TIME-VARYING YIELD CURVE DYNAMICS AND INTERACTIONS FOR TURKISH SOVEREIGN BONDS

EMRE SOYKÖK

BOĞAZİÇİ UNIVERSITY

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TIME-VARYING YIELD CURVE DYNAMICS AND INTERACTIONS FOR TURKISH SOVEREIGN BONDS

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DECLARATION OF ORIGINALITY

I, Emre Soykök, certify that

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ABSTRACT

Time-Varying Yield Curve Dynamics and Interactions

for Turkish Sovereign Bonds

Sovereign bond yields are one of the fundamental indicators that investors use as benchmark for other interest rates and draw inferences regarding future situation of the economy. Early interpretation of the relationship between term to maturity and bond yields depended on the expectations hypothesis which proposes that long-term interest rates reflect expected future short-term interest rates except a possible constant liquidity preference premium. However, researchers documented that holding returns on investing long-term bonds over short-term bonds have a time-varying forecastable component suggesting that investors demand a premium as compensation for interest rate risk they bear. A growing body of research aims to understand time-variation in term premium. This study contributes to the topic by identifying determinants of term premium in Turkish treasury yields. First part of the thesis employs a Nelson Siegel class model to generate continuous yield estimates directly from the quotes in bond market. In the second part, the resulting yield curves are used as input to construct a three-factor ACM model which decomposes observed yields into risk-neutral and term premium components. Regression analyses suggest that a combination of measures representing market risk, liquidity risk, credit risk, and behavioral factors can explain the majority of the variation in term premia. Explanation power of the credit risk measure is found to increase while those of liquidity, volatility, and behavioral factors diminish with the maturity horizon.

ÖZET

Türk Devlet Tahvillerinin Zamanla Değişen Getiri Eğrisi Dinamikleri ve Etkileşimleri

Devlet tahvili getirileri, yatırımcıların diğer faiz oranları için gösterge olarak kullandığı ve ekonominin gelecekteki durumuna ilişkin çıkarımlarda bulunduğu temel göstergelerden biridir. Tahvil getirileri ve vadeler arasındaki ilişkinin ilk dönemlerdeki yorumu, uzun vadeli faiz oranlarının olası bir sabit likidite tercihi primi dışında gelecekte beklenen kısa vadeli faiz oranlarını yansıttığını öne süren beklentiler hipotezine dayanmıştır. Bununla birlikte, araştırmacılar, uzun vadeli tahvillerin kısa vadeli tahvillerin üstüne sağladığı getirilerin, yatırımcıların üstlendikleri faiz oranı riskinin telafisi olarak bir prim talep ettiğini gösteren, zamanla değişen tahmin edilebilir bir bileşeni içerdiğini belgelemişlerdir. Bu alanda süregelen araştırmalar vade priminin zaman içinde değişimini anlamayı amaçlamaktadır. Bu çalışma, Türk devlet tahvillerinin getirilerindeki vade primini belirleyen faktörleri tespit ederek konuya katkı sağlamaktadır. Tezin ilk bölümünde, doğrudan tahvil piyasasındaki kotasyonlardan sürekli getiri tahminleri üretmek için Nelson Siegel sınıfı bir model kullanılmıştır. İkinci bölümde ise elde edilen getiri eğrileri, gözlemlenen getirileri riskten bağımsız bir bileşene ve vade primine ayıran üç faktörlü bir ACM modeli oluşturmak için girdi olarak kullanılmıştır. Regresyon analizleri, piyasa riskini, likidite riskini, kredi riskini ve yatırımcı duyarlılığını temsil eden faktörlerin bir kombinasyonunun vade primlerindeki değişimin çoğunu açıklayabildiğini göstermektedir. Kredi riski ölçütlerinin vade primlerini açıklama gücünün vadeler ile birlikte arttığı fakat likidite, piyasa oynaklığı ve yatırımcı duyarlılığı ölçütlerinin açıklama gücünün ise azaldığı tespit edilmiştir.

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I would like to thank Dr. Cenk Karahan from the bottom of my heart for being my advisor and introducing me with the universe of asset pricing. He guided almost every part of my journey through this thesis, from assigning my earliest directed readings to revising my final manuscripts. He showed his support and understanding when I needed them the most. Perhaps most memorably, I gained irreplaceable insight from experiencing his critical thinking style that will be inspiring for me to come up with new research questions in my future studies. I would also like to express my gratitude to Serhat Çevikel for his endeavors to assist me in getting familiar with R software. Moreover, I am indebted to him for helping me to turn raw market quotes into a user-friendly format. I am also grateful to Professor Nesrin Okay for her valuable comments and providing me with academic literature sources on the topic of interest rate modeling. Last but not least, I specially thank the Scientific and Technological Research Council of Turkey (TÜBİTAK) for financially supporting my master's studies.

Dedicated to *Emma*, my cat who did not leave my desk alone a single night while I was writing this thesis.

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ABBREVIATIONS

ACM:	Adrian, Crump, and Moench
BIST:	Borsa Istanbul
CBRT:	Central Bank of the Republic of Turkey
CCI:	Consumer Confidence Index
CDS:	Credit Default Swap
EMBSI:	Emerging Markets Bonds Sentiment Index
JPS:	Joslin, Priebsch, and Singleton
JSZ:	Joslin, Singleton, and Zhu
OLS:	Ordinary Least Squares
VAR:	Vector Autoregression
VIF:	Variance Inflation Factor

CHAPTER 1

CONSTRUCTING TERM STRUCTURE

1.1 Introduction

The cost of borrowing in an economy is determined by policy rates of central banks for short periods. The policy rate for the Central Bank of the Republic of Turkey (CBRT) is the weekly repurchase agreement lending rate. Daily interest rates are further constrained by overnight borrowing and late liquidity lending rates which fluctuate around the policy rate. Firms, households, and governments borrow funds for longer terms to maturity, albeit with variant rates. This variation in interest rates is attributable to the riskiness of borrowers. Since the central government is, to some extent, considered exempt from idiosyncratic risks associated with various financial entities, the interest rates at which it borrows are more closely shaped by investors' expectations about macroeconomic factors. Thus, investors frequently keep track of these interest rates and draw inferences regarding future economic conditions.

The central government borrows from investors in the domestic market by issuing treasury bonds in auctions conducted by the Republic of Turkey Ministry of Treasury and Finance through the agency of the CBRT. A bond is a claim on future cash flows. Cash flows embedded in some treasury bonds are fixed at the time of issuance, whereas those in floating rate bonds are indexed or benchmarked to a currently unknown value such as interest rate to be determined in a bond auction or future inflation with a spread. After the auction takes place, investors may trade a bond in the secondary market until the maturity date when issuer pays par value of the bond to the bondholder. Then, portfolios of treasury bonds with different times to maturity may be formed in various ways by investors to hedge against interest rate risk, speculate on interest rate level, or profit from arbitrage opportunities.

Yields of fixed coupon treasury bonds in relation to their times to maturity form the yield curve which is also called the term structure of interest rates. Floating rate bonds are excluded from the term structure because their hedged cash flow structure makes their prices stay close to their par value and contain less information. As term structure depicts expectations about the future path of the economy, it is also a source of information for investors of markets other than bonds. For instance, term structure properties are known to anticipate future business cycle phases. The slope of term structure was shown to be a predictor of future growth rate of gross domestic product (Ang, Andrew & Piazzesi, 2006) and a better forecaster of growth than expectation surveys (Estrella & Hardouvelis, 1991). Ang, Bekaert, and Wei (2008) demonstrated that the upward slope of term structure in economic boom periods is mainly associated with an expected rise in inflation level. Chauvet and Senyuz (2016) contended that yield curve components have a better forecasting ability of the beginnings and endings of recessions than benchmark factors, with slope factor having the major forecast power.

Yields are calculated from zero-coupon bond prices. Using continuous compounding and annualized yields for simplicity, m-year yield is defined as y_m which satisfies:

$$P_m = e^{(-m y_m)} \tag{1}$$

where P_m denotes the price of a zero-coupon bond with time to maturity m and a par value of 1. Yields cannot be directly calculated from prices of coupon-bearing bonds. Instead, a coupon-bearing bond might be used to extract yield to maturity which is a constant interest rate providing that the bond’s trading price equal to present value of its coupons and par value. Yield to maturity of a bond is shown as y_{ytm} which satisfies:

$$P_{m,c} = \mathrm{e}^{(-m y_{\mathrm{ytm}})} + c \sum_{i=1}^{n} \mathrm{e}^{(-m_i y_{\mathrm{ytm}})} \quad (2)$$

where $P_{m,c}$ denotes the dirty price of a bond with coupon rate c, time to coupon payments m_i, time to maturity m, and a par value of 1. Notice that taking $c = 0$ would equate y_m in Equation 1 to y_{ytm} in Equation 2 for zero-coupon bonds.

Yield to maturity of a bond can be used to derive the Macaulay duration. Macaulay duration of a bond with a par value of 1 is defined as:

$$D_m = \frac{m\mathrm{e}^{(-m y_{\mathrm{ytm}})} + c \sum_{i=1}^{n} m_i \mathrm{e}^{(-m_i y_{\mathrm{ytm}})}}{P_m} \quad (3)$$

Notice that taking $c = 0$ again would equate D_m in Equation 3 to m for zero-coupon bonds.

Figure 1a depicts yields to maturity in relation to times to maturity of all bonds traded on a particular day when term structure had a steep shape. Yields to maturity of 1-year and 2-year coupon-bearing bonds seem to catch those in zero-coupon bonds with shorter times to maturity. On the other hand, Figure 1b displays that yields to maturity of coupon-bearing bonds are aligned with those in zero-coupon bonds having the same duration. This difference is related to the properties of yield to maturity.

Yield to maturity is a weighted average of yields and affected by yields for times to coupon payments that are shorter than time to maturity. Similarly, since the bond duration is a weighted average of times to coupon payments and time to maturity, yields to maturity of bonds with the same duration may be better aligned than those with the same time to maturity. However, the weights of yields which form yield to maturity are determined by the interaction between the shape of yield curve and cash flow structure of bonds. Therefore, one cannot always expect the alignment to hold true between yields and yields to maturity that are derived from zero-coupon bonds and coupon-bonds with the same duration, respectively. Even if there were no bond-specific illiquidity premium, different coupon rates on bonds with the same duration may make their yields to maturity uneven. This is a drawback of using yields to maturity of bonds to deduce market expectations. The issue is only aggravated when we consider that coupon-bearing bonds are issued with times to maturity up to 10 years, however, zero-coupon bonds are at most issued up to 2 years. For these reasons, one ought to use a more rigorous method to model short-term, medium-term, and long-term interest rate levels in the market separately.

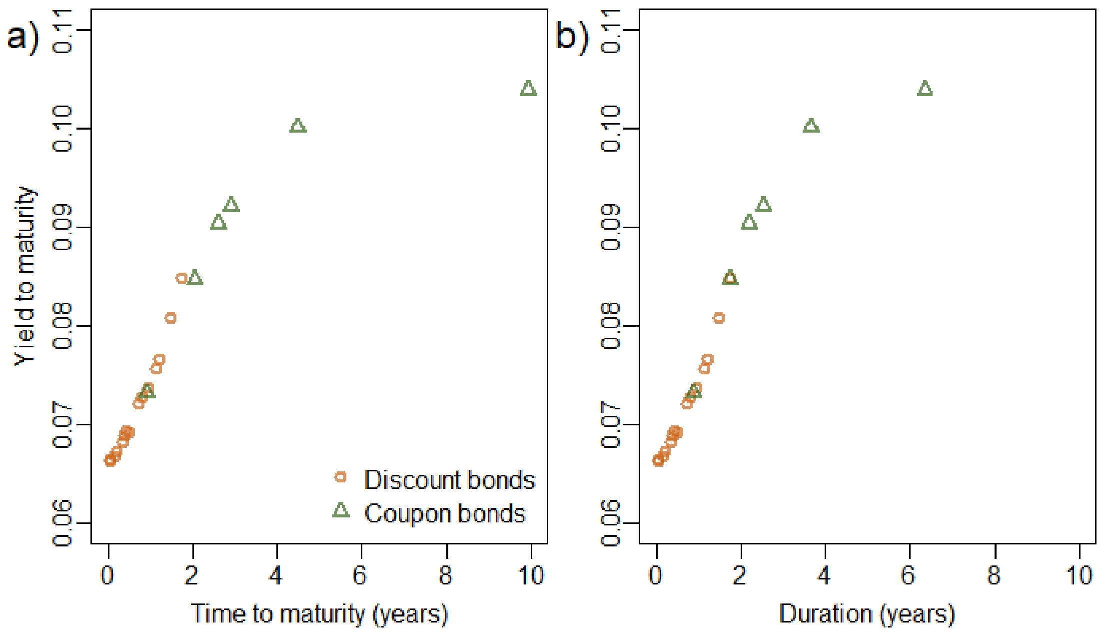

Figure 1. Yields to maturity of treasury bonds traded as of 18 February 2010

Coupon payments are made quarterly or semiannually depending on the bond. Hence, although we do not have bonds for each time to maturity, price of coupon-bearing bonds with long terms to maturity include information on yields that approximately span each moment until 10 years into the future. If we know m_1 and m_2-year yields, y_{m1} and y_{m2}, for times to maturity $m_2 > m_1$, we can derive the interest rate for borrowing m_1 years later for a period of $m_2 - m_1$ years. This interest rate is called the forward rate and defined as follows:

$$F_{m1,m2} = \frac{m_2 y_{m2} - m_1 y_{m1}}{m2 - m1} \quad (4)$$

When m_2 is chosen arbitrarily close to m_1, the resulting limit is the rate of change in y_{m1} for a marginal increase in time to maturity. This interest rate is called the instantaneous forward rate and defined as:

$$f(m_1) = \lim_{m_2 \to m_1} F_{m1,m2} \quad (5)$$

Yields for any time to maturity can be directly calculated as an average of instantaneous forward rates:

$$y(m) = \frac{\int_0^m f(t)dt}{m} \quad (6)$$

Therefore, a specification for instantaneous forward rate implies a term structure model. This chapter is devoted to employing a term structure model.

1.2 Literature review

Intending to model the yield curve as a smooth function, McCulloch (1971) proposed fitting polynomial splines which are connected at time to maturity knots of which two subsequent have an equal number of bonds between. Fisher, Nycha, and Zervos (1994) asserted that effective number of parameters in smoothing splines could be adaptively extracted by employing a roughness penalty instead of exogenously selecting them. They remarked that their method brought about more accurate and less biased estimates compared to that of McCulloch.

Fama and Bliss (1987) used a bootstrap method that iteratively identifies forward rates from discount function that is updated in each step with bonds of longer times to maturity. This method results in a piecewise linear yield curve with number of knots equaling number of bonds used. Bliss (1989) implemented a smoothed version of the model by fitting approximating functions to the yields that exactly price bonds.

Nelson and Siegel (1987) proposed a specification for the instantaneous forward rate which uses Laguerre functions. With the intention of keeping model parameters parsimonious, their suggestion was the identical roots solution of a second order differential equation that can generate various shapes for forward curves depending on two parameters and a constant asymptote equaling to a third parameter. Svensson (1994) argued that an extended version of the Nelson Siegel model might offer a more flexible estimation of yield curve that also fits more accurately. He proposed adding a fourth component with two additional parameters to account for a second hump.

In a comparison of the Nelson Siegel model with several spline-fitting models, Bliss (1997) stated that both unsmoothed Fama Bliss model and the Nelson Siegel model outperform other models. He also claimed that differences in estimates of these two models are impractical and the flexibility of splines is only useful at the long end of yield curve.

Vasicek (1977) posited that instantaneous yield, also called the short rate, follows a mean reverting stochastic process with constant variance. As usual in the no-arbitrage pricing framework, he exploited the possibility of forming a riskless portfolio to show that the resulting partial differential equation has a solution for yields that is linear in model parameters. However, since equilibrium conditions in the economy are pre-specified in Vasicek model, solution for a yield curve represents theoretical interest rates which do not exactly fit observed bond prices. Cox, Ingersoll, and Ross (1985) posited that short rate follows a square root diffusion process, that is variance is linear in short rate. Although this change switches the distribution of yields to chi-square instead of normal, yields are still linear in model parameters. Duffie and Kan (1996) formulized a general framework for affine term structure models which presents earlier works as special cases. Duffie (2002) asserted that while affine models fit observed yield curve shapes well, they do so by giving up ability to represent empirically observed patterns in excess holding period returns on bonds. As a result, he claimed that forecasts depending on affine models underestimate future returns especially when the slope factor is high. He compared the forecasting performance of various multifactor affine models and concluded that three state variables formation comprised of level, slope, and curvature factors together with an excess holding return specification is the only model that beats simple random walk.

Ho and Lee (1986) devised an arbitrage-free term structure model which is based on binomial trees. Since their motivation is to find accurate prices of interest rate derivatives that conform to observed term structure, their model takes the term structure as exogenous which in turn exactly prices bonds. Heath, Jarrow, and Morton (1992) provided a general framework for arbitrage-free term structure models which take observed yield curve as given and formed the basis of market models for pricing interest rate derivatives.

Gurkaynak, Sack, and Wright (2007) used prices of coupon-bearing bonds to model U. S. treasury yield curves over a long observation period starting from 1961. They asserted that both Nelson Siegel and Svensson models successfully capture yield characteristics, despite their parsimonious structures. On the other hand, they suggested yield curve models require a second hump factor to capture a convexity effect that pulls down interest rates on bonds with times to maturity longer than 20 years. Therefore, they used the Nelson Siegel model for yield curves before 1980, when 20-year bonds were introduced, and the Svensson model for later yield curves.

Diebold and Li (2006) propounded that a forecasting approach which uses Nelson Siegel model parameters results in consistent out of sample estimates that outperform benchmark methods. They showed that Nelson Siegel parameters might be construed as level, slope and, curvature factors of the yield curve which could be accurately identified thanks to the composition of factor loadings enforced by the model. While implementing the model, they suggested fixing the λ parameter where hump factor peaks approximately at two and a half years to attain smooth time series of model parameters by ensuring factor loadings would not change day by day. They also made an adjustment to the yield curve parameter setup in order to both foster that parameter estimates indicate meaningful economic explanations and provide that they do not intrinsically include coherence which might create multicollinearity.

De Pooter (2007) attested that forecasts generated by Nelson Siegel class of models outperform those of autoregressive and vector autoregression (*VAR*) models of yields. Koopman, Mallee, and Wel (2010) argued that no-arbitrage models of term structure are unsuitable for estimation purposes since they are based on cross sectional fit, while forecasts generated from affine term structure models do not yield better fits than a random walk. They advocated the use of dynamic Nelson Siegel model in which parameters are treated as latent processes with a state space representation and identified with maximum likelihood estimation.

Christensen, Diebold, and Rudebusch (2011) proposed a three-factor no-arbitrage affine Nelson Siegel model to benefit from theoretical consistency of no-arbitrage models and superior fit of Nelson Siegel model at the same time. They empirically showed that while factor structure imposed by the Nelson Siegel model eases parameter estimation, out of sample predictions might be ameliorated by no-arbitrage condition especially at the long end of yield curve.

Annaert, Claes, Ceuster, and Zhang (2013) contended that methods based on spline fitting to model the term structure result in unappealing characteristics that lack financial intuition. They asserted that the prevalence of the Nelson Siegel class of models among both practitioners and researchers stems from the models' superior performance against their counterparts. Nevertheless, they maintained that the multicollinearity problem in estimating model parameters had not been universally fixed, although the previous applications of the model had solved it case by case basis by fixing the lambda parameter. They suggested that employing a ridge regression framework generates better estimates at the long end of yield curve whenever fixing lambda parameter results in immoderate collinearity between curvature and slope parameters.

Steeley (2014) claimed that the dynamic Nelson Siegel model produces superior out of sample forecasts against benchmark models after-2008 period when interest rates are close to zero. Wahlstrom, Paraschiv, and Schurle (2021) asserted that the most stable and intuitive parameters are generated from the Nelson Siegel model in a comparison with more flexible models. They endorsed using the Nelson Siegel model over the Svensson model due to economically inexplainable interaction between two curvature parameters in the latter, regardless of the shape of yield curve or regime phases such as financial crises.

Alper, Akdemir, and Kazimov (2004) compared the performance of McCulloch model with that of Nelson Siegel model for Turkish treasury bonds which went on the secondary market in June 1991. They estimated monthly yield curves using solely discount bonds traded between 1992 and 2003 and remarked that the Nelson Siegel model outperforms the McCulloch method in terms of out of sample forecasting performance. In a second study (2007), authors forecasted yields from one month to one year ahead using discount bonds. They claimed a generalized autoregressive conditional heteroskedasticity specification of Nelson Siegel parameters is superior to random walk for all periods and terms to maturity. They observed that during financial crises in 1994 and 2001, decreases in the explanatory power of level factor were accompanied by increases in those of slope and curvature factors, keeping total explanatory power of the model unchanged. Thus, they concluded that a three-factor specification is required to capture yield movements.

Akinci, Gurcihan, Gurkaynak, and Ozel (2007) estimated daily Svensson yield curves between February 2005 and October 2006, including 5-year treasury coupon bonds in the analysis. Authors argued that using coupon-bearing bonds would enhance the accuracy of yield curve estimation since they contain information on longer times to maturity. Cepni and Kucuksarac (2017) investigated the performance of different calibration methods for the Svensson model between January 2011 and May 2016. They found that using a price minimization algorithm produces parameters that fit better to in sample data than those generated by a yield minimization algorithm. They also noted that inclusion of repo rates with times to maturity up to three days worsens the fit. Ertan, Karahan, and Temucin (2020) compared the dynamic Nelson Siegel model with a heuristically fixed λ parameter and the Svensson model between February 2005 and December 2018. They maintained that although monthly estimates of both models similarly catch yield curve movements, fitting performance of the dynamic Nelson Siegel model is superior. They attributed underperformance of the Svensson model to a local minima problem that non-linear least squares algorithm suffers.

Hu, Pan, and Wang (2013) proposed that discrepancies between observed yields of bonds and fitted Nelson Siegel yields capture a market-wide liquidity information that correlates with other liquidity proxies but extends them. Since prices of bonds turn out to be insensitive to yield changes at the short end of yield curve, they suggested using root mean square yield errors of bonds with times to maturity longer than 1 year as an illiquidity risk factor. They claimed that exposure to the illiquidity factor consistently explains cross-sectional excess returns of currency carry trades and hedge funds. Driessen, Joost, and Nijman (2018) investigated the effect of a change in illiquidity factor in short-term and long-term bonds separately and avouched that while a segmentation based on liquidity occurs, its impact is economically negligible. In particular, they professed that an increase of one basis point in illiquidity factor of short-term bonds implies on average an increase of 0.75 basis points in illiquidity factor of long-term bonds.

1.3 Data and methodology

I use the Nelson Siegel model to extract yield curves from prices of treasury bonds for three reasons. For one thing, yield curve is represented by a smooth continuous function that extrapolates reasonably well towards the long end of the curve after the maximum time to maturity of bonds of which prices it fits. Secondly, Laguerre functions are sufficiently flexible to approximate various shapes forward curve takes during the period they will be fitted. Third reason is that recent literature averred promising results when Nelson Siegel yield curves are used as input to construct more complex no-arbitrage affine term structure models, which is the subject of Chapter 2.

The postulated specifications for instantaneous forward curve and yield curve bring about equal yields in the Nelson Siegel and Diebold Li models. Since estimated parameters are slightly more intuitive in the latter, I follow the factorization of Diebold and Li (2006). In other words, instantaneous forward rate is represented as:

$$f^{\mathrm{NS}}(m) = \beta_1 + \beta_2 \mathrm{e}^{-\lambda m} + \beta_3 \lambda m \mathrm{e}^{-\lambda m} \quad (7)$$

where β_1, β_2, β_3, and λ are Nelson Siegel parameters with λ being positive. Using Equations 6 and 7, yield curve can be represented as:

$$y^{\mathrm{NS}}(m) = \beta_1 + \beta_2 \left(\frac{1-\mathrm{e}^{-\lambda m}}{\lambda m}\right) + \beta_3 \left(\frac{1-\mathrm{e}^{-\lambda m}}{\lambda m} - \mathrm{e}^{-\lambda m}\right) \quad (8)$$

This formulation allows us to separately account for three attributes of yields. First, β_1 has a constant loading. Thus, it may be interpreted as a long-term component considering that loadings of other components vanish for long terms to maturity. Second, β_2 has an exponential loading that monotonically decreases as time to maturity increases. Hence, it differentiates long-term yields from short-term yields and may be construed as a slope component. Third, β_3 has another exponential loading that monotonically decreases after peaking at a middle term to maturity, generating a hump shape. It is deemed as a curvature component seeing that it differentiates medium-term yields from others. The location of the peak generated by curvature factor is solely determined by λ parameter. For a specified $\hat{\lambda}$ value, the location of this peak can be identified by equating the derivative of curvature loading to zero in Equation 8. For instance, Diebold and Li (2006) fixed λ at 0.7308, which conditions that impact of curvature component on yield curve is paramount at approximately 2.5 years.

Nelson Siegel parameters can be estimated using non-linear least squares optimization. To do so, first a simplification is needed as prices of both discount and coupon-bearing bonds will be used. Since the intention is to estimate yields as a function of time to maturity, before calibrating the model to bond prices, I deconstruct cash flows pertaining to coupon-bearing bonds and represent them as a portfolio of discount bonds. This is because only when coupon payments are assumed to be priced separately can we estimate yields for their terms to maturity. Namely for any bond, one ought to think a vector of cash flows $C = (c_1, c_2, \dots, c_T)$ which have respective times to maturity vector $M = (m_1, m_2, \dots, m_T)$. Needless to say, the last element of cash flow vector is coupon plus par value for a coupon-bearing bond, while cash flow and time to maturity vectors only have one element for discount bonds. Then, from Equation 1, price of a bond is defined as:

$$P_{C,M} = \sum_{i=1}^{T} c_i \mathrm{e}^{(-m_i y_{m_i})} \qquad (9)$$

Similarly, Nelson Siegel price of a bond is defined as:

$$P_{C,M}^{\mathrm{NS}} = \sum_{i=1}^{T} c_i \mathrm{e}^{(-m_i y^{\mathrm{NS}}(m_i))} \qquad (10)$$

Using specifications for observed prices in Equation 9 and theoretical prices in Equation 10, objective function can be established. Nelson Siegel parameters on day t minimize the t-day sum of squared price errors which is defined as:

$$S_t^2 = \sum_{k=1}^{n_t} \left(P_{C,M,k} - P_{C,M,k}^{\mathrm{NS}}\right)^2 \qquad (11)$$

where n_t denotes the number of bonds traded on day t, $P_{C,M,k}$, and $P_{C,M,k}^{\mathrm{NS}}$denote respectively observed and Nelson Siegel prices of bond k traded on day t. Fitting performance of the model on day t can be measured by root mean square error which is defined as:

$$RMSE_t = \sqrt{\frac{S_t^2}{n_t}} \quad (12)$$

Moreover, I define $\hat{\lambda}$ to replace λ in Equation 7 as a constant which minimizes sum of squared errors in the observation period which is defined as:

$$S^2 = \sum_{k=1}^{N_t} \left(P_{C,M,k} - P_{C,M,k}^{\mathrm{NS}}\right)^2 \quad (13)$$

where N_t denotes the number of bond transaction quotes in the observation period.

Recent literature provided evidence which demonstrates fitting errors in the Nelson Siegel model contain valuable information on financial markets. When a shock to bond prices cannot be absorbed by arbitrage capital in the market, fitting errors stay high for a series of days or even months. That is to say, insufficient trade volumes in bond market make room for arbitrage opportunities. Following Hu, Pan, and Wang (2013), I define a market-wide illiquidity measure which tracks episodes of liquidity crises as mean root square yield error of bonds with times to maturity longer than 1-year. Since yields on coupon-bearing bonds are not observed, I define Nelson Siegel yield to maturity of a bond as $y_{\mathrm{ytm}}^{\mathrm{NS}}$ which is a constant interest rate generating its Nelson Siegel price:

$$P_{C,M}^{\mathrm{NS}} = \sum_{i=1}^{T} c_i \mathrm{e}^{(-m_i y_{ytm}^{\mathrm{NS}})} \quad (14)$$

Next, from Equations 2 and 14, market illiquidity on day t is defined as:

$$Illiquidity_t = \sqrt{\frac{\sum_{k=1}^{l_t}\left(y_{ytm,k} - y_{ytm,k}^{NS}\right)^2}{l_t}} \quad (15)$$

where l_t denotes the number of bonds with times to maturity longer than 1-year traded on day t, $y_{ytm,k}$ and $y_{ytm,k}^{\mathrm{NS}}$denote respectively yield to maturity and Nelson Siegel yield to maturity of bond k traded on day t.

I use historical price data of all bonds that were traded on Borsa Istanbul (BIST) for all trading days between 6 January 2005 and 31 March 2021. BIST Data

Platform provides historical data of bonds without differentiating them. For the reasons specified in Chapter 1.1, I discard all bonds that were issued by firms, denominated in a foreign currency, stripped from their coupons or principals, or indexed to currently unknown variables which make their coupon rates floating. As a result, the remaining is the data of all discount and fixed coupon-bearing Turkish treasury bonds. For every trading day in the observation period, it displays weighted average prices, accrued coupons, times to maturity, and time to the next coupon payments. Day count convention is to assume every year is 364 days, therefore semi-annual and quarterly coupons are paid respectively once in every 182 and 91 days. Accordingly, I calculate cash flow and time to maturity matrices for all bonds using these quotes.

An important thing to mention is that 10-year treasury bonds were first issued on 27 January 2010. Thus, the highest time to maturity for a bond was 5 years before this date. Table 1 reports summary statistics after and before the issuance of 10-year bonds. Observation period includes a total of 4085 trading days. Number of bonds traded each day varied between 9 and 32. Table 1a shows that after January 2010, on average five discount bonds were traded on a day while corresponding figure was 17 for coupon-bearing bonds. In fact, no discount bond was traded for 33 days. On the other hand, discount bonds dominated treasury bond market before January 2010. Table 1b shows that on average only two coupon-bearing bonds were traded each day while this figure was 16 for discount bonds. Furthermore, no coupon-bearing bond was traded for 10 days. Introduction of 10-year bonds also changed average times to maturity of bonds traded each day. While mean of this figure was approximately 1 year before January 2010, later it tripled to approximately 3 years.

Meanwhile, corresponding impact was slightly weaker in average durations of bonds traded each day with mean figures jumping from 0.87 years to 2.32 years.

Table 1. Summary Statistics of Treasury Bonds

Daily Observation	Mean	Standard deviation	Minimum	Maximum
Panel a. 2812 trading days between 27.01.2010 and 31.03.2021				
Number of bonds	22.37	3.67	9	32
Number of discount bonds	5.12	3.4	0	15
Number of coupon bonds	17.25	5.78	2	27
Average years to maturity	2.97	0.75	1.05	4.41
Highest years to maturity	9.28	0.73	4.27	10
Average duration	2.32	0.53	0.97	3.38
Highest duration	6.04	0.62	3.45	7.32
Panel b. 1273 trading days between 06.01.2005 and 26.01.2010				
Number of bonds	19.62	2.53	14	26
Number of discount bonds	16.89	3.18	12	23
Number of coupon bonds	2.73	1.07	0	6
Average years to maturity	0.97	0.24	0.53	1.43
Highest years to maturity	3.97	1.05	1.49	5
Average duration	0.87	0.19	0.53	1.28
Highest duration	2.98	0.68	1.49	3.98

This table shows summary statistics for treasury bonds traded between 6 January 2005 and 31 March 2021.

Figure 2 displays daily average and highest years to maturity of traded bonds. It is observed from Figure 2a that after 27 January 2010, the highest daily times to maturity hardly depleted below 8 years due to consecutive issuances of new 10-year bonds. This figure varied between 1.5 and 10 years with a mean of 7.62 years. Figure 2b shows that average daily times to maturity of traded bonds mostly lied between 2 and 4 years after January 2010. Moreover, Figure 2a exhibits that, for some of the trading days, daily highest times to maturity oscillated between 2 and 4 years until 27 January 2010 when 10-year bonds were introduced. That is because for these days, illiquidity in long term bonds pulled daily highest time to maturity down. Long-term yields must be extrapolated after the last available time to maturity for such days. Arbitrage capital is expected to ensure deviations between extrapolated long-term yields stay in line with long-term yields estimated on days when long-term bonds were traded. A measure of deficiency of arbitrage capital in this aspect is given by Equation 15. All in all, it is expected that yields for times to maturity at least up to 2 years should be consistently estimated during the whole observation period, as generally there is vast information on bonds with years to maturity up to this point each day.

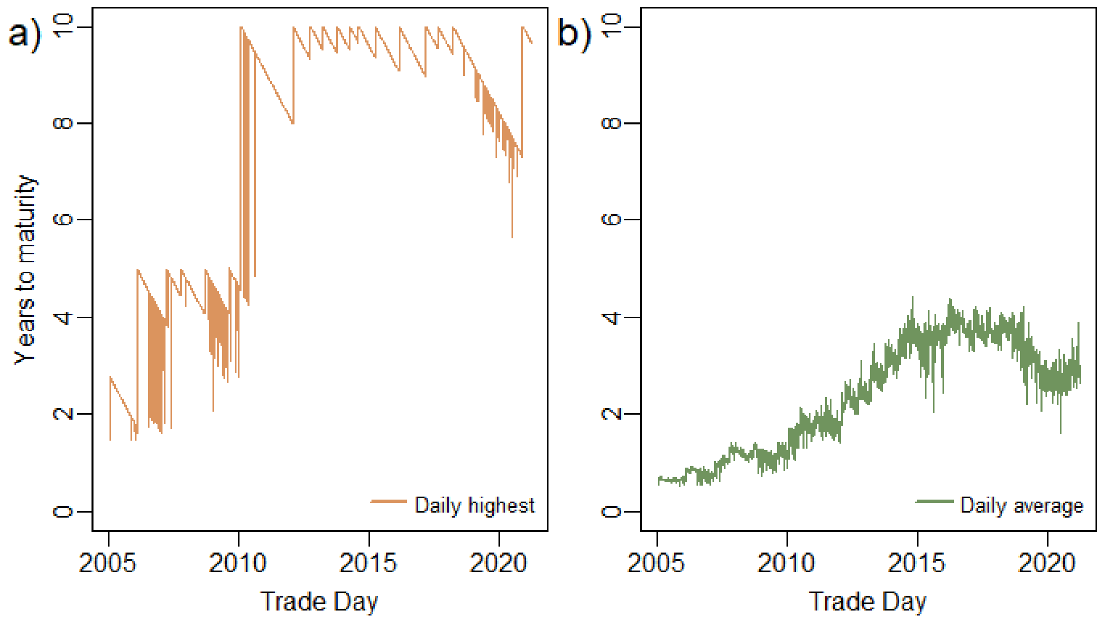

Figure 2. Daily highest and average years to maturity of traded treasury bonds

1.4 Results

Preliminary evaluations of the non-linear optimization algorithm with the objective function defined in Equation 11 show that extending Equation 7 with a fourth β and a second λ parameter to create a second hump in yield curve, such as Svensson (1994) did, results in extreme collinearity between two curvature components. Namely, β_3 and β_4 parameters take high values with opposite signs while λ_2 stays in the neighborhood of λ_1, with the sum of β_3 and β_4 having smooth time series. This unappealing impact of adding a fourth component is best seen in days when number of bonds traded is relatively low or yield curve has a flat shape. Thus, I continue with the specification for three components. Nelson Siegel components do not generate smooth time series with the specification in Equation 7 either. To fix this hurdle in parameter estimation, Diebold and Li (2006) heuristically selected a constant $\hat{\lambda}$ that locates the pinnacle of curvature loading in the middle of 2 and 3 years. Instead, I execute a grid search between 0.5 and 1.5 to estimate $\hat{\lambda}$ that minimizes the objective function in Equation 13. Figure 3 displays the results of this search.

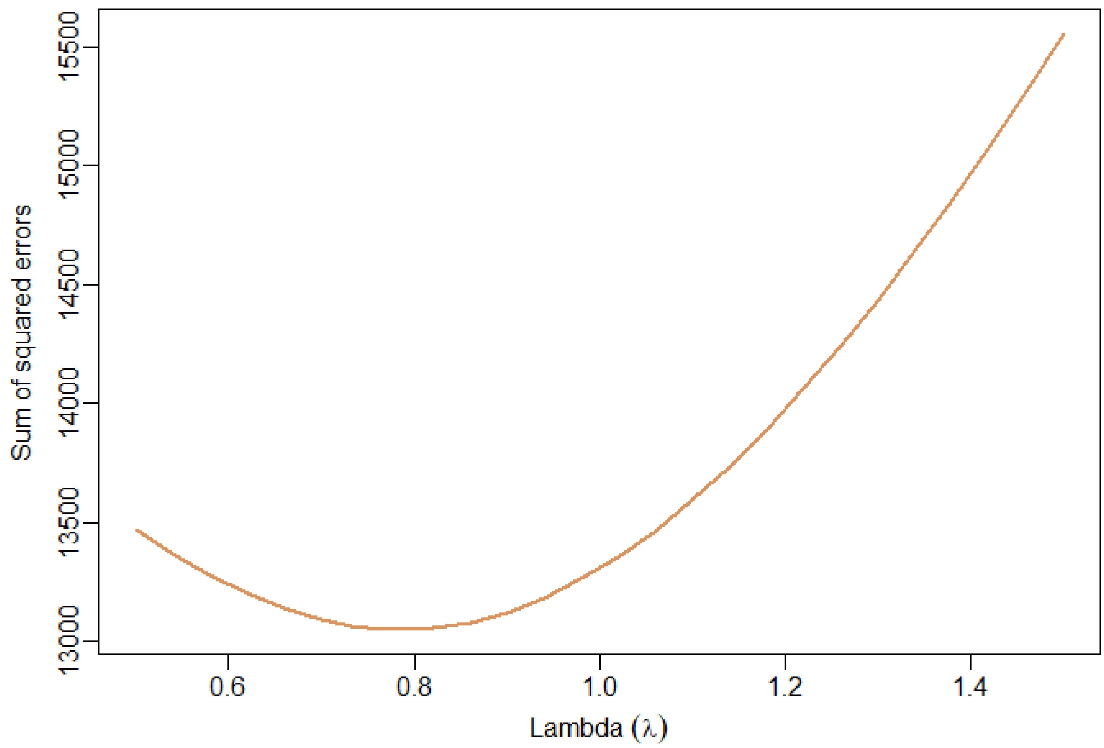

Figure 3. Sum of squared price deviations for different fixed lambda values

Precise up to four digits after decimal, $\hat{\lambda}$ is estimated as 0.7865. This figure conditions loading of curvature component to peak at 2.28 years for yield curves between 6 January 2005 and 31 March 2021. This method requires extra time to optimize curvature loading. However, replacing variable λ in Equation 7 with the constant $\hat{\lambda}$ switches the optimization problem with objective function defined in Equation 11 into a linear one. Doing so immensely saves time for solving 4085 optimization problems, one for each trading day in the observation period. Now, Equation 8 turns into:

$$y_t^{\mathrm{NS}}(m) = \beta_1^t + \beta_2^t \left(\frac{1-\mathrm{e}^{-0.7865m}}{0.7865m}\right) + \beta_3^t \left(\frac{1-\mathrm{e}^{-0.7865m}}{0.7865m} - \mathrm{e}^{-0.7865m}\right) \quad (16)$$

I estimate Nelson Siegel parameters β_1^t, β_2^t, and β_3^t which minimize the objective function in Equation 11, using Equations 9, 10, and 16. Table 2 reports sample statistics of estimated parameters while Figure 4a, 4b, and 4c demonstrate time series of long-term, time-decay, and curvature components respectively.

Table 2. Sample Statistics of Nelson Siegel Parameters

Nelson Siegel parameter	Mean	Standard deviation	Skewness	Kurtosis
Long-term component (β_1)	0.11	0.03	0.59	1.31
Time-decay component (β_2)	0	0.03	0.7	2.27
Curvature component (β_3)	0.03	0.06	2.36	7.34

This table shows summary statistics of estimated Nelson Siegel parameters between 6 January 2005 and 31 March 2021.

It seems from Figure 4 that all three time series became smoother after January 2010. I attribute this change to the introduction of 10-year bonds, since parameters stopped fluctuating as they did when the highest time to maturity of bonds were 5 years, even in the periods when they exhibited jumps. Long-term yield component has an increasing trend between 2005 and 2009 when it ascended to as high as 0.25. Later, it diminished to 0.1, its sample mean, in 2010 and stayed close to this point until 2018. From the summer of 2018, it became more volatile and fluctuated between 0.1 and 0.15 until it surpassed 0.15 in March 2021. As loading of β_2 is the highest in short-term, it is expected that the parameter increases when market experiences liquidity crises. Figure 4b shows that it jumped to extraordinarily high levels in crises of 2007 and 2018. Curvature component β_3 differentiates medium-term yields from other yields. As expected, it peaked in the summer of 2018 when medium-term interest rates rose because of elevated inflation expectations.

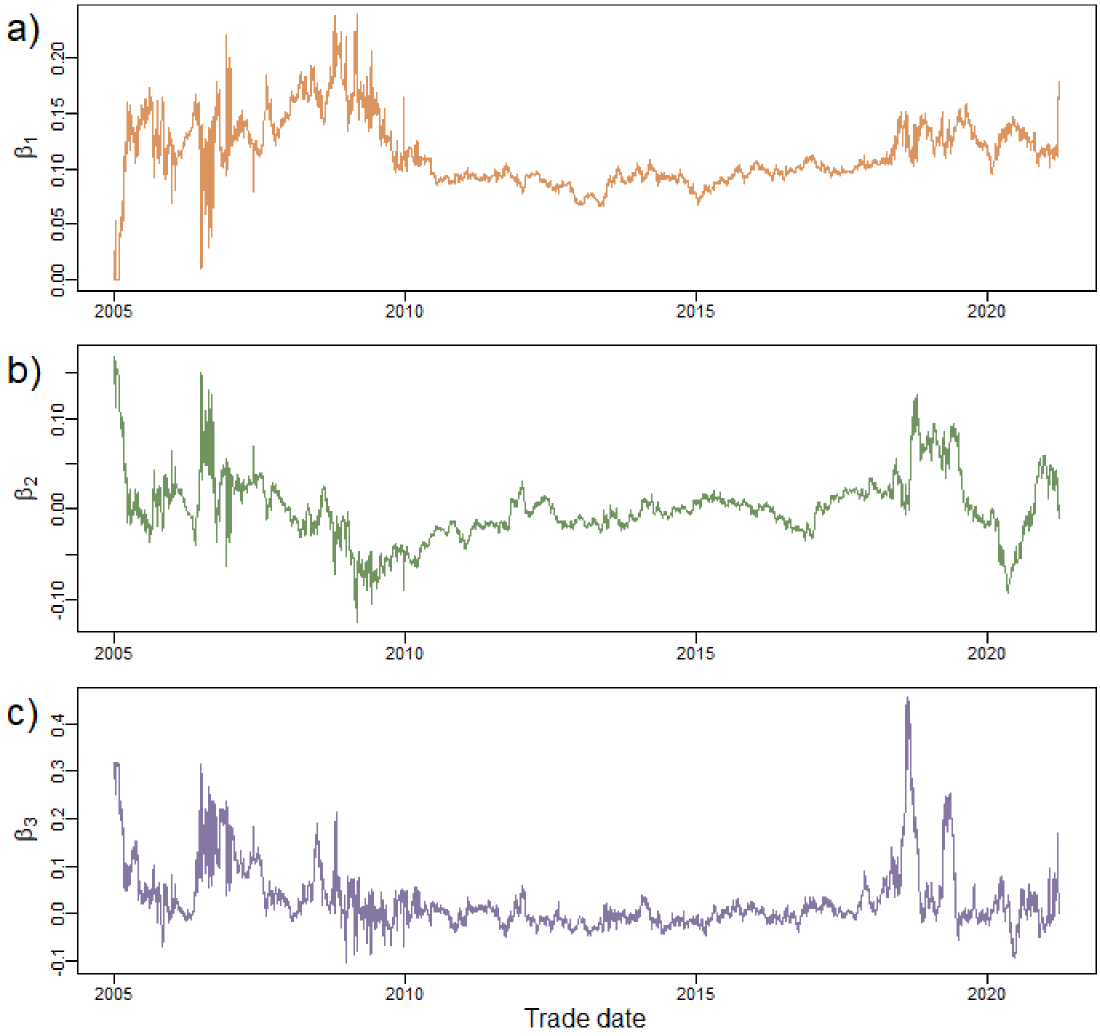

Figure 4. Time series of Nelson Siegel parameters

Table 3 reports summary statistics for selected Nelson Siegel yields. It is observed that both means and standard deviations of yields peak at the middle terms to maturity. Daily observed maximum yields peak at the middle terms to maturity as well, while daily observed minimum yields in the observation period increase with term to maturity.

Table 3. Summary Statistics of Estimated Nelson Siegel Yields

Statistic	Nelson Siegel yields						
	1-month	3-month	6-month	1-year	2-year	5-year	10-year
Mean	0.114	0.116	0.117	0.12	0.121	0.118	0.104
St. dev.	0.04	0.041	0.042	0.043	0.042	0.036	0.023
Skewness	0.561	0.56	0.577	0.627	0.669	0.81	0.989
Kurtosis	-0.507	-0.613	-0.66	-0.584	-0.423	-0.111	0.641
Minimum	0.041	0.041	0.042	0.044	0.048	0.055	0.06
Maximum	0.24	0.242	0.244	0.243	0.259	0.238	0.189

This table shows summary statistics of daily Nelson Siegel yields from 6 January 2005 to 31 March 2021 for times to maturity up to 5 years. Statistics for 10-year yields pertain to observation period starting from 27 January 2010.

Figure 5 displays estimated yield curves for each trading day in the observation period. It is seen that for trading days when the market experienced a financial turmoil such as in 2008 and 2018, the level of yields was high and yield curve had a prominently humped shape.

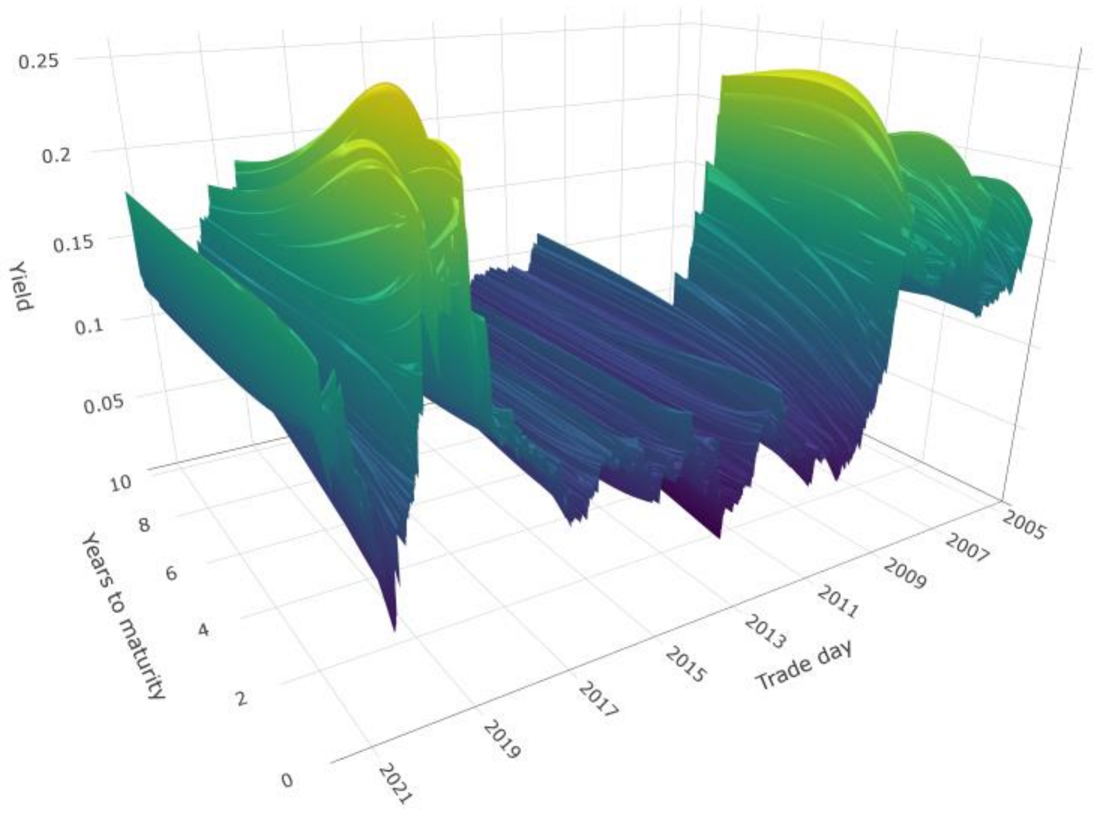

Figure 5. Estimated Nelson Siegel yield surface

Figure 6a displays time series for monthly and annual yields while Figure 6b shows time series of 2 and 5-year yields. Monthly yield can be deemed as short-term interest rate that is approximately explained by a weighted average of β_1 and β_2. It has a smoother series compared to those of individual parameters because it does not fluctuate when parameters move in opposite directions. Time series of annual and 2-year yields show sharp spikes on trading days when β_3 parameter jumped. Because 5-year yield is close to the long end of yield curve, related time series follows the shape of curve in Figure 4a more closely compared to other yields. Figure 2a shows that fluctuations in highest daily time to maturity of bonds correspond to fluctuations in the time series of 5-year yield between 2005 and 2010.

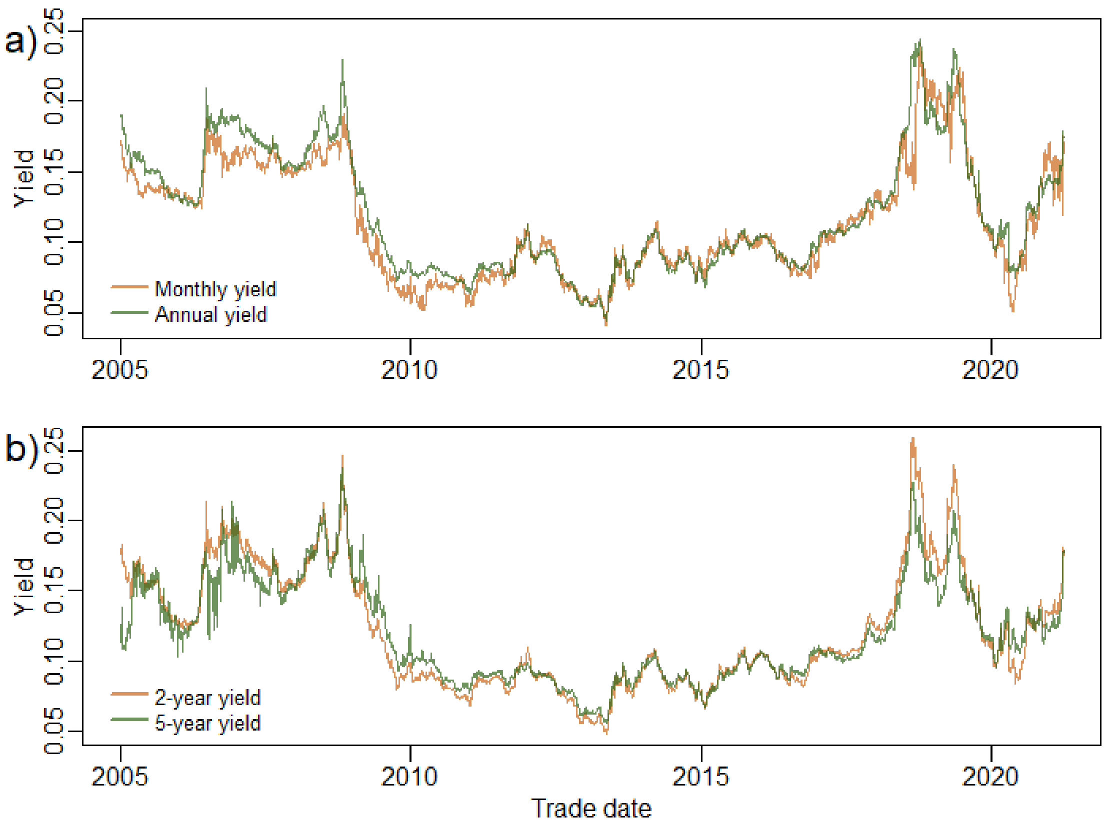

Figure 6. Time series of Nelson Siegel yields

I calculate daily root mean square price deviations between estimated Nelson Siegel prices and observed prices in market as defined in Equation 12 to evaluate fitting performance of the model on individual bonds grouped according to their terms to maturity. Table 4 reports resulting sample statistics. It seems that fitting performance of the Nelson Siegel model deteriorates with increasing time to maturity of bonds. Keeping in mind that each treasury bond has an equal par value of 100, mean *RMSE* of the last group that is composed of bonds with times to maturity at least 5 years is 0.36 that is nearly three times the mean of the first group composed of bonds with times to maturity at most 1 year. Meanwhile, standard deviations increase more drastically with times to maturity of bonds. Sample standard deviation of daily *RMSE* of the last group is more than four times that of the first group. The change in these figures is negligible when sample period is started from January 2010, the date for introduction of 10-year treasury bonds. This observation show that prices of long-term bonds are more volatile and harder to estimate, possibly due to factors such as illiquidity.

Table 4. Summary Statistics for Root Mean Square Bond Pricing Errors

Bonds	Mean	Standard deviation	Minimum	Maximum
Years to maturity up to 1 year	0.13	0.11	0.01	0.99
Years to maturity at least 1 year and up to 2 years	0.21	0.19	0.01	2.8
Years to maturity at least 2 year and up to 5 years	0.27	0.3	0	6.81
Years to maturity at least 5 years	0.36	0.48	0	5.98

This table shows summary statistics for daily root mean square price deviation between estimated and observed prices of Turkish treasury bonds grouped according to times to maturity and traded between 6 January 2005 and 31 March 2021.

Figure 7 demonstrates market-wide illiquidity measure of treasury bonds as defined in Equation 15. Illiquidity jumps to exorbitant levels when a new information brings about rapid changes in bond prices and insufficient trading volumes leave room for arbitrage opportunities. As shown in Figure 7, illiquidity measure periodically stays high for a duration, indicating that liquidity crises continue for a period of time. The highest illiquidity in the observation period was recorded in August 2018. Later, the measure stayed abnormally high until February 2019, when it jumped again carrying the effect until October 2019. The last two weeks of March 2021 also recorded abnormally high illiquidity in consecutive trading days.

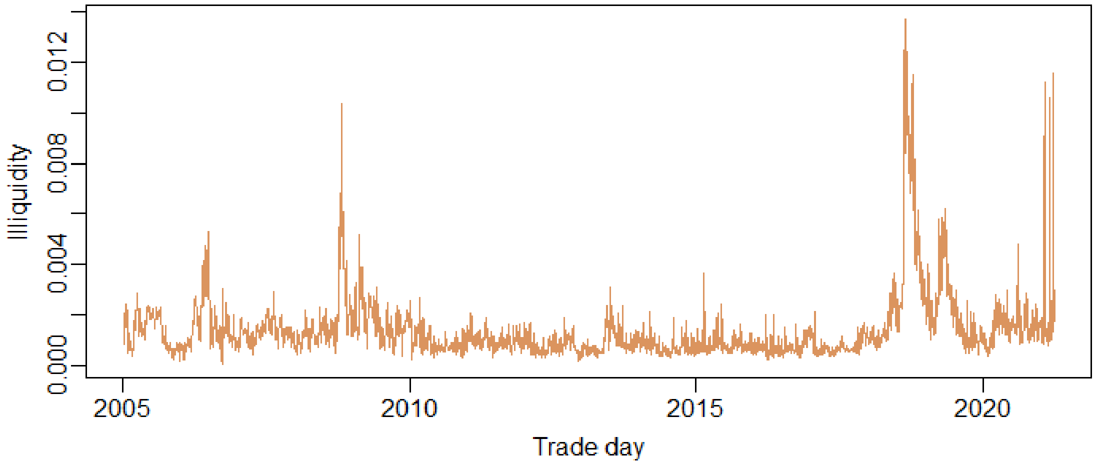

Figure 7. Time series of market-wide illiquidity

Figure 8a displays the fitted yield curve and yields to maturity of individual bonds on 20 August 2020, the day with the highest illiquidity in observation period. It can be observed that individual observed yields do not indicate a reasonable pattern for a smooth function to approximate. For the same day, Figure 8b instead shows yields to maturity of individual bonds according to their durations. Calculated assuming continuous compounding, Macaulay duration is equal to the modified duration which refers to the response of a bond price to a change in its yield to maturity. Therefore, six bonds with yields to maturity between 0.19 and 0.22, each having a duration of 4.5 years, indicate an arbitrage opportunity. That is, selling the bond with the lowest yield to maturity and using the proceedings to buy the bond with the highest yield to maturity would generate a non-negative return that is largely hedged against interest rate risk. On the other hand, low trading volumes limited profiting from existing arbitrage opportunities. Thus, illiquidity measure peaked on this day.

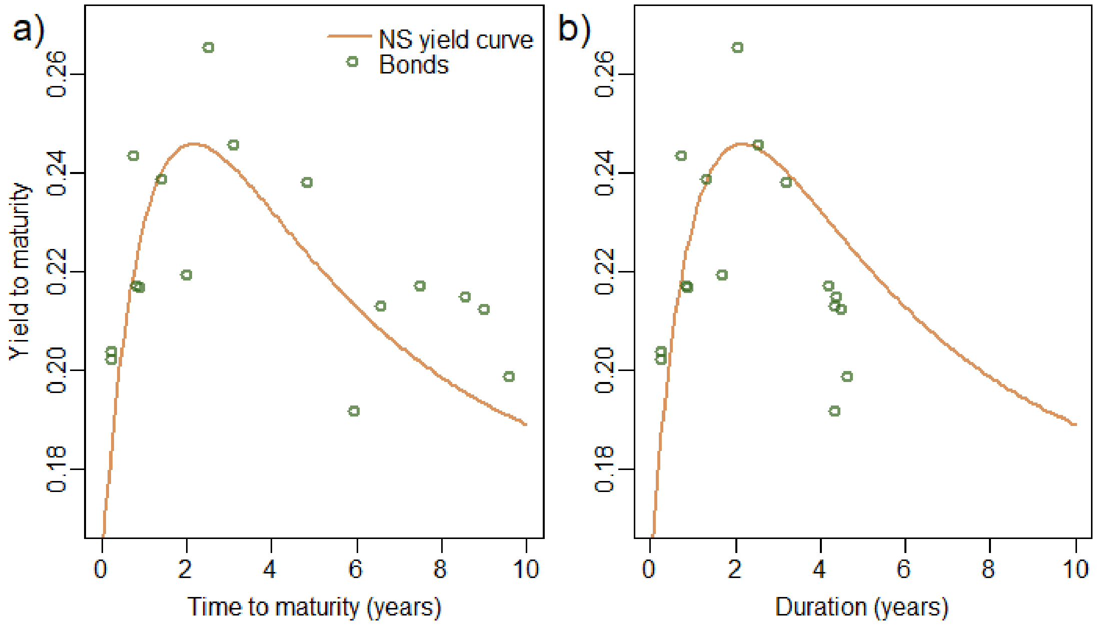

Figure 8. Nelson Siegel yield curve and yields to maturity as of 20 August 2018

1.5 Conclusion

The methodology I used in deriving yield curves for Turkish treasury bonds differs from previous works on the subject in three aspects. Firstly, I executed a grid search to identify λ parameter and found its optimal value, whereas other researchers calibrating the Diebold Li model to bond prices opted for heuristically assigning a value to it. The result indicates that a factor affecting treasury yields culminates at 27 months. Secondly, the observation period of the Turkish bond market in this study is the longest among all yield curve fitting attempts. Thus, sample statistics of estimated yields should be meaningful to describe their characteristics. In contrast with what is usually assumed, average yield curve does not turn out to be upwardsloping. In fact, middle-term debt is the most expensive for treasury, while long-term is the cheapest. Whether observed high yields in the middle of term structure are related to expected life-time of inflation shocks, or perhaps behavioral attitudes of investors towards country-specific risks is a matter of future research. All in all, optimally calibrating the model to rich data grants that loadings for factors affecting yield movements are meaningfully estimated. This holistic identification of factor structures is required for effectively decomposing yields into expectation and risk premium components that is investigated in Chapter 2. Thirdly, fitting errors were solely used to measure performance of different yield curve models. Instead, I estimated a market-wide illiquidity factor, utilizing fitting errors in an enterprising way.

CHAPTER 2

ESTIMATING TERM PREMIUM

2.1 Introduction

Expectations hypothesis for the term structure of interest rates, which is credited to Fisher, Hicks, and Lutz (Wood, 1964), assumes that expected future short rates are identical to instantaneous forward rates. Then, since instantaneous forward rates are derived from bond prices, return on investing in a long-term bond would be equal to expected return on investing the same amount on a short-term bond and rolling over this investment until the maturity of the long-term bond. Despite simplicity of the hypothesis, usual arbitrage arguments cannot show the equivalence of these two investments even though default risk and convexity adjustment are disregarded. That is because at any point in time, possibly until the last rolling period, investment in a long-term bond has greater duration than that in a short-term bond, thus greater exposure to interest rate risk. Risk-averse investors would demand compensation for bearing any such risk. Contrary to the expectations hypothesis, term structure models allowing this compensation to exist differentiate observed yield curves from risk-neutral yield curves by time-varying term premia.

Assume term premium on a short period is negligible for simplicity. Then, term premium on n-period yield at date t is tp_n^t which satisfies:

$$y_n^t = \frac{1}{n} E_t\left(\sum_{i=0}^{n-1} r^{t+i}\right) + tp_n^t \qquad (17)$$

where r^t denotes the risk-free interest rate for a short period and E_t denotes expectation taken according to the information set in t. The first term on the right-hand side accounts for expected return on rolling over one-period discount bonds for the next n periods, with the first term in the summation being only observed rate.

The second term is the interest rate risk premium component in n-period yield which is in excess of the average expected one-period rates. It is beneficial to denote the term premium using excess holding period returns on bonds. To do this, first define one-period holding return on n-period discount bond as the ex-post realized return on buying an n-period discount bond at date t and selling it after a short period as an $n-1$ period discount bond:

$$r_{n-1}^{t+1} = \ln P_{n-1}^{t+1} - \ln P_n^t \tag{18}$$

One-period excess holding return on n-period discount bond is the difference between one-period holding return on n-period discount bond and return on investing in a one-period discount bond:

$$rx_{n-1}^{t+1} = r_{n-1}^{t+1} - r^t \tag{19}$$

Then, using Equations 17 and 19, n-period term premium is defined as the average of expected one-period excess holding returns of declining times to maturity for the next $n-1$ periods:

$$tp_n^t = \frac{1}{n-1} E_t\left(\sum_{i=1}^{n-1} rx_{n-i}^{t+i}\right) \tag{20}$$

For estimation purposes, effective size of samples increases with expressing term premium as excess holding period returns on bonds since short holding periods diminish the hindrance of overlapping observations (Kim & Orphanides, 2007).

It should be noted that instantaneous forward rates derived from bond prices always differ from expected future short rates when investors are assumed to be risk-neutral. Gilles (1996) gave an arbitrage opportunity for the contrary case with two possible states of the world. For a general result, consider that if instantaneous forward rates are equal to expected future short rates, using Equations 1 and 6, price of m-year discount bond is:

$$P_m = \mathrm{e}^{E_t\left(-\int_0^m s(t)dt\right)} \tag{21}$$

where $s(t)$ denotes the short rate t-year later. On the other hand, risk-neutrality of investors conditions prices to be equal to their expected values. Therefore, price of m-year discount bond is:

$$P_m = E_t\left(\mathrm{e}^{-\int_0^m s(t)dt}\right) \tag{22}$$

Since bond price is a convex function of short rates, Jensen's inequality indicates that bond price in Equation 22 is always higher than bond price in Equation 21 under uncertain market conditions. Thus, instantaneous forward rates always underestimate expected future short rates when investors are risk-neutral. When the source of uncertainty in the diffusion of short rates is a Brownian motion with volatility σ_t, Ito's lemma indicates that a convexity adjustment to bond price in Equation 21 equates it to bond price in Equation 22 as:

$$E_t\left(\mathrm{e}^{-\int_0^m s(t)dt}\right) = \mathrm{e}^{E_t\left(-\int_0^m s(t)dt\right)+E_t\left(-\frac{1}{2}\int_0^m \sigma_t^2 dt\right)} \tag{23}$$

The effect of convexity adjustment always increases with the volatility of short rate. Some term structure models include convexity adjustment in term premium whereas others only count pure term premium which stems from risk aversion of investors. As the effect of convexity adjustment is usually small, two definitions generate highly collinear term premia estimates.

2.2 Literature review

Earlier works testing the expectations hypothesis relied on treasury bills due to the lack of an appropriate term structure model which could reconcile irregular issuances of long-term bonds. Fama (1984) argued that yields peak at 8 months instead of monotonically increasing toward 12 months and the term spread, namely the slope of yield curve, can forecast excess holding period returns on bonds. He asserted that these observations are inconsistent with both expectations hypothesis and liquidity preference hypothesis which states that gap between yields and average expected short rates should monotonically increase with time to maturity and stay constant through time.

In one of the first works suggesting long-term yields be composed of expectation and term premium components, Fama and Bliss (1987) regressed the difference between one-year spot rates at year $t + m$ and t on forward-spot spread at t which is the difference between one-year forward rate to borrow at $t + m$ and one-year spot rate at t. They contended that forward-spot spread can forecast the change in one-year spot rate in m years and the prediction power of this forecast increases with time to maturity. Authors also regressed one-year excess holding period returns on bonds with times to maturity up to 5 years on forward-spot spreads and claimed that long-term bond prices have a risk component which vary through time. They attributed the variation to business cycles because expected excess holding period returns are mostly negative during recessions and positive during booms.

Campbell and Shiller (1991) tested the expectations hypothesis by regressing excess holding period returns on term spreads for various times to maturity up to 10 years. They claimed that for almost any two times to maturity, high term spread predicts a declining yield for long-term bond through the life of short-term bond. They argued that this observation indicates a deviation from the expectations hypothesis, possibly resulting from time-varying risk premia in bond yields.

Hardouvelis (1994) argued that widened term spread predicts declining long-term bond yields for most of the Group of Seven countries, but the evidence is found less robust compared to tests for the US bond market. Gurkaynak and Wright (2012) asserted that expectations hypothesis tests generally result in strong rejection for observation periods with high inflation uncertainty or countries where central banks stabilized interest rates. They claimed that affine term structure framework may reasonably explain yield curve anomalies by imposing no-arbitrage conditions and decomposing yields into term premia and expectation components.

Duffie (2002) claimed that affine term structure models produce poor forecasts because model framework is unable to reflect aberrations from expectations hypothesis. In particular, he asserted that the previous applications of affine models underestimated future excess holding period returns on bonds when the slope of yield curve is abnormally high. He remarked that shocks to the slope component emerge from business cycle shifts which are the main factor determining excess bond returns, while shocks to level and curvature components represent permanent changes in interest rates and ephemeral flight-to-safety indicators that are loosely correlated with excess returns. He proposed that forecasting performance can be enhanced when term premium component in yield curve is modeled in a way that it could vary independently of the yield volatility.

Similarly, Dai and Singleton (2002) suggested that affine term structure models could incorporate deviations from the expectation hypothesis if risk factors are parametrized to determine market prices of risk directly. They asserted that a three-factor affine model with state-dependent Gaussian specification for market prices of risk consistently matches risk premium adjusted yield changes while models including state variables which follow square root diffusions are impotent to represent risk premiums since they are constrained to be proportional to factor volatilities.

Cochrane and Piazzesi (2005) claimed that regression of one-year excess holding returns on bonds with times to maturity up to 5 years on a linear combination of forward rates has an explanation power exceeding that of the first three yield curve factors combined. They regressed this combination of forward rates on the first five principal components of yields and noted that the slope and the fourth yield curve factor explain most of the variance. Therefore, they asserted that despite its narrow contribution in yields, fourth principal component may explain excess holding period returns on bonds together with the slope factor of which influence on excess bond returns is well-documented. In another work, Cochrane and Piazzesi (2009) used the return forecasting factor they had defined with the first three principal components of yields as state variables to form a four-factor affine model. They also restricted market prices of risk by setting all prices to zero except contribution of the return forecasting factor and a constant in the price of level risk. On the other hand, Gutierrez, Hevia, and Sola (2020) claimed that this return forecasting factor outperforms first three principal components of yields for only one-year excess holding return regressions, but for longer holding periods three principal components have more than triple explanatory power. Therefore, they concluded that fitting a single return forecasting factor is inconsistent with observations since short and long-term risk determinants are different.

Joslin, Singleton, and Zhu (JSZ, 2011) proposed a model with observed yields as state variables of which parameters can be estimated with ordinary least squares (*OLS*) regression while other model parameters require maximum likelihood estimation. They also contended that imposing the no-arbitrage condition does not alter out of sample yield forecasts in Gaussian affine term structure models. They asserted that forecasting performance improvement in these models must emerge from constraints on the distribution of pricing factors such as number of factors determining risk premia and link between risk-neutral and historical drift of factors. Similarly, Joslin and Le (2021) contended that no-arbitrage condition in affine term structure models affects yield forecasts and volatility dynamics only when interest rate volatility is assumed to be stochastic.

In addition to yield curve factors, several studies included macroeconomic factors as state variables in Gaussian affine term structure models. Ang and Piazzesi (2003) attributed a significant portion of the explanatory power of inflation and economic growth factors in yields to their correlations with level and slope components. In compliance with general equilibrium asset pricing perspective, Wright (2011) argued that a positive term premium requires that bond returns be negatively correlated with marginal utility of investors. He asserted that if inflation is negatively correlated with consumption growth, bond prices should decrease when marginal utility is high, producing a positive term premium. Then, a decrease in term premium may stem from a corresponding decrease in inflation uncertainty due to the reduced covariance between inflation and consumption growth. He regressed term premia estimates, which are both calculated from interest rate expectation surveys and generated from an affine model, on inflation uncertainty measures for 10 developed countries and reported a strong positive relationship explaining a substantial part of term premia. Joslin, Priebsch, and Singleton (JPS, 2014) used a Gaussian affine term structure model which includes macroeconomic factors that are unspanned, namely factors which are not perfectly correlated with yield curve factors, to estimate their predictive power on excess bond returns. They claimed that inflation risk has a crucial impact on term premium despite the fact that a large proportion of its variation is already spanned by bond prices.

De Graeve, Emiris, and Wouters (2009) asserted that flexibility in reduced form affine term structure models may bring about misspecification due to the inadequate description of macroeconomy. They proposed a more structured dynamic stochastic equilibrium model to estimate term premium and future interest rate expectations of investors dependent on eight exogenous processes, each describing a macroeconomic factor such as aggregate demand or government spending. Authors claimed that resulting interest rate expectation component can explain approximately 90% of fluctuations in yields, indicating that the need for a time-varying risk premium may be lower than expected. Dewachter, Iania, and Lyrio (2011) utilized a dynamic general equilibrium model to decompose yield curve into expectation and term premium components. They assumed investors have imperfect information regarding long-term inflation target of the central bank and explicitly formed a constant gain adaptive learning model of which dynamics are estimated by a Bayesian framework. They remarked that resulting estimates of future interest rate expectations of investors explain more of the variance in long-term yields compared to other models, however, they reported evidence to reject expectations hypothesis. Implementing a similar structural macroeconomic model, Crump, Eusepi, and Moench (2016) also noted that incorporating the time-varying term premium is a necessary condition to explain the majority of variation in middle and long-term yields.

Adrian, Crump, and Moench (ACM, 2013) proposed a Gaussian affine term structure model which uses principal components of yields as state variables to price interest rate risk using computationally fast three-step linear regressions to estimate all parameters. They allowed each factor to contribute to market prices of risk, thus adapting to various risk specifications that the shape of yield curve may imply. Risk-neutral coefficients in the Gaussian *VAR* specification of factors are found by taking one-month yields as risk-free rates, while risk prices are identified by regressing monthly excess holding returns on long-term bonds on lagged factors and contemporaneous factor innovations. Their model also adjusts yields derived from bond prices for convexity, and therefore estimates pure term premium. Authors asserted that their model has superior forecasting performance compared to four-factor model of Cochrane and Piazzesi (2008), while two models are equally successful in explaining in sample term premia dynamics. Authors also argued that the first five yield curve factors specification of their model has better fit to the cross section of yields than macro-finance specification of the JPS model with inflation factor, while the latter significantly contributes to the price of slope risk, indicating the impact of an unspanned factor.

Kim and Orphanides (2012) noted that since bond yields have a persistent time series, identification of market prices of risk dynamics is obstructed by a small sample problem. They proposed incorporating forecasting surveys as an additional input to affine term structure models and claimed that this would result in a more reasonable and stable estimation of long-horizon term structure dynamics. On the other hand, Li, Meldrum, and Rodriguez (2017) avouched that while using surveys may produce estimates which are robust to changing the observation period, these estimates may not be better in practice. For example, they reported that regressing term premium estimates generated from the ACM model on unemployment gap suggests that term premium be countercyclical as expected. However, they noted that term premium estimated by affine models using survey forecasts as input has no significant relationship with the unemployment gap.

Aydin and Ozel (2019) estimated monthly time series of term premia in Turkish treasury yields between May 2005 and February 2019 by using both ACM and JSZ models. They asserted that both models generate similar term premium estimates with a correlation coefficient as high as 0.99 and fit the yield curve well. They also claimed that future interest rate estimates derived from affine models trace market conditions more intimately than expectation surveys which tend to stay flat and react to changing conditions after a time lag. They remarked the countercyclical behavior of term premium and attributed a significant portion of its variance to foreign holdings in treasury bond market. Adopting the ACM framework, Ozbek and Talasli (2020) estimated monthly time series of term premia in treasury yields between January 2010 and October 2018 for 16 emerging markets including Turkey. They asserted that market-wide liquidity and foreign exchange volatility are significant determinants of term premia while inflation and economic surprise indices are positively correlated with it. However, authors did not share information about portion of variance explained by these factors.

2.3 Data and methodology

I use ACM model to estimate term premium in Turkish treasury yield curve because the model only requires using Nelson Siegel yields as input. However, using only yield curve factors does not restrict model's capability to generate economic interpretation since estimated term premium can then be regressed on macroeconomic factors (Wright, 2011), which is the aim of this chapter.

Under Gaussian affine term structure framework, series of assumptions are made to ensure that yields are affine functions of state variables as in Duffie (2002). Dynamics of K state variables are assumed to follow a $VAR(1)$ model under both physical and risk-neutral probability measures with normally distributed shocks and common variance covariance matrix Σ. Thus, physical evolution of state variables is given by:

$$X_{t+1} = \mu + \phi X_t + v_{t+1} \tag{24}$$

Similarly, risk-neutral evolution is given by:

$$X_{t+1} = \tilde{\mu} + \tilde{\phi} X_t + \tilde{v}_{t+1} \tag{25}$$

where μ, $\tilde{\mu}$, v, $\tilde{v}$, and X_t are $K \times 1$ matrices while ϕ and $\tilde{\phi}$ are $K \times K$ matrices. Therefore, time-varying market prices of risk are also linear in state variables:

$$\lambda_t = \Sigma^{-\frac{1}{2}}(\lambda_0 + \lambda_1 X_t) \tag{26}$$

where transition between physical and risk-neutral measures are defined by parameters $\lambda_0 = \Sigma^{-\frac{1}{2}}(\mu - \tilde{\mu})$ and $\lambda_1 = \Sigma^{-\frac{1}{2}}(\phi - \tilde{\phi})$. Risk-free interest rate is assumed to be linear in state variables:

$$r_t = \delta_0 + \delta_1' X_t \tag{27}$$

where δ_0 is a constant and δ_1 is a $K \times 1$ matrix.

Imposing no-arbitrage condition indicates the existence of a stochastic discount factor M which consistently prices bonds:

$$P_n^t = E_t(M_{t+1} P_{n-1}^{t+1}) \tag{28}$$

Equations 24 and 27 imply that stochastic discount factor is exponentially affine and lognormally distributed:

$$M_{t+1} = \mathrm{e}^{-r_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\Sigma^{-\frac{1}{2}} v_{t+1}} \tag{29}$$

Assuming excess holding period returns rx_{n-1}^{t+1} and shocks to the state variables v_{t+1} are jointly normally distributed and using Equations 19, 26, 28, and 29, Adrian et al. (2013) found that unexpected excess holding period return can be decomposed into a component correlated with v_{t+1} and another that is orthogonal:

$$rx_{n-1}^{t+1} - E_t(rx_{n-1}^{t+1}) = \beta_{n-1}' v_{t+1} + e_{n-1}^{t+1} \tag{30}$$

where $\beta_{n-1}' = Cov_t(rx_{n-1}^{t+1}, v_{t+1}')\Sigma^{-1}$ is a $1 \times K$ matrix and e_{n-1}^{t+1} denotes the return pricing error with variance σ^2. Then, excess holding period returns can be decomposed into four components:

$$rx_{n-1}^{t+1} = \beta_{n-1}'(\lambda_0 + \lambda_1 X_t) - \frac{1}{2}\left(\beta_{n-1}'\Sigma\beta_{n-1} + \sigma^2\right) + \beta_{n-1}' v_{t+1} + e_{n-1}^{t+1} \tag{31}$$

where the first and second components stand for expected return and convexity adjustment while the third and fourth components represent priced return innovation and return pricing error, respectively. When times to maturity and dates are piled into matrices, Equation 31 turns into:

$$rx = \beta'(\lambda_0 \iota_T' + \lambda_1 X_L) - \frac{1}{2}(B^* vec(\Sigma) + \sigma^2 \iota_N)\iota_T' + \beta' V + E \tag{32}$$

where T denotes the number of trading days in the observation period, N denotes the number of bonds with different times to maturity of which returns are observed every trading day, rx is a $N \times T$ matrix denoting excess holding period returns, β is a $K \times N$ matrix denoting factor loadings, $B^* = \left[vec\left(\beta_1\beta_1'\right) \ldots vec(\beta_N\beta_N')\right]'$ is a $N \times K^2$ matrix, ι_T and ι_N are $1 \times T$ and $1 \times N$ all-ones matrices, X_L is a $K \times T$ matrix denoting one-period lagged state variables, V is a $K \times T$ matrix denoting *VAR* residuals, and E is a $N \times T$ matrix denoting return pricing errors.

Adrian et al. (2013) used a three-step regression approach to extract market prices of risk from observed excess holding period returns decomposed as in Equation 32. First, Equation 24 is estimated by *OLS* to break down state variables into a predictable component and an innovation component. An estimator of variance covariance matrix is obtained by the resulting innovation matrix as: $\hat{\Sigma} = \hat{V}\hat{V}/T$. Secondly, excess holding period returns are stated as the linear combination of a constant, innovation matrix, and one-period lagged state variables:

$$rx = \alpha \iota_T' + \beta' \hat{V} + cX_L + E \tag{33}$$

where $\alpha = \beta' \lambda_0 - \frac{1}{2}(B^* vec(\Sigma) + \sigma^2 \iota_N)$ and $c = \beta' \lambda_1$ from Equation 32. Equation 33 is estimated using *OLS,* and the resulting mean squared error estimates the variance: $\hat{\sigma}^2 = tr(\hat{E}\hat{E}')/NT$. Then, estimators for market prices of risk are given as:

$$\widehat{\lambda_0} = \left(\hat{\beta}\hat{\beta}'\right)^{-1} \hat{\beta} \left(\hat{\alpha} + \frac{1}{2}\left(\hat{B}^* vec\left(\hat{\Sigma}\right) + \hat{\sigma}^2 \iota_N\right)\right) \tag{34}$$

and

$$\widehat{\lambda_1} = \left(\hat{\beta}\hat{\beta}'\right)^{-1} \hat{\beta}\hat{c} \tag{35}$$

Using estimated parameters, log bond prices can be expressed as affine functions of state variables:

$$\ln P_n^t = A_n + B_n' X_t + u_n^t \tag{36}$$

where u_n^t is an error term. Yields are found by annualizing the negative of right-hand side in Equation 36. A_1 and B_1 are equal to $-\delta_0$ and $-\delta_1$ in Equation 27 and can be estimated by regressing one-period interest rates on state variables. Using Equations 19, 31, and 36, Adrian et al. (2013) showed that long-term yields can be recursively calculated from difference equations:

$$A_n = A_{n-1} + B_{n-1}'(\mu - \lambda_0) + \frac{1}{2}(B_{n-1}' \Sigma B_{n-1} + \sigma^2) - \delta_0 \tag{37}$$

and

$$B_n' = B_{n-1}'(\phi - \lambda_1) - \delta_1' \tag{38}$$

Yields derived from Equations 36, 37, and 38 satisfy no-arbitrage conditions. Convexity-adjusted expected average future short rate component in yields can be found by equating market prices of risk to zero, accordingly, the resulting interest rates denote the risk-neutral yield curve. The remaining component in yields denotes the compensation that investors demand for bearing interest rate risk, thus term premium.

Affine term structure models do not restrain number of state variables to be used for estimating model parameters. State variables also do not have to be yield curve components. For instance, one may select Nelson Siegel parameters directly or macroeconomic variables such as inflation and output growth. I use principal components of daily Nelson Siegel yields which are derived from the eigenvalue decomposition of the covariance matrix of yields as state variables since they are more commonly used than Nelson Siegel parameters. I also standardize state variables before using them as daily pricing factors so that interpretation of model implied factor loadings would be more intuitive. First principal component explains 95.5% of the total variance in yields while corresponding figures are 4.1% and 0.4% for second and third principal components, respectively. I use the first three principal components since they combined explain almost all of the variation in yields. Table 5 reports descriptive statistics of pricing factors. Slope component is negatively skewed, indicating that the observation period includes some trading days with extreme negative slope values. Figure 9 displays daily pricing factors together with their yield loadings. Time series of the level component seems smoother than those of other components.

Table 5. Summary Statistics of Pricing Factors

Statistic	Pricing Factors		
	Level (X_1)	Slope (X_2)	Curvature (X_3)
Mean	0	0	0
Standard deviation	1	1	1
Skewness	0.58	-0.53	0.84
Kurtosis	-0.62	4.64	6.00
Minimum	-1.82	-5.68	-4.07
Maximum	3.15	4.61	7.27
Portion of variance explained	0.955	0.041	0.004

This table shows summary statistics of daily pricing factors for 4085 trading days between 6 January 2005 and 31 March 2021.

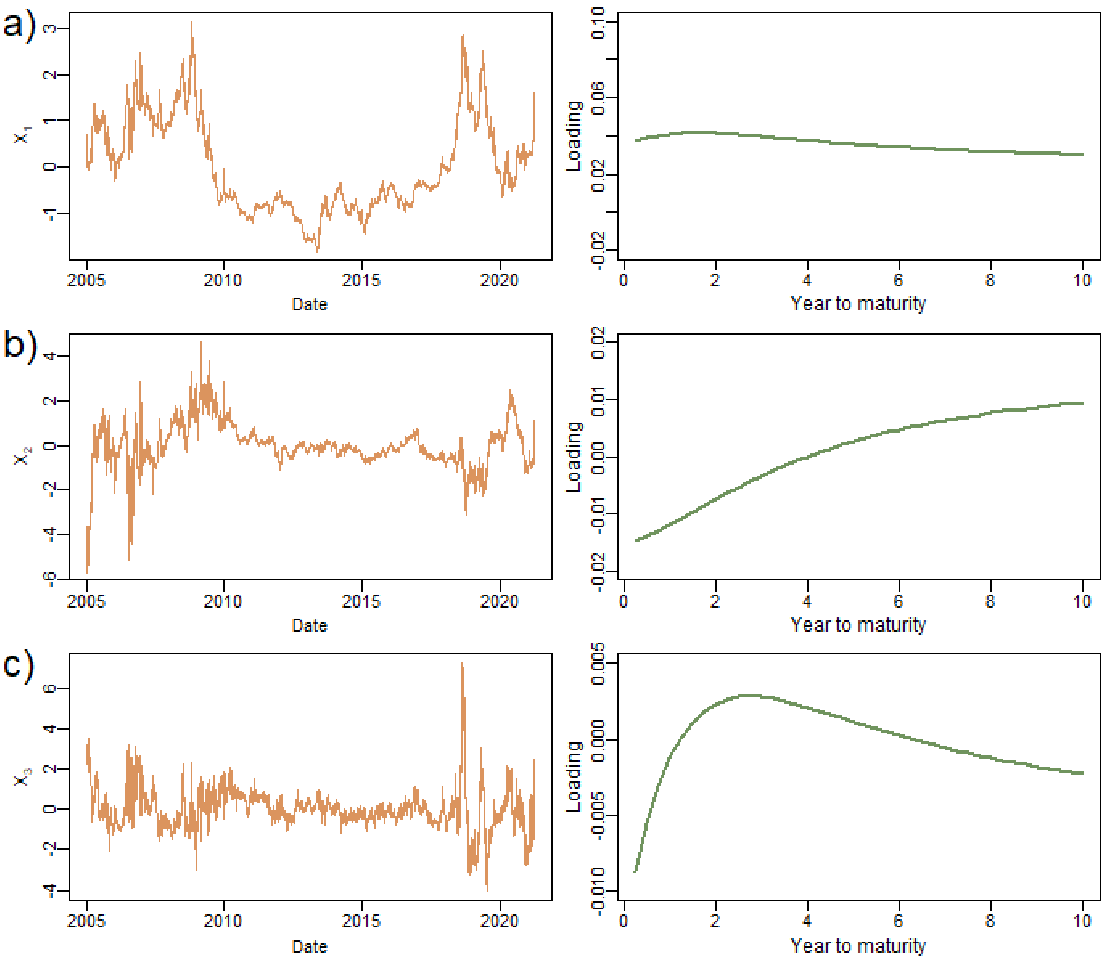

Figure 9. Time series of pricing factors and yield loadings

Following the baseline specification in the ACM model, I take one-month yield as the risk-free rate and calculate excess returns for $n = 6, 12, \ldots, 60, 84, 120$ month discount bonds, making a total of 12 times to maturity. However, I do not use individual bonds since their limited numbers put a limit on available times to maturity. As model parameters are estimated by regressions at monthly frequency, Adrian et al. (2013) suggested aggregating yields either by selecting end of month values or calculating monthly averages. I use monthly averages since the resulting model implied yields fit better to input yields. Table 6 reports sample statistics for one-month excess holding returns calculated using monthly average yield curves.

First, I estimate model parameters using monthly observations. Then, I impute term premia at daily frequency using daily pricing factors.

Table 6. Sample Statistics for One-Month Excess Holding Returns

Time to maturity (n)	Statistics for excess holding period returns (rx_{n-1})			
	Mean(%)	St. dev.(%)	Minimum(%)	Maximum(%)
6 months	0.05	0.4	-1.4	1.4
12 months	0.08	0.9	-3.7	3.8
18 months	0.09	1.5	-5.9	6.2
24 months	0.08	2.1	-8.5	8.4
30 months	0.07	2.7	-11.1	10.1
36 months	0.04	3.2	-13.2	11.3
42 months	0.02	3.6	-14.9	13.0
48 months	-0.01	4.1	-16.2	14.6
54 months	-0.04	4.5	-17.2	16.1
60 months	-0.07	5.0	-18.7	17.5
84 months	-0.21	4.76	-18.81	12.94
120 months	-0.26	5.89	-19.75	12.77

This table shows summary statistics for one-month excess holding returns calculated from monthly average yield curves pertaining to observation period of 194 months between February 2005 and March 2021 for times to maturity up to five years and 134 months between February 2010 and March 2021 for longer terms to maturity.

2.4 Results

Firstly, implementation of the ACM model and the resulting term premia estimates are presented. Then, estimated term premia are regressed on a variety of exogenous factors in order to identify their determinants. I focus on term premium in 2-year yield since this time to maturity point is used as a benchmark for the interest rate market. Factors are selected to represent market risk, liquidity risk, credit risk, and behavioral factors.

2.4.1 Implementation of the ACM model

I estimate the *VAR*(1) model in Equation 24 for monthly averages of pricing factors. Table 7 reports the resulting coefficient estimates. It should be seen from calculating autocorrelations that a shock to level component persists through 24 months, while those to slope and curvature components decay after 12 months and 2 months, respectively. Since level component in yields is particularly persistent, a linear combination of one-month lagged state variables explains more than 93% of the variation in its level. Both its own and slope component's lagged values significantly contribute to this combination. Though approximately 75% of the variation in slope component is explained by one-month lagged state variables, its own lagged value is the only significant one. Lastly, one-month lagged slope and curvature values significantly predict contemporary curvature component. *VAR* coefficients, residuals, and variance covariance matrix are obtained from results.

Table 7. *VAR*(1) Model of Monthly State Variables

Regressor	State Variables		
	Level (X_1)	Slope (X_2)	Curvature (X_3)
Intercept	0.004	0.025	-0.016
	(0.198)	(0.768)	(-0.344)
Level ($X_{1,L}$)	0.968 ***	-0.003	-0.066
	(53.053)	(-0.107)	(-1.418)
Slope ($X_{2,L}$)	-0.044 **	0.795 ***	0.148 ***
	(-2.357)	(23.624)	(3.106)
Curvature ($X_{3,L}$)	-0.008	-0.035	0.654 ***
	(-0.404)	(-0.977)	(13.021)
Observations	194	194	194
R squared	0.937	0.746	0.491

This table shows coefficient estimates for regressions of monthly state variables on their one-month lagged values. Robust t-statistics are shown in parentheses. *** and ** indicate significance levels of 0.01 and 0.05, respectively.

I estimate the coefficients in Equation 33 by regressing one-month excess holding returns calculated from monthly average yield curves as in Equation 19 on one-month lagged state variables and contemporaneous *VAR* residuals. Results are reported in Table 8. An increase in level factor is associated with a corresponding increase in expected one-month holding period returns and the related coefficient is increasing with time to maturity of the bond. An increase in curvature factor predicts decreasing excess returns at the short and long terms to maturity albeit increasing excess returns for middle terms to maturity. *VAR* residuals explain unpredictable component in excess returns.

It is worth pointing out that, compatible with the literature, the sturdiest predictor of future excess holding period returns of long-term bonds is identified as the slope component. In particular, one standard deviation positive shock to the slope component increases expected one-month excess holding return of 10-year bond by 3.8%. Regression intercepts and coefficients of regressors are collected from the results.

Table 8. Excess Holding Period Return Regressions

Regressor	Excess holding period returns (rx_{n-1})					
Panel a. Short terms to maturity	n					
	6 months	12 months	18 months	24 months	30 months	36 months
Intercept	0.001	0.001	0.001	0.001	0.001	0.000
V_1	-0.016	-0.037	-0.059	-0.079	-0.098	-0.116
V_2	0.006	0.011	0.014	0.015	0.014	0.011
V_3	0.003	0.002	-0.001	-0.004	-0.007	-0.008
Level ($X_{1,\text{L}}$)	0.001	0.002	0.002	0.003	0.004	0.004
Slope ($X_{2,\text{L}}$)	0.000	0.000	0.000	0.001	0.002	0.004
Curvature ($X_{3,\text{L}}$)	-0.000	0.001	0.002	0.003	0.004	0.004
Observations	194	194	194	194	194	194
Panel b. Long terms to maturity	n					
	42 months	48 months	54 months	60 months	84 months	120 months
Intercept	0.000	-0.000	-0.000	-0.001	-0.002	-0.004
V_1	-0.132	-0.147	-0.162	-0.176	-0.226	-0.298
V_2	0.007	0.001	-0.005	-0.012	-0.043	-0.093
V_3	-0.009	-0.009	-0.008	-0.006	0.004	0.022
Level ($X_{1,\text{L}}$)	0.004	0.005	0.005	0.005	0.006	0.007
Slope ($X_{2,\text{L}}$)	0.005	0.007	0.009	0.012	0.022	0.038
Curvature ($X_{3,\text{L}}$)	0.005	0.005	0.005	0.005	0.003	-0.002
Observations	194	194	194	194	134	134

This table shows coefficient estimates of one-month excess holding period returns on one-month lagged state variables and contemporaneous *VAR* residuals. Excess holding period returns for times to maturity longer than 5 years are observed between February 2010 and March 2021, however, coefficients stay stable if observation period begins in February 2005 as it does for shorter times to maturity. All coefficients are significant at 0.001 level.

I estimate market prices of risk using Equations 34 and 35 together with coefficient estimates in the excess return regressions. Table 9 reports the resulting estimates. Level risk has a negative constant component, indicating that investors on average demand positive expected excess return for holding the level portfolio since excess return betas, B_n, are negative multiples of yield loadings, $b_n = -\frac{12}{n} B_n$. The loading of level risk, -0.058, is negative and has the highest absolute value in slope factor, indicating that expected excess returns are mostly affected by slope factor and increase with it. Similarly, slope factor, -0.195, is negative and has the highest absolute value among loadings of slope risk, which implies that a portfolio that is long in long-term bonds and short in short-term bonds has an expected excess return increasing with the slope. Curvature risk has a positive loading, 0.026, in level factor, meaning that a portfolio which is long in middle-term bonds and short in both long and short-term bonds has an expected excess return decreasing with interest rate level.

Table 9. Market Prices of Risk

Component	Risk prices			
	λ_0	$\lambda_{1,1}$	$\lambda_{1,2}$	$\lambda_{1,3}$
Level (X_1)	-0.009	-0.034	-0.058	-0.020
Slope (X_2)	0.044	0.034	-0.195	0.028
Curvature (X_3)	0.081	0.026	0.125	-0.228

This table shows the prices of factors that contribute to three risk components in the ACM model.

Before bonds can be priced in ACM model, risk-free rate should be regressed on state variables to estimate coefficients in Equation 27. Table 10 reports the resulting coefficients when holding return on one-month discount bond is taken as the risk-free rate. The resulting coefficients show that risk-free rate is increasing with the level factor and decreasing with slope and curvature factors as it is expected for one-month yield.

Table 10. Risk-free Rate Regression

Regressor	Risk-free rate (r)
Intercept	0.0095
Level (X_1)	0.0030
Slope (X_2)	-0.0013
Curvature (X_3)	-0.0009
Observations	195

This table shows coefficient estimates for regression of return on one-month discount bonds calculated from monthly average yield curves as the risk-free rate on state variables.

Now, with daily pricing factors and all of the model parameters in hand, I use Equations 36, 37, and 38 to recursively extract affine model yields for months to maturity 1 to 120. Risk-neutral yields are derived using the same equations by equating all market prices of risk to zero. The difference between physical and risk-neutral yields gives an estimate of term premium. From Equation 36, annualized n-month yield is estimated as:

$$y_n^t = -\frac{12}{n}(A_n + B_n' X_t) \tag{39}$$

Consequently, model implied yield loadings for pricing factors can be derived as $-\frac{12}{n}B_n$. Moreover, from the first term in Equation 31, model implied expected one-month excess return loadings can be calculated as $B_n'\lambda_1$. Figure 10a displays model implied yield loadings which could be construed as the response of n-month yield to a standard deviation increase in the respective pricing factor. Loading of the level factor is distinctly higher than loadings of slope and curvature factors and its sign is positive regardless of times to maturity in contrast with those of other factors. Also, it demonstrates a slightly hump-like shape similar to the average yield curve identified in Chapter 1. Figure 10b shows expected excess return loadings that can be considered as the response of expected one-month excess holding return on n-month bond to a standard deviation shock to the respective factor. It seems that slope factor dominates the predictable part of excess returns for long terms to maturity.

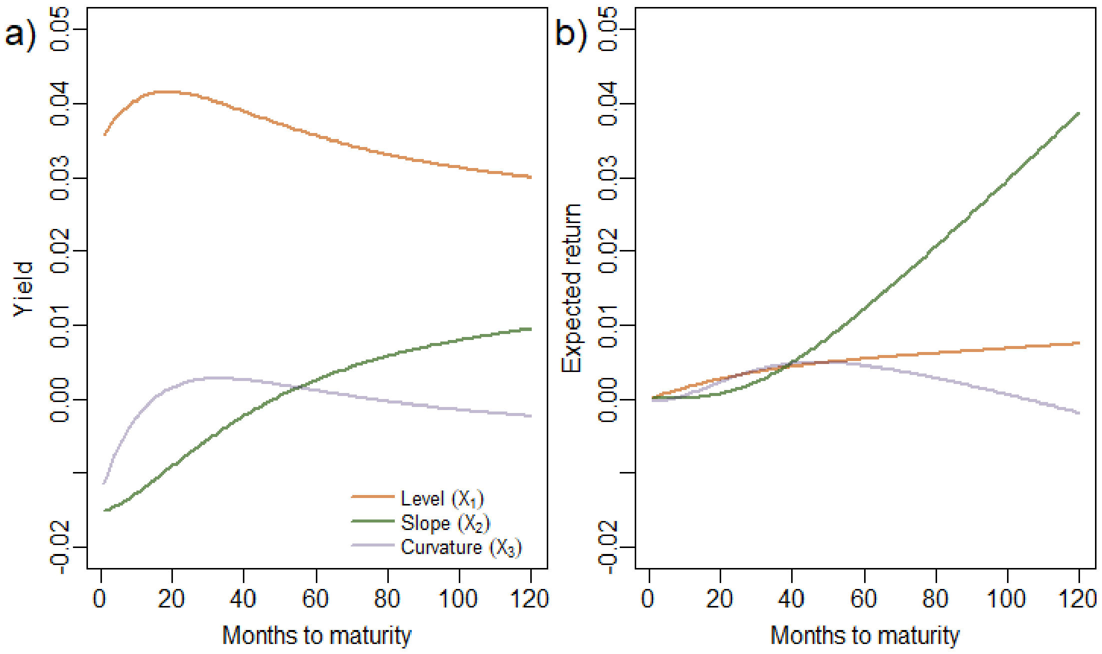

Figure 10. Model implied yield and expected one-month excess holding return loadings

Cross-sectional fitting performance of the ACM model is comparable with that of the Nelson Siegel model. Figure 11a shows that affine model successfully fits input Nelson Siegel yields at middle terms to maturity and slightly understates short and long terms to maturity. On the other hand, Figure 11b depicts that affine model's performance in fitting volatility of yields is indistinguishable from that of the Nelson Siegel model.

Sample statistics for term premia at selected times to maturity are reported in Table 11. It is observed that means of term premium at different times to maturity have a humped shape and peak at approximately 30 months. Standard deviations of term premia, on the other hand, increase with times to maturity of bonds.

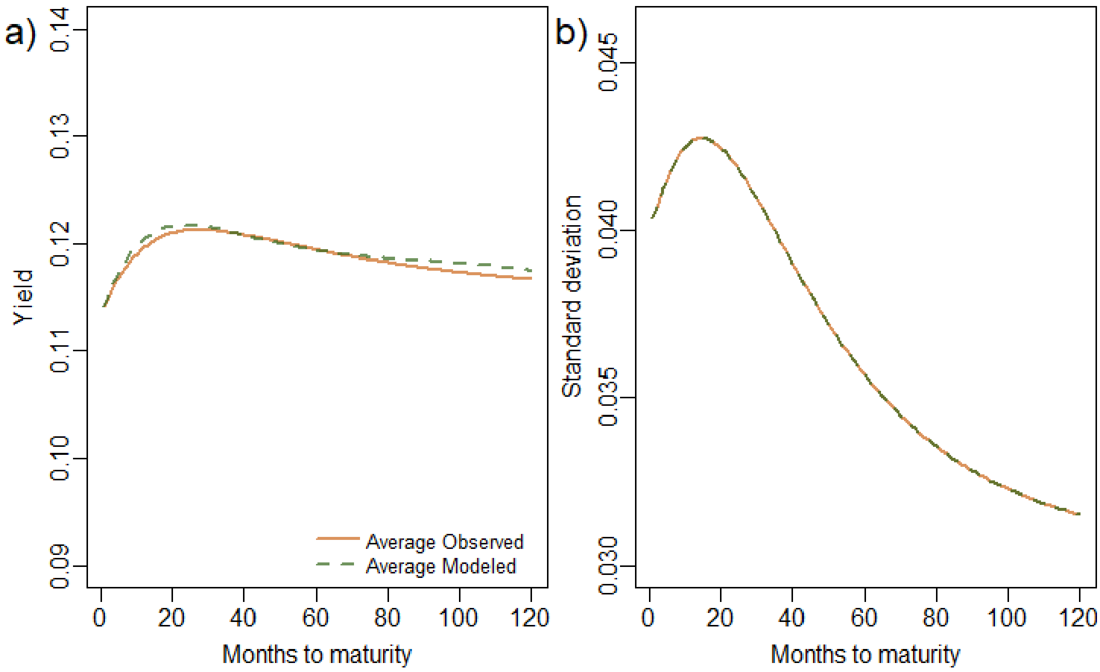

Figure 11. Means and standard deviations of observed and modeled yields

I focus on 1, 2, 5, and 10-year yields to evaluate their decomposition into expectation and term premium components. Figure 12 displays charts of corresponding four time series. Time series of 1-year yield components show that expected average short rates are highly correlated with yields for short-terms.

Table 11. Sample Statistics for Term Premia

Time to maturity (n)	Statistics for term premia (tp_n)			
	Mean(%)	St. dev.(%)	Minimum(%)	Maximum(%)
6 months	0.41	0.50	-0.54	1.93
12 months	0.72	0.95	-0.92	4.18
18 months	0.88	1.28	-1.26	6.32
24 months	0.95	1.51	-1.55	7.49
30 months	0.97	1.66	-1.78	7.98
36 months	0.96	1.76	-1.96	8.05
42 months	0.94	1.85	-2.10	7.88
48 months	0.92	1.91	-2.22	8.15
54 months	0.91	1.97	-2.31	8.53
60 months	0.90	2.02	-2.86	8.86
84 months	-0.02	1.47	-2.67	6.04
120 months	-0.10	1.53	-2.98	5.74

This table shows summary statistics for term premia pertaining to observation period of 4085 trading days between 6 January 2005 and 31 March 2021 for times to maturity up to five years and 2812 trading days between 27 January 2010 and 31 March 2021 for longer terms to maturity.

It is observed that term premia in long-term bonds are more volatile and explain a higher fraction of the variation in yields. After the introduction of 10-year bonds in 27 January 2010, 10-year term premia preserved a certain degree of smoothness even during volatile periods in 2018.

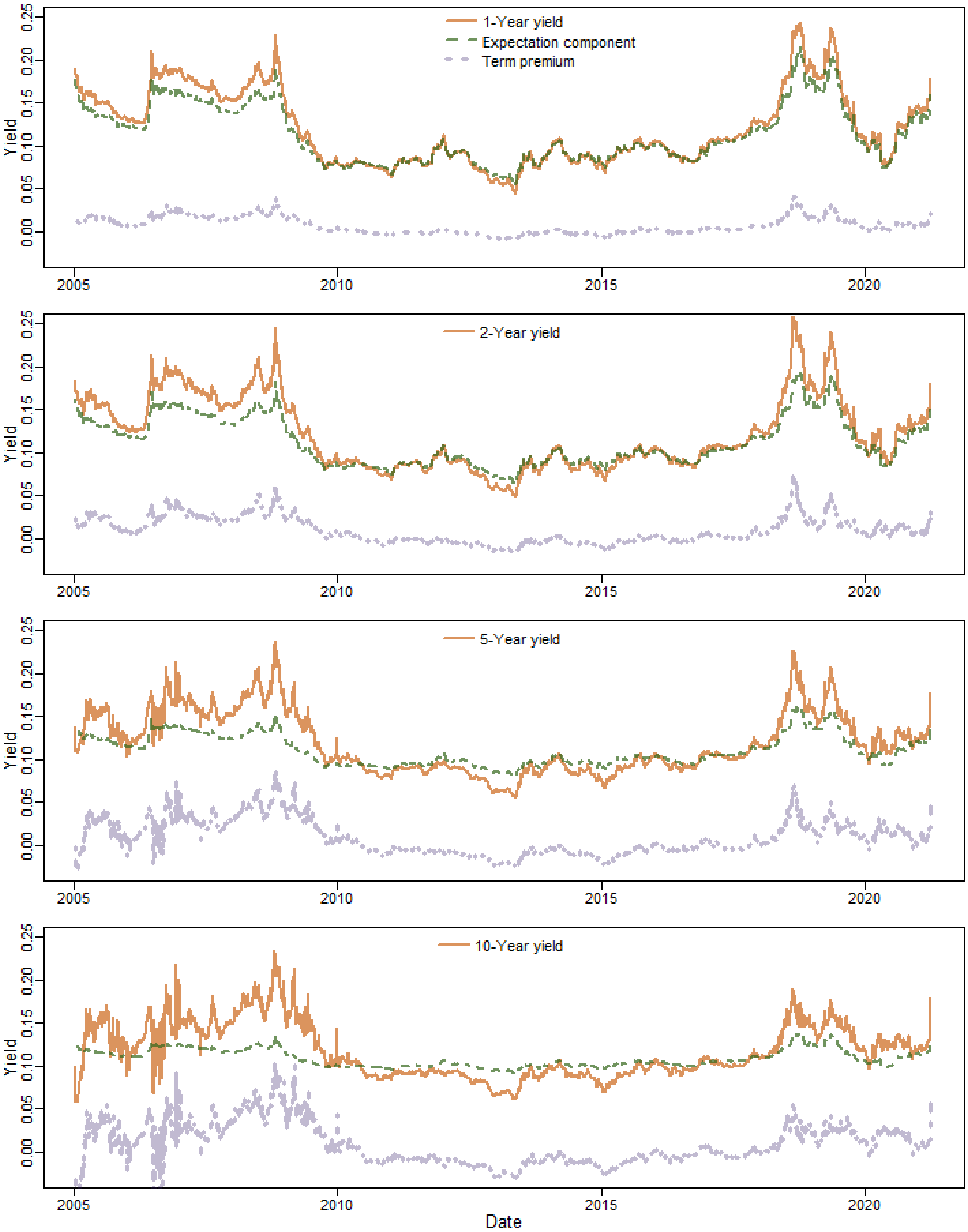

Figure 12. Time series of term premium and expectation components

2-year term premium peaked in August 2018 when it was argued in Chapter 1 that bond market illiquidity measure also peaked. Figure 13 shows decomposition of yields for the day in which 2-year term premium achieved its maximum value. It seems that the curvature component in yields is abnormally high besides the level component. Since short and long-term yields are not compatible with elevated yields for times to maturity close to 2 years, the ACM model attributes a significant portion of corresponding yields to the risk compensation. This instance suggests that macroeconomic factors such as volatility and ambiguity of foreign exchange parity play major roles as determinants of term premium, seeing that expected future foreign exchange parity is a major component of investors' inflation expectations, and therefore current interest rates.

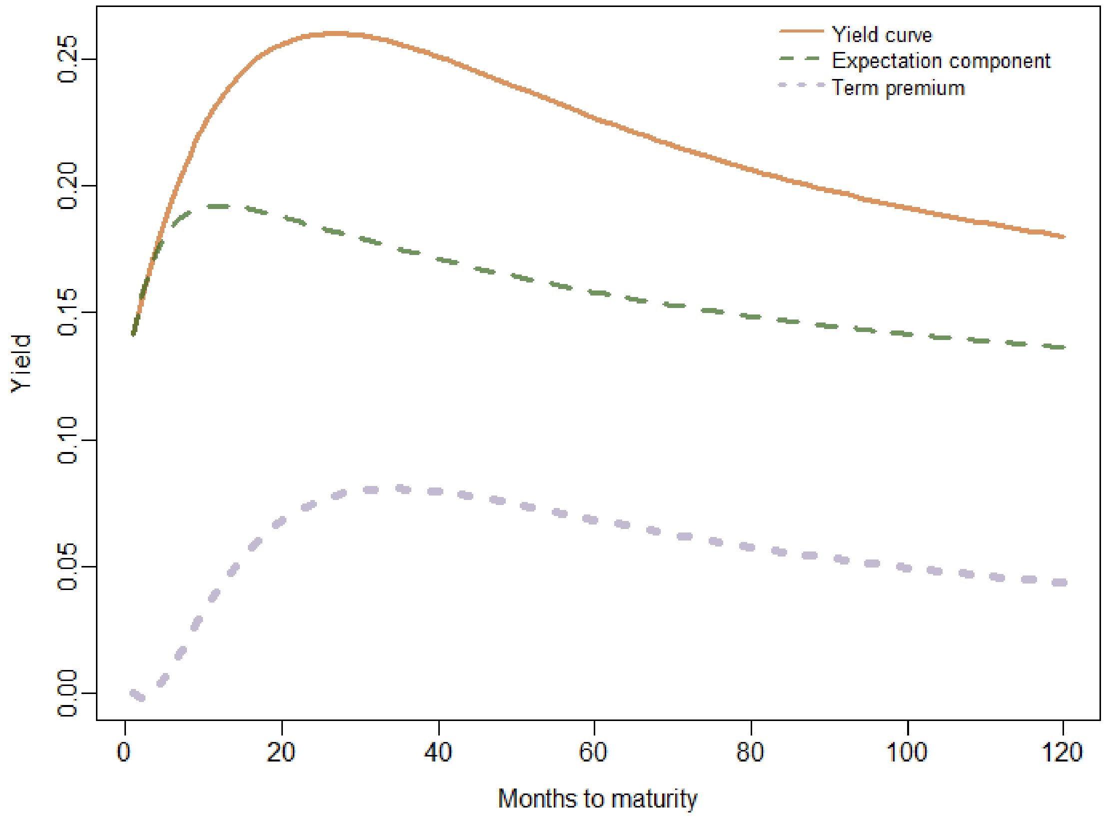

Figure 13. Decomposition of yield curve as of 17 August 2018

2.4.2 Determinants of the term premium

A major factor determining yields is inflation expectation. However, inflation expectation alone cannot be a determinant of term premium as it is almost always assumed in finance that risk-averse investors demand compensation for the second moment of underlying variables. Thus, a measure of future inflation uncertainty is needed. Aydin and Ozel (2019) proposed using dispersion of responses in inflation expectation surveys, however, surveys are not available on daily basis and their reliability is questionable. Instead, I assume the existence of a credible central bank which aims to keep short-run real interest rates more or less constant. Therefore, implied volatility derived from foreign exchange options reveals a measure of inflation uncertainty, especially when it is provided that the inflation for foreign currency is stabilized. Thus, I use daily observations of one-month implied volatility for USDTRY options. As a robustness check for this measure, I also calculate 21 days rolling window estimate of one-month yield volatility. Furthermore, I use bond market illiquidity measure derived in Chapter 1 because term premium and illiquidity peak at the same time and seem related.

Although governments are usually assumed to be default-free in their domestic currency debt, credit default swap (CDS) premia on their bonds might rise aloft. I use the CDS premium in domestic currency denominated bonds to test whether credit risk premium is related to term premium. I also use CDS premium in USD denominated Turkish treasury bonds as a robustness check.

Aydin and Ozel (2019) suggested that the percentage of foreign holdings in treasury bonds market be a significant determinant of term premium. Thus, changes in sentiment towards emerging market bonds might be a determinant of term premium. To test this, I use monthly shocks to the Emerging Markets Bonds

Sentiment Index (EMBSI) published by Sentix. As a robustness check of this sentiment measure, I use monthly shocks to the Consumer Confidence Index (CCI) published by the Turkish Statistical Institute.

Another possible behavioral determinant of term premium might be ambiguity, though empirical works contributing a measurable factor for it are rarely found on asset pricing in general and interest rate modeling in specific. Guidolin and Rinaldi (2013) presented a survey on the asset pricing implications of ambiguity. The limited number of studies on ambiguity and interest rates focus mostly on theoretical models and ambiguity on macroeconomic variables such as inflation. In one of the earlier works in the field, Gagliardini, Porchia, and Trojani (2009) modeled a utility function based on ambiguity aversion and documented how excess bond returns reflect a premium for ambiguity on top of the risk premium. Ulrich (2013) documented inflation ambiguity and its impact on term structure of interest rates in the US bond market. In a more recent study, Zhao (2020) offered an equilibrium bond pricing model using ambiguity about inflation and growth to explain the term structure of bond returns. This study takes a more practical view of ambiguity to empirically test its contribution to term premia. Izhakian (2020) developed the foundation of an ambiguity measure as volatility of probabilities. Research of Brenner and Izhakian (2018) is the first in a line of studies empirically testing the said measure on the stock market. This measure is conceptually simple, intuitive, and applicable for the empirical measurement of the degree of ambiguity across a wide range of markets. To the best of our knowledge, this will be the first study using this novel ambiguity measure in explaining interest rates.

In this study, the methodology developed by Brenner and Izhakian (2018) is employed to measure volatility of probabilities as a consistent measure of ambiguity in currency market, namely USDTRY exchange rate. It is implicitly assumed that the currency ambiguity encompasses all the ambiguity information regarding inflation and other macroeconomic variables and by extension the ambiguity of Turkish bond market and interest rates. As ambiguity is taken to be uncertainty of the probabilities of returns, following Izhakian (2020) ambiguity is defined as:

$$\mho^2[\mathrm{r}] = \int \mathrm{E}[\varphi(\mathrm{r})]\mathrm{Var}[\varphi(\mathrm{r})]\mathrm{dr} \tag{40}$$

where r is the return and $\varphi(\mathrm{r})$ is the marginal probability of returns. While risk can be measured by the volatility of returns, this new measure empirically quantifies ambiguity by the volatility of probabilities.

Following Brenner and Izhakian (2018), 5-minute returns of the exchange rates are retrieved. The website of Forexite provides data for USDTRY which is available between 29 November 2010 and 31 March 2021. The foreign exchange market is open from Australasian market opening on Monday to North American market closing on Friday. This allows us to consider a 5-day week with full 24 hours or 288 observations of 5-minute returns each day. For each day, a histogram of returns is derived. Specifically, 5-minute returns are divided into 62 bins, 60 bins for returns between -1% and 1%, and two bins for extreme values which are above 1% and below -1%. Second, the probability of returns being in each bin is estimated as the frequency of 5-minute returns within a day. Then, the mean and variance of the probability for each of the 62 bins are calculated. Finally, the degree of foreign exchange ambiguity can be estimated as:

$$\mho^2[r] = E[\phi(r_0; \mu, \sigma]Var[\phi(r_0; \mu, \sigma] + \tag{41}$$

$$\sum_{i=1}^{60} E[\phi(r_i; \mu, \sigma) - \phi(r_{i-1}; \mu, \sigma)]Var\,[\phi(r_i; \mu, \sigma) - \phi(r_{i-1}; \mu, \sigma)] +$$

$$E[1 - \phi(r_{60}; \mu, \sigma)]Var[1 - \phi(r_{60}; \mu, \sigma)]$$

where r_0 and r_{60} are the lowest and highest returns (±1%) and ϕ denotes the cumulative probability. Interested readers can refer to the original study for the details of the methodology. Our ambiguity measure is computed by a rolling window calculation of the Equation 41 for 30 days with a minimum of 20 days of data at initiation. This rolling window ambiguity measure is then used as an ambiguity estimate for the subsequent day to be incorporated into term premia estimation. Figure 14 displays time series of ambiguity.

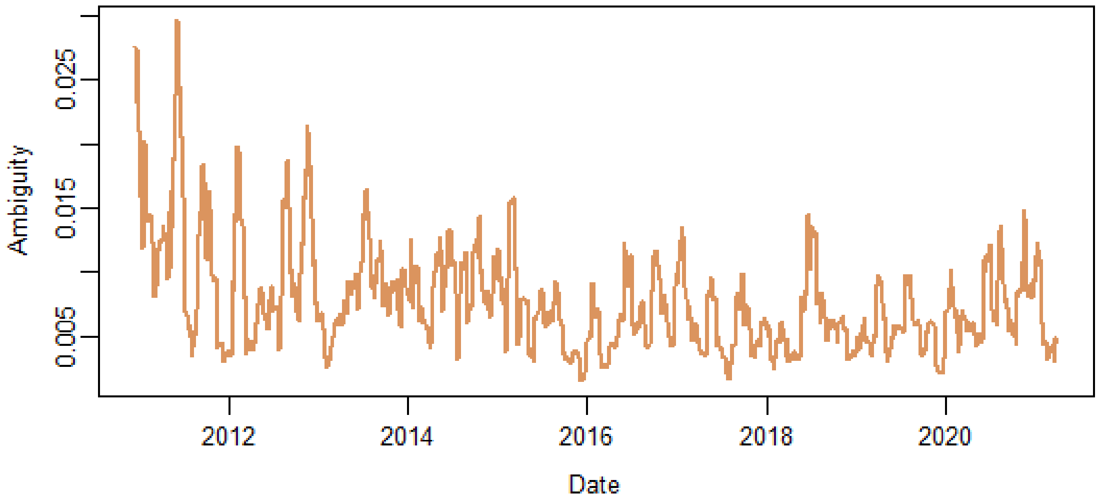

Figure 14. Time series of USDTRY ambiguity

Descriptive statistics for all eight factors are reported in Table 12. Statistics for each factor pertain to different observation periods. Implied volatilities in one-month USDTRY currency options are observed between 3 February 2016 and 21 August 2020. Since I used 21-day rolling window to estimate one-month bond yield volatility, the factor is observed between 8 February 2005 and 31 March 2021. Bond market illiquidity is available for each trading day between 6 January 2005 and 31 March 2021 and measured according to the methodology described in Chapter 1. CDS premia for treasury bonds denominated in domestic currency and foreign currency are observed until 31 March 2021 starting from 7 May 2015 and 8 August 2008, respectively. EMBSI and CCI are updated once per month according to survey responses. Therefore, values of the monthly shocks to these indices stay the same during each month. These factors are observed until 31 March 2021 starting from 30 April 2007 and 31 January 2007, respectively. Ambiguity is observed between 21 December 2010 and 31 March 2021. All observed values are used in univariate regressions on factors while the longest observation period which is common in all factors are used in multivariate regressions. Figure 15 displays time series of factors.

Table 12. Sample Statistics for Factors

Factor	Statistic					
	Mean	St. dev.	Skew.	Kurt.	Min.	Max.
Implied Vol.	15.08	5.60	1.40	2.55	6.38	42.32
Bond Yield Vol.	0.004	0.003	2.43	9.41	0.001	0.028
Illiquidity	0.001	0.001	4.42	26.78	0.000	0.014
2-Year CDS (TL)	149.9	92.3	1.02	0.09	35.6	439.0
2-Year CDS (USD)	187.1	122.1	1.56	1.99	49.7	834.9
Shocks to EMBSI	0.03	5.88	-1.15	3.55	-24.50	13.25
Shocks to CCI	-0.04	2.37	0.06	1.12	-7.04	8.93
Ambiguity	0.008	0.004	1.51	3.47	0.001	0.030

This table shows summary statistics of factors that are selected to test whether they explain variation in term premia.

First, 2-year term premium is regressed on factors one by one. Table 13 reports the resulting coefficient estimates all of which turn out to be significant at 0.001 level. Implied volatility derived from currency options explains 70% of the variation in 2-year term premium. As expected, it has a positive coefficient since uncertainty in macroeconomic factors, especially inflation, should increase compensation for risk that investors demand. CDS premium for TL denominated bonds explains the second highest variance in 2-year term premium. Its positive coefficient shows that a significant portion of sources of credit risk is common in interest rate risk. Bond market illiquidity explains a similar portion of the variance in term premium. It also has a positive slope indicating that exposure to the noise in bond prices affects interest rate risk component in yields.

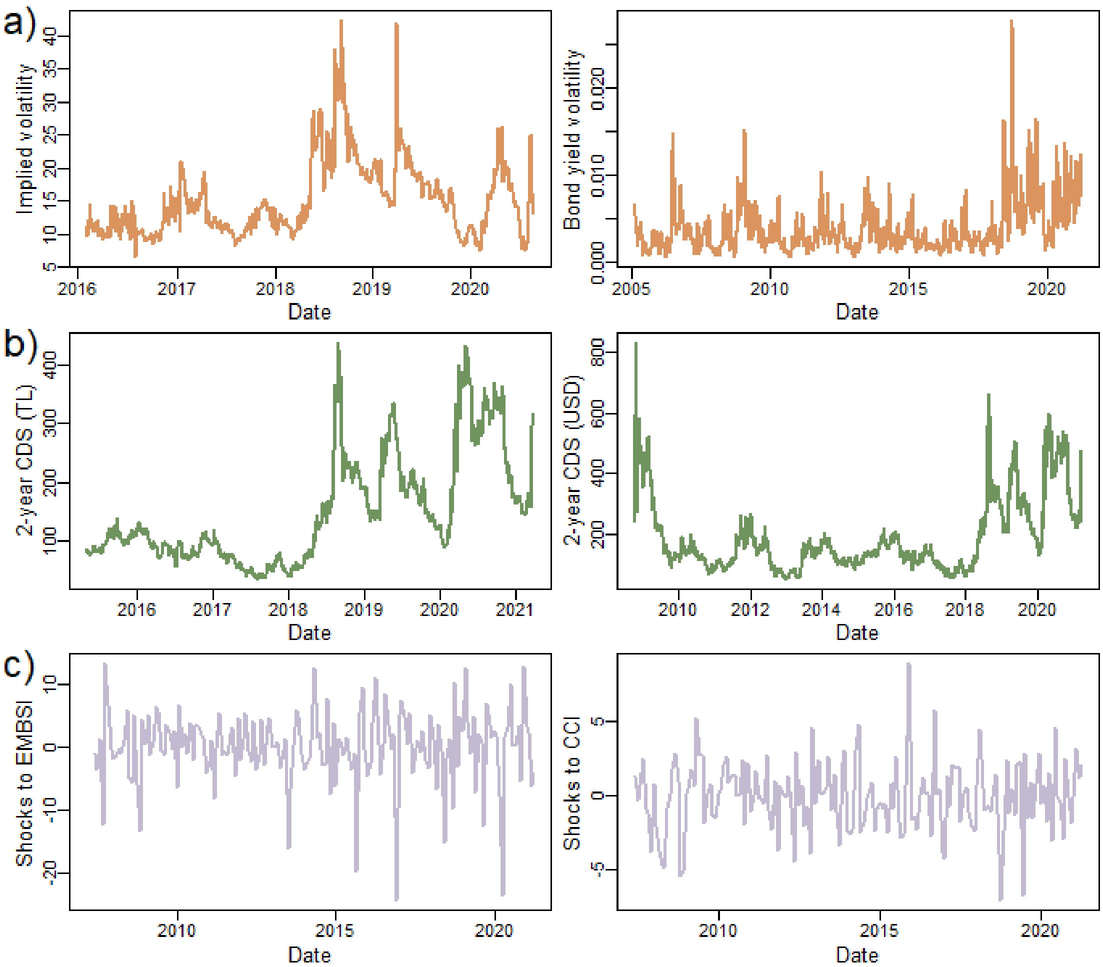

Figure 15. Time series of factors

Monthly shocks to the Emerging Markets Bonds Sentiment Index (EMBSI) factor has a negative coefficient that is typical for sentiment measures. On the other hand, the factor explains as low as 1% of the variation in term premia, which is expected since surveys respond to the market developments after a time lag. Lastly, an increase in ambiguity predicts decreasing 2-year term premium. During times of elevated ambiguity in the economy, ambiguity-averse investors would expect worst-case scenario and forego higher return in exchange for less risk. Although treasury bonds lose a portion of their value in large scale market downturns, a sovereign default is an unlikely event. Therefore, compared to other financial instruments such as equities, treasury bonds are generally considered a safe haven investment. For this reason, the negative sign of ambiguity coefficient might stem from flight-to-safety to treasury bonds from other assets. Nevertheless, the assumption of ambiguity aversion requires further justification to devise reliable explanations for the interaction between ambiguity and other variables.

Table 13. Univariate Term Premium Regressions

Regressor	2-Year Term Premium			
	Coefficient	Constant	N	R^2
Implied Volatility	2.09 ***	-0.02 ***	1141	0.70
	(32.69)	(-23.28)		
CDS Premium	8.64 ***	-0.00 ***	1485	0.38
	(19.65)	(-5.88)		
Bond Market Illiquidity	7.15 ***	-0.00	4085	0.36
	(24.89)	(-0.59)		
EM Bonds Sentiment Shock	-1.45 ***	0.01 ***	3502	0.01
	(-3.11)	(29.47)		
Ambiguity	-7.46 ***	0.01 ***	2588	0.06
	(-16.33)	(19.36)		

This table shows coefficient estimates for univariate regressions of 2-year term premium on various factors measured daily. Factors were scaled with powers of 10 in order to ensure that they have the same order of magnitude. Robust t-statistics are shown in parentheses. *** indicates a significance level of 0.001.

Coefficient estimates in multivariate regressions require a robustness control when factors are suspected of multicollinearity. I calculate variance inflation factors (VIF) for each coefficient and report them in brackets. A coefficient estimate with a VIF measure higher than 5 is considered unreliable. I regress 2-year term premium on the five factors at the same time. Table 14 reports the results. It is seen that VIF measures do not indicate a severe multicollinearity problem. Thus, I proceed with interpreting coefficients and their significance levels. The first regression shows that factors in Table 13 combined explain 81% of the variation in 2-year term premium.

All of the coefficients are still significant at 0.01 level. Signs of all coefficients are identical to those in univariate regressions. Thus, explanations made there are supported by multivariate regression.

The majority of the explanation power stems from option implied volatility. Therefore, second regression substitutes it with bond yield volatility measure to see how other factors perform. Bond yield volatility has a positive coefficient that is significant at 0.01 level while proportion of the variance explained drops to 70%. Another change is that the significance of ambiguity measure drops to 0.05 level. Since implied volatility includes inflation uncertainty information which is unspanned by other factors, discarding it diminishes the need for adjusting it according to investors' sentiments. Third regression substitutes CDS premium on USD denominated bonds with that on TL denominated bonds. Fraction of the explained variance increases a bit while coefficients of other factors stay the same. Increases in both the explanation power of regression and the significance of related regressor coefficient suggest that CDS premium for debt in foreign currency include information that is unspanned by that in domestic currency.

The last regression substitutes monthly shocks to the consumer confidence with monthly shocks to the EM bonds sentiment. Coefficient of the monthly shocks to the CCI is negative and significant at 0.01 level, as well. Therefore, it can be concluded that two sentiment factors carry similar information regarding 2-year term premium. I run regressions to test how factors determine term premia for shorter and longer times to maturity. Table 15 reports the results. VIF measures indicate that there is no severe multicollinearity problem for regressions of 1, 5, and 10-year term premia. Hence, the coefficients of regressors and their significance levels can be meaningfully interpreted.

Table 14. Multivariate 2-Year Term Premium Regressions

Regressor	2-Year Term Premium			
	(1)	(2)	(3)	(4)
Market Risk Measures				
Option Implied	1.11 ***		1.09 ***	1.10 ***
Volatility	(16.45)		(16.42)	(16.80)
	[2.85]		[2.85]	[2.85]
Bond Yield		0.36 ***		
Volatility		(4.47)		
		[1.78]		
Liquidity Risk Measure				
Bond Market	3.13 ***	4.45 ***	3.04 ***	2.94 ***
Illiquidity	(14.04)	(15.08)	(13.84)	(13.29)
	[2.65]	[1.63]	[2.70]	[2.76]
Credit Risk Measures				
2-Year CDS Premium	1.87 ***	3.45 ***		1.90 ***
TL Denominated	(5.97)	(10.13)		(6.16)
	[1.75]	[1.62]		[1.75]
2-Year CDS Premium			1.51 ***	
USD Denominated			(7.07)	
			[1.85]	
Behavioral Factors				
EM Bonds Sentiment	-1.78 ***	-1.39 ***	-1.78 ***	
Shock	(-6.42)	(-5.55)	(-6.4)	
	[1.00]	[1.01]	[1.00]	
Consumer Confidence				-4.16 ***
Shock				(-4.33)
				[1.13]
Ambiguity	-3.36 ***	-2.27 **	-3.42 ***	-3.18 ***
	(-4.23)	(-2.48)	(-4.33)	(-3.92)
	[1.05]	[1.08]	[1.05]	[1.06]
Constant	-0.01 ***	-0.00 ***	-0.01 ***	-0.01 ***
	(-13.76)	(-5.58)	(-13.91)	(-13.45)
Observations	1141	1485	1141	1141
R squared	0.81	0.70	0.82	0.81

This table shows coefficient estimates for four term premium regressions on different combinations of factors which are measured daily. Factors were scaled with powers of 10 in order to ensure that they have the same order of magnitude. Robust t-statistics are shown in parentheses. Variance Inflation Factors are shown in brackets. *** and ** indicate significance levels of 0.01 and 0.05, respectively.

First regression shows that CDS premium does not significantly contribute to 1-year term premium while other factors are significant determinants of it at 0.001 level. It is observed that both significance level and coefficient of CDS premia as a determinant of term premia increase with the horizon. This contrasts with volatility measures which lose some part of their significance as determinants of term premia with increasing horizon. On the other hand, sentiment-related measures lose their significance completely as determinants of term premia for the endmost time to maturity. Shocks to the EM bonds sentiment factor is insignificant for 10-year term premium while ambiguity does not significantly contribute to 5 and 10-year term premia. All in all, five factors combined explain 78% and 79% of the variations in 1 and 5-year term premia, respectively. Since impacts of market risk, liquidity, and behavioral measures on term premia decline at the long end of maturity horizon, only 69% of the variation in 10-year term premium can be explained. Moreover, negative sign of the intercept for 10-year term premium indicates that another factor might be a determinant of term premium for the longest terms to maturity.

Table 15. Multivariate Term Premia Regressions

Regressor	Term Premium		
	1-year	5-year	10-year
Market Risk Measure			
Option Implied	0.76 ***	0.98 ***	0.59 ***
Volatility	(17.64)	(14.27)	(8.66)
	[2.91]	[2.81]	[2.93]
Liquidity Risk Measure			
Bond Market	2.07 ***	2.25 ***	1.37 ***
Illiquidity	(13.31)	(10.89)	(6.54)
	[2.74]	[2.58]	[2.52]
Credit Risk Measures			
1-Year CDS Premia	0.39		
TL Denominated	(1.50)		
	[2.02]		
5-Year CDS Premia		4.68 ***	
USD Denominated		(21.19)	
		[1.53]	
10-Year CDS Premia			8.45 ***
USD Denominated			(24.39)
			[1.51]
Behavioral Factors			
EM Bonds Sentiment	-0.71 ***	-1.71 ***	-0.46
Shock	(-3.77)	(-5.56)	(-1.34)
	[1.00]	[1.00]	[1.01]
Ambiguity	-3.32 ***	-0.98	0.64
	(-6.13)	(-1.08)	(0.63)
	[1.05]	[1.06]	[1.06]
Constant	-0.00 ***	-0.02 ***	-0.03 ***
	(-8.62)	(-24.12)	(-27.21)
Observations	1141	1141	1141
R squared	0.78	0.79	0.69

This table shows coefficient estimates for 1-year, 5-year, and 10-year term premia regressions on various factors measured daily. Factors were scaled with powers of 10 in order to ensure that they have the same order of magnitude. Robust t-statistics are shown in parentheses. Variance Inflation Factors are shown in brackets. *** indicates a significance level of 0.01.

2.5 Conclusion

In this study, I decomposed Turkish treasury yield curve into expectation and interest risk premium components. I derived the yield data directly from quotes in bond market. For every trading day in the observation period, I fitted Nelson Siegel curves to obtain continuous functions which reasonably estimate yields for times to maturity that are unspanned by traded bonds. Resulting yield curves are used as input to construct a three-factor ACM affine term structure model. Model parameters are estimated by linear regressions, and results are used with daily pricing factors to obtain estimates of risk-neutral yields. Term premia for every months to maturity are estimated by subtracting risk-neutral yields from physical yields.

Literature on the topic lacks a study which identifies the determinants of term premium in Turkish treasury yields using daily variations. To contribute in this aspect, I select three groups of factors to test whether they determine term premia. I form a bond market illiquidity measure from the discrepancies in bond prices and define it as a volatility measure along with the option implied volatility and the bond yield volatility. I use the CDS premia for domestic and foreign currency denominated bonds as credit risk measures. Lastly, I use an ambiguity measure derived from foreign exchange market as a sentiment measure together with the shocks to market sentiment and consumer confidence indices.

Putting the emphasis on term premium in benchmark 2-year yield, I regress term premia at different horizons on factors. Results indicate that all these factors significantly contribute to term premia and can explain more than 80% of its variation, with the major explanation power pertaining to the implied volatility in currency options. Effect of sentiment measures in determining term premia in the longest-term bonds cease to exist while significance of volatility measures diminish and that of credit risk measures increase with the maturity horizon. Another study on the same topic may construct a dynamic stochastic general equilibrium model which estimates term premium dependent on exogenously specified macroeconomic factors that are unspanned by yield curve components.

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