	451034.0256	619109.7801	769316.0397	900762.0966
	13	231922.6671	408512.184	569846.8636	714630.0075	842722.397
	14	200633.2176	369624.6064	524545.1331	664068.1965	788143.6804
	15	172095.7581	333991.4428	482841.1619	617308.5135	737260.4532

Table A.5. Profit of the first airline when it is the market leader, with identical fleets (cont.).

		$\overline{fare_2}$				
		6	7	8	9	10
$\overline{fare_1}$	1	1232948.172	1256559.798	1274389.688	1288216.371	1299229.289
	2	1404247.461	1443207.69	1472184.319	1494389.889	1511621.887
	3	1490384.465	1542353.757	1582925.475	1615320.891	1641090.52
	4	1510083.217	1580939.432	1636521.807	1678026.906	1710892.83
	5	1483539.225	1565998.662	1632233.404	1684535.98	1728238.306
	6	1437194.515	1525575.227	1598737.75	1659048.167	1708414.094
	7	1371476.744	1470837.398	1550404.027	1615302.55	1667265.559
	8	1296743.881	1400131.89	1486554.859	1558208.022	1616192.911
	9	1224482.616	1326714.152	1414049.034	1488931.542	1551603.384
	10	1151148.126	1255629.335	1342610.629	1416815.054	1480682.258
	11	1082128.22	1183745.696	1272046.601	1347395.418	1410265.081
	12	1016694.371	1116504.171	1202755.264	1277326.9	1342700.568
	13	954483.3277	1052822.239	1137341.367	1210573.108	1273851.025
	14	897281.2823	992132.6576	1075661.171	1147704.668	1210250.941
	15	843286.3423	935988.7417	1016757.127	1087842.3	1149714.224

Table A.5. Profit of the first airline when it is the market leader, with identical fleets (cont.).

		$\overline{fare_2}$				
		11	12	13	14	15
$\overline{fare_1}$	1	1308123.695	1315282.684	1321235.857	1326056.803	1330092.982
	2	1525802.021	1537295.801	1546587.48	1554127.002	1560645.373
	3	1661989.085	1678615.032	1692265.941	1703892.849	1713938.865
	4	1738249.584	1759993.997	1777435.09	1792051.275	1804188.9
	5	1764655.605	1794446.767	1818950.929	1838974.222	1855363.859
	6	1749404.025	1783240.13	1812019.386	1836587.58	1857481.843
	7	1712695.66	1751063.608	1783003.24	1810379.997	1833578.054
	8	1664214.627	1703974.913	1738539.47	1768555.765	1794191.953
	9	1603993.605	1647960.113	1685382.811	1717074.775	1743512.884
	10	1535990.84	1582210.197	1622314.922	1655699.172	1684628.265
	11	1465256.295	1512948.839	1554231.463	1589836.337	1620085.72
	12	1397704.842	1444550.491	1485390.505	1521088.318	1552535.105
	13	1329840.696	1378455.83	1419768.116	1454878.632	1486121.252
	14	1264395.538	1311556.006	1353500.568	1390122.844	1422357.542
	15	1202763.844	1249084.462	1289931.179	1325450.994	1357282.796

Table A.6. Profit of the second airline when the first airline is the market leader, with identical fleets.

		$\overline{fare_1}$				
		1	2	3	4	5
$\overline{fare_2}$	1	740048.6294	880868.1065	974009.5731	1036220.881	1078968.606
	2	723032.2189	893240.6641	1019596.464	1109425.974	1173799.409
	3	702951.9313	861091.2039	1007043.42	1116187.207	1197663.063
	4	674431.2283	806883.874	960369.3496	1083230.403	1176601.675
	5	670494.7974	751311.6239	900299.1407	1026893.165	1127665.84
	6	616541.7001	703095.9381	835735.865	965258.5439	1069339.241
	7	607651.4766	647283.4682	777241.8513	899766.8779	1004759.191
	8	657706.8225	630838.8535	719127.8582	834794.7842	939877.7884
	9	512478.8831	617726.6867	697323.1373	774243.6122	875315.8908
	10	613589.0235	568789.4694	623953.6552	717913.2949	814089.2031
	11	568340.5167	555454.6382	606871.1409	662921.2485	756349.8568
	12	526088.8334	540977.758	556930.4313	624604.6543	702783.8545
	13	519560.1528	514419.2494	533088.7783	592694.9471	662459.4646
	14	656105.9193	506335.78	499864.1339	553579.2043	607000.1301
	15	630289.8629	588052.0682	535534.4027	543885.3984	568084.1652

Table A.6. Profit of the second airline when the first airline is the market leader, with identical fleets (cont.).

		$\overline{fare_1}$				
		6	7	8	9	10
$\overline{fare_2}$	1	1110185.431	1133229.945	1150380.145	1163213.51	1173488.029
	2	1220918.667	1258077.851	1287371.638	1310573.42	1329305.347
	3	1261893.478	1310835.324	1348433.955	1378166.493	1402445.712
	4	1250883.644	1310969.081	1359053.727	1398322.26	1431110.374
	5	1210091.156	1277139.651	1332143.547	1377408.52	1414759.7
	6	1154052.274	1225114.428	1283742.652	1332420.166	1373370.522
	7	1092988.155	1166241.36	1227149.9	1278861.704	1321632.974
	8	1027813.488	1101949.959	1164460.223	1217436.84	1261918.154
	9	962842.253	1036572.687	1099604.485	1153371.926	1199187.741
	10	899480.9565	972617.4261	1035149.457	1088379.096	1134439.541
	11	839897.9194	911355.6965	972687.8186	1025340.631	1070615.957
	12	782899.6304	852967.4393	913172.353	965069.3384	1009971.374
	13	729403.1128	797425.9624	856101.6437	906946.58	951127.8394
	14	678995.8818	745068.1334	802278.3153	851928.175	895058.7336
	15	631952.5638	695914.457	751423.5503	799701.8363	841801.1291

Table A.6. Profit of the second airline when the first airline is the market leader, with identical fleets (cont.).

		$\overline{fare_1}$				
		11	12	13	14	15
$\overline{fare_2}$	1	1182090.105	1189291.328	1195270.975	1200027.436	1204033.863
	2	1344488.775	1356736.246	1366989.849	1375711.301	1383144.533
	3	1422847.278	1440190.777	1454825.082	1467037.848	1477500.49
	4	1457994.365	1479563.291	1497169.907	1512991.963	1525997.688
	5	1445883.811	1472436.989	1494931.28	1513802.554	1529983.711
	6	1408102.274	1437659.093	1462989.18	1484681.242	1503399.467
	7	1358193.085	1388913.42	1415524.671	1438784.01	1459072.772
	8	1300213.697	1333191.991	1361573.124	1386551.696	1408177.339
	9	1238673.134	1272479.835	1301481.082	1327008.485	1349064.106
	10	1174196.986	1208654.426	1238371.108	1264489.856	1287227.004
	11	1110105.718	1144554.209	1174615.093	1200947.726	1224101.465
	12	1048968.216	1082872.223	1112241.495	1138373.809	1161403.081
	13	989637.5112	1023226.652	1052656.56	1078557.329	1101126.666
	14	932682.6708	965665.6143	994635.7527	1020166.751	1042744.447
	15	878618.3204	910914.5059	939334.0585	964422.1173	986639.9921

Table A.7. Profit of the first airline when it is the market leader, with different fleets.

		$\overline{fare_2}$				
		1	2	3	4	5
$\overline{fare_1}$	1	847567.4245	993530.8742	1087877.35	1154928.837	1200478.871
	2	833057.9186	1043936.266	1188121.051	1282584.226	1352417.743
	3	769117.308	1011104.719	1191255.316	1322504.232	1420582.039
	4	700614.3392	944786.8347	1146546.503	1301206.474	1418253.224
	5	630203.6409	874762.6429	1079960.389	1248958.622	1380055.677
	6	563035.8308	802620.6633	1007867.808	1179461.884	1321765.06
	7	500717.1779	731209.6223	933213.1285	1105566.67	1249399.332
	8	443913.6762	665543.9906	861393.5478	1030588.214	1175865.089
	9	392546.7678	604573.9356	793102.3506	959615.5543	1101478.41
	10	346161.0665	548640.4278	730774.302	891090.9318	1031193.243
	11	304251.4066	497598.4228	672700.795	827943.0446	964072.9668
	12	266323.3176	451031.0256	619106.7801	769313.0397	900759.0966
	13	231919.6671	408509.184	569843.8636	714627.0075	842719.397
	14	200630.2176	369621.6064	524542.1331	664065.1965	788140.6804
	15	172092.7581	333988.4428	482838.1619	617305.5135	737257.4532

Table A.7. Profit of the first airline when it is the market leader, with different fleets (cont.).

		$\overline{fare_2}$				
		6	7	8	9	10
$\overline{fare_1}$	1	1232943.172	1256554.798	1274386.74	1288211.371	1299213.584
	2	1404240.461	1443202.69	1472133.531	1494382.889	1511613.887
	3	1490382.465	1542349.757	1582917.475	1615313.891	1641083.52
	4	1510077.217	1580935.432	1636517.807	1678022.906	1710886.83
	5	1483531.225	1565991.662	1632246.467	1684530.98	1728235.306
	6	1437189.515	1525571.227	1598729.75	1659041.167	1708396.271
	7	1371472.744	1470833.398	1550400.027	1615298.55	1667261.559
	8	1296739.881	1400127.89	1486550.859	1558204.022	1616188.911
	9	1224478.616	1326711.152	1414045.034	1488927.542	1551599.384
	10	1151144.126	1255625.335	1342607.629	1416812.054	1480678.258
	11	1082124.22	1183741.696	1272042.601	1347391.418	1410262.081
	12	1016691.371	1116500.171	1202751.264	1277322.9	1342696.568
	13	954480.3277	1052819.239	1137337.367	1210569.108	1273847.025
	14	897278.2823	992129.6576	1075658.171	1147701.668	1210246.941
	15	843283.3423	935985.7417	1016754.127	1087839.3	1149711.224

Table A.7. Profit of the first airline when it is the market leader, with different fleets (cont.).

		$\overline{fare_2}$				
		11	12	13	14	15
$\overline{fare_1}$	1	1308114.355	1315282.243	1321231.857	1326057.144	1330088.982
	2	1525794.021	1537287.801	1546459.228	1554119.002	1560637.373
	3	1661916.103	1678537.407	1692260.941	1703889.849	1713933.865
	4	1738242.584	1759986.882	1777426.09	1792042.275	1804179.9
	5	1764650.605	1794440.767	1818944.929	1838968.222	1855358.859
	6	1749398.025	1783234.13	1812015.386	1836583.58	1857477.843
	7	1712689.66	1751057.608	1782995.24	1810371.997	1833570.054
	8	1664210.627	1703970.913	1738535.47	1768551.765	1794187.953
	9	1603989.605	1647956.113	1685378.811	1717070.775	1743508.884
	10	1535986.84	1582206.197	1622310.922	1655695.172	1684624.265
	11	1465253.295	1512945.839	1554227.463	1589832.337	1620081.72
	12	1397700.842	1444546.491	1485386.505	1521085.318	1552532.105
	13	1329836.696	1378451.83	1419764.116	1454874.632	1486117.252
	14	1264391.538	1311552.006	1353496.568	1390118.844	1422353.542
	15	1202759.844	1249080.462	1289927.179	1325446.994	1357278.796

Table A.8. Profit of the second airline when the first airline is the market leader, with different fleets.

		$\overline{fare_1}$				
		1	2	3	4	5
$\overline{fare_2}$	1	683611.353	809447.1556	893011.6664	946986.9871	984761.1925
	2	687567.2518	832336.6874	944741.7328	1027916.593	1085974.77
	3	683380.5378	823478.6941	950933.1824	1045809.479	1117602.722
	4	667300.2975	788680.5087	925920.4946	1035464.143	1117027.83
	5	681245.0026	748175.0224	883598.0012	997512.6398	1088825.848
	6	624804.2284	732115.9883	834048.1536	952169.0652	1047355.2
	7	628448.8596	667331.1648	789017.8599	901924.7635	998117.5008
	8	690601.8119	673927.8562	735728.752	845452.1215	945790.8764
	9	593214.6475	660893.4602	723747.6833	792689.9057	888932.7893
	10	646016.1741	607009.4029	682307.2379	745734.6021	835014.3227
	11	609003.6107	595859.5545	616857.0909	698286.5758	785142.9484
	12	627247.3473	585139.9591	595810.6076	662934.4401	738193.0051
	13	560525.6007	607369.4105	578265.351	633166.8702	700799.0909
	14	531498.8336	548285.2455	621971.4436	594696.1051	647285.0936
	15	669966.0483	578320.1618	682425.5042	585996.166	618905.1507

Table A.8. Profit of the second airline when the first airline is the market leader, with different fleets (cont.).

		$\overline{fare_1}$				
		6	7	8	9	10
$\overline{fare_2}$	1	1011301.548	1031407.725	1046690.041	1058865.507	1068473.219
	2	1129834.969	1164102.082	1190930.7	1211772.952	1228294.33
	3	1175913.863	1221477.261	1256241.683	1284633.301	1307884.842
	4	1182612.304	1235919.828	1278960.667	1314262.257	1344225.597
	5	1162797.212	1222391.265	1271731.841	1312565.539	1346606.877
	6	1125451.686	1190029.779	1242927.895	1286427.616	1323445.44
	7	1078888.974	1146702.346	1202814.482	1249576.754	1288972.541
	8	1026967.818	1095265.379	1152407.545	1201497.691	1243062.586
	9	971517.3176	1041040.362	1098794.901	1148738.929	1191602.686
	10	917154.424	986507.0223	1044810.443	1093732.875	1136611.927
	11	863465.1744	931568.553	990818.7055	1041036.386	1083461.313
	12	812482.8338	879096.8979	936644.3523	986786.8998	1030252.582
	13	765605.6674	828709.7934	884198.8708	932648.2939	975215.136
	14	718128.33	782301.4757	836465.9363	882625.1331	923545.1373
	15	672486.4738	735993.1902	790091.5	836495.8076	876611.5913

Table A.8. Profit of the second airline when the first airline is the market leader, with different fleets (cont.).

		$\overline{fare_1}$				
		11	12	14	14	15
$\overline{fare_2}$	1	1075856.989	1081881.994	1087000.866	1091372.776	1095059.136
	2	1241345.251	1251907.352	1260856.769	1268419.957	1274812.809
	3	1327387.271	1343659.747	1357078.749	1368487.524	1378269.328
	4	1368726.682	1388497.123	1405108.025	1419664.755	1431884.594
	5	1374958.912	1398791.291	1419101.223	1436617.234	1451559.702
	6	1355042.385	1381956.276	1405018.036	1425075.048	1442513.19
	7	1322846.866	1351042.289	1375360.493	1396757.72	1415360.445
	8	1278825.252	1309192.774	1335279.093	1358461.975	1378423.857
	9	1228362.563	1260044.363	1286845.462	1310999.607	1331713.456
	10	1173119.385	1205147.218	1233027.013	1257380.067	1278686.105
	11	1119695.957	1151040.47	1178708.912	1202947.414	1224723.872
	12	1067964.878	1100118.552	1128143.985	1152420.663	1173450.598
	13	1012784.388	1045462.033	1074179.538	1098989.209	1120790.412
	14	959394.3734	991264.1875	1019438.332	1044454.954	1066671.203
	15	911148.0919	941547.4322	968615.9037	992517.5667	1014306.876

REFERENCES

1. Bazargan, M., *Airline Operations and Scheduling*, Ashgate Publishing, Ltd., Surrey, 2012.

2. Barnhart, C., T. S. Kniker and M. Lohatepanont, “Itinerary-Based Airline Fleet Assignment”, *Transportation Science*, Vol. 36, No. 2, pp. 199–217, 2002.

3. Sherali, H. D., E. K. Bish and X. Zhu, “Airline Fleet Assignment Concepts, Models, and Algorithms”, *European Journal of Operational Research*, Vol. 172, No. 1, pp. 1–30, 2006.

4. Barnhart, C., P. Belobaba and A. R. Odoni, “Applications of Operations Research in the Air Transport Industry”, *Transportation science*, Vol. 37, No. 4, pp. 368–391, 2003.

5. Belobaba, P., A. Odoni and C. Barnhart, *The Global Airline Industry*, Vol. 23, John Wiley & Sons, West Sussex, 2009.

6. Lohatepanont, M. and C. Barnhart, “Airline Schedule Planning: Integrated Models and Algorithms for Schedule Design and Fleet Assignment”, *Transportation Science*, Vol. 38, No. 1, pp. 19–32, 2004.

7. Clarke, L. W., C. A. Hane, E. L. Johnson and G. L. Nemhauser, “Maintenance and Crew Considerations in Fleet Assignment”, *Transportation Science*, Vol. 30, No. 3, pp. 249–260, 1996.

8. Gopalan, R. and K. T. Talluri, “The Aircraft Maintenance Routing Problem”, *Operations Research*, Vol. 46, No. 2, pp. 260–271, 1998.

9. Klabjan, D., E. L. Johnson, G. L. Nemhauser, E. Gelman and S. Ramaswamy, “Solving Large Airline Crew Scheduling Problems: Random Pairing Generation

and Strong Branching", *Computational Optimization and Applications*, Vol. 20, No. 1, pp. 73–91, 2001.

10. Gamache, M., F. Soumis, G. Marquis and J. Desrosiers, "A Column Generation Approach for Large-Scale Aircrew Rostering Problems", *Operations research*, Vol. 47, No. 2, pp. 247–263, 1999.

11. McGill, J. I. and G. J. Van Ryzin, "Revenue Management: Research Overview and Prospects", *Transportation science*, Vol. 33, No. 2, pp. 233–256, 1999.

12. Talluri, K. and G. Van Ryzin, "An Analysis of Bid-Price Controls for Network Revenue Management", *Management Science*, Vol. 44, No. 11-part-1, pp. 1577–1593, 1998.

13. Belobaba, P. P. and A. Farkas, "Yield Management Impacts on Airline Spill Estimation", *Transportation Science*, Vol. 33, No. 2, pp. 217–232, 1999.

14. Bish, D. R., E. K. Bish, L. Liao and J. Liu, "Revenue Management with Aircraft Reassignment Flexibility", *Naval Research Logistics (NRL)*, Vol. 58, No. 2, pp. 136–152, 2011.

15. Abara, J., "Applying Integer Linear Programming to the Fleet Assignment Problem", *Interfaces*, Vol. 19, No. 4, pp. 20–28, 1989.

16. Berge, M. E. and C. A. Hopperstad, "Demand Driven Dispatch: A Method for Dynamic Aircraft Capacity Assignment, Models and Algorithms", *Operations research*, Vol. 41, No. 1, pp. 153–168, 1993.

17. Hane, C. A., C. Barnhart, E. L. Johnson, R. E. Marsten, G. L. Nemhauser and G. Sigismondi, "The Fleet Assignment Problem: Solving a Large-Scale Integer Program", *Mathematical Programming*, Vol. 70, No. 1-3, pp. 211–232, 1995.

18. Rushmeier, R. A. and S. A. Kontogiorgis, "Advances in the Optimization of Airline

Fleet Assignment", *Transportation science*, Vol. 31, No. 2, pp. 159–169, 1997.

19. Sandhu, R. and D. Klabjan, "Integrated Airline Fleeting and Crew-Pairing Decisions", *Operations Research*, Vol. 55, No. 3, pp. 439–456, 2007.

20. Kniker, T. S., *Itinerary-Based Airline Fleet Assignment*, Ph.D. Thesis, Massachusetts Institute of Technology, 1998.

21. Farkas, A., *The Influence of Network Effects and Yield Management on Airline Fleet Assignment Decisions*, Ph.D. Thesis, Massachusetts Institute of Technology, 1996.

22. Wang, D. D., D. Klabjan and S. Shebalov, "Attractiveness-Based Airline Network Models with Embedded Spill and Recapture", *Journal of Airline and Airport Management*, Vol. 4, No. 1, 2014.

23. Atasoy, B., M. Salani and M. Bierlaire, "An Integrated Airline Scheduling, Fleeting, and Pricing Model for a Monopolized Market", *Computer-Aided Civil and Infrastructure Engineering*, Vol. 29, No. 2, pp. 76–90, 2014.

24. Garrow, L. A., *Discrete Choice Modelling and Air Travel Demand: Theory and Applications*, Ashgate Publishing, Ltd., Surrey, 2012.

25. Coldren, G. M., F. S. Koppelman, K. Kasturirangan and A. Mukherjee, "Modeling Aggregate Air-Travel Itinerary Shares: Logit Model Development at a Major US Airline", *Journal of Air Transport Management*, Vol. 9, No. 6, pp. 361–369, 2003.

26. Atasoy, B. and M. Bierlaire, *An Air Itinerary Choice Model Based on a Mixed RP/SP Dataset*, Tech. rep., 2012.

27. Borenstein, S., "The Evolution of US Airline Competition", *The Journal of Economic Perspectives*, pp. 45–73, 1992.

28. Levine, M. E., "Airline Competition in Deregulated Markets: Theory, Firm Strat-

egy, and Public Policy", *Yale J. on Reg.*, Vol. 4, p. 393, 1986.

29. Kanafani, A. and A. A. Ghobrial, "Airline Hubbing—Some Implications for Airport Economics", *Transportation Research Part A: General*, Vol. 19, No. 1, pp. 15–27, 1985.
30. Hansen, M., "Airline Competition in a Hub-Dominated Environment: An Application of Noncooperative Game Theory", *Transportation Research Part B: Methodological*, Vol. 24, No. 1, pp. 27–43, 1990.
31. Adler, N., "Competition in a Deregulated Air Transportation Market", *European Journal of Operational Research*, Vol. 129, No. 2, pp. 337–345, 2001.
32. Takebayashi, M. and A. Kanafani, "Network Competition in Air Transportation Markets: Bi-Level Approach", *Research in Transportation Economics*, Vol. 13, pp. 101–119, 2005.
33. Adler, N., "Hub-Spoke Network Choice under Competition with an Application to Western Europe", *Transportation Science*, Vol. 39, No. 1, pp. 58–72, 2005.
34. Hong, S. and P. T. Harker, "Air Traffic Network Equilibrium: Toward Frequency, Price and Slot Priority Analysis", *Transportation Research Part B: Methodological*, Vol. 26, No. 4, pp. 307–323, 1992.
35. Vaze, V. and C. Barnhart, "Competitive Airline Scheduling under Airport Demand Management Strategies", *Massachusetts Institute of Technology*, 2010.
36. Wei, W. and M. Hansen, "Impact of Aircraft Size and Seat Availability on Airlines' Demand and Market Share in Duopoly Markets", *Transportation Research Part E: Logistics and Transportation Review*, Vol. 41, No. 4, pp. 315–327, 2005.
37. Wei, W. and M. Hansen, "Airlines' Competition in Aircraft Size and Service Frequency in Duopoly Markets", *Transportation Research Part E: Logistics and Trans-*

portation Review, Vol. 43, No. 4, pp. 409–424, 2007.

38. Hansen, M. and Y. Liu, “Airline Competition and Market Frequency: A Comparison of the S-Curve and Schedule Delay Models”, *Transportation Research Part B: Methodological*, Vol. 78, pp. 301–317, 2015.

39. Battal, Ü., H. Yılmaz and S. S. Ateş, “Türkiye’de İç Hatlarda Serbestleşme ve Geleceği”, *Kayseri VI. Havacılık Sempozyumu*, 2006.

40. Gurobi Optimization, I., “Gurobi Optimizer Reference Manual”, http://www.gurobi.com, 2015, [Accessed May 2015].