T H E S I S

Ferroresonance

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Appendix

The author is deeply indebted to Prof. Roland Schinzinger without whos help this work would not have been performed.

Object:

The object of this thesis is to study the phenomena known as ferroresonnance. The problem is to find a method to predict the behaviour of a ferroresonnant circuit.

What is ferroresonnance:

A circuit is said to be ferroresonnant when it contains capacitance and an iron cored reactor. The fundamental equation of a series ferroresonnant circuit is:

$$\frac{d}{dt}(Li) + Ri + \frac{1}{C}\int i\,dt = E_m \sin \omega t \qquad (1)$$

which is the more general case of the well known equation dealing with RLC series circuits:

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = E_m \sin \omega t \qquad (2)$$

The coefficients of equation (2) are constant parameters, therefore independent of the variable I. Such an equation could be easily solved by the usual mathematical tools.

Equation (1) involved the term $\frac{d}{dt}(Li)$ where L is a function of i. Such an equation is cumbersome to solve mathematically.

Why ferroresonnance occurs:

The presence of the term $\frac{d}{dt}(Li)$ is due to the concept of inductance itself

$$L = N\frac{d\phi}{dI} \qquad (3)$$

where ϕ is flux in webers

I the current in amperes

N number of turns which link flux ϕ

L inductance in henrys.

When the circuit contains an iron cored inductor, the flux ϕ is related to the magnetomotive force (mmf) by means of the magnetization curve of the iron core.

The term $\frac{d\phi}{dI}$ appears as the slope of the magnetization curve of the magnetic material. Since the relation ϕ vs I is a nonlinear one, the slope of this curve is not constant and therefore related to I.(fig 1)

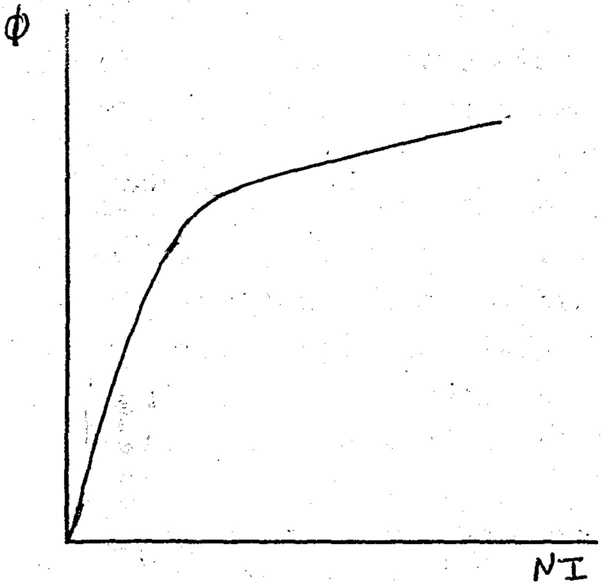


fig. I Magnetisation curve.

$$\phi = F(I) \qquad (4)$$

$$L = N\,F'(I) \qquad (5)$$

How ferroresonance occurs:

The effect of a ferroresonant circuit is a non linear volt ampere characteristic, with an unstable part. By unstable part is meant a section on the volt ampere curve where for a single value of voltage two values of current appear. The volt ampere relation of such a circuit is shown in fig. 2 and graph 1. Referring to fig 2 it is seen that for value of voltages

$$V_1 < V < V_2$$

the current has two different values. Increasing the voltage from 0 to V_2 makes the current follow the curve 0a; then a very small increase of voltage will make the current jump to b. A further increase of voltage causes the current to follow the portion of the curve labelled bc. When the voltage is reduced down to $V = V_1$, the current follows the curve cd, where at d it jumps directly to the point e of the curve oa. With further decrease of V to V = 0 the current follow the curve e0. In the practical set up studied (graph 1) the current at V_2 = I7 Volt, I jump from 0.I7 ampere to 0.75 ampere, and at V_1 = I5.6 volt, I is reduced from 0.60 ampere to 0.I3 ampere.

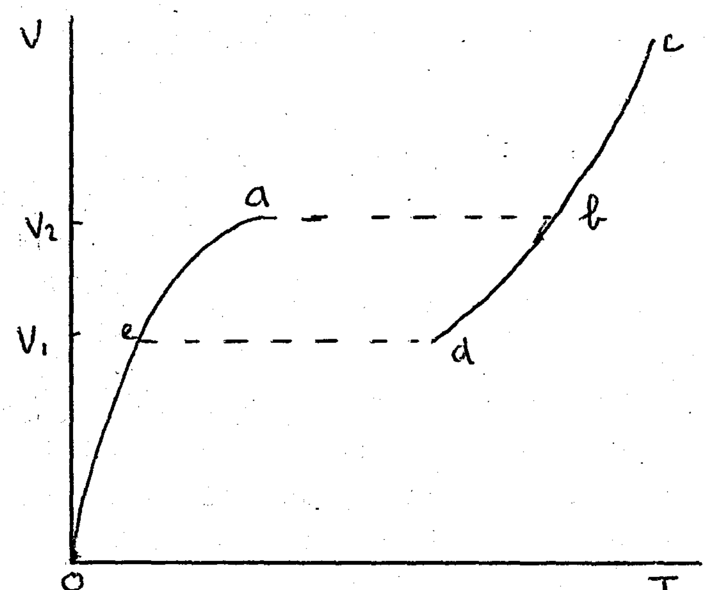


fig 2. Volt ampere relation of a ferroresonnant circuit.

Effect of ferroresonance:

The presence of ferroresonance could be desirable or undesirable. It could be desirable in the field of relays and control circuits, where an appreciable change of current for an infinitesimal change of voltage could have many practical applications

As undesirable phenomena, one may state its presence in power distribution systems. In power systems condensers are frequently used in order to correct the power factor or to reduce voltage drops on long line[1]. Transformers could be considered as saturable inductors, the loads are highly resistive, therefore all the conditions of ferroresonance are satisfied, resulting in the unexpected presence of high currents which could damage the installations.

In order to show the importance of such currents a simple case will be considered[5]. The circuit studied does not contain any condenser, however it will be shown that the initial current could be many time as big as the full load current. One can well imagine the serious consequence of capacitance.

Solution of a circuit containing R and L(i).

The differential equation of the circuit shhown (fig. 3) is:

$$L(i)\frac{di}{dt} + Ri = e(t) \qquad (6)$$

Substituing (3) into (6)

$$N\frac{d\phi}{di}\cdot\frac{di}{dt} + Ri = e(t)$$

$$N\frac{d\phi}{dt} + Ri = e(t) \qquad (7)$$

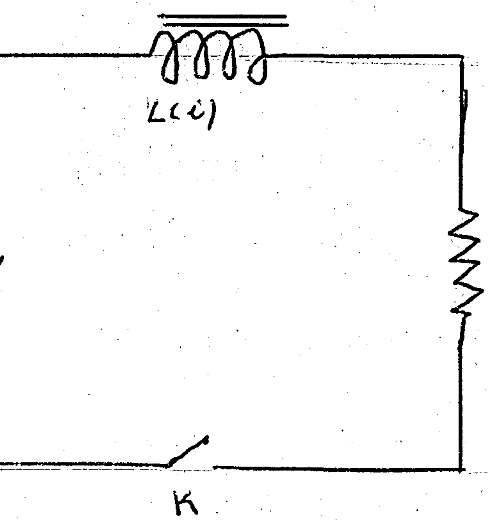


fig. 3 R, L(i) circuit.

ϕ is related to i through the magnetisation curve of the transformer; furthermore it is necessary to eliminate one of the variables i or ϕ. Since Ri is small ϕ is essentially sinusoidal and it is better to eliminate i of equation (7). In transformers where R is small we could state:

$$Ri \leq L\frac{di}{dt} \qquad Ri \leq N\frac{d\phi}{dt}$$

(1), (5) The bracketed numbers indicate references listed in the appendix.

$$Ri \ll \omega L_{av} i \qquad (8)$$

and it is possible to approximate the value of i in the Ri term only by the relation:

$$i \approx \frac{N\phi}{L_{av}} \qquad (9)$$

This i could not be used in the $L\frac{di}{dt}$ term, we did not mean that L_{av} is equal to L, this is a trick used in order to simplified the mathematical derivation. Equation (7) becomes

$$N\frac{d\phi}{dt} + \frac{R}{L_{av}} N\phi = e(t) \qquad (I0)$$

but e being a sine function we get

$$e(t) = \Re e\left[E_m \varepsilon^{j\omega t}\right] \qquad (II)$$

where $E_m = E_{m1} + jE_{m2}$, E_{m1} and E_{m2} being real constants. The steady state solution of (I0) is given by:

$$\phi_s = \Re e\left[\Phi_m \varepsilon^{j\omega t}\right] \qquad (I2)$$

where

$$\Phi_m = \frac{E_m L_{av}}{N(R + j\omega L_{av})} \qquad (I3)$$

The transient solution is

$$\phi_t = \phi_t \, \varepsilon^{-\frac{R}{L_{av}}t} \qquad (I4)$$

and the complete solution becomes:

$$\phi = \Re e\left[\phi_m \varepsilon^{j\omega t} + \phi_t \varepsilon^{-\frac{R}{L_{av}}t}\right] \qquad (I5)$$

introducing the initial conditions t=0 ,i = 0, $\phi = \phi_r$ (Residual flux)

$$\phi_t = \phi_R - \Re e\left[\phi_m\right] \qquad (I6)$$

and finally

$$\phi = \mathrm{Re}\left[\Phi_m \varepsilon^{j\omega t}\right] + \left\{\Phi_r - \mathrm{Re}\left[\Phi_m\right]\right\} \varepsilon^{-\frac{R}{L_{av}} t} \qquad (17)$$

If, when $t = 0, E_{m1} = 0, E_{m2} = -E_m$, then from equation (13)

$$\Phi_m = \frac{-jL_{av}E_m}{N(R + j\omega L_{av})} \qquad (18)$$

from (13)

$$\Phi_m = -\frac{E_m}{N\omega}\angle 0^\circ \qquad (19)$$

which, substitued in equation (17), gives for the switching instant

$$\phi = -\frac{E_m}{N\omega}\cos\omega t + \left(\frac{E_m}{N\omega} + \phi_r\right)\varepsilon^{-\frac{R}{L_{av}}t} \qquad (20)$$

The first maximum of this last equation occurs when $t = \frac{\pi}{\omega}$, assuming $\frac{R}{\omega L_{av}} = 0.05$

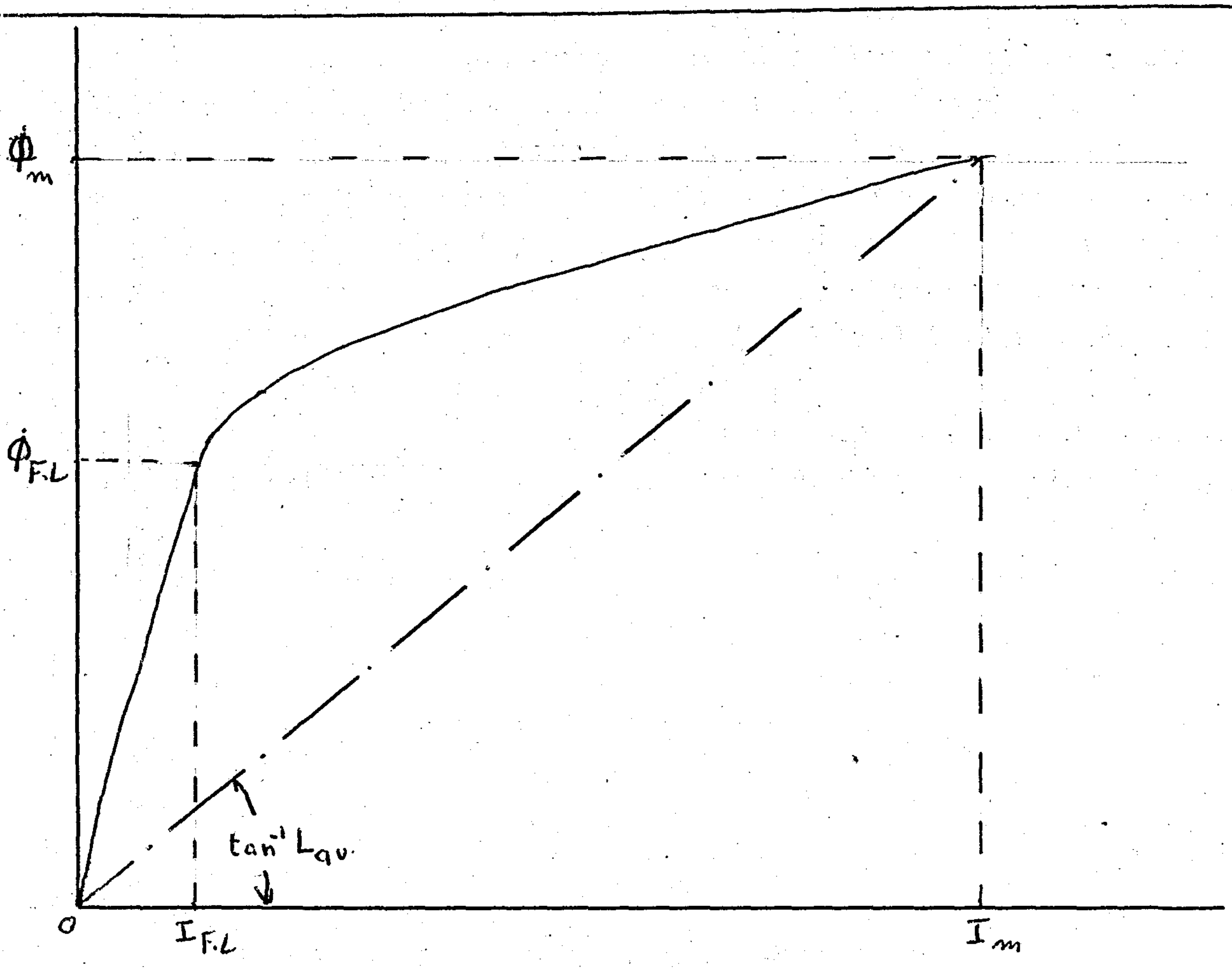


fig. 4 When $\phi_m = 1.8\ \phi_{fl}, I_m = 8\ I_{fl}$.

$$\Phi_m = \Phi_r\,\varepsilon^{-0.05\pi} + \frac{E_m}{N\omega}\left(1 + \varepsilon^{-0.05\pi}\right) =$$

$$\Phi_m = 0.86\,\Phi_r + 1.86\,\frac{E_m}{N\omega} \qquad (21)$$

Under this condition the maximum flux is almost twice the normal flux amplitude. Taking into consideration that usually a transformer is rated in such a way as to work just below the knee of its magnetization curve, it is easily seen that twice the rate flux means many times the rated current (fig. 4), sometimes the maximum inrush current could be as high as 9 or 10 times the full load current. This would result in producing considerable electri and magnetic forces, which could damage the installation.

The necessity to be able to predict the behaviour of circuits containing such inductive properties is now evident.

How to predict the behaviour of a ferroresonant circuit:

Suit[I] in his work has shown empirically that ferroresonance occurs nearly on the knee of the magnetisation curve, which could be expected, because here L is swept through a wide range of values. Therfore, when the voltage across the transformer in a powe system is a little higher than the working voltage, ferroresonance could occur. However, ferroresonance is also closely associated with the value of capacitance in the circuit, fortunately this value of capacitance is quite high and difficult to reach in a power system. In spite of this, more detailed and accurate tools for predicting the critical values (E, I and C) in such a circuit are needed.

Weber[2] has developped a graphical approximation method introduced first by Margrand, and succeeded in analysing ferroreson circuits.

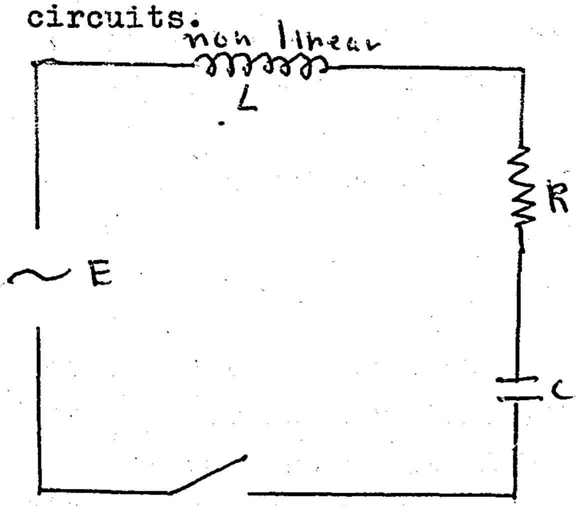


fig. 5 Ferroresonant circuit.

Weber starts from the basic

$$E = IZ \qquad (22)$$

Since

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \qquad (23)$$

we obtain

$$E = \sqrt{I^2R^2 + \left(\omega L I - \frac{I}{\omega C}\right)^2} \qquad (24)$$

If we called

$$E_L = \omega L I = \qquad (25)$$

L being L(i).
Equation (24) becomes

$$E = \sqrt{(IR)^2 + \left(E_L - \frac{I}{\omega C}\right)^2} \qquad (26)$$

Solving for E_L

$$E^2 = (IR)^2 + \left(E_L - \frac{I}{\omega C}\right)^2 \qquad (27)$$

$$E_L = \pm\sqrt{E^2 - (IR)^2} + \frac{I}{\omega C} \qquad (28)$$

The first member on the right side og equation (28) forms an ellipse :

$$\pm\sqrt{E^2 - (IR)^2}$$

whose principal axes have values E and E/R respectively. The addition of the term $\frac{I}{\omega C}$ to this ellipse has for result another ellipse whose axes did not coincide with the coordinate axes.

This rotated ellipse represents the voltage drop across the inductor, from the circuit point of view. But the volt ampere characteristic of the inductor which will be called F(i), represe also the voltage drop occuring in the inductor when a certain current I flow through it. The intersection of these two curves E_L and F(i), should be a solution of the circuit. The position of such points should give indication about the stability of the system.

From equation (28) we get

$$F(i) = \pm\sqrt{E^2 - (IR)^2} + I/\omega C \qquad (29)$$

Weber had shown that the positive sign should be used when the characteristic curve cross the upper part of the ellipse.

Any such point is stable if for a slight change of current F(i) correspondingly changes at a greater rate than the right hand member of (29) (Point I fig 6), taking cognizance of the ser of variation of the current.

Thus for an increment of I, the ordinate of the reactor chara

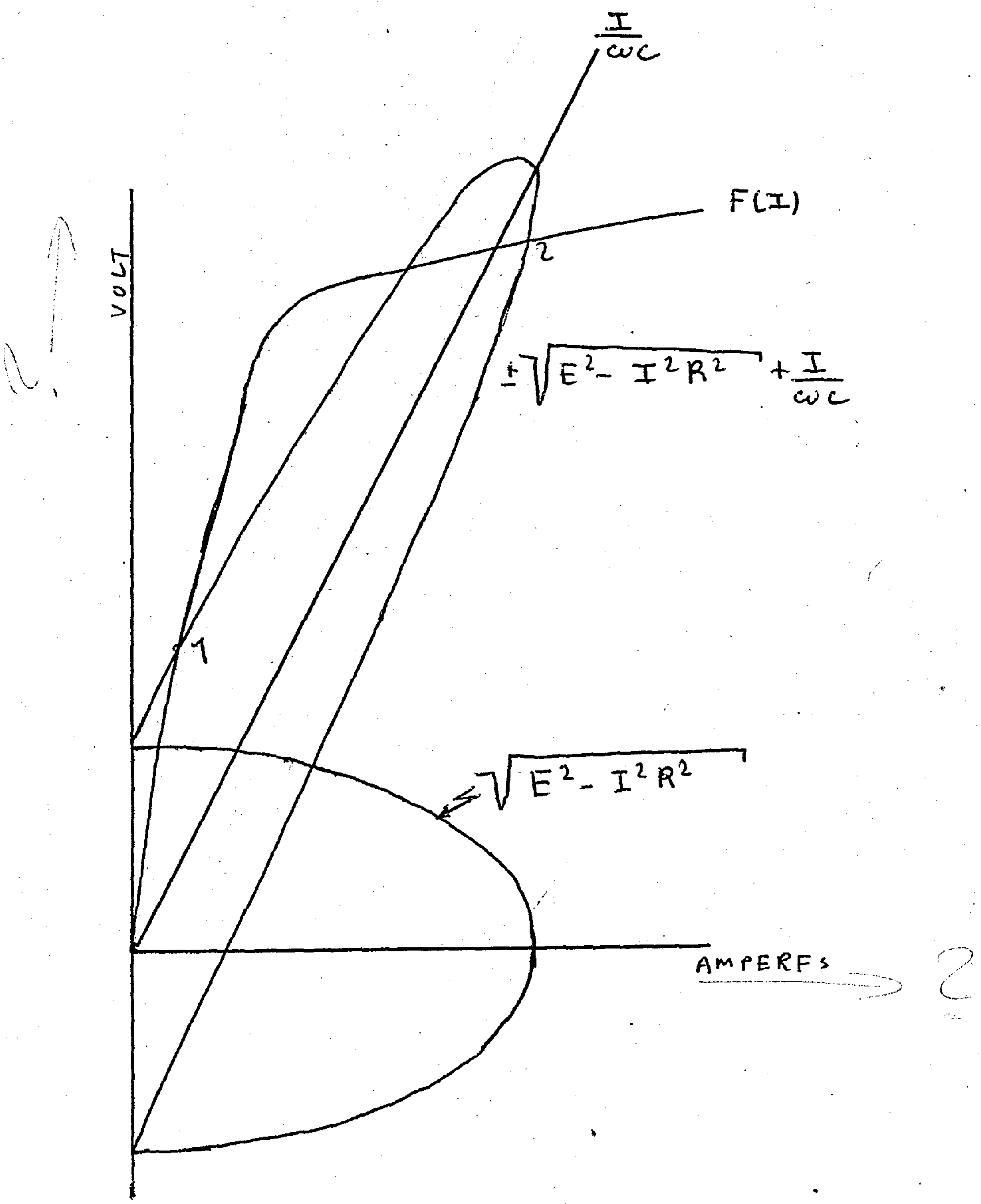


fig. 6: Webers solution of ferroressonnant circuit.

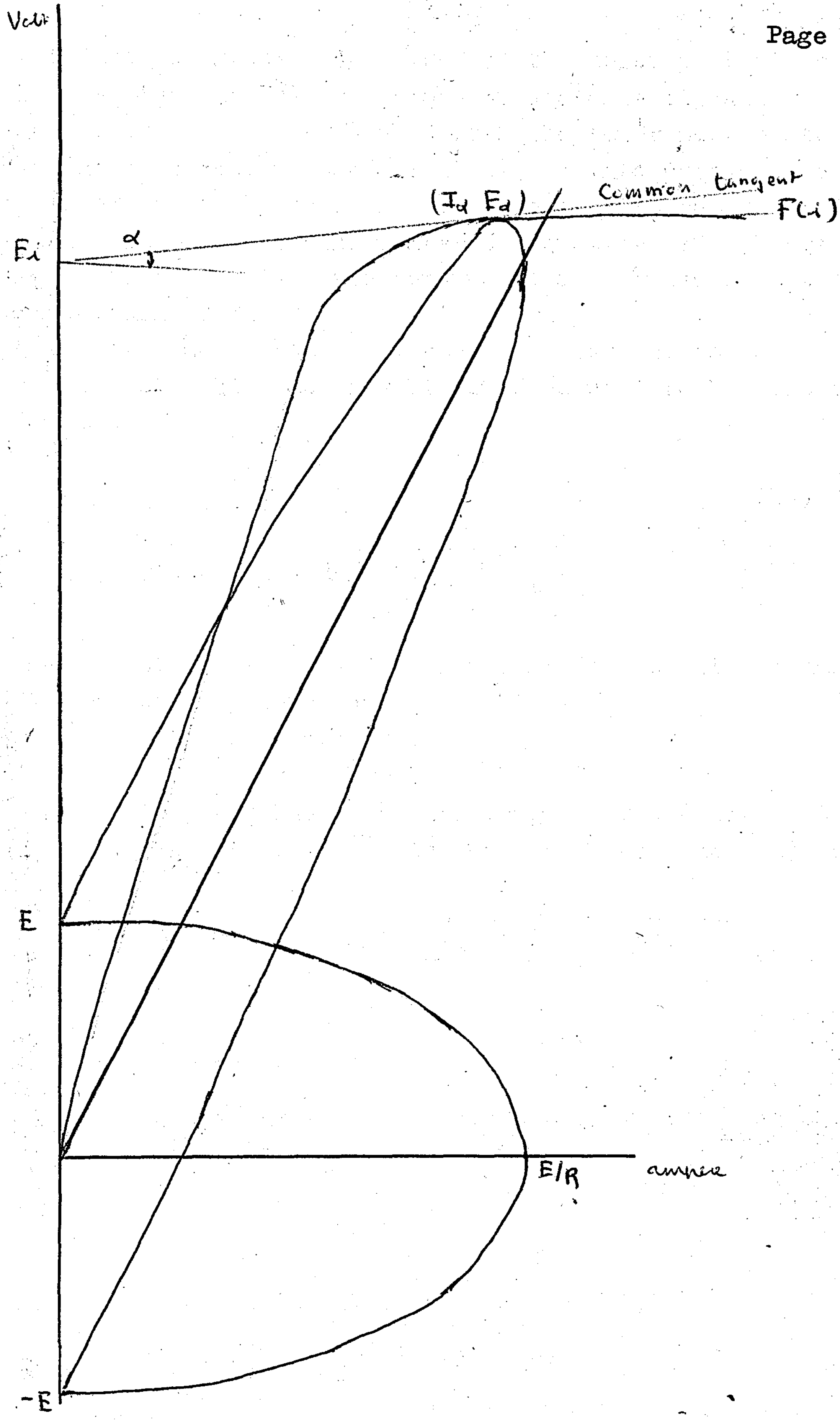


fig 7:Critical condition in ferroresonnance.

teristic must be greater than that of the upper part of the elli and less if the current is assumed to decrease slightly.

For intersections associated with the lower part of the elli the graphical solution is given by taking into account the minus sign. Inthis case the condition of stability is reversed; a poin is stable if the right hand member of equation (29) change at a greater rate than F(i) corresponding to an assumed deviation of current. (Point 2 fig. 6)

Similarly the current a function of frequencies may be adequ ly described if the reactor voltage is assumed to be directly pr portional to

$$E_L = \omega L I \tag{30}$$

$$f(I) = \pm \frac{1}{\omega}\sqrt{E^2 - (IR)^2} + I/\omega^2 C \tag{31}$$

Now an attempt to determine critical values will be made (fi 7). For instability to occure the ellipse

$$\pm\sqrt{E^2 - (IR)^2} + I/\omega C$$

and the reactor must have a common tangent. The point of tangenc is (I_α E_α). The intercept of the common tangent and the voltage axis is E_i

$$E_i = E_\alpha - I_\alpha\left(\frac{dE_L}{dI}\right)_\alpha \tag{32}$$

where

$$\left(\frac{dE_L}{dI}\right)_\alpha = \tan\alpha \tag{33}$$

With E_α and I_α being the critical values of voltage and current, Equation (28) appears as

$$E_\alpha = \sqrt{E^2 - (I_\alpha R)^2} + \frac{I_\alpha}{\omega C} \tag{34}$$

$$\left(\frac{dE}{dI}\right)_\alpha = \frac{-R^2 I_\alpha}{\sqrt{E^2 - (I_\alpha R)^2}} + \frac{1}{\omega C} \tag{35}$$

Inserting (33), (34) in (32) solving for I

$$I_\alpha = \frac{E}{R}\sqrt{1 - \left(\frac{E}{E_i}\right)^2} \tag{36}$$

C is determined from (33), (35) and (36)

$$C_\alpha = \frac{1}{\omega\left(\tan\alpha + R\sqrt{\left(\frac{E_i}{E}\right)^2 - 1}\right)} \qquad (37)$$

By making use of the expression for current and capacitance the reactive voltages are obtainedw

$$E_{L\alpha} = E\left\{\frac{E}{E_i} + \sqrt{1-\left(\frac{E}{E_i}\right)^2}\left(\frac{\tan\alpha}{R} + \sqrt{\left(\frac{E_i}{E}\right)^2 - 1}\right)\right\} \qquad (38)$$

$$E_{C\alpha} = E\sqrt{1-\left(\frac{E}{E_i}\right)^2}\left(\frac{\tan\alpha}{R} + \sqrt{\left(\frac{E_i}{E}\right)^2 - 1}\right) \qquad n(39)$$

The similarity between $E_{L\alpha}$ and $E_{C\alpha}$ reminds us of the fact that in ferroresonnance, the voltage drop in the inductor equals the voltage drop in the capacitance, during one part of the cycle.

Weber had succeeded to predict graphically the critical values of voltage and current in ferroresonnant circuits.

McCrumm[3] on the other hand studied the effect of ferroresonance from the subharmonic currents point of view. However, a study of the ferroresonant phenomena from this approach would be beyond the scope of this thesis.

Simulation on the computer:

Another solution would be to simulate the physical circuit on an analog computer. However, to be able to simulate this circuit on the computer one needs to know the relation between L and I.

This relation should be simple and capable of being generated on the computer itself.

L versus I:

The proposed solution to this problem is based on the volt ampere (rms) characteristic of the reactor. If we study the circuit of fig. 8 we get the basic relation:

$$V = IZ \qquad (40)$$

where

$$Z = R + jX_L \qquad (41)$$

If

$$X_L \gg R \qquad (42)$$

then

$$X_L = \frac{V}{I} \qquad (42)$$

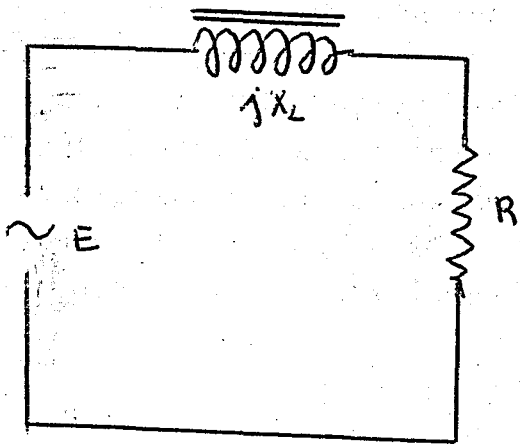


fig.8 Series L R circuit.

It should be understood that the current is 90° out of phase with the voltage.

Referring to fig 9 it is readily seen that, for different values of rms volts: V_1, V_2, and V_3 we get the relations:

$$X_{L1} = \frac{V_1}{I_1}, \quad X_{L2} = \frac{V_2}{I_2} \quad X_{L3} = \frac{V_3}{I_3}$$

If we examine one of these term, we see that it represent the equation of a straight line passing through the origin, and it is in a way the average value of the variation of X_L during one cycle of current. Plotting now the value of X_L versus I_{rms} we get a curve representing $X_L = F(I_{rms})$ but

$$I_{rms} = \sqrt{\frac{I_1^2 + I_2^2 + \cdots + I_n^2}{2}} \qquad (43)$$

where I_1, I_2 and I_n represents the maximum values of the components of the current expressed in Fourier series. Now if we neglect the effect of Harmonics we get:

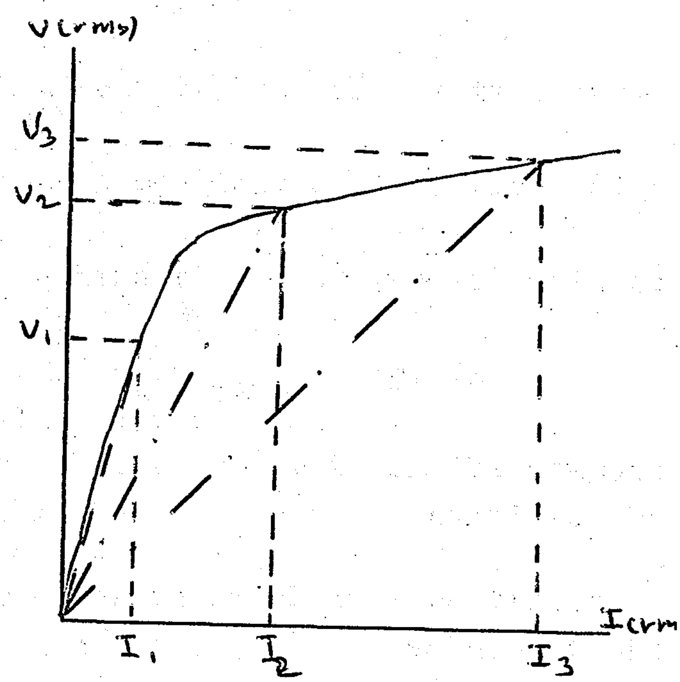


fig9 V. versus I.

$$I_{rms} = \frac{I_m}{\sqrt{2}} \qquad (44)$$

I_m being the maximum value of the current. So it is possible to shift from X_L versus I_{rms} to X_L versus I_m, and furthermore we get also L vs I_m curve. Another approximation is made in saying that the magnitude of L for a certain I_m is the same as its instantaneous value for a certain instantaneous value of I, having the same magnitude as I_m. Therefore it is easy to plot the curve representing L as a function of I. After the curve being traced, it is necessary to find a mathematical relation to fit this curve. This is known as curve fitting. One of the easiest methods is to first guess from the shape of the curve the general form of the mathematical expression needed, then put values from the experiment in it and check if it coincides. From the general shape of our curve, (appendix . Graph 2) an expression of the form:

$$L = \frac{1}{ai^2 + bi + c} \tag{45}$$

should be expected to fit the curve. In the appendix will be found different expression for the constants a, b, and c. The resulting curves are also plotted with the experimental ones. (appendix Graphs 2 and 3).

Solution on the computer:

The basic operations which an electronic differential analyzer can perform are:

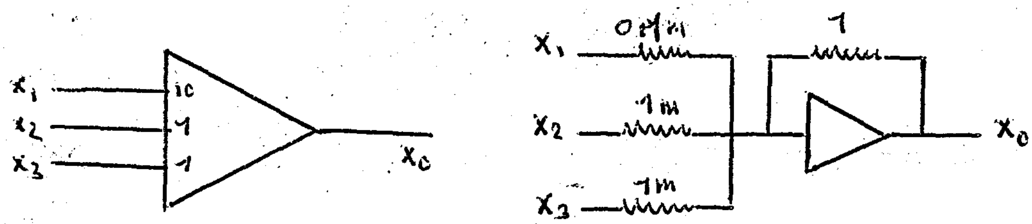
a) Summation, with multiplication by constant coefficients. Sign changes. See fig II a
b) Integration with respect to machine time from initial values, coupled with operation (a). See fig. II b
c) Multiplication by constant coefficients. See fig. IIc
d) Multiplication of two variables. See fig IId

Using these setups it is possible to generate on the computer voltages which would be related in such a way as to simulate the physical relation involved in the problem.

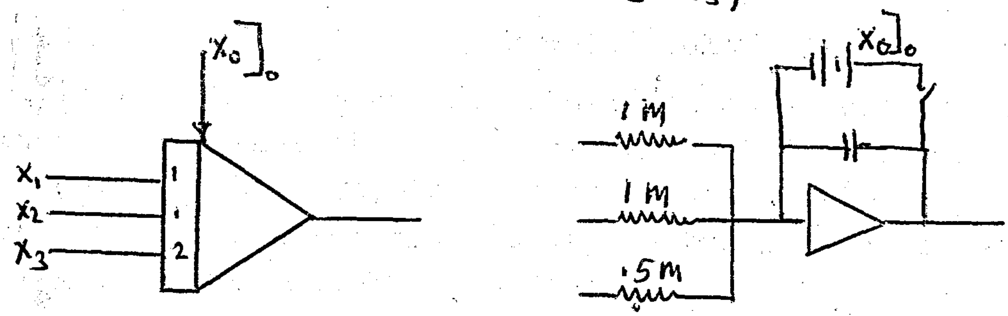
The point is now to pass from the physical differential equation to the machine equation, and to relate the values of the different functions and parameters of the physical system to voltages on the computer. This operation is known as scaling.

The Limitation of the computer:

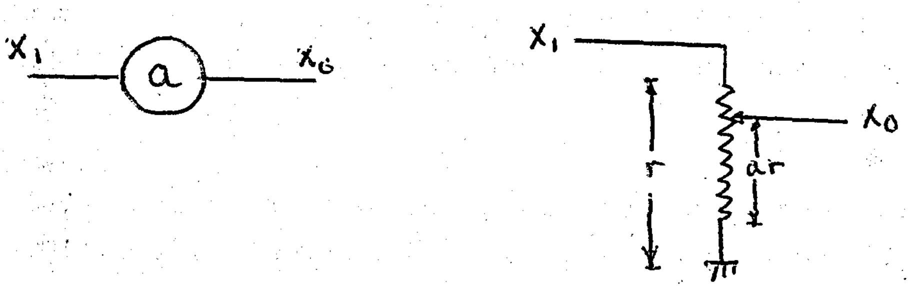
Before introducing scaling it is necessary to say something about the computer and its limitations. The computer used for this study is a DONNER Model 3000 Analog Computer, equipped with a Model 307I Multiplier. I0 operational amplifiers are available. The output is recorded on a Brush Mark II Recorder.



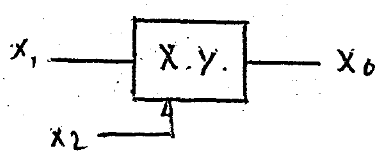
a) $X_0 = -(10X_1 + X_2 + X_3)$



b) $X_0 = -\frac{1}{p}(X_1 + X_2 + 2X_3)$



c) $X_0 = aX_1$



d) $X_0 = \frac{-X_1 X_2}{100}$

Fig II. Operations performed on the computer

It was said that each function appears on the computer as an output voltage of an operational amplifier. This voltage, to be kept in the range of linearity, should never exceed I00 volts. This applies also to the electronic function multiplier, where also the output should never exceed 100 volts. In order that two voltages of 100 volts each fed into the multiplier should not give a product of 10,000 volts, the output of the multiplier in volts is equal to the product of the two inputs divided by hundred .

Example:

$V_1 = 50$ volts $\qquad V_o = \frac{50 \times 20}{100} = 10$ volts

$V_2 = 20$ volts

Due to these limitations it was necessary to introduce the ~~new~~ concept of Machine Unit (MU).[6] If by definition we state that:

$$1 \text{ MU} = 100 \text{ volts} \qquad (46)$$

the limitation of an output of maximum amplitude equal to 100 volts becomes that the output of any amplifier should never exceed 1MU. Furthermore, the output of the multiplier using relation (46) becomes the product of the two inputs expresses in MU. Taking the same example as before

$V_1 = 0.5$ MU $\qquad V_o = 0.5 \times 0.2 = 0.1$ MU

$V_2 = 0.2$ MU

Therefore the introduction of the concept of Machine Unit is justified, since it shows at once the limitation of the operational amplifiers, and simplifies the output of the function multiplier.

Scaking:

The problem is now to relate the different functions involved in the physical system to voltages (expressed as MU)on the computer in such a way as not to have any output greater than 1. The solution is to use the so-called transformation equations. A transformation equation is defined as the relation which changes 1 unit of a function in the physical system to one unit on the computer.

Example:

If we ~~got~~ have a certain variable x, this variable would appear on the computer as a voltage $\mathcal{X}$ related to the physical variab by the relation

$$\mathcal{X} = \alpha_x X. \qquad (47)$$

the term α_x is called the scale factor.

Furthermore it is necessary to introduce also a time factor, that would relate the time in seconds of the physical system to the time on the computer. The use of such a time factor is necessary because the computer can not handle responses of relatively high frequencies.

$$\int \sin \omega t \, dt = -\frac{1}{\omega} \cos \omega t$$

wolud complicate the setup due to the presence of the factor ω
To avoid this difficulty it is advisible to use a transformation equation similar to equation (47)

$$T = \alpha_t t \qquad (48)$$

where t = real time, T equal computer time.

It is possible by choosing properly the term α_t to get only relations in the form

$$\int \sin t \, dt = -\cos t$$

without the appearance of ω.

In order to keep the amplifiers in the range of linearity the output of any amplifier should not exceed 1 MU, as stated previously. The scale factors should then be choosen accordingly. Korn and Korn(6) gives a formula for determining the magnitude of the scale factors:

$$\alpha_x = \frac{1}{\text{Maximum value of } x.} \qquad (49)$$

The general procedure in solving differential equation on an analog computer can be summarised as:

a) Write down the differential equations describing the physical phenomena.
b) Determine all the functions involved in the physical system.
c) Relate them to voltages on the computer by means of scale factors.
d) Write the machine equations.
e) Develop the setup that would be used on the computer. (Block diagram).
f) Determine if any additional, or auxillary, functions appear.
g) If yes, choose their scale factors.
h) Determine the relation between different scale factors.
i) Determine the maximum values of all the functions that would appear on the computer.
j) Choose the magnitude of the scale factors according to restrictions (i) and (h).

the highest differential

Practical setup:

Now that the general theory has been outlined the actual procedure used in setting the differential equation (1) on the computer will be explained.

As a first step equation (1) is rewritten:

$$\frac{d}{dt}(li) + Ri + \frac{1}{c}\int i\,dt = E_m \sin \omega t \qquad (1)$$

Substituing operator $p = \frac{d}{dt}$

$$p(li) + Ri + \frac{i}{pC} = E_m \sin \omega t \qquad (50)$$

assuming C initially uncharged.

The functions will appear in the machine equation with script letters. So the machine equation would be of the form:
(choosing $T = \omega t$)

$$P(\mathscr{L}\mathscr{I}) + \mathscr{R}\mathscr{I} + \frac{\mathscr{I}}{P\mathscr{C}} = \mathscr{E}_m \sin T \qquad (51)$$

As a first approach we assume that there is no capacitance in the physical circuit. Assuming that we have available a voltage representing the term $P(\mathscr{L}\mathscr{I})$, and the following block diagram is suggested:

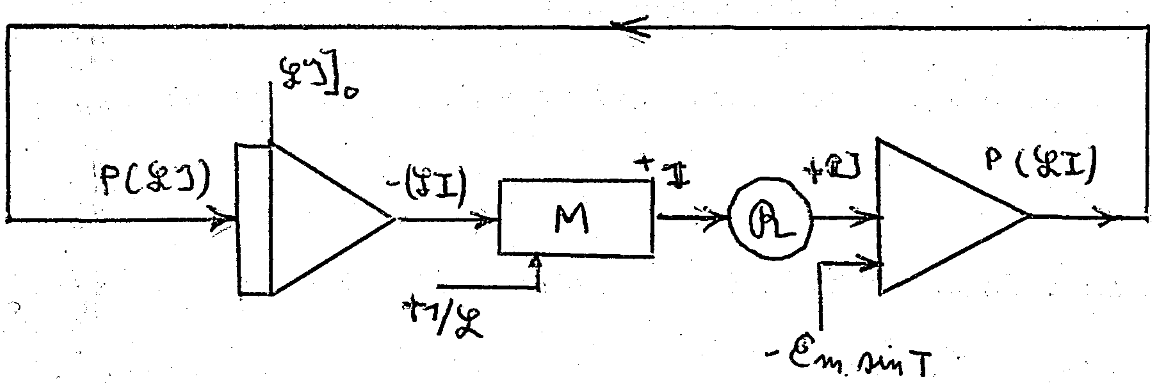


fig. I2 Solution of equation (51)

It is seen that, knowing the term $(\mathscr{L}\mathscr{I})$ for getting $\mathscr{I}$ it is necessary to multiply it by $1/\mathscr{L}$? So on the computer we need to generate the function $1/\mathscr{L}$. In the physical system L was given by equation (45), or

$$1/L = a i^2 + b i + c \qquad (52)$$

Equation (52) would appear in the machine equation as:

$$1/\mathcal{L} = B_1 J^2 + |B_2 I| + B_3 \tag{53}$$

The block diagram of this equation would be:

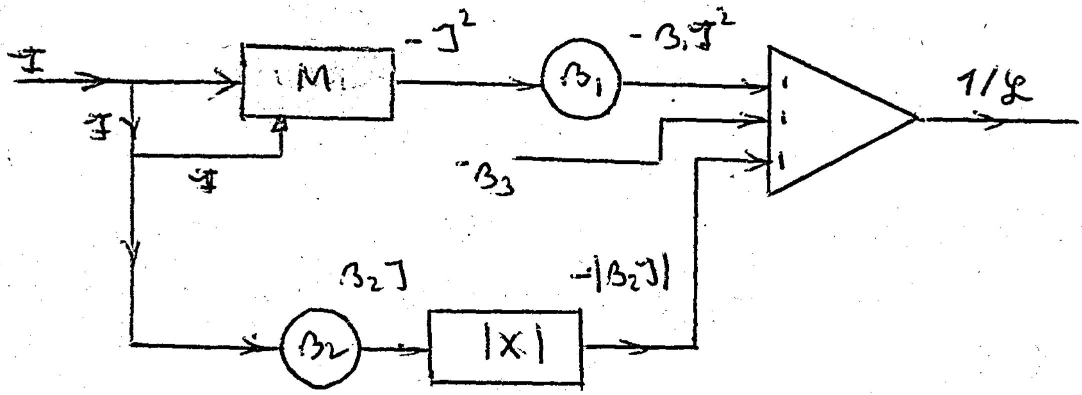


fig. I3 Solution of equation (53)

It is also necessary to generate a sine wave of the form

$$y = \mathcal{E}_m \sin T \tag{54}$$

Such a sine wave is the solution of the differantial equation

$$y'' + y = 0 \qquad \text{at } t = 0 \begin{cases} y = 0 & y' = \mathcal{E}_m \\ & y'' = 0 \end{cases} \tag{55}$$

The block diagram of the sine wave generator is therefore

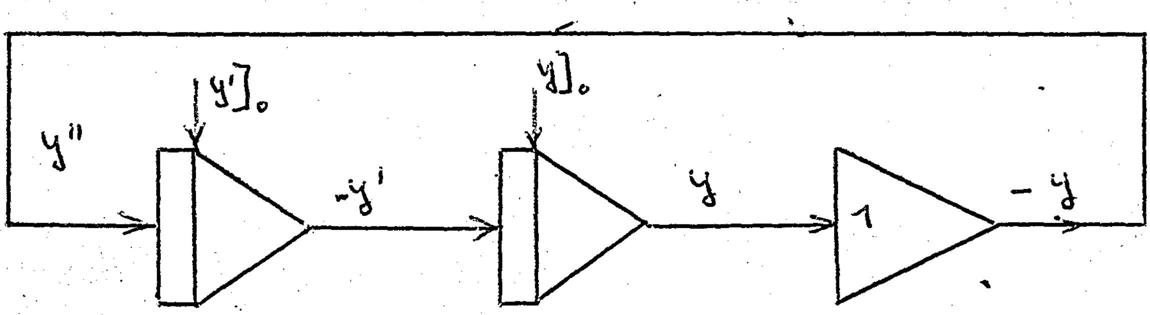


fig. I4 Solution of equation (55)

Introducing capacitance in the physical circuit would have as a result the presence of the term:

$$\mathcal{I}/p\mathcal{C} \quad . \tag{56}$$

In the machine equation, this term should be added to the summing amplifier (fig. I2) with the same sign as $\mathcal{R}\mathcal{I}$. The block diagram of the capacitance voltage drop is therefore:

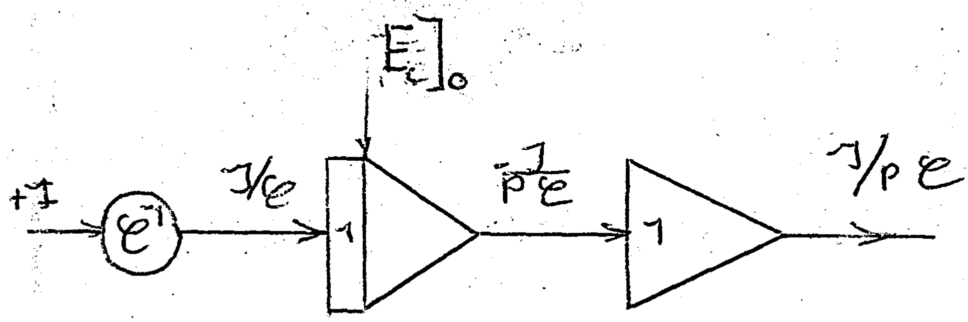


fig. I5 Solution of equation (56).

The point is now to determine the magnitude of the scaling factors, and to calculate the coefficients which would appears on the machine equation.

The functions which appear on the physical system are: I, (LI) and t; therefore on the machine equation three scale factors would appear.

$$\alpha_i \qquad \alpha_{li} \qquad \alpha_t \tag{57}$$

On the other hand we generate on the computer the function $1/\mathcal{L}$ Therefore another scale factor is needed; and the final scale factors are

$$\alpha_{1/L} \qquad \alpha_i \qquad \alpha_{li} \qquad \alpha_t \tag{58}$$

However the relation LI x 1/L would appear as $\mathcal{L}\mathcal{I} \times 1/\mathcal{L}$ therefore we get the relation

$$\mathcal{L}\mathcal{I} \times 1/\mathcal{L} = \mathcal{I} \qquad \qquad \frac{\mathcal{I}}{I} = \alpha_i$$

$$(LI) \times 1/L = I$$

$$\alpha_{(li)} \times \alpha_{(1/l)} = \alpha_i \qquad (59)$$

On the other hand the function 1/L given by the relation (56) is related to the relation (57) by the scale factor $\alpha_{1/l}$. This scale factor affect directly the parameters

$$\beta_1 \; , \; \beta_2 \; , \; \beta_3$$

following the relation:

$$1/\mathcal{L} = \beta_1 \mathcal{I}^2 + \beta_2 \mathcal{I} + \beta_3 \qquad 1/L = a I^2 + b I + c$$

$$\frac{1/\mathcal{L}}{1/L} = \alpha_{1/L} = \frac{\beta_1 \mathcal{I}^2 + \beta_2 \mathcal{I} + \beta_3}{\frac{a I^2}{\alpha_i^2} + \frac{b I}{\alpha_i} + c} \quad \text{but } I = \mathcal{I}/\alpha_i$$

$$\alpha_{1/L} = \frac{\beta_1 \alpha_i^2}{a} = \frac{\beta_2 \alpha_i}{b} = \frac{\beta_3}{c} \qquad (60)$$

Now that wa have determined the relation between the various scale factors, it is necessary to find the maximum expected values of the function involved in order to find the magnitude of the different scale factors.

The maximum values of the different functions are determined in the appendix. According to the different requirements the mag-nitude of the scale factors are choosen as

$$\alpha_i = 0.1 \qquad \alpha_{(li)} = 5 \qquad \alpha_{1/L} = 0.020$$

$$\alpha_t = 100\pi \qquad (61)$$

Now it is possible to pass from equation (50) to equation (51), but it would be interesting to note the effect of the time factor on the differential operator P .

$$\alpha t = T \qquad t = \frac{T}{\alpha} \qquad dt = \frac{dT}{\alpha}$$

$$\frac{d}{dt} = p \qquad \frac{\alpha d}{dT} = p$$

$$p = \alpha_t P \qquad (62)$$

Therefore in passing from (50) to (51) we get:

$$p(LI) + RI + \frac{1}{pC} I = E_m \sin \omega t.$$

$$\alpha_t \frac{P(\mathcal{L}\mathcal{J})}{\alpha_{(LI)}} + \frac{\mathcal{R}\mathcal{J}}{\alpha_i} + \frac{1}{p\mathcal{C}} \frac{\mathcal{J}}{\alpha_t \alpha_i} = E_m \sin \frac{\omega T}{\alpha_t}$$

$$P(\mathcal{L}\mathcal{J}) + \mathcal{R}\mathcal{J} \frac{\alpha(\ell i)}{\alpha_t \alpha_i} + \frac{1}{P\mathcal{C}} \frac{\mathcal{J} \cdot \alpha(\ell i)}{\alpha_t^2 \alpha_i} = \frac{\alpha(\ell i)}{\alpha_t} E_m \sin \omega T/\alpha_t. \qquad (63)$$

Equating identically equation (63) and equation (51) we get

$$\mathcal{R} = R \frac{\alpha(\ell i)}{\alpha_t \alpha_i} = \frac{R}{2\pi}$$

$$\mathcal{C}^{-1} = \frac{1}{C} \cdot \frac{\alpha(\ell i)}{\alpha_t^2 \alpha_i} = \frac{5.06 \times 10^{-4}}{C} \qquad \frac{\omega}{\alpha_t} = 1.$$

$$\mathcal{E}_m = \frac{\alpha(\ell i)}{\alpha_t} E_m = \frac{E_m}{20\pi} \qquad (64)$$

Using relation (60) we get also

$$\beta_1 = \frac{a\,\alpha(1/L)}{\alpha_i^2} = 2a \qquad \beta_2 = \frac{a\,\alpha\,1/L}{\alpha_1} = 0.2\ell.$$

$$\beta_3 = \frac{\alpha\,1/L}{c} = 0.02c \qquad (65)$$

Following the requirements given by (64) and (65) we get:

R = 1 ohm	$\mathcal{R}$ = 0.1595
R = 6.28 ohm	$\mathcal{R}$ = 1.
C = 88 x 10^{-6} Farad	$\mathcal{C}^{-1}$ = 5.75
a = 6.82	β_1 = 13.64
b = 2.20	β_2 = 0.44
c = 2.27	β_3 = 0.0454 MU

<u>Singularities in the practical setup</u>:

Referring to fig 13, generation of the function $1/\mathcal{L}$, we see that the coefficient β_1 has an order of magnitude of 13.64, this would require a potentiometer setting of 0.1364 and an amplification of 100 on the summing amplifier (fig 11 d and a). This could produce unbalance and overloading, furthermore the multiplier works linearly only if the input is nearer ~~than~~ 100 volts than a fraction of a volt. So the following block diagram is suggested:

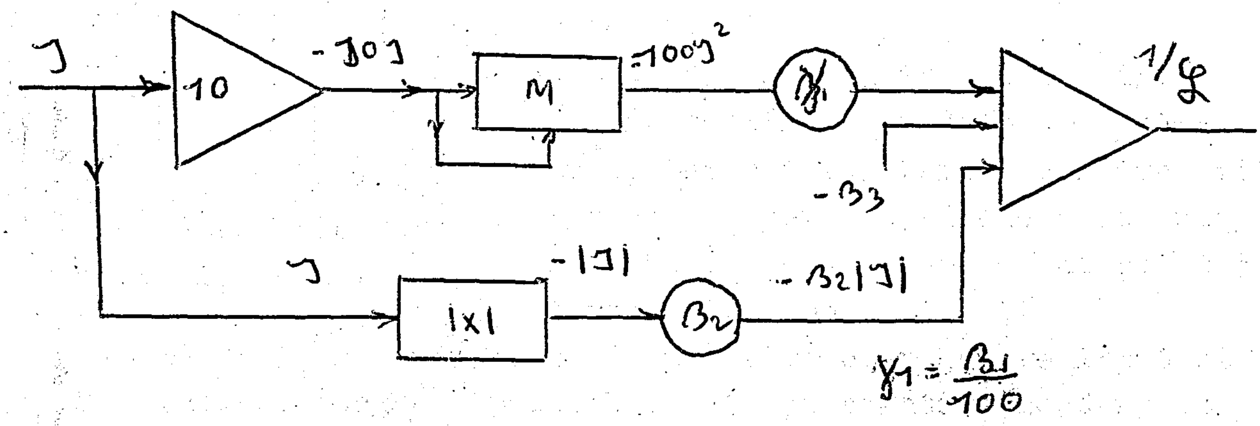


fig I6 Improvement of fig I3

This setup would require an additional operational amplifier, however an economy could be made on fig I5 (Capacitance voltage drop.), due to the fact that $\mathcal{E}^{-1} = 5.75$ which would require a potentiometer setting of 0.575 and an amplification of I0 somewhere. The following block diagram is suggested

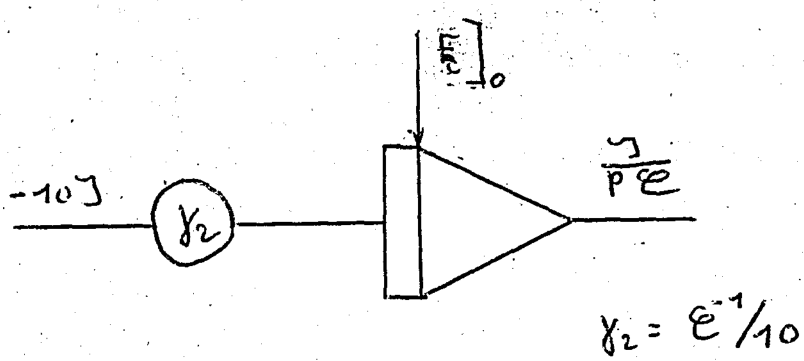


fig .I7 Amelioration of fig. I5

Generation of the absolute value of $\mathcal{J}$:

Referring to fig I8 we see that a box has an input of $\mathcal{J}$ and an output of $-|\mathcal{J}|$. The connection used needs a little ex-planation. Referring to fig I8, we see that if the input V_1

is positive the voltage at point 2, V_2, is negative and equal to V_1 providing the ratio of amplification $\frac{R_{fb}}{R_1} = 1$. The voltage V_2 being negative, the diode (b) act as a short circuit, and the voltage V_2 appears at point 3, point 3 is now negative with respect to 1, and the diode (a) acts as an open circuit, and V_1 cannot appear at 3 through the upper circuit.

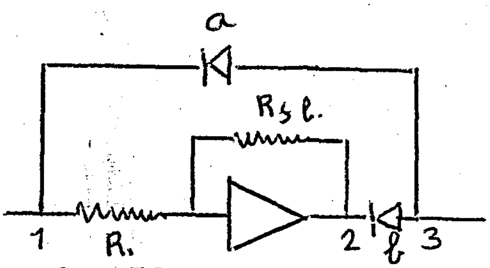


fig I8. Generation of an absolute value.

If V_1 is negative, the point 2 V_2 is positive, diode (b) does not conduct; but point 3 being positive with respect to point 1, diode (a) conducts, and the voltage V_1 appears at point 3 through the upper circuit.

It is seen that for either a positive or negative input at point 1, the voltage at point 3 is always negative leading to the relation:

$$V_3 = -|V_1| \tag{66}$$

If the voltage V_1 is proportional to $\mathfrak{J}$, the output of the amplifier, taking at point 3 is equal to $-|\mathfrak{J}|$.

Now everything is ready to draw the final connection diagram, fig I9. The circuit shown in this figure is supposed to simulate the physical ferroresonnant circuit relative to equation (1).

<u>Note</u>: In fig I9

a) When no initial condition appears on an integrator, that means that at t= 0 V = 0.

b) Amplifiers I and 2 with the multiplier XY correspond to figI2

c) - 7, 8 and IO - UV - - figI6

d) - 9 - - figI7

e) - 3, 4 and 5 - - figI4

Points:

(a) is the output of the $1/\mathfrak{J}$ generator

(b) - - input - - - -

(c) - - input - - capacitance term.

(d) - - output - - - -

(e) - - - - - sine wave generator.

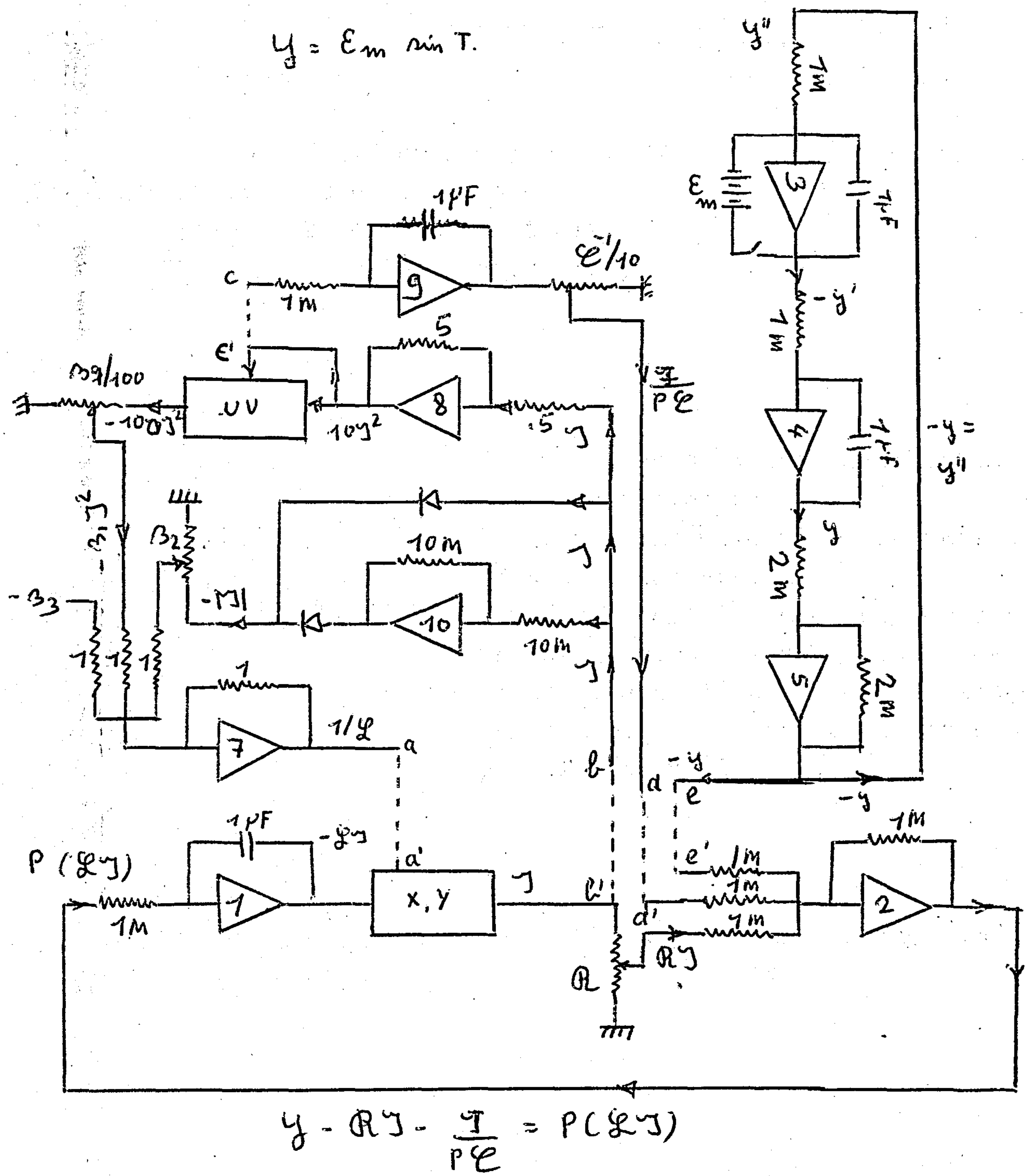


Fig. I9 . Solution of equation (51)

Results:

Part A: Study of a LR circuit, R = I ohm, L = f(I).

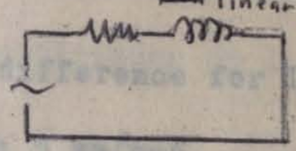
Physical Circuit

E_{rms} = I2.5 V.

E_m = I7.6 V

I_{rms} = 0.I amp.

I_m = 0.14 amp.



Computer

$\mathcal{E}_{rms}$ = .I98 MU ≅ 12.5 V.

$\mathcal{E}_m$ = .28 MU ≅ 17.6 V

$\mathcal{I}_{rms}$ = .0I06 MU ≅ 0.106 amp

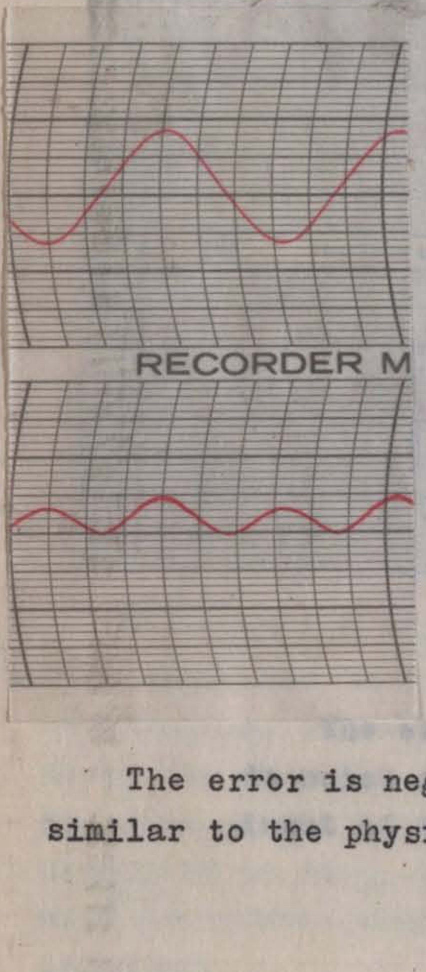
$\mathcal{I}_m$ = .0I5 MU ≅ 0.106 amp

% difference for I = 6%

Oscillogram 1: Speed: 5mm/sec

scale: Current, .2 volt per chart line

L^{-1} , .2 volt per chart line



The upper part of the oscillogram shows the current ($\mathcal{I}$)
The lower part shows the inverse of the inductance. ($\mathcal{L}^{-1}$)

Irregularities and unsymmetrie in the $\mathcal{L}^{-1}$ part is due to the presence of a small dc ~~current~~ voltage which appear as a noise at the output of the multiplier.

The error is negligible, and the shape of the current is similar to the physical current.

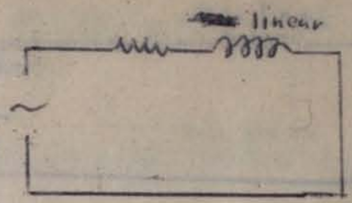
Physical Circuit

E_{rms} = 17.6 V.

E_m = 24.9 V

I_{rms} = 0.16 amp.

I_m = 0.227 amp.



computer

$\mathcal{E}_{rms}$ = .268 MU = 17.6 V

$\mathcal{E}_m$ = .38 MU = 24.9 V

$\mathcal{I}_{rms}$ = .0162 ~~MU~~ = 0.16a

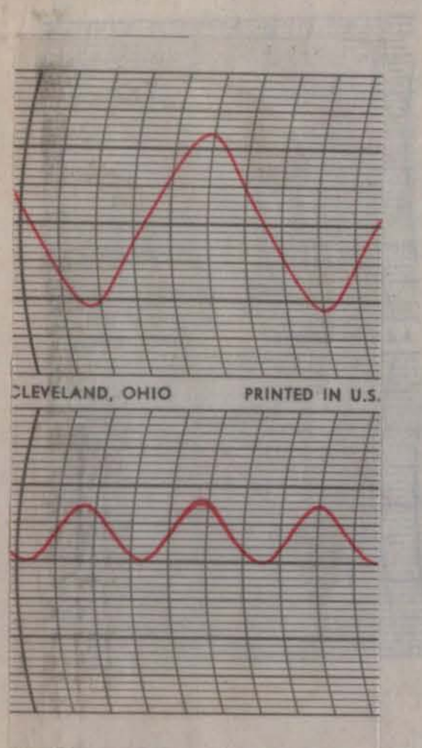
$\mathcal{I}_m$ = .023 MU = 0.23 a

% difference for I = 2%

Oscillogram 2 : Speed: 5 mm/sec

scale: Current, .2 volt per chart line

.2 volt per chart line



Current ($\mathcal{I}$)

Same remark as for oscillogram I

inductance^{-1} ($\mathcal{L}^{-1}$)

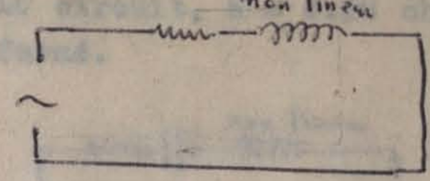
The error is less due to the fact that the relative dc noise of the multiplier is reduced, because the input of the amplifier is increased.

Physical Circuit

E_{rms} = 22.2 V.

E_m = 31.4 V.

I_{rms} = 0.55 amp.



Computer

$\mathcal{E}_{rms}$ = .352 MU $\equiv$ 22.2 Vol

$\mathcal{E}_m$ = .50 MU = 31.4 vol

$\mathcal{J}_{rms}$ = .0247 MU = 0.247

$\mathcal{J}_m$ = .0425 MU = 0.4 am

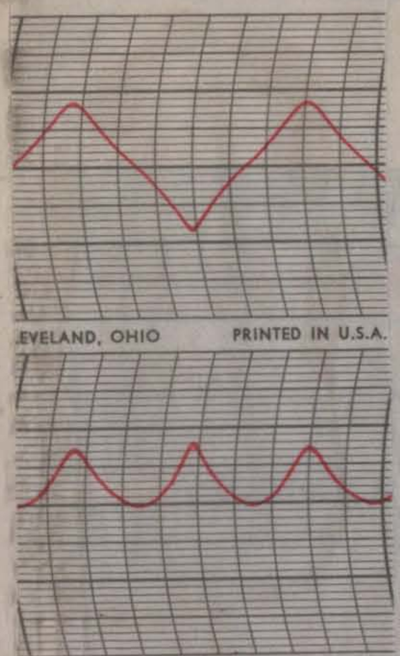
% difference in I = 45.6%

$I_{rms} \approx \frac{I_m}{\sqrt{3}}$

Oscillogram 3 : Speed: 5 mm/sec

scale: Current, .2 volt per chart line

$\mathcal{L}^{-1}$ .5 volt per chart line



Current $(\mathcal{J})$

Inductance^{-1} $(\mathcal{L}^{-1})$

The error is big because the effect of the harmonics have been neglected in approximating L, but as it is seen from the oscillogram, there is a strong harmonice current in the current wave. The shape is similar as seen on the oscilloscope when the current of the physical circuit is examined. However it is necessary to say, that the peaks of the physical current are more pronounced than those in the oscillogram taking from the computer.

Part B: Study of a RLC circuit, R = 6.28 ohm, L = f(I)
C = 88 microfarad.

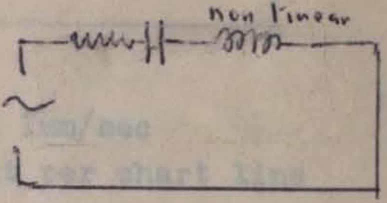
Physical Circuit

E_{rms} = 9.75 V.

E_m = 13.8 V.

I_{rms} = 0.08 amp.

I_m = 0.113 amp.



Computer

$\mathcal{E}_{rms}$ = .155 MU ≈ 9.75 V

$\mathcal{E}_m$ = .22 MU ≅ 13.8 V

I_{rms} = .093 MU ≡ 0.09

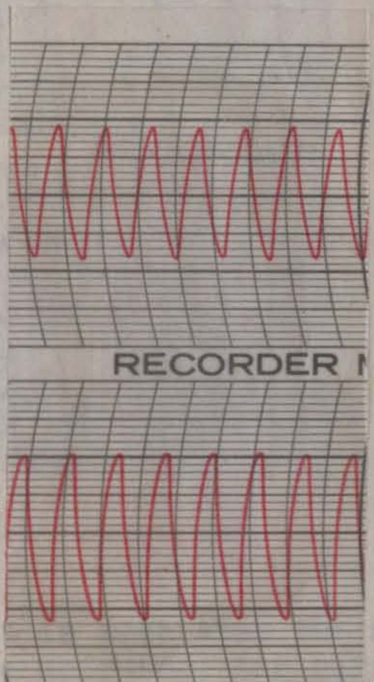
I_m = .016 MU ≡ .16

% difference in I = 7% ~~621.5~~ % $I_{rms} = \frac{I_m}{\sqrt{3}}$

Oscillogram 4 speed: 1 mm/sec

scale: Current, .2 Volt per chart line

Voltage, 2 volt per chart line



Current I

Voltage $\mathcal{E}_m$

The error is due to a first approximation of L, an appreciable source of error is also the fact that the multiplier, XY, drives three amplifiers causing an appreciable voltage drop.

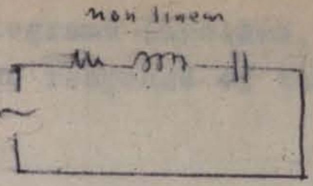
Physical Circuit

E_{rms} ≐ 9.8 Volts

E_m = 13.9 Volts

I_{rms} = 0.09 amp

I_m =



Computer

$\mathcal{E}_{rms}$ = .161 MU ≅ 9.8 V.

$\mathcal{E}_m$ = .23 MU ≅ 13.9 V

$\mathcal{I}_{rms}$ = .354 MU ≅ .125 am

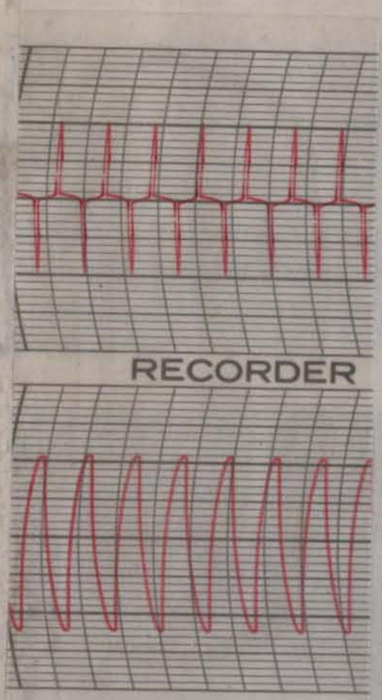
$\mathcal{I}_m$ = .50 MU ≅ .5 am

Oscillogram 5 speed Imm/sec

scale: Current, 5 volt per chart line

voltage, 2 volt per chartlline

$I_{rms} \approx \frac{I_m}{4}$



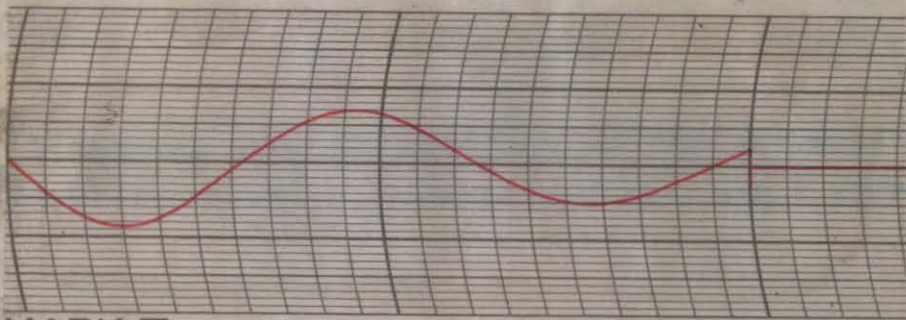
current ($\mathcal{I}$)

voltage ($\mathcal{E}$)

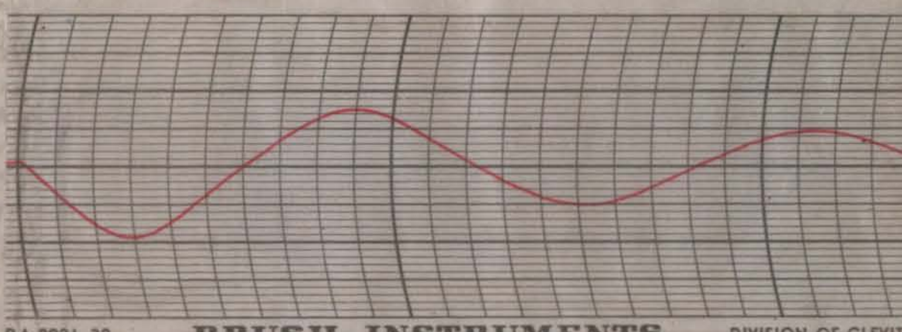
It is seen that the ferroresonnance phenomena is accentued on the computer, the current jumping for a lower voltage (9.8 volts instead of I6) to a much higher value (3.54 amp instead of .6 amp). Furthermore the current on the computer is in reality even higher than the values given now, because overloading of amplifiers 10, and 7 reduce the value of $\mathcal{I}$ .The only reason for not having similar results on the physical circuit and on the computer is a wrong approximation of L, however the fact that the multiplier XY drive three amplifiers should not be overlooked.

The following oscillograms labelaed, oscillogram 6, 7, 8 and 9 respectively, show the response of the network to a dc step impulse.

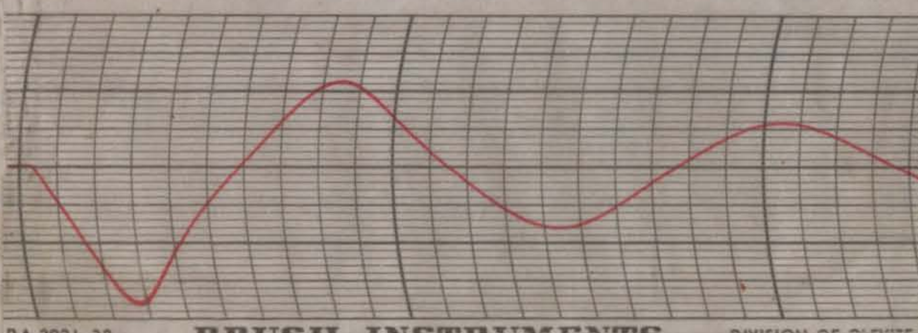
oscillogram	speed	scale	ε	V_{dc}
N°	mm/sec	Volt/ch. Li.	MU	Volt
6	5	0.I	0.1	6.28
7	5	0.2	0.2	I2.56
8	5	0.2	0.3	18.84
9	5	2	0.4	25.12



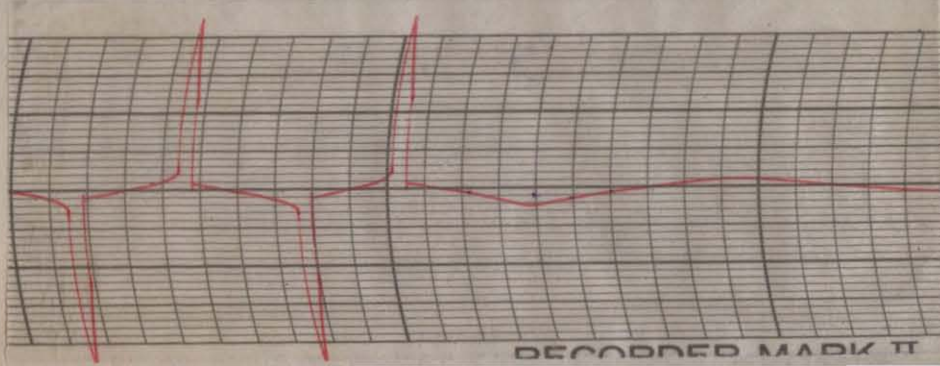
Oscil.6



Oscil. 7



Oscil. 8



Oscil. 9

From these oscillograms it is seen that the natural frequenciy of oscillation is not constant, but varies with the applied dc step impulse. For example, the first oscillation in oscillogram 6 has a period of I2 seconds. The first oscillation in oscillogram 9 last 5.6 sec. Furthermore it is seen that the period of oscillation is not constant at the beginning and the end of the same as oscillogram. Refërring to oscillogram 8, it is seen that the first cycle last 11 sec., the following cycle duration is 12 sec.

The reason of this behaviour is that the natural oscillation frequency of a circuit is given by the relation

$$f = \frac{1}{2\pi\sqrt{LC}} \qquad (67)$$

L varying with the magnitude of the current; as the applied dc impulse increase, L becomes smaller and f increase.

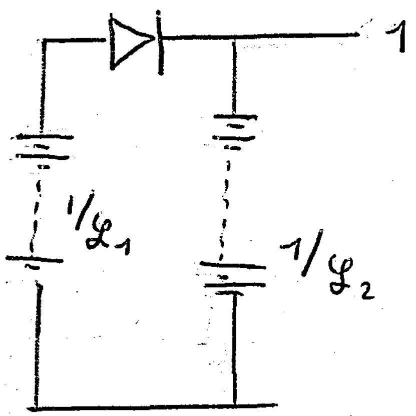
Conclusion:

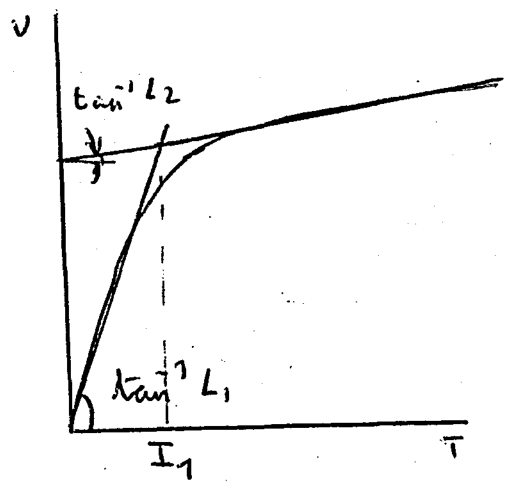
The results are encouraging as a first attempt. It is seen that it is possible to simulate a ferroresonant circuit on an analog computer. The quality of the results depends on the accurac of the approximation of L.

For further studies of ferroresonant circuits, the following steps are suggested.

a) Improved approximation of L, not neglecting the harmonics in the currant wave. Another approach would be to take the slope of the magnetization curve of the inductor, at different values ofI.

b) Approximate L by two straight lines, the circuit suggested is the following:





Take I at point b' of fig I9. This I should closed the contact C, when it reach a predetermined value I_1. For I smaller than I_1, contact C is open and the voltage at point 1 is equal to $1/\mathcal{L}_1$ When I is greater than I_1, contact C is closed, the voltage $1/\mathcal{L}_2$ being greater than $1/\mathcal{L}_1$ the diode does not conduct, and $1/\mathcal{L}_2$ appea at point 1. If point 1 is connected to point a' of fig I9 the computer would simulate a circuit, having a certain inductance L_1 for $I < I_1$, and an inductance L_2 for $I > I_1$

c) Study parallel resonnance, its application as voltage stabilize

Faruk Sarc
B.S. in EE

A P P E N D İ X

The saturable reactor used is an autotransformer

Powerstat

Type 116	Phasel
Pri. V. 120	50/60 cps
Out. V. 0 - I40	
Max. amp. 7.5	max. KVA 1

The Superior Electric Company

Bristol Connecticut U.S.A.

This transformer is uded during the experiments its dial setted on 20 (twenty)., feeded from the output, primary winding open cipcuited.

Its internan dc resistance is 0.5 ohm.

Determination of the magnetisation curve and of L:

The following values had been obtained:

V volt rms	I amp. rms	X_L ohms	L henrys	I amp. max.
6.4	0.05	I28	0.406	0.0707
I0	0.08	I25	0.398	0.II3
I2	0.I0	I20	0.382	0.I4I
I4	0.I2	II6.6	0.37I	0.I70
I6.8	0.I5	II2	0.357	0.2I2
I9.2	0.20	96	0.3055	0.283
20.7	0.30	69	0.22	0.424
2I	0.34	6I.7	0.I965	0.48I
2I.4	0.4	53.5	0.I705	0.565
22	0.5	44	0.I40	0.707
22.5	0.6	37.3	0.II9	0.848
22.8	0.7	32.6	0.I04	0.99
23.2	0.8	29	0.0923	I.II
23.6	0.9	26.2	0.0835	I.27
24	I.0	24	0.0763	I.4I

The circuit used is the circuit of fig. 8 with an ammeter (Evershed N° 108323) in series, and a voltmeter in parallel (Weston Model 433 N° I7302I)

The values of the table were calculated using equations (42) and (44).

Approximation of L vs I:ü

From the graph of L vs I_m some suitable points are taken and substitued in the equation

$$L = \frac{1}{aI^2 + bI + c}$$

Sample calculation:

$L_1 = \frac{I}{aI^2 + bI + c}$ in this approximation we take a = 0

the points chosen are $I = 0$ $L = 0.46$

$I = 0.707$ $L = 0.I4$

we get

$$0.46 = \frac{1}{c} \quad \text{therefore } c = 2.18$$

$$0.14 = \frac{1}{.707 \times b + 2.18} \qquad a = 7.05$$

and $L_1 = \frac{1}{7.05\ I + 2.18}$

5 approximations for L were found, the best one chosen; these approximations are shown on tha accompanying graphs.

$I = 0$ $L = 0.46$
$I = 0.707$ $L = 0.I4$

$$L_1 = \frac{1}{7.05\ I + 2.I8}$$

$I = 0$ $L = 0.46$
$I = 0.2$ $L = 0.354$
$I = 1$ $L = 0.I02$

$$L_2 = \frac{1}{5.5\ I^2 + 2.22\ I + 2.I8}$$

$I = 0$ $L = 0.4I9$
$I = 0.707$ $L = 0.I4$

$$L_3 = \frac{1}{6.74\ I + 2.39}$$

$$L_4^{-1} = \frac{1}{2}(L_3^{-1} + L_2^{-1})$$

$$L_4 = \frac{1}{2.75\ I^2 + 4.34\ I + 2.28}$$

$L = 0$ $L = 0.44$
$I = 0.3$ $L = 0.294$
$I = 0.7$ $L = 0.14$

$$L_5 = \frac{1}{6.82\ I^2 + 2.2\ I + 2.27}$$

Maximum values of the functions appearing on the computer:

$0 \leq I \leq 2$ $\alpha_i \leq 0.5$

$0 \leq L^{-1} \leq 30$ $\alpha_{1/L} \leq 0.033$

$0 \leq (LI) \leq 1/7.5$ $\alpha_{LI} \leq 7.5$

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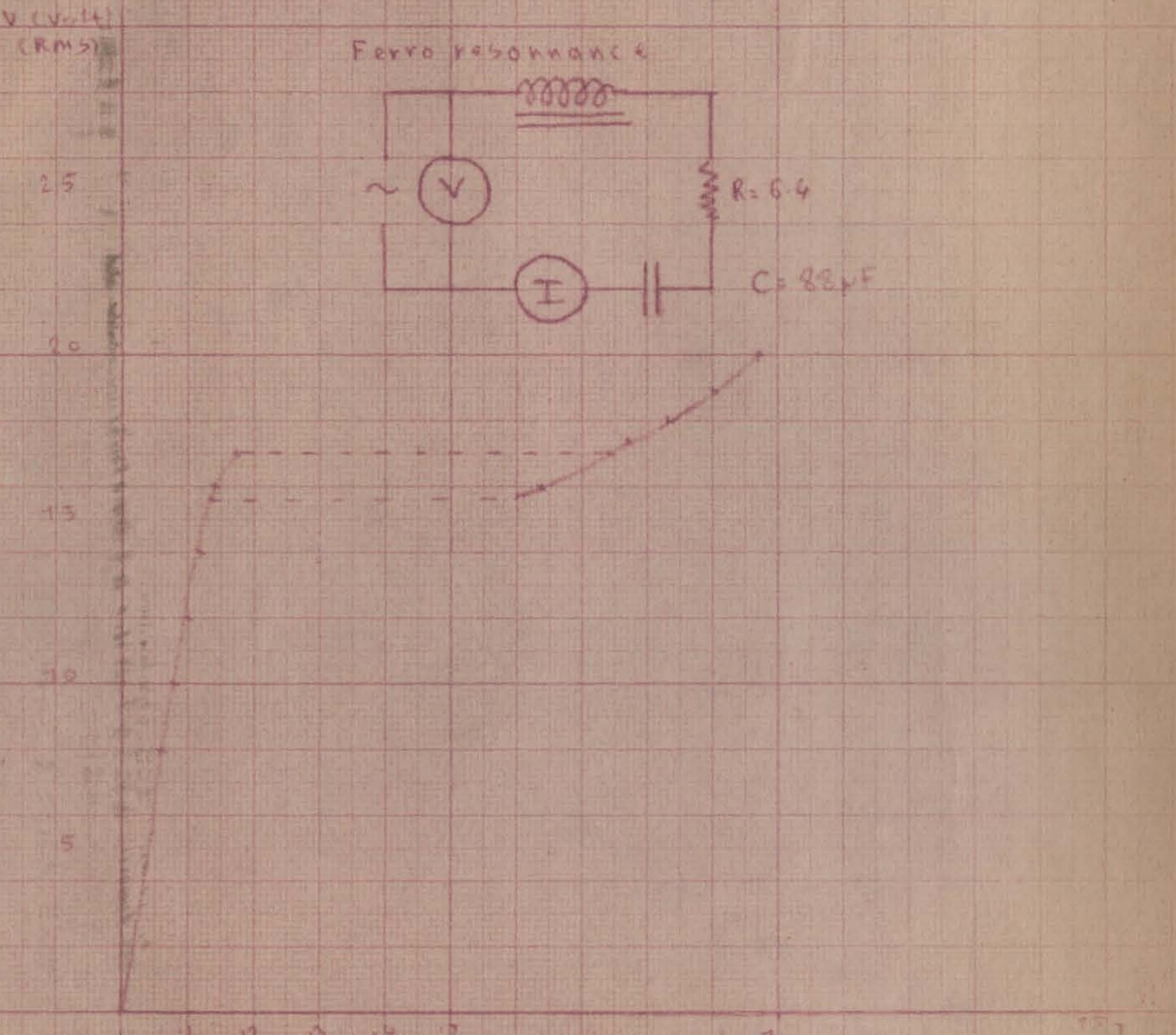
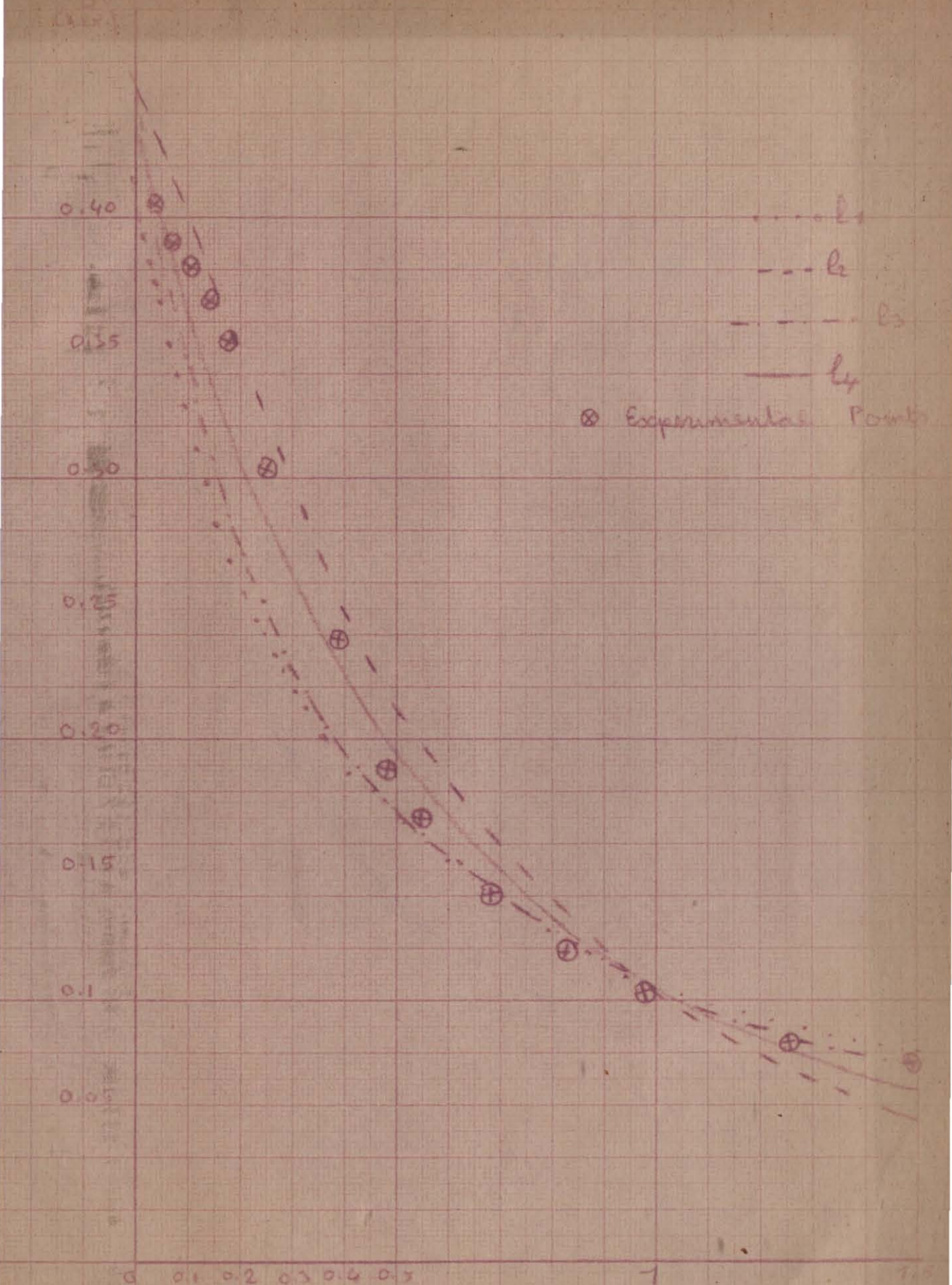
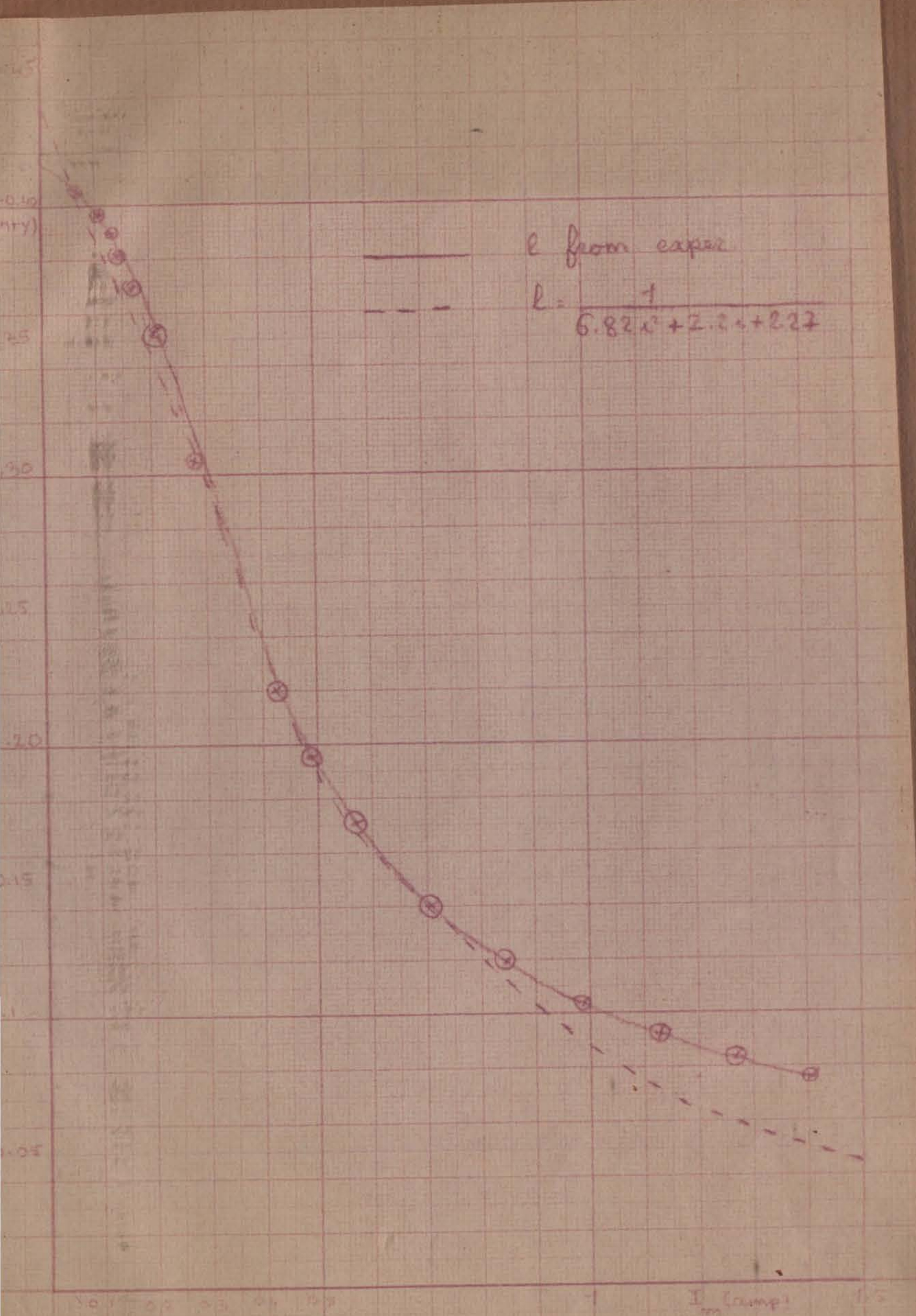


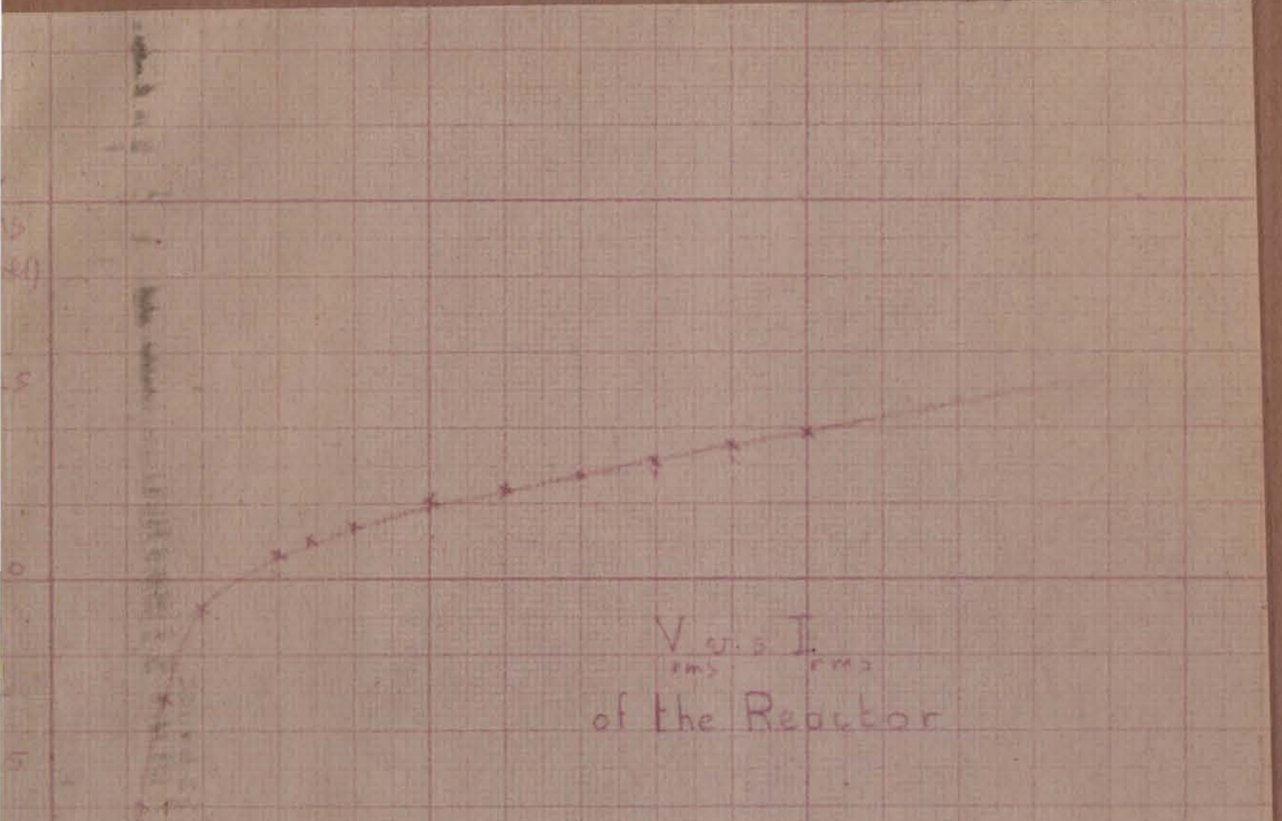
Graph 1


l1
l2
l3
l4
⊗ Experimental Points
0.40
0.35
0.30
0.25
0.20
0.15
0.1
0.05
0
0.1
0.2
0.3
0.4
0.5
1

Graph 2



Graph 13



Graph 4