INTERSECTIONS OF S-BRANES WITH WAVES AND MONOPOLES

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ABSTRACT

INTERSECTIONS OF S-BRANES WITH WAVES AND MONOPOLES

In this thesis we study static and time dependent solutions of supergravity theories. We discuss p-branes, plane waves, Kaluza-Klein monopoles and time dependent S-brane solutions. We then proceed to describe the Kaluza-Klein dimensional reduction procedure and discuss how theories in lower dimensions can be obtained from theories in higher dimensions. As the main result of this thesis we present new solutions of supergravity theories involving intersections of S-branes with plane waves and Kaluza-Klein monopoles. We find that configurations involving intersections of S-branes with waves are restricted in that the wave can be placed only on the transverse space of the S-brane and the transverse space must be flat. We also find that a larger number of configurations involving intersections of S-branes with Kaluza-Klein monopoles exist. As a potential application we consider adding an S-brane to ten dimensional solutions that describe extremal black holes in lower dimensions, which in turn could lead to black holes in a time dependent background. However, we find that in these configurations the conditions imposed on the integration constants render the metric time independent.

ÖZET

S-ZARLARIN DALGALAR VE MONOPOLLERLE KESİŞİMLERİ

Bu tezde süperçekim kuramlarının zamandan bağımsız ve zamana bağlı çözümlerini inceleyeceğiz. Dalgalari, p-zarlarini, Kaluza-Klein monopollerini ve zamana bağlı S-zar çözümlerini tartışacağız. Bunun ardından Kaluza-Klein boyutsal indirgeme kuramını çalışacağız ve düşük boyutlardaki kuramların bu yolla yüksek boyutlardaki kuramlardan nasıl elde edildiğini göreceğiz. Temel sonuç olarak süperçekim kuramlarının S-zarlarının dalgalarla ve Kaluza-Klein monopolleriyle kesişimlerini içeren yeni çözümlerini sunacağız. Bulgularımız S-zarlarının dalgalarla kesişimlerinde dalganın S-zarının sadece dış uzayına yerleştirilebileceğini ve dış uzayın düz seçilmesi gerektiğini gösteriyor. S-zarlarının Kaluza-Klein monopolleriyle kesişimlerindeyse daha fazla sayıda seçeneğin bulunduğunu gösterecegiz. Olası bir uygulama olarak düşük boyutlarda uç kara delikleri tasvir eden on boyutlu çözümlere S-zarı eklemeyi düşüneceğiz; bu olası çözüm zamana bağlı bir arka planda bir kara delik çözümü verirdi. Ancak göreceğiz ki bu çözümlerde integral sabitlerine gelen koşullar metriği zamandan bağımsız kılacak.

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1. INTRODUCTION

Two very successful theories revolutionized our understanding of nature in the twentieth century: general relativity and quantum field theory. These theories have been very successful in their domains of applicability; general relativity in explaining large scale phenomena (planets, galaxies, black holes, evolution of the universe) and quantum field theory in explaining small scale phenomena (atoms, nuclei, elementary particles). The necessity to unify these theories reveals itself in many facets, among them black holes where spacetime gets highly curved in a small area and quantum mechanical effects become important [1]. String theory attempts to reconcile these two theories. According to string theory, fundamental objects of nature are tiny strings, various modes of oscillation of which give rise to the particles we are observing.

The critical dimension for the five superstring theories which contain in their spectra the bosons and fermions we observe is ten; this is required for flat space to be a solution of these theories [2]. All five superstring theories can be obtained from the conjectured M-theory which lives in eleven spacetime dimensions and as such it is considered as the fundamental theory. The discovery that revealed eleven dimensional supergravity as the low energy limit of M-theory gave it a prominent role in the search for a quantum theory of gravity. Low energy limits of Type IIA and Type IIB string theories are the Type IIA and Type IIB supergravity theories [3].

In this thesis we review static and time dependent solutions of ten and eleven dimensional supergravity theories with large amount of symmetry. An important class of solutions of a D-dimensional supergravity theory comprises the p-branes, extended objects of p spatial dimensions and $d = p + 1$ dimensional worldvolume that have $(\text{Poincaré})_d \times \text{SO(D-d)}$ symmetry [2,4,5]. Associated with a p-brane is a q-form field strength that obeys the Bianchi identity $dF_{[q]} = 0$. Thus, p-branes carry charge. There exist also purely geometric solutions: the Kaluza-Klein monopole and the plane wave [6-8].

A class of time dependent solutions of supergravity theories comprise the S-branes. An Sp-brane can be viewed as an object that has a purely spacelike $p+1$ dimensional worldvolume and exists for an instant in time. Like p-branes, S-branes carry charge and they are identified by hyperbolic functions of time [9,10].

Having presented the p-brane, Sp-brane, Kaluza-Klein monopole and plane wave solutions of supergravity theories we proceed to discuss Kaluza-Klein dimensional reduction. The universe as we observe it has four spacetime dimensions and what relates the string/M-theory solutions to the physics of our world is the dimensional reduction of these theories over a compact space. Reduction of $D=11$ M-theory solutions to $D=10$ string theory solutions are performed over a circle, S^1, and in Section 6.1.1 we discuss how solutions of Type IIA supergravity can be obtained from eleven dimensional supergravity via such a procedure. We discuss in Section 6.1.2 how S-brane solutions in $D=10$ can be obtained by dimensional reduction from S-brane solutions in $D=11$.

An important topic in this thesis is intersections of branes. Intersections of p-branes, Sp-branes, waves and monopoles is a vast subject and many allowed configurations have been catalogued [6,11-15]. Intersecting p-brane solutions obey a simple *harmonic function rule* and various intersections can be obtained using this rule as discussed in Chapter 8. Among these configurations are those that give rise to extremal black holes in $D=5$ and $D=4$ upon dimensional reduction [1,16].

In Chapter 7 we present intersections of S-branes with waves and Kaluza-Klein monopoles in $D=11$. We find that a wave can be added only on the transverse space of an S-brane, restriction due to a condition on the constants of integration brought about by the wave and roots in the structure of the metric. A single S-brane can have a constant curvature spatial transverse space, however, when a wave is added to this configuration only flat transverse space is allowed. Several configurations involving intersections with Kaluza-Klein monopoles exist and these are listed in Section 7.2.

In Section 7.3 we consider some of the multiple intersections that involve S-branes, waves and monopoles. A natural application of interest is configurations that may reduce in lower dimensions to black holes in time dependent backgrounds. However, we find that, for example in the SD1-D1-D5-Wave intersection, the wave condition is too strong and the charge of the SD1-brane has to be set to zero and the metric loses its time dependence. Having found that these new configurations merely impose a condition on the constants of integration in the S-brane solution, it does not seem likely that they could lead to new cosmological solutions upon dimensional reduction.

2. THE ACTION AND THE FIELD EQUATIONS

The general action in D dimensions describing the bosonic sector of various supergravity theories comprising the metric g_{MN}, a $(q-1)$-form gauge potential $A_{[q-1]}$ with corresponding field strength $F_{[q]} = dA_{[q-1]}$ and a dilaton scalar field ϕ coupled to the form field with the coupling constant a is given in the Einstein frame as

$$I = \int d^D x \sqrt{-g} \left[R - \frac{1}{2} \partial_M \phi \partial^M \phi - \frac{1}{2q!} e^{a\phi} F^2_{[q]} \right] . \qquad (2.1)$$

Varying the action (1.1) with respect to the fields g_{MN}, $A_{[q-1]}$ and ϕ gives the field equations as

$$R_{MN} - \frac{1}{2}\partial_M \phi \partial_N \phi - \frac{e^{a\phi}}{2(q-1)!} \left[F_{MA_2 \ldots A_q} F_N{}^{A_2 \ldots A_q} - \frac{q-1}{q(D-2)} F^2_{[q]} g_{MN} \right] = 0, \qquad (2.2)$$

$$\partial_M \left(\sqrt{-g} e^{a\phi} F^{MA_1 \ldots Aq-1} \right) = 0, \qquad (2.3)$$

$$\frac{1}{\sqrt{-g}} \partial_M \left(\sqrt{-g} \partial^M \phi \right) - \frac{a}{2q!} e^{a\phi} F^2_{[q]} = 0. \qquad (2.4)$$

It is sometimes more illuminating to write the action in the language of differential forms

$$I = \int \left[R * 1 - \frac{1}{2} d\phi \wedge * d\phi - \frac{1}{2q!} e^{a\phi} F_{[q]} \wedge * F_{[q]} \right] . \qquad (2.5)$$

The Bianchi identity and the field equation for the form field, the latter of which is written in component form in Equation 2.3, are $dF = 0$ and $d[e^{a\phi} * F] = 0$. However, we should note that we have omitted in the action a term of the form $F \wedge F \wedge A$, called the Chern-Simons term. This term is required by local supersymmetry and in the presence of such a term the equations for the form field are modified as

$$dF = 0, \qquad (2.6)$$

$$d[e^{a\phi} * F] = F \wedge F. \qquad (2.7)$$

For the type of solutions of the field equations we are going to study the $F \wedge F \wedge A$ term and its variation vanish, that is, the system is equivalent to the one given in Equations 2.2-2.4. In the absence of this term it is easy to see that the action is invariant under the following discrete transformation, called the S-duality:

$$g_{MN} \to g_{MN}, \qquad F \to e^{-a\phi} * F, \qquad \phi \to -\phi. \tag{2.8}$$

3. THE BRANE SOLUTIONS OF STRING/M-THEORY

In this chapter we present the classical p-brane solutions of the supergravity equations of motion. For this basic class of solutions we make an ansatz requiring (Poincaré)$_d\times$SO(D-d) symmetry. One may view the solutions this ansatz brings as flat $d = p+1$ dimensional hypersurfaces embedded in the ambient D-dimensional spacetime and these hypersurfaces may be viewed as histories, or worldvolumes, of p-dimensional spatial surfaces [2]. The hyperspace with SO(D-d) symmetry is called the *transverse space* of the p-brane.

In accordance with the stated ansatz we split the spacetime coordinates into two as $x^M = (x^\mu, y^m)$ where x^μ $(\mu = 0, 1, ..., p = d-1)$ are the worldvolume coordinates with (Poincaré)$_d$ symmetry and y^m $(m = 1, ..., D-d)$ are the coordinates transverse to the worldvolume. A metric ansatz respecting the (Poincaré)$_d\times$SO(D-d) symmetry is

$$ds^2 = e^{A(r)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2B(r)}\delta_{mn}dy^m dy^n \tag{3.1}$$

where $r = \sqrt{\delta_{mn}y^m y^n}$. The ansatz for the dilaton is simply $\phi = \phi(r)$.

For the antisymmetric field strength, $F_{[q]}$, we face a bifurcation of possibilities for the ansatz, the two possibilities being related by the S-duality described in Equation 2.8. The first possibility, called the *electric* ansatz, is naturally expressed in terms of the gauge potential $A_{[q-1]}$. Just as in four dimensions the gauge potential, a one-form, couples to the worldline of a charged particle, a 0-brane, $A_{[q-1]}$ couples to the worldvolume of a $p = d-1 = (q-1)-1$ dimensional charged object, the p-brane. Thus in the electric ansatz $A_{[q-1]}$ supports a $d_{el} = q-1$ dimensional worldvolume and its nonzero components are

$$A^{(el)}_{\mu_1\ldots\mu_{(q-1)}} = \tilde{\epsilon}_{\mu_1\ldots\mu_{(q-1)}}e^{C(r)}. \tag{3.2}$$

Here $\tilde{\epsilon}_{\mu_1 \ldots \mu_{(q-1)}}$ denotes the antisymmetric symbol and is related to the antisymmetric tensor, denoted without a tilde, as $\epsilon_{\mu_1 \ldots \mu_{(q-1)}} = \sqrt{|^x g|}\tilde{\epsilon}_{\mu_1 \ldots \mu_{(q-1)}}$ where $^x g$ denotes the determinant of the $\{x^\mu\}$-space. The corresponding field strength reads

$$F^{(el)}_{m\mu_1 \ldots \mu_{(q-1)}} = \tilde{\epsilon}_{\mu_1 \ldots \mu_{(q-1)}} \partial_m e^{C(r)}. \tag{3.3}$$

In light of the S-duality the second possibility of relating the rank q of $F_{[q]}$ to the worldvolume dimension of an extended object emerges. The symmetry of the Bianchi identity and the field equation for the form field suggest that the underlying gauge potential for $*F$, which is guarenteed to exist locally by courtesy of Equation 2.7 with its right hand side equal to zero, would naturally couple to a $d_{mag} = D - q - 1$ dimensional worldvolume since $*F$ is a $(D - q)$-form. Instead of writing this dualized gauge potential which would be nonlocally related to the other fields, we write the *magnetic* ansatz for the field strength $F_{[q]}$ which has nonvanishing components only in the transverse directions,

$$F^{(mag)}_{m_1 \ldots m_q} = \lambda \tilde{\epsilon}_{m_1 \ldots m_q p} \frac{y^p}{r^{q+1}} \tag{3.4}$$

where λ, the magnetic charge parameter, is a constant. The power of r in the magnetic ansatz is fixed by requiring $F_{[q]}$ to satisfy the Bianchi identity, $dF = 0$. This guarantees the existence of a gauge potential for $F_{[q]}$ which is necessary; we claim to have carried on with the variation of the action with respect to this gauge potential.

One should observe that the worldvolume dimensions of the electric and magnetic solutions are related by $d_{mag} = \hat{d}_{el} \equiv D - d_{el} - 2$ and this relation is idempotent; $\hat{\hat{d}} = d$.

The Ricci tensor for the p-brane metric is given in Appendix B.1. Denoting the derivative with respect to r with a prime, Einstein equations and the dilaton equation for the electric and magnetic ansäzte are

$$
\begin{aligned}
A'' + d(A')^2 + \tilde{d}A'B' + \frac{(\tilde{d}+1)}{r}A' &= \frac{\tilde{d}}{2(D-2)}S^2, \\
B'' + dA'B' + \tilde{d}(B')^2 + \frac{(2\tilde{d}+1)}{r}B' + \frac{d}{r}A' &= -\frac{d}{2(D-2)}S^2, \\
\tilde{d}B'' + dA'' - 2dA'B' + d(A')^2 - \tilde{d}(B')^2 - \frac{\tilde{d}}{r}B' - \frac{d}{r}A' + \frac{1}{2}(\phi')^2 &= \frac{1}{2}S^2, \\
\phi'' + dA'\phi' + \tilde{d}B'\phi' + \frac{\tilde{d}+1}{r}\phi' &= -\frac{1}{2}\zeta a S^2
\end{aligned}
\tag{3.5}
$$

where the source appearing on the right hand sides of these equations is

$$
S = \begin{cases} (e^{\frac{1}{2}a\phi - dA + C}(C') & \text{electric: } d = q-1, \quad \zeta = +1, \\ \lambda(e^{\frac{1}{2}a\phi - \tilde{d}B})r^{-\tilde{d}-1} & \text{magnetic: } d = D-q-1, \quad \zeta = -1. \end{cases}
\tag{3.6}
$$

The following condition simplifies the field equations

$$
dA' + \tilde{d}B' = 0. \tag{3.7}
$$

After eliminating $B(r)$ using Equation 3.7 the independent equations become

$$
\nabla^2\phi = -\frac{1}{2}\zeta a S^2 \tag{3.8}
$$

$$
\nabla^2 A = \frac{\tilde{d}}{2(D-2)}S^2 \tag{3.9}
$$

$$
d(D-2)(A')^2 + \frac{1}{2}\tilde{d}(\phi')^2 = \frac{1}{2}\tilde{d}S^2 \tag{3.10}
$$

where $\nabla^2 \equiv \delta_{mn}y^m y^n$ is the Laplacian in the transverse space and for spherically symmetric functions it is given as $\nabla^2\phi = \phi'' + (\tilde{d}+1)r^{-1}\phi'$.

Equations 3.8 and 3.9 suggest we impose

$$
\phi' = \frac{-\zeta a(D-2)}{\tilde{d}}A'. \tag{3.11}
$$

It is useful to define a new variable, Δ,

$$a^2 = \Delta - \frac{2d\tilde{d}}{(D-2)}. \tag{3.12}$$

With this new variable Equation 3.10 becomes

$$S^2 = \frac{\Delta(\phi')^2}{a^2} \tag{3.13}$$

and the equation for the dilaton becomes $\nabla^2\phi + \frac{\zeta\Delta}{2a}(\phi')^2 = 0$. This can be put into the form of a Laplace equation, defining a harmonic function, as

$$\nabla^2 e^{\frac{\zeta\Delta}{2a}\phi} = 0. \tag{3.14}$$

With the boundary condition $\phi|_{r\to\infty} = 0$, the spherically symmetric solution of the Laplace equation in $D - d = \tilde{d} + 2$ dimensions is given as

$$e^{\frac{\zeta\Delta}{2a}\phi} \equiv H(r) = 1 + \frac{k}{r^{\tilde{d}}}. \tag{3.15}$$

For the electric ansatz, with $\zeta = +1$, it remains to solve for $C(r)$ appearing in the ansatz for $F_{[q]}$. It follows from Equations 3.6 and 3.13

$$S^2 = e^{a\phi - 2dA}(C'e^C)^2 = \frac{\Delta(\phi')^2}{a^2}. \tag{3.16}$$

With $a < 0$, this gives

$$(e^C)' = -\frac{\sqrt{\Delta}}{a}\phi' e^{-\frac{1}{2}a\phi + Ad} \tag{3.17}$$

Indeed, this relation is in consistence with the field equation for $F[q]$, Equation 2.3,

$$\nabla^2 C + (C')^2 + \tilde{d}C'B' - dC'A' + aC'\phi' = 0. \tag{3.18}$$

Fixing the constants of integration by the requirement that the solution tends to flat empty space at transverse infinity, that is, $A|_{r\to\infty} = B|_{r\to\infty} = 0$ we summarize the solution

$$\begin{aligned} ds^2 &= H^{\frac{-4\tilde{d}}{\Delta(D-2)}} dx^\mu dx^\nu \eta_{\mu\nu} + H^{\frac{4d}{\Delta(D-2)}} dy^m dy^n \delta_{mn}, && (3.19)\\ e^\phi &= H^{\frac{2a}{\zeta\Delta}}\ , \qquad \zeta = \begin{cases} +1, & \text{electric} \\ -1, & \text{magnetic} \end{cases} && (3.20)\\ H(r) &= 1 + \frac{k}{r^{\tilde{d}}}. && (3.21) \end{aligned}$$

In the electric case we have

$$e^C = \frac{2}{\sqrt{\Delta}} H^{-1}. \qquad (3.22)$$

In the magnetic case the constant of integration, k, is related to the magnetic charge parameter λ by

$$k = \frac{\sqrt{\Delta}}{2\tilde{d}} \lambda. \qquad (3.23)$$

3.1. Brane Solutions in Eleven and Ten Dimensions

In this section we present the described brane solutions in eleven and ten dimensions. The low energy effective action of M-theory is that of eleven dimensional supergravity. The bosonic fields of $D = 11$ supergravity are the metric and the three form gauge potential. The action can thus be obtained from Equation 2.1 by setting $q = 4$ and setting $a = 0$ since there is no dilaton field in the theory.

The solutions are the electric M2-brane and the magnetic M5-brane. The M2-brane has a three dimensional worldvolume and the solution can be read from Equations 3.19-3.23 by setting $d = 3,\ \zeta = +1$ and the M5-brane has a six dimensional worldvolume with $d = 6,\ \zeta = -1$.

Brane solutions in ten dimensions are richer. The low energy limits of Type IIA and Type IIB superstring theories are the Type IIA and Type IIB supergravity theories, respectively. Setting the fermionic fields to zero the bosonic Type II supergravity action can be written in terms three sectors: the Neveu-Schwarz-Neveu-Schwarz sector (NSNS), the Ramond-Ramond sector (RR) and the Chern-Simons sector (CS). The antisymmetric tensors, $F_{[q]} = dA_{[q-1]}$, belonging to the solutions we are going to discuss, including the intersecting brane solutions in Chapter 8, have $F \wedge F = 0$ and the CS sector involving terms proportional to $F \wedge F \wedge A$ has no effect on the equations of motion as discussed in Chapter 2.

The NSNS sector is common to both Type IIA and IIB supergravity theories and involves a three form, $H_{[3]} = dB_{[2]}$, which couples to the electric 1-brane called the fundamental string and denoted by F1, and to the magnetic 5-brane called the solitonic fivebrane and denoted by NS5.

In the RR sector we have Dp-branes. The RR sector of Type IIA theory involves forms of even rank, $F_{[2]}$ and $F_{[4]}$, and the RR sector of Type IIB theory involves forms of odd rank, $F_{[1]}$, $F_{[3]}$, $F_{[5]}$. Each q-form field strength $F_{[q]}$ couples to an electric D(q-2)-brane and a magnetic D(8-q)-brane as discussed in the first part of Chapter 3.

The action that describes both Type II theories can be written, in the Einstein frame, as

$$S = \int d^{10}x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2.3!}e^{-\phi}H_{[3]}^2 - \frac{1}{2}\sum_n \frac{1}{q!}e^{\frac{5-q}{2}\phi}F_{[q]}^2\right]. \qquad (3.24)$$

4. KALUZA-KLEIN MONOPOLES AND PLANE WAVES

Kaluza Klein monopoles and plane waves are purely gravitational solutions of eleven dimensional supergravity. They also have counterparts in lower dimensions. We denote the eleven dimensional wave and the Kaluza-Klein monopole by $\mathcal{W}$ and $\mathcal{KK}$, respectively and their ten dimensional counterparts by W and KK in the subsequent chapters.

The plane wave in D dimensions is given by the metric

$$ds^2 = (H-2)dt^2 + Hdz^2 + 2(1-H)dtdz + (dx_1^2 + ... + dx_{(D-2)}^2). \tag{4.1}$$

Vacuum Einstein equations, which describe a purely gravitational solution, require H to be harmonic function in the variables $\{t+z, x_1, ..., x_{(D-2)}\}$. One can impose some of these coordinates as isometry directions and still have a solution. The metric for the plane wave can be rewritten as

$$ds^2 = -H^{-1}dt^2 + H\big[(H^{-1}-1)dt + dz\big]^2 + (dx_1^2 + ... + dx_{(D-2)}^2) \tag{4.2}$$

where construction of an orthonormal frame is now easy.

The metric for the Kaluza-Klein monopole in D dimensions reads, for $i = 1, 2, 3$,

$$ds^2 = -dt^2 + dx_1^2 + ... + dx_{(D-5)}^2 + H^{-1}(dz + J_i dy_i)^2 + H(dy_1^2 + dy_2^2 + dy_3^2) \tag{4.3}$$

where H and $\{J_i\}$ depend on $\{y_i\}$, H is a harmonic function and the relation between H and $\{J_i\}$ is

$$F_{ij} \equiv \partial_i J_j - \partial_j J_i = \epsilon_{ijk}\partial_k H. \tag{4.4}$$

A special case of the given configuration is when one of the y_i's correspond to an isometry direction. Suppose y_1 is taken as an isometry direction. Then Equation 4.4 becomes

$$\begin{aligned} \partial_2 J_3 - \partial_3 J_2 &= 0 \\ \partial_3 J_1 &= \partial_2 H \\ -\partial_2 J_1 &= \partial_3 H \end{aligned} \tag{4.5}$$

In this case one can set J_1 and J_2 equal to zero by a coordinate transformation; they can be gauged away. One should no longer assume spherical symmetry in the $\{y_i\}$-space and observe that both H and J_1 are harmonic in the variables y_2 and y_3.

5. S-BRANES

S-branes are time dependent solutions of supergravity theories and they can be pictured as analogs of the static p-branes with spacelike worldvolumes [10,17]. We write the metric ansatz for an S(p-1)-brane in d-dimensions with a p-dimensional Euclidean worldvolume and a transverse space comprising of a k-dimensional constant curvature hyperspace, $\Sigma_{k,\sigma}$ and a $(q-k)$ dimensional flat space,

$$ds^2 = -e^{2A}dt^2 + e^{2B}(dx_1^2 + ... + dx_p^2) + e^{2C}d\Sigma_{k,\sigma}^2 + e^{2D}(dy_1^2 + ... + dy_{q-k}^2) \qquad (5.1)$$

parametrized by four time dependent functions $A(t), B(t), C(t), D(t)$. The hyperspace $\Sigma_{k,\sigma}$ is the k-dimensional flat space, sphere and hyperbolic space for $\sigma = 0, +1, -1$, respectively, described as

$$d\Sigma_{k,\sigma}^2 = \begin{cases} d\psi^2 + \sinh^2\psi\ d\Omega_{k-1}^2, & \sigma = -1, \\ d\psi^2 + \psi^2 d\Omega_{k-1}^2, & \sigma = 0, \\ d\psi^2 + \sin^2\psi\ d\Omega_{k-1}^2, & \sigma = +1. \end{cases} \qquad (5.2)$$

Referring to geometric quantities on $\Sigma_{k,\sigma}$ using a tilde sign, with respect to an orthonormal frame on $\Sigma_{k,\sigma}$ we have

$$\tilde{R}_{ab} = \sigma(k-1)\delta_{ab}. \qquad (5.3)$$

The following ansatz for the field strength $F_{[q]}$ solves the corresponding field equation

$$F_{[q]} = Q_s \mathrm{vol}(\Sigma_{k,\sigma}) \wedge dy_1 \wedge ... dy_{q-k} \qquad (5.4)$$

where Q_s, a constant, is the field strength parameter and $\mathrm{vol}(\Sigma_{k,\sigma})$ denotes the volume element on the hyperspace $\Sigma_{k,\sigma}$. The ansatz for the dilaton field is simply $\phi = \phi(t)$.

The calculation of the Ricci tensor is given in Appendix B.2. The Ricci tensor is largely simplified when we impose the gauge condition, where dots denote derivatives with respect to time,

$$-\dot{A} + p\dot{B} + k\dot{C} + (q-k)\dot{D} = 0. \tag{5.5}$$

With the stated gauge, Einstein equations reduce to

$$\begin{aligned}
-\ddot{A} + \dot{A}^2 - p\dot{B}^2 - k\dot{C}^2 - (q-k)\dot{D}^2 - \frac{1}{2}\dot{\phi}^2 - \frac{(q-1)Q_s^2}{2(d-2)}e^{a\phi+2pB} &= 0, && (5.6)\\
\ddot{B} + \frac{(q-1)Q_s^2}{2(d-2)}e^{a\phi+2pB} &= 0, && (5.7)\\
\ddot{C} + \sigma(k-1)e^{2A-2C} - \frac{pQ_s^2}{2(d-2)}e^{a\phi+2pB} &= 0, && (5.8)\\
\ddot{D} - \frac{pQ_s^2}{2(d-2)}e^{a\phi+2pB} &= 0. && (5.9)
\end{aligned}$$

The dilaton equation reads

$$\ddot{\phi} + \frac{aQ_s^2}{2}e^{a\phi+2pB} = 0. \tag{5.10}$$

Equations 5.7, 5.9 and 5.10 have a similar structure. Defining c_1 and c_2 as constants of integration one can write

$$\phi = \frac{a(d-2)}{q-1}B + c_1 t + c_2. \tag{5.11}$$

Similarly, in the most general form Equations 5.7 and 5.9 enable us to relate $B(t)$ and $D(t)$, the exponents in the conformal factors multiplying the worldvolume and a portion of the transverse space, respectively, with the introducion of two constants of integration. Indeed, when we proceed to the intersection of a wave with an S-brane the first constant of integration will be required to be nonvanishing for the solution to support the charged S-brane. These two constants need to be nonvanishing also for the dimensional reduction of eleven dimensional S-branes to ten dimensions to be possible.

In both these tasks we are going to write, introducing c_3 and c_4

$$D = -\frac{p}{q-1}B + c_3 t + c_4 \qquad \text{(in general).} \tag{5.12}$$

However, we follow in this chapter the analysis in [10] where these constants are simply set to zero,

$$D = -\frac{p}{q-1}B. \tag{5.13}$$

Setting the constant that is introduced upon the integration of the gauge condition in Equation 5.5 to zero and with Equations 5.8 and 5.13 we are left with two independent functions $f(t), g(t)$ in terms of which $A(t), B(t), C(t), D(t)$ can be parametrized as

$$A = kg - \frac{p}{q-1}f, \quad B = f, \quad C = g - \frac{p}{q-1}f, \quad D = -\frac{p}{q-1}f. \tag{5.14}$$

With further simplifying notation as

$$\chi = 2p + \frac{a^2(d-2)}{q-1} \tag{5.15}$$

the equations of motion reduce to

$$\ddot{f} + \frac{(q-1)Q_s^2}{2(d-2)}e^{\chi f + ac_1 t + ac_2} = 0, \tag{5.16}$$

$$\ddot{g} + \sigma(k-1)e^{2(k-1)g} = 0, \tag{5.17}$$

$$\frac{p}{q-1}\ddot{f} - k\ddot{g} + k(k-1)\dot{g}^2 - \frac{(d-2)\chi}{2(q-1)}\dot{f}^2 - \frac{1}{2}c_1^2 - \frac{ac_1(d-2)}{(q-1)}\dot{f} - \frac{(q-1)Q_s^2}{2(d-2)}e^{\chi f + ac_1 t + ac_2} = 0. \tag{5.18}$$

Defining $h(t)$, the terms linear in t can be absorbed into f

$$f(t) = h(t) - \frac{ac_1}{\chi}t - \frac{ac_2}{\chi}. \tag{5.19}$$

In terms of $h(t)$ the equations of motion read

$$\begin{aligned}
\ddot{h} + \frac{(q-1)Q_s^2}{2(d-2)} e^{\chi h} &= 0, && (5.20)\\
\ddot{g} + \sigma(k-1)e^{2(k-1)g} &= 0, && (5.21)\\
\frac{p}{q-1}\ddot{h} - k\ddot{g} + k(k-1)\dot{g}^2 - \frac{(d-2)\chi}{2(q-1)}\dot{h}^2 - \frac{pc_1^2}{\chi} - \frac{(q-1)Q_s^2}{2(d-2)} e^{\chi h} &= 0. && (5.22)
\end{aligned}$$

Equations 5.20-5.22 are equivalent to the two first order equations

$$\begin{aligned}
\dot{h}^2 + \frac{(q-1)Q_s^2}{2(d-2)} e^{\chi h} &= \alpha^2, && (5.23)\\
\dot{g}^2 + \sigma e^{2(k-1)g} &= \beta^2, && (5.24)
\end{aligned}$$

provided the integration constants α and β satisfy

$$\frac{pc_1^2}{\chi} + \frac{(d-2)\chi\alpha^2}{2(q-1)} - k(k-1)\beta^2 = 0. \qquad (5.25)$$

This system of equations can be integrated easily and the solution in terms of $f(t)$ and $g(t)$ is given as

$$f(t) = \frac{2}{\chi}\ln\left(\frac{\alpha}{\cosh\left[\frac{\chi\alpha}{2}(t-t_0)\right]}\right) + \frac{1}{\chi}\ln\left(\frac{(d-2)\chi}{(q-1)Q_s^2}\right) - \frac{ac_1}{\chi}t - \frac{ac_2}{\chi}, \qquad (5.26)$$

$$g(t) = \begin{cases} \frac{1}{k-1}\ln\left(\frac{\beta}{\sinh\left[(k-1)\beta(t-t_1)\right]}\right), & \sigma = -1,\\ \pm\beta(t-t_1), & \sigma = 0,\\ \frac{1}{k-1}\ln\left(\frac{\beta}{\cosh\left[(k-1)\beta(t-t_1)\right]}\right), & \sigma = +1. \end{cases} \qquad (5.27)$$

In the displayed form the solution seems to depend on six parameters $t_0, t_1, c_1, c_2, Q_s, \beta$. However, two of them can be eliminated. One can set $\beta = 1$ by a rescaling of the time coordinate $t \to \beta^{-1}t$ together with a suitable rescaling of the other coordinates. One can also set $t_1 = 0$ by a shift of t. Hence the solution depends on four parameters.

6. KALUZA-KLEIN DIMENSIONAL REDUCTION

String theories and M-theory live in ten and eleven dimensions, respectively, as we have discussed. The universe as we observe it, however, has four spacetime dimensions. The implications of these higher dimensional theories on the physics of our world are obtained by Kaluza-Klein dimensional reduction over a compact space. All string theories in ten dimensions can be obtained from M-theory in eleven dimensions. In the present chapter we show that solutions of Type IIA theory can be obtained from M-theory via dimensional reduction. In the process we are going to observe that theories with complicated field content can be obtained from simpler theories in higher dimensions.

We begin with the dimensional reduction of the gravitational field and continue with that of the form field. In $(D+1)$ dimensions the Einstein-Hilbert Lagrangian is given as

$$\mathcal{L} = \sqrt{-\hat{g}}\hat{R} \tag{6.1}$$

where we use hats to denote $(D+1)$-dimensional quantities. Let the $(D+1)$-dimensional spacetime, denoted M_{D+1}, in which our theory of Einstein gravity lives have topology $M_{D+1} = M_D \times S^1$ where S^1 is a circle of radius L and let the corresponding coordinate be denoted by z and M_D by x^μ ($\mu = 1, ..., D$). Since the $(D+1)$-dimensional metric is periodic in z we can Fourier expand it

$$\hat{g}_{MN}(x^\mu, z) = \sum_n \hat{g}^{(n)}_{MN}(x^\mu)e^{inz/L} \tag{6.2}$$

which describes an infinite number of fields in D dimensions upon compactification over z, denoted by the Fourier mode number n.

It turns out that the $n \neq 0$ modes are associated with massive fields and the $n = 0$

mode describes a massless field. To illustrate this we are going to give the example of a massless scalar field in $(D+1)$-dimensional flat space;

$$\hat{\triangle}\hat{\phi} = \left[\triangle + \frac{\partial^2}{\partial z^2}\right]\hat{\phi} = 0 \tag{6.3}$$

where $\hat{\triangle} = \partial^M \partial_M$ and $\triangle = \partial^\mu \partial_\mu$. Fourier expanding $\hat{\phi}$

$$\hat{\phi}(x^\mu, z) = \sum_n \phi_n(x^\mu) e^{inz/L} \tag{6.4}$$

and compactifying on z we see the D-dimensional fields satisfy

$$\triangle\phi_n - \frac{n^2}{L^2}\phi_n = 0. \tag{6.5}$$

This equation describes a scalar field of mass $|n|/L$.

The Kaluza-Klein dimensional reduction idea holds that the radius of the compactified coordinate, L, is very small so that it is not visible in D dimensions. A very small radius of compactification corresponds to scalar fields of very large mass; they can be neglected from a physical point of view because they would be inaccessible to our detectors. Thus, as is usually done, we are going to restrict our attention to the massless sector.

Setting now $n = 0$, our Kaluza-Klein reduction ansatz is to simply take the $(D+1)$-dimensional metric to be indepenedent of z, that is, $\hat{g}_{MN} = \hat{g}_{MN}(x^\mu)$. We are now in a position to split the $(D+1)$-dimensional metric into $\hat{g}_{\mu\nu}$, $\hat{g}_{\mu z}$ and $\hat{g}_{zz}$ which describe the field content of the D-dimensional theory: the metric, a one-form and a scalar field, respectively. In light of this we write the $(D+1)$-dimensional metric in the form

$$d\hat{s}^2 = e^{2\alpha\phi} ds^2 + e^{2\beta\phi}(dz + \mathcal{A}_\mu dx^\mu)^2 \tag{6.6}$$

where α and β are constants and all the fields are independent of z. The $(D+1)$-dimensional metric has the components

$$\hat{g}_{\mu\nu} = e^{2\alpha\phi} g_{\mu\nu} + e^{2\beta\phi} \mathcal{A}_\mu \mathcal{A}_\nu, \qquad \hat{g}_{\mu z} = e^{2\beta\phi} \mathcal{A}_\mu, \qquad \hat{g}_{zz} = e^{2\beta\phi} \tag{6.7}$$

where the right hand sides of the equations above give the listed field content of the D-dimensional theory. The determinants of the $(D+1)$-dimensional and D-dimensional metrics are related as

$$\sqrt{-\hat{g}} = e^{(\beta + D\alpha)\phi} \sqrt{-g}. \tag{6.8}$$

The vielbein for the reduction ansatz is given as

$$\hat{E}^a = e^{\alpha\phi} E^a, \qquad \hat{E}^z = e^{\beta\phi} (dz + \mathcal{A}_{[1]}) \tag{6.9}$$

where $a = 0, 1, ..., D-1$ and E^a is an orthonormal frame on the D-dimensional spacetime. Calculating the curvature of the $(D+1)$-dimensional metric we observe that the associated D-dimensional Lagrangian, obtained by integrating over z, is of the form $\mathcal{L} = e^{[\beta + (D-2)\alpha]\phi} \sqrt{-g} R + ...$ To preserve the Einstein-Hilbert form of the action in terms of the D-dimensional fields, that is, to reach an action in the Einstein frame, we write $\beta = -(D-2)\alpha$. With the given choice for β the Lagrangian reads

$$\mathcal{L} = \sqrt{-\hat{g}} \hat{R} = \sqrt{-g} \left[R - (D-1)(D-2)\alpha^2 (\partial\phi)^2 - \frac{1}{4} e^{-2(D-1)\alpha\phi} \mathcal{F}^2 \right] \tag{6.10}$$

where we dropped a term proportional to $\triangle\phi$ since it gives a total derivative in the Lagrangian, having no effect on the field equations.

Another property desired of the D-dimensional theory is for the kinetic term of the scalar field to acquire the proper normalisation, meaning a term of the form

$-\frac{1}{2}\sqrt{-g}(\partial\phi)^2$ in the Lagrangian. This fixes α and our constants assume

$$\alpha^2 = [2(D-1)(D-2)]^{-1}, \qquad \beta = -(D-2)\alpha. \tag{6.11}$$

We now proceed to analyse how an antisymmetric field strength is dimensionally reduced in the Kaluza-Klein ansatz. Suppose we have an n-form field strength with the corresponding gauge potential of rank $(n-1)$ in the $(D+1)$-dimensional theory; $\hat{F}_{[n]} = d\hat{A}_{[n-1]}$. In terms of components, after compactification over S^1, there will be two types of potentials in D dimensions: one with all its $(n-1)$ indices belonging to the D-dimensional spacetime and one with $(n-2)$ of indices lying in the D-dimensional spacetime and its remaining index being in the direction of S^1. Using differential forms notation this decomposition is expressed as

$$\hat{A}_{[n-1]}(x^\mu, z) = A_{[n-1]}(x^\mu) + A_{[n-2]}(x^\mu) \wedge dz. \tag{6.12}$$

The field strength then decomposes as

$$\hat{F}_{[n]} = dA_{[n-1]} + dA_{[n-2]} \wedge dz. \tag{6.13}$$

However, the constituents in the above decomposition of the field strength are not the most convenient quantities to work with since upon dimensional reduction they lead to a Chern-Simons like term in the action. The metric in $(D+1)$ dimensions couples to all fields; the undifferentiated Kaluza Klein one-form $\mathcal{A}_{[1]}$ couples to $dA_{[n-1]}$ and $dA_{[n-2]}$ and as such it is not practical to identify these as the D-dimensional field strengths. Instead, we add and subtract a term in Equation 6.13

$$\begin{aligned} \hat{F}_{[n]} &= dA_{[n-1]} - dA_{[n-2]} \wedge \mathcal{A}_{[1]} + dA_{[n-2]} \wedge (dz + \mathcal{A}_{[1]}) \\ &\equiv F_{[n]} + F_{[n-1]} \wedge (dz + \mathcal{A}_{[1]}) \end{aligned} \tag{6.14}$$

where we defined the D-dimensional field strengths

$$F_{[n]} = dA_{[n-1]} - dA_{[n-2]} \wedge \mathcal{A}_{[1]}, \qquad F_{[n-1]} = dA_{[n-2]}. \tag{6.15}$$

As we calculate $\hat{F}^2_{[n]}$ we are going to see how it leads to the desired kinetic terms $F^2_{[n]}$ and $F^2_{[n-1]}$ which are indeed convenient to identify as D-dimensional field strengths, and no Chern-Simons cross terms. To do this, we write $\hat{F}_{[n]}$ in the orthonormal basis written in Equation 6.9 and express it in terms of $F_{[n]}$ and $F_{[n-1]}$,

$$\begin{aligned}
\hat{F}_{[n]} &= \frac{1}{n!}\hat{F}_{A_1\ldots A_n}\hat{E}^{A_1}\wedge\ldots\wedge\hat{E}^{A_n} \\
&= \frac{e^{n\alpha\phi}}{n!}\hat{F}_{a_1\ldots a_n}E^{a_1}\wedge\ldots\wedge E^{a_n} + \frac{e^{[(n-1)\alpha+\beta]\phi}}{(n-1)!}\hat{F}_{a_1\ldots a_{n-1}z}E^{a_1}\wedge\ldots\wedge E^{a_{n-1}}\wedge(dz+\mathcal{A}_{[1]}) \\
&\equiv \frac{1}{n!}F_{a_1\ldots a_n}E^{a_1}\wedge\ldots\wedge E^{a_n} + \frac{1}{(n-1)!}F_{a_1\ldots a_{n-1}}E^{a_1}\wedge\ldots\wedge E^{a_{n-1}}\wedge(dz+\mathcal{A}_{[1]})
\end{aligned} \tag{6.16}$$

Thus we reach the identifications

$$\hat{F}_{a_1\ldots a_n} = e^{-n\alpha\phi}F_{a_1\ldots a_n}\ , \qquad \hat{F}_{a_1\ldots a_{n-1}z} = e^{(D-n-1)\alpha\phi}F_{a_1\ldots a_{n-1}} \tag{6.17}$$

where we used Equation 6.11 to express β in terms of α. Having expressed the field strengths in the orthonormal basis, calculating $\hat{F}^2_{[n]}$ is easy since it just involves the Minkowski metric, η_{AB}. Making use of the relation between the determinants of the $(D+1)$ and D-dimensional metrics given in Equation 6.8 we can now express the kinetic term for the $(D+1)$-dimensional field strength in the action for the $(D+1)$-dimensional theory in terms of the field content of the D-dimensional theory as

$$\mathcal{L} = -\frac{\sqrt{-\hat{g}}}{2n!}\hat{F}^2_{[n]} = -\frac{\sqrt{-g}}{2n!}e^{-2(n-1)\alpha\phi}F^2_{[n]} - \frac{\sqrt{-g}}{2(n-1)!}e^{2(D-n)\alpha\phi}F^2_{[n-1]}. \tag{6.18}$$

We have worked out the dimensional reduction of the Einstein-Hilbert action and the form field kinetic term. Dimensional reduction of a scalar field is trivial. We can now sum all the constituents up and perform the dimensional reduction of our general action in Equation 2.1

$$\hat{I} = \int d^{D+1}x\sqrt{-\hat{g}}\left[R(\hat{g}) - \frac{1}{2}(\partial_M\psi)(\partial^M\psi) - \frac{1}{2n!}e^{\hat{a}\psi}\hat{F}^2_{[n]}\right] \tag{6.19}$$

to find in D dimensions

$$I = \int d^Dx\sqrt{-g}\left[R(g) - \frac{1}{2}(\partial_M\psi)(\partial^M\psi) - \frac{1}{2}(\partial_M\phi)(\partial^M\phi) - \frac{1}{4}e^{-2(D-1)\alpha\phi}\mathcal{F}^2_{[2]}\right.$$
$$\left. - \frac{1}{2n!}e^{-2(n-1)\alpha\phi+\hat{a}\psi}F^2_{[n]} - \frac{1}{2(n-1)!}e^{2(D-n)\alpha\phi+\hat{a}\psi}F^2_{[n-1]}\right] \tag{6.20}$$

In Kaluza Klein dimensional reduction a *consistent truncation* is a restriction one poses on the D-dimensional fields such that any solution of the D-dimensional theory involving these fields is also a solution of the $(D+1)$-dimensional theory. For the Einstein-Hilbert action in $(D+1)$ dimensions, the choice of making the metric independent of z is a consistent truncation. For the reduction of the $(D+1)$-dimensional field strength $\hat{F}_{[n]}$, keeping just one of the three gauge fields $\{\mathcal{A}_{[1]}, A_{[n-1]}, A_{[n-2]}\}$ while setting the other two to zero and retaining simultaneously the scalar field combination appearing in the exponential prefactor before the corresponding kinetic term in the action in Equation 6.20 is a consistent truncation. In this manner one can obtain brane solutions in $D = 10$ by dimensional reduction of an appropriate solution in $D + 1 = 11$.

6.1. Dimensional Reduction of Eleven Dimensional Solutions

We have outlined the Kaluza-Klein monopole, plane wave, p-brane and S-brane solutions of supergravity theories in the previous sections. In the present section we perform the dimensional reduction of some of the eleven dimensional solutions. We have two p-brane solutions in eleven dimensions, namely the electric M2-brane and the magnetic M5-brane, and two S-brane solutions, the SM2-brane and the SM5-brane. The Kaluza-Klein reduction ansatz from $D + 1 = 11$ dimensions is, reading from

Equation 6.6,

$$ds_{11}^2 = e^{\frac{1}{6}\phi} ds_{10}^2 + e^{-\frac{4}{3}\phi}(dz + \mathcal{A}_\mu dx^\mu)^2 \tag{6.21}$$

where we used Equation 6.11 for the values of the constants α and β.

6.1.1. Reduction of Static Solutions

We first consider the dimensional reduction of a Kaluza-Klein monopole in eleven dimensions the metric for which is given in Equation 4.3. Here t, x^μ $(\mu = 1, ..., 6)$ and z are isometry directions. To perform dimensional reduction over x^μ we bring the metric in eleven dimensions into the form of the Kaluza Klein reduction ansatz in Equation 6.6 by setting $\mathcal{A}_{[1]} = 0$ and $\alpha = \beta = 0$. It is clear then that we have a Kaluza-Klein monopole in ten dimensions with Kaluza-Klein vector J_i inherited from eleven dimensions with its corresponding harmonic function.

Proceeding with a reduction over z we observe that the Kaluza-Klein monopole metric is already in the form of the reduction ansatz with $\mathcal{A}_{[1]} = J_{[1]}$ where $J_{[1]}$ is the Kaluza Klein vector. All there is to do is to identify the harmonic function of the monopole as $H_{KK} \equiv e^{\frac{4}{3}\phi}$. Then in ten dimensions,

$$ds_{10}^2 = H_{KK}^{-1/8} \eta_{\mu\nu} dx^\mu dx^\nu + H_{KK}^{7/8} \delta_{ij} dy^i dy^j \tag{6.22}$$

where $\mu = 0, 1, ..., 6$ and $i = 1, 2, 3$. This is the metric for the magnetic D6-brane as can be seen in Equation 3.19 with $a = -\frac{3}{2}$, $\Delta = 4$, $d = 7$, $\tilde{d} = 1$. The harmonic function of the KK monopole is identified with that of the D6-brane, $H_{KK} \equiv H_{D6}$ with the KK vector directions as the transverse directions of D6. The dilaton field of the D6-brane is inherited from the Einstein-Hilbert action in eleven dimensions, correctly described as $e^\phi = H_{D6}^{3/4}$ as witnessed in Equation 3.20. The two-form field strength of the D6-brane is $F_{[2]} = dJ_{[1]}$. To see this explicitly, we write the field strength tensor for the D6-brane from Equation 3.4 and set it equal to $dJ_{[1]}$ using Equation 4.4,

$$F_{ij} = \lambda \tilde{\epsilon}_{ijk} \frac{y^k}{r^3} = \tilde{\epsilon}_{ijk} \partial_k H_{KK}. \tag{6.23}$$

This equation is correctly solved by the harmonic function H_{KK} and relates the constant that describes it by the magnetic charge parameter of the D6-brane, λ. Here we should pause for a moment and observe how a consistent truncation this is: a solution of the lower dimensional theory, the D6-brane described by a harmonic function H_{D6}, is also a solution of the higher dimensional theory, the Kaluza-Klein monopole also described by a harmonic function, and the relation between them as simple as $H_{D6} = H_{KK}$.

For the plane wave in Equation 4.2, when z is imposed as an isometry direction, it is easy to see via a similar procedure that in ten dimensions the solution corresponds to a D0-brane. The vector field carrying the charge, of which only the time component does not vanish, can be read as $A_t = H_W^{-1} - 1$ where H_W is the harmonic function of the wave. One can also impose one of the x^μ ($\mu = 1, ..., 9$) as an isometry direction, reduction over which corresponds to a plane wave in ten dimensions.

One can perform the dimensional reduction of a p-brane over one of its worldvolume coordinates or one of its transverse coordinates. We have two p-brane solutions in eleven dimensions: the electric M2-brane and the magnetic M5-brane. One can perform the dimensional reduction of these solutions over one of their transverse coordinates by imposing their harmonic functions to be independent of that coordinate. The resulting solutions in ten dimensions are the D2-brane and the NS5-brane for M2 and M5, respectively.

We are going to perform the dimensional reduction of the M5-brane over one of its transverse coordinates; this gives the solitonic fivebrane, NS5, in ten dimensions.

To see this we write the metric of the M5-brane in the form of the reduction ansatz

$$ds_{11}^2 = H_{M5}^{-1/3}\eta_{\mu\nu}dx^\mu dx^\nu + H_{M5}^{2/3}(dy_1^2 + ...dy_4^2 + dz^2) \tag{6.24}$$

where $\mu = 0, 1, ..., 5$ and we imposed $H_{M5} = H_{M5}(y^m)$. Reducing over z we get in ten dimensions

$$ds_{10}^2 = H_{M5}^{-1/4}\eta_{\mu\nu}dx^\mu dx^\nu + H_{M5}^{3/4}\delta_{mn}dy^m dy^n \tag{6.25}$$

where $\mu = 0, 1, ..., 5$ and $m = 1, ..., 4$ and $e^\phi = H_{M5}^{-1/2}$. This is the NS5 solution as can be seen in Equations 3.19-3.21 with $a = 1$, $\Delta = 4$, $d = 6$, $\tilde{d} = 2$ identifying its harmonic function as $H_{NS5} \equiv H_{M5}$. The kinetic term for the three-form field strength of NS5-brane can be read in the reduced action in Equation 6.18 substituting $\alpha = 12$, $D = 10$, $\hat{a} = 0$ and it correctly gives the coupling with the dilaton as $a = 1$. To see why no four-form is inherited from $D + 1 = 11$ we write the four-form of the magnetic M5-brane given in Equation 3.4 in differential forms notation

$$F_{[4],M5} = *d\left[H_{M5}^{-1}(y^m)\text{vol}(X)\right] \tag{6.26}$$

where vol(X) denotes the volume form on $\{x^\mu\}$-space in Equation 6.24. It is evident that $\hat{F}_{[4]} = F_{[4],M5}$ has a leg in the compactified direction z in which case we have $dA_{[3]} = 0$ in Equations 6.13 and 6.15. Since we have $\mathcal{A}_{[1]} = 0$, Equation 6.15 tells us $F_{[3]} = 0$ in $D = 10$.

Reduction of the M5-brane over one of its worldvolume coordinates yields the D4-brane in ten dimensions. The metric for the M5-brane reads

$$ds_{11}^2 = H_{M5}^{-1/3}(-dt^2 + dx_1^2 + ...dx_4^2 + dz^2) + H_{M5}^{2/3}\delta_{mn}dy^m dy^n \tag{6.27}$$

where $m = 1, ..., 5$ and $H_{M5}(y^m)$ is the harmonic function of the M5-brane. We identify this eleven dimensional metric with the Kaluza-Klein reduction ansatz with the S^1

coordinate denoted by z as usual, $\mathcal{A}_{[1]} = 0$, and then just read the corresponding ten dimensional metric as

$$ds_{10}^2 = H_{M5}^{-3/8} \eta_{\mu\nu} dx^\mu dx^\nu + H_{M5}^{5/8} \delta_{mn} dy^m dy^n \tag{6.28}$$

where $\mu = 0, 1, ..., 4$ and the scalar field in the reduction ansatz reads $e^\phi = H_{M5}^{1/4}$. This is just the metric for the magnetic D4-brane with $a = -\frac{1}{2}$, $\Delta = 4$, $d = 5$, $\tilde{d} = 3$ identifying its harmonic function as $H_{D4} \equiv H_{M5}$. The dimensionally reduced action contains the Einstein-Hilbert part, the kinetic term for a dilaton field which satisfies $e^\phi = H_{D4}^{1/4}$, and a four-form. The four-form in $D+1 = 11$ has no leg in the compactified direction z since it is a worldvolume direction for the magnetic M5-brane. This means that $\hat{F}_{[4]} = F_{[4],M5}$ decomposes in Equation 6.13 with $dA_{[2]} = 0$ in which case Equation 6.15 tells us no three-form exists in the reduced theory. Reading again from Equations 6.13 and 6.15 we have in ten dimensions $F_{[4],D4} \equiv F_{[4],M5}$ that has non-vanishing components in the $\{y^m\}$ directions. The dilaton-field strength coupling reads $a = -2\alpha(n-1) = -1/2$. The dimensionally reduced theory is correctly described by the D4-brane solution of the Type IIA theory.

Similarly, reduction of the M2-brane over a worldvolume direction and over a transverse direction gives the fundamental string and the D2-brane, respectively. We have thus shown that solutions of Type IIA theory listed in Section 3.1 can be obtained from M-theory.

6.1.2. Reduction of S-branes

In this section we perform the reduction of M-theory S-branes; the SM2-brane and the SM5-brane as outlined in [18]. Reduction on transverse directions yield in ten dimensions the SD2 and the SNS5 solutions and reduction on worldvolume directions yield the SNS1 and the SD4 solutions in ten dimensions for the SM2-brane and the SM5-brane, respectively. As mentioned in Chapter 5 we need to modify the solution given in that chapter for this procedure. In terms of this solution S-branes in ten dimensions are characterized by four parameters whereas S-branes in eleven dimensions are characterized by two parameters. The reason for this is that the dilaton field is zero in the eleven dimensional theory, that is, $a = c_1 = c_2 = 0$. It can be seen in Equations 5.14 and 5.26 that this means the exponents of the metric functions do not involve terms linear in t.

We first perform dimensional reduction over a transverse coordinate. In order for the reduction to yield a dilaton field with a term linear in t as in Equation 5.11, we are going to write the eleven dimensional solutions in a delocalized form. We start with an Sp-brane in d dimensions, writing its metric from Equation B.9 with $(q-k) = 1$,

$$ds_d^2 = -e^{2\hat{A}}dt^2 + e^{2\hat{B}}(dx_1^2 + ... + dx_{p+1}^2) + e^{2\hat{C}}d\Sigma_{q-1,\sigma}^2 + e^{2\hat{D}}dy^2 \tag{6.29}$$

where the y coordinate is called a delocalized direction and the solution is going to be reduced on this direction. We are interested in eleven dimensional solutions, thus the dilaton is zero, $\phi = 0$, and the q-form field strength is given as $\hat{F}_{[q]} = \hat{Q}_s \mathrm{vol}(\Sigma_{q-1,\sigma}) \wedge dy$. The field equations give the metric functions as

$$\hat{A} = (q-1)\hat{g} - \frac{p+1}{q-1}\hat{f} + q(\hat{c}_1 t + \hat{c}_2), \tag{6.30}$$

$$\hat{B} = \hat{f}, \tag{6.31}$$

$$\hat{C} = \hat{g} - \frac{p+1}{q-1}\hat{f} + \hat{c}_1 t + \hat{c}_2, \tag{6.32}$$

$$\hat{D} = -\frac{p+1}{q-1}\hat{f} + \hat{c}_1 t + \hat{c}_2, \tag{6.33}$$

where $\hat{f}(t)$ and $\hat{g}(t)$ are given as

$$\hat{f}(t) = \frac{2}{\chi}\ln\left(\frac{\hat{\alpha}}{\cosh\left[\frac{\chi\hat{\alpha}}{2}(t-t_0)\right]}\right) + \frac{1}{\chi}\ln\left(\frac{(d-2)\chi}{(q-1)Q_s^2}\right), \tag{6.34}$$

$$\hat{g}(t) = \begin{cases} \frac{1}{q-2}\ln\left(\frac{\hat{\beta}}{\sinh[(q-2)\hat{\beta}t]}\right) - \frac{q-1}{q-2}(\hat{c}_1 t + \hat{c}_2), & \sigma = -1, \\ \pm\hat{\beta}t - \frac{q-1}{q-2}(\hat{c}_1 t + \hat{c}_2), & \sigma = 0, \\ \frac{1}{q-2}\ln\left(\frac{\beta}{\cosh[(q-2)\beta(t-t_1)]}\right) - \frac{q-1}{q-2}(\hat{c}_1 t + \hat{c}_2), & \sigma = +1. \end{cases} \tag{6.35}$$

where t_1 in Equation 5.27 has been set to zero with a shift in t. An important difference with the solution in Equations 5.26 and 5.27 is that c_3 and c_4 in Equation 5.12 which were set to zero for simplicity in Equation 5.13 are taken as nonvanishing constants in the present solution; $c_3 = \hat{c}_1$, $c_4 = \hat{c}_2$. In regard to this $\hat{g}(t)$ differs from $g(t)$ in Equation 5.27 as demanded by the gauge condition. As discussed, this is necessary for a dimensional reduction to ten dimensions. Einstein equation for R_{tt} gives the relation between the displayed constants as

$$\frac{q-1}{q-2}\hat{c}_1^2 + \frac{(p+1)(d-2)}{q-1}\hat{\alpha}^2 - (q-1)(q-2)\hat{\beta}^2 = 0. \tag{6.36}$$

In $d = 11$, Equations 6.30-6.35 describe the delocalized SM2-brane solution for $p = 2$, $q = 7$, $\chi = 6$ and the delocalized SM5-brane solution for $p = 5$, $q = 4$, $\chi = 12$.

We show explicitly the dimensional reduction of the SM2-brane over the delocalized transverse coordinate y, the procedure is similar for the SM5-brane. We observe in Equation 6.11 that one can write $\alpha \to -\alpha$, $\beta \to -\beta$ and still have the correctly normalized kinetic term for the dilaton in the reduced theory. Then the Kaluza Klein reduction ansatz from eleven dimensions to ten becomes

$$ds_{11}^2 = e^{-\phi/6} ds_{10}^2 + e^{4\phi/3}(dz + \mathcal{A}_\mu dx^\mu)^2. \tag{6.37}$$

Comparing the SM2-brane metric in (6.29) with this, we read the dilaton in ten dimensions as

$$\phi = -\frac{1}{4}\ln\left(\frac{\hat{\alpha}}{\cosh\left[3\hat{\alpha}(t-t_0)\right]}\right) - \frac{1}{8}\ln\left(\frac{9}{Q_s^2}\right) + \frac{3}{2}(\hat{c}_1 t + \hat{c}_2). \tag{6.38}$$

The dilaton for the SD2-brane in ten dimensions can be read in Equation 5.11 written in general for an $S(p-1)$-brane, with $p = 3$, $a = -1/2$, $\chi = 32/5$, as

$$\phi = -\frac{1}{4}\ln\left(\frac{\alpha}{\cosh\left[\frac{16}{5}\alpha(t-t_0)\right]}\right) - \frac{1}{8}\ln\left(\frac{256}{25Q_s^2}\right) + \frac{15}{16}(c_1 t + c_2). \tag{6.39}$$

Comparing Equations 6.38 and 6.39 the relation between constants in ten and eleven dimensions can be read as

$$\hat{\alpha} = \frac{16}{15}\alpha, \quad \hat{c}_1 = \frac{5}{8}c_1, \quad \hat{c}_2 = \frac{5}{8}c_2. \tag{6.40}$$

Comparing $\hat{g}(t)$ with $g(t)$ in Equation 5.27 we find $\hat{\beta} = \beta$ and with this identification the condition for the parameters in eleven dimensions for SM2 in Equation 6.36 reduce correctly to the condition for parameters for the SD2-brane solution:

$$\frac{6}{5}\hat{c}_1^2 + \frac{9}{2}\hat{\alpha}^2 - 30\hat{\beta}^2 = 0 \quad \Rightarrow \quad \frac{15}{32}c_1^2 + \frac{128}{25}\alpha^2 - 30\beta^2 = 0. \tag{6.41}$$

The ten dimensional metric can be read from Equation 6.37 as

$$ds_{10}^2 = -e^{\phi/6+2\hat{A}}dt^2 + e^{\phi/6+2\hat{B}}(dx_1^2 + ... + dx_3^2) + e^{\phi/6+2\hat{C}}d\Sigma_{6,\sigma}^2. \tag{6.42}$$

Comparing Equations 6.30-6.35 and 6.38 with Equations 5.14, 5.26 and 5.27, the metric functions are correctly identified with the SD2-brane solution as

$$\begin{aligned}
\frac{1}{6}\phi + 2\hat{A} &= -\frac{9}{8}\hat{f} + 12\hat{g} + \frac{57}{4}(\hat{c}_1 t + \hat{c}_2) = 12g - \frac{6}{5}f = 2A, \\
\frac{1}{6}\phi + 2\hat{B} &= \frac{15}{8}\hat{f} + \frac{1}{4}(\hat{c}_1 t + \hat{c}_2) = 2f = 2B, \\
\frac{1}{6}\phi + 2\hat{C} &= -\frac{9}{8}\hat{f} + 2\hat{g} + \frac{9}{4}(\hat{c}_1 t + \hat{c}_2) = 2g - \frac{6}{5}f = 2C.
\end{aligned} \tag{6.43}$$

The four-form field strength from eleven dimensions is also equal to the field strength for the SD2-brane. Thus, reduction of an SM2-brane over a delocalized transverse direction in eleven dimensions yields an SD2-brane in ten dimensions. Starting with an SM5-brane one obtains the SNS5-brane in such a reduction.

We continue with reduction over one of the worldvolume directions. Comparing the eleven dimensional solutions in Equations 5.14, 5.26, 5.27 with the Kaluza-Klein reduction ansatz, it can be seen that the reduction of the SM-branes with isotropic worldvolumes cannot yield a dilaton field involving a term linear in t in ten dimensions.

The remedy is an anisotropic worldvolume:

$$ds_d^2 = -e^{2\hat{A}}dt^2 + e^{2\hat{B}}(dx_1^2 + ... + dx_p^2) + e^{2\hat{D}}dy^2 + e^{2\hat{C}}d\Sigma_{q,\sigma}^2 \tag{6.44}$$

where $y \equiv x_{p+1}$ is identified as the S^1 direction. This metric is of the same form as Equation B.9 and the corresponding Ricci tensor given in Appendix B.2 is simplified with the gauge condition

$$-\dot{\hat{A}} + p\dot{\hat{B}} + \dot{\hat{D}} + q\dot{\hat{C}} = 0. \tag{6.45}$$

Setting the dilaton field to zero and surviving the constants of integration in the equation $\ddot{B} = \ddot{D}$ we can parametrize the metric functions in terms of two functions $\hat{f}(t)$ and $\hat{g}(t)$ as

$$\begin{aligned} \hat{A} &= q\hat{g} - \frac{p+1}{q-1}\hat{f} + (q+1)(\hat{c}_1 t + \hat{c}_2), && (6.46) \\ \hat{B} &= \hat{f}, && (6.47) \\ \hat{C} &= \hat{g} - \frac{p+1}{q-1}\hat{f} + \hat{c}_1 t + \hat{c}_2, && (6.48) \\ \hat{D} &= \hat{f} + \hat{c}_1 t + \hat{c}_2. && (6.49) \end{aligned}$$

With the field strength given as $\hat{F}_{[q]} = \hat{Q}_s \mathrm{vol}(\Sigma_{q,\sigma})$ the field equations are solved by

$$\hat{f}(t) = \frac{2}{\chi}\ln\left(\frac{\hat{\alpha}}{\cosh\left[\frac{\chi\hat{\alpha}}{2}(t-t_0)\right]}\right) + \frac{1}{\chi}\ln\left(\frac{(d-2)\chi}{(q-1)Q_s^2}\right) - \frac{1}{p+1}\hat{c}_1 t + \hat{c}_2, \tag{6.50}$$

$$\hat{g}(t) = \begin{cases} \frac{1}{q-1}\ln\left(\frac{\hat{\beta}}{\sinh\left[(q-1)\hat{\beta}t\right]}\right) - \frac{q}{q-1}(\hat{c}_1 t + \hat{c}_2), & \sigma = -1, \\ \pm\hat{\beta}t - \frac{q}{q-1}(\hat{c}_1 t + \hat{c}_2), & \sigma = 0, \\ \frac{1}{q-1}\ln\left(\frac{\beta}{\cosh\left[(q-1)\beta(t-t_1)\right]}\right) - \frac{q-1}{q-2}(\hat{c}_1 t + \hat{c}_2), & \sigma = +1. \end{cases} \tag{6.51}$$

provided the constants satisfy

$$\frac{p}{p+1}\hat{c}_1^2 + \frac{(p+1)(d-2)}{q-1}\hat{\alpha}^2 - q(q-1)\hat{\beta}^2 = 0. \tag{6.52}$$

In $d = 11$ these equations describe the anisotropic SM2-brane for $p = 2$, $q = 7$, $\chi = 6$ and the anisotropic SM5-brane for $p = 5$, $q = 4$, $\chi = 12$. Comparing the metric in Equation 6.44 with the Kaluza Klein reduction ansatz in Equation 6.37 we read the dilaton field in the reduced theory as

$$\phi = \frac{3}{2}\hat{f} + \frac{3}{2}(\hat{c}_1 t + \hat{c}_2). \tag{6.53}$$

We proceed to perform the reduction SM2 $\to$ SNS1, the reduction of the SM5 $\to$ SD4 is similar. The dilaton field emerging in ten dimensions in this case, with $p = 2$, $q = 7$, $\chi = 6$ is

$$\phi \;=\; \frac{1}{2}\ln\left(\frac{\hat{\alpha}}{\cosh\left[3\hat{\alpha}(t-t_0)\right]}\right) + \frac{1}{4}\ln\left(\frac{9}{Q_s^2}\right) + \hat{c}_1 t + \hat{c}_2. \tag{6.54}$$

The SNS1-brane solution in ten dimensions with a flat transverse space is given in Chapter 5 for $p = 2$, $q = k = 7$, $a = 1$, $\chi = 16/3$. Its dilaton field is given as

$$\phi \;=\; \frac{1}{2}\ln\left(\frac{\alpha}{\cosh\left[\frac{8}{3}\alpha(t-t_0)\right]}\right) + \frac{1}{4}\ln\left(\frac{64}{9Q_s^2}\right) + \frac{3}{4}(c_1 t + c_2). \tag{6.55}$$

Comparing Equation 6.54 with Equation 6.55 we identify the constants in eleven and ten dimensions as

$$\hat{\alpha} = \frac{8}{9}\alpha, \quad \hat{c}_1 = \frac{3}{4}c_1, \quad \hat{c}_2 = \frac{3}{4}c_2. \tag{6.56}$$

Comparing $\hat{g}(t)$ in Equation 6.51 with $g(t)$ in Equation 5.27 we find $\hat{\beta} = \beta$. The condition for parameters in eleven dimensions in Equation 6.52 lead correctly to the condition for the parameters of the SNS1-brane solution:

$$\frac{2}{3}\hat{c}_1^2 + \frac{9}{2}\hat{\alpha}^2 - 42\hat{\beta}^2 = 0 \quad \Rightarrow \quad \frac{3}{8}c_1^2 + \frac{32}{9}\alpha^2 - 42\beta^2 = 0. \tag{6.57}$$

The ten dimensional metric can be read from Equations 6.37, 6.44 as

$$ds_{10}^2 = -e^{\phi/6+2\hat{A}}dt^2 + e^{\phi/6+2\hat{B}}(dx_1^2 + dx_2^2) + e^{\phi/6+2\hat{C}}d\Sigma_{6,\sigma}^2. \tag{6.58}$$

Using Equations 5.14, 5.26, 5.27, 6.46-6.51, 6.54, we find that the reduced metric describes exactly the SNS1-brane solution,

$$\frac{1}{6}\phi + 2\hat{A} = -\frac{3}{4}\hat{f} + 14\hat{g} + \frac{65}{4}(\hat{c}_1 t + \hat{c}_2) = 14g - \frac{2}{3}f = 2A, \tag{6.59}$$

$$\frac{1}{6}\phi + 2\hat{B} = \frac{9}{4}\hat{f} + \frac{1}{4}(\hat{c}_1 t + \hat{c}_2) = 2f = 2B, \tag{6.60}$$

$$\frac{1}{6}\phi + 2\hat{C} = -\frac{3}{4}\hat{f} + 2\hat{g} + \frac{9}{4}(\hat{c}_1 t + \hat{c}_2) = 2g - \frac{2}{3}f = 2C. \tag{6.61}$$

The field strength from eleven dimensions is equal to the field strength in ten dimensions. Thus, we have shown that the reduction of an anisotropic SM2-brane in eleven dimensions yields an SNS1-brane in ten dimensions.

6.1.3. Cosmology from S-branes

Astronomical observations have shown that the universe is undergoing accelerated expansion. It is thus desirable to find a model that incorporates this feature and is derivable from a fundamental theory and one candidate for such a theory is the String/M-theory. It has been shown [19] that cosmologies with accelerated expansion can arise from a time dependent solution of vacuum Einstein equations in $(n + 4)$ dimensions compactified on n-dimensional hyperbolic space. For $n = 7$, the time dependent S-brane solutions in eleven dimensions provide this framework [20]. It has also been shown [21] that the 7-space can be chosen as the flat space or the sphere, for a special case of the eleven dimensional solution in the latter.

For simplicity, we consider the SM2-brane solution with flat transverse space as given in Equation B.9 for $p = 3$, $q = k = 7$, $a = c_1 = c_2 = 0$, $\chi = 6$. The metric reads

$$ds_{11}^2 = -e^{2A}dt^2 + e^{2B}(dx_1^2 + dx_1^2 + dx_3^2) + e^{2C}d\Sigma_{7,\sigma=1}^2 \tag{6.62}$$

where $d\Sigma^2_{7,\sigma=1}$ is the Euclidean metric on the 7-space. Using Equation 5.14 we write the metric in terms of the functions $f(t)$ and $g(t)$,

$$ds^2_{11} = -e^{14g-f}dt^2 + e^{2f}(dx_1^2 + dx_2^2 + dx_3^2) + e^{2g-f}(dz_1^2 + ...dz_7^2). \tag{6.63}$$

Using Equation 5.25 we see that $\alpha = 1$ for $\beta = \frac{1}{2}\sqrt{\frac{3}{7}}$. With this choice $f(t)$ and $g(t)$ can be read from Equations 5.26, 5.27 as

$$f(t) = \frac{1}{6}\ln\left(\frac{9}{Q_s^2\cosh^2[3t]}\right), \qquad g(t) = \frac{1}{2}\sqrt{\frac{3}{7}}t. \tag{6.64}$$

To reduce this solution on z_1 to ten dimensions we compare it with the KK ansatz with $\mathcal{A}_{[1]} = 0$

$$ds^2_{11} = e^{\frac{1}{6}\phi_1}ds^2_{10} + e^{-\frac{4}{3}\phi_1}dz_1^2 \tag{6.65}$$

and read the ten dimensional metric, with $\phi_1 = (3f - 6g)/4$,

$$ds^2_{10} = -e^{\frac{57}{4}g-\frac{9}{8}f}dt^2 + e^{\frac{1}{4}g+\frac{15}{8}f}(dx_1^2 + dx_2^2 + dx_3^2) + e^{\frac{9}{4}g-\frac{9}{8}f}(dz_2^2 + ...dz_7^2) \tag{6.66}$$

Continuing the process of dimensional reduction in this manner over all the z_i coordinates we reach in four dimensions

$$ds^2_4 = -S^6dt^2 + S^2(dx_1^2 + dx_2^2 + dx_3^2) \tag{6.67}$$

where $S(t)$ is defined as

$$S(t) = \exp\left(\frac{7}{2}g - \frac{3}{4}f\right). \tag{6.68}$$

Defining the proper time $d\eta = S^3dt$ we get

$$ds^2_E = -d\eta^2 + S^2(\eta)(dx_1^2 + dx_2^2 + dx_3^2). \tag{6.69}$$

This is the FLRW metric with a flat 3-space, scale factor $S(\eta)$ and the speed of light set equal to 1. The universe described by this metric is expanding if $\frac{dS}{d\eta} > 0$, which implies

$$\tanh[3t] + \sqrt{\frac{7}{3}} > 0. \tag{6.70}$$

The expansion will be accelerating if $\frac{d^2S}{d\eta^2} > 0$, that is, if

$$\frac{\sqrt{2}}{\cosh[3t]} - \tanh[3t] - \sqrt{\frac{7}{3}} > 0 \tag{6.71}$$

These conditions are simultaneously satisfied for a period of time in the region $t < 0$.

7. INTERSECTIONS OF S-BRANES WITH WAVES AND MONOPOLES IN ELEVEN DIMENSIONS

S-brane solutions are described by hyperbolic functions of time and carry charge, Kaluza-Klein monopoles and plane waves are purely geometric solutions described by harmonic functions. In this chapter we investigate intersections of S-branes with these purely geometric solutions in eleven dimensions. Intersection rules in an arbitrary number of dimensions are similar with an additional constant of integration introduced by the presence of the dilaton. As to be anticipated, the crucial point in constructing these intersections is the structure of the metric.

7.1. Intersections of S-branes with Plane Waves

A plane wave can be added on the transverse space of an S-brane. One has to reparametrize the wave in both cases as will be displayed below. The harmonic function of the wave depends on the spatial tranverse coordinates of the eleven dimensional S-brane, denoted as the SM-brane.

It was seen that the S-brane ansatz supported a sphere or a hyperbolic space on its transverse space. However, in the presence of a wave this is not possible and the transverse space must be flat for the separability of the time dependent functions and the spatial function of the wave.

The effect on the time dependent functions in the metric of adding a wave to the S-brane is a linear condition involving the time derivatives of exponents in the metric multiplying the time part, the z coordinate of the wave, and the space on which the harmonic function of the wave depends. Since by definition the time coordinate is transverse to the S-brane, this condition requires the wave to be placed on the transverse space of the S-brane and the harmonic function of the wave to depend on a number of the transverse coordinates.

We write the metric as

$$ds^2 = -e^{2A-2G}dt^2 + e^{2B}(dx_1^2 + ...dx_p^2) + e^{2C+2G}\left[(e^{-2G}-1)e^{A-C}dt + dz\right]^2 + e^{2D}(dy_1^2 + ...dy_k^2) + e^{2E}(dw_1^2 + ...dw_{(q-k)}^2). \quad (7.1)$$

The metric above is parametrized by five time dependent functions $A(t), B(t), C(t), D(t), E(t)$ and $G = G(y^a)$. For SM5 we have $p = 6,\ q = 3$ and SM2 we have $p = 3,\ q = 6$.

With the coordinates $x^A = (t, x^\mu, z, y^a, w^i)$ the vielbein reads

$$E^{\bar{t}} = e^{A-G}dt,\ E^{\bar{\mu}} = e^B dx^\mu,\ E^{\bar{a}} = e^D dy^a,\ E^{\bar{i}} = e^E dw^i,\ E^{\bar{z}} = (e^{A-G} - e^{A+G})dt + e^{C+G}dz. \quad (7.2)$$

Writing the antisymmetric tensor in the orthonormal frame and with charge parameter Q_s, for the SM5-brane we have

$$F_5 = 6Q_s e^{6B-A+G} E^{\bar{z}} \wedge E^{\bar{y}_1} \wedge ... \wedge E^{\bar{y}_k} \wedge E^{\bar{w}_1} \wedge ... \wedge E^{\bar{w}_{(3-k)}}. \quad (7.3)$$

For the SM2-brane we have

$$F_2 = 6Q_s e^{3B-A+G} E^{\bar{t}} \wedge E^{\bar{x}_1} \wedge E^{\bar{x}_2} \wedge E^{\bar{x}_3}. \quad (7.4)$$

The Ricci tensor for the given metric largely simplifies with the gauge condition

$$-\dot{A} + p\dot{B} + \dot{C} + k\dot{D} + (q-k)\dot{E} = 0 \quad (7.5)$$

The terms in the Ricci tensor of the form $\partial_a\partial_a G + 2(\partial_a G)(\partial_a G)$ vanish as one imposes $e^{G(y^a)}$ to be a harmonic function, that is,

$$\partial_a\partial_a e^{2G} = 0. \quad (7.6)$$

The field equations for the time dependent functions are the usual ones for an S-brane with z appearing as a delocalized transverse direction except that one has to account for the new Ricci tensor components brought about by the harmonic function of the wave

$$
\begin{aligned}
R_{ta} &= e^{2G}(\partial_a G)\left[-p\dot{B}-(q-k)\dot{E}-2\dot{C}-(q-2)\dot{D}\right], && (7.7)\\
R_{za} &= e^{2G+C-A}(\partial_a G)\left[p\dot{B}+(q-k)\dot{E}+2\dot{C}+(q-2)\dot{D}\right]. && (7.8)
\end{aligned}
$$

Since in the metric neither t nor z mix with the y^a's the given Ricci components must vanish on their own. Simplifying the expression in square brackets using the gauge condition in Equation 7.5, it is seen that these components vanish if

$$\dot{A}+\dot{C}=2\dot{D} \tag{7.9}$$

which we are going to call the *wave condition*. In the absence of the wave, that is, when no such condition as Equation 7.9 exists the solution reads

$$
\begin{aligned}
C &= -\frac{p}{q}B+c_1 t, && (7.10)\\
D &= -\frac{p}{q}B+c_2 t, && (7.11)\\
E &= -\frac{p}{q}B+c_3 t, && (7.12)\\
A &= -\frac{p}{q}B+[c_1+kc_2+(q-k)c_3]t, && (7.13)\\
B &= \frac{1}{p}\ln\left(\frac{M}{\cosh\left[pM(t-t_0)\right]}\right)+\frac{1}{2p}\ln\left(\frac{2p(q+p)}{qQ_s^2}\right) && (7.14)
\end{aligned}
$$

where the constants satisfy

$$M^2-\frac{q}{p(q+p)}[(k^2-k)c_2^2+((q-k)^2-(q-k))c_3^2+2kc_1c_2+2k(q-k)c_2c_3+2(q-k)c_1c_3]=0. \tag{7.15}$$

Adding the wave brings, by the wave condition in Equation 7.9,

$$2c_1 + (k-2)c_2 + (q-k)c_3 = 0. \tag{7.16}$$

In describing these configurations we adopt the convention employed in [6]. We denote a worldvolume direction by a $\times$ symbol and a transverse direction by a $-$ symbol. We denote the portion of the metric that involves the term $H_{\mathcal{W}}\left[(H_{\mathcal{W}}^{-1}-1)dt+dz\right]^2$ by the z coordinate of the wave.

Table 7.1. Intersections of SM-branes with waves.

$SM5$	t	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$-$	$-$	$-$	$-$
$\mathcal{W}$	t	$-$	$-$	$-$	$-$	$-$	$-$	z	$-$	$-$	$-$
$SM2$	t	$\times$	$\times$	$\times$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$\mathcal{W}$	t	$-$	$-$	$-$	z	$-$	$-$	$-$	$-$	$-$	$-$

These configurations are in contrast with intersections of waves with p-branes [22]. In the latter the wave is suitably placed inside the worldvolume of the p-brane; translations in time are isometries of the generic p-brane solutions and time direction is inside the worldvolume of the p-brane. However, an S-brane has a spacelike worldvolume and the metric is time dependent. As such, the wave must be placed on the transverse space.

7.2. Intersections of S-branes with Kaluza Klein Monopoles

Several configurations are possible for an intersecting S-brane and a Kaluza-Klein monopole. We denote the portion of the metric that involves the term $H_{\mathcal{KK}}^{-1}(dz+J_i dy_i)^2$ by the z coordinate of the Kaluza Klein monopole and each of the y_i directions, which appear with the line element $H_{\mathcal{KK}}dy_i^2$ in the metric, by the corresponding J_i. We call these $\{z, J_i\}$ directions transverse to $\mathcal{KK}$ and the rest of the coordinates as worldvolume directions of the $\mathcal{KK}$.

When the full Kaluza-Klein metric with no additional isometry directions assumed is to be implemented, the $\{z, J_i\}$ part of the metric must have the same overall time dependent factor. When an isometry direction is assumed, say y_1, one can write the metric with different time factors multiplying the $\{z, J_1\}$ and $\{J_2, J_3\}$ parts.

In studying the single S-brane solution we have seen how the S-brane ansatz supported a constant curvature spatial hyperspace in its transverse space. Unlike the S-brane and wave intersection, the S-brane and Kaluza-Klein monopole intersection allows a constant curvature hyperspace of arbitrary dimesionality in the relative transverse space of the S-brane, that is, in the directions those are transverse to the S-brane and are worldvolume directions for the Kaluza-Klein monopole. In the table below we list these configurations.

Table 7.2. Intersections of SM-branes with Kaluza-Klein monopoles.

$SM5$	t	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$-$	$-$	$-$	$-$
$\mathcal{KK}$	t	J_1	J_2	J_3	z	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
$SM5$	t	$-$	$-$	$-$	$-$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
$\mathcal{KK}$	t	J_1	J_2	J_3	z	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
$SM5$	t	$\times$	$-$	$-$	$\times$	$\times$	$\times$	$\times$	$\times$	$-$	$-$
$\mathcal{KK}$	t	J_1	J_2	J_3	z	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
$SM5$	t	$-$	$\times$	$\times$	$-$	$\times$	$\times$	$\times$	$\times$	$-$	$-$
$\mathcal{KK}$	t	J_1	J_2	J_3	z	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
$SM2$	t	$-$	$-$	$-$	$-$	$\times$	$\times$	$\times$	$-$	$-$	$-$
$\mathcal{KK}$	t	J_1	J_2	J_3	z	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
$SM2$	t	$-$	$\times$	$\times$	$-$	$\times$	$-$	$-$	$-$	$-$	$-$
$\mathcal{KK}$	t	J_1	J_2	J_3	z	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
$SM2$	t	$\times$	$-$	$-$	$\times$	$\times$	$-$	$-$	$-$	$-$	$-$
$\mathcal{KK}$	t	J_1	J_2	J_3	z	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$

We write the metric and the form field explicity for two of the given configurations for clarity. In the following, we write the harmonic function of the Kaluza-Klein monopole in an exponential form, $H_{\mathcal{KK}}(y^i) \equiv e^{2G(y^i)}$. For the first configuration the metric reads

$$ds^2 = -e^{2A}dt^2 + e^{2B}\left[\delta_{\mu\nu}dx^\mu dx^\nu + e^{-2G}(dz + J_i dy^i)^2 + e^{2G}\delta_{ij}dy^i dy^j\right] \\ +e^{2C}d\Sigma_{k,\sigma}^2 + e^{2D}\delta_{ab}dv^a dv^b \tag{7.17}$$

where $\mu = 1, 2$; $i = 1, 2, 3$; $a = 1, ..., (4-k)$; $k > 1$ and $\Sigma_{k,\sigma}$ is a k-dimensional constant curvature hyperspace; sphere for $\sigma = 1$, flat space for $\sigma = 0$, hyperbolic space for $\sigma = -1$.

The form field is exactly the same as it is with a single SM5, that is,

$$F_{[4]} = Q_s \mathrm{vol}(\Sigma_{k,\sigma}) \wedge dv_1 \wedge ...dv_{4-k}. \tag{7.18}$$

The time dependent equations are the same as they are with a single SM5-brane, too, which can be read from the solution given in Chapter 5 directly by setting $p = 6$, $q = 4$, $a = c_1 = c_2 = 0$ with the dilaton field set to zero. Effectively, since the Kaluza-Klein metric is Ricci flat given its harmonic function, placing it in the worldvolume of an S-brane does not alter other fields than the metric.

We also write explicitly the third configuration. The metric reads

$$ds^2 = -e^{2A}dt^2 + e^{2B}\left[\delta_{\mu\nu}dx^\mu dx^\nu + e^{-2G}(dz + J_1 dy^1)^2 + e^{2G}(dy^1)^2\right] \\ +e^{2C+2G}\left[(dy^2)^2 + (dy^3)^2\right] + e^{2D}d\Sigma_{2,\sigma}^2 \tag{7.19}$$

where $\mu = 1, ..., 4$ and $\Sigma_{2,\sigma}$ is a 2-dimensional sphere, flat space or hyperbolic space. For this configuration we write the form field in explicit component form

$$F_{ijab} = Q_s e^{-A+6B+2C+2D+2G}\tilde{\epsilon}_{ij}\tilde{\epsilon}_{ab} \tag{7.20}$$

where $\{i, j\}$ refer to the coordinate basis in the $\{y^i\}$-space and $\{a, b\}$ refer to an orthonormal basis on the hyperspace $\Sigma_{2,\sigma}$.

7.3. Multiple Intersections

Having established intersection rules for an S-brane with a plane wave and a KK monopole, we proceed to construct multiple intersections of interest in eleven dimensions. In a multiple intersection each pair obtained by removing one of the constituents must be one of the basic pairs we described in previous chapters. Removing here should be understood for S-branes as setting the charge parameter equal to zero and adjusting α in Equation 5.26 such that $f(t)$ remains finite and for waves, Kaluza-Klein monopoles and p-branes as setting the corresponding harmonic function equal to one.

One can add several SM2-branes to a given time-dependent solution in eleven dimensions using Lunin-Maldacena deformations as described in [23]. What is required of the original solution in eleven dimensions is that it entails three or more commuting isometry directions which do not mix in the metric with other coordinates and that its four-form field strength has at most one overlapping with these three coordinates. Addition of one or more SM2-branes to a given pair intersection can be carried out using this method.

At most two SM5-branes can intersect in eleven dimensions [17], restriction due to the six dimensional worldvolume of an SM5-brane. Their common worldvolume is four-dimensional, two relative transverse directions and an overall transverse direction is present for each. One can add a wave to this intersection with the z coordinate of the wave as one of the overall transverse directions. The harmonic function of the wave depends on the single overall transverse coordinate. Addition of a Kaluza Klein monopole to an SM5-SM5 pair is subject to the multiple intersection rule that poses each SM5-$\mathcal{KK}$ pair should be one of those listed in Section 7.2.

Multiple intersections involving the $\mathcal{W}$-$\mathcal{KK}$ pair are restricted by the existence of a single possible configuration for this pair where the z_1 part of the wave and the $\{z_2, J_i\}$ part of the Kaluza Klein monopole do not overlap and both harmonic functions depend on the overall transverse coordinates, that is, the J_i directions. The SM2-$\mathcal{KK}$-$\mathcal{W}$ intersection is of interest and we write its metric explicitly,

$$\begin{aligned} ds_{11}^2 = -e^{2\hat{A}} H_{\mathcal{W}}^{-1} dt^2 + e^{2\hat{B}}(dx_1^2 + dx_2^2 + dx_3^2) + e^{2\hat{C}} H_{\mathcal{W}} \Big[(H_{\mathcal{W}}^{-1} - 1) e^{\hat{A}-\hat{C}} dt + dz_1 \Big]^2 \\ + e^{2\hat{D}} \left[H_{\mathcal{KK}}^{-1} (dz_2 + J_i dy_i)^2 + H_{\mathcal{KK}} (dy_1^2 + dy_2^2 + dy_3^2) \right] + e^{2\hat{E}} (dv_1^2 + dv_2^2) \end{aligned} \tag{7.21}$$

where $H_{\mathcal{W}}$ and $H_{\mathcal{KK}}$ are harmonic functions in the variables $\{y_i\}$ and $\{x^\mu\}$-space ($\mu = 1, 2, 3$) describes the worldvolume of the SM2-brane. Field equations for the time dependent functions are solved in the usual fashion, surviving the constants of first integration in the solution of the equations

$$\ddot{\hat{B}} = -2\ddot{\hat{C}} = -2\ddot{\hat{D}} \tag{7.22}$$

and imposing on them the wave condition

$$\dot{\hat{A}} + \dot{\hat{C}} = 2\dot{\hat{D}} \tag{7.23}$$

We are interested in the reduction of this solution to ten dimensions over z_2 and having written $\hat{D}(t)$ in a form where it involves a term linear in t we are allowed to proceed to dimensional reduction as discussed in Section 6.1.2. In ten dimensions the reduced theory is described by the SD2-D6-W solution

$$\begin{aligned} ds_{10}^2 = H_{D6}^{-1/8} \left\{ -e^{2A} H_{\mathcal{W}}^{-1} dt^2 + e^{2C} H_{\mathcal{W}} \big[(H_{\mathcal{W}}^{-1} - 1) e^{A-C} dt + dz_1 \big]^2 + e^{2E} (dv_1^2 + dv_2^2) \right. \\ \left. + e^{2B} (dx_1^2 + dx_2^2 + dx_3^2) \right\} + H_{D6}^{7/8} e^{2D} (dy_1^2 + dy_2^2 + dy_3^2) \end{aligned} \tag{7.24}$$

where we identified the harmonic function of the D6-brane as $H_{D6} \equiv H_{\mathcal{KK}}$. We observe that the wave condition is preserved under dimensional reduction, that is, Equation 7.23 with hatted functions in eleven dimensions implies in ten dimensions $\dot{A} + \dot{C} = 2\dot{D}$.

We shall write the time dependent functions explicitly. The gauge condition is

$$-\dot{A} + 3\dot{B} + \dot{C} + 3\dot{D} + 2\dot{E} = 0. \tag{7.25}$$

With this gauge we have the parametrization

$$\dot{C} = -\frac{3}{5}\dot{B} + \lambda, \tag{7.26}$$
$$\dot{D} = -\frac{3}{5}\dot{B} + \sigma, \tag{7.27}$$
$$\dot{E} = -\frac{3}{5}\dot{B} + \mu, \tag{7.28}$$
$$\dot{A} = -\frac{3}{5}\dot{B} + \lambda + 2\mu + 3\sigma, \tag{7.29}$$
$$\phi_s = -\frac{4}{5}B + \gamma t + \Omega. \tag{7.30}$$

In terms of

$$h(t) \equiv B(t) - \frac{5}{64}\gamma t - \frac{5}{64}\Omega \tag{7.31}$$

the Einstein equations and the dilaton equation reduce to

$$\dot{h}^2 + \frac{25}{256}Q_s^2 e^{32h/5} = M^2 \tag{7.32}$$

where Q_s is the charge parameter for the SD2-brane and

$$M^2 = \frac{25}{128}\left(2\mu^2 + 6\sigma^2 + 12\sigma\mu + 4\lambda\mu + 6\lambda\sigma - \frac{15}{32}\gamma^2\right). \tag{7.33}$$

The field equation for the two-form field strength of the D6-brane and the wave condition bring, respectively,

$$\dot{\phi}_s - \frac{4}{3}\dot{D} = 0, \tag{7.34}$$
$$\dot{A} + \dot{C} - 2\dot{D} = 0. \tag{7.35}$$

Equation 7.32 is similar in form with Equation 5.23 and the solution is given in Equation 5.26. The constants satisfy

$$\gamma = \frac{4}{3}\sigma, \quad \sigma = -2\lambda - 2\mu \quad \Rightarrow \quad M^2 = \frac{25}{128}\left[\left(\sqrt{\frac{13}{6}}\sigma + \sqrt{\frac{24}{13}}\mu\right)^2 - \frac{50}{13}\mu^2\right]. \tag{7.36}$$

We continue with the SD1-D5-W intersection which cannot be obtained via dimensional reduction. We write the metric ansatz as

$$\begin{aligned} ds^2_{10} = H_{D5}^{-1/4}\big\{-e^{2A}H_{\mathcal{W}}^{-1}dt^2 + e^{2B}H_{\mathcal{W}}\left[(H_{\mathcal{W}}^{-1} - 1)e^{A-B}dt + dz_1\right]^2 + e^{2C}(dx_1^2 + dx_2^2) \\ + e^{2D}(dv_1^2 + dv_2^2)\big\} + H_{D5}^{3/4}e^{2E}(dr^2 + r^2 d\Omega_3^2) \end{aligned} \tag{7.37}$$

where $d\Omega_3^2$ is the metric on S^3 and the four dimensional transverse space of the D5-brane is written is spherical coordinates and the $\{x^\mu\}$-space ($\mu = 1, 2$) describes the worldvolume of the SNS1-brane. With this ansatz and the corresponding three-form field strengths for the SNS1-brane and the D5-brane, the field equations tell us that $H_{\mathcal{W}}(r)$ and $H_{D5}(r)$ are harmonic functions. The field equation for the field strength associated with the SD1-brane is the same as the gauge condition that simplifies time dependent equations,

$$-\dot{A} + \dot{B} + 2\dot{C} + 2\dot{D} + 4\dot{E} = 0. \tag{7.38}$$

With this gauge, the time dependent functions take the form

$$\dot{B} = -\frac{\dot{C}}{3} + \lambda, \tag{7.39}$$
$$\dot{D} = -\frac{\dot{C}}{3} + \mu, \tag{7.40}$$
$$\dot{E} = -\frac{\dot{C}}{3} + \sigma, \tag{7.41}$$
$$\dot{A} = -\frac{\dot{C}}{3} + \lambda + 2\mu + 4\sigma, \tag{7.42}$$
$$\phi_s = -\frac{4}{3}C + \gamma t + \Omega. \tag{7.43}$$

In terms of

$$h(t) \equiv C(t) - \frac{3}{16}\gamma t - \frac{3}{16}\Omega \tag{7.44}$$

Einstein equations and the dilaton equation reduce to

$$\dot{h}^2 + \frac{9}{64}Q_s^2 e^{16h/3} = M^2 \tag{7.45}$$

where Q_s is the charge parameter for the SD1-brane and

$$M^2 = \frac{9}{16}\left(\mu^2 + 6\sigma^2 + 8\sigma\mu + 4\lambda\mu + 8\lambda\sigma - \frac{3}{8}\gamma^2\right). \tag{7.46}$$

The field equation for the three-form field strength of the D5-brane and the wave condition bring, respectively,

$$\dot{\phi}_s - 4\dot{E} = 0, \tag{7.47}$$
$$\dot{A} + \dot{B} - 2\dot{E} = 0. \tag{7.48}$$

Solving Equations 7.45-7.48 together we find

$$\dot{h}(t) = M = Q_s = 0, \quad \gamma = 4\mu = 4\sigma = -2\lambda. \tag{7.49}$$

We have thus shown that the SD1-D5-W intersection in ten dimensions is a solution only if the SD1-brane is chargeless.

8. BLACK HOLES FROM INTERSECTING BRANES

In the previous chapters we discussed time independent and time dependent solutions of string/M-theory. An important class of solutions are those involving intersections of these objects. We have already considered various intersections involving S-branes, however, left out intersections of time independent p-brane solutions. Intersecting brane solutions in $D = 11$ and $D = 10$ provide a unified viewpoint since many other solutions can be obtained via dimensional reduction and duality transformations [16], the latter of which we shall not discuss in this thesis. In the present chapter we give a preliminary account of this vast subject, focusing briefly on an important application: intersecting p-brane solutions that give rise to black hole spacetimes upon dimensional reduction [1]. We then investigate if by adding an S-brane to these configurations it would be possible to obtain a black hole in a time dependent background and find that starting with the ansätze we propose the restrictions imposed on the integration constants render the metric time independent.

We start with an example and use it to introduce some terminology and to outline general features of intersecting branes. Two M2-branes intersect over a point and the metric is given as

$$ds_{11}^2 = H_{(1)}^{1/3} H_{(2)}^{1/3} \left[H_{(1)}^{-1} H_{(2)}^{-1} dt^2 + H_{(1)}^{-1} dx_{(1-2)}^2 + H_{(2)}^{-1} dy_{(1-2)}^2 + dz_{(1-6)}^2 \right] \tag{8.1}$$

where we introduced the notation $dx_{(1-n)}^2 \equiv dx_1^2 + ... + dx_n^2$. Here $H_{(1)}$ and $H_{(2)}$ are harmonic functions in the varibles $\{z_i\}$. The nonvanishing components of the form fields are given as

$$F^{(1)}{}_{z_i t x_1 x_2} = \partial_{z_i} H_{(1)}^{-1}, \qquad F^{(2)}{}_{z_i t y_1 y_2} = \partial_{z_i} H_{(2)}^{-1} \tag{8.2}$$

In Equations 3.19-3.21 we can see in the metric that describes a single M2-brane that the worldvolume of the M2-brane is multiplied by $H_{M2}^{-2/3}$ and its transverse space is multiplied by $H_{M2}^{1/3}$ where H_{M2} is the harmonic function of the M2-brane. This feauture is preserved in the intersection of two M2 branes in the following sense: The worldvolume and the transverse space of each M2-brane is multiplied by the appropriate power of the corresponding harmonic function, the resulting solution sometimes referred to as the *harmonic superposition* of two M2-branes.

Identifying $H_{(1)}$ and $H_{(2)}$ as the harmonic functions of the two M2-branes in Equation 8.1, t is called the *common worldvolume* coordinate, the $\{x_i\}$-space and $\{y_i\}$-space are *relative transverse* spaces meaning that they are worldvolume directions for one M2-brane are transverse directions for the other, and the $\{z_i\}$-space is the *overall transverse* space meaning that it is transverse to both M2-branes. The described intersection of two M2-branes over a point is denoted as $(0|M2, M2)$. The notation $(q|p_1, p_2)$ specifies the intersection of a p_1-brane and a p_2-brane with common worldvolume of q spatial dimensions.

The structure of the metric describing the $(0|M2, M2)$ intersection reveals a common feature for the multiple intersections we are going to study which is described by the *harmonic function rule*: In a multiple intersection the harmonic function of each constituent brane depends only on the overall transverse space and in the conformal frame where the overall transverse space emerges as the flat space with no multiplicative factor a coordinate x that is in the worldvolume of several branes appears in the metric with the inverse powers of the harmonic functions of these branes multiplying dx^2.

It is also exemplified in the $(0|M2, M2)$ intersection that the q-form field strengths of constituent branes are just summed up as differential forms. A rule that limits possible intersecting configurations is in order: p-branes of the same type can intersect only over a (p-2)-brane Ref[Tseyt] as witnessed above in the intersection of two M2-branes over a 0-brane. It can be seen that this is required by the equations of motion for the contituent field strengths.

Having outlined the general intersection rules we proceed directly to an important example in $D = 10$ that reduces in $D = 5$ to a black hole, the D1-D5-W system. The metric is

$$ds_{10}^2 = H_{D5}^{3/4} H_{D1}^{1/4} \Big[H_{D5}^{-1} H_{D1}^{-1} \big(-H_W^{-1} dt^2 + H_W \big[(H_W^{-1} - 1) dt + dz \big]^2 \big) + H_{D5}^{-1} (dx_1^2 + ... + dx_4^2) + (dr^2 + r^2 d\Omega_3^2) \Big]. \tag{8.3}$$

where harmonic functions are taken to be isotropic in the overall transverse space.Denoting the coordinates as $x^A = (t, z, x^\mu, \Sigma_{1,3})$ where $\Sigma_{1,3}$ identifies the 3-sphere we have

$$F_{tzr}^{(D1)} = \partial_r H_{D1}^{-1} \ , \quad F_{ijk}^{(D5)} = (\partial_r H_{D5}) r^3, \quad e^{2\phi} = H_{D1} H_{D5}^{-1} \tag{8.4}$$

where the three form for D5 is written with respect to an orthonormal frame on $\Sigma_{1,3}$.

The harmonic functions depend isotropically on the flat 4-space

$$H_{D1} = 1 + \frac{Q_1}{r^2} \ , \quad H_{D5} = 1 + \frac{Q_5}{r^2} \ , \quad H_W = 1 + \frac{Q_W}{r^2}. \tag{8.5}$$

Dimensionally reducing this solution over the $z, x_1, ..., x_4$ directions we obtain in five dimensions

$$ds_5^2 = \lambda^{-2/3} dt^2 + \lambda^{1/3} (dr^2 + r^2 d\Omega_3^2) \tag{8.6}$$

where

$$\lambda = H_{D1} H_{D5} H_W \tag{8.7}$$

This is a 3-charge extremal black hole in five dimensions written in isotropic coordinates. The horizon is located at $r = 0$ and the area of the horizon can be calculated to be finite.

We proceed to investigate if one can add an S-brane to the D1-D5-W system. We have seen in Section 7.3 that one can have an SD1-D5-W intersection with the wave placed on the worldvolume of the D5-brane and on the transverse space of the chargeless SD1-brane; this configuration seems to be compatible with the D1-D5-W system. If the SD1-D1-D5-W solution existed, it would yield upon dimensioal reduction a black hole in a time dependent background in five dimensions. We write the metric ansatz for the SD1-D1-D5-W intersection as

$$\begin{aligned} ds_{10}^2 =& H_{D5}^{-1/4}\Big(H_{D1}^{-3/4}\big\{-e^{2A}H_{\mathcal{W}}^{-1}dt^2 + e^{2B}H_{\mathcal{W}}\big[(H_{\mathcal{W}}^{-1}-1)e^{A-B}dt + dz_1\big]^2\big\} \\ &+ H_{D1}^{1/4}\big\{e^{2C}(dx_1^2+dx_2^2) + e^{2D}(dv_1^2+dv_2^2)\big\}\Big) + H_{D5}^{3/4}H_{D1}^{1/4}e^{2E}(dr^2+r^2d\Omega_3^2) \end{aligned} \tag{8.8}$$

where harmonic functions are taken to be isotropic in the four dimensional overall transverse space and field strength q-forms and dilaton fields of constituent branes are simply summed up.

Field equations yield exactly the same solution as in the SD1-D5-W system with the additional constraint on the constants of integration in Equation 7.49 coming from the equation of motion for the field strength of the D1-brane

$$4\dot{C} + 4\dot{D} + 4\dot{E} + \dot{\phi}_s = 0 \qquad \Rightarrow \qquad \gamma = -4\sigma - 4\mu. \tag{8.9}$$

Along with Equation 7.49 we see that the metric reduces to be time independent and the SD1 charge is zero; the system we have is just the D1-D5-W solution.

We continue with another intersection configuration that gives rise to a black hole in four dimensions. To have a black hole solution in four dimensions that has a regular horizon, four independent charges are required. The D2-D6-NS5-W intersection in ten dimensions is given as

$$\begin{aligned} ds_{10}^2 = H_{D2}^{3/8}H_{D6}^{7/8}H_{NS5}^{3/4}\Big\{&H_{D2}^{-1}H_{D6}^{-1}H_{NS5}^{-1}\big(-H_{\mathcal{W}}^{-1}dt^2 + H_{\mathcal{W}}\big[(H_{\mathcal{W}}^{-1}-1)dt + dx_1\big]^2 + dx_2^2\big) \\ &+H_{D6}^{-1}H_{NS5}^{-1}\ dx_{(3-5)}^2 + H_{D6}^{-1}dx_6^2 + dx_{(7-9)}^2\Big\} \end{aligned} \tag{8.10}$$

where harmonic functions depend on the overall transverse coordinates x_7, x_8, x_9 and the harmonic function rule is apparent. Assuming isotropicity in the overall transverse space and defining $r = \sqrt{x_7^2 + x_8^2 + x_9^2}$ each harmonic function is of the form $H = 1 + Qr^{-1}$. Upon dimensional reduction over the isometry directions $x_1, ..., x_6$ the metric in Einstein frame in four dimensions reads

$$ds_4^2 = -dt^2 (H_{D2} H_{D6} H_{NS5} H_{\mathcal{W}})^{-\frac{1}{2}} + (dr^2 + r^2 + d\Omega_2^2)(H_{D2} H_{D6} H_{NS5} H_{\mathcal{W}})^{\frac{1}{2}}. \quad (8.11)$$

Given the r^{-1} dependence of the harmonic functions this is an extremal black hole in four dimensions which reduces to the extremal Reissner-Nordstrom solution in a certain limit.

Following a similar line of reasoning, we wonder if one can add an S-brane to the D2-D6-NS5-W configuration which could lead to a time dependent black hole in four dimensions. We have the SD2-D6-W solution in Equations 7.26-7.36. Adding a D2-brane to this configuration, with one of its spatial worldvolume coordinates coinciding with the z-direction of the wave and all of its worldvolume inside the worldvolume of the D6-brane and on the transverse space of the SD2-brane, we find that equations of motion reduce to those of the SD2-D6-W system with an additional condition on the constants of integration in Equation 7.36 brought by the equation of motion for the field strength of the D2-brane

$$\gamma = -4\sigma - 8\mu \quad (8.12)$$

and along with Equations 7.26-7.36 it is found that this condition renders the SD2-brane chargeless and the metric time independent; what we have is the D2-D6-W solution.

9. CONCLUSION

The main result of this thesis is the new solutions of ten and eleven dimensional supergravity theories involving intersections of S-branes with plane waves and Kaluza-Klein monopoles. We find that a wave can be placed only on the transverse space of an S-brane and the transverse space is required to be flat. This restriction is due to the structure of the metric: S-brane worldvolume is spacelike and the metric is time dependent. A larger number of configurations is allowed for intersections with Kaluza-Klein monopoles and these have been listed in Section 7.2.

Before describing these solutions we discussed supergravity theories in general. Eleven dimensional supergravity is the low energy limit M-theory from which all five superstring theories can be obtained and it follows that Type IIA and IIB supergravity theories are the low energy limits of the corresponding superstring theory. These features attribute fundamental importance to supergravity theories. We restricted our attention to purely bosonic supergravity and described the NSNS, RR and CS sectors. p-branes constitute an important class of solutions of these theories. We discussed the p-brane ansatz and established that p-branes carry charge via field strength tensors obeying Bianchi identities. The symmetry between the Bianchi identity and the equation of motion for a field strength q-form enables us to distinguish electric and magnetic p-branes which are in turn dual solutions.

We proceeded to describe S-branes in general. These solutions are described by hyperbolic functions of time and have a spacelike worldvolume, hence they exist at only an instant in time.

Kaluza-Klein dimensional reduction is a central topic. Some of the ten dimensional solutions we discussed can be obtained from eleven dimensional solutions via dimensional reduction. In particular, we showed that one can obtain Type IIA theory by wrapping M-theory around on a circle. Dimensional reduction of M-theory S-branes is slightly more involved. In essence, one must match the numbers of parameters that characterize the higher dimensional and dimensionally reduced S-brane solutions. An application of dimensional reduction of S-branes is cosmology. We showed briefly that one can obtain FLRW cosmology with acceleated expansion by reducing M-theory S-branes down to four spacetime dimensions.

Intersections of branes is a vast subject and we briefly discussed general features of intersecting p-branes. We presented ten dimensional intersecting brane solutions involving waves that give rise to extremal five and four dimensional black holes. As a potential application for intersecting S-branes and waves we discussed if S-branes could be added to the p-brane intersections that reduce to black holes. However, we found that, following our ansätze, in each of the would-be SD1-D1-D5-W and SD2-D2-D6-W solutions the wave condition is too strong and renders the S-brane chargeless and the metric time independent.

APPENDIX A: VARIATION OF THE SUPERGRAVITY ACTION

We perform the variation of the supergravity action

$$S = \int d^D x \sqrt{-g}\left[R - \frac{1}{2}\partial_M\phi\partial^M\phi - \frac{1}{2q!}e^{a\phi}F_{[q]}^2\right] \tag{A.1}$$

with respect to the metric g^{MN}, the $(q-1)$-form potential $A_{[q-1]}$, and the dilaton ϕ. The third term in the action is the field strength for the potential, $F_{[q]} = dA_{[q-1]}$.

First we vary the action with respect to the metric. We have

$$\begin{aligned} \delta(\sqrt{-g}) &= -\frac{1}{2}\sqrt{-g}g_{MN}\delta g^{MN}, & \text{(A.2)} \\ \delta(\sqrt{-g}R) &= \sqrt{-g}G_{MN}\delta g^{MN} & \text{(A.3)} \end{aligned}$$

where G_{MN} is the Einstein tensor; $G_{MN} = R_{MN} - \frac{1}{2}Rg_{MN}$. It is easy to perform the variations with respect to the metric of the kinetic terms for the field strength and the dilaton. Setting $\delta S[g^{MN}] = 0$ gives the Einstein equation as

$$\begin{aligned} R_{MN} - \frac{1}{2}Rg_{MN} - \frac{1}{2}\partial_M\phi\partial_N\phi + \frac{1}{4}g_{MN}(\partial\phi)^2 \\ - \frac{e^{a\phi}}{2(q-1)!}F_{MA_1\ldots A_{q-1}}F_N{}^{A_1\ldots A_{q-1}} + \frac{e^{a\phi}}{4q!}F^2 g_{MN} = 0. \end{aligned} \tag{A.4}$$

Contracting this with g^{MN} we get the Ricci scalar R. Sustituting for it in Equation A.4 we reach the final form of the Einstein equation

$$R_{MN} - \frac{1}{2}\partial_M\phi\partial_N\phi - \frac{e^{a\phi}}{2(q-1)!}F_{MA_1\ldots A_{q-1}}F_N{}^{A_1\ldots A_{q-1}} + \frac{(q-1)e^{a\phi}}{2q!(D-2)}F^2 g_{MN} = 0 \tag{A.5}$$

Extremizing the action with respect to the gauge potential $A_{[q-1]}$ we get

$$\begin{aligned} \delta S[A_{[q-1]}] &= 0 \quad \Rightarrow \\ \delta\left[-\frac{\sqrt{-g}}{2q!}e^{a\phi}F^2\right] = -\frac{\sqrt{-g}}{q!}e^{a\phi}F^{M_1\ldots M_q}\delta F_{M_1\ldots M_q} &= 0 \quad \Rightarrow \\ \sqrt{-g}e^{a\phi}F^{M_1\ldots M_q}\delta(\partial_{[M_1}A_{M_2\ldots M_q]}) &= 0. \end{aligned} \tag{A.6}$$

Integrating by parts we reach the field equation for $F_{[q]}$,

$$\partial_{M_1}(\sqrt{-g}e^{a\phi}F^{M_1\ldots M_q}) = 0. \tag{A.7}$$

Variation of the action with respect to the dilaton leads to

$$\delta\left[-\frac{1}{2}\sqrt{-g}\partial_M\phi\partial^M\phi\right] = -\sqrt{-g}(\partial_M\delta\phi)\partial^M\phi. \tag{A.8}$$

Integrating this term by parts gives $\partial_M(\sqrt{-g}\partial^M\phi)\delta\phi$. The kinetic term for the field strength also involves the dilaton

$$\delta\left[-\frac{\sqrt{-g}}{2q!}e^{a\phi}F^2\right] = -\frac{\sqrt{-g}}{2q!}ae^{a\phi}F^2\delta\phi. \tag{A.9}$$

Then $\delta S[\phi] = 0$ gives the dilaton equation as

$$\frac{1}{\sqrt{-g}}\partial_M\left(\sqrt{-g}\partial^M\phi\right) - \frac{a}{2q!}e^{a\phi}F_{[q]}^2 = 0. \tag{A.10}$$

APPENDIX B: CALCULATION OF CURVATURE

We use Cartan's formalism to calculate the curvature of a spacetime given its metric. The vielbein is provides an orthonormal frame as

$$ds^2 = g_{MN}dx^M dx^N = \eta_{\bar{M}\bar{N}}E^{\bar{M}}E^{\bar{N}} \tag{B.1}$$

where orthonormal frame indices are barred versions of world indices. The torsion free connnection one-forms, $\omega^{\bar{M}}{}_{\bar{N}}$, are obtained using the following first structure equation

$$dE^{\bar{M}} + \omega^{\bar{M}}{}_{\bar{N}} \wedge E^{\bar{N}} = 0 \tag{B.2}$$

Using the second structure equation one gets the curvature two-forms, $\Omega^{\bar{M}}{}_{\bar{N}}$,which are related to the Riemann tensor components in the orthonormal frame as

$$\Omega^{\bar{M}}{}_{\bar{N}} = d\omega^{\bar{M}}{}_{\bar{N}} + \omega^{\bar{M}}{}_{\bar{L}} \wedge \omega^{\bar{L}}{}_{\bar{N}} = \frac{1}{2}R^{\bar{M}}{}_{\bar{N}\bar{K}\bar{L}}E^{\bar{K}} \wedge E^{\bar{L}}. \tag{B.3}$$

We write the Ricci tensor as $R_{\bar{N}\bar{L}} = R^{\bar{M}}{}_{\bar{N}\bar{M}\bar{L}}$ and one goes to the world indices from the orthonormal indices using $R_{MN} = R_{\bar{K}\bar{L}}E^{\bar{K}}{}_M E^{\bar{L}}{}_N$.

B.1. p-brane metric

For the p-brane metric

$$ds^2 = e^{A(r)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2B(r)}\delta_{mn}dy^m dy^n, \tag{B.4}$$

we have the vielbein

$$E^{\bar{\mu}} = e^A dx^\mu, \qquad E^{\bar{m}} = e^B dy^m \tag{B.5}$$

where $\mu = 0, 1, ..., d-1$ and $m = 1, ..., \tilde{d}+2$. Connection one-forms are

$$\omega^{\bar{\mu}}{}_{\bar{n}} = e^{-B}(\partial_n A)E^{\bar{\mu}}, \quad \omega^{\bar{m}}{}_{\bar{n}} = e^{-B}(\partial_n B)E^{\bar{m}} - e^{-B}(\partial_m B)E^{\bar{n}}. \tag{B.6}$$

The Ricci tensor components are given as

$$\begin{aligned} R_{\mu\nu} &= -\eta_{\mu\nu}e^{2A-2B}\delta^{kl}\big[d(\partial_k A)(\partial_l A) + \tilde{d}(\partial_k A)(\partial_l B) + (\partial_k\partial_l A)\big], \\ R_{mn} &= -\delta_{mn}\delta^{kl}\big[\partial_k\partial_l B + d(\partial_k A)(\partial_l B) + \tilde{d}(\partial_k B)(\partial_l B)\big] - d(\partial_m\partial_n A) - \tilde{d}(\partial_m\partial_n B) \\ &\quad - d(\partial_m A)(\partial_n A) + \tilde{d}(\partial_m B)(\partial_n B) + d(\partial_m A)(\partial_n B) + d(\partial_m B)(\partial_n A) \end{aligned} \tag{B.7}$$

where $\partial_m \equiv \partial/\partial y^m$. Defining the radial coordinate $r = \sqrt{\delta_{mn}y^m y^n}$ we have, for an isotropic function $f(r)$, $\partial_m f = f' y^m r^{-1}$ where prime denotes derivative with respect to r. Then in terms of $A(r)$, $B(r)$ we have

$$\begin{aligned} R_{\mu\nu} &= -\eta_{\mu\nu}e^{2A-2B}\left[A'' + d(A')^2 + \tilde{d}A'B' + \frac{(\tilde{d}+1)}{r}A'\right], \\ R_{mn} &= -\delta_{mn}\left[B'' + dA'B' + \tilde{d}(B')^2 + \frac{(2\tilde{d}+1)}{r}B' + \frac{d}{r}A'\right] \\ &\quad - \frac{y^m y^n}{r^2}\left[\tilde{d}B'' + dA'' - 2dA'B' + d(A')^2 - \tilde{d}(B')^2 - \frac{\tilde{d}}{r}B' - \frac{d}{r}A'\right] \end{aligned} \tag{B.8}$$

B.2. S-brane metric

For the S-brane metric

$$ds^2 = -e^{2A}dt^2 + e^{2B}(dx_1^2 + ... + dx_p^2) + e^{2C}d\Sigma_{k,\sigma}^2 + e^{2D}(dy_1^2 + ... + dy_{q-k}^2) \tag{B.9}$$

the vielbein reads

$$E^{\bar{t}} = e^A dt, \quad E^{\bar{\mu}} = e^B dx^\mu, \quad E^{\bar{a}} = e^C \tilde{e}^a, \quad E^{\bar{i}} = e^D dy^i \tag{B.10}$$

where $\tilde{e}^a$ specifies an orthonormal frame on the constant scalar curvature hyperspace $\Sigma_{k,\sigma}$. With respect to this frame

$$\tilde{R}_{ab} = \sigma(k-1)\delta_{ab}. \tag{B.11}$$

Connection one-forms are then

$$\omega^{\bar{\mu}}{}_{\bar{t}} = e^{-A}\dot{B}E^{\bar{\mu}}, \quad \omega^{\bar{a}}{}_{\bar{b}} = \tilde{\omega}^a{}_b, \quad \omega^{\bar{a}}{}_{\bar{t}} = e^{-A}\dot{C}E^{\bar{a}}, \quad \omega^{\bar{i}}{}_{\bar{t}} = e^{-A}\dot{D}E^{\bar{i}}, \tag{B.12}$$

The Ricci tensor reads

$$\begin{aligned}
R_{tt} &= -p(\ddot{B}+\dot{B}^2-\dot{A}\dot{B}) - k(\ddot{C}+\dot{C}^2-\dot{A}\dot{C}) - (q-k)(\ddot{D}+\dot{D}^2-\dot{A}\dot{D}),\\
R_{\mu\nu} &= e^{2B-2A}\left[\ddot{B}-\dot{A}\dot{B}+p\dot{B}^2+k\dot{B}\dot{C}+(q-k)\dot{B}\dot{D}\right]\delta_{\mu\nu},\\
R_{ij} &= e^{2D-2A}\left[\ddot{D}-\dot{A}\dot{D}+p\dot{B}\dot{D}+k\dot{D}\dot{C}+(q-k)\dot{D}^2\right]\delta_{ij},\\
R_{ab} &= e^{2C-2A}\left[\ddot{C}-\dot{A}\dot{C}+p\dot{B}\dot{C}+k\dot{C}^2+(q-k)\dot{C}\dot{D}\right]\delta_{ab}+\sigma(k-1)\delta_{ab}.
\end{aligned} \tag{B.13}$$

With the gauge condition

$$-\dot{A}+p\dot{B}+k\dot{C}+(q-k)\dot{D} = 0 \tag{B.14}$$

the Ricci tensor simplifies as

$$R_{tt} = -\ddot{A}+\dot{A}^2-p\dot{B}^2-k\dot{C}^2-(q-k)\dot{D}^2, \tag{B.15}$$

$$R_{\mu\nu} = e^{2B-2A}\ddot{B}\ \delta_{\mu\nu}, \tag{B.16}$$

$$R_{ij} = e^{2D-2A}\ddot{D}\ \delta_{ij}, \tag{B.17}$$

$$R_{ab} = [e^{2C-2A}\ddot{C}+\sigma(k-1)]\delta_{ab}. \tag{B.18}$$

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