

INVERSION FOR ELASTICITY TENSOR OF FOCAL REGION USING
MACHINE LEARNING ALGORITHMS

by

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ABSTRACT

INVERSION FOR ELASTICITY TENSOR OF FOCAL REGION USING MACHINE LEARNING ALGORITHMS

The moment tensor is an essential tool in seismology to examine the structure of seismic sources. The deformation at the focal region, using combinations of force couples arranged in a 3×3 matrix, is represented by the moment tensor. The moment tensor can be expressed as a linear combination of the eigenvectors of the anisotropic focal region's elasticity tensor. The eigenvalues of a vertically transversely isotropic (VTI) elasticity tensor from a occurring moment tensor of a focal region can be obtained, and the precision of this determination depends on the degree of anisotropy.

Machine learning optimization involves iteratively enhancing a machine learning model's precision by reducing the error level. Choosing an algorithm that can effectively sample the search space and identify optimal solutions is necessary to optimise a function. Many algorithms are available for function optimization, but it is crucial to set a baseline to determine the practicable solutions for a given problem.

This thesis defines a new objective function (misfit function). The function is proposed for obtaining the elastic parameters of an anisotropic focal region, and these parameters are calculated by using machine learning algorithms such as Grid Search, Random Search, Simulated Annealing, and Nelder-Mead.

ÖZET

MAKİNE ÖĞRENMESİ ALGORİTMALARI KULLANARAK DEPREM ODAK BÖLGESİNİN ESNEKLİK TENSÖRÜNÜN ÇÖZÜLMESİ

Moment tensörü, 3×3 matriste düzenlenmiş genelleştirilmiş kuvvet çiftlerine dayalı olarak kaynaktaki deformasyonu tanımlayan önemli bir araçtır. Bir moment tensörü, anizotropik odak bölgesinin esneklik tensörünün özvektörlerinin doğrusal bir kombinasyonu olarak ifade edilebilir. Bir odak bölgesinin belirli bir moment tensörü için dikey enine izotropik (VTI) esneklik tensörünün özdeğerleri, sabit bir faktör dışında elde edilebilir ve bu belirlemenin kesinliği, anizotropinin derecesine bağlıdır.

Makine öğrenimi optimizasyonu, hata düzeyini azaltarak bir makine öğrenimi modelinin kesinliğini yinelemeli olarak geliştirmeyi içerir ve bir fonksiyonu optimize etmek için, arama uzayını etkili bir şekilde örnekleyebilen ve optimal çözümleri tanımlayabilen bir algoritma seçmek oldukça önemlidir. Fonksiyon optimizasyonu için çok sayıda algoritma mevcuttur, ancak belirli bir problem için uygulanabilir çözümleri belirlemek için bir temel oluşturmak çok önemlidir.

Bu tez çalışmasında, anizotropik bir odak bölgesinin elastik parametrelerini elde etmek için yeni bir metod önerilmiş ve bu metod yoluyla parametreler makine öğrenmesi algoritmaları içerisinde yer alan Grid Search, Random Search, Simulated Annealing ve Nelder-Mead gibi optimizasyon yöntemleri kullanılarak hesaplanmıştır.

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LIST OF SYMBOLS

C^{iso}	Isotropic elasticity tensor
C_{ijkl}	Elasticity tensor
$\mathbf{d}$	Source vector
D_{ij}	Source tensor
$\hat{d}^{iso}$	Aproximate source vector
$\hat{D}$	Source tensor obtained from unit slip and unit normal vectors
$\hat{\mathbf{d}}$	Source vector obtained from unit slip and unit normal vectors
$\hat{\mathbf{D}}^{iso}$	Isotropic approximation of source tensor
$\hat{\mathbf{e}}$	Unit eigenvectors of the moment tensor
$\mathbf{m}$	Moment vector
M_{kl}	Moment tensor
$\mathbf{n}$	Normal vector
$\hat{\mathbf{n}}$	Slip normal vector
$\hat{\mathbf{n}}^{iso}$	Isotropic approximation of fault vector
$\mathbf{u}$	Slip vector
$\hat{\mathbf{u}}$	Unit normal vector
$\hat{\mathbf{u}}^{iso}$	Isotropic approximation of slip vector
ϵ_{kl}	Strain tensor
λ	Eigenvalue
λ_2^{iso}	Eigenvalue of the closest isotropic elasticity tensor
γ	Ratio of the stiffness
μ	Shear modulus
Σ	Eigenvector
Σ^{iso}	Eigenvector in deviatoric space
σ_{ij}	Stress tensor
$\otimes$	Tensor product (or dyadic product)

LIST OF ACRONYMS/ABBREVIATIONS

GS	Grid Search
ISO	Isotropic
NM	Nelder-Mead
RS	Random Search
SA	Simulated Annealing
TI	Transversely isotropic
VTI	Vertical transversely isotropic

1. INTRODUCTION

Data processing in seismology is, in general, applied under the assumption of isotropy. This supposition leads to reduced accuracy because the subsurface layer is actually anisotropic. Furthermore, the inclusion of anisotropy values in seismic data processing significantly impacts the resulting outputs. One of the main reasons for seismic anisotropy is that seismic wave velocity is in the direction of propagation. In other words, two parameters are used to represent isotropic source regions. These parameters that can be used to represent isotropic source regions include velocities V_p and V_s , Lamé parameters, or the two eigenvalues of the elasticity tensor. Although this situation simplifies certain inversion operations, it keeps our ability to observe the distinct typical structures of the seismic source regions. On the other hand, Transversely isotropic (TI) source location has five parameters. Using only two parameters to represent the focal region instead of five can conceal certain information about the physical characteristics of fractures.

The origin or cause of seismic activity is represented by the moment tensor. Furthermore, a moment tensor reports the deformation at the source that produces seismic waves. The source type plays an essential role in determining the type of moment tensor. For instance, shear sources exclusively display double-couple components, tensile sources have isotropic components, and volcanic sources have significant CLVD-components. In other words, the presence of anisotropic focal regions can also result in certain non-dc elements for shear sources [1]. Consequently, it may be more complex to determine whether these non-DC components arise from the source properties or the anisotropic focal region [2].

The orthotropic elasticity tensor parameters of the focal region were inverted by Vavrycuk (2004) [3]. Apart from determining the elasticity parameters, he also calculates three Euler angles to specify the direction of the orthotropic tensor. However, this thesis treats the orientation of the TI elasticity tensor as vertical. In order to

solve this non-linear problem, Vavrycuk (2004) [3] utilizes a group of moment tensors. Like the approach described in this thesis, Vavrycuk (2004) [3] also fixes two elastic parameters during the inversion process and focuses on considering the ratios of eigenvalues. He states that it's not possible to invert the entire group of tensor parameters because elastic parameters, slip, and fault area are interdependent. For this thesis, we are using a similar idea of inverting the ratios of eigenvalues. However, we distinguish ourselves by not treating slip and fault areas as distinct elements. Instead, we focus on potency as a singular variable. We introduce a method to inverse the parameters of anisotropic focal regions that varies substantially from the approaches outlined in Vavrycuk's(2004) [3] and Li et al.(2018) [4].

We introduce a misfit function for calculating the elastic parameters in an anisotropic focal region. We will make the assumption that the seismic source region is VTI (vertically transversely isotropic) and perform an inversion process on the five elastic parameters of the VTI elasticity tensor. To be more specific, our intention is to perform an inversion on the ratio between the eigenvalue of VTI (vertically transversely isotropic) and the eigenvalue (specifically shear) of the nearest isotropic elasticity tensor. The primary purpose of this study is to define a new misfit function regarding elasticity tensor parameters and to minimize it by using machine learning algorithms such as Grid Search, Random Search, Simulated Annealing, and Nelder-Mead. We use different optimization algorithms to observe the performance differences between these algorithms and find the best results for the objective function (misfit function).

Knowing the elastic parameters of the focal region can give us a lot of information about the stress level and also about the rupture. For example, the radiation pattern data can be interpreted more accurately if one assumes an anisotropic focal region. One can also check whether the elastic parameters change before and after an earthquake. In other words, the variation of stress level (stress drop) may lead to an alteration in the elastic parameters. Moreover, introducing the anisotropy of the focal region can also be considered as a linear perturbation from a simpler model, namely from isotropy. Hence this first-degree perturbation can better interpret the observed data.

The physical and geometric meanings of the eigenvalues of elasticity tensor are more evident than the elastic parameters. The contribution of the eigenvalues to the non-dc components of the resulting moment tensor can be directly observed. The same argument is also true for the parameter γ where it characterizes whether the focal region is filled with liquid or not. This parameter can also be related to the pore pressure of the source.

In Chapter 2, the algorithms of the methods we will use to minimize the misfit function are defined. Then, we present the expressions of moment, elasticity, and source tensors in Kelvin notation. These expressions allow for the use of linear algebraic manipulation tools in the decomposition process. Chapter 3 explains the methods needed to develop the misfit function. In Chapter 4, minimization results are plotted for the percentage of ratios on the 10° gridded stereographic projection. Moreover, we plot the solutions for each parameter in histograms and fit them to a normal distribution to calculate the solutions' mean and observe how the results are distributed around the mean. When calculating these values, we excluded the degenerated parts from the calculation. Then, we briefly discuss the results.

2. THEORETICAL BACKGROUND

In this chapter, the theoretical foundation of this research is presented. In particular, the definitions of the elasticity tensor, source tensor, Grid search algorithm, Random search algorithm, Nelder-Mead algorithm, Simulated Annealing algorithm, moment tensor decomposition, and representation theorem are stated.

2.1. Function Optimization

Many problems in applied seismology comprise finding the appropriate solution to understand earthquake structures. In general, the main aim is to detect the maximum or minimum value of a function. Identifying the fitting function, namely the objective function or misfit function, and determining the function's parameters where the maximum or minimum achieves the first step to resolving such problems.

Function optimization plays an essential role in nearly all machine learning algorithms and estimating modelling. The idea of function optimization can be imagined as investigating a search space. In addition, it can be mentioned that costs, potential solutions, and objective functions are the three main components of function optimization.

2.1.1. Grid Search Algorithm

Grid Search (GS) method computes the performance for every possible solution and picks the optimum value supplied by the model parameters. The technique in a GS algorithm is to select the minimum value from a regular grid in the search space.

In other words, we define a specific range or boundary of values for all unknown parameters in the objective function and calculate the objective function for each value in these ranges. Then a regular grid comprises each value returned from the objective

function. As a result, the minimum value on this grid gives us the optimum solution for the objective function parameters. The total number of unknown parameters determines the dimension of the grid.

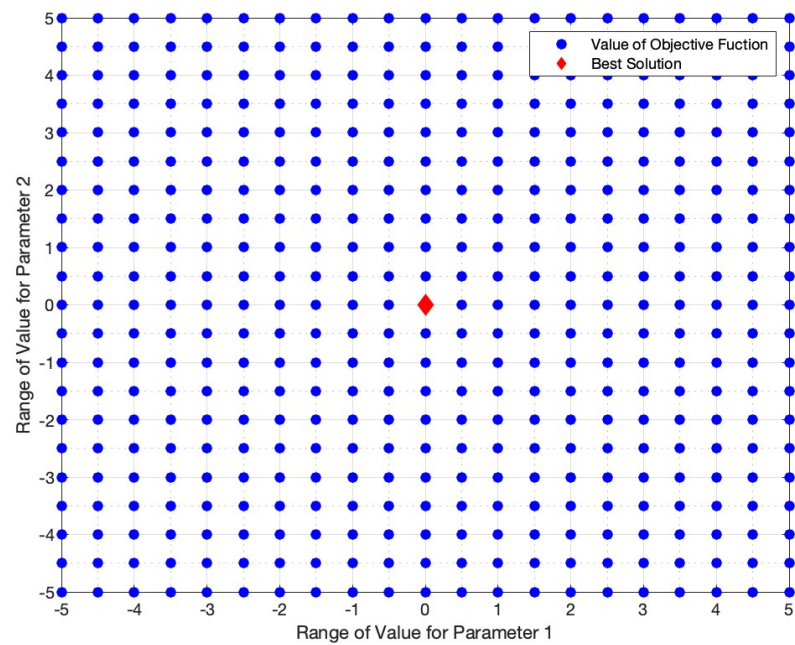


Figure 2.1: Grid Search for two unknown parameters

Algorithm 1 Grid Search Method

```

function OBJECTIVE FUNCTION( $x_1, x_2, x_3$ )
     $score \leftarrow f(x_1, x_2, x_3)$ 
    return  $score$ 
end function

 $bestScore$  (Initial Value)
 $x_1Min \leq bounderOf x_1 \leq x_1Max$ 
 $x_2Min \leq bounderOf x_2 \leq x_2Max$ 
 $x_3Min \leq bounderOf x_3 \leq x_3Max$ 

for  $i \leftarrow length(bounderOf x_1)$  do
    for  $j \leftarrow length(bounderOf x_2)$  do
        for  $k \leftarrow length(bounderOf x_3)$  do
             $score \leftarrow objectiveFunction$ 
            if  $bestScore < score$  then
                 $bestScore = score$ 
            end if
        end for
    end for
end for

```

2.1.2. Random Search Algorithm

The Random Search (RS) method in function optimization contains sampling and evaluating random inputs to the objective function. The random search algorithm is commonly preferred in optimization problems when the wide search space has high dimensionality.

The first step in the RS method is to define the search space. In other words, we determine the range or boundaries of the unknown parameters of the objective function. After the first step, the number of random samples or iterations performed by the algorithm is specified. The next step is to generate random solutions and evaluate the objective function. To be more specific, we randomly pick potential candidates for optimal solutions from the search space. Then we evaluate the objective function with

these random inputs. The last step is that we update the optimum solution for each iteration and return the best solution. In other words, after each iteration, we compare the value with the previous iteration value and keep the optimum result.

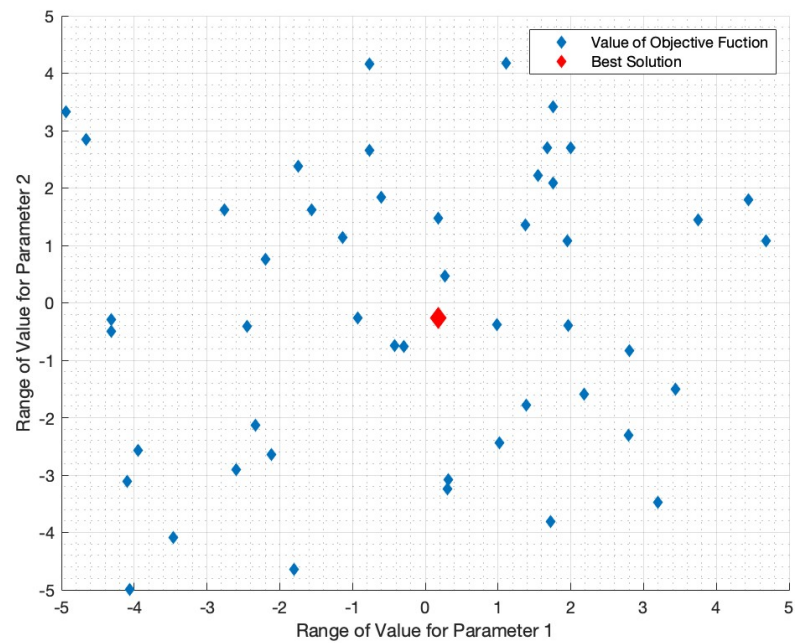


Figure 2.2: Random Search for two unknown parameters

Algorithm 2 Random Search Method

```

function OBJECTIVE FUNCTION( $x_1, x_2, x_3$ )
     $score \leftarrow f(x_1, x_2, x_3)$ 
    return  $score$ 
end function

```

```

 $IterationNumber$ 
 $bestScore \leftarrow (InitialValue)$ 
 $x_1Min \leq bounderOf x_1 \leq x_1Max$ 
 $x_2Min \leq bounderOf x_2 \leq x_2Max$ 
 $x_3Min \leq bounderOf x_3 \leq x_3Max$ 

 $ValueOf x_1 = rand(IterationNumber)$ 
 $ValueOf x_2 = rand(IterationNumber)$ 
 $ValueOf x_3 = rand(IterationNumber)$ 

for  $i \leftarrow length(ValueOf x_1)$  do
    for  $j \leftarrow length(ValueOf x_2)$  do
        for  $k \leftarrow length(ValueOf x_3)$  do
             $score \leftarrow objectiveFunction$ 
            if  $bestScore < score$  then
                 $bestScore = score$ 
            end if
        end for
    end for
end for

```

2.1.3. Nelder-Mead Algorithm

The Nelder-Mead (NM) [5] method is a pattern search (direct search) method to calculate an objective function's minimum (or maximum) in a multi-dimensional space. Pattern search methods do not require or use the gradient information of the objective function being optimized.

For an n -dimensional optimization problem, the NM method uses simplex, a geometrical shape with $n + 1$ vertices, as a tool to achieve optimal solutions in search

space. A Simplex is a line in one dimension and a triangle in two dimensions. NM algorithm starts with a randomly-generated set of simplex points in the parameter space.

The primary stages of the NM algorithm involve the following steps:

- (i) **Ordering:** After evaluating the objective function at each vertex of the simplex, they are sorted based on their function value. Let's say the vertices are indicated as V_0 (best), V_1 , V_2 , ..., and V_n (worst).
- (ii) **Reflection:** In this step, the reflection point (V_r) is computed.

$$V_r = c + \alpha(c - V_n) \quad (2.1)$$

where α is the reflection parameter, and c , namely centroid, is the average of the n best vertices' positions

$$c = \frac{1}{n} \sum_{i \neq h} x_i. \quad (2.2)$$

V_r is a point situated on the line connecting c and V_n , but it is positioned apart from V_n . This step aims to shift the simplex in a direction that leads away from the region around V_n . Note that α is usually equal to 1.

- (a) If the objective function's value at V_r is better than the value at V_0 (the best point), then move to the next iteration.
 - (b) If the objective function's value at V_r is worse than the second-worst value (V_{n-1}), perform contraction step.
 - (c) If the objective function's value at V_r is better than the value at V_n (the worst point), replace V_n with V_r in the simplex.
- (iii) **Expansion:** If the objective function's value at V_r is better than the value at V_0 ,

we compute an expansion point V_e . The expanded point is defined as:

$$V_e = c + \gamma(V_r - c) \quad (2.3)$$

If the objective function's value at V_e is better than the value at V_0 , replace V_n with V_e in the simplex; otherwise, replace V_n with V_r in the simplex. Expansion consistently results in a larger simplex, and this allows to facilitate greater exploration of the search space.

- (iv) **Contraction:** If the objective function's value at V_r is worse than the second-worst value V_{n-1} , we need to understand that the direction to V_r might not be the optimal choice for motion. Hence, we calculate the contraction point V_c in the simplex towards the centroid. The contraction point V_c is defined as:

$$V_c = c + \beta(V_r - c) \quad (2.4)$$

where β is the contraction parameter, usually equal to 0.5, if the value of the objective function at V_c is better than the value at V_n , replace V_n with V_c . However, if this is not the case, we move to the last step: the Shrink contraction.

- (v) **Shrink:** If the preceding steps do not result in substantial enhancement, we redefine the entire simplex. We redefine the entire simplex if previous steps do not lead to significant improvement or convergence. In other words, we only keep the best point V_0 (the current best point) and shrink the whole simplex towards it. The new point V_j is now defined as:

$$V_j = V_0 + \delta(V_j - V_0) \quad (2.5)$$

(vi) **Termination:** In this iteration, we repeat previous steps until a termination criterion is satisfied, such as reaching a maximum number of iterations and achieving the minimum size limit of the simplex or parameter values. These criteria can be as follows:

- (a) The current best solution reaches the boundary.
- (b) The algorithm reaches a specific number of iterations.
- (c) The simplex reaches a certain minimum size threshold.

Algorithm 3 Nelder-Mead Method

```

function OBJECTIVE FUNCTION( $x_1, x_2, x_3$ )
     $score \leftarrow f(x_1, x_2, x_3)$ 
    return  $score$ 
end function

```

initialization: Choose an initial simplex of vertices
 $V_{initial} = V_0, V_1, V_2, \dots, V_n$ evaluate f at the points in $V_{initial}$

Restrictions:

$$x_1Min \leq bounderOf x_1 \leq x_1Max$$

$$x_2Min \leq bounderOf x_2 \leq x_2Max$$

$$x_3Min \leq bounderOf x_3 \leq x_3Max$$

```

for  $i = 0, 1, \dots$  do
    Set  $V = V_i$ 
    Order
    Reflect
    Expand
    Contract
    Shrink
end for

```

2.1.4. Simulated Annealing Algorithm

Simulated Annealing (SA) [6] method is a stochastic optimization algorithm. In other words, randomness is integrated into the search procedure to find global solutions in the SA algorithm. Due to its incorporation of randomness, the algorithm is well-suited for nonlinear objective functions. Furthermore, using the SA algorithm, one can

solve combinatorial optimization problems, aiming to find the optimal solution from a wide set of possible solutions.

In SA, the search procedure starts with an initial solution and progressively reduces a control parameter until it reaches the optimal solution. SA algorithm can be defined as following steps:

- (i) **Generate an initial solution:** We randomly choose an initial solution (S). Subsequently, we set a control parameter (α) to specify how fast the temperature (T) value reduces during the annealing.
- (ii) **Initializing Temperature:** The process's primary step is defining an initial temperature (T) value. T audits the probability of accepting worse solutions and is typically assigned a high value.
- (iii) **A new solution is generated in the region surrounding the current solution:** In this step, we generate a new solution (S') in the neighbourhood of the current solution (S). We can acquire S' by applying minor random changes to the S .
- (iv) **Evaluation of a new solution:** After computing the objective function value for both S and S' , we calculate the difference ($\Delta E = f(S') - f(S)$) between the $f(S)$ and $f(S')$. If $\Delta E < 0$, we accept S' as the new current solution. On the other hand, if $\Delta E \geq 0$, we accept S' with a probability base on the following equation: $p(\text{accept}) = \exp(-\Delta E/T)$.
- (v) **Reducing the temperature gradually:** We decrease the temperature according to the control parameter: $T = T * \alpha$.
- (vi) **Repeating steps 2–6 until a stopping criterion is satisfied:** We decrease the temperature according to the control parameter: $T = T * \alpha$.

Algorithm 4 Simulated Annealing Method

function OBJECTIVEFUNCTION(x_1, x_2, x_3) $score \leftarrow f(x_1, x_2, x_3)$ **return** $score$ **end function**Select initial solution x_0 $\triangleright S : x_0$ Select temperature T Select control parameter α **while** termination criterion is not satisfied **do** **for** number of new solution **do** Select a new solution: $x_0 + \Delta x$ $\triangleright S' : x_0 + \Delta x$ $\Delta E = \text{ObjectiveFunction}(x_0 + \Delta x) - \text{ObjectiveFunction}(x_0)$ **if** $\Delta E < 0$ **then** $x_0^{new} = x_0 + \Delta x$ **else** random $r(0, 1)$ **if** $r < \exp(-\Delta E/T)$ **then** $x_0^{new} = x_0 + \Delta x$ **else** $x_0^{new} = x_0$ **end if** **end if** $x_0 = x_0^{new}$ Reduce tempeprature $T = T * \alpha$ **end for****end while**

2.2. Tensor Applications

In this section, the theoretical foundation of tensor applications is presented. In particular, the definitions of the elasticity tensor, the source tensor, the moment tensor, and the Kelvin notation theorem are stated.

2.2.1. Elasticity Tensor

The elasticity tensor, namely the elastic modulus tensor or stiffness tensor, is a tensor of the fourth-rank that defines the linear relation between stress and strain in a linear elastic material. This relationship is defined by Hooke's law.

In contrast to stress and strain, elasticity is an inherent characteristic of a material and the property of a matter to restore its original form under external stress.

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl}, \quad (2.6)$$

where σ_{ij} and ϵ_{kl} are stress tensor and strain tensor, respectively.

2.2.2. Source Tensor

The non-elastic strain acting at the source is stated by the source tensor. The definition of the source tensor D_{ij} is as follows:

$$D_{ij} = \frac{1}{2}([u_i]n_j + [u_j]n_i), \quad (2.7)$$

where u_i and n_j are components of slip and normal vectors, respectively.

2.2.3. Moment Tensor

The non-elastic stresses acting at the source are stated by the moment tensor. It is a symmetric tensor that represents nine pairs of equivalent dipole forces, which can occur at the source of an earthquake. It is a fundamental measurement calculated for earthquakes of all magnitudes, ranging from small acoustic waves to significant and destructive earthquake waves. It is defined as [7]

$$M_{kl} = C_{ijkl}[u_i]n_j \quad (2.8)$$

$$= C_{ijkl}D_{ij} \quad (2.9)$$

since elasticity tensor is symmetric w.r.t. the indices i and j .

2.2.4. Kelvin Notation

Using Kelvin notation, we can express tensors as vectors. Therefore, the source and moment tensors are represented as

$$\begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{12} & M_{22} & M_{23} \\ M_{13} & M_{23} & M_{33} \end{bmatrix} \rightarrow \begin{bmatrix} M_{11} \\ M_{22} \\ M_{33} \\ \sqrt{2}M_{23} \\ \sqrt{2}M_{13} \\ \sqrt{2}M_{12} \end{bmatrix} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ \sqrt{2}m_4 \\ \sqrt{2}m_5 \\ \sqrt{2}m_6 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{13} & D_{23} & D_{33} \end{bmatrix} \rightarrow \begin{bmatrix} D_{11} \\ D_{22} \\ D_{33} \\ \sqrt{2}D_{23} \\ \sqrt{2}D_{13} \\ \sqrt{2}D_{12} \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \sqrt{2}d_4 \\ \sqrt{2}d_5 \\ \sqrt{2}d_6 \end{bmatrix}, \quad (2.10)$$

In Kelvin notation, each matrix element is evaluated based on the number of elements in the tensor. More precisely, since non-diagonal components seem twice in the moment tensor, the coefficient of components has square roots.

$$C = \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & \sqrt{2}C_{1123} & \sqrt{2}C_{1113} & \sqrt{2}C_{1112} \\ C_{1122} & C_{2222} & C_{2233} & \sqrt{2}C_{2233} & \sqrt{2}C_{2213} & \sqrt{2}C_{2212} \\ C_{1133} & C_{1133} & C_{3333} & \sqrt{2}C_{3323} & \sqrt{2}C_{3313} & \sqrt{2}C_{3312} \\ \sqrt{2}C_{1123} & \sqrt{2}C_{2233} & \sqrt{2}C_{3323} & 2C_{2323} & 2C_{2313} & 2C_{2312} \\ \sqrt{2}C_{1113} & \sqrt{2}C_{2213} & \sqrt{2}C_{3313} & 2C_{2313} & 2C_{1313} & 2C_{1312} \\ \sqrt{2}C_{1112} & \sqrt{2}C_{2212} & \sqrt{2}C_{3312} & 2C_{2312} & 2C_{1312} & 2C_{1212} \end{bmatrix} \quad (2.11)$$

When the indices of the tensors are altered following [8]

$$(1, 1) \rightarrow 1, (2, 2) \rightarrow 2, (3, 3) \rightarrow 3,$$

$$(2, 3) \rightarrow 4, (1, 3) \rightarrow 5, (1, 2) \rightarrow 6.$$

The norm of both tensors and vectors is maintained by the Kelvin notation. In

summary, Equation (2.9) can be written as a vector equation

$$\begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ \sqrt{2}m_4 \\ \sqrt{2}m_5 \\ \sqrt{2}m_6 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & \sqrt{2}C_{14}\sqrt{2}C_{15}\sqrt{2}C_{16} \\ C_{12} & C_{22} & C_{23} & \sqrt{2}C_{24}\sqrt{2}C_{25}\sqrt{2}C_{26} \\ C_{13} & C_{23} & C_{33} & \sqrt{2}C_{34}\sqrt{2}C_{35}\sqrt{2}C_{36} \\ \sqrt{2}C_{14}\sqrt{2}C_{24}\sqrt{2}C_{34} & 2C_{44} & 2C_{45} & 2C_{46} \\ \sqrt{2}C_{15}\sqrt{2}C_{25}\sqrt{2}C_{35} & 2C_{45} & 2C_{55} & 2C_{56} \\ \sqrt{2}C_{16}\sqrt{2}C_{26}\sqrt{2}C_{36} & 2C_{46} & 2C_{56} & 2C_{66} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \sqrt{2}d_4 \\ \sqrt{2}d_5 \\ \sqrt{2}d_6 \end{bmatrix}. \quad (2.12)$$

To be more specific, the transformation described in Equation (2.10) is designed to ensure the norms of the tensors and vectors that are transformed into each other are equal [9] [10].

After using kelvin notation, we can view the elasticity matrix as a symmetric linear mapping in the six-dimensional vector space $\mathbb{R}^6$. Therefore, we clearly specify the spaces corresponding to the eigenvalues of the elasticity matrix. Also, the application of eigendecomposition could be implemented on the elasticity tensor. In simpler terms, the eigendecomposition of elasticity tensor can be shown as

$$\mathbf{C} = \lambda_1^C (\boldsymbol{\Sigma}_1 \otimes \boldsymbol{\Sigma}_1) + \lambda_2^C (\boldsymbol{\Sigma}_2 \otimes \boldsymbol{\Sigma}_2) + \lambda_3^C (\boldsymbol{\Sigma}_3 \otimes \boldsymbol{\Sigma}_3) + \lambda_4^C (\boldsymbol{\Sigma}_4 \otimes \boldsymbol{\Sigma}_4) + \lambda_5^C (\boldsymbol{\Sigma}_5 \otimes \boldsymbol{\Sigma}_5) + \lambda_6^C (\boldsymbol{\Sigma}_6 \otimes \boldsymbol{\Sigma}_6), \quad (2.13)$$

where $\boldsymbol{\Sigma}_i$'s are the six-unit eigenvectors with the corresponding eigenvalues λ_i^C and $\otimes$ denotes the tensor product (or dyadic product) of two vectors. For more detail see [11].

3. DATA AND METHOD

This chapter explains the methods needed to develop the misfit function. To be more precise, this misfit function is a tool for the inversion of the elasticity tensor.

3.1. VTI Elasticity Tensors

This section expresses the vertical transversely isotropic (VTI) elasticity tensor as eigenvalues and eigenspaces.

3.1.1. Decomposition of Moment Tensor

We can decompose a moment tensor using (2.12) and (2.13). This decomposition is into the eigenspaces of the transversely isotropic (TI) elasticity tensor.

$$\mathbf{m} = \lambda_1^{TI}(\boldsymbol{\Sigma}_1 \cdot \mathbf{d})\boldsymbol{\Sigma}_1 + \lambda_2^{TI}(\boldsymbol{\Sigma}_2 \cdot \mathbf{d})\boldsymbol{\Sigma}_2 + \lambda_3^{TI}((\boldsymbol{\Sigma}_3 \cdot \mathbf{d})\boldsymbol{\Sigma}_3 + (\boldsymbol{\Sigma}_4 \cdot \mathbf{d})\boldsymbol{\Sigma}_4) + \lambda_5^{TI}((\boldsymbol{\Sigma}_5 \cdot \mathbf{d})\boldsymbol{\Sigma}_5 + (\boldsymbol{\Sigma}_6 \cdot \mathbf{d})\boldsymbol{\Sigma}_6), \quad (3.1)$$

where $\mathbf{d}$ is the source tensor and $\mathbf{m}$ is the moment vector in $\mathbb{R}^6$; λ_i^{TI} 's are the eigenvalues of the TI elasticity tensor of the focal region and $\boldsymbol{\Sigma}_i$'s are the corresponding unit eigenvectors.

Table 3.1: The eigenvalues and eigenvectors of a VTI elasticity tensor.

Eigenvectors of VTI tensor	TI Eigenvalues
$\boldsymbol{\Sigma}_1 = (\frac{1}{\sqrt{2+\gamma^2}})(1, 1, \gamma, 0, 0, 0)$	λ_1^{TI}
$\boldsymbol{\Sigma}_2 = (\frac{1}{\sqrt{4+2\gamma^2}})(\gamma, \gamma, -2, 0, 0, 0)$	λ_2^{TI}
$\boldsymbol{\Sigma}_3 = (0, 0, 0, 1, 0, 0)$	λ_3^{TI}
$\boldsymbol{\Sigma}_4 = (0, 0, 0, 0, 1, 0)$	λ_3^{TI}
$\boldsymbol{\Sigma}_5 = (0, 0, 0, 0, 0, 1)$	λ_5^{TI}
$\boldsymbol{\Sigma}_6 = (\frac{1}{\sqrt{2}})(1, -1, 0, 0, 0, 0)$	λ_5^{TI}

In Table 3.1, five independent parameters are evident: $\lambda_1^{TI}, \lambda_2^{TI}, \lambda_3^{TI}, \lambda_5^{TI}$, and γ . These parameters characterize a VTI elasticity tensor [8]. Nevertheless, they are coordinate-independent parameters, while the ingredients of the elasticity tensor rely on the coordinate system.

Herein $\gamma \in \mathbb{R}$ expresses the stiffness ratio throughout the vertical axis, denoted by z , to any axis on the horizontal plane, denoted by xy , [8]. It is important to note that in a medium with a transversely isotropic (TI) elasticity tensor, all directions in the xy -plane are regarded as similar. For comparison, take account of the eigenvectors and eigenvalues of an isotropic (ISO) elasticity tensor. It is worth noting that comparing the eigenvectors of TI elasticity tensors, γ is set to 1, as all directions are considered equal in isotropy.

Additionally, the approximate relationship between the eigenvalues of a transversely isotropic (TI) elasticity tensor and the eigenvalues of the closest isotropic elasticity tensor to a given TI tensor can be expressed as

$$\lambda_1^{iso} \approx \lambda_1^{TI} \quad (3.2)$$

$$\lambda_2^{iso} \approx \frac{\lambda_2^{TI} + 2\lambda_3^{TI} + 2\lambda_5^{TI}}{5}, \quad (3.3)$$

If $\gamma \neq 1$, then Σ_2 does not qualify as a deviatoric tensor. To obtain the closest isotropic tensor, the TI tensor is projected orthogonally onto the linear subspace of elasticity tensors that are isotropic [12] [13]. Then, for a transversely isotropic elasticity tensor, the proximity to isotropy can be determined by assessing the differences between the TI-eigenvalues and the eigenvalues of the nearest isotropic elasticity tensor, as follows:

$$\left(\frac{\lambda_1 - \lambda_1^{iso}}{\lambda_1}\right)^2 + \left(\frac{\lambda_2 - \lambda_2^{iso}}{\lambda_2}\right)^2 + 2\left(\frac{\lambda_3 - \lambda_2^{iso}}{\lambda_3}\right)^2 + 2\left(\frac{\lambda_5 - \lambda_2^{iso}}{\lambda_5}\right)^2 \quad (3.4)$$

Table 3.2: The eigenvalues and distance to isotropy values for the elasticity tensors.

$\lambda_3 < \lambda_2$						
Rocks	λ_1	λ_2	$\lambda_3 = \lambda_4$	$\lambda_5 = \lambda_6$	γ	Distance to Isotropy
Slate	185.65	49.45	42.2	92.	0.77	0.89
Shale-II	54.58	15.52	10.8	21.2	0.67	0.77
Schist	159.01	73.49	59.8	83.6	0.65	0.3558
Water-Filled Cracks	98.05	35.58	28.56	35.72	0.99	0.2525
$\lambda_2 < \lambda_3$						
Shale-I	88.72	9.05	26.46	47.08	0.77	2.53
Sandstone	90.50	26.68	32.52	39.35	0.95	0.34
Dry Cracks	78.09	26.56	28.56	35.72	0.55	0.29

Table 3.3: Eigenvectors and eigenvalues of the elasticity tensor in an isotropic focal region.

Eigenvectors of ISO tensor	ISO Eigenvalues
$\Sigma_1 = (\frac{1}{\sqrt{3}})(1, 1, 1, 0, 0, 0)$	$\lambda_1^{iso} = 3\lambda + 2\mu$
$\Sigma_2 = (\frac{1}{\sqrt{6}})(1, 1, -2, 0, 0, 0)$	$\lambda_2^{iso} = 2\mu$
$\Sigma_3 = (0, 0, 0, 1, 0, 0)$	$\lambda_2^{iso} = 2\mu$
$\Sigma_4 = (0, 0, 0, 0, 1, 0)$	$\lambda_2^{iso} = 2\mu$
$\Sigma_5 = (0, 0, 0, 0, 0, 1)$	$\lambda_2^{iso} = 2\mu$
$\Sigma_6 = (\frac{1}{\sqrt{2}})(1, -1, 0, 0, 0, 0)$	$\lambda_2^{iso} = 2\mu$

Thus, as stated in Equation (3.1), any moment tensor of a seismic source is formed in an anisotropic focal region can be represented as a linear combination of the eigenvectors of the elasticity tensor of the focal region. The combination's coefficients, which are the eigenvalues of the elasticity tensor and the projection of $\mathbf{d}$ onto the eigenspaces, i.e., the value of $(\Sigma_i \cdot \mathbf{d})$, define the set of all potential moment tensors found in an anisotropic focal region. More precisely, considering an elasticity tensor, the structures of the resulting moment tensors (such as eigenvalues, norms, and traces) alter as the orientation of $\mathbf{d}$ changes. This change is due to strike, dip, and rake angle variations.

3.1.2. Derivation of approximate source tensor $\hat{\mathbf{d}}^{iso}$

Assume that the focal region has a slight degree of anisotropy; the eigenvectors of the moment tensor $\mathbf{M}$ are approximately equal to the eigenvectors of the source tensor

D. In other words, the eigenvectors of the source tensor $\mathbf{D}$ do not undergo significant changes affected by the elasticity tensor $\mathbf{C}$ when the linear relation $M_{ij} = C_{ijkl}D_{kl}$ is applied.

$$\hat{\mathbf{D}}^{\text{iso}} = \frac{1}{2}(\hat{\mathbf{u}}^{\text{iso}} \otimes \hat{\mathbf{n}}^{\text{iso}} + \hat{\mathbf{n}}^{\text{iso}} \otimes \hat{\mathbf{u}}^{\text{iso}}), \quad (3.5)$$

where

$$\hat{\mathbf{u}}^{\text{iso}} = \frac{1}{\sqrt{2}}(\hat{\mathbf{e}}_1 + \hat{\mathbf{e}}_2), \quad (3.6)$$

$$\hat{\mathbf{n}}^{\text{iso}} = \frac{1}{\sqrt{2}}(\hat{\mathbf{e}}_1 - \hat{\mathbf{e}}_2). \quad (3.7)$$

In this case, $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_2$ denote the unit eigenvectors of the moment tensor $\mathbf{M}$, which correspond to the two largest eigenvalues in terms of magnitude. (It should be noted that the second eigenvalue is nearly zero.). Because of the auxiliary solution, it becomes possible to replace the expressions of slip and normal in Equations (3.6) and (3.7), resulting in the same $\hat{\mathbf{D}}^{\text{iso}}$ would be obtained.

3.2. Inversion for Determining the Rates of Eigenvalues

With an occurring moment tensor, Equation (3.1) can be utilized to determine both γ and the rates of the eigenvalues $\frac{\lambda_i}{\lambda_2^{\text{iso}}}$ for $i \in 1, 2, 3, 4$. Here, λ_2^{iso} represents the second eigenvalue of the nearest isotropic elasticity tensor, namely $\mathbf{C}^{\text{iso}}$, to a given TI elasticity tensor.

First note that we can acquire the coefficients in Equation (3.1) as:

$$\lambda_i(\mathbf{d} \cdot \boldsymbol{\Sigma}_i) = \mathbf{m} \cdot \boldsymbol{\Sigma}_i \quad (3.8)$$

As $\mathbf{m}$ can be expressed as a linear combination of fundamental elements, namely $\boldsymbol{\Sigma}_i$'s. Subsequently, Equations (3.8) can be used to determine the rates of the eigenvalues of $\mathbf{C}$ and $\mathbf{C}^{\text{iso}}$ as follows

$$\frac{\lambda_i}{\lambda_2^{\text{iso}}} = \frac{\frac{\mathbf{m} \cdot \boldsymbol{\Sigma}_i}{\mathbf{d} \cdot \boldsymbol{\Sigma}_i}}{\frac{\lambda_2^{\text{iso}}}{\lambda_2^{\text{iso}}}} = \frac{\frac{\mathbf{m} \cdot \boldsymbol{\Sigma}_i}{\|\mathbf{u}\| \|\mathbf{n}\| (\hat{\mathbf{d}} \cdot \boldsymbol{\Sigma}_i)}}{\lambda_2^{\text{iso}}} \approx \frac{\frac{\lambda_2^{\text{iso}}(\mathbf{m} \cdot \boldsymbol{\Sigma}_i)}{\|\mathbf{m}\| \sqrt{2} (\hat{\mathbf{d}}^{\text{iso}} \cdot \boldsymbol{\Sigma}_i)}}{\lambda_2^{\text{iso}}} = \frac{\mathbf{m} \cdot \boldsymbol{\Sigma}_i}{\|\mathbf{m}\| \sqrt{2} (\hat{\mathbf{d}}^{\text{iso}} \cdot \boldsymbol{\Sigma}_i)}, \quad (3.9)$$

In the second equality, the source vector is written as $\mathbf{d} = \|\mathbf{u}\| \|\mathbf{n}\| \hat{\mathbf{d}}$, where $\hat{\mathbf{d}}$ refers to the source vector obtained from unit slip $\hat{\mathbf{u}}$ and unit normal $\hat{\mathbf{n}}$, i.e $\hat{\mathbf{D}} = \frac{1}{2}(\hat{\mathbf{u}} \otimes \hat{\mathbf{n}} + \hat{\mathbf{n}} \otimes \hat{\mathbf{u}})$. Subsequently, two approaches are employed, the first one concerning the projection of the source tensor onto an eigenspace, namely for $(\hat{\mathbf{d}} \cdot \boldsymbol{\Sigma}_i)$ which is evident in the second part of the expression (3.9). The other approach pertains to the potency $\|\mathbf{u}\| \|\mathbf{n}\|$.

Both of these approaches rely on the hypothesis of the focal region exhibiting isotopy. Expressly, the first approach assumes that

$$\hat{\mathbf{d}} \cdot \boldsymbol{\Sigma}_i \approx \hat{\mathbf{d}}^{\text{iso}} \cdot \boldsymbol{\Sigma}_i, \quad (3.10)$$

where, $\hat{\mathbf{d}}^{\text{iso}}$ represents the source tensor derived from the eigenvectors of the moment tensor $\mathbf{M}$ using the widely recognized formula for slip and normal vectors (refer to Equations 3.6 to 3.7). Remember that in the case of an isotropic focal region, the eigenvectors of the moment tensor and the source tensor are identical. However, this

equivalence is not valid for anisotropic focal regions.

The second approach concerns calculating the potency, namely $||\mathbf{u}||||\mathbf{n}||$. In the case of an isotropic focal region, the seismic moment can be calculated as the dot product of $\mathbf{d}$ and Σ_2^{iso} , namely the deviatoric space, and multiplying it by the corresponding eigenvalue λ_2^{iso} anisotropic-focal region. More precisely, the seismic moment can be described as:

$$||\mathbf{m}|| = \lambda_2^{\text{iso}}(\mathbf{d} \cdot \Sigma_2^{\text{iso}}), \quad (3.11)$$

where Σ_2^{iso} is the vector space of deviatoric tensors. Remember that this space is formed by five eigenvectors. Consequently, shear source tensors, which include slip and normal vectors that are orthogonal to each other, are part of the eigenspace Σ_2^{iso} due to the sum of its diagonal elements is zero. Thus, $(\mathbf{d} \cdot \Sigma_2^{\text{iso}}) = ||\mathbf{d}||$ and this mention $||\mathbf{m}|| = \frac{\lambda_2^{\text{iso}} ||\mathbf{u}||||\mathbf{n}||}{\sqrt{2}}$.

The approximation of potency theorizes that a shear source in an anisotropic focal region generates non-dc components of unimportant magnitude. Subsequently, the norm of the moment tensor for shear sources that occur in an anisotropic focal region can be approximated as

$$||\mathbf{m}|| \approx \lambda_2^{\text{iso}}(\hat{\mathbf{d}} \cdot \Sigma_2^{\text{iso}}) ||\mathbf{u}||||\mathbf{n}|| \Rightarrow ||\mathbf{u}||||\mathbf{n}|| \approx \frac{||\mathbf{m}||}{\lambda_2^{\text{iso}}(\hat{\mathbf{d}} \cdot \Sigma_2^{\text{iso}})} = \frac{||\mathbf{m}||\sqrt{2}}{\lambda_2^{\text{iso}}}, \quad (3.12)$$

The final equation is derived from that the sum of diagonal elements $\hat{\mathbf{d}}$ is zero, namely traceless. Note that $\hat{\mathbf{d}} \in \Sigma_2^{\text{iso}}$, and $||\hat{\mathbf{d}}|| = \frac{1}{\sqrt{2}}$.

3.2.1. Evaluation of the parameters γ , $\frac{\lambda_1}{\lambda_2^{iso}}$ and $\frac{\lambda_2}{\lambda_2^{iso}}$

The eigenspaces Σ_1 and Σ_2 contain parameter γ (see Equation 3.1). Therefore, the rates $\frac{\lambda_1}{\lambda_2^{iso}}$ in Equation (3.9), which is used for inversion, also include the unidentified parameter γ . Thus, Equation (3.9) alone is insufficient for determining these two ratios, which means that one needs additional equations to invert these ratios successfully. To analyse the $\frac{\lambda_1}{\lambda_2^{iso}}$, $\frac{\lambda_2}{\lambda_2^{iso}}$ and γ , we will focus on using the trace and the M_{33} entry of the moment tensor. These specific measurements depend on these ratios and are not influenced by the other ratios. Note that there are alternative equations available, apart from using the trace and the M_{33} entry, for inverting the ratios $\frac{\lambda_1}{\lambda_2^{iso}}$, $\frac{\lambda_2}{\lambda_2^{iso}}$ and γ . These alternative equations can involve the determinant of the moment tensor, the sum of second-order minors, or the other components of the moment tensor. However, these equations cover additional parameters, namely $\frac{\lambda_3}{\lambda_2^{iso}}$ and $\frac{\lambda_4}{\lambda_2^{iso}}$. We intentionally select the equation involving the trace and the M_{33} entry not to move the inversion error of $\frac{\lambda_3}{\lambda_2^{iso}}$ and $\frac{\lambda_4}{\lambda_2^{iso}}$ to the other parameters. Furthermore, the inverse problem becomes more difficult due to the nonlinearity of determinants and minors.

Using Equations (3.10) and (3.12), an approximation of a moment tensor can be expressed in relation to the eigenvalues derived from Equation (3.1) as

$$\begin{aligned} \mathbf{m} \approx & \|\mathbf{m}\| \sqrt{2} \left(\frac{\lambda_1^{TI}}{\lambda_2^{iso}} (\Sigma_1 \cdot \hat{\mathbf{d}}^{iso}) \Sigma_1 + \frac{\lambda_2^{TI}}{\lambda_2^{iso}} (\Sigma_2 \cdot \hat{\mathbf{d}}^{iso}) \Sigma_2 \right. \\ & + \frac{\lambda_3^{TI}}{\lambda_2^{iso}} \left((\Sigma_3 \cdot \hat{\mathbf{d}}^{iso}) \Sigma_3 + (\Sigma_4 \cdot \hat{\mathbf{d}}^{iso}) \Sigma_4 \right) \\ & \left. + \frac{\lambda_4^{TI}}{\lambda_2^{iso}} \left((\Sigma_5 \cdot \hat{\mathbf{d}}^{iso}) \Sigma_5 + (\Sigma_6 \cdot \hat{\mathbf{d}}^{iso}) \Sigma_6 \right) \right) \end{aligned} \quad (3.13)$$

Subsequently, the trace and the M_{33} -component of the moment tensor can be

estimated by considering the trace and the M_{33} -component on both sides as

$$Tr(\mathbf{m}) \approx \frac{\|\mathbf{m}\|}{\sqrt{2}} \left(\frac{\lambda_1}{\lambda_2^{iso}} (\hat{\mathbf{d}}^{iso} \cdot \boldsymbol{\Sigma}_1) Tr(\boldsymbol{\Sigma}_1) + \frac{\lambda_2}{\lambda_2^{iso}} (\hat{\mathbf{d}}^{iso} \cdot \boldsymbol{\Sigma}_2) Tr(\boldsymbol{\Sigma}_2) \right), \quad (3.14)$$

$$M_{33} \approx \frac{\|\mathbf{m}\|}{\sqrt{2}} \left(\frac{\lambda_1}{\lambda_2^{iso}} (\hat{\mathbf{d}}^{iso} \cdot \boldsymbol{\Sigma}_1) (\boldsymbol{\Sigma}_1)_{33} + \frac{\lambda_2}{\lambda_2^{iso}} (\hat{\mathbf{d}}^{iso} \cdot \boldsymbol{\Sigma}_2) (\boldsymbol{\Sigma}_2)_{33} \right), \quad (3.15)$$

$$Tr(\boldsymbol{\Sigma}_1) = \frac{1}{\sqrt{2 + \gamma^2}} (2 + \gamma), \quad (3.16)$$

$$Tr(\boldsymbol{\Sigma}_2) = \frac{1}{\sqrt{4 + 2\gamma^2}} (2\gamma - 2). \quad (3.17)$$

On the other hand, because of that the sum of $\boldsymbol{\Sigma}_3, \boldsymbol{\Sigma}_4, \boldsymbol{\Sigma}_5, \boldsymbol{\Sigma}_6$ diagonal elements are equal to zero, these eigenspaces have no impact on the M_{33} . Moreover, these equations consist of three undefined, the rates $\frac{\lambda_1}{\lambda_2^{iso}}, \frac{\lambda_2}{\lambda_2^{iso}}$ and γ . Therefore, we can create an objective function by merging the trace Equation (3.14), the M_{33} -entry Equation (3.15), and the equations of the ratios for $\frac{\lambda_1}{\lambda_2^{iso}}, \frac{\lambda_2}{\lambda_2^{iso}}$ as

$$\begin{aligned} Misfit \left(\frac{\lambda_1}{\lambda_2^{iso}}, \frac{\lambda_2}{\lambda_2^{iso}}, \gamma \right) &= \left(Tr(\mathbf{M}) - \frac{\|\mathbf{m}\|}{\sqrt{2}} \left(\frac{\lambda_1}{\lambda_2^{iso}} (\hat{\mathbf{d}}^{iso} \cdot \boldsymbol{\Sigma}_1) Tr(\boldsymbol{\Sigma}_1) + \frac{\lambda_2}{\lambda_2^{iso}} (\hat{\mathbf{d}}^{iso} \cdot \boldsymbol{\Sigma}_2) Tr(\boldsymbol{\Sigma}_2) \right) \right)^2 \\ &+ \left(M_{33} - \frac{\|\mathbf{m}\|}{\sqrt{2}} \left(\frac{\lambda_1}{\lambda_2^{iso}} (\hat{\mathbf{d}}^{iso} \cdot \boldsymbol{\Sigma}_1) (\boldsymbol{\Sigma}_1)_{33} + \frac{\lambda_2}{\lambda_2^{iso}} (\hat{\mathbf{d}}^{iso} \cdot \boldsymbol{\Sigma}_2) (\boldsymbol{\Sigma}_2)_{33} \right) \right)^2 \\ &+ \left(\frac{\lambda_1}{\lambda_2^{iso}} - \frac{\mathbf{m} \cdot \boldsymbol{\Sigma}_1}{\|\mathbf{m}\| \sqrt{2} (\hat{\mathbf{d}}^{iso} \cdot \boldsymbol{\Sigma}_1)} \right)^2 + \left(\frac{\lambda_2}{\lambda_2^{iso}} - \frac{\mathbf{m} \cdot \boldsymbol{\Sigma}_2}{\|\mathbf{m}\| \sqrt{2} (\hat{\mathbf{d}}^{iso} \cdot \boldsymbol{\Sigma}_2)} \right)^2. \end{aligned}$$

Subsequently, we can minimize the misfit function to perform the inversion of $\frac{\lambda_1}{\lambda_2^{iso}}, \frac{\lambda_2}{\lambda_2^{iso}}$ and γ .

As a result, we are interested in the minimization of misfit function parameters in this study. We artificially generate moment tensors for all 10° orientations of slip and normal vectors using elasticity tensors of slate, water-filled, and dry cracks of focal regions. Subsequently, the inversion of the ratios of the eigenvalues of the elasticity tensors will be performed by minimizing the misfit function (objective function). We will use the four different optimization methods described in the second chapter to minimize the misfit function, namely grid search, random search, simulated annealing and Nelder-Mead methods. After minimization, we compare the results and methods.

4. RESULTS

In this chapter, the minimization results were plotted for the percentage of the ratios on 10° gridded stereographic projection where each point corresponds to particular dip and rake angles, see Figure 4.1 and 4.2 (see [11] for the detailed description).

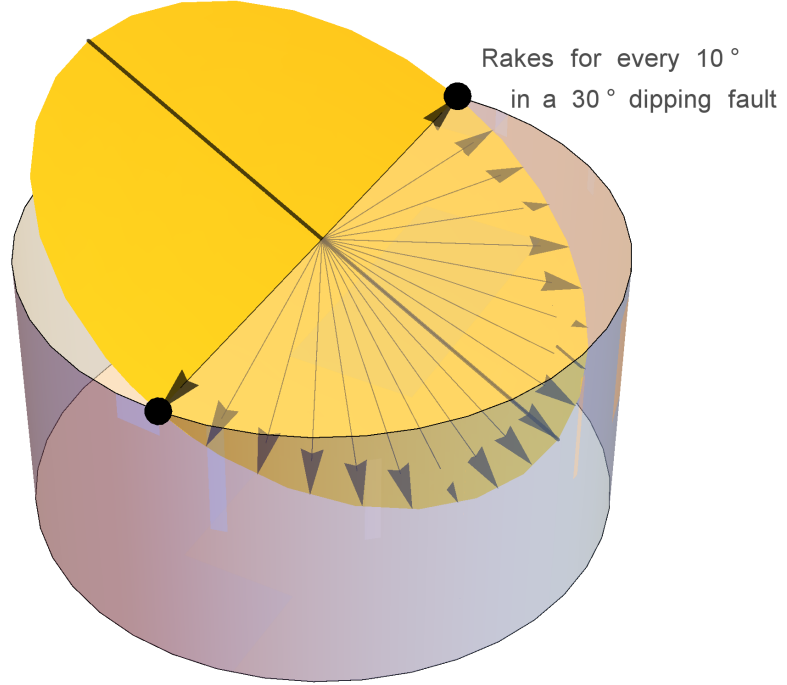


Figure 4.1: A TI medium with a vertical infinite-rotational axis is represented by a cylinder. The dipping fault in the TI fault zone is shown by the circle-shaped plane. For a TI medium with a vertical infinite-fold axis, strike direction does not change the structure of the resulting moment tensors. Moreover, the invariants of the moment tensors remain the same for all rakes that are mirror images of themselves w.r.t the strike or w.r.t the line perpendicular to the strike on the fault plane (thick line). Hence, it is enough to plot only the first quadrant to show any invariant of the moment tensor.

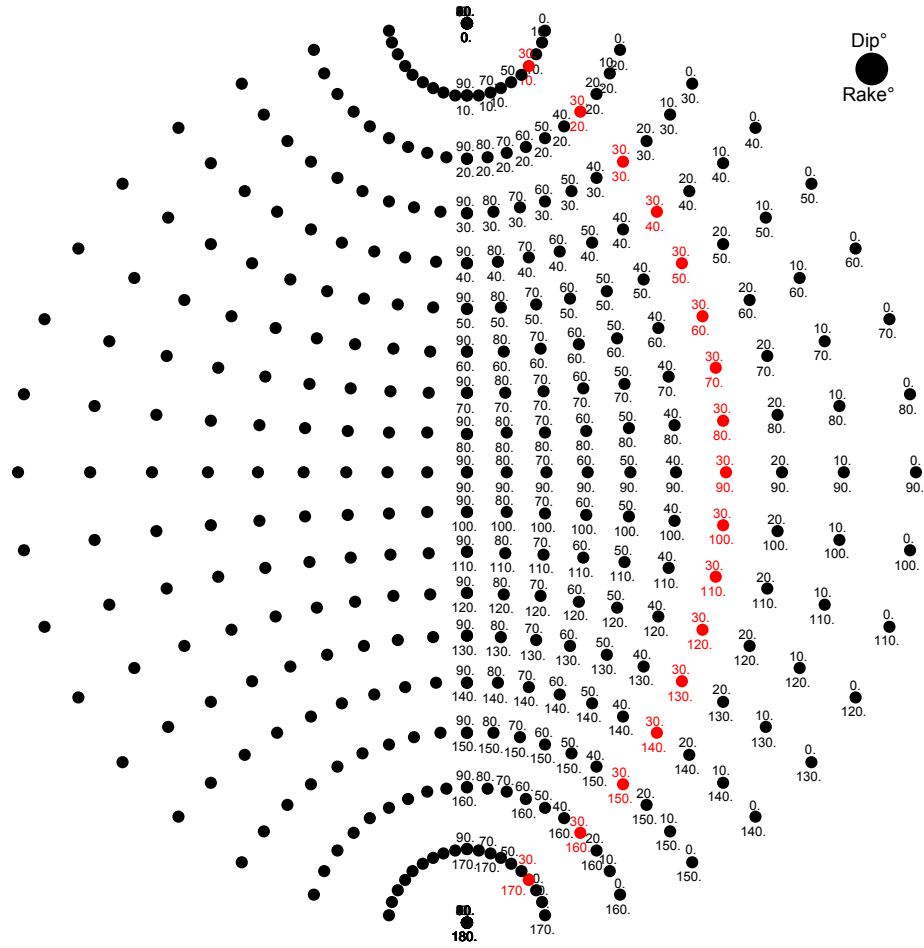


Figure 4.2: The red fault corresponds to the fault depicted in Figure 4.1, wherein each red dot represents different rake angles indicated by the arrows.

We displayed the solutions for each parameter in histograms for a better interpretation. Moreover, we fitted them to a normal distribution to calculate the solutions' mean and to observe how the results were distributed around the mean. While computing these values, we excluded the degenerated parts from the calculation.

The inversion results of the parameter γ for all optimization methods yield nearly identical to the original values for dry cracks and water-filled cracks. The decrease in accuracy is only observed in Slate, a highly anisotropic focal region. Rock types are listed in Table 3.2.

The stability of the optimization relies on the strength of anisotropy because the eigenvalue $\frac{\lambda_1}{\lambda_2^{iso}}$ is dependent on the γ value. Furthermore, as the γ value approaches unity, it can be observed that the solution quality of $\frac{\lambda_1}{\lambda_2^{iso}}$ for all methods becomes very low, particularly for the elasticity tensor of water-filled cracks where $\gamma = 0.99$. The resolution of $\frac{\lambda_1}{\lambda_2^{iso}}$ is related to the percentage of the isotropic component of the resulting moment tensor. There is no isotropic component if γ is equal to unity.

4.1. Results of Inversion for Slate

Considering Table 4.1, one can observe that SA and NM provide the best solution for the parameter γ . Furthermore, in Figure 4.7, it can be seen that the closest values to the real value are calculated by the SA algorithm.

On the other hand, considering Figure 4.12, 4.17 and Table 4.1, there are slight deviations in the calculation of $\frac{\lambda_1}{\lambda_2^{iso}}$ and $\frac{\lambda_2}{\lambda_2^{iso}}$ values calculated by the algorithm. This may be due to the anisotropic structure of the slate.

Table 4.1: The value of mean and standard deviation computed by algorithms for slate.

Slate			
	$\frac{\lambda_1}{\lambda_2^{iso}}$	$\frac{\lambda_2}{\lambda_2^{iso}}$	γ
Original Value	2.9203	0.77794	0.768458
Simulated Annealing	2.643014	1.00173	0.568117
	0.3269	0.2786637	0.195147
Nelder-Mead	2.4546	0.99974	0.539522
	0.45847	0.2890262	0.15596
Random Search	2.50220	0.9913043	0.5163614
	0.389206	0.275047	0.20467
Grid Search	2.742465	0.94452054	0.578767
	0.176318	0.1981794	0.216813

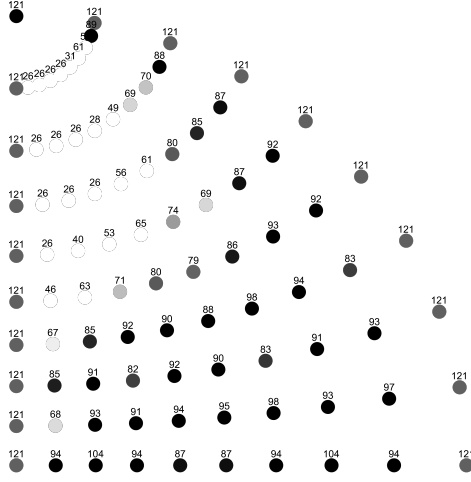


Figure 4.3: Inversion of γ value using simulated annealing algorithm for slate.

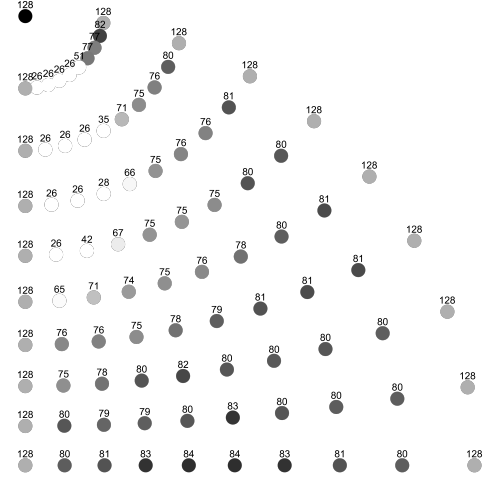


Figure 4.4: Inversion of γ value using Nelder-Mead algorithm for slate.

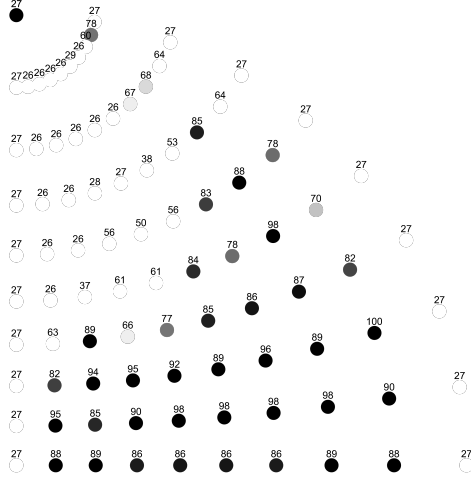


Figure 4.5: Inversion of γ value using random search algorithm for slate.

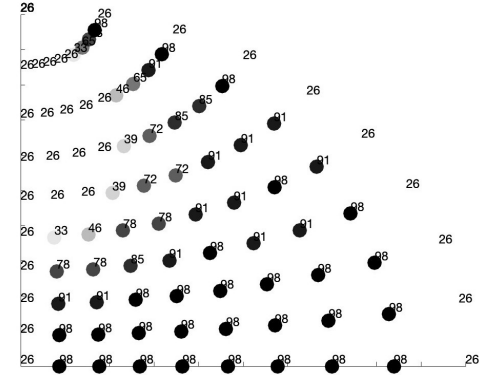


Figure 4.6: Inversion of γ value using grid search algorithm for slate.

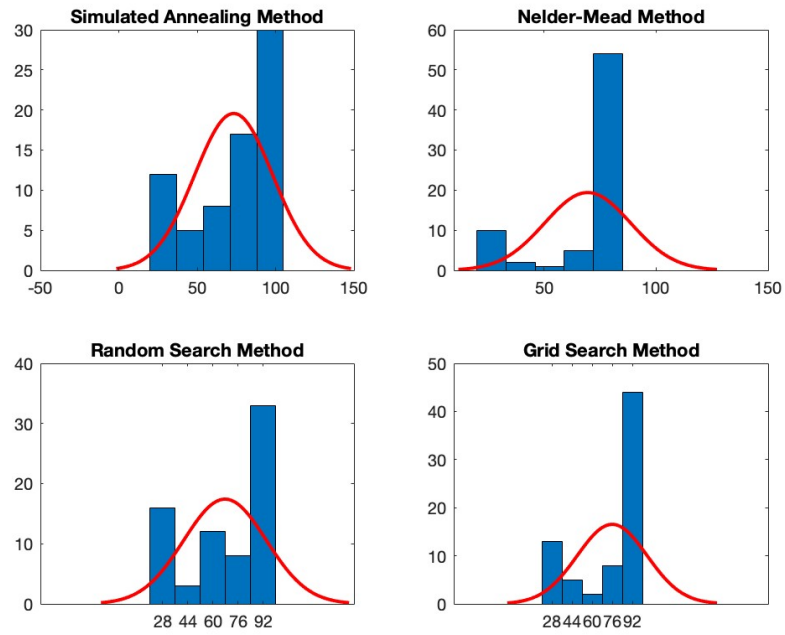


Figure 4.7: The histograms of the percentage of γ for slate. The true value is %100.

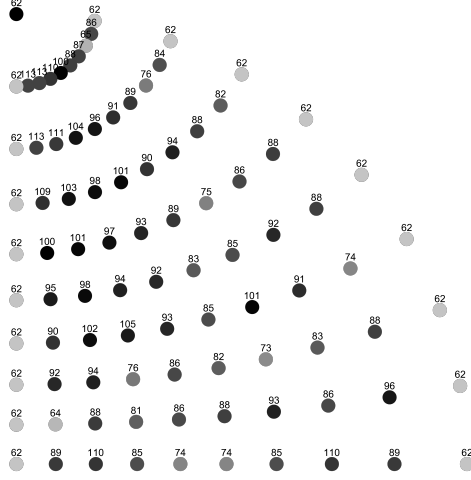


Figure 4.8: Inversion of $\frac{\lambda_1}{\lambda_2^{iso}}$ value using simulated annealing algorithm for slate.

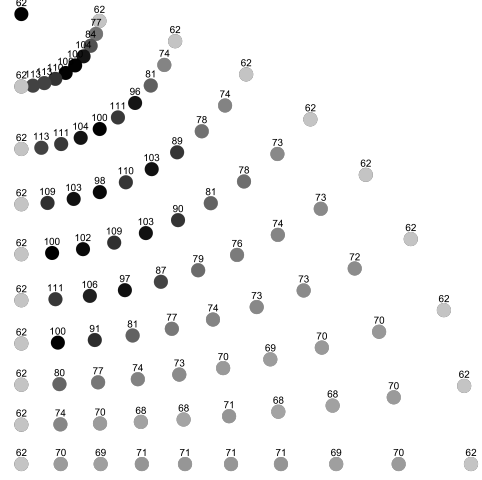


Figure 4.9: Inversion of $\frac{\lambda_1}{\lambda_2^{iso}}$ value using Nelder-Mead algorithm for slate.

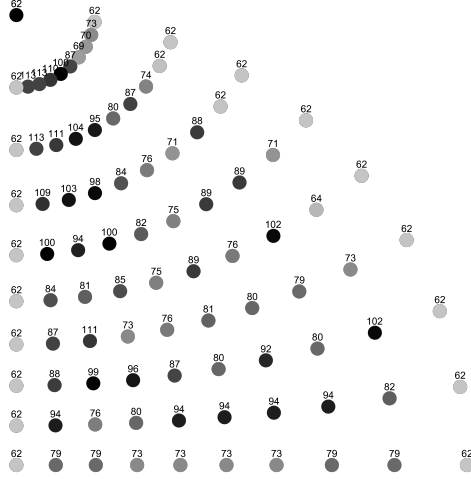


Figure 4.10: Inversion of $\frac{\lambda_1}{\lambda_2^{iso}}$ value using random search algorithm for slate.

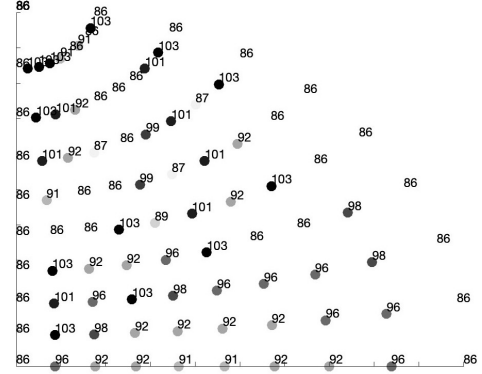


Figure 4.11: Inversion of $\frac{\lambda_1}{\lambda_2^{iso}}$ value using grid search algorithm for slate.

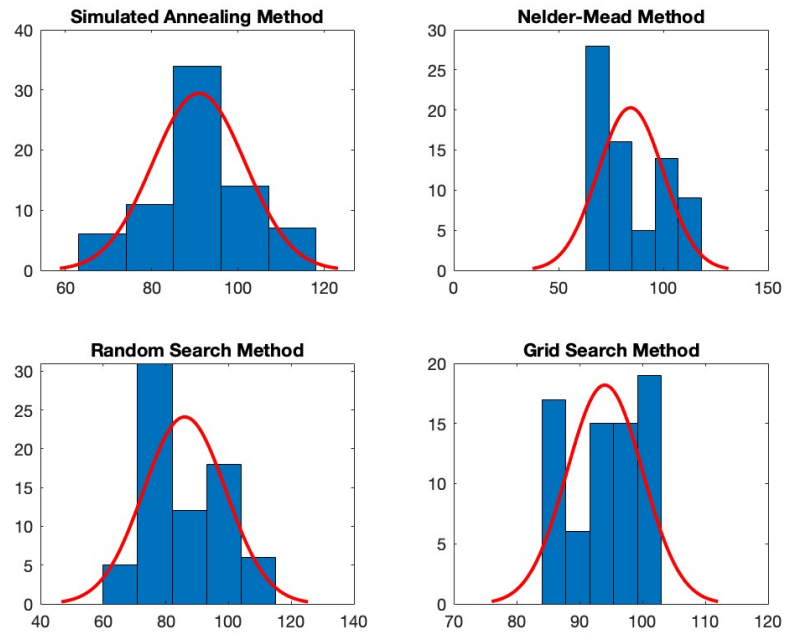


Figure 4.12: The histograms of the percentage of $\frac{\lambda_1}{\lambda_2^{iso}}$ for slate. The true value is %100.

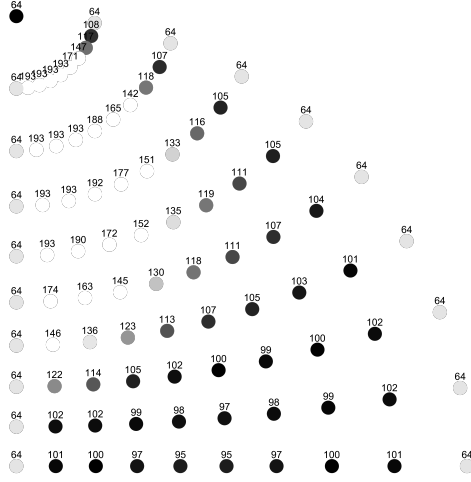


Figure 4.13: Inversion of $\frac{\lambda_2}{\lambda_2^{iso}}$ value using simulated annealing algorithm for slate.

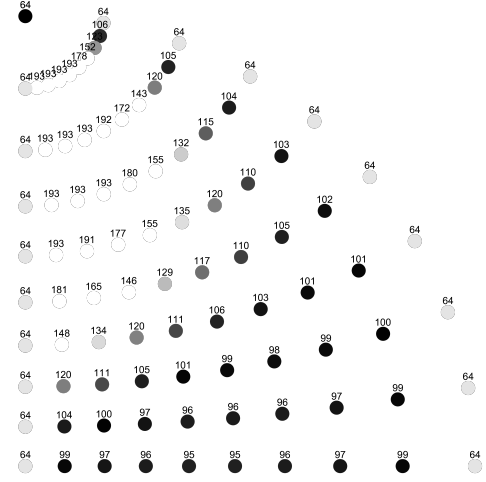


Figure 4.14: Inversion of $\frac{\lambda_2}{\lambda_2^{iso}}$ value using Nelder-Mead algorithm for slate.

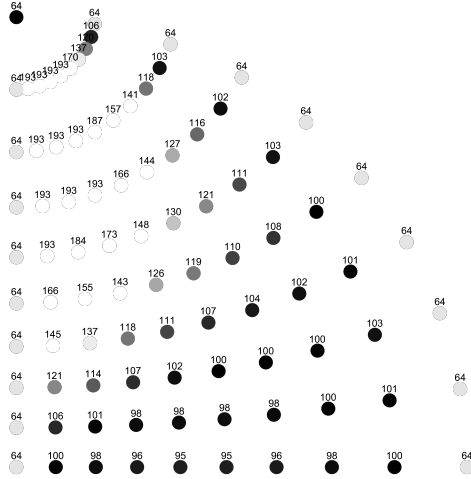


Figure 4.15: Inversion of $\frac{\lambda_2}{\lambda_2^{iso}}$ value using random search algorithm for slate.

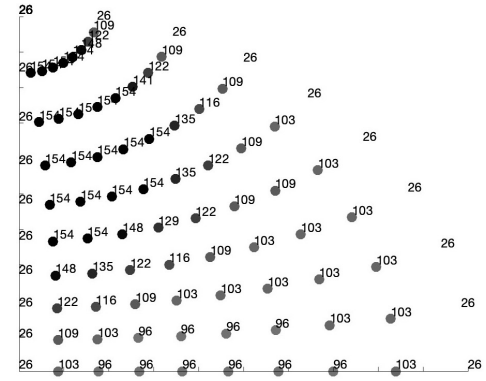


Figure 4.16: Inversion of $\frac{\lambda_2}{\lambda_2^{iso}}$ value using grid search algorithm for slate.

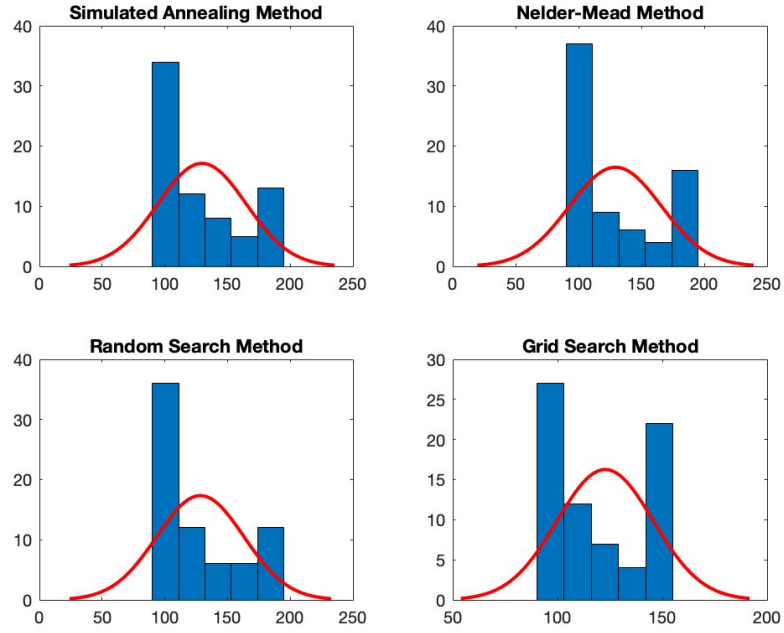


Figure 4.17: The histograms of the percentage of $\frac{\lambda_2}{\lambda_2^{iso}}$ for slate. The true value is %100.

4.2. Results of Inversion for Water-Filled Cracks

According to the data in Table 6, the results produced by optimization methods are close to the true value for γ . Figure 4.18, 4.19, 4.20, 4.21, and 4.22 also support this inference. However, the value of $\frac{\lambda_1}{\lambda_2^{iso}}$ computed by optimization methods is quite far from the real result. Unlike the parameter $\frac{\lambda_1}{\lambda_2^{iso}}$, the inversion of $\frac{\lambda_2}{\lambda_2^{iso}}$ is close to its original value.

The anisotropic structure of the water-filled crack focal region may have made it difficult to solve the $\frac{\lambda_1}{\lambda_2^{iso}}$ and $\frac{\lambda_2}{\lambda_2^{iso}}$ parameters. On the other hand, the dependency of these two parameters is one factor that makes the minimization of the objective function more challenging.

Table 4.2: The value of mean and standard deviation computed by algorithms for water-filled cracks.

Water-Cilled Cracks			
	$\frac{\lambda_1}{\lambda_2^{iso}}$	$\frac{\lambda_2}{\lambda_2^{iso}}$	γ
Original Value	2.98685	1.08379	0.989472
Simulated Annealing	1.97716	1.0221633	0.967137
	0.36584	0.1179712	0.054367
Nelder-MMead	1.5811828	1.0246136	0.9513624
	0.052649	0.11688	0.09146
Random Search	1.64738	1.024617	0.95237362
	0.268250	0.11689	0.092017
Grid Search	2.50000	1.0369863	0.9438356
	0.0000	0.121343	0.052668

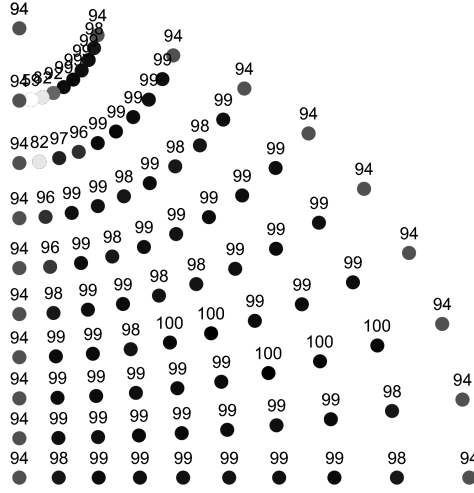


Figure 4.18: Inversion of γ value using simulated annealing algorithm for water-filled cracks.

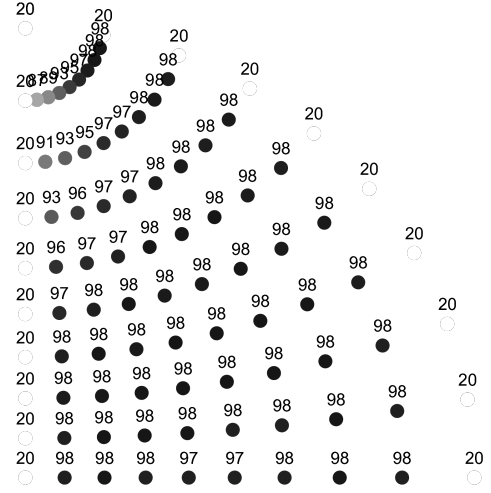


Figure 4.19: Inversion of γ value using Nelder-Mead algorithm for water-filled cracks.

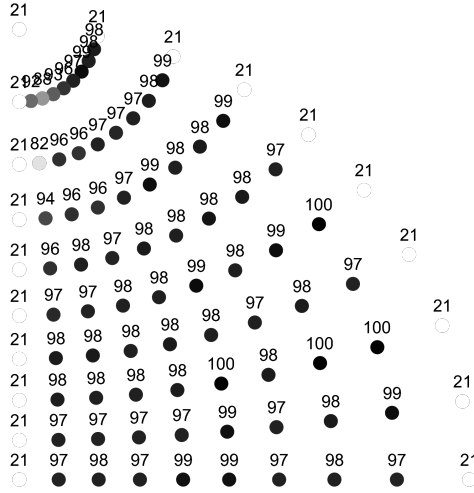


Figure 4.20: Inversion of γ value using random search algorithm for water-filled cracks.

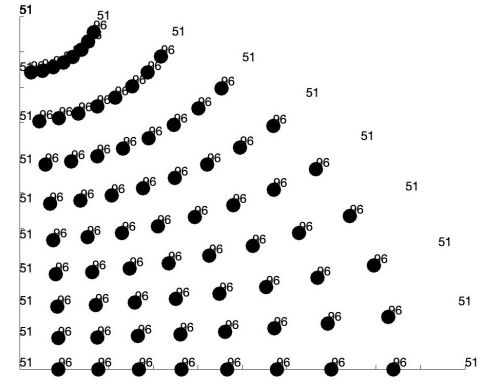


Figure 4.21: Inversion of γ value using grid search algorithm for water-filled cracks.

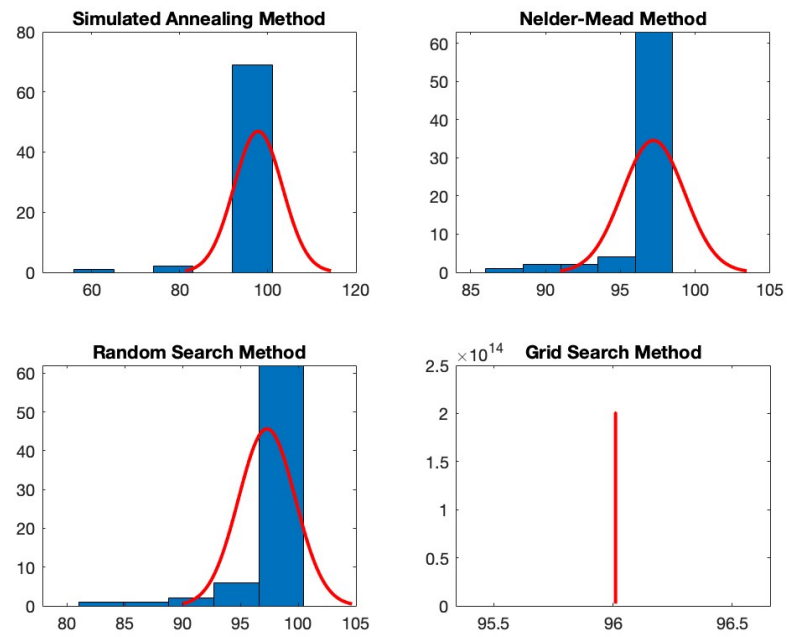


Figure 4.22: The histograms of the percentage of γ for water-filled cracks. The true value is %100.

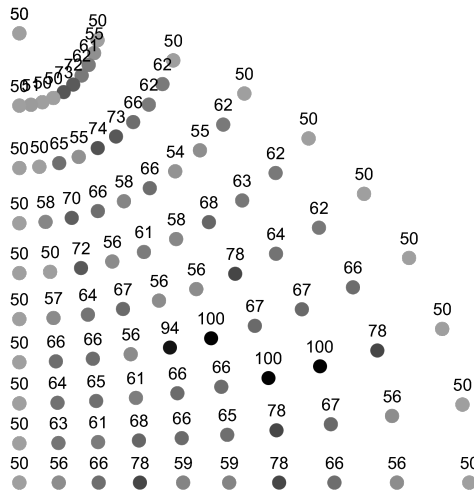


Figure 4.23: Inversion of $\frac{\lambda_1}{\lambda_2^{iso}}$ value using simulated annealing algorithm for water-filled cracks.

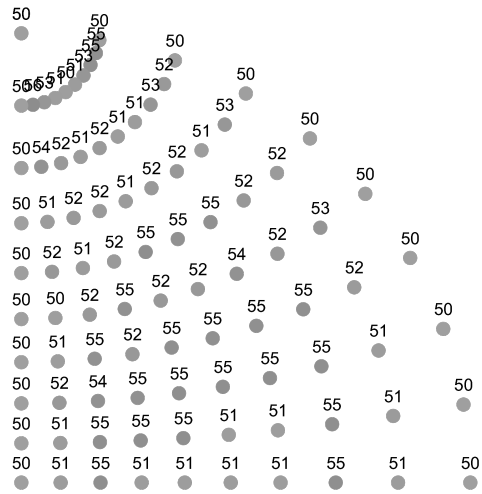


Figure 4.24: Inversion of $\frac{\lambda_1}{\lambda_2^{iso}}$ value using Nelder-Mead algorithm for water-filled cracks.

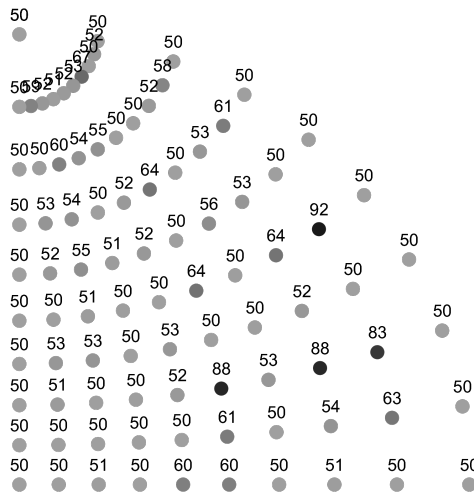


Figure 4.25: Inversion of $\frac{\lambda_1}{\lambda_2^{iso}}$ value using random search algorithm for water-filled cracks.

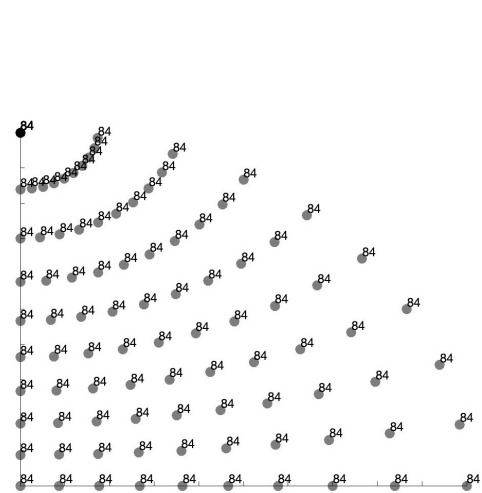


Figure 4.26: Inversion of $\frac{\lambda_1}{\lambda_2^{iso}}$ value using grid search algorithm for water-filled cracks.

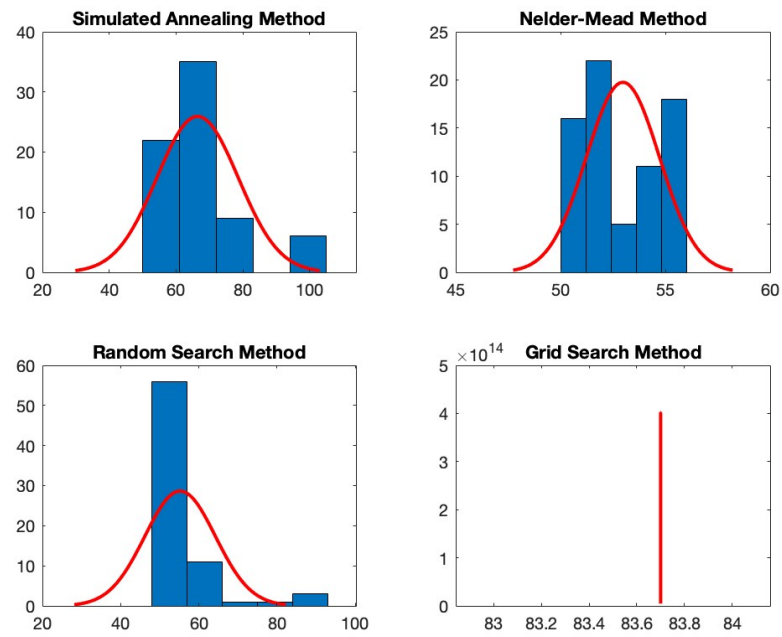


Figure 4.27: The histograms of the percentage of $\frac{\lambda_1}{\lambda_2^{iso}}$ for water-filled cracks. The true value is %100.

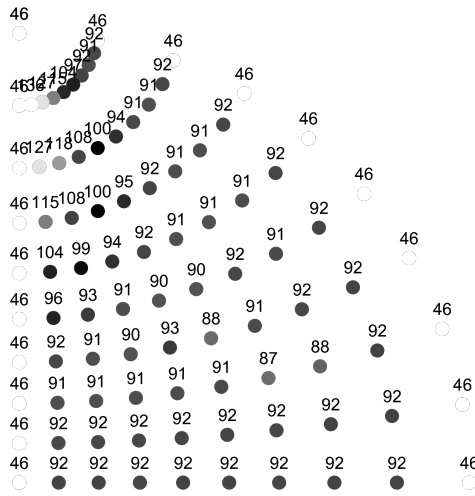


Figure 4.28: Inversion of $\frac{\lambda_2}{\lambda_2^{iso}}$ value using simulated annealing algorithm for water-filled cracks.

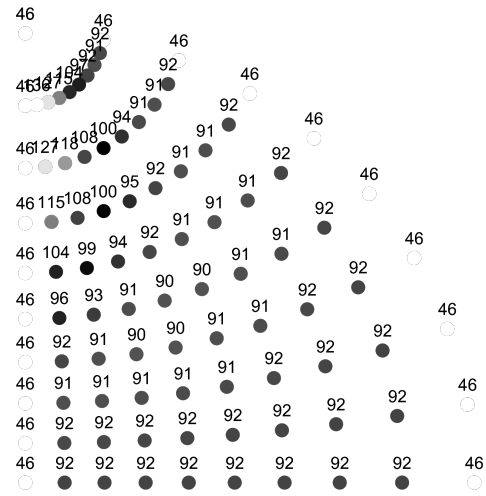


Figure 4.29: Inversion of $\frac{\lambda_2}{\lambda_2^{iso}}$ value using Nelder-Mead algorithm for water-filled cracks.

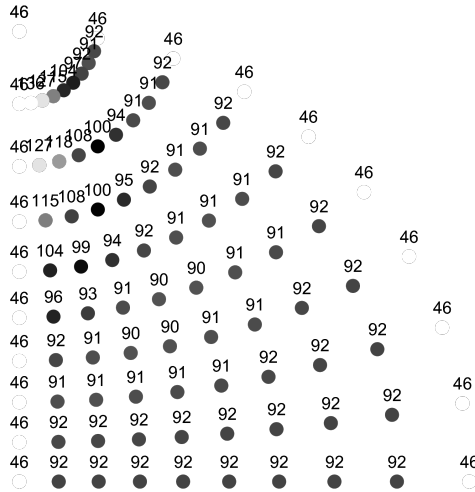


Figure 4.30: Inversion of $\frac{\lambda_2}{\lambda_2^{iso}}$ value using random search algorithm for water-filled cracks.

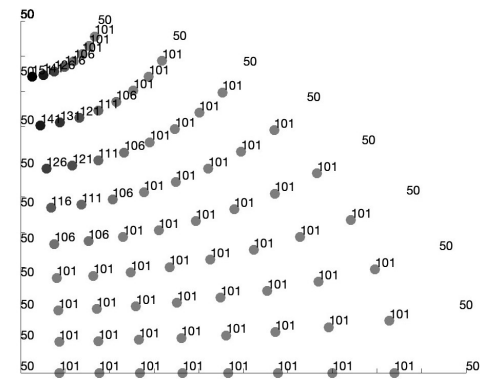


Figure 4.31: Inversion of $\frac{\lambda_2}{\lambda_2^{iso}}$ value using grid search algorithm for water-filled cracks.

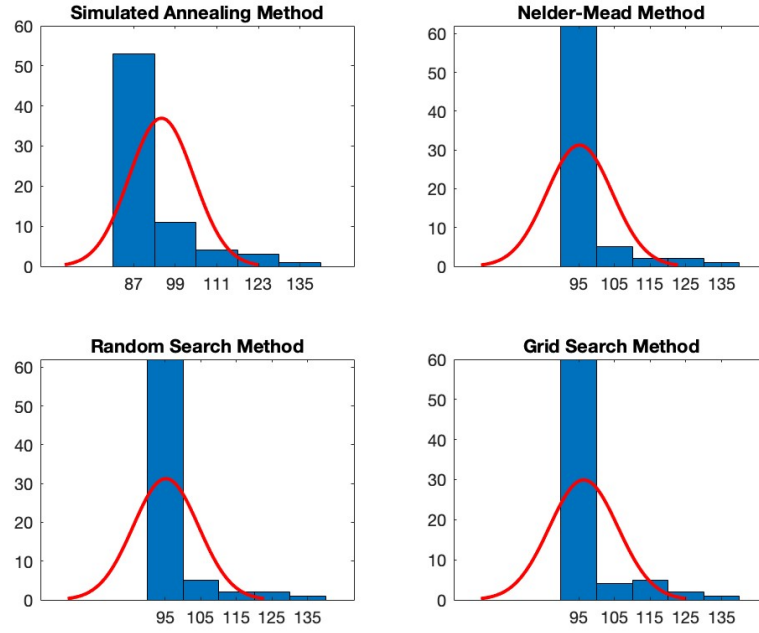


Figure 4.32: The histograms of the percentage of $\frac{\lambda_2}{\lambda_2^{iso}}$ for water-filled cracks. The true value is %100.

4.3. Results of Inversion for Dry Cracks

Considering Table 4.3 and Figure 4.37, the SA method yields highly accurate results and closely approximates the actual value for γ . However, the standard deviation of the results of the NM algorithm for γ is lower, which shows us that it produces more realistic results for all angles.

Table 4.3: The value of mean and standard deviation computed by algorithms for dry cracks.

Dry Cracks			
	$\frac{\lambda_1}{\lambda_2^{iso}}$	$\frac{\lambda_2}{\lambda_2^{iso}}$	γ
Original Value	2.51688	0.920552	0.55079
Simulated Annealing	2.347245	0.854218	0.4784
	0.2944648	0.07774	0.116265
Nelder-Mead	2.61933	0.878860	0.55384881
	0.22435	0.082310	0.05676
Random Search	2.15726	0.831336	0.39193
	0.32978	0.078727	0.131300
Grid Search	2.439726	0.8616438	0.4945205
	0.310237	0.079721	0.098093

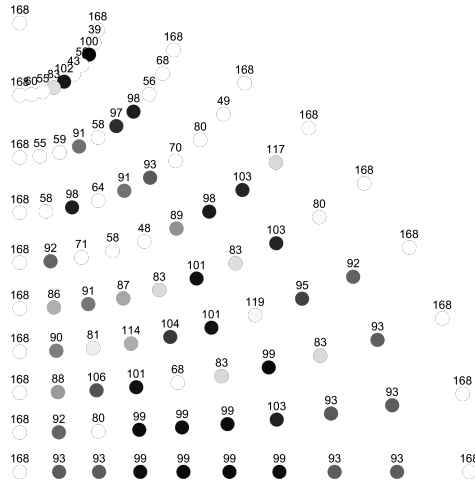


Figure 4.33: Inversion of γ value using simulated annealing algorithm for dry cracks

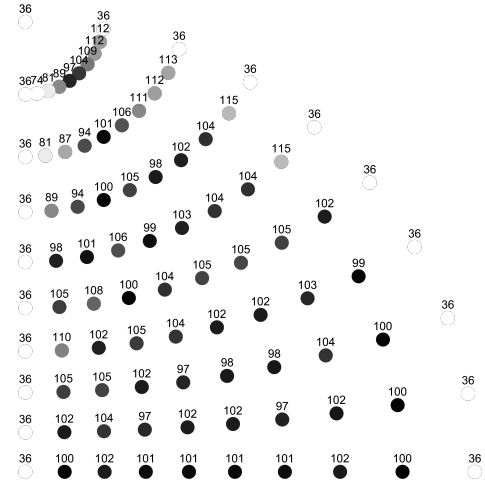


Figure 4.34: Inversion of γ value using Nelder-Mead algorithm for dry cracks

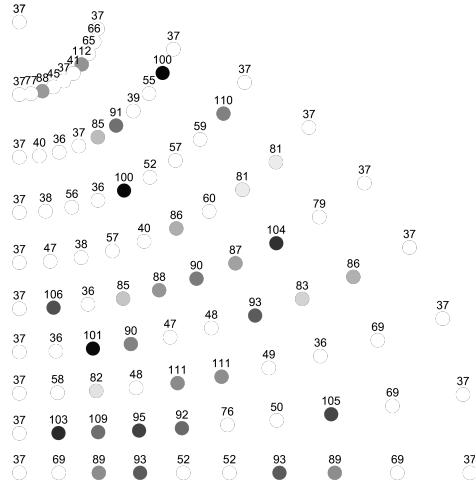


Figure 4.35: Inversion of γ value using random search algorithm for dry cracks.

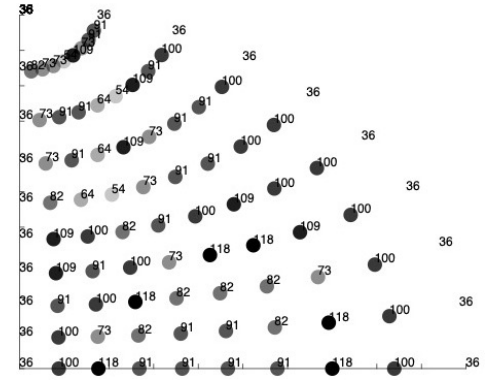


Figure 4.36: Inversion of γ value using grid search algorithm for dry cracks

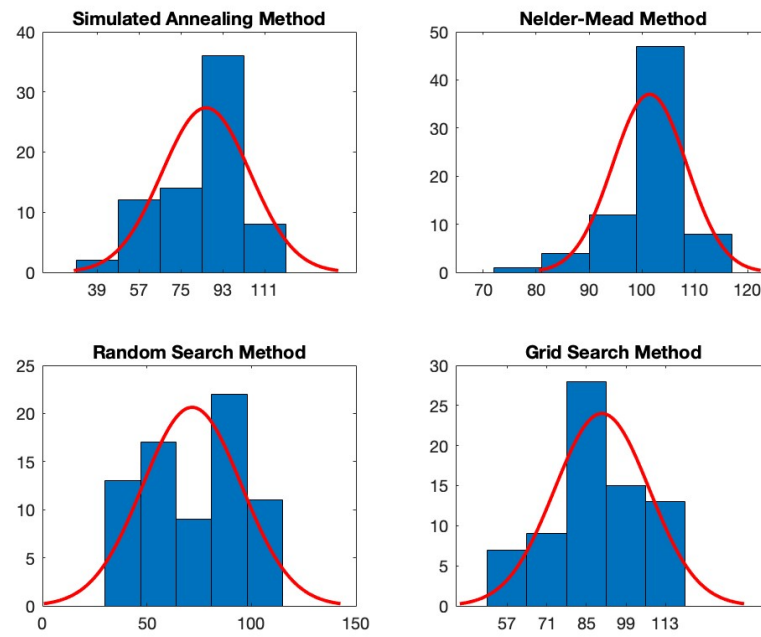


Figure 4.37: The histograms of the percentage of γ for dry cracks. The true value is %100.

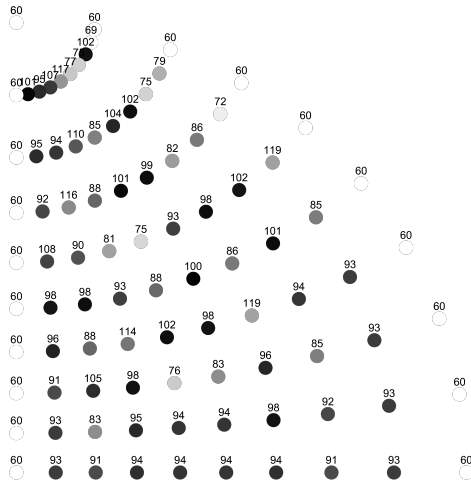


Figure 4.38: Inversion of $\frac{\lambda_1}{\lambda_2^{iso}}$ value using simulated annealing algorithm for dry cracks.

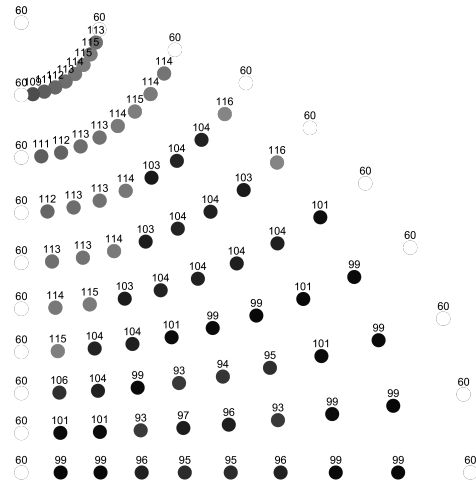


Figure 4.39: Inversion of $\frac{\lambda_1}{\lambda_2^{iso}}$ value using Nelder-Mead algorithm for dry cracks.

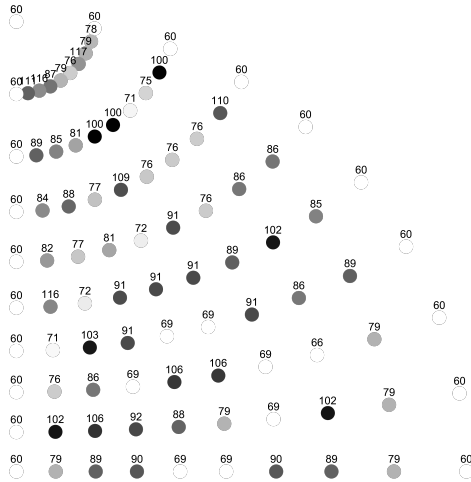


Figure 4.40: Inversion of $\frac{\lambda_1}{\lambda_2^{iso}}$ value using random search algorithm for dry cracks.

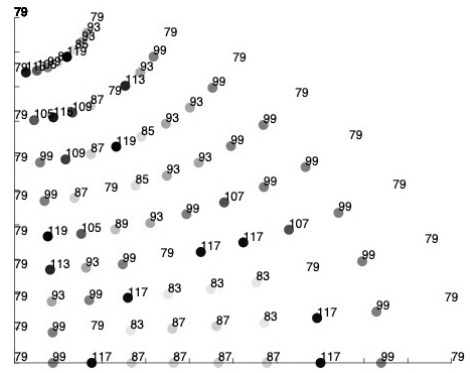


Figure 4.41: Inversion of $\frac{\lambda_1}{\lambda_2^{iso}}$ value using grid search algorithm for dry cracks.

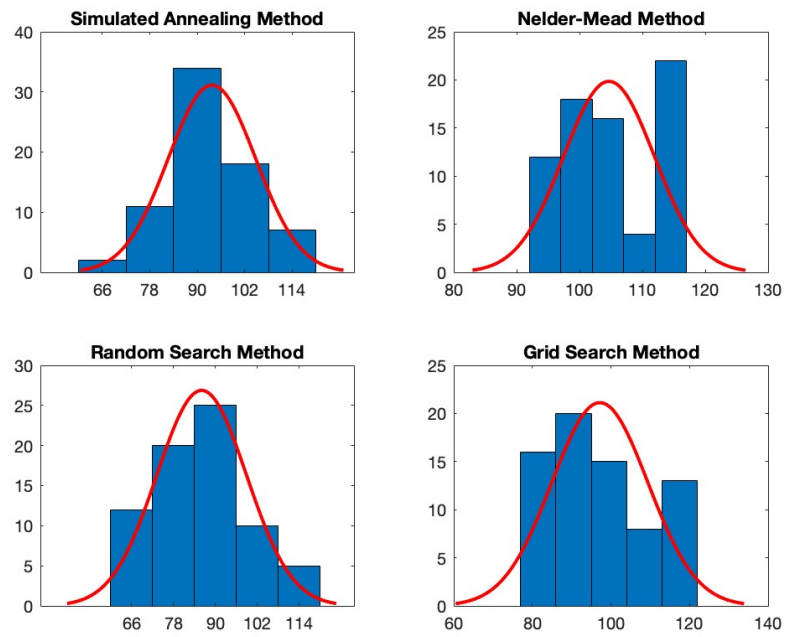


Figure 4.42: The histograms of the percentage of $\frac{\lambda_1}{\lambda_2^{iso}}$ for dry cracks. The original value of the potency is %100.

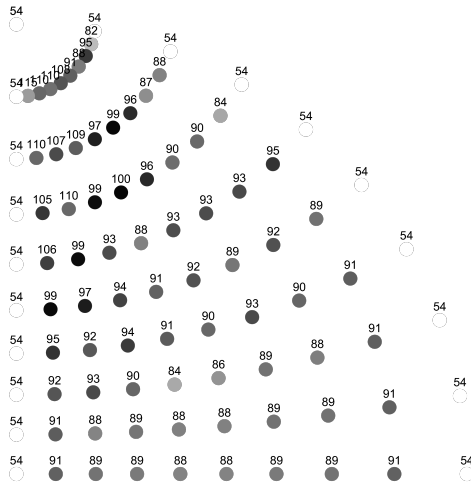


Figure 4.43: Inversion of $\frac{\lambda_2}{\lambda_2^{iso}}$ value using simulated annealing algorithm for dry cracks.

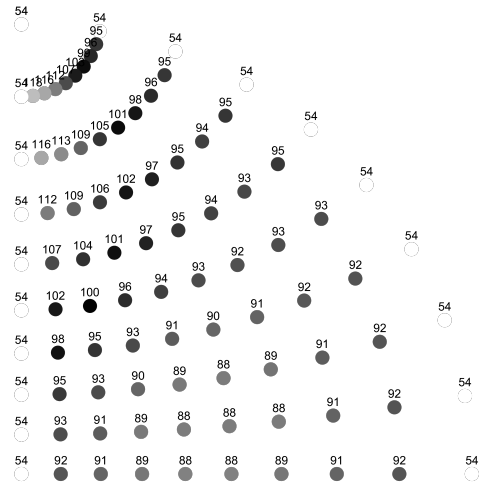


Figure 4.44: Inversion of $\frac{\lambda_2}{\lambda_2^{iso}}$ value using Nelder-Mead algorithm for dry cracks.

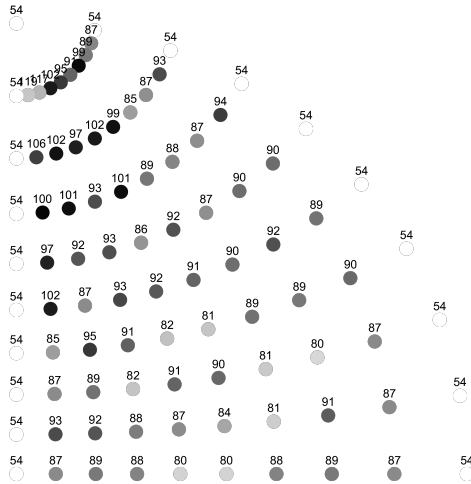


Figure 4.45: Inversion of $\frac{\lambda_2}{\lambda_2^{iso}}$ value using random search algorithm for dry cracks.

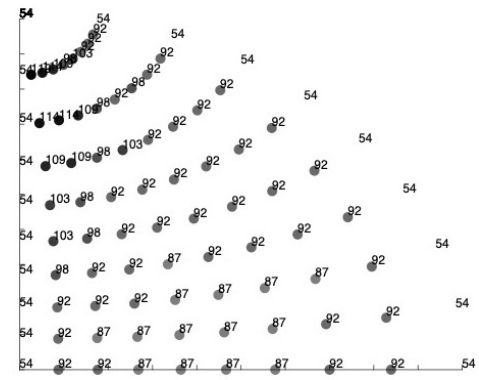


Figure 4.46: Inversion of $\frac{\lambda_2}{\lambda_2^{iso}}$ value using grid search algorithm for dry cracks.

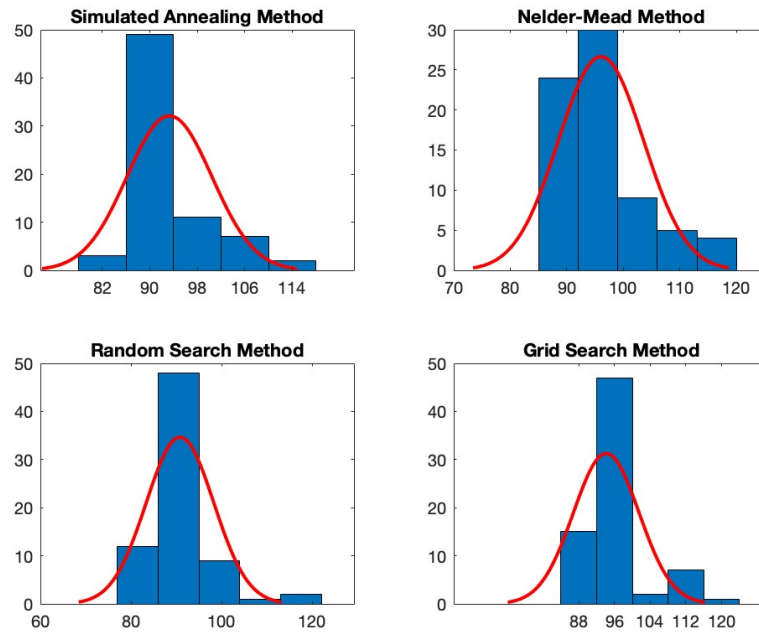


Figure 4.47: The histograms of the percentage of $\frac{\lambda_2}{\lambda_2^{iso}}$ for dry cracks. The true value is %100.

5. CONCLUSION

This thesis describes a new misfit function for the inversion of the elasticity tensor parameters of the focal region. The main purpose of this inversion is to obtain the ratios of the eigenvalues of the elasticity tensor, which generates the moment tensor. We use four different optimization methods in machine learning to perform the inversion process. Moreover, these methods produced different results when solving parameters. These differences between results are due to many reasons, e.g. the structure of the algorithms used in the inversion.

The Grid Search (GS) algorithm can compute γ parameters with a high percentage. However, the same high percentage is not seen in the solution of $\frac{\lambda_1}{\lambda_2^{iso}}$ and $\frac{\lambda_2}{\lambda_2^{iso}}$. In particular, $\frac{\lambda_1}{\lambda_2^{iso}}$ is inverted with a low resolution by GS. The ratio $\frac{\lambda_1}{\lambda_2^{iso}}$ depends on γ parameter in the misfit function (Equation 3.18). This is the reason for the low resolution in the calculation of $\frac{\lambda_1}{\lambda_2^{iso}}$ by GS. The GS method treats each parameter independently. In other words, it does not consider their potential relations or dependencies. The incapability of GS to calculate a solution for water-filled cracks may arise from the high dependence on parameters in this focal region. Moreover, the parameter values are limited to the user-defined grid. GS could not pick the optimal configuration if the optimal values are between the grid points. GS can be a sensible selection if computational resources are limited or if the parameter space is not very high-dimensional. Hence, other methods are more effective for complex problems and more extensive search spaces.

Similar to GS, Random Search (RS) treats parameters independently. Therefore, RS calculated the $\frac{\lambda_1}{\lambda_2^{iso}}$ values at a low resolution. Moreover, the iteration number can be set to a high-value so that RS can generate a higher resolution. Then, the computation time takes longer. Unlike grid search, the random search is more adaptable to different search spaces. Because of that, it does not require a defined grid of parameter values. Besides, RS can make use of the probability distributions for parameters. This enables

RS to do more intelligent sampling in the search space.

Although Simulated Annealing (SA) can solve the parameters with a high resolution, it solves the $\frac{\lambda_1}{\lambda_2^{iso}}$ with a low percentage if the strength of anisotropy of the focal region is high, except for some dip and rake angles. The global minimum could not be found because of the increase in the dependence between the parameters due to the increase in anisotropy and the approximations included in the misfit function. On the other hand, SA produces better optimal solutions than other algorithms, and the run-time is also shorter. The probabilistic structure of the SA algorithm allows it to accept worse solutions initially to escape local optima. Thus it explores the search space more broadly. As a result, SA is a robust optimization algorithm for global optimization problems, mainly in case objective functions are highly non-linear.

While Nelder-Mead (NM) algorithm outperforms the SA algorithm for focal regions with low anisotropy, it falls behind the SA algorithm when the strength of anisotropy increases in the focal region. This is because the NM algorithm does not adapt its step size or simplex design during the minimization process. This causes inadequate investigation of the search space.

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