PRESERVING PRIORITIES IN THE CASE OF THE RE-PLACEMENT OF MEDICAL RESIDENTS IN TURKEY

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DECLARATION OF ORIGINALITY

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ABSTRACT

Preserving Priorities in the Case of
the Re-Placement of Medical Residents in Turkey

In this paper, we analyze the re-placement mechanism used in the Examination of Specialty in Medicine in Turkey, following the correction of a miscalculation in the scores. We observe that the preservation of vested interests, together with the limited capacities lead to violation of fairness. Furthermore, we show that the choice functions create additional capacities at programs, even to those doctors, who would not get into the programs if the score calculation was correct in the beginning. In line with preventing the creation of those unnecessary capacities, we define a new notion of fairness, capacity respecting fairness (QRF), such that a candidate, who is ranked within the original capacity of a program among the application pool is never rejected. We also define a QRF-Adjusted Modified Choice Function for the programs, and show that it is the choice function, which minimizes the deviation from the target capacities whilst preserving the vested interests. Furthermore, we also show that the QRF-Adjusted Modified Choice Function induced by the Deferred Acceptance Algorithm, minimizes the deviation of the outcome from the target capacities of the programs.

ÖZET

Türkiye'de Tıp Asistanlarının Yeniden Yerleştirilmesinde Önceliklerin Korunması

Bu çalışmada, Türkiye'deki Tıpta Uzmanlık Sınavının puanlarının hesaplanmasında yapılan yanlışlığın düzeltilmesinin ardından uygulanan yeniden yerleştirme mekanizmasını inceledik. Kazanılmış hakların korunması ve kapasitelerin limitli olmasının adalet ilkesini ihlal ettiğini gözlemledik. Ayrıca, kullanılan seçim fonksiyonlarının; puan hesaplaması en başta doğru olsaydı o programa giremeyecek olan adaylara bile ekstra kontenjan açılmasına sebep olduğunu gösterdik. Bu gerekli olmayan kontenjanların açılmasını önlemek için; bir aday havuzu içerisindeki sıralaması o programın kapasitesinden düşük olan adayların asla reddedilmediği yeni bir adalet kavramı tanımladık: Kapasiteye Duyarlı Adalet (KDA). Bunun yanında, KDA-Ayarlı Modifiye Seçim Fonksiyonu'nu da tanımlayarak, bu seçim fonksiyonunun kazanılmış hakları korurken hedef kapasitelerden sapmayı en aza indiren seçim fonksiyonu olduğunu; ve bu foksiyonun Gecikmeli Kabul Algoritması ile birlikte kullanılmasıyla ortaya çıkan yerleştirmelerin, programların hedef kapasitelerinden sapmayı minimize ettiğini de kanıtladık.

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CHAPTER 1

INTRODUCTION

After completing 6 years of undergraduate education, the medical students who want to continue their education with specialization, take an exam which is called the "Examination for Specialty in Medicine" (ESM) and is conducted twice a year, one in the spring and one in the fall. The exam consists of two parts, basic medical sciences and clinical medicine, and the residential candidates receive a two-category score at the end[1]. After they receive their scores, the candidates submit a preference list of residency programs. The Student Selection and Placement Center (SSPC) in Turkey receives the preferences and places the residential candidates to programs. The problem of matching the residential candidates to the programs sets an example for the application of the Matching Theory, which is a subfield of the Game Theory.

The matching literature started off with two-sided one-to-one matching problems. These problems are usually referred to as marriage problems since the marriage market serves as a perfect example with two sides (women and men), and each woman (*man*) being allowed to be matched to one man (*woman*). The prevalent opinion is that the work of Gale and Shapley (1962) constitutes the beginning of the literature, where they have defined the famous Deferred Acceptance Algorithm in the search for stable marriages. A stable marriage is where no woman (*man*) would rather stay single instead of being married to her (*his*) husband (*wife*) and there is no pair of a woman and man, who prefer each other to their matched partners. In their work, Gale and

[1]Meaning the candidates receive two different scores: Basic Medical Sciences and Clinical Medical Sciences Scores, in which the weights of the tests differ in calculating the scores.

Shapley prove the existence of a stable matching in the marriage problem and they define a way to achieve that, which is the Deferred Acceptance Algorithm.

Later on, other early works by Alvin E. Roth (Roth, 1982, Roth, 1984 and Roth 1985) widened the scope of the matching theory by introducing many-to-one matchings. Many-to-one matching is usually referred to as *College Admissions Problem* and differs from one-to-one (marriage) problem in the sense that it allows one party (schools-colleges) to be matched with many (students), whereas the other party (students) can be matched to only one (school). In many-to-one matchings, however, stability is not automatically achieved. We need an extra assumption about the schools' preferences which is called *responsiveness* to guarantee the existence of a stable matching (Roth, 1985). That is, if two assignments are different from each other in only one student, the school must prefer the assignment which contains the more preferred student. Roth also wrote another paper on many-to-one matchings, where he analyzed the National Residency Matching Program used in the US (Roth, 1984). Those early works were considered as mathematical or political issues as it can be understood from the journals in which they were published. However, when Roth and Sotomayor (1990) wrote their book where they have analyzed two-sided matchings, they established a new era for the Matching Theory. The niche field of Game Theory became so popular afterwards, that in 2012, The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel was awarded jointly to Alvin E. Roth and Lloyd S. Shapley "for the theory of stable allocations and the practice of market design".

A many-to-one matching problem has many components such as the students, the colleges, the colleges' preferences, the students' preferences, capacities of the colleges etc. The factor which determines the preferences of colleges gives the problem its name. In the absence of a centralized score, as seen in many examples in the US (Boston, Minneapolis etc.), the problem where the students are placed to schools via their exogenously determined priorities (walk-zone, sibling) is called the School Choice Problem and the mechanism used widely in those districts has been named after Boston and was analyzed by Abdulkadiroğlu and Sönmez (2003a). The two economists have analyzed the mechanism and showed its weaknesses, namely the mechanism failing strategy-proofness which means that the students being able to manipulate the mechanism via submitting insincere preferences and elimination of justified envy, which is the equivalent notion of stability in that problem.

However, with the existence of a finite set of residential candidates, a finite set of programs, a capacity and a category for each program, an exam score and a preference profile for each student, the placement of residents matching problem is an example of the "Student Placement Problem" described by Balinski and Sönmez (1999)[2]. It is an example of many-to-one matchings, where a residential candidate can be matched to only one program, whereas a program might be matched to many residents.

The ESM of Fall 2016 and the placements afterwards were exceptional. Following the exam, SSPC was sued by some doctors, claiming that some questions in the exam were problematic. The court decided that some of the questions had to be canceled. The scores of the doctors were than calculated

[2]The SSPC places the residential candidates using the Multicategory Serial Dictatorship Mechanism, where only about %3 of the total capacities are used by the programs which accept residences according to their basic medical sciences rankings.

based on the remaining accurate questions, and after the students received their scores and submitted their preferences, the system of placement worked just as usual.

However, SSPC filed an appeal to vindicate the accuracy of the problematic questions. The State Council decided that the canceled questions were in fact not problematic, thus revoked the cancellation. The scores of the candidates were recalculated.

For the sake of providing those with fairness, whose rankings have risen after the recalculation, SSPC announced that there was going to be a re-placement. Before this re-placement procedure, the doctors could update their preference list that they had submitted before, but this was not mandatory. By updating her preference list, a candidate would practically empty her occupied slot, and the re-placements would be done according to the already empty or emptied capacities.

This system would provide some fairness to those who have a higher ranking than before. However, another issue was that the candidates were already placed and started working according to their original scores and preference submissions. The candidates who updated their list but assigned a worse alternative than their original assignment could use their vested interest and stay at their program, independent of being reassigned to a new program. We found that the existence of the vested interests and the capacity constraints resulted in a violation of fairness.

Furthermore, the modified choice function used by SSPC during the re-placement procedure creates additional capacities. Since the original capacities of the residency programs were adjusted according to the needs of the programs and the country, this unintended expansion of the choice function would disturb the balance and result in inefficiencies. Thus, admitting that the existence of vested interests makes the choice function vulnerable to fairness, we need an alternative definition. After finding that reasonable interpretation of fairness, we need to minimize the deviation from the target capacities in order to prevent inefficiencies in the market.

The paper is organized as follows: Chapter 2 introduces the model and explains the notation. Chapter 3 describes the regular placement procedure, explains the current compensation procedure and discusses its weaknesses. Chapter 4 presents an alternative notion of fairnes, introduces an alternative choice function and compares it with the current one. Chapter 5 analyzes the properties and compares the two choice functions. Chapter 6 concludes.

CHAPTER 2

MODEL

Before starting to describe the model, we first need to define and describe the notation to be used throughout the paper:

For the doctors, $\mathcal{D}$ denotes set of all doctors such that d denotes a doctor, whereas D denotes set of all doctors such that $\forall d \in D$ have updated their preference list before re-placements. We denote the preference relation of a doctor over programs by P^d.

At the hospital side, H denotes set of all residency programs such that h denotes a program. Y_h denotes the set of doctors in the application pool of hospital h. Different than the doctors, programs have preferences over sets of doctors, denoted by P^h.

Furthermore, we need to define the choice functions of doctors and hospitals. C_d denotes the choice function of doctor d. Since doctors have unit demand, their choice function is driven from a strict preference relation P^d over H, $C_d : 2^H \rightarrow H$ and $C_d(X) = max_{P^d}(X)$, where $X \subset H$. Notice that $\emptyset \subset H$, so that a doctor might prefer the outside option among the choices.

C_h denotes choice function of program h. Different than the doctors, programs choose from application pools sets of doctors, thus $C_h : 2^D \rightarrow 2^D$, and for any $D' \subset (D)$, $C_h(D') \subset D'$. Similar to C_d, C_h allows the program to choose no doctor from its pool.

Once C_h is defined, it makes sense to define $R_h(Y_h)$ as well, which is the rejection function and denotes the not chosen doctors in the application pool of hospital h. Those doctors are referred to as rejected doctors.

s_d denotes the originally calculated score of doctor d, and s'_d her new score. s_d will be used only in the placement problem of the residents, whereas s'_d will be a component of the re-placement problem. Furthermore, the ranking of doctor d in an application pool is defined as a function $z(d|s_{D'}, D') : D' \rightarrow \mathbb{N}^+$, such that z_d is weakly decreasing with s_d. Similar to s_d and s'_d, $z'(d|s'_{D'}, D')$ is d's new ranking after the recalculation.

b_h denotes the cut-off score of program h, and b'_h the new cut-off score of h after the re-placement.

q_h denotes the exogenously determined capacity of program h. Throughout the paper, q_h will also be referred to as "the target capacity", since deviations from it will arise during the re-placement.

After the placements and before the re-placements, two additional notations are needed such that m_h denotes the number of doctors currently placed at program h and have not updated their preferences after the recalculation of the scores. E_h denotes the set of existing doctors of program h, who were originally placed at hospital h.

A matching μ is a set of doctor-program (d, h) pairs such that each doctor d appears in at most one pair.

Let $\mathcal{X}$ denote the set of all possible matchings. A direct mechanism is then a function $\phi : \mathcal{P}^D \rightarrow \mathcal{X}$ that selects a matching for each preference profile of the doctors. In this paper, we denote the matching, which is the outcome of the direct mechanism ϕ as μ^ϕ. Similarly, $\mu(d)$ and $\mu(h)$ denote respectively the match of the doctor d and program h at matching μ.

A resident matching problem is then a tuple $(\mathcal{P}^D, D, H, q_H)$.

The minimal requirement that we expect from a matching that it is stable. Stability is satisfied whenever no parties can individually or mutually be better off by being out of the system.

Formally, a matching is stable if it is:

1. individually rational, $C_i(\mu(i)) = \mu(i)$ for all $i \in (D \cup H)$.

2. not blocked, $\nexists(d, h)$ pair such that $\mu(d) \neq h$, where $C_d(\mu(d) \cup h) = h$ and $d \in C_h(\mu(h) \cup d)$.

CHAPTER 3

PLACEMENT AND RE-PLACEMENT OF THE RESIDENTS MATCHING PROBLEM

3.1 Placement of the residents matching problem

Since only %3 of the total capacities accept their candidates from the basic medical sciences category, and the vast majority being placed in clinical programs, we simply focus on the clinical category programs and their candidates. As in many exams and placement mechanisms run by SSPC, in ESM, doctors are ranked according to their scores. Any tie between them is broken arbitrarily. Then, they pick programs starting from the doctor on top of the list. Any program, whose capacity is full is out of the program list. The second doctor then picks her favorite program among the remaining programs. In this special framework, with program preferences being responsive, this simple serial dictatorship mechanism is equivalent to the student proposing deferred acceptance algorithm of Gale and Shapley (1962), which works as follows after ranking the candidates according to their scores s_d.

Step 1: Each doctor applies to her favorite program. Each program which received applications tentatively accepts the top ranked q_h doctors and reject others in its application pool.

Step k: The candidates who were rejected in step $k-1$ apply to their second best choices. Each program which received applications tentatively accepts the top ranked q_h doctors among the new applicants and the ones it has chosen in Step $k-1$.

The mechanism stops when no further applications are made and match the programs to the doctors whose applications it is holding.

3.2 Re-placement of the residents matching problem

After the ESM of 2016, scores were calculated according to the not-canceled questions. Then, placements were done by the above described mechanism. Nonetheless, as stated before, the State Council revoked the cancellation, which lead to a difference in the scores and hence the rankings of the doctors. Thus, the original placement was not fair to some students, especially to those whose rankings have increased after the recalculation of the scores.

As a compensation, it was announced that there was going to be a re-placement procedure, which aimed to provide fairness to the students with increased rankings. However, as a legal responsibility, the choice function had to preserve the vested interests of already existing residents, meaning that it had to choose the existing candidates who are in the application pool. Formally, preservation of the vested interests is defined as follows:

Preservation of Vested Interests: A choice function preserves the vested interests of the existing residents if and only if:

$\forall h \in H, \forall Y \subset D, d \in (Y \cap E_h) \implies d \in C_h(Y)$

The following definitions of previous studies were defined in the matching with contracts framework. However, we will be using them according to our matching design in the following chapters:

Substitutes: Elements of Y are substitutes for program h if for all subsets $Y' \subset Y'' \subset Y$ we have $R_h(Y') \subset R_h(Y'')$.(Hatfield and Milgrom, 2005)

Law of Aggregate Demand (LAD): The preferences of program h satisfy the law of aggregate demand if for all $Y' \subset Y'' \subset Y$, $|C_h(Y')| \leq |C_h(Y'')|$. (Hatfield and Kojima, 2010)

Irrelevance of Rejected Contracts (IRC): Given a set of doctors D, a choice function satisfies the irrelevance of rejected contracts if and only if:

$\forall Y \subset D, \quad \forall z \in D \setminus Y$

$z \notin C(Y \cup \{z\}) \implies C(Y) = C(Y \cup \{z\})$.(Aygün and Sönmez, 2013)

As shown by Aygün and Sönmez (2013), Substitutes and IRC together imply the existence of a stable matching. Furthermore, Substitutes and LAD together imply IRC, thus resulting in the existence of a stable matching as well.

Similar to Sönmez and Switzer (2013) choice functions in the Cadet-Branch Matching problem, the modified choice function, is a selection rule of a program from an application pool and works as follows:

1. Rank the doctors in Y_h according to their recalculated scores.
2. Add all doctors $d \in (E_h \cap Y_h)$ to $C_h(Y_h)$.
3. Based on their recalculated rankings, add from remaining doctors $d \in (Y_h \setminus E_h)$ one-by-one to $C_h(Y_h)$ until either $(q_h - m_h)$ doctors from $(Y_h \setminus E_h)$ are added to $C_h(Y_h)$ or all doctors are considered.
4. Terminate the procedure, add all other doctors to $R_h(Y_h)$.

The modified choice function used by SSPC preserves the vested interests by definition, since it chooses all the existing residents from the application pool. Furthermore, it satisfies substitutes and law of aggregate demand (LAD). These two conditions together imply the irrelevance of rejected contracts (IRC) and thus guarantee the existence of a stable matching (Aygün & Sönmez, 2013). In the introduction, we have mentioned that stability of a matching is equivalent to the concept of elimination of justified envy. With the above mentioned properties of the modified choice function, the stability of the matching which is be the outcome of the Deferred Acceptance Algorithm induced by the modified choice function is guaranteed.

The number of the selected candidates depends on the number of existing residents in the program who have updated their preferences. $|C_h(Y_h)| \leq 2q_h$. If all the existing residents of hospital h update their preferences, and are in the application pool, then the program doubles its capacity.

The most important aim in the re-placement procedure is to restore the fairness of the system. To analyze its success, first, the fairness of a choice function has to be defined.

General Fairness (GF): A choice function satisfies general fairness if and only if:

$\nexists d \in Y_h$ such that $s'_d > b'_h$ and $d \notin C_h(Y_h)$, $\forall h \in H$ and $\forall Y_h \subset D$.

Claim 1: The modified choice function used by SSPC violates General Fairness.

Proof: Consider the following example:

Let $q_h = 1$. $\mathcal{D} = Y_h = \{a, x, y\}$, recalculated scores of the doctors respectively: $S' = \{80, 95, 90\}$, $\{a\}$ an existing resident at h, $\{x, y\}$ the new candidates. Then $C_h(Y_h) = \{a, x\}$, $R_h(Y_h) = \{y\}$, $b'_h = 80$. With $s'_y = 90 > 80 = b'_h$ and $y \notin C_h(Y_h)$, the choice function violates general fairness. Independent from the increase in the capacities, as long as the realized capacity of a program is less than the number of all candidates, the vested interests endanger general fairness.

Result: Existence of vested interests (the obligation for the choice function to choose any existing candidate independent of her recalculated score) is the main problem which results in the violation of general fairness.

Furthermore, consider the following example:

Let $q_h = 4$. $\mathcal{D} = \{m, a, b, c, x, y, z, t\}$, recalculated scores of the doctors respectively: $S' = \{92, 90, 85, 80, 82, 80, 79, 78\}$ and $Y_h = \mathcal{D} \setminus \{c, m\}$, where m is the candidate who did not update her preference list, $\{c\}$ left for another program, $\{a, b\}$ the existing candidates and $\{x, y, z, t\}$ new applicants. Then, $C_h(Y_h) = \{a, b, x, y, z\}$. However, if the score calculation was correct in the very beginning, with c not wanting h anymore, the original choice function would have chosen $\{a, b, x\}$. In that case, $\{y, z\}$ would not have been chosen by the program.

The modified choice function aims to restore fairness. However, as shown above, it is doomed to fail fairness as long as it preserves the vested interests. Moreover, in order to preserve the vested interests of the existing residents, a choice function might have to deviate from its capacity. Nevertheless, as in the second example, the choice function chooses 2 more candidates from the new applicants, $\{y, z\}$, after it meets its capacity. The reason for this situation is open to debate.

The unintended acceptance of these doctors results in an efficiency in many ways. First, from governmental point of view, if they were not placed at the original placements, they are harming the government's budget. Second, from program h's point of view, if the program was optimally designed for 4 residents that term, then the additional 2 residents may reduce the quality of the education. Third, if y and z have already started with another residency programs, they being accepted by h means a loss for their original assignments. Depending on the presence and quality of the other doctors in their application pools, those programs might face other complications. Additionally, as the young resident candidates usually prefer the programs in the urban areas rather than the rural ones, this unintended creation of additional capacities are likely to disturb the balance between rural and urban hospitals in terms of the number of residents employed, who are an important chain in the middle of the health industry.

CHAPTER 4

AN ALTERNATIVE TO THE CURRENT SYSTEM

4.1 An alternative notion of fairness

Admitting that it won't be possible to satisfy the general notion of fairness in the presence of vested interests, we still want a choice function to minimize the deviation from the carefully optimized original capacities of the hospitals to preserve the balance among the programs. Thus, an alternative notion of fairness needs to be defined.

Capacity Respecting Fairness (QRF): A choice function satisfies capacity respecting fairness if and only if:

$\nexists d \in Y_h$ such that $z'(d|s'_{Y_h}) \leq q_h$ and $d \notin C_h(Y_h)$, $\forall h \in H$ and $\forall Y_h \subset D$.

Intuitively, QRF suggests that a choice function is fair to doctors, as long as it chooses a doctor from its application pool, whose ranking in that pool is within the capacity of the program. Observe that QRF is a weaker condition than the General Fairness. QRF allows a choice function to admit one doctor with the lower score and reject the one with the higher score, if the candidate with the higher score is ranked higher than q_h in the application pool. Furthermore, we need to observe that the modified choice function used by SSPC satisfies QRF.

4.2 The QRF-Adjusted Modified Choice Function

Recall the example, where we showed that the modified choice function chooses candidates who would not be chosen if the score calculation had been correct in the beginning. With the intent of inhibiting the creation of excess additional capacities, we will now define a new choice function, which also satisfies QRF.

The QRF-Adjusted Modified Choice Function (QRF-Adjusted MCF) defines another selection rule and proceeds as follows:

1. Rank all the doctors in Y_h according to their recalculated scores.

2. Based on their recalculated rankings, add doctors one-by-one to $C_h(Y_h)$ until q_h is full or all doctors are considered.

3. Add all the remaining doctors such that $d \in (E_h \cap Y_h)$ to $C_h(Y_h)$.

4. Terminate the procedure, add all other doctors to $R_h(Y_h)$.

(Number of doctors chosen by $C_h(Y_h)$, shown by $|C_h(Y_h)|$, is the capacity of that hospital plus the number of the existing candidates who are not ranked within the application pool. Formally:

$|C_h(Y_h)| = q_h + |d \in (E_h \cap Y_h) : z'_d > q_h|$.)

Minimum Deviation: The first observation to make is that the deviation of the QRF-Adjusted Choice Function is the number of the existing residents, whose recalculated ranking is below the target capacity of the hospital. For this reason, the QRF-Adjusted Choice Function is the Choice Function, which minimizes deviation from the target capacities whilst preserving the vested interests of the existing residents and satisfying QRF.

Substitutes: Second observation is to prove that it satisfies substitutes. Any violation of substitutes would require the existence of doctor d such that $d \in Y_h$, $d \notin C_h(Y_h)$ but $d \in C_h(D'_h)$ for some D'_h such that $Y_h \subset D'_h$. All existing residents are chosen by the choice function, $\forall d \in (E_h \cap Y_h)$, $d \in C_h(Y_h)$. So our violation of substitutes, if any, must stem from the new candidates in the application pool. Suppose $d \in (Y_h \setminus E_h)$, $d \notin C_h(Y_h)$. Then $z'_d > q_h$, intuitively meaning d has not a high enough ranking in Y_h. Clearly, doctor d's ranking in the set Y_h weakly decreases while the set expands by the addition new doctors. As a consequence, d will still not be chosen from any set D'_h such that $Y_h \subset D'_h$ either. Thus, substitutes condition is satisfied.

Independence of Rejected Contracts(IRC): Suppose the choice function chooses $C_h(Y_h)$ from the application pool Y_h, where $d \notin C_h(Y_h)$. As above, $d \in (Y_h \setminus E_h)$ and $z'_d > q_h$. Then, removing d from Y_h would have no effect on the chosen set, namely $\forall d \in Y_h$ such that $d \notin C_h(Y_h)$, $C_h(Y_h) = C_h(Y_h \setminus \{d\})$. Thus the choice function satisfies IRC.

With substitutes and IRC being satisfied, the existence of a stable matching and its achievability via the deferred acceptance algorithm is guaranteed.

CHAPTER 5

PROPERTIES OF THE QRF-ADJUSTED MODIFIED CHOICE FUNCTION

5.1 Domination

We will now compare the two choice functions, C_h^{SSPC} and C_h^{QRF} in terms of the number of the candidates they choose and in terms of the realized cut-off score after their acceptance.

(Simple) Domination: A choice function C_h dominates C'_h iff

$min\{q_h, |Y_h|\} \leq |C_h(Y_h)| \leq |C'_h(Y_h)|, \qquad \forall Y_h \subset D.$

Claim 2: C_h^{QRF} dominates C_h^{SSPC}.

Proof: Suppose both choice functions are choosing from the set Y_h. Then,

$|C_h^{QRF}| = q_h + |\{d \in (E_h \cap Y_h) : z'_d > q_h\}|$, whereas

$|C_h^{SSPC}| = q_h + |d \in (E_h \cap Y_h)|$.

It is clear that $\forall Y_h \subset D$ and $\forall h \in H$, $|C_h^{QRF}(Y_h)| \leq |C_h^{SSPC}(Y_h)|$. Thus, $C_h^{QRF}(Y_h)$ dominates $C_h^{SSPC}(Y_h)$.

Cut-Off Score Domination: A choice function C_h dominates C'_h (cut-off-wise) iff

$(b_h|C_h(Y_h)) \geq (b_h|C'_h(Y_h)), \qquad \forall Y_h \subset D.$

Claim 3: C_h^{QRF} dominates C_h^{SSPC} (cut-off-wise).

Proof: Since C_h^{QRF} dominates C_h^{SSPC} in terms of the number of candidates it chooses, and the candidates are ranked in a decreasing order in terms of their scores, then a choice function which chooses weakly less candidates (C_h^{QRF}) results in a weakly higher cut-off score than the other one (C_h^{SSPC}).

Result: Suppose for two choice functions:

1. $\forall d \in E_h, \quad d \in Y_h \Rightarrow d \in C_h(Y_h)$.

2. $\forall d, d' \in (Y_h \setminus E_h), \quad d, d' \in Y, \quad d \in C_h(Y_h) \quad \& \quad z_d > z_{d'} \Rightarrow d' \in C_h(Y_h)$.

When for any two choice functions, the above two criteria is satisfied, the cut-off score domination is equivalent to the number of the new doctors chosen by the functions. Since all existing candidates are chosen, equivalence also holds for the numbers of all doctors chosen.

Let us now review the advantages of the definition and the implication of the QRF. First of all, both the modified choice function and the QRF-Adjusted MCF satisfy substitutability and Law of Aggregate Demand. Thus, by Aygün and Sönmez (2013), we can conclude that they both satisfy IRC and hence the Deferred Acceptance Algorithm induced by both functions guarantee the achievement of a stable outcome and eliminate justified envy. Admitting that, we were interested in the excess capacities created by the modified choice function rather than stability. In the claims and result, we show that QRF-Adjusted MCF chooses less new doctors than the modified choice function used by SSPC. Thus, it creates less additional capacities in a single step. In the next chapter, we will analyze the properties of the Deferred Acceptance Algorithm induced by the QRF-Adjusted MCF.

5.2 Deviation

As we admit that the choice function we propose (QRF-Adjusted MCF) minimizes the deviation from the target capacities for a given application pool, we still need to analyze its behavior along with the Deferred Acceptance Algorithm, which was proposed by Gale and Shapley (1962) and widely used by SSPC in many placement problems.

QRF (Mechanism): A mechanism ϕ satisfies capacity respecting fairness iff $\nexists(d,h)$ such that $h \succ_d \mu^{\phi}(d)$ & $z(d|h, \mu(h) \cup \{d\}) \leq q_h$.

Non-Wastefulness: A mechanism ϕ is non-wasteful iff $\forall(d,h) \in (D \times H)$, $h \succ_d \mu^{\phi}(d) \implies |\mu^{\phi}(h)| \geq q_h$.

Preservation of Vested Interests (Mechanism): A mechanism ϕ preserves the vested interests of the existing residents iff

$\nexists d \in D$ such that $d \in E_h$ for some h and $h \succ_d \mu^{\phi}(d)$.

Deviation: Being interested only in the excessive employments in the programs, we calculate the deviation of an outcome from the original target capacity of a program as follows: $max\{\mu(h) - q_h, 0\}$.

Proposition 1: QRF(Mechanism) implies non-wastefulness.

Proof: Suppose the mechanism ϕ violates non-wastefulness. Then $\exists(d,h) \in (DxH)$ such that $h \succ_d \mu^{\phi}(d)$ and $|\mu^{\phi}(h)| < q_h$. Then, $z(d|h, \mu^{\phi}(h) \cup \{d\}) \leq q_h$ which means ϕ also violates QRF(Mechanism).

Proposition 2: For every re-placement of residents matching problem, among the mechanisms which satisfy QRF (and thus non-wastefulness), and the mechanisms which preserve the vested interests of the existing doctors, the deferred acceptance algorithm induced by the QRF-Adjusted MCF (QRF-Adjusted DA), whose outcome is denoted by μ^{QRF}, results in an outcome, which deviates the least from the target capacities of the programs.

Proof: Suppose there exists a mechanism μ^{ϕ}, which satisfies the above listed criteria and creates less deviation from the target capacities. Formally, this means $\exists$ a residency matching problem where $\forall h \in H$, $\mu^{\phi}(h) \leq \mu^{QRF}$ and $\exists h \in H$, $\mu^{\phi}(h) < \mu^{QRF}$. We will proceed this proof after proving Lemma 1.

Lemma 1: Since μ^{ϕ} results in less deviation, there are some doctors who are worse-off under μ^{ϕ} than under μ^{QRF}.

Proof: Since there is less deviation with μ^{ϕ}, μ^{QRF} and μ^{ϕ} are different matchings. Suppose no resident is worse-off under μ^{ϕ}. Since they are different and no resident is worse-off, there exists at least one resident who is better-off under μ^{ϕ}. Let x be resident whose ranking is the highest among the better-off residents and let $\mu^{QRF}(x) = h$ and $\mu^{\phi}(x) = h'$. QRF-Adjusted DA did not place x to h', and since it satisfies QRF, it has placed $q_{h'}$ other residents who have higher rankings to h', which are by construction as well-off under μ^{ϕ}, thus matched to the same program under μ^{QRF} and μ^{ϕ}. μ^{ϕ} matching both those $q_{h'}$ doctors and x to h' contradicts with the initial assumption that μ^{ϕ} creates less deviation from the target capacities.

Proof of Proposition 2-Continued: Let $(a, b, c...)$ be the doctors who are worse off under μ^{ϕ} and ranked according to their recalculated scores. Suppose $\mu^{QRF}(a) = h$ and $\mu^{\phi}(a) = h'$. Since we assumed a is worse-off under μ^{ϕ}, $h \succ_a h'$.

Since ϕ satisfies QRF and non-wastefulness, it has employed at least q_h residents who are ranked higher than a according to their recalculated scores. Those residents are at least as well-off under μ^{ϕ}, since a is the highest ranked doctor among the under μ^{ϕ} worse-off doctors. However, since μ^{QRF} is an outcome of the Deferred Acceptance Algorithm, it must have employed the same q_h candidates as in μ^{ϕ}, which leads to the final contradiction.

After proving Proposition 2, we can now conclude that: In a (re)placement problem with the existence of the vested interests, if the deviation from the target capacities is undesirable, the Deferred Acceptance Algorithm induced by QRF-Adjusted MCF needs to be implemented.

CHAPTER 6

CONCLUSION

The Student Selection and Placement Center in Turkey uses the serial dictatorship mechanism in many of its exams, and in Examination for Specialty in Medicine as well. The candidates obtain a single-category score after the exam, they submit their preferences and get assigned to programs accordingly. This mechanism in this case is equivalent to the deferred acceptance algorithm, where the doctors apply to the programs and the programs choose doctors up to its capacity according to their rankings.

After the exam of Fall 2016, some questions were canceled and the scores of the residential candidates were calculated based on the remaining questions. The candidates were placed according to the system. However, this cancellation was revoked by a superior court. Thus, to restore the fairness whose rankings would rise after the recalculation, SSPC announced that re-placements were going to be held according to a modified choice function.

An analysis of the modified choice function used by SSPC showed that this choice function satisfies substitutes and law of aggregate demand, thus guarantees the existence of a stable matching. However, as it was the very first aim to restore fairness, it easily violates the general notion of fairness. Indeed, any choice function which preserves the vested interests and has binding capacities fail general fairness.

Furthermore, the modified choice function creates additional and redundant capacities for the new candidates. However, since the initial capacities were calculated elaborately according to the needs of the programs and the country, any deviation from them is undesirable. Thus we define an alternative notion of fairness, which also respects the initial capacities (QRF) and a QRF-Adjusted Choice Function to be used instead. We show that the QRF-Adjusted Choice Function is the choice function which minimizes the deviation from the target capacities, while preserving the vested interests of the existing residents. It also satisfies substitutes and law of aggregate demand at a given matching problem, so it also guarantees the existence of a stable matching, and the achievability of it via the deferred acceptance algorithm. Furthermore, the QRF-Adjusted Choice Function dominates the Choice Function used by SSPC in terms of both the additional candidates it is admitting and in terms of the cut-off score resulting from the choice function.

REFERENCES

Abdulkadiroğlu, A., & and Sönmez, T. (2003). School choice: A mechanism design approach. *American Economic Review, 93*(3), 729-747.

Aygün, O., & Sönmez, T. (2013). Matching with contracts: Comment. *American Economic Review, 103*(5), 2050-51.

Balinski, M., & Sönmez, T. (1999). A tale of two mechanisms: student placement. *Journal of Economic Theory, 84*(1), 73-94.

Echenique, F., & Yenmez, M. B. (2015). How to control controlled school choice. *American Economic Review, 105*(8), 2679-94.

Gale, D., & Shapley, L. S. (1962). College admissions and the stability of marriage. *The American Mathematical Monthly, 69*(1), 9-15.

Hatfield, J. W., & Milgrom, P. R. (2005). Matching with contracts. *American Economic Review, 95*(4), 913-935.

Hatfield, J. W., & Kojima, F. (2010). Substitutes and stability for matching with contracts. *Journal of Economic Theory, 145*(5), 1704-1723.

Roth, A. E. (1982). The economics of matching: Stability and incentives. *Mathematics of Operations Research, 7*(4), 617-628.

Roth, A. E. (1984). The evolution of the labor market for medical interns and residents: a case study in game theory. *Journal of Political Economy, 92*(6), 991-1016.

Roth, A. E. (1985). The college admissions problem is not equivalent to the marriage problem. *Journal of Economic Theory, 36*(2), 277-288.

Roth, A. E., & Sotomayor, M. (1990). *Two-sided matching: a study in game-theoretic modeling and analysis.* Cambridge, England: Cambridge University Press.

Sönmez, T., & Switzer, T. B. (2013). Matching With (BranchofChoice) Contracts at the United States Military Academy. *Econometrica, 81*(2), 451-488.