# IDLE SPEED CONTROL OF A DIESEL ENGINE

by

Bahadır Akyıldız

B.S., Mechanical Engineering, Middle East Technical University, 2004

Submitted to the Institute for Graduate Studies in
Science and Engineering in partial fulfillment of
the requirements for the degree of
Master of Science

Graduate Program in Mechanical Engineering
Boğaziçi University
2007

## ACKNOWLEDGEMENTS

I would like to thank my supervisor Assoc. Prof. İ Emre Köse for his guidance and support throughout this thesis. His willing to teach and his enthusiasm have always been stimulating factors for me.

Also I cordially thank my co-supervisor Prof. Levent Güvenç for his support throughout this work and I appreciate the opportunity to work in one of the best automotive control research centers in Turkey.

All of this work is funded by EU FP6 project AUTOCOM (INCO-16426). Their generous support is gratefully acknowledged.

All the experiments in this thesis were conducted in Measurement and Control Laboratory (IMRT), ETH Zürich. I would like to thank Prof. Lino Guzzella and other IMRT staff for their hospitality during my stay in Switzerland. I especially thank Ezio Alfieri for his support during implication of the controller.

I appreciate the support of my colleagues at İTU MEKAR Laboratory, this work would not have been so fun without them.

Finally and most importantly, I have to thank my family for simply everything and I feel like I have to thank anyone who has indirect contributions to this study.

# ABSTRACT


## IDLE SPEED CONTROL OF A DIESEL ENGINE

In this thesis, model based idle speed control of a diesel engine is studied. Since diesel engine models derived from first principles are very complicated, a mean-value diesel engine model which captures the dynamics of the engine to a reasonable accuracy is used. Different operating conditions of a diesel engine are expressed as system uncertainties in the model. Appropriate weighting functions are proposed for expressing these system uncertainties.

A robust controller is designed via $\mu$ synthesis with $D - K$ iteration. An order reduction algorithm based on Hankel singular values is performed for the implication of the controller. Two different types of idle speed control problem is investigated. It is concluded that both simulations and experiments give satisfactory results for the two problems.

# ÖZET

## DİZEL MOTOR RÖLANTİ DEVRİ KONTROLÜ

Bu tezde bir dizel motorun model tabanlı rölanti devri kontrolü calışılmıştır. Birinci prensiplerden türetilen dizel motor modellerinin komplike olmasından dolayı, motorun dinamiğini makul bir doğrulukta yansıtan bir ortalama-değer dizel motor modeli kullanılmıştır. Bir dizel motorun değişik çalışma koşulları sistem belirsizlikleri olarak ifade edilmiştir. Ağırlık filtreleri kullanlılarak bu belirsizlikler ifade edilmiştir.

$D-K$ iterasyonu ile $\mu$ sentezi sonucunda bir dayanıklı kontrolcü tasarlanmıştır. Kontrolcünün gerçeklenebilmesi için Hankel tekil değerleri tabanlı bir derece azaltma algoritması uygulanmıştır. İki farklı rölanti devri kontrolü problemi incelenmiştir. İki problem için de, hem simulasyon sonuçlarının hem de deneysel sonuçların tatmin edici olduğu sonucuna varılmıştır.

# TABLE OF CONTENTS

# LIST OF FIGURES

# LIST OF TABLES

# LIST OF SYMBOLS/ABBREVIATIONS

| | |
|---|---|
| $a$ | Parameter for the friction torque affine fit |
| $A_{cont}$ | Controller matrix |
| $A_p$ | Piston area |
| $b$ | Parameter for the friction torque affine fit |
| $B$ | Bore of the engine |
| $B_0$ | Parameter for friction mean effective pressure |
| $B_{cont}$ | Controller matrix |
| $c_m$ | Mean piston speed |
| $C_{cont}$ | Controller matrix |
| $D_{cont}$ | Controller matrix |
| $e$ | Efficiency expression |
| $e_0$ | Parameter for efficiency expression |
| $e_1$ | Parameter for efficiency expression |
| $e_2$ | Parameter for efficiency expression |
| $H_l$ | The lower heating value of fuel |
| $J_e$ | Engine inertia |
| $k_1$ | Parameter for friction mean effective pressure |
| $k_2$ | Parameter for friction mean effective pressure |
| $k_3$ | Parameter for friction mean effective pressure |
| $K$ | Controller |
| $K_{red}$ | Reduced order controller |
| $m_i$ | Injected fuel |
| $N$ | Number of cylinders |
| $p_{me}$ | Brake mean effective pressure |
| $p_{mf}$ | Fuel mean effective pressure |
| $p_{mr}$ | Friction mean effective pressure |
| $\mathbb{R}$ | The real line |
| $S$ | Stroke of the engine |
| $T_e$ | Engine torque |

| | |
|---|---|
| $T_l$ | Load torque |
| $\tilde{\mathcal{T}}_{ef}$ | Fueling to torque production term |
| $\tilde{\mathcal{T}}_{ef,nom}$ | Nominal fueling to torque production term |
| $\lvert\delta_{\tilde{\mathcal{T}}_{ef}}\rvert$ | The weighting function for time delay |
| $\tilde{T}_{er}$ | Friction torque affine fit |
| $W_{del}$ | The weighting function for time delay |
| $W_{dif}$ | The weighting function for approximate integral action |
| $W_{low}$ | The weighting function for disturbance input |
| $\mathcal{W}_e$ | Net engine work |
| $V_d$ | Engine displacement volume |
| | |
| $\gamma$ | Closed loop $\mathcal{H}_\infty$ norm |
| $\delta_1$ | Dynamic uncertainty related with torque production |
| $\Delta_1$ | Dynamic uncertainty related with time delay |
| $\eta$ | Parameter for friction mean effective pressure |
| $\mu$ | Structured singular value |
| $\Pi_d$ | Boost ratio |
| $\tau$ | Input to power-stroke delay |
| $\tau_{max}$ | Maximum input to power-stroke delay |
| $\omega_{dif}$ | Engine speed difference in $rad/s$, i.e. $\omega_r - \omega_e$ |
| $\omega_e$ | Engine speed in $rad/s$ |
| $\omega_r$ | Reference engine speed in $rad/s$ |
| | |
| EGR | Exhaust Gas Recirculation |
| LMI | Linear Matrix Inequality |
| LPV | Linear Parameter Varying |
| LVS | Linear Vector Space |
| LTI | Linear Time Invariant |
| VGT | Variable Geometry Turbine |

# 1. INTRODUCTION

As most extensively used transportation vehicles, automobiles play an important role in our daily life. Although there is a considerable research on other prime movers, internal combustion engines are far most widely used ones. There are two main types of internal combustion engines namely spark ignited or gasoline and compression ignited or diesel engines.

## 1.1. Technological Advances in Diesel Engines

Diesel engines have been the choice for commercial and heavy duty vehicles due to their better fuel economy and high torque output. But in the past, diesel engines had not been used in passenger cars very commonly because of their low speed, high noise and vibration characteristics and low power to weight ratio. However, in the last two decades, diesel engines have subjected to major technological advances such as:

- **Effective use of turbocharging in diesel engines:** Diesel engines work in lean conditions. Thus they need more air than a gasoline engine. Atmospheric pressure is generally sufficient to meet the air demand of gasoline engines whereas for diesel engines it is inadequate. Therefore, turbocharging is the solution for meeting the excessive air demand of diesel engines.

  But it is very difficult to match the air supply capacity of a turbocharger with the air demand of a diesel engine. If the turbocharger is designed for low speed conditions, at high speeds its air pumping capacity is extremely high that it will damage the engine and conversely if the turbocharger is designed for high speed conditions, at low speeds it will have no use at all.

  The first proposed solution to this problem was designing the turbocharger for low speed conditions and discharging excessive air through a waste gate valve at high speeds. But later on a much better control action was introduced. By means of changing a turbocharger's turbine blade angles it has become possible to match the air demand of a diesel engine in its full operating region. This type of turbochargers are known as variable geometry turbochargers.

- **New fuel injection techniques:** With the introduction of common rail system together with high pressure fuel pumps and injectors, better atomization of the fuel has become possible. This results in a more homogeneous injection which increases the efficiency of the combustion and low emissions.

- **Advances in electronic control units:** Indeed this has become mandatory in order to use the benefits of above mentioned advancements successfully. While turbochargers are capable of meeting the air demand of the engine it is not possible to match the air demand of the engine without a good control system. Also the effect of fuel pressure and fuel ignition timing over combustion is severe and this can be satisfied only with a very good control system which is capable of providing very precise control action.

With these technological advances, diesel engines have discarded their drawbacks and their use in automobiles has increased significantly. Main objectives of diesel engine control system can be stated as follows:

- Low fuel consumption,
- High torque output,
- Low emissions,
- Good drivability.

These objectives are highly conflicting and a good controller should perform a compromising solution. Among many important control problems(air to fuel ratio control, speed and torque control), in this study idle speed control is investigated. Indeed idle speed control is a specific version of speed control which the operating region of the engine is very limited.

## 1.2. Importance and Objectives of Idle Speed Control

It should be noted that all vehicles spend some time in idling. Start-ups and waiting in traffic lights are obvious examples for idling. But especially in the crowded cities, the idling period of a vehicle may significantly increase due to the traffic conditions which makes this problem inherently important. Particularly, the objectives of idle speed control can be stated as follows:

- Comfort,
- Low fuel consumption,
- Low emissions.

In idling an engine encounters torque loads from different components. These are generally electrical accessories such as air conditioning, electric windows, headlights etc. For comfort, an idle speed controller should be able to keep the engine speed as close to idle speed set value as possible when it encounters torque loads. On the other hand for low fuel consumption the idle speed set value should be as low as possible but the idle speed controller should prevent engine from stalling when it encounters torque loads. Also a lower idle speed set value is preferable in terms of emissions as well.

So a good idle speed controller is the one which is able to keep the engine close to a lower idle speed set value when under torque disturbances.

## 1.3. Idle Speed Control Strategies in Literature

Basically, there are two different strategies for idle speed control, these are static gain controllers (maps) and model based controllers.

### 1.3.1. Static Gain Controllers (Maps)

In this methodology, different controllers are designed for different operating conditions. Generally, a PID controller structure is preferred for this methodology. For each operating condition, a controller is designed based on test bench experiments together with an intuitive starting point for the proportional, integral and derivative term coefficients of the controller. Then, these designed coefficients are merged into a look up table which is called as a *"map"*. This methodology has certain drawbacks:

- In order to increase the performance of the controller, number of designed controllers should be increased. Since obtaining these controller coefficients rely on mostly experiments, this methodology is time consuming.
- Since these controllers are designed for a specific engine, mostly with the help of experiments, it is not possible to directly use these designed controllers for another engine. In most of the situations, previous designed controllers can only be used for starting point for new controller design processes.
- From theoretical point of view, it is extremely difficult to prove the stability and performance criteria of this *"switched"* system. Experiments are generally used for performance investigation of the controller during transients. It is obvious that this approach can give a proof neither for stability nor for performance.

Although this methodology has obvious drawbacks, it is still the solution for most of the vehicles due to its low computation power requirements.

### 1.3.2. Model Based Controllers

With the advances in electronic control units, computing abilities of the processors are increased, thus they allow more complicated controllers to be implemented. In this methodology, first a mathematical model for a diesel engine is obtained. Then, a controller is designed based on different analytic methods. The main advantages of this approach is:

- The number of experiments can be reduced significantly based on the controller generation method.
- Generally it is easy to express different engines with the same mathematical model with different parameters. Since the controller design relies on the mathematical

model of the diesel engine, it is possible to apply a controller design methodology for different engines with ease.

In [1], Kuang, Wang and Tan designed an $\mathcal{H}_\infty$ output feedback controller for a diesel engine which is modeled as a plant with an unknown time delay and some uncertain parameters. By considering the time delay, the problem is converted into feasibility of three linear matrix inequalities (LMIs). It is proven that the designed controller is robust against the deviations in time delay but the robustness against other uncertain parameters are only shown through simulations.

In [2], Memering and Meckl developed two self-tuning adaptive algorithms for a heavy-duty diesel engine. A similar diesel engine model with [1] is used. The minimum variance self-tuning regulator and pole placement self-tuning regulators are the techniques that Memering and Meckl used for adaptive control algorithms.

In [3], Outbib *et al.* used a nonlinear approach to control engine speed using injected fuel. The engine model is composed of both inertial dynamics as well as manifold dynamics. Apart from engine speed regulation, possible use of this controller for emission controls are outlined.

In [4], Karray and Conrad investigated artificial neural network based controller, fuzzy controller and neuro-fuzzy controllers.

In [5], Song and Grigoriadis applied linear parameter varying (LPV) approach to modeling of a diesel engine. The design method was formulated in terms of linear matrix inequalities (LMIs) that was solved using convex optimization algorithms. The engine speed control problem was formulated as an $\mathcal{L}_2$ gain optimization problem for a simplified LPV engine model. The model contains a speed dependent engine friction and a speed dependent time delay. The variable time delay was approximated using a first-order Pade approximation. The designed dynamic controller was scheduled based on the real-time measurements of the engine speed. The performance of the proposed speed controller was validated both by a nonlinear engine model and also by hardwarein-the-loop configuration.

In [6], Balluchi *et al.* proposed a new cycle detailed hybrid model of an internal combustion engine that captures the interactions between the discrete phenomena of torque generation and spark ignition, and the continuous evolution of the powertrain and air dynamics. Although spark ignition engines were under investigation, the modeling approach is new for idle speed control purposes and can be extended to diesel engine idle speed control problem as well.

In [7], Mohamed and Koivo proposed a genetic algorithm self tuning PID controller based on indirect estimation of the time delay and recursive least squares parameter estimation. The estimated engine parameters and time delay were used in tuning the PID controller. It was shown that the algorithm is robust with respect to variations in the engine time delay.

In [8], Roy, Malik and Hope obtained an explicit estimate of the plant parameters and time delay. The estimate of the plant model then be used in a predictive controller, which is variable according to the estimated time delay.

In [9], Jiang developed a generalized gain scheduling control mechanism based on off-line optimization techniques for a two cylinder diesel engine. A set of linearized models were obtained for the engine operating at three different speeds and a total of fifteen load conditions. By the help of experimental data analysis, it is concluded that the behavior of the engine can best be characterized by a set of fifth order difference equations with appropriate time delays. Optimal controllers with a PID structure are then designed by off-line numerical optimization using these mathematical models to minimize the integral square error of the engine speed deviation subject to a step change command. The designed controller was implemented and tested on a diesel engine.

In [10], Karaman *et al.* proposed a robust controller based on parameter space approach. The engine model used is similar to [1] and PI controller structure is chosen in order to satisfy stability and performance criteria. Simulation results based on a nonlinear engine model were given.

Also in [11], preliminary version of this thesis with a different expression for time delay was published.

## 1.4. Conclusions of Literature Survey

As stated previously, diesel engine idle speed control problem is very important because of its effects on comfort, fuel consumption and emissions. Throughout the possible idle speed control strategies, model based control approach has significant advantageous over static gain controllers and considerable research was conducted for possible model based solutions of this problem. In this thesis, a diesel engine idle speed controller will be designed based on $\mu$ synthesis with $D-K$ iteration approach. In the next section overview of this thesis can be found.

## 1.5. Outline of the Thesis

### Chapter 2: Diesel Engine Modeling

This chapter presents the mean-value modeling of the engine. Starting from nonlinear differential equations, affine fits for the torque production and friction torque of the engine are obtained. Different operating points are expressed as system uncertainties. With real engine data it is shown that derived mathematical model shows a good characterization of the diesel engine.

### Chapter 3: Controller Synthesis

In this chapter, first appropriate weighting functions are designed for expressing the system uncertainties. Also additional weighting functions are proposed for zero steady state error and reducing the conservatism related with reference input tracking. $\mu$ synthesis with $D-K$ iteration is performed which ensures robust stability and robust performance conditions. For implementation of the controller an order reduction algorithm based on Hankel singular values of the controller is performed.

**Chapter 4: Results**

The designed controllers are tested in two different types of situations. These are: torque disturbance for a reference engine speed and change in reference speed without any torque disturbance. It is observed that, the designed controllers give good results both in simulations and experiments.

**Chapter 5: Summary and Conclusions**

A brief summary of this thesis can be found in this chapter. Also possible future work is highlighted.

# 2. DIESEL ENGINE MODELING

Diesel engines have substantial difficulties in modeling because of highly transient characteristics of thermodynamic boundary conditions and combustion process. For the purpose of engine development, models relying on first order principles are used. Although these models can express the combustion process quite accurately [12], they are very complicated to use for control purposes.

For controller design, models reflecting input-output behavior of the diesel engine subsystems are used. These models are typically composed of non-linear differential equations and provide reasonable accuracy with low computational complexity. However, even though these models are less complicated compared to first principle models, for certain control problems they can be further simplified for controller design.

The diesel engine model in this thesis is mainly adapted from [13] and [14]. Starting from the proposed non-linear, high order diesel engine model in [13] and [14], and taking into account that the engine is working in a very limited operating region in idling, the engine model becomes a first order system with time-delay and system uncertainties. The advantage of this simplification is obvious in designing robust controllers. In this chapter details of above mentioned simplification in the engine model can be found.

## 2.1. Mean Value Modeling of Diesel Engine

As stated before the engine model in this thesis reflects input-output behavior of the engine rather than dealing with the combustion process explicitly. A further simplification in modeling approach is using the mean properties of all cylinders instead of using the dynamics of individual cylinders. Although this can seem as a very crude approach, for certain applications, like idle speed control, air-path control etc., it captures the dynamics of the engine reasonably.

Using Newton's $2^{nd}$ law, it is possible to express engine angular acceleration in terms of net torque as

$$T_e(t) - \tilde{T}_l(t) = J_e \dot{\omega}_e(t), \tag{2.1}$$

where $T_e(t)$ is the mean engine torque and $\tilde{T}_l(t)$ stands for all the load torque that the engine have to stand for. In idle conditions typical loadings come from air-conditioning system, electric windows, headlights etc. Also in equation (2.1) $J_e$ is the inertia of the engine and $\omega_e(t)$ is the engine speed in $rad/s$.

The engine torque can be represented as

$$T_e(t) = \tilde{T}_e(t - \tau), \tag{2.2}$$

where $\tau$ is *"Input to power-stroke delay"* and it represents the time delay arising from the discrete cylinder firing. For a four-stroke engine it is approximated as

$$\tau \approx \frac{4\pi}{N\omega_e}, \tag{2.3}$$

where $N$ is number of cylinders.

An important performance measure of an internal combustion engine is the *"Brake mean effective pressure"* $p_{me}$. It is the pressure that the piston has to undergo in a full expansion stroke in order to produce the same amount of work as the real engine does in one complete thermodynamic cycle. (Two engine revolutions for a four-stroke engine) Using the definition of $p_{me}(t)$, net engine work can $\mathcal{W}_e$ can be expressed as

$$\mathcal{W}_e(t) = p_{me}(t) \cdot A_p \cdot S \cdot N, \tag{2.4}$$

where $A_p$ is the piston area and $S$ is the stroke of the engine. On the other hand in terms of mean engine torque $\tilde{T}_e(t)$ it is possible to express $\mathcal{W}_e(t)$ as

$$\mathcal{W}_e(t) = \tilde{T}_e(t) \cdot 4\pi. \tag{2.5}$$

Using equations (2.4) and (2.5), it is possible to express $\tilde{T}_e(t)$ as

$$\tilde{T}_e(t) = \frac{p_{me}(t)V_d}{4\pi}, \tag{2.6}$$

where $V_d$ is displacement volume.

Another important concept in internal combustion engines is the *"Fuel mean effective pressure"* $p_{mf}$. It is the pressure that the piston has to undergo in a full expansion stroke in order to produce the same amount of work as the ideal engine with a 100% efficiency does in one complete thermodynamic cycle. It is given by

$$p_{mf}(t) = \frac{H_l m_i(t)}{V_d}, \tag{2.7}$$

where $H_l$ is the lower heating value of the fuel and $m_i(t)$ is the injected fuel. In [15], it was observed that an affine relation between brake mean effective pressure and fuel mean effective pressure captures the dynamic characteristics of a diesel engine quite satisfactorily as

$$p_{me}(t) = e(\omega_e)p_{mf}(t) - p_{mr}(t) \tag{2.8}$$

In equation (2.8), $e(\omega_e)$ is related with efficiency of the engine and it will be mentioned in the sequel and $p_{mr}(t)$ stands for various friction losses, thus, $p_{mr}(t)$ can be interpreted as the *"Friction mean effective pressure"*. Based on real diesel engine data, an empirical relationship for $p_{mr}(t)$ was given in [14] as

$$p_{mr}(t) = k_1 \left( [0.5 + k_2 c_m^{\eta}]\Pi_d + k_3 p_{me}(t) \right) \sqrt{\frac{B_0}{B}}. \tag{2.9}$$

Equation (2.9) relies on experiments and $k_1, k_2, k_3, \mu, B_0$ are the parameters to be found. Typical values for these parameters for passenger car four-stroke engines which are given in [14] can be found in table (2.1).

Table 2.1. Typical values for parameters in equation (2.9)

| Constant | Value |
|---|---|
| $k_1$ | $1.4 \cdot 10^5$ |
| $k_2$ | $8.6 \cdot 10^{-3}$ |
| $k_3$ | $2.15 \cdot 10^{-7}$ |
| $\eta$ | 1.8 |
| $B_0$ | 0.075 |

Also in equation (2.9), $\Pi_d$ is the boost ratio of the engine, i.e. the ratio of intake manifold pressure over ambient pressure and $B$ is the bore of the engine. The term $c_m$ in equation (2.9) is the mean piston speed and it is defined as

$$c_m := \frac{S \cdot \omega_e}{\pi}, \tag{2.10}$$

where $S$ is the stroke of the engine.

Going back to equation (2.8), it is obvious that $e(\omega_e)$ is related with the efficiency of the engine. In fact $e$ is a function of engine speed, Exhaust Gas Recirculation (EGR) rate, air to fuel ratio etc. SO the engine speed, $\omega_e$ is not the only parameter affecting efficiency. But the effect of $\omega_e$ is more dominant on $e$ especially in idle conditions [13]. Later on, in the next section, tolerances for $e$ will be kept larger, in order to cover the minor effects of EGR rate, air to fuel ratio etc. Proposed form for $e(\omega_e)$ is

$$e(\omega_e) = e_2\omega_e^2 + e_1\omega_e + e_0, \tag{2.11}$$

where $e_2, e_1, e_0$ are the parameters to be found. In table (2.2), typical values for these parameters which is given in [14] can be found.

Table 2.2. Typical values for coefficients in equation (2.11)

| Constant | Value |
|---|---|
| $e_2$ | $-10^{-6}$ |
| $e_1$ | $6.5 \cdot 10^{-4}$ |
| $e_0$ | 0.35 |

## 2.2. Input-Output Behavior of the Diesel Engine Model

Equations (2.6) to (2.11) give an input-output relationship between $m_i(t)$ and $\tilde{T}_e(t)$, where the former is the controlled input and the latter together with $\tilde{T}_l(t)$ are the main terms affecting controlled output $\omega_e(t)$ through equations (2.1) to (2.3).

Putting together all the equations from (2.6) to (2.11), it is possible to express $\tilde{T}_e(t)$ in the form of

$$\tilde{T}_e(t) = \tilde{T}_{ef}(\omega_e) \cdot m_i(t) - \tilde{T}_{er}(\omega_e). \tag{2.12}$$

Using the data given in table (2.1), it is possible to characterize $\tilde{T}_{er}(\omega_e)$ as an affine function of $\omega_e$. Figure (2.1) shows that an affine relation in the form of

$$\tilde{T}_{er}(\omega_e) = a \cdot \omega_e + b \tag{2.13}$$

reflects the characteristics of $\tilde{T}_{er}(\omega_e)$ astonishingly well. Constants for the affine fit, i.e. equation (2.13) are given in table (2.3) where $\omega_e$ in $rad/s$.

Table 2.3. Constants for the equation (2.13)

| Constant | Value |
|---|---|
| $a$ | 0.0375 |
| $b$ | 19.6339 |

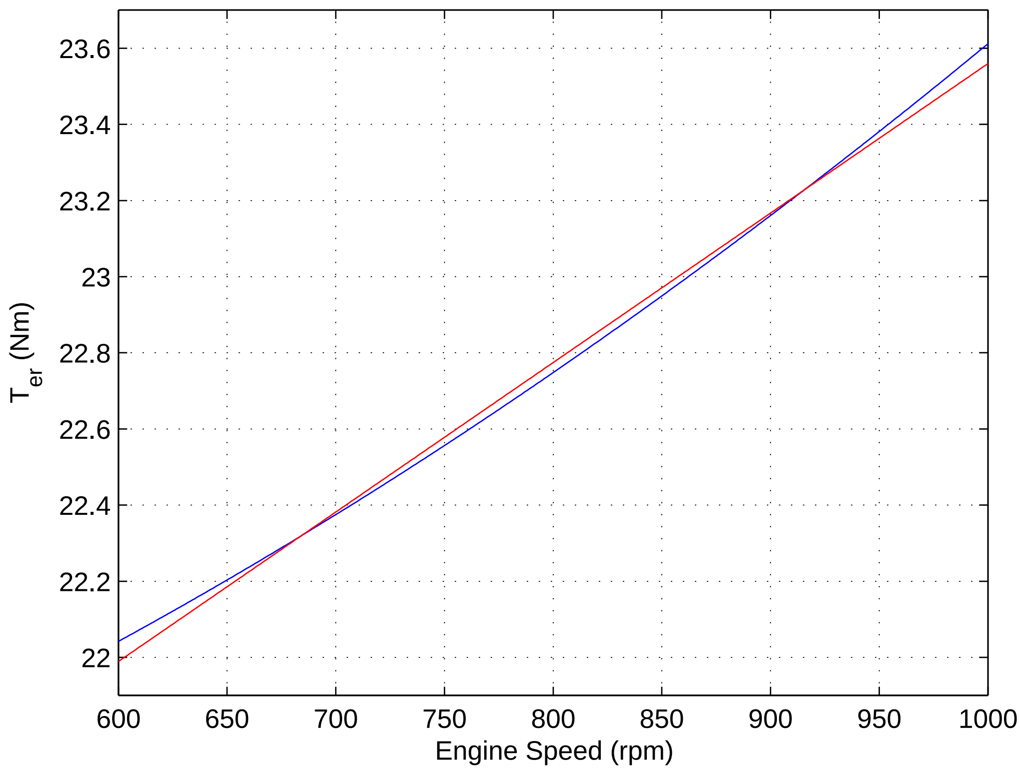


Figure 2.1. $\tilde{T}_{er}$ based on the data given in table (2.1) (blue), affine fit to $\tilde{T}_{er}$ (red)

Going back to equation (2.12) and using the data given in table (2.2), $\tilde{\mathcal{T}}_{ef}(\omega_e)$ is shown in figure (2.2). Although it is possible to construct an affine relation for $\tilde{\mathcal{T}}_{ef}(\omega_e)$, a nominal value for $\tilde{\mathcal{T}}_{ef}(\omega_e)$ and deviation about that nominal value will be considered, advantage of this approach will be evident in controller design. Thus

$$\tilde{\mathcal{T}}_{ef}(\omega_e) = \tilde{\mathcal{T}}_{ef,nom} + \delta_{\tilde{\mathcal{T}}_{ef}}, \tag{2.14}$$

where $\tilde{\mathcal{T}}_{ef,nom}$ is determined according to figure (2.2). Also $\delta_{\tilde{\mathcal{T}}_{ef}}$ is considered as 10% of $\tilde{\mathcal{T}}_{ef,nom}$ such that

$$|\delta_{\tilde{\mathcal{T}}_{ef}}| \leq 0.1 \cdot \tilde{\mathcal{T}}_{ef,nom} \tag{2.15}$$

The values of $\tilde{\mathcal{T}}_{ef,nom}$ and $\delta_{\tilde{\mathcal{T}}_{ef}}$ based on the figure (2.2) can be found in table (2.5).

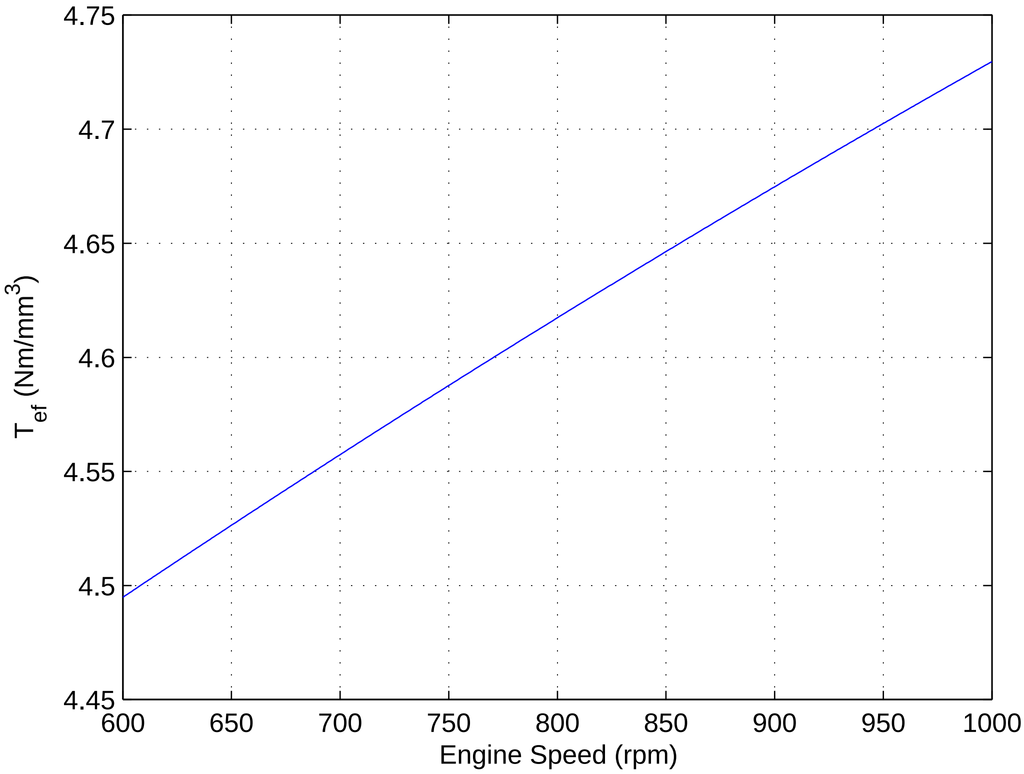


Figure 2.2. $\tilde{\mathcal{T}}_{ef}$ based on the data given in table (2.2)

Table 2.4. Constants for the equation (2.14) based on figure (2.2)

| Constant | Value |
|---|---|
| $\tilde{\mathcal{T}}_{ef,nom}$ | 4.61 |
| $\max\lvert\delta_{\tilde{\mathcal{T}}_{ef}}\rvert$ | 0.46 |

In fact, with looking at figure (2.2), it is clear that, 10% is more than enough but as it is noted earlier, $\delta_{\tilde{\mathcal{T}}_{ef}}$ is chosen in order to cover minor effects of EGR rate, air to fuel ratio, etc.

### 2.3. Validation of the Diesel Engine Model

Using equations (2.13) and (2.14), it is possible to rewrite equation (2.12) as:

$$\tilde{T}_e(t) = (\tilde{\mathcal{T}}_{ef,nom} + \delta_{\tilde{\mathcal{T}}_{ef}}) \cdot m_i(t) - a \cdot \omega_e(t) - b \tag{2.16}$$

At 800 *rpm* equation (2.16) becomes

$$\tilde{T}_e = (4.61 \mp 0.46) \cdot m_i - 22.77 \tag{2.17}$$

In the figure (2.3), engine torque characteristics of a diesel engine at 880 *rpm* can be found.

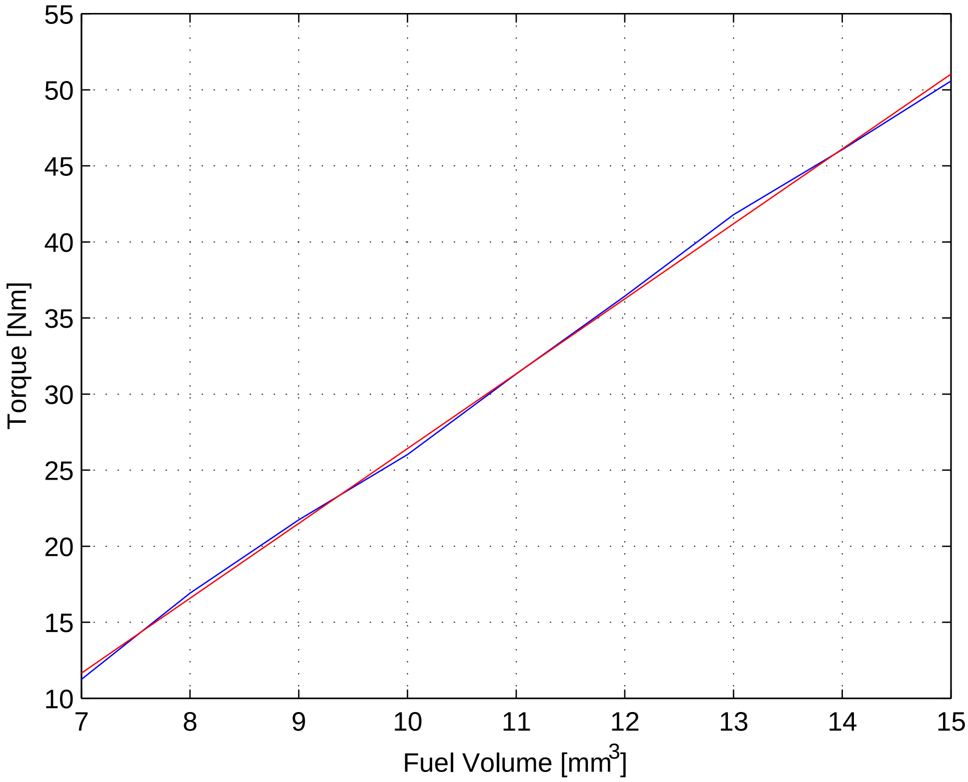


Figure 2.3. Engine torque at 880 *rpm*, (blue) measured engine data, (red) affine fit for the measured data

Again, an affine fit is constructed to express torque production. Note that mentioned affine fit expresses the torque production characteristics very accurately. In table (2.5), the constants for this affine fit can be found. It is observed that, constants in table (2.5) are covered by equation (2.17), so it is concluded that equation (2.16) gives a good characterization of the torque production of the engine.

Table 2.5. Constants for the affine fit for the real diesel engine data (2.14) based on figure (2.3)

| Constant | Value |
|---|---|
| $\tilde{\mathcal{T}}_{ef,meas}$ | 4.9219 |
| $\tilde{T}_{er,meas}$ | $-22.7962$ |

# 3. CONTROLLER SYNTHESIS

The engine model obtained in chapter (2) is shown as a block diagram in figure (3.1).

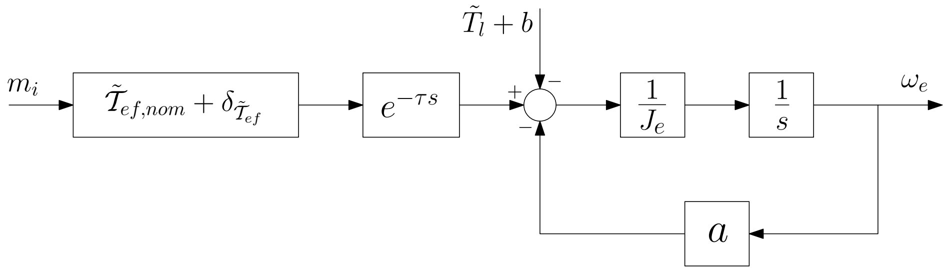


Figure 3.1. Diesel engine model

Note that $\tilde{T}_{er}(\omega_e)$ has a constant term $b$ which acts like a constant load torque, so it is possible to redefine the load torque as

$$T_l(t) := \tilde{T}_l(t) + b. \tag{3.1}$$

Obviously, in order to regulate the speed of the engine, a reference input $\omega_r(t)$ is needed. Also defining speed difference:

$$\omega_{dif}(t) := \omega_r(t) - \omega_e(t) \tag{3.2}$$

From a system point of view, $\omega_r(t)$ and $T_l(t)$ are the inputs to the system and $\omega_{dif}(t)$ is the main output of the system. Engine model with introduction of reference speed $\omega_r(t)$ can be found in figure (3.2).

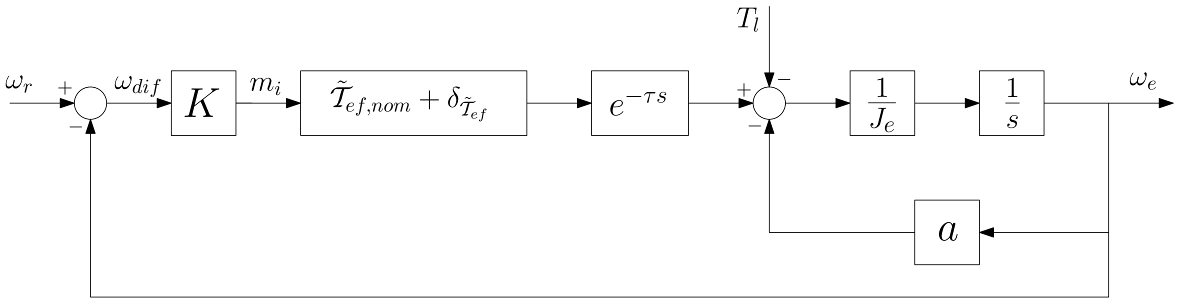


Figure 3.2. Diesel engine model with reference input

The controller $K$ will be implemented by closing the loop between $\omega_{dif}(t)$ and $m_i(t)$, performance objective of the controller will be discussed clearly in the sequel but for now, we only state that it should be able keep $\omega_{dif}(t)$ small against the changes in $\omega_r(t)$ and $T_l(t)$.

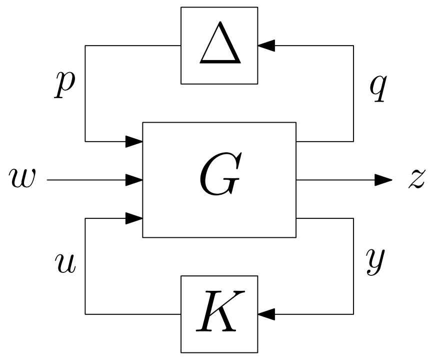


Figure 3.3. General plant framework

For controller design purposes, it is desirable to put the system into general plant framework figure (3.3). Therefore, uncertainty in torque production and time-delay should be represented accordingly with norm bounded perturbation blocks. This procedure is referred as *"pulling out the deltas"* in the literature. In the following two sections, proper application of mentioned approach for torque production and time-delay can be found.

### 3.1. Uncertainty Representation for Torque Production

Torque production submodel of the engine is shown in figure (3.4). It is easy to

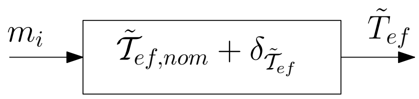


Figure 3.4. Torque production submodel of the engine

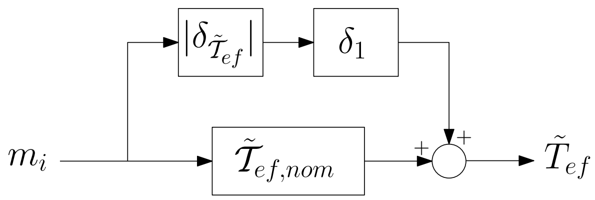


Figure 3.5. Torque production submodel of the engine with uncertainty $\delta_1$

express figure (3.4) as figure (3.5) for

$$\|\delta_1\|_\infty \leq 1$$

So, torque production is represented by a nominal value $\tilde{\mathcal{T}}_{ef,nom}$, a weighting function $\max \delta_{\tilde{\mathcal{T}}_{ef}}$ and a norm bounded dynamic uncertainty $\delta_1$.

### 3.2. Uncertainty Representation for Time-Delay

A broad literature exists on analysis and synthesis problems for time-delay systems. Generally, time-delay causes degradation of controller performance, even worse, for some cases it can cause instability. A very detailed overview of time-delay systems can be found in [16]. In this study time-delay will be represented by a linear weighting function and a norm bounded dynamic uncertainty.

The time delay part of the diesel engine model is shown in figure(3.6) which is equivalent to figure (3.7). The objective is to find a proper weighting function which represents the characteristics of time delay as good as possible.

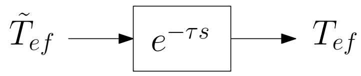


Figure 3.6. Time-delay part of the engine

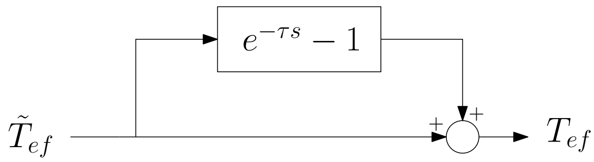


Figure 3.7. Time-delay part of the engine

It should be remembered that the time-delay (equation (2.3)) is a function of engine speed. Since it is inversely proportional to engine speed, the maximum time-delay occurs in minimum engine speed. Considering the idle conditions, minimum attainable engine speed is assumed as 600 *rpm*. Using equation (2.3) maximum time-delay is calculated as

$$\tau_{max} \approx 0.05. \tag{3.3}$$

Maximum time-delay will be primary concern in the design of the above mentioned weighting function. In [17] an appropriate weighting function is proposed for $\tau \in [0, 0.1]$ as

$$W_{del}(s) = \frac{0.21s}{0.1s + 1}. \tag{3.4}$$

Using this weighting function, it is possible to express figure (3.7) by figure (3.8) for

$$\|\Delta_1\|_\infty \leq 1.$$

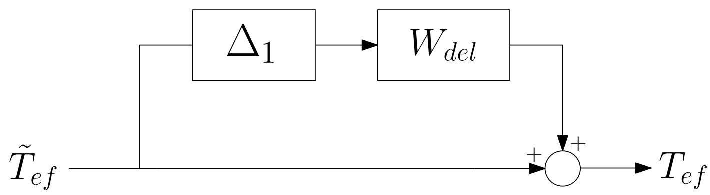


Figure 3.8. Time-delay part of the engine with uncertainty $\Delta_1$

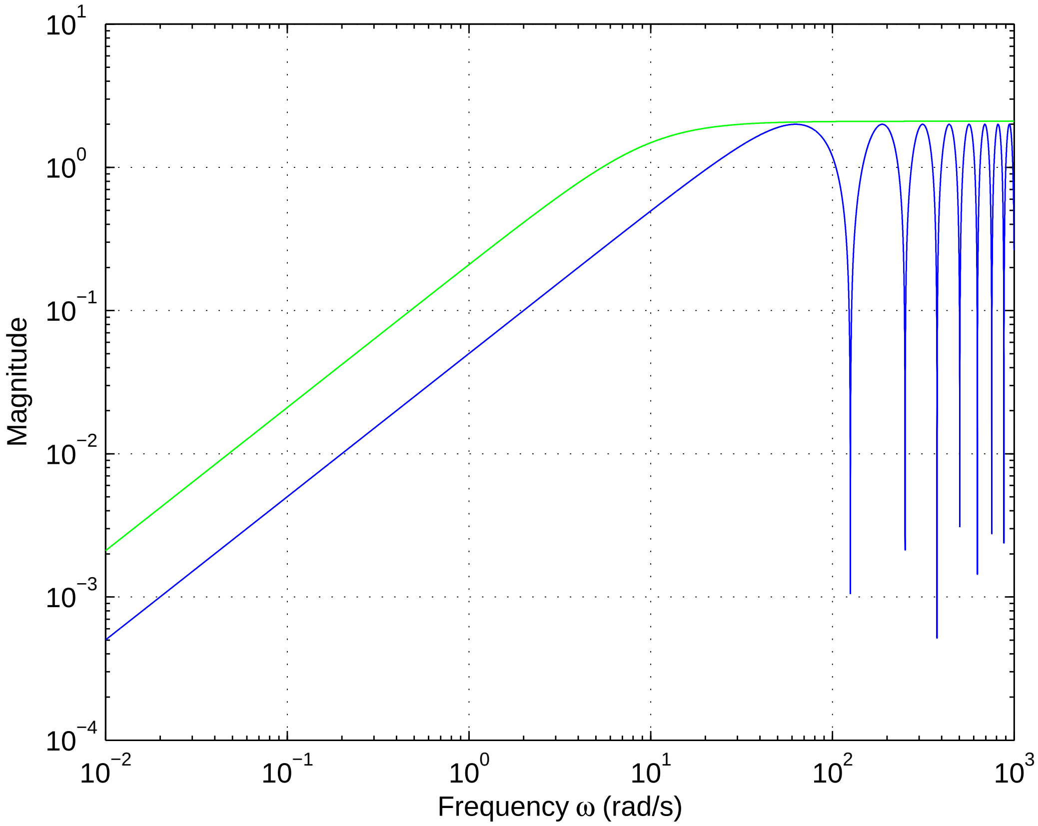


Figure 3.9. Comparison of $|e^{-\tau_{max}j\omega} - 1|$ (blue) and $|W_{del}(j\omega)|$ (green)

As it can be seen from figure (3.9), $W_{del}$ defines an upper bound for the original time-delay part of the system. The closed loop simulation results for controller designed using equation (3.4) as $W_{del}$ can be found in [11].

It is obvious that lowering the upper bound of the weighting function in high frequency region ($\omega \geq 100\ rad/s$) is not possible but the time-delay representation performance of the weighting function will be enhanced in low frequency region.

Using the recommended multiplier in [18], it is possible to obtain a proper weighting function for the time-delay part of the system:

$$W_{del}(s) = \frac{0.2828\tau_{max}^2\ s^2 + \tau_{max}\ s}{0.1414\tau_{max}^2\ s^2 + 0.6419\tau_{max}\ s + 1} \tag{3.5}$$

It should be noted that the maximum time-delay is used in this weighting function.

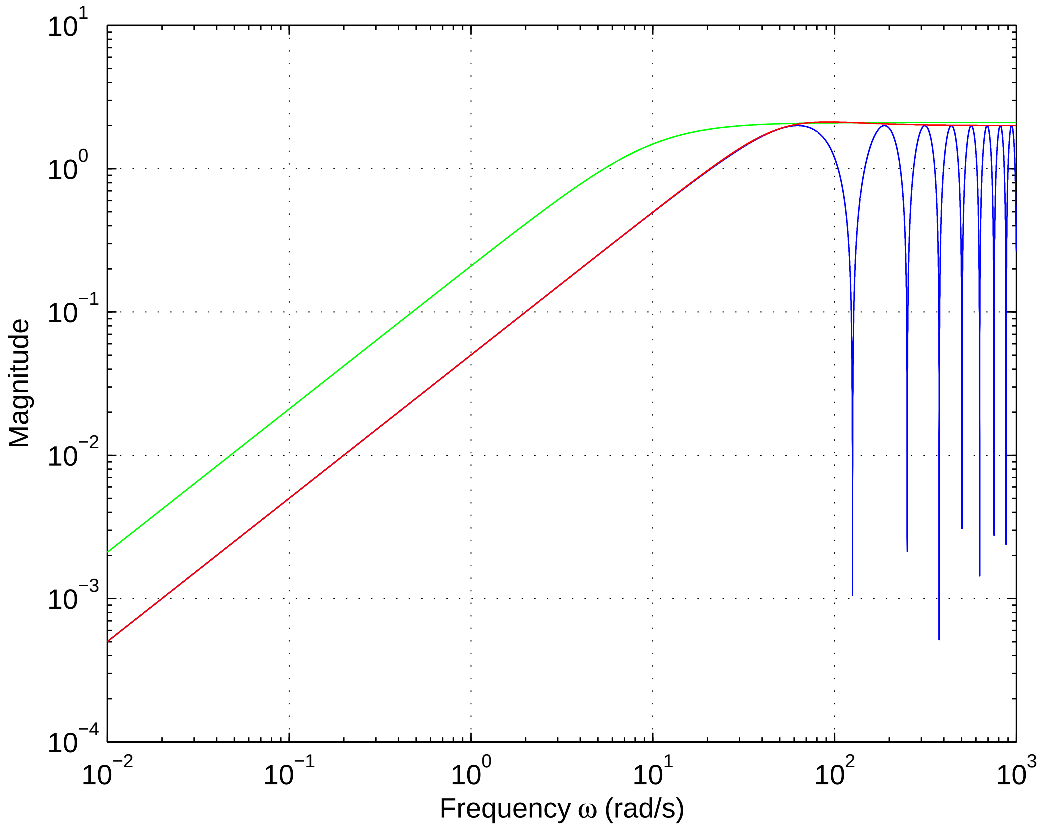


Figure 3.10. Comparison of $|e^{-\tau_{max}j\omega} - 1|$ (blue), former $|W_{del}(j\omega)|$ (green) and latter $|W_{del}(j\omega)|$ (red)

Frequency response of this new function together with the previous one can be found in figure (3.10). Although nothing changes in high frequencies, in low frequency region, the new function provide a better representation of time-delay. In fact, it represents time-delay up to 70 $rad/s$ astonishingly well. So the time-delay part of the system is represented by figure (3.8) where $W_{del}$ is given by equation (3.5) and $\Delta_1$ is a norm bounded dynamic uncertainty.

### 3.3. Design of Weighting Functions for Performance Enhancement

It is known that use of appropriate weighting functions during controller design stage significantly increases the performance of the closed loop system. With the help of these functions it is possible to take into account of frequency content of the disturbances, limitations in controlled input and controlled output. In the preceding subsections, the use of weighting functions in order to decrease steady state error in engine speed $\omega_{dif}(t)$ and taking into account of the frequency content of the disturbance input $\omega_r(t)$ is shown.

### 3.3.1. Integral Action for Engine Speed Difference $\omega_{dif}(t)$

Integral action gives zero steady state error for step input therefore it is generally desirable to have an integral action in controller synthesis. In this study, $\mu$ synthesis approach is used for controller design. Since $\mu$ synthesized controllers do not give an integral action, the system is enforced to have an integral action by means of a weighting function.

However, standard integral term $1/s$ gives an uncontrollable imaginary axis pole which violates assumptions of $\mu$ synthesis theory. Thus, using standard integral term as a weighting function in controller design is not possible. In [19] and [20] it is shown that an approximate integral action can be achieved by

$$W_{dif}(s) = \frac{1}{s+\epsilon} \tag{3.6}$$

for sufficiently small $\epsilon > 0$. Implication of this weighting function in controller synthesis is given in figure (3.11).


Figure 3.11. Weighting function $W_{dif}$ for approximate integration

In this study $\epsilon$ is chosen as 0.001. It is observed that this value of $\epsilon$ satisfactorily decreases the steady state error in engine speed ($\omega_{dif}(t)$).

### 3.3.2. Frequency Content Representation of Disturbance Input $\omega_r(t)$

One of the most important advantages of the $\mu$ synthesis approach comes out in taking into account of frequency contents of disturbance inputs. One can use appropriate weighting functions that reflects the characteristics of the frequency content of the different signals or more specifically weighting functions can be introduced to emphasize the frequencies that are interested.

In this thesis, only one weighting function is used for reference engine speed $\omega_r(t)$. It is assumed that the changes in reference engine speed can not be in high frequencies. In fact, in a real diesel engine, reference engine speed or idle speed is determined by engine ECU, so it is possible to enforce this assumption. So during controller design procedure a low pass filter is used to emphasize the low frequency changes in the reference input $\omega_r(t)$. The implication of this filter is shown in figure (3.12).


Figure 3.12. Low pass filter $W_{low}$

Although it is possible to introduce a weighting function for the other disturbance input $T_l(t)$, in order not to bring optimism in controller design it is not preferred.

### 3.4. General Plant Framework

Previously designed weighting functions, i.e. $|\delta_{\tilde{\mathcal{T}}_{ef}}|$ for uncertainty representation of torque production, $W_{del}$ (equation (3.5)) for uncertainty representation of time-delay, $W_{dif}$ for decreasing the steady state error in engine speed ($\omega_{dif}(t)$) and $W_{low}$ for taking into account frequency content of the disturbance input $\omega_r(t)$ are all shown in figure (3.13).

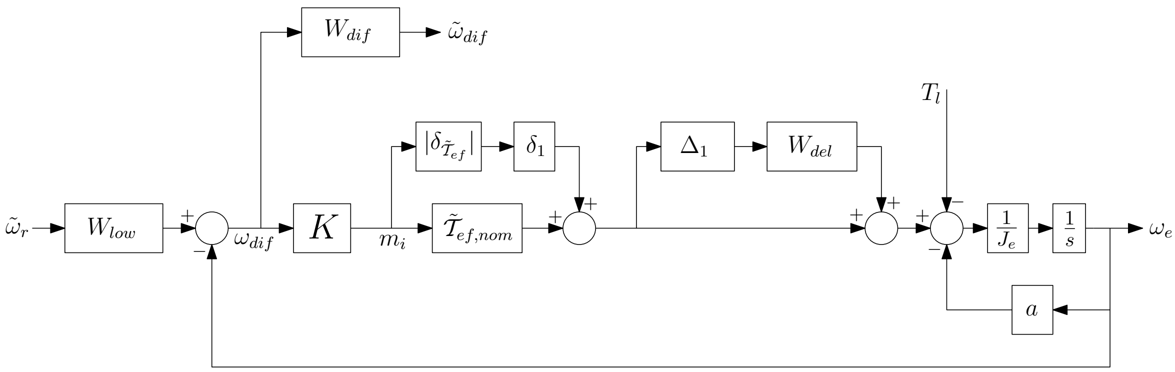


Figure 3.13. Diesel engine model with reference input and weighting functions

It is possible to put diesel engine model (3.13) into general plant framework as in figure (3.14).

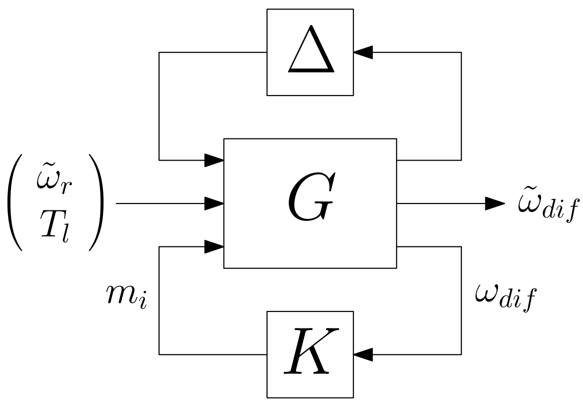


Figure 3.14. Diesel engine model in general plant framework

where $\Delta$ is defined as

$$\Delta = \begin{pmatrix} \delta_1 & 0 \\ 0 & \Delta_1 \end{pmatrix}.$$

So, in figure (3.14), all the uncertainties are collected in $\Delta$ block, $K$ is the controller to be designed and $G$ is the generalized nominal plant. Note that, in this form $G$ includes all the weighting functions as well.

### 3.5. $\mu$ Synthesis

$\mu$ synthesis with $D-K$ iteration is performed. Reader is invited to appendix A for an overview of $\mu$ synthesis. Details of $\mu$ synthesis can be found in [19] and [20]. $\mu$ *Analysis and Synthesis Toolbox* for Matlab (with Matlab 6.5.1) was used for controller design in early stages of this thesis, where *Robust Control Toolbox* for Matlab (with Matlab 7.0.4) was used in final work.

In table (3.1) $D-K$ iteration summary can be found. In the $3^{rd}$ iteration, peak $\mu$ value is obtained as 0.344 and with a closed loop $\mathcal{H}_\infty$ norm of 0.344.

Table 3.1. $D-K$ iteration summary

| Iteration # | 1 | 2 | 3 |
|---|---|---|---|
| Controller Order | 6 | 12 | 14 |
| Total D-Scale Order | 0 | 6 | 8 |
| $\gamma$ Achieved | 1.773 | 0.397 | 0.344 |
| Peak $\mu$-Value | 0.964 | 0.367 | 0.344 |

In figure (3.15) $\mu$ plot for the $3^{rd}$ iteration can be found. Since $\mu < 1$, robust stability is guaranteed and and since closed loop $\mathcal{H}_\infty$ norm is less than 1, robust performance is guaranteed as well.

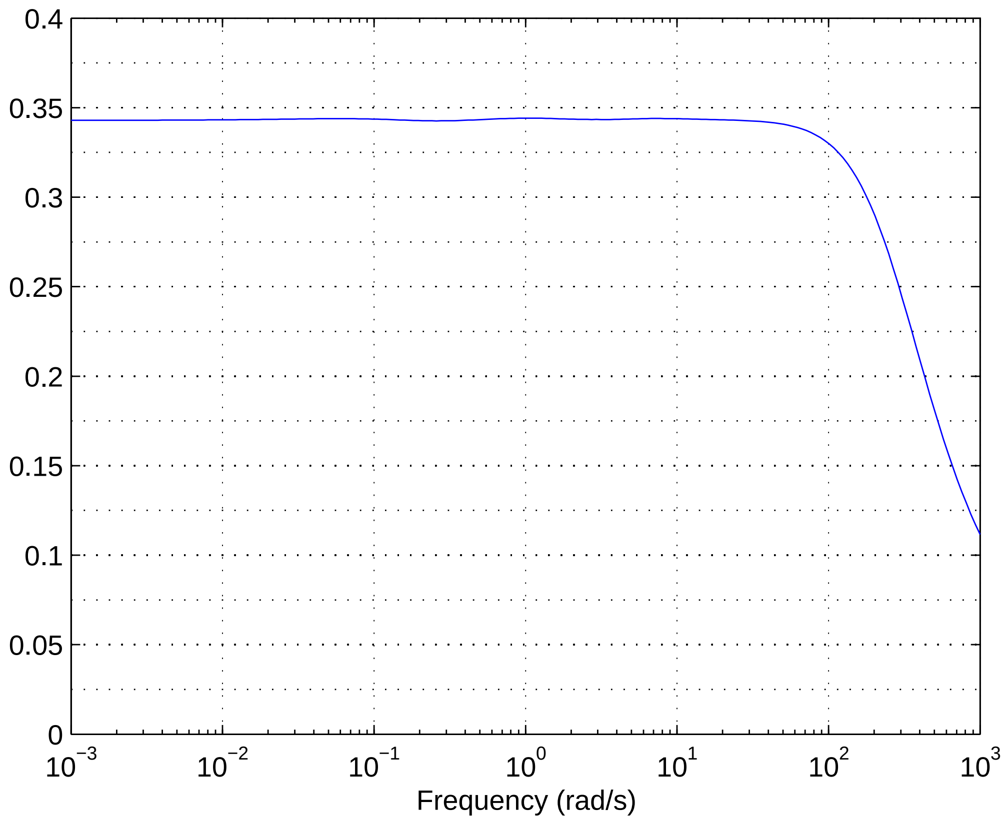


Figure 3.15. $\mu$ plot for the $3^{rd}$ iteration

Designed controller is an $14^{th}$ order controller, with single input engine speed difference $\omega_{dif}(t)$ and single output injected fuel $m_i(t)$. In figure (3.16), Bode plot of the designed controller can be found.

### 3.6. Controller Order Reduction

The drawback of the $\mu$ synthesis approach is that it generally gives higher order controllers. A controller with 14 states is very complicated for implementation. Therefore an order reduction procedure is needed.

A reduced order controller is designed based on the Hankel singular values of the controller. Hankel singular values of a stable system indicates the respective state energy of the system. Therefore order to be reduced can be determined from Hankel singular value plot of a system. In figure (3.17) Hankel singular values for the 14 state

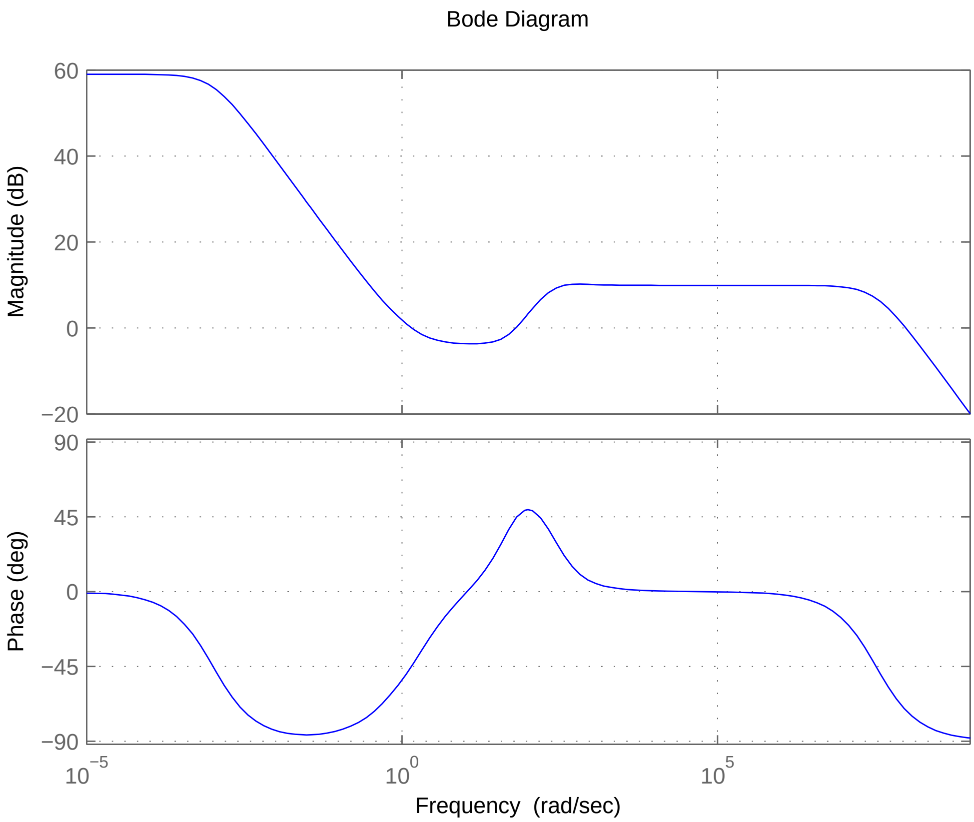


Figure 3.16. Bode plot of the controller with 14 states

controller can be found. As a performance criteria, additive error is used as

$$\min \|K - K_{red}\|_{\infty}, \tag{3.7}$$

where $K$ is the original controller and $K_{red}$ is the reduced one.

Looking at (3.17) it will be concluded that a first order controller can be very satisfactory but it was not the case, an additive error bound of 6.7723 is found for a first order controller. However, for a $3^{rd}$ order controller the additive error bound is found as 0.3845 which is much more accurate and still easy to implement. In figure (3.18) Bode plots of the reduced order controller together with original $14^{th}$ order controller can be found. It is concluded that reduced order controller captures the frequency response of the original controller very efficiently.

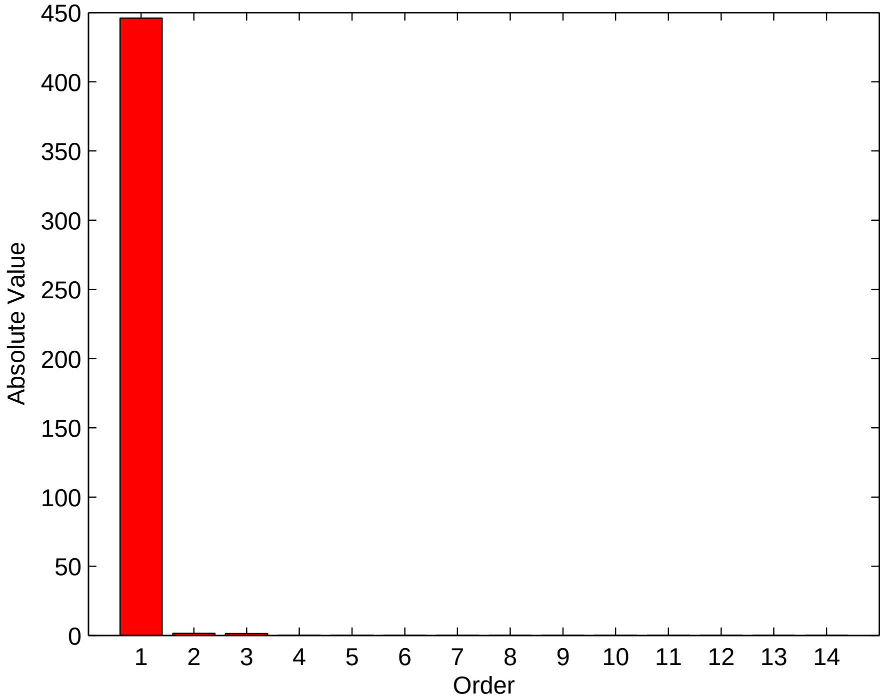


Figure 3.17. Hankel singular values of the controller

Resulting controller matrices after order reduction are as follows:

$$A_{cont} = \begin{bmatrix} -0.001 & -21.296 & -0.004 \\ -21.260 & -3.22 \cdot 10^7 & -6.97 \cdot 10^4 \\ 0.022 & 6.39 \cdot 10^4 & -0.952 \end{bmatrix},$$

$$B_{cont} = \begin{bmatrix} -0.946 \\ -10^4 \\ -0.079 \end{bmatrix},$$

$$C_{cont} = \begin{bmatrix} -0.948 & -10^4 & -1.660 \end{bmatrix},$$

$$D_{cont} = [0].$$

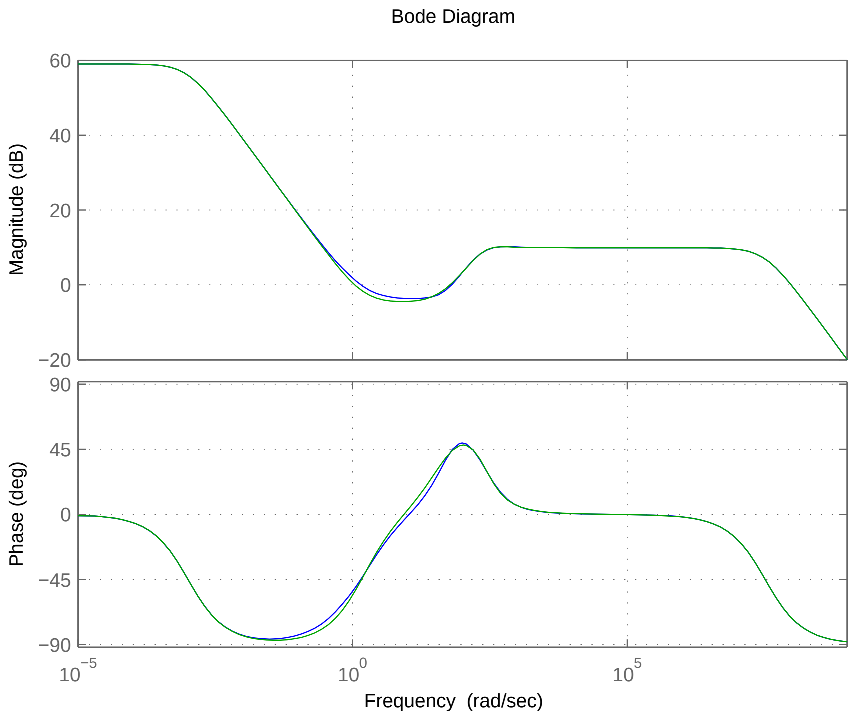


Figure 3.18. Bode plot of controllers: 14 order $\mu$ synthesized (blue), 3 order reduced (green)

# 4. RESULTS

In the preceding pages, the response of the system for the designed controllers can be found. Designed controllers are tested both by simulations and experiments.

For simulations the engine model derived in chapter (2) was used together with the original $14^{th}$ order controller.

All of the experiments of this thesis were conducted in Measurement and Control Laboratory of ETH Zürich, Switzerland. For experiments, a very modern passenger car turbocharged diesel engine with EGR and VGT was used. The details of the engine is not available due to the confidentiality agreement. In the experiments, the engine was mounted on a test bench and the discretized version of the reduced order controller was used . The controller was implemented by the help of a dSPACE DS 1005 board.

Two different types of situations are investigated. These are:

- Torque disturbance for a reference engine speed
- Change in reference speed with no torque disturbance

## 4.1. Torque Disturbance for a Reference Engine Speed

The first type of experiment simulates the primary idle speed problem. In normal idling conditions, diesel engine is subjected to torque loads from air conditioning, electric windows, headlights and other electrical accessories. Performance of the idle speed controller for this application can be evaluated in terms of its deviation from reference speed and response time. A low deviation from reference speed together with a fast response is preferable.

### 4.2. Change in Reference Speed with no Torque Disturbance

The second type of experiment simulates the secondary idle speed problem. Especially in cold starts, diesel engine will be subjected to variable reference speeds. Performance of the idle speed controller for this application can be evaluated in similar terms. A small overshoot together with a low rise time is preferable.

### 4.3. Comments on Results

As it is mentioned previously, in the simulations, original $14^{th}$ order controller is used whereas for experiments the discretized version of the $3^{rd}$ order reduced one is used. The results for the simulations and experiments are generally very similar and satisfactory. Only considerable inconsistency between simulation and experiment occurs in figure (4.3) where measured maximum speed deviation is greater than simulated one. In a very low speed, engine will get closer to stalling region so that the actual engine dynamics will deviate from the mathematical model of the engine. This will be a possible reason for this inconsistency.

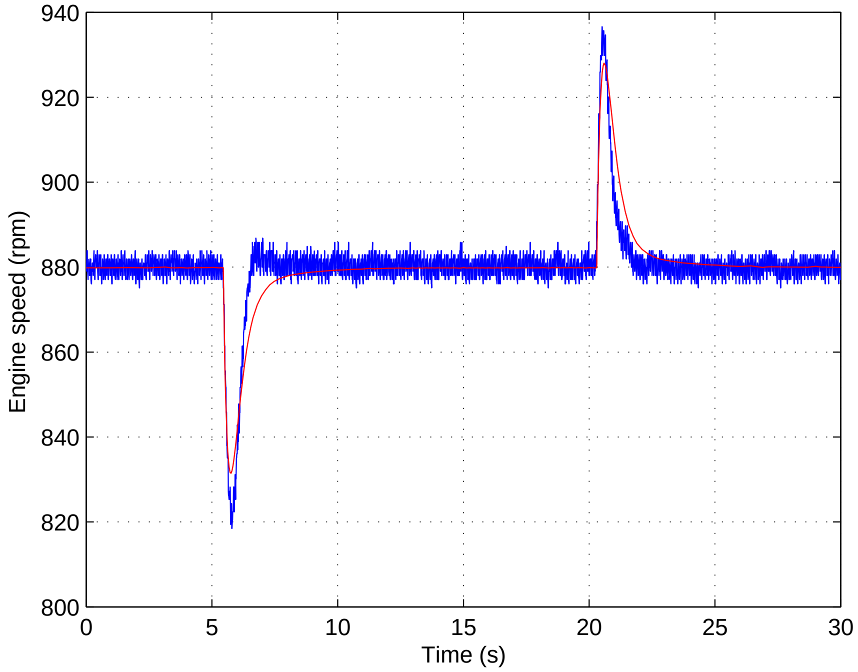


Figure 4.1. Reference speed 880 $rpm$, $\pm 12$ $Nm$ torque disturbance, (blue) measured, (red) simulated

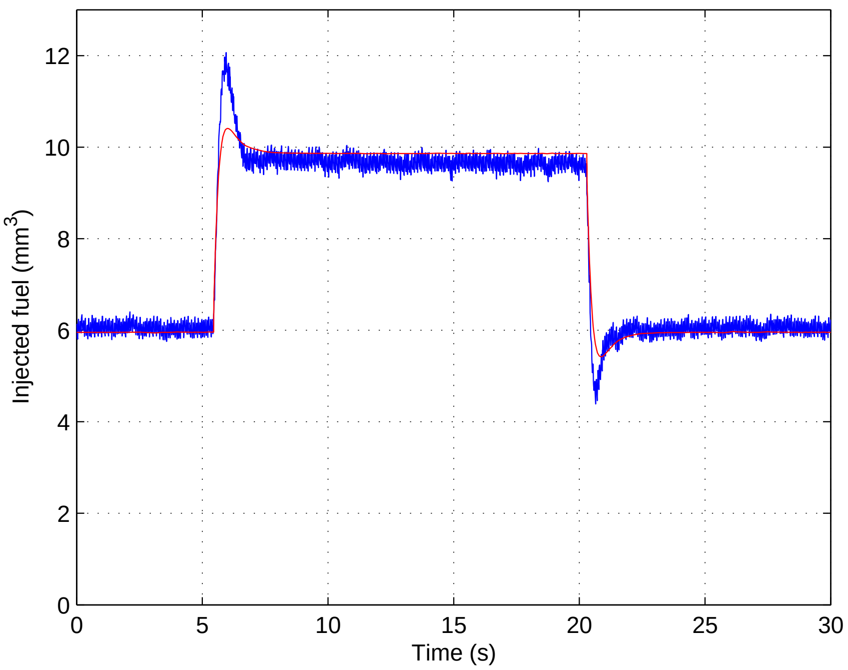


Figure 4.2. Reference speed 880 $rpm$, $\pm 12$ $Nm$ torque disturbance, (blue) measured, (red) simulated

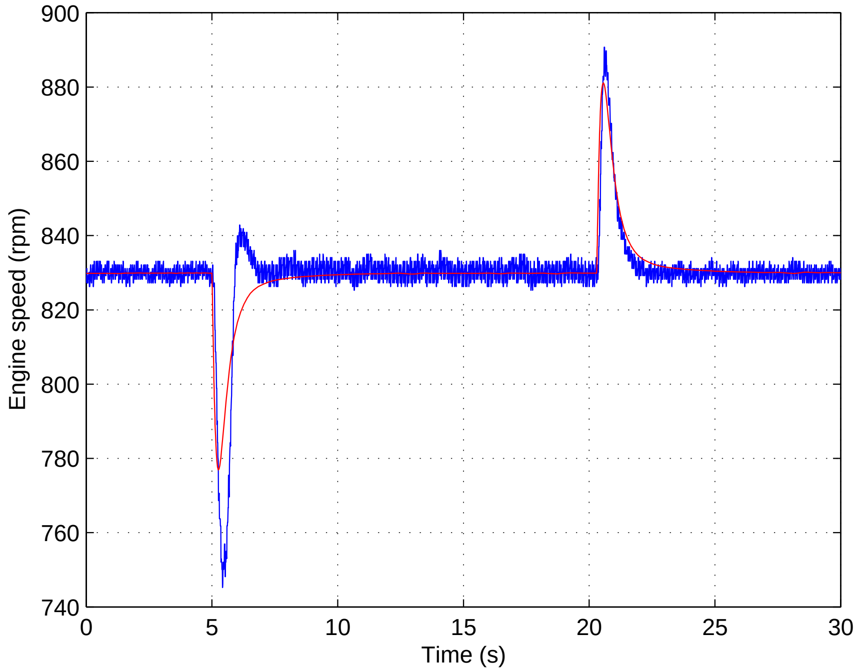


Figure 4.3. Reference speed 830 $rpm$, $\pm 12$ $Nm$ torque disturbance, (blue) measured, (red) simulated

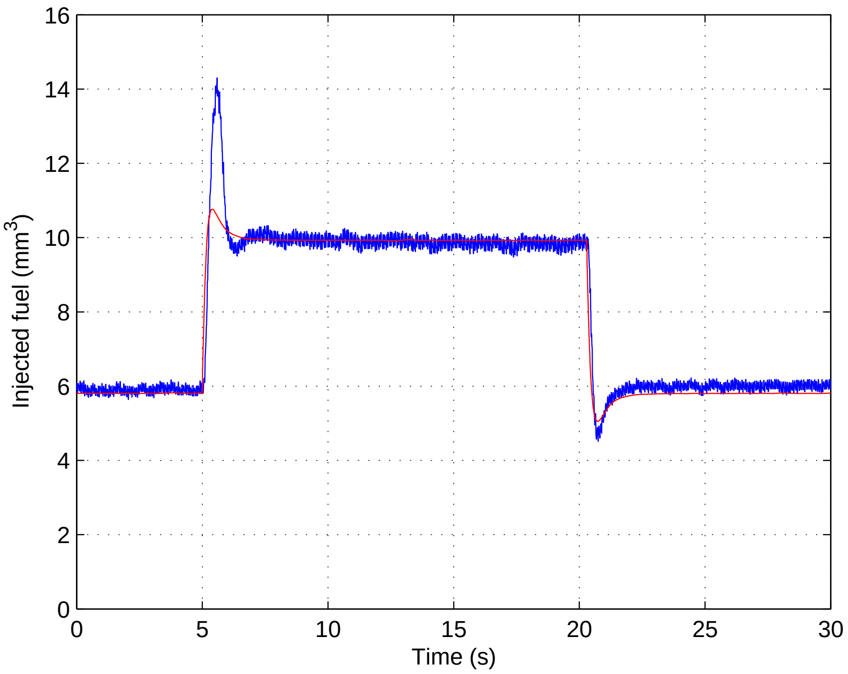


Figure 4.4. Reference speed 830 $rpm$, $\pm 12$ $Nm$ torque disturbance, (blue) measured, (red) simulated

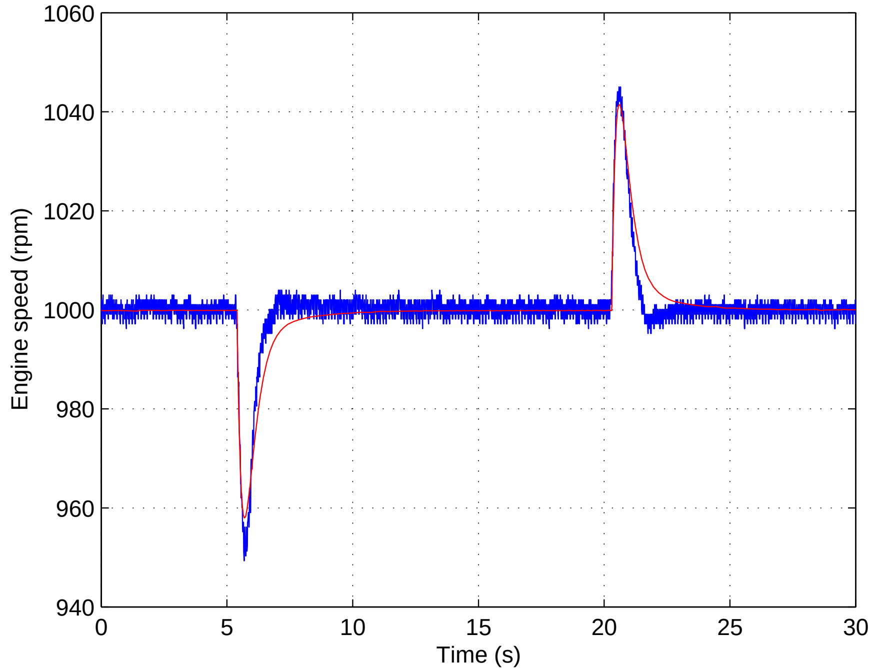


Figure 4.5. Reference speed 1000 $rpm$, $\pm 12$ $Nm$ torque disturbance, (blue) measured, (red) simulated

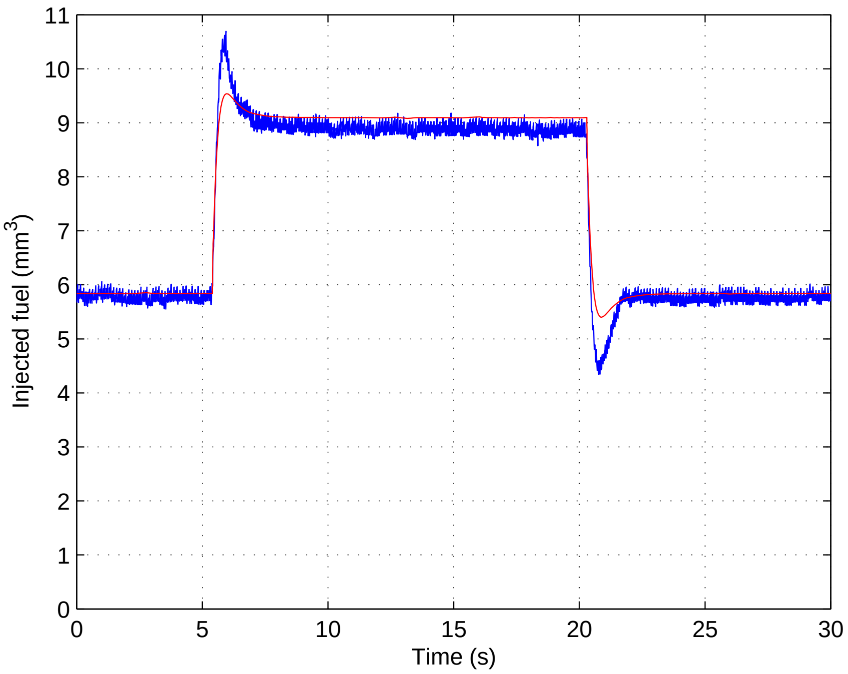


Figure 4.6. Reference speed 1000 $rpm$, $\pm 12$ $Nm$ torque disturbance, (blue) measured, (red) simulated

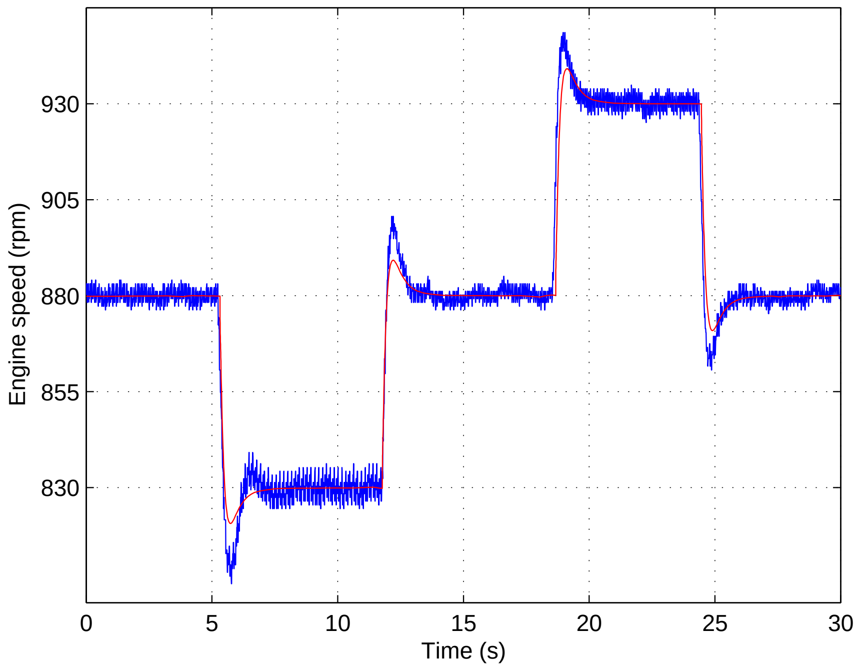


Figure 4.7. Reference speed 880 $rpm$, $\pm 50$ $rpm$, (blue) measured, (red) simulated

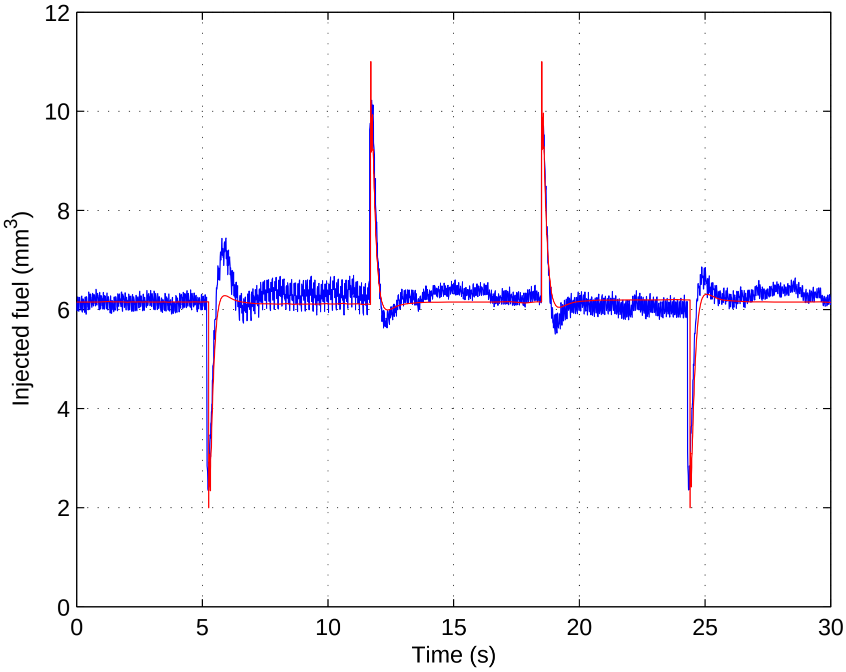


Figure 4.8. Reference speed 880 $rpm$, $\pm 50$ $rpm$, (blue) measured, (red) simulated

# 5. SUMMARY AND CONCLUSIONS

In this thesis idle speed control problem of a diesel engine was investigated. All vehicles spend some time in idling, especially in crowded cities, vehicles will spend considerable time in idling which makes this problem inherently important.

A model based control approach was performed in this thesis. Hence, a mathematical model of a diesel engine was needed first. Diesel engines have very complicated dynamics due to the transient characteristics of thermodynamic boundary conditions and combustion process. Although models relying on first principles are essential for some applications, these models are not very useful for controller synthesis.

Thus, a mean-value engine model was used for controller design. This model reflects the input-output behavior of the engine with a reasonable accuracy with very low computational complexity. Different operating conditions were modeled as system uncertainties. Two different weighting functions were proposed for these system uncertainties.

In order to assure zero steady state error, an approximate integrator was used in controller design procedure and to reduce conservatism a low pass filter was used for emphasizing low frequency changes in reference engine speed.

$\mu$ synthesis with $D-K$ iteration was performed. Robust stability and robust performance conditions were satisfied with the designed controller which has 14 states. In order to implement the controller, an order reduction algorithm was carried out based on the Hankel singular values. The resultant controller was a $3^{rd}$ order controller which captures the dynamics of the actual controller quite satisfactorily.

Designed controllers were tested both in simulations and experiments. Two different types of problems were investigated. First one was torque disturbance for a constant reference speed and the second one was variable reference speed for no torque disturbance. The results were very satisfactory.

One improvable point in this thesis is that the time delay in engine model was assumed to be unknown. Only the upper limit of this time delay was considered during the controller design. But this time delay is a function of the engine speed. So a controller design approach which considers the relationship between the time delay and the engine speed will give better results.

Also in this thesis, the effects of EGR, VGT and injection timing on torque production were considered to be very limited. Although this assumption is reasonable for idle speed conditions it will be very optimistic for entire engine operating region. But the obvious advantage of this approach for idle speed conditions is that it allows EGR, VGT and injection timing control independent of speed regulation problem to some extent. Noting that these variables have extreme importance in emissions, one can design the emission controller independent of the speed controller. Definitely the multi objective structure of the combination of the two control problems have clear benefits but it is a much more challenging task.

# APPENDIX A: ROBUST CONTROL THEORY

This appendix is an introductory material to provide the reader with the main topics in robust control theory. For more detailed information, [25] is a very good starting point as being very pedagogic. Additional concepts like model order reduction together with some examples in different topics can be found in [19]. Also [20] being a broader version of [19] is a comprehensive reference in robust control theory.

*μ Analysis and Synthesis Toolbox* for Matlab (with Matlab 6.5.1) was used for controller design in early stages of this thesis, where *Robust Control Toolbox* for Matlab (with Matlab 7.0.4) was used in final work. The manuals of these toolboxes [26] for the former and [27] for the latter are good references for implementation of controller design procedure with vast amount of examples.

## A.1. Systems and Signals

### A.1.1. System Representations

A system is defined as a mapping between inputs and outputs. A continuous time LTI system can be represented in state space as:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{A.1}$$

with the states $x(t) \in \mathbb{R}^n$, the inputs $u(t) \in \mathbb{R}^m$ and the outputs $y(t) \in \mathbb{R}^l$ and system matrices $A$, $B$, $C$, $D$ of suitable size. Applying Laplace transformation, transfer matrix $G(s)$ can be obtained as:

$$G(s) = C(sI - A)^{-1}B + D \tag{A.2}$$

and for zero initial condition, i.e. $x(0) = 0$, another alternative representation for (A.1) is

$$Y(s) = G(s)U(s) \tag{A.3}$$

where $U(s)$ and $Y(s)$ are Laplace transforms of $u(t)$ and $y(t)$ respectively.

### A.1.2. Signal Norms

**Definition.** *($\mathcal{L}_p$ Spaces)*

*The LVS $\mathcal{L}_p[0, \infty)$, for $1 \leq p \leq \infty$, is defined as the collection of all measurable functions $x(t)$ such that*

$$\int_0^\infty |x(t)|^p \, dt < \infty$$

*The space $\mathcal{L}_p[0, \infty)$ is equipped with the norm*

$$\|x\|_p := \left( \int_0^\infty |x(t)|^p \, dt \right)^{1/p}.$$

Throughout this thesis, it is assumed that all the signals belong to $\mathcal{L}_2$.

### A.1.3. System Norms

**Definition.** *($\mathcal{H}_\infty$ Spaces)*

*The space $\mathcal{H}_\infty$ is a closed subspace of $\mathcal{L}_\infty$ with functions that are analytic and bounded in the open right-half plane. The space $\mathcal{H}_\infty$ is equipped with the norm*

$$\|G\|_\infty := \sup_{\omega \in \mathbb{R}} \bar{\sigma}[G(j\omega)].$$

The real rational subspace of $\mathcal{H}_\infty$ is denoted by $\mathcal{RH}_\infty$. Thus, it consists of all real rational proper transfer matrices.

## A.2. Stability and Performance

### A.2.1. Structured Singular Value - $\mu$

**Definition.** *The structured singular value ($\mu$) of the matrix $M$ with respect to the set $\mathbf{\Delta}$ is defined as*

$$\mu_{\mathbf{\Delta}}(M) := \frac{1}{min\{\bar{\sigma}(\Delta) : \Delta \in \mathbf{\Delta}, det(I - M\Delta) = 0\}}.$$

Calculation of exact $\mu$ value is not possible except for some special cases. But it has been shown that $\mu$ is bounded as

$$\rho(M) \leq \mu_{\mathbf{\Delta}}(M) \leq \bar{\sigma}(M).$$

These bounds can be arbitrarily large. However, there exists transformations which affect $\rho$ and $\bar{\sigma}$ but do not affect $\mu$. Next theorem addresses a set of transformations.

**Theorem.** *[20] Consider the sets*

$$\mathcal{D} = \{diag[D_1, \ldots, D_n] : D_i = D_i^* > 0\}, \tag{A.4}$$

$$\mathcal{Q} = \{\Delta \in \mathcal{B}_\Delta : \Delta^* \Delta = I\}, \tag{A.5}$$

*then for all $D \in \mathcal{D}$ and $Q \in \mathcal{Q}$,*

$$\mu_{\mathbf{\Delta}}(QM) = \mu_{\mathbf{\Delta}}(MQ) = \mu_{\mathbf{\Delta}}(M) = \mu_{\mathbf{\Delta}}(DMD^{-1}).$$

**Corollary.** *[20] For $D \in \mathcal{D}$ and $Q \in \mathcal{Q}$, the bounds for $\mu$ can be tightened as:*

$$\max_{Q \in \mathcal{Q}} \rho(QM) \leq \mu_{\mathbf{\Delta}}(M) \leq \inf_{D \in \mathcal{D}} \bar{\sigma}(DMD^{-1}).$$

This corollary plays an important role in design of $\mu$ synthesized controllers.

### A.2.2. Robust Stability

**Definition.** *Given the description of uncertainty model set $\mathcal{G}$ suppose $G \in \mathcal{G}$ is the nominal design model and $K$ is the resulting controller. If $K$ internally stabilizes every plant belonging to $\mathcal{G}$ then the closed loop feedback system is said to have robust stability.*

Next theorem gives the relation between robust stability and $\mu$.

**Theorem.** *[20] The feedback interconnection shown in figure (A.1) is stable for all $\Delta \in \mathbf{\Delta}$ with $\|\Delta\|_\infty < 1$ if and only if*

$$\sup_{\omega \in \mathbb{R}} \mu_{\mathbf{\Delta}}(G(j\omega)) \leq 1.$$

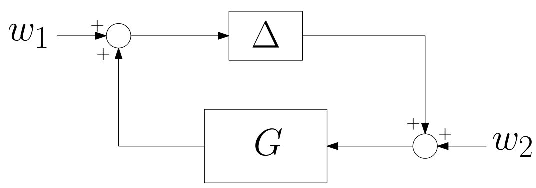


Figure A.1. Feedback interconnection

### A.2.3. Robust Performance

**Definition.** *Given the description of uncertainty model set $\mathcal{G}$ and a set of performance objectives, suppose $G \in \mathcal{G}$ is the nominal design model. If all the performance objectives are satisfied for every plant belonging to $\mathcal{G}$ then the closed loop feedback system is said to have robust performance.*

Next theorem gives the relation between robust performance and $\mu$.

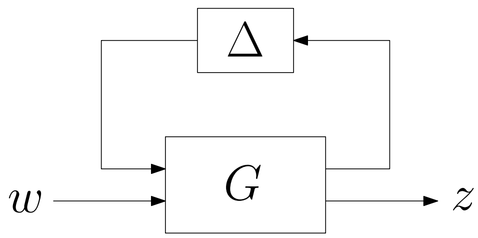


Figure A.2. Feedback interconnection

**Theorem.** *[20] For all $\Delta \in \boldsymbol{\Delta}$ with $\|\Delta\|_\infty < 1$, the feedback interconnection shown in figure (A.2) is stable and $\|\mathcal{F}_u(G,\Delta)\|_\infty \leq 1$ if and only if*

$$\sup_{\omega\in\mathbb{R}} \mu_{\boldsymbol{\Delta}_p}(G(j\omega)) \leq 1,$$

*where $\boldsymbol{\Delta}_p$ is defined as*

$$\boldsymbol{\Delta}_p := \left\{ \begin{pmatrix} \Delta & 0 \\ 0 & \Delta_f \end{pmatrix} : \Delta \in \boldsymbol{\Delta} \right\}.$$

Throughout this thesis $\|\mathcal{F}_u(G,\Delta)\|_\infty \leq 1$ is considered as the performance criteria in accordance with the figure (A.2).

### A.3. $\mu$ Synthesis with $D-K$ iteration

Above mentioned $D$ scaling matrices (equation (A.4)) can be used in controller synthesis in an iterative manner.

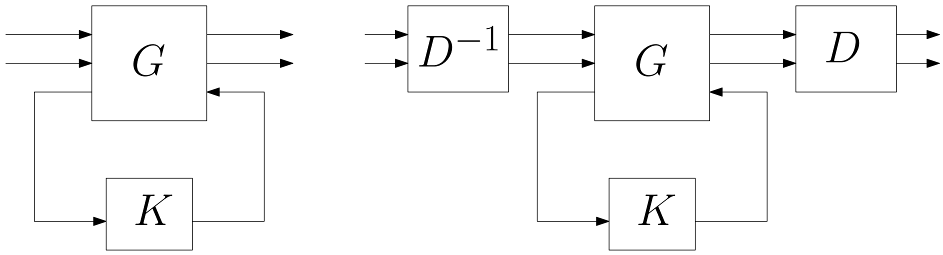


Figure A.3. $G-K$ feedback interconnection with and without $D$ scales

Noting that $\mu$ value is not changing for the feedback interconnections shown in figure (A.3) below iterative procedure can be used to design a controller [20].

- Holding $D$ fixed, design $K$ to minimize $\|D\mathcal{F}_L(G,K)D^{-1}\|_\infty$
- Holding $K$ fixed, find the new optimal $D$ scalings
- If not satisfactory, proceed to first step

This iterative process is known as $D-K$ iteration. There are certain theoretical drawbacks of this method. Namely, global minimum can not be guaranteed for $D-K$ iteration, and worse the method gives no guarantee for convergence to any local minimum. But in spite of theoretical shortcomings, $D-K$ iteration is used commonly in the design of robust controllers due to the fact that it gives satisfactory results in most of the practical applications.

As stated earlier $\mu$ *Analysis and Synthesis Toolbox* for Matlab (with Matlab 6.5.1) [26] was used for controller design in early stages of this thesis, where *Robust Control Toolbox* for Matlab (with Matlab 7.0.4) [27] was used in final work.

# REFERENCES

1. Kuang, B., Y. Wang and Y. L. Tan, “An $H_\infty$ Controller Design for Diesel Engine Systems”, *Proceedings of International Conference on Power System Technology*, Perth, WA, Australia, 4 December-7 December 2000, Vol 1, pp 61-66, IEEE, 2000.

2. Memering, D. W. and P. H. Meckl, “Comparison of Adaptive Control Techniques Applied to Diesel Engine Idle Speed Regulation”, *Journal of Dynamic Systems, Measurement, and Control*, Vol 124, pp 682-688, 2002.

3. Outbib, R., X. Dovifaaz, A. Rachid and M. Ouladsine, “Speed Control of a Diesel Engine: A Nonlinear Approach”, *Proceedings of the American Control Conference*, Anchorage, AK, USA, 8 May-10 May 2002, pp 3293-3294, 2002.

4. Karray, F. and E. Conrad, “Design of Intelligent Controllers for Electronic Speed Regulation of a Diesel Engine”, *Proceedings of the First International Conference of Knowledge-Based Intelligent Electronic Systems*,21 May-23 May 1997, Adelaide, Australia, 1997.

5. Song, Q. and K. M. Grigoriadis, “Diesel Engine Speed Regulation Using Linear Parameter Varying Control”, *Procedings of the American Control Conference*, 4 June-6 June, Denver, Colorado, USA, 2003.

6. Balluchi, A., L. Benvenutti, M. D. Di Benedetto and T. Villa, “Hybrid Controller Synthesis for Idle Speed Management of an Automotive Engine”, *Proceedings of the American Control Conference*, 28 June-30 June, Chicago, Illinois, USA, 2000.

7. Mohamed, F. A. and H. N. Koivo, “Diesel Engine Systems with Generic Algorithm Self Tuning PID Controller”, *International Conference on Future Power Systems*, 16 Nov-18 Nov, Amsterdam, Netherlands, 2005.

8. Roy, S., O. P. Malik and G. S. Hope, “An Adaptive Control Scheme for Speed

Control of Diesel Driven Power-plants", *IEEE Transactions on Energy Conversion*, Vol. 6, No. 4, 1991.

9. Jiang, J., "Optimal Gain Scheduling Controller for a Diesel Engine", *IEEE Control Systems Magazine*, Vol. 14, No. 4, pp 42-48, 1994.

10. Karaman, S., L. Güvenç, R. Ergen, B. Uygun and E. Güraslan, "Dizel Motoru Rölanti Devri Dayanıkli Kontrolü", *Proceedings of 3.rd Automotive Technologies Congress*, Bursa, Turkey, 26-28 June 2006.

11. Akyıldız B., İ. E. Köse and L. Güvenç, "$H_\infty$ Yaklaşımıyla Dizel Motor Rölanti Devri Kontrolü", *Proceedings of 3.rd Automotive Technologies Congress*, Bursa, Turkey, 26-28 June 2006.

12. Heywood, J. B., *Internal Combustion Engine Fundamentals*, McGraw-Hill, New York, 1988.

13. Guzzella, L. and C. H. Onder, *Introduction to Modeling and Control of Internal Combustion Engine Systems*, Springer, Berlin, 2004.

14. Guzzella, L. and A. Amstutz, "Control of Diesel Engines", *IEEE Control Systems Magazine*, Vol 18, No.5, pp 53-71, 1998.

15. Greene, A. B. and G. G. Lucas, *The Testing of Internal Combustion Engines*, English University Press Ltd., 1969.

16. Richard, J. P., "Time-delay systems: an overview of some recent advances and open problems", *Automatica*, Vol 39, pp 1667-1694, 2003.

17. Doyle, J. C., B. Francis and A. Tannenbaum, *Feedback Control Theory*, Macmillan, 1990.

18. Megretski A. and A. Rantzer, "System Analysis via Integral Quadratic Constraints", *IEEE Transactions on Automatic Control*, Vol 42, No.6, pp 819-830,

1997.

19. Zhou, K. and J. C. Doyle, *Essentials of Robust Control*, Prentice-Hall, New Jersey, 1998.

20. Zhou, K., J. C. Doyle and K. Glover, *Robust and Optimal Control*, Prentice-Hall, New Jersey, 1996.

21. Jensen, J.P., A.F. Kristensen, S.C. Sorenson, N. Houbak and E. Hendricks, "Mean Value Modeling of a Small Turbocharged Diesel Engine", *SAE Paper*, No. 910070, 1991.

22. Jung, M. and K. Glover, "Calibratable Linear Parameter-Varying Control of a Turbocharged Diesel Engine", *IEEE Transactions on Control Systems Technology*, Vol 14, No.1, pp 45-62, 2006.

23. Jung, M., K. Glover and U. Christen, "Comparison of Uncertainty Parameterisations for $H_\infty$ Robust Control of Turbocharged Diesel Engines", *Control Engineering Practice*, Vol 13, No. ??, 2005.

24. Jung, M., *Mean-Value Modelling and Robust Control of the Airpath of a Turbocharged Diesel Engine*, Ph. D. Thesis, Cambridge University, 2003.

25. Scherer, C. *Theory of Robust Control*, http://www.dcsc.tudelft.nl/ cscherer/, 2001.

26. Balas, G. J., J. C. Doyle, K. Glover, A. Packard and R. Smith, *$\mu$ Analysis and Synthesis Toolbox*, MUSYN Inc. and The MathWorks Inc., June 1998.

27. Balas, G., R. Chiang, A. Packard and Michael Safonov, *Robust Control Toolbox*, The MathWorks Inc., September 2006.