REWEIGHTED ROBUST DISPERSION ESTIMATION METHODS FOR UNIVARIATE S-CHARTS

by

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ABSTRACT

REWEIGHTED ROBUST DISPERSION ESTIMATION METHODS FOR UNIVARIATE S-CHARTS

Maintaining the quality of manufactured products at a desired level depends on the stability of process dispersion and location parameters and detection of perturbations in these parameters as promptly as possible. In the application of S-Charts, which are one of the most widely used techniques to monitor process variability in statistical process monitoring, sample standard deviation and sample mean are known to be the most efficient traditional estimators in determining process parameters, based on the assumption of independent and normally distributed datasets. In the cases of estimated process parameters from Phase I data clouded with outliers, efficiency of traditional estimators is significantly reduced, and performance of S-Charts are undesirably low. The aim of this thesis is to propose various robust estimators and reweighting procedures to increase the performance of S-Charts in Phase II monitoring. Three dispersion estimators: sample standard deviation, median absolute deviation and scale M-estimator, and three location estimators: sample mean, Harrell-Davis q^{th} quantile estimator and location M-estimator, are employed to directly construct the Phase II control limits of S-Charts, and also reweighted via different methods. Phase I efficiency of the proposed estimators and Phase II performance of S-Charts constructed from these estimators are assessed both under normality and against diffuse-localized and symmetric-asymmetric contaminations at different contamination density and magnitudes using 50,000-100,000 Monte Carlo simulations. As a result, scale M-estimator combined with Harrell-Davis 0.5^{th} quantile estimators yield parameter estimates with the highest efficiency, and reweighting at skipping level 2-4% using a common location estimate in individuals charts to screen outlier subgroups, and individual observations are found to improve the Phase II performance of the S-Charts.

ÖZET

TEK DEĞİŞKENLİ S-GRAFİKLERİ İÇİN YENİDEN AĞIRLIKLANDIRMALI SAĞLAM YAYILIM TAHMİN YÖNTEMLERİ

Elde edilen ürünlerin kalitesinin istenilen seviyede olması ve bu seviyenin korunması, üretim sürecinin konum ve yayılım parametrelerinin kararlılığına ve bu parametrelerdeki değişimlerin mümkün olabildiğince hızlı tespit edilmesine bağlıdır. Sürecin yayılımınındaki değişimleri denetleyebilmek için, istatistiksel süreç kontrolünün en yaygın araçlarından biri olan S kontrol grafikleri kullanılmaktadır. Örneklenmiş evrenlerin normal dağılıma uygunluğu varsayıldığında, örneklem ortalaması ve standart sapması, konum ve yayılım tahmin etmek için kullanılan en verimli istatistiklerdir. Öte yandan süreç parametrelerinin tahmin edildiği durumlarda ve örneklemelerde uçdeğer verilerin varlığında, geleneksel örneklem ortalaması önemli ölçüde değişmekte ve örneklem standart sapmasının ani yükselişi kalitesiz ürünlerin tespitini geciktirmektedir. Bu çalışmada, süreç yayılımını tahmin etmek için örneklem standart sapması, medyan mutlak sapma ve M-ölçek tahmin ediciler; ve sürecin konumunu tahmin etmek için ise örneklem ortalaması, Harrell-Davis ve M-konum tahmin ediciler kullanılmış, ve elde edilen istatistikler 2. Aşama kontrol limitlerinin oluşturulmasında doğrudan ve yeniden ağırlıklandırılarak kullanılmıştır. Önerilen tahmin edicilerin 1. Aşama verimlilikleri ve 2. Aşama güçleri, normal dağılımda ve dağılmış-bölgesel, simetrik-asimetrik bozulmalara karşı farklı bozulma miktar ve yoğunluklarında 50,000-100,000 mertebesinde sayılarında yapılan Monte Carlo benzetimleri ile kıyaslanmıştır. Sonuç olarak, %2-4 kırpma miktarı ile yeniden ağırlıklandırılan M-ölçek ve Harrell-Davis konum tahmin edicilerinin beraber kullanımı ve ortak konum parametre tahmini ile çizilen tekil gözlem grafikleri yardımıyla uç değerlerden ayıklanmış yayılım tahmini kullanılarak oluşturulan S kontrol grafiklerinin 2. Aşama performansının yükseldiği belirlenmiştir.

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LIST OF SYMBOLS

ARL	Average run length
$ARFS$	Average rate of false signal
ARS	Average rate of signal
ave	Sample average
b_n	Unbiasing coefficient of MAD_n
c	Tuning constant of ML estimator
C	Constant to set allowable distance from the central line
$cARL$	Conditional average run length
$cARL_{.1}$	$cARL$ for 10% quantiles of dispersion estimate distributions
$cARL_{.5}$	$cARL$ for 50% quantiles of dispersion estimate distributions
$cARL_{.9}$	$cARL$ for 90% quantiles of dispersion estimate distributions
cov	Covariance
c_4	Unbiasing coefficient of S_n
d	Contamination magnitude
d_2	Unbiasing coefficient of R
d_3	Coefficient in defining relationship between $\hat{\sigma}_R$ and $\hat{\sigma}$
D_3	Upper control limit coefficient of R-chart
D_4	Upper control limit coefficient of R-chart
F_i	Event that ${\hat{\sigma}_i}^{II}$ falls beyond CLs
G_i	i^{th} subgroup of Phase I sample
H_i	Logical value of screened outliers in individuals chart
HD_q	Harrell-Davis q^{th} quantile estimator
$HD_{.5}$	Harrell-Davis median estimator
${I_i}^L$	Logical value of screened samples due to location estimates
${I_i}^S$	Logical value of screened samples due to dispersion estimates
K	Contamination density
L_n	Lower control limit coefficient
${L_n}^I$	Lower control limit coefficient estimate

$L_n{}^*$	Known lower control limit coefficient
LCL^*	Lower control limit
m	Subgroup (sample) size
MAD_n	Median absolute deviation
$mARL$	Marginal (unconditional) average run length
$mARL_0$	Marginal average run length when process is in-control
ML	Location M-estimator
MS_n	Scale M-estimator
MSE	Mean square error
n	Number of individual observations in a subgroup
N_{run}	Number of Monte Carlo simulation
O^L	Outlier subgroups which have different location estimate
O^S	Outlier subgroups which have different dispersion estimate
p	Probability of giving false signal
R_i	Range of i^{th} sample
R_i	Individual outliers in i^{th} sample
RL	Run length
$\bar{S}$	Grand mean of standard deviations of subgroup
S_i	Standard deviation of i^{th} sample
S_n	Sample standard deviation
$SDRL$	Standard deviation of run length
U_{ij}	Logical value to indicate screened individual outliers
U_n	Upper control limit coefficient
$U_n{}^I$	Upper control limit coefficient estimate
$U_n{}^*$	Known upper control limit coefficient
UCL^*	Upper control limit
W_i	Weights of observations in HD_q
$\bar{X}_i$	Mean of i^{th} sample
Y_{ij}	Phase II data
Z_{ij}	Phase I data
$Z_k{}^*$	Pooled observations after screening outlier subgroup and individual outliers

α	Type I error rate
α_1	Skipping level of subgroup screening
α_2	Skipping level of after screening individual outliers
γ	Standard deviation of Phase I sample
Γ	Gamma function
Θ_i	Bias correction term
μ	Process mean
$\hat{\mu}$	Process mean estimate
$\hat{\mu}^I$	Process location estimate of Phase I sample
$\hat{\mu_i}^I$	Location estimate of i^{th} Phase I sample
μ_0	In-control process mean
σ	Process standard deviation
$\hat{\sigma}$	Process standard deviation estimate
$\hat{\sigma}_R$	Process standard deviation estimate using range
$\hat{\sigma}^I$	Process standard deviation estimate of Phase I sample
$\hat{\sigma_i}^I$	Sample standard deviation estimate of i^{th} Phase I sample
$\hat{\sigma_i}^{II}$	Sample standard deviation estimate of i^{th} Phase II sample
ψ	Psi function
ω	Measurement related to quality characteristics

LIST OF ACRONYMS/ABBREVIATIONS

ACM 1	Autoregressive process with normal data
ACM 2	Autoregressive process with diffuse variance contamination
ACM 3	Autoregressive process with diffuse mean contamination
AR	Autoregressive process
BDP	Breakdown point
CM 1	Diffuse symmetric variance contamination model
CM 2	Diffuse asymmetric variance contamination model
CM 3	Localized variance contamination model
CM 4	Diffuse mean contamination model
CM 5	Localized mean contamination model
CL	Control limit
IF	Influence function
i.i.d	Independently and identically distributed
LCL	Lower control limit
MC	Monte Carlo
SCM 1	Diffuse and localized variance contamination model
SCM 2	Diffuse and localized mean contamination model
SCM 3	Diffuse variance and mean contamination model
SCM 4	Localized variance and mean contamination model
SCM 5	Diffuse variance and localized mean contamination model
SCM 6	Localized variance and diffuse mean contamination model
SPC	Statistical process control
SPM	Statistical process monitoring
SQM	Statistical quality control
UCL	Upper control limit

1. INTRODUCTION

Competition in consumer market has affected manufacturers to move towards three major interrelated arenas: quality, cost, and responsiveness. The concepts of cost and responsiveness have been defined so as to encompass financial and non-financial aspects of the process requirements and management, respectively. The word of quality came into existence many years ago with different definitions. Fitness for the purpose, degree of customer satisfaction, accuracy in meeting the specification or design, meeting the standards or norms, and degree of excellence are some of these definitions [1]. Although its meaning changes from time to time, it is generally used to symbolize excellence of products or services referring to certain physical characteristics defined by performance, serviceability, durability, reliability, aesthetics, features, and/or conformance to standards of these products and services. First and foremost, customer needs must be recognized to define and manage quality [2]. While subjectivity and polysemy of quality makes the concept difficult to comprehend, quality should be attained and kept at a satisfactory level as much as possible, regardless of its definition [3].

Rendering consistent customer satisfaction through high quality products and services makes it possible to survive in market, therefore manufacturers need quality control techniques to monitor their processes and improve product quality by reducing process variability. A variety of tools are used in many disciplines, such as chemistry, medicine, environment [4], industry, economics [5, 6], informatics, even in traffic management [7], for employing quality control. Effective control techniques reduce scrap and rework, increase production capacity and decrease in production cost via keeping the process under control.

Statistical process monitoring (SPM) can be defined as the collection of tools and techniques to reduce variation of quality characteristics of target product by recognizing the causes behind variation, and bringing the process into a state of control. The most common and valuable tool in SPM is control charts, devised by Dr. Shewhart in 1920s [8]. A control chart is basically a graphical representation of the variation of

quality characteristics over time; extensive use and popularity of control charts are due to their effectiveness and simplicity. Different control charts, such as S charts, X-bar charts, EWMA and CUSUM charts, and methods are used by practitioners in the cases of univariate and multivariate processes to monitor the change in process mean and variance.

Efficiency of a process is related with its stability, commonly measured by its dispersion and location parameters, and prompt detection of changes in these parameters is crucial in SPM applications. In many cases, in-control process parameters would be unknown, therefore control charts are to be constructed using estimated control limits using Phase I data. In SPM, sample mean and standard deviation statistics are known to be the most efficient traditional estimators to determine the process location and dispersion parameters, respectively. The use of traditional estimators rely on the assumption of independent and normally distributed data sets. On the other hand, real-world process data may not be normally distributed in the presence of outliers, which contaminate samples. In such cases, efficiency of conventional estimators may be unacceptably dampened. Phase I efficiency of the proposed estimators are commonly evaluated using mean square error for both normal and non-normal processes. In the literature, outliers within in-control data are usually modeled via contaminated normal distributions; Phase I data are dominated by clean observations, which come from an in-control process, while contaminated by outliers. Both parameter estimation and existence of outliers in Phase I data deteriorate the performance of control charts, so it is an important task in SPM literature to propose estimators with high robustness and efficiency. Phase II performance of control charts constructed various estimators are generally evaluated using a large number of Monte Carlo simulations and metrics, such as average run length, conditional run length, average rate of signals.

This study aims to introduce novel robust approaches to construct univariate S-Charts, which have higher power compared to traditional dispersion estimation methods, and assess the Phase II performance of S-Charts based on the proposed estimators using computer simulations and a wide palette of statistical measures. The outline of the thesis is the following. In the current thesis, historical development, theoretical

concepts and practical applications of control charts are discussed in Chapter 2. Chapter 3 consists of an overall discussion of dispersion estimators, i.e. sample standard deviation, median absolute deviation and scale M-estimator, and location estimators, i.e. sample average, Harrell-Davis q^{th} quantile estimator and M-Huber location estimator. Step-by-step explanation of the proposed Phase I estimators and the process contamination models, utilized in the current study, are also given in same chapter. Efficiency and performance of S-Charts, application of the proposed methods on two real life examples, flow width measurements of hard bake process and syringe lengths, are in Chapter 4. Finally, conclusions and recommendations for future researchers are stated.

2. STATISTICAL PROCESS MONITORING

Quality of products and services plays an important role in decision making processes of different customer segments. Maintaining quality at the desired level, though may be challenging, is imperative in achieving high level of customer satisfaction, as well as, maximizing revenue and market share, rendering elimination of waste product for companies and prolongation of product loyalty for the customers. The second law of thermodynamics states that all systems move towards a state of greater entropy (or disorderliness). This phenomenon is observed also in manufacturing processes: operators may make a mistake, machines may break down, etc., when equipments are not controlled or regulated. Therefore, quality of a process is strongly affected by its stability and repeatability. Although it is impossible to prevent the inherent tendency towards disorderliness of processes, various techniques may reduce the process variability sufficiently. This is why quality control and improvement is an important concern to industries.

Reduction in variability of quality characteristics is achieved via online statistical process monitoring (SPM) techniques. SPM may be defined as the engineering tools and statistical techniques applied to measure and observe the quality characteristics of a process for monitoring and quality improvement. Process monitoring may be achieved via seven quality tools, which are histogram plots, pareto charts, cause and effect diagrams, defect-concentration diagrams, scatter diagrams, sampling inspections and check sheets, which yield limited discrete knowledge about the process rather than distinguishing the variation over a period (Figure 2.1a). Control charts, on the other hand, offer continuous information about the process, and provide an opportunity to apply systematic approaches to decrease the variability in quality characteristics (Figure 2.1b) [9].

The concept of quality control was introduced by Dr. Walter Andrew Shewhart, a physicist, during his service in Bell Telephone Laboratories in 1920s. He needed to decrease the frequency of failures and repairs of transmission systems, so he employed

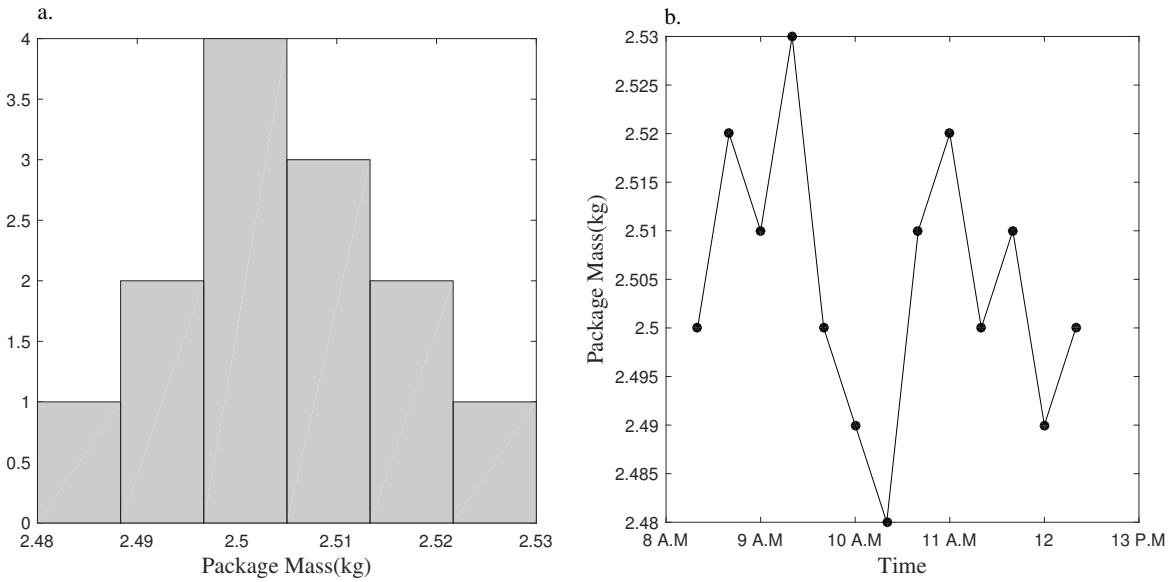

Figure 2.1. Representation of process variation via a) histogram b) control charts.

control charts. In the following years, quality control drew the attention of researchers and practitioners. Deming, who is another widely known proponent of statistical quality control (SQC), brought a new perspective to the application of control charts by emphasizing the responsibilities of the top management. He developed his complete philosophy of management and popularized the Deming PDCA cycle based on the idea to "plan the necessities", "do what is needed", "check whether it works or not", and "act to prevent abnormalities". Juran defined the quality as "fitness for use" to emphasize that achievement of desired quality cannot happen accidentally, it should be planned and controlled. He advocated a triology for quality management as planning, improvement and control. Usage of pareto charts to prioritize variables was his suggestion, as well. Ishikawa ideology, on the other hand, focused on the importance of total quality control of an organization, rather than just focusing on products and/or services. He urged managers to resist becoming content with merely improving a product's quality, insisting that quality improvement can always go one step further. He emphasized the importance of "internal customers"; customer would continue receiving service even after receiving the product. This service would extend across the company in all levels of management, and even beyond the company to everyday lives of those in-

volved. Importance of quality control, however, was not widely recognized particularly by the U.S. chemical industries. After World War II, due to necessity of improving the manufacturing efficiency, application areas of SQC were highly extended in chemical industries. Numerious studies demonstrated the positive relationship between quality control and manufacturing efficiency. In 1980s, an ingenious perception of Taguchi showed the importance of design of experiments in SQC. In the following years, further research was conducted in statistical techniques more efficiently. [8, 10, 11].

Aim of online application of control charts is not to decrease inherent variability in a process regardless of design and maintenance, but to detect unusual changes. Shewhart, Deming and Juran view the unusual changes as a result of local faults/special causes and sporadic problems, while they see inherent variation as a consequence of chance causes, system faults/common causes and chronic problems. Process is defined as set of causes and conditions, which come together in different combinations to transform inputs (chemical compounds, information, etc.) into outputs (chemical product, service, behaviour, etc.). If one samples products in equal time intervals, these measurements are subjected to variation which may stem from common and/or special causes. When variation is only caused by common causes, the process is stable and statistically in-control, since variability of common causes generates a fluctuation within an acceptable and predictable bandwidth. Common causes do not have a prominent effect on equilibrium point, because the underlying probability distributions of mean and standard deviation estimates of a process result remain stable over time [12]. Special causes, on the other hand, are the causes that stem from assignable sources and indicators of problems, which need to be solved. Special causes may be due to extraordinary conditions, incidents or lack of attention, or a new supplier for raw material, etc. A process, in which outputs are affected by both common and special causes, is called an unstable, or statistically out-of-control process. The magnitude of the variation and change in the quality characteristics are unpredictable. Consequently, special causes should be identified and removed for maintaining a stable process. Deming [13] illustrated the necessity of a stable process as follows.

- A stable process has an identity, and its performance is foreseeable; therefore, it allows rational planning.
- Maximum productivity and minimum costs are obtained in the existence of stability in a process.
- Effect of changes for a stable process can be identified in a more reliable way. In an unstable process, it is difficult to distinguish special causes from common causes. If the inherent variability of a process is not identified separately, it would be more challenging to know when a change results in improvement.

As importance of quality management has been accepted over years, application areas of control charts have been widened in manufacturing processes [14–16]. The earliest modifications of control charts, e.g. cumulative sum, acceptance control, and multivariate characteristic charts, were first published in the studies by Gibra [17], Vance [18] and Vardeman and Cornell [19], respectively. Saniga and Shirland [20] and Faltin et al. [21] discussed industrial applications of control charts in their studies. Incorporation of statistical process methods into industrial practice, relationship between hypothesis testing and control charting, and modelling of performance were discussed by Woodall, Montgomery and their co-workers [9, 12, 22–25].

2.1. Shewhart Control Charts

Typical control charts consist of central line, which is the expected value of the corresponding statistics representing the quality characteristics of the in-control process, and two horizontal lines, named upper control limit (UCL) and lower control limit (LCL). These control limits (CLs) represent the acceptance boundaries of natural variability of the in-control process. If the measured or computed characteristics falls outside these limits, then the process is deemed to be out-of-control, and one should try to define the source of the problem. To illustrate the connection of hypothesis testing and control charts, suppose that the central line in Figure 2.2 is the process mean $\mu = \mu_0$, and the current sample average ($\bar{x}_i$) is within the CLs, i.e. process mean is in-control. When $\bar{x}_i$ exceeds the CLs and gives false signal, μ is deemed to

be not equal to μ_0 anymore, i.e. process is out-of-control. Consequently, being within CLs is equivalent to failing to reject the null hypothesis, i.e. process is assumed to be in-control [9].

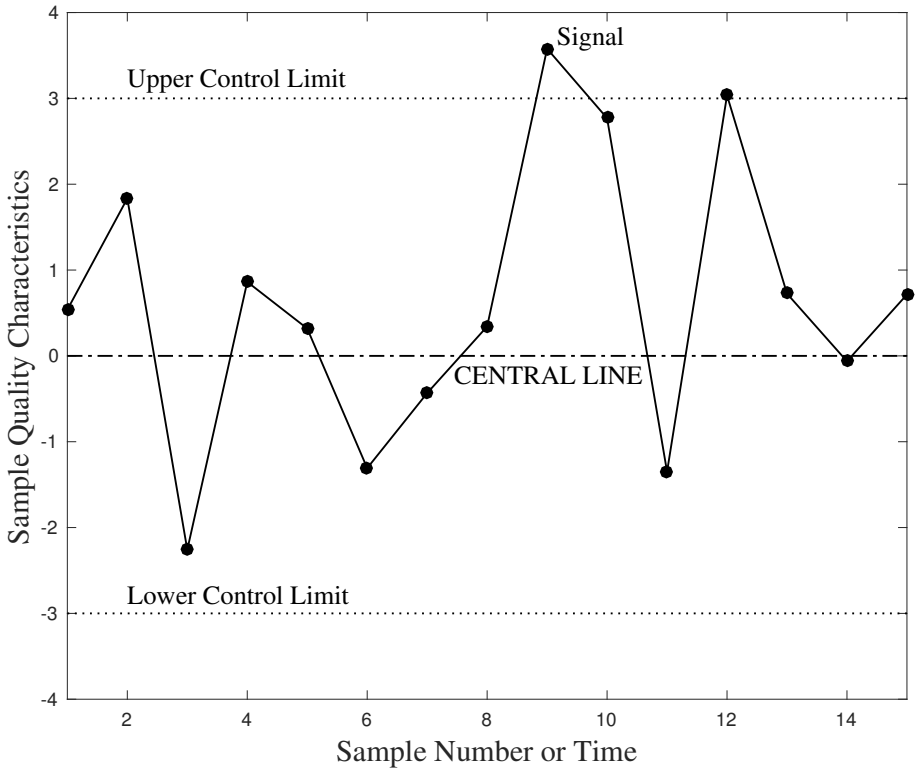

Figure 2.2. A typical Shewhart $\bar{X}$ control chart.

A process may be subjected to different causes which may perturb mean, or variability, or both parameters of the process. For example, process mean and variation may shift instantaneously, then remain stable at that level, which is called "sustained shift", or process parameters may shift abruptly for a short time, and turn back to the previous in-control value, or there may be a systematic or gradual increasing/decreasing change in the process mean or variation, or they may vary from time to time in erratic fashion. Usual hypothesis testing framework can deal especially with sustainable shifts, therefore the use of hypothesis testing in inference of control chart performance is more reasonable [8]. In the analysis of the performance of a control chart, hypothesis testing perspective moreover gives information about the probability of type I and type II errors, i.e. the false positive and false negative signals, respectively, for a control chart.

Control charts are classified with respect to the measurability of the quality characteristics. If quality characteristics can be expressed and measured quantitatively, such as temperature, length, concentration, etc., it is common practice to use variables control chart, in which estimates of location and dispersion parameters of the quality characteristics are plotted; otherwise, attributes control charts, in which the number of nonconformities, are used to make qualitative judgements, such as pass/fail, good/bad, yes/no, etc (Figure 2.3). Main purpose of both control charts is to make inferences from the sampled data about the performance of the process, or quality of a product. Variables control charts are widely used in industrial applications because quality characteristics of the processes are generally identified with numerical measurements.

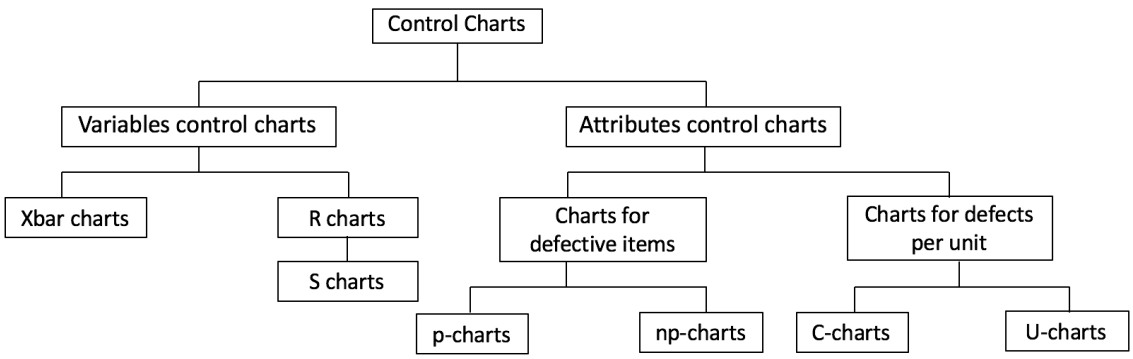

Figure 2.3. Types of control charts.

A prevailing technique used to monitor the quality of products is to take measurements generally as about 20-50 samples of 1-9 units, and analyze the process using control charts such as and $\bar{X}$ and S, or $\bar{X}$ and R charts. For a general model, let ω be a measurement related quality characteristics. Then central line, UCL and LCL become as follows:

$$UCL = \mu_\omega + C\sigma_\omega \tag{2.1}$$

$$CL = \mu_\omega \tag{2.2}$$

$$LCL = \mu_\omega - C\sigma_\omega \tag{2.3}$$

where C is the allowable distance from the central line. From this point on, the subscript ω will not be used, and μ and σ will represent the population parameters of the measurement related with the quality characteristic.

This approach may be applied to all $\bar{X}$, S/R control charts to check whether there is a significant shift in the level of process mean and variation, respectively. For a normally distributed process with mean μ and standard deviation σ, the CL, LCL and UCL of $\bar{X}$ chart may be shown as follows:

$$UCL = \mu + 3\sigma/\sqrt{n} \tag{2.4}$$

$$CL = \mu \tag{2.5}$$

$$LCL = \mu - 3\sigma/\sqrt{n} \tag{2.6}$$

here, n denotes the sample size. Sample averages of new measurements are computed and checked whether they fall within CLs, or not. Central line of S-Charts is taken to be equal to $c_4\sigma$, with a constant c_4, rather than σ itself, because S is a biased estimator of σ. Furthermore, standard deviation of S is equal to $\sigma\sqrt{1-{c_4}^2}$, so 3-sigma limits of S-chart are approximated as in Equation 2.7-9. In prospective analysis, sample standard deviation of new measurements are found, and checked whether they fall within these CLs, or not.

$$UCL = c_4\sigma + 3\sigma\sqrt{1-{c_4}^2} \tag{2.7}$$

$$CL = c_4\sigma \tag{2.8}$$

$$LCL = c_4\sigma - 3\sigma\sqrt{1-{c_4}^2} \tag{2.9}$$

here, a numerical example is given to illustrate how $\bar{X}$ and S-Charts are constructed. Hardness measurement from a manufacturing process of allow is assumed to represent the quality characteristics, and a sample consists of five hardness measurements ($n = 5$). Furthermore, mean and standard deviation of the hardness measurements are known to be equal to 45 and 0.1 mm, respectively. Standard deviation of the $\bar{X}$ is

determined as:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{0.1}{\sqrt{5}} = 0.0447 \tag{2.10}$$

Based on the assumption of normally distributed in-control process, $100(1-\alpha)\%$ of sample mean ($\bar{X}$) is expected to fall within $45 - z_{\alpha/2}(0.0447)$ and $45 + z_{\alpha/2}(0.0447)$ mm. It is common practice to use the constant C as 3, therefore $z_{\alpha/2}$ is customarily chosen as 3. Then, UCL and LCL become as:

$$UCL = 45 + 3(0.0447) = 45.134 \tag{2.11}$$

$$LCL = 45 - 3(0.0447) = 44.866 \tag{2.12}$$

If a future sample mean falls in (44.866, 45.134) interval, process is deemed to be in-control, otherwise process is assumed to be out-of-control. It should be noted that, 3-sigma approach is approximately equivalent to that $\alpha = 0.0027$; that is, the probability of exceeding CLs is equal to 0.0027, when process is in-control. After determining CLs, a hypothesis testing procedure is applied for each future measurement sequentially over time.

$$H_0 : \mu = 45 \tag{2.13}$$

$$H_1 : \mu \neq 45 \tag{2.14}$$

Similarly for S-Charts, the constant c_4 is found to be equal 0.94 (the values for different sample sizes are given in Table A.1), therefore 3-sigma control limits of S-chart determined as follows:

$$UCL = 0.94(0.1) + 3(0.1)\sqrt{1 - 0.94^2} = 0.196 \tag{2.15}$$

$$LCL = 0.94(0.1) - 3(0.1)\sqrt{1 - 0.94^2} \approx 0 \tag{2.16}$$

2.2. Two Phases of Data Collection: Phase I and Phase II

It is straightforward to apply control charts when process parameters are known, as discussed in the preceding section. However, when the process parameters are unknown, construction of control charts may be challenging. The parameters would be accurately determined if it were possible to collect a large volume of data during in-control operation [26], but it is difficult to take in-control measurements for a long period of time. Therefore, one may assume that parameters of most processes are practically unknown, and should be determined empirically. To solve this problem, process monitoring is predominantly handled in two consecutive phases: Phase I and Phase II. In Phase I, which is known as the retrospective phase, samples are collected in rational subgroups. Stability of the process should be validated prior to parameter estimation using Phase I samples. In Phase II, which is known as the prospective phase, control limits are determined using parameter estimates from Phase I, and hypothesis tests are continuously performed for new observations. Here, the aim is to detect occurance of a disturbance on the process promptly, using subgroup statistics and Phase II CLs. [24, 27]. Additional investigation may be applied to understand whether the alarm is false or not. In the presence of a valid alarm, changes in process operating conditions may be required to bring the process in-control [25].

Phase I control charts are used to check the process stability and ensure that collected samples represent the variability of the process: quality characteristics are decided, rational subgrouping strategy is determined, long-term/short-term stability is assessed, and nature of the process variation is understood. When the process is established as not stable, one should first stabilize the process by analyzing historical data and eliminating the variation stemming from special causes. Once the process is deemed to be in-control, a control chart is constructed based on Phase I data. If any sample among Phase I samples exceeds the CLs, then potential special causes are identified and eliminated, and revised CLs are determined. It should be noted that a more efficient Phase I sampling yields a more powerful Phase II monitoring [25].

2.3. Rational Subgrouping

Subgroups (samples) should be collected in a way to yield observations within subgroups being subjected to only common causes, and differing subgroups if any special cause exists. In other words, subgroups should be constructed so as to minimize variability that stems from common (natural) causes within subgroups and maximize the variability that stems from special (assignable) causes between subgroups. This procedure is called rational subgrouping. When rational subgrouping is employed conveniently, CLs will show the acceptable borders within which all common cause variability is included, and any abnormalities due to special causes will tend to give a signal [9, 10, 28]. Sample size and sampling frequency are important parameters in a rational sampling scheme, and these parameters should be determined carefully. Sample size is generally taken to be equal to, 4 to 8 [26, 29–31]. The sample size should be small enough to be economically appealing to be collected. On the other hand, subgroups should also be large enough for the sample mean to be normally distributed, particularly for non-normal distributed observations. Additionally, subgroups should have high sensitivity in prompt detection of special causes, and sensitivity is increased with larger sample size. It is stated that longer sampling periods may yield delays in detection of an out-of-control status of a process [32]. Furthermore, outliers in a process, which has small sample size, have a relatively higher effect on process monitoring than that in a process having larger sample size [32–34].

The homogeneity of rational subgroups is commonly guaranteed via two different methods. In the first one, measurements in a single subgroup are collected approximately at the same time, thus common cause variability can be included via consecutive sampling. The level of process mean and variation, however, may abruptly change for a short time and turn back to the in-control state between samples, and samples include outlier(s). In such a case, this method would not be feasible. The second approach states that the measurements should be collected randomly instead of simultaneous sampling, thus possibility of occurrence of abrubt shifts in level of process mean and variation stem from special causes can be minimized [9, 10].

In additional to sample size, sampling frequency should also be carefully considered. If a process has not been monitored using control charts and shows somewhat erratic behaviour, samples should be collected as frequently as possible to detect abnormalities and develop a strategy quickly. When process does not allow quality characteristic to be collected frequently, individual measurements could be grouped and sampling frequency may be decreased as the process shows less erratic behaviour. Another important point is to detect the events which may create special causes in a process: sampling interval may be readjusted with respect to to the probability of occurrence of special causes, i.e. adjustment in process equipments, change in operators or raw materials may result the need to increase sampling frequency. In all these considerations, cost of sampling should also be taken into account [10, 35].

Rational subgrouping is an essential concept in monitoring a process, and its importance is generally underestimated. In many applications, it is appealing to combine the data from different machines and different raw materials, which are assumed to be nearly identical and to use a single control chart to monitor these samples. The variations of stratification and mixing are problematic. In stratification of subgroups and mixing observations, each of operators, machines and raw materials, and so on, yields an observation in a subgroup. In this case, each sample represents different machines, raw materials, etc, equally. This unavoidably leads to an increase of variance within subgroups, and reduces the power of control charts [3, 10]. Therefore, sampling should be carefully designed to obtain useful information from the control charts [3, 9, 28, 35].

2.4. Construction of Shewhart Control Charts for Processes with Unknown Parameters

Construction of control limits and control charts for known process parameters has been discussed in Section 2.1. In real world, however, it is almost impossible to know the true parameters of a process, so process monitoring is handled in two consecutive phases, as discussed in Section 2.2. Process parameters (μ, σ) should be estimated reliably from Phase I sampling, and known process parameters are replaced

with their estimates $(\hat{\mu}, \hat{\sigma})$ for computing central line, UCL and LCL for $\bar{X}$ chart as:

$$UCL = \hat{\mu} + C\hat{\sigma} \tag{2.17}$$

$$CL = \hat{\mu} \tag{2.18}$$

$$LCL = \hat{\mu} - C\hat{\sigma} \tag{2.19}$$

and for S chart as

$$UCL = U_n\hat{\sigma} \tag{2.20}$$

$$CL = \hat{\sigma} \tag{2.21}$$

$$LCL = L_n\hat{\sigma} \tag{2.22}$$

Based on the assumption of normally distributed statistic, CLs of $\bar{X}$ charts are determined according to 3-sigma approach using the location estimate $(\hat{\mu})$ and dispersion estimate $(\hat{\sigma})$ from range or standard deviation. Let $\bar{X}_i$ be the mean of i^{th} sample, $\hat{\mu}$ can be determined by the grand mean of m available samples, and is equal to $\hat{\mu} = \bar{\bar{X}} = \frac{1}{m}\sum_{i=1}^{m}\bar{X}_i$. Dispersion parameter σ can be estimated using range or standard deviation of the samples. Taking R_i as the range of i^{th} sample, $\bar{R}$ is the average sample range and equals $\bar{R} = \frac{1}{m}\sum_{i=1}^{m}R_i$. Since the unbiased estimate of standard deviation is $\hat{\sigma} = \bar{R}/d_2 = \bar{S}/c_4$. Central line, UCL and LCL are computed as follows:

$$UCL = \bar{\bar{X}} + \frac{3}{d_2\sqrt{n}}\bar{R} \tag{2.23}$$

$$= \bar{\bar{X}} + \frac{3}{c_4\sqrt{n}}\bar{S} \tag{2.24}$$

$$CL = \bar{\bar{X}} \tag{2.25}$$

$$LCL = \bar{\bar{X}} - \frac{3}{d_2\sqrt{n}}\bar{R} \tag{2.26}$$

$$= \bar{\bar{X}} - \frac{3}{c_4\sqrt{n}}\bar{S} \tag{2.27}$$

Variations in process location and dispersion parameters are monitored simultaneously in real world applications of control charts. Standard deviation of sample range is proportional to the process standard deviation, and computed with the Equation 2.28.

$$\hat{\sigma}_R = d_3\hat{\sigma} = d_3\frac{\bar{R}}{d_2} \tag{2.28}$$

Hence, central line, UCL and LCL of R-chart are computed as follows:

$$UCL = \bar{R} + \frac{3d_3}{d_2}\bar{R} \tag{2.29}$$

$$CL = \bar{R} \tag{2.30}$$

$$LCL = \bar{R} - \frac{3d_3}{d_2}\bar{R} \tag{2.31}$$

If constants are defined as $D_3 = 1 - 3d_3/d_2$ and $D_4 = 1 + 3d_3/d_2$ (the constant D_3 and D_4 are tabulated for different sample sizes in Table A.1), then CLs of R-chart can be expressed as follows:

$$UCL = D_4\bar{R} \tag{2.32}$$

$$CL = \bar{R} \tag{2.33}$$

$$LCL = D_3\bar{R} \tag{2.34}$$

Instead of sample range, standard deviation can also be used as a dispersion estimator. Taking S_i to be equal to standard deviation of the i^{th} sample, the unbiased standard deviation estimate ($\hat{\sigma}$) is equal to $\hat{\sigma} = \bar{S}/c_4$, where $\bar{S}$ is the grand mean of standard deviations of subgroups, and equals $\bar{S} = \frac{1}{m}\sum_{i=1}^{m} S_i$. If constants are defined as $L_n = 1 - \sqrt{1 - {c_4}^2}/c_4$ and $U_n = 1 - \sqrt{1 + {c_4}^2}/c_4$, the central line and control limits

of S-chart can be now expressed as follows:

$$UCL = \bar{S} + 3\frac{\bar{S}}{c_4}\sqrt{1 - {c_4}^2} \tag{2.35}$$

$$= U_n \bar{S} \tag{2.36}$$

$$CL = \bar{S} \tag{2.37}$$

$$LCL = \bar{S} - 3\frac{\bar{S}}{c_4}\sqrt{1 - {c_4}^2} \tag{2.38}$$

$$= L_n \bar{S} \tag{2.39}$$

Two different approaches, namely 3-sigma limits and probability limits, are used in practice to determine the constants C, U_n and L_n. Although 3-sigma approach ($C = 3$) is commonly used in industrial applications, the probability limits approach is preferred particularly. In probability limits approach, control limit coefficients are adjusted to obtain a desired type I error probability. Taking $\hat{\sigma}^I = \bar{S}/c_4(n)$ as the standard deviation estimate of the process using Phase I samples, and ${\hat{\sigma}_i}^{II}$ is the sample standard deviation of the i^{th} Phase II sample, F_i is defined as the event that ${\hat{\sigma}_i}^{II}$ falls beyond the control limits, and $P(F_i) = p$ is the probability of occurrance of F_i (false alarm probability). U_n and L_n are adjusted to yield a type I error rate α, generally taken to be equal to 0.0027.

$$P(F_i|\hat{\sigma}^I) = P(L_n\hat{\sigma}^I \leq {\hat{\sigma}_i}^{II} \leq \ U_n\hat{\sigma}^I) = 1 - \alpha \tag{2.40}$$

2.5. Performance Measures of Control Charts

Suppose that Z_{ij} (i=1,2,...,m and j=1,2,...,n) represents the m subgroups of n individual measurements, assumed to be sampled from a stable in-control process with $N(\mu_0,{\sigma_0}^2)$ distribution. Phase II data, on the other hand, consist of an undefined number of subgroups with the same sample size (n), denoted by Y_{ij}, (i=1,2,... and j=1,2,...,n). Y_{ij} is similarly assumed to be independent, but $N(\mu_0,(\gamma\sigma_0)^2)$ distributed,

where γ is a constant. When γ=1, process is in-control; otherwise it is exposed to disturbances.

Run length (RL) is defined as the number of successive subgroups, which have not fallen beyond the limits until the first signal. If the plotted statistics are independent and identically distributed random variables, the events F_i are independent and thus, RL has a geometric distribution with parameter $p = P(F_i) = \alpha$. The performance of control charts are evaluated using the average RL (ARL), and the standard deviation of RL ($SDRL$). When the process parameters are known, ARL is given by $1/p$ and $SDRL$ is by $\sqrt{1-p}/p$.

$$ARL = E(RL) = \frac{1}{P(F_i)} = \frac{1}{p} = \frac{1}{\alpha} \tag{2.41}$$

In the case of unknown process parameters, control limits are determined using Equation 2.35-2.40. For a given $\hat{\sigma}^I$, the distribution of RL is geometric with parameter $P(F_i|\hat{\sigma}^I)$ [36, 37]. Consequently, the conditional ARL ($cARL$) is given by

$$cARL = E(RL|\hat{\sigma}^I) = \frac{1}{P(F_i|\hat{\sigma}^I)} \tag{2.42}$$

Contrary to conditional RL distribution, unconditional RL distribution is not geometric with parameter $P(F_i) = p$, since events F_i are not independent [38, 39]; they will be independent only when conditioned on a specific value of $\hat{\sigma}^I$. Consequently, the marginal (unconditional) $mARL$ may be obtained by averaging a large number of $cARL$ over all possible values of ${\hat{\sigma}_i}^I$.

$$mARL = E\{E(RL|\hat{\sigma}^I)\} = E\left\{\frac{1}{P(F_i|\hat{\sigma}^I)}\right\} \tag{2.43}$$

and $SDRL$ is given by [40]

$$SDRL = \sqrt{var(RL)} = \sqrt{E(var(RL|\hat{\sigma}^I) + var(E(RL|\hat{\sigma}^I)))} \tag{2.44}$$

$$= \sqrt{2E\Big(\frac{1}{p(F_i|\hat{\sigma}^I)}\Big)^2 - \Big(E\frac{1}{p(F_i|\hat{\sigma}^I)}\Big)^2 - E\Big(\frac{1}{p(F_i|\hat{\sigma}^I)}\Big)} \tag{2.45}$$

Although $mARL$ is an accepted performance measure of control charts due to its simple interpretation for researchers, it is peculiar for practitioners since it could be realized only by extremely long runs [33, 37]. It is suggested to use a more intuitive metric, which is called average rate of false signal ($ARFS$), since control charts are constructed to be used for a certain period of time. In order to clarify the difference between these performance measures, the nature of RL distribution is elaborated as follows. $SDRL$ exceeds ARL, when the parameters are unknown [25, 26, 32, 37, 39, 41–45], i.e. large number of quite short runs are balanced by a very few number of long runs. If $ARFS$ is taken to be equal to 0.0027 for determining CLs, then all these runs will be equally weighted, whereas short runs have higher importance (weight) when $mARL$ is taken to be ≈ 370 to determine CLs. This is why $ARFS$ is higher than 0.0027, when CLs are determined with respect to intended $mARL$ value [32, 33].

2.6. Robust Statistics

Statistical inferences and estimation methods are based on samples from population, and certain assumptions are required for their underlying distributions. i.e. independently and identically distributed (i.i.d), or normally distributed observations. There is no guarantee that these assumptions will be actually realized, however their use may be justified with the help of continuity and stability principle: a minor error in the model is likely to yield small errors in conclusions. Unfortunately, this is not always true; sensitivity of traditional methods to minor deviations from assumed models sometimes necessitiates use of alternative robust procedures [43, 46]. There is a subtle difference between robustness and outlier resistance. The term outlier resistant is used to imply that appropriate statistics is minimally affected in the existence of

outliers, e.g. median is one of the commonly used outlier resistant estimators. Robustness, on the other hand, indicates insensitivity of a procedure despite the deviations in underlying assumptions [46, 47].

Concept of robustness may be clarified emphasizing its difference from non-parametric methods. In non-parametric methods, no assumption is made about the underlying joint distribution. Robustness, on the other hand, is related with the strength against the deviations from the assumptions [46]. Models are based on assumptions, which hold for the majority of observations, however a fraction of observations may have a different pattern than the majority, or may have no pattern. These observations are called as outliers [48]. Deviations may be handled via nonparametric methods, but it is possible to use parametrized set of underlying distribution for the majority of the observations and arbitrary distribution for the outliers via robust statistics [49].

In the presence of outliers, traditional methods are not efficient in determining process parameters, due to increase in bias and variance, therefore outlier resistant statistics are employed to remove outliers in Phase I sampling before estimating CLs [9, 43, 50]. Howell proposed to use midrange to estimate location, and range to estimate dispersion, and determine the Phase II control limits, accordingly [51]. Ferrell proposed to use median of median [52], while Howell proposed to use mean of median as the measurement of central line. Moreover, Clifford proposed to bare CLs are based on the median of subgroup range [53]. Langenberg and Iglewicz suggested trimmed mean and trimmed range to compute central line and Phase II CLs [54]. Tatum showed that efficiency of reweighted estimators is higher than that of traditional methods in process parameter estimation against diffuse and localized disturbances, in which outliers are equally likely to affect the observations in all subgroup, and are allowed only to effect specific subgroups, respectively [39]. Schoonhoven, Does and Riaz, furthermore, improved the performance of control charts with the use of adaptive skipping and screening of individual observations using outlier resistant estimators [37, 40, 44, 45]. Although, these methods are relatively more resistant against outliers, the problem of skewed process distribution has not been thoroughly studied. Robust control charts using median [55, 56], sample range [57] and median absolute

deviations from the median [34, 58] also showed higher efficiency and higher power, mainly due to insensitivity to deviations from normality assumption rather than traditional estimators. Robust estimators are found to be more efficient both against diffuse-localized and symmetric-asymmetric contaminations, and have higher power in detecting disturbances, compared to non-reweighted and reweighted traditional estimators [26, 43, 50, 55–57, 59–63].

3. METHODOLOGY

In SPM, identical statistics (generally sample mean and sample standard deviation/range) are generally used to estimate the dispersion parameter in both Phase I (σ^I) and Phase II (σ^{II}). In the current study, shifts in level of process mean and variation (outliers) are assumed to contaminate only Phase I samples, so Phase I estimators should be robust against contaminations and efficient under normal situations, while Phase II estimator should be sensitive to process disturbances. Hence, robust estimator and/or reweighting procedures are better suited to be employed in determining process parameters using Phase I data [24, 37]. Robustness of an estimator can be measured by the local stability assessed via influence function (IF), and global reliability via breakdown point (BDP). Asymptotic BDP measures the fraction of contamination, which is capable to make the estimate useless [64], while IF measures effect of a single observation, varying from $-\infty$ to $+\infty$, on the estimate [65]. A robust estimator is expected to have a bounded and small IF and high BDP to resist against outliers in a dataset [64, 66].

Robustness properties of three dispersion estimators: sample standard deviation (S_n), median absolute deviation (MAD_n) and scale M-estimator (MS_n), and three location estimators: sample mean (ave), Harrell-Davis q^{th} quantile estimator (HD_q) and location M-estimator (ML) are investigated in Section 3.1. Reweighting procedures employed on these estimators; the algorithms of different reweighting procedures to construct CLs of S-Charts, used to monitor Phase II data are discussed in Section 3.2. Robustness of the non-reweighted and reweighted estimators are elucidated against various Phase I contaminations in Section 3.3.

3.1. Proposed Phase I Estimators

In the following discussion, it is assumed that Z_{ij} ($i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$) denotes the j^{th} individual observation of the i^{th} subgroup (sample), while G_i ($i = 1, 2, \ldots, m$) represents the i^{th} subgroup comprising a set of n individual ob-

servations in Phase I sampling. In the design of S control charts for an in-control process, standard deviation estimates of rational subgroups ($\hat{\sigma_i}^I$) are determined using dispersion estimators, then process standard deviation is estimated ($\hat{\sigma}^I$) via location estimation of $\hat{\sigma_i}^I$s of m subgroups. Although process location estimation is not needed in the simple application of S control charts, employing individuals charts for reweighting procedure requires determining location parameters of rational subgroups ($\hat{\mu_i}^I$) and the process location estimate ($\hat{\mu}^I$) (Figure 3.1).

$$Z_{ij} = \begin{array}{llll} Z_{11} & Z_{21} \dots Z_{n1} & = G_1 & \Rightarrow \hat{\sigma_1}^I, \hat{\mu_1}^I \\ Z_{12} & Z_{22} \dots Z_{n2} & = G_2 & \Rightarrow \hat{\sigma_2}^I, \hat{\mu_2}^I \\ \vdots & & & \\ Z_{1m} & Z_{2m} \dots Z_{nm} & = G_m & \Rightarrow \hat{\sigma_m}^I, \hat{\mu_m}^I \end{array} \Rightarrow \hat{\sigma}^I \text{ and } \hat{\mu}^I$$

Figure 3.1. Steps in computing $\hat{\sigma}^I$ and $\hat{\mu}^I$.

3.1.1. Dispersion Estimators

Sample standard deviation S_n is known to be most efficient traditional dispersion estimator for normally distributed datasets.

$$S_n(G_i) = \sqrt{\frac{1}{n-1} \sum_{j=1} (Z_{ij} - ave_{j=1}^n Z_{ij})^2} \tag{3.1}$$

here, $ave_{j=1}^n Z_{ij}$ represents the sample mean of i^{th} subgroup and n denotes the number of observations in a subgroup. It should be noted that S_n is biased estimator of σ^I, thus a correction factor is applied ($\sigma = E(S)/c_4(n)$) [9], that is $\hat{\sigma_i}^I = S_n(G_i)/c_4(n)$, where $c_4(n)$ is a constant (Table A.1). Besides having 0% BDP, S_n has unbounded IF, i.e. when a single observation is replaced with an arbitrary number, dispersion estimate is affected. The next estimator is a well known robust estimator, median absolute deviation [67]:

$$MAD_n(G_i) = MAD_{j=1}^n Z_{ij} = \frac{1}{\phi^{-1}(3/4)} med_{j=1}^n |Z_{ij} - med_{j=1}^n Z_{ij}| \tag{3.2}$$

here, $med_{j=1}^{n} Z_{ij}$ represents the median of i^{th} subgroup, and ϕ is the cumulative normal distribution function. In case of normality, $\phi^{-1}(3/4) = 1.4826$ [46]. Since MAD_n is a biased estimator of σ^I, a correction factor is required to obtain an unbiased dispersion parameter estimate, $E(MAD_n(G_i)) = \hat{\sigma_i}^I / b_n$, where b_n is a constant (computed via simulations). Although MAD_n has bounded IF and 50% BDP, having low asymptotic efficiency (37%) and discontinuities in IF make the estimator less favourable especially in small samples [66]. The next robust dispersion estimator is scale M-estimator with auxiliary location. MS_n is a maximum likelihood-like estimator, which minimizes $\sum_{i=1}^{n} \rho(x_i, \theta)$, where θ is the corresponding statistics. Having bounded and smooth IF, and high BDP ($= \kappa/\rho(\infty)$), MS_n is minimized using the following constraint:

$$\frac{1}{n} \sum_{j=1}^{n} \rho\Big(\frac{Z_{ij} - \hat{\mu}_i}{MS_n(G_i)}\Big) = \kappa \tag{3.3}$$

$$\kappa = E_\phi\{\rho\} \tag{3.4}$$

here, $med_{j=1}^{n} Z_{ij}$ and MAD_n estimates are used for $\hat{\mu}_i$ and initialization of $MS_n(G_i)$, respectively. The fully iterated M-estimator with logistic ϕ function (ρ function is shown below), which yields $\kappa = 0.5$ and $\rho(\infty) = 1$, is used in this study, to give a BDP of 50% [66].

$$\rho(x) = \Big(\frac{2}{1 + e^{\frac{-x}{0.3739}}} - 1\Big)^2 \tag{3.5}$$

$MS_n(G_i)$ is a biased estimate of σ^I, and its correction factor is determined via computer simulations.

After subgroup standard deviation estimates ($\hat{\sigma_i}^I$) are determined via using dispersion estimators discussed above, location estimators discussed in following subsection are used to determine the process dispersion estimate ($\hat{\sigma}^I$). Location estimators are additionally used to determine location estimates of rational subgroups and the process.

3.1.2. Location Estimators

Sample mean (ave), which is the average value of observations, is the most widely used location estimator. Process dispersion and location estimates are calculated using Equation 3.6 and 3.7, respectively:

$$\hat{\sigma}^I = ave(\hat{\sigma_i}^I) = \frac{1}{m}\sum_{i=1}^{m}\hat{\sigma_i}^I \tag{3.6}$$

$$\hat{\mu}^I = ave(\hat{\mu_i}^I) = \frac{1}{m}\sum_{i=1}^{m}\left(\frac{1}{n}\sum_{j=1}^{n}Z_{ij}\right) \tag{3.7}$$

Similar to S_n, ave is sensitive to outliers in a dataset, and has asymptotic BDP of 0% (its finite sample BDP is $1/k$), and unbounded IF. The next estimator is distribution-free Harrell-Davis quantile estimator (HD_q), which uses the weighted average of all order statistics [68, 69]:

$$\hat{\sigma}^I = HD_q(\hat{\sigma_i}^I) = \sum_{i=1}^{m}W_i\hat{\sigma_i}^I \tag{3.8}$$

$$\hat{\mu}^I = HD_q(\hat{\mu_i}^I) = \sum_{i=1}^{m}W_i\hat{\mu_i}^I \tag{3.9}$$

where weights (W_i , $i = 1, 2, \ldots, m$) are obtained via computing $P(g)$ as:

$$W_i = P(\frac{i-1}{m} \leq G_i \leq \frac{i}{m}) \tag{3.10}$$

here, probability density function G_i, a beta random variable with parameters $\alpha = (m+1)q$ and $\beta = (m+1)(1-q)$, where q is the quantile ($q = 0.5$ is used in this study). Weights (W_i , $i = 1, 2, \ldots, m$) are obtained via computing $P(g)$ as

$$P(g) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)+\Gamma(\beta)}g^{\alpha-1}(1-g)^{\beta-1} \tag{3.11}$$

where Γ represents gamma function. Using all observations in a dataset rather than the order statistics makes HD_q highly efficient, however its efficiency is lowered in the existence of relatively extreme quantiles. It was recommended to be used in the estimation of sample median due to its high asymptotic efficiency, when sample size is small [70], therefore $HD_{.5}$ is used in this study.

Another robust location estimator is location M-estimator (ML), which has been first proposed by Huber [46]. Huber ψ function is used instead of logistic ψ function due to its superiority against asymmetrically located (one-sided) outliers. Denoting X_i as the representation of ${\hat{\sigma}_i}^I$ $(i = 1, 2, \ldots, m)$, solution of the equation yields the unbiased ML estimate of $\hat{\sigma}^I$

$$\sum_{i=1}^{m} \psi\Big(\frac{X_i - ML}{\hat{\sigma}_X}\Big) = 0 \tag{3.12}$$

here, initial estimate of ML $(= \hat{\sigma}^I)$ is obtained using $med_{j=1}^{n} Z_{ij}$, and $\hat{\sigma}_X$ is replaced by MAD_n estimator for $\hat{\sigma}_X$. ML estimator has BDP of 50%, and bounded and linear IF with $\psi(x) = min(c, max(x - c))$ and the tuning constant $c = 1.5$.

In order to obtain process location parameter, ${\hat{\mu}_i}^I$ and $\hat{\mu}^I$ are determined. Assuming that Y_i denotes ${\hat{\mu}_i}^I$ $(i = 1, 2, \ldots, m)$, which is estimated using the following equation, then $\hat{\mu}^I$ is determined using the following equations respectively:

$$\sum_{j=1}^{n} \psi\Big(\frac{Z_{ij} - \hat{\mu}_{ij}^I}{\hat{\sigma}_i^I}\Big) = 0 \tag{3.13}$$

$$\sum_{i=1}^{m} \psi\Big(\frac{Y_i - \hat{\mu}_i}{\hat{\sigma}_Y}\Big) = 0 \tag{3.14}$$

3.2. Algorithms of Phase I Estimators

Traditional estimators, S_n, ave, and robust estimators MAD_n, MS_n, and $HD_{.5}$, ML are employed both directly, and in a reweighted manner using skipping/screening

procedures to screen outliers in Phase I sampling to determine final dispersion estimates. Although asymptotic properties of reweighted estimators were found to be questionable, finite sample properties of reweighted estimators were found to be desirable [71]. In reweighted estimators, subgroups may be screened to eliminate localized variance contaminations at a desired type I probability ${\alpha_1}^I$ using $P(F_i) = P({L_n}^I\hat{\sigma}^I \leq \hat{\sigma_i}^I \leq {U_n}^I\hat{\sigma}^I) = 1 - \alpha_1$, and in some of methods, to eliminate localized mean contaminations with desired type I probability (α_2) using $P(F_i) = P(\hat{\mu}^I - {C_n}^I\hat{\sigma}^I/\sqrt{n} \leq \hat{\mu_i}^I \leq \hat{\mu}^I + {C_n}^I\hat{\sigma}^I/\sqrt{n}) = 1 - \alpha_2$. The main purpose here is to detect the subgroups which are subjected to localized contaminations. Furthermore, individuals charts may be constructed to eliminate diffuse variance and mean contaminations with the criteria $P(F_i) = P(\hat{\mu}^I - {C_n}^I\hat{\sigma}^I \leq Z_{ij} \leq \hat{\mu}^I + {C_n}^I\hat{\sigma}^I) = 1 - \alpha_2$. All upper (${U_n}^I$) and lower (${L_n}^I$) Phase I CLs, ${C_n}^I$ coefficients, and required bias correction terms Θ_i ($i = 1, 2, \ldots, 5$) are determined via 100,000 computer simulations. The coefficients and multipliers for $m = 50$ are shown below, while those for $m = 20$ can be found in Table A.2.

Five main reweighting methods, shown with Roman numerals, are studied at different trimming amounts. These methods are denoted with upper case letters to represent the subtle differences between sub-methods, and with numbers to represent different skipping levels. Traditional estimators are used in Method I.A, while reweighting with same estimators is employed in Method I.B. Skipping level increases from Method I.B1 to I.B4. Robust estimators and their reweighted versions are used in Method II.A and II.B, respectively. Reweighting procedure is extended in Methods III-V, and individuals charts are constructed to screen individual outlier observations in addition to outlier subgroups. These three methods differ among themselves in application of individuals charts. In Method III, each subgroup location parameter ${\hat{\mu}_i}^I$ is estimated and placed as the central line of individuals charts, separately. In Method IV.A, on the other hand, a single robust location estimate of the process $\hat{\mu}^I$ is used, and a single individuals chart is employed for all subgroups. In Method IV.B, process location estimates are reweighted and a robustified $\hat{\mu}^I$ is used. In Method V, after screening subgroups via robust dispersion and location estimates, individual observations are pooled without their subgroup information and robust $\hat{\sigma}^I$ and $\hat{\mu}^I$ are used in the individuals charts.

In the following subsections, detailed algorithms of five main methods are discussed in detail to explain underlying mechanisms of the reweighting procedures. Here, ${\alpha_1}^I$ and ${\alpha_2}^I$ represent the skipping level when subgroups and (additionally) individual observations are screened, respectively, while α^I shows the total skipping level.

3.2.1. Method I

Traditional estimators, and common procedure of estimation methods are applied in Method I.A (Figure 3.2). First, subgroup standard deviations ${\hat{\sigma}_i}^I$ are estimated from S_n using Equation 3.1, then ${\hat{\sigma}_i}^I$s are multiplied with bias correction factor Θ_1; since n equals 5, $\Theta_1 = 1/c_4(n) = 1.064$. Process dispersion parameter estimate $\hat{\sigma}^I$ is obtained by taking the average of dispersion estimates using Equation 3.6. Bias correction is not needed ($\Theta_2 = 1$), since *ave* is an unbiased estimator of location.

i. Compute $\mathbf{S_n(G_i)}$ as defined in Equation 3.1,
ii. Employ bias correction: $\hat{\sigma}_\mathbf{i}^\mathbf{I} = \mathbf{\Theta_1} \times \mathbf{S_n(G_i)}$,
iii. Compute $\hat{\sigma}^\mathbf{I}$ as defined in Equation 3.6.

Figure 3.2. Algorithm of Method I.A.

In Method I.B, S_n and *ave* are used in an adaptive skipping scheme (Figure 3.3). Similar to that performed in Method I.A, ${\hat{\sigma}_i}^I$s are estimated using S_n, and multiplied with Θ_1 to obtain unbiased standard deviation estimates of rational subgroups. The difference from Method I.A lies in the next step; outlier subgroups are eliminated until all the remaining subgroups fall within the computed Phase I CLs. ${U_n}^I$ and ${L_n}^I$ are determined with respect to desired skipping level α_1, which is taken to be equal to 0.0027, 0.010, 0.025, 0.050. used in Method I.B1-B4, respectively. As seen in Figure 3.3, outlier subgroups set O^S is initialized as a null set, then subgroups are added to O^S if they exceed ${L_n}^I\hat{\sigma}^I$, or ${U_n}^I\hat{\sigma}^I$. In each iteration, new $\hat{\sigma}^I$, ${L_n}^I$ and ${U_n}^I$ are computed, and the average of the remaining subgroups is multiplied with Θ_3 to obtain

an unbiased $\hat{\sigma}^I$. The Phase I CLs (${L_n}^I$ and ${U_n}^I$), and bias correction terms (Θ_1, Θ_2 and Θ_3) for $m = 50$ are shown in Table 3.1. While Θ_2 and Θ_3 are available in literature, other parameters are determined via computer simulations. As skipping level increases, higher ${L_n}^I$ and lower ${U_n}^I$ are employed to render tighter CLs.

i. Employ Method I.A. Set $\mathbf{O^S} = \{\}$,

ii. Compute the following for $\mathbf{i^{th}}$ subgroup, which satisfy ($i \in M$ to $i \notin O^S$), using $\alpha_\mathbf{1}$ =(0.0027, 0.010, 0.025, 0.050),

$$I_i{}^S = \begin{cases} 1 & \text{if } {L_n}^I \hat{\sigma}^I \leq \hat{\sigma_i}^I \leq {U_n}^I \hat{\sigma}^I \\ 0 & \text{if otherwise} \end{cases}$$

$$O_{new} \leftarrow O^S \cup \{i | I_i{}^S = 0\}$$

$$\hat{\sigma}^I = \frac{1}{m - |O_{new}|} \sum_{i=1}^{m} \hat{\sigma_i}^I I_i{}^S$$

if $O_{new} \neq O^S$ **then**

$O^S = O_{new}$ and go to Step **ii**,

else

Continue.

end if

iii. Employ bias correction as $\hat{\sigma}^\mathbf{I} = \mathbf{\Theta^{III}} \times \hat{\sigma}^\mathbf{I}$.

Figure 3.3. Algorithm of Method I.B.

Table 3.1. Phase I CLs and bias correction factors of Method I for $m = 50$.

Method	Θ_1	Θ_2	${L_n}^I$	${U_n}^I$	Θ_3
I.A	1.064	1.000	-	-	-
I.B1	1.064	1.000	0.175	2.236	1.001
I.B2	1.064	1.000	0.244	2.048	1.002
I.B3	1.064	1.000	0.309	1.907	1.003
I.B4	1.064	1.000	0.371	1.789	1.004

3.2.2. Method II

Instead of traditional estimators, robust estimators are used without reweighting in Method II.A. Three different combinations of dispersion and location estimators are employed, which are $MS_n + ML$, $MS_n + HD_{.5}$ and $MAD_n + HD_{.5}$ in Method II.A1-II.A3, respectively (Figure 3.4). Biased ${\hat{\sigma}_i}^I$s are calculated using MS_n in Method II.A1 and II.A2 and using MAD_n in Method II.A3. The unbiased ${\sigma_i}^I$s are estimated via multiplying with bias correction term Θ_1. Unbiased process dispersion estimate $\hat{\sigma}^I$ is further determined by calculating the location of ${\hat{\sigma}_i}^I$s using ML for Method II.A1, and using $HD_{.5}$ for Method II.A2 and II.A3, followed by multiplication with bias correction term Θ_2.

Use one of the estimators below, and compute $\hat{\sigma}_\mathbf{i}$ **and** $\hat{\sigma}^\mathbf{I}$ using

i. Eq. 3.3-3.5 **and** Eq. 3.12 for Method II.A1,

ii. Eq. 3.3-3.5 **and** Eq. 3.8, 3.10-3.11 for Method II.A2,

iii. Eq. 3.2 **and** Eq. 3.8, 3.10-3.11 for Method II.A3.

iv. Employ bias correction: $\hat{\sigma}_\mathbf{i} = \mathbf{\Theta^I}\hat{\sigma}_\mathbf{i}$ **and** $\hat{\sigma}^\mathbf{I} = \mathbf{\Theta^{II}}\hat{\sigma}^\mathbf{I}$.

Figure 3.4. Algorithm of Method II.A.

In Method II.B, one-step reweighting is performed on the same combinations of estimators employed in Method II.A1-II.A3, respectively (Figure 3.5). The main difference in reweighting method employed in Method II.B from one in Method I.B is that skipping is employed only once (one-step skipping) rather than adaptive skipping. Additionally, skipping level α_1 is constant at 0.025 for Method II.B1-3, while it varies in Method I.B. First, Method II.A procedure is performed, then outlier subgroups O^S, which fall beyond the ${L_n}^I\hat{\sigma}$ and ${U_n}^I\hat{\sigma}$, are determined and eliminated. Averaging the remaining subgroup dispersion estimates and bias correction yield $\hat{\sigma}^I$.

i. Use Method II.A1, II.A2, or II.A3, corresponding to Methods II.B1, II.B2, or II.B3, respectively,

ii. Set $\mathbf{O^S} = \{\}$,

iii. Compute the following for $\mathbf{i^{th}}$ subgroup, using $\alpha_1 = 0.025$,

$$I_i{}^S = \begin{cases} 1 & \text{if } L_n{}^I \hat{\sigma}^I \leq \hat{\sigma}_i{}^I \leq U_n{}^I \hat{\sigma}^I \\ 0 & \text{if otherwise} \end{cases}$$

$$O^S = \{i | I_i{}^S = 0\}$$

$$\hat{\sigma}^I = \frac{1}{m - |O^S|} \sum_{i=1}^{m} \hat{\sigma}_i{}^I I_i{}^S$$

iv. Employ bias correction as $\hat{\sigma}^{\mathbf{I}} = \mathbf{\Theta^{III}} \hat{\sigma}^{\mathbf{I}}$.

Figure 3.5. Algorithm of Method II.B.

The Phase I CLs and bias correction terms of Method II estimators for $m = 50$ are given in Table 3.2. Since reweighting is not employed in Method II.A algorithm, $L_n{}^I$, $U_n{}^I$ and Θ_3 terms correspond to empty cells. Θ_1 and Θ_2 are identical to those for Methods II.A1 and II.B1, II.A2 and II.B2, and II.A3 and II.B3, since identical combinations of estimators are used.

Table 3.2. Phase I CLs and bias correction factors of Method II for $m = 50$.

Method	Θ_1	Θ_2	$L_n{}^I$	$U_n{}^I$	Θ_3
II.A1	1.319	1.024	-	-	-
II.A2	1.319	1.055	-	-	-
II.A3	1.803	1.094	-	-	-
II.B1	1.319	1.024	0.137	2.325	1.008
II.B2	1.319	1.055	0.136	2.369	1.008
II.B3	1.803	1.094	0.106	2.656	1.013

3.2.3. Method III

Outliers, which are localized in a subgroup, may be, to a certain extent, eliminated via subgroup screening used in Method I.B and II.B, but diffuse outliers cannot be detected by this method since robust estimators used on subgroups dampen the effect of individual outliers. Reweighting procedure is extended in Method III using individuals charts to screen individual observations in the remaining subgroups. $MS_n + ML$ estimators are used in Method III.1-III.3, while $MS_n + HD_{.5}$ are used for Method III.4-III.6. Skipping level in the first subgroup screening stage (α_1) are taken to be equal to 0.0027, 0.010 and 0.025, and total skipping level after individuals chart (α) is adjusted to be equal to 0.005, 0.020 and 0.040 for Method III.1 and III.4, III.2 and III.5, and III.3 and III.6, respectively. First, Method II.B algorithm is performed using different α_1, and intermediate $\hat{\sigma}^I$s are computed. Then, ML is used for Method III.1-III.3, while $HD_{.5}$ is used for Method III.4-III.6 as location estimators. For each remaining subgroup m different individuals charts, with a common dispersion estimate and different location estimates are constructed, i.e. each individuals chart has different central line ($\hat{\mu_i}^I$), and identical acceptance interval ($\hat{\sigma}^I$). Then n observations are placed on corresponding individuals chart and observation(s), which are deemed to be outlier(s), are determined (R_i). Since even robust estimation methods may fail in the presence of high fraction of outliers in determining all the outliers, if the number of outlier observations in a subgroup is found to be higher than $n/2$, then that subgroup is deemed to be subjected to localized contamination, and eliminated, otherwise only individual outlier(s) is/are screened. After all outlier subgroups and individual observations are eliminated, unbiased $\hat{\sigma}^I$s are computed from the remaining subgroups via first computing S_n using the remaining individual observations for each subgroup, then by taking their average (Figure 3.6). The Phase I CLs and bias correction terms used in Method III estimators for $m = 50$ are given in Table 3.3. Since the same procedure is repeated in subgroup screening, Θ_1 and Θ_2 are similar to those in Method II. ${L_n}^I$, ${U_n}^I$ and Θ_3 terms are computed via computer simulations. Columns of ${C_n}^I$ and Θ_4 represent the coefficients used in individuals chart and bias correction terms after screening individual observations.

i. Use Method II.A1, or Method II.A2 corresponding to Method III.1-III.3, or Method III.4-III.6, respectively,

ii. Set $\mathbf{O^S} = \{\}$,

iii. Compute the following for $\mathbf{i^{th}}$ subgroup, using $\alpha_1 = 0.0027, 0.010, 0.020$,

$$I_i{}^S = \begin{cases} 1 & \text{if } L_n{}^I \hat{\sigma}^I \leq \hat{\sigma}_i{}^I \leq U_n{}^I \hat{\sigma}^I \\ 0 & \text{if otherwise} \end{cases}$$

$$O^S = \{i | I_i{}^S = 0\}$$

$$\hat{\sigma}^I = \frac{1}{m - |O^S|} \sum_{i=1}^{m} \hat{\sigma}_i{}^I I_i{}^S$$

iv. Employ bias correction as $\hat{\sigma}^{\mathbf{I}} = \mathbf{\Theta^{III}} \hat{\sigma}^{\mathbf{I}}$,

v. Compute $\hat{\mu_i}^{\mathbf{I}}$ using Eq. 3.13-3.14, or Eq. 3.9-3.11, $(i \in M, i \notin O^S)$, for Method III.1-3, or Method III.4-6, respectively,

vi. Compute the following for the individual observations using $\alpha = 0.005, 0.020, 0.040$,

$$U_{ij} = \begin{cases} 1 & \text{if } \hat{\mu}_i{}^I - C_n{}^I \hat{\sigma}^I \leq Z_{ij} \leq \hat{\mu}_i{}^I + C_n{}^I \hat{\sigma}^I \\ 0 & \text{if otherwise} \end{cases}$$

vii. Remove the individual outliers, and use Eq. 3.1 on the remaining data,

$$R_i = \{j | U_{ij} = 0\}$$

$$H_i = \begin{cases} 1 & \text{if } |R_i| \geq n/2 \\ 0 & \text{if otherwise} \end{cases}$$

$$O^S \leftarrow O^S \cup \{i | H_i = 0\}$$

$$\hat{\mu}_i = \frac{1}{n - R_i} \sum_{j=1}^{n} Z_{ij} U_{ij},\ i \notin O^S$$

$$\hat{\sigma}_i{}^I = \sqrt{\frac{\sum_{j=1}^{n} (Z_{ij} - \hat{\mu}_i)^2 U_{ij}}{(n - |R_i| - 1)}} / c_4(n - |R_i|),\ i \notin O^S$$

$$\hat{\sigma}^I = \frac{1}{m - |O^S|} \sum_{i \notin O^S}^{m} \hat{\sigma}_i{}^I$$

viii. Employ bias correction as $\hat{\sigma}^{\mathbf{I}} = \mathbf{\Theta^{IV}} \hat{\sigma}^{\mathbf{I}}$.

Figure 3.6. Algorithm of Method III.

Table 3.3. Phase I CLs and bias correction factors of Method III for $m = 50$.

Method	Θ_1	Θ_2	${L_n}^I$	${U_n}^I$	Θ_3	${C_n}^I$	Θ_4
III.1	1.319	1.024	0.045	2.893	1.002	2.991	1.015
III.2	1.319	1.024	0.087	2.570	1.005	2.462	1.049
III.3	1.319	1.024	0.137	2.325	1.008	2.298	1.070
III.4	1.319	1.055	0.045	2.953	1.002	3.040	1.013
III.5	1.319	1.055	0.087	2.622	1.004	2.481	1.050
III.6	1.319	1.055	0.137	2.361	1.008	2.353	1.069

3.2.4. Method IV

In Method III, individuals charts are employed to eliminate diffuse outliers in Phase I sampling by estimating subgroup location estimates separately (${\hat{\mu}_i}^I$), and constructed using the limits ${\hat{\mu}_i}^I \pm {C_n}^I \hat{\sigma}^I$, however ${\hat{\mu}_i}^I$ is not an efficient location estimator for processes with small sample size (n). Use of single intermediate process location estimate ($\hat{\mu}^I$) is proposed to remedy this problem, and identical CLs ($\hat{\mu}^I \pm {C_n}^I \hat{\sigma}^I$) are used for all individuals charts in Method IV.A. First, subgroups which are deemed to have different standard deviations are eliminated, and $\hat{\sigma}^I$ is computed as an intermediate process dispersion estimate (Figure 3.7). Robust location estimators are then employed to calculate ${\hat{\mu}_i}^I$ from the remaining subgroups, and followed by estimating a single μ^I as an intermediate process location estimate. $MS_n + ML$ dispersion and location estimators are used in Method IV.A1-IV.A3, and $MS_n + HD_{.5}$ are used for Method IV.A4-IV.A6. Skipping level α_1 takes the values 0.0027, 0.010 and 0.025, and α is taken to be equal to 0.005, 0.020 and 0.040 for Method IV.A1, IV.A4, IV.A2, IV.A5 and IV.A3, IV.A6, respectively.

The difference of Method IV.B from Method IV.A lies in reweighting the process location estimate and screening subgroups with different locations. Here, $MS_n + HD_{.5}$ dispersion and location estimators are employed. After intermediate $\hat{\mu}^I$ and $\hat{\sigma}^I$ are calculated, subgroups with process location estimates, which fall beyond the CLs de-

fined by $\hat{\mu}^I \pm {C_n}^I \hat{\sigma}^I/\sqrt{n}$, are eliminated. Then individuals charts are constructed for the remaining subgroups using the CLs $\hat{\mu}^I \pm {C_n}^I \hat{\sigma}^I$. It should be noted that the same coefficient C_n is used for both subgroup screening charts and individuals charts to yield α to be equal to 0.005, 0.020 and 0.040 for Method IV.B1-IV.B3, respectively. The Phase I CLs and bias correction terms of Method IV.A and IV.B estimators for $m = 50$ are given in Table 3.4.

Table 3.4. Phase I CLs and bias correction factors of Method IV for $m = 50$.

Method	Θ_1	Θ_2	${L_n}^I$	${U_n}^I$	Θ_3	${C_n}^I$	Θ_4
IV.A1	1.319	1.024	0.045	2.893	1.002	3.070	1.011
IV.A2	1.319	1.024	0.087	2.570	1.005	2.578	1.038
IV.A3	1.319	1.024	0.137	2.325	1.008	2.411	1.057
IV.A4	1.319	1.055	0.045	2.953	1.002	3.092	1.011
IV.A5	1.319	1.055	0.087	2.622	1.004	2.577	1.039
IV.A6	1.319	1.055	0.137	2.361	1.008	2.420	1.056
IV.B1	1.319	1.055	0.045	2.953	1.002	3.030	1.013
IV.B2	1.319	1.055	0.087	2.622	1.004	2.913	1.019
IV.B3	1.319	1.055	0.137	2.361	1.008	2.762	1.030

3.2.5. Method V

$MS_n + HD_{.5}$ dispersion and location estimators are employed, and skipping level α_1 takes the values 0.0027, 0.010 and 0.025, and α is taken to be equal to 0.005, 0.020 and 0.040 for Method V.1-V.3, respectively. After skipping subgroups deemed to be outliers, as in Method IV.B, intermediate $\hat{\mu}^I$ and $\hat{\sigma}^I$ are determined via pooling of remaining observations in subgroups without their subgroup information. Corresponding robust dispersion and location estimators are employed to determine $\hat{\mu}^I$ and $\hat{\sigma}^I$ from the pooled observations in the remaining subgroups and individuals chart is constructed with the CLs defined by $\hat{\mu}^I \pm {C_n}^I \hat{\sigma}^I$. The Phase I CLs and bias correction terms of Method V estimators for $m = 50$ are given in Table 3.5.

i. Use Method II.A1 and II.A2 corresponding to Method IV.A1-IV.A3, or Method IV.A4-IV.A6, respectively,

ii. Set $\mathbf{O^S} = \{\}$,

iii. Compute the following for $\mathbf{i^{th}}$ subgroup, using $\alpha_1 = 0.0027, 0.010, 0.020$,

$$I_i{}^S = \begin{cases} 1 & \text{if } L_n{}^I \hat{\sigma}^I \leq \hat{\sigma}_i{}^I \leq U_n{}^I \hat{\sigma}^I \\ 0 & \text{if otherwise} \end{cases}$$

$$O^S = \{i | I_i{}^S = 0\}$$

$$\hat{\sigma}^I = \frac{1}{m - |O^S|} \sum_{i=1}^{m} \hat{\sigma}_i{}^I I_i{}^S$$

iv. Employ bias correction as $\hat{\sigma}^{\mathbf{I}} = \mathbf{\Theta^{III}} \hat{\sigma}^{\mathbf{I}}$,

v. Compute $\hat{\mu}_{\mathbf{i}}{}^{\mathbf{I}}$ **and** $\hat{\mu}^{\mathbf{I}}$ using Eq. 3.13-3.14, or Eq. 3.9-3.11, $(i \in M, i \notin O^S)$, for Method IV.A1-IV.A3, or Method IV.A4-IV.A6, respectively,

vi. Compute the following for the individual observations, $(i \in M, i \notin O^S)$, using $\alpha = 0.005, 0.020, 0.040$. Note that, contrary to step vi of Method III, a single location estimate $\hat{\mu}^{\mathbf{I}}$ is used,

$$U_{ij} = \begin{cases} 1 & \text{if } \hat{\mu}^I - C_n{}^I \hat{\sigma}^I \leq Z_{ij} \leq \hat{\mu}^I + C_n{}^I \hat{\sigma}^I \\ 0 & \text{if otherwise} \end{cases}$$

vii. Remove the individual outliers, and use Eq. 3.1 on the remaining data,

$$R_i = \{j | U_{ij} = 0\}$$

$$H_i = \begin{cases} 1 & \text{if } |R_i| \geq n/2; \\ 0 & \text{if otherwise.} \end{cases}$$

$$O^S \leftarrow O^S \cup \{i | H_i = 0\}$$

$$\hat{\mu}_i = \frac{1}{n - R_i} \sum_{j=1}^{n} Z_{ij} U_{ij},\ i \notin O^S$$

$$\hat{\sigma}_i{}^I = \sqrt{\frac{\sum_{j=1}^{n} (Z_{ij} - \hat{\mu}_i)^2 U_{ij}}{(n - |R_i| - 1)}} / c_4(n - |R_i|),\ i \notin O^S$$

$$\hat{\sigma}^I = \frac{1}{m - |O^S|} \sum_{i \notin O^S}^{m} \hat{\sigma}_i{}^I$$

viii. Employ bias correction as $\hat{\sigma}^{\mathbf{I}} = \mathbf{\Theta^{IV}} \hat{\sigma}^{\mathbf{I}}$.

Figure 3.7. Algorithm of Method IV.A.

i. Repeat Method II.A2, and set $\mathbf{O^S} = \{\}$,

ii. Compute the following for $\mathbf{i^{th}}$ subgroup, using $\alpha_1 = 0.0027, 0.010, 0.020$,

$$I_i{}^S = \begin{cases} 1 & \text{if } L_n{}^I \hat{\sigma}^I \leq \hat{\sigma}_i{}^I \leq U_n{}^I \hat{\sigma}^I \\ 0 & \text{if otherwise} \end{cases}$$

$$O^S = \{i | I_i{}^S = 0\}$$

$$\hat{\sigma}^I = \frac{1}{m - |O^S|} \sum_{i=1}^{m} \hat{\sigma}_i{}^I I_i{}^S$$

iii. Employ bias correction as $\hat{\sigma}^{\mathbf{I}} = \mathbf{\Theta^{III}} \hat{\sigma}^{\mathbf{I}}$,

iv. Compute $\hat{\mu}_{\mathbf{i}}{}^{\mathbf{I}}$ **and** $\hat{\mu}^{\mathbf{I}}$ using Eq. 3.9-3.11, $(i \in M, i \notin O^S)$, and set $\mathbf{O^L} = \{\}$,

v. Compute the following for $\mathbf{i^{th}}$ subgroup $(i \in M, i \notin O^S)$,

$$I_i{}^L = \begin{cases} 1 & \text{if } \hat{\mu}^I - C_n{}^I \hat{\sigma}^L / \sqrt{n} \leq \hat{\mu}_i{}^I \leq \hat{\mu}^I + C_n{}^I \hat{\sigma}^L / \sqrt{n} \\ 0 & \text{if otherwise} \end{cases}$$

$$O^L = \{i | I_i{}^L = 0\} \textbf{ and } O^S \leftarrow O^S \cup O^L$$

vi. Re-estimate the process location parameter using the following reweighted estimator,

$$\hat{\mu} = \frac{\sum_{i=1}^{m} \hat{\mu}_i I_i{}^L I_i{}^S}{m - |O^L| - |O^S|},$$

vi. Compute the following for the individual observations, $(i \in M, i \notin O^S)$, using $\alpha = 0.005, 0.020, 0.040$,

$$U_{ij} = \begin{cases} 1 & \text{if } \hat{\mu}^I - C_n{}^I \hat{\sigma}^I \leq Z_{ij} \leq \hat{\mu}^I + C_n{}^I \hat{\sigma}^I \\ 0 & \text{if otherwise} \end{cases}$$

viii. Remove the individual outliers and use Eq. 3.1 on the remaining data,

$$R_i = \{j | U_{ij} = 0\}$$

$$H_i = \begin{cases} 1 & \text{if } |R_i| \geq n/2 \\ 0 & \text{if otherwise} \end{cases}$$

$$O^S \leftarrow O^S \cup \{i | H_i = 0\}$$

$$\hat{\mu}_i = \frac{1}{n - R_i} \sum_{j=1}^{n} Z_{ij} U_{ij},\ i \notin O^S$$

$$\hat{\sigma}_i{}^I = \sqrt{\frac{\sum_{j=1}^{n} (Z_{ij} - \hat{\mu}_i)^2 U_{ij}}{(n - |R_i| - 1)}} / c_4(n - |R_i|),\ i \notin O^S$$

$$\hat{\sigma}^I = \frac{1}{m - |O^S|} \sum_{i \notin O^S}^{m} \hat{\sigma}_i{}^I$$

ix. Employ bias correction as $\hat{\sigma}^{\mathbf{I}} = \mathbf{\Theta^{IV}} \hat{\sigma}^{\mathbf{I}}$.

Figure 3.8. Algorithm of Method IV.B.

i. Repeat Method II.A2, and set $\mathbf{O^S} = \{\}$,

ii. Compute the following for $\mathbf{i^{th}}$ subgroup, using $\alpha_1 = 0.0027, 0.010, 0.020$,

$$I_i{}^S = \begin{cases} 1 & \text{if } L_n{}^I \hat{\sigma}^I \leq \hat{\sigma}_i{}^I \leq U_n{}^I \hat{\sigma}^I \\ 0 & \text{if otherwise} \end{cases}$$

$$O^S = \{i | I_i{}^S = 0\}$$

$$\hat{\sigma}^I = \frac{1}{m - |O^S|} \sum_{i=1}^{m} \hat{\sigma}_i{}^I I_i{}^S$$

iii. Employ bias correction as $\hat{\sigma}^{\mathbf{I}} = \mathbf{\Theta^{III}} \hat{\sigma}^{\mathbf{I}}$,

iv. Compute $\hat{\mu}_{\mathbf{i}}^{\mathbf{I}}$ **and** $\hat{\mu}^{\mathbf{I}}$ using Eq. 3.9-3.11, $(i \in M, i \notin O^S)$, and set $\mathbf{O^L} = \{\}$,

v. Compute the following for $\mathbf{i^{th}}$ subgroup $(i \in M, i \notin O^S)$,

$$I_i{}^L = \begin{cases} 1 & \text{if } \hat{\mu}^I - C_n{}^I \hat{\sigma}^L / \sqrt{n} \leq \hat{\mu}_i{}^I \leq \hat{\mu}^I + C_n{}^I \hat{\sigma}^L / \sqrt{n} \\ 0 & \text{if otherwise} \end{cases}$$

$$O^L = \{i | I_i{}^L = 0\} \textbf{ and } O^S \leftarrow O^S \cup O^L$$

vi. Pool remaining observations **and** compute process parameter estimates,

$$Z_k^* = \{Z_{ij} | i \in M \backslash O^S, j = 1, 2, \ldots, n\}$$

$$\hat{\mu}^I = HD_{.5}(Z_k^*) \textbf{ and } \hat{\sigma}^I = MS_n(Z_k^*)$$

vii. Employ bias correction as $\hat{\sigma}^{\mathbf{I}} = \mathbf{\Theta^V} \hat{\sigma}^{\mathbf{I}}$,

viii. Compute the following for the individual observations, $(i \in M, i \notin O^S)$, using $\alpha = 0.005, 0.020, 0.040$,

$$U_{ij} = \begin{cases} 1 & \text{if } \hat{\mu}^I - C_n{}^I \hat{\sigma}^I \leq Z_{ij} \leq \hat{\mu}^I + C_n{}^I \hat{\sigma}^I \\ 0 & \text{if otherwise} \end{cases}$$

ix. Remove the individual outliers, and use Eq. 3.1 on the remaining data,

$$R_i = \{j | U_{ij} = 0\}$$

$$H_i = \begin{cases} 1 & \text{if } |R_i| \geq n/2 \\ 0 & \text{if otherwise} \end{cases}$$

$$O^S \leftarrow O^S \cup \{i | H_i = 0\}$$

$$\hat{\mu}_i = \frac{1}{n - R_i} \sum_{j=1}^{n} Z_{ij} U_{ij},\ i \notin O^S$$

$$\hat{\sigma}_i{}^I = \sqrt{\frac{\sum_{j=1}^{n} (Z_{ij} - \hat{\mu}_i)^2 U_{ij}}{(n - |R_i| - 1)}} / c_4(n - |R_i|),\ i \notin O^S$$

$$\hat{\sigma}^I = \frac{1}{m - |O^S|} \sum_{i \notin O^S}^{m} \hat{\sigma}_i{}^I$$

x. Employ bias correction as $\hat{\sigma}^{\mathbf{I}} = \mathbf{\Theta^{IV}} \hat{\sigma}^{\mathbf{I}}$.

Figure 3.9. Algorithm of Method V.

Table 3.5. Phase I CLs and bias correction factors of Method V for $m = 50$.

Method	Θ_1	Θ_2	${L_n}^I$	${U_n}^I$	Θ_3	${C_n}^I$	Θ_4	Θ_5
V.1	1.319	1.055	0.045	2.953	1.002	3.030	1.013	1.0009
V.2	1.319	1.055	0.087	2.622	1.004	2.913	1.019	1.019
V.3	1.319	1.055	0.137	2.361	1.008	2.762	1.030	1.017

3.3. Contamination Models

Phase I data (Z_{ij}) is composed of $m = 20$ or $m = 50$ subgroups (samples) with $n = 5$ observations in each sample. G_i denotes the i^{th} subgroup of Phase I data. Samples consist of normally distributed data clouded with outliers described with various models in this section. Five main contamination models (CM 1-5), suggested by Tatum [39], are used, and outlier density (K) and magnitude of contamination (d) are adjusted to devise more realistic scenarios similar. Furthermore, these main contamination models are superimposed (SCM 1-6) to test the efficiency and performance of the proposed estimators more rigorously. In each case, in-control phase I data are assumed to be consist of standardized Gaussian variable $Z_G \sim N(0, 1)$, while outliers are represented with another random variable Z_C, describing contamination models, which correspond to temporary out-of-control states of the out-of-control process. In CM 1-5 and SCM 1-6 models, observations are assumed to be i.i.d ($corr(Z_{ij}, Z_{ik} = 0), j \neq k$), whereas autoregressive contamination models (ACM 1-3) exclude the independence assumption and assumes an unstable process.

Outliers in Phase I samples are drawn either from another normal distribution, which has a mean different than the main normal distribution to represent shifts in process mean, or from a distribution, which has a different dispersion to correspond to the changes in process variation. In some cases, outliers are spread in all subgroups, i.e. observations in subgroups have equal probability of coming from Z_C (diffuse contamination models, mainly CM 1, 2 and 4), while, in other case, outliers are concentrated in certain subgroups (localized contamination models, mainly CM 3 and 5), i.e. observations in subgroups come from Z_C, and the remaining subgroups are clean.

Denoting the Bernoulli distributed indicator with I_{ij} ($I_{ij} \sim B(1, K)$) to represent the contaminated observations in diffuse contamination models, each I_{ij} ($i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$) has two outcomes: success (outlier) with probability of K and failure (clean) with probability of $1 - K$. The indicator I_i, ($i = 1, 2, \ldots, m$) is used to represent contaminated subgroups in localized contamination models.

An example of simulated normally distributed ($Z_{ij} \sim N(0, 1)$) Phase I data is shown in Figure 3.10. Here, there are neither individual outliers within subgroups, nor outlier subgroups.

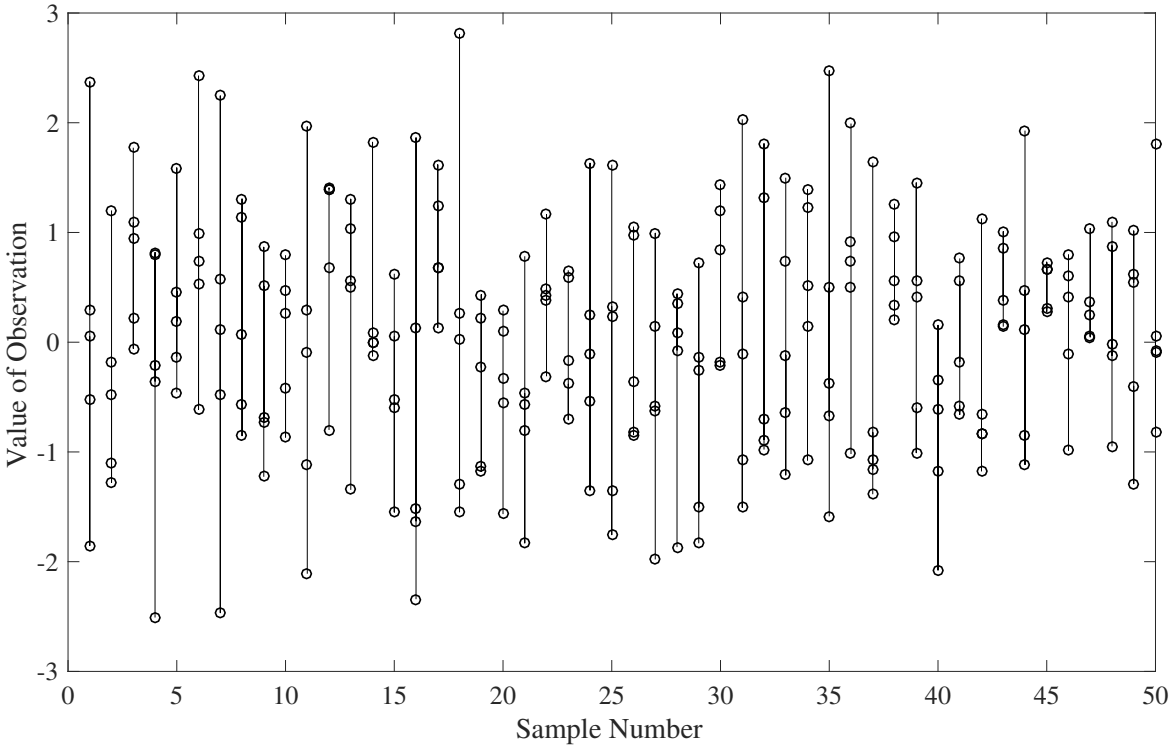

Figure 3.10. An example of normally distributed Phase I data.

3.3.1. Main Contamination Models

Five different contamination models are discussed in this section. In CM 1, 3-5, parameters are taken to be equal to $K = (0.10, 0.15, \text{ and } 0.20)$ and $d = (0.5, 1, \ldots, 6)$ for Phase I efficiency analysis, and $(K, d) = (0.10, 2; 0.15, 3; 0.20, 3)$ for Phase II performance analysis. Only in CM 2, $K = (0.10, 0.15, \text{ and } 0.20)$ and $d = (0.25, 0.50, \ldots, 3)$

for Phase I efficiency, and $(K, d) = (0.10, 0.75; 0.15, 1; 0.20, 1)$ for Phase II performance are studied.

3.3.1.1. CM 1: Diffuse Symmetric Variance Contamination Model. Each observation in each subgroups has K probability of being drawn the population of $Z_C \sim N(0, d^2)$ and $1-K$ probability of coming from the normal distribution $Z_G \sim N(0, 1)$. Therefore, observations may be described by the following probability distribution:

$$Z_{ij} = \begin{cases} \sim N(0,1) & \text{if } I_{ij} = 0 \\ \sim N(0, d^2) & \text{if } I_{ij} = 1 \end{cases} \tag{3.15}$$

A simulated example of Phase I sampling under CM 1 is shown in Figure 3.11, in which observations denoted with empty circles are clean observations, i.e. $Z_G \sim N(0, 1)$ $(I_{ij} = 0)$, while ones with black filled circles are outliers, i.e. $Z_C \sim N(0, 3^2)$ $(I_{ij} = 1)$.

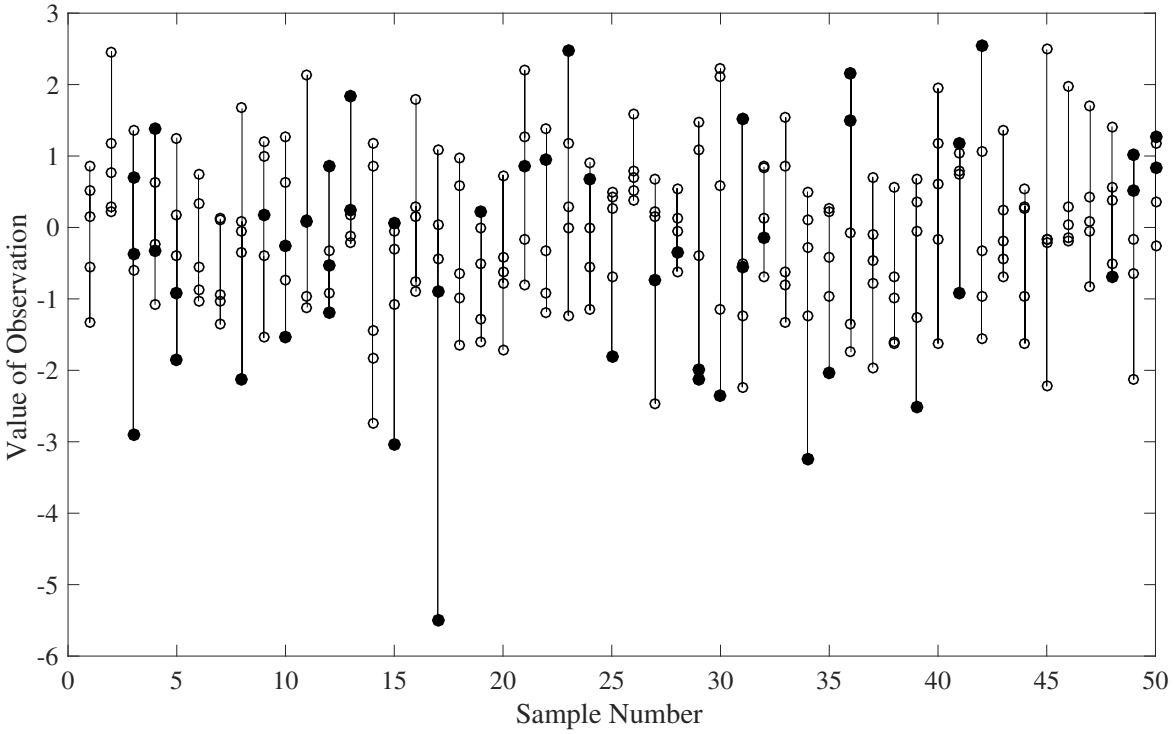

Figure 3.11. Phase I sampling subjected to CM 1 (K=20%, d=3).

3.3.1.2. CM 2: Diffuse Asymmetric Variance Contamination Model. Each observation in each subgroup has K probability of being drawn from a chi-square distribution with one degree of freedom, $Z_C \sim d{\chi_1}^2$, and $1-K$ probability of being drawn from the normal distribution $Z_G \sim N(0,1)$.

$$Z_{ij} = \begin{cases} \sim N(0,1) & \text{if } I_{ij} = 0 \\ \sim d{\chi_1}^2 & \text{if } I_{ij} = 1 \end{cases} \tag{3.16}$$

A simulated example of Phase I sampling under CM 2 is shown in Figure 3.11, in which observations denoted with empty circles are clean observations, i.e. $Z_G \sim N(0,1)$ ($I_{ij} = 0$), while ones with black filled circles are outliers, i.e. $Z_C \sim 1{\chi_1}^2$ ($I_{ij} = 1$). Compared to Figure 3.11, one can clearly see the asymmetry of distribution.

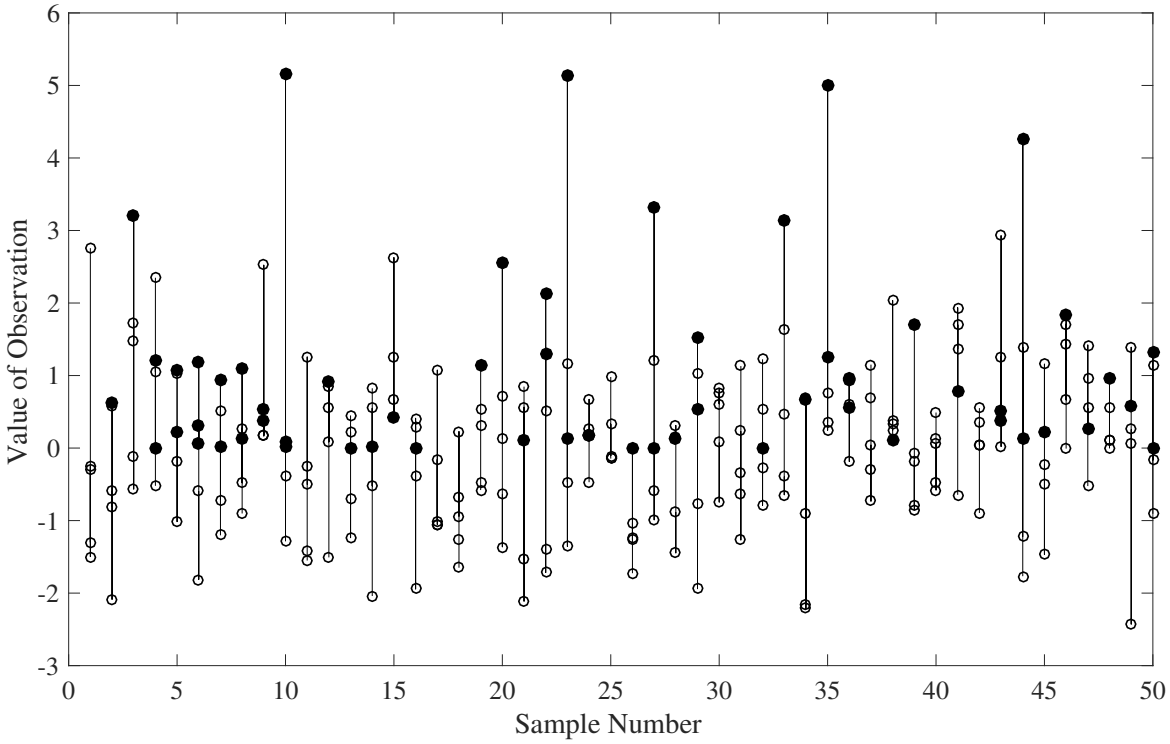

Figure 3.12. Phase I sampling subjected to CM 2 (K=20%, d=1).

3.3.1.3. CM 3: Localized Variance Contamination Model. All observations in the first $K \times m$ subgroups are sampled from $Z_C \sim N(0, d^2)$, while the rest of the subgroups are

free from the outliers.

$$Z_{ij} = \begin{cases} \sim N(0, d^2) & \text{if } 1 \leq i \leq \text{round}(K \times m) \\ \sim N(0, 1) & \text{if round}(K \times m) + 1 \leq i \leq m \end{cases} \tag{3.17}$$

A simulated example of Phase I sampling under CM 3 is shown in Figure 3.13. While observations in the first 10 subgroups, shown with black filled circles are outliers, i.e. $Z_C \sim N(0, 3^2)$, the rest of the subgroups, denoted with empty circles are clean, i.e. $Z_G \sim N(0, 1)$.

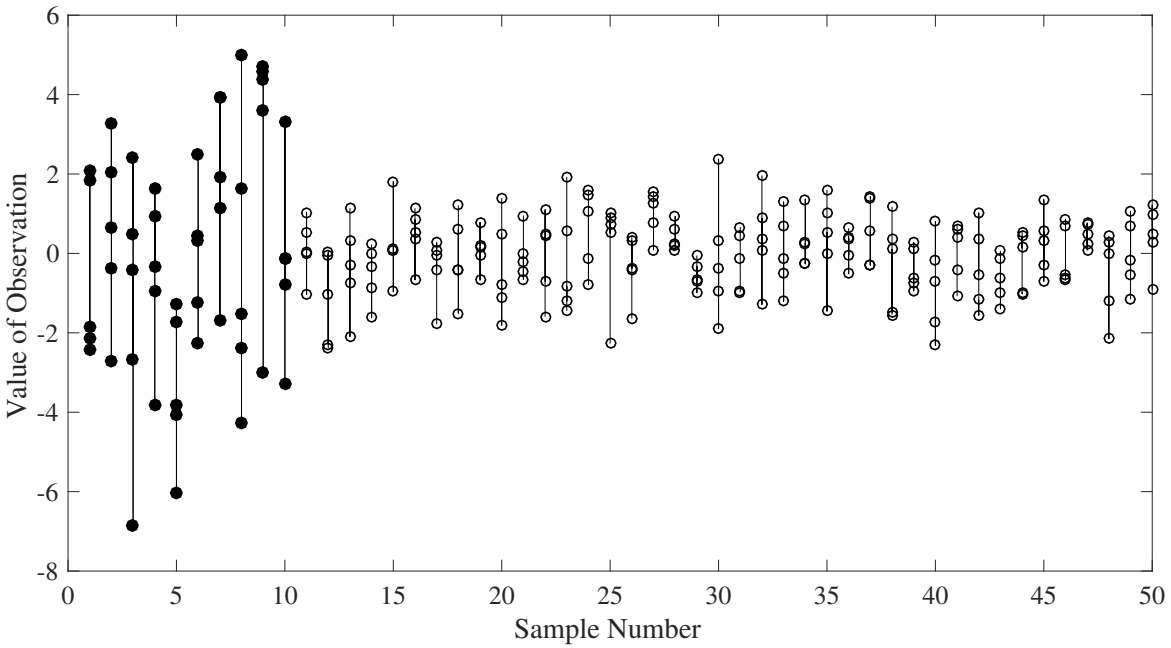

Figure 3.13. Phase I sampling subjected to CM 3 (K=20%, d=3).

3.3.1.4. CM 4: Diffuse Mean Contamination Model. In this case, each observation in each subgroup has K probability of being drawn from the population of $Z_C \sim N(d, 1)$, and $1 - K$ probability of from normal distribution $Z_G \sim N(0, 1)$.

$$Z_{ij} = \begin{cases} \sim N(0, 1) & \text{if } I_{ij} = 0 \\ \sim N(d, 1) & \text{if } I_{ij} = 1 \end{cases} \tag{3.18}$$

A simulated example of Phase I sampling under CM 4 is shown in Figure 3.14, in which observations denoted with empty circles are clean observations, i.e. $Z_G \sim N(0,1)$ ($I_{ij} = 0$), while ones with black filled squares are outliers, i.e. $Z_C \sim N(3,1)$ ($I_{ij} = 1$). Asymmetric distribution of observations is similar to that seen under CM 2, but outliers are less obvious, since they are concentrated at smaller positive values.

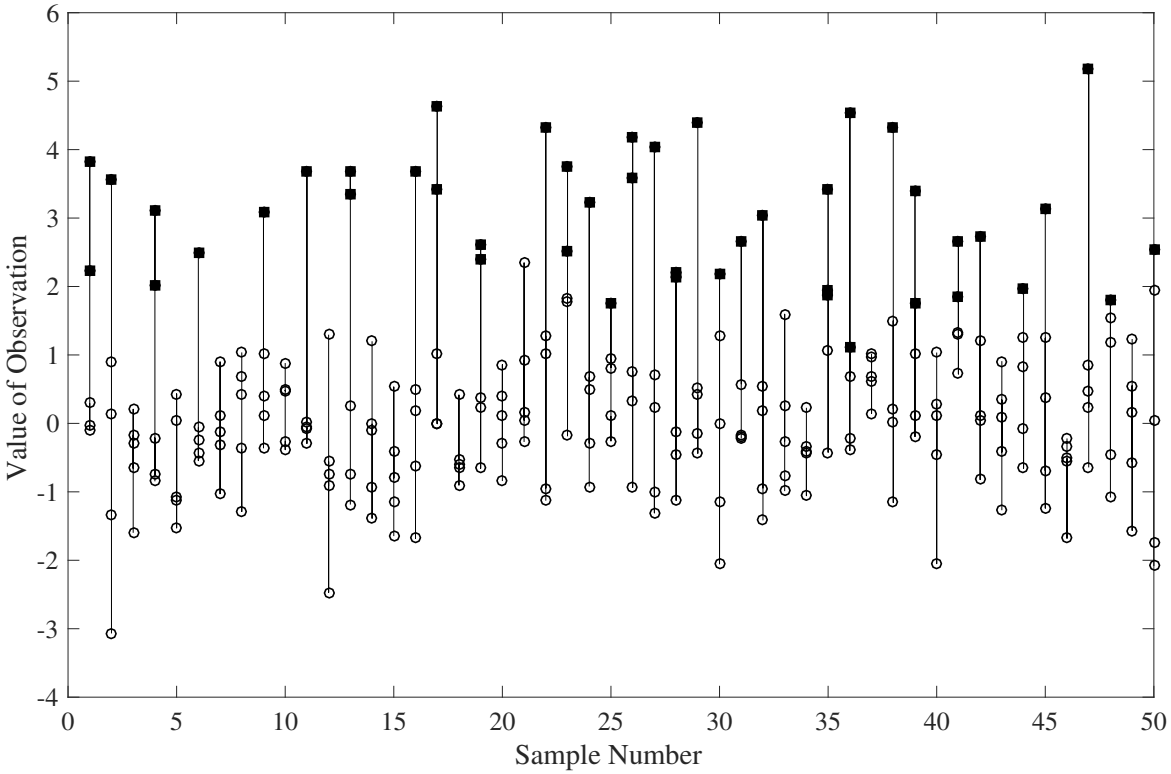

Figure 3.14. Phase I sampling subjected to CM 4 (K=20%, d=3).

3.3.1.5. CM 5: Localized Mean Contamination Model. This contamination model is similar to CM 3; the only difference is the shift in process mean instead of that in process variation. In this case, first $K \times m$ subgroups are sampled from $Z_C \sim N(d,1)$, while the rest of the subgroups are free from the outliers.

$$Z_{ij} = \begin{cases} \sim N(d,1) & \text{if } 1 \leq i \leq \text{round}(K \times m) \\ \sim N(0,1) & \text{if round}(K \times m) + 1 \leq i \leq m \end{cases} \tag{3.19}$$

A simulated example of Phase I sampling under CM 5 is shown in Figure 3.15, in which observations denoted with empty circles are clean observations, i.e. $Z_G \sim N(0,1)$, while ones with black filled squares are outliers, i.e. $Z_C \sim N(3,1)$.

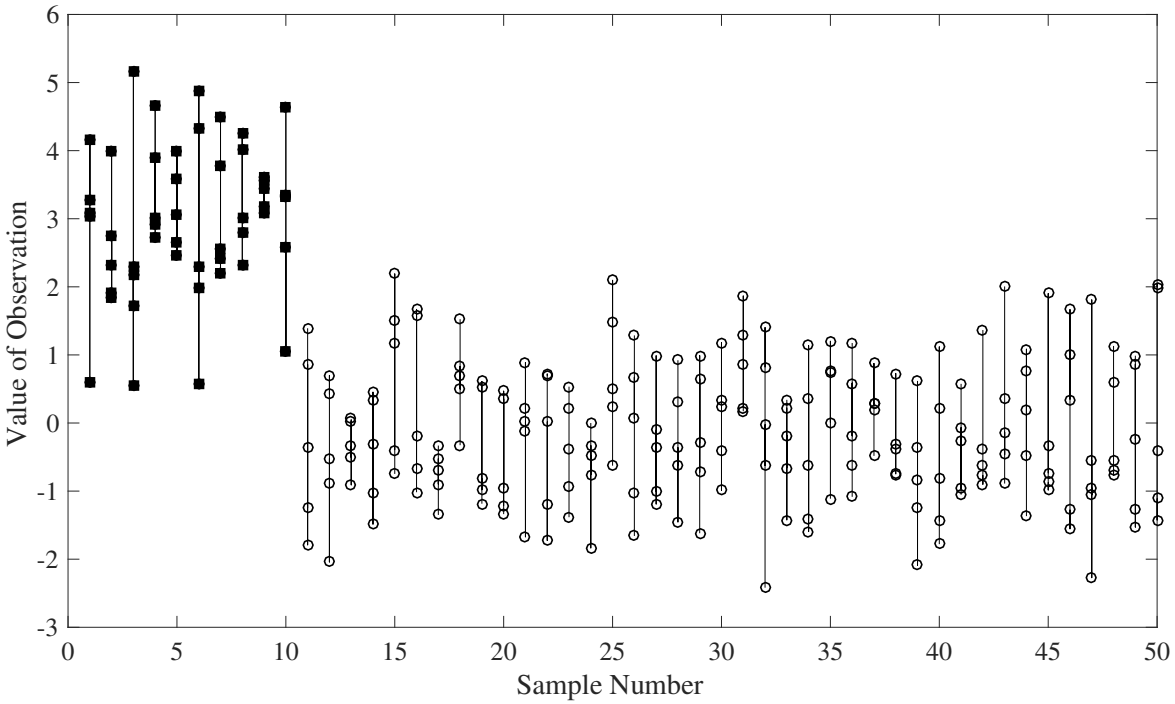

Figure 3.15. Phase I sampling subjected to CM 5 (K=20%, d=3).

3.3.2. Superimposed Contamination Models

Six superimposed CMs (SCM) are discussed here to examine the effect of simultaneous perturbation in process mean and variation, and/or local and diffuse contaminations. In SCM 1-6, single contamination magnitude is taken to be equal to ($d = 3$), while contamination density K takes the values $K = 0.08$ and 0.10 for $m = 50$ and $m = 20$ to yield overall outlier densities to be equal to 0.16 and 0.20 for $m = 50$ and $m = 20$, respectively.

3.3.2.1. SCM 1: Diffuse + Localized Variance Contamination Model. The first $K \times m$ subgroups are sampled from normal distribution $Z_C \sim N(0, 3^2)$, and each observations in the rest of subgroups has K probability of being drawn from the same Z_C.

3.3.2.2. SCM 2: Diffuse + Localized Mean Contamination Model. $K \times m$ subgroups are sampled from normal distribution $Z_C \sim N(3,1)$, and each observations in the rest of subgroups has K probability of being drawn from the same population Z_C.

3.3.2.3. SCM 3: Diffuse Variance + Mean Contamination Model. Each observations in each subgroups has K probability of being drawn the population of $Z_C \sim N(3,1)$, and K probability of being drawn the population of $Z_C \sim N(0,3^2)$ simultaneously.

3.3.2.4. SCM 4: Localized Variance + Mean Contamination Model. $K \times m$ subgroups are sampled from normal distribution $Z_C \sim N(3,1)$, and another $K \times m$ subgroups from another normal distribution $Z_C \sim N(0,3^2)$, while the rest of the subgroups are normally distributed.

3.3.2.5. SCM 5: Diffuse Variance + Localized Mean Contamination Model. The first $K \times m$ subgroups are sampled from normal distribution $Z_C \sim N(3,1)$, and observations in the rest of the subgroups have a probability of K to be drawn from another normal distribution $Z_C \sim N(0,3^2)$.

3.3.2.6. SCM 6: Localized Variance + Diffuse Mean Contamination Model. The first $K \times m$ subgroups are sampled from normal distribution $Z_C \sim N(0,3^2)$, and observations in the rest of the subgroups have a probability of K to be drawn from another normal distribution $Z_C \sim N(3,1)$.

3.3.3. Autoregressive Contamination Model

Phase I estimators are used to detect shifts in process mean and variation, and determine whether process is in statistically control or not. Autoregressive contamination models (ACM) assumes that the process is out-of-control and clouded with outliers. Phase I sampling under ACM will be discussed only to examine the efficiency of estimators in detecting of out-of-control status of the process, and not further be

examined in comparison of Phase II performance. A first order autoregressive (AR) process is employed to model process mean, as follows.

$$\mu_i = \phi\mu_{i-1} + e_i \tag{3.20}$$

in which e_i is normally distributed ($e_i \sim N(0, {\sigma_e}^2)$); the location of the i^{th} subgroup is correlated with the location parameter of the $(i-1)^{th}$ subgroup, Phase I samples are modeled as:

$$Z_{ij} = \mu_i + \epsilon_{ij} \tag{3.21}$$

3.3.3.1. ACM 1: Autoregressive Process with Normal Data. In this model, the process does not contain outliers, and two different parameter value sets $\phi = (.4, .8)$ and ${\sigma_e}^2 = (.5, 1)$ are used to simulate out-of-control processes.

3.3.3.2. ACM 2: Autoregressive Process with CM 1. Out-of-control process is described with the parameter values $\phi = .8$ and ${\sigma_e}^2 = 1$. ϵ_{ij} in Equation 3.21 is modeled by a contaminated normal distribution, corresponding to diffuse variance contamination in Equation 3.15 with $K = .15$ and $d = 3$.

3.3.3.3. ACM 3: Autoregressive Process with CM 4. Same parameters of $\phi = .8$ and ${\sigma_e}^2 = 1$ are used, but ϵ_{ij} in Equaton 3.21 is modeled by a contaminated normal distribution, corresponding to diffuse mean contamination in Equation 3.18 with $K = .15$ and $d = 3$.

4. RESULTS

In this chapter we investigate the Phase I efficiency of the proposed estimation methods and Phase II performance of S-Charts constructed from these methods. Phase I data, which are denoted with Z_{ij} ($i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$), consist of m subgroups with n individual observations, and Phase II data, which are denoted with Y_{ij} ($i = 1, 2, \ldots,$ and $j = 1, 2, \ldots, n$), consist of an identical number of individual observations (n), while k depends on the length of time that the control chart is used. In robust application of control charts, Phase I estimator should be efficient under normal distribution and robust against contaminations, while Phase II estimator should be sensitive to disturbances [24]. Therefore, the proposed Phase I estimators are used to determine $\hat{\sigma}^I$, and the dispersion estimate of the i^{th} sample of Phase II data is computed by $\hat{\sigma_k}^{II} = S_n(Y_{ij})/c_4(n)$, and checked using Phase II CLs.

This chapter is structured as follows. Phase I performance of estimators is evaluated in Section 4.1, when uncontaminated normal and contaminated situations are encountered. The results for $m = 50$ are given at moderate contamination density ($K = 0.15$) in the same section (Figure 4.3-4.10), while those for low ($K = 0.10$) and high contamination density ($K = 0.20$) are given in Appendix B (Figure B.1-B.4). The results for $m = 20$ under all CMs are given in Appendix B (Figure B.5-B.10). In Section 4.2, derivation of Phase II CLs for S-Charts using the proposed estimation methods is presented. Next, Phase II performances of S-Charts using the proposed estimators for $m = 50$ are shown in Section 4.3 under normal distribution and in the presence of contaminations with various outlier densities. For easier inspection, visual presentations of the results for $K = 0.15$ are presented in Figure 4.11-4.18. Phase II results for $m = 50$ are given in (Table C.1-C.44), while those for $m = 20$ are tabulated in Appendix D (Table D.1-D.44). Phase II performance metrics are obtained via 50,000 computer simulations. In Section 4.4, Phase I performance of the proposed estimators is studied on out-of-control processes. Finally, control charts are tested on real-data examples in Section 4.5.

4.1. Evaluation of Phase I Efficiency of Proposed Estimation Methods

Phase I efficiency of dispersion estimators is assessed by mean square error (MSE), which is equal to the expected value of the squared difference between the unbiased estimate and the true value:

$$MSE = \frac{1}{N_{run}} \sum_{q=1}^{N_{run}} (\hat{\sigma}^I_{\{q\}} - \sigma_0)^2 \tag{4.1}$$

here, $\hat{\sigma}^I_{\{q\}}$ represents the standard deviation estimate of q^{th} Monte Carlo (MC) simulation, and N_{run} is the number of MC simulations, and is equal to 100,000.

MSE values under normality are shown for $m = 50$ and $m = 20$ in Figure 4.1 and 4.2, respectively. In both figures, x axes denote the estimator number, and dashed lines separate proposed estimation methods from Methods I to V, for which subgroup statistics and trimming amounts are given in Table 4.1. In the absence of outliers, traditional sample standard deviation (S_n) is the most efficient estimator, and its efficiency slightly deteriorates as trimming amount increases (Method I). Gaussian efficiency of non-reweighted and reweighted robust estimators (Method II) is far from desirable, especially when MAD_n is used as dispersion estimator (Estimator No. 8 and 11). Reweighting procedure increases the efficiency of Method II.A2 and II.A3 estimators, in which $HD_{.5}$ is used as the location estimator, but decreases the efficiency of Method II.A1 estimator, in which ML is utilized. Employing individuals charts within subgroups in Method III-V improves the Phase I efficiency of robust estimators under normality. Method V yields the minimum MSEs, followed by Method IV.A, IV.B and III, respectively. While the increase in skipping level (α) deterioriates the Gaussian efficiency of all estimators, it should be noted that the efficiency loss is minimum in Method V and Method IV.B (Figure 4.1). Similar trends are observed when the sample size is relatively low ($m = 20$), the only difference is that MSE values are relatively higher, because low sample size inflates the variance of estimators (Figure 4.2).

Table 4.1. Summary of proposed estimators.

Est. No.	**Method**	**Subgroup Statistics**		α_1	α
1	I.A	S_n	ave	-	-
2	I.B1	S_n	ave	0.0027	-
3	I.B2	S_n	ave	0.010	-
4	I.B3	S_n	ave	0.025	-
5	I.B4	S_n	ave	0.050	-
6	II.A1	MS_n	ML	-	-
7	II.A2	MS_n	$HD_{.5}$	-	-
8	II.A3	MAD_n	$HD_{.5}$	-	-
9	II.B1	MS_n	ML	0.025	-
10	II.B2	MS_n	$HD_{.5}$	0.025	-
11	II.B3	MAD_n	$HD_{.5}$	0.025	-
12	III.1	MS_n	ML	0.0027	0.005
13	III.2	MS_n	ML	0.010	0.020
14	III.3	MS_n	ML	0.025	0.040
15	III.4	MS_n	$HD_{.5}$	0.0027	0.005
16	III.5	MS_n	$HD_{.5}$	0.010	0.020
17	III.6	MS_n	$HD_{.5}$	0.025	0.040
18	IV.A1	MS_n	ML	0.0027	0.005
19	IV.A2	MS_n	ML	0.010	0.020
20	IV.A3	MS_n	ML	0.025	0.040
21	IV.A4	MS_n	$HD_{.5}$	0.0027	0.005
22	IV.A5	MS_n	$HD_{.5}$	0.010	0.020
23	IV.A6	MS_n	$HD_{.5}$	0.025	0.040
24	IV.B1	MS_n	$HD_{.5}$	0.0027	0.005
25	IV.B2	MS_n	$HD_{.5}$	0.010	0.020
26	IV.B3	MS_n	$HD_{.5}$	0.025	0.040
27	V.1	MS_n	$HD_{.5}$	0.0027	0.005
28	V.2	MS_n	$HD_{.5}$	0.010	0.020
29	V.3	MS_n	$HD_{.5}$	0.025	0.040

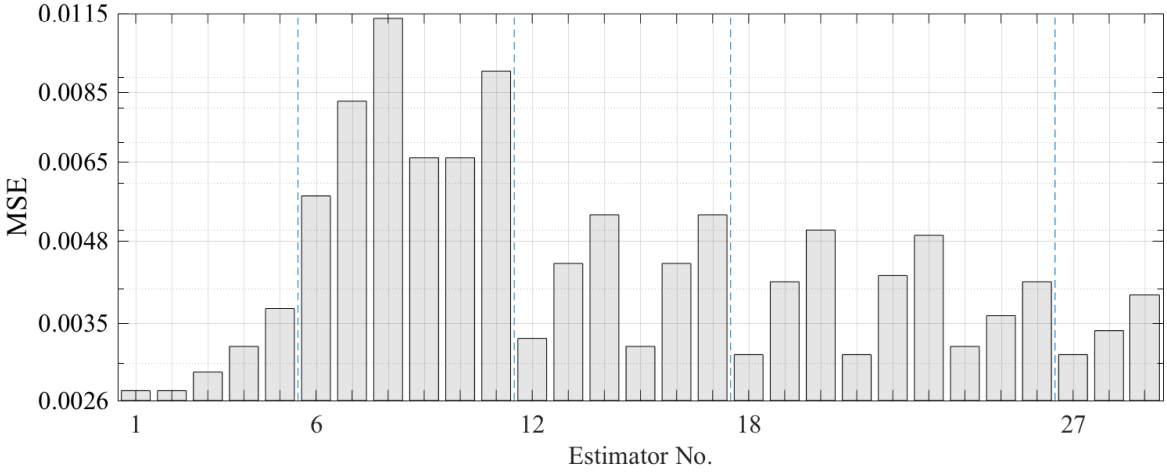

Figure 4.1. Phase I efficiency of the proposed estimators for $m = 50$ under normal distribution.

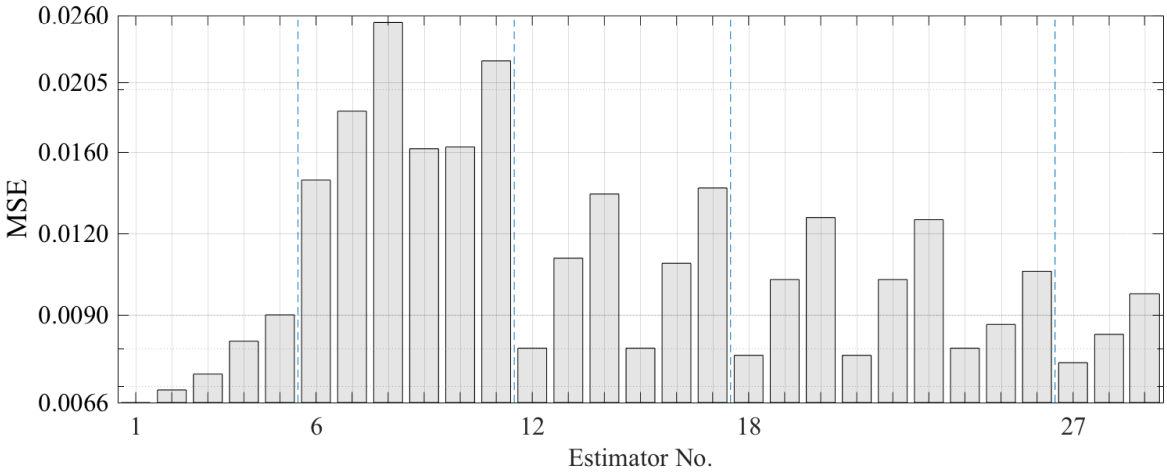

Figure 4.2. Phase I efficiency of the proposed estimators for $m = 20$ under normal distribution.

Figure 4.3 shows the MSE values when Phase I samples is subjected to CM 1. Method I estimator, having the highest Gaussian efficiency, promptly loses its superiority to robust estimators as contamination magnitude (d) increases. Although Method II estimators are more robust compared to Method I estimators against CM 1, their performances are worse than Method III-V estimators. It is not expected from Method I and II estimators to have high efficiencies, since these methods ignore individual outliers, and are based on subgroup screening only. Among the estimation methods

in which individuals charts are employed, maximum efficiency is observed in Method V, closely followed by Method IV.A6, IV.B3 and III.6, sorted with respect to the decreasing efficiencies. It should be noted that Method III estimator falls short slightly at low and moderate contamination densities, and its inferiority is more apparent at higher contamination density (Figure B.1b). This can be explained on the basis of differences in reweighting algorithms: individuals chart are constructed using location estimates of each subgroup, which are estimated separately in Method III, and can still be influenced by outliers, while a single location estimate is used in Method IV-V. It should further be noted that Method IV.A estimator has higher efficiency than Method IV.B, possibly because partioning the total skipping level into screening subgroups and individual observations in Method IV.B yields wider CLs, which seems to decrease its efficiency against diffuse outliers.

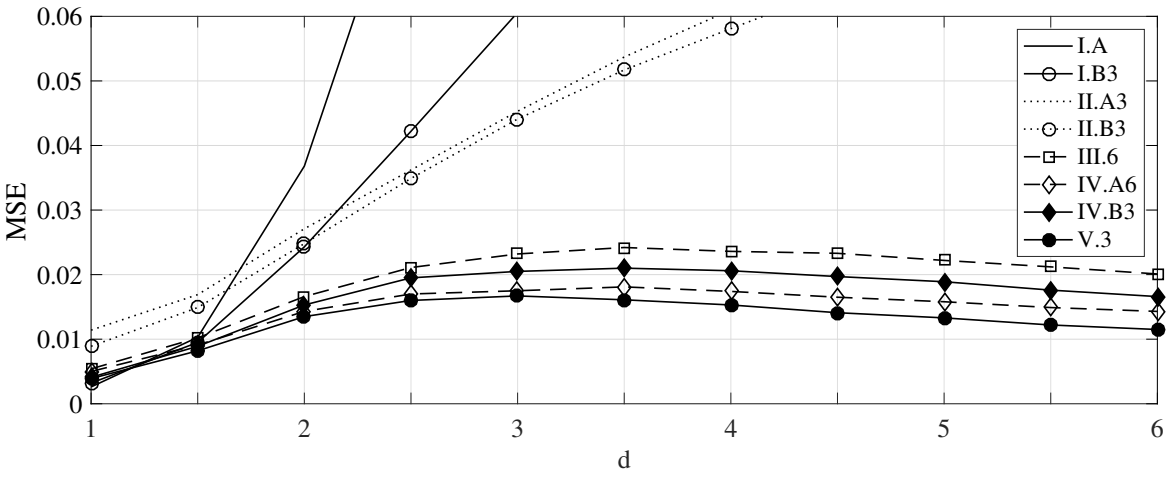

Figure 4.3. Phase I efficiency of the proposed estimators for $m = 50$ under CM 1 with K = 0.15.

The effect of skipping level on Phase I efficiency of Method IV and V estimators can be seen in Figure 4.4. Although lower skipping level yields higher Gaussian efficiency for both methods (Figure 4.1), efficiency of the estimators under contamination is increased when skipping level is inflated, recognized even at low contamination magnitude ($d \approx 2$). Method V estimator is affected less by skipping level, rendering this estimator more favorable.

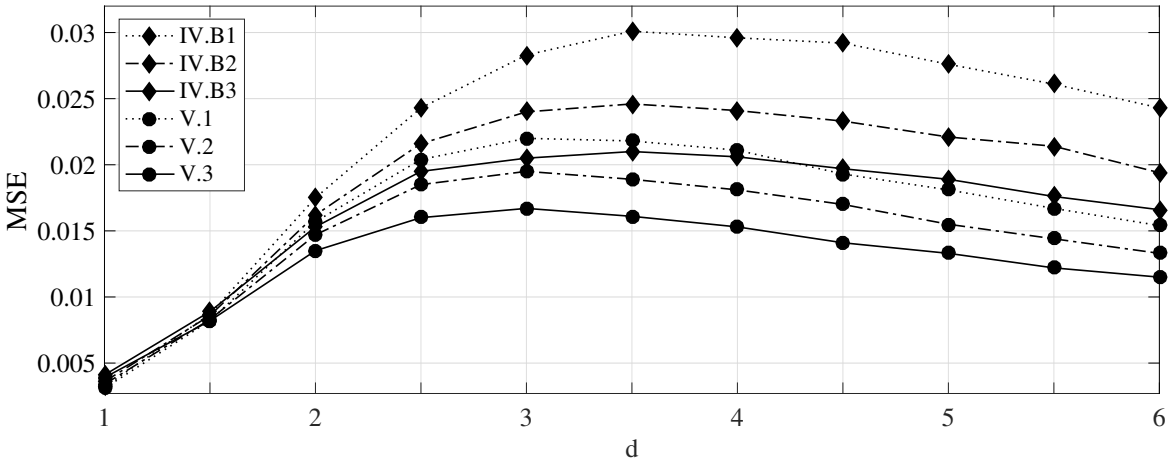

Figure 4.4. Phase I efficiency of the Method IV and V estimators for $m = 50$ under CM 1 with K = 0.15.

When Phase I sampling is under CM 2, results are similar to those under CM 1. The main difference is seen in the marginally superior efficiency of Method IV.B compared to that of Method IV.A, particulary for $d \lesssim 1.5$, which is a realistic range for the outliers in practice. (Figure 4.5 and Figure B.2). A similar picture emerges also for $m = 20$; efficiency of Method IV.A is higher than that of Method IV.B only at the highest contamination level (Figure B.6).

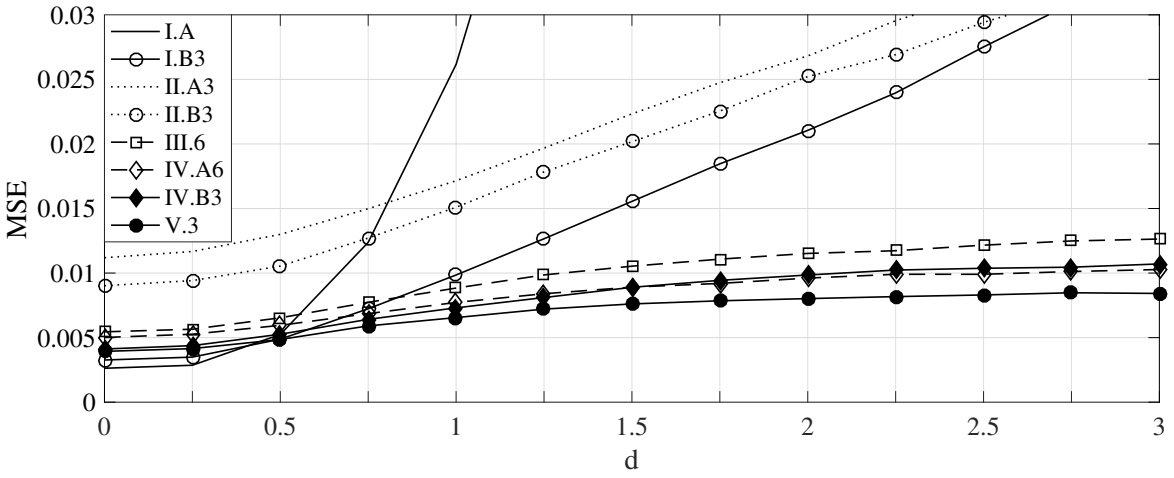

Figure 4.5. Phase I efficiency of the proposed estimators for $m = 50$ under CM 2 with K = 0.15.

Figure 4.6 shows the efficiency of estimators when outliers are allowed only to effect specific subgroups in Phase I samples (CM 3). Contrary to that seen in the presence of diffuse outliers, reweighting significantly improves the efficiency of traditional estimators, since Method I.B estimators are mainly based on screening the subgroups which have different dispersion estimates. Method V and IV.B estimators, on the other hand, yield very close MSE values to those of Method I.B estimator against CM 3 until contamination magnitude $d \lesssim 4$, and will be more efficient than that of Method I.B as contamination magnitude is further increased. Similar behaviour is observed at different contamination densities (Figure B.3), and for smaller number of subgroups (Figure B.7).

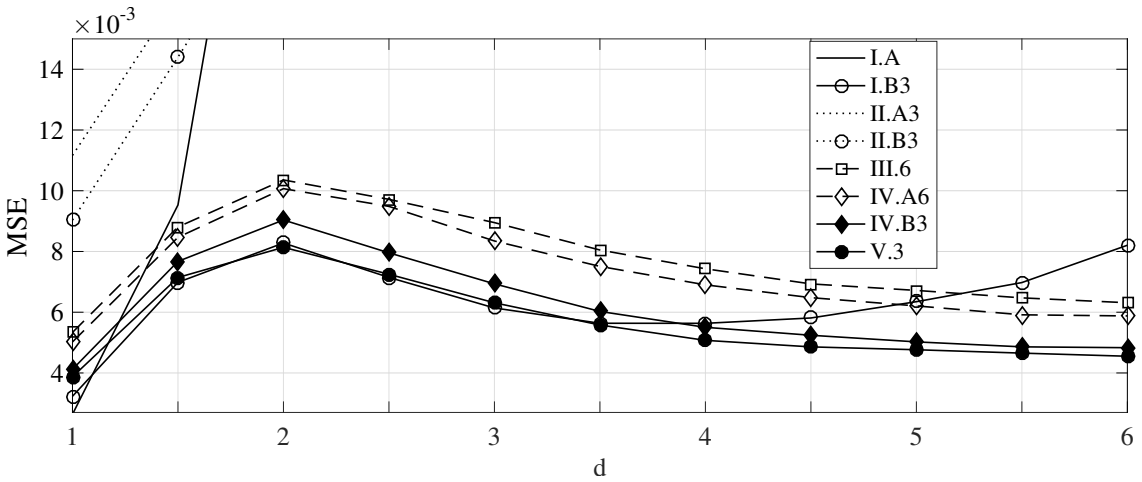

Figure 4.6. Phase I efficiency of the proposed estimators for $m = 50$ under CM 3 with K = 0.15.

When diffuse mean contaminations (CM 4) are present in Phase I samples (Figure 4.7), Method I estimators perform far worse than the other estimators. Method II.A and II.B estimators perform significantly better, but not as well as the estimators using inidividuals chart. Even though all robust estimators have close MSE values until $d \approx 2$ at moderate contamination density, Method III-V estimators will be more resistant to outliers in contrast with Method II as d increases, i.e. MSE values of Method II.A and II.B remain approximately the same when contamination magnitude is further increased $d \gtrsim 3.5$, but those of Method III-V decrease with increasing contamination

magnitude. Against CM 4, Phase I efficiency of Method IV.B estimator is slightly lower for $m = 50$ compared to other estimators, which utilize individuals charts, while being superior to both Method III and IV.A when m equals 20 (Figure B.8).

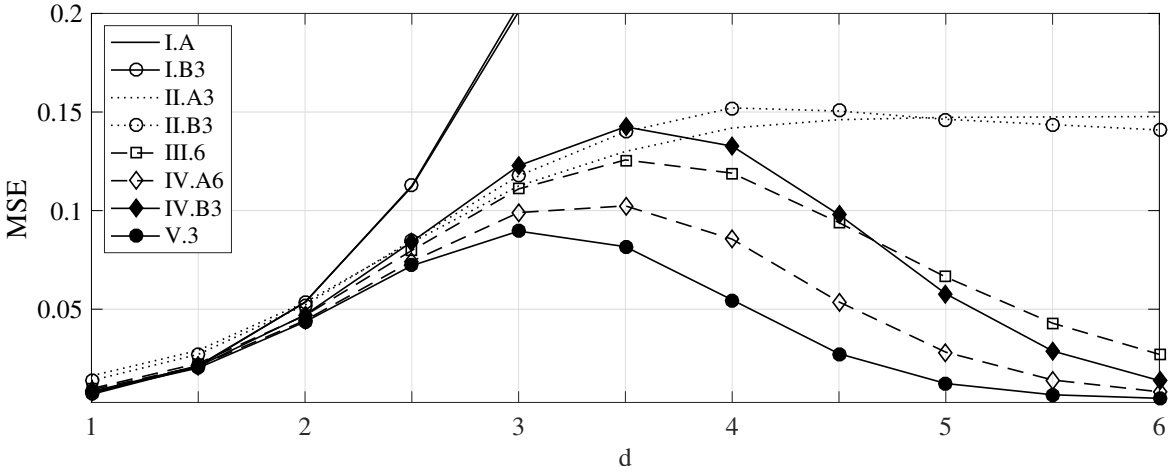

Figure 4.7. Phase I efficiency of the proposed estimators for $m = 50$ under CM 4 with K = 0.15.

Method I-III estimators are not affected by localized mean contaminations, since reweighting steps are based on only subgroup statistics in these estimators. When individuals charts are constructed using a single location estimate, on the other hand, between-subgroup variation may affect the final process dispersion estimate, hence efficiency of only Method IV.A, IV.B and V estimators against CM 5 are examined at $K = 0.15$ (Figure 4.8), $K = 0.10$ and $K = 0.20$ (Figure 4.9a and 4.9b, respectively). While Method IV.B and V estimators are not perturbed significantly, there seems to be a peak around ($d \approx 2$) in the MSE profile of Method IV.A estimator; weakness of Method IV.A estimator lies in including subgroups with different location estimates in individuals charts. It should, further, be noted that MSE of Method III estimators are higher than 5.0x10^{-3} under normal distribution (Figure 4.1), hence efficiencies of Method IV.B and V estimators are still higher than that of Method III estimator under CM 5.

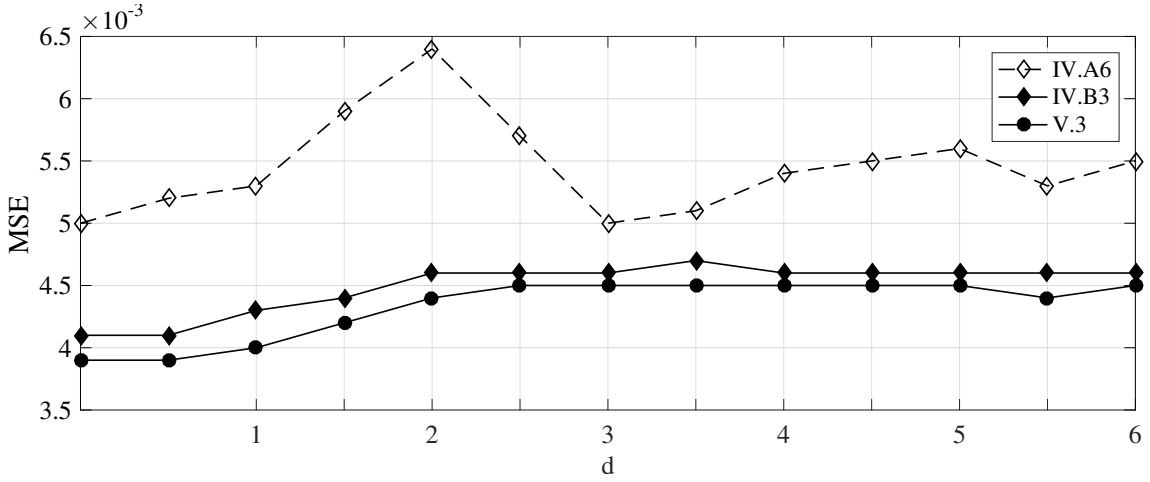

Figure 4.8. Phase I efficiency of the proposed estimators for $m = 50$ under CM 5 with K = 0.15.

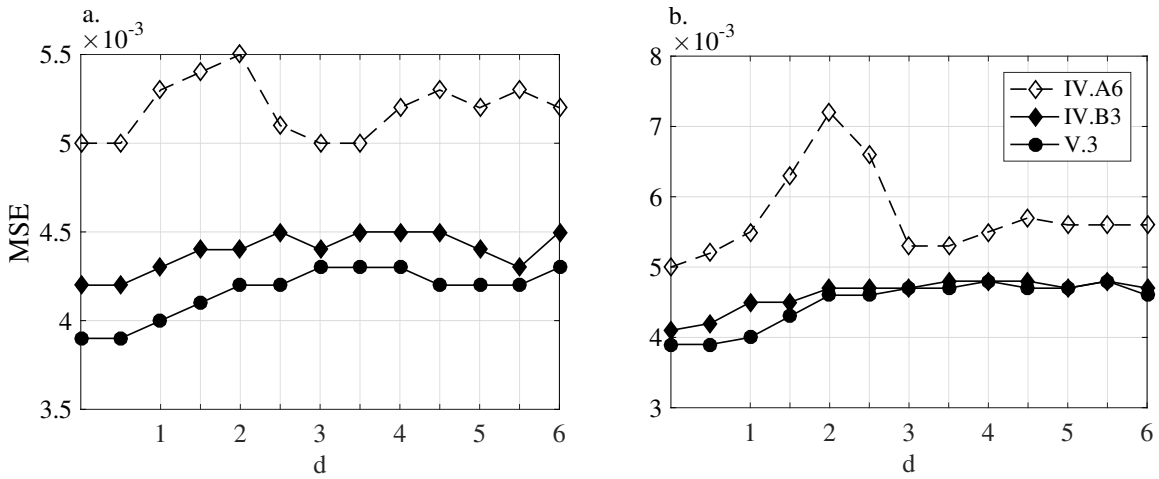

Figure 4.9. Phase I efficiency of the proposed estimators for $m = 50$ under CM 5 with a. K = 0.10 and b. K = 0.20.

Figure 4.10 shows the efficiency of estimators when Phase I samples are subjected to superimposed contamination models (SCM 1-6). Under all SCMs, Method I estimators has the lowest efficiency with the highest MSE. Efficiencies of Method I.B and II estimators vary with respect to contaminated model. While, Method I.B estimators are more efficient when localized variance disturbance is present in SCM (Figure 1a, 1d, 1e), Method II estimators give lower MSE in the presence of individual outliers in subgroups (Figure 1b, 1c, 1f). Except in the presence of SCM 4, efficiency of Method

III-V estimators are higher than those of Method I and II estimators. Comparing estimators using individuals charts, Method V estimator yields the lowest MSE against SCM 1, 3 and 6, while Method IV.A has the highest efficiency against SCM 2 and SCM 5. It should be noted that increase in skipping level improves the robustness of the estimators, and maximum efficiency is observed at $\alpha = 0.04$ for $m = 50$, and frequently shifts to $\alpha = 0.02$ when m equals 20 (Figure B.10).

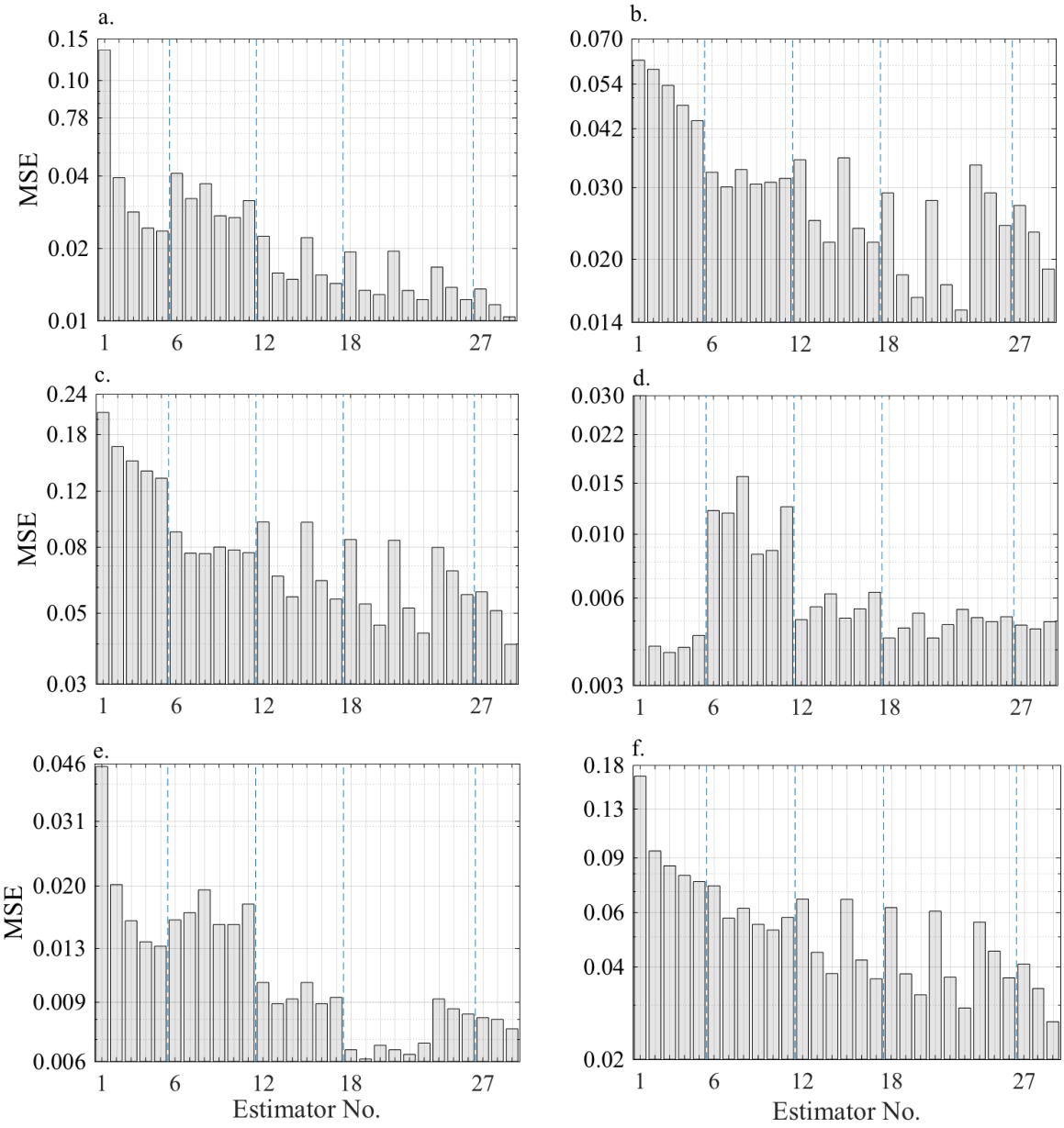

Figure 4.10. Phase I efficiency of the proposed estimators for $m = 50$ under a. SCM 1, b. SCM 2, c. SCM 3, d. SCM 4, e. SCM 5, f. SCM 6.

4.2. Derivation of Phase II Control Limits

Phase II CLs for the proposed Phase I estimators are determined using Equation 2.40. Process dispersion parameter in Phase II samples is estimated by $S_n/c_4(n)$ for all charts. U_n and L_n adjusted so as to make $ARFS$ equal to 0.0027 with equal probability of exceeding the upper and lower control limits. 50,000 computer simulations are used to determine multipliers. Tables 4.2 and 4.3 show U_n and L_n for $m = 50$ and $m = 20$, respectively.

Table 4.2. Multipliers U_n and L_n to determine Phase II control limits.

	$m = 50$		$m = 20$	
Method	L_n	U_n	L_n	U_n
I.A	0.172	2.285	0.171	2.356
I.B1	0.172	2.289	0.171	2.362
I.B2	0.172	2.295	0.171	2.372
I.B3	0.172	2.300	0.171	2.398
I.B4	0.172	2.310	0.170	2.415
II.A1	0.171	2.344	0.169	2.507
II.A2	0.171	2.384	0.169	2.585
II.A3	0.171	2.441	0.168	2.709
II.B1	0.172	2.358	0.169	2.559
II.B2	0.171	2.358	0.169	2.581
II.B3	0.170	2.407	0.168	2.703
III.1	0.172	2.302	0.171	2.398
III.2	0.172	2.302	0.170	2.467
III.3	0.171	2.342	0.170	2.550
III.4	0.172	2.296	0.171	2.396
III.5	0.172	2.325	0.170	2.472
III.6	0.172	2.345	0.170	2.546

Table 4.3. Multipliers U_n and L_n to determine Phase II control limits cont.

	$m = 50$		$m = 20$	
Method	L_n	U_n	L_n	U_n
IV.A1	0.172	2.296	0.171	2.391
IV.A2	0.172	2.317	0.171	2.451
IV.A3	0.172	2.332	0.171	2.504
IV.A4	0.172	2.300	0.171	2.391
IV.A5	0.172	2.317	0.170	2.453
IV.A6	0.172	2.338	0.170	2.515
IV.B1	0.172	2.295	0.171	2.413
IV.B2	0.172	2.306	0.171	2.413
IV.B3	0.172	2.321	0.171	2.467
V.1	0.172	2.298	0.171	2.383
V.2	0.172	2.302	0.171	2.403
V.3	0.172	2.318	0.171	2.442

4.3. Comparison of Phase II Performances of Proposed Estimators

Phase II performances of the estimators are assessed by $ARFS$ (marginal probability of a false signal from an in-control process), $mARL_0$ (marginal average run length of an in-control process), ARS (average rate of signal under a disturbance in Phase II), and $mARL$ (marginal average run length under a disturbance in Phase II). When the process is in-control, i.e $\gamma = 1$, it is desired to obtain $mARL_0$ equal to or higher than ~ 370, and $ARFS$ equal to or lower than 0.0027. Furthermore, it is desired that $SDRL$ to be close to its intended value ~ 370 for an in-control process. In the out-of-control situation, i.e. $\gamma = 1.5$, the change in the process variation is desired to be detected promptly, so $mARL$ should be as low as possible with high ARS. In addition to marginal performance metrics, $cARL$ is computed for $0.10, 0.50$ and 0.90 quantiles of the $\hat{\sigma}^I$ distributions, denoted with $cARL_{.1}$, $cARL_{.5}$ and $cARL_{.9}$, respectively, to examine the best, moderate and worst performance scenarios. Visual representation

of the performance measures for $K = 0.15$ and $m = 50$ are given in Figure 4.11-4.18, in which bar plots indicate the $mARL_0/mARL$ values, and lines denote $ARFS/ARS$ values. Phase II performances of S-Charts using all estimation methods are shown for $m = 50$ in Appendix C (Table C.1-C.44), for $m = 20$ in Appendix D (Table D.1-D.44)

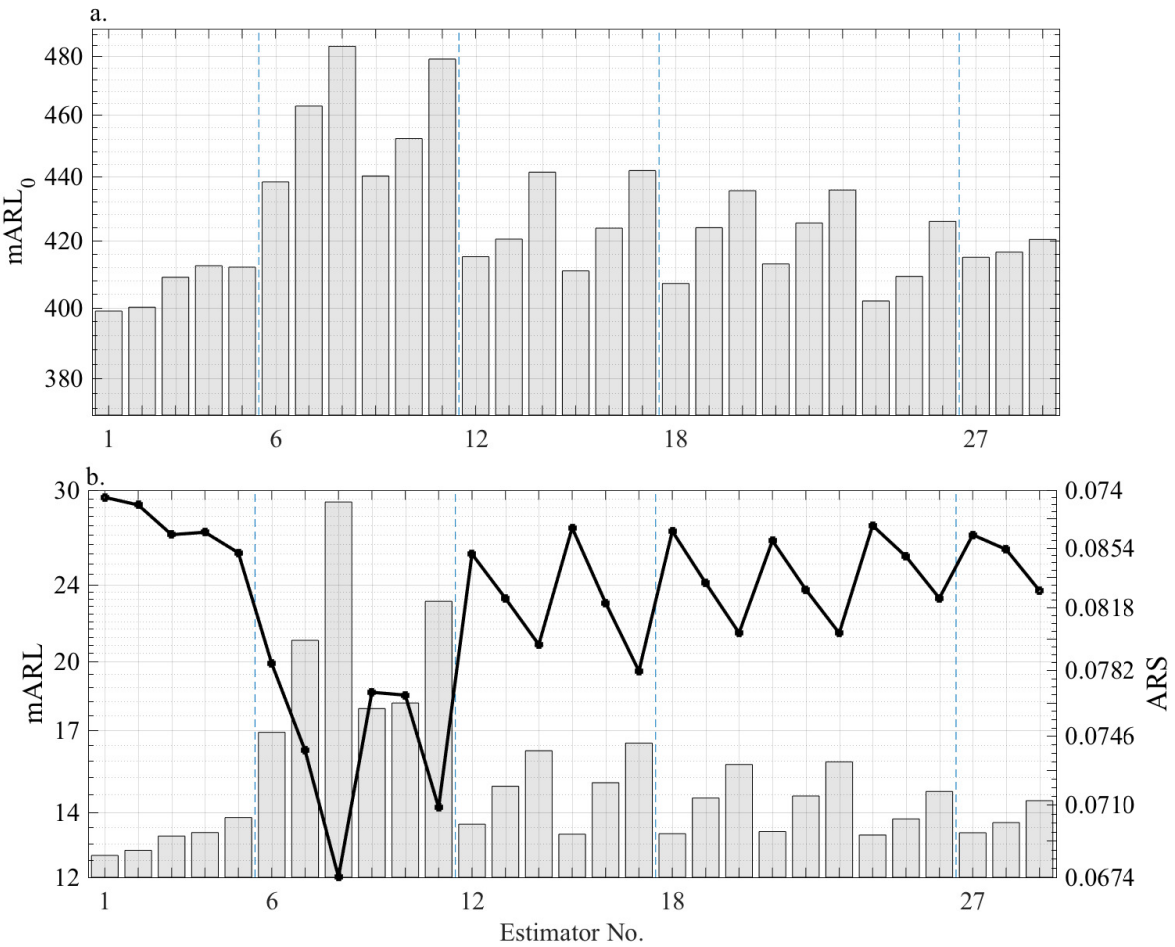

Figure 4.11. Phase II performance of S-Charts for $m = 50$ when a. $\gamma = 1$ and b. $\gamma = 1.5$ using normally distributed Phase I data.

First, the case, in which the process follows a normal distribution and the Phase I samples are not contaminated, is considered. Regardless of the sample size, $mARL_0$ values of all estimators are higher than 370 (Figure 4.11.a). When the process is out-of-control in Phase II, power of S_n estimator is slightly decreased via reweighting. It is seen that reweighting procedure improves the power of S-Charts using Method II estimators with $HD_{.5}$ estimator, but not with ML estimator. S-Charts using MAD_n

yield low power and high $SDRL$, hence MAD_n estimator is not found to be convenient, particularly for low subgroup size (Table D.2). Utilizing individuals charts in Method III-V estimators gives sensitivity (Figure 4.11a, Table C.1) and specificity (Figure 4.11b, Table C.2) values comparable to those by Method I.A, while Method II estimators are remarkably less powerful. When m is equal to 20, higher $mARL_0$ and $SDRL$ values are obtained, since variance of the average run lengths inflates due to low sample size (Table D.1, D.2).

In the presence of outliers in Phase I samples, S-Charts constructed using standard deviation give $mARL_0$ higher than 370 only when contamination density and magnitude are low, i.e. $K = 0.10$ and $d = 2$ for CM 1, 3-5 and $d = 0.75$ for CM 2 (Table C.3, C.9, C.15, C.27) , but $mARL_0$ decreases to values lower than 370, when contamination is increased (except under CM 2). Reweighting traditional estimators, on the other hand, has a significantly positive effect on the sensitivity of S-Charts, when outliers come from an asymmetric long-tailed distribution (CM 2), or specific subgroups are affected by outliers (CM 3), e.g. $mARL_0$ values are equal to 225.2, 418.4, 259.4 and 187.3 when Method I.A estimator is employed, and equal to 368.5, 452.4, 452.4 and 195.8, when Method I.B4 estimator is used in S-Charts using Phase I data under CM 1-4, respectively, at moderate contamination density (Table 4.4). Increasing skipping level in Method I.B generally yields higher $mARL_0$ and lower $ARFS$ (except under CM 4), and the best performance is attained at $\alpha = 0.025 - 0.050$ for both $m = 50$ and $m = 20$. Reweighting has a substantial effect also on the power of S-Charts constructed by S_n in detecting shifts in the process variation (except under CM 4), particularly against CM 1 and CM 3, e.g. mARL of S-Charts using Phase I data under CM1 and CM 3 with moderate contamination is equal to 246.5 and 159.1 when Method I.A estimator is used, but decreases to 136.0 and 20.0 even with the low skipping amount ($\alpha = 0.0027$, Table 4.4). It should be noted that S-Charts constructed using Method I estimators may become ARL-biased when Phase data is under CM 4 for the given contamination level, i.e. $mARL_0$ may be lower than $mARL$ [24]. Reweighting S_n estimator has no curative effect on the status of ARL-biased S-Charts, and increase in power is negligibly small in the presence of CM 4 in Phase I sampling (Table 4.4).

Table 4.4. Phase II performance of S-Charts constructed from Method I estimators for $m = 50$ under CM 1- CM 4 (d=3, K=0.15 for CM 1,CM 3-CM 4 and d=1, K = 0.15 for CM 2).

CM	Method	$ARFS$	$mARL_0$	$SDRL$	ARS	$mARL$	$SDRL$
CM 1	**I.A**	0.0048	225.2	244.2	0.0063	246.5	312.7
	I.B1	0.0037	301.8	328.6	0.0150	136.0	211.3
	I.B2	0.0033	331.8	357.6	0.0193	109.1	180.5
	I.B3	0.0031	354.0	378.3	0.0226	93.1	160.8
	I.B4	0.0030	368.5	393.6	0.0232	90.4	157.8
CM 2	**I.A**	0.0025	418.4	428.4	0.0357	40.4	59.9
	I.B1	0.0023	451.0	458.0	0.0508	25.5	33.5
	I.B2	0.0023	456.7	464.4	0.0533	24.3	31.6
	I.B3	0.0023	451.2	460.1	0.0551	23.7	31.0
	I.B4	0.0023	452.4	461.9	0.0549	24.3	32.3
CM 3	**I.A**	0.0041	259.4	271.8	0.0089	159.1	204.9
	I.B1	0.0025	430.5	443.7	0.0660	20.0	27.1
	I.B2	0.0024	440.2	455.7	0.0697	18.2	23.2
	I.B3	0.0024	444.8	458.9	0.0679	18.5	23.2
	I.B4	0.0023	452.4	463.6	0.0624	20.4	25.8
CM 4	**I.A**	0.0056	187.3	196.4	0.0038	323.7	364.8
	I.B1	0.0056	188.8	199.3	0.0038	329.7	372.7
	I.B2	0.0056	189.4	202.1	0.0040	328.7	374.4
	I.B3	0.0056	193.9	210.0	0.0041	337.9	387.8
	I.B4	0.0056	195.8	216.1	0.0044	338.1	394.2

When S-Charts are based on Method II estimators, $ARFS$ values do not exceed 0.0027 at low contamination level (Table C.3, C.9, C.15, C.21), while they may be higher or lower than 0.0027, depending on the underlying distribution of contamination in Phase I samples, at higher outlier densities. When outliers come from an asymmetric long-tailed distribution (CM 2) as seen in Table C.11 and C.13, or specific subgroups are affected by outliers (CM 3) as seen in Table C.17 and C.19, at relatively higher contamination densities, $ARFS$ is generally lower than 0.0027. While S-Charts with higher $mARL_0$ values than the nominal value do not present a problem in application,

power of these charts are undesirably low compared to those constructed by Method III-V estimators. S-Charts based on Method II estimators yield higher power than those based on Method I.B estimators in presence of CM 1 and CM 4 in Phase I samples (Table 4.5).

Table 4.5. Phase II performance of S-Charts constructed from Method II estimators for $m = 50$ under CM 1- CM 4 (d=3, K=0.15 for CM 1,CM 3-CM 4 and d=1, K = 0.15 for CM 2).

CM	**Method**	$ARFS$	$mARL_0$	$SDRL$	ARS	$mARL$	$SDRL$
CM 1	**II.A1**	0.0028	382.0	405.4	0.0209	90.1	146.0
	II.A2	0.0026	412.8	441.2	0.0228	97.6	170.2
	II.A3	0.0027	421.3	460.0	0.0213	131.4	238.6
	II.B1	0.0028	389.6	414.5	0.0238	87.2	147.8
	II.B2	0.0027	398.8	424.0	0.0245	87.2	150.2
	II.B3	0.0026	421.5	454.4	0.0231	111.7	201.2
CM 2	**II.A1**	0.0022	475.1	488.5	0.0490	30.7	46.4
	II.A2	0.0023	470.9	490.6	0.0480	37.3	65.1
	II.A3	0.0023	486.2	516.1	0.0440	52.8	107.4
	II.B1	0.0023	461.3	477.8	0.0502	31.6	49.3
	II.B2	0.0023	467.0	483.9	0.0504	32.2	51.0
	II.B3	0.0023	489.9	515.4	0.0464	42.3	76.7
CM 3	**II.A1**	0.0025	422.1	440.8	0.0262	64.9	101.5
	II.A2	0.0023	461.8	482.4	0.0341	56.6	97.8
	II.A3	0.0024	456.4	488.4	0.0283	90.6	172.0
	II.B1	0.0024	456.6	475.3	0.0477	36.9	63.2
	II.B2	0.0024	462.8	484.4	0.0510	34.9	60.8
	II.B3	0.0024	468.9	498.7	0.0411	59.3	119.9
CM 4	**II.A1**	0.0042	263.7	291.6	0.0073	246.2	329.3
	II.A2	0.0039	293.6	331.1	0.0088	248.0	350.2
	II.A3	0.0039	303.9	349.9	0.0089	285.4	403.4
	II.B1	0.0042	267.7	299.7	0.0083	245.8	336.7
	II.B2	0.0041	278.1	313.0	0.0091	236.4	330.1
	II.B3	0.0039	301.6	346.8	0.0091	273.2	385.9

Reweighting Method II.A estimators at $\alpha = 0.025$ yields relatively higher ARS values, but improvement is not remarkable (maximum relative percentage improvement $\approx 8.3\%$ except against CM 3). Performances of robust estimators can be ordered as $MS_n + HD_{.5}, MS_n + ML, MAD_n + HD_{.5}$, from best to the worse. MAD_n is neglected for further study due to its low power and high $SDRL$. It should be noted that, even though S-Charts are very efficient under CM 4 at low level of contamination, their efficiency are lost when contamination magnitude and density are increased. S-Charts, which are constructed using Method II estimators, are not ARL-biased at moderate contamination density, but not satisfactory, because the power of S-Charts and efficiency of estimators against CM 4 are very low (Table 4.5).

Utilizing individuals chart to screen individual outliers in subgroups generally yields higher $mARL_0$ compared to Method I.B estimators, and slightly lower compared to Method II estimators, but significantly improves the power of S-Charts compared to those constructed using Method I and II estimators (Table 4.6). There is no remarkable difference between the methods which use $HD_{.5}$ and ML estimators, but $HD_{.5}$ and MS_n location and dispersion estimators are used in S-Charts based on Method IV.B and V estimators due to having relatively higher Phase I efficiency (Table 4.6, 4.7). Using single location estimate in the construction of individuals chart in Method IV-V, rather than using subgroup location estimates in Method III, yields higher $mARL_0$ and lower $mARL$ against all CMs. In all CMs, the highest power is attained by Method V estimator, at generally high skipping level, except against CM 3. Method III estimator, on the other hand, always falls behind the Method IV and V estimators. Superiority of Method IV.B over IV.A, on the other hand, varies with respect to contamination model, i.e. Method IV.A estimator attains higher power when Phase I sampling is subjected to CM 1 and CM 4; however, Method IV.B estimator is more powerful against CM 2 and CM 3. Phase II performances of Method III-V estimators are comparable to the results for $m = 20$. Although the highest power is again ranked from best the worst as Method V, IV and III, $\alpha = 0.020$ is more favorable for $m = 20$. Consequently, higher skipping level is suggested when subgroup size is high, i.e. $m = 50$, however skipping level should be decreased for datasets having smaller subgroup size, i.e $m = 20$, yet still an α value much higher than .0027 is recommended (Table 4.8).

Table 4.6. Phase II performance of S-Charts constructed from Method III estimators for $m = 50$ under CM 1- CM 4 (d=3, K=0.15 for CM 1,CM 3-CM 4 and d=1, K = 0.15 for CM 2).

CM	**Method**	$ARFS$	$mARL_0$	$SDRL$	ARS	$mARL$	$SDRL$
CM 1	**I.B4**	0.0030	368.5	393.6	0.0232	90.4	157.8
	II.B2	0.0027	398.8	424.0	0.0245	87.2	150.2
	III.1	0.0027	392.0	408.4	0.0288	59.4	96.8
	III.2	0.0025	418.4	432.9	0.0360	47.2	77.1
	III.3	0.0024	442.6	459.9	0.0368	47.8	78.7
	III.4	0.0027	387.9	404.1	0.0288	59.0	95.2
	III.5	0.0025	423.7	438.2	0.0365	46.6	75.6
	III.6	0.0024	434.7	451.1	0.0372	47.8	79.1
CM 2	**I.B4**	0.0023	452.4	461.9	0.0549	24.3	32.3
	II.B2	0.0023	467.0	483.9	0.0504	32.2	51.0
	III.1	0.0023	457.4	465.6	0.0575	22.1	27.9
	III.2	0.0023	458.8	472.2	0.0598	22.4	29.3
	III.3	0.0023	469.5	486.5	0.0588	24.0	33.1
	III.4	0.0023	449.9	458.5	0.0587	21.5	27.0
	III.5	0.0023	454.4	467.6	0.0592	22.6	29.5
	III.6	0.0023	465.9	482.3	0.0576	24.6	34.0
CM 3	**I.B4**	0.0023	452.4	463.6	0.0624	20.4	25.8
	II.B2	0.0024	462.8	484.4	0.0510	34.9	60.8
	III.1	0.0023	450.7	462.5	0.0552	25.3	35.2
	III.2	0.0024	450.9	467.7	0.0606	23.3	32.7
	III.3	0.0023	469.5	486.5	0.0588	24.0	33.1
	III.4	0.0024	444.0	456.5	0.0578	24.0	33.5
	III.5	0.0024	452.0	469.2	0.0619	23.0	32.5
	III.6	0.0024	457.0	476.5	0.0614	23.8	34.7
CM 4	**I.B4**	0.0056	195.8	216.1	0.0044	338.1	394.2
	II.B2	0.0041	278.1	313.0	0.0091	236.4	330.1
	III.1	0.0049	221.8	242.9	0.0060	272.1	340.0
	III.2	0.0043	264.8	295.8	0.0094	220.6	307.0
	III.3	0.0040	289.4	325.8	0.0111	209.1	305.9
	III.4	0.0050	220.8	242.0	0.0062	265.0	331.3
	III.5	0.0043	266.1	298.3	0.0098	218.7	307.6
	III.6	0.0040	287.0	323.3	0.0116	210.0	310.1

Table 4.7. Phase II performance of S-Charts constructed from Method IV.A estimators for $m = 50$ under CM 1- CM 4 (d=3, K=0.15 for CM 1,CM 3-CM 4 and d=1, K = 0.15 for CM 2).

CM	Method	$ARFS$	$mARL_0$	$SDRL$	ARS	$mARL$	$SDRL$
CM 1	**III.6**	0.0024	434.7	451.1	0.0372	47.8	79.1
	IV.A1	0.0025	414.8	428.7	0.0346	46.5	73.7
	IV.A2	0.0024	432.4	444.3	0.0413	37.8	58.8
	IV.A3	0.0024	441.7	454.8	0.0421	37.7	58.9
	IV.A4	0.0024	441.7	454.8	0.0421	37.7	58.9
	IV.A5	0.0024	436.1	448.3	0.0413	37.9	58.6
	IV.A6	0.0023	453.2	468.2	0.0431	37.6	59.3
CM 2	**III.6**	0.0023	465.9	482.3	0.0576	24.6	34.0
	IV.A1	0.0023	453.3	462.4	0.0613	20.3	25.1
	IV.A2	0.0023	458.2	472.0	0.0621	20.9	26.6
	IV.A3	0.0024	458.1	473.3	0.0616	22.1	29.8
	IV.A4	0.0023	451.9	460.8	0.0608	20.4	25.3
	IV.A5	0.0023	463.5	477.2	0.0617	21.2	27.2
	IV.A6	0.0024	462.3	478.6	0.0610	22.4	29.7
CM 3	**III.6**	0.0024	457.0	476.5	0.0614	23.8	34.7
	IV.A1	0.0023	446.0	457.6	0.0560	24.9	35.0
	IV.A2	0.0024	442.3	456.8	0.0610	22.9	31.7
	IV.A3	0.0024	453.2	470.3	0.0612	23.1	32.5
	IV.A4	0.0024	439.4	451.1	0.0564	24.9	35.2
	IV.A5	0.0025	441.8	458.1	0.0628	22.4	31.4
	IV.A6	0.0024	452.3	471.1	0.0627	23.0	33.1
CM 4	**III.6**	0.0040	287.0	323.3	0.0116	210.0	310.1
	IV.A1	0.0049	223.7	246.4	0.0064	262.7	329.9
	IV.A2	0.0042	268.4	300.5	0.0106	212.3	303.4
	IV.A3	0.0039	292.8	328.3	0.0130	192.0	290.0
	IV.A4	0.0048	227.1	249.4	0.0066	260.8	330.6
	IV.A5	0.0041	276.2	309.9	0.0115	200.8	293.0
	IV.A6	0.0037	308.3	346.7	0.0146	179.1	278.3

Table 4.8. Phase II performance of S-Charts constructed from Method IV.B and V estimators for $m = 50$ under CM 1- CM 4 (d=3, K=0.15 for CM 1,CM 3-CM 4 and d=1, K = 0.15 for CM 2).

CM	**Method**	$ARFS$	$mARL_0$	$SDRL$	ARS	$mARL$	$SDRL$
CM 1	**III.6**	0.0024	434.7	451.1	0.0372	47.8	79.1
	IV.A6	0.0023	453.2	468.2	0.0431	37.6	59.3
	IV.B3	0.0024	435.0	448.4	0.0410	39.7	63.8
	V.3	0.0023	445.2	456.8	0.0450	33.5	50.3
CM 2	**III.6**	0.0023	465.9	482.3	0.0576	24.6	34.0
	IV.A6	0.0024	462.3	478.6	0.0610	22.4	29.7
	IV.B3	0.0023	460.5	474.2	0.0613	21.3	27.3
	V.3	0.0023	453.9	467.1	0.0637	20.1	25.4
CM 3	**III.6**	0.0024	457.0	476.5	0.0614	23.8	34.7
	IV.A6	0.0024	452.3	471.1	0.0627	23.0	33.1
	IV.B3	0.0025	443.5	461.5	0.0666	20.1	26.9
	V.3	0.0025	440.6	459.0	0.0695	18.9	24.9
CM 4	**III.6**	0.0040	287.0	323.3	0.0116	210.0	310.1
	IV.A6	0.0037	308.3	346.7	0.0146	179.1	278.3
	IV.B3	0.0042	271.0	306.5	0.0113	210.9	303.5
	V.3	0.0036	313.3	346.6	0.0157	153.5	242.8

Visual representations of the Phase II performances of the proposed estimators are given in Figure 4.12-4.15 for Phase I samples under main CMs, and in Figure 4.16 and 4.17 for Phase I samples under SCMs. While S-Charts based on Method I.A estimator yield $mARL_0$ values lower than the nominal value, reweighting used in Method I.B increases its specificity. Power of S-Charts constructed using Method I.B estimators is considerably low when Phase I samples are subjected to CM 4 (Figure 4.15b, 4.16d, 4.16f, 4.17f), and expectedly high in the presence of CM 3 (Figure 4.14b, 4.16b, 4.17b).

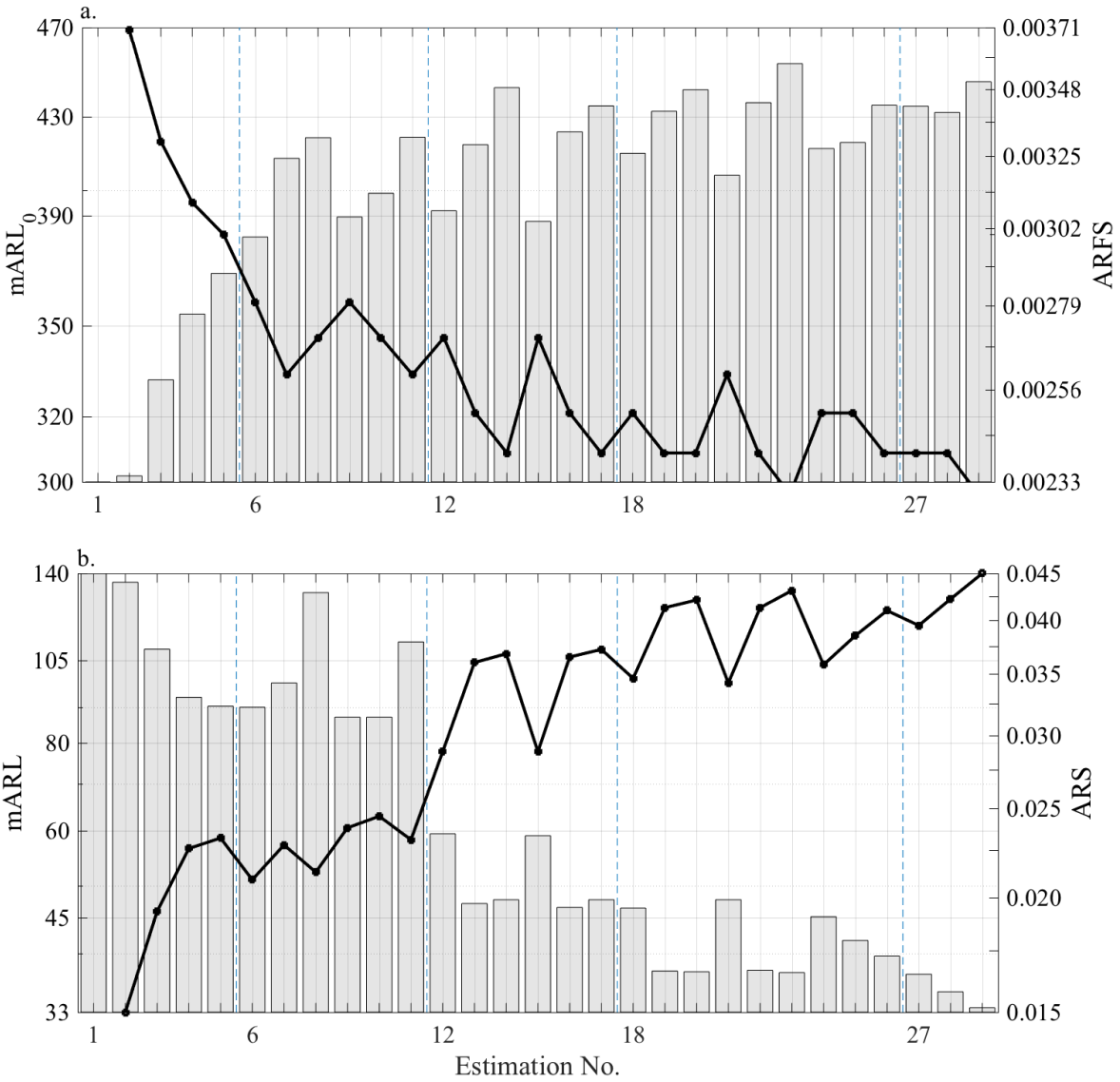

Figure 4.12. Phase II performance of S-Charts for $m = 50$ when a. $\gamma = 1$ and b. $\gamma = 1.5$ using Phase I data under CM 1 (d=3, K=0.15).

It should be noted that, S-Charts constructed using Method I.B estimators are not efficient in detecting perturbations in variance using Phase I data under diffuse mean contamination at even very low contamination density; for 92% of observations in the subgroups equally likely (at a probability of 0.08) to come from a distribution, which have higher mean than zero in SCM 6, power of Method I.B estimators is unacceptably low, rendering application of reweighted traditional estimators in constructing S-Charts useless.

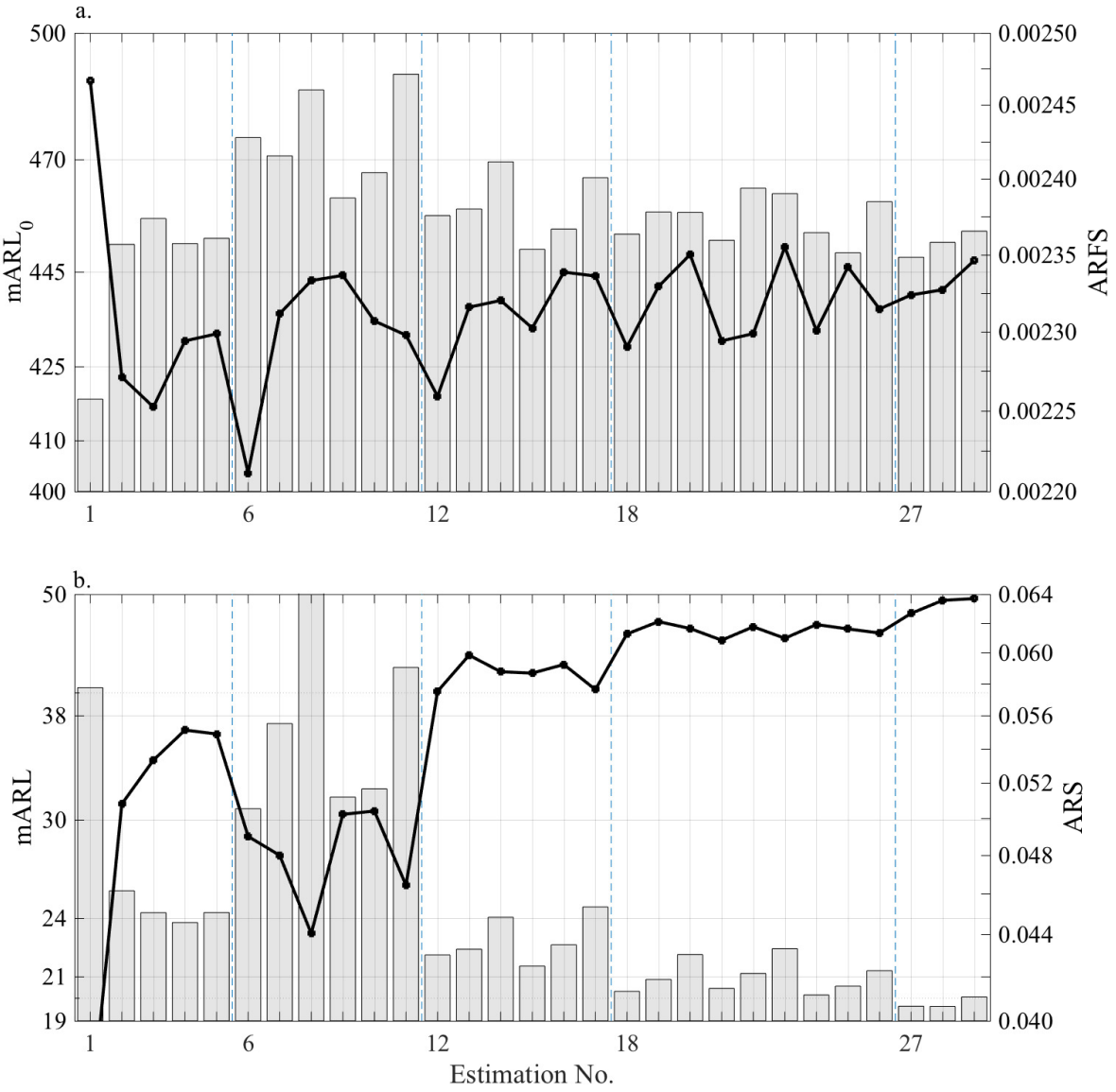

Figure 4.13. Phase II Performance of S-Charts for $m = 50$ when a. $\gamma = 1$ and b. $\gamma = 1.5$ using Phase I data under CM 2 (d=1, K=0.15)

S-Charts based on Method II estimators yield lower $mARL$ values than those based on Method I.B estimators, when Phase I samples are under only diffuse symmetric (Figure 4.12), diffuse mean contaminations (Figure 4.15), their combinations (Figure 4.16f), but higher $mARL$ values higher than those based on Method I.B estimators in the presence of CM 2 (Figure 4.13). Variation in the power of S-Charts based on Method II and Method I.B estimators with respect to different contaminations is possibly due to difference in tail length of the asymmetric distributions; distribution of

Phase I samples under CM 2 and CM 4 are long- and short-tailed, respectively, showing that reweighted traditional estimators may be more effective compared to robust estimators in long tailed asymmetric error distributions.

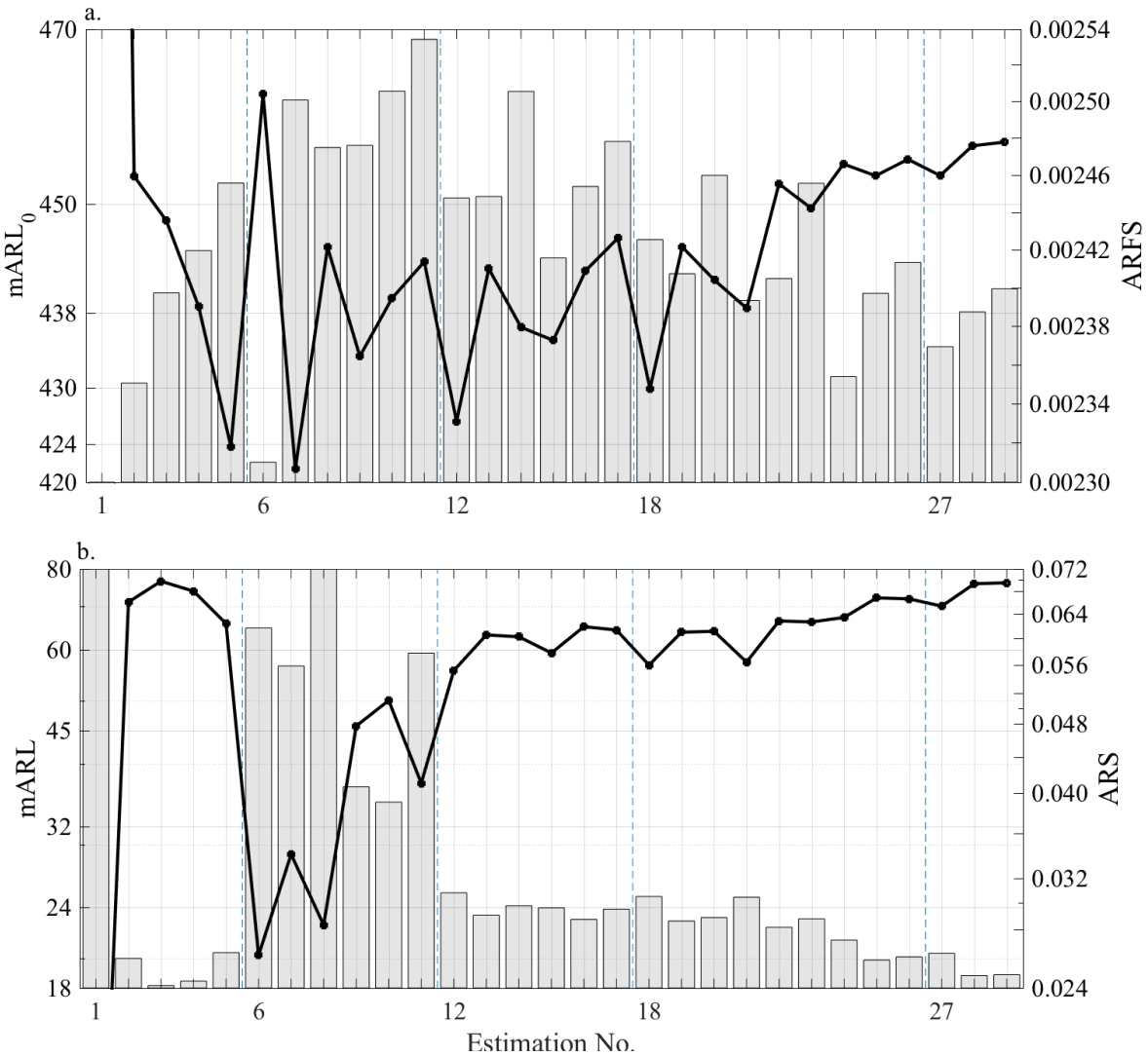

Figure 4.14. Phase II performance of S-Charts for $m = 50$ when a. $\gamma = 1$ and b. $\gamma = 1.5$ using Phase I data under CM 3 (d=3, K=0.15).

S-Charts based on individuals charts yield $mARL_0$ values, which are always higher than the nominal value, at high skipping level in the presence of SCM 1-6 in Phase I samples. It is seen that use of single process location estimate instead of subgroup location estimates in individuals charts increases the power of S-Charts

against all SCMs. On the other hand, superiority of Method IV.B over IV.A varies with respect to contamination model: Method IV.A yields lower $mARL$ in the existence of light localized mean contamination. (Figure 4.16d, 4.17b, 4.17d).

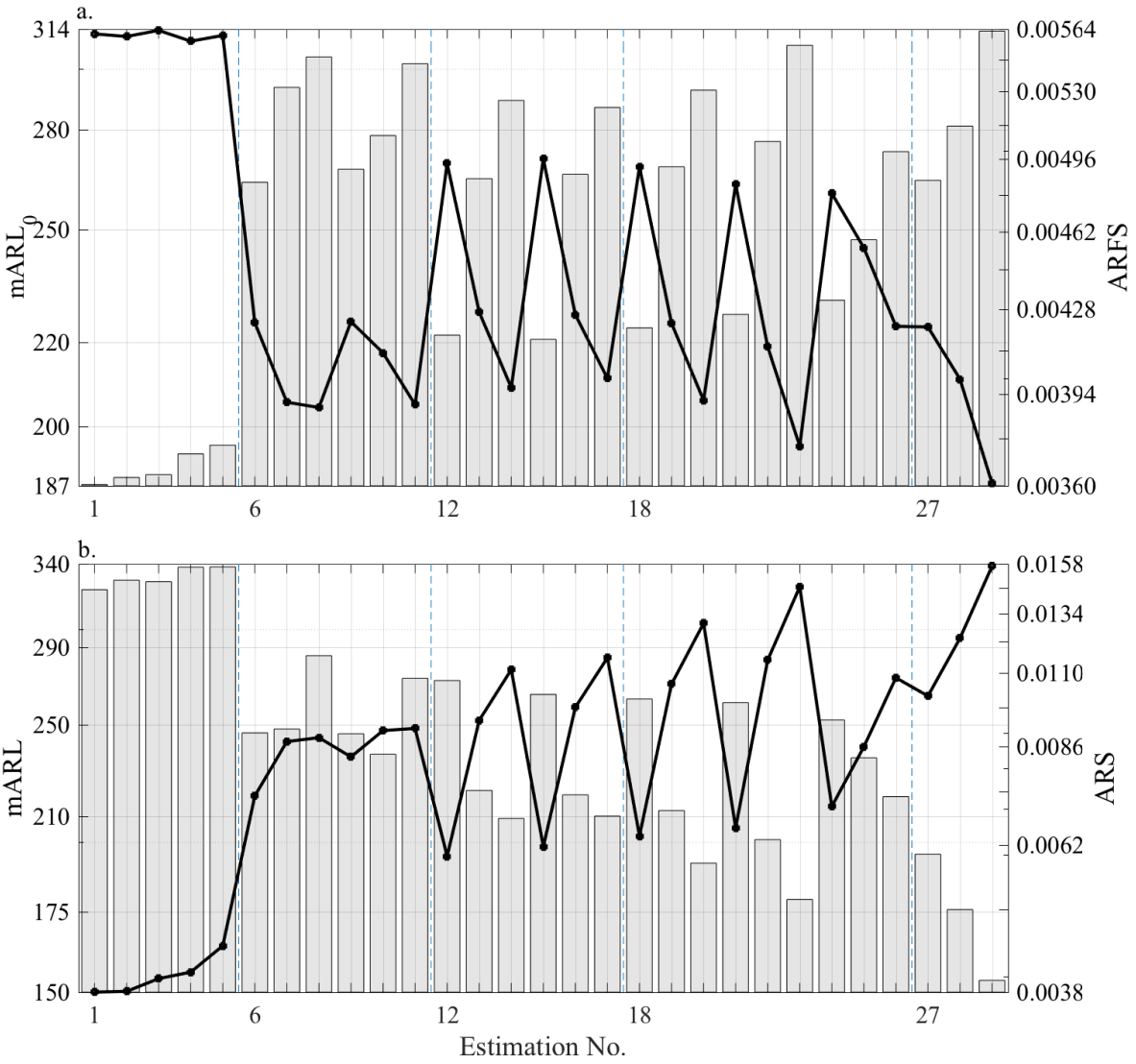

Figure 4.15. Phase II performance of S-Charts for $m = 50$ when a. $\gamma = 1$ and b. $\gamma = 1.5$ using Phase I data under CM 4 (d=3, K=0.15).

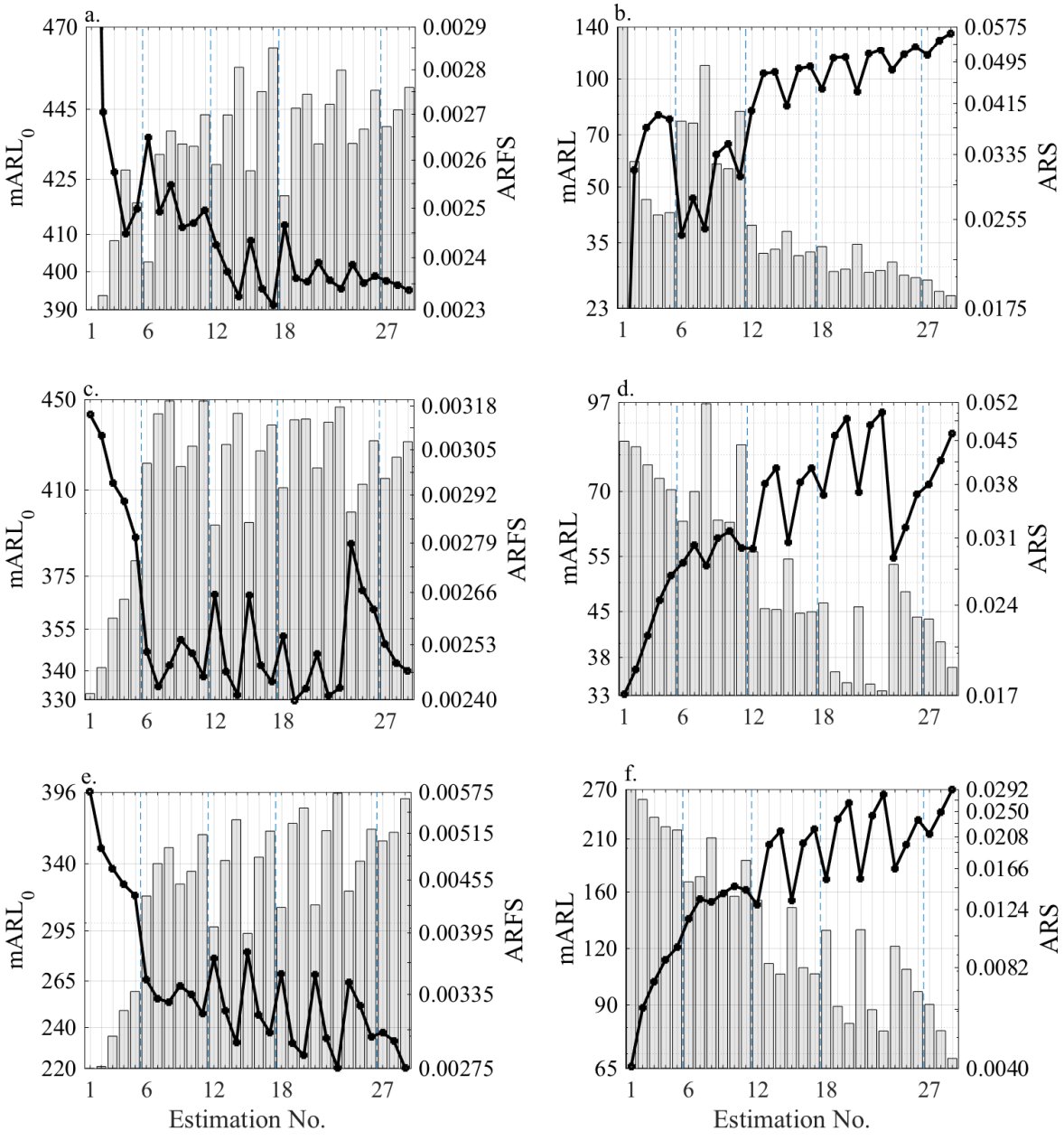

Figure 4.16. Phase II performance of S-Charts for $m = 50$ when a. $\gamma = 1$ and b. $\gamma = 1.5$ using Phase I data under a. SCM 1 ($\gamma = 1$) b. SCM 1 ($\gamma = 1.5$) c. SCM 2 ($\gamma = 1$) d. SCM 2($\gamma = 1.5$) e. SCM 3 ($\gamma = 1$) f. SCM 3 ($\gamma = 1.5$).

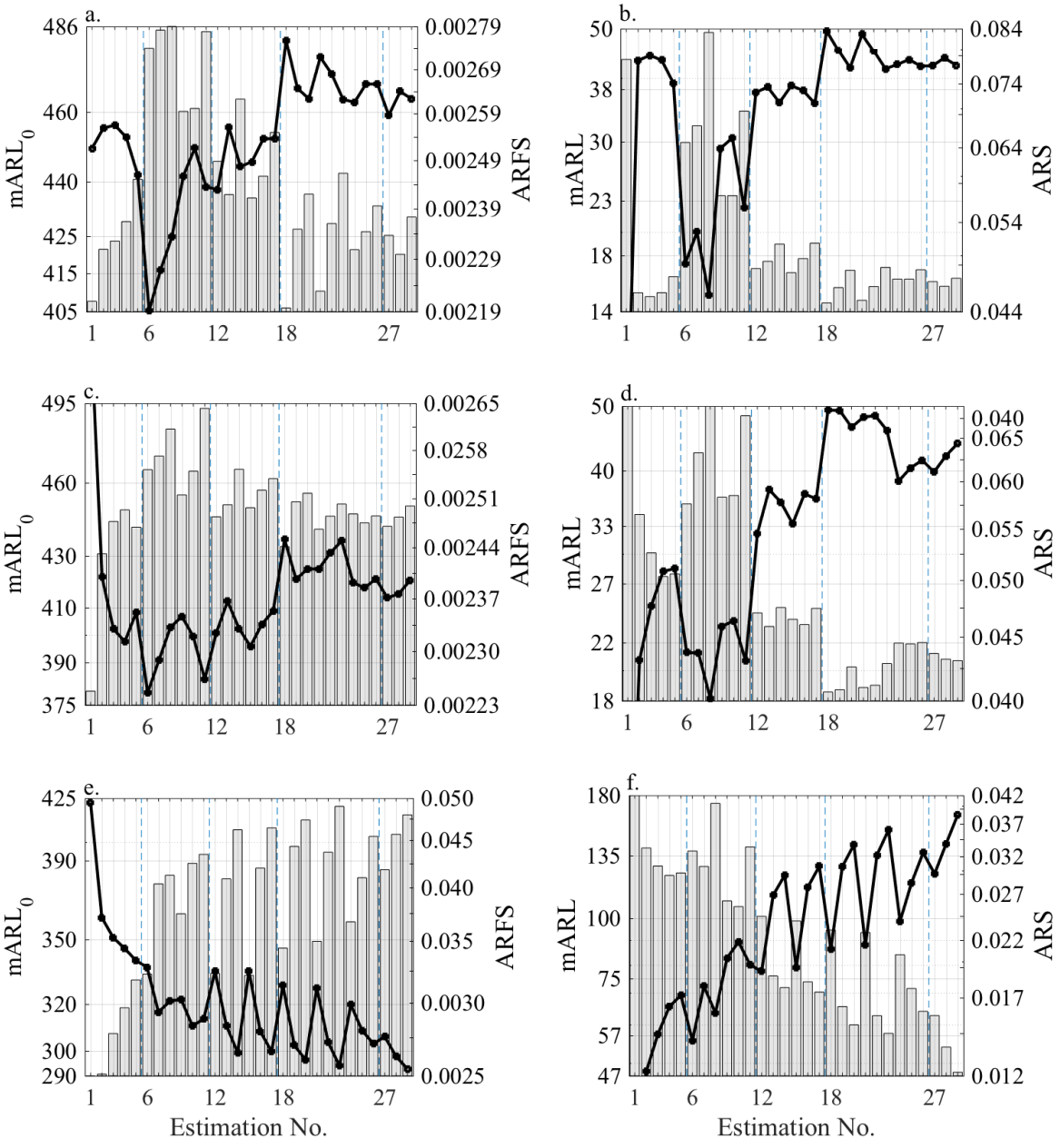

Figure 4.17. Phase II performance of S-Charts for $m = 50$ when a. $\gamma = 1$ and b. $\gamma = 1.5$ using Phase I data under a. SCM 4 ($\gamma = 1$) b. SCM 4 ($\gamma = 1.5$) c. SCM 5 ($\gamma = 1$) d. SCM 5($\gamma = 1.5$) e. SCM 6 ($\gamma = 1$) f. SCM 6 ($\gamma = 1.5$).

Effect of between-subgroup variation on the Phase II performance of S-Charts based on Method IV and V estimators can be seen in Figure 4.18 and Table C.27-C.32 for $m = 50$. Performance of S-Charts using Method I-III estimators against CM 5 is identical to those under normally distributed data (Figure 4.11), since reweighting steps are based on only subgroup statistics in these estimators. $ARFS$ values obtained from Method IV.B and V estimators stay almost constant at 0.0027-0.0028, while those from Method IV.A exceed 0.0030. The results show that, while S-Charts constructed using Method IV.A estimators have higher power for some cases Method IV.B and V estimators are negligibly affected from between-subgroup variation, and less likely to give false signals.

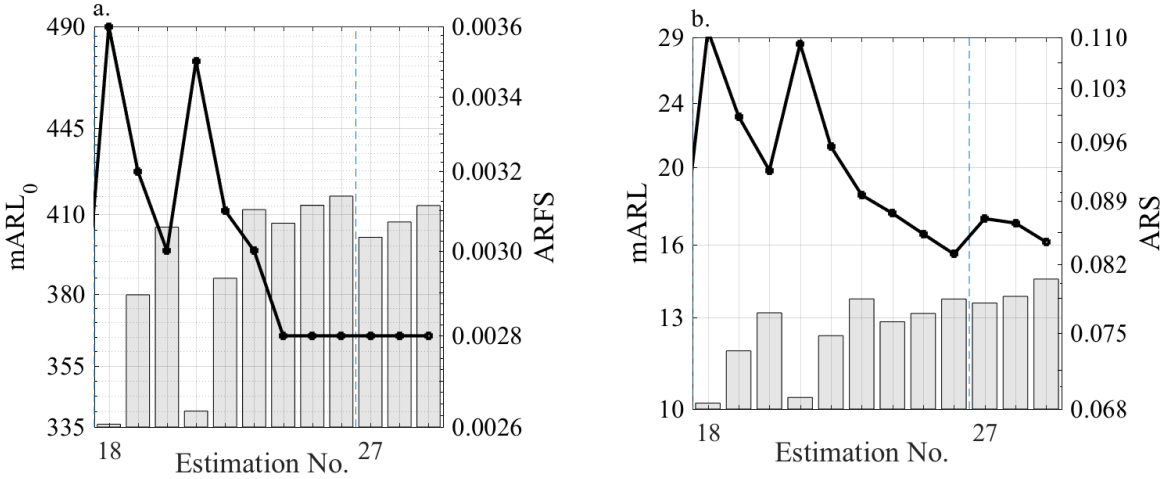

Figure 4.18. Phase II performance of S-Charts for $m = 50$ when a. $\gamma = 1$ and b. $\gamma = 1.5$ using Phase I data under CM 5 (d=3, K=0.15).

4.4. Performance of the Estimators on an out-of-Control Process

The current study focuses on designing robust Phase I estimators used to compute CLs of S-Charts for efficient Phase II monitoring, hence S-Charts have been evaluated for independent observations in Section 4.3. Autocorrelated observations, on the other hand, are prevalent in industrial applications. Some studies have found that such autocorrelation renders S-Charts misleading if dependency of observations is ignored

[72], and application of S-Charts to the correlated data affects the $ARFS$, and decreases its performance undesirably [73]. In this section, autoregressive models of order one are studied to determine the effect of between-subgroup variation on the proposed estimators and to investigate the capability of the estimators in determining whether process is in-control using correlated Phase I data.

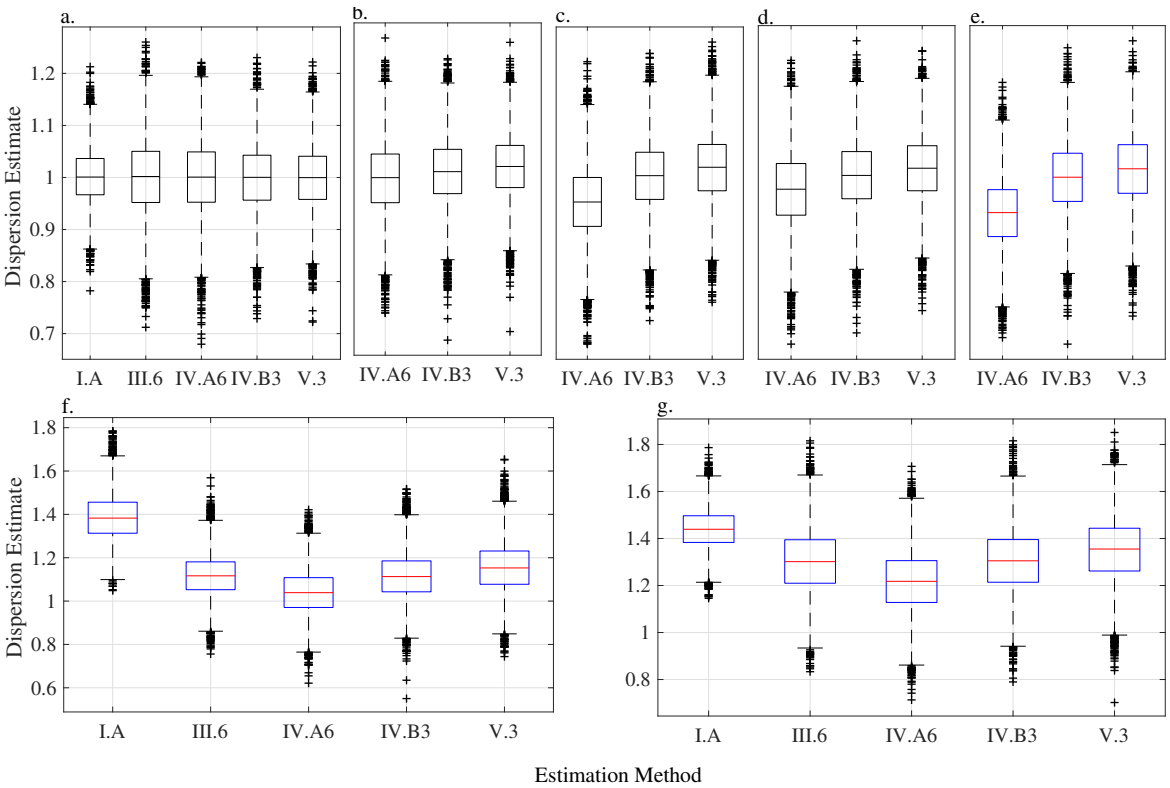

Figure 4.19. Box plots of dispersion estimates via various estimators for $m = 50$ in the presence of ACM 1 with a. $\phi = 0$, ${\sigma_e}^2 =$ b. $\phi = 0.4$, ${\sigma_e}^2 = 0.5$ c. $\phi = 0.4$, ${\sigma_e}^2 = 1$ d. $\phi = 0.8$, ${\sigma_e}^2 = 0.5$ e. $\phi = 0.4$, ${\sigma_e}^2 = 1$, and in the presence of f. ACM 2 g. ACM 3.

In Figure 4.19a, box plots of the most promising dispersion estimates (along with the traditional standard deviation estimator) are shown, when process is in-control. As ϕ and σ_e are changed (Figure 4.19b, 4.19c, 4.19d, 4.19e), Method I-III estimators are not affected. Increase in ACM 1 parameters yields a negative bias by Method IV.A estimator, since individual observations, which are distant from bulk of the data, are likely to be screened due to tight CLs of individuals chart in that method. In Method

IV.B and V, on the other hand, this effect is mostly eliminated, and nearly unbiased dispersion estimates are obtained, since most of the subgroups which have substantially different location estimates are already screened in these methods. When diffuse outliers exist in autocorrelated Phase I sampling (Figure 4.19f, 4.19g), Method I.A yields highly positively biased dispersion estimates, thus wider CLs would be constructed, and it would be difficult to detect that the process is out-of-control. Method V estimator yields dispersion estimates smaller than that of Method I.A, yet still having a have high positive bias. This shows that shifts in the process mean slightly decreases the efficiency of the screening methods. Method IV.B estimator seems convenient in detecting out-of-control status of the process, since estimates are not negatively biased in the absence of outliers, and not highly positively biased in the presence of outliers.

4.5. Real Data Example

In this section, implementation of the control charts constructed using the proposed estimators to real-world process data is studied. Two different examples are discussed: hard-bake [8] and syringe [74] production process. For both examples, dispersion estimates are computed from the first 20 subgroups using algorithms given in Section 3.2; σ^I, upper and lower CLs are estimated using ${U_n}^I$ and ${L_n}^I$ shown in Table 4.2, and remaining subgroups are monitored. It is not possible to determine the best dispersion estimate from such a limited dataset, but comparing the dispersion estimates and CLs of S-Charts constructed from proposed estimators gives an overall idea about the performance and potential of the estimation methods.

4.5.1. Hard-bake Flow Width Example

A hard-bake process is used in conjunction with photolithography in semiconductor manufacturing. First twenty samples of flow width measurements (in microns), which are assumed to be collected when process is in-control, are shown in Table 4.9. Each sample consists five observations, which are sampled every hour. Dispersion estimates of hard-bake process using proposed Phase I estimators are shown in Table 4.10, and they are used to determine Phase II upper and lower CLs, which are given

in the same table, e.g. $\hat{\sigma}^I$ based on S_n is equal to 0.1511, and the corresponding lower and upper CL coefficients are 0.171 and 2.356, the Phase II CLs are 0.0258 and 0.3559. Observations and dispersion estimates of the last twenty-five samples and dispersion parameters which are obtained using S_n are shown in Table 4.11. $\hat{\sigma}_i^{II}$ values are used to check whether they fall between the pre-determined CLs, deemed to represent the variability of the in-control process.

Table 4.9. Flow width measurements (microns) for the hard-bake process.

	Observations				
Sample No.	1	2	3	4	5
1	1.3235	1.4128	1.6744	1.4573	1.6914
2	1.4314	1.3592	1.6075	1.4666	1.6109
3	1.4284	1.4871	1.4932	1.4324	1.5674
4	1.5028	1.6352	1.3841	1.2831	1.5507
5	1.5604	1.2735	1.5265	1.4363	1.6441
6	1.5955	1.5451	1.3574	1.3281	1.4198
7	1.6274	1.5064	1.8366	1.4177	1.5144
8	1.4190	1.4303	1.6637	1.6067	1.5519
9	1.3884	1.7277	1.5355	1.5176	1.3688
10	1.4039	1.6697	1.5089	1.4627	1.5220
11	1.4158	1.7667	1.4278	1.5928	1.4181
12	1.5821	1.3355	1.5777	1.3908	1.7559
9	1.3884	1.7277	1.5355	1.5176	1.3688
10	1.4039	1.6697	1.5089	1.4627	1.522
11	1.4158	1.7667	1.4278	1.5928	1.4181
12	1.5821	1.3355	1.5777	1.3908	1.7559
13	1.2856	1.4106	1.4447	1.6398	1.1928
14	1.4951	1.4036	1.5893	1.6458	1.4969
15	1.3589	1.2863	1.5996	1.2497	1.5471
16	1.5747	1.5301	1.5171	1.1839	1.8662
17	1.368	1.7269	1.3957	1.5014	1.4449
18	1.4163	1.3864	1.3057	1.621	1.5573
19	1.5796	1.4185	1.6541	1.5116	1.7247
20	1.7106	1.4412	1.2361	1.382	1.7601

In the original reference, from which this example is taken [8] (pp. 232-234), no out-of-control signal is obtained in R and $\bar{X}$ charts for any of the subgroups, hence one may assume that the in-control process data is free from outliers. In this case, mean of the standard deviations of the subgroups is likely to yield the best dispersion estimate.

Figure 4.20 shows the S-Chart, in which CLs are computed via Method I.A and methods using individuals charts. All of the Phase II dispersion parameter estimates fall between CLs, showing that the proposed estimators yield similar results to the original source.

Since process is in-control and Phase I samples are not contaminated, it is expected from S-Charts based on Method I.A to have maximum power with tighter CLs. Method I.B estimators yield close dispersion estimates and upper and lower CLs, followed by Method V, IV.A, IV.B and III. Method II estimators, on the other hand, yield dispersion parameter estimates with low efficiency, and wider CLs are observed in S-Charts constructed from these estimators.

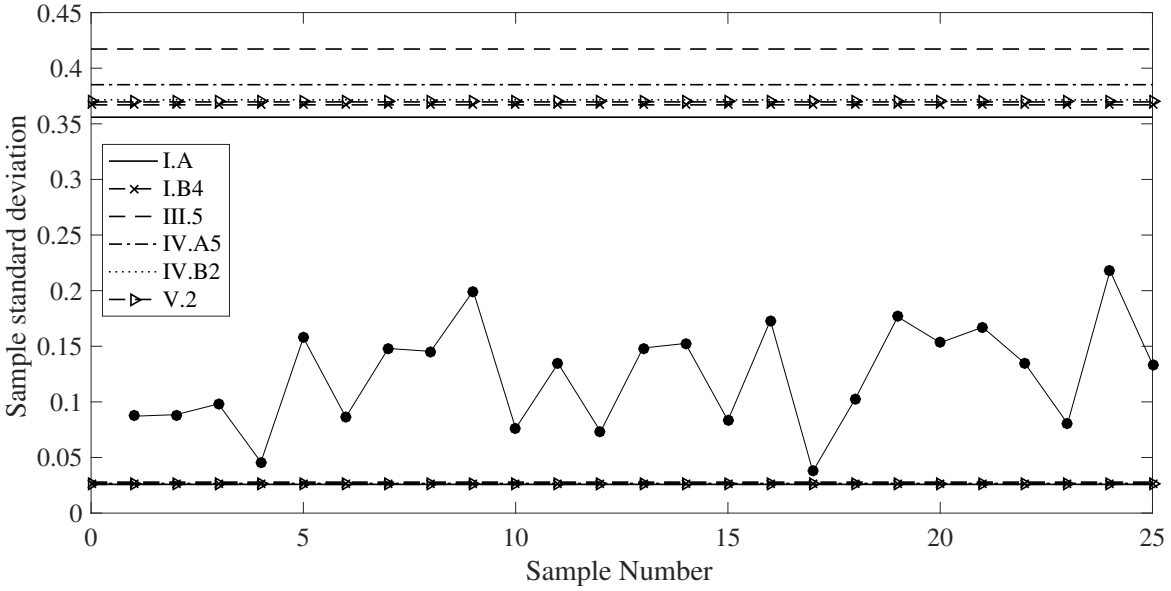

Figure 4.20. S-Charts constructed using Method I.A, III.6, IV.A6, IV.B3 and V.3 estimators.

Table 4.10. Process dispersion estimates of hard-bake process and CLs of S-Charts.

Method	$\hat{\sigma}^I$	UCL	LCL
I.A	0.1511	0.3559	0.0258
I.B1	0.1512	0.3572	0.0259
I.B2	0.1514	0.3591	0.0259
I.B3	0.1517	0.3637	0.0259
I.B4	0.1520	0.3671	0.0258
II.A1	0.1636	0.4100	0.0276
II.A2	0.1724	0.4456	0.0291
II.A3	0.1973	0.5344	0.0331
II.B1	0.1625	0.4163	0.0275
II.B2	0.1625	0.4195	0.0275
II.B3	0.1815	0.4905	0.0305
III.1	0.1531	0.3670	0.0262
III.2	0.1581	0.3901	0.0269
III.3	0.1637	0.4175	0.0278
III.4	0.1531	0.3668	0.0262
III.5	0.1581	0.3909	0.0269
III.6	0.1639	0.4172	0.0279
IV.A1	0.1528	0.3653	0.0261
IV.A2	0.1568	0.3844	0.0268
IV.A3	0.1615	0.4043	0.0276
IV.A4	0.1529	0.3656	0.0262
IV.A5	0.1570	0.3852	0.0267
IV.A6	0.1609	0.4047	0.0274
IV.B1	0.1531	0.3662	0.0261
IV.B2	0.1540	0.3716	0.0263
IV.B3	0.1562	0.3853	0.0267
V.1	0.1530	0.3646	0.0262
V.2	0.1538	0.3697	0.0263
V.3	0.1559	0.3808	0.0267

Table 4.11. Additional flow width measurements (micron) for monitoring.

	Observations					
Sample No.	1	2	3	4	5	$\hat{\sigma}_i^{II}$
21	1.4371	1.5051	1.3485	1.5670	1.4880	0.0871
22	1.4738	1.5936	1.6583	1.4973	1.4720	0.0885
23	1.5917	1.4333	1.5551	1.5295	1.6866	0.0981
24	1.6399	1.5243	1.5705	1.5563	1.5530	0.0459
25	1.5797	1.3663	1.6240	1.3732	1.6887	0.1576
26	1.4483	1.5458	1.4538	1.4303	1.6206	0.0863
27	1.5435	1.6899	1.5830	1.3358	1.4187	0.1478
28	1.5175	1.3446	1.4723	1.6657	1.6661	0.1454
29	1.5454	1.0931	1.4072	1.5039	1.5264	0.1997
30	1.4418	1.5059	1.5124	1.4620	1.6263	0.0762
31	1.4301	1.2725	1.5945	1.5397	1.5252	0.1345
32	1.4981	1.4506	1.6174	1.5837	1.4962	0.0733
33	1.3009	1.5060	1.6231	1.5831	1.6454	0.1484
34	1.4132	1.4603	1.5808	1.7111	1.7313	0.1525
35	1.3817	1.3135	1.4953	1.4894	1.4596	0.0833
36	1.5765	1.7014	1.4026	1.2773	1.4541	0.1732
37	1.4936	1.4373	1.5139	1.4808	1.5293	0.0376
38	1.5729	1.6738	1.5048	1.5651	1.7473	0.1028
39	1.8089	1.5513	1.8250	1.4389	1.6558	0.1765
40	1.6236	1.5393	1.6738	1.8698	1.5036	0.1532
41	1.4120	1.7931	1.7345	1.6391	1.7791	0.1671
42	1.7372	1.5663	1.4910	1.7809	1.5504	0.1344
43	1.5971	1.7394	1.6832	1.6677	1.7974	0.0805
44	1.4295	1.6536	1.9134	1.7272	1.4370	0.2179
45	1.6217	1.8220	1.7915	1.6744	1.9404	0.1339

4.5.2. Syringe Length Example

In this example, syringe length samples manufactured by a midwest pharmaceutical company, are used for both determining CLs in Phase I and monitoring in Phase II. First twenty samples of syringe length measurements (in inch), which consist five observations, are sampled at every 15 minutes, and the measurements are shown in Table 4.12.

Table 4.12. Syringe length (inch) samples.

	Observations				
Sample No.	1	2	3	4	5
1	4.960	4.946	4.950	4.956	4.958
2	4.958	4.927	4.935	4.940	4.95
3	4.971	4.929	4.965	4.952	4.938
4	4.94	4.982	4.97	4.953	4.96
5	4.964	4.95	4.953	4.962	4.956
6	4.969	4.951	4.955	4.966	4.954
7	4.96	4.944	4.957	4.948	4.951
8	4.969	4.949	4.963	4.952	4.962
9	4.984	4.928	4.96	4.943	4.955
10	4.97	4.934	4.961	4.94	4.965
11	4.975	4.959	4.962	4.971	4.968
12	4.945	4.977	4.95	4.969	4.954
13	4.976	4.964	4.97	4.968	4.972
14	4.97	4.954	4.964	4.959	4.968
15	4.982	4.962	4.968	4.975	4.963
16	4.961	4.943	4.95	4.949	4.957
17	4.98	4.97	4.975	4.978	4.977
18	4.975	4.968	4.971	4.969	4.972
19	4.977	4.966	4.969	4.973	4.97
20	4.975	4.967	4.969	4.972	4.972

In the original study, it was stated that the first 15 subgroups were sampled from the first shift, while the rest of the subgroups were sampled from the new shift, which started at 4:00 p.m. of the same day. UCL and LCL are computed for $\bar{X}$ and R charts. When the corresponding statistics of first 15 samples are placed on the $\bar{X}$ and R control charts, these charts show that process is already out-of-control in both center and variation. The operators from the first shift continued to collect 15 samples, each of size 5 and plotted the sample means and ranges in $\bar{X}$ and R charts using the CLs obtained from the first set of 15 samples. These charts also showed clearly the center of the process is to be out-of-control.

In the current study, process dispersion estimates are computed from the first 20 samples to determine the CLs of the corresponding S-Charts (Table 4.13), and the remaining 27 samples (Table 4.14) are monitored. Methods using individual charts are found to give lower $\hat{\sigma}^I$ at higher skipping levels. In the cited reference, the first 15 subgroups were said to be collected from an already out-of-control process; hence monitoring the rest of the observations using CLs obtained from an out-of-control process is not very meaningful. Nevertheless, it should be expected for some of the future samples collected from an in-control process to be below the dispersion LCL. In the cited reference, the lower limit of R-chart was taken to be equal to zero, hence it was not possible to determine future samples, with a smaller dispersion parameter estimate. Tighter CLs are obtained in S-Charts constructed using Method IV and V estimators (Figure 4.21a). Here, sample standard deviation of the 23^{th} sample (the third sample in Figure 4.21) is found to be equal to 0.0013, which is lower than the LCL of the S-charts, constructed via all estimators excepted reweighted standard deviation estimator. Also considering that Method II estimators give remarkably high UCLs, it may suggested that Method III, IV, and V estimators are capable of determining a future sample, which has a significantly lower dispersion than that seen in Phase I data. (Figure 4.21b).

Table 4.13. Process dispersion estimates of syringe manufacturing process and CLs of S-Charts.

Method	$\hat{\sigma}^I$	UCL	LCL
I.A	0.0094	0.0222	0.0016
I.B1	0.0088	0.0208	0.0015
I.B2	0.0071	0.0169	0.0012
I.B3	0.0069	0.0166	0.0012
I.B4	0.0067	0.0163	0.0012
II.A1	0.0114	0.0285	0.0019
II.A2	0.0102	0.0263	0.0017
II.A3	0.0117	0.0316	0.0020
II.B1	0.0108	0.0277	0.0018
II.B2	0.0108	0.0280	0.0018
II.B3	0.0116	0.0313	0.0020
III.1	0.0096	0.0230	0.0016
III.2	0.0095	0.0235	0.0016
III.3	0.0090	0.0220	0.0015
III.4	0.0096	0.0229	0.0016
III.5	0.0090	0.0223	0.0015
III.6	0.0090	0.0229	0.0015
IV.A1	0.0096	0.0228	0.0016
IV.A2	0.0094	0.0230	0.0016
IV.A3	0.0091	0.0227	0.0016
IV.A4	0.0096	0.0229	0.0016
IV.A5	0.0090	0.0221	0.0015
IV.A6	0.0091	0.0229	0.0015
IV.B1	0.0097	0.0232	0.0017
IV.B2	0.0090	0.0216	0.0015
IV.B3	0.0091	0.0225	0.0016
V.1	0.0097	0.0231	0.0017
V.2	0.0092	0.0220	0.0017
V.3	0.0091	0.0222	0.0016

Table 4.14. Additional syringe length measurements (inch) for monitoring.

	Observations					
Sample No.	1	2	3	4	5	$\hat{\sigma}_i^{II}$
21	4.980	4.960	4.974	4.965	4.971	0.0083
22	4.979	4.967	4.976	4.970	4.978	0.0056
23	4.973	4.976	4.974	4.974	4.973	0.0013
24	4.978	4.970	4.976	4.975	4.971	0.0036
25	4.979	4.967	4.972	4.977	4.970	0.0053
26	4.987	4.963	4.979	4.974	4.977	0.0093
27	4.979	4.971	4.975	4.977	4.978	0.0034
28	4.974	4.970	4.973	4.972	4.971	0.0017
29	4.993	4.974	4.980	4.988	4.980	0.0080
30	4.992	4.961	4.971	4.986	4.970	0.0135
31	4.988	4.976	4.981	4.986	4.979	0.0053
32	4.987	4.975	4.980	4.984	4.979	0.0049
33	4.975	4.945	4.952	4.968	4.960	0.0128
34	4.981	4.956	4.966	4.974	4.968	0.0099
35	4.980	4.956	4.971	4.966	4.962	0.0097
36	4.965	4.958	4.960	4.962	4.960	0.0028
37	4.974	4.960	4.966	4.971	4.964	0.0059
38	4.973	4.962	4.969	4.964	4.967	0.0046
39	4.960	4.974	4.968	4.965	4.968	0.0054
40	4.967	4.956	4.962	4.959	4.961	0.0043
41	4.964	4.968	4.967	4.966	4.965	0.0017
42	4.971	4.963	4.966	4.969	4.966	0.0033
43	4.966	4.959	4.963	4.961	4.961	0.0028
44	4.953	4.967	4.960	4.958	4.957	0.0055
45	4.969	4.951	4.958	4.963	4.959	0.0071
46	4.970	4.950	4.962	4.957	4.966	0.0083
47	4.965	4.957	4.958	4.963	4.962	0.0036

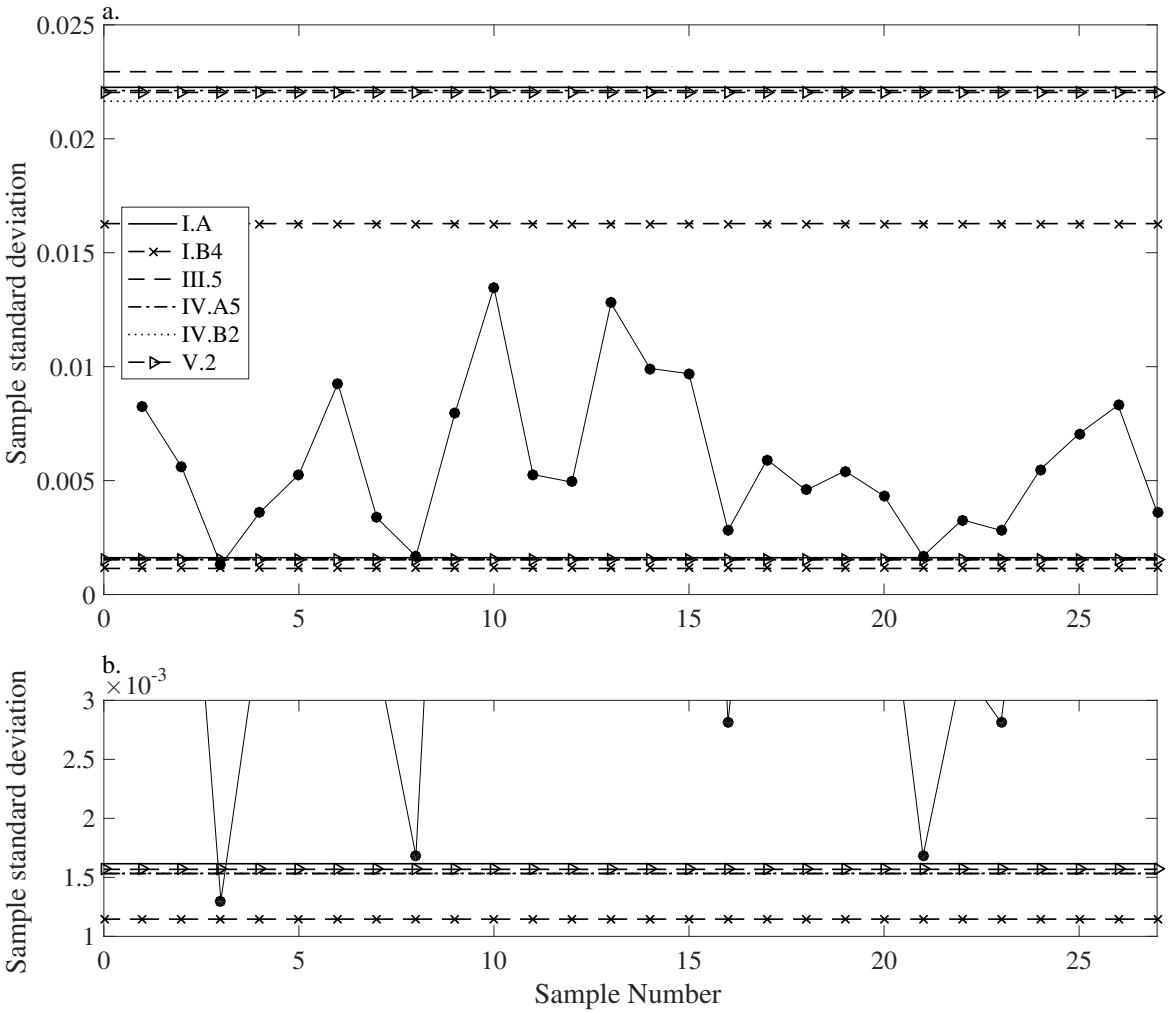

Figure 4.21. S-Charts constructed using Method I.A, III.6, IV.A6, IV.B3 and V.3 estimators.

5. CONCLUSIONS AND RECOMMENDATIONS

Based on the assumption of independent and normally distributed dataset, sample standard deviation and sample mean are known to be most efficient traditional estimators to determine process parameters, and S-Charts based on these estimators yield high power under normality. However, the efficiency of traditional estimators is significantly diminished, and power of S-Charts is undesirably low when process parameters are estimated, and outliers are present in sampling [25, 37, 40, 43–45, 50]. Inability to promptly detect disturbances in a process causes an inefficient process monitoring and a serious delay in detection of inferior products. One way of overcoming this problem is reweighting traditional estimators, based on screening the outlier subgroups. While reweighted traditional estimators are highly efficient against outliers that affect only certain subgroups, their efficiency is significantly low against outliers that come from short-tailed asymmetric distributions. An improved method is to use robust statistics for parameter estimation in Phase I. Application of robust estimators may be valuable in the increasing performance of control charts, but robust estimators have low efficiency under normality. Furthermore, screening outlier subgroups does not significantly increase the efficiency and performance of robust estimators. It should be noted that power of S-Charts based on reweighted traditional estimators (Method I.B), non-reweighted (Method II.A) / reweighted robust estimators (Method II.B) are higher than those based on traditional estimators (Method I.A) when outliers exist in samples, but the performance against diffuse mean contamination (CM 4) may be undesirably low; S-Charts may become ARL-biased. To address this problem, the current study focuses on using individuals charts in reweighting robust estimators, being a method recommended by Does and co-workers [37, 40].

In the current study, a number of improvements over the currently existing reweighting methods are proposed to increase Phase II performance of S-Charts in the presence of outliers, keeping high power and efficiency under normality. First of all, good efficiency and performance of scale M-estimator with logistic influence function MS_n is to be noted. Average distance to the median (ADM_n) [37], interquartile

range (IQR) [43, 50], Rousseeuw's and Croux' Q_n [29] estimator are also frequently used in the literature for SPM applications. Although performance of Q_n was found to be superior to ADM_n and IQR [75], MAD_n and MS_n showed more satisfactory performance in our preliminary studies [76]. For these reasons, S_n, MAD_n and MS_n are the dispersion estimators focused in this study. In conclusion, MS_n is shown to be a convenient robust dispersion estimator, while MAD_n yields low power due to inflated variance of parameter estimation. Robust location estimators $HD_{.5}$ and ML do not show significant difference in parameter estimates and performance of S-Charts.

Utilizing individuals chart to screen outlier observations in subgroups (Method III-V) increases the efficiency of estimators, and S-Charts based on these estimators are found to be more powerful against all CMs. Difference of reweighting procedures in Methods III-V lies in obtaining the initial and intermediate parameter estimates of the subgroups. In Method III, suggested by Does and co-workers, each subgroup location parameter ($\hat{\mu}_i^{II}$) is estimated separately, and individual charts are constructed using subgroup location estimates [37]. It should be noted that, ($\hat{\mu}_i^{II}$) is not likely to be an efficient estimator of process location, since it can still be affected by outliers in a sample with a small size of n observations. To deal with this problem, a single robust location estimate based on subgroup statistics (Method IV.A, IV.B and V) is proposed. Efficiency of individual charts in detecting outliers is significantly increased using single location estimate. In Method IV.A, subgroups, which have different location estimates than the others, are still included in determining the CLs of individual charts, hence the final standard deviation estimates of subgroups with a different location parameter are likely to be lower than the in-control process standard deviation. To remedy this problem, subgroups, which have different location estimates, are screened before constructing individuals chart in Method IV.B and V. In Method V, intermediate location ($\hat{\mu}^{II}$) and dispersion ($\hat{\sigma}^{II}$) estimates are determined via pooling all the individual observations, regardless of their subgroups in Method V. The results show that Method III estimator falls short at low and moderate contamination densities, and its inferiority is more apparent at higher contamination density, compared to Method IV-V estimators. It should further be noted that Method V estimators yield the best parameter estimation against all CMs, and generally followed by Method IV.B and

IV.A. Method IV.A estimator has higher efficiency than Method IV.B against CM 1 and CM 4, possibly because partioning the total skipping level into screening subgroups and individual observations in Method IV.B yields wider CLs, decreasing its efficiency against diffuse outliers, but Method IV.B is more efficient against CM 2 and CM 3.

Another important point in the current study is the effect of skipping level on the S-chart performance using various estimators. Higher skipping levels have been shown to increase the performance of S-Charts against contaminations in Phase I samples, and maximum power is observed at $\alpha = 0.04 - 0.02$ for $m = 50$, and $\alpha = 0.02 - 0.005$ for $m = 20$, both being much higher than 0.0027. The loss of efficiency and power due to increasing skipping level under normality is found to be acceptably low, when S-Charts are constructed from Method IV.B and V estimators.

In this study, novel reweighted robust dispersion estimation methods are proposed, and their Phase I efficiency and Phase II performance are assessed both under normality and against various CMs at different contamination magnitude and densities, which are adjusted to better represent the real-world data. It is found that Method IV.B and V estimators are the most promising dispersion estimators, and found to be efficient at high skipping levels. Estimation methods may be extended performing different combination of dispersion and location estimators on subgroups and individual observations, i.e. while $HD_{.5}$ estimators is used to determine initial process location estimate in subgroup screening, ML may be used to determine intermediate process location estimate in screening individual outliers. Phase II power analysis of Method IV.B and V estimators may further be studied using ML as a location estimator, since it has high BDP. A previous study suggested to use MS_n estimator with bisquare ρ function [77], further study may be performed to compare the power and efficiency of MS_n with different ρ functions using proposed reweighting procedures. Higher skipping levels can also be further studied, and may be optimized for estimation methods with respect to sample size. Proposed reweighting procedures can be studied in $\bar{X}$ control charts and multivariate processes as well.

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APPENDIX A: BIAS CORRECTION FACTORS AND COEFFICIENTS

Table A.1. Bias correction factors are used in the calculation of process dispersion estimate.

n	c_4	d_2	d_3	D_3	D_4
1	-	-	1	0.820	1
2	2.205	0.603	1.128	0.853	0.954
3	2.016	0.463	1.693	0.888	1.588
4	0.921	0.389	2.059	0.879	1.978
5	0.940	0.341	2.326	0.864	2.257
6	0.952	0.308	2.534	0.848	2.472
7	0.959	0.282	2.704	0.833	2.645
8	0.965	0.262	2.847	0.820	2.791
9	0.969	0.246	2.970	0.808	2.915
10	0.973	0.232	3.078	0.797	3.024
11	0.975	0.221	3.173	0.787	3.121
12	0.978	0.211	3.258	0.779	3.207
13	0.979	0.202	3.336	0.770	3.285
14	0.981	0.194	3.407	0.763	3.356
15	0.982	0.187	3.472	0.756	3.422
16	0.983	0.181	3.532	0.750	3.482
17	0.985	0.175	3.588	0.744	3.538
18	0.985	0.170	3.640	0.739	3.591
19	0.986	0.165	3.689	0.734	3.640
20	0.987	0.161	3.735	0.729	3.686
21	0.988	0.157	3.778	0.724	3.73
22	0.988	0.153	3.819	0.720	3.771
23	0.989	0.150	3.858	0.716	3.811
24	0.989	0.147	3.895	0.712	3.847
25	0.9904	0.144	3.931	0.708	3.883

Table A.2. Phase I CLs and bias correction factors for $m = 20$.

Method	Θ_1	Θ_2	${L_n}^I$	${U_n}^I$	Θ_3	${C_n}^I$	Θ_4	Θ_5
I.A	1.064	1	-	-	-	-	-	-
I.B1	1.064	1	0.179	2.216	1.001	-	-	-
I.B2	1.064	1	0.247	2.032	1.002	-	-	-
I.B3	1.064	1	0.314	1.892	1.004	-	-	-
I.B4	1.064	1	0.376	0.775	1.006	-	-	-
II.A1	1.319	1.022	-	-	-	-	-	-
II.A2	1.319	1.055	-	-	-	-	-	-
II.A3	1.803	1.083	-	-	-	-	-	-
II.B1	1.319	1.022	0.14	2.335	1.008	-	-	-
II.B2	1.319	1.055	0.138	2.406	1.007	-	-	-
II.B3	1.803	1.083	0.108	2.724	1.012	-	-	-
III.1	1.319	1.022	0.047	2.978	1.002	3.090	1.013	-
III.2	1.319	1.022	0.088	2.602	1.005	2.511	1.047	-
III.3	1.319	1.022	0.127	2.393	1.008	2.221	1.084	-
III.4	1.319	1.055	0.046	3.063	1.002	3.089	1.013	-
III.5	1.319	1.055	0.087	2.668	1.004	2.548	1.047	-
III.6	1.319	1.055	0.127	2.446	1.008	2.261	1.085	-
IV.A1	1.319	1.022	0.047	2.978	1.002	3.142	1.011	-
IV.A2	1.319	1.022	0.088	2.602	1.005	2.620	1.038	-
IV.A3	1.319	1.022	0.127	2.393	1.008	2.350	1.069	-
IV.A4	1.319	1.055	0.046	3.063	1.0016	3.151	1.012	-
IV.A5	1.319	1.055	0.087	2.668	1.004	2.628	1.039	-
IV.A6	1.319	1.055	0.127	2.446	1.008	2.371	1.065	-
IV.B1	1.319	1.055	0.046	3.063	1.002	3.110	1.013	-
IV.B2	1.319	1.055	0.087	2.668	1.004	3.010	1.019	-
IV.B3	1.319	1.055	0.127	2.446	1.008	2.743	1.034	-
V.1	1.319	1.055	0.046	3.063	1.002	3.111	1.013	1.016
V.2	1.319	1.055	0.087	2.668	1.004	3.001	1.018	1.019
V.3	1.319	1.055	0.127	2.446	1.008	2.734	1.024	1.024

APPENDIX B: PHASE I EFFICIENCIES OF PROPOSED ESTIMATORS

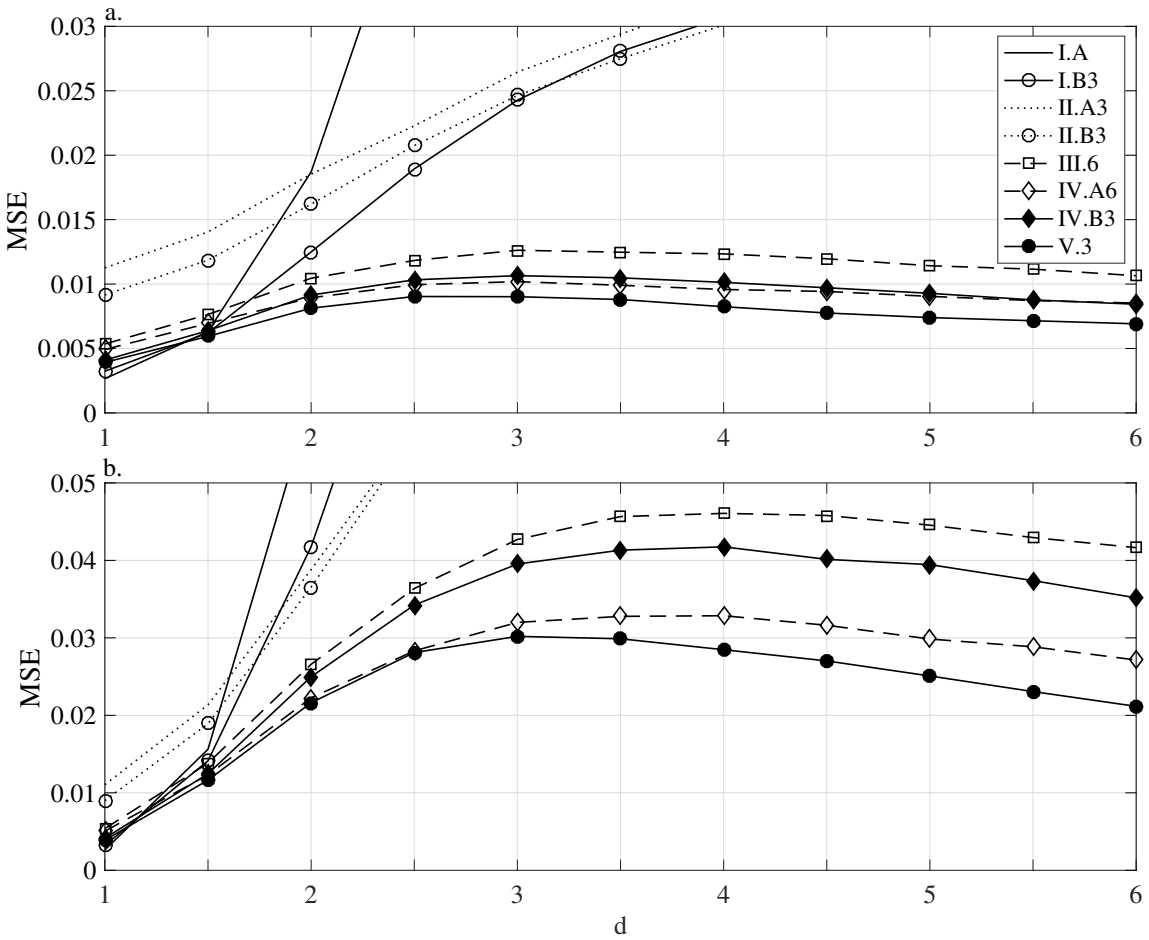

Figure B.1. Phase I efficiency of the proposed estimators for $m = 50$ under CM 1 at a. K = 0.10 b. K = 0.20.

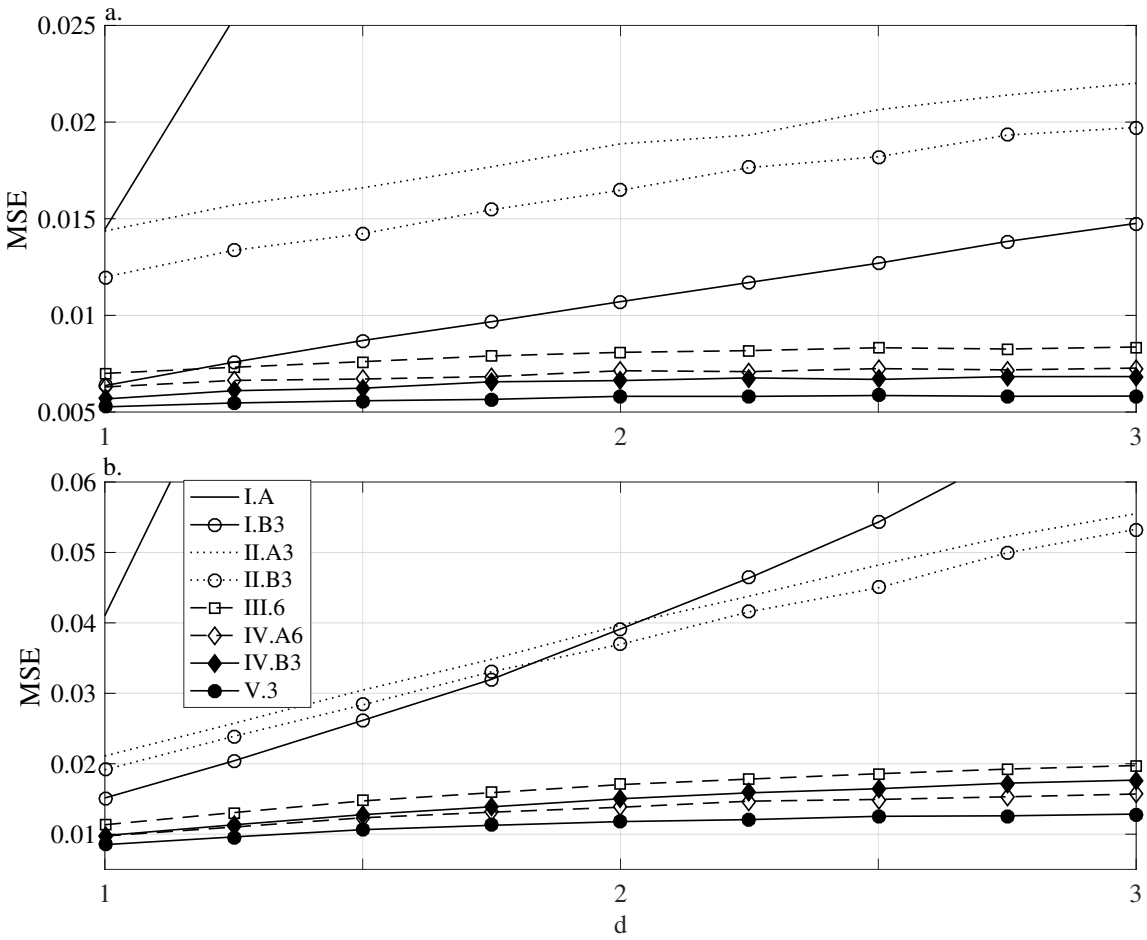

Figure B.2. Phase I efficiency of the proposed estimators for $m = 50$ under CM 2 at a. K = 0.10 b. K = 0.20.

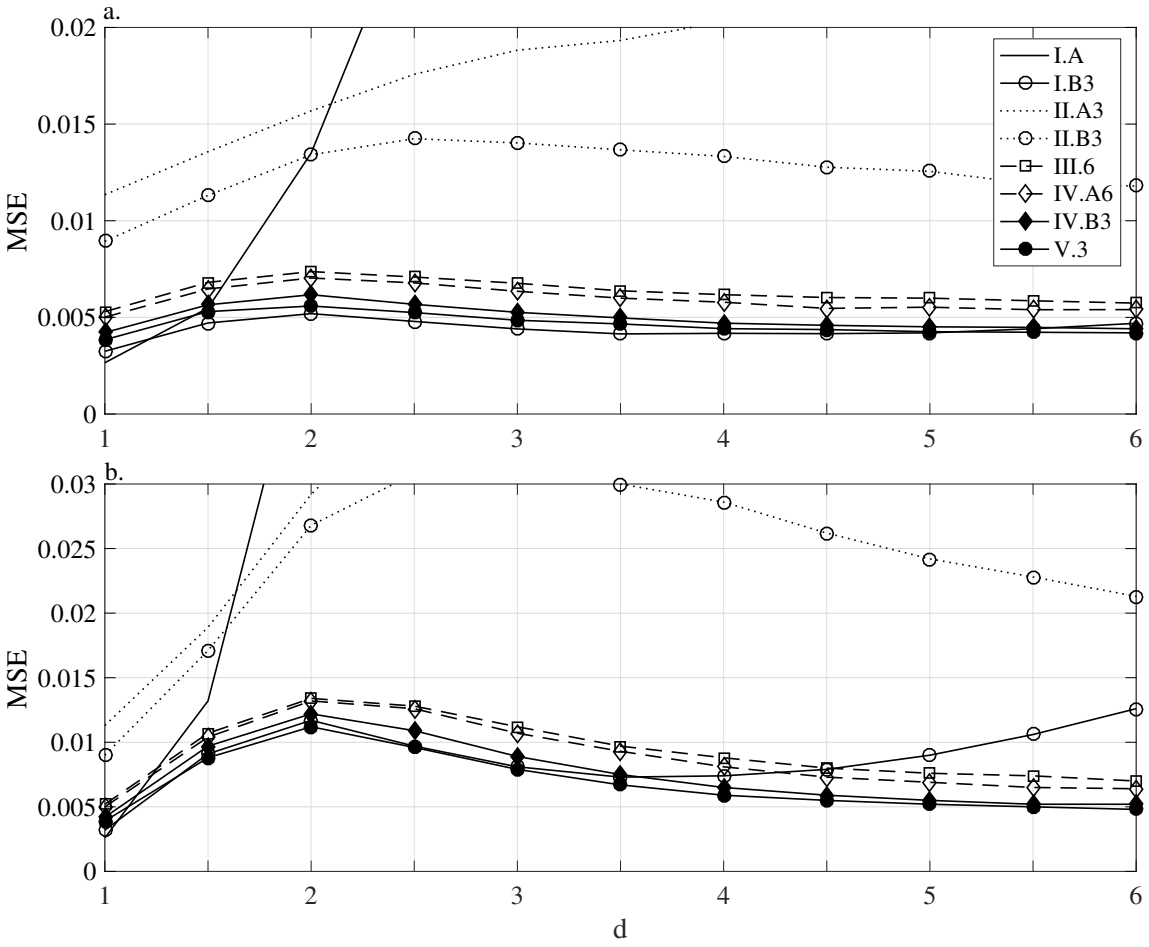

Figure B.3. Phase I efficiency of the proposed estimators for $m = 50$ under CM 3 at a. K = 0.10 b. K = 0.20.

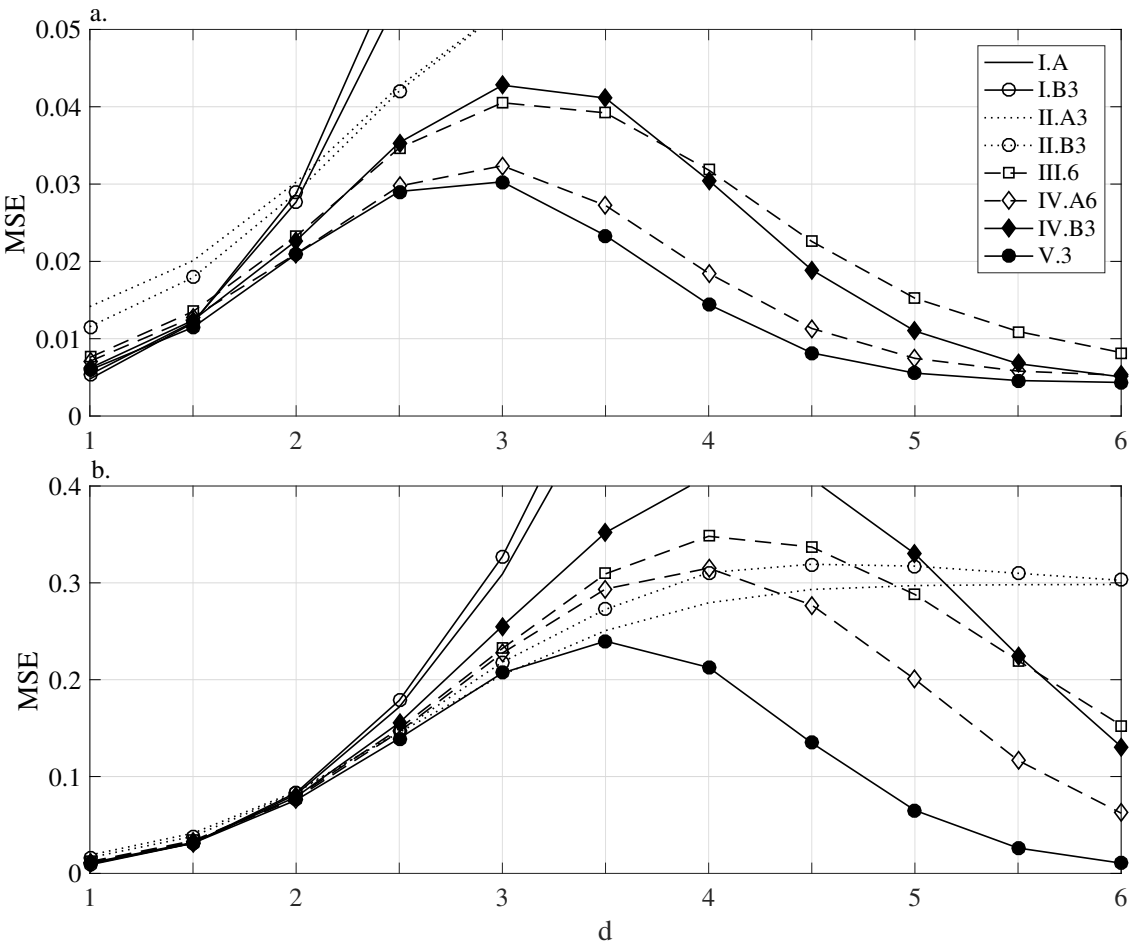

Figure B.4. Phase I efficiency of the proposed estimators for $m = 50$ under CM 4 at a. K = 0.10 b. K = 0.20.

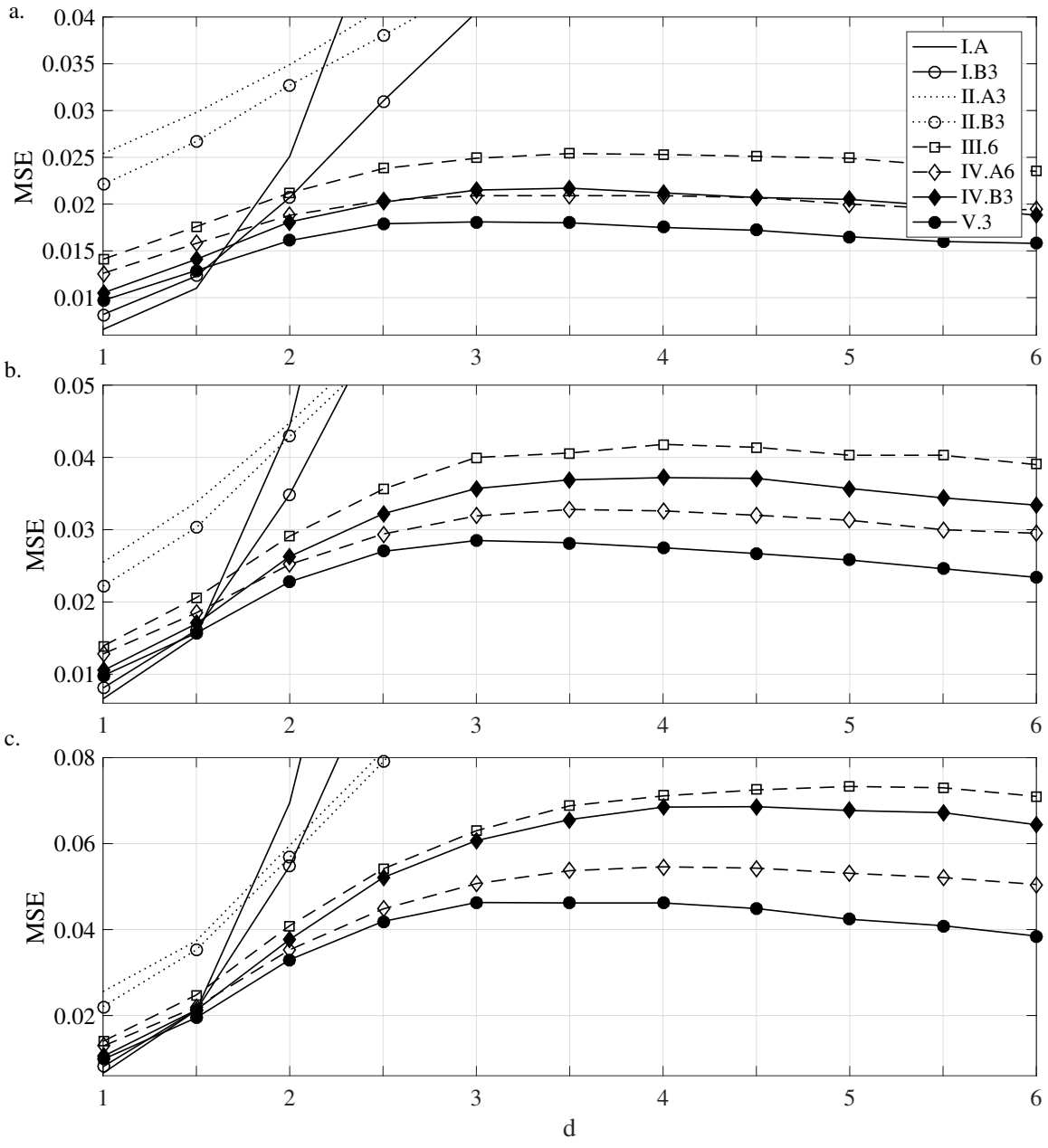

Figure B.5. Phase I efficiency of the proposed estimators for $m = 20$ under CM 1 at a. K = 0.10 b. K = 0.15 c. K = 20.

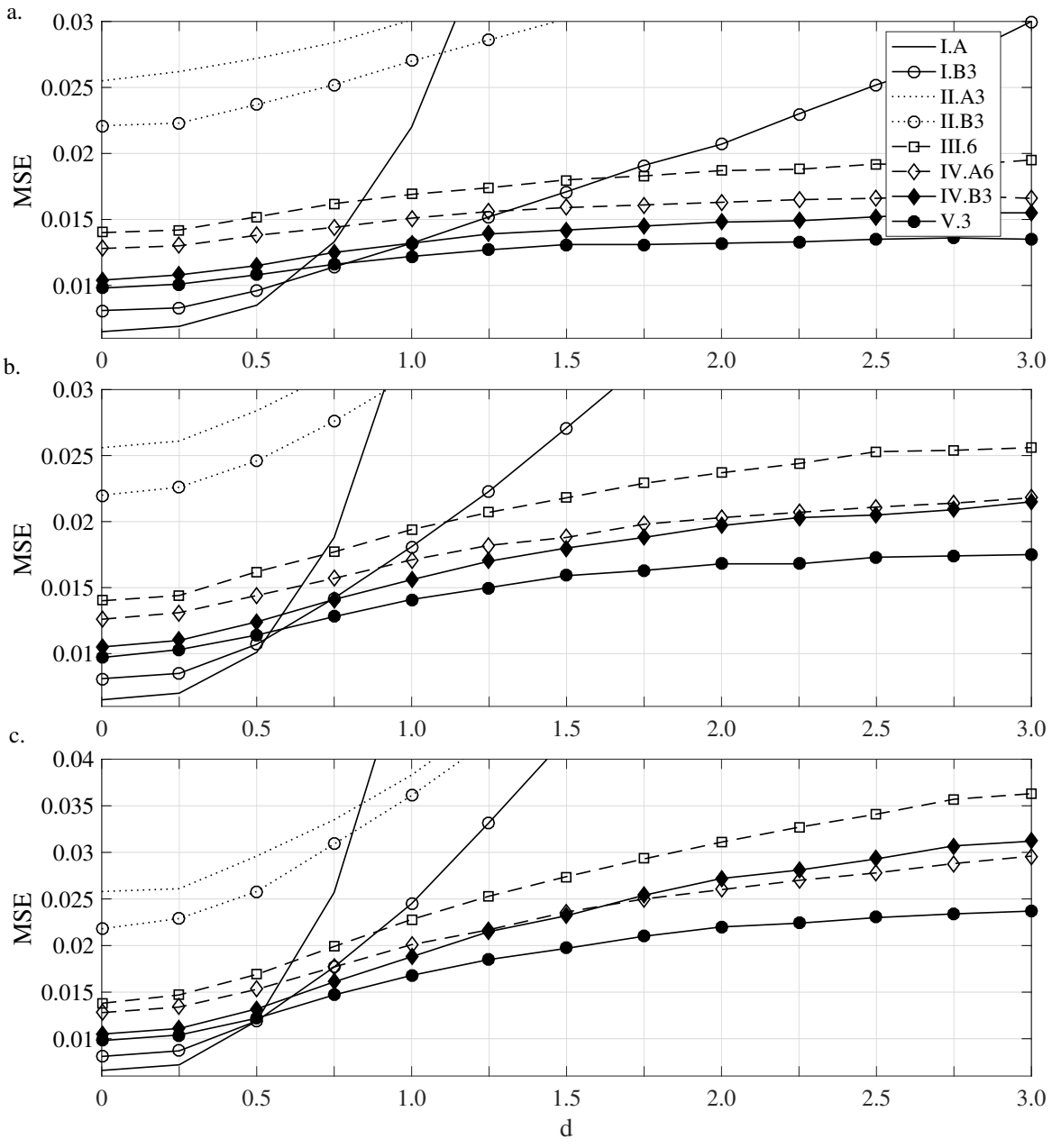

Figure B.6. Phase I efficiency of the proposed estimators for $m = 20$ under CM 2 at a. K = 0.10 b. K = 0.15 c. K = 20.

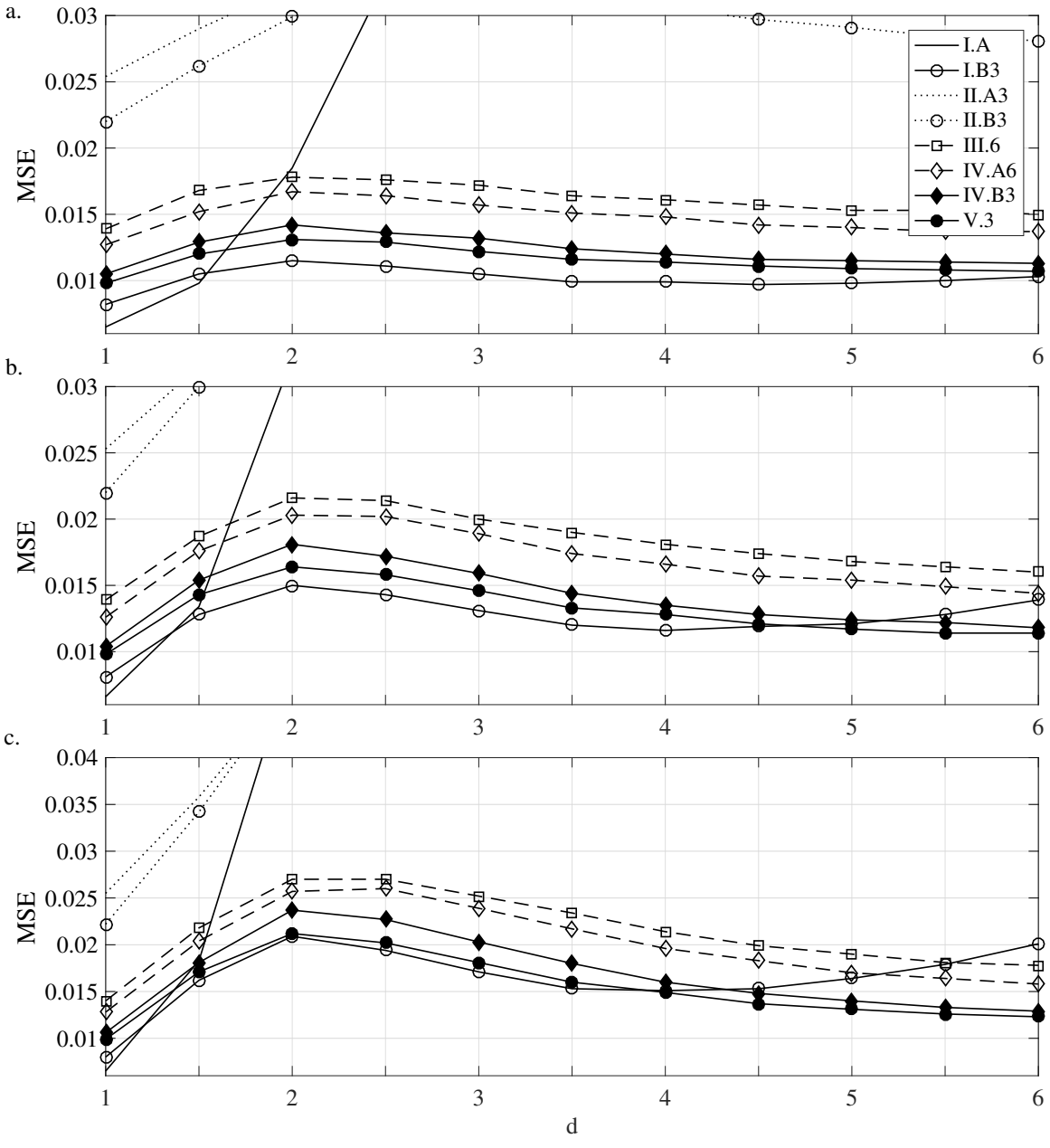

Figure B.7. Phase I efficiency of the proposed estimators for $m = 20$ under CM 3 at a. K = 0.10 b. K = 0.15 c. K = 20.

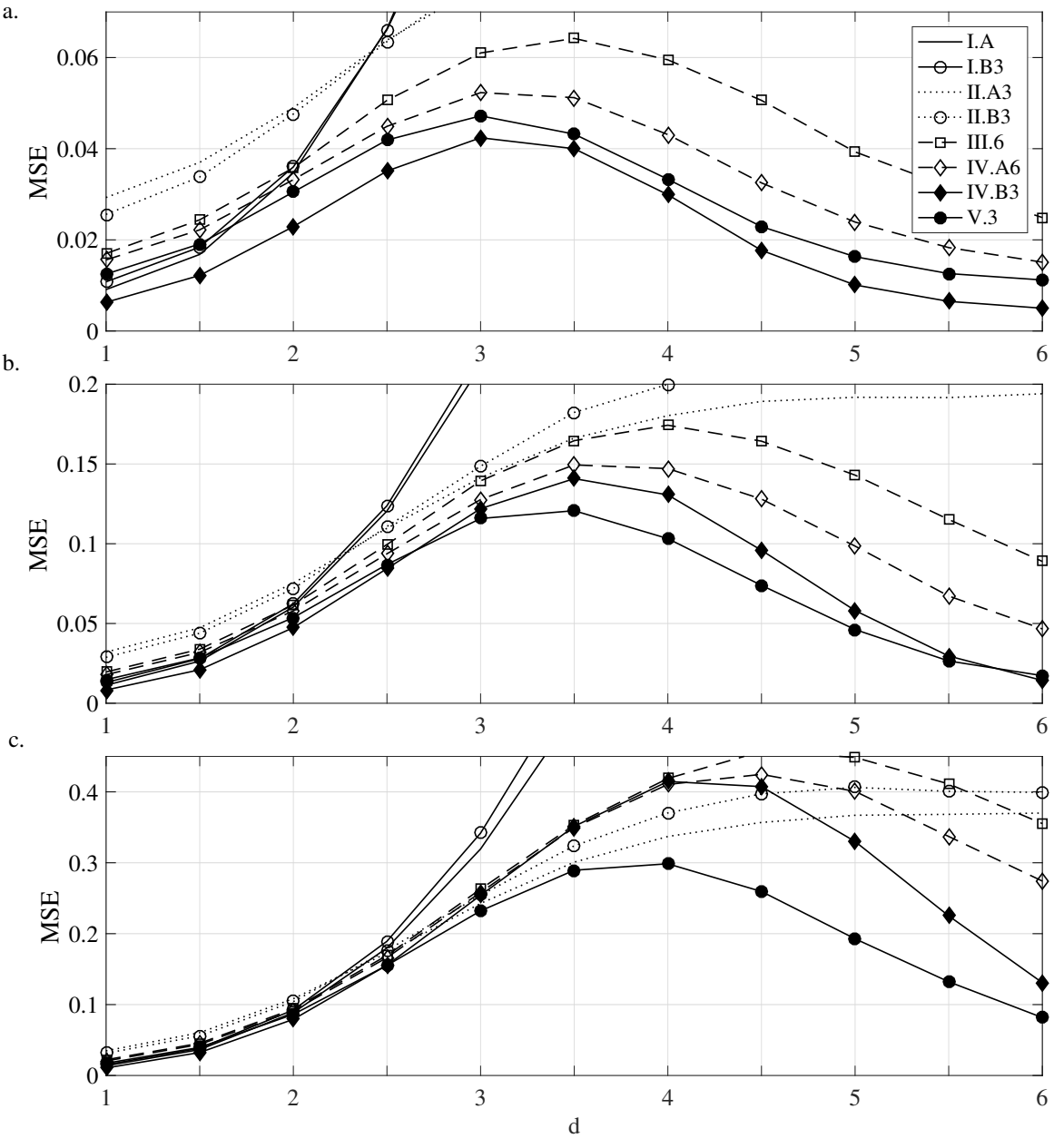

Figure B.8. Phase I efficiency of the proposed estimators for $m = 20$ under CM 4 at a. K = 0.10 b. K = 0.15 c. K = 20.

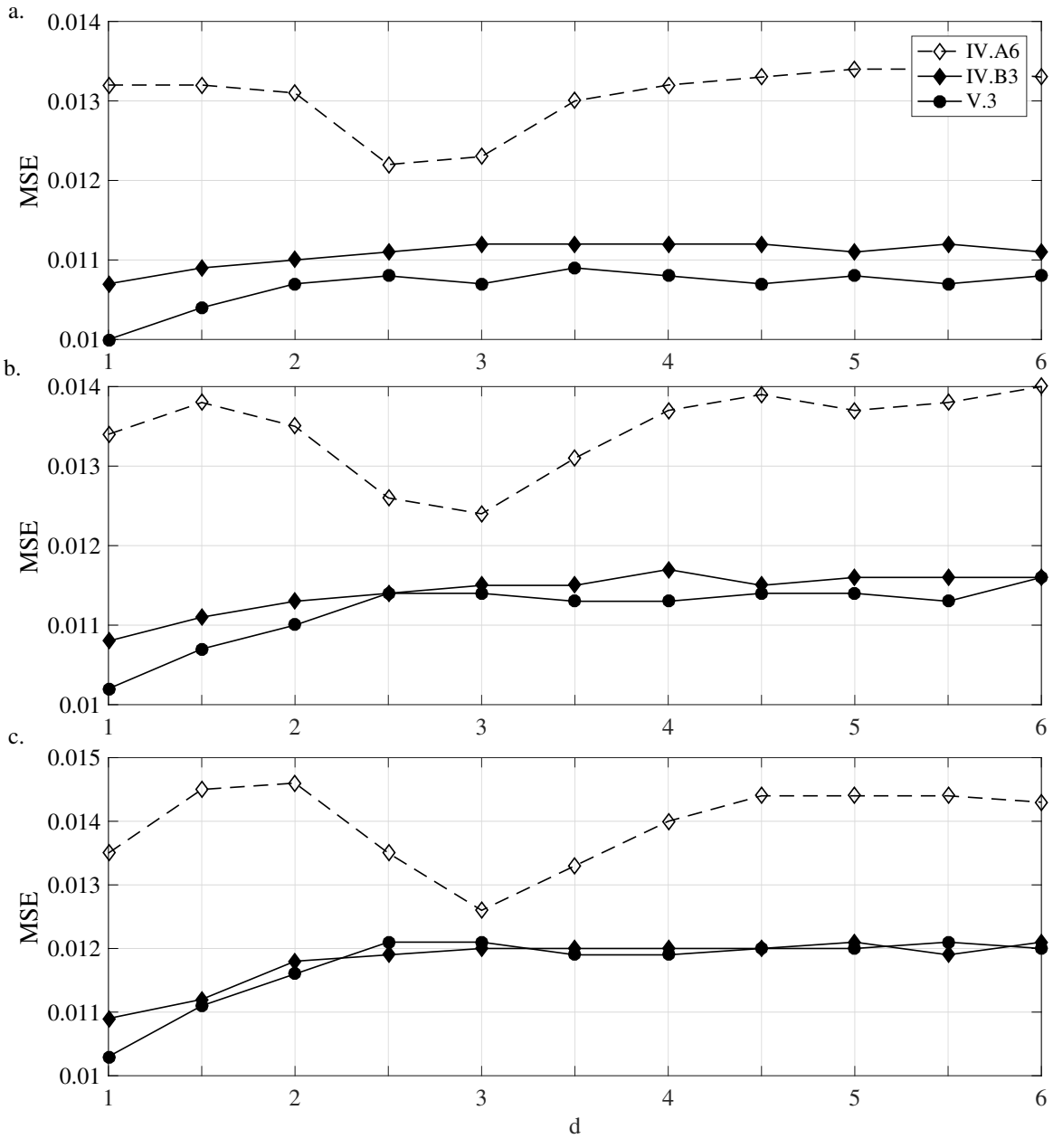

Figure B.9. Phase I efficiency of the proposed estimators for $m = 20$ under CM 5 at a. K = 0.10 b. K = 0.15 c. K = 20.

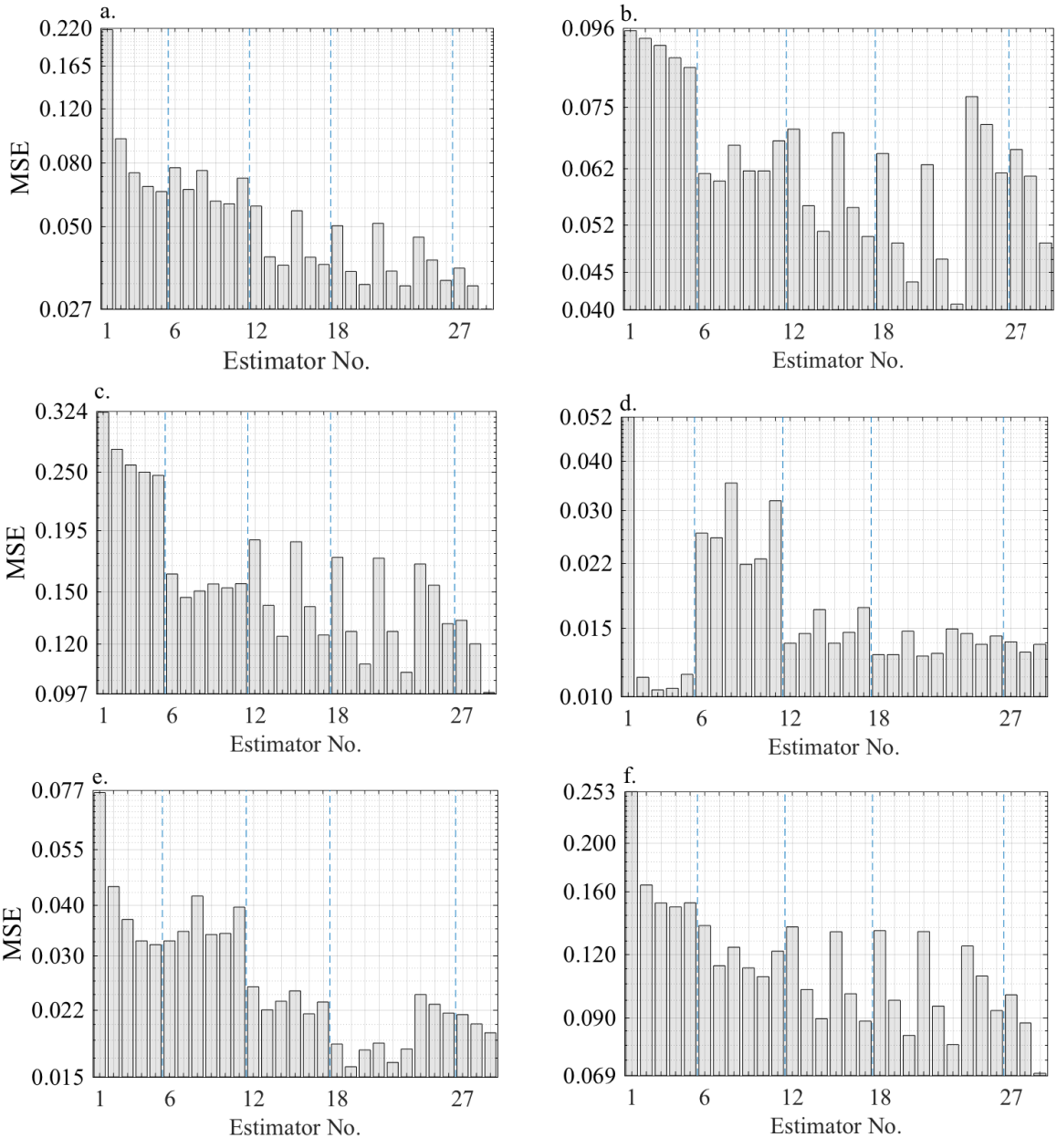

Figure B.10. Phase I efficiency of the proposed estimators for $m = 20$ under a. SCM 1 b. SCM 2 c. SCM 3 d. SCM 4 e. SCM 5 f. SCM 6.

APPENDIX C: PHASE II PERFORMANCES OF S-CHARTS FOR m=50

Phase II performances of S-Charts using proposed estimation methods (Method I-V) are shown for $m = 50$ under normal distribution and in the presence of contaminations with various outlier densities.

Table C.1. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using normally distributed Phase I data.

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0027	399.0	419.0	258.2	430.7	491.5
I.B1	0.0027	400.1	420.7	257.2	431.0	495.1
I.B2	0.0027	408.9	430.5	260.1	441.9	503.3
I.B3	0.0027	412.4	435.7	256.4	452.5	504.5
I.B4	0.0027	411.9	435.4	254.4	455.4	497.5
II.A1	0.0027	438.2	468.3	245.1	514.1	500.6
II.A2	0.0027	463.0	499.8	243.8	572.2	489.6
II.A3	0.0027	483.5	527.1	246.3	628.8	464.9
II.B1	0.0027	440.1	472.2	240.8	523.7	497.8
II.B2	0.0027	452.2	487.3	239.9	538.6	514.6
II.B3	0.0027	479.1	519.2	245.9	604.4	493.9
III.1	0.0027	415.1	437.5	259.4	453.7	506.3
III.2	0.0027	420.4	446.9	246.8	476.2	500.4
III.3	0.0027	441.3	471.9	246.7	510.6	515.7
III.4	0.0027	410.8	434.2	253.4	447.5	504.6
III.5	0.0027	423.7	450.4	251.0	478.5	501.5
III.6	0.0027	441.8	471.3	252.3	512.1	506.3
IV.A1	0.0027	407.1	429.3	254.3	445.0	496.3
IV.A2	0.0027	423.9	450.0	252.1	475.5	509.8
IV.A3	0.0027	435.4	464.2	249.7	499.0	509.3
IV.A4	0.0027	412.9	435.7	258.6	452.2	503.7
IV.A5	0.0027	425.3	451.6	253.0	477.7	509.5
IV.A6	0.0027	435.6	464.0	250.7	500.6	504.6
IV.B1	0.0027	409.6	432.6	256.0	449.4	502.2
IV.B2	0.0027	416.5	440.8	254.2	462.2	503.9
IV.B3	0.0027	424.7	450.0	258.6	479.4	500.1
V.1	0.0027	414.9	437.1	261.8	452.4	506.6
V.2	0.0027	416.4	440.6	255.8	458.4	507.9
V.3	0.0027	420.3	444.7	255.6	472.5	502.1

Table C.2. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using normally distributed Phase I data.

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0885	12.6	13.7	7.8	11.8	18.4
I.B1	0.0881	12.8	14.0	7.8	11.9	18.8
I.B2	0.0862	13.2	14.7	7.9	12.2	19.8
I.B3	0.0864	13.3	14.9	7.7	12.3	20.2
I.B4	0.0851	13.8	15.8	7.7	12.6	21.5
II.A1	0.0786	16.9	22.4	7.5	14.1	29.3
II.A2	0.0738	21.0	33.4	7.4	15.7	39.5
II.A3	0.0674	29.2	58.5	7.4	18.4	59.4
II.B1	0.0770	17.9	24.6	7.5	14.6	31.6
II.B2	0.0768	18.1	24.9	7.4	14.8	32.4
II.B3	0.0709	23.1	37.5	7.5	17.0	44.5
III.1	0.0850	13.6	15.3	7.8	12.5	20.7
III.2	0.0823	14.9	17.7	7.6	13.3	24.0
III.3	0.0797	16.2	20.3	7.5	14.0	27.5
III.4	0.0866	13.3	14.9	7.7	12.2	20.1
III.5	0.0821	15.0	18.0	7.6	13.3	24.5
III.6	0.0782	16.5	20.6	7.6	14.3	27.8
IV.A1	0.0864	13.3	14.8	7.7	12.2	20.1
IV.A2	0.0833	14.5	17.0	7.6	13.0	22.9
IV.A3	0.0803	15.7	19.1	7.6	13.8	26.0
IV.A4	0.0858	13.4	14.9	7.8	12.3	20.3
IV.A5	0.0829	14.5	17.0	7.7	13.0	23.2
IV.A6	0.0803	15.8	19.3	7.5	13.8	26.4
IV.B1	0.0863	13.3	14.9	7.7	12.3	20.2
IV.B2	0.0846	13.9	15.8	7.8	12.6	21.4
IV.B3	0.0821	14.7	17.3	7.8	13.1	23.4
V.1	0.0862	13.3	14.9	7.8	12.2	20.2
V.2	0.0853	13.7	15.5	7.7	12.5	21.0
V.3	0.0828	14.4	16.7	7.8	13.0	22.7

Table C.3. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under CM 1 (d=2, K=0.10).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0023	439.2	445.2	485.0	462.7	364.9
I.B1	0.0023	442.0	448.2	460.7	474.5	375.1
I.B2	0.0023	443.7	450.2	445.4	484.4	386.8
I.B3	0.0023	448.0	456.0	438.7	492.5	392.1
I.B4	0.0023	457.0	466.1	436.9	507.4	401.7
II.A1	0.0023	465.9	478.8	447.7	525.9	387.5
II.A2	0.0023	479.4	499.7	441.0	557.2	381.9
II.A3	0.0023	490.5	521.1	466.3	574.7	355.2
II.B1	0.0023	458.6	474.1	433.4	523.4	375.4
II.B2	0.0023	465.7	482.5	438.1	536.3	381.4
II.B3	0.0023	485.1	509.4	459.1	566.1	375.3
III.1	0.0023	451.5	459.0	426.0	497.8	406.5
III.2	0.0023	449.4	461.0	390.5	509.3	409.6
III.3	0.0023	470.0	485.6	396.7	543.9	417.2
III.4	0.0023	447.4	454.7	422.5	496.0	401.9
III.5	0.0023	454.5	466.0	394.4	515.0	414.8
III.6	0.0023	462.2	476.4	399.9	529.7	411.0
IV.A1	0.0023	455.3	463.5	404.1	506.8	420.5
IV.A2	0.0023	450.9	462.1	379.4	508.8	423.6
IV.A3	0.0023	456.1	469.8	373.9	524.4	425.0
IV.A4	0.0023	449.3	457.2	407.8	500.7	410.1
IV.A5	0.0023	454.9	466.4	382.4	515.6	428.0
IV.A6	0.0023	468.0	483.6	379.5	539.1	431.0
IV.B1	0.0023	450.7	459.1	401.7	500.0	418.2
IV.B2	0.0023	456.7	466.8	395.8	514.6	425.7
IV.B3	0.0023	457.9	470.2	381.8	522.5	426.6
V.1	0.0022	460.3	468.6	406.0	511.8	434.4
V.2	0.0023	451.5	460.7	386.9	507.6	425.4
V.3	0.0023	459.2	470.7	383.6	519.6	433.6

Table C.4. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under CM 1 (d=2, K=0.10).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0404	31.8	40.6	15.0	27.1	53.6
I.B1	0.0453	28.6	36.8	13.3	24.3	48.7
I.B2	0.0479	27.4	36.1	12.5	23.1	47.0
I.B3	0.0501	26.6	35.3	11.8	22.3	46.2
I.B4	0.0503	27.0	36.5	11.5	22.4	47.4
II.A1	0.0469	32.2	48.6	11.6	24.7	60.5
II.A2	0.0461	39.3	68.2	10.9	27.0	80.3
II.A3	0.0415	57.4	115.8	11.1	33.1	128.5
II.B1	0.0474	33.9	53.1	11.1	25.3	65.9
II.B2	0.0477	34.3	54.4	11.1	25.4	66.8
II.B3	0.0442	44.9	81.9	11.0	29.5	93.9
III.1	0.0521	24.8	31.8	11.5	21.3	42.1
III.2	0.0547	25.2	34.2	10.4	20.9	44.5
III.3	0.0538	27.1	38.3	10.3	21.8	49.9
III.4	0.0524	24.7	31.6	11.5	21.1	41.6
III.5	0.0548	25.1	33.8	10.4	20.8	44.4
III.6	0.0533	27.4	38.6	10.3	22.0	50.6
IV.A1	0.0557	22.9	28.8	10.9	19.7	38.5
IV.A2	0.0579	22.9	30.1	10.2	19.4	39.3
IV.A3	0.0572	24.1	32.4	10.0	20.0	42.6
IV.A4	0.0550	23.2	29.5	11.0	19.8	38.9
IV.A5	0.0575	23.1	30.2	10.2	19.5	39.8
IV.A6	0.0573	24.4	33.4	9.9	20.1	43.5
IV.B1	0.0560	22.8	28.9	10.8	19.6	38.4
IV.B2	0.0569	22.8	29.3	10.5	19.6	38.7
IV.B3	0.0573	23.6	31.4	10.2	19.8	41.1
V.1	0.0572	21.9	27.3	10.8	19.1	36.1
V.2	0.0579	22.0	27.7	10.4	19.0	37.1
V.3	0.0586	22.3	28.8	10.1	19.1	38.1

Table C.5. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under CM 1 (d=3, K=0.15).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0048	225.2	244.2	317.6	215.3	147.6
I.B1	0.0037	301.8	328.6	443.9	291.4	186.8
I.B2	0.0033	331.8	357.6	478.5	328.6	206.6
I.B3	0.0031	354.0	378.3	497.5	359.0	225.0
I.B4	0.0030	368.5	393.6	514.4	374.9	233.4
II.A1	0.0028	382.0	405.4	533.8	381.8	253.3
II.A2	0.0026	412.8	441.2	585.5	418.5	264.6
II.A3	0.0027	421.3	460.0	632.4	424.2	249.6
II.B1	0.0028	389.6	414.5	546.0	393.1	256.8
II.B2	0.0027	398.8	424.0	556.3	404.3	261.7
II.B3	0.0026	421.5	454.4	609.8	431.3	262.7
III.1	0.0027	392.0	408.4	504.1	404.7	277.0
III.2	0.0025	418.4	432.9	493.5	447.8	312.1
III.3	0.0024	442.6	459.9	509.8	481.0	326.9
III.4	0.0027	387.9	404.1	499.0	399.2	275.3
III.5	0.0025	423.7	438.2	487.9	456.5	317.6
III.6	0.0024	434.7	451.1	498.4	472.7	323.8
IV.A1	0.0025	414.8	428.7	498.2	438.8	304.7
IV.A2	0.0024	432.4	444.3	467.9	474.7	337.4
IV.A3	0.0024	441.7	454.8	468.1	487.5	350.9
IV.A4	0.0026	406.0	420.0	494.4	427.5	299.2
IV.A5	0.0024	436.1	448.3	470.7	476.5	343.3
IV.A6	0.0023	453.2	468.2	474.8	507.9	356.7
IV.B1	0.0025	416.8	430.0	490.7	443.9	310.1
IV.B2	0.0025	419.3	432.2	478.7	451.2	319.9
IV.B3	0.0024	435.0	448.4	474.2	479.2	336.6
V.1	0.0024	434.6	445.4	488.6	467.2	340.7
V.2	0.0024	431.9	442.6	465.3	470.8	343.1
V.3	0.0023	445.2	456.8	458.3	492.7	361.7

Table C.6. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under CM 1 (d=3, K=0.15).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0063	246.5	312.7	78.3	224.6	461.6
I.B1	0.015	136.0	211.3	30.6	95.6	317.0
I.B2	0.0193	109.1	180.5	24.2	72.2	254.7
I.B3	0.0226	93.1	160.8	21.2	59.2	213.7
I.B4	0.0232	90.4	157.8	20.3	57.2	205.8
II.A1	0.0209	90.1	146.0	23.3	63.0	194.4
II.A2	0.0228	97.6	170.2	20.2	62.4	222.7
II.A3	0.0213	131.4	238.6	20.1	75.7	337.6
II.B1	0.0238	87.2	147.8	19.9	58.3	192.0
II.B2	0.0245	87.2	150.2	19.1	57.0	195.9
II.B3	0.0231	111.7	201.2	19.2	67.6	268.4
III.1	0.0288	59.4	96.8	17.8	42.6	119.2
III.2	0.036	47.2	77.1	14.4	34.2	93.0
III.3	0.0368	47.8	78.7	13.8	33.9	95.6
III.4	0.0288	59.0	95.2	17.9	42.4	118.0
III.5	0.0365	46.6	75.6	14.1	34.0	91.9
III.6	0.0372	47.8	79.1	13.5	34.0	96.7
IV.A1	0.0346	46.5	73.7	15.3	34.3	90.9
IV.A2	0.0413	37.8	58.8	13.0	28.5	71.7
IV.A3	0.0421	37.7	58.9	12.7	28.4	71.7
IV.A4	0.0342	47.8	76.3	15.5	34.8	94.4
IV.A5	0.0413	37.9	58.6	13.0	28.7	71.8
IV.A6	0.0431	37.6	59.3	12.3	27.8	73.3
IV.B1	0.0358	45.2	71.7	14.8	33.1	87.9
IV.B2	0.0385	41.8	66.5	13.9	30.9	81.3
IV.B3	0.041	39.7	63.8	12.9	29.3	76.8
V.1	0.0395	37.4	55.4	14.1	29.1	68.8
V.2	0.0422	35.3	52.1	13.1	27.4	66.3
V.3	0.0450	33.5	50.3	12.3	25.9	62.7

Table C.7. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under CM 1 (d=3, K=0.20).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0067	163.8	178.0	229.4	155.8	107.2
I.B1	0.0052	220.2	248.2	334.5	205.2	129.9
I.B2	0.0046	249.8	281.6	387.8	234.1	142.6
I.B3	0.0042	273.6	306.9	423.0	259.0	154.2
I.B4	0.004	286.9	321.9	446.8	272.5	160.1
II.A1	0.0035	312.5	340.3	457.3	300.2	196.3
II.A2	0.0032	350.2	385.3	531.9	336.5	208.8
II.A3	0.0032	358.2	402.8	571.2	340.3	198.1
II.B1	0.0034	327.9	359.0	494.4	314.0	201.3
II.B2	0.0033	339.9	371.6	515.7	328.1	208.4
II.B3	0.0032	362.0	401.6	566.7	350.6	208.4
III.1	0.0034	324.9	349.9	467.6	318.0	206.3
III.2	0.0029	370.0	392.2	511.4	375.9	243.0
III.3	0.0027	397.7	422.4	548.1	404.2	262.7
III.4	0.0034	321.2	345.6	463.0	314.3	205.4
III.5	0.0029	376.4	398.5	517.9	379.9	251.4
III.6	0.0028	392.6	416.0	535.3	402.2	260.6
IV.A1	0.0031	354.0	377.1	496.4	351.5	231.0
IV.A2	0.0027	392.8	411.2	511.9	407.0	272.4
IV.A3	0.0026	407.7	426.2	520.0	427.9	284.8
IV.A4	0.0031	347.8	370.2	487.8	347.0	227.6
IV.A5	0.0027	397.6	416.2	515.1	413.2	275.2
IV.A6	0.0025	420.1	440.2	532.5	443.9	291.3
IV.B1	0.0030	356.6	378.6	495.5	358.8	234.2
IV.B2	0.0028	381.3	402.7	513.2	388.1	254.7
IV.B3	0.0027	393.9	413.9	514.9	410.4	268.5
V.1	0.0027	389.1	406.8	511.8	395.9	272.6
V.2	0.0027	394.2	410.7	506.1	407.0	279.6
V.3	0.0025	414.8	431.1	511.7	437.9	300.6

Table C.8. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under CM 1 (d=3, K=0.20).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0032	376.3	410.1	189.0	430.1	468.3
I.B1	0.0069	266.5	344.3	67.2	250.5	502.0
I.B2	0.0094	223.9	312.8	46.6	183.8	488.6
I.B3	0.0115	196.3	290.1	37.8	147.1	467.4
I.B4	0.0126	183.0	277.6	34.4	130.4	447.5
II.A1	0.0120	163.0	244.0	38.7	121.0	363.4
II.A2	0.0137	167.9	264.9	31.9	117.7	402.8
II.A3	0.0133	205.8	326.2	31.6	139.2	523.6
II.B1	0.0143	153.3	239.7	31.6	107.7	360.1
II.B2	0.0151	151.3	241.9	29.7	103.5	361.0
II.B3	0.0146	181.4	293.1	28.9	120.0	457.7
III.1	0.0168	118.2	188.3	28.0	81.6	267.6
III.2	0.0238	83.2	139.3	20.1	56.4	182.5
III.3	0.0253	81.8	142.1	18.8	53.4	180.9
III.4	0.0173	115.3	184.3	27.2	79.7	262.3
III.5	0.0243	82.4	139	19.8	55.5	180.5
III.6	0.0261	79.8	137.3	18.2	52.9	177.6
IV.A1	0.0212	90.2	147.5	22.8	62.1	195.6
IV.A2	0.0289	63.2	106.7	17.3	43.7	131.5
IV.A3	0.0307	60.7	103.3	16.1	41.7	126.7
IV.A4	0.0212	90.9	148.8	23.0	62.5	198.6
IV.A5	0.0290	63.7	107.7	17.2	43.9	132.2
IV.A6	0.0315	60.0	102.7	15.6	41.0	124.9
IV.B1	0.0225	86.0	142.9	21.6	58.4	188.2
IV.B2	0.0257	75.0	127.5	19.0	50.5	160.7
IV.B3	0.0291	67.2	116.2	16.8	44.7	144.1
V.1	0.0269	63.5	101.4	19.0	45.8	127.8
V.2	0.0301	57.1	92.7	17.1	40.7	115.4
V.3	0.0331	52.2	85.7	15.5	37.3	104.4

Table C.9. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under CM 2 (d=0.75, K=0.10).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0023	450.2	457.3	388.9	496.7	438.3
I.B1	0.0023	450.9	460.5	359.0	499.5	467.1
I.B2	0.0024	442.6	453.3	342.0	492.5	463.6
I.B3	0.0024	448.2	461.6	334.1	503.9	469.7
I.B4	0.0024	451.2	466.0	334.1	510.0	470.0
II.A1	0.0023	463.7	482.2	336.2	542.2	453.4
II.A2	0.0024	474.3	499.7	331.2	578.0	436.1
II.A3	0.0024	489.8	522.8	342.7	613.5	409.0
II.B1	0.0024	456.9	477.4	326.3	542.5	437.6
II.B2	0.0024	466.8	489.6	324.8	558.3	446.7
II.B3	0.0024	485.6	514.7	335.2	598.7	430.7
III.1	0.0024	446.1	458.8	334.7	496.8	471.5
III.2	0.0025	444.6	462.6	309.9	510.1	473.0
III.3	0.0024	460.7	482.2	309.2	536.7	474.1
III.4	0.0024	441.0	454.0	326.9	492.4	467.5
III.5	0.0024	449.1	467.3	315.3	517.1	470.6
III.6	0.0025	456.9	477.7	311.4	533.8	468.8
IV.A1	0.0024	443.2	457.5	320.2	494.0	480.8
IV.A2	0.0025	442.6	459.6	311.6	504.3	468.3
IV.A3	0.0025	448.7	469.4	299.2	520.7	472.9
IV.A4	0.0024	441.2	454.9	324.0	494.0	473.4
IV.A5	0.0025	439.9	457.0	305.8	500.5	468.7
IV.A6	0.0025	451.0	472.3	298.5	524.1	475.4
IV.B1	0.0024	441.9	456.3	319.6	493.9	477.9
IV.B2	0.0024	440.6	456.4	312.9	495.5	475.6
IV.B3	0.0025	445.4	464.2	308.7	512.4	472.2
V.1	0.0024	445.9	460.3	323.3	497.9	484.9
V.2	0.0025	435.6	450.4	312.3	486.9	475.4
V.3	0.0024	443.2	459.9	310.0	503.5	476.2

Table C.10. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under CM 2 (d=0.75, K=0.10).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0601	20.2	24.6	10.6	17.7	32.4
I.B1	0.0672	17.8	21.1	9.6	15.9	28.0
I.B2	0.0682	17.6	20.8	9.4	15.6	27.9
I.B3	0.0691	17.5	21.0	9.2	15.6	28.3
I.B4	0.0684	18.1	22.1	9.1	15.9	29.6
II.A1	0.0628	22.3	31.4	9.1	18.0	40.0
II.A2	0.0593	28.0	47.0	8.9	20.1	54.5
II.A3	0.0546	39.2	79.8	8.8	23.6	83.1
II.B1	0.0627	23.6	34.7	8.8	18.5	43.7
II.B2	0.0627	23.7	34.6	8.8	18.6	43.9
II.B3	0.0579	30.7	53.0	8.8	21.6	61.3
III.1	0.0696	17.3	20.3	9.2	15.4	27.6
III.2	0.0696	18.3	22.8	8.6	15.9	30.9
III.3	0.0679	19.9	26.3	8.5	16.8	34.9
III.4	0.0710	16.9	20.0	9.0	15.1	26.8
III.5	0.0693	18.5	23.3	8.6	16.1	31.2
III.6	0.0664	20.4	26.8	8.7	17.2	35.6
IV.A1	0.0724	16.4	19.2	8.9	14.7	25.9
IV.A2	0.0715	17.4	21.1	8.6	15.3	28.7
IV.A3	0.0698	18.7	23.9	8.5	16.0	31.8
IV.A4	0.0717	16.6	19.4	9.0	15.0	26.3
IV.A5	0.0709	17.6	21.3	8.7	15.4	28.9
IV.A6	0.0692	19.1	24.4	8.5	16.3	32.8
IV.B1	0.0723	16.5	19.3	8.9	14.8	25.9
IV.B2	0.072	17.0	20.3	8.7	15.1	27.4
IV.B3	0.0705	17.7	21.6	8.7	15.6	29.3
V.1	0.0727	16.3	18.9	8.9	14.6	25.6
V.2	0.073	16.5	19.5	8.8	14.6	26.2
V.3	0.0715	17.3	20.9	8.7	15.3	28.1

Table C.11. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under CM 2 (d=1, K=0.15).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0025	418.4	428.4	490.2	440.6	323.0
I.B1	0.0023	451.0	458.0	436.4	495.0	397.9
I.B2	0.0023	456.7	464.4	427.2	503.7	413.2
I.B3	0.0023	451.2	460.1	406.2	503.7	410.1
I.B4	0.0023	452.4	461.9	405.5	505.5	413.2
II.A1	0.0022	475.1	488.5	439.6	538.3	401.7
II.A2	0.0023	470.9	490.6	424.9	549.2	379.9
II.A3	0.0023	486.2	516.1	435.8	581.8	361.7
II.B1	0.0023	461.3	477.8	417.6	531.5	388.4
II.B2	0.0023	467.0	483.9	410.8	540.9	398.4
II.B3	0.0023	489.9	515.4	442.4	576.4	386.3
III.1	0.0023	457.4	465.6	405.4	507.0	430.8
III.2	0.0023	458.8	472.2	368.4	525.0	438.9
III.3	0.0023	469.5	486.5	365.0	546.0	442.1
III.4	0.0023	449.9	458.5	391.6	501.0	426.3
III.5	0.0023	454.4	467.6	370.5	520.1	428.3
III.6	0.0023	465.9	482.3	369.0	542.5	435.5
IV.A1	0.0023	453.3	462.4	379.2	505.1	445.3
IV.A2	0.0023	458.2	472.0	359.5	524.7	445.4
IV.A3	0.0024	458.1	473.3	353.5	529.9	441.6
IV.A4	0.0023	451.9	460.8	381.3	502.5	439.5
IV.A5	0.0023	463.5	477.2	362.7	529.7	454.8
IV.A6	0.0024	462.3	478.6	356.2	535.0	443.2
IV.B1	0.0023	453.6	463.6	371.8	507.5	446.6
IV.B2	0.0023	449.2	460.5	361.3	506.9	439.3
IV.B3	0.0023	460.5	474.2	360.5	526.3	448.2
V.1	0.0023	448.2	457.9	367.3	498.6	445.6
V.2	0.0023	451.5	462.6	365.1	504.8	451.1
V.3	0.0023	453.9	467.1	354.4	516.3	451.6

Table C.12. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under CM 2 (d=1, K=0.15).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0357	40.4	59.9	15.8	31.5	73.4
I.B1	0.0508	25.5	33.5	11.9	21.5	43.1
I.B2	0.0533	24.3	31.6	11.3	20.6	40.9
I.B3	0.0551	23.7	31.0	10.8	20.0	40.7
I.B4	0.0549	24.3	32.3	10.7	20.4	41.7
II.A1	0.049	30.7	46.4	11.2	23.7	57.4
II.A2	0.048	37.3	65.1	10.6	25.6	75.7
II.A3	0.044	52.8	107.4	10.7	30.5	116.7
II.B1	0.0502	31.6	49.3	10.6	23.8	60.3
II.B2	0.0504	32.2	51.0	10.5	23.9	62.4
II.B3	0.0464	42.3	76.7	10.5	28.1	87.7
III.1	0.0575	22.1	27.9	10.6	19.0	36.9
III.2	0.0598	22.4	29.3	9.8	18.8	39.1
III.3	0.0588	24.0	33.1	9.6	19.7	43.2
III.4	0.0587	21.5	27.0	10.4	18.6	35.9
III.5	0.0592	22.6	29.5	9.8	19.2	39.5
III.6	0.0576	24.6	34.0	9.7	20.2	44.2
IV.A1	0.0613	20.3	25.1	10.1	17.7	33.5
IV.A2	0.0621	20.9	26.6	9.7	17.9	35.6
IV.A3	0.0616	22.1	29.8	9.4	18.4	38.7
IV.A4	0.0608	20.4	25.3	10.2	17.8	33.4
IV.A5	0.0617	21.2	27.2	9.7	18.1	36.1
IV.A6	0.061	22.4	29.7	9.4	18.9	39.2
IV.B1	0.0619	20.1	24.9	10.0	17.6	33.0
IV.B2	0.0616	20.6	25.6	9.9	17.8	34.2
IV.B3	0.0613	21.3	27.3	9.7	18.2	36.1
V.1	0.0627	19.6	23.9	10.0	17.2	31.9
V.2	0.0636	19.6	24.2	9.8	17.1	32.3
V.3	0.0637	20.1	25.4	9.5	17.3	33.7

Table C.13. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under CM 2 (d=1, K=0.20).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0027	385.3	400.5	496	391.8	275.9
I.B1	0.0023	440.0	448	483.6	470.8	356.7
I.B2	0.0023	439.2	446.5	460.4	473.0	368.8
I.B3	0.0023	446.9	455.9	452.2	490.4	374.1
I.B4	0.0023	451.4	460.9	454.7	498.0	380.0
II.A1	0.0023	460.0	473.2	487.1	507.8	363.4
II.A2	0.0023	468.5	488.4	484.9	527.2	355.7
II.A3	0.0023	479.8	509.8	509.9	548.2	335.2
II.B1	0.0023	452.0	467.3	463.1	506.4	355.2
II.B2	0.0023	461.9	478.9	469.4	519.5	364.0
II.B3	0.0023	486.7	512.2	494.4	557.2	361.5
III.1	0.0023	452.0	459.3	440.7	495.6	397.0
III.2	0.0023	454.6	465.5	409.0	510.5	408.7
III.3	0.0023	467.2	481.5	413.0	535.6	411.9
III.4	0.0023	447.8	455.3	436.0	493.8	392.7
III.5	0.0023	456.5	467.9	415.7	513.1	406.9
III.6	0.0023	462.8	476.9	410.9	529.2	405.1
IV.A1	0.0023	455.9	463.6	419.5	505.7	413.7
IV.A2	0.0023	453.3	463.3	403.5	507.5	414.4
IV.A3	0.0023	459.9	473.5	389.2	527.5	417.4
IV.A4	0.0023	452.7	460.2	423.3	498.8	409.3
IV.A5	0.0023	450.6	461.1	393.8	509.0	414.7
IV.A6	0.0023	460.1	474.3	387.2	528.2	418.4
IV.B1	0.0023	451.3	459.2	415.5	501.9	408.1
IV.B2	0.0023	459.1	468.4	413.5	512.9	420.3
IV.B3	0.0023	455.5	466.7	394.9	512.7	416.0
V.1	0.0022	459.6	467.4	413.2	512.3	423.1
V.2	0.0023	449.6	458.0	392.2	500.8	422.7
V.3	0.0023	454.8	465.2	391.9	511.7	425.6

Table C.14. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under CM 2 (d=1, K=0.20).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0265	58.7	90.2	20.4	43.9	112.1
I.B1	0.0414	33.4	47.0	14.1	27.0	58.8
I.B2	0.0445	30.8	43.0	13.0	25.2	54.1
I.B3	0.0466	29.8	41.7	12.3	24.3	53.0
I.B4	0.0466	30.2	42.3	12.2	24.6	54.2
II.A1	0.0415	37.9	58.9	12.9	28.6	72.1
II.A2	0.0412	45.9	82.2	12.0	30.7	94.1
II.A3	0.0375	64.6	128.1	12.2	37.0	147.5
II.B1	0.0431	38.6	62.0	12.1	28.5	76.1
II.B2	0.0433	39.5	64.8	11.9	28.6	79.2
II.B3	0.0404	52.1	96.1	11.8	33.3	112.9
III.1	0.0496	26.6	35.0	11.9	22.4	45.8
III.2	0.0528	26.3	35.8	10.8	21.7	47.0
III.3	0.0524	28.0	39.9	10.5	22.4	51.4
III.4	0.0507	25.9	34.0	11.8	21.8	44.6
III.5	0.0524	26.5	36.1	10.9	22.0	47.3
III.6	0.0513	28.7	41.3	10.8	23.0	53.1
IV.A1	0.0536	24.1	31.0	11.2	20.5	41.0
IV.A2	0.0554	24.2	31.9	10.6	20.3	41.8
IV.A3	0.0553	25.4	35.2	10.2	20.8	45.3
IV.A4	0.0533	24.2	31.2	11.4	20.6	40.9
IV.A5	0.0552	24.4	32.6	10.5	20.5	42.6
IV.A6	0.055	25.6	35.1	10.2	21.3	45.7
IV.B1	0.0545	23.6	30.2	11.1	20.2	39.6
IV.B2	0.055	23.9	31.2	10.8	20.2	41.0
IV.B3	0.055	24.7	33.1	10.5	20.7	43.2
V.1	0.0555	22.8	28.8	11.0	19.6	38.2
V.2	0.0568	22.6	28.8	10.6	19.4	38.2
V.3	0.0574	23.0	30.1	10.4	19.4	39.3

Table C.15. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under CM 3 (d=2, K=0.10).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0022	453.2	457.3	481.2	477.8	393.5
I.B1	0.0023	444.0	454	354.4	495.1	449.2
I.B2	0.0024	437.7	450.6	326.4	492.7	461.9
I.B3	0.0024	443.8	458.6	322.3	499.7	473.4
I.B4	0.0024	450.5	466.9	321.0	511.8	480.4
II.A1	0.0022	471.4	484.5	426.5	534.6	407.9
II.A2	0.0023	476.7	497.9	393.2	567.9	403.0
II.A3	0.0023	492.2	522.3	414.4	594.5	376.4
II.B1	0.0024	457.3	476.4	350.7	540.2	417.8
II.B2	0.0024	466.1	488.6	340.7	562.2	425.0
II.B3	0.0024	489.6	518.5	381.0	595.5	400.5
III.1	0.0023	455.1	466.6	360.3	513.1	454.0
III.2	0.0024	448.2	465.3	322.6	518.9	452.0
III.3	0.0024	470.1	491.3	326.2	552.2	470.6
III.4	0.0023	448.1	459.3	355.0	505.3	447.5
III.5	0.0024	456.3	473.9	329.1	529.3	458.3
III.6	0.0025	455.0	475.2	319.6	536.8	448.6
IV.A1	0.0023	450.0	460.7	361.6	504.7	448.7
IV.A2	0.0024	453.7	469.8	335.2	520.9	457.9
IV.A3	0.0024	462.6	481.8	330.5	540.4	461.4
IV.A4	0.0024	439.7	449.8	356.8	492.1	438.9
IV.A5	0.0024	443.5	458.6	328.0	509.1	450.2
IV.A6	0.0024	459.2	479.3	322.1	540.0	459.8
IV.B1	0.0024	441.2	453.9	333.2	498.6	449.6
IV.B2	0.0024	445.0	460.2	324.0	509.0	457.7
IV.B3	0.0025	441.7	458.6	313.8	505.8	459.5
V.1	0.0024	447.6	459.9	344.8	503.9	458.1
V.2	0.0024	442.5	456.1	332.8	499.1	462.1
V.3	0.0025	440.7	457.3	310.6	502.5	463.0

Table C.16. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under CM 3 (d=2, K=0.10).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0456	26.4	31.5	14.0	23.6	42.1
I.B1	0.0652	18.7	22.5	9.7	16.5	30.1
I.B2	0.069	17.7	21.2	9.1	15.7	28.4
I.B3	0.0711	17.2	20.7	8.8	15.2	27.9
I.B4	0.0702	17.7	21.6	8.8	15.5	29.0
II.A1	0.0506	29.0	42.3	10.9	22.9	53.5
II.A2	0.0522	32.9	55.9	9.8	23.4	65.5
II.A3	0.0459	49.3	100.3	10.3	29.0	106.4
II.B1	0.058	26.6	40.7	9.3	20.2	50.8
II.B2	0.0589	26.7	41.5	9.1	20.1	51.0
II.B3	0.0523	37.8	70.9	9.3	24.7	78.0
III.1	0.0631	19.7	24.1	9.8	17.3	32.5
III.2	0.0652	20.3	26.2	9.0	17.3	34.9
III.3	0.0649	21.2	28.5	8.8	17.6	37.8
III.4	0.0641	19.3	23.5	9.7	17.0	31.6
III.5	0.0658	20.0	25.9	8.9	17.1	34.4
III.6	0.0647	21.6	29.4	8.7	17.8	38.6
IV.A1	0.0635	19.5	23.8	9.8	17.1	32.0
IV.A2	0.0656	19.7	25.0	9.1	17.0	33.4
IV.A3	0.0653	20.6	27.1	8.9	17.3	35.8
IV.A4	0.0628	19.7	24.1	9.8	17.4	32.3
IV.A5	0.0653	20	25.6	9.1	17.1	34.1
IV.A6	0.0658	20.7	27.4	8.7	17.4	36.5
IV.B1	0.0662	18.8	22.9	9.3	16.4	31.0
IV.B2	0.0681	18.5	22.9	9.0	16.1	30.7
IV.B3	0.0682	19.0	24.1	8.8	16.3	32.2
V.1	0.0667	18.5	22.4	9.4	16.2	30.3
V.2	0.0691	17.9	22.0	9.0	15.7	29.4
V.3	0.0697	18.2	22.8	8.7	15.8	30.4

Table C.17. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under CM 3 (d=3, K=0.15).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0041	259.4	271.8	338.7	251.9	190.8
I.B1	0.0025	430.5	443.7	326.8	494.3	427.2
I.B2	0.0024	440.2	455.7	319.3	502.8	457.2
I.B3	0.0024	444.8	458.9	328.7	506.8	456.4
I.B4	0.0023	452.4	463.6	366.9	511.0	439.9
II.A1	0.0025	422.1	440.8	553.8	432.8	296.0
II.A2	0.0023	461.8	482.4	548.4	499.2	329.9
II.A3	0.0024	456.4	488.4	604.4	483.9	292.6
II.B1	0.0024	456.6	475.3	423.7	532.5	364.8
II.B2	0.0024	462.8	484.4	392.6	547.8	380.7
II.B3	0.0024	468.9	498.7	470.1	545.1	331.3
III.1	0.0023	450.7	462.5	390.2	513.6	405.6
III.2	0.0024	450.9	467.7	345.8	529.9	426.1
III.3	0.0024	462.8	481.6	348.7	545.5	436.2
III.4	0.0024	444.0	456.5	370.4	509.9	406.0
III.5	0.0024	452.0	469.2	338.7	530.8	432.1
III.6	0.0024	457.0	476.5	333.0	538.0	435.8
IV.A1	0.0023	446.0	457.6	390.5	509.2	400.2
IV.A2	0.0024	442.3	456.8	347.5	512.8	419.5
IV.A3	0.0024	453.2	470.3	346.4	531.5	427.7
IV.A4	0.0024	439.4	451.1	376.8	504.2	397.4
IV.A5	0.0025	441.8	458.1	339.1	513.4	419.4
IV.A6	0.0024	452.3	471.1	327.8	532.7	436.1
IV.B1	0.0025	431.1	444.6	336.0	495.6	416.3
IV.B2	0.0025	440.2	456.8	317.8	509.9	447.0
IV.B3	0.0025	443.5	461.5	313.4	516.2	449.4
V.1	0.0025	434.4	448.7	327.9	500.1	433.9
V.2	0.0025	438.1	455.4	309.2	506.3	458.1
V.3	0.0025	440.6	459.0	304.8	510.1	459.3

Table C.18. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under CM 3 (d=3, K=0.15).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0089	159.1	204.9	61.6	137.0	291.7
I.B1	0.0660	20.0	27.1	9.1	16.4	34.3
I.B2	0.0697	18.2	23.2	8.9	15.5	30.1
I.B3	0.0679	18.5	23.2	9.1	15.9	30.4
I.B4	0.0624	20.4	25.8	9.8	17.5	34.2
II.A1	0.0262	64.9	101.5	19.6	47.9	129.3
II.A2	0.0341	56.6	97.8	14.4	38.3	120.2
II.A3	0.0283	90.6	172.0	15.8	52.3	218.9
II.B1	0.0477	36.9	63.2	10.8	25.6	74.1
II.B2	0.0510	34.9	60.8	10.0	24.2	70.5
II.B3	0.0411	59.3	119.9	11.2	34.1	133.1
III.1	0.0552	25.3	35.2	10.4	20.4	45.4
III.2	0.0606	23.3	32.7	9.4	18.9	42.0
III.3	0.0603	24.1	34.6	9.2	19.2	44.3
III.4	0.0578	24.0	33.5	10.0	19.4	42.3
III.5	0.0619	23.0	32.5	9.1	18.6	41.3
III.6	0.0614	23.8	34.7	9.1	18.8	43.7
IV.A1	0.056	24.9	35.0	10.3	20.0	44.9
IV.A2	0.0610	22.9	31.7	9.4	18.5	40.6
IV.A3	0.0612	23.1	32.5	9.2	18.7	41.7
IV.A4	0.0564	24.9	35.2	10.1	19.9	44.7
IV.A5	0.0628	22.4	31.4	9.1	18.0	39.9
IV.A6	0.0627	23.0	33.1	8.9	18.3	42.0
IV.B1	0.0634	21.4	29.5	9.3	17.4	37.4
IV.B2	0.0668	19.9	26.5	8.9	16.5	34.3
IV.B3	0.0666	20.1	26.9	8.9	16.7	34.7
V.1	0.0653	20.4	27.3	9.1	16.8	35.5
V.2	0.0693	18.8	24.7	8.7	15.9	31.9
V.3	0.0695	18.9	24.9	8.6	15.8	32.2

Table C.19. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under CM 3 (d=3, K=0.20).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0051	207.9	218.3	271.8	201.7	152.9
I.B1	0.0025	427.2	440.1	350.5	495.3	387.6
I.B2	0.0024	441.9	455.9	340.7	510.4	429.8
I.B3	0.0024	448.3	460.5	358.9	511.1	434.4
I.B4	0.0023	453.9	463.3	402.9	508.2	412.8
II.A1	0.0029	372.3	396.1	521.6	366.8	248.0
II.A2	0.0025	434.2	458.7	581.7	447.7	290.8
II.A3	0.0027	419.2	455.3	625.4	423.1	255.1
II.B1	0.0025	439.6	460.5	483.5	491.1	318.7
II.B2	0.0024	451.7	473.7	449.6	518.3	340.5
II.B3	0.0025	446.7	478.7	533.7	494.0	290.1
III.1	0.0024	439.2	451.6	447.5	491.4	352.7
III.2	0.0024	448.2	463.4	391.7	519.6	385.3
III.3	0.0023	462.2	478.9	388.7	540.2	403.4
III.4	0.0024	436.2	448.7	416.7	493.1	360.5
III.5	0.0024	450.5	466.2	377.9	521.9	395.2
III.6	0.0024	456.5	474.1	370.8	534.6	405.8
IV.A1	0.0024	432.7	445.4	441.6	482.5	343.3
IV.A2	0.0024	437.8	451.3	386.6	503.3	376.8
IV.A3	0.0024	451.1	466.6	385.4	523.2	393.1
IV.A4	0.0024	428.9	441.4	416.3	484.1	350.8
IV.A5	0.0024	439.0	454.2	371.4	510.6	383.3
IV.A6	0.0024	450.3	468.3	355.5	530.7	407.8
IV.B1	0.0024	437.3	450.4	382.6	504.9	381.4
IV.B2	0.0024	440.1	454.5	350.5	510.2	412.8
IV.B3	0.0024	443.1	458.6	340.6	514.7	423.1
V.1	0.0024	435.2	447.7	362.1	502.8	399.2
V.2	0.0024	440.6	456.1	330.7	512.5	428.8
V.3	0.0024	442.9	459.5	322.6	516.2	437.6

Table C.20. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under CM 3 (d=3, K=0.20).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.005	264.6	312.9	111.9	253.8	438.8
I.B1	0.0595	24.3	36.6	9.6	18.6	44.5
I.B2	0.0643	20.8	28.7	9.3	17.0	36.0
I.B3	0.0625	20.9	27.6	9.6	17.5	35.5
I.B4	0.0555	23.7	31.3	10.8	19.9	40.5
II.A1	0.0179	102.2	158.6	27.2	74.3	216.8
II.A2	0.0265	77.2	133.9	17.9	51.1	167.4
II.A3	0.0212	126.4	226.8	20.3	75.3	316
II.B1	0.0392	50.7	92.4	12.4	32.8	106.7
II.B2	0.0434	45.4	83.2	11.3	29.7	95.2
II.B3	0.0334	80.9	161.1	13.3	44.7	196.0
III.1	0.0454	33.9	51.3	11.9	25.8	64.9
III.2	0.0531	28.6	43.4	10.3	22.0	53.6
III.3	0.0535	28.8	43.6	10.1	22.1	54.9
III.4	0.0489	30.9	46.6	11.2	23.7	57.8
III.5	0.0546	28.0	42.6	10.0	21.4	53.0
III.6	0.0553	28.0	42.5	9.7	21.4	53.4
IV.A1	0.0452	34.6	53.6	11.8	26	66.5
IV.A2	0.0531	28.7	44.2	10.3	21.8	54.3
IV.A3	0.0539	28.3	42.6	10.1	21.7	53.1
IV.A4	0.0478	32.7	50.9	11.2	24.6	62.2
IV.A5	0.0561	26.8	40.4	9.8	20.6	50.1
IV.A6	0.0577	26.6	40.9	9.5	20.3	49.9
IV.B1	0.0549	27.2	41.5	10.1	20.7	50.7
IV.B2	0.0598	23.7	34.3	9.5	18.8	43.0
IV.B3	0.0615	22.9	32.4	9.3	18.3	41.0
V.1	0.0581	24.4	35.1	9.8	19.3	44.4
V.2	0.0636	21.6	30.4	9.1	17.4	38.0
V.3	0.0647	21.1	29.1	9.0	17.2	37.1

Table C.21. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under CM 4 (d=2, K=0.10).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0025	409.7	418.0	489.4	416.7	325.2
I.B1	0.0025	411.7	420.6	498.0	418.6	324.9
I.B2	0.0025	415.1	424.4	504.8	424.0	325.5
I.B3	0.0025	415.5	425.9	509.5	425.0	321.7
I.B4	0.0025	419.5	430.5	512.1	431.6	324.4
II.A1	0.0023	449.5	466.0	558.7	472.4	326.8
II.A2	0.0024	451.9	474.6	563.7	479.0	312.3
II.A3	0.0024	457.8	490.9	601.2	488.7	291.1
II.B1	0.0024	434.4	452.2	536.9	459.6	312.4
II.B2	0.0024	444.7	463.6	546.7	472.8	318.8
II.B3	0.0024	461.1	488.3	589.5	494.2	311.1
III.1	0.0024	422.0	431.5	508.7	434.0	331.8
III.2	0.0024	430.7	442.9	513.9	451.5	332.2
III.3	0.0023	448.0	463.3	538.6	477.0	336.9
III.4	0.0024	420.4	429.3	499.2	433.9	331.1
III.5	0.0024	434.9	447.4	516.4	458.4	333.6
III.6	0.0024	437.9	453.4	518.3	468.5	326.2
IV.A1	0.0025	418.5	427.4	497.6	431.6	329.9
IV.A2	0.0024	435.7	446.7	505.9	463.1	340.7
IV.A3	0.0024	438.5	452.1	502.2	470.4	336.3
IV.A4	0.0024	427.6	437.0	509.1	441.6	337.1
IV.A5	0.0024	435.2	446.5	504.2	460.6	338.6
IV.A6	0.0024	434.8	448.2	496.6	469.0	334.6
IV.B1	0.0024	424.7	433.8	500.1	441.9	335.4
IV.B2	0.0024	424.1	434.4	503.0	443.3	331.3
IV.B3	0.0024	428.2	439.6	497.6	451.7	332.8
V.1	0.0024	430.7	439.3	505.9	450.2	342.1
V.2	0.0024	425.5	434.9	494.3	447.8	334.5
V.3	0.0023	443.7	454.7	509.7	470.6	347.6

Table C.22. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under CM 4 (d=2, K=0.10).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0300	42.4	53.2	20.2	36.8	71.3
I.B1	0.0302	42.9	54.5	19.6	37.2	72.9
I.B2	0.0304	43.7	56.7	19.3	37.1	75.7
I.B3	0.0311	44.1	58.5	18.4	37.1	78.4
I.B4	0.0317	45.0	61.9	17.6	36.6	81.7
II.A1	0.0317	51.5	80.7	16.5	38.3	100.6
II.A2	0.0311	65.0	115.6	15.4	42.6	139.9
II.A3	0.0288	89.7	171.1	15.4	51.6	215.2
II.B1	0.0321	55.3	89.4	15.6	40.0	112.8
II.B2	0.0324	55.9	91.4	15.4	40.4	114.5
II.B3	0.0302	74.1	134.6	15.3	46.9	166.5
III.1	0.0325	42.2	56.3	17.5	35.3	74.7
III.2	0.0349	43.2	61.9	15.5	34.6	80.4
III.3	0.0350	46.8	70.9	14.8	35.7	91.7
III.4	0.0333	41.1	54.4	17.2	34.4	73.1
III.5	0.0348	43.7	63.7	15.5	34.5	82.0
III.6	0.0346	47.7	73.0	14.9	36.4	93.3
IV.A1	0.0337	40.4	53.7	17.1	33.9	71.6
IV.A2	0.0364	40.3	56.8	15.1	32.6	74.5
IV.A3	0.0371	42.6	63.9	14.3	32.9	81.4
IV.A4	0.0337	40.4	53.3	17.1	34.0	71.4
IV.A5	0.0365	40.5	57.6	15.0	32.5	74.9
IV.A6	0.0368	43.4	65.0	14.1	33.5	84.1
IV.B1	0.0345	39.7	52.9	16.6	33.2	70.5
IV.B2	0.0349	40.6	55.3	16.0	33.7	73.4
IV.B3	0.0359	41.8	59.6	15.2	33.4	78.1
V.1	0.0357	38.0	50.4	16.1	31.8	66.9
V.2	0.0366	37.9	51.0	15.6	31.4	67.9
V.3	0.0376	38.8	55.0	14.7	31.1	71.1

Table C.23. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under CM 4 (d=3, K=0.15).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0056	187.3	196.4	244.1	181.3	137.5
I.B1	0.0056	188.8	199.3	248.9	182.3	136.7
I.B2	0.0056	189.4	202.1	256.8	180.9	133.7
I.B3	0.0056	193.9	210.0	268.0	183.6	132.6
I.B4	0.0056	195.8	216.1	279.8	181.8	129.0
II.A1	0.0042	263.7	291.6	387.2	248.2	163.7
II.A2	0.0039	293.6	331.1	456.6	273.4	169.4
II.A3	0.0039	303.9	349.9	493.3	281.2	164.1
II.B1	0.0042	267.7	299.7	409.8	248.9	160.1
II.B2	0.0041	278.1	313.0	431.6	258.6	163.1
II.B3	0.0039	301.6	346.8	491.2	276.8	165.0
III.1	0.0049	221.8	242.9	317.4	208.6	145.3
III.2	0.0043	264.8	295.8	404.4	247.2	157.7
III.3	0.004	289.4	325.8	461.6	270.5	166.2
III.4	0.0050	220.8	242.0	315.8	206.8	145.0
III.5	0.0043	266.1	298.3	408.7	247.1	157.6
III.6	0.0040	287.0	323.3	456.9	269.5	164.1
IV.A1	0.0049	223.7	246.4	325.9	208.9	145.0
IV.A2	0.0042	268.4	300.5	415.4	249.9	159.3
IV.A3	0.0039	292.8	328.3	466.3	275.6	167.6
IV.A4	0.0048	227.1	249.4	327.5	213.0	146.9
IV.A5	0.0041	276.2	309.9	435.1	257.9	161.0
IV.A6	0.0037	308.3	346.7	498.6	291.2	173.3
IV.B1	0.0047	234.3	259.5	345.5	217.7	148.4
IV.B2	0.0046	247.0	276.4	377.2	228.9	150.4
IV.B3	0.0042	271.0	306.5	435.8	251.5	155.0
V.1	0.0042	264.3	291.5	394.5	248.8	164.7
V.2	0.004	281.0	311.3	433.3	265.2	169.1
V.3	0.0036	313.3	346.6	489.4	299.9	183.9

Table C.24. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under CM 4 (d=3, K=0.15).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0038	323.7	364.8	154.4	330.9	484.9
I.B1	0.0038	329.7	372.7	152.4	339.1	492.6
I.B2	0.0040	328.7	374.4	144.2	341.3	493.9
I.B3	0.0041	337.9	387.8	137.1	359.0	509.1
I.B4	0.0044	338.1	394.2	120.7	366.2	512.0
II.A1	0.0073	246.2	329.3	64.1	213.1	499.8
II.A2	0.0088	248.0	350.2	50.2	200.7	543.2
II.A3	0.0089	285.4	403.4	47.5	239.0	623.2
II.B1	0.0083	245.8	336.7	53.9	210.2	515.1
II.B2	0.0091	236.4	330.1	48.7	196.2	510.3
II.B3	0.0091	273.2	385.9	46.2	228.7	596.3
III.1	0.0060	272.1	340.0	83.0	261.5	488.0
III.2	0.0094	220.6	307.0	47.5	183.5	468.1
III.3	0.0111	209.1	305.9	39.0	161.7	477.8
III.4	0.0062	265.0	331.3	80.7	254.1	476.4
III.5	0.0098	218.7	307.6	45.7	181.3	474.1
III.6	0.0116	210.0	310.1	36.9	160.5	485.3
IV.A1	0.0064	262.7	329.9	77.3	253.4	473.9
IV.A2	0.0106	212.3	303.4	41.3	170.6	469.7
IV.A3	0.0130	192.0	290.0	32.8	141.9	459.1
IV.A4	0.0066	260.8	330.6	73.6	248.1	479.3
IV.A5	0.0115	200.8	293.0	38.1	156.0	457.2
IV.A6	0.0146	179.1	278.3	28.7	125.5	439.5
IV.B1	0.0073	249.2	322.8	64.3	230.3	471.9
IV.B2	0.0090	232.0	315.2	49.8	203.6	471.4
IV.B3	0.0113	210.9	303.5	37.5	170.6	468.4
V.1	0.0102	195.2	274.7	44.9	159.8	418.6
V.2	0.0123	175.6	259.4	36.2	132.9	399.3
V.3	0.0157	153.5	242.8	27.8	105.0	374.9

Table C.25. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under CM 4 (d=3, K=0.20).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0074	142.1	148.1	182.6	138.4	106.5
I.B1	0.0074	142.0	148.4	183.9	137.7	105.9
I.B2	0.0075	140.8	147.9	184.0	135.9	103.6
I.B3	0.0077	138.1	146.2	182.8	132.9	100.0
I.B4	0.0078	137.6	147.9	186.8	130.7	96.4
II.A1	0.006	186.1	206.6	270.8	174.3	117.1
II.A2	0.0056	208.6	240.5	322.3	192.0	117.5
II.A3	0.0055	219.2	260.1	353.9	197.7	114.7
II.B1	0.0061	187.8	213.1	282.0	173.2	112.1
II.B2	0.0059	195.5	223.9	298.0	179.0	114.4
II.B3	0.0056	212.9	251.1	339.8	191.0	115.6
III.1	0.0070	154.6	166.8	211.0	146.3	107.0
III.2	0.0064	175.4	197.9	259.1	161.0	109.1
III.3	0.006	193.7	224.3	300.0	174.7	111.6
III.4	0.0070	153.3	165.2	209.3	144.9	106.7
III.5	0.0063	178.8	203.4	268.0	163.6	108.5
III.6	0.0061	192.1	225.3	302.2	171.7	110.0
IV.A1	0.0072	149.9	161.7	204.0	141.3	104.8
IV.A2	0.0066	172.0	195.7	255.8	156.4	106.0
IV.A3	0.0062	186.6	218.3	293.7	166.9	106.7
IV.A4	0.0071	152.4	165.3	209.4	143.5	105.5
IV.A5	0.0064	178.5	205	271.2	161.3	107.4
IV.A6	0.0060	196.8	231.6	314.9	175.3	109.9
IV.B1	0.0070	155.7	169.7	215.4	146.3	106.2
IV.B2	0.0067	164.6	184.8	239.0	150.7	106.2
IV.B3	0.0065	176.2	203.7	268.8	158.0	106.5
V.1	0.0063	175.8	195.4	254.1	162.4	113.6
V.2	0.0061	184.9	209.6	276.8	168.6	113.7
V.3	0.0057	203.6	235.4	320.0	183.8	117.1

Table C.26. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under CM 4 (d=3, K=0.20).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0025	427.4	438.8	321.5	470.5	462.1
I.B1	0.0024	440.8	452.5	333.6	489.4	469.5
I.B2	0.0024	447.4	459.1	341.8	497.4	470.6
I.B3	0.0024	443.4	456.0	340.7	498.8	454.8
I.B4	0.0024	452.7	466.8	349.7	516.5	452.7
II.A1	0.0033	400.7	448.3	170.1	466.5	522.1
II.A2	0.0039	394.7	459.6	128.1	470.9	530.4
II.A3	0.0041	423.7	499.3	121.8	528.9	547.8
II.B1	0.0037	391.8	444.5	148.3	467.7	507.2
II.B2	0.0039	398.0	458.3	135.3	477.0	531.9
II.B3	0.0041	415.5	484.7	127.0	512.2	536.3
III.1	0.0028	417.1	442.1	249.6	479.7	477.6
III.2	0.0037	385.9	432.5	155.0	459.1	492.1
III.3	0.0043	381.9	442.7	120.2	462.8	507.6
III.4	0.0028	417.2	442.7	246.7	480.7	478.7
III.5	0.0037	386.9	435.2	150.1	463.5	491.4
III.6	0.0044	380.9	441.9	120.3	461.1	505.1
IV.A1	0.0027	420.7	443.8	263.0	483.2	472.5
IV.A2	0.0037	388.7	432.7	159.7	471.6	479.2
IV.A3	0.0043	376.8	434.6	120.7	464.9	486.7
IV.A4	0.0028	410.4	434.2	247.2	470.3	465.7
IV.A5	0.0039	386.1	437.9	140.2	467.5	491.2
IV.A6	0.0050	363.8	431.1	97.4	436.8	503.4
IV.B1	0.0029	410.7	439.9	225.0	479.2	480.7
IV.B2	0.0033	395.7	431.2	191.9	470.2	471.7
IV.B3	0.0041	383.1	433.0	140.4	468.6	480.5
V.1	0.0037	375.0	420.0	156.2	434.7	494.1
V.2	0.0043	354.4	408.8	120.7	409.6	494.4
V.3	0.0058	325.5	396.1	83.1	362.8	503.8

Table C.27. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under CM 5 (d=2, K=0.10).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
IV.A1	0.0031	379.3	410.6	207.9	407.2	504.0
IV.A2	0.0032	387.2	424.9	192.9	427.2	519.2
IV.A3	0.0031	401.5	441.8	192.9	452.9	526.5
IV.A4	0.0031	380.3	410.9	210.5	412.3	499.6
IV.A5	0.0032	384.8	421.8	193.9	420.9	515.7
IV.A6	0.0031	401.6	441.7	191.9	449.2	528.8
IV.B1	0.0028	405.2	430.1	245.4	446.0	500.7
IV.B2	0.0027	412.9	438.5	246.3	459.1	503.2
IV.B3	0.0027	420.8	447.9	247.3	477.4	499.5
V.1	0.0028	402.9	426.8	246.5	439.8	498.8
V.2	0.0027	409.9	435.0	246.1	453.5	503.2
V.3	0.0027	415.6	442.0	244.3	466.6	498.9

Table C.28. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under CM 5 (d=2, K=0.10).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
IV.A1	0.0964	12.0	13.4	6.9	11.0	18.2
IV.A2	0.0965	12.4	14.5	6.6	11.1	19.7
IV.A3	0.0932	13.3	16.1	6.7	11.7	21.7
IV.A4	0.0957	12.1	13.5	7.0	11.0	18.4
IV.A5	0.0950	12.6	14.7	6.7	11.3	20.0
IV.A6	0.0928	13.5	16.5	6.6	11.8	22.2
IV.B1	0.0872	13.3	15.0	7.6	12.2	20.5
IV.B2	0.0855	13.8	15.9	7.6	12.5	21.6
IV.B3	0.0831	14.7	17.4	7.5	13.2	23.6
V.1	0.087	13.3	15.1	7.6	12.1	20.5
V.2	0.0863	13.7	15.7	7.5	12.4	21.3
V.3	0.0841	14.4	16.9	7.5	12.9	22.9

Table C.29. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under CM 5 (d=3, K=0.15).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
IV.A1	0.0036	335.9	374.9	166.6	346.1	494.0
IV.A2	0.0032	379.8	418.3	187.9	412.1	521.8
IV.A3	0.0030	405.0	442.9	204.0	454.0	527.1
IV.A4	0.0035	340.1	377.7	172.6	353.1	490.7
IV.A5	0.0031	385.8	421.2	201.4	419.6	519.1
IV.A6	0.0030	411.8	447.9	212.4	460.6	527.6
IV.B1	0.0028	406.5	431.7	244.0	450.2	499.9
IV.B2	0.0028	413.5	440.2	242.1	462.5	503.8
IV.B3	0.0028	417.1	447.7	241.9	472.3	496.8
V.1	0.0028	401.1	426.2	239.4	441.0	496.9
V.2	0.0028	407.0	433.6	238.7	451.1	501.4
V.3	0.0028	413.4	441.1	239.1	467.5	496.2

Table C.30. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under CM 5 (d=3, K=0.15).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
IV.A1	0.1110	10.2	11.1	6.2	9.4	15.0
IV.A2	0.0993	11.8	13.4	6.6	10.8	18.3
IV.A3	0.0926	13.2	15.6	6.8	11.7	21.3
IV.A4	0.1091	10.3	11.2	6.3	9.6	15.3
IV.A5	0.0955	12.3	14.1	6.9	11.2	19.2
IV.A6	0.0897	13.7	16.4	6.9	12.2	22.1
IV.B1	0.0876	13.4	15.4	7.5	12.2	20.8
IV.B2	0.0853	14.0	16.4	7.5	12.6	22.3
IV.B3	0.0831	14.9	17.9	7.5	13.1	24.2
V.1	0.0870	13.5	15.6	7.5	12.3	21.1
V.2	0.0865	13.8	16.1	7.5	12.3	21.9
V.3	0.0844	14.5	17.4	7.4	12.9	23.4

Table C.31. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under CM 5 (d=3, K=0.20).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
IV.A1	0.0041	309.6	354.9	139.7	309.3	481.8
IV.A2	0.0035	354.4	397.3	165.4	375.5	515.1
IV.A3	0.0033	380.8	423.0	178.2	417.2	522.6
IV.A4	0.0038	323.8	367.8	153.0	325.6	495.1
IV.A5	0.0033	372.2	411.8	182.1	399.2	518.5
IV.A6	0.0031	395.3	434.5	194.2	436.2	526.5
IV.B1	0.0028	402.3	429.0	232.7	446.7	500.6
IV.B2	0.0028	409.5	437.0	235.6	457.0	503.9
IV.B3	0.0028	418.4	447.6	235.0	477.0	499.2
V.1	0.0028	403.8	430.6	234.7	448.6	500.0
V.2	0.0028	411.7	440.2	234.3	460.3	509.7
V.3	0.0028	419.8	449.7	234.8	474.8	507.0

Table C.32. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under CM 5 (d=3, K=0.20).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
IV.A1	0.1193	9.5	10.3	5.8	8.7	14.0
IV.A2	0.1065	11.0	12.4	6.2	10.0	16.9
IV.A3	0.1001	12.1	14.2	6.3	10.8	19.4
IV.A4	0.1168	9.6	10.4	5.9	8.9	14.1
IV.A5	0.1015	11.5	13.1	6.5	10.5	17.8
IV.A6	0.0941	13.0	15.3	6.7	11.6	21.0
IV.B1	0.0879	13.4	15.5	7.4	12.1	20.9
IV.B2	0.0857	14.1	16.6	7.5	12.6	22.5
IV.B3	0.0833	14.9	18.2	7.4	13.1	24.5
V.1	0.0874	13.6	15.8	7.4	12.2	21.3
V.2	0.0868	13.8	16.2	7.4	12.4	21.9
V.3	0.084	14.7	17.8	7.4	13.1	24.1

Table C.33. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under SCM 1.

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0045	240.8	257.2	328.0	232.2	164.6
I.B1	0.0027	393.6	411.7	500.8	413.6	271.3
I.B2	0.0026	408.1	422.3	469.8	441.3	300.6
I.B3	0.0024	427.6	441.0	474.0	467.4	323.7
I.B4	0.0025	418.4	431.1	474.5	455.0	317.8
II.A1	0.0026	402.4	423.2	542.9	406.8	275.4
II.A2	0.0025	432.0	456.0	569.7	450.4	291.5
II.A3	0.0025	438.7	473.9	619.9	454.0	269.4
II.B1	0.0025	434.9	455.7	532.7	465.5	305.2
II.B2	0.0025	434.3	454.5	511.8	473.1	309.8
II.B3	0.0025	443.4	472.1	560.9	480.4	293.0
III.1	0.0024	429.1	441.2	477.4	466.2	331.2
III.2	0.0024	443.4	456.1	436.8	496.2	365.7
III.3	0.0023	457.5	472.0	435.6	517.0	382.8
III.4	0.0024	427.3	439.1	468.1	465.2	332.4
III.5	0.0023	450.2	463.2	435.1	508.5	375.3
III.6	0.0023	463.3	479.3	437.4	530.1	381.9
IV.A1	0.0025	420.4	430.4	439.5	461.1	339.5
IV.A2	0.0024	445.4	457.6	412.4	505.2	379.6
IV.A3	0.0024	449.4	462.3	413.0	511.1	386.4
IV.A4	0.0024	435.0	446.2	461.1	478.6	347.0
IV.A5	0.0024	446.5	458.8	411.3	507.6	382.8
IV.A6	0.0023	456.6	471.7	403.8	525.5	393.1
IV.B1	0.0024	435.1	445.5	430.9	485.2	360.4
IV.B2	0.0024	439.2	450.2	409.6	496.0	379.0
IV.B3	0.0023	450.6	463.7	403.0	513.5	391.4
V.1	0.0024	439.9	449.4	411.9	495.7	381.1
V.2	0.0023	444.8	455.3	396.1	503.2	398.4
V.3	0.0023	451.5	464.2	382.7	520.0	407.8

Table C.34. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under SCM 1.

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0075	206.5	270.2	68.2	177.7	403.4
I.B1	0.0314	58.8	102.5	16.0	39.2	123.7
I.B2	0.0375	46.2	79.8	13.8	31.9	91.8
I.B3	0.0396	41.8	70.9	13.5	29.5	81.1
I.B4	0.0388	42.4	71.1	13.7	30.4	81.9
II.A1	0.0238	76.2	123.8	20.9	53.8	158.2
II.A2	0.0278	75.2	133.8	17.1	48.9	165.0
II.A3	0.0245	108.9	201.7	17.9	63.5	267.2
II.B1	0.0335	58.0	102.6	14.8	38.7	121.4
II.B2	0.035	56.2	99.4	13.8	37.3	117.7
II.B3	0.0305	81.2	153.2	14.8	48.0	190.5
III.1	0.0403	39.1	62.2	13.3	29.1	74.0
III.2	0.0471	32.7	49.9	11.5	24.8	61.8
III.3	0.0474	33.6	52.8	11.3	25.1	63.7
III.4	0.0411	37.6	58.2	13.2	28.3	71.5
III.5	0.0481	32.2	49.3	11.2	24.6	60.7
III.6	0.0486	33.0	52.0	10.8	24.9	62.9
IV.A1	0.0441	34.1	51.7	12.5	26.1	63.6
IV.A2	0.0504	29.1	42.5	11.0	22.8	53.7
IV.A3	0.0506	29.5	43.9	10.9	23.0	54.6
IV.A4	0.0437	34.7	52.8	12.6	26.6	64.9
IV.A5	0.0512	28.9	43.1	10.8	22.5	53.4
IV.A6	0.0521	29.3	44.4	10.4	22.6	54.9
IV.B1	0.0478	30.9	46.2	11.7	23.9	57.0
IV.B2	0.0512	28.4	41.8	11.0	22.2	51.9
IV.B3	0.0525	28.0	41.5	10.7	21.9	51.3
V.1	0.051	27.5	39.2	11.2	22.1	49.6
V.2	0.0543	25.6	36.2	10.6	20.6	45.1
V.3	0.056	24.9	34.8	10.3	20.2	44.6

Table C.35. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under SCM 2.

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0032	332.0	346.0	431.3	326.7	244.8
I.B1	0.0031	341.0	357.1	454.9	334.2	245.3
I.B2	0.003	358.9	377.3	488.5	354.2	252.5
I.B3	0.0029	366.0	384.3	494.8	367.8	253.7
I.B4	0.0028	380.9	400.6	509.9	390.2	260.1
II.A1	0.0025	421.2	440.0	546.0	435.2	295.7
II.A2	0.0024	443.3	467.2	568.7	468.6	302.9
II.A3	0.0025	449.3	482.9	604.7	475.5	279.6
II.B1	0.0025	419.8	439.9	539.1	439.1	292.4
II.B2	0.0025	428.7	450.7	545.1	454.9	297.7
II.B3	0.0025	449.2	479.1	592.5	474.7	293.4
III.1	0.0027	395.1	410.0	502.8	408.0	287.2
III.2	0.0025	429.5	445.8	494.1	466.6	318.6
III.3	0.0024	443.5	461.1	471.7	499.3	335.9
III.4	0.0027	396.2	411.5	504.1	410.1	284.0
III.5	0.0025	426.6	442.4	485.2	462.8	319.1
III.6	0.0024	438.2	455.8	469.9	488.2	330.7
IV.A1	0.0026	410.7	424.7	479.8	443.0	305.6
IV.A2	0.0024	440.6	455.3	442.9	496.3	349.5
IV.A3	0.0024	440.9	457.0	412.1	507.3	358.6
IV.A4	0.0025	419.2	434.4	495.9	449.1	306.7
IV.A5	0.0024	439.5	453.6	427.7	497.2	356.4
IV.A6	0.0024	446.4	463.7	400.2	521.7	370.7
IV.B1	0.0026	400.5	417.1	493.8	424.9	283.9
IV.B2	0.0026	412.2	428.7	482.1	446.5	300.2
IV.B3	0.0025	431.4	450.0	457.8	486.1	320.9
V.1	0.0025	414.7	429.0	480.1	446.7	308.4
V.2	0.0025	423.9	438.6	463.1	470.1	319.9
V.3	0.0025	430.7	445.7	428.9	489.5	340.5

Table C.36. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under SCM 2.

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0171	84.0	114.5	32.4	68.8	155.5
I.B1	0.0188	82.3	117.2	28.2	65.5	158.3
I.B2	0.0214	77.0	114.0	23.9	59.7	152.9
I.B3	0.0244	73.3	114.3	20.2	53.8	150.8
I.B4	0.0269	70.3	114.9	18.1	49.3	148.9
II.A1	0.0282	62.6	101.8	17.9	44.8	126.9
II.A2	0.0301	69.8	125.4	15.6	44.9	153.1
II.A3	0.0279	96.5	185.2	15.7	54.4	235.5
II.B1	0.0309	62.9	108.3	15.8	42.5	133.2
II.B2	0.0318	62.3	107.3	15.2	42.4	132.9
II.B3	0.0298	82.9	155.3	15.1	49.7	194.6
III.1	0.0298	55.9	86.9	17.3	41.5	111.4
III.2	0.0381	45.4	74.4	13.4	32.8	91.1
III.3	0.0405	45.2	78.0	12.4	31.2	92.2
III.4	0.0304	54.5	84.4	17.0	40.5	108.6
III.5	0.0383	44.6	72.9	13.4	32.1	89.2
III.6	0.0404	44.9	76.2	12.4	31.3	91.3
IV.A1	0.0365	46.3	74.7	14.1	33.5	92.8
IV.A2	0.0458	36.0	58.9	11.5	26.0	70.0
IV.A3	0.0489	34.6	58.1	10.7	24.6	68.0
IV.A4	0.0369	45.7	73.3	14.0	32.9	91.1
IV.A5	0.0477	34.4	56.4	11.1	24.9	66.5
IV.A6	0.0501	33.5	56.0	10.5	24.1	65.3
IV.B1	0.0334	53.4	88.0	15.0	37.4	110.4
IV.B2	0.0373	48.3	81.3	13.5	33.5	99.0
IV.B3	0.0418	44.0	76.1	12.0	29.9	90.2
V.1	0.0380	43.7	70.7	13.8	31.4	86.7
V.2	0.0416	40.1	65.7	12.6	28.9	79.7
V.3	0.0462	36.6	61.3	11.3	26.0	72.3

Table C.37. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under SCM 3.

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0058	186.5	199.6	255.3	179.3	128.1
I.B1	0.0049	220.8	240.4	312.5	209.9	144.4
I.B2	0.0047	235.6	259.0	338.8	222.3	150.7
I.B3	0.0045	248.8	275.9	371.4	232.4	154.2
I.B4	0.0044	259.0	289.1	393.3	242.4	154.5
II.A1	0.0035	317.4	346.1	462.6	304.5	199.5
II.A2	0.0033	340.1	375.0	517.0	326.8	202.8
II.A3	0.0033	351.8	393.9	558.2	336.4	197.6
II.B1	0.0034	325.6	356.8	489.0	313.2	198.4
II.B2	0.0033	334.5	367.5	509.3	320.4	202.2
II.B3	0.0032	361.7	404.4	579.6	347.1	204.5
III.1	0.0037	297.2	322.3	432.2	285.6	190.4
III.2	0.0032	342.4	369.0	499.8	337.2	214.6
III.3	0.0029	373.3	401.4	540.7	375.2	233.2
III.4	0.0038	293.1	318.2	428.0	281.1	185.9
III.5	0.0032	344.8	370.0	499.4	340.7	220.1
III.6	0.0030	364.4	392.4	533.3	365.8	228.0
IV.A1	0.0035	309.7	335.2	450.8	298.4	197.1
IV.A2	0.0029	370.4	395.3	521.4	375.3	238.0
IV.A3	0.0028	382.8	407.7	526.7	394.5	247.0
IV.A4	0.0035	311.5	337.5	456.3	300.3	196.3
IV.A5	0.0030	364.8	389.0	513.9	368.4	234.9
IV.A6	0.0028	395.0	419.6	531.4	411.5	255.8
IV.B1	0.0034	320.7	347.8	473.2	309.6	200.9
IV.B2	0.0032	341.8	368.4	503.5	337.2	214.7
IV.B3	0.0030	365.8	393.1	524.2	369.4	229.9
V.1	0.0030	356.9	379.3	499.3	356.9	234.7
V.2	0.0030	363.5	384.5	498.8	367.9	240.9
V.3	0.0028	390.4	411.4	511.7	408.4	262.6

Table C.38. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under SCM 3.

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0041	321.5	368.1	136.2	335.5	488.0
I.B1	0.0061	256.3	322.3	80.9	239.1	473.0
I.B2	0.0074	234.0	305.7	64.2	209.2	457.5
I.B3	0.0087	222.9	303.8	52.7	190.3	463.3
I.B4	0.0095	219.2	304.8	47.5	182.3	467.9
II.A1	0.0116	168.3	249.5	40.5	126.7	374.1
II.A2	0.0133	172.8	271.1	32.9	120.0	417.2
II.A3	0.0131	210.7	334.4	32.0	143.2	540.6
II.B1	0.0139	160.1	248.3	32.1	114.0	374.5
II.B2	0.0146	156.4	246.9	30.4	109.3	373.9
II.B3	0.0143	187.8	300.3	29.5	127.3	469.0
III.1	0.0128	153.2	228.4	36.5	114.9	342.8
III.2	0.0197	111.0	183.6	23.0	74.2	258.2
III.3	0.0216	105.2	179.9	21.0	67.7	245.5
III.4	0.0132	147.6	219.9	35.3	111.0	327.2
III.5	0.0199	108.6	178.7	23.0	72.9	249.0
III.6	0.022	105.3	181.1	20.6	66.5	249.4
IV.A1	0.0154	131.4	202.2	30.1	94.8	295.1
IV.A2	0.0236	89.2	152.0	19.8	58.5	201.9
IV.A3	0.0265	81.7	144.1	17.5	52.1	186.5
IV.A4	0.0154	131.8	204.0	29.7	94.4	295.7
IV.A5	0.0241	87.7	150.1	19.3	57.4	198.7
IV.A6	0.0282	78.6	140.9	16.6	49.6	177.3
IV.B1	0.0169	121.1	190.2	27.4	85.9	271.8
IV.B2	0.0201	107.7	177.1	22.9	72.3	250.9
IV.B3	0.0234	96.1	165.9	19.5	61.9	223.6
V.1	0.0212	90.0	145.6	22.7	63.0	195.0
V.2	0.0248	78.7	131.8	19.5	53.4	170.0
V.3	0.0292	68.4	117.7	16.5	45.5	147.9

Table C.39. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under SCM 4.

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0025	407.7	416.2	490.9	413.9	322.0
I.B1	0.0026	421.4	439.1	282.7	471.7	478.3
I.B2	0.0026	423.7	442.7	279.1	471.7	491.3
I.B3	0.0025	429.0	448.1	286.6	479.7	490.6
I.B4	0.0025	440.7	458.3	303.9	496.3	490.4
II.A1	0.0022	479.4	492.4	445.4	541.1	410.0
II.A2	0.0023	484.9	506.5	389.9	576.5	414.4
II.A3	0.0023	486.0	515.3	429.0	581.5	366.1
II.B1	0.0025	460.2	483.8	312.4	553.9	441.2
II.B2	0.0025	461.1	487.7	293.2	559.2	450.9
II.B3	0.0024	484.5	514.6	334.6	598.4	420.7
III.1	0.0024	445.8	463.5	309.9	505.7	484.8
III.2	0.0026	436.4	457.6	283.5	501.1	475.7
III.3	0.0025	463.9	489.2	292.1	549.0	491.4
III.4	0.0025	435.5	453.0	297.0	492.0	477.8
III.5	0.0025	441.6	462.7	291.4	510.5	472.4
III.6	0.0025	454.1	478.9	284.9	535.4	483.0
IV.A1	0.0028	405.9	429.6	245.7	455.1	485.1
IV.A2	0.0026	426.9	451.3	262.0	483.9	497.5
IV.A3	0.0026	436.5	461.9	264.7	507.0	488.9
IV.A4	0.0027	410.3	433.3	250.6	461.0	485.5
IV.A5	0.0027	428.4	455.0	252.9	490.1	499.1
IV.A6	0.0026	442.4	469.5	262.9	517.7	493.9
IV.B1	0.0026	421.5	441.2	277.5	481.1	470.3
IV.B2	0.0026	426.1	446.5	274.7	484.0	478.4
IV.B3	0.0026	433.3	456.2	271.9	499.0	480.5
V.1	0.0026	425.2	445.7	275.6	483.0	480.4
V.2	0.0026	420.1	441.2	270.2	474.9	481.7
V.3	0.0026	430.2	454.2	265.5	493.6	486.7

Table C.40. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under SCM 4.

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0298	43.5	55.7	20.1	37.2	74.5
I.B1	0.078	15.2	17.9	8.3	13.5	24.1
I.B2	0.079	15.0	17.4	8.2	13.5	23.4
I.B3	0.0781	15.2	17.8	8.3	13.7	23.9
I.B4	0.0741	16.4	19.6	8.6	14.6	26.3
II.A1	0.0491	29.9	43.8	11.2	23.6	55.4
II.A2	0.0528	32.3	53.9	9.9	23.0	63.9
II.A3	0.0457	49.2	97.9	10.3	29.3	106.8
II.B1	0.0639	23.6	36.3	8.6	18.1	43.7
II.B2	0.0654	23.6	36.7	8.3	17.8	44.4
II.B3	0.0557	34.5	64.7	8.9	22.7	70.3
III.1	0.0726	17.0	20.7	8.6	14.9	27.7
III.2	0.0734	17.6	22.2	8.2	15.1	29.7
III.3	0.0709	19.0	25.0	8.2	16.0	33.3
III.4	0.0737	16.7	20.3	8.5	14.6	27.1
III.5	0.0729	17.8	22.6	8.2	15.3	30.1
III.6	0.0708	19.0	25.2	8.3	16.0	33.1
IV.A1	0.0836	14.6	17.6	7.6	12.8	23.5
IV.A2	0.0800	15.6	19.1	7.7	13.7	25.8
IV.A3	0.0768	16.9	21.4	7.8	14.4	28.7
IV.A4	0.0829	14.7	17.9	7.6	12.9	23.8
IV.A5	0.0798	15.7	19.3	7.7	13.7	25.9
IV.A6	0.0766	17.1	22.0	7.7	14.6	29.1
IV.B1	0.0764	16.2	19.9	8.2	14.1	26.3
IV.B2	0.077	16.2	19.9	8.0	14.1	26.7
IV.B3	0.0754	16.9	21.4	8.1	14.5	28.3
V.1	0.0771	16.0	19.6	8.1	14.0	26.2
V.2	0.0785	15.7	19.1	8.0	13.8	25.5
V.3	0.0772	16.3	200	7.9	14.2	27.0

Table C.41. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under SCM 5.

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0028	379.9	396.1	493.4	383.8	270.1
I.B1	0.0024	431.0	441.2	461.5	472.2	342.3
I.B2	0.0023	444.0	453.8	446.4	490.1	367.5
I.B3	0.0023	448.8	459.0	422.9	506.1	385.4
I.B4	0.0024	441.7	451.5	415.6	492.5	385.0
II.A1	0.0022	465.7	478.6	478.1	519.9	377.5
II.A2	0.0023	471.6	491.6	462.9	543.0	365.5
II.A3	0.0023	483.4	514.0	481.0	560.7	343.1
II.B1	0.0023	455.0	471.1	441.1	515.0	366.5
II.B2	0.0023	465.0	483.1	442.5	532.9	375.7
II.B3	0.0023	492.7	518.8	472.9	575.2	373.0
III.1	0.0023	445.9	454.9	400	499.6	406.6
III.2	0.0024	450.9	464.3	368.2	518.1	422.5
III.3	0.0023	465.9	482.6	365.8	542.2	435.8
III.4	0.0023	449.7	458.7	400.0	503.7	411.7
III.5	0.0023	456.9	470.8	372.6	525.2	428.9
III.6	0.0024	461.9	478.3	370.9	536.2	423.7
IV.A1	0.0025	433.9	447.7	321.7	494.0	446.8
IV.A2	0.0024	452.2	469.0	321.7	517.7	470.5
IV.A3	0.0024	455.7	474.6	320.3	530.2	460.1
IV.A4	0.0024	440.9	454.7	330.1	502.8	450.0
IV.A5	0.0024	446.2	462.7	321.4	510.6	463.2
IV.A6	0.0024	451.2	470.1	318.0	526.0	455.2
IV.B1	0.0023	447.1	458.2	371.8	508.3	425.5
IV.B2	0.0024	443.1	455.3	360.1	505.2	424.2
IV.B3	0.0024	446.2	461.0	347.8	514.7	428.2
V.1	0.0024	442.0	453.0	362.4	499.5	423.4
V.2	0.0024	445.8	458.8	351.1	510.4	434.1
V.3	0.0024	450.4	465.9	338.7	519.3	447.5

Table C.42. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under SCM 5.

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0248	63.2	96.2	21.4	47.6	121.9
I.B1	0.0431	34.4	52.4	12.9	26.2	63.8
I.B2	0.0477	30.1	44.3	11.9	23.5	54.4
I.B3	0.0508	27.7	39.8	11.3	22.1	49.8
I.B4	0.0511	28.0	41.1	11.2	22.1	50.0
II.A1	0.0437	35.6	54.8	12.2	27.0	67.9
II.A2	0.0437	42.5	75.6	11.5	28.7	86.9
II.A3	0.0402	59.9	120.0	11.5	34.5	134.3
II.B1	0.0459	36.5	59.4	11.3	26.6	71.6
II.B2	0.0464	36.7	60.1	11.1	26.4	72.2
II.B3	0.0431	48.4	89.8	11.1	31.1	103.8
III.1	0.0545	24.4	32.8	10.9	20.4	42.2
III.2	0.0591	23.3	31.7	9.8	19.2	41.5
III.3	0.0578	24.9	35.3	9.7	20.0	45.2
III.4	0.0555	23.9	31.9	10.7	19.9	41.0
III.5	0.0586	23.4	31.8	9.9	19.3	41.5
III.6	0.0581	24.8	35.1	9.6	20.0	45.1
IV.A1	0.0685	18.6	24.0	9.0	15.7	30.9
IV.A2	0.0684	18.7	23.7	8.9	16.1	31.5
IV.A3	0.0664	20.2	26.7	8.8	17.0	35.0
IV.A4	0.0676	18.9	24.2	9.1	16.0	31.6
IV.A5	0.0678	19.0	24.2	8.9	16.3	32.1
IV.A6	0.066	20.5	27.2	8.8	17.1	35.8
IV.B1	0.0598	22.0	29.0	10.0	18.4	37.6
IV.B2	0.0608	21.9	29.1	9.7	18.4	37.9
IV.B3	0.0617	22.1	30.0	9.4	18.2	39.0
V.1	0.0611	21.2	27.5	9.9	18.0	35.8
V.2	0.0629	20.8	27.0	9.5	17.6	35.7
V.3	0.0644	20.7	27.5	9.2	17.3	35.9

Table C.43. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under SCM 6.

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0049	213.8	225.5	282.8	207.4	154.1
I.B1	0.0037	290.7	311.3	405.4	279.9	195.0
I.B2	0.0035	307.4	330.5	441.0	296.0	202.9
I.B3	0.0034	318.5	344.0	465.8	307.3	204.7
I.B4	0.0033	330.9	359.0	491.5	322.9	205.4
II.A1	0.0033	333.6	360.1	480.6	323.2	214.1
II.A2	0.0029	377.7	409.3	559.3	371.0	233.3
II.A3	0.0030	382.2	426.2	613.8	367.5	216.6
II.B1	0.0030	362.5	390.1	531.4	358.3	229.5
II.B2	0.0028	388.5	418.3	568.4	390.1	243.8
II.B3	0.0029	393.4	431.5	602.9	390.4	231.7
III.1	0.0032	333.9	357.0	477.5	327.4	219.1
III.2	0.0028	380.3	402.3	513.3	389.8	253.6
III.3	0.0026	406.9	429.4	535.1	425.8	273.9
III.4	0.0032	333.0	355.1	468.9	327.1	219.7
III.5	0.0028	386.0	408.3	521.2	398.8	258.9
III.6	0.0027	408.0	431.7	529.5	431.1	273.3
IV.A1	0.0031	345.8	368.8	491.1	341.3	225.3
IV.A2	0.0027	397.7	418.2	509.7	419.9	270.3
IV.A3	0.0026	412.4	433.0	504.2	446.5	285.0
IV.A4	0.0031	349.0	373.2	498.3	346.9	225.4
IV.A5	0.0027	394.5	414.6	499.6	417.5	271.0
IV.A6	0.0026	420.2	441.1	495.9	459.0	294.1
IV.B1	0.003	358.5	380.5	497.6	362.5	234.9
IV.B2	0.0028	380.9	401.6	505.9	396.1	255.2
IV.B3	0.0027	403.2	424.2	502.3	434.8	275.6
V.1	0.0028	385.2	403.0	498.5	398.1	268.9
V.2	0.0026	404.4	422.1	498.2	430.0	285.5
V.3	0.0025	415.3	430.9	465.3	457.5	308.5

Table C.44. Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under SCM 6.

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0052	257.0	306.4	105.7	246.1	431.6
I.B1	0.0122	139.9	197.6	40.5	111.5	286.0
I.B2	0.0144	128.3	189.3	33.2	98.6	268.2
I.B3	0.0163	122.7	188.0	28.5	89.5	268.4
I.B4	0.0172	124.1	197.6	26.5	86.1	287.9
II.A1	0.0141	137.9	211.7	33.6	100.4	303.7
II.A2	0.0179	128.1	212.7	25.0	84.6	301.0
II.A3	0.0159	173.2	289.3	26.5	108.9	446.9
II.B1	0.0203	108.6	182.3	22.7	72.0	248.3
II.B2	0.0218	105.7	181.1	20.9	68.5	246.4
II.B3	0.0197	140.6	244.1	21.7	86.0	360.5
III.1	0.0191	100.9	159.2	24.9	71.9	221.4
III.2	0.0269	75.8	130.0	17.6	50.5	165.9
III.3	0.0294	71.7	126.9	16.1	46.4	158.4
III.4	0.0195	98.7	155.5	24.7	70.3	215.0
III.5	0.0278	73.7	126.8	17.0	49.0	162.5
III.6	0.0306	70.2	125.9	15.4	44.6	155.4
IV.A1	0.0211	94.4	154.0	22.3	65.2	207.8
IV.A2	0.0306	65.5	114.6	15.8	42.6	142.9
IV.A3	0.0337	60.0	107.8	14.3	38.8	128.5
IV.A4	0.0215	93.4	151.1	21.7	64.9	207.6
IV.A5	0.0322	62.7	111.6	15.1	40.7	135.6
IV.A6	0.036	57.6	106.8	13.5	36.3	123.7
IV.B1	0.0242	83.3	138.8	19.4	56.5	183.9
IV.B2	0.0283	72.4	124.8	16.7	47.4	160.4
IV.B3	0.0322	64.2	115.4	14.9	41.2	139.8
V.1	0.0296	62.7	105.1	16.6	42.9	131.8
V.2	0.0339	53.9	91.7	14.9	37.0	111.2
V.3	0.0385	47.8	83.2	13.0	32.4	98.6

APPENDIX D: PHASE II PERFORMANCES OF S-CHARTS FOR m=20

Phase II performances of S-Charts using proposed estimation methods (Method I-V) are shown for $m = 20$ under normal distribution and in the presence of contaminations with various outlier densities.

Table D.1. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using normally distributed Phase I data.

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0027	449.9	484.4	240.4	538.5	507.9
I.B1	0.0027	444.2	478.2	239.8	533.3	498.2
I.B2	0.0027	453.4	487.7	243.7	547.9	494.9
I.B3	0.0027	476.5	513.6	253.7	590.6	497.7
I.B4	0.0027	492.8	532.3	263.5	619.1	500.6
II.A1	0.0027	526.5	579.7	268.6	704.6	455.9
II.A2	0.0027	565.3	631.6	298.3	762.7	431.7
II.A3	0.0027	607.8	698.2	336.6	821.6	394.7
II.B1	0.0027	558.4	617.8	301.0	743.3	444.9
II.B2	0.0027	572.1	632.6	326.2	753.6	448.1
II.B3	0.0027	635.6	722.1	410.3	820.9	421.8
III.1	0.0027	472.0	507.4	257.4	578.1	497.9
III.2	0.0027	516.2	559.4	280.0	658.4	483.7
III.3	0.0027	547.2	599.0	311.7	708.8	451.7
III.4	0.0027	473.4	510.2	255.4	584.5	502.5
III.5	0.0027	532.3	576.4	295.9	677.6	497.3
III.6	0.0027	558.6	611.1	324.9	723.1	460.9
IV.A1	0.0027	464.2	498.8	256.6	564.8	494.8
IV.A2	0.0027	500.6	542.0	268.4	635.2	481.8
IV.A3	0.0027	522.1	569.0	286.0	677.1	459.5
IV.A4	0.0027	468.3	503.3	255.9	575.4	489.2
IV.A5	0.0027	518.5	560.6	285.1	656.5	494
IV.A6	0.0027	546.8	596.2	304.0	709.3	479.2
IV.B1	0.0027	471.3	506.6	258.5	582.2	496.5
IV.B2	0.0027	487.1	525.1	265.1	608.1	492.7
IV.B3	0.0027	505.8	545.3	289.3	637.3	474.3
V.1	0.0027	460.9	495.7	251.9	561.7	493.9
V.2	0.0027	475.4	511.7	259.9	588.7	490.2
V.3	0.0027	501.8	542.0	275.0	638.7	490.5

Table D.2. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using normally distributed Phase I data.

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0778	17.9	25.2	7.4	14.4	32.0
I.B1	0.0768	18.5	26.5	7.4	14.7	33.5
I.B2	0.0753	19.4	28.2	7.4	15.3	35.4
I.B3	0.0713	21.7	33.2	7.6	16.5	40.9
I.B4	0.0697	23.4	37.2	7.5	17.4	45.0
II.A1	0.0601	42.8	98.1	7.6	23.1	92.9
II.A2	0.0537	67.8	166.0	7.9	29.2	165.2
II.A3	0.0457	127.7	310.6	8.4	43.3	380.7
II.B1	0.0543	54.5	123.8	8.0	27.8	124.7
II.B2	0.0527	58.5	134.0	8.2	29.6	135.6
II.B3	0.0441	115.8	277.4	9.0	44.7	319.9
III.1	0.0715	21.4	32.2	7.6	16.5	40.1
III.2	0.0634	30.1	53.2	7.8	20.7	61.3
III.3	0.0540	46.8	94.9	8.4	27.7	103.4
III.4	0.0723	21.2	32.4	7.5	16.4	39.6
III.5	0.0621	30.7	53.8	7.9	21.2	62.6
III.6	0.0543	46.3	93.3	8.3	27.6	102.9
IV.A1	0.0725	20.8	31.2	7.5	16.2	38.9
IV.A2	0.0646	28.1	48.3	7.8	19.9	56.1
IV.A3	0.0591	37.4	71.4	7.9	23.9	79.1
IV.A4	0.0718	21.0	31.5	7.6	16.4	39.2
IV.A5	0.0636	28.6	49.0	7.9	20.1	57.6
IV.A6	0.0576	39.0	74.6	8.1	24.7	84.0
IV.B1	0.0718	21.2	31.9	7.6	16.4	39.8
IV.B2	0.0685	23.5	36.8	7.7	17.9	44.8
IV.B3	0.0622	30.2	52.4	8.0	21.2	61.1
V.1	0.0733	20.6	30.8	7.5	15.9	38.3
V.2	0.0704	22.3	34.4	7.6	17.0	42.2
V.3	0.0661	26.5	44.2	7.7	19.0	52.8

Table D.3. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under CM 1 (d=2, K=0.10).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0024	451.4	471.8	515.6	496.4	323.9
I.B1	0.0024	451.1	473.2	474.8	509.7	329.4
I.B2	0.0025	446.7	469.2	445.7	516.4	333.6
I.B3	0.0024	463.6	490.5	462.4	536.7	336.9
I.B4	0.0024	483.3	514.2	474.3	561.7	350.6
II.A1	0.0023	516.8	561.7	533.8	598.2	339.4
II.A2	0.0024	536.5	597.6	602.9	613.0	320.1
II.A3	0.0024	580.5	672.2	689.8	651.0	300.0
II.B1	0.0024	523.7	577.4	577.5	595.7	326.6
II.B2	0.0025	534.3	592.3	588.9	602.3	331.4
II.B3	0.0025	579.1	665.8	729.6	641.0	310.8
III.1	0.0024	467.7	492.7	444.5	542.5	353.2
III.2	0.0025	495.3	530.9	456.0	582.1	364.1
III.3	0.0025	522.8	569.7	519.3	610.7	354.5
III.4	0.0024	470.2	496.0	446.9	546.8	356.6
III.5	0.0024	501.0	537.9	473.7	590.0	360.2
III.6	0.0025	521.3	569.4	518.1	607.8	349.0
IV.A1	0.0024	474.5	500.8	431.5	560.4	362.9
IV.A2	0.0024	491.9	524.1	432.8	583.8	372.7
IV.A3	0.0025	509.2	550.7	451.7	604.2	366.1
IV.A4	0.0024	472.7	498.2	430.2	557.9	363.9
IV.A5	0.0024	502.7	536.7	441.5	599.6	380.7
IV.A6	0.0025	520.5	563.2	469.9	618.6	374.0
IV.B1	0.0024	468.9	494.3	416.7	553.3	367.5
IV.B2	0.0024	482.6	511.7	429.7	575.3	369.2
IV.B3	0.0024	502.4	537.1	466.0	594.2	371.1
V.1	0.0024	472.5	497.5	411.8	560.9	373.4
V.2	0.0024	478.9	505.6	408.2	573.3	380.3
V.3	0.0024	494.6	527.4	412.2	600.5	381.4

Table D.4. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under CM 1 (d=2, K=0.10).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0366	53.9	98.5	13.4	34.9	114.0
I.B1	0.0411	50.1	94.7	11.8	31.8	106.1
I.B2	0.0427	49.4	94.9	11.3	30.8	105.3
I.B3	0.0419	53.7	104.5	11.3	32.6	118.0
I.B4	0.0421	56.5	114.0	11.1	33.0	124.4
II.A1	0.0359	91.8	196.0	11.9	45.0	231.1
II.A2	0.0330	130.0	277.6	12.1	57.2	369.7
II.A3	0.0286	224.9	456.3	13.0	89.0	775.7
II.B1	0.0343	115.1	244.6	11.9	54.0	307.7
II.B2	0.0328	125.0	261.8	12.4	59.0	337.7
II.B3	0.0279	212.3	418.7	13.7	93.5	676.8
III.1	0.0439	48.1	92.0	11.0	30.2	102.9
III.2	0.0423	61.6	125.0	10.5	35.5	139.0
III.3	0.0372	89.7	188.6	11.5	46.0	223.1
III.4	0.0447	47.5	90.1	10.8	29.9	101.9
III.5	0.0415	62.9	126.9	10.9	36.3	143.1
III.6	0.0374	89.8	187.3	11.5	46.4	224.4
IV.A1	0.0465	43.5	81.5	10.5	28.0	91.6
IV.A2	0.0448	52.6	104.7	10.3	31.7	115.1
IV.A3	0.0417	68.3	141.1	10.6	37.5	160.0
IV.A4	0.0463	44.4	84.4	10.4	28.4	93.4
IV.A5	0.0445	53.4	105.4	10.5	32.0	116.9
IV.A6	0.0409	70.3	144.3	10.8	38.7	165.1
IV.B1	0.0469	43.5	82.0	10.3	28.0	91.7
IV.B2	0.0458	47.5	91.2	10.5	29.8	101.7
IV.B3	0.0434	58.1	117.1	10.5	34.0	129.9
V.1	0.0484	40.3	74.2	10.2	26.3	83.8
V.2	0.0480	42.8	81.1	10.1	27.3	89.9
V.3	0.0472	47.8	93.6	10.0	29.3	103.7

Table D.5. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under CM 1 (d=3, K=0.15).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0050	244.4	288.4	407.4	222.6	123.3
I.B1	0.0040	311.9	364.0	550.6	296.3	148.5
I.B2	0.0037	333.7	385.1	568.6	329.6	159.6
I.B3	0.0035	352.2	403.9	586.6	352.6	173.0
I.B4	0.0034	371.1	425.8	614.3	372.9	180.8
II.A1	0.0029	417.9	477.1	712.1	405.2	215.5
II.A2	0.0028	442.6	512.6	767.3	430.8	218.1
II.A3	0.0028	488.6	594.8	964.1	465.6	208.0
II.B1	0.0029	435.4	503.8	761.3	425.9	218.0
II.B2	0.0029	441.1	515.1	780.2	434.2	215.3
II.B3	0.0029	471.4	571.2	926.8	447.1	209.3
III.1	0.0030	391.9	434.4	577.1	414.4	217.1
III.2	0.0027	447.1	495.3	608.3	481.2	257.0
III.3	0.0026	476.2	533.7	676.1	511.7	268.3
III.4	0.003	395.8	437.5	580.5	414.6	223.1
III.5	0.0028	442.2	488.0	604.7	477.7	259.2
III.6	0.0027	477.3	535.6	672.0	511.1	269.4
IV.A1	0.0028	411.7	450.2	549.4	445.6	241.0
IV.A2	0.0026	448.9	489.5	555.6	495.7	277.6
IV.A3	0.0026	469.1	515.0	586.4	520.0	286.0
IV.A4	0.0029	403.7	440.8	531.9	433.4	239.1
IV.A5	0.0026	454.9	495.4	554.0	507.3	283.7
IV.A6	0.0026	479.5	528.2	589.4	530.6	293.4
IV.B1	0.0028	409.5	447.1	541.4	443.4	243.5
IV.B2	0.0028	424.0	464.0	534.1	469.5	254.1
IV.B3	0.0027	451.5	496.4	562.5	500.0	270.7
V.1	0.0027	425.7	459.2	517.3	471.7	269.8
V.2	0.0026	436.5	470.8	501.7	494.4	282.3
V.3	0.0026	455.0	492.2	495.8	518.5	299.5

Table D.6. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under CM 1 (d=3, K=0.15).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0073	300.9	395.4	58.1	301.5	534.6
I.B1	0.0162	215.3	341.3	24.1	143.4	567.3
I.B2	0.0198	187.5	315.7	19.8	109.6	544.4
I.B3	0.0210	183.2	317.9	18.8	100.9	558.5
I.B4	0.0207	182.1	318.0	19.1	98.6	563.4
II.A1	0.0170	221.5	377.4	23.1	129.3	636.2
II.A2	0.0171	262.5	443.3	22.4	153.7	751.8
II.A3	0.0154	367.9	592.9	24.5	241.2	917.6
II.B1	0.0184	242.6	413.0	21.2	141.9	700.3
II.B2	0.0179	267.0	454.3	21.9	153.0	770.5
II.B3	0.0155	372.1	592.5	25.2	256.7	923.4
III.1	0.0256	131.4	246.0	16.2	68.9	372.8
III.2	0.0288	128.7	253.4	14.3	63.4	363.1
III.3	0.0266	163.0	315.7	15.1	79.8	482.9
III.4	0.0261	129.2	244.5	15.9	66.9	368.0
III.5	0.0287	129.8	257.1	14.3	64.0	366.5
III.6	0.0272	159.2	308.8	14.8	78.8	470.1
IV.A1	0.0300	106.7	212.0	14.2	53.9	292.7
IV.A2	0.0329	100.0	205.2	12.9	49.7	261.1
IV.A3	0.0311	116.5	236.3	13.3	57.5	317.4
IV.A4	0.0296	107.3	210.4	14.2	54.7	288.4
IV.A5	0.0326	102.1	208.5	13.0	51.3	268.0
IV.A6	0.0314	120.0	245.3	13.3	58.3	324.2
IV.B1	0.0303	106.6	212.3	14.1	53.6	288.6
IV.B2	0.0321	104.6	212.5	13.2	51.7	282.5
IV.B3	0.0321	110.3	224.2	13.2	54.1	299.3
V.1	0.0341	81.6	163.9	13.0	43.5	200.7
V.2	0.0356	81.5	167.8	12.3	42.6	199.2
V.3	0.0377	82.0	171.7	11.7	41.6	203.4

Table D.7. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under CM 1 (d=3, K=0.20).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0069	180.5	217.9	297.0	162.2	89.2
I.B1	0.0056	238.7	297.7	447.4	208.5	102.5
I.B2	0.0051	262.8	324.2	501.5	233.7	110.0
I.B3	0.0048	280.5	346.0	538.9	252.8	114.5
I.B4	0.0046	296.7	363.6	560.1	270.9	120.2
II.A1	0.0036	349.0	413.4	617.4	320.1	166.5
II.A2	0.0034	379.8	461.1	719.9	342.7	168.8
II.A3	0.0034	413.7	519.5	838.2	367.5	166.4
II.B1	0.0034	378.8	459.3	726.4	343.0	170.3
II.B2	0.0034	382.5	463.7	739.0	349.1	170.8
II.B3	0.0034	408.9	515.4	843.7	361.0	166.0
III.1	0.0038	328.9	382.9	578.0	312.7	158.6
III.2	0.0032	392.2	449.7	656.9	391.3	197.5
III.3	0.003	424.8	493.0	747.9	421.5	209.4
III.4	0.0037	331.6	384.6	575.9	319.1	161.5
III.5	0.0032	394.4	452.5	672.1	394.4	198.4
III.6	0.0031	423.0	489.8	730.8	420.6	209.7
IV.A1	0.0035	350.9	399.3	577.6	347.7	177.0
IV.A2	0.0030	403.8	455.1	625.6	413.9	216.2
IV.A3	0.0028	433.3	488.8	668.9	452.2	231.1
IV.A4	0.0035	349.5	398.7	582.3	343.9	176.6
IV.A5	0.0030	410.5	462.0	633.7	424.8	220.3
IV.A6	0.0029	437.5	495.5	675.9	451.8	235.7
IV.B1	0.0034	354.6	403.7	586.0	352.5	181.1
IV.B2	0.0033	377.6	430.5	611.2	382.4	191.5
IV.B3	0.0031	401.5	456.3	641.1	413.6	207.7
V.1	0.0031	382.4	424.1	579.1	393.1	210.4
V.2	0.0030	390.2	432.3	572.4	409.5	218.5
V.3	0.0028	424.5	469.5	588.6	452	245.6

Table D.8. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under CM 1 (d=3, K=0.20).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0040	390.8	446.8	135.4	526.8	418.8
I.B1	0.0090	315.5	415.9	45.8	362.7	483.9
I.B2	0.0113	285.4	396.7	34.1	285.3	505.4
I.B3	0.0124	282.9	407.2	31.4	255.8	540.9
I.B4	0.0125	286.2	417.0	31.2	250.2	563.7
II.A1	0.0104	317.0	463.2	38.5	259.6	703.4
II.A2	0.0111	352.0	517.9	35.9	292.9	754.9
II.A3	0.0102	461.1	662.3	39.4	459.1	797.6
II.B1	0.0119	341.4	506.0	33.0	281.2	745.8
II.B2	0.0122	356.4	527.3	32.9	295.7	760.9
II.B3	0.0107	461.7	652.9	38.8	489.3	779.5
III.1	0.0162	217.1	345.2	24.6	144.6	575.9
III.2	0.0200	202.5	349.3	19.8	113.7	598.2
III.3	0.0192	235.0	401.3	20.4	135.0	683.3
III.4	0.0167	213.0	341.2	23.7	140.0	575.0
III.5	0.0202	204.0	352.5	19.8	115.9	603.4
III.6	0.0200	231.7	399.0	19.9	131.7	683.2
IV.A1	0.0198	180.0	306.9	20.1	103.3	521.4
IV.A2	0.0241	158.7	293.4	16.7	81.6	476.7
IV.A3	0.0234	172.0	315.1	17.2	90.2	508.9
IV.A4	0.0197	180.2	308.6	20.2	103.8	528.7
IV.A5	0.0239	161.7	297.6	16.7	84.0	483.2
IV.A6	0.0238	178.1	329.9	16.7	91.6	541.8
IV.B1	0.0205	175.6	302.9	19.3	100.0	511.8
IV.B2	0.0223	172.6	307.1	17.9	92.3	521.2
IV.B3	0.0237	173.7	316.7	16.8	89.4	523.1
V.1	0.0239	137.2	251.1	17.3	74.2	387.0
V.2	0.0257	134.9	252.5	16.0	70.9	383.6
V.3	0.0285	127.0	247.5	14.5	64.1	357.0

Table D.9. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under CM 2 (d=0.75, K=0.10).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0024	463.3	484.2	370.6	554.6	396.4
I.B1	0.0024	467.2	490.8	336.8	566.4	428.0
I.B2	0.0025	469.2	495.6	325.6	574.8	426.0
I.B3	0.0025	478.7	506.7	339.0	588.5	425.9
I.B4	0.0024	499.6	531.8	343.6	619.6	435.2
II.A1	0.0024	529.1	575.0	368.2	672.1	402.6
II.A2	0.0025	553.6	614.2	410.7	699.4	372.6
II.A3	0.0025	598.5	683.3	496.5	740.2	348.8
II.B1	0.0025	551.3	604.9	412.6	690.1	392.0
II.B2	0.0024	565.9	625.2	444.3	698.0	388.9
II.B3	0.0025	611.3	696.2	552.1	732.6	365.0
III.1	0.0024	480.1	506.8	338.5	585.1	433.1
III.2	0.0025	520.3	558.3	356.8	646.4	435.4
III.3	0.0025	539.4	587.4	407.6	663.7	400.8
III.4	0.0025	475.0	501.7	334.9	580.2	429.0
III.5	0.0025	525.3	563.6	367.8	651.2	435.7
III.6	0.0026	547.0	594.8	402.3	682.1	415.9
IV.A1	0.0025	470.9	498.5	315.5	577.7	433.2
IV.A2	0.0025	496.4	529.1	342.6	613.8	429.0
IV.A3	0.0026	530.3	573.9	368.5	663.9	420.7
IV.A4	0.0025	473.1	499.7	326.1	575.6	432.2
IV.A5	0.0025	517.8	554.3	352.2	646.1	437.1
IV.A6	0.0026	533.1	576.7	373.6	669.0	420.6
IV.B1	0.0025	480.4	509.0	321.1	593.5	443.3
IV.B2	0.0025	484.3	514.8	330.2	600.1	429.7
IV.B3	0.0025	515.0	551.6	361.5	635.2	432.3
V.1	0.0025	470.6	497.9	316.7	578.4	433.8
V.2	0.0025	478.2	507.1	318.3	589.7	437.0
V.3	0.0025	497.3	531.9	328.0	623.9	433.5

Table D.10. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under CM 2 (d=0.75, K=0.10).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0540	32.6	60.8	9.6	22.0	63.6
I.B1	0.0594	28.1	47.6	8.9	20.0	54.2
I.B2	0.0597	28.4	48.4	8.7	20.2	55.7
I.B3	0.0572	31.6	57.1	8.9	21.7	63.0
I.B4	0.0561	34.0	63.3	8.9	22.8	68.9
II.A1	0.0478	61.1	138.1	9.2	31.0	140.8
II.A2	0.0432	94.1	219.5	9.4	39.5	245.7
II.A3	0.0371	164.9	367.6	10.3	59.3	537.5
II.B1	0.0442	76.4	171.1	9.6	37.0	184.5
II.B2	0.0426	83.5	186.4	9.9	40.0	204.9
II.B3	0.0361	155.2	341.8	10.8	61.1	477.2
III.1	0.0585	29.6	50.2	8.8	21.1	58.4
III.2	0.0531	40.5	76.4	9.0	25.9	86.5
III.3	0.0462	61.7	128.5	9.5	34.3	142.4
III.4	0.0585	29.5	49.9	8.8	21.0	58.3
III.5	0.0523	41.6	79.7	9.1	26.3	88.9
III.6	0.0465	61.2	126.9	9.5	34.2	141.2
IV.A1	0.0602	28.2	47.7	8.7	20.3	54.7
IV.A2	0.0553	36.1	66.0	8.8	24.0	74.8
IV.A3	0.0503	48.8	98.1	9.1	29.0	107.3
IV.A4	0.0601	28.2	47.9	8.7	20.3	54.8
IV.A5	0.0549	37.0	67.5	8.8	24.5	76.4
IV.A6	0.0497	50.0	100.4	9.2	29.8	110.8
IV.B1	0.0599	28.5	48.1	8.6	20.5	55.8
IV.B2	0.0577	31.3	54.8	8.8	21.9	62.7
IV.B3	0.0529	39.7	74.9	9.1	25.8	83.5
V.1	0.0618	26.9	44.1	8.5	19.6	52.2
V.2	0.0598	29.0	49.2	8.6	20.6	56.9
V.3	0.0572	34.1	61.1	8.6	22.8	70.9

Table D.11. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under CM 2 (d=1, K=0.15).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0026	421.0	448	514.6	462.4	273.1
I.B1	0.0025	454.7	478.8	428.4	536.0	342.9
I.B2	0.0025	456.3	480.7	405.3	545.1	353.7
I.B3	0.0024	469.9	497.4	425.9	555.6	356
I.B4	0.0024	484.0	513.5	436.3	574.3	366.5
II.A1	0.0023	512.0	554.9	504.9	603.0	343.5
II.A2	0.0024	537.9	599.2	560.2	626.2	325.5
II.A3	0.0024	578	665.8	646.3	664.8	307.3
II.B1	0.0024	527.7	580.9	536.1	616.6	334.6
II.B2	0.0024	534.2	590.4	566.8	614.9	338.6
II.B3	0.0024	584.7	669.7	708.1	652.1	322.1
III.1	0.0024	478.2	505.4	406.1	571.7	374.3
III.2	0.0025	504.9	541.1	418.5	611.0	384.4
III.3	0.0025	532.4	580.2	482.3	634.8	370.0
III.4	0.0024	473.9	500.1	404.1	568.1	376.5
III.5	0.0025	510.2	546.0	433.5	612.5	387.6
III.6	0.0025	532.9	581.3	480.4	629.0	370.8
IV.A1	0.0024	468.5	493.2	384	561.7	384.1
IV.A2	0.0024	499.9	533.4	402.7	607.5	393.2
IV.A3	0.0026	507.7	548.2	429.6	611.7	374.4
IV.A4	0.0024	481.1	507.1	392.9	579.1	393.6
IV.A5	0.0025	497.9	531.3	395.6	606.3	393.5
IV.A6	0.0025	518.2	560.9	427.8	625.5	383.4
IV.B1	0.0024	472.6	499.1	385.3	572.8	382.9
IV.B2	0.0025	474.1	502.2	378.6	572.5	385.0
IV.B3	0.0025	503.6	537.9	419.7	606.9	388.9
V.1	0.0024	474.3	499.1	372.4	572.1	397.7
V.2	0.0024	480.6	508.2	373.7	582.4	398.5
V.3	0.0025	491.6	524.5	379.1	602.6	392.8

Table D.12. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under CM 2 (d=1, K=0.15).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0340	73.0	147.8	13.5	39.4	171.5
I.B1	0.0456	46.3	93.1	10.7	28.0	98.2
I.B2	0.0478	43.8	87.8	10.2	26.8	90.8
I.B3	0.0461	47.7	97.4	10.4	28.5	101.1
I.B4	0.0456	50.5	104.1	10.4	29.7	108.1
II.A1	0.0377	85.1	183.8	11.3	41.9	208.8
II.A2	0.035	125.5	272.0	11.5	53.5	358.4
II.A3	0.0302	207.2	422.2	12.3	82.1	707.0
II.B1	0.0361	106.7	229.1	11.3	49.4	284.4
II.B2	0.0349	115.0	245.2	11.7	53.3	308.3
II.B3	0.0293	200.2	404.7	13.2	85.9	639.6
III.1	0.0481	42.4	81.0	10.2	27.1	88.0
III.2	0.0454	54.2	108.7	10.1	31.9	120.6
III.3	0.0405	79.2	167.8	10.7	41.5	191.0
III.4	0.0486	41.3	78.4	10.1	26.5	85.1
III.5	0.0451	54.9	110.6	10.2	32.3	122.4
III.6	0.0405	78.1	163.0	10.6	41.7	187.3
IV.A1	0.0508	37.8	70.4	9.8	25.0	77.4
IV.A2	0.0485	46.0	89.5	9.7	28.5	98.9
IV.A3	0.0442	60.8	124.6	10.0	34.8	138.3
IV.A4	0.0508	37.9	70.8	9.8	25.0	77.7
IV.A5	0.0476	47.4	93.5	9.9	29.3	101.9
IV.A6	0.0440	62.3	127.4	10.0	35.3	143.4
IV.B1	0.0514	37.6	70.2	9.6	25.0	76.7
IV.B2	0.0504	41.2	78.7	9.7	26.3	86.1
IV.B3	0.0470	50.7	101.1	9.8	30.6	110.8
V.1	0.0535	34.6	63.0	9.5	23.3	70.3
V.2	0.0525	36.8	67.4	9.5	24.3	75.7
V.3	0.0510	42.0	81.0	9.4	26.7	88.7

Table D.13. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under CM 2 (d=1, K=0.20).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0029	395.3	428.4	556.0	412.5	233.6
I.B1	0.0025	437.5	463.6	486.7	493.6	298.1
I.B2	0.0025	443.8	470.7	467.0	509.4	309.7
I.B3	0.0025	453.8	482.4	473.5	521.9	316.6
I.B4	0.0024	472.6	505.0	490.0	544.2	327.1
II.A1	0.0024	502.1	547.3	569.9	565.7	316.3
II.A2	0.0024	521.9	583.9	644.2	583.5	298.1
II.A3	0.0024	560.0	648.1	757.1	617.7	283.5
II.B1	0.0024	516.3	571.5	615.2	578.6	314.9
II.B2	0.0025	525.7	588.1	659.5	577.4	306.7
II.B3	0.0025	562.3	651.8	804.1	597.4	291.4
III.1	0.0024	462.7	487.6	463.3	532.2	344.0
III.2	0.0024	501.3	538.2	489.2	587.1	356.5
III.3	0.0025	514.5	563.3	547.6	587.8	337.0
III.4	0.0025	458.3	483.4	457.2	531.8	339.4
III.5	0.0024	504.9	542.4	489.2	588.6	358.4
III.6	0.0025	523.3	571.4	543.2	608.6	345.1
IV.A1	0.0024	459.5	484.4	430.4	539.8	348.3
IV.A2	0.0024	483.4	513.8	452.7	568.4	360.7
IV.A3	0.0025	512.6	555.3	489.2	606.1	354.4
IV.A4	0.0024	460.0	484.3	444.6	535.0	348.8
IV.A5	0.0024	502.1	536.8	464.9	595.7	368.4
IV.A6	0.0025	515.0	557.8	492.1	610.1	357.4
IV.B1	0.0024	463.5	488.6	431.9	542.1	352.3
IV.B2	0.0025	469.7	498.7	430.3	554.3	352.1
IV.B3	0.0025	478.8	512.3	459.5	558.2	348.1
V.1	0.0024	462.1	486.1	417.7	543.7	359.3
V.2	0.0024	469.6	495.6	414.4	556.8	366.7
V.3	0.0025	486.7	519.0	424.4	584.8	369.7

Table D.14. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under CM 2 (d=1, K=0.20).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0259	104.0	197.7	17.0	56.6	269.9
I.B1	0.0380	62.9	127.3	12.3	35.5	142.6
I.B2	0.0401	59.1	120.1	11.7	34.1	131.6
I.B3	0.0394	62.8	128.3	11.7	35.3	141.0
I.B4	0.0387	66.9	139.6	11.8	36.9	153.6
II.A1	0.0323	108.5	227.1	12.8	51.9	287.2
II.A2	0.0300	151.6	313.0	13.1	67.0	452.6
II.A3	0.0261	239.9	466.3	14.2	101.6	815.5
II.B1	0.0309	130.4	267.2	13.1	62.2	363.2
II.B2	0.0301	141.5	288.5	13.4	66.4	403.7
II.B3	0.0257	237.0	454.9	14.7	106.7	771.0
III.1	0.0420	53.0	103.9	11.3	32.3	115.0
III.2	0.0404	65.8	133.2	11.1	37.6	151.6
III.3	0.0358	95.3	197.4	11.8	49.4	240.8
III.4	0.0424	52.2	102.0	11.2	31.8	113.3
III.5	0.0398	67.8	138.1	11.3	38.0	155.9
III.6	0.0361	95.0	195.9	11.6	49.3	237.8
IV.A1	0.0449	47.3	92.6	10.7	29.3	100.4
IV.A2	0.0431	56.0	110.7	10.7	33.3	124.1
IV.A3	0.0397	73.2	151.8	11.0	39.9	171.5
IV.A4	0.0447	47.2	92.4	10.8	29.3	99.3
IV.A5	0.0428	57.2	114.5	10.7	33.8	126.6
IV.A6	0.0396	74.3	152.9	11.1	40.7	175.6
IV.B1	0.0449	47.4	91.7	10.7	29.4	100.1
IV.B2	0.0447	50.2	98.5	10.6	30.6	108.2
IV.B3	0.0419	61.8	125.9	10.9	35.4	139.6
V.1	0.0475	42.1	79.3	10.3	27.2	87.0
V.2	0.0469	44.6	85.6	10.3	28.2	93.6
V.3	0.046	50.4	99.6	10.1	30.4	109.5

Table D.15. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under CM 3 (d=2, K=0.10).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0023	463.7	478.7	497.8	511.8	358.3
I.B1	0.0025	457.9	483.2	328.0	558.1	403.7
I.B2	0.0025	467.8	497.3	307.0	579.8	420.1
I.B3	0.0025	472.9	502.8	319.9	585.3	420.1
I.B4	0.0025	489.7	521.9	327.8	610.8	432.1
II.A1	0.0023	527.6	571.4	520.1	622.0	356.0
II.A2	0.0024	541.2	597.6	501.5	653.5	348.3
II.A3	0.0024	585.9	672.8	617.8	678.8	320.6
II.B1	0.0025	539.2	595.6	441.8	667.8	360.1
II.B2	0.0026	549.1	609.2	462.1	670.8	361.0
II.B3	0.0025	581.3	666.0	588.9	676.8	331.0
III.1	0.0024	481.6	510.1	369.2	584.8	398.9
III.2	0.0025	511.0	550.5	360.4	640.5	407.9
III.3	0.0025	541.0	590.7	422.2	666.1	391.9
III.4	0.0025	469.9	496.9	358.7	568.7	393.9
III.5	0.0025	508.6	547.0	371.9	629.2	403.6
III.6	0.0025	539.2	587.5	426.0	657.0	392.2
IV.A1	0.0024	476.6	503.4	365.7	579.3	401.4
IV.A2	0.0025	497.5	533.6	365.2	616.9	398.7
IV.A3	0.0026	515.4	560.3	377.8	643.1	387.4
IV.A4	0.0025	471.1	497.6	371.6	572.3	389.2
IV.A5	0.0025	502.5	538.5	368.1	624.6	405.8
IV.A6	0.0025	531.4	577.4	392.8	658.9	401.8
IV.B1	0.0025	471.6	500.1	347.0	577.2	400.2
IV.B2	0.0026	475.8	507.2	334.2	592.3	404.5
IV.B3	0.0025	500.2	536.0	363.5	621.0	405.9
V.1	0.0025	469.0	496.2	335.8	572.8	408.0
V.2	0.0025	474.8	505.5	328.0	587.1	410.2
V.3	0.0026	489.6	524.5	331.1	613.9	415.1

Table D.16. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under CM 3 (d=2, K=0.10).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0403	41.9	68.4	12.8	30.2	82.6
I.B1	0.0580	30.2	51.0	8.8	21.4	60.5
I.B2	0.0615	28.7	49.9	8.3	19.9	57.5
I.B3	0.0593	30.9	54.9	8.5	21.2	61.7
I.B4	0.0576	32.9	60.2	8.7	22.1	66.6
II.A1	0.0391	79.9	173.1	11.0	39.8	193.7
II.A2	0.0382	107.4	239.4	10.6	47.0	288.7
II.A3	0.0317	197.5	408.7	11.7	77.0	668.5
II.B1	0.0419	90.5	203.8	9.9	41.0	232.9
II.B2	0.0404	98.3	217.5	10.1	44.7	258.2
II.B3	0.0324	185.7	385.8	11.7	74.3	604.4
III.1	0.0525	36.4	64.0	9.4	24.6	75.4
III.2	0.0511	46.3	91.6	9.0	28.0	101.0
III.3	0.0449	69.0	146.0	9.6	36.8	163.2
III.4	0.0536	35.5	62.6	9.2	24.2	72.4
III.5	0.0496	48.0	94.8	9.3	28.9	106.2
III.6	0.0449	68.2	142.8	9.6	36.6	162.1
IV.A1	0.0532	35.5	63.0	9.4	24.1	72.5
IV.A2	0.0514	42.9	81.6	9.1	26.9	92.0
IV.A3	0.0478	56.7	118.4	9.4	31.7	129.3
IV.A4	0.0529	36.0	63.7	9.3	24.4	74.1
IV.A5	0.0506	44.8	86.7	9.2	27.9	97.3
IV.A6	0.0478	58.2	121.4	9.3	32.6	133.5
IV.B1	0.0545	34.8	61.9	9.1	23.6	72.0
IV.B2	0.0553	36.4	67.5	8.8	23.9	76.0
IV.B3	0.0516	44.3	87.0	9.1	27.2	95.4
V.1	0.0571	32.0	56.1	8.8	22.0	64.8
V.2	0.0575	33.2	59.5	8.6	22.3	68.8
V.3	0.0554	38.0	72.9	8.8	24.1	80.0

Table D.17. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under CM 3 (d=3, K=0.15).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0038	297.0	329.3	446.6	279.8	179.6
I.B1	0.0027	437.7	469.7	284.3	559.7	371.6
I.B2	0.0027	445.0	476.7	282.0	564.8	395.2
I.B3	0.0025	479.1	508.8	341.1	591.2	413.2
I.B4	0.0024	492.1	521.4	383.3	600.0	401.4
II.A1	0.0025	468.1	518.6	702.9	481.7	266.8
II.A2	0.0024	513.0	574.6	717.4	550.1	283.6
II.A3	0.0025	538.7	635.8	906.3	544.4	251.2
II.B1	0.0025	523.5	583.6	505.7	628.1	317.4
II.B2	0.0026	523.9	587.2	485.3	625.9	320.9
II.B3	0.0027	555.7	651.5	700.6	616.3	275.5
III.1	0.0027	450.1	483.2	358.3	555.3	337.3
III.2	0.0026	500.8	543.8	358.9	632.5	380.1
III.3	0.0026	531.8	582.1	440.5	648.8	371.1
III.4	0.0027	453.5	486.7	355.5	560.9	345.2
III.5	0.0026	495.6	537.8	363.4	618.6	373.0
III.6	0.0026	529.0	579.2	419.1	646.7	374.8
IV.A1	0.0026	451.0	483.7	367.8	554.6	334.2
IV.A2	0.0027	476.8	515.1	358.6	597.0	361.9
IV.A3	0.0026	502.0	545.5	391.7	622.7	367.6
IV.A4	0.0026	452.1	486.4	357.8	561.1	335.6
IV.A5	0.0026	490.7	532.4	346.8	619.9	377.0
IV.A6	0.0027	511.9	559.2	382.0	640.8	376.2
IV.B1	0.0027	450.6	484.0	326.1	566.8	352.9
IV.B2	0.0027	466.2	502.3	311.9	597.2	380.5
IV.B3	0.0026	491.8	530.3	356.4	613.8	383.7
V.1	0.0027	447.6	481.1	304.0	569.7	363.5
V.2	0.0027	455.5	489.4	299.4	579.0	387.1
V.3	0.0027	484.6	522.9	311.3	618.9	407.4

Table D.18. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under CM 3 (d=3, K=0.15).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0100	207.7	300.0	44.9	162.3	464.2
I.B1	0.0606	35.0	73.0	8.1	20.6	73.9
I.B2	0.0627	30.9	61.0	8.0	19.6	62.9
I.B3	0.0575	33.9	65.3	8.6	21.9	68.6
I.B4	0.0515	39.3	75.8	9.5	25.1	81.4
II.A1	0.0228	153.7	287.1	18.0	82.9	426.6
II.A2	0.0260	166.8	325.6	15.3	79.0	497.1
II.A3	0.0204	291.5	522.0	18.4	146.6	911.1
II.B1	0.0369	118.7	261.7	10.9	49.7	338.1
II.B2	0.0377	126.4	277.3	10.5	51.5	377.0
II.B3	0.0280	247.5	477.3	13.3	105.2	846.6
III.1	0.0488	51.7	108.3	9.4	28.5	116.0
III.2	0.0495	56.4	122.6	9.0	29.9	129.2
III.3	0.0431	78.3	173.0	9.9	38.6	190.7
III.4	0.0499	49.9	105.3	9.2	27.8	111.6
III.5	0.0488	57.2	123.6	9.2	30.4	131.0
III.6	0.0440	77.1	169.2	9.7	38.1	190.1
IV.A1	0.0491	51.4	108.6	9.3	28.1	115.7
IV.A2	0.0495	53.2	115.1	9.2	28.9	119.1
IV.A3	0.0465	65.3	143.0	9.3	33.5	153.4
IV.A4	0.0496	51.7	110.1	9.2	27.9	118.6
IV.A5	0.0502	53.5	116.7	9.0	28.7	120.8
IV.A6	0.0471	65.5	144.5	9.2	33.3	154.1
IV.B1	0.0538	45.3	97.9	8.7	24.8	99.8
IV.B2	0.0558	41.6	88.7	8.6	23.8	89.2
IV.B3	0.0513	48.5	104.1	9.0	27.3	105.5
V.1	0.0577	38.9	82.4	8.3	22.4	84.2
V.2	0.0586	37.5	78.9	8.2	22.1	78.8
V.3	0.0569	40.1	83.2	8.4	23.5	86.0

Table D.19. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under CM 3 (d=3, K=0.20).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0050	227.2	256.2	342.3	211.5	134.7
I.B1	0.0028	424.3	459.6	306.9	546.2	313.0
I.B2	0.0027	440.9	474.0	302.1	564.7	355.3
I.B3	0.0025	472.5	503.1	370.3	580.7	375.5
I.B4	0.0023	483.5	511.8	451.3	569.3	362.3
II.A1	0.0029	408.8	468.0	700.1	389.0	212.9
II.A2	0.0026	465.6	530.6	782.2	463.7	243.5
II.A3	0.0027	494.2	597.0	954.2	465.8	216.5
II.B1	0.0026	494.9	562.4	609.1	550.7	267.4
II.B2	0.0027	493.6	559.5	569.7	560.0	273.7
II.B3	0.0029	506.2	609.1	822.9	521.2	222.5
III.1	0.0028	423.8	460.6	444.9	492.6	274.1
III.2	0.0027	477.8	522.7	428.0	579.7	321.2
III.3	0.0026	510.3	562.9	502.9	594.7	329.0
III.4	0.0028	427.4	463.6	416.0	505.9	283.7
III.5	0.0027	486.1	532.2	410.1	593.2	333.6
III.6	0.0027	509.4	562.4	484.7	603.6	331.9
IV.A1	0.0028	416.6	452.8	432.6	483.9	270.3
IV.A2	0.0027	456.1	496.9	406.7	551.9	307.5
IV.A3	0.0026	487.4	534.6	438.6	589.5	321.4
IV.A4	0.0028	423.4	461.6	421.9	500.4	273.1
IV.A5	0.0027	470.7	513.7	388.0	579.5	327.8
IV.A6	0.0027	496.8	545.3	426.8	609.8	334.9
IV.B1	0.0028	430.5	466.9	370.0	527.9	296.5
IV.B2	0.0027	451.8	489.3	346.1	565.5	330.3
IV.B3	0.0026	483.2	524.0	392.5	598.7	347.8
V.1	0.0028	434.1	468.3	340.8	538.2	316.8
V.2	0.0027	453.7	490.6	320.5	577.0	348.1
V.3	0.0027	469.2	508.7	327.5	597.7	361.5

Table D.20. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under CM 3 (d=3, K=0.20).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0051	330.8	406.6	94.4	342.2	551.1
I.B1	0.0539	51.5	117.8	8.5	24.7	119.9
I.B2	0.0569	41.2	91.1	8.4	22.6	88.5
I.B3	0.0511	43.7	91.0	9.4	25.5	93.3
I.B4	0.0442	51.7	106.7	10.8	30.1	110.9
II.A1	0.0147	238.8	390.7	27.4	151.1	646.2
II.A2	0.0186	229	402.7	20.9	125.7	675.7
II.A3	0.0146	367.3	586.7	25.8	249.4	918.9
II.B1	0.0298	169.2	339.1	13.0	73.2	536.1
II.B2	0.0312	170.8	347.5	12.3	71.9	554.2
II.B3	0.0234	310.5	545.2	15.5	160.6	933.2
III.1	0.0392	82.4	172.8	10.8	40.3	211.8
III.2	0.0423	78.5	170.5	10.1	38.4	194.1
III.3	0.0376	103.6	223.3	10.9	47.8	275.7
III.4	0.0406	76.8	161.6	10.6	38.4	194.2
III.5	0.0429	78.2	171.4	10.0	37.9	190.8
III.6	0.0388	100.9	219.6	10.6	46.7	265.6
IV.A1	0.0393	83.1	175.6	10.8	40.2	215.6
IV.A2	0.0429	75.7	165.3	10.0	36.6	189.0
IV.A3	0.0406	86.9	188.6	10.3	41.4	223.3
IV.A4	0.041	79.3	167.8	10.3	38.6	202.7
IV.A5	0.0441	74.8	166.6	9.7	35.7	187.0
IV.A6	0.0422	85.9	192.7	10.0	39.6	217.2
IV.B1	0.046	66.3	146.0	9.5	32.0	162.9
IV.B2	0.0497	58.0	130.2	9.1	28.5	138.1
IV.B3	0.0471	63.3	142.7	9.5	31.3	148.6
V.1	0.0496	55.7	122.5	9.0	28.6	130.1
V.2	0.0529	49.3	109.9	8.7	25.8	110.7
V.3	0.0529	50.5	112.5	8.7	26.5	114.3

Table D.21. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under CM 4 (d=2, K=0.10).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0026	417.7	439.3	558.4	428.4	284.4
I.B1	0.0026	419.2	440.9	556.6	435.1	286.3
I.B2	0.0026	421.9	446.5	568.2	437.1	284.7
I.B3	0.0026	426.5	455.3	585.7	442.3	279.8
I.B4	0.0026	437.2	468.6	602.1	455.3	284.0
II.A1	0.0025	471.1	517.9	675.8	494.7	275.3
II.A2	0.0025	491.8	557.4	743.5	510.3	264.3
II.A3	0.0025	532.2	629.6	901.6	538.9	246.6
II.B1	0.0025	483.7	542.4	726.4	502.4	271.3
II.B2	0.0025	489.4	551.9	761.2	503.1	271.3
II.B3	0.0026	523.5	615.4	913.1	520.1	254.6
III.1	0.0025	431.0	457.1	577.0	450.5	291.4
III.2	0.0025	458.9	499.5	643	479.9	287.4
III.3	0.0026	479.7	533.4	708.6	497.3	281.8
III.4	0.0025	434.9	463.7	586.2	455.3	286.3
III.5	0.0026	456.8	496.0	643.1	480.2	288.3
III.6	0.0026	468.7	521.1	689.6	487.6	277.1
IV.A1	0.0025	432.8	459.2	569.4	457.6	292
IV.A2	0.0026	446.6	481.7	597.4	473.2	289.8
IV.A3	0.0026	451.8	496.5	615.7	479.4	276.3
IV.A4	0.0025	432.5	459.6	570.7	456.0	288.8
IV.A5	0.0026	455.4	492.6	607.8	484.8	294.1
IV.A6	0.0026	472.7	519.2	650.8	504.5	289.1
IV.B1	0.0025	432.7	459.0	568.1	457.4	291.5
IV.B2	0.0026	433.7	464.1	579.0	456.8	285.6
IV.B3	0.0026	449.6	488.7	624.0	473.6	284.9
V.1	0.0025	431.3	456.2	551.9	462.1	293.3
V.2	0.0025	440.8	469.8	566.2	470.2	293.9
V.3	0.0026	450.0	484.5	572.0	484.1	295.9

Table D.22. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under CM 4 (d=2, K=0.10).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0269	70.7	118.9	18.2	48.5	150.0
I.B1	0.0269	73.2	123.7	17.9	50.2	155.1
I.B2	0.0271	77.2	132.6	17.4	51.6	168.4
I.B3	0.0267	87.1	153.9	17.1	55.5	201.1
I.B4	0.027	95.4	175.1	16.5	58.5	222.9
II.A1	0.0245	139.7	266.3	16.9	73.6	380.5
II.A2	0.0232	196.2	373.2	16.9	94.6	603.9
II.A3	0.0205	297.5	524.6	18.2	153.1	905.0
II.B1	0.0233	178.1	334.6	17.1	91.9	515.0
II.B2	0.0228	190.8	355	17.5	99.3	559.9
II.B3	0.0196	302.7	529.1	19.4	165.9	904.0
III.1	0.0279	81.0	143.3	16.6	52.1	181.2
III.2	0.0275	109.5	204.7	15.5	63.5	271.9
III.3	0.0248	160.4	303.5	16.5	85.5	443.9
III.4	0.0281	81.0	143.6	16.5	52.1	181.7
III.5	0.0269	112.5	209.8	16.0	65.5	278.9
III.6	0.0249	154.2	288.4	16.6	83.2	422.8
IV.A1	0.029	78.1	139.8	16.0	50.1	174.5
IV.A2	0.0293	97.3	183.2	14.9	56.8	236.6
IV.A3	0.028	123.8	237.9	14.8	66.7	327.1
IV.A4	0.0289	78.2	139.9	16.2	49.7	176.5
IV.A5	0.0289	100.2	190.3	15.1	57.8	244.5
IV.A6	0.0276	127.1	243.7	15.1	68.6	334.5
IV.B1	0.0292	78.1	139.4	15.8	49.8	176.5
IV.B2	0.0292	86.0	156.3	15.4	53.5	198.9
IV.B3	0.0281	106.4	199.4	15.3	61.6	262.1
V.1	0.0306	72.0	128.6	15.1	46.5	159.5
V.2	0.0309	77.1	141	14.9	47.6	177.1
V.3	0.0313	88.0	167.5	14.2	51.0	212.1

Table D.23. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under CM 4 (d=3, K=0.15).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0056	201.0	227.6	299.8	185.9	120.1
I.B1	0.0056	204.7	234.8	310.6	187.6	119.7
I.B2	0.0057	205.3	239.4	322.1	185.1	116.3
I.B3	0.0057	210.6	252.9	344.6	185.4	113.0
I.B4	0.0056	220.3	271.7	381.5	190.5	111.3
II.A1	0.0043	298.1	363.9	531.7	262.8	138.4
II.A2	0.0041	326.7	410.1	622.4	286.1	138.5
II.A3	0.0041	356.5	466.3	725.3	299.2	134.7
II.B1	0.0043	312.0	393.2	605.2	268.8	135.0
II.B2	0.0043	316.2	400.5	619.7	271.5	135.9
II.B3	0.0042	346.3	459.9	729.9	286.9	131.7
III.1	0.0051	240.4	288.3	409.6	211.8	125.3
III.2	0.0045	286.4	352.6	556.1	250.6	130.3
III.3	0.0043	320.1	403.5	656.1	278.8	134.3
III.4	0.0052	236.0	283.4	405.8	207.4	121.9
III.5	0.0045	287.0	353.4	553.2	252.2	131.3
III.6	0.0043	320.3	402.8	660.1	279.0	135.3
IV.A1	0.0051	239.9	290.0	425.4	208.9	122.0
IV.A2	0.0046	289.1	357.8	574.0	253.7	128.0
IV.A3	0.0043	313.4	388.4	639.7	281.4	133.1
IV.A4	0.0051	241.8	292.7	428.7	210.4	122.8
IV.A5	0.0044	297.9	368.2	597.0	263.2	132.7
IV.A6	0.0042	331.6	411.9	680.5	300.1	138.5
IV.B1	0.0050	247.1	299.8	443.6	215.3	122.8
IV.B2	0.0049	260.9	321.2	504.5	225.5	123.4
IV.B3	0.0046	290.3	361.7	597.9	253.4	127.4
V.1	0.0045	275.1	329.3	505.1	246.8	134.7
V.2	0.0044	287.1	345.4	544.8	258.5	135.6
V.3	0.0040	324.4	390.1	628.8	303.9	145.7

Table D.24. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under CM 4 (d=3, K=0.15).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0040	381.9	444.9	130.1	436.3	544.7
I.B1	0.0043	382.1	444.1	125.1	448.1	526.8
I.B2	0.0047	381.4	445.9	116.0	454.4	523.5
I.B3	0.0049	407.6	478.9	111.0	514.8	524.8
I.B4	0.0055	415.5	496.4	96.3	537.8	523.5
II.A1	0.0069	397.0	520.3	61.7	441.1	641.8
II.A2	0.0076	426.6	569.5	55.6	492.8	658.4
II.A3	0.0071	517.8	678.9	61.6	718.5	638.1
II.B1	0.0077	432.3	568.3	56.8	522.3	634.7
II.B2	0.008	443.5	584.1	54.8	548.7	641.7
II.B3	0.0075	535.5	691.7	64.5	780.6	626.6
III.1	0.0069	355.7	448.1	67.5	405.2	554.4
III.2	0.0099	359.4	485.4	42.3	391.3	598.7
III.3	0.0109	385.4	530.4	38.2	427.4	628.5
III.4	0.0071	356.0	450.5	64.8	403.8	564.5
III.5	0.0099	362.8	489.2	42.9	394.0	599.7
III.6	0.0114	383.2	530.2	36.2	418.4	625.7
IV.A1	0.0076	345.1	438.7	60.4	392.8	546
IV.A2	0.0117	328.7	455.7	34.8	337.8	583.8
IV.A3	0.0134	332.5	477.3	29.2	318.2	611.9
IV.A4	0.0079	346.7	443.2	57.9	391.9	551.6
IV.A5	0.012	340.1	476.4	33.3	339.5	616.5
IV.A6	0.0142	337.0	492.6	27.4	305.0	649.6
IV.B1	0.0083	342.7	442.6	53.5	385.1	559.2
IV.B2	0.0102	338.6	449.1	41.7	378.5	559.8
IV.B3	0.0127	336.2	467.8	31.6	353.9	581.6
V.1	0.0111	286.3	398.9	36.7	265.8	567.2
V.2	0.0133	280.6	405.3	29.8	242.5	588.1
V.3	0.0172	265.5	409.9	21.8	194.2	622.0

Table D.25. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under CM 4 (d=3, K=0.20).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0074	152.3	170.6	222.5	141.7	94.2
I.B1	0.0074	151.6	170.9	222.0	140.8	92.6
I.B2	0.0075	151.0	173.2	223.2	138.4	91.0
I.B3	0.0077	151.4	179.5	231.9	135.6	87.1
I.B4	0.0077	154.7	188.6	243.8	135.6	85.4
II.A1	0.0060	216.3	273.4	374.2	186.0	99.2
II.A2	0.0058	240.7	317.9	445.5	201.6	97.3
II.A3	0.0056	266.7	370.5	524.7	214.2	95.8
II.B1	0.0060	224.5	295.7	411.2	185.9	95.6
II.B2	0.0060	231.0	309.9	434.4	188.3	96.6
II.B3	0.0058	256.3	359.0	506.6	202.3	94.4
III.1	0.0070	169.5	205.3	270.0	148.5	93.6
III.2	0.0065	200.6	262.6	368.3	163.5	91.9
III.3	0.0063	226.5	310.1	454.2	178.8	91.3
III.4	0.0071	167.4	202.6	266.0	146.3	92.5
III.5	0.0065	202.1	263.8	369.5	165.7	93.5
III.6	0.0062	226.1	307.3	450.5	180.2	91.5
IV.A1	0.0071	167.8	204.4	269.3	146.0	91.5
IV.A2	0.0068	193.5	255.9	355.8	157.0	88.8
IV.A3	0.0065	217.1	294.6	434.7	173.0	88.0
IV.A4	0.0071	167.7	205.4	267.0	146.1	92.2
IV.A5	0.0065	203.6	268.9	380.8	165.7	92.2
IV.A6	0.0062	226.7	307.4	468.9	180.0	91.4
IV.B1	0.0071	169.8	209.4	279.4	145.7	91.4
IV.B2	0.0070	180.3	231.8	311.7	150.2	90.9
IV.B3	0.0066	201.4	267.8	386.4	161.2	90.3
V.1	0.0066	188.5	235.3	323.9	160.5	96.1
V.2	0.0064	199.9	255.7	361.5	166.1	95.6
V.3	0.0060	227.8	298.1	462.0	185.1	97.4

Table D.26. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under CM 4 (d=3, K=0.20).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0025	457.6	482.4	302.6	553.5	441.9
I.B1	0.0025	460.7	486.4	306.0	560.9	441.8
I.B2	0.0026	465.7	492.7	313.9	566.6	434.2
I.B3	0.0027	473.7	504.0	333.1	583.4	411.9
I.B4	0.0028	483.9	519.3	336.6	602.0	402.9
II.A1	0.0035	502.7	577.2	166.7	718.6	477.3
II.A2	0.0038	520.3	610.6	150.2	781.9	459.7
II.A3	0.0040	586.3	701.9	154.8	918.1	456.0
II.B1	0.0040	511.6	593.2	157.5	752.3	453.7
II.B2	0.0041	530.3	615.5	159.3	777.5	466.9
II.B3	0.0042	595.1	703.9	174.5	895.4	449.9
III.1	0.0034	453.4	496.9	215.6	586.2	439.3
III.2	0.0048	462.6	532.7	133.9	660.4	447.5
III.3	0.0053	483.6	571.2	118.5	730.5	433.8
III.4	0.0033	453.1	496.2	216.0	587.8	438.0
III.5	0.0047	463.7	532.9	140.2	661.2	438.4
III.6	0.0056	481.2	571.1	113.4	736.7	429.7
IV.A1	0.0034	456.9	500.6	222.2	592.4	439.6
IV.A2	0.0050	439.2	507.1	125.0	632.0	420.7
IV.A3	0.0062	444.5	530.0	91.1	673.9	419.3
IV.A4	0.0035	447.8	491.2	210.1	582.8	436.3
IV.A5	0.0054	445.0	517.7	115.6	642.6	437.3
IV.A6	0.0067	452.6	545.9	84.6	699.3	441.7
IV.B1	0.0038	444.6	492.4	190.8	587.0	436.9
IV.B2	0.0045	444.9	502.2	156.3	608.5	431.7
IV.B3	0.0060	437.4	511.1	105.8	639.6	423.2
V.1	0.0049	413.4	477.0	117.1	560.7	451.8
V.2	0.0057	409.3	481.2	97.1	577.2	447.6
V.3	0.0081	390.4	485.4	58.0	564.1	460.7

Table D.27. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under CM 5 (d=2, K=0.10).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
IV.A1	0.0030	446.8	492.7	199.4	550.5	510.8
IV.A2	0.0031	482.3	534.4	205.1	618.7	513.7
IV.A3	0.0031	513.5	569.7	224.0	683.8	494.8
IV.A4	0.0030	452.4	497.4	206.4	554.1	514.5
IV.A5	0.0030	499.9	554.1	214.3	652.3	521.9
IV.A6	0.0030	531.6	588.2	239.7	704.4	505.5
IV.B1	0.0028	462.9	500.8	241.8	577.0	487.9
IV.B2	0.0028	477.2	517.2	249.2	603.1	485.0
IV.B3	0.0028	504.0	545.7	280.1	643.5	472.4
V.1	0.0028	458.4	496.5	235.4	568.9	490.3
V.2	0.0028	466.1	505.3	239.4	581.2	488.7
V.3	0.0028	496.4	541.2	247.5	639.8	486.9

Table D.28. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under CM 5 (d=2, K=0.10).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
IV.A1	0.0811	19.0	29.2	6.7	14.5	35.5
IV.A2	0.0754	23.6	40.7	6.7	16.7	47.3
IV.A3	0.0676	31.5	60.8	7.1	20.0	66.2
IV.A4	0.0806	19.1	29.1	6.7	14.6	35.9
IV.A5	0.0736	24.3	42.3	6.8	17.1	48.4
IV.A6	0.0663	32.5	63.1	7.1	20.6	68.5
IV.B1	0.0729	21.5	33.4	7.4	16.3	40.7
IV.B2	0.0702	23.6	38.2	7.5	17.5	46.0
IV.B3	0.0630	30.8	55.1	7.8	21.0	62.8
V.1	0.0748	20.6	31.5	7.2	15.8	38.8
V.2	0.072	22.6	36.1	7.4	16.8	43.8
V.3	0.0673	27.2	47.5	7.4	19.0	54.9

Table D.29. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under CM 5 (d=3, K=0.15).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
IV.A1	0.0033	425.1	479.7	167.3	506.4	542.2
IV.A2	0.0031	477.8	528.1	206.9	605.5	523.9
IV.A3	0.0029	511.1	561.0	245.3	668.9	491.0
IV.A4	0.0033	424.4	475.5	174.3	506.4	534.7
IV.A5	0.003	497.5	548.9	220.4	632.2	535.6
IV.A6	0.0029	541.6	594.5	260.1	705.2	511.2
IV.B1	0.0029	459.3	499.1	230.5	576.6	479.8
IV.B2	0.0028	471.9	512.9	239.2	597.8	479.5
IV.B3	0.0028	507.9	551.6	273.4	647.0	471.9
V.1	0.0029	450.5	491.0	221.3	560.3	486.0
V.2	0.0029	462.2	502.5	233.6	583.6	475.1
V.3	0.0030	477.9	522.6	236.1	622.7	460.0

Table D.30. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under CM 5 (d=3, K=0.15).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
IV.A1	0.0910	16.1	24.0	6.1	12.5	29.7
IV.A2	0.0764	22.2	36.9	6.8	16.1	43.0
IV.A3	0.0663	31.3	58.4	7.2	20.5	65.6
IV.A4	0.0896	16.3	24.2	6.3	12.7	29.9
IV.A5	0.0739	23.1	37.9	7.0	16.9	45.4
IV.A6	0.0642	32.7	61.4	7.4	21.4	68.4
IV.B1	0.0730	22.2	36.1	7.2	16.5	42.1
IV.B2	0.0703	24.5	40.9	7.3	17.6	48.2
IV.B3	0.0631	31.3	57.3	7.8	21.0	64.5
V.1	0.0754	21.3	33.7	7.1	15.9	40.8
V.2	0.0730	23.3	38.9	7.1	16.8	45.4
V.3	0.0684	27.8	50.7	7.3	19.0	56.6

Table D.31. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under CM 5 (d=3, K=0.20).

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
IV.A1	0.0038	401.2	465.3	138.0	465.2	560.7
IV.A2	0.0034	471.1	533.1	171.1	597.9	556.2
IV.A3	0.0032	510.5	568.6	208.3	674.6	521.5
IV.A4	0.0037	410.2	472.7	144.6	474.9	560.7
IV.A5	0.0031	482.0	537.3	197.0	609.6	542.9
IV.A6	0.0031	528.3	583.8	234.3	692.4	518.2
IV.B1	0.0030	448.3	488.9	218.9	566.9	472.6
IV.B2	0.0030	463.0	504.1	231.3	588.4	463.2
IV.B3	0.0029	501.2	547.5	259.4	654.2	460.5
V.1	0.0030	454.1	497.6	212.1	572.3	486.1
V.2	0.0030	458.0	502.1	215.7	591.4	468.6
V.3	0.0031	477.4	525.5	221.4	627.0	458.1

Table D.32. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under CM 5 (d=3, K=0.20).

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
IV.A1	0.1001	14.7	22.3	5.6	11.3	27.0
IV.A2	0.0844	19.7	32.2	6.2	14.4	38.2
IV.A3	0.0719	27.8	51.2	6.8	18.6	57.6
IV.A4	0.0975	15.0	22.3	5.8	11.7	27.3
IV.A5	0.0798	20.9	34.2	6.5	15.3	40.3
IV.A6	0.0684	30.0	55.9	7.1	19.9	62.8
IV.B1	0.0739	22.5	37.2	7.1	16.4	43.7
IV.B2	0.0709	25.0	42.8	7.2	17.5	49.9
IV.B3	0.0637	32.1	60.8	7.6	21.1	66.3
V.1	0.0759	21.9	36.4	7.0	15.9	42.4
V.2	0.0730	23.9	41.1	7.0	17.0	47.5
V.3	0.0696	28.4	52.7	7.0	18.8	58.7

Table D.33. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under SCM 1.

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0057	212.2	248.9	337.6	193.8	111.7
I.B1	0.0037	337.7	389.1	559.3	338.8	161.6
I.B2	0.0034	367.5	415.2	542.9	389.8	187.0
I.B3	0.0032	386.6	434.3	561.9	414.6	198.6
I.B4	0.0030	402.3	452.1	599.5	424.4	208.9
II.A1	0.0031	394.1	456.3	685.4	372.3	197.1
II.A2	0.0029	436.1	509.1	783.2	416.8	210.0
II.A3	0.0029	465.5	570.3	928.9	432.4	194.8
II.B1	0.0029	441.9	512.4	753.7	439.3	215.3
II.B2	0.0029	448.9	523.4	784.3	446.6	218.9
II.B3	0.0029	474.5	576.4	918.3	454.3	203.4
III.1	0.0031	388.5	431.5	561.4	410.7	215.2
III.2	0.0027	451.8	500.7	568.0	499.8	263.7
III.3	0.0027	477.9	535.1	637.5	517.3	273.3
III.4	0.0030	390.1	432.0	546.1	415.8	219.6
III.5	0.0027	448.8	496.2	572.3	493.9	262.4
III.6	0.0027	482.4	541.3	636.1	529.4	272.7
IV.A1	0.003	400.6	441.8	531.1	434.8	231.0
IV.A2	0.0028	436.3	477.9	517.5	487.6	266.3
IV.A3	0.0026	466.4	512.9	560.7	521.6	285.6
IV.A4	0.0029	404.8	446.6	532.4	440.6	231.9
IV.A5	0.0027	457.3	502.9	540.6	514.5	278.6
IV.A6	0.0026	483.8	535.0	566.6	547.5	293.9
IV.B1	0.0029	403.7	443.3	510.8	446.5	237.2
IV.B2	0.0028	428.1	469.4	498.7	487.0	258.1
IV.B3	0.0027	457.9	501.1	525.5	521.9	284.9
V.1	0.0028	416.3	451.1	464.5	474.6	263.8
V.2	0.0027	439.6	476.7	464.7	510.1	284.5
V.3	0.0026	461.7	500.7	455.3	541.5	310.0

Table D.34. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under SCM 1.

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0049	353.9	426.8	96.0	412.4	511.0
I.B1	0.0226	176.0	305.9	17.1	95.0	536.1
I.B2	0.0269	146.4	274.1	14.8	69.7	465.9
I.B3	0.0269	143.4	273.5	15.0	67.5	461.3
I.B4	0.0254	149.6	285.3	15.9	71.4	475.6
II.A1	0.0138	258.4	411.3	28.6	172.9	680.5
II.A2	0.0153	280.8	463.1	25.4	172.6	778.5
II.A3	0.0133	396.4	613.6	28.4	302.3	906.1
II.B1	0.0198	239.5	417.1	19.3	130.1	724.0
II.B2	0.0203	245.9	429.4	18.8	133.2	742.4
II.B3	0.0166	372.0	595.7	23.1	254.5	915.8
III.1	0.0274	131.1	249.2	14.9	65.9	388.1
III.2	0.0311	124.6	253.9	13.2	58.0	359.1
III.3	0.0284	152.6	302.3	14.1	73.2	449.7
III.4	0.0280	127.6	245.2	14.7	63.4	368.2
III.5	0.0314	123.9	250.8	13.0	58.9	354.8
III.6	0.0295	150.5	300.6	13.5	70.8	448.1
IV.A1	0.0304	115.2	227.5	13.5	56.5	327.3
IV.A2	0.0339	102.8	214.3	12.5	49.1	274.3
IV.A3	0.0326	115.9	238.4	12.8	55.6	319.2
IV.A4	0.0307	115.2	227.1	13.4	56.1	328.9
IV.A5	0.0340	104.7	218.0	12.3	50.0	282.8
IV.A6	0.0327	119.9	247.8	12.5	56.1	334.2
IV.B1	0.0328	105.0	211.9	12.8	50.3	292.8
IV.B2	0.0354	97.2	202.3	11.9	46.0	261.8
IV.B3	0.0347	101.8	214.6	12.2	48.0	273.5
V.1	0.0364	83.8	173.3	11.8	42.2	214.0
V.2	0.0388	79.1	168.6	11.2	39.1	196.9
V.3	0.0400	77.5	167.5	11.0	38.5	192.4

Table D.35. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under SCM 2.

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0036	312.1	345.9	473.4	296.5	185.8
I.B1	0.0036	318.0	355.7	510.5	299.8	185.1
I.B2	0.0036	325.9	367.4	540.4	309.6	183.1
I.B3	0.0036	333.9	380.2	572.3	316.4	180.2
I.B4	0.0034	356.0	407.9	617.2	344.9	185.8
II.A1	0.0028	424.5	482.0	708.2	413.9	222.4
II.A2	0.0028	451.8	523.7	784.4	446.0	221.9
II.A3	0.0028	481.4	582.1	923.9	461.9	208.1
II.B1	0.0029	438.6	508.4	766.4	433.5	219.0
II.B2	0.0029	445.9	520.7	789.3	435.5	218.0
II.B3	0.0029	472.1	573.0	917.9	448.7	207.0
III.1	0.0032	366.4	410.1	596.0	362.3	205.1
III.2	0.0030	417.2	470.9	633.4	433.4	226.0
III.3	0.0029	444.3	508.8	695.6	460.4	232.2
III.4	0.0032	366.8	410.4	591.0	363.4	203.1
III.5	0.0030	414.6	468.4	632.0	433.0	221.2
III.6	0.0030	446.4	509.7	686.2	468.8	233.9
IV.A1	0.0032	370.0	413.8	567.0	378.9	205.9
IV.A2	0.0030	417.2	468.2	538.0	458.0	232.0
IV.A3	0.0029	447.0	505.8	559.2	499.5	242.1
IV.A4	0.0031	383.7	429.1	580.2	396.8	212.0
IV.A5	0.0030	426.6	478.4	539.5	472.8	240.1
IV.A6	0.0029	461.8	519.7	551.9	520.1	259.7
IV.B1	0.0034	357.3	406.8	589.2	355.9	187.7
IV.B2	0.0034	362.3	411.9	571.0	371.7	191.4
IV.B3	0.0032	398.7	455.7	575.0	424.9	205.7
V.1	0.0032	370.3	415.9	556.0	382.6	201.5
V.2	0.0032	381.3	427.9	535.2	406.8	209.4
V.3	0.0031	414.4	465.5	509.3	463.2	231.9

Table D.36. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under SCM 2.

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0116	189.9	284.9	38.2	139.2	437.8
I.B1	0.0133	192.5	294.4	32.8	139.0	458.5
I.B2	0.0156	191.2	300.8	26.9	133.1	478.4
I.B3	0.0167	200.9	322.2	24.2	133.6	521.1
I.B4	0.0186	206.4	340.7	21.1	129.6	558.8
II.A1	0.0178	212.1	365.2	22.5	123.2	612.8
II.A2	0.0178	256.2	438.8	21.6	145.8	752.1
II.A3	0.0161	357.1	582.2	23.4	228.3	916.9
II.B1	0.0186	250.0	427.2	20.9	143.6	730.0
II.B2	0.0187	259.6	440.3	21.0	151.6	747.8
II.B3	0.0161	368.9	588.6	23.6	251.5	910.4
III.1	0.0202	163.5	280.6	20.2	98.7	439.6
III.2	0.0251	166.4	306.1	15.7	86.7	492.3
III.3	0.0245	199.8	369.6	15.9	99.7	623.6
III.4	0.0209	157.9	270.6	19.7	95.0	426.3
III.5	0.0248	169.5	310.9	16.0	88.5	498.9
III.6	0.0250	197.5	364.1	15.7	99.1	612.9
IV.A1	0.0248	149.0	268.8	16.2	81.6	418.5
IV.A2	0.0313	136.9	272.1	12.7	62.5	416.0
IV.A3	0.0318	143.7	289.1	12.5	63.5	445.8
IV.A4	0.0254	145.8	265.2	15.7	78.3	414.7
IV.A5	0.0318	133.7	268.2	12.6	61.0	402.5
IV.A6	0.0321	143.8	292.6	12.4	63.0	437.4
IV.B1	0.0220	175.3	304.2	17.7	99.8	502.3
IV.B2	0.0249	171.7	305.5	15.6	93.1	507.2
IV.B3	0.0276	171.3	319.1	14.0	82.7	540.7
V.1	0.0262	145.8	268.8	15.1	76.0	429.3
V.2	0.0291	141.6	269.7	13.4	68.7	424.2
V.3	0.0332	132.2	265.9	11.9	58.8	408.5

Table D.37. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under SCM 3.

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
textbfI.A	0.0074	161.5	191.1	256.4	146.1	86.2
I.B1	0.0066	187.5	228.4	311.8	166.0	93.4
I.B2	0.0063	199.0	246.0	340.7	173.3	96.4
I.B3	0.0061	211.7	266.2	375.3	182.0	97.8
I.B4	0.006	219.7	280.8	406.0	185.3	97.2
II.A1	0.0045	288.0	354.8	510.3	254.1	130.8
II.A2	0.0042	321.6	407.2	611.6	277.4	134.6
II.A3	0.0041	352.6	463.9	718.3	295.2	131.1
II.B1	0.0044	304.3	384.3	581.8	263.5	129.4
II.B2	0.0043	314.5	399.5	618.0	271.1	130.6
II.B3	0.0043	344.1	455.4	714.6	287.3	128.1
III.1	0.0052	246.0	300.6	443.0	216.2	116.7
III.2	0.0043	306.5	376.2	597.6	274.5	134.7
III.3	0.0040	339.6	421.1	683.8	303.3	143.7
III.4	0.0051	248.7	304.5	448.7	218.0	117.6
III.5	0.0043	306.6	377.0	599.1	272.1	135.2
III.6	0.004	340.7	423.3	691.6	304.7	142.9
IV.A1	0.0049	257.4	312.4	469.5	227.7	122.1
IV.A2	0.0042	312.9	378.5	602.2	285.7	140.0
IV.A3	0.0039	346.0	419.5	670.2	322.6	149.3
IV.A4	0.0050	256.5	312.7	474.6	226.4	120.0
IV.A5	0.0041	319.9	388.4	621.7	291.7	141.7
IV.A6	0.0038	358.9	436.9	706.5	333.3	155.3
IV.B1	0.0048	267.1	325.5	498.8	236.1	124.0
IV.B2	0.0047	276.5	336.9	531.1	246.1	126.6
IV.B3	0.0043	314.4	384.9	626.5	284.8	136.4
V.1	0.0043	294.0	348.9	538.1	269.3	140.2
V.2	0.0041	313.0	371.1	578.8	291.0	147.5
V.3	0.0037	348.1	410.2	629.3	337.2	162.0

Table D.38. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under SCM 3.

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0031	416.1	453.7	212.8	541.3	398.6
I.B1	0.0043	394.6	455.1	125.9	523.3	444.1
I.B2	0.0051	389.6	461.9	100.4	521.9	460.9
I.B3	0.0058	385.2	462.5	87.8	526.0	453.1
I.B4	0.0061	401.3	487.1	82.0	562.7	461.8
II.A1	0.0063	412.9	533.4	68.1	484.8	620.0
II.A2	0.0070	438.8	575.2	61.1	539.2	627.4
II.A3	0.0068	529.7	690.8	65.1	759.0	624.0
II.B1	0.0078	422.8	552.8	55.5	526.9	610.1
II.B2	0.0080	442.9	580.8	56.0	559.8	614.8
II.B3	0.0072	537.4	695.0	65.8	789.0	616.2
III.1	0.0080	336.4	434.2	54.5	382.8	525.1
III.2	0.0115	332.3	469.2	35.2	317.9	625.1
III.3	0.0121	356.8	509.7	32.7	346.7	660.3
III.4	0.0082	334.0	433.2	52.7	380.1	532.7
III.5	0.0116	327.7	460.5	35.9	316.9	607.7
III.6	0.0128	350.6	505.0	31.3	328.2	658.5
IV.A1	0.0093	322.9	432.8	44.9	340.2	560.8
IV.A2	0.0137	285.7	419.2	28.5	241.9	603.4
IV.A3	0.0155	288.7	439.3	24.9	221.0	646.4
IV.A4	0.0096	314.5	420.4	44.3	331.7	542.7
IV.A5	0.0142	292.5	433.6	27.2	240.5	629.3
IV.A6	0.0157	300.7	462.0	24.4	226.2	685.9
IV.B1	0.0102	312.4	423.8	40.3	320.7	550.4
IV.B2	0.0121	306.6	430.3	33.3	294.9	579.0
IV.B3	0.0144	301.9	444.2	26.9	2610	614.6
V.1	0.0132	256.5	377.6	30.3	207.5	579.2
V.2	0.0151	251.7	384.1	25.7	185.2	602.0
V.3	0.0187	231.2	375.5	20.4	146.6	623.3

Table D.39. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under SCM 4.

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0028	384.9	413.7	557.1	380.4	243.6
I.B1	0.0027	450.0	482.4	266.5	565.1	427.7
I.B2	0.0027	457.2	490.4	265.9	573.9	443.4
I.B3	0.0026	474.6	506.0	295.8	591.2	445.8
I.B4	0.0024	497.2	528.8	335.2	617.4	442.1
II.A1	0.0023	510.7	555.2	590.6	576.9	324.5
II.A2	0.0024	539.7	597.7	564.7	629.1	331.9
II.A3	0.0024	574.8	662.3	717.9	642.3	299.7
II.B1	0.0025	539.9	597.1	424.3	671.1	363.8
II.B2	0.0026	549.4	611.4	416.0	690.5	365.1
II.B3	0.0026	587.1	675.9	592.5	685.4	323.2
III.1	0.0026	460.2	491.0	310.6	575.7	395.9
III.2	0.0026	511.9	554.9	323.6	656.6	423.3
III.3	0.0026	544.9	594.8	395.2	683.6	408.4
III.4	0.0027	463.0	495.3	308.6	583.6	398.8
III.5	0.0026	514.9	558.1	334.7	652.7	415.9
III.6	0.0027	540.1	590.7	375.5	681.7	407.2
IV.A1	0.0029	449.9	489.5	236.8	583.2	426.3
IV.A2	0.0028	492.0	535.9	267.3	641.6	443.0
IV.A3	0.0028	513.6	561.8	302.1	671.9	421.5
IV.A4	0.0029	450.7	490.6	237.0	584.2	420.7
IV.A5	0.0028	493.5	536.9	273.0	641.1	442.0
IV.A6	0.0028	534.8	584.2	316.6	690.6	442.4
IV.B1	0.0027	462.0	499.1	277.6	594.7	406.0
IV.B2	0.0028	467.4	505.9	276.8	600.2	409.1
IV.B3	0.0027	495.4	535.6	319.8	631.7	412.6
V.1	0.0028	449.2	486.0	255.5	580.0	405.9
V.2	0.0028	462.8	501.9	258.5	600.2	420.7
V.3	0.0028	481.3	523.2	273.0	625.4	422.7

Table D.40. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under SCM 4.

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0203	102.5	170.3	23.1	68.6	227.8
I.B1	0.0666	26.3	47.6	7.8	17.8	51.7
I.B2	0.0673	25.2	43.4	7.8	17.8	49.3
I.B3	0.0624	28.2	49.8	8.2	19.7	55.7
I.B4	0.0577	32.4	59.0	8.7	21.9	65.6
II.A1	0.0328	98.4	204.9	13.0	49.8	246.3
II.A2	0.0339	123.5	265.5	11.8	54.9	342.2
II.A3	0.0276	226.0	453.3	13.5	92.3	771.2
II.B1	0.0433	87.7	201.4	9.6	38.5	227.0
II.B2	0.0423	95.9	218.9	9.7	42.0	253.6
II.B3	0.0335	192.2	405.5	11.4	72.9	647.1
III.1	0.0581	35.0	68.0	8.4	22.1	73.4
III.2	0.0551	42.9	88.7	8.5	25.4	93.4
III.3	0.0472	63.0	136.9	9.2	33.3	147.5
III.4	0.058	34.4	65.9	8.5	21.9	71.8
III.5	0.0537	44.4	91.4	8.6	26.2	96.1
III.6	0.0477	63.7	138.9	9.1	33.6	148.6
IV.A1	0.069	29.0	58.9	7.2	17.9	60.0
IV.A2	0.0626	34.2	68.1	7.7	21.1	71.9
IV.A3	0.0549	45.7	94.9	8.3	26.3	100.8
IV.A4	0.0676	30.4	62.4	7.3	18.5	62.6
IV.A5	0.0616	35.6	73.5	7.8	21.6	74.1
IV.A6	0.0545	47.1	100.3	8.4	26.7	104
IV.B1	0.0611	35.0	71.8	7.9	21.0	73.6
IV.B2	0.0615	34.7	71.2	7.9	21.0	72.7
IV.B3	0.0563	41.9	87.6	8.4	24.5	89.8
V.1	0.0641	32.1	64.9	7.6	19.8	66.3
V.2	0.0641	32.2	65.0	7.6	19.9	67.2
V.3	0.0618	35.9	73.5	7.7	21.8	76.3

Table D.41. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under SCM 5.

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0033	353.3	393.3	554.0	347.3	194.6
I.B1	0.0028	403.7	438.4	500.9	447.0	243.4
I.B2	0.0028	417.8	450.5	463.7	477.8	268.0
I.B3	0.0026	440.1	476.0	490.5	505.1	280.7
I.B4	0.0026	453.2	490.1	494.2	520.8	292.2
II.A1	0.0024	487.8	536.3	631.5	532.8	289.0
II.A2	0.0024	521.0	587.2	688.6	567.6	286.0
II.A3	0.0025	557.7	651.0	797.3	596.7	272.8
II.B1	0.0025	492.4	548.1	618.1	543.4	288.1
II.B2	0.0026	505.7	568.0	672.9	547.8	288.3
II.B3	0.0025	552.3	643.1	814.7	583.0	280.4
III.1	0.0025	455.4	485.7	459.6	531.9	318.8
III.2	0.0025	485.3	522.8	454.6	578.4	343.9
III.3	0.0025	519.2	568.4	518.5	603.9	343.8
III.4	0.0026	447.0	476.5	446.5	521.5	316.0
III.5	0.0025	497.5	536.9	469.0	589.5	352.6
III.6	0.0025	520.9	570.2	520.9	609.2	346.1
IV.A1	0.0026	458.9	491.5	343.1	575.7	358.4
IV.A2	0.0026	489.4	525.8	357.3	613.0	388.5
IV.A3	0.0026	513.6	558.1	409.7	632.1	374.4
IV.A4	0.0026	455.9	488.2	339.2	570.1	355.9
IV.A5	0.0025	499.9	537.3	375.0	622.2	391.4
IV.A6	0.0025	524.2	568.9	423.8	640.8	387.4
IV.B1	0.0026	452.3	483.9	411.1	540.9	325.1
IV.B2	0.0026	453.6	486.5	404.2	547.8	327.3
IV.B3	0.0026	484.4	524.5	428.3	586.2	342.6
V.1	0.0026	449.3	478.8	386.8	542.7	335.6
V.2	0.0026	457.4	489.8	379.7	556.8	343.5
V.3	0.0026	481.3	520.3	374.1	599.0	356.1

Table D.42. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under SCM 5.

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0189	149.9	259.7	22.0	87.6	408.7
I.B1	0.0333	91.2	186.1	13.1	44.7	240.9
I.B2	0.0375	79.5	168.1	11.8	39.3	198.5
I.B3	0.0371	80.6	171.8	11.8	40.0	201.1
I.B4	0.0367	82.0	176.3	12.0	40.7	202.2
II.A1	0.0296	121.8	248.8	13.9	59.4	332.4
II.A2	0.0282	163.9	329.3	14.1	73.1	498.6
II.A3	0.0247	260.1	492.2	15.0	115.9	873.4
II.B1	0.0293	145.6	295.0	13.6	67.4	422.2
II.B2	0.0285	157.7	317.2	14.0	73.2	459.8
II.B3	0.0239	256.1	480.2	15.9	120.6	829.5
III.1	0.0409	63.0	131.7	11.2	34.5	145.0
III.2	0.0415	68.7	145.6	10.7	36.7	161.0
III.3	0.0367	97.9	208.9	11.5	47.6	249.4
III.4	0.0421	60.3	125.7	11.0	33.2	135.4
III.5	0.0410	70.7	151.0	10.8	37.6	164.8
III.6	0.0372	95.5	202.9	11.3	47.3	241.5
IV.A1	0.053	44.5	96.6	9.0	24.6	96.7
IV.A2	0.0512	47.1	100.0	9.1	27.0	102.1
IV.A3	0.0459	60.9	129.4	9.8	33.4	139.2
IV.A4	0.0530	44.4	96.6	9.0	24.9	95.2
IV.A5	0.0502	48.9	105.1	9.3	27.9	106.0
IV.A6	0.0454	64.1	137.4	9.8	34.3	149.2
IV.B1	0.0447	58.1	124.0	10.2	31.4	133.3
IV.B2	0.0457	59.3	128.6	9.9	31.4	134.5
IV.B3	0.0436	67.9	147.9	10.1	35.3	159.5
V.1	0.0476	50.7	106.6	9.8	28.6	111.6
V.2	0.0482	52.1	110.8	9.6	28.7	116.2
V.3	0.0486	55.2	119.3	9.3	30.0	123.7

Table D.43. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under SCM 6.

Method	$ARFS$	$mARL_0$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0063	181.9	207.3	275.9	167.7	106.4
I.B1	0.0048	249.9	293.4	418.4	225.8	131.2
I.B2	0.0046	264.3	312.9	464.3	236.7	136.1
I.B3	0.0046	270.5	324.3	490.7	240.0	133.6
I.B4	0.0045	279.5	337.4	515.5	248.6	133.2
II.A1	0.0041	307.7	373.1	545.4	274.5	144.2
II.A2	0.0037	357.1	438.1	674.2	318.7	157.9
II.A3	0.0037	386.7	498.0	790.1	332.6	148.1
II.B1	0.0037	354.2	432.7	687.6	318.6	156.2
II.B2	0.0037	367.0	450.2	722.9	334.1	158.4
II.B3	0.0038	390.4	504.8	835.0	338.7	147.2
III.1	0.0043	286.8	340.8	522.0	260.2	138.9
III.2	0.0037	351.0	417.5	658.5	331.5	162.8
III.3	0.0035	385.2	465.2	746.8	364.1	170.6
III.4	0.0043	287.5	341.2	527.3	261.3	140.0
III.5	0.0037	348.7	415.9	661.3	326.9	161.6
III.6	0.0035	386.7	464.4	737.5	366.6	174.1
IV.A1	0.0043	287.6	341.4	533.2	261.6	140.0
IV.A2	0.0037	351.3	415.8	641.0	337.9	161.3
IV.A3	0.0035	376.1	445.6	679.6	367.1	171.0
IV.A4	0.0043	293.9	349.6	546.0	267.5	141.8
IV.A5	0.0036	360.5	426.6	652.1	350.7	165.4
IV.A6	0.0034	399.7	472.4	697.4	400.4	182.4
IV.B1	0.0041	305.2	362.1	569.5	280.6	146.8
IV.B2	0.0039	323.4	382.0	599.2	306.0	152.8
IV.B3	0.0037	359.7	427.2	656.1	349.8	163.8
V.1	0.0038	326.8	378.9	575.1	315.8	161.8
V.2	0.0036	346.2	399.8	591.9	342.8	170.4
V.3	0.0033	385.8	443.1	594.2	404.0	193.0

Table D.44. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under SCM 6.

Method	ARS	$mARL$	$SDRL$	$cARL_{.1}$	$cARL_{.5}$	$cARL_{.9}$
I.A	0.0034	407.9	456.2	166.4	501.0	494.4
I.B1	0.0081	302.8	401.6	53.4	297.6	560.8
I.B2	0.0097	291.0	395.2	43.8	278.8	560.7
I.B3	0.0102	306.7	421.2	41.4	296.2	591.3
I.B4	0.0105	315.8	433.9	39.0	312.6	586.6
II.A1	0.0078	374.4	506.2	53.0	378.4	665.4
II.A2	0.0090	395.0	552.4	44.8	385.4	726.6
II.A3	0.0082	497.1	673.6	51.8	618.7	705.9
II.B1	0.0113	362.2	524.8	34.9	325.2	712.0
II.B2	0.0118	372.9	540.1	33.7	341.3	726.3
II.B3	0.0103	484.6	670.3	40.8	585.9	713.9
III.1	0.0118	281.5	402.6	34.3	245.8	592.4
III.2	0.0168	268.1	421.2	23.1	187.8	669.6
III.3	0.0167	296.8	468.1	23.1	205.2	725.2
III.4	0.0124	277.0	398.1	32.7	239.7	591.6
III.5	0.0164	271.3	425.0	23.3	190.2	675.2
III.6	0.0176	289.0	460.0	21.8	195.2	727.4
IV.A1	0.0133	268.1	389.4	29.5	229.0	573.3
IV.A2	0.0187	241.8	390.7	20.2	157.7	635.5
IV.A3	0.0202	240.8	403.8	18.6	140.4	682.7
IV.A4	0.0134	268.6	391.4	29.3	227.4	584.9
IV.A5	0.0196	236.6	388.3	19.4	146.1	640.6
IV.A6	0.0214	241.2	409.0	17.7	137.9	695.2
IV.B1	0.0150	253.2	379.5	26.1	197.8	581.5
IV.B2	0.0177	241.9	381.2	21.5	168.4	610.9
IV.B3	0.0203	240.5	397.7	18.6	147.5	661.5
V.1	0.0188	207.7	338.8	20.5	131.5	565.4
V.2	0.0220	192.5	328.7	17.2	109.8	558.4
V.3	0.0259	176.7	321.4	15.0	89.4	554.0