DESIGN OF COMPOSITE STRUCTURES FOR MINIMUM WEIGHT

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To my family

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ABSTRACT

DESIGN OF COMPOSITE STRUCTURES FOR MINIMUM WEIGHT

The goal of this study is to develop methodologies to optimize the structural design of composite materials to achieve the minimum weight. Mainly three different problem areas were considered. Firstly, weight minimization of laminated composite plates subjected to in-plane loading; secondly, as an extension of the first problem, weight minimization of laminated composites subjected to in-plane and out-of-plane loads; thirdly, optimal design of laminated composite plates with notches using progressive failure approach.

Considering that composite materials are generally used in applications where weight is critical and considering their high cost, in this study, designs with minimum material use were aimed. Fiber orientation angle and number of plies in each lamina were used as design variables. The maximum stress and Tsai-Wu criteria were used individually or together to predict static failure. Different geometries and loading conditions were considered.

Because the problems considered in this study contain numerous local optimums, a global search algorithm, Simulated Annealing, was used as the optimization algorithm. A number of modifications were proposed to improve the reliability of the algorithm.

ÖZET

KOMPOZİT YAPILARIN EN AZ AĞIRLIK İÇİN TASARIMI

Bu çalışmanın amacı en az ağırlığı elde etmek için kompozit malzemelerin yapısal tasarımlarını en iyileyecek metodolojilerin geliştirilmesidir. Temelde üç farklı problem alanı göz önünde bulundurulmuştur, ilk olarak düzlem içi yüklemeye maruz katmanlı kompozit plakaların ağırlık azaltması; ikincisi, ilk problemin bir uzantısı olarak, hem düzlem içi hem de düzlem dışı yüklemeye maruz katmanlı kompozitlerin ağırlık azaltması ve üçüncü olarak, çentik ihtiva eden katmanlı kompozit plakaların progresif hasar yaklaşımı kullanarak optimum tasarımıdır.

Kompozit malzemelerin genellikle ağırlığın kritik olduğu uygulamalarda kullanıldığını ve yüksek maliyetlerini düşünerek, bu çalışmada, en az malzeme kullanan tasarımlar amaçlanmıştır. Fiberlerin oryantasyon açıları ve her bir laminedeki tabaka sayıları tasarım değişkenleri olarak kullanılmıştır. Statik hasarı tespit edebilmek için maksimum gerilme ve Tsai-Wu kriterleri ayrı ayrı ya da birlikte kullanılmıştır. Farklı geometriler ve yükleme koşulları göz önünde bulundurulmuştur.

Bu çalışmada ele alınan problemler çok sayıda yersel optimumlar içerdiği için, global bir arama algoritması olan, Tavlama Simülasyonu optimizasyon algoritması olarak kullanılmıştır. Algoritmanın güvenilirliğini iyileştirmek için birçok değişiklikler önerilmiştir.

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LIST OF SYMBOLS/ABBREVIATIONS

a	First coefficient of the Tsai-Wu quadratic expression
b	Second coefficient of the Tsai-Wu quadratic expression
c_i	Coefficients used in objective functions
d	hole diameter
f	Cost value in DSA
f_h	Highest cost in the current set in DSA
f_l	Lowest cost in the current set in DSA
f_t	Cost of newly generated configuration
h	The height of the laminate
k	Lamina number counted from the bottom
m	The number of distinct fiber orientation angles
m	The number of distinct lamina
n	Number of design variables
n_k	Number of plies
$n_k^{'}$	Ply number of the newly generated configurations
r_i	Randomly chosen real number in DSA
t	Thickness of the laminate
t_0	Thickness of an individual ply
w_i	Suitable coefficients for safety factors

A_t	Acceptability of a newly generated trial configuration in DSA
A_{ij}	Components of extensional stiffness matrix
B_{ij}	Bending extension coupling stiffness components
D_{ij}	Bending stiffness components
E_1	Modulus of elasticity in 1 direction
E_2	Modulus of elasticity in 2 direction
F	Value of the cost function
G_{12}	Shear Modulus
L^j	The number of trials executed in the j^{th} Markow chain
L_a^j	The number of accepted configurations
L	Number of initial configurations
M_{xx}	Bending moment in x direction
M_{yy}	Bending moment in y direction
M_{xy}	Twisting moment in xy direction
N_{xx}	Uniformly distributed in plane load in x direction
N_{yy}	Uniformly distributed in plane load in y direction
N_{xy}	Uniformly distributed in plane load in xy direction
P_{MS}	Penalty value calculated based on maximum stress
P_{TW}	Penalty value calculated based on Tsai-Wu
Q_{ij}	Principal stiffness components

$\overline{Q_{ij}}$	Off-axis stiffness components
SF_{MS}	Safety value maximum stress
SF_{MS}^{k}	Safety factor maximum stress for the kth laminate
SF_{TW}	Safety value Tsai-Wu
SF_{TW}^{k}	Safety factor Tsai-Wu for the kth laminate
S	Shear strength
T	Current temperature in DSA
T_j	Temperature (or control) parameter in DSA
X_c	In-plane normal strength in compression
X_t	In-plane normal strength in tension
Y_c	In-plane transverse strength in compression
Y_t	In-plane transverse strength in tension
$\Delta n \max$	Maximum variation of laminate thickness
$\Delta\theta_{\max}$	Maximum variation of fiber orientation
α_j	The temperature reduction factor
ε_{xx}	Strain in x direction
ε_{xx}^{0}	Mid-plane strain in x direction
ε_{yy}	Strain in y direction
ε_{yy}^{0}	Mid-plane strain in y direction
ϕ	The angles between consecutive angles

γ_{xy}	Strain in xy direction
γ_{xy}^{0}	Mid-plane strain in xy direction
κ_{xx}	Curvature in x direction
κ_{yy}	Curvature in y direction
κ_{xy}	Curvature in xy direction
ν	Poisson's ratio
θ	Fiber orientation angle
θ_k	Fiber orientation angle in the k^{th} layer in DSA
θ_k'	Fiber orientation angle of newly generated configuration in DSA
σ_{11}	Stress at principal direction 11
σ_{22}	Stress at principal direction 22
τ_{12}	Stress at principal direction 12
σ_{xx}	Stress component in x direction
σ_{xx}^{k}	Off-axis stress component in x direction
σ_{yy}	Stress component in y direction
σ_{yy}^{k}	Off-axis stress component in y direction
σ_{xy}^{k}	Off-axis tress component in xy direction
τ_{xy}	Stress component in xy direction
CLT	Classical Lamination Theory

DSA	Direct Simulated Annealing
FEM	Finite Element Method
SA	Simulated Annealing

1. INTRODUCTION

In the past decades, use of laminated composite materials has increased in industry especially in the aerospace industry due to their high specific strength and stiffness. Despite the low density, because continuous fiber reinforced composites are demanded in weight-critical application, there is still a strong need to further minimize the weight of the composite structures by optimizing their designs. Besides, considering that composites have very high cost, significant savings in material cost could be achieved by minimizing the weight. In this context, the minimum weight design of composite laminates has been studied by many researchers [1- 23]. In the present study, an attempt was made to minimize the weight of the laminated composite structures having various geometrical patterns and subject to in - plane and out - of - plane static loading conditions.

During their life cycles, composite structures may undergo several types of loading and failure mechanisms. Buckling[23- 26], vibration[9, 27], fatigue, static failure [1-3] are the main failure mechanisms. Within the scope of this study, only static failure of composite laminates is taken into consideration. Accordingly, two most widely used static failure criteria, maximum stress and Tsai-Wu, were used. Lamina thickness and the orientation angles of the fibers in each lamina were chosen as the optimization variables. In many studies, layer thickness [1 - 4, 7, 9-12, 15, 17, 28] and ply angles [3, 4, 7, 9-11, 13, 15, 17, 24, 29-34] were considered to be continuous design variables. However in practice, composite laminates are fabricated using prepregs with a specific thickness. Besides, fiber orientations are chosen from a finite set of angles during the design process because of the difficulty of exactly orienting fibers along a given direction. If layer thickness and fiber orientation angles are taken as continuous variables in an optimization process, the optimum values should be converted to the nearest discrete manufacturable values. In that case, the resulting design may not be optimal; besides constraints may be violated. Due to these manufacturing constraints, the design variables for a fiber angle or layer thickness should take only discrete values. As opposed to a zero order search algorithm, a gradient based optimization procedure may fail to cope with the discrete nature of such problems. The main difficulty in composite optimization problems is the existence of immense number of locally optimum designs. With a high number of design variables, local optimums dramatically increase in number. Since a locally optimum design may significantly be worse than the globally optimal design, an effective optimization procedure should aim to find the global optimum. Deterministic local search algorithms [4, 9-12, 19, 21, 35-46], which may be coupled with a multi-start optimization approach [47], analytical methods [48] and parametric studies [49] are not viable approaches unless the design variables are scarce or just an improvement over the current design is desired. In some of the previous studies, global search algorithms were used like genetic algorithms [18, 19, 28, 29, 49-53], ant colony optimization [54] and branch and bound [55]. On the other hand, simulated annealing (SA), which is known to be one of the most reliable search algorithms in locating the globally optimal point, found few applications in composites optimization [20, 25, 27, 56, 57]. In this study, a variant of the simulated annealing algorithm known as direct simulated annealing (DSA) was used as search algorithm. As discussed in detail in the following sections, certain modifications were also introduced to DSA algorithm in order to increase its effectiveness. In addition to regular rectangular laminates, composite structures containing circular holes were also investigated. Contrary to regular rectangular laminates it was observed that a new failure approach is needed to effectively minimize the weight of notched composites laminates. In this respect, a new algorithm using progressive failure approach was introduced in this study.

In the first part of the study, which is presented in Chapter 4, weight (or thickness) of laminated composite plates subject to in-plane loading were minimized. Structural analysis of laminates was carried out using Classical Lamination Theory (CLT). The structure was a thin rectangular laminated plate containing no shape irregularities and the loading condition was only in-plane normal and shear forces. Therefore CLT was selected as the most appropriate stress analysis method. Layer thicknesses and the orientation angles in each lamina were considered as the optimization variables.

As the objective of the problem is to minimize the thickness (or weight) of the laminate, the decrease of the load carrying capacity of the composite structure is inevitable during optimization. For this reason, a constraint is imposed to avoid failure of the final design. Because the fracture of even a single layer in a composite laminate should be considered as a significant damage even if it does not lead to collapse of the structure, first ply failure approach was adopted. The previous researchers adopted the first-ply-failure approach using Tsai-Wu [18, 19, 29, 41-45, 47, 49, 51, 53, 58-61], Tsai-Hill [4, 11, 35, 40, 50], the maximum stress [49, 61], or the maximum strain [62] static failure criteria. In this study two static failure criteria, Tsai-Wu and the maximum stress, were tried. However, individual use of these criteria turned out to yield unreliable results. Tsai-Wu criterion, which is one of the most reliable static failure criteria, as well as the maximum stress criterion lead to false optimum designs for some loading cases and lay-ups due to the particular feature of their failure envelopes. In some studies [30, 31], in order to avoid this condition, the compressive strength of the material was taken the same as its tensile strength. In a design process, assuming the material to be weaker than it actually is leads to overly conservative designs. For this reason, this assumption is against the purpose of design optimization. In order to overcome this difficulty, it is proposed in this study that Tsai-Wu criterion be employed together with the maximum stress criterion and satisfaction of both criteria be observed.

In the second part of the study, which is presented in Chapter 5, design optimization of laminated composites subject to in-plane and out-of-plane loads was considered. The main difference between the first part of the study and this part is that not only in-plane forces but also out-of-plane loads, bending and twisting moments, were taken into account. While minimizing the weight or thickness of composite structures, the designers need to consider all the design parameters, loading conditions, failure modes and computational assumptions. In typical engineering applications, composite structures are under various types of loading conditions, not only in-plane loads but also out-of-plane loads such as auto-body chassis or airplane fuselage and wings. In this respect, a model accounting for only in-plane loads fails to capture the physics of the phenomena; as a result it is essential to consider all the loading types in the analysis and design optimization of composite plates. However, in most of the studies on the optimization of laminated composite plates only in-plane loads were considered. The studies accounting for out-of-plane loads [4, 10-12, 18, 19, 28, 29, 35-55, 58-64], either being bending and/or twisting moments [18, 28, 35, 42-45, 48, 51, 52, 55, 58, 60-63] or transverse loads [4, 10-12, 19, 29, 36-41, 43, 46, 47, 49, 50, 53, 54 , 59, 64], were relatively rare. In some of these studies the objective was to minimize thickness [4, 10-12, 19, 41, 42, 47, 58], weight [35, 36, 39, 43, 52, 59,60], cost and weight [46, 50, 61], thickness and change in the strain energy [62], or maximize the static strength of composite laminates for a given thickness [10, 28, 29, 40, 44, 50, 53, 55], strength-to-weight ratio [28], stiffness [12, 28, 36, 37, 38, 39, 41, 45, 48, 54, 63], energy absorption capacity [49] twisting angle at plate tip to reduce aerodynamic loading [18], or maximizing static strength while minimizing weight [51]. In the present study, laminate thickness was minimized; but by modifying the objective function, the same optimization procedure could be applied to design optimization problems in which different criteria are used for the effectiveness of the laminate design. Although plain laminates are considered in this study, the algorithm can be applied to hybrid laminates or sandwich plates by introducing minor changes to the optimization algorithm. In the second part of this study, again the classical lamination theory was used in the structural analysis. When a laminate is subject to out-of-plane loads not only existence of a layer with a specific thickness and fiber orientation, but also its position in the laminate affects its structural response. This means that stacking sequence of the laminate should also be optimized. In that case, the difficulty of the optimization problem is considerably increased; so a highly reliable global search algorithm is needed. For this purpose, many features of the DSA algorithm were modified to increase its reliability. After these modifications, the search algorithm developed in this study can be considered by itself a new variant of the classical simulated annealing algorithm. Basic features of SA, which are the use of a temperature parameter to control convergence and evaluation of acceptability based on Boltzmann distribution [65], were adopted. Besides, a population of current configurations was used instead of a single current configuration as in the direct search simulated annealing (DSA) [66]. On the other hand, a number of modifications were introduced in the generation mechanism of new configurations, replacement scheme of accepted configurations, and the reduction scheme (or cooling schedule) for the temperature parameter. Through these changes, first of all, the reliability and efficiency of SA algorithm were increased. Secondly, convergence was made dependent on a single parameter. SA like the other stochastic global search algorithms requires many trials in order to thoroughly search the feasible domain. Effectiveness of the search process depends on the values of the parameters defining the cooling schedule, replacement scheme, and generation mechanism. Finding appropriate values for the parameters suitable for the problem at hand is time consuming. By making all the parameters dependent on a single parameter, adaptation of the algorithm to other problem areas is made easier. After introducing these changes, global optimization of composite laminates with a much larger number of distinct laminae, i.e. with a much larger design domain, in comparison to the previous studies, can reliably be achieved.

The last part of this study is on the optimal design of laminated composite plates with notches using progressive failure approach, which is covered in Chapter 6. In this part of the study, a more complicated geometrical shape, a rectangular laminated plate with a circular hole, was considered. There were studies, which considered the design of laminated plates with circular opening. Anlas and Tüzer [67] studied the design of a symmetric laminated composite plate with a circular opening subject to in-plane loading. In the calculation of failure loads the concept of characteristic length was used in that study. This method has some limitations and difficulties such that the characteristic length around the hole depends on the laminate dimensions, stacking sequence, material properties and also loading. Even after the correction of this length on the basis of the above mentioned parameters, the optimization procedure and the calculation scheme is valid mainly for uniaxial loading. Khosravi and Sedaghati [68] presented a study for the optimum design of a rectangular plate with a central hole. In this study Tsai-Hill failure criterion was used without adopting a progressive failure approach. In another study, Cho and Rowlands [69], succeeded in reducing tensile stress concentrations in a perforated laminated composite by using a genetic algorithm based optimization method to optimally orient the fibers locally. Whitney and Nuismer [70] developed two related criteria based on normal stress distribution for predicting the uniaxial tensile strength of laminated composites containing through-the-thickness discontinuities. Here the loading was restricted to only uniaxial. Huang and Haftka [71] optimized locally the orientation of fibers around the hole in a composite plate to obtain superior load carrying capacity. Since in many real engineering applications, the designers encounter practical usage of riveting the composite plates, bolting hole openings of filament winded composite pipes or in many other applications, it is essential to apply mechanical stress calculations and to minimize the material expenditures through the weight reduction. In order to serve the above mentioned purposes and achieve weight minimization in composite structures including circular holes, an optimization procedure is presented in this study. Through the introduction of combinational usage of DSA (Direct simulated annealing) optimization scheme, progressive failure approach and finite element method a new synthesis is made, which does not exist in any of the previously mentioned studies in the literature.

Because of the local nature of the circular notch in the composite plate, layers do not undergo a total fracture but just local damage occurs around the notch, therefore use of the first-ply failure approach is not appropriate. The number of studies dealing with the optimization of composite laminates including notch [67-72] is quite limited. And among them only a few [72, 73] benefitted from the progressive failure approach. In the study [72] by Venkataraman progressive failure analysis of composite laminate was carried out. The idea presented in the paper is to load the composite structure until a fracture occurs then the stiffness of the failed plies is degraded using the discrete ply discounting method discussed in that study. The stiffness reduction leads to a change in ply stress due to load redistribution. Following the degradation, the structure is reloaded and the plies are checked for further failure, this procedure goes on until no more plies fail. Although the basic idea is similar to the approach in the present study, its applicability is quite restricted as no hole or other types of notches is taken into consideration. When these types of geometrical irregularities exist there is no possibility of removing only the failed portions of the plies since in that study the ply as a whole is eliminated from the structure. Harik [73] also benefitted from progressive ply failure approach. In that study progressive laminate damage is modeled with a maximum strain-based ply failure criteria and a ply modulus discount method. When a strain failure in a ply is detected, the load carried by the ply is removed and subsequently redistributed to unfailed plies in the laminate. Incremental loading is continued until the number of plies in the laminate reaches a point to where the laminate cannot sustain load without undergoing excessive deformation or strain. As to the present study, the geometry involves circular discontinuities and multi-axial load application is possible. A new algorithm for the application of progressive failure approach was developed and applied in composite optimization. In order to investigate the effect of the circular notch in the weight minimization process, a computer program was developed using ANSYS parametric design language to implement the optimization algorithm and carry out the structural analyses through finite element method as the geometric complexity did not allow the CLT to be benefitted from.

There are also design constraints that should be satisfied while minimizing the total weight of the composite structure. The stress state occurring throughout the composite plate has to be kept below acceptable levels. The allowable stress state is computed through Tsai-Wu and it is taken into account in the objective function by means of penalty function method. To the extent of the authors knowledge, it is the first time with this study that a progressive failure approach instead of first ply failure criterion is adopted and implemented in the Tsai-Wu stress failure calculation in an optimization scheme. Although this approach is valid for the composite laminates without any discontinuity and with smooth and uniform stress distributions, it is almost necessary to employ Progressive failure approach for structures undergoing non-uniform stress states due to geometric discontinuities inside them. By means of progressive failure criterion, just small perturbations in the whole laminate under the effect of high stress levels over the strength level is eliminated and the remaining large portion continues to carry the applied load. This way it is more likely to end up with thinner laminates and obtain lighter designs.

2. ANALYSIS OF LAMINATES USING CLASSICAL LAMINATION THEORY

2.1 Analysis of Composite Laminates Subject to in-Plane and out-of-Plane Loads

The classical lamination theory is used to analyze the mechanical behavior of the composite laminate. We assume that plane stress condition is valid for each ply. Accordingly, out-of-plane stress components are taken as zero. Even though the laminate is subject to out-of-plane loads, each ply is assumed to be under plane stress condition and, therefore, out-of-plane stress components are zero ($\sigma_{zz} = \tau_{xz} = \tau_{yz} = 0$). In-plane loads induce a uniform strain distribution through the thickness, ε_{ij}^{0}, with respect to the global coordinates, while out-of-plane loads induce a strain state varying linearly through the thickness and depending on the curvature. Superimposing them, one may express the strain state in the laminate with respect to the coordinate system shown in Figure 2.1 as

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xx}^{0} \\ \varepsilon_{yy}^{0} \\ \gamma_{xy}^{0} \end{Bmatrix} + z \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} \tag{2.1}$$

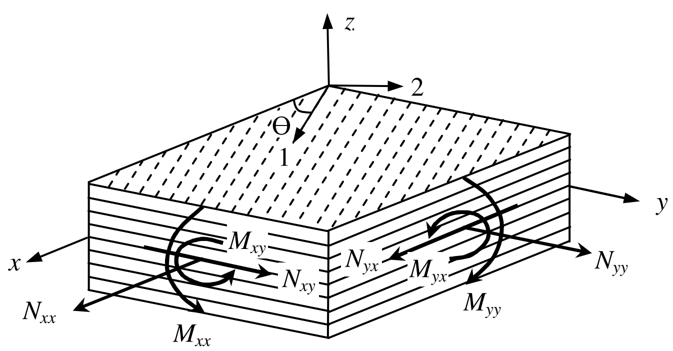

Figure 2.1. A schematic of the composite structure and the loading in this study.

where ε_{xx}^0, ε_{yy}^0, and γ_{xy}^0 are the mid-plane strains, κ_{xx}, κ_{yy}, and κ_{xy} are the curvature terms. The in-plane stress components are related to the strain components as given in Eq.(2.2)

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}_k \tag{2.2}$$

The stress components in the k^{th} lamina can be expressed in terms of the in-plane strains and the curvatures by substituting Eq. (2.1) into Eq. (2.2).

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \left(\begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \right) \tag{2.3}$$

Because the stress components, σ_{ij}, depend on the z coordinate, not only lamina thickness and lamina angles but also stacking sequence of the laminae affects the mechanical response of the laminate in the case of out-of-plane loading unlike in-plane loading. Accordingly, not only the existence of a lamina with a certain fiber angle but also its location is important during optimization.

Stress resultants (forces and bending moments per unit lateral length of a cross section) are obtained by through-the-thickness integration of the stresses in each ply.

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} z\,dz = \sum_{k=1}^{2m} \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}_k z\,dz \tag{2.4}$$

Here m is the number of distinct laminae in one of the symmetric portions above or below the mid-plane. Substituting the stress-strain relation given by Eq. (2.3) into Eqs. (2.4) and (2.5), we get

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} dz = 2\sum_{k=1}^{m} n_k t_o \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}_k \tag{2.5}$$

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^0_{xx} \\ \varepsilon^0_{yy} \\ \gamma^0_{xy} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} \tag{2.6}$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^0_{xx} \\ \varepsilon^0_{yy} \\ \gamma^0_{xy} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} \tag{2.7}$$

where A_{ij} are membrane stiffness components, D_{ij} are bending stiffness components, and B_{ij} are bending-extension coupling stiffness components given by

$$A_{ij} = \sum_{k=1}^{2m} \left(\overline{Q}_{ij}\right)_k \left(z_k - z_{k-1}\right) \tag{2.8}$$

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{2m} \left(\overline{Q}_{ij}\right)_k \left(z_k^2 - z_{k-1}^2\right) \tag{2.9}$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{2m} \left(\overline{Q}_{ij}\right)_k \left(z_k^3 - z_{k-1}^3\right) \tag{2.10}$$

As seen in Eqs. Eq. (2.6) and (2.7), the in-plane response and out-of plane response of the laminate are coupled for nonzero B_{ij}. However, because only symmetric laminates are considered in this study, B_{ij} terms become zero as Eq. (2.9) implies. Accordingly, the response of the laminate to membrane and bending forces is uncoupled.

Given the in-plane loading, N_{xx}, N_{yy}, and N_{xy}, and out-of-plane loading, M_{xx}, M_{yy}, and M_{xy}, one may obtain the mid-plane strains ε_{xx}^0, ε_{yy}^0, and γ_{xy}^0, and the curvature terms, κ_{xx}, κ_{yy}, are κ_{xy}, using Eqs. (2.6) and (2.7). Then, the off-axis stress components in each ply σ_{xx}^k, σ_{yy}^k, and τ_{xy}^k can be calculated using Eq. (2.3). After that, the principal stress components can be calculated using the transformation given in Eq.(2.11)

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{Bmatrix}_k = \begin{bmatrix} \cos^2\theta_k & \sin^2\theta_k & 2\cos\theta_k \sin\theta_k \\ \sin^2\theta_k & \cos^2\theta_k & -2\cos\theta_k \sin\theta_k \\ -\cos\theta_k \sin\theta_k & \cos\theta_k \sin\theta_k & \cos^2\theta_k - \sin^2\theta_k \end{bmatrix}_k \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}_k \qquad (2.11)$$

Based on the principal stress components, σ_{11}^k, σ_{22}^k, and τ_{12}^k, one may judge whether the k^{th} ply will fail or not using appropriate static failure criteria.

3. FAILURE ANALYSIS OF LAMINATES

3.1 Static Failure Criteria

Weight minimization of composite structures necessarily involves strength constraints, because decreasing number of load carrying plies eventually leads to failure. The structure must be able to withstand the imposed loads without suffering any failure. In this study, only the static failure modes are assumed to be critical for the laminates. The other failure modes, low stiffness, buckling, delamination etc. are assumed not to be critical.

3.1.1 Maximum Stress Criterion

In order to check the feasibility of a configuration generated by the search algorithm during an optimization process, one need to use reliable failure criteria. A common approach is to use a limit theory such as the maximum stress criterion. According to this criterion, failure is predicted whenever one of the principal stress components exceeds its corresponding strength. The failure envelope for a ply under in-plane normal and shear stresses is then defined by

$$\sigma_{11} < X_t \quad \text{and} \quad \sigma_{11} > X_c \quad \text{and} \quad \sigma_{22} < Y_t \quad \text{and} \quad \sigma_{22} > Y_c \quad \text{and} \quad |\tau_{12}| < S \qquad (3.1)$$

where "*X*" and "*Y*" denote the strength along the fiber direction and transverse to it, respectively; the subscripts "*t*" and "*c*" signify the tensile and compressive strengths; *S*, on the other hand, is the ultimate in-plane shear strength of a laminate under pure shear loading. Adopting the first - ply - failure criterion, the whole laminate is assumed to have failed, if one of these inequalities is not satisfied for any one of the laminae. Once the stress state in the principal coordinates (σ_{11}, σ_{22}, and τ_{12}) for each lamina is determined, it is straightforward to apply this failure criterion.

Although the maximum stress criterion is easy to apply, it does not account for the interaction between the effects of different stress components. Figure 3.1 shows the safety factor calculated using this criterion for a laminate subject to uniaxial loading (only $N_{xx} \neq 0$) for various fiber orientation angles, θ. Two different laminate lay-up configurations were considered. One is a balanced and symmetric laminate, $[\theta_{25} / -\theta_{25}]_s$, the other is a unidirectional laminate, $[\theta_{51}]_s$. One may observe sudden changes in the trend line as one of the inequalities in Eq. 3.1 becomes inactive while one of the others becomes active due to a small change in θ. This does not conform to the empirically observed trends. The reason for this lies in the incapability of the maximum stress criterion to reflect the interactive effects. We may also observe that for the balanced laminate, $[\theta_{25} / -\theta_{25}]_s$, the criterion correctly predicts that the laminate is strongest for $\theta = 0^\circ$, in which fibers are oriented along the loading direction. The safety factor for this case is less than one; but is the highest of all. However, for the unidirectional laminate, $[\theta_{51}]_s$, the criterion falsely predicts the highest safety factor for $\theta = 5^\circ$. This means that an optimization process in which failure is assessed based on the maximum stress criterion may stick to a spurious optimum design for an unbalanced laminate. Although, coupling between normal and shear strains occurs in an unbalanced laminate, for many applications, this may be tolerated. A general design optimization procedure should then be able to optimize unbalanced laminates. Accordingly, failure analysis should not solely be based on this criterion.

3.1.2 Tsai-Wu Criterion

The Tsai-Wu failure criterion is one of the most reliable static failure criteria as it provides a simple analytical expression taking into account the competing interactive effects among the stress components. Its general form for orthotropic materials under plane stress assumption is expressed as [74, 79].

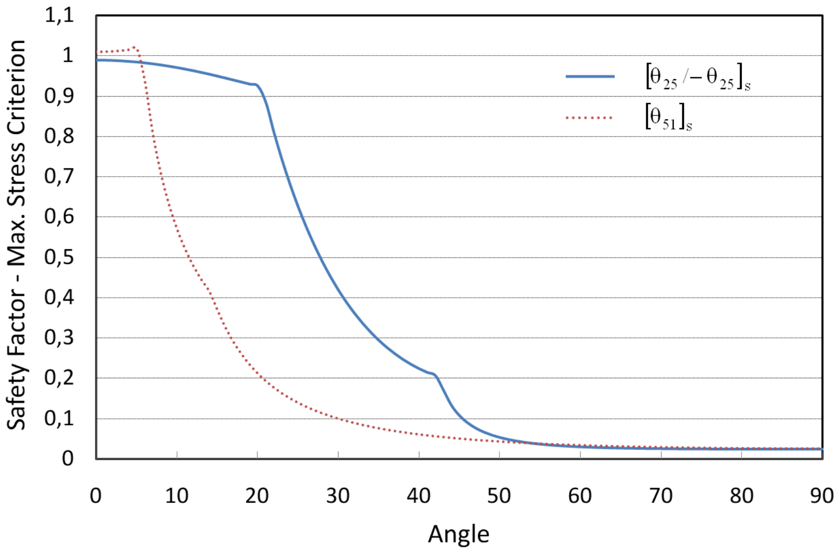

Figure 3.1 Safety factor calculated for a laminate subjected to uniaxial loading (only $N_{xx} \neq 0$) for various fiber orientation angles, θ, using the maximum stress criterion.

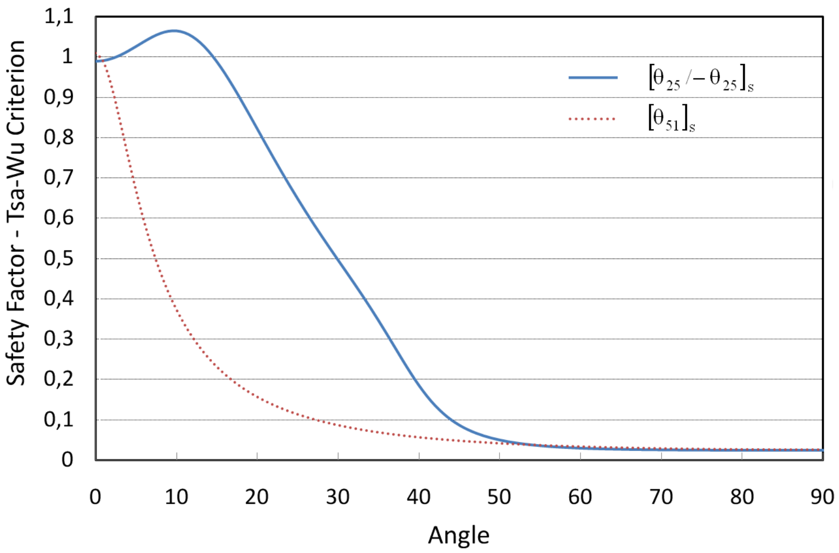

Figure 3.2 Safety factor calculated for a laminate subjected to uniaxial loading (only $N_{xx} \neq 0$) for various fiber orientation angles, θ, using the Tsai-Wu criterion.

$$\frac{\sigma_{11}^{\ 2}}{X_t|X_c|}+\frac{\sigma_{22}^{\ 2}}{Y_t|Y_c|}+\frac{\tau_{12}^{\ 2}}{S^2}-\frac{\sigma_{11}\sigma_{22}}{\sqrt{X_tX_cY_tY_c}}+\left(\frac{1}{X_t}-\frac{1}{|X_c|}\right)\sigma_{11}+\left(\frac{1}{Y_t}-\frac{1}{|Y_c|}\right)\sigma_{22}<1 \quad (3.2)$$

Here the coefficient in front of $\sigma_{11}\sigma_{22}$, which explains the interaction among normal stress components, is expressed in terms of the available uniaxial strengths. Since in this form it does not require data obtained through biaxial stress tests, the Tsai-Wu criterion is as easy to apply as the maximum stress criterion. Figure 3.2 shows the safety factor calculated using the Tsai-Wu criterion. For the unidirectional laminate, $[\theta_{51}]_s$, the criterion correctly predicts that the laminate is strongest for $\theta = 0^\circ$. The safety factor quickly decays with increasing θ. However, for the balanced laminate, $[\theta_{25}/-\theta_{25}]_s$, the criterion falsely estimates the highest safety factor as 1.065 at $\theta = 10^\circ$. Actually, the laminate is expected to fail. One may conclude that the Tsai-Wu criterion will also lead to false optimum designs in an optimization process.

Considering the predictions of these two failure criteria for the two different laminate designs, each criterion seems to compensate the deficiencies of the other. By enforcing the satisfaction of both criteria, one may find the optimum design in both cases. In this study, both the maximum stress and the Tsai-Wu criteria are used to assess the load bearing capacity of a composite laminate with the hope that false optimum designs will be avoided for any laminate configuration.

As for the out of plane loading case the behavior of Maximum stress and Tsai-Wu is investigated as follows;

The safety factor of a structure is an indication of its load carrying capacity. Values less than 1.0 indicate failure. In order to calculate the safety factor of a laminate based on the maximum stress criterion, first, the principal stresses (σ_{11}^k, σ_{22}^k, and τ_{12}^k) in each lamina are determined; the safety factor for each failure mode is calculated; then the minimum of them is denoted as the safety factor of the lamina, SF_{MS}^k. Eq.(4.4)

As Figure 3.3 shows, the highest safety factors are calculated at angles other than $\theta = 0^\circ$ for a laminate subjected to bending moment M_{xx}. Therefore the maximum stress criterion by itself is not suitable for a general laminate design optimization procedure.

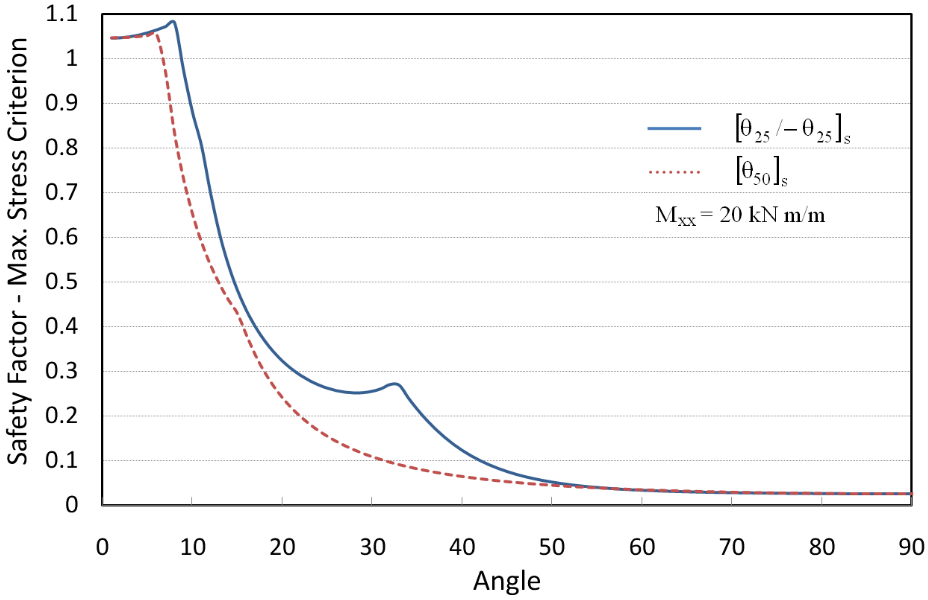

Figure 3.3 Safety factors calculated for a laminate subject to one component of bending moment (only $M_{xx} \neq 0$) for various fiber orientation angles, θ, using the maximum stress criterion.

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Tsai-Wu failure criterion is one of the most reliable static failure criteria as it provides a simple analytical expression accounting for the competing interactive effects among the stress components.

The safety factor for the k^{th} lamina, SF_{TW}^{k}, according to the Tsai-Wu criterion is defined as the multiplier of the stress components at lamina *k*, σ_{ij}^{k}, that makes the right hand side of Eq. (3.2) equal to 1.0.

The root of Eq. (4.8) gives the safety factor. Because a negative safety factor is not physically meaningful, the absolute value of the first root is considered as the actual safety factor.

As opposed to the maximum stress criterion, the Tsai-Wu criterion correctly estimates that the laminate is strongest for $\theta = 0^\circ$ if is subjected to bending moment M_{xx} as shown in Figure 3.4.

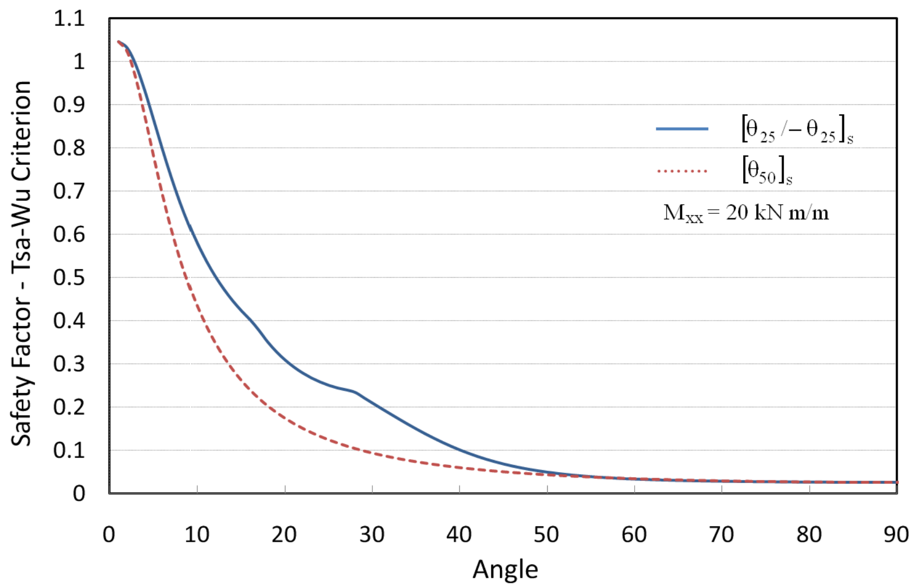

Figure 3.4 Safety factor calculated for a laminate subject to one component of bending moment (only $M_{xx} \neq 0$) for various fiber orientation angles, θ, using the Tsai-Wu criterion.

Considering the predictions of these two failure criteria for the two different laminate designs under various loads, each criterion seems to compensate the shortcomings of the other. If one of them incorrectly predicts the trend of strength for a given laminate configuration, the other correctly predicts. By enforcing the satisfaction of both criteria, one may correctly find the optimum designs. In this study, both the maximum stress and the Tsai-Wu criteria are used to assess the load bearing capacity of a composite laminate with the expectation that false optimum designs will be avoided for any laminate configuration.

4. OPTIMUM DESIGN OF COMPOSITE LAMINATES FOR MINIMUM THICKNESS

4.1 Problem Formulation

4.1.1 Problem Statement

The structure to be optimized is a symmetric 2-D multilayered structure reinforced by continuous fibers subject to in-plane normal and shear loading as shown in Figure 2.1. Accordingly no bending and twisting moments are considered in the analysis of its mechanical behavior.

The laminate consists of plies having the same thickness. The objective is to find the optimum design of the laminate to attain the minimum possible laminate thickness with the condition that it does not fail.

$$\text{Minimize } \; t \tag{4.1}$$

where t is the thickness of the laminate.

The number of distinct fiber orientation angles, m, is given. The orientation angles, θ_k, and how many plies, n_k, are oriented along each angle are to be determined in the design process. Accordingly, the number of design variables is $2m$. The laminate thickness can be expressed as

$$t = 2t_o \sum_{k=1}^{m} n_k \tag{4.2}$$

where t_o is the thickness of an individual ply and n_k is the number of plies with fiber angle θ_k. The factor '2' appears because of the symmetry condition for the laminate with respect to its middle plane. Because the plies are made of the same material, minimizing thickness leads to the same optimum configuration as the minimization of weight.

The orientation angles take discrete values; they are chosen from a given set of angles. According to the manufacturing precision, the interval between the consecutive angles may be 15°, 10°, 5°, 1°, 0.5° or even smaller.

4.2 Methodology

4.2.1 Formulation of the Objective Function

Failure of any ply signals inception of failure of the whole structure, even though its ultimate load bearing capacity may not be exceeded. For this reason, this is considered as a design limit. Accordingly, the first-ply failure approach is adopted in the design optimization and safety of each lamina in a laminate design generated during the optimization process is checked using the Tsai-Wu and maximum stress failure criteria. Failure is predicted if one of the inequalities in Eqs. (3.1) and (3.2) is not satisfied for one of the laminae. If a configuration generated during the optimization procedure leads to failure according to the failure criteria, a penalty value is calculated and added to the cost function. The overall cost function may then be expressed as

$$F = 2t_o \sum_{k=1}^{m} n_k + w_1 P_{MS} + w_2 P_{TW} - w_1 \, \mathrm{SF}_{MS} - w_2 \, \mathrm{SF}_{TW} \tag{4.3}$$

where the first term represents the total thickness of the composite structure as given in Eq. (4.2); n_k is the number of plies in the k^{th} lamina, in which the orientation angle is θ_k; m is the total number of distinct laminae; the second and third terms represent the penalty values introduced to increase the value of the objective function for designs for which failure is predicted and thus to restrict the search to the feasible design space; P_{MS} and P_{TW}

are penalty values calculated based on the maximum stress criterion and the Tsai-Wu criterion, respectively. SF_{MS} and SF_{TW} are equal to the safety factors according to the maximum stress and Tsai-Wu criteria, respectively, if they are greater than 1.0, otherwise these terms are equal to zero; w_i are suitable coefficients. The reason that the objective is reduced for safe designs is that there may be many feasible designs with the same minimum thickness. Of these designs, the optimum was defined as the one with the largest failure load. Accordingly, the objective function was linearly reduced in proportion to the failure margin as in ref. [5]. Similarly in another study [21], the margins to initial failure were maximized with the minimum feasible number of laminae.

The safety factor of the laminate according to the maximum stress criterion, SF_{MS}, is calculated as follows: First, the principal stresses (σ_{11}^k, σ_{22}^k, and τ_{12}^k) in each lamina are determined; the safety factors for each failure mode are calculated; then the minimum of them is denoted as the safety factor of the lamina, SF_{MS}^k.

$$\mathrm{SF}_{MS}^k = \min \mathrm{of} \begin{cases} \mathrm{SF}_X^k = \begin{cases} X_t/\sigma_{11} & \text{if } \sigma_{11} > 0 \\ X_c/\sigma_{11} & \text{if } \sigma_{11} < 0 \end{cases} \\ \mathrm{SF}_Y^k = \begin{cases} Y_t/\sigma_{22} & \text{if } \sigma_{22} > 0 \\ Y_c/\sigma_{22} & \text{if } \sigma_{22} < 0 \end{cases} \\ \mathrm{SF}_S^k = S/\left|\tau_{12}\right| \end{cases} \tag{4.4}$$

Then, the minimum of SF_{MS}^k is chosen among the safety factors of m number of different laminae as SF_{MS}.

$$\mathrm{SF}_{MS} = \min \mathrm{of}\ \mathrm{SF}_{MS}^k \quad \text{for } k = 1, 2, ..., m-1, m \tag{4.5}$$

The penalty value due to the violation of the maximum stress criterion is calculated as follows: If a principal stress component exceeds its respective strength, a penalty value is calculated.

$$P_X^k = \begin{cases} 0 & \text{if} \quad \mathrm{SF}_X^k \geq 1.0 \\ \left(1/\mathrm{SF}_X^k\right) - 1 & \text{if} \quad \mathrm{SF}_X^k < 1.0 \end{cases}$$

$$P_Y^k = \begin{cases} 0 & \text{if} \quad \mathrm{SF}_Y^k \geq 1.0 \\ \left(1/\mathrm{SF}_Y^k\right) - 1 & \text{if} \quad \mathrm{SF}_Y^k < 1.0 \end{cases} \tag{4.6}$$

$$P_S^k = \begin{cases} 0 & \text{if} \quad \mathrm{SF}_S^k \geq 1.0 \\ \left(1/\mathrm{SF}_S^k\right) - 1 & \text{if} \quad \mathrm{SF}_S^k < 1.0 \end{cases}$$

The total penalty value for the laminate due to the violation of the maximum stress criterion is then calculated by summing up the penalty values calculated for each lamina.

$$P_{MS} = \sum_{k=1}^{m} P_X^k + P_Y^k + P_S^k \tag{4.7}$$

The safety factor for the k^{th} lamina, SF_{TW}^k, according to the Tsai-Wu criterion is defined as the multiplier of the stress components at lamina k, σ_{ij}^k, that makes the right hand side of Eq. (3.2) equal to 1.0. Eq. (3.2) then becomes

$$a\left(\mathrm{SF}_{TW}^k\right)^2 + b\left(\mathrm{SF}_{TW}^k\right) - 1 = 0 \tag{4.8}$$

where

$$a = \frac{\left(\sigma_{11}^k\right)^2}{X_t|X_c|} + \frac{\left(\sigma_{22}^k\right)^2}{Y_t|Y_c|} + \frac{\left(\tau_{12}^k\right)^2}{S^2} - \frac{\left(\sigma_{11}^k\right)\left(\sigma_{22}^k\right)}{\sqrt{X_t X_c Y_t Y_c}} \tag{4.9}$$

$$b = \left(\frac{1}{X_t} - \frac{1}{|X_c|}\right)\sigma_{11}^k + \left(\frac{1}{Y_t} - \frac{1}{|Y_c|}\right)\sigma_{22}^k \tag{4.10}$$

The root of Eq. (4.8) gives the safety factor. Because a negative safety factor is not physically meaningful, the absolute value of the first root is considered as the actual safety factor.

$$\mathrm{SF}_{TW}^k = \left|\frac{-b + \sqrt{b^2 + 4a}}{2a}\right| \tag{4.11}$$

Then, the minimum of SF_{TW}^{k} is chosen as the safety factor of the laminate

$$\mathrm{SF}_{TW} = \min \text{of } \mathrm{SF}_{TW}^{k} \quad \text{for } k = 1, 2, ..., m-1, m \tag{4.12}$$

A penalty value is calculated and added to the objective function, if the Tsai-Wu criterion is violated at a lamina.

$$P_{TW}^{k} = \begin{cases} 0 & \text{if} \quad \mathrm{SF}_{TW}^{k} \geq 1.0 \\ \left(1/\mathrm{SF}_{TW}^{k}\right) - 1 & \text{if} \quad \mathrm{SF}_{TW}^{k} < 1.0 \end{cases} \tag{4.13}$$

The total penalty value for the laminate due to the violation of the Tsai-Wu criterion is then found by summing up the penalty values calculated for each lamina.

$$P_{TW} = \sum_{k=1}^{m} P_{TW}^{k} \tag{4.14}$$

4.2.2 Optimization Procedure

In this study, a variant of simulated annealing (SA) algorithm called “direct search simulated annealing” (DSA) [66] was used to minimize the thickness of laminated composite structures subject to in-plane loading. The application of DSA search algorithm to optimization of composite materials was explained in a previous study [25]. In this study, a number of improvements were introduced to increase the reliability of the algorithm.

In DSA unlike ordinary SA, a set of current configurations rather than a single current configuration is maintained during the optimization process. Accordingly, unlike the standard SA algorithm where only the neighborhood of a single point is searched, DSA searches the neighborhood of all the current points in the set. At the start of the optimization process, *N* number of initial configurations are randomly created within the design domain by randomly selecting values for the design variables. *N* is equal to $7(2m + 1)$, where $2m$ is the number of design variables as mentioned before. The design variables are the number of plies in the k^{th} lamina, n_k, and the orientation angle of the fibers in these plies, θ_k. A number among (0, 1, 2,…,20) is randomly chosen for n_k, and among (-90, -90 + ϕ,…,-2ϕ, -ϕ, 0, ϕ, 2ϕ,…, 90-ϕ, 90) for θ_k. Here ϕ is the interval between consecutive angles, which may be 0.5°, 1°, 5°, 10°, 15°, 30°, 45°. If zero ply number is chosen, this means that no material exists for the respective lamina and this lamina does not contribute to the load carrying capacity of the laminate. After the initial laminate configurations are randomly chosen, their objective functions are calculated. DSA like SA requires random generation of a new configuration in each iteration. A configuration in the neighborhood of one of the current configurations is randomly generated as follows: First, one of the current configurations is randomly chosen. Then, random differences are introduced to the ply numbers and fiber angles.

$$\begin{aligned} n'_k &= n_k + r_1 \Delta n_{\max} \\ \theta'_k &= \theta_k + r_2 \Delta\theta_{\max} \end{aligned} \qquad (4.15)$$

Here n_k and θ_k are the ply number and fiber angle in the k^{th} lamina of the randomly chosen current configuration; n'_k and θ'_k are the ply number and fiber angle of the newly generated configuration; r_i are randomly chosen real numbers within [-1, 1]; $\Delta n_{\max}$ and $\Delta\theta_{\max}$ are the maximum variations that may be introduced to n_k and θ_k to generate a new lay-up configuration. The lower limit for n'_k is zero; if n'_k is negative, a new random number, r_1, is generated. There is no upper limit for n'_k. The lower and upper limits for θ'_k are -90° and 90°, respectively. If a number greater than 90 is generated for θ'_k, 180 is subtracted from this number. If it is less than -90, 180 is added. Acceptability of a newly generated trial configuration, A_t, depends on the value of its cost, f_t, which is calculated by

$$A_t = \begin{cases} 1 \text{....if} \quad f_t \le f_h \\ \exp((f_h - f_t)/T_j) \text{...if} \quad f_t > f_h \end{cases} \qquad (4.16)$$

Here f_h is the highest cost in the current set. This means every new design having a cost lower than the cost of the worst design is accepted. But, if the cost is higher, the trial configuration may be accepted depending on the value of A_t. If it is greater than a randomly generated number, P_r, the trial configuration is accepted, otherwise it is rejected. If the trial design is accepted, it replaces the worst configuration. Iterations during which the value of the temperature (or control) parameter, T_j, is kept constant are called j^{th} Markov chain (or inner loop). After a certain number of iterations, the temperature parameter, T, is reduced, a new inner loop begins. As Eq. (4.16) implies, when T is decreased, the probability that a worse configuration is accepted becomes lower. At low values of temperature parameter, acceptability becomes low; thus, acceptance of worse configurations is unlikely, just as the atoms become stable, and do not tend to change their arrangements at low temperatures.

In order to find the globally optimal design, one should be able to search a large solution domain. For this reason, instead of giving small perturbations to the current configuration to obtain a new configuration in its near neighborhood, one should allow a large variance in the current configurations. For this reason, the magnitudes of $\Delta n_{\max}$ and $\Delta\theta_{\max}$ were taken as 15 and 50, respectively. This means that the neighborhood of a current configuration where a new configuration is generated is initially quite large. This can also be considered as a logical consequence of simulating the physical annealing process, where mobility of atoms is large at high temperatures, and thus the probability that atoms may form a quite different configuration is high. Also, as in the physical process, where mobility of atoms decreases as the temperature is lowered, variations in n_k and θ_k are also reduced as the temperature parameter is decreased; but the reduction scheme does not directly depend on temperature. The configuration that is worse than all current configurations except the worst one is defined as the worse configuration, and if no improvement is obtained in the worse configuration during a Markow chain, $\Delta n_{\max}$ and $\Delta\theta_{\max}$ are reduced. For other details regarding the optimization procedure, one may refer to ref. [25].

4.3 Numerical Results and Discussions

Two graphite/epoxy materials were considered in the lay-up sequence optimization. One is T300/5308 with the material properties given in Table 4.1 and the other is T300/5208 presented in Table 4.2.

Table 4.1 The Material Properties of T300/5308

E_{11} = 40.91 GPa	X_T = 779 MPa
E_{22} = 9.88 GPa	X_C = −1134 MPa
G_{12} =2.84 GPa	Y_T = 19 MPa
ν_{12} = 0.292	Y_C = −131 MPa
	S = 75 MPa

Table 4.2 The Material Properties of T300/5208

E_{11} = 181 GPa	X_T = 1500 MPa
E_{22} = 10.3 GPa	X_C = −1500 MPa
G_{12} = 7.17 GPa	Y_T = 40 MPa
ν_{12} = 0.28	Y_C = −246 MPa
	S = 68 MPa

As discussed before, relying on just one failure criterion may lead to false optimal designs. Use of a particular failure criterion will have impact on the safety and optimality of the resulting laminate design. Table 4.3 shows the dependence of optimal designs obtained by applying the aforementioned optimization procedure on the chosen failure criterion. The loading is uniaxial (N_{xx} = 100 × 10^6 N/m) and two distinct fiber orientations are permitted. The interval between the angles is chosen to be 1°. If only the Tsai-Wu criterion is used, the optimal design is almost balanced and the angle imparting the highest strength is predicted around ±10°, following the trend shown in Figure 3.2. According to the maximum stress criterion, however, this configuration is unsafe. If only the maximum stress criterion is used, the optimal laminate is unidirectional with a fiber orientation angle of 5°, conforming to the trend shown in Figure 3.1. According to the Tsai-Wu criterion, this design is highly nonconservative. If the Tsai-Wu and maximum stress criteria are used together, the optimal lay-up design conforms to the empirical observations; i.e. a laminate under uniaxial loading is strongest if all the fibers are aligned along the load direction.

Table 4.3 Dependence of optimal designs on the chosen failure criteria for the loading Nxx = 100, Nyy = Nxy = 0 MPa·m, and for two distinct fiber angles ($0 \leq j \leq 51$).

Failure criteria used to check feasibility	Optimal lay-up	Half laminate thickness	Safety factor for Tsai-Wu	Safety factor for max.
Only Tsai-Wu criterion	$[-9_{25}/10_{22}]_s$	47	1.0007	0.9142
Only max stress criterion	$[5_{51}/\theta_0]_s$	51	0.6688	1.0168
Both Tsai-Wu and max. stress	$[0_j/0_{51-j}]_s$	51	1.0091	1.0091

For some other load cases, the optimal lay-ups having minimum thickness were obtained using the Tsai-Wu and maximum stress criteria together. A range of values were tried for the number of distinct fiber orientations. Table 4.4 shows the optimum angles, the number of plies oriented along these angles, and the total number of plies for a biaxially loaded laminate (N_{xx} =10, N_{yy} =10, N_{xy} = 0 MPa·m) made of T300/5308. For this loading case, quite a number of multiple globally optimum lay-up designs were found. For two distinct fiber angles, the optimization algorithm found $[90_{47}/0_{47}]_s$, $[89_{47}/1_{47}]_s$, $[88_{47}/2_{47}]_s$,…, $[45_{47}/-45_{47}]_s$,…, $[1_{47}/-89_{47}]_s$ as the optimal lay-ups having the same objective function value. This means that the strength of a laminate having $[90_{47}/0_{47}]_s$ lay-up is the same for all in-plane biaxial loads having equal magnitude applied along any arbitrary *x-y* directions. For this loading case, tensile stresses transverse to the fibers are critical.

Because for all these lay-ups, $[\theta_{47}/\theta\text{-}90_{47}]_s$, transverse tensile stress in each ply is the same, they have the same safety factor, and thus the same objective function value. Among the globally optimal designs, $[90_{47}/0_{47}]_s$ and $[45_{47}/\text{-}45_{47}]_s$ are balanced lay-up sequences; hence they do not have shear-extension coupling. For T300/5208, optimal fiber orientations were found to be the same, $[\theta_7/\theta\text{-}90_7]_s$; but the total number of plies is 14 as opposed to 94, because its tensile strengths are larger. Stacking sequence does not affect the strength of a symmetric laminate subject to in-plane loads since none of the equations, Eq. (2.2), Eqs. (2.3-2.7), depend on z coordinates of the plies. Therefore, $[40_{47}/\text{-}50_{47}]_s$ and $[50_{47}/\text{-}40_{47}]_s$, for example, have the same strength. In view of that, alternative stacking sequences are excluded from the results given in the tables.

Table 4.4 The optimum lay-ups for the loading Nxx = 10, Nyy = 10, Nxy = 0 MPa·m, and for various numbers of distinct fiber angles. The material is T300/5308. (α, θ, β… are arbitrary angles between -90° and 90°, k and n are arbitrary positive integers less than 48).

Number of distinct fiber angles	Optimum lay-up sequences	Half laminate thickness	Safety factor for Tsai-Wu	Safety factor for max. stress
2	$[\theta_{47}/\theta\text{-}90_{47}]_s$: $[90_{47}/0_{47}]_s$, $[60_{47}/\text{-}30_{47}]_s$,…	94	1.0009	1.0050
4	$[\theta_{47}/\alpha_0/\beta_0/\theta\text{-}90_{47}]_s$: $[45_{47}/83_0/32_0/\text{-}45_{47}]_s$… $[\theta_n/\theta_{47\text{-}n}/\theta\text{-}90_k/\theta\text{-}90_{47\text{-}k}]_s$: $[20_{40}/20_7/\text{-}70_{20}/\text{-}70_{27}]_s$ $[\theta_n/\theta_{47\text{-}n}/\alpha_0/\theta\text{-}90_{47}]_s$: $[20_{40}/20_7/42_0/\text{-}70_{47}]_s$… $[\theta_n/\theta\text{-}90_n/\alpha_{47\text{-}n}/\alpha\text{-}90_{47\text{-}n}]_s$: $[60_{20}/\text{-}30_{20}/90_{27}/0_{27}]_s$	94	1.0009	1.0050
8	$[\text{-}11_{26}/79_{26}/\text{-}12_{20}/78_{20}/\text{-}17_1/73_1/\alpha_0/\beta_0]_s$	94	1.0009	1.0050
16	$[\text{-}38_1/52_1/\text{-}35_{28}/55_{28}/\text{-}36_6/54_6/\text{-}37_{12}/53_{12}/\alpha_0/\beta_0/\theta_0/\beta_0/\lambda_0/\varphi_0/\delta_0/\gamma_0]_s$	94	1.0009	1.0050

Increasing the number of distinct fiber angles did not yield better lay-up designs. All of them have the same thickness (94 plies) and the same safety factor. However, larger numbers of distinct angles offered alternative designs. Among the optimal designs obtained using four distinct fiber angles, some of them are the same like $[65_{47}/39_0/13_0/\text{-}25_{47}]_s$, $[45_{20}/45_{27}/\text{-}45_{32}/\text{-}45_{47}]_s$, but some are different like $[90_{30}/0_{30}/45_{27}/\text{-}45_{27}]_s$. Although the optimal designs do not show any difference according to the fitness criterion adopted in this study, some of the designs may have more resistance to other forms of failure like buckling, fatigue, resonance, or better thermal properties, which may become critical for some applications. For this reason, being able to obtain all or most of the alternative optimal designs is important for a more comprehensive design process.

The algorithm can find a global design or a near global design in every run even with a large number of optimization variables. All the optimal designs had the same thickness of 94 plies with a safety factor of at least 1.00089 for Tsai-Wu, 1.0049 for the maximum stress criterion. This shows the reliability of the algorithm in finding the best solution among countless local optimums. Moreover, because the DSA algorithm uses *N* number of current configurations, it may find many multiple globally optimum designs in a single run.

Table 4.5 shows the results for the case of pure shear loading for two distinct fiber angles. Using a larger number of distinct fiber angles did not result in a different lay-up sequence. For different materials, different optimal lay-up sequences were obtained. For these loading cases, because the safety factor for Tsai-Wu, SF_{TW}, was much smaller than that of the maximum stress criterion, SF_{MS}, the weight for the Tsai-Wu criterion (w_2) was increased. Otherwise, the algorithm might choose laminate designs with a larger SF_{MS}, but a lower SF_{TW}.

Table 4.5 The optimum lay-ups obtained using two distinct fiber angles for the loading $N_{xx} = 0$, $N_{yy} = 0$, $N_{xy} = 40$ MPa·m.

Material	Optimum lay-up sequences	Half laminate thickness	Safety factor for Tsai-Wu	Safety factor for max. stress
T300/5208	$[-45_{14}/45_{27}]_s$	41	1.0150	1.4577
T300/5308	$[-63_{27}/48_{54}]_s$, $[-27_{27}/42_{54}]_s$	81	1.0032	1.1637

Table 4.6 shows the results for the case of pure shear loading; but this time the magnitudes of the compressive strengths are taken the same as the tensile strengths, ($X_c = -X_t$, $Y_c = -Y_t$). This is the way some researchers adopt to avoid false optimums when the Tsai-Wu criterion is used. However, this approach may yield extremely conservative designs as seen in the table. The optimal design for T300/5308 is $[0_n/90_{210-n}]_s$. Because shear stresses are critical and the magnitudes of the shear stresses in 0° and 90° plies are the same, any combination of 0° and 90° plies with a total number of 210 plies is optimal. This thickness is far larger than the thickness of the real optimum laminate, which is 81 plies as given in Table 4.5. The optimal design for T300/5208, on the other hand, is [-4523/4523]s. Because, the compressive and tensile strengths along the fiber direction are the same and the tensile stresses transverse to the fibers are critical, taking $Y_c = -Y_t$ did not much affect the optimal configuration. Since the Tsai-Hill criterion does not use compressive strengths of laminates, it also yield the same nonconservative results.

Table 4.6 The optimum lay-ups obtained using only tensile strengths ($X_c = -X_t$, $Y_c = -Y_t$) for the loading $N_{xx} = 0$, $N_{yy} = 0$, $N_{xy} = 40$ MPa·m.

Material	Optimum lay-up sequences	Half laminate thickness	Safety factor for Tsai-Wu	Safety factor for max. stress
T300/5208	$[-45_{23}/45_{23}]_s$	43	1.0206	1.4615
T300/5308	$[0_n/90_{210-n}]_s$: $[0_{105}/90_{105}]_s$, $[0_{10}/90_{200}]_s$,	210	1.0001	1.0001

Table 4.7 shows the optimal laminate designs for various biaxial loading cases obtained using two distinct fiber angles. For the loading case N_{xx} = 10, N_{yy} = 5, N_{xy} = 0 MPa·m, the optimal lay-up is $[37_{27}/\text{-}37_{27}]_s$ with 54-ply thickness. When N_{xx} is increased to 20 MPa·m, strangely the thickness of the optimal laminate becomes smaller. This counter intuitive result can be explained by considering the differences in the stress states. When N_{xx} is increased to 20 MPa·m and the laminate design is changed to $[31_{23}/\text{-}31_{23}]_s$, ε_{xx} increases from 0.323×10^{-2} to 0.729×10^{-2}, ε_{yy}, on the other hand, turns from tension to (8.39×10^{-5}) compression (-0.237×10^{-2}) due to Poisson's effect. The stress transverse to the fibers then decreases from 18.49 MPa to 15.85 MPa, while the other principal stresses (shear stress and normal stress along the fiber direction) increase. Because, the transverse tensile stresses are critical, a thinner laminate could carry a larger load. When N_{xx} is increased to 40 MPa·m, the same trend continues. However, when it is increased to 80 or 120 MPa·m, a thicker laminate is required.

Table 4.7 The optimum lay-ups obtained using two distinct fiber angles for various biaxial loading cases. The material is T300/5308.

Loading: $N_{xx}/N_{yy}/N_{xy}$ (MPa.m)	Optimum lay-up sequences	Half laminate thickness	Safety factor for Tsai-Wu	Safety factor for max. stress
10 / 5 / 0	$[37_{27}/\text{-}37_{27}]_s$	54	1.0068	1.0277
20 / 5 / 0	$[31_{23}/\text{-}31_{23}]_s$	46	1.0208	1.1985
40 / 5 / 0	$[26_{20}/\text{-}26_{20}]_s$	40	1.0190	1.5381
80 / 5 / 0	$[21_{25}/\text{-}19_{28}]_s$	53	1.0113	1.2213
120 / 5 / 0	$[17_{35}/\text{-}17_{35}]_s$	70	1.0030	1.0950

Table 4.8 shows the optimal laminate designs for load cases in which shear load is increased. One interesting loading case is N_{xx} = 10, N_{yy} = 10, N_{xy} = 10 MPa·m. In comparison to the thickness of the optimal laminate for the loading case N_{xx} = 10, N_{yy} = 10, N_{xy} = 0 MPa·m, which is 94 plies as given in Table 4.4, the thickness is quite low (11 plies). The reason is that the application of N_{xy}, causes the principal shear and transverse

stresses disappear for a fiber oriented with an angle of 45°. Only tensile stresses along the fiber direction remain, for which the plies are strongest. When the shear loading is increased, the thickness increases, because this condition is disrupted. As seen in the table, for three distinct angles a better design is obtained, which have the same thickness but a larger safety factor.

Table 4.8 The effect of increasing shear load. The material is T300/5308.

Loading: N_{xx} / N_{yy} / N_{xy} (MPa.m)	Optimum lay-up sequences	Half laminate thickness	Safety factor for Tsai-Wu	Safety factor for max. stress
10 / 10 / 10	$[45_{11}/\alpha_0]_s$	11	1.0883	1.0883
10 / 10 / 20	$[45_{30}/-12_4]_s$, $[45_{30}/-78_4]_s$	34	1.0127	1.1489
	$[44_{27}/43_3/-76_4]_s$, $[46_{27}/47_3/-14_4]_s$	34	1.0132	1.1256
10 / 10 / 40	$[45_{60}/-22_{18}]_s$, $[45_{60}/-68_{18}]_s$	78	1.0103	1.2076
	$[45_3/44_{56}/-21_{19}]_s$, $[45_3/46_{56}/-69_{19}]_s$	78	1.0121	1.2093
10 / 10 / 80	$[43_{113}/-24_{47}]_s$, $[47_{113}/-66_{47}]_s$	160	1.0053	1.1853
	$[43_{111}/44_2/-24_{47}]_s$, $[47_{111}/46_2/-66_{47}]_s$	160	1.0063	1.1850

Table 4.9 shows the optimal laminate designs for various load cases in which compressive load is increased. With increasing load, the required minimum thickness also increases. For the loading case N_{xx} = 10, N_{yy} = -80, N_{xy} = 0 MPa·m, a better design with a smaller thickness was obtained with four distinct lamina angles in comparison to the design with two distinct lamina angles.

Table 4.9 The effect of increasing compressive load. The material is T300/5308.

Loading: N_{xx} / N_{yy} / N_{xy} (MPa.m)	Optimum lay-up sequences	Half laminate thickness	Safety factor for Tsai-Wu	Safety factor for max. stress
10 / -10 / 0	$[3_{14}/72_{7}]_s$	21	1.0404	1.2068
10 / -20 / 0	$[3_{18}/82_{12}]_s$	30	1.0156	1.1253
	$[4_{1}/5_{15}/12_{1}/80_{13}]_s$	30	1.0172	1.1352
10 / -40 / 0	$[1_{20}/89_{25}]_s$	45	1.0219	1.1104
	$[0_{3}/-1_{17}/87_{2}/-89_{23}]_s$	45	1.0226	1.1112
10 / -80 / 0	$[3_{18}/90_{54}]_s$	72	1.0150	1.1026
	$[-4_{18}/-87_{2}/89_{1}/90_{50}]_s$	71	1.0007	1.0871
10 / -10 / 10	$[19_{18}/-48_{11}]_s$, $[26_{18}/-87_{11}]_s$	29	1.0096	1.1855
	$[21_{15}/22_{4}/-50_{10}]_s$, $[23_{4}/24_{15}/-85_{10}]_s$	29	1.0145	1.1888
10 / -20 / 10	$[20_{21}/-84_{15}]_s$	36	1.0056	1.1331
	$[20_{4}/21_{16}/-85_{16}]_s$	36	1.0058	1.1410
10 / -40 / 10	$[9_{23}/-76_{26}]_s$	49	1.0095	1.1020
10 / -80 / 10	$[5_{20}/-84_{54}]_s$	74	1.0050	1.0915
	$[4_{20}/-83_{25}/-84_{29}]_s$	74	1.0056	1.0924

Table 4.10 shows the optimal designs obtained using various intervals between the fiber orientation angles for the loading N_{xx} = 40, N_{yy} = 5, N_{xy} = 0 MPa·m. As the results indicate, with a smaller interval, one may obtain better designs. Choosing large intervals may lead to gravely inferior designs. If only 0°, ±45°, and 90° angles are allowed, almost three times thick laminates are required to bear the applied load.

Table 4.10 The optimum lay-ups for the loading Nxx = 40, Nyy = 5, Nxy = 0 MPa·m, and for various numbers of distinct fiber angles. The material is T300/5308.

Interval between orientation angles	Optimum lay-up sequences	Half laminate thickness	Safety factor for Tsai-Wu	Safety factor for max. stress
1°	$[26_{20}/-26_{20}]_s$	40	1.0190	1.5380
5°	$[25_{20}/-25_{20}]_s$	40	1.0112	1.6914
10°	$[30_{17}/-20_{26}]_s$	43	1.0100	1.8556
15°	$[30_{23}/-30_{23}]_s$	46	1.0397	1.2318
30°	$[30_{23}/-30_{23}]_s$	46	1.0397	1.2318
45°	$[0_{107}]_s$	107	1.0093	1.0328

4.3.1 Comparison with the Results of a Nonlinear Programming

As far as the author know, there is no study that formulated the problem as in the present study. An investigation conducted by Wang and Karihaloo [7] is similar to this study in that they considered composite laminates subject to in-plane static loads and they used lamina thickness and orientation angles as design variables. On the other hand, they aimed at maximizing the strength of a laminate rather than minimizing its total thickness. Because they used a deterministic local search algorithm that employed a nonlinear programming, comparison of their results with the results obtained using the algorithm proposed in the present study may indicate the advantage gained by global search algorithms. The present algorithm was modified to maximize the safety factor and optimal lay-ups were obtained for a number of load case. Table 0 and 0 show optimal lay-up sequences for four and eight distinct lamina angles, respectively. Lamina thickness is not varied and equal to four as in some of the example problems that Wang and Karihaloo [7] considered. Although many multiple globally optimum lay-ups were obtained, only one lay-up was given for each load case. As seen in the tables, the global search algorithm managed to find better lay-up sequences. The algorithm found $[45_n]$ as the optimal laminate design for the loading case of $N_{xx} = N_{yy} = N_{xy}$. This design has a very large safety factor. This is because Wang and Karihaloo [7] used a failure criterion based on fracture mechanics taking into account transverse cracks that are susceptible to cause failure under shear stress and transverse tensile normal stress only. Accordingly, a laminate with [45_n] lay-up sequence subject to $N_{xx} = N_{yy} = N_{xy}$ has theoretically infinite strength since only normal stresses along the fiber direction exist. Therefore, the result of the optimization for this loading case is not surprising. . Wang and Karihaloo [7] considered lamina thickness as a continuous optimization variable in some other design optimization problems. Because the present algorithm was developed considering thickness as a discrete variable, it was not possible to compare the results of the optimization problems in which fiber angles together with thickness were varied.

Table 4.11 Comparison with the results of a nonlinear programming (Wang & Karihaloo [7]). Lamina thickness is constant and equal to four plies. No of distinct fiber angles is four.

Loading: N_{xx} / N_{yy} / N_{xy} (kPa.m)	Optimum lamina orientations		Safety factor	
	Wang & Karihaloo [7]	Present study	Wang & Karihaloo [7]	Present study
200/200/0	[62.68_4/-54.20_4/ 81.23_4/-1.96_4]$_s$	[50.80_4/-49.80_4/ 26.59_4/-49.73_4]$_s$	1.59	2.14
200/0/200	[32.30_4/-56.61_4/ -7.78_4/33.87_4]	[31.72_{16}]	1.86	4.84
400/200/0	[-27.46_4/57.58_4/ -43.61_4/20.11_4]	[30.98_4/-36.57_4/ 37.67_4/-37.20_4]	1.34	1.64
200/200/200	[45.45_4/51.72_4/ 43.38_4/39.58_4]	[45.00_{16}]	10.21	1.11×10^{16}

Table 4.12 Comparison with the results of a nonlinear programming (Wang & Karihaloo [7]). Lamina thickness is constant and equal to four plies. No of distinct fiber angles is eight.

Loading: N_{xx} / N_{yy} / N_{xy} (kPa.m)	Optimum lamina orientations		Safety factor	
	Wang & Karihaloo [7]	Present study	Wang & Karihaloo [7]	Present study
400/400/0	$[-3.08_4/-90.00_8/-28.51_4/88.00_4/-28.49_4/-27.59_4/40.05_4]_s$	$[-73.68_4/33.57_4/-50.69_4/30.89_4/-51.55_8/30.75_8]_s$	1.70	2.15
400/0/400	$[36.66_4/-57.68_4/36.47_4/90.00_4/37.31_4/24.58_4/-57.77_4/31.65_4]_s$	$[31.72_{16}]$	2.22	4.84
800/400/0	$[-26.94_4/49.26_4/-37.54_4/40.09_4/-42.78_4/55.36_4/-24.16_4/10.86_4]_s$	$[-29.94_4/35.60_4/-36.69_4/36.31_{12}/-36.75_8]_s$	1.40	1.65
400/400/400	$[49.29_8/49.27_4/26.60_4/49.42_4/11.91_4/50.14_4/49.33_4]_s$	$[45.00_{16}]$	3.62	3.86×10^{15}

5. DESIGN OPTIMIZATION OF LAMINATED COMPOSITES SUBJECTED TO IN-PLANE AND OUT-OF-PLANE LOADS

5.1 Problem Formulation

5.1.1 Problem Statement

Consider a structure made of orthotropic layers perfectly bonded together and reinforced by continuous fibers. The structure is symmetric with respect to its mid-plane. This multilayered structure is subjected to in-plane normal (N_{xx} and N_{yy}) and shear (N_{xy} and N_{yx}) loading as well as bending (M_{xx} and M_{yy}) and twisting moment resultants (M_{xy} and M_{yx}) as shown in Figure. 2.1.

The objective is to minimize the laminate thickness, *t*, with the condition that it does not fail under the applied static loads. The problem can be stated in general terms as

$$\text{Minimize } t = \text{thickness} \qquad \text{Subject to failure criteria} \tag{5.1}$$

The thickness of each ply in the laminate is the same and not varied during the optimization, because laminates are usually made of prepregs with a given thickness. The number of distinct fiber orientation angles, *m*, is specified by the designer, while the orientation angles, θ_k, and how many plies, n_k, are oriented with angle θ_k are to be determined during the optimization process. A stack of contiguous plies oriented in the same direction is called lamina. θ_1 is the orientation angle of the outermost lamina and θ_m is the innermost lamina below the mid-plane as shown in Figure 5.1. The laminate thickness can be expressed as

$$t = 2t_o \sum_{k=1}^{m} n_k \tag{5.2}$$

where t_o is the thickness of an individual ply, n_k is the number of plies with a fiber orientation of θ_k. The factor '2' appears because of the symmetry with respect to the middle plane. Considering that the plies are made of the same material, minimizing the thickness leads to the same optimum configuration as the minimization of weight.

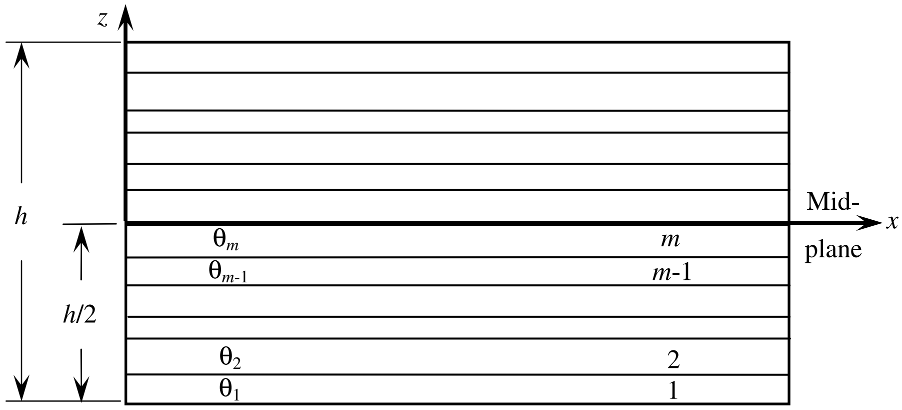

Figure 5.1 Stacking sequence of a symmetric laminate.

The orientation angles take discrete values; they are chosen from a given set of angles. According to the manufacturing precision, the interval between the consecutive angles may be 15°, 10°, 5°, 1°, or even smaller.

5.2 Methodology

5.2.1 Formulation of the Objective Function

The same formulation of the Objective function employed in Chapter 4 (4.2.1) was also used for this part of the study.

5.2.2 Optimization Procedure

In order to search for the globally optimum laminate designs, a variant of simulated annealing (SA) algorithm is proposed. This is similar to the direct search simulated annealing (DSA) [66] discussed in 4.2.2 in detail, in that a set of current configurations rather than a single current configuration is maintained during the optimization process unlike ordinary SA. Accordingly, unlike the standard SA algorithm, where only the neighborhood of a single point is searched, the neighborhood of all the current points in the set is searched. This feature resembles population in genetic algorithms. If a newly generated configuration is accepted, it replaces a current configuration other than the best configuration; thus the best current configuration is not lost. In this way, one of the most important drawbacks of the standard SA is avoided, where many good current solutions are replaced by worse solutions especially at the early stages of the optimization process.

In DSA, if a configuration generated during iterations is accepted, it replaces the worst current configuration. This is the one of the drawbacks of DSA that becomes especially apparent with a high number of design variables. For instance, if one optimizes a laminate with 16 distinct lamina angles and thicknesses using DSA, the number of current configurations will be $8(2\times16+1) = 264$. After the start of the optimization process, the current configurations quickly gather around local minimums except the worst one, which is frequently updated by higher-cost configurations at high temperatures. Because new configurations are generated in the neighborhood of the current ones, search becomes restricted to a small portion of the feasible domain except for the trials in which the new configuration is generated around the worst current configuration, which are very few. In order to remedy this, a different replacement scheme is adopted. The current configurations are ordered with respect to their objective function value. If a new configuration is accepted, it replaces a current configuration randomly chosen among $4(2m + 1)$ worst configurations instead of the worst one. Thus, half of the current configurations having low cost remains in the set unless better ones are found through iterations. The other half, on the other hand, may be replaced by higher cost configurations with a probability depending on the temperature parameter. In this way, $4(2m + 1)$ number of configurations become dispersed in the feasible domain at high temperatures and thus a thorough search of the feasible domain can be achieved. In calculating the acceptability using Eq. (0), the objective function value of the best of the worst $4(2m + 1)$ current configurations is used for f_h.

In order to find the globally optimal design, one should be able to search a large solution domain. For this reason, instead of giving small perturbations to the current configuration to obtain a new configuration in its near neighborhood, one should allow a large variance in the current configurations. For this reason, the magnitudes of $\Delta n_{\max}$ and $\Delta\theta_{\max}$ are taken as 15 and 50, respectively. This means that the neighborhood of a current configuration where a new configuration is generated is initially quite large. This can also be considered as a logical consequence of simulating the physical annealing process, where mobility of atoms is large at high temperatures, and thus the probability that atoms may form a quite different configuration is high. Also, as in the physical process, where the mobility of atoms decreases as the temperature is lowered, variations in n_k and θ_k are also reduced as the temperature parameter is decreased; but the reduction scheme does not directly depend on temperature. If no improvement is obtained in the worst of the best $4(2m + 1)$ current configurations during a Markow chain, $\Delta n_{\max}$ and $\Delta\theta_{\max}$ are reduced by multiplying with a factor to make the searched region smaller and thus increase the likelihood of finding a better design.

The temperature parameter, T, controls the convergence of the optimization process just like the temperature controls micro-structural changes in the physical annealing process. At the initial stages of the optimization, temperature should be high enough for the algorithm to accept almost any arbitrarily generated configuration regardless of the objective function value. At the final stages, it should take such low values that a new configuration that is worse than the current configurations is almost never accepted. These correspond to the melting and freezing temperatures, respectively, in the physical annealing process. The reduction scheme is as follows

$$T_j = \alpha_j T_{j-1} \tag{5.3}$$

where T_j and T_{j-1} are the values of the temperature parameter in the j^{th} and $(j-1)^{\text{th}}$ Markow chains, respectively, and α_j is temperature reduction factor. The other parameter controlling convergence is the maximum variation in the optimization variables, $\Delta n_{\max}$ or $\Delta\theta_{\max}$. At the beginning, large variations in the optimization variables are allowed so that the feasible domain is thoroughly searched for optimum designs. Towards the end of the optimization process, only close neighborhood of the current configurations is searched in order to exactly determine the value of the optimal point. As mentioned before the reduction scheme in $\Delta\theta_{\max}$ depends on whether improvement is achieved or not in a Markow chain. If a configuration is not found that is better than at least one of the best $4(2m + 1)$ current configurations, $\Delta\theta_{\max}$ is reduced. At this point, there is a similarity to the physical annealing process. Reduction in the temperature should be slow enough to allow time - dependent micro-structural changes to occur and to reach equilibrium. In simulated annealing, non-improvement in the current set implies that further changes are unlikely with the current values of $\Delta\theta_{\max}$ or $\Delta n_{\max}$. For this reason, a reduction scheme different from SA or DSA algorithms is adopted for the temperature parameter. Instead of reducing T_j by a specific ratio, it is made dependent on the reduction in $\Delta\theta_{\max}$ or $\Delta n_{\max}$, because it is a better indication of equilibrium at a specific temperature level. The temperature reduction factor, α_j in Eq. (5.3), is calculated as

$$\alpha_j = \begin{cases} \alpha_{\max} & \text{if } \; L_a^j / L^j < c_1(\Delta\theta_{\max} - 0.5\phi)/(\Delta\theta_{i\max} - \phi) + c_2 \\ \alpha_{\min} & \text{else} \end{cases} \tag{5.4}$$

where L^j is the number of trials executed in the j^{th} Markow chain and L_a^j is the number of accepted configurations. Accordingly, L_a^j / L^j ratio is a measure of acceptability in a Markow chain. ϕ is the interval between consecutive angles, $\Delta\theta_{i\max}$ is the initial maximum variation in the lamina angles. $\alpha_{\max}$ and $\alpha_{\min}$ are taken to be 0.9999 and 0.99; c_1 and c_2 are constant coefficients having values 1.2 and 0.01. Through the use of this equation, acceptability of Markow chains, L_a^j / L^j, is made closer to the ratio of the reduced and initial maximum variance, $\Delta\theta_{\max} / \Delta\theta_{i\max}$. When the acceptability is higher, temperature parameter is reduced at a faster rate, α_{max}; otherwise at a lower rate, α_{min}. At the initial stages of the optimization, the right hand side of the inequality is close to 1.0; accordingly, the temperature level is kept high such that the ratio of the number of accepted trials to the total number of trials is close to 1.0. When $\Delta\theta_{max}$ approaches 0.5ϕ at the end of the optimization process, temperature parameter also approaches zero; acceptability of a new configuration, A_t in Eq. (4.16), then comes close to zero. Iterations are continued until the difference between the values of the best and the worst current configurations becomes small.

5.3 Numerical Results and Discussions

The numerical results were obtained for a graphite/epoxy material, T300/5308, with the following material properties: E_{11} = 40.91 GPa, E_{22} = 9.88 GPa, G_{12} = 2.84 GPa, ν_{12} = 0.292, X_T = 779 MPa, X_C = −1134 MPa, Y_T = 19 MPa, Y_C = −131 MPa, S = 75 MPa. Thickness of the plies is 0.127 mm.

The optimal laminate design obtained through an optimization algorithm depends on the failure criterion. Table 5.1 shows the optimal designs obtained by applying the aforementioned optimization procedure for various uses of failure criteria. The loading is uniaxial (N_{xx} = 10 × 10^6 N/m) and two distinct fiber orientations are used. The interval between the angles is taken to be 1°. If only the Tsai-Wu criterion is used, the optimal design is almost balanced and the angle imparting the highest strength is predicted around ±10°, conforming to the trend shown in Figure 3.2, which is obtained using only a single angular parameter, θ. According to the maximum stress criterion, however, this configuration is unsafe. If only the maximum stress criterion is used, the optimal laminate is unidirectional with 5° fiber orientation angle, conforming to the trend shown in Figure 3.1. According to the Tsai-Wu criterion, however, this design is highly nonconservative. These results imply that relying on just one failure criterion may lead to spurious optimal designs. If the Tsai-Wu and maximum stress criteria are used together, the optimal lay-up design agrees with the empirical observations; i.e. a laminate under uniaxial loading is strongest if all the fibers are aligned along the load direction. Table 5.2 shows the optimal designs of laminates under bending loading for various uses of failure criteria. The optimum laminates are not unidirectional according to both criteria. For $[0_{43}]_s$, the safety factor is 1.0325. The optimum designs, on the other hand, have a higher safety factor. When both criteria are used, the weight of the terms associated with the Tsai-Wu criterion in Eq. (4.3) is taken to be much higher for bending (w_1 = 0.1, w_2=0.001). In this way, another optimal design is found with a safety factor (1.1124) very close to the one found by using only Tsai-Wu; but the safety factor based on the maximum stress criterion is larger (1.0123 in comparison to 1.0115). One may conclude that use of the two failure criteria together shows a potential in composite optimization.

Table 5.1 Dependence of the optimal designs on the chosen failure criteria for the loading $N_{xx} = 10 \times 10^6$, $N_{yy} = N_{xy} = 0$ N/m, and for two distinct fiber angles. $M_{xx} = M_{yy} = M_{xy} = 0$ Nm/m.

Failure criteria used to check feasibility	Optimal lay-up	Total number of plies	Safety factor for Tsai-Wu	Safety factor for max. str.
Only Tsai-Wu criterion	$[-9_{25}/10_{22}]_s$	94	1.0007	0.9142
Only max stress criterion	$[5_{51}]_s$	102	0.6688	1.0168
Both Tsai-Wu and max. stress	$[0_{51}]_s$	102	1.0091	1.0091

Table 5.2 Dependence of the optimal designs on the chosen failure criteria for the loading M_{xx} = 15, M_{yy} = M_{xy} = 0 kNm/m, and for two distinct fiber angles. N_{xx} = N_{yy} = N_{xy} = 0 N/m.

Failure criteria used to check feasibility	Optimal lay-up	Total number of plies	Safety factor for Tsai-Wu	Safety factor for max. str.
Only Tsai-Wu criterion	$[3_{15}/-10_{28}]_s$	86	1.1124	1.0115
Only max stress criterion	$[-12_4/4_{39}]_s$	86	1.0102	1.0563
Both Tsai-Wu and max. stress	$[2_{21}/-16_{22}]_s$	86	1.1124	1.0123

For some out-of-plane loading cases, the optimal lay-ups having minimum thickness were obtained using the Tsai-Wu and maximum stress criteria together in order to see the effectiveness of the optimization algorithm proposed in this study. A range of values were tried for the number of distinct laminae. Tables 5.3 – 5.6 show the optimum angles, the number of plies oriented along these angles, and the total number of plies for laminates subjected to various out-of-plane loads. For the cases presented in the tables, in-plane loads are zero, N_{xx} = N_{yy} = N_{xy} = 0 N/m, unless otherwise stated. Since stacking sequence affects the laminate response for out-of-plane loading, only the adjacent plies are shown by a single symbol, e.g. $[90/90/0_2/0/90_3/0]_s$ may be shown as $[90_2/0_3/90_3/0]_s$, but not as $[90_5/0_4]_s$, because they lead to different stress and strain states under the same out-of-plane loading. Furthermore, if the optimum number of plies in a lamina is found to be zero, it is not shown in the results because no material exists in that lamina.

For biaxial bending (M_{xx} = 15, M_{yy} = 15, M_{xy} = 0), quite a number of multiple globally optimum lay-up configurations were found as shown in Table 5.4. For two distinct fiber angles, the optimization algorithm found $[-90_{27}/0_{150}]_s$, $[-89_{27}/1_{150}]_s$,…, $[-1_{27}/89_{150}]_s$, $[0_{27}/90_{150}]_s$, $[1_{27}/-89_{150}]_s$,…, $[90_{27}/0_{150}]_s$ as optimal configurations; all of them had the same objective function value. This means that the strength of a laminate having $[-90_{27}/0_{150}]_s$ lay-up is the same for all biaxial bending loads having equal magnitude applied along any arbitrary *x-y* directions. For this loading case, tensile stresses transverse to the fibers are critical. Because for all these lay-ups, $[\theta_{27}/90+\theta_{150}]_s$, transverse tensile stress in the outermost ply is the same, they have the same safety factor, and thus the same objective function value. Because, the material is weakest for tensile loads applied transverse to the fibers, the optimum laminates have thickness much higher than the one subject to only one component of bending moment ($M_{xx} = 15$, $M_{yy} = M_{xy} = 0$ kNm/m).

By increasing the number of distinct fiber angles, better lay-up designs, i.e. either thinner laminates or laminates with a larger safety factor, can be obtained. One should also note that when a larger design domain is chosen, the number of local optimums and the complexity of the design domain may dramatically increase. In that case, a reliable global search algorithm should be used to be able to locate the globally optimum design(s). With 16-distinct laminae, i.e. 32 optimization variables, and one-degree interval between possible fiber angles, $180^{32} \cong \cong 1.5 \times 10^{72}$ number of different lay-up configurations exist. In addition to multiple global optimums, numerous near global optimum designs exist. Obtaining an improved result with such a large solution domain attests the reliability of the search algorithm proposed in this study. Improvement is more pronounced for the biaxial bending (Table 5.4) and pure twisting loads (Table 5.6). Even for cases in which improvement is insignificant, many alternative designs can be obtained by enlarging the design domain.

Table 5.3 The optimum lay-ups for the loading $M_{xx} = 15$, $M_{yy} = M_{xy} = 0$ kNm/m for various numbers of distinct fiber angles.

Number of distinct fiber angles	Optimum lay-up sequences	Total number of plies	Safety factor for Tsai-Wu	Safety factor for max. stress
1	$[0_{43}]_s$	86	1.0325	1.0325
2	$[2_{21}/\text{-}16_{22}]_s$	86	1.1124	1.0123
4	$[0_2/\text{-}6_9\ /7_8/6_{24}]_s$	86	1.1164	1.0122
8	$[0_2/\text{-}6_3\ /\text{-}7_5/6_{31}/\text{-}14_2]_s$	86	1.1165	1.0122
16	$[0_1/\text{-}5_2\ /\text{-}6_4/7_7/6_4/\text{-}5_{12}\ /7_{12}/26_1]_s$	86	1.1165	1.0122

Table 5.4 The optimum lay-ups for the loading M_{xx} = 15, M_{yy} = 15, M_{xy} = 0 kNm/m for various numbers of distinct fiber angles. (α, θ, β… are arbitrary integer angles between -90° and 90°.

Number of distinct fiber angles	Optimum lay-up sequences	Total number of plies	Safety factor for Tsai-Wu	Safety factor for max. stress
1	$[\theta_{272}]_s$: $[90_{272}]_s$, $[89_{272}]_s$, $[88_{272}]_s$, …	544	1.0073	1.0076
2	$[\theta_{27}/90+\theta_{150}]_s$: $[-90_{27}/0_{150}]_s$, $[-89_{27}/1_{150}]_s$, …	354	1.0052	1.0115
4	$[\theta_{10}/\theta+29_{11}/\theta-45_{16}/\theta+87_{138}]_s$, $[\theta_{10}/\theta-29_{11}/\theta+45_{16}/\theta-87_{138}]$	350	1.0113	1.0171
8	$[-39_4/-57_5/-66_6/-6_5/-82_6/10_8/74_{12}/37_{127}]_s$	346	1.0001	1.0030
16	$[-27_5/-45_4/-2_4/4_5/11_5/-72_5/25_3/30_3/$ $-90_5/78_{33}/71_{23}/49_{26}/79_{38}/13_{14}]_s$	346	1.0024	1.0065

Table 5.5 The optimum lay-ups for the loading $M_{xx} = M_{yy} = M_{xy}$ = 15 kNm/m for various numbers of distinct fiber angles.

Number of distinct fiber angles	Optimum lay-up sequences	Total number of plies	Safety factor for Tsai-Wu	Safety factor for max. stress
1	$[45_{60}]_s$	120	1.0052	1.0052
2	$[38_{13}/52_{47}]_s$, $[52_{13}/38_{47}]_s$	120	1.0523	1.0009
4	$[53_9/39_{22}/53_6/36_{23}]_s$	120	1.0563	1.0000
8	$[37_4/38_6/52_{25}/38_7/40_3/44_4/43_{11}]_s$	120	1.0564	1.0000
16	$[53_7/39_1/51_5/38_{10}/38_9/38_{11}/45_1/38_5/52_3/36_4$ $/56_1/40_3]_s$	120	1.0564	1.0000

Table 5.6 The optimum lay-ups for the loading $M_{xx} = M_{yy} = 0$, $M_{xy} = 15$ kNm/m for various numbers of distinct fiber angles.

Number of distinct fiber angles	Optimum lay-up sequences	Total number of plies	Safety factor for Tsai-Wu	Safety factor for max. stress
1	$[0_{137}]_s$, $[90_{137}]_s$	274	1.0091	1.0091
2	$[6_{63}/-77_{71}]_s$, $[84_{63}/-13_{71}]_s$	268	1.0127	1.0144
4	$[-82_{35}/-78_{19}/14_{38}/27_{38}]_s$	260	1.0070	1.0079
8	$[8_{30}/-79_{17}/14_{11}/16_{10}/-72_{12}/-68_{11}/29_{11}/-42_{25}]_s$	254	1.0111	1.0114
16	$[-82_{16}/10_{16}/12_{15}/-76_{10}/16_{5}/-73_{10}/20_{6}/-68_{6}/$ $-65_{6}/-61_{4}/-57_{4}/39_{3}/-35_{15}/54_{2}/-36_{8}]_s$	252	1.0066	1.0067

Table 5.7 shows the optimal designs obtained using various intervals between the consecutive fiber orientation angles for the loading $M_{xx} = M_{yy} = 0$, $M_{xy} = 15$ kNm/m. As the results indicate, better designs can be obtained within a design domain enlarged by choosing a smaller interval. For other loading cases, choosing a large interval may lead to gravely inferior designs. For this reason, one should choose the minimum interval that the manufacturing method allows.

Table 5.7 The optimum lay-ups for the loading $M_{xx} = M_{yy} = 0$, $M_{xy} = 15$ kNm/m, and for various numbers of distinct fiber angles.

Interval between orientation angles	Optimum lay-up sequences	Total number of plies	Safety factor for Tsai-Wu	Safety factor for max. stress
90°	$[0_i, 90_{137-i}]_s$: $[90_{137}]_s$, $[0_1, 90_{136}]_s$,... $[90_i, 0_{137-i}]_s$: $[0_{137}]_s$, $[90_1, 0_{136}]_s$,...	274	1.0091	1.0091
45°	The same as above	274	1.0091	1.0091
30°	$[0_{106}/\text{-}30_{31}]_s$, $[0_{106}/30_{31}]_s$, $[90_{106}/60_{31}]_s$, $[90_{106}/\text{-}60_{31}]_s$,	274	1.0130	1.0356
15°	$[0_{86}/15_{49}]_s$, $[0_{86}/\text{-}15_{49}]_s$, $[90_{86}/75_{49}]_s$, $[90_{86}/\text{-}75_{49}]_s$	270	1.0068	1.0144
10°	$[0_{70}/10_{65}]_s$, $[0_{70}/\text{-}10_{65}]_s$, $[90_{70}/80_{65}]_s$, $[90_{70}/\text{-}80_{65}]_s$	270	1.0128	1.0156
5°	$[85_{74}/\text{-}15_{60}]_s$, $[\text{-}85_{74}/15_{60}]_s$, $[5_{74}/\text{-}75_{60}]_s$, $[\text{-}5_{74}/75_{60}]_s$	268	1.0081	1.0125
1°	$[6_{63}/\text{-}77_{71}]_s$, $[84_{63}/\text{-}13_{71}]_s$	268	1.0127	1.0144

Table 5.8 shows the optimal laminate designs for various biaxial bending loading cases obtained using two distinct fiber angles. For the loading case $M_{xx} = 10$, $M_{yy} = 5$, $M_{xy} = 0$ kNm/m, the optimal lay-up is $[31_{20}/\text{-}43_{88}]_s$ with 216-ply thickness. When M_{xx} is increased to 20 kNm/m, strangely the optimal laminate becomes thinner. This counter intuitive optimal design can be explained by considering the differences in the stress states. When M_{xx} is increased to 20 kNm/m and the laminate design is changed to $[30_{19}/\text{-}33_{80}]_s$, σ_{11} increases from 71.4 MPa to 166.6 MPa and ε_{11} increases from 0.16×10^{-2} to 0.40×10^{-2}; ε_{22}, on the other hand, decreases from 0.14×10^{-2} to 0.05×10^{-2} due to Poisson's effect. The stress transverse to the fibers then decreases from 18.6 MPa to 16.2 MPa, while the other principal stresses (shear stress and normal stress along the fiber direction) increase. Because, the transverse tensile stresses are critical, a thinner laminate could carry a larger load. If M_{xx} is increased up to 40 kNm/m, the same trend continues. However, when it is increased to 50 kNm/m, a thicker laminate is required.

Table 5.8 The optimum lay-ups obtained using two distinct fiber angles for various loading cases.

Loading: M_{xx} / M_{yy} / M_{xy} (kNm/m)	Optimum lay-up sequences	Total number of plies	Safety factor for Tsai-Wu	Safety factor for max. stress
5 / 5 / 0	$[\theta_{16}/\theta\text{-}90_{87}]_s$: $[0_{16}/90_{87}]_s$, $[1_{16}/-89_{87}]_s$…	206	1.0182	1.0245
10 / 5 / 0	$[31_{20}/-43_{88}]_s$, $[-31_{20}/43_{88}]_s$	216	1.0044	1.0195
20 / 5 / 0	$[30_{19}/-33_{80}]_s$, $[-30_{19}/33_{80}]_s$	198	1.0197	1.1698
30 / 5 / 0	$[26_{19}/-30_{74}]_s$, $[-26_{19}/30_{74}]_s$	186	1.0238	1.5369
40 / 5 / 0	$[25_{18}/-27_{74}]_s$, $[-25_{18}/27_{74}]_s$	184	1.0060	1.4522
50 / 5 / 0	$[20_{21}/-26_{74}]_s$, $[-20_{21}/26_{74}]_s$	190	1.0105	1.3196

6. OPTIMAL DESIGN OF LAMINATED COMPOSITE PLATES WITH A NOTCH USING PROGRESSIVE FAILURE APPROACH

6.1 Problem Formulation

6.1.1 Problem Statement

The structure to be optimized is a symmetric 2-D multilayered laminate with a notch in the form of a circular hole in the middle. The laminate is reinforced by continuous fibers subject to in-plane normal and shear loading as shown in Figure 6.1. Accordingly no bending and twisting moments are considered in the analysis of its mechanical behavior.

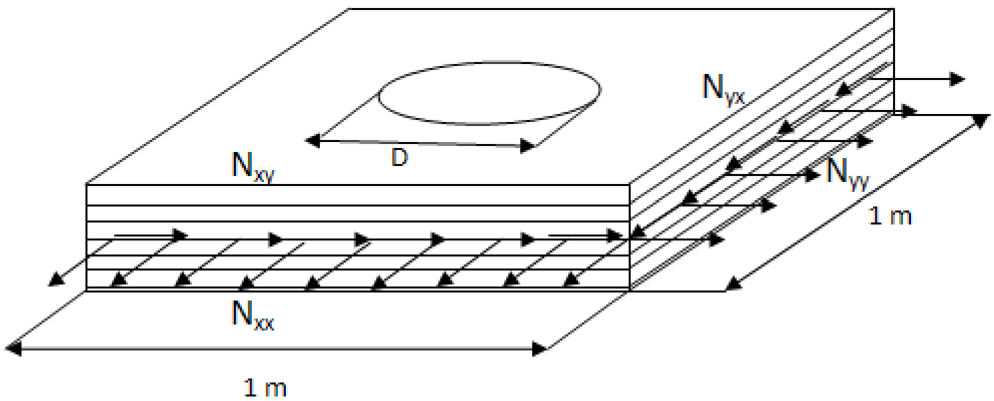

Figure 6.1 A scheme of the composite structure considered in this study.

The laminate consists of plies having the same thickness. The objective is to find the optimum design of the laminate to attain the minimum possible laminate thickness with the condition that it does not fail as stated in Eq(4.1).

The number of distinct fiber orientation angles, m, is given. The orientation angles, θ_k, and how many plies, n_k, are oriented along each angle are to be determined in the design process. Accordingly, the number of design variables is 2*m*. The laminate thickness can be expressed as in Eq(4.2).

Because the plies are made of the same material, minimizing thickness leads to the same optimum configuration as the minimization of weight.

The orientation angles take discrete values; they are chosen from a given set of angles. According to the manufacturing precision, the interval between the consecutive angles may be 15°, 10°, 5°, 1°, 0.5° or even smaller.

6.1.2 Analysis of a Laminated Composite Plate

The structural calculation was performed using the Finite Element Analysis (FEA) program of ANSYS to analyze the mechanical behavior of the composite laminate. Two-dimensional (2-D) mesh was generated to optimize computational time for solution convergence.

SHELL 181 [80]. Figure 6.2 was used as the element type for the composite structure to be analyzed. SHELL 181 is commonly used for 2-D shell-structural modeling. SHELL181 is suitable for analyzing thin to moderately-thick shell structures. It is a 4-node element with six degrees of freedom at each node: translations in the x, y, and z directions, and rotations about the x, y, and z-axes. (If the membrane option is used, the element has translational degrees of freedom only). SHELL181 may be used for layered applications for modeling laminated composite shells or sandwich construction. The accuracy in modeling composite shells is governed by the first order shear deformation theory (usually referred to as Mindlin-Reissner shell theory).

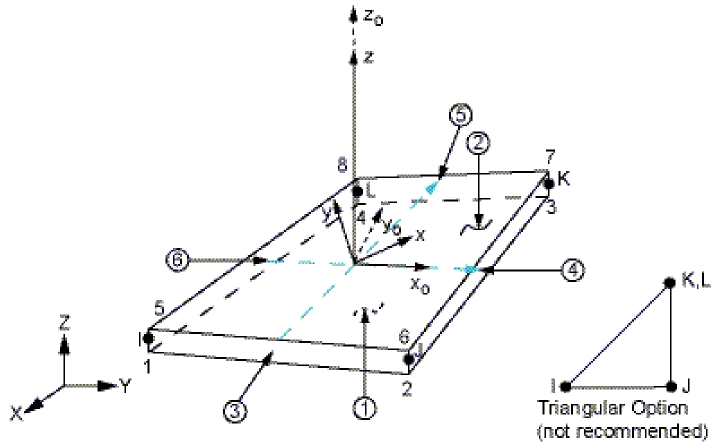

Figure 6.2 General geometry of SHELL 181 element [80].

6.1.3 Static Failure Criteria

Weight minimization of composite structures necessarily involves strength constraints, because decreasing number of load carrying plies eventually leads to failure. The structure must be able to withstand the imposed loads without suffering any failure. In this study, only the static failure modes are assumed to be critical for the laminates. The other failure modes, low stiffness, buckling, delamination etc. are assumed to be not critical.

In order to check the feasibility of a configuration generated by the search algorithm during an optimization process, one needs to use reliable failure criteria. In this study limit theory Tsai-Wu criterion was used.

6.2 Methodology

6.2.1 Progressive Failure Method

While studying the effect of the radius of the hole in the rectangular plate, it was noticed that when the normal load magnitudes same as much as the amount applied in the examples of the previous studies were utilized, the thickness of the laminate to be required for resisting the loading condition gets enormously thicker (>> 600) ,which makes the assumption of classical lamination theory or finite element shell approach useless. This circumstance results from the fact that the whole laminate is assumed to have failed when one of the plies bearing the applied load fails as a requirement of the first-ply failure approach. But in fact, because of the discontinuity created by the hole, the stress distribution is not uniform and in many cases only a small portion of the laminate goes through a real fracture. Therefore a new failure approach known as progressive failure approach was adopted to render the only fractured portion of the laminate ineffective and eliminate from the whole structure. This way, less thicker laminates was able to carry the same amount of loads.

In order to apply progressive failure criteria, material properties used in the analysis were defined for two cases, firstly the real material properties and secondly the pseudo material properties (having very low strength values). Each element in the composite structure is checked according to Tsai-Wu failure criterion for every layer of the plate. When a failure mode is determined, the material properties of the layer pertaining to the related element is changed by the pseudo material properties, thereby this part of the elements are rendered ineffective and they don’t contribute to the stiffness of the whole structure. Since the stress distribution is not uniform throughout the plate and severe only around the discontinuities, only elements and layers undergoing unacceptably high stress states are removed, contrary to the first-ply approach in which the whole layer would be eliminated.

6.2.2 Formulation of the Objective Function

As a requirement of the progressive failure approach, failure of the whole laminate is assumed to take place when the number of failed elements *kk* at all layers reaches the critical amount *failure_number* set at the beginning of the analysis. Unlike the first ply failure element, the crack in a single element of any layer does not lead to elimination of that particular layer. The whole procedure is illustrated in the following figure. As could be understood from the flow chart, the calculation of the objective function is an iterative procedure involving the multiple analysis runs. Here nn stands for the iteration number and Fu(nn) represents the number of cracked elements in a single layer. The configuration i.e. the number of lamina (thickness) and the orientation angles corresponding to each laminate are used in the ANSYS finite element calculation. After the finite element calculation is carried out, p (the number of cracked elements at all layers) and r (the number of cracked elements in a single layer) are set to zero. And an iteration is initiated from one to TotElemNo(total element number), in which the cracked elements at all layers p and at individual layers r are counted by stepping from one layer to another. In order to avoid excessive iteration numbers nn a check is made at the beginning by means of the predefined value nc(limit iteration number), if the limit iteration number is exceeded, the iteration is stopped and p and r values are used in the computation of the penalty value then the value of the cost function is defined and the optimization scheme is activated for determining the new optimization parameters. If nc value is not exceeded then the absolute value of the difference between consecutive Fu(nn) (the number of cracked elements in a single layer) and p (the number of cracked elements at all layers) are compared with limit numbers 2 and *failure_number* on the condition that the difference is greater than 2 and p less than *failure_number* , the iteration is repeated vice versa it comes to a halt and cost function is computed. It is worth mentioning about the importance of *failure_number* as it determines the accuracy of the progressive failure approach, when *failure_number* is taken as high number (more than 20 percent of the TotElemNo) that means larger portions of the laminate is allowed to undergo failure and this causes excessively damaged laminates which cannot be considered to be sound structures. On the other case when *failure_number* is set to be very small, then the structure is considered to go through failure under very limited number of failed elements and progressive failure approach resembles and simulates first ply approach. The whole procedure is illustrated in Figure 6.3.

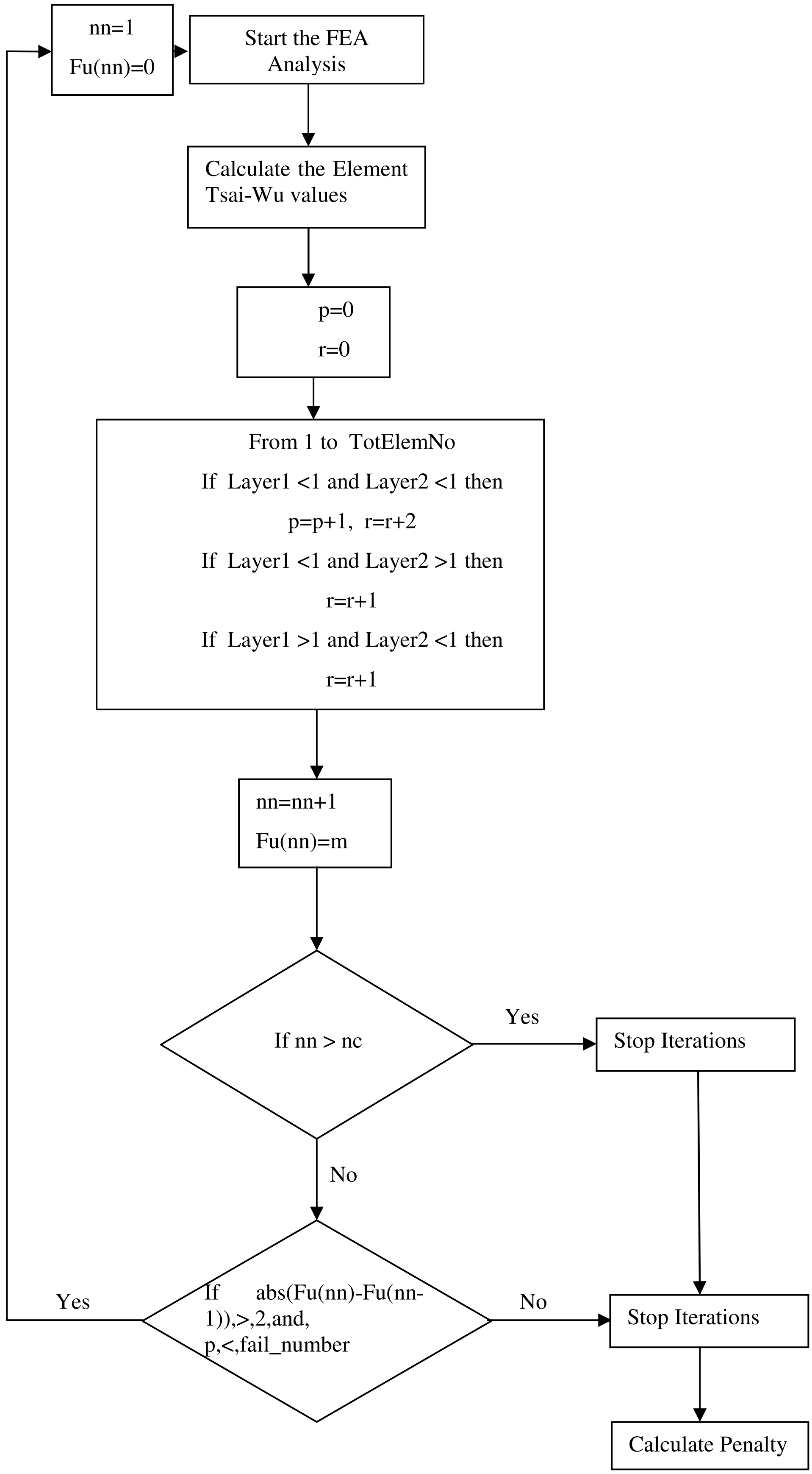

Figure 6.3 Cost Function Calculation Scheme.

The total value of the cost function F is given in Eq. (6.1) the first term represents the total thickness of the composite structure as given in Eq. (4.2); n_k is the number of plies in the k^{th} lamina, in which the orientation angle is θ_k; m is the total number of distinct laminae;

$$F = 2t_o \sum_{k=1}^{m} n_k + Penalty \tag{6.1}$$

The Penalty value is calculated depending on the value of the number of failed elements p at all layers, as shown in the following algorithm part.

```
If k > fail_number, then
    Penalty=PG*p
      else                                  (6.2)
    penalty=PK*p
      endif
penalty=penalty + PM*r
```

where PG is a number large enough to artificially increase the penalty value, PK and PM appropriately selected small numbers to adjust the relative penalty magnitudes.

6.2.3 Optimization Procedure

The same optimization procedure employed in Chapter 4 (4.2.2) was also used for this part of the study.

6.3 Numerical Results

The results of the developed code were compared with the experimental and theoretical outputs of another study [47] from literature. In that study a rectangular plate without hole was applied a point load right in the middle perpendicular to the plate plane. The material Properties pertaining to the experimental work is presented in Table-6.1. ANSYS Calculations reveal that experimental first ply failure loads of Study [47] are quite close to the results of shell approach adopted in the present study. Table-6.2 shows the concordance among the outputs of both studies for various types of laminate configurations.

Table-6.1 Material Properties of Graphite/Epoxy

Material Constants		**Strengths**	
E_1,Gpa	132.5	X_T, Mpa	1690.75
E_2,Gpa	7.90	X_C,Mpa	1893.64
E_3,Gpa	7.90	Y_T=Z_T,Mpa	41.06
G_{12}=G_{13}Gpa	4.20	Y_C=Z_C,Mpa	41.06
G_{23}	1.02	S=T,Mpa	58.7
v_{12}	0.28		
v_{23}	0.25		
t cm	0.015		

Table-6.2 Comparison of first ply failure loads of ANSYS with experimental data

	Laminate	Theoretical Force (N) [**47**]	Experimental Force (N) [**47**]	Theoretical Force (N) Present Study	Error percentage between experimental value and the present study(%)
1	$[0/90]_{6s}$	1794.5	1702.9	1648.0	3.2
2	$[0_6/90_6]_s$	1250.5	1041.5	972.0	6.7
3	$[45_6/-45_6]_s$	1279.2	1345.1	867.0	35.5
4	$[45_3/-45_9]_s$	1703.1	1572.7	1430.0	9.1
5	$[45/-45_2/45_9]_s$	1968.1	1835.7	1690.0	7.9

In order to check the validity of the code prepared in ANSYS parametric design language, formerly prepared Delphi code [57] was used. A comparison between the optimization parameters of both studies was carried out and the outputs were presented in Table-6.4. The material properties pertaining to graphite /epoxy T300/5308 table 6.3 were used in the calculations.

Table-6.3 Material Properties of Graphite/Epoxy T300/5308

Material Constants		**Strengths**	
E_1,Gpa	40.91	X_T, Mpa	779
E_2,Gpa	9.88	X_C,Mpa	1134
E_3,Gpa	9.88	Y_T=Z_T,Mpa	19
G_{12}=G_{13}Gpa	2.84	Y_C=Z_C,Mpa	131
v_{12}	0.292	S=T,Mpa	75
v_{23}	0.25		
t mm	0.127		

When the two results of both finite element and Classiscal Lamination theory is investigated, it is clearly seen that exactly the same outputs are obtained. In order to see the effect of a hole introduced in the middle of the plate with a diameter of 0.03 m a third finite element result is also presented. As could be seen from the given values, introduction of a single Hole to the plate decreases the strength of it in such a way that the minimum amount of ply numbers to bear the same load grows up to 68. Here it is also important to note that first ply failure approach was used in the failure mechanism.

Table-6.4 Comparison between Classical Lamination Theory (CLT) and Finite Element Method (FEM) for Nxx=4E5 Pa.m Nyy=1E5 Pa.m

Method	Optimal Lay-up	Half Laminate Thickness	Safety Factor Tsai-Wu	Geometry
CLT	$[31_5/-31_5]_s$	10	1.1095	Plate without Hole
FEM	$[-31_5/31_5]_s$	10	1.1095	Plate without Hole
FEM	$[-11_{68}]_s$	68	1.0117	Plate with Hole D=0.03

In table 6.5 the optimum lay-up sequences and the minimum lay-up configurations are presented for the geometry including a hole in the middle having a diameter of 0.03 m. For the whole results progressive failure approach were used. As could be figured out from the Fracture Ratios employed in the problem formulations, when Fracture ratio is increased from 1% up to 50% the minimum half-laminate thickness gets smaller values. This could be explained on the fact that as fracture ratios gets higher values, we allow more elements to be eliminated from the structure and as a result smaller thickness values are achieved.

Table 6.5- The Optimum Lay-up for the loading Nxx=4 × 10^5N/m, Nyy= 1 × 10^5N/m Nxy=0, Hole Diameter=0.03 m

Optimal Lay-Up	Half Laminate Thickness	Fracture Ratio(%)
$[-6_{59}/-65_2]_s$	61	1
$[-31_{11}/34_{10}]_s$	21	20
$[-32_{10}/34_{10}]_s$	20	30
$[32_{10}/-34_{10}]_s$	20	50

7. CONCLUSIONS

The aim of this study is to develop methodologies to optimize the structural design of composite materials to obtain the minimum weight. In the first part of the study, an optimization methodology for weight minimization of composite plates under in-plane loading was presented. Methods were proposed in order to overcome the difficulties faced by the previous researchers. The direct simulated annealing algorithm (DSA) was adopted as search algorithm. For some loading cases, many global or near-global optimum designs were found to exist. The algorithm proved to be quite reliable in locating these designs. In a single optimization run, the algorithm could find one or more of them even with a large number of design variables.

When the Tsai-Wu or maximum stress failure criteria are used individually, an optimization algorithm is lead to false optimal designs because of the particular features of their failure envelopes. On the other hand, when they are used together, false optimums are avoided. Use of the Tsai-Hill criterion, which includes only tensile strengths, or taking compressive strengths equal to the tensile strengths leads to overly conservative designs.

For different materials, fiber orientation angles in the optimal lay-up designs may be different for the same loading case. Thus, one may not generalize the results obtained for a particular material to others.

In some cases, the optimal designs can be counter-intuitive. Sometimes, when one component of loading is increased, load bearing capacity of the optimized laminate may increase. Therefore, a design process for composite materials should not be based on intuition or experience.

Usually, choosing only two distinct fiber angles is sufficient to obtain the best possible design. In some load cases, different lay-up sequences with the same objective function value were obtained with a large number of distinct fiber angles. In some others, however, better designs were obtained with three or four distinct angles, which have the same thickness but a larger safety factor. In one case, a thinner laminate was obtained with four distinct angles.

If the available fiber orientations are scarce, extremely inferior designs may be obtained. This is the case, if only 0°, ±45°, and 90° angles are allowed. For this reason, the interval between the available angles should be small enough to be able to find the best design.

In the second part of the study a methodology is presented to optimize composite laminates subject to both in-plane and out-of-plane loading for minimum thickness. A variant of the simulated annealing algorithm is proposed to search the globally optimum design(s). Many multiple global or near global optimums were found to exist. The algorithm proved to be quite reliable in locating these designs. The search algorithm yielded consistent and reliable results in all the optimization runs.

By increasing the number of distinct lamina angles and the range of values they may take, one obtains a larger design domain, i.e. more lay-up configurations become possible. In this way, the complexity of the design domain and the number of local minima greatly increase; but existence of a better global optimum becomes also more likely. In this study, up to sixteen distinct lamina thicknesses and angles with 1° angle increments were used as design variables to optimize laminates. To the knowledge of the author, optimization with such a large solution domain was not attempted in the previous studies. With a larger domain, it was possible to obtain better optimum designs. Unlike in-plane loading, using two or three distinct lamina angles is not sufficient to obtain the best possible design for laminates under out-of-plane loading. For all the out-of-plane loading cases considered in this study, optimum lay-up configurations with 16-distinct laminae turned out to be better than the ones with 8-distinct laminae.

In the last part of the study, optimal design of laminated composite plates with stress concentration was covered. In this study, a more complicated geometrical shape was considered and a stress concentration discontinuity in the form of a circular hole was introduced in the middle of the rectangular plate. In order to investigate the effect of circular notch in the weight minimization process, a computer program was developed using ANSYS parametric design language. Instead of Classical Lamination Theory, finite element method was employed. The finite element model to be used in the analysis was also tested by using the comparisons made between the structure of the model and those found in the related literature.

Apart from the first two studies, progressive failure approach was employed along with the static failure criterion Tsai-Wu. It was clearly demonstrated that when progressive failure approach was used, the minimum half laminate thickness gets much smaller values compared to the first ply failure approach. This can be explained on the basis that first ply failure approach assumes that the whole laminate should be considered to have failed when any of the laminae fails, but in the progressive failure approach only the failed elements in the structure are eliminated and the rest remains to support the applied loads.

Thermal residual stresses resulting from thermal expansion mismatch in adjacent plies in the laminates during cool down from the stress-free state at the cure temperature were not accounted for in the predictions. It is acknowledged that the inclusion of thermal residual stresses will have some effect on the ultimate laminate strength predictions. The exact effect, however, will depend on the specific laminate architecture and loading considered.

REFERENCES

1. Schmit, LA. and B. Farshi, "Optimum laminate design for strength and stiffness", *International Journal for Numerical Methods in Engineering*, Vol. 7: pp. 519-536, 1973.

2. Schmit LA. and B. Farshi, "Optimum design of laminated fibre composite plates", *International Journal for Numerical Methods in Engineering*, Vol. 11: pp. 623-640, 1977.

3. Fukunaga, H. and GN. Vanderplaats, "Strength Optimization of Laminated Composites with respect to layer thickness and/or layer orientation angle", *Computers & Structures*, Vol. 40(6): pp. 1429-1439, 1991.

4. Soares CMM, VF. Correia and H. Mateus, J. Herskovits, "A discrete model for the optimal design of thin composite plate – shell type structures using a two – level approach", *Composite Structures*, Vol. 30: pp. 147-157, 1995.

5. Le Riche R. and RT. Haftka, "Improved genetic algorithm for minimum thickness composite laminate design", *Composites Engineering*, Vol. 5(2): pp. 143-161, 1995.

6. Jayatheertha C, JPH Webber and SK. Morton, "Application of artificial neural networks for the optimum design of a laminated plate", *Computers & Structures*, Vol. 59(5): pp. 831-845, 1996.

7. Wang J. and B.L. Karihaloo, "Optimum In Situ Strength Design of Composite Laminates. Part II Optimum Design", *Journal of Composite Materials*, Vol. 30: pp. 1338, 1996.

8. Adali S and VE. Verijenko, "Minimum cost design of hybrid composite cylinders with temperature dependent properties", *Composite Structures*, Vol. 38: pp. 623-630, 1997.

9. Correia VMF, CMM Soares and CAM. Soares, "Higher order models on the eigenfrequency analysis and optimal design of laminated composite structures", *Composite Structures*, Vol. 39(3-4): pp. 237-253, 1997.

10. Soares CMM, CAM Soares and VMF Correia, "Optimization of multilaminated structures using higher-order deformation models", *Computer Methods in Applied Mechanics and Engineering*, Vol. 149: pp. 133-152, 1997.

11. Abu-Odeh AY and HL. Jones, "Optimum design of composite plates using response surface method", *Composite Structures*, Vol. 43: pp. 233-242, 1998.

12. Lombardi M and RT. Haftka, "Anti-optimization technique for structural design under load uncertainties", *Computer Methods in Applied Mechanics and Engineering*, Vol. 157: pp. 19-31, 1998.

13. Le Riche R and J. Gaudin, "Design of dimensionally stable composites by evolutionary optimization", *Composite Structures*, Vol. 41: pp. 97-111, 1998.

14. Sivakumar K and NGR Iyengar, K. Deb, "Optimum design of laminated composite plates with cutouts using a genetic algorithm", *Composite Structures*, Vol. 42: pp. 265-279, 1998.

15. Barakat SA and GA. Abu-Farsakh, "The use of an energy-based criterion to determine optimum configurations of fibrous composites", *Composite Science and Technology*, Vol. 59: pp. 1891-1899,1999.

16. Richard F and D. Perreux, "A reliability method for optimization of $[+\varphi,-\varphi]_n$, fiber reinforced composite pipes", *Reliability Engineering & System Safety*, Vol. 68: pp. 53-59, 2000.

17. Moita JS and JI Barbosa, CMM Soares, CAM Soares, "Sensitivity analysis and optimal design of geometrically non-linear laminated plates and shells", *Computers & Structures*, Vol. 76: pp. 407-420,2000.

18. Soremekun G and Z Gurdal, RT Haftka, "Watson LT. Composite laminate design optimization by genetic algorithm with generalized elitist selection", *Computers & Structures*, Vol. 79: pp. 131-143, 2001.

19. Walker M and RE. Smith, "A technique for the multiobjective optimisation of laminated composite structures using genetic algorithms and finite element analysis", *Composite Structures*, Vol. 62: pp. 123-128, 2003.

20. Sciuva MD and M Gherlone, D Lomario, "Multiconstrained optimization of laminated and sandwich plates using evolutionary algorithms and higher-order plate theories", *Composite Structures*, Vol. 59: pp. 149-154, 2003.

21. Kere Pand M Lyly, J. Koski, "Using multicriterion optimization for strength design of composite laminates", *Composite Structures*, Vol. 62: pp. 329-333, 2003.

22. Park CH and WI Lee, WS Han, A. Vautrin, "Weight minimization of composite laminated plates with multiple constraints", *Composite Science and Technology*, Vol. 63: pp. 1015-1026, 2003.

23. Kang JH and CG. Kim, "Minimum-weight design of compressively loaded composite plates and stiffened panels for post buckling strength by genetic algorithm", *Composite Structures*, Vol. 69: pp. 239-246, 2005.

24. Adali S and A Richter, VE. Verijenko, "Optimization of shear-deformable laminated plates under buckling and strength criteria", *Composite Structures*, Vol. 39(3-4): pp. 167-178 1997.

25. Erdal O and FO. Sonmez, "Optimum design of composite laminates for maximum buckling load capacity using simulated annealing", *Composite Structures*, Vol. 71: pp. 45-52, 2005.

26. Victor M. and Franco Correia , Crist_ovao M. Mota Soares , Carlos A. Mota Soares, "Buckling optimization of composite laminated adaptive structures", *Composite Structures*, Vol. 62: pp. 315–321, 2003.

27. Moita J.M.S. and V.M.F. Correia, PG. Martins, "Optimal design in vibration control of adaptive structures using a simulated annealing algorithm", *Composite Structures*, Vol. 75: pp. 79-87, 2006.

28. Callahan KJ and GE. Weeks, "Optimum design of composite laminates using genetic algorithms", *Composites Engineering*, Vol. 2(3): pp. 149-160, 1992.

29. Park JH and JH Hwang, CS Lee, W. Hwang, "Stacking sequence design of composite laminates for maximum strength using genetic algorithms", *Composite Structures*, Vol. 52: pp. 217-231, 2001.

30. Kim CW and W Hwang, HC Park, KS. Han, "Stacking sequence optimization of laminated plates", *Composite Structures*, Vol. 39(3-4): pp. 283-288, 1997.

31. Kim CW and W. Hwang, "Optimal stacking sequence design of laminated composite shells", *Science and Engineering of Composite Materials*, Vol. 8(3): pp. 159-174, 1999.

32. Tabakov PY and M. Walker, "A Technique for optimally designing engineering structures with manufacturing tolerances accounted for", *Engineering Optimization*, Vol. 39(1): pp. 1-15, 2007.

33. Oh JH and YG Kim, DG. Lee, "Optimum bolted joints for hybrid composite materials", *Composite structures*, Vol. 38: pp. 329-341, 1997.

34. Wang J and BL. Karihaloo, "Optimum in situ strength design of laminates under combined mechanical and thermal loads", *Composite Structures*, Vol. 47: pp. 635-641, 1999.

35. Martin P.M.J.W., "Optimum design of anisotropic sandwich panels with thin faces", *Engineering Optimization*, Vol.11: pp. 3-12, 1987.

36. Adali S. and E.B. Summers, V.E. Verijenko, "Minimum weight and deflection design of thick sandwich laminates via symbolic computation", *Composite Structures*, Vol. 29: pp. 145-160, 1994.

37. Kam T.Y. and F.M. Lai, "Maximum stiffness design of laminated composite plates via a constrained global optimization approach", *Composite Structures*, Vol.32: pp. 391-398, 1995.

38. Huang C. and B. Kroplin, "On the optimization of composite laminated plates", *Engineering Computations*, Vol. 12: pp. 403-414, 1995.

39. Huang C. and B. Kröplin, "Optimum design of composite laminated plates via a multi-objective function", *Int. J. Mech. Sci.*, Vol. 37: pp. 317-326, 1995.

40. Song S.R. and W. Hwang, H.C. Park, "Optimum stacking sequence of composite laminates for maximum strength", *Mechanics of Composite Materials*, Vol. 31(3): pp. 290-300, 1995.

41. Walker M. and T. Reiss, S. Adali, “Optimal design of symmetrically laminated plates for minimum deflection and weight”, *Composite Structures*, Vol. 39: pp. 337-346, 1997.

42. Haridas B. and W.K. Rule, “A modified interior penalty algorithm for the optimization of structures subjected to multiple independent load cases”, *Computers & Structures*, Vol.65(1): pp. 69-81, 1997.

43. Yamazaki K. and N. Tsubosaka, “A stress analysis technique for plate and shell built-up structures with junctions and its application to minimum-weight design of stiffened structures”, *Structural Optimization*, Vol. 14: pp. 173-183, 1997.

44. Kere P. and J. Koski, “Multicriterion stacking sequence optimization scheme for composite laminates subjected to multiple loading conditions”, *Composite Structures*, Vol. 54: pp. 225-229, 2001.

45. Bruyneel M. and C. Fleury, “Composite structures optimization using sequential convex programming”, *Advances in Engineering Software*, Vol. 33: pp. 697-711, 2002.

46. Kasprzak J. and M. Ostwald, “Multicriterion optimization of hybrid composite plates with various supports under transverse load”, *Proceedings in Applied Mathematics and Mechanics*, Vol. 5: pp. 747-748, 2005.

47. Kam T.Y. and F.M. Lai, S.C. Liao, “Minimum weight design of laminated composite plates subject to strength constraint”, *AIAA Journal*, Vol. 34(8): pp. 1699-1708, 1996.

48. Avalle M. and G. Belingardi, “A theoretical approach to the optimization of flexural stiffness of symmetric laminates”, *Composite Structures*, Vol. 31: pp. 75-86, 1995.

..49. Jadhav P.and P.R. Mantena, “Parametric optimization of grid-stiffened composite panels for maximizing their performance under transverse loading”, *Composite Structures*, Vol. 77: pp. 353-363, 2007.

50. Deka D.J. and G. Sandeep, D. Chakraborty, “Multiobjective optimization of laminated composites using finite element method and genetic algorithm”, *Journal of Reinforced Plastics and Composites*, Vol. 24(3): pp. 273-285, 2005.

51. Pelletier J.L. and S.S. Vel, “Multi-objective optimization of fiber reinforced composite laminates for strength, stiffness and minimal mass”, *Computers & Structures*, Vol. 84: pp. 2065-2080, 2006.

52. Park C.H. and W.I. Lee, W.S. Han, “Improved genetic algorithm for multidisciplinary optimization of composite laminates”, *Computers & Structures*, Vol. 86: pp. 1894-1903, 2008.

53. Kim J.S., “Development of a user-friendly expert system for composite laminate design”, *Composite Structures*, Vol. 79: pp. 76-83, 2007.

54. Aymerich E. and M. Serra, “An ant colony optimization algorithm for stacking sequence design of composite laminates”, *Computer Modeling in Engineering & Sciences*, Vol. 13(1): pp. 49-65, 2006.

55. Todoroki A. and N. Sasada, M. Miki, “Object-oriented approach to optimize composite laminated plate stiffness with discrete ply angles”, *Journal of Composite Materials*, Vol. 30: pp. 1020-1041, 1996.

56. Rao A.N.and C. Ratnam, J. Srinivas, A. Premkumar, “Optimum design of multilayer composite plates using simulated annealing”, *Proceedings of the Institution of Mechanical Engineers Part L: Journal of Materials: Design and Applications*, Vol. 216(3): pp. 193-197, 2002.

57. Akbulut M. and F.O. Sonmez, “Optimum design of composite laminates for minimum thickness”, *Computers & Structures*, Vol. 86: pp. 1974–1982, 2008.

58. Massard T.N., “Computer sizing of composite laminates of strength”, *Journal of Reinforced Plastics and Composites*, Vol. 3: pp. 300-327, 1984.

59. Soerio A.V. and C.A.C. Antonio, AT. Marques, “Multilevel optimization of laminated composite structures”, *Structural Optimization*, Vol. 7: pp. 55-60, 1994.

60. Fang C. and G.S. Springer, “Design of composite laminates by a Monte Carlo method”, *Journal of Composite Materials*, Vol. 27(7): pp. 721-753, 1993.

61. Omkar S.N. and R. Khandelwal, S. Yathindra, G.N. Naik, S. Gopalakrishnan, Artificial immune system for multi-objective design optimization of composite structures”, *Engineering Applications of Artificial Intelligence*, Vol. 21: pp. 1416-1429, 2008.

62. Watkins RI and Morris AJ., “A multicriteria objective function optimization scheme for laminated composites for use in multilevel structural optimization schemes”, *Computer Methods in Applied Mechanics and Engineering*, Vol. 60: pp. 233-251, 1987.

63. Tauchert T.R. and S. Adibhatla, “Design of laminated plates for maximum stiffness”, *Journal of Composite Materials*, Vol. 18: pp. 58-69, 1984.

64. Verijenko V.E. and E.B. Summers, S. Adali, “Minimum stress design of transversely isotropic sandwich plates based on higher-order theory”, *Structural Optimization*, Vol. 15: pp. 114-123, 1998.

65. Kirkpatrick S and CD Gelatt, MP. Vecci, “Optimization by Simulated Annealing”, *Science*, Vol. 220: pp. 671-80, 1983.

66. Ali MM and A Törn, S. Viitanen, “A Direct Search Variant of the Simulated Annealing Algorithm for Optimization Involving Continuous Variables”, *Computer Operation Research*, Vol. 29: pp. 87-102, 2002.

67 . Anlas G. and Ö. Tüzer, “Design of Laminated Composite Plates Containing a Hole under In-Plane Loadings”, *Journal of Reinforced Plastics and Composites*, Vol.20: pp. 1024, 2001.

68 . Khosravi P. and R. Sedaghati, “Design of laminated composite structures for Optimum fiber direction and layer thickness, using optimality criteria”, *Struct Multidisc Optim*, Vol.36: pp. 159-167, 2008.

69 . Cho 1 H.K. and R.E. Rowlands, “Reducing tensile stress concentration in perforated hybridlaminate by genetic algorithm”, *Composites Science and Technology*, Vol.67: pp. 2877–2883, 2007.

70. Whitney J.M. and R.J.Nuismer, “Stress Fracture Criteria for Laminated Composites Containing Stress Concentrations”, *Journal of Composite Materials*, Vol. 8: pp. 253, 1974.

71 . Huang J. and R.T. Haftka, “Optimization of fiber orientations near a hole for increased load-carrying capacity of composite laminates”, *Struc Multidisc Optim*, Vol. 30: pp. 335-341, 2005.

72. Venkataraman S. and P. Salas, “Optimization of Composite Laminates for Robust and Predictable Progressive Failure Response”, *AIAA Journal*, Vol. 45: pp. 1113-1125, 2007.

..73. Harik V.M., “Optimization of Structural designs for a safe failure pattern: layered material systems”, *Materials & Design*, Vol. 22: pp. 317-324, 2001.

74. Jones RM. Mechanics of Composite Materials, 2nd edition. Taylor & Francis. 1999.

75. Borovkov A and V Palmov, N Banichuk, V Saurin, F Barthold, E. Stein, "Macro-failure criterion for the theory of laminated composite structures with free edge delaminations" *Composite and Structures*, Vol. 76: pp. 195-204, 2000.

76. Tabakov PY. ,"Multi-dimensional design optimisation of laminated structures using an improved genetic algorithm", *Composite Structures*, Vol. 54: pp. 349-354, 2001.

77. Soremekun G and Z Gürdal, C Kassapoglou, D. Toni, "Stacking sequence blending of multiple composite laminates using genetic algorithms", *Composite Structures*, Vol. 56: pp. 53-62, 2002.

78. Groenwold AA and RT. Haftka, "Optimization with non-homogeneous failure criteria like Tsai-Wu for composite laminates", *Struct. Multidisc. Optim.*, Vol. 32: pp. 183-190, 2006.

79. Gürdal Z and RT Haftka, P. Hajela, "Design and Optimization of Laminated Composite Materials". *John Wiley & Sons*. 1999.

80. Release 11.0 Documentation for ANSYS Commercial Package, 2008.