EFFECT OF DIMENSIONAL DISCREPANCIES ON THE DETERMINATION OF SAFETY FACTOR OF REINFORCED CONCRETE COLUMNS

THESIS

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SYNOPSIS

The object of this thesis will be to determine the effects of dimensional discrepancies due to construction of concrete columns, with respect to design values; on the safety of the structural member, and to determine a tolerable limit for these discrepancies.

After a discussion of safety factors in general, and the factors that affect it, the effect of dimensional discrepancies will be considered in particular. Statistical data is analyzed to check the contribution of tolerances to the safety factor. Certain examples taken from the local practice will illustrate that better results for the safety factors may be obtained if the controlling on the site is done properly.

LIST OF SYMBOLS

A : Initial cost of structure
C_F : Cost of failure
C_S : Cost of unserviceability
L : Load
$\bar{L}$: Mean Load
ΔL : Change in mean load
N : Safety Factor
$\bar{N}$: Probability factor
P_F : Probability of failure
P_F^* : Probability of survival
P_L : Probability of occurance of an extreme load
P_L^* : Probability of occurance of a load intensity less than L
P_S : Probability of occurance of a minimum strength
P_S : Probability of unserviceability
P_S^* : Probability of occurance of the ultimate load $S > L$
$P(B_i)$: Probability of occurance
S : Strength
$\bar{S}$: Mean Strength
ΔS : Change in mean strength
X : Quality characteristic
$\bar{X}$: Mean value
X_m : Mode value
m : Factor of ignorance
n : Number of occurances
p_e : Constant probability
r : Difference of strength and loading
t_p : Confidence limit parameter
α : Angle included between x-axis and the line of balance
σ : Standard deviation
σ_L : Standard deviation of load
σ_S : Standard deviation of strength

CHAPTER I

INTRODUCTION

Engineering can be defined as the art and science of controlling and adapting the forces of nature so that more comfortable and better living conditions are attained for the human society. In this capacity, engineers have to deal with natural phenomena and forces, and must overcome them. However natural forces are random in character and though a pattern may be observed in some phenomena from time to time, the engineer must rely on his "engineering judgement", and make assumptions pertaining to the action of such natural forces and base his work on these. This shows that engineers are working in a degree of uncertainty. This uncertainty, this relative ignorance of natural forces with which the engineers must work, have caused them to introduce allowances when they are dealing with such forces. The civil engineer, probably, is the unluckiest among his collegues, having to design for forces of which in most cases he is completely uncertain, and has to make large allowances from time to time.

These allowances have come to be recognized as factors of safety. In the past, factors of safety were assigned quite arbitrarily; stemming either from past experience with a similar sort of problem, or from excessive and extreme caution. At the present as we have come to learn the behaviour of our materiels, and as our design methods have improved through the ages from trial and error methods onwards, it has been possible to make great reductions in our safety factors, thus achieving more and more economy in time, energy, and materiel. It indeed is a long way from the ziggurats of the Babylonians, and the massive construction style of the world's greatest engineers, - The Romans, to the modern skyscraper, yet the best is not achieved.

With the introduction of the computer to solve structural problems, it has been possible to refine design criteria and methods even further; but with respect to the procedure, the assumptions that are to be made prior to design lag far behind, thus depriving us of achieving real economy that could otherwise be attained by our brand new systems.

One possibility to achieve better results, is to accept the probabilistic attitude towards the factor of safety. Though many engineers still refuse to recognize or admit it, determination of safety factors by statistical analysis, is a better method than our previous solutions, and is here to stay until still better methods can be developed.

The core of the method can very briefly be summarized as follows. Many variable factors contribute to the uncertainty in our assumptions prior to design. These variables however mostly obey the normal or Gaussian distribution of probability, and furthermore can be combined with one another, thus giving rise to a single factor of safety, which is by far the more the more reliable and economic safety factor, and which gives a better insight to the whole problem of design in comparison with the empirical factors of safety that are in use today.

Tolerances or dimensional discrepancies in construction is one of the variables which contribute to the final safety factor. Up to date no one has really bothered to deal with this aspect; assuming, perhaps quite correctly, that the effect of tolerances is quite unimportant and negligable to the final safety factor. The object of the present study is to investigate the contribution of dimensional discrepancies to the final result, and thus to fill a gap which is existant in the engineering literature, and to justify or to refuse this assumption.

The measurement of tolerances, however requires a great amount of time, due to the experimental and statistical nature of the work, and so far very little has been done in this field. The idea of the present study was, therefore, to start as a first step, with a simple case. As this simple case the effect of construction tolerances on the safety of reinforced concrete columns has been selected. This selection is made primarily because:

1- The failure of columns is of primary importance, since it costs more in terms of human life and materiel.

2- The ultimate strength of columns can be expressed with simpler equations.

3- The measurement of column dimensions is much easier than that of any other structural element.

The results of the study are presented below, following a literature review and analysis on the factors which affect the factor of safety, and in conclusion a better insight has been gained as to where construction tolerances stand with respect to other factors.

It is observed from Tables 2 and 3 that if the effects of dimensional discrepancies are neglected, the percent of error in the safety factor varies from 2.76 %

for the case where p is 0.005 to 5.26 for the case where p is equal to 0.030.

It will be observed in Chapter IA, Part I that allowing an error of $(1+\Delta)$ cm, where Δ is a very small quantity, the error in the safety factor is bound to be about five or six percent. Furthermore, in order to prevent the error to exceed 5 % in the safety factor, the errors in the dimensions of the columns must be about 0.7 cm. However, with increasing column dimensions this limit may also be somewhat extended, still yielding a 5 % error in the safety factor.

CHAPTER II

A LITERATURE SURVEY AND ANALYSIS ON SAFETY FACTORS, CONSTRUCTION TOLERANCES AND STATISTICAL METHODS

A- SEQUENCE OF DESIGN

The work of design involves the provision of a structure with a decided level of safety, at a minimum cost. According to Freudenthal(6) and Brown (14) the sequence consists of three steps:

1- Determination of design loads, i.e. the specification of the standard load pattern, and its relationship with the actual; the evaluation of the probability of occurance of various intensities of loads, and the analysis of the time dependence of the standard load, and its variability.

2- Determination of the strength of the structure, i.e. structural analysis. This step includes the consideration of the strength of the building materiels, the selection of sections, and the structural arrangement of them and the analysis of the structure in the light of the aforementioned. In this step the maximum intensity of standard load which can be applied to the idealized structure without causing failure during the period of service must be determined, and its probability distribution evaluated.

3- Determination of a quantitative value of safety, i.e.

safety analysis. This consists of two steps:

a- Formulation of the rules for the selection of the design conditions of the structure with respect to the unserviceability and failure, and choice of the optimal rule. This is formulated on the basis of two fundamentally different approaches: 1- Direct probability approach which specifies numerical values for the probabilities with which both failure and unserviceability are to be avoided, and 2- The economic approach which compares the cost of failure to the cost of increasing the carrying capacity of the structure.

b- Determination of the intensities of the standard load associated with the acceptable probabilities of failure and of unserviceability that have resulted from the selected decision rules.

B- SAFETY FACTORS

The fundamental, conventional concept of "allowable stress" involves a comparison between a computed maximum strain, and the strength of the materiel, and implies the existance of a margin between the two. However the justification of this margin has been questioned by most engineers. Freudenthal (1) points out that as its conventional name "margin of safety" suggests, it reveals the subjective striving on the part of the designer for an adequate measure of safety, as well as a consciousness of the limitations of his knowledge, and the arbitrariness of his assumptions; that its real character has remained obscure, however and its magnitude is generally estimated on the basis of subjective judgement rather than objective fact, and that very little has been done to establish a criterion for its determination on a more rational basis than the experience and judgement of the designer. Even the most refined design is thus deprived of its merits, the designer being compelled to choose the fundamental assumptions of his design largely on the basis of subjective arguments, without ascertaining their validity by the identification of the objective conditions. Research in the sphere of new materiels cannot be expected to bear its full weight upon the economy of structures if the safety factor can be fixed rather arbitrarily "between 1.25 and 4.00 or even higher". Therefore an analysis of the safety factor to identify its true character and to determine its objective values has become of increasing urgency. The conventional safety factor N, can be stated as:

$$N = \frac{\text{yield stress}}{\text{allowable stress}} \qquad (1)$$

whereas the new definitions (5),(20) are as follows:

$$N = \frac{\text{ultimate load}}{\text{working load}} \qquad (2)$$

and as Freudenthal (1) points out the true character of safety factor is disclosed by the introduction of a statistical concept of physical qualities, according to which the individual properties composing the structural phenomena of strain and resistance are presented by frequency distributions instead of by individual, i.e. minimum or maximum values. The safety factor, correlating the induced strain with the resistance of the structure is derived from observable and measurable physical properties and phenomena.

The correlation of strain and resistance requires a careful analysis of all features of structural design, both from the points of view of basic assumptions, and of statistical interpretation, even though the available experimental data is still far from complete.

The principle underlying the concept of the factor of safety can be understood best by reviewing the fundamental difficulty of structural design. The structural characteristic "strain" is a result of calculations, whereas "resistance" is derived from the observations of physical properties, and hence the result of material conception. The two characteristics, therefore, are uncompatible with each other, and cannot be equated. Consequently strain may only be correlated to the resistance by a relation of inequality, which is expressed in terms of a probable correlation range. This range itself is a function of the degree of perfection in the concept It should therefore provide for the following:

1- The imperfection of human observation, uncertainty
2- The imperfection of design procedures, ignorance

The range therefore is a function of the two factors cited above. The improving methods of design have done much to decrease or eliminate the second factor, however the uncertainty in observation, though it can be reduced due to changing of circumstances that are causing it, can never be removed completely. Hence the safety factor is a measure of uncertainty rather than of ignorance. Today we are able to reduce it partly because of refined methods of design, but mostly because of excellent control of production, and standardization of engineering materiels.

The laws of structural design are derived from the laws and principles of classical mechanics. However, most unfortunately, but inevitably a certain number of parameters of these equations represent observable or measurable physical properties or phenomena. The application of the equations to structural design requires the introduction of real values of such properties under all possible conditions of practicle importance. Some of this values must be predicted or estimated since the observation and measurement of these under all relevant conditions are impracticable. Such prediction is entirely different from laws of classical mechanics, and it can only refer to past experience; making the effect of chance an absolute necessity. Systems of chance thus give rise to statistical laws represented by frequency distributions. Prediction based on a statistical casualty, therefore, may be expressed only in terms of the probability that a certain event will occur within definite limits.

The value of the safety factor N may be derived from the condition that the maximum load L or "strain" induced in the structure by actual service conditions must never cause such damage as to impede its fitness for service, even were this maximum load to coincide with the lowest value of the structure's strength, or "resistance" S. Thus:

$$N = \frac{S}{L} > 1 \qquad (3)$$

If $\pm\Delta L$ denotes the maximum range of fluctuation of actual load about the expected value $\bar{L}$, and $\pm\Delta S$ the maximum range of fluctuation of the strength of the structure about its expected value $\bar{S}$, the maximum load will be

$$(\bar{L} + \Delta L) \qquad (4)$$

and the minimum strength will never be less than

$$(\bar{S} - \Delta S) \qquad (5)$$

So by eq. 3 , the condition to just prevent the failure of the structure

$$\bar{L} + \Delta L = \bar{S} - \Delta S \qquad (6)$$

factoring $\bar{S}$ and $\bar{L}$, and dividing both sides with the factor $(1 + \Delta L/L_0)$ we obtain

$$\bar{L} = \frac{\left(1 - \frac{\Delta S}{\bar{S}}\right)}{\left(1 + \frac{\Delta L}{\bar{L}}\right)} \bar{S} \qquad (7)$$

in which by definition

$$N = \frac{1 - \Delta S/\bar{S}}{1 + \Delta L/\bar{L}} \qquad (8)$$

and eventually

$$\bar{L} = N\bar{S} \qquad (9)$$

At this point it would be advisable to consider the Ratio Method of safety factor determination (14) . Brown begins by defining stress and load factors. He has defined the stress factor to be the ratio of the ultimate stress of materiel to the greatest strength induced by design loads into the materiel i.e. the design stress. He also defines the load factor as the ratio of the ultimate load to cause failure to the working load. Nowadays load factors have penetrated the codes of design practice for steel and concrete, yet they also have their own shortcomings. Though Torroja, chairman of ICBR Committee advocated their use (9) other authoritative names on the subject of safety, such as Freudenthal, Herzog, Rosenblueth, and Collier (12),(13) defend the view that load factors are really of no help to the engineer.

Brown (14) defines factor of safety as the ratio of resistance of structure S to the design load effect L. S stands for maximum stress at the critical section at failure in the stress factor, and the load applied just to cause failure in the load factor, whereas L stands for maximum stress created at the critical section by the design load in the stress factor, and the design load in the load factor.

The design load L is generally the greatest reasonably anticipated load in use, and the resistance S is generally the lowest reasonably anticipated strength of the structure. Both the load L, and the strength S can be selected at any point on their respective curves.(Fig.1) However the selection of lower values of L, and higher values of S apparently give higher values of safety factor N.

The intention is considered to be to give a ratio of the lowest reasonable anticipated resistance of the structure to the highest reasonably anticipated loading*. From the preceding the safety factor is finally defined as

* Baker, L.L.A.; "The Work of the European Committee on Concrete" Structural Engineering, vol 36, January 1958

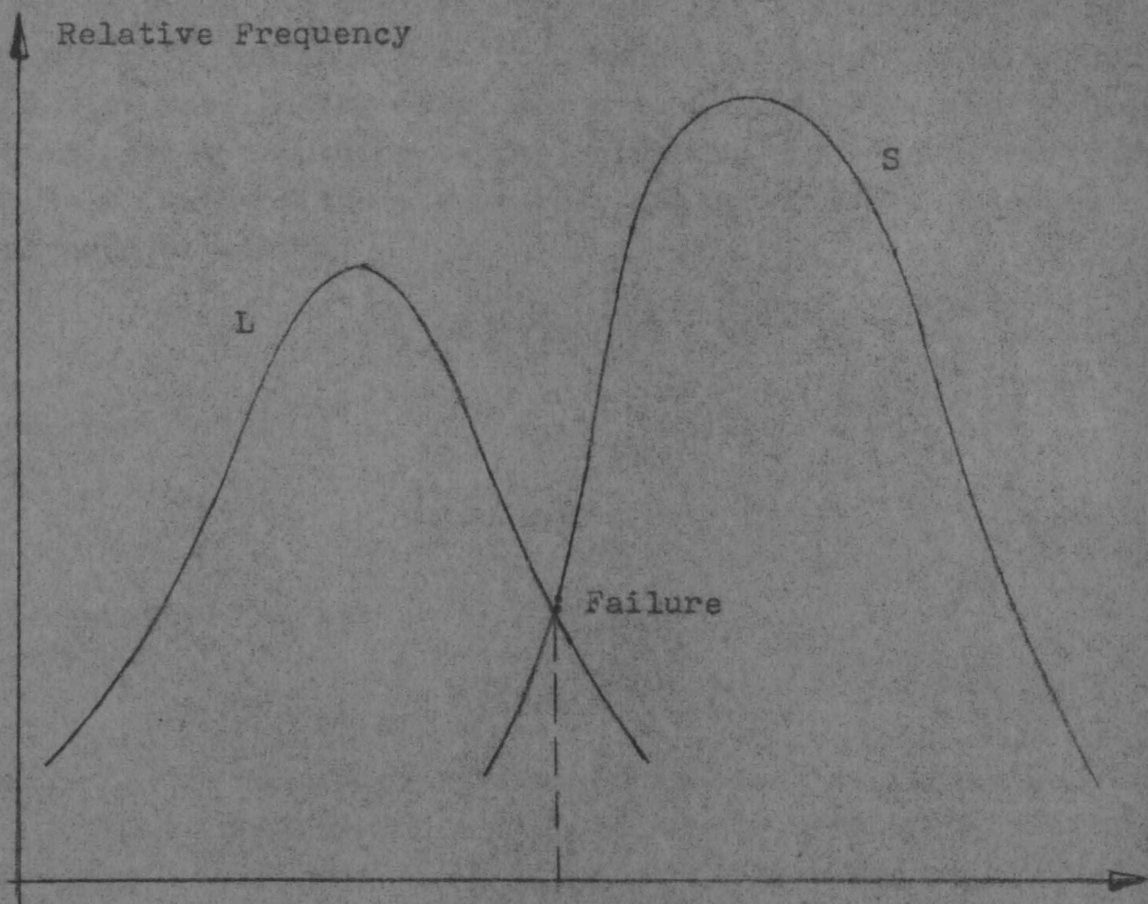


Fig. 1

$$N = \frac{S}{L} > 1 \qquad (3)$$

which is apparently the same equation as given by Freudenthal.

It should now be appropriate to recall that if there are a certain number of events, with their individual chances of occurance not affecting each other, then the probability of them all occuring is equal to the product of all the separate probabilities.(23)(24)(26)

$$P(B_1, B_2, B_3, \ldots B_r) = \prod_{i=1}^{r} P(B_i) \qquad (10)$$

Under the light of this explanation, and referring back to Brown (14), it is observed that L is multiplied by a factor i which varies according to the certainty and adequacy of the loading date, and S is divided by a factor j , which varies according to the certainty and adequacy of the strength data of the structure. Then in order to obtain a balanced situation,

$$iL = \frac{S}{j} \qquad (11)$$

and cross multiplying

$$\frac{S}{L} = ij \qquad (11a)$$

and substituting eq. 3

$$N = ij \qquad (11b)$$

which is the factor of safety from the engineering viewpoint. In order to satisfy the social factor of safety the provided strength S is increased by dividing it by a factor k. Then the balanced situation is

$$iL = \frac{S}{jk} \qquad (12)$$

and cross multiplication yields

$$\frac{S}{L} = ijk \qquad (12a)$$

and substituting eq. 3

$$N = ijk \qquad (12b)$$

where N is the final factor of safety. The values of i, j, and k are necessarily greater than unity for normal selections of reasonable loads and strengths.

C- COSTS

The acceptable probability of failure or of unserviceability can be specified arbitrarily, in relation to the expected number of load applications, or on the basis of an economic balance between the cost of increasing safety, and the cost of failure (6) . The choice of the specification of the probability of failure depends on the importance and cost of the structure. It is evident that the probability of failure of an important structure or of a structure the failure of which would endanger human life should be practically zero. Monumental structures, too,(14) which are built with a long span of life in consideration should also have high safety factors. Unless the design (6) could be based on an absolute upper limit of the load intensity, and on a distribution of the significant strength parameter known to be definitely limited at a minimum value; structures based on such bases would however be inefficient and uneconomical. It is, moreover, not quite logical to attempt to design for zero probability of failure or unserviceability as other risks to a structure, fire or earthquake are accepted as inevitable. Although all measures are taken to reduce their incidence, or the damage associated with their occurance, the risk remains real, though quite small, and its magnitude finds numerical expression in terms of insurance rates or of that part of the rates which represent the pure risk. Hence comparison of the risk of failure or unserviceability with other risks of similar consequences may provide a first rough rule for the specification of an acceptable probability of failure or of unserviceability.

A more rational rule can be derived from the requirement that the combination of an improbably high load intensity with an improbably low carrying capacity, i.e. strength or ultimate load, which would produce failure of the structure should not be expected to occur during the lifetime of the structure. Hence the probability of its occurance should be very small; and the return period of such a combination should be very much longer than the life of the structure, or its return number should be much higher than the expected total number of load applications.

The economic approach to the specification of an acceptable failure risk is based on the assumption that the cost of failure or of unserviceability can be considered as a charge against the structure equal to the capitalized total cost of failure C_F or of unserviceability C_S, multiplied by the probability of its occurance P_F or P_S. As the cost of failure, and to a lesser extend, the cost of unserviceability are necessarily functions of the probability of failure P_F or of unserviceability P_S, for which the structure has been designed, the optimal economic probabilities of failure or of unserviceability should make the sum of the initial cost of the structure A, and the capitalized cost of failure or of unserviceability a minimum. Hence the conditions for the optimal probabilities are:(17)

$$A + P_S C_S \text{ ---- a minimum} \qquad (13)$$

and

$$A + P_F C_F \text{ ---- a minimum} \qquad (14)$$

so differentiating with respect to respective probabilities

$$\frac{dA}{dP_S} + C_S + P_S \frac{dC_S}{dP_S} = 0 \qquad (15)$$

and also

$$\frac{dA}{dP_F} + C_F + P_F \frac{dC_F}{dP_F} = 0 \qquad (16)$$

The total cost of failure is made up of two components, one which is independent of P_F and includes all direct and indirect losses resulting from the failure, as well as the cost of repair, if such repair is possible, and the other which represents the cost of reconstruction when repair is impossible, and depends on P_F as does the initial cost.

Thus the economically optimal risk of failure or of unserviceability for which the structure should be designed decreases with increasing ratio of cost of such failure or unserviceability to the initial cost, as well as with increasing service life and decreasing interest rate.

D- FACTORS AFFECTING SAFETY

The influences that affect the factor of safety have been divided into two groups, bearing relatively upon load and carrying capacity 1 5 12 . In addition to these principal groups, an intermediate group embodying influences of the method and procedure of the computation of strain has been introduced.

The fundamental types of influences are enumerated in the following outline, separated into classes and groups in conformity with the aforementioned divisions.

Group A:

Causes of Fluctuation of Loads

1- Uncertainty and variability of loading conditions
   a- Dead load
   b- Live load with dynamic effects

2- Uncertainty and variability of external conditions that are independent of the load
   a- Change of temperature
   b- Wind forces
   c- Uncertainty of behaviour of subsoil

3- Variation of rigidity

4- Imperfection of methods andshortcomings of assumptions
   a- Accuracy of method and the tolerances of numerical calculation
   b- Inadequacy of assumptions concerning initial and boundary conditions

Group B:

Causes of Fluctuations of the Carrying Capacity

5- Uncertainty and inaccuracy of the assumed mechanism of carrying capacity
    a- Inaccuracy or inadequacy of the conceived mechanism
    b- Variability of resistance limits of materiels

6- Variation of structural dimensions

It is deemed necessary to go deeper into some of the aforementioned items in order to obtain a better understanding of the mechanism of safety factor.

1- Uncertainty and variability of loading conditions:
The fluctuations of the dead load values are caused by the variability of the specific weight of the materiels concerned, and of the dimensions of the component parts of the structure. Therefore, the combined consideration of the tolerances of weight and dimensions will lead to a reasonably accurate estimate of the variation of the dead load.
On the part of the live loads, however, it can be said that among the many instances of inconsistency between the accuracy and the refinement of design methods, and the inadequacy and vagueness of the underlying assumptions, specifications concerning live loads are the most conspicuous. The values recommended are arbitrary and for the most part, have no structural significance, emanating often from non-engineering quarters. A clear cut distinction between everyday conditions of service and fictitious conditions is noticeably lacking. It is

imperative that the service conditions be identifed as reliably as the strain or resistance of the structure. It must be admitted, however, that the difficulties of defining the service load on which a judicious and rational design can be based are impressive, owing to the considerable fluctuations of static weights as well as to the variability of the accompanying dynamic effects. The design live load should:

a- Represent the most probable actual service conditions of reasonably high frequency of occurance, and
b- Provide for such increase of the service load as may be expected, reasonably, during the assumed period of service of the structure, considering the general trend as well as local circumstances.

Allowance should be made for possible, yet comparatively infrequent, adverse conditions of service, in the form of appropriate ranges of fluctuations about the design value, and this allowance should be embodied in the factor of safety.

In general the conventional specifications do not fulfill any of the foregoing requirements, the design load stipulated represents for the most part, highly unfavourable conditions, the occurance of which is not only infrequent, but also improbable. Safety factors currently adopted are neither concerned with, nor do they refer to , the probability that such design live loads may actually occur, so that the numerical safety attributed to the design has no relation to the actual safety of the structure.

Live load specifications should distinguish between two groups of buildings with regard to the purpose they are expected to serve; namely buildings for human occupancy, and buildings for industrial purposes and storage. The principal difference between these two groups is the character of the service loads. In buildings for human occupancy, the intensity of dead loads and live loads fluctuate widely, 20 whereas in industrial, and especially in storage buildings, there is an almost constant maximum load. Since the simultaneous occurance of such load on all parts of the structure represents actual service conditions, the probability that other conditions may occur is irrelevant.

2- Changes in temperature:

Temperature changes in a structure involve chance fluctuations. The probability that extreme temperatures will occur is comparatively small. Changes of temperature within a restricted range, however will occur frequently enough to justify their inclusion in any study concerning service conditions.

3- Wind forces:

The magnitude and distribution of wind forces are chance events. Within a certain range of wind pressures the frequency is comparatively high, and consequently, the structure can be expected to withstand wind pressure permanently and with a degree of safety equal to that required for the principal service loads. Extreme values of wind forces, such as those, that occur during severe storms, need only be expected on rare occasions. Hence wind forces should be introduced into the actual design only as far as moderate and frequently occuring values are concerned, whereas maximum effects should be anticipated in the factor of safety. Long time records of wind velocities will furnish the materiel for the preparation of distribution curves from which design values and fluctuations can be derived.

4- Uncertainty in the behaviour of subsoil:

The foundation is an integral part of the structure, and its behaviour, mainly depending on the behaviour of the subsoil, exerts considerable influence on the state of strain in statically indeterminate structures. The strain resulting from such movement in the structure is either entirely neglected in the conventional design, being based mostly on the assumption that the supports always remain fixed, or that they settle uniformly by equal amounts, or it is regarded as of secondary importance. The movement of subsoil being a reality the attitude is not justified.

5- Variation of structural rigidity:

The rigidity of a structural member is expressed as the the product of a sectional value; such as the moment of inertia, or the area, and the respective modulus of elasticityof the materiel divided by the length of the member. The degree of rigidity will vary, therefore, with changes of either the modulus of elasticity or the sectional value, or both, fluctuations in length being insignificant.

6- Accuracy of methods and tolerances of numerical computation:

Methods of computing strain characteristics are generally based upon the assumption of perfect elasticity. If short cut methods are applied as substitutes for a strict theoretical analysis, the range of error must be compensated by an appropriate increase in the general factor of safety. The functional dependence of the safety factor on the relative crudeness of the design method is thus established. The maximum range of possible error must be ascertained reliably and subsequently introduced as a constituent chance fluctuation.

7- Boundary conditions:

Boundary conditions are expressed in terms of displacement, and the angular deflection of end sections. The boundary conditions for the supports may be derived from observations of the movement of the foundations and the subsoil. There are cases, however, in which the boundary conditions are chosen rather arbitrarily, or to simlify the mathematics, without satisfactory evidence that the choice is rational.
The manner in which the boundary conditions for end sections are assumed will considerably influence the actual safety of the members subject to bending and buckling. It is doubtful that any assumed degree of fixity at the ends will ever be realized precisely in the actual structure. Therefore, minimum and maximum values of the probable angular deflection at the ends should be estimated, and the mean of these values should be selected as the design value, the range of variation about the mean being considered a constituent part of the safety factor.

8- Uncertainty and inaccuracy of the mechanism of resistance:

Structural resistance is defined, generally, with referance to the state of strain which delimits the fitness for service of a structure. It is principally a function of two variables, a sectional characteristic, and the resistance, -strength- of the materiel. For practically every structural materiel this resistance is influenced by the rate at which strain,-load-is applied, so that the same materiel which appears permanently tough and deformable under low strain rates, will appear perfectly elastic and brittle under impact strain, -load- and will show increased strength. In certain cases resistance is not a function of sectional strength alone, but also of structure's stability, as in members subject to axial compression.

9- Uncertainty and variation of resistance limits of structural materiels:

The material limits of the engineering materiels is affected by the strain field, the rate and duration of the load, the amplitude of load cycles, and their number. The influence of the strain field is generally expressed by the conceived mechanism of resistance. Effects of local concentrations of strain due to notches, welds or rivets are not embodied in the general mechanism of strength -resistance- but are considered by empirically reducing the resistance of the materiel in the undisturbed field of strain.

E- TOLERANCES

The issue of tolerances and dimensional errors(1)(10)(12)(16)(22) is one of the factors that influence the determination of safety factor, though its effect is almost negligable in standard procedure. According to Collier(12)" there is but one source of dimensional errors: construction. None are significant except sometimes in reinforcements." Dimensional adjustments would be included as specific instructions where applicable. Whereas Rosenblueth(12) states that the total depths of members may be expected to be on the average 0.2 inches greater than the specified depths with normal distribution, and a standard deviation of 0.4 inches, widths should be subject to lesser variation. For columns the cross sectional area would be expected to be systematically a little larger than nominal, with deviations independent of size. Rosenblueth also goes on to add that effects of average workmanship, and average accuracy of computations are better accounted for, by the use of additive constants than multiplicative correction factors.

According to Freudenthal(1) the ranges of variation of the sectional resistance values can be derived from the tolerances ofthe linear dimensions. Since these tolerances do not generally exceed 2% to 3% the fluctuations of areas, and section moduli are within the range of 4% to 6% about the mean.

Design as a process of prediction, however, is not concerned only with the initial state of the structure, but also with its change in the future owing to deterioration and corrosion. These influences must be considered, therefore, in the computation of the initial safety factor.

TABLE 1

ITEM		Nichols	Com. 622
1- Variation from the vertical			
a- in the lines and surfaces of columns, piers and walls of arrises in general	in 10 ft		1/4"
	in 20 ft	1/2"	3/8"
	in 40 ft		3/4"
	in 50 ft	1"	
b- for exposed corner columns, control joints, and grooves	in 20 ft	1/4"	1/4"
	in 40 ft	1/2"	1/2"
2- Variation from straight, or correct position in plan of building lines, position of columns, and wall dimensions.	in 20 ft	1/2"	1/2"
	in 40 ft	1"	1"

Furthermore John R. Nichols(22), has stated that tolerances must be related on the one hand to their reasonableness, to the cost of building within them; and on the other hand to the need for, and the value of close adherance to the indicated line and grade. In judging any proposed tolerance, therefore, one must inquire first if it is necessary or sufficient to build within this tolerance in order that the structure may have suitable appearance, may satisfy the purpose for which it is erected, and may be structurally safe; and secondly if such accuracy can be obtained reasonably, that is without unjustified cost.

Nichols(16) also gives certain values for tolerable limits in construction. The tolerable limit is 1/2 inches of deviation, according to Nichols, in the construction of reinforced concrete columns. However the ACI Committee 622 gives a limit between 1/4 inches to 3/4 inches. Table 1 makes a complete comparison of values given by Nichols and Committee 622 for various structural members.

F- STATISTICAL DEFINITIONS

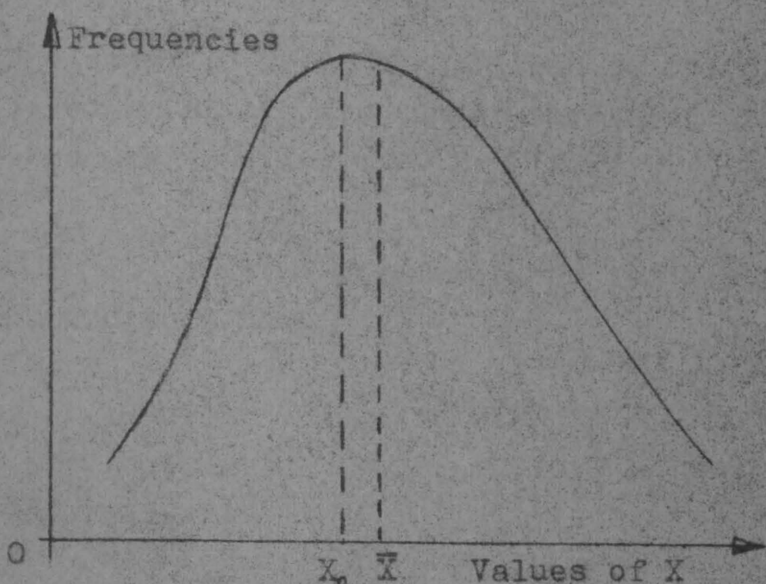


Fig. 2

In order that data and statistical information can be used effectively (1), they will have to be condensed and presented in the form of a frequency distribution. Subsequently the frequency distribution should be approximated by an algebraic distribution function $f(\overline{X},X,\sigma)$ of the respective quality characteristic X (Fig. 2), containing the three principal statistics of the frequency distribution computed from the n observations X_n of the characteristic X; namely its expected or mean value is

$$\overline{X} = \frac{\sum_{n=1}^{\infty} X_n}{n} \tag{17}$$

The mean value divides the area under the frequency distribution into two equal halves, whereas X_o, the mode value gives the maximum frequency the distribution attains. The standard deviation of the quality characteristic X is given as:

$$\sigma = \sqrt{\frac{\sum_{n=1}^{\infty}(X_n - \overline{X})^2}{n}} \qquad (18)$$

The standard deviation (σ) can be defined as the radius of gyration of the area under the theoretical probability curve about the center, i.e. $\overline{X}$. (7)(9)

If the parameters of the distribution function are chosen such that

$$\int_{-\infty}^{+\infty} f(X,\overline{X},\sigma)dX = 1 \qquad (19)$$

the integral

$$\int_{a}^{b} f(X,\overline{X},\sigma)dX$$

is a measure of the probability that an individual value X_n will occur within the limits X = a, and X = b.

The shape of the frequency distribution of the material property is an indication of the conditions under which this property has been or is being produced. In manufacturing processes an effort is generally made to attain a definite value of characteristic property. This objective can be expressed either by stipulating a minimum value of the property or by imposing a limit on the fluctuations about its most probable value. In either case the objective cannot be achieved in any degree without imposing controls on the manufacturing process, in order to insure the systematic elimination of assignable causes of fluctuation, thus reducing the range of dispersion of the individual values. The visible effect of such control is to produce a unimodal frequency distribution of the characteristic bell shape, which in the ideal case of maximum control, may be expressed by the normal or Gaussian law

$$f(X,\overline{X},\sigma)=\phi_o(z)=\frac{1}{\sqrt{2\pi}}e^{-0.5z^2} \qquad (20)$$

where

$$z = \frac{X_n - \overline{X}}{\sigma} \qquad (21)$$

The distribution functions of real properties generally deviate more or less from the ideal form expressed by eq. 20 even if the degree of production control imposed is high. They are mostly asymmetric and the most probable or peak value of the distribution, its modal value X_o, differs from the mean value $\overline{X}$.(Fig. 2)

When imposing a standard of control on the manufacturing process of a given physical object, the question necessarily arises as to whether the value aimed at should be stipulated as the minimum limit or whether the inevitable dispersion of values should be controlled about an expected average. It can easily be shown that the setting of minimum limits, although still the prevalent conventional procedure, is considerably less effective in safeguarding such a limit, and less reliable than the control of fluctuations about an expected mean or most probable value. The current opinion that adequate safety can be insured only by specifying minimum limits is unfounded.(1)

Even under adequate conditions of control the occurance of values lower than the stipulated minimum is still possible; this probability is computable by a statistical interpretation of the tests. The only possibility of reducing this probability practically to zero is by controlling the dispersion of values about an expected mean value, thus defining the minimum limit in terms of the probable extreme range of fluctuations below this mean value. The distribution functions reproducing the results of the check tests determine the range of fluctuation $\pm j\sigma$ about the mean, or the most probable value of X, in such a manner as to enable the designer to make the probability of an individual value smaller than $\bar{X} - j\sigma$, lower than any arbitrarily chosen figure.

Experience should be represented by the mean value $\bar{X}$ of the observations, together with its standard deviation σ. Only in cases in which the available data are unsuitable for statistical interpretation owing to deficiencies in scope and character will it be necessary to stipulate extreme limits for all possible values of the phenomenon or the characteristic considered. Only than will it be necessary to limit the maximum range of fluctuation by assumptions based on the collateral information and to introduce the midpoint between the extremes as a tentative approximation of the most probable value.(1)

G- MATHEMATICAL ANALYSISOF SAFETY FACTOR*

If L denotes the intensity of the applied loads under specified operating conditions, and S the intensity of same type of load that would produce failure of the structure or unserviceability i.e. strength, the following relationship (6)(17)(18)

$$r = S - L \leqslant 0 \qquad (22)$$

determines the conditions of failure or of unserviceability; the inequality $r > 0$ is the condition of survival. If a "factor of ignorance" $m > 1$ is introduced, eq. 22 becomes

$$r = S - mL \leqslant 0 \qquad (23)$$

Because the statistical variations of S and L are independent, the probability of failure P_F is the product of the probability P_L of occurance of a load intensity exceeding L, and the probability P_S of occurance of a failure load $S \leqslant L$. (23) (24) (26) Conversely the probability of survival is the probability P'_L of occurance of a load intensity less than L, and the probability P'_S of occurance of a failure load $S > L$. Both probabilities can be defined either with respect to failure or with respect to unserviceability; provided that the failure load i.e. ultimate load S is related to actual failure or to unserviceability of the structure respectively.

When L and S are normally distributed with means $\bar{L}$ and $\bar{S}$ and standard deviations σ_L and σ_S and if two variables x and y are defined by

* The analysis presented is a more detailed expansion of the study given by Freudenthal 6 .

$$x = \frac{L - \bar{L}}{\sigma_L} \qquad (24)$$

and

$$y = \frac{\bar{S} - S}{\sigma_S} \qquad (25)$$

so that the distribution functions of x and y are according to eq. 20

$$p(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} x^2\right) \qquad (26a)$$

$$p(y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} y^2\right) \qquad (26b)$$

Hence the probability of any combination of values (x, y) defining a certain value of the variable can be obtained as follows: From eq.s 24 and 25

$$r = S - L = (\bar{S} - y\sigma_S) - (x\sigma_L - \bar{L})$$

and regrouping

$$r = S - L = (\bar{S} - \bar{L}) - (x\sigma_L - y\sigma_S) \qquad (27)$$

and the probability is given by

$$p(x) \cdot p(y) = \frac{1}{2\pi} \exp\left[-\frac{1}{2}(x^2 + y^2)\right] \qquad (28)$$

and the lines of constant probability are therefore given as:

$$p(x) \cdot p(y) = p_o \qquad (29)$$

$$p_o = \frac{1}{2\pi} \exp\left[-\frac{1}{2}(x^2 + y^2)\right] \qquad (30)$$

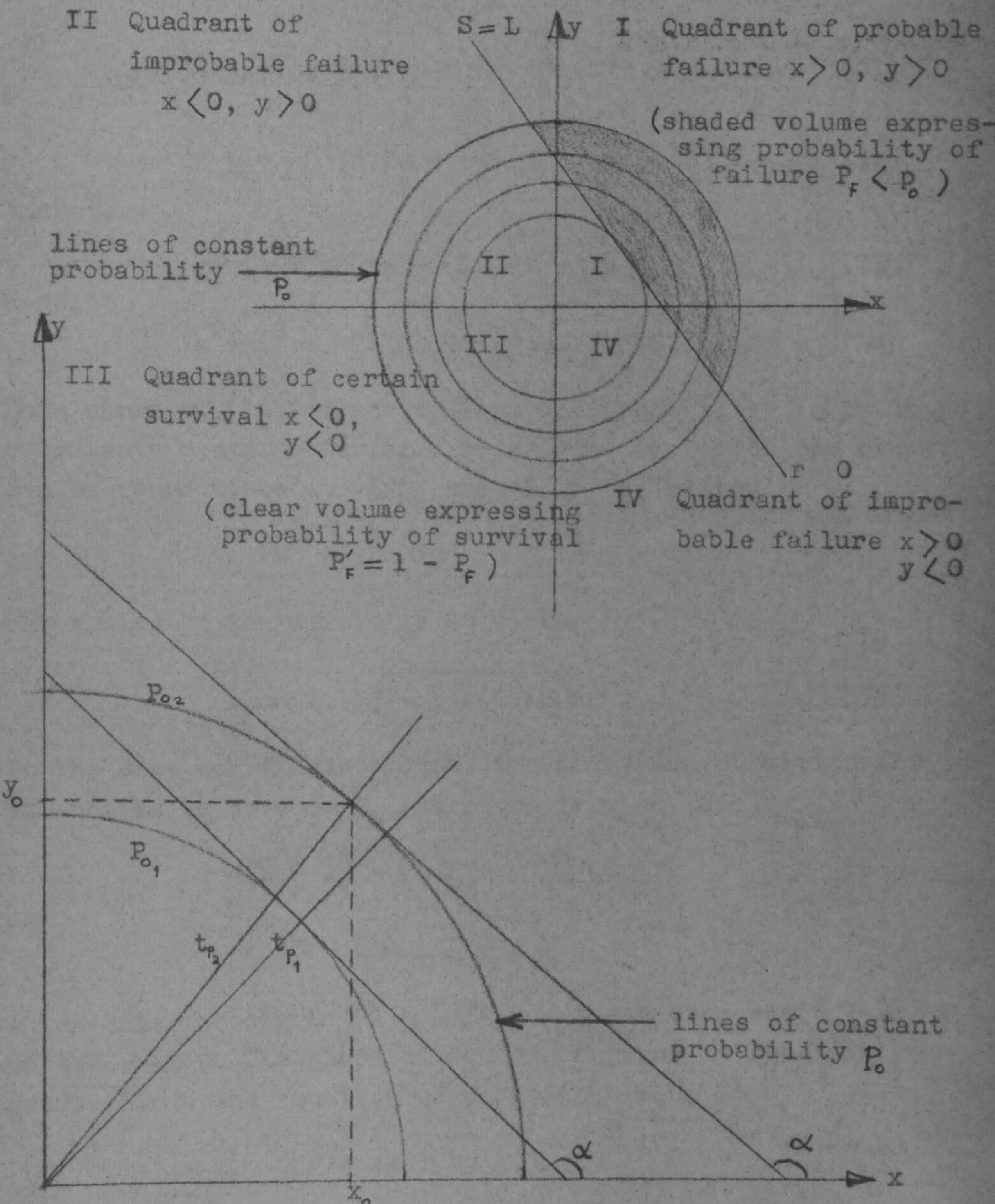


Fig. 3

In Fig. 3 the x and y axes represent the eq.s 24 and 25.

taking the logarithms of both sides of eq. 30

$$\ln p_o = \ln 1 - \ln 2\pi - \frac{1}{2}(x^2 + y^2)\ln e \qquad (31)$$

$$\ln p_o = -\ln 2\pi - \frac{1}{2}(x^2 + y^2) \qquad (32)$$

therefore

$$\frac{1}{2}(x^2 + y^2) = -(\ln 2\pi + \ln p_o) \qquad (33)$$

hence

$$x^2 + y^2 = -2\ln(2\pi p_o) = -4.606\log(2\pi p_o) \qquad (34)$$

This shows that the lines of constant probability in the (x,y) coordinate system describe circles. The center of the circle can be found to be at $\bar{S}/\sigma_S$ and $\bar{L}/\sigma_L$ by letting

$$x^2 + y^2 = 0$$

The radius of the circle is equal to

$$t_p = 2.15\sqrt{-\log(2\pi p_o)} \qquad (35)$$

In the same coordinate system, the necessary condition for balance is to have $r = 0$. Therefore from eq. 27

$$r = (\bar{S} - \bar{L}) - (x\sigma_L + y\sigma_S) = 0 \qquad (36)$$

and

$$\bar{S} - \bar{L} = x\sigma_L + y\sigma_S \qquad (37)$$

which cuts the length $(\bar{S} - \bar{L})/\sigma_L$ on the x-axis and $(\bar{S} - \bar{L})/\sigma_S$ on the y-axis. The slope of eq. 37 is found by analytic geometry. Since the equation of a straight line is

$$y = mx + b \qquad (38)$$

with a slope equal to m. Therefore eq. 37 can be written as

$$y\sigma_S = -x\sigma_L + (\bar{S} - \bar{L}) \qquad (39)$$

and dividing eq. 39 by σ_S

$$y = -\frac{\sigma_L}{\sigma_S}x + \frac{(\bar{S} - \bar{L})}{\sigma_S} \qquad (40)$$

Hence the slope of eq. 37 is $(-\sigma_L/\sigma_S)$. The condition that this line of balance eq. 40 be tangent to the circle of constant probability eq. 34 is that the radius of the circle is perpendicular to the line of balance at the point of tangency. Defining the angle, the lines of balance form counter-clockwise with respect to the x-axis, as α, the complementary angle is found to be $(\pi - \alpha)$ Fig.s 3 and 4.

From Fig. 4, with the knowledge that angle(1) is equal to 90 degrees, angle(2) is found to be

$$180 - [90 - 180 + \alpha] = \alpha - 90$$

and from there angle(3) is found tobe

$$90 - (\alpha - 90) = \pi - \alpha$$

The cosine for angle(3) is found to be

$$\cos(\pi - \alpha) = \frac{b}{a} \qquad (41)$$

At $x = 0$ eq.37 gives y as equal to $(\bar{S} - \bar{L})/\sigma_S$ which is equal to the quantity (a) in Fig. 4 . Since (b) is the radius of the circle of constant probability, it can be written as t_p. Therefore eq.41 becomes

$$\frac{\bar{S} - \bar{L}}{\sigma_S} \quad \frac{t_p}{\cos(\pi - \alpha)} \qquad (42)$$

From Fig. 4 it can be discerned that the area of the triangle AOB can be found by two wholly different methods, yielding identical answers, i.e.

$$\text{Area} = ac = b\,\overline{AB} = bd$$

At $y = 0$ eq.37 gives x as equal to $(\bar{S} - \bar{L})/\sigma_L$ which is in turn

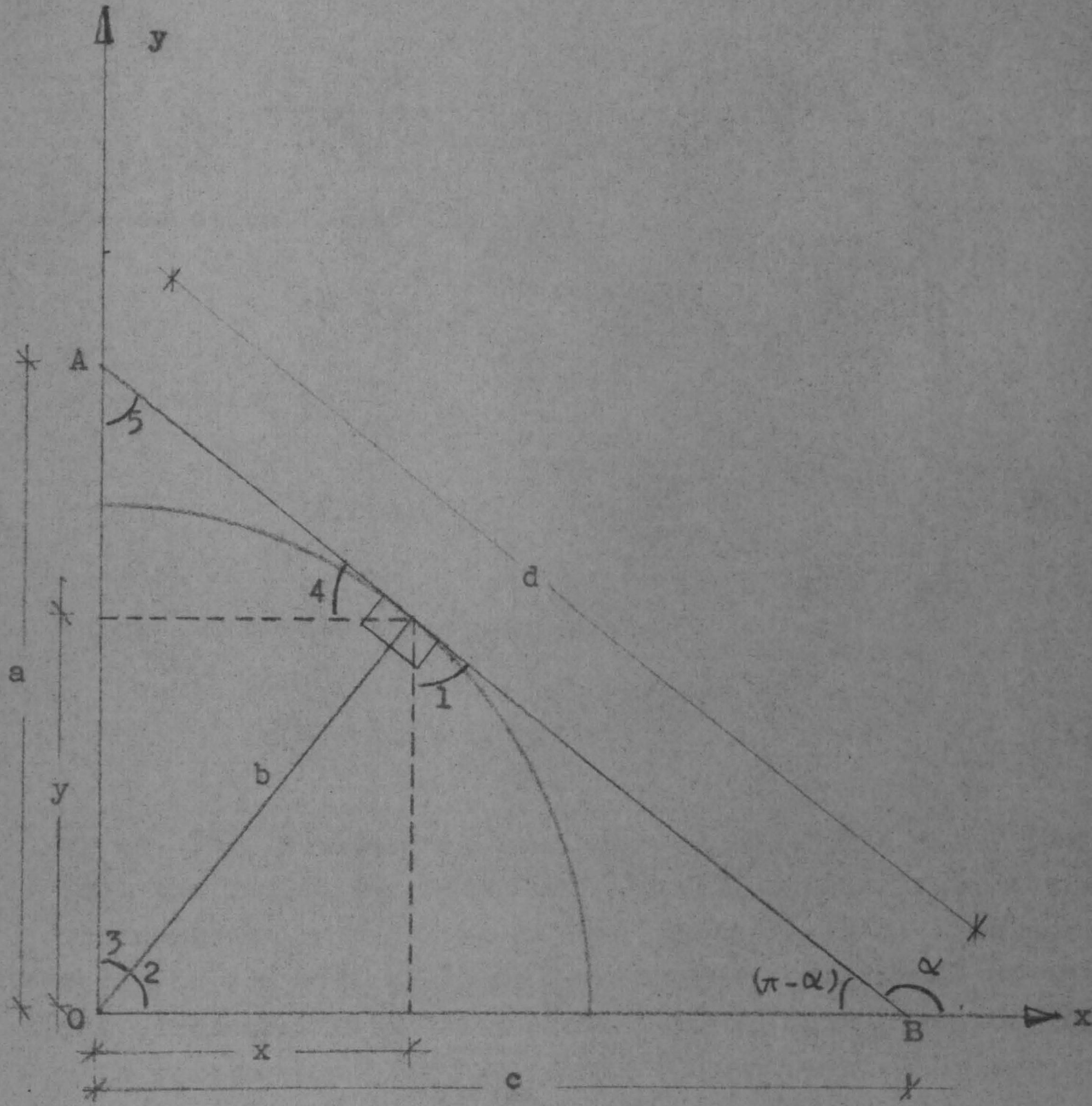


Fig. 4

equal to (c). Then (d) can be evaluated by the following formul

$$d = \overline{AC} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Therefore (d) is given as

$$\left(\frac{\bar{S} - \bar{L}}{\sigma_S}\right) \cdot \left(\frac{\bar{S} - \bar{L}}{}\right) = t_p \sqrt{(\bar{S} - \bar{L})^2 \left[\frac{1}{\sigma_L^2} + \frac{1}{\sigma_S^2}\right]}$$

Simplifying

$$\frac{(\bar{S}-\bar{L})^2}{\sigma_L . \sigma_S} = t_p(\bar{S}-\bar{L}\sqrt{\frac{\sigma_L^2+\sigma_S^2}{\sigma_L^2 \cdot \sigma_S^2}}$$

the equation transforms into

$$(\bar{S}-\bar{L}) = t_p\sqrt{\sigma_L^2+\sigma_S^2}$$

and dividing by σ_S

$$\frac{(\bar{S}-\bar{L})}{\sigma_S} = t_p\sqrt{1+\left(\frac{\sigma_L}{\sigma_S}\right)^2} \qquad (43)$$

which determines the relationship

$$x\sigma_L - y\sigma_S = t_p\sigma_S\sqrt{1+\left(\frac{\sigma_L}{\sigma_S}\right)^2} \qquad (44)$$

by eq. 36 , eq. 44 separates the region of "failure" $(r<0)$ from the region of "survival" $(r>0)$. On this line, the failure combination with the highest probability of occurance, which is equal to p_o, is defined by the coordinates of the point of contact with the circle, i.e. x_o and y_o can be found with reference to Fig. 4 . Angle (2) and angle (5) are both equal to $(\alpha - 90)$. The cosine of angle (2) is

$$\cos(\alpha-90) = \frac{x_o}{t_p}$$

and of angle (5) is

$$\cos(\alpha-90) = \frac{a}{d}$$

i.e.

$$\frac{x_o}{t_p} = \frac{\dfrac{\bar{S} - \bar{L}}{\sigma_s}}{\sqrt{(\bar{S} - \bar{L})^2 \left[\dfrac{1}{\sigma_L^2} + \dfrac{1}{\sigma_s^2} \right]}} \qquad (45a)$$

and

$$\frac{x_o}{t_p} = \frac{(\bar{S} - \bar{L})}{\sigma_s (\bar{S} - \bar{L}) \sqrt{\left[\dfrac{1}{\sigma_L^2} + \dfrac{1}{\sigma_s^2} \right]}}$$

cancelling

$$x_o = \frac{t_p}{\sqrt{1 + \left(\dfrac{\sigma_s}{\sigma_L} \right)^2}} \qquad (46a)$$

and the sine of angle (2) is

$$\sin (\alpha - 90) = \frac{y_o}{t_p}$$

and of angle (5) is

$$\sin (\alpha - 90) = \frac{c}{d}$$

i.e.

$$\frac{y_o}{t_p} = \frac{\dfrac{\bar{S} - \bar{L}}{\sigma_L}}{\sqrt{(\bar{S} - \bar{L})^2 \left[\dfrac{1}{\sigma_L^2} + \dfrac{1}{\sigma_s^2} \right]}} \qquad (45b)$$

$$\frac{y_o}{t_p} = \frac{\bar{S} - \bar{L}}{\sigma_L (\bar{S} - \bar{L}) \sqrt{\left[\dfrac{1}{\sigma_L^2} + \dfrac{1}{\sigma_s^2} \right]}}$$

cancelling

$$y_o = \frac{t_p}{\sqrt{1 + \left(\frac{\sigma_L}{\sigma_S}\right)^2}} \qquad (46b)$$

and since the x_o, and y_o values from eq.s 46a and 46b can be substituted into eq.s 24 and 25 in place of x and y one obtains

$$\sigma_S y_o = \bar{S} - S \qquad (47a)$$

and

$$\sigma_L x_o = L - \bar{L} \qquad (47b)$$

The probability of failure P_F for all combinations (L,S) along the line r=0 is given by that part of the volume under the surface of rotation p(x,y) which is cut off by a plane along the line r=0, perpendicular to the x-y plane. Because the critical failure combination (x_o, y_o) has the highest probability of occurence along the line r=0, it represents the combination to be used in design. In a hypothetical design based on the mean values

$$L = \bar{L} = S = \bar{S}$$

for which the line r=0 according to eq. 37 degenerates into an arbitrary straight line of negative slope passing through the origin, the chances of failure or survival would be even and equal to 0.5 as in this case the plane along r=0 bisects the volume of P(x,y).

The difference $(\bar{S} - \bar{L})$ represents the "margin of safety", and the ratio $\bar{S}/\bar{L}$ is the factor of safety with respect to the mean values of the applied load and failure load, associated with the design for the critical "failure combination

$$L = \bar{L} + x_o\sigma_L \qquad (47c)$$

and

$$S = \bar{S} - y_o\sigma_S \qquad (47d)$$

If $\overline{L}, \sigma_L, \sigma_s$ and p_o are given, the required mean value $\overline{S}$, and the associated safety factor, which should be defined rather as the probability factor, $\overline{N}$ where $\overline{N} = \overline{S}/\overline{L}$, can be computed from eq.s 42 and 43. This is the core of the design problem, and conversely if either $\overline{S}, \sigma_L, \sigma_s$, and p_o or $\overline{S}, \overline{L}, \sigma_L, \sigma_s$, are given, the permissable mean value of $\overline{L}$ or the probability of failure can be computed, which represents the core of the rating problem

Eq. 43 can be rewritten so as to permit the direct evaluation of the probability factor $\overline{N}$.

$$\frac{\overline{S} - \overline{L}}{\sigma_s} = t_p \sqrt{1 + \left(\frac{\sigma_L}{\sigma_s}\right)^2} \qquad (43)$$

squaring both sides

$$\frac{\overline{S}^2 - 2\,\overline{S}.\overline{L} + \overline{L}^2}{\sigma_s^2} = t_p^2 \left[1 + \left(\frac{\sigma_L}{\sigma_s}\right)^2\right] \qquad (48)$$

multiplying by σ_s^2

$$\overline{S}^2 - 2\,\overline{S}.\overline{L} + \overline{L}^2 = t_p^2.\sigma_s^2 + t_p^2.\sigma_L^2 \qquad (48a)$$

regrouping eq. 48a

$$\overline{S}^2 - t_p^2.\sigma_s^2 - 2\,\overline{S}.\overline{L} + \overline{L}^2 - t_p^2.\sigma_L^2 = 0 \qquad (48b)$$

such that

$$\overline{S}^2\left(1 - t_p^2.\frac{\sigma_s^2}{\overline{S}^2}\right) - 2\,\overline{S}.\overline{L} + \overline{L}^2\left(1 - t_p^2.\frac{\sigma_L^2}{\overline{L}^2}\right) = 0 \qquad (48c)$$

and dividing eq. 48c by $\overline{L}^2$

$$\frac{\overline{S}^2}{\overline{L}^2}\left(1 - t_p^2.\frac{\sigma_s^2}{\overline{S}^2}\right) - 2\,\frac{\overline{S}}{\overline{L}} + \left(1 - t_p^2.\frac{\sigma_L^2}{\overline{L}^2}\right) = 0 \qquad (48d)$$

Since the coefficient of variation V is defined as (23) (24) (26)

$$V = \frac{\sigma}{\bar{X}} \qquad (49)$$

and

$$\bar{N} = \frac{\bar{S}}{\bar{L}}$$

Eq. 48d can be written as (6) (17) (18)

$$(1 - t_p^2 V_s^2)\bar{N}^2 - 2\bar{N} + (1 - t_p^2 V_L^2) = 0 \qquad (50)$$

Eq. 50 enables us to calculate the ratio $\bar{N}$, the probability factor in terms of the coefficients of variation of strengths (V_s) and loads (V_L) and the confidance limit parameter t_p. Tables relating this parameter to the probability are given in the Appendix (Tables B and C) . According to eq.50 if the average strength $\bar{S}$ is kept $\bar{N}$ times larger than the average load $\bar{L}$, the probability of failure will be less than p.

Direct use of probability factor would call for a change in engineering design procedures, because it is based on mean loads and mean strengths. In practice, an estimated upper limit L for the load and an estimated lower limit S for the strength are specified as design values. Thus there is already a certain amount of safety, and the probability factor must be decreased in the following manner, to obtain the design factor of safety N such that

$$N = \frac{S}{L} = \frac{\bar{S}(1 - t_s V_s)}{\bar{L}(1 - t_L V_L)} \qquad (51)$$

and

$$N = \bar{N}.\frac{(1 - t_s V_s)}{(1 - t_L V_L)} \qquad (52)$$

Here the relation between N and $\bar{N}$ has been obtained, by writing design loads and design strengths as confidance limits which are taken from Tables A and E in the Appendix. In eq. 52 the numerator is called the understrength factor, and the denominator is called the overload factor.

CHAPTER III

EFFECT OF CONSTRUCTION TOLERANCES ON THE SAFETY OF REINFORCED CONCRETE COLUMNS

A- THEORETICAL DERIVATIONS

The main objective of the present study was to investigate the effect of dimensional discrepancies caused in construction on the structural safety.

Many factors affect and cause deviations in either the carrying capacity of the structure or in the load that the structure is required to carry, as a result of which changes occur in the safety factor. One of these factors is the dimensional discrepancies of a structure.

To analyze their effect on structural safety, a study on the dimensional discrepancies of reinforced concrete columns was conducted. Columns were chosen particularly, because;

1- The failure of columns is of primary importance, since they cost more in terms of material losses and human life.

2- The ultimate strength of columns can be expressed by a simpler equation.

3- Measurement of column dimensions is easiest, since columns are not integrated into other structural units.

Eq.s 50 and 52 are going to be used in this text in conjunction with the ultimate load equation for reinforced concrete columns which follows.

$$P = 0.85\, f_c' A_c + f_y A_s \qquad (53)$$

This equation represents the load the column can carry. Here

P is the load carrying capacity
f_c' is the ultimate compressive strength of concrete
A_c is the area of concrete
f_y is the yield strength of steel reinforcement and
A_s is the area of steel reinforcement

The equation contains four independent variables: f_c', f_y, A_c, and A_S. If $A_c f_c'$ is denoted by C, and $A_s f_y$ by Y ; then,

$$P = C + Y$$

and the mean value of P is the sum of mean values of C and Y. Hence

$$\overline{P} = \overline{C} + \overline{Y} = 0.85\, \overline{f}_c' \overline{A}_c - \overline{f}_y \overline{A}_s \qquad (54)$$

The coefficient of variation of C, V_c is the coefficient of variation of the product of two variables, and as given in Appendix, Table A, for such relations

$$V_c = \sqrt{V_{f_c'}^2 - V_{A_c}^2} \qquad (55)$$

The coefficient of variation of Y, V_y is analogous to V_c and is also obtained from Table A as follows

$$V_y = \sqrt{V_{f_y}^2 - V_{A_s}^2} \qquad (56)$$

From eq.54, P may be considered to be equal to the sum of two variables, C and Y. Table A also gives the equation of coefficient of variation of a variable, which is equal to the sum of two other variables. Hence the coefficient of variation V_p of P may be written as follows:

$$V = \sqrt{\left(\frac{V_c \cdot \bar{C}}{\bar{C} + \bar{Y}}\right)^2 + \left(\frac{V_y \cdot \bar{Y}}{\bar{C} + \bar{Y}}\right)^2} \qquad (57)$$

Substituting the values of V_c and V_y from eq.s 55, and 56 into eq.57, V_p transforms into

$$V = \sqrt{\frac{(V_{f'_c}^2 + V_{A_c}^2)\bar{C}^2 + (V_{f_y}^2 + V_{A_s}^2)\bar{Y}^2}{(\bar{C} + \bar{Y})^2}} \qquad (58)$$

Considering $\bar{C}^2/(\bar{C} - \bar{Y})^2$ alone, it is obvious that by dividing both numerator and denominator by $\bar{C}^2$ one obtains

$$\frac{1}{\left(1 + \frac{\bar{Y}}{\bar{C}}\right)^2} \qquad (59a)$$

and also dividing the numerator and denominator of $\bar{Y}^2/(\bar{C} - \bar{Y})^2$ by $\bar{Y}^2$ one obtains

$$\frac{1}{\left(1 + \frac{\bar{Y}}{\bar{C}}\right)^2} \qquad (59b)$$

but the ratio $\bar{Y} / \bar{C}$ is the ratio of A_s to A_c, i.e. the percentage of steel p. Therefore, putting these back into eq.58

$$V = \sqrt{(V_{f'_c}^2 - V_{A_c}^2)\frac{1}{(1 + p)^2} + (V_{f_y}^2 - V_{A_s}^2)\frac{p^2}{(1 + p)^2}} \qquad (60)$$

B- DETERMINATION OF STATISTICAL DATA

1- Measurements were made for 400 reinforced concrete columns. For each column three measurements of the cross section, one at the top of the column, one approximately at the center and the last at the toe of the column in question, were taken; and the smallest dimensions were considered. Then the ratio of the measured, i.e. the actual cross sectional area, to the intended, i.e. design cross sectional area was calculated for each column. The results are presented in Appendix, Table F.

The mean value of this ratio, A_A / A_D , where A_A stands for the actual cross sectional area, and A_D stands for the design cross sectional area; was determined by summing up the 400 individual values, and dividing the result by 400.

Next, the standard deviation σ of A_A / A_D was calculated, using eq. 18. Knowing the mean value, and the standard deviation of the aforementioned ratio, the coefficient of variation, V was computed by the following equation.

$$V = \frac{\sigma}{\bar{X}}$$

The same procedure was repeated for 250 pieces of steel reinforcement. Following the directions given in the above mentioned paragraphs, the coefficient of variation for the ratios of concrete, and steel areas are found out to be:

Coefficient of variation of concrete areas:

$$V_{A_c} = 0.026932$$

Coefficient of variation of steel areas :

$$V_{A_s} = 0.029178$$

2- The afore mentioned values were substituted into eq.60 where the necessary values for $V_{f_c'}$, and V_{f_y} were taken from Appendix Table D. Both of thesevalues are assumed to equal 0.15, since this value corresponds to the worse limits of good quality control of concrete, and of steel reinforcement.

Since in the Turkish reinforced concrete practice, the percentage of steel to be used varies from a minimum of 0.5 % to a maximum of 3.0 %, there are a set of values, ranging from 0.005 to 0.03, which could be substituted in place of p in eq. 60. Because of this variation of p, eq. 60 was solved six times for V_p , by assuming a different value of p at each solution, varying from 0.005 to 0.03. Values of p were taken such that they were all multiples of 0.005. The results are presented in Table 2, and Fig. 8 .

3- V_s in eq. 50 is analogous to V_p with which it has been dealt up to this point. The probability factor $\overline{N}$ was calculated from eq. 50. In this process two probabilities of failure, namely

$$p_{o1} = 10^{-4}$$

and

$$p_{o2} = 10^{-5}$$

were considered.

In eq. 50, t_p is the confidence limit parameter, and from Tables B and C in the Appendix, one determines that t_p equals 3.72 and 4.27 for the constant probability of failures of p_{o1} and p_{o2} respectively. The coefficient of variation of load V_L is taken from Table E in Appendix as 0.14, since there are no traffic loads, and by choosing the coefficient of variation of live load, instead of the dead load, a mistake on the safe side is assured. Solving eq. 50 for n different values of V_p obtained from eq.60, one obtains 2n different values of the probability factor $\overline{N}$, since two constant probabilities of failure are assumed. The resulting 12 $\overline{N}$ values that were obtained by the six V_p values afore mentioned are also tabulated in Table 2 and plotted in Fig. 9 .

4- The last step is to calculate the safety factor N, and this was done accordingly from eq. 52. For each value of $\overline{N}$, eq. 52 was re-evaluated. Here again, for each $\overline{N}$ the appropriate value of V_s (V_p) that is obtained in step two is used. V_L is the same value that was used in step three, i.e. it is equal to 0.14 . t_s and t_L are the confidance limit parameters for the variation of strength and load respectively. A probability that the strength could be 10 % below, and the load could be 10 % above their respective design values was assumed. On the basis of this assumption, t_s and t_L are taken equal to 1.28 from Table 2, Appendix. With these values in hand, eq.52 is solved for N, and 12 values, each corresponding to a different $\overline{N}$ value, are obtained. The results are again presented in Table 2, and Fig.10 .

5- In order to determine the effect of dimensional discrepancies, and to be able to make comparisons, the procedure that was disclosed in steps 1 to 4 inclusive, was repeated again. This time, however, V was taken to be equal to zero in eq.60 . Steps 2, 3, and 4 experience no other change, and the resulting values are presented in Table 3 and Fig.s 11, 12, and 13 respectively.

C- RESULTS

The detailed derivation of all the results can be found and followed through, in the calculations, Chapter AI of Appendix. Below a resume of some of the most important results is presented in a concise form. Explanatory Tables and Figures are provided in the following pages.

Mean value of the ratio A_{A_c}/A_{D_c} of concrete areas $\overline{X}_{A_c}$: 0.98893

Mean value of the ratio A_{A_s}/A_{D_s} of steel areas $\overline{X}_{A_s}$: 1.05098

Standard deviation of the ratio A_{A_c}/A_{D_c} of concrete areas, σ_{A_c}

σ_{A_c}: 0.02664

Standard deviation of the ratio A_{A_s}/A_{D_s} of steel areas, σ_{A_s}

σ_{A_s} : 0.03067

Coefficient of variation of the ratio A_{A_c}/A_{D_c} of concrete areas V_{A_c} : 0.02643

Coefficient of variation of the ratio A_{A_s}/A_{D_s} of steel areas V_{A_s} : 0.02918

The percentage of error between Tables 2 and 3 for a constant probability of p_{o1} at $p = 0.005$: 1.78 %

The percentage of error between Tables 2 and 3 for a constant probability of p_{o1} at $p = 0.030$: 3.44 %

The percentage of error between Tables 2 and 3 for a constant probability of p_{o2} at $p = 0.005$: 2.76 %

The percentage of error between Tables 2 and 3 for a constant probability of p_{o2} at $p = 0.030$: 5.26 %

TABLE 2

% of steel (p)		0.005	0.010	0.015	0.020	0.025	0.030
Coefficient of Variation of Strengths V_p		0.15165	0.15091	0.15018	0.14946	0.14875	0.14804
Probability Coefficient $\overline{N}$	Probability of Failure 10^{-4} Confidence Limit Parameter t=3.72	2.50737	2.49355	2.48026	2.46730	2.45469	2.44240
	Probability of Failure 10^{-5} Confidence Limit Parameter t=4.27	3.08568	3.06108	3.03745	3.01468	2.99234	2.97089
Safety Factor N	Probability of Failure 10^{-4} Confidence Limit Parameter t=3.72	1.71359	1.70615	1.69901	1.69206	1.68531	1.67875
	Probability of Failure 10^{-5} Confidence Limit Parameter t=4.27	2.10882	2.09354	2.08070	2.06746	2.05444	2.04201

EFFECT OF TOLERANCES CONSIDERED

TABLE 3

% of steel (p)		0.005	0.010	0.015	0.020	0.025	0.030
V_ρ Coefficient of Variation of Strengths		0.14851	0.14705	0.14562	0.14420	0.14282	0.14146
Probability Coefficient $\overline{N}$	Probability of Failure = 10^{-4} Confidence Limit Parameter t = 3.72	2.45055	2.42502	2.40134	2.37745	2.35561	2.33411
	Probability of Failure = 10^{-5} Confidence Limit Parameter t = 4.27	2.98565	2.94071	2.89980	2.85892	2.82185	2.78558
Safety Factor N	Probability of Failure = 10^{-4} Confidence Limit Parameter; t = 3.72	1.68311	1.66943	1.65685	1.64401	1.63244	1.62099
	Probability of Failure: 10^{-5} Confidence Limit Parameter t = 4.27	2.05063	2.02444	2.00076	1.97694	1.95554	1.93453

EFFECT OF TOLERANCES NEGLECTED

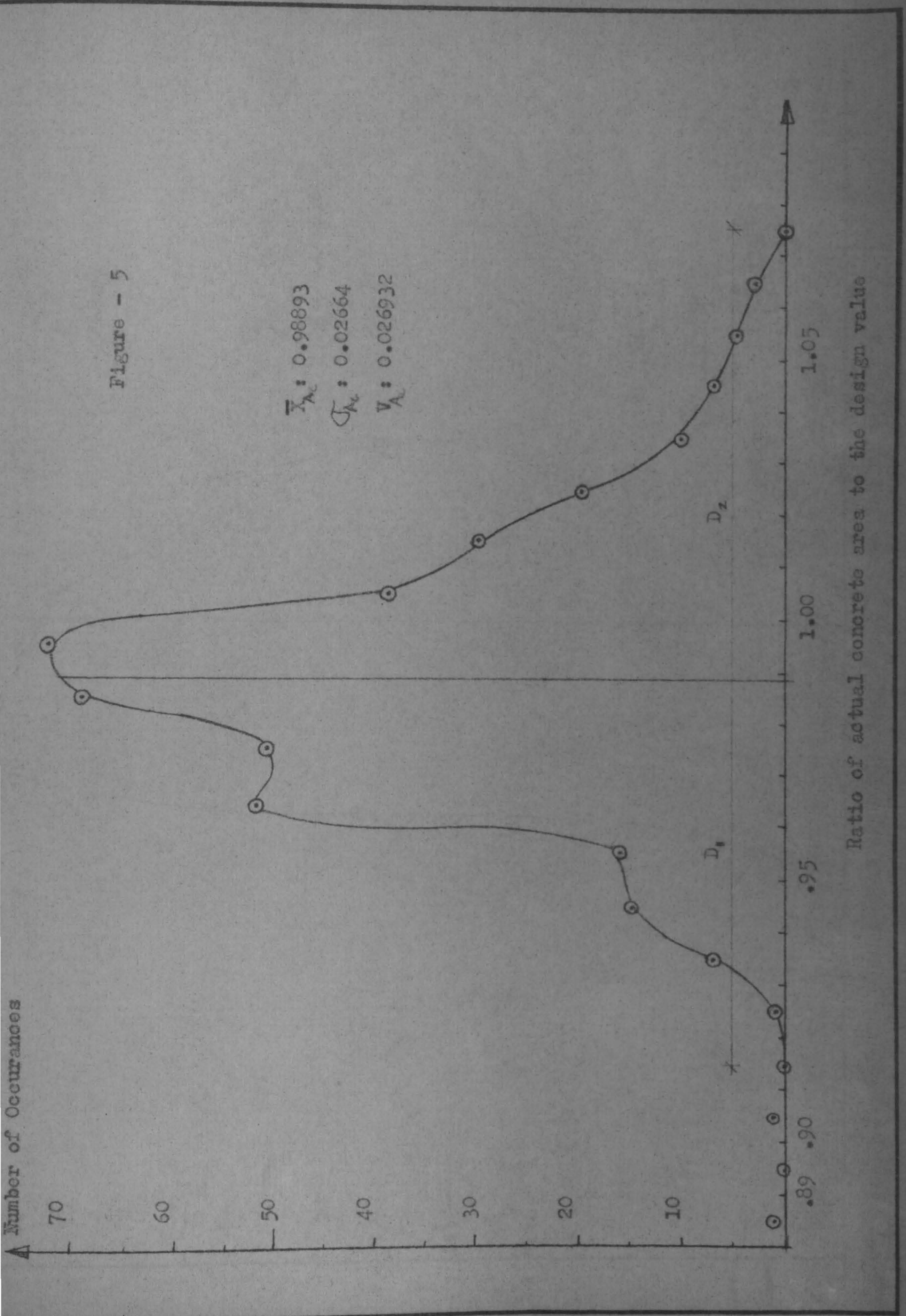

Number of Occurances
70
60
50
40
30
20
10
.89
.90
.95
1.00
1.05
D_1
D_2
$\overline{X}_{A_c}$: 0.98893
σ_{A_c} : 0.02664
V_A : 0.026932
Ratio of actual concrete area to the design value

Figure - 5

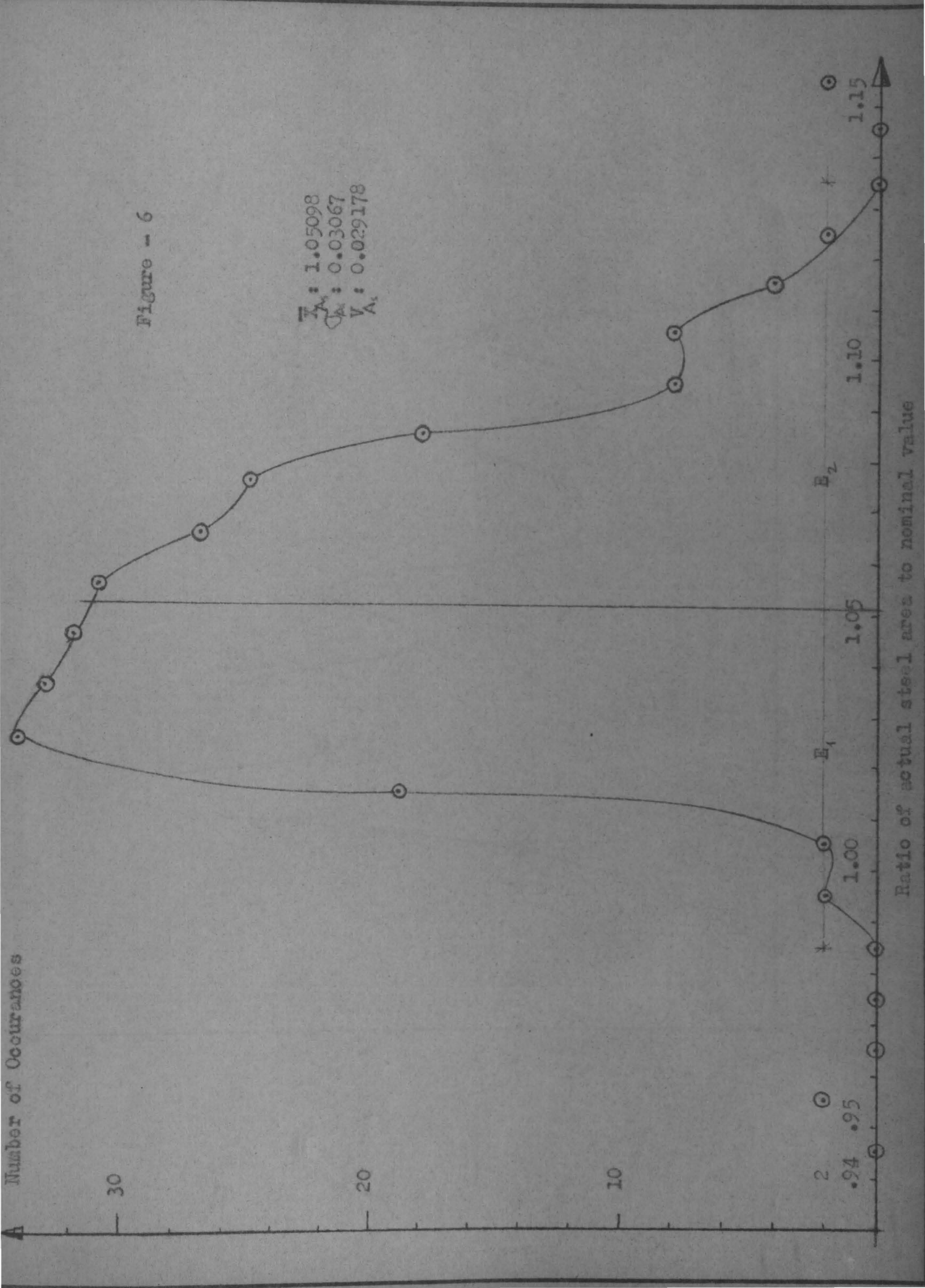

Number of Occurances
30
20
10
2
.94
.95
1.00
1.05
1.10
1.15
E_1
E_2
$\bar{X}_{A_s}$: 1.05098
σ_{A_s} : 0.03067
V_{A_s} : 0.029178
Ratio of actual steel area to nominal value

Figure - 6

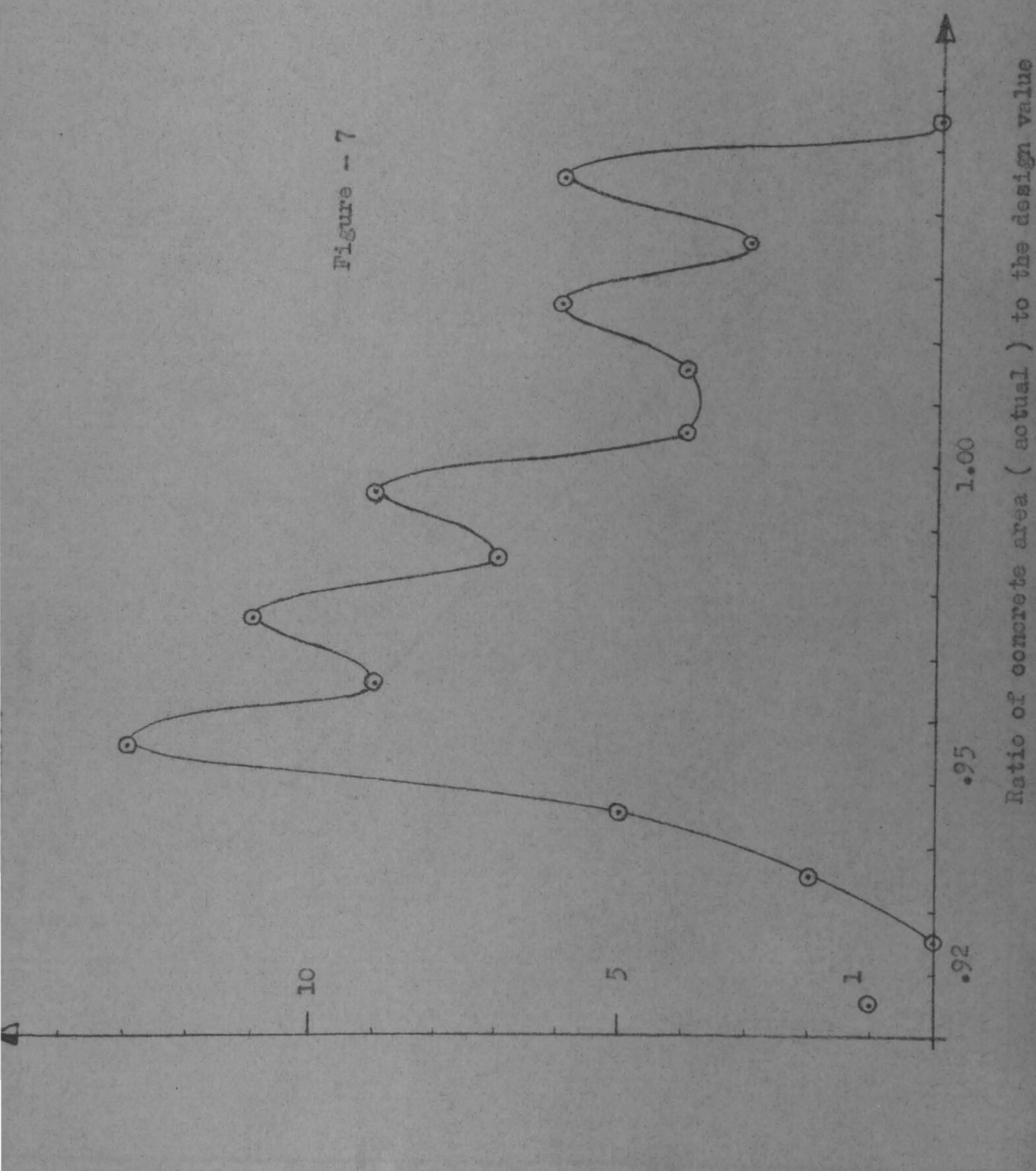

10
5
1
.92
.95
1.00
Ratio of concrete area (actual) to the design value

Figure - 7

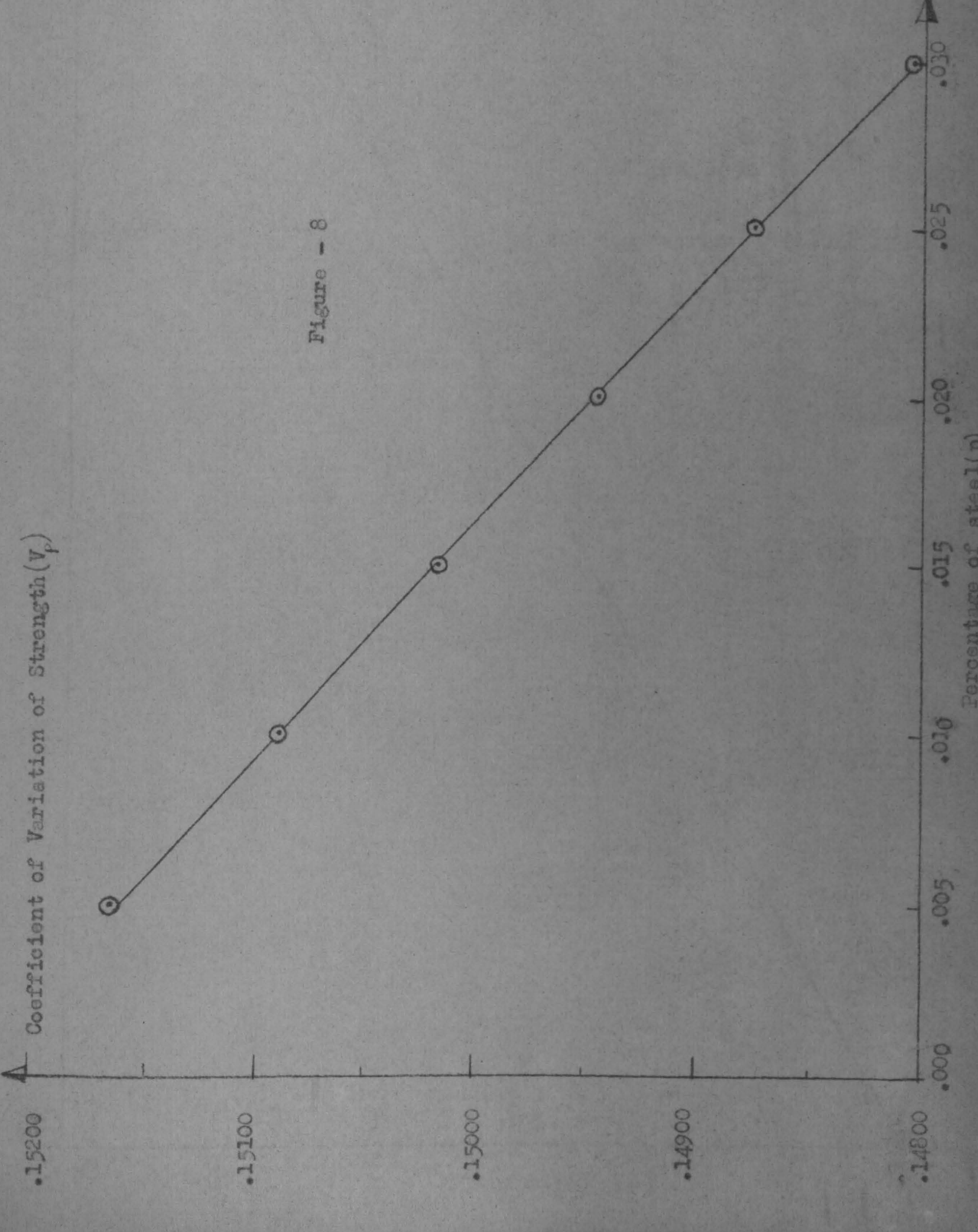

Coefficient of Variation of Strength (V_p)
.15200
.15100
.15000
.14900
.14800
.000
.005
.010
.015
.020
.025
.030
Percentage of steel (p)

Figure - 8

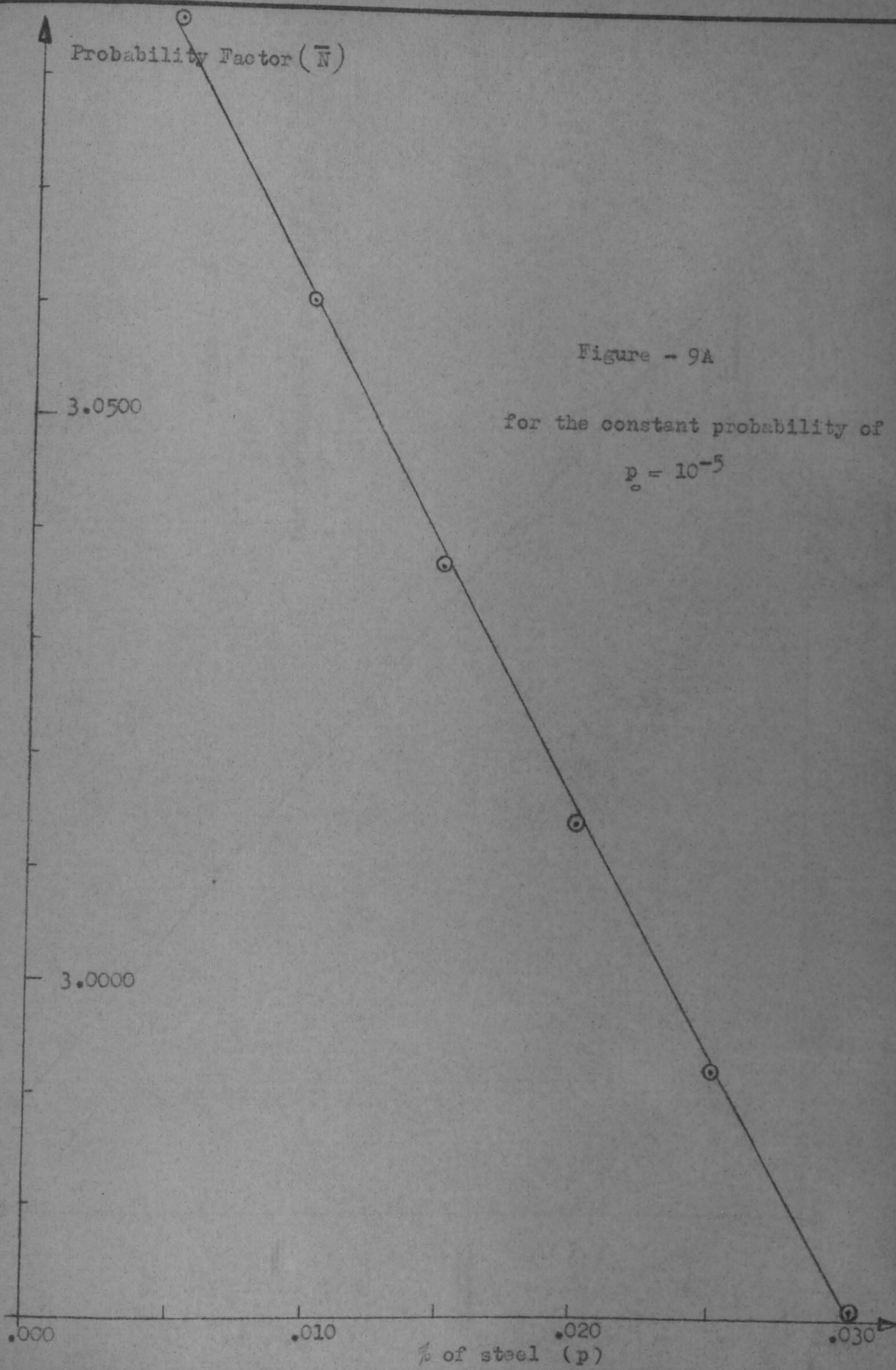


Figure - 9A

for the constant probability of

$p_o = 10^{-5}$

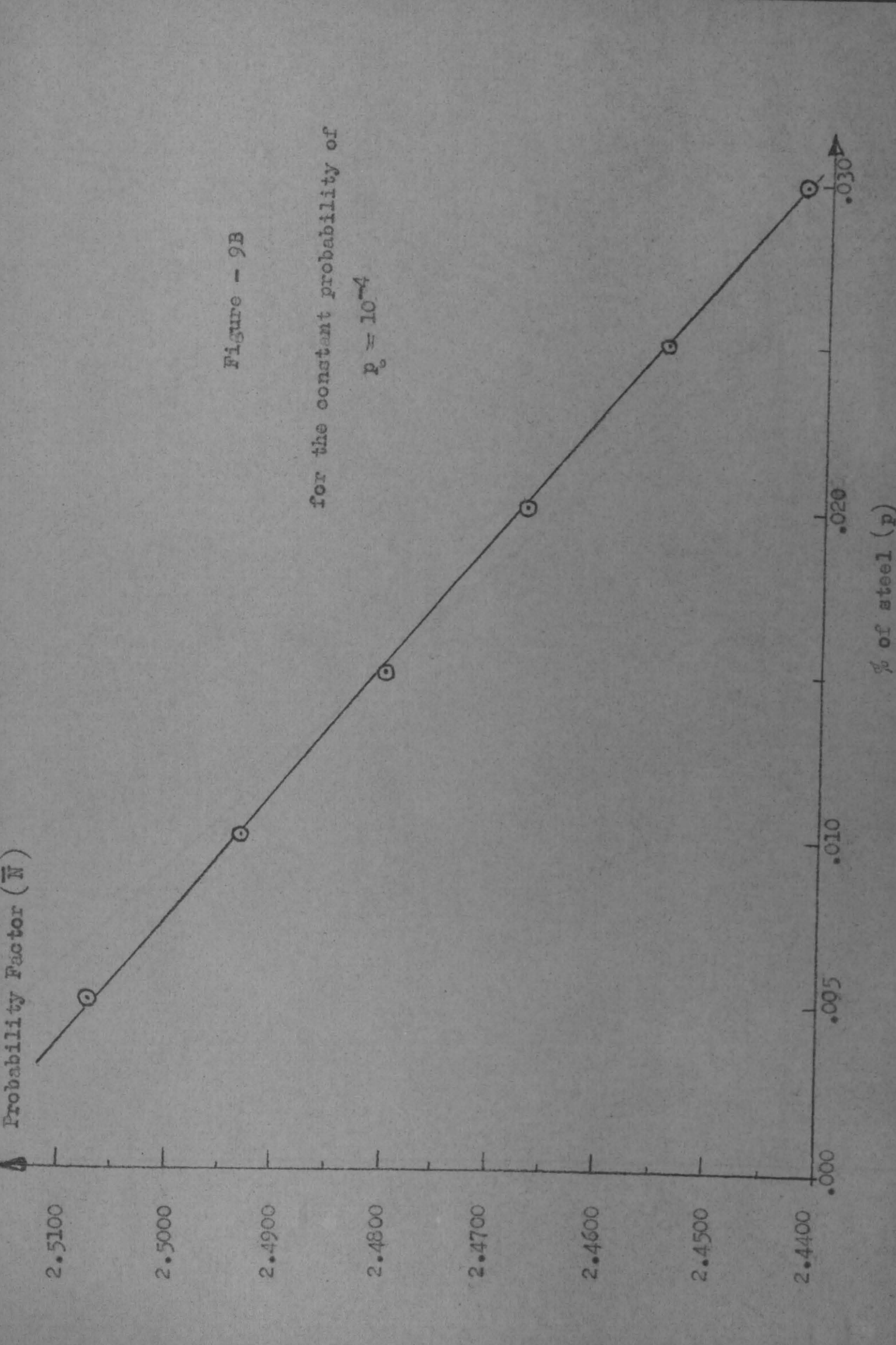


Figure - 9B

for the constant probability of

$p_c = 10^{-4}$

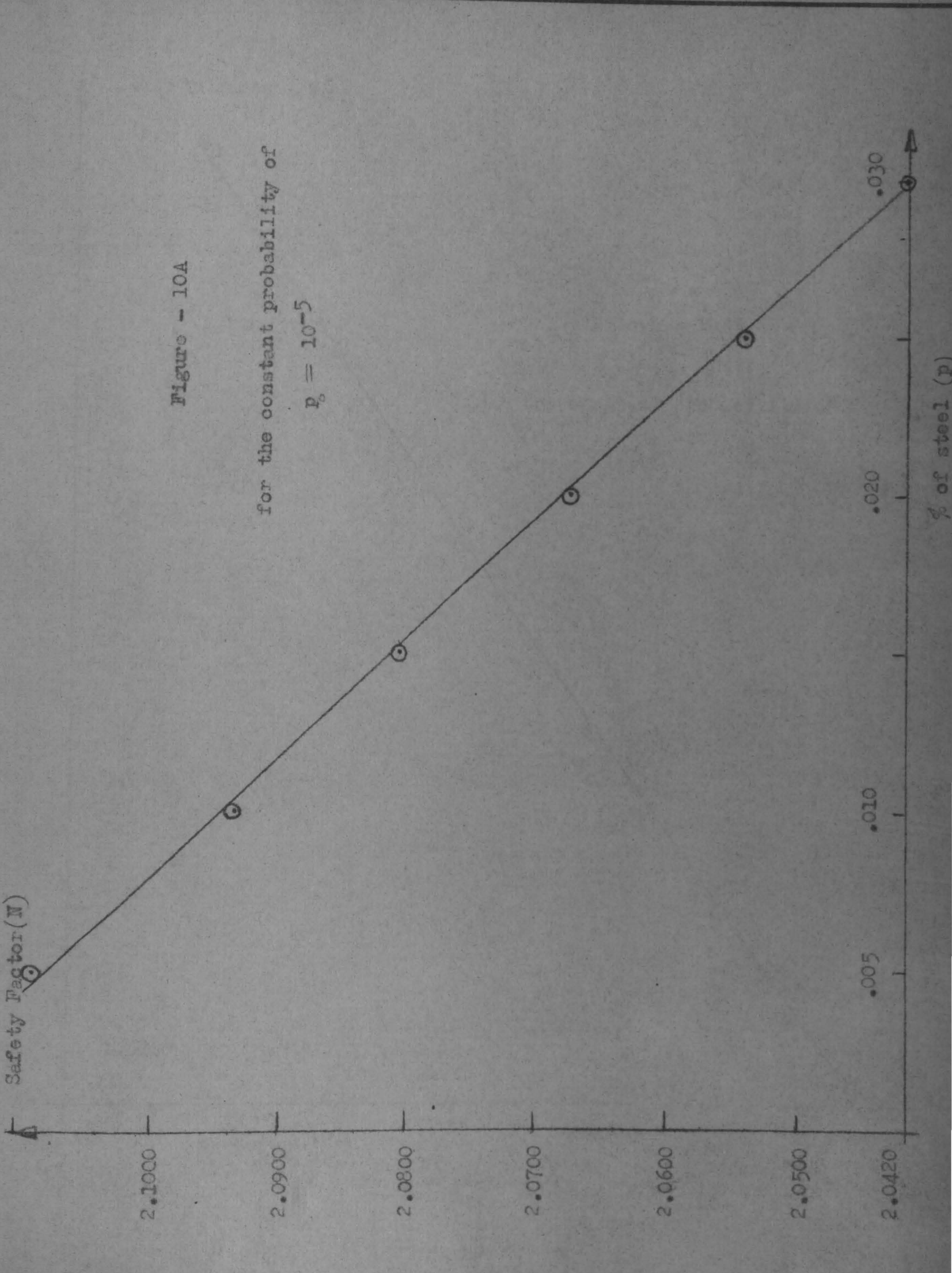
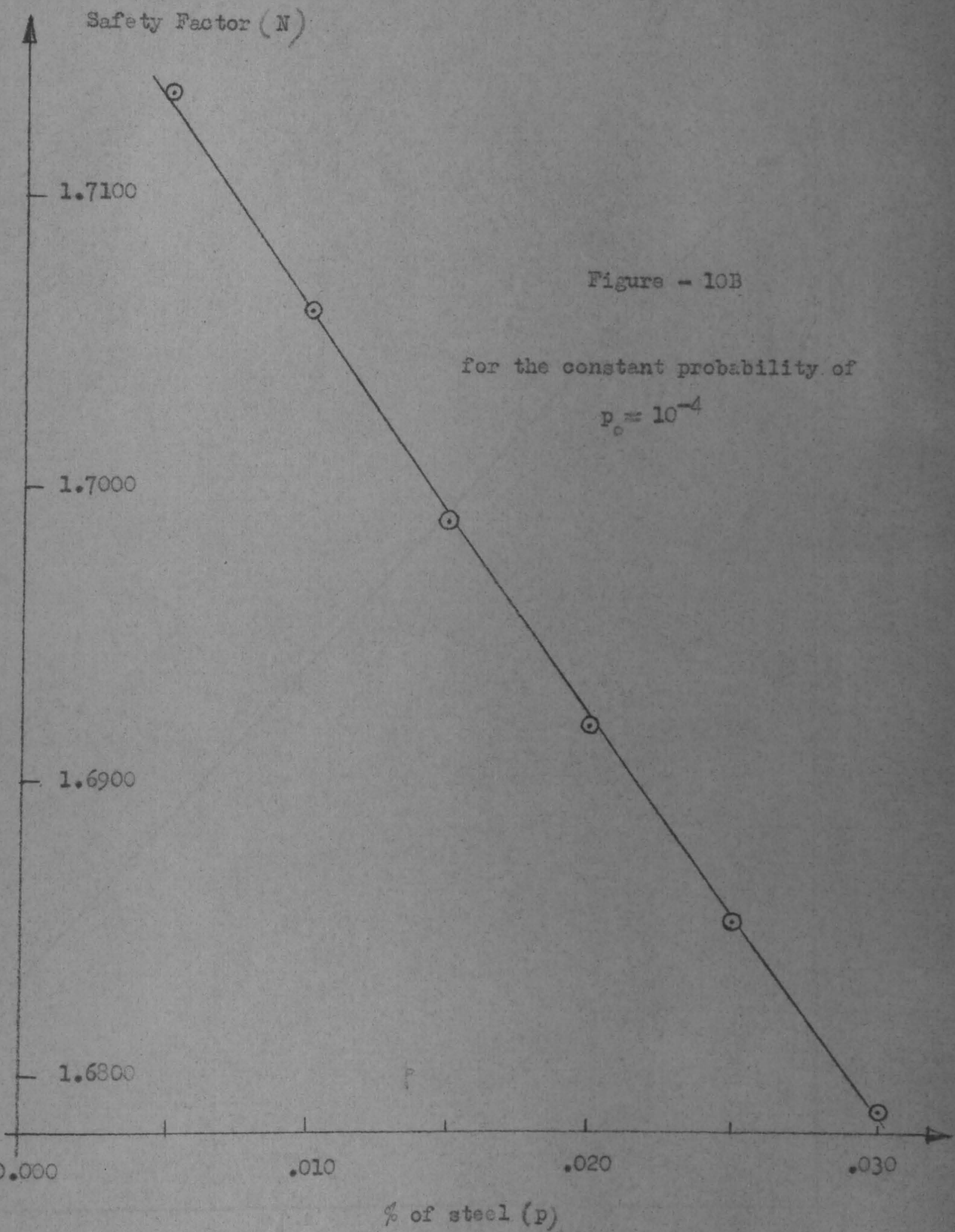


Figure - 10A
for the constant probability of
$p_s = 10^{-5}$
Safety Factor(N)
2.1000
2.0900
2.0800
2.0700
2.0600
2.0500
2.0420
.005
.010
.020
.030
% of steel (p)


Safety Factor (N)
1.7100
1.7000
1.6900
1.6800
Figure - 10B
for the constant probability of
$p_o = 10^{-4}$
0.000
.010
.020
.030
% of steel (p)

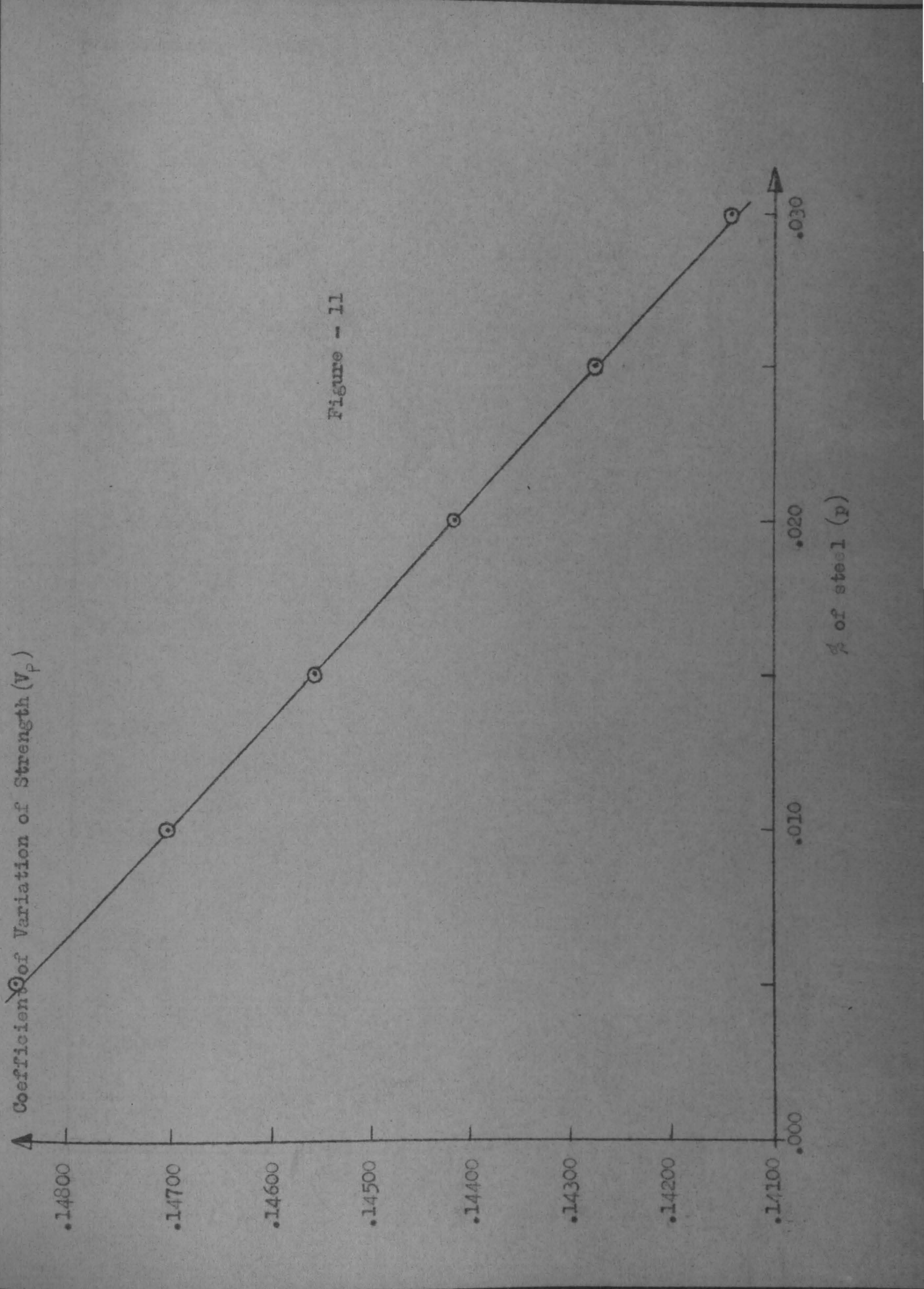
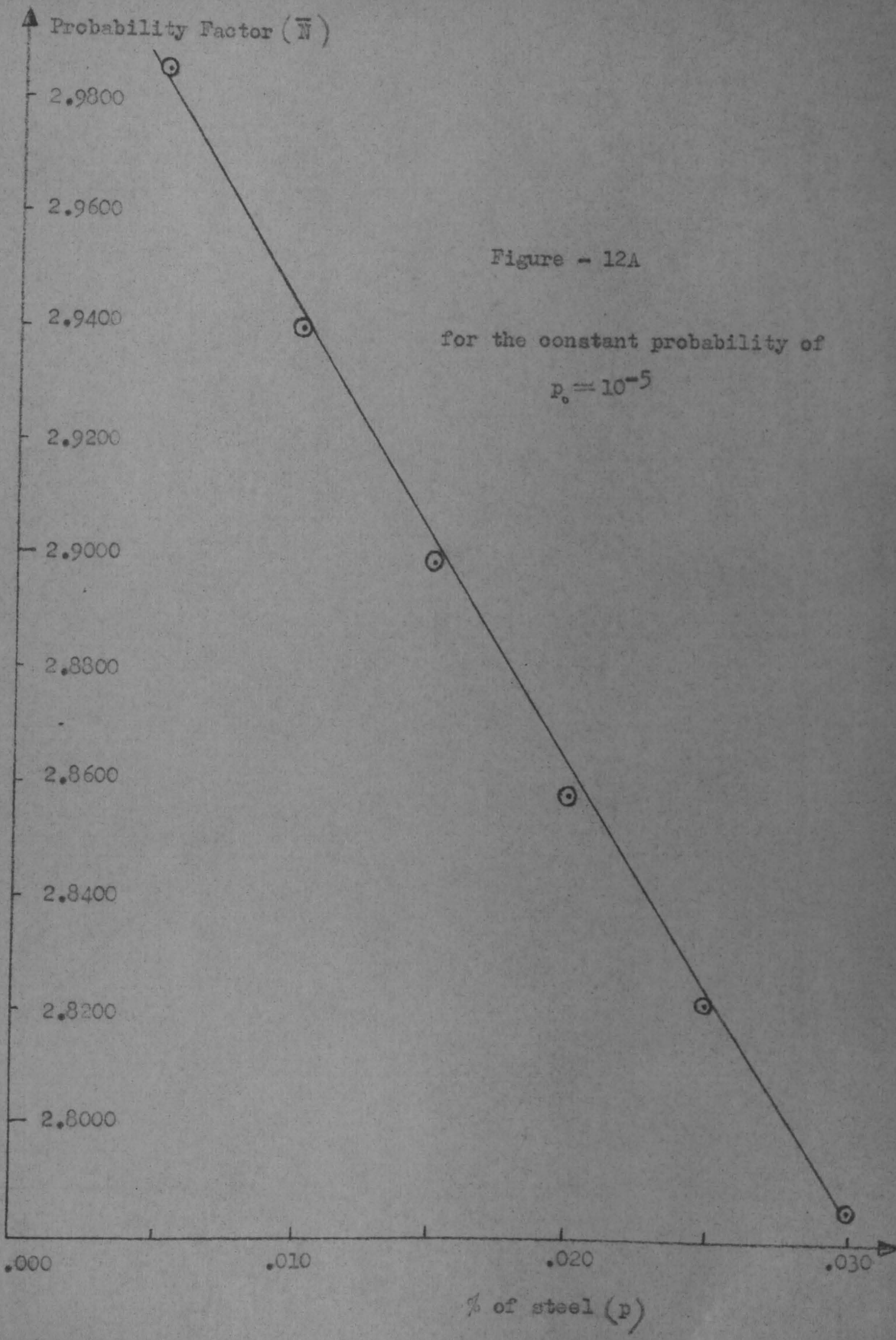

Coefficient of Variation of Strength (V_p)
.14800
.14700
.14600
.14500
.14400
.14300
.14200
.14100
.000
.010
.020
.030
% of steel (p)

Figure - 11


Probability Factor ($\overline{N}$)
2.9800
2.9600
2.9400
2.9200
2.9000
2.8800
2.8600
2.8400
2.8200
2.8000
.000
.010
.020
.030
% of steel (p)
Figure - 12A
for the constant probability of
$p_o = 10^{-5}$

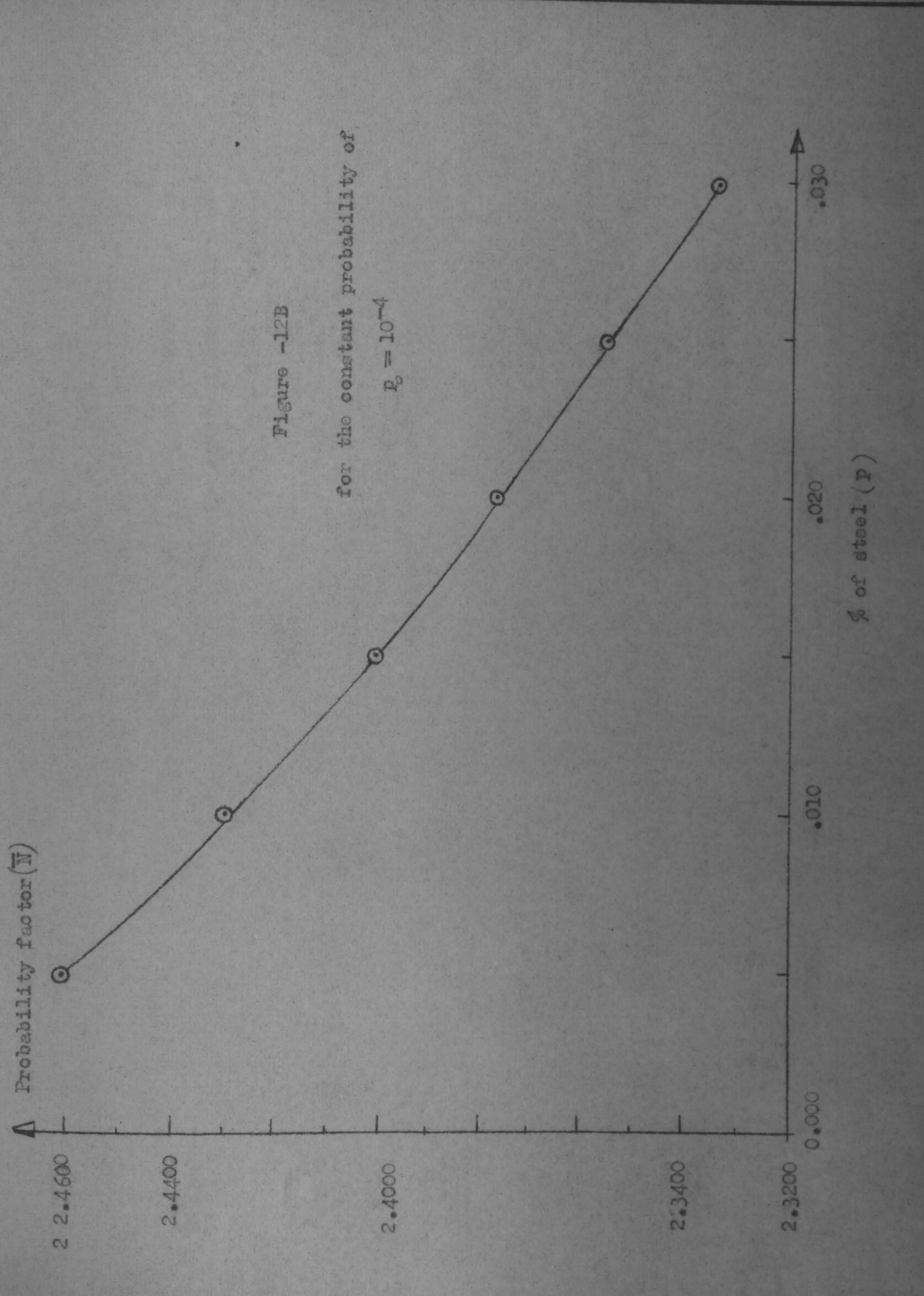


Figure -12B

for the constant probability of

$p_o = 10^{-4}$

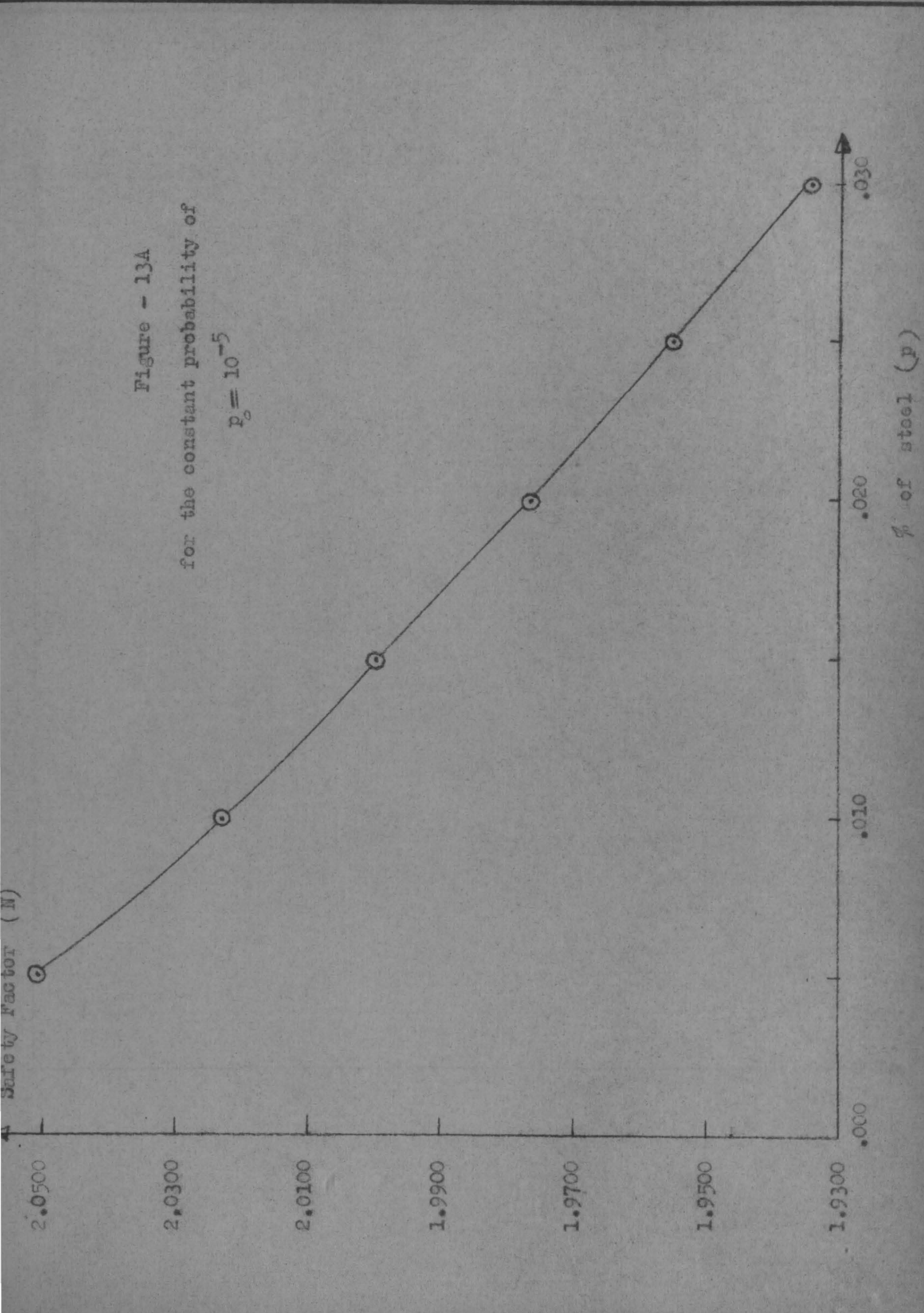


Figure - 13A
for the constant probability of
$p_o = 10^{-5}$

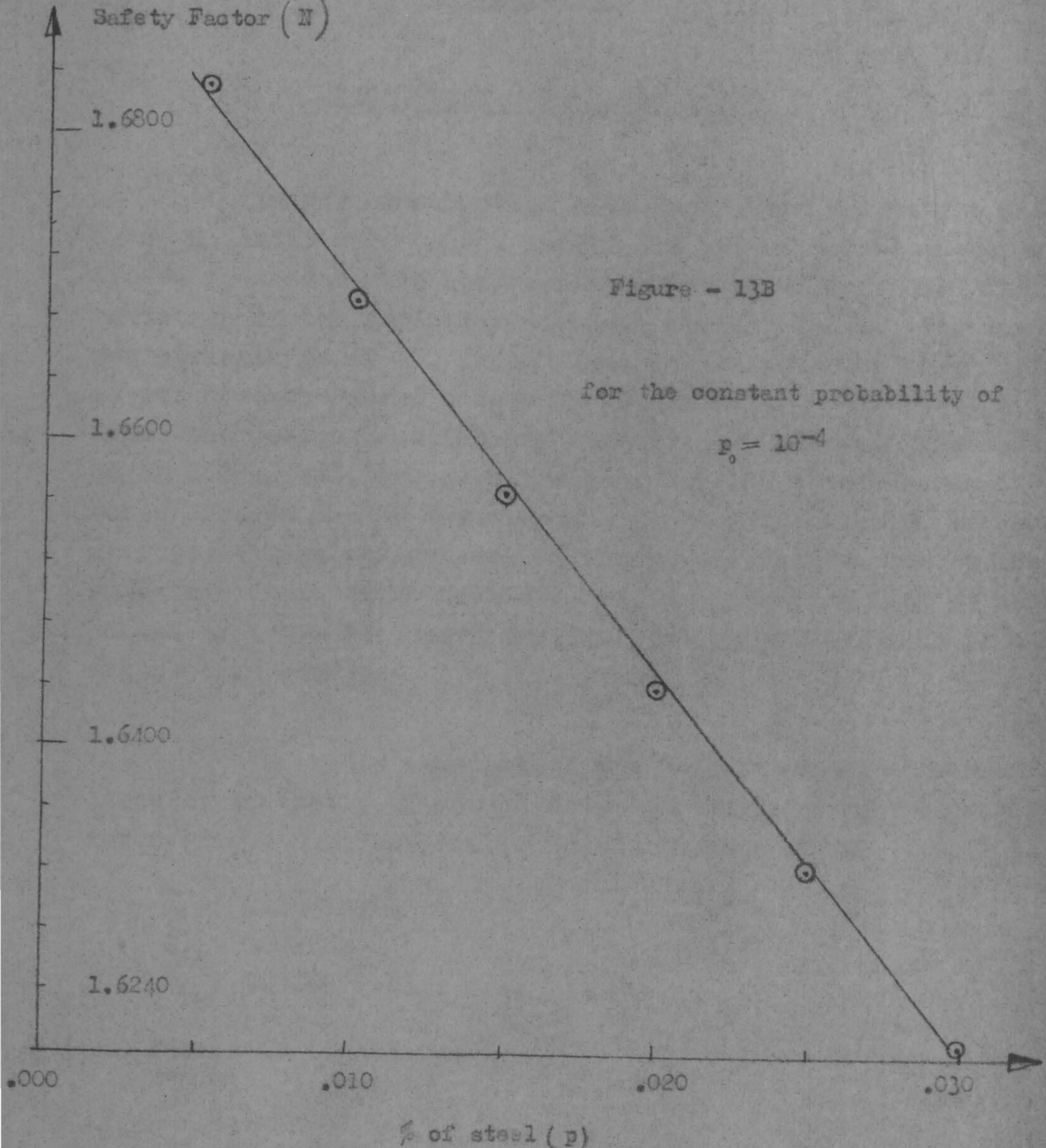


Figure - 13B

for the constant probability of

$p_0 = 10^{-4}$

CHAPTER IV

DISCUSSION AND CONCLUSION

In the construction site from where values for Table F, G, H, and I were taken, thirty odd concrete test cubes were broken to measure the compressive strength of concrete. The variation of the seven day strength was 153 kg/cm^2, the maximum strength being 350 kg/cm^2, and the minimum 197 kg/cm^2. It is not however deemed necessary to consider this variation, since the design, i.e."28-day" strength of concrete was assumed to be 160 kg/cm^2, and even the lowest value of the seven day cubes exceeds the 28 day strength by 37 kg/cm^2. This, of course directly causes an increase in the safety factor, but at the same time it is quite needless, and represents a loss of money caused by the unnecessary zeal and excessive diligence of the engineer in charge.

It is now appropriate that the frequency distributions of the ratio of actual areas to design areas be considered.(Fig.s 5 and 6) For ratio of concrete areas:

$\overline{X}_{A_c}$: 0.98893
σ_{A_c} : 0.02664
V_{A_c} : 0.02693

and for ratio of steel areas:

$\overline{X}_{A_s}$: 1.05098
σ_{A_s} : 0.03067
V_{A_s} : 0.02918

These results are in accordance with the results obtained below by approximate rule of thumb methods. For instance in the case of the ratio of concrete areas:

$$D_1 = 0.98893 - 0.91000 = 0.07893$$

and

$$D_2 = 1.07000 - 0.98893 = 0.08107$$

$$\frac{D_1 + D_2}{2} = \frac{0.07893 + 0.08107}{2} = 0.08000$$

and since,

$$\sigma_{A_c} = \frac{(D_1 + D_2)/2}{3} = \frac{0.08000}{3} = 0.02667$$

which means that, there is only an error of 0.1 %. One can also run through the same procedure for the ratio of areas of steel to obtain:

$$E_1 = 1.05098 - 0.98000 = 0.07098$$

and

$$E_2 = 1.13000 - 1.05098 = 0.07902$$

$$\frac{E_1 + E_2}{2} = \frac{0.07098 + 0.07902}{2} = 0.07500$$

and since,

$$\sigma_{A_s} = \frac{(E_1 + E_2)/2}{3} = \frac{0.07500}{3} = 0.02500$$

which means that in this case the error is about 15 % .

It is also interesting to point out that, although the mean for the ratio of areas of concrete is below unity, i.e. the constructed areas are in general, smaller than their design values, the mean for ratio of steel areas is above unity which means that steel is produced in larger than nominal dimensions. These discrepancies have their own effects. The deviation of the steel area means that more steel is to be used in the construction than is indicated in the design, which in turn comes to mean that more money than necessary is to be spent, without visible benefit, whereas the discrepancy in the concrete area means that the actual areas are smaller than anticipated ones which results in a theoretical decrease of the carrying capacity of the structure in question.

The ratio $\overline{Y}/\overline{C}$ (Chapter IV, Section A) was defined to be the ratio of concrete area to steel area, whereas it is rather the ratio of the gross cross sectional area of the column, to the area of the steel. However since the values of concrete columns (Appendix, Table F) include the cross sectional areas of steel in them, and since the percentage of steel, which does not exceed 3 % at most is quite small, this assumption is not without foundation.

By observation one can deduce that eq. 60 will result in a hyperbola if it were plotted. However, the curve is so flat, with a radius of curvature of such great magnitude, that within the limits p 0.005 to p 0.030, it may be approximated to a straight line (Fig.s 8 and 11). The equation of this straight line calculated from Fig. 8 by the formula

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

is

$$V_p = \frac{3.05 - 2.9\,p}{20}$$

It will be recalled that the values $V_{f_c'}$ and V_{f_y} were obtained from (Appendix, Table D) both as being equal to 0.15. This value represents the maximum variation in steel reinforcement, and it was chosen in order to obtain the largest variation possible in V_P , which in turn gives the smallest, and of course the most critical safety factor. It also represents the maximum variation in good controlled concrete, or the minimum variation in fair controlled concrete, and is both a probable and sensible choice.

In eq.52 t_S and t_L are confidance limit parameters of the probability of strength being below design strength, and the probability of load being above design load respectively. Both of these probabilities are assumed to be 10 %, (Table D) although the probability that strength will be 5 % below the anticipated value is the assumed practice in Europe. The choice of 10 % is based on the fact that in Turkey , the quality control is poorer, and that 10 % is a mistake on the safe side, yielding a still smaller factor of safety. From Tables B and C one can determine that both t_S and t_L are equal to 1.28 .

It is also considered worthwhile to investigate the effect of the second expression under the radical sign in eq.60. It represents the effect of steel and is multiplied by the factor

$$\frac{p^2}{(1 + p)^2}$$

where the maximum value of p can only go up to 0.030 . Therefore, $p^2 = 0.0009$ and the effect of steel is about 9/10000 of the effect of concrete. It could have been left out of calculations completely, and very little change would occur, which probably would be somewhere about 3 % in the coefficient of variation of strength V_P ; and this is for the maximum value of p, which is equal to 0.030 . Numerically

$$V_p = \sqrt{0.021897 - 0.000020}$$

where the first term represents concrete, and the second term represents steel.

The question at once arises as to what the effect of tolerances are, and what would have happened, had they not been considered? The effect of tolerances is represented by V_{A_c}, the coefficient of variation of the concrete area. Assuming this to be equal to zero, and dropping the V_{A_c} term, a new set of values of V_p (V_s), of probability factor $\bar{N}$, and of safety factor N are calculated and presented in Table 3. It is natural to await the new set of safety factors to be smaller than the previous set, presented in Table 2; and the results confirm this observation. The deviations between the two sets become more pronounced as the percentage of steel p is increased. However, at most the deviation adds up to 5.26 % . This shows, therefore, that although the inclusion of the effect of tolerances in the calculation of safety factor provides an added refinement to the design, the exclusion of it does not have a great effect on the overall picture.

It will also be noticed that when the effects of the dimensional discrepancies were neglected, only V_{A_c} was considered to be zero. Strictly and theoretically, V_{A_s} should have also been considered as zero during this process. However, as it was mentioned earlier, when its effect is a maximum, i.e. when $p = 0.030$, the effect of the steel area in eq.60 is only 9/10000 of what concrete contributes. Therefore , practically nothing is changed by keeping the coefficient of variation of steel as it is, instead of reducing it to zero. This, however, saves one, a lot of unnecessary burden, and by keeping V the way it is, a great economy of time is achieved.

In order to check what the situation is where there is none or very little quality control, eighty more columns were measured. These measurements were made at two wholly different building sites, and forty columns were measured at each site. The measurements were taken as the previous 400 were made; i.e. three measurements were taken from each column, one from the top, where it is integrated into the ceiling above, one approximately at the center of the column and one at its toe. The smallest dimensions were chosen as is customary, and an actual cross sectional area was determined. Then the ratio of these actual cross sectional areas to intended "design" cross sectional areas was calculated. The result is presented in Fig.7 . As it can be observed in the figure, the distribution curve thus obtained for the above mentioned columns is extremely skew. Having made the readings at two different sites may be one of the greatest reasons for this skewness; but the poor quality control exercised at both of the sites is certainly an outstanding reason. One might therefore, point out that, better the quality control exercised at the building site, the more one can expect from the computations executed under the influence of the probability theory. This is based on the fact that good quality control decreases the skewness of the distribution curve and also causes it to become narrower, and thus better results are obtained.

Committee 622, and R. Nichols have given certain values for deviations, and these are presented in Table 1. While these are more or less held to be true for the majority of the readings (Table F, Appendix) which do not exceed a tolerance limit of 2 cm, in some cases the deviations are rather large.

What Nichols states in Chapter II, Section E, comes under fire from Anderson 22 . Anderson criticizes Nichols as being outdated. "Nichols" he has written "was thinking of cast-in-place concrete construction". It is true that today concrete construction is moving towards prefabrication of structural members, and becoming an industrialized process in which mass production of factory manufactured structural elements will prevail. However, it is also true that these changes are taking place in highly developedand industrialized regions, and in Turkey, as in most parts of Europe, the predominant practice is unfortunately cast-in-place construction. Both the strength variations and the dimensional deviations are more pronounced in this type of construction. Therefore, what Nichols has stated in 1940 is still true and acceptable in Turkey today.

To conclude, it might be added that; by adopting the principle that neither design loads, nor safety factors, and permissable stresses should be specified arbitrarily, it will be possible not only to eliminate inadequate design, but frequently to achieve considerable economy. It might be possible, moreover to determine correct safety factors and permissable stresses for unconventional structures, or new structural forms and materiels.

A P P E N D I X

CHAPTER IA

CALCULATIONS

A- CALCULATION OF COEFFICIENT OF VARIATION OF CONCRETE V_{A_c} :

Calculation of the mean value of the ratio of the actual cross sectional areas of concrete to the design cross sectional areas:

Sum total of the ratio of actual to design areas ΣX_{A_c}: 395.57119
Number of readings taken(n): 400
Therefore, mean value of the ratio of the areas: $\bar{X}_{A_c} = \Sigma X/n$

$$\bar{X}_{A_c} = \frac{395.57119}{400} = 0.98893$$

Calculation of the standard deviation σ_{A_c} of the ratio of the actual cross sectional areas to the design cross sectional areas of concrete :

$\Sigma(X_{A_c} - \bar{X}_{A_c})^2 = 0.283789370$
Therefore, the standard deviation σ_{A_c} :

$$\sigma_{A_c} = \sqrt{\frac{\Sigma(X - \bar{X})^2}{n}}$$

$$\sigma_{A_c} = \sqrt{\frac{0.283789370}{400}} = 0.026636$$

Calculation of the coefficient of variation V_{A_c} of the ratio of the actual cross sectional areas to design cross sectional areas of concrete :

Coefficient of variation is equal to $V_{A_c} = \sigma_{A_c}/\bar{X}$

V	0.026636 0.9889	0.026932

B- CALCULATION OF COEFFICIENT OF VARIATION OF STEEL V_{A_s}:

Calculation of the mean value of the ratio of the actual cross sectional areas of concrete to the design cross sectional areas:

Sum total of the ratio of actual to design areas: ΣX_{A_s} : 262.74569
Number of readings taken (n) : 250
Therefore, mean value of the ratio of the areas: $\bar{X}_{A_s} = \Sigma X_{A_s}/n$

$$\bar{X}_{A_s} = \frac{262.74569}{250} = 1.05098$$

Calculation of the standard deviation σ_{A_s} of the ratio of the actual cross sectional areas to the design cross sectional areas of concrete :

$\Sigma(X_{A_s} - \bar{X}_{A_s})^2 = 0.235115711$
Therefore, the standard deviation σ_{A_s}:

$$\sigma_{A_s} = \sqrt{\frac{\Sigma(X - \bar{X})^2}{n}}$$

$$\sigma_{A_s} = \sqrt{\frac{0.235115711}{400}} = 0.030666$$

Calculation of the coefficient of variation V_{A_s} of the ratio of the actual cross sectional areas to stipulated cross sectional areas of steel :

Coefficient of variation is equal to $V_{A_s} = \sigma_{A_s}/\bar{X}_{A_s}$

$$V_{A_s} = \frac{0.030666}{1.0510} = \underline{0.029178}$$

C- CALCULATION OF THE COEFFICIENT OF VARIATION OF STRENGTH :

In this section eq.60 will be solved several times for V_p with changing values of p . Values of $V_{f_c'}$ and V_{f_y} are both taken from Table D, Appendix, to be equal to 0.15. For V_{A_c} and V_{A_s} the values obtained in sections A and B, i.e. 0.026932 and 0.029178 are going to be used. Eq. 60 is

$$V_p = \sqrt{\left(V_{A_c}^2 - V_{f_c'}^2\right)\frac{1}{(1+p)^2} + \left(V_{A_s}^2 - V_{f_y}^2\right)\frac{p^2}{(1+p)^2}}$$

and substituting the following values in :

V_{A_c} : 0.026932

V_{A_s} : 0.029178

$V_{f_c'}$: 0.15

V_{f_y} : 0.15

the equation becomes :

$$V_p = \sqrt{\left(\overline{0.15}^2 - \overline{0.027}^2\right)\frac{1}{(1+p)^2} + \left(\overline{0.15}^2 - \overline{0.029}^2\right)\frac{p^2}{(1+p)^2}}$$

which can be simplified into:

$$V_p = \sqrt{0.02323\,\frac{1}{(1+p)^2} + 0.02334\,\frac{1}{(1+p)^2}}$$

Now, values of p are going to be substituted into this equation

Case 1 : p : 0.005

$$V_p = \sqrt{0.02332\,\frac{1}{(1+0.005)^2} + 0.02334\,\frac{0.005^2}{(1+0.005)^2}}$$

$V_p = 0.15165$

Case 2 : p : 0.010

$$V_p = \sqrt{0.02323 \frac{1}{(1 + 0.010)^2} + \frac{0.010^2}{(1 + 0.010)^2} 0.02334}$$

$V_p = 0.15091$

Case 3 : p : 0.015

$$V_p = \sqrt{0.02323 \frac{1}{(1 + 0.015)^2} + 0.02334 \frac{0.015^2}{(1 + 0.015)^2}}$$

$V_p = 0.15018$

Case 4 : p : 0.020

$$V_p = \sqrt{0.02323 \frac{1}{(1 + 0.020)^2} + 0.02334 \frac{0.020^2}{(1 + 0.020)^2}}$$

$V_p = 0.14946$

Case 5 : p : 0.025

$$V_p = \sqrt{0.02323 \frac{1}{(1 + 0.025)^2} + 0.02334 \frac{0.025^2}{(1 + 0.025)^2}}$$

$V_p = 0.14875$

Case 6 : p : 0.030

$$V_p = \sqrt{0.02323 \frac{1}{(1+0.030)^2} - 0.02354 \frac{0.030^2}{(1+0.030)^2}}$$

$V_p = 0.14804$

D- CALCULATION OF PROBABILITY FACTOR $\overline{N}$:

In this section eq.50 will be solved several times, for changing confidance limit parameter t_p, and coefficient of variation V_p. The value of V_L is taken from Table E, Appendix, and is equal to 0.14 . Eq. 50 is

$$(1 - t_p^2 V_p^2)\overline{N}^2 - 2\,\overline{N} + (1 - t_p^2 V_L^2) = 0$$

and substituting the value of V_L : 0.14, it becomes,

$$(1 - t_p^2 V_p^2)\overline{N}^2 - 2\,\overline{N} + (1 - t_p^2\, 0.0196) = 0$$

Case 1 V_p : 0.15165, t_p : 3.72

$$(1 - \overline{3.72}^2\, \overline{0.15165}^2)\overline{N}^2 - 2\,\overline{N} + (1 - \overline{3.72}^2\, \overline{0.0196}) = 0$$

$$0.68173\,\overline{N}^2 - 2\,\overline{N} + 0.72877 = 0$$

$$\underline{\overline{N} = 2.50737}$$

Case 2 V_p : 0.15091, t_p : 3.72

$$(1 - \overline{3.72}^2\, \overline{0.15091}^2)\overline{N}^2 - 2\,\overline{N} + (1 - \overline{3.72}^2\, \overline{0.0196}) = 0$$

$$0.68486\,\overline{N}^2 - 2\,\overline{N} + 0.72877 = 0$$

$$\underline{\overline{N} = 2.49355}$$

Case 3 V_p : 0.15018, t_p : 3.72

$$(1 - \overline{3.72}^2\, \overline{0.15018}^2)\overline{N}^2 - 2\,\overline{N} + (1 - \overline{3.72}^2\, \overline{0.0196}) = 0$$

$$0.68790\,\overline{N}^2 - 2\,\overline{N} + 0.72877 = 0$$

$$\underline{\overline{N} = 2.48026}$$

<u>Case 4</u> V_p : 0.14946, t_p : 3.72

$$\left(1 - \overline{3.72^2}\;\overline{0.14946^2}\right)\overline{N}^2 - 2\,\overline{N} + \left(1 - \overline{3.72^2}\;\overline{0.0196}\right) = 0$$

$$0.69089\,\overline{N}^2 - 2\,\overline{N} + 0.72877 = 0$$

$$\underline{\overline{N} = 2.46730}$$

<u>Case 5</u> V_p : 0.14875, t_p : 3.72

$$\left(1 - \overline{3.72^2}\;\overline{0.14875^2}\right)\overline{N}^2 - 2\,\overline{N} + \left(1 - \overline{3.72^2}\;\overline{0.0196}\right) = 0$$

$$0.69383\,\overline{N}^2 - 2\,\overline{N} + 0.72877 = 0$$

$$\underline{\overline{N} = 2.45469}$$

<u>Case 6</u> V_p : 0.14804, t_p : 3.72

$$\left(1 - \overline{3.72^2}\;\overline{0.14804^2}\right)\overline{N}^2 - 2\,\overline{N} + \left(1 - \overline{3.72^2}\;\overline{0.0196}\right) = 0$$

$$0.69670\,\overline{N}^2 - 2\,\overline{N} + 0.72877 = 0$$

$$\underline{\overline{N} = 2.44240}$$

<u>Case 7</u> V_p : 0.15165, t_p : 4.27

$$\left(1 - \overline{4.27^2}\;\overline{0.15165^2}\right)\overline{N}^2 - 2\,\overline{N} + \left(1 - \overline{4.27^2}\;\overline{0.0196}\right) = 0$$

$$0.58066\,\overline{N}^2 - 2\,\overline{N} + 0.64264 = 0$$

$$\underline{\overline{N} = 3.08568}$$

Case 8 V_p : 0.15091, t_p : 4.27

$$\left(1 - \overline{4.27}^2\ \overline{0.15091}^2\right)\overline{N}^2 - 2\,\overline{N} + \left(1 - \overline{4.27}^2\ \overline{0.0196}\right) = 0$$

$$0.58478\ \overline{N}^2 - 2\,\overline{N} + 0.64264 = 0$$

$$\underline{\overline{N} = 3.06108}$$

Case 9 V_p : 0.15018, t_p : 4.27

$$\left(1 - \overline{4.27}^2\ \overline{0.15018}^2\right)\overline{N}^2 - 2\,\overline{N} + \left(1 - \overline{4.27}^2\ \overline{0.0196}\right) = 0$$

$$0.58879\ \overline{N}^2 - 2\,\overline{N} + 0.64264 = 0$$

$$\underline{\overline{N} = 3.03745}$$

Case 10 V_p : 0.14946, t_p : 4.27

$$\left(1 - \overline{4.27}^2\ \overline{0.14946}^2\right)\overline{N}^2 - 2\,\overline{N} + \left(1 - \overline{4.27}^2\ \overline{0.0196}\right) = 0$$

$$0.59271\ \overline{N}^2 - 2\,\overline{N} + 0.64264 = 0$$

$$\underline{\overline{N} = 3.01468}$$

Case 11 V_p : 0.14875, t_p : 4.27

$$\left(1 - \overline{4.27}^2\ \overline{0.14875}^2\right)\overline{N}^2 - 2\,\overline{N} + \left(1 - \overline{4.27}^2\ \overline{0.0196}\right) = 0$$

$$0.59660\ \overline{N}^2 - 2\,\overline{N} + 0.64264 = 0$$

$$\underline{\overline{N} = 2.99234}$$

Case 12 V_p : 0.14804, t_p : 4.27

$$\left(1 - \overline{4.27}^2\ \overline{0.14804}^2\right)\overline{N}^2 - 2\,\overline{N} + \left(1 - \overline{4.27}^2\ \overline{0.0196}\right) = 0$$

$$0.60039\,\bar{N}^2 - 2\cdot\bar{N} + 0.64264 = 0$$

$$\underline{\bar{N} = 2.97089}$$

E- CALCULATION OF THE SAFETY FACTOR N :

In this section eq.52 will be solved several times, for changing values of coefficient of variation of strength V_p (V_s) , and probability factor $\bar{N}$. In this equation confidance limit parameters for strength and load, t_s and t_L, are both taken to be 1.28 from Tables B and C, Appendix, for constant probabilities of 10 %. V_L is still equal to 0.14 . Eq. 52 is

$$N = \bar{N} \frac{(1 - t_s V_s)}{(1 - t_L V_L)}$$

and substituting the given values into the equation, it becomes

$$N = \bar{N} \frac{(1 - 1.28\ V_s)}{(1 - [1.28]\ [0.14])}$$

Case 1 V_p : 0.15165, $\bar{N}$: 2.50737

$$N = 2.50737\ \frac{1 - (1.28)(0.15165)}{1.17920} = \underline{1.71359}$$

Case 2 V_p : 0.15165, $\bar{N}$: 3.08568

$$N = 3.08568\ \frac{1 - (1.28)(0.15165)}{1.17920} = \underline{2.10882}$$

Case 3 V_p : 0.15091, $\bar{N}$: 2.49355

$$N = 2.49355\ \frac{1 - (1.28)(0.15091)}{1.17920} = \underline{1.70615}$$

<u>Case 4</u> V_p : 0.15091, $\bar{N}$: 3.06108

$$N = 3.06108 \; \frac{1 - (1.28)(0.15091)}{1.17920} = \underline{2.09354}$$

<u>Case 5</u> V_p : 0.15018, $\bar{N}$: 2.48026

$$N = 2.48026 \; \frac{1 - (1.28)(0.15018)}{1.17920} = \underline{1.69901}$$

<u>Case 6</u> V_p : 0.15018, $\bar{N}$: 3.03745

$$N = 3.03745 \; \frac{1 - (1.28)(0.15091)}{1.17920} = \underline{2.08070}$$

<u>Case 7</u> V_p : 0.14946, $\bar{N}$: 2.46730

$$N = 2.46730 \; \frac{1 - (1.28)(0.14946)}{1.17920} = \underline{1.69206}$$

<u>Case 8</u> V_p : 0.14946, $\bar{N}$: 3.01468

$$N = 3.01468 \; \frac{1 - (1.28)(0.14946)}{1.17920} = \underline{2.06746}$$

<u>Case 9</u> V_p : 0.14875, $\bar{N}$: 2.45469

$$N = 2.45469 \; \frac{1 - (1.28)(0.14875)}{1.17920} = \underline{1.68531}$$

<u>Case 10</u> V_p : 0.14875, $\bar{N}$: 2.99234

$$N = 2.99234 \; \frac{1 - (1.28)(0.14875)}{1.17920} = \underline{2.05444}$$

<u>Case 11</u> V_p : 0.14804, $\overline{N}$: 2.44240

$$N = 2.44240 \, \frac{1 - (1.28)(0.14804)}{1.17920} \approx \underline{1.67875}$$

<u>Case 12</u> V_p : 0.14804, $\overline{N}$: 2.97089

$$N = 2.97089 \, \frac{1 - (1.28)(0.14804)}{1.17920} = \underline{2.04201}$$

F- CALCULATION OF THE COEFFICIENT OF VARIATION OF STRENGTH WITH THE EFFECT OF TOLERANCES NEGLECTED :

In this section, as in section C, eq.60 will be solved several times for V_p with changing values of p. However this time both V_{A_c} and V_{A_s} are taken to be equal to zero. $V_{f_c'}$ and V_{f_y} as before are taken to be equal to 0.15 from Table D, Appendix . Eq. 60, therefore transforms into :

$$V_p = \sqrt{V_{f_c'}^2 \frac{1}{(1+p)^2} + V_{f_y}^2 \frac{p^2}{(1+p)^2}}$$

Substituting the given values and simplifying,

$$V_p = \frac{0.15}{(1+p)} \sqrt{1+p^2}$$

Now, into this new equation, the different values of p are substituted.

Case 1 p : 0.005

$$V_p = \frac{0.15}{(1+0.005)} \sqrt{1+\overline{0.005}^2}$$

$V_p = 0.14851$

Case 2 p : 0.010

$$V_p = \frac{0.15}{(1+0.010)} \sqrt{1-\overline{0.010}^2}$$

$\underline{V_p = 0.14705}$

<u>Case 3</u> p : 0.015

$$V_p = \frac{0.15}{(1 + 0.015)} \sqrt{1 + \overline{0.015}^2}$$

$\underline{V_p = 0.14562}$

<u>Case 4</u> p : 0.020

$$V_p = \frac{0.15}{(1 + 0.020)} \sqrt{1 + \overline{0.020}^2}$$

$\underline{V_p = 0.14420}$

<u>Case 5</u> p : 0.025

$$V_p = \frac{0.15}{(1 + 0.025)} \sqrt{1 + \overline{0.025}^2}$$

$\underline{V_p = 0.14282}$

<u>Case 6</u> p : 0.030

$$V_p = \frac{0.15}{(1 + 0.030)} \sqrt{1 + \overline{0.030}^2}$$

$\underline{V_p = 0.14146}$

G- CALCULATION OF THE PROBABILITY FACTOR $\overline{N}$, WITH THE EFFECT OF TOLERANCES NEGLECTED :

In this section as in section D, eq. 50 will be solved several times, i.e. once for each value of V . Everything is the same; except the V values, which are taken from the preceding chapter. As given in section D,

$$\left(1 - t_p^2 V_p^2\right)\overline{N}^2 - 2\,\overline{N} + \left(1 - t_p^2\,\overline{0.0196}\right) = 0$$

Case 1 V_p : 0.14851, t_p: 3.72

$$\left(1 - \overline{3.72}^2\,\overline{0.14851}^2\right)\overline{N}^2 - 2\,\overline{N} + \left(1 - \overline{3.72}^2\,\overline{0.0196}\right) = 0$$

$$0.69479\,\overline{N}^2 - 2\,\overline{N} + 0.72877 = 0$$

$$\overline{N} = 2.45055$$

Case 2 V_p : 0.14705, t_p: 3.72

$$\left(1 - \overline{3.72}^2\,\overline{0.14705}^2\right)\overline{N}^2 - 2\,\overline{N} + \left(1 - \overline{3.72}^2\,\overline{0.0196}\right) = 0$$

$$0.70081\,\overline{N}^2 - 2\,\overline{N} + 0.72877 = 0$$

$$\overline{N} = 2.42502$$

Case 3 V_p : 0.14562, t_p: 3.72

$$\left(1 - \overline{3.72}^2\,\overline{0.14562}^2\right)\overline{N}^2 - 2\,\overline{N} + \left(1 - \overline{3.72}^2\,\overline{0.0196}\right) = 0$$

$$0.70649\,\overline{N}^2 - 2\,\overline{N} + 0.72877 = 0$$

$$\overline{N} = 2.40134$$

Case 4 V_p : 0.14420, t_p: 3.72

$$\left(1 - \overline{3.72}^2\ \overline{0.14420}^2\right)\bar{N}^2 - 2\,\bar{N} + \left(1 - \overline{3.72}^2\ \overline{0.0196}\right) = 0$$

$$0.71230\ \bar{N}^2 - 2\,\bar{N} + 0.72877 = 0$$

$$\underline{\bar{N} = 2.37745}$$

Case 5 V_p : 0.14282, t_p: 3.72

$$\left(1 - \overline{3.72}^2\ \overline{0.14282}^2\right)\bar{N}^2 - 2\,\bar{N} + \left(1 - \overline{3.72}^2\ \overline{0.0196}\right) = 0$$

$$0.71770\ \bar{N}^2 - 2\,\bar{N} + 0.72877 = 0$$

$$\underline{\bar{N} = 2.35561}$$

Case 6 V_p : 0.14146, t_p: 3.72

$$\left(1 - \overline{3.72}^2\ \overline{0.14146}^2\right)\bar{N}^2 - 2\,\bar{N} + \left(1 - \overline{3.72}^2\ \overline{0.0196}\right) = 0$$

$$0.72309\ \bar{N}^2 - 2\,\bar{N} + 0.72877 = 0$$

$$\underline{\bar{N} = 2.33411}$$

Case 7 V_p : 0.14851, t_p: 4.27

$$\left(1 - \overline{4.27}^2\ \overline{0.14851}^2\right)\bar{N}^2 - 2\,\bar{N} + \left(1 - \overline{4.27}^2\ \overline{0.0196}\right) = 0$$

$$0.59778\ \bar{N}^2 - 2\,\bar{N} + 0.64264 = 0$$

$$\underline{\bar{N} = 2.98565}$$

Case 8 V_p : 0.14705, t_p: 4.27

$$\left(1 - \overline{4.27}^2\, \overline{0.14705}^2\, \overline{N}^2\right) - 2\,\overline{N} + \left(1 - \overline{4.27}^2\, \overline{0.0196}\right) \approx 0$$

$$0.60580\, \overline{N}^2 - 2\,\overline{N} + 0.64264 = 0$$

$$\underline{\overline{N} = 2.94071}$$

Case 9 V_p : 0.14562, t_p: 4.27

$$\left(1 - \overline{4.27}^2\, \overline{0.14562}^2\right)\overline{N}^2 - 2\,\overline{N} + \left(1 - \overline{4.27}^2\, \overline{0.0196}\right) \approx 0$$

$$0.61328\, \overline{N}^2 - 2\,\overline{N} + 0.64264 = 0$$

$$\underline{\overline{N} = 2.89980}$$

Case 10 V_p : 0.14420, t_p: 4.27

$$\left(1 - \overline{4.27}^2\, \overline{0.14420}^2\right)\overline{N}^2 - 2\,\overline{N} + \left(1 - \overline{4.27}^2\, \overline{0.0196}\right) \approx 0$$

$$0.62094\, \overline{N}^2 - 2\,\overline{N} + 0.64264 = 0$$

$$\underline{\overline{N} = 2.85892}$$

Case 11 V_p : 0.14282, t_p: 4.27

$$\left(1 - \overline{4.27}^2\, \overline{0.14282}^2\right)\overline{N}^2 - 2\,\overline{N} + \left(1 - \overline{4.27}^2\, \overline{0.0196}\right) \approx 0$$

$$0.62805\, \overline{N}^2 - 2\,\overline{N} + 0.64264 = 0$$

$$\underline{\overline{N} = 2.82185}$$

Case 12 V_p : 0.14146, t_p : 4.27

$$\left(1 - \overline{4.27}^2\ \overline{0.14146}^2\right)\overline{N}^2 - 2\,\overline{N} + \left(1 - \overline{4.27}^2\ \overline{0.0196}\right) = 0$$

$$0.63516\ \overline{N}^2 - 2\,\overline{N} + 0.64264 = 0$$

$$\underline{\overline{N} = 2.78558}$$

H- CALCULATION OF THE SAFETY FACTOR N, WITH THE EFFECT OF TOLERANCES NEGLECTED :

In this section, as in section E, eq.52 will be solved several times, each time for a new value of $\bar{N}$, and all other values are the same as in section E, i.e. t_s and t_L both are equal to 1.28, and V_L is equal to 0.14 . The following equation therefore is used.

$$N = \bar{N} \frac{1 - 1.28\ V_s}{[1 - (1.28)(0.14)]}$$

Case 1 V_s : 0.14851, $\bar{N}$: 2.45055

$$N = 2.45055 \frac{1 - (1.28)(0.14851)}{1.17920} = \underline{1.68311}$$

Case 2 V_s : 0.14851, $\bar{N}$: 2.98565

$$N = 2.98565 \frac{1 - (1.28)(0.14851)}{1.17920} = \underline{2.05063}$$

Case 3 V_s : 0.14705, $\bar{N}$: 2.42502

$$N = 2.42502 \frac{1 - (1.28)(0.14705)}{1.17920} = \underline{1.66943}$$

Case 4 V_s : 0.14705, $\bar{N}$: 2.94071

$$N = 2.94071 \frac{1 - (1.28)(0.14705)}{1.17920} = \underline{2.02444}$$

Case 5 V_s : 0.14562, $\bar{N}$: 2.40134

$$N = 2.40134 \frac{1 - (1.28)(0.14562)}{1.17920} = \underline{1.65685}$$

Case 6 V_s : 0.14562, $\bar{N}$: 2.89980

$$N = 2.89980 \frac{1 - (1.28)(0.14562)}{1.17920} = \underline{2.00076}$$

Case 7 V_s : 0.14420, $\bar{N}$: 2.37745

$$N = 2.37745 \frac{1 - (1.28)(0.14420)}{1.17920} = \underline{1.64401}$$

Case 8 V_s : 0.14420, $\bar{N}$: 2.85892

$$N = 2.85892 \frac{1 - (1.28)(0.14420)}{1.17920} = \underline{1.97694}$$

Case 9 V_s : 0.14282, $\bar{N}$: 2.35561

$$N = 2.35561 \frac{1 - (1.28)(0.14282)}{1.17920} = \underline{1.63244}$$

Case 10 V_s : 0.14282, $\bar{N}$: 2.82185

$$N = 2.82185 \frac{1 - (1.28)(0.14282)}{1.17920} = \underline{1.95554}$$

Case 11 V_s : 0.14146, $\bar{N}$: 2.33411

$$N = 2.33411 \frac{1 - (1.28)(0.14146)}{1.17920} = \underline{1.62099}$$

<u>Case 12</u> V_s : 0.14146, $\overline{N}$: 2.78558

$$N = 2.78558 \, \frac{1 - (1.28)(0.14146)}{1.17920} = \underline{1.93453}$$

I- NOTES ON THE LIMITS OF ALLOWABLE ERROR :

Given a rectangular column, with the design, i.e. intended dimensions as (a) and (b), in width and depth; there is positively going to exist a discrepancy, as described in the previous chapters. Denoting the discrepancies as Δa and Δb in (a) and (b) respectively, and defining both Δa and Δb as negative, so as to insure the worst probable case, the design and cross sectional areas can be written as follows:

Design Area : $a.b$

Actual Area : $(a - \Delta a)(b - \Delta b) = a.b - a\Delta b - b\Delta a + \Delta a \Delta b$

Neglecting the last term, since it is of second order, the above expression for the actual area can be written as:

Actual Area : $a.b - a\Delta b - b\Delta a$

and dividing the actual area by the design area;

$$\frac{\text{Actual Area}}{\text{Design Area}} = 1 - \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right) \qquad (A)$$

where the expression after (1) stands for the coefficient of variation of the areas, $V_{A_c}/3$. To magnify the existant errors to the utmost, a square column of the smallest cross section must be considered. For instance a 20 x 20 can be considered, with allowances of 1 cm on each side. Then equation A becomes :

$$\text{Ratio of areas} = 1 - 2\frac{\Delta a}{a}$$

and with the given values the coefficient of variation comes out to be 0.033, whence the percent of error in the safety factor barely passes five or six percent. To obtain the coefficient of variation of concrete areas, i.e. 0.026932, for a 20 cm by 20 cm column cross section, from the above equation, one observes that the ratio $\Delta a/a$ must not exceed 0.8 cm.

CHAPTER IIA

KEY TO TABLES

1- Table A : This table gives formulae for the mean, standard deviation, coefficient of variation for various functions and is obtained from Reference, number 18 .

2- Tables B and C : These two tables give various values for areas under a normal distribution curve, and are obtained from Reference, numbers 23 , 26 .

3- Tables D and E : These two tables give various values for the coefficient of variation of strength of reinforcing steel and of concrete, and also they give the coefficient of variation of load. They were taken from References, number 17 .

4- Table F : This table was constructed by the data obtained at a construction site, and with the help of this Fig. 5 was plotted. It gives, basically, the ratio of the actual concrete area to the intended or design value..

5- Table G : This table was also constructed by the data obtained at the same construction site, and Fig.6 was plotted using this table. It gives the ratio of the actual steel area to the nominal area, and the mean values for concrete and steel ratios were calculated by the help of Tables F and G.

6- Tables H and I : These tables show the calculation procedure for the obtainment of the standard deviations for ratios of concrete and steel areas respectively and are constructed by the help of data obtained from Tables F and G. In these tables X represent the ratio of concrete areas, and X' the ratio of steel areas.

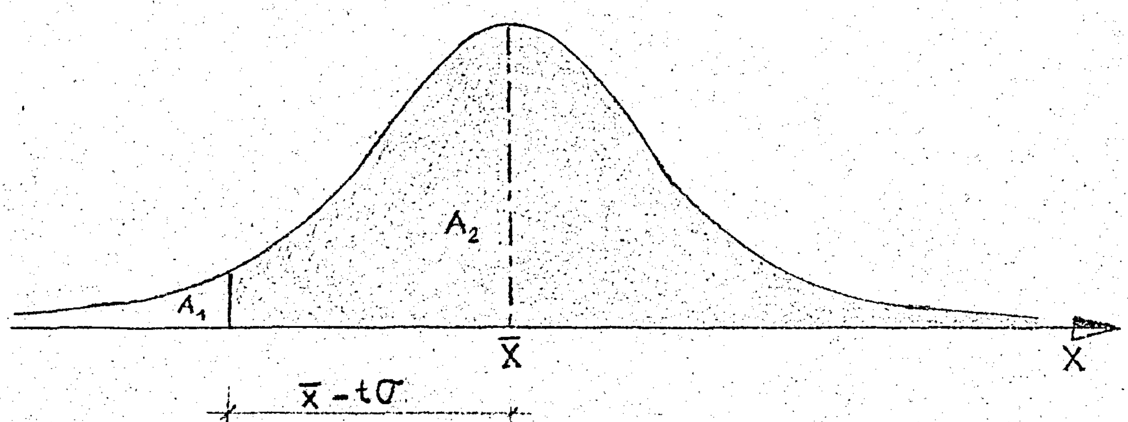
TABLE A

Statistical Value(X)	$X = x$	$X = x_1 + x_2$	$X = x_1 . x_2$
Arithmetic Average ($\bar{X}$)	$\frac{1}{n} \sum x$	$\bar{x}_1 + \bar{x}_2$	$\bar{x}_1 \cdot \bar{x}_2$
Variance σ_x^2	$\frac{1}{n} \sum (x - \bar{x})^2$	$\sigma_1^2 + \sigma_2^2$	$\bar{x}_2 \sigma_1^2 + \bar{x}_1 \sigma_2^2$
Coefficient of Variation	$\frac{\sigma_x}{\bar{x}}$	W *	$\sqrt{V_1^2 + V_2^2}$

*

$$W = \sqrt{V_1\left(\frac{\bar{x}_1}{\bar{x}_1 + \bar{x}_2}\right)^2 + V_2\left(\frac{\bar{x}_2}{\bar{x}_1 + \bar{x}_2}\right)^2}$$

TABLE B



With $\bar{X}$ as the mean value, there are two probabilities of observation in a normal distribution curve.

1. An observation may be smaller than $\bar{X} - \sigma t$, in which case it will be denoted as A_1

2. An observation may be greater than $\bar{X} - \sigma t$, in which case it will be denoted as A_2

In Table B which occupies the next two pages, values of A_2 is given for corresponding values of t, and in Table C which is on the third following page, values of A_1 is given for the corresponding values of t .

tp	.00	.01	.02	.03	.04
.0	.5000	.5040	.5080	.5120	.5160
.1	.5398	.5438	.5478	.5517	.5557
.2	.5793	.5832	.5871	.5910	.5948
.3	.6179	.6217	.6255	.6293	.6331
.4	.6554	.6591	.6628	.6664	.6700
.5	.6915	.6950	.6985	.7019	.7054
.6	.7257	.7291	.7324	.7357	.7389
.7	.7580	.7611	.7642	.7673	.7703
.8	.7881	.7910	.7939	.7967	.7995
.9	.8159	.8186	.8212	.8238	.8264
1.0	.8413	.8438	.8461	.8485	.8508
1.1	.8643	.8665	.8686	.8708	.8729
1.2	.8849	.8869	.8888	.8907	.8925
1.3	.90320	.90490	.90658	.90824	.90988
1.4	.91924	.92073	.92220	.92364	.92507
1.5	.93319	.93448	.93574	.93699	.93822
1.6	.94520	.94630	.94738	.94845	.94950
1.7	.95543	.95637	.95728	.95818	.95907
1.8	.96407	.96485	.96562	.96638	.96712
1.9	.97128	.97193	.97257	.97320	.97381
2.0	.97725	.97778	.97831	.97882	.97932
2.1	.98214	.98257	.98300	.98341	.98382
2.2	.98610	.98645	.98679	.98713	.98745
2.3	.98928	.98956	.98983	$.9^2 0097$	.9 0358
2.4	.9 1802	.9 2024	.9 2240	.9 2451	.9 2656
2.5	.9 3790	.9 3963	.9 4132	.9 4297	.9 4457
2.6	.9 5339	.9 5473	.9 5604	.9 5731	.9 5855
2.7	.9 6533	.9 6636	.9 6736	.9 6833	.9 6928
2.8	.9 7445	.9 7523	.9 7599	.9 7673	.9 7744
2.9	.9 8134	.9 8193	.9 8250	.9 8305	.9 8359
3.0	$.9_3 8650$	.9 8694	.9 8736	.9 8777	.9 8817
3.1	$.9^3 0324$	.9 0646	.9 0957	.9 1260	.9 1553
3.2	.9 3129	.9 3363	.9 3590	.9 3810	.9 4024
3.3	.9 5166	.9 5335	.9 5499	.9 5658	.9 5811
3.4	.9 6631	.9 6752	.9 6869	.9 6982	.9 7091
3.5	.9 7674	.9 7759	.9 7842	.9 7922	.9 7999
3.6	.9 8409	.9 8469	$.9_4 8527$	.9 8583	.9 8637
3.7	.9 8922	.9 8964	$.9^4 0039$	.9 0426	.9 0799
3.8	.9 2765	.9 3052	.9 3327	.9 3593	.9 3848
3.9	.9 5190	.9 5385	.9 5573	.9 5753	.9 5926
4.0	.9 6833	.9 6964	.9 7090	.9 7211	.9 7327

t_p	.05	.06	.07	.08	.09
.0	.5199	.5239	.5279	.5319	.5359
.1	.5596	.5636	.5675	.5714	.5753
.2	.5987	.6026	.6064	.6103	.6141
.3	.6368	.6406	.6443	.6480	.6517
.4	.6736	.6772	.6808	.6844	.6879
.5	.7088	.7123	.7157	.7190	.7224
.6	.7422	.7454	.7486	.7517	.7549
.7	.7734	.7764	.7794	.7823	.7852
.8	.8023	.8051	.8078	.8106	.8133
.9	.8289	.8315	.8340	.8365	.8389
1.0	.8531	.8554	.8577	.8599	.8621
1.1	.8749	.8770	.8790	.8810	.8830
1.2	.8944	.8962	.8980	.8997	.90147
1.3	.91149	.91309	.91466	.91621	.91774
1.4	.92647	.92785	.92922	.93056	.93189
1.5	.93943	.94062	.94179	.94295	.94408
1.6	.95053	.95154	.95254	.95352	.95449
1.7	.95994	.96080	.96164	.96246	.96327
1.8	.96784	.96856	.96926	.96995	.97062
1.9	.97441	.97500	.97558	.97615	.97670
2.0	.97982	.98030	.98077	.98124	.98169
2.1	.98422	.98461	.98500	.98537	.98574
2.2	.98778	.98809	.98840	.98870	.98899
2.3	.9^{2}0613	.9 0863	.9 1106	.9 1344	.9 1576
2.4	.9 2857	.9 3053	.9 3244	.9 3431	.9 3613
2.5	.9 4614	.9 4766	.9 4915	.9 5060	.9 5201
2.6	.9 5975	.9 6093	.9 6207	.9 6319	.9 6427
2.7	.9 7020	.9 7110	.9 7197	.9 7282	.9 7365
2.8	.9 7814	.9 7882	.9 7948	.9 8012	.9 8074
2.9	.9 8411	.9 8462	.9 8511	.9 8559	.9 8605
3.0	.9 8856	.9 8893	.9 8930	.9 8965	.9 8999
3.1	.9^{3}1836	.9 2112	.9 2378	.9 2636	.9 2886
3.2	.9 4230	.9 4429	.9 4623	.9 4810	.9 4991
3.3	.9 5959	.9 6103	.9 6242	.9 6376	.9 6505
3.4	.9 7197	.9 7299	.9 7398	.9 7493	.9 7585
3.5	.9 8074	.9 8146	.9 8215	.9 8282	.9 8347
3.6	.9 8639	.9 8739	.9 8787	.9 8834	.9 8879
3.7	.9^{4}1158	.9 1504	.9 1838	.9 2159	.9 2468
3.8	.9 4094	.9 4331	.9 4558	.9 4777	.9 4988
3.9	.9 6092	.9 6253	.9 6406	.9 6554	.9 6696
4.0	.9 7439	.9 7546	.9 7649	.9 7748	.9 7843

TABLE C

Here the values given below are for A_1 :

t_p	3.0	4.0	5.0	6.0
.00	.00135	$.0^{4}317$	$.0^{6}281$	$.0^{9}987$
.01	$.0^{3}968$	$.0^{4}207$	$.0^{6}170$	$.0^{9}530$
.02	$.0^{3}687$	$.0^{4}133$	$.0^{7}996$	$.0^{9}282$
.03	$.0^{3}483$	$.0^{5}854$	$.0^{7}579$	$.0^{9}149$
.04	$.0^{3}337$	$.0^{5}541$	$.0^{7}333$	$.0^{10}777$
.05	$.0^{3}233$	$.0^{5}340$	$.0^{7}190$	$.0^{10}402$
.06	$.0^{3}159$	$.0^{5}211$	$.0^{7}107$	$.0^{10}206$
.07	$.0^{3}108$	$.0^{5}130$	$.0^{8}599$	$.0^{10}104$
.08	$.0^{4}723$	$.0^{6}793$	$.0^{8}332$	$.0^{11}523$
.09	$.0^{4}481$	$.0^{6}479$	$.0^{3}182$	$.0^{11}260$

TABLE D

Suggested Coefficients of Variation and Tolerance Limits for some Structural Materiels are given below :

Materiel	Coefficient of Variation	Probability of Strength Being Below Design Strength
Steel Reinforcement, f_y	0.10 – 0.15	0.05 – 0.10
Concrete, f_c excellent control good control fair control poor control	 0.05 – 0.10 0.10 – 0.15 0.15 – 0.20 0.20 and above	0.10 in U.S.A. 0.50 in Europe
Structural Steel, f_y	0.08 – 0.10	0.01 – 0.05

Remark : This Table is taken from (17) in References

TABLE E

Ratio of Failure Losses to Construction Cost	Permissable Probability of Failure p	Coefficient of Variation of Loads V_L		Values of the safety factor N — V_S Coefficient of Variations of Strength: Excellent Control 0.05	Good Control 0.10	Fair Control 0.15	Poor Control 0.20
1	10^{-3}	Dead Loads	0.08	1.22	1.30	1.52	1.92
		Live Loads	0.14	1.16	1.22	1.38	1.72
		Traffic Loads	0.24	1.27	1.30	1.41	1.66
10	10^{-4}	Dead Loads	0.08	1.16	1.38	1.82	2.82
		Live Loads	0.14	1.26	1.38	1.70	2.54
		Traffic Loads	0.24	1.40	1.41	1.73	2.38
100	10^{-5}	Dead Loads	0.08	1.34	1.52	2.25	4.85
		Live Loads	0.14	1.35	1.52	2.05	4.30
		Traffic Loads	0.24	1.50	1.60	2.09	4.40

Probability of strength being below design strength : 0.10
Probability of load being above design load : 0.10
Based on normal distributions

TABLE F

Minimum Actual Dimensions in cm	Design Dimensions in cm	Actual Area in cm^2	Design Area in cm^2	Ratio of Areas
29.80 x 77.72	30.00 x 80.00	2316.056	2400.000	0.96502
29.85 x 97.21	30.00 x 100.00	2901.719	3000.000	0.96724
29.41 x 80.32	30.00 x 80.00	2362.211	2400.000	0.98425
28.52 x 89.34	30.00 x 90.00	2547.977	2700.000	0.94370
30.31 x 99.12	30.00 x 100.00	3004.327	3000.000	1.00144
29.85 x 98.73	30.00 x 100.00	2947.091	3000.000	0.98236
60.31 x 58.91	60.00 x 60.00	3552.862	3600.000	0.98691
58.55 x 59.25	60.00 x 60.00	3469.088	3600.000	0.96364
30.14 x 97.64	30.00 x 100.00	2942.870	3000.000	0.98096
29.16 x 114.84	30.00 x 110.00	3348.734	3300.000	1.01477
39.62 x 89.55	40.00 x 90.00	3547.971	3600.000	0.98555
29.35 x 109.42	30.00 x 110.00	3211.477	3300.000	0.97317
29.65 x 80.41	30.00 x 80.00	2384.157	2400.000	0.99340
30.28 x 108.71	30.00 x 110.00	3291.739	3300.000	0.99750
29.78 x 49.74	30.00 x 50.00	1481.257	1500.000	0.98750
20.36 x 151.31	20.00 x 150.00	3080.627	3000.000	1.02688
38.97 x 108.22	40.00 x 110.00	4217.333	4400.000	0.95848
29.83 x 109.76	30.00 x 110.00	3274.141	3300.000	0.99216
29.84 x 99.67	30.00 x 100.00	2974.153	3000.000	0.99138
19.71 x 171.02	20.00 x 170.00	3370.804	3400.000	0.99141
20.31 x 154.46	20.00 x 150.00	3137.083	3000.000	1.04569
30.01 x 99.76	30.00 x 100.00	2993.798	3000.000	0.99793
31.64 x 100.47	30.00 x 100.00	3178.871	3000.000	1.05962
29.88 x 79.84	30.00 x 80.00	2385.619	2400.000	0.99401
30.15 x 109.74	30.00 x 110.00	3308.661	3300.000	1.00262
38.84 x 110.56	40.00 x 110.00	4294.150	4400.000	0.97594
29.87 x 109.43	30.00 x 110.00	3268.674	3300.000	0.99051
29.89 x 101.76	30.00 x 100.00	3041.606	3000.000	1.01387
59.48 x 59.23	60.00 x 60.00	3523.000	3600.000	0.97861
60.28 x 60.27	60.00 x 60.00	3633.076	3600.000	1.00919

Minimum Actual Dimensions in cm	Design Dimensions in cm	Actual Area in cm^2	Design Area in cm^2	Ratio of Areas
30.08 x 98.64	30.00 x 100.00	2967.091	3000.000	0.98903
29.64 x 114.26	30.00 x 110.00	3386.666	3300.000	1.02626
39.52 x 88.80	40.00 x 90.00	3509.376	3600.000	0.97483
29.74 x 109.74	30.00 x 110.00	3263.668	3300.000	0.98899
29.84 x 79.84	30.00 x 80.00	2382.426	2400.000	0.99268
30.69 x 98.73	30.00 x 100.00	3030.024	3000.000	1.01001
29.79 x 79.41	30.00 x 80.00	2365.624	2400.000	0.98568
29.62 x 88.94	30.00 x 90.00	2653.970	2700.000	0.98295
28.65 x 100.43	30.00 x 100.00	2877.320	3000.000	0.95911
29.84 x 100.44	30.00 x 100.00	2997.130	3000.000	0.99904
20.37 x 79.57	20.00 x 80.00	1620.841	1600.000	1.01302
20.96 x 89.31	20.00 x 90.00	1871.938	1800.000	1.03997
20.09 x 70.15	20.00 x 70.00	1409.314	1400.000	1.00665
20.31 x 69.55	20.00 x 70.00	1412.561	1400.000	1.00897
20.71 x 88.98	20.00 x 90.00	1842.776	1800.000	1.02376
21.12 x 89.54	20.00 x 90.00	1891.085	1800.000	1.05060
45.21 x 59.52	45.00 x 60.00	2690.899	2700.000	0.99663
50.47 x 59.74	50.00 x 60.00	3015.078	3000.000	1.00503
21.11 x 89.78	20.00 x 90.00	1895.256	1800.000	1.05292
20.69 x 89.52	20.00 x 90.00	1852.169	1800.000	1.02898
30.58 x 89.67	30.00 x 90.00	2742.109	2700.000	1.01556
21.06 x 110.52	20.00 x 110.00	2327.551	2200.000	1.05798
20.33 x 70.04	20.00 x 70.00	1423.913	1400.000	1.01708
21.39 x 99.62	20.00 x 100.00	2130.872	2000.000	1.06544
20.42 x 49.43	20.00 x 50.00	1009.361	1000.000	1.00936
20.35 x 141.12	20.00 x 140.00	2871.792	2800.000	1.02564
34.68 x 80.32	35.00 x 80.00	2785.498	2800.000	0.99482
20.99 x 99.76	20.00 x 100.00	2093.962	2000.000	1.04698
20.43 x 89.62	20.00 x 90.00	1830.937	1800.000	1.01719
19.59 x 169.12	20.00 x 170.00	3313.061	3400.000	0.97443
20.63 x 141.20	20.00 x 140.00	2912.956	2800.000	1.04034
20.86 x 102.41	20.00 x 100.00	2136.273	2000.000	1.06814
21.33 x 103.54	20.00 x 100.00	2208.508	2000.000	1.10425
19.98 x 69.34	20.00 x 70.00	1385.413	1400.000	0.98958
20.50 x 99.43	20.00 x 100.00	2038.315	2000.000	1.01916

Minimum Actual Dimensions in cm	Design Dimensions in cm	Actual Area in cm^2	Design Area in cm^2	Ratio of Areas
34.65 x 80.38	35.00 x 80.00	2785.167	2800.000	0.99470
20.53 x 99.86	20.00 x 100.00	2050.129	2000.000	1.02506
20.22 x 89.54	20.00 x 90.00	1810.489	1800.000	1.00583
45.14 x 59.98	45.00 x 60.00	2707.497	2700.000	1.00278
50.24 x 59.36	50.00 x 60.00	2982.246	3000.000	0.99408
20.21 x 89.25	20.00 x 90.00	1803.743	1800.000	1.00208
20.36 x 90.42	20.00 x 90.00	1840.951	1800.000	1.02278
30.23 x 89.09	30.00 x 90.00	2693.191	2700.000	0.99748
21.32 x 109.46	20.00 x 110.00	2333.687	2200.000	1.06077
20.35 x 70.33	20.00 x 70.00	1431.216	1400.000	1.02230
19.74 x 89.85	20.00 x 90.00	1773.639	1800.000	0.98536
20.64 x 69.84	20.00 x 70.00	1441.498	1400.000	1.02964
19.95 x 70.64	20.00 x 70.00	1409.268	1400.000	1.00662
20.42 x 90.24	20.00 x 90.00	1842.701	1800.000	1.02372
21.06 x 89.64	20.00 x 90.00	1887.818	1800.000	1.04879
19.74 x 79.83	20.00 x 80.00	1576.042	1600.000	0.98503
21.01 x 80.29	20.00 x 80.00	1686.893	1600.000	1.05431
19.66 x 69.38	20.00 x 70.00	1364.011	1400.000	0.97429
20.81 x 70.18	20.00 x 70.00	1460.446	1400.000	1.04318
20.09 x 79.22	20.00 x 80.00	1591.530	1600.000	0.99471
20.28 x 69.94	20.00 x 70.00	1418.383	1400.000	1.01313
20.52 x 99.81	20.00 x 100.00	2048.101	2000.000	1.02405
29.41 x 88.51	30.00 x 90.00	2603.079	2700.000	0.96410
19.92 x 79.69	20.00 x 80.00	1587.425	1600.000	0.99214
19.65 x 79.39	20.00 x 80.00	1560.014	1600.000	0.97501
45.04 x 59.74	45.00 x 60.00	2690.690	2700.000	0.99655
39.92 x 59.95	40.00 x 60.00	2393.204	2400.000	0.99717
20.21 x 79.74	20.00 x 80.00	1611.545	1600.000	1.00722
20.99 x 98.82	20.00 x 100.00	2074.232	2000.000	1.03712
35.14 x 75.04	35.00 x 75.00	2640.658	2625.000	1.00596
19.82 x 128.16	20.00 x 130.00	2540.131	2600.000	0.97697
20.19 x 50.89	20.00 x 50.00	1027.469	1000.000	1.02747
19.25 x 99.89	20.00 x 100.00	1922.883	2000.000	0.96144
19.36 x 69.68	20.00 x 70.00	1349.005	1400.000	0.96358
20.32 x 89.07	20.00 x 90.00	1809.902	1800.000	1.00550

Minimum Actual Dimensions in cm	Design Dimensions in cm	Actual Area in cm^2	Design Area in cm^2	Ratio of Area
20.73 x 89.52	20.00 x 90.00	1855.750	1800.000	1.03097
19.86 x 129.12	20.00 x 130.00	2564.323	2600.000	0.98629
19.93 x 168.14	20.00 x 170.00	3351.030	3400.000	0.98560
20.20 x 79.81	20.00 X 80.00	1612.162	1600.000	1.00760
20.75 x 99.43	20.00 x 100.00	2063.173	2000.000	1.03159
34.49 x 74.22	35.00 x 75.00	2559.848	2625.000	0.97518
20.53 x 99.07	20.00 x 100.00	2033.907	2000.000	1.01695
20.44 x 69.73	20.00 x 70.00	1425.281	1400.000	1.01806
19.90 x 99.12	20.00 x 100.00	1972.488	2000.000	0.98624
29.73 x 89.42	30.00 x 90.00	2658.457	2700.000	0.98461
20.41 x 79.95	20.00 x 80.00	1631.780	1600.000	1.01986
20.79 x 79.55	20.00 x 80.00	1653.845	1600.000	1.03365
45.04 x 60.74	45.00 x 60.00	2735.730	2700.000	1.01323
39.92 x 58.52	40.00 x 60.00	2336.118	2400.000	0.97338
19.84 x 80.05	20.00 x 80.00	1588.192	1600.000	0.99262
20.80 x 79.28	20.00 x 80.00	1649.024	1600.000	1.03064
19.98 x 69.32	20.00 x 70.00	1385.014	1400.000	0.98930
20.10 x 69.11	20.00 x 70.00	1389.111	1400.000	0.99222
19.95 x 79.91	20.00 x 80.00	1594.205	1600.000	0.99638
20.00 x 69.51	20.00 x 70.00	1390.200	1400.000	0.99300
20.82 x 79.20	20.00 x 80.00	1648.944	1600.000	1.03059
20.88 x 79.82	20.00 x 80.00	1666.642	1600.000	1.04165
20.24 x 69.45	20.00 x 70.00	1405.668	1400.000	1.00405
20.36 x 69.28	20.00 x 70.00	1410.541	1400.000	1.00753
20.81 x 79.68	20.00 x 80.00	1658.141	1600.000	1.03634
20.20 x 69.39	20.00 x 70.00	1401.678	1400.000	1.00120
20.38 x 99.33	20.00 x 100.00	2024.345	2000.000	1.01217
30.28 x 88.69	30.00 x 90.00	2685.533	2700.000	0.99464
20.68 x 79.12	20.00 x 80.00	1636.202	1600.000	1.02263
18.10 x 78.32	20.00 x 80.00	1417.592	1600.000	0.88600
43.93 x 59.12	45.00 x 60.00	2597.142	2700.000	0.96190
39.81 x 59.70	40.00 x 60.00	2376.657	2400.000	0.99027
20.35 x 79.61	20.00 x 80.00	1620.064	1600.000	1.01254
20.04 x 99.31	20.00 x 100.00	1990.172	2000.000	0.99509
34.72 x 75.18	35.00 x 75.00	2610.250	2625.000	0.99538

Minimum Actual Dimensions in cm	Design Dimensions in cm	Actual Area in cm^2	Design Area in cm^2	Ratio of Areas
20.70 x 99.60	20.00 x 100.00	2061.720	2000.000	1.03086
20.20 x 69.14	20.00 x 70.00	1396.628	1400.000	0.99759
19.79 x 89.51	20.00 x 90.00	1771.403	1800.000	0.98411
20.65 x 89.59	20.00 x 90.00	1850.034	1800.000	1.02780
20.28 x 128.16	20.00 x 130.00	2599.085	2600.000	0.99965
19.83 x 168.14	20.00 x 170.00	3334.216	3400.000	0.98065
20.26 x 79.96	20.00 x 80.00	1619.990	1600.000	1.01249
20.15 x 99.39	20.00 x 100.00	2002.709	2000.000	1.00135
35.22 x 74.81	35.00 x 75.00	2634.808	2625.000	1.00374
19.81 x 128.91	20.00 x 130.00	2553.707	2600.000	0.98220
19.94 x 50.54	20.00 x 50.00	1007.768	1000.000	1.00777
20.31 x 97.17	20.00 x 100.00	1973.523	2000.000	0.98676
19.80 x 69.49	20.00 x 70.00	1375.902	1400.000	0.98279
20.75 x 98.72	20.00 x 100.00	2048.440	2000.000	1.02422
30.28 x 89.72	30.00 x 90.00	2716.722	2700.000	1.00619
20.00 x 79.61	20.00 x 80.00	1592.200	1600.000	0.99513
20.90 x 79.19	20.00 x 80.00	1655.071	1600.000	1.03442
44.67 x 59.71	45.00 x 60.00	2667.246	2700.000	0.98787
40.79 x 59.72	40.00 x 60.00	2435.979	2400.000	1.01499
20.29 x 79.11	20.00 x 80.00	1605.142	1600.000	1.00321
20.23 x 79.74	20.00 x 80.00	1613.140	1600.000	1.00821
19.48 x 69.44	20.00 x 70.00	1352.691	1400.000	0.96651
19.54 x 69.48	20.00 x 70.00	1357.639	1400.000	0.96974
19.11 x 78.91	20.00 x 80.00	1507.970	1600.000	0.94248
20.21 x 70.06	20.00 x 70.00	1415.913	1400.000	1.01137
29.61 x 99.24	30.00 x 100.00	2938.496	3000.000	0.97950
28.82 x 98.94	30.00 x 100.00	2851.451	3000.000	0.95048
29.72 x 90.12	30.00 x 90.00	2678.366	2700.000	0.99199
30.64 x 79.52	30.00 x 80.00	2436.493	2400.000	1.01521
30.38 x 98.62	30.00 x 100.00	2996.076	3000.000	0.99869
29.46 x 78.80	30.00 x 80.00	2321.448	2400.000	0.96727
29.14 x 109.65	30.00 x 110.00	3195.201	3300.000	0.96824
39.28 x 88.72	40.00 x 90.00	3484.922	3600.000	0.96803
30.08 x 98.36	30.00 x 100.00	2958.669	3000.000	0.98622
29.92 x 99.96	30.00 x 100.00	2990.803	3000.000	0.99693

Minimum Actual Dimensions in cm	Design Dimensions in cm	Actual Area in cm^2	Design Area in cm^2	Ratio of Areas
59.13 x 59.21	60.00 x 60.00	3501.087	3600.000	0.97252
58.66 x 59.82	60.00 x 60.00	3509.043	3600.000	0.97474
30.13 x 99.32	30.00 x 100.00	2992.512	3000.000	0.99750
29.71 x 109.12	30.00 x 110.00	3241.955	3300.000	0.98241
40.48 x 110.52	40.00 x 110.00	4473.850	4400.000	1.01678
29.92 x 108.46	30.00 x 110.00	3245.123	3300.000	0.98337
29.78 x 79.61	30.00 x 80.00	2370.786	2400.000	0.98783
29.71 x 99.59	30.00 x 100.00	2958.819	3000.000	0.98627
30.52 x 99.82	30.00 x 100.00	3046.506	3000.000	1.01550
19.77 x 168.13	20.00 x 170.00	3323.930	3400.000	0.97763
19.81 x 148.61	20.00 x 150.00	2943.964	3000.000	0.98132
30.28 x 98.81	30.00 x 100.00	2991.967	3000.000	0.99732
30.58 x 109.76	30.00 x 110.00	3356.461	3300.000	1.01711
40.15 x 109.61	40.00 x 110.00	4400.842	4400.000	1.00019
20.24 x 147.45	20.00 x 150.00	2984.388	3000.000	0.99480
30.12 x 49.44	30.00 x 50.00	1489.133	1500.000	0.99276
30.72 x 108.26	30.00 x 110.00	3325.747	3300.000	1.00780
29.39 x 79.36	30.00 x 80.00	2332.390	2400.000	0.97183
29.88 x 109.19	30.00 x 110.00	3262.597	3300.000	0.98867
39.72 x 88.61	40.00 x 90.00	3519.589	3600.000	0.97766
29.87 x 99.18	30.00 x 100.00	2962.507	3000.000	0.98750
30.31 x 98.42	30.00 x 100.00	2983.110	3000.000	0.99434
59.32 x 59.44	60.00 x 60.00	3525.981	3600.000	0.97944
58.70 x 59.61	60.00 x 60.00	3499.107	3600.000	0.97197
30.14 x 100.12	30.00 x 100.00	3017.617	3000.000	1.00587
30.14 x 99.52	30.00 x 100.00	2999.533	3000.000	0.99984
30.48 x 78.81	30.00 x 80.00	2402.129	2400.000	1.00089
30.09 x 79.81	30.00 x 80.00	2401.483	2400.000	1.00062
30.39 x 98.39	30.00 x 100.00	2989.768	3000.000	0.99658
29.72 x 78.49	30.00 x 80.00	2332.723	2400.000	0.97197
29.46 x 79.41	30.00 x 80.00	2339.419	2400.000	0.97476
30.04 x 98.91	30.00 x 100.00	2971.256	3000.000	0.99042
30.35 x 78.64	30.00 x 80.00	2386.724	2400.000	0.99447
28.74 x 88.12	30.00 x 90.00	2532.569	2700.000	0.93799
29.84 x 99.81	30.00 x 100.00	2978.330	3000.000	0.99277

Minimum Actual Dimensions in cm	Design Dimensions in cm	Actual Area in cm^2	Design Area in cm^2	Ratio of Areas
29.42 x 99.42	30.00 x 100.00	2924.936	3000.000	0.97465
58.16 x 59.86	60.00 x 60.00	3481.458	3600.000	0.96707
58.64 x 58.86	60.00 x 60.00	3451.550	3600.000	0.95876
28.69 x 99.71	30.00 x 100.00	2860.680	3000.000	0.95356
30.46 x 108.81	30.00 x 110.00	3314.353	3300.000	1.00435
39.21 x 88.72	40.00 x 90.00	3478.711	3600.000	0.96631
29.64 x 108.81	30.00 x 110.00	3225.128	3300.000	0.97731
29.64 x 81.27	30.00 x 80.00	2408.843	2400.000	1.00368
29.81 x 108.72	30.00 x 110.00	3240.943	3300.000	0.98210
29.96 x 48.17	30.00 x 50.00	1443.173	1500.000	0.96212
19.61 x 147.27	20.00 x 150.00	2887.965	3000.000	0.96266
38.52 x 108.43	40.00 x 110.00	4176.724	4400.000	0.94926
29.17 x 109.17	30.00 x 110.00	3184.489	3300.000	0.96500
30.26 x 97.24	30.00 x 100.00	2942.482	3000.000	0.98083
19.81 x 168.91	20.00 x 170.00	3346.107	3400.000	0.98415
20.96 x 149.61	20.00 x 150.00	3135.826	3000.000	1.04528
29.26 x 97.62	30.00 x 100.00	2856.361	3000.000	0.95212
29.87 x 98.27	30.00 x 100.00	2935.325	3000.000	0.97844
28.34 x 78.42	30.00 x 80.00	2214.581	2400.000	0.92274
28.86 x 110.91	30.00 x 110.00	3200.863	3300.000	0.96996
39.26 x 110.45	40.00 x 110.00	4336.267	4400.000	0.98552
29.14 x 110.85	30.00 x 110.00	3230.169	3300.000	0.97884
29.46 x 98.68	30.00 x 100.00	2907.113	3000.000	0.96904
57.43 x 58.81	60.00 x 60.00	3377.458	3600.000	0.93818
58.45 x 59.47	60.00 x 60.00	3476.022	3600.000	0.96556
29.74 x 97.64	30.00 x 100.00	2903.814	3000.000	0.96760
28.73 x 108.73	30.00 x 100.00	3123.813	3300.000	0.94661
38.65 x 89.68	40.00 x 90.00	3466.132	3600.000	0.96281
30.08 x 110.92	30.00 x 110.00	3336.474	3300.000	1.01105
29.64 x 79.84	30.00 x 80.00	2366.458	2400.000	0.98602
30.15 x 98.67	30.00 x 100.00	2974.901	3000.000	0.99163
30.34 x 78.43	30.00 x 80.00	2379.565	2400.000	0.99149
29.89 x 88.67	30.00 x 90.00	2650.346	2700.000	0.98161
29.64 x 99.67	30.00 x 100.00	2954.219	3000.000	0.98474
29.82 x 99.12	30.00 x 100.00	2955.758	3000.000	0.98525

Minimum Actual Dimensions in cm	Design Dimensions in cm	Actual Area in cm^2	Design Area in cm^2	Ratio of Areas
19.86 x 78.96	20.00 x 80.00	1568.146	1600.000	0.98009
19.76 x 89.74	20.00 x 90.00	1773.262	1800.000	0.98515
19.84 x 69.69	20.00 x 70.00	1382.650	1400.000	0.98761
19.16 x 69.31	20.00 x 70.00	1327.980	1400.000	0.94856
19.24 x 88.98	20.00 x 90.00	1711.975	1800.000	0.95110
19.12 x 89.31	20.00 x 90.00	1707.607	1800.000	0.94867
45.54 x 59.74	45.00 x 60.00	2720.560	2700.000	1.00761
49.17 x 60.57	50.00 x 60.00	2978.227	3000.000	0.99274
20.58 x 88.99	20.00 x 90.00	1831.414	1800.000	1.01745
20.09 x 89.78	20.00 x 90.00	1803.680	1800.000	1.00204
30.04 x 89.31	30.00 x 90.00	2682.872	2700.000	0.99366
20.33 x 110.76	20.00 x 110.00	2251.751	2200.000	1.02352
19.97 x 69.85	20.00 x 70.00	1394.905	1400.000	0.99636
19.18 x 98.97	20.00 x 100.00	1898.245	2000.000	0.94912
19.24 x 49.37	20.00 x 50.00	949.879	1000.000	0.94988
19.86 x 139.61	20.00 x 140.00	2772.655	2800.000	0.99023
34.09 x 79.65	35.00 x 80.00	2715.269	2800.000	0.96974
19.86 x 98.96	20.00 x 100.00	1965.346	2000.000	0.98267
20.89 x 88.67	20.00 x 90.00	1852.316	1800.000	1.02906
19.98 x 169.71	20.00 x 170.00	3390.806	3400.000	0.99730
20.61 x 139.41	20.00 x 140.00	2873.240	2800.000	1.02616
20.53 x 98.96	20.00 x 100.00	2031.649	2000.000	1.01582
20.35 x 100.84	20.00 x 100.00	2052.094	2000.000	1.02605
20.22 x 69.72	20.00 x 70.00	1409.738	1400.000	1.00696
20.63 x 99.34	20.00 x 100.00	2049.384	2000.000	1.02469
34.08 x 78.96	35.00 x 80.00	2690.957	2800.000	0.96106
20.09 x 99.34	20.00 x 100.00	1995.741	2000.000	0.99787
19.85 x 89.31	20.00 x 90.00	1772.804	1800.000	0.98489
44.28 x 58.97	45.00 x 60.00	2611.192	2700.000	0.96711
48.98 x 58.69	50.00 x 60.00	2874.636	3000.000	0.95821
19.86 x 89.76	20.00 x 90.00	1786.634	1800.000	0.99035
19.73 x 89.02	20.00 x 90.00	1756.365	1800.000	0.97576
29.07 x 89.98	30.00 x 90.00	2615.719	2700.000	0.96878
20.03 x 109.10	20.00 x 110.00	2185.273	2200.000	0.99331
19.86 x 68.71	20.00 x 70.00	1364.581	1400.000	0.97470

Minimum Actual Dimensions in cm	Design Dimensions in cm	Actual Area in cm^2	Design Area in cm^2	Ratio of Areas
19.25 x 90.07	20.00 x 90.00	1733.848	1800.000	0.99632
19.41 x 68.71	20.00 x 70.00	1333.661	1400.000	0.95265
20.15 x 68.36	20.00 x 70.00	1377.454	1400.000	0.98390
19.41 x 89.71	20.00 x 90.00	1741.271	1800.000	0.96737
19.41 x 90.53	20.00 x 90.00	1757.187	1800.000	0.97622
29.91 x 79.61	30.00 x 80.00	2381.135	2400.000	0.99214
29.22 x 99.61	30.00 x 100.00	2910.604	3000.000	0.97020
29.38 x 79.82	30.00 x 80.00	2345.112	2400.000	0.97713
29.86 x 88.51	30.00 x 90.00	2642.909	2700.000	0.97886
29.47 x 100.54	30.00 x 100.00	2962.914	3000.000	0.98764
29.27 x 99.83	30.00 x 100.00	2922.024	3000.000	0.97401
58.97 x 59.73	60.00 x 60.00	3522.278	3600.000	0.97841
58.22 x 59.74	60.00 x 60.00	3478.063	3600.000	0.96613
29.65 x 97.74	30.00 x 100.00	2897.991	3000.000	0.96600
28.22 x 108.91	30.00 x 110.00	3073.440	3300.000	0.93135
37.02 x 87.64	40.00 x 90.00	3244.433	3600.000	0.90123
28.43 x 108.91	30.00 x 110.00	3096.311	3300.000	0.93828
29.69 x 78.93	30.00 x 80.00	2343.432	2400.000	0.97643
29.87 x 108.86	30.00 x 110.00	3251.648	3300.000	0.98535
28.93 x 48.83	30.00 x 50.00	1412.652	1500.000	0.94177
19.13 x 148.73	20.00 x 150.00	2845.205	3000.000	0.94840
39.87 x 108.98	40.00 x 110.00	4345.033	4400.000	0.98751
30.47 x 109.87	30.00 x 110.00	3347.739	3300.000	1.01447
28.86 x 98.96	30.00 x 100.00	2855.986	3000.000	0.95200
18.97 x 169.13	20.00 x 170.00	3208.396	3400.000	0.94365
19.26 x 149.87	20.00 x 150.00	2886.496	3000.000	0.96217
28.93 x 98.71	30.00 x 100.00	2855.680	3000.000	0.95189
29.97 x 99.89	30.00 x 100.00	2993.703	3000.000	0.99790
30.37 x 78.93	30.00 x 80.00	2397.104	2400.000	0.99879
28.69 x 109.41	30.00 x 110.00	3138.973	3300.000	0.95120
40.25 x 110.73	40.00 x 110.00	4456.883	4400.000	1.01293
28.99 x 108.99	30.00 x 110.00	3159.620	3300.000	0.95746
29.86 x 99.01	30.00 x 100.00	2956.439	3000.000	0.98548
59.81 x 60.03	60.00 x 60.00	3590.394	3600.000	0.99733
58.99 x 59.21	60.00 x 60.00	3492.798	3600.000	0.97022

Minimum Actual Dimensions in cm	Design Dimensions in cm	Actual Area in cm^2	Design Area in cm^2	Ratio of Areas
29.26 x 98.91	30.00 x 100.00	2894.107	3000.000	0.96470
30.04 x 109.00	30.00 x 110.00	3274.360	3300.000	0.99223
39.31 x 89.19	40.00 x 90.00	3506.059	3600.000	0.97391
29.11 x 77.74	30.00 x 80.00	2262.234	2400.000	0.94260
29.27 x 98.73	30.00 x 100.00	2889.827	3000.000	0.96328
29.76 x 79.94	30.00 x 80.00	2379.014	2400.000	0.99126
30.36 x 88.36	30.00 x 80.00	2682.610	2700.000	0.99356
29.97 x 97.64	30.00 x 100.00	2926.271	3000.000	0.97542
29.83 x 99.91	30.00 x 100.00	2980.315	3000.000	0.99344
19.52 x 79.76	20.00 x 80.00	1556.915	1600.000	0.97307
19.63 x 88.64	20.00 x 90.00	1740.003	1800.000	0.96667
19.82 x 69.13	20.00 x 70.00	1370.157	1400.000	0.97868
19.76 x 68.37	20.00 x 70.00	1350.991	1400.000	0.96499
18.84 x 89.13	20.00 x 90.00	1679.209	1800.000	0.93289
19.39 x 89.81	20.00 x 90.00	1741.416	1800.000	0.96745
44.09 x 59.21	45.00 x 60.00	2610.569	2700.000	0.96688
49.73 x 59.86	50.00 x 60.00	2976.838	3000.000	0.99228
19.86 x 89.03	20.00 x 90.00	1768.136	1800.000	0.98230
19.91 x 89.54	20.00 x 90.00	1782.741	1800.000	0.99041
30.17 x 89.90	30.00 x 90.00	2712.283	2700.000	1.00455
19.68 x 109.46	20.00 x 110.00	2154.173	2200.000	0.97917
20.14 x 69.71	20.00 x 70.00	1403.959	1400.000	1.00283
19.16 x 99.07	20.00 x 100.00	1898.181	2000.000	0.94909
19.62 x 49.32	20.00 x 50.00	967.658	1000.000	0.96766
19.42 x 138.65	20.00 x 140.00	2692.583	2800.000	0.96164
34.81 x 78.89	35.00 x 80.00	2746.161	2800.000	0.98077
19.41 x 98.97	20.00 x 100.00	1921.008	2000.000	0.96050
19.35 x 89.38	20.00 x 90.00	1729.503	1800.000	0.96084
19.52 x 168.97	20.00 x 170.00	3298.294	3400.000	0.97009
19.67 x 138.96	20.00 x 140.00	2733.343	2800.000	0.97619
18.96 x 98.91	20.00 x 100.00	1875.334	2000.000	0.93767
19.74 x 99.93	20.00 x 100.00	1972.618	2000.000	0.98631
19.63 x 69.81	20.00 x 70.00	1376.370	1400.000	0.98312
19.86 x 98.83	20.00 x 100.00	1962.764	2000.000	0.98138
35.34 x 80.35	35.00 x 80.00	2839.569	2800.000	1.01413

Minimum Actual Dimensions in cm	Design Dimensions in cm	Actual Area in cm^2	Design Area in cm^2	Ratio of Areas
19.96 x 99.43	20.00 x 100.00	1984.623	2000.000	0.99231
20.48 x 89.24	20.00 x 90.00	1827.635	1800.000	1.01535
44.21 x 59.13	45.00 x 60.00	2614.137	2700.000	0.96820
49.21 x 59.21	50.00 x 60.00	2913.724	3000.000	0.97124
19.68 x 89.77	20.00 x 90.00	1766.673	1800.000	0.98149
19.92 x 88.62	20.00 x 90.00	1765.310	1800.000	0.98073
28.64 x 89.16	30.00 x 90.00	2553.542	2700.000	0.94576
19.79 x 108.93	20.00 x 110.00	2155.755	2200.000	0.97983
19.17 x 69.53	20.00 x 70.00	1332.890	1400.000	0.95206
19.51 x 89.62	20.00 x 90.00	1748.486	1800.000	0.97138
19.64 x 69.91	20.00 x 70.00	1373.032	1400.000	0.98074
19.89 x 69.85	20.00 x 70.00	1389.317	1400.000	0.99237
19.76 x 90.35	20.00 x 90.00	1785.316	1800.000	0.99184
18.84 x 89.67	20.00 x 90.00	1689.383	1800.000	0.93855
29.96 x 79.69	30.00 x 80.00	2387.512	2400.000	0.99480
29.69 x 99.76	30.00 x 100.00	2961.874	3000.000	0.98729
29.12 x 79.83	30.00 x 80.00	2324.650	2400.000	0.96860
30.89 x 88.71	30.00 x 90.00	2740.252	2700.000	1.01491
29.81 x 99.24	30.00 x 100.00	2958.344	3000.000	0.98611
28.58 x 98.91	30.00 x 100.00	2826.848	3000.000	0.94228
58.97 x 59.31	60.00 x 60.00	3497.511	3600.000	0.97153
59.62 x 60.48	60.00 x 60.00	3605.818	3600.000	1.00162
29.28 x 98.17	30.00 x 100.00	2874.418	3000.000	0.95814
30.34 x 109.11	30.00 x 110.00	3310.397	3300.000	1.00315
39.22 x 89.91	40.00 x 90.00	3526.270	3600.000	0.97952
28.81 x 110.08	30.00 x 110.00	3171.405	3300.000	0.96103
29.86 x 78.93	30.00 x 80.00	2356.850	2400.000	0.98202
30.09 x 109.13	30.00 x 110.00	3283.722	3300.000	0.99507
29.76 x 49.13	30.00 x 50.00	1462.109	1500.000	0.97474
19.69 x 148.96	20.00 x 150.00	2933.022	3000.000	0.97767
38.91 x 109.11	40.00 x 110.00	4245.470	4400.000	0.96488
28.97 x 109.38	30.00 x 110.00	3168.739	3300.000	0.96022
29.63 x 98.71	30.00 x 100.00	2924.777	3000.000	0.97493
19.93 x 169.10	20.00 x 170.00	3370.163	3400.000	0.99122
19.86 x 149.13	20.00 x 150.00	2961.722	3000.000	0.98774

Minimum Actual Dimensions in cm	Design Dimensions in cm	Actual Area in cm^2	Design Area in cm^2	Ratio of Areas
30.18 x 98.91	30.00 x 100.00	2985.104	3000.000	0.99503
29.02 x 99.95	30.00 x 100.00	2900.549	3000.000	0.96685
29.81 x 78.89	30.00 x 80.00	2351.711	2400.000	0.97988
29.91 x 109.37	30.00 x 110.00	3271.257	3300.000	0.99129
39.27 x 108.99	40.00 x 110.00	4280.037	4400.000	0.97274
29.65 x 110.14	30.00 x 110.00	3265.651	3300.000	0.98959
30.09 x 99.86	30.00 x 100.00	3004.787	3000.000	1.00160
58.76 x 60.09	60.00 x 60.00	3530.888	3600.000	0.98080
59.67 x 59.81	60.00 x 60.00	3556.929	3600.000	0.98804
29.87 x 99.81	30.00 x 100.00	2981.325	3000.000	0.99378
28.91 x 109.81	30.00 x 110.00	3174.607	3300.000	0.96200
39.41 x 89.69	40.00 x 90.00	3534.683	3600.000	0.98186
28.96 x 109.31	30.00 x 110.00	3165.618	3300.000	0.95928
29.97 x 78.82	30.00 x 80.00	2362.235	2400.000	0.98426
28.93 x 99.86	30.00 x 100.00	2888.950	3000.000	0.96298
29.17 x 80.97	30.00 x 80.00	2361.895	2400.000	0.98412
29.08 x 89.91	30.00 x 90.00	2614.583	2700.000	0.96836
29.12 x 98.91	30.00 x 100.00	2880.259	3000.000	0.96009
30.26 x 98.09	30.00 x 100.00	2968.203	3000.000	0.98940
29.65 x 97.74	30.00 x 100.00	2897.991	3000.000	0.96600

TABLE G

Nominal Diameter in mm : 12.0

Nominal Area in cm^2 : 1.13098

Actual Diameter in mm	Actual Area in cm^2	Ratio of Areas	Actual Diameter in mm	Actual Area in cm^2	Ratio of Areas
12.2	1.16899	1.03361	12.4	1.20763	1.06777
12.3	1.18823	1.05062	12.5	1.22719	1.08507
12.3	1.18823	1.05062	12.5	1.22719	1.08507
12.4	1.20763	1.06777	12.3	1.18823	1.05062
12.3	1.18823	1.05062	12.4	1.20763	1.06777
12.5	1.22719	1.08507	12.5	1.22719	1.08507
12.5	1.22719	1.08507	12.4	1.20763	1.06777
12.1	1.14990	1.01673	12.3	1.18823	1.05062
12.1	1.14990	1.01673	12.5	1.22719	1.08507
12.2	1.16899	1.03361	12.5	1.22719	1.08507
12.2	1.16899	1.03361	12.3	1.18823	1.05062
12.5	1.22719	1.08507	12.1	1.14990	1.01673
12.6	1.24690	1.10250	12.3	1.18823	1.05062
12.5	1.22719	1.08507	12.5	1.22719	1.08507
12.1	1.14990	1.01673	12.5	1.22719	1.08507
12.2	1.16899	1.03361	12.6	1.24690	1.10250
12.3	1.18823	1.05062	12.7	1.26677	1.12006
12.3	1.18823	1.05062	12.3	1.18823	1.05062
12.9	1.30698	1.15562	12.5	1.22719	1.08507
12.3	1.18823	1.05062	12.4	1.20763	1.06777
12.4	1.20763	1.06777	12.9	1.30698	1.15562
12.3	1.18823	1.05062	12.4	1.20763	1.06777
12.5	1.22719	1.08507	12.4	1.20763	1.06777
12.5	1.22719	1.08507	12.4	1.20763	1.06777
12.4	1.20763	1.06777	12.4	1.20763	1.06777

Nominal Diameter in mm : 14.0
Nominal Area in cm^2 : 1.53938

Actual Diameter in mm	Actual Area in cm^2	Ratio of Areas	Actual Diameter in mm	Actual Area in cm^2	Ratio of Areas
14.5	1.65130	1.07270	14.5	1.65130	1.07270
14.7	1.69717	1.10250	14.1	1.56145	1.01434
14.7	1.69717	1.10250	14.5	1.65130	1.07270
14.5	1.65130	1.07270	14.5	1.65130	1.07270
14.8	1.72034	1.11755	14.7	1.69717	1.10250
14.1	1.56145	1.01434	14.2	1.58368	1.02878
14.5	1.65130	1.07270	14.5	1.65130	1.07270
14.5	1.65130	1.07270	14.1	1.56145	1.01434
14.1	1.56145	1.01434	14.3	1.60606	1.04332
14.5	1.65130	1.07270	14.5	1.65130	1.07270
14.5	1.65130	1.07270	14.1	1.56145	1.01434
14.2	1.58368	1.02878	14.1	1.56145	1.01434
14.3	1.60606	1.04332	14.2	1.58368	1.02878
14.2	1.58368	1.02878	14.3	1.60606	1.04332
14.2	1.58368	1.02878	14.3	1.60606	1.04332
14.2	1.58368	1.02878	14.4	1.62861	1.05796
14.3	1.60606	1.04332	14.3	1.60606	1.04332
14.4	1.62861	1.05796	14.5	1.65130	1.07270
14.1	1.56145	1.01434	14.3	1.60606	1.04332
14.4	1.62861	1.05796	14.5	1.65130	1.07270
14.4	1.62861	1.05796	14.4	1.62861	1.05796
14.7	1.69717	1.10250	14.8	1.72034	1.11755
14.3	1.60606	1.04332	14.6	1.67416	1.08755
14.5	1.65130	1.07270	14.5	1.65130	1.07270
14.3	1.60606	1.04332	14.5	1.65130	1.07270

Nominal Diameter in mm : 16.0
Nominal Area in cm^2 : 2.01062

Actual Diameter in mm	Actual Area in cm^2	Ratio of Areas	Actual Diameter in mm	Actual Area in cm^2	Ratio of Areas
16.4	2.11241	1.05063	16.4	2.11241	1.05063
16.3	2.08673	1.03785	16.5	2.13825	1.06348
16.2	2.06120	1.02516	16.6	2.16425	1.07641
16.5	2.13825	1.06348	16.5	2.13825	1.06348
16.2	2.06120	1.02516	16.3	2.08673	1.03785
16.4	2.11241	1.05063	16.6	2.16425	1.07641
16.3	2.08673	1.03785	16.6	2.16425	1.07641
16.5	2.13825	1.06348	16.6	2.16425	1.07641
16.2	2.06120	1.02516	16.3	2.08673	1.03785
16.4	2.11241	1.05063	16.3	2.08673	1.03785
16.3	2.08673	1.03785	17.0	2.26981	1.12891
16.9	2.24318	1.11567	16.9	2.24318	1.11567
15.5	1.88692	0.93848	15.6	1.91135	0.95063
16.5	2.13825	1.06348	16.5	2.13825	1.06348
16.2	2.06120	1.02516	16.2	2.06120	1.02516
16.2	2.06120	1.02516	16.3	2.08673	1.03785
16.1	2.03584	1.01254	16.5	2.13825	1.06348
16.5	2.13825	1.06348	16.5	2.13825	1.06348
16.8	2.21671	1.10250	16.7	2.19040	1.08942
16.8	2.21671	1.10250	16.6	2.16425	1.07641
16.3	2.08673	1.03785	16.2	2.06120	1.02516
16.3	2.08673	1.03785	16.4	2.11241	1.05063
16.1	2.03584	1.01254	16.3	2.08673	1.03785
16.7	2.19040	1.08942	16.5	2.13825	1.06348
16.3	2.08673	1.03785	16.5	2.13825	1.06348

Nominal Diameter in mm : 18.0
Nominal Area in cm^2 : 2.54470

Actual Diameter in mm	Actual Area in cm^2	Ratio of Areas	Actual Diameter in mm	Actual Area in cm^2	Ratio of Areas
18.2	2.60156	1.02234	18.2	2.60156	1.02234
18.3	2.63023	1.03361	18.6	2.71717	1.06778
18.3	2.63023	1.03361	18.3	2.63023	1.03361
18.4	2.65905	1.04494	18.7	2.74647	1.07929
18.2	2.60156	1.02234	18.2	2.60156	1.02234
18.3	2.63023	1.03361	18.2	2.60156	1.02234
18.2	2.60156	1.02234	18.4	2.65905	1.04494
18.6	2.71717	1.06778	18.4	2.65905	1.04494
18.7	2.74647	1.07929	18.4	2.65905	1.04494
18.3	2.62023	1.03361	18.8	2.77592	1.09086
18.4	2.65905	1.04494	18.2	2.60156	1.02234
18.3	2.63023	1.03361	18.5	2.68803	1.05632
18.3	2.63023	1.03361	18.3	2.63023	1.03361
18.5	2.68803	1.05632	18.4	2.65905	1.04494
18.4	2.65905	1.04494	18.6	2.71717	1.06778
18.2	2.60156	1.02234	18.3	2.63023	1.03361
18.4	2.65905	1.04494	18.2	2.60156	1.02234
18.3	2.63023	1.03361	18.3	2.63023	1.03361
18.4	2.65905	1.04494	18.5	2.68803	1.05632
18.1	2.57305	1.01114	18.1	2.57305	1.01114
18.4	2.65905	1.04494	18.8	2.77592	1.09086
18.1	2.57305	1.01114	18.1	2.57305	1.01114
18.2	2.60156	1.02234	18.2	2.60156	1.02234
18.2	2.60156	1.02234	18.2	2.60156	1.02234
18.4	2.65905	1.04494	18.5	2.68803	1.05632

Nominal Diameter in mm : 20.0
Nominal Area in cm^2 : 3.14160

Actual Diameter in mm	Actual Area in cm^2	Ratio of Areas	Actual Diameter in mm	Actual Area in cm^2	Ratio of Areas
20.2	3.20475	1.02010	20.3	3.23655	1.03022
20.4	3.26852	1.04040	20.4	3.26852	1.04040
20.4	3.26852	1.04040	20.3	3.23655	1.03022
20.2	3.20475	1.02010	20.3	3.23655	1.03022
20.2	3.20475	1.02010	20.4	3.26852	1.04040
20.4	3.26852	1.04040	20.4	3.26852	1.04040
20.7	3.36536	1.07122	20.0	3.14160	1.00000
20.1	3.17309	1.01002	20.9	3.43071	1.09203
20.7	3.36536	1.07122	19.9	3.11026	0.99002
20.1	3.17309	1.01002	19.9	3.11026	0.99002
20.2	3.20475	1.02010	20.9	3.36536	1.09203
20.4	3.26852	1.04040	20.6	3.33292	1.06090
20.7	3.36536	1.07122	20.4	3.26852	1.04040
20.6	3.33292	1.06090	20.8	3.39795	1.08160
20.9	3.43071	1.09203	20.3	3.23655	1.03022
20.9	3.43071	1.09203	20.2	3.20475	1.02010
20.2	3.20475	1.02010	20.4	3.26852	1.04040
20.4	3.26852	1.04040	20.5	3.30064	1.05062
19.5	2.98648	0.95062	20.4	3.26852	1.04040
20.5	3.30064	1.05062	20.0	3.14160	1.00000
20.2	3.20475	1.02010	20.3	3.23655	1.03022
20.4	3.26852	1.04040	20.5	3.30064	1.05062
20.5	3.30064	1.05062	20.1	3.17309	1.01002
20.3	3.23655	1.03022	20.3	3.23655	1.03022
20.5	3.30064	1.05062	20.9	3.43071	1.09203

TABLE H

X_{A_L}	$(X_{A_L} - \bar{X}_{A_L})$	$(X_{A_L} - \bar{X}_{A_L})^2 x\ 10^{-4}$
0.88600	0.10293	105.94585
0.90123	0.08770	76.91290
0.92274	0.06619	43.81116
0.93135	0.05758	33.15456
0.93289	0.05604	31.40482
0.93767	0.05126	26.27583
0.93799	0.05094	25.94884
0.93818	0.05075	25.75563
0.93828	0.05065	25.65423
0.93855	0.05038	25.38144
0.94177	0.04716	22.24066
0.94228	0.04665	21.76222
0.94248	0.04645	21.57603
0.94260	0.04633	21.46469
0.94365	0.04528	20.50278
0.94370	0.04523	20.45753
0.94576	0.04317	18.63649
0.94661	0.04232	17.90982
0.94840	0.04053	16.42681
0.94856	0.04037	16.29737
0.94867	0.04026	16.20868
0.94909	0.03984	15.87226
0.94912	0.03981	15.84836
0.94926	0.03967	15.73709
0.94988	0.03905	15.24903
0.95048	0.03845	14.78403
0.95110	0.03783	14.31109
0.95120	0.03773	14.23555
0.95189	0.03704	13.71962
0.95200	0.03693	13.63825

X_{A_t}	$X_{A_t} - \bar{X}_{A_t}$	$(X_{A_t} - \bar{X}_{A_t})^2 \times 10^{-4}$
0.95206	0.03687	13.59397
0.95212	0.03681	13.54976
0.95262	0.03631	13.18416
0.95356	0.03537	12.51037
0.95746	0.03147	9.90361
0.95814	0.03079	9.48024
0.95821	0.03072	9.43718
0.95848	0.03045	9.27203
0.95876	0.03017	9.10229
0.95911	0.02982	8.89232
0.95928	0.02965	8.79123
0.96009	0.02884	8.31746
0.96022	0.02871	8.24264
0.96050	0.02843	8.08265
0.96084	0.02809	7.89048
0.96103	0.02790	7.78410
0.96106	0.02787	7.76737
0.96144	0.02749	7.55700
0.96164	0.02729	7.44744
0.96190	0.02703	7.30621
0.96200	0.02693	7.25225
0.96212	0.02681	7.18776
0.96217	0.02676	7.16098
0.96266	0.02627	6.90113
0.96281	0.02612	6.82254
0.96298	0.02595	6.73403
0.96325	0.02568	6.59462
0.96328	0.02565	6.57923
0.96358	0.02535	6.42623
0.96364	0.02529	6.39584
0.96410	0.02483	6.16529
0.96460	0.02433	5.91949
0.96470	0.02423	5.87093
0.96488	0.02405	5.78403
0.96499	0.02394	5.73124

X_{Ac}	$X_{Ac} - \overline{X}_{Ac}$	$(X_{Ac} - \overline{X}_{Ac})^2 \times 10^{-4}$
0.96500	0.02393	5.72645
0.96502	0.02391	5.71688
0.96556	0.02337	5.46157
0.96600	0.02293	5.25785
0.96613	0.02280	5.19840
0.96621	0.02272	5.16193
0.96631	0.02262	5.11664
0.96667	0.02226	4.95508
0.96685	0.02208	4.87526
0.96688	0.02205	4.86203
0.96707	0.02186	4.77860
0.96711	0.02182	4.76112
0.96724	0.02169	4.70456
0.96727	0.02166	4.69156
0.96737	0.02156	4.64834
0.96745	0.02148	4.61390
0.96760	0.02133	4.54969
0.96766	0.02127	4.52413
0.96803	0.02090	4.36810
0.96820	0.02073	4.29733
0.96824	0.02069	4.28076
0.96836	0.02057	4.23125
0.96860	0.02033	4.13309
0.96878	0.02015	4.06023
0.96904	0.01989	3.95612
0.96974	0.01919	3.68256
0.96974	0.01919	3.68256
0.96996	0.01897	3.59861
0.97009	0.01884	3.54946
0.97020	0.01873	3.50813
0.97022	0.01871	3.50064
0.97124	0.01769	3.12936
0.97138	0.01755	3.08003
0.97153	0.01740	3.02760
0.97183	0.01710	2.92410

X_{A_c}	$X_{A_c} - \bar{X}_{A_c}$	$(X_{A_c} - \bar{X}_{A_c})^2 \times 10^{-4}$
0.97197	0.01696	2.87642
0.97197	0.01696	2.87642
0.97252	0.01641	2.69288
0.97274	0.01619	2.62116
0.97307	0.01586	2.51540
0.97317	0.01576	2.48378
0.97338	0.01555	2.41803
0.97391	0.01502	2.25600
0.97401	0.01492	2.22606
0.97429	0.01464	2.14330
0.97443	0.01450	2.10250
0.97465	0.01428	2.03918
0.97470	0.01423	2.02493
0.97474	0.01419	2.01356
0.97474	0.01419	é.01356
0.97476	0.01417	2.00789
0.97483	0.01410	1.98810
0.97493	0.01400	1.96000
0.97501	0.01392	1.93766
0.97518	0.01375	1.89063
0.97542	0.01351	1.82520
0.97576	0.01317	1.73449
0.97594	0.01299	1.68740
0.97619	0.01274	1.62308
0.97622	0.01271	1.61544
0.97643	0.01250	1.56250
0.97697	0.01196	1.43042
0.97713	0.01180	1.39240
0.97731	0.01162	1.35024
0.97763	0.01130	1.27690
0.97766	0.01127	1.27013
0.97767	0.01126	1.26788
0.97841	0.01052	1.10670
0.97844	0.01049	1.10040
0.97861	0.01032	1.06502

X_{A_t}	$X_{A_t} - \bar{X}_{A_t}$	$(X_{A_t} - \bar{X}_{A_t})^2 \times 10^{-4}$
0.97868	0.01025	1.05063
0.97884	0.01009	1.01808
0.97886	0.01007	1.01405
0.97917	0.00976	0.95258
0.97944	0.00949	0.90060
0.97950	0.00943	0.88925
0.97952	0.00941	0.88548
0.97988	0.00905	0.81903
0.97988	0.00905	0.81903
0.98009	0.00884	0.78146
0.98065	0.00828	0.68558
0.98073	0.00820	0.67240
0.98074	0.00819	0.67076
0.98077	0.00816	0.66586
0.98080	0.00813	0.66097
0.98083	0.00810	0.65610
0.98096	0.00797	0.63521
0.98132	0.00761	0.57912
0.98138	0.00755	0.57003
0.98149	0.00744	0.55354
0.98151	0.00732	0.53582
0.98186	0.00707	0.49985
0.98202	0.00691	0.47748
0.98210	0.00683	0.46649
0.98220	0.00673	0.45293
0.98230	0.00663	0.43957
0.98236	0.00657	0.43165
0.98241	0.00652	0.42510
0.98267	0.00626	0.39188
0.98279	0.00614	0.37700
0.98295	0.00598	0.35760
0.98312	0.00581	0.33756
0.98337	0.00556	0.30914
0.98390	0.00503	0.25301
0.98411	0.00482	0.23232

X_{A_c}	$X_{A_c} - \overline{X}_{A_c}$	$(X_{A_c} - \overline{X}_{A_c})^2 \times 10^{-4}$
0.98412	0.00481	0.23136
0.98415	0.00478	0.22848
0.98425	0.00468	0.21902
0.98426	0.00467	0.21809
0.98461	0.00432	0.18662
0.98474	0.00419	0.17556
0.98489	0.00404	0.16322
0.98503	0.00390	0.15210
0.98515	0.00378	0.14288
0.98525	0.00368	0.13542
0.98535	0.00358	0.12816
0.98536	0.00357	0.12745
0.98548	0.00345	0.11903
0.98552	0.00341	0.11628
0.98555	0.00338	0.11424
0.98560	0.00333	0.11089
0.98568	0.00325	0.10563
0.98602	0.00291	0.08468
0.98611	0.00282	0.07952
0.98622	0.00271	0.07344
0.98624	0.00269	0.07236
0.98627	0.00266	0.07076
0.98629	0.00264	0.06970
0.98631	0.00262	0.06864
0.98676	0.00217	0.04709
0.98691	0.00202	0.04080
0.98724	0.00169	0.02856
0.98729	0.00164	0.02290
0.98750	0.00143	0.02045
0.98750	0.00143	0.02045
0.98751	0.00142	0.02016
0.98761	0.00132	0.01742
0.98764	0.00129	0.01664
0.98783	0.00110	0.01210
0.98787	0.00106	0.01124

X_{A_c}	$X_{A_c} - \overline{X}_{A_c}$	$(X_{A_c} - \overline{X}_{A_c})^2 \times 10^{-4}$
0.98804	0.00089	0.00792
0.98867	0.00026	0.00068
0.98899	0.00006	0.00004
0.98903	0.00010	0.00010
0.98930	0.00037	0.00137
0.98940	0.00047	0.00221
0.98958	0.00065	0.00422
0.98959	0.00066	0.00436
0.99023	0.00130	0.01690
0.99027	0.00134	0.01796
0.99035	0.00142	0.02016
0.99041	0.00148	0.02190
0.99042	0.00149	0.02220
0.99051	0.00158	0.02496
0.99122	0.00229	0.05244
0.99126	0.00233	0.05429
0.99129	0.00236	0.05570
0.99138	0.00245	0.06003
0.99141	0.00248	0.06150
0.99149	0.00256	0.06554
0.99163	0.00270	0.07290
0.99184	0.00291	0.08468
0.99199	0.00306	0.09364
0.99214	0.00321	0.10304
0.99214	0.00321	0.10304
0.99216	0.00323	0.10433
0.99222	0.00329	0.10824
0.99223	0.00330	0.10890
0.99228	0.00335	0.11223
0.99231	0.00338	0.11424
0.99237	0.00344	0.11834
0.99262	0.00369	0.13616
0.99268	0.00375	0.14063
0.99274	0.00381	0.14516
0.99276	0.00383	0.14669

X_{Ac}	$X_{Ac} - \bar{X}_{Ac}$	$(X_{Ac} - \bar{X}_{Ac})^2 \times 10^{-4}$
0.99277	0.00384	0.14746
0.99300	0.00407	0.16565
0.99331	0.00438	0.19184
0.99340	0.00447	0.19981
0.99344	0.00451	0.20340
0.99356	0.00463	0.21437
0.99366	0.00473	0.22373
0.99378	0.00485	0.23523
0.99401	0.00508	0.25806
0.99408	0.00515	0.26523
0.99434	0.00541	0.29268
0.99438	0.00545	0.29702
0.99447	0.00554	0.30692
0.99464	0.00571	0.32604
0.99470	0.00577	0.33293
0.99471	0.00578	0.33408
0.99480	0.00587	0.34457
0.99480	0.00587	0.34457
0.99482	0.00589	0.34692
0.99503	0.00610	0.37210
0.99507	0.00614	0.37700
0.99509	0.00616	0.37946
0.99513	0.00620	0.38440
0.99636	0.00743	0.55205
0.99638	0.00745	0.55502
0.99655	0.00762	0.58064
0.99658	0.00765	0.58522
0.99663	0.00770	0.59790
0.99693	0.00800	0.64000
0.99717	0.00824	0.67897
0.99730	0.00837	0.70057
0.99732	0.00839	0.70392
0.99733	0.00840	0.70560
0.99748	0.00855	0.73102
0.99750	0.00857	0.73445

X_{Ac}	$X_{Ac} - \bar{X}_{Ac}$	$(X_{Ac} - \bar{X}_{Ac})^2 \times 10^{-4}$
0.99750	0.00857	0.73445
0.99759	0.00866	0.74996
0.99787	0.00894	0.79924
0.99790	0.00897	0.80461
0.99793	0.00900	0.81000
0.99869	0.00976	0.95258
0.99879	0.00986	0.97220
0.99904	0.01011	1.02212
0.99965	0.01072	1.14918
0.99984	0.01091	1.19028
1.00019	0.01126	1.26788
1.00062	0.01169	1.36656
1.00089	0.01196	1.43042
1.00120	0.01227	1.50553
1.00135	0.01242	1.54256
1.00144	0.01251	1.56500
1.00160	0.01267	1.60529
1.00162	0.01269	1.61036
1.00204	0.01311	1.71872
1.00208	0.01315	1.72923
1.00262	0.01369	1.87416
1.00273	0.01385	1.91822
1.00283	0.01390	1.93210
1.00315	0.01422	2.02208
1.00321	0.01428	2.03918
1.00368	0.01475	2.17562
1.00374	0.01481	2.19336
1.00405	0.01512	2.28614
1.00435	0.01542	2.37776
1.00455	0.01562	2.43984
1.00503	0.01610	2.59210
1.00550	0.01657	2.74565
1.00583	0.01690	2.85610
1.00587	0.01694	2.86964
1.00596	0.01703	2.96021

x_{A_c}	$x_{A_c} - \bar{x}_{A_c}$	$(x_{A_c} - \bar{x}_{A_c})^2 \times 10^{-4}$
1.00619	0.01726	2.97908
1.00662	0.01769	3.12936
1.00665	0.01772	3.13998
1.00696	0.01803	3.25081
1.00722	0.01829	3.34524
1.00753	0.01860	3.45960
1.00760	0.01867	3.48569
1.00761	0.01868	3.48942
1.00777	0.01884	3.54946
1.00780	0.01887	3.56077
1.00821	0.01928	3.71718
1.00897	0.02004	4.01602
1.00919	0.02026	4.10463
1.00936	0.02043	4.17385
1.01001	0.02108	4.44366
1.01105	0.02212	4.89294
1.01137	0.02244	5.03554
1.01217	0.02324	5.40098
1.01249	0.02356	5.55074
1.01254	0.02361	5.57432
1.01293	0.02400	5.76000
1.01302	0.02409	5.80328
1.01313	0.02420	5.85640
1.01323	0.02430	5.90490
1.01387	0.02494	6.22004
1.01413	0.02520	6.35040
1.01447	0.02554	6.52292
1.01477	0.02584	6.67706
1.01491	0.02598	6.74960
1.01499	0.02606	6.79124
1.01521	0.02628	6.90638
1.01535	0.02642	6.98016
1.01550	0.02657	7.05965
1.01556	0.02663	7.09157
1.01582	0.02689	7.23072

$\bar{X}_{Ac}$	$(X_{Ac} - \bar{X}_{Ac})$	$(X_{Ac} - \bar{X}_{A})^2 \times 10^{-4}$
1.01678	0.02785	7.75623
1.01695	0.02802	7.85720
1.01708	0.02815	7.92422
1.01711	0.02818	7.94112
1.01719	0.02826	7.98628
1.01745	0.02852	8.13390
1.01806	0.02913	8.48557
1.01916	0.03023	9.13853
1.01986	0.03093	9.56665
1.02230	0.03337	11.13557
1.02263	0.03370	11.35690
1.02278	0.03385	11.45822
1.02352	0.03459	11.96468
1.02372	0.03479	12.10344
1.02376	0.03483	12.13129
1.02405	0.03512	12.33414
1.02422	0.03529	12.45384
1.02469	0.03576	12.78778
1.02506	0.03613	13.05377
1.02564	0.03671	13.47624
1.02605	0.03712	13.77894
1.02616	0.03723	13.86073
1.02626	0.03733	13.93529
1.02688	0.03795	14.40203
1.02747	0.03854	14.85332
1.02780	0.03887	15.10877
1.02898	0.04005	16.04002
1.02906	0.04013	16.10417
1.02964	0.04071	16.57304
1.03059	0.04166	17.35556
1.03064	0.04177	17.39724
1.03086	0.04193	17.58125
1.03097	0.04204	17.67362
1.03159	0.04266	18.19876
1.03365	0.04472	19.99878

X_{A_c}	$X_{A_c} - \bar{X}_{A_c}$	$(X_{A_c} - \bar{X}_{A_c})^2 \times 10^{-4}$
1.03442	0.04549	20.69340
1.03634	0.04741	22.47708
1.03712	0.04819	23.22276
1.03997	0.05104	26.05082
1.04034	0.05141	26.42988
1.04165	0.05272	27.79398
1.04318	0.05425	29.43063
1.04528	0.05635	31.75322
1.04569	0.05676	32.21698
1.04698	0.05805	33.69803
1.04819	0.05986	35.83220
1.05060	0.06167	38.03189
1.05292	0.06399	40.94720
1.05431	0.06538	42.74544
1.05798	0.06905	47.67903
1.05962	0.07069	49.97076
1.06077	0.07184	51.60986
1.06544	0.07651	53.53780
1.06814	0.07921	62.74224
1.10425	0.11532	132.98702
Σ 395.57119		Σ 2837.89370

TABLE I

X_{A_s}	n	$X_{A_s} - \overline{X}_{A_s}$	$n(X_{A_s} - \overline{X}_{A_s})^2 \times 10^{-4}$
0.93848	1	0.11250	126.56250
0.95062	2	0.10036	201.44260
0.99002	2	0.06096	74.32244
1.00000	2	0.05098	51.97920
1.01002	3	0.04096	50.33166
1.01114	4	0.03984	63.48904
1.01254	2	0.03844	29.55268
1.01434	6	0.03664	80.54940
1.01673	4	0.03425	46.92252
1.02010	7	0.03088	66.75018
1.02234	13	0.02864	106.63250
1.02515	7	0.02582	46.66704
1.02878	7	0.02220	34.49880
1.03022	7	0.02076	30.16846
1.03361	15	0.01737	45.25755
1.03785	11	0.01313	18.96357
1.04040	12	0.01058	13.43232
1.04332	9	0.00766	5.28084
1.04494	11	0.00604	4.01302
1.05062	22	0.00036	0.02860
1.05632	4	0.00534	1.14064

X_{A_s}	n	$X_{A_s} - \overline{X}_{A_s}$	$n(X_{A_s} - \overline{X}_{A_s})^2 \times 10^{-4}$
1.05796	5	0.00698	2.43600
1.06090	2	0.00992	1.96812
1.06348	11	0.01250	17.18750
1.06778	14	0.01680	39.51360
1.07122	2	0.02024	8.19316
1.07270	16	0.02172	75.48128
1.07641	5	0.02543	32.33425
1.07929	2	0.02831	16.02912
1.08160	1	0.03062	9.37584
1.08507	14	0.03409	162.69792
1.08755	1	0.03657	13.37365
1.08942	2	0.03844	29.55268
1.09086	2	0.03988	31.80828
1.09203	6	0.04105	101.10612
1.10250	8	0.05152	212.34480
1.11567	2	0.06469	83.69592
1.11755	2	0.06657	88.63130
1.12006	1	0.06908	47.72046
1.12891	1	0.07793	60.73085
1.15562	2	0.10464	218.99060
Σ262.74569	Σ250		Σ2351.15711

CHAPTER IIIA

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