OPTIMIZATION OF LPG DELIVERY PROCESS FOR AN OIL COMPANY

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Submitted to the Institute for Graduate Studies in
Science and Engineering in partial fulfillment of
the requirements for the degree of
Master of Science

Graduate Program in Industrial Engineering

Boğaziçi University

2019

ACKNOWLEDGEMENTS

I would first like to express my gratitude to my thesis advisor Prof. Necati Aras for his endless support, patience, and encouragement. I have learned many precious things from him throughout the entire thesis study and this study would not end up without his endless guidance.

I would like to thank Prof. Ümit Bilge and Prof. Tülin Aktin for being a part of my thesis committee, and their valuable comments.

I would like to extend many thanks to all of my friends for their understanding and patience. I would like to thank Aylin, Yasemin Aylin, and Ruçhan for being encouraging and supportive, to Canan and Emre for being with me even in the most pessimistic moments, and to Mert Akbaş for being there whenever I need any help.

I would like to express my gratitude to my beloved parents and my brother Özgür; for their never-ending support and understanding throughout my life. I am very grateful for their existence.

ABSTRACT

OPTIMIZATION OF LPG DELIVERY PROCESS FOR AN OIL COMPANY

Science and technology, which have shown a great improvement in recent years, has driven companies to diversify their activities and to analyze their processes in order to sustain their existence. In this context, many companies which are operating in large geographic regions are conducting process analysis on supply chain management. The main reason behind is that supply chain activities consist of various decision-making mechanisms and any improvement will be effective on the overall system. In this study, the LPG delivery system is analyzed for an oil company that also active in Turkey. The system contains supplier selection and vehicle routes. In the current system, LPG procurement and delivery processes are carried out by two independent departments. There are annual agreements with suppliers and LPG price is the only concern for the procurement department. These suppliers are then shared with the distribution department. Since the location of suppliers are not considered, an extra transportation cost may occur. For this reason, a solution approach that integrates both procurement and distribution components are suggested. The number of suppliers, parking lots and stations yields us to a three-phase solution approach. In the first two stages, stations were clustered and suppliers and parking lots are assigned to each cluster with using mathematical models. In the last phase, an Inventory Routing Problem that works for 3-days period is created and routes are generated for each cluster.

ÖZET

BİR AKARYAKIT FİRMASI İÇİN LPG DAĞITIM SİSTEMİNİN OPTİMİZASYONU

Son yıllarda büyük bir gelişme gösteren bilim ve teknoloji, işletmeleri varlıklarını sürdürmek için çalışmalarını çeşitlendirmeye ve süreçlerini analiz etmeye yönlendirmiştir. Bu kapsamda, büyük coğrafi bölgelerde faaliyet gösteren birçok işletme tedarik zinciri yönetimi konusunda süreç analiz çalışmaları yapmaktadır. Bunun temel sebebi tedarik zinciri faaliyetlerinin birçok karar mekanizmasını barındırması ve yapılacak iyileştirmelerin daha kapsamlı olacağının düşünülmesidir. Bu çalışmada Türkiye'de faaliyet gösteren bir akaryakıt firmasının LPG dağıtım süreci tedarikçi seçimi, araç deposu seçimi ve LPG transfer süreçlerini de içerecek şekilde ele alınmaktadır. Mevcut sistemde tedarikçi seçimi ve LPG dağıtım süreçleri iki ayrı departman tarafından, birbirinden bağımsız bir şekilde yürütülmektedir. Senelik olarak yenilenen anlaşmalarla, sadece LPG fiyatı göz önünde bulundurularak tedarikçi seçimi yapılmaktadır. Sonrasında bu seçimler dağıtım planlama departmanıyla paylaşılmaktadır. Tedarikçi seçiminde dağıtım maliyetinin dikkate alınmaması planlanmayan ekstra maliyetlere sebep olmaktadır. Bu nedenle, bu iki sürecin kısıtlarını göz önünde bulunduran bir çözüm önerisi hedeflenmektedir. İşletmenin çalıştığı tedarikçi, araç deposu ve akaryakıt istasyonu sayısı problem üç aşamalı bir çözüm önerisi sunulmasına sebep olmuştur. İlk iki aşamada matematiksel modeller yardımıyla istasyonlar gruplanmış, tedarikçi seçimi yapılmıştır. Son aşamada ise bir Envanter Rotalama Problemi oluşturulmuş ve her küme için çalıştırılan bu model ile periyodik rotalar oluşturulmuştur.

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a	Capacity of a vehicle that serves for a particular cluster
b_l	Capacity of supplier l
c_i	Service point capacity
d_{kl}	Distance between cluster k and supplier l
d'_{lm}	Distance between supplier l and parking lot m
d''_{km}	Distance between cluster k and parking lot m
d_{ij}	Distance between point i and point j
F_{it}	A continuous variable for sub-tour elimination constraints
h_{it}	Demand of service point i at time period t
h_k	Demand of cluster k
$\overline{d}_{ij}$	Distance between service point i and service point j
$\overline{h}$	A big number
i	Service points
gi_{i0}	Initial inventory level of point i
G_{it}	Initial inventory level of point i
j	Service points
k	Clusters
l	Suppliers
m	Parking lots
n_c	Number of clusters
n_m	Number of trucks assigned to parking lot m
n'_m	Number of lorries assigned to parking lot m
p	Number of service points in a cluster
r_t	Amount of LPG available at the supplier in time period t
t	Time periods
U_{kl}	Binary variable for supplier cluster assignment
V_{kl}^m	Amount of LPG sent to cluster k from supplier l with trucks in parking lot m

V'^{m}_{kl}	Amount of LPG sent to cluster k from supplier l with trucks in parking lot m
Q_{it}	Inventory of service point i at the end of time period t
W_{ijt}	Binary variable if customer j is visited after customer i at time period t
X_{ij}	Binary variable of assigning the service point i to cluster j
Y_j	Binary variable of selecting the service point j as cluster center
α	Travelling cost per one kilometer
β	Exchange rate of TRY-USD
θ_l	LPG liter price
γ	Vehicle capacity of a lorry
γ'	Vehicle capacity of a truck

LIST OF ACRONYMS/ABBREVIATIONS

BPGS	Branch-and-Price Guided Search
BSS	Bike Sharing System
EMRA	Energy Market Regulatory Authority of Turkey
EOQ	Economic Order Quantity
IP	Integer Program
IRP	Inventory Routing Problem
IRPT	Inventory Routing Problem with Transshipment
LNG	Liquefied Natural Gas
LPG	Liquefied Petroleum Gas
MINLP	Mixed Integer Nonlinear Program
MIRP	Multi-vehicle Inventory Routing Program
MIP	Mixed Integer Program
PVRP	Periodic Vehicle Routing Problem
RVND	Randomized Variable Neighborhood Descent
SCM	Supply Chain Management
TDP	Truck Dispatching Problem
VMI	Vendor Managed Inventory
VNS	Variable Neighborhood Search
VRP	Vehicle Routing Problem
VRPTW	Vehicle Routing Problem with Time Window
TSI	Turkish Statistical Institute
TSP	Traveling Salesman Problem
WLPGA	World LPG Agency

1. INTRODUCTION

Science and technology, which have shown a great improvement in recent years, gave rise to a competitive environment where companies have to examine their operational activities in order to sustain their profitability. Companies that wish to survive in this globally challenging environment should take actions to improve themselves on various issues.

Supply chain management (SCM) is one of the operations existing in almost every company, which provides considerable potential for improvement in terms of sustainable profitability. The first examples of SCM include production systems where each business unit such as production planning, inventory, procurement, and routing is handled separately. In most of the initial studies, each business unit tries to individually solve its problems considering its own objective. For example, a problem may be solved with an aim to minimize traveling cost between customers and production site, therefore problem solver tries to find the most suitable place for a newly opened facility. On the other side, another problem may be solved to minimize production cost and does not consider transportation activities between customers and production site. However, in recent years, depending on changing conditions around the world, companies tend to work with different approaches where integration of various systems provides a comprehensive response to a wide variety of commercial purposes. These novel approaches reflect the combinations of different systems and consider various perspectives in order to find a balanced solution.

In traditional SCM, applied methodologies do not ensure a collaborative relationship between its components such as customers, suppliers, and producers. Each component has its own decision-making mechanism that works almost independently of others. Since the decision of each component is proved to be not effective on others' operations, the applicability of such methodologies has distinctly decreased. Hence, more implementable approaches are developed in order to create comprehensive solutions. Inventory routing problem (IRP) is one of these complex problems in which inventory decisions and routing decisions are integrated and solved in a collaborative manner. First examples of IRP in literature are seen under the name of Vendor Managed Inventory (VMI) problems where mutually beneficial relations are created for a supplier and customer. In VMI the supplier is responsible for inventory planning decisions of the customer. Instead of the classic way where each customer follows its own inventory level and orders the product, this concept makes the supplier responsibility for monitoring the customers' stock levels and making a delivery plan that prevents them from being out of stock at any time. This concept drives researchers to work on a specific replenishment policy known as IRP which associates transportation and inventory decisions in one objective.

IRP can be seen as a special version of the traditional Vehicle Routing Problem (VRP) where customer inventory management is involved. The purpose of the IRP is to create a solution that minimizes the overall cost of vehicle routing and inventory holding. These problems involve decisions related to inventory delivery frequency and quantity to customer sites and the design of vehicle routes. The main restriction of the IRP solutions is to create routes in which any of the customers cannot be stock-out at any time. It is seen from the first examples that in these problems, distribution and inventory replenishment decisions are made by the supplier for each customer. In this way, suppliers can decide on the timing and quantity of products delivered to customers and create a cost-effective solution for themselves. As a result, the inventory control and routing decisions made by the supplier lead to a reduction in the logistics cost of the suppliers and eliminate the need to monitor their own stock levels for customers. These features make IRP a win-win solution for both suppliers and customers.

IRP provides effective solutions by creating a chance to control activities such as purchasing, production, inventory, and routing. Therefore, it is preferred to be applied in many industrial areas. One of these industrial areas where IRPs are implemented is frequently is the gas and petroleum industry. There is a considerable amount of studies that involve the distribution of gas and petroleum products between storage locations and gas stations through a repeated distribution plan.Usage tendency of oil and petroleum products differs among countries. Factors like geographical location, and economic status have significant effects on vehicle usage. According to the Turkish Statistical Institute (TSI), the number of registered vehicles has dramatically increased over the years. In Figure 1.1, the change in the number of vehicles during the years 2007-2017 is seen. Also, the number of registered vehicles per capita is stated as nearly 150.

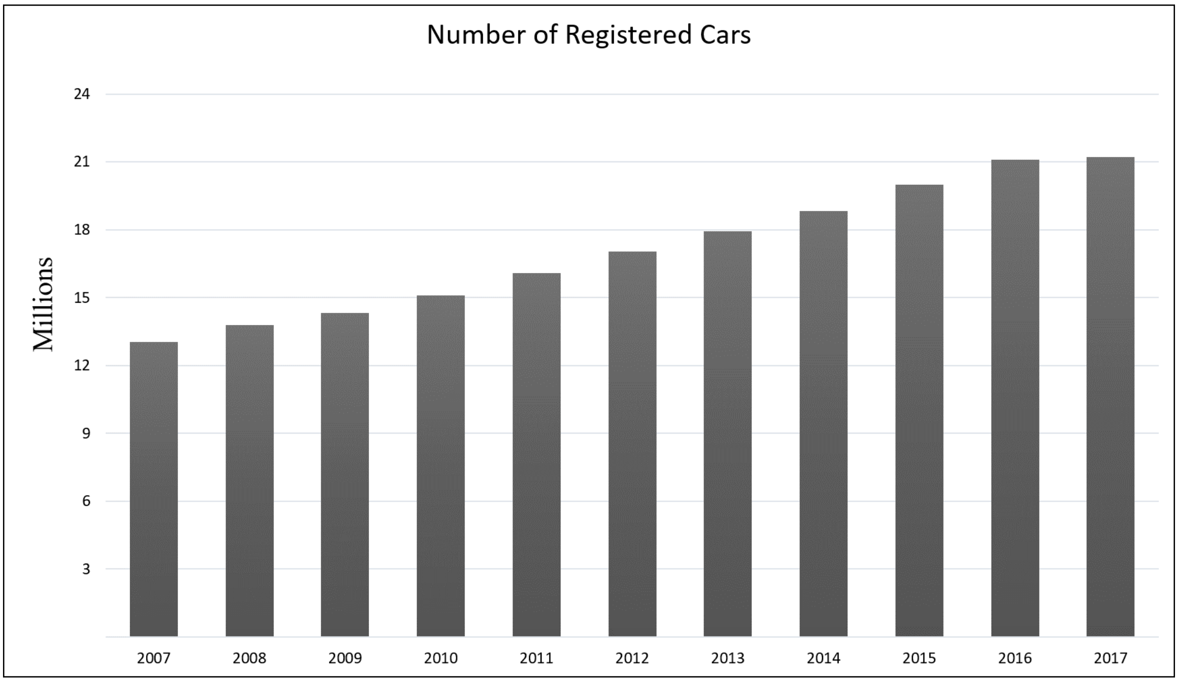

Figure 1.1. Number of road motor vehicles in Turkey between 2007 and 2017.

The increasing use of vehicles worldwide causes competition in numerous areas. The increased competition provides customers the opportunity to choose their products among different options. In the automotive industry, the fuel type of vehicle has an effect on the customer decisions. At this stage, one of the most important factors affecting the customer's decision is evaluated as fuel price. Autogas - Liquefied Petroleum Gas (LPG) offers customers relatively competitive prices, which in turn causes customers to change their fuel usage habits. According to the World LPG Agency's (WLPGA) 2018 report, there are more than 27 million vehicles using LPG as a transport fuel and it is expected that this number will continue to increase over the following years. In 2017, the total use of LPG in countries South Korea, Turkey, Russia, Poland, and Italy was equal to almost half of autogas use worldwide. Even though autogas use is concentrated in specific countries, it is still used in all regions in the world. Turkey is one of the key LPG markets and due to the effects of exchange rate and taxes on fuel prices this market is expected to grow in the forthcoming years. Figure 1.1 shows the rise in the number of vehicles that use LPG as the main transport fuel between the years 2007-2017 in Turkey according to TSI's annual report. According to these numbers, approximately 40 percent of the vehicles use autogas as fuel and this situation yields oil companies to examine their facilities on this fuel type.

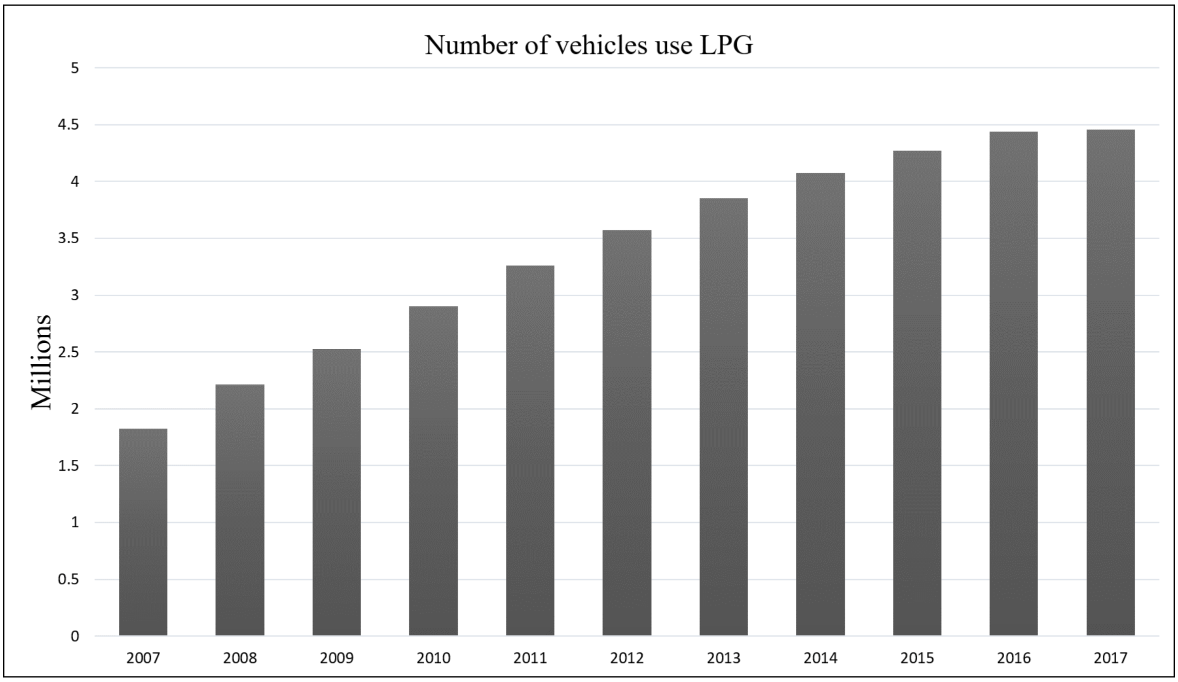

Figure 1.2. Number of vehicles use LPG as transport fuel in Turkey between 2007 and 2017.

In this study, a problem arising in an LPG purchasing and distribution system is considered for a facility owned by a global oil company. The system has two main components: procurement and distribution. The procurement involves all activities related to the purchase of autogas from supplier sites including annual agreements, daily purchase amounts, and supplier selection. The distribution activities, on the other hand, contain all transportation-related decisions in the system. These two components work separately although their decisions have an impact on each other's activities. The relation between these two components is not currently involved in the decisions and this may prevent the company from working efficiently. Both components should work with different business partners and for some of the partners, long-term decisions are required. In the current distribution network, there are 54 suppliers which have different features in terms of location, price policy, and daily capacity. The procurement department makes annual contracts with these suppliers based on their pricing policies. After suppliers are selected, the distribution department creates daily routes considering the locations and capacity restrictions. It is an obvious fact that activities of the distribution department are mainly affected by the outcomes of the agreement made by the procurement department. Since they work independently with different business objectives, the logistics system of the company does not consider the integration of these units. Thus, there exists a need for a solution which combines the decisions of the two departments. The aim of the study is creating an implementable solution considering the company's current constraints and restrictions.

To the best of our knowledge, there are no single study in the IRP field that involves both purchasing and routing decisions together. To create a distribution system in which company's all supply chain management system components are considered, the problem is separated in multiple stages. First two stages are called as strategic stages, where gas stations are grouped together, and a mathematical model is proposed in order to make supplier selection considering different factors, not only price. Results of these stages give which stations are gathered as a group and which supplier is assigned to each station group. In the third stage, for each group and corresponding supplier, routing decisions made for a particular number of time periods. Thus, a solution procedure in which procurement and distribution decisions are considered in an integrated manner is developed regarding the company's objective.

In the second chapter, there is a detailed review on studies seen in the literature related to the inventory routing problems. The deterministic and heuristic solution approaches are analyzed which are used to solve both industrial case applications and benchmark problems. In the following chapter, we define the problem in detail and the aim of the study is clearly given.

After defining the problem, detailed information of solution procedure and developed models are stated. Model assumptions and constraints are explained in Chapter 4. Additionally, used decomposition technique is expressly stated.

In Chapter 5, the suggested solution approach is analyzed and the results of the study are evaluated. Also, the advantages of the suggested method are given. The last chapter mainly covers the outcomes of the study and gives future study opportunities.

2. LITERATURE REVIEW

The importance of decision-making for a company arises from its effects on improvements in operational efficiency and the increase in profit and performance quality. Even in small companies, there are different levels where decision-making take place. A strategic level decision is always made with considering little or none information transmitted from tactical or operational level. However, each decision made in the strategic level has effects on lower level decisions and eventually, these effects are seen in the lower levels. The tactical and operational decisions are restricted because of this consecutive relation among the levels. As a result, making decisions which does not consider other business units may cause sub-optimal or even infeasible solutions that can only be avoided by an appropriate integration of different decision levels. Such integration may result in complex and large-scale problems that are difficult to solve. [1]

Our review of the literature shows that in the field of the oil industry, there is no previous study, in which strategic, tactical and operational decisions are combined. In most of the early studies, each layer has its own problem which analyzed and solved individually. Also, there is no prior study that combines the objectives of different levels.

During the review of the studies in the literature, we mainly focus on studies related to IRP literature arises in different areas. Although most of the studies provide competitive results to benchmark problems through heuristic approaches, there exist some examples which are conducted as industrial application studies. In further explanation, we analyze studies regarding their types (studies on benchmark problems and industrial applications) and give some important examples respectively.

The classical VRP belongs to the class of NP-hard problems. It solves a distribution or a collection problem between a central location and several customer points and aims to find the minimal total traveling distance, minimal traveling cost or the minimal number of vehicles to complete the task. Although the objectives of the problems are similar, there are different variations in terms of constraints and problem structure. Since new fields of study emerge as a result of requirements, VRP and its variations are still being studied by many researchers in different areas.

The first study seen in the VRP field belongs to Dantzig and Ramster [2]. The study mainly aims to create optimized routing decisions in a gas distribution system. The problem issued in the study is referred as The Truck Dispatching Problem (TDP) and involves all distribution activities from a central depot to gas stations supplied by the depot. This problem is derived from the Traveling Salesman Problem (TSP). A new mathematical model is proposed with an algorithmic solution approach. After this initial study, Clarke and Wright [3] have improved TDP by considering multiple demand and truck capacity and proposed a heuristic solution approach. Since then, many studies have been established on similar problems with different restrictions and methodologies. In order to respond to the problems which have different specifications, variations of VRP are developed with different problem definitions, objectives, constraints and solution approaches. Weise *et al.* [4] classify traditional VRP as shown in Figure 2.1.

One of the variations of VRP is called as Vehicle Routing Problem with Time Window (VRPTW) where capacitated vehicles are used and time is added to the model as a constraint. Since time is a restriction in most of the real cases, time of delivery is considered by the suggested mathematical model and obtained solution guarantees that service at any customer starts and finishes within a given time interval. Since VRPTW is an extension of classical VRP which is NP-hard, the early studies established in this field are small-sized case study instances. Another problem type derived from VRP is Periodic Vehicle Routing Problem (PVRP) which is introduced by Beltrami and Bodin [5] in a seminal paper. The PVRP find solution to problems where vehicle routes are constructed over multiple time periods. During a given time period, a fleet of capacitated vehicles travels along routes that begin and end at the central depot. The objective of the PVRP is to determine vehicle routes that minimizes total travel cost while satisfying operational constraints. Although PVRP provides solutions to the routing decisions for each time period, it is not applicable to the problems seen in a system which has inventory aspect. In this context, Vendor-Managed Inventory-Routing Problem arises as a new approach that considers the relation between inventory and routing costs in an integrated manner.

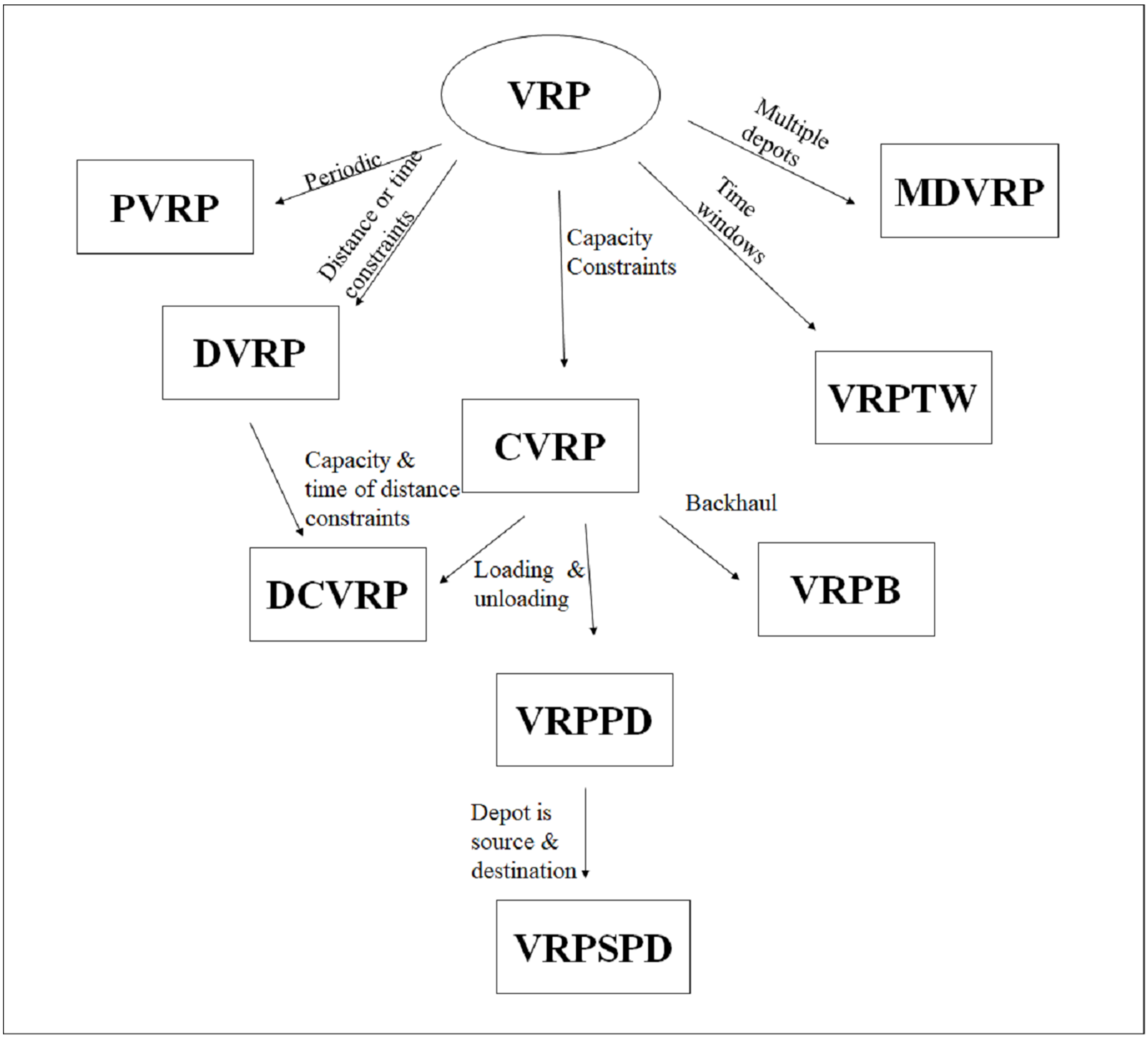

Figure 2.1. Vehicle Routing Problem types

One of the variations of VRP is called as Vehicle Routing Problem with Time Window (VRPTW) where capacitated vehicles are used and time is added to the model as a constraint. Since time is a restriction in most of the real cases, time of delivery is considered by the suggested mathematical model and obtained solution guarantees that service at any customer starts and finishes within a given time interval. Since VRPTW is an extension of classical VRP which is NP-hard, the early studies established in this field are small-sized case study instances. Another problem type derived from VRP is Periodic Vehicle Routing Problem (PVRP) which is introduced by Beltrami and Bodin [5] in a seminal paper. The PVRP find solution to problems where vehicle routes are constructed over multiple time periods. During a given time period, a fleet of capacitated vehicles travels along routes that begin and end at the central depot. The objective of the PVRP is to determine vehicle routes that minimizes total travel cost while satisfying operational constraints. Although PVRP provides solutions to the routing decisions for each time period, it is not applicable to the problems seen in a system which has inventory aspect. In this context, Vendor-Managed Inventory-Routing Problem arises as a new approach that considers the relation between inventory and routing costs in an integrated manner.

Importance of VMI is widely accepted as it provides effective managerial practices and improves the service level of supply chains. In an SCM system that takes VMI principals into account, the supplier is selected as the decision maker and responsible for all inventory decisions, such as when and how much to deliver and how to integrate deliveries into vehicle routes on a finite planning horizon. The VMI implementation results in a win-win situation for both customer and supplier. In such an SCM, customers are not required to use their own resources for inventory checks and suppliers are able to create a distribution system that controls overall distribution cost. The needs of creating this type of distribution systems arise IRP since it solves optimization problems with integration of inventory management and multi-period routing decisions.

In earlier supply chain practices where IRP methodologies are implemented, two types of system structures exist: one-to-many and many-to-many. In a one-to-many structure, products are transported to multiple customer sites from one supplier. In a many-to-many structure, there are more than one supplier sites and products are delivered to the customer points from these suppliers. The common feature of these two structures is that vehicles directly travel between suppliers and customer points, not through a warehouse. However, in the real-life supply chain structures, there exist different components which need to be considered. When the number of these components increases, the complexity of the problem increases. The more complex structures are seen in large companies working with more than one supplier or production facility and require to control the upstream and the downstream flows. Although such supply chain structures allow large companies to follow different product flows and improve their distribution systems while decreasing overall transportation and inventory costs, there exist few studies which consider such problems. [6].

The idea of gathering inventory and scheduling decisions in the same problem is first seen in a study established by Bell *et al.* [7]. This study includes the delivery of industrial gases to customers and the generation of vehicle schedules. The study is motivated by the improvement options for the decision-making system of a company. The optimization approach used in the study is based on a Lagrangian relaxation algorithm that aims to give near-optimal solutions to mixed integer programs (MIP) which involve a considerable amount of variables and constraints. This study is accepted as a pioneer in the fields of IRP and emerges a new research field based on the idea of integrated inventory management and route scheduling. Coelho and Laporte [8] give some fundamental examples of IRP in their review study and state that, the following questions should be addressed in a classical IRP:

- When products should be delivered to a specific customer?
- How much to deliver to this specific customer when there is a delivery?
- In what order should customers be visited?

Since the IRP has numerous application areas, system requirements and problem structures vary considerably. Andersson *et al.* [9] classifies IRP studies in the literature according to some criteria listed in the Table 2.1. Due to the different problem structures encountered, the solution methods are also differentiated for the subjects presented in the table below.

Any possible combination of these criteria can be seen in a real life supply chain system, and this encourages researchers to address different combinations rather than focusing on a particular type of problem. However, there are some application areas

Table 2.1. Variations of Inventory Routing Problem

Characteristic	**Options**			
Time	Finite	Infinite	Instant	
Structure	One-to-one	Many-to-many	One-to-many	
Routing	Direct	Multiple	Continuous	
Fleet size	Single	Multiple	Unconstrained	
Fleet composition	Homogeneous	Heterogeneous		
Demand	Deterministic	Stochastic		
Inventory	Fixed	Stock-out	Lost sale	Back-order

where IRP is mostly implemented due to the high consumption rate of the products dealt in the problem. In the following part of the section, we first focus on industrial cases regarding their applications areas. Later, we focus on further studies where different problem structures and solution methods are used.

2.1. Literature Review on Industrial Applications of IRP

The first study on IRP field is established by Bell *et al.* [7] for an industrial gases distribution system and the aim of the study is to create a decision-support system that contains inventory management and fleet route scheduling. The proposed solution approach firstly includes generation of all possible routes and then, assignments of the delivery amount for each route. The study considers only transportation costs and the demand at customer sites are assumed to be stochastic. A Lagrangian relaxation algorithm is used in order to obtain a near optimal solution due to the number of constraints and variables. After this pioneering study, a few similar studies are conducted by the same authors.

Another study is presented by Federgruen and Zipkin [10]. Demands of the customers are assumed to be random. The total system cost involves inventory holding and shortage costs along with the transportation cost. A mathematical programming model is given and a separation algorithm with a generalized Benders' decomposition is introduced. The authors establish this study as an extension of the classical VRP.

Miller [11] presents a study that consists of vehicle scheduling and inventory management problem faced by a chemical company. The aim of the study is to utilize a fleet of tankers to deliver gasoline to distribution sites and customers all over the world. The scheduling problem considers decisions on route generation, arrival/departure times, and inventory replenishment policies. The problem is issued as a network flow model and a MIP model is described. Then, the components of the decision support system are generated in order to create schedules with the integration of inventory management strategies.

Blumenfeld *et al.* [12] brings a scientific approach to the distribution system of one of the most known automobile and truck production company. The company needs the development of a method which defines the most suitable delivery size, delivery frequency, and fleet routes. This need encourages authors to develop a system in which trade-off between transportation and inventory costs are considered. In order to manage inventory balance, they propose a method that allows them to find optimal shipping strategy based on economic order quantity. As a result of their study, cost-saving opportunities are seen by the company and further implementations are conducted.

Bard *et al.* [13] study a decomposition technique to solve an IRP problem faced by a satellite facility. The proposed solution approach is distinguished from the prior studies since it examines two periods at a time and identify all customers whose optimal delivery day falls within this time frame instead of dividing customers into separate groups. The identification of deliveries made in each period centers on the fact that, there exists an optimal frequency for each customer. Therewith, the delivery schedule can be created regarding the frequency levels of customers. The idea behind the determining schedules is based on whether the next scheduled visit falls within the planning horizon the customer or not. Three heuristics are developed to solve the vehicle routing problem with satellite facilities: Randomized Clarke–Wright, GRASP, Modified Sweep.

Christiansen [14] presents a solution approach to a real-life shipping problem for transporting ammonia from production sites to consumption harbors. The study reflects the combination of inventory management and routing problems with time period. The information of inventory levels at each consumption site, production amounts, and ship capacities determine the number of possible deliveries to each consumption site for each time windows. It mainly means that for each consumption site, visit frequency differs and some sites can be visited multiple times in a time period. This inventory management problem contains pickups and deliveries and called as the Inventory Pickup and Delivery Problem with Time Windows (IPDPTW). Because of the complexity of the problem, decomposition methods are used in the solution approach. The problem is separated into subproblems for each ship and internal harbor. Later, these subproblems are solved by using a Dantzig-Wolfe decomposition.

One of the industrial application studies belongs to Campbell and Savelsbergh [15]. They propose a two-phase solution approach based on a decomposition method to an IRP problem seen in an international industrial gases company. Firstly, they create a delivery schedule with using a clustering implementation in order to reduce the number of possible schedules. Later, they construct a set of delivery routes. The first phase of the suggested method contains an integer programming (IP) while the second phase contains an insertion heuristic approach. They state the aim of the study as creating an implementable solution approach which deals with large-scale industrial problems and gives computational results which show their proposed IP-based model promising for further studies.

Gaur and Fisher [16] propose a study to solve a problem faced by a supermarket chain in the Netherlands. The study provides help on decision-making in both tactical and strategic level. They simplify the problem with using a fixed partition policy and gather all the stores in a region into a cluster. They use a three-phased solution approach. The first phase is called as inventory routing phase where delivery times of stores and vehicle routes are determined by using a periodic IRP. In the second phase, truck assignment decisions are made and the aim is to reduce total transportation time regarding fleet size and time restrictions. In the final phase, they analyze departure times and involve time windows restriction to the solution and adjust workload balance among distribution centers.

Since from the first applications, one of the most common application areas for IRP is maritime logistics. As companies produce their products at overseas facilities and deliver them to end customers through distribution centers, a detailed logistics system is required. Persson and Lundgren [17] study a shipment planning problem where ships are transporting product from refineries to the storage locations. For the problem, a solution method based on column generation is proposed. They state that classical branch-and-bound approach is not proper to solve a large-sized problem as in this study. Instead, another strategy that limits branch search is recommended in which column generation and branching is considered. Valid inequalities are involved in the solution approach and a branch-and-bound search for selecting the best combination of generated columns improves the suggested solution.

Hemmelmayr *et al.* [18] works on an IRP in a different area. The study aims to generate a cost-effective delivery optimization for an Austrian blood bank. They propose three different solution approaches and evaluate the results using different test samples. The first approach is accepted as a basic approach where neither a mathematical model nor a heuristic algorithm is used. The second approach includes an integer programming where a mathematical model is proposed and different branching strategies are involved. The final approach suggested in the study is using a variable neighborhood search (VNS) based on PVRP with tour length constraints. A comparison of these three approaches is given where the basic approach gives a poor solution while other approaches give similar solutions.

In the paper proposed by Al-Khayyal and Hwang [19], a mixed-integer nonlinear program (MINLP) for route planning and pickup delivery schedules of a ship fleet for a finite horizon is defined. The aim of the study is to minimize total travel cost and setup costs. The proposed model differs from the existing models since ships in the problem have different compartments and each compartment can carry different type of products. The model deals with nonlinearities and they use some novel linearizing schemes obtained from global optimization theory.

Savelsbergh *et al.* [20] introduce a problem with continuous moves and present a study for two significant real-life challenges: limitations of product availability at facility locations and customers which cannot be served using out-and-back tours. They design routes for multiple time periods and consider large geographic regions comprises different product collection facilities. They develop an innovative randomized greedy algorithm which contains a linear programming based post-processing technology, and demonstrate the effectiveness of their method with an extensive computational study.

Gronhaug *et al.* [21] study on a maritime IRP for liquefied natural gas (LNG) distribution. The proposed model aims to generate routes for the fleet and monitor inventory levels for both liquefaction plants and regasification terminals. The problem is distinguished among other maritime IRP studies because of its complicated structure that involves following:

- A constant rate of the cargo evaporates each day and is used as fuel during transportation,
- A variable production and consumption of LNG,
- A variable number of tanks unloaded at the regasification terminals.

The problem is solved using a branch-and-price approach and upper bounds in the search tree are obtained by using column generation.

Another case study in the oil transportation is carried out by Shen *et al.* [22]. In the problem, there is a transport system in which crude oil is transported from a supplier to customer facilities. A heterogeneous fleet of vehicles is owned and operated by a third-party company. A pipeline and multiple route types are considered and inventory and storage capacities have limitations for each customer. The purpose of this study is to determine the number of tankers required for each period over a finite horizon and to create routes including delivery quantities per customer. After formulating the problem as an MIP, a Lagrangian relaxation approach is developed and a near optimal solution is obtained. The approach is also applied to another variant of the problem that allows to transfer crude oil with full or partially loaded tankers. Numerical experiments show that the proposed formulation and the solution approach is superior to an existing meta-heuristic algorithm, especially for large-scale samples.

Christiansen *et al.* [23] study a maritime logistics problem seen in a cement producing company. The company works with a heterogeneous fleet and cement product is transported from a production site to regional customer sites located along the coast of Norway. For both customer and production sites, there exist inventory restrictions and also the fleet size is limited which causes cement shortage in peak seasons. Additionally, different products may be transported together however, they should not be mixed. A construction heuristic embedded in a genetic algorithmic framework is proposed considering model restrictions. The proposed solution approach is used to solve real-life problems within a reasonable solution time with near-optimal solutions.

Later, Benoist *et al.* [24] analyze a real-life problem with a different approach in which short-term and long-term objectives are firstly defined individually and, then combined. In this problem, constraints are defined realistically in terms of components such as inventory, driver,and production. The objective function is defined which helps to deal with short-term optimization with a rolling horizon. The main challenge in the problem is the difficulty of implementing short-term and long-term decisions in a reliable objective function. Therefore, short-term decisions are stated as an objective of the model and local lower bounds are used to ensure a long-term optimization.

Stalhane *et al.* [25] conducts another study on a routing problem for a company that works in the field of LNG. In the proposed model, a delivery process for multiple products is considered. The aim of the problem is to generate a delivery schedule for a one-year period and ensure that a contract is signed with a producer at minimum cost level while revenue maximization is satisfied. They suggest using a construction and improvement heuristic that is a multi-start local search heuristic. In the used heuristic approach, a set of solutions are generated with using a greedy insertion process. Obtained solutions are supported by a first-descent neighborhood search and a branch-and-bound on the mathematical formulation. They apply the proposed solution approach on real-time problems faced by the same company and show the capabilities of the suggested solution approach.

Popovic *et al.* [26] focus on a problem seen in fuel delivery transportation and define the problem as an extension of classical VRP where inventory management is involved. The aim of the study is to create a solution that minimizes the total system cost involving vehicle routing and inventory management. They propose a multi-product multi-period IRP for fuel delivery with multi-compartment homogeneous vehicles. Deterministic consumption rates that varies among petrol stations and fuel types are used. They develop a VNS and compare the results of using a stochastic VNS, a MILP and a deterministic compartment transfer heuristic.

Another industrial application study is carried by Michel and Vanderbeck [27] in which an inventory management problem is considered as a tactical level decision. The study concerns a single product system with pickups, and inventory levels are considered as deterministic. Problem is handled in two phases: tactical and operational. In the tactical level, sites are grouped and assigned to vehicles. In the second phase of the problem, vehicle schedules are obtained and periodically repeated. In order to solve the problem, they propose a solution approach based on a truncated branch-and-price-and-cut algorithm combined with rounding and local search heuristics that yield both primal solutions and dual bounds.

Aksen *et al.* [28] observe a problem faced by a biodiesel production company during waste oil collection processes. The company uses collected oil as raw material for biodiesel production. The problem addresses questions such as: which of the presented source points should be visited, which vehicle routes should be used each day, how many vehicles are required. The aim of the study is to create a solution that minimizes total collection, inventory and purchasing costs. A selective and periodic inventory routing problem is defined with two different formulations. A comparison among suggested formulations are made and the one that gives better solutions is applied to different scenarios and promising results are stated.

Hewitt *et al.* [29] consider a real life maritime IRP in which vessels load product at one or more supply ports, then cross the ocean and discharge the product at one or more consumption ports. To solve the problem efficiently, they first propose a Branch-and-Price Guided Search (BPGS) and apply it to a set of computational experiments to show the quality of solutions. Later, they combine BPGS with a local search scheme to obtain qualified solutions in less time.

Soysal *et al.* [30] apply IRP methodologies to a problem seen in an uncommon field. The problem is faced by a supermarket's supply chain department where food waste reduction and some environmental effects are highly concerned. They propose a multi-period IRP model in which distribution cost is included based on truck routes and involves CO2 emission and fuel consumption, perishability is considered. A service level constraint for meeting uncertain demand. A real-world problem seen in the distribution of vegetable is analyzed and a promising decision support system is developed.

Brinkmann*et al.* [31] apply the fundamentals of IRP to develop a new approach to bike sharing system (BSS) used in a city. In a BSS system, operators must provide a sufficient number of bikes and empty bike racks at each station. Therefore, some bikes should be moved between stations in order to balance the number of bikes parked and required. This operation needs a routing decision considers rentals and returns occur in a time period where a significant amount of activities take place unpredictably and spontaneously. A stochastic IRP is defined to clarify a short-term relocation strategy of BSS and a case study is used to test the outcomes of the proposed model.

Peres *et al.* [32] propose a solution approach to a problem faced by a large Brazilian retail company with using a multi-period, multi-product Inventory-Routing Problem with planned Transshipment (IRPT). In the solution phase, they propose a meta-heuristic algorithm together with an exact method. The meta-heuristic algorithm is a hybrid Randomized Variable Neighborhood Descent (RVND) which searches over large neighborhoods mostly based on the IRP literature allowing non-improvement moves to include diversity.

Cho *et al.* [33] study an LNG problem which is related to vessel routing decisions under challenging weather conditions. Weather conditions are important for an LNG facility because extreme situations require rearrangements of loading /unloading operations in order to prevent accidents. The authors formulate two mathematical models where possible weather conditions are considered. The first model is a stochastic MIP with two-stages that maximize the overall expected revenue while minimizing the cost caused by the uncertain impact of weather disruptions. The other model is designated to help decision maker in order to evaluate risks. Two methods have been created to deal with these models and computational results are given based on a case study.

All studies given in this section are based on industrial cases and show how IRP implemented on different industrial problems. The most common feature of the problems is the high demand rate of the products in the analyzed system. In Table 2.2, a brief summary of industrial applications is given with problem specifications.

Table 2.2 Review on IRP applications for industrial cases

Author	**Year**	**Industry**	**Method**
Bell *et al.*	1983	Industrial gases	Lagrangian relaxation
Federgruen and Zipkin	1984		Generalized Bender's decomposition
Miller	1987	Chemicals	Network flow and MIP
Blumenfeld *et al.*	1987	Automotive	EOQ based strategy
Bard *et al.*	1998	Satellite facility	Randomized Clarke-Wright, GRASP
Christiansen	1999	Chemical	Dantzig-Wolfe decomposition
Campbell and Savelsbergh	2004	Industrial gases	Insertion heuristic
Gaur and Fisher	2004	Supermarket	Decomposition to clusters
Persson and Lundgren	2006	Oil product	Column generation

Table 2.2 Review on IRP applications for industrial cases (cont.)

Author	**Year**	**Industry**	**Method**
Hemmelmayr *et al.*	2006	Blood	Variable neighborhood search
Al-Khayyal and Hwang	2007	Chemicals	Global optimization theory
Savelsbergh *et al.*	2008	Chemicals	Randomized greedy algorithm
Gronhaug *et al.*	2010	Industrial gas	Column generation
Shen *et al.*	2011	Industrial gas	Lagrangian relaxation
Christiansen *et al.*	2011	Cement	Genetic algorithm
Benoist *et al.*	2011		Local lower bounds
Stalhane *et al.*	2012	Industrial gas	First descent neighborhood
Popovic *et al.*	2012.	Petroleum	Variable neighborhood search
Michel and Vanderbeck	2012	Not stated	Approximate solution with clustering
Aksen *et al.*	2012	Waste oil	Partial linear relaxation on lower bounds
Hewitt *et al.*	2013	Maritime logistic	Branch-and-price
Soysal *et al.*	2015	Supermarket	Robust MIP and branch-and-cut
Brinkmann*et al.*	2015	Bike sharing	Stochastic IRP
Peres *et al.*	2017	Retail products	Randomized variable neighborhood descent
Cho *et al.*	2018	Vessel shipment	Stochastic IRP

2.2. Literature Review on Other IRP Applications

Beside industrial cases of IRP, we focus on IRP literature regarding specific features of studies. As it is stated by Andersson *et al.* [9], different variants of IRP can be seen in the literature. In the following part of this section, main studies are mentioned chronologically.

Aghezzaf *et al.* [34] propose an IRP model that covers a distribution system where transportation activities between a distribution center and sale points are considered as cost components. Since economic order quantity (EOQ) based inventory management models bring nonlinearity to problems, they propose an IRP where demand rates are constant over a planning horizon and EOQ-like policies are used for managing customer inventory levels. A column generation-based approximation method is developed. The resulting sub-problems are solved using a savings-based approximation method. Even though initial results show the solution approach give important solutions, more improvement areas are left open by the authors.

Savelsbergh and Song [20] introduce an IRP formulation to extend existing problems and propose an implementable solution approach to real-life complexities. Limited product availability at source nodes and not serving customers with out-and-back tours are the differences aspects of the study. In this study, a single supplier does not have availability to meet the demand of its customers because of its location. Therefore, the problem includes multiple suppliers and trips those last more than one period. They develop a randomized greedy algorithm based on a linear program.

Michel and Vanderbeck *et al.* [27] focus on a pick-up problem with inventory control decisions at customer locations. They work on a problem in which customers have deterministic stock levels and are visited when all their stock is emptied. The company has a limitation on fleet size, the number of vehicles is given but it can decrease. A truncated branch-and-price-and-cut algorithm is integrated with rounding and local search heuristic. The model gives regional clusters, and routes are created for each vehicle.

Abdelmaguid *et al.* [35] study an IRP which consist of multi-period vehicle routes, inventory management, and backlogging decisions. They propose a constructive and improvement heuristics. The constructive heuristic aims to minimize a single allocation cost which is estimated for each customer. Cost estimations for each customer are made by using two subproblems provided in the study. In addition to the constructive heuristic that considers only routing management, an improvement heuristic is introduced based on the observations of the effects of delivery amounts between periods.

Boudia *et al.* [36] consider a multi-period production and distribution system with three different cost components: production setup cost, inventory cost, and distribution cost. They propose an algorithm in order to make production and distribution decisions simultaneously instead of the classical two-phase decision methods where production and distribution decisions are made consecutively. They use a memetic algorithm with population management to solve the problem easily. The proposed metaheuristic's results are compared with the results of classical two-phase heuristic and, also greedy randomized adaptive search procedure. The major contribution of the study is explained as creating an improved way that integrates production and distribution decisions.

Solyalı *et al.* [37] introduce an exact solution algorithm to solve a deterministic IRP that allows backlogging. The problem is formulated as an MIP and a branch-and-cut algorithm is developed. The study has a robustness aspect that takes demand uncertainty into account. They give two different robust MIP formulations. While one of the formulations includes more constraints and variables, the other formulation is close to the classical IRP formulation with demand uncertainty. The proposed branch-and-cut algorithm is applied to both two problems, and robustness regarding unknown demand is obtained.

Archetti and Bertazzi [38] work on a multi-period IRP where a set of customers are served by a given supplier. Two replenishment strategies are analyzed. They develop a heuristic that combines tabu search scheme with ad hoc designed mixed integer programming model. The introduced hybrid heuristic applied whenever the best incumbent solution is updated. This type of MIP explores in depth the neighborhood of the incumbent solution.

Coelho *et al.* [39] introduce a new concept to the IRP in order to increase service level and create an effective solution algorithm. The new concept is called as consistency and it has different aspects such as quantity consistency, filling rate consistency, order-up-to consistency, driver consistency, partial driver consistency, and visiting space. These features are related to the problem specifications. Some consistency types may not be applicable for each IRP instances. They formulate the problem as a multi-vehicle IRP for both considering and without considering consistency situations and work with a metaheuristic for the solutions. The proposed metaheuristic is an adaptive large neighborhood search, and it can solve classical multi-vehicle IRP (MIRP) and its some extensions with some consistency features.

Coelho and Laporte [40] introduce an IRP where transshipment between all nodes are allowed and it is called as Inventory Routing Problem with Transshipment (IRPT). In classical IRP, products are delivered to customers through suppliers. In IRPT, in addition to the flow from supplier to customers, product delivery among customers is allowed. They formulate the problem with transshipment feature and propose an adaptive large neighborhood search heuristic to solve the problem. The heuristic works on route generation. To solve the transshipment part of the study, a network flow algorithm is used. The heuristic approach solves different variants of the classical IRP where transshipment and different inventory policies are considered.

Coelho and Laporte [41] propose a branch-and-cut algorithm for a multi-product multi-vehicle IRP considering two consistency features those are driver partial consistency and visit spacing. For driver consistency, they associate some customers to some drivers and guarantee that these customers are always visited by the same driver. In terms of visit spacing, the applied consistency creates a temporal space between consecutive visits to the same customer. They propose MIRP and add valid inequalities to strengthen the formulation. Additionally, the formulation involves symmetry-breaking constraints to ensure that any vehicle, let us say vehicle k, cannot leave the depot if the vehicle k-1 is not departed before. They propose a branch-and-cut algorithm that solves the suggested problem with considerable instance size.

Another study established by Coelho and Laporte [42] focus on improvement opportunities for IRP using valid inequalities and input ordering. Besides well-known valid inequalities used in the VRP and IRP literature, they introduce three inequality sets in order to consider the relationship between customer demands and production capacities. They also state importance of the input data order based on its effects on the linear relaxation of the proposed IRP formulation. They show that if the critical customers are prioritized, for the other customers the flexibility may decrease, and lower bound can increase. They propose a branch-and-cut algorithm to solve relatively large problem instances. They test the proposed model by working on the benchmark instances where 249 instances do not have optimal solutions. The proposed model finds an optimal solution for 11 instances and improve the lower bound of more 90 instances among these instances.

Bertazzi *et al.* [43] work on a problem which contains demand stochasticity. They first describe a deterministic formulation, then a new stochastic dynamic programming formulation is introduced with different aspects. This formulation allows some small problems to find an optimal solution. To solve more realistic problems with large sizes, a matheuristic approach which integrates a roll-out algorithm and an optimal solution of MIP models are developed.

Avella *et al.* [44] bring a new reformulation to the vendor managed IRP based on single item IRP. They produce additional constraints and valid inequalities instead of defining new decision variables. In addition to the new single item IRP formulation, two new cutting plane families are used which come from the interaction between routing and lot-sizing aspects. As a primal heuristic, a classical branch-and-cut algorithm is used and lower bounds are obtained. The results are compared to the results given by Coelho *et al.* [42] and it is stated that some improvements have been done with the new constraints and inequalities.

Desaulniers *et al.* [45] introduce a new mathematical formulation for the IRP and use a branch-and-cut algorithm to solve it. Their suggested algorithm includes new valid inequality sets and new capacity inequalities. They introduce four types of valid inequalities based on following condition: minimum number of visits per customer, minimum number of routes per time interval, minimum number of sub-deliveries,and capacity. In the branch-and-cut algorithm, lower bounds are added by using column generation, and cutting planes are added dynamically to tighten the linear relaxations. They test suggested approach on a benchmark problem where more than 200 instances do not have an optimal solution. Proposed approach provide optimal solutions to 54 of the open problems.

Lefever *et al.* [46] propose an IRP with transshipment. A branch-and-cut formulation is proposed with different improvements. Firstly, they define new valid inequalities to improve the strengthens of the problem's relaxation. Then, bounds on the continuous delivery variables are improved and they are used to tighten the inventory constraints. At the next step, a new formulation is given to create routes for direct shipment extension of the IRP. Some of the variables used in the proposed mathematical model are proved to be eliminated. Hence, the computational burden of solving the problem decreases. Their proposed model gives an optimal solution to two benchmark instances which are not optimally solved previously.

In classical IRP, a distribution problem which is seen between customer and supplier or depot location is analyzed. Guimaraes *et al.* [6] extend the classical problem with adding one more echelon to the problem. In the system they work on, there are 3 layers and the layer between supplier and customers is responsible for pickup of inputs and delivery of final product to customers. They give a mathematical formulation that is capable of handling all decisions of the system and design a branch-and-cut algorithm to solve it. To solve the proposed model, they give a metaheuristic which combines the mathematical model with and adaptive large neighborhood search and gives considerable results for even large problem sets.

A brief summary of IRP studies in the literature is shown in Table 2.3, with main characteristics and distinctive aspects of the studies.

Table 2.3. Review on IRP studies with the distinctive aspects

Author	**Year**	**Feature**
Aghezzaf *et al.*	2006	Column generation-based approximation
Savelsbergh and Song	2008	Randomized greedy algorithm
Michel and Vanderbeck	2008	Branch-and-price-and-cut algorithm
Abdelmaguid *et al.*	2009	A constructive heuristic with backlogging
Boudia *et al.*	2009	Memetic algorithm
Solyalı *et al.*	2012	Branch-and-cut for robust solution
Coelho *et al.*	2012	Adaptive large neighborhood search
Coelho and Laporte	2012	Adaptive large neighborhood search with transshipment
Coelho and Laporte	2013	Branch-and-cut with consistency feature
Coelho and Laporte	2014	New valid inequalities
Bertazzi *et al.*	2014	Matheuristic
Archetti and Bertazzi	2015	Tabu search
Avella *et al.*	2015	Cutting plane and branch-and-cut
Desaulniers *et al.*	2016	Branch-and-cut and column generation
Lefever *et al.*	2018	Branch-and-cut

3. PROBLEM DEFINITION

Car ownership has increased sharply worldwide as a consequence of technological and economic growth. This increase leads to a demand rise for fuels such as petroleum, diesel, and LPG. The high level of demand encourages oil companies to improve their operations in order to create sustained profitability. One of the most important operations of such companies is the management of distribution networks and there exist the activities in both operational and tactical business levels. All operations related to procurement, transportation, and sales can be directly or indirectly involved in distribution networks. In this context, supply chain management gains importance because it involves all logistics activities and combines different decision segments in both operational and tactical level.

This study is conducted to create an efficient and implementable solution to problems faced by an LPG distribution network of a global oil company also active in Turkey. In the current distribution network, there are two significant components: procurement and management of distribution network. The first component is responsible for selecting suppliers spread all over Turkey. There are 54 suppliers in different regions, however the majority of them are located close to major cities like İstanbul, Ankara, and İzmir. Suppliers differ in terms of pricing policy, capacity, and closeness to transportation. One most important selection criterion in the current situation is the pricing policy. In Turkey, the base price per liter is determined by the Energy Market Regulatory Authority of Turkey (EMRA). In addition to the base price, some extra charges can be issued by the suppliers. This situation causes LPG price to change based on current market conditions and creates a competitive environment. There exist annual agreements which is signed by taking only purchasing prices into account. On the other hand, the second component of the distribution network in charge of all activities related to the distribution of LPG from suppliers to gas stations. The company performs the distribution operations with its heterogeneous fleet. In the fleet of trucks and lorries, each vehicle is allocated to a parking lot where it departs and returns at the end of its route. Trucks and lorries are different in terms of capacity. In total, there are 11 parking lot spread all over Turkey except in some specific regions. At the same time, the company serves with its 810 gas stations located in different regions of Turkey. In Figure 3.1 all gas stations are given.

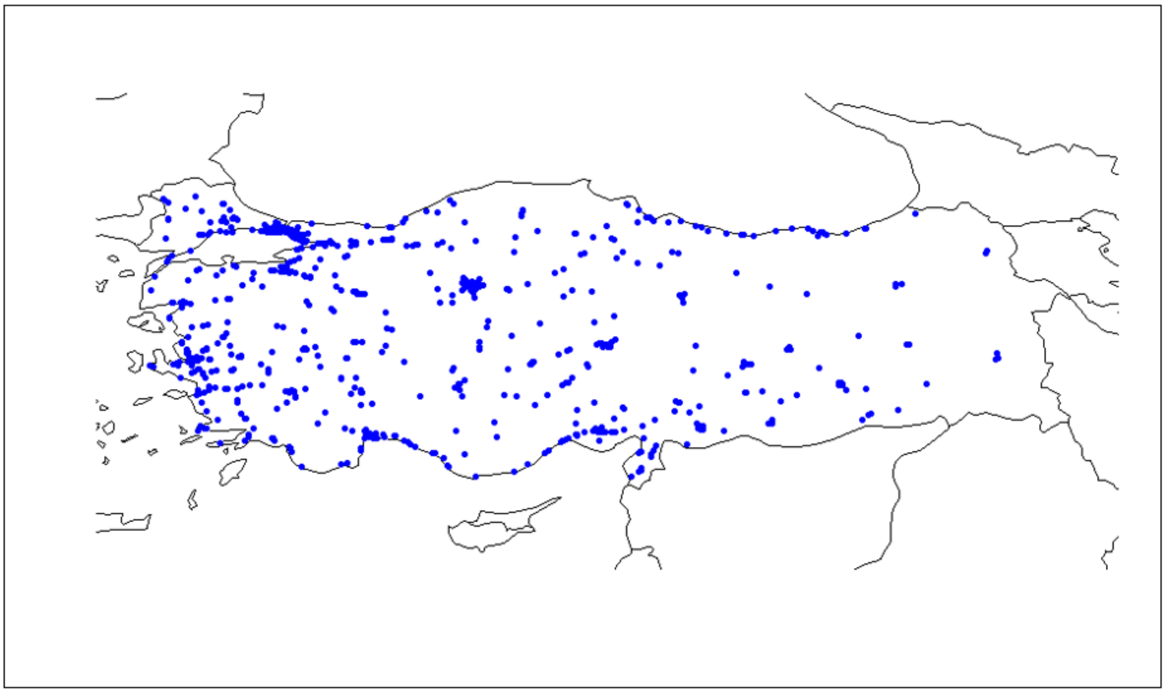

Figure 3.1. Illustration of the company's gas stations on a map.

In the distribution network, vehicles are assigned a different route each day. When a vehicle is assigned to any route, it departs from the parking lot, delivers the required amount of LPG to specified gas stations and returns to the same parking lot when the route is completed. The main reason of creating different routes is the different demand structures and changing the delivery requirements of the stations. Figure 3.2 gives an illustration for a route. While generating a route, inventory levels of stations are controlled, and some of them are prioritized regarding their levels. Except this, there are particular restrictions which should be considered while creating a vehicle route. Supplier capacities and locations are among the most significant restrictions that affect route creation. It mainly shows that decisions made by the first component have a vital impact on activities in distribution management.

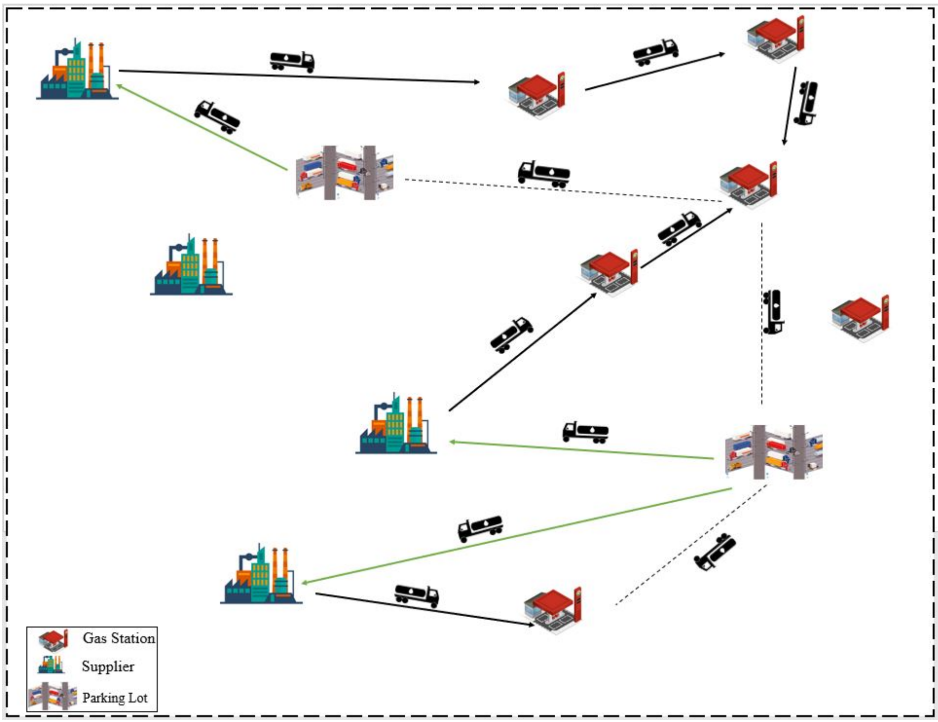

Figure 3.2. LPG distribution system overview.

In Turkey, the vast majority of the population lives in big cities. As a result of the concentration of the population in certain regions, the transportation networks in these areas are well developed. However, some of the suppliers are located in areas with a low population density due to different reasons. This situation causes them to be preferred less comparing to the suppliers which have an easy access to the transportation. Some suppliers who want to extinguish this disadvantage aim to reach the fuel companies with the price advantage they provide. As in the problem dealt with in this study, fuel companies may consider only LPG prices and select suppliers who offer more affordable prices even though their location is not close to the facility sites. Therefore, companies work with some suppliers located far away from the majority of the gas stations and face an additional transportation cost as a result. Since there is no balance between provided price and additional transportation cost, the overall cost of the distribution network may be more than the company expectations because of the supplier selection. In Figure 3.3 locations of gas stations and suppliers are shown.

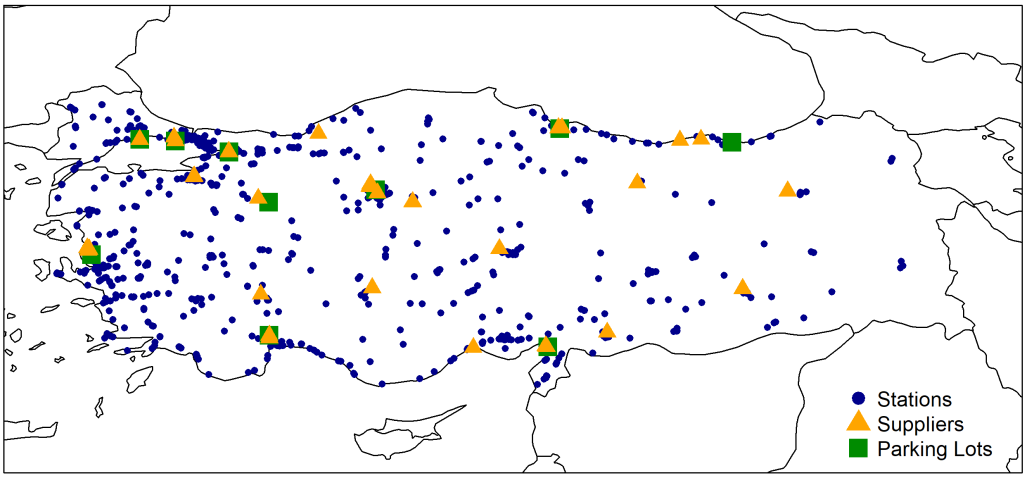

Figure 3.3. Illustration of the company's suppliers on a map.

The aim of the study is to create a solution methodology that will decide which suppliers to be worked with and generate vehicle routes for the distribution network for a given time period. In this context, an integrated solution approach is developed that will balance the procurement and the transportation costs.

To come up with an implementable solution approach to this industrial problem, different alternatives have been evaluated. Based on the literature review stated in Chapter 2, it should be noted that there are no previous single study on a problem that involves more than 800 customers with multiple supplier and depots. However, there are a few studies that deal with the considerably big amount of data with multiple-phased solution approaches. It is mostly seen that in these previous studies, problems are divided into two groups: clustering and scheduling. This mainly shows that the number of parameters and constraints involved in the model developed has a significant effect on the suggested solution method. On the other hand, in similar studies, there are a pre-determined supplier and depot sites, and these models generate routes accordingly. However, our problem also includes supplier selection which causes an additional phase to deal with. Therefore, we have decided to create a solution methodology that contains clustering, supplier and parking lot assignment, and routing phases.

The clustering phase of our model is restricted by different factors. Firstly, it is been known that the company fleet is constant and purchasing vehicles can take a long time. As a consequence, there is a fleet size restriction that our model should consider. Therefore, we restrict the cluster size and fix the number of clusters under this circumstance. After stations are grouped into clusters, a mathematical model is developed that considers two main cost components; transportation cost and procurement cost. As a result of the second phase, supplier and parking lots are determined for each cluster. In the last phase, for each cluster, supplier and parking lot assignment, a mathematical model is used for each group to create periodic routes where inventory levels of the stations are found.

4. MODEL DEVELOPMENT

In this study, mathematical models are developed to make supplier selection and route generation decisions to a distribution network. In the network, the most challenging issue is the presence of a high number of gas stations. To reduce the complexity of the mathematical models, we analyze the network in detail and notice that some stations are located close to each other. Hence, such stations are combined at this step and considered as a single service point in the following phases. This implies that the total demand of these stations is also aggregated. To determine which stations to combine we use a hierarchical clustering algorithm.

The main idea of the suggested approach is to take some decisions in the first two phases so as to find out which stations should be in the same route and which supplier should be used for this route. Then, vehicle routes are designed for each time period considering stations' inventory levels. There are some general assumptions that are made in the models as developed:

- Some stations are combined into service points in order to reduce the the size of the problem and the demand of new service points are considered as the total demand of combined stations.
- Demand and capacity of each service point is known and constant over planning horizon.
- The purchasing price of LPG is known for each supplier and it is constant through the year.
- Capacity restrictions are exist for each supplier and assumed to be constant through the year.
- Location of service points, suppliers and parking lots are known.
- Distance d between two locations (φ_1, λ_1) and (φ_2, λ_2) is calculated using the below formula:
  $a = \sin^2((\varphi_2 - \varphi_1)/2) + \cos\varphi_1 \cos\varphi_2 sin^2((\lambda_2 - \lambda_1)/2)$
  $c = 2 \arctan 2(\sqrt{a}, \sqrt{1-a})$

$d = Rc$

where φ is latitude, λ is longitude and R is earth's radius.

The following sections include the three phases of the suggested solution approach.

4.1. Phase 1: Clustering

Based on our review of the IRP literature, it turns out that neither exact nor heuristic approaches have been developed to solve a problem of this size. This encourages us to proceed with dividing the problem into subproblems, i.e., clusters of smaller sizes. There are multiple ways of generating clusters that can be used in different problems. However, we mainly focus on three clustering methods as follows:

(i) K-means clustering algorithm
(ii) Hierarchical clustering algorithm
(iii) Mathematical model-based clustering approaches

It is clear that each clustering method has its own advantages and disadvantages based on the application area. Even though each algorithm creates clusters using similarities, they have been developed with different aims. The most popular clustering algorithm is the k-means algorithm since it facilitates implementation and interpretation. In the k-means algorithm, the data set is partitioned into K clusters. Initially, each data point is assigned to the clusters randomly. Then for each data point, its distance from the cluster centers is calculated. If the data point is already assigned to its nearest cluster, it remains in this cluster. However, if the data point is not assigned to its closest cluster, it is moved into the closest cluster center. This procedure is repeated for each data point until all data points are assigned to their closest clusters.

Hierarchical clustering is divided into two groups: divisive or agglomerative. The divisive way known as a top-down clustering method, all data points are regarded as a single cluster initially and then clusters are separated by identifying most dissimilar clusters. Then, the least similar clusters are partitioned until all data points are assigned as a single cluster. In the agglomerative method, which is called as a bottom-top clustering method, each data item is assumed to be a cluster at the beginning of the procedure. Later, pairwise distances between clusters are calculated and the closest pair of clusters are gathered until all data points are gathered into a single cluster. [47]

Our proposed model is sensitive to the cluster size. However, in both clustering algorithms the cluster size is variable. Differences in cluster sizes prevent us to apply the same solution approach to every cluster in subsequent phases. Hence, rather than implementing k-means or hierarchical clustering algorithm, we proceed to this and, a mathematical model based on p-median problem is formulated as explained in below:

4.1.1. A Mathematical Model for Clustering Service Points

Sets, parameters and decision variables for the p-median model are given in Table 4.1. This model is developed with following assumptions:

- Previously generated service points are used.
- Coordinates of service points are assumed to be the average longitudes and latitudes of combined stations.
- The number of clusters is restricted by the fleet size.
- Cluster size is predefined in order to make the implementation easier.

Table 4.1. Sets and parameters for clustering phase.

Sets	**Definition**
I	Service point
Indices	**Definition**
i,j	Service point indices (i,j∈I)
Parameters	**Definiton**
n	Number of service points in a cluster
p	Number of clusters
$\overline{d}_{ij}$	Distance between service points i and j

$$X_{ij} = \begin{cases} 1, & \text{If service point } i \text{ is assigned to cluster } j, \\ 0, & \text{otherwise} \end{cases}$$

$$Y_j = \begin{cases} 1, & \text{If service point } j \text{ is selected as a cluster center,} \\ 0, & \text{otherwise} \end{cases}$$

$$\min \quad \sum_{i \in I} \sum_{j \in I} \overline{d}_{ij} X_{ij} \tag{4.1}$$

$$\sum_{j \in I} X_{ij} = 1 \qquad i \in I \tag{4.2}$$

$$\sum_{j \in I} Y_j = p \tag{4.3}$$

$$\sum_{i \in I} X_{ij} = n \qquad j \in I \tag{4.4}$$

$$X_{ij} \leq Y_j \qquad i, j \in I \tag{4.5}$$

$$X_{ij} \geq 0 \qquad i, j \in I \tag{4.6}$$

$$Y_j \in \{0, 1\} \qquad j \in I \tag{4.7}$$

The objective function (4.1) minimizes the sum of geographical distances between service points and their assigned cluster centers. Constraints (4.2) ensure that each service point is assigned to a single cluster. Constraint (4.3) gives the number of clusters. Constraints (4.4) state the number of service points assigned to each cluster center and cannot exceed n. Constraints (4.5) prevent assignment of service points to a nonexisting clusters.

The solution of this model gives p clusters each of which consists of at most n service points. The centroids of the clusters are considered as one single cluster point in the second phase where supplier and parking lot assignments are made.

4.2. Phase 2: Supplier Selection and Cluster Assignment Problem

In this phase, a mathematical model is solved in order to integrate decisions of procurement and distribution network components. We create the model with considering the following assumptions in addition to the general assumptions:

- The number of vehicles and vehicle types assigned to each parking lot is given and constant during a year.
- Travelling cost involves driver fee and other additional charges. It is assumed to be proportional to the distance traveled by the vehicle.
- In the currently used SCM system, different currencies are used for procurement activities and transportation costs. While converting US dollars to Turkish Lira a constant exchange rate provided by the company is used.
- The total cluster demand is assumed to be the sum of the demand of service points found in the cluster.
- One supplier can serve different clusters, but one cluster can only be served by a supplier.
- The cluster demand should be satisfied.
- It is assumed that purchased LPG is delivered directly to the cluster center. Distribution among service points is ignored. It means that there exist three transportation activities carried by vehicles:
  - (i) Parking lot - Supplier
  - (ii) Supplier - Cluster center

(iii) Cluster center - Parking lot

4.2.1. Mathematical Model for the Supplier Selection and Cluster Assignment Problem

The sets, parameters and decision variables used in the Supplier Selection and Cluster Assignment are given in Table 4.2, Table 4.3,and Table 4.4.

Table 4.2. Sets and indices for the supplier selection and cluster assignment problem

Sets	**Definition**
K	Clusters
L	Suppliers
M	Parking lots
Indices	**Definition**
k	Index set for clusters ($k \in K$)
l	Index set for suppliers ($l \in L$)
m	Index set for parking lots ($m \in M$)

Table 4.3. Parameters for the supplier selection and cluster assignment problem

Parameters	**Definition**
h_k	Demand of cluster k
b_l	Capacity of supplier l
d_{kl}	Distance between cluster k and supplier l
d'_{lm}	Distance between supplier l and parking lot m
d''_{km}	Distance between cluster k and parking lot m
$\overline{h}$	A big number
n_m	Number of lorries at parking lot m
n'_m	Number of trucks at parking lot m
α	Travelling cost per km in TL
β	Exchange rate of TL/USD
θ_l	LPG price per liter at supplier l (USD)
γ	Capacity of lorries
γ'	Capacity of trucks

Table 4.4. Decision variables for the supplier selection and cluster assignment problem

Decision variables
$U_{kl} = \begin{cases} 1, & \text{If demand of cluster } k \text{ is satisfied by supplier } l, \\ 0, & \text{otherwise.} \end{cases}$
V_{klm} = Amount of LPG sent from supplier l to cluster k with trucks in parking lot m
V'_{klm} = Amount of LPG sent from supplier l to cluster k with lorries in parking lot m

$$\min \quad \beta \sum_k \sum_l \sum_m \theta_l (V_{klm} + V'_{klm}) + \alpha \sum_k \sum_l \sum_m (d_{kl} + d'_{lm} + d''_{km})(\frac{V_{klm}}{\gamma} + \frac{V'_{klm}}{\gamma'}) \tag{4.8}$$

$$\sum_m \sum_l (V_{klm} + V'_{klm}) \geq h_k \qquad k \in K \tag{4.9}$$

$$\sum_k \sum_m (V_{klm} + V'_{klm}) \leq b_l \qquad l \in L \tag{4.10}$$

$$\sum_k \sum_l V_{klm} \leq 365 n_m \gamma \qquad m \in M \tag{4.11}$$

$$\sum_k \sum_l V'_{klm} \leq 365 n'_m \gamma' \qquad m \in M \tag{4.12}$$

$$\sum_m (V_{klm} + V'_{klm}) \geq \overline{h} U_{kl} \qquad k \in K, l \in L \tag{4.13}$$

$$\sum_l U_{kl} = 1 \qquad k \in K \tag{4.14}$$

$$V_{klm}, V'_{klm} \geq 0 \qquad k \in K, l \in L, m \in M \tag{4.15}$$

$$U_{kl} \in \{0, 1\} \qquad k \in K, l \in L \tag{4.16}$$

The objective function of (4.8) minimizes the total cost consisting of procurement and delivery activities. The first component represents the LPG procurement cost, while the second cost component is the delivery cost consisting of the vehicle travel from parking lots to suppliers, from suppliers to cluster centers and from cluster centers to parking lots. There is an approximation for number of deliveries. The number of total deliveries in a year is assumed to be proportional to the ratio of the LPG amount to be delivered to the vehicle capacities for both trucks and lorries separately. Constraints (4.9) ensure that for each cluster, demand is satisfied by using either trucks or lorries.

Constraints (4.10) guarantee that supplier capacities are not exceeded. Constraints (4.11) and constraints (4.12) give the amount of LPG delivered by different vehicle types(trucks and lorries). Constraints (4.13) ensure that the total system demand is satisfied. The last constraint (4.13) shows that each cluster is working with only one supplier.

As a result of the second phase, supplier selection and cluster-to-supplier and cluster-to-parking lot assignments are made. More specifically, at the end of the first two phases the following issues are resolved:

(i) Which service points are in the same cluster,
(ii) Which suppliers should provide LPG for each cluster,
(iii) Which vehicle type and which parking lot should be used to each cluster.

4.3. Phase 3: Inventory Routing Problem

The third phase involves the development of a mathematical model to generate routes for each cluster by respecting the inventory level at each service point in each time period. The following assumptions are made:

- Initial inventory levels are known and periodic demands are given for each service point.
- Capacities of the suppliers are sufficient enough to satisfy cluster demand for a 3-day period.
- Each service point is visited once in a time period.
- Service point demands should be satisfied, backlogging and loss of sale is not allowed.

4.3.1. Mathematical Model for Inventory Routing Problem

Sets, parameters, and decision variables for IRP model is explained in Table 4.5, Table 4.6, and Table 4.7

Table 4.5. Sets and indices for Inventory Routing Problem.

Sets	**Definition**
IC	Set of service points
I	Set of service points and suppliers
T	Set of time periods
Indices	**Definition**
i,j	Index for service points and supplier (i,$j \in I$)
t	Index set for time periods ($t \in T$)

Table 4.6. Parameters for Inventory Routing Problem

Parameters	**Definition**
a	Vehicle capacity
c_i	Capacity of service point i
d_{ij}	Distance between service points i and j
h_{it}	Demand of service point i in time period t
g_{i0}	Initial inventory of service point i
r_t	Amount of LPG available at the supplier in time period t

Table 4.7. Decision Variables for Inventory Routing Problem

Decision Variables
$W_{ijt} = \begin{cases} 1, & \text{if service point } j \text{ is visited after service point } i \text{ in time period } t, \\ 0, & \text{otherwise.} \end{cases}$
G_{it} = Inventory of service point or supplier i at the end of time period t, $i \in I$, $t \in T$
Q_{it} = Amount recieved to station i in time period t, $i \in IC$, $t \in T$
F_{it} = Load of vehicle when leaving point i

$$\min \quad \sum_i \sum_j \sum_t d_{ij} W_{ijt} \tag{4.17}$$

$$G_{it} = g_{i0} + r_t - \sum_{i \in IC} Q_{it} \qquad t \in T, i \in I \backslash IC \tag{4.18}$$

$$G_{it} = G_{it-1} + Q_{it} - h_{it} \qquad t \in T, i \in IC \tag{4.19}$$

$$G_{it} \leq c_i \qquad t \in T, i \in I \tag{4.20}$$

$$Q_{it} \leq c_i \sum_{j \in I, i \neq j} W_{ijt} - G_{it-1} \qquad t \in T, i \in IC \tag{4.21}$$

$$\sum_{i \in IC} Q_{it} \leq a \qquad t \in T \tag{4.22}$$

$$\sum_{i \in I, i \neq j} W_{ijt} = \sum_{i \in I, i \neq j} W_{jit} \qquad t \in T, j \in I \tag{4.23}$$

$$\sum_{i \in I, i \neq j} W_{ijt} = 1 \qquad t \in T, j \in I \backslash IC \tag{4.24}$$

$$F_{jt} - F_{it} + aW_{ijt} \leq a - Q_{jt} \qquad t \in T, i \in I, i \in IC \tag{4.25}$$

$$F_{it} \leq a \qquad t \in T, i \in I \tag{4.26}$$

$$W_{ijt} \in \{0, 1\} \qquad i \in I, j \in I, t \in T \tag{4.27}$$

$$Q_{it}, F_{it} \geq 0 \qquad i \in IC, t \in T \tag{4.28}$$

$$G_{it} \geq 0 \qquad i \in I, t \in T \tag{4.29}$$

The decisions made in the first two phases are used as parameters in this model. The objective function of the model aims to minimize the total cost of traveling activities of delivering LPG from supplier to service points for 3-days time period. Constraints (4.18) and (4.19) are defined to show the inventory balance at supplier and service points, respectively. Constraints (4.20) make sure that inventory amount at any service point should be less than the capacity of the service point. Constraints (4.21) ensure that delivery amount to a service point should be less than empty tank capacities of the service point in that specific time period. Constraints (4.22) state that the amount of delivery in a time period should not exceed the vehicle capacity. Constraints (4.23) guarantee that if a vehicle comes to service point j from any service point, it should depart from service point j to any service point in a time period. Constraints (4.24) identify the number of vehicles that return to supplier. Constraints (4.25) and (4.26) are defined as subtour elimination constraints.

5. RESULTS

As mentioned earlier, we analyze the problem in three phases: clustering, supplier and parking lot assignment and inventory routing. In each phase, the models are developed within the GAMS software 25.1.3 and solved in using Cplex 12.8.0.

Before proposing mathematical models, we use hierarchical clustering algorithm to reduce problem size. The effects of the the hierarchical algorithm can be seen in Figure 5.1. We obtain 535 new service points out of 810 gas station existing in the problem with using the algorithm.

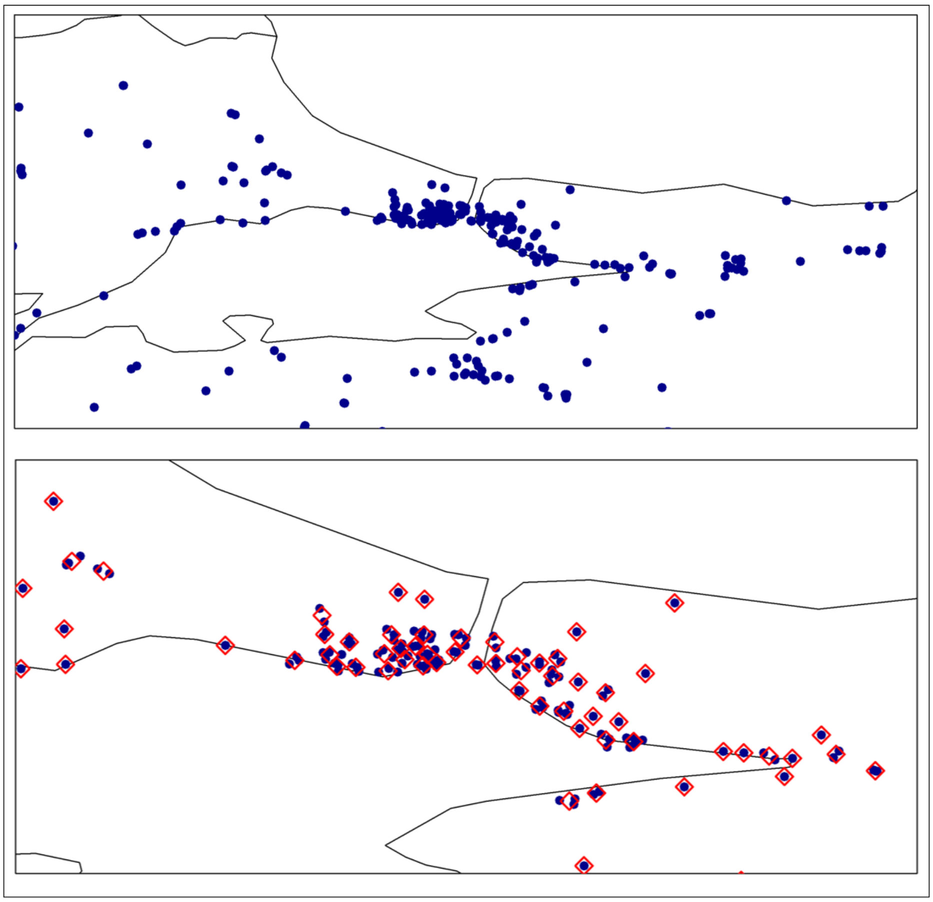

Figure 5.1. Reducing the problem size using a hierarchical clustering algorithm.

The first model is solved optimally and we obtain 107 clusters out of 535 service points existing at the beginning of the clustering phase of the problem. In Figure 5.2, each color represent a cluster in the Marmara Region. The resulting clusters can be seen in the figure for the region where the highest number of gas stations exists . After we obtain all clusters, we use the supplier and parking lot assignment model in order to proceed further with assignments. The model proposed in the second phase is solved for 36000 seconds and the result is obtained with 4.48% GAP. In Table 5.1, detailed information on clusters is given, i.e., which service points are gathered in each cluster, which suppliers provide LPG for each cluster, which vehicle type and which parking lot should be used to each cluster.

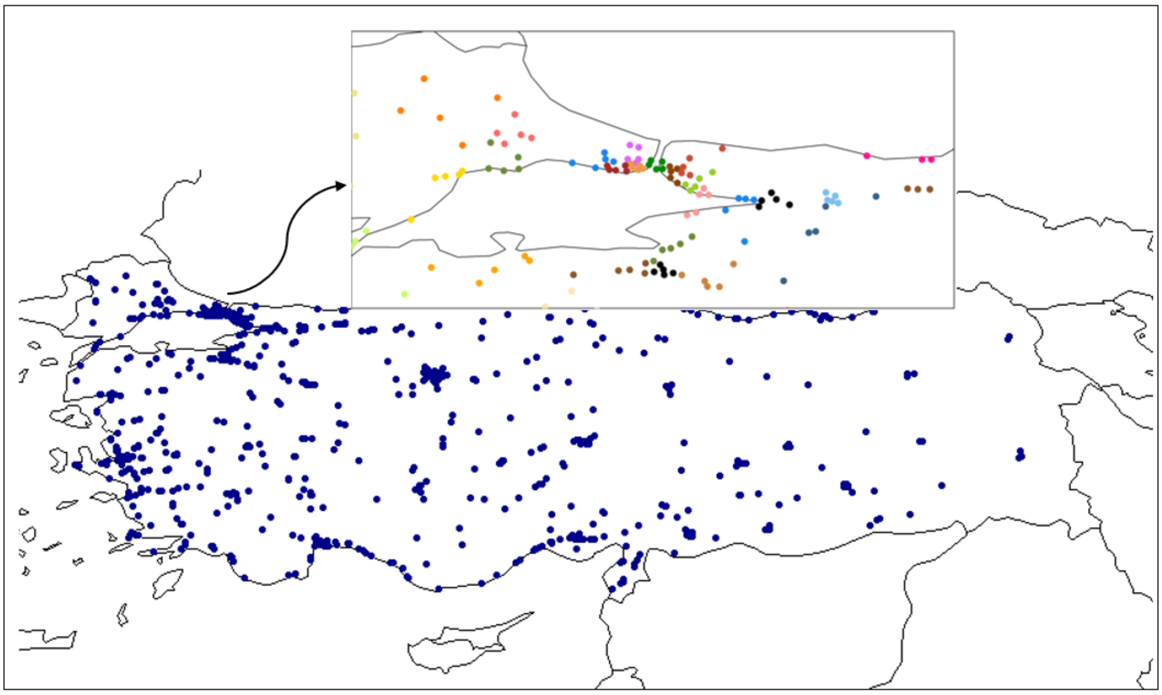

Figure 5.2. Clusters in Marmara Region created in the clustering phase.

Table 5.1 Result of the Supplier Selection and Cluster Assignment Problem

Cluster ID	Service Points	Supplier	Parking Lot	Vehicle Type
1	1,270,336,343,519	44	2	Lorry
2	5,10,15,33,276	41	1	Lorry
3	11,137,156,375,479	41	1	Lorry
4	19,346,450,499,521	20	6	Truck
5	24,39,210,267,361	17	6	Truck
6	26,123,180,475,515	20	6	Truck
7	29,34,42,208,382	20	6	Lorry
8	8,35,147,150,437	41	1	Lorry
9	27,36,175,381,525	17	6	Truck
10	47,124,255,359,423	5	11	Lorry
11	50,53,55,148,389	34	4	Truck
12	52,117,294,303,397	5	11	Lorry
13	30,57,250,415,439	5	11	Truck
14	17,25,58,178,310	17	6	Truck
15	2,59,192,232,535	44	2	Truck
16	49,60,63,311,446	34	4	Truck
17	61,128,316,435,481	41	1	Lorry
18	62,368,426,469,486	41	1	Lorry
19	65,102,291,351,365	12	6	Lorry
20	72,97,328,429,490	44	2	Lorry
21	80,96,109,112,355	41	1	Lorry
22	88,400,407,413,513	25	3	Truck
23	89,201,223,260,261	20	8	Lorry
24	95,195,222,330,487	25	3	Truck
25	79,106,253,360,472	5	11	Lorry
26	98,99,116,231,409	25	3	Truck
27	14,119,145,273,440	20	6	Truck

Table 5.1. Result of the Supplier Selection and Cluster Assignment Problem (cont.)

Cluster ID	**Service Points**	**Supplier**	**Parking Lot**	**Vehicle Type**
28	118,125,372,464,496	48	1	Lorry
29	132,216,268,269,374	20	6	Lorry
30	77,138,170,233,436	20	6	Lorry
31	129,139,183,213,337	20	8	Lorry
32	142,143,272,364,444	17	10	Lorry
33	135,149,455,483,504	41	1	Lorry
34	78,152,456,461,491	41	1	Lorry
35	86,153,157,246,362	5	11	Lorry
36	22,159,202,219,322	20	8	Lorry
37	31,94,104,161,168	5	11	Lorry
38	21,158,162,275,405	17	6	Truck
39	163,248,252,416,524	17	6	Truck
40	6,12,166,188,295	20	6	Truck
41	171,179,182,193,289	17	10	Lorry
42	32,40,181,287,473	5	11	Truck
43	3,4,141,190,277	44	2	Lorry
44	164,191,421,432,485	23	3	Truck
45	197,297,332,347,378	17	10	Lorry
46	172,207,309,366,476	41	1	Lorry
47	199,211,212,235,325	54	2	Lorry
48	66,177,214,259,417	20	8	Lorry
49	18,185,198,200,215	41	1	Lorry
50	218,224,237,324,335	20	6	Lorry
51	74,220,299,484,509	44	2	Lorry
52	126,221,229,312,420	25	3	Truck
53	131,133,136,230,274	20	8	Lorry
54	107,236,240,428,506	5	11	Lorry
55	54,67,73,241,242	5	11	Lorry

Table 5.1. Result of the Supplier Selection and Cluster Assignment Problem (cont.)

Cluster ID	**Service Points**	**Supplier**	**Parking Lot**	**Vehicle Type**
56	243,278,292,408,470	53	1	Lorry
57	184,247,399,445,460	25	3	Truck
58	251,258,395,451,453	5	11	Lorry
59	203,225,266,284,334	20	6	Lorry
60	43,227,279,302,477	44	2	Lorry
61	20,130,280,339,349	41	1	Lorry
62	173,244,283,285,340	20	6	Truck
63	288,305,376,443,480	44	2	Truck
64	155,169,293,320,505	5	11	Truck
65	194,300,301,454,526	41	1	Lorry
66	144,314,323,329,357	44	2	Lorry
67	176,245,331,503,517	17	10	Lorry
68	37,84,204,209,333	20	6	Lorry
69	121,151,338,462,465	41	1	Lorry
70	167,234,296,342,391	44	2	Lorry
71	48,304,345,379,508	5	11	Lorry
72	350,377,404,452,489	41	1	Lorry
73	83,140,217,341,352	44	2	Lorry
74	44,45,146,353,356	34	4	Truck
75	85,115,358,449,518	41	1	Lorry
76	82,90,108,319,363	25	3	Truck
77	254,263,370,433,528	20	6	Truck
78	16,189,371,393,510	41	1	Lorry
79	290,321,373,402,493	5	11	Truck
80	69,71,76,383,467	41	1	Lorry
81	111,308,384,387,401	5	11	Truck
82	281,386,431,468,533	48	1	Lorry
83	56,68,390,392,530	5	11	Lorry

Table 5.1. Result of the Supplier Selection and Cluster Assignment Problem (cont.)

Cluster ID	**Service Points**	**Supplier**	**Parking Lot**	**Vehicle Type**
84	91,92,93,394,534	23	3	Truck
85	127,187,388,396,447	41	1	Lorry
86	23,103,165,315,410	17	8	Lorry
87	38,114,367,398,411	5	11	Lorry
88	64,239,307,318,422	44	2	Lorry
89	105,256,317,425,438	20	6	Lorry
90	28,46,113,122,457	5	11	Truck
91	238,424,442,458,459	5	11	Lorry
92	154,430,434,448,463	44	2	Lorry
93	9,174,262,265,466	20	6	Truck
94	120,354,403,471,482	5	11	Lorry
95	7,13,326,474,501	20	6	Lorry
96	160,226,257,327,478	17	6	Truck
97	228,419,488,522,523	53	2	Lorry
98	134,306,344,492,498	5	11	Lorry
99	186,196,427,495,500	17	10	Lorry
100	41,205,298,385,502	17	8	Lorry
101	101,110,286,414,507	41	1	Lorry
102	87,282,313,380,512	20	8	Lorry
103	70,81,100,511,514	20	6	Lorry
104	271,348,369,406,516	5	11	Lorry
105	51,75,206,249,520	5	11	Lorry
106	264,418,527,529,532	20	6	Truck
107	412,441,494,497,531	53	1	Lorry

For the phase three, we apply the proposed model for each cluster-supplier-parking lot pairs. We run the model for each cluster for 3 day time period and set the time limit as 360 seconds. The results are given in Table 5.2 for all clusters. It is seen that the proposed model gives optimal solution for most of the clusters in the allowed time limit of 360 seconds.

It is seen that, some the clusters cannot be solved optimally. Therefore, we increase the allowed time limit and set as 3600 seconds. Obtained results are shown in Table blahblah.

As a result of these three phased solution approach, an integrated decision making mechanism is developed. The aim of the multi-phased solution approach is preventing occurrence of hidden cost components. With this approach, any interaction among the procurement and delivery components are considered and vehicle routes are generated for company's fleet.

Table 5.2 Result of the Inventory Routing Phase

Cluster ID	**Supplier**	**Parking Lot**	**Vehicle Type**	**Time(s)**	**GAP**
1	44	2	Lorry	0.2	-
2	41	1	Lorry	167.8	-
3	41	1	Lorry	475.5	-
4	20	6	Truck	0.3	-
5	17	6	Truck	360.0	12.2%
6	20	6	Truck	1.3	-
7	20	6	Lorry	212.7	-
8	41	1	Lorry	475.5	-
9	17	6	Truck	360.0	15.1%
10	5	11	Lorry	90	-
11	34	4	Truck	306.3	-
12	5	11	Lorry	473.8	-

Table 5.2. Result of the Inventory Routing Phase (cont.)

Cluster ID	Supplier	Parking Lot	Vehicle Type	Time(s)	GAP
13	5	11	Truck	0.2	-
14	17	6	Truck	360.0	16.9%
15	44	2	Truck	0.3	-
16	34	4	Truck	0.2	-
17	41	1	Lorry	354.4	-
18	41	1	Lorry	360.0	27.5%
19	12	6	Lorry	0.2	-
20	44	2	Lorry	0.2	-
21	41	1	Lorry	0.1	-
22	25	3	Truck	0.2	-
23	20	8	Lorry	277.1	-
24	25	3	Truck	23.2	-
25	5	11	Lorry	0.1	-
26	25	3	Truck	0.4	-
27	20	6	Truck	41.3	-
28	48	1	Lorry	360.0	10.7%
29	20	6	Lorry	360.0	27.2%
30	20	6	Lorry	346.6	-
31	20	8	Lorry	313	-
32	17	10	Lorry	360.0	29.0%
33	41	1	Lorry	360.0	20.0%
34	41	1	Lorry	14.6	-
35	5	11	Lorry	40.6	-
36	20	8	Lorry	360.0	20.1%
37	5	11	Lorry	1.9	-
38	17	6	Truck	360.0	23.4%
39	17	6	Truck	28.4	-
40	20	6	Truck	0.2	-

Table 5.2. Result of the Inventory Routing Phase (cont.)

Cluster ID	**Supplier**	**Parking Lot**	**Vehicle Type**	**Time(s)**	**GAP**
41	17	10	Lorry	360.0	28.4%
42	5	11	Truck	131.3	-
43	44	2	Lorry	14.1	-
44	23	3	Truck	28.6	-
45	17	10	Lorry	0.1	-
46	41	1	Lorry	360.0	12.6%
47	54	2	Lorry	0.1	-
48	20	8	Lorry	155.8	-
49	41	1	Lorry	0.2	-
50	20	6	Lorry	242.5	-
51	44	2	Lorry	23	-
52	25	3	Truck	8.9	-
53	20	6	Lorry	312.1	-
54	5	11	Lorry	244.5	-
55	5	11	Lorry	290.3	-
56	53	1	Lorry	0.2	-
57	25	3	Truck	450.7	-
58	5	11	Lorry	453.7	-
59	20	6	Lorry	468.1	-
60	44	2	Lorry	0.2	-
61	41	1	Lorry	360.0	14.1%
62	20	6	Truck	297.5	-
63	44	2	Truck	0.2	-
64	5	11	Truck	6.1	-
65	41	1	Lorry	310.2	-
66	44	2	Lorry	0.2	-
67	17	8	Lorry	360.0	5.5%
68	20	6	Lorry	19.1	-

Table 5.2. Result of the Inventory Routing Phase (cont.)

Cluster ID	**Supplier**	**Parking Lot**	**Vehicle Type**	**Time(s)**	**GAP**
69	41	1	Lorry	157.3	-
70	44	2	Lorry	0.1	-
71	5	11	Lorry	0.2	-
72	41	1	Lorry	432.1	-
73	44	2	Lorry	360.0	10.1%
74	34	4	Truck	298.2	-
75	41	1	Lorry	0.1	-
76	25	3	Truck	0.4	-
77	20	6	Truck	91.9	-
78	41	1	Lorry	0.2	-
79	5	11	Truck	35.9	-
80	41	1	Lorry	202.3	-
81	5	11	Truck	0.4	-
82	48	1	Lorry	0.2	-
83	5	11	Lorry	0.2	-
84	23	3	Truck	0.1	-
85	41	1	Lorry	360.7	-
86	17	8	Lorry	360.0	22.8%
87	5	11	Lorry	0.1	-
88	44	2	Lorry	0.2	-
89	20	6	Lorry	49.2	-
90	5	11	Truck	379.8	-
91	5	11	Lorry	159.2	-
92	44	2	Lorry	360.0	25.8%
93	20	6	Truck	360.0	16.4%
94	5	11	Lorry	0.2	-
95	20	6	Lorry	107.4	-
96	17	6	Truck	198.3	-

Table 5.2. Result of the Inventory Routing Phase (cont.)

Cluster ID	**Supplier**	**Parking Lot**	**Vehicle Type**	**Time(s)**	**GAP**
97	53	1	Lorry	63.8	-
98	5	11	Lorry	138.5	-
99	17	10	Lorry	0.3	-
100	17	8	Lorry	155.9	-
101	41	1	Lorry	0.3	-
102	20	8	Lorry	231.8	-
103	20	6	Lorry	242.5	-
104	5	11	Lorry	360.0	19.2%
105	5	11	Lorry	54.9	-
106	20	6	Truck	7.2	-
107	53	1	Lorry	0.2	-

6. CONCLUSION AND FUTURE RESEARCHES

The main motivation of this study is a problem faced by a global oil company's Turkey facility. The company wants to implement an integrated decision-making mechanism for their LPG procurement and distribution system. To the best of our knowledge, there are no similar studies in the literature that combine acquisition and distribution decisions for any industrial application. In this respect, the problem will also be a pioneer for other industrial application problems where it is necessary to consider the effect of more than one decision mechanism on each other. Additionally, the problem has more than 50 suppliers, 11 parking lot, 105 vehicles, and more than 800 stations, where all have different features, and the number of these elements increase the size of the problem. Since the problem has a considerable amount of parameters and variables, the exact solution methods and even most of the heuristic applications are insufficient to provide a viable solution. We decided to divide the main problem into subproblems in order to facilitate the application of the solution approach.

To reduce the problem size, we create clusters with gathering some of the service points together, and these clusters are considered as individual subproblems in following phases. In the second phase of the solution approach, an assignment model is proposed where a supplier and parking lot assignment is done for each cluster. In the final phase, a mathematical model is proposed and it is run for a 3-days period for each supplier-cluster-parking lot match.

The main aim of the company is to run the proposed IRP model before planning daily routes with the last inventory information obtained from gas stations and create vehicle routes for its fleet. To have a reasonable computation time for each run, this study is carried with small-sized clusters. This cause us to achieve a sub-optimal solution. However, if the effect of the cluster size on the computation time is decreased, more cost-efficient solution can be obtained. Therefore, further analyses in the can be done by changing the size of the clusters. When the cluster size is increased, another heuristic solution approaches may be applied in order to combine three phases of the problem. Hence, possible heuristic approaches which can solve IRP for larger clusters can be adapted.

There is a demand rate change due to some reasons such as weekends, summer months, and holidays. However, proposed models neglect the variances on demand and demand of stations are assumed to be constant for a time period. A robust solution approach which consists demand uncertainty can be proposed as an extension to the currently proposed model.

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