THE KINDERGARTEN RULE OF SUSTAINABLE GROWTH:

A SCHUMPETERIAN APPROACH

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Thesis Abstract

Gül Bahar Şenol, "Kindergarten Rule of Sustainable Growth: A Schumpeterian Approach"

In this thesis, I construct a dynamic economic growth model that takes into account environmental concerns, and search for a difference in the pattern of environmental quality when technological progress is taken to be endogenous. In order to do so, I extend Brock and Taylor's (2003) model. In their model, Brock and Taylor define pollution production and abatement explicitly by considering pollution as an output of both production and abatement activities, and suggest a new approach to pollution convergence, which they name as the "Environmental Catch-up Hypothesis". Their theoretical framework relies on AK production function where production increase depends significantly on capital increase. Since pollution is generated via production, analyzing capital stock as a source of pollution is necessary which raises attention to the types of capital such as physical and knowledge capital where the latter is cleaner compared to the former. I add to the model of Brock and Taylor (2003) by applying endogenous technological progress using the Schumpeterian approach, where I include those two types of capital separately in the functional form. With this approach, I observe that the possibility to attain a sustainable balanced growth path increases as technology improves steadily. Moreover, I show that the rise in technological growth causes pollution growth to decline. Furthermore, I solve the Environmental Catch-up formulation which illustrates that as income grows, environmental quality deteriorates much more in initially poor countries compared to the rich ones, and as economic growth proceeds, the differences in their environmental quality narrow and converge to each other in the long run.

Tez Özeti

Gül Bahar Şenol, "Sürdürülebilir Büyümenin Anaokulu Kuralı:

Schumpeterci Yaklaşım"

Bu tezde, teknolojik ilerlemenin içsel olduğunu varsayarak, çevresel sorunları ve çevresel nitelik değişikliklerini ele alan dinamik bir model kurdum. Bu amaçla, Brock ve Taylor'ın (2003) modelini baz alarak bu modeli genişlettim. Brock ve Taylor, modellerinde kirliliğin hem üretimden hem de kirliliğin azaltılması faaliyetlerinden kaynaklandığını belirgin bir şekilde ifade ediyorlar. Ayrıca, ülkeler arasında kirliliğin yakınsaması üzerine "Çevresel Yakınsama Hipotezi" adlı yeni bir yaklaşım sunuyorlar. Kurdukları modelde, teorik yaklaşımları, üretimin özellikle sermaye artışına bağlı olduğu AK üretim fonksiyonuna dayanıyor. Kirlilik, üretime bağlı artış gösterdiğinden, fiziksel ve görece daha temiz olan bilişsel olarak ikiye ayırabileceğimiz sermaye stokunu kirliliğin sebebi olarak analiz edebiliriz. Bu bağlamda, Brock ve Taylor'un (2003) modelindeki üretim fonksiyonuna bu ikili sermaye yapısını ekleyerek, içsel teknolojik gelişim varsayımı altında Schumpeterci yaklaşımı uyguladım. Bu yaklaşımla, teknolojik ilerleme arttıkça sürdürülebilir dengeli büyümeye erişmenin daha olası olduğunu gözledim. Ayrıca, teknolojik büyümenin kirliliğin büyümesinin azalışına sebep olduğunu gösterdim. Bunun yanında, gelir arttıkça çevrenin ilk aşamada görece daha fakir olan ülkelerde daha fazla zarar gördüğünü, ekonomik büyümeyle zengin ülkelerle aralarındaki çevresel farkın azaldığını ve uzun vadede yakınsadığını gösteren Çevresel Yakınsama formülünü çözdüm.

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CHAPTER 1

INTRODUCTION and LITERATURE REVIEW

The interaction between economic growth and the environment is one of the fundamental issues for long term economic growth analysis. Indeed, this relationship has two dimensions which contradict each other. On one hand, as the economy grows, pollution is generated as a by-product of production which deteriorates the level of environmental quality. On the other hand, as the economy continues to grow, the resources for pollution abatement and environment-friendly technologies accrue which help to reduce pollution stock and improve environmental quality (Butter and Hofkes, 1998). Accordingly, it is questionable whether economic growth and healthy environmental conditions will be compatible in the long run. This challenging question generated one branch of the vast sustainable development literature.

The term sustainable development is defined as “development that meets the needs of the present without compromising the ability of future generations to meet their own needs” in report of World Commission on Environment and Development (1987,Chp 2). Since then it is said to be a vague term that could be interpreted differently for various goals and specializations. In our analysis, we perceive sustainability as continuous economic growth with non-declining environmental quality with the help of improving technological progress. This statement is significantly related with ecological modernization which asserts the possibility of reducing pollution by technological progress in production (Andersen and Massa, 2000). In other words, ecological modernization incorporates the innovation and diffusion of cleaner technologies that assures decline in environmental intensity of each unit of output while providing advantage for economic development. Additionally, it maintains more rational and holistic method to environmental protection and policies (Gouldson and Murphy, 1996).

To maintain sustainable growth considering environmental constraints is a challenge since the 1970s. The earlier models incorporate the impacts of natural resource extraction within the neoclassical approach (Brock and Taylor, 2005). In these models, being the source of growth, technology is taken to be exogenous to the system. Within this realm, the Green Solow model (Brock and Taylor, 2004) serves as a benchmark for our purpose. In this analysis, Brock and Taylor augment the Solow model by incorporating technological progress in pollution abatement. They distinguish the impacts of technological progress in abatement and goods production on environmental quality. While the technical progress in abatement leads to a technique effect, decreasing emissions, the technical progress in goods production leads to a scale effect, increasing emissions. However, taking technology as exogenous, this type of models could only "assume" growth as they do not take into account the progress in technology which is a major determinant in economic growth (Gradus and Smulders, 1993).

In the 1990s, economic growth models with endogenous technical progress became popular. Since technological innovation significantly determines the path of economic growth dynamics, endogenous growth models with evolving technological progress are characteristically more appropriate to examine sustainability (Aghion and Howitt,1998). Indeed, in the context of sustainability, the effect of technological progress is twofold. On one hand, it is a threat to sustainability provided that it evolves in the direction of energy or resource using technologies. On the other hand, it is a solution when it is a progress through clean or resource-saving technologies. Consequently, in order to conjecture sustainability, the direction of technological improvement is needed to be analyzed (Bretscher and Smulders, 2007).

Combining this idea with environmental concerns, Smulders (1999) claims that permanent economic growth is attainable through continual knowledge accumulation without rising natural resource extraction and pollution generation. Using endogenous growth specifications, Hofkes (1996) contributes with a model which includes abatement activities. Among endogenous growth studies, Bovenberg and Smulders (1995) are the first that analyze pollution augmenting technological change in a two sector endogenous growth model. Nevertheless, since Stokey (1998), Aghion and Howitt (1998) and Brock and Taylor (2003) are theoretically closer to our work, we will focus more on these models.

Stokey (1998) uses both Cobb-Douglas and AK production technologies and adjusts them with an emission index that defines the techniques of production. She argues that since abatement and economic activity both use the same scarce resources, an increase in abatement intensity would result in a decline in economic growth. Therefore when abatement increases it decreases pollution stock both through technique and scale effects. In this model, Stokey uses the assumption of declining marginal utility of consumption in order to increase environmental quality. Accordingly, when the marginal utility of consumption declines, marginal disutility of environmental pollution relatively increases which raises the incentives for pollution abatement investments.

Using AK production function, Brock and Taylor (2003) contribute to the literature in two fundamental ways. First, they define pollution production and abatement explicitly by considering pollution as an output of both production and abatement activities. Second, they suggest a new approach to convergence, which is the "Environmental Catch-up Hypothesis" in relation with the Environmental Kuznets Curve (EKC). For the first one, they argue that technological progress eliminates diminishing returns to both capital accumulation and abatement. They embrace zero emission technologies that enable pollution to be cleaned up when it is created. With technological progress, they challenge the assumption of rapidly declining marginal utility of consumption which is needed by Stokey (1998) as we mentioned above. However, with this assumption, it is possible for the ratio of investments in abatement to total investments to reach one in the limit. This is not plausible concerning the real data on abatement costs which stay as a small share of GDP (Brock and Taylor, 2004). Therefore, the way to match empirics and theory could be to assume technological progress in abatement in order to achieve sustainable growth. For the second contribution, Brock and Taylor (2003) introduce a new hypothesis which they call as the Environmental Catch-up. Accordingly, as income grows, the environmental quality deteriorates much more in initially poor countries compared to the rich ones. Additionally, as economic growth proceeds, the differences in their environmental quality narrow and converge to each other in the long run.

Despite the novel assumptions and innovations of the Kindergarten Rule model, the theoretical framework relies on a simple AK production function where production increase depends significantly on capital increase. Since pollution is generated via production, analyzing capital stock as a source of pollution is necessary which raises attention to the types of capital such as physical and knowledge capital where the latter is cleaner compared to the former. However constructing a one-sector endogenous growth model, Brock and Taylor (2003) and Stokey(1998) do not separate the physical capital from the knowledge capital and perceive capital as a single term. Therefore it is worthy to extend their model including those two types separately in the functional form. Here, the aim is to construct a two sector economic growth model that analyzes the impact of endogenous technological change and pollution abatement on sustainable balanced growth path. To do so, we extend the Kindergarten Rule Model of Brock and Taylor (2003) incorporating endogenous technological change through the Schumpeterian approach. By this framework we also show an increase in the possibility of attaining sustainable growth. In this context, the decline in consumption growth through the rise in abatement intensity can be offset by the rise in improving technological progress which could be another motivation for our work.

In this context, Aghion and Howitt (1998) apply the Schumpeterian approach in structuring the capital stock and environmental quality stock equations distinguishing tangible capital goods (physical capital) from the intangible capital, technology. They modify Stokey's (1998) model inducing Schumpeterian production function while holding multiplicative measure of the dirtiness of techniques. In this thesis, incorporating knowledge capital that grows with technological progress, we add to the model of Brock and Taylor (2003) while keeping the abatement intensity framework the same. We develop an endogenous growth model similar to the model of Aghion and Howitt (1998). Throughout, we use the Schumpeterian approach with pollution constraint. Additionally, the theoretical construction concerning labor input and technological progress in the production function are more detailed and specified as in the model of Aghion and Howitt (1998).

Different from the earlier models, such as Stokey (1998) and Aghion and Howitt (1998), in our model we do not specify a pollution index that adjusts the production function with "dirtiness" of techniques. Instead, we adjust the production function with abatement intensity where an increase decreases the dirtiness of the production. In line with Brock and Taylor (2003), with the Schumpeterian approach, we search for differences in feasible and optimal sustainable growth conditions as well as Environmental Catch-up Hypothesis dynamics.

In this context, we extend the paper of Brock and Taylor (2003) in two fields. First, we substitute the AK specification with the Schumpeterian growth model by which we become able to distinguish the cleaner and dirtier factors of production where the former is technology and the latter is capital. Second, we increase the possibility of attaining sustainable growth in the balanced growth path; while the decline in consumption growth through the rise in abatement intensity can be offset by the rise in improving technological progress.

Our findings are as follows. We find conditions for feasible and optimal sustainable growth and the conditions for countries to switch to active abatement. In addition to the findings of Brock and Taylor (2003), we conclude that the rise in technological growth causes decline in pollution growth. In contrast to the level effect in AK specification we find that the technological growth is effective to decrease pollution growth by this specification. Additionally, we analyze data to check whether real observations support our theoretical results. We see that the ratio of R&D expenditures to GDP and the ratio of pollution to GDP for CO_2 are negatively related for thirteen of twenty OECD countries. However, we could not observe any relationship for abatement intensity and pollution intensity at least for the case of CO_2.

The rest of this thesis is organized as follows. In the second chapter, we set our model and solve for the conditions of the "Environmental Catch-up Hypothesis" and deduce our propositions. Additionally, we discuss the emprical implications with a special focus on CO_2 emissions. In the third chapter, we state our results. Finally, the appendices cover the balance growth path solution of the model, proofs of propositions and figures.

CHAPTER 2

THE MODEL and ENVIRONMENTAL CATCH-UP HYPOTHESIS

We use an infinitely lived representative agent model with the impatience factor of ρ. As Brock and Taylor (2003), we use the constant elasticity formulation for the utility function. Accordingly, the utility of the consumer increases with consumption, C, and decreases with the pollution stock, X. We also define a constant B as the impact of local pollution on an individual, considering the variables like population density and land size of the country that are effective in determination of the disutility of pollution stock. The resulting utility function is as follows:

$$U(C,X) = \frac{C^{1-\varepsilon}}{1-\varepsilon} - \frac{BX^{\gamma}}{\gamma} \tag{1}$$

where $\gamma \geq 1$ and $\varepsilon > 0$.

The gross output, Y, is assumed to be used either for investment, consumption or pollution abatement purposes. The factors of production are capital, K, labor, L and technology, T. Assuming that the population growth is zero, the labor factor in the economy is divided into two, represented as L and n which sum up to 1, where the latter is used in research activities and the former is used to produce the goods for consumption and investment. Incorporating the Schumpeterian framework of Aghion and Howitt (1998), we assume that the production of final output is done with labor and a continuum of distinct intermediate goods, M. The productivity levels of M are determined by $T(i)$ which indicates the quality of the intermediate good i. Hence the production function is given by:

$$Y = L^{1-\alpha} \int_0^{\infty} T(i) M(i)^{\alpha} di \tag{2}$$

Intermediate goods are produced using $K(i)$ and $T(i)$ through the production function $M(i) = K(i)/T(i)$. The optimal amount of each M is found by maximizing $Y = L^{1-\alpha} \int_0^1 T(i)M(i)^\alpha di$ subject to $\int_0^1 T(i)M(i)di = K$. Writing the maximization problem yields:

$$\mathcal{L} = L^{1-\alpha} \int_0^1 T(i)M(i)^\alpha di - \lambda(\int_0^1 T(i)M(i)di - K) \tag{3}$$

Solving the first order condition and taking the derivative with respect to i, we obtain the following equation:

$$\alpha L^{1-\alpha} T(i)M(i)^{\alpha-1} = \lambda T(i) \tag{4}$$

$$M(i) = L\left(\frac{\lambda}{\alpha}\right)^{\frac{1}{\alpha-1}} = M \tag{5}$$

Substituting M into $\int_0^1 T(i)M(i)di = K$ constraint we find $M\int_0^1 T(i)di = K$ where $\int_0^1 T(i) \equiv T$ is the average quality. Thus the optimal amount of each intermediate good is $M(i) = M = K/T$. Since $\int_0^1 T(i)M(i)^\alpha di = \int_0^1 T(i)M^\alpha di = M^\alpha \int_0^1 T(i)di = M^\alpha T = (K/T)^\alpha T = K^\alpha T^{1-\alpha}$. Therefore the production function is:

$$Y = F(K, TL) = K^\alpha (T(1-n))^{1-\alpha} \tag{6}$$

Technology stock is determined through the Schumpeterian framework as in Aghion and Howitt (1998) where we assume there is a leading edge technology being the maximum level of quality, $T(i)^{max}$ which is updated by each innovation generated by sector i. We also assume that the technological change is positively related with the maximum level of quality, $T(i)^{max}$, a positive rate, σ, that the innovations shifts the technological frontier of the economy and a positive parameter, η, that comes from a Poisson distribution and indicates the arrival rate of innovations to a single research worker, resulting in the following technological growth equation:

$$\dot{T} = \sigma\eta n T$$

We define the pollution stock, X , in the same manner as Brock and Taylor (2003) so that it increases with production and decreases with abatement and nature's constant regeneration ability factor, β. We define the economy's abatement intensity, θ, which is Y^A/Y where Y^A is the quantity of output used for pollution abatement. We illustrate abatement as an increasing and linearly homogeneous function, a that depends on Y and Y^A. Thus the pollution stock equation is:

$$\dot{X} = Y - a(Y, Y^A) - \beta X$$

Brock and Taylor (2003) do not precisely determine the type and definition of pollution. Here, we specify pollution as local, since the pollution stock is determined through local drivers such as regional output and regeneration capacity. Indeed, in our specification, the pollution accumulation does not depend on a pollution stock variable or variables that capture the pollution that diffuses from other polluter countries. In addition, environmental quality is defined similar to Bovenberg and Smulders (1995). Accordingly, the environmental quality is a natural capital stock, which improves through regeneration and deteriorates through pollution.

Furthermore, we introduce an abatement technology, T^A, that raises the marginal productivity of abatement by augmenting the total production, Y. When T^A is higher, greater level of pollution abatement is realized. T^A is assumed to be proportional to the average abatement intensity, θ where the factor proportionality is taken to be 1. (i.e. if the output allocated for abatement increases, the level of abatement technology increases by the same proportion). Hence, T^A is equal to θ

which is defined to be Y^A/Y . Adjusting the total output by pollution abatement technology the equation becomes:

$$\dot{X} = Y - a(YT^A, Y^A) - \beta X \tag{7}$$

Using the linear homegeneity property of the abatement function it is possible to write:

$$\dot{X} = Y(1 - T^A a(1, Y^A/YT^A)) - \beta X \tag{8}$$

Substituting the definition of abatement intensity gives us:

$$\dot{X} = Y(1 - T^A a(1,1)) - \beta X \ where\ \theta = Y^A/Y = T^A \tag{9}$$

As Brock and Taylor (2003) designated, θ is less than or equal to $1/a$ since output increase causes rise in pollution stock (i.e.$(1 - \theta a(1,1)) > 0$) and abatement decreases the pollution growth.

For the resource constraint we follow the traditional assumptions and specify the capital accumulation equation as usual. The only difference is that we adjust output by deducting the output allocated to abatement. The resulting resource constraint is as follows:

$$\dot{K} = Y(1 - \theta) - \delta K - C \tag{10}$$

In our model, the social planner maximizes the lifetime utility of the representative household with a discount factor of ρ subject to the resource constraint and pollution accumulation. The problem is stated to maximize:

$$\int_0^\infty \left(\frac{C^{1-\varepsilon}}{1-\varepsilon} - \frac{BX^\gamma}{\gamma}\right)e^{-\rho t}dt \tag{11}$$

subject to

$$\dot{K} = K^{\alpha}(T(1-n))^{1-\alpha}(1-\theta) - \delta K - C \tag{12}$$

$$\dot{X} = K^{\alpha}(T(1-n))^{1-\alpha}(1-\theta a) - \beta X$$

$$\dot{T} = \sigma\eta n T$$

$$K(0) = K_0, X(0) = X_0, T(0) = T_0 \ \ and \ \ \theta \leq 1/a$$

The current value Hamiltonian equation can be written as:

$$\rho W(K,X,T) = Max\{H = \frac{C^{1-\varepsilon}}{1-\varepsilon} - \frac{BX^{\gamma}}{\gamma} + \lambda_1[K^{\alpha}(T(1-n))^{1-\alpha}(1-\theta) - \delta K - C]$$

$$+\lambda_2[K^{\alpha}(T(1-n))^{1-\alpha}(1-\theta a) - \beta X] + \lambda_3[\sigma\eta n T]\} \tag{13}$$

where λ_1 is the positive shadow value of capital, λ_2 is the negative shadow cost of pollution and λ_3 is the positive shadow value of technology. The control variables are C, n and θ and the state variables are K, X and T. The solution of the Hamiltonian equation, the corresponding first order conditions and the growth rates are presented in Appendix A.

In order to set the optimal level of abatement we focus on the terms including θ in the Hamiltonian function. The simplified form of these are shown below:

$$Max\left\{K^{\alpha}(T(1-n))^{1-\alpha}\theta\left[-\lambda_1 - a\lambda_2\right]\right\} \ \ s.t. \ 0 \leq \theta \leq 1/a \tag{14}$$

As Brock and Taylor (2003) we define the term $S = -\lambda_1 - a\lambda_2$ and examine the model whether S is positive, negative or zero. While $S > 0$, the shadow cost of pollution is higher than the shadow value of capital which leads to maximum abatement. On the contrary, while $S < 0$, the shadow value of capital is higher than the shadow cost of pollution which results in zero abatement. Lastly, while $S = 0$, the shadow cost of pollution and the shadow value of capital are equalized which conduces to active but not necessarily maximum abatement.

When $S > 0$, the abatement intensity, θ, is at its maximum level which is named as the Kindergarten Rule level by Brock and Taylor (2003), $\theta = \theta^K = 1/a$. The equations become:

$$\begin{aligned}
S &> 0,\ \theta = \theta^K = 1/a \qquad (15)\\
\dot{\lambda}_1 &= \left[(\rho+\delta) - \alpha(1-\theta^K)\frac{Y}{K}\right]\lambda_1\\
\dot{\lambda}_2 &= (\rho+\beta)\lambda_2 + BX^{\gamma-1}\\
\dot{\lambda}_3 &= (\rho-\sigma\eta n)\lambda_3 - (1-\alpha)(1-1/a)\frac{Y}{T}\lambda_1\\
\dot{K} &= K^\alpha(T(1-n))^{1-\alpha}(1-\theta^K) - \delta K - C, \quad K(0) = K_0\\
\dot{X} &= K^\alpha(T(1-n))^{1-\alpha}(1-\theta^K a) - \beta X, \quad X(0) = X_0\\
\dot{T} &= \sigma\eta n T, \quad T(0) = T_0
\end{aligned}$$

When $S = 0$, θ is between 0 and $1/a$ and the dynamics are as follows:

$$\begin{aligned}
S &= 0,\ 0 < \theta \leq 1/a \qquad (16)\\
\dot{\lambda}_1 &= \left[(\rho+\delta) - \alpha(1-\theta)\frac{Y}{K}\right]\lambda_1\\
\dot{\lambda}_2 &= (\rho+\beta)\lambda_2 + BX^{\gamma-1}\\
\dot{\lambda}_3 &= (\rho-\sigma\eta n)\lambda_3 - (1-\alpha)(1-1/a)\frac{Y}{T}\lambda_1\\
\dot{K} &= K^\alpha(T(1-n))^{1-\alpha}(1-\theta) - \delta K - C, \quad K(0) = K_0\\
\dot{X} &= K^\alpha(T(1-n))^{1-\alpha}(1-\theta a) - \beta X, \quad X(0) = X_0\\
\dot{T} &= \sigma\eta n T, \quad T(0) = T_0
\end{aligned}$$

When $S < 0$, θ is at its minimum level which leads to the dynamics:

$$\begin{aligned}
S &< 0, \quad \theta = 0 \\
\dot{\lambda}_1 &= \left[(\rho + \delta) - \alpha\frac{Y}{K}\right]\lambda_1 - \alpha\lambda_2\frac{Y}{K} \\
\dot{\lambda}_2 &= (\rho + \beta)\lambda_2 + BX^{\gamma-1} \\
\dot{\lambda}_3 &= (\rho - \sigma\eta n)\lambda_3 - (1-\alpha)(1-1/a)\frac{Y}{T}\lambda_1 \\
\dot{K} &= K^{\alpha}(T(1-n))^{1-\alpha} - \delta K - C, \quad K(0) = K_0 \\
\dot{X} &= K^{\alpha}(T(1-n))^{1-\alpha} - \beta X, \quad X(0) = X_0 \\
\dot{T} &= \sigma\eta n T, \quad T(0) = T_0
\end{aligned} \tag{17}$$

With maximum abatement intensity, pollution is cleaned up at the same time as it is generated. In this case, the pollution stock decelerates due to the coefficient β, the regeneration capacity of nature. When abatement intensity is interior, pollution is partially abated and pollution stock is determined both by the abatement intensity and the regeneration coefficient. Without abatement, the pollution stock rises continuously with production and diminishes only with its own dynamics.

Capital accumulation and the shadow value of capital is also determined through the level of abatement intensity. Capital accumulation with abatement is less than that without abatement while some part of capital is allocated for abatement activities in the former. The shadow value of capital for active abatement case declines with a constant exponential rate given that $\alpha(1 - 1/a)\frac{Y}{K} > \rho + \delta$ which is shown in Appendix A.

$$\frac{\dot{\lambda}_1}{\lambda_1} = \rho + \delta - \alpha(1 - 1/a)\frac{Y}{K} < 0 \tag{18}$$

In all cases the technological growth is determined by its own dynamics where the shadow value of technology is partially linked to the shadow value of capital.

When abatement intensity is constant, the growth rates of capital, output, technology and consumption are found to be the same in the balanced growth path. Indeed, basically there are two possibilities for θ to be constant. The first one occurs when the abatement intensity is at its maximum level, where it is set to be a constant, θ^K. In this case the environment continuously improves as $\dot{X} = -\beta X$ and the growth rates are equalized at the balanced growth path which are shown below:

$$g_Y = g_K = g_T = g_C = \frac{\sigma\eta - \rho}{\varepsilon} \text{ and } g_X = -\beta < 0 \tag{19}$$

The second one occurs when θ is between zero and θ^K. In this case the dynamics could converge to the balanced growth path with equalized growth rates if θ becomes constant which could be attained in the limit when θ approaches to θ^K. In order to check whether this happens, as Brock and Taylor (2003) we define the deviation of θ from θ^K as follows:

$$D(\theta) = \frac{\theta^K - \theta}{\theta^K} \tag{20}$$

Substituting $D(\theta)$ to pollution accumulation equation we obtain:

$$\frac{\dot{X}}{X} = \frac{K^\alpha (T(1-n))^{1-\alpha} D(\theta)}{X} - \beta \tag{21}$$

With this formulation the growth rate of the pollution stock is determined through growth rates of capital, technology and $D(\theta)$. If the sum of these growth rates is less than zero, then pollution growth will be negative which can be shown as:

$$\alpha g_K + (1-\alpha) g_T + g_D = g_X < 0 \tag{22}$$

In equation (22) the deceleration of g_D is substantial so that the convergence of θ to θ^K is very rapid. Thence, it more than offsets the growth in K and T which after all led the inequality to be satisfied.

In this set up, while pollution stock accumulates through production, it decays with the environment's ability to regenerate. The implications of this could be seen by analysing $S = 0$ dynamics where rate of change of shadow cost of pollution must fall over time at the same rate the shadow value of capital falls over time. That could be written as:

$$\dot{\lambda_1} = -a\dot{\lambda_2} \tag{23}$$

Substituting $\dot{\lambda_1}$ and $\dot{\lambda_2}$ equations we end up with:

$$-\frac{1}{a} = \frac{BX^{\gamma-1}C^{\varepsilon}}{\rho - \beta - \alpha(1-1/a)\frac{Y}{K}} \tag{24}$$

Taking the derivative of both sides with respect to time we get the following negative growth rate:

$$\frac{\dot{X}}{X} = \frac{-\varepsilon}{\gamma-1}\frac{\dot{C}}{C} = \frac{\rho - \sigma\eta}{\gamma - 1} < 0 \tag{25}$$

As can be seen in Appendix A, $\sigma\eta$ determines the growth rate of technology. In equation (25) we find that increase in technological growth causes pollution growth to decrease.

In order to see the deviation dynamics of θ we rewrite equation (21) as:

$$D(\theta) = \frac{X}{K^{\alpha}(T(1-n))^{1-\alpha}}\left[\frac{\beta(\gamma-1) - (\sigma\eta - \rho)}{(\gamma-1)}\right] \tag{26}$$

$D(\theta)$ should be higher than or equal to zero where pollution and output should be positive which implies:

$$\beta(\gamma - 1) > \sigma\eta - \rho \tag{27}$$

This condition is satisfied if $\gamma \neq 1$ and β is adequately large relative to the term $\sigma\eta - \rho$. When we divide both sides of equation (27) by ε, the right hand side becomes the growth rate of output ($\frac{\beta(\gamma-1)}{\varepsilon} > \frac{\sigma\eta-\rho}{\varepsilon}$)which is shown in Appendix A. (Equation (69)). That signifies, when β is adequately large to satisfy this condition, then the balanced growth path could be attained with high regeneration capacity of nature and relatively low output growth which we call as the high regeneration-slow growth scenario. In this case since regeneration is active in pollution reduction, applying maximum abatement will not be necessary in the beginning of the growth process. As economy grows, abatement intensity approaches to its maximum value asymptotically. On the contrary if regeneration is limited ($\beta(\gamma - 1) < \sigma\eta - \rho$) then the balanced growth path could be satisfied with high level of abatement activities denoting that θ should be equal to θ^K.

Thus far we have analysed the implications of active or maximum abatement cases where we search for reliable conditions for sustainable economic growth. In order to complete the statement we should also examine the scenario without abatement. When $\theta = 0$, the pollution accumulation equation becomes $\dot{X} = K^{\alpha}(T(1-n))^{1-\alpha} - \beta X$ where $\dot{X}$ stabilizes when the growth rates of output and pollution are equalized. ($g_x = g_Y > 0$). According to this condition, shadow cost of pollution rises steadily which in some point in time provides incentives for abatement. Furthermore, the shadow value of capital declines as the shadow cost of pollution increases (as $\dot{\lambda_1}$ in equation (17)) which again direct the process to active abatement. Consequently, a balanced growth path without abatement is not the optimal one. In sum, we attain the proposition below:

Proposition 1 : Assuming $\sigma\eta - \rho > 0$, sustainable economic growth with continuously decreasing pollution stock is possible and optimal

i) When $\beta(\gamma - 1) > \sigma\eta - \rho$, abatement intensity approaches to its maximum level, θ^K, asymptotically.

ii) When $\beta(\gamma - 1) < \sigma\eta - \rho$, abatement intensity is always equal to θ^K through the balanced growth path.

iii) When the abatement intensity is equal to zero, the balanced growth path we attained is not feasible.

The proof of proposition 1 can be found in Appendix B.

The necessary condition for sustainable growth, $\sigma\eta - \rho > 0$, is also the required condition for positive growth in Aghion and Howit's model (1998). Accordingly, $\sigma\eta$ is larger than ρ, which means technological growth is higher than the impatience factor. This means, research activity is sufficiently productive to prevent the solution with very low or without technological development which would end economic growth gradually in the future. Provided that the impatience factor is sufficiently high, economic growth would eventually cease, however that high impatience factor would not be chosen by a rational individual. As a result, this is a reasonable condition which is also a common requirement for Schumpeterian growth models (Aghion and Howitt, 1998).

When $\beta(\gamma - 1) < \sigma\eta - \rho$, in the fast growth-slow regeneration scenario, environmental degradation occurs. This could be attributed to the scale effect, as the environment deteriorates through the rise in economic activity. However, in our analysis, we offset this effect through introducing abatement intensity, the pollution abatement costs per output, which in the limit approaches to a positive constant, the Kindergarten Rule level. This brings about decline in the pollution stock which could be realized by allocating more output to abatement activities which triggers technological progress in abatement. Besides, after a certain income level, as composition and technique effects start to dominate, the environmental quality begins to improve. Hence, the degradation occurred in the first stage might be stemmed from unavailable or nonexistent environment-friendly technologies.

Consequently, with this proposition, we show that; when the abatement intensity is equal to θ^K, the sustainable balanced growth path is possible and optimal. In order to generate this result, Brock and Taylor (2003) need abatement intensity to be productive, noting that $1/a$ should not approach to 1. They do not specify the bounds of a, but only ascertain it to be higher than 1 for sustainable growth. However, this may lead us to incorrect results for consumption growth and therefore sustainability dynamics when the production function is in AK form. Accordingly, using the AK approach, we may not obtain a sustainable growth path. In order to show this, we analyse the consumption growth equation in the sustainable case of Brock and Taylor's model. That is:

$$\frac{\dot{C}}{C} = \frac{1}{\varepsilon}\left[A\left(1-\theta^K\right)-\delta-\rho\right] > 0 \tag{28}$$

where $A\left(1-\theta^K\right) > \delta+\rho$ for sustainable growth and $\theta^K = 1/a$. Considering $a > 1$, as a approaches to 1, we observe substantial declines in consumption growth. Therefore, the share of output allocated to abatement activities climbs to the level which would eventually reduce the share of output to be consumed and invested. This may cause consumption growth to decline sharply below the level needed for sustainable growth.

In our case, however, the consumption growth is solved as in equation (56) as below (See Appendix A):

$$\frac{\dot{C}}{C} = \frac{1}{\varepsilon}\left[\alpha\left(1-\frac{1}{a}\right)\frac{Y}{K}-\delta-\rho\right] > 0 \tag{29}$$

where obviously $\alpha\left(1-\frac{1}{a}\right)\frac{Y}{K}=\alpha\left(1-\frac{1}{a}\right)\left(\frac{TL}{K}\right)^{1-\alpha}$. This term can remain unchanged forever as the technological progress, T, grows faster than the capital, K, which would be able to offset the decline in the term $\left(1-\frac{1}{a}\right)$. Therefore, inserting an ever improving endogenous technological change, the system is able to settle in a balanced growth path, offseting any possible negative effect that might result from a rise in abatement intensity. This practically clarifies the reason to prefer the Schumpeterian approach in our analysis.

Environmental Catch-up Hypothesis

Our model is built upon the role and the magnitude of abatement intensity in order to curb pollution stock while technological progress counterbalances diminishing returns in production factors, and makes abatement more productive. In this context, in this section we will expand the Environmental Catch-up Hypothesis that was introduced by Brock and Taylor (2003).

Environmental Catch-up Hypothesis refers to the convergence of environmental quality between the rich and the poor countries at a point in time. It is highly related with EKC which relates rising per capita income with deteriorating environmental quality, while the relation is reversed after certain level of per capita income is reached (Barbier, 1997). In Environmental Catch-up Hypothesis, the poor country, C_p, is the one with lower initial capital stock whereas the rich country, C_R, corresponds to the one that has higher initial capital stock. Except their initial capital stocks, both of these countries are identical and have a pristine environment in the beginning, which we name as Stage I. In this stage, while capital stock rises, pollution stock climbs, hence environmental quality degenerates. This pattern continues until the economy encounters to the switching locus and head to Stage II. In Stage II, while the capital stock continues to increase, pollution abatement activities are initiated and hence the pollution stock begins to decline. Therefore, the pollution stock levels of the poor and the rich countries initially diverge and then converge which generates the Environmental Catch-up Hypothesis (Brock and Taylor, 2003). (See Data and Empirics)

Initial divergence can be attributed to high abatement costs and high shadow price of capital for the poor country. In the first stage, the poor country features economic growth, production and consumption, and puts back environmental concerns. Hence, regeneration of the nature remains limited and environment degenerates abruptly in the poor country. In contrast, with lower shadow price of capital, and relatively lower abatement costs, natural regeneration in the rich country could be able to offset the pollution generated by production. Therefore, the environmental status in which the rich country switches to the second stage is better than the one in which the poor country switches. In this switching process, for both of the countries it is necessary to have the rate of return on capital to be lower or equal to the rate of return on pollution abatement.

Applying the procedure of Brock and Taylor (2003) to our model, we continue our analysis assuming that we are in fast growth-slow regeneration scenario i.e. $\beta(\gamma-1) < \sigma\eta-\rho$ where the abatement intensity always equals to the Kindergarten Rule level. In the following section we show the existence and the characteristics of the switching locus and Stage II.

Switching Locus and Stage II

The economy switches to Stage II when the shadow value of capital equals the shadow cost of pollution. After this point, as environmental costs exceeds the benefits from capital accumulation, it becomes reasonable to begin pollution abatement. In our analysis, this switching time is denoted as t^* and the levels of capital, pollution and technology at $t^*$are depicted as K^*, X^*, and T^* respectively. When these values are attained and equation (23) is satisfied, the economy reaches to the condition below:

$$\frac{\lambda_1(K^*X^*T^*)}{\lambda_2(K^*X^*T^*)} = a \tag{30}$$

At this point, marginal abatement costs equalized to marginal damage from pollution. When equation (30) is satisfied, while pollution decreases utility, it becomes rational to start abatement activities as in $S > 0$ dynamics. In order to find this starting point, the switching locus values, we need to solve $\lambda_1{}^*$ and $\lambda_2{}^*$. For the shadow value of capital and the corresponding capital accumulation equation we have:

$$\lambda_1(t) = \lambda_1(t^*)e^{-gt} \tag{31}$$

$$K(t) = e^{-Jt}\left[e^{-Jt^*}K(t^*) + \lambda_1(0)^{-1/\varepsilon}\left(\frac{e^{(J+g/\varepsilon)t} - e^{(J+g/\varepsilon)t^*}}{J+g/\varepsilon}\right)\right] \tag{32}$$

where $J = \delta - \left(\frac{K(t)}{T(t)(1-n)}\right)^{\alpha-1}$ which is found to be a constant. To solve the value of $\lambda_1(t^*)$ we apply the TVC of $\lim_{t\to\infty}\lambda_1(t)K(t)e^{-\rho t} = 0$.

Substituting equation (31) and (32) we obtain:

$$\lim_{t\to\infty} e^{-\rho t}\lambda_1(t^*)e^{-gt}e^{-Jt}\left[e^{-Jt^*}K(t^*) + \lambda_1(0)^{-1/\varepsilon}\left(\frac{e^{(J+g/\varepsilon)t} - e^{(J+g/\varepsilon)t^*}}{J+g/\varepsilon}\right)\right] = 0 \tag{33}$$

Taking $t^* = 0$ for simplicity, we get:

$$\lim_{t\to\infty} e^{-(\rho+g+J)t}\lambda_1(0)\left[K(0) + \lambda_1(0)^{-1/\varepsilon}\left(\frac{e^{(J+g/\varepsilon)t} - 1}{J+g/\varepsilon}\right)\right] = 0 \tag{34}$$

where the sign of $(\rho + g + J)$ is negative and the sign of $J + g/\varepsilon$ should be negative for TVC to hold. Therefore λ_1^* is found to be:

$$\lambda_1(t^*) = [-K(t^*)(J + g/\varepsilon)]^{-\varepsilon} \tag{35}$$

Equation (35) indicates that the shadow value of capital is only determined by the capital stock. Accordingly, when economic growth is faster than the nature's regeneration capacity, the shadow value of capital declines which rationalizes the maximum abatement intensity.

For the shadow cost of pollution and pollution accumulation we have:

$$\lambda_2(t) = e^{(\beta+\rho)(t-t^*)} \left[\lambda_2(t^*) + BX(t^*)^{\gamma-1} \left[\frac{e^{\phi(t-t^*)} - 1}{-\phi} \right] \right] \tag{36}$$

$$X(t) = X(t^*)e^{-\beta(t-t^*)} \tag{37}$$

where $\phi = \rho + \gamma\beta$

Invoking the TVC of $\lim_{t\to\infty} \lambda_2(t)X(t)e^{-\rho t} = 0$ and substituting equation (36) and (37) we end up with:

$$\lim_{t\to\infty} e^{-\rho t^*} X(t^*) \left[\lambda_2(t^*) + BX(t^*)^{\gamma-1} \left(\frac{e^{\phi(t-t^*)} - 1}{-\phi} \right) \right] = 0 \tag{38}$$

where we solve $\lambda_2(t^*)$ as:

$$\lambda_2(t^*) = -\frac{BX(t^*)^{\gamma-1}}{\rho + \gamma\beta} \tag{39}$$

Taking equations (31) and (39) and implementing equation (23) we obtain the switching locus formulation as follows:

$$MAC = \frac{1}{a} = \frac{BX^{*\gamma-1}\left[-K(t^{*})(J+g/\varepsilon)\right]^{\varepsilon}}{\rho+\gamma\beta} = MD \tag{40}$$

where the left hand side of the equation represents the marginal abatement costs and the right hand side represents the marginal damage of pollution.

When $\gamma > 1$, as pollution stock rises, marginal damage increases. Moreover, income elasticity of marginal damage is $\varepsilon/\alpha > 0$.Higher values of ε and lower values of α lead to higher income effect. Additionally, lower α values also increases the factor share of technology in total production.

Whether every country reaches to the switching locus and passes to Stage II is still a question to be answered. Furthermore despite an economy without abatement is found to be non-optimal, we have to show that an economy that switches to Stage II, stays in Stage II forever.

Proposition 2: Assuming that $\sigma\eta - \rho > 0$ and $\beta(\gamma - 1) < \sigma\eta - \rho$,

(i) every economy that is in Stage I must switch to Stage II in finite time.

(ii) every economy in Stage II stays there forever.

(iii) The switching point is where MAC=MD as in equation (40)

The proof of proposition 2 and switching locus graph can be found in

Appendix B and Figure 1 respectively.

Data and Empirics: Focus on Carbon Dioxide

In this section, we provide the analysis of data on the role of technological progress on pollution intensity and a limited survey on the empirical studies on convergence of pollution among different countries. Here, we focus primarily on carbon dioxide (CO_2) emissions as the pollutant. As denoted by Brock and Taylor (2004), the data on carbon dioxide emissions are available for many countries and for a long time period. Moreover, carbon data seem to be an outlier for the implications of EKC, since the turning point for carbon are at very high levels or it may not exhibit an EKC pattern for some countries.

In order to show the relationship between CO_2 emissions and R&D activities, we use the data on CO_2 emissions from energy use (million tones), sizes of GDP and research and development expenditure as a share of GDP where the first two is taken from OECD Factbook 2008: Economic, Environmental and Social Statistics (from Air and Land, Emissions of Carbon Dioxide and Macroeconomic Trends, Sizes of GDP respectively) and the latter from World Development Indicators (WDI Online). The data are collected annually for the period between 1990 and 2004 for OECD countries such as Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Korea, Mexico, Netherlands, Poland, Portugal, Slovak Republic, Spain, Sweden, Switzerland, Turkey, UK, and USA. Using the data, first we draw scatter graphs of carbon dioxide emission intensities (CO_2/GDP) and R&D expenditures per Gross Domestic Products (R&D/GDP) of the countries. As can be seen in Figure 3, for thirteen of twenty countries the CO_2/GDP and R&D/GDP are negatively related.

However, when the same graphs are drawn with CO_2 on the y-axis, this pattern disappears and we observe no trend in those graphs.Moreover, we graph abatement intensity with CO_2 or CO_2/GDP. For abatement intensity we use the pollution abatement costs and expenditures data in OECD Environmental Data Compendium 2006/2007, Environmental Expenditures and Taxes. Again we see that there is no specific trend in data for OECD countries. (See Figure 4 and Figure 5) With the data in use, we conclude that even if it is theoretically possible to prove, it is not empirically observable yet that the rise in abatement intensity would cause a decline in pollution (CO_2) or pollution per output. In this analysis, the main difficulty is the absence of long term data for abatement intensities for OECD countries and nonexistent data for lower or middle income countries. Therefore comparision between different countries is impossible for the moment.

To check whether Environmental Catch up Hypothesis is empirically valid, we survey the convergence studies on CO_2 emissions among several countries. In this line of research, Strazizich and List (2003) find that for twenty-one industrial countries the conditional convergence in CO_2 emissions exists for the years between 1960 and 1997. Alvares, Gustavo and Puch (2004) claim that according to European Environment Agency data, countries having high level of emissions reduced their emission levels more rapidly compared to the ones that have lower emission levels. Aldy (2006) also observe CO_2 emissions convergence between 23 OECD countries; however divergence appears for 88 countries in global sample for the years between 1960 and 2000. In line with this result, Alvarez, Gustavo and Puch (2004) denote that the heterogeneity in economic development and output growth are the major determinants for the convergence dynamics.

Consequently, we can argue that pollution convergence among countries is possible; however the patterns may differ through the country-specific characteristics. In our analysis, we consider that the countries are identical except their capital stock levels. Therefore, it seems reasonable to have Environmental Catch up theoretically. Nevertheless, it would be nearly impossible to eliminate heterogeneity in practice.

CHAPTER 3
CONCLUSION

Whether the economic growth and environmental quality will be compatible in the long run is a challenging question for economists and ecologists as well as any other specialists that are interested in sustainable growth. In this thesis, we have tried to answer this question using the tools of endogenous growth theory.

Previous attempts to model economic growth considering environmental quality have taken technological progress as exogenous. Since the technological progress is the determinant factor in output growth, this type of explanation will not be satisfactory in order to explain economic growth. Moreover, in the context of sustainability, the technological progress has a large impact. First of all, it is a relatively cleaner factor of production compared to capital and secondly, it could help pollution abatement techniques to develop which would provide a better environmental quality. Therefore, taking technology as endogenous and specify its progress in detail would probably be a better approach in order to analyze sustainability.

In this thesis, we construct a dynamic economic growth model that takes into account environmental concerns, and search for a difference in the pattern of environmental quality when the technological progress is taken to be endogenous. We apply the Schumpeterian approach which was introduced by Aghion and Howitt (1998). In our analysis, we aim to discuss whether the rise in technological progress, abatement intensity and pollution abatement technologies would help to decrease pollution stock. We try to find the conditions for feasible and optimal balanced growth path. In order to check this, we mainly follow the line of thought in the paper of Brock and Taylor (2003). Additionally we solve the Environmental Catch-up dynamics using the Schumpeterian framework.

We add to the paper of Brock and Taylor (2003) in two important ways. First, we replace the simple endogenous growth model (AK model) with the Schumpeterian growth specification. With this method, we become able to distinguish the cleaner and dirtier factors of production where the former is technology and the latter is capital. Second, by this formulation we increase the possibility of attaining sustainable growth in the balanced growth path. In this context, the decline in consumption growth through the rise in abatement intensity can be offset by the rise in improving technological progress.

We find conditions for feasible and optimal sustainable growth and the conditions for countries to switch to active abatement. In addition to the findings of Brock and Taylor (2003), we conclude that the rise in technological growth causes decline in pollution growth. In contrast to the level effect in AK specification we find that the growth is effective to decrease pollution growth by this specification. Moreover, in Environmental Catch up formulation, we find the countries with higher technological progress are able to switch earlier to pollution abatement as technological progress quickens the fall in shadow value of capital. Additionally, we analyze data to check whether real observations support our theoretical results. We see that the ratio of R&D expenditures to GDP and the ratio of pollution to GDP for CO_2 are negatively related for thirteen of twenty OECD countries. However, we could not observe any specific pattern for abatement intensity and pollution intensity at least for the case of CO_2.

In our work, we solve our model for fast growth-slow regeneration scenario and do not conduct an analysis for the reverse case, since the former seem more reasonable in practice. Slow growth-fast regeneration scenario can be studied as an extension. More importantly, it is essential to support these results by real observations. However, to find data on environmental quality as well as environmental technologies and pollution abatement intensities is a difficult task which unfortunately put limits on empirical studies. Another research area could be econometric investigations of these results.

To conclude, it is more reasonable to study sustainable growth with endogenous growth models such as the Schumpeterian approach by which we can show an increase in possibility of attaining sustainable balanced growth path. Using this framework, we also show that the rise in technological growth leads to a decline in pollution growth.

APPENDICES

Appendix A: The Solution of The Model

The current value Hamiltonian is:

$$\rho W(K,X,T) = Max\{H = \frac{C^{1-\varepsilon}}{1-\varepsilon} - \frac{BX^{\gamma}}{\gamma} + \lambda_1[K^{\alpha}(T(1-n))^{1-\alpha}(1-\theta) - \delta K - C]$$

$$+\lambda_2[K^{\alpha}(T(1-n))^{1-\alpha}(1-\theta a) - \beta X] + \lambda_3[\sigma\eta nT]\}$$

The First order conditions are:

$$\frac{\partial H}{\partial C} = C^{-\varepsilon} - \lambda_1 = 0 \tag{41}$$

$$\frac{\partial H}{\partial \theta} = -(\lambda_1 + a\lambda_2)[K^{\alpha}(T(1-n))^{1-\alpha}] = 0 \tag{42}$$

$$\frac{\partial H}{\partial n} = (\alpha-1)[\lambda_1(1-\theta) + \lambda_2(1-\theta a)]\frac{K^{\alpha}(T(1-n))^{1-\alpha}}{(1-n)} + \lambda_3\sigma\eta T = 0 \tag{43}$$

$$\frac{\partial H}{\partial K} = \alpha[\lambda_1(1-\theta a) + \lambda_2(1-\theta a)]\frac{K^{\alpha}(T(1-n))^{1-\alpha}}{K} - \lambda_1\delta = \rho\lambda_1 - \dot{\lambda_1} \tag{44}$$

$$\frac{\partial H}{\partial X} = -BX^{\gamma-1} - \lambda_2\beta = \rho\lambda_2 - \dot{\lambda_2} \tag{45}$$

$$\frac{\partial H}{\partial T} = (1-\alpha)[\lambda_1(1-\theta) + \lambda_2(1-\theta a)]\frac{K^{\alpha}(T(1-n))^{1-\alpha}}{T} + \lambda_3\sigma\eta n = \rho\lambda_3 - \dot{\lambda_3} \tag{46}$$

From equation (42):

$$\lambda_1 = -a\lambda_2 \tag{47}$$

From equation (43):

$$(1-\alpha)[\lambda_1(1-\theta)+\lambda_2(1-\theta a)]\frac{Y}{1-n} = \lambda_3\sigma\rho T \tag{48}$$

From equation (44), (45) and (46) respectively:

$$\dot{\lambda_1} = (\rho+\delta)\lambda_1 - \alpha[\lambda_1(1-\theta)+\lambda_2(1-\theta a)]\frac{Y}{K} \tag{49}$$

$$\dot{\lambda_2} = (\rho+\beta)\lambda_2 + BX^(\gamma - 1) \tag{50}$$

$$\dot{\lambda_3} = (\rho-\sigma\eta n)\lambda_3 - (1-\alpha)[\lambda_1(1-\theta)+\lambda_2(1-\theta a)]\frac{Y}{T} \tag{51}$$

where $\lambda_1(1-\theta)+\lambda_2(1-\theta a) = \lambda_1(1-1/a) = \lambda_2(1-a)$

Transversality conditions are:

$$\lim_{t\to\infty} e^{-\rho t}\lambda_1(t)K(t) = 0, \;\; \lim_{t\to\infty} e^{-\rho t}\lambda_2(t)X(t) = 0, \;\; \lim_{t\to\infty} e^{-\rho t}\lambda_3(t)T(t) = 0 \tag{52}$$

From equation (49):

$$\frac{\dot{\lambda_1}}{\lambda_1} = (\rho+\delta) - \alpha(1-1/a)\frac{Y}{K} \tag{53}$$

From equation (41):

$$C^{-\varepsilon} = \lambda_1$$

$$\dot{\lambda_1} = -\varepsilon C^{-\varepsilon-1}\dot{\lambda_1}$$

$$\frac{\dot{\lambda_1}}{\lambda_1} = -\varepsilon\frac{\dot{C}}{C} \tag{54}$$

Use equation (53) and (54):

$$\frac{\dot{C}}{C} = \frac{1}{\varepsilon}((\alpha(1-1/a)\frac{Y}{K} - \rho - \delta)) \tag{55}$$

From equation (48):

$$\lambda_1(1-\alpha)(1-1/a)\frac{Y}{1-n} = \lambda_3\sigma\eta T$$

$$\frac{(1-\alpha)(1-1/a)}{1-n}(\dot{Y}\lambda_1 + \dot{\lambda_1}Y) = \sigma\eta(\dot{\lambda_3}T + \dot{T}\lambda_3)$$

$$\frac{(1-\alpha)(1-1/a)}{1-n}(\dot{Y}\lambda_1 - \varepsilon\lambda_1\dot{\lambda_Y}) = \sigma\eta T(\dot{\lambda_3} + \sigma\eta\lambda_3)$$

Substitute equation (54) and $\dot{T}$ equation:

$$\frac{(1-\alpha)(1-1/a)}{1-n}\dot{Y}\lambda_1(1-\varepsilon) = \sigma\eta T(\dot{\lambda_3} + \sigma\eta\lambda_3) \tag{56}$$

From equation (51):

$$\dot{\lambda_3} = (\rho - \sigma\eta n)\lambda_3 - (1-\alpha)(1-1/a)\frac{Y}{T}\lambda_1$$

$$\dot{\lambda_3} + \sigma\eta n\lambda_3 = \rho\lambda_3 - (1-\alpha)(1-1/a)\frac{Y}{T}\lambda_1$$

Substitute equation (56) and divide by $\frac{(1-\alpha)(1-1/a)\lambda_1}{Y}$:

$$\frac{\dot{Y}}{Y} = -\frac{1-n}{1-\varepsilon}\sigma\eta + \frac{\lambda_3\sigma\eta T\rho(1-n)}{(1-\alpha)(1-1/a)(1-\varepsilon)\lambda_1 Y} \tag{57}$$

$$g_n = 0 \tag{58}$$

$$g_K = g_Y = g_C \tag{59}$$

$$\frac{\dot{K}}{K} = (1-\theta)\frac{Y}{K} - \frac{C}{K} - \delta \tag{60}$$

$$\frac{\dot{Y}}{Y} = \alpha\frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{T}}{T} + (1-\alpha)\frac{\dot{n}}{n}$$

Since $g_n = 0$:

$$g_Y = \alpha g_K + (1-\alpha)\sigma\eta n$$

Substitute equation (59) and divide both sides by $(1-\alpha)$:

$$g_Y = \sigma\eta n = g_T \tag{61}$$

From equation (47):

$$\frac{\dot{\lambda_1}}{\lambda_1} = -a\frac{\dot{\lambda_2}}{\lambda_2}$$

Therefore $g_{\lambda_1} = g_{\lambda_2}$ where:

$$-\varepsilon g_C = g_{\lambda_2} \tag{62}$$

From equation (50):

$$g_{\lambda_2} = \frac{\dot{\lambda_2}}{\lambda_2} = \rho + \beta + \frac{BX^{(\gamma-1)}}{\lambda_2} \tag{63}$$

Differentiate with respect to time:

$$0 = \frac{(\gamma-1)BX^(\gamma-2)\dot{X}\lambda_2 - \dot{\lambda_2}BX^{(\gamma-1)}}{\lambda_2^2}$$

$$(\gamma-1)\frac{\dot{X}}{X} = \frac{\dot{\lambda_2}}{\lambda_2} \tag{64}$$

Differentiate equation (57) with respect to time and rearrange:

$$0 = (\sigma\eta\rho(1-n)(\dot{\lambda_3}T + \dot{T}\lambda_3)(1-\alpha)(1-1/a)(1-\varepsilon)\lambda_1 Y$$

$$-\sigma\eta\rho(1-n)\lambda_3 T(1-\alpha)(1-1/a)(1-\varepsilon)(\dot{\lambda_1}Y + \dot{Y}\lambda_1))((1-\alpha)(1-1/a)(1-\epsilon)\lambda_1 Y)^{-2}$$

$$\dot{\lambda_3} + \sigma\eta n\lambda_3 = (1-\varepsilon)\lambda_3\frac{\dot{Y}}{Y}$$

$$\frac{\dot{\lambda_3}}{\lambda_3} + \sigma\eta n = (1-\varepsilon)\frac{\dot{Y}}{Y}$$

$$g_{\lambda_3} + \sigma\eta n = (1-\varepsilon)g_Y \tag{65}$$

From equation (48) and (51):

$$\lambda_1(1-1/a)(1-\alpha)\frac{Y}{T} = \lambda_3\sigma\eta(1-n)$$

$$\dot{\lambda_3} = (\rho - \sigma\eta n)\lambda_3 - (1-\alpha)\lambda_1(1-1/a)\frac{Y}{T}$$

$$\frac{\dot{\lambda_3}}{\lambda_3} = (\rho - \sigma\eta) = g_{\lambda_3} \tag{66}$$

From equation (51):

$$\frac{\dot{\lambda_3}}{\lambda_3} = (\rho - \sigma\eta) - (1-\alpha)(1-1/a)\frac{Y}{T}\frac{\lambda_1}{\lambda_3}$$

Differentiate the equation above with respect to time:

$$0 = -(1-\alpha)(1-1/a)\frac{(\dot{\lambda_1}Y + \dot{Y}\lambda_1)(T\lambda_3) - (\dot{T}\lambda_3 + T\dot{\lambda_3})\lambda_1 Y}{(\lambda_3 T)^2}$$

$$(\dot{\lambda_1}Y + \dot{Y}\lambda_1)\lambda_3 T = (\dot{T}\lambda_3 + T\dot{\lambda_3})\lambda_1 Y$$

$$\dot{Y}(1-\varepsilon)\lambda_1\lambda_3 T = (\dot{T}\lambda_3 + T\dot{\lambda_3})\lambda_1 Y$$

$$\frac{\dot{Y}}{Y}(1-\varepsilon) = \frac{\dot{T}}{T} + \frac{\dot{\lambda_3}}{\lambda_3}$$

$$g_{\lambda_3} = g_Y(1-\varepsilon) - g_T \tag{67}$$

From equation (61):

$$g_{\lambda_3} = -\varepsilon g_Y = \rho - \sigma\eta \tag{68}$$

$$g_Y = \frac{\sigma\eta - \rho}{\varepsilon} \tag{69}$$

Appendix B: Proof of Proposition 1

We show that the balanced growth path with positive capital, technology, output and consumption growth in Appendix A. Now the optimality of these growth rates should be proved for (i) $(S = 0)$ and (ii) $(S > 0)$ for constant θ.

To prove (i) we write the pollution accumulation equation, $\dot{X} = K^{\alpha}(T(1-n))^{1-\alpha}(1-\theta a) - \beta X$, in terms of θ such as:

$$\theta = \frac{1}{a}\left(1 - \frac{X(0)e^{Nt}(N+\beta)}{Y}\right) \tag{70}$$

where we define $N = -(\sigma\eta - \rho)/(\gamma - 1)$.

Rewriting the capital accumulation equation we obtain:

$$\dot{K} = K^{\alpha}(T(1-n))^{1-\alpha}(1 - \frac{1}{a}\left(1 - \frac{X(0)e^{Nt}(N+\beta)}{Y}\right)) - \delta K - C(t) \tag{71}$$

where $X(t) = X(0)e^{[(\rho-\sigma\eta)/(\gamma-1)]t}$ and $C(t) = C(0)e^{(N(1-\gamma)/\varepsilon)t}$.

Solving equation (71) we get:

$$K(t) = \pi_1 X(0)e^{Nt} - \pi_2 C(0)^{(N(1-\gamma)/\varepsilon)t} + \pi_3 e^{Ft} \tag{72}$$

As $\beta(\gamma-1) > \sigma\eta - \rho$, $\pi_1 = \frac{(N+\beta)}{N-F} < 0$, $\pi_2 = \frac{1}{N(1-\gamma)/\varepsilon - F} = \frac{-\varepsilon}{\rho\varepsilon - g(1-\varepsilon)} < 0$ where

$F = \frac{Y}{K}(1 - \frac{1}{a}) - \delta$

The TVC is set to be $\lim_{t\to\infty} \lambda_1(t)K(t)e^{-\rho t} = 0$. Substituting $\lambda_1(t)$ and $K(t)$ we attain:

$$\lim_{t\to\infty} \lambda_1(0)e^{(\rho-\sigma\eta)t}e^{-\rho t}\left[\pi_1 X(0)e^{Nt} - \pi_2 C(0)^{(N(1-\gamma)/\varepsilon)t} + \pi_3 e^{Ft}\right] = 0 \tag{73}$$

The exponent of the first term goes to zero as $-\sigma\eta + N = -\sigma\eta + \frac{\rho-\sigma\eta}{1-\gamma} < 0$. For the second term to satisfy the TVC $-\sigma\eta + N(1-\gamma)/\varepsilon < 0$ should be met. Together with the balanced growth path solution this implies that $\sigma\eta(1-\varepsilon) < \rho < \sigma\eta$ which is consistent with the balanced growth path solution of $\lim_{t\to\infty} \lambda_3(t)T(t)e^{-\rho t} = 0$ (Appendix A). The exponent of the third term is $-\sigma\eta + F = -\sigma\eta + \frac{Y}{K}(1-\frac{1}{a}) - \delta$ which is higher than zero as $\frac{Y}{K}(1-\frac{1}{a}) - \delta - \rho > 0$. Consequently π_3 should be zero for TVC to hold. Writing capital accumulation function again we get:

$$K(t) = e^{(N(1-\gamma)/\varepsilon)t}\left[\pi_1 X(0)e^{(N-N(1-\gamma)/\varepsilon)t} - \pi_2 C(t)e^{-(N(1-\gamma)/\varepsilon)t}\right] \tag{74}$$

Considering equation (74) in the limit, as time goes to infinity we observe that the exponent in front of the term $\pi_1 X(0)$ goes to zero since $N < 0$ and $K(t)$ converges to consumption i.e. the capital growth approaches to consumption growth ($\dot{K}(t) = \dot{C}(t)$.

This convergence is observed in the balanced growth path where the abatement intensity approaches to its maximum level, θ^K. In this process the magnitude of $N = -(\sigma\eta - \rho)/(\gamma - 1)$ is determinant for the speed of the convergence. That is, as N increases in absolute value the speed of convergence increases. When $\sigma\eta - \rho$ is

larger, technological growth is far larger than the discount factor, meaning that as the technological growth increases the speed of convergence of θ to θ^K increases.

When γ is larger, this means that the increase in pollution decreases utility more than before which raises incentives to abate pollution quickly.

Lastly, the TVC, $\lim_{t\to\infty} \lambda_2(t)X(t)e^{-\rho t} = 0$ must be checked for the balanced growth path. To do, we solve the shadow value of pollution as:

$$\lambda_2(t) = \left[\lambda_2(0) + \frac{BX(0)^{\gamma-1}}{g+\rho+\beta}\left(1 - e^{-(g+\rho+\beta)t}\right)\right]e^{(\rho+\beta)t} \tag{75}$$

Substituting $\lambda_2(t)$ and $X(t)$ we obtain:

$$\lim_{t\to\infty} X(0)\left[\lambda_2(0) + \frac{BX(0)^{\gamma-1}}{g+\rho+\beta}\left(1-e^{-(g+\rho+\beta)t}\right)\right]e^{[\beta+(\rho-\sigma\eta)/(\gamma-1)]t} = 0 \tag{76}$$

Since $\beta + (\rho - \sigma\eta)/(\gamma - 1) > 0$ in this case, the term multiplying $X(0)\lambda_2(0)$ goes to plus infinitity. Therefore in order to satisfy the TVC the terms in brackets must be equal to zero in the limit which in turn implies that:

$$\lambda_2(0) = -\frac{BX(0)^{\gamma-1}}{g+\rho+\beta} \tag{77}$$

This ends the proof of part (i). To prove part (ii) we assume $\beta(\gamma - 1) < \sigma\eta - \rho$ and show that $S > 0$ and $\theta = \theta^K$ dynamics as in equation (15) are relevant for this case. In the switching locus section we assumed the following in order to switch to the active abatement:

$$-a\lambda_2(t) > \lambda_1(t)\ \forall\ t > t^s : \text{switching time} \tag{78}$$

For $S > 0$, the shadow value of capital and the shadow cost of pollution are found as:

$$\lambda_1(t) = \lambda_1(0)e^{-gt},\ \ \lambda_1(0) = [hK(0)]^{-\varepsilon}\ \ and\ h = -J - g/\varepsilon \tag{79}$$

$$\lambda_2(t) = \frac{-BX(0)^{\gamma-1}e^{\beta(1-\gamma)t}}{\rho+\gamma\beta} \tag{80}$$

Substituting $\lambda_1(t)$ and $\lambda_2(t)$ into (78) we obtain:

$$\frac{aBX(0)^{\gamma-1}e^{\beta(1-\gamma)t}}{\rho+\gamma\beta} > [hK(0)]^{-\varepsilon}\, e^{-gt}\ for\ \forall\ t > t^s \tag{81}$$

When $t = t^s$, $\frac{aBX(0)^{\gamma-1}e^{\beta(1-\gamma)t}}{\rho+\gamma\beta} = [hK(0)]^{-\varepsilon} e^{-gt}$. If we multiply both sides by the switching locus equation we get:

$$1 > e^{[-g-\beta(1-\gamma)](t-t^s)} \tag{82}$$

This condition is satisfied for $t > t^s$ if and only if $\beta(\gamma-1) < g = \sigma\eta - \rho$ which is the assumption we made for the fast growth slow regeneration case. Since we are in the $S > 0$ zone and θ is always equal to θ^K.

For part (iii), we prove that a balanced growth path with no abatement is not possible. In this case the growth rate of pollution equal to the growth rate of output as $D(\theta) = 1$. The shadow value of pollution is equal to:

$$\lambda_2(t) = \lambda_2(0)e^{(\rho+\beta)t} + \frac{BX(0)^{\gamma-1}}{g+\rho+\beta}e^{(\rho+\beta)t} - \frac{BX(0)^{\gamma-1}}{g+\rho+\beta}e^{-gt} \tag{83}$$

The transversality condition can be written as:

$$\lim_{t\to\infty} X(0)e^{[(\rho-\sigma\eta)/(\gamma-1)]t}\left[\lambda_2(0)e^{(\rho+\beta)t} + \frac{BX(0)^{\gamma-1}}{g+\rho+\beta}e^{(\rho+\beta)t} - \frac{BX(0)^{\gamma-1}}{g+\rho+\beta}e^{-gt}\right]e^{-\rho t} = 0$$

The exponent of the term multiplying $\left(-\frac{BX(0)^{\gamma-1}}{g+\rho+\beta}\right)$ is

$-g-\rho+(\rho-\sigma\eta)/(\gamma-1) < 0$ as $g = \sigma\eta - \rho > 0$. The TVC holds if we apply the condition of $\lambda_2(t) = -\frac{BX(0)^{\gamma-1}}{g+\rho+\beta}e^{-gt}$ which requires $\beta + (\rho-\sigma\eta)/(\gamma-1) = 0$. Considering equation (83), the shadow cost of pollution decreases exponentially as time goes to infinity. In addition, $X(t)$ declines exponentially as time evolves since $(\gamma-1)g_x = g_{\lambda_2} = g_{\lambda_1} = -\varepsilon g_c = \rho - \sigma\eta = -g$. This creates a contradiction; since as the shadow cost of pollution declines the incentives will be on the direction of positive pollution generation which in turn decreases the utility and at some point in time raises concerns for abatement. Therefore the balanced growth path without abatement is not optimal.

Appendix C: Proof of Proposition 2

Part (*ii*) is proved in proof of proposition 1 and (*iii*) is shown in the text. In order to prove part (*i*) we will use a contradiction as Brock and Taylor (2003) do. We assume that it will not be optimal to leave Stage I in finite time and $S < 0$ for all t meaning that:

$$0 \leq -a\lambda_2(t) < \lambda_1(t) \tag{84}$$

We bound $\lambda_1(t)$ from below and $\lambda_2(t)$ from above. While proving part (*i*) we will generate a contradiction that violates this condition. Since we assumed $S < 0$, the shadow value of capital should be as below:

$$\lambda_1(t) < \lambda_1(0)e^{-gt} \tag{85}$$

For $\lambda_2(t)$ we already have the following solution:

$$\lambda_2(t) = e^{(\rho+\eta)t}\left[\lambda_2(0) + \int_0^t BX(s)^{\gamma-1}e^{-(\rho+\eta)s}ds\right] \tag{86}$$

Using the TVC of $\lim_{t\to\infty} e^{-\rho t}\lambda_2(t)X(t) = 0$ we solve $\lambda_2(0)$ as:

$$\lambda_2(0) = -BX(0)^{\gamma-1} = -\int_0^\infty BX(s)^{\gamma-1}e^{-(\rho+\eta)s}ds \tag{87}$$

Together with equation (87), equation (86) becomes:

$$\lambda_2(t) = e^{(\rho+\eta)t}\left[-\int_t^\infty BX(s)^{\gamma-1}e^{-(\rho+\eta)s}ds\right] \tag{88}$$

The corresponding bounded pollution stock when $S < 0$ is:

$$X(t) \geq X(0)e^{-\eta t} \tag{89}$$

Substituting pollution stock equation into the solution of $\lambda_2(t)$ we obtain:

$$-\lambda_2(t) \geq -\lambda_2(t^*) = \; e^{(\rho+\eta)t}\left[\int_t^{\infty} BX(s)^{\gamma-1}e^{-(\rho+\eta)s}ds\right] \tag{90}$$

$$-\lambda_2(t^*) = e^{(-\eta(\gamma-1))t}\frac{BX(0)^{\gamma-1}}{\rho+\eta\gamma}$$

Now including the above solutions we rewrite equation (84) and get:

$$e^{(g-\eta(\gamma-1))t}\frac{aBX(0)^{\gamma-1}}{\rho+\eta\gamma} < \lambda_1(0) \tag{91}$$

which will not be satisfied for a sufficiently large t, since $\beta(\gamma-1) < \sigma\eta - \rho = g$.

Appendix D. Figures

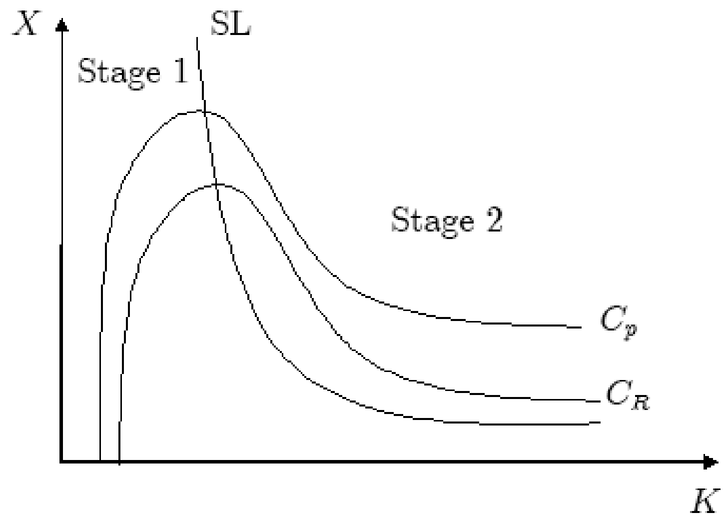

Figure 1: Switching Locus and Environmental Catch-up

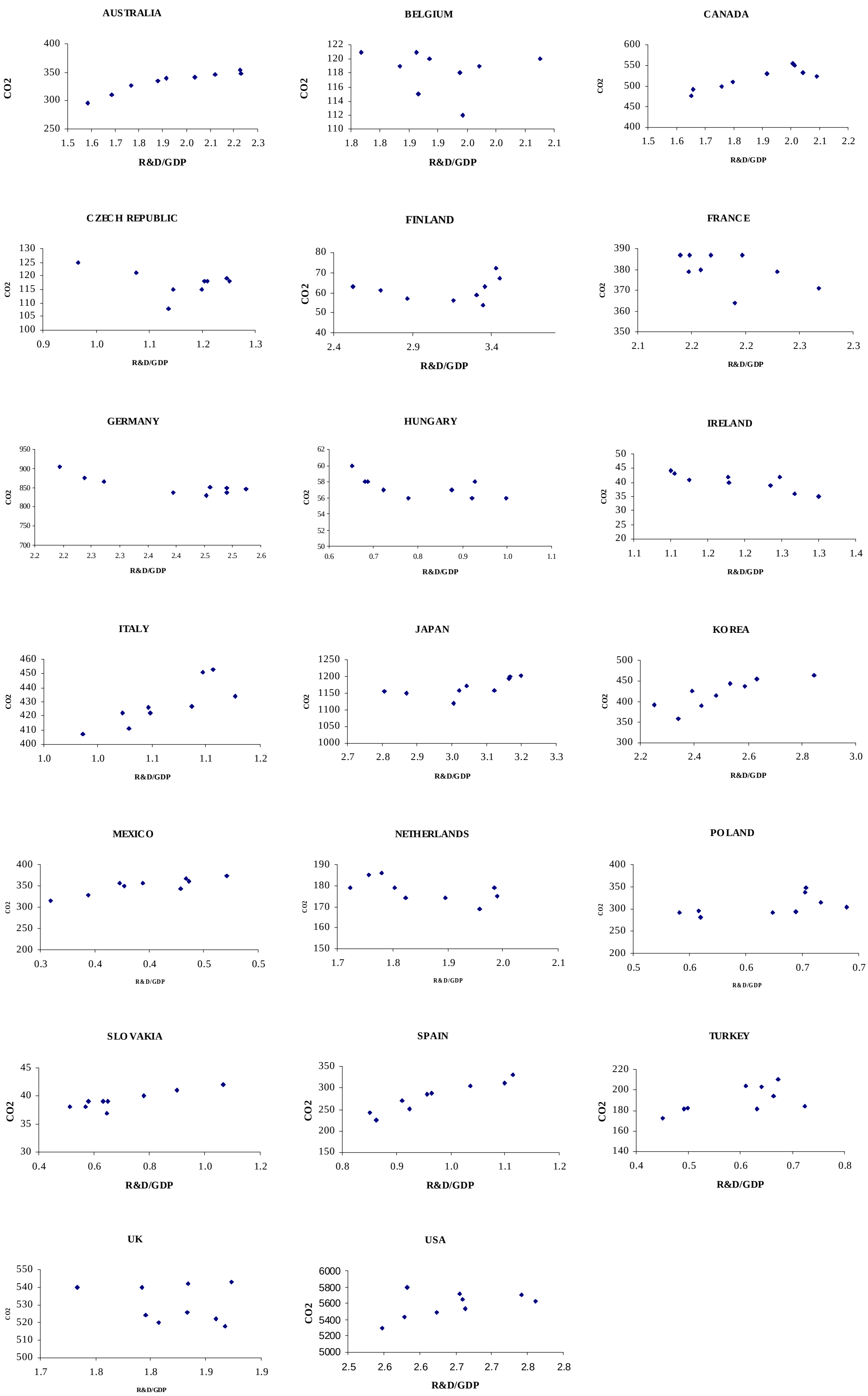

Figure 2: CO_2/GDP vs R&D/GDP for OECD countries

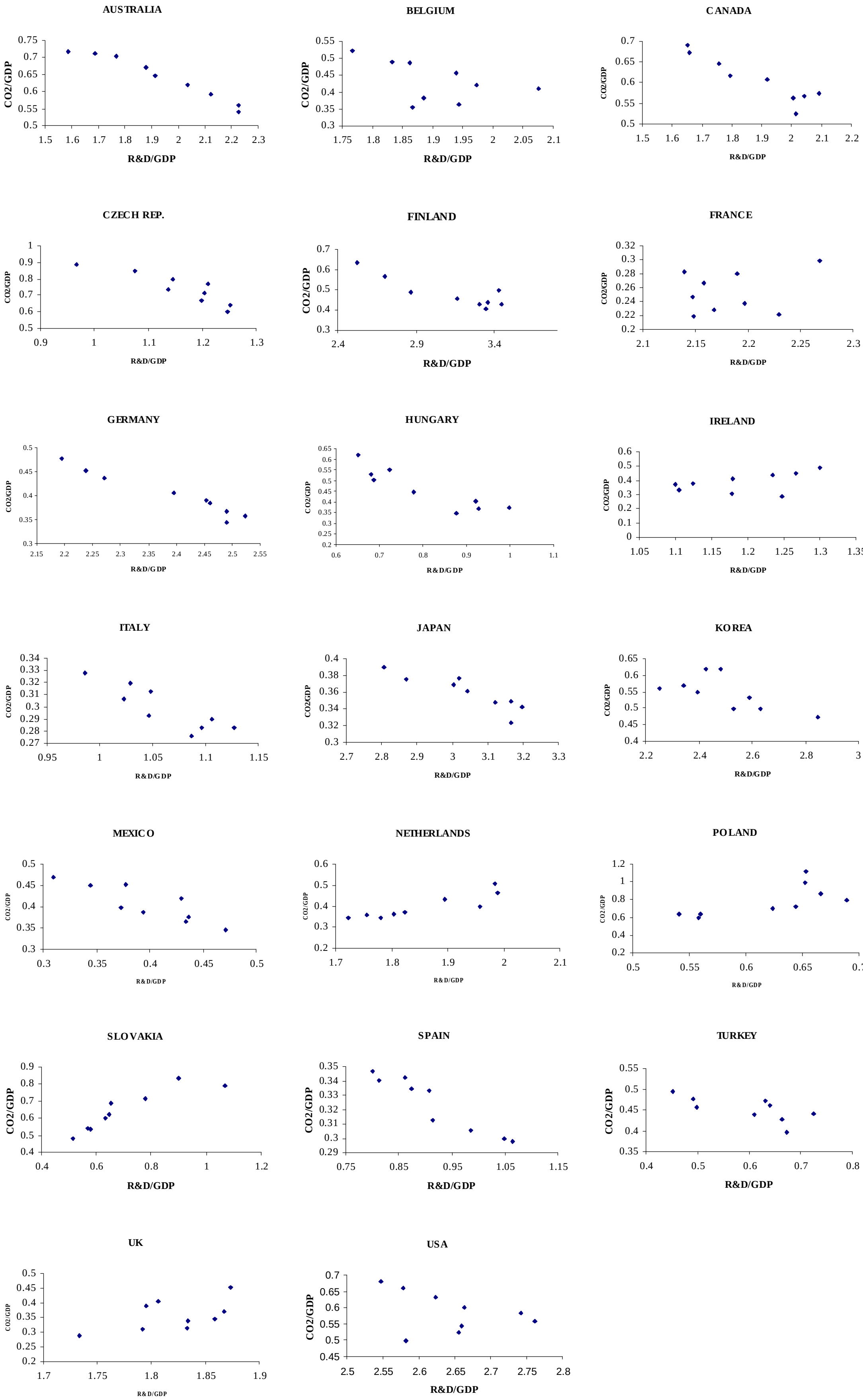

Figure 3: CO_2 vs R&D/GDP for OECD countries

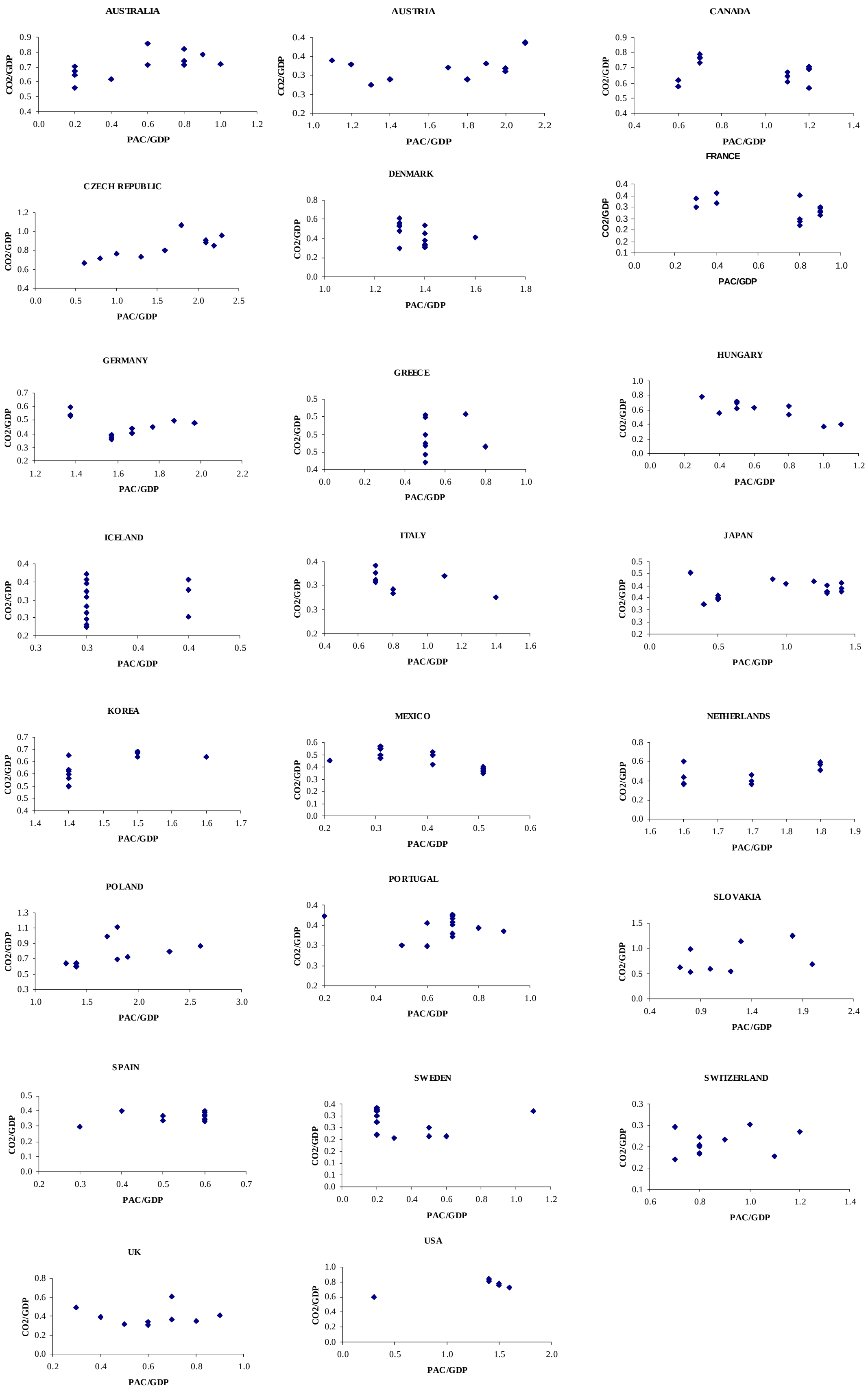

Figure 4: CO_2/GDP vs PAC/GDP for OECD countries

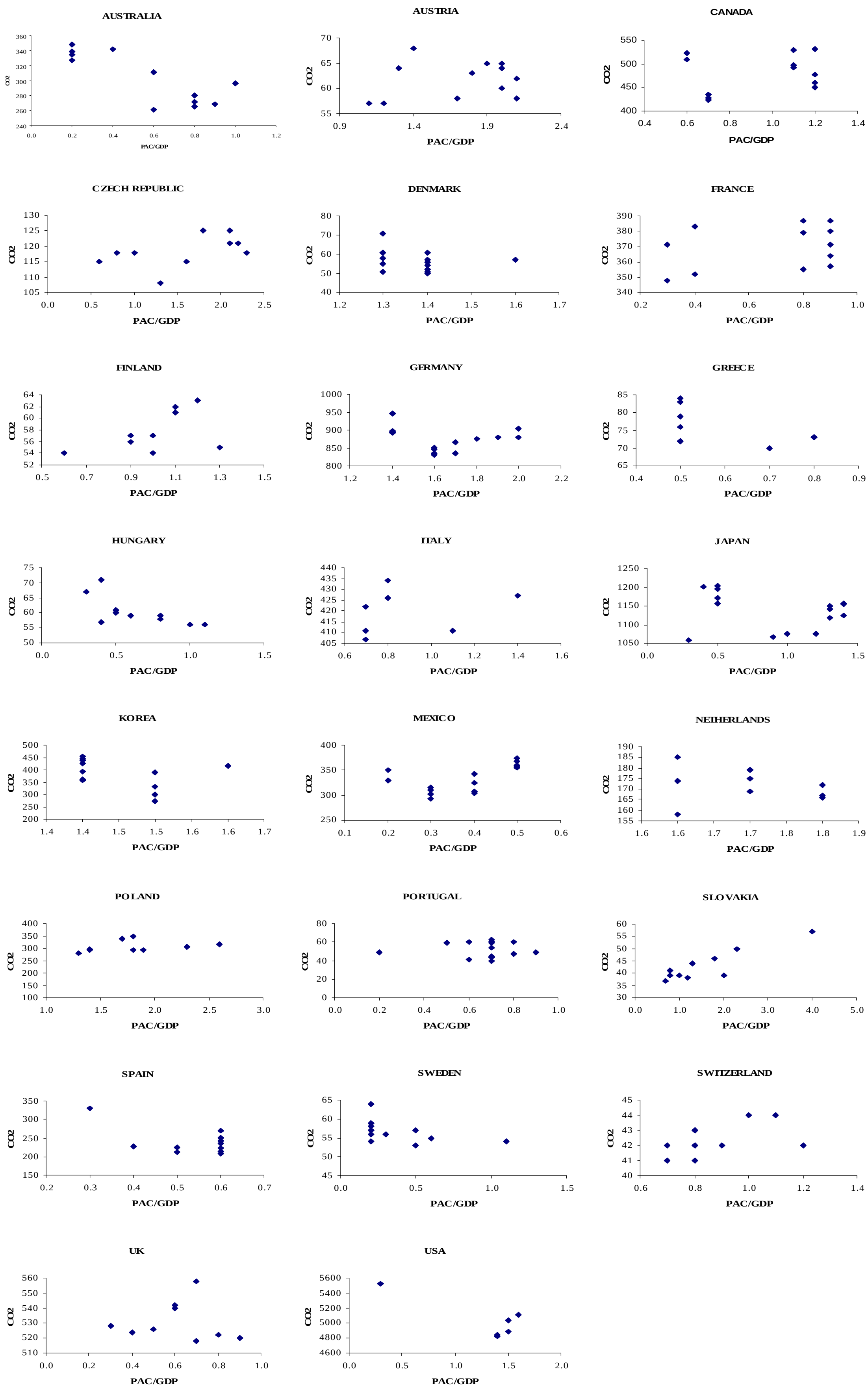

Figure 5: CO_2 vs PAC/GDP for OECD countries

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