MEASURING PRE-SERVICE TEACHERS' MATHEMATICS CONTENT KNOWLEDGE AND MATHEMATICS PEDAGOGICAL CONTENT KNOWLEDGE ON RATIO AND PROPORTION

by

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I dedicate this work to my growing family, my husband and my yet unborn baby girl...

ABSTRACT

MEASURING PRE-SERVICE TEACHERS' MATHEMATICS CONTENT KNOWLEDGE AND MATHEMATICS PEDAGOGICAL CONTENT KNOWLEDGE ON RATIO AND PROPORTION

This study investigated pre-service teachers' Mathematics Content Knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK) on ratio and proportion. MCK and MPCK instruments (Ekawati, Lin, & Yang, 2015) were translated to Turkish to conduct the study with senior pre-service middle school and secondary school mathematics teachers from twelve different universities including both the secondary and primary education departments. Data were gathered from 590 senior pre-service teachers in total. Data was analyzed both descriptively and also using confirmatory and exploratory factor analysis. Results from confirmatory factor analysis aligned with (Ekawati *et al.*, 2015) showed that there were three factors for MCK-T and results from the exploratory factor analysis showed that there were three factors for MPCK-T. Results regarding MCK of both preservice middle and secondary school mathematics teachers showed that they were very successful at questions requiring procedural answers. However, they were not able to reason on questions requiring to distinguish proportional situations from non-poportional situations as well as the questions having figural representations. Similarly, results regarding MPCK of both pre-service middle school and secondary school mathematics teachers showed that although they were able construct problems related to ratio and proportion, they were not able to take into consideration students? reasoning and back-ground knowledge. By the same token, neither they were able to determine students? misconceptions nor were they able to provide feedback based on their misconceptions. Results also suggested that all pre-service teachers knew few student strategies.

ÖZET

MATEMATİK ÖĞRETMEN ADAYLARININ ORAN VE ORANTI KONUSUNDA SAHİP OLDUKLARI ALAN BİLGİSİNİN VE PEDAGOJİK ALAN BİLGİSİNİN ÖLÇÜLMESİ

Bu çalışma, matematik öğretmen adaylarının, oran ve orantı konusunda sahip oldukları alan bilgisini ve pedagojik alan bilgisini araştırmayı amaçlamıştır. Bu amaç için Ekawati ve arkadaşlarının (Ekawati, Lin, & Yang, 2015) oran ve orantı üzerine tasarlamış oldukları Matematik Alan Bilgisi (MAB) ve Matematik Pedagojik Alan Bilgisi (MPAB) ölçekleri Türkçe?ye çevrilerek, Türkiye?de ilköğretim ve ortaöğretim matematik öğretmenliği bölümlerini aynı anda bulunduran on iki üniversitenin 590 son sınıf ilköğretim ve ortaöğretim matematik öğretmen adayına uygulanmıştır. Toplanan veri betimsel istatistiklerle ve faktör analizleri ile incelenmiştir. MAB ölçeğinin Türkçe çevirisinin verileri ile yapılan doğrulayıcı faktör analizi sonuçları, Ekawati ve arkadaşlarının (2015) çalışması ile uyum göstermiştir ve aynı üç faktörü ortaya çıkmıştır. MPAB ölçeğinin Türkçe çevirisinin verileri ile yapılan açımlayıcı faktör analizi sonuçlarına göre de üç faktör ortaya çıkmıştır. Matematik öğretmen adaylarının MAB sonuçları değerlendirildiğinde öğretmen adayları yöntemsel bilgi gerektiren sorularda başarılı bulunmuşlardır. Fakat sonuçlar, orantılı durumları orantısız durumlardan ayıran sorularda ve şekilsel sorularda muhakeme edemediklerini göstermiştir. MPAB ölçeğinin sonuçlarına göre ise adayların, oran ve orantı ile ilgili problem tasarlayabilmelerine rağmen öğrenci düşüncesini ve öğrencinin önceki bilgilerini göz önünde bulunduramadıkları ve öğrenci kavram yanılgılarını belirleyememekle beraber bu kavram yanılgılarını ortadan kaldıracak geri dönütleri veremedikleri gözlenmiştir. Ek olarak sonuçlar, matematik öğretmen adaylarının oran ve orantı ile ilgili çok az strateji bilgisine sahip olduklarını göstermektedir.

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LIST OF ACRONYMS/ABBREVIATIONS

CFA	Confirmatory Factor Analysis
CFI	Comparative Fit Index
COACTIV	Cognitively Activating Instruction Study
EFA	Exploratory Factor Analysis
MCK	Mathematics Content Knowledge
MCK-T	Turkish Translation Version of MCK Instrument
MKT	Mathematical Knowledge for Teaching
MPCK	Mathematics Pedagocical Content Knowledge
MPCK-T	Turkish Translation Version of MPCK Instrument
NCTM	National Council of Teachers of Mathematics
PCK	Pedagogical Content Knowledge
PMMT	Pre-service Middle School Mathematics Teacher
PRED	Primary (middle) School Mathematics Education Program
PSMT	Pre-service Secondary School Mathematics Teacher
RMSEA	Root Mean Square Error of Approximation
SCED	Secondary School Mathematics Education Program
SMK	Subject Matter Knowledge
TEDS-M	Teacher Education and Development Study in Mathematics Study
TLI	Tucker-Lewis index

1. INTRODUCTION

Teachers are required to have a broad and deep understanding of the mathematics they teach (Ball & Cohen, 1999; Ma, 1999). Teachers' content knowledge affects the mathematics they teach in their classroom by the tasks they decide to use and the type of discussions they initiate. Therefore, teacher knowledge is the most important component of teaching which affects what is done in classrooms and ultimately what students learn (Fennema & Franke, 1992). This suggests that teacher education should be taken into consideration attentively.

When the history of teacher education in Turkey was examined, teacher education policies have been changed constantly (Özoğlu, Gür, & Çelik, 2010). In Turkey, there are two ways to be a mathematics teacher: one might graduate from a teacher education program, namely middle school mathematics education or secondary school mathematics education program or one might graduate from a faculty of art or science department but needs to further complete a teaching certificate program. Therefore, teacher candidates are educated differently because of the different content and different requirements of teacher education programs and undergraduate programs in mathematics. This again necessiates consideration fo the quality of teacher education and teachers deeply (Özoğlu *et al.*, 2010).

Though, international and national studies point that measuring teacher quality is a complex phenomenon (Wilson, 2007). Still, there is a tendency to measure teacher knowledge as an indicator of teacher quality. Ball and McDiarmid (1990) asserted that subject matter knowledge and pedagogical content knowledge are crucial elements of teacher knowledge. Results from studies focusing on secondary school mathematics teachers' subject matter knowledge showed that although they have more advanced mathematics courses, knowledge they have of elementary mathematics topics is limited (Ball, 1988; Even, 1993; Even & Tirosh, 1995; Wilson, 1994). Previous research also pointed that pre-service elementary teachers having procedural skills have limited conceptual knowledge of mathematics (Cramer & Lesh, 1988).

Among all the concepts in mathematics, some deserve particular attention by teachers, researchers, and teacher educators. Among all, the ability to reason multiplicatively is the most essential one for students' mathematical development (Harel & Confrey, 1994); therefore, teachers are required to know how to reason in multiplicative situations (Post, Harel, Behr, & Lesh, 1988; Sowder, Armstrong, Lamon, Simon, Sowder & Thompson, 1998). Ratio and proportion is the cornerstone of multiplicative reasoning and constitutes an area of mathematics that could be problematic to teachers at all levels (including elementary, middle and secondary) (Cramer, Post, & Currier, 1993; Heinz, 2000; Post, Harel, Behr, & Lesh, 1991; Simon & Blume, 1994; Smith, Silver, Leinhardt, & Hillen, 2003; Sowder *et al.*, 1998). In fact, since ancient times, ratio and proportion have been considered as one of the most important mathematics topics and given great importance and attention. The application of ratio and proportion in solving daily life problems can be even observed in the Rhind Mathematical Papyrus (1600 B.C.). Then, a thousand years later, using the concept of ratio, "Thales discovered how to obtain the height of pyramids and all other similar objects, namely, by measuring the shadow of the object at a time when a body and its shadow are equal in length" (Heath, 1981). Similarly, National Council of Teachers of Mathematics (NCTM) Curriculum and Evaluation Standards (1989), asserted that "the ability to reason proportionally develops in students throughout grades 5-8. It is of such great importance that it merits whatever time and effort that must be expended to assure its careful development" (p.82). However, previous research has shown that not only high school students have some lack of knowledge regarding the use of proportional reasoning (Post, Behr, & Lesh, 1988; Ben-Chaim, Fay, Fitzgerald, Benedetto, & Miller, 1998) but also pre-service and in-service elementary mathematics teachers' pedagogical knowledge for teaching is not enough (Sowder *et al.*, 1998).

Therefore, the main purpose of the current study was to measure mathematics content knowledge and mathematics pedagogical content knowledge of pre-service mathematics teachers on ratio and proportion. Results might shed light on what pre-service teachers know and do not know as well as the difficulties they might entail regarding the concept of ratio and proportion.

2. LITERATURE REVIEW

In this section, firstly, I explain the teacher knowledge in general and mathematical knowledge in particular for teaching. Then, I present the previous research studies about the assessment of teacher knowledge by reporting the findings of studies like TEDS-M and COACTIV. After that, I describe the theoretical framework of the concept of ratio by focusing on the strategies students use on ratio and proportion problems; the task variables and some conceptions of ratio. Lastly, I mention the previous research of teacher knowledge especially on the topics ratio, proportion and proportional reasoning.

2.1. Teacher Knowledge

In the last three decades, many researchers have interested in teacher education and in particular teachers' knowledge of mathematics. Shulman (1986) and Ball, Thames and Phelps (2008) have developed their own models. Many researchers have been influenced from their ideas. In the following paragraphs, I briefly explain those two models firstly. Then, I point to the mathematical knowledge for teaching.

Shulman (1986), has defined the components of teachers' content knowledge by categorizing it in three parts: Subject matter knowledge, pedagogical content knowledge and curricular knowledge. He defined the subject matter knowledge in the following way:

"To think properly about content knowledge requires going beyond knowledge of the facts or concepts of a domain. It requires understanding the structures of the subject matter [...]. Content knowledge refers to the amount and organization of knowledge per se in the mind of teacher. This involves the understanding beyond the structure of the subject matter (p. 9)".

Further clarifying this definition, Shulman stated that "the teacher needs not only understand that something is so, the teacher must further understand why it is so" (p. 9). Therefore, content knowledge describes a teacher's understanding of the structures of his or her subject. On the other hand, pedagogical content knowledge is described as: "that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding" (p. 8). This could be interpreted that the teachers should know the best and alternative ways of any explanation, demonstration, analogy or illustration, and the most appropriate exercise of a topic to make the topic comprehensible for students. And also, it is about the knowledge of difficult and easy parts of the subject.

According to Shulman (1986), the last category of the content knowledge is curricular knowledge. It is about the knowledge of alternative instruction materials. In addition, Shulman mentioned two other important dimensions of curricular knowledge as lateral curriculum knowledge and vertical curriculum knowledge. Lateral knowledge is about the learning of students in other subject areas. Vertical knowledge can be defined as "familiarity with the topics and issues that have been and will be taught in the same subject area during the preceding and later years in school, and the materials that embody them" (Shulman, 1986, p. 10).

After Shulman's conceptualization of teacher knowledge, other researchers used this categorizations as a basis for their research. Specifically, in the field of mathematics education, Ball *et al.*, (2008) developed Mathematical Knowledge for Teaching (MKT). Because of it's importance for this study, in the following section I explain MKT model in detail.

2.2. Mathematical Knowledge for Teaching and It's Assessment

As stated, MKT model of teacher content knowledge is based on Shulman's (1986) categorizations. In particular, building on Shulman's (1986) SMK and PCK, Ball and her colleagues (2008) focused on what teachers need to know and need to do for effecive mathematics teaching. They argued that subject matter knowledge requires knowledge beyond the mathematical knowledge such that the mathematical practices specific to the subject are also regarded as important. By the same token, they considered pedagogical content knowledge as related to the knowledge of a teacher used in the classroom during mathematics teaching.

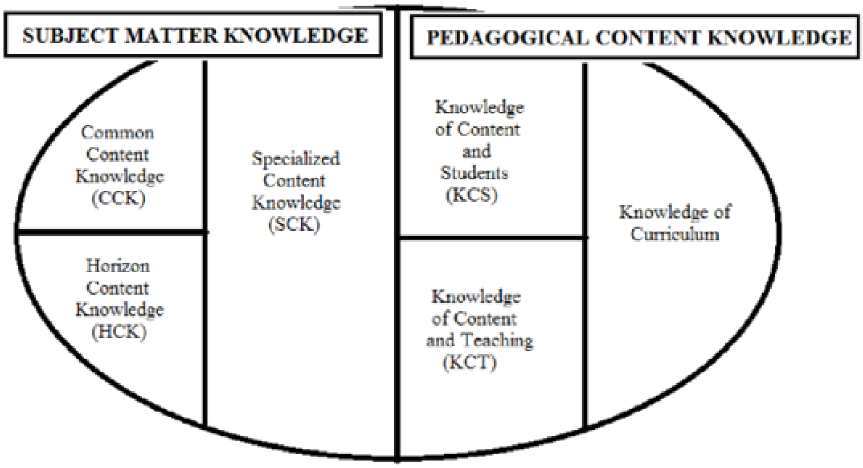


Figure 2.1. Mathematical knowledge for teaching (Ball *et al.*, 2008, p. 403).

Ball and her colleagues, subdivided subject matter knowledge and the pedagogical content knowledge into six categories: Common content knowledge (CCK), specialized content knowledge (SCK) and horizon content knowledge (HCK) were included in subject matter knowledge; whereas, knowledge of content and students (KCS), knowledge of content and teaching (KCT) and knowledge of curriculum were included in pedagogical content knowledge (See Figure 2.1 showing the MKT model as refinement of Shulman's categorizations).

As the first component of the subject matter knowledge, Ball and her colleagues defined CCK as "the mathematical knowledge and skill used in settings other than teaching" (Ball *et al.*, 2008, p. 399). They argued that it is about using terms and notations, solving a question correctly, noticing wrong solutions or missing definitions of students. From their perspective, CCK is essential; but is not specific to teachers or teaching since for example any physics teacher or an engineer can have CCK as well. Ball and her colleagues considered the second component of subject matter knowledge as specialized content knowledge (SCK). They argued that although teachers may never directly teach their SCK to their students, they are expected to know more than factual and procedural knowledge. That is, they need to know why procedures work, how mathematical facts and claims are justified, and how to derive formulas (Ball, 2003). Also, they need to use SCK especially when they are analyzing the solutions of students or while answering the "why" questions of students. While distinguishing CCK from SCK, they argued that any educated person having CCK might compute for instance, division on fractions; however, a teacher has to recognize the reasons for the process of division by considering why the divisor is reversed and then multiplied with the dividend fraction.

Ball and her colleagues (2008) considered the last component of the subject matter knowledge as the horizon content knowledge (HCK). It is described as "an awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (p. 403). Moreover, HCK is defined as knowledge requiring mathematics that would go beyond the grade levels teachers need to teach (Ball & Bass, 2009). That is, this knowledge is about the totality of mathematics in itself such that by using this knowledge, teachers can design an instruction focusing on both pre-requisite knowledge and also further knowledge of students.

As Shulman (1986) defined pedagocial content knowledge, "amalgam of content and pedagogy", Ball (2003) redefined the subcategories of pedagocial content knowlegde. The first subcategory, the knowledge of content and students (KCS), is about predicting how students might think or what they might find confusing as well as being aware of preconceptions and misconceptions of students. Besides, it requires teachers to be careful about choosing examples and to be more motivating according to grade level of students.

As the second subcategory of pedagogical content knowledge, knowledge of content and teaching (KCT) is about knowing appropriate instructional designs and teaching strategies according to students' levels. Having such knowledge, teachers also determine the sequence of particular content for instruction while choosing appropriate examples to start with, focusing on different representations of the content and useful examples to take students' attention to the content more deeply.

In Ball and her colleagues' model, the last component of pedacogical content knowledge is the knowledge of curriculum. This knowledge is the same with the curricular knowledge in Shulman's model such that it is about knowing relation of mathematical content and curriculum at the same time.

Using Ball and her colleagues' model, numerous studies have been done on the assessment of teachers' subject matter knowlegde (i.e., mathematics content knowledge, MCK) and pedagogical content knowledge (i.e., mathematics pedagocical content knowledge MPCK) (Hill *et al.*, 2004; Krauss, *et al.*, 2008; Tatto*et al.*, 2008). Some of these assessment studies were MKT measure, and Teacher Education and Development Study in Mathematics Study (TEDS-M) and Cognitively Activating Instruction Study (COACTIV). While MKT measure (e.g., Hill, Rowan, and Ball, 2005; Delaney, Ball, Schilling, & Zopf, 2008; Aslan-Tutak, 2009; Çopur Gençtürk, 2012) focused on knowledge of elementary and middle school teachers, COACTIV study only focused on the knowledge of secondary school mathematics techers and TEDS-M's focus was on elementary and secondary pre-service teachers' preparation, knowledge and beliefs (Tatto *et al.*, 2014).

Among these MKT measure focused mainly on the (i) number concepts, (ii) operations, (iii) patterns, functions, and algebra by regarding MCK; and, the two domains of MPCK, (i) knowledge of content, (ii) knowledge of content and students (Hill *et al.*, 2004). On the other hand, TEDS-M included items measuring mathematics content knowledge (MCK) within the four content areas of mathematics (number, algebra, geometry and data) and three cognitive dimensions (knowing, applying and reasoning); and mathematics and pedagogical content knowledge (MPCK) consisting of knowledge of curricula, and planning, and interactive knowledge about how to enact mathematics for teaching and learning. Though researchers stated that MCK and MPCK items are not disjoint since "it is impossible to construct disjoint sub-domains because the solution of an item in the domain MPCK generally requires MCK" (Döhrmann, Kaiser & Blömeke, 2012, p. 336).

Finally, COACTIV study, as a longitudinal assessment in Germany, was about investigating the relationship between students' performance in PISA 2003/2004 and competencies of teachers (Krauss *et al.*, 2008). The instrument consisted of both MPCK test and MCK test to examine the connectedness of the two knowledge categories in teachers who have different mathematical expertise (Krauss, Brunner, *et al.*, 2008). Particularly, the items regarding MPCK included knowledge of mathematical tasks, knowledge of student misconceptions and difficulties, and knowledge of mathematics-specific instructional strategies. Also, MCK test items covered secondary school content areas in order to measure teachers' conceptual or procedural skills (Krauss *et al.*, 2008).

Since the instrument used in this study measuring the teachers' MCK and MPCK reagarding ratio and proportion was based on COACTIV study, the rationale behind the use of COACTIV study will be discussed further in the procedure section. Therefore, prior to discussing previous research on the teacher knowledge regarding ratio and proportion, in the following section, I explain the concepts and constructs of ratio and proportion.

2.3. Ratio and Proportion

2.3.1. Definitions of Ratio, Rate, Proportion and Proportional Reasoning

2.3.1.1. Ratio. In the literature, although there has been a large amount of research on the conception of ratio (Lamon, 2012; Lobato *et al.*, 2010; Karagoz Akar, 2007; 2015; Thompson, 1994; Vergnaud, 1988, Lesh *et al.*, 1988); there is no consensus among the researchers in the use of terminology about ratio such that some researchers even use different names for the same concepts in this domain (Lesh *et al.*, 1988). For instance, while Lesh *et al.* defined ratio as binary relations, which involve extensive, intensive or scalar types of ordered pairs of quantities (Lesh *et al.*, 1988), Hart claimed, "ratio is a statement of the numerical relationship between two entities" (Hart, 1988, p. 198). Likewise, Ohlsson defined ratio as the relationship of how much there is one quantity in relation to another quantity (Ohlsson, 1988).

Some researchers prefer to use the terms within-state and between-state ratios in their studies (Lamon 1993, Lamon 1994, Noelting, 1980b, Karagöz Akar, 2007; 2015). According to Noelting, the within-state ratios refers to quantities coming from two different measure spaces, on the other hand and between-state ratios refers to the quantities coming from the same measure space in two different situations. For instance, "There are 5 bananas for every 3 oranges in a basket, how many bananas would be if there are 15 oranges in the basket" If the student thinks about the problem in the ratios 3 oranges: 15 oranges and 5 bananas: x bananas, then this is called between-state ratios. On the other hand, if they use the 5 bananas: 3oranges and x bananas: 15 oranges, then this is called within-state ratios. Lamon asserted that "a student is said to be using a within strategy or a scalar method when s/he equates two internal or within-measure space ratios and uses the sameness of scalar operators to determine the missing term" (Lamon, 1994, p. 95).

Furthermore, Heinz (2000) uses different terms of the conceptions of ratio rather than within-state and between-state ratios. One of them identical groups conception and the other one is raito-as-quantity. The identical groups conception refer to the ratio as two extensive quantities coming from different measure spaces, namely, within-state ratios. Ratio-as-quantity represents the ratio as an intensive quantity measuring the relative size. So, the structure of ratio-as-quantity can be used for both within and between-state ratios.

Similarly, Vergnaud used different terms for the types of ratios. He used "functional ratios and scalar ratios" instead of within and between-state ratios respectively. Scalar ratio has no dimension because it is regarded as the quotient of quantities of the same kind (Vergnaud, 1988).

On the other hand, Kaput and West (1994) have considered the concept of ratio as particular ratio (internalized) and rate-ratio (interiorized). For instance, the particular ratio of "3 pounds per 4 dolars" represents a particular purchase, however, rate-ratio is the linear function which is defined between pounds and dollars.

Lastly, Thompson defined ratio as "the result of comparing two quantities multiplicatively" (Thompson, 1994). There are three common ratio comparisons: part-part (for example, two glass of milk and five spoons of sugar or 2/5), part-whole (for example, nine of the eleven employee or 9/11); and scaling, whole-whole (comparing wholes to wholes, where 2 dm on the map equals 500 dm on the ground) (Suggate *et al.*, 2006).

<u>2.3.1.2. Rate.</u> Ratio and rate are inherently related terms. According to Thompson (1994), ratio is an expression of specific situations and rate can be defined as constant ratio. So, for the generalized situations rate is reflectively derived from ratio (Thompson, 1994). Thompson (1994) stated that:

"...rate is (from my point of view) a linear function that can be instantiated with the value of an approximately conceived structure. To say that an object travels at 50 miles/hour quantifies the objects' motion, but it says nothing about a distance traveled nor about a duration traveled at that speed (Schwartz, 1988, cited in Thompson, 1994). However, conceiving speed of travel in relation to an amount of time traveled produces a specific value for the distance traveled" (p. 192).

On the other hand, Lesh and colleagues defined rate as intensive quantities, which are recognizable by the "per" in their unit labels such as 2 miles-per-hour (Lesh, Post, & Behr, 1988). Also, Ohlsson stated rate as the numerical representation of change over time. So, it can be defined as the simplest ratio between the quantity and a period of time (Ohlsson, 1988). To sum up, ratio is used in particular situations; however, rate goes beyond the particular situation such that it represents the multiplicative relationship between the quantities.

<u>2.3.1.3. Proportion and Proportional Reasoning.</u> Proportion is simply defined as a relationship between four numbers or quantities in which the ratio of the first pair equals the ratio of the second pair and written as a/b = c/d (Ekawati *et al.*, 2015, p.515). Also, Karplus and his friends (1983) introduced proportions to make a definition for linear functional relationships. They viewed that the multiplier m in the linear function

y=mx can be an intensive quantity (Karplus *et al.*, (1983).

On the other hand, proportional reasoning describes any kind of reasoning that focuses on the relationship between two ratios (Vergnaud, 1983). The ability of proportional reasoning is needed for both teachers and students because this ability is important to develop mathematical thinking and also it is considered as the cornerstone of higher mathematics (Ekawati *et al.*, 2015; Lamon, 2012; Lobato, Ellis, 2010).

Calling them as proportional thinkers, Lamon (2012) concurred that when students reason proportionally, they show some similarities. Some of those common characteristics point that they can think in terms of complex units, develop strategies and show greater competencies when they are solving proportional problems. Besides, they can distinguish proportional situations from non-proportional situations and apply an appropriate algorithm for these situations. In addition, they can explain their thinking in proportional situations by using necessary vocabularies (Lamon, 2012, p.259).

Similarly, proportional reasoning can be considered as a second-order relationship which means the relationship between two relationships rather than simply a relationship between two concrete objects (Piaget & Inhelder as cited in Lesh, Post & Behr, 1988). By the same token, proportional reasoning ability is determined by focusing on how a student can recognize the correct relationhips between quantities and also which strategies s/he might use when solving problems (Vergnaud, 1983).

Therefore, in the following section I focus on some different informal and formal strategies students possibly use when they are solving problems related to ratio as well as the task variables affecting students' ways of reasoning. Then, I focus on different conceptions of ratio students might possibly have.

2.3.2. Strategies

According to Lamon, firstly, the building-up strategy is formed by successive multiples of some other units (Lamon, 1993). Although the students use multiplication in this strategy, the main focus is additive reasoning. In other words, in this strategy the repeated addition version of multiplication is used which is based on replications of the identified units and counting them (Kaput & West, 1994). For instance, the problem "Ali, Ahmet, and Mehmet bought three jelly tots and paid 2.00$ for all three. They decided to go back to the store and buy enough jelly tots for everyone in their class. How much did they pay for 24 jelly tots" is solved by students using the building-up strategy in the following way: The student would make a table and say:

\item2	4	6	8	10	12	14	16	(\$paid)
\item3	6	9	12	15	18	21	24	(jellytots)

As it is seen, the student replicates both 2 and 3 until the desired number of jelly tots is reached. In this strategy, the final situation is based on the replication of the original quantities (Heinz, 2000). So, briefly, the identical copies of the original situation is regarded as repeated addition of multiplication. However, it is asserted that the use of building up strategy does not require an understanding of the multiplicative relationship regarding the conception of ratio (Heinz, 2000). So it may be the evidence that some students who successfully solve some ratio problems by using the building-up strategy can be unsuccessful in solving other problems that involve proportional relationship (Lamon 1993; Lesh *et al.*, 1988). Also Kaput and West (1994) stated that if a problem involves the ratio 3:8 and one is asked to find the first quantity by corresponding to 58 as the second quantity, then some of the students may not be successful in solving this problem because of the divisibility failure, 58 divided by 8 is not an integer.

Another strategy students might use is the abbreviated build-up strategy. It can be regarded more complicated than the building-up strategy because if students use abbreviated build-up strategy than it is expected to be able to multiply both the first quantity and second quantity in the ratio by the same factor (Kaput & West, 1994). On the other hand, although it is based on a multiplicative structure between ratios, this multiplicative structure is based also on the repeated addition (Heinz, 2000; Karagöz

Akar, 2007). This strategy is similar to the ratio-unit/ build-up method. According to Lo and Watanabe, if a student can develop an “x elements for y elements” relationship and apply this relationship for the whole-number sense, than this strategy used is called as ratio-unit/buildingup strategy (Lo & Watanabe, 1997).

Heinz (2000) asserted that there are some other strategies used to deal with a divisibility failure like unit factor strategy, incorrect addition strategy, multiplicative strategies, within and between strategies and so on. Kaput and West (1994) claimed that those strategies are based on more advanced conception of ratio because of the need for understanding of proportional relationship.

The first strategy as mentioned above is called as the unit factor approach. It is claimed that in this approach, students can use division to find the unit factor and then multiply this factor by the given quantity (Kaput & West, 1994). Actually this is the main difference from the building-up strategy. Also, unit-factor approach is based on the use of partitive division scheme, on the other hand in the abbreviated build-up process, quotitive division scheme is used (Kaput & West, 1994). Heinz (2000) also introduced the unit factor approach as a “per-one strategy” because the unit factor strategy involves a reduction of the given ratio such that one of the components is 1 and is based on the association between the two quantities (Heinz, 2000).

The second one, the incorrect additon strategy, can be introduced as the constant difference strategy which is used by students who do not understand the conception of ratio. Heinz (2000) asserted that there are two main reasons for the comman use of the incorrect addition strategy. One of them is that students are more familiar with the additive comparisons in their early education. The second one is that multiplication is generally introduced as repeated addition (Kaput & West, 1994). So, by considering those claims Heinz (2000) pointed out that students may develop additive conceptions rather than multiplicative in their nature.

Heinz (2000, p.44) has also presented the definiton of multiplicative strategies: “Multiplicative strategies are those in which the relationship between two quantities are appropriately represented with a ratio, and then the ratio is applied as needed to the remainder of the applicable data in the problem to either find missing values or to make comparisons across cases in a multiplicative situation".

To solve missing value proportion problems and ratio comparison problems, ratios can be formed in three strategies such as within strategy, between strategy and proportion equation strategy. If a ratio is composed of two quantities within one measure space and between two situations, the approach is called as the within strategy. On the other hand, if a ratio is composed of quantities between two measure spaces and within one situation, the approach is referred as the between strategy (Heinz, 2000). For instance, assume that one is using 3 cans of yellow paint and 6 cans of red paint. The other one wants to form the same colour by using 5 cans of yellow paint. To find the number of red cans, if the ratio (3 yellow:5 yellow) is used, the second quantity would contain 5/3 times the first quantity, so red cans will be 6*(5/3), 10 red cans. This approach is the within strategy and also called a scalar strategy because within a measure space, one quantity is a scalar multiple of the other. If one uses the ratio (3 yellow: 6 red) to find the number of red cans for 5 cans of yellow paint by considering that each yellow can need to be mixed with 2 red cans, there should be 5*2=10 cans of red paints. This approach is the between strategy and also called as the functional strategy since there is a functional relationship between quantities in different measure spaces (Vernaugh, 1988). Moreover, Lamon (1989) asserted that one strategy is not more "natural" than the other. So we cannot say that students were more likely to use within or between strategies. On the other hand, in the literature it is asserted that students are more likely to choose strategies which exists an integer ratio between quantities. For instance 2:8 and 24:6 are integer ratios because quantities are composed of integer multiple of each other (Kaput, 1985).

As a third multiplicative strategy we can mention proportion equation strategy. It is called as the cross-multiplication strategy also. It is about understanding the proportional relationships that exists between or within ratios. However, if one can use the cross multiplication strategy, we can not necessarily say that the one can make proportional reasoning (Kaput & West, 1994).

2.3.3. Task Variables

The strategies which are used on ratio and proportion problems and the difficulty level of such problems depend on the different task variables. Kaput and West (1994) has classified the task variables as numerical features of the problem, semantic features of the problem and context of the problem.

In the literature, it is asserted that problems which include integer ratios are easier than problems which include noninteger ratios. For instance, multiplication by 2, 3, or taking half were considered as easy problems. On the other hand, fraction operation was regarded as complex number structure. (Kaput & West, 1994; Noelting, 1980b; Hart, 1988). Also, the use of numbers whose magnitudes are relatively close to each other in making up the quantities of ratio could facilitate the use of additive reasoning (Heinz, 2000; Kaput & West, 1994). For instance, Heinz presented in her study that most of the students used additive thinking while working on the ratio pairs of 3 lemons to 2 limes and 4 lemons to 3 limes (Heinz, 2000).

Moreover, in the Kaput & West study, the building-up strategy was generally applied when the quantities in the problem were explicitly associated with a “for every” or “for each” statement. Because of the role of unit formation, problems are getting easier for students (Heinz, 2000; Kaput & West, 1994).

In addition, Lamon (1993) have distinguished four types of tasks according to their context. Those are well-chunked measures, part-part-whole problems, associated sets, and stretchers and shrinkers. For instance, similarity problems or geometry tasks can be given as examples for stretchers and shrinkers. Speed and price can be considered as well-chunked measures. The context of the ratio of number of girls to the number of boys in a class can be given part-part-whole problems. On the other hand, Tourniaire & Pulos (1985), make a different classification as physical, rate, mixture, and probability problems. It can be said that different context situations may influence different students’ way of thinking or reasoning strategies.

The other variable can be introduced as quantity type. Quantities can be classified as discrete or continuous. For instance marbles may regarded as discrete and the amounts of liquids may regarded as continuous (Alatorre & Figueras, 2005).

Furthermore, Ekawati and her colleagues (2013) have grouped task variables into three as context situations, task type, and number structures. The task type can be a missing value problem or a ratio comparison problem.

In summary, there are many task variables that affect both the problem difficulty and the strategies which students use. However, there is a complex relationship between task variables and student answers (Lesh *et al.*, 1987; Vergnaud, 1988). In the current study, the instruments include many type of task variables to analyze student answers from different contexts.

2.3.4. Conceptions of Ratio

In the literature, some of the conceptions of ratio have already been discussed. Lamon (1993) stated that, some of the students who were proportional reasoners solved different ratio and proportion problems using different conceptions. Those conceptions can be regarded as identical groups conception, ratio-as-measure, between-ratio and within-ratio conceptions (Heinz, 2000; Kaput & West, 1994; Behr, Harel, Post, & Lesh, 1993; Karagöz-Akar, 2007).

Firstly the identical groups conception can be considered in two ways. One of them is as the collection of sets of the extensive quantities in ratio; and, the second one is as breaking down the ratio into equal parts. Students with this conception think of ratio as a particular combination of two quantities rather than the difference between quantities. In other words, students understand that the association between the extensive quantities remains invariant since the initial extensive quantities given in the ratio is repeated together until one of the known quantities is reached (Heinz, 2000). For instance, when asked how many apples are needed for 20 children if there are 4 apples for every 5 children, an identical groups conception makes someone think that the amount of apples per child is determined by the combination of 4 apples and 5 children. In here students use the building-up strategy rather than finding the difference between quantities (Heinz, 2000).

On the other hand, the related process with breaking down is partitive division within the elements of a given ratio. In other words, the elements of the first extensive quantity are distributed over the elements of the second extensive quantity in the partitive division process. The strategy students use for breaking down the quantities using partitive division is called per-one. In this process, ratio expresses how much of one extensive quantity associates with one unit or n units of the other extensive quantity (Heinz, 2000). It can be asserted that both for the per-one [per n] strategy and for building-up strategy as an implication of identical groups conception, the goal is to keep the structure the same, which can be regarded as the same quality of interest. The difference is that in building-up students try to make twins, triples'n tiples of the same structure; whereas, in per-one strategy students try to make smaller groups of the given ratio.

Heinz (2000) claimed that students with identical groups conception are not able to solve all types of ratio tasks such as tasks with a divisibility failure. For instance, lets think about the question "How many apples are needed for 62 children if there are 5 apples for every 8 children". In this question students may not know what to do after accumulating the groups 5 apples and 8 children and arrive at 35 apple and 56 children if they are in the identical groups conception.

As a second conception of ratio, I will mention about the ratio-as-measure. It is defined as the quantification of a given attribute. If the ratio itself is considered as a quantity that measures a particular attribute of a situation such as lemoniness, squareness, density, sweetness, etc. then it is called as ratio-as-measure conception (Simon & Blume, 1994). For instance, if the two people's ages are compared, the measure is direct because the relation is expressed as a difference between two extensive quantities. However, sweetness can not be expressed by the absolute values of sugar and water in the situation. So, the sweetness is an indirect measure. It can be said that the relativeness comes from the fact that both quantities covary simultaneously on two levels.

The difference between identical groups conception and ratio-as-measure is that in the identical groups conception the ratio is made up of two extensive quantities. However, in the ratio-as-measure, ratio is expressed as an intensive quantity (Heinz, 2000). In ratio-as-measure conception, ratio is assumed to be the multiplicative relationship between two quantities. Besides, in identical groups conception of ratio, students recognize that a quality of interest is expressed as invariant. However, in ratio-as-measure understanding, students realize that they can measure the invariant quality of interest by an invariant quantity, ratio (Heinz, 2000).

The last conceptions of ratio are the conceptions of between-state and within-state ratios (Karagöz Akar, 2007; 2015). As opposed to the definitions given about within and between ratios under the strategies section, Karagöz Akar (2007) considered Noelting (1980a) and Lamon (1993)'s distinctions regarding between-state ratios and within-state ratios. That is, a within-state (or internal) ratio compares two quantities from different measure spaces and a between-state (or external) ratio compares two quantities from the same measure space in two different situations. Working with three pre-service mathematics teachers, Karagöz Akar (2007, 2015) proposed these conceptions as distinct from both identical groups and the ratio as measure conceptions.

A between-state ratio conception differs from the identical groups conception in such a way that students having this conception use one of the multiplicative strategies, such as within strategy in two ways. Kaput and West (1994) proposed, students using multiplicative strategies might be at higher stages of knowing in ratio. This is explained further below. Another difference is that, a between-state ratio conception allows students to deal with the divisibility failure. Though, interestingly, students having the between-state ratio might understand ratio in two ways. First, they might understand ratio as an expression of association between two extensive quantities. That is, they do not understand ratio as a multiplicative relationship between two quantities although they use within ratio strategy. In particular, a student with between-state ratio conception might think of the problem “A mixture of juice calls for 6 apples for 2 bananas. How many apples would be needed for 7 bananas, if we want the mixture of juice to taste the same” in two different ways as follows:

She might think of 7/2 as a between-state ratio and multiply it by 6. So the number of apples needed would be 21. However, as stated, at this level, 7/2 would be understood as a ratio expressing the associaton between two extensive quantities (bananas from two different mixtures) operated upon another extensive quantity (apples).

At another level, 7/2 would be understood as a multiplicative relationship between two quantities expressing how much one is increased or enlarged. That is, 7/2 would be understood as 35% increase in the quantities such that 6 (apples) would be increased as much too. Though, students at these two levels of between-state ratios might not still have any idea about the per-one strategy (the unit factor approaches). That is, a final distinction between identical groups conception and the between-state ratio conception is that between-state ratio conception does not require an understanding of per-one approach. In particular, a student with a between-state ratio conception might utilize division for the quantities in the original ratio situation. For instance, dividing 6 (apples) by 2 (bananas) . Still, as opposed to the identical groups conception, these students do not make sense of such number, 3 (apples per one banana), as the association between so many units of one quantity and one unit of another quantity. On the other hand, an understanding of within-state ratios may develop to some degree independently from the understanding of between-state ratios (Karagöz Akar, 2007). For example, a student having within-state ratios might use the per-one strategy at two levels again: First, she might want to find out how much of one quantity there exists for one unit of another quantity. For instance, dividing 6 by 2, she might make sense of the result, 3, as the ratio representing an association between two extensive quantities. However, as opposed to the between-state ratio conception, she might not at all make sense of the between-state ratio, 7/2, as neither a ratio representing the association between two extensive quantities (bananas from two mixtures) nor as the multiplicative relationship such as a percent increase-decrease. At a second level, the student might want to divide 6 by 2 to find out 3. Though at this level, the result of the division, 3, is understood as the multiplicative relationship between quantities such that it is stated mathematically as y=3x, where y is the number of apples and the x is the number of bananas for all the mixtures of the same taste.

2.4. Research on Teacher Knowledge on Ratio and Proportion

In the literature, there is a consensus that pre-service and in-service teachers demonstrate some misconceptions and lack of knowledge about the concepts of ratio, proportion and proportional reasoning (Simon & Blume, 1994; Livy & Vale, 2011; Livy & Herbert, 2013; Çıkla, & Duatepe, 2002; Johnson, 2013; Karagoz *et al.*, 2015). In particular, Simon and Blume (1994) claimed that pre-service elementary teachers may prefer to use additive strategies rather than multiplicative strategies as appropriately in some ratio and proportion problems. Furthermore, results from research on first year primary pre-service teachers (Livy &Vale, 2011) on second year primary pre-service teachers (Livy & Herbert, 2013) have found that the great majority of pre-service teachers need to develop their understanding of mathematical structure and mathematical connections within the problem contexts especially related to ratio and proportion. Moreover, although pre-service mathematics teachers can solve ratio and proportion problems, they have difficulties in giving definitions of the concepts of ratio and proportion and have lack of conceptual knowledge about these terms (Çıkla, & Duatepe, 2002). Similarly, pre-service teachers show limited flexibility to use multiple solution methods when they are solving ratio related problems. (Berk *et al.*, 2009). Additionaly, results from research on pre-service teachers' interpretations and responds to student errors in some ratio and proportion tasks related to similar rectangles showed that a lot of pre-service teachers considered that the errors were related to procedural aspects of similarity although the errors regarded conceptual aspects of similarity (Ji-Won Son, 2013). In addition, Johnson (2013) has mentioned four assumptions and misconceptions of pre-service teachers regarding proportional reasoning. These four assumptions included: reasoning quantitatively, recognizing ratios as measurement, misconceptions about the concept of ratio and fraction, and the obstacle of linearity. Johnson also asserted that once teachers are in need of further development of proportional reasoning skills they may struggle with engaging students in tasks involving ratios and may have difficulty in leading discussions about proportional reasoning. Furthermore, previous research also pointed that pre-service mathematics teachers may have understanding of between-state ratios although not even having a per-one meaning for within-state ratios (Karagöz Akar, 2007). On the other hand pre-service teachers with an understanding of within-state ratios may not have meaning for the between-state ratios. That is, they might not have abstracted that between-state ratios represent the change factor from one ratio situation to the other (Karagöz Akar, 2007; 2015). So it can be said that understanding one of the conceptions of ratio does not mean understanding the other conceptions or all of the conceptions of ratio. Furthermore, it was shown that the relationships between quantities both in within-state ratio and between-state ratio are not recognized by pre-service teachers (Valverde & Castro, 2012). Also, because of the pre-service teachers' extensive reliance on cross-multiplication and lack of familiarity with different student reasoning, pre-service teachers have a tendency to consider students' reasoning as incorrect (Valverde & Castro, 2012).

Concerning previous research on ratio done with in-service mathematics teachers Ekawati and her colleagues (2015) have worked with primary in-service teachers and have analyzed their MCK and MPCK scores by factorizing the questions into six. They asserted that some teachers have difficulties in the factor of figural representations and they performed best on factor of number structure in situations. Furthermore, some teachers had challenges on knowing students' conceptual understanding. They needed more sensitivity of students understanding such as misconceptions and different student strategies in solving proportional problems. That is, for some teachers giving feedback and evaluating and analysing student solutions were considered as the most difficult items. In addition, so many teachers had misunderstanding in distiguishing non-proportional and proportional situations. Most teachers also tended to present a single solution strategy by comparing ratios; they considered one kind of number arrangement in ratio that could be interpreted as within ratio strategy (Ekawati *et al.*, 2015). In particular, especially stretcher and shrinker situations were found to be the most difficult by both primary teachers (Ekawati *et al.*, 2014) and students (Kaput & West, 1994).

Moreover, the mathematics knowledge has been defined '..as a thorough understanding of the breadth and depth of the relevant topics in mathematics, including awareness of their interconnections' (Ekawati *et al.*, 2014, p.3). It was suggested that pre-service teachers' knowledge of mathematics plays an important role in their usage of materials, assessments students' progress, sequencing the lessons etc. (Ball *et al.*, 2005).

In sum, for both pre-service and in-service mathematics teachers, recent findings have showed that they have lack of knowledge of multiplicative reasoning, some ratio conceptions, and some strategies. Researchers asserted that the focus should be on conceptual knowledge of teachers rather than procedural knowledge. Also, for both pre-service and inservice teachers, giving feedback, analysing student misconceptions and using different solution strategies are considered to be among challenging issues regarding the concepts of ratio and proportion.

3. SIGNIFICANCE OF THE STUDY

The knowledge and competencies that a teacher has, is lying at the core of effective teaching (Walshaw, 2012). Particular to mathematics, the domain-specific knowledge that a teacher has is considered to have a great importance for the quality of the instruction (Ball, Lubienski, and Mewborn, 2001). Among those topics, ratio and proportion are introduced as the most complex and the most difficult for both teachers to teach and students to learn (Behr *et al.*, 1992; Lamon, 2007). Also, proportional reasoning has been considered as the cornerstone for achievement in upper mathematics and as the precursor to algebraic thinking (Lamon, 1999). Besides, ratio and proportion are core topics in elementary as well as secondary mathematics education (NCTM, 1989, 2000).

Regarding the importance of ratio and proportion, in this study MCK and MPCK instruments will be used to analyze senior pre-service mathematics teachers' content knowledge and pedagogical content knowledge on the topic of ratio and proportion. Ekawati and her colleagues (2015) have used these instruments, in their study, however participants were in-service primary mathematics teachers with various education background. On the other hand, "ratio and proportion" is one of the important topics of secondary school also. There are learning outcomes in the 9th grade curriculum such as "Students can solve problems includes concepts of ratio and proportion". Besides in so many topics, students are expected to use ratio concepts like similarity in triangles, problems including sets, exponents, velocity, age, mixture and speed etc. In other areas like physics and chemistry, students are also introduced topics needed to use ratio and proportion. By regarding those aims of the high school curriculum, participants of this study were selected from both elementary and secondary school pre-service mathematics teachers as seniors to provide further discussion beyond the original research done by Ekawati and her colleagues (2015). Besides,the studies related to the concept of ratio were generally conducted in small samples, however this study was conducted with large samples of pre-service mathematics teachers in a quantitative way.

4. STATEMENT OF THE PROBLEM

The current study strives to investigate pre-service teachers' Mathematics Content Knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK) specifically on ratio and proportion. Both SCED and PRED pre-service mathematics teachers are registered as a sample for the study. However, in Turkey, there are twelve universities which have both SCED and PRED departments at the same time. Therefore, this study aims to examine the Mathematics Content Knowledge and Mathematics Pedagogical Content Knowledge of students from those twelve universities. In accordance with this purpose, two instruments were used as Mathematics Content Knowledge Instrument (MCK-T) and the Mathematics Pedagogical Content Knowledge Instrument (MPCK-T) which are Turkish translated versions of MCK and MPCK instruments which were developed by Ekawati and her colleagues (2015).

Furthermore, since this study is a cross-sectional study it is not possible to measure participants' progresses in different times but it allows comparing different population groups at a single point in time.

4.1. Research Questions

(i) How is pre-service mathematics teachers' mathematics content knowledge (MCK) on ratio and proportion?

- How is pre-sevice middle school mathematics teachers' mathematics content knowledge (MCK) on ratio and proportion?
- How is pre-service secondary school mathematics teachers' mathematics content knowledge (MCK) on ratio and proportion?

(ii) How is pre-service mathematics teachers' mathematics pedagogical content knowledge (MPCK) on ratio and proportion?

- How is pre-sevice middle school mathematics teachers' mathematics pedagogical content knowledge (MPCK) on ratio and proportion?
- How is pre-service secondary mathematics teachers' mathematics pedagogical content knowledge (MPCK) on ratio and proportion?

(iii) How do pre-service mathematics teachers perform on the sub-dimensions of mathematics content knowledge (MCK)?

(iv) How do pre-service mathematics teachers perform on the subdimensions of mathematics pedagogical content knowledge (MPCK) on ratio and proportion?

5. METHOD

5.1. Design

The current study aims to examine Mathematics Content Knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK) of pre-service senior secondary scool and pre-service middle school mathematics teachers particularly on the topic of ratio and proportion. Data were collected from eleven different universities of Turkey. Since data were collected from selected participants at a single point in time, cross-sectional survey design was used (Gay, Mills, & Airasian, 2011). The data were gathered from all participants by using both Mathematics Content Knowledge Instrument in Turkish (MCK-T) and Mathematics Pedagogical Content Knowledge Instrument in Turkish (MPCK-T). These instruments were translated from the study "Primary Teacher's knowledge for teaching ratio and proportion in Mathematics: The case of Indonesia" (Ekawati *et al.*, 2015).

5.2. Sampling and Participants

The target population of this study are pre-service senior university students who were studying Primary Mathematics Education and Secondary Mathematics Education departments in Turkey. In Turkey, there are totally twelve universities having Secondary School Mathematics Education Program. In all those twelve universities, pre-service students studying currently on the Secondary School Mathematics Education Program (SCED) were selected as the sample of the study. Also, the pre-service students studying currently on the Primary (middle) School Mathematics Education Program (PRED) in all these universities are also selected conveniently as the sample of the study. So the PRED students are determined according to the universities, which include both SCED and PRED programs. After this section, the abbreviation PMMTs will be used for Pre-service Middle (Primary) School Mathematics Teachers or PRED students. Also, the abbreviation PSMTs will be used for Pre-service Secondary School Mathematics Teachers or SCED students as it is mentioned above.

The instruments were given to all senior students from those twelwe universities. Those students were asked to voluntarily participate to the study however one of the universities did not want to participate in the study. So, totally eleven universities, 140 PSMTs and 450 PMMTs students participated in this study. The numbers of PMMTs and PSMTs who were expected to participate to the study and also real numbers of PMMTs and PSMTs who have already participated to the study are shown in the Table 5.1.

Table 5.1. The Expected and Participated Real Numbers of PSMTs and PMMTs.

University	PMMT (Expected)	PSMT (Participated)	PSMT (Expected)	PMMT (Participated)
U1	16	13	100	65
U2	30	30	80	45
U3	8	8	48	30
U4	11	11	66	31
U5	20	16	100	33
U6	20	12	100	25
U7	21	12	71	64
U8	20	17	99	81
U9	19	13	55	42
U10	30	0	129	0
U11	8	5	48	21
U12	12	3	57	13
Total	215	140	953	450

5.3. Instruments

In this study the aim is to investigate Mathematics Content Knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK) of PSMTs and PMMTs on the topic of ratio and proportion. According to this aim, MCK-T and MPCK-T instruments were used (Appendix C and Appendix D). These intruments were the Turkish translation of MCK and MPCK intruments (Appendix A and Appendix B), which were designed to investigate Indonesian In-service Primary Teachers' understanding of ratio and proportion by Ekawati and her colleagues (2015).

The conceptualizations of MCK and MPCK in the original study (Ekawati *et al.*, 2015) were adapted from the COACTIV study. There were four levels of MCK described by Krauss *et al.*, (2013) in COACTIV such as (1) Everyday mathematics knowledge required by the average adult; (2) A reasonable command of school-level mathematical knowledge; (3) A profound understanding of the content of the secondary school mathematics curriculum; (4) University-level knowledge of mathematics. The fourth level was not included because it was beyond the scope of the original study (Ekawati *et al.*, 2015, p.515).

In terms of MPCK, synthesizing the stream of the description of MPCK by Cheang *et al.*, (2007) there were four parts of MPCK used in the original study such as Knowledge about teaching the concept (include giving feedback); Knowledge about students' understanding of the concept; Knowledge about level of task and Knowledge about the appropriate teaching approach for students understanding (Ekawati *et al.*, 2015, p.515).

Furthermore, three important variables were included for the items of MCK such as context situations, task type and number structure. Recipe context, geometrical enlargement, well-chunked problems, stretchers and shrinkers can be given as example of context situations. In terms of the task type, missing value problems and ratio comparison problems were included to see the different reasoning strategies that might be used. In addition, number structure variable consisted of integer multiple and non-integer multiple number structures. On the other hand, MPCK instrument includes the followings: the knowledge how teachers make the content (ratio and proportion) understandable for students; the suitable teaching method for students that emphasized the topic proportional reasoning; the cognitive demand of mathematics task for students; as well as the understanding of students' errors and misconceptions.

In addition, item problems in the MCK and also MPCK were designed in three forms such as Multiple Choice (MC), Complex Multiple Choice (CMC) and Open Problem (See Table 5.2 and Table 5.3). Also, the maximum score of each MCK and MPCK items was considered as 1 (See Appendix E and for the rubric for MCK and MPCK).

Table 5.2. Item Forms and Problem Definitions in MCK.

Code	Problem Overview	Item Format
MCK1	Rate missing value problem about the use of fuel with non-integer number structure	MC
MCK2	Innate ratio in the congruency of two geometrical figure	MC
MCK3	Reasoning of congruency of two geometrical figure	OP
MCK4	Non-integer scaling and number structure (Mr.Short&Mr.Tall problem)	OP
MCK5	Proportional& Non-proportional situations	CMC
MCK6	Ratio relation of two different objects (speed context)	OP
MCK7	Meaning of proportional relation in situation	OP
MCK8	Meaning of equivalence sign in proportional relation	OP
MCK9	The conditional statement of two proportional statements within speed context	OP
MCK10	Drawing the enlargement figure with integer number structure scaling	OP
MCK11	Missing value problem with non-integer scale factor enlargement	OP
MCK12	Ratio relation in Cartesian coordinate system	MC

Table 5.3. Item Forms and Problem Definitions in MPCK.

Code	Problem Overview	Item Format
MPCK1	Develop proportional problem that fit to primary level	OP
MPCK2	Identify students' responses on contextual proportional problem	OP
MPCK3	Analyze and interpret students' misconception for solving proportional problem	OP
MPCK4	Encourage/guides students to aware of their misconception	OP
MPCK5	Choose appropriate teaching method for students understanding and reasoning ability.	MC
MPCK6	Analyzing of teaching unitary method for ratio and proportion problem	OP
MPCK7	Provide appropriate feedback for students misconception	OP
MPCK8	Evaluate students' mathematics solution on providing different number arrangement of proportional problem.	MC
MPCK9	Analyze the different students strategy in solving proportional problem	OP
MPCK10	Identify tasks' difficulty level for students based on their cognitive demand	CMC
MPCK11	Analyze the more demanding task compares to other	OP

In the original study, Ekawati and her colleagues (2015) worked with 271 in-service primary teachers of grades one to six. To analyze their data, item analysis and factor analysis methods were used.For the acceptable internal reliability, the Cronbach Alpha values were found as 0.651 and 0.641 for MCK and MPCK respectively (Hair *et al.*, 1998) and indicated a reliable measure of both categories. Afterwards, the exploratory factor analysis was done. They have used the eigenvalues greater than 1 for the number of factors and they formed the factors by Oblimin rotation with loadings of greater than 0.3. Finally, the three underlying factors for each MCK and MPCK instruments were identified by using Oblimin and Kaiser Normalization's rotation method (Kaiser 1970,1974).

In the original study, items of intruments have been grouped into six different factors. In particular, MCK items were categorized into three factors as F1, F2 and F3. F1 represents the meaning of proportional and non-proportional situations. It was about understanding the meaning of ratio relations in proportional patterns within situations. F2 was about number structures in situations and measuring the ability to find quantitative mathematical solutions for proportional problems. F3 considered ratio relations in geometrical figures and in representations. For MPCK instrument, teacher knowledge was researched in the sense of how teachers make the content (ratio and proportion) understandable for students. Factors for MPCK have been grouped into three as well. Knowing students conceptual understanding was regarded as F4 which was about students' misconceptions and thinking on ratio and proportion. F5 was the ratio and proportion task level feature. It was about the identification of task level difficulty and the underline reason of the hierarchy level. Finally, F6 was about teaching problem solving strategy of ratio and proportion. It was about teachers' understanding of strategies to teach ratio and proportion concept (Ekawati *et al.*, 2015).

In the current study, regarding the research questions 3 and 4, Confirmatory Factor Analysis (CFA) and Exploratory Factor Analysis (EFA) were done to analyze the sub-dimensions of MCK-T and MPCK-T. In the followings headings, both of the factor analyses were explained in detail.

5.3.1. Confirmatory Factor Analysis

Confirmatory Factor Analysis (CFA) is used to validate a proposed theory or potential relationships among variables. In other words, by using CFA, the new collected data is checked with the hypothesized model (Kline, 1998).

In order to have valid CFA results, using statistical or graphical methods, some assumptions need to be established: multivariate normality, linearity, continuity of variables and the ratio of sample size to number of variable. For the normality, the skewness and kurtosis values should be seen in between $\pm$ 2 to get acceptable values (Tabachnick & Fidell, 2001).

In the current study, Ekawati and her colleagues' model (2015) was specified as the hypothesized model. After that, to assess the difference between sample covariance matrix and the population covariance matrix values, some fit indices were used. The aim was to find the difference between these matrices as minimum in order to claim a "good fit". In fact, there are various fit indices to test the hypothesized model. Among these, in the current study, the Root Mean Square Error of Approximation (RMSEA), the Comparative Fit Index (CFI) and Tucker-Lewis index (TLI) were examined specifically.

Particularly, the root mean square error of approximation (RMSEA) estimates the lack of fit by comparing perfect model and estimated model using degrees of freedom. RMSEA ranges from 0 to 1 and values less than 0.06 means a good fitting model (Ullman, 2001). Similarly, if RMSEA value is higher than 0.10, this means a poor fitting model (Browne and Cudeck, 1993).

Comparative fit index (CFI) are also mostly advised comparative fit indices in which both of them include comparison of independence model and estimated model with using degrees of freedom. In Both of them, the range from 0 to 1 and values over 0.95 means a good fitting model (Ullman, 2001). Moreover, Tucker-Lewis index (TLI) is interpreted in large samples to assess the relative drop in noncentrality per degree of freedom. To have a good fitting model, the suggestion for TLI value is to be over 0.9 (Bentler, 1990).

Though, when the fit of the implied model is not satisfactory according to several fit indices, then the Exploratory Factor Analysis might be conducted in the same data to understand the relationship among the variables.

5.3.2. Exploratory Factor Analysis

Exploratory factor analysis is used to explore the main dimensions in a test. Actually it is exploratory in nature. In particular, EFA can be considered in three ways: First of all, factor analysis can be used to decrease the number of variables into a smaller set of variables. Secondly, EFA can be used in the formation and refinement of a theory. And thirdly, EFA can be used to show the construct validity of the scales (Williams, Onsman, & Brown, 2010).

To conduct the Exploratory Factor Analysis, there are three main steps to consider. The first issue is about the sample size. Tabachnick and Fidel (1996) suggested having at least 300 cases for factor analysis. In the current study, the sample was 590 pre-service mathematics teachers. So, the sample size was considered as very good to conduct EFA. Besides, there are some other criterions which are considered suitable for factor analysis: the correlation matrix showing at least some correlations of $r = .3$ or greater; Bartlett's test of sphericity being statistically significant at $p < .05$ and the Kaiser-Meyer-Olkin value being .6 or above (Pallant, 2001).

Second step is factor extraction, which involves determining the smallest number of factors that can be used to best represent the interrelationships among the set of variables. In the current study, principal axis factoring method was used for factor extraction and also Kaiser's criterion was used to determine the number of factors. Using this rule, only factors with an eigenvalue of 1.0 or more were considered for further investigation. The eigenvalue of a factor represents the amount of the total variance explained by that factor (Pallant, 2001).

Last step is factor rotation and interpretation. There are two main approaches to rotation: orthogonal rotation and oblique rotation. The orthogonal rotation produce factor structures that are uncorrelated. On the other hand, oblique rotation produce factors that are correlated. In the curent study, the orthogonal rotation was used. The most commonly used orthogonal approach is the Varimax method, which attempts to minimise the number of variables that have high loadings on each factor. Lastly, for interpretation step, the researcher examined which variables were attributable to a factor, and gave that factor a name or theme (Pallant, 2001).

5.3.3. Translation of MCK and MPCK instruments and Validity and Reliability Issues

With the increase in the number of multi-national and multi-cultural research, the need for the adaptation and translation is increased. This process of adaptation requires a unique methodology to provide the equivalence of target language and the original source (Beaton *et al.*, 2000). In this study, Beaton and her colleagues' (2000) guidelines for the cross-cultural adaptation process were taken into consideration. These guidelines included translation, synthesis, back translation, expert committee review and pretesting.

Firstly, MCK and MPCK Instruments were translated in Turkish before using them to collect data for the current study. Turkish translated versions of instruments were named as MCK-T and MPCK-T respectively. The method which was used while translating for each of the two instruments consisted of five stages: Firstly, two people from different professions, one of them mathematics teacher and the other one a mechanical engineer, made translation by using forward translation technique. Source language was English and target language was Turkish which is mother tongue for both translators. Those translations were coded as T1 and T2.

After the translation process, one expert in mathematics teacher education and two expert translators made comparisons item by item to create a common synthesis of T1 and T2 as T12 by resolving discrepencies. And then to check the validity and also looking for the conceptual errors inT12, back translation method was used. One translator (different from the earlier ones) and one secondary school English teacher translated T12 back to the source language (English). After that, to produce final version of the translation, three different experts in mathematics teacher education examined the original instrument and all translations (T12 and two back translations). To achive equivalance between original source and translations, experts focused on four different areas such as semantic equivalance, idiomatic equaivalance, experiential equivalance and conceptual equivalance. After reaching on agreement about all items, final version was constructed. Then a Turkish teacher checked the grammer of final version of the intstruments. For the final stage, 35 pre-service mathematics and science teachers which are native in Turkish and fluent in English took the instruments to establish emprical evidence for the equivalance of the instruments. Participants were administered firstly the original instrument (English version) in their class time. After approximately 1 month later, they took the Turkish version of the instrument.

For the equivalence of the instruments, after scoring some descriptives were analyzed by using Statistical Package for Social Sciences (SPSS) software. Skewness and Kurtosis values were seen as -.17 and -.68 respectively. These values were in between -2 and +2, so they were considered acceptable in order to prove normal univariate distribution (George & Mallery, 2010). Then, the correlation coefficient of participants' total points that they got from English and Turkish versions were calculated. Pearson correlations were used because of the normal univariate distribution. The Pearson correlation results showed that two versions of MCK instrument were highly correlated ($r(35) = .81$, $p < .05$) and two versions of MPCK instrument were also highly correlated ($r(35) = .75$, $p < .05$). Besides, $R^2 \leq 0.01$, small effect size; R2 around 0.09, medium effect size; $R^2 \geq 0.25$, large effect size are recommended by Cohen and Cohen (1983). So, in the current study R2 of MCK is 0.65 and R2 of MPCK is 0.56 which are considered as large effect size (Cohen J. & Cohen P., 1983).

After analysis of the equivalence of the instruments, all of the items were checked in detail again and the revisions of translation were completed according to reviews and comments. In particular, for the items MCK2, MCK10,MPCK4 and MPCK8, some revisions were done in personal communications with the researcher (Rooselyna Ekawati) of the original study. According to the factor analysis results, Ekawati and her colleagues had removed the item MCK2 though leaving the item MCK3 in the instrument. During personal communication with her, she proposed to have a new question for the current study as it is shown MCK-T(Appendix C). Therefore, the rubric was also revised for the item MCK2. In addition, in the items MCK10 and MPCK4, some extra explanations or phrases were addded according to participants' understandings in the pilot study. In the item MCK 10, the extra explanation regarding the gap given in the question was added to make it understandable because in the rubric the gap is scored by 0.5 points also. In the item MPCK4, participants were expected to design a question for Ina so that she would notice her misconception. Writing it as it is, in the pilot study, it was recognized that participants have tried to write a different word problem for Ina. Thoug, in the rubric for MPCK what was expected from the participants to write as a feedback short questions for bringing awareness for Ina. So, it was decided to use a different word to describe it. In addition,in the item MPCK8, the ratio arrangements in the third option were changed to make clear the understanding of the participants' thinking in part-part and part-whole ratio comparisons. The rubric was revised for MPCK8 also.These differences were determined and corrected.

Furthermore, to check the reliability of the instruments, Cronbach's alpha (or Coefficient alpha) value on the scores from both the pre-service middle school and secondary school teachers in the main study were used. In reliability analysis, the aim was to check whether all the items in the instrument measured the same construct. Ideally, the Cronbach alpha value of an instrument should be above 0.7 (George &Mallery, 2001). However, the number of items in the instruments might affect the Cronbach alpha values. Specifically, in the short scales (e.g.,scales with less than ten items) it is common to find low Cronbach alpha values (e.g., .5) (Pallant, 2001). Moreover, Alpha values between 0.5 and 0.75 are generally accepted as indicating moderately reliable tests(Brownlow, C., McMurray, I., & Cozens, B., 2004). In the current study, the Cronbach's Alpha of MCK-T was found as 0.470 and MPCK-T as 0.563 for reliability. So, the MPCK-T instrument was considered as moderately reliable. Though, the MCK-T instrument might need further examinations for its reliability, the low number of items or some easy items in MCK-T might explain the reason behind low Cronbach's Alpha value.

5.4. Data Collection

After the equivalence of the intsruments study process and formation of final version of the instruments, universities which include both the secondary school mathematics education and middle (primary) school mathematics education departments at the same time were determined. As the first phase, the instructors were reached from both of the departments secondary education and primary education (SCED & PRED). According to the number of students in their classes, instruments were sent by cargo. Then, under the supervision of the instructors, during classroom hours, the senior pre-service middle school and secondary school mathematics teachers took both of the instruments at one session. The sessions ranged from 60 minutes to 75 minutes. Then, the instructors sent the answered-instruments back again by cargo.

5.5. Data Analysis

In this section, data analysis methods used for research questions were explained separately. For the first and second research questions "How is pre-service mathematics teachers' mathematics content knowledge (MCK) on ratio and proportion" and "How is pre-service mathematics teachers' mathematics pedagogical content knowledge (MPCK) on ratio and proportion?", descriptive statistics were conducted and student answers were analyzed separately for every item of MCK-T instrument and for every item of MPCK-T instrument. For the descriptive statistics, the minimum, maximum values and also the mean and the standard deviation of the total scores from each participant both in MCK-T and MPCK-T were calculated. Also, to further examine how pre-service mathematics teachers' mathematical content knowledge (MCK) and mathematical pedagogical content knowledge (MPCK) were reflected in the specific items, the percentages of correct, incorrect and the partial scores as well as the frequencies for every item in MCK-T and MPCK-T were depicted. As the rubric indicated (see APPENDIX E&F), if the participants correctly answered the items 1,2,3,4,6,7,8,11 in

MCK-T or items 1,4,7,9,11 in MPCK-T, they got 1 point and if they did not answer these items correctly or if they left them blank, they got 0 points. Similarly, based on the rubric, for the items 5,9,10,12 in MCK-T or 2,3,5,6,8,10 in MPCK-T, the participants' answers were calculated partially depending on the specificity of their answers.

For the third and fourth research questions, which were related to the sub-dimensions of MCK-T and MPCK-T, the factor analysis were done. Firstly, the confirmatory factor analysis was done by using The Mplus program. Once, the dimensions of MCK-T were identified as similar to the sub-dimensions in Ekawati *et al.* (2015), it was reported which of the items in MCK-T were clustered as a factor. Also, the confirmatory factor analysis (CFA) was conducted for MPCK-T. However, the results of CFA did not fit the exploratory factor analysis (EFA) results of MPCK by Ekawati and her colleagues (2015). Then, a new exploratory factor analysis (EFA) was conducted to form new factors using IBM SPSS Statistics 22. This way, the sub-dimensions of MPCK-T were able to be identified.

6. RESULTS

The Turkish translated versions of Mathematical Content Knowledge Instrument (MCK-T) and Mathematics Pedagogical Content Knowledge Instrument (MPCK-T) were given to a total number of 590 senior pre-service mathematics teachers. 140 of them were from secondary school mathematics education departments and 450 of them were from the middle school mathematics education departments at eleven universities in Turkey.

In the following three sections, results regarding each research question will be presented. In the first section, MCK-T results will be presented by considering pre-service middle school and secondary school mathematics teachers separately and then their results will be presented together as pre-service mathematics teachers. In the second part, the factor analysis results related to MCK-T will be presented. In the third part, MPCK-T results will be presented by considering pre-service middle school and secondary school mathematics teachers separately again. And then, all of the pre-service mathematics teachers' MPCK-T results will be presented. Lastly, the factor analysis results related to MPCK-T will be presented also.

6.1. Pre-sevice Mathematics Teachers' MCK on Ratio and Proportion

6.1.1. Pre-service Middle School Mathematics Teachers' MCK on Ratio and Proportion

Data were gathered from 450 senior pre-service middle school mathematics teachers (PMMTs). They were from PRED departments at eleven universities in Turkey. The mean, standard deviation, minimum and maximum scores were calculated for the PMMTs' total test scores obtained from MCK-T. Table 6.1 shows descriptive statistics obtained from the MCK-T insturument. Also, the distribution of the percentages of total scores of all PMMTs for MCK-T were illustrated in Figure 6.1.

Table 6.1. Descriptive Results for MCK-T of PMMTs.

	N	M	SD	Min.	Max.
Total score for MCK-T	450	8.4334	1.57030	4.00	12.00

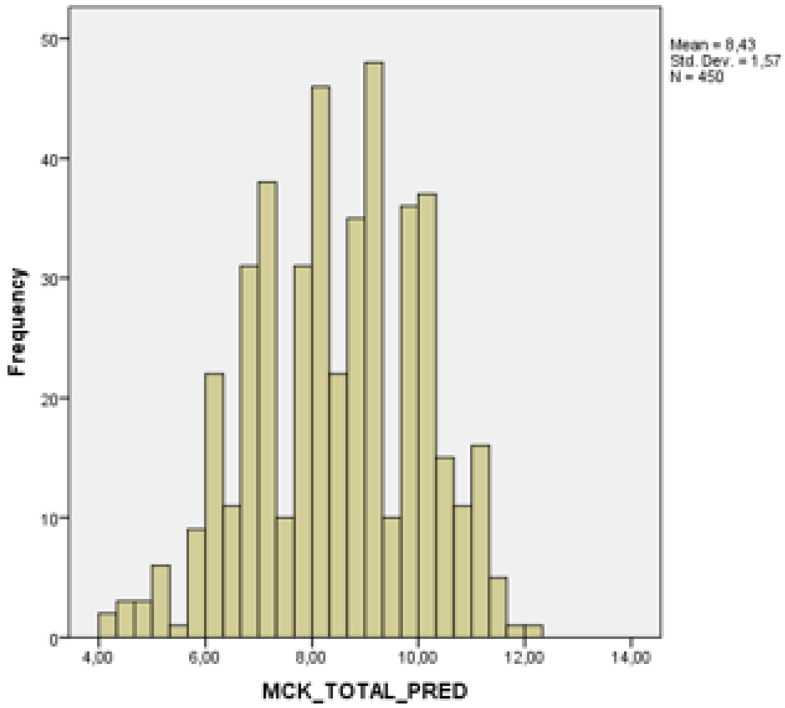


Figure 6.1. Distribution of MCK-T Scores of PMMTs.

MCK-T consists of 12 items in total. So the maximum score is expected to be 12 points from MCK-T. As the data showed in Figure 6.1, the maximum total score is calculated as 12 points and the minimum score was 4 points for PMMTs. As shown in Table 6.1, the mean value of the total scores was 8.4334 and the standard deviation was 1.57030. This suggested to some extent that PMMTs' mathematical content knowledge on ratio and proportion was high. Still, due to the low number of participants having the maximum points or minimum points and the participants' responses to the partially scored items, further examination of the total scores per item is needed.

Specifically, as will be shown in Figure 6.2, percentages of the MCK scores for each item is shared and partially scored items, MCK 5-9-10 and 12 are given further attention.

Additionally, since some items are partially scored, the results are presented in three parts as incorrect, partial and correct percentages.

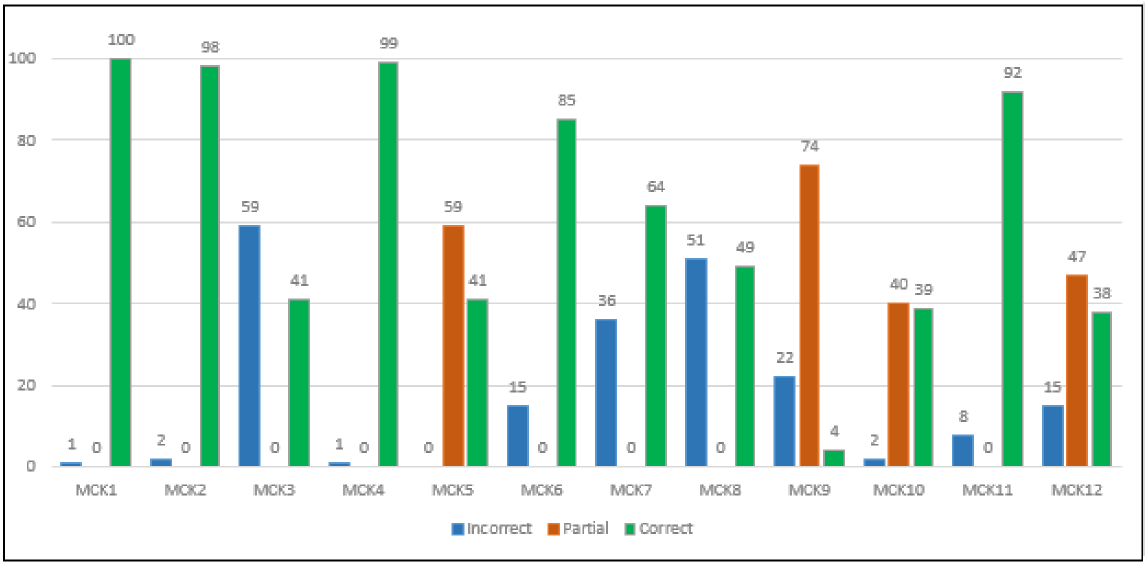


Figure 6.2. PMMTs' Response Percentages for MCK-T Instrument.

As the data showed, items MCK1, MCK2, MCK4 and MCK11 were correctly answered by almost everyone at the percentages of 100%, 98%, 97% and 90% respectively. Particularly, MCK1, MCK4 and MCK11 are different number structured problems within different situations. For example, MCK1 is a missing value problem and asked to find out the liters of fuel which will the machine use in 100 hours if it continues to use fuel at the same rate. Similarly, MCK2 is a question regarding the congruency of two figures familiar to the participants. In addition, in MCK4, the number of paperclips needed to measure Mr. Tall's height is asked. Finally, in MCK 11, participants were expected to find the new width of the rectangle after the enlargement. Therefore, these items are designed to measure the ability to find numerical solutions for proportional problems. The high percentages of responses regarding these items showed that PMMTs could easily solve proportional problems requiring just procedural knowledge.

However, in the item MCK3, participants were expected to explain the reason behind the congruency of the two given figures in MCK2. The percentage of correct answers was only 37%, while the percentage of incorrect answers was 63% . Therefore, with the 98% percentage of the participants stating that the two figures in MCK2 were congruent, 63% of the percentage of the participants' not being able to explain the reason for the congruency in MCK3 also suggested that pre-service teachers were able to procedurally solve the problems without having a rationale behind it. In fact, so many participants asserted that the two figures were congruent because the lengthof the edges of the figures were equal. However, they were expected to assert the equality of the diagonals and angles of the figures. This result suggested that they were not able to make connections between the concept of ratio and the concept of congruency in geometrical figures. In addition to MCK 2 and MCK3, in the item MCK10, participants were expected to reason about the similarity of two figures. Results showed that 32% of the participants answered the question correctly, while 16% of them answered the question incorectly and 52% of them answered it partially. Partially answering the question meant that the participants ignored expanding the given gap in the figure by considering the ratio. In the Figure 6.3 an example of a participant's partially correct answer is shown.

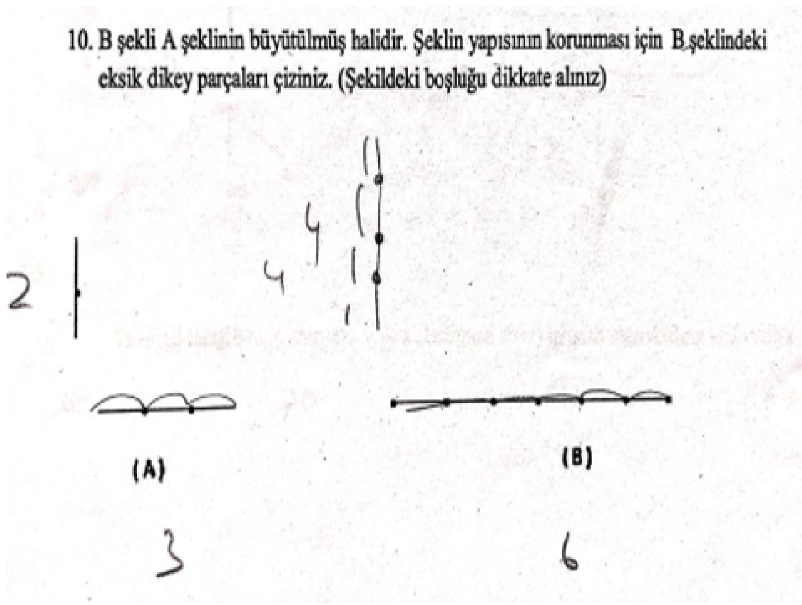


Figure 6.3. An example of a participant's answer for MCK10.

As shown, in the figure, participants are expected to reason that all units have to be doubled, both the lines and the gap, to keep the figure's configuration the same. However, more than half, 52%, of the PMMTs made calculations similar to the figure above (see Figure 6.3) by ignoring the gap in the figure and 16% of them could not asnwer the question at all. These results together with the acknowledgement that their attention had been taken to the gap in the figure, showed that 66% of the participants seemed to have ignored the gap in the question. These results in juxtaposition with the results from MCK3 might suggest that pre-service middle school mathematics teachers have difficulties in figural representations.

Similar to MCK 3 and MCK 10, MCK12 was also a partially scored item related to graphical representation of ratio relations (See Figure 6.4). As one representative example was shared in Figure 6.4, 13% of pre-service middle school students answered the question incorrectly, while 43% of them answered it partially and 44% of them answered it correctly. According to the rubric, partial scores stemmed from choosing the option A only or choosing the option B only or choosing the options A-B-C together or the options A-B-D together.

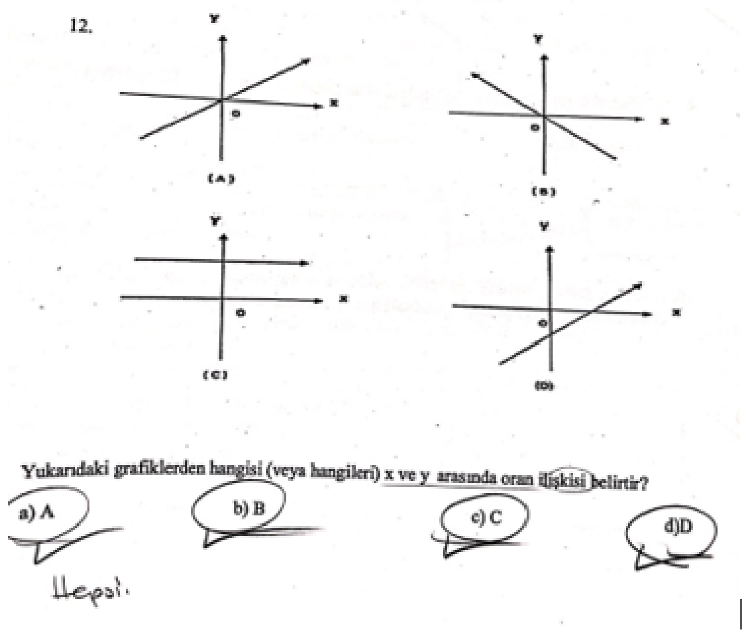


Figure 6.4. An example of a PRED student's answer for MCK12.

These results showed that both the participants partially answering the question and and the participants incorrectly answering the question were not able to realize the multiplicative relationship between x and y in the graphs. In other words,in total, 56% of the participants were not able to recognize the ratio concept depicted in the graphical representation. These results further suggested that 56% of the PMMTs were not able to differentiate the concept of ratio and the concept of linearity.

In sum, for the items MCK3, MCK12 and MCK10, the percentages of the participants who answered these three questions correctly were much smaller than the percentages of the participants who answered these three questions incorrectly or partially. Considering these three questions as related to the figural representation of ratio and proportion, results suggested that PMMTs had difficulties in figural representations regarding ratio and proportion concepts. Particularly, results from all these three items showed that PMMTs had difficulty in connecting the concept of ratio with the concepts of congruency, similarity and linearity.

MCK5 was another partially scored item. PMMTs were expected to correctly match 6 problems with proportional or non-proportional situations. As seen in the Table 6.2. in detail, out of 450 PMMTs, approximately only 30% were able to match all situations correctly. Assuming that the participants who were able to match at least five situations correctly might be considered as participants differentiating proportional and non proportional situations, then 68% of the participants could be considered in this group. Nevertheless, approximately 150 participants out of 450 were able to match at most 4 situations correctly. This suggested that PMMTs had difficulty in being aware of the proportional and non-proportional situations.

Table 6.2. Descriptive statistics of the item MCK5 for PMMTs.

MCK5

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	0.00	1	0.2	0.2	0.2
	0.17	2	0.4	0.4	0.7
	0.33	9	2.0	2.0	2.7
	0.5	43	9.6	9.6	12.2
	0.67	90	20.0	20.0	32.2
	0.83	171	38.0	38.0	70.2
	1.00	134	29.8	29.8	100.0
	Total	450	100.0	100.0	

Finally, items MCK6-7-8 and 9 were interrelated questions regarding the meaning of proportion within the context of velocity. Results showed that 78% of the participants were able to state that given the ratio, distance over time, referred to velocity. This might have stemmed from their familiarity with the velocity concept from their early physics classes. However, only approximately 50% of the participants correctly answered the items MCK7 and MCK8 asking for explaining the meaning of equivalence sign in the proportions, $\frac{s_1}{t_1} = \frac{s_2}{t_2}$ and $\frac{s_1}{s_2} = \frac{t_1}{t_2}$. This showed that half of the pre-service middle school mathematics teachers were not able to explain the proportional relations. Lastly, for the item MCK9, it is expected to consider the proportions $\frac{s_1}{t_1} = \frac{s_2}{t_2}$ and $\frac{s_1}{s_2} = \frac{t_1}{t_2}$ conditionally. In other words, it is expected to write and prove why if the first proportional condition is true, then the second proportional condition has to be true. Only 5% of the participants were able to answer the question correctly while 65% of them were able to answer it partially. This showed that only 5% of them were able to explain the reasoning underlying the equality. Actually, the reason for high percentages of partial answers is that so many PMMTs have tried to explain the conditional statement, if MCK7 is true than MCK8 is true, by using cross-multiplication to prove equality of two proportions. These results suggested that PMMTs were not able to provide the reasons behind proportionality.

In sum, for the items MCK5-6-7-8 and 9, PMMTs were expected to think and reason about proportional and non-proportioal situationsand provide their reasons behind proportionality. However, results showed that although PMMTs were able to determine the given ratio representing some concepts familiar to them like velocity and use cross-multiplication as a particular reason behind proportionality, they had difficulty in explaining why and how the equal sign showing proportionality held.

6.1.2. Pre-service Secondary School Mathematics Teachers' MCK on Ratio and Proportion

Data were gathered from 140 senior pre-service secondary school mathematics teachers (PSMT) who were from secondary school science and mathematics education (SCED) departments at eleven universities in Turkey. The mean, standard deviation, minimum and maximum scores were calculated for the PSMTs' total test scores obtained from MCK-T Table 6.3. shows descriptive statistics obtained from the MCK-T insturument. Also, the distribution of the percentages of total scores of all PSMTs for MCK-T were illustrated in Figure 6.5.

Table 6.3. Descriptive Results for MCK-T of PSMTs.

	N	M	SD	Min.	Max.
Total score for MCK-T	140	8.7171	1.46546	4.50	11.50

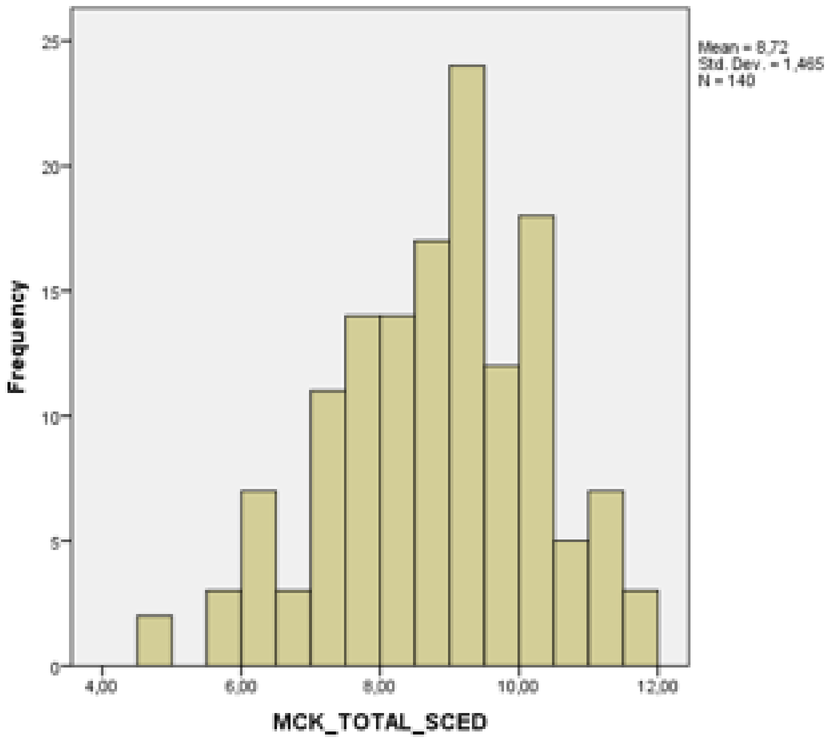


Figure 6.5. Distribution of MCK-T Scores of PSMTs.

As the data showed in Figure 6.5 for PSMTs, the maximum total score was calculated as 11.50 points and minimum score was 4.50 points. Also, as shown in Table 6.2, the mean value of the total scores was 8.7171 and the standard deviation was 1.46546. This suggested to some extent that PSMTs' mathematical content knowledge on ratio and proportion was high. Still, due to the participants' responses to the partially scored items, further examination of the total scores per item is needed. As shown in Figure 6.6, results are presented in three parts: incorrect, partial and correct percentages.

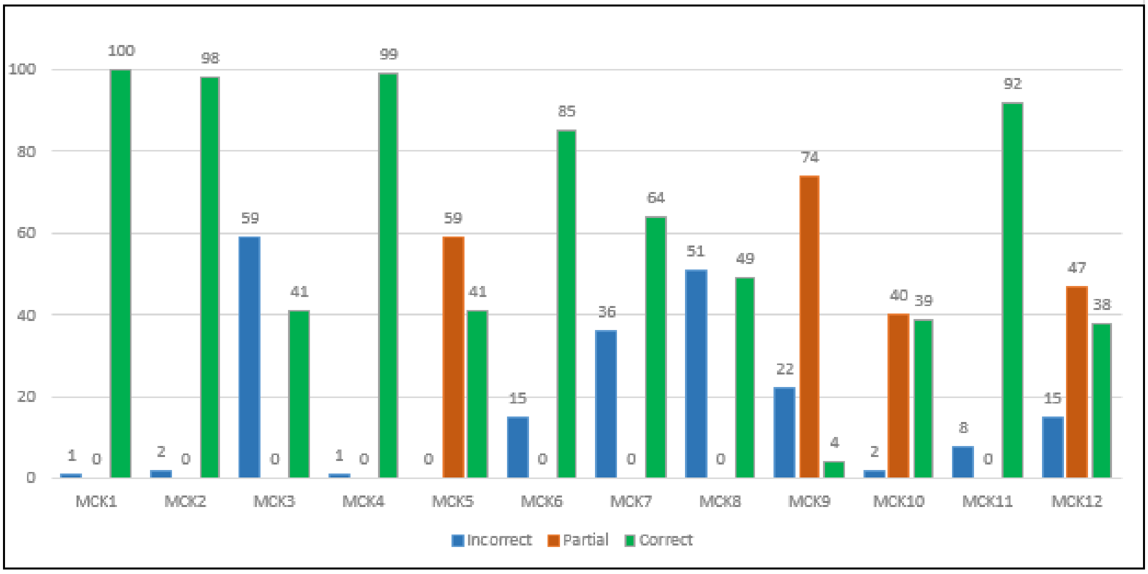


Figure 6.6. PSMTs' Response Percentages for MCK-T Instrument.

As the results indicated, items MCK1, MCK2, MCK4 and MCK11 were correctly answered by almost everyone at the percentages 96%, 98%, 99% and 92% respectively. As already mentioned earlier, these questions required procedura answer. Therefore, these results suggested that PSMTs had required procedural knowledge to solve proportional problems in different number structures. Also, similar to the results from PMMTs, the high percentages in MCK2 might have stemmed from PSMTs' familiarity with the congruency problems.

On the other hand, the items MCK3, MCK12 and MCK10, PSMTs' scores pointed to low percenteges as in PMMTs. As stated earlier, those items are related to figural representations of ratio concept. In particular, in MCK3 related with MCK2 participants are expected to explain the reason of the congruency given in MCK2. Results showed that approximately 60% of the PSMTs couldn't explain the reason pointing to the sameness of the ratio of the side lenghts representing the relation between the two figures although approximately 98% of them asserted the congruency in MCK2. This suggests that PSMTs also did not relate the concepts of ratio and congruency in geometrical figures.

As also mentioned above, in MCK10, participants are expected to expand both the given lines and the gap in the original figure to construct the new figure. Although again the attention was taken to the gap given in the original figure, 61% of the PSMTs ignored expanding the gap given in the original figure. Only, 39% of them were able to answer the question correctly. This suggests that more than half of the PSMTs were not able to completely apply proportional reasoning in figural representations.

The item MCK12 is also related to the graphical representation of ratio concept. Similar to the results from the PMMTs, the percentage of the partial and incorrect answers was 62% for PSMTs. This again further could be considered as the evidence of PSMTs' not being proportional reasonsers: Their not being able to answer MCK12 pointed to the fact that they were not able to recognize the multiplicative relationship between x and y in ratio represented in graphical representation in the Coordinate Plane. This result also suggested that PSMTs were not able to differentiate the concept of ratio from the concept of linearity: Had they been able to differentiate these concepts, they would all have known that in the figural form, a line, not always represents the mutliplicative realitionship between two variables. Similarly, they would have known that linearity corresponds to the relationship between the two variables such that each dependent variable is equal to the some multiple of each independent variable with the addition of a constant value.

As mentioned earlier, items MCK5-6-7-8 and 9 were the questions related with the meaning of proportional and non-proportional situations. For instance in MCK5, participants were expected to distinguish proportional and non-proportional situations given in six different problem context. According to Table **??**, although 76% of the PSMTs in total were able to match at least five problems correctly, the remaining 24% of them were able to match at most four problems correctly. This suggested that almost 25% of the PSMTs had difficulty in being aware of the proportional and non-proportional situations.

Table 6.4. Descriptive statistics of the item MCK5 for PSMTs..

MCK5

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	0.17	1	0.7	0.7	0.7
	0.33	3	2.1	2.1	2.9
	0.50	10	7.1	7.1	10.0
	0.67	19	13.6	13.6	23.6
	0.83	50	35.7	35.7	59.3
	1.00	57	40.7	40.7	100.0
	Total	140	100.0	100.0	

Similarly, items MCK6-7-8 and 9 were related to the meaning of proportion within the context of velocity. Results showed that, 85% of the PSMTs were able to state that given the ratio, distance over time, referred to velocity. The velocity concept is introduced since the middle school years; therefore, the high percentage could be explained PSMTs' familiarity with the concept. In the items MCK7 and MCK8, the correct answer percentages were 64% and 49% respectively. This suggested that more than 50% of the participants answering the item MCK7 and more than %35 of the participants' answering the MCK8 incorrectly asking for explaining the meaning of equivalence sign in the proportions, $\frac{s_1}{t_1} = \frac{s_2}{t_2}$ and $\frac{s_1}{s_2} = \frac{t_1}{t_2}$

This showed many of the PSMTs were not able to explain the proportional relations. In particular to the incorrect anwers, they have generally written what they have seen in what is given (See Figure 6.7).

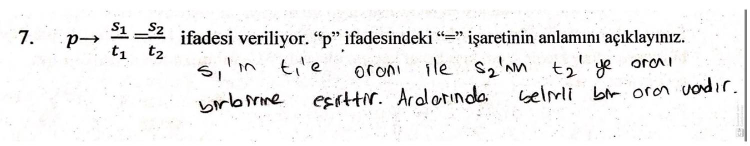


Figure 6.7. An example of a PSMT's answer for MCK7.

As the data indicated, although they have been asked to explain what the equality meant, participants have written: "distance1 over time1 is equal distance2 over time2" as an answer for MCK7.

Lastly, in the item MCK9, the proof and the explanation of the conditional statement "If MKC7 is true than MCK8 is true" were expected. 74% of the PSMTs have taken partial point from this item: They were easily able to prove the conditional statement by stating that "When the cross-multiplication is done", both expressions refer to "s1 x t2 = s2 x t1". Although this could be regarded as a proof, it is not an explanation for the proportional relations. Therefore, in total 96% of the PSMTs were not able to explain the reasons underlying this equality. Had they been proportional reasoners they would have explained that both expression in the two parts of the equation represented the multiplicative relationship between the time and the corresponiding distance. Also, they would have been able to state that for the velocity to be the same in the original situation and the second situation, the multiplicative relationship between the times and the distances in two situations have to be the same.

In sum, for the items MCK5-6-7-8 and 9, the PSMTs were expected to think and reason about proportional and non-proportioal situations and provide their reasons behind proportionality. However, results showed that although they were able to determine the given ratio representing velocity and use cross-multiplication as a particular reason behind proportionality, they had difficulty in explaining what the equal sign in proportionality meant and were not able to explain the reason behind the equal sign.

In the next section, I share the results from MCK instrument taking into consideration of both the pre-service middle and secondary school mathetamatics teachers' responses in total.

6.1.3. Pre-service Mathematics Teachers' MCK on Ratio and Proportion

Data were gathered from 590 senior pre-service mathematics teachers at eleven universities in Turkey. The mean, standard deviation, minimum and maximum scores were calculated for pre-service mathematics teachers' total test scores obtained from MCK-T. Table 6.5 shows descriptive statistics obtained from the MCK-T insturument. Also, the distribution of total scores for all pre-service teachers for MCK-T were illustrated in Figure 6.8.

Table 6.5. Descriptive Results for MCK-T of Pre-service mathematics teachers.

	N	M	SD	Min.	Max.
Total score for MCK-T	450	8.5007	1.54956	4.00	12.00

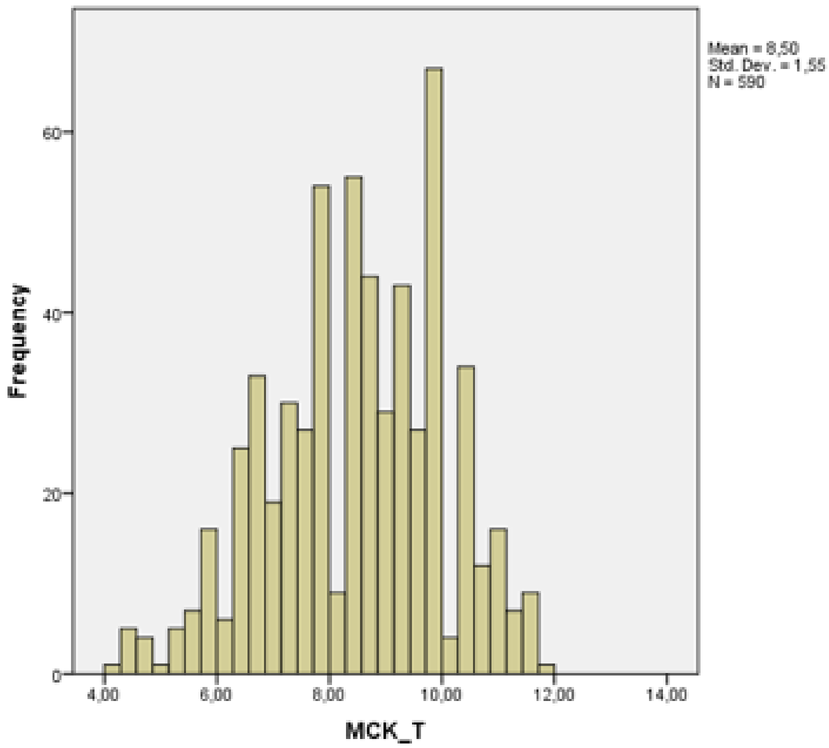


Figure 6.8. Distribution of Total MCK-T Scores of Pre-service Mathematics Students.

MCK-T consists of 12 items in total. So the maximum points is expected to be 12 points from MCK-T. As the data showed in Figure 6.8 and Table 6.5, for all pre-service teachers, the maximum total score was calculated as 12 points and minimum score was 4 points. Also, as shown in Table6.5, the mean value of the total scores was 8.5007 and the standard deviation was 1.54956. This suggested to some extent that pre-service teachers' mathematical content knowledge on ratio and proportion was high. Still, due to the participants' responses to the partialy scored items, further examination of the total scores per item is needed. As shown in Figure 6.9, results are presented in three parts: incorrect, partial and correct percentages. When the table of descriptive statistics, the histogram of score distributions and the graph of response percentages of pre-service mathematics teachers wer examined and compared to results from both pre-service middle and secondary school mathematics teachers, all results correlate.

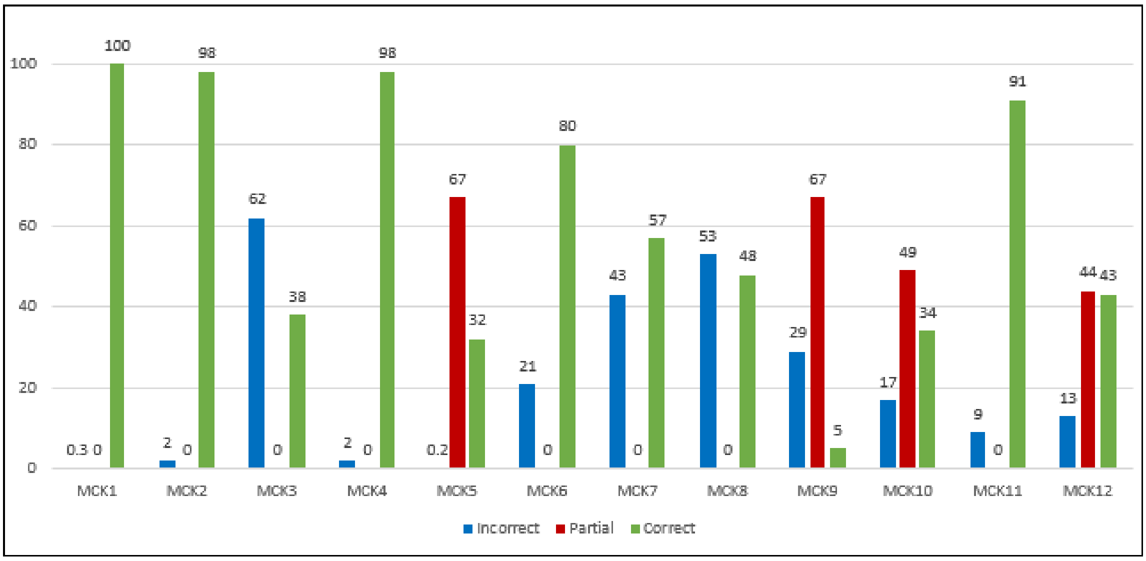


Figure 6.9. Pre-service Mathematics Teachers' Response Percentages for MCK-T.

In particular, the items MCK1-2-4 and 11were answered at high percentages; 100%, 98%, 98% and 91% respectively. These results suggested that pre-service mathematics teachers were able to solve different number structure problems requiring just procedural knowledge. Considering Turkish National Educational System in which learner have been familiar to those types of problems, the high percentages seems to be reasonable to expect. Though, in the items MCK 3-12-10, participants were expected to reason about the figural representation of ratio concept. When the incorrect and partial scores were examined, results indicated that pre-service mathematics teachers had some difficulties in explaining the concept of congruency in relation to the concept of ratio; expanding figures in propotonal relationship; and, recognizing the concept of ratio in graphical representation in the Coordinate Plane and differentiating it from the concept of linearity.

Lastly, items MCK5-6-7-8 and 9 were related to the recognition of proportional and non-proportional situations. The percentage of the partial anwers in the item MCK5 and MCK 9 as 67% and 67% ; the percentage of the incorrect answers in MCK7, MCK 8 and MCK 9 as 43%, 53% and 29% respectively indicated that pre-service mathematics teachers had difficulty in distinguishing proportional and non-proportional situations and also explaining the reasoning behind the proportionality in different situations.

6.2. Factor Analysis on Pre-service Mathematics Teachers' MCK-T Scores

In this study, confirmatory factor analysis (CFA) was performed to test the previous model which was conducted by Ekawati and her colleagues (2015). In the proposed model, the 11 MCK items were categorized into three factors that interpreted: the meaning of proportional and non-proportional situations (F1), number structures in situation (F2) and figural representation (F3) (Ekawati *et al.*, 2014). In the Table 6.6, these factors and items in each factor were presented.

Table 6.6. Factors of MCK in the proposed model.

Factors	Name of the factors	Items in the factors
F1	The meaning of proportional and non-proportional situations	MCK5, MCK6, MCK7, MCK8, MCK9
F2	The number structures	MCK1, MCK4, MCK11
F3	Figural representations	MCK3, MCK10 ,MCK12

According to CFA results for MCK-T, multivariate normality assumption was met; the skewness values and kurtosis values were within an excellent range as -.258 and -.433 respectively (George & Mallery, 2001). Also, in this dataset, there were no missing data, no outliers, and data was considered as continuous. With the use of summated scales, scores were continuous. In addition, there were 590 participants and 12 observed variables and the ratio of participant to observed variable was 590:12 which was appropriate.

When the fit indices were examined, the Root-Mean-Squared Error of Approximation (RMSEA) of the model was 0.045 which was smaller than 0.06, therefore, showed a sign of good fit to the data. Also, the 90 percent confidence interval for RMSEA was 0.032 and 0.057. The Comparative Fit Index (CFI) of the model was 0.93, which means moderate fit to the data. Lastly, Tucker-Lewis index (TLI) of the model was 0.91,which was higher than the values of 0.90 for a good fit.

As a result, the fit of the implied model was found satisfactory according to several fit indices given above. As it was shown in the Table 6.2.1, MCK2 was not included in any factor because in the proposed model, Ekawati and her colleagues (2015) asserted that the item MCK2 was deleted since its score correlation to the total score and the factor loading value of this item was calculated as less than 0.3. However, according to descriptive analysis of MCK-T in the Section 6.1., the MCK2 was considered with the items MCK1, MCK4 and MCK11. So, for the current study, the MCK2 was also taken into consideration in Factor 2 .

6.3. Pre-Sevice Mathematics Teachers' MPCK on Ratio and Proportion

6.3.1. Pre-service Middle School Mathematics Teachers' MPCK on Ratio and Proportion

Data were gathered from 450 senior PMMTs who were from PRED departments at eleven universities in Turkey. The mean, standard deviation, minimum and maximum scores were calculated for the PMMTs' total test scores obtained from MPCK-T instrument. Table 6.7 shows descriptive statistics obtained from the MPCK-T insturument. Also, the distribution of the percentages of total scores of all PMMTs for MPCK-T were illustrated in Figure 6.10.

Table 6.7. Descriptive Results for MPCK-T of PMMTs.

	N	M	SD	Min.	Max.
Total score for MPCK-T	450	6.4829	1.57030	1.00	10.26

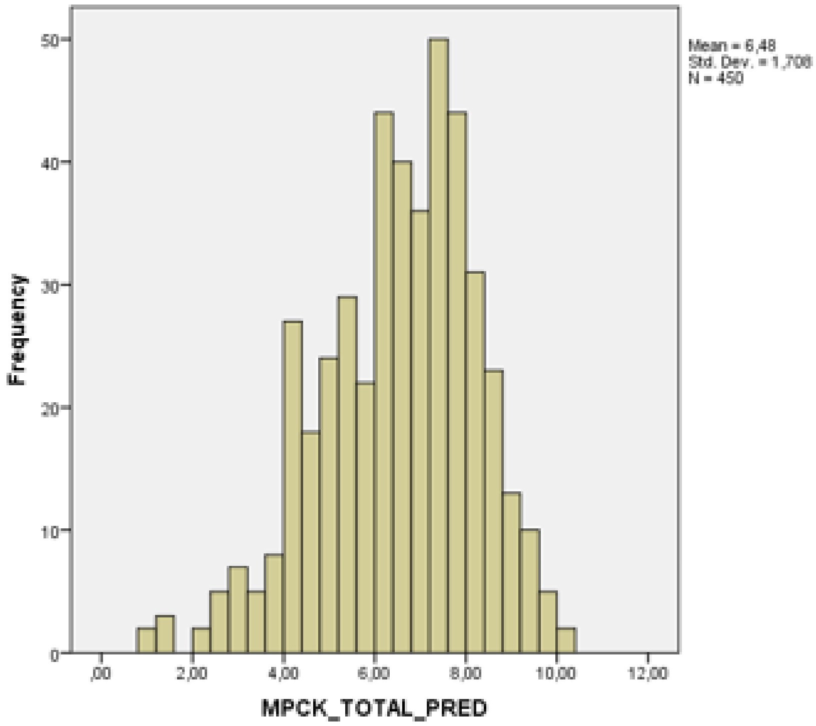


Figure 6.10. Distribution of MPCK-T Scores of PMMTs.

MPCK-T consists of 11 items in total. So the maximum score is expected to be 11 points from MPCK-T. As the data showed in Table 6.7 and Figure 6.10, the maximum total score was calculated as 10.26 points and minimum score was 1 point for PMMTs.

Also, as shown in Table 6.7, the mean value of the total scores was 6.4829 and the standard deviation was 1.57030. This suggested to some extent that PMMTs' mathematical pedagogical content knowledge on ratio and proportion was low. Further examination of the total scores per item also pointed to the same results.
Particularly, in the Figure 6.11, percentages of the MPCK-T scores of PMMTs were

shown. As shown, some of the items, MPCK1-4-7-9-11, were scored as correct and incorrect; and, some items were scored also as partially correct, MPCK2-3-5-6-8-10.

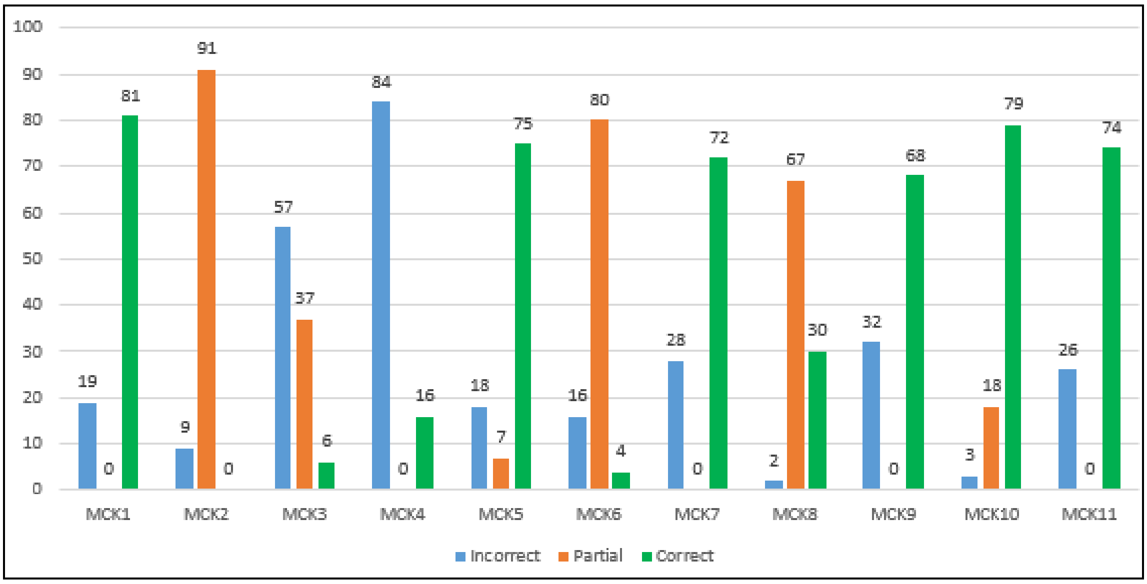


Figure 6.11. PMMTs' Response Percentages for MPCK-T Instrument.

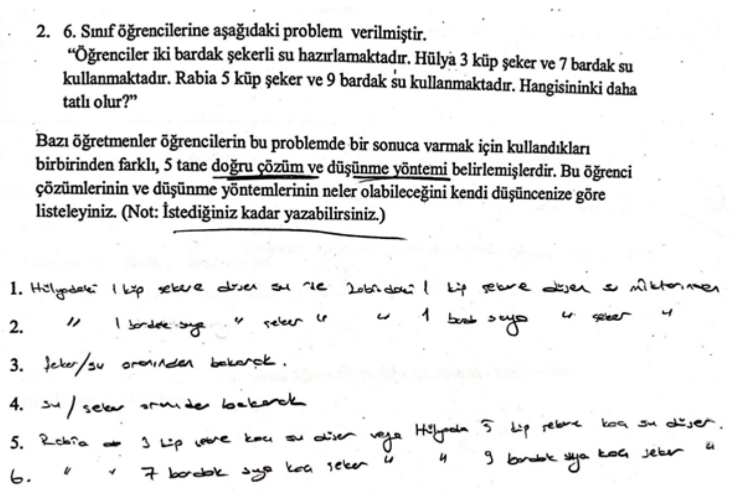
As the data indicated, PMMTs have taken high correct percentages from the items MPCK1-7-10 and 11 in general. Particularly, in the MPCK1, participants were expected to develop a different question involving proportionality that would fit to middle school students' knowledge level. Besides, the problem was asked to be more easier than the problem given in MCK1. Results showed that although, 81% of the participants were able to design an easier problem for middle school level, 19% were not able to construct a problem appropriate to the middle school students' knowledge base. This showed that one-fifth of the PMMTs were not able to consider the student knowledge according to student's grade level.

Like MPCK1, the item MPCK7 was also related to thinking about student understanding and also giving feedback for a student's answer to the problem given in MCK11. As already mentioned earlier, in MCK11, there was a rectangle with 3cm width and 5cm base and the rectangle was to be enlarged so that the new base would be equal to 12cm. In MPCK7, a hypothetical student solution was provided as follows: The student had taken the difference between 5 and 12 and by considering the difference as 7, the student decided that the new width would be 7 more than 3, and the answer would be 10 cm width for t he new shape. And then in MPCK7, participants were asked to give feedback on the student's answer by considering the student's thinking. Although the feedback was expected to focus on the multiplicative relationship between the base and the width or the multiplicative relationship between the base 5 cm and the new base 12 cm, data showed that 28%, 126 out of 450 PMMTs were not able to give appropriate feedback. Some of the participants asserted that the student had to keep the area of the rectangle the same rather than pointing to multiplicative relationship between the width and the base represented in ratio. Some of them asserted that there was no problem in the student's solution strategy. Moreover, some of them explained that they would state the correct answer of the problem to the student as feedback. This showed that although there was a high percentage of correct feedbacks, 72%, so many PMMTs were not able to focus on the fact that the student's wrong answer stemmed from his/her using the incorrect solution strategy. The incorrect solution strategy shows that the student could not focus on the meaning of the ratio as a multiplicative relationship. Therefore, almost one third of the PMMTs were not aware of the fact that the student was using additive reasoning in proportional situations. This also suggested that their feedback was not related to the enlargement of the rectangle's width and base by the same ratio.

The items MPCK10 and 11was related items such that in MPCK10, participants were asked to think about which proportion problems were more harder to solve for six graders. Similarly, in MPCK 11, participants were asked to explain the reason of their selection in MPCK10. Thus, both of the items focused on thinking of student knowledge. The correct percentages were 79% and 74% respectively. This showed that so many pre-service middle school mathematics teachers were able to select the correct problems, Problem 2 and 3, as harder problems pointing to the reason: the numbers in those problems as non-integer ratios. On the other hand, while providing explanation on why those problems would be harder for 6th grade students, especially one fourth of the PMMTs were not able to make a relation between the non-integer ratios and the difficulty of the problems by considering the level of students.

In sum, the items MPCK1-7-10 and 11 were the items the PMMTs answered correctly in high percentages. However, results showed that approximately 25% of the were not really able to focus on student knowledge or students' way of thinking to either provide feedback or to select appropriate problems according to students' levels.

The items MPCK2-5-6-8 and 9 focused on some strategies students possibly might use about the ratio problems. Particularly, in MPCK2, participants were asked to write different correct strategies a student might use by solving the given ratio problem in the Figure 6.12.



2. 6. Sınıf öğrencilerine aşağıdaki problem verilmiştir.
"Öğrenciler iki bardak şekerli su hazırlamaktadır. Hülya 3 küp şeker ve 7 bardak su kullanmaktadır. Rabia 5 küp şeker ve 9 bardak su kullanmaktadır. Hangisininki daha tatlı olur?"

Bazı öğretmenler öğrencilerin bu problemde bir sonuca varmak için kullandıkları birbirinden farklı, 5 tane doğru çözüm ve düşünme yöntemi belirlemişlerdir. Bu öğrenci çözümlerinin ve düşünme yöntemlerinin neler olabileceğini kendi düşüncenize göre listeleyiniz. (Not: İstediğiniz kadar yazabilirsiniz.)

1. Hülyadaki 1 küp şekere düşen su ile Rabiadaki 1 küp şekere düşen su miktarından
2. " 1 bardak suya " şeker " " 1 bardak suya " şeker "
3. şeker/su oranından bakarak.
4. su/şeker oranından bakarak
5. Rabia 3 küp şekere kaç su düşer veya Hülyada 5 küp şekere kaç su düşer.
6. " " 7 bardak suya kaç şeker " " 9 bardak suya kaç şeker "

Figure 6.12. An example of a PMMT' answer for MPCK2.

In the Table 6.8, the number of partially correct answers were presented. Every correct strategy was scored as 0.2 points. So, there was no PMMT who was able to write five different strategies. Also, only 23% of the them were able to mention three or four strategies. On the other hand, the remaining 77% were able to write at most two correct strategies. Particularly, those strategies included the abbreviated build-up strategy, unit factor approach, within and between strategies. In addition, so many PMMTs have considered the comparison of part-part and part-whole ratios as different strategies.The low percentages showed that so many PMMTs were able to consider just a couple of strategies related to ratio and proportion.

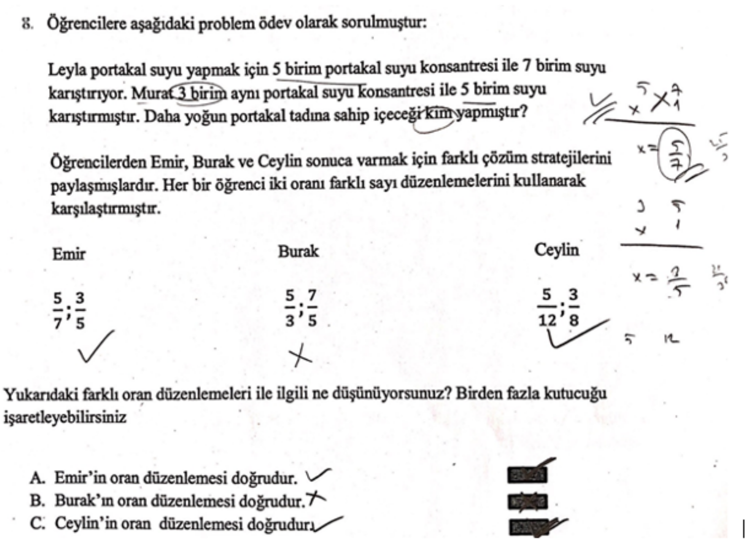
Table 6.8. Descriptive statistics of the item MPCK2 for PMMTs.

MPCK2

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	0.00	42	9.3	9.3	9.3
	0.20	142	31.6	31.6	40.9
	0.40	164	36.4	36.4	77.3
	0.60	85	18.9	18.9	96.2
	0.80	17	3.8	3.8	100.0
	Total	450	100.0	100.0	

The items MPCK5 and 6 were related to the unit factor strategy in particular. In both questions, unit factor strategy was introduced as the most appropriate strategy for students' better understanding and thinking. In item MPCK5, 75% of the PMMTs selected the unit factor strategy rather than abbreviated build-up and cross-multiplication for students' better understanding and reasoning. However in MPCK6, they were expected to mention the reason of their selection such as unit factor strategy rather than cross-multiplication. So many PMMTs asserted that unit factor strategy was easier than the other strategies such that by using this strategy students might understand the ratio concept. They further commented that by using cross-multiplication strategy, students might not understand the ratio concept in a meaningful way. However, according to the rubric, they are expected to mention about students' prior knowledge as a reason for starting with unit factor strategy. Though, the high percentage, 80% of PMMTs taking partial points, showed that they ignored to take into consideration of the prior knowledge of students related to ratio concept. Therefore, this showed that although pre-service middle school mathematics teachers were able to recognize the importance and also the simplicity of the unit factor strategy, they were not able to consider prior knowledge of students related to unit factor strategy.

The items MPCK8 and 9 were also interrelated items. In the item MPCK8, correct ratio comparisons were asked according to the given problem. Only 30% of the PMMTs were able to select all options. First option was a part-part comparison including within strategy, second option was a part-part comparison including between strategy and third option was a part-whole comparison including within strategy as shown in Figure 6.13.


8. Öğrencilere aşağıdaki problem ödev olarak sorulmuştur:

Leyla portakal suyu yapmak için 5 birim portakal suyu konsantresi ile 7 birim suyu karıştırıyor. Murat 3 birim aynı portakal suyu konsantresi ile 5 birim suyu karıştırmıştır. Daha yoğun portakal tadına sahip içeceği kim yapmıştır?

Öğrencilerden Emir, Burak ve Ceylin sonuca varmak için farklı çözüm stratejilerini paylaşmışlardır. Her bir öğrenci iki oranı farklı sayı düzenlemelerini kullanarak karşılaştırmıştır.

Emir	Burak	Ceylin
$\frac{5}{7}; \frac{3}{5}$	$\frac{5}{3}; \frac{7}{5}$	$\frac{5}{12}; \frac{3}{8}$

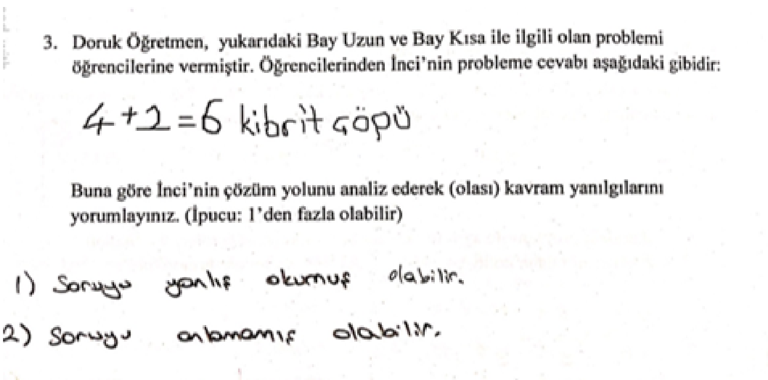
Yukarıdaki farklı oran düzenlemeleri ile ilgili ne düşünüyorsunuz? Birden fazla kutucuğu işaretleyebilirsiniz

A. Emir'in oran düzenlemesi doğrudur.
B. Burak'ın oran düzenlemesi doğrudur.
C. Ceylin'in oran düzenlemesi doğrudur.

Figure 6.13. An example of a PMMT's answer for MPCK8.

48% of the participants have selected the two options mostly: the first and the third. This showes that they were not able to consider between strategy when they were solving ratio comparison problems. Moreover, MPCK9 focused on analysing the first and the third ratio comparisons. 32% percent of the PMMTs were not able to explain the difference between those two comparisons as part-part and part-whole comparison. In addition, although some of them were able to explain the difference, they asserted that one of them was correct but the other was not.

In sum, considering the items MPCK2-5-6-8 and 9 altogether, results showed that PMMTs had lack of knowledge both about strategies students might possibly utilze while solving ratio problems and also the differences between those strategies. All these suggested that PMMTs were not able to analyze student thinking in ratio and proportion problems . Lastly, the item MPCK3 and 4 were also related to students' misunderstandings regarding ratio and proportion. Results showed that PMMTs score the highest incorrect percentages in those items: 57% and 83% respectively. Particularly, participants expressed the misreading or misunderstanding the given question as misconception (See Figure 6.14. for MPCK3). However, they were expected to explain that the misconceptions might have stemmed from the misunderstanding of the multiplicative relationship in ratio and proportion; and, also stemmed from ignoring the different units, stick and paperclip as different units.


3. Doruk Öğretmen, yukarıdaki Bay Uzun ve Bay Kısa ile ilgili olan problemi öğrencilerine vermiştir. Öğrencilerinden İnci'nin probleme cevabı aşağıdaki gibidir:

4+2=6 kibrit çöpü

Buna göre İnci'nin çözüm yolunu analiz ederek (olası) kavram yanılgılarını yorumlayınız. (İpucu: 1'den fazla olabilir)

1) Soruyu yanlış okumuş olabilir.

2) Soruyu anlamamış olabilir.

Figure 6.14. An example of a PMMT's answer for MPCK3.

Furthermore, in the item MPCK4, they were expected to write a feedback question in order to raise the student's awareness of the misconception in MPCK3. In this item, so many PMMTs wrote a different problem including ratio and proportion. Also, some of them explained the given question again as examples of providing feedback by asking a question. All these results showed that, PMMTs were not able to recognize particular student misconceptions about ratio and proportion. Similarly, they were not able to write any question to provide feedback related to those misconceptions either.

6.3.2. Pre-service Secondary School Mathematics Teachers' MPCK on Ratio and Proportion

Data were gathered from 140 senior PSMTs who were from SCED departments at eleven universities in Turkey. The mean, standard deviation, minimum and maximum scores were calculated for the PSMTs' total test scores obtained from MPCK-T Table 6.9. shows descriptive statistics obtained from the MPCK-T insturument. Also, the distribution of total scores of all PSMTs for MPCK-T were illustrated in Figure 6.15.

Table 6.9. ADescriptive Results for MPCK-T of PSMTs.

	N	M	SD	Min.	Max.
Total score for MPCK-T	140	6.1191	1.61148	2.33	9.60

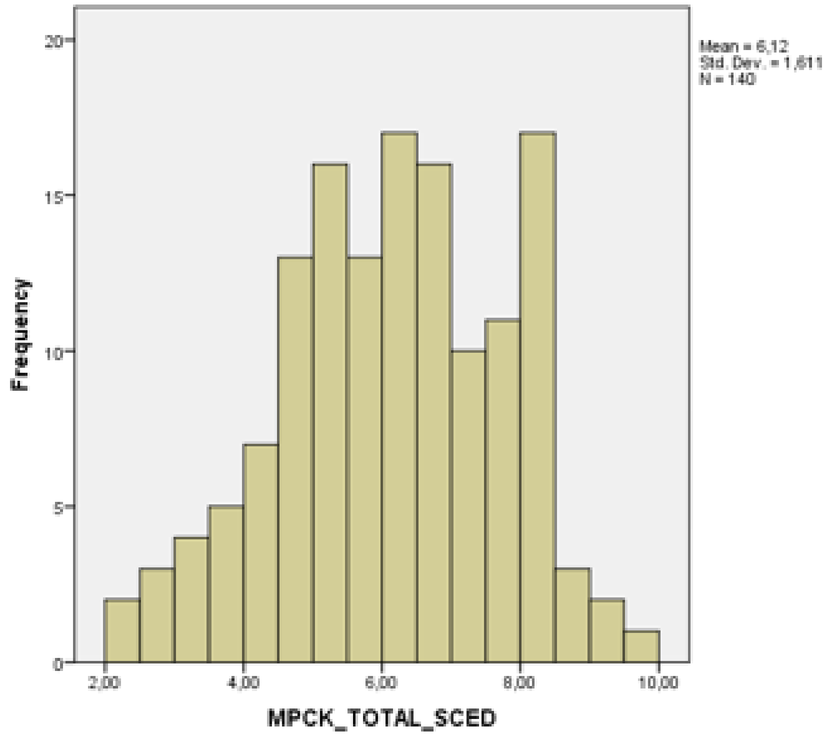


Figure 6.15. Distribution of MPCK-T Scores of PSMTs.

As the data showed, the maximum total score was calculated as 9.60 points and minimum score was 2.33 points for PSMTs. Also, as shown in Table 6.9, the mean value of the total scores was 6.1191 and the standard deviation was 1.61148. This suggested to some extent that PSMTs'mathematical pedagogical content knowledge on ratio and proportion was low. Further examination of the total scores per item also pointed to the same results.

In the Figure 6.16, percentages of the MPCK-T scores were shown.The items MPCK2-3-5-6-8-10 were designed as partially scored itesm and the items MPCK1-4-7-9-11 were designed as only correct and incorrect scored items. Also, the results from PMMTs and PSMTs were very similar:

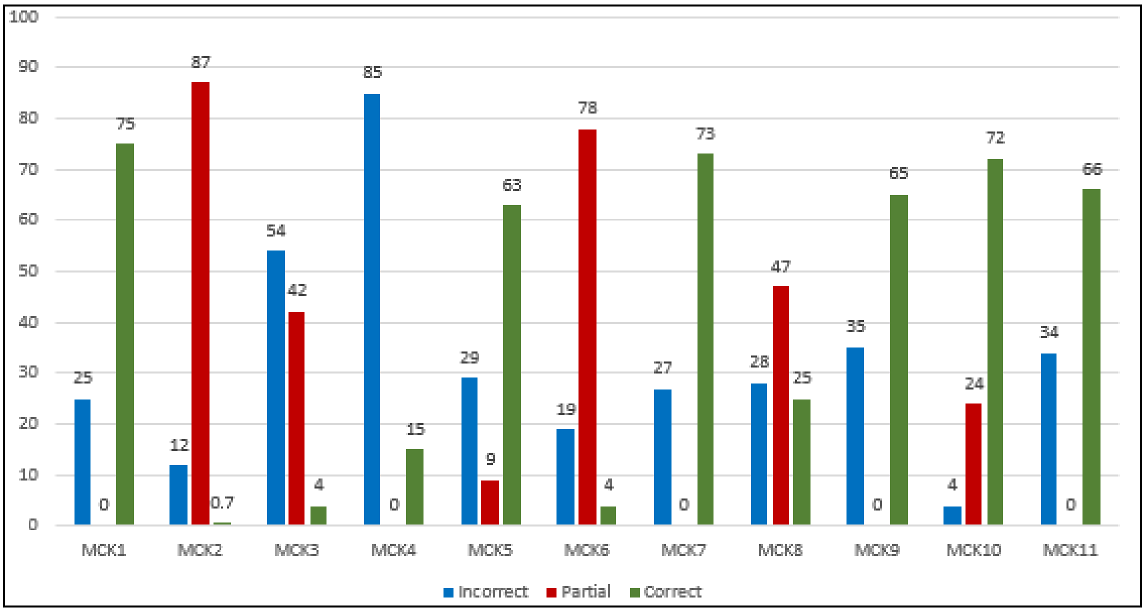


Figure 6.16. PSMTs' Response Percentages for MPCK-T Instrument.

Firstly, similar to PMMTs, PSMTs scored relatively high in the items MPCK1-7-10 and 11. The corect percentages were 75%, 73%, 72% and 66% respectively. However, when incorrect and partial scores were examined further, results pointed to the following: First of all, these items were related to thinking about student knowledge and determinining appropriate problems according to student knowledge. Particularly, in item MPCK1, one fourth of the PSMTs were not able to design a proportional problem which was expected to be more easier than the given problem in MCK1 for the middle school level. This suggested that they were not aware of the level of the student thinking nor that they were able to construct a problem taking into consideration of student thinking given the grade level.

In the item MPCK7, a student solution strategy for a proportional problem was given and the appropriate feedback to this solution was asked. Again in this item, although so many PSMTs were able to give correct feedback, one fourth of them were not able to provide their feedbacks in a correct way. In particular, they were expected to focus on the multiplicative relation in the ratio concept when giving feedback to the student's incorrect additive solution strategy. However, some of them just explained the correct answer for the given problem, some of them asserted that the additive relation was true and some of them asserted that to protect the shape of the rectangle, the area of the rectangle should be kept the same rather than providing explanation pointing to the ratio between the sides of the rectangle. This showed that more than one fourth of the PSMTs had lack of knowldge about the concept of ratio such that neither they were able to determine the misconception of the student given in MCK7 nor were they able to to provide appropriate feedback.

Items MPCK10 and 11 were about ratio and proportion task level feature. Participants were expected to find the harder problems according to the given student levels and the explanation of the reason behind their choise. Similar to the results from PMMTs, 72% of the PSMTs were able to select the problems: Problem 2 and 3, as harder problems in MPCK10. However, in MPCK11, 34% of them were not able to explain the reason of difficulty regarding the problems asked in MCK10. These results suggested that PSMTs were not able to make a connection between student knowledge and the problem difficulty related to the ratio concept. In particular, results showed that they were not aware of the fact that for six grade students problems including non-integer ratios might be considered as harder than the problems with integer ratios.

The items MPCK2-5-6-8 and 9 focused on some strategies students possibly might use in ratio and proportion problems. The number of different correct strategies a PSMT might explain were examined in the item MPCK 2. As it was shown in the Table 6.10, every correct strategy was scored as 0.2 points.

Table 6.10. Descriptive statistics of the item MPCK2 for a PSMT.

MPCK2					
		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	0.00	17	12.1	12.1	12.1
	0.20	32	22.9	22.9	35.0
	0.40	60	42.9	42.9	77.9
	0.60	27	19.3	19.3	97.1
	0.80	3	2.1	2.1	99.3
	1.00	1	0.7	0.7	100.0
	Total	140	100.0	100.0	

The table showed that, smilar to the results from PMMTs, totally 78% of the PSMTs were able to explain at most two correct strategies. Moreover, some participants pointed to the same strategy as if they were explaining different strategies. For instance, when they examined the ratio of sugar to water as 3/7, they were not able to recognize that they have found the sugar cubes per one glass of water as 0.42. In other words, they have asserted that the ratio 3/7 and the number 0.42 were referring to two different strategies. In addition, some of them have introduced the ratios 3/7 and 7/3 as different strategies. These examples and the low number of strategies pointed by the particants showed: Firstly, PSMTs were not able to use multiple solution strategies for ratio comparison problems in particular. Secondly, they might have some misconceptions about the strategies students might use in ratio problems.

In the item MPCK5, partcipants were expected to think that the unit factor strategy might be prefered for better understanding of students. Results showed that 63% of the PSMTs were able to select the unit factor strategy, while the remaining claimed cross-multiplication and abbreviated build-up strategy. In the item MPCK6, the reasoning behind the use unit factor strategy in teaching was asked. Again, in MPCK6, so many participants pointed to the simplicity of the unit factor strategy. However, they were not able to think about other factors such as the pre-knowledge of students, prerequisites of the ratio concept etc.. So, for the items MPCK5 and 6, results indicated that although PSMTs, 63%, were aware of the simplicity and the importance of the unit factor, they were not able to relate the choice of using unit factor in teaching neither by focusing on student knowledge nor focusing on the different levels of the concept of ratio.

Furthermore, the related items MPCK8 and 9 were also about strategies students might possibly use in the concept of ratio. Participants were expected to select the three different types of ratio arrangements: part-part comparison including within strategy, part-part comparison including between strategy and part-whole comparison including within strategy. Only 25% of the PSMTs were able to select all of the strategies correctly. Moreover, in the item MPCK9, the difference between the part-part and part-whole ratio comparisons was asked. Although 65% of the PSMTs were able explain the reason, 35% of them were not able to explain the reason. These results showed that so many PSMTs might have the lack of knowledge about the different levels of the conception of ratio, between ratio conception in particular. Again this was because 75% of them were not able to select part-part comparison including between ratio in MPCK8. However they were able to mention that first one was part-part comparison and third one was part-whole comparison without using the words “part-part” and “part-whole”.

In sum, results regarding the items MPCK2-5-6-8 and 9 showed that the PSMTs have the lack of knowledge on the strategies students might use in solving ratio and proportion problems. Particularly, they do not realize that between ratios is appropriate to represent the quantities in ratio.

Lastly, in the items MPCK3 and 4, participants were expected to evaluate the misconceptions in the given student answer regarding ratio concept and also were expected to write a question to help the student to realize the misconception. In fact, the student strategy given in MPCK3 was related to the incorrect addition strategy. So, in MPCK 3, the participants were expected to comment that the misconception of the student might have stemmed from the confusion of different units in the question or the use of additive reasoing rather than recognizing the multiplicative relationship between the quantities in the ratio. Results showed that only 4% of the PSMTs were able to recognize those two misconceptions and mentioned the incorrect addition strategy in ratio concept together with the student's unawareness of different units. This suggested that most of the PSMTs were not able to determine the student misconception. This further suggested that they might not have realized that ratio represents the multiplicative relationship between the quantities. In MPCK4, expecting to focus on the multiplicative relationships of the quantities as well as referring to the different units in the question, 85% of the PSMTs were not able to write a question as an appropriate feedback to correct the misconception of the student given in MPCK3. Similar to the results from the PMMTs, so many of them wrote that they would explain the question to the student again. Also, some of them solved the question in detail as feedback to the student. These results showed that the PSMTs were not able to recognize the student misconceptions related to ratio and proportion. Also, they were not able to create appropriate feedback questions to correct the misconceptions.

6.3.3. Pre-service Mathematics Teachers' MPCK on Ratio and Proportion

Data were gathered from 590 senior pre-service mathematics teachers at eleven universities in Turkey. The mean, standard deviation, minimum and maximum scores were calculated for pre-service mathematics teachers' total test scores obtained from MPCK-T Table 6.11. Shows descriptive statistics obtained from the MPCK-T instusrument. Also, the distribution of total scores for MPCK-T were illustrated in Figure 6.17.

Table 6.11. Descriptive Results for MPCK-T of Pre-service mathematics teachers.

	N	M	SD	Min.	Max.
Total score for MPCK-T	590	6.3965	1.69138	1.00	10.26

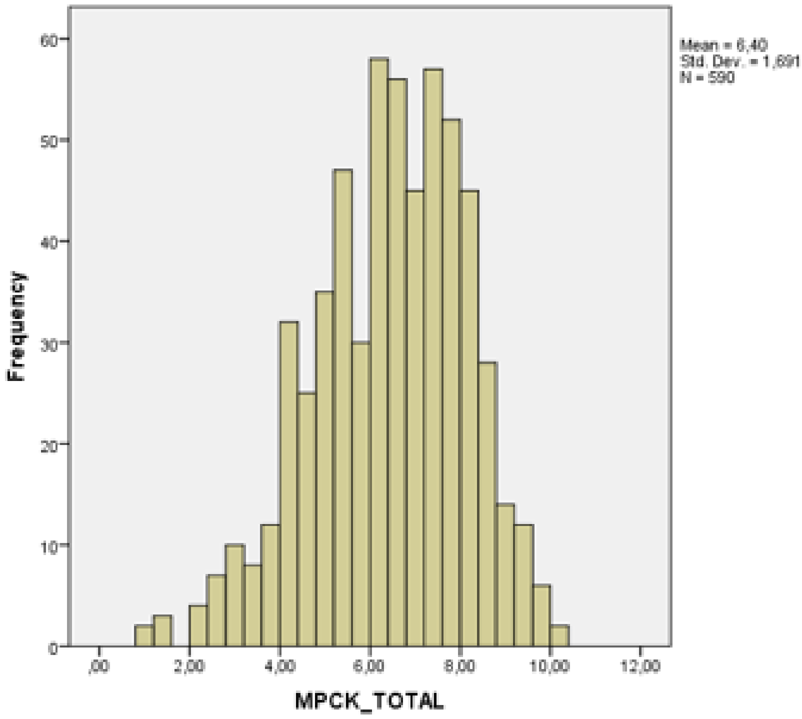


Figure 6.17. Distribution of Total MPCK-T Scores of Pre-service Mathematics Students.

As the data showed, out of 11 items in MCK-T, the maximum score was calculated as 10.26 points and the minimum score was 1 point for pre-service mathematics teachers. Also, as shown in Table 6.11, the mean value of the total scores was 6.3965 and the standard deviation was 1.69138. This suggested to some extent that pre-service teachers' mathematical pedagocial content knowledge on ratio and proportion was low.

In the Figure 6.18, percentages of the total MPCK-T scores of pre-service mathematics teachers were shown. When the table of descriptive statistics, the histogram of score distributions and the graph of response percentages of pre-service mathematics teachers were examined and compared to results from both the pre-service middle school mathematics teachers and the pre-service secondary school mathematics teachers, all of the results correspond to each other.

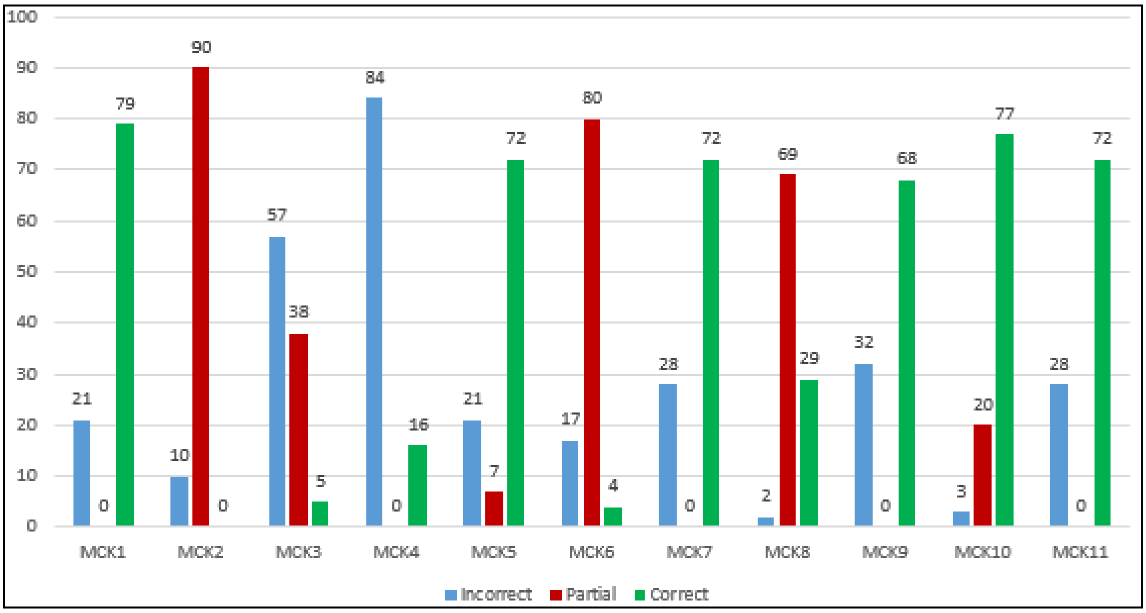


Figure 6.18. Pre-service Mathematics Teachers' Response Percentages for MPCK-T.

Particularly, the items MPCK1-7-10 and 11 were designed to evaluate student knowledge and to determine the problems according to students' level of knowledge. Although the correct percentages were high, 79%, 72%, 77% and 72% respectively, when incorrect and partial scores were examined further, results pointed to the following: First, although pre-service mathematics teachers could design a problem related to ratio and proportion, they had lack of knowledge about the simplicity of the problems related to the student level of knowledge. Besides, the pre-service mathematics teachers were not able to evaluate the student responses for any ratio problem such that their feedbacks have become irrelevant to the student knowledge. Moreover, they were not able to make a relation between the task variables and the level of the student knowledge.That is, they were not able to relate the difficulty of the problem level for 6th graders pointing to the number structures in the problems.

In the items MPCK2-5-6-8 and 9, the number of different correct strategies related to ratio concept such as unit factor approach in particular, and within and between strategies were examined. Results showed that 77% of pre-service mathematics teachers were able to write at most two different strategies to solve ratio and proportion problems. This suggests that they have lack of knowledge for the multiple solution strategies students might use in ratio and proportion problems. In particular, they were not able to use and explain the between ratio strategies in their solutions.

Lastly, the items MPCK3 and 4 were interrelated items. Some misconceptions and feedbacks related to ratio concept were considered in these items. Particularly, the incorrect addition strategy was presented in the given example in MPCK3 and the possible feedbacks were expected to be stated by the participants in MPCK4 . Results showed that the correct answer percentages were very low, only 5% and 16% respectively. This showed that so many pre-service mathematics teachers were not able to explain the misconception of a student regarding the use of incorrect addition in ratio concept. Similarly, so many of them were not able to construct appropriate feedback related to the provided misconception. These results also suggested that pre-service mathematics teachers might not have realized that ratio represents the multiplicative relationship between the quantities. Thus, this might refer to pre-service mathematics teachers' lack of conceptual knowledge about ratio and proportion.

6.4. Factor Analysis on Pre-service Mathematics Teachers' MPCK-T Scores

For the MPCK-T, the confirmatory factor analysis was also applied firstly. In the proposed model, the 11 MPCK items were categorized into three factors that interpreted: knowing students conceptual understanding (F4), ratio and proportion task level feature (F5) and teaching problem solving strategy of ratio and proportion (F6). In the Table 6.12, the factors of MPCK in the proposed model and related items were presented. As seen from the table, MPCK 5 was deleted by Ekawati and her colleagues (2015) because its score correlation to the total score and the factor loading value of this item were less than 0.3 (Ekawati *et al.*, 2015).

Table 6.12. Factors of MPCK in the proposed model.

Factors	Name of the factors	Items in the factors
F4	Knowing students conceptual understanding	MPCK1, MPCK2, MPCK3, MPCK4
F5	Ratio and proportion task level feature	MPCK10, MPCK11
F6	Teaching problem solving strategy of ratio and proportion	MPCK6, MPCK7, MPCK8, MPCK9

However, the fit of the implied model (Ekawati *et al.*, 2015) was not found satisfactory according to several fit indices like the root mean square error of approximation (RMSEA), the comparative fit index (CFI) and Tucker-Lewis index (TLI). After that, in order to understand how items were grouped and to provide empirical support for what was measured by MPCK-T, exploratory factor analysis was conducted.

Kaiser-Meyer-Olkin measure of sampling adequacy value of 0.60 which was equal to the recommended value of 0.6 (Kaiser, 1974) and the Bartlett's test of sphericity (Bartlett, 1954) reached statistical significance, supporting the factorability of the data. Principal axis factoring with varimax rotation showed that there were five components which had eigenvalues larger than 1.00. Moreover, approximately 65% of the variance was explained by this factor structure. The Table 6.13 showed rotated factor loadings for MPCK-T items. In this table, loadings less than 0.25 awere omitted for the clarity of the matrix.

Table 6.13. Rotated Factor Loadings of MPCK-T items.

Rotated Factor Matrixa					
	Factor				
	1	2	3	4	5
MPCK9	0.836				
MPCK8	0.764				
MPCK2	0.261				
MPCK11		0.808			
MPCK10		0.774			
MPCK3			0.611		
MPCK4			0.396		
MPCK7				0.370	
MPCK1				0.357	
MPCK6					0.435
MPCK5					0.327
Extraction Method: Principal Axis Factoring.					
Rotation Method: Varimax with Kaiser Normalization.a					
a. Rotation converged in 5 iterations.					

Exploratory factor analysis results in Table 6.13 indicated that five main dimensions were measured for MPCK-T items. However, when the descriptive and qualitative analysis of items were done, it was plausible that the items MPCK1 and MPCK7, as fourth factor, could be included in the second factor with the items MPCK10 and MPCK11. This was because all of the four items MPCK1-7-10 and 11 showed similar correct percentages and they were related to taking into consideration students' reasoning and background knowledge. Besides, based on the descriptive and qualitative analysis of the items, the items MPCK5 and MPCK6 was getting similar to the items MPCK2-8 and 9. So, it was also plausible that the items MPCK5 and MPCK6, as fifth factor, could be included in the first factor with the items MPCK2-8 and MPCK9. This reason was again all of the five items were related to the knowledge of student strategies. Therefore, the factors and the related items were shown in the Table 6.14.

Table 6.14. Factors of MPCK-T according to EFA results.

Factors	Name of the factors	Items in the factors
F4	Knowledge of student strategies	MPCK2, MPCK5, MPCK6, MPCK8, MPCK9
F5	Recognition and evaluation of student reasoning	MPCK1, MPCK7, MPCK10, MPCK11
F6	Knowledge of student misconceptions	MPCK3, MPCK4

In sum, by considering the mean values of the factors which was given in the Table 6.15, the pre-service mathematics teachers' scores on the factors F1, F2 and F3 related to MCK-T, could be considered as higher than their scores on the factors F4,F5 and F6 related to MPCK-T. These results might be suggested that pre-service teachers' content knowledge could be considered as higher than their pedagogical content knowledge on ratio and proportion. Moreover, when the factor F2(number structures in situations) was omitted, the total mean value got lower for MCK-T, which was 0.57 approximately. Besides, when the factor F5(recognition and evaluation of student reasoning) was omitted, the total mean value of MPCK-T got lower , which was 0.39 approximately. These new mean values might be suggested that senior pre-service mathematics teachers have lack of both the content knowledge and pedagogical content knowledge on the topic ratio and proportion.

Table 6.15. Descriptive Statistics of Factors for MCK-T and MPCK-T.

Descriptive Statistics					
	N	Minimum	Maximum	Mean	Std. Deviation
F1	590	0.07	1.00	0.6056	0.23590
F2	590	0.50	1.00	0.9648	0.09515
F3	590	0.00	1.00	0.5379	0.24022
F4	590	0.07	0.92	0.5774	0.18905
F5	590	0.00	1.00	0.7767	0.24642
F6	590	0.00	1.00	0.2013	0.26355
Valid N (listwise)	590				

7. DISCUSSION AND CONCLUSION

This study investigated mathematical content knowledge and mathematical pedagogical content knowledge of pre-service middle school and secondary school teachers on ratio and proportion. Also, sub-dimensions of pre-service mathematics teachers' mathematics content knowledge and mathematics pedagogical content knowledge were scrutinized. Results regarding mathematical content knowledge of both preservice middle and secondary school mathematics teachers showed that they were successful at questions requiring procedural answers. However, they were not able to reason on questions requiring to distinguish proportional situations from non-poportional situations as well as the questions having figural representations. Results regarding MPCK of both pre-service middle and secondary school mathematics teachers showed that although they were able construct problems related to ratio and proportion, they were not able to take into consideration students' reasoning and back ground knowledge. Similarly, they were not able to determine students' misconceptions nor were they able to provide feedback based on their misconceptions. Furthermore, results suggested that all pre-service teachers knew few student strategies. Regarding the sub-dimensions of pre-service mathematics teachers' MCK, results showed that the factors (F1) the meaning of proportional and non-proportional situations, (F2) number structures in situation and (F3) figural representations were in alignment with previous research conducted by Ekawati her colleagues (2015). Finally, regarding the subdimensions of pre-service mathematics teachers' MPCK, the exploratory factor analysis results pointed to three factors: (F4) Knowledge of student strategies, (F5) knowledge of student misconceptions and (F6) recognition and evaluation of student reasoning. Considering that there are twelve universities having secondary school mathematics education departments with a total number of 204 senior pre-service teachers, results of this study are generalizable to the population of pre-service secondary school mathematics teachers in Turkey. Additionaly, considering that within these twelve universities there are a total number of 953 pre-service middle school mathematics teachers, results of this study are also generalizable to the target population of pre-service middle school teachers in these twelve universities. Also, results both from descriptive examination and also factor analysis in juxtaposition to each other pointed to similar results. Therefore, in the following paragraphs, I explain the factor analysis and desrcriptive results together pointing to both the results from the pre-service middle school and secndary mathematics teachers.

7.1. Results from MCK-T Instrument

Results of MCK-T was related to mathematics content knowledge of pre-service mathematics teachers on ratio and proportion. Pre-service mathematics teachers' knowledge on ratio and proportion is vital in their instructional roles such as using materials, assessing students' progress and sequencing the lessons etc. (Ball, Hill, & Bass, 2005). In this study, results from confirmatory factor analysis pointed to the same three factors in Ekawati *et al.* (2015): F1 (the meaning of proportional and non-proportional situations, F2 (number structures in situations) and F3 (figural representation).

Regarding Factor1 in MCK instrument, the items MCK5-6-7-8 and MCK9 were related to the meaning of proportional and non-proportional situations. Results showed that pre-service middle school and secondary mathematics teachers showed some lack of knowledge to distinguish proportional situations from non-proportional situations. For instance, for MCK5, approximately 150 PRED students out of 450 were able to match at most 4 situations out of 6 situations correctly as proportional or non-proportional. Similarly, almost 25% of the pre-service secondary school mathematics teachers had difficulty in being aware of the proportional and non-proportional situations. Particularly, taking into consideration the task variables Lamon (1993), the propotional situations in MCK5 were related to the well-chunked (such as price problem) and associated sets (such as recipe problem). Also, the number combinations used in the proportional situations were mostly integer multipliples of each other. Previous research results point that problems including integer ratios are easier than problems including non-integer ratios; and problems involving "for every" statements are easier for students (Heinz, 2000; Kaput & West, 1994). Therefore, in this study, one third of the pre-service middle school teachers' matching at most four situations correctly shows that they were not able to distinguish proportional situations from non-proportional situations.

Moreover, approximately 50% of pre-service middle school teachers were not able to explain the meaning of the equivalence signs in proportions in MCK 7 and MCK 8. Similarly, more than 50% of the pre-service secondary school mathematics teachers answered the item MCK7 and more than 35% of them answered the MCK8 incorrectly. In addition, only 5% of pre-service middle school teachers were able to explain proportional relations in an appropriate way in MCK9, while 65% of them explained such relations just applying the cross-multiplication procedure. By the same token, 74% of the pre-service secondary mathematics teachers had extensive reliance on cross-multiplication procedure to show the equality of the two proportions, while 22% of them were not able to provide any justification. This suggested that in total 96% of the pre-service secondary school mathematics teachers were not able to explain the reasons underlying the equality in a proportion. Results altogether showed that many of the pre-service mathematics teachers were not able to explain the proportional relations: Had they been proportional reasoners they would have explained that both expression in the two parts of the equation in MCK 7- 8-9 represented the multiplicative relationship between the time and the corresponding distance. Also, they would have been able to state that for the velocity to be the same in the original situation and the second situation, the multiplicative relationship between the times and the distances in two situations would have to be the same. Previously, reseachers emphasized that proportional reasoners distinguish proportional from non-proportional situations and not blindly apply an algorithm if the situation does not involve proportional relationships and also to explain their thinking in proportional situation (Lamon, 2012). Therefore, aligning with Johnson (2013), these results point that both pre-service middle school and secondary school mathematics teachers' proportional reasoning need further development using different problem contexts (Livy & Herbert, 2013). Additionaly, although these items were not missing value problems, results alling with earlier research such that both preservice and inservice teachers favor cross-multiplication rules in missing value problems (Ekawati *et al.*, 2014; Lobato *et al.*, 2011; Valverde & Castro, 2012). Regarding Factor 2, number structures, in the items MCK1-2-4 and 11, both pre-service middle school and secondary school mathematics teachers' results showed high correct response percentages. This suggested that pre-service mathematics teachers were able to solve different number structure problems within different situations by using just procedural knowledge. These results alligned with previous research such that both pre-service (Çıkla, O. A., & Duatepe, A., 2002) and in-service (Ekawati *et al.*, 2015) mathematics teachers might solve different number structure problems.

On the other hand, previous research has shown that both primary teachers (Ekawati *et al.*, 2014) and students (Kaput & West, 1994) have found to reason on especially stretcher and shrinker situations as difficult. Similar to the previous research results (Ekawati *et al.*, 2014; Kaput & West, 1994), data in this study from the items, MCK3, MCK10 and MCK12, related to the figural representation of ratio concept also showed that pre-service middle school mathematics teachers' correct response percentages were less than half. These results suggested the following: First of all, pre-service mathematics teachers were not able to connect the concepts of ratio and congruency and similarity in geometrical figures. Pre-service teachers' mathematical knowledge on ratio and propotion (Lamon, 2007) including their awareness of the interconnections between the ratio concepts and others (Ekawati, Lin, & Yang, 2014) is important. Secondly, pre-service teachers were not able to completely apply proportional reasoning in figural representation. Lastly, they were not able to recognize the multiplicative relationship between x and y in ratio depicted in graphical representation in the Coordinate Plane. Particular to MCK12, additionaly, results suggested that pre-service mathematics teachers were not able to differentiate the concept of ratio from the concept of linearity: Had they been able to differentiate these concepts, they would all have known that in the figural form, a line, not always represent the mutliplicative realitionship between two variables. Similarly, they would have known that linearity corresponds to the relationship between the two variables such that each dependent variable is equal to the some multiple of each independent variable with the addition of a constant value.

7.2. Results from MPCK-T Instrument

Results of MPCK-T was related to mathematics pedagogical content knowledge of pre-service mathematics teachers in general. Results from the explanatory factor analysis pointed to three factors: F4 (Knowledge of student strategies), F5 (Recognition and evaluation of student reasoning) and F6 (Knowledge of student misconception).

Regarding F4, the items MPCK2-5-6-8 and MPCK9, were related to the knowledge of student strategies previous research pointed to: the building-up strategy, the abbreviated build-up strategy, the unit factor approach, within strategy, between strategy and cross-multiplication strategy (Lamon, 1993; Kaput & West, 1994; Heinz, 2000; Karagöz Akar, 2007). Specifically in the item MPCK2, pre-service teachers could have used all of these strategies. Similarly, items MPCK5 and MPCK6 included pre-service teachers' reasoning on the unit factor strategy, abbreviated build-up and cross-multiplication strategy.

Results from MPCK2 showed that 77% of pre-service middle school and 78% of pre-service secondary school mathematics teachers were able to write at most two correct strategies to solve a ratio comparison problem. Moreover, in the answers pre-service mathematics teachers have considered the comparison of part-part and part-whole ratios as different strategies. Therefore, as previous research indicated, in this study pre-service mathematics teachers showed limited flexibility regarding the ability to use multiple solution methods for a set of problems or the ability to solve the same problem using multiple methods (Berk, Taber, Gorowara, & Poetzl, 2009; Ekawati *et al.*, 2015). Particularly, in the items MPCK 5 and MPCK6, a relatively high percentage, 75%, of the pre-service middle school and 63% of the pre-service secondary school mathematics teachers selected the unit factor strategy as the easiest strategy, while the remaining claimed cross-multiplication and abbreviated build-up strategies. Though, the high percentage, 80% of pre-service middle school and 78% of the pre-service secondary school mathematics teachers taking partial points in MPCK6 showed that they were not able to explain the reasoning behind why the unit factor strategy would be easy for students to understand and use by pointing to the factors such as the prior knowledge of students and the different levels of the concept of ratio. This showed that although pre-service middle school and secondary school mathematics teachers were able to recognize the importance and also the simplicity of the unit factor strategy, they were not able to consider different strategies and also student levels simultaneously. Similarly, in the item MPCK 8, 70% of the pre-service middle school and 75% of the pre-service secondary school mathematics teachers were not able to consider between ratio strategy when they were solving the ratio comparison problem. Previous research showed that pre-service mathematics teachers with an understanding of within-state ratios may not have the meaning for the between-state ratios (Karagöz Akar, 2007; 2010, 2011, 2015) and also they do not recognize the relationships between quantities neither in within-state nor in between-state ratios (Valverde & Castro, 2012). Therefore, results from F4, the knowledge of student strategies, pointed that pre-service mathematics teachers were not knowlegable about different strategies students might utilize while solving ratio and proportion problems. Even if they knew few strategies, they were not able to provide reasons as to why students might possibly use them.

Results from the items MPCK1-7-10 and 11 which were related to the recognition and evaluation of student reasoning pointed to high correct percentages. However when the partial and incorrect answers were examined, results pointed to the following: First, in MPCK1, 19% of the pre-service middle school and 25% of the pre-service secondary school mathematics teachers were not able to consider student knowledge according to student's grade level in oder to design an appropriate problem. Similarly, in MPCK7, 28% of the pre-service middle school teachers and 27% of the pre-service secondary school mathematics teachers were not able to give an appropriate feedback based on the incorrect addition strategy. More specifically, results pointed to three main categories regarding pre-service middle school and secondary school mathematics teachers' explanations on providing feedback to the student: explaining the correct answer for the given problem, asserting that the additive relation is true, asserting that keeping the area of the rectangle, the shape of the figure would be kept the same. This showed that more than one fourth of the pre-service mathematics teachers were not able to focus on the fact that the student's wrong answer stemmed from his/her using the incorrect addition strategy. This suggested that pre-servive teachers did not realize that the use of incorrect addition strategy showed that the student couldn't focus on the meaning of the ratio as a multiplicative relationship. Therefore, almost one third of the pre-service mathematics teachers were not aware of the fact that the student was using additive reasoning in proportional situations. This even further might suggest that these pre-service mathematics teachers might have had the same reasoning such that they were not aware of the proportional situations. Previous research on in-service teachers' MPCK on ratio and proportion also has shown that giving feedback and evaluating and analysing student solutions were considered as difficult for some in-service mathematics teachers (Ekawati *et al.*, 2015).

Finally, results from items MPCK3 and MPCK4, designed to assess the knowledge of student misconceptions showed the highest incorrect percentages for both pre-service middle school and secondary school mathemartics teachers. Particularly, rather than considering the multiplicative relationship in the quantities and pointing to the incorrect addition of different units, pre-service mathematics teachers expressed that the misreading and misunderstanding of the given question referred to the misconception. Besides, in MPCK4, 85% of pre-service secondary school mathematics teachers and 84% of pre-service middle school mathematics teachers were not able to ask a question to raise the awareness of the misconceptions on the student's part. Similarly, when the feedbacks were examined they either have written a different problem including ratio and proportion or explained the given question to the student again. These results might have suggested to the following: First, pre-service mathematics teachers were not able to recognize the misconceptions related to ratio and proportion regarding the incorrect addition strategy. Second, they had the lack of knowledge about giving or designing appropriate feedback in general. Third, they might have had the same misconception. Johnson (2013) asserted that pre-service teachers have some misconceptions about the concepts of ratio and fraction. Additionally, previous research pointed that although the students' errors came from conceptual aspects of ratio, majority of pre-service teachers identified the errors as stemming from procedural aspects of ratio (Ji-Won Son, 2013).

8. LIMITATIONS AND IMPLICATIONS FOR FURTHER RESEARCH

In this section, the limitations of the study and the suggestions for further research will be presented. First of all, in this study MCK instrument was translated toTurkish and the factors of MCK (Ekawati *et al.*, 2015) were fitted with the data gathered from Turkish pre-service mathematics teachers by using confirmatory factor analysis (CFA). These results suggest that the MCK instrument might be interpreted in, adjusted or tranlated to different languages and might be used in different countries in order to assess what pre-service and inservice mathematics teachers know on ratio and proportion. This might further shed light on what pre-service and inservice teachers know and do not know, what diffifulties they have about ratio and proportion.

On the other hand, the factors which has been defined by exploratory factor analysis (EFA) on MPCK-T pointed to different factors than MPCK (Ekawati et. al., 2015). I propose doing further research on the MPCK in different countries or with different populations to further determine the patterns in teachers' pedagogical content knowledge.

Considering the items and the related answers in the instruments, results suggested to the following: First, there were six proportional and non-proportional situations in the MCK instrument. Moreover, the results showed that only 32% of pre-service mathematics teachers were able to distinguish all proportional situations from non-proportional situations. Also, the proportional situations included only the well-chunked and associated set problems. Therefore, further research might be done including proportional and non-proportional situations in different contexts with different numbers of problems. These results also suggest the need to focus on differentiating proportional and non-proportional situations in teaching methods and content courses during preparation of teachers. Similarly, aligning with previous research (Berk,Taber, Gorowara, & Poetzl, 2009; Ekawati *et al.*, 2015), this study has shown that pre-service teachers were able to use few strategies which students possible use in solving problems related to the concept of ratio. This suggests that pre-service teachers need to become knowledgeable about different strategies. Specially, aligning with earlier research (Karagoz Akar, 2011; 2015, Valverde & Castro, 2012), results pointed that pre-service teachers were not aware of especially the between ratios as appropriate for the representation of the quantities in proportional situations. Furthermore, results have shown that pre-service teachers were not able to determine the concept of ratio in graphical representation. This suggested that they might not have been aware of the fact that ratio is the representation of the multiplicative relationship between the quantities in proportional situaions. This also suggested to their inability to differentiate linearity from the concept of ratio. Therefore, teacher education courses need to focus on pre-service mathematics teachers' awareness on the different student strategies as well as their knowledge of student and content in the context of ratio and proportion and linearity. Similarly pre-service teachers' awareness and development of the different concepts of ratio and different student strategies might be investigated rather than focusing solely on what they do not know about strategies.

Lastly, the Cronbach Alpha values of the intruments were calculated as 0.470 and 0.563 for MCK-T and MPCK- T respectively, which were considered as lower than 0.7 as an acceptable value. Although, the values bigger than 0.5 could be considered as moderately reliable (Brownlow, C., McMurray, I., & Cozens, B., 2004), the reasons behind the low reliability values might have stemmed from the simplicity of some group of items like MCK1-2-4 and 11. Besides, the instruments could be considered as short scales to get high reliability values. Therefore, although results of this study have provided valuable information on pre-service teachers' content knowledge and pegagogical content knowledge on ratio and proportion, results points to the following: Considering some of the questions and factors in Ekawati *et al.* (2015) and including further questions, developing instruments and measuring preservice and inservice teachers' content knowledge and pedagogical content knowledge on ratio a proportion is needed.

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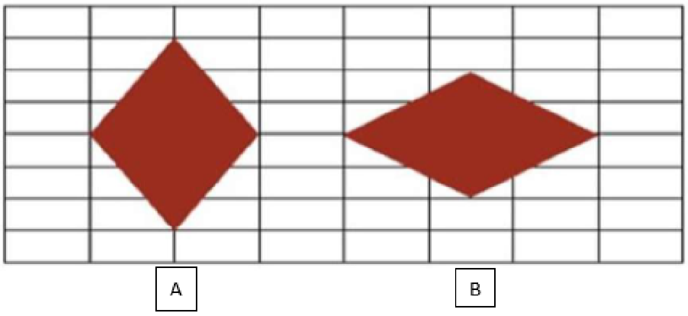
APPENDIX A: ORIGINAL MCK

A.1. Mathematics Content Knowledge Scale

1. A machine uses 2.4 liters of fuel for every 30 hours of operation. How many liters of fuel will the machine use in 100 hours if it continues to use fuel at the same rate?

   a) 7.2 b) 8.0 c) 8.4 d) 9. 6

2. There are two rhombuses figures on the grid paper with size 3cm x 2cm below.



   Are A and B congruent?

   a. Yes, A and B are congruent
   b. No, A and B aren't congruent

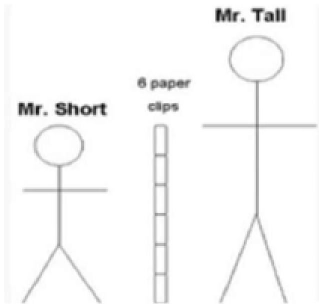
3. Explain the reason of your chosen answer above! (in 2nd question)

Figure A.1. Mathematics Content Knowledge Scale 1.

4. You can see the height of Mr. Short measured with paper clips is 6 paper clips. Mr. Short has a friend Mr. Tall. When we measure their heights with matchsticks:

   Mr. Short's height is four matchsticks,
   Mr. Tall's height is six matchsticks.

How many paperclips are needed to measure Mr. Tall's height? Explain your answer!



5. Match the situational problems on the left side with corresponding situations on the right side by drawing lines!

A) Robot Lia and Robot Matt did running at the same speed. Lia started before Matt. When Robot Lia had run for 4 minutes, Robot Matt had run 2 minutes. How long had Robot Matt run when Robot Lia had run 12 minutes?	Non-Proportional
B) 4 tents can house 12 campers. How many tents are needed for 30 campers?	Proportional
C) When Dina is 1 yearold, Budi is 2 years old. What will be Budi's age when Dina's age is 10 years old?	
D) A recipe of onion soup for 6 people such as 5 onions, 4 gingers and 8 chicken broth. How many gingers do we need to make soup for 4 people?	
E) A video store charges TRY25 per month for unlimited rentals. Sally rented 5 videos last month. This month she rented 6 videos. How much Sally paid for 6 videos she rented?	
F) Heny bought 3 candies for TRY24. How much should Henny pay if she wants to buy 4 more for her brother?	

Figure A.2. Mathematics Content Knowledge Scale 2.

6. Given s_1= distance 1, s_2= distance 2, t_1= time 1, t_2= time 2

What does it mean by $\frac{s_1}{t_1}$?

7. Given statement $p \rightarrow \frac{s_1}{t_1} = \frac{s_2}{t_2}$,explain the meaning of sign "=" in the statement p!

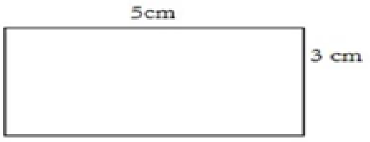
8. Given statement $q \rightarrow \frac{s_1}{s_2} = \frac{t_1}{t_2}$,explain the meaning of sign "=" in the statement q!

9. Explain why if the statement p correct,then the statement q is also correct!

10. Picture B is the enlargement of picture A. Draw the missing vertical line in picture B so that it keeps the same.

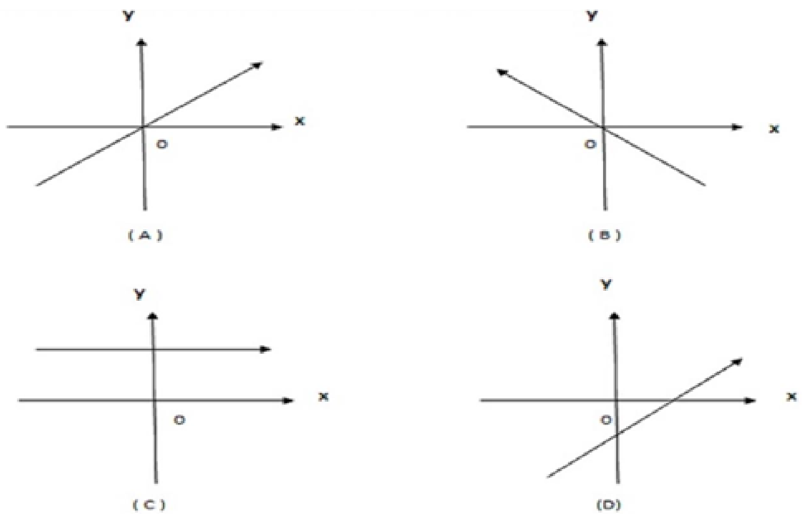
Figure A.3. Mathematics Content Knowledge Scale 3.

11. Below is the figure of rectangle with size 5 cm length and 3 cm width.



Dina wants to enlarge the above rectangle so that the new base becomes 12 cm long. What will be the new width? Explain how you attain the answer?

12.



Which of the graphic representation above would present the ratio relationship of x and y?

a) A b) B c) C d) D

Figure A.4. Mathematics Content Knowledge Scale 4.

APPENDIX B: ORIGINAL MPCK

B.1. Mathematics Pedagogical Content Knowledge Scale

Mathematics Pedagogical Content Knowledge Scale

A machine uses 2.4 liters of fuel for every 30 hours of operation. How many liters of fuel will the machine use in 100 hours if it continues to use fuel at the same rate?

1. Create a different problem of the same type as the problem above (same processes/operations) that is EASIER for <primary> students to solve.

2. The following problem was given to 6th grade students "Some children are making two glasses of sweetened water. Henny uses 3 sugar cubes and 7 glasses of water. Ria uses 5 sugar cubes and 9 glass of water. Which one is sweeter?

   Some teachers have identified five different students' solutions and reasoning methods to come to the conclusion for that problem.According to you, list all the students' solutions and reasoning method! (Note: Based on you, you may write as many as possible)

1.

2.

3.

4.

5.

3. Mr. Dodi, (as teacher) gave the problem in a(problem of Mr. Tall and Mr. Short) above to students. One of his students, Ina, answered and shared strategy as follow

   4+2 =6matchsticks

   Analyze the strategy that Ina show and interpret all Ina's misconceptions based her solution? (Hint: It may be more than 1)

Figure B.1. Mathematics Pedagogical Content Knowledge Scale 1.

4. Design a question for Ina so that she awares of her misconception!

5. There is a problem in a textbook as follow:
"Indah and Gana want to paint wall together. They want to use each exactly the same color. Indah uses 3 cans of yellow paint and 6 cans of red paints. If Gana uses 5 cans of yellow paint, how much red paint does Gana needs?" If there are three teaching strategies in solving the problem above described as below

Indah uses 3 cans of yellow paint and 6 cans of red paint.

Therefore, if she uses 1 can of yellow paint, so there will be 2 cans of red paint.

If Gana wants to make the same color and uses 5 yellow paint, so that the red paint will be 5cans x2 = 10 cans of red paint.

A

Indah uses 3 cans of yellow paint and 6 cans of red paint.

To find the number of the red cans with 5 yellow cans of Gana

(x5) $(x\frac{1}{3})$

3 yellow → 15 yellow → 5 yellow (Gana)

(x5) $(x\frac{1}{3})$

5red → 30 red → Gana's red cans will be 10.

B

Indah uses 3 cans of yellow paint and 6 cans of red paint. If Gana has 5 yellow paints

Indah	3 yellow	6 red
Gana	5 yellow	x? red

(cross multiplication) 3x=30, x=30/3 = 10

C

Among A, B and C teaching strategies, which strategy will you choose for students' **better understanding and reasoning**?

a) A b) B c) C

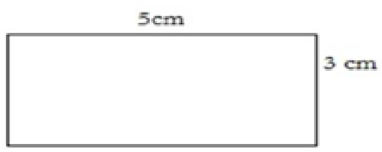
Figure B.2. Mathematics Pedagogical Content Knowledge Scale 2.

6. In teaching ratio and proportion, Mr Indra shared an example of problem to students to lead them to understand it as follow.:
 "A company can produce in avarage 21 birdcages in 3 hours. How many birdcages that can be produced in 5 hours?"
 She prefers to begin the teaching by using concept 'every one'(finding the one unit of 'hour')
 Given two reasons she could have for preferring to do this rather than simply teaching the children use cross multiplication strategy?

 Reason 1:

 Reason 2:

The rectangle given below is 5 cm length and 3 cm width



Dina wants to enlarge the rectangle in a way that the larger side (length) becomes 12 cm. So what will the new width be? Explain.

7. One of the students solve the problem and concentrate on the difference 12 and 5. The difference is 7, so the new width is 7 more than 3, the answer is 10. How do you give feedback on this students' solution strategy?

Figure B.3. Mathematics Pedagogical Content Knowledge Scale 3.

8. Students are asked to solved the problem below as homework:
Luis mixed 5 ounces of orange juice concentrate with 7 ounces of water to make orange juice. Martin mixed 3 ounces of the same orange juice concentrate with 5 ounces of water. Who made the drink with the stronger orange flavor?
Students Amin, Brandon and Charlie shared different strategies to come to the result. Each student compares two ratios by stating different number arrangements.

Amir	Brandon	Charlie
$\frac{5}{7};\frac{3}{5}$	$\frac{5}{3};\frac{7}{5}$	$\frac{5}{5};\frac{3}{7}$

What do you think about the different ratio arrangements above? You may check more than one box.

A. Amin's ratio number arrangement is correct. ☐
B. Brandon's ratio number arrangement is correct ☐
C. Charlie's ratio number arrangement is correct. ☐

9. Analyze Amin's and Charlie's different number arrangements, write your analysis below!

10. A <6 grade> teacher asks her students to solve the following four story problems, in any way they like, including using materials if they wish.

a. The teacher notices that two of the problem are more difficult for her students than the other two.Identify the TWO problems which are likely to be more DIFFICULT to solve for <6th grade> students.

Problem . . . . and Problem . . . .

Figure B.4. Mathematics Pedagogical Content Knowledge Scale 4.

Problem 1: To make chicken soup for 8 people need some ingredient such as 2 glasses of water, 6 onions and 4 chicken cubes. How many chicken cubes needed for 4 people soup?

Problem 2 : To make chicken soup for 8 people need some ingredient such as 2 glasses of water, 6 onions and 4 chicken cubes. How many chicken cubes needed for 6 people soup?

Problem 3 : What is the size of the missing vertical line enlargement below!

Problem 4 : What is the size of the missing vertical line enlargement below!

11. Explain why those two tasks you choose above are more difficult than others for your sixth grade students.

Figure B.5. Mathematics Pedagogical Content Knowledge Scale 5.

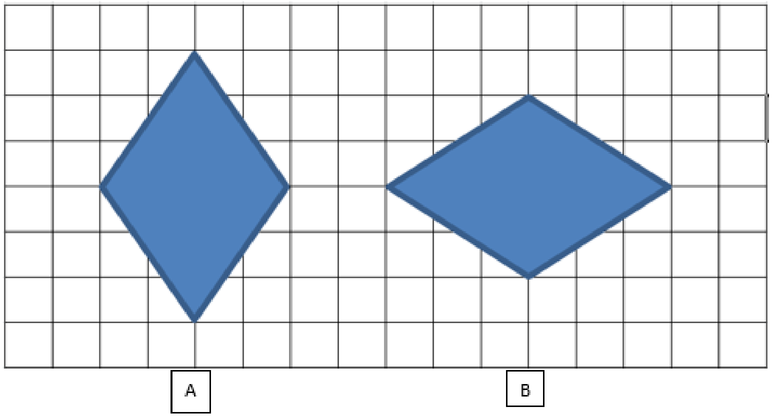
APPENDIX C: MCK-T

C.1. Matematik Bilgisi Ölçeği

1. Bir makina çalıştığı her 30 saat için 2,4 litre yakıt kullanıyor. Eğer makine aynı oranda yakıt kullanmaya devam ederse 100 saatte kaç litre yakıt kullanacaktır?

a)7,2 b) 8,0 c) 8,4 d)9,6

2. Bir birimlik karelerden oluşan bir kağıt üzerinde iki adet eşkenardörtgen figürü vardır.



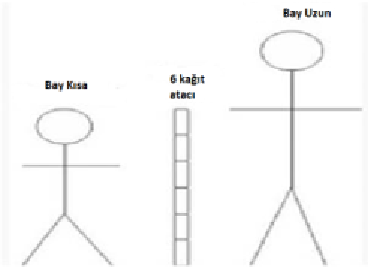
A ve B eş midir?

c. Evet, A ve B eştir.
d. Hayır, Ave B eş değildir.

3. Yukarıda seçtiğiniz cevabın nedenini açıklayınız.

Figure C.1. Matematik Bilgisi Ölçeği 1.

4. Kağıt atacı ile ölçülen Bay Kısa'nın boyunun 6 kağıt atacı olduğu görülmektedir. Bay Kısa'nın Bay Uzun isminde bir arkadaşı vardır. İkisinin boylarını kibrit çöpleri ile ölçtüğümüzde;
Bay Kısa'nın boyu 4 kibrit çöpü,
Bay Uzun'un boyu 6 kibrit çöpüdür.
Bay Uzun'un boyunu ölçmek için kaç tane kağıt atacına ihtiyaç vardır? Cevabınızı açıklayınız.



5. Sol tarafta bulunan problem durumlarını onlara karşılık gelen sağ taraftaki durumlarla çizgiler çizerek eşleştiriniz.

A) Robot Alya ve Robot Mehmet aynı hızda koşuyorlardı. Alya Mehmet'ten önce başladı. Robot Alya 4 dakika koştuğunda, Robot Mehmet 2 dakika koşmuştu. Robot Alya 12 dakika koştuğunda Robot Mehmet kaç dakika koşmuş olacaktır?

B) 4 çadırda 12 kampçı kalabiliyor. 30 kampçı için kaç çadıra ihtiyaç vardır?

C)Derya 1 yaşındayken, Banu 2 yaşındadır. Derya 10 yaşına geldiğinde Banu'nun yaşı kaç olacaktır?

D) 6 kişilik soğan çorbası tarifinde 5 adet soğan, 4 adet zencefil ve 8 tavuk suyu tableti vardır. 4 kişilik çorba yapmak için kaç adet zencefile ihtiyaç vardır?

E) Bir video mağazası, sınırsız kiralama için aylık 25TL ücret almaktadır. Serap geçen ay 5 video kiralamıştır. Bu ay 6 adet video kiralamıştır. Serap kiraladığı 6 adet video için ne kadar ücret ödemiştir?

F) Hülya 24 TL karşılığında 3 şeker satın almıştır. Eğer erkek kardeşine 4 tane daha şeker almak isterse ne kadar ödemelidir?

Orantısız Durum

Orantılı Durum

Figure C.2. Matematik Bilgisi Ölçeği 2.

s_1= uzaklık 1, s_2= uzaklık 2, t_1= zaman 1, t_2= zaman 2 olarak verilmiştir.
Aşağıdaki 6., 7., 8. ve 9. soruları bu bilgilere göre cevaplayınız.

6. $\frac{s_1}{t_1}$ 'in anlamı nedir? Açıklayınız.

7. $p \rightarrow \frac{s_1}{t_1} = \frac{s_2}{t_2}$ ifadesi veriliyor. "p" ifadesindeki "=" işaretinin anlamını açıklayınız.

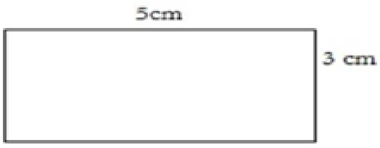
8. $q \rightarrow \frac{s_1}{s_2} = \frac{t_1}{t_2}$ ifadesi veriliyor. "q" ifadesindeki "=" işaretinin anlamını açıklayınız.

9. "p" ifadesi doğru olduğunda "q" ifadesinin de doğru olmasının sebebini açıklayınız.

10. B şekli A şeklinin büyütülmüş halidir. Şeklin yapısının korunması için B şeklindeki eksik dikey parçaları çiziniz. (Şekildeki boşluğu dikkate alınız)

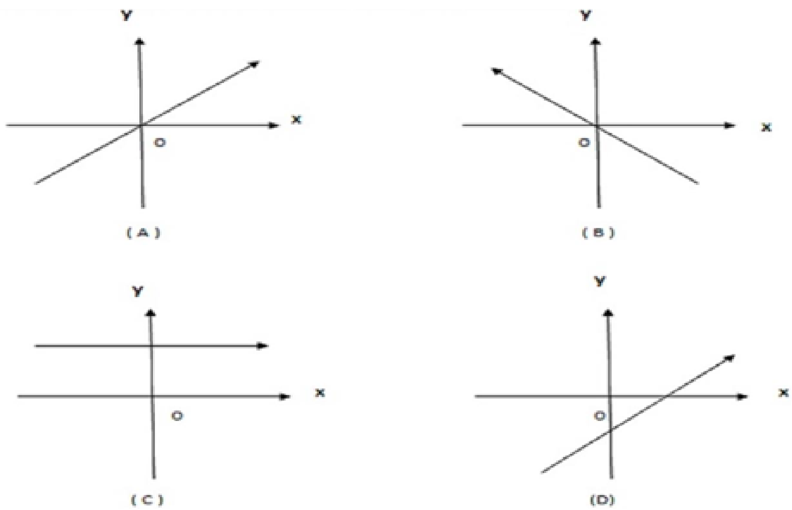
Figure C.3. Matematik Bilgisi Ölçeği 3.

11. Aşağıdaki dikdörtgen 5 cm uzunluğunda ve 3 cm genişliğindedir.



Derya yukarıdaki dikdörtgeni, şeklin yapısını koruyarak, uzun kenarı 12 cm olacak şekilde büyütmek istemektedir. Yeni kısa kenar kaç cm olacaktır? Cevabınıza nasıl vardığınızı açıklayınız.

12.



Yukarıdaki grafiklerden hangisi (veya hangileri) x ve y arasında oran ilişkisi belirtir?

a) A b) B c) C d) D

Figure C.4. Matematik Bilgisi Ölçeği 4.

APPENDIX D: MPCK-T

D.1. Matematik Öğretme Bilgisi Ölçeği

"Bir makine çalıştığı her 30 saat için 2,4 litre yakıt kullanmaktadır. Eğer makine aynı oranda yakıt kullanmaya devam ederse 100 saatte kaç litre yakıt kullanacaktır?"

a)7,2 b) 8,0 c) 8,4 d)9,6

1. Yukarıdaki problem ile aynı türde olan (aynı süreçler/işlemler) ve ilkokul öğrencilerinin daha kolay çözebileceği farklı bir problem tasarlayınız.

2. 6. Sınıf öğrencilerine aşağıdaki problem verilmiştir.
   "Öğrenciler iki bardak şekerli su hazırlamaktadır. Hülya 3 küp şeker ve 7 bardak su kullanmaktadır. Rabia 5 küp şeker ve 9 bardak su kullanmaktadır. Hangisininki daha tatlı olur?"

Bazı öğretmenler öğrencilerin bu problemde bir sonuca varmak için kullandıkları birbirinden farklı, 5 tane doğru çözüm ve düşünme yöntemi belirlemişlerdir. Bu öğrenci çözümlerinin ve düşünme yöntemlerinin neler olabileceğini kendi düşüncenize göre listeleyiniz. (Not: İstediğiniz kadar yazabilirsiniz.)

1.
2.
3.
4.
5.

Figure D.1. Matematik Öğretme Bilgisi Ölçeği 1 .

3. Doruk Öğretmen, yukarıdaki Bay Uzun ve Bay Kısa ile ilgili olan problemi öğrencilerine vermiştir. Öğrencilerinden İnci'nin probleme cevabı aşağıdaki gibidir:

   4+2=6 kibrit çöpü

   Buna göre İnci'nin çözüm yolunu analiz ederek (olası) kavram yanılgılarını yorumlayınız. (İpucu: 1'den fazla olabilir)

4. İnci'ye (olası) kavram yanılgılarını fark edebileceği bir soru sorunuz.

Figure D.2. Matematik Öğretme Bilgisi Ölçeği 2.

5. Ders kitabında aşağıdaki gibi bir problem vardır:
"Ömer ve Gamze bir duvarı beraber boyamak istiyorlar. İkisi de tam olarak aynı rengi oluşturmak istiyor. Ömer 3 kutu sarı boya ve 6 kutu kırmızı boya kullanıyor. Eğer Gamze 5 kutu sarı boya kullanırsa, Gamze ne kadar kırmızı boyaya ihtiyaç duyar?"
Eğer yukarıdaki problemin çözümü için aşağıda ifade edildiği gibi üç öğretim stratejisi varsa,

Ömer 3 kutu sarı boya ve 6 kutu kırmızı boya kullanıyor.

Buna göre, eğer Ömer 1 kutu sarı boya kullanırsa, demek ki 2 kutu kırmızı boya olacaktır.

Eğer Gamze aynı rengi oluşturmak isterse ve 5 sarı boya kullanırsa, demek ki 5x2=10 kutu kırmızı boya olacaktır.

A

Ömer 3 kutu sarı boya ve 6 kutu kırmızı boya kullanıyor.

Gamze'nin 5 sarı kutu ile kullanacağı kırmızı kutu sayısını bulmak için;

x5 x$\frac{1}{3}$

3 sarı → 15 sarı → 5 sarı (Gamze)

x5 x$\frac{1}{3}$

6 kırmızı → 30 kırmızı → Gamze'nin kırmızı kutuları 10 olacak.

B

Ömer 3 kutu sarı boya ve 6 kutu kırmızı boya kullanıyor. Eğer Gamze'nin 5 sarı boyası varsa

Ömer	3 sarı	6 kırmızı
Gamze	5 sarı	x? kırmızı

(Çapraz çarpım) 3x=30, x=30/3 =10

C

A, B ve C öğretim stratejileri arasından, öğrencilerin daha iyi anlama ve akıl yürütmesi için hangi stratejiyi seçerdiniz?

a) A b) B c) C

Figure D.3. Matematik Öğretme Bilgisi Ölçeği 3.

6. Oran ve orantıyı öğretirken, Bay İdris öğrencileri ile aşağıdaki problemi paylaşmıştır:
   "Bir fabrika 3 saatte ortalama 21 kuş kafesi üretebiliyor. 5 saatte kaç kuş kafesi üretilebilir?"
   Bay İdris öğretmeye 'Birim' kavramını kullanarak başlamayı tercih eder.
   (birim saat başına düşen miktarı bularak)

   Bay İdris'in, çocuklara basitçe çapraz çarpma stratejisini kullanmayı öğretmek yerine, bunu yapmayı tercih etmesinin iki sebebini belirtiniz.

   Sebep1:

   Sebep2:

Aşağıdaki dikdörtgen 5 cm uzunluğunda ve 3 cm genişliğindedir.

Derya yukarıdaki dikdörtgeni, şeklin yapısını koruyarak, uzun kenarı 12 cm olacak şekilde büyütmek istemektedir. Yeni kısa kenar ne olacaktır? Bu sonuca nasıl vardığınızı açıklayınız.

7. Öğrencilerden biri, 12 ile 5 arasındaki farka odaklanarak yukarıdaki problemi çözer.
   "Fark 7'dir, bu yüzden de yeni kısa kenar 3'ten 7 fazladır, cevap 10dur."
   Bu öğrencinin çözüm stratejisine nasıl dönüt verirsiniz?

Figure D.4. Matematik Öğretme Bilgisi Ölçeği 4.

8. Öğrencilere aşağıdaki problem ödev olarak sorulmuştur:

Leyla portakal suyu yapmak için 5 birim portakal suyu konsantresi ile 7 birim suyu karıştırıyor. Murat 3 birim aynı portakal suyu konsantresi ile 5 birim suyu karıştırmıştır. Daha yoğun portakal tadına sahip içeceği kim yapmıştır?

Öğrencilerden Emir, Burak ve Ceylin sonuca varmak için farklı çözüm stratejilerini paylaşmışlardır. Her bir öğrenci iki oranı farklı sayı düzenlemelerini kullanarak karşılaştırmıştır.

Emir	Burak	Ceylin
$\frac{5}{7}; \frac{3}{5}$	$\frac{5}{3}; \frac{7}{5}$	$\frac{5}{12}; \frac{3}{8}$

Yukarıdaki farklı oran düzenlemeleri ile ilgili ne düşünüyorsunuz? Birden fazla kutucuğu işaretleyebilirsiniz

D. Emir'in oran düzenlemesi doğrudur. ☐
E. Burak'ın oran düzenlemesi doğrudur. ☐
F. Ceylin'in oran düzenlemesi doğrudur. ☐

9. Emir ve Ceylin'in farklı sayı düzenlemelerini analiz ediniz. Analizinizi aşağıya yazınız.

10. Bir 6. sınıf öğretmeni, öğrencilerinden aşağıdaki dört problemi istedikleri yoldan, diledikleri materyalleri kullanarak çözmelerini istemiştir.

a. Öğretmen problemlerden ikisinin, öğrencileri için diğer ikisinden daha zor olduğunu fark etmiştir. 6. sınıf öğrencileri için çözümü daha zor olan bu iki problemi belirleyiniz.

Problem . . . . ve Problem . . . .

Figure D.5. Matematik Öğretme Bilgisi Ölçeği 5.

Problem 1: 8 kişilik tavuk çorbası yapmak için gerekenler; 2 bardak su, 6 soğan ve 4 tavuk suyu tabletidir. 4 kişilik çorba için kaç tane tavuk suyu tableti gerekir?

Problem 2 : 8 kişilik tavuk çorbası yapmak için gerekenler; 2 bardak su, 6 soğan ve 4 tavuk suyu tabletidir. 6 kişilik çorba için kaç tane tavuk suyu tableti gerekir?

Problem 3 : Aşağıdaki I. şekil büyütüldüğünde oluşan II. şekildeki eksik dikey parçanın uzunluğu ne kadardır?

Problem 4 : Aşağıdaki I. Şekil büyütüldüğünde oluşan II. Şekildeki eksik dikey parçanın uzunluğu ne kadardır?

11.Yukarıda seçtiğiniz iki problemin, 6. sınıf öğrencileri için neden diğerlerinden daha zor olduğunu açıklayınız.

Figure D.6. Matematik Öğretme Bilgisi Ölçeği 6.

APPENDIX E: MCK-T

E.1. MCK-T RUBRIC

Item code	Responses	Response code	Score
MCK1	Incorrect make mistake in calculation the missing value (choose A,C,D)	70	0
	No answer	99	0
	Correct can correctly find the missing value within the context (choose B)	20	1
MCK2	Incorrect (choose non-congruent)	70	0
	Incorrect (choose both congruent and incongruent)	71	0
	No answer	99	0
	Correct (choose congruent)	20	1
MCK3	Incorrect. The two rhombus are congruent/incongruent due to:		
	The visual reason	70	0
	The area of two rhombus are the same	71	0
	The area of grid which cover the two rhombus are the same	72	0
	- No answer	99	0
	- Correct. The two rhombuses are congruent due to: The ratio of corresponding diagonals are equivalent	20	1
MCK4	Misplaced the number structure in the proportional so that make wrong calculation	70	0
	Do addition strategy and get wrong result	71	0
	No answer	99	0
	- Calculate and apply multiplicative - on proportional problem 6/p = 4/6 p = 9	20	1
	Make correct arrangement of number structure in proportional statement but miscalculate the answer	21	1
MCK5	Wrongly match the statement	70	0

Figure E.1. MCK-T Rubric 1.

	with proportion and non-proportional situation		
	No matching line	71	0
	Correctly match 1 situation to corresponding situations	10	0.17
	Correctly match 2 situation to corresponding situations	11	0.33
	Correctly match 3 situation to corresponding situations	12	0.5
	Correctly match 4 situation to corresponding situations	13	0.67
	Correctly match 5 situation to corresponding situations	14	0.83
	Correctly match 6 situation to corresponding situations	20	1
	No Answer	99	0
MCK6	Distance divided by time	70	0
	No answer	99	0
	The ratio that represents how long it takes to do something, such as traveling a certain distance	20	1
	Speed	21	1
MCK7	Equivalent sign means 'same'	70	0
	No answer	99	0
	Equivalent speed (speed 1 = speed 2)	20	1
MCK8	The ratio are proportional	70	0
	No answer	99	0
	The ratio of distance equivalence with the ratio of time	20	1
	The ratio of similar travelling distance travel will impact on the ratio of time	21	1
MCK9	If it is combined, the result will be same	70	0
	p and q have the same result	71	0

Figure E.2. MCK-T Rubric 2.

	No answer	99	0
	The same speed refers that the ratio of distance is equivalence to the ratio of time	10	0.5
	$\frac{s1}{t1} = \frac{s2}{t2} \rightarrow \frac{s1}{s2} = \frac{t1}{t2}$ s1 x t2 = s2 x t1 → s1xt2=s2 x t1	11	0.5
	Given $\frac{s1}{t1} = \frac{s2}{t2}$ and will show $\frac{s1}{s2} = \frac{t1}{t2}$ $\frac{s1}{t1}\ x\ t1 = \frac{s2}{t2} x t1$ (multiply both sides by t1) s1 =s2 x $\frac{t1}{t2}$ $\frac{1}{s2}\ x\ s1 = \frac{1}{s2}\ x\ s2\ x\ \frac{t1}{t2}$ (multiply both sides by $\frac{1}{s2}$) $\frac{s1}{s2} = \frac{t1}{t2}$	20	1
MCK 10	Missing (No drawing, perform calculation only)	70	0
	Incorrect (The position of drawing vertical line is not the same as figure A)	71	0
	Incorrect (The drawing scale is not twice as the vertical line of A)	72	0
	Missing	99	0
	Partially Correct (Drawing the vertical line with correct enlargement factor but not enlarge the gap between the vertical and horizontal line	10	0.5
	Correct (The drawing vertical line in B is twice as vertical line A and the gap between vertical and horizontal line is also enlarged with appropriate enlargement factor (shown by the separated dot or measure with ruler)	20	1

Figure E.3. MCK-T Rubric 3.

MCK11	Incorrect calculation of relatively new proportional line length	70	0
	Addition strategy and result incorrect new width	71	0
	Missing	99	0
	Can answer and calculate missing line length of rectangle figure 5/3 =12/x X= 7.2 cm	20	1
	Able to write the proportional statement correctly but miscalculation	21	1
MCK12	Incorrect Choose 4 representation (A, B,C,D)	70	0
	Incorrect Choose representation C and D	71	0
	Incorrect Choose representation C or D only	72	0
	Incorrect Choose representation (A,C,D) or (B,C,D)	73	0
	Not choose any representations	99	0
	Choose representation A only	10	0.5
	Choose representation B only	11	0.5
	Choose representations (A, B, C) or (A,B,D)	12	0.5
	Correct (Choose representation A and B)	99	1

Figure E.4. MCK-T Rubric 4.

APPENDIX F: MPCK-T

F.1. MPCK-T Rubric

Item code	Responses	Response code	Score
MPCK1	A different problem of the **same type** (same processes/operations) but is **NOT easier** to solve. (Note: Items judged to be of the **same** level of difficulty are NOT easier.) *Examples:* • *A machine uses 2 liters of fuel for every 30 hours of operation. How many liters of fuel will the machine use in 100 hours? (2 is not divisible by 3)* • *A tap drips 2 liters of water every day. How many ml is this per second? (the metric knowledge required and computationa l load is significantly higher)*	70	0
	Other incorrect (including crossed out, erased, stray marks, illegible, or off task) *Example:* • *Questions that are not meaningful/have no answer*	71	0
	No answer	99	0
	Correct A different problem of the **same type** (same processes/operations) but is **easier** to solve. *Example:* • *A machine uses 3 liters of fuel for every 30 hours of operation. How many liters of fuel will the machine use in 100 hours?* • *A car uses 2.4 liters of fuel for every 50 km. How many liters of fuel will the car use in 100 km?*	20	1
MPCK2	Incorrect - Students do - experiment - 3/7:3/9=3/7.9/3=9:7 - By addition 3+7=10 dan 3+9=12	70	0
	No answer	99	0
	Participants could answer 5 out of 6 possible	20	0.2-1

Figure F.1. MPCK-T Rubric 1.

	students' solution method. Each correct answer scored 0.2 C = compare 1 part (common sense) P = compare 2 part in each person U = compare 2 part in each unit W = compare part – whole S = compare reminder R = ratio on 1 glass sugar water		
MPCK3	Incorrect - Only stated that those - students strategies wrong - Directly interpret the - meaning of problem	70	0
	No answer	99	0

	Correct (each correct misconception score 0.5)
	- Students were not consider
	- the different ***unit***
	  - The difference between
	  - the stik and paperclip is same
	  - 1 clip is not the same
	  - as 1 paperclip
	  - The measurement of stik
	  - and paperclip are different
	  - so it can not be added Stik
	  - dan klip berbeda ukuran shg
	  - tidak bisa dijumlahkan
	- Students do not understand
	- ***multiplicative relation*** in ratio
	- and proportion
		20	0.5-1

MPCK4	Incorrect (i.e) - Design similar question - as the problem - Why you add 2?	70	0
	No answer	99	0

	Correct
	- Question about unit :
	  - See the measurement
	  - unit, how is the measure
	  - of stik and paper clip?Is
	  - that different?
	  - What is the length of 1 stick
	  - represent how many clip?
		20	1

MPCK5	Incorrect - write C only - write both B and C	70	0
	No answer	99	0
	A: write A only	20	1
	B: write B only	10	0.5
	AB: write both A and B	11	0.75
	AC: write both A and C	12	0.25
	ABC: write all	13	0.5

Figure F.2. MPCK-T Rubric 2.

MPCK6	Incorrect i.e ▪ Mr. Indra strategy is more ▪ contextual for students ▪ With cross multiplication, ▪ it will not known the number ▪ of unit cage. ▪ Unit method is not simple ▪ as cross multiplication	70	0
	No answer	99	0
	Correct Pre-knowledge About Division (Unit) ▪ By explored the unit is ▪ easier to find the number ▪ unit bird cage every hour. ▪ Unit strategy is easier for ▪ students regarding their pre ▪ understanding than cross ▪ multiplication. Algorithm isn't simple ▪ If students do not understand ▪ the concept, there is a risk ▪ in posit the proper number ▪ (consider the indirect proportion) ▪ The concept could not ▪ understood by students since ▪ the orientation of fast ▪ solution with cross ▪ multiplication.	20	0.5-1
MPCK7	Incorrect - Directly give the right - answer - Ask students to directly - use cross multiplication strategy	70	0
	No answer	99	0
	Correct (the answer could be one of the following) The feedback related to enlargement ratio - Students are asked about - the enlargement of the base - and then the height - Remind about the main point - of multiplicative relation - in proportion Ratio on the drawing - Asked students to draw - and analyze the diference - regarded the ratio. - By using the drawing, - students were explained about - the properties of two similar figures.	20	1

Figure F.3. MPCK-T Rubric 3.

MPCK8	No answer	99	0
	Choose A or B or C	10	0.33
	Choose AB, AC or BC	11	0.66
	Choose ABC	20	1
MPCK9	Incorrect - Charlie's strategy is - wrong since if we cross - multiplied, the result is not - the same as others - Charlie was using - indirect proportion	70	0
	No answer	99	0
	Correct - Charlie compares part of - whole and Amin - compare part by part	20	1
MPCK10	Incorrect Choose 1 or 4	99	0
	No answer	99	0
	Correct	20	
	Answer 2 or 3	10	0.5
	Answer both 2 and 3	11	1
MPCK11	Incorrect (i.e) - Reason: the problem is too difficult - The story problem was too - long so that students can - not answer	70	0
	No answer	99	0
	Correct (i.e) -The number structure is not integer multiple/not the factor	20	1

Figure F.4. MPCK-T Rubric 4.