CROSS-SECTIONAL STUDY ON GRADES, GEOMETRY ACHIEVEMENT AND VAN HIELE GEOMETRIC THINKING LEVELS

by

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ABSTRACT

CROSS-SECTIONAL STUDY ON GRADES, GEOMETRY ACHIEVEMENT AND VAN HIELE GEOMETRIC THINKING LEVELS

This study was conducted to determine whether there are differences in van Hiele geometric thinking levels of students and to investigate whether there is a correlation between geometry academic achievement and van Hiele Geometry Test scores of students at different grade levels. The van Hiele Geometry Test was examined on the basis of Turkish geometry curricula and contents of the curricula were examined on the basis of van Hiele geometric thinking levels in order to see whether they are aligned to geometric thinking levels of the students. The study was conducted at one of the private schools in Istanbul. The students were at 7^{th}, 8^{th}, 10^{th} and 11^{th} grades. 10^{th} and 11^{th} grades were either at science high school or Anatolian high school. University students were at SCED and PRED departments at Boğaziçi University. The van Hiele geometric thinking levels and scores of the students were determined by the van Hiele Geometry Test which was developed by Usiskin (1982) and has been used widely in educational research studies. The Turkish version which was translated by Duatepe (2000) was administered in the study. The design of the study was causal comparative research design. The data were analyzed by SPSS 17.0. Comparison of van Hiele Geometry Test scores was carried out by ANOVA Dunnet-C Test and the correlation analyses were done by Pearson-r. Multiple comparison analyses revealed that there are significant differences between the van Hiele Geometry Test scores of students at various grade levels. Results of the correlation analyses showed that there are significant correlations between the van Hiele Geometry Test scores and geometry achievement scores of the students at 7^{th}, 8^{th}, 10^{th} and 11^{th} grade levels. The results of the study showed that the geometric thinking levels of the students may not necessarily depend on the age or maturation but may depend more on the geometry curriculum and the geometric thinking levels are correlated with the geometry achievement levels of students.

ÖZET

SINIF DÜZEYLERİ, GEOMETRİ AKADEMİK BAŞARISI VE VAN HIELE GEOMETRİK DÜŞÜNME DÜZEYLERİ ÜZERİNE KESİTSEL ÇALIŞMA

Bu çalışma farklı sınıf düzeylerindeki öğrencilerin van Hiele geometrik düşünme düzeyleri arasında fark olup olmadığını ve van Hiele Geometri Testi puanları ile geometri başarı puanları arasında korelasyon olup olmadığını belirlemek için gerçekleştirilmiştir. Öğrencilerin geometrik düşünme düzeylerine uygun olup olmadığını belirlemek üzere, Van Hiele Geometri Testi, Türk geometri müfredatları bazında ve müfredat içeriği van Hiele geometrik düşünme düzeyleri bazında incelenmiştir. Çalışma İstanbul'daki bir özel okulun 7., 8., 10. ve 11. sınıf öğrencileriyle ve Boğaziçi Üniversitesi Ortaöğretim ve İlköğretim Matematik Öğretmenliği bölümlerindeki öğrencilerle gerçekleştirilmiştir. 10. ve 11. sınıf öğrencileri fen lisesi veya Anadolu lisesi öğrencileridirler. Öğrencilerin van Hiele geometrik düşünme düzeyleri ve puanları, Usiskin (1982) tarafından geliştirilen ve eğitim araştırmalarında yaygın olarak kullanılan van Hiele Geometri Testi ile belirlenmiştir. Çalışmada testin Duatepe (2000) tarafından çevrilen Türkçe versiyonu uygulanmıştır. Çalışmanın deseni nedensel karşılaştırmalı araştırma desenidir. Veriler SPSS 17.0 ile analiz edilmiştir. Karşılaştırma analizleri ANOVA Dunnett-C, korelasyon analizleri ise Pearson-r kullanılarak gerçekleştirilmiştir. Çoklu karşılaştırma analizleri farklı sınıf düzeylerinde van Hiele Geometri Testi puanları arasında anlamlı farklılıklar olduğunu göstermiştir. Korelasyon analizlerinin sonucunda, 7., 8., 10. ve 11. sınıf düzeylerinde van Hiele Geometri Test puanları ile geometri başarı puanları arasında anlamlı bir korelasyon görülmüştür. Çalışmanın sonuçları öğrencilerin geometri düşünme düzeylerinin yaşa veya olgunlaşmaya bağlı olmayabileceğini, daha çok geometri deneyimlerine bağlı olabileceğini ve geometri başarısı ile van Hiele geometri düzeyleri arasında ilişki olduğunu göstermiştir.

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LIST OF SYMBOLS

I	Grade Level
J	Grade Level
N	Number
r_1	Average of mathematics achievement scores for grade 7
r_2	Average of mathematics achievement scores for grade 8
r_3	Average of mathematics achievement scores for grade 10
r_4	Average of mathematics achievement scores for grade 11
$\bar{x}_1$	The Mean Score of 7^{th} -8^{th} Grades from the van Hiele Geometry Test
$\bar{x}_2$	The Mean Score of 10^{th}-11^{th} grades (Anatolian high school) from the van Hiele Geometry Test
$\bar{x}_3$	The Mean Score of 10^{th}-11^{th} grades (science high school) from the van Hiele Geometry Test
$\bar{x}_4$	The Mean Score of SCED Students from the van Hiele Geometry Test
$\bar{x}_5$	The Mean Score of PRED Students from the van Hiele Geometry Test
$\bar{x}_6$	The Mean Score of 7^{th} Grades from the van Hiele Geometry Test
$\bar{x}_7$	The Mean Score of 8^{th} Grades from the van Hiele Geometry Test
$\bar{x}_8$	The Mean Score of 10^{th} Grades from the van Hiele Geometry Test
$\bar{x}_9$	The Mean Score of 11^{th} Grades from the van Hiele Geometry Test

LIST OF ACRONYMS/ABBREVIATIONS

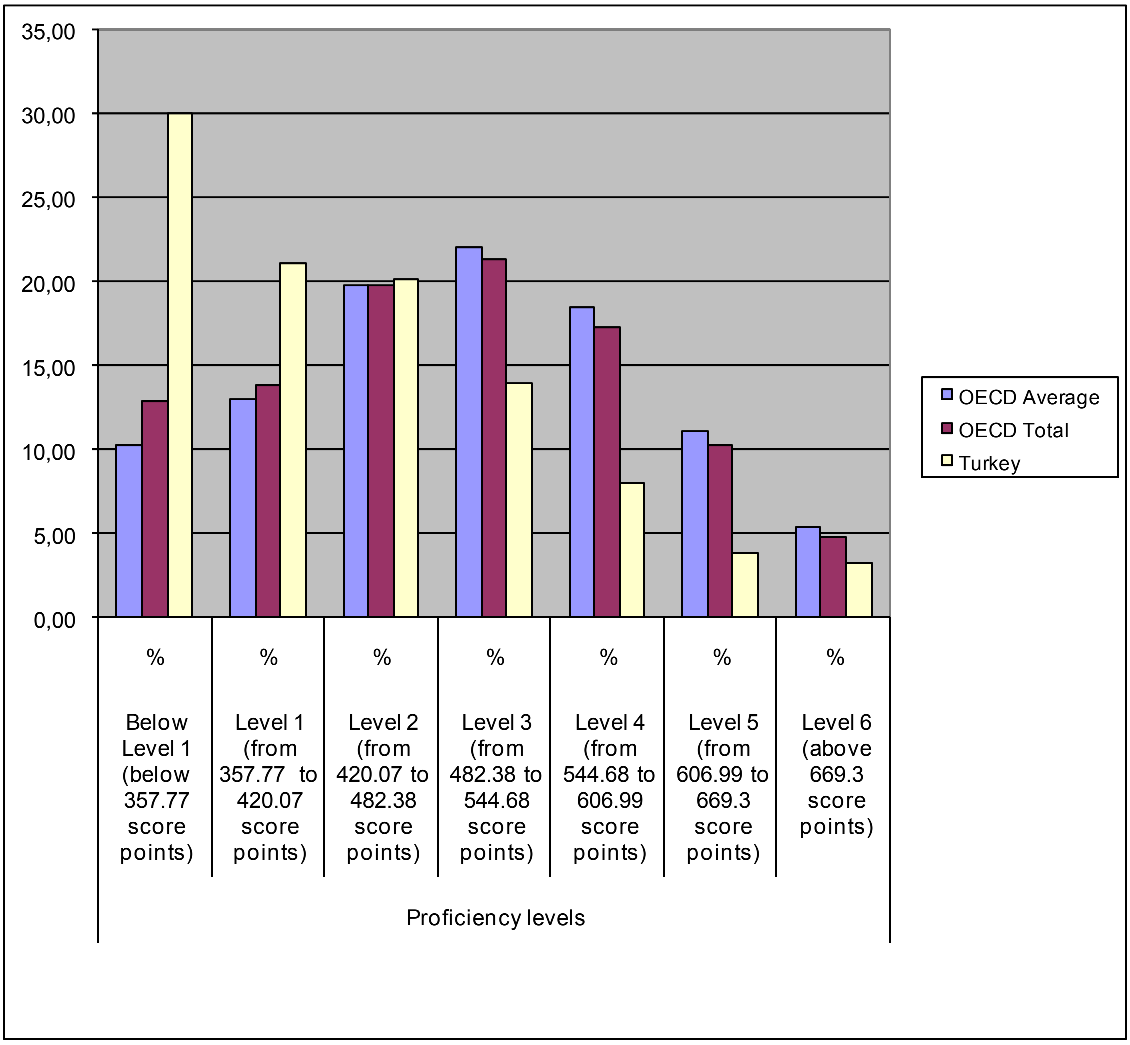
ANOVA	Analysis of Variance
CDASSG	Cognitive Development and Achievement in Secondary School Geometry
MEB	Milli Eğitim Bakanlığı
NCTM	National Council of Teachers of Mathematics
OECD	Organisation for Economic Co-operation and Development
PISA	Programme for International Student Assessment
PRED	Primary Education
SCED	Secondary School Science and Mathematics Education
Sig.	Significant Value
SPSS	Statistical Package for the Social Sciences
Std. Error	Standard Error
TIMSS	Third International Mathematics and Science Study
USA	United States of America

1. INTRODUCTION

Geometry is one of the most important parts of mathematics. By the help of geometry students can develop skills related to spatial relations, problem solving and critical thinking. Understanding of geometric shapes and their properties can help students in realizing the world around them. It is important to apply properties of shapes to real world conditions and problems in different content areas (National Council of Teachers of Mathematics [NCTM], 2000). Simple and fundamental concepts of geometry can be taught at early ages but more difficult and complicated concepts should be taught as the students grow up. In spite of its importance in curriculum students have low success levels in geometry. Clements and Battista (1992) stated that elementary and middle school students cannot develop an adequate understanding of geometric concepts, geometric reasoning and geometric problem solving. So, they are unprepared to study the deeper geometric concepts and proofs. Other studies on geometry achievement reveal that students cannot learn geometry in a degree that they are expected (Usiskin, 1982; Duatepe-Paksu and Ubuz, 2009).

International studies as Third International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA) have shown that geometry achievement levels of students in Turkey are lower than the international average (OECD, 2004). PISA included change/ relationships domain, space/ shape domain and students find it difficult to analyze those relationships, as well as making a strategic plan in solving geometry questions. In TIMSS geometry items included symmetry, congruence and similarity. For answering these items students are expected to visualize geometric figures and demonstrate what they have understood from the properties of figures (Battista, 1999). In PISA and TIMSS students were required to visualize geometric figures and demonstrate their understanding of the properties of figures in both studies. As Mullis *et al.* (2000) stated Turkish students had the lowest scores from the geometry area in Third International Mathematics and Science Study among eighth- grade students from 38 countries because they are not used to handle such geometry items.

In geometry it is important to analyze the change and relationships in geometric figures to solve problems. All geometric concepts are related to each other and geometric figures have common properties. So, it is important to know the properties and relationships in geometry. Figure 1.1 shows the proficiency levels of Turkish students in PISA 2003 in the change and relationships domain and compares their levels with OECD total and OECD average proficiency levels. Proficiency levels are formed according to the range of scores.

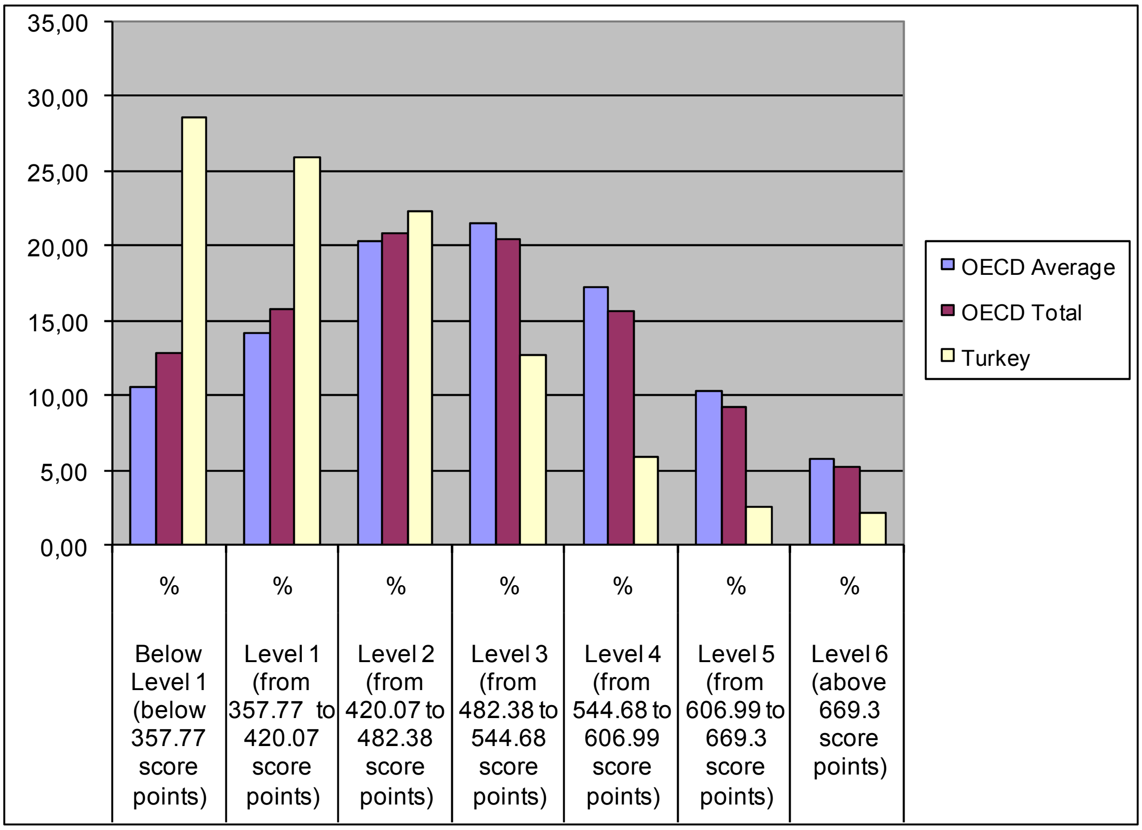


Adapted from www.pisa.oecd.org

Figure 1.1. PISA 2003 mathematics results by levels (change/ relationships domain).

Figure 1.1 shows that proficiency levels of 30% of Turkish students are below level 1, 21% are at level 1, 20% are at level 2, 14% are at level 3, 8% are at level 4, 4% are at level 5, 3% are at level 6. They are above the OECD average at 3rd, 4th, 5th and 6th levels which shows that to a large extend they are not capable of demonstrating their understandings at change and relationships.

In geometry it is also important to analyze the shapes and to analyze the figures in three dimensional fields. It is crucial to visualize the figure or a solid in different perspectives and to apply space and shape relationships in problems. Figure 1.2 shows the proficiency levels of Turkish students compared to OECD total and OECD average in 2003 PISA "space and shape" domain.



Adapted from www.pisa.oecd.org

Figure 1.2. PISA 2003 mathematics results by levels (space/ shape domain).

As in change and relationships domain, most Turkish students are below level 1(29%) or at level 1 (26 %). Only 14% are at level 4, 6.5 % are at level 5 and 2% are at level 6 which shows that they are not good at higher level space and shape domain items.

Figure 1.3 shows the general proficiency levels of Turkish students in PISA mathematics domain in 2006 and 2009 and compares the proficiency levels of Turkish students with OECD total and OECD average proficiency levels. As it can be seen from the figure although the proficiency levels of Turkey increased in 2006 and 2009 PISA when compared to 2003 PISA, higher percentage of students are still at below level 1, level 1 and level 2. Turkey's performance in PISA mathematics domain has increased since 2003 but students still show low performance when compared to OECD average. Therefore problem in mathematics and geometry continued in those years for Turkish students.

Adapted from www.pisa.oecd.org

Figure 1.3. Proficiency levels in PISA 2006-PISA 2009 (mathematics domain).

It is revealed in Third International Mathematics and Science Study (TIMSS) data that performance of Turkish students in geometry and measurement areas were below the average of thirty eight countries in 1999. According to mathematics results Turkey ranked

31 among 38 countries. International average mean score was 487 and Turkey's score was 429 in that year. In 2007 TIMSS, the data revealed that Turkey' score was 432 where TIMSS scale average was 500 and ranked 30 among 48 countries. The average score for geometry was 411 in 2007 TIMSS (http://nces.ed.gov//timss). Table 1.1 shows the average scores in geometry and measurement area in TIMSS 1999 compared with the international average for year 1999.

Table 1.1. Turkey's proficiency level in geometry and measurement areas.

Domains	**National Average**	**International Average**
Geometry	428 SD* =5.7	487 SD=0.7
Measurement	436 SD=6.5	487 SD= 0.7

Adapted from www.earged.meb.gov.tr

When the data of the international assessments are examined, it would be stated that there were low achievement levels of Turkish students in geometry for eight grade and secondary school students. There is also a decrease in geometry performance levels of students from 1999 to 2007 in TIMSS. Students' performances in geometry and measurement do not seem to develop between 1999 and 2007 according to TIMSS results. It would be thought that the students in Turkey do not focus on mathematical and geometrical thinking and analytical ways of reasoning. The reason for this may be the fact that geometry is often taught by rote memorization or in a way that do not require any student involvement in the learning process as it is mentioned in Fuys *et al.*'s (1988) study. It is stated that there are mostly very little student explanation or argument on the content material so this would prevent students from thinking about the concepts in geometry. The result of the memorization is forgetting or confusing the concepts, relationships and spatial properties in geometry which leads to again low achievement in geometry (Faucett, 2007). Another reason for the low achievement levels might be related to the nature of geometry itself, curriculum, curricular activities and materials and cognitive developmental levels of students. Cognitive factors such as spatial visualization as well as van Hiele levels of geometrical thinking have been two of the areas of research for the educators to explain students' difficulties in geometry (Cannizzaro and Menghini, 2006; Faucett, 2007).

One of the ways to solve the problem of low achievement level in geometry may be to take geometry teaching into account because students' understanding of geometry and geometric concepts depends on the way of teaching geometry (Battista *et al.*, 1982). It is also stated that geometric thinking skills of students can be developed by appropriate instruction. Improving geometric thinking levels of students should be one of the most critical aims of the mathematics education. Various studies have been conducted by researchers to see whether appropriate and relevant materials and different approaches can make it possible to make the students become more aware of the world of geometry and construct their knowledge in geometry (Güven, 2006; Yıldırım, 2009; Kılıç, 2003; Matthews, 2004; Tutak, 2008; Accascina and Rogora, 2006; Anghileri, 2006; Hannafin *et al.*, 2008; Fyhn, 2010). All these studies are based on the theory of Pierre van Hiele. Pierre van Hiele was a Dutch mathematics teacher and during 1950s he and his wife Dina van Hiele-Geldof began to study on difficulties their students have while studying geometry. He established a theory on five levels of geometric thinking. These five levels which are called as visualization, analysis, informal deduction, formal deduction and rigor describe the processes which take place in geometric thinking (Usiskin, 1982). According to the theory, a student goes through the levels sequentially. At visualization level the figures are considered according to their appearance and at analysis level geometric properties are used rather than appearance. Students at informal deduction level order the properties logically and at formal deduction level development of logical reasoning and conceptual understanding occurs. At rigor level, students are able to work in both Euclidean and non-Euclidean geometric systems (Usiskin, 1982). Moreover, van Hiele focused on understanding geometry and examined geometric insight of students which is critical to create geometry content which is aligned with the geometry thinking levels of students and to design the geometry instruction by considering levels of students. So, the theory suggests teaching methods to organize geometry instruction and to improve learning by manipulation of materials and content of the geometry course (Kemp, 1990).

This study aims to find out whether there are differences in van Hiele geometric thinking levels of students at different grade levels as measured by the van Hiele

Geometry Test which was developed by Usiskin (1982) and was translated into Turkish by Duatepe (2000). It also aims to find whether there is a correlation between students' geometry achievement scores and the van Hiele Geometry Test scores. The geometry textbooks were also examined on the basis of van Hiele levels to see whether they are aligned with the geometric thinking levels of students.

2. LITERATURE REVIEW

This chapter overviews the theory in which this study is grounded. It also consists of the reviews from the literature which are relevant to the study. There are three main parts. The first part focuses on the van Hiele Level Theory on geometric thinking and geometric reasoning which is a cognitive model supporting constructivist belief of mathematics education (Faucett, 2007). This part begins with general information about the theory, continues with the properties, levels and phases which are stated in the theory and lasts with the review of the research studies on van Hiele Level Theory. The second part deals with the geometry achievement and review of the studies conducted on geometry achievement and the last part deals with the geometry instruction in Turkey and the content of the geometry courses by focusing on the geometry textbooks that are provided to all schools by National Ministry of Education (MEB).

2.1. The van Hiele Level Theory

Geometric thinking abilities of students should be considered to design an appropriate and relevant learning environment. Meeting students' needs for focusing on the geometric concepts or theorems may help them to be successful in geometry. The van Hiele Level Theory not only offers focusing on cognitive abilities of students to help them gain geometry reasoning abilities but also shows the ways of organizing effective instruction (Kemp, 1990).

The van Hiele Level Theory is the study of two Dutch mathematicians Pierre van Hiele and Dina van Hiele-Geldof on the issue of learning and teaching geometry which was ground breaking in terms of providing the first model to understand learners' stages of development in geometric thinking and to offer suggestions for improvement of the geometry instruction (Clements and Battista, 1992). They explained their ideas on geometric levels of thinking in their dissertations. The dissertation of Dina van Hiele-Geldof was titled as "The Didactics of Geometry in the Lowest Class of Secondary School" and Pierre M. van Hiele's was "The Problem of Insight in Connection with School Children's Insight into the Subject- Matter of Geometry". Montessori Theory was the base for the van Hiele Level Theory. Montessori Theory is sequential in nature and focuses on the significance of developmental learning. This theory also mentions that children can learn through their senses, by manipulating the concrete objects. This is parallel to claims of the van Hiele Level Theory. Therefore van Hiele Level Theory and Montessori Theory both include developmental theory and classroom experience. Van Hiele Level Theory also includes the claim that knowledge can be gained by construction rather than accumulation of information from external channels (Sears, 1981 as cited in Kemp, 1990). Therefore the theory has influenced geometry education and geometry curriculum studies in many countries especially in Russia and in USA after 1970s and it has been used worldwide since 1984 (Olkun and Toluk, 2003).

The most important characteristic of the theory is that it explains the development of geometric thinking in five related levels which are called as van Hiele geometric thinking levels. Each of these levels defines the way that learners think and their different kinds of geometric ideas. The theory claims that van Hiele geometric thinking level of a student does not depend on the age of the student, instead it is expected to increase as the student learn and experience geometry in a learning environment with different teaching and learning methods (van Hiele, 1957, 1999). Therefore it provides a guide for the geometry instruction. The theory also states that the geometric thinking levels of students develop hierarchically as they progress in developing geometric ideas and provides five levels starting from visual level to proof level (van Hiele, 1957; Clements and Battista, 1992).

2.1.1. The van Hiele Geometric Thinking Levels

There are five geometric thinking levels defined in the theory and these levels are numbered through 0 to 4 by van Hieles. Dina van Hiele-Geldof named the levels as the basic level, the aspect of geometry, the essence of geometry, insight into the theory of geometry and scientific insight to the theory of geometry (van Hiele-Geldof, 1958). The names, visualization, analysis, informal deduction, formal deduction and rigor, are suggested by Burger and Shaughnessy (1986) and mostly used in research studies. Therefore the names which are suggested by Burger and Shaughnessy (1986) are used in this study. In the theory these levels are described in general and in behavioral terms.

At level 0 which is named as the *visualization level*, there is a nonverbal thinking (van Hiele, 1999). The student is able to learn the names of the shapes and to recognize the shapes as a whole. Visual and mental representation of the figures can be formed but class inclusions, properties and generalizations cannot be understood. For example the student in this level knows that squares and rectangles are different from each other (Usiskin, 1982). However this differentiation is because of the appearance. So, students recognize the squares, rectangles, paralellograms and triangles but this recognition is because of their shapes. The student might say “it is a rectangle because it looks like a door”. They are not aware of the class of figures or common properties of figures. The problems that can be solved at this level are simple and routine problems which require operating on shapes rather than using properties (Gül-Toker, 2008).

Level 1 is called as the *analysis level*. The students at this level not only use the visual appearance of the figure or describe the figure by using something else which it looks like but also describe the figure by certain properties without logically ordering them (van Hiele, 1999). They can analyze figures in terms of their components, find relationships among them and recognize rules and properties of a class of shapes by doing experiments such as folding, measuring, using diagrams or grids (Fuys and Geddes, 1984). At this level they cannot see the relationships between properties and cannot recognize class inclusion. For example students at analysis level can classify rectangle by properties of it, such as having four right interior angles since they see that some certain shapes have certain properties but they do not know the relationship between square and rectangle. As another example, students can list the properties of a rectangle as a rectangle has four right interior angles, four sides where opposite sides are equal, opposite sides are parallel and so on. But a square is not a rectangle because all sides of the square are equal to each other, not just the opposite ones. This is same as saying that a rectangle is not a parallelogram because it has four right interior angles but interior angles of a paralellogram are not right angles. All the examples above show that the students cannot inter relate the properties of figures at this level of geometric thinking. Van Hiele (1999) mentioned that at analysis level language is important in explaining the geometric shapes.

Level 2 is named as the *informal deduction level*. At that level “the student can order figures and relationships but does not operate within a mathematical system” (Usiskin, 1982, p. 4). That is why the students at informal deduction level are not successful in Euclidean geometry (van Hiele, 1999). At this level the student can include figures to classes of figures, see interrelations of properties, make informal deductions, follow simple deductions but still cannot understand the proof and cannot form formal deduction (Usiskin, 1982; Çeziktürk, 2003). Moreover, they can link the properties by focusing on the similar properties and characteristics of the figures. For example a student can state the properties of a rectangle and a parallelogram, can recognize that they have common properties and finds out that they are not separate class of figures. Also students at this level can recognize the relationships between the properties of the same figure. For example they can state that the lengths of the diagonals of the rectangle are equal to each other because opposite sides of the rectangle are congruent to each other and interior angles of the rectangle are 90^0.

Level 3, *formal deduction level*, requires students to make formal proofs of theorems and establish interrelationships among theorems (Fuys and Geddes, 1984). As van Hiele (1959) stated, different from analysis and informal deduction levels, students are able to think the meaning of deduction and the converse of a theorem and state necessary and sufficient conditions in proofs and understand the axioms. Therefore, they can understand that proofs, theorems and postulates have roles in deduction. They can also understand the role of axioms, importance of definitions and theorems in geometry (Usiskin, 1982). Therefore, they can write the proofs with understanding at this level of geometric thinking. Mistretta (2000) stated that students can state a logical argument and can form facts that support the argument and can use this logical argument to prove a geometrical concept. As De Villiers (2003) stated they can also start to form longer sequences of geometrical statements which lead to proofs and they can understand the significance of deduction in geometry world.

Level 4 is named as *level of rigor* which includes scientific insight to the geometric theories (van Hiele-Geldof, 1958). This level of geometric thought meets the standards of the Hilbertian standard of rigor. Students at this level go through concrete to abstract nature of geometrical objects. Therefore students at this level are able to develop a theory without making any concrete interpretation (Wirszup, 1976). At this level it is also possible to make abstract deductions and work outside the Euclidean geometry. Students can explore other systems of axioms, compare and analyze these systems. This level of geometric thinking can be seen at university or higher levels of education because students who are operating at rigor level are able to make highly abstract geometry and work in non Euclidean systems of geometry (Usiskin, 1982; Fuys and Geddes, 1984). Therefore this level is not relevant to the high school geometry in general and is not needed to meet the standards of Euclidean geometry. Van de Walle (2007) stated that the products of this level contain thoughts of comparisons and contrasts and they belong to various axiomatic systems of geometry. Interrelations between these various axiomatic systems of geometry are also possible because geometry has broader applications and more general character at this level.

To summarize, in van Hiele Level Theory, it is stated that the geometric thinking goes through five developmental levels and at each level, the students' approaches to the geometric issues are limited because they can demonstrate certain characteristics at each level. At visualization level, only concrete objects and figures are considered, next level is analysis level, where the properties of figures emerge, relationships between the properties can be understood at the informal deduction level, students can distinguish a proposition and its converse at the formal deduction level and the level of rigor is hardly attainable until the high school/ college because logical thinking is the base of that level. These characteristics of levels indicate that students can handle geometric issues to a degree at each level so it could be important to consider the students' geometric thinking levels and to face them with various geometric experiences and to form appropriate activities for instruction of geometry. Therefore the instruction should be aligned with students' levels of geometric thought. As it can be understood from the theory, it might be beneficial to use the same language with the student by considering the levels of their geometric thinking. When the characteristics of the levels are examined it can be seen that these levels have some properties. Examining the levels not only by considering the characteristics of the levels but also by detecting the related properties should be important. The summary of the characteristics of van Hiele geometric thinking levels are given in Table 2.1.

Table 2.1. Characteristics of the van Hiele geometric thinking levels.

Levels	Characteristics
Level 0 Visualization	General recognition Appearance of figures Student recognize shapes in objects Properties of shapes are not considered
Level 1 Analysis	Geometric properties are used rather than appearance of the shape Distinguish properties of shapes Properties not organized Necessary conditions are discovered Sufficient conditions are not considered
Level 2 Informal Deduction	Order The properties are ordered logically Relationships between properties are recognized Construct figures from properties Significance of deduction is not understood
Level 3 Formal Deduction	Sufficient conditions are considered Compose families of geometric figures Meaningful geometric proofs can be created Logical reasoning ability and conceptual understanding development Distinguishment of a proposition and its converse
Level 4 Rigor	Analyze families of figure Theorems are analyzed in different geometry systems other than Euclidean geometry, such as Hilbertian. Formal logic Symbols loose the original significance Thinking is the main subject

2.1.2. Properties of van Hiele Levels

Van Hiele (1958) discussed some properties to clarify the thinking processes. These properties are formed by some assumptions underlying the van Hiele Level Theory. The assumptions underlying the properties would be considered as important for the fact that they can be used for making instructional plans and decisions. According to van Hiele (1959) there are three properties of geometric thinking levels. These properties are given by their names which were assigned by Usiskin (1982). These are adjacency, distinction and separation properties. After examining the van Hiele Level Theory, Usiskin (1982) formed two other properties which are called as fixed sequence and attainment properties. The properties are described and exemplified in this part.

The first property is the property of adjacency as it is called by Usiskin (1982). "At each level what was intrinsic in the preceding level becomes extrinsic in the current level" (Usiskin, 1982, p. 5). For example at recognition level the student would say that the window is a rectangle because it looks like a rectangle but at analysis level the student would say that the window is a rectangle because it has right angles and it has two pairs of equal sides. Therefore each level consists of extensions of thought of the previous levels.

Second property is distinction which implies that each level has unique symbols and relationships which connect these symbols. In geometry learning process these unique symbols and the relationships play important role (Kemp, 1990). For example a relationship which is correct at one level would not be correct at the higher level. A student would think that a rhombus and a parallelogram are different geometric figures at one level, at the other level a student can be aware of properties of these figures and at the next level can think that a rhombus is also a parallelogram and rhombus is the element of the set of parallelograms. Thus, the relationships between rhombus and parallelogram change according to the level of the student.

Third property is separation property which means that two persons at different levels cannot understand each other. This is because each level has its own symbols, terminology and properties. This property is also one of the explanations for the fact that students cannot be successful in geometry if the level of the activities is higher than the level of the students (Usiskin, 1982). For example if the student is at formal deduction level but the teacher teaches at the level of rigor and makes a proof, then the student will just follow the proof while the teacher is doing but cannot do the same without guidance of the teacher. The student can follow the proof because making proof is intrinsic at formal deduction level, but cannot prove anything until it becomes extrinsic at level of rigor.

Fourth property in the van Hiele Level Theory is that students have to go through the levels in a specific and predetermined order. This is called fixed sequence property. The student cannot reach to formal deduction level without going through analysis level (Mayberry, 1981; Kemp, 1990). There are some studies which confirmed that the levels of geometric thinking are hierarchical in nature (Fuys and Geddes, 1984; Usiskin, 1982; Senk, 1989; Mayberry, 1981; Burger and Shaughnessy, 1986). Sequence of the van Hiele geometric thinking level of a student does not depend on the age, instead the level is expected to increase as the student learns and experiences geometry in an appropriate learning environment with appropriate methods which might help the student to achieve higher levels of geometric thinking (Fuys and Geddes, 1984). If geometry instruction does not support this sequential nature then a gap might occur which affects the performance of the student at the current level. Crowley (1987) concluded that if the student does not master the current level and the instruction continues with the next level then the student would only make algorithmic operations on the next level. So if a student cannot perform the requirements of the level of the instruction, then rote memorization of the content of the instruction could occur. Memorization in geometry could result in decline in the process of reasoning and geometrical thinking. So, it might be important to have students be part of the lesson by encouraging them to discover the facts and implications and to participate in the process of geometrical proof and reasoning stages during the instruction. Also, the textbooks should be prepared by considering this sequence property.

The last property is the property of attainment as it is called by Usiskin (1982). The attainment of the levels can be accelerated by the proper instruction and are not consequences of biological maturation. Therefore learning process should be designed to help the student achieve complete understanding. This is the way of attaining van Hiele geometric thinking levels. In her dissertation Dina van Hiele-Geldof (1958) introduced and explained five phases to design the learning activities which lead to complete understanding and attainment in geometry courses. These phases are explained in the next section.

2.1.3. Phases of Instruction in Learning Geometry

Dina van Hiele- Geldof (1958) stated that students can learn geometry if the teachers let them to use their minds and experience the power of exact thinking. For this purpose she introduced five phases of learning geometry in her studies. These phases show the way of determining the teaching techniques which are appropriate for the geometry curriculum. The students' geometric thinking would be accelerated by motivating them to discover the world of geometry and guide them during this discovery. These phases are information, directed orientation, explicitation, free orientation, integration (van Hiele-Geldof, 1957, 1958).

In information phase, teacher uses discussions to learn students' prior knowledge about the concepts to be learned. The main aim of the teacher is to discover what knowledge the student brought to the classroom. These discussions can also be used to design appropriate activities by determining the levels of students. Students learn to discover field of investigation and the structures in the subject matter which is presented by the teacher (Clements and Battista, 1992; Kemp, 1990). Students also have the chance to discover intrinsic order of the subject material which is presented by the teacher and which aspect of their geometric experiences are dealt with in the subject of the course (van Hiele-Geldof, 1958). Teacher can consider this phase as a way to understand whether the students are capable of the following purposeful action of the geometry course or not.

By the conversations which are managed by the teacher, the students can be brought into a new phase which is the phase of directed orientation. The aim of this phase is to have students look at the figures in a certain and conscious way (van Hiele-Geldof, 1958). In this phase the teacher develops activities at the appropriate level to motivate students to become more familiar with the concepts which are being taught. Manipulation of plane figures is more apparent in that phase. Teachers can manipulate materials and geometric figures to make them more understandable for the students. These manipulations help students to discover spatial figures and their properties by making it. By this way, the students also notice the basic structures and understand the direction of the content. The exploration of the subject matter becomes more oriented in that phase (Clements and Battista, 1992; Kemp, 1990).

Phase of explicitation is the phase where transition between reliance on the teacher and the students' self-reliance occurs. The teacher introduces the necessary terminology and the students express the observations by themselves. The student tries to form relationships partially in that phase by building links between the acquired experiences and the linguistic symbols of the subject matter (Clements and Battista, 1992; Kemp, 1990). The students try to explain their experiences with manipulated objects by words and exchange their experiences to find the properties of geometric objects and figures they have worked with (van Hiele-Geldof, 1958).

In free orientation phase, teacher pays attention to the students while they are inventing geometric ways to accomplish given tasks. In that phase tasks can be approached by various methods and students have the opportunity for self-orientation in domain of geometric figures (Clements and Battista, 1992; Kemp, 1990). Teacher should give opportunity to students to experience the figures or objects empirically. For example students should create new figures or objects by using translation and rotation and explore the properties of the new figures (van Hiele-Geldof, 1958). In that phase the students are not in a real problem conditions because they only follow the order of the manipulations they have carried out.

The phase of free orientation is expected to lead the students to be oriented in the domain of symbols where the symbols have the characteristics of the field that is being studied (van Hiele-Geldof, 1958). At integration phase, the students are able to do operations with the properties of figures they have worked with, summarize the content of the concepts to be learned and arrive at a generalization at the end of the analyses. If students understand the manipulations, then they would become capable of conducting further operations. Teacher only provides global concepts. That is the phase where the new level of thought is reached by the students (van Hiele-Geldof, 1958; Clements and Battista, 1992; Kemp, 1990).

To summarize, the van Hiele Level Theory is one of the cognitive developmental theories in geometry which states five levels of geometric thinking. Complete understanding of geometry requires students to go sequentially through these thinking levels which are characterized by certain thinking processes and do not depend only on acquiring geometry knowledge. To motivate the students to acquire understanding of geometry, teachers should utilize appropriate teaching techniques and plan geometry instructions by following five phases which are defined by van Hieles. The properties of the geometric thinking levels expose that if two persons are at different stages, then this will mean they use different terminology and different relationships. This difference makes it difficult for these persons to understand each other. Therefore geometry instruction should be aligned with the level of the students to lead to understanding of the geometry.

2.2. Research on van Hiele Geometric Thinking Levels

This part of the literature review includes studies conducted on the van Hiele geometric thinking levels. These studies are reviewed in two sections. First section reviews the studies conducted to characterize the geometric thinking levels of students at various grade levels and to examine the possible correlation between van Hiele levels and students' geometry achievement levels. This may be considered as the basic review part of the study because this study also deals with the van Hiele geometric thinking levels of students and does not consider the effects of different instructional methods or dynamic geometry environments on van Hiele geometric thinking levels. Second section reviews the studies which examined the effects of different instructional methods, different classroom environments and dynamic geometry approaches on van Hiele geometric thinking levels.

2.2.1. Characterizing the Geometric Thinking Levels

At this part of the literature review, several studies which dealt with the characterization of the van Hiele geometric thinking levels of students are presented.

One of the earlier major studies with van Hiele Level Theory is the Cognitive Development and Achievement in Secondary School Geometry project (CDASSG) which was developed by Zalman Usiskin and Sharon Senk in 1982. As stated by Usiskin (1982,

p. 8) “Major aim of the study was to test the ability of the van Hiele Level Theory to describe and predict the performance of students in secondary school geometry”. The study investigated whether there is a relationship between van Hiele geometric thinking levels and geometry knowledge of tenth grade students. In that project a multiple choice test was developed which consisted of 25 geometry items to measure students’ van Hiele geometric thinking levels. The reason for developing the test was to see whether this test could predict geometry achievement levels of students at secondary school level. In that study 2700 tenth grade students took the test at the beginning of the geometry course. They took the same test at the end of the course together with a standardized geometry achievement test. Results of the study revealed that there was a correlation between the performance on the standardized geometry test and the van Hiele levels of students. At the beginning of the semester this correlation was .52, at the end of the semester it was .67. The correlations were significant at 0.0001 level (Usiskin, 1982). This correlation indicated strong relationship between van Hiele Test and standardized geometry achievement test. Therefore it was stated that there is a strong relationship between the concurrent geometry knowledge and van Hiele levels. Usiskin (1982) concluded that the low performance of students on geometry test was associated with being at the low van Hiele geometric thinking levels. As a result of the study he also found that the students were at visualization or analysis levels and not ready for the secondary school geometry since secondary school geometry requires informal deduction level.

Shaughnessy and Burger (1985) stated that attainment of van Hiele geometric thinking levels does not happen quickly, instead it requires several months or several years. During this time period the students go through low, intermediate and high levels of acquisition of van Hiele geometric thinking levels. Therefore, it is recommended that the geometry should be introduced by using more activities that lead students attain informal deduction level before secondary school. There is another study by Burger and Shaughnessy (1986) which was conducted on van Hiele geometric thinking levels. The subjects of the study varied in grade from kindergarten to college. They conducted the study to get the answers to three research questions related to van Hiele Level Theory. First question was whether van Hiele Level Theory was appropriate for classifying students’ levels in geometry. Another question was about whether there exist specific indicators of students’ reasoning which are also the indicators of levels. Thirdly, they questioned whether certain levels of geometric thought were more effective in students' thinking and reasoning for a certain task. So, they examined students' cognitive processes when dealing with certain geometric tasks through interviews. As a result of this study, it was found that students' behaviors were aligned with van Hiele geometric thinking levels. They also identified specific characteristics of students' thinking in geometry as based on van Hiele geometric thinking levels. They concluded these levels can be used in describing students' thinking processes on geometry and the levels of the students can be determined operationally by observing students' behaviors while doing the tasks.

As Usiskin (1982) stated, if the geometry instruction is directed toward the students' geometry level, then students' level of understanding geometry would increase. So van Hiele based instruction would yield to growth in geometry understanding. In a similar study Çelebi-Akkaya (2006) investigated whether the activities developed by considering the van Hiele geometric thinking levels have effect on 6^{th} grade students' achievement levels in geometry. The van Hiele Geometry Test and Geometry Achievement Test were administered before and after the treatment. The results of the study showed that the activities based on van Hiele Level Theory lead to higher scores for the students in the experimental group. In the control group the instruction were traditional and the students in the control group remained stable in terms of achievement scores in both tests. Moreover, the van Hiele geometric thinking levels of students who were in visualization level before the treatment in the experimental group have risen to analysis level after the treatment. The results of the study are in accordance with the similar ones by Mistretta (2000), Choi-Koh (1999), Usiskin (1982), De Villiers (2003).

The study of Mistretta (2000) would be examined in the same manner. In that study the researcher aimed to increase eighth grade students' van Hiele geometric thinking levels. Before the study, to assess students' levels, a pretest, consisted of short answer and multiple choice items of visualization, analysis and informal deduction levels was administered. As a result of the pretest 5 students (22 %) were classified as being at the visualization level, 7 students (30%) at the analysis level and 8 students (35 %) were non classifiable. 3 students (13%) were at informal deduction level but none of the students could be classified at the formal deduction level or level of rigor. Interviews were conducted with the students and it was reported that students did not have understandings of concepts of area and perimeter of geometric shapes especially when the shapes are irregular. It was also stated that students were not aware of the properties of polygons and relationships between properties of different kinds of polygons. Then a unit of study which aimed to encourage students to use higher order thinking skills was developed. The unit of study included three types of activities. These were concept cards, finding areas of irregular shapes and analyzing three dimensional geometric models. After finishing the unit, a posttest was administered and the results showed that 1 (4%) student was at visualization level, 6 (26%) at analysis level, 16 (70%) were at informal deduction level. There were no non classifiable students at the end of the study. As a result of the study it was concluded that for making students accept the challenges of analytical thinking, the geometry lessons should be designed to encourage the students develop their cognitive skills.

The study which considered other several variables affecting van Hiele geometric thinking levels of students was Fidan's (2009) where the levels of fifth grade students in terms of gender, computer use, parents' education levels and having pre-school education were examined. The results of the study indicated that there was a significant difference between average scores of girls and average scores of boys in van Hiele geometric thinking levels. The average scores on geometric thinking levels of girls were higher than the scores of boys. Moreover the study showed that there was a significant difference between scores of students who use computer and who do not use it. Geometric thinking scores of students who use computers were higher than those of students who do not use computers. Moreover, the results of the study indicated that geometric thinking levels of students were positively correlated with parents' levels of education in favor of parents having higher levels of education. Also geometric thinking levels of students were showed to be related to pre-school education in this study. Students who had pre-school education had higher scores in the geometric thinking levels test than the students who did not have pre-school education.

Baima (2010) compared honor level and regular high school students in terms of their geometry performances and van Hiele geometric thinking levels. The results of the study revealed significant differences between geometry performances of the two groups of students. There were no significant differences with respect to the van Hiele geometric thinking levels of students in terms of gender or different ethnic groups. It was mentioned that honor level students were better at explaining their reasoning while solving geometry problems than the regular group students and geometric thinking levels of students in honor level were higher than the regular students.

In another study Fuys *et al.* (1988) characterized the geometric thinking levels of students at sixth and ninth grades. They interviewed the students in six to eight instructional assessment sessions with 45 minutes duration. At the end of the instruction, they provided information on students' progress within and between van Hiele geometric thinking levels and stated that sixth grade students had not learned much geometry in school. Similarly Senk (1989) found that at the end of the secondary school most of the students could reach analysis or informal deduction level. She also found that progressing from the informal deduction level to the level of rigor was very slow. According to Senk (1989), the results of the study showed that the entry geometric thinking level of the student was important and had crucial role in students' success level in high school geometry.

Bobango (1987) examined van Hiele geometric thinking levels and student achievement in standard content which was measured by "Cooperative Mathematics Test: Geometry" which consisted of 40 geometry items. The participants of the study were students from 9th, 10th, 11th and 12th grades. As a result, a significant correlation between students' van Hiele geometric thinking levels and their achievement levels was found. The researcher also investigated whether there was a significant correlation between the van Hiele geometric thinking levels of students which were determined by interviews and by the van Hiele Geometry Test. The results indicated a significant correlation between the geometric thinking levels of students determined by interviews and by the van Hiele Geometry Test.

To conclude, the studies which were conducted to characterize the van Hiele geometric thinking levels of students and to find whether there is a correlation between the geometric thinking and geometry achievement levels of students show that there is a significant correlation between these two variables. It can be stated that the entry van Hiele levels of the students play important role for success in geometry.

2.2.2. Factors Affecting the van Hiele Geometric Thinking Levels

There are studies which examine the effects of different instructional methods and classroom environments on van Hiele geometric thinking levels of students. Some other studies examine only effects of dynamic geometry environments on the levels.

One of the studies examining the effects of different instructional methods on van Hiele geometric thinking levels was conducted by Duatepe-Paksu and Ubuz (2009). In this study, the effects of drama based instruction on students' geometric thinking levels were examined (Duatepe-Paksu and Ubuz, 2009). The analysis of the study failed to detect any significant differences in van Hiele geometric thinking levels between the students in the experimental group where drama based instruction was used and the students in the control group at sixth grade level. This result was attributed to the fact that longer period of time is needed for the increase in the geometric thinking levels.

The results of Matthews' (2004) study also confirmed the results of the study reviewed above. In that study Matthews (2004) compared effects of different methods of instruction on fifth grade students' van Hiele geometric thinking levels. The methods which were tested in that study were phase-based instruction, traditional textbook instruction and no instruction. The results of the study indicated a significant increase in levels of students who received phase-based instruction and traditional textbook instruction when compared to the students in no instruction group. On the other hand, there were no significant differences between phase-based instruction group and traditional textbook instruction group in terms of van Hiele geometric thinking levels at the end of the treatment. Inclusion of instruction at various levels and longer period of instruction time were recommended as a result of the study.

There are some studies which examine the effects of dynamic geometry approaches on van Hiele geometric thinking levels of students. For example studies of Ubuz *et al.* (2009) and Yıldırım (2009) showed that dynamic geometry environment affected students' learning in geometry and it was efficient for students who have low geometric thinking levels. Since students who are operating at low van Hiele geometric thinking levels prefer relying on prototypical examples in geometric arguments and problem solving, this inhibits their proper generalization and is a cause of misconceptions in geometry. To develop students thinking and problem solving levels in geometry, technologically appropriate classroom environments are needed (Schwarz and Hershkowitz, 1999). As Ubuz *et al.* (2009) stated students, whose visual awareness and spatial ability through figures are high, will operate at high van Hiele geometric thinking levels. Also technology use such as dynamic geometry programs and sketchpad or other visual materials is one of the critical ways of developing such abilities of students.

Other study which examined the effects of dynamic geometry software while teaching by guided discovery method is Gül-Toker's (2008) study. Gül-Toker (2008) conducted the study with sixth grade students and found that the van Hiele geometric thinking levels of students did not increase significantly between pretest and posttest process. It was concluded that significant increase in van Hiele geometric thinking levels of students requires time which confirms the results of the studies by Duatepe-Paksu and Ubuz (2009) and Matthews (2004). On the other hand, when the performance of students in the experimental and control group was compared, it was found that the students in the experimental group who were taught by guided discovery method performed significantly better on the van Hiele Geometry Test than the students in the control group who were not taught by guided discovery method. This result was interpreted as van Hiele geometric thinking levels were more dependent on instruction then dynamic geometry approaches.

Patsiomitou and Emvalotis (2010) conducted a research study to examine teaching and learning processes in geometry. They used Geometer's Sketchpad dynamic geometry software to teach quadrilaterals to 14 students at grade 10. They hypothesized that the research process helps students develop geometrical thinking in accordance with the van Hiele geometric thinking levels. The results of the study indicated that the students were able to transfer the skills and reformulate the problem when working with Sketchpad and they developed thinking processes needed to describe the relationships. Moreover researchers stated that the link between visual and formal abilities which are essential for the transition from lower to upper van Hiele geometric thinking levels occurred and as a result the van Hiele Geometry post-test results showed a significant increase in the geometric thinking levels of students.

In summary, van Hiele geometric thinking levels of students may be affected by the quality of geometry instruction or by different teaching-learning techniques such as phase-based instruction and by different approaches as dynamic-geometry programs if required time period is devoted (Duatepe-Paksu and Ubuz, 2009; Matthews, 2004; Gül-Toker, 2008). On the contrary, some of the studies showed that the levels could be increased in suitable classroom environments and with different teaching methods (Ubuz, Üstün and Erbaş, 2009; Yıldırım, 2009; Schwarz and Herschkowitz, 1999; Patsiomitou and Emvalotis, 2010; Çelebi and Akkaya, 2006; Choi-Koh, 1999; De Villiers, 2003; Mistretta, 2000; Fidan, 2009).

2.3. Geometry Achievement

This part of the literature review consists of research on geometry instruction, geometry textbooks and curriculum which are considered as some of the variables affecting geometry achievement levels of students.

2.3.1. Geometry Instruction

As van Hiele (1957) stated the increase in students' van Hiele geometric thinking levels does not depend on biological maturation or age. Increase in students' geometric thinking levels can occur when the students gather experience in geometry learning in a learning environment aligned with their current levels. Therefore geometry instruction might be considered as an important and one of the crucial factors affecting students' thinking levels in geometry.

Some studies state that students' low performance levels in geometry is related to the van Hiele level of geometric thinking (Usiskin, 1982; Faucett, 2007; Kemp, 1990; Mayberry, 1981; Burger and Shaughnessy, 1986; Battista, 1999). Battista (1999) stated that in international assessments, geometry items assess the range of students' progress through the van Hiele geometric thinking levels. These items also displays whether the students use their decision on visual appearance (visualization level) or by demonstrating the properties and measurement issues while solving the geometric problems (analysis level). In that manner, Usiskin (1982) mentioned that to perform well on geometry tests the student must be at a higher van Hiele level, and the converse is also true, that is, to be at a higher van Hiele level, the student must have knowledge of standard geometry topics. Also Kemp (1990) mentioned that students enter geometry courses at low levels and since geometry courses are taught at formal deduction level or level of rigor, students can not achieve success in geometry. Therefore the students' entering van Hiele level is mentioned to be important to succeed in geometry courses.

Students' performances in geometry fail when the tasks require recognition, recall and manipulation of geometric concepts. They perform better on items which can be solved visually or which are based on informal methods and on basic knowledge, not requiring abstract thinking and higher levels of understanding. Simple deduction and analysis are difficult for students in geometry (Brown *et al.*, 1988). As Senk (1983) stated they also have conceptual difficulties in proof writing and following logical arguments in geometric proofs. Students need more experience in geometry before entering the field of deductive reasoning and proof. The same argument was stated by Battista (1999) that the results of the international and national assessments on geometry shows that students need more experience and work on spatial visualization and three dimensional geometry. Swafford *et al.* (1997) stated that although there was significant increase in arithmetics and measurement parts of mathematics, there was no significant increase in geometry achievement levels of students in 1990s and the reason for this is stated as the teachers' low level of knowledge of students' cognition and geometric thinking levels.

According to the van Hiele Level Theory, student growth and success in geometry depend on the instruction which is directed toward the students' geometric thinking levels (Choi-Koh, 1999). The activities used in the instruction period should be appropriate to the students' levels and the activities should be ordered for leading the students toward more sophisticated levels of geometric thinking (Carroll, 1998). According to Carroll (1998) these activities should emphasize applications of geometric concepts and relationships and should be relevant to daily life situations. The instruction should also emphasize reasoning, problem solving, discussion and group studies. Bishop (1986) emphasizes that these activities should include different shapes, modeling, patterns, paper folding and sketching. They should also be formed in a way that emphasizes geometrical properties and relationships between geometrical forms. Because it might be important to have the students be able to visualize geometrical constructions and use simple geometric figures to form complex designs. This can be the way of being successful in Euclidean geometry and become familiar with the deductive reasoning which the Euclidean geometry is based on.

Similarly, Erdoğan *et al.* (2009) stated that the geometry instruction should consider the student- centered approaches such as cooperative learning and learning by doing. Teachers should consider the importance of geometry experiences of students. Therefore geometry instruction should be organized in a way that leads using concrete materials, orienting students to discuss and think on a geometrical issue, searching for different geometric concepts and sharing ideas.

Unfortunately, traditional geometry instruction provides students cumulative content to be learned and to be used on various forms of assessment such as tests and examinations. Mostly these content materials are forced to the students without considering their readiness or perceptual levels (De Villiers, 2003). It is stated that when geometric skills and knowledge are taught without any blank period for student conceptualizations, then there does not emerge any of geometric thinking skills (Roh, 2003). Therefore one of the problems in geometry instruction is that content is presented in a way that is not suitable for the students' operation levels so students cannot understand what the teacher explains. Students fail to establish networks of relationships between concepts in geometry which leads to just application of algorithms in mathematics to geometric tasks without understanding why and also leads to forgetting the learned material in a short period of time (De Villiers, 2003; Faucett, 2007). As a way of solving this problem it is recommended that the instructions should not be like solving problems on the blackboard. Instead students should be encouraged to do geometry. For example the concepts of space and distance should be taught by letting the students measure things around, for example their height, the length of their desks, the height of the classroom door. By this way the geometry instruction provides settings which contribute students to engage actively in the learning process (Shaffer, 1993).

Another problem in geometry instruction which is similar to the one mentioned is that traditional geometry course is based on Euclidean geometry, which requires use of deductive logic. Deductive logic is needed for writing proofs and to analyze the theorems in geometry. Geometry textbooks, curriculum and the instructors of geometry expect students to be ready for making formal proofs by logical reasoning which is difficult for the students. Therefore, further applications on Euclidean geometry makes it complicated to learn geometry content. That is why the educators should give importance what Choi-Koh (1999) suggested. According to Choi-Koh (1999) geometry instruction would be more effective when the teacher identifies the van Hiele geometric thinking level of the student correctly and involves the student to the activities which are suitable for the student's level of thought. Teachers should create geometry instruction so as to help student to move to the next van Hiele level progressively. Frerking (1994) stated that rather than teaching geometry as a cumulative terminology and properties to be memorized, geometry can be approached from problem solving perspective. So, teachers could emphasize geometric reasoning and connections between topics of geometry to have students increase their geometric thinking levels.

To summarize, there are studies showing that low achievement levels of students in geometry are related to their levels of the van Hiele geometry thinking levels. So, to be successful in geometry courses, thinking levels of students should be high (Usiskin, 1982; Faucett, 2007; Kemp, 1990; Mayberry, 1981; Burger and Shaughnessy, 1986; Battista, 1999). Since experiences in geometry have effects on increasing thinking levels of students, geometry instructions should be designed by including activities which encourage students to make and learn by actively involving in geometry (Choi-Koh, 1999; Shaffer, 1993; Erdoğan *et al.* 2009; Carroll, 1998; Bishop, 1986). Unfortunately, research on geometry instructions shows that these instructions are not aligned with the levels of the students and they just become explaining terminology and properties in order to have students memorize without understanding. Therefore the geometry experiences of students do not lead to increase in thinking levels (Frerking, 1994; De Villiers, 2003; Faucett, 2007; Roh, 2003).

2.3.2. Use of Geometry Textbooks

Faucett (2007) stated that geometry teachers depend on the geometry textbooks and follow the books for the content and sequence in planning instructions. Therefore geometry learning includes following the textbooks. Research which is related to the geometry textbooks provides insights into the instruction process in geometry courses. The major objectives in geometry courses are to present the subject matter and to complete the exercises at the end of the section. This sequence of topic, exercises, new topic, exercises is an approach for the geometry course which is not suitable for the aim of the course (Faucett, 2007). The aim of the geometry course should be to have students develop logical thinking ability.

Since geometry research emphasize on geometry textbooks, analyses of textbooks become part of the research studies. One of such studies was conducted by Scandura (1971). In that study four geometry textbook series were analyzed and it was reported that for early grades it is important to add exercises which lead to experiences with concrete objects for early grades and the major emphasis in textbooks are stated to be presenting the material rather than leading recognition or classification by analysis. Another study is by Fuys *et al*. (1985). In that study three textbook series for grades 6 to 8 were analyzed by considering van Hiele geometric thinking levels. These analyses showed that there was an inconsistency between the texts and exercises and test questions in terms of van Hiele levels. The texts were at a higher level than the exercises and tests. The exercises and tests were at visualization or analysis levels but the texts were at analysis and informal deduction levels. If the students' geometry level is not suitable for the presentation of topic, utilizing these textbooks would cause insufficient learning. As it is known that learning depends on prior knowledge (Hailikari *et al.*, 2008; Harackiewicz *et al.*, 2002; Dochy *et al.*, 1999; Deutsch and Tobias, 1980), results of Fuys *et al.* (1985)'s study on textbooks is generalized as the geometry background of students and students' prior knowledge are insufficient for the high school geometry course and students lack higher order geometric thinking skills in terms of van Hiele geometric thinking levels.

Fuys *et al.*'s (1988) another study on textbooks provides insight into different textbooks. After examining the textbooks, they stated that very few questions required students to write the justification of their answers. Moreover the exercises on topics were not ordered to use the relationships and properties that are learned. So the content of the books were found to be rote and not useful for the geometry instruction that leads an increase in students' thinking levels because in many cases the teachers are asked to present geometric concepts and the relationships between these concepts in a way that leads the student memorize rather than leading them to explore the relationships by themselves. Students are not expected to make interpretations and applications in exercises. As a result it is recommended that the exercises in the textbooks could be replaced in different parts of the text material and present conflicting views about relationships between geometric figures and shapes. Steele (2006) stated that the current geometry textbooks were like what Fuys *et al.* (1988) found out. According to the researcher, the textbooks still make the students just practice basic computational skills, however, richer conceptual ideas and discussions are very limited. Moreover, low level problems in the geometry textbooks require little cognitive operations and content of the textbooks repeat itself with unconnected sequence and unrelated exercises as Steele (2006) mentioned.

Fuys and Geddes (1984) also examined geometry textbooks in terms of their richness and continuity of instruction. They also examined the activities and examples in the books. They concluded that the non-examples are not presented in a line with the examples which makes it difficult for the students to describe the shape or concept. Moreover, the textbooks were insufficient in leading the students formulate the properties of the shapes, the text itself generalized the properties just in one way. They also added that the formulas and applications are not asked to the students which lead students to rote memorization of the formulas and rules. This lack of thinking and discussing style of the textbooks lead the students to work at the same thinking level. It supports the van Hiele’s (1957) argument that the students have difficulty in secondary school geometry because they do not have the chance of experiencing analysis level sufficiently at previous grade levels. The gap between visualization and informal deduction levels is a disadvantage for the students and they cannot develop their thinking levels enough to be successful at geometry topics which require informal deduction.

Carroll (1998) examined geometry textbooks and stated that the textbooks are not sequenced to lead students explore geometry at their own thinking level. According to this study, textbooks are not focused on investigations of geometric figures and the relationships between them. It is stated in the same study that the textbooks should include activities which move from simple identification of geometric figures and objects to investigations of relationships and properties. Steele (2006) examined the exercises in the elementary school geometry textbooks and stated that the exercises do not require a level of thinking beyond visualization and differentiation of shapes. It is also mentioned that the topics are repeated each year but the content do not develop significantly, that is, the content is circular instead of being spiral which means that they are repeated every year but not in a developing way.

To conclude, the textbooks are thought as being the primary resources for the geometry and mathematics teachers. The content and sequence of a textbook are mentioned to have an influence on what is being taught to the students. Research has shown that geometry textbooks do not serve to the purpose of leading students discuss the concepts, share the ideas, investigate the properties or relationships, apply the investigated concept or relationship to various problem situations and by that way increase their geometric thinking levels. Concrete objects are useful for students at lower grade levels to understand the rules and properties of certain objects but the textbooks do not provide an insight to concrete objects. Therefore the textbooks seem to be leading students to memorization without understanding, they also seem to be far from leading students achieve higher levels of thinking and learn the geometric concepts permanently. Researchers recommend placing activities which are aligned with the students' thinking levels and sequencing the material in textbooks from simple to complex.

2.3.3. Geometry Curriculum and Instruction in Turkey

Mathematics curriculum in grades 1 to 8 and then mathematics and geometry curriculum in grades 9 to 12 have been changed step by step beginning from 2004 as a result of the curriculum project by MEB. The basis of this project was changes and innovations in the field of mathematics and geometry which were mentioned in standards of National Council of Teachers of Mathematics (NCTM). NCTM standards consist of wide range of approaches and theories. In developing the field of geometry on the basis of NCTM standards van Hiele Level Theory was taken into account (NCTM, 2000).

In grades 6 to 8, mathematics curricula are divided into five learning areas. These areas are named as numbers, geometry, measurement, probability/ statistics and algebra (MEB, 2009). So geometry curricula are included in mathematics curricula at these grade levels. In implementing the reformed curriculum, it is recommended that to achieve the objectives which are decided for geometry field, geometry instructions should be based on student-centered methods. It is mentioned to be important to develop higher level thinking skills of students by using appropriate models and methods (Erdoğan *et al.*, 2009). In that sense the aims of the geometry learning area are stated as follows:

- analyze the properties of the geometric figures and objects and develop the relationships between these properties

- classify the geometric objects by using minimum number of their characteristics

- use spatial ability to analyze the dimensional properties of objects

- use the concepts of congruence, similarity, symmetry, transformation and rotation to construct geometric figures and objects

- utilize materials for measurement purposes

- understand the importance of estimation and estimate surface areas, perimeters, volumes of geometric objects without measuring and use these estimation strategies in daily life problems (MEB, 2009).

As it can be noticed inductive and deductive reasoning are not stated among the aims of the geometry area in grade levels 6 to 8. Also students are not expected to make proofs or generalizations. The students are expected to learn how to measure, to use geometry materials and to estimate length, area, volume of figures and objects in geometry courses. Properties of geometric figures and examining the relationships between the figures are also expected. These aims might show that the aims of the geometry lessons may be having students learn geometry topics by doing.

When secondary school geometry education is considered, according to the missions of MEB, aims of the geometry lessons in secondary schools are having students to be able to investigate relationships between geometric objects and to examine these objects by drawing, visualizing, transforming, comparing and measuring. It also aims to develop inductive and deductive reasoning abilities of students and to have students make connections between geometry and other branches of mathematics, especially algebra, and also between other disciplines. In elementary school years students are expected to learn two and three dimensional shapes and besides finding perimeter, area and volume. Since secondary school geometry continues after elementary and middle school geometry topics, geometry in secondary schools is positioned to develop students' measurement skills in terms of comparing and generalizing measurements and find the relationships between properties of general geometric shapes.

Students are also required to elaborate relationships between measurements and move from particular to general inside the world of geometric shapes. Making proofs in vectorial, analytical and synthetic geometry and examining the differences between vectorial, analytical and synthetic geometric approaches are also aimed at Turkish geometry curricula. In that sense students are expected to use geometric terminology and to study geometry in a formal way (MEB, 2010a, b).

Secondary school geometry education requires higher levels of geometric thinking than elementary school because mental abilities such as inductive and deductive reasoning, using different branches of mathematics in geometry; making connections between geometry and other subject areas; making generalizations; examining vectorial, analytical and synthetic geometry require higher levels of geometric thinking (MEB, 2010a, b).

To conclude, the curricula for mathematics from 6^{th} to 8^{th} grade levels and geometry from 9^{th} to 12^{th} grade levels were changed as a result of a project of MEB on the basis of NCTM standards and the van Hiele Level Theory was taken into account in this project. Therefore, geometry objectives in the curriculum were formed on the basis of geometric thinking levels of students. For example the curricula from 6^{th} to 8^{th} grade levels do not include making inferences or deduction and induction since these students are not expected to reach formal deduction level or level of rigor. On the other hand, secondary school geometry curricula include objectives which require higher levels of geometric thinking. Making proofs, deduction, induction, making generalizations are taken into account in the secondary school geometry curricula.

To sum up the review of the related literature, Van Hiele Level Theory states that the geometric thinking goes through five levels and at each level the approaches of students to the geometric issues are limited. This limitation is because of the certain characteristics that the students can demonstrate at each level. The theory mentions about some properties of the levels and suggests phases of instruction for geometry learning environments. The research studies and the articles which are related to the van Hiele Level Theory have shown that there is a significant correlation between the geometry achievement levels and van Hiele geometric thinking levels. Students' entry van Hiele geometric thinking levels are mentioned to be important for success in geometry and these levels are stated to be affected by the quality of the instruction not by maturation or age. Therefore different approaches of teaching geometry were examined and concluded that different approaches as phase-based instruction, dynamic geometry environments could affect the levels of the students if sufficient time is devoted. Moreover, quality of the instruction is mentioned to be affected by the use of textbooks since the textbooks are thought as the primary resources of instruction by geometry teachers. Therefore, the content and sequence of the textbooks are said to have influence on what is being taught and how it is taught to the students. Unfortunately, research has shown that geometry textbooks do not serve to the purpose of increasing the geometric thinking levels of students.

3. SIGNIFICANCE OF THE STUDY

The learners go through experiences in a certain learning area and these are expected to affect the learners' understandings in that area. If these do not affect the learner, then this might mean that the effect of learning area is not sufficient to create a difference and therefore the impact is limited. The learner's geometric thinking develops progressively and this development depends on quality and impact of the environment and the education which is suitable to the level of the learner (Usiskin, 1982; Clements and Battista, 1992).

In Turkish education system, at grades 6 to 8, there is not a separate geometry curriculum but it is included in mathematics curriculum. At the secondary school level students select a domain of study either in science-mathematics, Turkish-mathematics or social sciences after ninth grade. There are geometry and mathematics courses at all grade levels in science-mathematics and Turkish-mathematics domains. In social sciences domain, mathematics and geometry are selective courses. The learners, except the ones who select social sciences, have the chance to experience and develop geometric/ spatial thinking in geometry courses in both elementary and secondary school levels. Such experiences are expected to lead to increase learners' ability in geometric and spatial thinking. Also at higher education/ college levels students at mathematics related departments are expected to experience higher level mathematics.

This study investigated levels of geometric thinking which are achieved by students in their present educational area. It also examines the correlation between the van Hiele Test scores and geometry achievement levels. If there is a positive correlation, then this study could support Usiskin's (1982) statement that students' performance on the van Hiele Geometry Test is a predictor of performance on geometry courses. This study is also carried out to see whether geometry curriculum from middle school to university create any difference in geometric thinking levels of learners. This difference was tested by the van Hiele Geometry Test which is widely used for similar purposes. As another part of the study, the textbooks for the grade levels 7, 8, 10 and 11 were examined on the basis of van Hiele geometric thinking levels to see whether there is an alignment between the levels of textbooks and students. Since geometry curriculum is covered by these textbooks and instruction depend on textbooks then this analysis should show that whether the topics covered and exercises on textbooks are aligned with the students' geometric thinking levels.

4. STATEMENT OF THE RESEARCH PROBLEM

Students' geometric thinking levels have been considered as important issues that contribute to the achievement levels. The low level of performance of Turkish students in international assessment studies has shown that there are some problems that need to be analyzed. In this study assigning the state of geometric thinking levels of students has been thought as one of the ways of approaching to that issue.

This study aims to examine whether there is a correlation between geometric thinking levels and geometry achievement levels of students. It also aims to compare the performances of group of students at grade levels 7- 8, 10-11 and at university on the basis of van Hiele geometric thinking levels. The textbooks used at 7^{th}, 8^{th}, 10^{th} and 11^{th} grade levels were also examined in this study to see whether the textbooks are aligned with the students' geometric thinking levels. The geometric thinking levels of students were measured by the van Hiele Geometry Test and achievement levels were measured by geometry/mathematics achievement tests and geometry/mathematics examinations in academic school years 2009-2010 and 2010-2011. 10^{th} and 11^{th} grade students were either at Anatolian high school or at science high school. University students were chosen from Secondary School Science and Mathematics Education Department (SCED)(1^{st} -5^{th} years) and Primary Education Department (PRED) (1^{st}- 4^{th} years) of Boğaziçi University. They were studying teaching mathematics undergraduate programs in their related departments.

The main purpose of the study is to examine whether geometry curricula studied affect the geometric thinking levels of the students or not. Different grade levels were chosen to examine the progress in geometry achievement and the difference between achievement levels in geometry and the proficiency levels in the van Hiele Geometry Test which measures the levels of students in geometric concepts and geometrical proof and thinking. Moreover, the correlations between the achievement levels of school geometry and achievement in the van Hiele Geometry Test were also examined. This correlation study was conducted to see whether the van Hiele Geometry Test shows the levels of students in geometric thinking and in what way the test is related to the school geometry achievement. To see the effect of geometry curriculum on geometry achievement and geometric thinking levels, three different groups were formed according to the grade levels. First group consists of students at 7th and 8th grade levels, second group consists of students at 10th and 11th grade levels both at Anatolian high school and science high school, third group consists of university students. Students at SCED and PRED teaching mathematics undergraduate programs were chosen for the university part because these programs are related to mathematics and geometry.

4.1. Variables and Operational Definitions

The variables investigated in this study were the Van Hiele Geometry Test scores, achievement levels of students in mathematics/geometry and grade levels.

The grade levels in this study are 7, 8, 10, 11 and university. 10th and 11th grade students are at Anatolian high school and science high school. Science high school students are selected by the national examination named as Seviye Belirleme Sınavı (SBS). Anatolian high school students are from mathematics-science and Turkish-mathematics domains. Social science domain students were not considered in that study because the students in those domains do not have the same geometry courses as other students. The content of the geometry course in social science domain is quite different than the other geometry contents. Science high school students study at mathematics-science domain. University students are from Boğaziçi University at SCED and PRED departments. These grade levels were grouped as 7th-8th, 10th-11th, SCED-PRED.

Geometry achievement levels of students at 7th and 8th grade levels were determined by the mean scores of six mathematics examination results and eight mathematics achievement test results because there is not a separate geometry examination or geometry achievement test for these grade levels. The mathematics examinations include 5 mathematics and 5 geometry open-ended questions. The mathematics achievement tests include 10 mathematics and 10 geometry multiple choice items. The mathematics achievement tests cover wider range of mathematics and geometry topics than mathematics examinations. Both have forty minutes duration. The mean score of each student were computed for both 2009-2010 and 2010-2011 academic school years. The achievement records for the previous school year were taken into consideration because research studies confirm that previous achievement levels of students affect the current achievement levels (Hailikari *et al.*, 2008; Harackiewicz *et al.*, 2002; Deutsch and Tobias, 1980; Carmichael *et al.*, 2010). Mathematics achievement tests were taken into account besides mathematics examinations because the achievement tests covered the larger parts of the content than usual examinations and the questions were more analytical than the usual examinations. To determine geometry achievement mean scores of 10^{th} and 11^{th} grade students' six geometry examination results from both 2009-2010 and 2010-2011 academic school years were used.

The van Hiele Geometry Test scores are the scores obtained from the implementation of the van Hiele Geometry Test which was developed by Usiskin (1982) and involves 25 multiple choice geometry items. The possible minimum and maximum scores range from 0 to 25. In that study the scores were computed by assigning 1 to correct items and 0 to incorrect and unanswered items. Table 4.1 explains the classification of the variables in the study.

Table 4.1. Classification of the variables in the study.

Name	**Type of Variable**	**Type of Value**	**Data Collection Instrument**
Grade level	Independent	Categorical	Demographic Information Part of the test
Geometry achievement levels	Independent	Interval	Student records for 7^{th}-8^{th}-10^{th}-11^{th} grade level students from their schools
The van Hiele Geometry Test scores	Dependent	Interval	The van Hiele Geometry Test (Usiskin, 1982) translated by Duatepe (2000)

4.2. Research Questions

The following research questions are leading the current research study:

(i) Are there any significant differences between means of the van Hiele Geometry Test scores of the students at grade levels 7-8, 10-11 Anatolian high school and science high school students and university students?

(ii) Is there any alignment between the textbooks of the grade levels 7, 8, 10, 11 and the students in terms of van Hiele geometric thinking levels?

(iii) Are there any correlations between means of the van Hiele Geometry Test scores and mathematics-geometry achievement scores of students at grade levels 7, 8, 10 and 11?

4.3. Hypotheses

The hypotheses for the corresponding research questions are the following:

(i) There are significant differences between the means of the Van Hiele Geometry Test scores of the students at grade levels 7-8, 10-11 Anatolian high school and science high school students and university students.

(ii) There is an alignment between the textbooks of the grade levels 7, 8, 10, 11 and the students in terms of van Hiele geometric thinking levels.

(iii) There are significant correlations between the means of the Van Hiele Geometry Test scores and the geometry achievement mean scores of students at grade levels 7, 8, 10 and 11.

In order to answer the research questions following null hypotheses are used:

(i) There will be no significant differences between the means of the Van Hiele Geometry Test scores of the students at grade levels 7-8, 10-11 Anatolian high school and science high school students and university students.

(ii) There is no alignment between the textbooks of the grade levels 7, 8, 10, 11 and the students in terms of van Hiele geometric thinking levels.

(iii) There will be no significant correlations between the means of the Van Hiele Geometry Test scores and the geometry achievement mean scores of students at grade levels 7, 8, 10 and 11.

5. METHODOLOGY

This study compares the van Hiele Geometry Test scores of students from different grade levels and examines whether there is a correlation between the van Hiele Geometry Test scores and geometry achievement scores at their related grades. Van Hiele geometric thinking levels of students are also examined in order to see whether the levels of the students are aligned with the van Hiele levels of the content of geometry topics in textbooks.

5.1. Design

The design of the study is causal-comparative research design. There are three groups in the study. First group includes 7th and 8th grade students, second group includes 10th and 11th grade students who study at either Anatolian high school or science high school, third group includes university students. In that study it is examined whether these groups differ in terms of van Hiele Geometry Test scores. The correlation between geometry achievement scores and van Hiele Geometry Test scores of the students in first and second groups were also examined. Moreover, content analysis was carried out to examine the textbooks of related grade levels on the basis of van Hiele geometric thinking levels. The aim was to determine whether the geometric thinking levels of students were aligned with the van Hiele geometric thinking levels of the presentation of the geometry topics in the textbooks.

5.2. Sample

The study was conducted during 2010-2011 academic school year and the sample of the study had the following characteristics:

The sample included 208 7th, 226 8th, 149 10th and 106 11th grade students. There are also 79 students at SCED and 117 students at PRED departments at Boğaziçi University. All of the 7th, 8th, 10th and 11th grade students are from one of the private schools in Istanbul, Beşiktaş. This private school that took part in this study has kindergarten, elementary school (1st to 5th grade levels), middle school (6th -8th grade levels), Anatolian high school (9^{th} to 12^{th} grade levels) and science high school (9^{th} to 12^{th} grade levels) and is among the most well known schools for its high academic standards. The students in that school have similar characteristics in terms of socio-ecomic background and general achievement levels. Also in that private school all secondary school students and 6-8 grade level students have mathematics and geometry courses. Convenience sampling method was used to determine the participants of the study. Table 5.1 shows the number of students at each grade level.

Table 5.1. Number of students at each grade level.

		Number of students		
Grade level		**Girls**	**Boys**	**Total**
First group	7	94	114	208
	8	110	116	226
Second group	10	68	81	149
	11	40	66	106
Third group	SCED	49	30	79
	PRED	79	38	117
Total		440	445	885

5.3. Instruments

5.3.1. The van Hiele Geometry Test

The van Hiele geometry test was administered to the groups during a single class period (40 minutes) to determine the geometric thinking test scores of students. This test was developed by Usiskin (1982) and consists of 25 multiple choice items. It was translated into Turkish and reliability studies were conducted by Duatepe (2000). In the test the first five items represent visualization level, second five items represent analysis level, third five items represent informal deduction level, fourth five items represent formal deduction level and last five items represent level of rigor. The reliability measures of the van Hiele geometry test according to Usiskin's (Usiskin administered the test twice in his study), Duatepe's administrations and current administration are given in the Table 5.2.

Table 5.2. Reliability measures of the Van Hiele Geometry Test for Usiskin (1982), Duatepe (2000) and current administrations.

Van Hiele Level	**Usiskin**		**Duatepe**	**Current Study**
	1st	2nd		
Level 0 (visualization)	0.31	0.39	0.82	0.45
Level 1(analysis)	0.44	0.55	0.51	0.40
Level 2 (informal deduction)	0.49	0.56	0.70	0.50
Level 3 (formal deduction)	0.13	0.30	0.72	0.36
Level 4 (rigor)	0.10	0.26	0.59	0.24

Usiskin (1982) stated that low reliabilities at formal deduction level and level of rigor represent the lack of specification and abstractness of van Hiele Level Theory at these levels. Duatepe (2000) mentioned that the reliability of the results is similar to the Usiskin's results and the test worked in the same way as the original test.

In this study, Cronbach alpha was used for reliability measures. As it can be seen from the Table 5.2, the reliability coefficients are low. This would be because of the fact that each subtest contains five items and five items would not be enough to get high reliability coefficients. Therefore, in this study five subtests are considered to determine van Hiele geometric thinking levels of the students and for the geometry thinking scores of the students the test was administered as a whole and students received same scores (1 from each correct answer, 0 from blank or incorrect answer) from all 25 multiple choice items. So the students in that study were assigned van Hiele geometry test scores and van Hiele geometric thinking levels. The reliability coefficient for the whole test was computed and found as 0.725.

The van Hiele Geometry Test was scored by assigning 1 point for each correct answer and 0 for each incorrect or blank answer. The van Hiele geometric thinking levels assigned to the students by using one of the criteria defined by Usiskin (1982). It was considered that a student is at level “n”, if 3 of 5 items were correctly responded at level “n” and at all preceeding levels by the student.

5.3.2. Mathematics/ Geometry Achievement Instruments

The geometry achievement scores of students at 7th and 8th grade levels were determined by mathematics course examinations and mathematics achievement tests which were administered by the school mathematics teachers throughout the school years 2009-2010 and 2010-2011. Therefore for each student were assigned two mean scores one from 2009-2010 academic school year, the other from 2010-2011academic school year. Mathematics examinations and mathematics achievement tests which include both mathematics and geometry items were used for these grade levels because there do not exist any geometry achievement test or geometry examination because geometry topics are placed into mathematics curricula of these levels. For grade levels 10 and 11 means of achievement scores of geometry examinations for the same school years 2009-2010 and 2010-2011 were used. Therefore each student was assigned two mean scores from two academic school years.

The mathematics examinations at grades 7 and 8 include 5 geometry and 5 mathematics questions and administered six times throughout the school year. These questions require applying what has been taught in the lessons until the examination period. The mathematics examination is prepared and the answer key is scored by the group of mathematics teachers. The analysis of each examination is carried out by the measurement and evaluation department of the school. Mathematics achievement tests are administered 8 times throughout the school year. These tests consist of multiple choice items on topics which have been taught from the beginning of the school year up to the test period. The number of topics which are included in the test increases from 1st test to 8th test. The items in the test are selected from an item pool of the school which was created by mathematics teachers and analyzed by the measurement and evaluation department of the school. The mathematics achievement tests include wider range of topics than examinations.

The geometry examinations at 10^{th} and 11^{th} grade levels include 10 geometry questions. Solving these questions require applying what have been taught in geometry lessons according to geometry curriculum. These examinations are prepared and scored by mathematics teachers. The analysis of the examinations is carried out by the measurement and evaluation department of the school.

5.4. Analysis of Data

The Turkish version of the van Hiele Geometry Test was administered to all groups and the duration of the administration was 40 minutes. The students' highest score from the test was 25 and lowest score was 0. The mean score was computed for each group. This mean score again was 0 at least and 25 at most. One way ANOVA was used to see whether there is a significant difference between mean scores of the groups in the study. Since there was significant difference between group means a post-hoc test was run to see which specific groups differed. As a post-hoc test Dunnet-C was used because there was no homogeneity of variances. To do further comparison analyses the grade levels 10 and 11 were divided into two groups as Anatolian high school and science high school and university students were divided into two groups as SCED and PRED. These groups were also compared with the other groups. Because there were significant differences between Anatolian high school students' mean scores and science high school students' mean scores and between SCED and PRED students. The correlation between mathematics achievement levels and means of van Hiele Geometry Test scores was analyzed by Pearson r Correlation Coefficients. Pearson r was used in correlation analyses because of the fact that the variables (geometry achievement scores and the van Hiele Geometry Test scores) were at interval level and linear relationship was being measured (Tabachnick and Fidell, 1996). $\overline{x}_1$ is the mean score of 7^{th} -8^{th} grades from the van Hiele Geometry Test, $\overline{x}_2$ is the mean score of 10^{th}-11^{th} grades (Anatolian high school), $\overline{x}_3$ is the mean score of 10^{th} -11^{th} grades (science high school), $\overline{x}_4$ is the mean score of SCED students and $\overline{x}_5$ is the mean score of PRED students. In correlation analyses 7^{th} grades' average mathematics score was shown by r_1, 8^{th} grades' by r_2, 10^{th} grades' average geometry score was shown by r_3 and 11^{th} grades' by r_4. Moreover, 7^{th} grades' van Hiele Geometry Test mean score was shown by $\bar{x}_6$, 8th grades' by $\bar{x}_7$, 10th grades by $\bar{x}_8$ and 11th grades' by $\bar{x}_9$. Table 5.3 and table 5.4 show the procedure of the analyses which were utilized in the study.

Table 5.3. Design of the comparison analyses in the study.

Groups	**Van Hiele Geometry Test Scores**
7th -8th grade levels	$\bar{x}_1$
10th -11th grade levels (Anatolian high school)	$\bar{x}_2$
10th -11th grade levels (science high school)	$\bar{x}_3$
SCED	$\bar{x}_4$
PRED	$\bar{x}_5$
Analyses	ANOVA Dunnet-C

Table 5.4. Design of the correlation analyses in the study.

Groups	Correlation
7^{th} grade level	$r_{1\sim}\ \overline{x}_6$
8^{th} grade level	$r_{2\sim}\ \overline{x}_7$
10^{th} grade level	$r_{3\sim}\ \overline{x}_8$
11^{th} grade level	$r_{4\sim}\ \overline{x}_9$
Analysis	Pearson r correlation coefficients

6. RESULTS

Results of the study are organized to address the research questions and consist of three parts. First part shows the results of comparison analyses, second part includes descriptive analyses which explains whether the levels of the students are aligned with the van Hiele levels of the presentation of geometry topics in textbooks. Also the van Hiele Geometry Test was analyzed on the basis of Turkish geometry curricula for grade levels 7, 8, 10 and 11 to see whether the curricula provides enough knowledge of topics to answer the items in the test, third part shows the results of correlation analyses.

6.1. Group Comparison Analyses

In relation to the research question “Are there any significant differences between means of the van Hiele Geometry Test scores of the students at grade levels 7-8, 10-11 Anatolian high school and science high school students and university students?” differences between mean scores were investigated. This analysis was done by SPSS 17.0 using ANOVA and the significance level was kept on 0.05. The results are shown in Table 6.1.

Table 6.1. Results of oneway ANOVA for the van Hiele Geometry Test scores.

	Sum of Squares	**Df**	**Mean Square**	**F**	**Sig.**
Between Groups	2772.272	2	1386.136	125.455	0.000
Within Groups	9745.107	882	11.049		
Total	12517.379	884			

ANOVA results from the van Hiele Geometry Test revealed a significant difference between groups of the study, $F(2, 882)=125.455$, $p< 0.05$. To see which groups differ from each other in terms of the van Hiele Geometry Test scores, post-hoc analysis was carried out using Dunnet-C Test. Table 6.2 shows the results of multiple comparisons between groups.

Table 6.2. Multiple comparisons of the van Hiele Geometry Test scores.

(I) Groups	(J) Groups	Mean Difference (I-J)	Std. Error	95% Confidence Interval	
				Lower Bound	Upper Bound
7-8	10-11	-2.82858*	0.26629	-3.4558	-2.2013
	University	-4.16870*	0.28053	-4.8303	-3.5071
10-11	7-8	2.82858*	0.26629	2.2013	3.4558
	University	-1.34012*	0.31482	-2.0830	-0.5972
University	7-8	4.16870*	0.28053	3.5071	4.8303
	10-11	1.34012*	0.31482	0.5972	2.0830

*. $p < 0.05$

Multiple comparisons indicated that the differences between groups were significant for all pair-wise comparisons in the study ($p < 0.05$). In the analyses it was seen that 10^{th} and 11^{th} grade students' mean scores were different from each other and the mean scores of students at SCED and PRED were different from each other. As further analyses these two groups were divided in terms of school and department. Second group were divided as Anatolian high school students and science high school students, third group were divided as SCED and PRED students. To see whether these differences between mean scores of the van Hiele Geometry Test are significant or not multiple comparisons were done for these new groups by Dunnet-C Test. Table 6.3 shows the results of these multiple comparisons for 10^{th} and 11^{th} grade Anatolian high school students.

Table 6.3. Multiple comparisons for 10th-11th grades at Anatolian high school.

(I) group	(J) group	Mean Difference (I-J)	Std. Error	95% Confidence Interval	
				Lower Bound	Upper Bound
10-11 Anatolian high school	7-8	2.18264*	0.27647	1.4230	2.9423
	10-11 Science high school	-3.66032*	0.39945	-4.7845	-2.5361
	SCED	-2.79963*	0.43228	-4.0022	-1.5970
	PRED	-1.47369*	0.36527	-2.4831	-0.4643

*. $p < 0.05$

As it is seen from Table 6.3, there is a significant difference between van Hiele Geometry Test mean scores of 10th grade Anatolian high school students and the other groups of students ($p < 0.05$). Mean scores of the 10th-11th grade Anatolian high school students are higher than the mean scores of 7th and 8th grade students and lower than the 10th-11th grade science high school students, SCED and PRED students.

The differences between means of the van Hiele Geometry Test scores for the science high school group were given in Table 6.4. As it is seen from the Table 6.4, there is a significant difference between mean scores of the groups 10th-11th grades science high school students and 10th-11th grades Anatolian high school students, students at 7th-8th grades and PRED students ($p < 0.05$). However, there is not a significant difference between 10th-11th grade science high school students and SCED students. Mean score of the SCED students is slightly lower than the mean score of 10th-11th grade science high school students.

Table 6.4. Multiple comparisons for 10^{th}-11^{th} grades at science high school.

(I) group	(J) group	Mean Difference (I-J)	Std. Error	95% Confidence Interval	
				Lower Bound	Upper Bound
10-11 Science high school	7-8	5.84296*	0.36554	4.8106	6.8754
	10-11 Anatolian high school	3.66032*	0.39945	2.5361	4.7845
	SCED	0.86068	0.49402	-0.5304	2.2518
	PRED	2.18663*	0.43659	0.9587	3.4145

*. $p < 0.05$

The differences in mean scores between SCED and other groups are shown in Table 6.5. This comparison reveals a significant difference between mean scores of SCED students with the 7^{th}-8^{th} grades, 10^{th}-11^{th} grade Anatolian high school and PRED ($p < 0.05$). There is not a significant difference between mean scores of students at SCED and at 10^{th}-11^{th} grade science high school as mentioned before.

Table 6.5. Multiple comparisons for SCED students.

(I) group	(J) group	Mean Difference (I-J)	Std. Error	95% Confidence Interval	
				Lower Bound	Upper Bound
SCED	7-8	4.98228*	0.40116	3.8651	6.0995
	10-11 Anatolian high school	2.79963*	0.43228	1.5970	4.0022
	10-11 Science high school	-0.86068	0.49402	-2.2518	0.5304
	PRED	1.32595*	0.46681	0.0259	2.6260

*. $p < 0.05$

Table 6.6. Multiple comparisons for PRED students.

(I) group	(J) group	Mean Difference (I-J)	Std. Error	95% Confidence Interval	
				Lower Bound	Upper Bound
PRED	7-8	3.65633*	0.32785	2.7504	4.5623
	10-11 Anatolian high school	1.47369*	0.36527	0.4643	2.4831
	10-11 Science high school	-2.18663*	0.43659	-3.4145	-0.9587
	SCED	-1.32595*	0.46681	-2.6260	-0.0259

*. $p < 0.05$

Table 6.6 shows the results of the comparison analysis for PRED students' mean van Hiele Geometry Test scores. It is revealed that there is a significant difference between mean scores of PRED students and mean scores of other groups ($p < 0.05$). Moreover, the mean scores of SCED and 10^{th}-11^{th} grade science high school students are higher than the mean scores of students at PRED.

To conclude, comparison analyses were done by ANOVA to see whether there are significant differences between the van Hiele Geometry mean scores of the groups 7^{th}-8^{th} grades, 10^{th}-11^{th} grade at Anatolian and science high schools and university students at SCED and PRED. As a result of ANOVA it is revealed that there is a difference between the mean scores of the groups ($p < 0.05$). Then, post-hoc analysis was done to see which groups create the significant difference specifically. Dunnet-C Test was utilized for this analysis. While doing the analyses it was seen that science high school students and Anatolian high school students differ from each other on the basis of mean scores, also SCED and PRED students differ from each other on the basis of their mean scores from the van Hiele Geometry Test. Therefore, to see whether this difference is significant or not, these groups were divided into two subgroups and further analyses were carried out and multiple comparisons were done for the total of five groups. The results of the further analyses revealed that the difference between mean scores of 10^{th}-11^{th} grade Anatolian high school students and science high school students was significant ($p < 0.05$). Moreover, the difference between mean scores of SCED and PRED students was also significant ($p < 0.05$). Multiple comparison analyses also revealed that the difference between mean scores of 10^{th}-11^{th} grade science high school students and SCED students was not significant and mean scores of science high school students was higher than both the mean scores of SCED and PRED students.

6.2. Descriptive Analyses

Van Hiele geometric thinking levels of the subjects were determined by the van Hiele Geometry Test. Some studies verify that the levels are hierarchical (Burger and Shaughnessy, 1986; Fuys and Geddes, 1984; Senk, 1989; Usiskin, 1982). Therefore if a student is assigned a level, it requires that the responses of that student satisfy not only the current level but also all the preceding levels. That is to say, if a student answered at least three items out of the five items at a certain level on the van Hiele Geometry Test and this fact is satisfied for the preceding levels, then the student is thought as being at that level. This is one of the assignment methods which were used by Senk (1985) and Usiskin (1982). Table 6.7 shows the number and percentage of the subjects at each van Hiele level.

Table 6.7. The van Hiele levels of the students.

The van Hiele Levels / Grade Level	Visualization (Level 0)	Analysis (Level 1)	Informal Deduction (Level 2)	Formal Deduction (Level 3)	Rigor (Level 4)	Sum
7-8	284 (65 %)	78 (18 %)	72 (17 %)	0	0	434
10-11 Anatolian high school	110 (52 %)	45 (21 %)	51 (24 %)	3 (2 %)	1(1%)	210
10-11 Science high school	11 (24 %)	10 (22 %)	17 (38 %)	6 (13%)	1(2 %)	45
SCED	21 (27%)	13 (16%)	39 (49%)	4 (5%)	2 (3%)	79
PRED	50 (43%)	14 (12%)	45 (38%)	8 (7%)	0	117
Total	476	160	224	21	4	885

As it is seen from Table 6.7, 65% of the students at 7^{th} -8^{th} grades, 52% at 10^{th} -11^{th} grades (Anatolian high school) and 24% at 10^{th}-11^{th} grades (science high school) are at visualization level. At university level 27 % of the SCED students and 43% of the PRED students are at visualization level. 18 % of the 7^{th}-8^{th} grades, 21% of the 10^{th}-11^{th} grades Anatolian high school students, 22 % of the 10^{th}-11^{th} grades science high school students, 16 % of SCED and 12 % of PRED students are at analysis level. 17% of 7^{th}-8^{th} grades, 24 % of 10^{th}-11^{th} grades at Anatolian high school, 38 % of 10^{th}-11^{th} grades at science high school, 49% of SCED and 38 % of PRED students are at informal deduction level. At formal deduction level there exists 2 % of the students at 10^{th}-11^{th} grade levels of Anatolian high school, 13% of students at 10^{th}-11^{th} grades of science high school, 5% of students at SCED, 7 % of PRED students. When level of rigor is considered 1% of the

10^{th}-11^{th} grade Anatolian high school students, 2% of 10^{th}-11^{th} grade science high school students and 3% of SCED students are at that level. None of the students at 7^{th}-8^{th} grades and at PRED is at level of rigor. The percentage of van Hiele geometric thinking levels on the basis of grade levels can be seen in Figure 6.1 below.

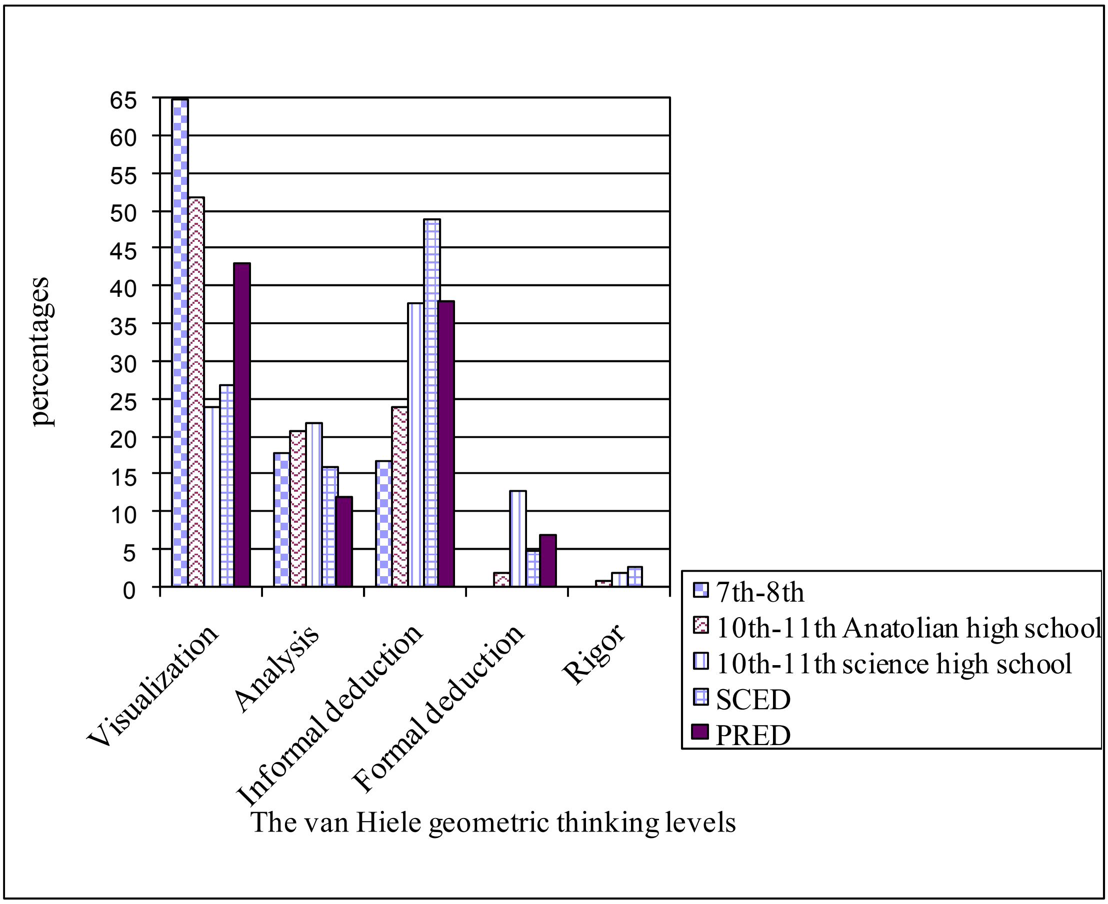


Figure 6.1. Percentages of van Hiele levels at different grade levels.

6.2.1. The Analysis of the van Hiele Geometry Test on the Basis of Turkish Geometry Curricula

After determining the van Hiele levels of students, the van Hiele Geometry Test was examined according to the curricula of the grade levels 7, 8, 10 and 11, to see whether the students at these grade levels are familiar with the content of the items of the test or not, also to see whether the topics which the items of the test belong to are taught in geometry courses at these grade levels or not. The items in the test was divided according to the van

Hiele geometric thinking levels and the subject matter which is required to respond the item correctly is stated and then the subject matter was placed into the curriculum from 7^{th} grade to 11^{th} grade. The result of the analysis of the van Hiele Geometry Test on the basis of geometry curricula of 7^{th}, 8^{th}, 10^{th} and 11^{th} grades are shown in Table 6.8, 6.9, 6.10, 6.11 and 6.12 for each class of items in the test.

Table 6.8. The van Hiele Geometry Test items of visualization level and related topics according to grade levels.

The van Hiele Geometry Test Item	**Topic Related to the Item**	**Grade 7**	**Grade 8**	**Grade 10^***	**Grade 10^{**}**	**Grade 11**
1	Square	+		+		
2	Triangle	+	+	+		+
3	Rectangle	+		+		
4	Square	+		+		
5	Paralellogram	+		+		

10^*: Previous curriculum 10^{**}: New curriculum

According to Table 6.8, to answer the questions of visualization level, it is sufficient to know the seventh grade geometry topics. Seventh grade geometry topics were revisited at previous tenth grade curriculum. This revision took place in detail and included proofs and generalizations which were not in the seventh grade curriculum. But visualization items do not require any generalizations or strategies of proof so all of the students in the current study should have knowledge to answer the items. Triangles are also covered at eighth and eleventh grades but different aspects of triangles are in concern at these grade levels such as similarity, theorems, construction of triangles and so on. For visualization level items it is enough to know that a triangle has three sides and three corners. Figure 6.2 shows the percentages of the students at visualization on the basis of grade levels.

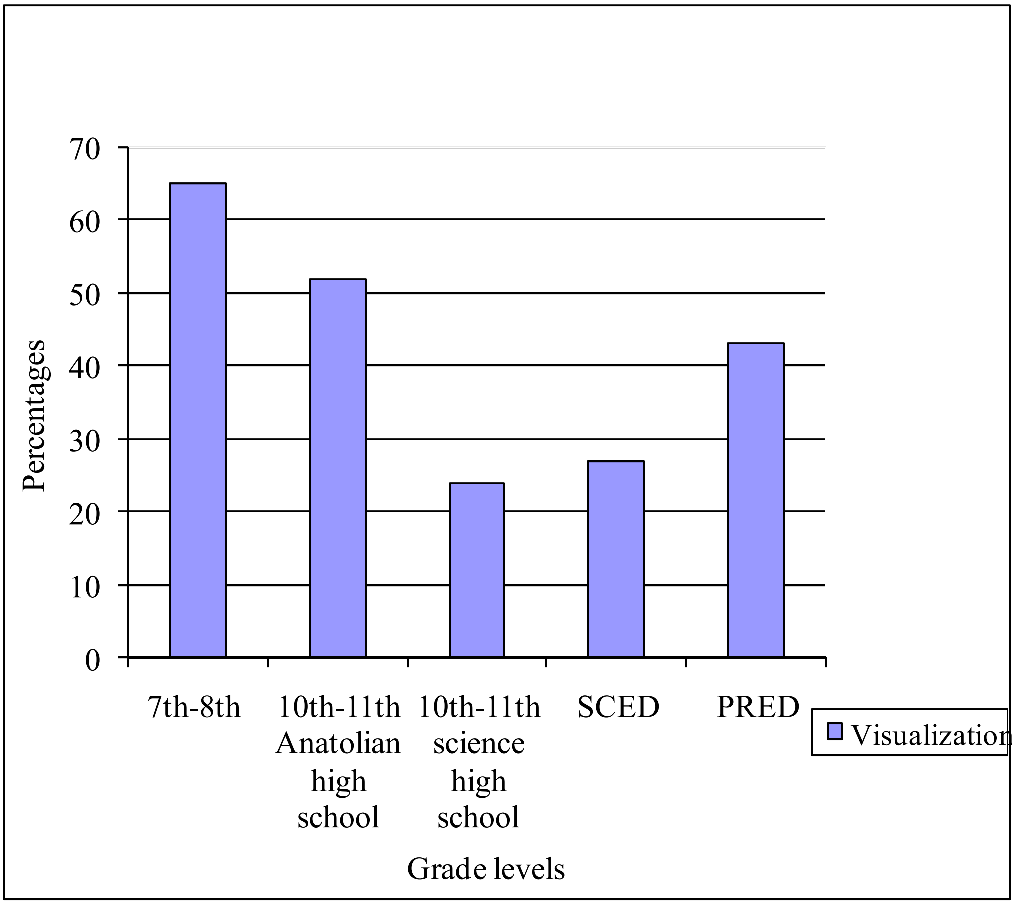


Figure 6.2. Percentages of the students at visualization level.

Table 6.9. The van Hiele Geometry Test analysis level items and related topics according to grade levels.

Van Hiele Geometry Test Item	**Topic Related to the Item**	**Grade 7**	**Grade 8**	**Grade 10***	**Grade 10****	**Grade 11**
6	Properties of a Square	+		+		
7	Properties of a Rectangle	+		+		
8	Properties of a Rhombus	+		+		
9	Properties of a Isosceles Triangle	+	+			
10	Properties of a Kite			+		

Second part of the test includes items of analysis level. As it can be seen from Table 6.9, the items require the knowledge of properties of geometric figures. To answer four of the five items the students need to know the seventh grade geometry topics. Only kite is not in seventh grade geometry, it is the part of previous tenth grade curriculum. Therefore students at grades 7 and 8 are expected to answer four items in that part. As stated in the theory, students at analysis level, know the properties of the figures but not the relationships between these figures. Figure 6.3 shows the percentages of the students at analysis on the basis of grade levels.

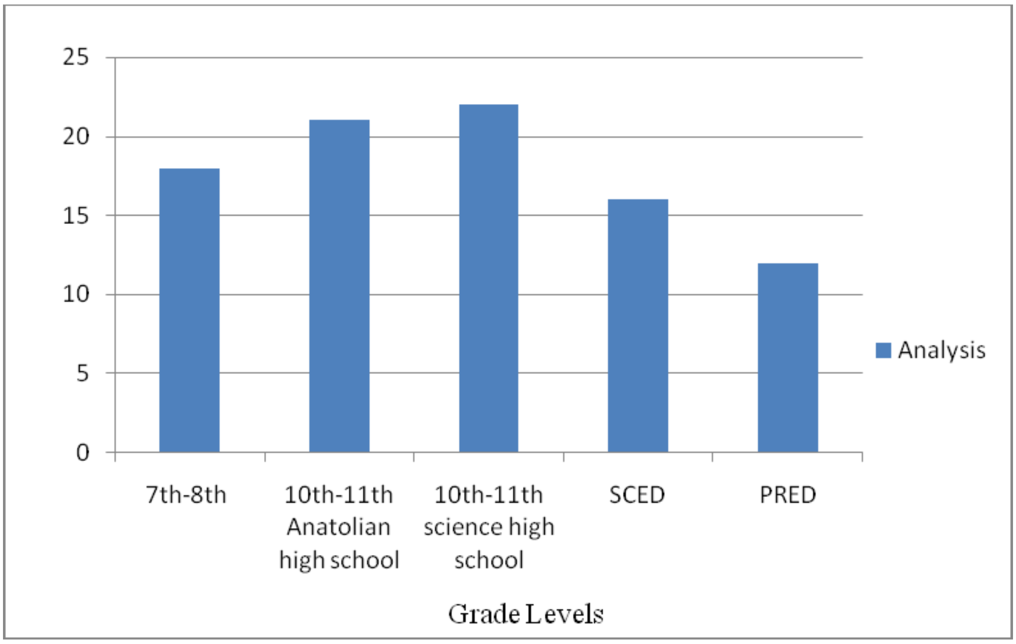


Figure 6.3. Percentages of the students at analysis level.

Table 6.10. The van Hiele Geometry Test informal deduction level items and related topics according to grade levels.

Van Hiele Geometry Test Items	Topic Related to the Item	Grade 7	Grade 8	Grade 10*	Grade 10**	Grade 11
11	Properties of Triangles	+	+			
12	Properties of a Rectangle and a Triangle	+		+		
13	Rectangle	+		+		
14	Comparison of a Rectangle and a Parallelogram	+		+		
15	Comparison of a Rectangle and a Square	+		+		

10*: Previous curriculum 10**: New curriculum

At informal deduction level, comparisons between the properties of geometric figures are needed. For that part students should know seventh grade and tenth grade geometry topics. Figure 6.4 shows the percentages of the students at informal deduction level according to the grade levels.

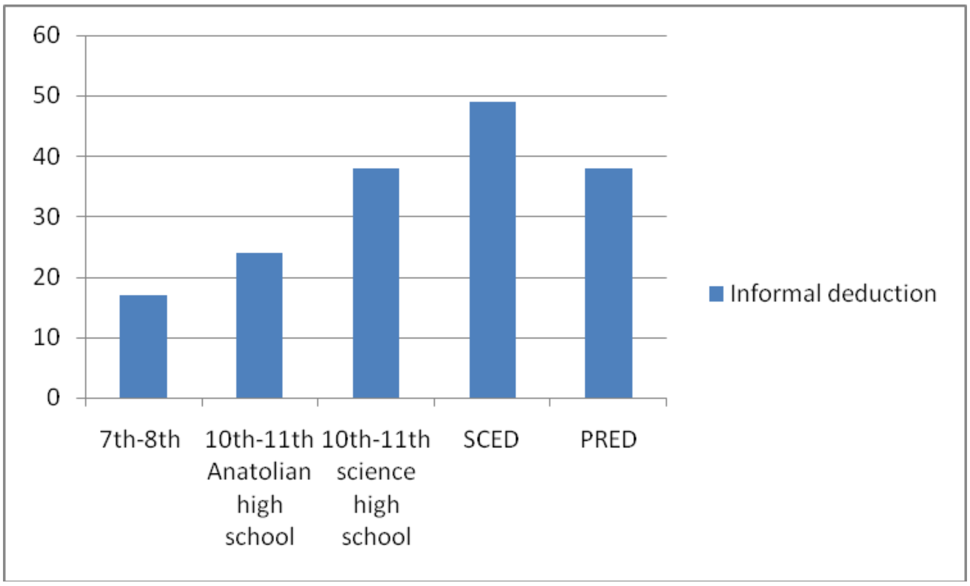


Figure 6.4. Percentages of the students at informal deduction level.

Table 6.11. The van Hiele Geometry Test formal deduction level items and related topics according to grade levels.

Van Hiele Geometry Test Items	Topic Related to the Item	Grade 7	Grade 8	Grade 10*	Grade 10**	Grade 11
16	Deduction From a Proof			+	+	+
17	Making Informal Proofs			+	+	+
18	Parallel and Perpendicular Lines	+			+	+
19	Relationships Between Rectangle and Square	+		+		
20	Making Inferences			+	+	+

10*: Previous curriculum 10**: New curriculum

Formal deduction level items require higher order geometric reasoning and deduction abilities as it can be seen from Table 6.11. Grade 7 and grade 8 curricula might not be sufficient to have the students develop such geometric abilities so it could be difficult for seventh and eighth grade students to answer formal deduction level items but students at grades 10 and 11 and university students should respond correctly. Figure 6.5 shows the percentages of the students at this level according to the grade levels.

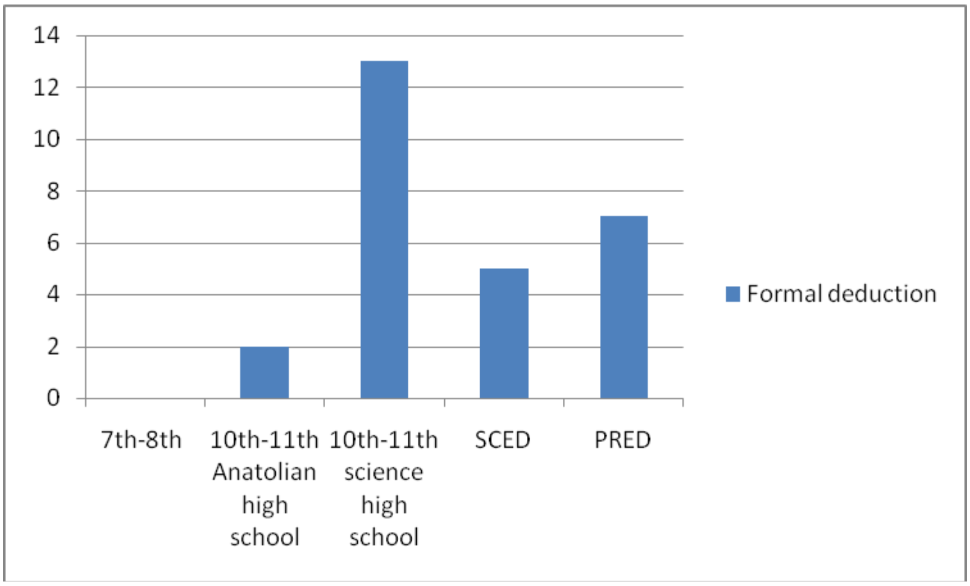


Figure 6.5. Percentages of the students at formal deduction level.

Table 6.12. The van Hiele Geometry Test level of rigor items and related topics according to grade levels.

Van Hiele Geometry Test Items	Topic Related to the Item	Grade 7	Grade 8	Grade 10^{*}	Grade 10^{**}	Grade 11
21	Deduction From a Proof			+	+	+
22	Making Inferences			+	+	+
23	Making Inferences			+	+	+
24	Making Inferences			+	+	+
25	Deduction From a Proof			+	+	+

10^{*}: Previous curriculum 10^{**}: New curriculum

Level of rigor items seem to be more discrete than formal deduction level items. All of the items require making inferences and knowledge of proof strategies. The tables for formal deduction level and level of rigor confirms Usiskin's (1982) results that secondary school geometry courses are at formal deduction level and at level of rigor. So students should be these to succeed at geometry at secondary school and beyond.

6.2.2. The Analyses of Geometry Curricula of 7th, 8th, 10th and 11th Grades on the Basis of the van Hiele Geometric Thinking Levels

Geometry instruction is based on geometry textbooks which are prepared according to the geometry curricula. Teachers follow the outline of the book and prepare the lesson plans based on the explanations and exercises in the textbook. Therefore it might important to examine the content of the textbooks to see whether the content is appropriate to the geometric thinking levels of students. The textbooks which are published by MEB are used in most schools. Therefore for this study, MEB textbooks were exemined. Table 6.13, 6.14, 6.15 and 6.16 show the subject matters and the required van Hiele geometric thinking levels for learning for each grade level 7, 8, 10 and 11 respectively.

Table 6.13. Seventh grade geometry curriculum and required van Hiele levels.

Subjects	**The van Hiele levels**
Reflection, Translation and Rotation	Analysis
Tesselations	Analysis
Lines and Angles	Analysis
Angle Measurement	Analysis
Polygons	Analysis
Congruence and Similarity	Analysis
Areas of Quadrilateral Regions	Informal deduction
Properties of a Circle and a Circular Region	Analysis
Measuring Arcs of a Circle	Analysis
Measuring Areas of Circular Regions	Informal deduction
Area and volume of a Cylinder	Informal deduction

Table 6.14. Eighth grade geometry curriculum and required van Hiele levels.

Subjects	**The van Hiele Levels**
Fractals	Analysis
Reflection, Translation and Rotation	Informal deduction
Triangles (congruence and similarity, theorems, Trigonometry)	Informal deduction
Symmetry	Analysis
Prisms, pyramids, sphere and cone (properties)	Analysis
Prisms, pyramids, sphere and cone (surface area)	Informal deduction
Prisms, pyramids, sphere and cone (volume)	Informal deduction
Projection and perspective	Analysis

The explanations, properties and exercises in the 7^{th} and 8^{th} grade textbooks may not be aligned with the geometric thinking levels of students since 65 % of the students are at visualization level. Although the activities in these textbooks are at visualization level, the examples which follow the activities are mostly at analysis level. For the students at grade level 8, it might be difficult to understand the content of the geometry because of the fact that the level of the topics are mostly at analysis or informal deduction levels and the students at these levels from both 7^{th} and 8^{th} grades are 35 %. At grade eight students are expected to recognize the properties, to see the relationships and to construct the geometric figures by using geometry material such as protractor, compass and ruler. The levels of these types of activities in the book are again appropriate for analysis level.

Table 6.15. Tenth grade geometry curriculum and required van Hiele levels.

Subjects	**The van Hiele Levels**
Basic Elements and Forms of Proof	Formal deduction
Point, Line and Vectors in Plane	Informal deduction
Coordinate Plane	Informal deduction
Lines	Informal deduction
Triangles (Properties, Measurements)	Informal deduction
Triangles (Theorems)	Formal deduction
Reflection, Rotation and Translation (Theorems)	Formal deduction

Table 6.16. Eleventh grade geometry curriculum and required van Hiele levels.

Subjects	**The van Hiele Levels**
Quadrilaterals (Proof and applications)	Informal or formal deduction
Special Quadrilaterals (Proof and applications)	Informal or formal deduction
Polygons (deriving rules/properties)	Informal or formal deduction
Circle (Theorems and applications)	Informal or formal deduction
Conic Sections	Informal or formal deduction

10^{th} and 11^{th} grade geometry textbooks might be appropriate for the students at science high school but not at Anatolian high school. Especially in textbook for 11^{th} grade, the students are expected to follow the proofs and apply them. The topics also include deriving rules and properties and applying these properties in geometry problems which should not be appropriate for the students who are not at informal or formal deduction levels. Since 73% of the 10^{th}-11^{th} grade Anatolian high school students are at analysis level or below, it might be difficult for them to follow the topics in the curriculum.

6.3. Correlation Analyses

This study focused on the relationships between the achievement levels in geometry and in the van Hiele Geometry Test. Correlational analyses part only describes whether there exist any correlations, not any causal relationships between the variables which are mentioned. In that sense the aim of the correlational analyses in this study is to find out at what degree these two variables are related to each other. The main purpose is to explain the degree of association between the variables of the study. Correlational analyses were carried out by SPSS 17.0, using Pearson r Correlational Coefficients.

The analyses were carried out to see whether there is a correlation between the van Hiele Geometry Test scores and geometry achievement scores of the students at grades 7, 8, 10 and 11. For the procedure, geometry achievement scores of 7^{th} and 8^{th} grade students were computed by using 6 mathematics examination scores and 8 mathematics achievement test scores. Since the geometry curriculum is placed in mathematics curriculum at these grades, the geometry topics are not measured by a geometry examination or by a geometry achievement test. Therefore for these grade levels mathematics scores were used. For the secondary school students six geometry examination scores were used. For each student the scores were computed for both academic school years 2009-2010 and 2010-2011. In relation to the question "Are there any correlations between means of the van Hiele Geometry Test scores and geometry achievement scores of students at grade levels 7, 8, 10 and 11?" the correlation coefficients are shown in the following tables. Table 6.17 shows the correlation between students' van Hiele Geometry Test scores and average scores for the school year 2009-2010 and 2010-2011 for the whole sample.

Table 6.17. Correlations between the van Hiele Geometry Test scores and average scores.

		Geometry achievement 2009- 2010	The van Hiele Geometry Test
Geometry achievement 2009-2010	Pearson Correlation	1	0.203**
	Sig. (2-tailed)		0.000
	N	689	689
Geometry achievement 2010-2011	Pearson Correlation	0.864**	0.150**
	Sig. (2-tailed)	0.000	0.000
	N	689	689

**. $p < 0.01$

It can be seen from Table 6.17 that there is a significant correlation between geometry achievement scores of students for the school year 2009-2010 and 2010-2011 ($r= 0.864$, $p< 0.01$). Also there is a significant correlation between geometry achievement scores for the school year 2009-2010 and the van Hiele Geometry Test scores ($r= 0.203$, $p< 0.01$). There is also a significant correlation between geometry achievement scores for the school year 2010-2011 and the van Hiele Geometry Test scores ($r= 0.150$, $p< 0.01$).

The correlation between geometry achievement scores and the van Hiele Geometry Test scores is lower than the correlation between geometry achievement scores for the two academic school years. To search for the reason, correlations for each grade levels were analyzed. Table 6.18, 6.19, 6.20, 6.21 show the correlations for the grade levels 7, 8, 10 and 11 respectively.

Table 6.18. Correlations for grade 7.

		Geometry achievement 2009- 2010	Geometry achievement 2010- 2011
The van Hiele Geometry Test	Pearson Correlation	0.564**	0.567**
	Sig. (2-tailed)	0.000	0.000
	Covariance	26.684	28.309

**. p < 0.01

There is a significant correlation between geometry achievement scores of students at 2009-2010 academic school year and the van Hiele Geometry Test scores (r= 0. 564, p< 0.01). The correlation between geometry achievement scores of students at 2010-2011 academic school year and the van Hiele Geometry Test scores is also significant (r= 0.567, p< 0.01).

Table 6.19. Correlations for grade 8.

		Geometry achievement 2009- 2010	Geometry achievement 2010- 2011
The van Hiele Geometry Test	Pearson Correlation	0.564**	0.509**
	Sig. (2-tailed)	0.000	0.000
	Covariance	27.946	27.722
	N	226	226

**. p < 0.01

There is a significant correlation between geometry achievement scores of students at 2009-2010 academic school year and the van Hiele Geometry Test scores (r= 0. 564, p< 0.01). The correlation between geometry achievement scores of students at 2010-2011 academic school year and the van Hiele Geometry Test scores is also significant (r= 0.509, p< 0.01).

Table 6.20. Correlations for grade 10.

		Geometry achievement 2009- 2010	**Geometry achievement 2010- 2011**
The van Hiele Geometry Test	Pearson Correlation	0.110	0.219**
	Sig. (2-tailed)	0.184	0.007
	Covariance	5.591	11.051
	N	149	149

**. $p < 0.01$

There is a significant correlation between geometry achievement scores of students at 2010-2011 academic school year and van Hiele Geometry Test scores ($r= 0.219$, $p< 0.01$). The correlation is lower than the correlations at grades 7 and 8. There is not a significant correlation between the geometry achievement scores for the academic school year 2009-2010 and the van Hiele Geometry Test scores.

Table 6.21. Correlations for grade 11.

		Geometry achievement 2009- 2010	**Geometry achievement 2010- 2011**
The van Hiele Geometry Test	Pearson Correlation	0.463**	0.542**
	Sig. (2-tailed)	0.000	0.000
	Covariance	31.613	35.829
	N	104	104

**. $p < 0.01$

There is a significant correlation between geometry achievement scores of students at 2009-2010 academic school year and the van Hiele Geometry Test scores ($r= 0.463$, $p< 0.01$). The correlation between geometry achievement scores of students at 2010-2011 academic school year and the van Hiele Geometry Test scores is also significant ($r= 0.542$, $p< 0.01$).

To conclude the results of the correlation analyses, there are significant correlations between geometry achievement scores and the van Hiele Geometry Test scores of students at grade levels 7, 8 and 11. The correlation for grade 10 is only significant at 2010-2011 academic year and lower than the correlation for the other grade levels.

7. DISCUSSION AND CONCLUSION

Research studies reveal that the geometry thinking levels of students are affected by their learning environment (Usiskin, 1982; Clements and Battista, 1992; Kemp, 1990; Hodanbosi, 2001; Johnson, 1988). The learning environment includes curriculum, instruction and classroom activities. The instruction is based on the curriculum as mentioned at the previous sections of the study. So, this study was done to examine the effects of the geometry curriculum on thinking levels of students at different grade levels. The grade levels were chosen diversely to see whether the geometric thinking levels of students differ as they progress on geometry courses in their schooling years. To examine the factors affecting on geometry thinking levels, the textbooks were examined on the basis of van Hiele geometric thinking levels and the van Hiele Geometry Test was analyzed to detect whether the geometry topics in the curriculum is included in the test and the students have the content knowledge to answer the items in the test. To examine whether there exists any correlation between geometric thinking levels of students and their achievement levels in geometry was also one of the aims of the study. Therefore, the research questions leading this study are the following:

(i) Are there any significant differences between means of the van Hiele Geometry Test scores of the students at grade levels 7-8, 10-11 Anatolian high school and science high school students and university students?

(ii) Is there any alignment between the textbooks of the grade levels 7, 8, 10, 11 and the students in terms of van Hiele geometric thinking levels?

(iii) Are there any correlations between means of the van Hiele Geometry Test scores and mathematics- geometry achievement scores of students at grade levels 7, 8, 10 and 11?

To achieve the purposes of the study, causal comparative research design was conducted on 885 students at grade levels 7, 8, 10, 11 and at university. The students at grade levels 10 and 11 were either at Anatolian high school or science high school. University students were selected from SCED and PRED at Boğaziçi University. All the university students were studying at teaching mathematics undergraduate program either at primary or secondary school teaching. To do analyses of difference and correlation, the van Hiele Geometry Test mean scores and geometry achievement mean scores were used. The van Hiele Geometry Test is also used to determine the students' van Hiele geometric thinking levels. Moreover, geometry textbooks for the mentioned grade levels were examined and a content analysis was employed on the basis of van Hiele Theory and characteristics of van Hiele levels. In comparison analyses the students were grouped according to the grade levels. So, the first group of the study was 7^{th} and 8^{th} grade students, 10^{th} and 11^{th} grades were grouped as second group and university students were the third group. In content and correlation analyses the levels were taken as 7^{th}, 8^{th}, 10^{th}, 11^{th} grades.

7.1. Comparison of van Hiele Geometric Thinking Scores

The comparisons of the mean scores received from the van Hiele geometry test were done in SPSS 17.0 by ANOVA Dunnett-C Test. First the sample was grouped as 7^{th}-8^{th} grades, 10^{th}-11^{th} grades and university students. The analyses by these groups revealed that the differences between the mean scores of the groups were significant. Also the results revealed that the mean scores of the 10^{th}-11^{th} grades were different from each other depending on the school type, Anatolian high school or science high school, also for university students the mean scores differed according to their departments so to do further multiple comparisons the sample was divided into 5 groups. These were 7^{th}-8^{th} grades, 10^{th}-11^{th} grades at Anatolian high school, 10^{th}-11^{th} grades at science high school, SCED and PRED students.

For 10^{th}-11^{th} grade Anatolian high school students, the differences between the mean score of this group and 7^{th}-8^{th}, SCED and PRED students' mean scores were significant as hypothesized. The mean score of this group was higher than the mean score of 7^{th}-8^{th} grade group and lower than the mean scores of the other groups. Since the van Hiele Geometry Test includes most of the topics of the 10^{th} grade curriculum, 10^{th} grade and 11^{th} grade students could have more correct responses than 7^{th} and 8^{th} grade students. Therefore this could be an expected result. On the other hand, science high school students, SCED and PRED students might have more knowledge on the geometry topics because of the fact that they have gone through selection processes since they enter their schools as a result of national examinations. To study at science high school and at a top ranking university, it is required that the students get high scores from these examinations. This might influence their studying periods and as a result they might have more geometry knowledge on the basis of the preparation period. For other possible reasons these students' geometry backgrounds and their attitudes towards geometry might be examined.

The differences between mean scores of 10^{th} -11^{th} grade science high school students and mean scores of Anatolian high school 10^{th}-11^{th} grades, 7^{th} and 8^{th} grades and PRED were significant. The mean score s for 10^{th}-11^{th} grade science high school students were higher than mean scores of the students at 7^{th}-8^{th} grades, 10^{th}-11^{th} grades at Anatolian high school and at PRED. This could be expected for the first two groups but not expected for the PRED group. This may be because of the fact that science high school students are more familiar with the geometry curriculum and their geometry learning process are still going on so they could be better at analyzing the geometric facts and figures and at making geometric proofs than PRED students. On the contrary, PRED students might not have developed their geometry knowledge levels at university because of the fact that there is not any geometry course other than PRED 350 Teaching Geometry in their curriculum. This could mean that they are still at the same geometric knowledge level as they were at high school. There was not a significant difference between the mean scores of the science high school group and the mean scores of SCED students. As it is stated for PRED students, this could be because of the fact that science high school students might be more familiar with the geometry curriculum since their geometry learning processes still continue. The van Hiele Geometry Test consists of the topics of their geometry curriculum and their geometry instructions follow this curriculum. The content of first 22 items in the van Hiele Geometry Test is covered at in 10^{th} grade geometry curriculum, especially more intensively at science high school. At science high school their curriculum is mostly based on mathematics and science which might make them familiar with geometric proofs and analytical and geometric thinking strategies. 11^{th} grade curriculum covers the last 10 items of the test which include "making inferences" and "deduction from a proof" topics. So, they might be used to solving geometry questions covering same kind of topics as in the van Hiele Geometry Test. On the contrary SCED students might not have developed their geometry knowledge levels at university because of the fact that SCED curriculum does not contain any geometry course.

The comparison results for SCED students revealed that there are significant differences between means of the van Hiele Geometry Test scores of students at SCED, PRED, Anatolian high school 10^{th} and 11^{th} grades, 7^{th} and 8^{th} grades. These might be the expected results since they have completed the geometry curriculum before other groups. The difference between SCED and PRED students might show the fact that the geometry knowledge could differ in favor of SCED students. Another reason may be that SCED students enter the university with higher mathematics scores than PRED students. For further analyses, other mathematical abilities such as making proofs and deduction-induction should be taken into account for these groups. The unexpected result of the comparison was the result that revealed there is no significant difference between mean scores of students at SCED and 10^{th}-11^{th} grades science high school students. Students at science high school are chosen according to their mathematics/ science achievement mean scores and SBS results. In that sense they might be among the more talented students than others. There might be more emphasis on geometry curriculum at science high school since the general curriculum is based on mathematics and science. SCED students might be seen as disadvantageous when compared to science high school students because they do not have geometry courses at their undergraduate program in which they might have chance to continue to make proofs or to develop their geometric thinking.

Other comparison analysis was done between PRED department and other groups. The differences were significant for 10^{th} and 11^{th} grade Anatolian high school students, 7^{th}-8^{th} grade students as expected. Mean scores of the students at 10^{th}-11^{th} grade science high school were higher than the mean scores of PRED students. These results may reveal that the science high school students are more familiar with the geometric concepts and proof methods since they are still having geometry courses. On the other hand, after graduating from secondary school, PRED students might not have repeated geometry curriculum at the university level. They only have teaching geometry course in which the content might not be consisting making geometric proofs and deduction or induction. So it could be stated that their geometric knowledge may not have developed.

As a result of the comparison analyses, the ranking of the differences is as follows; SCED, 10^{th}-11^{th} grades science high school, PRED, 10^{th}-11^{th} grades Anatolian high school, 11^{th} grade Anatolian high school, 7^{th}-8^{th} grades. It might be said that one of the factors which created these differences in geometric thinking performances is the learning, not the age or the grade level alone. Although PRED and SCED students do not have geometry lessons during their undergraduate studies, their familiarity with mathematics or geometry and the fact that they have completed the geometry curriculum in their education period and their studying period for the university entrance examinations could have created a basis for their geometry thinking performances. Also selection process might be taken as an important factor as a result of this study. 10^{th} -11^{th} grades science high school students are placed in that program because of their high academic achievement scores in science and mathematics. Although mathematics and geometry curricula are the same as in Anatolian high schools, the courses they have might constitute to the development of higher mathematical abilities and geometrical thinking levels when compared to the students at the same grade levels in other types of programs. Students at science high school might prefer to discover and solve the geometry problems instead of waiting for the teacher to guide them or expecting the teacher to solve the problem. They might be more self-confident because of their learning environment or interactions with the teachers or peers. There is also a selection process for PRED and SCED students which is done through national examinations. SCED department requires higher score than PRED department so SCED students perform better in university entrance examinations.

The results of the comparison studies might be thought as similar to the results of the studies which concluded that the van Hiele levels of the students depend on the geometry learning they have at school (Usiskin, 1982; Carroll, 1998; Corley, 1990; Halat, 2007). Carroll (1998) mentioned that the structure and the sequence of the geometry lessons should be shaped to have students explore the relationships and increase their levels of geometric reasoning. Moreover it is stated that with an appropriate level of geometry content, the levels of the students might be developed. The fact that there are not significant differences between students at 10^{th}-11^{th} grade at science high school and SCED students might be showing that van Hiele geometric thinking levels of students do not necessarily depend on the age or maturation but depend more so on the knowledge of the geometry curriculum. This is also supported by the literature (Usiskin, 1982; Burger and Shaughnessy, 1986; Johnson, 2002). Frykholm (1994) conducted a study with students from 8^{th} to 11^{th} grade levels and searched for the relationships between van Hiele Geometry Test performance and geometry achievement scores, age and gender. It was found that the students at 10^{th} grade performed better than the students at other grade levels. This might be the confirmation of the results which revealed that the mean scores of the students at 10^{th}-11^{th} grades were higher than the mean scores of the students at 7^{th}-8^{th} grades and this does not change according to whether they are at Anatolian high school or science high school. Mayberry's (1981) study also supported the fact that it is not the biological maturation that results in higher geometric thinking levels it is the learning process including direct instruction and student exploration which takes place in the course of geometry education (Fuys and Geddes, 1984; Fuys et. al. 1984).

7.2. The van Hiele Geometric Thinking Levels

None of the 7^{th}-8^{th} grade level students in the study went beyond informal deduction level. Most of the students' van Hiele geometric thinking levels were visualization or analysis levels. This result might be thought as being in accordance with the findings of the studies by Burger and Shaughnessy (1986), Fuys *et. al*. (1988) and Crowley (1987). These studies stated that in grades K-8, students' reasoning stays generally at analysis level. What Idris (1998) stated may be also a confirmation of the result of the study. According to the study conducted by Idris (1998), it is found that most of the students at middle school were at visualization or analysis levels. Therefore these students are able to identify the geometric objects and figures but they cannot distinguish the properties of them. The students who are at visualization level are not able to recognize the geometric shapes as a whole, they just use the visual appearance of the figures to approach to the geometric problems.

When the textbooks of grade 7 and 8 were analyzed in terms of van Hiele levels, it is seen that content of geometry parts in the books may be appropriate for analysis or informal deduction levels. Only the activities at the introduction of the topic and in the explanation part can be thought as being appropriate for the levels of the students. The exercises were sequenced from simple to difficult, however in most of the exercises students are required to make connections between geometric shapes and rules, to use relationships which might not be aligned with the students' levels.

When the van Hiele Geometry Test items were examined to see which item corresponds to which topic in all grade levels, it was seen that first 15 questions in the test which corresponds to the van Hiele geometric thinking levels of visualization, analysis and informal deduction, cover the seventh grade curriculum. Therefore it may be expected that seventh and eighth grade students might be successful in responding these 15 items of the test. This was not the case in the results of the study. Only 17% of 7^{th}-8^{th} grades answered at most the first 15 items of the test. 65% of the 7^{th}-8^{th} grades could answer at most first 5 items in the test. This might be because the curricular activities they have which are included in the geometry instructions. It may be suggested that there is a need to change the approach to teaching geometry. This could be done by exposing the students to different activities which foster geometric understanding and make them familiar with the geometric terms and relationships. That may help them to learn the geometric content in middle school and to be successful in geometry in secondary school.

At secondary school, the levels of the students should be examined separately depending on their studying at science high school and Anatolian high school. Levels of science high school students were higher than the levels of students at Anatolian high school. The students at science high school were selected by a central exam named as Seviye Belirleme Sınavı (SBS) which is administered by MEB. So these students had a selection process on the basis of their SBS scores and they are among the ones who have highest scores on SBS. Therefore one of the explanations for their difference may come from their learning and studying geometry curriculum during their education at 6^{th}-8^{th} grades. Science high school students might have learned the topics in the geometry curriculum more than their peers.

The textbooks for the tenth and eleventh grade levels were aligned with informal or formal deduction levels. Especially at grade 11, students are required to follow the proofs and to apply the proofs in the problems. There are some theorems in the 10^{th} grade geometry book but other topics require students to explore the properties and do measurements which are suitable for informal deduction level. These textbooks might be appropriate for science high school students on the basis of van Hiele levels but for Anatolian high school students the level of the instruction based on textbooks might be higher than their levels of geometric thinking. Since they are secondary school students, they may not be expected to answer last five items in the test but the subject matters in the 10th and 11th grade geometry curricula could be considered as being sufficient for students to respond all the items in the test. Therefore, these students' low levels of geometric thinking at Anatolian high school might be attributed to the quality of geometry instruction, level of geometry instruction and geometry curriculum.

The curricula from 7th to 11th grades follow a hierarchy in terms of thinking levels. For example, at grade 7 students learn congruence and similarity as corresponded to analysis level. The activities at the beginning of the chapter are at visualization level, students are only expected to recognize the figures which have the same appearance. After the activities, congruence and similarity of polygons are stated. In the exercises part, first few exercises are appropriate to visualization level, then it moves on with the ones appropriate to analysis level. 7th grade congruence and similarity topic does not require forming relationships, only using properties to compare figures. At 8th grade, same topic is introduced to the students but only for triangles. At this grade level topic include rules to utilize for finding similar triangles and state the similarity ratio, but the theorems of congruence and similarity are not stated. Students are expected to construct similar triangles and to explore the relationships between congruent or similar triangles. Therefore the congruence and similarity topic taught at 8th grade may be aligned with the informal deduction level. 10th grade curriculum also includes congruence and similarity of triangles. This topic at grade 10 includes properties, theorems, proofs of theorems and measurements. Students are expected to perform simple proofs and to carry out applications of proofs. This is why the level of the congruence and similarity topic might be taken as aligned with the informal or formal deduction level.

The van Hiele geometric thinking levels of SCED students vary from visualization level to the level of rigor. 23 % of them are at visualization level. The percentage of the students at informal deduction level (49 %) is more than the percentage of other levels. 57% of the students are at informal deduction level or above. When the levels of PRED students are considered, 43% are at the visualization level and 45% are at the informal deduction level or above. None of the students at PRED is at the level of rigor. Levels of SCED students might be thought as similar to the levels of science high school students and levels of PRED students might be thought as similar to the Anatolian high school students when the results of the study are examined. For further analysis on geometry performance of the students at PRED and SCED, the studies based on interviews or proof making processes should be conducted.

7.3. Correlation Between Geometry Achievement and van Hiele Geometry Test Performance

The correlation analyses were carried out using SPSS 17.0 by Pearson r Correlation Coefficients. The mean scores of students from the van Hiele Geometry Test and geometry achievement mean scores were used to see whether there is a correlation between the Van Hiele scores and achievement scores. These analyses do not mention causal relationships between the variables. For the correlation analyses the sample was divided into groups on the basis of the grade levels 7, 8, 10 and 11.

The results of the correlation analyses show that there are significant correlations between means of the geometry achievement scores (mathematics achievement scores for grade levels 7 and 8) and van Hiele Geometry test scores for all grade levels in the study. The correlation coefficients are lower at grades 10 and 11 than grades 7 and 8. The teachers of the mathematics department stated that the content of the geometry courses at grade levels 10 and 11 are the same as the content of the textbooks of that grade levels and they are expected to follow the textbooks during the lessons. But the problems which are solved in the geometry lessons include the ones which are more suitable for the university entrance examinations and they are mostly multiple choice types of questions. Although the students are familiar with the multiple choice questions and direct application questions, in the examinations they are required to solve harder questions which need interpretation and formal deduction. Therefore the possible imbalance between the content of the course and the content of the examinations might be a disadvantage for the students. This might be the reason for lower correlation coefficients for those grade levels.

The results of the correlation studies are consistent with the other research studies which showed that there is a significant correlation between achievement levels and levels of geometric thinking (Usiskin, 1982; Burger and Shaughnessy, 1986; Bobango; 1987; Fuys et. al., 1988; Johnson, 1988; Senk, 1989; Corley, 1990; Frerking, 1994; Frykholm, 1994; Idris, 1998, Coşkun, 2009). Usiskin (1982) stated that there is a significant relationship between the performance of students on van Hiele Geometry test and their achievement scores in geometry. For example Corley (1990) found a strong association between the scores of the van Hiele Geometry Test and the achievement scores of the students assigned by the course teacher and also stated that this association grew at the end of the school year which shows that the thinking levels of students change as long as they learn geometry. In the same study, it is found that the van Hiele Test is reliable to predict the students' achievement levels in a traditional high school geometry course which supports the results of the current study. Moreover Frykholm (1994) found geometry achievement levels of students as the best predictors of van Hiele Geometry levels of students and mentioned that there is strong correlation between geometry achievement scores and van Hiele Geometry Test scores of students at 8^{th} to 11^{th} grade levels.

7.4. Limitations

There are some limitations which restrict the generalization of findings of the study to different settings, classes and situations. First of all the sample of the study was not randomly selected, instead convenient sampling was used. Students in middle and secondary school were not selected, all students who are at school on the day of administration of the test were participated in the study. At the university level, the researcher administrated the test on more than one day in order to reach as many students as possible. The data of the study is limited with the students in one of the private schools in Istanbul and with the undergraduate students at SCED and PRED departments for the academic year 2010-2011.

The instruments used in the study were all paper-pencil tests, the geometry achievement scores were the results of the examinations and multiple choice tests which students have had throughout the academic school years 2009-2010 and 2010-2011. The geometry achievement scores of 7^{th} and 8^{th} grade students could not be accessed since there was not an examination or a test just including geometry topics. The geometry curricula at these grades are placed in the mathematics curricula so the examinations or tests include topics from both mathematics and geometry. This is another limitation of the study. To determine van Hiele levels and to get van Hiele mean scores, the van Hiele

Geometric Test was used which is again a multiple choice test. More in-depth interviews with the selected students to assign van Hiele levels could have improved the research.

Moreover, all the participants at SCED department assumed to have similar educational backgrounds and same factor is assumed for PRED students. The middle school and secondary school students are also assumed to have similar geometry backgrounds since most of them are at the same school throughout their schooling years. It was also assumed that all participants answered the test honestly.

7.5. Implications

This part includes implications and recommendations which are based on the interpretation of the results of the study and the possible extensions of the study in the field of van Hiele Level Theory.

It may be thought that if the teacher knows more on students' way of learning and cognitive factors such as van Hiele geometric thinking levels, they could plan the instruction and remediations more effectively. This might help development of students' geometric thinking. Therefore, mathematics teachers might be trained on van Hiele Level Theory and the applications of the theory. Since the geometry instruction may be thought as being important in development of van Hiele geometric thinking levels, teachers might be guided to apply phase-based geometry instruction, to prepare lesson plans and activities on the basis of van Hiele Level Theory and conduct assessment to determine the geometry thinking levels of their students.

A longitudinal study may be conducted to see the stages of increase in van Hiele geometric thinking levels, by using various teaching methods. Also the correlations between van Hiele geometric thinking levels and other variables such as prior knowledge, individual learning differences, social interactions, computer use, reading habits, attitude towards strategy games, motivation levels, teacher qualifications, proficiency levels of preservice teachers might be analyzed.

In this study, the van Hiele Geometry Test was used to determine the van Hiele levels of the students. Using interviews might be recommended to assign van Hiele geometric thinking levels. These interviews might also be formed as to identify the particular parts of the geometry that the students do not understand.

APPENDIX A: THE VAN HIELE GEOMETRY TEST

VAN HIELE GEOMETRİ TESTİ

YÖNERGE

Bu test 25 sorudan oluşmaktadır. Sizden testteki her soruyu bilmeniz beklenmemektedir. Kitapçığı açtığınızda;

1- Bütün soruları dikkatlice okuyun.

2- Doğru olduğunu düşündüğünüz seçenek üzerinde düşünün. Her soru için tek bir doğru cevap vardır. Cevap kağıdına doğru olduğunu düşündüğünüz seçeneği işaretleyin.

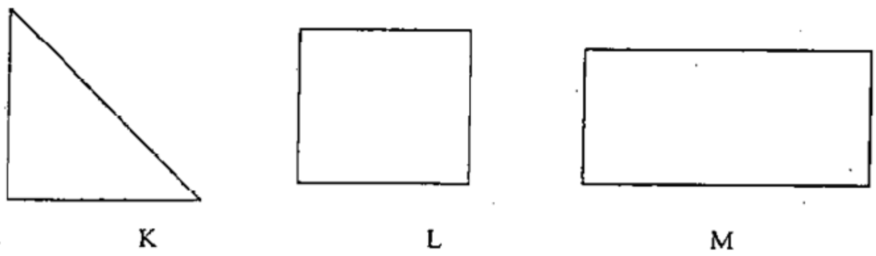
3- Lütfen soru kağıdının üzerine her hangi bir işaret koymayın. Cevap kağıdındaki boşlukları çizim yapmak için kullanabilirsiniz.

4- İşaretlemiş olduğunuz cevabı değiştirmek isterseniz, ilk işareti tamamen siliniz.

5- Bu test için size verilecek süre 35 dakikadır.

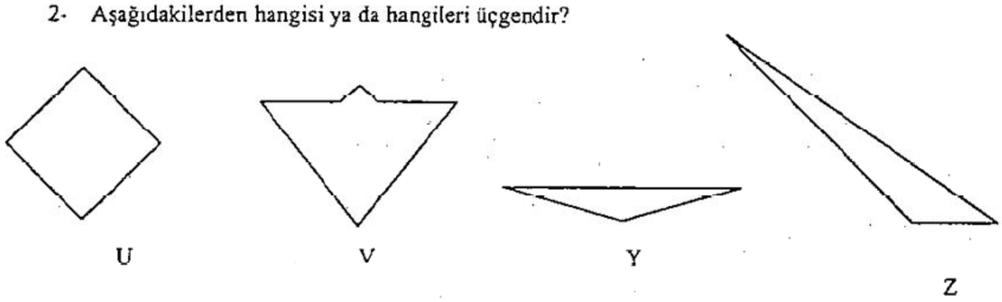
VAN HIELE GEOMETRİ TESTİ

1- Aşağıdakilerden hangisi ya da hangileri karedir?



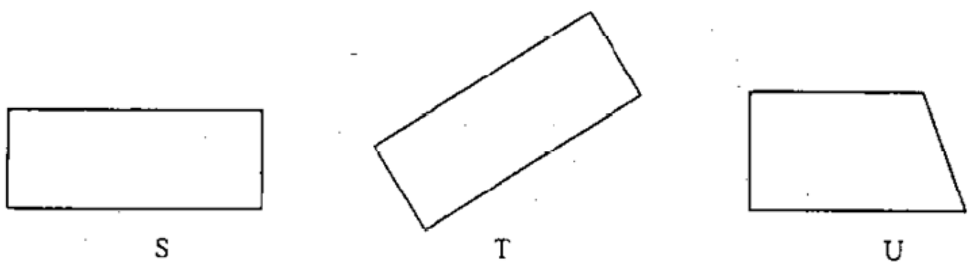
A) Yalnız K
B) Yalnız L
C) Yalnız M
D) L ve M
E) Hepsi karedir.

2- Aşağıdakilerden hangisi ya da hangileri üçgendir?



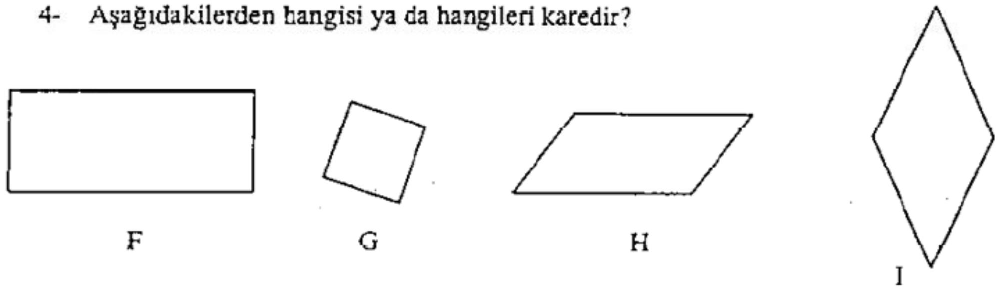
A) Hiçbiri üçgen değildir
B) Yalnız V
C) Yalnız Y
D) Y ve Z
E) V ve Y

3- Aşağıdakilerden hangisi ya da hangileri dikdörtgendir?



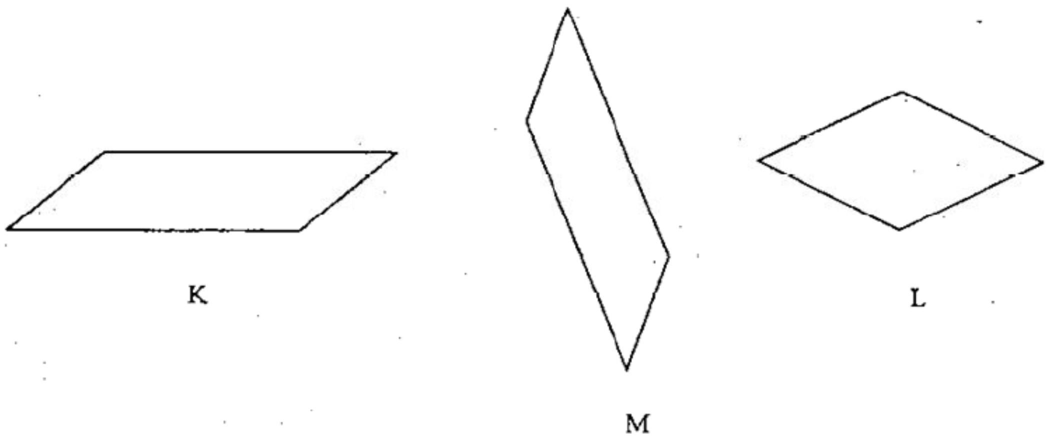
A) Yalnız S
B) Yalnız T
C) S ve T
D) S ve U
E) Hepsi dikdörtgendir.

4- Aşağıdakilerden hangisi ya da hangileri karedir?



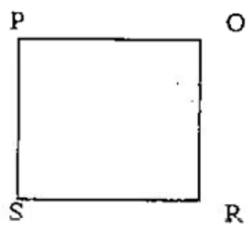
A) Hiçbiri kare değildir.
B) Yalnız G
C) F ve G
D) G ve I
E) Hepsi karedir.

5- Aşağıdakilerin hangisi ya da hangileri paralel kenardır?



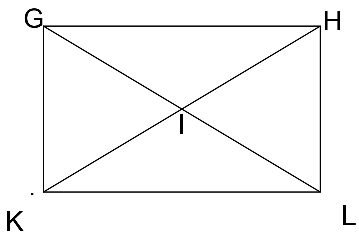
A) Yalnız K
B) Yalnız L
C) K ve M
D) Hiçbiri paralel kenar değildir
E) Hepsi paralel kenardır.

6- PORS bir karedir.
Aşağıdakilerden hangi özellik her kare için doğrudur?



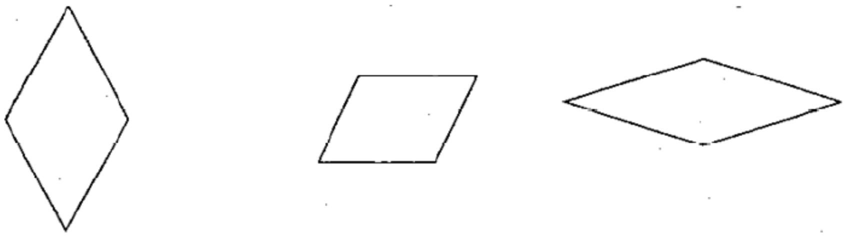
A) [PR] ve [RS] eşit uzunluktadır.
B) [OS] ve [PR] diktir.
C) [PS] ve [OR] diktir.
D) [PS] ve [OS] eşit uzunluktadır.
E) O açısı R açısından daha büyüktür.

7- Bir GHJK dikdörtgeninde, [GL] ve [HK] köşegendir. Buna göre aşağıdakilerden hangisi her dikdörtgen için doğru değildir?



A) Dört dik açısı vardır
B) Dört kenarı vardır
C) Köşegenlerinin uzunlukları eşittir
D) Karşılıklı kenarların uzunlukları eşittir
E) [GI], [GH] den kısadır

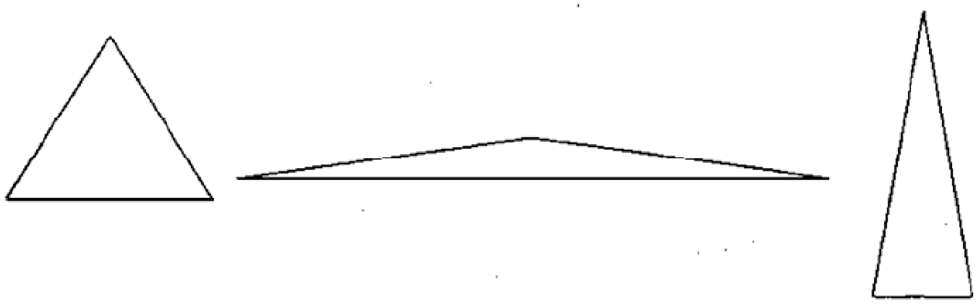
8- Eşkenar dörtgen tüm kenar uzunlukları eşit olan, 4 kenarlı bir şekildir. Aşağıda 3 tane eşkenar dörtgen verilmiştir.



Aşağıdaki seçeneklerden hangisi her eşkenar dörtgen için doğru değildir?

A) İki köşegenin uzunlukları eşittir
B) Her köşegen aynı zamanda açı ortaydır.
C) Köşegenler birbirine diktir.
D) Karşılıklı açılarının ölçüleri eşittir.
E) Ardışık köşelerdeki açıları bütünlerdir.

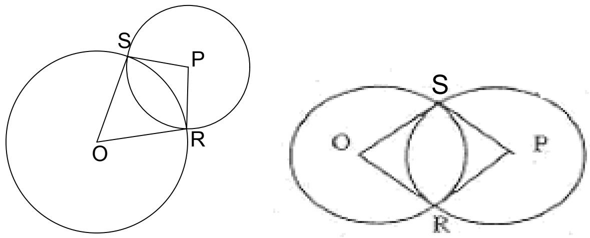
9- İkizkenar üçgen, iki kenarı eşit olan üçgendir. Aşağıda üç ikiz kenar üçgen verilmiştir.



Aşağıdaki seçeneklerinden hangisi her ikizkenar üçgen için doğrudur?

A) Üç kenarı eşit uzunlukta olmalıdır.
B) Bir kenarının uzunluğu diğerinin iki katı olmalıdır
C) Ölçüsü eşit olan en az iki açısı olmalıdır.
D) Üç açısının da ölçüsü eşit olmalıdır
E) Seçeneklerden hiç biri her ikizkenar üçgen için doğru değildir.

10. Merkezleri P ve O olan iki çember 4 kenarları PROS şeklini oluşturmak üzere R ve S noktalarında kesişirler. Aşağıda iki örnek verilmiştir.



Aşağıdaki seçeneklerinden hangisi her zaman doğru değildir?

A) PROS şeklinin iki kenarı eşit uzunlukta olacaktır.
B) PROS şeklinin en az iki açısının ölçüsü eşit olacaktır.
C) [PO] ve [RS] dik olacaktır
D) P ve O açılarının ölçüleri eşit olacaktır.
E) [PO], [OR] den daha uzundur.

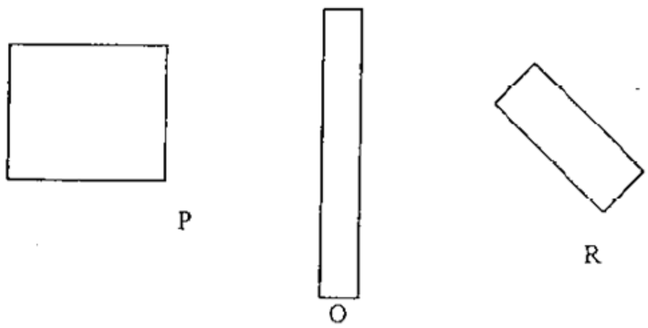
11. Önerme S: ABC üçgeninin üç kenarı eşit uzunluktadır.
Önerme T: ABC üçgeninde, B ve C açılarının ölçüleri eşittir.
Buna göre aşağıdakilerden hangisi doğrudur?

A) S ve T önermeleri aynı anda doğru olamaz
B) Eğer S doğruysa T de doğrudur
C) Eğer T doğruysa S de doğrudur
D) Eğer S yanlışsa T de yanlıştır
E) Yukarıdaki seçeneklerin hiçbiri doğru değildir.

12. Önerme 1: F şekli bir dikdörtgendir.
Önerme 2: F şekli bir üçgendir.
Bu iki önermeye göre aşağıdakilerden hangisi doğrudur?

A) Eğer 1 doğruysa 2 de doğrudur
B) Eğer 1 yanlışsa 2 doğrudur
C) 1 ve 2 aynı anda doğru olamaz
D) 1 ve 2 aynı anda yanlış olamaz
E) Yukarıdaki seçeneklerin hiçbiri doğru değildir.

13. Aşağıdaki şekillerden hangisi ya da hangileri dikdörtgen olarak adlandırılabilir?



A) Hepsi
B) Yalnız O
C) Yalnız R
D) P ve O
E) O ve R

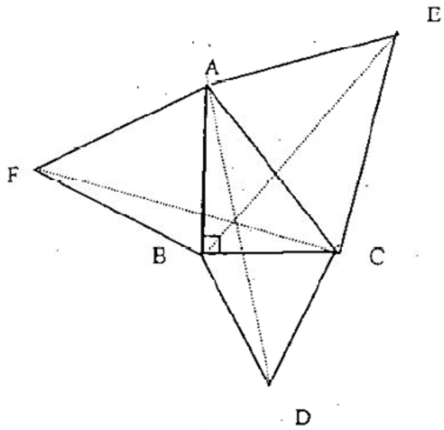
14. Tüm dikdörtgenlerde olup, bazı paralel kenarlarda olmayan özellik nedir?

A) Karşılıklı kenarları eştir
B) Köşegenleri eştir
C) Karşılıklı kenarlar paraleldir
D) Karşılıklı açıları eştir
E) Yukarıdaki seçeneklerin hiçbiri doğru değildir

15- Aşağıdakilerden hangisi doğrudur?

A) Dikdörtgenlerin tüm özellikleri tüm kareler için geçerlidir
B) Karelerin tüm özellikleri tüm dikdörtgenler için geçerlidir
C) Dikdörtgenlerin tüm özellikleri tüm paralel kenarlar için geçerlidir
D) Karelerin tüm özellikleri tüm paralel kenarlar için geçerlidir
E) Yukarıdaki seçeneklerin hiçbiri doğru değildir

16- Aşağıda bir ABC dik üçgeni verilmiştir. ABC üçgeninin kenarları üzerinde; ACE, ABF ve BCD eşkenar üçgenleri çizilmiştir.



Bu bilgilerden [AD], [BE] ve [CF] ortak bir noktadan geçtikleri kanıtlanabilir. Bu kanıt size neyi ifade eder?

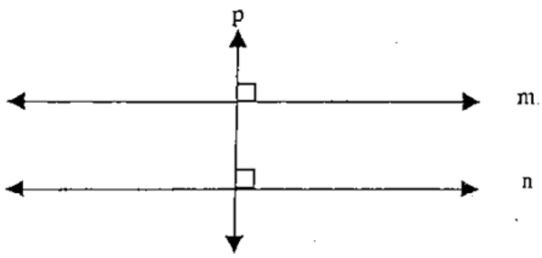
A) Yalnızca bu üçgen için [AD], [BE] ve [CF] nin ortak bir noktası olduğundan emin olabiliriz.
B) Sadece bazı dik üçgenlerde [AD], [BE] ve [CF] nin ortak bir noktası vardır.
C) Herhangi bir dik üçgende [AD], [BE] ve [CF] nin ortak bir noktası vardır.
D) Herhangi bir üçgende [AD], [BE] ve [CF] nin ortak bir noktası vardır.
E) Herhangi bir eşkenar üçgende [AD], [BE] ve [CF] nin ortak bir noktası vardır.

17- Aşağıda iki önerme verilmiştir.
I- Eğer bir şekil dikdörtgense, köşegenleri birbirini ortalayarak keser.
II- Eğer bir şeklin köşegenleri birbirini ortalayarak kesiyorsa şekil dikdörtgendir.
Buna göre aşağıdakilerden hangisi doğrudur?

A) I in doğru olduğunu kanıtlamak için II nin doğru olduğunu kanıtlamak yeterlidir.
B) II nin doğru olduğunu kanıtlamak için I in doğru olduğunu kanıtlamak yeterlidir.
C) II nin doğru olduğunu kanıtlamak için, köşegenleri birbirini ortalayan bir dikdörtgen bulmak yeterlidir.
D) II nin yanlış olduğunu kanıtlamak için köşegenleri birbirini ortalayan dikdörtgen olmayan bir şekil bulmak yeterlidir.
E) Yukarıdaki seçeneklerin hiç biri doğru değildir.

18- Aşağıdaki üç ifadeyi inceleyin.
{1} Aynı doğruya dik olan iki doğru paraleldir.
{2} İki paralel doğrudan birine dik olan doğru, diğerine de diktir.
{3} Eğer iki doğru eş uzaklıktaysa paraleldir.

Aşağıdaki şekilde, m ve p, n ve p doğrularının birbirine dik olduğu verilmiştir. Buna göre yukarıdaki cümlelerden hangisi ya da hangileri m doğrusunun n doğrusuna paralel olmasının nedeni olabilir?



A) Yalnız {1}
B) Yalnız {2}
C) Yalnız {3}
D) {1} ya da {2}
E) {2} ya da {3}

19- Aşağıda bir şeklin üç özelliği verilmiştir.
Özellik D: Köşegenleri eşit uzunluktadır.
Özellik S: Bir karedir.
Özellik R: Bir dikdörtgendir.
Bu özellikler dikkate alındığında aşağıdakilerden hangisi doğrudur?

A) D gerektirir S, o da gerektirir R
B) D gerektirir R, o da gerektirir S
C) S gerektirir R, o da gerektirir D
D) R gerektirir D, o da gerektirir S
E) R gerektirir S, o da gerektirir D

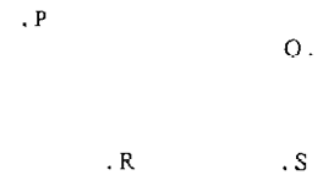
20- Aşağıdaki ifadelerden hangisi doğrudur?
Geometride,

A) Her terim tanımlanabilir ve her önermenin doğru olduğu kanıtlanabilir
B) Her terim tanımlanabilir ama bazı önermelerin doğru olduğunu varsaymak gerekir
C) Bazı terimler tanımsız kalmalıdır, ama bütün doğru önermelerin doğruluğu kanıtlanabilir
D) Bazı terimler tanımsız kalmalıdır ve doğru olduğu var sayılmış bazı önermelere gerek vardır
E) Yukarıdaki seçeneklerden hiçbiri doğru değildir.

21- Bir açıyı üçlemek demek onu üç eşit parçaya bölmek demektir. 1847 yılında, P.L. Wantzel bir açının yalnızca pergel ve işaretlenmemiş cetvel kullanarak üçlenemeyeceğini kanıtlamıştır. Bu kanıttan nasıl bir sonuca varabilirsiniz?

A) Açılar yalnızca pergel ve işaretlenmemiş cetvel kullanarak iki eş parçaya ayrılamazlar
B) Açılar yalnızca pergel ve işaretlenmiş cetvel kullanarak üçlenemezler
C) Açılar herhangi bir çizim aracı kullanarak üçlenemezler
D) Gelecekte birinin yalnız pergel ve işaretlenmiş cetvel kullanarak açıları üçlemesi mümkün olabilir
E) Hiç kimse açıları yalnızca pergel ve işaretlenmemiş cetvel kullanarak üçleyecek genel bir yöntem bulamayacaktır.

22- F geometrisinde, her şey alışık olduklarımızdan farklıdır. Burada sadece dört nokta ve 6 doğru vardır Her doğru iki nokta içerir. Eğer P, O, R ve S nokta ise, {P,O}, {P,R}, {P,S}, {O,R}, {O, S} ve {R, S} doğrulardır.



Kesişme ve paralel terimlerinin F- geometrisindeki kullanımı şöyledir: {P, O} ve {P,R} doğruları P' de kesişirler çünkü P {P, O} ve {P,R} ın ortak noktasıdır. {P, O} ve {R, S} doğruları paraleldir çünkü ortak hiçbir noktaları yoktur.
Buna göre, aşağıdakilerden hangisi doğrudur?

A) {P, R} ve {O, S} kesişirler
B) {P, R} ve {O, S} paraleldir
C) {O, R} ve {R, S} paraleldir
D) {P, S} ve {O, R} kesişirler
E) Yukarıdaki seçeneklerin hiçbiri doğru değildir

23- Ali adlı bir matematikçinin kendi tanımladığı geometriye göre, aşağıdaki önerme doğrudur. Bir üçgenin iç açılarının ölçüsü toplamı 180 dereceden azdır.
Buna göre aşağıdakilerden hangisi doğrudur?

A) Ali üçgenin açılarını ölçerken hata yapmıştır
B) Ali mantıksal bir hata yapmıştır
C) Ali doğru sözcüğünün anlamını bilmiyordur
D) Ali bilinen geometridekilerden farklı varsayımlarla başlamıştır
E) Yukarıdaki seçeneklerin hiçbiri doğru değildir

24- İki ayrı geometri kitabı 'dikdörtgen' sözcüğünü iki farklı şekillerde tanımlamıştır. Buna göre aşağıdakilerden hangisi doğrudur?

A) Kitaplardan birinde hata vardır
B) Tanımlardan biri yanlıştır, dikdörtgen için iki farklı tanım olamaz
C) Bir kitapta tanımlanan dikdörtgenin özellikleri diğer kitaptakinden farklı olmalıdır
D) Bir kitapta tanımlanan dikdörtgenin özellikleri diğer kitaptakiyle aynı olmalıdır
E) Kitaplarda tanımlanan dikdörtgenlerin farklı özellikleri olabilir.

25- Varsayalım aşagıdaki önerme I ve II yi kanıtladınız.
I. Eğer p ise q dir.
II. Eğer s ise q dir.
Buna göre önerme I ve II den aşağıdakilerden hangisi çıkartılabilir?

A) Eğer s ise, p değildir
B) Eğer p değil ise q değildir
C) Eğer p veya q ise s dir
D) Eğer p ise s dir
E) Eğer s değil ise p dir.

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