

ERASMUS STUDENT EXCHANGE PROGRAMME:
A PLACEMENT PROBLEM

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Thesis Abstract

Ekrem Akkaya, “Erasmus Student Exchange Programme: A Placement Problem”

I analyze the placement mechanism used at Boğaziçi University in order to place students to positions offered by partner institutions according to the Erasmus Student Exchange Programme. First, I define the placement problem in this context and then I describe the mechanism used to place students to positions. I show that the mechanism satisfies individual rationality and strategy-proofness, whereas it fails to satisfy non-wastefulness, fairness and Pareto efficiency. I make three proposals to upgrade the mechanism so that the modified mechanism satisfies all of the desired properties mentioned above. Finally, using actual data of 2009-2010, I illustrate the improvement obtained by modifying the mechanism according to one of the proposals I make.

Tez Özeti

Ekrem Akkaya, “Erasmus Öğrenci Değişim Programı: Bir Yerleştirme Problemi”

Erasmus Öğrenci Değişim Programı çerçevesinde, Boğaziçi Üniversitesi’nde öğrencileri partner kurumlar tarafından açılan pozisyonlara yerleştirmek için kullanılan yerleştirme mekanizmasını analiz ediyorum. İlk olarak, bu bağlamdaki yerleştirme problemini tanımlıyorum ve ardından öğrencileri pozisyonlara yerleştirmek için kullanılan mekanizmayı tarif ediyorum. Mekanizmanın bireysel rasyonellik ve strateji-dayanıklılığı sağladığını, ancak ziyankar olmama, adillik ve Pareto verimliliği sağlamadığını gösteriyorum. Mekanizmayı düzeltmek adına üç öneride bulunuyorum, öyle ki değiştirilmiş mekanizma yukarıda bahsedilen istenen özelliklerin hepsini sağlıyor. Son olarak, 2009-2010’un gerçek verisini kullanarak, mekanizmanın yaptığım önerilerden biri doğrultusunda değiştirilmesiyle elde edilen gelişmeyi örnekliyorum.

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CHAPTER 1

INTRODUCTION

Erasmus Programme (European Region Action Scheme for the Mobility of University Students) is an international exchange programme for students and teaching staff of higher education institutions. According to this programme, students have the opportunity to study abroad as exchange students and professors can benefit from the programme by contacting other professors at partner institutions so that they can share recent developments on their research areas.

Boğaziçi University is one of the institutions participating in this programme. Focusing on the student exchange part of the programme, Boğaziçi University places its students to positions offered by partner institutions and welcomes exchange students in return every year. This placement procedure of students to positions constitutes a placement problem.

In a placement problem, the market is constituted by two sides; on the one side there are “active” agents (such as students, interns, employees etc.) with their preferences over the agents from the other side and on the other side there are “passive” agents (such as schools, hospitals, offices etc.). The problem in such an environment is to place active agents on the one side of the market to the passive agents on the other side. In Erasmus exchange programme, active agents are students and passive agents are positions at partner institutions.

For a placement problem, a corresponding matching problem can be obtained by constructing preferences for the passive agents of the market. Doing this, a market is obtained where agents on both sides of the market are active. Instead of placing active agents to the passive agents as in the environment of a placement

problem, the problem in such an environment is to match agents from opposite sides of the market with each other.

A mechanism is used in a placement problem in order to select agents from the active side of the market and place them to the agents from the other side. This mechanism can be centralized in that there is a central authority acting as a mediator between the two sides of the market. This central authority decides which active agents are to be placed to passive agents and realizes the placements in the market accordingly. The mechanism can also be decentralized where the two sides of the market communicate with each other directly without a central authority acting as a mediator.

The mechanism in a placement problem can result in a many-to-many placement, as well as a many-to-one or a one-to-one placement. In a many-to-many placement, each active agent can be placed to more than one passive agent and to each passive agent more than one active agent can be placed. In a many-to-one placement, each active agent can be placed to one passive agent only and to each passive agent more than one active agent can be placed. Finally, in a one-to-one placement, each active agent can be placed to one passive agent only and to each passive agent only one active agent can be placed.

The mechanism in a placement problem is desired to satisfy some properties. Among them, there is individual rationality which means that no active agent is placed to a passive agent which is not acceptable for her. Non-wastefulness is another property which guarantees that an active agent is placed to a passive agent which she prefers if there is a room for the active agent at that passive agent. Another property is fairness which avoids a situation where a “better” active agent (with

higher scores, higher priority etc) is not placed to a passive agent which she prefers but a “worse” active agent is placed to this passive agent. A further property is Pareto efficiency which enables efficient placement of active agents to passive agents such that no active agent is worse-off and some active agents are better-off in comparison to any other placements. Another property to mention is strategy-proofness which ensures that no active agent can make herself better-off by misrepresenting her preferences and hence revealing true preferences is of best interest for each active agent.

In this paper, I consider the student exchange part of the Erasmus Programme. I focus on the mechanism used at Boğaziçi University in order to place students of Boğaziçi University to positions offered by partner institutions. I analyze the mechanism used for this purpose in that I discuss if the mechanism satisfies some of the desired properties of a placement mechanism which are mentioned above. The paper is designed as follows: Chapter 1 is the introduction part. In chapter 2 the literature review on placement problems is given and in chapter 3 the exchange market is introduced. Chapter 4 presents the preliminaries part where first the definition of a placement problem is given, next some of the desired properties of a placement and hence some of the desired properties of a mechanism used in a placement problem are defined, then the corresponding matching problem for a placement problem is described and lastly some of the desired properties of the corresponding matching problem for a placement problem are defined. In chapter 5 the analysis of the mechanism used at Boğaziçi University for this placement problem is made in that first the mechanism is defined, then it is discussed which of the desired properties defined in chapter 4 are satisfied and lastly some proposals are

made in order to upgrade the mechanism. In chapter 6 actual data of 2009-2010 is employed where first some statistical facts about the realized placements are given and then the placements which are realized by making use of the current mechanism are compared to the placements obtained by making use of the mechanism which is modified according to one of the proposals made in chapter 5 in order to observe the improvement. Finally chapter 7 concludes.

CHAPTER 2

LITERATURE REVIEW

The model of the placement procedure used at Boğaziçi University where students are placed to positions is designed to deal with a placement problem. Such a model originates from the college admissions model (Gale & Shapley, 1962). In a college admissions model, agents from both sides of the market are active in that they have preferences over the agents from the other side. So, a college admissions model is designed to deal with a matching problem where agents from different sides of the market are matched with each other. In a placement problem, on the other hand, only one side of the market is active and agents from this active side are placed to the passive agents from the other side. In this sense, the model of the placement procedure used at Boğaziçi University is more closely related to models of school choice problems.

In a school choice model, students are assigned to schools by using some placement mechanisms. Among many other papers about the school choice problem, Abdulkadiroğlu and Sönmez (2003) considers the school choice problem as a mechanism design problem and shows the deficiencies of the mechanisms used in some states in the USA for school choice plans. Ergin and Sönmez (2006) discusses the shortcomings of Boston mechanism which is used in some school districts in the USA. These mechanisms are designed to place students who have preferences over schools to schools which don't have preferences but priority orderings over students and a limited number of seats. The deficiencies of these mechanisms have been removed in some states of the USA including Boston and New York City by redesigning the mechanisms following the proposals in some papers among them

there is Abdulkadiroğlu and Sönmez (2003), Ergin and Sönmez (2006), and Abdulkadiroğlu et al. (2005a, 2005b).

Tie-breaking has been an important issue in school choice problems. Since active agents do not necessarily have strict preferences over the agents from the other side of the market, they may well be indifferent between some passive agents. Erdil and Ergin (2008) discusses that randomly tie-breaking causes loss of efficiency and proposes a *stable improvement cycle* in order to obtain efficiency gains. In the model, students are assumed to have strict preferences over schools and schools are allowed to have weak priority orderings over students. Abdulkadiroğlu et al. (2009) discusses the trade-off between strategy-proofness and efficiency by the presence of weak preferences of schools over students. The effect of breaking ties randomly by the presence of weak preferences of schools over students in terms of ex-ante efficiency is discussed in Abdulkadiroğlu et al. (2008).

In this paper, the model I consider is closely related to school choice models. The difference though is that in the model I consider, the passive agents of the market (positions) do not have priority orderings over the active agents (students). They only have their quotas which limit the number of students to be placed to them. Also, I consider a model where the active agents of the market are allowed to have weak preferences over some of the passive agents. In this sense, this paper belongs to the category of school choice problems where passive agents are purely to consume (without preferences and priority orderings) as a difference from other papers in this category. In addition, I allow the active agents of the market to have weak preferences over some of the passive agents and consider tie-breaking in

such an environment as opposed to other papers about tie-breaking which let passive agents of the market have weak preferences over the active agents.

CHAPTER 3

THE EXCHANGE MARKET

In this section, I will introduce the exchange market. I will present the agents in the market and I will describe the placement environment.

The market of the Erasmus student exchange programme is constituted by two sides: On the one side there are the students of Boğaziçi University and on the other side there are positions offered by partner institutions. The active side of the market is constituted by students. Students have their preferences over positions and hence are the active agents of the market to be placed to the passive agents which are the positions offered by partner institutions. Positions do not have preferences over students and hence constitute the passive side of the market. In addition, each student has a *total university grade*¹ and each position has its quota so that students are placed to positions according to their preferences without violating the quota restriction of positions.

The exchange market is a *centralized placement market* in that there is a central authority at Boğaziçi University –the Office of International Relations of Boğaziçi University– which realizes all the placements of the students of Boğaziçi University to positions offered by partner institutions. Positions submit their quotas to the central authority, the central authority announces the positions along with their quotas, and students submit their strict preferences to the central authority. Having received the preference lists of the students, the central authority ranks the students

¹ Each student has an interview with the Erasmus coordinators of her department. She receives an interview grade and an English proficiency grade after this interview. In addition, each student receives a university grade related to her GPA. These three grades add up to her total university grade.

according to their total university grades in decreasing order and realizes placements without violating the quota restriction of positions.

The mechanism used for the placement procedure results in a many-to-one placement where each student is placed to one position only and to each position more than one student can be placed. The quantity of the students placed to a position cannot exceed the quota of that position.

Positions offered by partner institutions to which students of Boğaziçi University can be placed include colleges and possibly faculties, and/or departments of the same colleges. The reason for this is that there are different types of agreements and hence different types of relevant quotas between Boğaziçi University and partner institutions. There is a *pool agreement* which is between Boğaziçi University and the partner colleges, and an associated *pool quota*. To positions which are associated with a pool agreement, any student can be placed irrespective of her department or her faculty at Boğaziçi University. There is a *faculty agreement* between the faculties of Boğaziçi University and the relevant faculties of partner colleges, and an associated *faculty quota*². A student can be placed to a position which is associated with a faculty agreement if she is a student from that faculty of Boğaziçi University. Finally, there is a *department agreement* between the departments of Boğaziçi University and the relevant departments of partner colleges, and an associated *department quota*. A student can be placed to a position which is associated with a department agreement if she is a student from that department of Boğaziçi University.

² Here, the term faculty is used as a school of a college instead of its teaching staff as in the Faculty of Economics and Business Administration.

Students are asked to submit strict preferences to the central authority for the selection procedure. However, a rational student should be indifferent between a college and the relevant faculty and the relevant department of that college, since each of these different positions leads the student to the relevant department of that college. In other words, a student should be indifferent³ between a college, the relevant faculty of that college and the relevant department of that college which are three different positions if all three types of agreements exist.

The existence of one type of agreements does not prohibit the existence of another type of agreements. In other words, if there is a pool agreement between Boğaziçi University and a college, this does not avoid a faculty of that college to have a faculty agreement with the relevant faculty of Boğaziçi University. Similarly, a department of that college is not prevented from having a department agreement with the relevant department of Boğaziçi University.

Each position has its own quota; so if a student is placed to a position which is a college and hence which is associated with a pool agreement, only the quota of this position is affected by this placement. The quota of the position which is a faculty of this college and hence which is associated with a faculty agreement is not affected by this placement. Similarly, the quota of the position which is a department of this college and hence which is associated with a department agreement is not affected by this placement.

³ Actually, a student could also be indifferent between positions which are offered by different partner institutions. I want to focus on the indifferences for a student which occur between the positions offered by the same partner institution.

CHAPTER 4
PRELIMINERIES

In this section, I will give the definition of a placement problem and introduce some of the desired properties of a placement on which I will focus while analyzing the mechanism used at Boğaziçi University in order to place its students to positions offered by partner institutions.

Placement Problem

A placement problem consists of

1. A set of students $S = \{s_1, s_2, \dots, s_n\}$,
2. A set of positions $X = \{x_1, x_2, \dots, x_r\}$ which is the union of the set $C = \{c^k\}_{k=1, \dots, K}$ of the colleges receiving students, the set of all faculties of all receiving colleges $F = \{f_j^k\}_{j=1, \dots, J} = \bigcup_j F_j$ with $F_j = \{f_j^k\}_{k=1, \dots, K}$ being the set of the particular faculty j of all receiving colleges, and the set of all departments of all receiving colleges $D = \{d_i^k\}_{i=1, \dots, I} = \bigcup_i D_i$ with $D_i = \{d_i^k\}_{k=1, \dots, K}$ being the set of the particular department i of all receiving colleges,
3. A no-position option x_0 such that if a student is not placed to a position in X she is assumed to be placed to x_0 ,
4. A capacity vector $q = (q_{x_1}, q_{x_2}, \dots, q_{x_r})$ where q_{x_i} is the capacity of the receiving college, or the capacity of the faculty of a college, or the

capacity of the department of a college corresponding to the i^{th} element x_i of the set of positions X ,

5. A list of strict student preferences $P_S = \{P_{s_1}, P_{s_2}, \dots, P_{s_n}\}$ where P_{s_i} is the strict preference list of student s_i over the set of positions X with for x_i and x_j on P_{s_i} , $r_{s_i}(x_i) < r_{s_i}(x_j)$ implies $x_i \succ_{s_i} x_j$, \succ_{s_i} representing the strict preference relation of the student s_i over the set of positions X union the no-position option x_0 and $r_{s_i} : X \rightarrow \aleph^+$ representing the rank function for each student s_i ,

6. A preference relation \succsim_{s_i} for each student which represents the at-least-as-good-as relation for a student s_i over the set of positions X union the no-position option x_0 such that $x_j \succsim_{s_i} x_k$ if and only if $x_j \succ_{s_i} x_k$ or $x_j \sim_{s_i} x_k$ where \sim_{s_i} is the indifference relation for s_i over $X \cup \{x_0\}$,

7. A function $g : S \rightarrow \mathfrak{R}$ which associates each student with a total university grade,

8. A function $\rho : S \rightarrow D^0$ which associates each student with its department where $D^0 = \{d_1, d_2, \dots, d_l\}$ is the set of all departments of the home institution sending its students to the receiving positions,

9. A function $\sigma : S \rightarrow F^0$ which associates each student with the faculty of its department where $F^0 = \{f_1, f_2, \dots, f_J\}$ is the set of all faculties of the home institution sending its students to the receiving positions.

A position $x \in X$ is said to be applicable for a student s if and only if $x \in C$ or $x \in F_j$ when $\sigma(s) = f_j$ or $x \in D_i$ when $\rho(s) = d_i$; acceptable for a student s if and only if $x \succ_s x_0$.

A *placement* is a function $p : S \rightarrow X \cup \{x_0\}$ which assigns each student to a position such that $\forall s \in S$ with $\rho(s) = d_i \in D^0$,

$$p(s) \in C \cup F_j \cup D_i \cup \{x_0\} \subset X \cup \{x_0\} \text{ and } |p^{-1}(x)| \leq q_x \quad \forall x \in X.$$

Desired Properties of a Placement and a Mechanism

There are properties which are desired to be satisfied by a placement. In this subsection I will define some of these desired properties which are basic in this context. In chapter 5 I will discuss which of the properties defined in this subsection are satisfied by the mechanism used at Boğaziçi University.

Definiton (IR): A placement p is said to be *individually rational* if the placement of each student is acceptable for her; i.e. ,

$$p(s) \succ_s x_0 \quad \forall s \in S.$$

Definition (NW): A placement p is said to be *non-wasteful* if it is not the case that a student prefers a position to her placement and the quota of that position is not full; i.e. ,

$$x \succ_s p(s) \Rightarrow |p^{-1}(x)| = q_x \quad \forall s \in S.$$

Definition (F): A placement p is said to be *fair* if each student who prefers an applicable position to her placement has a lower total university grade than any student who is placed to that position; i.e. ,

$\forall s, s' \in S$ with $x' = p(s')$ such that $\rho(s) = d_m \in D^0$, $\rho(s') = d_{m'} \in D^0$,
 $\sigma(s) = f_n \in F^0$, $\sigma(s') = f_{n'} \in F^0$, and $x' \in [(C \cup F_n \cup D_m) \cap (C \cup F_{n'} \cup D_{m'})] \subset X$ it
is the case that $x' \succ_s p(s) \Rightarrow g(s') > g(s)$.

Definition (PD): A placement p *Pareto dominates* another placement p' if
no student is worse-off and at least one student is better-off by p ; i.e. ,

$$p(s) \succeq_s p'(s) \quad \forall s \in S \quad \text{and} \quad p(s') \succ_s p'(s') \quad \text{for at least one } s' \in S.$$

Definition (PE): A placement p is said to be *Pareto efficient* if it is not
Pareto dominated by another placement.

Definition (SP): A placement p is said to be *strategy-proof* if no student
can make herself better-off by misrepresenting her preferences; i.e. ,

$\forall s \in S$ $p(s) \succeq_s p'(s)$ where $p(s)$ is her placement under p by submitting
her true preference list P_s and $p'(s)$ is her placement under p by submitting
another preference list P'_s .

A mechanism is a procedure which picks a placement for each placement
problem. A mechanism is said to be individually rational if it always picks an
individually rational placement, non-wasteful if it always picks a non-wasteful
placement, fair if it always picks a fair placement, Pareto dominates another
mechanism if no student ever prefers the placement picked by the other mechanism
to the placement picked by the mechanism and at least one student prefers in at least
one placement problem the placement picked by the mechanism to the placement
picked by the other mechanism, Pareto efficient if it always picks a Pareto efficient
mechanism, and strategy-proof if it always picks a strategy-proof placement.

Corresponding Matching Problem for a Placement Problem

For a placement problem, the *corresponding college admissions problem* is constructed by defining strict preferences for positions⁴ over the set of students in that the student with the highest total university grade is assumed to be the most preferred student by all positions and the lower the total university grade of a student is the less she is assumed to be preferred by the positions.

As mentioned earlier, not each position is applicable for each student. Similarly, from the point of view of the positions, a student s is said to be *eligible* for a position $x \in X$ if $x \in C$ or $\sigma(s) = f_j$ when $x \in F_j$ or $\rho(s) = d_i$ when $x \in D_i$. Obviously, a position is applicable for a student if and only if the student is eligible for that position. Hence, in the corresponding college admissions problem, positions are assumed to have common preferences over the set of students as long as the eligibility is satisfied.

Similar to the no-position option, there is the no-student option s_0 such that a position is assumed to be assigned to the no-student option if no student is assigned to that position. A student s is acceptable for a position x if and only if $s \succ_x s_0$ where \succ_x represents the strict preference relation for position x over the set of the students union the no-student option. Since the preferences of the positions are constructed rather than revealing true choices, it is assumed that each student is acceptable for each position as long as eligibility is satisfied.

⁴ Actually, for the preferences of the positions, indifferences can also be allowed and a tie-breaking rule can be used. However, in this problem I want to focus on the preferences of the students.

A *matching* is a function $\mu: S \rightarrow X \cup \{x_0\}$ corresponding to the placement p which matches each student with a position in X or with the no-position option x_0 such that $\forall s \in S$ with $\rho(s) = d_i \in D'$, $\mu(s) \in C \cup F_j \cup D_i \cup \{x_0\} \subset X \cup \{x_0\}$ and $|\mu^{-1}(x)| \leq q_x \quad \forall x \in X$.

A student-position pair (s, x) is said to constitute a *blocking pair* if $x \succ_s p(s)$ and $s \succ_x \hat{s}$ where $\hat{s} \in \mu(x)$ and both s and \hat{s} are eligible for x .

Properties of the Corresponding Matching Problem for a Placement Problem

The definitions given in chapter 4 as the desired properties of a placement on which I will focus are carried over as the desired properties of the corresponding matching and there is one additional property I want to mention.

Definition (S): A matching is said to be *stable* if it is both individually rational and there does not exist a blocking pair.

The following proposition associates individual rationality, fairness and non-wastefulness of a placement in a placement problem with stability of a matching in the corresponding matching problem.

Proposition 1: A placement p is individually rational, fair and non-wasteful if and only if the corresponding matching μ is stable.

This proposition is also mentioned in Balinski and Sönmez (1999). Here comes the proof in this context:

Proof: Let p be a fair and individually rational placement satisfying non-wastefulness and μ be the corresponding matching. Let s and \hat{s} be two students

who are both eligible for a position x such that $g(s) = \min_{s' \in p^{-1}(x)} (g(s'))$, $s \in p^{-1}(x)$, $\hat{s} \notin p^{-1}(x)$, and $x \succ_{\hat{s}} p(\hat{s})$. Since p is fair, $g(s) > g(\hat{s})$ should hold. Then, \hat{s} and x cannot constitute a blocking pair in the corresponding matching μ , since x prefers each one of her current matches to \hat{s} . Since p is also individually rational, no student is placed to a position which is not acceptable for her. In addition, since p is non-wasteful, there is no position x' which is not fully occupied and which is preferred by \hat{s} to $p(\hat{s})$; hence \hat{s} and x' cannot constitute a blocking pair. By assumption, each student is acceptable for each position as long as the eligibility of the student is given. Thus, stability is satisfied.

Let now μ be a stable matching corresponding to the placement p . Since μ is stable, μ is individually rational and hence no student is matched with a position which is not acceptable for her. So, individual rationality of p follows. Assume that p fails to be non-wasteful and suppose that $x \succ_{\hat{s}} \mu(\hat{s})$ for a position x and a student \hat{s} , such that $|p^{-1}(x)| < q_x$. Then, x and \hat{s} constitute a blocking pair and this contradicts the stability of μ . Thus, p must be non-wasteful. Now let s and \hat{s} be 2 students such that they are both eligible for a position x with $\mu(s) = x$ and $\mu(\hat{s}) \neq x$. Assume that $x \succ_{\hat{s}} \mu(\hat{s})$. Since μ is stable, there does not exist a blocking pair. So, if $g(\hat{s}) > g(s)$, then (\hat{s}, x) constitute a blocking pair which contradicts stability of μ . Hence, $g(s) > g(\hat{s})$ which shows that p is fair. Q.E.D.

CHAPTER 5

THE MECHANISM USED AT BOĞAZIÇI UNIVERSITY

In this section, I will first introduce the mechanism used at Boğaziçi University in order to place students of Boğaziçi University to positions offered by partner institutions and mention the selection procedure used by the mechanism. Then, I will discuss which of the properties mentioned in section 4 are satisfied by the mechanism. Finally, I will make some proposals in order to make the mechanism satisfy those properties which it fails to satisfy otherwise.

The Mechanism

The selection procedure used at Boğaziçi University in order to place students to positions at partner institutions is a two-staged process. According to this procedure, positions submit their quotas and students submit their strict preferences over the positions to the Office of International Relations of Boğaziçi University. Having ranked the students according to their total university grades in decreasing order, the central authority applies *serial dictatorship* in order to run the first stage.

After the placements are realized, students who are placed to a position are given a time period to guarantee that they are going to attend to the programme having observed the positions they are placed to. Some of the students who are placed to a position in the first stage may simply change their minds and decide not to participate in the programme or they may become unacceptable because of the GPA criterion⁵. Then these students are removed from the market and the positions

⁵ A student is removed from the market due to the GPA criterion if her GPA falls down to a level which is less than 2.5 over 4.0 after the placements are realized.

occupied by them become vacant. The Office of International Relations of Boğaziçi University announces these positions along with their quotas and the second stage takes place. Students participate in the second stage only if they are not placed to a position in the first stage. Such students are asked to submit new preference lists to the central authority following the positions announced by the Office of International Relations of Boğaziçi University as vacant after the first stage. As in the first stage, serial dictatorship is applied in the second stage.

Serial dictatorship which is applied in both stages works as follows:

Step 1: Only the first-ranked student's preferences are considered. The student is placed to her top choice and the quota of this position is decreased by 1.

Step 2: Only the second-ranked student's preferences are considered. The student is placed to her top choice, if there is still room at this position. If this position is fully occupied, then her second choice is considered and she is placed to her second choice. Following this placement, the quota of this position is decreased by 1.

⋮

Step k : Only the preferences of the k^{th} student are considered. She is placed to her top choice, if this position is not fully occupied and the quota of this position is decreased by 1. If there is no room left at this position, then her second choice is considered and she is placed to her second choice, if there is still room for her and the quota is decreased by 1. If not, then her third choice is considered and continuing this way she is placed to one of her choices, if that position is not fully occupied and the quota is decreased by 1. If each one of her choices is fully occupied, then she is placed to the no-position option x_0 .

The procedure terminates either if all students have their turn or if all the positions are fully occupied so that considering the preferences of the remaining students is not going to enable new placements due to quota restrictions.

The following example illustrates how the mechanism works:

Example:

Let $S = \{s_1, s_2, s_3, s_4, s_5\}$ be the set of the students and $X = \{x_1, x_2, x_3\}$ be the set of the positions. Let $q = (1, 1, 1)$ and assume that x_1 is applicable for s_1, s_2, s_5 ; x_2 is applicable for s_3, s_4 ; x_3 is applicable for all students. Suppose that the total university grades of the students decrease as the subscripts of them increase such that s_1 has the highest total university grade and s_5 the lowest. Let the submitted preference lists of the students be as follows:

$$P_{s_1} : x_1, x_3 \quad P_{s_2} : x_1, x_3 \quad P_{s_3} : x_3, x_2 \quad P_{s_4} : x_3, x_2 \quad P_{s_5} : x_1, x_3$$

In the first stage, first s_1 's preferences are considered and she is placed to x_1 . The quota of this position is decreased by 1 and hence becomes 0 so that this position is fully occupied. Next, it is s_2 's turn. She cannot be placed to her top choice due to quota restrictions and thus she is placed to x_3 . Continuing this way, the first stage results in the following placements:

$$(s_1, x_1) \quad (s_2, x_3) \quad (s_3, x_2) \quad (s_4, x_0) \quad (s_5, x_0)$$

Suppose that s_3 gives up on participating in the programme. Then the position x_2 becomes vacant and students who are not placed to a position are expected to submit new preference lists. Since s_5 does not apply to the only position which has still

room for a student, namely x_2 , s_4 is the only student who submits her new preference list as follows:

$$P'_{s_4} : x_2$$

Then, s_4 is placed to x_2 and the second stage ends. Hence, the mechanism yields the following placements:

$$(s_1, x_1) \quad (s_2, x_3) \quad (s_3, x_0) \quad (s_4, x_2) \quad (s_5, x_0)$$

One crucial observation is necessary: Students are asked to submit strict preferences to the central authority and the placements are realized according to these strict preferences. However, due to the rationality assumption, students are actually indifferent between different positions which lead them to the same place. Hence, students are asked to break indifferences before submitting their preferences to the Office of International Relations of Boğaziçi University.

Students can break the indifferences by applying a *pool priority rule* which means that students prefer a position associated with a pool quota over another position which is associated with the relevant department quota of the same partner institution.

Another way of breaking the indifferences is to apply a *department priority rule* which is to favour the position associated with the relevant department quota of a partner institution over the position associated with the pool quota of the same partner institution.

Finally, students can randomize between these two rules in that they do not follow a special rule and simply break the indifferences by, say, throwing a coin which is referred as the *random rule*.

I do not focus on faculty agreements and hence do not mention a faculty priority rule, since comparing the pool priority rule with the department priority rule is similar to and stronger than comparing the department priority rule with the faculty priority rule. So, the faculty agreements and hence the faculty quotas will not be evaluated.

Satisfied Properties by the Mechanism Used at Boğaziçi University

I will discuss, if the desired properties for a placement which are mentioned in section 4 are satisfied in this environment following serial dictatorship as the selection procedure. While considering Pareto efficiency, I will evaluate all 3 types of rules for breaking the indifferences, since applying different rules favours different students and hence different departments in terms of Pareto efficiency.

Proposition 2: IR is satisfied.

Proof: Since students are assumed to be rational, they are not expected to prefer a position which is not acceptable for them and since they are placed only according to their preferences, IR is proven to be satisfied, i.e. ,

$s \in p^{-1}(x) \leftrightarrow x \in P_s$ by the construction of the selection procedure, and

$x \in P_s \leftrightarrow x \succ_s x_0$ by the rationality assumption for student s . Q.E.D.

Proposition 3: NW fails to be satisfied.

Proof: NW fails to be satisfied because of two reasons: The first reason is that students are restricted to submit a certain length of preference list to the central authority. Because of this, a student may not be able to submit all acceptable positions for her. Now, if during the selection process she remains unplaced to a position, i.e if all of her choices are considered and she could not be placed to one of

these choices due to quota restrictions and is placed to the no-position option, and if there is a position which is acceptable for her but is not on her preference list, then this student certainly would prefer to be placed to this position. If there is still room for her at this position, then NW fails to be satisfied, i.e. ,

Let P_s be the submitted preference list of s such that $x \notin P_s$ and $x \succ_s x_0$.

Assume that $p(s) = x_0$ and $|p^{-1}(x)| < q_x$ after the selection process. $x \succ_s x_0$ and $p(s) = x_0$ give $x \succ_s p(s)$, and $|p^{-1}(x)| < q_x$ then shows that NW fails to be satisfied.

The second reason is that students who are placed to a position in the first stage are not allowed to join to the second stage. Now, if a student is placed to a position in the first stage and if another position which she prefers more than her current placement becomes vacant and is not fully occupied in the second stage by students with higher total university grades than her, then NW fails to be satisfied. Formally,

let s_i be a student who is placed to $p(s_i)$ in the first stage. Let x be a position which becomes vacant after the first stage due to the removal of a student from the market who was placed to x such that $r_{s_i}(x) < r_{s_i}(p(s_i))$. Assume that $|p^{-1}(x)| < q(x)$ after the second stage. Then, NW fails to be satisfied. Q.E.D.

Proposition 4: If there is no restriction on the quantity of submitted choices of students and if all the students who are not removed from the market are allowed to attend the second stage, then NW is satisfied.

Proof: If there was no such restriction, then each student would have each acceptable position on her preference list. Since the placements are made according

to these preferences, a student would be placed to a position on her preference which she prefers less than another position on her preference list, if the quota of the position she favours more is fully occupied, i.e. ,

Let P_s be the submitted preference list of s such that $x \in P_s$ for all $x \succ_s x_0$.

Now, $x \succ_s p(s)$ if and only if x has a higher rank than $p(s)$ on P_s , i.e.

$r_s(x) < r_s(p(s))$. This means that during the selection process, the central authority tries to place s to x before $p(s)$. Since s is not placed to x , $|p^{-1}(x)| = q_x$ should be the case.

Furthermore, if a student who is placed to a position in the first stage was allowed to attend the second stage, then she would be placed to a position which becomes vacant after the first stage and which she favours more than her current place, if this position is not fully occupied by other students with higher total university grades. Formally,

let s be a student such that $x \succ_s p(s)$. Assume that $|p^{-1}(x)| < q_x$ after the second stage by not allowing s to attend the second stage. Then, if s is allowed to attend the second stage s will be placed to x . Hence, making these arrangements, NW is satisfied. Q.E.D.

Hence, putting restriction on the quantity of submitted student preferences may cause wastefulness. The reason of such a restriction is to avoid students from submitting each applicable position to the central authority. However, if each applicable position is also acceptable for a student or if there are more acceptable positions than the restricted quantity of the submitted preferences, then NW may fail to be satisfied. Also, disabling the students who are placed to a position in the first

stage to attend the second stage causes the failure of NW which can be overcome by allowing them to join the second stage.

Proposition 5: In a one-staged placement, F is satisfied.

Proof: According to the serial dictatorship, the preferences of a student who has a higher total university grade are considered before another student's preferences with a lower total university grade. Also, each student's preferences are considered according to the ranking submitted by the student, hence a position which is favoured by a student to another one is considered before. Now, if a student with a higher total university grade is placed to a position which is not her top choice, then the positions on her preference list which are ranked by her before the position she is placed to should already be fully occupied. Thus, another student with a lower total university grade cannot be placed to one of these positions, either and hence, F is guaranteed. Formally,

let s, s' be 2 students such that $\rho(s) = d_i$, $\rho(s') = d_{i'}$, $\sigma(s) = f_j$, and $\sigma(s') = f_{j'}$. Suppose that $x' = p(s')$ and $x' \in [(C \cup F_j \cup D_{ij}) \cap (C \cup F_{j'} \cup D_{i'j'})]$ so that the position x' is applicable for both students. Now assume that $x' \succ_s p(s)$, then due to rationality x' should have a higher rank than $p(s)$ on P_s . So, the central authority should try to place s to x' before $p(s)$. Since $x' \neq p(s)$, it follows that $|p^{-1}(x')| = q_{x'}$. Hence if $g(s') < g(s)$, then $x' \neq p(s')$ by $|p^{-1}(x')| = q_{x'}$ and this gives a contradiction. Thus, $g(s') > g(s)$ should hold, which proves that F is satisfied. Q.E.D.

Proposition 6: The mechanism used at Boğaziçi University fails to satisfy F.

Proof: The problem with the fairness is caused by the second stage. Since only those students participate in the second stage who have not been placed to a position in the first stage, F may fail to be satisfied, if a student is placed to a position in the second stage which is preferred by another student with a higher total university grade who is already placed to another position which she favours less. Let's construct such a situation and hence show the failure of F:

Let $S = \{s_1, s_2, s_3\}$ and $X = \{x_1, x_2\}$ such that $q = (1,1)$ and each position is not only applicable but also acceptable for each student. Assume that $g(s_1) > g(s_2) > g(s_3)$ and let the students have common preferences such that their submitted preference lists are as follows:

$$P_{s_1} = P_{s_2} = P_{s_3} : x_1, x_3$$

After the first stage, the following placements are realized:

$$(s_1, x_1) \quad (s_2, x_2) \quad (s_3, x_0)$$

Now assume that s_1 gives up on participating in the programme and hence x_1 becomes vacant. Since s_3 is the only student who is not placed to a position, she is the only student who takes part in the second stage and she only submits x_1 on her preference list so that she is placed to this position. Thus, after the second stage, the following placements are realized:

$$(s_1, x_0) \quad (s_2, x_2) \quad (s_3, x_1)$$

Obviously, F fails to be satisfied. So, for $s_i, s_j \in S$ with $g(s_i) > g(s_j)$, it can be the case that s_i is placed to $p(s_i)$ and s_j is placed to $p(s_j)$ and $p(s_j) \succ_{s_i} p(s_i)$, since

s_i 's placement is realized in the first stage and s_j 's placement is realized in the second stage. Thus, F fails to be satisfied. Q.E.D.

Proposition 7: Given strict student preferences (as if each position would lead a student to a different place than another position), PE is satisfied.

Proof: Assume that there are no indifferences for the students. So, each student favours the position associated with the pool quota of a partner institution to the position associated with the relevant department quota of the same partner institution or vice versa, because these positions lead a student to different places.

Let p be the placement obtained by serial dictatorship and let p' be another placement. Assume for a contradiction that p' Pareto dominates p . Then $p'(s) \succ_s p(s)$ or $p'(s) \sim_s p(s) \quad \forall s \in S$ with at least for one $s' \in S$ $p'(s') \succ_{s'} p(s')$.

$p'(s') \succ_{s'} p(s')$ means that $p'(s') \in P_{s'}$ and $p'(s')$ has a higher rank than $p(s')$ on $P_{s'}$. So, during the selection process, the central authority must have tried to place s' to $p'(s')$ and the reason why this placement is not realized can only be that $|p^{-1}(p'(s'))| = q_{p'(s')}$.

Now, s' is placed to $p'(s')$ implies that for some $\tilde{s} \in p^{-1}(p'(s'))$, \tilde{s} is placed to a position $p'(\tilde{s})$ which she favours to $p(\tilde{s})$, since otherwise Pareto domination is violated. Then, the same argumentation applies and there must be some $\tilde{s}' \in p^{-1}(p'(\tilde{s}))$, who should be placed to a position which she favours to $p(\tilde{s}')$. Going this way, we arrive at a position which is preferred by all the students placed to this position as their top choice and hence an improvement for the students being placed to this position is not possible. Thus, we get a contradiction and hence the

placement obtained by serial dictatorship must be Pareto efficient under these circumstances. Q.E.D.

Proposition 8: The mechanism used at Boğaziçi University fails to satisfy PE.

Proof: Now, taking the indifferences into account, the situation changes. Let's compare the pool priority rule with the department priority rule to see if breaking the indifferences by applying one of these rules results in a placement which Pareto dominates the placement obtained by applying the other one.

Let s_1, s_2 , and s_3 be 3 students and let x_1, x_2 , and x_3 be 3 positions such that x_1 and x_2 are the positions of the same partner institution with x_1 being the position associated with the pool quota and x_2 being the position associated with a department quota, and x_3 is a position of another partner institution associated with a department quota. Assume that $q_{x_1} = q_{x_2} = q_{x_3} = 1$, x_3 is not applicable for s_1 and s_3 but applicable for s_2 , and x_2 is not applicable for s_2 but is applicable for s_1 and s_3 . Suppose also that $g(s_1) > g(s_2) > g(s_3)$ and let the preferences of the students be as follows: $x_1 \sim_{s_1} x_2$, $x_1 \succ_{s_2} x_3$, and $x_1 \sim_{s_3} x_2$. Now, if students apply the pool priority rule, then the submitted preferences of the students are;

$$P_{s_1} : x_1, x_2 \qquad P_{s_2} : x_1, x_3 \qquad P_{s_3} : x_1, x_2 .$$

Serial dictatorship places then s_1 to x_1 , s_2 to x_3 , and s_3 to x_2 ; so the resulting placements are as follows:

$$(s_1, x_1) \qquad (s_2, x_3) \qquad (s_3, x_2)$$

On the other hand, if the students apply the department priority rule, then the submitted preferences are;

$$P_{s_1} : x_2, x_1 \qquad P_{s_2} : x_1, x_3 \qquad P_{s_3} : x_2, x_1 .$$

Serial dictatorship places this time s_1 to x_2 , s_2 to x_1 , and s_3 is unplaced to a position; i.e. the following placements are realized:

$$(s_1, x_2) \qquad (s_2, x_1) \qquad (s_3, x_0)$$

s_1 is indifferent between the 2 rules, whereas s_2 prefers the department priority rule to the pool priority rule and s_3 prefers the pool priority rule to the department priority rule. Either s_2 gets better-off at the expense of s_3 or vice versa and hence neither of the rules results in a Pareto efficient placement. Q.E.D.

As observed, applying the pool priority rule favours some students at the expense of some other students and applying the department priority rule reverses the situation. By applying the pool priority rule, students from a department with high total university grades hinder other students from different departments in that they occupy positions which are associated with pool quotas of partner institutions. Doing this, they enable students from the same department to occupy positions associated with the relevant department quotas of the same partner institutions.

Applying the department priority rule, on the other hand, has an opposite effect. Students from a department with high total university grades hinder other students from the same department with lower total university grades by occupying positions which are associated with relevant department quotas of partner institutions. Other students from different departments with higher total university

grades than the hindered students occupy then positions which associated with pool quotas of the same partner institutions.

Proposition 9: SP is satisfied.

Proof: Let P_s be the true preferences of s and let x be the position she is placed to. Now, by misrepresenting her true preferences, s aims to be placed to a position which is ranked by her in a higher position than x on P_s . However, since each position she favours more than x is already occupied by other students with higher total university grades, she cannot be placed to a position which she prefers more than x . Thus, by submitting another preference list, x is the best position for s to be placed to. Hence, she cannot make herself better-off by misrepresenting her true preferences and therefore SP is satisfied. Q.E.D.

However, applying different types of rules for breaking the indifferences favours different students. As mentioned earlier, applying the pool priority rule or applying the department priority rule favours some students at the expense of some other students. This fact suggests the possibility of *coalitional strategic manipulation*. Coalitional strategic manipulation simply means that for a set of agents (here students), some agents are better-off and the remaining ones are not worse-off by misrepresenting their preferences.

The result I obtained while discussing PE seems to be related to this concept, however it actually yields a different conclusion. Students in this placement problem do not misrepresent their preferences in order to make other students better-off while they are not worse-off. They just change the type of the rule they apply for breaking the indifferences and hence (deliberately or not) favour different students. So, a set of students do not generate a coalition and misrepresent their preferences in

order to strategically manipulate the mechanism used in this placement problem for making some among them better-off while the others are not worse-off.

Still, some students can be made better-off by other students at the expense of some other students. This fact can be used by departments for manipulating the mechanism in this placement problem. If the objective of a department is to place as many of its students as possible to positions offered by partner institutions and if this department has students with high total university grades besides others with lower total university grades, then this department may advise its students to apply the pool priority rule due to the reasoning given while discussing PE.

Proposals

Now, I will make some proposals in order to obtain some improvements. I will offer some upgrades by eliminating the reasons for the failure of the satisfaction of the desired properties which are mentioned in section 4 so that the modified mechanism satisfies all of these properties.

Proposal 1: There should not be a restriction on the quantity of the submitted preferences of students.

Discussion: As shown, such a restriction may cause the failure of NW and removing this restriction enables NW to be satisfied. Practically, it may not be that easy to allow students to submit as many preferences as they wish. If there are too many applicable positions which are also acceptable for too many students, then dealing with the submitted preferences of the students can be extremely hard.

However, theoretically letting students free in the quantity of submitted preferences results in a placement satisfying NW. As mentioned, the reason of such a restriction

is to avoid students from submitting each applicable position as if each of these positions was also acceptable. In other words, the main hesitation of the central authority about this issue is that the rationality of the students cannot be taken as granted. So, a student can simply submit her preference list without considering if a position is acceptable for her or not. Still, this restriction may cause NW fail to be satisfied.

Proposal 2: In the first stage, the placements should be tentative so that each student should participate in the second stage after some (if any) of the students give up on participating in the programme or become unacceptable. Then, students will not be asked new preference lists and the preference lists they already submitted will be considered.

Discussion: As shown, the second stage causes the failure of F and NW. If there was only one stage, then F would be satisfied and NW would be satisfied depending on the removal of the restriction on the quantity of the submitted preferences. Now, making the placements tentative which are realized in the first stage makes the selection procedure work as a one-staged process. The first stage will then remove all the students who change their minds and give up on participating in the programme or who become unacceptable, and the second stage will be the real selection stage. Since the total university grades of the students do not change in the second stage and since no additional student is included, the rank of a student either remains the same or gets higher in comparison to her rank before some students are removed and hence the student is either placed to the position she is tentatively placed in the first stage or to a position which she prefers more than her tentative placement. Furthermore, no student can occupy a position which is

preferred by another student with a higher total university grade to her placement, since all the students participate in the second stage. Thus, F is satisfied thanks to such an upgrade.

Proposal 3: There should not be both a pool quota and a department quota of a partner institution simultaneously. There should be either only a pool quota or department quotas without a pool quota.

Discussion: As mentioned (respecting the rationality assumption of students), students are indifferent among different positions offered by a partner institution which lead them to the same place, namely the relevant department of that partner institution. Since they are expected to submit strict preferences to the central authority, they can apply different types of rules for breaking the indifferences. As shown, different rules of breaking these indifferences favours some students over the other ones and hence some departments over the other ones such that PE fails to be satisfied. Also, some students can be made better-off by other students at the expense of some other students by the presence of these indifferences. In order to enable PE to be satisfied and in order to avoid students from having the option of making other students better-off (and hence some other worse-off), no indifferences should be allowed. So, a partner institution should only offer a pool quota or department quotas without a pool quota simultaneously.

CHAPTER 6

DATA ANALYSIS

In this section, I will employ actual data of 2009-2010. First, I will give some facts and observations about the data and the placements which are realized. Then, I will apply one of our proposals where I will consider the placements which are realized in the first stage as tentative and let all students who are not removed from the market attend the second stage and compare the resulting placements with the current placements in order to see the improvement.

Facts

Considering actual data of 2009-2010, one observation immediately emerges: The rationality assumption for students fails to hold for some students. 19 students out of 481 preferred only the pool quota or the relevant department quota (without violating the eligibility criterion) of a partner institution but not both positions simultaneously, although as mentioned these positions lead students to the same places, namely to the relevant department of that partner institution. These students did not submit 12 choices (which is the maximum quantity allowed), so this behaviour cannot be justified by the restriction put on the quantity of submitted preferences. Since these students should be indifferent between these two positions, according to the rationality assumption they should submit both of these positions and what is more these two positions should be submitted in consecutive orders. There are 2 more students who submitted both of these positions but not in consecutive orders.

Another observation which shows the failure of the rationality assumption is that some students preferred some positions which are not applicable for them. Ignoring

applicability, or equivalently eligibility, caused them to waste one (sometimes even more) choice(s). Hence, the rationality assumption seems to fail to hold for some students.

Turning our attention to statistical facts about the data, we make the following observations: There are 634 quotas at 274 different positions in 194 partner institutions. 481 students applied to the programme in order to be an exchange student. 368 of them are placed to a position in the first stage and the remaining 113 are not placed to a position in the first stage. 296 students out of 368 who are placed to a position in the first stage guaranteed their participation in the programme and the remaining 72 out of 368 either changed their minds or became unacceptable and hence are removed from the market. So, the quantity of the positions exceed the quantity of students applied to the programme, but there are still students who are not placed to a position and there are positions which are not associated with a student. This means that some positions are not preferred by any student at all.

59 students out of 113 who are not placed to a position in the first stage participated in the second stage and the remaining 54 did not submit new preference lists for the second stage. 46 students out of 59 are placed to a position in the second stage and 13 of them remained unplaced. 15 students out of 46 either gave up on being an exchange student or became unacceptable and hence are removed from the market and the remaining 31 guaranteed their study abroad.

Altogether, 327 students are placed to a position, 296 in the first stage and 31 in the second stage, and 154 students are either not placed to a position or removed from the market.

78 students submitted 12 choices to the central authority (which is the maximum quantity allowed) and 10 of them are not placed to a position in the first stage. The reason of this can be traced in the restriction put on the quantity of the submitted preferences. 8 students out of this 10 participated in the second stage and 2 of them are not placed to a position in the second stage, either.

With 50 students placed to a position, the department of management is the leader among all the departments. 32 students out of these 50 are placed to positions through department agreements, 13 students are placed through pool agreements and the remaining 5 are placed through faculty agreements. The department of political science and international relations takes the second place with 40 students placed to a position. 27 students out of these 40 are placed to positions through department agreements, 9 students are placed through pool agreements, and the remaining 4 students are placed through faculty agreements.

The department of political science and international relations is the department with the highest quantity of department quotas arranged. There are 58 department quotas at 20 relevant departments of different partner institutions for the department of political science and international relations. The department of management is the second department in this ranking with 45 department quotas at 18 relevant departments of different partner institutions.

Comparison

The 3 proposals above are aimed to improve the mechanism in that they will enable the mechanism to satisfy the desired properties mentioned in section 4 which it fails to satisfy currently.

Proposal 1 is aimed to guarantee the satisfaction of NW. As mentioned, if for a student there are more acceptable positions than she is allowed to prefer, then NW may fail to be satisfied. Proposal 3 tries to modify the mechanism in order to enable PE and in order to avoid students from making other students better-off at the expense of some others by removing indifferences among positions for students.

The improvement obtained by modifying the mechanism according to proposal 1, can be observed by randomly assigning additional positions to the submitted preference lists of students respecting applicability. Especially those 10 students who already submitted 12 choices but are not placed to a position in the first stage should be observed to see if some of them are placed to a position after the modification of the mechanism. However, while randomly assigning additional positions to the submitted preference lists, acceptability may fail to be satisfied. Assigning randomly an applicable position does not guarantee acceptability of that position for the student. So, in order to support the theoretical result with actual data, true preferences of the students are required.

There is only one partner institution which offers both a pool position and some department positions simultaneously. In order to observe the improvement obtained by modifying the mechanism according to proposal 3, either the pool position or the department positions of this partner institution should be ignored (the quota of the pool position can be distributed among the existent department positions or vice versa). However, ignoring the pool quota may require a new department position offered by this partner institution, since this partner institution may welcome not only those students from departments with existent department agreement but

also students from different departments. So, the partner institution's and its departments' decision on this issue is required in order to observe the improvement.

I will focus on proposal 2 in order to compare the recently used mechanism with the one modified accordingly. So, I will consider the placements as tentative which are realized in the first stage. Then, having removed those students from the market who changed their minds and gave up on participating in the programme or who became unacceptable, I will apply serial dictatorship once again including all students who are not removed from the market for the second stage and hence realize the final placements. All the students who are placed to a position in the first stage and then are removed from the market, and all the students who are not placed to a position in the first stage and who did not participate in the second stage will be considered as eliminated. Finally, I will compare the resulting placements with the placements obtained by the current mechanism in order to observe the improvement.

Before comparing the resulting placements, let's mention one of the situations which will enable improvement in terms of fairness in order to observe how the new placements which result by using the modified mechanism are obtained. Students who are ranked as the thirty-third and fortieth students, are placed to University of Amsterdam in the first stage and then are removed from the market. So, the central authority announced 2 quotas for this position before the second stage. One hundred and seventy-seventh student is the only student placed to this position in the second stage. Seventy-fourth and eighty-fourth students are placed to Utrecht University and Fontys University of Applied Sciences, respectively. Utrecht University is the third choice of the seventy-fourth student and Fontys University of Applied Sciences is the third choice of the eighty-fourth

student. Since both students are placed to their third choices in the first stage, they couldn't participate in the second stage. However, they both prefer University of Amsterdam more than their current placements. Seventy-fourth student prefers this position as her second choice and eighty-fourth students prefers it as her top choice. So, they both would like to be placed to this position instead of being placed to their current placements. Also, they both have higher total university grades than the one hundred and seventy-seventh student. Hence, the failure of fairness is noticeable. In addition, the failure of non-wastefulness is also to be noticed, since University of Amsterdam is announced to have 2 quotas before the second stage and only 1 student is placed to this position. While applying the modification, we will make the placements obtained in the first stage of the seventy-fourth and eighty-fourth students tentative, so that they will be placed to University of Amsterdam in the second stage and fairness will be satisfied. Having placed them to University of Amsterdam, the positions they tentatively occupy will have 1 additional quota and other students will be able to be placed to these positions according to their preferences.

I will not give all the placements which are realized by the mechanism currently used and by the modified mechanism. I will focus on the students whose placements are changed having applied the modification and consider their placements in order to observe the improvement.

Table 1 shows the improvements obtained by modifying the mechanism.

Table 1

| Student Rank | Current Placement (CP) | Rank of CP | New Placement (NP) | Rank of NP |
|--------------|-------------------------|------------|-------------------------|------------|
| 74 | Utrecht Uni. | 3 | Uni. of Amsterdam | 2 |
| 84 | Fontys Uni. | 3 | Uni. of Amsterdam | 1 |
| 125 | Heinrich Heine Uni. | 4 | Uni. of Erfurt | 2 |
| 149 | Tilburg Uni. | 2 | HEC Uni. | 1 |
| 177 | Uni. of Amsterdam | 3 | Uni. of Erfurt | 6 |
| 184 | Uni. of Aarhus | 6 | Uni. of Erfurt | 2 |
| 198 | Heinrich Heine Uni. | 7 | Uni. of Aarhus | 4 |
| 205 | Michigan State Uni. | 6 | Utrecht Uni. | 1 |
| 218 | Uni. of Gent | 6 | Utrecht Uni. | 4 |
| 227 | Uni. Catholique Louvain | 3 | Tilburg | 2 |
| 230 | Uni. Catholique Louvain | 4 | Tilburg | 3 |
| 239 | Roskilde Uni. | 4 | Utrecht | 3 |
| 249 | Uni. of Maastricht | 4 | Uni. of Gent | 1 |
| 251 | Uni. Catholique Louvain | 4 | Tilburg | 3 |
| 263 | Jönköping Uni. | 7 | Uni. of Gent | 5 |
| 272 | Uni. North Carolina | 6 | Uni. of Aarhus | 4 |
| 292 | Uni. Jyvaskyla | 9 | Uni. of Gent | 7 |
| 295 | Euromed Marseille Uni. | 8 | Uni. Catholique Louvain | 7 |
| 305 | Uni. Maalardalen | 9 | Uni. Catholique Louvain | 5 |
| 323 | Uni. of Aarhus | 4 | Uni. of Uppsala | 3 |
| 325 | Tallinn Uni. | 5 | Uni. of Aarhus | 4 |
| 329 | Uni of Erfurt | 4 | Not nominated | |
| 338 | Victoria Business Uni. | 6 | Erasmus Uni. | 4 |
| 372 | Uni. North Carolina | 5 | Uni. of Aarhus | 2 |
| 382 | Uni. of Gent | 3 | Uni. Catholique Louvain | 5 |
| 383 | Utrecht Uni. | 1 | Uni. of Maastricht | 2 |
| 384 | Uni. of Maastricht | 5 | Not nominated | |
| 390 | West Virginia Uni. | 6 | Michigan State Uni. | 4 |
| 393 | Uni. of Alberta | 3 | Uni. of Aarhus | 1 |
| 420 | Jönköping | 4 | ZHW | 1 |
| 425 | Uni. of Gent | 2 | Stockholm Uni. | 4 |
| 429 | Tilburg | 1 | Uni. Köln | 3 |

We observe that 32 students' placements are changed applying the modification to the data. 25 of them are better-off and 7 of them are worse off. Although an improvement in terms of Pareto efficiency is not the case, an improvement in terms of fairness is obtained.

Students whose current and new placements are highlighted in the table above are the ones who are placed in the second stage of the mechanism currently used. They are worse-off by the modified mechanism, since the failure of fairness was caused by them.

In short, modifying the mechanism as mentioned enables fairness to be satisfied.

CHAPTER 7

CONCLUSION

I have discussed the mechanism used at Boğaziçi University in order to place students to positions at partner institutions according to the Erasmus student exchange programme. The placement procedure used for this purpose is introduced as a placement problem where students constitute the only active side of the market and positions constitute the passive side. The mechanism is designed to select students and to place them to positions in a centralized environment. The Office of International Relations of Boğaziçi University is the central authority which realizes the placements. I wanted to observe if the mechanism satisfies individual rationality, non-wastefulness, fairness, Pareto efficiency and strategy-proofness. I discussed that the mechanism which is currently used satisfies individual rationality and strategy-proofness, but fails to satisfy non-wastefulness, fairness, and Pareto-efficiency.

Individual rationality is satisfied by the mechanism, so no student is placed to a position which she does not prefer. Also, strategy-proofness is satisfied which means that no student can make herself better-off by misrepresenting her preferences.

The reason why non-wastefulness fails to be satisfied appears to be the restriction put on the quantity of the submitted preferences of students, as well as the second stage. I offered, at least theoretically, that letting students submit as many preferences as they wish (respecting applicability of a position and hence equivalently eligibility of a student) enables to overcome this problem. Although, as mentioned it sounds as it is, namely extremely hard to deal with especially considering the fact that students do not seem to be rational. The failure of the

rationality of students, which is observed in actual data, makes this proposal hard to be applicable.

In addition to causing the failure of non-wastefulness, the second stage also causes the violation of fairness. As discussed, only those students are allowed to participate in the second stage who are not placed to a position in the first stage. The placements which are realized in the second stage may then cause the violation of fairness as shown and supported by actual data. The proposal to overcome this situation is to make the placements tentative which are realized in the first stage. Doing this, the selection procedure will be as a one-stage process, so that fairness will hold as shown.

Pareto efficiency is another desired property which is not satisfied by the mechanism. The reason is that the mechanism relies on strict preferences of students over positions. However this does not need to be the case. Especially, if a partner institution offers both a pool position which can be preferred by all students and some department positions which can only be preferred by students of the relevant department, Pareto efficiency cannot be satisfied. As mentioned, applying different rules for breaking these indifferences to submit strict preferences to the central authority, favours different students and hence different departments.

A similar argumentation is valid for strategy-proofness. Although strategy-proofness is satisfied, students can make some students better-off at the expense of some others because of the existence of indifferences for students. As shown, a student cannot make herself better-off by applying different rules for breaking the indifferences, however the rule she applies affects other students and hence

departments may manipulate the mechanism by advising their students possibly to their best interest which is in order to send more students to more desired positions.

As a proposal to overcome both of these difficulties, I mentioned that the central authority could arrange all partner institutions to offer either a pool position or department positions but not both types of positions simultaneously.

Finally, I supported one of the proposals I made in order to upgrade the mechanism by making use of actual data 2009-2010. To improve the placements in terms of fairness, I made the placements which are realized in the first stage tentative, so that the first stage acts just like an elimination phase. Then, all the students who are not removed from the market are taken into account in the second stage and the resulting placements exemplified the improvement. This improvement is as discussed in terms of fairness and not Pareto efficiency, since some students became better-off at the expense of some other students which was expected.

There are some issues which are still to be discussed in this context. One of these issues is that students can be indifferent between positions offered by different partner institutions. In this paper, I considered only indifferences of students between positions offered by the same partner institution. I showed that the existence of such indifferences causes the violation of Pareto efficiency. As a proposal to overcome this problem, I offered to allow only one type of agreements (either a pool agreement or a department agreement, but not both types of agreements simultaneously) with each partner institution so that each partner institution can offer only one type of positions. However, if students are allowed to be indifferent between positions offered by different partner institutions, then this proposal does not work. So a different approach to this problem will be necessary.

Existence of different types of agreements yields another issue to be discussed. As shown, indifferences of students between positions offered by the same partner institutions caused by the existence of different types of agreements enables students to make some students better-off at the expense of some other students. I showed that avoiding a partner institution to offer different types of positions simultaneously solves this problem. It can be discussed if this problem can still be solved without putting a restriction to the types of positions offered by partner institutions.

A final issue I want to mention which can be discussed is the historical path of the model for the placement problem I considered. It can be discussed if different types of agreements have existed since the beginning of the placement problem or if the model became the current state as a consequence of a supergame between the departments of Boğaziçi University. The departments whose students have lower total university grades can have experienced lack of quantity of students placed to positions. So, such departments may have the incentive to arrange department agreements with partner institutions in order to guarantee at least some of their students to be placed to positions through department quotas. Then, it can also be discussed if the current state of the model is an equilibrium of the supergame.

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