COMPETITIVE SUPPLY CHAIN MANAGEMENT AMONG NON-IDENTICAL SUPPLIERS UNDER CAPACITY CONSTRAINT

by

Ozan Candeniz

B.S., Mathematical Engineering, Yıldız Technical University, 2013

Submitted to the Institute for Graduate Studies in
Science and Engineering in partial fulfillment of
the requirements for the degree of
Master of Science

Graduate Program in Industrial Engineering
Boğaziçi University
2018

ACKNOWLEDGEMENTS

I would like to express the deepest appreciation to my thesis supervisor Prof. Taner Bilgiç for his keen support, guidance and patience. He is not only a great instructor, but also a wonderful mentor and a good friend. This thesis would not have been possible without his great tolerance, trust and endless help during the work this thesis. It has been an honor and a privilege for me to work with him.

I also would like to thank Prof. Kadri Özçaldıran for encouraging and motivating me to start master program in Boğaziçi University.

I am grateful to my family, Canan and Bahattin for their endless love and support to me throughout my life as well as this thesis.

I am also grateful to my wife Öznur for her patience and tolerance towards me during the hardest times of this thesis.

Last but not least, I really appreciate to my friends, Havva, Antoni, Kadir, Mert, Bahar and Elif for their help and support during my master program.

ABSTRACT

COMPETITIVE SUPPLY CHAIN MANAGEMENT AMONG NON-IDENTICAL SUPPLIERS UNDER CAPACITY CONSTRAINT

A supply chain problem is considered that consists of a major manufacturer and two non-identical suppliers who supply a special product to the manufacturer. The aim of both suppliers is to work with the manufacturer by offering better contract. One of the suppliers is large (possibly works for other manufacturers) and the other one is small (possibly works exclusively for the manufacturer). The market demand of the manufacturer is a known random distribution. By its infrastructure and experience, the big supplier has the production cost advantage compared to the small supplier. The manufacturer constitutes a small part of the business among the big supplier's many customers while she is the main customer of the small supplier. Thus, the big supplier may not have an adequate capacity to satisfy the entire order from the manufacturer. For all practical purposes, it is assumed that the small supplier does not have a capacity restriction. In this study, all events are analyzed from the point of the big supplier, and how the equilibrium behavior of each player are examined when the production capacity of the big supplier is limited. For example, if the capacity of the big supplier is less than the optimal order quantity of the manufacturer to the small supplier, the big supplier does not offer a contract to the manufacturer because of getting negative profit. In other words, the manufacturer prefers to work with the small supplier. Moreover, while the expected profit of the small supplier is equal to zero under full information, the small supplier gets a positive expected profit (as information rent) when his production cost is private. Furthermore, the capacitated big supplier demands a premium because of information asymmetry on the processing cost of the manufacturer. However, this increases the wholesale price that the big supplier offers to the manufacturer, so the big supplier faces the risk of losing the opportunity to work with the manufacturer.

ÖZET

KAPASİTE KISITI ALTINDA ÖZDEŞ OLMAYAN TEDARİKÇİLER ARASINDAKİ REKABETÇİ TEDARİK ZİNCİRİ YÖNETİMİ

Bir ana üretici ve bu üreticiye özel bir ürün sağlayan özdeş olmayan iki tedarikçiden oluşan bir tedarik zinciri problemi ele alınmıştır. Her iki tedarikçinin amacı da daha iyi bir sözleşme önererek üretici ile çalışmaktır. Bunlardan biri büyük (muhtemelen başka üreticilerle de çalışıyor) diğeri ise küçük (muhtemelen yalnızca bir üretici için çalışacak) birer tedarikçidir. Üreticinin pazar talebi ise belirli bir rastgele dağılımdır. Büyük tedarikçi sahip olduğu altyapı ve tecrübe ile küçük tedarikçiye göre üretim maliyeti avantajına sahiptir. Üretici, büyük tedarikçinin birçok müşterisi arasındaki işinin küçük bir kısmını oluştururken, küçük tedarikçinin ise ana müşterisidir. Bu nedenle büyük tedarikçi, üreticiden gelen her siparişi karşılayacak kapasiteye sahip olmayabilir. Tüm pratik amaçlar için, küçük tedarikçinin kapasite kısıtlaması olmadığı varsayılmaktadır. Bu çalışmada olaylar büyük tedarikçi açısından incelenmekte olup, büyük tedarik-çinin üretim kapasitesinin sınırlı olması durumunda her bir aktörün denge davranışı irdelenmiştir. Örneğin, büyük tedarikçinin kapasitesi, üreticinin küçük tedarikçiden talep edeceği en iyi sipariş miktarından küçükse, büyük tedarikçi negatif gelir elde edeceği için üreticiye sözleşme önermez. Bundan başka, tam bilgi altında küçük tedarikçinin beklenen geliri sıfıra eşitken, küçük tedarikçinin üretim maliyeti bilgisi gizliyken beklenen geliri pozitif olmaktadır (information rent). Dahası, üreticinin işlem maliyeti üzerindeki bilginin gizli olması durumunda, kapasitesi sınırlı olan büyük tedarikçi üreticiden bir prim talep eder. Fakat bu durumda büyük tedarikçinin üreticiye sunacağı toptan fiyat artar. Bu yüzden büyük tedarikçi üretici ile çalışma şansını kaybetmek ile yüzleşir.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF SYMBOLS

c_B	Production cost of the big supplier
c_H	Production cost of cost-inefficient small supplier
c_L	Production cost of cost-efficient small supplier
c_s	Production cost of the small supplier
D	Single period market demand to the product
$f()$	Density function of the demand
$F()$	CDF of a random demand for the product
$\bar{F}()$	Inverse function of $F()$
k	Processing cost of the manufacturer
k_H	Processing cost of high-cost manufacturer
k_L	Processing cost of low-cost manufacturer
p_H	Probability of cost-inefficient small supplier - high cost manufacturer
p_L	Probability of cost-efficient small supplier - low cost manufacturer
Q	Order quantity of big supplier with capacity constraint
Q_H	Order quantity of big supplier with high-capacity
Q_L	Order quantity of big supplier with low-capacity
r	Retail price for the product
t_s	Transfer payment for the small supplier
t_{sH}	Transfer payment for the cost-inefficient small supplier
t_{sL}	Transfer payment for the cost-efficient small supplier
T_B	Transfer payment for the big supplier
$T_B(Q)$	Optimal transfer payment for the big supplier with capacity constraint under full information
$T_B^{A1}(Q)$	Optimal transfer payment for the big supplier with capacity constraint when the production cost of the small supplier is private

$T_{BH}^{A2}(Q)$	Optimal transfer payment for the big supplier that offers to the high-cost manufacturer with capacity constraint when the processing cost of the manufacturer is private
$T_{BL}^{A2}(Q)$	Optimal transfer payment for the big supplier that offers to the low-cost manufacturer with capacity constraint when the processing cost of the manufacturer is private
y	Order quantity
w_B	Wholesale price of the big supplier
$w_B(Q)$	Optimal wholesale price of the big supplier with capacity constraint under full information
$w_B^{A1}(Q)$	Optimal wholesale price of the big supplier with capacity constraint when the production cost of the small supplier is private
$w_{BH}^{A2}(Q)$	Optimal wholesale price of the big supplier that offers to the high-cost manufacturer with capacity constraint when the processing cost of the manufacturer is private
$w_{BL}^{A2}(Q)$	Optimal wholesale price of the big supplier that offers to the low-cost manufacturer with capacity constraint when the processing cost of the manufacturer is private
w_s	Wholesale price of the small supplier
w_{sH}	Wholesale price of the cost-inefficient small supplier
w_{sL}	Wholesale price of the cost-efficient small supplier
$\overline{w}_s$	Optimal wholesale price of the manufacturer that offers to the small supplier when the production cost of the small supplier is private

1. INTRODUCTION

There are various dynamics regarding a supplier selection for a buyer (or a customer) in supply chain problems. For different business sectors, these dynamics can be listed such as cost or quality of a product, production capacity of a supplier, timing, technology, confidence, service quality, and even various combinations of these individual variables and so on.

Supply chain relations between a buyer and suppliers cause some problems as well. These problems not only exist somewhere in the world, but also appear on media. Apple, for example, who is one of the most famous electronic device manufacturers, has a trouble with her major processor supplier Qualcomm. Apple claims Qualcomm to demand excessive royalty rate which is five times more than Apple's other suppliers. Federal Trade Commission (FTC) blames Qualcomm to be a monopoly by restraining his competitors. Moreover, Qualcomm counterclaims Apple due to limiting his processors' performance. Therefore, he asserts that iPhones with Qualcomm processors fall behind iPhones with Intel processors in terms of performance. Qualcomm also accuses Apple hiding this information from customers. However, Apple cannot stop working with Qualcomm even if she is damaged materially and morally. All these interactions between conceptual scope and real world motivate us to find the source of problems like Apple – Qualcomm case and then to come up with solutions.

The crucial point in the case between Apple and Qualcomm is that the latter wants to take advantage of being the main supplier of Apple. Probably, Qualcomm's experience and infrastructure makes him a main or a big supplier. Thus, his experience or infrastructure may provide a cost advantage to Qualcomm. As can be seen in the Apple - Qualcomm case, being a big supplier (Qualcomm) provides a power to him over a manufacturer (Apple), and the supplier uses his power in order to increase his profit. On the other hand, the manufacturer also may have outside options which are small suppliers. They perhaps have a cost disadvantage due to lack of experience and infrastructure. Thus, the manufacturer can dictate her contract terms to a small supplier by using the cost disadvantage of the small supplier. This is the environment which we model in general.

In our study, we particularly examine single buyer or a manufacturer (she) and competing suppliers (he) cases. She selects only one of the possible suppliers which are non-identical in terms of production cost. Moreover, we analyze power of an asymmetric information between actors in the chain. In other words, what private information brings to the players as advantages or disadvantages. It is important to note that we mainly focus on the interaction between players if one of the suppliers has limited production capacity. For this reason, we review the related research in Chapter 2 in order to specify scope of the study which we work through. In Chapter 3, we introduce the model and represent it mathematically. Furthermore, the order of events is expressed with the picture of the model that is crucial to understand which actor has the power over the other. Chapter 4 deals with the problem of the big supplier and finds optimal solution to the players in the chain under full information. We analyze capacity effect of the big supplier on supplier selection of the manufacturer. Chapter 5 consists of two different information asymmetry scenarios. The first one occurs between the manufacturer and the small supplier when the small supplier's production cost is undisclosed. The second private information is the manufacturer's processing cost and the information asymmetry is between the manufacturer and the big supplier. We mainly focus on the interaction of these two asymmetric information with the capacity of the big supplier, separately. Finally, we state concluding remarks in Chapter 6.

2. LITERATURE REVIEW

Supplier selection by manufacturer has been studied in many aspects which can be based on types of suppliers (homogeneous or heterogeneous), procurement (single or multiple), market demand (deterministic or stochastic). Moreover, we overview the literature which highlight asymmetric information and effect of the capacity which essential for our study.

Ha *et al.* (2003) study the case of two suppliers who compete to supply a product to a single customer. Suppliers are homogenous and the customer's aim is to minimize her cost by triggering competition between suppliers. The suppliers' competition is based on delivery frequency (indirectly price) which is crucial for the customer since lower interarrival times between deliveries decrease the holding cost of the customer. Therefore, the more frequent supplier gets larger amount of the total order quantity from the customer. However, the transfer cost of suppliers increases as the delivery frequency increases. This dilemma initiates the competition between suppliers over the delivery frequency. The model is as follows: The customer first announces the price for each of the supplier, and then the suppliers specify their delivery frequencies according to the proclaimed prices. Finally, the customer distributes her total order quantity considering the suppliers' delivery frequencies. In addition to this, the problem is solved by backward method in three steps. When the cost function of the customer is written as a function of the demand allocation rate, an exact solution is found for this rate as a function of delivery frequency at the third step. At the second step, the profit functions of each supplier are written as a function of delivery frequencies; so Nash equilibrium of the delivery competition occurs. The optimal delivery frequency solutions for the suppliers follow functions of the prices. At the first step, the price decisions can be found numerically for each supplier due to there is no exact solution to this particular part of the problem. In the same way, the whole problem is solved by fixing the delivery frequencies for each supplier at the beginning of the problem, and then the suppliers compete over the price. The final step is allocating the total order quantity of the customer to the suppliers as usual. As a result, the customer interestingly gets more profit if the competition is over the delivery frequency instead of the price. Furthermore, the competition over the delivery frequency provides control to the customer in order to reach the product with lower prices.

Cachon and Zhang (2007) examine a similar feature in their study which involves two suppliers and a buyer. Cachon and Zhang (2007) is distinguished from Ha *et al.* (2003) in terms of suppliers' form in their research. The latter has two identical suppliers whereas the suppliers are acted as servers in the former one. Another difference between the both investigations is structure of the models. Stochastic models are introduced in the problem of Cachon and Zhang while Ha *et al.* work on deterministic demand. The remaining concept of the problems is very similar. The buyer's purpose is to diminish average delivery lead time of received services from the suppliers by encouraging them to increase their production capacity. In other words, the buyer's order allocation is based on efficiency of the suppliers who determine their capacities or production rates in advance. It is found that investing in productivity of suppliers provides shorter lead time to the buyer. Due to optimal arguments in closed form are hard to reach by using threshold allocation, a simpler allocation is used instead of it. Even if the benchmark allocation is used, it allows the suppliers to produce their maximum.

Delivery frequency is also analyzed from different perspectives. Cachon and Zhang (2006) investigate how lead time speed of suppliers affects a buyer's total cost. The problem consists of a buyer and a group of suppliers. The purpose of the buyer is minimizing her total cost which is sum of purchasing cost of a material and operating cost. The first one is directly related to the price of the component which is delivered by suppliers, and the second one is implicitly related to the delivery lead time of the suppliers. It is important to note that, increase in delivery lead time increases the buyer's holding cost as well as operating cost. Moreover, the buyer is responsible for supplier selection and specifying contract terms. The supplier selection is performed via open bidding in order to dictate the buyer's conditions, so she may decrease her total cost by forcing the chosen supplier to decrease his lead time. The tenor of events are as follows: The buyer knows customer's demand distribution of a product. First of all, she declares her supplier choosing procedure and terms of contract. In the second place, one of the suppliers is selected. According to the scenario, the selected supplier takes the contract, and then he decides his capacity. In the third place, the buyer determines her base stock level according to the supplier's expected capacity. However, certain information regarding the capacity costs is not known by the buyer whereas the suppliers know their costs definitely. Thus, a case of information asymmetry occurs. Finally, the buyer faces with procurement and operating costs as a result of the business and she minimizes her total cost. The optimal sourcing structure is hard to evaluate since it involves nonlinear functions. In order to find some simpler structures, two methods are implemented: a late fee mechanism and a lead time mechanism. By those simpler mechanisms, the supplier who has the lowest capacity cost among the others gets all the information rents. In other words, the most efficient supplier in terms of capacity cost takes the advantage of information asymmetry and the total cost of the buyer does not improve. By the optimal structure, the buyer gets some of the information rents, but the amount is insignificant. As a result, even if the sourcing structure is optimal, the buyer's total cost does not change significantly.

Competition between heterogeneous suppliers takes an important place in the supply chain literature like homogeneous suppliers' practices. Li *et al.* (2015) survey one of its special implementation that involves a retailer and two suppliers. The suppliers are non-identical in terms of their production infrastructures. One of the suppliers is flexible and the other one is efficient. The latter is highly automated; therefore he has high fixed cost and low variable cost. On the other hand, the flexible supplier has low fixed cost and high variable cost due to his less developed production conditions compared to the efficient one. Feature of the retailer can be either low volume or high volume, and this information is not known by the suppliers. The suppliers' objective is to offer the retailer the best menu of contract in order to win the game whereas the retailer wants to maximize her expected profit by using the power of private information. The sequence of events are as follows: The retailer realizes her type based on customer demand, and she keeps this information private. Each supplier offers two different contracts to the retailer in order to capture different types of the retailer. The retailer accepts the contract which maximizes her expected profit, and then she determines her order quantity according to the chosen contract. The acquired supplier delivers the ordered quantity. Finally, the supplier realizes the retailer's volume, and then payments are done with reference to the selected contract. The findings show that the efficient supplier's fixed cost has a great impact on the equilibrium contract menus for both of the high volume and the low volume retailer. The efficient supplier has obvious variable cost advantage compared to the flexible one, if the subject is high volume retailer. For the low volume retailer, the fixed cost of the efficient supplier will be determinative (i.e. if the fixed cost of the efficient one is adequately large, the flexible one may take mean cost advantage). An another important result is that the flexible supplier should focus on acquiring the low volume retailer since low fixed cost and high variable cost are compatible with her type.

Özer and Raz (2011) take the case of heterogeneous suppliers from a different point of view. A supply chain problem has been tackled that consists of a major manufacturer and two non-identical suppliers who supply a special product to the manufacturer. The aim of both suppliers is to work with the manufacturer. One of the suppliers is big (well-known) and the other one is small (unfamiliar). The market demand of the manufacturer is a known random distribution. By its infrastructure and experience, the big supplier has the production cost advantage compared to the small supplier. The manufacturer is candidate to constitute a small part of the business among the big supplier's many customers while she is candidate to be main customer of the small supplier. The order of events are as follows: The big supplier offers a contract to the manufacturer with a wholesale price and a transfer payment. While this bidding procedure, he also considers the manufacturer's other option, the small supplier. The manufacturer can either accept or reject the contract. In the case of acceptance, she orders the quantity which maximizes her expected profit to the big supplier. He produces the order quantity and the reciprocal payments are made at the end of period. If the manufacturer rejects the contract which is offered by the big supplier, then the manufacturer offers a contract to the small supplier with a wholesale price and a transfer payment. The small supplier takes it or leaves it. In this study, all events are analyzed from the point of the big supplier, and how the equilibrium behaviors of each player change are examined in case of information asymmetry on the production cost of the small supplier and the processing cost of the manufacturer, respectively.

Our model can be considered to be a direct extension of this model by imposing a capacity constraint on the big supplier. We concentrate on the effects of capacity on supply chain activities.

3. THE BASE MODEL

In this chapter we present the base model which is common to extended models of Chapters 4 - 5. The manufacturer produces a single product and needs to acquire a specific part of this product from two possible suppliers. One of the suppliers is a large manufacturer of this specific part which we call the "big supplier" and the other one is small. The big supplier acts as a leader and offers a contract to the manufacturer as $(w_B(Q),T_B(Q))$ where $w_B(Q)$ is a wholesale price, $T_B(Q)$ is a transfer payment and Q represents capacity constraint of the big supplier. In the same way, the small supplier's contract consists of w_s and t_s as a wholesale price and a fixed price, respectively. The big supplier has a unit cost advantage compared to the small supplier as $c_B<c_s$, however the big supplier has a limited production capacity. The manufacturer can either choose $(w_B(Q),T_B(Q))$ or (w_s,t_s). Moreover, if the small supplier keeps his cost information private, c_s can be either c_L or c_H where $c_L<c_H$ with probabilities p_L and $p_H = 1 - p_L$. However, the big supplier still has cost advantage as $c_B<c_L<c_H$. The manufacturer also faces unit processing cost k to produce the final product. In a similar manner to the production cost, the unit processing cost can be either k_L for a low cost manufacturer or k_H for a high cost manufacturer if the manufacturer keeps her cost information private. The final product's retail price is r which is fixed and exogenous to the model. The single period market demand of the manufacturer is a random variable with a known continuous distribution F. f and F^{-1} are the probability density function and inverse function of F, respectively.

3.1. Order of Events

The order of events is as follows: The big supplier offers a menu of contract to the manufacturer with a wholesale price and a transfer payment. While building the contract(s), he also considers the manufacturer's other option, the small supplier. The manufacturer can either accept or reject the contract. In the case of acceptance, she orders the quantity which maximizes her expected profit to the big supplier. He produces the order quantity within the bounds of his capacity and the reciprocal payments are made at the end of period. If the manufacturer rejects the contract which is offered by the big supplier, then the manufacturer offers a menu of contract to the small supplier with a wholesale price and a transfer payment that maximizes the manufacturer's expected profit whereas the small supplier gets non-negative profit.

The picture of the model is shown in Figure 3.1.

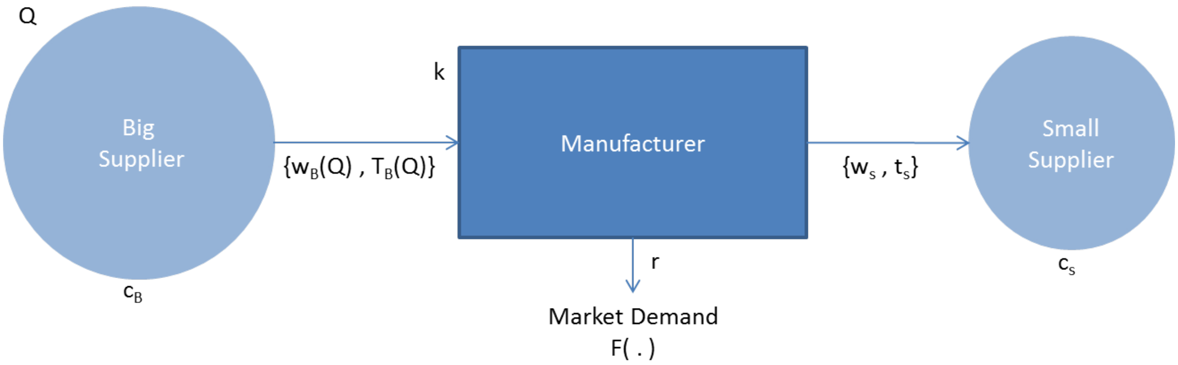

Figure 3.1. Picture of The Model

3.2. Mathematical Representation of The Model

We develop the expected profit of the manufacturer, and then optimize her expected profit.

Expected profit function of the manufacturer is

$$
\begin{aligned}
\Pi_M(y) &= -(w+k)\,y + r\int_0^{\infty} min(u,y)\,f(u)\,du - T \\
&= -(w+k)\,y + r\int_0^{y} u\,f(u)\,du + r\int_y^{\infty} y\,f(u)\,du - T \\
&= [r-(w+k)]\,y - r\int_0^{y} F(u)\,du - T
\end{aligned} \tag{3.1}
$$

where w is the wholesale price and r is the retail price of a specific product,respectively. In addition, k is the processing cost and y is the order quantity of manufacturer, respectively. T is transfer payment from manufacturer to the supplier.

In order to obtain the optimal order quantity of the manufacturer, the first derivative of (3.1) is taken with respect to y and the result is equaled to zero.

The first derivative of $\Pi_M(y)$ with respect to y is

$$\frac{d\Pi_M(y)}{dy} = [r - (w + k)] - r\,F(y) \tag{3.2}$$

Optimal order quantity of the manufacturer is

$$y^*(w,k) = F^{-1}\left(\frac{r - (w + k)}{r}\right) \tag{3.3}$$

The second derivative of $\Pi_M(y)$ with respect to y is

$$\frac{d^2\Pi_M(y)}{dy^2} = -r\,f(y)<0 \tag{3.4}$$

Due to (3.4), $\Pi_M(y)$ is strictly concave in y and therefore $y^*(w,k)$ is a unique maximizer.

Optimal expected profit of the manufacturer is obtained by substituting (3.3) in (3.1). The result is shown below.

$$\Pi_M(y^*(w,k)) - T \tag{3.5}$$

where

$$\Pi_M(y^*(w,k)) = [r - (w + k)]\,y^*(w,k) - r\int_0^{y^*(w,k)} F(u)\,du \tag{3.6}$$

4. CAPACITATED MODEL UNDER FULL INFORMATION

In this chapter, both the production cost of the small supplier (c_s) and the processing cost of the manufacturer (k) are known by the big supplier. The big supplier takes the advantage of this public information. He knows that aim of the small supplier is to get non-negative profit while the manufacturer wants to maximize her expected profit.

4.1. Problem of the Big Supplier

In this section we are going to develop the problem of the big supplier step by step. In order to understand the problem of the big supplier, we have to define what the manufacturer gains if she procures from the small supplier by the manufacturer.

Expected profit of the manufacturer is given as follows by using (3.5):

$$\Pi_M(y^*(w_s, k)) - t_s \tag{4.1}$$

where t_s is transfer payment to the small supplier and w_s is the wholesale price to be paid to the small supplier. It is important to note that the manufacturer has full power over the trade with the small supplier and can dictate w_s and t_s as she wishes.

The manufacturer wants to minimize her costs in order to maximize her expected profit. Therefore, the manufacturer wants to minimize the profit of the small supplier by offering "just enough" profit to stay in the game.

$$\min \Pi_s(w_s, k, t_s) = (w_s - c_s)\, y^*(w_s, k) + t_s$$

s.t.

$$(w_s - c_s)\, y^*(w_s, k) + t_s \geq 0$$

which means the small supplier gets non-negative profit.

As an optimal solution to the problem above, parameters of the best contract that is zero solution for the small supplier from the point of view of the manufacturer is obtained as follows:

$$w_s = c_s \quad \& \quad t_s = 0 \tag{4.2}$$

By using (4.2) expected profit of the manufacturer is $\Pi_M(y^*(c_s, k))$.

Now we are able to write problem of the big supplier with his objective function and constraints as follows. Thereafter, we are going to maximize objective function of the big supplier, so he decides optimal the $w_B(Q)$ and $T_B(Q)$ in order to work with the manufacturer as a function of its capacity Q. In other words, the aim of the big supplier is to offer a better price so that the manufacturer does not prefer to work with the small supplier. The first constraint (4.4) forces the manufacturer's working condition with the big supplier instead of the small supplier. Moreover, the manufacturer cannot order more than the capacity of big supplier which is ensured by the second constraint (4.5).

$$\max_{w_B(Q), T_B(Q)} \Pi_B(w_B(Q), k, T_B(Q)) = (w_B(Q) - c_B)\, y^*(w_B(Q), k) + T_B(Q) \tag{4.3}$$

s.t.

$$\Pi_M(y^*(w_B(Q), k)) - T_B(Q) \geq \Pi_M(y^*(c_s, k)) \tag{4.4}$$

$$y^*(w_B(Q), k) \leq Q \tag{4.5}$$

4.2. Results

This section contains optimal solution of the big supplier's problem and optimal expected profit of the actors in the chain.

4.2.1. Optimal Solution to the Big Supplier's Problem

The optimal solution to the problem above simply can be achieved by making equalities in (4.4) and (4.5) instead of inequalities, if and only if the objective function of the supplier as a function of Q is strictly concave for all Q. We show that this is the case at the end of this section.

$$\Pi_M(y^*(w_B(Q),k)) - T_B(Q) = \Pi_M(y^*(c_s,k)) \tag{4.6}$$

$$y^*(w_B(Q),k) = Q \tag{4.7}$$

Substituting (4.7) into (4.6) and then rewriting $T_B(Q)$ in terms of the profit functions' difference of the manufacturer, we get:

$$\Pi_M(Q) - T_B(Q) = \Pi_M(y^*(c_s,k))$$

$$T_B(Q) = \Pi_M(Q) - \Pi_M(y^*(c_s,k)) \tag{4.8}$$

This determines the transfer payment, the big supplier is willing to offer to the manufacturer.

In order to determine $w_B(Q)$, we use (3.3) and then leave $w_B(Q)$ alone.

$$\begin{aligned}
y^*(w_B(Q),k) = Q = F^{-1}\left(\frac{r-(w_B(Q)+k)}{r}\right) \\
F(Q) = \frac{r-(w_B(Q)+k)}{r} \\
F(Q) = 1 - \frac{w_B(Q)+k}{r} \\
w_B(Q)+k = r[1-F(Q)] \\
w_B(Q) = r[1-F(Q)] - k \qquad (4.9)
\end{aligned}$$

If the demand is more than the capacity of the big supplier, he increases the wholesale price.

We can write objective function of the supplier as a function of Q by substituting (4.8) and (4.9) as follows.

$$\begin{aligned}
\Pi_B(w_B(Q),k,T_B(Q)) &= (w_B(Q)-c_B)\, y^*(w_B(Q),k) + T_B(Q) \\
\Pi_B(Q) &= (r\,[1-F(Q)] - k - c_B)\, Q + \Pi_M(Q) - \Pi_M(y^*(c_s,k)) \qquad (4.10)
\end{aligned}$$

where

$$\begin{aligned}
\Pi_M(Q) &= [r-(w_B(Q)+k)]\, Q - r\int_0^Q F(u)\, du \text{ from (3.6)} \\
&= [r-(r[1-F(Q)]-k+k)]\, Q - r\int_0^Q F(u)\, du \\
&= r\, F(Q)\, Q - r\int_0^Q F(u)\, du
\end{aligned}$$

The first derivative of $\Pi_B(Q)$ with respect to Q is

$$\begin{aligned}
\frac{d\Pi_B(Q)}{dQ} &= -Q\, r\, f(Q) + (r[1-F(Q)] - (c_B+k)) + \frac{d\Pi_M(Q)}{dQ} \\
\frac{d\Pi_B(Q)}{dQ} &= -Q\, r\, f(Q) + (r[1-F(Q)] - (c_B+k)) + r[Q\, f(Q) + F(Q)] - rF(Q) \\
\frac{d\Pi_B(Q)}{dQ} &= r[1-F(Q)] - (c_B+k) \qquad (4.11)
\end{aligned}$$

The second derivative of $\Pi_B(Q)$ with respect to Q is

$$\frac{d^2\Pi_B(Q)}{d^2Q} = -r\,f(Q) < 0 \tag{4.12}$$

Since $\Pi_B(Q)$ is strictly concave for all Q, (4.8) and (4.9) are optimal solutions to the big supplier's problem as follows.

$$w_B(Q) = r[1 - F(Q)] - k \quad \& \quad T_B(Q) = \Pi_M(Q) - \Pi_M(y^*(c_s, k)) \tag{4.13}$$

It is important to note that $w_B(Q) = c_B$ when Q goes to infinity. The big supplier optimally offers the wholesale price as c_B like he does not have a capacity constraint, and gets a positive transfer payment. When $y^*(c_s, k) < Q < y^*(w_B, k)$, $w_B(Q)$ becomes greater than c_B. On the other hand, the transfer payment decreases as long as $\Pi_B(w_B(Q), k, T_B(Q)) > 0$ and $\Pi_M(y^*(w_B(Q), k)) - T_B(Q) > \Pi_M(y^*(w_s, k)) - t_s$ are satisfied. Last but not least, when Q is less than $y^*(c_s, k)$, $\Pi_B(w_B(Q), k, T_B(Q)) > 0$ and $\Pi_M(y^*(w_B(Q), k)) - T_B(Q) > \Pi_M(y^*(w_s, k)) - t_s$ conditions cannot be satisfied together where $\Pi_M(y^*(w_B(Q), k)) - T_B(Q)$ and $\Pi_M(y^*(w_s, k)) - t_s$ are the expected profits when the manufacturer works with the big supplier and the small supplier, respectively.

4.2.2. Optimal Expected Profit of the Actors in the Chain

In order to find the optimal expected profit of the manufacturer and the big supplier, optimal solution in (4.13) is substituted into (3.5) and (4.3).

Optimal expected profit of the manufacturer is

$$\begin{aligned}\Pi_M(y^*(w, k)) - T &= \Pi_M(y^*(w_B(Q), k)) - T_B(Q)\\ &= \Pi(Q) - [\Pi(Q) - \Pi_M(y^*(c_s, k))]\\ &= \Pi_M(y^*(c_s, k))\end{aligned} \tag{4.14}$$

Optimal expected profit of the big supplier is

$$\begin{aligned}\Pi_B(w_B(Q), k, T_B(Q)) &= (w_B(Q) - c_B)\, y^*(w_B(Q), k) + T_B(Q) \\ &= (w_B(Q) - c_B)\, Q + \Pi(Q) - \Pi_M(y^*(c_s, k)) \qquad (4.15)\end{aligned}$$

4.3. Effect of Capacity

In this section, we investigate the effect of the capacity constraint of the big supplier. We examine how capacity constraint of the big supplier affects supplier choice of the manufacturer. In other words, under which conditions the big supplier is able to work to work with the manufacturer. This is our contribution to the literature under this problem settings. First of all, we show how optimal order quantity of the manufacturer changes as wholesale price increases. Using (3.3) we can write:

$$F(y^*(w, k)) = \frac{r - (w + k)}{r} \qquad (4.16)$$

And then taking the first derivative of both sides of (4.16) with respect to w we have:

$$\frac{\partial F(y^*(w, k))}{\partial w} = -\frac{1}{r} \qquad (4.17)$$

As a result, (4.17) shows optimal order quantity of the manufacturer decreases as w increases. To put this information with cost advantage of the big supplier over the small supplier as $c_B < c_s$, makes the big supplier more profitable than the small supplier for the manufacturer unless the big supplier has capacity constraint.

What capacity levels of the big supplier makes him attractive to work with the manufacturer? We illustrate this is in Figure 4.1.

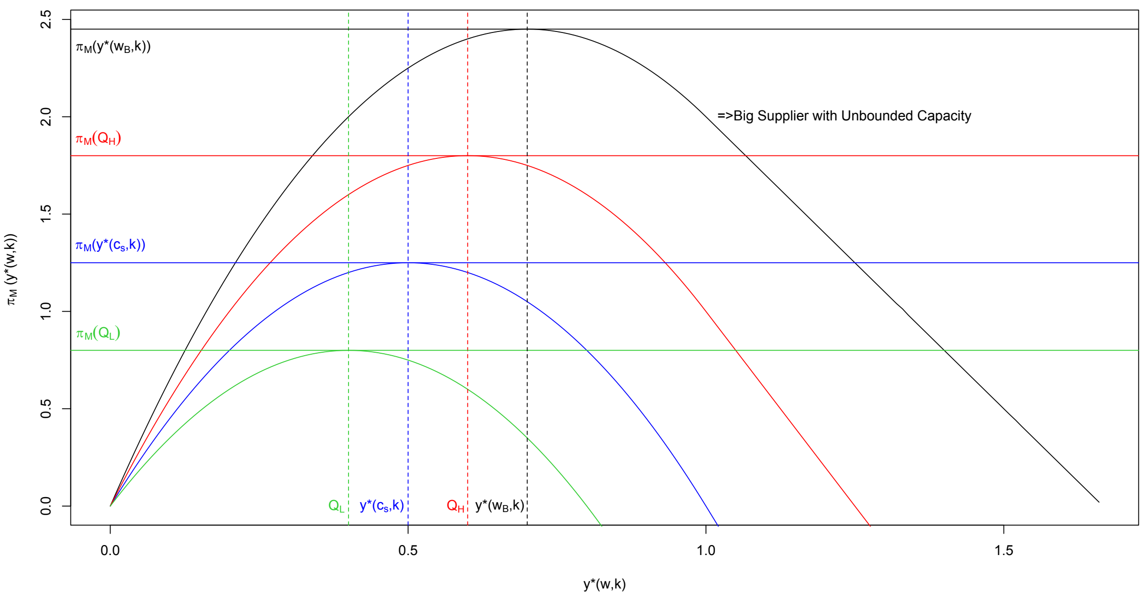

Figure 4.1. Optimal Order Quantity and Expected Profit of The Manufacturer Under Full Information

The manufacturer's optimal expected profit is $\Pi_M(y^*(c_s, k))$ when she optimally acquires $y^*(w_s, k)$ from the small supplier. In a similar way, when the big supplier does not have a capacity constraint, the optimal order quantity and the optimal expected profit of the manufacturer are $y^*(w_B, k)$ and $\Pi_M(y^*(c_B, k))$, respectively. This is also the best possible condition among others as it is illustrated in Figure 4.1.

There are three possible cases for capacity intervals of the big supplier.

4.3.1. Case 1

First of all, when the production capacity of the big supplier is less than $y^*(w_s, k)$, then the small supplier is selected by the manufacturer.

$$
\begin{gathered}
Q_L < y^*(c_s, k) \\
\Pi_M(Q_L) < \Pi_M(y^*(c_s, k))
\end{gathered}
\tag{4.18}
$$

Result: The manufacturer prefers to work with the small supplier because capacity constraint of the big supplier leads to price disadvantage for him. For this case, $w_B(Q_L) > w_s$ where $w_B(Q_L) = r[1 - F(Q_L)] - k$ and $w_s = c_s$. Please see Figure 4.1

4.3.2. Case 2

Secondly, when the big supplier's production capacity is Q_H which is somewhere between $y^*(w_s, k)$ and $y^*(w_B, k)$, the manufacturer acquires goods from the big supplier as whole quantity of Q_H at a price of $w_B(Q_H)$.

$$\begin{gathered} y^*(c_s, k) < Q_H < y^*(w_B, k) \\ \Pi_M(Q_H) > \Pi_M(y^*(c_s, k)) \end{gathered} \tag{4.19}$$

Result: $w_B(Q_H) = r[1 - F(Q_H)] - k$ and $w_s = c_s$ are optimal wholesale prices of the big supplier and the small supplier, respectively and $w_B(Q_H) < w_s$. Since the big supplier is the better option for the manufacturer, he has the right to work with the manufacturer. Please see Figure 4.1

4.3.3. Case 3

Lastly, when capacity of the big supplier goes beyond $y^*(w_B, k)$, the manufacturer obviously prefers to work with the big supplier and she acquires the unconstrained $y^*(w_B, k)$ since the optimal order quantity is obtained at that point.

$$\begin{gathered} Q > y^*(w_B, k) \\ \Pi_M(Q) = \Pi_M(y^*(w_B, k)) \end{gathered} \tag{4.20}$$

Result: When capacity constraint of the big supplier Q is greater than $y^*(w_B, k)$, the optimal order quantity is equal to $y^*(w_B, k)$. Thus, production capacity of the big supplier becomes redundant, and he works with the manufacturer. Please see Figure 4.1.

5. CAPACITATED MODEL UNDER ASYMMETRIC INFORMATION

In this chapter, we examine how asymmetric information affects optimal expected profits of the actors in the chain and supplier selection of the manufacturer when the big supplier has limited capacity. In order to see that, we focus on two different types of asymmetric information, separately: The production cost of the small supplier (c_s) is private and the processing cost of the manufacturer (k) is private. The big supplier still wants to win the game while the manufacturer and the small supplier can use the power of their private information.

5.1. The Production Cost of the Small Supplier is Private

In this section, we consider the case where the processing cost of the manufacturer (k) is known, production cost of the small supplier (c_s) is private by the big supplier and the manufacturer. In Chapter 4, the manufacturer sets her objective to give zero profit to the small supplier by the help of public c_s, easily. Therefore, the big supplier takes the advantage of the small supplier's public information regarding production cost. The big supplier designs his problem settings to leave better profit to the manufacturer than the small supplier. When c_s is known, it is easy to set a constraint to pass what the manufacturer gets if she uses this constant production cost (actually wholesale price). However, c_s is unknown in this chapter by the big supplier and the manufacturer, so c_s cannot be taken constant anymore. Without loss of generality, we assume two type of small suppliers are possible in terms of production cost. One type has low production cost (c_L) with probability p_L, and the second type has high production cost (c_H) with probability p_H where $p_L + p_H = 1$. It is important to note that, $c_B < c_L < c_H$. Following this, the manufacturer should consider these different cost types of small supplier. Thus, she has to design a menu of contract which consists of a wholesale price and a fixed transfer payment for both types of small supplier. The manufacturer offers (w_{sL}, t_{sL}) for the low cost small supplier with probability p_L and (w_{sH}, t_{sH}) for the high cost small supplier with probability p_H.

The picture of the model when c_s is private is shown in Figure 5.1.

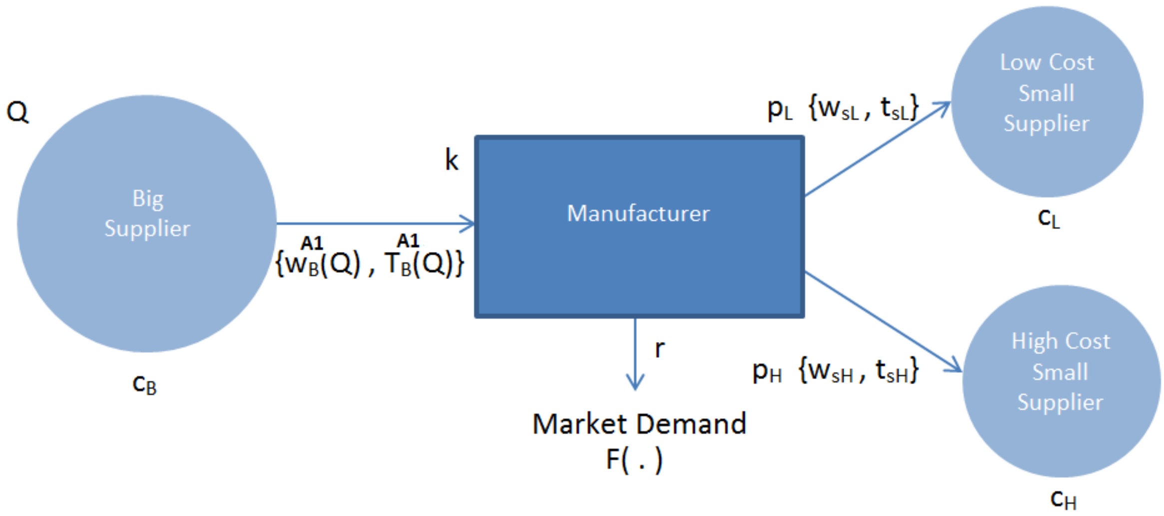

Figure 5.1. Picture of The Model When c_s is Private

5.1.1. Problem of the Manufacturer

We develop the manufacturer's problem. Her objective consists of two possible contracts with their relevant probabilities. Moreover, she should make sure that both type of small suppliers be in the game and none of them should choose the contract which has been designed for the other one.

The manufacturer wants to maximize her expected profit as follows:

$$\max_{w_{sL},w_{sH},t_{sL},t_{sH}} \Pi_M[(w_{sL},t_{sL}),(w_{sH},t_{sH})] = p_L[\Pi_M(y^*(w_{sL},k)) - t_{sL}] + p_H[\Pi_M(y^*(w_{sH},k)) - t_{sH}] \tag{5.1}$$

s.t.

$$(w_{sL} - c_L)\ \ y^*(w_{sL}, k) + t_{sL} \geq 0 \tag{5.2}$$

$$(w_{sH} - c_H)\ \ y^*(w_{sH}, k) + t_{sH} \geq 0 \tag{5.3}$$

$$(w_{sL} - c_L)\ \ y^*(w_{sL}, k) + t_{sL} \geq (w_{sH} - c_L)\ \ y^*(w_{sH}, k) + t_{sH} \tag{5.4}$$

$$(w_{sH} - c_H)\ \ y^*(w_{sH}, k) + t_{sH} \geq (w_{sL} - c_H)\ \ y^*(w_{sL}, k) + t_{sL} \tag{5.5}$$

where (5.2) and (5.3) are participation constraints. In other words, both low cost and high cost small suppliers get non-negative profits. Moreover, (5.4) and (5.5) ensure that each type of small supplier choose the contract that is designed for himself (incentive compatibility constraints).

In order to solve the problem above, first redundant constraints should be eliminated. Since $c_L{<}c_H$ together with (5.3), then the right hand side of (5.4) is greater than zero. To put $(w_{sH}{-}c_L)\ \ y^*(w_{sH}, k){+}t_{sH}\ > \ 0$ with (5.2), (5.2) becomes redundant since it is not a border of feasible region anymore.

Now we have two constraints which are (5.3) and (5.4) at hand. The optimal solution to the problem above simply can be obtained by making equalities in (5.3) and (5.4) instead of inequalities. This is justified by the fact that the manufacturer wants to give as little as possible to the inefficient (high cost) supplier and wants to work with the efficient (low cost) supplier. Therefore, the transfer payments can be left alone, and then the objective function can be written as a function of the wholesale prices as follows.

$$(w_{sH} - c_H)\ \ y^*(w_{sH}, k) + t_{sH} = 0 \tag{5.6}$$

$$(w_{sL} - c_L)\ \ y^*(w_{sL}, k) + t_{sL} = (w_{sH} - c_L)\ \ y^*(w_{sH}, k) + t_{sH} \tag{5.7}$$

Let us leave transfer payments alone and plug them into the objective function of the manufacturer.

$$
\begin{aligned}
t_{sH} &= (c_H - w_{sH})\, y^*(w_{sH}, k) \ \text{ which is equal to negative information rent} \qquad (5.8)\\
t_{sL} &= (w_{sH} - c_L)\, y^*(w_{sH}, k) + t_{sH} - (w_{sL} - c_L)\, y^*(w_{sL}, k) \ \text{ where } t_{sH} \text{ is shown above}\\
&= (w_{sH} - c_L)\, y^*(w_{sH}, k) + (c_H - w_{sH})\, y^*(w_{sH}, k) - (w_{sL} - c_L)\, y^*(w_{sL}, k)\\
&= (c_H - c_L)\, y^*(w_{sH}, k) - (w_{sL} - c_L)\, y^*(w_{sL}, k) \qquad (5.9)
\end{aligned}
$$

where $(c_H - c_L)\, y^*(w_{sH}, k)$ in (5.9) is information rent paid to the efficient (low cost) supplier.

We are able to write the objective function of the manufacturer as a function of w_{sL} and w_{sH} by using (5.8) and (5.9) as follows.

$$
\begin{aligned}
\Pi_M[(w_{sL}), (w_{sH})] =& p_L[\Pi_M(y^*(w_{sL}, k)) - (c_H - c_L)\, y^*(w_{sH}, k) + (w_{sL} - c_L)\, y^*(w_{sL}, k)]\\
&+ p_H[\Pi_M(y^*(w_{sH}, k)) - (c_H - w_{sH})\, y^*(w_{sH}, k)] \qquad (5.10)
\end{aligned}
$$

Now let us rewrite (5.10) as a function of w_{sL} and w_{sH} separately, and then maximize following equations in order to find optimal w_{sL} and w_{sH}. The problem is separated for two types of suppliers:

$$
\begin{aligned}
\max_{w_{sL}} \Pi_M[(w_{sL})] =& p_L[\Pi_M(y^*(w_{sL}, k)) + (w_{sL} - c_L)\, y^*(w_{sL}, k)] \qquad (5.11)\\
\max_{w_{sH}} \Pi_M[(w_{sH})] =& p_H[\Pi_M(y^*(w_{sH}, k)) + (w_{sH} - c_H)\, y^*(w_{sH}, k)]\\
&+ p_L[(c_L - c_H)\, y^*(w_{sH}, k)] \qquad (5.12)
\end{aligned}
$$

In order to find the optimal wholesale price of the low cost small supplier, let us take the derivative of (5.11) with respect to w_{sL}, and then equate it to zero as follows.

$$\begin{aligned}
\frac{\partial \Pi_M[(w_{sL})]}{\partial w_{sL}} &= p_L \left[\frac{\partial \Pi_M(y^*(w_{sL},k))}{\partial w_{sL}} + \frac{\partial[(w_{sL}-c_L)\, y^*(w_{sL},k)]}{\partial w_{sL}} \right] \\
&= p_L \left[\frac{\partial \Pi_M(y^*(w_{sL},k))}{\partial w_{sL}} + y^*(w_{sL},k) + \frac{\partial y^*(w_{sL},k)}{\partial w_{sL}}(w_{sL}-c_L) \right] \\
&= p_L \left[-y^*(w_{sL},k) + y^*(w_{sL},k) - \frac{1}{r\, f(y^*(w_{sL},k))}(w_{sL}-c_L) \right] \\
&= -\frac{p_L\,(w_{sL}-c_L)}{r\, f(y^*(w_{sL},k))} = 0
\end{aligned}$$

where $p_L > 0$, $r > 0$ and $f(y^*(w_{sL},k)) > 0$. The only part make this equation equal to zero is $(w_{sL}-c_L)$. As a result,

$$w_{sL} = c_L \tag{5.13}$$

In similar way, in order to find optimal wholesale price of the high cost small supplier, let us take the derivative of (5.12) with respect to w_{sH}, and then equate it to zero as follows. It important to note that in (5.12), $(w_{sH}-c_H)\, y^*(w_{sH},k)$ is information rent paid to the efficient supplier.

$$\begin{aligned}
\frac{\partial \Pi_M[(w_{sH})]}{\partial w_{sH}} =& p_L \left[(c_L-c_H)\frac{\partial y^*(w_{sH},k)}{\partial w_{sH}} \right] \\
&+ p_H \left[\frac{\partial \Pi_M(y^*(w_{sH},k))}{\partial w_{sH}} + y^*(w_{sH},k) + \frac{\partial y^*(w_{sH},k)}{\partial w_{sL}}(w_{sH}-c_H) \right] \\
=& p_L(c_L-c_H)\frac{\partial y^*(w_{sH},k))}{\partial w_{sH}} + p_H \frac{\partial y^*(w_{sH},k)}{\partial w_{sL}}(w_{sH}-c_H) \\
=& \frac{\partial y^*(w_{sH},k))}{\partial w_{sH}}[p_L(c_L-c_H) + p_H(w_{sH}-c_H)] \\
=& \frac{1}{r\, f(y^*(w_{sH},k))}[p_L(c_L-c_H) + p_H(w_{sH}-c_H)] = 0
\end{aligned}$$

where $r > 0$ and $f(y^*(w_{sH}, k)) > 0$. Therefore, $[p_L(c_L - c_H) + p_H(w_{sH} - c_H)] = 0$. When the wholesale price is left alone on the left hand side,

$$w_{sH} = \frac{(c_H - c_L)}{p_H} + c_L \;>\; w_{sL} \tag{5.14}$$

In order to develop transfer payments of the low cost and high cost small suppliers, first of all let us plug $w_{sL} = c_L$ into (5.9) that follows:

$$t_{sL} = (c_H - c_L)\, y^*(w_{sH}, k) \tag{5.15}$$

And then, t_{sH} is obtained by substituting (5.14) in (5.8), so the result as follows:

$$t_{sH} = -\frac{(c_H - c_L)}{p_H}\, p_L\, y^*(w_{sH}, k) \;<\; 0 \tag{5.16}$$

The optimal wholesale price of the manufacturer is somewhere between c_L and c_H since optimal expected profit of the manufacturer $\Pi_M(y^*(w,k))$ is convex in w. In other words, the optimal expected profit of the manufacturer is decreasing in w. In order to indicate $\Pi_M(y^*(w,k))$ is convex, let us derivate it with respect to w. We know $\Pi_M(y^*(w,k)) = [r - (w+k)]\, y^*(w,k) - r \int_0^{y^*(w,k)} F(u)\, du$ from (3.6) .

$$\begin{aligned}
\frac{\partial \Pi_M(y^*(w,k))}{\partial w} =& r\, \frac{\partial y^*(w,k)}{\partial w} - k\, \frac{\partial y^*(w,k)}{\partial w} - \left[y^*(w,k) + w\, \frac{\partial y^*(w,k)}{\partial w} \right] \\
& - r\, F(y^*(w,k))\, \frac{\partial y^*(w,k)}{\partial w} \\
=& \frac{\partial y^*(w,k)}{\partial w} [r - (w+k)] - y^*(w,k) - r\, F(y^*(w,k))\, \frac{\partial y^*(w,k)}{\partial w}
\end{aligned}$$

where $y^*(w,k) = F^{-1}\left(\frac{r-(w+k)}{r}\right)$ from (3.3) and thus, $F(y^*(w,k)) = \frac{r-(w+k)}{r}$. The equation above as follows:

$$\begin{aligned}\frac{\partial \Pi_M(y^*(w,k))}{\partial w} &= \frac{\partial y^*(w,k)}{\partial w}[r-(w+k)] - y^*(w,k) - r\left(\frac{r-(w+k)}{r}\right)\frac{\partial y^*(w,k)}{\partial w} \\ &= -y^*(w,k) \qquad (5.17)\end{aligned}$$

Second derivative of $\Pi_M(y^*(w,k))$ is as follows:

$$\frac{\partial^2 \Pi_M(y^*(w,k))}{\partial w^2} = -\frac{\partial y^*(w,k)}{\partial w} < 0$$

Since $y^*(w,k) = F^{-1}\left(\frac{r-(w+k)}{r}\right)$,

$$\frac{\partial^2 \Pi_M(y^*(w,k))}{\partial w^2} > 0 \qquad (5.18)$$

Due to the second derivative of $\Pi_M(y^*(w,k))$ with respect to the w is greater than zero, $\Pi_M(y^*(w,k))$ is convex in w. That demonstrates the optimal wholesale price of the manufacturer, $\overline{w}_s$, is between c_L and c_H.

The optimal expected profit of the manufacturer is $\Pi_M(y^*(\overline{w}_s,k))$ as follows:

$$\Pi_M(y^*(\overline{w}_s,k)) = p_L[\Pi_M(y^*(w_{sL},k)) - t_{sL}] + p_H[\Pi_M(y^*(w_{sH},k)) - t_{sH}] \qquad (5.19)$$

In order to abbreviate the equation above, (5.13), (5.15) and (5.16) are substituted into (5.19).

$$\begin{aligned}\Pi_M(y^*(\overline{w}_s,k)) =& p_L[\Pi_M(y^*(w_{sL},k)) - t_{sL}] + p_H[\Pi_M(y^*(w_{sH},k)) - t_{sH}] \\ =& p_L[\Pi_M(y^*(c_L,k)) - (c_H - c_L)\, y^*(w_{sH},k)] \\ &+ p_H\left[\Pi_M(y^*(w_{sH},k)) + \frac{(c_H - c_L)}{p_H}\, p_L\, y^*(w_{sH},k)\right] \\ =& p_L \Pi_M(y^*(c_L,k)) + p_H \Pi_M(y^*(w_{sH},k)) \qquad (5.20)\end{aligned}$$

where $\overline{w}_s \in (c_L, c_H)$.

5.1.2. Problem of the Big Supplier

In this section, we develop the big supplier's problem where the manufacturer faces the problem of Section 5.1.1. And then, we maximize his objective function, hence the big supplier determines optimal $w_B^{A1}(Q)$ and $T_B^{A1}(Q)$ in order to provide better profit than the small supplier to the manufacturer as long as the capacity of the big supplier is sufficient.

$$\max \Pi_B(w_B^{A1}(Q), k, T_B^{A1}(Q)) = (w_B^{A1}(Q) - c_B)\, y^*(w_B^{A1}(Q), k) + T_B^{A1}(Q) \tag{5.21}$$

s.t.

$$\Pi_M(y^*(w_B^{A1}(Q), k)) - T_B^{A1}(Q) \geq \Pi_M(y^*(\overline{w}_s, k)) \tag{5.22}$$

$$0 \leq y^*(w_B^{A1}(Q), k) \leq Q \tag{5.23}$$

where (5.22) and (5.23) are working constraint of manufacturer with the big supplier and capacity constraint of the big supplier, respectively.

5.1.3. Optimal Solution to the Big Supplier's Problem

In order to find the optimal solution to the big supplier's problem, inequalities in (5.22) and (5.23) can be changed to equalities because of concavity. It is important to note that for the wholesale price to be optimal, the big supplier's objective function has to be concave over Q. This is shown at the end of this section. The only difference between Chapters 4 and 5 is right hand side of (4.6) and (5.22). The small supplier does not have any information advantage in (4.6) and the small supplier is given only his production cost by the manufacturer whereas the small supplier takes the advantage of private cost information in (5.22), and thus he receives better profit if the manufacturer decides to work with the small supplier.

The constraints become:

$$\Pi_M(y^*(w_B^{A1}(Q),k)) - T_B^{A1}(Q) = \Pi_M(y^*(\overline{w}_s,k)) \tag{5.24}$$

$$y^*(w_B^{A1}(Q),k) = Q \tag{5.25}$$

Substituting (5.25) into (5.24) and then rewriting $T_B^{A1}(Q)$ in terms of the profit functions' difference of the manufacturer, we get:

$$\Pi_M(Q) - T_B^{A1}(Q) = \Pi_M(y^*(\overline{w}_s,k))$$

$$T_B^{A1}(Q) = \Pi_M(Q) - \Pi_M(y^*(\overline{w}_s,k)) \tag{5.26}$$

where $T_B^{A1}(Q)$ is the transfer payment that the big supplier is willing to pay to the manufacturer.

In order to determine $w_B^{A1}(Q)$, we use (3.3) and then leave $w_B^{A1}(Q)$ alone.

$$y^*(w_B^{A1}(Q),k) = Q = F^{-1}\left(\frac{r-(w_B^{A1}(Q)+k)}{r}\right)$$

$$F(Q) = \frac{r-(w_B^{A1}(Q)+k)}{r}$$

$$F(Q) = 1 - \frac{w_B^{A1}(Q)+k}{r}$$

$$w_B^{A1}(Q)+k = r[1-F(Q)]$$

$$w_B^{A1}(Q) = r[1-F(Q)] - k \tag{5.27}$$

Herein if the manufacturer's demand is greater than capacity of the big supplier, he increases the wholesale price as long as $w_B^{A1}(Q) < \overline{w}_s$.

The objective function of the big supplier can be written as a function of Q by substituting (5.26) and (5.27) as follows. Moreover, the big supplier determines the wholesale price according to the manufacturer orders the entire capacity of the big supplier since concavity.

$$\Pi_B(w_B^{A1}(Q), k, T_B^{A1}(Q)) = (w_B^{A1}(Q) - c_B)\, y^*(w_B^{A1}(Q), k) + T_B^{A1}(Q)$$
$$\Pi_B(Q) = (r\,[1 - F(Q)] - k - c_B)\, Q + \Pi_M(Q) - \Pi_M(y^*(\overline{w}_s, k)) \tag{5.28}$$

where

$$\begin{aligned}\Pi_M(Q) &= [r - (w_B^{A1}(Q) + k)]\, Q - r\int_0^Q F(u)\, du \text{ from (3.6)}\\ &= [r - (r[1 - F(Q)] - k + k)]\, Q - r\int_0^Q F(u)\, du\\ &= r\, F(Q)\, Q - r\int_0^Q F(u)\, du\end{aligned}$$

The first and the second derivatives of $\Pi_B(Q)$ with respect to Q are same as (4.11) and (4.12), and the results are as follows, respectively.

$$\frac{d\Pi_B(Q)}{dQ} = r[1 - F(Q)] - (c_B + k) \quad \& \quad \frac{d^2\Pi_B(Q)}{d^2Q} = -r\, f(Q) \tag{5.29}$$

Since $\Pi_B(Q)$ is strictly concave for all Q, (5.26) and (5.27) are optimal solutions to the big supplier's problem as follows.

$$w_B^{A1}(Q) = r[1 - F(Q)] - k \quad \& \quad T_B^{A1}(Q) = \Pi_M(Q) - \Pi_M(y^*(\overline{w}_s, k)) \tag{5.30}$$

It is crucial to say that $w_B^{A1}(Q) = c_B$ when Q goes to infinity. The big supplier acts like he has unbounded capacity and sets the wholesale price to c_B, and gains a positive transfer payment. When $y^*(\overline{w}_s, k) < Q < y^*(w_B, k)$, $w_B(Q)$ becomes grater than c_B. On the other hand, the transfer payment decreases (it can even be negative) as long as $\Pi_B(w_B^{A1}(Q), k, T_B^{A1}(Q))$ is greater than zero and $\Pi_M(y^*(w_B^{A1}(Q), k)) - T_B^{A1}(Q) > \Pi_M(y^*(\overline{w}_s, k)) - t_s$ conditions are satisfied. Last but not least, when Q is less than $y^*(\overline{w}_s, k)$, $\Pi_B(w_B^{A1}(Q), k, T_B^{A1}(Q)) > 0$ and $\Pi_M(y^*(w_B^{A1}(Q), k)) - T_B^{A1}(Q) > \Pi_M(y^*(\overline{w}_s, k)) - t_s$ conditions cannot be satisfied together where $\Pi_M(y^*(w_B^{A1}(Q), k)) -$

$T_B^{A1}(Q)$ and $\Pi_M(y^*(\overline{w}_s,k)) - t_s$ are the expected profits when the manufacturer works with the big supplier and the small supplier, respectively.

5.1.4. Optimal Expected Profit of the Actors in the Chain

In order to find the optimal expected profit of the manufacturer and the big supplier, optimal solution in (5.30) is substituted into (3.5) and (5.21).

The manufacturer's optimal expected profit is

$$\begin{aligned}\Pi_M(y^*(w,k)) - T &= \Pi_M(y^*(w_B^{A1}(Q),k)) - T_B^{A1}(Q)\\ &= \Pi(Q) - [\Pi(Q) - \Pi_M(y^*(\overline{w}_s,k))]\\ &= \Pi_M(y^*(\overline{w}_s,k)) \qquad (5.31)\end{aligned}$$

where $\Pi_M(y^*(\overline{w}_s,k)) = p_L\Pi_M(y^*(c_L,k)) + p_H\Pi_M(y^*(w_{sH},k))$ from (5.20) and $\overline{w}_s \in (c_L,c_H)$.

The big supplier's optimal expected profit is

$$\begin{aligned}\Pi_B(w_B^{A1}(Q),k,T_B^{A1}(Q)) &= (w_B^{A1}(Q) - c_B)\, y^*(w_B^{A1}(Q),k) + T_B^{A1}(Q)\\ &= (w_B^{A1}(Q) - c_B)\, Q + \Pi(Q) - \Pi_M(y^*(\overline{w}_s,k)) \qquad (5.32)\end{aligned}$$

The first part of the expected profit of the big supplier, $(w_B^{A1}(Q) - c_B)$ is always greater than or equal to zero. The big supplier sets higher wholesale price in order to use his whole the capacity since the concavity of the big supplier's objective function over Q. However, the big supplier gets lower transfer payment, $\Pi(Q) - \Pi_M(y^*(\overline{w}_s,k))$ which can be even negative as long as his expected profit stays a positive. In any case, the capacitated big supplier's expected profit cannot be grater than when he does not have capacity constraint which means $\Pi_B(w_B,k,T_B)$ is always greater than or equal to $\Pi_B(w_B^{A1}(Q),k,T_B^{A1}(Q))$.

In case of the big supplier does not offer a contract to the manufacturer because of getting a negative profit (actually limited capacity), the small supplier's optimal expected profit is as follows.

$$\begin{aligned}\Pi_s[(w_{sL}, k, t_{sL}), (w_{sH}, k, t_{sH})] =& p_L[(w_{sL} - c_L)\, y^*(w_{sL}, k) + t_{sL}] \\ &+ p_H[(w_{sH} - c_H)\, y^*(w_{sH}, k) + t_{sH}]\end{aligned}$$

by substituting (5.13), (5.14), (5.15) and (5.16) into the equation above, we get:

$$\begin{aligned}\Pi_s[(w_{sL}, k, t_{sL}), (w_{sH}, k, t_{sH})] =& p_L[(c_L - c_L)\, y^*(w_{sL}, k) + t_{sL}] \\ &+ p_H[(w_{sH} - c_H)\, y^*(w_{sH}, k) + t_{sH}] \\ =& p_L\, t_{sL} + p_H[(w_{sH} - c_H)\, y^*(w_{sH}, k)] + p_H\, t_{sH} \\ =& p_L\, t_{sL} + p_H[(w_{sH} - c_H)\, y^*(w_{sH}, k)] \\ &- (c_H - c_L)\, p_L\, y^*(w_{sH}, k) \\ =& p_L\, t_{sL} + p_H[(w_{sH} - c_H)\, y^*(w_{sH}, k)] - p_L\, t_{sL} \\ =& p_H[(w_{sH} - c_H)\, y^*(w_{sH}, k)] \qquad (5.33)\end{aligned}$$

or

$$\begin{aligned}\Pi_s[(w_{sL}, k, t_{sL}), (w_{sH}, k, t_{sH})] =& p_H[(w_{sH} - c_H)\, y^*(w_{sH}, k)] \\ =& p_H\left[\left(\frac{(c_H - c_L)}{p_H} + c_L - c_H\right) y^*(w_{sH}, k)\right] \\ =& (c_H - c_L)\,(1 - p_H)\, y^*(w_{sH}, k) \\ =& p_L\, t_{sL} \qquad (5.34)\end{aligned}$$

So the small supplier gets a positive expected profit (as information rent).

5.1.5. Effect of Capacity

We examine the effect of the big supplier's capacity when the small supplier's cost information is private in this section. We have already shown that optimal order quantity of the manufacturer decreases as wholesale price increases in (4.17). As long as the big supplier takes cost advantage compared to the small supplier as $c_B < c_L < c_H$, the manufacturer acquires goods from the big supplier when he does not have capacity limitation.

Figure 5.2 demonstrates which capacity points of the big supplier makes him more profitable than the small supplier for the manufacturer.

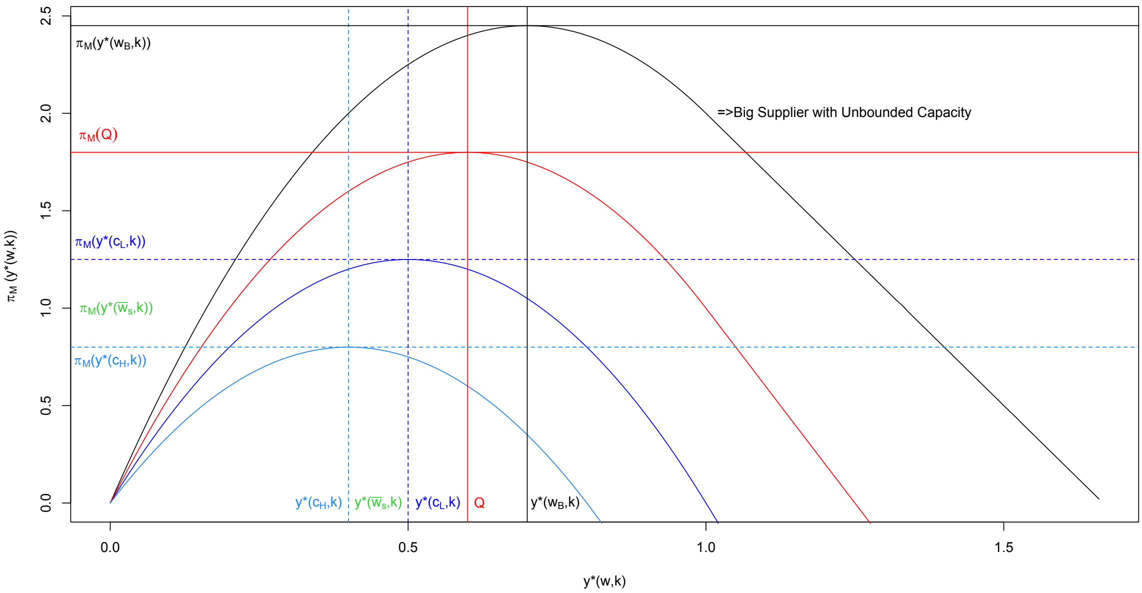

Figure 5.2. Optimal Order Quantity and Expected Profit of The Manufacturer When c_s is Private

The manufacturer's optimal expected profit is $\Pi_M(y^*(\overline{w}_s, k))$ when she optimally acquires $y^*(\overline{w}_s, k)$ from the small supplier. Herein, $\Pi_M(y^*(c_H, k)) < \Pi_M(y^*(\overline{w}_s, k)) < \Pi_M(y^*(c_L, k))$. Moreover, the optimal order quantity, $y^*(\overline{w}_s, k)$, is also between $y^*(c_L, k)$ and $y^*(c_H, k)$ where $y^*(c_H, k) < y^*(c_L, k)$. These are natural outcomes of (3.3) due to $c_L < c_H$. Last but not least, the best possible profit, $\Pi_M(y^*(c_B, k))$, in terms of the manufacturer is obtained when the big supplier delivers by $y^*(w_B, k)$ which means the big supplier does not have capacity constraint.

The possible conditions for the big supplier's capacity is in different intervals are listed below.

In the first case, limited production capacity forces the big supplier to determine high wholesale price as $w_B^{A1}(Q) > \overline{w}_s$. Therefore, the manufacturer prefers to work with the small supplier due to his cost advantage.

$$Q < y^*(\overline{w}_s, k)$$
$$\Pi_M(Q) < \Pi_M(y^*(\overline{w}_s, k)) \tag{5.35}$$

Result: $w_B^{A1}(Q) > \overline{w}_s$ where $w_B^{A1}(Q) = r[1 - F(Q)] - k$ and $\overline{w}_s \in (c_L, c_H)$.The small supplier is procured by the manufacturer.

In the second case, if capacity of the big supplier, Q, is somewhere between $y^*(\overline{w}_s, k)$ and $y^*(w_B, k)$, the manufacturer orders the entire capacity of the big supplier.

$$\Pi_M(y^*(\overline{w}_s, k)) < Q < y^*(w_B, k)$$
$$\Pi_M(Q) > \Pi_M(y^*(\overline{w}_s, k)) \tag{5.36}$$

Result: The manufacturer works with the big supplier since $w_B^{A1}(Q) < \overline{w}_s$ where $w_B^{A1}(Q)$ $= r[1 - F(Q)] - k$ and $\overline{w}_s \in (c_L, c_H)$.

In the third case, since the big supplier's capacity overreaches $y^*(w_B, k)$ which is the optimal order quantity of unconstrained big supplier, the big manufacturer chooses the big supplier.

$$Q > y^*(w_B, k)$$
$$\Pi_M(Q) = \Pi_M(y^*(w_B, k)) \tag{5.37}$$

Result: When capacity constraint of the big supplier Q is greater than $y^*(w_B, k)$, the optimal order quantity is equal to $y^*(w_B, k)$ which means $y^*(w_B^{A1}(Q), k) = y^*(w_B, k)$.

Thus, the big supplier wins the game.

5.1.6. Relaxing the Assumption on the Big Supplier's Production Cost (c_B)

Because of the big supplier's experience and infrastructure our general assumption is $c_B < c_s$ through Chapters 3, 4 and 5. Now, we investigate what if $c_B > c_s$ where c_B and c_s are the production costs of the big supplier and the small supplier, respectively.

The manufacturer's expected profit is concave in y by (3.4) and $y^*(w, k)$ decreases as w increases (as well as production cost) by (4.17). In other words, a cost advantage makes a supplier more profitable than the other for the manufacturer. As a result, the following cases occur:

In the first case, $c_B < c_L < c_H$ which is also our general assumption, means the manufacturer prefers to work with the big supplier.

In the second case, if $c_L < c_H < c_B$, then the big supplier does not offer any contract to the manufacturer. Otherwise, he gets a negative profit. Therefore, the manufacturer works with the small supplier.

In the third case, if $c_L < c_B < c_H$, then the big supplier offers a contract to the manufacturer if $c_B < \overline{w}_s$ where $\overline{w}_s \in (c_L, c_H)$.

All three cases are illustrated in Figure 5.3.

5.2. The Processing Cost of the Manufacturer is Private

In this section, we deal the case where the production cost of the the small supplier (c_s) is known by the manufacturer while the processing cost of the manufacturer (k) is unknown by the big supplier. The concept in this section is similar to in Section 5.1. The only difference between Sections 5.1 and 5.2 is that the information asymmetry is between the big supplier and the manufacturer in stead of the manufacturer and the

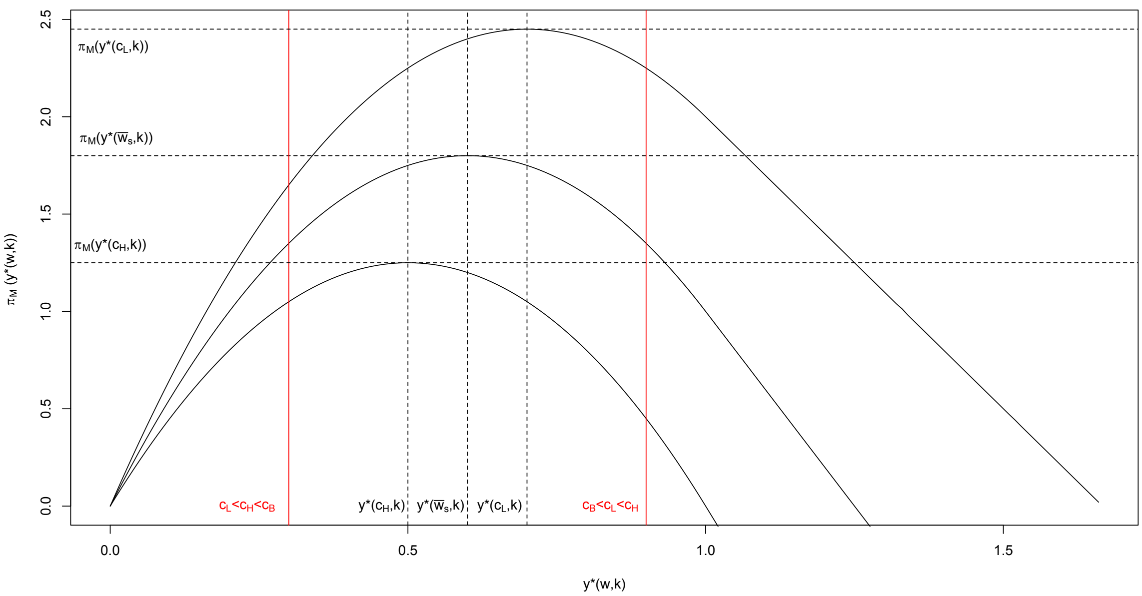

Figure 5.3. Relaxing The Assumption on c_B

small supplier. Without loss of generality, the big supplier should design a contract for two type of manufacturers in terms of the processing cost, a low processing cost manufacturer and a high processing cost manufacturer. The low cost manufacturer has (k_L) as the processing cost with probability p_L and the high cost manufacturer has (k_H) with probability p_H where $p_L + p_H = 1$. The big supplier offers $(w_{BL}^{A2}(Q), T_{BL}^{A2}(Q))$ for the low cost manufacturer with probability p_L and $(w_{BH}^{A2}(Q), T_{BH}^{A2}(Q))$ for the high cost manufacturer with probability p_H. Moreover, the big supplier aims to leave better profit than the small supplier (the manufacturer's outside option) to the manufacturer in order to win the tender bid.

The picture of the model when k is private is shown in Figure 5.4.

5.2.1. Problem of the Manufacturer

The manufacturer's problem is similar to the one in Chapter 4. We develop optimal solutions to both low processing cost manufacturer and high processing cost

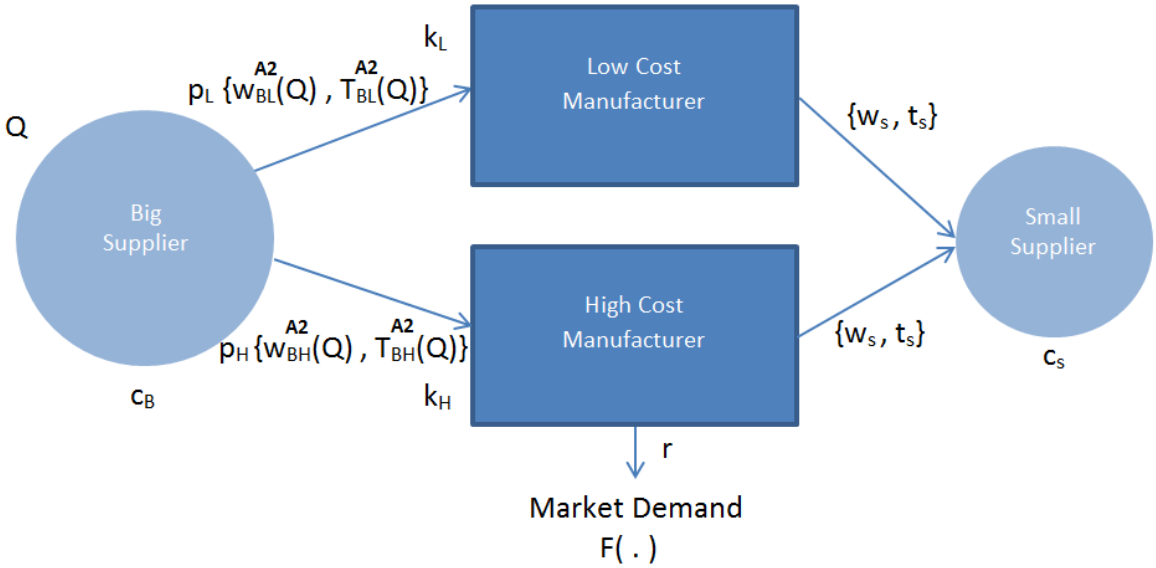

Figure 5.4. Picture of The Model When k is Private

manufacturer. Expected profit of the low processing cost manufacturer is given as follows by using (3.5):

$$\Pi_M(y^*(w_s, k_L)) - t_s \tag{5.38}$$

where k_L is processing cost for the low cost manufacturer, t_s is transfer payment to the small supplier and w_s is the wholesale price to be paid to the small supplier.

The low processing cost manufacturer wants to maximize her expected profit by minimizing her costs. Hence, she minimizes the small supplier's profit by offering "just enough" profit to stay in the game.

$$\min \Pi_s(w_s, k_L, t_s) = (w_s - c_s)\, y^*(w_s, k_L) + t_s$$

s.t.

$$(w_s - c_s)\, y^*(w_s, k_L) + t_s \geq 0$$

which ensures the small supplier gets a non-negative profit.

As an optimal solution to the low cost manufacturer's problem, parameters of the best contract that is zero profit for the small supplier is obtained as follows:

$$w_s = c_s \quad \& \quad t_s = 0 \tag{5.39}$$

By substituting (5.39) in (5.38), expected profit of the low processing cost manufacturer is $\Pi_M(y^*(c_s, k_L))$. In a similar manner, expected profit of the high processing cost manufacturer is obtained as $\Pi_M(y^*(c_s, k_H))$ by using k_H in stead of k_L, straightforwardly.

5.2.2. Problem of the Big Supplier

Different from Chapter 4 and Section 5.1, the big supplier faces two different types of manufacturer because of the private k. Therefore, we consider both the low cost and the high cost manufacturers by developing big supplier's problem. He determines optimal $w_{BL}^{A2}(Q)$, $w_{BH}^{A2}(Q)$, $T_{BL}^{A2}(Q)$ and $T_{BH}^{A2}(Q)$ by taking into consideration the small supplier as long as the big supplier has enough capacity.

$$\begin{aligned}\Pi_B\left[(w_{BL}^{A2}(Q), T_{BL}^{A2}(Q)), (w_{BH}^{A2}(Q), T_{BH}^{A2}(Q))\right] &= p_L[(w_{BL}^{A2}(Q) - c_B)\, y^*(w_{BL}^{A2}(Q), k_L) \\ &+ T_{BL}^{A2}(Q)] + p_H\left[(w_{BH}^{A2}(Q) - c_B)\, y^*(w_{BH}^{A2}(Q), k_H) + T_{BH}^{A2}(Q)\right]\end{aligned} \tag{5.40}$$

s.t.

$$\Pi_M(y^*(w_{BL}^{A2}(Q),k_L)) - T_{BL}^{A2}(Q) \geq \Pi_M(y^*(c_s,k_L)) \tag{5.41}$$

$$\Pi_M(y^*(w_{BH}^{A2}(Q),k_H)) - T_{BH}^{A2}(Q) \geq \Pi_M(y^*(c_s,k_H)) \tag{5.42}$$

$$\Pi_M(y^*(w_{BL}^{A2}(Q),k_L)) - T_{BL}^{A2}(Q) \geq \Pi_M(y^*(w_{BH}^{A2}(Q),k_L)) - T_{BH}^{A2}(Q) \tag{5.43}$$

$$\Pi_M(y^*(w_{BH}^{A2}(Q),k_H)) - T_{BH}^{A2}(Q) \geq \Pi_M(y^*(w_{BL}^{A2}(Q),k_H)) - T_{BL}^{A2}(Q) \tag{5.44}$$

$$y^*(w_{BL}^{A2}(Q),k_L) \leq Q \tag{5.45}$$

$$y^*(w_{BH}^{A2}(Q),k_H) \leq Q \tag{5.46}$$

where (5.41) and (5.42) are participation constraints of the low cost manufacturer and the high cost manufacturer with the big supplier, respectively. Furthermore, (5.43) and (5.44) are incentive compatibility constraints. (5.45) and (5.46) are capacity constraints of the big supplier.

5.2.3. Optimal Solution to the Big Supplier's Problem

In order to find the optimal solution to the big supplier problem, inequalities (5.45) and (5.46) are written in the objective function by Lagrangian multipliers.

As a result, the big supplier's problem and the constrains become:

$$\begin{aligned}
\max_{w_{BL}^{A2}(Q),w_{BH}^{A2}(Q),T_{BL}^{A2}(Q),T_{BH}^{A2}(Q),\lambda_1,\lambda_2} \quad & L\left(w_{BL}^{A2}(Q),w_{BH}^{A2}(Q),T_{BL}^{A2}(Q),T_{BH}^{A2}(Q),\lambda_1,\lambda_2\right) \\
=& p_L\left[\left(w_{BL}^{A2}(Q)-c_B\right)y^*(w_{BL}^{A2}(Q),k_L)+T_{BL}^{A2}(Q)\right] \\
&+ p_H\left[\left(w_{BH}^{A2}(Q)-c_B\right)y^*(w_{BH}^{A2}(Q),k_H)+T_{BH}^{A2}(Q)\right] \\
&+ \lambda_1[Q-y^*(w_{BL}^{A2}(Q),k_L)] \\
&+ \lambda_2[Q-y^*(w_{BH}^{A2}(Q),k_H)]
\end{aligned} \tag{5.47}$$

s.t.

$$\Pi_M(y^*(w^{A2}_{BL}(Q),k_L)) - T^{A2}_{BL}(Q) \geq \Pi_M(y^*(c_s,k_L)) \tag{5.48}$$

$$\Pi_M(y^*(w^{A2}_{BH}(Q),k_H)) - T^{A2}_{BH}(Q) \geq \Pi_M(y^*(c_s,k_H)) \tag{5.49}$$

$$\Pi_M(y^*(w^{A2}_{BL}(Q),k_L)) - T^{A2}_{BL}(Q) \geq \Pi_M(y^*(w^{A2}_{BH}(Q),k_L)) - T^{A2}_{BH}(Q) \tag{5.50}$$

$$\Pi_M(y^*(w^{A2}_{BH}(Q),k_H)) - T^{A2}_{BH}(Q) \geq \Pi_M(y^*(w^{A2}_{BL}(Q),k_H)) - T^{A2}_{BL}(Q) \tag{5.51}$$

$$\lambda_1, \lambda_2 \geq 0 \tag{5.52}$$

Since $k_L < k_H$, the right hand side of (5.50) is greater than the left hand side of (5.49). Hence, $\Pi_M(y^*(w^{A2}_{BL}(Q),k_L)) - T^{A2}_{BL}(Q)$ becomes greater than or equal to $\Pi_M(y^*(c_s,k_H))$. To put this information with (5.48), (5.48) becomes redundant because $\Pi_M(y^*(c_s,k_L))$ is greater than or equal to $\Pi_M(y^*(c_s,k_H))$.

Now we have two constraints which are (5.49) and (5.50) at hand. The optimal solution to the problem above simply can be obtained by making equalities in (5.49) and (5.50) instead of inequalities. Therefore, the transfer payments can be left alone, and then the objective function can be written as a function of the wholesale prices as follows.

$$\Pi_M(y^*(w^{A2}_{BH}(Q),k_H)) - T^{A2}_{BH}(Q) = \Pi_M(y^*(c_s,k_H)) \tag{5.53}$$

$$\Pi_M(y^*(w^{A2}_{BL}(Q),k_L)) - T^{A2}_{BL}(Q) = \Pi_M(y^*(w^{A2}_{BH}(Q),k_L)) - T^{A2}_{BH}(Q) \tag{5.54}$$

We leave transfer payments alone and plug them into (5.47).

$$T^{A2}_{BH}(Q) = \Pi_M(y^*(w^{A2}_{BH}(Q),k_H)) - \Pi_M(y^*(c_s,k_H)) \tag{5.55}$$

$$\begin{aligned} T^{A2}_{BL}(Q) =& \Pi_M(y^*(w^{A2}_{BL}(Q),k_L)) - \Pi_M(y^*(w^{A2}_{BH}(Q),k_L)) + T^{A2}_{BH}(Q) \\ =& \Pi_M(y^*(w^{A2}_{BL}(Q),k_L)) - \Pi_M(y^*(w^{A2}_{BH}(Q),k_L)) + \Pi_M(y^*(w^{A2}_{BH}(Q),k_H)) \\ & - \Pi_M(y^*(c_s,k_H)) \end{aligned} \tag{5.56}$$

We are able to write objective function, L, as a function of $w_{BL}^{A2}(Q)$, $w_{BH}^{A2}(Q)$, λ_1 and λ_2 by using (5.55) and (5.56) as follows.

$$\begin{aligned}
&L(w_{BL}^{A2}(Q), w_{BH}^{A2}(Q), \lambda_1, \lambda_2) \\
=&p_L[(w_{BL}^{A2}(Q) - c_B)\, y^*(w_{BL}^{A2}(Q), k_L) + \Pi_M(y^*(w_{BL}^{A2}(Q), k_L)) - \Pi_M(y^*(w_{BH}^{A2}(Q), k_L)) \\
&+ \Pi_M(y^*(w_{BH}^{A2}(Q), k_H)) - \Pi_M(y^*(c_s, k_H))] + p_H[(w_{BH}^{A2}(Q) - c_B)\, y^*(w_{BH}^{A2}(Q), k_H) \\
&+ \Pi_M(y^*(w_{BH}^{A2}(Q), k_H)) - \Pi_M(y^*(c_s, k_H))] + \lambda_1[Q - y^*(w_{BL}^{A2}(Q), k_L)] \\
&+ \lambda_2[Q - y^*(w_{BH}^{A2}(Q), k_H)] \qquad (5.57)
\end{aligned}$$

Now, let us take the derivative of (5.57) with respect to $w_{BL}^{A2}(Q)$, $w_{BH}^{A2}(Q)$,λ_1 and λ_2, and then equate them to zero, respectively, in order to find optimal solution.

In order to develop the optimal wholesale price to the efficient manufacturer by the big supplier:

$$\begin{aligned}
\frac{\partial L}{\partial w_{BL}^{A2}(Q)} =&p_L\left[y^*(w_{BL}^{A2}(Q), k_L) + \frac{\partial y^*(w_{BL}^{A2}(Q), k_L)}{\partial w_{BL}^{A2}(Q)}(w_{BL}^{A2}(Q) - c_B) - y^*(w_{BL}^{A2}(Q), k_L)\right] \\
&- \lambda_1 \frac{\partial y^*(w_{BL}^{A2}(Q), k_L)}{\partial w_{BL}^{A2}(Q)} \\
=&p_L\left[-\frac{1}{r\ f(y^*(w_{BL}^{A2}(Q), k_L))}(w_{BL}^{A2}(Q) - c_B)\right] + \lambda_1 \frac{1}{r\ f(y^*(w_{BL}^{A2}(Q), k_L))} \\
=&-\frac{1}{r\ f(y^*(w_{BL}^{A2}(Q), k_L))}\left[p_L(w_{BL}^{A2}(Q) - c_B) - \lambda_1\right] = 0
\end{aligned}$$

where $p_L > 0$, $f(y^*(w_{BL}^{A2}(Q), k_L)) > 0$ and $r > 0$. So, the only part make this equation equal to zero is $p_L(w_{BL}^{A2}(Q) - c_B) - \lambda_1$. As a result,

$$w_{BL}^{A2}(Q) = c_B + \frac{\lambda_1}{p_L} \qquad (5.58)$$

The optimal wholesale price which is offered by the capacitated big supplier to the efficient manufacturer in (5.58) shows that private information on k together with the capacity constraint on the big supplier prevent the big supplier to use his cost advantage. In other words, if the big supplier does not have a capacity constraint ($\lambda_1 = 0$),

$w^{A2}_{BL}(Q)$ would be c_B which provides a cost advantage to the big supplier. However, the big supplier demands a premium as $\frac{\lambda_1}{p_L}$ because of existence of the capacity which causes the big supplier to loss his cost advantage. Moreover, when the optimal order quantity of the low cost manufacturer ($y^*(w^{A2}_{BL}(Q), k_L)$) is greater than the capacity of the big supplier (Q) then the expected profit of the big supplier decreases as λ_1 increases. Last but not least, as the probability of encountering with efficient manufacturer (p_L) increases $w^{A2}_{BL}(Q)$ decreases. Therefore, the big supplier can keep his cost advantage as long as $c_B < w^{A2}_{BL}(Q) < c_s$ is provided.

In order to develop the optimal wholesale price to the inefficient manufacturer by the big supplier:

$$\begin{aligned}\frac{\partial L}{\partial w^{A2}_{BH}(Q)} =& p_L\left[y^*(w^{A2}_{BH}(Q), k_L) - y^*(w^{A2}_{BH}(Q), k_H)\right] + p_H\Big[y^*(w^{A2}_{BH}(Q), k_H) \\ &+ \frac{\partial y^*(w^{A2}_{BH}(Q), k_H)}{\partial w^{A2}_{BH}(Q)}(w^{A2}_{BH}(Q) - c_B) - y^*(w^{A2}_{BH}(Q), k_H)\Big] \\ &- \lambda_2\frac{\partial y^*(w^{A2}_{BH}(Q), k_H)}{\partial w^{A2}_{BH}(Q)} \\ =& p_L\left[y^*(w^{A2}_{BH}(Q), k_L) - y^*(w^{A2}_{BH}(Q), k_H)\right] \\ &+ p_H\left[-\frac{1}{r\ f(y^*(w^{A2}_{BH}(Q), k_H))}(w^{A2}_{BH}(Q) - c_B)\right] + \lambda_2\frac{1}{r\ f(y^*(w^{A2}_{BH}(Q), k_H))} \\ =& p_L\left[y^*(w^{A2}_{BH}(Q), k_L) - y^*(w^{A2}_{BH}(Q), k_H)\right] \\ &- \frac{1}{r\ f(y^*(w^{A2}_{BH}(Q), k_H))}\left[p_H(w^{A2}_{BH}(Q) - c_B) - \lambda_2\right] = 0\end{aligned}$$

We consider whole the equation above in order to find the optimal wholesale price of the inefficient manufacturer. When $w^{A2}_{BH}(Q)$ is left alone, it is equal to as follows.

$$w^{A2}_{BH}(Q) = c_B + \frac{\lambda_2}{p_H} + \frac{p_L}{p_H}r\ f(y^*(w^{A2}_{BH}(Q), k_H))\left[y^*(w^{A2}_{BH}(Q), k_L) - y^*(w^{A2}_{BH}(Q), k_H)\right] \tag{5.59}$$

Similar to the optimal wholesale price of the efficient manufacturer ($w^{A2}_{BL}(Q)$), the big supplier wants a premium as $\frac{\lambda_2}{p_H}$ from the inefficient manufacturer. Since $p_L > 0$, $p_H > 0$, $f(y^*(w^{A2}_{BH}(Q), k_H)) > 0$, $r > 0$ and $\left[y^*(w^{A2}_{BH}(Q), k_L) - y^*(w^{A2}_{BH}(Q), k_H)\right] > 0$ (because of $y^*(w,k)$ is decreasing where $k_L < k_H$), whole the part, $\frac{p_L}{p_H}\ r\ f(y^*(w^{A2}_{BH}(Q),k_H))$ $\left[y^*(w^{A2}_{BH}(Q),k_L)\ -y^*(w^{A2}_{BH}(Q),k_H)\right]$, is strictly positive. This means the big supplier demands extra premium from the inefficient manufacturer which is expectable. In a similar manner to the efficient manufacturer case, the expected profit of the big supplier decreases as λ_2 increases if the optimal order quantity of the high cost manufacturer ($y^*(w^{A2}_{BL}(Q),k_L)$) is greater than the capacity of the big supplier.

When we try to find optimal Lagrangian multipliers, the optimal order quantities of the low cost manufacturer (efficient) and the high cost manufacturer (inefficient) are achieved as follows.

$$\frac{\partial L}{\partial \lambda_1} = Q - y^*(w^{A2}_{BL}(Q),k_L) = 0$$

$$y^*(w^{A2}_{BL}(Q),k_L) = Q \tag{5.60}$$

$$\frac{\partial L}{\partial \lambda_2} = Q - y^*(w^{A2}_{BH}(Q),k_H) = 0$$

$$y^*(w^{A2}_{BH}(Q),k_H) = Q \tag{5.61}$$

In this section, since the capacity of the big supplier (Q) is less than the optimal order quantity of the manufacturer to the uncapacitated big supplier ($y^*(w_B,k)$) that the case we work, the manufacturer always orders as whole the capacity of the big supplier (Q) as an optimal order quantity (e.g. $y^*(w^{A2}_{BL}(Q),k_L)$ for the efficient manufacturer). Otherwise, if the capacity constraint of the big supplier (Q) goes beyond the optimal order quantity of the uncapacitated big supplier ($y^*(w_B,k)$), the manufacturer obviously acquires $y^*(w_B,k)$ from the big supplier as an optimal order quantity which is less than Q. In other words, if Q is greater than $y^*(w_B,k)$, both of $y^*(w^{A2}_{BL}(Q),k_L)$ and $y^*(w^{A2}_{BH}(Q),k_H)$ is equal to $y^*(w_B,k)$ in stead of Q. It is important to note that, these are the local optimal points (FOC).

As a result, the optimal solution to the big supplier's problem as follows.

$$
\begin{aligned}
w_{BL}^{A2}(Q) =& c_B + \frac{\lambda_1}{p_L} \\
w_{BH}^{A2}(Q) =& c_B + \frac{\lambda_2}{p_H} + \frac{p_L}{p_H} r\ f(y^*(w_{BH}^{A2}(Q), k_H)) \left[y^*(w_{BH}^{A2}(Q), k_L) - y^*(w_{BH}^{A2}(Q), k_H) \right] \\
T_{BL}^{A2}(Q) =& \Pi_M(y^*(w_{BL}^{A2}(Q), k_L)) - \Pi_M(y^*(w_{BH}^{A2}(Q), k_L)) + \Pi_M(y^*(w_{BH}^{A2}(Q), k_H)) \\
& - \Pi_M(y^*(c_s, k_H)) \\
T_{BH}^{A2}(Q) =& \Pi_M(y^*(w_{BH}^{A2}(Q), k_H)) - \Pi_M(y^*(c_s, k_H)) \\
& y^*(w_{BL}^{A2}(Q), k_L) = Q \\
& y^*(w_{BH}^{A2}(Q), k_H) = Q
\end{aligned} \tag{5.62}
$$

where $w_{BL}^{A2}(Q)$ and $w_{BH}^{A2}(Q)$ are the wholesale prices that the big supplier offers to the low cost manufacturer and the high cost manufacturer, respectively. Similarly, $T_{BL}^{A2}(Q)$ and $T_{BH}^{A2}(Q)$ are the transfer payments that the low cost manufacturer and the high cost manufacturer pay to the big supplier, respectively. Lastly, $y^*(w_{BL}^{A2}(Q), k_L)$ and $y^*(w_{BH}^{A2}(Q), k_H)$ are the optimal order quantity for the low cost manufacturer and the high cost manufacturer, respectively.

6. CONCLUSION

Throughout this study, we examine a supply chain problem which consists of a manufacturer and two non-identical suppliers. It is important to note that, the problem is studied from the aspect of the big supplier whose aim is to work with the manufacturer when she has an outside option, the small supplier. However, the big supplier has limited production capacity since he has many other customers. Our main goal is to provide that the manufacturer works with the big supplier under full information and different asymmetric information cases.

In Chapter 4, we show that capacity has a direct effect of supplier selection of the manufacturer. Three possible cases for capacity intervals are shown under full information. When the capacity of the big supplier is between the optimal order quantity of the manufacturer to the small supplier and the optimal order quantity of the manufacturer to the uncapacitated big supplier, the manufacturer prefers to work with the big supplier supplier. However, the expected profit of the capacitated big supplier is less than uncapacitated big supplier. Moreover, if the capacity of the big supplier is greater than the optimal order quantity to the uncapacitated big supplier, the big supplier's capacity constraint becomes redundant since the manufacturer orders less than the capacity. Lastly, when the capacity is less than the optimal order quantity of the manufacturer to the small supplier, the manufacturer procures from the small supplier.

Chapter 5 is divided into two main sections. In the first one, we study the case when the small supplier's production cost is private. This private cost information provides an advantage to the small supplier as well as the manufacturer. The expected profit of the small supplier is zero under full information while his expected profit becomes positive as information rent under the private information assumption. Furthermore, the capacity affects supplier procurement of the manufacturer similar to in Chapter 4. The only difference is that we are able to compare the capacity with the certain optimal order quantities in Chapter 4 whereas we face a quantity range for the order quantity of the manufacturer to the small supplier in Section 5.1. The optimal order quantity of the manufacturer to the small supplier occurs somewhere in this range. Therefore, when the capacity of the big supplier is less than the optimal order quantity of the manufacturer to the small supplier, the manufacturer acquires goods from the small supplier. In other words, the production cost of the big supplier is greater than both the efficient and inefficient small suppliers. Otherwise, the manufacturer chooses the big supplier to work same as in Chapter 4.

We examine the case of the manufacturer's processing cost is undisclosed in Section 5.2. Different from Chapter 4 and Section 5.1, we use Lagrangian multipliers in order to find the optimal solution to the big supplier's problem. Reason of using different solution methods between Chapter 4 - Section 5.1 and Chapter 5.2 is that the information asymmetry is between the manufacturer and the small supplier in Chapter 4 and Section 5.1 whereas the information asymmetry is between the manufacturer and the big supplier in Section 5.2. The information asymmetry together with the capacity cause higher wholesale price for the efficient manufacturer. In other words, the capacitated big supplier demands a premium because of his capacity. Similarly, the inefficient manufacturer incurs extra premium due to the fact that the capacity. Therefore, the capacitated big supplier faces the risk of missing the opportunity to work with the manufacturer. Moreover, as the probability of working with the efficient manufacturer increases, the optimal wholesale price decreases. Thus, the manufacturer selects the big supplier as long as the optimal wholesale price of the capacitated big supplier is less than the optimal wholesale price of the small supplier.

Another interesting result is that if the manufacturer sets her problem to give a positive profit to the small supplier instead of a non-negative profit, the right and side of the constraint of manufacturer's minimization problem in Chapter 4, changes from greater than or equal to zero to greater than zero. As a result, the expected profit of the manufacturer increases and the big supplier sets his constraint according to this new expected profit of the manufacturer. Intuitively, the manufacturer orders whole the capacity of the big supplier as an optimal order quantity as usual and the big supplier offers a larger wholesale price to the manufacturer in order to get positive expected profit. However, if this new wholesale price which is offered to the manufacturer by the big supplier is greater than the production cost of the small supplier, the big supplier faces the risk of losing the opportunity to work with the manufacturer.

For further research, a computational result can be studied in order to generalize our findings. Investigating multiple periods market demand of the manufacturer in stead of the single period market demand can be another research option. Moreover, the capacity constraint can be applied to the small supplier together with the big supplier.

REFERENCES

Bernstein, F. and A. Federgruen, "Decentralized supply chains with competing retailers under demand uncertainty", *Management Science*, Vol. 51, No. 1, pp. 18–29, 2005.

Cachon, G. P., "Supply chain coordination with contracts", *Handbooks in operations research and management science*, Vol. 11, pp. 227–339, 2003.

Cachon, G. P. and F. Zhang, "Procuring fast delivery: Sole sourcing with information asymmetry", *Management Science*, Vol. 52, No. 6, pp. 881–896, 2006.

Cachon, G. P. and F. Zhang, "Obtaining fast service in a queueing system via performance-based allocation of demand", *Management Science*, Vol. 53, No. 3, pp. 408–420, 2007.

Chen, F., "Information sharing and supply chain coordination", *Handbooks in operations research and management science*, Vol. 11, pp. 341–421, 2003.

Çakanyıldırım, M., Q. Feng, X. Gan and S. P. Sethi, "Contracting and coordination under asymmetric production cost information", *Production and Operations Management*, Vol. 21, No. 2, pp. 345–360, 2012.

Elmaghraby, W. J., "Supply contract competition and sourcing policies", *Manufacturing & Service Operations Management*, Vol. 2, No. 4, pp. 350–371, 2000.

Ha, A. Y., L. Li and S.-M. Ng, "Price and delivery logistics competition in a supply chain", *Management Science*, Vol. 49, No. 9, pp. 1139–1153, 2003.

Laffont, J.-J. and D. Martimort, *The theory of incentives: the principal-agent model*, Princeton university press, 2009.

Li, L., "Information sharing in a supply chain with horizontal competition", *Management Science*, Vol. 48, No. 9, pp. 1196–1212, 2002.

Li, Z., J. K. Ryan, L. Shao and D. Sun, "Supply contract design for competing heterogeneous suppliers under asymmetric information", *Production and Operations Management*, Vol. 24, No. 5, pp. 791–807, 2015.

Özer, Ö. and W. Wei, "Strategic commitments for an optimal capacity decision under asymmetric forecast information", *Management science*, Vol. 52, No. 8, pp. 1238–1257, 2006.

Özer, Ö. and G. Raz, "Supply chain sourcing under asymmetric information", *Production and Operations Management*, Vol. 20, No. 1, pp. 92–115, 2011.

Salanié, B., *The economics of contracts: a primer*, MIT press, 2005.

Savaskan, R. C. and L. N. Van Wassenhove, "Reverse channel design: the case of competing retailers", *Management science*, Vol. 52, No. 1, pp. 1–14, 2006.

Tsay, A. A., S. Nahmias and N. Agrawal, "Modeling supply chain contracts: A review", *Quantitative models for supply chain management*, pp. 299–336, Springer, 1999.

AnadoluAjansı, *Apple, tedarikçisine 1 milyar dolarlık dava açtı*, 2017, `http://www.bloomberght.com/haberler/haber/1977169-apple-tedarikcisine-1-milyar-dolarlik-dava-acti`, accessed at May 2018.

Haberturk.com, *Qualcomm, Apple'a karşı dava açtı*, 2017, `http://www.bloomberght.com/haberler/haber/2002356-qualcomm-applea-karsi-dava-acti`, accessed at May 2018.