EFFECTS OF PRICING AND FLEET STRUCTURE ON THE AIRLINE FLEET ASSIGNMENT PROBLEM

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ABSTRACT

EFFECTS OF PRICING AND FLEET STRUCTURE ON THE AIRLINE FLEET ASSIGNMENT PROBLEM

The fleet assignment problem constitutes a significant part of airline costs, given that flight operating costs are directly related to it and the fleeting decision affects passenger revenues. The basic fleet assignment problem overlooks itinerary demands and revenue management. In this study, a mixed integer planning model for fleet assignment that incorporates itinerary-level demands, the Itinerary-based Fleet Assignment Model is solved, utilizing real data from a Turkish and a European airline. A multi-factorial experimental design is constructed to examine the interactions of itinerary-based fares, demand and fleet structure based on their effect on the fleet assignment decision. The design defines levels for each factor, representing a percent change in the fare and demand parameters, and different fleet compositions for the fleet structure. The model is solved to optimality for three datasets, for each level of the factors fare, demand and fleet structure. The results are analyzed by a statistical software with respect to different response variables and the effect of each factor to fleet assignment is investigated. Results indicate that fleeting contribution and passenger losses are strongly related to itinerary pricing, demand, and fleet structure.

ÖZET

FİYATLANDIRMA VE FİLO YAPISININ HAVAYOLU FİLO ATAMA PROBLEMİNE ETKİSİ

Filo atama problemi uçuş işletme maliyetleri ile doğrudan ilişkili olması ve filo yapısı kararının yolcu gelirlerini etkilemesi göz önüne alındığında, havayolu maliyetlerinin önemli bir bölümünü oluşturmaktadır. Temel filo atama problemi güzergah talepleri ve gelir yönetimini göz ardı etmektedir. Bu çalışmada, bir Türk ve Avrupa havayolunun gerçek verileri kullanılarak, filo atama için bir karışık-tamsayı programlama modeli olan Güzergah Tabanlı Filo Atama Modeli çözülmüştür. Filo atama kararı üzerindeki etkisine dayanarak, güzergah fiyatı, talebi ve filo yapısı etmenlerinin etkileşimini incelemek için bir çok etkenli deney tasarımı inşa edilmiştir. Tasarımda, fiyat ve talep etkenleri için yüzde değişim, filo yapısı için ise farklı filo bileşimleri olmak üzere her etken için düzey belirlenmiştir. Model üç veri seti ile fiyat, talep ve filo yapısı etkenlerinin her düzeyi için eniyiliğe çözülmüştür. Model sonuçları farklı bağımlı değişkenler için deney tasarımına uygun olarak bir istatistiksel yazılım ile analiz edilmiş ve her etkenin etkisi araştırılmıştır. Sonuçlar filo atama kararının ve yolcu kayıplarının güzergah fiyatı, talebi ve havayolu filo yapısı ile yakından ilişkili olduğunu ortaya koymuştur.

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LIST OF SYMBOLS

A	Set of airports or station, indexed by *o*.
b_p^r	Recapture rate from itinerary *p* to itinerary *r*.
$c_{k,i}$	The cost of assigning fleet type *k* to leg *i* as a function of operating, carrying and spill costs
CL(*k*)	Set of flight legs for fleet type k that pass the count time.
D_p	Unconstrained demand of itinerary *p*
$fare_p$	Average fare for itinerary *p*
$f_{k,i}$	Binary variable indicating whether fleet type *k* is assigned to flight leg *i* or not
i	Index for flight legs
I(*k*,*o*,*t*)	Set of inbound flight legs to node {*k*,*o*,*t*}
$i \in I_s$	Index for alternative itinerary
I_s	Choice set of alternative itineraries
k	Index for fleet types
K	Set of different fleet types, indexed by *k*
L	Set of flight legs, indexed by *i*
$morning_i$	Dummy variable, which is 1 if itinerary *i* is a morning itinerary departing between 07:00-11:00, 0 otherwise
N	Set of nodes indexed by {*k*,*o*,*t*}
N_k	The number of aircraft in fleet type *k*
$non\text{-}stop_i$	Dummy variable, which is 1 if itinerary *i* is a non-stop itinerary, 0 otherwise
o	Index for airports
O(*k*,*o*,*t*)	Set of outbound flight legs to node {*k*,*o*,*t*}
P	Set of itineraries in a market, indexed by *p* or *r*
p,*r*	Indices for itineraries
$price_i$	The price of itinerary *i*, normalized by 100
Q_i	Unconstrained demand on leg *i*
$s \in S$	Index of market segments
$Seats_k$	Number of seats available on aircraft of type *k*

$stop_i$	Dummy variable, which is 1 if itinerary *i* is a one-stop itinerary, 0 otherwise
t -, t+	The time preceding and succeeding event time *t* in the time line
T	Sorted set of all event (departure or availability) times at all airports, indexed by t_j
t_p^-	Number of passengers spilled from itinerary *p* and not recaptured on any other itinerary
t_p^r	Number of passengers spilled from itinerary *p* but redirected to itinerary *r*
$time_i$	The duration of itinerary *i* in hours
V_i	Utility of alternative *i*
$\mathbf{x}_i$	Vector of attributes, explanatory variables
$y_{k,o,t,t+}$	The number of aircraft of fleet type *k* on the ground at airport *o* between event times *t* and *t*+
$\boldsymbol{\beta}$	Vector of parameters
δ_i^p	Binary parameter indicating if itinerary *p* includes leg *i*

LIST OF ACRONYMS/ABBREVIATIONS

AOR	Average Occupancy Rate
ASR	Average Spill Rate
FAM	Fleet Assignment Model
FAP	Fleet Assignment Problem
IFAM	Itinerary-based Fleet Assignment Model
OC	Operating Cost
O-D	Origin-Destination
PMM	Passenger Mix Model
QSI	Quality of Service Index
SP	Spill Cost

1. INTRODUCTION

The airline industry becomes an increasingly competitive marketplace with each passing year as new low-fare airlines are born. Low profit margins, rising fuel costs and fluctuating demands cause the planning stage of airlines become extremely complex as well as vital for revenue management. Due to the thin profit margins, airlines use enhanced modeling and optimization techniques to keep fares low and ensure profitability by top utilization of their resources [1]. In this context, airlines are in need of proper scheduling and fleet management for financial viability. Furthermore, the only product of an airline is aircraft seats, which may be categorized as “perishable” goods, i.e., seats that are not purchased until the flight departs are lost. Hence, airlines must also try to balance demand and supply in such a way that neither seats nor passengers are lost [2]. This requires more accurate models and better solution methodologies with narrow error margins [3].

Airlines usually follow sequential stages for planning their processes and operations, in which each process is dependent on the preceding one for input and delivers output to the subsequent operation. There are four fundamental operational planning problems in aviation, which can be seen from Figure 1.1.

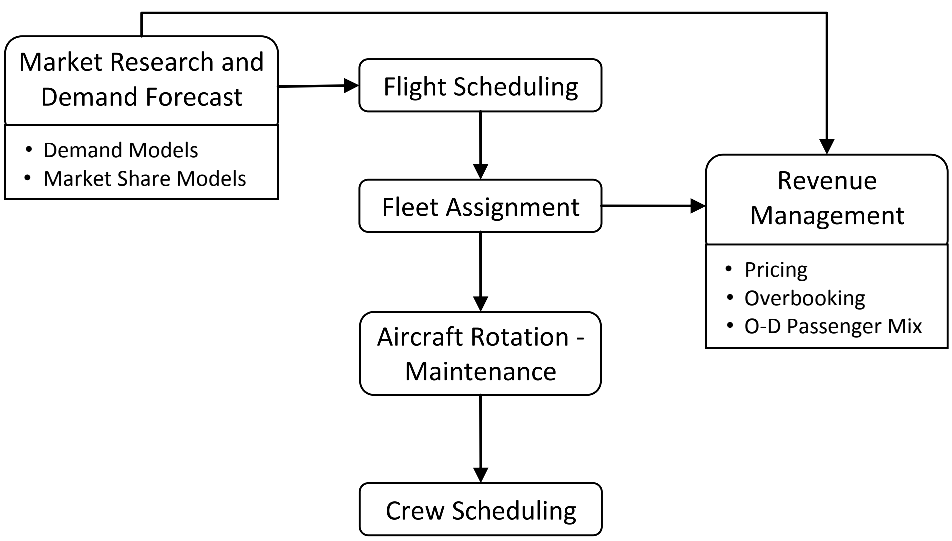

Figure 1.1. Airline operations.

Airlines first solve the *flight scheduling* problem to determine the flight origins, destinations and times in a given period and establish the flight network. This schedule is the basis for the *fleet assignment* problem, in which aircraft types are assigned to scheduled flights, subject to cover considerations and the conservation of flow. Next, the fleet assignments are fed to the aircraft *maintenance routing* problem to assign routes to aircraft that ensure maintenance constraints. The *crew scheduling* problem is both dependent on the fleeting and routing decisions. Additional aspects such as *revenue management* and *demand modeling* are also interdependent with these processes. The overall airline planning process is potentially an intractable problem [2,4].

Market research and demand forecast determine how many passengers want to fly and which itineraries (defined as a flight or sequence of flights) they prefer. *Market share models* are used to predict the percentage of travelers who select each itinerary in an airport pair. Demand models provide input to both flight scheduling and revenue management, and indirectly affect the subsequent airline operations. Given a schedule and estimated demand and fares, revenue management aims to gain maximum revenue through price, demand and capacity control. The *O-D passenger mix* problem is a sub-process of revenue management that allocates seat inventory on each flight among one-flight and connecting itinerary passengers. Thus, the optimum passenger flow that ensures revenue-maximization is obtained.

In this thesis, we study the interactions of itinerary-based fares, demand and fleet mix based on their effect on the fleet assignment decision. The basic fleet assignment problem overlooks itinerary demands and revenue management. Market share models and the passenger flow problem are solved separately from fleet assignment. Consequently, airlines may lose passengers due to capacity restrictions of some flights. Therefore, the relation of demand and fare to fleet assignment is worth investigating. We also concentrate on the effect of fleet structure, which is a strategic decision about the fleet mix and capacity. Using real-life data from two major Turkish and European airlines, the Itinerary-based Fleet Assignment Model (IFAM), a mixed integer programming model for fleet assignment that incorporates itinerary-level demands, is solved for different levels of fare, demand and fleet structure. Our motivation is to determine how fleeting contribution is effected by itinerary-based demands, fares and the fleet structure of an airline. In order to do so, we need to answer the question of “how does the solution of the fleet assignment problem changes when any of the parameters itinerary fare, demand or the fleet structure is varied independently?” Therefore we construct an experimental design where we define different levels for each factor and six response variables to measure the performance of IFAM. Then, we solve the model for each level of the factors and perform statistical analysis on the results to observe the main effects of the factors and their interactions on the response variables.

The rest of this thesis is organized as follows: Chapter 2 contains a literature review about the fleet assignment models, their extensions related to other airline operations, and itinerary-based models. Then, in Chapter 3 we give the formulations of the basic fleet assignment model, and explain the itinerary-based fleet assignment model and itinerary choice model that we employ in our study. An experimental design is proposed in Chapter 4, in order to explore the effects of fare, demand and fleet structure on the fleet assignment decision. Next, we give the computational results and analysis of our experimental setting in Chapter 5.

2. LITERATURE REVIEW

The Fleet Assignment Problem (FAP) has always been a challenging problem to solve and has attracted many researchers both to seek new approaches for solving methods and additional considerations of the problem. Earlier mathematical models proposed at the end of the 20th century mainly considered FAP as a stand-alone problem, ignoring the interdependency of any other airline operation such as maintenance routing, crew scheduling or revenue management. Many traditional models hold several assumptions such as daily flight schedules (as opposed to weekly schedules), approximate revenue and cost estimates, and flight-based static demands. In this thesis, we mainly focus on cost estimation and capturing network effects with itinerary based demands in FAP, while investigating the influence of itinerary pricing and fleet structure. In our review we will draw attention to several optimization-based studies that are relevant to this thesis.

Contrary to the assumptions of earlier basic models, FAP is actually an interconnected problem, as it depends on the scheduling process and contributes to the maintenance routing and crew scheduling problems. Even though an integrated optimization solution for the whole airline system of operations is complex and would probably be intractable, many researchers progressively focused on integration and additional considerations involving several aspects of airline management. Recent developments in computer technology, and computational power and algorithmic advances allowed FAP to become an integrated problem with other airline operations and revenue management, and encouraged researchers to develop superior solution techniques.

2.1. Terminology

Some terminology that is used with airline planning requires explanation. The *fleet type* is a set of aircraft that have the same operating costs and capacity. A *flight leg* is a non-stop trip from an origin airport to a destination airport starting at a specific departure time. A *market* is an ordered origin-destination (O-D) airport pair, in which passengers want to fly. An *itinerary* (*path*) in a particular market is a sequence of one or more flight legs between the specified origin and destination, starting at a specific departure time [5,6]. The overall set of aircraft that an airline has is called the *fleet structure/composition/mix*, which consists of one or more fleet types.

The fleet assignment problem accounts for a major part of airline costs, since flight operating costs are directly related to it and the fleeting decision highly impacts passenger revenues. If an aircraft that has a smaller capacity than demand is assigned to a flight, some customers will be lost (*spill*) because of insufficient capacity. There is a possibility that a portion of these spilled passengers are *recaptured* to alternative itineraries. The assignment of a larger aircraft, on the other hand, will cause most likely higher operating costs and empty seats (*spoil*) with a carrying cost but no revenues. In addition, the demand of a connecting passenger is dependent on the available number of seats on all related flights. This flight leg interdependencies of itinerary demands is called the *network effect* [2].

2.2. Basic Fleet Assignment Models

Typically, fleet assignment models (FAM) formulate the problem as a multi-commodity network flow problem. Two approaches are employed in the literature to construct this problem; *connection networks* and *time-space networks*. Though the structures and the formulations of these two approaches are somewhat different, the main constraints are the same. The *cover constraints* require that a fleet type is assigned to each flight in the network. The *balance constraints* ensure the conservation of flow. The *count (availability) constraints* limit the number of assigned aircraft to the available number per fleet.

In connection networks, nodes correspond to events in time when flights arrive or depart. The network enumerates the possible connections that link the nodes, which are represented by flight connection arcs. Each *flight connection arc* links two events (flights) which can be performed by the same aircraft successively in time [7]. This requires all feasible connection opportunities between pairs of flight legs to be specified, which means higher number of variables and consequently may increase the size of the network [2]. There is also an additional constraint named *schedule balance constraint* that ensures the daily repeating schedule. An advantage of the connection network structure is incorporating additional considerations to the model, such as maintenance and crew planning at a basic level.

Time-space networks, on the other hand, depict the flight legs as arcs in the network, instead of connections. This network actually overlays a collection of networks for each fleet type, in which a time-line is associated with each airport. Nodes representing the arrival or departure event of a flight are sequentially lined up on each timeline. *Flight arcs* represent the flight legs between airports and *ground arcs* represent aircraft staying on the airport between two events. A third type called *wrap-around arc* connects the last node of each airport timeline to its first node, enabling to repeat the schedule every day. This formulation has lower number of decision variables, yet it is not as flexible as the connection network on specific decisions about connections.

One of the first fleet assignment models is introduced by Abara [8] who uses a connection network and limits the number of feasible connections to keep the model tractable. The solution approach of Abara involves relaxing the schedule balance and availability constraints with corresponding penalty terms. Rushmeier and Kontogiorgis [7] also employ a similar network model and formulation, and avoid limiting connections by partitioning events in each station into *connecting complexes*, a subset of incoming and outgoing flights. They also address several resource constraints like maintenance and crew pairing, i.e. penalizing aircraft waiting times on the ground and flying hours of the crews.

Berge and Hopperstad [9] and Hane *et al.* [10] alternatively use the time-space network structure. Berge and Hopperstad consider dynamically re-assigning aircraft of each family to flight legs, and thus adapting to quick market changes. Their demand-driven refleeting model uses demand forecasts that improve as the departure time comes near. The basic FAM of Hane *et al.* [10] is a multi-commodity network model that solves the basic fleet assignment problem on a time-space network. They propose a number of preprocessing methods (i.e. node aggregation, creating islands, eliminating missed connections) to decrease the size of the model and improve the solution efficiency. They apply a dual steepest edge simplex routine and branch-and-bound enhancement on cover constraints, and compare with an interior-point algorithm. Following them, time-space network model has become the preferred choice in many fleet assignment models.

2.3. Extensions on the Fleet Assignment Model

These basic fleet assignment models made noteworthy improvements in monetary benefits and had a significant effect on airline planning research by developing frequently used preprocessing techniques. Since then, more extensive fleeting models have been developed that consider additional components of the airline planning processes. Researchers gradually focused on the integration of airline operations and tried to improve the overall sub-optimality of sequential airline planning, and to accurately represent passenger choices by itinerary-based models.

Focusing on the aircraft maintenance requirements and crews, Clarke *et al.* [11] extends the basic FAM to include several aspects of maintenance routing and crew pairing considerations, i.e. maintenance time windows and the avoidance of *lonely overnights* (when a crew has to stay overnight at a non-base airport because there is no appropriate aircraft type to be flown by them after rest period.) To satisfy the maintenance constraints, Clarke *et al.* add two types of maintenance arcs to the time-space network, which the aircraft that require these two types of maintenance checks are assigned to. They utilize a similar arc called rest arc to limit the number of lonely overnights of crews, which encourages aircraft to remain with the crew. These additions increase the size of the model considerably, thus Clarke *et al.* apply dual steepest-edge simplex to the LP and prioritize variables to improve the efficiency of the branching.

From a different perspective, Barnhart *et al.* [12] propose a string-based model and a branch-and-price algorithm that solves a weekly fleet assignment problem along with the aircraft routing problem. They define an *augmented string* as a sequence of connected flights beginning and ending at maintenance stations, the last flight including the minimum maintenance time required. The main idea is to specify the set of strings and assign fleet types to them instead of flight legs. This way, a solution to the model gives fleet-specific rotations, from which maintenance feasible aircraft routings can be derived. An interesting aspect of their research is the concept of *through revenues*, which might be considered as a step towards an itinerary-based model. Through means a sequence of flights flown by the same aircraft, where similar to an itinerary there is demand from the origin of the first flight to the destination of the last one. However, this concept does not play a role in the fleet assignment except as a better estimation of revenue. As expected, the number of strings is high; therefore Barnhart *et al.* apply a column generation algorithm, in which the subproblem is cast as a constrained shortest path problem on a connection network. They prefer the connection network because they assert that through revenues and maintenance costs cannot be captured in a time-space network.

Other researchers such as Desaulniers *et al.* [13] and Rexing *et al.* [14] focused on integrating scheduling to fleeting by varying the departure times of given flights within certain time windows. Desaulniers *et al.* [13] develop two different models; one defines a bi- nary variable for each possible schedule, resulting in a large Set Partitioning type problem, and the other is a time constrained multi-commodity network flow formulation where a binary variable represents a possible connection between two flights. They propose a column generation technique for the first model, and a Dantzig-Wolfe decomposition approach for the second via time constrained shortest path subproblems. Rexing *et al.* [14] propose a generalized fleet assignment model where a discretized time-window is associated to each flight in order to optimize flight departure times. Each possible departure time is represented by a copy of the flight arc; however this increases the number of variables and as a result the problem size grows remarkably. They present two solution approaches; a direct solution procedure where all the flight arcs are included, and an iterative decomposition technique where extra flight arc copies are included at each iteration if they help improve the current solution. They observe significant solution improvements by incorporating schedule flexibility to FAM.

Following these attempts to integrate one of the airline planning problems into FAP, Sandhu and Klabjan [15] propose an integrated fleeting, aircraft routing (partial integration) and crew scheduling model and combine these three stages of the airline planning process to a single solution framework. They present two solution procedures involving Lagrangean relaxation along with delayed column generation, and Benders decomposition. The results indicate higher profits and they state the Lagrangean relaxation approach to be more robust and practical, though it requires more computational time. Similarly, Papadakos [16] integrated fleet assignment, maintenance routing and crew scheduling problems and applied an enhanced Benders decomposition method combined with accelerated column generation. There are more papers in this area worth mentioning, such as Sandhu and Klabjan [17], Gao *et al.* [18], and Ruther [19], that deals with the integration of routing and crew pairing problems. These works fall beyond the scope of our study, since we focus on the revenue management aspect in fleet assignment.

2.4. Itinerary-Based Models

Another research interest was capturing the network effects in the fleet assignment problem. The assumptions in the objective function of traditional FAM (especially for spill costs) were investigated further to incorporate revenue management aspects into FAM. The cost term used in traditional models was based on the related fleet type capacity and flight leg demand forecasts; and the spill costs were only roughly estimated [2]. This is an approximation of the reality, as passenger demand occurs as O-D pairs, not by flight legs. Moreover, spill and recapture costs were estimated while ignoring the capacity of other flight legs besides the one considered, or the interdependencies of legs [20]. In our study, we also rely on these observations to incorporate network effects to FAM.

In the light of these remarks, Farkas [21] underlines the impact of revenue management on FAM and uses a Monte Carlo revenue management simulation approach that decides on the passenger traffic from different itineraries for each flight leg, after the capacity is assigned. This approach seems to be impractical for real world problems of considerable size, as it requires solving the basic FAM repeatedly to determine the capacity. Nonetheless, Farkas shows that incorporating network and yield management effects into the fleet decision process can bring significantly higher profits.

Realizing these contributions, Kniker [6] integrates the passenger mix model (PMM) introduced by Glover *et al.* [22] and FAM. Passenger mix model determines the number of passengers on each flight leg after spill and recapture is taken into account. Kniker uses key-path variables introduced by [23], representing the desired itinerary for a passenger to reduce the problem, and considers partial recapture. The solution is obtained by using column generation, with each column representing passengers spilled from one itinerary and recaptured on another.

The new integrated itinerary-based fleet assignment model (IFAM) is investigated further by Barnhart *et al.* [5] to improve the traditional leg-based FAM by explicitly including the network and recapture effects to the fleeting model. IFAM integrates the aforementioned PMM with FAM, utilizing the spill variables (key-path variables) and together with the two demand constraints of PMM, to model passenger flow simultaneously with fleet assignment. An explicit explanation of this model will be given later, since we utilize this formulation in our study as well. They define recapture rates for the spill variables by Quality of Service Index (QSI), whereas in our model we make use of a logit itinerary choice model from literature. Insertion of these additional variables and constraints has a significant effect on the problem size and complexity; therefore Barnhart *et al.* propose an intricate heuristic solution algorithm that involves column and row generation and a branch-and-price-and-cut algorithm. Such a solution approach is necessary in order to solve large datasets with up to 2044 flights and 76641 itineraries, whereas not required in our study as the size of our data is manageable. Their computational results based on these datasets indicate significant savings when both network and recapture effects are considered in fleet assignment.

Adopting another approach, Jacobs *et al.* introduce the concept of O&D FAM in [24], and in a follow-up paper [25], which uses revenue management mechanisms that are incorporated into fleeting decisions. They use Benders decomposition to solve an O&D Revenue Management subproblem and the master problem FAM iteratively. Even though their case study is small-sized, the solution approach is efficient and fleet assignments from O&D FAM show significant improvement in expected profit over traditional FAM. From a different perspective, Yan and Tseng [26] incorporate passenger demands with their flight scheduling and fleeting model. They use two types of networks; a time-space network for fleet flow and a time-space network for each O-D pair that represents passenger flow. The solution algorithm uses Lagrangian relaxation along with a sub-gradient method to solve the proposed integer multiple commodity network flow problem. The result of their case study indicates the algorithm performs well, converging within an error gap of 3%.

There were also extensions to the model of Barnhart *et al.* [5], mostly by utilizing itinerary-based demands in integrated airline planning. Lohatepanont and Barnhart [27] also consider path demands when they include leg selection decisions into FAM and thus integrate scheduling and fleeting processes via a different approach. Sherali, Bae, and Haouari [28] integrate scheduling to fleet assignment by allowing optional flight legs, itinerary-based demands and multiple fare classes and applied a polyhedral analysis procedure along with Benders decomposition. In a later publication, Sherali *et al.* [1] investigate integrated airline planning further by considering additional aspects such as flight retiming, schedule balance and demand recapture, along with itinerary-based demands.

2.5. Market Share Models

Market share models are used to model the itinerary choice of passengers who are traveling in an O-D market. These models estimate the probability of a passenger selecting an itinerary among the set of itineraries between an airport pair. In our study, an itinerary choice model is used to calculate the recapture rates in IFAM, and to determine itinerary demands when itinerary fares are changed in our experimental design.

There are different types of models proposed in literature which can be classified into two main groups; models based on the Quality of Service Index (QSI) and models employing a discrete choice (logit) approach. QSI models define the *quality* of an itinerary based on several service characteristics of the itinerary, and compute the corresponding preference weights of these attributes by statistical techniques and intuition. Then the QSI of an itinerary is expressed as a function of its service characteristics and preference weights. There are some drawbacks of the QSI models. These models omit the interactions between service attributes and the dynamic competitive nature of itineraries in a market [29].

Discrete choice models, on the other hand, simultaneously estimate parameters and capture trade-offs among alternatives, and thus have improved forecast accuracy. Discrete choice models take into account the attributes of the alternatives that were not chosen as well as the attributes of the chosen itinerary. Moreover, discrete choice models are useful if the availability or the characteristics of alternatives tend to change, for example in predicting recapture rates or modeling fare-demand interactions [30]. Nonetheless, discrete choice modeling proves to be a difficult problem since it involves analyzing large volume of data. In this study, a logit itinerary choice model proposed by Atasoy and Bierlaire [31] is modified to estimate recapture rates in IFAM and model demand in our experimental design. Atasoy and Bierlaire make use of two datasets to model passenger choices and propose a price elastic demand model.

In light of these studies, it is clear that the traditional leg-based fleet assignment model is no longer sufficient to capture the revenue management aspects and network effects. Therefore our study utilizes itinerary-based demands to incorporate network effects to FAM. This research investigates the effects of itinerary fare-demand interactions on fleeting decisions, by means of an experimental design. Even though there were studies involving stochastic demand, these mostly involved the scheduling stage, as in [32] and [33]. Hence our study will offer an insight into the interactions of pricing, demand and fleet assignment. Moreover, we also consider the impact of the fleet structure by solving our model for different fleet structures with different fleet types.

3. PROBLEM DEFINITION

In this chapter, the mixed-integer programming formulations of the basic fleet assignment model and the itinerary-based fleet assignment model are presented. We explain the assumptions of the basic model and the additional parameters and decision variables of the itinerary-based model. Lastly, an itinerary share model is described, which is used to accurately represent the spill and recapture, and for building the fare-demand interactions in our experimental design in Chapter 4.

3.1. Basic Fleet Assignment Model

The basic fleet assignment model maximizes profitability, or equivalently, minimizes the total of fleeting and spill costs subject to three primary constraints; cover, balance and plane count. Given a flight schedule and a set of aircraft, it aims to assign different aircraft types to the scheduled flights. As explained by Hane *et al.* [10], a time-space network underlies this formulation.

The time-space network is essentially a set of networks, one for each fleet type. Each event of a flight's arrival or departure is represented by a *node* at the corresponding station. As seen in Figure 3.1, three types of arcs are present in this network: flight arcs, ground arcs and wrap-around arcs. A flight arc, representing a flight leg, originates from the node representing the departure airport and time of the flight and ends at the node that represents the arrival airport and the arrival time plus *turn time* of the flight. Turn time is the minimum time necessary for an aircraft to be prepared for another flight. A ground arc connects two consecutive nodes in an airport's timeline and represents the number of aircraft on the ground at that airport during the time spanned between those two events. A wrap-around arc is a variant of a ground arc, which connects the last event of the day to the first event, ensuring that the schedule repeats every day. To count the number of aircraft in the network, a point in time called the *count time* is selected. All flight arcs and the wrap-around arcs that span the count time are said to cross the count time.

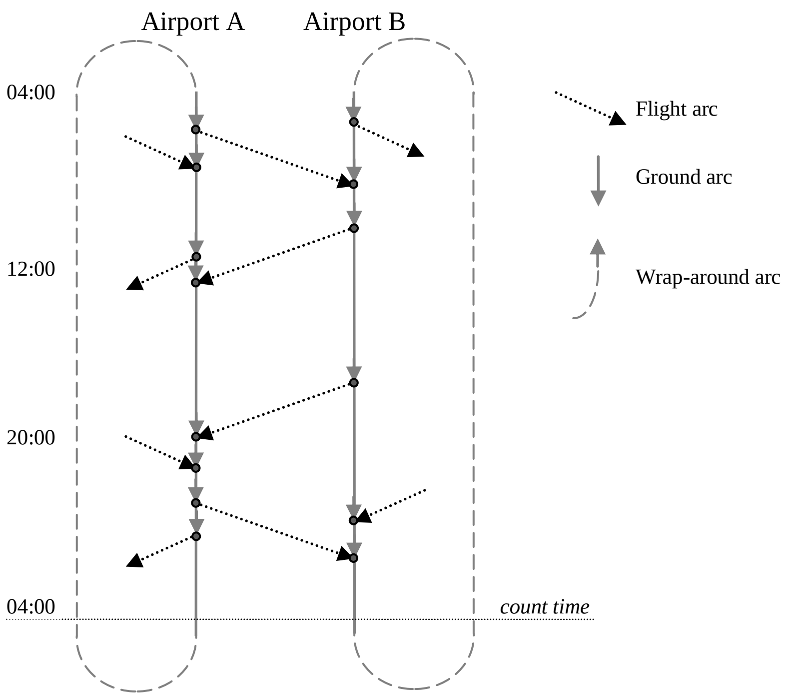

Figure 3.1. Network representation.

In connection with the time-space network discussed above, the following notation is used in the formulation of the basic FAM, which is similar to the ones used in Barnhart *et al.* [5] and Sherali *et al.* [2].

Table 3.1. Indices and sets in basic FAM.

Sets	**Definition**
A	Set of airports, indexed by *o*.
L	Set of flight legs, indexed by *i*.
K	Set of different fleet types, indexed by *k*.
T	Sorted set of all event times (departure or arrival) at all airports, indexed by t_j. Supposing $\|T\|=m$, t_1 is the time of the first event and t_m is the time of the last event before the count time.
N	Set of nodes indexed by $\{k,o,t\}$.
CL(k)	Set of flight legs for fleet type *k* that cross the count time.
I(k,o,t)	Set of inbound flight legs to node $\{k,o,t\}$.
O(k,o,t)	Set of outbound flight legs to node $\{k,o,t\}$.
t^-, t^+	The time preceding and succeeding event time *t* in the time line.

Table 3.2. Variables in basic FAM.

Variables	Definition
$f_{k,i}$	Binary variable indicating whether fleet type *k* is assigned to flight leg *i* or not.
$y_{k,o,t,t+}$	The number of aircraft of fleet type *k* on the ground at airport *o* between event time *t* and t^+.

Table 3.3. Parameters in basic FAM.

Parameters	Definition
$c_{k,i}$	The cost of assigning fleet type *k* to leg *i* as a function of operating, carrying and spill costs.
N_k	The number of aircraft in fleet type *k*.

The count time is used as the beginning of the timeline where the series of events are represented. The mathematical formulation of basic FAM is given as follows.

$$\text{Min} \quad \sum_{i\in L}\sum_{k\in K} c_{k,i}\, f_{k,i} \tag{3.1}$$

subject to

$$\sum_{k\in K} f_{k,i} = 1, \qquad \forall i \in L, \tag{3.2}$$

$$y_{k,o,t^-,t} + \sum_{i\in I(k,o,t)} f_{k,i} - y_{k,o,t,t^+} - \sum_{i\in O(k,o,t)} f_{k,i} = 0, \qquad \forall k,o,t, \tag{3.3}$$

$$\sum_{o\in A} y_{k,o,t_m,t_1} + \sum_{i\in CL(k)} f_{k,i} \le N_k, \qquad \forall k \in K, \tag{3.4}$$

$$f_{k,i} \in \{0,1\}, \qquad \forall k \in K, \forall i \in L, \tag{3.5}$$

$$y_{k,o,t,t^+} \ge 0, \qquad \forall k,o,t. \tag{3.6}$$

The cover constraints (3.2) force a fleet type to be assigned to each leg in the network. The balance constraints (3.3) are flow conservation equations, requiring that the number of arrivals must be equal to the number of departures at each node, for each fleet type, station and event time. The plane count constraints (3.4) ensure that the assigned number of planes for each fleet type does not exceed the total number aircraft available, by counting the number of aircraft on the ground and in flight at count time. The fleet assignment variable f (3.5) has value 1 if corresponding fleet type is assigned to the flight leg, 0 otherwise. The variable y (3.6) represents the ground arcs, and they are defined as continuous because when the flight variables are integral, the flow on the ground arcs are forced to be integral as well. The representation of variables can be seen in Figure 3.2.

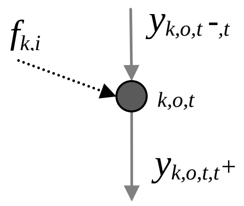

Figure 3.2. Node detail in time-space network.

The objective function (3.1) of the basic FAM includes a cost term which represents the cost of assigning fleet type k to flight leg i. This term includes the cost of spill and recapture to a basic level. The cost for spilled passengers are computed for each fleet-flight pair, using the projected demand, recapture rate of a flight, fare structure and available seats [10]. However, spill costs cannot be captured accurately in this way, as fares and demands are known by itineraries in an O-D market. Itinerary fares must be allocated to flight legs to compute leg-based spill costs. There are several approaches for this revenue allocation; however none of them are totally accurate in calculating spill costs.

Moreover, flight demand may consist of multiple-leg passengers, and thus the network interactions of different markets competing over seats on shared flight legs must be considered. The demand of multiple-leg passengers is dependent on the available number of seats of all the flights involved. This is called the *network effect*. The basic FAM also estimates recaptured revenue independent of the capacity of other flights, therefore ignoring the network effects.

3.2. Itinerary-Based Fleet Assignment Model

To incorporate network effects and passenger spill decisions into FAM, Barnhart *et al.* [5] formulated the itinerary-based fleet assignment model, where the aforementioned passenger mix model is integrated with FAM. Traditionally the PMM is solved after the FAM, in order to decide which passengers are flying on each leg, given the capacity of the flights and itinerary demands. The key idea of PMM is that passengers of different itineraries flying on the same flight are not identical in terms of revenue and resources. For example, multiple-leg passengers occupy seats in more than one flight leg and thus use more resources, but the revenue obtained from them may be lesser than one-leg passengers.

The basic fleet assignment model is extended to include passenger spill decisions via new variables and parameters, and the integrated FAM (IFAM) formulation is proposed. Additional notation is given in Table 3.4.

Table 3.4. Additional notation in IFAM.

Sets	**Definition**
P	Set of itineraries in a market, indexed by *p* or *r*.
Variables	**Definition**
t_p^r	Number of passengers spilled from itinerary *p* but are redirected to itinerary *r*.
t_p^-	Number of passengers spilled from itinerary *p* and not recaptured on any other itinerary.
Parameters	**Definition**
$Seats_k$	Number of seats available on aircraft of type *k*.
D_p	Unconstrained demand of itinerary *p*.
$fare_p$	Average fare for itinerary *p*.
δ_i^p	Binary parameter indicating if itinerary *p* includes leg *i*.
b_p^r	Recapture rate from itinerary *p* to itinerary *r*.
Q_i	Unconstrained demand on leg *i*.

Using the notation in Table 3.4 in addition to the FAM notation given in Tables 3.1, 3.2 and 3.3, the itinerary-based fleet assignment model is presented as a mixed-integer programming model.

$$\text{Min} \sum_{i\in L}\sum_{k\in K} c_{k,i} f_{k,i} + \sum_{p\in P}\sum_{r\in P: r\neq p} (fare_p - b_p^r fare_r) t_p^r \tag{3.7}$$

subject to

$$\sum_{k\in K} f_{k,i} = 1, \qquad \forall i \in L, \tag{3.8}$$

$$y_{k,o,t^-,t} + \sum_{i\in I(k,o,t)} f_{k,i} - y_{k,o,t,t^+} - \sum_{i\in O(k,o,t)} f_{k,i} = 0, \qquad \forall k,o,t, \tag{3.9}$$

$$\sum_{o\in A} y_{k,o,t_m,t_1} + \sum_{i\in CL(k)} f_{k,i} \le N_k, \qquad \forall k \in K, \tag{3.10}$$

$$\sum_{k\in K} Seats_k f_{k,i} + \sum_{p\in P}\sum_{r\in P: r\neq p} \delta_i^p t_p^r - \sum_{p\in P}\sum_{r\in P: r\neq p} \delta_i^p b_r^p t_r^p \ge Q_i, \qquad \forall i \in L, \tag{3.11}$$

$$\sum_{r\in P} t_p^r \le D_p, \qquad \forall p \in P, \tag{3.12}$$

$$f_{k,i} \in \{0,1\}, \qquad \forall k \in K, \forall i \in L, \tag{3.13}$$

$$y_{k,o,t} \ge 0, \qquad \forall k,o,t, \tag{3.14}$$

$$t_p^r \ge 0, \qquad \forall p,r \in P. \tag{3.15}$$

The objective function (3.7) minimizes the total operating cost and the total spill cost less recapture revenue. The total lost revenue because of spill and the regained revenue from recaptured passengers are formulated as in Equation 3.16. If the term $fare_p - b_p^r fare_r$ is positive, this means the revenue obtained from the desired itinerary *p* is higher. In this case, passengers are spilled and recaptured to some extent on itinerary *r* if there are capacity limitations only. On the other hand, if $fare_p - b_p^r fare_r$ is negative, it means that the revenue gained from the redirected itinerary *r* is higher, and thus, redirecting the passenger is preferred. However, this is usually not the case, as the recapture rate must be considerably high, along with the $fare_r$ value.

$$\sum_{p\in P}\sum_{r\in P: r\neq p} (fare_p - b_p^r fare_r) t_p^r \tag{3.16}$$

The unconstrained demand of an itinerary is the forecasted demand by the airline regardless of capacity. The unconstrained demand on flight leg i can be expressed as

$$Q_i = \sum_{p \in P} \delta_i^p D_p. \tag{3.17}$$

Constraints (3.8) to (3.10) are the original FAM constraints. Leg capacity constraints (3.11) ensure that the capacity assigned to a leg does not exceed the total demand of this leg minus the net spillage of this leg, which consists of both spilled and recaptured passengers. The number of passengers that requested but are spilled from the itineraries that contain leg i is formulated as in Equation 3.18. The number of passengers recaptured from other itineraries to the ones with the leg i is formulated in Equation 3.19. The difference between those two terms gives the net number of passengers lost in that leg. In other words, the number of available seats (with spill and recapture considered) in a flight must be greater than or equal to the aggregate demand of that flight leg. The leg capacity constraints form the connection between the PMM and FAM.

$$\sum_{p \in P} \sum_{r \in P: r \neq p} \delta_i^p t_p^r \tag{3.18}$$

$$\sum_{p \in P} \sum_{r \in P: r \neq p} \delta_i^p b_r^p t_r^p \tag{3.19}$$

Lastly, path demand constraints (3.12) require that spill from an itinerary cannot exceed the demand of that itinerary.

The new variables t_p^r (3.15) represent the passengers that requested an itinerary p but are redirected to another itinerary r and the variables t_p^- represent the passengers of itinerary p that are lost to the airline. The null spill variable (t_p^-) is just a special case where there is no recapture. In this case, there are no alternatives to itinerary p, so all spilled passengers are lost to the airline. The model formulates recapture by using these spill variables and the recapture rates, b_p^r. The recapture rate is an estimation of the fraction of passengers that wish to travel on itinerary p, but are willing to accept itinerary r when offered. Hence, the number of passengers traveling on itinerary r that originally desired itinerary p is given as $t_p^r b_p^r$. This term can be seen in leg capacity constraints (3.11) and the objective function. Estimating the recapture rates is in itself a difficult problem, where the itinerary choices of passengers must be taken into account.

3.3. Modeling Passenger Itinerary Choices

A discrete choice model is a random utility maximizing model that explains how customers choose an alternative from a finite set of mutually exclusive and collectively exhaustive alternatives. A customer chooses the alternative with the maximum utility, whose function is defined as:

$$V_i = \boldsymbol{\beta}\mathbf{x}_i. \tag{3.20}$$

where $\boldsymbol{\beta}$ is the vector of parameters associated with the vector of attributes $\mathbf{x}_i$, called the explanatory variables. The market segments for each O-D pair forms the set S and is indexed by s, where the choice set of itineraries is represented by I_s. The utility of each alternative itinerary $i \in I_s$ in the market segment s is represented by V_i. The explanatory variables are explained in Table 3.5. There are additional variables included in [31] but not in our model, as our data does not provide information about these variables.

Table 3.5. Explanatory variables of the Itinerary Choice Model.

Variables	**Definition**
$price_i$	The price of itinerary *i*, normalized by 100
$time_i$	The duration of itinerary *i* in hours
$non\text{-}stop_i$	Dummy variable, which is 1 if itinerary *i* is a non-stop itinerary, 0 otherwise
$stop_i$	Dummy variable, which is 1 if itinerary *i* is a one-stop itinerary, 0 otherwise
$morning_i$	Dummy variable, which is 1 if itinerary *i* is a morning itinerary departing between 07:00-11:00, 0 otherwise

In the model, the *time* and *price* variables are interacted with the number of stops (the dummy variables *non-stop* and *stop*) as there are strong correlations among the number of stops and the time and price of the itinerary. Moreover, it should be noted that the effect of the increase in price is not linear for different levels of the price. Hence the *price* variable is formulated as a log function. The model defines the choice probability for each itinerary *i* in market segment *s* as

$$P^s(i) = \frac{\exp(V_i)}{\sum_{j \in I_s} \exp(V_j)} \qquad s \in S, i \in I_s. \tag{3.21}$$

There are two alternatives in the choice set of the model; the first alternative is a non-stop itinerary and the second one is a one-stop itinerary. The utility functions of the two alternatives are given below. The β parameters are indexed by *NS* and *S*, representing non-stop and stop respectively.

$$V_1 = \beta^{NS}_{price} \times \ln(price_1 / 100) + \beta^{NS}_{time} \times time_1 + \beta_{morning} \times morning_1 \tag{3.22}$$

$$V_2 = \beta^{S}_{price} \times \ln(price_2 / 100) + \beta^{S}_{time} \times time_2 + \beta_{morning} \times morning_2 \tag{3.23}$$

Utilizing a combination of two datasets, Atasoy and Bierlaire [31] estimate parameters by maximum likelihood estimation and a price elastic demand model is obtained. Using these results, it is possible to calculate the recapture rate b_p^r by means of Equation 3.21. If the desired itinerary becomes unavailable to a passenger, they choose an alternative based on this probability.

Furthermore, it is possible to estimate the demand if the itinerary fares were to change, as we experiment to observe the effects of fares in IFAM. Keeping the total market demand constant, we can observe how the aggregate market demand is distributed among its itineraries when their fares are altered. To accomplish this, we re-calculate the utility of each itinerary based on the new modified fare values. Then, based on the new calculated utilities, we determine the choice probability of each itinerary, as in Equation 3.21. This probability also gives the market share of an itinerary, thus we are able to allocate the total market demand among the itineraries.

With this approach we analyze a monopolistic situation ignoring any competitive effect and we assume that the market size is fixed (i.e., everyone in the market will fly on their itinerary unless spilled by the airline). Although this is a strong assumption, it is necessary to isolate the effect of fare on revenue, from the demand and the fleet structure.

4. EXPERIMENTAL DESIGN

The fleet assignment problem with itinerary-based demands is dependent on many factors, such as the passenger demands, itinerary fares and fleet structure. In this section an experimental design is constructed in order to study the effects of these factors on fleeting decisions. The itinerary-based fleet assignment model is applied to three real data sets, two from a major Turkish airline and one from a European airline. The characteristics of the three data instances and the model parameters are provided in Section 4.1. The experimental setting is explained in Section 4.2, with which the effects of certain factors are measured.

4.1. Data Instances

The data sets used in our study each consists of a daily flight schedule and information about itineraries. The characteristics of the three data sets are provided in Table 4.1. The data set code identifies each instance; the following columns refer to the number of flights, itineraries, market segments, and airports respectively.

Table 4.1. Characteristics of the data sets.

Data Set	Flight Legs	Itineraries	Markets	Airports
P01	228	396	241	44
P02	228	865	511	44
A01	464	1059	301	35

The data obtained from a Turkish airline is labeled as P01, and it is an international hub-and spoke network that covers Turkey, Europe and a portion of Near East. The hub airport is the İstanbul Sabiha Gökçen Airport, as seen in Figure 4.1. This data set includes a flight schedule, the set of airports, and the itinerary information including the corresponding flight legs and average fares. This data does not include any information concerning the demand; therefore we assign the aggregate demand of an O-D market (D_s)

randomly and allocate the demand of its itineraries utilizing the itinerary share model. The choice probability of an itinerary defined in Equation 3.21, $P^s(i)$, is also the O-D market share ratio of that itinerary. Thus, the demand of itinerary i in market segment s is calculated as

$$D(i) = P^s(i) \times D_s \qquad s \in S, i \in I_s. \tag{4.1}$$

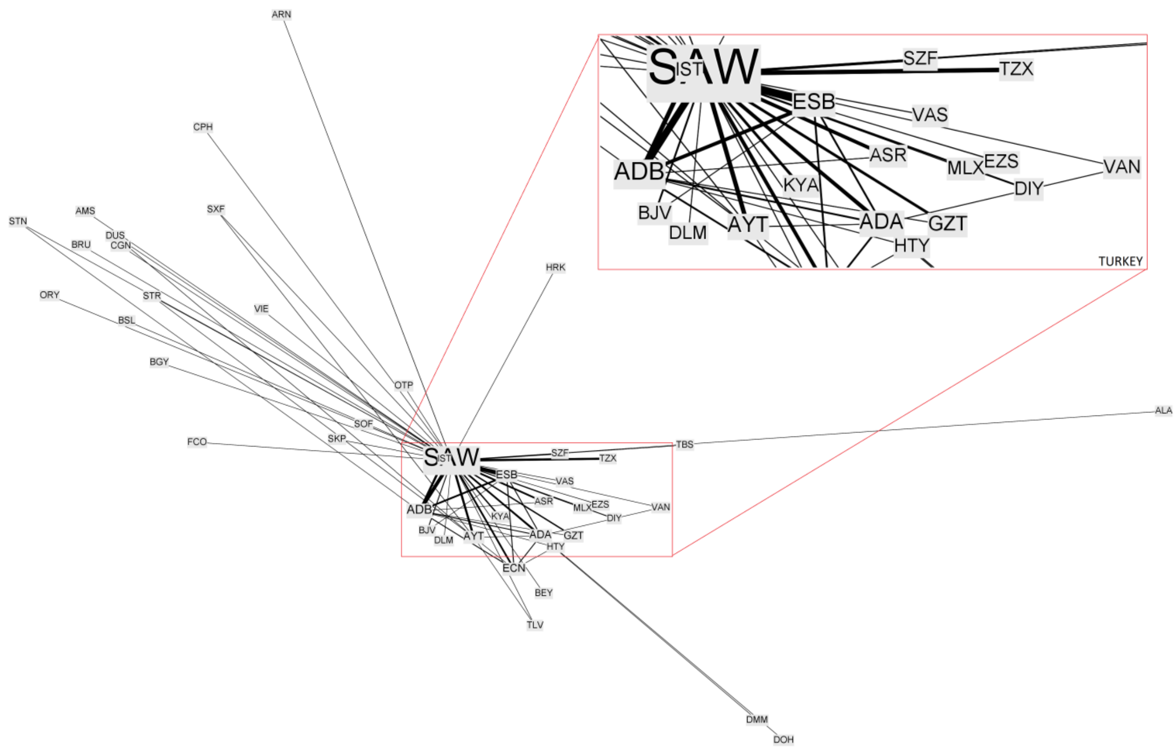

Figure 4.1. Flight network of data sets P01 and P02.

The data labeled P02 is based on the flight network of P01 and has the same set of airports. We use this information to enlarge our data set and generate a set of random itineraries in addition to the original set of itineraries of P01. For each flight, we find a consecutive flight leg that originates from the arrival airport of the given flight within a time period after its arrival time. The new itinerary set is generated by connecting these two flights. The randomly generated itineraries are one-stop itineraries whose layover time varies between 0:30 and 4:30 hours. Thus, the number of itineraries and the number of markets are considerably higher in P02.

The data from a European airline is labeled A01 and it is provided in the context of ROADEF Challenge 2009 [34]. This booking data provides the flight schedule, and the set of airports, aircraft and itineraries. It also includes the average fare and actual demand of the itineraries. A01 has a medium sized hub-and spoke flight network that covers France and a few neighboring airports (See Figure 4.2). The itineraries are either non-stop or one-stop, and some of them include return trips as well.

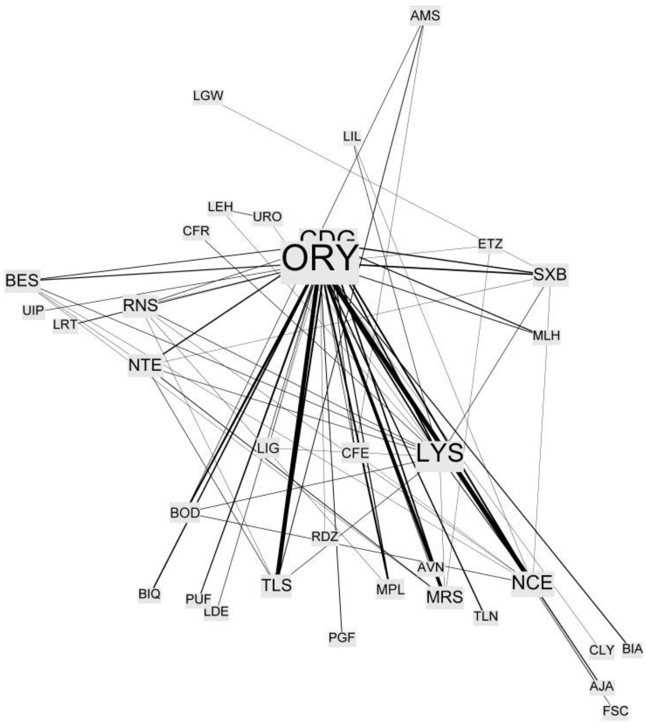

Figure 4.2. Flight network of data set A01.

The time-space network of IFAM is constructed with respect to the three data sets.

4.1.1. Recapture Rates

Passengers come to the airline requesting a specific itinerary. However, some of them cannot be accommodated on their desired itinerary due to capacity restrictions. Assumed that the airline has control over the flow of passengers, when capacity is exceeded the airline offers the passengers two options, an alternative itinerary if it exists or the null itinerary. Then, passengers either decide to fly on the alternative itinerary or try another airline, based on the recapture rate. The revenue of passengers that are offered the null itinerary and the ones that prefer another airline are lost to the airline. Modeling the spill and recapture effect enables the spill costs to be accurately calculated. Consequently, airlines gain insight about fleeting decisions in long term planning, since spill is closely related to aircraft capacities and thus to the fleeting decision.

Estimating the recapture rates is a challenging problem as it requires an understanding of a passenger's itinerary decision process. Thus, we utilize a logit itinerary share model for calculating the probability of passenger to accept an alternative itinerary among a set of alternatives, including the competitive airlines. We define the market share of the airline as h_s for each market segment $s \in S$ to define the probability of recapturing passengers of the airline among the competitive airlines in that market s. The set of alternatives the airline offers to the passenger include all of the itineraries of the airline in market segment s, denoted as I_s, but excluding the desired itinerary. Using the utility functions defined in Equations 3.22 and 3.23, the probability of passengers from itinerary p to accept traveling on itinerary r is:

$$\frac{\exp(V_r)}{\sum_{m \in I_s - \{p\}} \exp(V_m)} \qquad s \in S,\ p, r \in I_s. \tag{4.2}$$

The number of passengers that accept an alternative itinerary is found by multiplying the ratio given in Equation 4.2 with the number of spilled passengers. However, we know that the airline may only capture a fraction of these passengers, based on its market share. The other passengers are assumed to prefer competitors and are lost to the airline. Therefore we give the recapture rate of passenger spilled from itinerary p and are recaptured by the airline on itinerary r as

$$b_p^r = h_s \frac{\exp(V_r)}{\sum_{m \in I_s - \{p\}} \exp(V_m)} \qquad s \in S,\ p, r \in I_s. \tag{4.3}$$

Thus, the recapture rates are higher if the airline is dominant or monopolistic in a market. The market share parameter is assumed constant among all markets in our instances, as we lack sufficient data regarding the competitors.

4.2. Experimental Structure

The fleeting decision is dependent on several factors. The fare and demand parameters in IFAM are closely related to spill costs and thus to the fleeting decision. Moreover, there is interaction between fare and demand of an itinerary. It has been shown that itinerary fares have a significant effect on choices of customers and thus on itinerary demands [29]. In IFAM, fare values are assumed to have deterministic values, since predicting the change in fares at the planning stage is a difficult problem that involves many components. However due to the competitive nature of the airline market, airlines are bound to adjust fare values.

The fleeting decision relies on the flight network as well as on the composition of the fleet, which imposes a bound on the capacity of a flight leg. The fleet structure, i.e. the set of aircraft types that are utilized by the airline, also has a significant effect on the subsequent maintenance routing and crew scheduling problems, as each type has different maintenance requirements and crew. Thus, different types of fleet structures may be considered in fleet assignment problem in order to simplify the following maintenance and crew planning problems.

4.2.1. Factors and Levels

To observe the effects of these factors, we design an experiment that offers different levels of fare, demand and fleet composition. A factorial design is necessary to model the possible interactions between factors, in addition to the main effects of each factor. We employ the general multi-factorial design that enables each factor to have different number of levels. For the fare and demand variables, we define a percentage change for each level. For example, a 10% decrease in itinerary fares is represented by the change factor of 0.9.

<u>4.2.1.1. Fare.</u> The fare factor has four levels, each corresponding to a change factor. The levels constitute of 0.9, 1.0, 1.1, and 1.25, corresponding to a 10% decrease, no change, 10% increase, and 25% increase respectively. The value 1.0 for the change factor implies that the original fare data is used when IFAM is solved.

For each level of the fare factor except 1.0, we calculate the demands of itineraries in each market via the itinerary choice model presented in Chapter 3. Keeping the total market demand constant, we allocate the passengers to the itineraries based on their choice probability, as seen in Equation 4.1. Thus, we are able to capture the behavior of passengers when itinerary fares change, assuming that the total market demand is not affected.

4.2.1.2. Demand. The demand factor has three levels that correspond to the value of each change factor. The change factor is varied from 0.5 to 1.5 at a 0.5 step, equivalent to a 50% decrease or increase in itinerary demand values. Similar to the fare factor, the value 1.0 implies that the original demand data is used when IFAM is solved.

4.2.1.3. Fleeting. The fleet structure variable has five levels, each having a different fleet composition. The levels may be divided into three groups: one-fleet, similar fleeting and dissimilar fleeting. The level *Fleet1* consists of only one fleet type that has 123 seats. Similar fleeting means that there are multiple types of fleets having similar capacities. The levels *Sim3* and *Sim9* fall under this category, consisting of three and nine types of aircraft respectively. Dissimilar fleeting means the fleet composition includes multiple types of fleets with different capacities, i.e. the capacity of aircraft varies in a wide range. The levels *Dis3* and *Dis9* are included in this category, consisting of three and nine types of aircraft respectively. The details of each fleet structure level are given in Table 4.2. The number of aircraft available is given in parenthesis next to the capacity of the fleet type.

Table 4.2. Fleet structure for each factor level.

Fleet Structure Level	**Number of Fleet Types**	**Fleet Capacity Per Fleet Type (Number of Aircraft)**
Fleet1	1	123 (90)
Sim3	3	100 (30), 120 (30), 123 (30)
Sim9	3	90 (10), 95 (10), 100 (10), 110 (10), 120 (10), 123 (10), 132 (10), 135 (10), 140 (10)
Dis3	9	50 (30), 123 (30), 210 (30)
Dis9	9	30 (10), 50 (10), 72 (10), 90 (10), 100 (10), 123 (10), 142 (10), 170 (10), 210 (10)

We solve IFAM for each combination of the different level of the three factors. There are 60 runs for each data set, corresponding to the combinations of the factors. Then we perform analysis of variance (ANOVA) to analyze our multi-factorial design based on the response variables.

4.2.2. Response Variables

The fleeting decision can be assessed with respect to different concepts. There are different ways to analyze the performance of IFAM that is influenced by the factors described above. For this purpose, we define the following response variables:

- Profit: The difference between the unconstrained revenue and the total cost.
- Operating Cost (OC): The cost of assigning fleets to flights.
- Spill Cost (SP): The lost revenue of the passengers because of insufficient capacity, where gained recapture revenue is also taken into account.
- Null Spill: The total number of passengers that are directed to a null itinerary, i.e. lost to the airline.
- Average Occupancy Rate (AOR): The average ratio of occupied seats on a flight to the assigned capacity of that flight.
- Average Spill Rate (ASR): The average ratio of the number of spilled passengers from an itinerary to the demand of that itinerary.

The results of each run of each data set are given in Appendix A. The ANOVA computations are performed for each response variable to analyze the main effects and two-factor interactions on the responses.

5. RESULTS

5.1. Computational Environment

All results obtained by IFAM in this thesis are implemented and solved using IBM ILOG CPLEX 12.6 by C# environment on Microsoft Visual Studio 2010 with default options. The CPLEX runs are performed on a hardware with Microsoft Windows Server 2003 R2 Enterprise X64 Edition operating system and Intel Xeon @3.40 GHz processor with 32 GB RAM. Each data instance was able to be solved to optimality in a short amount of CPU time. The comparison of model sizes is given in Figure 5.1 for each data instance that is solved for the one-fleet composition.

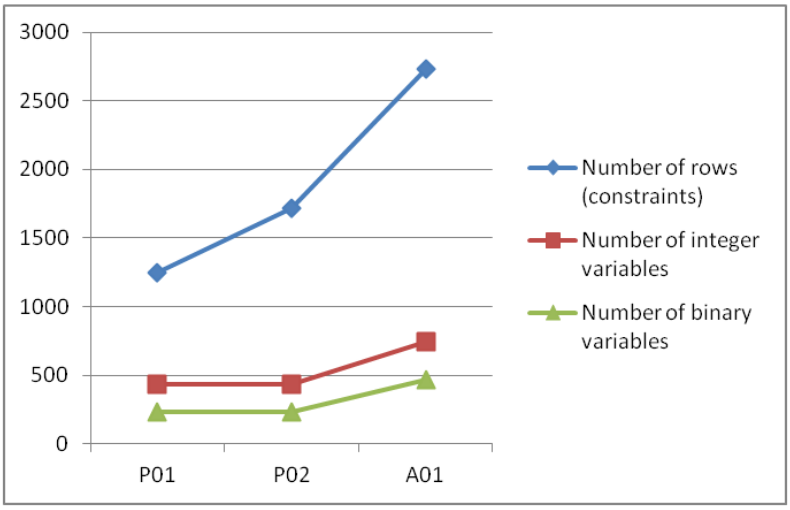

Figure 5.1. The number of variables and constraints for the data instances.

In our experimental design, ANOVA computations are performed by a statistics software named Design Expert ver. 7.0. Given the factors and the value of the response, Design Expert performs F tests on the model and the individual terms to confirm significance.

5.2. Data Analysis

We analyze the runs of each data set based on each response of the multi-factorial design. The ANOVA table for the two-factor interaction effects model for each response gives information about the significance of the main effects of factors and their interactions.

5.2.1. Profit

Profit, i.e. fleeting contribution in IFAM is calculated as the difference of the unconstrained revenue and the total assignment cost, which also the objective function of the model. The assignment cost is the combination of operating costs and spill costs.

$$\text{Profit} = D_p\, fare_p - \left(\sum_{i\in L} \sum_{k\in K} c_{k,i}\, f_{k,i} + \sum_{p\in P} \sum_{r\in P: r\neq p} (\, fare_p - b_p^r\, fare_r)\, t_p^r \right) \quad \forall p \in P \qquad (5.1)$$

The results of the ANOVA for all data sets are given in Tables 5.1 - 5.3. The Model F-values imply that the model is significant for each data set.

Table 5.1. ANOVA results for *Profit* response of P01.

Response 1 Profit ANOVA for selected factorial model Analysis of variance table [Classical sum of squares - Type II]						
Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**p-value Prob > F**	
Model	3.553E+013	35	1.015E+012	317.53	< 0.0001	significant
A-Fare	2.221E+013	3	7.405E+012	2316.14	< 0.0001	
B-Demand	5.983E+012	2	2.991E+012	935.66	< 0.0001	
C-Fleeting	5.436E+012	4	1.359E+012	425.05	< 0.0001	
AB	4.592E+011	6	7.653E+010	23.94	< 0.0001	
AC	2.945E+011	12	2.455E+010	7.68	< 0.0001	
BC	1.144E+012	8	1.429E+011	44.71	< 0.0001	
Residual	7.673E+010	24	3.197E+009			
Cor Total	3.561E+013	59				

Table 5.2. ANOVA results for *Profit* response of P02.

Response 1 Profit **ANOVA for selected factorial model** **Analysis of variance table [Classical sum of squares - Type II]**						
Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**p-value Prob > F**	
Model	6.88E+13	35	1.97E+12	1122.47	< 0.0001	significant
A-Fare	4.2E+13	3	1.4E+13	7997.51	< 0.0001	
B-Demand	3.24E+12	2	1.62E+12	925.21	< 0.0001	
C-Fleeting	2.14E+13	4	5.34E+12	3050.39	< 0.0001	
AB	2.32E+11	6	3.86E+10	22.04	< 0.0001	
AC	1.35E+12	12	1.12E+11	64.00	< 0.0001	
BC	5.99E+11	8	7.48E+10	42.72	< 0.0001	
Residual	4.2E+10	24	1.75E+09			
Cor Total	6.89E+13	59				

Table 5.3. ANOVA results for *Profit* response of A01.

Response 1 Profit **ANOVA for selected factorial model** **Analysis of variance table [Classical sum of squares - Type II]**						
Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**p-value Prob > F**	
Model	4.23E+14	35	1.21E+13	3985.655	< 0.0001	significant
A-Fare	4.39E+13	3	1.46E+13	4822.378	< 0.0001	
B-Demand	3.66E+14	2	1.83E+14	60243.64	< 0.0001	
C-Fleeting	4.02E+12	4	1E+12	330.6756	< 0.0001	
AB	5.53E+12	6	9.22E+11	303.7716	< 0.0001	
AC	6.68E+10	12	5.57E+09	1.834838	0.0994	
BC	4.18E+12	8	5.22E+11	172.0181	< 0.0001	
Residual	7.29E+10	24	3.04E+09			
Cor Total	4.24E+14	59				

Values of "Prob > F" greater than 0.10 indicate the model terms are not significant. In this case, the terms *Fare*, *Demand*, *Fleeting*, and their two-factor interactions are all significant, seeing that they all have small p-values. We can see the extent of the effect for each term from the model graphs.

5.2.1.1. Fare-Demand Interactions. The main effects of *Fare* and *Demand* on *Profit* are both positive. The one-factor graphs of *Fare* and *Demand* from data sets P01, P02 and A01 are given in Figure 5.2 - 5.4. The corresponding *Demand*/*Fare* level and the *Fleeting* level are set to average in all graphs.

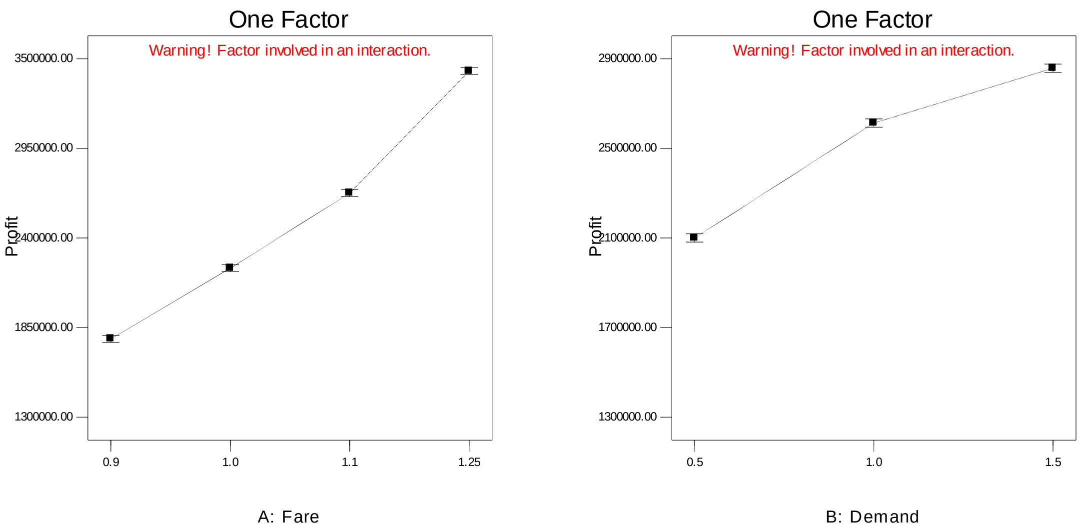

Figure 5.2. One-factor graphs for *Fare* and *Demand* to *Profit*, of P01.

The main effect of *Fare* appears linear, whereas *Demand* shows a rapid increase and then a slower increase. The same trend is observed in P02 and A01.

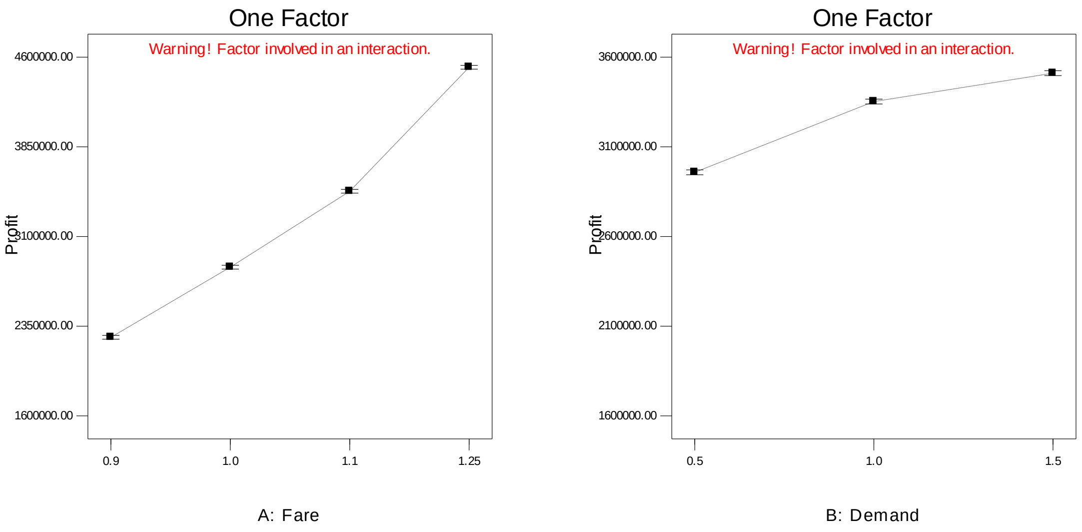

Figure 5.3. One-factor graphs for *Fare* and *Demand* to *Profit*, of P02.

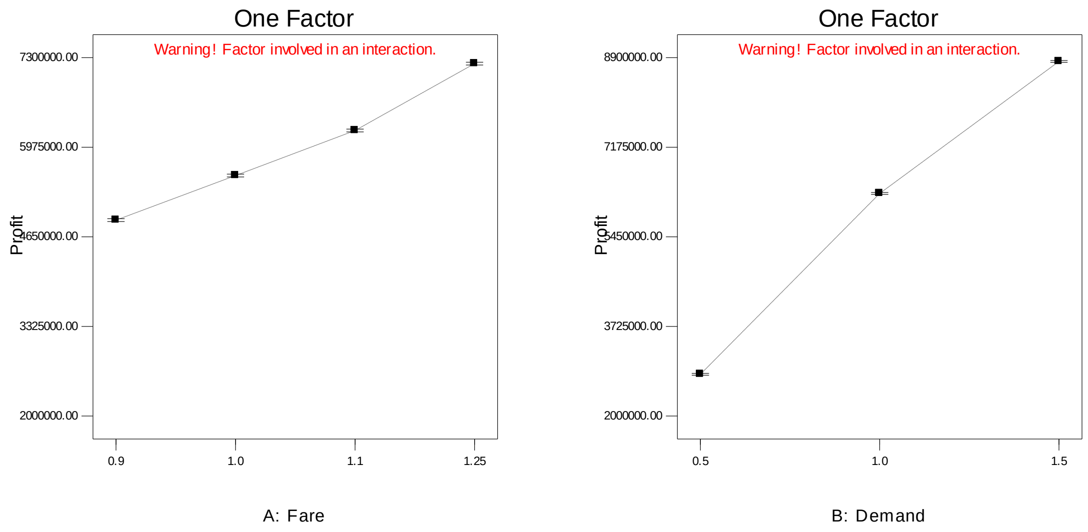

Figure 5.4. One-factor graphs for *Fare* and *Demand* to *Profit*, of A01.

It is also interesting to notice that the *Fare* term seems to have a stronger effect than *Demand* on average profit values of P01 and P02. However, in A01 *Demand* seems to have a stronger effect on *Profit* variable than *Fare* factor. This is due to the demand data used in A01, where the actual realized demand values are considered. Therefore, when the factor levels are 1.0, the network has sufficient capacity for the demand. Thus, spill is virtually nonexistent, and the spill costs are close to zero. The spill costs are highly relevant to profit. Consequently, the unsaturated network is more sensitive to demand values.

In P01 and P02, the networks both have excess forecasted demand. Therefore there is always some spill, no matter what the fare and demand values are. These saturated networks are less sensitive to demand. The response variable *Profit* is most insensitive to *Demand* in P02, where we observe the highest spill values.

Nevertheless, analyzing only the main effects may prove to be insufficient, since the two-factor interaction of the terms is also significant.

The two-factor interactions graphs of *Fare* and *Demand* are given in Figures 5.5 - 5.7. The x-axis represents *Fare* and the levels of *Demand* are represented by the colored lines. The y-axis values (profit) are scaled to see the difference between the data sets.

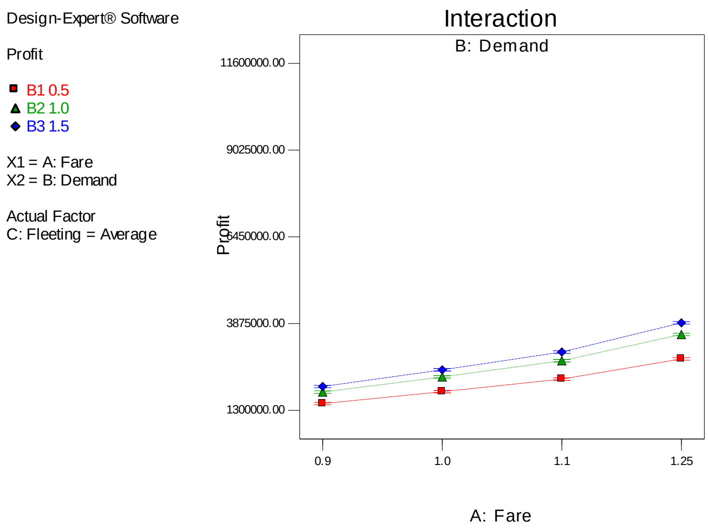

Figure 5.5. Two-factor interaction graph for *Fare* and *Demand* to *Profit*, of P01.

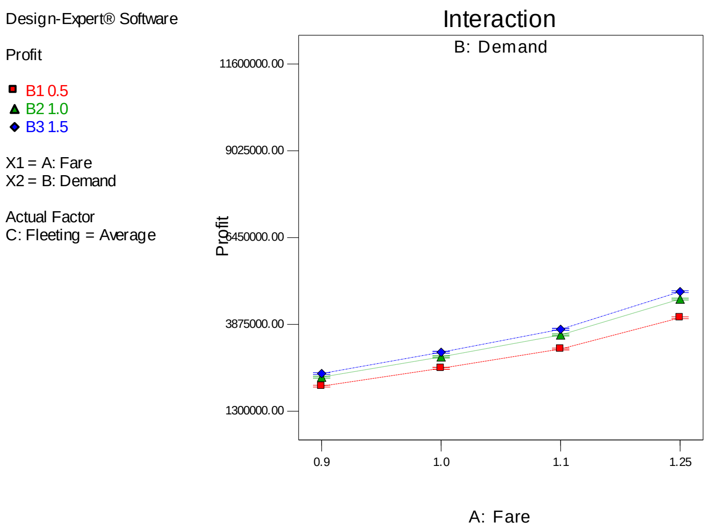

Figure 5.6. Two-factor interaction graph for *Fare* and *Demand* to *Profit*, of P02.

The two-way interactions of *Fare* and *Demand* show that there is a strong positive effect to *Profit*. We also observe that the difference between the three levels of *Demand* gets smaller as the data set becomes more saturated, as seen in Figures 5.7, 5.5 and 5.6 for A01, P01 and P02 respectively.

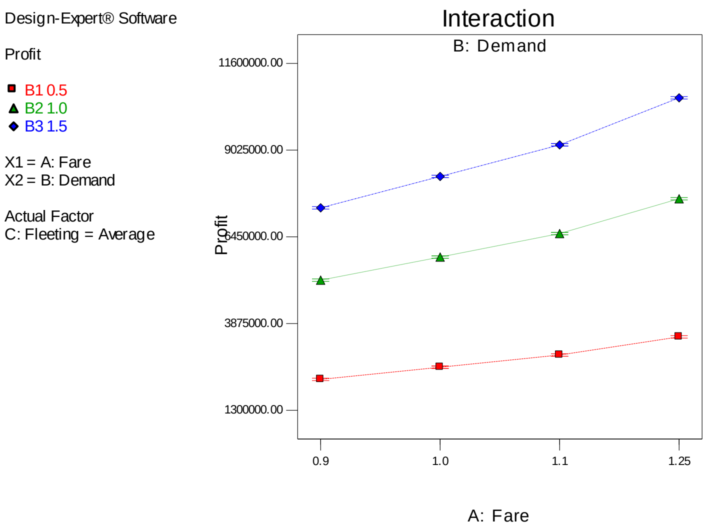

Figure 5.7. Two-factor interaction graph for *Fare* and *Demand* to *Profit*, of A01.

In addition, the data set A01 seems to be less effected from the two-factor term *Fare-Demand* with respect to the other data sets. The ascending trend is slightly slower.

<u>5.2.1.2. Fleeting.</u> The levels of the *Fleeting* factor consist of different fleet compositions where the capacities and the types of aircraft vary. The main effect of *Fleeting* to *Profit* is not linear; however there is a certain trend in the fleeting choices. The one-factor graphs of *Fleeting* are given in Figures 5.8 - 5.10.

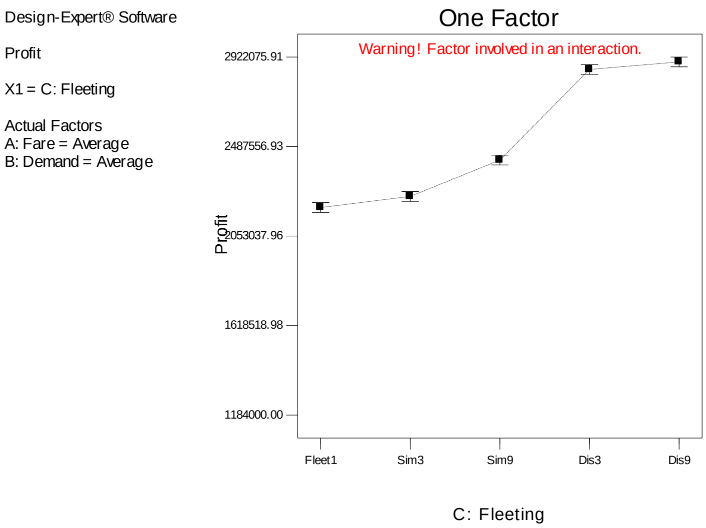

Figure 5.8. One-factor graph for *Fleeting* to *Profit*, of P01.

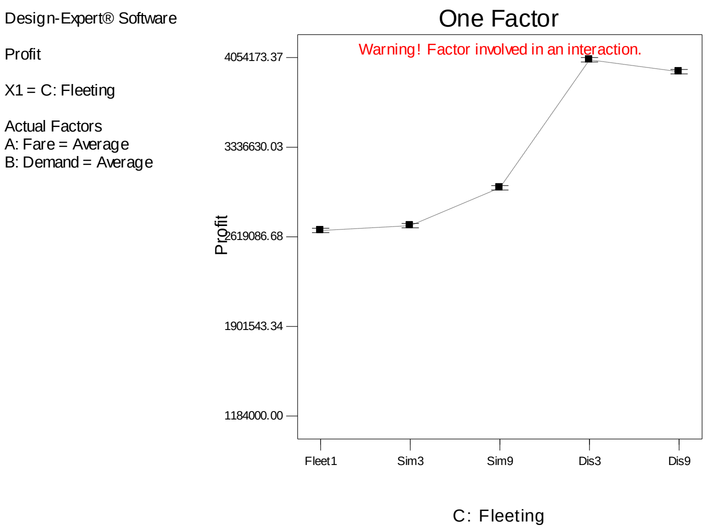

Figure 5.9. One-factor graph for *Fleeting* to *Profit*, of P02.

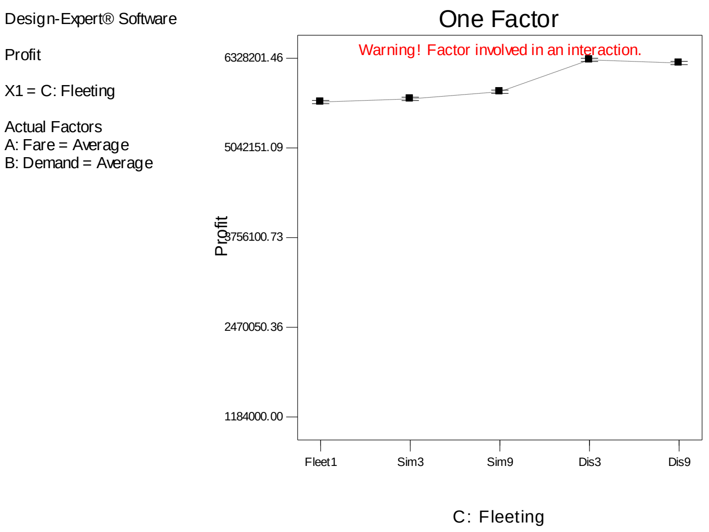

Figure 5.10. One-factor graph for *Fleeting* to *Profit*, of A01.

From Figures 5.8, 5.9 and 5.10, we see that the profit is higher when a dissimilar fleeting (*Dis3* or *Dis9*) is chosen. However there is no significant difference between the numbers of fleets to utilize. The profit is lowest when only one type of fleet is used, as in *Fleet1* level. Therefore, we conclude that introducing a certain level of flexibility to the capacity of fleeting has a positive effect on *Profit*. However, this flexibility comes at the cost of more complex maintenance and crew scheduling and planning.

Moreover, the effect of *Fleeting* on *Profit* is observed to be smaller in A01 data set. This may be due to the saturation of the network, as well as the structure of the network and flights.

The two-factor interactions of *Fleeting* and *Fare* support our conclusion that the dissimilar fleeting has a significant effect on *Profit*. The interaction graphs are given in Figures 5.11 - 5.13.

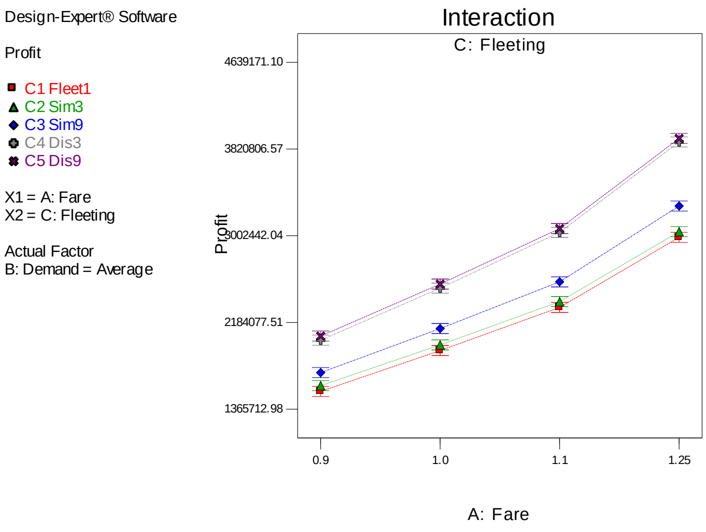

Figure 5.11. Two-factor interaction graph for *Fleeting* and *Fare* to *Profit*, of P01.

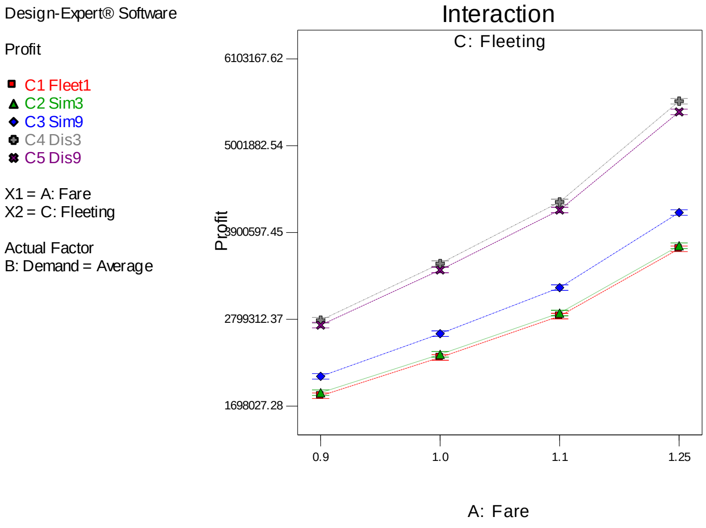

Figure 5.12. Two-factor interaction graph for *Fleeting* and *Fare* to *Profit*, of P02.

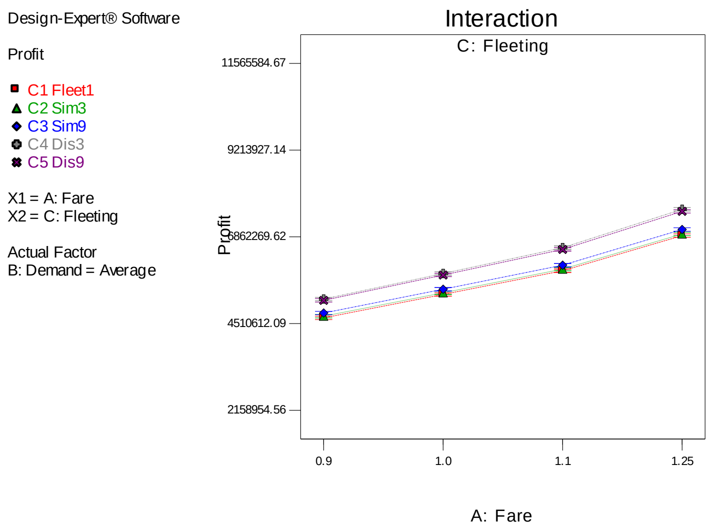

Figure 5.13. Two-factor interaction graph for *Fleeting* and *Fare* to *Profit*, of A01.

In each data set, dissimilar fleeting outperforms fleeting with one fleet type and fleet compositions with similar capacities. There is a linear increase observed that originates from *Fare*, however we see that no matter how fare values are changed, the influence of the fleet structure stays the same.

The two-factor interactions of *Fleeting* and *Demand* are represented by the graphs provided in Figures 5.14 - 5.16. The main effect of demand is observed also in these graphs, there is an increasing trend that starts to level off as demand values get higher.

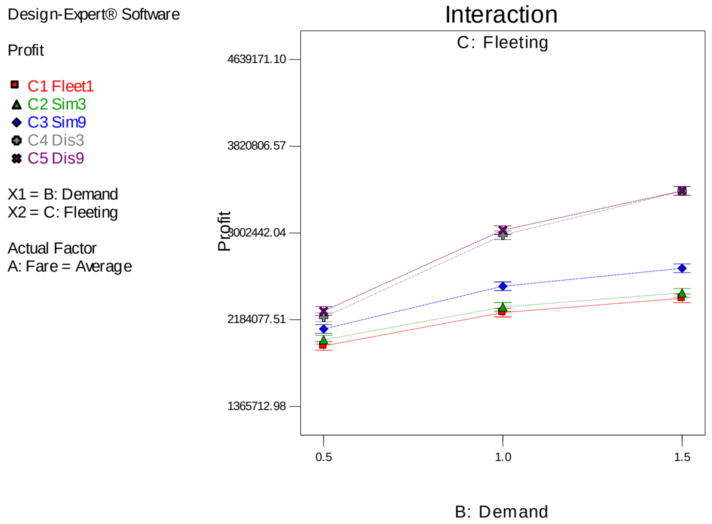

Figure 5.14. Two-factor interaction graph for *Fleeting* and *Demand* to *Profit*, of P01.

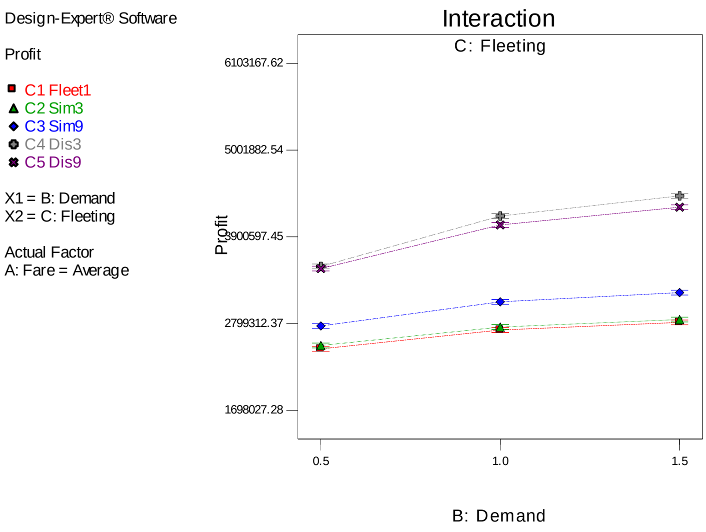

Figure 5.15. Two-factor interaction graph for *Fleeting* and *Demand* to *Profit*, of P02.

The dissimilar fleeting choices are preferable in each level of Demand in data sets P01 and P02. Still, the effects of the *Fleeting* vary for each *Demand* level. The influence of the dissimilar fleeting *Dis3* and *Dis9* increases more rapidly as demand gets higher, with respect to the other fleeting levels.

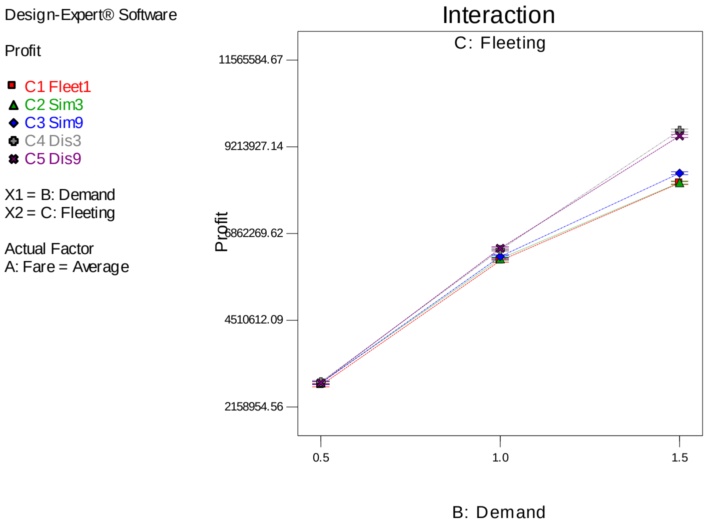

Figure 5.16. Two-factor interaction graph for *Fleeting* and *Demand* to *Profit*, of A01.

This is observed most clearly in the A01 data set, in Figure 5.16. In this data set, there is no significant effect of *Fleeting* to *Profit* when the *Demand* level is 0.5. The difference between dissimilar and similar fleeting is observed when the demand values are higher.

5.2.2. Operating Cost

The operating cost (OC) is the part of the objective function of IFAM where the cost of operating a fleet type on a flight leg is calculated. Unlike profit, the operating cost is not directly related to fare and demand, and it is a major part of the total assignment cost.

$$\text{Operating Cost} = \sum_{i \in L} \sum_{k \in K} c_{k,i} f_{k,i} \tag{5.2}$$

The ANOVA results for each data set are given in Tables 5.4 - 5.6. The Model F-values imply that the model is significant for each data set.

Table 5.4. ANOVA results for *Operating Cost* response of P01.

Response 2 Operating Cost ANOVA for selected factorial model Analysis of variance table [Classical sum of squares - Type II]						
Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**p-value Prob > F**	
Model	4.85E+10	35	1.38E+09	2643.571	< 0.0001	significant
A-Fare	14898708	3	4966236	9.481551	0.0003	
B-Demand	5.19E+09	2	2.59E+09	4952.464	< 0.0001	
C-Fleeting	3.93E+10	4	9.82E+09	18744.91	< 0.0001	
AB	6624644	6	1104107	2.107965	0.0898	
AC	15524322	12	1293694	2.469923	0.0287	
BC	3.96E+09	8	4.96E+08	946.2089	< 0.0001	
Residual	12570693	24	523778.9			
Cor Total	4.85E+10	59				

In table 5.5, the analysy of variance results of the P02 data set is given. The individual terms *Fare*, *Demand*, *Fleeting* have very small p-values, indicating each term factor to be significant.

Table 5.5. ANOVA results for *Operating Cost* response of P02.

Response 2 Operating Cost ANOVA for selected factorial model Analysis of variance table [Classical sum of squares - Type II]						
Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**p-value Prob > F**	
Model	6.2E+10	35	1.77E+09	2322.344	< 0.0001	significant
A-Fare	78755735	3	26251912	34.43829	< 0.0001	
B-Demand	1.26E+09	2	6.3E+08	826.2311	< 0.0001	
C-Fleeting	5.99E+10	4	1.5E+10	19644.09	< 0.0001	
AB	2572263	6	428710.5	0.562399	0.7558	
AC	53769727	12	4480811	5.878104	0.0001	
BC	6.68E+08	8	83470804	109.5003	< 0.0001	
Residual	18294922	24	762288.4			
Cor Total	6.2E+10	59				

Table 5.6. ANOVA results for *Operating Cost* response of A01.

Response 2 Operating Cost ANOVA for selected factorial model Analysis of variance table [Classical sum of squares - Type II]						
Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**p-value Prob > F**	
Model	1.15E+11	35	3.3E+09	10520.19	< 0.0001	significant
A-Fare	15076477	3	5025492	16.03521	< 0.0001	
B-Demand	4.3E+10	2	2.15E+10	68664.63	< 0.0001	
C-Fleeting	3.59E+10	4	8.97E+09	28635.73	< 0.0001	
AB	2197872	6	366311.9	1.168819	0.3553	
AC	26820812	12	2235068	7.131597	< 0.0001	
BC	3.64E+10	8	4.55E+09	14524.22	< 0.0001	
Residual	7521685	24	313403.5			
Cor Total	1.15E+11	59				

Values of "Prob > F" less than 0.05 indicate that the corresponding model terms are significant. The terms *Fare*, *Demand*, *Fleeting*, and the two-factor interactions *Fare-Fleeting* and *Demand-Fleeting* are significant, seeing that they all have small p-values. The

Fare-Demand interaction is not a significant term for the *Operating Cost* model, since the fare and demand data is not utilized when the operating cost is calculated.

We can see the extent of the effect for each term from the model graphs.

5.2.2.1. Fare-Demand Interactions. The one-factor graphs of *Fare* and *Demand* from data sets P01, P02 and A01 are given in Figure 5.17 - 5.19. The corresponding *Demand/Fare* level and the *Fleeting* level are set to average in all graphs.

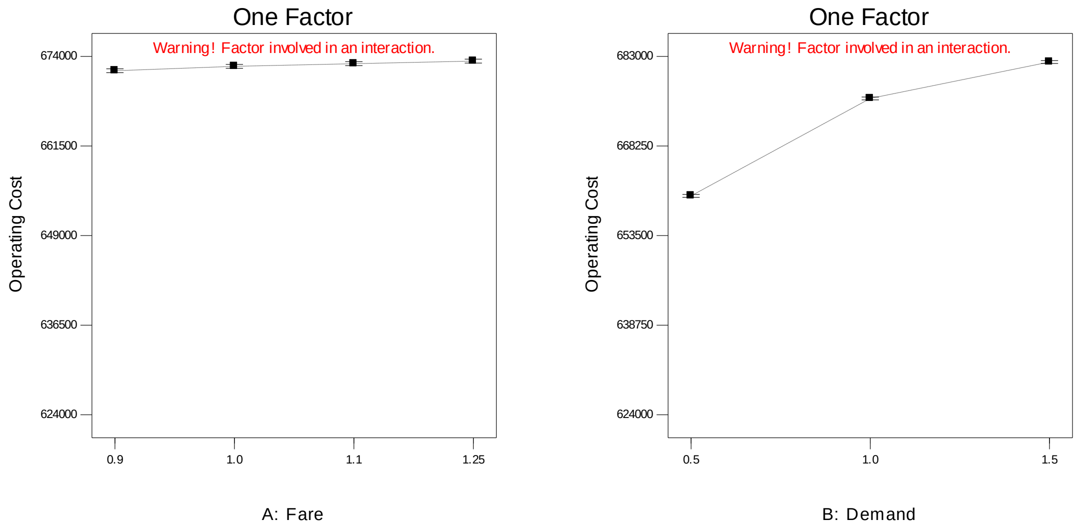

Figure 5.17. One-factor graphs for *Fare* and *Demand* to *OC*, of P01.

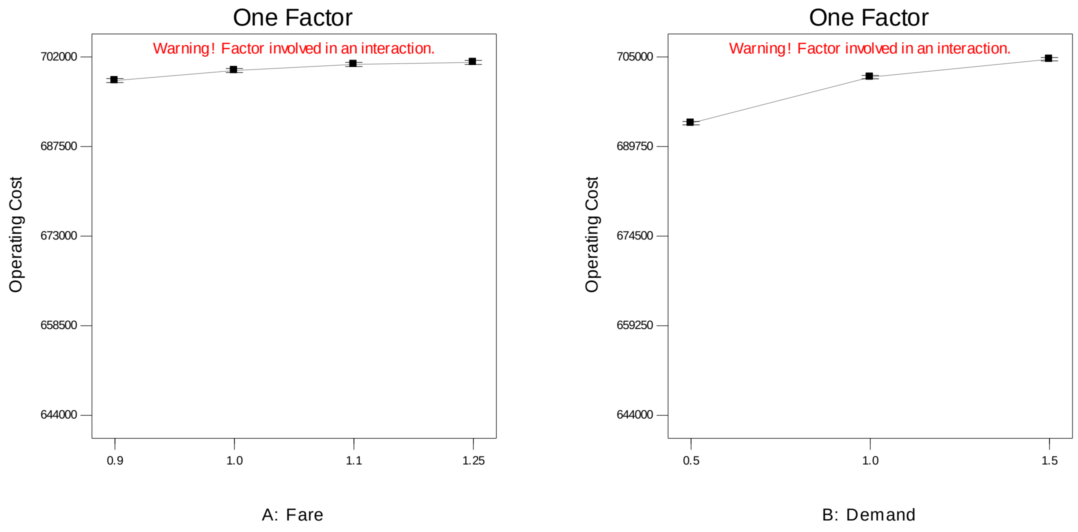

Figure 5.18. One-factor graphs for *Fare* and *Demand* to *OC*, of P02.

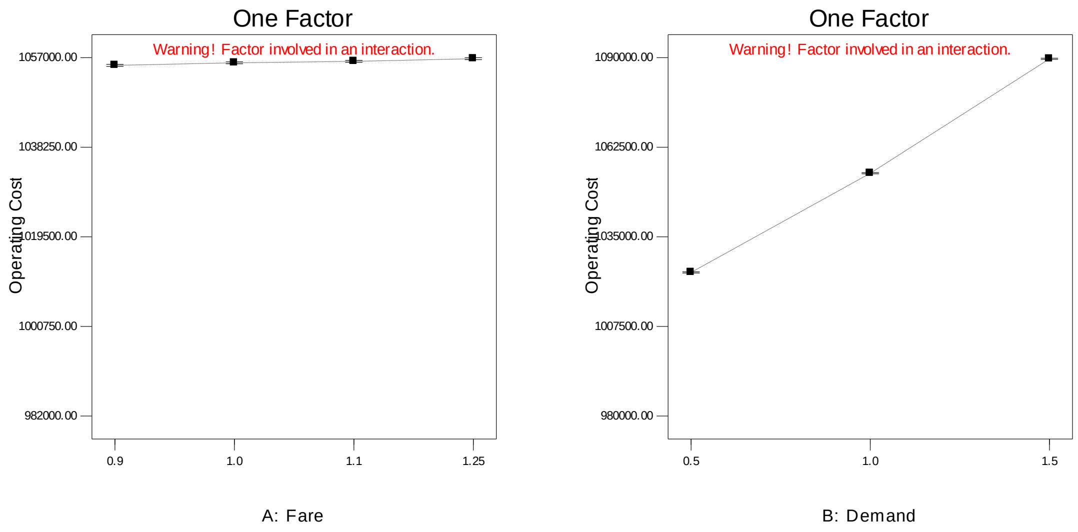

Figure 5.19. One-factor graphs for *Fare* and *Demand* to *OC*, of A01.

There are no visible main effects of *Fare* to *Operating Cost*. On the other hand, we observe an increase in operating cost value as demand gets higher. This is caused by the fleeting choices. When demand gets higher, IFAM tends to reduce the elevated spill costs. Hence the fleet types with more capacity are utilized, albeit the higher operating costs. Analyzing the main effects of *Demand* is not sufficient in this case, as it is closely related to *Fleeting* levels.

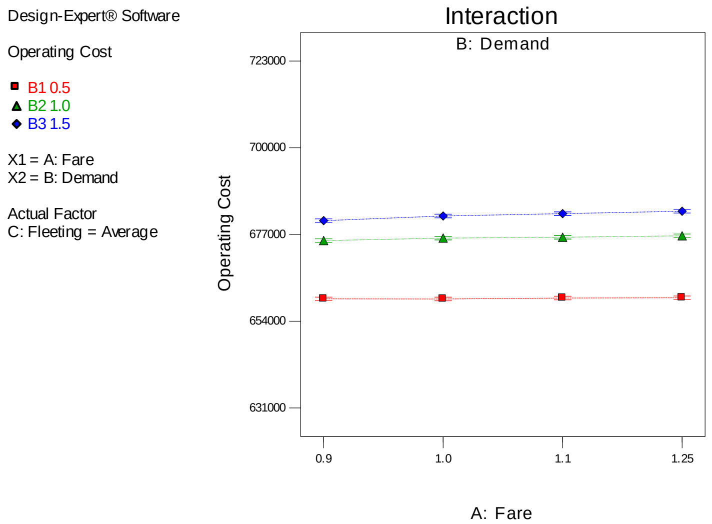

Figure 5.20. Two-factor interaction graph for *Fare* and *Demand* to *OC*, of P01.

The two-factor interactions graphs of *Fare* and *Demand* is given in Figures 5.20 - 5.22. As the model p-values suggest, the interaction of the two factors is insignificant.

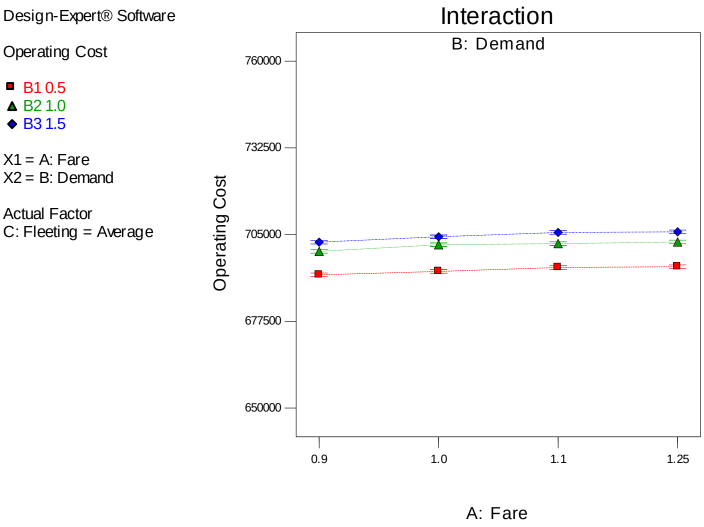

Figure 5.21. Two-factor interaction graph for *Fare* and *Demand* to *OC*, of P02.

The two-factor interaction graphs of *Fare* and *Demand* show that there is no considerable effect to *Operating Cost*. The only visible effect is the main effect of *Demand*, which raises operating cost values as demand increases.

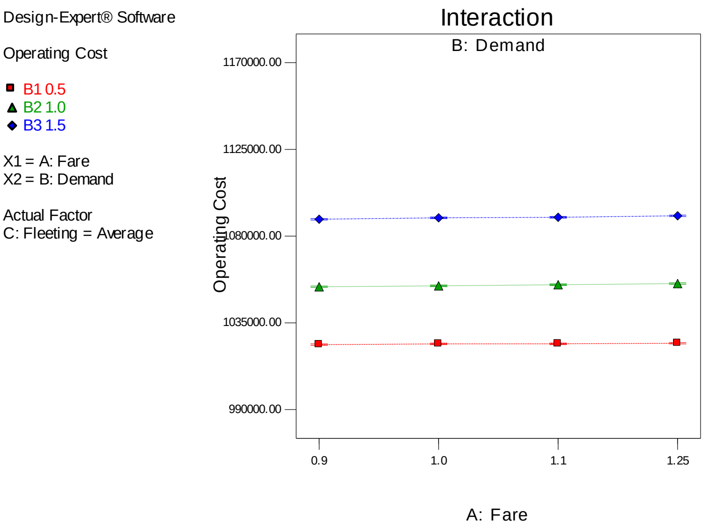

Figure 5.22. Two-factor interaction graph for *Fare* and *Demand* to *OC*, of A01.

In addition, it is seen that the effect of *Demand* is of different degree for each data set. This may be due to the network structure or demand saturation of the data sets. Similar to the *Profit* response variable, the *Operating Cost* variable is also more sensitive to *Demand* if the network is unsaturated.

<u>5.2.2.2. Fleeting.</u> The main effect of *Fleeting* to *Operating Cost* is irregular to some extent. The one-factor graphs of *Fleeting* are given in Figures 5.23 - 5.35.

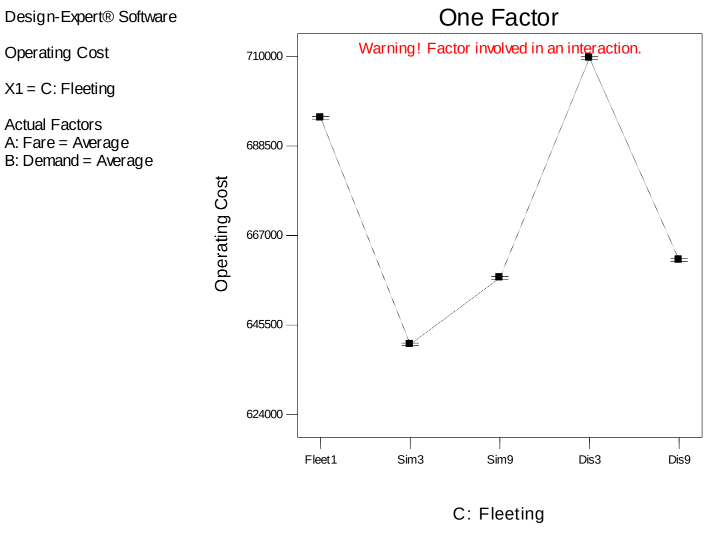

Figure 5.23. One-factor graph for *Fleeting* to *Operating Cost*, of P01.

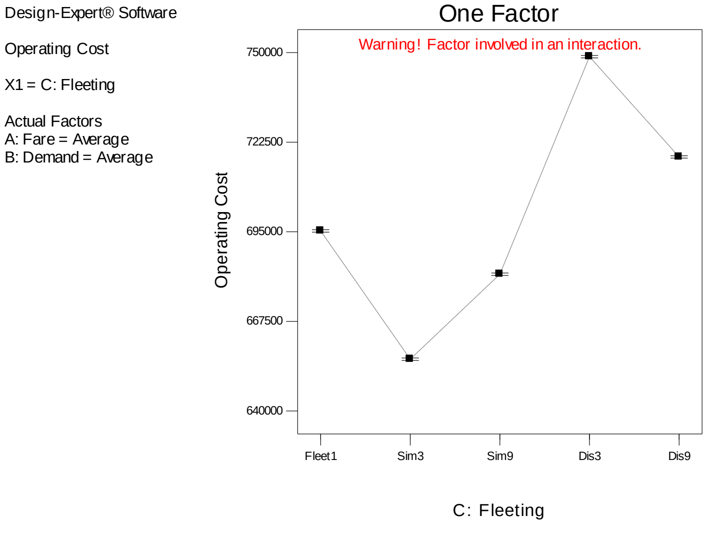

Figure 5.24. One-factor graph for *Fleeting* to *Operating Cost*, of P02.

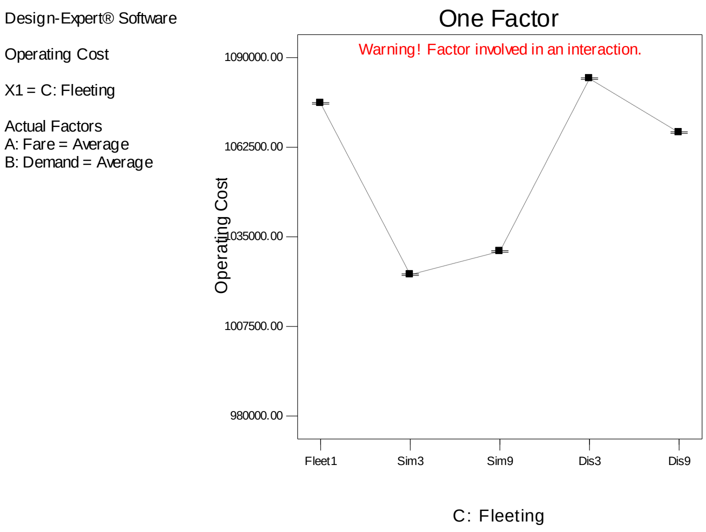

Figure 5.25. One-factor graph for *Fleeting* to *Operating Cost*, of A01.

From the Figures 5.23 - 5.35, it is observed that *Fleeting* is an important factor that influences *Operating Cost*. By average, the *Dis3* level of *Fleeting* has the highest operating cost for each data set. The similar fleet compositions have lower operating costs, whereas the *Fleet1* remains in between.

The two-factor interactions of *Fleeting* and *Fare* are given in the following Figures 5.26 - 5.28.

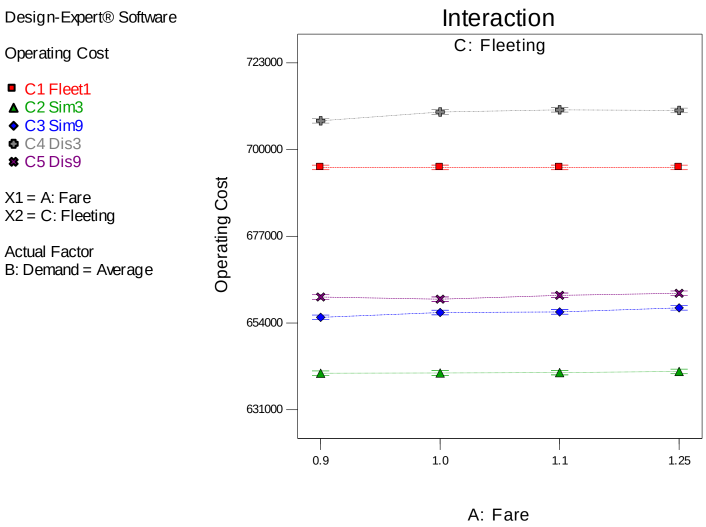

Figure 5.26. Two-factor interaction graph for *Fleeting* and *Fare* to *OC*, of P01.

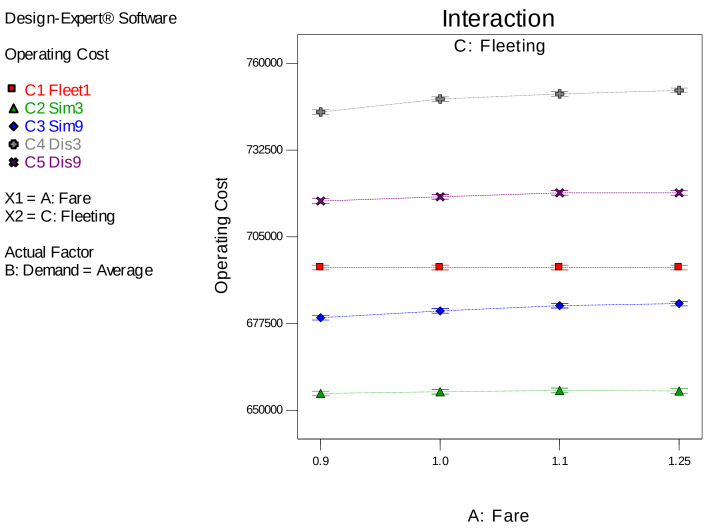

Figure 5.27. Two-factor interaction graph for *Fleeting* and *Fare* to *OC*, of P02.

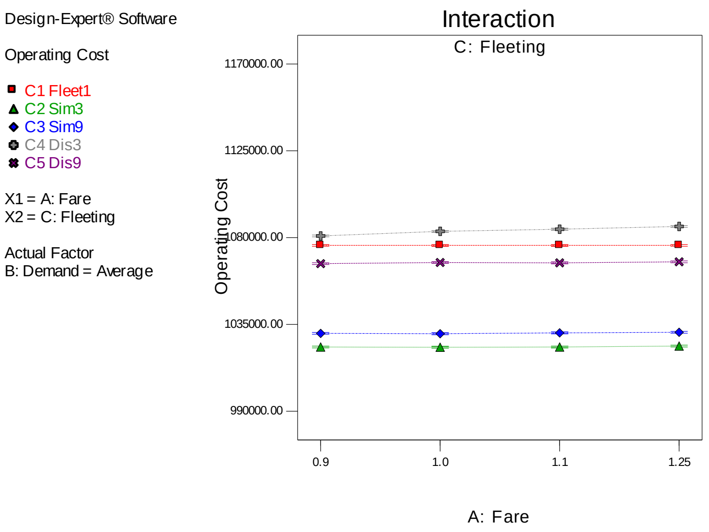

Figure 5.28. Two-factor interaction graph for *Fleeting* and *Fare* to *OC*, of A01.

The two-factor interactions of *Fleeting* and *Fare* support our conclusion that *Fleeting* has a significant effect on *Operating Cost*, though *Fare* has no major influence. The operating costs of each data set follow this pattern closely.

The two-factor interactions of *Fleeting* and *Demand* are represented by the graphs given in Figures 5.29 - 5.31.

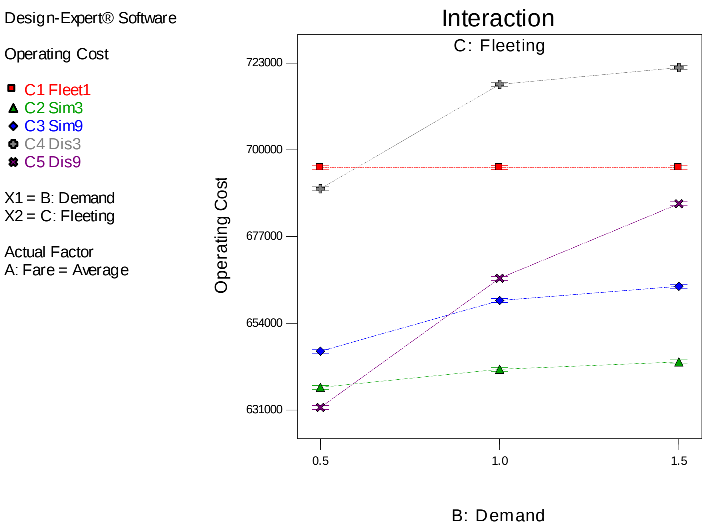

Figure 5.29. Two-factor interaction graph for *Fleeting* and *Demand* to *OC*, of P01.

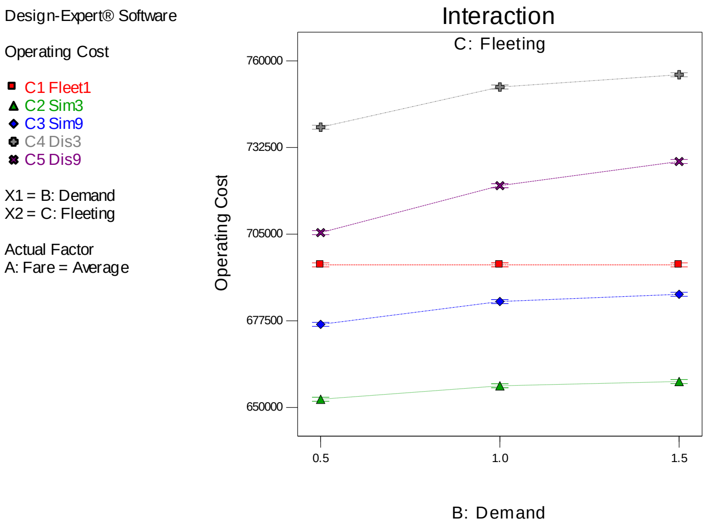

Figure 5.30. Two-factor interaction graph for *Fleeting* and *Demand* to *OC*, of P02.

From the two-factor interaction graphs of *Fleeting* and *Demand* to *Operating Cost*, it is seen that demand values influence fleet compositions differently. Demand has a major effect on dissimilar fleeting, and a less significant influence on similar fleet structures.

The operating cost values of dissimilar fleet structures are much more sensitive to demand, in regard of the similar fleet structures and the fleet composition with one type of fleet.

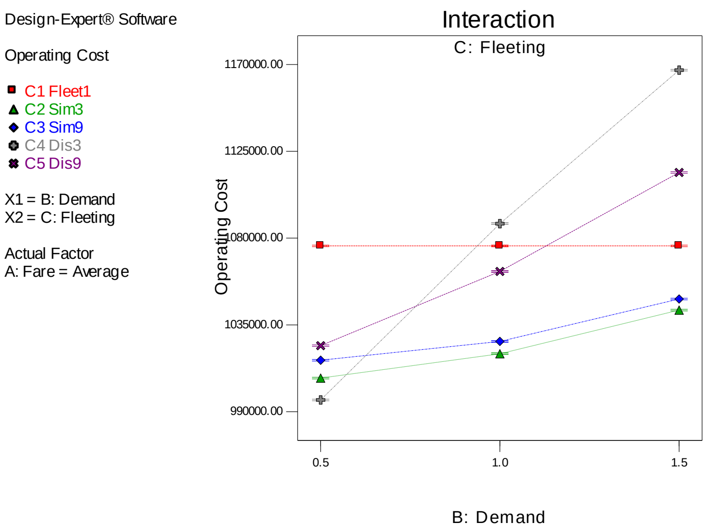

Figure 5.31. Two-factor interaction graph for *Fleeting* and *Demand* to *OC*, of A01.

5.2.3. Spill Cost

The spill cost (SC) consists of the lost revenue from spilled passengers minus the revenue gained from the passengers redirected to another itinerary. Together with the operating costs, it constitutes the objective function of IFAM.

$$\text{Spill Cost} = \sum_{p \in P} \sum_{r \in P : r \neq p} (fare_p - b_p^r fare_r) t_p^r \qquad (5.3)$$

We give the results of the ANOVA tests for each data set in the following tables. The F-values and the corresponding small p-value of the models indicate that they are significant.

The analysis of variance tables for each data set are given in Tables 5.7, 5.8 and 5.9. In these tables, it is seen that the p-values of each factor is smaller than 0.1. This indicates the one-factor effect of each term is significant for the spill cost.

Table 5.7. ANOVA results for *Spill Cost* response of P01.

Response 3 Spill Cost ANOVA for selected factorial model Analysis of variance table [Classical sum of squares - Type II]						
Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**p-value Prob > F**	
Model	4.85E+13	35	1.39E+12	429.6771	< 0.0001	significant
A-Fare	3.37E+12	3	1.12E+12	348.7184	< 0.0001	
B-Demand	3.59E+13	2	1.79E+13	5561.124	< 0.0001	
C-Fleeting	5.74E+12	4	1.44E+12	445.2913	< 0.0001	
AB	1.95E+12	6	3.25E+11	100.6541	< 0.0001	
AC	2.96E+11	12	2.47E+10	7.661638	< 0.0001	
BC	1.27E+12	8	1.58E+11	49.15834	< 0.0001	
Residual	7.74E+10	24	3.22E+09			
Cor Total	4.86E+13	59				

Table 5.8. ANOVA results for *Spill Cost* response of P02.

Response 3 Spill Cost ANOVA for selected factorial model Analysis of variance table [Classical sum of squares - Type II]						
Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**p-value Prob > F**	
Model	6.88E+13	35	1.97E+12	1122.47	< 0.0001	significant
A-Fare	4.2E+13	3	1.4E+13	7997.51	< 0.0001	
B-Demand	3.24E+12	2	1.62E+12	925.21	< 0.0001	
C-Fleeting	2.14E+13	4	5.34E+12	3050.39	< 0.0001	
AB	2.32E+11	6	3.86E+10	22.04	< 0.0001	
AC	1.35E+12	12	1.12E+11	64.00	< 0.0001	
BC	5.99E+11	8	7.48E+10	42.72	< 0.0001	
Residual	4.2E+10	24	1.75E+09			
Cor Total	6.89E+13	59				

The p-values of the terms indicate that all the terms *Fare, Demand*, *Fleeting*, and their two-factor interactions have significant effect to the spill cost. This is a reasonable outcome as the spill cost is directly related to fares and the number of spilled passengers is related to fleet structure and demand.

Table 5.9. ANOVA results for *Spill Cost* response of A01.

Response 3 Spill Cost ANOVA for selected factorial model Analysis of variance table [Classical sum of squares - Type II]						
Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**p-value Prob > F**	
Model	3.48E+13	35	9.95E+11	329.539	< 0.0001	significant
A-Fare	3.16E+11	3	1.05E+11	34.9547	< 0.0001	
B-Demand	2.47E+13	2	1.24E+13	4096.878	< 0.0001	
C-Fleeting	4.47E+12	4	1.12E+12	370.2052	< 0.0001	
AB	3.69E+11	6	6.14E+10	20.35598	< 0.0001	
AC	6.87E+10	12	5.73E+09	1.896916	0.0879	
BC	4.86E+12	8	6.07E+11	201.1909	< 0.0001	
Residual	7.24E+10	24	3.02E+09			
Cor Total	3.49E+13	59				

The model graph of each term shows the nature of its effect.

<u>5.2.3.1. Fare-Demand Interactions.</u> The one-factor graphs of *Fare* and *Demand* from data sets P01, P02 and A01 are given in Figures 5.32 - 5.34. The corresponding *Demand*/*Fare* level and the *Fleeting* level are set to average in all graphs.

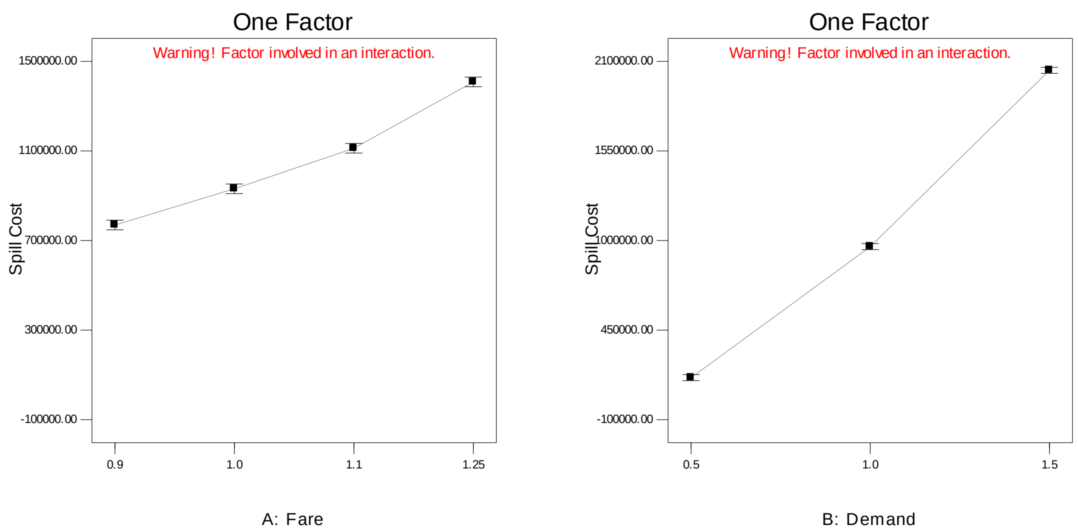

Figure 5.32. One-factor graphs for *Fare* and *Demand* to *Spill Cost*, of P01.

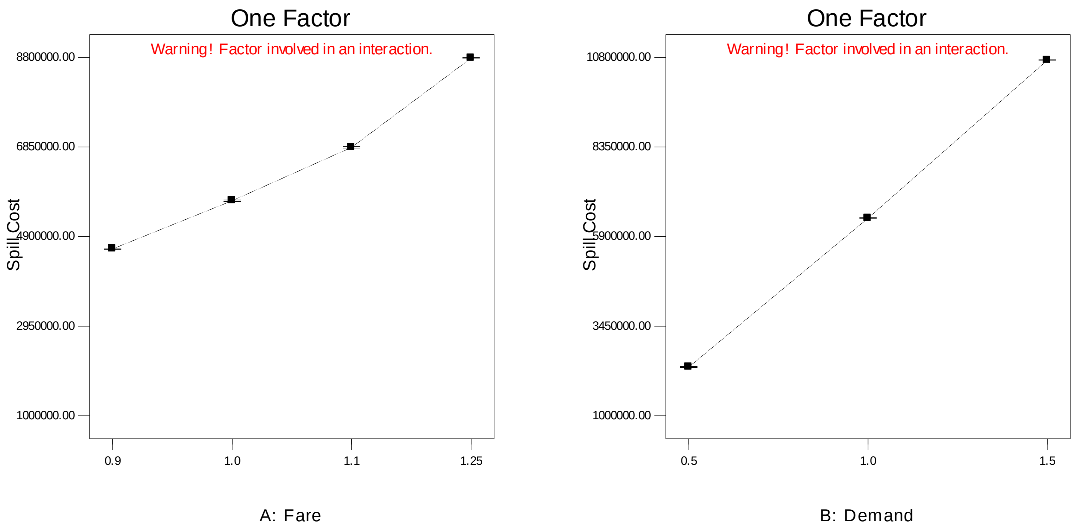

Figure 5.33. One-factor graphs for *Fare* and *Demand* to *Spill Cost*, of P02.

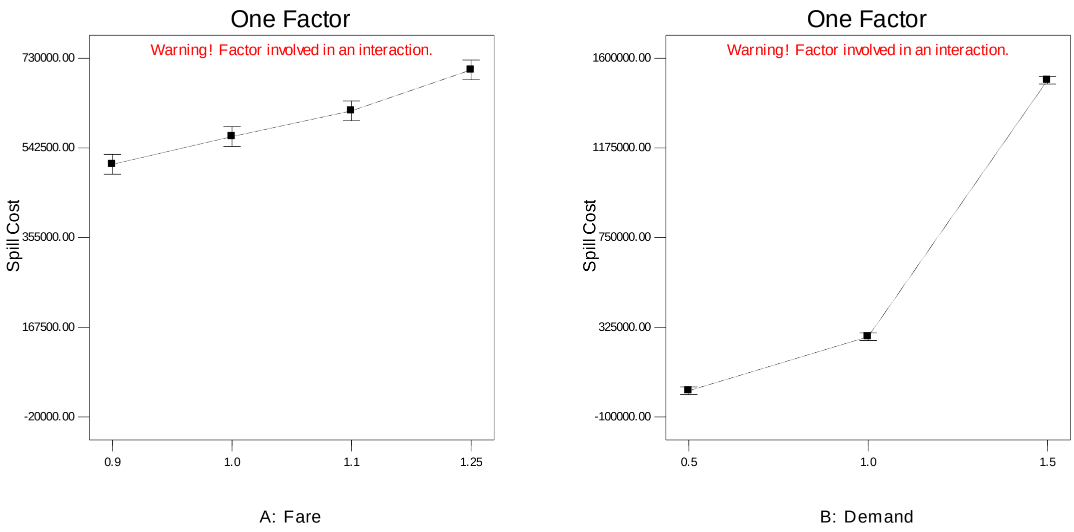

Figure 5.34. One-factor graphs for *Fare* and *Demand* to *Spill Cost*, of A01.

From the one-factor graphs, it is seen that both *Fare* and *Demand* have a positive effect to *Spill Cost*. Demand values are more influential than fares. The effect is mostly linear, though for the A01 data set the trend of *Demand* seems more like concave. This is probably related to the unsaturated nature of the demand data of A01. The original demand data (level 1.0) of A01 is close to the assigned capacities of flights and the spill is already small. Hence, when demand values are 50% less, we see that the spill costs are not much affected.

The two-factor interactions graphs of *Fare* and *Demand* is provided in Figures 5.35 - 5.37. In these graphs, we see how the *Fare* factor influences the spill cost in different levels of *Demand*.

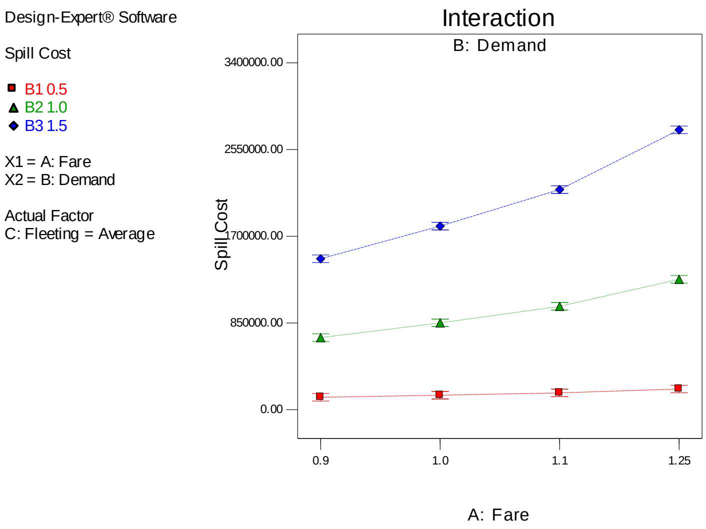

Figure 5.35. Two-factor interaction graph for *Fare* and *Demand* to *SC*, of P01.

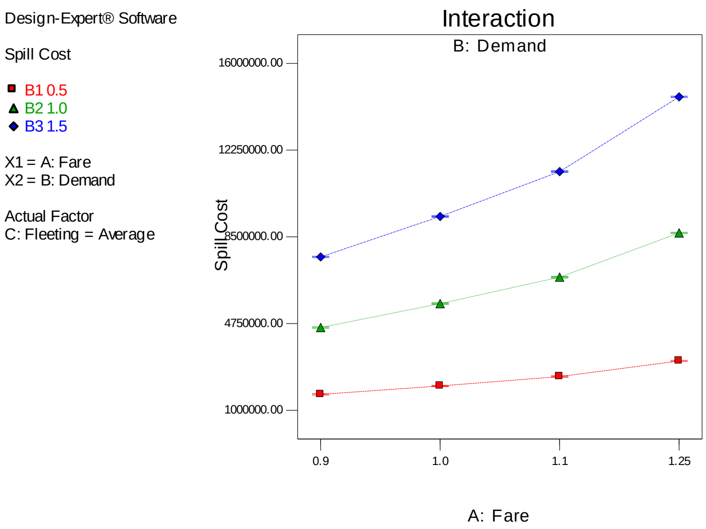

Figure 5.36. Two-factor interaction graph for *Fare* and *Demand* to *SC*, of P02.

The two-factor interaction graphs for each data set suggest that *Fare* causes a significant raise in spill costs when demand values are higher.

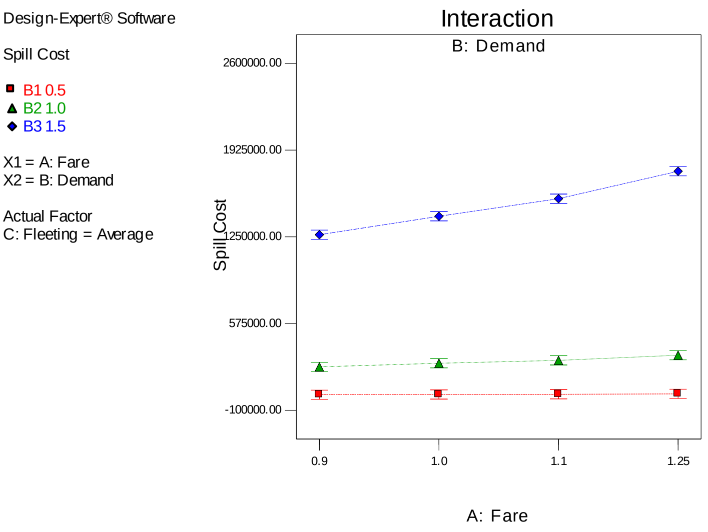

Figure 5.37. Two-factor interaction graph for *Fare* and *Demand* to *SC*, of A01.

We also see for A01 the effect of *Fare* is trivial when *Demand* is at levels 0.5 and 1.0 and the spill cost is closer to one another. We have also observed this result from the main effect graph of *Demand* in Figure 5.43.

5.2.3.2. Fleeting. The main effect of *Fleeting* to *Spill Cost* follows a particular trend, which can be observed from the one-factor graphs. The one-factor graphs of *Fleeting* are given in Figures 5.38 - 5.40.

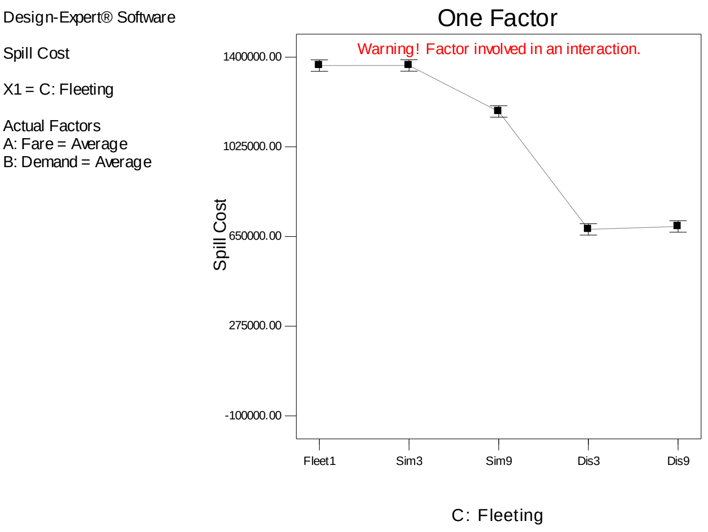

Figure 5.38. One-factor graph for *Fleeting* to *Spill Cost*, of P01.

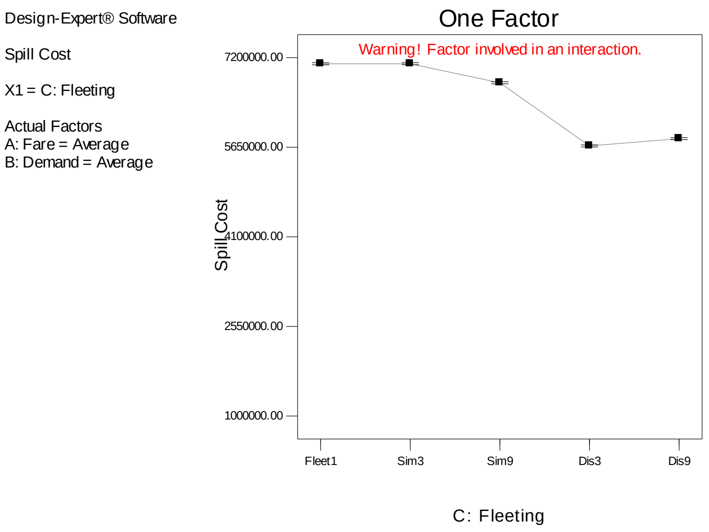

Figure 5.39. One-factor graph for *Fleeting* to *Spill Cost*, of P02.

From Figures 5.38 – 5.40, we observe that *Spill Cost* is significantly lower when a dissimilar fleet structure is chosen. There is no considerable difference between *Dis3* and *Dis9*; therefore we may conclude that the number of fleet types is insignificant. The spill cost can be lowered by utilizing a fleeting structure consisting of aircraft with different capacities.

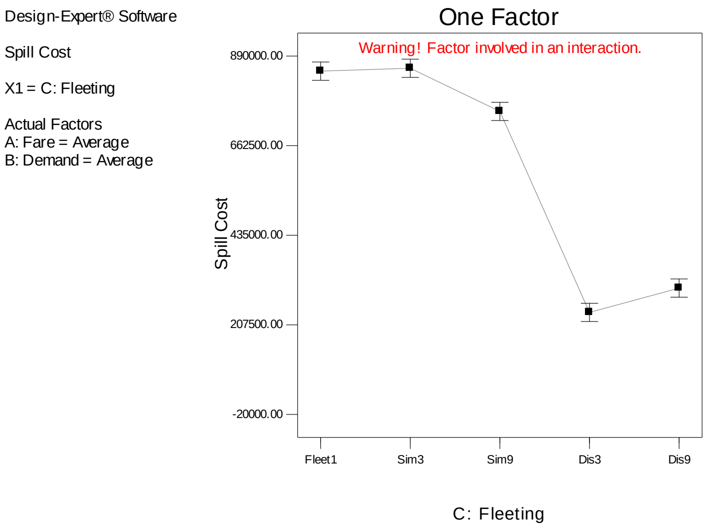

Figure 5.40. One-factor graph for *Fleeting* to *Spill Cost*, of A01.

When the two-factor interactions of *Fleeting* are analyzed, we observe the main effect of *Fleeting* as well as its joint effect with *Fare* and *Demand*.

The two-factor interactions of *Fleeting* and *Fare* are given in Figures 5.41 - 5.43.

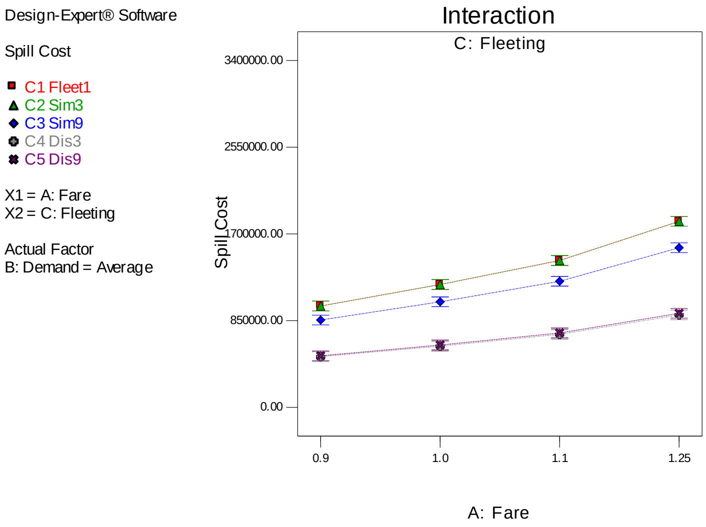

Figure 5.41. Two-factor interaction graph for *Fleeting* and *Fare* to *SC*, of P01.

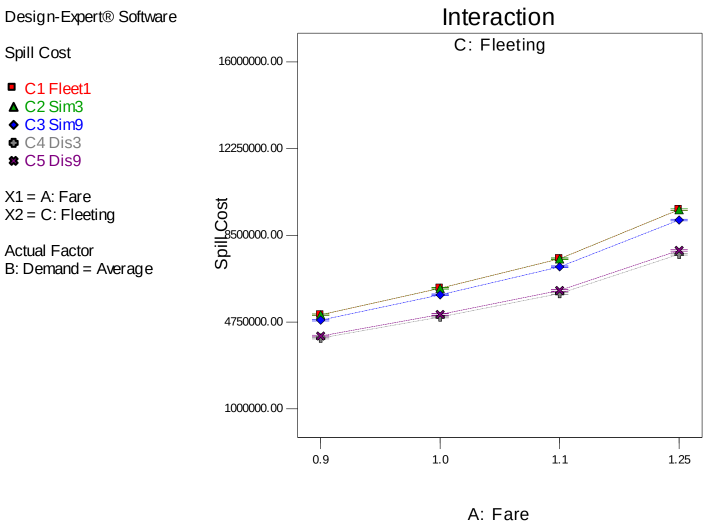

Figure 5.42. Two-factor interaction graph for *Fleeting* and *Fare* to *SC*, of P02.

In each data set, dissimilar fleeting leads to smaller spill costs, which support our conclusion about the main effect of *Fleeting*. Increasing the fare values raises the *Spill Cost* of each fleet composition similarly. In other words, the order of *Fleeting* levels with respect to their spill cost values is not affected by *Fare*.

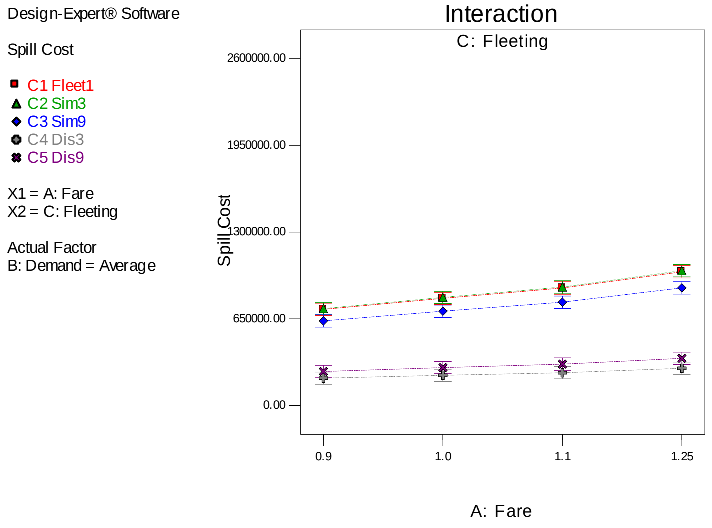

Figure 5.43. Two-factor interaction graph for *Fleeting* and *Fare* to *SC*, of A01.

Via the two-factor interaction graphs of *Fleeting* and *Demand*, we observe the behavior of *Spill Cost* under different *Fleeting* and *Demand* levels. The graphs are given in Figures 5.44 - 5. 46.

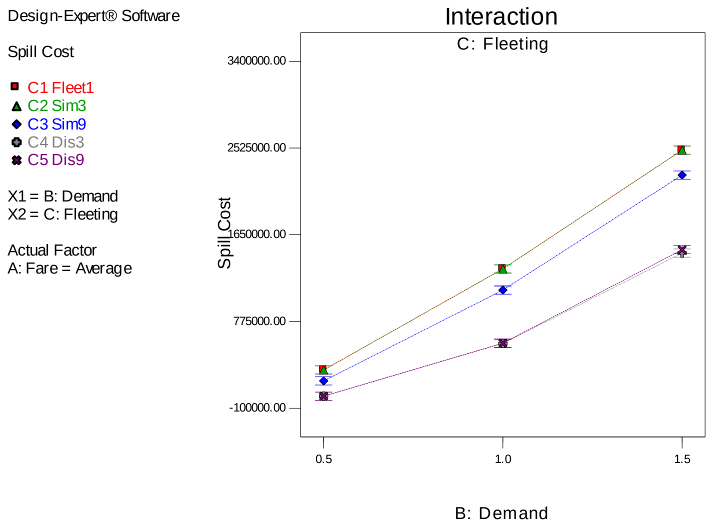

Figure 5.44. Two-factor interaction graph for *Fleeting* and *Demand* to *SC*, of P01.

From the Figures 5.44, 5.45 and 5.46, we see that the joint effect of *Fleeting* and *Demand* is positive on *Spill Cost*.

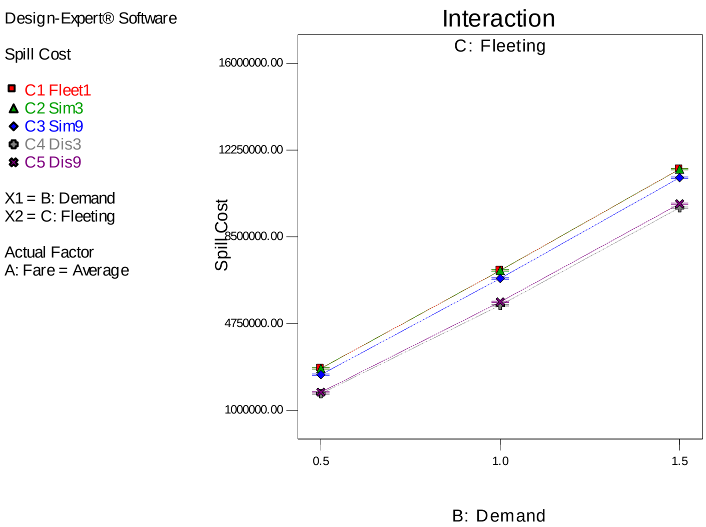

Figure 5.45. Two-factor interaction graph for *Fleeting* and *Demand* to *SC*, of P02.

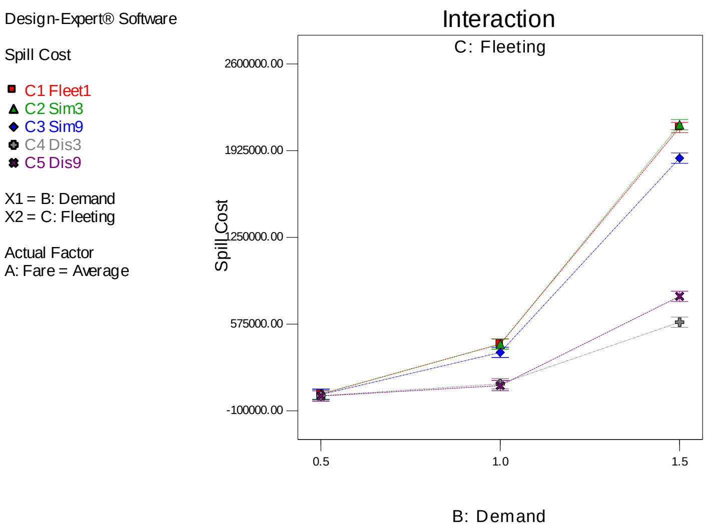

Figure 5.46. Two-factor interaction graph for *Fleeting* and *Demand* to *SC*, of A01.

However, it is noted that fleeting structures behave differently as demand increases for different data sets. In Figure 5.45, the trend is close to linear in data set P02, which is the data set with the most saturated demand, i.e. excess demand. In P01, refraction is observed when *Demand* is at level 1.0, especially in dissimilar *Fleeting* levels. In Figure 5.46, the trend lines are relatively concave, meaning that higher demand values leads to much higher spill costs if the network demand is unsaturated as in A01 data set.

Moreover, the spill costs of *Fleet1*, *Sim3* and *Sim9* are affected more than *Dis3* and *Dis9* as demand values increase. Thus, it may be inferred that dissimilar fleeting compositions are more successful to contain spill costs when demand values get higher.

5.2.4. Null Spill

The response variable *Null Spill* represents the total number of passengers that are lost to the airline due to insufficient capacity. For airlines, it is important to keep null spill to a minimum in order to maintain customer satisfaction as well as to minimize the lost revenue caused by spills.

The ANOVA results for each data set are given in Tables 5.10 - 5.12. The Model F-values imply that the model is significant for each data set.

Table 5.10. ANOVA results for *Null Spill* response of P01.

Response 4 Null Spill ANOVA for selected factorial model Analysis of variance table [Classical sum of squares - Type II]						
Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**p-value Prob > F**	
Model	3.16E+08	35	9020833	20590.16	< 0.0001	significant
A-Fare	30452.45	3	10150.82	23.16936	< 0.0001	
B-Demand	2.65E+08	2	1.32E+08	302342.3	< 0.0001	
C-Fleeting	42853762	4	10713440	24453.55	< 0.0001	
AB	23494.1	6	3915.683	8.937592	< 0.0001	
AC	16107.47	12	1342.289	3.06379	0.0094	
BC	7884596	8	985574.5	2249.585	< 0.0001	
Residual	10514.73	24	438.1139			
Cor Total	3.16E+08	59				

In Table 5.10, the terms *Fare*, *Demand*, *Fleeting*, and their two-factor interactions are all significant for P01 data set. Each term and their two-factor interactions all have small p-values.

Table 5.11. ANOVA results for *Null Spill* response of P02.

Response 4 Null Spill **ANOVA for selected factorial model** **Analysis of variance table [Classical sum of squares - Type II]**						
Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**p-value Prob > F**	
Model	4.34E+09	35	1.24E+08	55735.45	< 0.0001	significant
A-Fare	224297	3	74765.66	33.58669	< 0.0001	
B-Demand	4.19E+09	2	2.09E+09	941054.1	< 0.0001	
C-Fleeting	1.49E+08	4	37162361	16694.31	< 0.0001	
AB	61150.97	6	10191.83	4.578436	0.0031	
AC	48922.6	12	4076.883	1.831443	0.1001	
BC	3795523	8	474440.4	213.131	< 0.0001	
Residual	53425.2	24	2226.05			
Cor Total	4.34E+09	59				

Table 5.12. ANOVA results for *Null Spill* response of A01.

Response 4 Null Spill **ANOVA for selected factorial model** **Analysis of variance table [Classical sum of squares - Type II]**						
Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**p-value Prob > F**	
Model	1.69E+08	35	4835016	126.6808	< 0.0001	significant
A-Fare	58771.8	3	19590.6	0.513288	0.6770	
B-Demand	1.09E+08	2	54437871	1426.311	< 0.0001	
C-Fleeting	23150964	4	5787741	151.6429	< 0.0001	
AB	122814.8	6	20469.13	0.536306	0.7753	
AC	443838.2	12	36986.52	0.969073	0.5023	
BC	36573418	8	4571677	119.7812	< 0.0001	
Residual	916005.7	24	38166.9			
Cor Total	1.7E+08	59				

We know that p-values that are greater than 0.10 indicate the insignificant model terms. In the case of *Null Spill*, we see that ANOVA yields different results for the data sets. In Table 5.10, the terms *Fare*, *Demand*, *Fleeting*, and their two-factor interactions are all significant for P01 data set. From Table 5.11 it is seen that the interaction term *Fare-*

Fleeting is insignificant for the model of P02, though by a negligible difference. For the A01 data set, the difference is greater; only the terms *Demand*, *Fleeting* and their interaction *Demand-Fleeting* are significant.

The influence of the *Fleeting* and *Demand* factor are straightforward, since *Fleeting* decides on the capacity of flights and *Demand* is directly related to the number passengers flying on flights. The discrepancy between the data sets is seems to be the *Fare* factor. When we examine the model graphs there is some detectable influence of *Fare* to *Null Spill* for each data set, though not a strong one. This is due to the considerable difference between the main effects of *Demand* and *Fare* on *Null Spill*. Especially for the A01 data set, change in fares is much less ineffective on *Null Spill*, compared to changes in demand.

<u>5.2.4.1. Fare-Demand Interactions.</u> The one-factor graphs of *Fare* and *Demand* from data sets P01, P02 and A01 are given in Figures 5.47 - 5.49. The corresponding *Demand*/*Fare* level and the *Fleeting* level are set to average in all graphs.

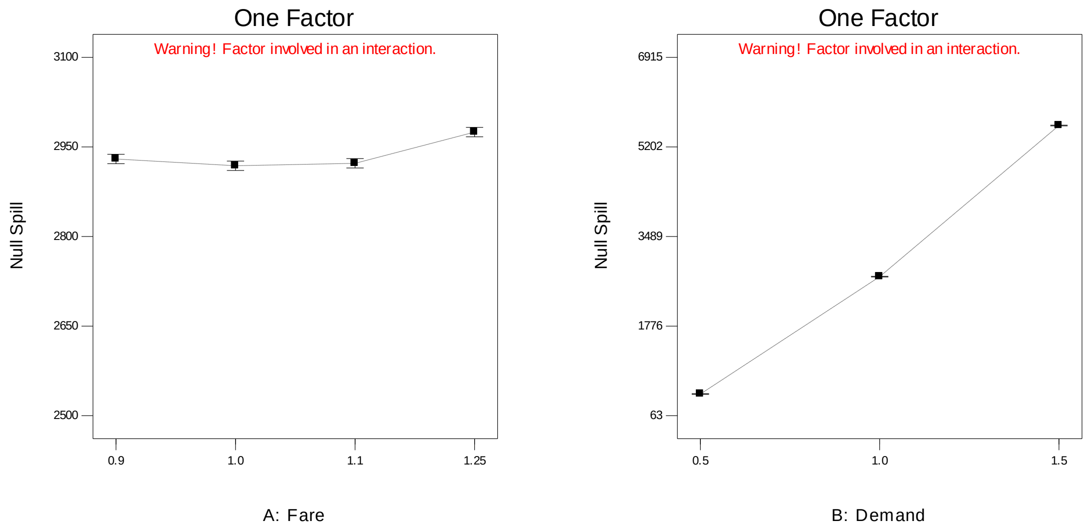

Figure 5.47. One-factor graphs for *Fare* and *Demand* to *Null Spill*, of P01.

From the one-factor graphs, it is observed that *Demand* is much more influential than *Fare*.

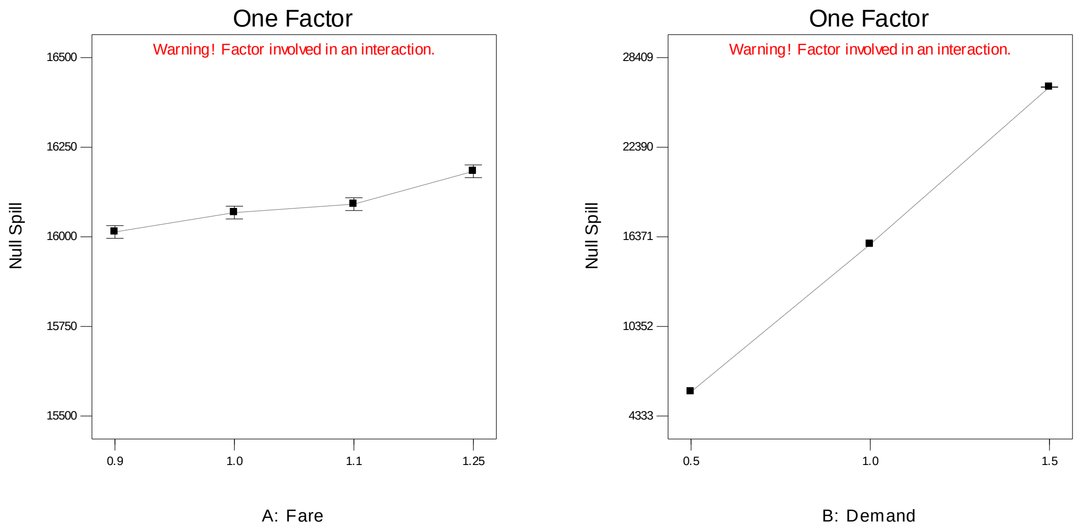

Figure 5.48. One-factor graphs for *Fare* and *Demand* to *Null Spill*, of P02.

The fluctuation in *Null Spill* caused by different *Fare* levels is in tens (as in A01) or hundreds (as in P01 and P02) at most, whereas *Demand* affects *Null Spill* in thousands. Therefore, we may conclude that the main effect of *Fare* is trivial for *Null Spill*.

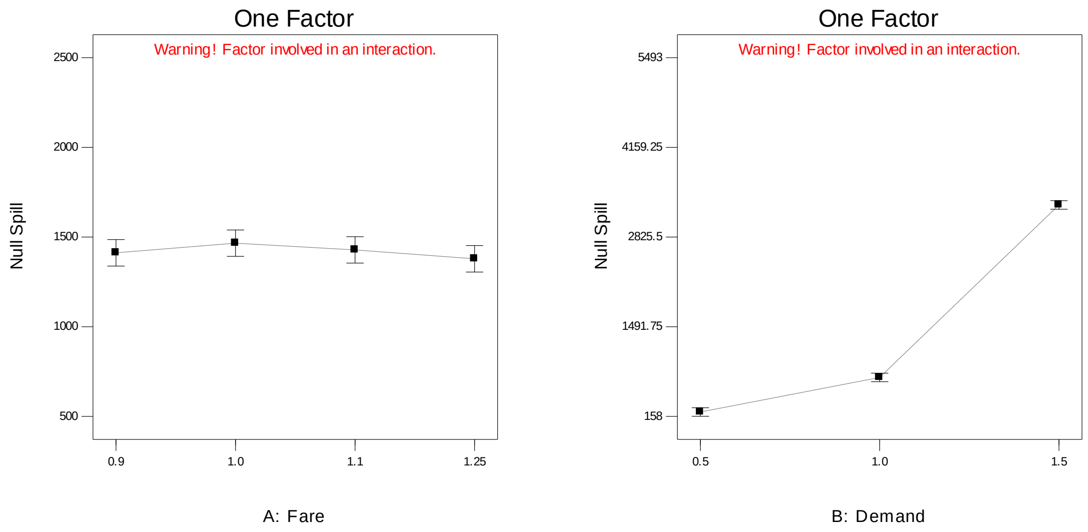

Figure 5.49. One-factor graphs for *Fare* and *Demand* to *Null Spill*, of A01.

When itinerary fares increase, it is expected that some of the passengers change their mind and prefer other airlines. This is closely related to the market share of an airline as well as customer behavior in a competitive market. However, in our model the passenger share model does not include competitive airlines and fare values only affect the choices of passengers when they decide among alternative itineraries of the airline only. We introduce market share in recapture rates as an arbitrary constant, therefore it is not affected by the change in fare values.

The main effect of *Demand* on *Null Spill* is clearly positive. Similar to the case of *Spill Cost* response variable, the trend appears more concave for the data set A01 from Figure 5.49. The number of passengers lost is less affected by the change in demand when the null spill numbers are already small.

The two-factor interactions graphs of *Fare* and *Demand* is provided in Figures 5.50 - 5.52.

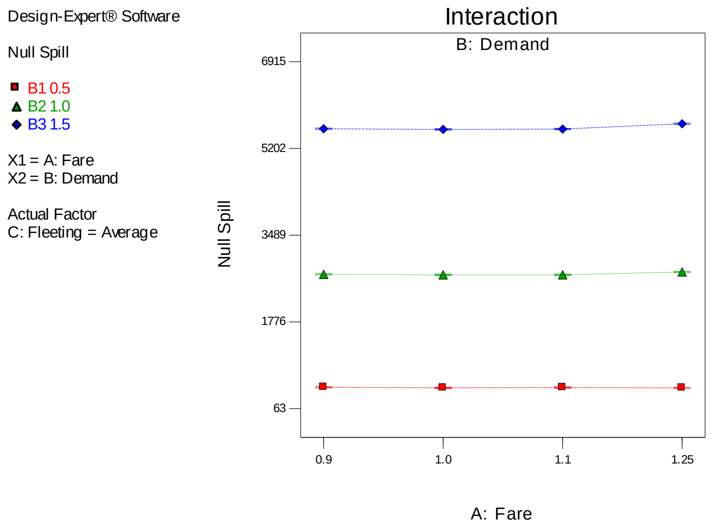

Figure 5.50. Two-factor interaction graph for *Fare* and *Demand* to *Null Spill*, of P01.

In Figure 5.50 we see the joint effect of the fare and the demand parameter to the null spill values of the itinerary-based fleet assignment model.

Form the Figures 5.50 - 5.52 it is observed that higher demand values lead to higher number of spilled passengers. On the other hand, even a 25% increase in fare values does not make an impact.

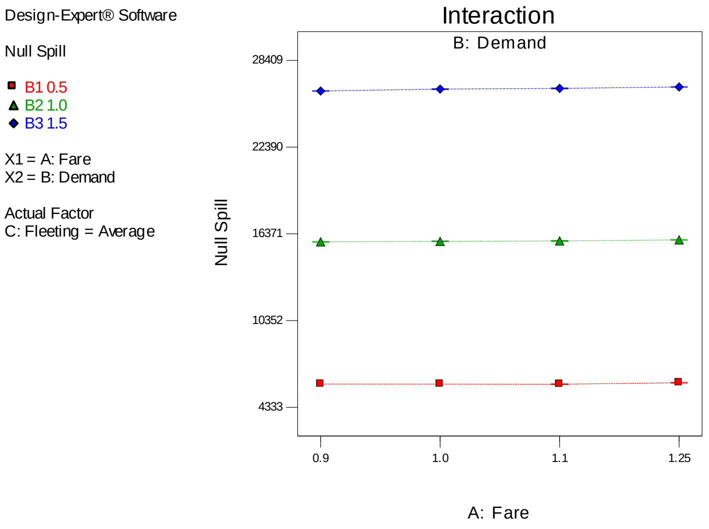

Figure 5.51. Two-factor interaction graph for *Fare* and *Demand* to *Null Spill*, of P02.

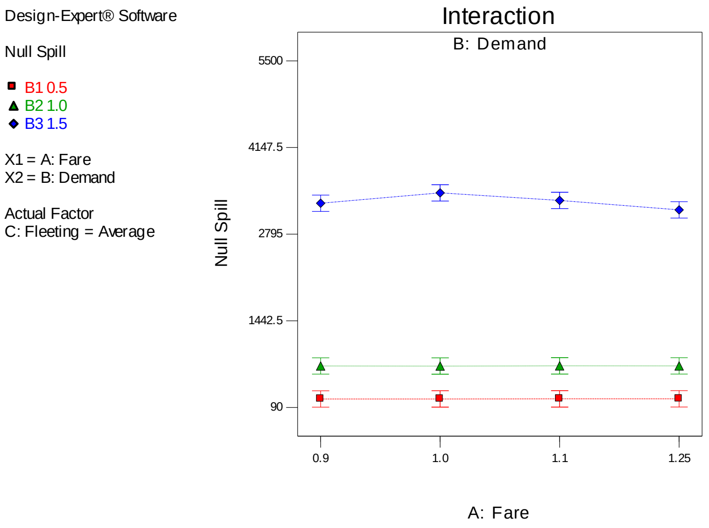

Figure 5.52. Two-factor interaction graph for *Fare* and *Demand* to *Null Spill*, of A01.

The two-factor interaction graphs of *Fare* and *Demand* show that *Demand* is very significant for *Null Spill*, while *Fare* does not cause a major effect with respect to *Demand*. Thus, in these graphs, we only observe the main effect of *Demand* on *Null Spill*.

5.2.4.2. Fleeting. The main effect of *Fleeting* to *Null Spill* follows a trend similar to *Spill Cost*, which can be seen from the one-factor graphs. The one-factor graphs of *Fleeting* for each data set are given in Figures 5.53 - 5.55.

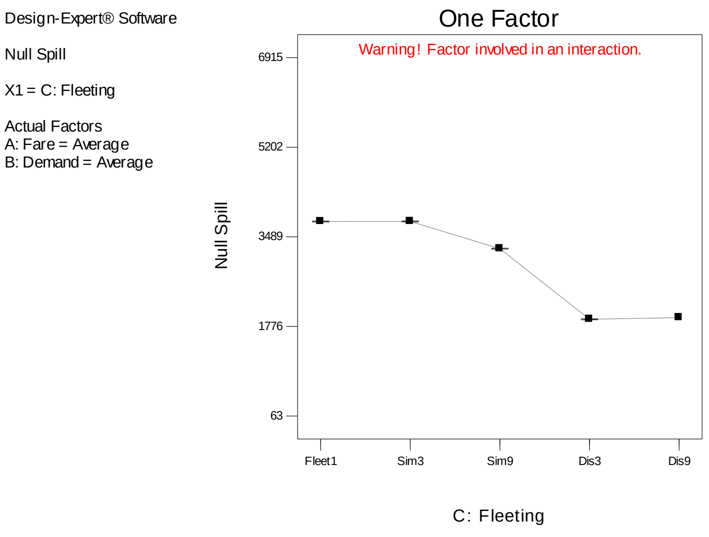

Figure 5.53. One-factor graph for *Fleeting* to *Null Spill*, of P01.

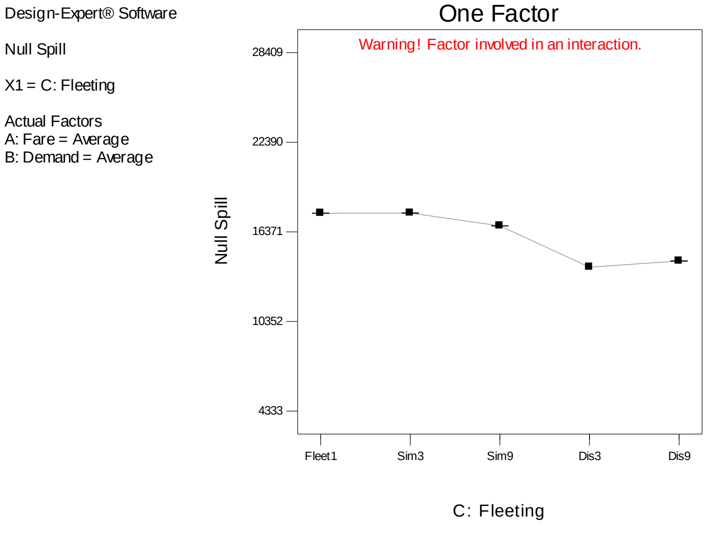

Figure 5.54. One-factor graph for *Fleeting* to *Null Spill*, of P02.

It is observed that when dissimilar fleet structures are utilized, the number of spilled passengers is significantly lower. There is no noticeable difference between using three or nine types of fleets. *Null Spill* is highest when only one type of fleet is used. Therefore, we may say that introducing some variety to the capacity of aircraft decreases spill significantly.

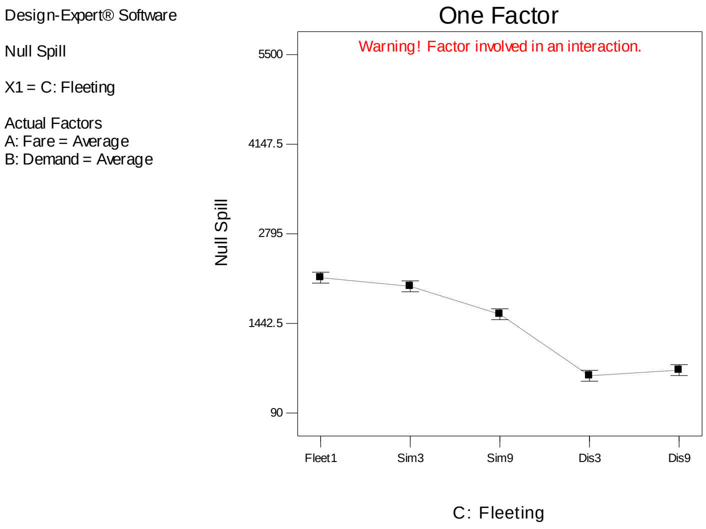

Figure 5.55. One-factor graph for *Fleeting* to *Null Spill*, of A01.

The two-factor interaction graphs point out how *Fleeting* affects *Null Spill* under different levels of *Fare* and *Demand*. First we analyze the interactions of *Fleeting* and *Fare*.

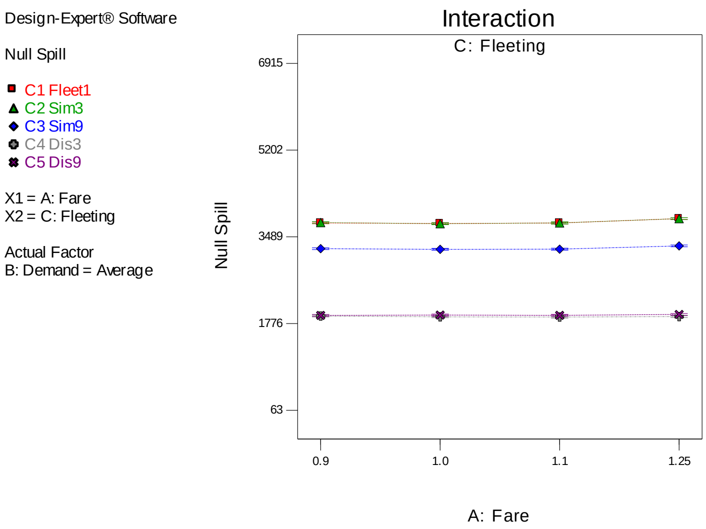

Figure 5.56. Two-factor interaction graph for *Fleeting* and *Fare* to *Null Spill*, of P01.

In Figures 5.56 - 5.58, the joint effect of the fleet structure and the fare parameter on the null spill values is presented for the three data sets.

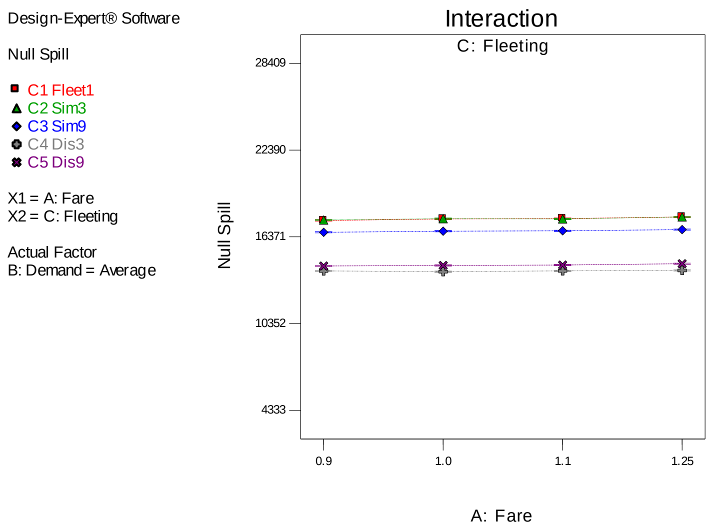

Figure 5.57. Two-factor interaction graph for *Fleeting* and *Fare* to *Null Spill*, of P02.

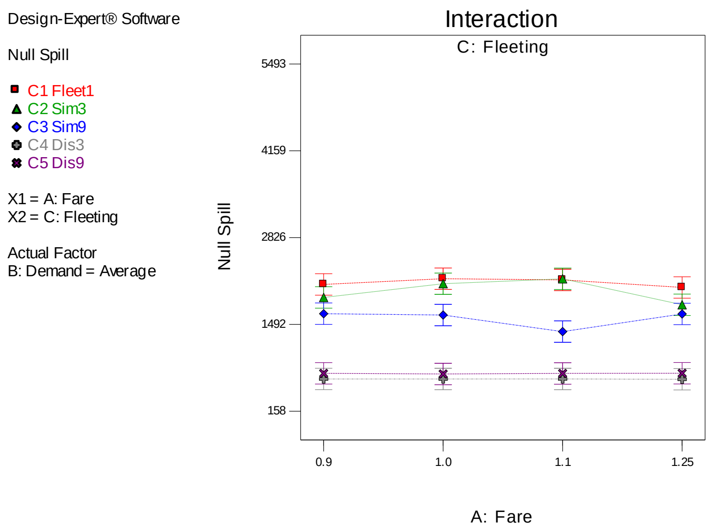

Figure 5.58. Two-factor interaction graph for *Fleeting* and *Fare* to *Null Spill*, of A01.

The interaction graphs provided above are consistent with the main effects of *Fare* and *Fleeting* on *Null Spill*. In each data set, dissimilar fleeting leads to lower spill values. There is no considerable impact of fares.

The two-factor interaction graphs of *Fleeting* and *Demand* are given in Figures 5.59 - 5.61. From the figures, we see that the combined effect of *Fleeting* and *Demand* differs slightly for each data set.

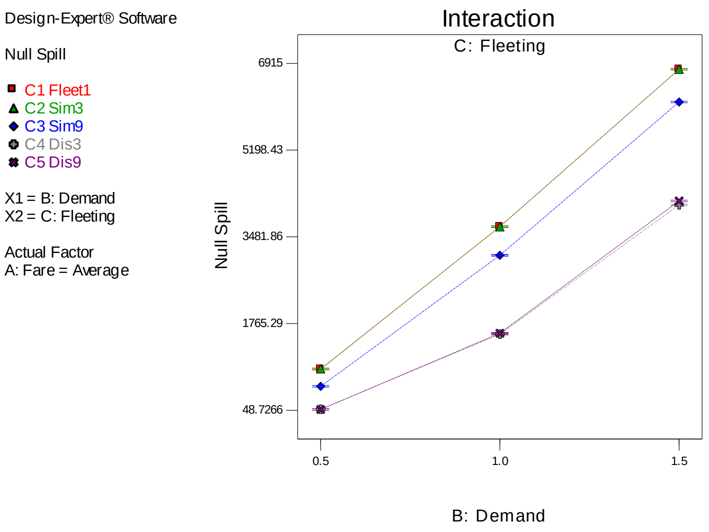

Figure 5.59. Two-factor interaction graph for *Fleeting* and *Demand* to *Null Spill*, of P01.

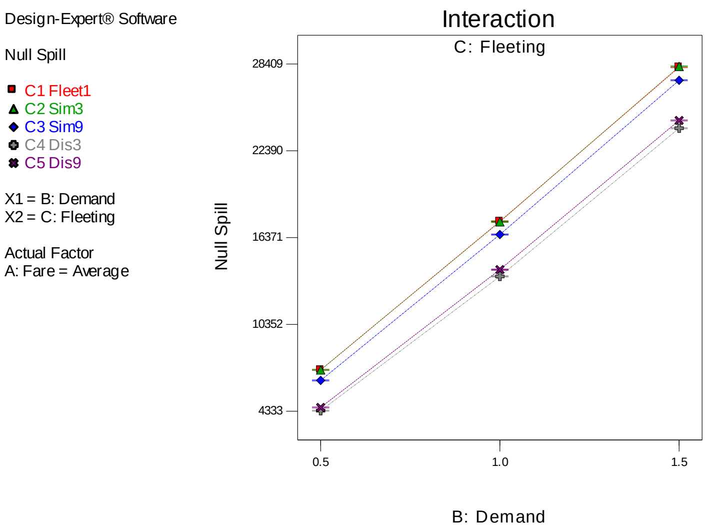

Figure 5.60. Two-factor interaction graph for *Fleeting* and *Demand* to *Null Spill*, of P02.

In Figure 5.59, the joint effect of *Fleeting* and *Demand* appears to be linear for the P02 data set. We observe from Figure 5.58 that, there's a slight bent on the lines of the dissimilar fleet structures, which suggests the *Null Spill* is less affected when demands decrease as opposed to when demands increase. In Figure 5.60 for the A01 data set, we see that the lines are much more concave-like. An increase in *Demand* results in a significant rise in *Null Spill* of similar fleet compositions and *Fleet1*. Relatively, the increase is much less for dissimilar fleet structures.

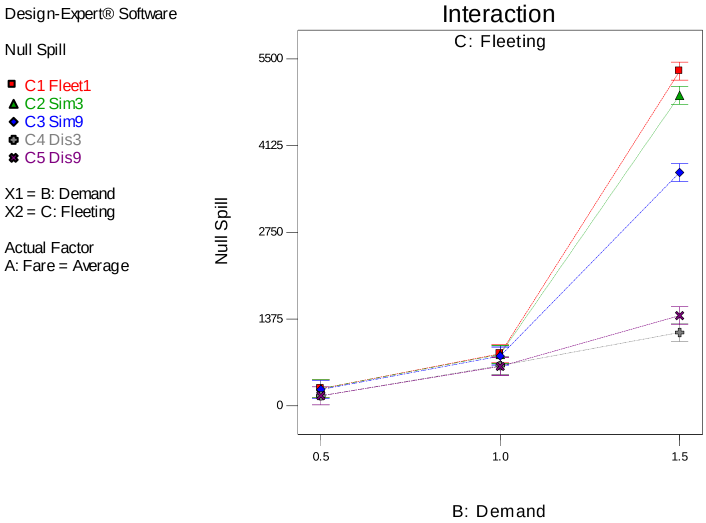

Figure 5.61. Two-factor interaction graph for *Fleeting* and *Demand* to *Null Spill*, of A01.

In addition, it is also noted that null spill values for each fleet structure are very close to each other when demand is lower. This is related to the unsaturated nature of demand data for the A01 data set.

5.2.5. Average Flight Occupancy

The number of passengers flying on flight *i* is

$$Q_i - \sum_{p \in P} \sum_{r \in P : r \neq p} \delta_i^p t_p^r + \sum_{p \in P} \sum_{r \in P : r \neq p} \delta_i^p b_r^p t_r^p \quad \forall i \in L. \tag{5.4}$$

The flight occupancy rate (also called passenger load factor) is the ratio of flying passengers to available seats on board a given flight. This ratio is between zero and one, indicating a flight is flown with an empty or full aircraft respectively.

The average occupancy rate (AOR) is the average of the load factor values of all flights. We give the results of the ANOVA tests for each data set in Tables 5.13 - 5.15. The F-values of the models indicate that they are significant.

From Table 5.13, we see that the terms *Fare*, *Demand*, *Fleeting*, and the two-factor interactions *Fare-Fleeting* and *Demand-Fleeting* are significant for the P01 data set.

Table 5.13. ANOVA results for *AOR* response of P01.

Response 5 Av. Occupancy ANOVA for selected factorial model Analysis of variance table [Classical sum of squares - Type II]						
Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**p-value Prob > F**	
Model	0.213172	35	0.006091	1571.212	< 0.0001	significant
A-Fare	0.00022	3	7.32E-05	18.88785	< 0.0001	
B-Demand	0.038692	2	0.019346	4990.783	< 0.0001	
C-Fleeting	0.17032	4	0.04258	10984.42	< 0.0001	
AB	3.33E-05	6	5.55E-06	1.431745	0.2437	
AC	0.000122	12	1.01E-05	2.6177	0.0216	
BC	0.003785	8	0.000473	122.0606	< 0.0001	
Residual	9.3E-05	24	3.88E-06			
Cor Total	0.213265	59				

Table 5.14. ANOVA results for *AOR* response of P02.

Response 5 Av. Occupancy ANOVA for selected factorial model Analysis of variance table [Classical sum of squares - Type II]						
Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**p-value Prob > F**	
Model	0.100772	35	0.002879	949.1911	< 0.0001	significant
A-Fare	1.12E-06	3	3.72E-07	0.122711	0.9458	
B-Demand	0.021492	2	0.010746	3542.687	< 0.0001	
C-Fleeting	0.077971	4	0.019493	6426.168	< 0.0001	
AB	4.1E-05	6	6.84E-06	2.254579	0.0724	
AC	0.000359	12	2.99E-05	9.870879	< 0.0001	
BC	0.000908	8	0.000113	37.41209	< 0.0001	
Residual	7.28E-05	24	3.03E-06			
Cor Total	0.100845	59				

From Table 5.14 for the P02 data set, we see that the terms *Demand, Fleeting,* and the two-factor interactions *Fare-Fleeting* and *Demand-Fleeting* are significant.

Table 5.15. ANOVA results for *AOR* response of A01.

Response 5 Av. Occupancy ANOVA for selected factorial model Analysis of variance table [Classical sum of squares - Type II]						
Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**p-value Prob > F**	
Model	1.909699	35	0.054563	35975.49	< 0.0001	significant
A-Fare	2.99E-05	3	9.98E-06	6.58	0.0021	
B-Demand	1.844203	2	0.922102	607979.11	< 0.0001	
C-Fleeting	0.02305	4	0.005763	3799.49	< 0.0001	
AB	7.77E-06	6	1.29E-06	0.85	0.5422	
AC	7.99E-05	12	6.66E-06	4.39	0.0010	
BC	0.042328	8	0.005291	3488.56	< 0.0001	
Residual	3.64E-05	24	1.52E-06			
Cor Total	1.909735	59				

In Table 5.15, the p-values of the terms indicate that the terms *Fare*, *Demand*, *Fleeting*, and the two-factor interactions *Fare-Fleeting* and *Demand-Fleeting* are significant. We can analyze the effect of each term in more detail using the model graphs.

5.2.5.1. Fare-Demand Interactions. The one-factor graphs of *Fare* and *Demand* from data sets P01, P02 and A01 are given in Figures 5.62 - 5.64. The corresponding *Demand*/*Fare* level and the *Fleeting* level are set to average in all graphs.

From the one-factor graphs of *Fare* and *Demand* terms, it can be observed that the fare parameter is not significant for the average load factor. This indicates the number of occupied seats is not directly related to the fare parameter.

It is also seen from the Figure 5.64 most clearly that higher itinerary demand values lead to the higher average flight occupancy rates. This is a reasonable outcome because when the total number of passengers who want to fly in a particular flight increases, the number of occupied seats also increases.

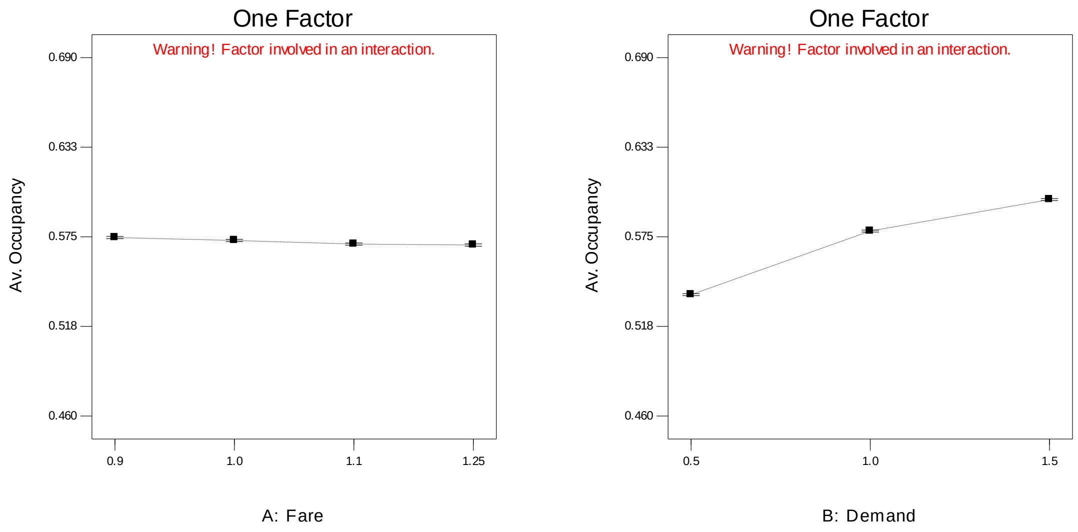

Figure 5.62. One-factor graphs for *Fare* and *Demand* to *AOR*, of P01.

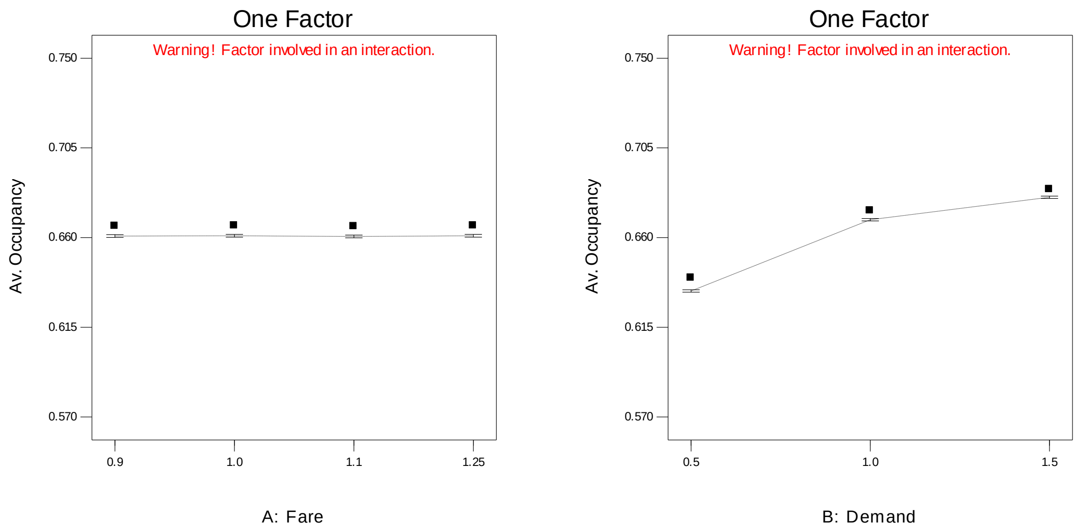

Figure 5.63. One-factor graphs for *Fare* and *Demand* to *AOR*, of P02.

From the one-factor graphs, it is seen that *Demand* has a positive effect on average occupancy rate, and *Fare* does not cause significant change.

We also observe that *AOR* starts to level off as demand values get higher. So higher demand values causes a smaller increase in average occupancy rate. For each data set, the trend of the average occupancy rate is similar.

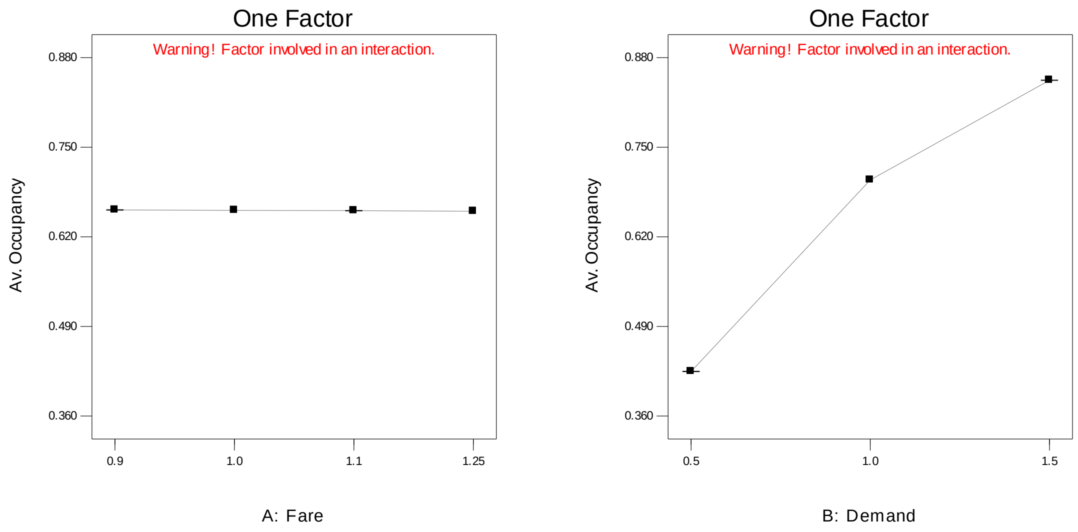

Figure 5.64. One-factor graphs for *Fare* and *Demand* to *AOR*, of A01.

The two-factor interactions graph of *Fare* and *Demand* is provided in Figures 5.65 - 5.67. These interaction graphs show the combined effect of the fare and the demand parameter on the load factor values of the itinerary- fleet assignment model, after a fleet type is assigned to each flight.

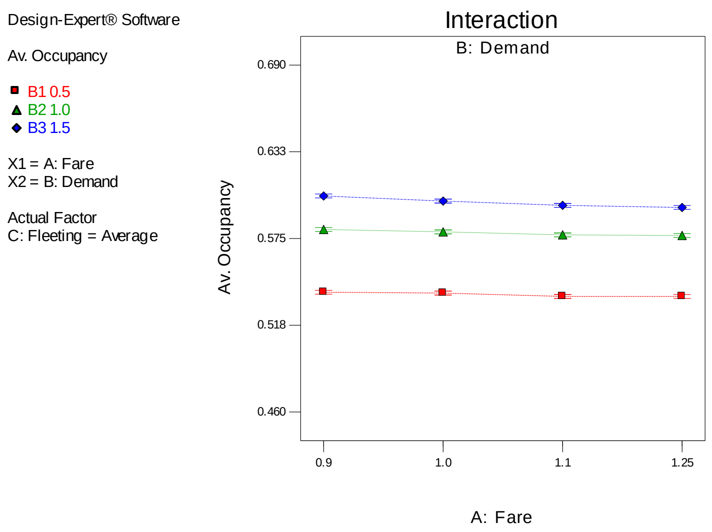

Figure 5.65. Two-factor interaction graph for *Fare* and *Demand* to *AOR*, of P01.

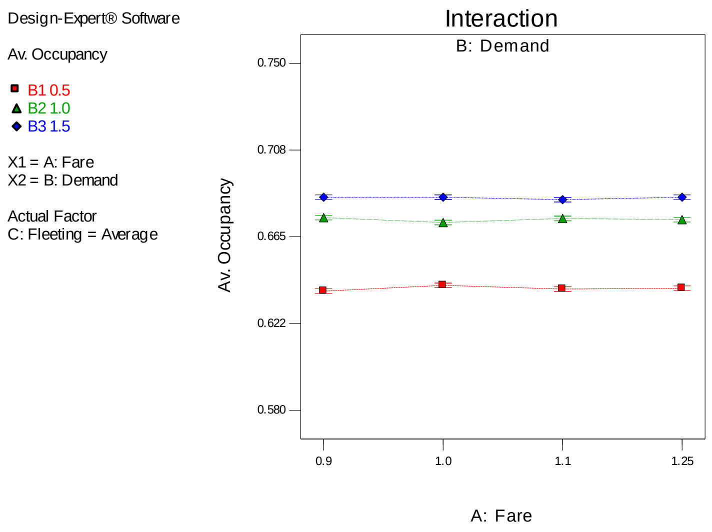

Figure 5.66. Two-factor interaction graph for *Fare* and *Demand* to *AOR*, of P02.

As the model suggests, there is no influence of the interaction of the two terms *Fare* and *Demand*. From the Figures 5.65 - 5.67, only the main effect of *Demand* can be observed.

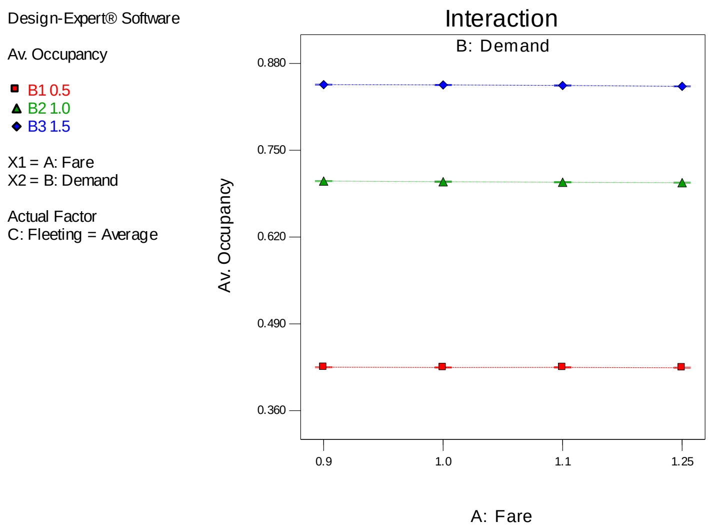

Figure 5.67. Two-factor interaction graph for *Fare* and *Demand* to *AOR*, of A01.

<u>5.2.5.2. Fleeting.</u> The main effect of *Fleeting* to *AOR* is significant, as seen in the one-factor graphs of *Fleeting* for each data set given in Figures 5.68 - 5.70.

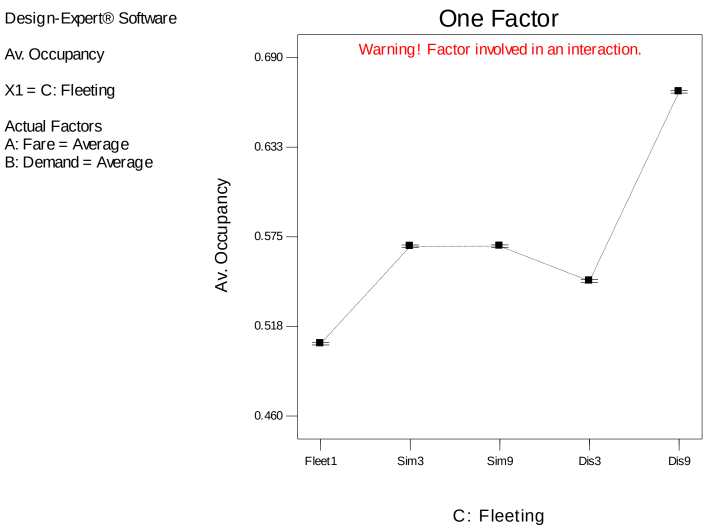

Figure 5.68. One-factor graph for *Fleeting* to *AOR*, of P01.

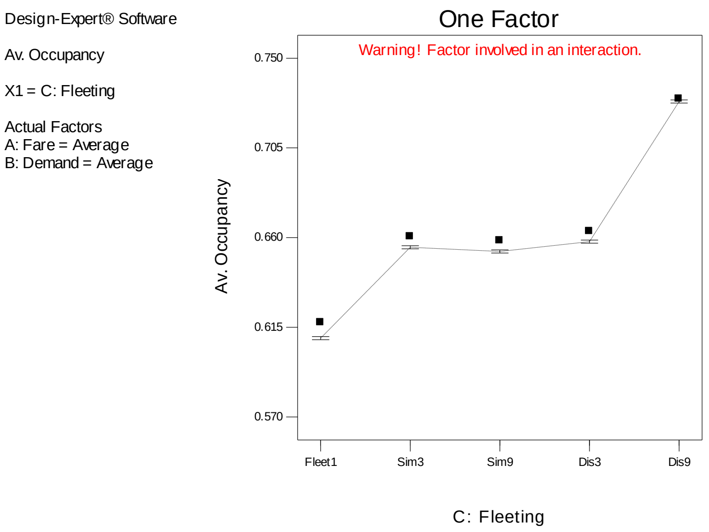

Figure 5.69. One-factor graph for *Fleeting* to *AOR*, of P02.

From the one-factor graphs given in Figures 5.68 and 5.69, we see that *Fleeting* has a considerable effect on average occupancy rate of all data sets.

The influence of the fleet structure is weaker for the A01 data set; however it is still visible and follows the same trend as the other two datasets.

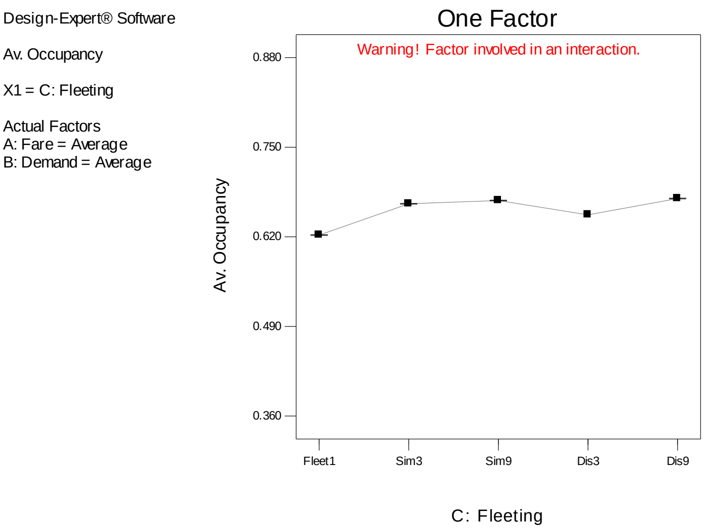

Figure 5.70. One-factor graph for *Fleeting* to *AOR*, of A01.

From the one-factor graphs we see that *Fleeting* has a considerable effect on average occupancy rate of all data sets. It is seen that *Dis9* surpasses all other fleet compositions with the highest average occupancy rate. Utilizing nine fleet types of different capacities allows IFAM to assign aircraft with capacities closer to the demand of the flights, and thus the number of empty seats drops.

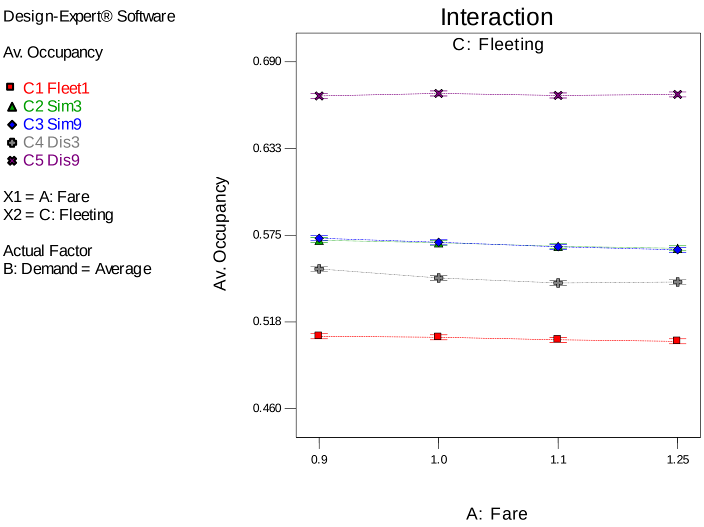

Figure 5.71. Two-factor interaction graph for *Fleeting* and *Fare* to *AOR*, of P01.

The two-factor interaction graphs indicate how *Fleeting* affects *AOR* under different levels of *Fare* and *Demand*. We analyze the interactions of *Fleeting* and *Fare* from the graphs provided in Figures 5.71 - 5.73.

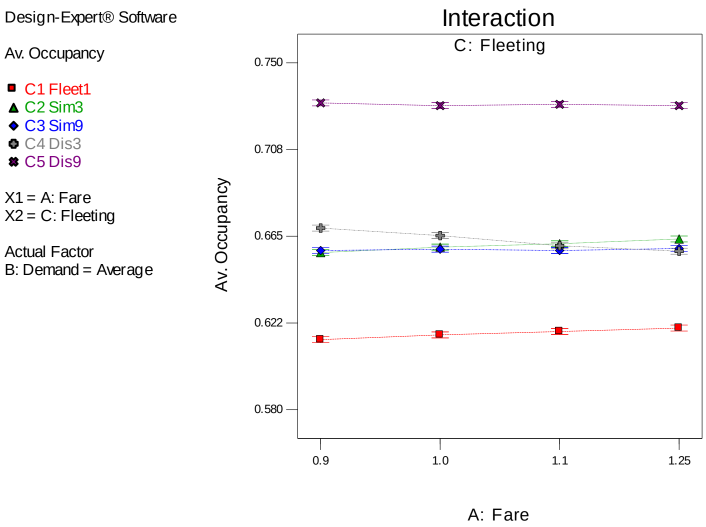

Figure 5.72. Two-factor interaction graph for *Fleeting* and *Fare* to *AOR*, of P02.

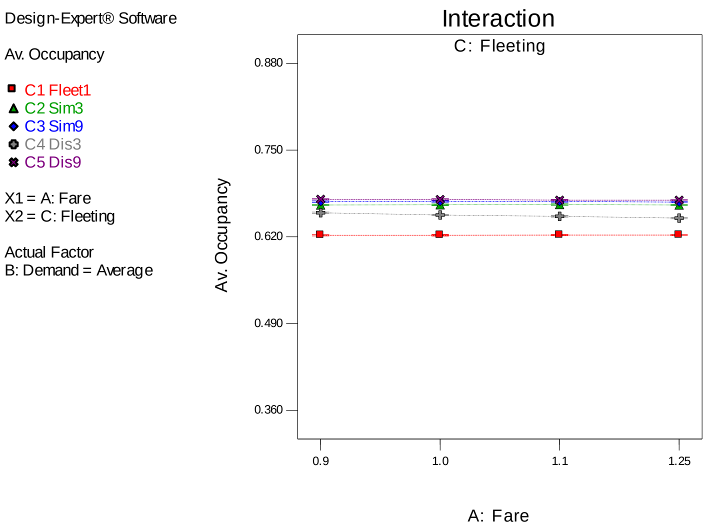

Figure 5.73. Two-factor interaction graph for *Fleeting* and *Fare* to *AOR*, of A01.

The two-factor interaction graphs for *Fleeting* and *Fare* confirms with the main effects of the individual terms. The effect of *Fare* remains trivial whereas *Fleeting* is significant for average occupancy rates. In each data set, level *Dis9* leads to highest average occupancy rates.

Next we provide the two-factor interaction graphs of *Fleeting* and *Demand* in Figures 7.74 - 5.76 for each data set.

From the two-factor graphs, it is observed that the joint effect of *Fleeting* and *Demand* is significant, in accordance with the analysis of variance results. However, the strength of the effect differs to some extent for each data set.

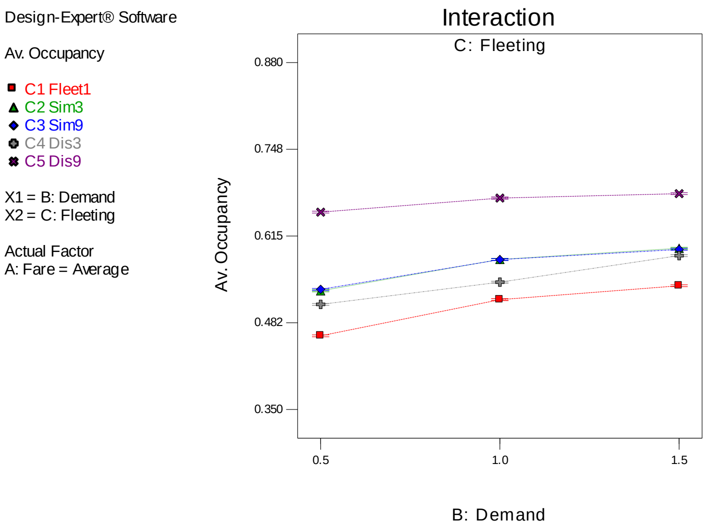

Figure 5.74. Two-factor interaction graph for *Fleeting* and *Demand* to *AOR*, of P01.

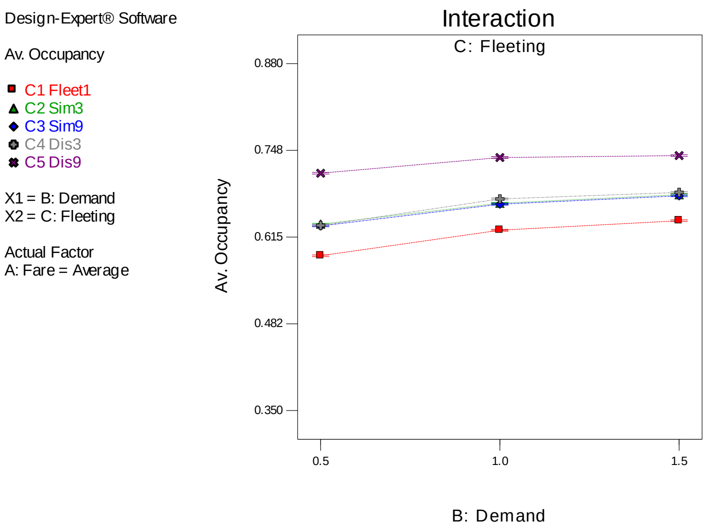

Figure 5.75. Two-factor interaction graph for *Fleeting* and *Demand* to *AOR*, of P02.

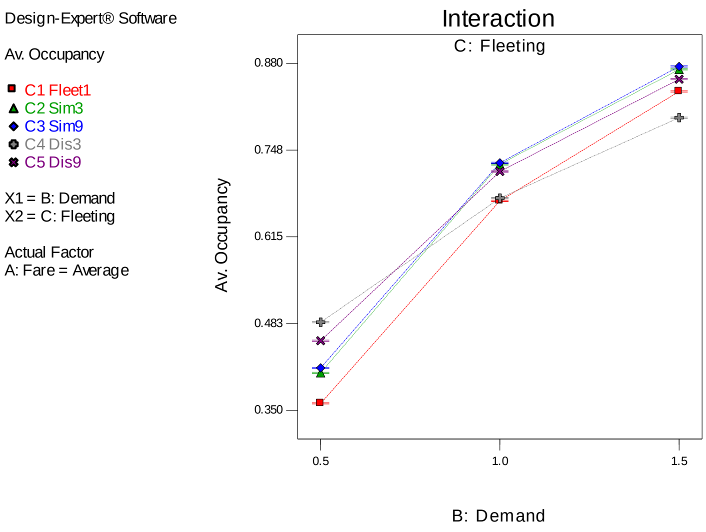

Figure 5.76. Two-factor interaction graph for *Fleeting* and *Demand* to *AOR*, of A01.

The two-factor interactions of *Fleeting* and *Demand* seem significant for all data sets. However, it is observed that the effect is strongest for the data set A01. In Figure 5.76, we also see the levels of *Fleeting* behave differently under different levels of *Demand*, though the difference between the values of *AOR* is negligible. This difference is much more visible in the data sets P01 and P02 in Figures 5.74 and 5.75 respectively.

5.2.6. Average Spill Rate

The average spill rate is another indicator of the spill performance of an airline. The spill rate is the ratio of the spill to the demand of an itinerary. This ratio is calculated for each itinerary in the data set and then the average of the ratios is defined as the response variable Average Spill Rate (ASR).

The results of the ANOVA tests for each data set are provided in Tables 5.16 - 5.18. The F-values of the models point out that they are significant.

In all data sets, the terms *Demand*, *Fleeting*, and the two-factor interaction *Demand-Fleeting* are significant.

Table 5.16. ANOVA results for *ASR* response of P01.

Response 6 Av. Spill Rate ANOVA for selected factorial model Analysis of variance table [Classical sum of squares - Type II]						
Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**p-value Prob > F**	
Model	0.249616	35	0.007132	22.37	< 0.0001	significant
A-Fare	0.00144	3	0.00048	1.50	0.2387	
B-Demand	0.176457	2	0.088228	276.68	< 0.0001	
C-Fleeting	0.060612	4	0.015153	47.52	< 0.0001	
AB	0.002271	6	0.000379	1.19	0.3463	
AC	0.003809	12	0.000317	1.00	0.4813	
BC	0.005027	8	0.000628	1.97	0.0951	
Residual	0.007653	24	0.000319			
Cor Total	0.25727	59				

Table 5.17. ANOVA results for *ASR* response of P02.

Response 6 Av. Spill Rate ANOVA for selected factorial model Analysis of variance table [Classical sum of squares - Type II]						
Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**p-value Prob > F**	
Model	0.310197	35	0.008863	4661.21	< 0.0001	significant
A-Fare	7.79E-05	3	2.6E-05	13.66	< 0.0001	
B-Demand	0.22305	2	0.111525	58654.53	< 0.0001	
C-Fleeting	0.083972	4	0.020993	11040.86	< 0.0001	
AB	0.000269	6	4.48E-05	23.54	< 0.0001	
AC	4.12E-05	12	3.43E-06	1.80	0.1056	
BC	0.002788	8	0.000348	183.27	< 0.0001	
Residual	4.56E-05	24	1.9E-06			
Cor Total	0.310243	59				

A discrepancy is observed from the data set P02 in Figure 5.17, where the term *Fare* and its interaction *Fare-Demand* also appear statistically significant. However, the effect of the two terms is not visible in model graphs.

Table 5.18. ANOVA results for *ASR* response of A01.

Response 6 Av. Spill Rate ANOVA for selected factorial model Analysis of variance table [Classical sum of squares - Type II]						
Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**p-value Prob > F**	
Model	0.105515	35	0.003015	97.01	< 0.0001	0.105515
A-Fare	5.32E-05	3	1.77E-05	0.57	0.6394	5.32E-05
B-Demand	0.053622	2	0.026811	862.79	< 0.0001	0.053622
C-Fleeting	0.018183	4	0.004546	146.28	< 0.0001	0.018183
AB	9.47E-05	6	1.58E-05	0.51	0.7963	9.47E-05
AC	0.000355	12	2.96E-05	0.95	0.5151	0.000355
BC	0.033207	8	0.004151	133.57	< 0.0001	0.033207
Residual	0.000746	24	3.11E-05			0.000746
Cor Total	0.106261	59				0.106261

This variation originates from the high demand values of P02. The market demand values are so high that when we calculate the demand of itineraries by the share model, the change in fares causes a bigger difference on the itinerary demands. The variation of itinerary demands is smaller in the data sets P01 and A01. Thus, for P02 we observe a change in demand at each level of *Fare*, independent of the *Demand* factor. The *Fare* factor seems significant due to its effect on demand values outside the context of the experimental design.

5.2.6.1. Fare-Demand Interactions. The one-factor graphs of *Fare* and *Demand* from data sets P01, P02 and A01 are provided in Figures 5.77 - 5.79. The corresponding *Demand*/*Fare* level and the *Fleeting* level are set to average in all graphs.

The main effect of *Fare* is trivial for *ASR* for all data sets. This result is in accordance with the *Null Spill* response variable. In this case, an increase or decrease in the itinerary fare value does not influence the average number of passengers that are spilled form the itinerary.

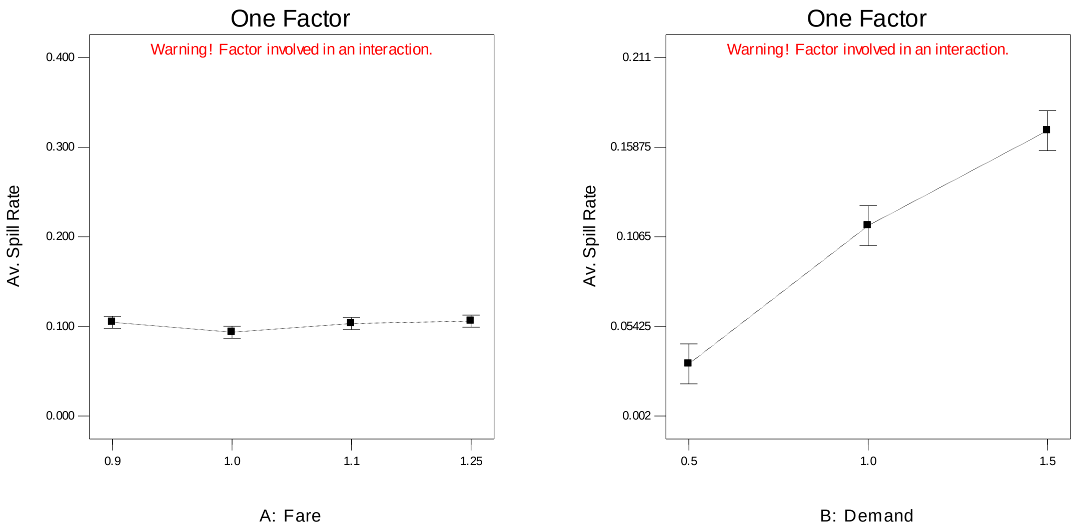

Figure 5.77. One-factor graphs for *Fare* and *Demand* to *ASR*, of P01.

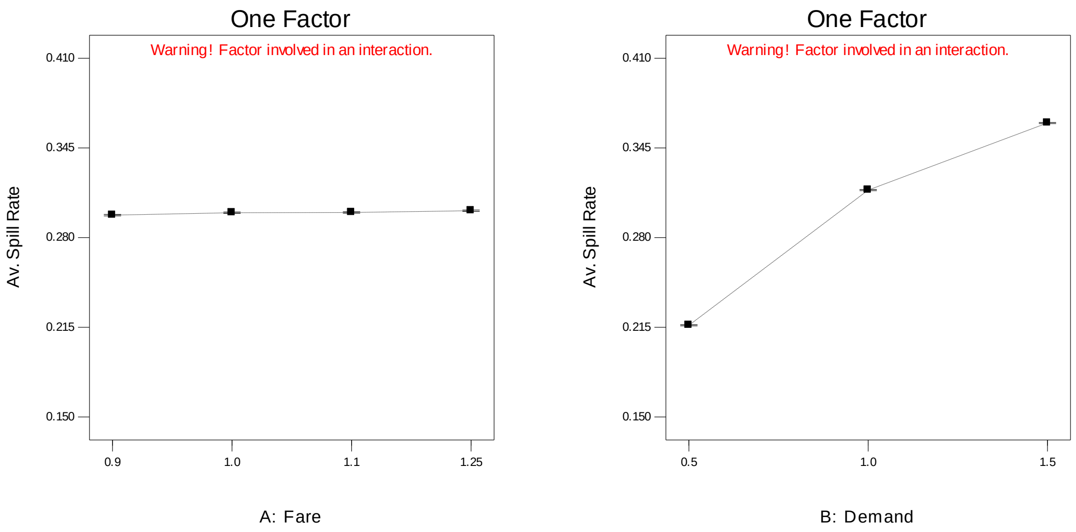

Figure 5.78. One-factor graphs for *Fare* and *Demand* to *ASR*, of P02.

As seen from the one-factor graphs, the main effect of *Demand* on *ASR* is plainly positive and close to linear. The trend appears more concave for the data set A01 from Figure 5.79, in accordance with the results of *Spill Cost* and *Null Spill* responses. The average spill rate remains close to zero when demand values are low.

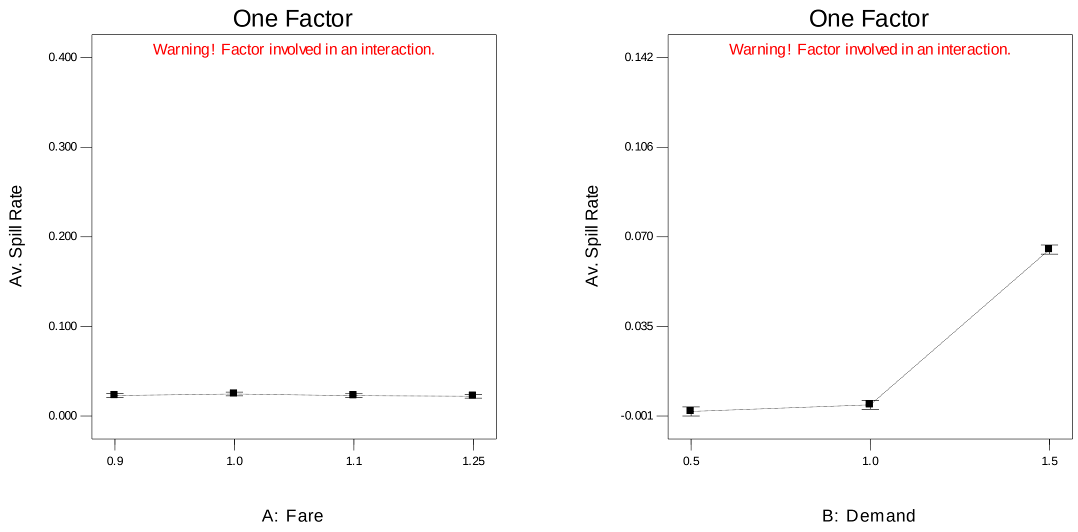

Figure 5.79. One-factor graphs for *Fare* and *Demand* to *ASR*, of A01.

In Figure 5.79, it is observed that the average spill rates are close to zero when demand values are low.

Next we analyze the joint effect of *Fare* and *Demand*. In Figures 5.80 - 5.81, two-factor interactions graphs of *Fare* and *Demand* are given.

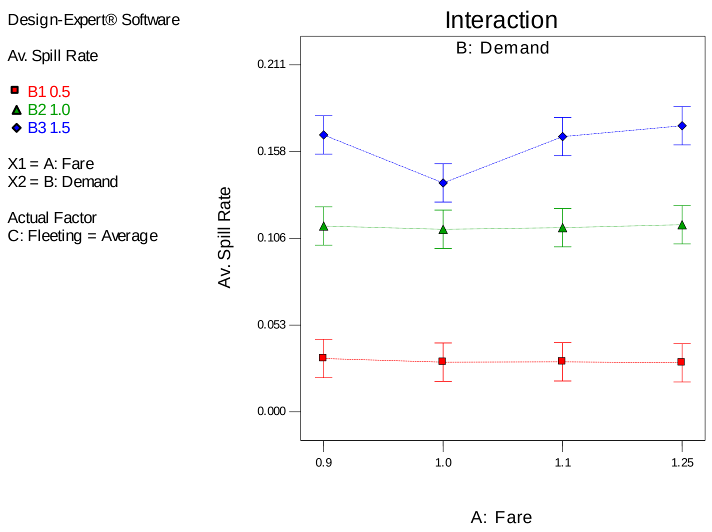

Figure 5.80. Two-factor interaction graph for *Fare* and *Demand* to *ASR*, of P01.

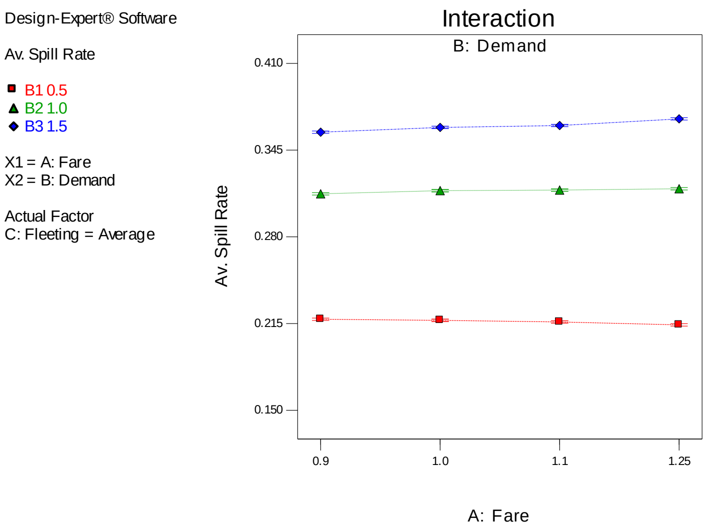

Figure 5.81. Two-factor interaction graph for *Fare* and *Demand* to *ASR*, of P02.

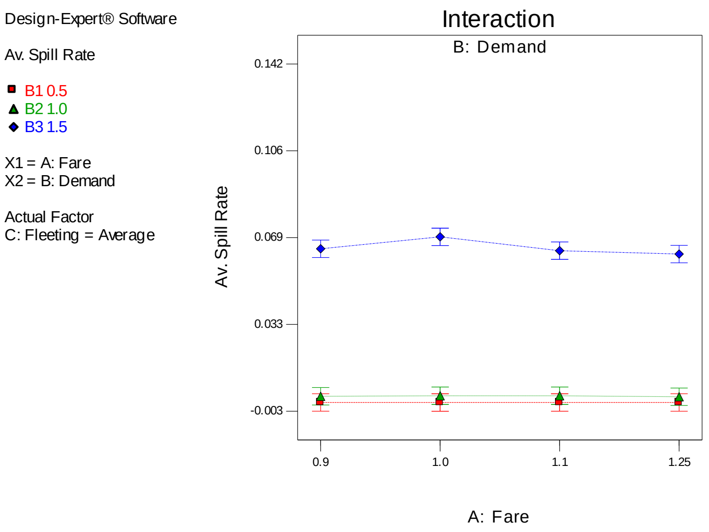

Figure 5.82. Two-factor interaction graph for *Fare* and *Demand* to *ASR*, of A01.

The two-factor interaction graphs of *Fare* and *Demand* show that *Demand* is very significant for *ASR*, while effect of *Fare* remains minimal. The main effect of *Demand* can be observed also from these graphs.

5.2.6.2. Fleeting. The main effect of *Fleeting* to *ASR* is similar to its effect on *Spill Cost* and *Null Spill*, as seen from the one-factor graphs.

The one-factor graphs of *Fleeting* for each data set are provided in Figures 5.83 - 5.85. From these figures; we see that the fleet structure of an airline is clearly significant for the average spill rates. This indicates capacity management decisions influence the performance of an airline.

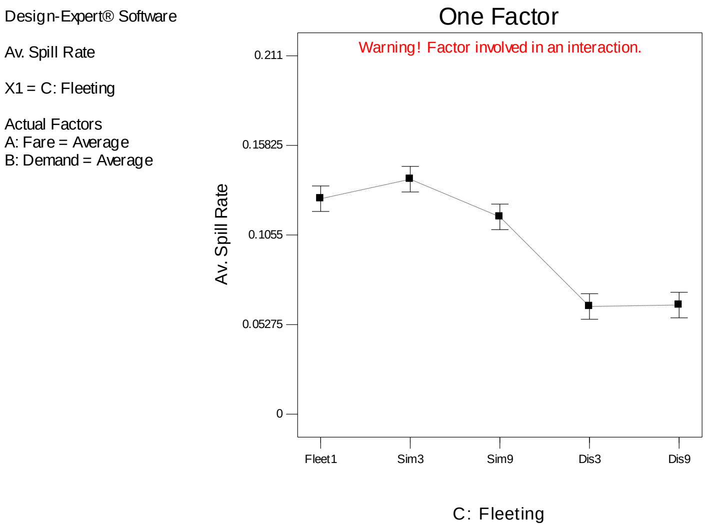

Figure 5.83. One-factor graph for *Fleeting* to *ASR*, of P01.

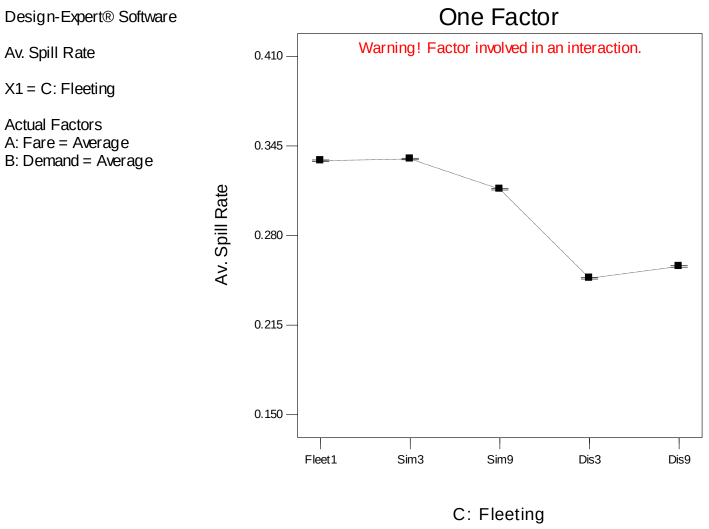

Figure 5.84. One-factor graph for *Fleeting* to *ASR*, of P02.

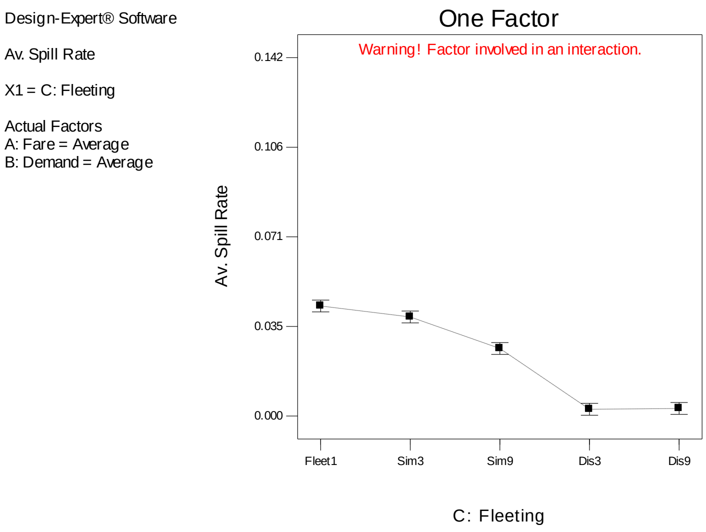

Figure 5.85. One-factor graph for *Fleeting* to *ASR*, of A01.

From Figures 5.83 – 5.85, we observe that the average spill rate is significantly lower when a dissimilar fleet structure is utilized. It is also seen that the number of fleet types is irrelevant, since the *ASR* of *Dis3* and *Dis9* is nearly the same.

Next, the two-factor interactions of *Fleeting* are analyzed; we observe the main effect of *Fleeting* as well as its joint effect with *Fare* and *Demand* on *ASR*. The two-factor interactions of *Fleeting* and *Fare* are given in Figures 5.86 - 5.88.

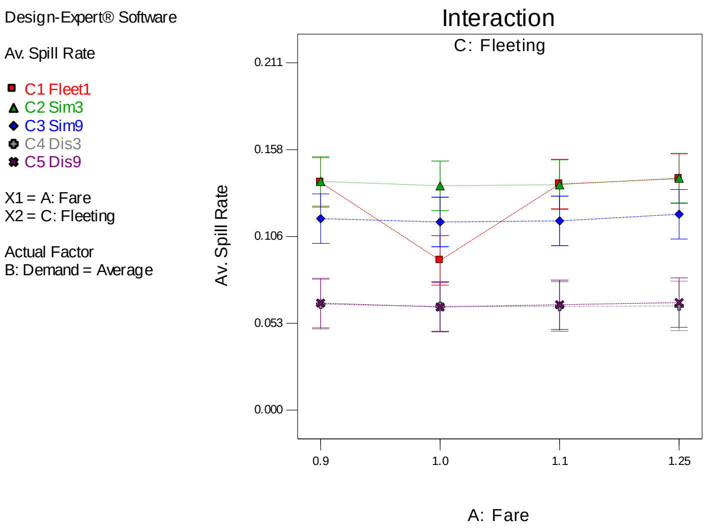

Figure 5.86. Two-factor interaction graph for *Fleeting* and *Fare* to *ASR*, of P01.

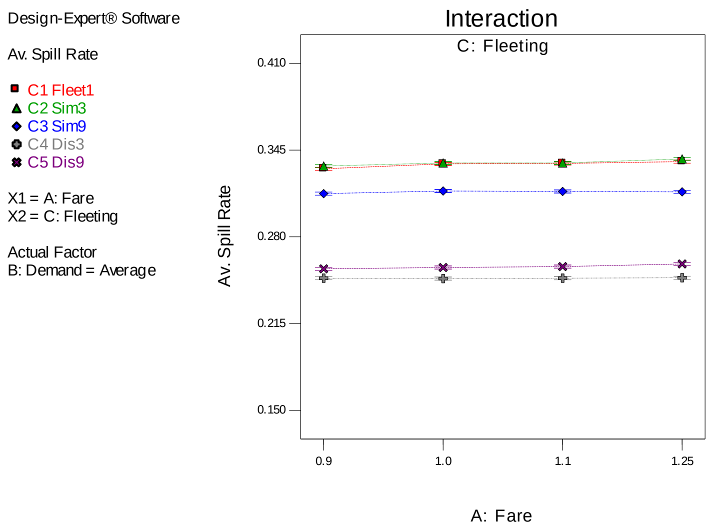

Figure 5.87. Two-factor interaction graph for *Fleeting* and *Fare* to *ASR*, of P02.

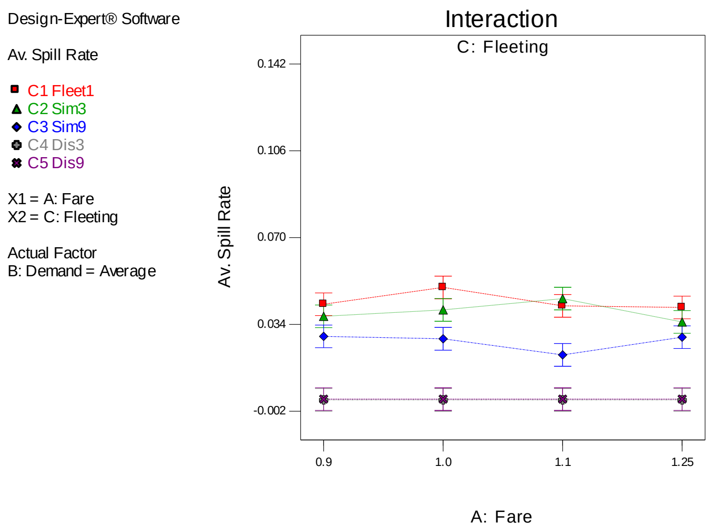

Figure 5.88. Two-factor interaction graph for *Fleeting* and *Fare* to *ASR*, of A01.

The interaction graphs provided above are consistent with the main effects of *Fleeting* on *ASR*. In each data set, dissimilar fleeting leads to lower average spill rate.

At different *Fare* levels some irregular fluctuation is observed in similar fleet compositions, though in small scale. Since the average spill rate values are between zero and one, these fluctuations are visible in the two- factor graphs.

The two-factor interaction graphs of *Fleeting* and *Demand* are given in Figures 5.89 - 5.91. For each data set the interaction effects of *Fleeting* and *Demand* are seen to be significant for the average spill rates.

From the figures, we see that the combined effect of *Fleeting* and *Demand* differs slightly for each data set. The trends are similar to the responses *Spill Cost* and *Null Spill*.

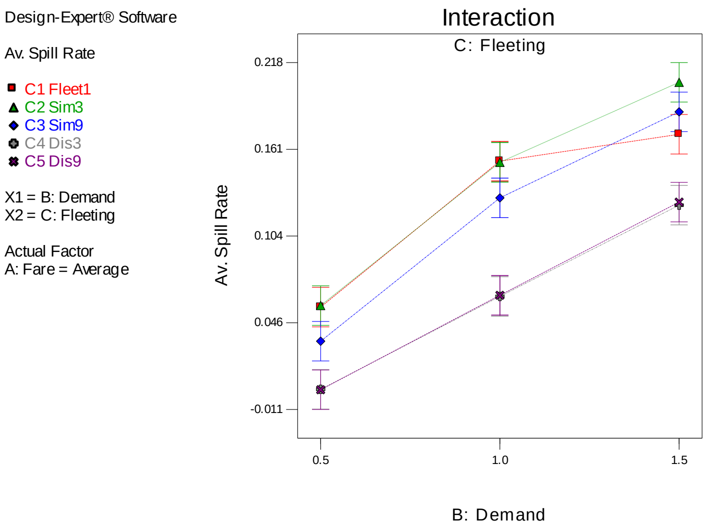

Figure 5.89. Two-factor interaction graph for *Fleeting* and *Demand* to *ASR*, of P01.

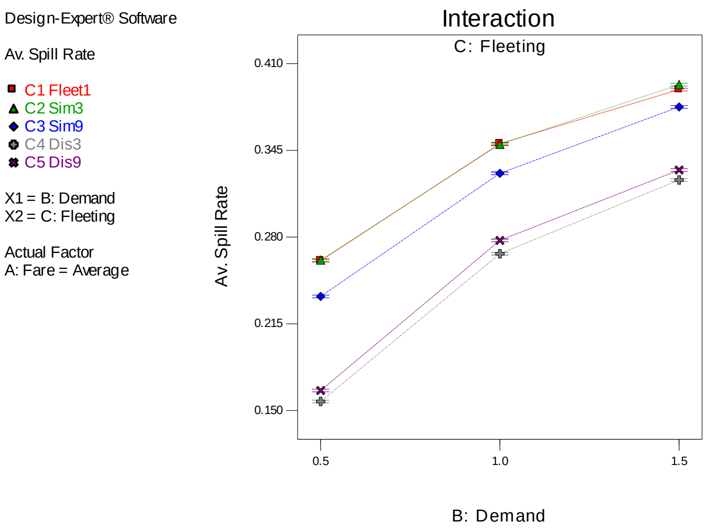

Figure 5.90. Two-factor interaction graph for *Fleeting* and *Demand* to *ASR*, of P02.

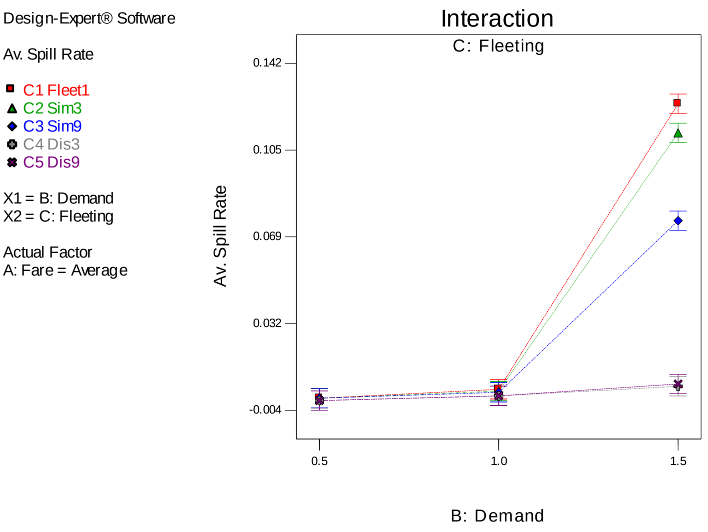

Figure 5.91. Two-factor interaction graph for *Fleeting* and *Demand* to *ASR*, of A01.

In Figure 5.91 for the A01 data set, an increase in *Demand* results in a significant rise in *ASR* of similar fleet compositions and *Fleet1*. Relatively, the increase is much less for dissimilar fleet structures. The effect appears more linear for the data sets P01 and P02.

6. CONCLUSION

The itinerary-based fleet assignment model is a mixed integer programming model that is able to capture network effects and more accurately estimate spill and recapture of passengers when fleet types are assigned to flights. In this thesis, we study the effect of fares, demands and the fleet structure on the fleet assignment. Our multi-factorial design defines levels for each factor, representing a percent change in the fare and demand parameters, and different fleet compositions for the fleet structure. Three different dataset are utilized for the experiment, where each dataset has different number of itineraries and different density of passenger demand. We solve IFAM for each level of the three factors and perform analysis of variance on the results.

It is observed that the main and interaction effects of the factors are mostly consistent for each dataset. Fare and demand values are clearly significant for the fleet assignment decision. The spill costs rise when fares and demands increase, though not much as the unconstrained revenue. Thus, profit also increases when fares and demands get higher. Demand also has a significant positive effect on the operating cost, null spill, and the flight occupancy and spill rates. Number of spilled passengers and average spill rates increase as demands get higher. As a result, we may conclude that itinerary fare is significant chiefly for the monetary output (i.e. profit, spill cost), whereas itinerary demand value has a major impact on the overall performance of fleet assignment. Hence, demand management appears to be a rather vital process that influences the fleet assignment decision.

There is a significant effect of the fleeting factor, which follows a similar trend for each dataset. For each response variable it is observed that dissimilar fleet structures are preferable, though the number of fleet types that are in the fleet mix is trivial. Introducing fleet diversity (regarding fleet capacity) enables airlines to meet the fluctuating demand with minimum loss, leading to smaller spill values and higher occupancy rates. Thus, airlines may lower the fleet assignment cost and obtain higher profit when they utilize a fleet composition with diverse capacity. Currently, the Turkish airline employs the *Fleet1* fleet structure for the P01 dataset. Our study indicates significant savings can be made if a dissimilar fleeting such as *Dis3* is utilized.

However, using multiple fleet types is not always desirable, considering the subsequent operations that follow fleet assignment. Since each fleet type has its own maintenance and crew requirements, airlines must also consider those constraints besides the capacity during fleet planning. Therefore, airlines should find a compromise between the two so that the cost of fleet assignment remains low and the routing and crew planning stages does not become complicated, and overly costly.

Some remarks can be made regarding the small discrepancies between datasets. Though many properties of the datasets are not comparable such as network flight construction and connectivity, we may compare the quantifiable characteristic demand data. An accurate demand forecast leads to an unsaturated network, meaning that the spill is lower and the capacity is more able to meet the demand. The A01 dataset consists of booked demand data, consequently the average number of spilled passengers, spill costs and rates are lower. From the results of the A01 dataset, it is seen that profit of an unsaturated network is more sensitive to demand. Moreover, the effect of demand appears concave for spill costs, null spill values and spill rates. These are close in numbers when demands are low in unsaturated networks. Similarly, the difference between fleet compositions is also trivial when demands are low. The main and interaction effects of the factors are observed at most when demands are higher, and the spill is much greater in numbers. For saturated networks such as P01 and P02 that result in noticeable spill numbers, profit and costs are more immune to changes in fares and demand. Moreover, the effects of the factors on spill related outputs mostly appear linear. The difference between levels can be clearly observed even when demands are low.

The multi factorial experiment allows us to examine how fleet assignment outputs are affected when itinerary fares and demands are altered. Nonetheless, it is not able to fully capture the interaction between itinerary fares and demands. The presented analysis shows that both fare and demand are important parameters for the fleet assignment problem. As future research, the demand parameter can be defined as a function of fare in order to fully integrate the demand modeling phase to fleet assignment. Thus airlines would be able to adjust fare values to experiment on the demand and the fleet assignment solution. However, it must be noted that if the fare parameter becomes a variable, the problem turns into a non-linear model.

Furthermore, another aspect of revenue management can also be incorporated to fleet assignment by introducing fare classes. In this case, passenger behavior must be exquisitely modeled to accurately predict the recapture rates, so that shifts between different fare classes are also included. Thus, airlines will be able to observe how demand changes with different pricing strategies and accurately calculate spill and recapture to obtain the optimum fleet assignment.

APPENDIX A: RESULTS OF THE EXPERIMENTAL DESIGN

Table A.1. Results of P01.

Factor A: Fare	Factor B: Demand	Factor C: Fleeting	Profit	Operating Cost	Spill Cost	Null Spill	Av. Occupancy	Av. Spill Rate
0.9	0.5	Fleet1	1365713	695250	218738	865	0.464	0.059
0.9	0.5	Sim3	1423320	636985	219396	887	0.533	0.061
0.9	0.5	Sim9	1497574	646316	135811	530	0.536	0.036
0.9	0.5	Dis3	1572052	688787	18862	73	0.515	0.003
0.9	0.5	Dis9	1630014	632263	17424	65	0.649	0.003
0.9	1.0	Fleet1	1570530	695250	955961	3664	0.519	0.152
0.9	1.0	Sim3	1623957	641448	956224	3662	0.583	0.152
0.9	1.0	Sim9	1760426	658169	803141	3115	0.584	0.129
0.9	1.0	Dis3	2098409	715144	408165	1576	0.551	0.066
0.9	1.0	Dis9	2146854	666875	407991	1555	0.668	0.066
0.9	1.5	Fleet1	1664074	695250	1805254	6762	0.541	0.204
0.9	1.5	Sim3	1714921	643589	1806066	6762	0.599	0.204
0.9	1.5	Sim9	1873736	662022	1628782	6118	0.599	0.184
0.9	1.5	Dis3	2379978	718951	1065555	4122	0.592	0.124
0.9	1.5	Dis9	2390963	683540	1090086	4190	0.685	0.126
1.0	0.5	Fleet1	1702734	695250	256321	854	0.463	0.056
1.0	0.5	Sim3	1760387	636924	256995	856	0.532	0.057
1.0	0.5	Sim9	1850248	646818	157240	519	0.535	0.033
1.0	0.5	Dis3	1944300	689120	20885	73	0.513	0.003
1.0	0.5	Dis9	2003302	631318	19685	65	0.651	0.002
1.0	1.0	Fleet1	1968576	695250	1153979	3649	0.519	0.152
1.0	1.0	Sim3	2021834	641825	1154036	3642	0.58	0.15
1.0	1.0	Sim9	2191497	659852	966396	3095	0.58	0.127
1.0	1.0	Dis3	2611023	717960	488791	1548	0.543	0.063
1.0	1.0	Dis9	2660522	665277	491977	1587	0.675	0.063
1.0	1.5	Fleet1	2084410	695250	2201682	6748	0.54	0.065
1.0	1.5	Sim3	2135380	643489	2202472	6748	0.598	0.202
1.0	1.5	Sim9	2337362	663669	1980191	6111	0.596	0.183
1.0	1.5	Dis3	2968300	722775	1290068	4104	0.584	0.123
1.0	1.5	Dis9	2974594	684335	1322267	4177	0.681	0.123
1.1	0.5	Fleet1	2056859	695250	298887	870	0.462	0.057
1.1	0.5	Sim3	2114626	636769	299601	872	0.531	0.057
1.1	0.5	Sim9	2222041	646637	182306	526	0.532	0.034
1.1	0.5	Dis3	2338979	690420	21597	63	0.507	0.002
1.1	0.5	Dis9	2398086	631640	21242	63	0.651	0.002

Table A.1. Results of P01 (cont).

1.1	1.0	Fleet1	2390447	695250	1374789	3648	0.517	0.152
1.1	1.0	Sim3	2443651	642046	1374789	3648	0.577	0.151
1.1	1.0	Sim9	2650662	660376	1149505	3093	0.577	0.128
1.1	1.0	Dis3	3161443	718418	580635	1550	0.541	0.064
1.1	1.0	Dis9	3209467	665254	585752	1581	0.675	0.065
1.1	1.5	Fleet1	2531975	695250	2643047	6769	0.538	0.203
1.1	1.5	Sim3	2582684	643739	2643849	6769	0.595	0.203
1.1	1.5	Sim9	2829935	663786	2376289	6116	0.593	0.183
1.1	1.5	Dis3	3602913	722775	1544249	4100	0.582	0.123
1.1	1.5	Dis9	3602745	687092	1580202	4170	0.677	0.125
1.25	0.5	Fleet1	2622287	695250	369739	859	0.461	0.055
1.25	0.5	Sim3	2679926	637334	370016	859	0.529	0.056
1.25	0.5	Sim9	2816174	646562	224540	516	0.531	0.034
1.25	0.5	Dis3	2971887	690420	24969	64	0.507	0.002
1.25	0.5	Dis9	3031536	631492	24248	63	0.655	0.002
1.25	1.0	Fleet1	3074713	695250	1734897	3771	0.516	0.156
1.25	1.0	Sim3	3128009	641934	1734767	3770	0.577	0.156
1.25	1.0	Sim9	3395568	661733	1447663	3158	0.575	0.131
1.25	1.0	Dis3	4052796	717918	734145	1545	0.543	0.062
1.25	1.0	Dis9	4099630	666339	738886	1569	0.674	0.064
1.25	1.5	Fleet1	3263800	695250	3363936	6915	0.537	0.211
1.25	1.5	Sim3	3314325	644201	3364423	6915	0.593	0.211
1.25	1.5	Sim9	3636249	665759	3020933	6254	0.59	0.192
1.25	1.5	Dis3	4639171	722775	1961097	4128	0.582	0.126
1.25	1.5	Dis9	4631352	687820	2003777	4235	0.676	0.13

Table A.2. Results of P02.

Factor A: Fare	Factor B: Demand	Factor C: Fleeting	Profit	Operating Cost	Spill Cost	Null Spill	Av. Occupancy	Av. Spill Rate
0.9	0.5	Fleet1	2158955	1075950	25670	269	0.36	0.001
0.9	0.5	Sim3	2227257	1007648	25670	269	0.406	0.001
0.9	0.5	Sim9	2221247	1016870	22458	255	0.414	0.001
0.9	0.5	Dis3	2249791	993746	17038	158	0.489	0
0.9	0.5	Dis9	2223549	1024303	12723	158	0.456	0
0.9	1.0	Fleet1	5024583	1075950	354972	823	0.67	0.004
0.9	1.0	Sim3	5080000	1019982	355064	809	0.726	0.004
0.9	1.0	Sim9	5128675	1026286	300349	796	0.728	0.004
0.9	1.0	Dis3	5274683	1084376	96131	636	0.679	0.002

Table A.2. Results of P02 (cont).

0.9	1.0	Dis9	5311357	1061974	81912	622	0.716	0.002
0.9	1.5	Fleet1	6807231	1075950	1780564	5225	0.837	0.122
0.9	1.5	Sim3	6823273	1042219	1799020	4640	0.871	0.107
0.9	1.5	Sim9	7037536	1047882	1579848	3917	0.876	0.082
0.9	1.5	Dis3	7998767	1164419	501805	1165	0.8	0.006
0.9	1.5	Dis9	7881048	1113199	670452	1439	0.857	0.007
1.0	0.5	Fleet1	2519508	1075950	29153	270	0.36	0.001
1.0	0.5	Sim3	2587961	1007347	29304	270	0.408	0.001
1.0	0.5	Sim9	2582605	1016595	25411	256	0.414	0.001
1.0	0.5	Dis3	2612152	996394	16065	158	0.484	0
1.0	0.5	Dis9	2586154	1024135	14322	158	0.457	0
1.0	1.0	Fleet1	5697133	1075950	396865	824	0.67	0.005
1.0	1.0	Sim3	5752575	1019982	396748	810	0.725	0.004
1.0	1.0	Sim9	5807450	1026330	335645	784	0.728	0.004
1.0	1.0	Dis3	5975925	1085890	107615	637	0.677	0.002
1.0	1.0	Dis9	6014062	1062796	92630	623	0.714	0.002
1.0	1.5	Fleet1	7680196	1075950	1982527	5486	0.837	0.142
1.0	1.5	Sim3	7694187	1042169	2004569	5270	0.871	0.115
1.0	1.5	Sim9	7933154	1047549	1759795	3866	0.877	0.079
1.0	1.5	Dis3	9017246	1167381	555837	1165	0.797	0.006
1.0	1.5	Dis9	8879546	1114337	746057	1409	0.857	0.007
1.1	0.5	Fleet1	2882411	1075950	31045	272	0.361	0.001
1.1	0.5	Sim3	2950848	1007347	31211	272	0.408	0.001
1.1	0.5	Sim9	2944814	1016623	27969	258	0.416	0.001
1.1	0.5	Dis3	2975467	996394	17545	158	0.484	0
1.1	0.5	Dis9	2949669	1024240	15497	158	0.455	0
1.1	1.0	Fleet1	6380133	1075950	432991	825	0.67	0.005
1.1	1.0	Sim3	6435522	1019982	433220	811	0.726	0.004
1.1	1.0	Sim9	6496918	1026303	365740	796	0.728	0.004
1.1	1.0	Dis3	6688320	1088648	111943	639	0.672	0.002
1.1	1.0	Dis9	6727741	1062994	98176	625	0.714	0.002
1.1	1.5	Fleet1	8562088	1075950	2178728	5430	0.837	0.119
1.1	1.5	Sim3	8573836	1042502	2200656	5493	0.871	0.129
1.1	1.5	Sim9	8837455	1048906	1929836	3089	0.875	0.059
1.1	1.5	Dis3	10041316	1167991	607475	1164	0.796	0.006
1.1	1.5	Dis9	9885823	1113464	817057	1432	0.856	0.007
1.25	0.5	Fleet1	3418553	1075950	35593	273	0.361	0.001
1.25	0.5	Sim3	3486968	1007347	35781	273	0.407	0.001
1.25	0.5	Sim9	3481577	1016621	31898	259	0.415	0.001
1.25	0.5	Dis3	3513910	997833	18353	158	0.481	0

Table A.2. Results of P02 (cont).

1.25	0.5	Dis9	4807476	706904	2385202	4607	0.711	0.164
1.25	1.0	Fleet1	3761278	695250	9540440	17585	0.628	0.352
1.25	1.0	Sim3	3799067	657142	9540638	17541	0.67	0.351
1.25	1.0	Sim9	4243805	686106	9064740	16659	0.664	0.328
1.25	1.0	Dis3	5744357	753049	7498390	13742	0.67	0.269
1.25	1.0	Dis9	5586117	721699	7688566	14232	0.735	0.28
1.25	1.5	Fleet1	3893537	695250	15514217	28354	0.643	0.398
1.25	1.5	Sim3	3929585	658722	15514897	28409	0.682	0.403
1.25	1.5	Sim9	4407628	687200	15006613	27400	0.68	0.382
1.25	1.5	Dis3	6103168	759940	13238552	23958	0.676	0.324
1.25	1.5	Dis9	5897268	728229	13476410	24677	0.741	0.335

Table A.3. Results of A01.

Factor A: Fare	Factor B: Demand	Factor C: Fleeting	Profit	Operating Cost	Spill Cost	Null Spill	Av. Occupancy	Av. Spill Rate
0.9	0.5	Fleet1	2158955	1075950	25670	269	0.36	0.001
0.9	0.5	Sim3	2227257	1007648	25670	269	0.406	0.001
0.9	0.5	Sim9	2221247	1016870	22458	255	0.414	0.001
0.9	0.5	Dis3	2249791	993746	17038	158	0.489	0
0.9	0.5	Dis9	2223549	1024303	12723	158	0.456	0
0.9	1.0	Fleet1	5024583	1075950	354972	823	0.67	0.004
0.9	1.0	Sim3	5080000	1019982	355064	809	0.726	0.004
0.9	1.0	Sim9	5128675	1026286	300349	796	0.728	0.004
0.9	1.0	Dis3	5274683	1084376	96131	636	0.679	0.002
0.9	1.0	Dis9	5311357	1061974	81912	622	0.716	0.002
0.9	1.5	Fleet1	6807231	1075950	1780564	5225	0.837	0.122
0.9	1.5	Sim3	6823273	1042219	1799020	4640	0.871	0.107
0.9	1.5	Sim9	7037536	1047882	1579848	3917	0.876	0.082
0.9	1.5	Dis3	7998767	1164419	501805	1165	0.8	0.006
0.9	1.5	Dis9	7881048	1113199	670452	1439	0.857	0.007
1.0	0.5	Fleet1	2519508	1075950	29153	270	0.36	0.001
1.0	0.5	Sim3	2587961	1007347	29304	270	0.408	0.001
1.0	0.5	Sim9	2582605	1016595	25411	256	0.414	0.001
1.0	0.5	Dis3	2612152	996394	16065	158	0.484	0
1.0	0.5	Dis9	2586154	1024135	14322	158	0.457	0
1.0	1.0	Fleet1	5697133	1075950	396865	824	0.67	0.005
1.0	1.0	Sim3	5752575	1019982	396748	810	0.725	0.004
1.0	1.0	Sim9	5807450	1026330	335645	784	0.728	0.004

Table A.3. Results of A01 (cont).

1.0	1.0	Dis3	5975925	1085890	107615	637	0.677	0.002
1.0	1.0	Dis9	6014062	1062796	92630	623	0.714	0.002
1.0	1.5	Fleet1	7680196	1075950	1982527	5486	0.837	0.142
1.0	1.5	Sim3	7694187	1042169	2004569	5270	0.871	0.115
1.0	1.5	Sim9	7933154	1047549	1759795	3866	0.877	0.079
1.0	1.5	Dis3	9017246	1167381	555837	1165	0.797	0.006
1.0	1.5	Dis9	8879546	1114337	746057	1409	0.857	0.007
1.1	0.5	Fleet1	2882411	1075950	31045	272	0.361	0.001
1.1	0.5	Sim3	2950848	1007347	31211	272	0.408	0.001
1.1	0.5	Sim9	2944814	1016623	27969	258	0.416	0.001
1.1	0.5	Dis3	2975467	996394	17545	158	0.484	0
1.1	0.5	Dis9	2949669	1024240	15497	158	0.455	0
1.1	1.0	Fleet1	6380133	1075950	432991	825	0.67	0.005
1.1	1.0	Sim3	6435522	1019982	433220	811	0.726	0.004
1.1	1.0	Sim9	6496918	1026303	365740	796	0.728	0.004
1.1	1.0	Dis3	6688320	1088648	111943	639	0.672	0.002
1.1	1.0	Dis9	6727741	1062994	98176	625	0.714	0.002
1.1	1.5	Fleet1	8562088	1075950	2178728	5430	0.837	0.119
1.1	1.5	Sim3	8573836	1042502	2200656	5493	0.871	0.129
1.1	1.5	Sim9	8837455	1048906	1929836	3089	0.875	0.059
1.1	1.5	Dis3	10041316	1167991	607475	1164	0.796	0.006
1.1	1.5	Dis9	9885823	1113464	817057	1432	0.856	0.007
1.25	0.5	Fleet1	3418553	1075950	35593	273	0.361	0.001
1.25	0.5	Sim3	3486968	1007347	35781	273	0.407	0.001
1.25	0.5	Sim9	3481577	1016621	31898	259	0.415	0.001
1.25	0.5	Dis3	3513910	997833	18353	158	0.481	0
1.25	0.5	Dis9	3488229	1024272	17595	158	0.457	0
1.25	1.0	Fleet1	7394558	1075950	493644	826	0.67	0.005
1.25	1.0	Sim3	7449888	1020199	493503	825	0.726	0.004
1.25	1.0	Sim9	7520630	1026755	416380	777	0.728	0.002
1.25	1.0	Dis3	7746709	1090993	126076	637	0.668	0.002
1.25	1.0	Dis9	7787715	1062994	113069	627	0.715	0.002
1.25	1.5	Fleet1	9871740	1075950	2477335	5078	0.837	0.117
1.25	1.5	Sim3	9880627	1043802	2503014	4281	0.87	0.1
1.25	1.5	Sim9	10180383	1049338	2197165	3920	0.873	0.083
1.25	1.5	Dis3	11565585	1168467	693336	1151	0.795	0.006
1.25	1.5	Dis9	11381186	1115070	930330	1438	0.853	0.007

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