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BEBEK, ISTANBUL

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BIAXIAL BENDING OF COLUMNS
WITH UNSYMMETRICAL T SECTIONS
(WORKING STRESS DESIGN BY COMPUTERS)

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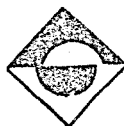
TAMER TUNCA

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ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

THESIS SUBMITTED TO THE GRADUATE
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ABSTRACT

A numerical method of working stress design of reinforced concrete columns with unsymmetrical T-shaped sections subjected to biaxial bending and axial force is presented. The design procedure, modified on the basis of Turkish Building Code requirements, is similar to Rüdinger's and Chu's iterative solution methods in which the stress at any point is found from a general formula for unsymmetrical bending of unsymmetrical sections. The formulation derived for T-sections is readily applicable to L-shaped as well as to rectangular sections. The theory includes the cases in which the eccentricities exceed the cracking limit of the section. A computer program is given and one complete numeric example is included.

THESIS

ROBERT COLLEGE GRADUATE SCHOOL
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TABLE OF CONTENTS

	Page
ACKNOWLEDGEMENT	
ABSTRACT	
NOTATIONS	1
CHAPTER 1 - INTRODUCTION	3
CHAPTER 2 - WORKING STRESS DESIGN	7
2.1 General Considerations	7
2.2 Small Eccentricities	7
2.3 Large Eccentricities	9
CHAPTER 3 - DESIGN PROCEDURE	11
3.1 Principles for Determining Stresses in an Uncracked Section	11
3.2 Location of Neutral Axis of Uncracked Section	13
3.3 First Approximate Location of Neutral Axis of Cracked Section	16
3.4 Expressions for Area, Centroid and Moments of Inertia of Cracked Section	19
3.5 Stresses in Cracked Section	26
3.6 Location of Neutral Axis of Cracked Section	27
3.7 Iteration Procedure	28
3.8 Summary of Design Procedure	28
CHAPTER 4 - COMPUTER PROGRAMS	31
CHAPTER 5 - CONCLUSIONS	33
APPENDIX A - Numeric Example	35
APPENDIX B - Data Instructions and Variable Definitions	40

THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

	Page
APPENDIX C - Positions of Neutral Axis and Parameters Used in Area, Centroid and Moments of Inertia Formulas for the Cracked Section	43
APPENDIX D - Computer Program Listings	53
REFERENCES	

NOTATIONS

A	- transformed area of the uncracked section
A'	- transformed area of the cracked section
a_{si}	- area of the i^{th} reinforcing bar
B_I, C_I	- distances to locate the neutral axis of the uncracked section
B, C	- distances to locate the neutral axis of the cracked section
e_x, e_y	- coordinates of the point of application of the normal force with respect to x, y axes
e_x', e_y'	- coordinates of the point of application of the normal force with respect to x', y' axes
F	- axial force
I_{xc}, I_{yc}, I_{xyc}	- moments of inertia of concrete of the uncracked section with respect to x, y axes
I_{xs}, I_{ys}, I_{xys}	- moments of inertia of steel of the uncracked section with respect to x, y axes
I_x, I_y, I_{xy}	- moments of inertia of the uncracked transformed section with respect to x, y axes
I_x', I_y', I_{xy}'	- moments of inertia of the cracked transformed section with respect to x', y' axes
n	- modular ratio (the ratio of the modulus of elasticity of steel to the modulus of elasticity of concrete)

THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE 2

- S, Z, P, O, R, Q - parameters used in area, centroid and moments of inertia formulas for the cracked section
- u, v - axes passing through the bottom-left corner of the section
- u_i, v_i - coordinates of the i^{th} reinforcing bar with respect to u, v axes
- u_o, v_o - coordinates of the centroid of the uncracked section with respect to u, v axes
- x, y - centroidal axes of the uncracked section
- x_o, y_o - coordinates of the centroid of the cracked section with respect to x, y axes
- x_i, y_i - coordinates of the i^{th} reinforcing bar with respect to x, y axes
- x', y' - centroidal axes of the cracked section
- x'_i, y'_i - coordinates of the i^{th} reinforcing bar with respect to x', y' axes
- $C(x)_x, C(x)_y$ - coordinates of the (x) corner of the section with respect to x, y axes
- $(\nabla_{Ic})_{c(x)}$ - stress in concrete when section is not cracked at the (x) corner
- $(\nabla_{Is})_i$ - stress in the i^{th} reinforcing bar when the section is not cracked
- $\nabla_c(x)$ - stress in concrete when the section is cracked at the (x) corner
- ∇_i - stress in the i^{th} reinforcing bar when the section is cracked
- θ - the angle between the neutral axis and the horizontal

CHAPTER 1

INTRODUCTION

It is generally a straightforward problem to determine the allowable capacity (under axial load and moments) of a given reinforced concrete section, providing that the position of the neutral axis and stresses are known. In practice, however, the procedure is reversed. Working loads are given and the sections as well as the reinforcements are required to support these loading. For this purpose, a trial section is selected and the maximum stresses are calculated. The allowable capacity of the selected section is reached when the calculated maximum stress in steel or concrete is equal to the allowable stress.

Columns with T or L-shaped sections are more effective and economical particularly in irregular column layouts, since most corner columns as well as some interior columns are subject to bending in two directions. Because of their larger moment of inertia properties, T and L-shaped columns can resist relatively bigger bending moments in both directions. Columns which are axially loaded occur rarely in buildings, simultaneous bending is almost always present. Bending moments are caused by continuity, i.e., by the fact that building columns are parts of monolithic frames, by transverse loads such as wind forces. Even when design calculations show a member to be loaded axially, inevitable imperfections of construction will introduce eccentricities and consequent bending in the member. Usually it is assumed that bending is present about only one of the two principal axes of the section. There are other situations, however, and they are by no means exceptional, in which axial compression is accompanied by simultaneous bending about both

principal axes of the section. Such is the case, for instance, in corner columns of tier buildings, where beams and girders frame into the column in the directions of both walls and transfer their end moments into the column in two perpendicular planes. Similar situations can occur with respect to interior columns, particularly in irregular column layouts, and in a variety of other structures. Further, T and I-shaped columns fit into the wall thicknesses and wall layout better than the rectangular columns. Therefore, they are also preferable for architectural reasons. We should bear in mind, however, that the design of even rectangular sections subject to biaxial bending requires complicated calculations.

Nevertheless, there are numerous papers published in Europe and the USA about the design of columns under biaxial bending. Most of the papers published in the USA are for ultimate strength design (1-12)*. J.Rüdinger (19) and Kuang-Han Chu (18) presented methods for the design of rectangular sections. Their methods are too lengthy. Moreover, Chu's formulas are for ultimate strength design.

L.S.Müller (13) presented a design method applicable to L-shaped columns symmetrical about a 45-deg. axis and assumed that the eccentricities do not exceed the limit of cracking of the section. K.Kammüller (22) and E.Mörsch (23) proposed design methods for any type of section using graphical methods.

B.Löser (24,25) presented three separate methods for the design of columns subject to biaxial bending. One of the methods is graphical and the other two are applicable only to rectangular sections. In one of the latter methods, the first location of neutral axis is determined, first, by finding two factors from a table which correspond to a previously calculated parameter

(*) Number in the brackets refer to the References at the end of the thesis.

THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE 5

and then substituting these into some equations. Generally speaking, all of his methods require lengthy computations.

Ouvrier (30) in his book about columns subjected to biaxial bending, gives design principles for square sections. Design procedures presented by Sager (26), Nolte (27,29), Ban (28) and Pucher (21) are applicable to rectangular sections only. Biaxial design of columns by these procedures require the use of auxiliary tables and diagrams prepared by the above mentioned authors. In Pucher's method, it is assumed that the reinforcing is present only on the tension side of the section.

As stated above all of the available design procedures are restricted to rectangular sections and further most of them require the use of design tables or charts. These tables and charts, although simplify the tedious calculations to some extent, due to the inevitable interpolations or extrapolations the results become approximate.

Cho-Liu Ang (14) and Eli Czerniak (15,16,17) have computurized the working stress design of reinforced concrete column sections based on ACI-Building Code, however, limiting their discussion to rectangular sections only.

The purpose of this paper is to develop a method applicable to the analysis of T,L-shaped and rectangular column sections with small and large eccentricities. It is also intended to develop a computer program for this method. The main difficulty in this problem is to locate the neutral axis of the cracked section. By means of this program the designer may determine the stresses in a tentatively selected section within about twenty-five minutes after supplying the dimensions of the section, coordinates and the diameters of the reinforcing bars to the IBM 1620 computer.

THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE 6

The method developed in this paper is similar to the iterative solution methods proposed by Rüdinger and Chu , modified on the basis of Turkish Building Code requirements. If the eccentrically applied force does not cause any tensile stress in concrete, in other words if the neutral axis does not intersect the section, the design is simple and straightforward. Once the transformed area of the section is obtained, the stress at any point may be calculated from a general formula derived in Strength of Material courses for unsymmetrical bending of unsymmetrical sections. But, if the section is cracked because of tensile stresses in concrete, the problem necessitates iterative solution and the basic difficulty lies in locating the neutral axis. This difficulty is eliminated to some extent by determining the first approximate location of the neutral axis from a formula proposed by Löser. After the neutral axis is determined the area, centroid and moments of inertia of the cracked section are calculated. To be able to write general expressions, six parameters are used. These parameters define the points of intersection of the neutral axis with the sides of the section. The exact location of the neutral axis is found by iteration and then the stresses are calculated by the formula mentioned above.

Due to the limited capacity of the IBM 1620 a column section is designed by running four separate programs which are linked together. The programs are so developed that one has to prepare data only for the first program. The first program tests the slenderness, minimum and maximum steel ratio of the section and determines whether the applied load exceeds the ultimate load under which the column fails or not.

CHAPTER 2

WORKING STRESS DESIGN

2.1 GENERAL CONSIDERATIONS

The vast majority of reinforced concrete structures have been proportioned based on a straight-line theory which is called working stress design. When using working stress design techniques, members are proportioned so that they may sustain the anticipated real loads induced (working or design loads) without the stresses in the concrete or reinforcing exceeding the proportional limits of the individual materials. Although the stress-strain diagram of concrete does not exhibit an initial straight-line portion, it is still assumed that Hooke's law does apply to concrete.

This leads to the basic assumptions in working stress design required for the development of design procedures:

1. Plane sections normal to the neutral plane remain plane after bending.
2. Tensile strength of concrete is neglected.
3. The strain in reinforcing bar is the same as that of the surrounding concrete.
4. Both the concrete and reinforcing steel obey Hooke's law.
5. Strain is proportional to the distance from the neutral axis.
6. All other basic assumptions of deformation and flexure of homogeneous sections are valid.

2.2 SMALL ECCENTRICITIES

At low stresses, up to about one-half the ultimate, eccen-

trically compressed members behave elastically. In elastic members the stresses may be calculated by simple addition of those caused by axial compression and those caused by bending. This will be the case in the design procedure presented in this paper.

If the eccentricities are small enough so that tension stresses in the concrete will not exist. Then the strain and stress distribution as shown in Fig. 2.1 is essentially the same as in an elastic, homogeneous member.

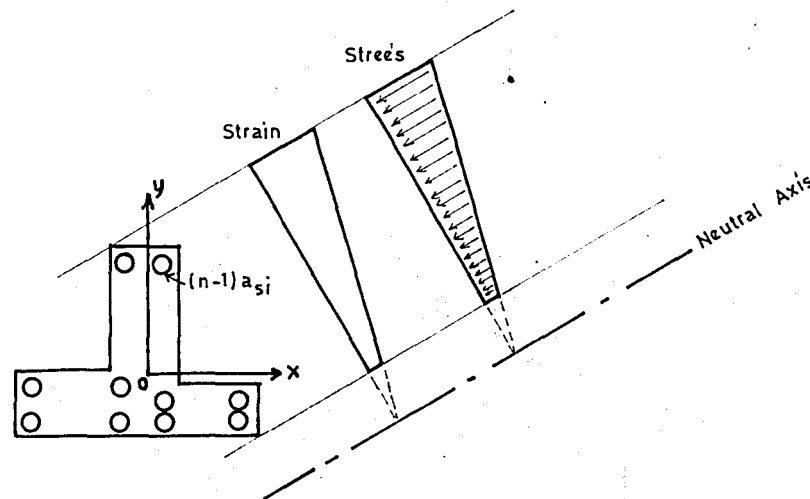


Fig.2.1 Stress-Strain in an Uncracked Section

Therefore, once the transformed section has been obtained, the usual method of analysis of elastic homogeneous members can be applied and the stresses in concrete are computed with equation 2.1 (34). Similarly the stresses in reinforcing bars are computed with Eq. 2.2 .

$$\sigma_c = \frac{F}{A} + \frac{M_x I_y + M_y I_{xy}}{I_x I_y - I_{xy}^2} y - \frac{M_y I_x + M_x I_{xy}}{I_x I_y - I_{xy}^2} x \quad 2.1$$

$$\sigma_s = n \sigma_c \quad 2.2$$

where F = axial force

M_x = bending moment about x axis

M_y = bending moment about y axis
 n = modular ratio (modulus of elasticity of steel/modulus of elasticity of concrete)

The sign convention is as follows:

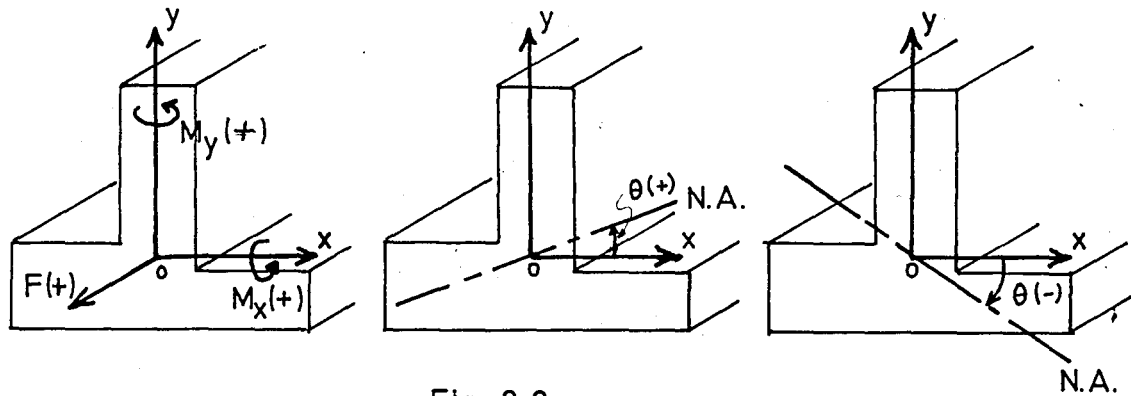


Fig. 2.2

Sign Convention

Axial load, F , and bending moments, M_x and M_y , can be replaced with a single eccentric force, F , of equal magnitude acting at e_x, e_y where $e_x = -M_y/F$ and $e_y = M_x/F$.

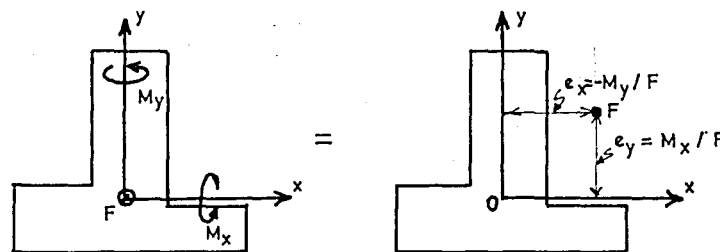


Fig. 2.3

Eccentric Loading

Then Eq.2.1 becomes

$$\sigma_c = F \left(\frac{1}{A} + \frac{e_y I_y - e_x I_{xy}}{I_x I_y - I_{xy}^2} y + \frac{e_x I_x - e_y I_{xy}}{I_x I_y - I_{xy}^2} x \right) \quad 2.3$$

2.3. LARGE ECCENTRICITIES

When the eccentricities are large, tension cracks will make part of the concrete ineffective and the elastic stresses are

distributed as in Fig.2.4 .

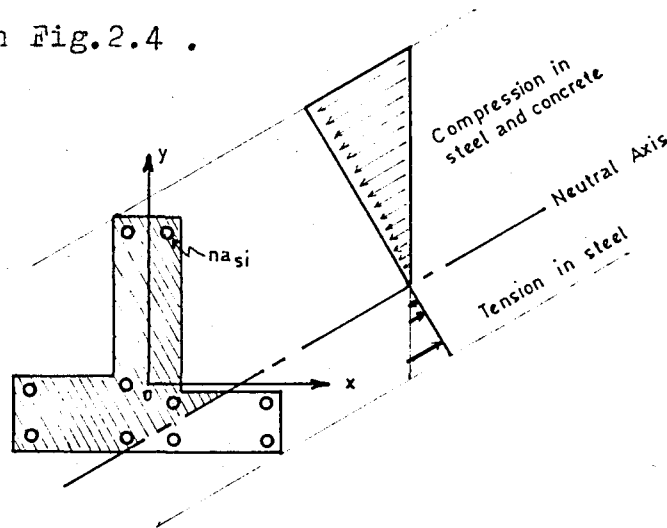


Fig.2.4. Stress Distribution in a Cracked Section

The shape of the cracked section is dependent on the magnitude of F and values of e_x, e_y . The possible shapes of the cracked section are shown in appendix C. If the position of the neutral axis is known, then the stresses are given by the following expressions.

Stresses in concrete:

$$\sigma_c = F \left(\frac{1}{A'} + \frac{e_y' I_x' - e_x' I_{xy}'}{I_x' I_y' - I_{xy}^2} y + \frac{e_x' I_x' - e_y' I_{xy}'}{I_x' I_y' - I_{xy}^2} x \right) \quad 2.4$$

Stresses in steel:

$$\sigma_s = n \sigma_c$$

CHAPTER 3

DESIGN PROCEDURE

3.1 PRINCIPLES FOR DETERMINING STRESSES IN AN UNCRACKED SECTION

Fig. 3.1 shows an unsymmetrical T section with an arbitrarily chosen reinforcement arrangement loaded by an eccentric force F .

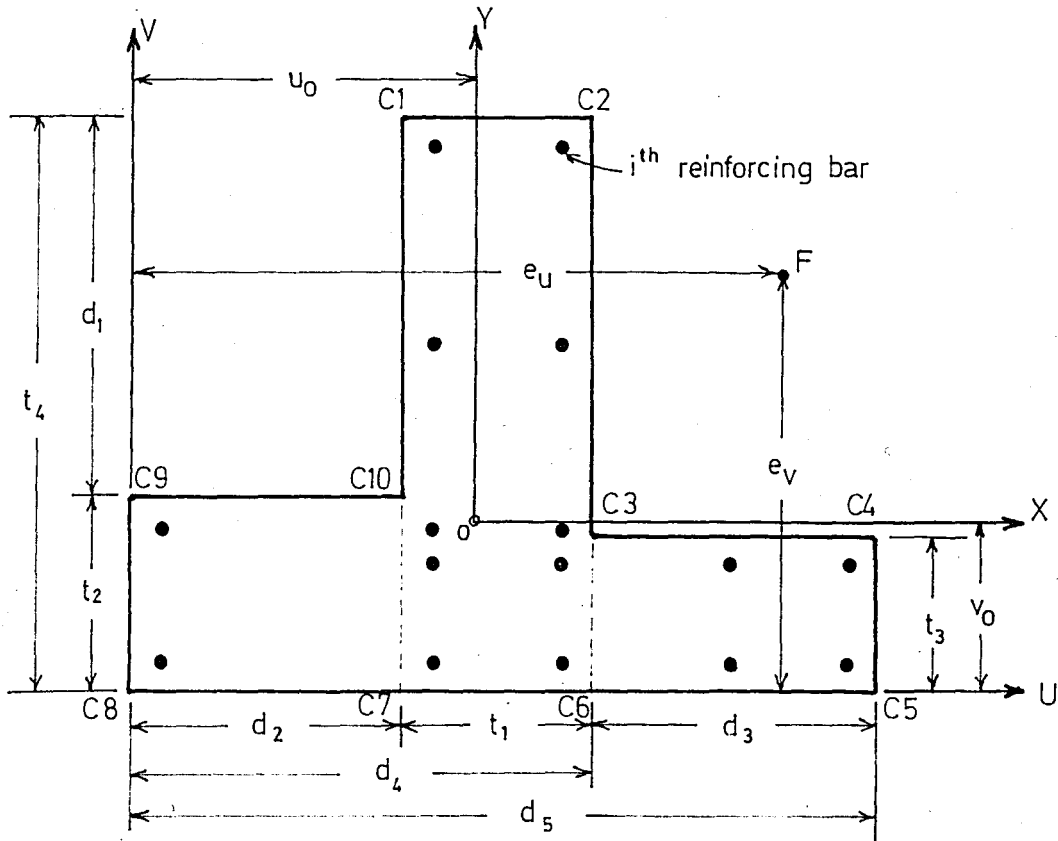


Fig. 3.1

A Typical Section

The area of the transformed section is:

$$A = t_1 t_4 + t_2 d_2 + t_3 d_3 + (n-1) \sum a_{si}$$

3.1

T H E S I S

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE 12

The coordinates of the centroid of the section with respect to the u, v axes will be:

$$U_0 = \frac{t_2 d_2^2 + 2t_1 t_4 d_2 + t_1^2 t_4 + 2t_3 d_3 d_4 + t_3 d_3^2 + 2(n-1) \sum a_{si} u_i}{2A} \quad 3.2$$

$$V_0 = \frac{t_2^2 d_2 + t_1 t_4^2 + d_3 t_3^2 + 2(n-1) \sum a_{si} v_i}{2A} \quad 3.3$$

Moments of inertia of concrete with respect to the x and y axes are determined from the following expressions:

$$I_{xc} = \frac{1}{12} d_2 t_2 (t_2^2 + 3(t_2 - 2V_0)^2) + \frac{1}{12} t_1 t_4 (t_4^2 + 3(t_4 - 2V_0)^2) + \frac{1}{12} d_3 t_3 (t_3^2 + 3(t_3 - 2V_0)^2) \quad 3.4$$

$$I_{yc} = \frac{1}{12} t_2 d_2 (d_2^2 + 3(d_2 - 2U_0)^2) + \frac{1}{12} t_1 t_4 (t_1^2 + 3(d_4 + d_2 - 2U_0)^2) + \frac{1}{12} t_3 d_3 (d_3^2 + 3(d_5 + d_4 - 2U_0)^2) \quad 3.5$$

$$I_{xyc} = \frac{1}{4} [t_2 d_2 (t_2 - 2V_0)(d_2 - 2U_0) + t_1 t_4 (t_4 - 2V_0)(d_4 + d_2 - 2U_0) + t_3 d_3 (t_3 - 2V_0)(d_5 + d_4 - 2U_0)] \quad 3.6$$

Moments of inertia of steel with respect to the x and y axes are determined from the following formulas:

$$I_{xs} = (n-1) \sum a_{si} y_i^2 \quad 3.7$$

$$I_{ys} = (n-1) \sum a_{si} x_i^2 \quad 3.8$$

$$I_{xys} = (n-1) \sum a_{si} x_i y_i \quad 3.9$$

The total moments of inertia of the section with respect to the x and y axes:

$$I_x = I_{xc} + I_{xs} \quad 3.10$$

$$I_y = I_{yc} + I_{ys} \quad 3.11$$

$$I_{xy} = I_{xyc} + I_{xys} \quad 3.12$$

The stresses in concrete at the corner points are determined from Eq. 2.3.

Let

$$K_y = \frac{e_y I_y - e_x I_{xy}}{I_x I_y - I_{xy}^2}$$

$$K_x = \frac{e_x I_x - e_y I_{xy}}{I_x I_y - I_{xy}^2}$$

then

$$\begin{aligned} (\sigma_{Ic})_{c1} &= F \left(\frac{1}{A} + K_y (t_4 - v_0) + K_x (d_2 - u_0) \right) \\ (\sigma_{Ic})_{c2} &= F \left(\frac{1}{A} + K_y (t_4 - v_0) + K_x (d_4 - u_0) \right) \\ (\sigma_{Ic})_{c3} &= F \left(\frac{1}{A} + K_y (t_3 - v_0) + K_x (d_4 - u_0) \right) \\ (\sigma_{Ic})_{c4} &= F \left(\frac{1}{A} + K_y (t_3 - v_0) + K_x (d_5 - u_0) \right) \\ (\sigma_{Ic})_{c5} &= F \left(\frac{1}{A} + K_y (-v_0) + K_x (d_5 - u_0) \right) \\ (\sigma_{Ic})_{c6} &= F \left(\frac{1}{A} + K_y (-v_0) + K_x (d_4 - u_0) \right) \\ (\sigma_{Ic})_{c7} &= F \left(\frac{1}{A} + K_y (-v_0) + K_x (d_2 - u_0) \right) \\ (\sigma_{Ic})_{c8} &= F \left(\frac{1}{A} + K_y (-v_0) + K_x (-u_0) \right) \\ (\sigma_{Ic})_{c9} &= F \left(\frac{1}{A} + K_y (t_2 - v_0) + K_x (-u_0) \right) \\ (\sigma_{Ic})_{c10} &= F \left(\frac{1}{A} + K_y (t_2 - v_0) + K_x (d_2 - u_0) \right) \end{aligned} \tag{3.13}$$

And stresses in reinforcing bars are determined from Eq. 2.2.

$$(\sigma_{Is})_i = nF \left(\frac{1}{A} + K_y (y_i) + K_x (x_i) \right) \tag{3.14}$$

3.2 LOCATION OF NEUTRAL AXIS OF UNCRACKED SECTION

The point of application of the force, F , has great importance because it determines the angle between the neutral axis and the horizontal and the place of the compression area. There

are four different cases concerning the position of the neutral axis and each case is further subdivided into two groups. These cases are summarized in Figs. 3.2 to 3.5.

Case (a) $\theta = 0^\circ$

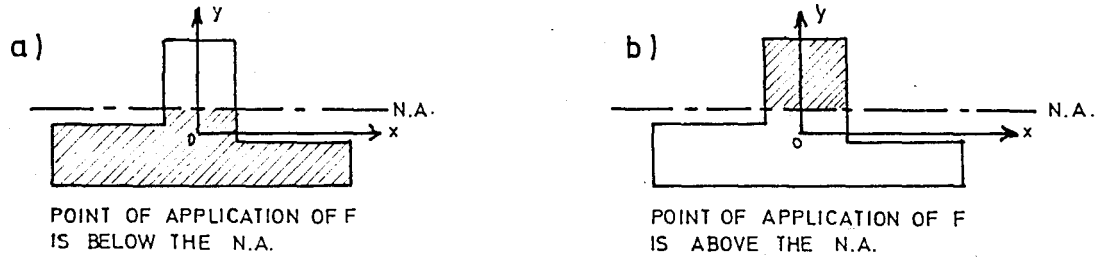


Fig. 3.2. Compression Areas When $\theta = 0^\circ$

Case (b) $\theta = 90^\circ$

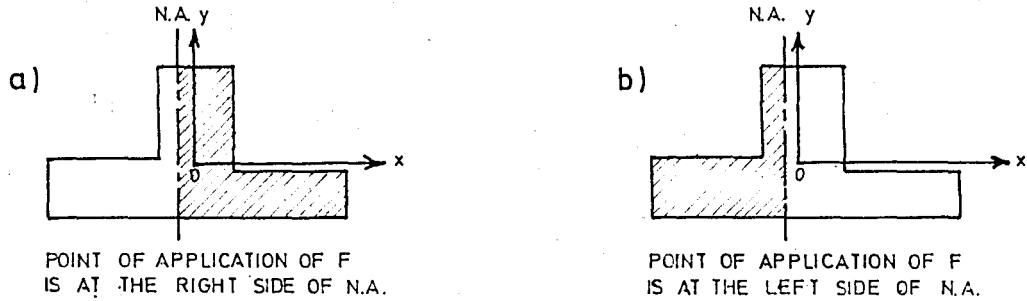


Fig. 3.3. Compression Areas When $\theta = 90^\circ$

Case (c) $0^\circ < \theta < 90^\circ$

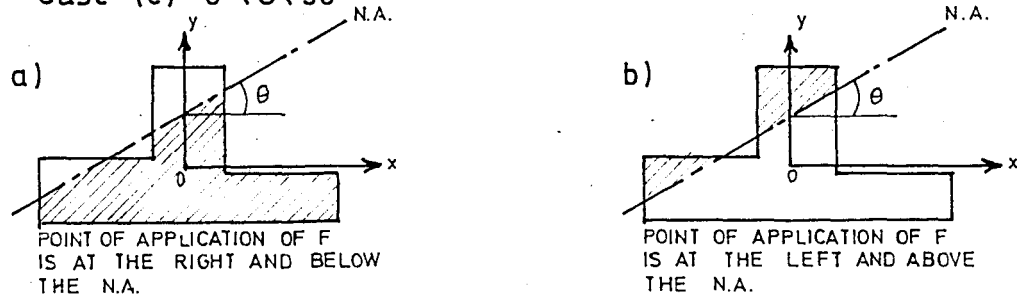


Fig. 3.4. Compression Areas When $0^\circ < \theta < 90^\circ$

Case (d) $0^\circ > \theta > -90^\circ$

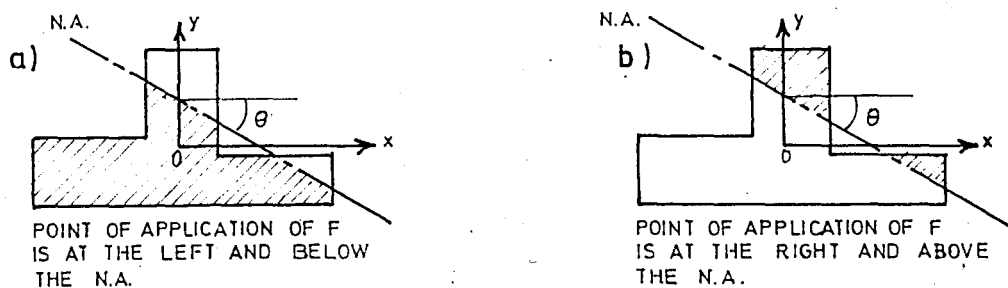


Fig. 3.5 Compression Areas When $0^\circ > \theta > -90^\circ$

Throughout this paper the location of neutral axis will be defined by distances B and C, as illustrated in Fig. 3.6 .

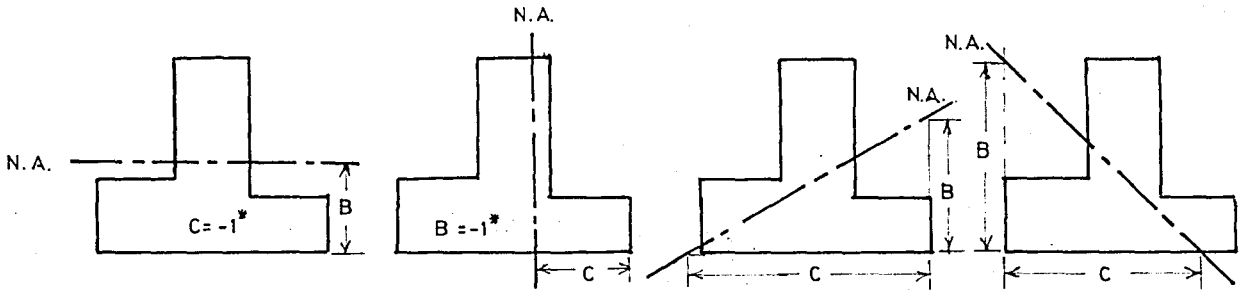


Fig.3.6. Distances B & C.

By definition neutral axis is the zero stress line, therefore, the equation of the neutral axis is obtained by setting $\nabla = 0$ in Eq. 2.3 .

$$0 = \frac{F}{A} + K_y (y) + K_x (x)$$

Case (a)

$$B_I = -\frac{1}{A} \cdot \frac{K_x}{K_y} + v_0 \quad 3.15$$

$$C_I = -1$$

Case (b)

$$B_I = -1 \quad 3.16$$

$$C_I = -\frac{1}{A} \cdot \frac{K_y}{K_x} + d_s - u_0$$

Case (c)

$$B_I = (d_s - u_0) \tan \theta - \frac{1}{A} \cdot \frac{K_x}{K_y} + v_0 \quad 3.17$$

where

$$\tan \theta = \frac{e_y I_{xy} - e_x I_x}{e_y I_y - e_x I_{xy}}$$

(*) Actually in these cases B or C are not defined but in the computer program they are identified as -1.

$$C_1 = \frac{V_o}{\tan \theta} + \frac{1}{A} \cdot \frac{K_y}{K_x} + d_s - U_o$$

Case (d)

$$B_1 = -U_o \tan \theta - \frac{1}{A} \cdot \frac{K_x}{K_y} + V_o$$

$$C_1 = -\frac{V_o}{\tan \theta} - \frac{1}{A} \cdot \frac{K_y}{K_x} + U_o$$

3.18

If there is no tension stresses in the concrete and if the calculated stresses do not exceed the allowable stresses, the section is uncracked and the design is completed provided that the steel ratio, slenderness and minimum transformed area satisfy the requirements of the Turkish Building Code. Also, if the calculated stresses are considerably smaller than the allowable stresses, a smaller section may be tried to make the design more economical.

3.3 FIRST APPROXIMATE LOCATION OF NEUTRAL AXIS OF CRACKED SECTION

If the section is cracked, the neutral axis must be moved toward the compression area as shown in Fig. 3.7 . In order to speed up the time-consuming trial and error procedure the first approximate location of the neutral axis will be determined by the Eqs. 3.19 to 3.22 proposed by Löser for rectangular sections.

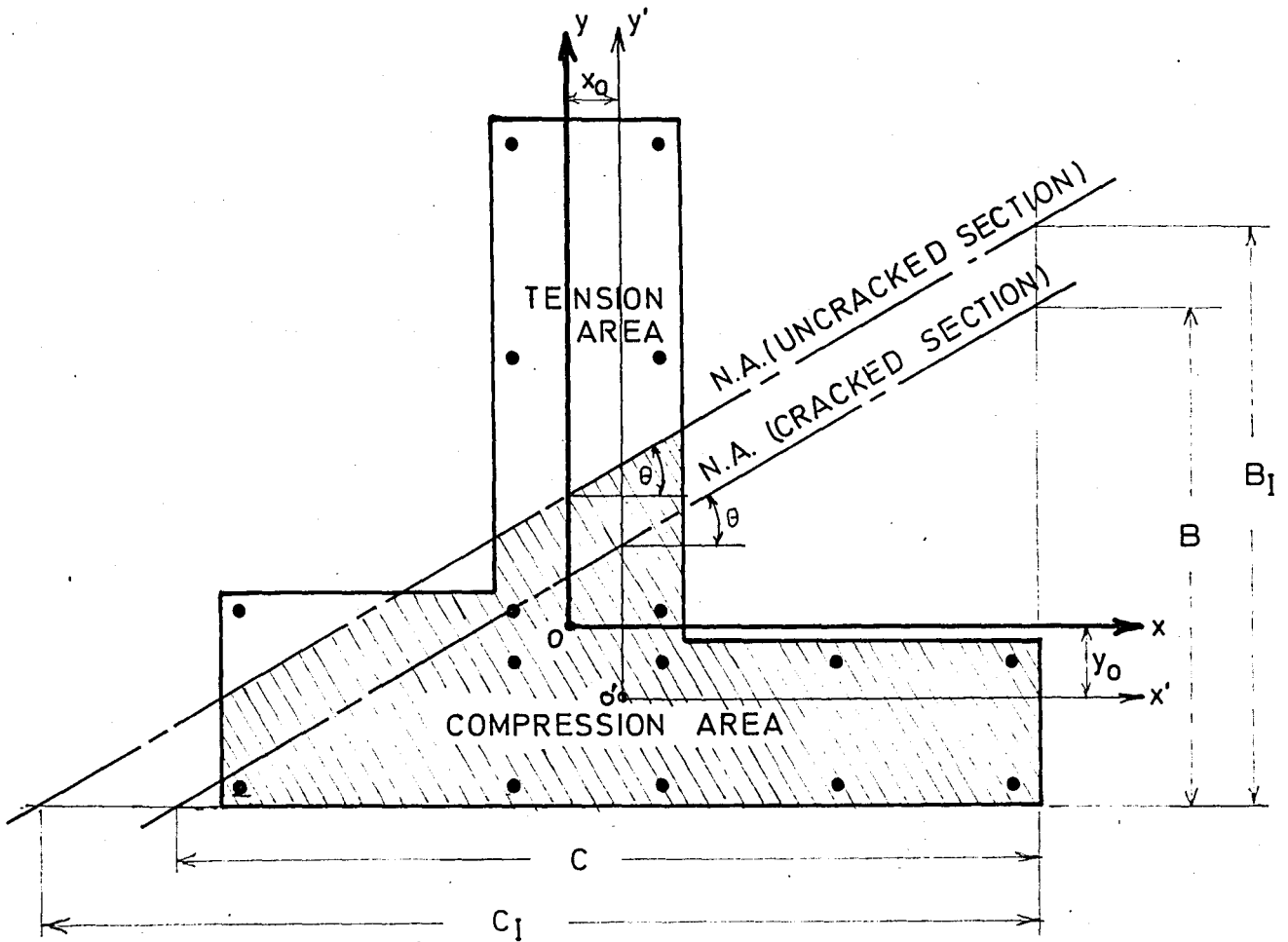
Case (a)

If the point of application of F is below the neutral axis

$$B = B_T \cdot \frac{(\nabla_{1c})_{c7} + 0.3 (\nabla_{1c})_{c1}}{(\nabla_{1c})_{c7}} ; C = -1 \quad 3.19 a$$

If the point of application of F is above the neutral axis

$$B = B_T \cdot \frac{(\nabla_{1c})_{c1} - 0.3 (\nabla_{1c})_{c7}}{(\nabla_{1c})_{c1}} ; C = -1 \quad 3.19 b$$



- x', y' - centroidal axis of cracked section
- o' - centroid of cracked section
- x, y - centroidal axis of uncracked section
- o - centroid of uncracked section

Fig. 3.7

A Typical Cracked Section

Case (b)

If the point of application of F is at the right side of the neutral axis

$$C = C_I \cdot \frac{(\nabla_{Ic})_{c4} + 0.3(\nabla_{Ic})_{c9}}{(\nabla_{Ic})_{c4}} ; B = -1 \quad 3.20 a$$

If the point of application of F is at the left side of the neutral axis

$$C = C_I \cdot \frac{(\nabla_{Ic})_{c9} - 0.3(\nabla_{Ic})_{c4}}{(\nabla_{Ic})_{c9}} ; B = -1 \quad 3.20 b$$

Case (c)

If the point of application of F is at the right and below the neutral axis

$$B = B_I \cdot \frac{(\nabla_{Ic})_{min.} + 0.3(\nabla_{Ic})_{max.}}{(\nabla_{Ic})_{min.}} ; C = \frac{B}{\tan \theta} \quad 3.21 a$$

If the point of application of F is at the left and above the neutral axis

$$B = B_I \cdot \frac{(\nabla_{Ic})_{min.} - 0.3(\nabla_{Ic})_{max.}}{(\nabla_{Ic})_{min.}} ; C = \frac{B}{\tan \theta} \quad 3.21 b$$

Case (d)

If the point of application of F is at the left and below the neutral axis

$$B = B_I \cdot \frac{(\nabla_{Ic})_{min.} + 0.3(\nabla_{Ic})_{max.}}{(\nabla_{Ic})_{min.}} ; C = -\frac{B}{\tan \theta} \quad 3.22 a$$

If the point of application of F is at the right and above the neutral axis

$$B = B_I \cdot \frac{(\nabla_{Ic})_{min.} - 0.3(\nabla_{Ic})_{max.}}{(\nabla_{Ic})_{min.}} ; C = -\frac{B}{\tan \theta} \quad 3.22 b$$

The values of B and C define one of the cases shown in appendix C.

3.4 EXPRESSIONS FOR AREA, CENTROID AND MOMENTS OF INERTIA OF CRACKED SECTION

In order to facilitate the derivation of area, centroid and moments of inertia formulas, typical expressions are supplied by means of parameters $S, Z, P, O, R,$ and Q introduced for each case in appendix C. These parameters are defined on the basis of the following discussion.

Let us consider a rectangular section as shown in Fig. 3.8a and find the area of the shaded portion. Divide the shaded portion into three parts (see Fig. 3.8b). The area will be the sum of these three parts.

$$A = (M2_x - S_x)(M2_y - M4_y) + (S_x - Z_x)(Z_y - M3_y) + \frac{1}{2} (S_x - Z_x)(S_y - Z_y)$$

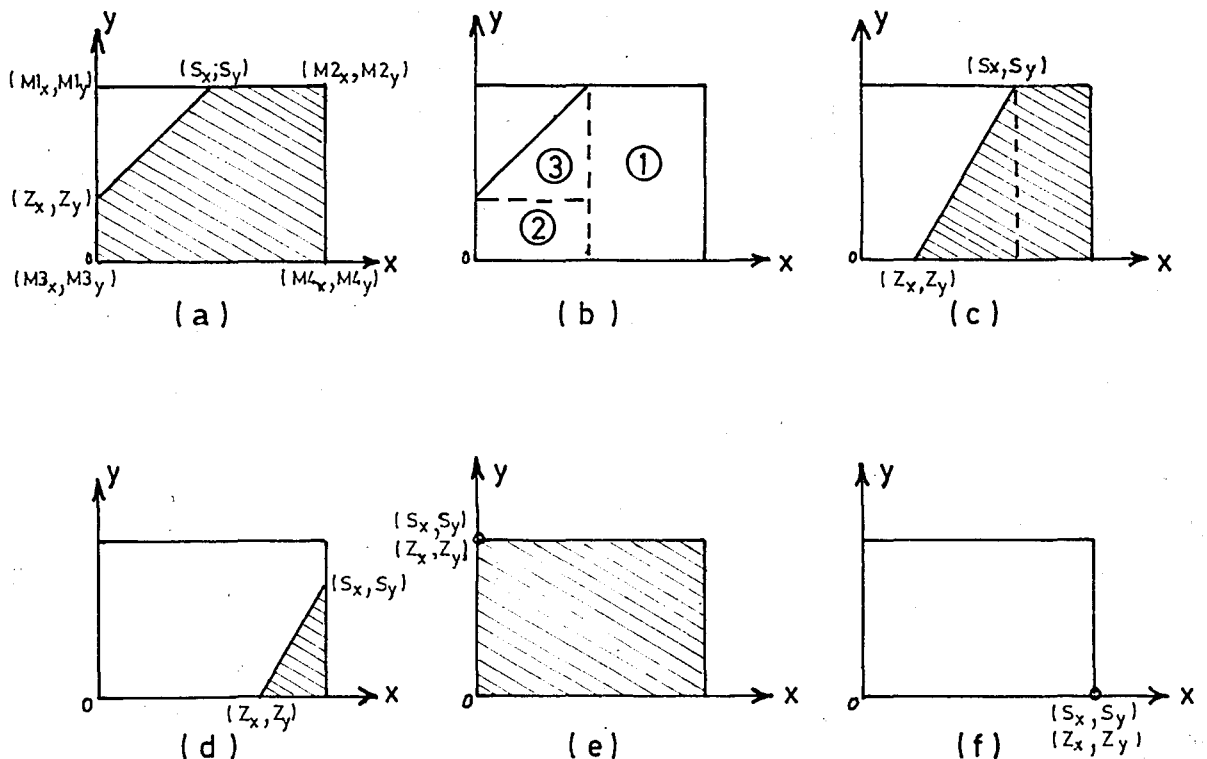


Fig.3.8 Different Compression Areas For a Rectangular Section

If the shaded portion is a trapezoid as shown in Fig. 3.8c, the above formula is still applicable. In this case $Z_y = M3_y$ therefore, $(S_x - Z_x)(Z_y - M3_y) = 0$.

Then

$$A = (M2_x - S_x)(M2_y - M4_y) + \frac{1}{2} (S_x - Z_x)(S_y - Z_y)$$

If the shaded portion is a triangle (Fig. 3.8d), $Z_y = M3_y$, $S_x = M2_x$ and

$$(S_x - Z_x)(Z_y - M3_y) = 0$$

$$(M2_x - S_x)(M2_y - M4_y) = 0$$

The expression for area becomes

$$A = \frac{1}{2} (S_x - Z_x)(S_y - Z_y)$$

If the shaded portion is a rectangle (Fig. 3.8e), $S_x = Z_x = M1_x$, $S_y = Z_y = M1_y$ and

$$(S_x - Z_x)(Z_y - M3_y) = 0$$

$$\frac{1}{2} (S_x - Z_x)(S_y - Z_y) = 0$$

Then

$$A = (M2_x - S_x)(M2_y - M4_y)$$

And, if there is no shaded portion, $S_x = Z_x = M4_x = M2_x$, $S_y = Z_y = M4_y$ and

$$(M2_x - S_x)(M2_y - M4_y) = 0$$

$$(S_x - Z_x)(Z_y - M3_y) = 0$$

$$\frac{1}{2} (S_x - Z_x)(S_y - Z_y) = 0$$

Then

$$A = 0$$

With the aid of these parameters four general equations are written for the area, eight equations for locating the centroid and twelve equations for the moments of inertia of the cracked section.

Case (1)

If the neutral axis is vertical or horizontal or has a positive angle with the horizontal and if the compression area

THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE 21

is below and/or to the right of the neutral axis, formulas for the area, centroid and moments of inertia will be

$$A' = A_1 + A_2 + \frac{1}{2} A_3 + A_4 + A_5 + \frac{1}{2} A_6 + A_7 + A_8 + \frac{1}{2} A_9 + n \sum a_{si} \quad 3.23$$

$$X_0 = \frac{3A_1B_1 + 3A_2B_2 + A_3B_3 + 3A_4B_4 + 3A_5B_5 + A_6B_6 + 3A_7B_7 + 3A_8B_8 + A_9B_9 + 6n \sum a_{si}x_i}{6A'} \quad 3.24a$$

$$Y_0 = \frac{3A_1B_{10} + 3A_2B_{11} + A_3B_{12} + 3A_4B_{13} + 3A_5B_{14} + A_6B_{15} + 3A_7B_{16} + 3A_8B_{17} + A_9B_{18} + 6n \sum a_{si}y_i}{6A'} \quad 3.24b$$

$$\begin{aligned} I'_x = & \frac{1}{12} A_1 ((C_{10y} - C_{7y})^2 + 3(B_{10} - 2Y_0)^2) + \frac{1}{12} A_2 ((Z_y - C_{8y})^2 + 3(B_{11} - 2Y_0)^2) \\ & + \frac{1}{36} A_3 ((S_y - Z_y)^2 + 2(B_{12} - 3Y_0)^2) + \frac{1}{12} A_4 ((C_{2y} - C_{7y})^2 + 3(B_{13} - 2Y_0)^2) \\ & + \frac{1}{12} A_5 ((P_y - C_{7y})^2 + 3(B_{14} - 2Y_0)^2) + \frac{1}{36} A_6 ((O_y - P_y)^2 + 2(B_{15} - 3Y_0)^2) \\ & + \frac{1}{12} A_7 ((C_{4y} - C_{5y})^2 + 3(B_{16} - 2Y_0)^2) + \frac{1}{12} A_8 ((Q_y - C_{6y})^2 + 3(B_{17} - 3Y_0)^2) \\ & + \frac{1}{36} A_9 ((R_y - Q_y)^2 + 2(B_{18} - 3Y_0)^2) + n \sum a_{si} y_i^2 \end{aligned} \quad 3.25a$$

$$\begin{aligned} I'_y = & \frac{1}{12} A_1 ((C_{10x} - S_x)^2 + 3(B_1 - 2X_0)^2) + \frac{1}{12} A_2 ((S_x - Z_x)^2 + 3(B_2 - 2X_0)^2) \\ & + \frac{1}{36} A_3 ((S_x - Z_x)^2 + 2(B_3 - 3X_0)^2) + \frac{1}{12} A_4 ((C_{2x} - O_x)^2 + 3(B_4 - 2X_0)^2) \\ & + \frac{1}{12} A_5 ((O_x - P_x)^2 + 3(B_5 - 2X_0)^2) + \frac{1}{36} A_6 ((O_x - P_x)^2 + 2(B_6 - 3X_0)^2) \\ & + \frac{1}{12} A_7 ((C_{4x} - R_x)^2 + 3(B_7 - 2X_0)^2) + \frac{1}{12} A_8 ((R_x - Q_x)^2 + 3(B_8 - 2X_0)^2) \\ & + \frac{1}{36} A_9 ((R_x - Q_x)^2 + 2(B_9 - 3X_0)^2) + n \sum a_{si} x_i^2 \end{aligned} \quad 3.25b$$

$$\begin{aligned} I'_{xy} = & \frac{1}{4} [A_1 (B_1 - 2X_0)(B_{10} - 2Y_0) + A_2 (B_2 - 2X_0)(B_{11} - 2Y_0) + A_4 (B_4 - 2X_0)(B_{13} - 2Y_0) \\ & + A_5 (B_5 - 2X_0)(B_{14} - 2Y_0) + A_7 (B_7 - 2X_0)(B_{16} - 2Y_0) + A_8 (B_8 - 2X_0)(B_{17} - 2Y_0)] \\ & + \frac{1}{72} A_3 (A_3 + 4(B_{12} - 3Y_0)(B_3 - 3X_0)) + \frac{1}{72} A_6 (A_6 + 4(B_{15} - 3Y_0)(B_6 - 3X_0)) \\ & + \frac{1}{72} A_9 (A_9 + 4(B_{18} - 3Y_0)(B_9 - 3X_0)) + n \sum a_{si} x_i^1 y_i^1 \end{aligned} \quad 3.25c$$

where:

$A_1 = (C_{10x} - S_x)(C_{10y} - C_{7y})$	$A_4 = (C_{2x} - O_x)(C_{2y} - C_{6y})$	$A_7 = (C_{4x} - R_x)(C_{4y} - C_{5y})$
$A_2 = (S_x - Z_x)(Z_y - C_{7y})$	$A_5 = (O_x - P_x)(P_y - C_{7y})$	$A_8 = (R_x - Q_x)(Q_y - C_{6y})$
$A_3 = (S_x - Z_x)(S_y - Z_y)$	$A_6 = (O_x - P_x)(O_y - P_y)$	$A_9 = (R_x - Q_x)(R_y - Q_y)$

$B_1 = C10_x + S_x$	$B_7 = C4_x + R_x$	$B_{13} = C2_y + C6_y$
$B_2 = S_x + Z_x$	$B_8 = R_x + Q_x$	$B_{14} = P_y + C7_y$
$B_3 = Z_x + 2S_x$	$B_9 = Q_x + 2R_x$	$B_{15} = O_y + 2P_y$
$B_4 = C2_x + O_x$	$B_{10} = C10_y + C7_y$	$B_{16} = C4_y + C5_y$
$B_5 = O_x + P_x$	$B_{11} = Z_y + C7_y$	$B_{17} = Q_y + C6_y$
$B_6 = P_x + 2O_x$	$B_{12} = S_y + 2Z_y$	$B_{18} = R_y + 2Q_y$

Case (2)

If the neutral axis is vertical or horizontal or has a positive angle with the horizontal and if the compression area is above and/or to the left of the neutral axis, formulas for the area, centroid and moments of inertia will be

$$A' = A_1 + A_2 + \frac{1}{2} A_3 + A_4 + A_5 + \frac{1}{2} A_6 + A_7 + A_8 + \frac{1}{2} A_9 + n \sum a_{si} \quad 3.26$$

$$X_0 = \frac{3A_1 B_1 + 3A_2 B_2 + A_3 B_3 + 3A_4 B_4 + 3A_5 B_5 + A_6 B_6 + 3A_7 B_7 + 3A_8 B_8 + A_9 B_9 + 6n \sum a_{si} x_i}{6A'} \quad 3.27a$$

$$Y_0 = \frac{3A_1 B_{10} + 3A_2 B_{11} + A_3 B_{12} + 3A_4 B_{13} + 3A_5 B_{14} + A_6 B_{15} + 3A_7 B_{16} + 3A_8 B_{17} + A_9 B_{18} + 6n \sum a_{si} y_i}{6A'} \quad 3.27b$$

$$\begin{aligned} I'_x = & \frac{1}{12} A_1 ((C10_y - C7_y)^2 + 3(B_{10} - 2Y_0)^2) + \frac{1}{12} A_2 ((C10_y - S_y)^2 + 3(B_{11} - 2Y_0)^2) \\ & + \frac{1}{36} A_3 ((S_y - Z_y)^2 + 2(B_{12} - 3Y_0)^2) + \frac{1}{12} A_4 ((C2_y - C6_y)^2 + 3(B_{13} - 2Y_0)^2) \\ & + \frac{1}{12} A_5 ((C2_y - O_y)^2 + 3(B_{14} - 2Y_0)^2) + \frac{1}{36} A_6 ((O_y - P_y)^2 + 2(B_{15} - 3Y_0)^2) \\ & + \frac{1}{12} A_7 ((C4_y - C5_y)^2 + 3(B_{16} - 2Y_0)^2) + \frac{1}{12} A_8 ((C4_y - R_y)^2 + 3(B_{17} - 2Y_0)^2) \\ & + \frac{1}{36} A_9 ((R_y - Q_y)^2 + 2(B_{18} - 3Y_0)^2) + n \sum a_{si} y_i^2 \end{aligned} \quad 3.28a$$

$$\begin{aligned}
 I'_y = & \frac{1}{12} A_1 ((Z_x - C_{B_x})^2 + 3(B_1 - 2X_0)^2) + \frac{1}{12} A_2 ((S_x - Z_x)^2 + 3(B_2 - 2X_0)^2) \\
 & + \frac{1}{36} A_3 ((S_x - Z_x)^2 + 2(B_3 - 3X_0)^2) + \frac{1}{12} A_4 ((P_x - C_{7_x})^2 + 3(B_4 - 2X_0)^2) \\
 & + \frac{1}{12} A_5 ((O_x - P_x)^2 + 3(B_5 - 2X_0)^2) + \frac{1}{36} A_6 ((O_x - P_x)^2 + 2(B_6 - 3X_0)^2) \\
 & + \frac{1}{12} A_7 ((O_x - C_{6_x})^2 + 3(B_7 - 2X_0)^2) + \frac{1}{12} A_8 ((R_x - Q_x)^2 + 3(B_8 - 2X_0)^2) \\
 & + \frac{1}{36} A_9 ((R_x - Q_x)^2 + 2(B_9 - 3X_0)^2) + n \sum a_{si} x_i'^2
 \end{aligned} \tag{3.28 b}$$

$$\begin{aligned}
 I'_{xy} = & \frac{1}{4} [A_1 (B_1 - 2X_0)(B_{10} - 2Y_0) + A_2 (B_2 - 2X_0)(B_{11} - 2Y_0) + A_4 (B_4 - 2X_0)(B_{13} - 2Y_0) \\
 & + A_5 (B_5 - 2X_0)(B_{14} - 2Y_0) + A_7 (B_7 - 2X_0)(B_{16} - 2Y_0) + A_8 (B_8 - 2X_0)(B_{17} - 2Y_0)] \\
 & + \frac{1}{12} A_3 (A_3 + 4(B_3 - 3X_0)(B_{12} - 3Y_0)) + \frac{1}{12} A_6 (A_6 + 4(B_6 - 3X_0)(B_{15} - 3Y_0)) \\
 & + \frac{1}{12} A_9 (A_9 + 4(B_9 - 3X_0)(B_{18} - 3Y_0)) + n \sum a_{si} x_i' y_i'
 \end{aligned} \tag{3.28 c}$$

where:

$$\begin{array}{lll}
 A_1 = (Z_x - C_{B_x})(C_{10_y} - C_{7_y}) & A_4 = (P_x - C_{7_x})(C_{2_y} - C_{6_y}) & A_7 = (Q_x - C_{6_x})(C_{4_y} - C_{5_y}) \\
 A_2 = (S_x - Z_x)(C_{10_y} - S_y) & A_5 = (O_x - P_x)(C_{2_y} - O_y) & A_8 = (R_x - Q_x)(C_{4_y} - R_y) \\
 A_3 = (S_x - Z_x)(S_y - Z_y) & A_6 = (O_x - P_x)(O_y - P_y) & A_9 = (R_x - Q_x)(R_y - Q_y)
 \end{array}$$

$$\begin{array}{lll}
 B_1 = Z_x + C_{B_x} & B_7 = Q_x + C_{6_x} & B_{13} = C_{2_y} + C_{6_y} \\
 B_2 = S_x + Z_x & B_8 = R_x + Q_x & B_{14} = C_{2_y} + O_y \\
 B_3 = Z_x + 2S_x & B_9 = Q_x + 2R_x & B_{15} = P_y + 2O_y \\
 B_4 = P_x + C_{7_x} & B_{10} = C_{10_y} + C_{7_y} & B_{16} = C_{4_y} + C_{5_y} \\
 B_5 = O_x + P_x & B_{11} = C_{10_y} + S_y & B_{17} = C_{4_y} + R_y \\
 B_6 = P_x + 2O_x & B_{12} = Z_y + 2S_y & B_{18} = Q_y + 2R_y
 \end{array}$$

Case (3)

If the neutral axis has a negative angle with the horizontal and if the compression area is below and to the left of the neutral axis, formulas for the area, centroid and moments of inertia will be

$$A' = A_1 + A_2 + \frac{1}{2} A_3 + A_4 + A_5 + \frac{1}{2} A_6 + A_7 + A_8 + \frac{1}{2} A_9 + n \sum a_{si} \quad 3.29$$

$$X_0 = \frac{3A_1 B_1 + 3A_2 B_2 + A_3 B_3 + 3A_4 B_4 + 3A_5 B_5 + A_6 B_6 + 3A_7 B_7 + 3A_8 B_8 + A_9 B_9 + 6n \sum a_{si} x_i}{6A'} \quad 3.30a$$

$$Y_0 = \frac{3A_1 B_{10} + 3A_2 B_{11} + A_3 B_{12} + 3A_4 B_{13} + 3A_5 B_{14} + A_6 B_{15} + 3A_7 B_{16} + 3A_8 B_{17} + A_9 B_{18} + 6n \sum a_{si} y_i}{6A'} \quad 3.30b$$

$$\begin{aligned} I'_x = & \frac{1}{12} A_1 ((C9_y - C8_y)^2 + 3(B_{10} - 2Y_0)^2) + \frac{1}{12} A_2 ((Z_y - C7_y)^2 + 3(B_{11} - 2Y_0)^2) \\ & + \frac{1}{36} A_3 ((S_y - Z_y)^2 + 2(B_{12} - 3Y_0)^2) + \frac{1}{12} A_4 ((C1_y - C7_y)^2 + 3(B_{13} - 2Y_0)^2) \\ & + \frac{1}{12} A_5 ((O_y - C6_y)^2 + 3(B_{14} - 2Y_0)^2) + \frac{1}{36} A_6 ((P_y - O_y)^2 + 2(B_{15} - 3Y_0)^2) \\ & + \frac{1}{12} A_7 ((C3_y - C6_y)^2 + 3(B_{16} - 2Y_0)^2) + \frac{1}{12} A_8 ((Q_y - C5_y)^2 + 3(B_{17} - 2Y_0)^2) \\ & + \frac{1}{36} A_9 ((R_y - Q_y)^2 + 2(B_{18} - 3Y_0)^2) + n \sum a_{si} y_i'^2 \end{aligned} \quad 3.31 a$$

$$\begin{aligned} I'_y = & \frac{1}{2} A_1 ((S_x - C9_x)^2 + 3(B_1 - 2X_0)^2) + \frac{1}{12} A_2 ((Z_x - S_x)^2 + 3(B_2 - 2X_0)^2) \\ & + \frac{1}{36} A_3 ((Z_x - S_x)^2 + 2(B_3 - 3X_0)^2) + \frac{1}{12} A_4 ((P_x - C1_x)^2 + 3(B_4 - 2X_0)^2) \\ & + \frac{1}{12} A_5 ((O_x - P_x)^2 + 3(B_5 - 2X_0)^2) + \frac{1}{36} A_6 ((O_x - P_x)^2 + 2(B_6 - 3X_0)^2) \\ & + \frac{1}{12} A_7 ((R_x - C3_x)^2 + 3(B_7 - 2X_0)^2) + \frac{1}{12} A_8 ((Q_x - R_x)^2 + 3(B_8 - 2X_0)^2) \\ & + \frac{1}{36} A_9 ((Q_x - R_x)^2 + 2(B_9 - 3X_0)^2) + n \sum a_{si} x_i'^2 \end{aligned} \quad 3.31 b$$

$$\begin{aligned} I'_{xy} = & \frac{1}{4} [A_1 (B_{10} - 2Y_0)(B_1 - 2X_0) + A_2 (B_{11} - 2Y_0)(B_2 - 2X_0) + A_4 (B_{13} - 2Y_0)(B_4 - 2X_0) \\ & + A_5 (B_{14} - 2Y_0)(B_5 - 2X_0) + A_7 (B_{16} - 2Y_0)(B_7 - 2X_0) + A_8 (B_{17} - 2Y_0)(B_8 - 2X_0)] \\ & + \frac{1}{72} A_3 (-A_3 + 4(B_{12} - 3Y_0)(B_3 - 3X_0)) + \frac{1}{72} A_6 (-A_6 + 4(B_{15} - 3Y_0)(B_6 - 3X_0)) \\ & + \frac{1}{72} A_9 (-A_9 + 4(B_{18} - 3Y_0)(B_9 - 3X_0)) + n \sum a_{si} x_i' y_i' \end{aligned} \quad 3.31 c$$

where:

$$\begin{array}{lll} A_1 = (S_x - C9_x)(C9_y - C8_y) & A_4 = (C1_y - C7_y)(P_x - C1_x) & A_7 = (R_x - C3_x)(C3_y - C6_y) \\ A_2 = (Z_x - S_x)(Z_y - C7_y) & A_5 = (O_x - P_x)(O_y - C6_y) & A_8 = (Q_x - R_x)(Q_y - C5_y) \\ A_3 = (Z_x - S_x)(S_y - Z_y) & A_6 = (O_x - P_x)(O_y - P_y) & A_9 = (Q_x - R_x)(R_y - Q_y) \end{array}$$

$$B_1 = S_x + C9_x$$

$$B_2 = Z_x + S_x$$

$$B_3 = Z_x + 2S_x$$

$$B_4 = P_x + C1_x$$

$$B_5 = O_x + P_x$$

$$B_6 = O_x + 2P_x$$

$$B_7 = R_x + C3_x$$

$$B_8 = Q_x + R_x$$

$$B_9 = Q_x + 2R_x$$

$$B_{10} = C9_y + C8_y$$

$$B_{11} = Z_y + C7_y$$

$$B_{12} = S_y + 2Z_y$$

$$B_{13} = C1_y + C7_y$$

$$B_{14} = O_y + C6_y$$

$$B_{15} = P_y + 2O_y$$

$$B_{16} = C3_y + C6_y$$

$$B_{17} = Q_y + C5_y$$

$$B_{18} = R_y + 2Q_y$$

Case (4)

If the neutral axis has a negative angle with the horizontal and if the compression area is above and to the right of the neutral axis, formulas for the area, centroid and moments of inertia will be

$$A' = A_1 + A_2 + \frac{1}{2} A_3 + A_4 + A_5 + \frac{1}{2} A_6 + A_7 + A_8 + \frac{1}{2} A_9 + n \sum a_{si} \quad 3.32$$

$$X_0 = \frac{3A_1B_1 + 3A_2B_2 + A_3B_3 + 3A_4B_4 + 3A_5B_5 + A_6B_6 + 3A_7B_7 + 3A_8B_8 + A_9B_9 + 6n \sum a_{si} x_i}{6A'} \quad 3.33a$$

$$Y_0 = \frac{3A_1B_{10} + 3A_2B_{11} + A_3B_{12} + 3A_4B_{13} + 3A_5B_{14} + A_6B_{15} + 3A_7B_{16} + 3A_8B_{17} + A_9B_{18} + 6n \sum a_{si} y_i}{6A'} \quad 3.33b$$

$$\begin{aligned} I'_x = & \frac{1}{12} A_1 ((C4_y - C5_y)^2 + 3(B_{10} - 2Y_0)^2) + \frac{1}{12} A_2 ((C3_y - R_y)^2 + 3(B_{11} - 2Y_0)^2) \\ & + \frac{1}{36} A_3 ((R_y - Q_y)^2 + 2(B_{12} - 3Y_0)^2) + \frac{1}{12} A_4 ((C1_y - P_y)^2 + 3(B_{13} - 2Y_0)^2) \\ & + \frac{1}{12} A_5 ((C2_y - C6_y)^2 + 3(B_{14} - 2Y_0)^2) + \frac{1}{36} A_6 ((P_y - O_y)^2 + 2(B_{15} - 3Y_0)^2) \\ & + \frac{1}{12} A_7 ((C9_y - S_y)^2 + 3(B_{16} - 2Y_0)^2) + \frac{1}{12} A_8 ((S_y - C7_y)^2 + 3(B_{17} - 2Y_0)^2) \\ & + \frac{1}{36} A_9 ((S_y - Z_y)^2 + 2(B_{18} - 3Y_0)^2) + n \sum a_{si} y_i^2 \end{aligned} \quad 3.34a$$

$$\begin{aligned}
 I'_y = & \frac{1}{12} A_1 ((C5_x - Q_x)^2 + 3(B_1 - 2X_0)^2) + \frac{1}{12} A_2 ((Q_x - R_x)^2 + 3(B_2 - 2X_0)^2) \\
 & + \frac{1}{36} A_3 ((Q_x - R_x)^2 + 2(B_3 - 3X_0)^2) + \frac{1}{12} A_4 ((O_x - P_x)^2 + 3(B_4 - 2X_0)^2) \\
 & + \frac{1}{12} A_5 ((C3_x - O_x)^2 + 3(B_5 - 2X_0)^2) + \frac{1}{36} A_6 ((O_x - P_x)^2 + 2(B_6 - 3X_0)^2) \\
 & + \frac{1}{12} A_7 ((C10_x - C9_x)^2 + 3(B_7 - 2X_0)^2) + \frac{1}{12} A_8 ((C7_x - Z_x)^2 + 3(B_8 - 2X_0)^2) \\
 & + \frac{1}{36} A_9 ((Z_x - S_x)^2 + 2(B_9 - 3X_0)^2) + n \sum a_{si} x_i'^2
 \end{aligned} \tag{3.34 b}$$

$$\begin{aligned}
 I'_{xy} = & \frac{1}{4} [A_1 (B_{10} - 2Y_0)(B_1 - 2X_0) + A_2 (B_{11} - 2Y_0)(B_2 - 2X_0) + A_4 (B_{13} - 2Y_0)(B_4 - 2X_0) \\
 & + A_5 (B_{14} - 2Y_0)(B_5 - 2X_0) + A_7 (B_{16} - 2Y_0)(B_7 - 2X_0) + A_8 (B_{17} - 2Y_0)(B_8 - 2X_0)] \\
 & + \frac{1}{72} A_3 (-A_3 + 4(B_{12} - 3Y_0)(B_3 - 3X_0)) + \frac{1}{72} A_6 (-A_6 + 4(B_{15} - 3Y_0)(B_6 - 3X_0)) \\
 & + \frac{1}{72} A_9 (-A_9 + 4(B_{18} - 3Y_0)(B_9 - 3X_0)) + n \sum a_{si} x_i' y_i'
 \end{aligned} \tag{3.34 c}$$

where:

$$\begin{array}{lll}
 A_1 = (C5_x - Q_x)(C4_y - C5_y) & A_4 = (O_x - P_x)(C1_y - P_y) & A_7 = (C10_x - C9_x)(C9_y - S_y) \\
 A_2 = (C3_y - R_y)(Q_x - R_x) & A_5 = (C3_x - O_x)(C2_y - C6_y) & A_8 = (C7_x - Z_x)(S_y - C7_y) \\
 A_3 = (Q_x - R_x)(R_y - Q_y) & A_6 = (O_x - P_x)(P_y - O_y) & A_9 = (Z_x - S_x)(S_y - Z_y)
 \end{array}$$

$$B_1 = C5_x + Q_x$$

$$B_2 = C10_x + C9_x$$

$$B_{13} = C1_y + P_y$$

$$B_3 = Q_x + R_x$$

$$B_8 = C7_x + Z_x$$

$$B_{14} = C2_y + C6_y$$

$$B_4 = 2Q_x + R_x$$

$$B_9 = 2Z_x + S_x$$

$$B_{15} = 2P_y + O_y$$

$$B_5 = C3_x + O_x$$

$$B_{10} = C4_y + C5_y$$

$$B_{16} = C9_y + S_y$$

$$B_6 = 2O_x + P_x$$

$$B_{11} = C3_y + R_y$$

$$B_{17} = S_y + C7_y$$

$$B_7 = 2O_x + P_x$$

$$B_{12} = 2R_y + Q_y$$

$$B_{18} = 2S_y + Z_y$$

3.5 STRESSES IN CRACKED SECTION

Once the area, centroid and moments of inertia of the cracked section are calculated, the stresses in concrete at the corner points of the section can be found from the following equations.

$$\begin{aligned} \sigma_{c1} &= F/A' + K_y' (t_4 - v_o - y_o) + K_x' (d_2 - u_o - x_o) \\ \sigma_{c2} &= F/A' + K_y' (t_4 - v_o - y_o) + K_x' (d_4 - u_o - x_o) \\ \sigma_{c3} &= F/A' + K_y' (t_3 - v_o - y_o) + K_x' (d_4 - u_o - x_o) \\ \sigma_{c4} &= F/A' + K_y' (-v_o - y_o) + K_x' (d_5 - u_o - x_o) \\ \sigma_{c5} &= F/A' + K_y' (-v_o - y_o) + K_x' (d_5 - u_o - x_o) \\ \sigma_{c6} &= F/A' + K_y' (-v_o - y_o) + K_x' (d_4 - u_o - x_o) \\ \sigma_{c7} &= F/A' + K_y' (-v_o - y_o) + K_x' (d_2 - u_o - x_o) \\ \sigma_{c8} &= F/A' + K_y' (-v_o - y_o) + K_x' (-u_o - x_o) \\ \sigma_{c9} &= F/A' + K_y' (t_2 - v_o - y_o) + K_x' (-u_o - x_o) \\ \sigma_{c10} &= F/A' + K_y' (t_2 - v_o - y_o) + K_x' (d_2 - u_o - x_o) \end{aligned}$$

3.35

where

$$K_y' = \frac{F e_y' I_y' + F e_x' I_{xy}'}{I_x' I_y' - I_{xy}'^2} ; \quad K_x' = \frac{F e_x' I_x' + F e_y' I_{xy}'}{I_x' I_y' - I_{xy}'^2}$$

And the stresses in reinforcing steel are given by the following equation.

$$\sigma_i = n [F/A' + K_y' (y_i') + K_x' (x_i')] \quad 3.36$$

3.6 LOCATION OF NEUTRAL AXIS OF CRACKED SECTION

The analysis of the cracked section continues with the determination of the location of the neutral axis by using the values found for $e_{x'}$, $e_{y'}$, I_x' , I_y' , I_{xy}' , and A' .

Case (a)

$$B = -\frac{F}{A' K_y'} + v_o + y_o ; \quad C = -1 \quad 3.37$$

Case (b)

$$B = -1 ; C = d_s - U_o - X_o + \frac{F}{A'K_{x'}} \quad 3.38$$

Case (c)

$$B = (d_s - U_o - X_o) \tan \theta' - \frac{F}{A'K_{y'}} + V_o + Y_o$$

$$C = (d_s - U_o - X_o) + \frac{F}{A'K_{x'}} - \frac{1}{\tan \theta'} (-V_o - Y_o) \quad 3.39$$

Case (d)

$$B = (-U_o - X_o) \tan \theta' - \frac{F}{A'K_{y'}} + V_o + Y_o$$

$$C = -\frac{F}{A'K_{x'}} - \frac{1}{\tan \theta'} (V_o + Y_o) + U_o + X_o \quad 3.40$$

where

$$\tan \theta' = -\frac{K_{x'}}{K_{y'}}$$

3.7 ITERATION PROCEDURE

The next step is to compare the values of B and B_I , C and C_I . If they coincide the analysis of the cracked section is completed, but if they do not, let $B_I = B$ and $C_I = C$. Find the area, centroid and moments of inertia of the cracked section whose neutral axis is defined by B and C. Then calculate the stresses and the new values of B and C. Compare them with the previously calculated B and C and repeat the procedure until they coincide. Then the analysis of the cracked section is completed.

3.8 SUMMARY OF DESIGN PROCEDURE

A reinforced concrete section is either given or tentatively selected. The loading (F, M_x, M_y) is known as the working loads. The problem is to find the stress-strain distribution, i.e., to determine whether the stresses exceed the allowable limits or not. The procedure for solving the problem is as follows:

1. Locate the centroid o , and compute I_x , I_y and I_{xy} of the uncracked section from Eqs.3.1 to 3.12.
2. Using equations 3.13, 3.14, determine the stresses in the concrete and in the reinforcing bars. If the calculated stresses are smaller than the allowable limits and if there is no tensile stress in the concrete, the problem is solved. But if there is tensile stress in the concrete, proceed as outlined in the following steps.
3. Determine the location of the neutral axis of the uncracked section. (If $e_x=0$, use Eq.3.15; if $e_y=0$, use Eq.3.16; if $e_x>0$ and $e_y<0$ or vice versa use Eq.3.17; if $e_x>0$ and $e_y>0$ or $e_x<0$ and $e_y<0$, use Eq.3.18)
4. Locate a new neutral axis by using Löser's approximate formulas (Eqs.3.19 to 3.22).
5. Locate the centroid of the new cracked section o' and compute I'_x , I'_y , and I'_{xy} of this new cracked section.
6. Using equations 3.35 and 3.36, determine the stresses in the concrete and reinforcing bars.
7. Determine the location of the neutral axis of the new cracked section by using the values found in step 5. If $e_x=0$, use Eq.3.37; if $e_y=0$, use Eq.3.38; if $e_x>0$ and $e_y<0$ or vice versa, use Eq.3.39; if $e_x>0$ and $e_y>0$ or $e_x<0$ and $e_y<0$, use Eq.3.40.
8. Compare the locations of the neutral axis found in step 7 and in step 4.
9. If the neutral axes coincide and the stresses calculated do not exceed the allowable, the problem is solved. If the neutral axes do not coincide repeat steps 5,6,7, and 8 assuming the location of the neutral axis as found in step 7.

IMPORTANT NOTE:

All the formulas derived in this paper using the sign convention shown in Fig.2.2 and labeling shown in Fig.3.1 apply to the cases where the right leg of the T section is thinner

than the left leg. If one wants to design a T section whose right leg is thicker than the left leg then, he must use the sign convention and labeling shown below.

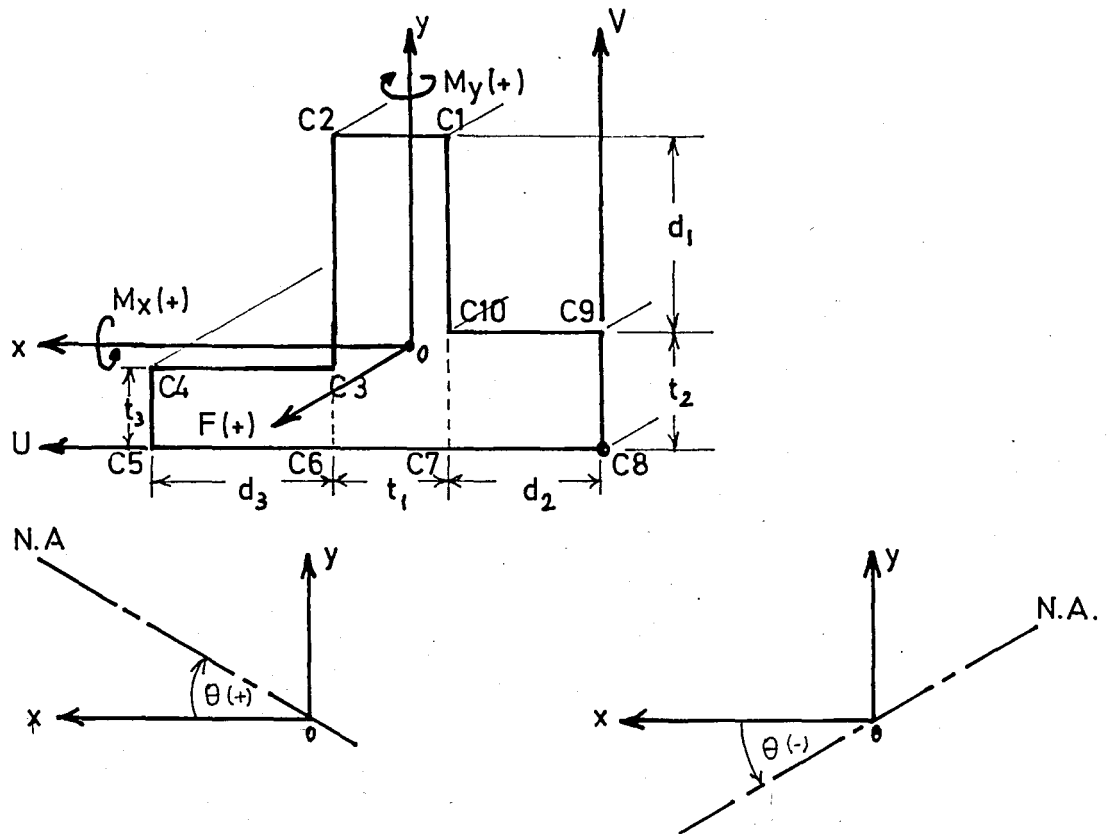


Fig.39 Sign Convention When Right Leg Is Thicker Than Left Leg.

CHAPTER 4

COMPUTER PROGRAMS

The computer program listings are given in appendix D.
For data instructions see appendix B.

The computer gives a message and terminates the operation in the following cases:

Case I

NORMAL FORCE GREATER THAN ALLOWABLE

This is printed when $F > (\sigma_c)_{al} A_c + (\sigma_s)_{al} A_s$

Case II

AREA TOO SMALL, CHANGE THE CROSS SECTION

This means that tentatively selected section is smaller than a section of 24cm x 24cm with a reinforcement of 4Ø14. This is the smallest section that can be designed according to the Turkish Building Code.

Case III

INCREASE THE LOAD WITH A FACTOR SPECIFIED IN THE CODE

This is printed when the column is slender. The load must be increased by the factor w given in Cetvel Va (page 55) in Betonarme Şartnamesi.

Case IV

CHANGE THE SECTION OR REINFORCEMENT

This means compressive stress in concrete exceeds the allowable stress.

Case V

CHANGE THE SECTION

This is printed when the first assumption of neutral axis falls outside or on the edge of the section. This means that the eccentricity of the load is too large and there is no compression area.

Case VI

N.A. FALLS OUTSIDE

See case V

Case VII

N.A. FALLS ON PT.4(1,5,8)

See case V

The other messages are self explanatory.

CHAPTER 5

CONCLUSIONS

As demonstrated by means of the numeric examples, the method presented determines theoretically the exact location of the neutral axis and the stress distribution of unsymmetrical or symmetrical column sections in biaxial bending with the eccentricities exceeding the limit of cracking. However, there may be approximations involved due to the assumptions inherent in the elastic theory of bending.

The method is programmed for purposes of electronic computation, instead of preparing design tables and charts. Therefore, the design time is reduced to bare minimum and moreover, more accurate results are obtained. Further, the errors that could easily be done in the manual design computations are avoided.

Türkiye Köprü ve İnşaat Cemiyeti Betonarme Şartnamesi requires that in biaxial bending, the calculated maximum tensile stress must not exceed 0.35 times the calculated maximum compressive stress in concrete. It gives the allowable compressive stress for B160 as 80 kg/cm² when moments are present in two directions. This means that concrete may take tensile stress up to 28 kg/cm². Since this is not reliable, tensile stress in concrete is altogether neglected. The other restrictions slenderness ratio, minimum area, steel ratio and allowable capacity of the section, are formulated and incorporated into the first computer program. Therefore, the designer need not spend time for testing these requirements.

The designer may use any type of steel arrangement. If the

reinforcement in the tentatively selected section is too much or too small the computer increases or decreases the diameter of reinforcement bars. On the other hand, if the section is too small, the computer prints out a message calling for the increase of the section. The section could be enlarged by the computer automatically but since the choice of the designer could be restricted by architectural considerations, the change of the section is left to the judgement of the designer.

This method is also applicable to the design of shear walls which have longitudinal reinforcement bars up to 50 since shear walls are designed as columns whose lengths are equal to the height of the building and have a uniformly spaced longitudinal reinforcement arrangement such as the reinforcement in a one way slab.

APPENDIX A

NUMERIC EXAMPLE

NUMERIC EXAMPLE

In order to show the results satisfy the equilibrium equations ($\Sigma F = 0, \Sigma M = 0$) easily, a T section symmetric about an axis (1-1) is chosen as shown in Fig. A.

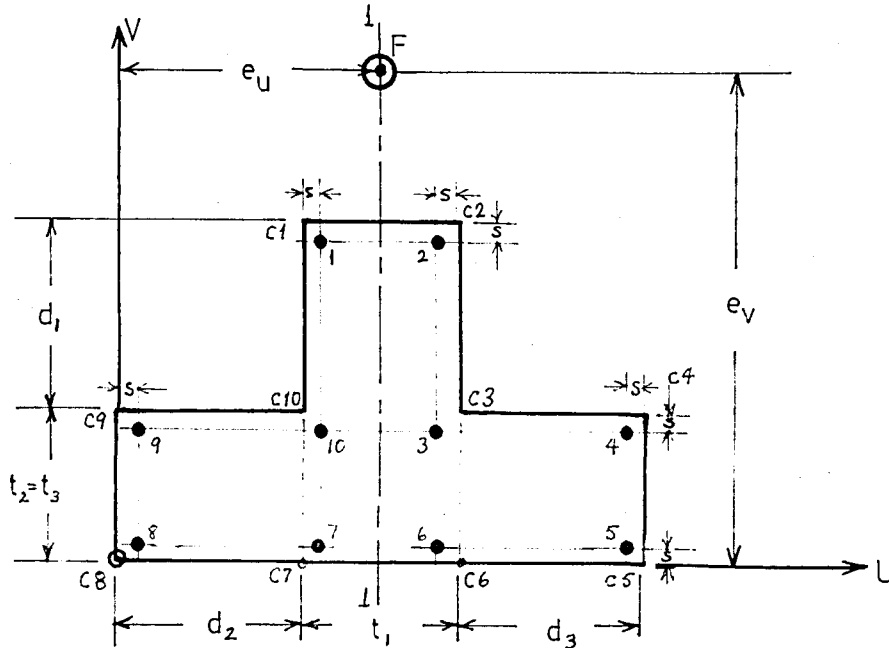


Fig. A

Let

F (axial force) = 5000 kg

$e_u = 35$ cm , $e_v = 90$ cm

$t_1 = t_2 = t_3 = 20$ cm

$d_1 = d_2 = d_3 = 25$ cm

$n = 15$

$s = 3$ cm

diameter of reinforcing bars = 14 mm

length of column = 300 cm

and assume B160 concrete and ST1 steel are used.

From Betonarme Şartnamesi

maximum steel ratio = 0.030

minimum steel ratio = 0.004

For B160 concrete allowable compressive stress is 80 kg/cm² and for ST1 steel allowable tensile and compressive stress is 1400 kg/cm².

According to the information given above, data for the computer is as follows:

EXAMPLE TO TEST THE PROGRAM

20. = T1	20. = T2	20. = T3	25. = D1	25. = D2	25. = D3	15.	10
1.4 = DS of the first bar	28. = ASU of the first bar	42. = ASV of the first bar	1.4	42.	42.	RM ↓ N ↓ DS, ASU, ASV of the second bar	
1.4	42.	17.	1.4	67.	17.		
1.4	67.	3.	1.4	42.	3.		
1.4	28.	3.	1.4	3.	3.		
1.4	3.	17.	1.4	28.	17.		
.004 = SRMI	.030 = SRMA						
	-5000. = F		35. = EV		90. = EV		
-80. = ACC	0. = ACS	-1400. = ATS	1400. = ATC				
300. = HI							

The solution

The computer prints out the solution in the following way:

WSD OF CC COLUMNS

EXAMPLE TO TEST THE PROGRAM

PART ONE, SECTION NOT CRACKED
ALL UNITS ARE IN KG AND CM

T1= 20.
T2= 20.
T3= 20.
D1= 25.
D2= 25.
D3= 25.
MODULAR RATIO= 15
NUMBER OF BARS= 10

MIN. STEEL RATIO= .00400
MAX. STEEL RATIO= .03000

NORMAL FORCE= -5000.
EU= 35.
EV= 90.

ALLOWABLE COMPRESSIVE STRESS IN CONCRETE= -80.000
ALLOWABLE TENSILE STRESS IN CONCRETE= 0.000
ALLOWABLE COMPRESSIVE STRESS IN STEEL= -1400.000
ALLOWABLE TENSILE STRESS IN STEEL= 1400.000

DIA. OF REINFORCEMENT BAR	U (HORIZONTAL)	V (VERTICAL)
1.40000	28.00000	42.00000
1.40000	42.00000	42.00000
1.40000	42.00000	17.00000
1.40000	67.00000	17.00000
1.40000	67.00000	3.00000
1.40000	42.00000	3.00000
1.40000	28.00000	3.00000
1.40000	3.00000	3.00000
1.40000	3.00000	17.00000
1.40000	28.00000	17.00000

NORMAL FORCE WITHIN ALLOWABLE RANGE

STEEL RATIO WITHIN ALLOWABLE RANGE

CENTROID OF THE SECTION

UO= 34.99999
VO= 15.96986

TIX= 303041.30000 TIY= 682981.79000 TIXY= -.01159

STRESSES IN CONCRETE

-37.822 -37.822 -7.286 -7.286 17.143
17.143 17.143 17.143 -7.286 -7.286

STRESSES IN BARS

-512.369 -512.369 -54.324 -54.324 202.180 202.180 202.180 202.180
-54.324 -54.324

MIN. STRESS IN CONCRETE= -37.822
MAX. STRESS IN CONCRETE= 17.143

MIN. STRESS IN BARS= 512.369
MAX. STRESS IN BARS= 202.180

PART TWO, SECTION CRACKED
ALL UNITS ARE IN KG AND CM

TANTET= 0.00

C= -1.000 B= 15.969

POINTS TO LOCATE N.A.

9.999	.000
-34.999	.000
9.999	.000
10.000	.000
35.000	.000
10.000	.000

XO= 0.0 YO= 8.816

CIXX= 69227.14600 CIYY= 131885.01000 CIXXYY= -.00025

SIXX= 63149.89100 SIYY= 101408.97000 SIXXYY= .00030

TOTAL MOMENT OF INERTIA WHEN SECTION CRACKED

TIXX=	132377.03000
TIYY=	233293.98000
TIXXYY=	.00005

TET= 0.0

N.A. HORIZONTAL, BA MEASURED FROM PT. 5

BA= 22.78314

CA= -1.00

STRESSES IN CONCRETE (CRACKED)

54.723
54.723
6.855
6.855
56.118
56.118
56.118
56.118
6.855
6.855

STRESSES IN BARS (CRACKED)

-710.014 -710.014 213.672 213.672 730.937 730.937 730.937 730.937
213.672 213.672

MIN. STRESS IN CONCRETE= -54.723
MAX. STRESS IN CONCRETE= 56.118

MIN. STRESS IN BARS= -710.014
MAX. STRESS IN BARS= 730.937

ALL STRESSES WITHIN ALLOWABLE RANGE

PART THREE, FINAL STRESSES
 ALL UNITS ARE IN KG AND CM

POINTS TO LOCATE N.A.

-34.999	4.030
-34.999	4.030
9.999	6.813
10.000	6.813
10.000	4.030
10.000	4.030

XO= 0. YO= 11.938

CIXX= 34182.35800 CIYY= 14811.23800 CIXXY= -.00015
 SIXX= 77497.44700 SIYY= 101408.97000 SIXXY= .00010

TOTAL MOMENT OF INERTIA WHEN SECTION CRACKED

TIXX= 111679.80000
 TIYY= 116220.20000
 TIXXY= -.00005

TEI= .0

N.A. HORIZONTAL, BA MEASURED FROM PT.5

BA= 25.24524
 CA= -1.00

STRESSES IN CONCRETE (CRACKED)

54.915
 54.915
 14.581
 14.581
 70.178
 70.178
 70.178
 70.178
 14.581
 14.581

STRESSES IN BARS (CRACKED)

-698.644 -698.644 343.812 343.812 927.588 927.588 927.588 927.588
 343.812 343.812

MIN. STRESS IN CONCRETE= -54.915
 MAX. STRESS IN CONCRETE= 70.178

MIN. STRESS IN BARS= 698.644
 MAX. STRESS IN BARS= 927.588

ALL STRESSES WITHIN ALLOWABLE RANGE

POINTS TO LOCATE N.A.

-34.999	4.030
-34.999	4.030
9.999	9.275
10.000	9.275
10.000	4.030
10.000	4.030

XO= 0. YO= 12.244

CIXX= 31701.74500 CIYY= 13169.83800 CIXXY= -0.00017

SIXX= 79147.64100 SIYY= 101408.97000 SIXXY= 0.00000

TOTAL MOMENT OF INERTIA WHEN SECTION CRACKED

TIXX= 110849.38000
TIYY= 114578.30000
TIXXY= -0.00017

TET= 0.0

N.A. HORIZONTAL, BA MEASURED FROM PT.5

BA= 25.34928

CA= -1.00

STRESSES IN CONCRETE (CRACKED)

54.764
54.764
14.907
14.907
70.645
70.645
70.645
70.645
14.907
14.907

STRESSES IN BARS (CRACKED)

-696.057 -696.057 349.029 349.029 934.277 934.277 934.277 934.277
349.029 349.029

MIN. STRESS IN CONCRETE= -54.764
MAX. STRESS IN CONCRETE= 70.645

MIN. STRESS IN BARS= -696.057
MAX. STRESS IN BARS= 934.277

ALL STRESSES WITHIN ALLOWABLE RANGE

POINTS TO LOCATE N.A.

-34.999	4.030
-34.999	4.030
9.999	9.379
10.000	9.379
10.000	4.030
10.000	4.030

XO= 0. YO= 12.254

CIXX=	31630.83700	CIYY=	13100.47300	CIXXYY=	-0.00017
SIXX=	79200.77600	SIYY=	101408.97000	SIXXYY=	0.00030

TOTAL MOMENT OF INERTIA WHEN SECTION CRACKED

TIXX=	110831.61000
TIYY=	114509.44000
TIXXYY=	0.00012

TET= .0

N.A. HORIZONTAL, BA MEASURED FROM PT. 5

BA= 25.34947
CA= -1.00

STRESSES IN CONCRETE (CRACKED)

54.764
54.764
14.908
14.908
70.646
70.646
70.646
70.646
14.908
14.908

STRESSES IN BARS (CRACKED)

-696.051 -696.051 349.037 349.037 934.287 934.287 934.287 934.287
349.037 349.037

MIN. STRESS IN CONCRETE= -54.764
MAX. STRESS IN CONCRETE= 70.646

MIN. STRESS IN BARS= 696.051
MAX. STRESS IN BARS= 934.287

ALL STRESSES WITHIN ALLOWABLE RANGE

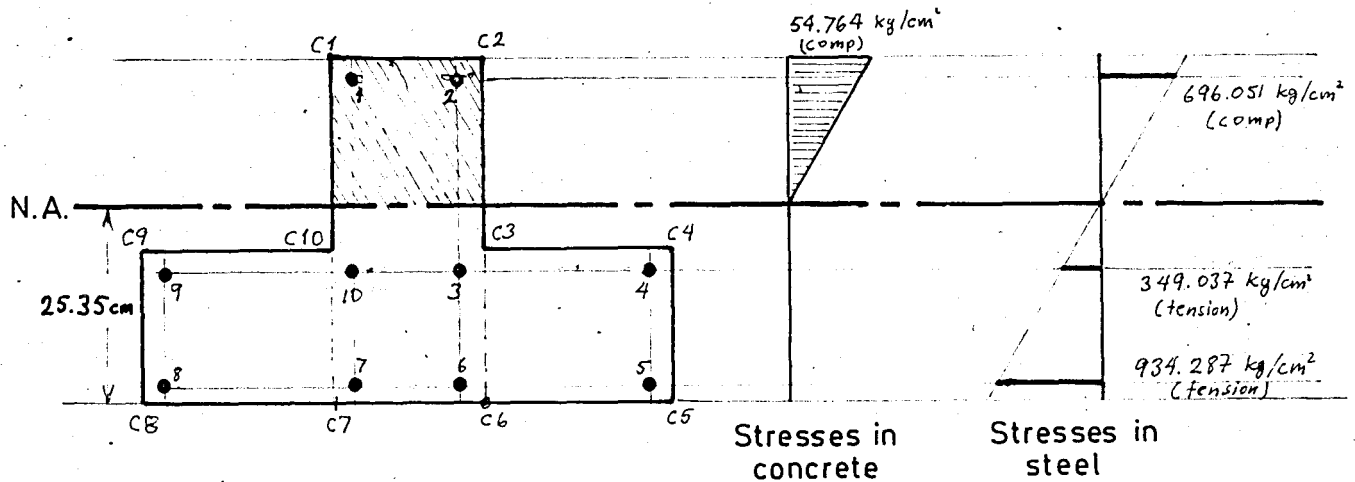
NEUTRAL AXIS LOCATED

STOP

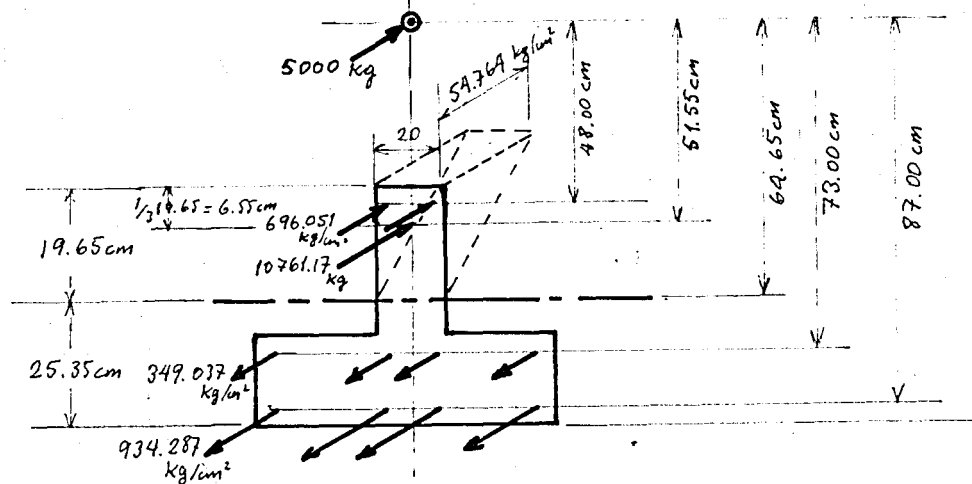
FINAL SOLUTION

Evaluation of the solution

The final solution is pointed out by a line on the left of the page. Representing the solution schematically



Tests for equilibrium:



Volume of compressive stresses in concrete

$$\frac{1}{2} \times 54.764 \times 19.65 \times 20 = -10761.17 \text{ kg}$$

Volume of compressive stresses in steel

$$2 \times 696.051 \times 1.539 = -2142.50 \text{ kg}$$

Compressive force developed in section

$$\underline{\underline{-12903.67 \text{ kg}}}$$

Tensile force developed in steel

$$349.037 \times 1.539 \times 4 + 934.287 \times 1.539 \times 4 = 7900.14 \text{ kg}$$

Equilibrium equations

$$F = F(\text{compression}) + F(\text{tension})$$

$$-5000.00 = -12903.67 + 7900.14 = -5003.53$$

and

$$\Sigma M = 0$$

Moment about a horizontal line passing through the point of application of F

$$0 = -10761.17 \times 51.55 - 2142.50 \times 48.00 + 2148.67 \times 73.00 \\ + 5751.47 \times 87.00 = 377.39$$

Since numbers are rounded off in calculations, we have some small discrepancies.

APPENDIX B

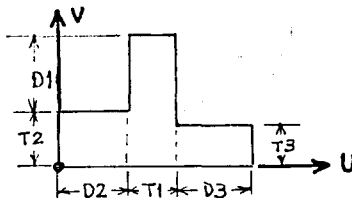
DATA INSTRUCTIONS AND VARIABLE DEFINITIONS

DATA INSTRUCTIONS

NO	VARIABLE	FORMAT
1	TITLE	1X,79H
2	T1, T2, T3, D1, D2, D3, RM, N	7F10.3, I5
3	{ DS, ASU, ASV -----	6F10.3
4	SRMI, SRMA	2F10.5
5	FN, EU, EV	3F20.5
6	ACC, ACS, ATS, ATC	4F10.3
7	HI	F10.2

VARIABLE DEFINITIONS

T1, T2, T3, D1, D2, D3 - Dimensions of the section (cm)



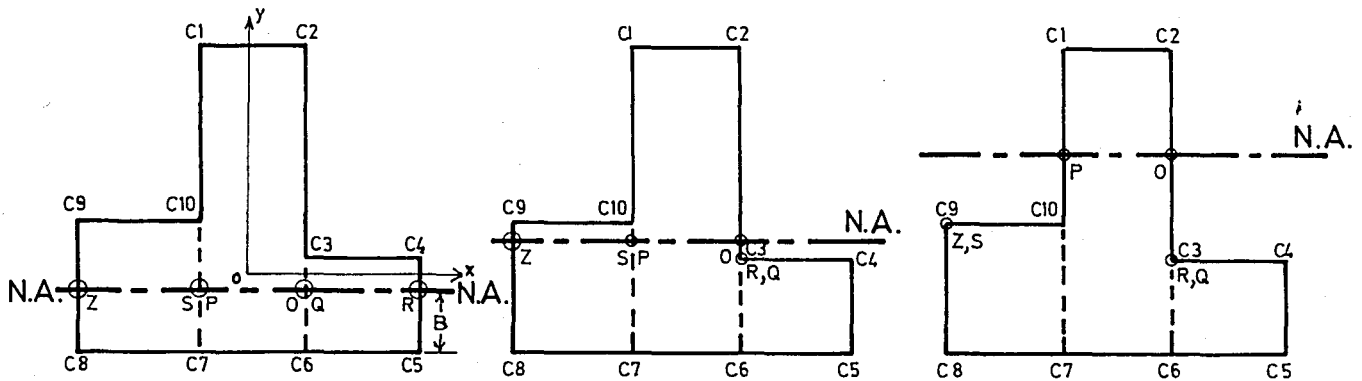
- RM - Modular ratio (modulus of elasticity of steel divided by modulus of elasticity of concrete)
- K - Number of reinforcing bars
- DS - Diameter of a reinforcing bar (cm)
- ASU, ASV - Coordinates of a reinforcing bar with respect to u, v axes (cm)
- SRMI - Minimum steel ratio specified by the Turkish Building Code according to the type of concrete and type of steel

SRMA	- Maximum steel ratio specified by the Turkish Building Code according to the type of concrete and type of steel
FN	- Design load (kg), if compressive (-)
EU, EV	- Coordinates of the point of application of FN with respect to u,v axes (cm)
ACC	- Allowable compressive stress in concrete (kg/cm ²)
ACS	- Allowable compressive stress in steel (kg/cm ²)
ATS	- Allowable tensile stress in steel (kg/cm ²)
ATC	- Allowable tensile stress in concrete (kg/cm ²)
HI	- Length of the column (cm)

APPENDIX C

POSITIONS OF NEUTRAL AXIS AND PARAMETERS USED IN AREA,
CENTROID AND MOMENTS OF INERTIA FORMULAS FOR THE
CRACKED SECTION.

Case (a) $\theta = 0^\circ$

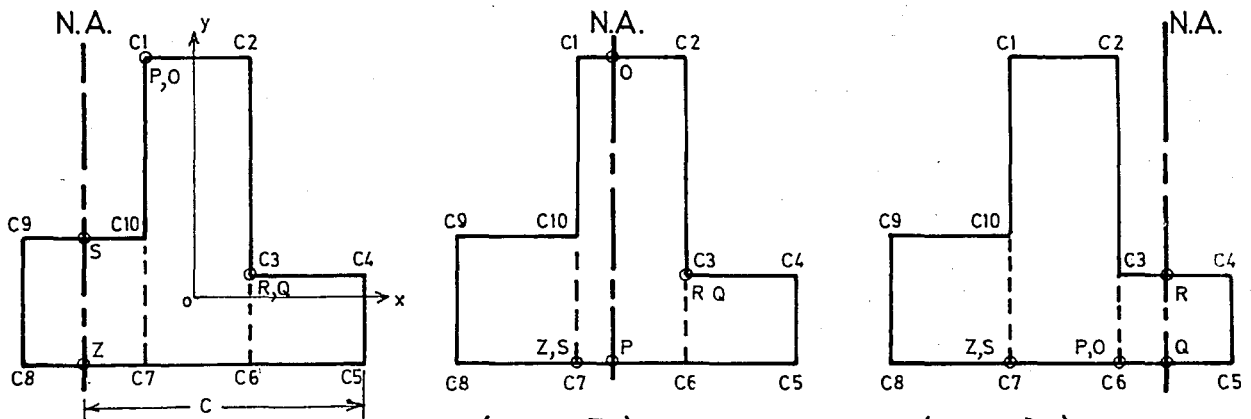


- S ($C_{10x}, B-V_0$)
- Z ($C_{9x}, B-V_0$)
- P ($C_{10x}, B-V_0$)
- O ($C_{3x}, B-V_0$)
- R ($C_{4x}, B-V_0$)
- Q ($C_{3x}, B-V_0$)

- S ($C_{10x}, B-V_0$)
- Z ($C_{9x}, B-V_0$)
- P ($C_{10x}, B-V_0$)
- O ($C_{3x}, B-V_0$)
- R (C_{3x}, C_{3y})
- Q (C_{3x}, C_{3y})

- S (C_{9x}, C_{9y})
- Z (C_{9x}, C_{9y})
- P ($C_{10x}, B-V_0$)
- O ($C_{3x}, B-V_0$)
- R (C_{3x}, C_{3y})
- Q (C_{3x}, C_{3y})

Case (b) $\theta = 90^\circ$

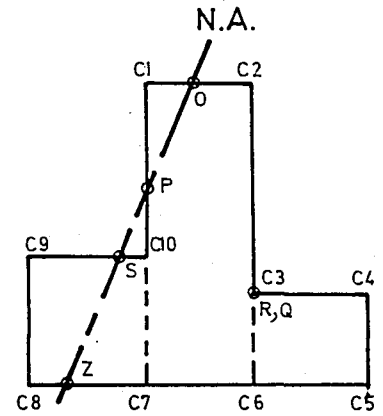
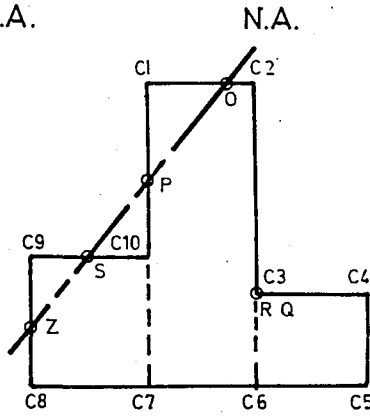
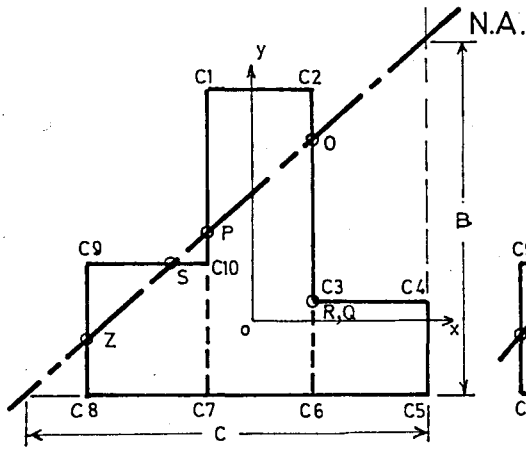


- S ($d_5 - U_0 - C, C_{9y}$)
- Z ($d_5 - U_0 - C, C_{8y}$)
- P (C_{1x}, C_{1y})
- O (C_{1x}, C_{1y})
- R (C_{3x}, C_{3y})
- Q (C_{3x}, C_{3y})

- S (C_{7x}, C_{7y})
- Z (C_{7x}, C_{7y})
- P ($d_5 - U_0 - C, C_{7y}$)
- O ($d_5 - U_0 - C, C_{1y}$)
- R (C_{3x}, C_{3y})
- Q (C_{3x}, C_{3y})

- S (C_{7x}, C_{7y})
- Z (C_{7x}, C_{7y})
- P (C_{6x}, C_{6y})
- O (C_{6x}, C_{6y})
- R ($d_5 - U_0 - C, C_{3y}$)
- Q ($d_5 - U_0 - C, C_{5y}$)

Case (c) $0^\circ < \theta < 90^\circ$

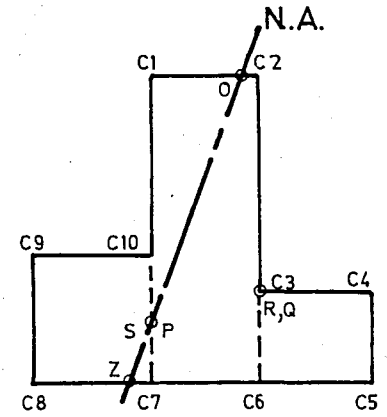
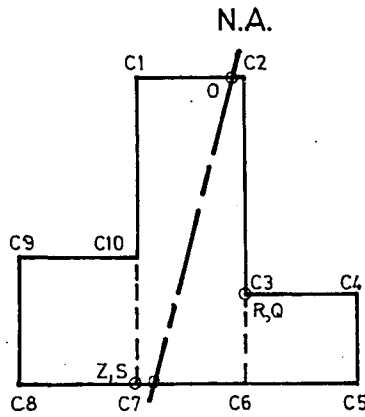
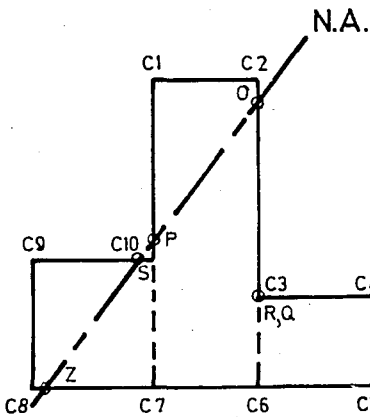


$$\begin{aligned} S & (d_5 - U_0 - C + t_2 / \tan \theta, C_{10y}) \\ Z & (C_8x, (C - d_5) \tan \theta - V_0) \\ P & (C_7x, (C - t_1 - d_3) \tan \theta - V_0) \\ O & (C_2x, (C - d_3) \tan \theta - V_0) \\ R & (C_3x, C_3y) \\ Q & (C_3x, C_3y) \end{aligned}$$

$$\begin{aligned} S & (d_5 - U_0 - C + t_2 / \tan \theta, C_{10y}) \\ Z & (C_8x, (C - d_5) \tan \theta - V_0) \\ P & (C_7x, (C - t_1 - d_3) \tan \theta - V_0) \\ O & (d_5 - U_0 - C + t_4 / \tan \theta, C_2y) \\ R & (C_3x, C_3y) \\ Q & (C_3x, C_3y) \end{aligned}$$

$$\begin{aligned} S & (d_5 - U_0 - C + t_2 / \tan \theta, C_{10y}) \\ Z & (d_5 - U_0 - C, C_8y) \\ P & (C_7x, (C - t_1 - d_3) \tan \theta - V_0) \\ O & (d_5 - U_0 - C + t_4 / \tan \theta, C_2y) \\ R & (C_3x, C_3y) \\ Q & (C_3x, C_3y) \end{aligned}$$

Case (c) cont.

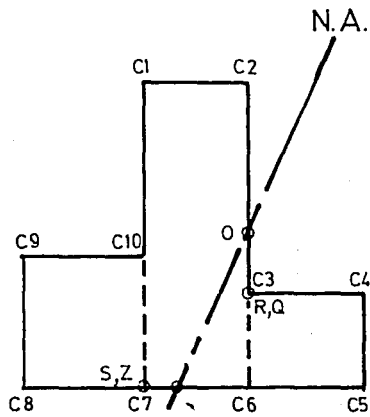


$$\begin{aligned} S & (d_5 - U_0 - C + t_2 / \tan \theta, C_{10y}) \\ Z & (d_5 - U_0 - C, C_8y) \\ P & (C_7x, (C - t_1 - d_3) \tan \theta - V_0) \\ O & (C_2x, (C - d_3) \tan \theta - V_0) \\ R & (C_3x, C_3y) \\ Q & (C_3x, C_3y) \end{aligned}$$

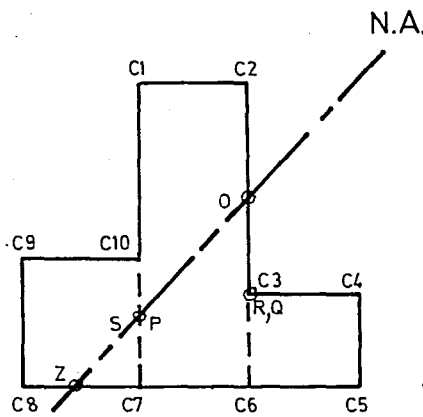
$$\begin{aligned} S & (C_7x, C_7y) \\ Z & (C_7x, C_7y) \\ P & (d_5 - U_0 - C, C_7y) \\ O & (d_5 - U_0 - C + t_4 / \tan \theta, C_2y) \\ R & (C_3x, C_3y) \\ Q & (C_3x, C_3y) \end{aligned}$$

$$\begin{aligned} S & (C_7x, (C - t_1 - d_3) \tan \theta - V_0) \\ Z & (d_5 - U_0 - C, C_7y) \\ P & (C_7x, (C - t_1 - d_3) \tan \theta - V_0) \\ O & (d_5 - U_0 - C + t_4 / \tan \theta, C_2y) \\ R & (C_3x, C_3y) \\ Q & (C_3x, C_3y) \end{aligned}$$

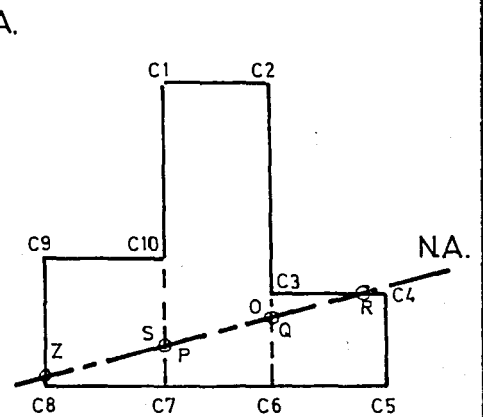
Case (c) cont.



$$\begin{aligned} S & (C7_x, C7_y) \\ Z & (C7_x, C7_y) \\ P & (d_5 - U_0 - C, C7_y) \\ O & (C2_x, (C - d_3) \tan \theta - V_0) \\ R & (C3_x, C3_y) \\ Q & (C3_x, C3_y) \end{aligned}$$

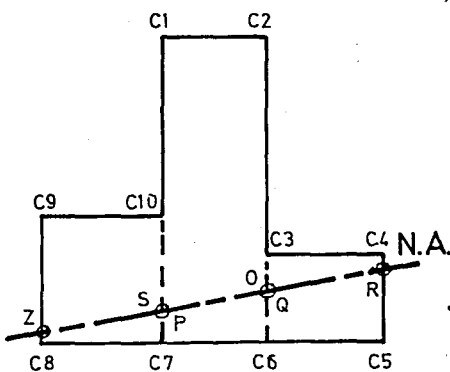


$$\begin{aligned} S & (C7_x, (C - t_1 - d_3) \tan \theta - V_0) \\ Z & (d_5 - U_0 - C, C7_y) \\ P & (C7_x, (C - t_1 - d_3) \tan \theta - V_0) \\ O & (C2_x, (C - d_3) \tan \theta - V_0) \\ R & (C3_x, C3_y) \\ Q & (C3_x, C3_y) \end{aligned}$$

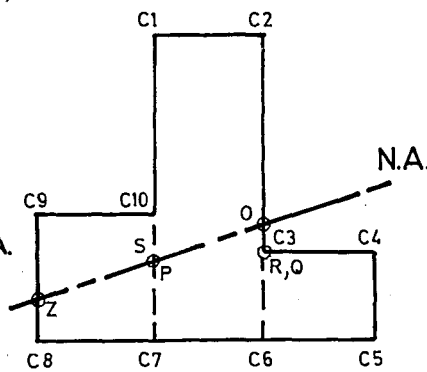


$$\begin{aligned} S & (C7_x, (C - t_1 - d_3) \tan \theta - V_0) \\ Z & (C8_x, (C - d_5) \tan \theta - V_0) \\ P & (C7_x, (C - t_1 - d_3) \tan \theta - V_0) \\ O & (C3_x, (C - d_3) \tan \theta - V_0) \\ R & (d_5 - U_0 - C + t_3 / \tan \theta, C3_y) \\ Q & (C3_x, (C - d_3) \tan \theta - V_0) \end{aligned}$$

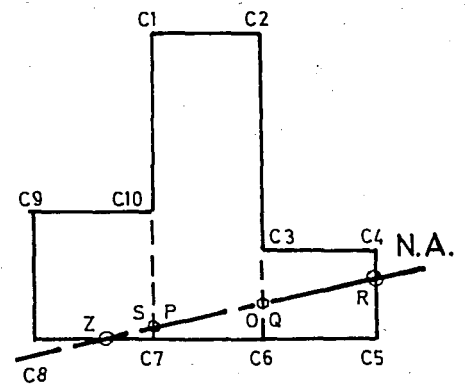
Case (c) cont.



$$\begin{aligned} S & (C7_x, (C - t_1 - d_3) \tan \theta - V_0) \\ Z & (C8_x, (C - d_5) \tan \theta - V_0) \\ P & (C7_x, (C - t_1 - d_3) \tan \theta - V_0) \\ O & (C2_x, (C - d_3) \tan \theta - V_0) \\ R & (C3_x, C \tan \theta - V_0) \\ Q & (C3_x, (C - d_3) \tan \theta - V_0) \end{aligned}$$

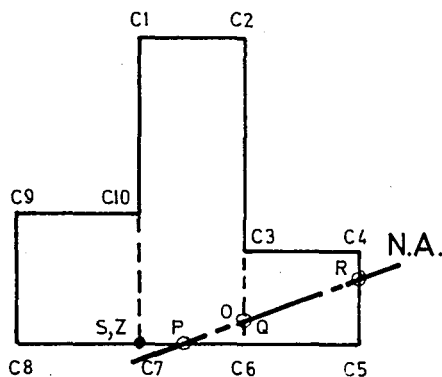


$$\begin{aligned} S & (C7_x, (C - t_1 - d_3) \tan \theta - V_0) \\ Z & (C8_x, (C - d_5) \tan \theta - V_0) \\ P & (C7_x, (C - t_1 - d_3) \tan \theta - V_0) \\ O & (C2_x, (C - d_3) \tan \theta - V_0) \\ R & (C3_x, C3_y) \\ Q & (C3_x, C3_y) \end{aligned}$$

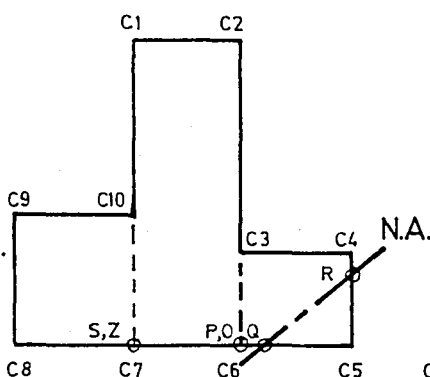


$$\begin{aligned} S & (C7_x, (C - t_1 - d_3) \tan \theta - V_0) \\ Z & (d_5 - U_0 - C, C7_y) \\ P & (C7_x, (C - t_1 - d_3) \tan \theta - V_0) \\ O & (C6_x, (C - d_3) \tan \theta - V_0) \\ R & (C4_x, C \tan \theta - V_0) \\ Q & (C6_x, (C - d_3) \tan \theta - V_0) \end{aligned}$$

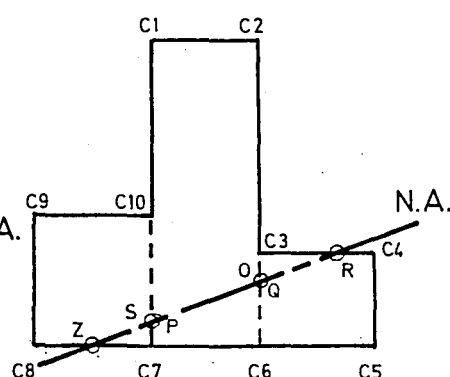
Case (c) cont.



- $S(C7_x, C7_y)$
- $Z(C7_x, C7_y)$
- $P(d_5 - U_0 - C, C6_y)$
- $O(C6_x, (C - d_3)\tan\theta - V_0)$
- $R(C4_x, C\tan\theta - V_0)$
- $Q(C6_x, (C - d_3)\tan\theta - V_0)$

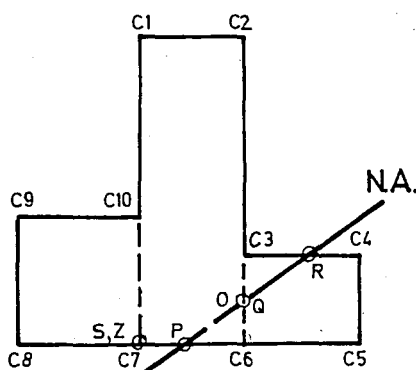


- $S(C7_x, C7_y)$
- $Z(C7_x, C7_y)$
- $P(C6_x, C6_y)$
- $O(C6_x, C6_y)$
- $R(C4_x, C\tan\theta - V_0)$
- $Q(d_5 - U_0 - C, C5_y)$

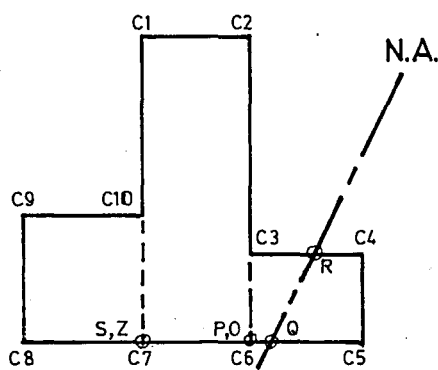


- $S(C7_x, (C - t_1 - d_3)\tan\theta - V_0)$
- $Z(d_5 - U_0 - C, C7_y)$
- $P(C7_x, (C - t_1 - d_3)\tan\theta - V_0)$
- $O(C3_x, (C - d_3)\tan\theta - V_0)$
- $R(d_5 - U_0 - C + t_3/\tan\theta, C4_y)$
- $Q(C3_x, (C - d_3)\tan\theta - V_0)$

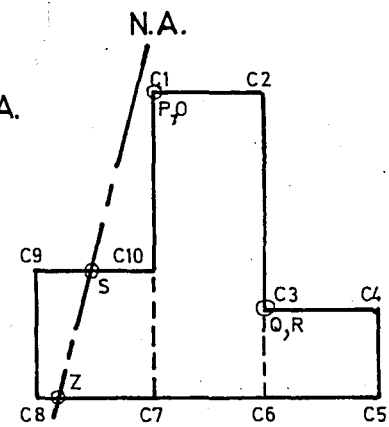
Case (c) cont.



- $S(C7_x, C7_y)$
- $Z(C7_x, C7_y)$
- $P(d_5 - U_0 - C, C6_y)$
- $O(C3_x, (C - d_3)\tan\theta - V_0)$
- $R(d_5 - U_0 - C + t_3/\tan\theta, C3_y)$
- $Q(C3_x, (C - d_3)\tan\theta - V_0)$

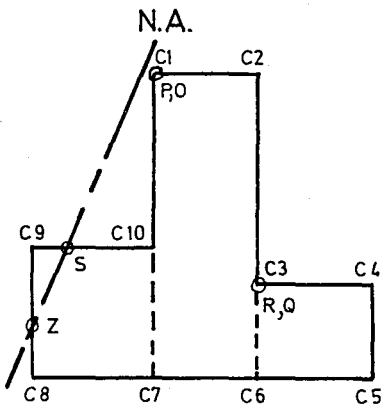


- $S(C7_x, C7_y)$
- $Z(C7_x, C7_y)$
- $P(C6_x, C6_y)$
- $O(C6_x, C6_y)$
- $R(d_5 - U_0 - C + t_3/\tan\theta, C3_y)$
- $Q(d_5 - U_0 - C, C5_y)$



- $S(d_5 - U_0 - C + t_2/\tan\theta, C10_y)$
- $Z(d_5 - U_0 - C, C8_y)$
- $P(C1_x, C1_y)$
- $O(C1_x, C1_y)$
- $R(C3_x, C3_y)$
- $Q(C3_x, C3_y)$

Case (c) cont.



$$S(d_5 - U_0 - C + t_2 / \tan \theta, C_{10y})$$

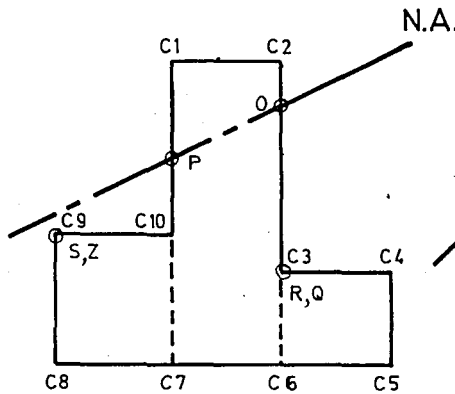
$$Z(C_{8x}, (C - d_5) \tan \theta - V_0)$$

$$P(C_{1x}, C_{1y})$$

$$O(C_{1x}, C_{1y})$$

$$R(C_{3x}, C_{3y})$$

$$Q(C_{3x}, C_{3y})$$



$$S(C_{9x}, C_{9y})$$

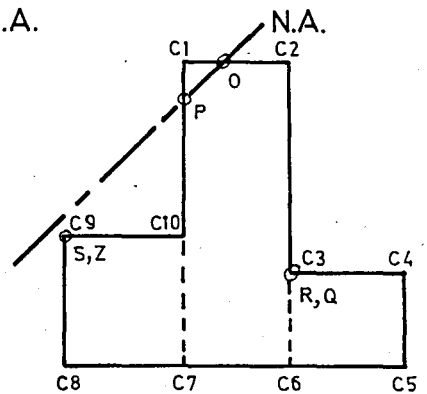
$$Z(C_{9x}, C_{9y})$$

$$P(C_{10x}, (C - t_1 - d_3) \tan \theta - V_0)$$

$$O(C_{2x}, (C - d_3) \tan \theta - V_0)$$

$$R(C_{3x}, C_{3y})$$

$$Q(C_{3x}, C_{3y})$$



$$S(C_{9x}, C_{9y})$$

$$Z(C_{9x}, C_{9y})$$

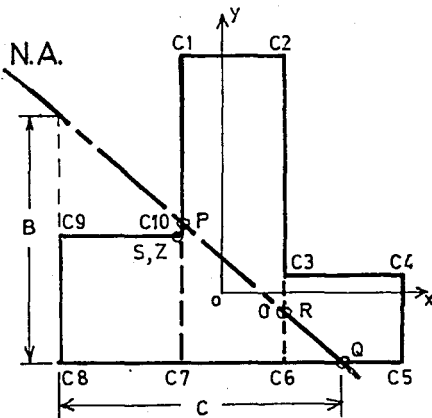
$$P(C_{10x}, (C - t_1 - d_3) \tan \theta - V_0)$$

$$O(d_5 - U_0 - C + t_4 / \tan \theta, C_{2y})$$

$$R(C_{3x}, C_{3y})$$

$$Q(C_{3x}, C_{3y})$$

Case (d) $0^\circ > \theta > -90^\circ$



$$S(C_{10x}, C_{10y})$$

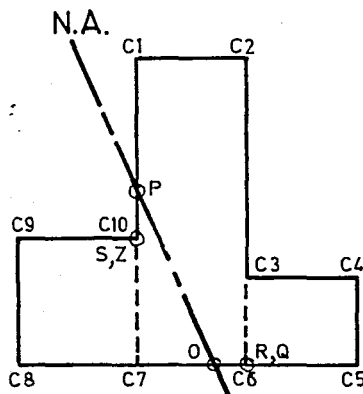
$$Z(C_{10x}, C_{10y})$$

$$P(C_{10x}, (C - d_2) \tan \theta - V_0)$$

$$O(C_{3x}, (C - d_4) \tan \theta - V_0)$$

$$R(C_{3x}, (C - d_4) \tan \theta - V_0)$$

$$Q(C - U_0, C_{6y})$$



$$S(C_{10x}, C_{10y})$$

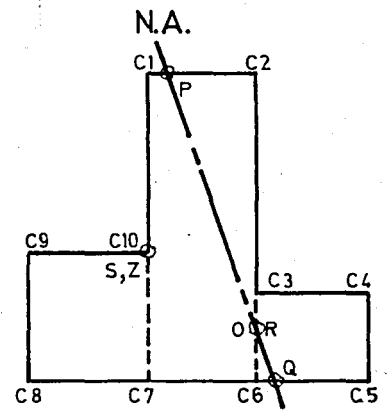
$$Z(C_{10x}, C_{10y})$$

$$P(C_{10x}, (C - d_2) \tan \theta - V_0)$$

$$O(C - U_0, C_{6y})$$

$$R(C_{6x}, C_{6y})$$

$$Q(C_{6x}, C_{6y})$$



$$S(C_{10x}, C_{10y})$$

$$Z(C_{10x}, C_{10y})$$

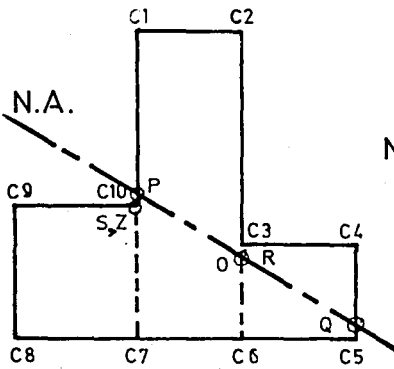
$$P(t_4 / \tan \theta + U_0 - C, C_{2y})$$

$$O(C_{3x}, (C - d_4) \tan \theta - V_0)$$

$$R(C_{3x}, (C - d_4) \tan \theta - V_0)$$

$$Q(C - U_0, C_{6y})$$

Case (d) cont.



$$S(C10_x, C10_y)$$

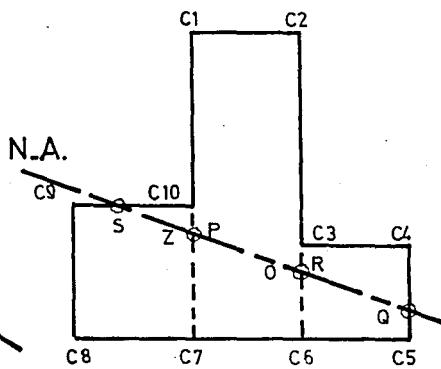
$$Z(C10_x, C10_y)$$

$$P(C10_x, (C-d_2)\tan\theta - V_0)$$

$$O(C3_x, (C-d_4)\tan\theta - V_0)$$

$$R(C3_x, (C-d_4)\tan\theta - V_0)$$

$$Q(C4_x, (C-d_5)\tan\theta - V_0)$$



$$S(C-t_2/\tan\theta - U_0, C10_y)$$

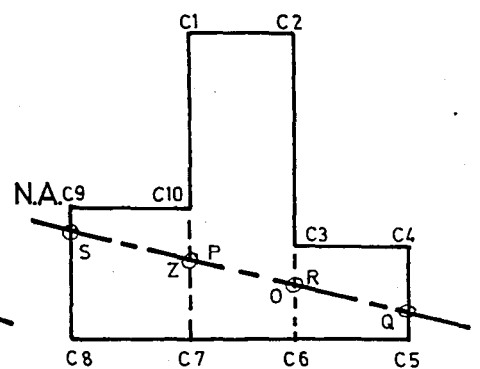
$$Z(C10_x, (C-d_2)\tan\theta - V_0)$$

$$P(C10_x, (C-d_2)\tan\theta - V_0)$$

$$O(C3_x, (C-d_4)\tan\theta - V_0)$$

$$R(C3_x, (C-d_4)\tan\theta - V_0)$$

$$Q(C4_x, (C-d_5)\tan\theta - V_0)$$



$$S(C9_x, C\tan\theta - V_0)$$

$$Z(C10_x, (C-d_2)\tan\theta - V_0)$$

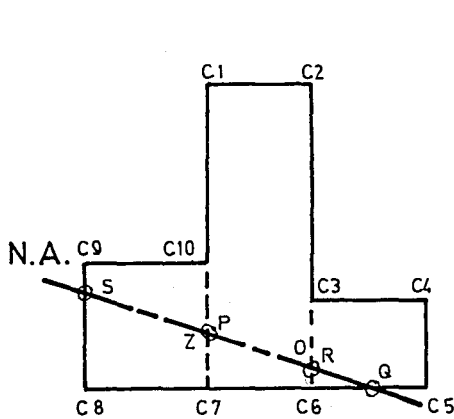
$$P(C10_x, (C-d_2)\tan\theta - V_0)$$

$$O(C3_x, (C-d_4)\tan\theta - V_0)$$

$$R(C3_x, (C-d_4)\tan\theta - V_0)$$

$$Q(C4_x, (C-d_5)\tan\theta - V_0)$$

Case (d) cont.



$$S(C9_x, C\tan\theta - V_0)$$

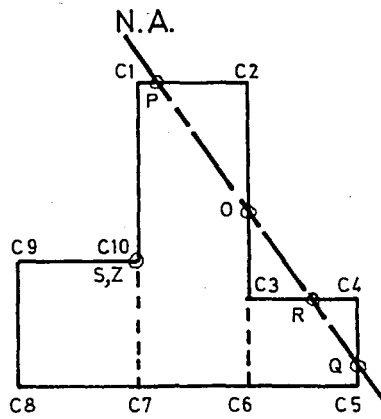
$$Z(C10_x, (C-d_2)\tan\theta - V_0)$$

$$P(C10_x, (C-d_2)\tan\theta - V_0)$$

$$O(C3_x, (C-d_4)\tan\theta - V_0)$$

$$R(C6_x, (C-d_4)\tan\theta - V_0)$$

$$Q(C-U_0, C6_y)$$



$$S(C10_x, C10_y)$$

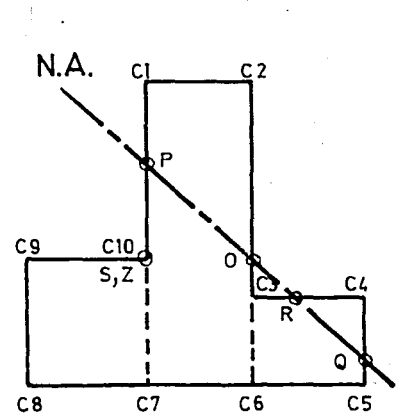
$$Z(C10_x, C10_y)$$

$$P(t_4/\tan\theta + U_0 - C, C2_y)$$

$$O(C3_x, (C-d_4)\tan\theta - V_0)$$

$$R(C-t_3/\tan\theta - U_0, C3_y)$$

$$Q(C4_x, (C-d_5)\tan\theta - V_0)$$



$$S(C10_x, C10_y)$$

$$Z(C10_x, C10_y)$$

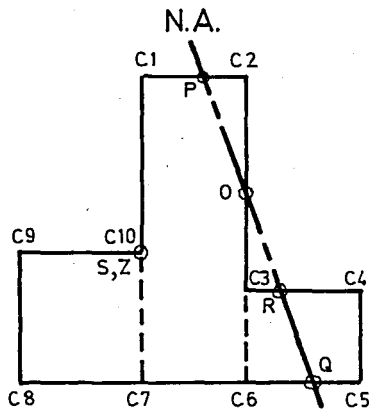
$$P(C10_x, (C-d_2)\tan\theta - V_0)$$

$$O(C3_x, (C-d_4)\tan\theta - V_0)$$

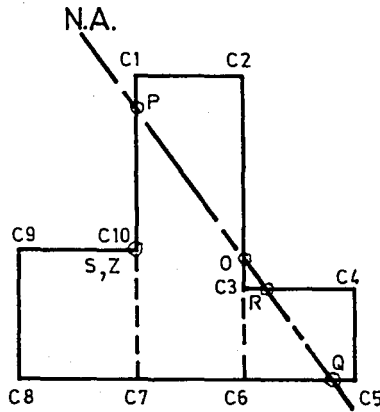
$$R(C-t_3/\tan\theta - U_0, C3_y)$$

$$Q(C4_x, (C-d_5)\tan\theta - V_0)$$

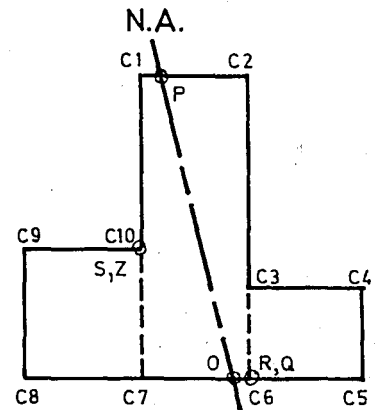
Case (d) cont.



$$\begin{aligned} &S(C10_x, C10_y) \\ &Z(C10_x, C10_y) \\ &P(t_4/\tan\theta + U_0 - C, C2_y) \\ &O(C3_x, (C-d_4)\tan\theta - V_0) \\ &R(C - t_3/\tan\theta - U_0, C4_y) \\ &Q(C - U_0, C5_y) \end{aligned}$$

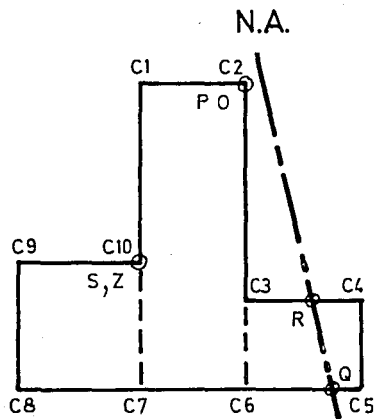


$$\begin{aligned} &S(C10_x, C10_y) \\ &Z(C10_x, C10_y) \\ &P(C10_x, (C-d_2)\tan\theta - V_0) \\ &O(C3_x, (C-d_4)\tan\theta - V_0) \\ &R(C - t_3/\tan\theta - U_0, C4_y) \\ &Q(C - U_0, C5_y) \end{aligned}$$

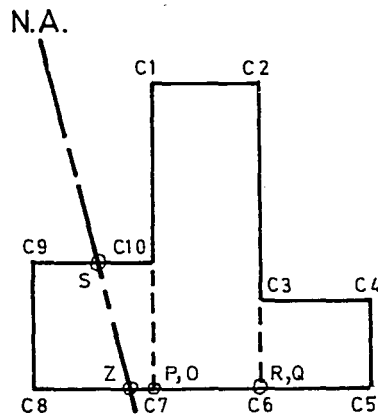


$$\begin{aligned} &S(C10_x, C10_y) \\ &Z(C10_x, C10_y) \\ &P(t_4/\tan\theta + U_0 - C, C2_y) \\ &O(C - U_0, C6_y) \\ &R(C6_x, C6_y) \\ &Q(C6_x, C6_y) \end{aligned}$$

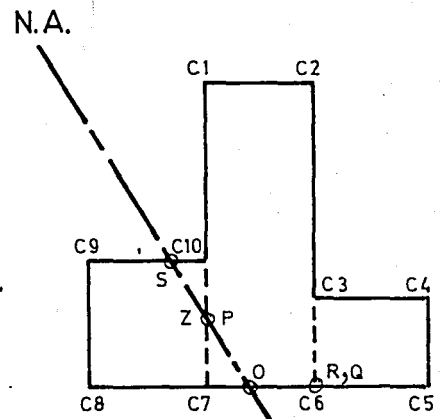
Case (d) cont.



$$\begin{aligned} &S(C10_x, C10_y) \\ &Z(C10_x, C10_y) \\ &P(C2_x, C2_y) \\ &O(C2_x, C2_y) \\ &R(C - t_3/\tan\theta - U_0, C4_y) \\ &Q(C - U_0, C5_y) \end{aligned}$$

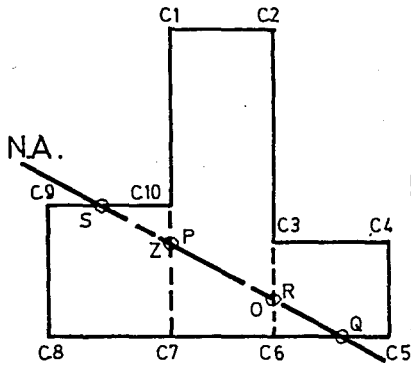


$$\begin{aligned} &S(C - t_2/\tan\theta - U_0, C10_y) \\ &Z(C - U_0, C8_y) \\ &P(C7_x, C7_y) \\ &O(C7_x, C7_y) \\ &R(C6_x, C6_y) \\ &Q(C6_x, C6_y) \end{aligned}$$

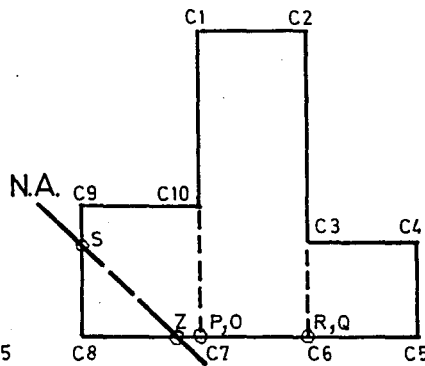


$$\begin{aligned} &S(C9_x, C\tan\theta - V_0) \\ &Z(C10_x, (C-d_2)\tan\theta - V_0) \\ &P(C10_x, (C-d_2)\tan\theta - V_0) \\ &O(C - U_0, C6_y) \\ &R(C6_x, C6_y) \\ &Q(C6_x, C6_y) \end{aligned}$$

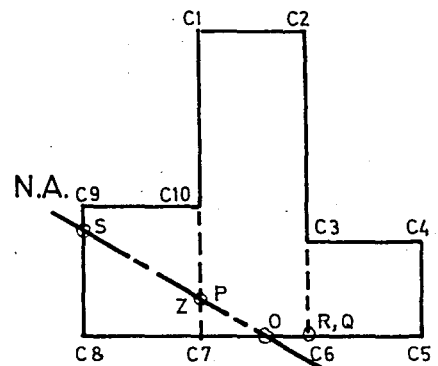
Case (d) cont.



$$\begin{aligned} S & (C-t_2/\tan\theta - U_0, C_{10y}) \\ Z & (C_{10x}, (C-d_2)\tan\theta - V_0) \\ P & (C_{10x}, (C-d_2)\tan\theta - V_0) \\ O & (C_{6x}, (C-d_4)\tan\theta - V_0) \\ R & (C_{6x}, (C-d_4)\tan\theta - V_0) \\ Q & (C-U_0, C_{6y}) \end{aligned}$$

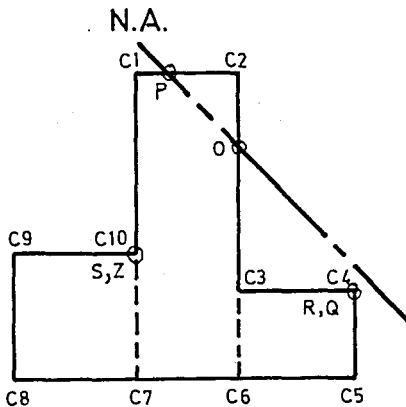


$$\begin{aligned} S & (C_{9x}, C\tan\theta - V_0) \\ Z & (C-U_0, C_{8y}) \\ P & (C_{7x}, C_{7y}) \\ O & (C_{7x}, C_{7y}) \\ R & (C_{6x}, C_{6y}) \\ Q & (C_{6x}, C_{6y}) \end{aligned}$$

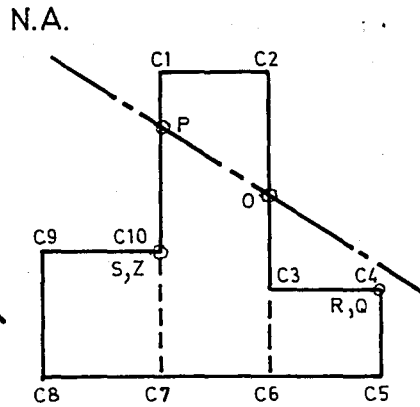


$$\begin{aligned} S & (C_{9x}, C\tan\theta - V_0) \\ Z & (C_{10x}, (C-d_2)\tan\theta - V_0) \\ P & (C_{10x}, (C-d_2)\tan\theta - V_0) \\ O & (C-U_0, C_{6y}) \\ R & (C_{6x}, C_{6y}) \\ Q & (C_{6x}, C_{6y}) \end{aligned}$$

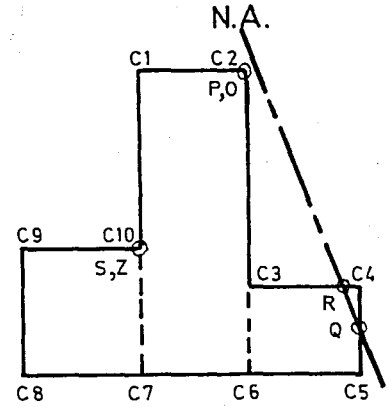
Case (d) cont.



$$\begin{aligned} S & (C_{10x}, C_{10y}) \\ Z & (C_{10x}, C_{10y}) \\ P & (t_4/\tan\theta + U_0 - C, C_{2y}) \\ O & (C_{2x}, (C-d_4)\tan\theta - V_0) \\ R & (C_{4x}, C_{4y}) \\ Q & (C_{4x}, C_{4y}) \end{aligned}$$

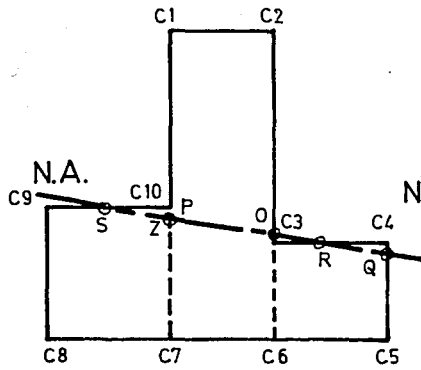


$$\begin{aligned} S & (C_{10x}, C_{10y}) \\ Z & (C_{10x}, C_{10y}) \\ P & (C_{1x}, (C-d_2)\tan\theta - V_0) \\ O & (C_{2x}, (C-d_4)\tan\theta - V_0) \\ R & (C_{4x}, C_{4y}) \\ Q & (C_{4x}, C_{4y}) \end{aligned}$$



$$\begin{aligned} S & (C_{10x}, C_{10y}) \\ Z & (C_{10x}, C_{10y}) \\ P & (C_{2x}, C_{2y}) \\ O & (C_{2x}, C_{2y}) \\ R & (C-t_3/\tan\theta - U_0, C_{4y}) \\ Q & (C_{4x}, (C-d_5)\tan\theta - V_0) \end{aligned}$$

Case (d) cont.



$$S(C - t_2 / \tan \theta - U_0, C10_y)$$

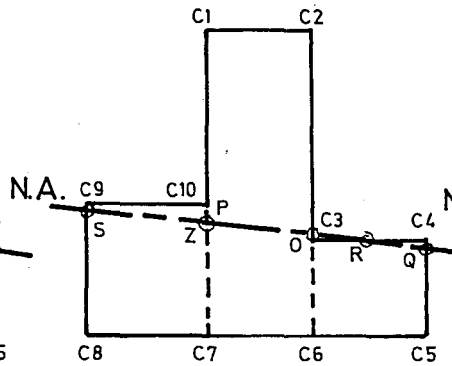
$$Z(C10_x, (C - d_2) \tan \theta - V_0)$$

$$P(C10_x, (C - d_2) \tan \theta - V_0)$$

$$O(C3_x, (C - d_4) \tan \theta - V_0)$$

$$R(C - t_3 / \tan \theta - U_0, C4_y)$$

$$Q(C4_x, (C - d_5) \tan \theta - V_0)$$



$$S(C9_x, C \tan \theta - V_0)$$

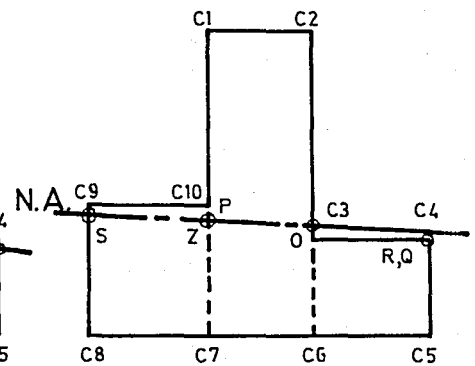
$$Z(C10_x, (C - d_2) \tan \theta - V_0)$$

$$P(C10_x, (C - d_2) \tan \theta - V_0)$$

$$O(C3_x, (C - d_4) \tan \theta - V_0)$$

$$R(C - t_3 / \tan \theta - U_0, C4_y)$$

$$Q(C4_x, (C - d_5) \tan \theta - V_0)$$



$$S(C9_x, C \tan \theta - V_0)$$

$$Z(C10_x, (C - d_2) \tan \theta - V_0)$$

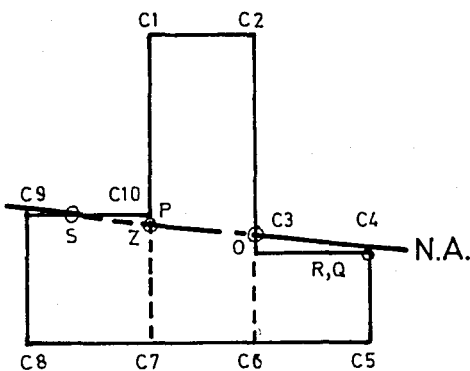
$$P(C10_x, (C - d_2) \tan \theta - V_0)$$

$$O(C3_x, (C - d_4) \tan \theta - V_0)$$

$$R(C4_x, C4_y)$$

$$Q(C4_x, C4_y)$$

Case (d) cont.



$$S(C - t_2 / \tan \theta - U_0, C10_y)$$

$$Z(C10_x, (C - d_2) \tan \theta - V_0)$$

$$P(C10_x, (C - d_2) \tan \theta - V_0)$$

$$O(C3_x, (C - d_4) \tan \theta - V_0)$$

$$R(C4_x, C4_y)$$

$$Q(C4_x, C4_y)$$

APPENDIX D

COMPUTER PROGRAM LISTINGS

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C WORKING STRESS DESIGN OF UNSYMMETRICAL COLUMN SECTIONS IN BIAXIAL
C BENDING PART ONE -SECTION NOT CRACKED-
C WRITTEN BY TAMER TUNCA (CE GRADUATE) OCT.25,1969
DIMENSION AS(50),ASU(50),ASV(50),ASX(50),ASY(50),STISK(50)
DIMENSION STICK(10),DS(50),DIA(50)
DEFINE DISK(10,4500)
PRINT 1
1 FORMAT(1H1,30X,14HMSD OF COLUMNS)
READ 11
PRINT 11
11 FORMAT(1X,79H
1
PRINT 12
12 FORMAT(1H0,29HPART ONE, SECTION NOT CRACKED/1H ,26HALL UNITS ARE I
IN KG AND CM)
READ 222,T1,T2,T3,D1,D2,D3,RM,N
222 FORMAT(7F10.3,I5)
KK=1
RECORD(KK) T1,T2,T3,D1,D2,D3,RM,N
PRINT 3,T1,T2,T3,D1,D2,D3,RM,N
3 FORMAT(/10X,3HT1=,F14.0/10X,3HT2=,F14.0/10X,3HT3=,F14.0/
110X,3HD1=,F14.0/10X,3HD2=,F14.0/10X,3HD3=,F14.0/
210X,14HMODULAR RATIO=,I5/10X,15HNUMBER OF BARS=,I5)
READ 4,(DS(I),ASU(I),ASV(I),I=1,N)
4 FORMAT(6F10.3)
READ 700,SRMI,SRMA
700 FORMAT(2F10.5)
PRINT 701,SRMI,SRMA
701 FORMAT(/10X,17HMIN. STEEL RATIO=,F10.5/10X,17HMAX. STEEL RATIO=,
1F10.5)
READ 19,FN,EU,EV
19 FORMAT(3F20.5)
PRINT 20,FN,EU,EV
20 FORMAT(/10X,13HNORMAL FORCE=,F10.3/10X,3HEU=,F10.3/
110X,3HEV=,F10.3)
READ 34,ACC,ATC,ACS,ATS
34 FORMAT(4F10.3)
KK=220
RECORD(KK) ACC,ATC,ACS,ATS
PRINT 35,ACC,ATC,ACS,ATS
35 FORMAT(/10X,41HALLOWABLE COMPRESSIVE STRESS IN CONCRETE=,F10.3/
110X,37HALLOWABLE TENSILE STRESS IN CONCRETE=,F10.3/
210X,38HALLOWABLE COMPRESSIVE STRESS IN STEEL=,F10.3/
310X,34HALLOWABLE TENSILE STRESS IN STEEL=,F10.3/)
999 CONTINUE
PRINT 5
5 FORMAT(/1X,25HDIA. OF REINFORCEMENT BAR,5X,13HU(HORIZONTAL),5X,
11HV(VERTICAL)/)
DO 61 I=1,N
PRINT 6,(DS(I),ASU(I),ASV(I),I=1,N)
61 CONTINUE
6 FORMAT(1H ,3F20.5)
C TOTAL AREA OF REINFORCEMENT
DO 71 I=1,N
71 AS(I)=((22./7.)*DS(I)**2)/4.
KK=1
RECORD(KK) (AS(I),I=1,N)
SUMAS=0.
DO 7 I=1,N
7 SUMAS=SUMAS+AS(I)
KK=65
RECORD(KK) SUMAS
C TRANSPOSED AREA OF THE CROSS-SECTION

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T4=T2+D1
D4=D2+T1
D5=D2+T1+D3
A=T1*T4+T2*D2+T3*D3+(RM-1.)*SUMAS
ACO=T1*T4+T2*D2+T3*D3
PEM=(ACC*ACO+ACS*SUMAS)/3.
IF(FN-PEM) 83,82,82
83 PRINT 84
84 FORMAT(/10X,41HNORMAL FORCE GREATER THAN ALLOWABLE RANGE)
STOP
82 CONTINUE
PRINT 85
85 FORMAT(/10X,35HNORMAL FORCE WITHIN ALLOWABLE RANGE)
IF(A-582.16) 90,91,91
90 PRINT 92
92 FORMAT(/10X,43HAREA IS TOO SMALL, CHANGE THE CROSS SECTION)
STOP
91 CONTINUE
PUR=SUMAS/ACO
IF(PUR-SRMI) 93,94,94
93 CONTINUE
PRINT 95
95 FORMAT(/10X,40HSTEEL RATIO SMALLER THAN ALLOWABLE RANGE/10X,
132HALL STEEL DIA. INCREASED BY 2 MM)
DO 100 I=1,N
100 DIA(I)=DS(I)+0.2
94 CONTINUE
IF(PUR-SRMA) 101,101,102
102 CONTINUE
PRINT 103
103 FORMAT(/10X,40HSTEEL RATIO GREATER THAN ALLOWABLE RANGE/10X,
132HALL STEEL DIA. DECREASED BY 2 MM)
DO 104 I=1,N
104 DIA(I)=DS(I)-0.2
IF(DIA(1)-DS(1)) 300,350,300
300 CONTINUE
DO 301 I=1,N
301 DS(I)=DIA(I)
GO TO 999
101 PRINT 105
105 FORMAT(/10X,34HSTEEL RATIO WITHIN ALLOWABLE RANGE)
350 CONTINUE
CALL CORDI(AS,ASU,ASV,RM,N,A,T1,T2,T3,T4,D1,D2,D3,D4,D5,
1C1X,C2X,C3X,C4X,C5X,C6X,C7X,C8X,C9X,C10X,C1Y,C2Y,C3Y,C4Y,C5Y,C6Y,
2C7Y,C8Y,C9Y,C10Y,ASX,ASY,UO,VO)
C MOMENT OF INERTIA OF CONCRETE
CIX=(1./12.)*(D2*T2*(T2*T2+3.*(T2-2.*VO)**2)+T1*T4*(T4*T4+3.*(T4-2.
1.*VO)**2)+D3*T3*(T3*T3+3.*(T3-2.*VO)**2))
CIY=(1./12.)*(T2*D2*(D2*D2+3.*(D2-2.*UO)**2)+T1*T4*(T1*T1+3.*(D4+D
12-2.*UO)**2)+T3*D3*(D3*D3+3.*(D5+D4-2.*UO)**2))
CIXY=(1./4.)*(T2*D2*(T2-2.*VO)*(D2-2.*UO)+T1*T4*(T4-2.*VO)*
1(D4+D2-2.*UO)+T3*D3*(T3-2.*VO)*(D5+D4-2.*UO))
FL=(CIX+CIY)/2.
FLA=SQRTF(((CIX-CIY)*(CIX-CIY)/4.)+(CIXY)**2)
BCIM=FL-FLA
BUCK=SQRTF(BCIM/ACO)
READ 990,HI
990 FORMAT(F10.2)
BFAC=HI/BUCK
IF(BFAC-50.) 991,991,992
992 PRINT 993,BFAC
993 FORMAT(/1X,10HH/D RATIO=,F10.3/1X,53HINCREASE THE LOAD WITH A FACT
10R SPECIFIED IN THE CODE/)

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STOP
991 CONTINUE
C MOMENT OF INERTIA OF REINFORCEMENT BARS
SIX=0.
DO 14 I=1,N
14 SIX=SIX+(RM-1.)*AS(I)*ASY(I)*ASY(I)
SIY=0.
DO 15 I=1,N
15 SIY=SIY+(RM-1.)*AS(I)*ASX(I)*ASX(I)
SIXY=0.
DO 16 I=1,N
16 SIXY=SIXY+(RM-1.)*AS(I)*ASX(I)*ASY(I)
TIX=CIX+SIX
TIY=CIY+SIY
TIXY=CIXY+SIXY
PRINT 18,TIX,TIY,TIXY
18 FORMAT(/1X,4HTIX=,F20.5,5X,4HTIY=,F20.5,5X,5HTIXY=,F20.5)
KK=7
RECORD(KK) TIX,TIY,TIXY
EX=EU-UO
EY=EV-VO
KK=74
RECORD(KK) FN,EX,EY
CALL STRESS(FN,EX,EY,TIX,TIY,TIXY,A,RM,N,ASX,ASY,T2,T3,T4,
1D2,D3,D4,D5,UO,VO,STICK,STISK)
CALL MINMA(STICK,STISK,N,CMIN,CMAX,SMIN,SMAX)
IF(ACC-CMIN) 41,41,40
40 PRINT 401
401 FORMAT(/1X,35HCHANGE THE SECTION OR REINFORCEMENT)
STOP
41 CONTINUE
IF(ACS-SMIN) 43,43,42
42 PRINT 421
421 FORMAT(/1X,50HMAX. COMPRESSIVE STRESS IN STEEL EXCEEDS ALLOWABLE/
11X,32HALL STEEL DIA. INCREASED BY 2 MM)
333 CONTINUE
DO 400 I=1,N
400 DS(I)=DS(I)+0.2
GO TO 999
43 CONTINUE
IF(ATS-SMAX) 45,44,44
45 PRINT 451
451 FORMAT(/1X,46HMAX. TENSILE STRESS IN STEEL EXCEEDS ALLOWABLE/
11X,32HALL STEEL DIA. INCREASED BY 2 MM)
GO TO 333
44 CONTINUE
IF(CMAX) 47,47,46
47 PRINT 471
471 FORMAT(/1X,17HSECTION UNCRACKED)
STOP
46 CONTINUE
AAA=EY*TIY-EX*TIXY
IF(AAA) 115,106,115
115 CONTINUE
IF(AAA-0.001) 200,200,201
200 AAA=0.0
GO TO 106
802 CONTINUE
106 TESTP=(FN/A)+((FN*EY*TIY-FN*EX*TIXY)/(TIX*TIY-TIXY*TIXY))*(-1.-
1VO)+((FN*EX*TIX-FN*EY*TIXY)/(TIX*TIY-TIXY*TIXY))*(-1.-UO)
GO TO 107
201 CONTINUE
TANDET=(EY*TIXY-EX*TIX)/(EY*TIY-EX*TIXY)

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```
TTT=ABS(F(TANDET))
IF (TTT-1000000.) 801,802,802
801 CONTINUE
IF (TANDET) 116,116,160
116 CONTINUE
IF (TANDET+0.001) 202,203,203
203 TANDET=0.0
GO TO 106
202 GO TO 106
160 CONTINUE
IF (TANDET-0.001) 203,203,204
204 CONTINUE
TESTP=(FN/A)+((FN*EY*TIY-FN*EX*TIXY)/(TIX*TIY-TIXY*TIXY))*(-10.-
105*TANDET)+((FN*EX*TIX-FN*EY*TIXY)/(TIX*TIY-TIXY*TIXY))*(-0.1-U0)
GO TO 107
107 CONTINUE
KK=9
RECORD(KK) TESTP
END
```

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C      SECOND PROGRAM
C      COMPRESSION BELOW NEUTRAL AXIS
      DIMENSION AS(50),ASX(50),ASY(50),STICK(10)
      DIMENSION AYY(50),AXX(50),STC(10),STS(50)
      DEFINE DISK(10,4500)
      KK=9
      FETCH(KK) TESTP
      IF(TESTP) 107,107,108
108    CONTINUE
      STOP
107    CONTINUE
      PRINT 1
1      FORMAT(1H1,25HPART TWO, SECTION CRACKED/1H ,26HALL UNITS ARE IN KG
1 AND CM)
      KK=1
      FETCH(KK) T1,T2,T3,D1,D2,D3,RM,N
      KK=1
      FETCH(KK) (AS(I),I=1,N)
      KK=65
      FETCH(KK) SUMAS
      KK=77
      FETCH(KK) UO,VO
      KK=8
      FETCH(KK) C1X,C2X,C3X,C4X,C5X,C6X,C7X,C8X,C9X,C10X,C1Y,C2Y,C3Y,
1C4Y,C5Y,C6Y,C7Y,C8Y,C9Y,C10Y
      KK=101
      FETCH(KK) (ASX(I),ASY(I),I=1,N)
      KK=7
      FETCH(KK) TIX,TIY,TIXY
      KK=74
      FETCH(KK) FN,EX,EY
      KK=201
      FETCH(KK) (STICK(II),II=1,10)
      KK=215
      FETCH(KK) CMIN,CMAX
      KK=220
      FETCH(KK) ACC,ATC,ACS,ATS
      T4=T2+D1
      D4=D2+T1
      D5=D2+T1+D3
      A=T1*T4+T2*T2+D3*D3+(RM-1.)*SUMAS
      AAA=EY*TIY-EX*TIXY
      IF(AAA) 24,125,24
24    CONTINUE
      FFF=ABSF(AAA)
      IF(FFF-0.001) 200,200,201
302   CONTINUE
200   AAA=0.0
      GO TO 125
125   WH=-(1./A)*(TIX*TIY-TIXY*TIXY)/(EX*TIX-EY*TIY)
      CI=D5-UO-WH
      C=CI*(STICK(4)+0.3*STICK(9))/STICK(4)
      B=-1.
      TANTET=1000000.
      PRINT 16,TANTET
      KK=5
      RECORD(KK) T,TANTET
      PRINT 15,C,B
15    FORMAT(/10X,2HC=,F15.3,5X,2HB=,F15.3)
      KK=6
      RECORD(KK) C,B
      IF(C-D5) 101,101,110
110   CONTINUE

```

```

CALL TINFI(C,UO,T1,D5,D3,C1X,C1Y,C7X,C7Y,C6X,C6Y,C3X,C3Y,C5Y,C8Y,
1SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY)
CALL CBELO(SUMAS,N,AS,ASX,ASY,RM,SX,SY,ZX,ZY,PX,PY,OX,OY,
1RX,RY,QX,QY,C2X,C2Y,C4X,C4Y,C5Y,C6Y,C7Y,C10X,C10Y,AA,XO,YO,
2A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11,
3B12,B13,B14,B15,B16,B17,B18)
CALL CBEXX(SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY,C2X,C2Y,C4X,
1C4Y,C5Y,C6Y,C7Y,C10X,C10Y,XO,YO,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,
2B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,B16,B17,B18,C1XX,
3C1YY,C1XXYY)
GO TO 888
201 CONTINUE
TANTET=(EY*TIXY-EX*TIX)/(EY*TIY-EX*TIXY)
TTT=ABSF(TANTET)
IF(TTT-1000000.) 301,302,302
301 CONTINUE
IF(TANTET) 21,22,23
21 CONTINUE
IF(TANTET+0.001) 901,900,900
900 TANTET=0.0
GO TO 22
23 CONTINUE
IF(TANTET-0.001) 800,800,801
800 TANTET=0.0
GO TO 22
22 PRINT 16,TANTET
16 FORMAT(/1X,7HTANTET=,F15.5)
KK=5
RECORD(KK) TANTET
WV=-((1./A)*((TIX*TIY-TIXY*TIXY)/(EY*TIY-EX*TIXY)))
BI=WV+VO
B=BI*(STICK(7)+0.3*STICK(1))/STICK(7)
C=-1.
PRINT 15,C,B
KK=6
RECORD(KK) C,B
IF(B) 101,101,115
115 CONTINUE
CALL THORI(B,VO,T2,T3,T4,C3X,C3Y,C4X,C9X,C9Y,C10X,SX,SY,ZX,ZY,
1PX,PY,OX,OY,RX,RY,QX,QY)
CALL CBELO(SUMAS,N,AS,ASX,ASY,RM,SX,SY,ZX,ZY,PX,PY,OX,OY,
1RX,RY,QX,QY,C2X,C2Y,C4X,C4Y,C5Y,C6Y,C7Y,C10X,C10Y,AA,XO,YO,
2A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11,
3B12,B13,B14,B15,B16,B17,B18)
CALL CBEXX(SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY,C2X,C2Y,C4X,
1C4Y,C5Y,C6Y,C7Y,C10X,C10Y,XO,YO,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,
2B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,B16,B17,B18,C1XX,
3C1YY,C1XXYY)
GO TO 888
C LOCATION OF NEUTRAL AXIS WHEN TANGEN* IS POSITIVE
801 CONTINUE
PRINT 16,TANTET
KK=5
RECORD(KK) TANTET
WV=TANTET*(D5-UO)-((1./A)*((TIX*TIY-TIXY*TIXY)/(EY*TIY-EX*TIXY)))
BI=WV+VO
WH=-((VO/TANTET)-((1./A)*((TIX*TIY-TIXY*TIXY)/(EX*TIX-EY*TIXY)))
CI=-WH+D5-UO
B=BI*(CMIN+0.3*CMAX)/CMIN
C=B/TANTET
PRINT 15,C,B
KK=6
RECORD(KK) C,B

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IF(B) 101,101,105
105 CONTINUE
CALL TPOSI(C,B,UO,VO,TANTET,D1,D2,D3,D4,D5,T1,T2,T3,T4,
1C2X,C2Y,C3X,C3Y,C4X,C4Y,C5X,C5Y,C6X,C6Y,C7X,C7Y,C8X,C8Y,C9X,C9Y,
2C10X,C10Y,C1X,C1Y,SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY)
CALL CBELO(SUMAS,N,AS,ASX,ASY,RM,SX,SY,ZX,ZY,PX,PY,OX,OY,
1RX,RY,QX,QY,C2X,C2Y,C4X,C4Y,C5Y,C6Y,C7Y,C10X,C10Y,AA,XO,YO,
2A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11,
3B12,B13,B14,B15,B16,B17,B18)
CALL CBEXX(SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY,C2X,C2Y,C4X,
1C4Y,C5Y,C6Y,C7Y,C10X,C10Y,XO,YO,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,
2B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,B16,B17,B18,C1XX,
3CIYY,C1XXYY)
GO TO 888
C LOCATION OF NEUTRAL AXIS WHEN TANGENT IS NEGATIVE
901 CONTINUE
PRINT 16,TANTET
KK=5
RECORD(KK) TANTET
WV=-UO*TANTET-(1./A)*((TIX*TIY-TIXY*TIXY)/(EY*TIY-EX*TIXY))
BI=WV+VO
WH=-((VO/TANTET)-((1./A)*((TIX*TIY-TIXY*TIXY)/(EX*TIX+EY*TIY)))
CI=UO+WH
B=BI*(CMIN+0.3*CMAX)/CMIN
C=-B/TANTET
PRINT 15,C,B
KK=6
RECORD(KK) C,B
IF(B) 101,101,100
101 PRINT 102
102 FORMAT(/10X,18HCHANGE THE SECTION)
STOP
100 CONTINUE
TAN=-TANTET
CALL TNEGA(C,B,TAN,UO,VO,T1,T2,T3,T4,D1,D2,D3,D4,D5,
1C1X,C1Y,C2X,C2Y,C3X,C3Y,C4X,C4Y,C5X,C5Y,C6X,C6Y,C7X,C7Y,C8X,C8Y,
2C9X,C9Y,C10X,C10Y,SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY)
CALL BELOW(SUMAS,N,AS,ASX,ASY,RM,SX,SY,ZX,ZY,PX,PY,OX,OY,
1RX,RY,QX,QY,C1X,C1Y,C3X,C3Y,C5Y,C6Y,C7Y,C8Y,C9X,C9Y,XO,YO,
2AA,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11,
3B12,B13,B14,B15,B16,B17,B18)
CALL BELXX(S,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY,C1X,C1Y,C3X,
1C5Y,C6Y,C7Y,C8Y,C9X,C9Y,XO,YO,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,
2B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,B16,B17,B18,C1XX,
3CIYY,C1XXYY,C3Y)
GO TO 888
888 CONTINUE
CALL STINE(N,XO,YO,RM,AS,ASX,ASY,C1XX,C1YY,C1XXYY,AYY,AXX,
1T1XX,T1YY,T1XXYY)
CALL CRACK(EY,EX,XO,YO,UO,VO,FN,AA,AYY,AXX,T1XX,T1YY,
1T1XXYY,T2,T3,T4,D2,D4,D5,RM,N,CA,BA,TET,STC,STS)
CALL CRAXX(STC,STS,ACC,ACS,ATS,N)
END

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C   THIRD PROGRAM
C   COMPRESSION ABOVE NEUTRAL AXIS
DIMENSION AS(50),ASX(50),ASY(50),STICK(10)
DIMENSION AYY(50),AXX(50),STC(10),STS(50)
DEFINE DISK(10,4500)
KK=900
FETCH(KK) TESTP
IF(TESTP) 107,107,108
107 CONTINUE
STOP
108 CONTINUE
PRINT 1
1   FORMAT(1H1,25HPART TWO, SECTION CRACKED/1H ,26HALL UNITS ARE IN KG
1   AND CM)
KK=1
FETCH(KK) T1,T2,T3,D1,D2,D3,RM,N
KK=10
FETCH(KK) (AS(I),I=1,N)
KK=65
FETCH(KK) SUMAS
KK=77
FETCH(KK) UO,VO
KK=80
FETCH(KK) C1X,C2X,C3X,C4X,C5X,C6X,C7X,C8X,C9X,C10X,C1Y,C2Y,C3Y,
1C4Y,C5Y,C6Y,C7Y,C8Y,C9Y,C10Y
KK=101
FETCH(KK) (ASX(I),ASY(I),I=1,N)
KK=70
FETCH(KK) TIX,TIY,TIXY
KK=74
FETCH(KK) EX,EY
KK=201
FETCH(KK) (STICK(II),II=1,10)
KK=215
FETCH(KK) CMIN,CMAX
KK=220
FETCH(KK) ACC,ATC,ACS,ATS
T4=T2+D1
D4=D2+T1
D5=D2+T1+D3
A=T1*T4+T2*T2+D3*D3+(RM-1.)*SUMAS
AAA=EY*TIY-EX*TIXY
IF(AAA) 24,125,24
24  CONTINUE
FFF=ABSF(AAA)
IF(FFF-0.001) 200,200,201
302 CONTINUE
200 AAA=0.0
GO TO 125
125 WH=-(1./A)*((TIX*TIY-TIXY*TIXY)/(EX*TIX-EY*TIY))
CI=D5-UO-WH
C=CI*(STICK(9)-0.3*STICK(4))/STICK(9)
B=-1.
TANTET=1000000.
PRINT 16,TANTET
KK=500
RECORD(KK) TANTET
PRINT 15,C,B
15  FORMAT (/10X,2HC=,F15.3,5X,2HB=,F15.3)
KK=600

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RECORD(KK) C,B
IF(C) 100,101,101
101 PRINT 102
102 FORMAT(10X,18HCHANGE THE SECTION)
STOP
100 CONTINUE
CALL TINFI(C,UO,T1,D5,D3,C1X,C1Y,C7X,C7Y,C6X,C6Y,C3X,C3Y,C5Y,C8Y,
1SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY)
CALL CABOV(SUMAS,N,AS,ASX,ASY,RM,SX,SY,ZX,ZY,PX,PY,OX,OY,
1RX,RY,QX,QY,C2Y,C4Y,C5Y,C6X,C6Y,C7X,C7Y,C8X,C10Y,AA,XO,YO,
2A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,
3B13,B14,B15,B16,B17,B18)
CALL CABXX(SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY,C2Y,C4Y,C5Y,
1C6X,C6Y,C7X,C7Y,C8X,C10Y,XO,YO,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,
2B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,B16,B17,B18,C1XX,
3C1YY,C1XXYY)
GO TO 888
201 CONTINUE
TANTET=(EY*TIXY-EX*TIX)/(EY*TIY-EX*TIXY)
TTT=ABSF(TANTET)
IF(TTT-1000000.) 301,302,302
301 CONTINUE
IF(TANTET) 21,22,23
21 CONTINUE
IF(TANTET+0.001) 901,900,900
900 TANTET=0.0
GO TO 22
23 CONTINUE
IF(TANTET-0.001) 800,800,801
800 TANTET=0.0
GO TO 22
22 PRINT 16,TANTET
16 FORMAT(/1X,7HTANTET=,F15.5)
KK=500
RECORD(KK) TANTET
WV=-(1./A)*((TIX*TIY-TIXY*TIXY)/(EY*TIY-EX*TIXY))
BI=WV+VD
B=BI*(STICK(I)-0.3*STICK(7))/STICK(I)
C=-1.
PRINT 15,C,B
KK=600
RECORD(KK) C,B
IF(B-T4) 103,101,101
103 CONTINUE
CALL THORI(B,VD,T2,T3,T4,C3X,C3Y,C4X,C9X,C9Y,C10X,SX,SY,ZX,ZY,
1PX,PY,OX,OY,RX,RY,QX,QY)
CALL CABOV(SUMAS,N,AS,ASX,ASY,RM,SX,SY,ZX,ZY,PX,PY,OX,OY,
1RX,RY,QX,QY,C2Y,C4Y,C5Y,C6X,C6Y,C7X,C7Y,C8X,C10Y,AA,XO,YO,
2A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,
3B13,B14,B15,B16,B17,B18)
CALL CABXX(SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY,C2Y,C4Y,C5Y,
1C6X,C6Y,C7X,C7Y,C8X,C10Y,XO,YO,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,
2B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,B16,B17,B18,C1XX,
3C1YY,C1XXYY)
GO TO 888
C LOCATION OF NEUTRAL AXIS WHEN TANGENT IS POSITIVE
801 CONTINUE
PRINT 16,TANTET
KK=500
RECORD(KK) TANTET

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WV=TAN TET*(D5-UO)-(1./A)*((TIX*TIY-TIXY*TIXY)/(EY*TIY-EX*TIXY))

BI=WV+VO

WH=- (VO/TAN TET)-(1./A)*((TIX*TIY-TIXY*TIXY)/(EX*TIX-EY*TIXY))

CI=-WH+D5-UO

B=BI*(CMIN-0.3*CMAx)/CMIN

C=B/TAN TET

PRINT 15,C,B

KK=600

RECORD(KK) C,B

IF(CMIN) 104,101,101

104 CONTINUE

CALL TPOSI(C,B,UO,VO,TAN TET,D1,D2,D3,D4,D5,T1,T2,T3,T4,
1C2X,C2Y,C3X,C3Y,C4X,C4Y,C5X,C5Y,C6X,C6Y,C7X,C7Y,C8X,C8Y,C9X,C9Y,
2C10X,C10Y,C1X,C1Y,SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY)

CALL CABOV(SUMAS,N,AS,ASX,ASY,RM,SX,SY,ZX,ZY,PX,PY,OX,OY,
1RX,RY,QX,QY,C2Y,C4Y,C5Y,C6X,C6Y,C7X,C7Y,C8X,C10Y,AA,XO,YO,
2A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,
3B13,B14,B15,B16,B17,B18)

CALL CABXX(SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY,C2Y,C4Y,C5Y,
1C6X,C6Y,C7X,C7Y,C8X,C10Y,XO,YO,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,
2B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,B16,B17,B18,C1XX,
3C1YY,C1XXYY)

GO TO 888

C LOCATION OF NEUTRAL AXIS WHEN TANGENT IS NEGATIVE

901 CONTINUE

PRINT 16,TAN TET

KK=500

RECORD(KK) TAN TET

WV=-UO*TAN TET-(1./A)*((TIX*TIY-TIXY*TIXY)/(EY*TIY-EX*TIXY))

BI=WV+VO

WH=- (VO/TAN TET)-(1./A)*((TIX*TIY-TIXY*TIXY)/(EX*TIX-EY*TIXY))

CI=UO+WH

B=BI*(CMIN-0.3*CMAx)/CMIN

C=-B/TAN TET

PRINT 15,C,B

KK=600

RECORD(KK) C,B

IF(CMIN) 105,101,101

105 CONTINUE

TAN=-TAN TET

CALL TNEGA(C,B,TAN,UO,VO,T1,T2,T3,T4,D1,D2,D3,D4,D5,
1C1X,C1Y,C2X,C2Y,C3X,C3Y,C4X,C4Y,C5X,C5Y,C6X,C6Y,C7X,C7Y,C8X,C8Y,
2C9X,C9Y,C10X,C10Y,SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY)

CALL ABOVE(SUMAS,N,AS,ASX,ASY,RM,SX,SY,ZX,ZY,PX,PY,OX,OY,
1RX,RY,QX,QY,C1Y,C2Y,C3X,C3Y,C4Y,C5X,C5Y,C6Y,C7X,C9X,C9Y,C10X,
2AA,XO,YO,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,
3B10,B11,B12,B13,B14,B15,B16,B17,B18)

CALL ABOXX(SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY,A1,A2,A3,A4,
1A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,
2B16,B17,B18,XO,YO,C1XX,C1YY,C1XXYY,C1Y,C2Y,C3X,C3Y,C4Y,C5X,C5Y,
3C6Y,C7X,C9X,C9Y,C10X)

GO TO 888

888 CONTINUE

CALL STINE(N,XO,YO,RM,AS,ASX,ASY,C1XX,C1YY,C1XXYY,AYY,AXX,
1T1XX,T1YY,T1XXYY)

CALL CRACK1(EY,EX,XO,YO,UO,VO,FN,AA,AYY,AXX,T1XX,T1YY,
1T1XXYY,T2,T3,T4,D2,D4,D5,RM,N,CA,BA,TET,STC,STS)

CALL CRAXX(STC,STS,ACC,ACS,ATS,N)

END

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C          FOURTH PROGRAM
C          EXACT POSITION OF NEUTRAL AXIS, CALCULATION OF FINAL STRESSES
          DIMENSION AS(50),ASX(50),ASY(50),STICK(10)
          DIMENSION AYY(50),AXX(50),STC(10),ST5(50)
          DEFINE DISK(10,4500)
          PRINT 1
1          FORMAT(1H1,26HPART THREE, FINAL STRESSES/1H ,26HALL UNITS ARE IN K
16 AND CM)
          KK=1
          FETCH(KK) T1,T2,T3,D1,D2,D3,RM,N
          KK=1
          FETCH(KK) (AS(I),I=1,N)
          KK=65
          FETCH(KK) SUMAS
          KK=77
          FETCH(KK) UO,VO
          KK=8
          FETCH(KK) C1X,C2X,C3X,C4X,C5X,C6X,C7X,C8X,C9X,C10X,C1Y,C2Y,C3Y,
1C4Y,C5Y,C6Y,C7Y,C8Y,C9Y,C10Y
          KK=101
          FETCH(KK) (ASX(I),ASY(I),I=1,N)
          KK=7
          FETCH(KK) TIX,TIY,TIXY
          KK=74
          FETCH(KK) FN,EX,EY
          KK=201
          FETCH(KK) (STICK(II),II=1,10)
          KK=215
          FETCH(KK) CMIN,CMAX
          KK=220
          FETCH(KK) ACC,ATC,ACS,ATS
          KK=5
          FETCH(KK) TANTET
          KK=6
          FETCH(KK) C,B
          KK=7
          FETCH(KK) TET
          KK=8
          FETCH(KK) BA,CA
          T4=T2+D1
          D4=D2+T1
          D5=D2+T1+D3
          A=T1*T4+T2*T2+D3*D3+(RM-1.)*SUMAS
999      CONTINUE
          IF(BA) 200,201,201
200      CONTINUE
          IF(B) 202,205,205
202      CONTINUE
          SUL=ABSF(CA-C)
          IF(SUL-0.01) 204,204,205
204      PRINT 206
206      FORMAT(710X,20HNEUTRAL AXIS LOCATED)
          STOP
205      C=CA
          B=-1.
          GO TO 50
201      CONTINUE
          IF(TANTET) 210,211,212
210      CONTINUE
          IF(TET) 213,214,217
213      SULE=ABSF(BA-B)
          IF(SULE-0.01) 216,216,225
216      CONTINUE

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GUL=ABSF(CA-C)
IF(GUL<0.01) 218,218,225
218 PRINT 206
STOP
217 B=BA
C=CA
GO TO 60
214 B=BA
C=-1.
GO TO 70
211 CONTINUE
IF(TET) 225,226,217
226 CONTINUE
FUSU=ABSF(BA-B)
IF(FUSU<0.01) 228,228,214
228 PRINT 206
STOP
225 C=CA
B=BA
GO TO 80
212 CONTINUE
IF(TET) 225,214,237
237 CONTINUE
FILIZ=ABSF(BA-B)
IF(FILIZ<0.01) 238,238,217
238 CONTINUE
BAL=ABSF(CA-C)
IF(BAL<0.01) 240,240,217
240 PRINT 206
STOP
50 CONTINUE
CALL TINFIT(C,CO,T1,D2,D3,C1X,C1Y,C7X,C7Y,C6X,C6Y,C3X,C3Y,C5Y,C5Y,
1SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY)
IF(STICK(4)) 4020,4030,4
4030 PRINT 4050
4050 FORMAT(/10X,18HN,A,FALLS ON PT.4)
STOP
4000 CONTINUE
CALL CABOV(SUMAS,N,AS,ASX,ASY,RM,SX,SY,ZX,ZY,PX,PY,OX,OY,
1RX,RY,QX,QY,C2Y,C4Y,C5Y,C6X,C6Y,C7X,C7Y,C8X,C10Y,AA,XO,YO,
2A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,
3B13,B14,B15,B16,B17,B18)
CALL CABXX(SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY,C2Y,C4Y,C5Y,
1C6X,C6Y,C7X,C7Y,C8X,C10Y,XO,YO,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,
2B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,B16,B17,B18,C1XX,
3CIYY,C1XXYY)
GO TO 888
4020 CONTINUE
CALL CBELO(SUMAS,N,AS,ASX,ASY,RM,SX,SY,ZX,ZY,PX,PY,OX,OY,
1RX,RY,QX,QY,C2X,C2Y,C4X,C4Y,C5Y,C6Y,C7Y,C10X,C10Y,AA,XO,YO,
2A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11,
3B12,B13,B14,B15,B16,B17,B18)
CALL CBEXX(SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY,C2X,C2Y,C4X,
1C4Y,C5Y,C6Y,C7Y,C10X,C10Y,XO,YO,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,
2B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,B16,B17,B18,C1XX,
3CIYY,C1XXYY)
GO TO 888
70 CONTINUE
CALL THORI(B,VO,T2,T3,T4,C3X,C3Y,C4X,C9X,C9Y,C10X,SX,SY,ZX,ZY,
1PX,PY,OX,OY,RX,RY,QX,QY)
IF(STICK(1)) 6510,6500,6490
6500 PRINT 6450
6450 FORMAT(/10X,18HN,A,FALLS ON PT.1)

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STOP
6510 CONTINUE
CALL CABOV(SUMAS,N,AS,ASX,ASY,RM,SX,SY,ZX,ZY,PX,PY,OX,OY,
1RX,RY,QX,QY,C2Y,C4Y,C5Y,C6X,C6Y,C7X,C7Y,C8X,C10Y,AA,XO,YO,
2A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,
3B13,B14,B15,B16,B17,B18)
CALL CABXX(SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY,C2Y,C4Y,C5Y,
1C6X,C6Y,C7X,C7Y,C8X,C10Y,XO,YO,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,
2B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,B16,B17,B18,C1XX,
3CIYY,C1XXYY)
GO TO 888
6490 CONTINUE
CALL CBELO(SUMAS,N,AS,ASX,ASY,RM,SX,SY,ZX,ZY,PX,PY,OX,OY,
1RX,RY,QX,QY,C2X,C2Y,C4X,C4Y,C5Y,C6Y,C7Y,C10X,C10Y,AA,XO,YO,
2A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11,
3B12,B13,B14,B15,B16,B17,B18)
CALL CBEXX(SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY,C2X,C2Y,C4X,
1C4Y,C5Y,C6Y,C7Y,C10X,C10Y,XO,YO,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,
2B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,B16,B17,B18,C1XX,
3CIYY,C1XXYY)
GO TO 888
60 CONTINUE
TANTET=TET
CALL TPOSI(C,B,UO,VO,TANTET,D1,D2,D3,D4,D5,T1,T2,T3,T4,
1C2X,C2Y,C3X,C3Y,C4X,C4Y,C5X,C5Y,C6X,C6Y,C7X,C7Y,C8X,C8Y,C9X,C9Y,
2C10X,C10Y,C1X,C1Y,SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY)
IF(STICK(5)) 5020,5010,5
5010 PRINT 5050
5050 FORMAT(/10X,18HN,A.FALLS ON PT.5)
STOP
5000 CONTINUE
CALL CABOV(SUMAS,N,AS,ASX,ASY,RM,SX,SY,ZX,ZY,PX,PY,OX,OY,
1RX,RY,QX,QY,C2Y,C4Y,C5Y,C6X,C6Y,C7X,C7Y,C8X,C10Y,AA,XO,YO,
2A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,
3B13,B14,B15,B16,B17,B18)
CALL CABXX(SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY,C2Y,C4Y,C5Y,
1C6X,C6Y,C7X,C7Y,C8X,C10Y,XO,YO,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,
2B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,B16,B17,B18,C1XX,
3CIYY,C1XXYY)
GO TO 888
5020 CONTINUE
CALL CBELO(SUMAS,N,AS,ASX,ASY,RM,SX,SY,ZX,ZY,PX,PY,OX,OY,
1RX,RY,QX,QY,C2X,C2Y,C4X,C4Y,C5Y,C6Y,C7Y,C10X,C10Y,AA,XO,YO,
2A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11,
3B12,B13,B14,B15,B16,B17,B18)
CALL CBEXX(SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY,C2X,C2Y,C4X,
1C4Y,C5Y,C6Y,C7Y,C10X,C10Y,XO,YO,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,
2B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,B16,B17,B18,C1XX,
3CIYY,C1XXYY)
GO TO 888
80 CONTINUE
TANTET=TET
TAN=-TANTET
CALL TNEGA(C,B,TAN,UO,VO,T1,T2,T3,T4,D1,D2,D3,D4,D5,
1C1X,C1Y,C2X,C2Y,C3X,C3Y,C4X,C4Y,C5X,C5Y,C6X,C6Y,C7X,C7Y,C8X,C8Y,
2C9X,C9Y,C10X,C10Y,SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY)
IF(STICK(8)) 8020,8010,8
8010 PRINT 8050
8050 FORMAT(/10X,18HN,A.FALLS ON PT.8)
STOP
8000 CONTINUE
CALL ABOVE(SUMAS,N,AS,ASX,ASY,RM,SX,SY,ZX,ZY,PX,PY,OX,OY,
1RX,RY,QX,QY,C1Y,C2Y,C3X,C3Y,C4Y,C5X,C5Y,C6Y,C7X,C9X,C9Y,C10X,

```

2AA,XO,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,
3B10,B11,B12,B13,B14,B15,B16,B17,B18)

CALL ABOXX(SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY,A1,A2,A3,A4,
1A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,
2B16,B17,B18,XO,YO,C1XX,C1YY,C1XXYY,C1Y,C2Y,C3X,C3Y,C4Y,C5X,C5Y,
3C6Y,C7X,C9X,C9Y,C10X)

GO TO 888

8020 CONTINUE

CALL BELOW(SUMAS,N,AS,ASX,ASY,RM,SX,SY,ZX,ZY,PX,PY,OX,OY,
1RX,RY,OX,OY,C1X,C1Y,C3X,C3Y,C5Y,C6Y,C7Y,C8Y,C9X,C9Y,XO,YO,
2AA,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11,
3B12,B13,B14,B15,B16,B17,B18)

CALL BELXX(SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY,C1X,C1Y,C3X,
1C5Y,C6Y,C7Y,C8Y,C9X,C9Y,XO,YO,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,
2B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,B16,B17,B18,C1XX,
3C1YY,C1XXYY,C3Y)

GO TO 888

888 CONTINUE

CALL STINE(N,XO,YO,RM,AS,ASX,ASY,C1XX,C1YY,C1XXYY,AYY,AXX,
1T1XX,T1YY,T1XXYY)

TANTET=TET

CALL CRACKLEY(LX,XO,YO,UO,VO,FN,AA,AYY,AXX,T1XX,T1YY,
1T1XXYY,T2,T3,T4,D2,D4,D5,RM,N,CA,BA,TET,STC,STS)

CALL CRAXX(STC,STS,ACC,ACS,ATS,N)

GO TO 999

END

SUB ROUTINE CORDI (AS,ASU,ASV,RM,N,A,T1,T2,T3,T4,D1,D2,D3,D4,D5,
C1X,C2X,C3X,C4X,C5X,C6X,C7X,C8X,C9X,C10X,C1Y,C2Y,C3Y,C4Y,C5Y,C6Y,
C7Y,C8Y,C9Y,C10Y,ASX,ASY,UO,VO)

C X AND Y COORDINATES OF CORNER POINTS

DIMENSION AS(50),ASU(50),ASV(50),ASX(50),ASY(50)

SUAS=0.

DO 800 I=1,N

800 SUAS=SUAS+AS(I)*ASU(I)

UO=(T2*D2*D2+2.*T1*T4*D2+T1*T1*T4+2.*T3*D3*D4+T3*D3*D3+2.*(RM-1.)

1*SUAS)/(2.*A)

SVAS=0.

DO 900 I=1,N

900 SVAS=SVAS+AS(I)*ASV(I)

VO=(T2*T2*D2+T1*T4*T4+D3*T3*T3+2.*(RM-1.)*SVAS)/(2.*A)

PRINT 10

10 FORMAT (/ / 30X, 23H CENTROID OF THE SECTION)

PRINT 11,UO,VO

11 FORMAT (/ 35X, 3HUO=, F10.5 / 35X, 3HVO=, F10.5)

KK=77

RECORD(KK) UO,VO

C1X=D2-UO

C2X=D4-UO

C3X=C2X

C4X=D5-UO

C5X=C4X

C6X=C2X

C7X=C1X

C8X=-UO

C9X=C8X

C10X=C1X

C1Y=T4-VO

C2Y=C1Y

C3Y=T3-VO

C4Y=C3Y

C5Y=-VO

C6Y=C5Y

C7Y=C5Y

C8Y=C5Y

C9Y=T2-VO

C10Y=C9Y

KK=80

RECORD(KK) C1X,C2X,C3X,C4X,C5X,C6X,C7X,C8X,C9X,C10X,C1Y,C2Y,C3Y,
C4Y,C5Y,C6Y,C7Y,C8Y,C9Y,C10Y

DO 12 I=1,N

12 ASX(I)=ASU(I)-UO

DO 13 I=1,N

13 ASY(I)=ASV(I)-VO

KK=101

RECORD(KK) (ASX(I),ASY(I),I=1,N)

RETURN

END

```

SUBROUTINE STRESS(FN,EX,EY,TIX,TIY,TIXY,A,RM,N,ASX,ASY,T2,T3,T4,
ID2,D3,D4,D5,UO,VO,STICK,STISK)
DIMENSION STICK(10),STISK(50),ASX(50),ASY(50)
C CALCULATION OF STRESSES IN CONCRETE
SKY=(FN*EY*TIY-FN*EX*TIXY)/(TIX*TIY-TIXY*TIXY)
SKX=(FN*EX*TIX-FN*EY*TIXY)/(TIX*TIY-TIXY*TIXY)
SK=FN/A
STICK(1)=SK+SKY*(T4-VO)+SKX*(D2-UO)
STICK(2)=SK+SKY*(T4-VO)+SKX*(D4-UO)
STICK(3)=SK+SKY*(T3-VO)+SKX*(D4-UO)
STICK(4)=SK+SKY*(T3-VO)+SKX*(D5-UO)
STICK(5)=SK+SKY*(-VO)+SKX*(D5-UO)
STICK(6)=SK+SKY*(-VO)+SKX*(D4-UO)
STICK(7)=SK+SKY*(-VO)+SKX*(D2-UO)
STICK(8)=SK+SKY*(-VO)+SKX*(-UO)
STICK(9)=SK+SKY*(T2-VO)+SKX*(-UO)
STICK(10)=SK+SKY*(T2-VO)+SKX*(D2-UO)
PRINT 25
25 FORMAT(/20X,20HSTRESSES IN CONCRETE/)
KK=201
RECORD(KK) (STICK(II),II=1,10)
PRINT 30,(STICK(II),II=1,10)
30 FORMAT(1H,5F10.3)
C CALCULATION OF STRESSES IN REINFORCEMENT BARS
DO 27 I=1,N
27 STISK(I)=RM*(SK+SKY*ASY(I)+SKX*ASX(I))
PRINT 129
129 FORMAT(/20X,16HSTRESSES IN BARS/)
DO 128 I=1,N
PRINT 28,(STISK(I),I=1,N)
128 CONTINUE
28 FORMAT(1H,8F10.3)
RETURN
END
```

```

SUBROUTINE MINMA(STICK,STISK,N,CMIN,CMAX,SMIN,SMAX)
DIMENSION STICK(10),STISK(50)
CMIN=STICK(1)
DO 29 II=2,10
IF(CMIN-STICK(II)) 29,29,30
30 CMIN=STICK(II)
29 CONTINUE
CMAX=STICK(1)
DO 32 II=2,10
IF(CMAX-STICK(II)) 31,32,32
31 CMAX=STICK(II)
32 CONTINUE
PRINT 33,CMIN,CMAX
33 FORMAT(/7/10X,24HMIN. STRESS IN CONCRETE=,F10.3/10X,24HMAX. STRESS
IN CONCRETE=,F10.3//)
KK=215
RECORD(KK) CMIN,CMAX
SMIN=STISK(1)
DO 36 I=2,N
IF(SMIN-STISK(I)) 36,36,37
37 SMIN=STISK(I)
36 CONTINUE
SMAX=STISK(1)
DO 38 I=2,N
IF(SMAX-STISK(I)) 39,38,38
39 SMAX=STISK(I)
38 CONTINUE
PRINT 341,SMIN,SMAX
341 FORMAT(/10X,20HMIN. STRESS IN BARS=,F10.3/10X,20HMAX. STRESS IN BAR
IS=,F10.3)
RETURN
END

```

```
      SUBROUTINE CABOV(SUMAS,N,AS,ASX,ASY, RM,SX,SY,ZX,ZY,PX,PY,OX,OY,  
      IRX,RY,QX,QY,C2Y,C4Y,C5Y,C6X,C6Y,C7X,C7Y,C8X,C10Y,AA,XO,YO,  
      2A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,  
      3B13,B14,B15,B16,B17,B18)  
      DIMENSION AS(50),ASX(50),ASY(50)
```

```
      C AREA OF TRANSPOSED CRACKED SECTION
```

```
      C COMPRESSION ABOVE N.A.
```

```
      A1=(ZX-C8X)*(C10Y-C7Y)
```

```
      A2=(SX-ZX)*(C10Y-SY)
```

```
      A3=(SX-ZX)*(SY-ZY)
```

```
      A4=(PX-C7X)*(C2Y-C6Y)
```

```
      A5=(OX-PX)*(C2Y-OY)
```

```
      A6=(OX-PX)*(OY-PY)
```

```
      A7=(QX-C6X)*(C4Y-C5Y)
```

```
      A8=(RX-QX)*(C4Y-RY)
```

```
      A9=(RX-QX)*(RY-QY)
```

```
      AAC=A1+A2+(1./2.)*A3+A4+A5+A6*(1./2.)+A7+A8+(1./2.)*A9
```

```
      AA=AA+RM*SUMAS
```

```
      C CENTROID OF THE CRACKED SECTION
```

```
      SXAS=0.
```

```
      DO1710 I=1,N
```

```
1710 SXAS=SXAS+AS(I)*ASX(I)
```

```
      B1=ZX+C8X
```

```
      B2=SX+ZX
```

```
      B3=ZX+2.*SX
```

```
      B4=PX+C7X
```

```
      B5=OX+PX
```

```
      B6=PX+2.*OX
```

```
      B7=QX+C6X
```

```
      B8=RX+QX
```

```
      B9=QX+2.*RX
```

```
      B10=C10Y+C7Y
```

```
      B11=C10Y+SY
```

```
      B12=ZY+2.*SY
```

```
      B13=C2Y+C6Y
```

```
      B14=C2Y+OY
```

```
      B15=PY+2.*OY
```

```
      B16=C4Y+C5Y
```

```
      B17=C4Y+RY
```

```
      B18=QY+2.*RY
```

```
      DUH=0./(.6.*AA)
```

```
      XO=(3.*A1*B1+3.*A2*B2+A3*B3+3.*A4*B4+3.*A5*B5+A6*B6+3.*A7*B7+  
      13.*A8*B8+A9*B9+6.*RM*SXAS)/(6.*AA)
```

```
      SYAS=0.
```

```
      DD 1810 I=1,N
```

```
1810 SYAS=SYAS+AS(I)*ASY(I)
```

```
      YO=(3.*A1*B10+3.*A2*B11+A3*B12+3.*A4*B13+3.*A5*B14+A6*B15+  
      13.*A7*B16+3.*A8*B17+A9*B18+6.*RM*SYAS)/(6.*AA)
```

```
      PRINT 1,XO,YO
```

```
1 FORMAT (710X,3HXO=,F10.3,10X,3HYO=,F10.3)
```

```
      RETURN
```

```
      END
```

SUBROUTINE CABXX(SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY,C2Y,C4Y,C5Y,
1C6X,C6Y,C7X,C7Y,C8X,C10Y,XO,YO,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,
2B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,B16,B17,B18,CIXX,
3CIYY,CIXXYY)

C MOMENT OF INERTIA OF CONCRETE

DUM=0./12.+0./36.+0./4.+0./72.

RRR=(1./12.)*A7*((C4Y-C5Y)**2+3.*(B16-2.*YO)**2)+(1./12.)*A8*

1((C4Y-RY)**2+3.*(B17-2.*YO)**2)+(1./36.)*A9*((RY-QY)**2+2.*(B18
2-3.*YO)**2)

SSS=(1./12.)*A5*((C2Y-OY)**2+3.*(B14-2.*YO)**2)+(1./36.)*A6*

1((OY-PY)**2+2.*(B15-3.*YO)**2)+RRR

CIXX=(1./12.)*A1*((C10Y-C7Y)**2+3.*(B10-2.*YO)**2)+(1./12.)*A2*

1((C10Y-SY)**2+3.*(B11-2.*YO)**2)+(1./36.)*A3*((SY-ZY)**2+2.*

2(B12-3.*YO)**2)+(1./12.)*A4*((C2Y-C6Y)**2+3.*(B13-2.*YO)**2)+SSS

WWW=(1./36.)*A6*((OX-PX)**2+2.*(B6-3.*XO)**2)+(1./12.)*A7*

1((OX-C6X)**2+3.*(B7-2.*XO)**2)+(1./12.)*A8*((RX-QX)**2+3.*

2(B8-2.*XO)**2)+(1./36.)*A9*((RX-QX)**2+2.*(B9-3.*XO)**2)

CIYY=(1./12.)*A1*((ZX-C8X)**2+3.*(B1-2.*XO)**2)+(1./12.)*A2*

1((SX-ZX)**2+3.*(B2-2.*XO)**2)+(1./36.)*A3*((SX-ZX)**2+2.*(B3-3.*

2XO)**2)+(1./12.)*A4*((PX-C7X)**2+3.*(B4-2.*XO)**2)+(1./12.)*A5*

3((OX-PX)**2+3.*(B5-2.*XO)**2)+WWW

WC=(1./72.)*A3*(A3+4.*(B3-3.*XO)*(B12-3.*YO))+(1./72.)*A6*(A6+

14.*(B6-3.*XO)*(B15-3.*YO))+(1./72.)*A9*(A9+4.*(B9-3.*XO)*(B18-
23.*YO))

CIXXYY=WC+(1./4.)*(A1*(B1-2.*XO)*(B10-2.*YO)+A2*(B2-2.*XO)*(B11

1-2.*YO)+A4*(B4-2.*XO)*(B13-2.*YO)+A5*(B5-2.*XO)*(B14-2.*YO)+A7*

2(B7-2.*XO)*(B16-2.*YO)+A8*(B8-2.*XO)*(B17-2.*YO))

PRINT 2,CIXX,CIYY,CIXXYY

2 FORMAT(/1X,5HCIXX=,F20.5,1X,5HCIYY=,F20.5,1X,7HCIXXYY=,F20.5)

RETURN

END

```
      SUBROUTINE CBELO(SUMAS,N,AS,ASX,ASY,RM,SX,SY,ZX,ZY,PX,PY,OX,OY,  
1RX,RY,QX,QY,C2X,C2Y,C4X,C4Y,C5Y,C6Y,C7Y,C10X,C10Y,AA,X0,Y0,  
2A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11,  
3B12,B13,B14,B15,B16,B17,B18)  
      DIMENSION AS(50),ASX(50),ASY(50)
```

```
C      AREA OF TRANSPOSED CRACKED SECTION
```

```
C      COMPRESSION BELOW N.A.
```

```
A1=(C10X-SX)*(C10Y-C7Y)
```

```
A2=(SX-ZX)*(ZY-C7Y)
```

```
A3=(SX-ZX)*(SY-ZY)
```

```
A4=(C2X-0X)*(C2Y-C6Y)
```

```
A5=(0X-PX)*(PY-C7Y)
```

```
A6=(0X-PX)*(OY-PY)
```

```
A7=(C4X-RX)*(C4Y-C5Y)
```

```
A8=(RX-QX)*(QY-C6Y)
```

```
A9=(RX-QX)*(RY-QY)
```

```
AAC=A1+A2+(1./2.)*A3+A4+A5+(1./2.)*A6+A7+A8+(1./2.)*A9
```

```
AA=AA+RM*SUMAS
```

```
C      CENTROID OF THE CRACKED SECTION
```

```
SXAS=0.
```

```
DO 710 I=1,N
```

```
710 SXAS=SXAS+AS(I)*ASX(I)
```

```
B1=C10X+SX
```

```
B2=SX+ZX
```

```
B3=ZX+2.*SX
```

```
B4=C2X+0X
```

```
B5=0X+PX
```

```
B6=PX+2.*0X
```

```
B7=C4X+RX
```

```
B8=RX+QX
```

```
B9=QX+2.*RX
```

```
B10=C10Y+C7Y
```

```
B11=ZY+C7Y
```

```
B12=SY+2.*ZY
```

```
B13=C2Y+C6Y
```

```
B14=PY+C7Y
```

```
B15=OY+2.*PY
```

```
B16=C4Y+C5Y
```

```
B17=QY+C6Y
```

```
B18=RY+2.*QY
```

```
DUM=0./(6.*AA)
```

```
X0=(3.*A1*B1+3.*A2*B2+A3*B3+3.*A4*B4+3.*A5*B5+A6*B6+3.*A7*B7+  
13.*A8*B8+A9*B9+6.*RM*SXAS)/(6.*AA)
```

```
SYAS=0.
```

```
DO 2810 I=1,N
```

```
2810 SYAS=SYAS+AS(I)*ASY(I)
```

```
Y0=(3.*A1*B10+3.*A2*B11+A3*B12+3.*A4*B13+3.*A5*B14+A6*B15+  
13.*A7*B16+3.*A8*B17+A9*B18+6.*RM*SYAS)/(6.*AA)
```

```
PRINT 500,X0,Y0
```

```
500 FORMAT (/10X,3HX0=,F10.3,10X,3HY0=,F10.3)
```

```
RETURN
```

```
END
```

SUBROUTINE CBEXX(SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY,C2X,C2Y,C4X,
C4Y,C5Y,C6Y,C7Y,C10X,C10Y,XO,YO,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,
B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,B16,B17,B18,C1XX,
C1YY,C1XXYY)

C MOMENT OF INERTIA OF CONCRETE

DUM=0./12.+0./36.+0./4.+0./72.

CCC=(1./12.)*A7*((C4Y-C5Y)**2+3.*(B16-2.*YO)**2)+(1./12.)*A8*

1*((QY-C6Y)**2+3.*(B17-2.*YO)**2)+(1./36.)*A9*((RY-QY)**2+2.*(B18-3.*
YO)**2)

YYY=(1./12.)*A5*((PY-C7Y)**2+3.*(B14-2.*YO)**2)+(1./36.)*A6*

1((OY-PY)**2+2.*(B15-3.*YO)**2)+CCC

C1XX=(1./12.)*A1*((C10Y-C7Y)**2+3.*(B10-2.*YO)**2)+(1./12.)*A2*

1((ZY-C8Y)**2+3.*(B11-2.*YO)**2)+(1./36.)*A3*((SY-ZY)**2+2.*
2(B12-3.*YO)**2)+(1./12.)*A4*((C2Y-C7Y)**2+3.*(B13-2.*YO)**2)+YYY

DDDD=(1./12.)*A7*((C4X-RX)**2+3.*(B7-2.*XO)**2)+(1./12.)*A8*

1((RX-QX)**2+3.*(B8-2.*XO)**2)+(1./36.)*A9*((RX-QX)**2+2.*
2(B9-3.*XO)**2)

XXX=(1./12.)*A5*((OX-PX)**2+3.*(B5-2.*XO)**2)+(1./36.)*A6*

1((OX-PX)**2+2.*(B6-3.*XO)**2)+DDDD

C1YY=(1./12.)*A1*((C10X-SX)**2+3.*(B1-2.*XO)**2)+(1./12.)*A2*

1((SX-ZX)**2+3.*(B2-2.*XO)**2)+(1./36.)*A3*((SX-ZX)**2+2.*

2(B3-3.*XO)**2)+(1./12.)*A4*((C2X-OX)**2+3.*(B4-2.*XO)**2)+XXX

EE=(1./72.)*A3*(A3+4.*(B12-3.*YO)*(B3-3.*XO))

DD=(1./72.)*A6*(A6+4.*(B15-3.*YO)*(B6-3.*XO))

FF=(1./72.)*A9*(A9+4.*(B18-3.*YO)*(B9-3.*XO))

C1XXYY=EE+DD+FF+(1./4.)*(A1*(B1-2.*XO)*(B10-2.*YO)+A2*(B2-2.*XO)*

1(B11-2.*YO)+A4*(B4-2.*XO)*(B13-2.*YO)+A5*(B5-2.*XO)*(B14-2.*YO)

2+A7*(B7-2.*XO)*(B16-2.*YO)+A8*(B8-2.*XO)*(B17-2.*YO))

PRINT 2,C1XX,C1YY,C1XXYY

2 FORMAT (/IX,5HC1XX=,F20.5,IX,5HC1YY=,F20.5,IX,7HC1XXYY=,F20.5)

RETURN

END

```
SUBROUTINE ABOVE(SUMAS,N,AS,ASX,ASY,RM,SX,SY,ZX,ZY,PX,PY,OX,OY,  
1RX,RY,QX,QY,C1Y,C2Y,C3X,C3Y,C4Y,C5X,C5Y,C6Y,C7X,C9X,C9Y,C10X,  
2AA,XO,YO,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,  
3B10,B11,B12,B13,B14,B15,B16,B17,B18)  
DIMENSION AS(50),ASX(50),ASY(50)
```

```
C AREA OF TRANSPOSED CRACKED SECTION
```

```
C COMPRESSION ABOVE NEUTRAL AXIS
```

```
A1=(C5X-OX)*(C4Y-C5Y)
```

```
A2=(C3Y-RY)*(QX-RX)
```

```
A3=(QX-RX)*(RY-QY)
```

```
A4=(OX-PX)*(C1Y-PY)
```

```
A5=(C3X-OX)*(C2Y-C6Y)
```

```
A6=(OX-PX)*(PY-OY)
```

```
A7=(C10X-C9X)*(C9Y-SY)
```

```
A8=(C7X-ZX)*(SY-C7Y)
```

```
A9=(ZX-SX)*(SY-ZY)
```

```
AAA=A1+A2+(1./2.)*A3+A4+A5+(1./2.)*A6+A7+A8+(1./2.)*A9
```

```
AA=AAA+RM*SUMAS
```

```
C CENTROID OF THE CRACKED SECTION
```

```
B1=C5X+OX
```

```
B2=QX+RX
```

```
B3=2.*QX+RX
```

```
B4=OX+PX
```

```
B5=C3X+OX
```

```
B6=2.*QX+PX
```

```
B7=C10X+C9X
```

```
B8=C7X+ZX
```

```
B9=2.*ZX+SX
```

```
B10=C4Y+C5Y
```

```
B11=C3Y+RY
```

```
B12=2.*RY+QY
```

```
B13=C1Y+PY
```

```
B14=C2Y+C6Y
```

```
B15=2.*PY+OY
```

```
B16=C9Y+SY
```

```
B17=SY+C7Y
```

```
B18=2.*SY+ZY
```

```
DUM=0./(6.*AA)
```

```
SXAS=0.
```

```
DO 1959 I=1,N
```

```
1959 SXAS=SXAS+AS(I)*ASX(I)
```

```
XO=(3.*A1*B1+3.*A2*B2+A3*B3+3.*A4*B4+3.*A5*B5+A6*B6+3.*A7*B7
```

```
1+3.*A8*B8+A9*B9+6.*RM*SXAS)/(6.*AA)
```

```
SYAS=0.
```

```
DO 1960 I=1,N
```

```
1960 SYAS=SYAS+AS(I)*ASY(I)
```

```
YO=(3.*A1*B10+3.*A2*B11+A3*B12+3.*A4*B13+3.*A5*B14+A6*B15+
```

```
13.*A7*B16+3.*A8*B17+A9*B18+6.*RM*SYAS)/(6.*AA)
```

```
PRINT 1,XO,YO
```

```
1 FORMAT (/10X,3HXO=,F10.3,10X,3HYO=,F10.3)
```

```
RETURN
```

```
END
```

SUBROUTINE ABOXX(SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY,A1,A2,A3,A4,
A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,
B16,B17,B18,XO,YO,C1XX,C1YY,C1XXYY,C1Y,C2Y,C3X,C3Y,C4Y,C5X,C5Y,
C6Y,C7X,C9X,C9Y,C10X)

C MOMENT OF INERTIA OF CONCRETE

DUM=0./12.+0./36.+0./4.+0./72.

ALI=(1./12.)*A1*((C4Y-C5Y)**2+3.*(B10-2.*YO)**2)+(1./12.)*A2*
1((C3Y-RY)**2+3.*(B11-2.*YO)**2)+(1./36.)*A3*((RY-QY)**2+2.*
2(B12-3.*YO)**2)

AII=(1./12.)*A4*((C1Y-PY)**2+3.*(B13-2.*YO)**2)+(1./12.)*A5*
1((C2Y-C6Y)**2+3.*(B14-2.*YO)**2)+(1./36.)*A6*((PY-OY)**2+2.*
2(B15-3.*YO)**2)

ALA=(1./12.)*A7*((C9Y-SY)**2+3.*(B16-2.*YO)**2)+(1./12.)*A8*
1((SY-C7Y)**2+3.*(B17-2.*YO)**2)+(1./36.)*A9*((SY-ZY)**2+2.*
2(B18-3.*YO)**2)

C1XX=ALI+AII+ALA

VAH=(1./12.)*A1*((C5X-QX)**2+3.*(B1-2.*XO)**2)+(1./12.)*A2*
1((QX-RX)**2+3.*(B2-2.*XO)**2)+(1./36.)*A3*((QX-RX)**2+2.*(B3-3.*
2XO)**2)

VAY=(1./12.)*A4*((OX-PX)**2+3.*(B4-2.*XO)**2)+(1./12.)*A5*
1((C3X-OX)**2+3.*(B5-2.*XO)**2)+(1./36.)*A6*((OX-PX)**2+2.*(B6-
23.*XO)**2)

HIT=(1./12.)*A7*((C10X-C9X)**2+3.*(B7-2.*XO)**2)+(1./12.)*A8*
1((C7X-ZX)**2+3.*(B8-2.*XO)**2)+(1./36.)*A9*((ZX-SX)**2+2.*
2(B9-3.*XO)**2)

C1YY=VAH+VAY+HIT

ES=(1./4.)*(A1*(B10-2.*YO)*(B11-2.*YO)+A2*(B11-2.*YO)*(B12-2.*YO))

ESE=(1./4.)*(A4*(B13-2.*YO)*(B14-2.*YO)+A5*(B14-2.*YO)*(B15-2.*YO))

ESEN=(1./4.)*(A7*(B16-2.*YO)*(B17-2.*YO)+A8*(B17-2.*YO)*(B18-2.*YO))

EMU=(1./72.)*A3*(-A3+4.*(B12-3.*YO)*(B3-3.*XO))

EMUT=(1./72.)*A6*(-A6+4.*(B15-3.*YO)*(B6-3.*XO))

EMUTLU=(1./72.)*A9*(-A9+4.*(B18-3.*YO)*(B9-3.*XO))

C1XXYY=ES+ESE+ESEN+EMU+EMUT+EMUTLU

PRINT 2,C1XX,C1YY,C1XXYY

2 FORMAT(/1X,5HC1XX=,F20.5,1X,5HC1YY=,F20.5,1X,7HC1XXYY=,F20.5)

RETURN

END

SUBROUTINE BELOW(SUMAS,N,AS,ASX,ASY,RM,SX,SY,ZX,ZY,PX,PY,OX,OY,
IRX,RY,QX,QY,C1X,C1Y,C3X,C3Y,C5Y,C6Y,C7Y,C8Y,C9X,C9Y,XO,YO,
2AA,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11,
3B12,B13,B14,B15,B16,B17,B18)
DIMENSION AS(50),ASX(50),ASY(50)

COMPRESSION BELOW NEUTRAL AXIS

C

AREA OF TRANSPOSED CRACKED SECTION

A1=(SX-C9X)*(C9Y-C8Y)

A2=(ZX-SX)*(ZY-C7Y)

A3=(ZX-SX)*(SY-ZY)

A4=(C1Y-C7Y)*(PX-C1X)

A5=(OX-PX)*(OY-C6Y)

A6=(OX-PX)*(PY-OY)

A7=(RX-C3X)*(C3Y-C6Y)

A8=(QX-RX)*(QY-C5Y)

A9=(QX-RX)*(RY-QY)

AAC=A1+A2+(1./2.)*A3+A4+A5+(1./2.)*A6+A7+A8+(1./2.)*A9

AA=AA+RM*SUMAS

C

CENTROID OF THE CRACKED SECTION

SXAS=0.

DO 7777 I=1,N

7777 SXAS=SXAS+AS(I)*ASX(I)

B1=SX+C9X

B2=ZX+SX

B3=ZX+2.*SX

B4=PX+C1X

B5=OX+PX

B6=OX+2.*PX

B7=RX+C3X

B8=QX+RX

B9=QX+2.*RX

B10=C9Y+C8Y

B11=ZY+C7Y

B12=SY+2.*ZY

B13=C1Y+C7Y

B14=OY+C6Y

B15=PY+2.*OY

B16=C3Y+C6Y

B17=QY+C5Y

B18=RY+2.*QY

DUM=0./(6.*AA)

XO=(3.*A1*B1+3.*A2*B2+A3*B3+3.*A4*B4+3.*A5*B5+A6*B6+3.*A7*B7+
13.*A8*B8+A9*B9+6.*RM*SXAS)/(6.*AA)

SYAS=0.

DO 4444 I=1,N

4444 SYAS=SYAS+AS(I)*ASY(I)

YO=(3.*A1*B10+3.*A2*B11+A3*B12+3.*A4*B13+3.*A5*B14+A6*B15+3.*
1A7*B16+3.*A8*B17+A9*B18+6.*RM*SYAS)/(6.*AA)

PRINT 500,XO,YO

500 FORMAT (/10X,3HXO=,F10.3,10X,3HYO=,F10.3)

RETURN

END

SUBROUTINE BELXX(SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY,C1X,C1Y,C3X,
C5Y,C6Y,C7Y,C8Y,C9X,C9Y,XO,YO,A1,A2,A3,A4,A5,A6,A7,A8,A9,B1,B2,
B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,B16,B17,B18,C1XX,
C1YY,C1XXYY,C3Y)

C MOMENT OF INERTIA OF CONCRETE

DUM=0./12.+0./36.+0./4.+0./72.

RE=(1./12.)*A1*((C9Y-C8Y)**2+3.*(B10-2.*YO)**2)+(1./12.)*A2*

1*((ZY-C7Y)**2+3.*(B11-2.*YO)**2)+(1./36.)*A3*((SY-ZY)**2+2.*(B12

2-3.*YO)**2)

REM=(1./12.)*A4*((C1Y-C7Y)**2+3.*(B13-2.*YO)**2)+(1./12.)*A5*

1((OY-C6Y)**2+3.*(B14-2.*YO)**2)+(1./36.)*A6*((PY-OY)**2+2.*(B15

2-3.*YO)**2)

REMA=(1./12.)*A7*((C3Y-C6Y)**2+3.*(B16-2.*YO)**2)+(1./12.)*A8*

1((QY-C5Y)**2+3.*(B17-2.*YO)**2)+(1./36.)*A9*((RY-QY)**2+2.*(B18

2-3.*YO)**2)

C1XX=REMA+REM+RE

AC=(1./12.)*A1*((SX-C9X)**2+3.*(B1-2.*XO)**2)+(1./12.)*A2*

1((ZX-SX)**2+3.*(B2-2.*XO)**2)+(1./36.)*A3*((ZX-SX)**2+2.*(B3-

23.*XO)**2)

ACU=(1./12.)*A4*((PX-C1X)**2+3.*(B4-2.*XO)**2)+(1./12.)*A5*

1((OX-PX)**2+3.*(B5-2.*XO)**2)+(1./36.)*A6*((OX-PX)**2+2.*(B6-

23.*XO)**2)

ACUN=(1./12.)*A7*((RX-C3X)**2+3.*(B7-2.*XO)**2)+(1./12.)*A8*

1((QX-RX)**2+3.*(B8-2.*XO)**2)+(1./36.)*A9*((QX-RX)**2+2.*(B9-

23.*XO)**2)

C1YY=AC+ACU+ACUN

UM=(1./72.)*A3*(-A3+4.*(B12-3.*YO)*(B3-3.*XO))+(1./72.)*A6*(-A6+

14.*(B15-3.*YO)*(B6-3.*XO))+(1./72.)*A9*(-A9+4.*(B18-3.*YO)*

2*(B9-3.*XO))

UMST=(1./4.)*(A1*(B10-2.*YO)*(B1-2.*XO)+A2*(B11-2.*YO)*(B2-2.*XO)

+A4*(B13-2.*YO)*(B4-2.*XO)+A5*(B14-2.*YO)*(B5-2.*XO)+A7*

2*(B16-2.*YO)*(B7-2.*XO)+A8*(B17-2.*YO)*(B8-2.*XO))

C1XXYY=UM+UMST

PRINT 2,C1XX,C1YY,C1XXYY

FORMAT(/1X,5HC1XX=,F20.5,1X,5HC1YY=,F20.5,1X,7HC1XXYY=,F20.5)

RETURN

END

2

```

SUBROUTINE CRACK(EY,EX,XO,YO,UO,VO,FN,AA,AYY,AXX,TIXX,TIYY,
IT,IXYY,T2,T3,T4,D2,D4,D5,RM,N,CA,BA,TET,STC,STS)
DIMENSION STC(10),AYY(50),AXX(50),STS(50)
EYY=EY-YO
EXX=EX-XO
SKYY=(FN*EYY*TIYY-FN*EXX*TIXXYY)/(TIXX*TIYY-TIXXYY*TIXXYY)
SKXX=(FN*EXX*TIXX-FN*EYY*TIXXYY)/(TIXX*TIYY-TIXXYY*TIXXYY)
SKK=FN/AA
IF(SKYY) 899,898,899
899 CONTINUE
UUU=ABSF(SKYY)
IF(UUU-0.001) 200,200,201
302 CONTINUE
200 SKYY=0.0
GO TO 898
898 WHH=-FN/(AA*SKXX)
CA=D5-UO-XO-WHH
BA=-1.
PRINT 897,BA,CA
897 FORMAT(/10X,36HN,A,VERTICAL,CA MEASURED FROM PT.5/10X,3HBA=,
F10.5/10X,3HCA=,F10.5)
KK=800
RECORD(KK) BA,CA
TET=1000000.
KK=700
RECORD(KK) TET
GO TO 50
201 TET=-SKXX/SKYY
TTT=ABSF(TET)
IF(TTT-1000000.) 301,302,302
301 CONTINUE
IF(TET) 1100,1110,1111
1100 CONTINUE
IF(TET+0.001) 901,900,900
900 TET=0.0
GO TO 1110
1111 CONTINUE
IF(TET-0.001) 800,800,801
800 TET=0.0
GO TO 1110
801 CONTINUE
PRINT 3,TET
KK=700
RECORD(KK) TET
WVV=TET*(D5-UO-XO)-FN/(AA*SKYY)
BA=WVV+VO+YO
WHH=-FN/(AA*SKXX)+(1./TET)*(-VO-YO)
CA=D5-UO-XO-WHH
PRINT 1121,BA,CA
1121 FORMAT(/10X,28HBA AND CA MEASURED FROM PT.5/10X,3HBA=,F10.5/
110X,3HCA=,F10.5)
KK=800
RECORD(KK) BA,CA
GO TO 50
1110 PRINT 3,TET
3 FORMAT(/10X,4HTET=,F10.5)
KK=700
RECORD(KK) TET
WVV=-FN/(AA*SKYY)
BA=WVV+VO+YO

```

```

CA=-1.
PRINT 1120,BA,CA
1120 FORMAT(/10X,38HN.A. HORIZONTAL, BA MEASURED FROM PT.5/10X,3HBA=,
1F10.5/10X,3HCA=,F10.5)
KK=800
RECORD(KK) BA,CA
GO TO 50
901 CONTINUE
PRINT 3,TET
KK=700
RECORD(KK) TET
WV= TET*(-UO-XO)-FN/(AA*SKYY)
BA=ABSF(WV+VO+YO)
WHH=-FN/(AA*SKXX)-(1./TET)*(VO+YO)
CA=ABSF(WHH+UO+XO)
PRINT 1122,BA,CA
1122 FORMAT(/10X,28HBA AND CA MEASURED FROM PT.8/10X,3HBA=,F10.5/
110X,3HCA=,F10.5)
KK=800
RECORD(KK) BA,CA
GO TO 50
C
50 CALCULATION OF STRESSES IN CONCRETE
STC(1)=SKK+SKYY*(T4-VO-YO)+SKXX*(D2-UO-XO)
STC(2)=SKK+SKYY*(T4-VO-YO)+SKXX*(D4-UO-XO)
STC(3)=SKK+SKYY*(T3-VO-YO)+SKXX*(D4-UO-XO)
STC(4)=SKK+SKYY*(T3-VO-YO)+SKXX*(D5-UO-XO)
STC(5)=SKK+SKYY*(-VO-YO)+SKXX*(D5-UO-XO)
STC(6)=SKK+SKYY*(-VO-YO)+SKXX*(D4-UO-XO)
STC(7)=SKK+SKYY*(-VO-YO)+SKXX*(D2-UO-XO)
STC(8)=SKK+SKYY*(-VO-YO)+SKXX*(-UO-XO)
STC(9)=SKK+SKYY*(T2-VO-YO)+SKXX*(-UO-XO)
STC(10)=SKK+SKYY*(T2-VO-YO)+SKXX*(D2-UO-XO)
PRINT 1968
1968 FORMAT(/1X,30HSTRESSES IN CONCRETE (CRACKED)/)
DO 1969 II=1,10
PRINT 1961,(STC(II),II=1,10)
1969 CONTINUE
1961 FORMAT(1H,5X,F10.3)
DO 1123 I=1,N
1123 STS(I)=RM*(SKK+SKYY*AYY(I)+SKXX*AXX(I))
PRINT 1970
1970 FORMAT(/10X,26HSTRESSES IN BARS (CRACKED)/)
DO 1971 I=1,N
PRINT 1972,(STS(I),I=1,N)
1971 CONTINUE
1972 FORMAT(1H,8F10.3)
RETURN
END

```

```

SUBROUTINE CRAXX(STC,STS,ACC,ACS,ATS,N)
DIMENSION STC(10),STS(50)
CMIN=STC(1)
DO 29 II=2,10
IF(CMIN-STC(II)) 29,29,30
30 CMIN=STC(II)
29 CONTINUE
CMAX=STC(1)
DO 32 II=2,10
IF(CMAX-STC(II)) 31,32,32
31 CMAX=STC(II)
32 CONTINUE
PRINT 33,CMIN,CMAX
33 FORMAT(/ /10X,24HMIN. STRESS IN CONCRETE=,F10.3/10X,24HMAX. STRESS
IN CONCRETE=,F10.3//)
SMIN=STS(1)
DO 36 I=2,N
IF(SMIN-STC(I)) 36,36,37
37 SMIN=STS(I)
36 CONTINUE
SMAX=STS(1)
DO 38 I=2,N
IF(SMAX-STC(I)) 39,38,38
39 SMAX=STS(I)
38 CONTINUE
PRINT 341,SMIN,SMAX
341 FORMAT(/ /10X,20HMIN. STRESS IN BARS=,F10.3/10X,20HMAX. STRESS IN BA
RS=,F10.3//)
IF(ACC-CMIN) 41,41,40
40 PRINT 401
401 FORMAT(/ /1X,35HCHANGE THE SECTION OR REINFORCEMENT)
STOP
41 CONTINUE
IF(ACS-SMIN) 43,43,42
42 PRINT 421
421 FORMAT(/ /1X,50HMAX. COMPRESSIVE STRESS IN STEEL EXCEEDS ALLOWABLE/)
STOP
43 CONTINUE
IF(ATS-SMAX) 45,44,44
45 PRINT 451
451 FORMAT(/ /1X,46HMAX. TENSILE STRESS IN STEEL EXCEEDS ALLOWABLE/)
STOP
44 PRINT 471
471 FORMAT(/ /1X,35HALL STRESSES WITHIN ALLOWABLE RANGE/)
RETURN
END

```

```

SUBROUTINE STINE(N,XO,YO,RM,AS,ASX,ASY,CIXX,CIYY,CIXXYY,AYY,AXX,
1T:IXX,T:IYY,T:IXXYY)
DIMENSION AXX(50),AYY(50),ASX(50),ASY(50),AS(50)
DO 777 I=1,N
777 AXX(I)=ASX(I)-XO
DO 888 I=1,N
888 AYY(I)=ASY(I)-YO
SIXX=0.
DO 2820 I=1,N
2820 SIXX=SIXX+RM*AS(I)*AYY(I)*AYY(I)
SIYY=0.
DO 2821 I=1,N
2821 SIYY=SIYY+RM*AS(I)*AXX(I)*AXX(I)
SIXXYY=0.
DO 2822 I=1,N
2822 SIXXYY=SIXXYY+RM*AS(I)*AXX(I)*AYY(I)
PRINT 500,SIXX,SIYY,SIXXYY
500 FORMAT (/1X,5HSIXX=,F20.5,1X,5HSIYY=,F20.5,1X,7HSIXXYY=,F20.5)
C
TOTAL MOMENT OF INERTIA
T:IXX=C:IXX+SIXX
T:IYY=C:IYY+SIYY
T:IXXYY=C:IXXYY+SIXXYY
PRINT 2823,T:IXX,T:IYY,T:IXXYY
2823 FORMAT (/10X,44HTOTAL MOMENT OF INERTIA WHEN SECTION CRACKED/
115X,7HT:IXX= ,F20.5/15X,7HT:IYY= ,F20.5/15X,7HT:IXXYY=,F20.5)
RETURN
END
```

SUBROUTINE TINFI(C,UO,T1,D5,D3,C1X,C1Y,C7X,C7Y,C6X,C6Y,C3X,C3Y,
C5Y,C8Y,SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY)

C NEUTRAL AXIS VERTICAL

SX=C7X

SY=C7Y

ZX=C7X

ZY=C7Y

PY=C7Y

OY=C1Y

RX=C3X

RY=C3Y

OX=C3X

OY=C3Y

IF(C-D3) 410,410,411

410 PX=C6X

OX=C6X

OY=C6Y

RX=D5-UO-C

OX=D5-UO-C

OY=C5Y

GO TO 1900

411 D10=D3+T1

IF(C-D10) 420,420,423

420 PX=D5-UO-C

OX=D5-UO-C

GO TO 1900

423 D11=D2+D10

IF(C-D11) 430,430,431

430 SX=D5-UO-C

SY=C9Y

ZX=D5-UO-C

PX=C1X

PY=C1Y

OX=C1X

GO TO 1900

431 PRINT 432

432 FORMAT(/1X,18HN.A. FALLS OUTSIDE)

STOP

1900 PRINT 500

500 FORMAT(/7/17X,21HPPOINTS TO LOCATE N.A./)

PRINT 1901,SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY

1901 FORMAT(1H,15X,F10.3,5X,F10.3)

RETURN

END

SUBROUTINE THORI(B,VO,T2,T3,T4,C3X,C3Y,C4X,C9X,C9Y,C10X,SX,SY,ZX,
ZY,PX,PY,QX,QY,RX,RY,QX,QY)

C NEUTRAL AXIS HORIZONTAL

SX=C10X

SY=B-VO

ZX=C9X

ZY=B-VO

PX=C10X

PY=B-VO

DX=C3X

OY=B-VO

RX=C3X

RY=C3Y

QX=C3X

QY=C3Y

IF(B-T3) 660,660,661

660 RX=C4X

RY=B-VO

QY=B-VO

GO TO 1500

661 T60=T2

IF(B-T60) 670,670,671

670 CONTINUE

GO TO 1500

671 T70=T4

IF(B-T70) 680,680,681

680 SX=C9X

SY=C9Y

ZY=C9Y

GO TO 1500

681 PRINT 682

682 FORMAT(/1X,18HN.A. FALLS OUTSIDE)

STOP

1500 PRINT 500

500 FORMAT(/17X,21HPPOINTS TO LOCATE N.A./)

PRINT 1501,SX,SY,ZX,ZY,PX,PY,QX,QY,RX,RY,QX,QY

1501 FURMAT(1H,15X,F10.3,5X,F10.3)

RETURN

END

SUB ROUTINE TPOSI(C,B,UO,VO,TANTET,D1,D2,D3,D4,D5,T1,T2,T3,T4,
1C2X,C2Y,C3X,C3Y,C4X,C4Y,C5X,C5Y,C6X,C6Y,C7X,C7Y,C8X,C8Y,C9X,C9Y,
2C10X,C10Y,C1X,C1Y,SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY)

SX=C7X

SY=(C-(T1+D3))*TANTET-VO

ZX=D5-UO-C

ZY=C7Y

PX=C7X

PY=(C-(T1+D3))*TANTET-VO

DX=C2X

OY=(C-D3)*TANTET-VO

RX=C3X

RY=C3Y

QX=C3X

QY=C3Y

IF(C-D3) 510,510,511

510 BB=B

IF(BB-T3) 520,520,521

520 SY=C7Y

ZX=C7X

PX=C6X

PY=C6Y

OY=C6Y

RX=C4X

RY=C*TANTET-VO

QX=D5-UO-C

QY=C5Y

GO TO 2800

521 SY=C7Y

ZX=C7X

PX=C6X

PY=C6Y

OY=C6Y

RX=D5-UO-C+T3/TANTET

RY=C4Y

QX=D5-UO-C

QY=C5Y

GO TO 2800

511 D20=D3+T1

IF(C-D20) 525,525,526

525 BC=B

IF(BC-T3) 530,530,531

530 SY=C7Y

ZX=C7X

PX=D5-UO-C

PY=C6Y

RX=C4X

RY=C*TANTET-VO

QY=(C-D3)*TANTET-VO

GO TO 2800

531 T20=T3+D3*TANTET

IF(BC-T20) 535,535,536

535 SY=C7Y

ZX=C7X

PX=D5-UO-C

PY=C6Y

RX=D5-UO-C+T3/TANTET

RY=C4Y

QY=(C-D3)*TANTET-VO

GO TO 2800

536 T21=T4+D3*TANTET
IF(BC-T21) 537,537,538

537 SY=C7Y
ZX=C7X
PX=D5-U0-C
PY=C7Y
GO TO 2800

538 SY=C7Y
ZX=C7X
PX=D5-U0-C
PY=C7Y
OX=(D5-U0)-C+T4/TANTET
OY=C2Y
GO TO 2800

526 D21=D2+D20
IF(C-D21) 540,540,541

540 BD=B
IF(BD-T3) 545,545,546

545 RX=C4X
RY=C*TANTET-V0
QX=C6X
QY=(C-D3)*TANTET-V0
GO TO 2800

546 T30=T3+D3*TANTET
IF(BD-T30) 550,550,551

550 RX=D5-U0-C+T3/TANTET
RY=C4Y
QY=(C-D3)*TANTET-V0
GO TO 2800

551 T31=T4+D3*TANTET
IF(BD-T31) 552,552,553

552 CZ=C-T1/TANTET
TD=T1+D3
IF(CZ-TD) 554,554,555

554 CONTINUE
GO TO 2800

555 SX=D5-U0-C+T2/TANTET
SY=C10Y
GO TO 2800

553 T32=T31+T1*TANTET
IF(BD-T32) 560,560,561

561 SX=-(C-T2/TANTET-D5+U0)
SY=C10Y
PY=C1Y
OX=C1X
OY=C1Y
GO TO 2800

560 CZZ=C-T2/TANTET
T=T1+D3
IF(CZZ-T) 565,565,562

565 OX=D5-U0-C+T4/TANTET
OY=C2Y
GO TO 2800

562 SX=D5-U0-C+T2/TANTET
SY=C10Y
OX=D5-U0-C+T4/TANTET
OY=C2Y
GO TO 2800

541 BF=B
IF(BF-T3) 570,570,571

570 SY=(C-(T1+D3))*TANDET-VO

ZX=C8X

ZY=(C-D5)*TANDET-VO

RY=C*TANDET-VO

QY=(C-D3)*TANDET-VO

GO TO 2800

571 T40=T3+D3*TANDET

IF(BF-T40) 572,572,573

572 ZX=C8X

ZY=(C-D5)*TANDET-VO

RX=D5-UO-C+T3/TANDET

QY=(C-D3)*TANDET-VO

GO TO 2800

573 T41=T4+D3*TANDET

IF(BF-T41) 575,575,576

575 CY=C-T2/TANDET

IF(CY-D5) 578,578,579

578 GU=T1+D3

IF(CY-GU) 853,853,854

853 ZX=C8X

ZY=(C-D5)*TANDET-VO

GO TO 2800

854 SX=D5-UO-C+T2/TANDET

SY=C10Y

ZX=C8X

ZY=(C-D5)*TANDET-VO

GO TO 2800

579 SX=C9X

SY=C9Y

ZX=C9X

ZY=C9Y

PY=(C-(T1+D3))*TANDET-VO

GO TO 2800

576 T42=T4+T1*TANDET

IF(BF-T42) 580,580,581

580 CY=C-T2/TANDET

IF(CY-D5) 900,900,910

910 SX=C9X

SY=C9Y

ZX=C9X

ZY=C9Y

DX=-(C-T4/TANDET-D5+UO)

OY=C2Y

GO TO 2800

900 SX=D5-UO-C+T2/TANDET

SY=C10Y

ZX=C8X

ZY=(C-D5)*TANDET-VO

QX=D5-UO-C+T4/TANDET

OY=C2Y

GO TO 2800

581 SX=-(C-T2/TANDET-D5+UO)

SY=C10Y

ZX=C8X

ZY=(C-D5)*TANDET-VO

PY=C1Y

QX=C1X

OY=C1Y

GO TO 2800

2800 PRINT 500

```
500  FORMAT (//17X,21HPPOINTS TO LOCATE .N. A./)
      PRINT 2801,SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,OX,OY
2801  FORMAT (1H ,15X,F10.3,5X,F10.3)
      RETURN
      END
```

SUBROUTINE TNEGA(C,B,TAN,UO,VO,T1,T2,T3,T4,D1,D2,D3,D4,D5,
1C1X,C1Y,C2X,C2Y,C3X,C3Y,C4X,C4Y,C5X,C5Y,C6X,C6Y,C7X,C7Y,C8X,C8Y,
2C9X,C9Y,C10X,C10Y,SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY)

SX=C10X

SY=C10Y

ZX=C10X

ZY=C10Y

PX=C7X

OX=C3X

OY=(C-D4)*TAN-VO

RX=C3X

RY=C3Y

QX=C4X

OY=C6Y

IF(C-D2) 807,807,808

807 CONTINUE

IF(B-T2) 809,809,810

809 SX=C9X

SY=C*TAN-VO

ZX=C-UO

ZY=C8Y

PY=C7Y

OX=C7X

OY=C7Y

RY=C6Y

QX=C6X

GO TO 3333

810 SX=C-T2/TAN-UO

ZX=C-UO

ZY=C8Y

PY=C7Y

OX=C7X

OY=C7Y

RY=C6Y

QX=C6X

GO TO 3333

808 DM=D2+T1

IF(C-DM) 811,811,812

811 CONTINUE

IF(B-T2) 813,813,814

813 SX=C9X

SY=C*TAN-VO

ZY=(C-D2)*TAN-VO

PY=(C-D2)*TAN-VO

OX=C-UO

OY=C6Y

RY=C6Y

QX=C6X

GO TO 3333

814 TM=T2+D2*TAN

IF(B-TM) 815,815,816

815 SX=C-T2/TAN-UO

ZY=(C-D2)*TAN-VO

PY=(C-D2)*TAN-VO

OX=C-UO

OY=C6Y

RY=C6Y

QX=C6X

GO TO 3333

816 TR=T4+D2*TAN

IF(B-TR) 817,817,818

817 PY=(C-D2)*TAN-VO

OX=C-UO

OY=C6Y

RY=C6Y

QX=C6X

GO TO 3333

818 PX=C-T4/TAN-UO

PY=C2Y

OX=C-UO

OY=C6Y

RY=C6Y

QX=C6X

GO TO 3333

812 CONTINUE

IF(C-D5) 819,819,820

819 CONTINUE

IF(B-T2) 821,821,822

821 SX=C9X

SY=C*TAN-VO

ZY=(C-D2)*TAN-VO

PY=(C-D2)*TAN-VO

RY=(C-D4)*TAN-VO

QX=C-UO

GO TO 3333

822 TT=TZ+D2*TAN

IF(B-TI) 823,823,824

823 SX=C-T2/TAN-UO

ZY=(C-D2)*TAN-VO

PY=(C-D2)*TAN-VO

RY=(C-D4)*TAN-VO

QX=C-UO

GO TO 3333

824 TS=T4+D2*TAN

IF(B-TS) 825,825,826

825 CM=C-T3/TAN

IF(CM-D4) 827,827,828

827 PY=(C-D2)*TAN-VO

RY=(C-D4)*TAN-VO

QX=C-UO

GO TO 3333

828 PY=(C-D2)*TAN-VO

RX=C-T3/TAN-UO

QX=C-UO

GO TO 3333

826 TQ=T4+TI*TAN+D2*TAN

IF(B-TQ) 829,829,830

829 CL=C-T3/TAN

IF(CL-D4) 831,831,832

831 PX=C-T4/TAN-UO

PY=C2Y

RY=(C-D4)*TAN-VO

QX=C-UO

GO TO 3333

832 PX=C-T4/TAN-UO

PY=C2Y

RX=C-T3/TAN-UO

QX=C-UO

GO TO 3333

830 PX=C2X

```

PY=C 2Y
OY=C 2Y
RX=C-T3/TAN-UO
OX=C-UO
GO TO 3333
820 CONTINUE
IF(B-T2) 833,833,834
833 CIP=C-T3/TAN
IF(CIP-D4) 835,835,836
835 SX=C 9X
SY=C*TAN-VO
ZY=(C-D2)*TAN-VO
PY=(C-D2)*TAN-VO
RY=(C-D4)*TAN-VO
QY=(C-D5)*TAN-VO
GO TO 3333
836 TIP=C-T3/TAN
IF(TIP-D5) 837,837,838
837 SX=C 9X
SY=C*TAN-VO
ZY=(C-D2)*TAN-VO
PX=C 10X
PY=(C-D2)*TAN-VO
RX=C-T3/TAN-UO
QY=(C-D5)*TAN-VO
GO TO 3333
838 SX=C 9X
SY=C*TAN-VO
ZY=(C-D2)*TAN-VO
PY=(C-D2)*TAN-VO
RX=C 4X
QY=C 4Y
GO TO 3333
834 BP=T2+D2*TAN
IF(B-BP) 839,839,840
839 SH=C-T3/TAN
IF(SH-D4) 841,841,842
841 SX=C-T2/TAN-UO
ZY=(C-D2)*TAN-VO
PY=(C-D2)*TAN-VO
RY=(C-D4)*TAN-VO
QY=(C-D5)*TAN-VO
GO TO 3333
842 SU=C-T3/TAN
IF(SU-D5) 843,843,844
843 SX=C-T2/TAN-UO
ZY=(C-D2)*TAN-VO
PY=(C-D2)*TAN-VO
RX=C-T3/TAN-UO
QY=(C-D5)*TAN-VO
GO TO 3333
844 SX=C-T2/TAN-UO
ZY=(C-D2)*TAN-VO
PY=(C-D2)*TAN-VO
RX=C 4X
QY=C 4Y
GO TO 3333
840 TA=T4+D2*TAN
IF(B-TA) 845,845,846
845 TAM=C-T3/TAN

```

```

      IF (TAM-D4) 847,847,848
847  PY=(C-D2)*TAN-VO
      RY=OY
      QY=(C-D5)*TAN-VO
      GO TO 3333
848  TAME=C-T3/TAN
      IF (TAME-D5) 849,849,850
849  PY=(C-D2)*TAN-VO
      RX=C-T3/TAN-UO
      QY=(C-D5)*TAN-VO
      GO TO 3333
850  PY=(C-D2)*TAN-VO
      RX=C4X
      QY=C4Y
      GO TO 3333
846  TU=T4+D2*TAN+T1*TAN
      IF (B-TU) 851,851,852
851  TUN=C-T3/TAN
      IF (TUN-D5) 853,853,854
853  PX=C-T4/TAN-UO
      PY=C2Y
      RX=C-T3/TAN-UO
      QY=(C-D5)*TAN-VO
      GO TO 3333
854  PX=C-T4/TAN-UO
      PY=C2Y
      RX=C4X
      QY=C4Y
      GO TO 3333
852  PX=C2X
      PY=C2Y
      OY=C2Y
      RX=C-T3/TAN-UO
      QY=(C-D5)*TAN-VO
      GO TO 3333
3333 PRINT 500
500  FORMAT (//17X,21HPPOINTS TO LOCATE N.A./)
      PRINT 3334,SX,SY,ZX,ZY,PX,PY,OX,OY,RX,RY,QX,QY
3334 FORMAT (1H ,15X,F10.3,5X,F10.3)
      RETURN
      END

```

THESIS

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