OPTIMAL CONTROL OF A SYSTEM WITH TWO SERVERS AND TWO CUSTOMER CLASSES UNDER A FLEXIBLE SERVER

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ABSTRACT

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Many organizations such as manufacturing companies, banks, hospitals, airlines, restaurants, telecommunication companies use queuing systems to allocate resources in order to manage demand effectively. Besides the efficiency, industries use queuing systems also to provide a high level of service quality. Industries use different types of queuing systems according to their needs. We consider a queuing system that can be used at branches of banks. At a branch, there is always arrival of customers needed to serve at different service points upon their transaction. Managing branches requires use of resources effectively. This leads to obtain flexible staff in order to manage trade off between service quality and efficiency. In general, flexible server is the person doing both back office and front office activities while dedicated server (teller) is just responsible of doing the front office activities. We consider a dynamic scheduling of a queuing system with two servers where one of the server is dedicated and the other server is flexible. There are also two types of customers arriving to the system. Server one is dedicated to give service to type-1 customer, while server two is flexible to give service to both types of customers. Customers arrive to the system according to a poisson process and service rates are exponentially distributed. The system consists of waiting and abandonment costs of customers. We try to optimize the system by minimizing the cost while assigning the servers to the customers in the system. In the optimal solution of the problem we observe different types of outputs from dynamic scheduling model which are mainly names as static, linear switching and threshold models. Samples from the experiment that reflect these patterns are shown. In addition, we also construct some heuristics and compare the results with optimal policy.

ÖZET

BİR KAYNAĞIN ESNEK OLARAK ÇALIŞTIĞI İKİ KAYNAKLI VE İKİ MÜŞTERİ TİPİNİN YER ALDIĞI SİSTEMDE OPTİMAL KONTROLÜ SAĞLAMA

Üretim firmaları, bankalar,hastaneler, havayolu şirketleri, restoranlar, telekomünikasyon firmaları gibi pek çok organizasyon, talepleri ve kaynakları etkin kullanmak, yüksek hizmet kalitesi sunabilmek için kuyruk sistemini kullanırlar. Bu çalışmada bankalarda kullanılabilecek bir sistemi konu edindik. Bankalarda, müşterilerin farklı hizmet noktalarında işlemlerini yürüttüğü bir sistem mevcuttur. Farklı hizmet noktalarındaki kaynakların verimli kullanılması önemlidir. Bankalar esnek kaynak kullanımını önemsemekte ve bu sayede hizmet kalitesiyle verimlilik arasındaki dengeyi sağlamaktadırlar. Bankalarda esnek çalışan personel hem ön ofis hem de arka ofiste çalışabilmektedir. Gişede çalışanlar ise sadece gişe hattındaki müşterileri karşılamaktadırlar. Çalışmamızda biri esnek, biri de gişe hattına atanmış iki personele yer vererek dinamik programlama yöntemiyle kuyruk yönetim sistemi çözümledik. Modelde iki müşteri tipi tanımlayarak, atanmış çalışanın birinci tip müşteriye, esnek çalışanın ise hem birinci hem de ikinci tip müşteriye hizmet verebileceği bir sistem tanımladık. Müşteriler Poisson dağılımla şubeye geliyorken, çalışanlar üssel dağılımla hizmet veriyorlar. Ayrıca müşteriler için kuyrukta bekleme ve sistemi terketme maliyetleri mevcuttur. Amacımız bu maliyetleri en aşağı seviyede tutarak esnek çalışanın doğru müşteri tipine hizmet vermesini sağlamaktır. Modelin çözümünde esnek çalışan için farklı karar matrisleri oluşmaktadır. Bu matrisler sistemde; öncelikli kendi müşterisine hizmet vermesi, belli bir değerden sonra kendi müşterisine hizmet vermesi, doğrusal bir karar noktasına göre kendi müşterisine hizmet vermesi şeklinde sıralanabilir. Farklı değişken değerlerini içeren deneylere yer vererek bu karar matrislerine ulaşılmıştır. En iyi çözümün yanı sıra farklı bulgular da analiz edilerek en iyi çözüm ile karşılaştırılmıştır.

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e	Unit vector
fd	Fully dedicated policy cost value
$fd\%$	Difference of cost values between fully dedicated policy and optimal policy in terms of percentage
opt	Optimal policy cost value
h	Holding cost of customers
hp	Cost value of Heuristics
$hp\%$	Difference of cost values between heuristics and optimal policy in terms of percentage
M	Upper bound for number of customer type 1 in the system
N	Upper bound for number of customer type 2 in the system
S	Sojourn time
sp	Static policy cost value
$sp\%$	Difference of cost values between static policy and optimal policy in terms of percentage
W	Waiting time
x	Number of customers in the system
$V(x_1, x_2)$	Cost function of the model
β	Abandonment cost of customers
γ	Abandonment rate of customers
λ	Customer Arrival Rate of Customers
Λ	Sum of arrival, service and abandonment rates
μ	Service Rate of Servers
ρ	fraction of time the server is working:$\rho = \frac{\lambda}{\mu}$

LIST OF ACRONYMS/ABBREVIATIONS

FCFS	First Come First Served
LCFS	Last Come First Served

1. INTRODUCTION

Queues are the lines for customers waiting to get service from servers which is essential to manage for the organizations in order to provide high service quality. We confront with the queues at many industries especially at service industry. Organizations attach importance to queue management, otherwise unsatisfied customers due to long waiting time can be a potential loss. In addition, excellent queue management also brings competitive advantage to industries.

Queuing systems have been processed for years with different models in terms of servers and queues. Any queuing system is governed by certain features like arrival pattern of customers, customers' decision to wait in the queue, type of queues, queue discipline and service process. Parameters of models can be grouped mainly as being single or multiple queues or servers and having parallel or tandem queues. Different policies at queue management are studied to get the best solution where threshold, exhaustive policies can be shown as examples. In addition, different attributes are defined for the queueing problems such as waiting costs for customers, rewards for service, reneging costs which are influential at constructing objective functions of models.

In many queuing systems for different industries, servers are faced with impatient customers who do not tolerate to wait at queues. This impatience leads to dissatisfaction of the service given by the company and even ending to work with the companies. This kind of reactions of customers attempt to implement methods to decrease levels of impatience of customers. This impatience can be defined in the model as waiting and abandonment costs of customers by setting abandonment rates.

The queuing problem studied here is motivated by techniques observed at branches of banks. Requests of customers differ that leads to being served at different service points at branches. When they arrive to the branch, they enter into the system by choosing the transaction type on the screen of a queuing machine. Machine gives ticket to the customer and record information about the customer in the system. At that time system knows customer, customer type, arrival time, and other needed information for the queuing algorithm. In this study, we design a queuing model related to the service point where operational requests of customers are done. In other words, we model the queueing system for the teller line and the back office of a branch.

There is a common trade-off experienced in queuing systems of banking industry which is balancing the need to keep waiting times low while minimizing the staffing required to serve at the serving points. This leads to use of flexible staff in order to manage trade off between service quality and efficiency. We consider this flexibility in this thesis by modelling the queuing system with a flexible server. Especially, we examine a queue system with two types of customers and two servers. One of the queues represents type-1 customers where customers are coming for activities that are able to be done at teller line. Besides, the other queue is for type-2 customers who exist for back office activities. Transactions like money withdraw, credit card payments, loan payments can be grouped as front office activities where customers arrive to the branches physically to make tellers do these transactions. Activities for type-2 customers represent mostly the transactions coming through e-mail or faxes. Type-2 customers arrive to the branch physically just for a few transactions. At branches, as time progresses, the queue lengths of two servers might become unbalanced such that flexible server (server doing back office activities in general) should switch from his own service point to the other one. We mostly confront with impatient customers also at branches of banks which can be resulted in abandonment of customers due to long waiting time. This reality of customer loss at the branches directed us to design queueing system by taking the abandonment rates of customers into account.

This queueing system motivated from branches of banks considers a dynamic scheduling problem with limited server and unlimited customer arrival under some defined parameters such as waiting and abandonment costs. Scheduling is allocation of customers to resources by using the information about system such as abandonments, arrivals, services and waiting. In the model, we define two types of customers with Poisson arrival streams. There are two servers, one of them serves only one type of customer while other server can serve both types of customers. Service rates are exponentially distributed that are independent of the customer types. In the model, all customers may abandon after an exponentially distributed period of time, with the abandonment rates allowed to be class dependent. Our goal is to provide an optimal switching model for the flexible server by using Markov decision process. We aim to minimize cost of the system where each queue has holding costs and abandonment costs per customer per unit time.

This study differs from other studies about the queueing systems done before, in terms of having some attributes together in one model. In other words, this study contributes to the literature by solving a dynamic scheduling problem where the system contains arrival of two types of customers and service of two servers where one of them is flexible. Besides, it optimizes a system by minimizing cost function that consists of both waiting and abandonment cost of customers together.

Problem is modelled as Markov Decision Problem with infinite horizon and solved by using value iteration algorithm. We analyse the system by considering value of cost function and decision matrix of flexible server. Different settings of parameters give different pattern of decision matrix. We investigate the experiments that result in different types of decision matrix such as priority based (exhaustive), with different threshold values and with linear switching curves. We analyse the states, parameter values that lead to these types of decision matrices for the flexible server. On the other hand, we construct some policies and heuristics such as static policy, fully dedicated policy, giving service to the customer type with higher holding and abandonment costs. Finally, we compare the cost values of these policies and heuristics with optimum policy. Static policy, in which server 2 gives service to type-1 customer in case of non existence of type-2 customer in the system, gives the best solution after optimal policy. In addition, we analyse the conditions that increase cost functions of the problem which are mainly decrease in service rates, increase in arrival rates, increase in holding costs and abandonment costs, and finally decrease in abandonment rates.

In the following sections, first a brief survey of related studies about single server, multiple server, parallel and tandem queues in the literature is presented in Chapter

2. where server types are varied like being dedicated or flexible. The model and the objectives of the problem are described in more detail in Chapter 3. The problem is modelled using Markov Decision Process to find optimal switching model. In Chapter 4, numerical results of experiments are analysed not only for optimal policy both also for some other policies and heuristics. In addition, observations from experiments and some managerial insights are summarized. Finally the conclusions are drawn in the Chapter 5.

2. LITERATURE REVIEW

Queueing systems with single or multiple resources and with single or multiple customer/demand types have been a focus of interest for researchers for decades. Several studies have been carried out in order to minimize cost or maximize reward while resources are effectively used. Depending on the properties of the environments, exact analytical solutions or approximation algorithms and heuristic methods are studied in the literature. In the literature, queueing systems are investigated with different attributes. The studies differ in terms of these attributes which are classified as below:

- Servers
    - (i) Single Server
    - (ii) Multiple Servers
        - (a) Flexible Servers
        - (b) Dedicated Servers
        - (c) Both Flexible and Dedicated Servers
- Queue Types
    - (i) Parallel Queues
    - (ii) Tandem Queues
- Customer Types
    - (i) Single Type of Customers
    - (ii) Multiple Types of Customers

In this thesis, queuing system with two servers and two queues is analysed where the system is modelled as a Markov Decision Process by using flexible and dedicated servers. Hence the literature is examined under three sections. The first section is the queues with single server. The second section contains studies about the queues with multiple servers. The last section includes the studies related to systems with both flexible and dedicated servers. At the end of literature survey, attributes that are examined in detail at other studies are summarized and compared with the thesis.

2.1. Queues with Single Server

Queues with single server have been studied for many years where the main objective is minimization of expected waiting cost of customers or maximizing the respond to the demands. One of the optimal policy studied is Smith rule or Weighted Shortest Processing Time ($c\mu$ rule) for geometric service time μ, pre-emptive service discipline and cost value c. This policy is investigated where customer with the largest $c\mu$ at the queue is served first (Buyukkoc et al., 1985).

In another study of M/G/1 queueing system, influence of threshold is investigated in a queueing system with single server and two classes of customers. In the model, one of the queues is served exhaustively while on the other hand, other queue is served until the first queue reaches to a threshold level. The analysis is done with and without a switch over time of server. Threshold policy is analysed by using performance indicators as queue lengths and waiting times of customer types (Boxma and Down, 1997).

In a more recent study, Markov Decision process is used for a single server queueing system where two types of customers exist. Customer impatience is included in the model by defining abandonment rates of customers. Customer arrivals are according to poisson process with different rates for each type and service rates are exponential. Goal of the study is to provide an optimal server assignment policy under reward and cost conditions. The problem is maximizing expected discounted or average rewards and minimizing expected discounted or average costs over an infinite horizon. For the first case, class-dependent reward is received for the customers whose service is successfully completed. It also makes decision between getting more rewards and avoiding idling. For the second case, cost is defined as holding cost of customers waiting in each queue and a class-dependent penalty for each customer that abandons (Down et al., 2011).

In the first section of study, Sarhangian and Balcıoğlu (2013) consider queueing model with one server and two types of customer where customers may abandon the system. There are high priority and low priority customer types where high priority customers are served exhaustively, in other words if a high priority customer exits it is served first. However, if customer with high priority enters to the system, he has to wait for the completion of the service of the low priority customer. The service time distributions and reneging rates of each type of customer differ from one another. Customers arrive to the system with poisson processes. At the end, they compute the steady-state system performance measures such as waiting time and utilization.

Çil et al. (2011) study a model with a single server and two types of customers in order to investigate an optimal sequencing and dynamic pricing problem. They aim to maximize profit earned from customers. They consider both dynamic pricing of customers with respect to total queue lengths and holding cost of customers, while calculating total expected discounted profit. Sequencing is optimized by giving priority to customers with higher holding costs. However for the pricing policy, simple dynamic pricing policies are effective where the total queue length based approach is proposed.

Preemptive assignment of a single server to two customer classes is studied in a queuing system. Arrival rates to both queues are according to poisson process and exponentially distributed service rates differ at each queue. Objective of the problem is preemptive dynamic assignment of the server to the queues while minimizing costs. The system consists of holding costs and switching costs. Holding cost is for customers waiting in the queue, and switching cost is incurred when server moves from one queue to the other. Besides optimal policy, threshold policy and other heuristics are investigated and compared with optimal policy. It analyses the optimal solution with different parameter values and obtains results related to queue lengths, switching costs and holding costs. Some conclusions are as following; (i) switching from queue 1 to queue 2 occurs only if there is no customer at queue 1, (ii) if queue 1 is empty, queue 2 is served exhaustively, (iii) if there is sufficient customers in queue 1 it switches to queue 1 from queue 2 in spite of switching cost. The last decision of conclusions occurs when serving queue 1 reduces the holding costs at a faster rate than by serving queue 2 (Koole, 1997).

In the study of Kim and Ward (2013), a dynamic scheduling of a single server over queues with multiple classes of customers are solved by formulation of an approximating Brownian control problem in a heavy traffic regime. The objective is to minimize average cost by choosing dynamically to which customer class the server should serve next. The problem consists of inter arrival rates, service rates and abandonment rates that differ for each customer class.

Iravani et al. (1997) study tandem queues with two stages where poisson arrivals and general service rates exist. Problem consists of holding costs for both stages and switching costs occurring by moving from one stage to the other. Optimal policy differs in terms of holding cost of the stages. For instance, if holding cost in the second stage is greater or equal to the cost in the first stage, then the optimal policy in the second stage is exhaustive. Heuristics are also studied where Triple-Threshold (TT) policy is found to be approximating the optimal policy. Triplet comes from the state which consists of the location of the server, the number of customers in the first stage and the number of customers in the second stage

There are also other studies that investigate different problems about queueing system with single server. Studies of Thangaraj and Vanitha (2010), Madan and Choudhury (2005) and Kalyanaraman and Murugan (2008) consider server vacations under a system of different service rates for customers. In addition Ayhan and Olsen (2000) studied single server queue in order to analyse mean throughput time by using two heuristics. Larranaga et al. (2014) investigates multi class queueing models with impatient customers while holding and abandonment costs of customers exist.

Set up time exists for the queuing systems of Duenyas et al (1998) and Hofri and Ross (1987) where objective is minimizing cost of both waiting and setup cost. Winkler (2011) studies a different case for queueing system where customers have repeated services in the system which is called as retrial policy. A reward system is studied for the queuing system (Tcha and Plisk, 1977) where customers change nodes or leave the system after completion of service.

2.2. Queues with Multiple Servers

Sleptchenko (2003) analyses queues with multiple types of items having different prioritization. Each items have their own arrival rates with poisson process and service rates of multiple servers are exponentially distributed. Servers can serve all types of items. Equations for steady state system are constructed for three cases which are (i) there is at least one high priority item in the queue, (ii) there is no high priority item and at least one low priority item in the queue, (iii) there is no item in the queue. The study is done in order to analyse the performance measures such as expected waiting times, expected queue length and expected postponement time.

In the study of Bruneel et al.(2013) we can see a discrete-time queueing model with dedicated servers and two types of customers where deterministic service rates and general arrival rates occur. Customers are served with the first come first served policy. All customers attend to a common queue and are served in their order of arrival, regardless of the class they belong to. Since one server could only serve one type of customer it differs from other queuing models with multi servers. The goal of the study is to see (i) impact of FCFS policy, (ii) effect of the relative distribution of the load amongst the customer classes on the system performance. They investigate when the value of mean arrival rate is smaller than 1, FCFS has little effect on the system effectiveness. However, having arrival rate more than value 1 (more loaded system) and having similar number of customers from each type in the system, system performance becomes more effective. In other words, at loaded system servers become rarely idle like flexible servers although they are dedicated servers.

In a study of queueing systems with multiple servers, it is aimed to assign servers to the different customer classes in order to optimize system performance, where servers are homogeneous and can give service to all kind of customers. It is tried to minimize number of staff and waiting cost of customers while maintaining performance goals like waiting time of customers. On the other hand, scheduling servers to different customer classes is also studied while optimizing the system performance. The asymptotically optimal routing policy is used by using priority policy with thresholds. A customer class is served according to a threshold value of number of idle servers in the system (Gurvich, 2004).

Jouini and Reibos (2014) study a queuing model with multiple servers and two classes of customers where customers are classified having high and low priority. Their arrival rates have poisson distribution with λ_1 and λ_2 and service rates are exponential with common μ. Abandonment rates for each type of customer are also considered in the model which are same for both types of customers. FCFS and LCFS policies are applied in order to compare the performance measures such as non conditional and conditional waiting times. LCFS policy is better for conditional expected waiting time given that customer will get service compared to FCFS policy. On the other hand LCFS policy gives better solution for the conditional expected waiting time given that customer will abandon.

Atar et al. (2010) study a queueing system with multiple servers and two types of customers where customers may abandon after waiting long time in the system. The goal is minimizing overall long run average holding cost of customers. In this study, optimal solution of routing policy is found as $c\mu/\Theta$ in a fluid and overload condition where c is holding cost per unit time, μ is service rate and Θ is abandonment rate. This model differs from our problem where there are no abandonment costs of customers and servers are all flexible. As a result, (i) $c\mu/\Theta$ rule reduces to the $c\mu$ rule when the abandonment rates and costs are same for all classes, (ii) similar to the $c\mu$ rule, the $c\mu/\Theta$ rule does not depend on the arrival rates of customers which leads robustness and simplicity of implementation as arrival rates are often varying and unpredictable, (iii) unlike the $c\mu$ rule which is demonstrated to be optimal for a variety of cost function including finite horizon and discounted costs, optimality $c\mu/\Theta$ rule is restricted to be optimal for long-term average cost.

Caban et al. (2016) study tandem queues for multiple servers. The model is based on maximizing revenue where servers are determined at each station in order to maximize reward. On the other hand, abandonment costs of customers also exit. The servers try to balance abandonments and prioritize customers with higher reward by using continuous-time Markov decision process. Study investigates that (i) optimal control ensures that all servers are busy, (ii) servers should work at another station if there are not enough customers at existing station (ii) servers should stay at their existing stations when there is adequate work to stay busy.

Ahn and Righter (2005) study reformulation of Markov decision processes with multiple actors in order to reduce the complexity of the problem considerably. Assumptions of the problem are (i) some actors differ in performance like being faster or slower than others, (ii) state transitions depend on the action chosen not on which actor chooses the action. These assumptions are parts of actions that should be chosen according to their marginal values. For instance, faster actors should be assigned to actions with higher marginal values. This tendency of the actors to take the same action considerably reduces the complexity of the problem.

The studies of Atar et al. (2004), Harrision and Zeevi (2004) investigate queueing models with impatient customers where they try to minimize their cost functions. Flexible servers give service to customers with abandonment rates. In addition, Hassenbein (2011) studies tandem queues and aimes to maximize throughput of the system. Ormeci et. al (2001) and Bhulai and Koole (2003) investigate customer admission or rejection in case of available servers in the system. Down and Lewis (2006) consider parallel queues with arrival of customers where customers can move from one queue to another. They use formulation of Markov decision process that store customers in the queue with lowest cost in order to minimize long-run average cost.

2.3. Queues with Both Flexible and Dedicated Servers

In this section, we investigate queues where both flexible and dedicated servers exist in the system. Kula (2004) model a production system as a Markov Decision Process where one of the machine produces product x and other machine produces either product x or y. It means machine 1 is dedicated server, on the other hand machine 2 is flexible server. The model targets to maximize the average profit by calculating both revenue obtained from products and inventory cost of products. Decision criteria of the model for machine 1 is producing product x or being idle, on the other hand decision criteria for machine two is producing product x or y or being idle. This study investigates the conditions to invest in dedicated and flexible machines. It finds a minimum limit for the value of flexibility (revenue of flexible server divided by revenue of dedicated server). In addition, value of flexibility decreases as capacity cost of flexible server and/or demand rate of product 2 increases. As a result, system with one dedicated and one flexible server is beneficial when the flexible capacity cost is moderate and the value of flexibility in the system is at a low level. This model differs from our model in terms of modelling strategy where we model a service policy based on minimizing total cost of customers in terms of abandonments and holding costs while deciding the type of customer to serve.

The queuing system with dedicated and flexible servers are studied also for tandem queues. A problem with given number of customers at two stations is analysed with Markov Decision Process where no arrival of customer occurs in the system. The objective of the model is minimizing expected total holding cost where there is a linear holding cost function for each stations. The aim is to use extra server in order to minimize that holding cost. Different cases are considered which are as following; an extra server may be switched between the two stations at any time, or it may be restricted to serve at just one of the station. Service rates of dedicated servers at two stations and flexible server are μ_1, μ_2 and μ respectively. Assigning flexible server to stations are related with holding costs of queues at those stations (Farrar, 1993).

In addition, Andradottir et al. (2007) study tandem queues where dynamic assignment of dedicated and flexible servers are considered. The goal of the study is maximizing the long-run average throughput. Moreover, effect of the number of flexible servers on the throughput is analysed. Furthermore, improvement of system by changing one dedicated server to flexible server or adding a new dedicated resource is investigated. As a result, optimal selection of the dedicated and flexible servers, assignment of dedicated servers to stations, and the thresholds for switching flexible servers from station 1 to station 2 depend on the buffer size of stations.

In another study of tandem queues, it is tried to solve different scenarios with zero buffer tandem queues. The objective is finding server allocations to maximize throughput. The study has four stages as (i) allocation of servers to stations when they are dedicated, (ii) introducing flexible servers to improve the throughput, (iii) existence of restricted flexible servers who can move between neighbouring stations rather than travelling through all stations, (iv) existence of flexibility only between parallel queues at tandem lines. Performance in terms of throughput is compared for those four stages. Trade-off between the cost and the throughput improvement of making servers flexible is also identified through the study (Yarmand, 2012).

Pandelis (2008) also studies tandem queues with dedicated and flexible servers. The goal is to find optimal dynamic assignment of flexible servers in two-stage tandem queuing systems where dedicated servers already exist at stations. It is analysed for both existence and non existence of customer arrivals. Service times are exponential and there exists linear holding costs of customers, and operating costs incurred by the servers at rates proportional to their speeds. They identify conditions for idling or non idling policies of flexible server, in terms of holding costs, operating costs, and service rates when there are jobs in stations. It investigates properties of the optimal policy when the conditions that ensure the optimality of non idling policies are satisfied.

Ormeci (2002) studies call center with Markov Decision Process model in order to derive the structure of dynamic admission policies that maximize the total expected discounted revenue over an infinite horizon as well as the long-run average revenue. The call center has two classes of customers that could be served at two stations with dedicated servers or at shared station with flexible servers where there are parallel servers at all three stations. There is no queue for waiting of customers which means customers are lost if servers are busy or decide not to serve. Service times at each station are exponential with different rates and they generate different rewards. The study concludes the fact that serving a customer in dedicated station is optimal if that station is available. This study also analyses sufficient condition for accepting customers to the shared station while the shared station is available in case of non availability of dedicated station. Optimal policy at giving service at shared station depends on the threshold value of number of customers at both stations.

Ahn et al. (2004) develop a model of a queuing system for two types of customers with two parallel servers where one server is dedicated to only one customer type, and other is flexible server who could serve both types of customers. The objective is to minimize the total holding costs of customers in a clearing system which means that there is no arrival of new customers to the system. The problem of minimizing total expected cost is formulated as a Markov Decision Process. They investigate the conditions of optimal policies of flexible server for clearing all type-2 jobs, switching to type-1 jobs in non-increasing manner and switching to type-1 jobs in non-decreasing manner. It also investigates the necessary and sufficient conditions under which each policy will be optimal. In addition it analyses the optimality of non idling policies, in other words, it is referred that idling of servers are optimal for some cases. Another optimality solution is found for some defined conditions of a system with existence of customer arrivals. This model differs from our study since there is no abandonment of customers in this problem.

A queueing system of a call center is also studied by Chaviler et al. (2004) where there exists dedicated servers giving service to one type of customer and flexible servers giving service to all types of customers. They investigated (i) the optimal routing rule for flexible servers in order to determine the types of customer and condition that flexible server will serve (ii) optimal number of both dedicated and flexible staff by considering personnel budget, loss rate of customers and reward obtained from servers. They analyse the service rates of flexible servers comparing to dedicated servers and also arrival rates of customers. However this model is an example of a loss system model, where they tries to minimize loss of customers while maximizing reward obtained from service completion.

The studies of Yarmand and Down (2012), Wu et al. (2005) represent queueing systems of tandem queues where each station has dedicated servers. They investigate for assigning flexible server to required stations in order to achieve optimal solution.

Table 2.1. Summary of Literature with Single Server

Study	**Queue Type**	**Cost Type**	**Reward**	**Customer Class**	**Event**
Buyukkoc et al. (1985)	Parallel	Waiting	-	Multi	Arrival, Service
Boxma and Down (1997)	Parallel	-	-	2	Arrival, Service, Switch
Down et al (2011)	Parallel	Waiting, Abandon	Yes	2	Arrival, Service, Abandon
Sarhangian and Balcıoğlu (2013)	Parallel	Waiting, Abandon	-	2	Arrival, Service, Abandonment
Çil et al. (2011)	Parallel	Waiting	Yes	2	Arrival, Service
Koole(1997)	Parallel	Waiting,Switching	-	2	Arrival, Service
Kim and Ward (2013)	Parallel	Abandon	-	Multi	Arrival, Service, Abandon
Iravani et al. (1997)	Tandem	Holding, Switching	-	2	Arrival, Service, Holding, Switching
Thangaraj and Vanitha (2010)	Parallel	-	-	2	Arrival, Service,Vacation
Madan and Choudhury (2005)	Tandem	-	-	1	Arrival, Service,Vacation
Kalyanaraman and Murugan (2008)	Single	-	-	1	Arrival, Service,Vacation
Ayhan and Olsen (2000)	Parallel	-	-	Multi	Arrival, Service
Duenyas et al(1998)	Tandem	Holding	-	Multi	Arrival, Service, Set-up
Hofri and Ross (1987)	Parallel	Holding, Set up	-	2	Arrival, Service, Set-up
Winkler (2011)	Parallel	Holding,	-	2	Arrival, Service, Repeated Service
Tcha and Plisk (1977)	Parallel	Holding,	Yes	Multi	Arrival, Service
Larranaga et al. (2014) (1977)	Parallel	Holding, Abandon	-	Multi	Arrival, Service, Abandon

Table 2.2. Summary of Literature with Multiple Servers

Study	Server Type	Queue Type	Cost Type	Reward	Customer Class	Event
Sleptchenko (2003)	Flexible	Parallel	-	-	Multi	Arrival, Service
Bruneel et al.(2013)	Dedicated	Parallel	-	-	2	Arrival, Service
Gurvich (2004)	Flexible	Parallel	Waiting, Staffing	-	Multi	Arrival, Service
Jouini and Reibos (2014)	Flexible	Parallel	Waiting	-	2	Arrival, Service, Abandon
Atar et al. (2010)	Flexible	Parallel	Holding	-	Multi	Arrival, Service, Abandon
Caban et al (2016)	Flexible	Tandem	Abandon	Yes	Multi	Arrival, Service, Abandon
Ahn and Righter(2005)	Flexible	Parallel	Abandon	Yes	2	Arrival, Service, Abandon
Atar et al. (2004)	Flexible	Parallel	-	-	Multi	Arrival, Service, Abandon
Harrision and Zeevi (2004)	Flexible	Parallel	Holding,Abandon	-	Multi	Arrival, Service, Abandon
Hassenbein (2011)	Flexible	Tandem	-	Yes	2	Service
Ormeci et. al (2001)	Flexible	Parallel	-	Yes	2	Arrival, Service, Failure
Bhulai and Koole (2003)	Flexible	Parallel	Holding,Waiting,Rejecting	-	Multi	Arrival, Service
Down and Lewis (2006)	Flexible	Parallel	Holding,Switching	-	Multi	Arrival, Service

Table 2.3. Summary of Literature with Dedicated and Flexible Servers

Study	Server Type	Queue Type	Cost Type	Reward	Customer Class	Event
Kula (2004)	1 Dedicated-1 Flexible	Parallel	Holding	Yes	2	Arrival, Service
Farrar (1993)	2 Dedicated and 1 Flexible	Tandem	Holding	-	2	Arrival, Service
Andradottir et al.(2007)	Multi Dedicated and Flexible	Tandem	-	-	Multi	Arrival, Service
Yarmand (2012)	Multi Dedicated and Flexible	Tandem	-	-	Multi	Service
Pandelis (2008)	2 Dedicated and 1 Flexible	Tandem	Operating, Holding	-	Multi	Arrival, Service
Ormeci (2002)	2 Dedicated and 1 Flexible	Parallel	-	Yes	2	Arrival, Service, Failure
Ahn et al. (2004)	1 Dedicated and 1 Flexible	Parallel	Holding	-	2	Service
Chaviler et al.(2004)	Multi Dedicated and Flexible	Parallel	Staffing	-	Multi	Arrival, Service, Loss
Yarmand and Down (2012)	2 Dedicated and 1 Flexible	Tandem	-	-	2	Service
Wu et al.(2005)	Multi Dedicated and Flexible	Tandem	Holding	-	2	Service, Failure
This Thesis	1 Dedicated and 1 Flexible	Parallel	Holding, Abandon	-	2	Arrival,Service, Abandon

Summary of literature analysis is shown in Tables 2.1, 2.2 and 2.3 where different systems are studied, different than our problem. Our study differs from the studies mentioned above in terms of attributes mentioned in the problem and the method used in order to get optimum solution. We have two types of customers where both holding and abandonment costs exist for those customers. There are two servers in the system where one of them is dedicated and the other one is flexible. Customers arrive to the system with different types of arrival rates. Besides, service rates are defined for each server. In addition, customers may abandon the system after waiting long time in a queue. The model is constructed by using Markov Decision process aiming to minimize average cost consisting of holding and abandonment costs. We construct an algorithm where we try to decide to the type of customer to serve for flexible server while maintaining the minimum cost.

3. MATHEMATICAL MODEL

In this study, we consider a dynamic scheduling of a queuing system with two types of customers and two parallel servers where one dedicated (server 1) and one flexible server (server 2) are defined. Server 1 is dedicated to give service to only type-1 customer while server 2 can serve both types of customers. The system is modelled as a Markov Decision Process where it is aimed to minimize average cost. The problem problem is solved optimally by using value iteration algorithm which is written in C Program Language. We solve the problem also with using some other policies and heuristics and then compare the results with optimum solution.

3.1. Base Model, Parameters and Assumptions

In the literature, parallel queues with dedicated and flexible servers are rarely studied. However, tandem queues with both flexible and dedicated servers are widely investigated in the literature. So, it is needed to do this study in parallel with the requirements of the industry accordingly. In banking sector, this type of model with two types of servers is widely used at the branches of banks. At the branches, tellers spend most of their time with giving service to the customers arriving to do financial operations. Tellers are generally dedicated to give service to those customers. On the other hand, number of dedicated tellers become inadequate to meet customer request especially at rush hours of branches. Therefore there are flexible staff at each branch to give service to the customers. However these flexible staff are also capable of doing back office activities. So, managers at branches are responsible to schedule these flexible staff appropriately in order to maintain service quality both for back office and front office (teller) activities. Main indicators for service quality are waiting time of customers and customer pleasure.

So, in order to meet these customers requests at branches, we consider two parallel queues with two parallel servers where two types of customers arrive following a Poisson process with rates λ_1 and λ_2. There are two servers who give service for defined customer types. Server 1 can process only type-1 customers with exponentially distributed rate μ_1. Server 2 can serve type-2 customers and also aid server 1 to give service to type-1 customers with exponentially distributed rate μ_2. Customers are served according to FCFS discipline, which means customers arrived earlier to his own queue is served in the first order. Hence, there is no priority within each queue. Another assumption is that one type of customer has a non-preemptive priority over the other type of customer. According to the non-preemptive discipline, the service that is already started should be completed without interruption of another customer. In other words, server 2 can not begin to serve other type of customer before completing the service of processing customer. In addition, there is no switching time or cost for flexible server while switching between customer types to serve.

In addition, there are also abandonment rates other that arrival and service rates. In existence of abandonment in the model, customers leave the system without getting service after waiting for a time in the queue. Our model with two parallel queues and two servers are shown in Figure 3.1

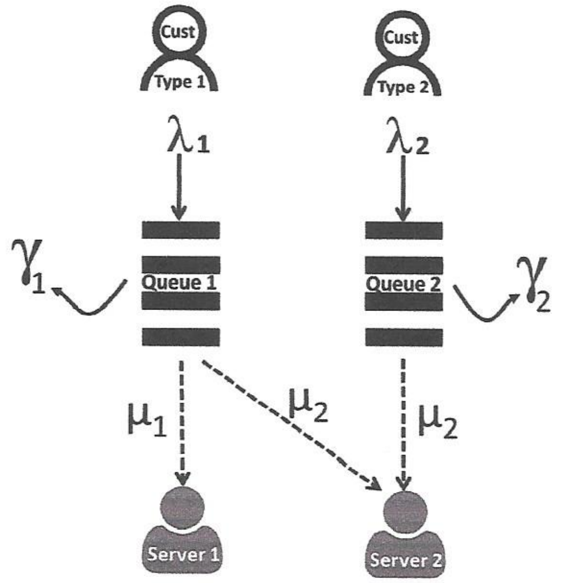

Figure 3.1. Model: One Dedicated - One Flexible Server.

Abandonment rates and costs for both types of customers differ. Besides, another cost parameter named as holding cost is defined for each type of customer waiting in the queue. We aim to minimize these holding costs and abandonment costs of customers by providing flexible server to decide on type of customer to serve for different conditions.

At the banks, abandonment rates of customers are approximately calculated on average for customers that leave the bank without getting service. Time for abandonment is calculated by getting information from the tickets of customers that are not given service. This abandonment may lead to loss of the customer which is ending working with the bank. Secondly, it can be a loss of a sales activity or deposit which are loss of money for the bank. Another cost that can be defined at banks is waiting/holding cost of customers. Holding cost of customers can also be given in terms of types of customers. Value of holding cost of customers getting service from teller line can be given as a multiple of holding cost of customers getting service from back office of a branch. The value of this coefficient can be defined by taking importance of customers and service types given at these different service points into consideration. In addition, value of abandonment cost can be described as a factor of holding cost while it depends on the decision that how much more critical of holding a customer than the abandonment of that customer.

In the first phase, we try to find optimal switching model for flexible server using Markov decision process. In the second phase, we define other policies and heuristics and compare them with optimum results of average cost function of the system.

3.2. Cost Function for Value Iteration Algorithm for Optimal Policy

Problem of a queueing system solved by a value iteration algorithm can be observed in the studies done by Down et al. (2011) and Ahn et al. (2004). However, Down et al. (2011) investigates a queuing system with one server and Ahn et al. (2004) investigates a closed system with no arrival and no abandonment rates and costs of customers.

As mentioned above, customers arrive to the system with λ_1 and λ_1 respectively.

In addition, rates of servers are μ_1 and μ_2. A holding cost, h_1 and h_2 is defined per customer per unit time in the system for type-1 customers and type-2 customers respectively. Abandonment rates and abandonment costs for each type of customers also exist with γ_1 and β_1 for type-1 customer and γ_2 and β_2 for type-2 customer respectively.

We formulate the problem of minimizing average cost as a Markov decision process. We let $x=(x_1, x_2)$ be the state of the system, where x_1 denotes the number of type-1 customers and x_2 denotes the number of type-2 customer in the system. The formulation of the model is as following, where $V(x_1, x_2)$ as average cost function that satisfies the following dynamic programming equation:

$$V(x_1, x_2) = min\left\{V^1(x_1, x_2), V^2(x_1, x_2)\right\}$$

$V^1(x_1, x_2)$ denotes the cost function that both server 1 and server 2 give service to type-1 customers, on the other hand $V^2(x_1, x_2)$ denotes the cost function that server 1 gives service to type-1 customers and server 2 gives service to type-2 customers. The formulation of $V^1(x_1, x_2)$ and $V^2(x_1, x_2)$ is as following;

$$\begin{aligned}
V^1(x_1, x_2) = \{& x_1 h_1 + x_2 h_2 + \lambda_1 V(x_1+1, x_2) + \lambda_2 V(x_1, x_2+1) + \mu_1 V(x_1-1, x_2) \\
& + \mu_2 V(x_1-1, x_2) + (x_1-2)\gamma_1 (V(x_1-1, x_2) + \beta_1) \\
& + x_2\gamma_2 (V(x_1, x_2-1) + \beta_2) + (M - x_1 + 2)\gamma_1 V(x_1, x_2) \\
& + (N - x_2)\gamma_2 V(x_1, x_2)\}\Lambda^{-1}
\end{aligned}$$

$$\begin{aligned}
V^2(x_1, x_2) = \{& x_1 h_1 + x_2 h_2 + \lambda_1 V(x_1+1, x_2) + \lambda_2 V(x_1, x_2+1) + \mu_1 V(x_1-1, x_2) \\
& + \mu_2 V(x_1, x_2-1) + (x_1-1)\gamma_1 (V(x_1-1, x_2) + \beta_1) + (x_2-1)\gamma_2 \\
& (V(x_1, x_2-1) + \beta_2) + (M - x_1 + 1)\gamma_1 V(x_1, x_2) \\
& + (N - x_2 + 1)\gamma_2 V(x_1, x_2)\}\Lambda^{-1}
\end{aligned}$$

where $\Lambda = \lambda_1 + \lambda_2 + \mu_1 + \mu_2 + M\gamma_1 + N\gamma_2$ which is sum of transition rates of arrival, service and abandonment.

Description for each part of the equation given above is as following: $x_1h_1 + x_2h_2$ denotes total holding cost of customers waiting in the queue; $\{\lambda_1 V(x_1 + 1, x_2)\}\Lambda^{-1}$ and $\{\lambda_2 V(x_1, x_2 + 1)\}\Lambda^{-1}$ can be interpreted as probability of arrival of type-1 customer and type-2 customer respectively; $\{\mu_1 V(x_1 - 1, x_2)\}\Lambda^{-1}$ is probability of giving service to type-1 customer by dedicated server (server 1). The main difference between equations $V^1(x_1, x_2)$ and $V^2(x_1, x_2)$ is the formula of server 2 giving service to type-1 customer or type-2 customer. Then formulation for abandonment of customers differs in each equation accordingly.

In equation $V^1(x_1, x_2)$, type-1 customer is served by flexible server which is defined as $\{\mu_2 V(x_1 - 1, x_2)\}\Lambda^{-1}$. In addition, probability of abandonment of type-1 customer in the system is formulated by $\{(x_1 - 2)\gamma_1(V(x_1 - 1, x_2) + \beta_1)\}\Lambda^{-1}$ where $(x_1 - 2)$ denotes the number of type-1 customer in the system not being served. Two of the type-1 customer are being served by dedicated and flexible servers that can not abandon the system. The equation proceeds with the probability of abandonment of type-2 customer which is formulated as $\{\ x_2\gamma_2(V(x_1, x_2 - 1) + \beta_2)\}\Lambda^{-1}$ where in this case x_2 denotes the number of type-2 customer in the system not being served. Besides, the probability of customers that do not abandon the system can be formulated by defining the customers who do not wait in the queue and do not exist in the system. The formulation for type-1 and type-2 customers are represented as $\{(M - x_1 + 2)\gamma_1 V(x_1, x_2)\}\Lambda^{-1}$ and $\{(N - x_2)\gamma_2 V(x_1, x_2)\}\Lambda^{-1}$ where $(M - x_1 + 2)$ and $(N - x_2)$ are number of customers that are not being served and do no wait in the queue. M and N represent maximum number of customers in the system for type-1 and type-2 customer respectively. The values of M and N are large enough in order to define the system with infinite capacity. In addition these large numbers provide the model to reach a steady state environment.

In equation $V^2(x_1, x_2)$, type-2 customer is served by flexible server which is defined as $\{\mu_2 V(x_1, x_2 - 1)\}\Lambda^{-1}$. In addition, probability of abandonment of type-1 customer in the system is formulated by $\{(x_1 - 1)\gamma_1(V(x_1 - 1, x_2) + \beta_1)\}\Lambda^{-1}$ where

$(x_1 - 1)$ denotes the number of type-1 customer in the system not being served since one of the type-1 customer are being served by dedicated server that can not abandon the system. The equation proceeds with the probability of abandonment of type-2 customer which is formulated as $\{(x_2 - 1)\gamma_2 (V(x_1, x_2 - 1) + \beta_2)\}\Lambda^{-1}$ where in this case $(x_2 - 1)$ denotes the number of type-2 customer in the system not being served. Unlike the formula in equation $V^1(x_1, x_2)$, the probability of customers that do not abandon the system can be formulated as $\{(M - x_1 + 1)\gamma_1 V(x_1, x_2)\}\Lambda^{-1}$ and $\{(N - x_2 + 1)\gamma_2 V(x_1, x_2)\}\Lambda^{-1}$ for type-1 and type-2 customers respectively in $V^2(x_1, x_2)$. In this case, $(M - x_1 + 1)$ and $(N - x_2 + 1)$ are number of customers that are not being served and do no wait in the queue.

We define an infinite horizon discrete time Markov decision process model where equations in the model are shown in detail in the following text. Dynamic programming equation is divided into sections for different conditions where these conditions depend on number of each type of customers in the system.

Different conditions result in difference in equations as shown in Figure 3.2 where x dimension refers to number of type-1 customer in the system whereas y dimension refers to number of type-2 customer in the system. Numbers at equations given below match with the numbers shown in the Figure 3.2.

First of all, we construct the equation for condition $x_1 = 0$, $x_2 = 0$ which means there is no customer in the system. We explicitly write cost equation as following. There is no holding cost because of no existence of customers in the system. Number of type-1 customer reaches to one in case of arrival of first type of customer, or number of type-2 customer reaches to 1 in case of its arrival to the system. There is no service or abandonments.

$$\begin{aligned} V^1(0,0) = \{&0h_1 + 0h_2 + \lambda_1 V(1,0) + \lambda_2 V(0,1) + \mu_1 V(0,0) + \mu_2 V(0,0) \\ &+ 0\gamma_1 (V(0,0) + \beta_1) + 0\gamma_2 (V(0,0) + \beta_2) + (M - 0)\gamma_1 V(0,0) \\ &+ (N - 0)\gamma_2 V(0,0)\}\Lambda^{-1} \end{aligned} \tag{3.1.1}$$

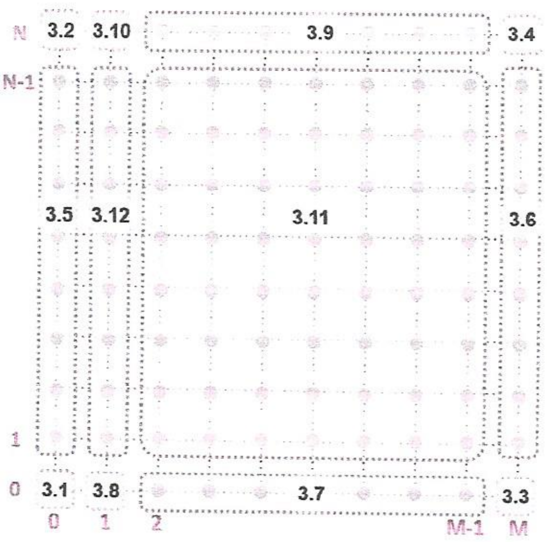

Figure 3.2. Visualization of Different Conditions.

$$V^2(0,0) = V^1(0,0) \tag{3.1.2}$$

Secondly, the equation for condition $x_1 = 0$, $x_2 = N$, where there is no type-1 customer in the system, however there are many type-2 customers in the system, is given as below:

$$\begin{aligned} V^1(0,N) = \{&0h_1 + Nh_2 + \lambda_1 V(1,N) + \lambda_2 V(0,N) + \mu_1 V(0,N) + \mu_2 V(0,N) \\ &+ 0\gamma_1(V(0,N) + \beta_1) + N\gamma_2(V(0,N-1) + \beta_2) + (M-0)\gamma_1 V(0,0) \\ &+ (N-N)\gamma_2 V(0,N)\}\Lambda^{-1} \end{aligned} \tag{3.2.1}$$

$$\begin{aligned} V^2(0,N) = \{&0h_1 + Nh_2 + \lambda_1 V(1,N) + \lambda_2 V(0,N) + \mu_1 V(0,N) + \mu_2 V(0,N-1) \\ &+ 0\gamma_1(V(0,N) + \beta_1) + (N-1)\gamma_2(V(0,N-1) + \beta_2) + (M-0)\gamma_1 V(0,0) \\ &+ (N-N+1)\gamma_2 V(0,N)\}\Lambda^{-1} \end{aligned} \tag{3.2.2}$$

Third equation consists of the condition $x_1 = M$, $x_2 = 0$, where there is no type-2 customer in the system, however there are many type-1 customers in the system, is given as below:

$$V^1(M,0) = \{Mh_1 + 0h_2 + \lambda_1 V(M,0) + \lambda_2 V(M,1) + \mu_1 V(M-1,0) \\ + \mu_2 V(M-1,0) + (M-2)\gamma_1 (V(M-1,0) + \beta_1) + 0\gamma_2 (V(M,0) + \beta_2) \\ + (M - M + 2)\gamma_1 V(M,0) + (N-0)\gamma_2 V(M,0)\}\Lambda^{-1} \tag{3.3.1}$$

$$V^2(M,0) = \{Mh_1 + 0h_2 + \lambda_1 V(M,0) + \lambda_2 V(M,1) + \mu_1 V(M-1,0) \\ + \mu_2 V(M,0) + (M-1)\gamma_1 (V(M-1,0) + \beta_1) + 0\gamma_2 (V(M,0) + \beta_2) \\ + (M - M + 1)\gamma_1 V(M,0) + (N-0)\gamma_2 V(M,0)\}\Lambda^{-1} \tag{3.3.2}$$

In the fourth order, the equation for condition $x_1 = M$, $x_2 = N$ is constructed, where there are so many type-1 and type-2 customers in the system.

$$V^1(M,N) = \{Mh_1 + Nh_2 + \lambda_1 V(M,N) + \lambda_2 V(M,N) + \mu_1 V(M-1,N) \\ + \mu_2 V(M-1,N) + (M-2)\gamma_1 (V(M-1,N) + \beta_1) \\ + N\gamma_2 (V(M,N-1) + \beta_2) + (M - M + 2)\gamma_1 V(M,N) \\ + (N-N)\gamma_2 V(M,N)\}\Lambda^{-1} \tag{3.4.1}$$

$$V^2(M,N) = \{Mh_1 + Nh_2 + \lambda_1 V(M,N) + \lambda_2 V((M,N) + \mu_1 V(M-1,N) \\ + \mu_2 V(M,N-1) + (M-1)\gamma_1 (V(M-1,N) + \beta_1) \\ + (N-1)\gamma_2 (V(M,N-1) + \beta_2) + (M - M + 1)\gamma_1 V(M,N) \\ + (N - N + 1)\gamma_2 V(M,N)\}\Lambda^{-1} \tag{3.4.2}$$

Equation continues with the condition $x_1 = 0$, $1 \leq x_2 \leq N-1$ where there is no type-1 customer in the system, and there is at least one type-2 customer in the system.

Formulation is given in detail as below:

$$V^1(0,x_2) = \{0h_1 + x_2h_2 + \lambda_1 V(1,x_2) + \lambda_2 V(0,x_2+1) + \mu_1 V(0,x_2) \\ + \mu_2 V(0,x_2) + 0\gamma_1 (V(0,x_2)+\beta_1) + x_2\gamma_2 (V(0,x_2-1)+\beta_2) \\ + (M-0)\gamma_1 V(0,x_2) + (N-x_2)\gamma_2 V(0,x2)\}\Lambda^{-1} \quad (3.5.1)$$

$$V^2(0,x_2) = \{0h_1 + x_2h_2 + \lambda_1 V(1,x_2) + \lambda_2 V(0,x_2+1) + \mu_1 V(0,x_2) \\ + \mu_2 V(0,x_2-1) + 0\gamma_1 (V(0,x_2)+\beta_1) + (x_2-1)\gamma_2 (V(0,x_2-1)+\beta_2) \\ + (M-0)\gamma_1 V(0,x_2) + (N-x_2+1)\gamma_2 V(0,x2)\}\Lambda^{-1} \quad (3.5.2)$$

Another part of the formulation is for condition $x_1 = M$, $1 \leq x_2 \leq N-1$ where number of type-1 customer is at high level in the system. However there exists at least one type-2 customer.

$$V^1(M,x_2) = \{Mh_1 + x_2h_2 + \lambda_1 V(M,x_2) + \lambda_2 V(M,x_2+1) + \mu_1 V(M-1,x_2) \\ + \mu_2 V(M-1,x_2) + (M-2)\gamma_1 (V(M-1,x_2)+\beta_1) \\ + x_2\gamma_2 (V(0,x_2-1)+\beta_2) + (M-M+2)\gamma_1 V(M,x_2) \\ + (N-x_2)\gamma_2 V(M,x_2)\}\Lambda^{-1} \quad (3.6.1)$$

$$V^2(M,x_2) = \{Mh_1 + x_2h_2 + \lambda_1 V(M,x_2) + \lambda_2 V(M,x_2+1) + \mu_1 V(M-1,x_2) \\ + \mu_2 V(M,x_2-1) + (M-1)\gamma_1 (V(M-1,x_2)+\beta_1) \\ + (x_2-1)\gamma_2 (V(0,x_2-1)+\beta_2) + (M-M+1)\gamma_1 V(M,x_2) \\ + (N-x_2+1)\gamma_2 V(M,x_2)\}\Lambda^{-1} \quad (3.6.2)$$

We observe the condition $2 \leq x_1 \leq M-1$, $x_2 = 0$ (there is no type-2 customer and there is at least 2 type-1 customer in the system) as following:

$$V^1(x_1,0) = \{x_1h_1 + 0h_2 + \lambda_1 V(x_1+1,0) + \lambda_2 V(x_1,1) + \mu_1 V(x_1-1,0)$$
$$+ \mu_2 V(x_1-1,0) + (x_1-2)\gamma_1(V(x_1-1,0)+\beta_1) + 0\gamma_2(V(x_1,0)+\beta_2)$$
$$+ (M-x_1+2)\gamma_1 V(x_1,0) + (N-0)\gamma_2 V(x_1,0)\}\Lambda^{-1} \quad (3.7.1)$$

$$V^2(x_1,0) = \{x_1h_1 + 0h_2 + \lambda_1 V(x_1+1,0) + \lambda_2 V(x_1,1) + \mu_1 V(x_1-1,0)$$
$$+ \mu_2 V(x_1,0) + (x_1-1)\gamma_1(V(x_1-1,0)+\beta_1) + 0\gamma_2(V(x_1,0)+\beta_2)$$
$$+ (M-x_1+1)\gamma_1 V((x_1,0) + ((N-0)\gamma_2 V(x_1,0)\}\Lambda^{-1} \quad (3.7.2)$$

Equation for the condition $x_1 = 1$, $x_2 = 0$ (there is no type-2 customer and there is one type-1 customer in the system) is written as follows:

$$V^1(1,0) = \{1h_1 + 0h_2 + \lambda_1 V(2,0) + \lambda_2 V(1,1) + \mu_1 V(1,0)$$
$$+ \mu_2 V(0,0) + (0)\gamma_1(V(0,0)+\beta_1) + 0\gamma_2(V(1,0)+\beta_2)$$
$$+ (M-1+1)\gamma_1 V(1,0) + (N-0)\gamma_2 V(1,0)\}\Lambda^{-1} \quad (3.8.1)$$

$$V^2(1,0) = \{1h_1 + 0h_2 + \lambda_1 V(2,0) + \lambda_2 V(1,1) + \mu_1 V(0,0)$$
$$+ \mu_2 V(1,0) + (0)\gamma_1(V(0,0)+\beta_1) + 0\gamma_2(V(1,0)+\beta_2)$$
$$+ (M-1+1)\gamma_1 V(1,0) + (N-0)\gamma_2 V(1,0)\}\Lambda^{-1} \quad (3.8.2)$$

The equations for the condition $2 \leq x_1 \leq M-1$, $x_2 = N$ is given as below:

$$\begin{aligned} V^1(x_1, N) = \{ & x_1 h_1 + N h_2 + \lambda_1 V(x_1+1, N) + \lambda_2 V(x_1, N) + \mu_1 V(x_1-1, N) \\ & + \mu_2 V(x_1-1, N) + (x_1-2)\gamma_1 (V(x_1-1, N) + \beta_1) \\ & + N\gamma_2 (V(x_1, N-1) + \beta_2) + (M - x_1 + 2)\gamma_1 V(x_1, N) \\ & + (N-N)\gamma_2 V(x_1, N)\}\Lambda^{-1} \end{aligned} \tag{3.9.1}$$

$$\begin{aligned} V^2(x_1, N) = \{ & x_1 h_1 + N h_2 + \lambda_1 V(x_1+1, N) + \lambda_2 V(x_1, N) + \mu_1 V(x_1-1, N) \\ & + \mu_2 V(x_1, N-1) + (x_1-1)\gamma_1 (V(x_1-1, N) + \beta_1) \\ & + (N-1)\gamma_2 (V(x_1, N-1) + \beta_2) + (M - x_1 + 1)\gamma_1 V(x_1, N) \\ & + (N-N+1)\gamma_2 V(x_1, N)\}\Lambda^{-1} \end{aligned} \tag{3.9.2}$$

Equation for two points $x_1 = 1$, $x_2 = N$ are formulated as following:

$$\begin{aligned} V^1(1, N) = \{ & 1h_1 + N h_2 + \lambda_1 V(2, N) + \lambda_2 V(1, N) + \mu_1 V(1, N) \\ & + \mu_2 V(0, N) + (0)\gamma_1 (V(0, N) + \beta_1) + N\gamma_2 (V(1, N-1) + \beta_2) \\ & + (M-1+1)\gamma_1 V(1, N) + (N-N)\gamma_2 V(1, N)\}\Lambda^{-1} \end{aligned} \tag{3.10.1}$$

$$\begin{aligned} V^2(1, N) = \{ & 1h_1 + N h_2 + \lambda_1 V(2, N) + \lambda_2 V(1, N) + \mu_1 V(0, N) \\ & + \mu_2 V(1, N-1) + (0)\gamma_1 (V(0, N) + \beta_1) + (N-1)\gamma_2 (V(1, N-1) + \beta_2) \\ & + (M-1+1)\gamma_1 V(1, N) + (N-N+1)\gamma_2 V(1, N)\}\Lambda^{-1} \end{aligned} \tag{3.10.2}$$

Equation for condition $2 \le x_1 \le M-1$, $1 \le x_2 \le N-1$ is given as follow:

$$\begin{aligned}V^1(x_1,x_2) = &\{x_1h_1 + x_2h_2 + \lambda_1 V(x_1+1,x_2) + \lambda_2 V(x_1,x_2+1) + \mu_1 V(x_1-1,x_2)\\ &+ \mu_2 V(x_1-1,x_2) + (x_1-2)\gamma_1 (V(x_1-1,x_2)+\beta_1)\\ &+ x_2\gamma_2 (V(x_1,x_2-1)+\beta_2) + (M-x_1+2)\gamma_1 V(x_1,x_2)\\ &+ (N-x_2)\gamma_2 V(x_1,x_2)\}\Lambda^{-1}\end{aligned} \tag{3.11.1}$$

$$\begin{aligned}V^2(x_1,x_2) = &\{x_1h_1 + x_2h_2 + \lambda_1 V(x_1+1,x_2) + \lambda_2 V(x_1,x_2+1) + \mu_1 V(x_1-1,x_2)\\ &+ \mu_2 V(x_1,x_2-1) + (x_1-1)\gamma_1 (V(x_1-1,x_2)+\beta_1) + (x_2-1)\gamma_2\\ &(V(x_1,x_2-1)+\beta_2) + (M-x_1+1)\gamma_1 V(x_1,x_2)\\ &+ (N-x_2+1)\gamma_2 V(x_1,x_2)\}\Lambda^{-1}\end{aligned} \tag{3.11.2}$$

Finally, for the condition $x_1 = 1$, $1 \le x_2 \le N-1$, the equation is written as following:

$$\begin{aligned}V^1(1,x_2) = &\{1h_1 + x_2h_2 + \lambda_1 V(2,x_2) + \lambda_2 V(1,x_2+1) + \mu_1 V(1,x_2)\\ &+ \mu_2 V(0,x_2) + (0)\gamma_1 (V(0,x_2)+\beta_1)\\ &+ x_2\gamma_2 (V(1,x_2-1)+\beta_2) + (M-1+1)\gamma_1 V(1,x_2)\\ &+ (N-x_2)\gamma_2 V(1,x_2)\}\Lambda^{-1}\end{aligned} \tag{3.12.1}$$

$$\begin{aligned}V^2(1,x_2) = &\{1h_1 + x_2h_2 + \lambda_1 V(2,x_2) + \lambda_2 V(1,x_2+1) + \mu_1 V(0,x_2)\\ &+ \mu_2 V(1,x_2-1) + (1-1)\gamma_1 (V(0,x_2)+\beta_1) + (x_2-1)\gamma_2\\ &(V(1,x_2-1)+\beta_2) + (M-1+1)\gamma_1 V(1,x_2)\\ &+ (N-x_2+1)\gamma_2 V(1,x_2)\}\Lambda^{-1}\end{aligned} \tag{3.12.2}$$

As can be seen from the formulations given above, we need to make a decision between two actions for any states which are allocating server 2 to a type-1 customer or allocating server 2 to a type-2 customer in order to minimize cost. Optimal solution of the process can be achieved by using value iteration algorithm where corner points, boundaries and non boundary points are formulated separately as shown at formulas given above. The model gives us a decision matrix for server 2 which includes serving type-1 customer or type-2 customer for different states (for the conditions with different number of customers in the system).

The pseudo-code that finds optimal solution to the problem is given in Figure 3.3 in which cost and decision matrices are calculated. Main body of algorithm is shown in this figure where pseudo-code of functions are shown in Figure 3.4 and 3.5.

In the main part, the function that calculates cost function is repeated by many iterations in order to reach steady state. Values of costs and decisions for different states are written on a file at the end of all iterations.

In the Figure 3.4, structure of cost function equations is shown and decision matrices of customer type to serve for flexible server at different states are calculated. First of all parameter set is assigned to the variables used at equations. Then cost values for server 1 and server 2 are calculated where two calculations are made for server 2. One of the calculations is for serving to type-1 customer where second one is for serving type-2 customer. Decision for customer type to serve is given by choosing the equation of cost function with minimum value. This calculation is done for all equations at different states.

```
Define variable IT to 10000, M to 100, N to 100
Declare Value and Decision Matrices with size M and N
Declare matrik variables called mat1, mat2, mat3
Declare parameter function
Declare initialize matrix function
Declare calculate matrix function
Declare difference matrix function
Declare display matrix function
Declare integer variables called counter,size,i,t
Declare a float variable called difference
Declare a float array called  parameter with size 13
file  <-Openfile
CALL initialize matrix (mat1, mat2, mat3)
WHILE end of file
    CALL parameter  (parameter,file)
    DISPLAY Parameters from 0 to 13
    FOR Counter 0 to IT-1
            Call calculate matrix (mat1, mat2, parameter,counter)
    END FOR
    difference<-Call difference matrix (mat1, mat2, mat3)
    Call display matrix (mat3, parameter,difference,mat2)
    Call initialize matrix (mat1, mat2, mat3)
```

Figure 3.3. Pseudo-Code for the Solution of the Problem.

FUNCTION calculate matrix (mat1, mat2, parameter,counter):

Declare float variables called h1,h2, ac1, ac2, ar1,ar2,abr1,abr2,s1,s21,s22,Lamda1,Lamda2

Declare float variable cost array

Assign values of variables from paramater array //h1:holding cost of customer type 1, h2:holding cost of customer type 1, ac1: abondonment cost of customer type 1, ac2: abondonment cost of customer type 2, ar1:arrival rate of customer type 1, ar2:arrival rate of customer type 2, abr1: abondonment rate of cutomer type 1, abr2: abondonment rate of cutomer type 2, s1:service rate of server 1, s21: service rate of server 2 for customer type 1, s22: service rate of server 2 for customer type 2

Assign Lamda1 = ar1+ar2+abr1*M+abr2*N+s1+s21; //Sum of rates when server 2 give service to customer type 1

Assign Lamda2 = ar1+ar2+abr1*M+abr2*N+s1+s22; // Sum of rates when server 2 give service to customer type 2

Find minimum of value matrix //value matrix is the cost function

Cost array[0]=mat1.Value Matrix^1 // Value matrix that server 2 gives service to customer type 1

Cost array[1]=mat1.Value Matrix^2 //Value matrix that server 2 gives service to customer type 2

Mat2.Value Matrix= minimum of mat1.Value Matrix^1 and mat1.Value Matrix^2

Mat2.Decision matrix=0 or 1 // 0:giving service to customer type 1 of server 2, 1:giving service to customer type 2 of server 2,

initialize cost array to 0

Repeat "finding minimum of value matrix for the points"

V[0][0], V[M][0], V[M][N], V[0][N], V[0][1]- V[0][N-1], V[M][1]- V[M][N-1],

V[2][0]- V[M-1][0], V[2][N]- V[M-1][N], V[1][0], V[1][N], V[2][1]- V[M-1][N-1], V[1][1]- V[1][N-1]

Figure 3.4. Pseudo-Code of Cost Function.

In the Figure 3.5, functions for (a) initialization of parameters, (b) reading parameters from file, (c) displaying outputs to a file and (d) taking difference of cost function between iterations are shown as pseudo-code.

```
FUNCTION initialize matrix (mat1, mat2, mat3):
    initialize value matrix to 0
    initialize decision matrix to 0
```

(a)

```
FUNCTION parameter (parameter,file):
    READ parameters from file
    WRITE  parameters to paramater array
```

(b)

```
FUNCTION display matrix (mat3, parameter,difference,mat2):
    Declare integers variables i and j
    Declare float variable difference
    f<-OPEN Outputfile.txt to APPEND
    WRITE Parameters on Outputfile
    FOR j N to 0
        FOR i 0 to M
          WRITE Value Matrix of Mat3 on Outputfile
          WRITE Decision Matrix of Mat3 on Outputfile
        END FOR
    END FOR
```

(c)

```
FUNCTION difference matrix (mat1, mat2, mat3)
    Declare integers variables i and j
    Declare float variable difference
    FOR  i 0 to M
        FOR j 0 to N
          Mat3.Value Matrix = Mat2.Value Matrix  - Mat1.Value Matrix
          Mat3.decision matrix = Mat2.decision matrix
        END FOR
    END FOR
    difference= Mat3.Value Matrix[M-2][N-2]  - Mat3. Value Matrix[2][2]
//Control if it reaches to steady stead condition
```

(d)

Figure 3.5. Pseudo-Code of Other Functions.

3.3. Heuristics and Other Policies

Optimal policy can be complicated since it contains so much parameters like arrival rates, service rates, abandonment rates and costs, holding costs. We also modelled different stationary policies besides optimal policy which are defined as below:

- (i) Fully Dedicated Policy: Two servers serve only to their customers.
- (ii) Static Policy: Server 2 serves type-1 customer if its own customer (type-2 customer) does not exist in the system.
- (iii) Heuristic 1: server 2 serves type-1 customer if its holding cost has higher value than type-2 customer .
- (iv) Heuristic 2: Server 2 serves customer with higher value of holding cost and

abandonment cost.

- (v) Heuristic 3: This takes waiting time E(W) into account where Server 2 serves customer with higher value of E(W).

3.4. Fully Dedicated Policy

In this policy, two servers give service only to their own customers, in other words flexible server behaves like dedicated server as well. Server 1 gives service only to type-1 customer, while server 2 gives service only to type-2 customer. They stay idle if their customer does not exist in the system. So that, we do not calculate $V^1(x_1, x_2)$ in the model since server 2 only serve type-2 customer whose cost is calculated with the equation $V^2(x_1, x_2)$. This is valid for all states (condition for different number of customers in the system). As a result, cost function $V^2(x_1, x_2)$ becomes as the following:

$$
\begin{aligned}
V^2(x_1, x_2) = \{ & x_1 h_1 + x_2 h_2 + \lambda_1 V(x_1+1, x_2) + \lambda_2 V(x_1, x_2+1) + \mu_1 V(x_1-1, x_2) \\
& + \mu_2 V(x_1, x_2-1) + (x_1-1)\gamma_1 (V(x_1-1, x_2) + \beta_1) + (x_2-1)\gamma_2 \\
& (V(x_1, x_2-1) + \beta_2) + (M - x_1 + 1)\gamma_1 V(x_1, x_2) \\
& + (N - x_2 + 1)\gamma_2 V(x_1, x_2)\} \Lambda^{-1}
\end{aligned}
$$

3.5. Static Policy

In static policy, server 2 gives service to type-1 customer if its customer (type 2) does not exist in the system. In other words, server 2 gives service to type-2 customer exhaustively, and then switches to type-1 customer if exists. So, cost function $V(x)$ becomes as the following:

The equation below is valid for the states that $0 \leq x_1 \leq M$, $1 \leq x_2 \leq N$ which means it serves type-2 customer at the existence of this type of customer. Servers giving service to their own customers is shown with the formula $V^2(x_1, x_2)$;

$$V^2(x_1, x_2) = \{x_1h_1 + x_2h_2 + \lambda_1 V(x_1 + 1, x_2) + \lambda_2 V(x_1, x_2 + 1) + \mu_1 V(x_1 - 1, x_2)$$
$$+ \mu_2 V(x_1, x_2 - 1) + (x_1 - 1)\gamma_1 (V(x_1 - 1, x_2) + \beta_1) + (x_2 - 1)\gamma_2$$
$$(V(x_1, x_2 - 1) + \beta_2) + (M - x_1 + 1)\gamma_1 V(x_1, x_2)$$
$$+ (N - x_2 + 1)\gamma_2 V(x_1, x_2)\}\Lambda^{-1}$$

The equation below is valid for the states $1 \leq x_1 \leq M$, $x_2 = 0$ which means flexible server gives service to type-1 customer at the absence of type-2 customer. This condition is given by the formula $V^1(x_1, x_2)$ given below.

$$V^1(x_1, 0) = \{x_1h_1 + 0h_2 + \lambda_1 V(x_1 + 1, 0) + \lambda_2 V(x_1, 1) + \mu_1 V(x_1 - 1, 0)$$
$$+ \mu_2 V(x_1 - 1, 0) + (x_1 - 2)\gamma_1 (V(x_1 - 1, 0) + \beta_1)$$
$$+ 0\gamma_2 (V(x_1, 0) + \beta_2) + (M - x_1 + 2)\gamma_1 V(x_1, 0)$$
$$+ (N - 0)\gamma_2 V(x_1, 0)\}\Lambda^{-1}$$

3.6. Heuristic 1

In Heuristic 1, if holding cost of type-1 customer is higher than holding cost of type-2 customer, server 2 gives service to type-1 customer, otherwise it serves type-2 customer. The condition is formulated by $x_1h_1 > x_2h_2$. In this equation, x_1 represents number of type-1 customer while x_2 represents number of type-2 customer in the system.

In the algorithm, we do not need to control the x_1h_1 or x_1h_2 for the cases $1 \leq x_1 \leq M$, $x_2 = 0$ or $1 \leq x_2 \leq N, x_1 = 0$ where flexible server gives service directly to type-1 customer for the first condition and type-2 customer for the second condition respectively. Other than these cases, we check if $x_1h_1 > x_2h_2$ or not.

In case of $x_1h_1 > x_2h_2$, the cost function becomes as following. We see by the formula $V^1(x_1, x_2)$ that server 2 gives service to type-1 customer as given in the following equation.

$$V^1(x_1,x_2) = \{x_1h_1 + x_2h_2 + \lambda_1 V(x_1+1,x_2) + \lambda_2 V(x_1,x_2+1) + \mu_1 V(x_1-1,x_2) \\ + \mu_2 V(x_1-1,x_2) + (x_1-2)\gamma_1 (V(x_1-1,x_2)+\beta_1) \\ + x_2\gamma_2 (V(x_1,x_2-1)+\beta_2) + (M-x_1+2)\gamma_1 V(x_1,x_2) \\ + (N-x_2)\gamma_2 V(x_1,x_2)\}\Lambda^{-1}$$

However, if $x_2h_2 > x_1h_1$, the following cost equation becomes valid where server 2 gives service to type-2 customer shown by the formula $V^2(x_1, x_2)$ given below.

$$V^2(x_1,x_2) = \{x_1h_1 + x_2h_2 + \lambda_1 V(x_1+1,x_2) + \lambda_2 V(x_1,x_2+1) + \mu_1 V(x_1-1,x_2) \\ + \mu_2 V(x_1,x_2-1) + (x_1-1)\gamma_1 (V(x_1-1,x_2)+\beta_1) + (x_2-1)\gamma_2 \\ (V(x_1,x_2-1)+\beta_2) + (M-x_1+1)\gamma_1 V(x_1,x_2) \\ + (N-x_2+1)\gamma_2 V(x_1,x_2)\}\Lambda^{-1}$$

3.7. Heuristic 2

In Heuristic 2, server 2 gives service to customer type with higher value of holding cost and abandonment cost. Server 2 gives service to type-1 customer if its value is higher than type-2 customer, otherwise it gives service to type-2 customer. The condition is formulated by

$$x_1\beta_1\gamma_1 + x_1h_1 > x_2\beta_2\gamma_2 + x_2h_2$$

This equation differs from heuristic 1 since it takes abandonment rate and abandonment cost into account. For the conditions $1 \leq x_1 \leq M$, $x_2 = 0$ or $1 \leq x_2 \leq N$, $x_1 = 0$, flexible server gives service directly to type-1 customer for the first condition and type-2 customer for the second condition respectively. For the conditions of non zero customer in the system, the equation given below is valid providing that type-1 customer has higher value of that policy. In this equation of cost function, server 2 gives service to type-1 customer by the formula $V^1(x_1, x_2)$ since type-1 customer has priority to be served for this heuristic.

$$\begin{aligned}
V^1(x_1, x_2) = \{&x_1 h_1 + x_2 h_2 + \lambda_1 V(x_1 + 1, x_2) + \lambda_2 V(x_1, x_2 + 1) + \mu_1 V(x_1 - 1, x_2) \\
&+ \mu_2 V(x_1 - 1, x_2) + (x_1 - 2)\gamma_1 (V(x_1 - 1, x_2) + \beta_1) \\
&+ x_2 \gamma_2 (V(x_1, x_2 - 1) + \beta_2) + (M - x_1 + 2)\gamma_1 V(x_1, x_2) \\
&+ (N - x_2)\gamma_2 V(x_1, x_2)\}\Lambda^{-1}
\end{aligned}$$

For the contrary conditions, that is, type-2 customer has higher value than type-1 customer of that policy, equation becomes as following. In this equation of cost function, server 2 gives service to type-2 customer by the formula $V^2(x_1, x_2)$ since type-2 customer has priority for this policy of heuristic.

$$\begin{aligned}
V^2(x_1, x_2) = \{&x_1 h_1 + x_2 h_2 + \lambda_1 V(x_1 + 1, x_2) + \lambda_2 V(x_1, x_2 + 1) + \mu_1 V(x_1 - 1, x_2) \\
&+ \mu_2 V(x_1, x_2 - 1) + (x_1 - 1)\gamma_1 (V(x_1 - 1, x_2) + \beta_1) + (x_2 - 1)\gamma_2 \\
&(V(x_1, x_2 - 1) + \beta_2) + (M - x_1 + 1)\gamma_1 V(x_1, x_2) \\
&+ (N - x_2 + 1)\gamma_2 V(x_1, x_2)\}\Lambda^{-1}
\end{aligned}$$

The condition for this case is given as below:

$$x_2 \beta_2 \gamma_2 + x_2 h_2 > x_1 \beta_1 \gamma_1 + x_1 h_1$$

3.8. Heuristic 3

In Heuristic 3, it takes waiting time into account in which the mean waiting time formulation, E(W), for M/M/1 queues is used as a policy. E(W) formula for one server queues (M/M/1 system) is as following (Adan and Resing, 2015):

$$E(W) = E(S) - 1/\mu = \frac{\rho/\mu}{1-\rho}$$

where W is waiting time, S is sojourn time and ρ is the fraction of time the server is working which is formulated by $\rho = \frac{\lambda}{\mu}$.

We multiplied this formula with both number of customers in queue and holding cost per unit customer. So, formulation for this condition becomes as following:

$$x_1 h_1 \frac{\frac{\lambda_1}{\mu_1^2}}{1-\frac{\lambda_1}{\mu_1}} > x_2 h_2 \frac{\frac{\lambda_2}{\mu_2^2}}{1-\frac{\lambda_2}{\mu_2}}$$

Server 2 serves customer with higher value of this control. Server 2 gives service to type-1 customer if its value is higher than customer type-2, otherwise it gives service to type-2 customer. The equation becomes as following for giving service to type-1 customer. This decision is shown by formula $V^1(x_1, x_2)$.

$$\begin{aligned} V^1(x_1,x_2) = \{ & x_1h_1 + x_2h_2 + \lambda_1 V(x_1+1,x_2) + \lambda_2 V(x_1,x_2+1) + \mu_1 V(x_1-1,x_2) \\ & + \mu_2 V(x_1-1,x_2) + (x_1-2)\gamma_1 (V(x_1-1,x_2)+\beta_1) \\ & + x_2\gamma_2 (V(x_1,x_2-1)+\beta_2) + (M-x_1+2)\gamma_1 V(x_1,x_2) \\ & + (N-x_2)\gamma_2 V(x_1,x_2)\}\Lambda^{-1} \end{aligned}$$

In case of giving service to type-2 customer, formula of $V^2(x_1, x_2)$ is used which is as following:

$$\begin{aligned} V^2(x_1,x_2) = \{ & x_1h_1 + x_2h_2 + \lambda_1 V(x_1+1,x_2) + \lambda_2 V(x_1,x_2+1) + \mu_1 V(x_1-1,x_2) \\ & + \mu_2 V(x_1,x_2-1) + (x_1-1)\gamma_1 (V(x_1-1,x_2)+\beta_1) + (x_2-1)\gamma_2 \\ & (V(x_1,x_2-1)+\beta_2) + (M-x_1+1)\gamma_1 V(x_1,x_2) \\ & + (N-x_2+1)\gamma_2 V(x_1,x_2)\}\Lambda^{-1} \end{aligned}$$

4. NUMERICAL RESULTS

In this chapter of the thesis, results of the experiments are analysed. The experiments are performed in C Program Language for the proposed model in order to find optimal solution with different sets of parameters. In other words, we investigate solutions with different design of parameters such as different arrival rates, service rates, abandonment rates, abandonment costs and holding costs. In addition to optimal model, we also analyse other policies in order to compare the results with optimal solution for different sets of parameters.

4.1. Parameter Settings

In the model, the algorithm is iterated for 10000 runs in order to reach a steady state. In addition, upper bound for number of customers are set to a big value in order to approximate to an infinite queue model. We test this approximation by calculating the difference between cost values of different states for iteration n and n+1. This value of cost functions should not differ too much between states in order to indicate the convergence.

We design experiments by using different settings of 11 independent parameters. All examples assume poisson arrival processes and exponential service processes with non-preemptive discipline. We set λ_2=5 and $\lambda = \lambda_1 + \lambda_2$ takes values from the set $\{10, 15, 20\}$. We set $\rho = \lambda/\mu$ that takes values $\{0.25, 0.5, 0.75\}$ where $\mu = \mu_1 + \mu_2$ and $\mu_1 = \mu_2$. We set holding cost of type-1 customer as h_1=1 and define h_2 as multiple of h_1 from the set $\{0.5, 1, 3\}$. Abandonment rates of customers differ in values of $(\gamma_1, \gamma_2) = \{(0,3), (0,3)\}$. Finally, values of abandonment costs are set as a multiple of holding costs which are defined as $h_i/\beta_i \in \{0.5, 1, 2\}$

4.2. Definition of Outputs

Model has two types of outputs which are decision matrix of flexible server (server 2) and value of optimal cost function for different set of experiments. First of all, we examine results of decision matrix for server 2 for type of customer to serve. This decision matrix is constructed with the goal of minimizing cost function. When we analyse the experiments for different combination of parameters and states we see mainly 4 types of decision matrices for flexible server which can be summarized as:

- priority based (exhaustive) decision matrices
- decision matrices with vertical threshold value
- linear decision matrices
- decision matrices with horizontal threshold value (similar behaviour with priority based decision matrices)

Priority based (exhaustive) model means that server 2 gives service to type-2 customer (its own customer) while type-2 customer exists in the system. Secondly, decision matrix with threshold value means server 2 decides to serve type-1 customer when number of type-1 customer in the system is above a limit. Thirdly, decision matrix of server 2 has also a linear switching line in order to begin to serve type-1 customer while number of customers changes in the system. Finally, in the horizontal type of threshold line, server 2 begins to give service to type-2 customer after type-2 customer reaches to a level in the system.

Besides, we investigate values of cost function as an output of experiments. The values of cost function are affected from different combination of parameter sets in an increasing or decreasing manner. This effect is analysed in detail in the further parts of this chapter.

4.3. Decision Matrix Types As an Output

Firstly, priority based decision matrix of the optimal solution can be seen in Figure 4.1. As seen in Figure 4.1, $x = (x_1, x_2)$ show the state of the system, where x_1 and x_2 are numbers of type-1 customers and type-2 customers respectively. In case of presence of type-2 customers, flexible server should serve these customers which is labelled in Figure 4.1 as 2, otherwise should serve type-1 customers which is labelled in Figure 4.1 as 1. In addition, 0 indicates there is no difference at the value of cost function at the state of (x_1, x_2) =$(0, 0)$. Examples from experiments that lead to this type of decision matrix are given in Table 4.1 where different combinations of parameters exist.

Table 4.1. A Sample of Experiments Giving Priority Based Decision Matrix

λ_1	λ_2	$\mu 1$	$\mu 2$	γ_1	γ_2	h_1	h_2	β_1	β_2
5	5	20	20	0	0	1	3	0	0
5	5	10	10	0	0	1	3	0	0
5	5	6.7	6.7	3	0	1	3	2	0
5	5	6.7	6.7	3	0	1	1	1	0
5	5	6.7	6.7	3	0	1	0.5	1	0
5	5	10	10	3	0	1	3	3	0
5	5	20	20	3	3	1	3	2	6
5	5	20	20	3	3	1	3	1	3
5	5	20	20	3	3	1	3	0.5	1.5
10	5	30	30	3	3	1	3	0	0
10	5	30	30	0	3	1	3	0	6
10	5	10	10	3	0	1	3	2	0

As seen in Table 4.1, when holding cost of type-2 customer is bigger than holding cost of type-1 customer, optimum model gives solution as to serve type-2 customer for server 2 in existence of type-2 customer in the system. Another inference from the table is, at low service rates, server 2 decides to serve type-2 customer although type-2 customer has less or equal holding cost compared to type-1 customer. In addition, existence of abandonment rate of type-1 customer is also influential at the decision matrix. Existence of abandonment rates for both customer types also leads to priority based solution when abandonment cost of type-2 customer is triple of type-1 customer.

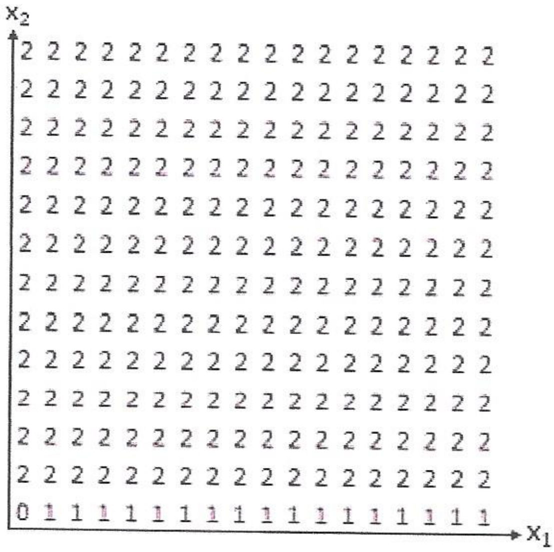

Figure 4.1. Priority Based Model.

Secondly, decision matrix with threshold value is analysed. This matrix occurs when server 2 decides to serve type-1 customer when number of this customer in the system is above a limit. Decision matrix is given in Figure 4.2. Some examples from experiments for different threshold values are given in Table 4.2.

As seen in Table 4.2, having abandonment rate for type-2 customer leads to serve type-1 customer for server 2 when type-1 customer is above a limit. Abandonment costs and holding costs are also effective for the construction of this decision matrix.

It is appeared that value of service rates has an effect on threshold value at decision matrix. An example of this condition is the set of experiments given in Table 4.3. Service rates are 20,10 and 6.7 respectively for set 1, while other parameters are fixed at the same value. For set 2, service rates also differs inside the experiments

Table 4.2. A Sample of Experiments Giving Decision Matrix with Threshold Value

λ_1	λ_2	$\mu 1$	$\mu 2$	γ_1	γ_2	h_1	h_2	β_1	β_2
5	5	6.67	6.67	0	3	1	0.5	0	1
5	5	10	10	0	3	1	1	0	2
5	5	6.67	6.67	0	3	1	1	0	2
5	5	20	20	3	3	1	0.5	2	1
5	5	10	10	3	3	1	0.5	2	1
5	5	6.67	6.67	3	3	1	0.5	1	0.5
5	5	20	20	0	3	1	0.5	0	0.25
5	5	20	20	0	3	1	0.5	0	0.5
10	5	30	30	0	3	1	0.5	0	1
10	5	30	30	3	3	1	0.5	2	1

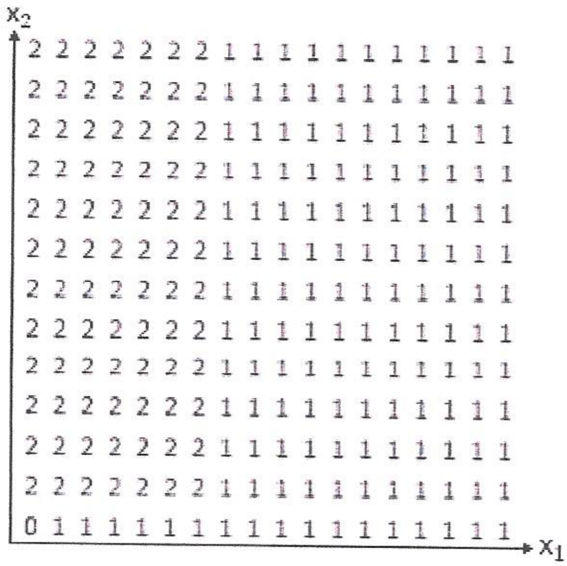

Figure 4.2. Matrix with Threshold Value.

of the set. In addition, set 2 differs in terms of holding costs and abandonment costs compared to parameters of set 1. While making comparison between sets, it is observed that, threshold value of set 2 experiments is higher than set 1 parameters. In other words, flexible server switches to type-1 customer when number of type-1 customer is at a higher level in set 2 compared to number of type-1 customer in set 1.

While we investigate the decision matrix within each set, we conclude that in case of decreasing service rates, flexible server switches to type-1 customer despite having less number of type-1 customer in the system. This could be explained as dedicated server could not serve its customers as fast as before that results in abandonment of customers, so that flexible server is more tend to help to dedicated server by serving type-1 customer.

Table 4.3. Service Rate Effect on Threshold Value

Set	λ_1	λ_2	$\mu1$	$\mu2$	γ_1	γ_2	h_1	h_2	β_1	β_2
1	5	5	20	20	0	3	1	0.5	0	0.25
	5	5	10	10	0	3	1	0.5	0	0.25
	5	5	6.67	6.67	0	3	1	0.5	0	0.25
2	5	5	20	20	0	3	1	3	0	1.5
	5	5	10	10	0	3	1	3	0	1.5
	5	5	6.67	6.67	0	3	1	3	0	1.5

In Figure 4.3 given below, decrease in service rate leads to threshold value becoming smaller. In the figure, (a) represents a decision matrix with the largest service rate, and (c) has the smallest service rate compared to (a) and (b).

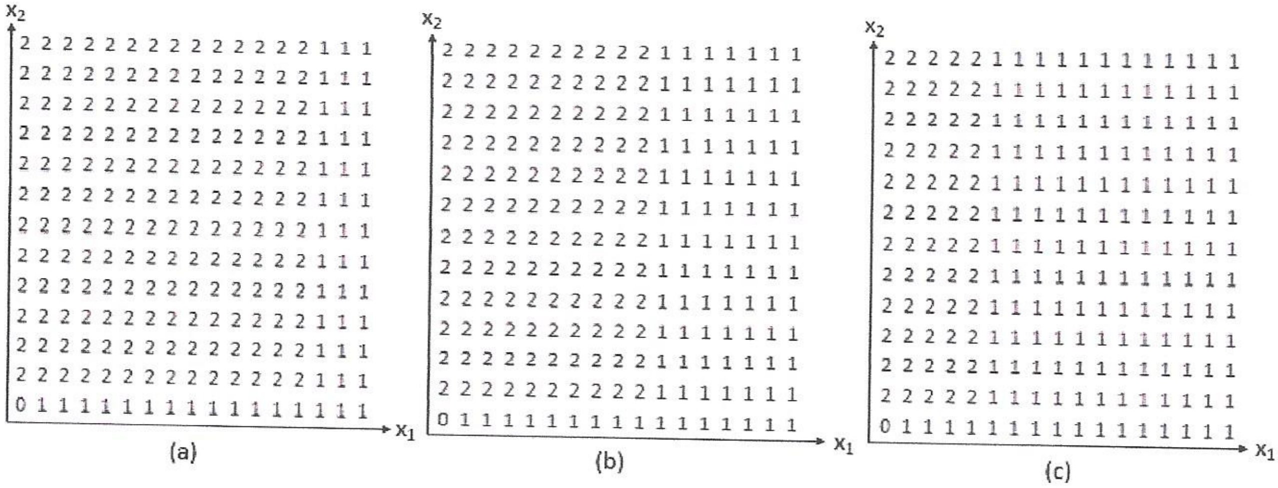

Figure 4.3. Threshold Values with different service rates

There is also a special case where a decision type with threshold value changes to a priority based decision type. Set of parameters for this condition is given in Table 4.4. First parameter set gives the threshold decision matrix, on the other hand second gives priority based decision matrix. Flexible server gives service to his own customers with increase in holding cost and abandonment cost of type-2 customer.

Table 4.4. From Threshold to Priority Based Experiments

λ_1	λ_2	$\mu1$	$\mu2$	γ_1	γ_2	h_1	h_2	β_1	β_2
10	5	30	30	3	3	1	0.5	2	1
10	5	30	30	3	3	1	3	2	6

Thirdly, decision matrix of server 2 has also linear switching line which shows moving from type-2 customers to type-1 customer as number of customers changes in the system. While number of type-1 customer is more than type-2 customer, server 2 is tending to serve type-1 customer for some experiments given in Table 4.5 in detail.

Parameters have same values except service rates in Table 4.5. Customers do not abandon, however their holding costs differ where holding cost of type-1 customer is

Table 4.5. A Sample of Experiments Giving a linear switching line

λ_1	λ_2	$\mu1$	$\mu2$	γ_1	γ_2	h_1	h_2	β_1	β_2
5	5	20	20	0	0	1	0.5	0	0
5	5	10	10	0	0	1	0.5	0	0
5	5	6.7	6.7	0	0	1	0.5	0	0
10	5	30	30	0	0	1	0.5	0	0
10	5	15	15	0	0	1	0.5	0	0
10	5	10	10	0	0	1	0.5	0	0

twice compared to holding cost of type-2 customer. Experiments for both arrival rates with values 5 and 10 give the same type of decision matrix. Decision matrix in Figure 4.4 belongs to experiment with minimum service rates where $\mu1=\mu2=6.7$ with having the highest utilization. As utilization rate decreases, linearity curve becomes steeper. In other words, server 2 begins to serve type-1 customer in case having less number of type-2 customer in the system comparing to higher utilization rates.

```
x2
2 2 2 2 2 2 2 2 2 2 2 2 2 1 1 1
2 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1
2 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1
2 2 2 2 2 2 2 2 2 2 2 1 1 1 1 1
2 2 2 2 2 2 2 2 2 2 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2 2 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 1 1 1 1 1 1 1 1 1 1
2 2 2 2 2 1 1 1 1 1 1 1 1 1 1 1
2 2 2 2 1 1 1 1 1 1 1 1 1 1 1 1
0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  x1
```

Figure 4.4. Matrix with Linear Switching line.

Another decision matrix extracted from the experiments are horizontal threshold type of matrix. In Figure 4.5, server 2 begins to serve type-2 customer after reaching to a value type-2 customer in the system. When we observe the values of parameters that

Table 4.6. A Sample of Experiments Giving Horizontal Threshold Decision Matrix

λ_1	λ_2	$\mu 1$	$\mu 2$	γ_1	γ_2	h_1	h_2	β_1	β_2
5	5	10	10	3	0	1	0.5	2	0
5	5	20	20	3	0	1	1	2	0
5	5	20	20	3	0	1	0.5	1	0
5	5	10	10	3	0	1	0.5	1	0
10	5	10	10	3	0	1	0.5	2	0
10	5	15	15	3	0	1	0.5	2	0
10	5	15	15	3	0	1	1	2	0
10	5	30	30	3	0	1	1	2	0

lead to that decision matrix, we observe that abandonment rates of type-1 customer are triple of abandonment rates of type-2 customers. There are differences at service rates, arrival rates, holding and abandonment costs that lead to this type of decision matrix as well.

As service rates are decreasing, this threshold value becomes narrower. In other words, when service rate of flexible server is lower, it begins to serve its own customer (customer type 2) earlier compared to higher service rate. That is, it is enough to reach a fewer number of type-2 customer in the system in order to begin to serve its own customer.

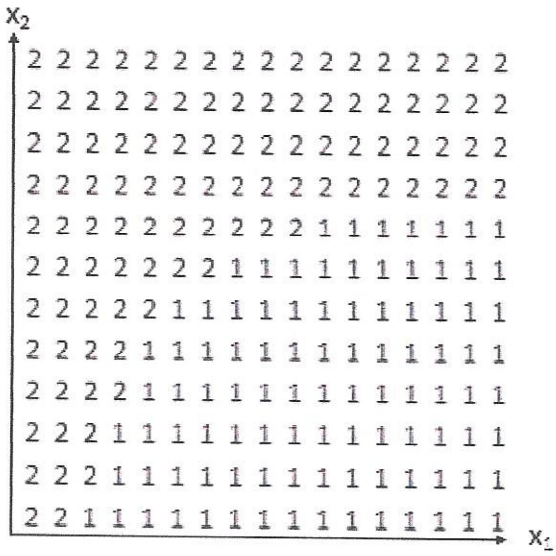

Figure 4.5. Matrix with horizontal threshold.

4.4. Values of Cost Function As an Output

When we examine values of cost function for different sets of parameters we can determine the factors that increase value of cost as below:

- decrease in service rates and increase in arrival rates
- increase in holding costs and abandonment costs
- decrease in abandonment rates

A sample of optimum values of cost function extracted from a set of experiments at different combination of parameters are given in the Table 4.7 where we can conclude with the inferences summarized above. The table shows results of 9 experiments for the state of the system beginning with no customer, in other words it gives values of cost function $V(0,0)$.

In the experiment 1 and 2, effect of service rate is seen obviously where decrease in service rate which means increase in utilization rate results in increase in cost of objective function. Secondly, we can compare experiment 1 and 3 to see the effect of holding

Table 4.7. Optimal values of cost function for different experiments

exp no	λ_1	λ_2	$\mu 1$	$\mu 2$	γ_1	γ_2	h_1	h_2	β_1	β_2	cost
1	5	5	20	20	3	0	1	1	2	2	17.9
2	5	5	10	10	3	0	1	1	2	2	59.0
3	5	5	20	20	3	0	1	3	2	6	37.9
4	5	5	20	20	3	3	1	1	2	2	15.7
5	5	5	20	20	3	3	1	1	1	1	12.2
6	15	5	20	20	3	0	1	1	2	2	53.8
7	15	5	20	20	3	0	1	3	2	6	82.4
8	15	5	13.3	13.3	3	0	1	1	2	2	140.5
9	15	5	13.3	13.3	3	3	1	1	2	2	97.8

cost. Holding cost of type-2 customer at experiment 3 is three times of the holding cost of type-2 customer at experiment 1 which leads to increase in cost of objective function. Thirdly, increase is abandonment cost leads to increase in objective function which can be observed from experiments 4 and 5. In experiment 5, both abandonment costs of customers are half of the abandonment costs of customers at experiment 4, which resulted in lower cost value of objective function. Fourthly, experiments 1 and 6, and experiments 3 and 7 show the effect of arrival rate to cost function. Experiments 6 and 7 have triple values of arrival rates compared to experiments 1 and 3 respectively which leads to higher value of cost function. Finally, experiments 8 and 9 show the effect of abandonment rates on value of cost function. We observe that abandonment rate exists for type-2 customer in experiment 9 contrary to experiment 8 which results in lower cost value. So we can conclude that abandonment rates result in decrease in values of cost function.

4.5. Comparison of Optimal Results with Other Policies

Values of cost function for different parameters are given in Table 4.8 where difference in cost values of other policies with optimal solution are showed in percentages.

The table shows results of 16 experiments for the state of the system beginning with no customer that is, $V(0,0)$. Values at the table show the difference of cost values in percentages between each policy ad optimum solution.

Column with $fd\%$ represents differences of cost values between fully dedicated policy and optimum solution which posses the worst values. The formulation of $fd\%$ is;

$$fd\% = \frac{fd - opt}{opt} \cdot 100$$

where fd is value of cost function of fully dedicated policy and opt is value of cost function of optimal policy. Values of $fd\%$ indicate that, in case of inflexibility of servers in the system leads to the highest cost of objective function.

$sp\%$ denotes differences of cost values in percentages between static policy and optimal policy where formulation is defined as following where sp is value of cost function of static policy ;

$$sp\% = \frac{sp - opt}{opt} \cdot 100$$

Contrary to $fd\%$, $sp\%$ has smaller values which indicates that values of cost function of this policy is close to the values of optimal policy. In other words, server 2 serves type-1 customer in case of non existence of type-2 customer which leads to better solution for objective function. However this policy is not sufficient which means that server 2 should serve type-1 customer in some conditions although it also has its own customer in the system. In addition, static policy has exactly same cost value for some sets of parameters which are seen at rows with 0.0 value at column of $sp\%$.

Difference of cost values between optimal policy and other three heuristics are also calculated in percentages which are represented as $hp_1\%$, $hp_2\%$ and $hp_3\%$. The

Table 4.8. Difference in cost values of other policies compared to optimum solution in terms of percentages

λ_1	λ_2	$\mu 1$	$\mu 2$	γ_1	γ_2	h_1	h_2	β_1	β_2	$fd\%$	$sp\%$	$hp_1\%$	$hp_2\%$	$hp_3\%$
5	5	20	20	3	0	1	1	2	2	2314.0	9.6	2.2	34.5	2.2
5	5	10	10	3	0	1	1	2	2	768.1	13.1	4.2	89.7	4.2
5	5	20	20	3	0	1	3	2	6	1088.3	1.8	1.8	35.7	1.8
5	5	10	10	3	0	1	3	2	6	353.0	0.2	0.2	47.2	0.2
5	5	20	20	3	3	1	1	2	2	1427.0	0.0	0.7	0.7	0.7
5	5	10	10	3	3	1	1	2	2	545.3	0.0	1.4	1.4	1.4
5	5	20	20	3	3	1	3	2	6	615.6	0.0	0.1	0.1	0.1
5	5	10	10	3	3	1	3	2	6	241.5	0.0	0.6	0.6	0.6
10	5	30	30	3	3	1	0.5	2	1	3106.5	6.4	43.3	43.3	63.4
10	5	30	30	0	3	1	0.5	0	0.5	3732.5	0.0	34.2	0.5	48.7
10	5	15	15	3	3	1	3	1	3	597.4	0.0	1.8	1.8	84.8
10	5	15	15	3	3	1	3	2	6	442.2	0.0	2.3	2.3	107.5
15	5	20	20	3	0	1	1	2	2	1247.3	28.4	6.6	50.7	55.9
15	5	20	20	3	0	1	3	2	6	801.1	6.1	5.5	49.4	49.7
15	5	20	20	3	3	1	1	2	2	859.5	0.0	1.7	1.7	85.6
15	5	20	20	3	3	1	3	2	6	579.0	0.0	3.9	3.9	180.2

formula for $hp_1\%$;

$$hp_1\% = \frac{hp_1 - opt}{opt} \cdot 100$$

where hp_1 is the value of cost function of heuristic 1

Formulas for $hp_2\%$ and $hp_3\%$ are as following where hp_2 and hp_3 represent values of cost functions for heuristic 2 and 3 respectively:

$$hp_2\% = \frac{hp_2 - opt}{opt} \cdot 100$$

$$hp_3\% = \frac{hp_3 - opt}{opt} \cdot 100$$

In general we observe heuristic $hp_1\%$ gives better solution compared to other two heuristics which has the rule of giving service to customers with higher holding cost in the system. Other heuristics represented by $hp_2\%$ and $hp_3\%$ follow policies as giving service to customers with higher holding and abandonment costs of customers in the system, and giving service to customers with higher waiting time. These heuristics may also give better values of cost function for some parameter sets.

The Figure 4.6 shows average, minimum and maximum values of differences between cost functions of policies and optimum solution in percentages. Best value in average and maximum for experiments belongs to $sp\%$ which is closest to optimum solution in average. In addition, $hp_1\%$ has the second best value of cost difference with optimum solution in average and in maximum value. Values of other policies and heuristics except fully dedicated policy are similar to each other however $fd\%$ has the worst value in average and has important difference in values compared to others.

For some set of experiments, objective functions of policies $sp\%$ and $hp_2\%$ give

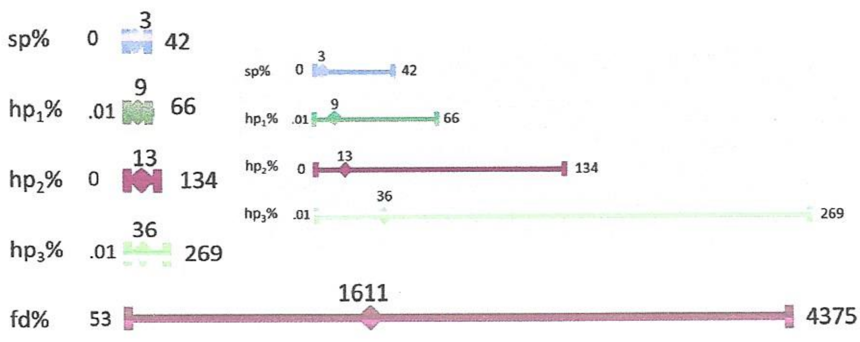

Figure 4.6. Comparison Graph of All Methodologies.

same value of cost with optimal solution. For 124 experiments, static policy has same values of cost function with optimal solution, which means optimal solution of these experiments are also solved with the same decision type. For the heuristic 2, costs values of 8 experiments are same as cost values of optimal solution.

In Table 4.9, we observe experiments from heuristic 2, which has the policy that flexible server gives service to customers with higher abandonment and holding cost. For some of experiments, objective function gives the same value with optimal cost in which type-2 customer has triple holding cost compared to type-1 customer. In addition, it is observed that abandonment rate of type-2 customer exists for all experiments.

Table 4.9. Same Cost Value of Heuristic 2 with Optimal Solution

λ_1	λ_2	$\mu 1$	$\mu 2$	γ_1	γ_2	h_1	h_2	β_1	β_2	**cost**
5	5	20	20	0	3	1	3	0	6	65.4
5	5	10	10	0	3	1	3	0	6	194.1
5	5	20	20	0	3	1	3	0	3	49.5
10	5	30	30	0	3	1	3	0	6	38.5
10	5	15	15	0	3	1	3	0	6	117.8
10	5	30	30	0	3	1	3	0	3	31.7
15	5	40	40	0	3	1	3	0	6	27.7
15	5	40	40	0	3	1	3	0	3	27.1

4.6. Comparision of Experiments with Low and High Values of Abandonment Rates

We also design different parameter sets where abandonment rates only differ and other parameters are fixed at the same value. We set abandonment rate with a lower value, 0.25, presented by $(\gamma_1, \gamma_2) = \{(0, 0.25), (0, 0.25)\}$. As we analyse the values of cost function, we observe that cost value becomes higher as abandonment rate decreases. Some examples from experiment set and comparison of values of objective function for the state $V(0,0)$ (begin to the system with none of customers) is given in Table 4.10.

Experiments given below all have parameter sets of abandonments $(0.25, 0.25)$ and $(3, 3)$ for cost values 1 and 2 respectively. Cost values with smaller abandonment rates (cost 1) are higher than the values with higher abandonment rates (cost 2).

Table 4.10. Value of objective function with different abandonment rates

λ_1	λ_2	$\mu 1$	$\mu 2$	h_1	h_2	β_1	β_2	**cost 1**	**cost 2**
5	5	20	20	1	0.5	2	1	45.4	10.0
5	5	10	10	1	0.5	2	1	160.1	30.0
5	5	10	10	1	3	2	6	518.7	108.6
5	5	6.67	6.67	1	3	2	6	1324.6	191.7
10	5	30	30	1	1	1	1	45.8	10.3
10	5	15	15	1	1	1	1	158.0	31.1

4.7. Observations and Managerial Insights

We made the following observations from experiments given at previous sections;

1) when holding cost of type-2 customer is bigger than holding cost of type-1 customer, giving service to type-2 customer by flexible server is generally chosen when both types of customers exist in the system.
2) when service rates becomes slower, server 2 decides to serve type-2 customer despite having lower holding cost for that customer type compared to other
3) when abandonment cost of type-2 customer is triple of type-1 customer, flexible server decides to give service to type-2 customer
4) decrease in service rates leads to smaller threshold value for switching to type-1 customer from type-2 customer in case of having decision matrix with threshold; in other words having less number of type-1 customers in the system is adequate to switch to serve to this type of customer.
5) increase in holding cost and abandonment cost of type-2 customer tends to give service to type-2 customers for flexible server
6) in cases of having decision matrix with linear switching curve, as service rates increase, linearity curve becomes steeper. In other words, server 2 tends to serve type-1 customer while having less number of type-2 customer in the system comparing to higher utilization rates.
7) for horizontal decision line, when service rate of flexible server is lower, it begins to serve type-2 customer after having a number of type-2 customer in the system which is smaller compared to higher service rate

Numerical studies generate certain managerial insights into related design and control issues. We have observed a number of interesting properties and gained some useful managerial insights in many aspects of such queueing systems.

1) Servers should not be idle while there exists customers in the system. In case of not having a tool for making decision which customer type to serve, it should serve its own customer. While there is no customers of flexible server, at that

time flexible server should give service to other customer type. So, continuous work of both servers result in better solutions compared to being idle.

2) At a queueing system, having only dedicated servers is the worst case since idle times should occur for servers in case of no arrival of customers to the system. So, system with both dedicated servers leads to much more higher cost. Having flexible server in the system is a cost efficient behaviour for the branches at banking sector.
3) Higher service rates of servers leads to lower cost at queueing system which shows the importance of seniority and ability of servers. Performance of servers in terms of service rates affects directly the system which should always be noticed by banks.
4) Increase in arrival rates of customers leads to increase in value of cost function which can be balanced with increase in number of servers in the system after a proper feasibility analysis.
5) Higher holding and abandonment costs lead to higher cost at queuing systems. So that, customer with more critical attributes should be served at the first place by defining a prioritization method at queueing system.

5. CONCLUSION

In this thesis, we investigate a queueing system motivated from the branches at banks. We consider a dynamic scheduling problem with two servers and unlimited arrivals of two types of customers. In the system, one of the servers are dedicated whereas other is enable to serve both types of customers. The objective of the problem is minimization of cost function while deciding on customer type to serve for flexible server. Cost of system consists of holding costs and abandonment costs of customers, while abandonment rates, arrival rates of customers and service rates of servers are defined in the problem.

In the literature, tandem queues are generally investigated for the case of both dedicated and flexible servers. One of the studies is close to our study however, it analyses a closed system where there is no arrival of customers (Ahn et al., 2004). This thesis also differs from other studies and contributes to the literature by solving a dynamic scheduling of a queueing system with two servers and with the attributes such as holding and abandonment cost of customers. The study is also an essential source for banking industry since it mentions to use personnel at high utilization level and to provide customer satisfaction.

Firstly, optimal model is designed by using value iteration algorithm which is coded in C Program Language. The algorithm for optimal solution and heuristics is provided to specify allocations for dedicated and flexible servers, respectively. Extensive numerical analysis was employed to study the algorithm and the heuristics. For optimal solution, cost function of the algorithm is calculated separately for different states, in other words, for different number of customers in the system. Because, cost equation differs for different points or intervals of number of customers in the system. In the pseudo-code it can be seen that, many iterations are done in the algorithm to make system reach steady state. Besides optimal policy, heuristics are modelled in order to see the effects of their results compared to optimal solution. Heuristics are constructed to gain insight for the managements of queuing system in banking sector. Some policies are applied for heuristics which can be summarized as following: (i) both servers give service to only their own customers, (ii) flexible server gives service to other customer type (type-1) in case of non-existence of type-2 customer in the system, (iii) flexible server gives service to the customer type with higher total holding cost in the system, (iv) flexible server gives service to the customer type with higher total holding and abandonment cost in the system and finally (v) flexible server gives service to the customer type with higher total waiting time in the system.

Furthermore, we evaluate impact of different values of parameters by constructing an experiment set of parameters. We observe from the experiments of optimal model that decision matrix for flexible server can be mainly grouped as priority based (exhaustive), matrix with threshold value, a linear decision matrix and decision matrix with horizontal threshold value. Firstly, priority based matrix referring to flexible server giving service to type-1 customer if and only if type-2 customer does not exist, is obtained for the conditions with high holding cost of type-2 customer and low service rates. Secondly, decision matrix of flexible server with a threshold value is resulted from having abandonment rate for type-1 customer. In addition, decrease in service rates result in lower threshold value for switching from type-2 customer to type-1 customer. In the third place, linear switching curve for flexible server exists for some parameter sets with no abandonment rates. In addition, linear curve becomes steeper with increase in service rates. Finally, another decision matrix has threshold value for type-2 customer in the system in order to switch from one customer type to another. This switching model is confronted with the cases of abandonment rates of type-1 customer being triple of type-2 customer. This threshold value diminishes with decrease in service rates.

Another observation is that some attributes for parameters affect cost value of the problem. Cost value of the problem increases provided that arrival rates, holding costs, abandonment costs increase and service rates, abandonment rates decrease.

In addition, experiments done in the study give some managerial insight that can be essential to use for queueing systems at banking sector. Servers should not become idle in case of having customer to serve in the system. Therefore, flexible or dedicated server should serve at least their own customers if there no a decision tool to use for the queueing system. Flexibility of servers lead to decrease in cost functions of the queueing model. For this reason, it is efficient for banks to train personnel and authorize for multiple kind of jobs. In addition, seniority and capability of personnel is also significant to decrease in costs for the queueing system since seniority leads to increase in service rates that result in lower cost function. On the other hand, increase in frequency at customer flow which means increase in arrival rates of customers leads to increase in cost function. Consequently, busy times at branches can be more costly to the bank with the same number of personnel. Because of this fact, banks should consider to increase number of servers for busy times such as having part time personnel under feasible conditions. Besides, customers with higher holding and abandonment costs result in higher cost at queuing systems. So banks should realize that having costly customers in the system should expose banks critical values of cost in case of having any fault at service.

In conclusion, the queueing model defined in this study is convenient for banking sector as it takes both service quality, efficiency into consideration. While taking costs of customers into account, we consider quality of service. On the other hand, while using flexible server in order to minimize cost function of the problem, we try to provide efficiency while giving service to the customers. Future research can develop this design function by including prioritization of customer types, increase in customer types and number of both dedicated and flexible server. The problem can also be enriched by adding costs of servers into the model.

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APPENDIX A: RESULTS OF EXPERIMENTS

Table A.1. Results of Experiments of $V(0,0)$ for λ_1=5, λ_2=5 and μ=μ1=μ2 where (1):Optimum,(2):Fully Dedicated,(3):Static,(4):Heuristic 1, (5):Heuristic 2, (6):Heuristic 3

Exp	μ	γ_1	γ_2	h_1	h_2	β_1	β_2	(1)	(2)	(3)	(4)	(5)	(6)
1	20	0	0	1	0.5	2	1	86	2884	87	96	96	96
2	10	0	0	1	0.5	2	1	389	4627	395	441	441	441
3	6.7	0	0	1	0.5	2	1	1433	3924	1449	1657	1657	1657
4	20	0	0	1	1	2	2	120	2917	120	121	121	121
5	10	0	0	1	1	2	2	562	4794	562	577	577	577
6	6.7	0	0	1	1	2	2	2088	4563	2088	2268	2268	2268
7	20	0	0	1	3	2	6	253	3051	253	254	254	254
8	10	0	0	1	3	2	6	1228	5460	1228	1237	1237	1237
9	6.7	0	0	1	3	2	6	4646	7121	4646	4801	4801	4801
10	20	0	3	1	0.5	2	1	17	421	17	26	17	26
11	10	0	3	1	0.5	2	1	49	483	49	71	49	71
12	6.7	0	3	1	0.5	2	1	92	391	93	117	92	117
13	20	0	3	1	1	2	2	27	431	27	29	27	29
14	10	0	3	1	1	2	2	78	512	78	88	78	88
15	6.7	0	3	1	1	2	2	143	442	143	162	144	162
16	20	0	3	1	3	2	6	65	470	65	66	65	66
17	10	0	3	1	3	2	6	194	628	194	198	194	198
18	6.7	0	3	1	3	2	6	345	643	345	364	345	364

Table A.1. Continued

Exp	μ	γ_1	γ_2	h_1	h_2	β_1	β_2	(1)	(2)	(3)	(4)	(5)	(6)
19	20	3	0	1	0.5	2	1	13	426	15	14	16	14
20	10	3	0	1	0.5	2	1	40	498	52	46	86	46
21	6.7	3	0	1	0.5	2	1	115	400	125	123	268	123
22	20	3	0	1	1	2	2	18	431	20	18	24	18
23	10	3	0	1	1	2	2	59	513	67	62	112	62
24	6.7	3	0	1	1	2	2	165	443	168	166	323	166
25	20	3	0	1	3	2	6	38	450	39	39	51	39
26	10	3	0	1	3	2	6	126	572	127	127	186	127
27	6.7	3	0	1	3	2	6	339	614	339	340	494	340
28	20	3	3	1	0.5	2	1	10	235	11	14	14	14
29	10	3	3	1	0.5	2	1	30	295	33	39	39	39
30	6.7	3	3	1	0.5	2	1	55	293	62	65	65	65
31	20	3	3	1	1	2	2	16	240	16	16	16	16
32	10	3	3	1	1	2	2	48	311	48	49	49	49
33	6.7	3	3	1	1	2	2	88	319	88	89	89	89
34	20	3	3	1	3	2	6	36	261	36	36	36	36
35	10	3	3	1	3	2	6	109	371	109	109	109	109
36	6.7	3	3	1	3	2	6	192	423	192	194	194	194
37	20	0	0	1	0.5	1	0.5	86	2884	87	96	96	96
38	10	0	0	1	0.5	1	0.5	389	4627	395	441	441	441
39	6.7	0	0	1	0.5	1	0.5	1433	3924	1449	1657	1657	1657
40	20	0	0	1	1	1	1	120	2917	120	121	121	121
41	10	0	0	1	1	1	1	562	4794	562	577	577	577
42	6.7	0	0	1	1	1	1	2088	4563	2088	2268	2268	2268
43	20	0	0	1	3	1	3	253	3051	253	254	254	254
44	10	0	0	1	3	1	3	1228	5460	1228	1237	1237	1237
45	6.7	0	0	1	3	1	3	4646	7121	4646	4801	4801	4801

Table A.1. Continued

Exp	μ	γ_1	γ_2	h_1	h_2	β_1	β_2	(1)	(2)	(3)	(4)	(5)	(6)
46	20	0	3	1	0.5	1	0.5	15	419	15	19	15	19
47	10	0	3	1	0.5	1	0.5	39	473	39	50	40	50
48	6.7	0	3	1	0.5	1	0.5	71	373	75	81	72	81
49	20	0	3	1	1	1	1	22	426	22	22	22	22
50	10	0	3	1	1	1	1	59	493	59	63	59	63
51	6.7	0	3	1	1	1	1	107	406	108	112	107	112
52	20	0	3	1	3	1	3	50	454	50	50	50	50
53	10	0	3	1	3	1	3	136	571	136	138	136	138
54	6.7	0	3	1	3	1	3	238	536	238	246	238	246
55	20	3	0	1	0.5	1	0.5	13	421	14	14	16	14
56	10	3	0	1	0.5	1	0.5	37	478	42	42	68	42
57	6.7	3	0	1	0.5	1	0.5	96	364	98	106	187	106
58	20	3	0	1	1	1	1	18	426	18	18	23	18
59	10	3	0	1	1	1	1	55	493	57	55	84	55
60	6.7	3	0	1	1	1	1	140	407	141	144	225	144
61	20	3	0	1	3	1	3	37	445	37	37	46	37
62	10	3	0	1	3	1	3	117	553	117	117	149	117
63	6.7	3	0	1	3	1	3	312	578	312	313	387	313
64	20	3	3	1	0.5	1	0.5	8	230	8	11	11	11
65	10	3	3	1	0.5	1	0.5	22	280	24	27	27	27
66	6.7	3	3	1	0.5	1	0.5	38	266	42	43	43	43
67	20	3	3	1	1	1	1	12	234	12	12	12	12
68	10	3	3	1	1	1	1	34	291	34	34	34	34
69	6.7	3	3	1	1	1	1	59	283	59	59	59	59
70	20	3	3	1	3	1	3	27	249	27	27	27	27
71	10	3	3	1	3	1	3	74	331	74	74	74	74
72	6.7	3	3	1	3	1	3	126	350	126	127	127	127

Table A.1. Continued

Exp	μ	γ_1	γ_2	h_1	h_2	β_1	β_2	(1)	(2)	(3)	(4)	(5)	(6)
73	20	0	0	1	0.5	0.5	0.25	86	2884	87	96	96	96
74	10	0	0	1	0.5	0.5	0.25	389	4627	395	441	441	441
75	6.7	0	0	1	0.5	0.5	0.25	1433	3924	1449	1657	1657	1657
76	20	0	0	1	1	0.5	0.5	120	2917	120	121	121	121
77	10	0	0	1	1	0.5	0.5	562	4794	562	577	577	577
78	6.7	0	0	1	1	0.5	0.5	2088	4563	2088	2268	2268	2268
79	20	0	0	1	3	0.5	1.5	253	3051	253	254	254	254
80	10	0	0	1	3	0.5	1.5	1228	5460	1228	1237	1237	1237
81	6.7	0	0	1	3	0.5	1.5	4646	7121	4646	4801	4801	4801
82	20	0	3	1	0.5	0.5	0.25	13	417	13	16	13	16
83	10	0	3	1	0.5	0.5	0.25	34	469	34	39	34	39
84	6.7	0	3	1	0.5	0.5	0.25	57	364	66	63	59	63
85	20	0	3	1	1	0.5	0.5	19	423	19	19	19	19
86	10	0	3	1	1	0.5	0.5	49	483	49	50	49	50
87	6.7	0	3	1	1	0.5	0.5	86	388	90	87	87	87
88	20	0	3	1	3	0.5	1.5	42	446	42	42	42	42
89	10	0	3	1	3	0.5	1.5	107	542	107	108	107	108
90	6.7	0	3	1	3	0.5	1.5	185	483	185	188	185	188
91	20	3	0	1	0.5	0.5	0.25	12	418	13	14	15	14
92	10	3	0	1	0.5	0.5	0.25	36	469	37	40	54	40
93	6.7	3	0	1	0.5	0.5	0.25	84	347	84	97	140	97
94	20	3	0	1	1	0.5	0.5	17	423	18	17	22	17
95	10	3	0	1	1	0.5	0.5	52	484	52	52	72	52

Table A.1. Continued

Exp	μ	γ_1	γ_2	h_1	h_2	β_1	β_2	(1)	(2)	(3)	(4)	(5)	(6)
96	6.7	3	0	1	1	0.5	0.5	127	389	127	133	181	133
97	20	3	0	1	3	0.5	1.5	37	442	37	37	37	37
98	10	3	0	1	3	0.5	1.5	112	543	112	112	118	112
99	6.7	3	0	1	3	0.5	1.5	299	561	299	300	323	300
100	20	3	3	1	0.5	0.5	0.25	7	228	7	9	9	9
101	10	3	3	1	0.5	0.5	0.25	18	273	19	21	21	21
102	6.7	3	3	1	0.5	0.5	0.25	29	252	32	33	33	33
103	20	3	3	1	1	0.5	0.5	10	231	10	11	11	11
104	10	3	3	1	1	0.5	0.5	26	280	26	27	27	27
105	6.7	3	3	1	1	0.5	0.5	44	264	44	44	44	44
106	20	3	3	1	3	0.5	1.5	23	243	23	23	23	23
107	10	3	3	1	3	0.5	1.5	57	311	57	57	57	57
108	6.7	3	3	1	3	0.5	1.5	93	313	93	93	93	93

Table A.2. Results of Experiments of $V(0,0)$ for λ_1=10, λ_2=5 and $\mu=\mu1=\mu2$ where (1):Optimum,(2):Fully Dedicated,(3):Static,(4):Heuristic 1, (5):Heuristic 2, (6):Heuristic 3

Exp	μ	γ_1	γ_2	h_1	h_2	β_1	β_2	(1)	(2)	(3)	(4)	(5)	(6)
1	30	0	0	1	0.5	2	1	60	2524	61	67	67	71
2	15	0	0	1	0.5	2	1	251	3762	263	284	284	420
3	10	0	0	1	0.5	2	1	812	10699	899	892	892	12797
4	30	0	0	1	1	2	2	75	2538	75	75	75	93
5	15	0	0	1	1	2	2	318	3818	318	325	325	518
6	10	0	0	1	1	2	2	1042	10842	1042	1097	1097	25211
7	30	0	0	1	3	2	6	128	2591	128	128	128	132
8	15	0	0	1	3	2	6	541	4040	541	552	552	827
9	10	0	0	1	3	2	6	1613	11412	1613	1778	1778	74873
10	30	0	3	1	0.5	2	1	14	508	14	23	14	26
11	15	0	3	1	0.5	2	1	42	514	42	67	42	84
12	10	0	3	1	0.5	2	1	92	504	95	123	93	148
13	30	0	3	1	1	2	2	19	513	19	21	19	41
14	15	0	3	1	1	2	2	57	530	57	72	57	142
15	10	0	3	1	1	2	2	123	533	123	156	124	256
16	30	0	3	1	3	2	6	38	532	38	39	38	45
17	15	0	3	1	3	2	6	118	591	118	127	118	291
18	10	0	3	1	3	2	6	238	647	238	286	238	688
19	30	3	0	1	0.5	2	1	13	533	16	15	16	16
20	15	3	0	1	0.5	2	1	43	669	60	48	68	63
21	10	3	0	1	0.5	2	1	108	628	143	119	235	443
22	30	3	0	1	1	2	2	16	536	18	17	21	20

Table A.2. Continued

Exp	μ	γ_1	γ_2	h_1	h_2	β_1	β_2	(1)	(2)	(3)	(4)	(5)	(6)
23	15	3	0	1	1	2	2	53	677	67	57	91	76
24	10	3	0	1	1	2	2	139	642	158	143	255	819
25	30	3	0	1	3	2	6	28	546	29	29	39	28
26	15	3	0	1	3	2	6	93	706	96	96	142	118
27	10	3	0	1	3	2	6	216	701	217	218	343	2324
28	30	3	3	1	0.5	2	1	9	298	10	13	13	15
29	15	3	3	1	0.5	2	1	31	371	35	41	41	50
30	10	3	3	1	0.5	2	1	65	365	77	77	77	90
31	30	3	3	1	1	2	2	13	301	13	13	13	24
32	15	3	3	1	1	2	2	44	380	44	44	44	80
33	10	3	3	1	1	2	2	92	380	92	94	94	144
34	30	3	3	1	3	2	6	23	311	23	23	23	26
35	15	3	3	1	3	2	6	76	412	76	78	78	158
36	10	3	3	1	3	2	6	152	440	152	159	159	363
37	30	0	0	1	0.5	1	0.5	60	2524	61	67	67	71
38	15	0	0	1	0.5	1	0.5	251	3762	263	284	284	420
39	10	0	0	1	0.5	1	0.5	812	10699	899	892	892	12797
40	30	0	0	1	1	1	1	75	2538	75	75	75	93
41	15	0	0	1	1	1	1	318	3818	318	325	325	518
42	10	0	0	1	1	1	1	1042	10842	1042	1097	1097	25211
43	30	0	0	1	3	1	3	128	2591	128	128	128	132
44	15	0	0	1	3	1	3	541	4040	541	552	552	827
45	10	0	0	1	3	1	3	1613	11412	1613	1778	1778	74873
46	30	0	3	1	0.5	1	0.5	13	507	13	18	13	20

Table A.2. Continued

Exp	μ	γ_1	γ_2	h_1	h_2	β_1	β_2	(1)	(2)	(3)	(4)	(5)	(6)
47	15	0	3	1	0.5	1	0.5	37	510	37	49	37	59
48	10	0	3	1	0.5	1	0.5	77	494	85	89	78	103
49	30	0	3	1	1	1	1	17	511	17	18	17	29
50	15	0	3	1	1	1	1	48	520	48	54	48	93
51	10	0	3	1	1	1	1	102	514	104	112	103	166
52	30	0	3	1	3	1	3	32	526	32	32	32	35
53	15	0	3	1	3	1	3	90	563	90	95	90	185
54	10	0	3	1	3	1	3	181	590	181	202	181	418
55	30	3	0	1	0.5	1	0.5	13	523	14	14	15	15
56	15	3	0	1	0.5	1	0.5	37	628	45	42	58	58
57	10	3	0	1	0.5	1	0.5	87	547	99	94	155	426
58	30	3	0	1	1	1	1	16	525	17	16	20	19
59	15	3	0	1	1	1	1	48	635	53	48	70	70
60	10	3	0	1	1	1	1	110	562	114	111	171	802
61	30	3	0	1	3	1	3	27	536	27	27	35	27
62	15	3	0	1	3	1	3	81	664	81	82	110	110
63	10	3	0	1	3	1	3	172	621	173	175	246	2307
64	30	3	3	1	0.5	1	0.5	8	292	8	10	10	11
65	15	3	3	1	0.5	1	0.5	23	347	25	29	29	33
66	10	3	3	1	0.5	1	0.5	45	318	51	51	51	58
67	30	3	3	1	1	1	1	10	294	10	10	10	17
68	15	3	3	1	1	1	1	31	352	31	32	32	52

Table A.2. Continued

Exp	μ	γ_1	γ_2	h_1	h_2	β_1	β_2	(1)	(2)	(3)	(4)	(5)	(6)
69	10	3	3	1	1	1	1	61	328	61	62	62	90
70	30	3	3	1	3	1	3	19	302	19	19	19	20
71	15	3	3	1	3	1	3	54	375	54	55	55	99
72	10	3	3	1	3	1	3	102	368	102	105	105	218
73	30	0	0	1	0.5	0.5	0.3	60	2524	61	67	67	71
74	15	0	0	1	0.5	0.5	0.3	251	3762	263	284	284	420
75	10	0	0	1	0.5	0.5	0.3	812	10699	899	892	892	12797
76	30	0	0	1	1	0.5	0.5	75	2538	75	75	75	93
77	15	0	0	1	1	0.5	0.5	318	3818	318	325	325	518
78	10	0	0	1	1	0.5	0.5	1042	10842	1042	1097	1097	25211
79	30	0	0	1	3	0.5	1.5	128	2591	128	128	128	132
80	15	0	0	1	3	0.5	1.5	541	4040	541	552	552	827
81	10	0	0	1	3	0.5	1.5	1613	11412	1613	1778	1778	74873
82	30	0	3	1	0.5	0.5	0.3	13	507	13	15	13	17
83	15	0	3	1	0.5	0.5	0.3	34	507	35	40	34	46
84	10	0	3	1	0.5	0.5	0.3	65	490	81	72	67	80
85	30	0	3	1	1	0.5	0.5	16	510	16	16	16	23
86	15	0	3	1	1	0.5	0.5	43	516	43	46	44	69
87	10	0	3	1	1	0.5	0.5	88	504	95	90	90	121
88	30	0	3	1	3	0.5	1.5	28	522	28	28	28	30
89	15	0	3	1	3	0.5	1.5	76	549	76	79	77	132

Table A.2. Continued

Exp	μ	γ_1	γ_2	h_1	h_2	β_1	β_2	(1)	(2)	(3)	(4)	(5)	(6)
90	10	0	3	1	3	0.5	1.5	152	562	153	161	153	282
91	30	3	0	1	0.5	0.5	0.3	12	518	13	14	15	15
92	15	3	0	1	0.5	0.5	0.3	34	607	38	38	49	55
93	10	3	0	1	0.5	0.5	0.3	74	507	77	82	113	418
94	30	3	0	1	1	0.5	0.5	15	520	16	15	19	19
95	15	3	0	1	1	0.5	0.5	44	614	45	44	60	68
96	10	3	0	1	1	0.5	0.5	91	522	92	95	135	794
97	30	3	0	1	3	0.5	1.5	26	531	26	26	27	27
98	15	3	0	1	3	0.5	1.5	74	643	74	75	80	107
99	10	3	0	1	3	0.5	1.5	150	581	150	154	180	2299
100	30	3	3	1	0.5	0.5	0.3	7	289	8	9	9	9
101	15	3	3	1	0.5	0.5	0.3	19	335	21	22	22	25
102	10	3	3	1	0.5	0.5	0.3	35	294	38	38	38	43
103	30	3	3	1	1	0.5	0.5	9	290	9	9	9	13
104	15	3	3	1	1	0.5	0.5	25	339	25	25	25	37
105	10	3	3	1	1	0.5	0.5	46	302	46	47	47	63
106	30	3	3	1	3	0.5	1.5	16	297	16	16	16	17
107	15	3	3	1	3	0.5	1.5	43	357	43	43	43	70
108	10	3	3	1	3	0.5	1.5	76	332	76	78	78	146

Table A.3. Results of Experiments of $V(0,0)$ for λ_1=15, λ_2=5 and μ=μ1=μ2 where (1):Optimum,(2):Fully Dedicated,(3):Static,(4):Heuristic 1, (5):Heuristic 2, (6):Heuristic 3

Exp	μ	γ_1	γ_2	h_1	h_2	β_1	β_2	(1)	(2)	(3)	(4)	(5)	(6)
1	40	0	0	1	0.5	2	1	47	2097	48	51	51	53
2	20	0	0	1	0.5	2	1	192	2852	202	214	214	299
3	13.3	0	0	1	0.5	2	1	605	17320	688	656	656	8747
4	40	0	0	1	1	2	2	55	2104	55	55	55	68
5	20	0	0	1	1	2	2	230	2880	230	233	233	430
6	13.3	0	0	1	1	2	2	752	17385	752	776	776	17141
7	40	0	0	1	3	2	6	83	2133	83	83	83	111
8	20	0	0	1	3	2	6	341	2991	341	350	350	671
9	13.3	0	0	1	3	2	6	1009	17642	1009	1145	1145	50718
10	40	0	3	1	0.5	2	1	13	526	13	20	13	23
11	20	0	3	1	0.5	2	1	38	488	38	63	39	78
12	13.3	0	3	1	0.5	2	1	92	938	94	125	94	149
13	40	0	3	1	1	2	2	16	529	16	18	16	36
14	20	0	3	1	1	2	2	47	497	47	63	48	131
15	13.3	0	3	1	1	2	2	112	956	112	152	114	251
16	40	0	3	1	3	2	6	28	541	28	28	28	73
17	20	0	3	1	3	2	6	85	535	85	96	85	313
18	13.3	0	3	1	3	2	6	185	1029	185	251	185	658
19	40	3	0	1	0.5	2	1	14	560	16	15	15	15
20	20	3	0	1	0.5	2	1	46	720	64	51	63	63
21	13.3	3	0	1	0.5	2	1	116	711	162	128	223	363
22	40	3	0	1	1	2	2	16	562	18	16	19	19

Table A.3. Continued

Exp	μ	γ_1	γ_2	h_1	h_2	β_1	β_2	(1)	(2)	(3)	(4)	(5)	(6)
23	20	3	0	1	1	2	2	54	724	69	57	81	84
24	13.3	3	0	1	1	2	2	141	720	171	148	236	638
25	40	3	0	1	3	2	6	24	569	25	25	32	30
26	20	3	0	1	3	2	6	82	743	88	87	123	123
27	13.3	3	0	1	3	2	6	203	754	205	206	306	1736
28	40	3	3	1	0.5	2	1	9	321	10	13	13	14
29	20	3	3	1	0.5	2	1	32	399	37	42	42	50
30	13.3	3	3	1	0.5	2	1	74	396	88	86	86	99
31	40	3	3	1	1	2	2	11	323	11	11	11	21
32	20	3	3	1	1	2	2	42	404	42	43	43	78
33	13.3	3	3	1	1	2	2	98	406	98	100	100	151
34	40	3	3	1	3	2	6	18	329	18	18	18	42
35	20	3	3	1	3	2	6	63	424	63	65	65	175
36	13.3	3	3	1	3	2	6	137	445	137	147	147	358
37	40	0	0	1	0.5	1	0.5	47	2097	48	51	51	53
38	20	0	0	1	0.5	1	0.5	192	2852	202	214	214	299
39	13.3	0	0	1	0.5	1	0.5	605	17320	688	656	656	8747
40	40	0	0	1	1	1	1	55	2104	55	55	55	68
41	20	0	0	1	1	1	1	230	2880	230	233	233	430
42	13.3	0	0	1	1	1	1	752	17385	752	776	776	17141
43	40	0	0	1	3	1	3	83	2133	83	83	83	111
44	20	0	0	1	3	1	3	341	2991	341	350	350	671
45	13.3	0	0	1	3	1	3	1009	17642	1009	1145	1145	50718
46	40	0	3	1	0.5	1	0.5	12	525	12	16	13	18

Table A.3. Continued

Exp	μ	γ_1	γ_2	h_1	h_2	β_1	β_2	(1)	(2)	(3)	(4)	(5)	(6)
47	20	0	3	1	0.5	1	0.5	35	485	35	48	36	56
48	13.3	0	3	1	0.5	1	0.5	80	932	89	93	81	106
49	40	0	3	1	1	1	1	15	528	15	16	15	26
50	20	0	3	1	1	1	1	42	492	42	50	43	87
51	13.3	0	3	1	1	1	1	99	945	101	112	101	165
52	40	0	3	1	3	1	3	24	537	24	24	24	49
53	20	0	3	1	3	1	3	69	519	69	75	69	196
54	13.3	0	3	1	3	1	3	151	995	151	181	152	402
55	40	3	0	1	0.5	1	0.5	13	547	14	14	14	14
56	20	3	0	1	0.5	1	0.5	38	659	47	42	53	55
57	13.3	3	0	1	0.5	1	0.5	90	589	109	97	146	338
58	40	3	0	1	1	1	1	15	548	16	15	18	18
59	20	3	0	1	1	1	1	46	664	52	47	64	76
60	13.3	3	0	1	1	1	1	110	598	117	110	157	613
61	40	3	0	1	3	1	3	23	556	23	23	29	29
62	20	3	0	1	3	1	3	70	683	70	71	95	115
63	13.3	3	0	1	3	1	3	152	632	152	155	216	1711
64	40	3	3	1	0.5	1	0.5	8	313	8	10	10	11
65	20	3	3	1	0.5	1	0.5	24	365	27	29	29	34
66	13.3	3	3	1	0.5	1	0.5	50	328	58	57	57	64
67	40	3	3	1	1	1	1	10	314	10	10	10	15
68	20	3	3	1	1	1	1	30	368	30	31	31	51

Table A.3. Continued

Exp	μ	γ_1	γ_2	h_1	h_2	β_1	β_2	(1)	(2)	(3)	(4)	(5)	(6)
69	13.3	3	3	1	1	1	1	65	335	65	66	66	94
70	40	3	3	1	3	1	3	15	320	15	15	15	29
71	20	3	3	1	3	1	3	45	383	45	46	46	108
72	13.3	3	3	1	3	1	3	91	362	91	97	97	215
73	40	0	0	1	0.5	0.5	0.3	47	2097	48	51	51	53
74	20	0	0	1	0.5	0.5	0.3	192	2852	202	214	214	299
75	13.3	0	0	1	0.5	0.5	0.3	605	17320	688	656	656	8747
76	40	0	0	1	1	0.5	0.5	55	2104	55	55	55	68
77	20	0	0	1	1	0.5	0.5	230	2880	230	233	233	430
78	13.3	0	0	1	1	0.5	0.5	752	17385	752	776	776	17141
79	40	0	0	1	3	0.5	1.5	83	2133	83	83	83	111
80	20	0	0	1	3	0.5	1.5	341	2991	341	350	350	671
81	13.3	0	0	1	3	0.5	1.5	1009	17642	1009	1145	1145	50718
82	40	0	3	1	0.5	0.5	0.3	12	525	12	14	12	15
83	20	0	3	1	0.5	0.5	0.3	34	484	34	40	34	45
84	13.3	0	3	1	0.5	0.5	0.3	70	930	86	77	71	85
85	40	0	3	1	1	0.5	0.5	14	527	14	15	14	21
86	20	0	3	1	1	0.5	0.5	40	490	40	43	41	65
87	13.3	0	3	1	1	0.5	0.5	89	939	95	92	91	123
88	40	0	3	1	3	0.5	1.5	22	535	22	23	22	38
89	20	0	3	1	3	0.5	1.5	62	511	62	65	62	137

Table A.3. Continued

Exp	μ	γ_1	γ_2	h_1	h_2	β_1	β_2	(1)	(2)	(3)	(4)	(5)	(6)
90	13.3	0	3	1	3	0.5	1.5	134	978	134	147	136	274
91	40	3	0	1	0.5	0.5	0.3	12	540	13	13	14	14
92	20	3	0	1	0.5	0.5	0.3	34	629	39	38	46	51
93	13.3	3	0	1	0.5	0.5	0.3	75	528	82	82	107	326
94	40	3	0	1	1	0.5	0.5	14	542	15	14	17	17
95	20	3	0	1	1	0.5	0.5	42	634	43	42	55	72
96	13.3	3	0	1	1	0.5	0.5	90	537	90	91	124	600
97	40	3	0	1	3	0.5	1.5	22	549	22	22	22	28
98	20	3	0	1	3	0.5	1.5	62	652	62	63	67	111
99	13.3	3	0	1	3	0.5	1.5	125	571	125	129	152	1699
100	40	3	3	1	0.5	0.5	0.3	7	309	7	9	9	9
101	20	3	3	1	0.5	0.5	0.3	20	348	21	23	23	26
102	13.3	3	3	1	0.5	0.5	0.3	38	295	43	42	42	47
103	40	3	3	1	1	0.5	0.5	9	310	9	9	9	12
104	20	3	3	1	1	0.5	0.5	24	351	24	25	25	37
105	13.3	3	3	1	1	0.5	0.5	48	300	48	49	49	66
106	40	3	3	1	3	0.5	1.5	13	315	13	13	13	22
107	20	3	3	1	3	0.5	1.5	36	362	36	37	37	75
108	13.3	3	3	1	3	0.5	1.5	69	320	69	72	72	143