

A MODEL WITH OPTIONS AND ORDER CANCELLATION
FOR IN-SEASON REPLENISHMENT

by

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ABSTRACT**A MODEL WITH OPTIONS AND ORDER
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We investigate a model with options and order cancellation in a buyer-supplier system for in-season replenishment. We analyse the model in two periods with correlated demand. We derive the appropriate prices that the supplier must choose for channel coordination, however individual rationality constraint of the supplier is violated. Finally, using a computational study, we quantify the value of options and order cancellation combination. We demonstrate that options and order cancellation combination provides flexibility to the buyer to respond to market changes thus increasing the total expected profit even more than the options-only model.

ÖZET

SEZON İÇİ SİPARİŞ İKMALİ İÇİN OPSİYONLU VE SİPARİŞ İPTALLİ BİR MODEL

Sezon içi sipariş ikmalı için, opsiyonlu ve sipariş iptalli bir model incelenmiştir. Model, bağımlı talepli iki dönemde analiz edilmiştir. Kanal koordinasyonu için tedarikçinin seçmesi gereken uygun fiyatlar çıkartılmış ancak tedarikçinin bireysel makulluk kısıtı bozulmuştur. Sonunda, hesapsal bir çalışmayla, opsiyon ve sipariş iptali birleşiminin değeri ölçülmüştür. Opsiyon ve sipariş iptali birleşiminin sağladığı esneklikle alıcının market değişimlerine yanıt verdiği ve toplam beklenen karın yalnız opsiyonlu modelden daha fazla arttığı gösterilmiştir.

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LIST OF SYMBOLS/ABBREVIATIONS

c_i	Unit wholesale price for orders to be delivered at the beginning of period i
c_e	Unit exercise price
c_o	Unit option price
c_p	Unit cancellation price
c_L	Unit cost of in-house production
c_r	Unit cost of raw material
γ	Additional unit cost of contractual agreement production in the expedited mode
h^s	Unit holding cost for finished goods at the supplier in the first period
h_i^b	Unit holding cost for finished goods at the buyer in period i
p_i	Unit shortage penalty cost incurred by the buyer in period i
r	Unit selling price of finished good
v_r^s	Unit salvage value of raw material for the supplier
v_f^s	Unit salvage value of finished goods for the supplier
v_f^b	Unit salvage value of finished goods for the buyer
I_2	On hand inventory before the order is updated at the beginning of the second period $X_2 - d_1$
I_c	On hand inventory at the end of the first period $X_1^c - d_1$ in CS
M	Number of options purchased at the beginning of the season
m^+	Number of options exercised at the beginning of the second period
m^-	Order quantity cancelled at the beginning of the second period
Q_i	Order quantity to be delivered at the beginning of period i
X_1	Q_1
X_2	$Q_1 + Q_2$

X_3	$Q_1 + Q_2 + M$
X_L	In-house production quantity
X_1^c	Shipment quantity to be delivered at the beginning of the first period in CS
X_2^c	Shipment quantity to be delivered at the beginning of the second period in CS
X_3^c	Raw material quantity purchased at beginning of the season in CS
X_L^c	In-house production quantity in CS
D_i	Demand variable for period i
d_i	Realized demand for period i
$F_{D_i}(\cdot)$	Distribution function of D_i
$f_{D_i}(\cdot)$	Density function of D_i
$\Phi(\cdot)$	Distribution function of standard normal
$\Phi^{-1}(\cdot)$	Inverse of standard normal distribution function
dd_i	$\mu_2 - \rho \frac{\sigma_2}{\sigma_1} + k_i \sigma_2 \sqrt{1 - \rho^2}$
μ_i	Mean of D_i
ν	$1 + \rho \frac{\sigma_2}{\sigma_1}$
ρ	Correlation coefficient of D_1 and D_2
σ_i	Standard deviation of D_i
CS	Centralized System
DS	Decentralized System
DSNO	Decentralized System with No Options
DSO	Decentralized System with Options
DSOC	Decentralized System with Options and Order Cancellation
VC	Value of coordination
VCC	Value of coordination for a system which uses option and order cancellation
VO	Value of option

VOC	Value of options and order cancellation combination
WTO	World Trade Organization

1. INTRODUCTION

In Turkey, the textile and apparel industry has been a very important driving force for the economy. Tan (2001) states that the textile and apparel industry accounts for 10.4 per cent of the GNP, 39 per cent of industrial output, 47.5 per cent of manufacturing output, 21 per cent of the total industrial employment, 10 per cent of the total employment, and 37.4 per cent of all the exports. Today, the textile and apparel industry has a major importance in Turkish economy.

The Turkish textile and apparel industry is currently facing a number of crucial challenges. The competitiveness of the industry is decreasing due to rising labor costs, inefficiencies, and the emergence of other low-cost suppliers in the Far East and Eastern Europe. On January 1, 2005, the quota system that had been limiting textile and apparel imports into the United States and other nations ended for all member countries of the World Trade Organization (WTO).

Abernathy *et al.* (2005) consider the impact of the removal of quota that had been limiting textile and apparel imports, in light of the new market forces, particularly changes in the relation of retail-apparel-textile supply chains. China Textile University (1999) has also prepared a report which expresses the potential development of the Chinese Apparel Industry.

The United State Trade Commission released its own report in January 2004 on the Assessment of the Competitiveness of Certain Foreign Suppliers to the U.S. Market concluding:

China is expected to become the “supplier of choice” for most U.S. importers (the large apparel companies and retailers) because of its ability to make almost any type of textile and apparel product at any quality level at a competitive price. Although many countries may see their share of the U.S. market decline, a large number of countries likely will become second-tier suppliers to U.S. apparel companies and retailers in niche goods and service (Abernathy *et al.* (2005)).

In addition to emergence of low-cost suppliers, Turkish textile and apparel industry has also other problems at the retailers' side. The changes that are occurring in the way that apparel is marketed to the consumer, have been considered by Oxborrow (2000). Because of the changes there are new pressures on retailers to find alternatives to their traditional strategy of cutting cost by finding the lowest cost suppliers.

The changes in the retailing industry and also in the customer demand are forcing the manufacturers in the supply chain to adopt new strategies to cope with the new environment. The retail apparel-textile channels are characterized by rapidly changing styles, uncertain customer demand, product proliferation, and long lead times.

In the apparel industry, the lead times from the retailer order to delivery are quite long because of the long procurement/production lead times. As a result of these long lead times, the risks of having too little or too much inventory increase if the retailers have to place the orders long before the season. Reducing these lead times has been the primary objective in the apparel industry. Moreover, delaying production and ordering decisions to incorporate revised demand forecasts in the production schedules reduces these risks (Fisher and Raman (1996)). As retailers become more concerned with the unpredictability of consumer demand, in-season replenishment is seen as a solution to reduce these risks.

With the emergence of low-cost suppliers, the industry has realized the need to compete with other features in addition to the cost. The only way to stay alive for experienced but high-cost suppliers is to reduce production lead times and become available for in-season replenishments. By this way they will help retailers to reduce their costs, due to unexpected demand fluctuations.

1.1. Literature Survey

For textile and apparel industry, new production policies and systems for decreasing lead times are investigated in literature. Solutions for demand volatility are also proposed such as subcontracting in single supplier systems and ordering flexibilities in

supplier-buyer systems. When ordering flexibilities are investigated for seasonal products particularly, models turn out to be in-season replenishment models with single or double periods instead of models with multiple periods.

New production philosophies for textile and apparel industry have been proposed such as ‘leagility’, which is a combination of lean and agile productions and has been examined by Bruce *et al.* (2004). Bouhia and Abernathy (2003) formulate production policies to maximize the performance of an apparel manufacturing system that replenishes basic items characterized by a flat average demand. The simulation-based model compares a number of production strategies and chooses an ordering and scheduling policy that increases the overall performance of the supply chain.

Decreasing lead times also became an important issue for textile and apparel research. Fisher and Raman (1996) examine a Quick Response system that aims to shorten lead times to allow a greater portion of production to be scheduled in response to initial demand. They solve a model, which has multiple products and capacity constraints with a lower bound approximation. They also apply this system for a major fashion skiwear firm and successfully reduce the costs.

Demand volatility has been a research area for textile and apparel industry, and subcontracting is proposed as a solution in single supplier systems. Tan (2002) considers a production system, which supplies products to meet a random demand that switches randomly between a high level and a low level. He investigates the strategy of increasing the production capacity temporarily through contractual agreements with short-cycle manufacturers. He determines the price of capacity options and investigates the effects of demand variability on capacity contracts. Tan and Gershwin (2004) try to find a feedback policy for a manufacturing firm that builds a product to stock to meet a random demand. A set of subcontractors, who have different costs and capacities, are available to supplement the firm’s production capacity. All two have modeled in continuous time. Arslan *et al.* (2001) consider analytical models for deciding when and how to expedite in a single-product make-to-order environment. They derive the structure of the optimal policy in both continuous and discrete time cases.

When we switch to supplier-buyer systems, ordering flexibilities are proposed against demand volatility, in literature. Bassok *et al.* (1997) examine a supplier-buyer system in which purchasing commitments for multiple periods are made at the beginning and the buyer has some flexibility for purchasing quantities that actually derive from the original commitments. In an environment with volatile demands for each period, the buyer has also flexibility to update following commitments as time passes. They develop a heuristic which is easy to implement and determines nearly optimal purchasing commitments and purchasing quantities.

When we focus on models which consider fashion goods with seasonal lives, there are buyer-supplier models which propose in-season replenishment against demand volatility. In literature, the season is investigated in two periods with correlated random demands or in one period with single demand realization and an early demand signal.

For two periods Eppen and Iyer (1997) present a backup agreement model. In this model, buyer states that he will purchase Q units of product totally. He purchases $Q(1 - \beta)$ of them for the first period at the price c per unit. After the first period demand is realized, he purchases $m < Q\beta$ of them for the second period at the price c per unit. He pays a penalty for each unit that he has not been purchased ($Q\beta - m$). Tsay and Lovejoy (1999) consider two period version of quantity flexibility contract. In this model the buyer states that he will purchase Q_1 units of product for the first period and Q_2 units of product for the second period. He purchases Q_1 units for the first period at the price c per unit. After the first period demand is realized he purchases a quantity which is at most $Q_2(1 + \alpha_u)$ and at least $Q_2(1 - \alpha_d)$ for the second period at the price c per unit. Barnes-Schuster *et al.* (2002) present a model with options in which buyer purchases Q_1 unit of products for the first period at the price c_1 per unit, Q_2 units of product for the second period at the price c_2 per unit and also he purchases M units of options at the price c_o per unit. After the first period demand is realized, buyer exercises $m < M$ options at the price c_e per unit, if he deems it is necessary.

In single period models in-season replenishment is done according to demand signal. In the model of Donohue (2000), while information is revealed in two stages,

demand is realized only once, and hence the model itself remains that of a single period. Her model is very similar to the model of Barnes-Schuster *et al.* (2002). Except for demand realization there is another difference between models. While Donohue (2000) limits her model to goods purchased entirely using options, Barnes-Schuster *et al.* (2002) allow both committed orders and options. Pay-to-Delay capacity reservation model which is proposed by Brown and Lee (1997) is also limited to a single period model in which demand is revealed in two stages but realized once. In this model the buyer makes total reservation z of which he is obligated to purchase at least $y < z$ units at a price c_f per unit, and he pays unit options cost of c_o for $z - y$ units. Additional units, up to $z - y$ can be exercised at a price c_e per unit. Additional units that exceeds the option quantity can be purchased at a unit cost of $c_p > c_o + c_e$.

1.2. Problem Definition

The aim of this thesis is to investigate in-season order replenishment by using options and order cancellation in a buyer-supplier system with two types of production: in-house production before the season starts and contractual agreement production with short-cycle manufacturers within the season.

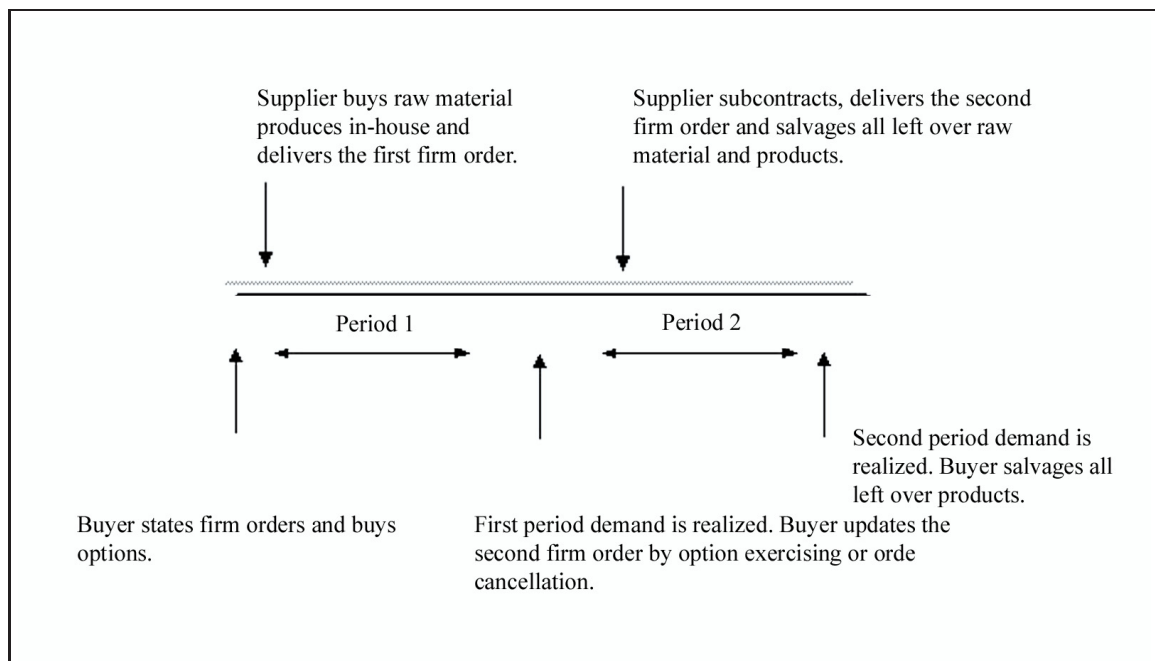


Figure 1.1. Time line for the model

At the beginning of the season the buyer states two firm orders for a single product, which can be sold only during that season because of rapidly changing styles. The buyer also buys options, which can be exercised later in the season if the demand turns out to be higher than expected.

The supplier buys enough raw material to produce both orders and options then she starts in-house production and delivers first firm order at the beginning of the season.

After the first period, the buyer updates the second firm order according to a demand signal received (the realized demand of the first period). The buyer can either exercise the options or can cancel some amount of the second firm order in order to update the second firm order.

After the buyer updates the second firm order, the supplier uses contractual agreement production if she needs and delivers the second firm order. Then she salvages all left-over raw materials and products. When the season ends, the buyer also salvages all left-over products. For the illustration please see Figure 1.1.

Table 1.1. Comparison of models with two periods

Models	Period 1	Period 2	Options	Cancellation	Prices				
	Quantity	Quantity	Quantity	Quantity					
Our Model	Q_1	Q_2	M	Q_2	c_1	c_2	c_o	c_e	c_p
B. Agreement	$Q(1 - \beta)$	$Q\beta$	-	$Q\beta$	c	c	-	-	b
Q. F. Contract	Q_1	Q_2	$Q_2\alpha_u$	$Q_2\alpha_d$	c	c	0	c	0
Options-only Model	Q_1	Q_2	M	-	c_1	c_2	c_o	c_e	-

We can see the correspondence between our model parameters and other models for in-season replenishment with two periods in Table 1.1. Our model is an extension of the (options-only) model proposed by Barnes-Schuster *et al.* (2002). Our model includes order cancellation in addition to options in order to have a more flexible structure. Backup agreement model corresponds to the order cancellation part of our model. Therefore we can say our model is a combination of the options-only model and

the backup agreement model. Quantity flexibility contract which has also both options and order cancellation has limitations according to our model. In our model, options quantity that can be purchased is M , however in quantity flexibility contract model it is limited to $Q_2\alpha_u$. In addition to this, in our model, cancellation can be up to Q_2 units however in quantity flexibility contract model it is limited to $Q_2\alpha_d$. Moreover, it has restrictions on the option, exercise and cancellation prices. All three models are the restricted version of our model. Therefore the performance of the supplier will be no worse with the model we propose.

2. MODEL WITH OPTIONS AND ORDER CANCELLATION

In our model there is a single buyer and a single supplier, as in Barnes-Schuster *et al.* (2002), and the product has a short life (one season). As there is an in-season replenishment, the season is investigated in two most probably unequal periods with correlated demands. Demand random variables are denoted as D_1 and D_2 . Realized demands are denoted as d_1 and d_2 . The demand random variables D_1 and D_2 are assumed to be normally distributed with means μ_1 and μ_2 and standard deviations σ_1 and σ_2 . Demand correlation is represented as ρ . Density functions are represented as $f_{D_1}(\cdot)$ and $f_{D_2}(\cdot)$ and distribution functions are represented as $F_{D_1}(\cdot)$ and $F_{D_2}(\cdot)$.

At the beginning of the season, the buyer makes his ordering decisions. He orders Q_1 units for the first period to be delivered at the beginning of the period. For the second period he orders Q_2 units to be delivered at the beginning of the period and purchases M units of options. With the help this M units of options, after the demand of the first period (d_1) is realized, he can update the second period order. One unit of option gives the buyer the right to buy one unit of good. He can exercise m units of options ($0 < m^+ \leq M$) and increase the second period order. Or he can cancel m units of second order ($-Q_2 \leq m^- < 0$) and decrease the second period order. This is where we differ from Barnes-Schuster *et al.* (2002). The fixed unit selling price is r for the buyer in the season. The unit holding costs of the buyer are h_1^b in the first period and h_2^b in the second period. The unit holding costs are allowed to be different in the two periods because there is no guarantee that the periods are in equal length. The unit shortage penalty costs are p_1 for the first period and p_2 for the second period. The unit shortage penalty costs are also allowed to be different for the periods. The difference in unit shortage penalty costs is due to loss of goodwill for backlogging of excess demand in the first period and for lost sales in the second period. Left-over finished goods at the end of the second period or in other words at the end of the season are salvaged at a per unit price of v_f^b . Unit salvage value v_f^b is an effective value which is discounted

with unit holding cost h_2^b .

At the beginning of the season the supplier announces unit wholesale prices c_1 and c_2 for the two periods, according to which the buyer will be charged for his order quantities of Q_1 and Q_2 , respectively. She also announces unit option price c_o , unit option exercise price c_e , and unit order cancellation price c_p . At c_o per unit, the buyer will buy M units of options. At c_e per unit, the buyer will exercise m^+ options. At c_p per unit, the buyer will decrease his second order quantity by m^- units. Owing to the long acquisition lead time for the raw material in apparel industry, the supplier has to purchase all sufficient raw materials for the maximal total quantity possibly requested which is $Q_1 + Q_2 + M$. Unit purchasing cost for raw material is denoted as c_r . There are two modes of production: First one is in-house production which is done before the first period after the buyer states firm orders and buys options. In this production the cost is c_L per unit. The second one is contractual agreement production which is done after the second period order is updated. In this production the cost is $c_L(1 + \gamma)$ per unit. The expensiveness of the second production mode can also be according to overtime or unscheduled in-house production. All three possibilities can be represented with an exogenous parameter γ in the model and it makes no difference in representation. Because the supplier has to produce the required quantity Q_1 for the first period, she schedules her in-house production which is the cheaper production mode, before the first period. In this production she can not only produce the first period order quantity Q_1 but also produce some units of the required quantity Q_2 for the second period. This extra production can fulfill the second period order quantity Q_2 in anticipation that no order cancellation will occur or can exceed the second period order quantity Q_2 in anticipation that options will be exercised. The in-house production quantity will be represented as X_L which is a decision variable. Every unit that will wait till the second period will incur a holding cost which is h^s per unit. In the second period after the buyer exercises some options m^+ or cancels some quantities m^- from the second period order, if there is still need for production, the supplier makes contractual agreement for production in other words; uses the expensive production mode. It is assumed that for both in-house production and contractual agreement production, there is no capacity restriction. After production is finished and the buyer's requests are fulfilled, left-

over raw material is salvaged for v_r^s per unit and left-over finished goods are salvaged for v_f^s per unit. In our model we do not account for the holding cost for the raw material. The reason for this is the fact that the cost mainly depends on how the raw materials are delivered. If the raw materials are delivered just in time for both modes of productions, there is no cost incurred by holding raw materials. As we mentioned before, the supplier purchases all sufficient raw material for the maximal total quantity possibly requested and there is no capacity restriction. Accordingly, there will not be any problem in fulfilling the buyer's requests and there will be no related cost.

2.1. Channel Structure

Our model has a decentralized system, and the buyer and the supplier play a Stackelberg game in which the supplier is the leader and the buyer is the follower. In this game, for the announced set of prices, the buyer determines the order quantities and the number of options purchased before the first period, and he updates the second period order after the first period demand is realized. While making these decisions, the buyer aims to maximize his profit. As we assume common knowledge for information structure, the supplier determines the optimal prices and production quantities in the way that maximize her profit.

We will denote our model as the Decentralized System with Options and Order Cancellation (DSOC) throughout the thesis.

2.2. Assumptions

- Demands are assumed to be normally distributed with means μ_1 and μ_2 and standard deviations with σ_1 and σ_2 .
- Demands are assumed to be correlated to each other with a correlation coefficient ρ which is assumed to be less than one. By this assumption we prevent our model from becoming a single period problem.
- Demand distributions in the market, selling price (r), salvage value for the buyer (v_f^b) of the product, and cost parameters of the buyer (p_1, p_2, h_1^b) which determine

the buyer's decisions are assumed to be known also by the supplier.

- The excess finished goods which are at the supplier side are salvaged by the supplier and which are at the buyer side are salvaged by the buyer, regardless of the salvage values. This is important for fair comparison of decentralized system with the centralized system.
- In the case of high demand realization, the supplier is obligated to supply goods against options purchased and in the case of low demand realization, she is obligated to admit the order cancellation for the second period.
- All goods, including possible exercised options, are delivered on time.
- Capacity in both production modes, in-house production and contractual agreement production are sufficient to fulfill the buyer's requirements.
- Raw materials have a long acquisition lead time and therefore it needs to be processed before the start of the season.
- Per unit holding cost of the finished goods for the supplier (h_s) is assumed to be less than the difference between contractual agreement production cost and in-house production cost (γc_L). By this assumption we ensure that two modes of production will occur.
- Per unit holding cost of the finished goods for the supplier (h^s) is assumed to be less than the cost for the buyer h_1^b . This is important for fair comparison of decentralized system with the centralized system.
- Unit salvage value of the raw material (v_r^s) is assumed to be less than the unit cost of the raw material (c_r). By this assumption we ensure that the profit in centralized system and the supplier's profit in decentralized system will be finite.
- Unit salvage value of the finished goods for the buyer (v_f^b) and unit salvage value of the finished goods for the supplier (v_f^s) are assumed to be less than the sum of raw material cost (c_r), in-house production cost (c_L) and per unit holding cost (h^s). By this assumption we ensure that the expected profit in centralized system and the expected profit of supplier in decentralized system will be finite.

3. ANALYSIS

In the analysis we first analyse our model (DSOC). In order to use in our model evaluation, we analyse centralized system and two more decentralized models. We compare our model with the centralized system and we examine the issue of channel coordination. We leave the comparison of our model with the other decentralized models to computational study (Chapter 4).

In the Centralized System (CS) there is one decision maker and he decides to maximize the total system profit. We call this profit the first best solution and it is the maximum profit that can be reached by our decentralized model. In this system the wholesale (c_1, c_2) , option (c_o) , exercise (c_e) and cancellation prices (c_p) do not affect the system profit, because they determine only the transition costs between the supplier and the buyer. The only decisions that affect the system profit are the quantity of raw material that is purchased, the shipment quantities for each period and production quantities under two modes, in-house and contractual agreement productions.

The first decentralized model is the one where there are no options to purchase or exercise and there can not be any order cancellation for the second period. We will call this system the Decentralized System with No Options (DSNO). The second decentralized model is the one where there are options to purchase and exercise but there can not be any order cancellation for the second period. It is the model that has been proposed by Barnes-Schuster *et al.* (2002). We will call this model the Decentralized System with Options (DSO).

3.1. Decentralized System with Options and Order Cancellation (DSOC)

3.1.1. Supplier's Problem

We define X_1 as the first period order quantity (Q_1). X_2 represents the sum of order quantities for both periods ($Q_1 + Q_2$), and I_2 represents an imaginary on-hand

inventory at the beginning of the second period ($X_2 - d_1$) before any option is exercised or any order cancellation is done. X_3 represents the quantity of raw materials that are purchased at the beginning of the season which is equal to the total of order quantities for both periods and the number of options that are purchased ($Q_1 + Q_2 + M$). We use X_1 , X_2 and X_3 instead of Q_1 , Q_2 and M because by this way, buyer's problem will be separated into three independent problems and this will make solution possible. X_L stands for the quantity of in-house production which has been made before the season starts. m^+ stands for the options that are exercised at the beginning of the second period by the buyer, while m^- stands for the quantity that is cancelled from the second period order quantity.

The expected profit function of the supplier can be written as:

$$\begin{aligned} \Pi_{d_{soc}}^S(X_L, c_1, c_2, c_o, c_e, c_p) = & E_{D_1}[c_1X_1 + c_2(X_2 - X_1) + c_o(X_3 - X_2) + c_em^+ \\ & + (c_2 - c_p)m^- - h_s(X_L - X_1) + v_r^s(X_3 - X_2 - \max(X_L - X_2, m)) \\ & + v_f^s(X_L - X_2 - m)^+ - c_rX_3 - c_LX_L - c_L(1 + \gamma)(m - (X_L - X_2))^+] \end{aligned}$$

The first term represents the revenue received from the transfer payment of the first period order. The second term represents the revenue received from the transfer payment of the second period order. The third term represents the revenue received from the sales of options. The fourth term represents the revenue received from the exercised options, if any. The fifth term represents the total of the revenue received from the order cancellation and the payback for cancelled quantity. The sixth term represents the holding cost of finished goods. The seventh term represents the revenue received from salvaging raw materials. If m is larger than $X_L - X_2$, it means that the buyer's total request ($X_2 + m$) will be reached with contractual agreement production by the supplier. Total use of raw material will be $X_2 + m$ and the left-over raw materials amount will be $X_3 - X_2 - m$. If m is less than $X_L - X_2$, it means that the buyer's total request ($X_2 + m$) has been exceeded with in-house production by the supplier. Total use of raw material is X_L and the left-over raw material amount is $X_3 - X_L$. The eighth term represents the revenue received from salvaging the finished goods. If m is

larger than $X_L - X_2$, it means that the buyer's total request ($X_2 + m$) will be reached with contractual agreement production by the supplier and there will be no left-over finished goods. If the m is less than $X_L - X_2$, it means that the buyer's total request ($X_2 + m$) has been exceeded with in-house production by the supplier, and the left-over finished goods amount is $X_L - X_2 - m$. The ninth term represents the total cost of raw materials. The tenth term represents the in-house production cost. The eleventh term represents the contractual agreement production cost. If m is larger than $X_L - X_2$, it means that the buyer's total request ($X_2 + m$) will be reached with contractual agreement production by the supplier, and contractual agreement production amount will be $X_2 + m - X_L$. If m is less than $X_L - X_2$, it means that the buyer's total request ($X_2 + m$) has been exceeded with in-house production by the supplier, and there will be no contractual agreement production.

After the supplier announces the wholesale (c_1, c_2), option (c_o), exercise (c_e), and cancellation (c_p) prices in the way that maximizes her profit, the buyer determines the order quantities (Q_1, Q_2) and the options quantity (M). In other words, X_1, X_2 , and X_3 are determined. We know that X_1, X_2 , and X_3 are consequences of the prices. Therefore total profit of the supplier depends on the prices and in-house production amount.

In order to optimize the profit, for a given set of prices, the supplier solves the following problem:

$$\max_{X_L} \Pi_{dsoc}^S(X_L | c_1, c_2, c_o, c_e, c_p) \quad \text{s.t.} \quad X_1 \leq X_L \leq X_3 \quad (3.1)$$

Proposition 3.1 For a given set of prices $\Pi_{dsoc}^S(X_L | c_1, c_2, c_o, c_e, c_p)$ is concave in X_L .

For the proof of Proposition 3.1, please refer to Section A.2 in the Appendix.

Depending on the relationship between c_e and $(c_2 - c_p)$ the expected m according to the first period demand, differs. In addition, some terms in the $\Pi_{dsoc}^S(X_L | c_1, c_2, c_o, c_e, c_p)$

are dependent on the relationship between X_L and X_2 . Therefore, we need to study first order conditions of four different cases.

For the case $c_e \geq (c_2 - c_p)$; if $X_2 < X_L < X_3$, the optimal in-house production quantity should satisfy the following equation:

$$F_{D_1}\left(\frac{X_L - dd_1}{\nu}\right) = \frac{(-h_s + c_L\gamma)}{(v_r^s + c_L(1 + \gamma) - v_f^s)} \quad (3.2)$$

where $\nu = 1 + \rho\frac{\sigma_2}{\sigma_1}$; $dd_1 = \mu_2 - \rho\frac{\sigma_2}{\sigma_1} + k_1\sigma_2\sqrt{1 - \rho^2}$; $k_1 = \Phi^{-1}\left(\frac{p_2+r-c_e}{p_2+r-v_f^b}\right)$.

For the case $c_e \geq (c_2 - c_p)$; if $X_1 < X_L < X_2$, the optimal in-house production quantity should satisfy the following equation:

$$F_{D_1}\left(\frac{X_L - dd_2}{\nu}\right) = \frac{(-h_s + c_L\gamma)}{(v_r^s + c_L(1 + \gamma) - v_f^s)} \quad (3.3)$$

where $dd_2 = \mu_2 - \rho\frac{\sigma_2}{\sigma_1} + k_2\sigma_2\sqrt{1 - \rho^2}$; $k_2 = \Phi^{-1}\left(\frac{p_2+r-(c_2-c_p)}{p_2+r-v_f^b}\right)$.

For the case $c_e < (c_2 - c_p)$; the buyer equates X_2 either to X_1 or X_3 , in order to maximize his total profit. We will comment on this later. For the case $c_e < (c_2 - c_p)$; if $X_2 = X_3$, the optimal in-house production quantity should satisfy the following equation:

$$F_{D_1}\left(\frac{X_L - dd_2}{\nu}\right) = \frac{(-h_s + c_L\gamma)}{(v_r^s + c_L(1 + \gamma) - v_f^s)} \quad (3.4)$$

For the case $c_e < (c_2 - c_p)$; if $X_2 = X_1$, the optimal in-house production quantity should satisfy the following equation:

$$F_{D_1}\left(\frac{X_L - dd_1}{\nu}\right) = \frac{(-h_s + c_L\gamma)}{(v_r^s + c_L(1 + \gamma) - v_f^s)} \quad (3.5)$$

For all the four cases, we have two different sets of first order conditions from which we can derive two different unconstrained X_L . We attribute them as uncon-

strained because they are the solution for the supplier's problem if they satisfy the boundary conditions.

Proposition 3.2 The first and the second unconstrained production quantities (X_L^1 and X_L^2) for a given set of prices can be written as:

$$\begin{aligned} X_L^1 &= dd_1 + \nu(\mu_1 + k_L\sigma_1) \\ X_L^2 &= dd_2 + \nu(\mu_1 + k_L\sigma_1) \end{aligned}$$

where $k_L = \Phi^{-1}\left(\frac{-h_s + c_L\gamma}{v_f^s + c_L(1+\gamma) - v_f^s}\right)$; $\nu = 1 + \rho\frac{\sigma_2}{\sigma_1}$; $dd_1 = \mu_2 - \rho\frac{\sigma_2}{\sigma_1} + k_1\sigma_2\sqrt{1 - \rho^2}$; $k_1 = \Phi^{-1}\left(\frac{p_2 + r - c_e}{p_2 + r - v_f^b}\right)$; $dd_2 = \mu_2 - \rho\frac{\sigma_2}{\sigma_1} + k_2\sigma_2\sqrt{1 - \rho^2}$; $k_2 = \Phi^{-1}\left(\frac{p_2 + r - (c_2 - c_p)}{p_2 + r - v_f^b}\right)$.

- For the case $c_e \geq (c_2 - c_p)$; $X_L^* = \min(\max(X_1, X_L^2), \max(X_2, X_L^1), X_3)$.
- For the case $c_e < (c_2 - c_p)$, and $X_2 = X_1$; $X_L^* = \max(X_2, \min(X_3, X_L^1))$.
- For the case $c_e < (c_2 - c_p)$, and $X_2 = X_3$; $X_L^* = \max(X_1, \min(X_2, X_L^2))$.

For the proof of Proposition 3.2, please refer to Section A.1 in the Appendix.

3.1.2. Buyer's Problem

After the supplier announces the wholesale (c_1, c_2) , option (c_o) , exercise (c_e) , and cancellation (c_p) prices, the buyer determines the order quantities (Q_1, Q_2) and the options quantity (M) . In other words X_1, X_2 and X_3 are determined. The expected profit of the buyer for the first period can be written as:

$$\begin{aligned} \Pi_{dsoc}^{1,B}(X_1, X_2, X_3) &= E_{D_1}[r \min(D_1, X_1) - c_o(X_3 - X_2) - c_1X_1 - c_2(X_2 - X_1) \\ &\quad - p_1(D_1 - X_1)^+ - h_1^b(X_1 - D_1)^+] \end{aligned}$$

The first term represents the revenue received from the sales in the first period. The second term represents the purchase cost of the options. The third term represents the purchase cost of the first period order. The fourth term represents the purchase

cost of the second period order. The fifth term represents the shortage penalty cost incurred in the first period. The sixth term represents the holding cost of finished goods.

The expected profit function of the buyer for the second period depends on the on-hand inventory at the beginning of the second period ($I_2 + m$) and exercised options or cancelled order quantity (m) for the the second period. It can be written as:

$$\begin{aligned} \Pi_{dsoc}^{2,B}(m, I_2) = & E_{D_2}[r \min(D_2, (I_2 + m)^+) + r \min((d_1 - X_1)^+, X_2 - X_1 + m) \\ & - c_e m^+ - (c_2 - c_p)m^- - p_2(D_2 - I_2 - m)^+ + v_f^b(I_2 + m - D_2)^+] \end{aligned}$$

The first term represents the revenue received from the sales of the second period. We have defined I_2 as an imaginary variable which would be on-hand inventory at the beginning of the second period if there were no order updating. Accordingly $I_2 + m$ will be the real on-hand inventory at the beginning of the second period. The second term represents the backorders satisfied in the second period. $X_2 + m - X_1$ stands for the amount of shipment for the second period. The third term represents the cost paid for exercised options, if any. The fourth term represents the sum of the cost paid for the order cancellation and the payback for cancelled quantity. The fifth term represents the shortage penalty cost incurred in the second period. The sixth term represents the revenue received from salvaging finished goods.

For a given I_2 , in order to optimize the profit in the second period, the buyer solves the following problem:

$$\max_m \Pi_{dsoc}^{2,B}(m|I_2) \quad \text{s.t.} \quad (X_1 - X_2) \leq m \leq (X_3 - X_2) \quad (3.6)$$

Proposition 3.3 For a given I_2 , $\Pi_{dsoc}^{2,B}(m|I_2)$ is concave in m .

For the proof of Proposition 3.3, please refer to Section A.4 in the Appendix.

The second period expected profit function of the buyer ($\Pi_{dsoc}^{2,B}(m, I_2)$) depends

on whether m represents order cancellation or exercised options. For each case we have a different first order condition to satisfy. If $0 < m < (X_3 - X_2)$, the exercised options quantity should satisfy the following equation:

$$F_{D_2|d_1}(I_2 + m) = \frac{(p_2 + r - c_e)}{(p_2 + r - v_f^b)} \quad (3.7)$$

If $(X_1 - X_2) < m < 0$, the cancelled order quantity should satisfy the following equation:

$$F_{D_2|d_1}(I_2 + m) = \frac{(p_2 + r - (c_2 - c_p))}{(p_2 + r - v_f^b)} \quad (3.8)$$

As illustrated in Figure 3.1 and Figure 3.2, we find two unconstrained m_u from the above equations (m_u^+ ; m_u^-). The first unconstrained m_u (m_u^+), can be the solution to the buyer's problem for d_1 values which are greater than $\frac{X_2 - dd_1}{\nu}$. The second unconstrained m_u (m_u^-), can be the solution to the buyer's problem for d_1 values which are smaller than $\frac{X_2 - dd_2}{\nu}$. The first unconstrained m_u can be written as $dd_1 + \nu d_1 - X_2$. The second unconstrained m_u can be written as $dd_2 + \nu d_1 - X_2$. Where $\nu = 1 + \rho \frac{\sigma_2}{\sigma_1}$; $dd_1 = \mu_2 - \rho \frac{\sigma_2}{\sigma_1} + k_1 \sigma_2 \sqrt{1 - \rho^2}$; $k_1 = \Phi^{-1}(\frac{p_2 + r - c_e}{p_2 + r - v_f^b})$; $dd_2 = \mu_2 - \rho \frac{\sigma_2}{\sigma_1} + k_2 \sigma_2 \sqrt{1 - \rho^2}$; $k_2 = \Phi^{-1}(\frac{p_2 + r - (c_2 - c_p)}{p_2 + r - v_f^b})$.

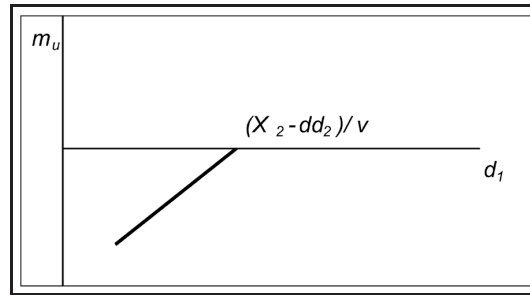
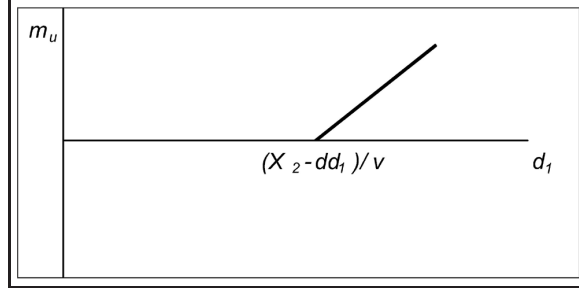


Figure 3.1. m_u^- related to d_1

For the case $c_e \geq (c_2 - c_p)$; at most one of the m_u becomes active related to the realized first period demand. It is because $\frac{X_2 - dd_2}{\nu}$ is smaller than $\frac{X_2 - dd_1}{\nu}$ for the case $c_e \geq (c_2 - c_p)$. As we see from Figure 3.3, for d_1 values less than $\frac{X_1 - dd_2}{\nu}$, m gets the value $X_1 - X_2$ which is the lower limit for it. For d_1 values between $\frac{X_1 - dd_2}{\nu}$ and $\frac{X_2 - dd_2}{\nu}$, m_u^- becomes the solution of the buyer's problem. For d_1 values between $\frac{X_2 - dd_2}{\nu}$

Figure 3.2. m_u^+ related to d_1

and $\frac{X_2 - dd_1}{\nu}$, neither one of the m_u becomes active and m has zero value. For d_1 values between $\frac{X_2 - dd_1}{\nu}$ and $\frac{X_3 - dd_1}{\nu}$, this time, m_u^+ becomes the solution of the buyer's problem. Lastly for d_1 values greater than $\frac{X_3 - dd_1}{\nu}$, m gets the value $X_3 - X_2$ which is the upper limit for it.

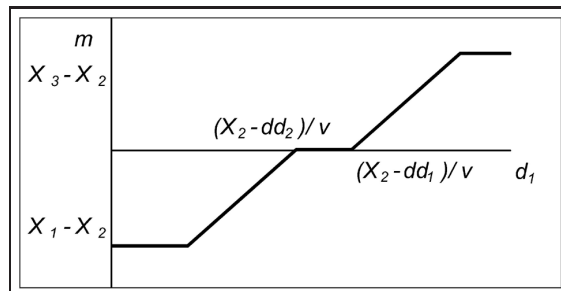
Proposition 3.4 The first and the second unconstrained m_u can be written as:

$$\begin{aligned} m_u^+ &= dd_1 + \nu d_1 - X_2 \\ m_u^- &= dd_2 + \nu d_1 - X_2 \end{aligned}$$

where $\nu = 1 + \rho \frac{\sigma_2}{\sigma_1}$; $dd_1 = \mu_2 - \rho \frac{\sigma_2}{\sigma_1} + k_1 \sigma_2 \sqrt{1 - \rho^2}$; $k_1 = \Phi^{-1}\left(\frac{p_2 + r - c_e}{p_2 + r - v_f^b}\right)$; $dd_2 = \mu_2 - \rho \frac{\sigma_2}{\sigma_1} + k_2 \sigma_2 \sqrt{1 - \rho^2}$; $k_2 = \Phi^{-1}\left(\frac{p_2 + r - (c_2 - c_p)}{p_2 + r - v_f^b}\right)$.

For the case $c_e \geq (c_2 - c_p)$; $m^* = \min(\max(X_1 - X_2, m_u^-), \max(0, m_u^+), X_3 - X_2)$.

For the proof of Proposition 3.4, please refer to Section A.3 in the Appendix and for the illustration please see Figure 3.3.

Figure 3.3. m related to d_1 for the case $c_e \geq (c_2 - c_p)$.

But for the case $c_e < (c_2 - c_p)$ it seems there will be two active m_u for the realized first period demand values (d_1) which are between $\frac{X_2 - dd_1}{\nu}$ and $\frac{X_2 - dd_2}{\nu}$. We can see the illustration in Figure 3.4.

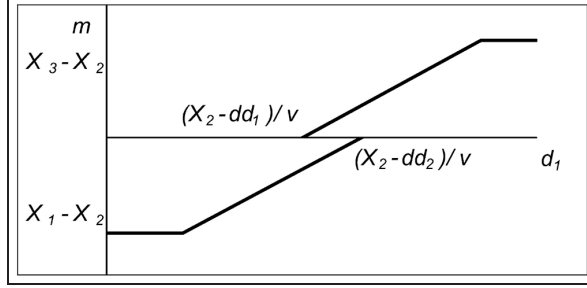


Figure 3.4. m related to d_1 for the case $c_e < (c_2 - c_p)$.

Let us define a value d_1^* which is between $\frac{X_2 - dd_1}{\nu}$ and $\frac{X_2 - dd_2}{\nu}$. It will be the value of d_1 where $\Pi_{dsoc}^{2,B}(m_u^-, I_2) = \Pi_{dsoc}^{2,B}(m_u^+, I_2)$. For $d_1 < d_1^*$, we will have $\Pi_{dsoc}^{2,B}(m_u^-, I_2) > \Pi_{dsoc}^{2,B}(m_u^+, I_2)$ and we will choose m_u^- for m^* . For $d_1 > d_1^*$, we will have $\Pi_{dsoc}^{2,B}(m_u^+, I_2) > \Pi_{dsoc}^{2,B}(m_u^-, I_2)$ and we will choose m_u^+ for m^* .

When we use the equation ' $\Pi_{dsoc}^{2,B}(m_u^-, X_2 - d_1^*) = \Pi_{dsoc}^{2,B}(m_u^+, X_2 - d_1^*)$ ' to determine the relationship between X_2 and d_1^* . We find that:

$$\frac{\partial X_2}{\partial d_1^*} = \frac{1}{\nu} \quad (3.9)$$

We can see this relationship also from the boundary conditions. For the derivation please refer to Section A.5 in the Appendix. This relationship plays a crucial role in the determination of the X_2 value for the case $c_e < (c_2 - c_p)$. In order to figure out the situation, let us have a look at the first and the second order conditions of the buyer's total expected profit.

If we write total expected profit of the buyer as:

$$\Pi_{dsoc}^B = \Pi_{dsoc}^{1,B}(X_1, X_2, X_3) + E_{I_2}[\Pi_{dsoc}^{2,B}(m, I_2)] \quad (3.10)$$

The first derivative of the profit according to X_2 for the case $c_e < (c_2 - c_p)$ will

be expressed as follows:

$$\frac{\partial \Pi_{dsoc}^B(X_1, X_2, X_3)}{\partial X_2} = (c_2 - c_p - c_e)F_{D_1}(d_1^*) + c_e + c_o - c_2 \quad (3.11)$$

Let us comment on the optimum value of X_2 , from this first order condition. We know that $c_e < (c_2 - c_p)$ because of the case we study. We see that, for $c_e + c_o > c_2$, first order condition will always be positive for any d_1^* . This means that X_2 will take the maximum value it can take which is X_3^* . We also see that for $c_e + c_o < c_2$, if $c_p > c_o$ first order condition will always be negative for any d_1^* . This means that X_2 will take the minimum value it can take which is X_1^* .

The second derivative of the profit according to X_2 will be expressed as follows:

$$\frac{\partial^2 \Pi_{dsoc}^B(X_1, X_2, X_3)}{\partial X_2^2} = \frac{f_{D_1}(d_1^*)}{\nu} (c_2 - c_p - c_e)$$

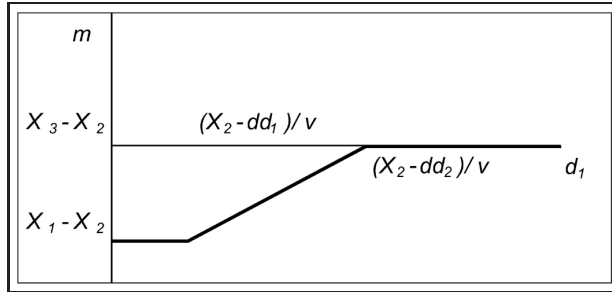


Figure 3.5. m related to d_1 for the case $c_e < (c_2 - c_p)$ if $X_2^* = X_3^*$.

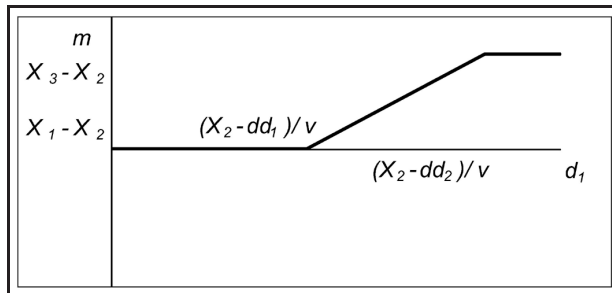


Figure 3.6. m related to d_1 for the case $c_e < (c_2 - c_p)$ if $X_2^* = X_1^*$.

We find out that total profit of the buyer is convex in X_2 which means that X_2

increases the optimum value either by increasing or decreasing. In other words, in the optimum solution X_2^* is either equal to X_1^* or X_3^* . For $c_e < (c_2 - c_p)$ the buyer turns the system out to be a position in which he can use either option (m_u^+) or order cancellation (m_u^-). Therefore he does not have a trade-off at the beginning of the second period because he has left only one chance for himself with his decision about X_2 at the beginning of the season. The resulted situations are illustrated in Figure 3.5 and Figure 3.6.

Proposition 3.5 The first and the second unconstrained m_u can be written as:

$$\begin{aligned} m_u^+ &= dd_1 + \nu d_1 - X_2 \\ m_u^- &= dd_2 + \nu d_1 - X_2 \end{aligned}$$

For the case $c_e < (c_2 - c_p)$;

- $m^* = \min(\max(X_1 - X_2, m_u^-), 0)$ if $X_2 = X_3$,
- $m^* = \max(\min(X_3 - X_2, m_u^+), 0)$ if $X_2 = X_1$.

For the proof of Proposition 3.5, please refer to Section A.5 in the Appendix.

As we have mentioned, total expected profit of the buyer can be written as:

$$\Pi_{dsoc}^B(X_1, X_2, X_3) = \Pi_{dsoc}^{1,B}(X_1, X_2, X_3) + E_{I_2}[\Pi_{dsoc}^{2,B}(m, I_2)]$$

For the case $c_e \geq (c_2 - c_p)$ we can rewrite it as:

$$\begin{aligned} \Pi_{dsoc}^{1,B}(X_1, X_2, X_3) + E_{I_2}[\Pi_{dsoc}^{2,B}(m, I_2)] &= J_1^b(X_1) + J_2^b(X_2) + J_3^b(X_3) \\ \text{where} \quad &0 \leq X_1 \leq X_2 \leq X_3 \end{aligned} \quad (3.12)$$

For the case $c_e < (c_2 - c_p)$, if the X_2^* is equal to X_1^* , we can rewrite the total

expected profit as:

$$\begin{aligned}\Pi_{dsoc}^B(X_1, X_3) &= J_1^b(X_1) + J_3^b(X_3) \\ \text{where} \quad &0 \leq X_1 \leq X_3\end{aligned}\tag{3.13}$$

For the case $c_e < (c_2 - c_p)$, if the X_2^* is equal to X_3^* , we can rewrite the total expected profit as:

$$\begin{aligned}\Pi_{dsoc}^B(X_1, X_3) &= J_1^b(X_1) + J_3^b(X_3) \\ \text{where} \quad &0 \leq X_1 \leq X_3\end{aligned}\tag{3.14}$$

Proposition 3.6 For the case $c_e \geq (c_2 - c_p)$; as $J_1^b(X_1)$, $J_2^b(X_2)$, and $J_3^b(X_3)$ are concave, $\Pi_{dsoc}^B(X_1, X_2, X_3)$ is jointly concave.

For the case $c_e < (c_2 - c_p)$, and the X_2^* is equal to X_1^* ; as $J_1^b(X_1)$ and $J_3^b(X_3)$ are concave, $\Pi_{dsoc}^B(X_1, X_3)$ is jointly concave.

For the case $c_e < (c_2 - c_p)$, if the X_2^* is equal to X_3^* ; as $J_1^b(X_1)$ and $J_3^b(X_3)$ are concave, $\Pi_{dsoc}^B(X_1, X_3)$ is jointly concave.

For the proof of Proposition 3.6, please refer to Section A.6 in the Appendix.

For the case $c_e < (c_2 - c_p)$, the buyer will equate X_2^* either to X_1^* or X_3^* . We have figured out that for $c_e + c_o > c_2$, the buyer will have $X_2^*=X_3^*$ and for $c_e + c_o < c_2$ and $c_p > c_o$, the buyer will have $X_2^*=X_1^*$. In other words, for the case where the marginal revenue of cancelling one unit for the second period ($c_2 - c_p$) is greater than the marginal cost of one extra unit for the second period (c_e), the buyer does not expect any gain from options if it is not cheaper than ordering ($c_e + c_o > c_2$). Also for this case, if using options is cheaper than ordering ($c_e + c_o < c_2$) and cancellation penalty is high ($c_p > c_o$), the buyer thinks there is no need to state an order that can be cancelled.

3.2. Centralized System (CS)

In the centralized system there is one decision maker who optimizes the total system profit. In this system, the wholesale (c_1, c_2) , option (c_o) , exercise (c_e) and cancellation prices (c_p) do not affect the system profit, as they determine only the transfer payment between the supplier and the buyer. The only decisions that affect the system profit are the quantity of raw material that are purchased, the shipment quantities for each period and production quantities under two modes which are in-house and contractual agreement production.

Let us define shipment quantities for both periods as X_1^c and X_2^c . Accordingly, our on-hand inventory at the end of the first period will be $I_c = X_1^c - d_1$. Let us also name the quantity of raw materials that are purchased at the beginning of the season as X_3^c and the quantity of in-house production which has been made before the season starts as X_L^c . Once we denote the variables, we can write the expected profit function of the central system for the first period as:

$$\begin{aligned} \Pi_{CS}^1(X_1^c, X_L^c, X_3^c) = & E_{D_1}[r \min(D_1, X_1^c) - c_r X_3^c - c_L X_L^c - p_1(D_1 - X_1^c)^+ \\ & - h_1^b(X_1^c - D_1)^+ - h_s(X_L^c - X_1^c)] \end{aligned}$$

The first term represents the revenue received from the sales of first period. The second term represents the total cost of raw materials. The third term represents the in-house production cost. The fourth term represents the shortage penalty cost incurred in the first period. The fifth term represents the holding cost of finished goods at the buyer's side, and the sixth term represents the holding cost of the finished goods at the supplier's side.

The expected profit of the centralized system for the second period is:

$$\begin{aligned} \Pi_{CS}^2(X_2^c, I_c) &= E_{D_2|d_1} [r \min(D_2, (I_c + X_2^c)^+) + r \min((d_1 - X_1^c), X_2^c) \\ &\quad - p_2(D_2 - I_c - X_2^c)^+ + v_f^b(I_c + X_2^c - D_2)^+ - c_L(1 + \gamma)(X_2^c - (X_L^c - X_1^c))^+ \\ &\quad + v_r^s(X_3^c - X_1^c - \max(X_2^c, (X_L^c - X_1^c))) + v_f^s(X_L^c - X_1^c - X_2^c)^+] \end{aligned}$$

The first term represents the revenue received from the sales of second period. The second term represents the revenue received from the backorders satisfied in the second period. The third term represents the shortage penalty cost incurred in the second period. The fourth term represents the revenue received from salvaging finished goods at the buyer's side. The fifth term represents the contractual agreement production cost. The sixth term represents the revenue received from salvaging of the raw materials. The seventh term represents the revenue received from salvaging finished goods at the supplier's side.

For a given I_c , in order to optimize the profit in the second period, the decision maker solves the following problem:

$$\max_{X_2^c} \Pi_{CS}^2(X_2^c|I_c) \quad \text{s.t.} \quad 0 \leq X_2^c \leq X_3^c - X_1^c \quad (3.15)$$

Proposition 3.7 For a given I_c , $\Pi_{CS}^2(X_2^c|I_c)$ is concave in X_2^c .

For the proof of Proposition 3.7, please refer to Section A.8 in the Appendix.

The optimal shipment quantity for the second period should satisfy the following equation:

$$F_{D_2|d_1}(I_c + X_2^c) = \frac{(p_2 + r - v_f^s) - (v_r^s - v_f^s + c_L(1 + \gamma)) \times 1_{X_2^c > X_L^c - X_1^c}}{(p_2 + r - v_f^b)} \quad (3.16)$$

We also have two different sets of first order conditions in CS. We find two unconstrained X_{2u}^c from these equations (X_{2u+}^c, X_{2u-}^c) . The first unconstrained X_{2u+}^c

can be written as $dd_3 + \nu d_1 - X_1^c$. The second unconstrained X_{2u-}^c can be written as $dd_4 + \nu d_1 - X_1^c$. Where $\nu = 1 + \rho \frac{\sigma_2}{\sigma_1}$; $dd_3 = \mu_2 - \rho \frac{\sigma_2}{\sigma_1} + k_3 \sigma_2 \sqrt{1 - \rho^2}$; $k_3 = \Phi^{-1}\left(\frac{p_2+r-v_r^s-c_L(1+\gamma)}{p_2+r-v_f^b}\right)$; $dd_4 = \mu_2 - \rho \frac{\sigma_2}{\sigma_1} + k_4 \sigma_2 \sqrt{1 - \rho^2}$; $k_4 = \Phi^{-1}\left(\frac{p_2+r-v_f^s}{p_2+r-v_f^b}\right)$.

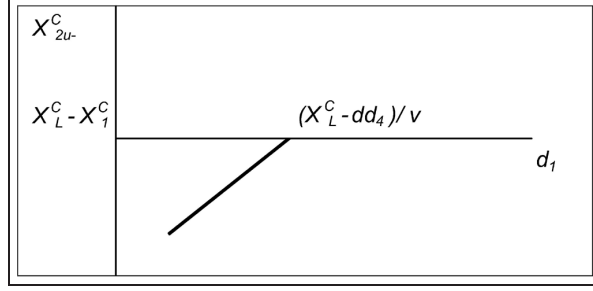


Figure 3.7. X_{2u-}^c related to d_1

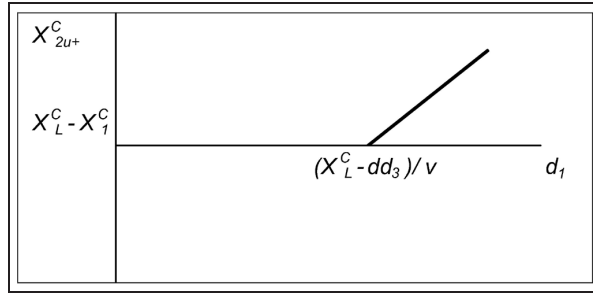


Figure 3.8. X_{2u+}^c related to d_1

For the illustration of the X_{2u-}^c and X_{2u+}^c please see Figure 3.7 and Figure 3.8. For the case $v_r^s + c_L(1 + \gamma) \geq v_f^s$; at most one of the X_{2u}^c becomes active related to the realized first period demand.

Proposition 3.8 The first and the second unconstrained X_{2u}^c can be written as:

$$X_{2u+}^c = dd_3 + \nu d_1 - X_1^c$$

$$X_{2u-}^c = dd_4 + \nu d_1 - X_1^c$$

where $\nu = 1 + \rho \frac{\sigma_2}{\sigma_1}$; $dd_4 = \mu_2 - \rho \frac{\sigma_2}{\sigma_1} + k_4 \sigma_2 \sqrt{1 - \rho^2}$; $dd_3 = \mu_2 - \rho \frac{\sigma_2}{\sigma_1} + k_3 \sigma_2 \sqrt{1 - \rho^2}$; $k_4 = \Phi^{-1}\left(\frac{p_2+r-v_f^s}{p_2+r-v_f^b}\right)$; $k_3 = \Phi^{-1}\left(\frac{p_2+r-(v_r^s+c_L(1+\gamma))}{p_2+r-v_f^b}\right)$;

For the case $v_r^s + c_L(1 + \gamma) \geq v_f^s$; $X_2^{c*} = \min(\max(0, X_{2u-}^c), \max(X_L^c - X_1^c, X_{2u+}^c), X_3^c - X_1^c)$.

For the proof of Proposition 3.8, please refer to Section A.7 in the Appendix and for the illustration of it please see Figure 3.9.

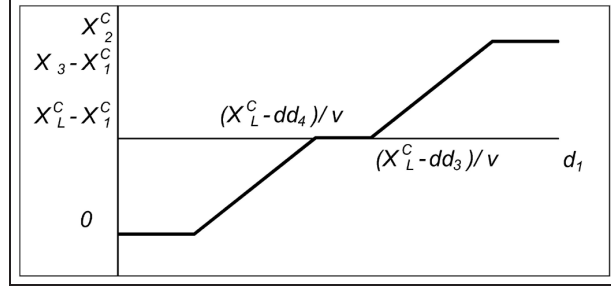


Figure 3.9. X_2^c related to d_1 for the case $v_r^s + c_L(1 + \gamma) \geq v_f^s$

However, for the case $v_r^s + c_L(1 + \gamma) < v_f^s$ it seems there will be two optimal X_{2u}^c for the realized first period demand values (d_1) which range between $\frac{X_L^c - dd_3}{\nu}$ and $\frac{X_L^c - dd_4}{\nu}$.

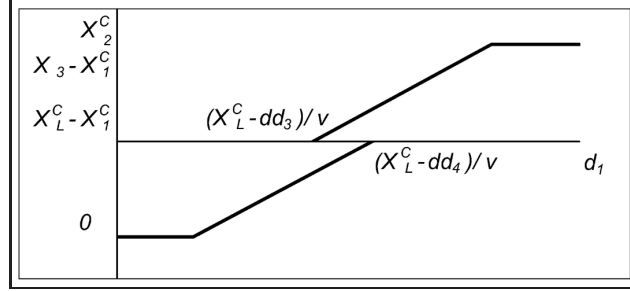


Figure 3.10. X_2^c related to d_1 for the case $v_r^s + c_L(1 + \gamma) < v_f^s$

If the salvage value of finished goods for the supplier or the salvage value of finished goods for the buyer is larger than the sum of salvage value of raw material and contractual agreement production cost ($v_f^s \geq v_r^s + c_L(1 + \gamma)$) or ($v_f^b \geq v_r^s + c_L(1 + \gamma)$) then the system can make a profit from every unit of raw material converted to finished goods. We have assumed, the holding cost for supplier is less than the cost difference between in-house and contractual agreement productions ($h_s \leq \gamma c_L$). So all raw materials will be converted to finished goods by using the in-house production mode. In other words assuming that the salvage value of finished goods for the supplier or the salvage value of finished goods for the buyer is larger than the sum of the salvage value of raw material, in-house production and holding cost ($v_f^s \geq v_r^s + c_L + h_s$) or ($v_f^b \geq v_r^s + c_L + h_s$), $X_L^c = X_3^c$, then we can rewrite the expected profit function of the

centralized system for the second period as follows:

$$\begin{aligned} \Pi_{CS}^2(X_2^c, I_c) &= E_{D_2|d_1}[r \min(D_2, (I_c + X_2^c)^+) + r \min((d_1 - X_1^c), X_2^c) \\ &\quad - p_2(D_2 - I_c - X_2^c)^+ + v_f^b(I_c + X_2^c - D_2)^+ + v_f^s(X_3^c - X_1^c - X_2^c)] \end{aligned}$$

Therefore in the case $v_r^s + c_L(1 + \gamma) < v_f^s$ the decision maker will not have a trade-off, because $X_L^c = X_3^c$.

Proposition 3.9 The unconstrained X_{2u}^c can be written as:

$$X_{2u}^c = dd_4 + \nu d_1 - X_1^c$$

For the case $v_r^s + c_L(1 + \gamma) < v_f^s$; $X_2^{c*} = \min(\max(0, X_{2u}^c), X_3^c - X_1^c)$

For the illustration of the Proposition 3.9, please see the Figure 3.11.

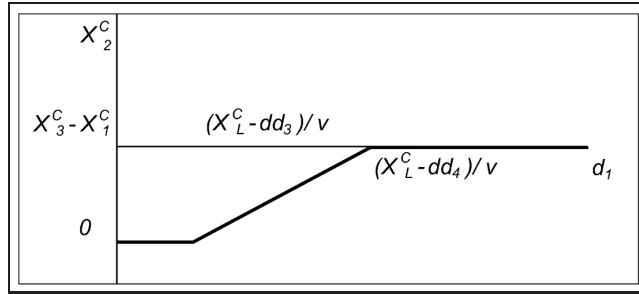


Figure 3.11. X_2^c related to d_1 for the case $v_r^s + c_L(1 + \gamma) < v_f^s$

The total expected profit of the centralized system can be written as:

$$\Pi_{CS}(X_1^c, X_L^c, X_3^c) = \Pi_{CS}^1(X_1^c, X_L^c, X_3^c) + E_{I_c}[\Pi_{CS}^2(X_2^{c*}, I_c)]$$

For the case $v_r^s + c_L(1 + \gamma) \geq v_f^s$, we can write the following:

$$\begin{aligned} \Pi_{CS}^1(X_1^c, X_L^c, X_3^c) + E_{I_c}[\Pi_{CS}^2(X_2^{c*}, I_c)] &= J_1^c(X_1^c) + J_2^c(X_L^c) + J_3^c(X_3^c) \\ \text{where } 0 \leq X_1^c \leq X_L^c \leq X_3^c & \quad (3.17) \end{aligned}$$

For the case $v_r^s + c_L(1 + \gamma) < v_f^s$, we can write the following:

$$\begin{aligned} \Pi_{CS}^1(X_1^c, X_3^c) + E_{I_c}[\Pi CS_2(X_2^{c*}, I_c)] &= J_1^c(X_1^c) + J_3^c(X_3^c) \\ \text{where } 0 \leq X_1^c \leq X_3^c & \end{aligned} \quad (3.18)$$

Proposition 3.10 For the case $v_r^s + c_L(1 + \gamma) \geq v_f^s$; $J_1^c(X_1^c)$, $J_2^c(X_L^c)$, and $J_3^c(X_3^c)$ are concave, therefore $\Pi_{CS}(X_1^c, X_L^c, X_3^c)$ is jointly concave.

For the case $v_r^s + c_L(1 + \gamma) < v_f^s$; $J_1^c(X_1^c)$, and $J_3^c(X_3^c)$ are concave, therefore $\Pi_{CS}(X_1^c, X_3^c)$ is jointly concave.

For the proof of Proposition 3.10, please refer to Section A.9 in the Appendix.

3.3. Channel Coordination

In this section we examine whether we can attain channel coordination with our decentralized model with options and order cancellation under linear pricing and the sufficient price conditions for it. Channel coordination is achieved if the mechanism yields the same result with the centralized system. In other words, when the total profit of the supplier and the buyer in the decentralized system is equal to the central system profit. We call the profit of the centralized system the first best solution which is the highest profit that can be gained from the channel. For instance channel coordination is achieved if the supplier sells the firm to the buyer.

In order to achieve channel coordination we need to make the buyer of the decentralized system and the decision maker of the centralized system behave in the same manner. For the buyer the decisions are taken according to linear price schemes but it is not the same for the decision maker, which is particularly obvious in the determination of the shipment quantity for the second period after the first period demand is realized. For instance if the buyer thinks that he needs more for the second period, the marginal cost is c_e for additional products. However for the decision maker the marginal cost is v_f^s for the quantities that have been produced in-house, $v_r^s + c_L(1 + \gamma)$ for the quantities

that have not been produced. Therefore it makes a difference for the decision maker if the unit has been produced in-house or not, but it does not matter for the buyer.

Conditions that coordinate the model will depend on the cost and salvage parameters. Therefore we examine the coordination according to those parameters. Channel coordination is achieved by equating production decisions (X_L, X_L^c) , raw material gatherings (X_3, X_3^c) , the first period shipment quantities (X_1, X_1^c) , and the second period shipment quantities $(X_2 - X_1 + m, X_2^c)$.

Proposition 3.11 The supplier gains zero profit when there is coordination.

For the proof of Proposition 3.11, please refer to Section A.10 in the Appendix.

Table 3.1 shows a summarized version of conditions that coordinates the model. We see that for each case c_1 is equal to $c_r + c_L$ which is the cost of supplying up to X_1 units of good. For the cases where there is a second period order ($X_2 \neq X_1$), we have the equality of $c_2 = c_r + c_L + h_s$. For these cases as $X_L \geq X_2$, the cost of supplying up to X_2 units of good is also $c_r + c_L + h_s$.

There can be exercised options in the cases where $X_2 \neq X_3$. In these cases we see that if the products have been produced in-house already ($X_L = X_3$), $c_o + c_e$ is equal to $c_r + c_L + h_s$. In addition to this, if the products have not been produced yet ($X_L = X_2$), $c_o + c_e$ is equal to $c_r + c_L(1 + \gamma)$. Therefore the revenue received from the exercised options, is equal to the cost of supplying that quantity.

For the unexercised options, the supplier gets $c_o + v_f^s (= c_r + c_L + h_s)$ if the products have been produced in-house already. For the raw material inventory at the end of the season, the supplier gets $c_o + v_r^s (= c_r)$. For the cancelled orders, the supplier gets $c_p + v_f^s (= c_r + c_L + h_s)$ as they are already produced in all cases.

We see that the revenue received by the supplier is equal to the cost of her. Therefore under the prices that are shown in the Table 3.1, channel can not coordinate

as the individual rationality of the supplier is violated. However prices that coordinates the model give all profit to the buyer for all cost conditions. Therefore we can say that in a structure where the buyer is the leader, channel coordination can be achieved for all cost conditions. This is not possible in the model of Barnes-Schuster *et al.* (2002).

Table 3.1. Prices at which DSOC behaves like CS for the given conditions

Conditions and Decisions	W. price (c_1)	W. price (c_2)	Option price (c_o)	Exercise price (c_e)	Can. Price (c_p)
$v_f^s \leq v_f^b$ and $v_r^s + c_L + h_s > v_f^b$ $X_1 = X_1^c, X_2 = X_L = X_L^c$ and $X_3 = X_3^c$	$c_r + c_L$	$c_r + c_L + h_s$	$c_r - v_r^s$	$v_r^s + c_L(1 + \gamma)$	$c_2 - c_p < v_f^b$
$v_f^s \leq v_f^b$ and $v_r^s + c_L + h_s \leq v_f^b$ $X_1 = X_2 = X_1^c$ and $X_3 = X_L = X_L^c = X_3^c$	$c_r + c_L$	$> c_r + c_L + h_s$	$c_r + c_L + h_s - c_e$	$< v_f^b$	$c_p > c_o$
$X_1 = X_1^c$ and $X_2 = X_3 = X_L = X_L^c = X_3^c$	$c_r + c_L$	$c_r + c_L + h_s$	$> c_r + c_L + h_s - c_e$	$< v_f^b$	$c_2 - c_p < v_f^b$
$X_1 = X_1^c$ and $X_3 = X_L = X_L^c = X_3^c$	$c_r + c_L$	$c_r + c_L + h_s$	$c_r + c_L + h_s - c_e$	$< v_f^b$	$c_2 - c_p < v_f^b$
$v_f^s > v_f^b$ and $v_r^s + c_L + h_s \leq v_f^s$ $X_1 = X_2 = X_1^c$ and $X_3 = X_L = X_L^c = X_3^c$	$c_r + c_L$	$> c_r + c_L + h_s$	$c_r + c_L + h_s - c_e$	v_f^s	$c_p > c_o$
$X_1 = X_1^c$ and $X_2 = X_3 = X_L = X_L^c = X_3^c$	$c_r + c_L$	$c_r + c_L + h_s$	$> c_r + c_L + h_s - c_e$	$< c_2 - c_p$	v_f^s
$v_f^s > v_f^b$ and $v_r^s + c_L + h_s > v_f^s$ $X_1 = X_1^c, X_2 = X_L = X_L^c$ and $X_3 = X_3^c$	$c_r + c_L$	$c_r + c_L + h_s$	$c_r - v_r^s$	$v_r^s + c_L(1 + \gamma)$	v_f^s

3.4. Decentralized System with Options (DSO)

As we mentioned before, this model was proposed by Barnes-Schuster *et al.* (2002). Similar to our model, also in this model, the buyer and the supplier play a Stackelberg game in which the supplier is the leader and the buyer is the follower. The difference of this model from ours that it is not allowed to cancel any amount from the second period order quantity. In this game for the announced set of prices which are wholesale (c_1, c_2) , option (c_o) , exercise (c_e) , the buyer places the order quantities (Q_1, Q_2) and determines the number of options purchased (M) . While determining these quantities the buyer aims to maximize his profit. As we have assumed common knowledge for the information structure, and the supplier's profit is affected by the buyer's order quantities, number of options purchased, number of options exercised, the supplier determines the optimal prices and production quantities in the way that maximize her profit.

As we have assumed the holding cost of finished goods is less than the difference between in-house production and contractual agreement production cost, the supplier produces at least quantity of X_2 as there is no order cancellation for the second period. According to the variables we have denoted, the expected profit function of the supplier can be written as:

$$\begin{aligned} \Pi_{dso}^S(X_L, c_1, c_2, c_o, c_e) = & E_{D_1}[c_1X_1 + c_2(X_2 - X_1) + c_o(X_3 - X_2) + c_em \\ & - h_s(X_L - X_1) + v_r^s(X_3 - X_2 - \max((X_L - X_2), m) + v_f^s(X_L - X_2 - m)^+ - c_rX_3 \\ & - c_LX_L - c_L(1 + \gamma)(m - (X_L - X_2))^+] \end{aligned}$$

The expected profit of the buyer for the first period can be written as:

$$\begin{aligned} \Pi_{dso}^{1,B}(X_1, X_2, X_3) = & E_{D_1}[r \min(D_1, X_1) - c_o(X_3 - X_2) - c_1X_1 \\ & - c_2(X_2 - X_1) - p_1(D_1 - X_1)^+ - h_1^b(X_1 - D_1)^+] \end{aligned}$$

After the first period demand is realized, the buyer exercises options for the

second period. The expected profit function of the buyer for the second period can be written as:

$$\begin{aligned} \Pi_{dso}^{2,B}(m, I_2) = & E_{D_2}[r \min(D_2, (I_2 + m)^+) + r \min((d_1 - X_1)^+, X_2 - X_1 + m) \\ & - c_e m - p_2(D_2 - I_2 - m)^+ + v_f^b(I_2 + m - D_2)^+] \end{aligned}$$

The solutions are available in Barnes-Schuster *et al.* (2002).

3.5. Decentralized System with No Options (DSNO)

Also in this model, the buyer and the supplier play a Stackelberg game in which the supplier is the leader and the buyer is the follower. The difference of this model from ours that it is not allowed to update the second period order quantity in other words there is no option or order cancellation. In this game for the announced wholesale prices (c_1, c_2) the buyer places the orders (Q_1, Q_2) . While determining these quantities the buyer aims to maximize his profit. As we have assumed common knowledge for the information structure, and the supplier's profit is affected by the buyer's order quantities, the supplier determines the optimal prices and production quantities in the way that maximizes her profit.

As we have assumed, the holding cost of finished goods is less than the difference between in-house production cost and contractual agreement production cost. It is also not allowed to update the second period order quantity in this model. Therefore, in-house production quantity is equal to X_2 for this model and there is no need for contractual agreement production. According to the variables we have denoted, the expected profit function of the supplier can be written as:

$$\Pi_{dsno}^S(c_1, c_2) = E_{D_1}[c_1 X_1 + c_2(X_2 - X_1) + -h_s(X_2 - X_1) - c_r X_2 - c_L X_2]$$

The expected profit of the buyer for the first period can be written as:

$$\begin{aligned}\Pi_{dsno}^{1,B}(X_1, X_2) &= E_{D_1}[r \min(D_1, X_1) - c_1 X_1 - c_2(X_2 - X_1) \\ &\quad - p_1(D_1 - X_1)^+ - h_1^b(X_1 - D_1)^+]\end{aligned}$$

As there is no order update in this model, the expected profit of the buyer for the second period depends only on I_2 . The expected profit function of the buyer for the second period can be written as:

$$\begin{aligned}\Pi_{dsno}^{2,B}(I_2) &= E_{D_2}[r \min(D_2, (I_2)^+) + r \min((d_1 - X_1)^+, X_2 - X_1) \\ &\quad - p_2(D_2 - I_2)^+ + v_f^b(I_2 - D_2)^+]\end{aligned}$$

We skip the solutions as they are self-evident.

4. COMPUTATIONAL STUDY

As we have discussed in the Channel Coordination (Section 3.3), our model does not coordinate because the prices that achieve channel coordination are not individually rational for the supplier. Questioning the value of the options and order cancellation combination, we examine the performance of the model by a computational study because the structural properties of the model are so complex for an analytical study. We assume that the supplier uses a single wholesale price for two periods for all DS models. This simplifies the computations.

For the channel we try to figure out the value of options, the value of options and order cancellation combination, the value of coordination and the potential gain that could be achieved from channel coordination from a system that already uses options and order cancellation. Π_{DSOC}^* , Π_{DSO}^* and Π_{DSNO}^* are the sum of optimal profits of the buyer and the supplier in the DSOC, the DSO and the DSNO, respectively. In the DSOC, the DSO and the DSNO, the supplier optimizes her expected profit subject to a constraint that the buyer's expected profit will be at least his reservation profit which is normalized at zero.

The value of options for the channel is denoted by VO and is given by percentage difference between Π_{DSO}^* and Π_{DSNO}^* . The value of options and order cancellation combination for the channel is denoted by VOC and is given by percentage difference between Π_{DSOC}^* and Π_{DSNO}^* . The value of coordination for the channel is denoted by VC and is given by percentage difference between Π_{CS}^* and Π_{DSNO}^* , and the value of coordination for a system that already uses options and order cancellation is denoted by VCC. We also compute the ratio VOC/VC as a fraction of total possible improvement

in the system (VC) captured by options and order cancellation combination (VOC).

$$VO = \frac{\Pi_{DSO}^* - \Pi_{DSNO}^*}{\Pi_{DSNO}^*} \times 100$$

$$VOC = \frac{\Pi_{DSOC}^* - \Pi_{DSNO}^*}{\Pi_{DSNO}^*} \times 100$$

$$VC = \frac{\Pi_{CS}^* - \Pi_{DSNO}^*}{\Pi_{DSNO}^*} \times 100$$

$$VCC = \frac{\Pi_{CS}^* - \Pi_{DSOC}^*}{\Pi_{DSOC}^*} \times 100$$

In our computational study the market is expressed in two categories. We use the names ‘fashion goods’ and ‘basic goods’ like Barnes-Schuster *et al.* (2002) have used in their study. Fashion goods are characterized by high margin ($r = 8$), low and medium salvage value for finished goods ($v_f^s \in \{2, 0.8\}$, $v_f^b \in \{1, -0.2\}$), low salvage value for raw material ($v_r^s = 1.5$), high level of coefficient of variation (CV) of demand ($\sigma/\mu = 0.25$) and high correlation ($\rho \in \{0.5, 0.75, 0.9\}$). Basic goods are characterized by low margin ($r = 5$), high salvage value for finished goods ($v_f^s = 4$, $v_f^b = 3$), high salvage value for raw material ($v_r^s = 3$), low and high level of coefficient of variation of demand ($\sigma/\mu \in \{0.125, 0.25\}$) and low correlation ($\rho \in \{0, 0.05, 0.1\}$). We assumed that the cost of raw material ($c_r = 3$) and in-house production cost ($c_L = 1$) are the same for both categories. Accordingly, the maximum margin ($r/(c_r + c_L)$) for fashion goods becomes 100 per cent, and it becomes 25 per cent for basic goods. The other common parameters are additional unit cost of contractual agreement production in the expedited mode ($\gamma = 0.5$), penalty costs ($p_1, p_2 \in \{0.25, 2, 3\}$), holding cost ($h_s = 0.4$) and means of demand distributions ($\mu_1 = 160$; $\mu_2 = 320$).

4.1. Effect of Demand Variability

We expect that while the demand variability increases, the probability of fitness of supply and demand decrease for both periods, which makes the coordination valuable. We observe the results from Table 4.1 that increasing CV of demand increases VC.

Which means that while the demand variability increases, the value of coordination increases.

We also observe that increasing CV of demand increases VO and VOC, which means that value of options or options and order cancellation combination increases as the demand variability increases. Therefore, by the help of ordering flexibility tools such as options or options and order cancellation, excess inventory or demand at the end of the first period is corrected and the mismatch of supply and demand for the second period is minimized. We also see that VOC is always greater than VO. It is obvious that options and order cancellation combination gives more flexibility.

When demand variability increases, order quantities decrease for low penalty cost and salvage values. We expect that order quantities of DS decrease more than order quantities of CS if the wholesale prices were fixed. However when order quantities decrease, in DS supplier will lower wholesale prices to increase the quantities. Therefore VCC increases as CV of demand increases. For high penalty cost and salvage values, we see the just the opposite effects. However double marginalization effect also increases and VCC increases as CV of demand increases.

Table 4.1. Effect of demand variability and demand correlation for basic goods

Demand $CV = 0.125$									
ρ	Π_{DSNO}	Π_{DSO}	Π_{DSOC}	Π_{CS}	VO	VOC	VC	VCC	VOC/VC
0	259.56	262.37	262.53	265.56	1.08	1.15	2.31	1.15	0.50
0.05	258.24	261.90	262.16	265.36	1.42	1.52	2.76	1.22	0.55
0.1	256.95	261.58	261.94	265.31	1.80	1.94	3.25	1.28	0.60

Demand $CV = 0.25$									
ρ	Π_{DSNO}	Π_{DSO}	Π_{DSOC}	Π_{CS}	VO	VOC	VC	VCC	VOC/VC
0	167.20	172.82	173.21	179.22	3.36	3.59	7.19	3.47	0.50
0.05	164.48	171.89	172.45	178.82	4.45	4.79	8.65	3.69	0.55
0.1	162.00	171.25	172.02	178.71	5.71	6.19	10.37	3.89	0.60

*with medium penalty costs.

4.2. Effect of Demand Correlation

Higher correlation signifies that the first period demand realization gives more information about the second period demand. So we observe the value of coordination (VC) increases as the correlation increases. The use of options or options and order cancellation combination are the ways to use this information for a decentralized system. So as correlation increases, VO and VOC increase as well. Results are listed in Table 4.1 and Table 4.2.

Increase in VO and VOC with correlation increases with the CV of demand. It means that more information is more valuable if demand variability is higher and you have a system that can react accordingly.

Effect of demand correlation to the demand variability changes according to the period. Before the season starts, total demand variability increases with the correlation and order commitments are made accordingly. After the first period demand realization, demand variability for the second period decreases and in-season replenishment is made accordingly. At the beginning VCC increases as the total demand variability increases, VCC later decreases as the demand variability for the second period decreases. The total effect on VCC changes according to the effect to dominate. In our case, for fashion goods the second effect dominates and VCC decreases with correlation. For basic goods first effect dominates and VCC increases with correlation.

As correlation increases, the ratio VOC/VC as a fraction of total possible improvement in the system, captured by options and order cancellation combination increases both for basic goods and fashion goods. For all ρ values, it is always more than 50 per cent for basic goods, but for fashion goods the ratio reaches that values only for high correlation.

Table 4.2. Effect of demand correlation and salvage risk for fashion goods

Medium Salvage ($v_f^s = 2, v_f^b = 1$)									
ρ	Π_{DSNO}	Π_{DSO}	Π_{DSOC}	Π_{CS}	VO	VOC	VC	VCC	VOC/VC
0.5	1275.30	1286.50	1288.10	1397.50	0.88	1.00	9.58	8.49	0.10
0.75	1243.30	1301.10	1304.90	1399.90	4.65	4.95	12.60	7.28	0.39
0.9	1224.90	1338.90	1345.10	1418.10	9.31	9.81	15.77	5.43	0.62

Low Salvage ($v_f^s = 0.8, v_f^b = -0.2$)									
ρ	Π_{DSNO}	Π_{DSO}	Π_{DSOC}	Π_{CS}	VO	VOC	VC	VCC	VOC/VC
0.5	1219.30	1240.90	1241.70	1346.60	1.77	1.84	10.44	8.45	0.18
0.75	1183.30	1263.50	1266.90	1358.70	6.78	7.06	14.82	7.25	0.48
0.9	1162.80	1311.10	1317.70	1388.90	12.75	13.32	19.44	5.40	0.69

*with medium penalty costs.

4.3. Effect of Salvage Risk

Salvage risk increases as the salvage value of finished good and raw material decreases. Increase in the risk, increases VC. When the salvage risk increases, the supplier under-produce and the buyer under-order respectively to avoid the risk of salvage. Flexibility in the system allows the supplier and the buyer to share the risk and increase order quantities and so VO and VOC increase when the salvage value decreases. We can observe these from Table 4.2.

In the absence of any pricing effect, increase in salvage values increase order quantities. Increase in order quantities in CS will be greater than DS. Therefore this will increase the difference between order quantities of two system and increase double marginalization. In DS, as salvage revenue of buyer increases, supplier will attempt to extract some revenue by increasing the prices. Increasing prices will also increase double marginalization and we will see an increase in VCC. Therefore we can conclude as salvage risk increases, VCC decreases.

As salvage risk increases, the ratio VOC/VC as a fraction of total possible improvement in the system, captured by options and order cancellation combination also increases.

4.4. Effect of Serviceability

We know that service level is proportional to the penalty costs. When penalty costs increase, in DS the buyer tends to buy more goods to satisfy a higher service level if the prices are assumed to be fixed. In the absence of pricing effect, in DS the buyer's profit decreases and supplier's profit increases and total profit may increase or decrease. In CS order quantities also increase and the increase in order quantities decreases the profit of CS. Increase in order quantities in DS is seen more than in CS. Therefore for fixed prices, double marginalization effect decreases.

Pricing can affect interaction in two different ways. If the buyer's profit has not bound to zero, the supplier increases prices knowing that buyer can not decrease order quantities due to high penalty costs. Therefore double marginalization effect increases and total profits in DSNO, DSO and DSOC decrease. If the buyer's profit has bounded to zero, the supplier can not increase price and even decreases prices in order to hold the buyer in the game. So double marginalization effect decreases and total profits in DSNO, DSO and DSOC increase. Total profits in DSNO, DSO and DSOC firstly depend on a combination of first and second effect, then they depend on a combination of first and third effect.

The behavior of VO, VOC and VC can not be figured out because of the uncertain first effect on profits of DS. However, the first effect on double marginalization is certain. In our case the first and the second effects are active and double marginalization range from decreasing to increasing. For fashion goods the decrease in VCC is diminishing and for basic goods it turns out to be increasing. We can observe the results from Table 4.3.

We observe that as serviceability increases, the ratio VOC/VC as a fraction of total possible improvement in the system, captured by options and order cancellation combination, increases.

Table 4.3. Effect of serviceability for fashion and basic goods

Fashion Goods									
Penalty Cost	Π_{DSNO}	Π_{DSO}	Π_{DSOC}	Π_{CS}	VO	VOC	VC	VCC	VOC/VC
low	1140.30	1206.00	1261.30	1470.60	5.76	10.61	28.97	16.59	0.37
medium	1243.30	1301.10	1304.90	1399.90	4.65	4.95	12.60	7.28	0.39
high	1229.60	1295.30	1299.40	1372.10	5.34	5.68	11.59	5.59	0.49
*with medium salvage value of finished goods, $\rho:0.75$, $CV:0.25$.									
Basic Goods									
Penalty Cost	Π_{DSNO}	Π_{DSO}	Π_{DSOC}	Π_{CS}	VO	VOC	VC	VCC	VOC/VC
low	242.10	242.70	242.80	253.10	0.25	0.29	4.54	4.24	0.06
medium	167.20	172.82	173.21	179.22	3.36	3.59	7.19	3.47	0.50
high	142.80	150.10	150.60	156.20	5.11	5.46	9.38	3.72	0.58
*with high salvage value of finished goods, $\rho:0.0$, $CV:0.25$.									

4.5. Effect of Supplier Flexibility

We know that as the additional unit cost of contractual agreement production in the expedited mode (γ) decreases, supplier flexibility increases. The profit in CS increases when the contractual agreement production cost decreases but the total profit in DSNO does not change because all production is done with in-house production. Total profit in DSO and DSOC also increases because as the contractual agreement cost decreases some of the production will shift to the second period and this will increase the match of the supply and the demand. As total profit in DSNO is fixed, increase in total profits of DSO, DSOC and CS make VO, VOC and VC increasing. We can observe these from Table 4.4.

When the supplier flexibility increases, if the prices are assumed to be fixed, order quantities increase in CS but it is not changed in DS. Therefore double marginalization effect increases. When the supplier flexibility increases, the wholesale prices are also affected. Therefore the double marginalization effect decrease while order quantities increase in DS. In our case the first effect dominates the second effect and VCC increases.

We also observe that VOC/VC is increasing which means that fraction of to-

tal possible improvement in the system, captured by options and order cancellation combination, increases when supplier flexibility increases.

Table 4.4. Effect of supplier flexibility for fashion goods

γ	Π_{DSNO}	Π_{DSO}	Π_{DSOC}	Π_{CS}	VO	VOC	VC	VCC	VOC/VC
0.8	1243.30	1290.40	1295.10	1383.20	3.79	4.17	11.25	6.80	0.37
0.7	1243.30	1294.00	1298.30	1387.00	4.08	4.42	11.56	6.83	0.38
0.6	1243.30	1297.60	1301.60	1392.30	4.37	4.69	11.98	6.97	0.39
0.5	1243.30	1301.10	1304.90	1399.90	4.65	4.95	12.60	7.28	0.39
0.4	1243.30	1304.70	1311.90	1413.60	4.94	5.52	13.70	7.75	0.40

*with medium penalty costs, medium salvage value of finished goods, $\rho:0.75$, $CV:0.25$.

5. CONCLUSION

In this thesis, we investigate a model with options and order cancellation in a buyer-supplier system for in-season replenishment which is seen as the only way to stay alive for experienced but high-cost suppliers in the textile and apparel industry. Our model is a decentralized system in which a buyer and a supplier play a Stackelberg game where the supplier is the leader and the buyer is the follower. We analyse the model in two periods with correlated demand. The second period order can be updated by exercising options or by cancelling some or all of the second period order. The production can be carried out under two modes in each period which are in-house and contractual agreement production. Contractual agreement production can be substantially more costly.

We also analyse the centralized system as a benchmark and derive appropriate prices for the channel coordination. We show that with coordinating prices individual rationality constraint of the supplier is violated. However in a structure where the buyer is the leader, channel coordination can be achieved which is not possible with the model of Barnes-Schuster *et al.* (2002).

We also analyse two more decentralized models which are “decentralized model with options” (DSO) and “decentralized model with no options” (DSNO). Options model is proposed by Barnes-Schuster *et al.* (2002) and our model extends it to accommodate order cancellations along with options.

Finally we make a computational study and we try to quantify the value of option, the value of option and order cancellation combination and the value of the coordination as a function of demand variability, demand correlation, salvage risk, serviceability and supplier flexibility. The computational study is limited but it clearly shows that our model has better results than the options model (DSO). Options and order cancellation combination which is offered by the supplier provides flexibility to the buyer to respond to market changes thus increasing the total profit.

As a future research, computational studies for this model can be extended. We have assumed common knowledge about the information structure. Information asymmetry can be also be an area for the future research.

APPENDIX A: PROOFS

A.1. Proof of Proposition 3.2

Derivation of (3.2):

$$\begin{aligned}
\frac{\partial \Pi_{d\text{soc}}^S(X_L)}{\partial X_L} &= -h_s - c_L + \frac{\partial}{\partial X_L} E_{D_1} [v_f^s(X_L - X_2 - m)^+ - v_r^s \max(X_L - X_2, m) \\
&\quad - c_L(1 + \gamma)(m - (X_L - X_2))^+] \\
&= -h_s - c_L + \frac{\partial}{\partial X_L} \left[\int_0^{\frac{X_1 - dd_2}{\nu}} (-v_r^s(X_L - X_2) + v_f^s(X_L - X_1)) dF_{D_1}(d_1) \right. \\
&\quad + \int_{\frac{X_1 - dd_2}{\nu}}^{\frac{X_2 - dd_2}{\nu}} (-v_r^s(X_L - X_2) + v_f^s(X_L - dd_2 - \nu d_1)) dF_{D_1}(d_1) \\
&\quad + \int_{\frac{X_2 - dd_2}{\nu}}^{\frac{X_2 - dd_1}{\nu}} (-v_r^s(X_L - X_2) + v_f^s(X_L - X_2)) dF_{D_1}(d_1) + \int_{\frac{X_2 - dd_1}{\nu}}^{\frac{X_L - dd_1}{\nu}} (-v_r^s(X_L - X_2) \\
&\quad + v_f^s(X_L - dd_1 - \nu d_1)) dF_{D_1}(d_1) + \int_{\frac{X_L - dd_1}{\nu}}^{\frac{X_3 - dd_1}{\nu}} (-v_r^s(dd_1 + \nu d_1 - X_2) \\
&\quad - c_L(1 + \gamma)(dd_1 + \nu d_1 - X_L)) dF_{D_1}(d_1) + \int_{\frac{X_3 - dd_1}{\nu}}^{\infty} (-v_r^s(X_3 - X_2) \\
&\quad \left. - c_L(1 + \gamma)(X_3 - X_L)) dF_{D_1}(d_1) \right] \\
&= -h_s - c_L + \int_0^{\frac{X_L - dd_1}{\nu}} (-v_r^s + v_f^s) dF_{D_1}(d_1) + \int_{\frac{X_L - dd_1}{\nu}}^{\infty} (c_L(1 + \gamma)) dF_{D_1}(d_1) \\
&= -h_s + \gamma c_L + (-v_r^s + v_f^s - c_L(1 + \gamma)) F_{D_1}\left(\frac{X_L - dd_1}{\nu}\right)
\end{aligned}$$

Derivation of (3.3):

$$\begin{aligned}
\frac{\partial \Pi_{dsoc}^S(X_L)}{\partial X_L} &= -h_s - c_L + \frac{\partial}{\partial X_L} E_{D_1} [v_f^s(X_L - X_2 - m)^+ - v_r^s \max(X_L - X_2, m) \\
&\quad - c_L(1 + \gamma)(m - (X_L - X_2))^+] \\
&= -h_s - c_L + \frac{\partial}{\partial X_L} \left[\int_0^{\frac{X_1 - dd_2}{\nu}} (-v_r^s(X_L - X_2) + v_f^s(X_L - X_1)) dF_{D_1}(d_1) \right. \\
&\quad + \int_{\frac{X_1 - dd_2}{\nu}}^{\frac{X_L - dd_2}{\nu}} (-v_r^s(X_L - X_2) + v_f^s(X_L - dd_2 - \nu d_1)) dF_{D_1}(d_1) \\
&\quad + \int_{\frac{X_L - dd_2}{\nu}}^{\frac{X_2 - dd_2}{\nu}} (-v_r^s(dd_2 + \nu d_1 - X_2) - c_L(1 + \gamma)(dd_2 + \nu d_1 - X_L)) dF_{D_1}(d_1) \\
&\quad + \int_{\frac{X_2 - dd_2}{\nu}}^{\frac{X_2 - dd_1}{\nu}} (-c_L(1 + \gamma)(X_2 - X_L)) dF_{D_1}(d_1) + \int_{\frac{X_2 - dd_1}{\nu}}^{\frac{X_3 - dd_1}{\nu}} (-v_r^s(dd_1 + \nu d_1 - X_2) \\
&\quad - c_L(1 + \gamma)(dd_1 + \nu d_1 - X_L)) dF_{D_1}(d_1) + \int_{\frac{X_3 - dd_1}{\nu}}^{\infty} (-v_r^s(X_3 - X_2) \\
&\quad \left. - c_L(1 + \gamma)(X_3 - X_L)) dF_{D_1}(d_1) \right] \\
&= -h_s - c_L + \int_0^{\frac{X_L - dd_2}{\nu}} (-v_r^s + v_f^s) dF_{D_1}(d_1) + \int_{\frac{X_L - dd_2}{\nu}}^{\infty} (c_L(1 + \gamma)) dF_{D_1}(d_1) \\
&= -h_s + \gamma c_L + (-v_r^s + v_f^s - c_L(1 + \gamma)) F_{D_1}\left(\frac{X_L - dd_2}{\nu}\right)
\end{aligned}$$

Derivation of (3.4):

$$\begin{aligned}
\frac{\partial \Pi_{dsoc}^S(X_L)}{\partial X_L} &= -h_s - c_L + \frac{\partial}{\partial X_L} E_{D_1} [v_f^s(X_L - X_2 - m)^+ - v_r^s \max(X_L - X_2, m) \\
&\quad - c_L(1 + \gamma)(m - (X_L - X_2))^+] \\
&= -h_s - c_L + \frac{\partial}{\partial X_L} \left[\int_0^{\frac{X_2 - dd_1}{\nu}} (-v_r^s(X_L - X_2) + v_f^s(X_L - X_2)) dF_{D_1}(d_1) \right. \\
&\quad + \int_{\frac{X_2 - dd_1}{\nu}}^{\frac{X_L - dd_1}{\nu}} (-v_r^s(X_L - X_2) + v_f^s(X_L - dd_1 - \nu d_1)) dF_{D_1}(d_1) \\
&\quad + \int_{\frac{X_L - dd_1}{\nu}}^{\frac{X_3 - dd_1}{\nu}} (-v_r^s(dd_1 + \nu d_1 - X_2) - c_L(1 + \gamma)(dd_1 + \nu d_1 - X_L)) dF_{D_1}(d_1) \\
&\quad \left. + \int_{\frac{X_3 - dd_1}{\nu}}^{\infty} (-v_r^s(X_3 - X_2) - c_L(1 + \gamma)(X_3 - X_L)) dF_{D_1}(d_1) \right] \\
&= -h_s - c_L + \int_0^{\frac{X_L - dd_1}{\nu}} (-v_r^s + v_f^s) dF_{D_1}(d_1) + \int_{\frac{X_L - dd_1}{\nu}}^{\infty} (c_L(1 + \gamma)) dF_{D_1}(d_1) \\
&= -h_s + \gamma c_L + (-v_r^s + v_f^s - c_L(1 + \gamma)) F_{D_1}\left(\frac{X_L - dd_1}{\nu}\right)
\end{aligned}$$

Derivation of (3.5):

$$\begin{aligned}
\frac{\partial \Pi_{dsoc}^S(X_L)}{\partial X_L} &= -h_s - c_L + \frac{\partial}{\partial X_L} E_{D_1} [v_f^s(X_L - X_2 - m)^+ - v_r^s \max(X_L - X_2, m) \\
&\quad - c_L(1 + \gamma)(m - (X_L - X_2))^+] \\
&= -h_s - c_L + \frac{\partial}{\partial X_L} \left[\int_0^{\frac{X_1 - dd_2}{\nu}} (-v_r^s(X_L - X_2) + v_f^s(X_L - X_1)) dF_{D_1}(d_1) \right. \\
&\quad + \int_{\frac{X_1 - dd_2}{\nu}}^{\frac{X_L - dd_2}{\nu}} (-v_r^s(X_L - X_2) + v_f^s(X_L - dd_2 - \nu d_1)) dF_{D_1}(d_1) \\
&\quad + \int_{\frac{X_L - dd_2}{\nu}}^{\frac{X_2 - dd_2}{\nu}} (-v_r^s(dd_2 + \nu d_1 - X_2) - c_L(1 + \gamma)(dd_2 + \nu d_1 - X_L)) dF_{D_1}(d_1) \\
&\quad \left. + \int_{\frac{X_2 - dd_2}{\nu}}^{\infty} (-c_L(1 + \gamma)(X_2 - X_L)) dF_{D_1}(d_1) \right] \\
&= -h_s - c_L + \int_0^{\frac{X_L - dd_2}{\nu}} (-v_r^s + v_f^s) dF_{D_1}(d_1) + \int_{\frac{X_L - dd_2}{\nu}}^{\infty} (c_L(1 + \gamma)) dF_{D_1}(d_1) \\
&= -h_s + \gamma c_L + (-v_r^s + v_f^s - c_L(1 + \gamma)) F_{D_1}\left(\frac{X_L - dd_2}{\nu}\right)
\end{aligned}$$

Under four conditions we attain two first order conditions. From these we find two unconstrained X_L .

A.2. Proof of Proposition 3.1

Concavity of (3.1):

$$\begin{aligned}
\frac{\partial^2 \Pi_{dsoc}^S(X_L)}{\partial X_L^2} &= \frac{\partial}{\partial X_L} \left[-h_s + \gamma c_L + (-v_r^s + v_f^s - c_L(1 + \gamma)) F_{D_1}\left(\frac{X_L - dd_1}{\nu}\right) \right] \\
&= 1/\nu (-v_r^s + v_f^s - c_L(1 + \gamma)) f_{D_1}\left(\frac{X_L - dd_1}{\nu}\right) \leq 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 \Pi_{dsoc}^S(X_L)}{\partial X_L^2} &= \frac{\partial}{\partial X_L} \left[-h_s + \gamma c_L + (-v_r^s + v_f^s - c_L(1 + \gamma)) F_{D_1}\left(\frac{X_L - dd_2}{\nu}\right) \right] \\
&= 1/\nu (-v_r^s + v_f^s - c_L(1 + \gamma)) f_{D_1}\left(\frac{X_L - dd_2}{\nu}\right) \leq 0
\end{aligned}$$

Both second order conditions are less than zero since $v_r^s + c_L(1 + \gamma) > v_f^s$ and $f_{D_1}(\cdot) \geq 0$.

A.3. Proof of Proposition 3.4

Derivation of (3.7):

$$\begin{aligned}
\frac{\partial \Pi_{dsoc}^{2,B}(m, I_2)}{\partial m} &= -c_e + \frac{\partial}{\partial m} E_{D_2/d_1} [r \min(D_2, (I_2 + m)^+) - p_2(D_2 - I_2 - m)^+ \\
&\quad + v_f^b(I_2 + m - D_2)^+] \\
&= -c_e + \frac{\partial}{\partial m} \left[\int_{X_2+m-d_1}^{\infty} (r(X_2 + m - d_1)) dF_{D_2|d_1}(d_2) \right. \\
&\quad + \int_0^{X_2+m-d_1} (rd_2) dF_{D_2|d_1}(d_2) - \int_{X_2+m-d_1}^{\infty} (p_2(-X_2 - m + d_1 + d_2)) dF_{D_2|d_1}(d_2) \\
&\quad \left. + \int_0^{X_2+m-d_1} (v_f^b(X_2 + m - d_1 - d_2)) dF_{D_2|d_1}(d_2) \right] \\
&= -c_e + (r + p_2)(1 - F_{D_2|d_1}(X_2 + m - d_1)) + v_f^b F_{D_2|d_1}(X_2 + m - d_1) \\
&= r + p_2 - c_e - F_{D_2|d_1}(X_2 + m - d_1)(r + p_2 - v_f^b)
\end{aligned}$$

Derivation of (3.8):

$$\begin{aligned}
\frac{\partial \Pi_{dsoc}^{2,B}(m, I_2)}{\partial m} &= -c_2 + c_p + \frac{\partial}{\partial m} E_{D_2/d_1} [r \min(D_2, (I_2 + m)^+) - p_2(D_2 - I_2 - m)^+ \\
&\quad + v_f^b(I_2 + m - D_2)^+] \\
&= -c_2 + c_p + \frac{\partial}{\partial m} \left[\int_{X_2+m-d_1}^{\infty} (r(X_2 + m - d_1)) dF_{D_2|d_1}(d_2) \right. \\
&\quad + \int_0^{X_2+m-d_1} (rd_2) dF_{D_2|d_1}(d_2) - \int_{X_2+m-d_1}^{\infty} (p_2(-X_2 - m + d_1 + d_2)) dF_{D_2|d_1}(d_2) \\
&\quad \left. + \int_0^{X_2+m-d_1} (v_f^b(X_2 + m - d_1 - d_2)) dF_{D_2|d_1}(d_2) \right] \\
&= -c_2 + c_p + (r + p_2)(1 - F_{D_2|d_1}(X_2 + m - d_1)) + v_f^b F_{D_2|d_1}(X_2 + m - d_1) \\
&= r + p_2 - c_2 + c_p - F_{D_2|d_1}(X_2 + m - d_1)(r + p_2 - v_f^b)
\end{aligned}$$

We find two unconstrained m from first order conditions whether m is greater than zero or not.

A.4. Proof of Proposition 3.3

Concavity of (3.6):

$$\frac{\partial^2 \Pi_{dsoc}^{2,B}(m, I_2)}{\partial m^2} = -f_{D_2/d_1}(X_2 + m - d_1)(r + p_2 - v_f^b) \leq 0$$

Second order condition is less than zero since $v_f^b < r$ and $f_{D_i}(\cdot) \geq 0$.

A.5. Proof of Proposition 3.5

Derivation of (3.9):

$$\begin{aligned} & \Pi_{dsoc}^{2,B}(m_u^+, X_2 - d_1^*) - \Pi_{dsoc}^{2,B}(m_u^-, X_2 - d_1^*) = 0 \\ & = E_{D_2/d_1^*}[r \min(D_2, (I_2 + m_u^+)^+) + r \min((d_1^* - X_1)^+, X_2 - X_1 + m_u^+) - c_e m_u^+ \\ & \quad - p_2(D_2 - I_2 - m_u^+)^+ + v_f^b(I_2 + m_u^+ - D_2)^+ - r \min(D_2, (I_2 + m_u^-)^+) \\ & \quad + r \min((d_1^* - X_1)^+, X_2 - X_1 + m_u^-) - (c_2 - c_p)m_u^- - p_2(D_2 - I_2 - m_u^-)^+ \\ & \quad + v_f^b(I_2 + m_u^- - D_2)^+] \\ & = (p_2 + r - v_f^b) \int_0^{I_2 + m_u^+} (d_2 - (I_2 + m_u^+)) dF_{D_2|d_1^*}(d_2) + (p_2 + r - c_e)m_u^+ \\ & \quad - (p_2 + r - v_f^b) \int_0^{I_2 + m_u^-} (d_2 - (I_2 + m_u^-)) dF_{D_2|d_1^*}(d_2) - (p_2 + r - c_2 + c_p)m_u^- \\ & = \int_{I_2 + m_u^-}^{I_2 + m_u^+} (d_2) dF_{D_2|d_1^*}(d_2) - F_{D_2|d_1^*}(I_2 + m_u^+)I_2 + F_{D_2|d_1^*}(I_2 + m_u^-)I_2 \\ & = \int_{I_2 + m_u^-}^{I_2 + m_u^+} F_{D_2|d_1^*}(d_2) \partial d_2 + F_{D_2|d_1^*}(I_2 + m_u^+)m_u^+ - F_{D_2|d_1^*}(I_2 + m_u^-)m_u^- = 0 \end{aligned}$$

$$\begin{aligned}
& \frac{\partial}{\partial d_1^*} \left[\int_{I_2+m_u^-}^{I_2+m_u^+} F_{D_2|d_1^*}(d_2) \partial d_2 + F_{D_2|d_1^*}(I_2+m_u^+)m_u^+ - F_{D_2|d_1^*}(I_2+m_u^-)m_u^- \right] \\
& + \frac{\partial}{\partial X_2} \left[\int_{I_2+m_u^-}^{I_2+m_u^+} F_{D_2|d_1^*}(d_2) \partial d_2 + F_{D_2|d_1^*}(I_2+m_u^+)m_u^+ - F_{D_2|d_1^*}(I_2+m_u^-)m_u^- \right] = 0 \\
& [(\nu-1)(F_{D_2|d_1^*}(I_2+m_u^+) - F_{D_2|d_1^*}(I_2+m_u^-)) - \rho \frac{\sigma_2}{\sigma_1} \int_{I_2+m_u^-}^{I_2+m_u^+} dF_{D_2|d_1^*}(d_2) \\
& + \nu(F_{D_2|d_1^*}(I_2+m_u^+) - F_{D_2|d_1^*}(I_2+m_u^-))] \partial d_1^* + [-F_{D_2|d_1^*}(I_2+m_u^+) \\
& + F_{D_2|d_1^*}(I_2+m_u^-)] \partial X_2 = 0
\end{aligned}$$

$$\begin{aligned}
(2\nu-1-\rho \frac{\sigma_2}{\sigma_1}) \partial d_1^* &= \partial X_2 \\
\frac{\partial X_2}{\partial d_1^*} &= \frac{1}{\nu}
\end{aligned}$$

Derivation of (3.10):

$$\begin{aligned}
\Pi_{dsoc}^B &= \Pi_{dsoc}^{1,B}(X_1, X_2, X_3) + E_{I_2}[\Pi_{dsoc}^{2,B}(m, I_2)] = E_{D_1}[r \min(D_1, X_1) - c_o(X_3 - X_2) \\
& - c_1 X_1 - c_2(X_2 - X_1) - p_1(D_1 - X_1)^+ - h_1^b(X_1 - D_1)^+] + E_{I_2}[\Pi_{dsoc}^{2,B}(m, I_2)] \\
& = -c_o X_3 - (c_2 - c_o)X_2 + (c_2 - c_1 + p_1)X_1 - p_1 \mu_1 \\
& - (h_1^b + p_1) \int_0^{X_1} (X_1 - d_1) dF_{d_1}(d_1) \\
& + E_{D_1}[\Pi_{dsoc}^{2,B}(m|d_1) + rD_1 - r(D_1 - X_1)^+]
\end{aligned}$$

$$\begin{aligned}
& E_{D_1}[\Pi_{dsoc}^{2,B}(m|d_1) + rD_1 - r(D_1 - X_1)^+] = E_{D_1}[E_{D_2}[r \min(D_2, (I_2 + m)^+ \\
& + r \min((d_1 - X_1)^+, X_2 - X_1 + m) - c_e m^+ - (c_2 - c_p)m^- - p_2(D_2 - I_2 - m)^+ \\
& + v_f^b(I_2 + m - D_2)^+]] \\
& = r \int_0^\infty \int_0^{X_2-d_1+m} (d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
& + r \int_0^{X_2+m} \int_{X_2-d_1+m}^\infty (X_2 - d_1 + m) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
& + r \int_{X_1}^{X_2+m} (d_1 - X_1) dF_{D_1}(d_1) + r \int_{X_2+m}^\infty (X_2 - X_1 + m) dF_{D_1}(d_1) \\
& - c_e \int_0^\infty (m^+) dF_{D_1}(d_1) - (c_2 - c_p) \int_0^\infty (m^-) dF_{D_1}(d_1) \\
& - p_2 \int_0^\infty \int_{X_2-d_1+m}^\infty (d_1 + d_2 - X_2 - m) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
& + v_f^b \int_0^\infty \int_0^{X_2-d_1+m} (X_2 - d_1 - d_2 + m) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) + r \int_0^\infty (d_1) dF_{D_1}(d_1) \\
& - r \int_{X_1}^\infty (d_1 - X_1) dF_{D_1}(d_1) = r \int_0^\infty \int_0^{X_2-d_1+m} (d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
& - r \int_0^\infty \int_0^{X_2-d_1+m} (X_2 - d_1 + m) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
& + r \int_0^{X_2+m} \int_0^\infty (X_2 - d_1 + m) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
& + r \int_{X_2+m}^\infty (X_2 - d_1 + m) dF_{D_1}(d_1) - c_e \int_0^\infty (m^+) dF_{D_1}(d_1) \\
& - (c_2 - c_p) \int_0^\infty (m^-) dF_{D_1}(d_1) \\
& - p_2 \int_0^\infty \int_0^\infty (d_1 + d_2 - X_2 - m) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
& + (v_f^b - p_2) \int_0^\infty \int_0^{X_2-d_1+m} (X_2 - d_1 - d_2 + m) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
& + r \int_0^\infty (d_1) dF_{D_1}(d_1) = r \int_0^\infty \int_0^{X_2-d_1+m} (d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
& - r \int_0^\infty \int_0^{X_2-d_1+m} (X_2 - d_1 + m) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
& + r \int_0^\infty (X_2 - d_1 + m) dF_{D_1}(d_1) - c_e \int_0^\infty (m^+) dF_{D_1}(d_1) \\
& - (c_2 - c_p) \int_0^\infty (m^-) dF_{D_1}(d_1) - p_2(\mu_1 + mu_2 - X_2) + p_2 \int_0^\infty (m) dF_{D_1}(d_1) \\
& + (v_f^b - p_2) \int_0^\infty \int_0^{X_2-d_1+m} (X_2 - d_1 - d_2 + m) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
& + r \int_0^\infty (d_1) dF_{D_1}(d_1)
\end{aligned}$$

$$\begin{aligned}
& E_{D_1}[\Pi_{dsoc}^{2,B}(m|d_1) + rD_1 - r(D_1 - X_1)^+] \\
&= rX_2 + (r - c_e + p_2) \int_0^\infty (m^+) dF_{D_1}(d_1) + (r - c_2 + c_p + p_2) \int_0^\infty (m^-) dF_{D_1}(d_1) \\
&+ (v_f^b - p_2 - r) \int_0^\infty \int_0^{X_2 - d_1 + m} (X_2 - d_1 - d_2 + m) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&- p_2(\mu_1 + \mu_2 - X_2)
\end{aligned}$$

$$\begin{aligned}
\Pi_{dsoc}^B &= -c_o X_3 + (r + p_2 - (c_2 - c_o))X_2 + (c_2 - c_1 + p_1)X_1 - p_1\mu_1 \\
&- (h_1^b + p_1) \int_0^{X_1} (X_1 - d_1) dF_{d_1}(d_1) + (r - c_e + p_2) \int_0^\infty (m^+) dF_{D_1}(d_1) \\
&+ (r - c_2 + c_p + p_2) \int_0^\infty (m^-) dF_{D_1}(d_1) - p_2(\mu_1 + \mu_2) \\
&+ (v_f^b - p_2 - r) \int_0^\infty \int_0^{X_2 - d_1 + m} (X_2 - d_1 - d_2 + m) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1)
\end{aligned}$$

Derivation of (3.11):

$$\begin{aligned}
\Pi_{dsoc}^B(X_1, X_2, X_3) &= -c_o X_3 + (r + p_2 - (c_2 - c_o))X_2 + (c_2 - c_1 + p_1)X_1 - p_1\mu_1 \\
&- (h_1^b + p_1) \int_0^{X_1} (X_1 - d_1) dF_{d_1}(d_1) - p_2(\mu_1 + \mu_2) \\
&+ (p_2 + r - c_2 + c_p) \int_0^{d_1^*} (dd_2 + \nu d_1 - X_2) dF_{D_1}(d_1) \\
&- (p_2 + r - c_2 + c_p) \int_0^{\frac{X_1 - dd_2}{\nu}} (dd_2 + \nu d_1 - X_1) dF_{D_1}(d_1) \\
&+ (p_2 + r - c_e) \int_{d_1^*}^\infty (dd_1 + \nu d_1 - X_2) dF_{D_1}(d_1) \\
&- (p_2 + r - c_e) \int_{\frac{X_3 - dd_1}{\nu}}^\infty (dd_1 + \nu d_1 - X_3) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_0^{\frac{X_1 - dd_2}{\nu}} \int_0^{X_1 - d_1} (X_1 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_{\frac{X_1 - dd_2}{\nu}}^{d_1^*} \int_0^{dd_2 + (\nu - 1)d_1} (dd_2 + (\nu - 1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_{d_1^*}^{\frac{X_3 - dd_1}{\nu}} \int_0^{dd_1 + (\nu - 1)d_1} (dd_1 + (\nu - 1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_{\frac{X_3 - dd_1}{\nu}}^\infty \int_0^{X_3 - d_1} (X_3 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Pi_{dsoc}^B(X_1, X_2, X_3)}{\partial X_2} &= (r + p_2 - c_2 + c_o) - (p_2 + r - c_2 + c_p) \int_0^{d_1^*} dF_{D_1}(d_1) \\
&+ \frac{\partial d_1^*}{\partial X_2} (p_2 + r - c_2 + c_p) m_u^- - (p_2 + r - c_e) \int_{d_1^*}^{\infty} dF_{D_1}(d_1) - \frac{\partial d_1^*}{\partial X_2} (p_2 + r - c_e) m_u^+ \\
&- \frac{\partial d_1^*}{\partial X_2} (p_2 + r - v_f^b) \int_0^{dd_2 + (\nu-1)d_1^*} (dd_2 + (\nu-1)d_1^* - d_2) dF_{D_2|d_1}(d_2) \\
&+ \frac{\partial d_1^*}{\partial X_2} (p_2 + r - v_f^b) \int_0^{dd_1 + (\nu-1)d_1^*} (dd_1 + (\nu-1)d_1^* - d_2) dF_{D_2|d_1}(d_2) \\
&(p_2 + r - c_2 + c_p) m_u^- - (p_2 + r - v_f^b) \int_0^{dd_2 + (\nu-1)d_1^*} (dd_2 + (\nu-1)d_1^* - d_2) dF_{D_2|d_1}(d_2) \\
&- (p_2 + r - c_e) m_u^+ + (p_2 + r - v_f^b) \int_0^{dd_1 + (\nu-1)d_1^*} (dd_1 + (\nu-1)d_1^* - d_2) dF_{D_2|d_1}(d_2) \\
&= 0
\end{aligned}$$

$$\frac{\partial \Pi_{dsoc}^B(X_1, X_2, X_3)}{\partial X_2} = (-c_e + c_2 - c_p) F_{D_1}(d_1^*) + c_e - c_2 + c_o$$

For the case $c_e < (c_2 - c_p)$, second order condition is positive. Therefore total profit of the buyer is convex and in the optimum solution X_2^* is either equal to X_1^* or X_3^* . We find two unconstrained m_u for the case $c_e < (c_2 - c_p)$.

A.6. Proof of Proposition 3.6

Concavity of (3.12):

$$\begin{aligned}
\Pi_{dsoc}^B(X_1, X_2, X_3) &= -c_o X_3 + (r + p_2 - (c_2 - c_o))X_2 + (c_2 - c_1 + p_1)X_1 - p_1 \mu_1 \\
&- (h_1^b + p_1) \int_0^{X_1} (X_1 - d_1) dF_{d_1}(d_1) - p_2(\mu_1 + \mu_2) \\
&+ (p_2 + r - c_2 + c_p) \int_0^{\frac{X_2 - dd_2}{\nu}} (dd_2 + \nu d_1 - X_2) dF_{D_1}(d_1) \\
&- (p_2 + r - c_2 + c_p) \int_0^{\frac{X_1 - dd_2}{\nu}} (dd_2 + \nu d_1 - X_1) dF_{D_1}(d_1) \\
&+ (p_2 + r - c_e) \int_{\frac{X_2 - dd_1}{\nu}}^{\infty} (dd_1 + \nu d_1 - X_2) dF_{D_1}(d_1) \\
&- (p_2 + r - c_e) \int_{\frac{X_3 - dd_1}{\nu}}^{\infty} (dd_1 + \nu d_1 - X_3) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_0^{\frac{X_1 - dd_2}{\nu}} \int_0^{X_1 - d_1} (X_1 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_{\frac{X_1 - dd_2}{\nu}}^{\frac{X_2 - dd_2}{\nu}} \int_0^{dd_2 + (\nu - 1)d_1} (dd_2 + (\nu - 1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_{\frac{X_2 - dd_2}{\nu}}^{\frac{X_2 - dd_1}{\nu}} \int_0^{X_2 - d_1} (X_2 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_{\frac{X_2 - dd_1}{\nu}}^{\frac{X_3 - dd_1}{\nu}} \int_0^{dd_1 + (\nu - 1)d_1} (dd_1 + (\nu - 1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_{\frac{X_3 - dd_1}{\nu}}^{\infty} \int_0^{X_3 - d_1} (X_3 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1)
\end{aligned}$$

$$\begin{aligned}
J_1^b(X_1) &= (c_2 - c_1 + p_1)X_1 - (h_1^b + p_1) \int_0^{X_1} (X_1 - d_1) dF_{d_1}(d_1) \\
&- (p_2 + r - c_2 + c_p) \int_0^{\frac{X_1 - dd_2}{\nu}} (dd_2 + \nu d_1 - X_1) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_0^{\frac{X_1 - dd_2}{\nu}} \int_0^{X_1 - d_1} (X_1 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&+ (p_2 + r - v_f^b) \int_0^{\frac{X_1 - dd_2}{\nu}} \int_0^{dd_2 + (\nu - 1)d_1} (dd_2 + (\nu - 1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1)
\end{aligned}$$

$$\begin{aligned}
J_2^b(X_2) &= (r + p_2 - (c_2 - c_o))X_2 \\
&+ (p_2 + r - c_2 + c_p) \int_0^{\frac{X_2 - dd_2}{\nu}} (dd_2 + \nu d_1 - X_2) dF_{D_1}(d_1) \\
&+ (p_2 + r - c_e) \int_{\frac{X_2 - dd_1}{\nu}}^{\infty} (dd_1 + \nu d_1 - X_2) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_0^{\frac{X_2 - dd_2}{\nu}} \int_0^{dd_2 + (\nu - 1)d_1} (dd_2 + (\nu - 1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_{\frac{X_2 - dd_1}{\nu}}^{\frac{X_2 - dd_2}{\nu}} \int_0^{X_2 - d_1} (X_2 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&+ (p_2 + r - v_f^b) \int_0^{\frac{X_2 - dd_1}{\nu}} \int_0^{dd_1 + (\nu - 1)d_1} (dd_1 + (\nu - 1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1)
\end{aligned}$$

$$\begin{aligned}
J_3^b(X_3) &= -c_o X_3 - (p_2 + r - c_e) \int_{\frac{X_3 - dd_1}{\nu}}^{\infty} (dd_1 + \nu d_1 - X_3) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_0^{\frac{X_3 - dd_1}{\nu}} \int_0^{dd_1 + (\nu - 1)d_1} (dd_1 + (\nu - 1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_{\frac{X_3 - dd_1}{\nu}}^{\infty} \int_0^{X_3 - d_1} (X_3 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial J_1^b(X_1)}{\partial X_1} &= -(h_1^b + p_1) F_{D_1}(X_1) + p_1 + c_2 - c_1 + (p_2 + r - c_2 + c_p) F_{D_1}\left(\frac{X_1 - dd_2}{\nu}\right) \\
&- (p_2 + r - v_f^b) \int_0^{\frac{X_1 - dd_2}{\nu}} F_{D_2|d_1}(X_1 - d_1) dF_{D_1}(d_1)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial J_2^b(X_2)}{\partial X_2} &= -c_2 + c_o + c_e - (p_2 + r - c_2 + c_p) F_{D_1}\left(\frac{X_2 - dd_2}{\nu}\right) \\
&+ (p_2 + r - c_e) F_{D_1}\left(\frac{X_2 - dd_1}{\nu}\right) - (p_2 + r - v_f^b) \int_{\frac{X_2 - dd_2}{\nu}}^{\frac{X_2 - dd_1}{\nu}} F_{D_2|d_1}(X_2 - d_1) dF_{D_1}(d_1)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial J_3^b(X_3)}{\partial X_3} &= -c_o + p_2 + r - c_e - (p_2 + r - c_e) F_{D_1}\left(\frac{X_3 - dd_1}{\nu}\right) \\
&- (p_2 + r - v_f^b) \int_{\frac{X_3 - dd_1}{\nu}}^{\infty} F_{D_2|d_1}(X_3 - d_1) dF_{D_1}(d_1)
\end{aligned}$$

$$F_{D_2|\frac{X_i-dd_j}{\nu}}(X_i - \frac{X_i-dd_j}{\nu}) = F_{D_2|\frac{X_i-dd_j}{\nu}}(I_2 + m) = \Phi(k_j)$$

for $i \in \{1, 2, 3\} j \in \{1, 2\}$

$$\frac{\partial^2 J_1^b(X_1)}{\partial X_1^2} = -(h_1^b + p_1)f_{D_1}(X_1)$$

$$-(p_2 + r - v_f^b) \int_0^{\frac{X_1-dd_2}{\nu}} f_{D_2/d_1}(X_1 - d_1)dF_{D_1}(d_1) \leq 0$$

$$\frac{\partial^2 J_2^b(X_2)}{\partial X_2^2} = -(p_2 + r - v_f^b) \int_{\frac{X_2-dd_2}{\nu}}^{\frac{X_2-dd_1}{\nu}} f_{D_2/d_1}(X_2 - d_1)dF_{D_1}(d_1) \leq 0$$

$$\frac{\partial^2 J_3^b(X_3)}{\partial X_3^2} = -(p_2 + r - v_f^b) \int_{\frac{X_3-dd_1}{\nu}}^{\infty} f_{D_2/d_1}(X_3 - d_1)df_{D_1}(d_1) \leq 0$$

Concavity of (3.13):

$$\begin{aligned} \Pi_{dsoc}^B(X_1, X_3) &= -c_o X_3 + (r + p_2 + p_1 - (c_1 - c_o))X_1 - p_1 \mu_1 \\ &- (h_1^b + p_1) \int_0^{X_1} (X_1 - d_1)dF_{D_1}(d_1) - p_2(\mu_1 + \mu_2) \\ &+ (p_2 + r - c_e) \int_{\frac{X_1-dd_1}{\nu}}^{\infty} (dd_1 + \nu d_1 - X_1)dF_{D_1}(d_1) \\ &- (p_2 + r - c_e) \int_{\frac{X_3-dd_1}{\nu}}^{\infty} (dd_1 + \nu d_1 - X_3)dF_{D_1}(d_1) \\ &- (p_2 + r - v_f^b) \int_0^{\frac{X_1-dd_1}{\nu}} \int_0^{X_1-d_1} (X_1 - d_1 - d_2)dF_{D_2|d_1}(d_2)dF_{D_1}(d_1) \\ &- (p_2 + r - v_f^b) \int_{\frac{X_1-dd_1}{\nu}}^{\frac{X_3-dd_1}{\nu}} \int_0^{dd_1+(\nu-1)d_1} (dd_1 + (\nu-1)d_1 - d_2)dF_{D_2|d_1}(d_2)dF_{D_1}(d_1) \\ &- (p_2 + r - v_f^b) \int_{\frac{X_3-dd_1}{\nu}}^{\infty} \int_0^{X_3-d_1} (X_3 - d_1 - d_2)dF_{D_2|d_1}(d_2)dF_{D_1}(d_1) \end{aligned}$$

$$\begin{aligned}
J_1^b(X_1) &= (r + p_2 + p_1 - (c_1 - c_o))X_1 \\
&- (h_1^b + p_1) \int_0^{X_1} (X_1 - d_1) dF_{d_1}(d_1) \\
&+ (p_2 + r - c_e) \int_{\frac{X_1 - dd_1}{\nu}}^{\infty} (dd_1 + \nu d_1 - X_1) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_0^{\frac{X_1 - dd_1}{\nu}} \int_0^{X_1 - d_1} (X_1 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&+ (p_2 + r - v_f^b) \int_0^{\frac{X_1 - dd_1}{\nu}} \int_0^{dd_1 + (\nu - 1)d_1} (dd_1 + (\nu - 1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial J_1^b(X_1)}{\partial X_1} &= -(h_1^b + p_1)F_{D_1}(X_1) + p_1 + c_o + c_e - c_1 + (p_2 + r - c_e)F_{D_1}\left(\frac{X_1 - dd_1}{\nu}\right) \\
&- (p_2 + r - v_f^b) \int_0^{\frac{X_1 - dd_1}{\nu}} F_{D_2|d_1}(X_1 - d_1) dF_{D_1}(d_1)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 J_1^b(X_1)}{\partial X_1^2} &= -(h_1^b + p_1)f_{D_1}(X_1) \\
&- (p_2 + r - v_f^b) \int_0^{\frac{X_1 - dd_1}{\nu}} f_{D_2|d_1}(X_1 - d_1) dF_{D_1}(d_1) \leq 0
\end{aligned}$$

Concavity of (3.14):

$$\begin{aligned}
\Pi_{dsoc}^B(X_1, X_3) &= (r + p_2 - c_2)X_3 + (p_1 + c_2 - c_1)X_1 - p_1\mu_1 \\
&- (h_1^b + p_1) \int_0^{X_1} (X_1 - d_1) dF_{d_1}(d_1) - p_2(\mu_1 + \mu_2) \\
&+ (p_2 + r - c_2 + c_p) \int_0^{\frac{X_3 - dd_2}{\nu}} (dd_2 + \nu d_1 - X_3) dF_{D_1}(d_1) \\
&- (p_2 + r - c_2 + c_p) \int_0^{\frac{X_1 - dd_2}{\nu}} (dd_2 + \nu d_1 - X_1) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_0^{\frac{X_1 - dd_2}{\nu}} \int_0^{X_1 - d_1} (X_1 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_{\frac{X_3 - dd_2}{\nu}}^{\frac{X_1 - dd_2}{\nu}} \int_0^{dd_2 + (\nu - 1)d_1} (dd_2 + (\nu - 1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_{\frac{X_3 - dd_2}{\nu}}^{\infty} \int_0^{X_3 - d_1} (X_3 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1)
\end{aligned}$$

$$\begin{aligned}
J_3^b(X_3) &= (r + p_2 - c_2)X_3 \\
&+ (p_2 + r - c_2 + c_p) \int_0^{\frac{X_3 - dd_2}{\nu}} (dd_2 + \nu d_1 - X_3) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_0^{\frac{X_3 - dd_2}{\nu}} \int_0^{dd_2 + (\nu - 1)d_1} (dd_2 + (\nu - 1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_{\frac{X_3 - dd_2}{\nu}}^{\infty} \int_0^{X_3 - d_1} (X_3 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial J_3^b(X_3)}{\partial X_3} &= -c_2 + p_2 + r - (p_2 + r - c_2 + c_p) F_{D_1}\left(\frac{X_3 - dd_2}{\nu}\right) \\
&- (p_2 + r - v_f^b) \int_{\frac{X_3 - dd_2}{\nu}}^{\infty} F_{D_2|d_1}(X_3 - d_1) dF_{D_1}(d_1)
\end{aligned}$$

$$\frac{\partial^2 J_3^b(X_3)}{\partial X_3^2} = -(p_2 + r - v_f^b) \int_{\frac{X_3 - dd_2}{\nu}}^{\infty} f_{D_2/d_1}(X_3 - d_1) dF_{D_1}(d_1) \leq 0$$

A.7. Proof of Proposition 3.8

Derivation of (3.16):

$$\begin{aligned}
\frac{\partial \Pi_{CS}^2(X_2^c, I_c)}{\partial X_2^c} &= \frac{\partial}{\partial X_2^c} E_{D_2/d_1} [r \int_{X_1^c + X_2^c - d_1}^{\infty} (X_1^c + X_2^c - d_1) dF_{D_2|d_1}(d_2) \\
&+ r \int_0^{X_1^c + X_2^c - d_1} (d_2) dF_{D_2|d_1}(d_2) - p_2 \int_{X_1^c + X_2^c - d_1}^{\infty} (d_1 + d_2 - X_1^c - X_2^c) dF_{D_2|d_1}(d_2) \\
&+ v_f^b \int_0^{X_1^c + X_2^c - d_1} (X_1^c + X_2^c - d_1 - d_2) dF_{D_2|d_1}(d_2) - c_L(1 + \gamma)(X_2^c - (X_L^c - X_1^c))^+ \\
&+ v_r^s(X_3^c - X_1^c - \max(X_2^c, (X_L^c - X_1^c))) + v_f^s(X_L^c - X_1^c - X_2^c)^+]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Pi_{CS}^2(X_2^c, I_c)}{\partial X_2^c} &= p_2 + r - v_f^s + (v_f^s - v_r^s - c_L(1 + \gamma)) 1_{X_2^c > X_L^c - X_1^c} \\
&- (p_2 + r - v_f^b) F_{D_2|d_1}(X_1^c + X_2^c - d_1)
\end{aligned}$$

We find two first order condition. Therefore we have two unconstrained X_{2u}^c .

A.8. Proof of Proposition 3.7

Concavity of (3.15):

$$\frac{\partial^2 \Pi_{CS}^2(X_2^c, I_c)}{\partial (X_2^c)^2} = -(p_2 + r - v_f^b) f_{D_2/d_1}(X_1^c + X_2^c - d_1) \leq 0$$

A.9. Proof of Proposition 3.10

Concavity of (3.17):

$$\begin{aligned} \Pi_{CS}(X_1^c, X_L^c, X_3^c) &= (p_2 + p_1 + h_s + r - v_f^s) X_1^c + (v_f^s - v_r^s - c_L - h_s) X_L^c \\ &+ (v_r^s - c_r) X_3^c - p_2(\mu_1 + \mu_2) - p_1 \mu_1 - (h_1^b + p_1) \int_0^{X_1^c} (X_1^c - d_1) dF_{d_1}(d_1) \\ &+ (p_2 + r - v_f^s) X_2^c + (v_f^s - v_r^s - c_L(1 + \gamma))(X_1^c + X_2^c - X_L^c)^+ \\ &- (p_2 + r - v_f^b) \int_0^{X_1^c + X_2^c - d_1} (X_1^c + X_2^c - d_1 - d_2) dF_{D_2|d_1}(d_2) \end{aligned}$$

$$\begin{aligned}
\Pi_{CS}(X_1^c, X_L^c, X_3^c) &= (p_2 + p_1 + h_s + r - v_f^s)X_1^c + (v_f^s - v_r^s - c_L - h_s)X_L^c \\
&+ (v_r^s - c_r)X_3^c - p_2(\mu_1 + \mu_2) - p_1\mu_1 - (h_1^b + p_1) \int_0^{X_1^c} (X_1^c - d_1)dF_{d_1}(d_1) \\
&+ (p_2 + r - v_f^s) \int_0^{\frac{X_L^c - dd_4}{\nu}} (dd_4 + \nu d_1 - X_L^c)dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^s) \int_0^{\frac{X_1^c - dd_4}{\nu}} (dd_4 + \nu d_1 - X_1^c)dF_{D_1}(d_1) \\
&+ (p_2 + r - v_r^s - c_L(1 + \gamma)) \int_{\frac{X_L^c - dd_3}{\nu}}^{\infty} (dd_3 + \nu d_1 - X_L^c)dF_{D_1}(d_1) \\
&- (p_2 + r - v_r^s - c_L(1 + \gamma)) \int_{\frac{X_3^c - dd_3}{\nu}}^{\infty} (dd_3 + \nu d_1 - X_3^c)dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_0^{\frac{X_1^c - dd_4}{\nu}} \int_0^{X_1^c - d_1} (X_1^c - d_1 - d_2)dF_{D_2|d_1}(d_2)dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_{\frac{X_1^c - dd_4}{\nu}}^{\frac{X_L^c - dd_4}{\nu}} \int_0^{dd_4 + (\nu - 1)d_1} (dd_4 + (\nu - 1)d_1 - d_2)dF_{D_2|d_1}(d_2)dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_{\frac{X_L^c - dd_4}{\nu}}^{\frac{X_L^c - dd_3}{\nu}} \int_0^{X_L^c - d_1} (X_L^c - d_1 - d_2)dF_{D_2|d_1}(d_2)dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_{\frac{X_L^c - dd_3}{\nu}}^{\frac{X_3^c - dd_3}{\nu}} \int_0^{dd_3 + (\nu - 1)d_1} (dd_3 + (\nu - 1)d_1 - d_2)dF_{D_2|d_1}(d_2)dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_{\frac{X_3^c - dd_3}{\nu}}^{\infty} \int_0^{X_3^c - d_1} (X_3^c - d_1 - d_2)dF_{D_2|d_1}(d_2)dF_{D_1}(d_1)
\end{aligned}$$

$$\begin{aligned}
J_1^c(X_1^c) &= (p_2 + p_1 + h_s + r - v_f^s)X_1^c \\
&- (h_1^b + p_1) \int_0^{X_1^c} (X_1^c - d_1)dF_{d_1}(d_1) \\
&- (p_2 + r - v_f^s) \int_0^{\frac{X_1^c - dd_4}{\nu}} (dd_4 + \nu d_1 - X_1^c)dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_0^{\frac{X_1^c - dd_4}{\nu}} \int_0^{X_1^c - d_1} (X_1^c - d_1 - d_2)dF_{D_2|d_1}(d_2)dF_{D_1}(d_1) \\
&+ (p_2 + r - v_f^b) \int_0^{\frac{X_1^c - dd_4}{\nu}} \int_0^{dd_4 + (\nu - 1)d_1} (dd_4 + (\nu - 1)d_1 - d_2)dF_{D_2|d_1}(d_2)dF_{D_1}(d_1)
\end{aligned}$$

$$\begin{aligned}
J_L^c(X_L^c) &= (v_f^s - v_r^s - c_L - h_s)X_L^c \\
&+ (p_2 + r - v_f^s) \int_0^{\frac{X_L^c - dd_4}{\nu}} (dd_4 + \nu d_1 - X_L^c) dF_{D_1}(d_1) \\
&+ (p_2 + r - v_r^s - c_L(1 + \gamma)) \int_{\frac{X_L^c - dd_3}{\nu}}^{\infty} (dd_3 + \nu d_1 - X_L^c) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_0^{\frac{X_L^c - dd_4}{\nu}} \int_0^{dd_4 + (\nu-1)d_1} (dd_4 + (\nu-1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_{\frac{X_L^c - dd_4}{\nu}}^{\frac{X_L^c - dd_3}{\nu}} \int_0^{X_L^c - d_1} (X_L^c - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&+ (p_2 + r - v_f^b) \int_0^{\frac{X_L^c - dd_3}{\nu}} \int_0^{dd_3 + (\nu-1)d_1} (dd_3 + (\nu-1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1)
\end{aligned}$$

$$\begin{aligned}
J_3^c(X_3^c) &= (v_r^s - c_r)X_3^c \\
&- (p_2 + r - v_r^s - c_L(1 + \gamma)) \int_{\frac{X_3^c - dd_3}{\nu}}^{\infty} (dd_3 + \nu d_1 - X_3^c) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_0^{\frac{X_3^c - dd_3}{\nu}} \int_0^{dd_3 + (\nu-1)d_1} (dd_3 + (\nu-1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&- (p_2 + r - v_f^b) \int_{\frac{X_3^c - dd_3}{\nu}}^{\infty} \int_0^{X_3^c - d_1} (X_3^c - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial J_1^c(X_1^c)}{\partial X_1^c} &= -(h_1^b + p_1)F_{D_1}(X_1^c) + p_1 + p_1 + h_s + (p_2 + r - v_f^s)F_{D_1}\left(\frac{X_1^c - dd_4}{\nu}\right) \\
&- (p_2 + r - v_f^b) \int_0^{\frac{X_1^c - dd_4}{\nu}} F_{D_2|d_1}(X_1^c - d_1) dF_{D_1}(d_1)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial J_2^c(X_L^c)}{\partial X_L^c} &= +c_L\gamma - h_s - (p_2 + r - v_f^s)F_{D_1}\left(\frac{X_L^c - dd_4}{\nu}\right) \\
&+ (p_2 + r - v_r^s - c_L(1 + \gamma))F_{D_1}\left(\frac{X_L^c - dd_3}{\nu}\right) \\
&- (p_2 + r - v_f^b) \int_{\frac{X_L^c - dd_4}{\nu}}^{\frac{X_L^c - dd_3}{\nu}} F_{D_2|d_1}(X_L^c - d_1) dF_{D_1}(d_1)
\end{aligned}$$

$$\begin{aligned} \frac{\partial J_3^c(X_3^c)}{\partial X_3^c} &= p_2 + r - c_r - c_L(1 + \gamma) - (p_2 + r - v_r^s - c_L(1 + \gamma))F_{D_1}\left(\left(\frac{X_3^c - dd_3}{\nu}\right)\right) \\ &- (p_2 + r - v_f^b) \int_{\frac{X_3^c - dd_3}{\nu}}^{\infty} F_{D_2|d_1}(X_3 - d_1) dF_{D_1}(d_1) \end{aligned}$$

$$\begin{aligned} F_{D_2|\frac{X_i^c - dd_j}{\nu}}(X_i^c - \frac{X_i^c - dd_j}{\nu}) &= F_{D_2|\frac{X_i^c - dd_j}{\nu}}(I_c + X_2^c) = \Phi(k_j) \\ \text{for } i \in \{1, L, 3\} j \in \{3, 4\} \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 J_1^c(X_1^c)}{\partial (X_1^c)^2} &= -(h_1^b + p_1)f_{D_1}(X_1) \\ &- (p_2 + r - v_f^b) \int_0^{\frac{X_1 - dd_4}{\nu}} f_{D_2/d_1}(X_1 - d_1) dF_{D_1}(d_1) \leq 0 \end{aligned}$$

$$\frac{\partial^2 J_2^c(X_L^c)}{\partial (X_L^c)^2} = -(p_2 + r - v_f^b) \int_{\frac{X_L^c - dd_4}{\nu}}^{\frac{X_L^c - dd_3}{\nu}} f_{D_2/d_1}(X_L - d_1) dF_{D_1}(d_1) \leq 0$$

$$\frac{\partial^2 J_3^c(X_3^c)}{\partial (X_3^c)^2} = -(p_2 + r - v_f^b) \int_{\frac{X_3^c - dd_3}{\nu}}^{\infty} f_{D_2/d_1}(X_3 - d_1) dF_{D_1}(d_1) \leq 0$$

Concavity of (3.18):

$$\begin{aligned}
\Pi_{CS}(X_1^c, X_3^c) &= (p_2 + p_1 + h_s + r - v_f^s)X_1^c + (v_f^s - c_r - c_L - h_s)X_3^c \\
&\quad - p_2(\mu_1 + \mu_2) - p_1\mu_1 - (h_1^b + p_1) \int_0^{X_1^c} (X_1^c - d_1)dF_{d_1}(d_1) \\
&\quad + (p_2 + r - v_f^s) \int_0^{\frac{X_3^c - dd_4}{\nu}} (dd_4 + \nu d_1 - X_3^c)dF_{D_1}(d_1) \\
&\quad - (p_2 + r - v_f^s) \int_0^{\frac{X_1^c - dd_4}{\nu}} (dd_4 + \nu d_1 - X_1^c)dF_{D_1}(d_1) \\
&\quad - (p_2 + r - v_f^b) \int_0^{\frac{X_1^c - dd_4}{\nu}} \int_0^{X_1^c - d_1} (X_1^c - d_1 - d_2)dF_{D_2|d_1}(d_2)dF_{D_1}(d_1) \\
&\quad - (p_2 + r - v_f^b) \int_{\frac{X_1^c - dd_4}{\nu}}^{\frac{X_3^c - dd_4}{\nu}} \int_0^{dd_4 + (\nu-1)d_1} (dd_4 + (\nu-1)d_1 - d_2)dF_{D_2|d_1}(d_2)dF_{D_1}(d_1) \\
&\quad - (p_2 + r - v_f^b) \int_{\frac{X_3^c - dd_4}{\nu}}^{\infty} \int_0^{X_3^c - d_1} (X_3^c - d_1 - d_2)dF_{D_2|d_1}(d_2)dF_{D_1}(d_1)
\end{aligned}$$

$$\begin{aligned}
J_3^c(X_3^c) &= (v_f^s - c_r - c_L - h_s)X_3^c \\
&\quad + (p_2 + r - v_f^s) \int_0^{\frac{X_3^c - dd_4}{\nu}} (dd_4 + \nu d_1 - X_3^c)dF_{D_1}(d_1) \\
&\quad - (p_2 + r - v_f^b) \int_0^{\frac{X_3^c - dd_4}{\nu}} \int_0^{dd_4 + (\nu-1)d_1} (dd_4 + (\nu-1)d_1 - d_2)dF_{D_2|d_1}(d_2)dF_{D_1}(d_1) \\
&\quad - (p_2 + r - v_f^b) \int_{\frac{X_3^c - dd_4}{\nu}}^{\infty} \int_0^{X_3^c - d_1} (X_3^c - d_1 - d_2)dF_{D_2|d_1}(d_2)dF_{D_1}(d_1)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial J_3^c(X_3^c)}{\partial X_3^c} &= v_f^s - c_r - c_L - h_s - (p_2 + r - v_f^s)F_{D_1}\left(\frac{X_3^c - dd_4}{\nu}\right) \\
&\quad - (p_2 + r - v_f^b) \int_{\frac{X_3^c - dd_4}{\nu}}^{\infty} F_{D_2|d_1}(X_3^c - d_1)dF_{D_1}(d_1)
\end{aligned}$$

$$\frac{\partial^2 J_3^c(X_3^c)}{\partial (X_3^c)^2} = -(p_2 + r - v_f^b) \int_{\frac{X_3^c - dd_4}{\nu}}^{\infty} f_{D_2|d_1}(X_3^c - d_1)dF_{D_1}(d_1) \leq 0$$

A.10. Proof of Proposition 3.11

Conditions that will coordinate the model will depend on the cost and salvage parameters. Therefore we will examine the coordination according to those parameters. Channel coordination is achieved by equating production decisions (X_L, X_L^c) , raw material gatherings (X_3, X_3^c) , the first period shipment quantities (X_1, X_1^c) , and the second period shipment quantities $(X_2 - X_1 + m, X_2^c)$. We first focus on to equating the second period shipment quantities as it is the hardest part to equate.

$$\text{Condition 1: } v_f^s \leq v_f^b \text{ and } v_r^s + c_L + h_s > v_f^b,$$

In the centralized system the shipment quantity for the second period can be expressed according to the cost conditions and first period demand realization as follows. The requirements that will make our model behave like the centralized system has been itemized below:

- If $0 \leq d_1 < \frac{(X_L^c - dd_3)}{\nu}$ then $X_2^{c*} = X_L^c - X_1^c$
- If $\frac{(X_L^c - dd_3)}{\nu} \leq d_1 < \frac{(X_3^c - dd_3)}{\nu}$ then $X_2^{c*} = dd_3 + \nu d_1 - X_1^c$
- If $\frac{(X_3^c - dd_3)}{\nu} \leq d_1 < \infty$ then $X_2^{c*} = X_3^c - X_1^c$

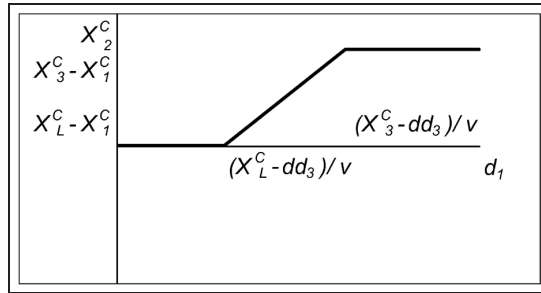


Figure A.1. X_2^c related to d_1 for the case $v_f^s \leq v_f^b$ and $v_r^s + c_L + h_s > v_f^b$.

In order to coordinate under this condition, we need to use either options or order cancellation because there is a single slope. As the X_2^c does not have zero value for any d_1 , in order to coordinate, we need to use options and no order cancellation.

- In order to equate the second period shipment quantity, $m + X_2 - X_1$ should

be equal to X_2^c . In order to ensure this we need to have $c_e = v_r^s + c_L(1 + \gamma)$, and $X_2 = X_L^c$. In order to make $X_2 = X_L^c$, we need to equate related first order conditions. Therefore, we need to have $c_o + c_e - c_2 = c_L\gamma - h_s$. In order not to have order cancellation we need to have $c_2 - c_p < v_f^b$.

- In-house production X_L in our model should be X_2 in order to equate in-house production quantities of our model and centralized system, because we have equated X_2 to the X_L^c above. Because of the boundary conditions, X_L has already been equal to X_2 .
- In order to equate the first period shipment quantities, we need to equate first order conditions related to the first period shipment quantities. Therefore we need to have $c_2 - c_1 = h_s$.
- In order to equate the total raw material gatherings, we need to have $c_o + c_e = c_r + c_L(1 + \gamma)$ which will equate first order conditions.

Condition 2: $v_f^s \leq v_f^b$ and $v_r^s + c_L + h_s \leq v_f^b$,

- For all d_1 , $X_2^{c*} = X_3^c - X_1^c$

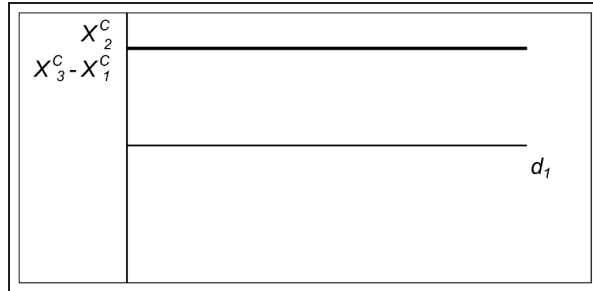


Figure A.2. X_2^c related to d_1 for the case $v_f^s \leq v_f^b$ and $v_r^s + c_L + h_s \leq v_f^b$.

In order to coordinate under this condition, three different cases can be used, as the second period shipment quantity can be purchased only by options, by order and options or only by order.

- The second period shipment quantity for centralized system is $X_3^c - X_1^c$ under this condition. In order to make it the same also in our model we need to have $c_e < v_f^b$ if X_2^c is not equal to X_3^c . Therefore the supplier will exercise all options.

In addition, in order not to have order cancellation, the revenue that the buyer gets when he cancels a unit of order quantity ($c_2 - c_p$) should be less than v_f^b if X_2^c is not equal to X_1^c or in other words if there is an order to cancel.

- In-house production is X_3^c for the centralized system. As $c_e < v_f^b$ if X_2^c is not equal to X_3^c , and we have prevented order cancellation, $X_L = X_3$ also in our model.
- In order to equate the first period shipment quantities we must define a difference first. If the quantities for the second period are attained only by using options ($c_e + c_o < c_2$ and $c_p > c_o$) then we need to have $c_o + c_e - c_1 = h_s$ in order to equate first order conditions related to the first period shipments. Otherwise ($c_e + c_o \geq c_2$) we need to have $c_2 - c_1 = h_s$.
- In order to equate the total raw material gatherings, we need to have $c_o + c_e = c_r + c_L + h_s$, if there is any option used. Otherwise, we need to have $c_2 = c_r + c_L + h_s$ in order to equate first order conditions.

Condition 3: $v_f^s > v_f^b$ and $v_r^s + c_L + h_s \leq v_f^s$,

- If $0 \leq d_1 < \frac{(X_1^c - dd_4)}{\nu}$ then $X_2^{c*} = 0$
- If $\frac{(X_1^c - dd_4)}{\nu} \leq d_1 < \frac{(X_3^c - dd_4)}{\nu}$ then $X_2^{c*} = dd_4 + \nu d_1 - X_1^c$
- If $\frac{(X_3^c - dd_3)}{\nu} \leq d_1 < \infty$ then $X_2^{c*} = X_3^c - X_1^c$

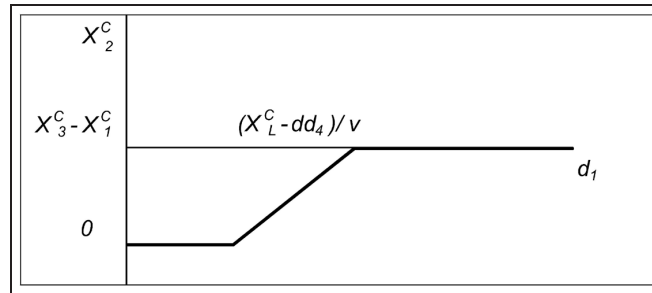


Figure A.3. X_2^c related to d_1 for the case $v_f^s > v_f^b$ and $v_r^s + c_L + h_s \leq v_f^s$.

In order to coordinate under this condition there are two different ways that can be used, as the slope of the second period shipment quantity which we see in the Figure A.3, can be made by only options (the first way) or by only order cancellation (second way).

- In order to equate second period shipment quantity, $m + X_2 - X_1$ should be equal to X_2^c . In order to do this, in the first way, we need to have $c_e = v_f^s$ and $X_2 = X_1$ ($(c_o + c_e) < c_2$ and $c_p > c_o$). In the second way in order to equate $m + X_2 - X_1$ and X_2^c , we need to have , $(c_2 - cp) = v_f^s$ and $X_2 = X_3$ ($(c_o + c_e) > c_2$ and $c_e < v_f^b$).
- In-house production is X_3^c for the centralized system, as $v_r^s + c_L + h_s \leq v_f^s$. For the same reason, X_L will also be the maximum value possible, which is X_3 .
- In the first way, in order to equate the first period shipment quantities, as the quantities for the second period are attained only by using options, we need to have $c_o + c_e - c_1 = h_s$. In the second way as there is no order used, we need to have $c_2 - c_1 = h_s$.
- In the first way, in order to equate the total raw material gatherings, as there is option used, we need to have $c_o + c_e = c_r + c_L + h_s$. In the second way as there is no option used we need to have $c_2 = c_r + c_L + h_s$.

Condition 4: $v_f^s > v_f^b$ and $v_r^s + c_L + h_s > v_f^s$,

- If $0 \leq d_1 < \frac{(X_1^c - dd_4)}{\nu}$ then $X_2^{c*} = 0$
- If $\frac{(X_1^c - dd_4)}{\nu} \leq d_1 < \frac{(X_L^c - dd_4)}{\nu}$ then $X_2^{c*} = dd_4 + \nu d_1 - X_1^c$
- If $\frac{(X_L^c - dd_4)}{\nu} \leq d_1 < \frac{(X_L^c - dd_3)}{\nu}$ then $X_2^{c*} = X_L^c - X_1^c$
- If $\frac{(X_L^c - dd_3)}{\nu} \leq d_1 < \frac{(X_3^c - dd_3)}{\nu}$ then $X_2^{c*} = dd_3 + \nu d_1 - X_1^c$
- If $\frac{(X_3^c - dd_3)}{\nu} \leq d_1 < \infty$ then $X_2^{c*} = X_3^c - X_1^c$

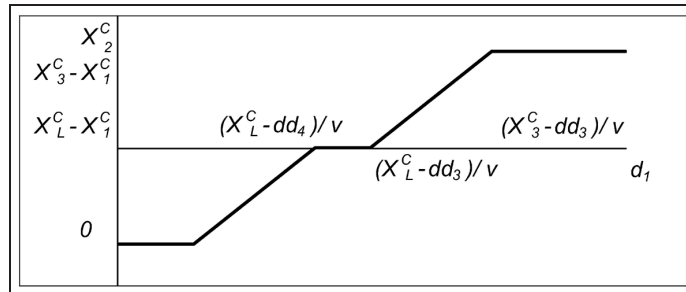


Figure A.4. X_2^c related to d_1 for the case $v_f^s > v_f^b$ and $v_r^s + c_L + h_s > v_f^s$.

- Under this condition, both options and order cancellation should be used. In order to equate the second period shipment quantity, $m + X_2 - X_1$ should be equal to X_2^c . In order to ensure this, we need to have $c_e = v_r^s + c_L(1 + \gamma)$ and

$$(c_2 - cp) = v_f^s.$$

- In order to equate in-house production quantities of our model and centralized system, we need to have $c_o + c_e - c_2 = c_L(\gamma) - h_s$.
- In order to equate the first period shipments, we need to have $c_2 - c_1 = h_s$.
- In order to equate the total raw material gatherings we need to have $c_o + c_e = c_r + c_L(1 + \gamma)$.

We see that for each case c_1 is equal to $c_r + c_L$ which is the cost of supplying up to X_1 units of good. For the cases where there is a second period order ($X_2 \neq X_1$), we have the equality of $c_2 = c_r + c_L + h_s$. For these cases as $X_L \geq X_2$, the cost of supplying up to X_2 units of good is also $c_r + c_L + h_s$.

There can be exercised options in the cases where $X_2 \neq X_3$. In these cases we see that if the products have been produced in-house already ($X_L = X_3$), $c_o + c_e$ is equal to $c_r + c_L + h_s$. In addition to this, if the products have not been produced yet ($X_L = X_2$), $c_o + c_e$ is equal to $c_r + c_L(1 + \gamma)$. Therefore the revenue received from the exercised options, is equal to the cost of supplying that quantity.

For the unexercised options, the supplier gets $c_o + v_f^s (= c_r + c_L + h_s)$ if the products have been produced in-house already. For the raw material inventory at the end of the season, the supplier gets $c_o + v_r^s (= c_r)$. For the cancelled orders, the supplier gets $c_p + v_f^s (= c_r + c_L + h_s)$ as they are already produced in all cases.

We see that the revenue received by the supplier is equal to the cost of her. Therefore under the prices that are shown in the Table 3.1, channel can not coordinate as the individual rationality of the supplier is violated.

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