A CRITICAL SURVEY OF THE EXISTING METHODS OF SLOPE STABILITY ANALYSIS

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ACKNOWLEDGEMENT

I submit my gratitude to Prof. Vahit Kumbasar, who has kindly advised the necessary books ,and provided every kind of information and help for the preparation of my thesis.

E.Ç.

June 1964

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I. INTRODUCTION

The sliding of slopes has always been a problem for the Civil Engineer. In Engineering History many railways, harbour and highway failures have occurred due to sliding of their embankement, so that studying the slope stability problem becomes extremely important.

The main characteristic of slopes is the uncertainty they present with respect to stability. For example a slope may be stable under existing conditions, but nobody can guarantee that it will be stable in future too. Therefore a satisfactory design can only be achieved after making a future estimate and introducing an appropriate factor of safety.

Another property of the slope is the difficulty that is encountered in stopping the failure after it had initiated. Therefore if possible the measures should be taken in advance.

For the reasons stated above it becomes evident that a slope stability analysis is necessary preliminary to the slope design.

The purpose of the work presented here is a comparison of the methods used in practice in order to determine the approach in a slope stability analysis, both for design and for check purposes. First the methods are classified according to their assumptions of the shape of the failure surface. Under the heading of "Circular Slip Surfaces " Swedish Method which is separated as Bishops Numerical Slices Method, Bishops Stability Coefficients, Conventional Method and also Friction Circle Method are explained. Under the heading of " Noncircular Slip Surfaces" full information is given on Culmann Method, Logarithmic Spiral Slip Surfaces, and Composite Slip Surfaces. Since pore-pressure, submergence, and seapage have a great effect on stability, they are explained in articles 5, 6, and 7. Article 9 is devoted to some criticism about the methods explained before, stating their good and bad properties and their applicability.

The application of the digital computer to the slope stability analysis is a completely new subject .Since it is an easy and fast process,may be in future the whole stability analysis will depend on computer. But arranging slope stability problems according to the needs of the computer is a special subject . Therefore some information is given in article 8 about the statement of a stability problem according to the needs of the computer.

2. INFORMATION ON THE ASSUMPTIONS OF SLOPE STABILITY ANALYSIS

For a slope to be stable or unstable is a matter determined by the magnitudes of two sets of forces:

Forces tending to produce failure and forces tending to prevent failure. The comparison of these forces and of their effects is called a stability analysis.

First of all, it must be stated, what is meant by the failure of a slope. Failure, as generally accepted isthe sliding of the mass of slope along some definite surface, called slip surface or failure surface. According to this definition, the first step in a stability analysis will be to assume some potential slip surface, along which the sliding of the slope-mass is possible.

After stating the necessity of a sliding surface, the second step of the analysis should be introduced: What will be the shape of the sliding surface.

The factor determining the shape of the slip surface is the shear strength of the soil. Generally the shear strength can be represented by the formula:

$$S = C + (\sigma - u) \tan \emptyset$$

At points having a lower shear strength than the rest, the failure condition will be reached earlier and the failure surface will pass through these weak points.

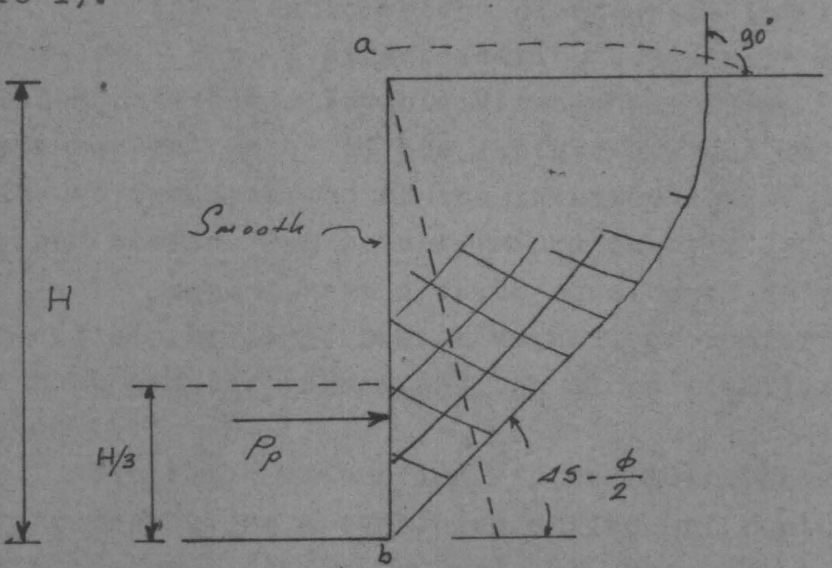
The slip surface may be of three different shapes.

1. Plane
2. Circular cylinder
3. General shape.

With regard to shape, the earliest assumption was a plane sliding surface. Coulomb and Rankine have used this assumption in the earth pressure theories.

In 1773, when Coulomb developed a formula for the normal ear pressure component on a retaining wall, for the fill behind the wall, assumed a plane rupture line passing through the foot of the wall and reaching the ground surface. Considering the wedge of soil bounded by the surface, the wall and the failure surface, he used the equilibrium condition of the forces acting on this wedge.

In 1857 Rankine considered the case of a semi-infinite cohesionless earth mass with sloping surface and making an angle β with the horizontal. He assumed that this earthmass was in a state of failu and his rupture figure he assumed a combination of two systems of straight, parallel lines, intersectivg each other at an angle of $90° \pm$ (Figure 1).



(Figure 1)

As a result Rankine's assumption gives a plane rupture surfac

As stated earlier the second possible shape for the sliding surface is a circular cylinder. This is the assumption used by the latest methods of slope stability analysis. In the circular cylinder method it is assumed that no movement occurs in the direction of the longitudinal axis of the cylinder. The sliding of the mass is along the boundaries of the circular cross-section.

The third possible shape of the sliding surface is a general shape. This can also be called a composite rupture line. When there i stratification in the soil or when the homogeneity of the soil is disturbed for some reason, the failure surface can not be of a single shape. At parts of stratification, the failure surface will have a tendency to follow the boundary. So, the failure surface will be rathe a series of different shapes, than a smooth, single curve.

The third step to be determined is the type of approach in a stability problem.

In general, four kinds of approaches are possible in a stability analysis.

1. Extreme Methods
2. Theories of Plasticity
3. Theories of Elasticity
4. Empirical Methods.

Theories of Plasticity and Theories of Elasticity are the most common ones, therefore a brief information will be given about them.

In the elastic method of analysis the procedure to be followed is the calculation of stressdistribution at the critical zones and its comparison with the allowable strength of soil. Here, it is assumed that the general shape of the fupture surface is known and that the boundary stresses acting at the intersection of the rupture surface with the ground surface can be determined exactly.

The second method to be considered is plastic method, which may be called the Limit Design Method, or Rupture Surface Method. This is the most popular method and the slope stability analysis is based mainly on it.

The basic assumption of the Limit Design Method is that the failure occurs along a continous surface of rupture. Therefore, a suitable slip surface is assumed, at every point of which a plastic state prevails. If a small darth element on the slip surface is considered, three unknown stresses and two equilibrium conditions are present. Relating these to each other a single equation may be obtained, showing the variation of stress along the rupture line.

$$\sigma_I = 2c \tan(45^\circ + \phi/2) + \sigma_{III} \tan^2 (45 + \phi) = 2c\sqrt{N\phi} + \sigma_{III} N\phi \quad \text{(2-1)}$$

$$N\phi = \tan^2 (45 + \phi/2) \quad \text{(2-2)}$$

$$\frac{\sigma_I + \sigma_{III}}{2} \sin\phi = \frac{\sigma_I - \sigma_{III}}{2} - \cos\phi \cdot c \quad \text{(2-3)}$$

$$\sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{xz}^2} - \frac{\sigma_z - \sigma_x}{2} \sin\phi = c \cos\phi \quad \text{(2-4)}$$

σ_I, σ_{II}, σ_{III} being the major, intermediate, minor principal stresses.

Theory of plasticity for soil is based on Mohr's Theory of rupture, Because no better substitute is found yet, describing the plastic properties of soil in a more satisfactory manner.

Usually following assumptions are made to apply limit design procedures.

1) Soil isconsidered to have uniform zones, each having constant properties.

2) Each zone's shear strength is:

$$S = C' + (\sigma - u) \tan \phi / R \quad \text{-------------------------(2-5)}$$

And shear mobilized is:

$$S_m = \frac{C}{F} + (\sigma - u) \frac{\tan \phi}{R} \quad \text{-------------------------(2-6)}$$

3) The stability problem is considered to be two dimensional ie: no shear strength is considered in a direction perpendicular to motion.

The fourth and last point to be explained is the introduction of a factor of safety consideration to the slppe stability analysis. As said at the beginning, the stability analysis is actually a comparison of driving and resisting forces. This gives us a dimentionless number called Factor of Safety, by means of which we are able to determine the stability of a slope.

There are various definitions of factor of safety:

1) Morgenstern and Bishop's definition: Factor of safety is that factor, by which the shear strength parameters in terms of effective stress C' and $\tan \phi'$ can be reduced before the slope is brought into a state of limiting equilibrium.

$$\text{Shear strength mobilized} = \tau = \frac{C'}{F.S} + (\sigma_n - u) \frac{\tan \phi'}{FS} \quad \text{------(2-7)}$$

2) Another definition of factor of safety is similar to the first one. Only there are two different factor of safeties. One is for cohesion and the other is for friction components.

$$\tau = \frac{C'}{FS_c} + (\sigma - u) \frac{\tan \phi'}{F.S_\phi} \quad \text{-------------------------(2-8)}$$

3) Nowadays the factor of safety is defined as:

$$F.S = \frac{Mr}{Md} = \frac{\text{Resisting moment}}{\text{Driving moment}}$$

The resisting moment is caused by Friction, Cohesion and normal forces about the centre. The driving moment is caused by Weight hydrostatic uplift, seapage pressure, and earthquake forces about the center.

After this statement of the main assumptions in a stability analysis, now the basic methods will be explained which can be applied for the solution of different problems. The methods are grouped according to the shape of the failure surface. First those assuming a circular slip surface will be explained, and later the ones with non-circular slip surfaces.

3. CIRCULAR SLIP SURFACES

3.1. SWEEDISH METHOD-, METHOD OF SLICES:

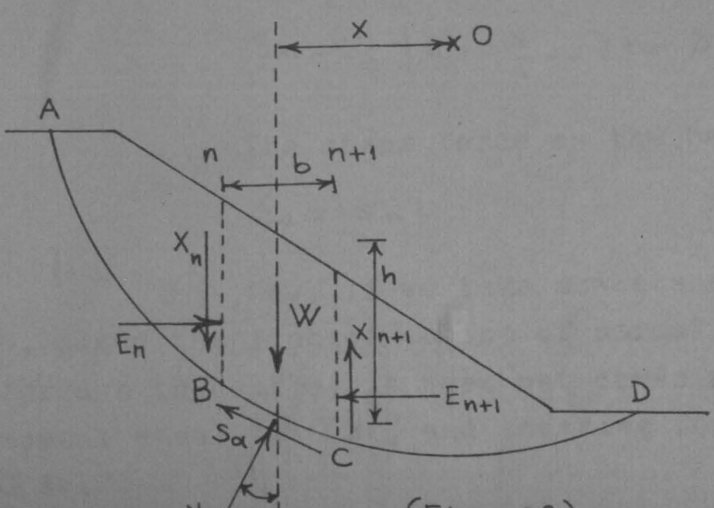
In 1913 a Geotechnic Commission was set up in Sweeden to investigate soil stability problems. The reason for this action was the occurance of a large number of harbour, railway and highway slides in soft clay soils. The work of the Geotechnical Commission led to the development of a slope stability analysis known as the Sweedish Cylindrical Surface Method.

The first use of theSweedish Method is proposed by K.E. Petterson, when he investigated the failure of the quaywall of Gothenburg Harbor (March 1916). This failure had occurred as the seawards slide of several hundred feet of quaywall, by a rotational movement of soil along a curved surface.

The principle of Petterson's cylindrical surface method is further developed by Fellenius. Using the same cylindrical slip surface Fellenius subdivided the mass above the failure surface into vertical slices for the purpose of a more accurate investigation. This is the reason why Sweedish Method may also be called Method of Slices.

Further work isdone on Sweedish Method since that time and its most important development is Bishop's Numerical Slices Method. It is considered to be most representative numerical presentation of the Slices Method.

3.1 a) Numerical Slices Method of Bishop:



(Figure 2)

In this method a circular slip surface is assumed and the soilmass above the critical surface is separated into vertical slices. Each slice is taken as a free body, being in equilibrium.

The acting forces at each slice are the weight W in a vertical direction, vertical shears X_n and X_{n+1} and resultants of the total horizontal forces on sections n and n+1, normal and shearing forces acting at the base of slice.

En, En+1 = Resultants of the total horizontal forces on sections n and n+1.

Xn, Xn+1 = Vertical shear forces on sections n and n+1.

W = Weight of the slice of soil.

N = Normal force acting on the base of the slice.

Sa = Shearing force acting on the base of the slice.

Sm = Shear strength mobilized or (developed).

h = Height of the slice.

b = width of the slice.

L = Length BC.

θ = Angle between BC and the horizontal.

X = Horizontal distance of slicefrom centre of rotation.

As known from the factor of safety analysis,

$$S_m = \frac{1}{F}\left\{ c' + (\sigma_n - u)\tan\phi' \right\} \qquad (3-1)$$

c' = cohesion } In terms of effective stress

ϕ' = angle of shearing resistence }

σ_n = total normal stress

u = pore pressure

$(\sigma_n - u)$ = σ' = effective normal stress.

The total normal stress is:

$$\sigma_n = \frac{N}{L} \qquad (3-2)$$

$$S_m = \frac{1}{F}\left\{ c' + \left(\frac{N}{L} - u\right)\tan\phi' \right\} \text{ and } F = \frac{1}{S_m}\left\{ c' + \left(\frac{N}{L} - u\right)\tan\phi' \right\} \qquad (3-3)$$

The shear force on the base of the considered slice is:

$$S_a = S_m L$$

Now, if we take moments of the acting forces about centre O, since the line of action of normal force N is along the radius and through thecentre, it does not cause any moment. Weight W causes a moment equal to ΣWX, and shearing force Sa causes a moment equal to $\Sigma S_m LR$.

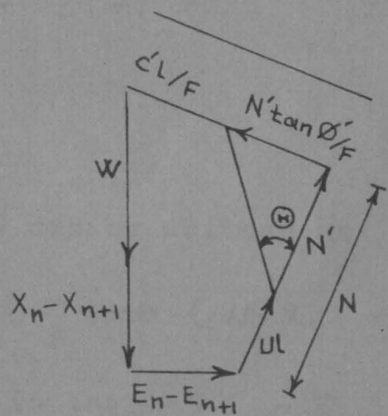
From the conditions of equilibrium it follows that:

$$\Sigma WX = \Sigma\ smLR \quad \text{------------------} \quad (3\text{-}4)$$

Therefore, $sm = \dfrac{\Sigma WX}{R\Sigma L}$, and

$$F = \frac{R}{\Sigma Wx}\Sigma[\,c'l + (N - ul)\tan\phi'\,] \quad \text{------------------} \quad (3\text{-}5)$$

$(N\text{-}ul) = N' =$ effective normal force (figure 3)



(Figure 3)

If we consider the equilibrium of the forces in a vertical direction, we get:

$$N' = \frac{W + X_n - X_{n+1} - L\left(u\cos\theta + \frac{c'}{F}\sin\theta\right)}{\cos\theta + \tan\phi'\sin\theta/F} \quad \text{------------} \quad (3\text{-}6)$$

also, $L = b\sec\theta$, $X = R\sin\theta$ and $u = \bar{B}\left(\frac{W}{b}\right)$ *
Therefore,

$$F = \frac{1}{\Sigma W\sin\theta}\Sigma\left[\left\{c'b + \tan\phi'\left(W(1-\bar{B}) + (X_n + X_{n+1})\right)\right\}\frac{\sec\theta}{1 + \frac{\tan\phi'\tan\theta}{F}}\right] \quad \text{---} \quad (3\text{-}7)$$

The first condition for the X values isthat:

$$\Sigma(X_n - X_{n+1}) = 0 \quad \text{------------------} \quad (3\text{-}8)$$

The values of X_n and X_{n+1} are found by successive approximation and after these values are found, $\Sigma(E_n - E_{n+1})$ can be determined from equation (3-7).

If we consider the equilibrium of the forces in a tangential direction, we get:

$$(W + X_n + X_{n+1}) \sin\theta + (E_n - E_{n+1}) \cos\theta = Sa$$

ie: $$(E_n - E_{n+1}) = Sa \sec\theta - (W + X_n - X_{n+1}) \tan\theta \quad \text{------------ (3-9)}$$

Equation (3-7) can be written as:

$$F = \frac{1}{\sum W \sin\theta} \sum(m) \quad \text{------------ (3-10)}$$

$$Sa = \frac{m}{F}$$

Now, equation (1-9) can be written as:

$$(E_n - E_{n+1}) = \sum\left[\frac{m}{F}\sec\theta - (W + X_n - X_{n+1})\tan\theta\right] \quad \text{------------ (3-11)}$$

The condition here is that $\sum E_n - E_{n+1} = 0$ ------------ (3-12)

ie: $$\sum \frac{m}{F} \sec\theta - (W + X_n - X_{n+1}) \tan\theta = 0 \quad \text{------------ (3-13)}$$

Equation (3-13) is the second condition that the X forces have to Satisfy.

For the application of the numerical slices method the following procedure is suitable:

First, $(X_n - X_{n+1}) = 0$ is assumed and by means of equation (3-7) an initial F- value is found. Then, another value of F is assumed and the variables of equation (3-7) are determined for each slice in a tabular form: (Table 1)

(1) Slice No	(2) b(ft)	(3) h(ft)	(4) W lb x 10^3	(5) θ	(6) $\sin\theta$	(7) $W\sin\theta$	(8) $c'b$	(9) $W(1-\bar{B}) \times \tan\phi$	(10) (8)+(9)	(11) $\sec\theta$	(12) $\tan\theta$	(13) *	(14) (10) x (13)
1	...	...	...										
2	...	...	...										
.													
.													
.													

(Table 1)

* (13) = $\sec\theta \Big/ 1 + \dfrac{\tan\emptyset' \tan\theta}{F}$

According to this table the computed factor of safety is:

$$F = \frac{\sum \text{column } 14}{\sum \text{column } 7} \quad \text{------------------} (3-14)$$

For a considerable accurate solution a number of trial F-values may be selected and the corresponding resulting F- values are found from equation (3-7) as shown above. The smallest factor of safety determines the critical surface.

3.1 b) Stability Coefficients Method of Bishop:

The tabular solution of Numerical Slices Method has to be done for each trial circle, which is a time consuming and repettitive task. Therefore, it is tried to solve the problem faster by means of stability charts prepared in such a way, that they can suit to the properties of different problems.

The first stability charts were prepared by Taylor, which wer based upon total stresses. After that, it is found out that stability charts should be based on effective stresses rather than on total stresses. As the pore pressure distribution plays a very important part in slope stability, it has to be considered too. This new kind o charts are prepared by A.W. Bishop and N. Morgenstern.

In a general solution, thereis an assumption that the pore pressure ratio r_u is constant over the whole cross-section. This kind of analysis is called a homogeneous pore pressure distribution.

The pore pressure ratio being constant, there are also a number of variables in the stability coefficients method. These are shown in Figure (4).

c' = cohesion intercept } In terms of effective stresses
$\emptyset'$ = shearing resistance }

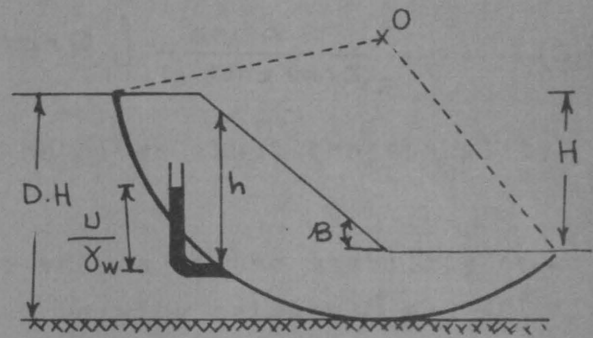
γ = bulk density

H= height of the slope

D.H. = depth of the first hard stratum below the crest of the section.

D= depth factor

β= angle of the slope.



(Figure 4)

When a definite dimensionles s number $\frac{C'}{\gamma H}$ is used to express the effect of cohesion on the stability of the slope, the factor of safety will depend totally on the geometry of the given section: ie:[D and cot β] and on the r_u and ϕ' values.

The stability charts are prepared for $\frac{C'}{\gamma H}$ values of 0, 0.025 and 0.05. This is the range, occuring in effectivestress analysis. In this range there is also the possibility of linear interpolation.

Depending on the information given above, the factor of safet can be expressed in form of a linear equation.

$$F = m - n r_u \quad \text{--} (3\text{-}15)$$

m and n being the stability coefficients.

Using the stability charts and Equation (3-15), the factor of safety can be determined.

The stability charts are prepared by means of an electronic digital computer, according to the programme of Price and Little. In this investigation more than 5000 trial circles are used. The slope inclinations are specified for the range of 2:1 to 5:1, the ϕ' values are between 10° and 40° and D is up to 1.5.

For the derivation of the stability coefficients, the computer uses this equation:

$$F = \frac{1}{\Sigma W \sin\theta} \Sigma\left[\left\{c'b + W(1-r_u)\tan\phi'\right\} \frac{\sec\alpha}{1 + \frac{\tan\alpha \tan\phi'}{F}}\right] \quad \text{--------------} (3\text{-}16)$$

But according to the needs of the machine equation (3-16) is somewhat changed and it becomes:

$$F = \frac{1}{\sum \frac{b}{H} \cdot \frac{h}{H} \sin\alpha} \sum \left[\left\{ \frac{c'}{\gamma H} \frac{b}{H} + \frac{b}{H} \cdot \frac{h}{H} (1 - r_u) \tan \phi' \right\} \frac{\sec\alpha}{1 + \tan\alpha \tan\phi'/F} \right] \quad \text{-------- (3-16a)}$$

Now, some information should be given about the stability charts themselves and about their use.

In the charts (Figure 5), the values of the stability coefficients are plotted against cot B , ϕ' varying between 0° and 40°. On each chart the usable range of D and $\frac{C'}{\gamma H}$ values are indicated.

The dark lines are stability coefficients m and n values at 10° intervals. The lighter lines arefound by interpolation.

As stated earlier the charts are prepared for a certain rang of $\frac{C'}{\gamma H}$ values. If the factor of safety outside this range is searched, a linear extra polation should be followed. But extrapolation can give an accurate factor of safety only fortherange of $0.05 < \frac{C'}{\gamma H} < 0.10$. Since the variationsof m and n with $\frac{C'}{\gamma H}$ are non linear outside this range, a linear extrapolation can't give accurate results.

On the stability charts there are also some broken lines shown by r_{ue}. They are called lines of equal r_u and their use is for the purpose of determining the critical depth factor D.

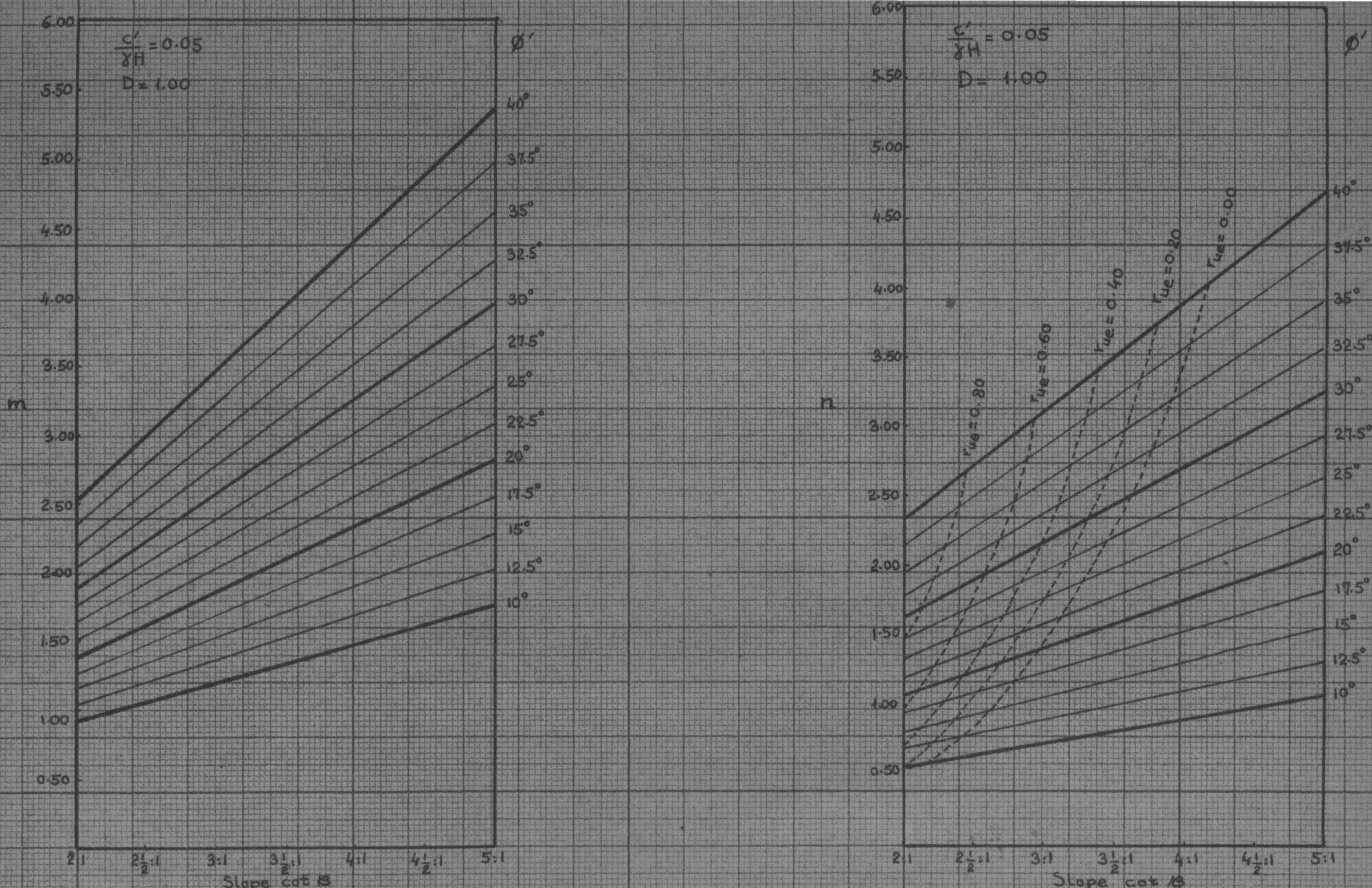
For a given set of parameters (B, ϕ', $\frac{C'}{\gamma H}$) there is an r_{ue} value, for which the factor of safety isthe same, whether D=1.00 or D=1.25 . This equality can be shown in such a way:

$$r_{ue} = \frac{m_{1.25} - m_{1.00}}{n_{1.25} - n_{1.00}} \quad \text{---------------- (3-17)}$$

For the specified parameters, if the pore pressure value used in design ishigher than r_{ue} of the section, the factor of safety with depth factor D=1.25 will be lower than the factor of safety with D=1.00 .

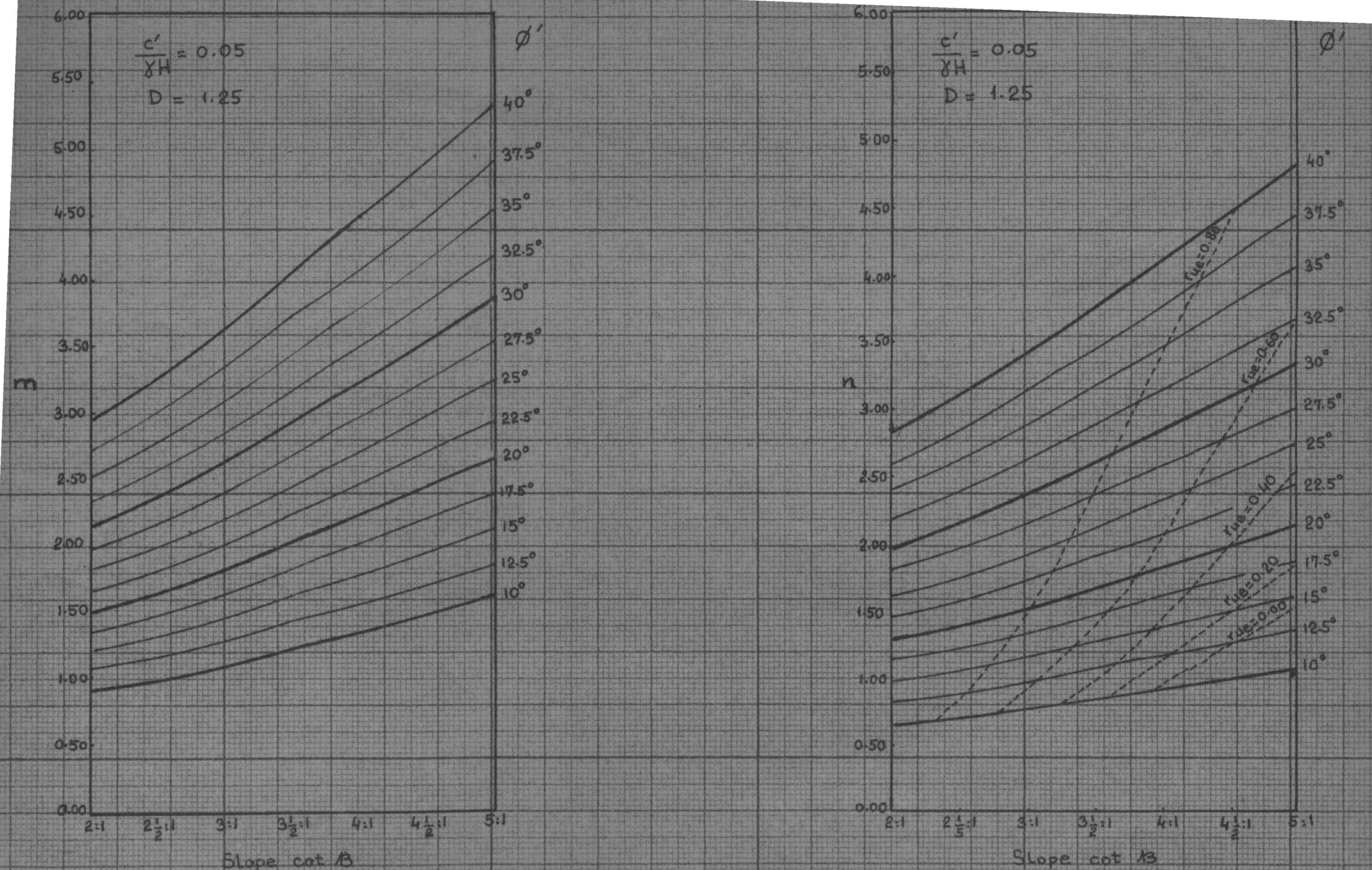
For a comparison between the factor of safeties with D=1.50 and D=1.25 we have:

$$r_{ue} = \frac{m_{1.50} - m_{1.25}}{n_{1.50} - n_{1.25}} \quad \text{---------------- (3-18)}$$



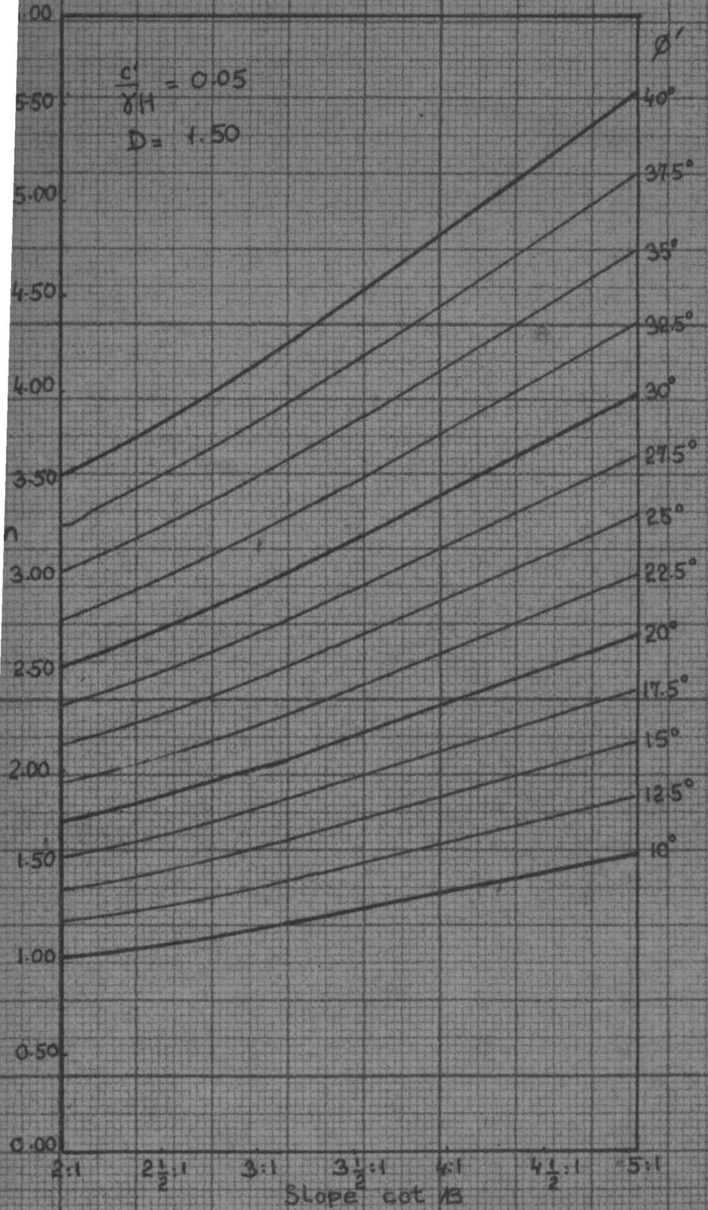
(Figure 5-a) Stability coefficients m and n for $\frac{c'}{\gamma H} = 0.05$ and $D = 1.00$

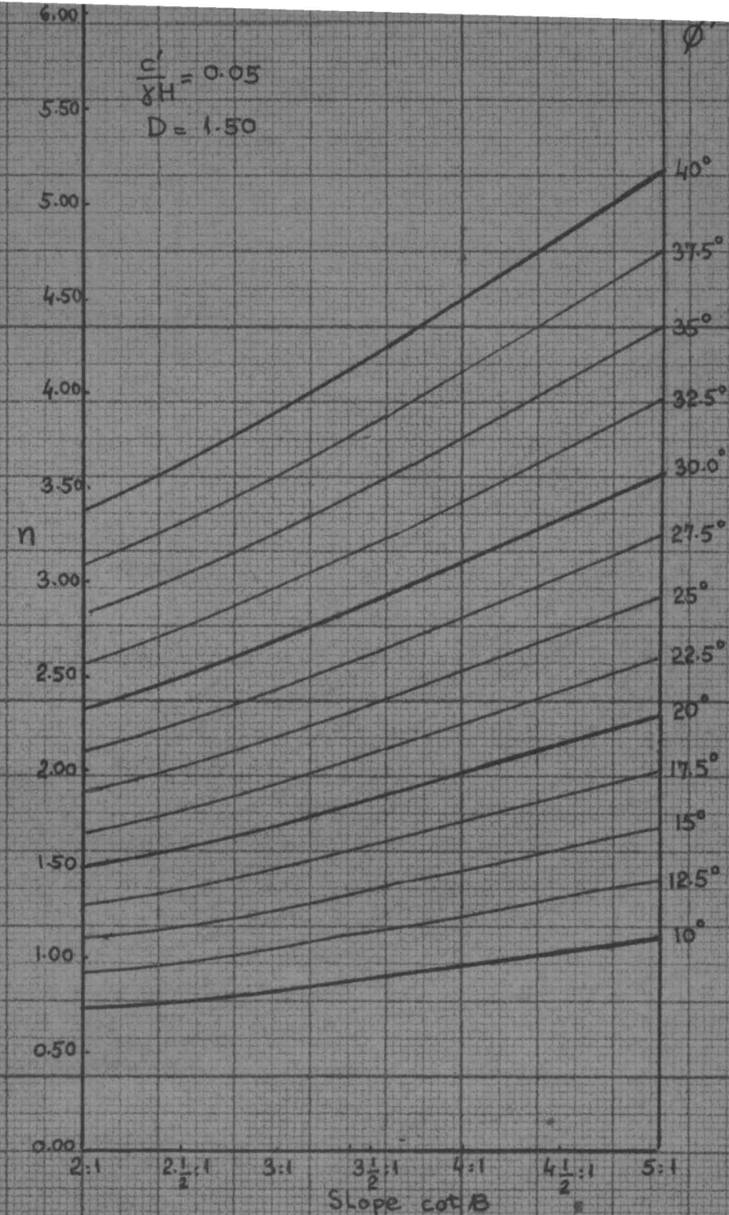
(A.W. Bishop - N. Morgenstern)



(Figure 5-b) Stability coefficients m and n for $\frac{c'}{\gamma H} = 0.05$ and D = 1.25

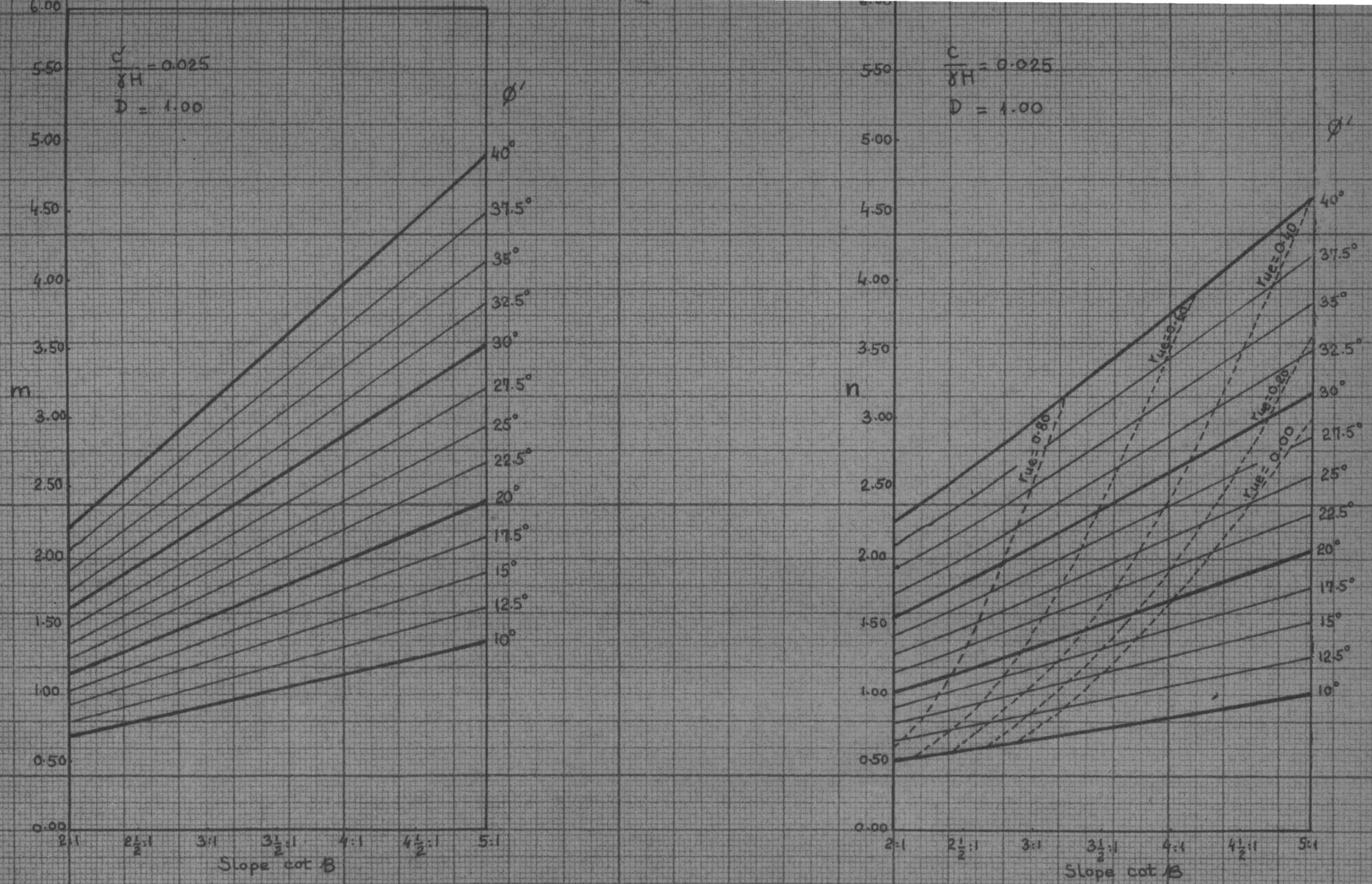
(A. W. Bishop - N. Morgenstern)





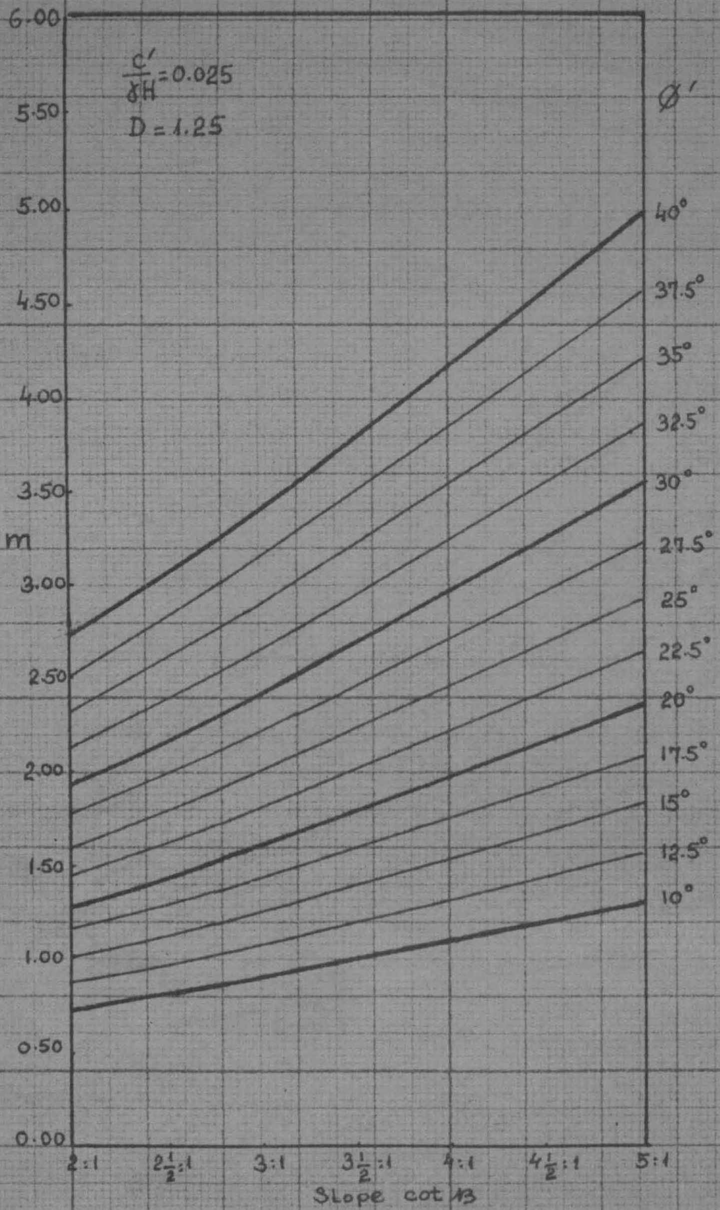
(Figure 5-c) Stability coefficients m and n for $\frac{c'}{\gamma H} = 0.05$ and D = 1.50

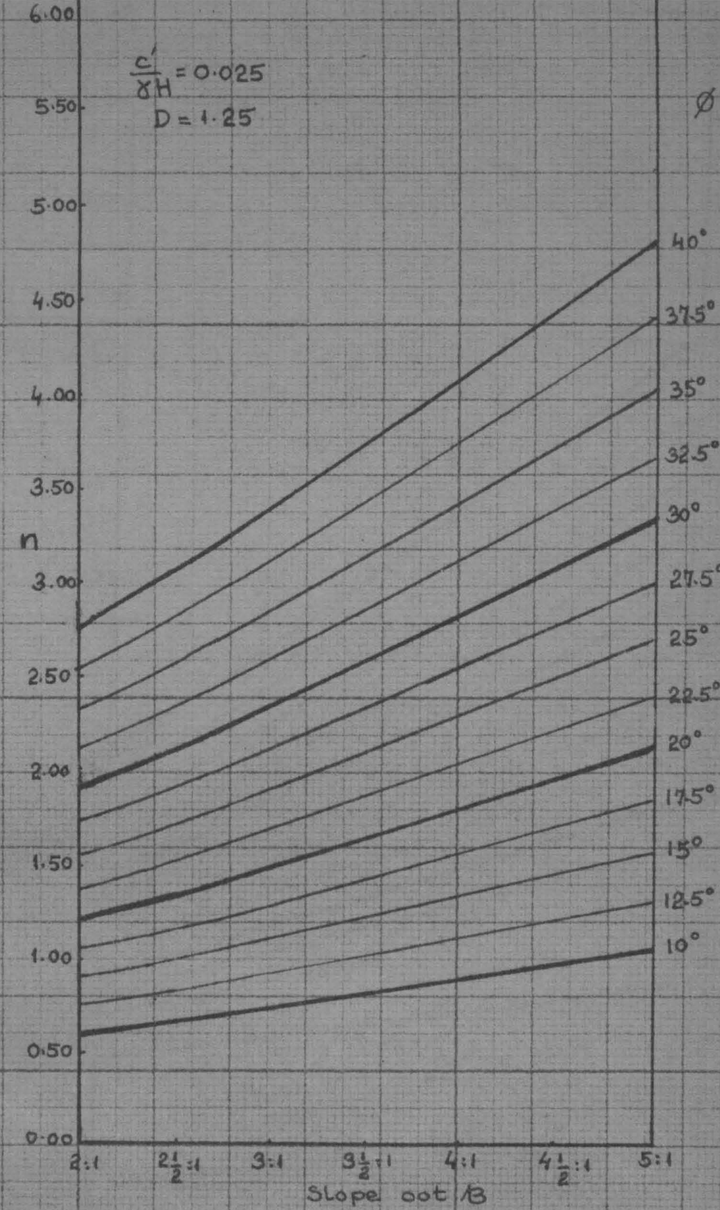
(A.W. Bishop – N. Morgenstern)



(Figure 5-d) Stability coefficients m and n for $\frac{c'}{\gamma H} = 0.025$ and D = 1.00

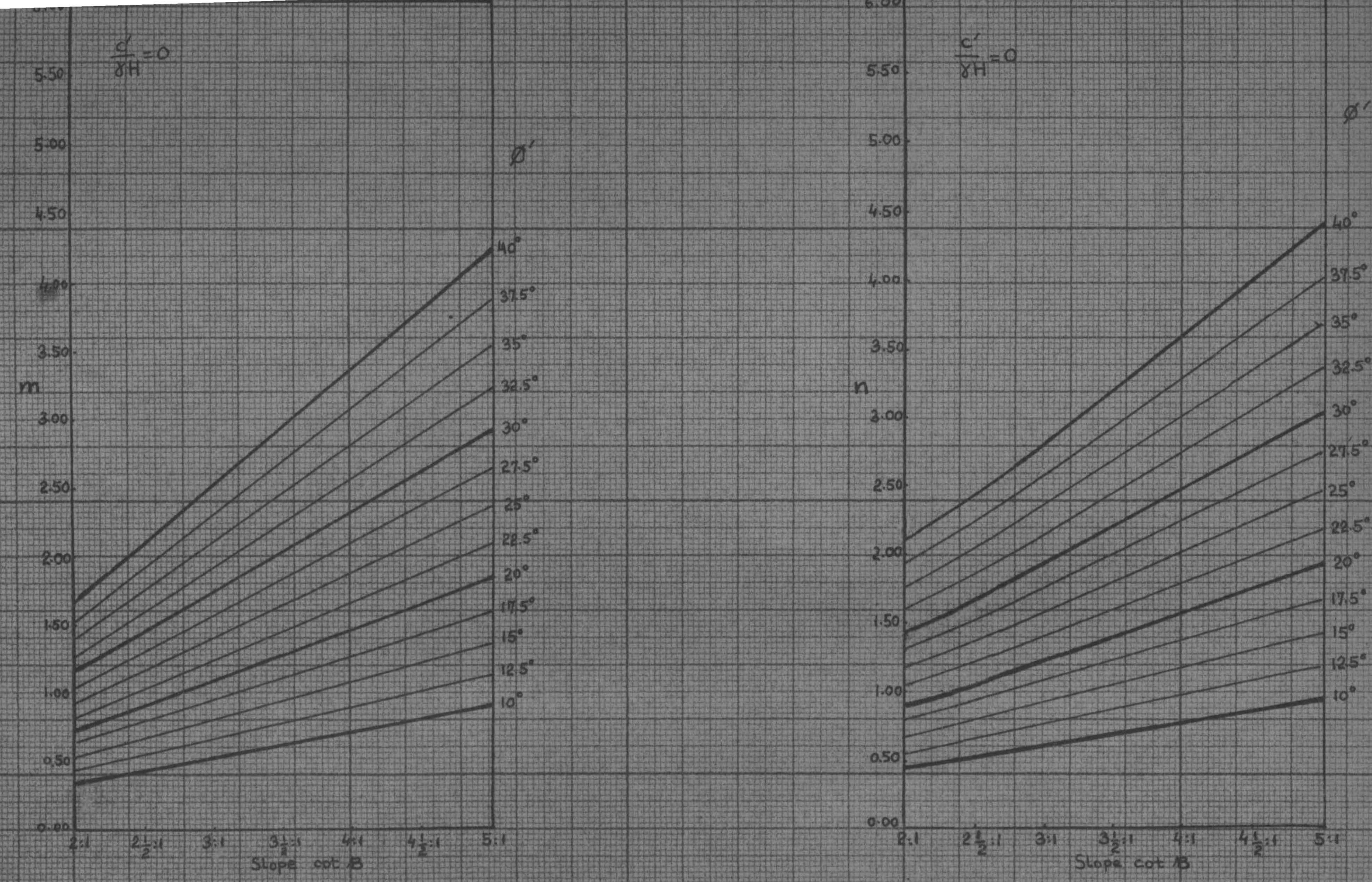
(A. W. Bishop - N. Morgenstern)





(Figure 5-e) Stability coefficients m and n for $\frac{c'}{\gamma H} = 0.025$ and D = 1.25

(A.W. Bishop _ N. Morgenstern)



(Figure 5-f) Stability coefficients m and n for $\frac{c'}{\gamma H} = 0$

(A. W. Bishop – N. Morgenstern)

In such a way the r_{ue} values shown on the charts could be drawn.

Chart (a) shows the comparison between r_{ue} values for D=1.25 and D=1.00, when $\frac{c'}{\gamma H} = 0.05$. And the same reasoning for the other charts.

Even if the sections are not directly on the hard stratum, one should begin the depth factor analysis by D=1.00 .

3.1c) Conventional Method:

To develope this method, first the acting forces are taken to be the same as those in Numerical Slices Method. (Figure 2) Therefore, equation (3-5) is applicable.

$$F = \frac{R}{Wx} \sum (c'l + (N - ul) \tan \phi')$$

If we isolate a strip of soilmass, considering the equlibrium of the acting forces in a direction perpendicular to the base, we get:

$$N - W \cos \theta \, (X_n - X_{n+1}) \cos \theta + (E_n - E_{n+1}) \sin \theta = 0$$

$$N = [W + (X_n - X_{n+1})] \cos \theta - (E_n - E_{n+1}) \sin \theta \text{ .... } (3\text{-}18)$$

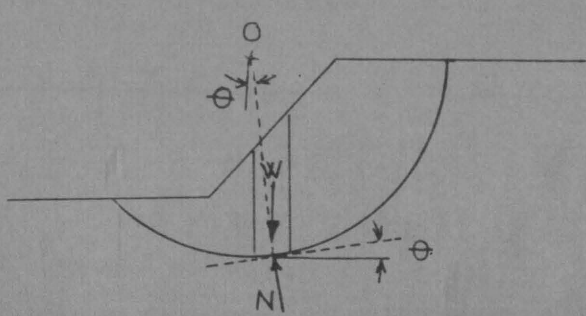
Therefore,

$$F = \frac{R}{\sum Wx} \sum [c'l + \tan\phi'(W\cos\theta - ul) + \tan\phi' \{(X_n - X_{n+1})\cos\theta - (E_n - E_{n+1})\sin\theta\}] \text{ .....} (3\text{-}19)$$

Terzaghi, Krey and May justify it, that the terms

$\sum \tan\phi' \{(X_n - X_{n+1})\cos\theta - (E_n - E_{n+1})\sin\theta$ may be neglected,

without introducing a great inaccuracy in the solution. According to this modification the present condition is shown in (Figure 6)



(Figure 6)

Equation (3-19) may now be written as:

$$F = \frac{1}{\Sigma W \sin\theta} \Sigma\left[c'l + \tan\phi'(W\cos\theta - ul)\right] \quad \text{------------------ (3-20)}$$

$u = \bar{B}\left(\frac{W}{b}\right)$ and $x = R\sin\theta$.

Finally, we get as factor of safety:

$$F = \frac{1}{\Sigma W \sin\theta} \Sigma\left[c'l + \tan\phi' W(\cos\theta - \bar{B}\sec\theta)\right] \quad \text{------------------ (3-21)}$$

As seen from equation (3-21) the advantage of conventional ethod is that it can be used for slopes composed of a number of layers f different soils with different C' and ϕ' values. The summation sign n equation (3-21) is an indication of this quality.

For a solution with conventional method againa tabular use f equation (3-21) is necessary. (Table 2)

(1) Slice No	(2) b(ft)	(3) h(ft)	(4) W lb $\times 10^3$	(5) θ	(6) $\sin\theta$	(7) $W\sin\theta$	(8) $\frac{1}{W\sin\theta}$	(9) $c'b$	(10) $\bar{B}\sec\theta$	(11) $\cos\theta$	(12) *	(13) (9)+(12)	(14) (8)x(13)
1 2 3 . . .													

(Table 2)

* $\tan\phi' W(\cos\theta - \bar{B}\sec\theta)$

After repeatingthis procedure for a number of F values, the allest F value determines the critical circle.

When we assume the pore pressure u to be equal to zero, $\bar{B}$ will come zero too, and equation (3-21) gets a simplified form, such as

$$F = \frac{\Sigma cl + \Sigma \tan\phi\, W\cos\theta}{W\sin\theta} \quad \text{------------------ (3-22)}$$

Equation (3-22), like equation (3-21) is applicable to slopes nsisting of different layers of soils.

When also uniform soil conditions (ie: constant C and ϕ throughut the slope) and a firm layerbelow some depth are present, the problem an be solved in another and very fast way. For this purpose Taylor has repared some stability charts, by means of which the critical slope ngle and depth can be determined.

These stability charts are prepared for two different cases.

ase 1: $\phi = 0$, C = constant.

$$\text{Shear strength} = S = C = \frac{1}{2}\, q_u \text{ ------------------------------ } (3\text{-}23)$$

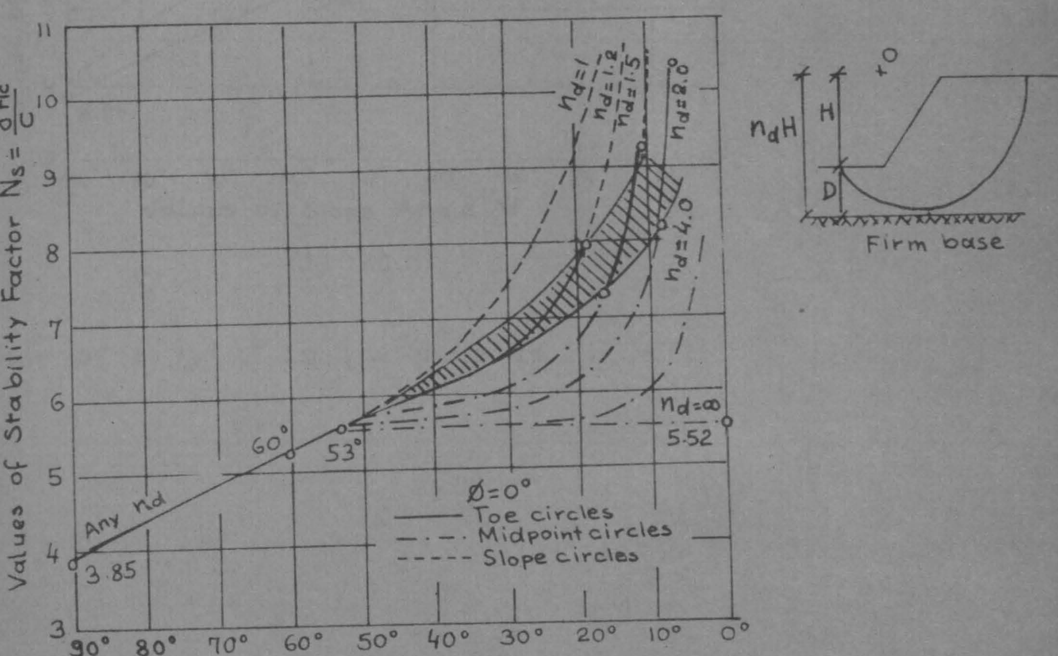
u being the unconfined compressive strength of clay.

If C is known, the critical height H_c of a slope of given β lope angle can be found by using equation (3-24).

$$H_c = N_s \frac{C_m}{\gamma} \text{ -- } (3\text{-}24)$$

Ns is called the stability factor, and its value is a function f β and depth factor n_d .

$$n_d = \frac{H + D}{D} \text{ -- } (3\text{-}25)$$



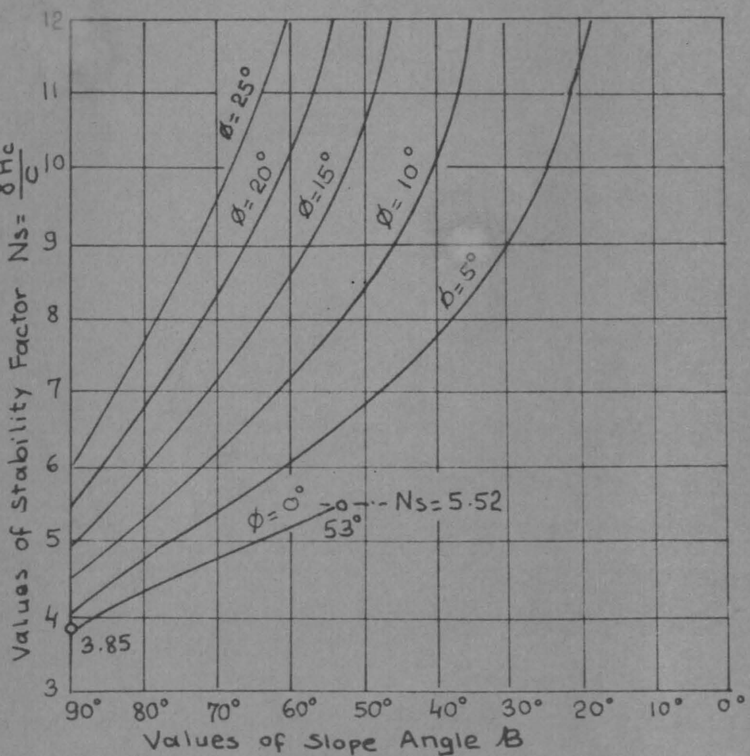
Values of slope angle β (Taylor)

(Figure 7)

When N_s and n_d are known the critical slope angle β can be determined by means of Figure 7.

Case 2: $\phi \neq 0$, $C \neq 0$

For a given value of ϕ, the critical height of slope will again be as in equation (3-24). The only difference is, that N_s is a function of β and ϕ this time. For $\phi = \beta$, N_s becomes infinite.



(Figure 8)

The use of Fig.(8) is the same as Fig.(7)

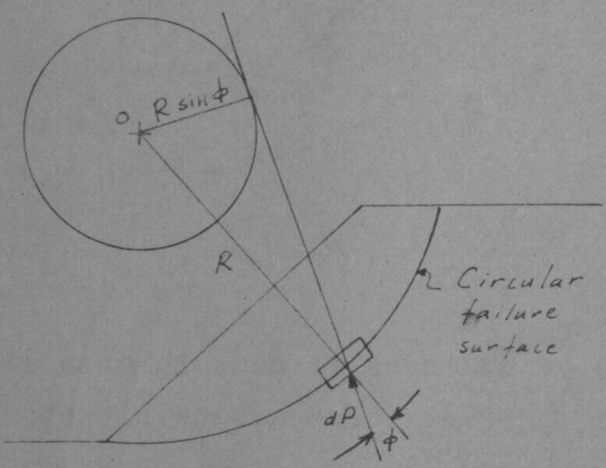
3.2. FRICTION CIRCLE METHOD- Ø CIRCLE METHOD:

The general requirements of this method is a thoroughly omogeneous slope with no stratification being present.

The friction circle method can deal with both Ø ≠0 and ≠ , and Ø =0, C≠ 0 cases.

3. 2a) Ø -circle Method for Ø ≠ 0:

Let us consider a slope with a circular failure surface, its centre being at point O and having a radius R.



(Figure 9)

A second concentric circle is drawn, its radius being RsinØ . As seen from Figure (9), at a unit element of the failure surface, the effective force dP is at Ø degrees obliquity and it is tangent to the inner circle.* This concentric circle with radius r_f=Rsin Ø is called the friction circle or the Ø - circle. Again, if we consider the mass of soil above the failure surface, there is Weight W acting through the centroid, the cohesive resisting forces are tangent to the failure surface and the effective forces are at an obliquity of $Ø_m$ to the failure surface.

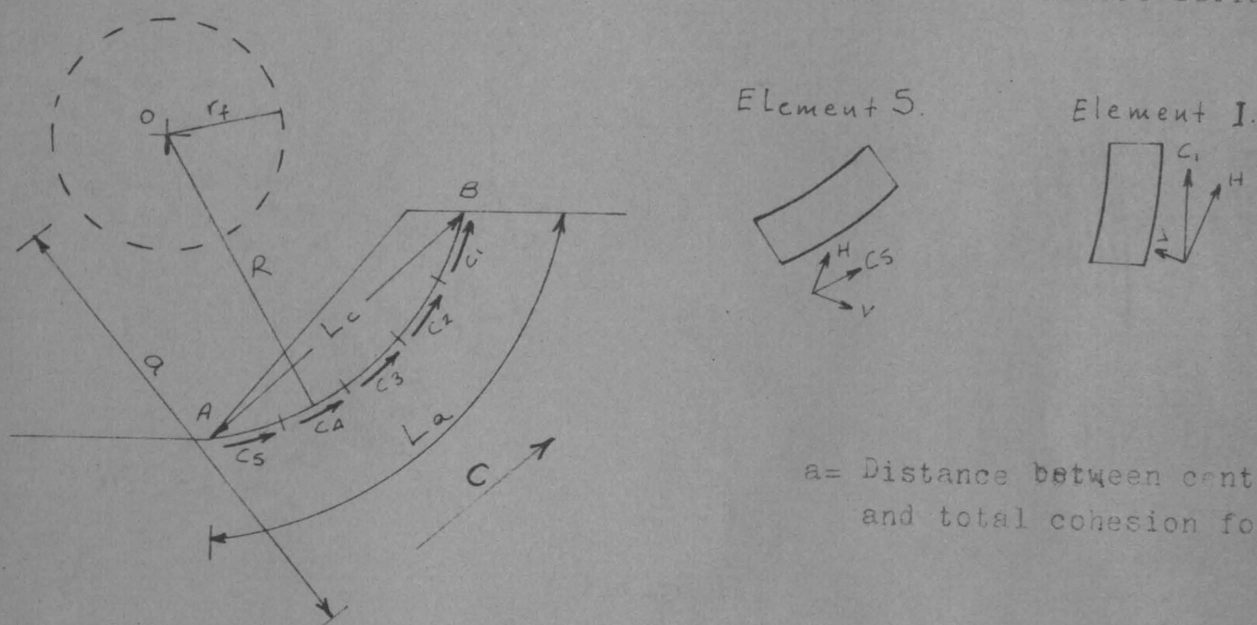
We can analyse these forces one by one.

Weight W acts through the centroid of the mass and it can be easily determined by planimetering the area of the considered cross-sec-

* Look at the correction discussed in modified Ø circle method.

tion. The easiest way to locate the centroid is to suspend the figure of the slope cut out of a cardboard from two points. The intersection of the vertical lines that are drawn while suspended gives the centroid.

The cohesion forces act tangentially to the failure surface,



a= Distance between center and total cohesion force.

(Figure 10)

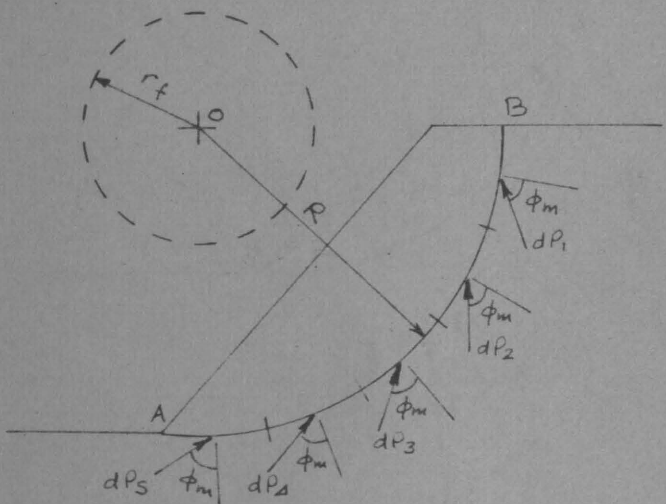
and each of them is equal to cm times a unit length of the arc. Their algebraic sum is equal to CmLa . But if, for example, we take the unit elements 5 and 1 on the failure surface, and resolve their cohesion force into components perpendicular and parallel to the chord AB (Figure 10), we see that only theparallel components are additive. The perpendicular components are in reverse directions and they only produce a counter-clockwise moment. Therefore, we have to consider only the parallel components and the cohesion forces can be added up to a total cohesion C parallel to the chord AB.

$$C = Cm\ Lc \quad - - - - (3\text{-}26)$$

Taking moments about center O, both representations of cohesion shown above should produce the same moment. ie: $CmLa \times R = CmLc\ a$

$$a = \frac{La}{Lc} R \quad - - - - (3\text{-}27)$$

The dR-forces at each element of the failure surface are force at Øm obliquity and as seen from figures (9) and (11), they have lines of action tangent to the Øm circle.



dP_1 , dP_2 = effective forces at each element of failure surface.

(Figure 11)

The total effective P-force too istangent to the Ø circle. A line of action outside the circle would mean an unsteable slope.

By means of the friction circle method used in a graphical way we can determine the stablity of a slope by following this procedure:

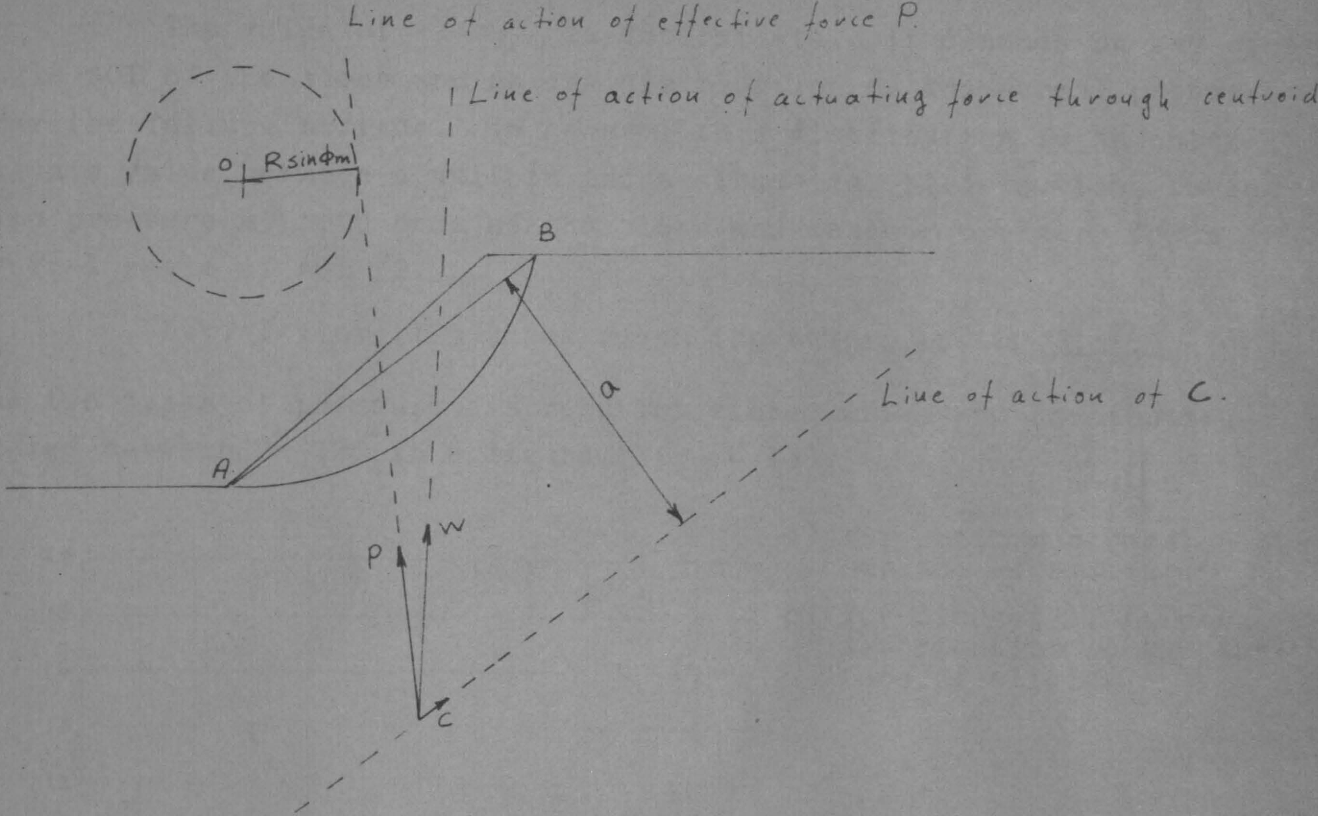
1) For the slope in question a trial failure surface with an assumed radius is drawn. With any reasonably assumed Cm value the corresponding friction circle is drawn too.

2) The weight W or its combination with neutral pressure considered as the actuating force is drawn through the centroid of the soil mass under consideration.

3) The line of action of cohesion force C is determined from equation (3-27) and it is drawn parallel to the chord. C and W cut eachother at point D.

4) The total effective force should pass through point D and should be tangent to the friction circle.

Since the lines of action of the three forces and the value of the actuating force are known, the values of C and P can be determined from the constructed force poligon. (Figure 12)



(Figure 12)

We measure the C vector parallel to chord and by means of equation (3-26) Cm value is determined.

$$F_c = \frac{C}{C_m} \qquad \text{and } F_\phi = \frac{\tan \phi}{\tan \phi_m} \quad ---- (3\text{-}28)$$

By different assumptions of ϕ_m value at the first step we can repeat this procedure for a number of times to find the smallest factor of safety.

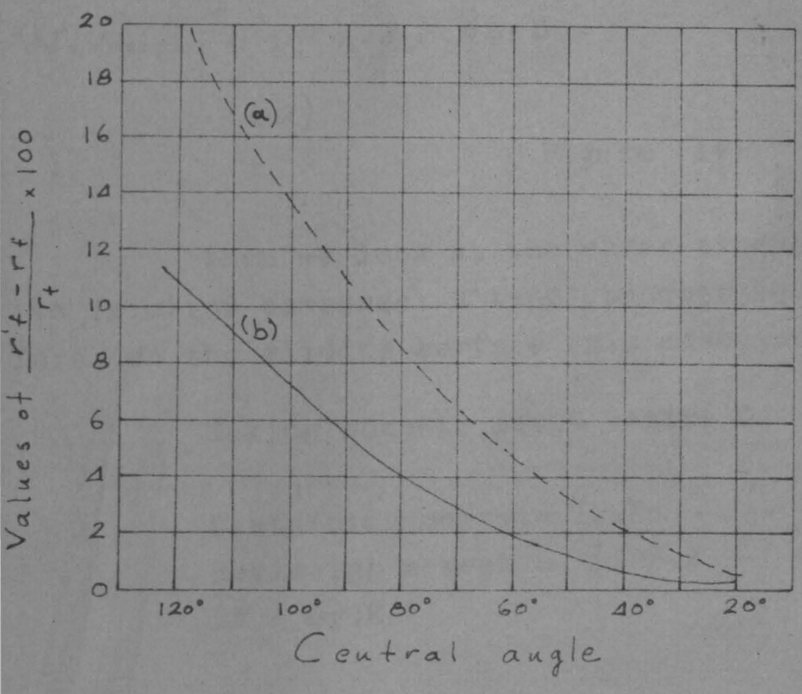
In order that the ϕ-circle method becomes more accurate, a s small modification in it is necessary.

Modified ϕ-Circle Method:

Although we have shown the effective forces at the failure surface to act tangent to the ϕ m circle, in reality they miss tangency by a small amount. Therefore the distance between the total P force and the center of the ϕ_m circle is not $r_f = R \sin \phi_m$, but $r_f' > r_f$.

The value of $r_f' - r_f / r_f$ is indeterminate. It depends on the central angle AOB of the slope and on the distribution of the normal pressure over the failure surface. In general this distribution is an intermediate value between a uniform and a sinusoidal distribution, having zero pressure at both ends of the slope and maximum pressure for a central angle of $\widehat{AOB}/2$.

D.W. Taylor (1937) has given the values of $100\frac{r_f' - r_f}{r_f}$ for the two types of pressure distribution stated above and for central angles between 0° -120° in a diagram(Figure 13).



a) For uniform stress on the surface of sliding.

b) For sinusoidal stress distribution on the surface of sliding.

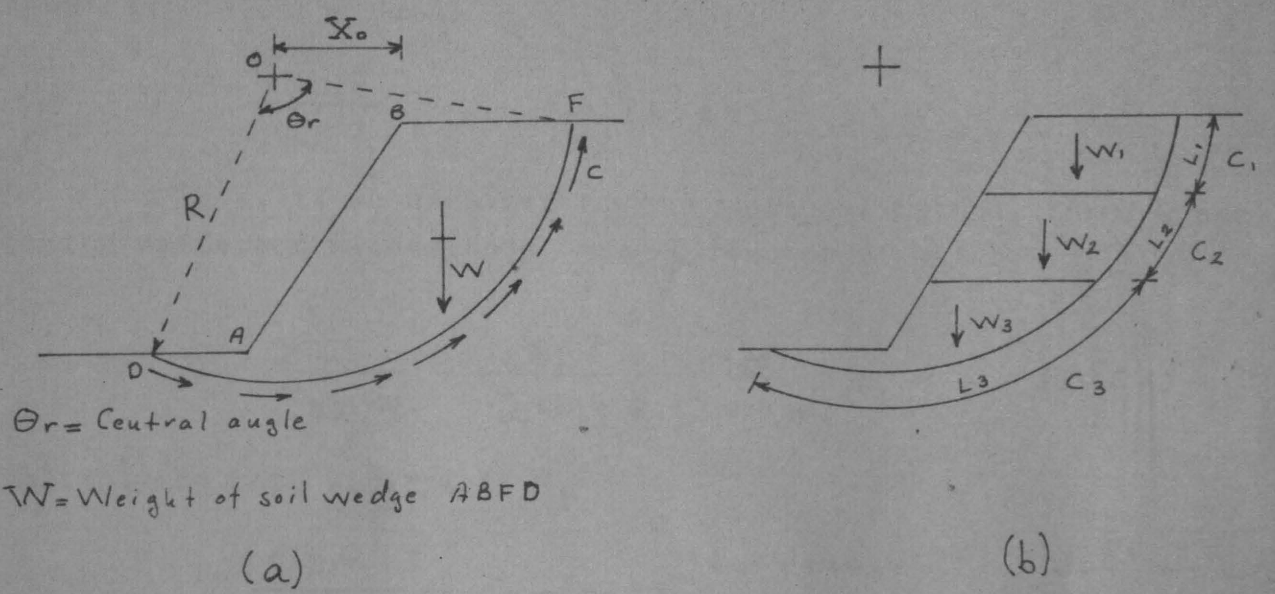
(Figure 13)

In general, curve b of the diagram isused. By multiplying the rf by the correction factor found from Figure 13, the true r_f' is found and the line of action of the total P-force is located after this small correction.

The Ø circle Method for Ø=0 :

In this method the critical surface of the slope is assumed to be a circle with centre O and radius R.

Since Ø = 0 , it follows that r_f = Rsin Ø = 0 too. A circle with zero radius becomes a point, therefore the friction circle at Ø=0 case becomes just a point. (Figure 14)



(Figure 14)

When we look at the shear strength expression, $s=c+\sigma \tan\phi$, its friction component $\sigma \tan\phi$ becomes equal to zero, since $\phi=0$. Therefore, on the sliding surface only cohesion forces are present.

Taking moments about centre O,

Disturbing moment= $W.X_o$ — — — — (3-29)
Resisting moment = $c.\overset{\frown}{DF}.R$ — — — — (3-30)
$\overset{\frown}{DF} = \theta r.R$

and,

$$F= \frac{\text{Resisting moment}}{\text{Disturbing moment}} = \frac{C\ \overset{\frown}{DF}\ R^2}{WX_o} \quad \text{———— (3-31)}$$

$$F= \frac{c.\theta r.R^2}{WX_o} \quad \text{———— (3-32)}$$

As seen until now, the basic assumption of ϕ-circle Method was a completely homogeneous slope with no stratification. But at this point it will be shown that for the case of $\phi = 0$, $c \neq 0$, also stratified slopes can be anaysed by means of the Friction Circle Method.

In $\phi = 0$ case the factor of safety was found to be:

$$F = \frac{C\ \theta r\ R^2}{WX_o}$$

Here, θrR can be replaced by the length of the arc, ie: $\theta rR=L_{DF}$, and equation (3-32) becomes:

$$F=\frac{C\ L_{DF}R}{WXo} \quad ----(3-33)$$

Now, if we consider the conventional method, for $\emptyset=0$ case, taking different slides, the general foactor of safety is:

$$F=\frac{\sum cl}{\sum w\sin\theta}=\frac{c\sum L}{\sum w\sin\theta}=\frac{c\ L_{DF}}{\sum w\sin\theta} \quad ----(3-34)$$

multiplying both sides with R,

$$F=\frac{c L_{DF}R}{\sum w R\sin\theta} \quad ----(3-35) \qquad \text{but } R\sin\theta=x \qquad F=\frac{c L_{DF}R}{\sum wx} \quad ----(3-36)$$

When we take $\sum wx$ over the entire length of the slip surface, $\sum wx$ = WXo and we get equation (3-32)

As seen from this derivation procedure, Conventional Method and $\emptyset$ circle Method for $\emptyset = 0$ case give the same result.

This fact can be used to analize stratified slopes. As seen from Figure (14 b) the only thing to be done in such a case is to take the different cohesion, w and X values of the strata into account and to apply a summation.

$$F=\frac{\sum C\ \theta\ r\ R^2}{\sum wX} \quad ----(3-37)$$

4. NONCIRCULAR SLIP SURFACES

4.1. PLANE FAILURE SURFACE:

a) Culmann Method:

The basic assumption is, that the maximum stressed surface of the slope is a plane, passing through the toe of the slope. In case of failure this plane will become the failure surface.

Let us represent the Culmann Method for a slope of H height and homogeneous soil.

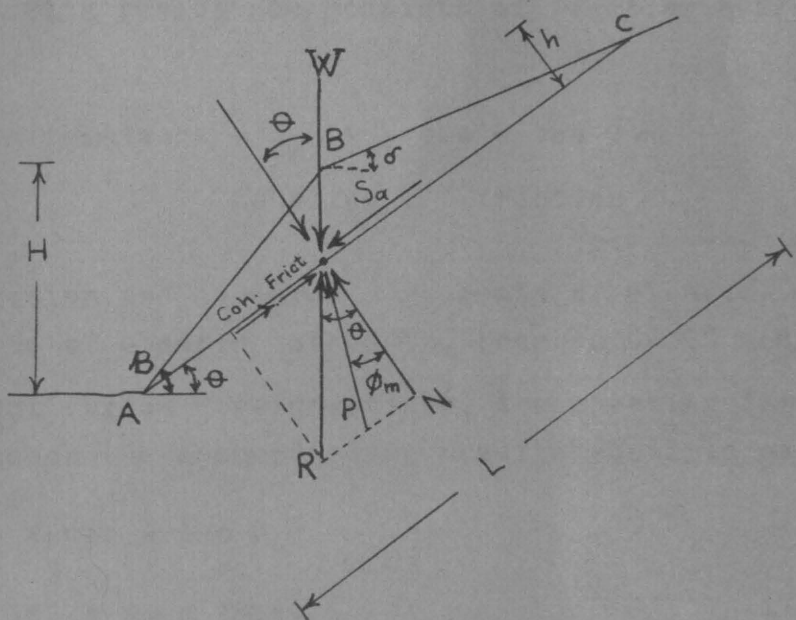


Figure (15)

AC= An arbitrarily assumed trial plane passing through the toe of the slope.

θ = Angle between the trial plane and the horizontal.

Sa= Shearing force on the AC plane

B = Slope angle.

W = Total weight of the soil mass above the trial plane.

γ = Unit weight of soil mass.

N = W cos θ = Normal component of the reaction Force R.

C_m= Developed cohesion, which is less than cohesion C.

ϕm = Developed friction angle, which is less than friction angle ϕ .

If we consider the wedge above the trial plane, the forces to be analyzed are these:

Due to the weight W of the mass, a shearing force, $S = W\sin\theta$ is developed parallel to the trial plane. It tries to slide the soil mass downwards.

Against this shearing force, a shearing resistance is developed on the assumed plane. The shearing resistance is always less than the shearing strength of the soil. Only at the boundary of failure, shearing resistance can become equal to the shearing strength of the soil.

The shearing resistance consists of friction and cohesion components.

$$\text{Shearing resistance} = \underbrace{c_m L}_{\text{cohesion}} + \underbrace{W \cos\theta \tan\phi_m}_{\text{friction}}$$

The friction and cohesion components of shearing resistance are less than those of shearing strength, because $C_m < C$ and $\phi_m < \phi$.

From equilibrium considerations, the shearing force and the shearing resistanceon the assumed plane must be equal to each other.

$$W \sin\theta = c_m L + W \cos\theta \tan\phi_m$$

From (Figure 15) it is seen that:

$$h = (H \csc\beta) \sin(\beta - \theta)$$

$$W = \frac{1}{2} L (H \csc\beta) \sin(\beta - \theta)$$

$$C_m = \frac{W}{L} (\sin\theta - \cos\theta \tan\phi_m)$$

$$C_m = \frac{1}{2} \gamma (H \csc\beta) \frac{\sin(\beta - \theta) \sin(\theta - \phi_m)}{\cos\phi_m} \qquad \text{---------------} (4-1)$$

For any $\phi_m < \phi$, when C_m found from equation (4-1) is the maximum C_m, the assumed trial plane is the real failure plane.

We can see from equation (4-1), that only θ is variable. It can be said therefore, that everything being constant, if θ becomes equal to a critical value θ_c , C_m becomes the maximum C_m, and the failure surface is thus determined.

The critical angle θ_c can be found in two ways:

1. By trial and error method: Different assumed θ values are substituted into equation (4-1). The one giving maximum C_m is the θ_c.

2. C_m is differentiated with respect to θ and set equal to zero. This gives:

$$\theta_c = \frac{1}{2}(B + \phi_m)$$

If we substitute θ_c into equation (4-1),

$$C_m = \frac{\gamma H\,[1 - \cos(B - \phi_m)]}{4 \sin B \cos \phi_m} \qquad (4\text{-}2)$$

and,

$$H = \frac{4 C_m \sin B \cos \phi_m}{\gamma\,[1 - \cos(B - \phi_m)]} \qquad (4\text{-}3)$$

The factor of safety of the slope is determined as follows:

F.S with respect to friction = $\dfrac{\tan \phi}{\tan \phi_m} = F\phi$.

F.S with respect to cohesion = $\dfrac{C}{C_m} = F_c$

Instead of determining the maximum height from equation (4-3 a graphical procedure may be followed:

1. A point A is taken to represent the toe of the slope.

2. A Line at an angle ϕ_m with the horizontal is drawn throu A.

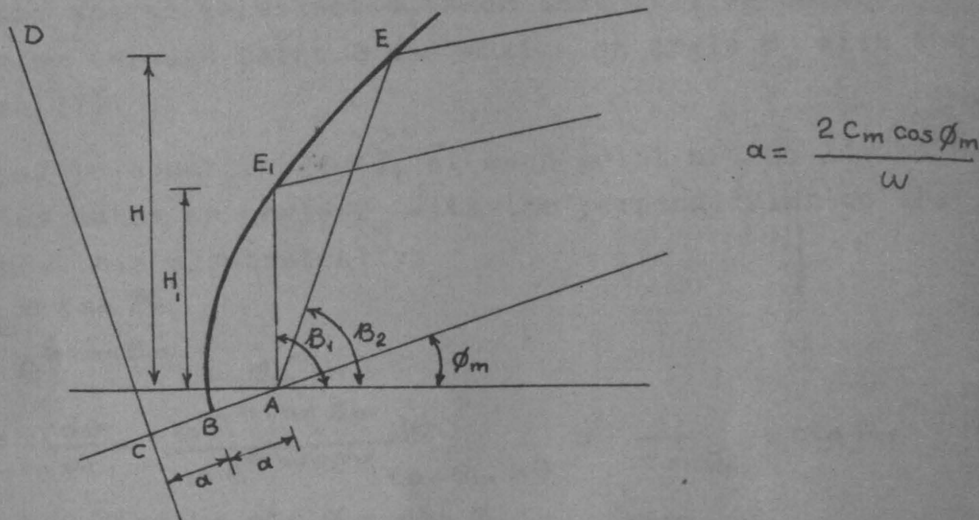
3. On this line point B is marked at a distance of $\dfrac{2C_m \cos\phi_m}{\gamma}$ from A, and point C is marked at a distance of $\dfrac{2C_m \cos\phi_m}{\gamma}$ from B. The scale used atthis step is the overall scale for the graph.

4. A perpendicular CD is drawn from point C.

5. A parabola is constructed, point A being the focus and line CD being the directrix.

6. A line AE at an angle B with the horizontal is drawn fro

point A. The distance between point E and the horizontal is the maximum safe H.

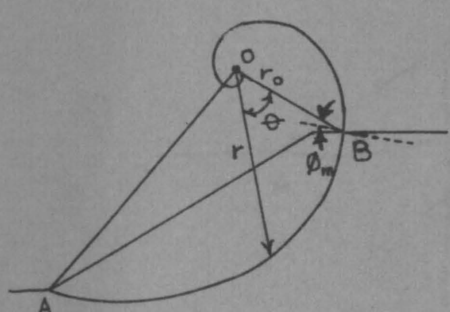


(Figure 16)

4.2. LOGARITHMIC SPIRAL FAILURE SURFACE:

The main assumption is that the maximum stressed surface is a logarithmic spiral, having the equation

$$r = ro^{\theta \tan \emptyset m}$$



(Figure 17)

θ = variable angle between r and ro vectors.

r = variable radius of the logarithmic spiral.

ro= radius vector at the beginning point of the arc BA.

$\emptyset_m$= Angle between radius and normal of the spiral. It is chosen such that it is equal to the angle of shearing resistance of the soil.

The forces present are the weight W of the soilmass acting through the centroid, the effective forces dP and the cohesive forces C along the failure surface or their resultants P and C. (Figure 18)

To apply the method, we begin by choosing an arbitrary point B on the horizontal plane, which represents the point where the logarithmic spiral cuts the horizontal. Through point B and Point A at the toe of the slope a logarithmic spiral AB is drawn. The center O of the logarithmic spiral is selected, such that it lies on a straight line passing through point B and making an angle ϕ_m with the horizontal. (Figure 17)

According to equation (4-4), at each point of the logarithmic spiral the radius makes an angle ϕ_m with the perpendicular to the spiral. We can show this algebraically:

$$r = r_o \, \epsilon^{\theta \tan \phi_m}$$

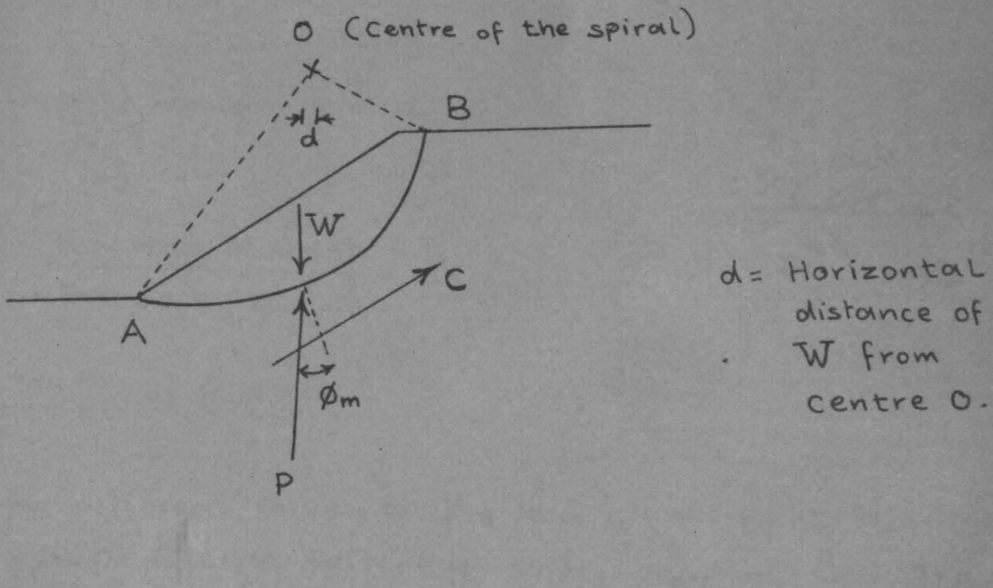
$$dr = r_o \, \epsilon^{\theta \tan \phi_m} \tan \phi_m \, dr$$

$$\tan \psi = \frac{r d\theta}{dr} = \frac{r_o \, \epsilon^{\theta \tan \phi_m} \, d\theta}{r_o \, \epsilon^{\theta \tan \phi_m} \tan \phi_m \, d\theta} = \frac{1}{\tan \phi_m} = ctg \, \phi_m$$

$\tan \psi = \tan(\alpha + 90) = ctg \, \alpha = ctg \, \phi_m$, and

$\alpha = \phi_m$, α being the angle between radius and normal.

The total effective force P is always at an angle of ϕ_m to the failure surface. Since this is also the angle for the radius, it means that the P force acts along theradius and through the center of the logarithmic spiral. (Figure 18)



(Figure 18)

We can find the factor of safety of the slope as a râtio of the moments about O, caused by resisting and disturbing forces. The disturbing force is W and the resisting forces are P and C.

Total effective force P passes through the center, therefore it does not cause any moment about O. Restoring moment is ~~is~~ caused only by cohesion.

When we consider a unit element ds along the logarithmic spiral the cohesion along it is cds. (Figure 19)

Therefore, the restoring moment about O is:

$$dM_c = c\, ds \times r \cos \phi$$

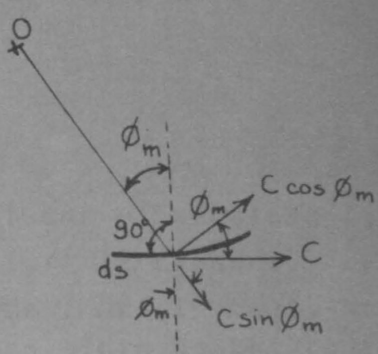
$$dM_c = rc \frac{r d\theta}{\cos\phi} \cos \phi = cr^2 d\theta$$

$$M_c = \int_0^\theta c r^2 d\theta = \int_0^\theta c r_o^2 e^{2\theta \tan\phi} d\theta$$

$$= \frac{c r_o^2}{2 \tan\phi} e^{2\theta \tan\phi} \Big|_0^\theta = c \left(\frac{r^2 - r_o^2}{2 \tan\phi} \right)$$

$$M_c = c \frac{(r - ro)}{2 \tan \phi} \qquad \text{(4-5)}$$

(Figure 19)



The disturbing moment is Wxd

(Figure 18)

$$\text{Factor of safety} = \frac{\text{Restoring Moment}}{\text{Disturbing Moment}}$$

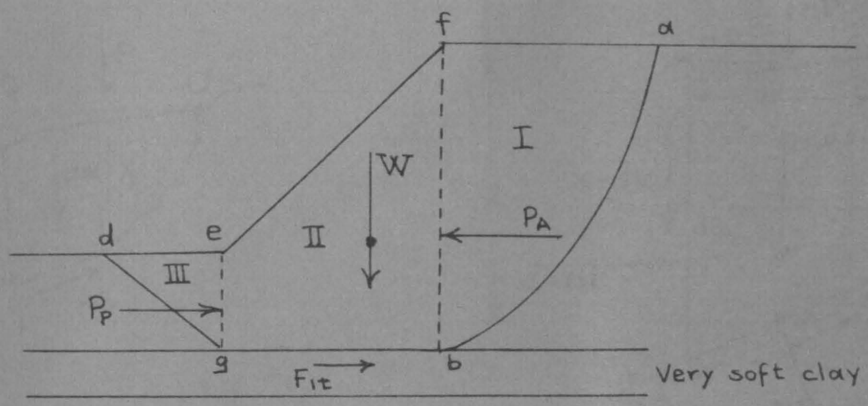
$$= \frac{c(r^2 - r_o^2)/2 \tan \phi}{W d} \qquad \text{(4-6)}$$

By assuming different values of ϕ_m and r_o, different logarithmic spirals and as a result different factors of safety may be found. The lowest factor of safety represents the maximum stressed logarithmic spiral. In case of failure it will become the failure surface of the slope.

4.3. COMPOSITE FAILURE SURFACES

In the methods dealt so far, althought a stratification of soil was allowed, the soils in adjacent strata were not very much different from each other, so that the failure surface could be a continuous curve.

But if for example one stratum is very thin and weak with respect to the others, part of the sliding will occur along the boundary of the weak stratum. (Figure 20). Therefore, the failure surface consists of a number of sections, joining each other in an abrupt way:



(Figure 20)

Figure (20) shows that the failure conditions of sections I, II and III are different. In section I active failure is present. By assuming a point b, where the boundaries of sections I and II meet, active earth pressure P_A can be determined. Section II is pushed to the left by the active pressure P_A. This movement is resisted by the passive earth pressure, cohesion and frictional resistance. By assuming a point g, where the boundaries of sections II and III meet, passive earth pressure P_p too can be determined.

The factor of safety against sliding is equal to the ratio between the sum of resisting forces and P_A.

$$F = \frac{\Sigma P_p + F_{it} + C\ (bg)}{P_A} \quad \text{------(4-7)}$$

P_p = passive pressure

P_A = active pressure

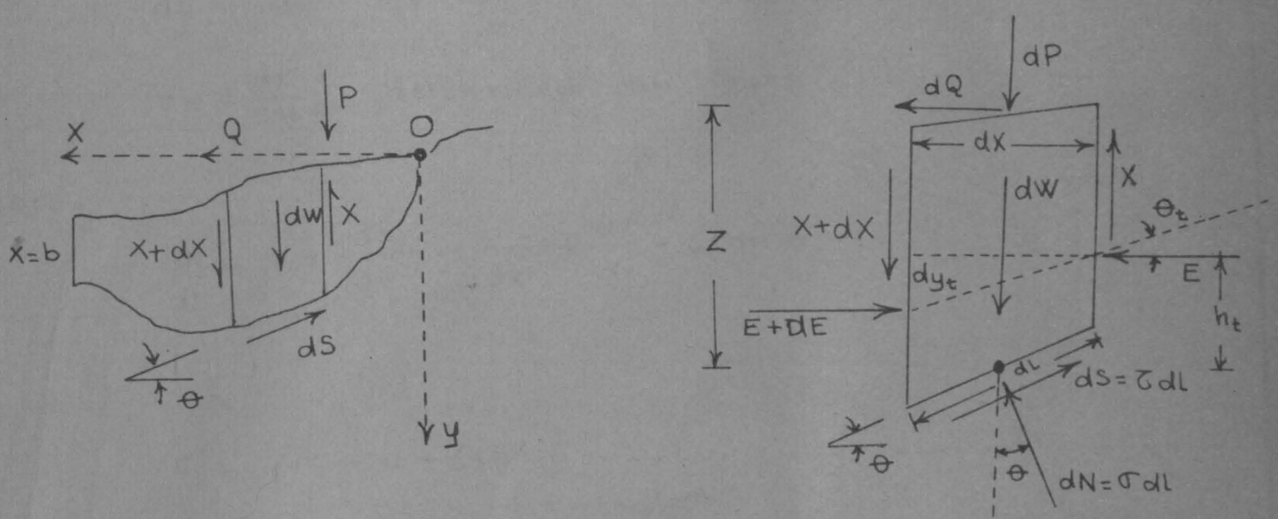
c = cohesion

Fit= frictional resistance.

By different assumptions of points g and b, different E_a values may be found. The smallest one determines the critical surface.

For the stability analysis of composite surfaces another, more elaborate method may be followed. Since the composite surface may be of any shape, a generalized slices method should be applicable to it. At the general slices method no simplifying assumptions are made with respect to the assumed sliding surface. Therefore it is applicable for any shape of the sliding surface.

As we have done in numerical slices method, the soil mass above the sliding surface of general shape is separated into vertical slices. (Figure 21).



(Figure 21)

If we isolate a slice, the following equations of equilibrium are present:

In vertical direction: $dW + dP + dX - dS \sin\theta - dN \cos\theta = 0$ ---- (4-8)

In horizontal direction: $dE - dQ + dS \cos\theta - dN \sin\theta = 0$ ---- (4-9)

Moment of the forces about the midpoint of the bottom of the slice is: $X\,dx + E\,dy_t - dE\,h_t + dQz = 0$ ---- (4-10)

For the stability criterion the overall equilibrium condition in the horizontal direction is used.

When we look at equation (4-9), except the dE force, forces dN and dS are unknown too. So, we should begin the procedure by ermining the unknowns first:

σ' is the effective normal stress and it is equal to:

$$\sigma' = \frac{dN}{dl} - u \quad \text{-------------------------------------} (4-11)$$

dN found from equation (4-8) is:

$$dN = \frac{dW}{\cos\theta} + \frac{dP}{\cos\theta} + \frac{dX}{\cos\theta} - dS\,\frac{\sin\theta}{\cos\theta} \quad \text{-----------} (4-12)$$

$$\frac{dN}{dl} = \frac{dW}{dl\cos\theta} + \frac{dP}{dl\cos\theta} + \frac{dX}{dl\cos\theta} - \frac{dS}{dl}\tan\theta \quad \text{----} (4-13)$$

$$dS = \tau\,dl \quad \text{and} \quad \frac{dS}{dl} = \tau \quad \text{------------------} (4-14)$$

$$dl = \frac{dx}{\cos\theta}$$ determined from Figure (21)

herefore,

$$\frac{dN}{dl} - u = \frac{dW}{dx} + \frac{dP}{dx} + \frac{dX}{dx} - \tau\tan\theta - u$$

$$\sigma' = p + t - u - \tau\tan\theta \quad \text{-----------------------} (4-15)$$

The shear stress developed at the bottom of the slice is:

$$\tau = c' + \sigma'\tan\varnothing' \quad \text{-------------------------------} (4-16)$$

$$\tau = c' + (p+t-u)\tan\varnothing' - \tau\tan\theta\tan\varnothing'$$

$$\tau(1+\tan\theta\tan\varnothing') = c' + (p+t-u)\tan\varnothing'$$

$$\tau = \frac{c' + (p+t-u)\tan\varnothing'}{(1+\tan\theta\tan\varnothing')} \quad \text{--------------------} (4-17)$$

Since the unknown forces dN and dS can be determined now by using equations (4-15) and (4-17), we return to our main problem of determining the stability in horizontal direction.

dE found from equation (4-9) is:

Equation (4-17) already gives the value of the developed (mobilized) shear. Therefore,

$$\tau_f = F\tau = F \frac{c + (p+t-u)\tan\phi'}{(1+\tan\theta\tan\phi')} \quad \text{------------------------}(4\text{-}21)$$

From equation (14), $\sum \tau \cos^{-2}\theta \, \Delta X = Q - E_b + \sum (p+t)\tan\theta \, \Delta X$

Therefore,

$$F = \frac{\sum \tau_f \cos^{-2}\theta \Delta X}{Q - Eb + \sum (p+t)\tan\theta \Delta X} \quad \text{------------------------}(4\text{-}22)$$

This is the factor of safety equation for a sliding surface of general shape.

Like the horizontal force E, also the vertical force X can be determined. From equation (4-10):

$$X\,dx = -E\,dyt + dE\,h_t - dQ\,z$$

$$X = -E\frac{dyt}{dx} + h_t\frac{dE}{dx} - z\frac{dQ}{dx}$$

$$X = -E\tan\theta_t + h_t\frac{dE}{dx} - z\frac{dQ}{dx} \quad \text{------------------------}(4\text{-}23)$$

The resultant force at the boundary face is

$$\int_0^b dX = X_b \quad \text{------------------------}(4\text{-}24)$$

In equations (4-21) and (4-22) a term $t = \frac{dX}{dx}$ is present when the actual stress condition is unknown, t becomes statically indeterminate. By assuming a reasonable position of the line of thrust, equations (4-18) and (4-23) give quite accurate values of E and X by means of successive approximations.

For example, first of all t=0 is assumed. This value substituted into ~~i the~~ equations (4-18) and (4-23), initial E and T

$$dE = dQ - dS\cos\theta + dN\sin\theta$$

$$dN = \frac{p+t}{\cos\theta}\,dx - dS\tan\theta$$

$$dE = dQ - dS\cos\theta - dS\cos^2\theta + (p+t)\,dx\sin\theta - dS\sin^2\theta$$

$$dE\cos\theta = dQ\cos\theta - dS + (p+t)\,dx\sin\theta$$

$$dE = dQ - \frac{dS}{\cos\theta} - (p+t)\,dx\tan\theta$$

$$dS = \tau\,dl = \tau\frac{dx}{\cos\theta}$$

$$dE = dQ - \tau\,dx\cos^{-2}\theta + (p+t)\,dx\tan\theta \quad \text{---------} \quad (4\text{-}18)$$

The resultant horizontal force at the boundary face $X=b$ is expressed as:

$$E_b = \int_0^b dE \quad \text{------------------------------} \quad (4\text{-}19)$$

$$E_b = Q - \int_0^b \tau\cos^{-2}\theta\,dx + \int_0^b (p+t)\tan\theta\,dx \quad \text{-----} \quad (4\text{-}20)$$

Equation (4-20) is the overall equilibrium condition in the horizontal direction.

After equation (4-20) is determined, the factor of safety analysis can be considered.

$$F = \frac{\text{available shear}}{\text{shear mobilized}} = \frac{\tau_f}{\tau}$$

values, ie: Eo and Xo are found. Xo value introduced into t term give E_1 and X_1 etc.

$$t=o \longrightarrow Eo, To$$

$$to = \frac{dTo}{dx} \longrightarrow E_1, T_1$$

$$t1 = \frac{dT_1}{dx} \longrightarrow E_2, T_2 \qquad \text{etc.}$$

When it is possible to estimate X in advance, the application is direct and there is not need for successive approximations.

5. THE IMPORTANCE OF PORE-PRESSURE FOR THE STABILITY OF SLOPES

When soil is saturated, its voids are filled with water. Soil and water particles both are incompressible materials, and for an increase in all round pressure a total volume change can occur only when drainage is present. If there are no drainage possiblities, some hydrostatic pressure is created in water, which is called pore-pressure.

The observed or predicted pore-pressure distribution and its change are very important for the long-term stability of slopes and earthdams. Therefore, these have to be included in a stability analysis.

Tests have shown a close relation between the factor of safety and the corresponding pore-pressure. As a close approximation it can be said that for a simple soil profile and specified strength parameters the factor of safety F varies linearly with the magnitude of the pore-pressure.

In order to present the results of stability analysis in a dimensionless way, the pore-pressure is expressed in terms of pore-pressure ratio r_u.

$$r_u = \frac{U}{\gamma h} \quad ---- (5-1)$$

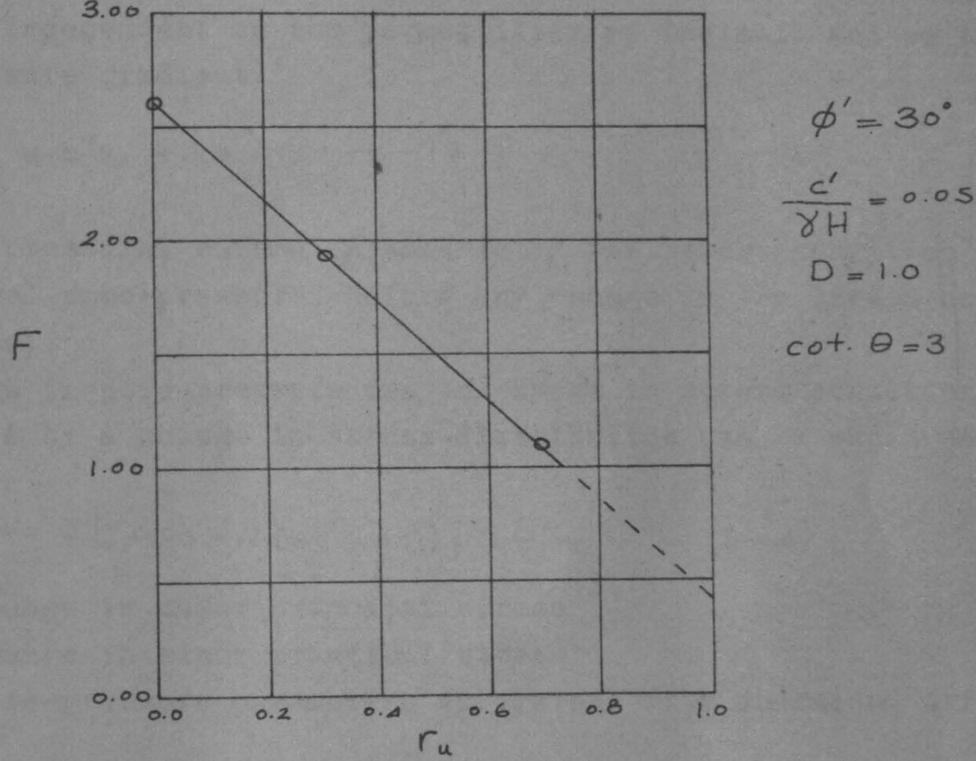
h= depth of the considered point
γ= bulk density of soil.

Especially for the range of r_u values between 0 and 0.7, which are the most often used values in practice, the linear relationship is followed almost exactly. Therefore,

$$F = m - n r_u \quad ---- (5-2)$$

m and n being the stability coefficients changing with slope and with soil properties.

A.W. Bishop and N.Morgenstern have shown the linear relationships between F and r_u graphicall. (Figure 22)



(Figure 22)

In equation (5-2) F and r_u are the variables, (The coordinate axes in Figure 22) m is the intersection of the line with the F-axis and n is its slope. When a zero pore-pressure case is present, the factor of safety is equal to m value. The slope isalways negative, because it represents the fact that all other parameters remaining constant, an increase of pore-pressure causes a decrease of the factor of safety.

After showing the general effect of pore-pressure on the factor of safety by means of equation (5-2) and Figure (22), some information should be given on the pore-pressure value u too.

The existing pore-pressure problems are of two types.

1) The pore-pressure is an independent variable, and it is controlled by groundwater level or by the flow pattern of the underground water. The pore-pressure distribution may be found by piezometer measurements, from flow-net or from the differential equation of steady flow of water through soils.

2) The pore-pressure depends on the stress-condition in the soil. For example rapid construction or excavation of soils having low

values of permeability cause stresses, creating instability in the soil. The pore-pressure is always changing with time to adjust itself to ultimate equilibrium with existing conditions. The rate of change of pore-pressure isdependent on the permeability of the soil and on the excess pore-pressure gradient.

$$u = u_0 + \Delta u \quad - - - - (5\text{-}3)$$

u = pore-pressure, at any time change in the stress condition of the soil

u_0 = initial pore-pressure, before any change in the stress condition of the soil

Δu = Change in pore-pressure due to change in stress conditions.

Δu caused by a change in stress-distribution can be shown as,

$$\Delta u = B\left[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)\right] \quad - - - - (5\text{-}4)$$

$\Delta\sigma_1$ = change in major principal stress

$\Delta\sigma_3$ = change in minor principal stress

A,B = Pore-pressure parameters determined from undrained triaxial test.

According to the original statement the change in pore-pressure can be related to the major principal stress as:

$$\frac{\Delta u}{\Delta\sigma_1} = B\left[\frac{\Delta\sigma_3}{\Delta\sigma_1} + A\left(1 - \frac{\Delta\sigma_3}{\Delta\sigma_1}\right)\right] = \bar{B} \quad - - - - (5\text{-}5)$$

A.W. Bishop and N.Morgenstern have shown the variation of pore-pressure parameter $\bar{B}$ with principal stress ratio and major principal stress graphically. (Figure 23)

$$\Delta u = \bar{B}\,\Delta\sigma_1 \quad - - - - (5\text{-}6)$$

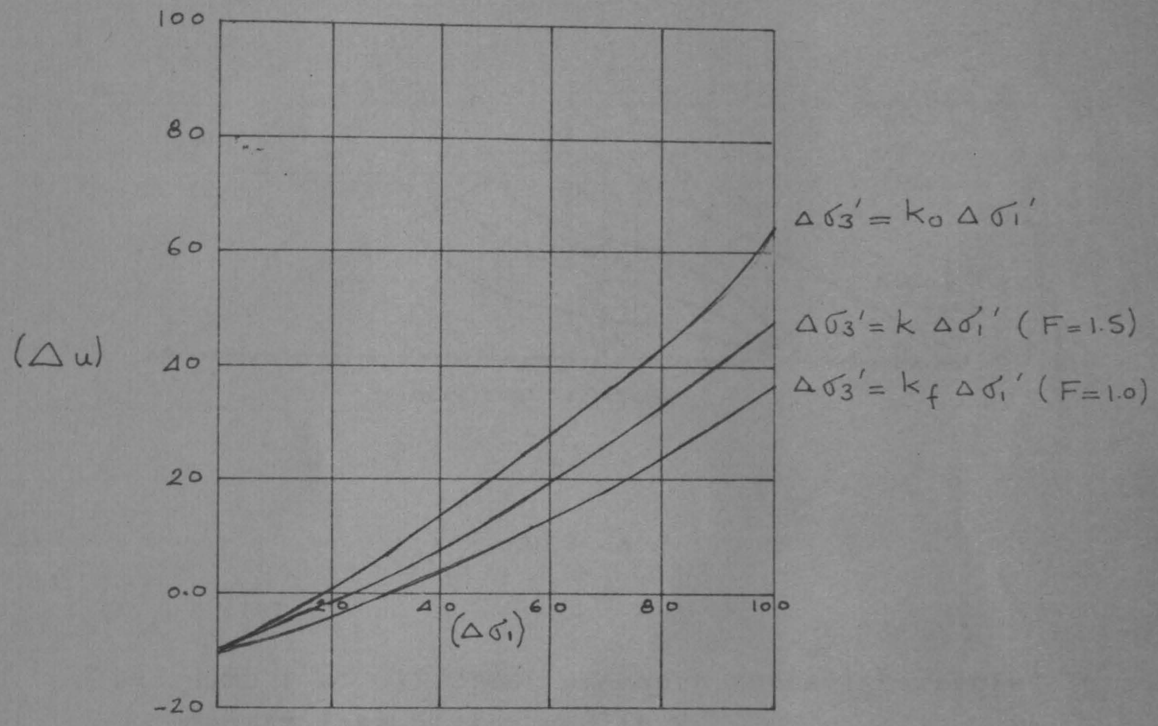
Therefore, from equation (1),

$$r_u = \frac{u_0 + \bar{B}\,\Delta\sigma_1}{\gamma h} = \frac{u_0}{\gamma h} + \bar{B}\,\frac{\Delta\sigma_1}{\gamma h} \quad - - - - (5\text{-}7)$$

The major principal stress at a point in an earthfill may be taken as equal to the weight of soil above it, ie: γh.

Therefore,

$$r_u = \frac{U_o}{\gamma h} + \bar{B} \quad ---- (5-8)$$



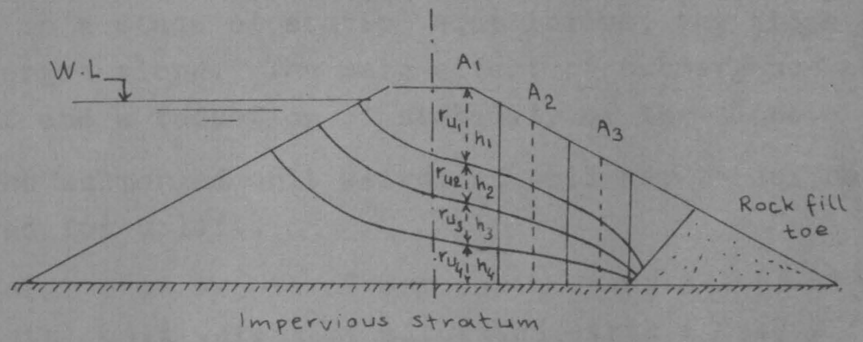
(Figure 23)

For general solutions mostly a homogeneous pore-pressure distribution is assumed and r_u is taken as constant throughout the cross section. But in reality the pore-pressure ratio r_u is not constant and in this case the use of an average r_u value gives quite satisfactory results. This possibility makes it possible to apply stability coefficients to the steady seapage case.

The method can be shown on a cross-section of an earthdam with different values. (Figure 24)

Section Area average $r_u = \dfrac{\sum_1^i h_i r_{u_i}}{\sum_1^i h_i}$ ---- (5-9)

Overall average $r_u = \dfrac{\sum_1^n A_n r_{u_n}}{\sum_1^n A_n}$ ---- (5-10)



(Figure 24)

i= number of different pressure zones intersected by the centre line of the sections.

n= number of sections.

6. THE EFFECT OF SUBMERGENCE

When a slope is under water, such that the water contained in its pores is in a state of static equilibrium, the slope is considered to be a submerged slope. The main effect of submergence on soil is an uplift effect and a reduction of stability of the slope.

The submerged unit weight of soil may be defined as its unit weight reduced for uplift.

Total unit weight of dry earth $= \gamma_{s+a} = (1-n)\gamma_s = \gamma$ ------ (6-1)

Total unit weight of saturated earth $= \gamma_{s+w} = (1-n)\gamma_s + n\gamma_w = \gamma$ ------ (6-2)

Submerged unit weight of saturated earth $= \gamma_{s+w-u} = (1-n)(\gamma_s - \gamma_w) = \gamma'$ ------ (6-3)

γ_w = specific gravity of water
γ_s = specific gravity of soil
n = porosity of soil

Therefore, in stability analysis the effective unit weight of soil should be taken as its submerged unit weight. The net effect of water pressure should be considered separately and added to the analysis.

The main factor causing submergence is the presence of a water table. Now, let us see how this can influence the stability of a slope:

1) When the ground-water level rises in the dry crust, the weight of any slice of the slope is increased and as a result the stability is somewhat reduced. But this is a negligible effect. For example, at Surte slide the effect of the increase in weight was found to be 0.2% of the weight of slice.

2) The total normal force acting on the slip surface is influenced by the water table. But this effect too, is negligible.

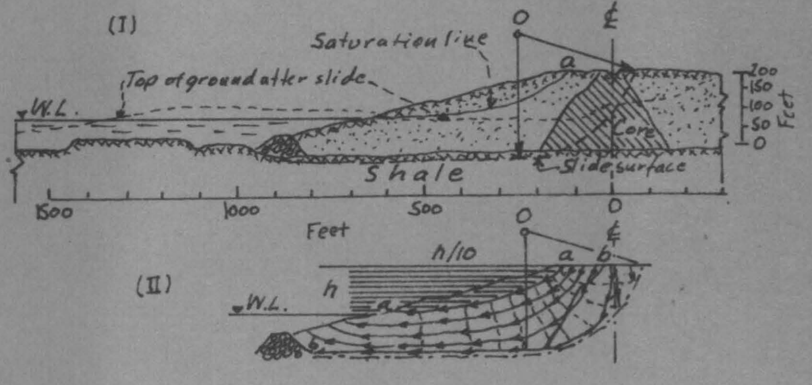
3) The main effect is that the shear force mobilized acting on the slip surface is very much influenced by the presence of the ground-water. For a high ground-water pressure, there develops a low effective pressure on the slip surface and the shear strength mobilized will be low too.

Takingthe stated effects into account, it can be said that submergence generally reduces the factor of safety of a slope.

7-THE EFFECT OF SEAPAGE

The gravitational flow of water through soil creates some forces in the soil, which are called seapage forces. Their direction is parallel to flow of watrr and they are proportional to the hydraulic gradient. As water always tends to bring itself to an equilibrium position, after a sudden draw-down it is to be expected that seapage will occur inside the slope.

The most effective way of determining the seapage forces is by means of constructing a flow net. The use of this principle can be explained best by means of Figure 26. It shows the general profile and the probable flow net prior to failure.



(Figure 26)

The saturation line is assumed to represent the upper stream line a-a. The sliding surface is taken to represent the lower stream line. The points from where the flowlines pass may be determined by means of the graphical procedure shown in Figure. 26

After the entire flownet is drawn the seapage force of each rectangle of the flownet can be estimated. From this estimation the increment of the overturning moment can be determined. When the increments of the overturning moments for the entire flownet are added, the total overturning moment due to seapage increases.

In general it can be said that the presence of seapage has an adverse effect on the stability of a slope.

8. SOME INFORMATION ON THE INTRODUCTION OF DIGITAL COMPUTER TO THE SLOPE STABILITY ANALYSIS

As it is seen up to now, the stability of a slope is governed by the minimum factor of safety, but to find the slip circle with the minimum factor of safety is a repetitive task. As explained earlier the factor of safety can be determined easily by means of stability charts for specified conditions. But nature almost never presents limited conditions. The problem is generally complicated by the presence of a number of soils and water table, so that those ideal charts can not be used in every case.

The repetitive pattern of stability analysis can beeasily handled by a digital computer and once the main programme is read into computer, it isready to solve any number of problems successively.

Forthe solution of a problem these main points should be followed:

1. The surface and strata division lines are specified by the coordinates of a number of points.

2. Two points chosen on the rigid base, are used for the calculation of the maximum radius. Also, another point is specified, which is used for the computation of a minimum radius.

3. A water table may be put anywhere under the condition that it is horizontal and at the same level both inside and outside the soil.

4. A number of uniformly distributed loads may be placed anywhere on the surface.

5. The cohesion and friction angles may be different for the soil strata present. The only condition is, that the cohesion must be less than 1 000 000 units and the friction angle must be less than 45°.

6. The cohesion may vary with depth by a $\frac{C}{P}$ ratio, which is variable.

7. A pore pressure coefficient $\bar{B}$ can be specified.

8. Any units may be used for parameters. But they have to be consistant under each other.

9. For each center point there are a number of failure circles. The computer choses such a radius, that the factor of safety,

for the corresponding center point is a minimum. After this, using a special searching technique the computer determines the center point, with minimum factor of safety among all center points. So, the center point and the radius with minimum factor of safety is determined. The process will occur faster, if the initial point of trial is given to the computer.

In order to present a slope stability method to the computer, some basic assumptions in the subject are necessary.

1. All forces acting on the sides of the slices are in equilibrium.

2. The failure surface is the arc of a circle.

3. If more than one soil are present, the unit weights are assumed to be the same.

4. The factor of safety is the ratio between resisting and overturning moments.

5. The resisting moment is due to the shear strength of soil and the overturning moment is due to the weight of the soil mass.

$$M_R = \text{Resisting Moment} = \int_A^B R\, s\, ds = \int_A^B R^2 s\, d\alpha \quad \text{--------------- (8-1)}$$

$$= \int_A^B [c + (p-u)\tan\phi]\, R^2 d\alpha \quad \text{--------------------------- (8-1a)}$$

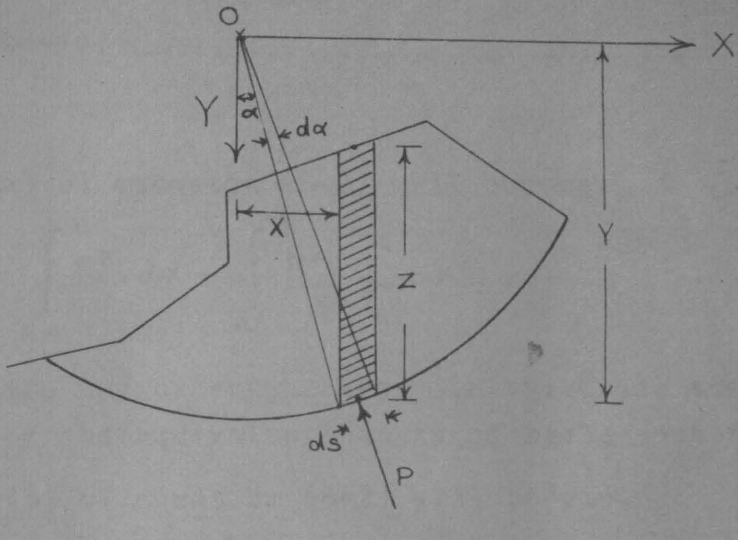
$$P = \text{Normal stress} = \gamma z \cos^2\alpha$$

$$M_R = \int_A^B R^2 c\, d\alpha + \int_A^B (\gamma z \cos^2\alpha)\tan\phi\, R^2 d\alpha - \int_A^B u \tan\phi\, R^2 d\alpha$$

$$R\cos\alpha = Y \qquad\qquad R\cos\alpha\, d\alpha = dx$$

$$R^2 d\alpha = R\, ds = R^2 \frac{dx}{Y}$$

$$M_R = \underbrace{\int_A^B \frac{R^2 c}{Y} dx}_{a} + \underbrace{\int_A^B \gamma \tan\phi\, Y z\, dx}_{b} - \underbrace{\int_A^B \frac{uR^2}{Y} \tan\phi\, dx}_{c} \quad \text{---------- (8-2)}$$



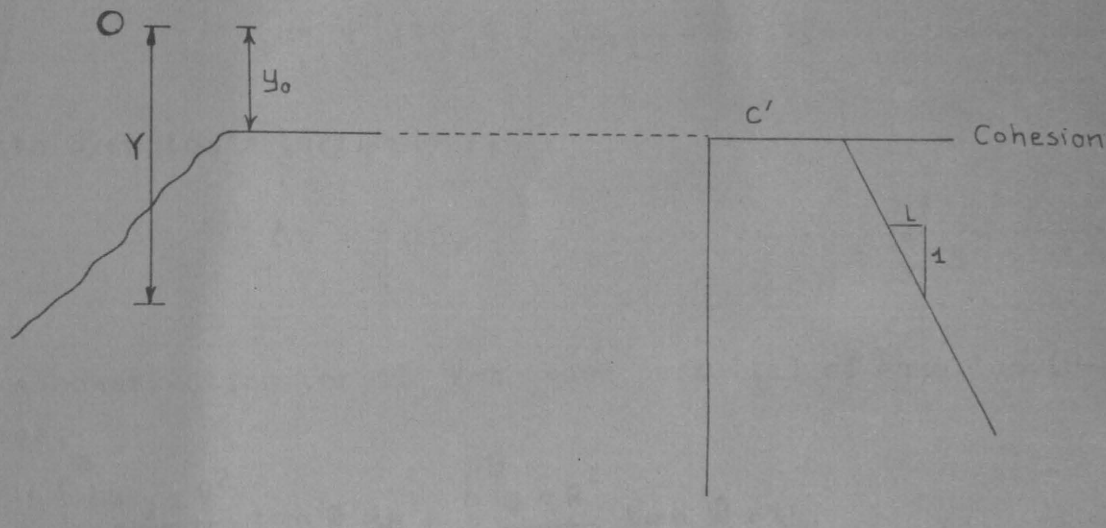
(Figure 27)

$$M_o = \text{Overturning moment} = \int_A^B \gamma z x \, dx \quad \text{------------------------ (8-3)}$$

6. Loads on the surface are carried vertically downwards into soil, with no lateral effects.

7. The shear strength of soil is equal to:
$s = c + (p-u) \tan \phi$

8. The cohesion may increase with depths (Figure 28).



(Figure 28)

In such a case forany point of the failure circle,

$$c = c + L\,(Y - y_o) \qquad (8-4)$$

$$L = \left(\frac{C}{p}\right)\gamma$$

Therefore, part (a) of equation (8-2) will become:

$$\int_A^B \frac{R^2 c'}{Y}\,dx + \int_A^B R^2 L\,dx - \int_A^B \frac{R^2 L}{Y}\,y_o\,dx$$

9. There may be surcharges: In this case the surcharge may be replaced by and equivalent height of soil, such that $z = z' + \frac{q}{\gamma}$

Therefore, part (b) of equation (8-2) will become,

$$\int_A^B \gamma \tan\phi\; Y z'\,dx + \int_A^B q \tan\phi\; Y\,dx$$

and equation (8-3) will become

$$M_o = \int_A^B \gamma z' X\,dx + \int_A^B q X\,dx$$

10. Effect of water table and pore pressures:

a. If no water table is present, the pore pressure at any point is a function of vertical pressure.

$$u = \Psi(\gamma z + q) \qquad (8-5)$$

or with Skempton's terminology,

$$\Delta u = B\,\Delta\sigma_v \qquad (8-6)$$

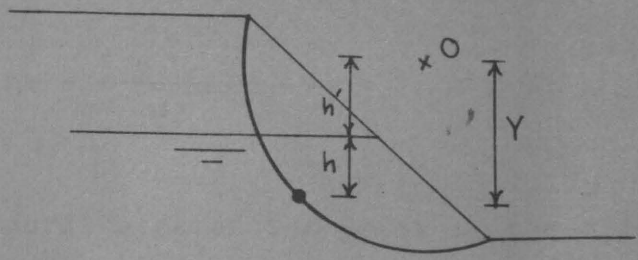
At the construction period $\Psi = B$ and part (c) of equation (8-2) becomes,

$$\int_A^B \frac{\Psi \gamma z R^2}{Y} \tan\phi\,dx + \int_A^B \frac{\Psi q R^2}{Y} \tan\phi\,dx$$

Only the resisting moment is effected by the pore pressures.

b. If water table is present, there happen two different effects.

The first effect is a decrease in shear strength and resisting moment.



(Figure 29)

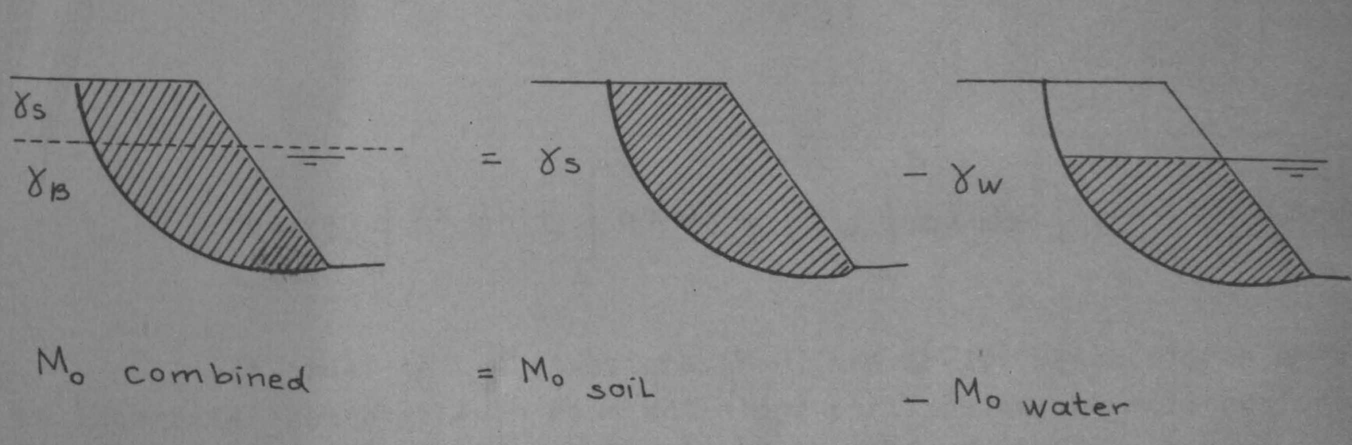
When we look at Figure (29), the pore pressure at any point below the water table is:

$$u = h\gamma w = (Y - h')\gamma w \quad \text{------------------------------} \quad (8-7)$$

Therefore part (c) of equation (8-2) will become:

$$\int_A^B R^2 \gamma_w \tan\phi \, dx - \int_A^B \frac{h' \gamma_w R^2}{Y} \tan\phi \, dx$$

The second effect is a lowering of the unit weight of soil by the buoyeant unit weight. The result of this will be a change in the overtumning moment. (Figure 30)



(Figure 30)

11. A rigid base may be present : in such a case automatically a maximum radius is specified for each center point, since the failure circle can not enter the rigid base. If in such a case the coordinate axes are considered to be located at the center point of critical circle the maximum radius will be given by the equation:

$$R_{max} = \frac{(y_1 - m\lambda_1)}{\sqrt{1 + m^2}} \quad \text{------------------------------------} (8-8)$$

λ and Y are the coordinates of the point on the rigid base and m is the slope of the rigid base.

12. Tension cracks: A soil with tension cracks can be represented by a soil strata having weight, but no shear strength.

After taking all specified points into consideration we get the general resisting and overturning moment equations.

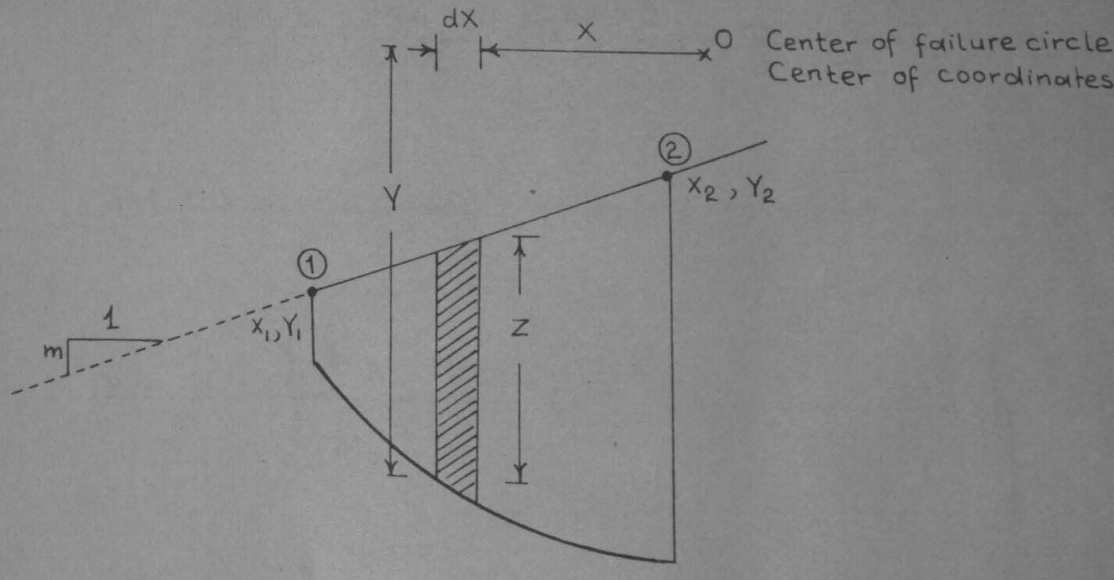
$$M_R = R^2 c \int_A^B \frac{1}{Y} dx + R^2 L \int_A^B dx - R^2 L y_o \int_A^B \frac{1}{Y} dx + \gamma \tan\phi \int_A^B YZ\, dx$$

$$+ \tan\phi \int_A^B qY\, dx - \Psi \gamma R^2 \tan\phi \int_A^B \frac{Z}{Y} dx - \Psi R^2 \tan\phi \int_A^B \frac{q}{Y} dx$$

$$- \left[R^2 \gamma_w \tan\phi \int_A^B dx - R^2 \gamma_w h' \tan\phi \int_A^B \frac{1}{Y} dx\right] \quad \text{------} (8-9)$$

$$M_o = \gamma \int_A^B zx\, dx + \int_A^B qx\, dx - \left[\gamma_w \int_A^B zx\, dx\right] \quad \text{-----------} (8-10)$$

In order to use equations (8-9) and (8-10) we should be able to integrate them over each constant slope portions of the soil surface. For this purpose the variables should be modified somewhat.



(Figure 31)

As shown in (Figure 31) points (1) and (2) are 2 points on a constant slope part of the surface. The partial overturning moment belonging to this section is:

$$M'_R = R^2 c\,\theta' + R^2 L\,(X_j - X_i) - R^2 L\,y_o\,\theta' + \gamma \tan\phi \Big[R^2 (X_j - X_i) - (X_j^3 - X_i^3) + \frac{m}{3}(A) + \frac{mX_i - y_i}{2}(B + R^2\theta')\Big] + q\,\frac{\tan\phi}{2}\,\big[B + R^2\theta'\big]$$
$$- \Psi q \tan\phi\, R^2 \phi' - \Psi\gamma \tan\phi\, R^2 \big[(X_j - X_i) + (mX_i - y_i)\theta' + mD\big]$$
$$- \big[R^2 \tan\phi\, \gamma_w (X_j - X_i) - R^2 \phi' \tan\phi\, \gamma_w h\big] \text{ ------------ } (8\text{-}11)$$

$$A = (R^2 - X_j^2)^{3/2} - (R^2 - X_i^2)^{3/2}$$

$$B = X_j (R^2 - X_j^2)^{1/2} - X_i (R^2 - X_i^2)^{1/2}$$

$$\theta' = \sin^{-1}\frac{X_j}{R} - \sin^{-1}\frac{X_i}{R}$$

$$D = (R^2 - X_j^2)^{1/2} - (R^2 - X_i^2)^{1/2}$$

And the partial overturning moment is equal to:

$$M_o' = \gamma \left[-\frac{A}{3} + \frac{m x_i - y_i}{2} (x_j^2 - x_i^2) - \frac{m}{3} (x_j^3 - x_i^3) \right] + \frac{a}{2} (x_j^3 - x_i^3) \quad \text{--------(8-12)}$$

And the overall factor of safety of the slope becomes:

$$F.S. = \frac{\sum_i^j M_R'}{\sum_i^j M_o'} \quad \text{-------------------------------- (8-13)}$$

9. DISCUSSION

It is seen up to now that the essential purpose of a stability analysis is to determine the critical surface and the factor of safety of the earthmass above the critical surface against sliding. The methods explained earlier assume plane, circular, logarithmic spiral and general shaped sliding surfaces. In order to come to a conclusion as to which approch has to be followed in a stability analysis, first the advantages and disadvantages of the existing methods should be stated.

The easiest approach in a slope stability analysis would be the assumption of a plane failure surface as in Culmann method. But there is an initial inaccuracy introduced in this method due to the error in its assumptions. As it is found out after studying a great number of actual slide data, the failure surface is a curved one, rather than being a plane. Espetially for relatively flatter planes this curvature becomes more pronounced. Therefore the use of a plane slip surface is only allowed for the case of steep slopes, where the actual curvature of the slip surface is not far from a straight line. This fact is stated numerically in Table 3. Here a comparison of $\beta=90^\circ$ and $\beta=15^\circ$ shows that the stability coefficient decreases very much for a flat slope.

β	ϕ_m	Culmann	Slices	ϕ circle	Log. Spiral
90°	0	0.250	0.261	0.261	0.261
	25	0.159	0.165	0.166	0.165
60°	0	0.144	0.219	0.219	0.219
	25	0.058	0.118	0.117	
15°	0	0.033	0.145	0.145	0.145
	10	0.004		0.023	

Table 3. (by Taylor)

The second reason why a plane sliding surface assumption is not justified comes from a consideration of probability. That is, whenever a large number of trials are possible ,the probability of hitting the right answer will be high too. When this principle is applied to the stability analysis the probability of coming closest to the critical surface will be large if the number of assumed trial planes is large too. But for the case of a plane failure surface the choice of reasonable trial planes is limited, compared to the other methods, so that the results will be on the unsafe side. This unsafety of the Culmann method can also be verified by means of Table 3. Here, the comparison of a plane sliding surface assumption with other methods gives the lowest factor of safety for the slope under consideration. Therefore a plane sliding surface assumption is not a justified way of solution and it should be used for only very steep slopes or for a fast analysis of the problem in order to get a general idea about the actual solution.

After a plane failure surface ,the second easiest assumption will be a circular failure surface. This assumption provides great mathematical simplicity and flexibility. As verified also by means of actual slide data this assumption is true ,especially for homogeneous slopes.

Bishop's numerical slices method is considered to be the most elaborate way of solving a stability problem with a circular failure surface assumption. Its main difference from other methods is that the effects of horizontal forces shown as En and En+1 in (figure 2) are taken into account, and this fact makes it more exact than the other ones. So, with respect to accuracy it is a good method but the calculation is too much time consuming and for a pratical Engineering purpose its application without any preliminary knowledge seems to be a lengthy job.

The lack of rapidity of the first method is overcome by the introduction of conventional method. It is an approach similar to Bishop's numerical Slices method, but with some simplifications in assumptions (shown in Fig 6). This is a rapid method, making a direct computation of factor of safety possible, by means of equations (3-21) and (3-22). But also some deficiencies are included in conventional method. For example the factor safety determined in this way is too conservative and leads to uneconomical design, especialy where deep slip surfaces are present, since the error changes with the central angle of the arc. (Figure 32) compares Conventionaland exact methods. As seen the Conventional method comes out to be always concervative with respect to factor of safety.

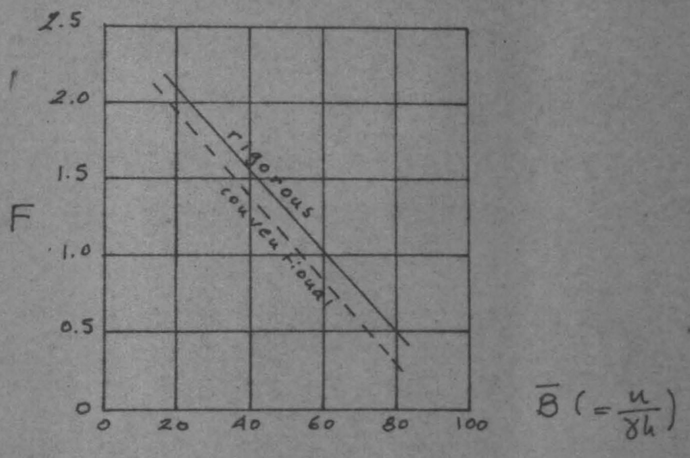


Figure 32

The effect of central angle on the factor of safety found by means of conventional method is shown in (figure 33). For the stated $\bar{B}$ values the ratio of $\frac{Fc}{Fr}$ decreases with increasing central arc.

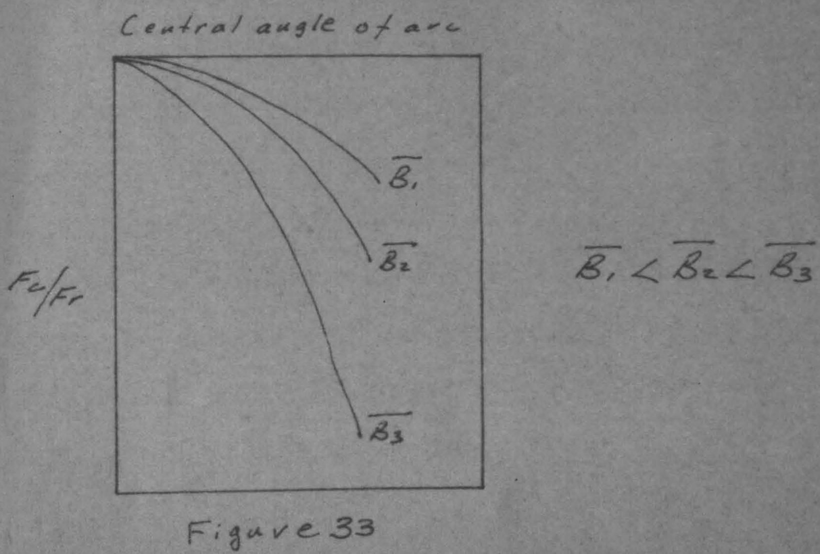


Figure 33

The third method using a circular failure plane is friction circle method. But also an initial uncertainty is present due to an assumption of the distribution of the normal stesses along the sliding surface. Since the exact distribution is unknown, an assumption for them should be introduced.

For the general shaped sliding surfaces the logarithmic spiral method may by used. Its advantage is that P-forces with obliquity Øm are directed towards the centre of the spiral. Therefore the analysis is statically determinate without an assumption relative to pressure distribution. But it has a disadvatage that the center of rotation is not definitely known.

When the failure surface consists of a number of shapes, the procedure to be followed is the generalized slices method. But some error may also be introduced due the assumption of the position of thrust.

10 - C O N C L U S I O N

As seen up to now, the most accurate and economical way of analysis is possible by means of Bishop's numerical slices method. But since it is very time consuming, to make many trials, the determination of the factor of safety is very difficult. Therefore such an approach may be suggested:

First a preliminary analysis can be done by means of friction circle method using weited average values of L, and C, and W, if stratification is present. After this initial determination of sliding surface, the trial values of Bishop's numerical slices method can be arranged accordingly, so that a few numbers of trials will be enough for the solution of the problem.

There is a trend in the slope stability analysis towards computer application to every problem encountered in practice. In order to echieve this goal, more importance should be laid to the develdpemont of methods used in computer analysis.

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