AGENT BASED MODELING OF SINGLE ASSET MARKETS

by

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ABSTRACT

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Understanding of the human behaviors has been a life long attempt along the history of science. This attempt has been accelerated along with the increasing speed of the computers which helps researchers to develop modern and more efficient modeling techniques. One of these recently developed approaches for modeling of the human behaviors is the simulation modeling approach. This thesis addresses the issue of the modeling of human behaviors (or decisions) by the application of the newly emerged simulation modeling technique called agent-based modeling. Agent-based modeling technique is a bottom-up simulation modeling approach through which human decisions are modeled and the global resultant dynamics emerged from the interactions of these humans, namely agents, possessing different decision mechanisms are tried to be understood.

In this thesis, we apply agent-based modeling technique in order to model two different single asset market models and to grasp the fundamental parameters of these models that correspond to the global human behaviors which are represented by some statistical facts that are derived from the statistical analysis of real stock market asset returns. In order to accomplish this task, first, models are described deeply and model parameters (decisions) that are expected to be crucial are examined through the sensitivity analysis considering those global statistical facts. The first model introduced is a simple single asset market model in which there is only one type of trader with different decision parameters. Second model described is again a single asset market model with a more realistic, but more sophisticated trader setting, in which there are two general trader types.

According to the investigation performed on these models, we could be able to reach important results on how humans behave while they are trading a single asset in a financial market.

ÖZET

TEK VARLIKLI PAZARLARIN AJAN TABANLI MODELLENMESİ

İnsan davranışlarının anlaşılmaya çalışılması bilim tarihinin hayat boyu süren uğraşlarından biri olmuştur. Bu uğraşö araştırmacılara daha etkin ve modern modelleme teknikleri geliştirmelerinde yardımcı olan bilgisayarların hızlanmasıyla birlikte süratlenmiştir. Bu yeni geliştirilmiş insan davranışlarını modelleme yaklaşımlarından birisi simulasyonla modellemedir. Bu çalışmada, kısa zaman önce ortaya çıkmış bir simulasyonla modelleme tekniği olan ajan tabanlı modelleme tekniği uygulanarak, insan davranışlarının (ya da kararlarının) modellenesine çalışılmıştır. Ajan tabanlı modelleme tekniği, insan kararlarının modellenmesine ve farklı karar mekanizmalarına sahip bu insanların birbirleriyle olan etkileşimlerinden ortaya çıkan genel dinamiğin anlaşılmaya çalışılmasına yardımcı olan aşağıdan yukarıya dogru simulasyon modellemesi yapan bir yaklaşımdır.

Bu çalışmada, iki farklı tek varlıklı pazar modeli oluşturmak için ve bu modellerdeki gerçek borsa varlıklarının fiyat kazanç oranlarının analizinden çıkartılan bazı istatiksel gerçeklerle temsil edilen genel insan davranışlarına karşılık gelen ana parametreleri iyice kavramak için ajan tabanlı modelleme tekniğini uyguladık. Bunu yerine getirebilmek için, ilk olarak modeller derinlemesine tanımlandı ve önemli görülen parametreler bahsedilen istatiksel gerçekler ışığında duyarlılık analizi uygulanarak incelendi. İlk tanıtılan model, farklı karar parametrelerine sahip tek tip borsa müşterilerinin alım satım yaptığı tek varlıklı bir market modelidir. İkinci model ise daha gerçekçi fakat daha karmaşık yapıdaki, iki farklı borsa müşteri tipinin bulundugu tek varlıklı pazar modelidir.

Bu modellerin incelenmesinden varılan sonuçlara göre, insanların finansal pazarlardaki tek varlık alışverişinde nasıl davrandıklarına ilişkin önemli sonuçlara ulaşabileceğimiz anlaşılmıştır.

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LIST OF SYMBOLS/ABBREVIATIONS

a	Scale parameter for capital assignment
c	Capital scaling parameter
D	Standard deviation of the common public signal
$d_{i,t}$	Demand of agent *i* at time period *t*
G(x)	Price impact function
I_t	External information at time period *t*
m_t	Mispricing at time period *t*, $m_t = p_t - v_t$
N	Number of agents
P_t	Price of an asset at period *t*
p_t	Logarithm of price at time period *t*, $p_t = \log P_t$
r_t	Return at time period *t*
$\vert r_t(\Delta)\vert$	Autocorrelation of the absolute returns
T	Entry threshold in the second model
$T_{\min}$	Minimum value for the entry threshold in the second model
$T_{\max}$	Maximum value for the entry threshold in the second model
S	Update frequency
$u_{i,t}$	Random number used in threshold updating mechanism
X_t	Excess demand computed in the first model
$x_{i,t}$	Position of the directional trader labeled *i* at time period *t*
Y_i	Measurement in a statistical study
Δ	Time lag used in autocorrelation computations
ε_t	Common signal received by all agents in the first model
η_t	Normally distributed IID random number used in value process
θ_i	Time delay of the i^{th} chartist in the second model
$\theta_{i,t}$	Decision threshold used by agent *i* at time period *t* in the first model
$\theta_{i,0}$	Initial decision threshold of agent *I* in the first model
$\theta_{\max}$	Maximum time delay in the second model
λ	Market depth

μ_η	Mean of the random term used in value process
v_i	Fixed random offset for the value perceived by agent *i*
$v_{i,t}$	Value perceived by the i^{th} trader at time *t*
v_{min}	Minimum value of the random offset
v_{max}	Maximum value of the random offset
v_t	Logarithm of the fundamental value
$\bar{v}_t$	Reference value process that follows $v_{t+1} = v_t + \eta_{t+1}$
ξ_t	Noise term (or random perturbations) in price process
σ_η	Standard deviation of the random term used in value process
σ_ξ	Standard deviation of the noise term in price process
τ	Exit threshold
τ_{max}	Maximum value for the exit threshold
τ_{min}	Minimum value for the exit threshold
$\omega_{i,t}$	Order (or demand) of agent *i* at time period *t*
ω_t	Net order at time period *t* for the second model, $\omega_t = \sum_i^N \omega_{i,t}$

ABS	Adaptive Belief Systems
ARCH	Autoregressive Conditional Heteroscedasticity
SFI	Santa Fe Institute
S&P	Standard and Poor's (index)
XP	Excess price where $Excess\,Price_i = Price(t) - Price(t - \theta_i)$

1. INTRODUCTION

1.1. Agent Based Modeling

Modeling may be seen as an abstraction of reality in order to investigate its properties. In order to understand the functioning of the world around us we have to model our environment. So, different modeling techniques are developed during the history. One of them which is called simulation modeling is the one emerged along with the increasing power of computers in the 21st century. Simulation modeling is a modeling approach which relies heavily on the computational power of computers while examining the complex dynamical problems where analytical solutions may not be possible.

Simulation modeling is applied when experimenting or prototyping with the real system of interest is impossible or too expensive. There are three main approaches in simulation modeling, namely discrete event modeling, agent based modeling and system dynamics modeling. In this study we prefer using agent based modeling technique in order to understand the crucial parameters behind the observed behaviors in real financial markets.

Agent based modeling can be applied to all complex dynamical problems including social, economical, ecological and political ones. It may be applied to examine the dynamics of a bird population, such as investigating the movement of a flock of birds, or it may be applied to examine the effect of traders in a market for goods. Agent-based modeling approach is preferred in modeling because even very simple agent based models may exhibit very complex behaviors and may provide useful information about the dynamics of the problem of interest.

In agent based modeling, a system is modeled as a collection of interacting autonomous entities called agents. Each agent in the model individually assesses his situation and makes decision according to his predetermined decision rule. Decisions that are made may represent buying or selling one share of a stock, or deciding on moving or staying inactive in traffic in accordance with the problems that are to be examined.

In addition, agents may be capable of evolving which may result in unanticipated behaviors. In more sophisticated agent based models, artificial intelligence applications may be seen. These are applied mostly in order to embed learning mechanisms, which allow evolution and adaptation to emerge, into the system which is a common property of systems under human influence. In order to represent this real world phenomenon they incorporate evolutionary algorithms, neural networks and other learning techniques into the model.

One of the most popular applications of agent based modeling is the modeling of the stock market which is also performed in this study. There are lots of models proposed in literature on this issue which will be discussed in the following subsection.

1.2. Agent Based Modeling of Financial Markets

Agent based modeling approach is applied widely in order to model financial stock markets. As it is discussed in Le Baron (2000), we can enumerate the reasons behind this as follows;

1) Financial time series contain many curious puzzles that are not well understood.
2) Issues of price and information aggregation tend to be sharper in financial settings where agent objectives tend to be clearer.
3) Financial markets provide a wealth of pricing and volume data at many different frequencies from annual to minute by minute that can be analyzed.
4) When evolution is considered, financial markets provide a good approximation to a crude fitness measure through wealth or return performance.
5) There are continuing developments in the area of experimental financial markets which give carefully controlled environments which can be compared with agent based model experiments.

Agent based models developed in order to simulate financial markets give emphasis on interactions among learning boundedly-rational agents. Agents (traders) in a market model gather information and each agent processes his information differently which leads to the heterogeneity among agents which is an important property of the financial agent based models. Besides heterogeneity, feedback mechanism through prices or returns or by any other information is also crucial in agent based market modeling. Feedback is a must in modeling of a complex dynamical system which is not only embedded in agent based models but also included in other modeling approaches. In financial market models, feedback is provided mostly by supplying agents with the prices or returns.

In addition to these properties, evolutionary learning mechanisms are included in more sophisticated agent based market models in literature through the use of genetic algorithms as well as neural networks using wealth or number of successful moves as a crude measure for the agents' success.

The most popular of the sophisticated market models is the Santa Fe artificial stock market which is developed at Santa Fe Institute (SFI). This market model is developed in late 1980's and early 1990's and published by Arthur et al. (1994). SFI market originated as a desire of Brian Arthur and John Holland to build a financial market driven by the interaction of trading strategies. In this market model, successful strategies (representing the agents) persist and replicate, whereas weak strategies give up existing creating places for the new strategies. So, market is ecology of strategies coevolving by inductive reasoning, representing the traders. They model market like this because they wanted the market to evolve by itself while not loading too much into the system exogenously. So, they let evolution to do most of the work. This model also incorporates the important tools for modeling learning, such as Holland's genetic algorithm and classifier system. SFI market simply functions as follows;

First of all a price is announced by the market maker to all traders and each agent finds a matching rule for the current market conditions and come to the market and buys or sells one share of the traded stock according to his strategy. Then, according to the excess demand new price is set by the market maker. As a result, strategies that are on the minority side with respect to the excess demand win and others lose. In SFI model strategies have points describing their success, and in relation to their successes and failures they survive or go away from the model. Strategies evolve by the learning mechanisms mentioned before, which will not be discussed here since it is out of the scope of this study.

In addition to the model proposed by SFI people, there are other important market models in literature that are worth to discuss. One of them is the model proposed by Brock and Hommes (1997) which they name it as "Adaptive Belief Systems", ABS. This early market model is an analytically tractable market model which is so simple that agent based modeling was not needed. However, they also mention in their paper that agent based modeling is a must for a more sophisticated market model with more trader types. In their model, traders are separated into two categories, fundamentalists and technical traders. These are also boundedly-rational traders who are allowed to switch between being a fundamentalist and a technical trader according to a fitness or performance measure based upon how their strategy perform in the generation of trading profits. They investigate results of the model analytically by the application of the chaos theory including strange attractor and bifurcation studies.

The last two market models which are also agent based market models are the models proposed by Farmer and Joshi (2002) and Ghoulmie et al. (2005). Since these two models are the ones investigated in this study, I am not going to mention the details here.

Before continuing to discuss what will be coming throughout this study in the next three sections, I will explain some of the statistical facts, called stylized facts which are going be referred in the text.

1.3. Some of the Important Stylized Facts investigated in this Study

There are several important statistical features observed in real financial markets. Here I will explain some of them which are mentioned in this study.

First one is the kurtosis (in this study we use excess kurtosis which will be explained in the following). Kurtosis is a measure of the peakedness of a distribution. Sometimes it's referred to as the "fourth movement."

Excess kurtosis is defined as the kurtosis of a distribution minus that of the normally distributed one. Kurtosis for a normal distribution is 3. So, 3 is subtracted from the kurtosis of a distribution in order to get the excess kurtosis of that distribution. Then,

$$\text{Excess Kurtosis} = \text{Kurtosis} - 3 \tag{1.1}$$

Kurtosis is computed with the following formula;

$$Kurtosis = \frac{\sum (X - \bar{X})^4}{\sigma_X^4} \tag{1.2}$$

As mentioned above, kurtosis of a normal distribution is 3 (excess kurtosis is 0) and is known as "mezokurtic"

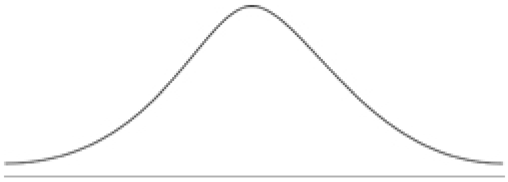

Figure 1.1. Mezokurtic Normal distribution

Conversely, if a distribution is tighter and taller than a normal distribution, the excess kurtosis will be a positive number. This kind of distribution is called "leptokurtic" because of its long tails.

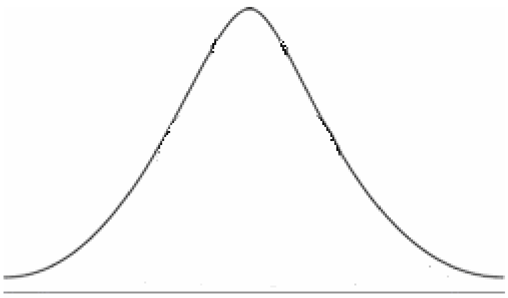

Figure 1.2. Leptokurtic distribution

Finally, if a distribution is flatter than the normal distribution, the excess kurtosis will be negative. This kind of distribution with short tails like a platypus is called "platykurtic".

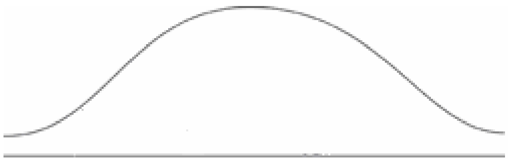

Figure 1.3. Platykurtic distribution

Leptokurtic distributed returns are observed in real financial markets. Because of this, we will be looking for the heavy tails with positive excess kurtosis from the returns generated by our agent based model.

Another important statistical fact is the volatility clustering encountered in real financial market data. This is also named as persistence of volatility. In financial stock markets, direction of returns is generally unpredictable which is expressed as the absence of autocorrelation. On the other hand, magnitude of the returns is often very predictable which is expressed as volatility clustering. Volatility clustering is investigated mostly by considering the significance of the autocorrelation of the absolute or squared returns.

So, before proceeding with the volatility clustering, autocorrelation function should be explained first.

Autocorrelation function is examined basically for two purposes in finance;

1) To detect non-randomness in a time series data
2) To identify an appropriate time series model if the data are not random

In our agent based market models we will investigate for the appropriate model parameters providing us with returns having zero autocorrelation in all lags. So, we use it for the first purpose in our study.

Autocorrelation function for time lag t is defined as follows;

$$r_t = \frac{\sum_{i=1}^{N-t}(Y_i - \bar{Y})(Y_{i+t} - \bar{Y})}{\sum_{i=1}^{N}(Y_i - \bar{Y})^2} \tag{1.3}$$

where Y_i, i=1,… , N are the measurements (in our study these correspond to returns) observed at equally spaced time periods.

Autocorrelation is a correlation coefficient but instead of the correlation between two different variables, it provides us the correlation between two values of the same variable at times i and i+t with time lag of t.

As I mentioned before, volatility clustering is examined by observing the autocorrelation of absolute returns. Volatility clustering as its name implies, is an indicator of whether there is a switching between periods of low activity and high activity. That is to say, whether the returns with low magnitudes are followed by returns with low magnitude and returns with high magnitudes are followed by the returns with similar high magnitudes. If this is the situation, then we can conclude that our model involves persistence of volatility which is a common property of most financial markets.

Finally, in the last part of this section I will summarize what have been done in this study.

1.4. What is performed in this study?

In the next section, I will introduce a simple single asset market model consisting of few variables which generates price returns with similar statistical properties of price returns observed in real financial markets. It is based on the model developed by Ghoulmie et al (2005).

In order to explain the certain facts that result in those statistical figures, model is constructed as a system of heterogeneous interacting agents. Agents possess a very simple decision rule to determine their demand at a certain period. All agents receive common public information, and compare it with their predetermined threshold values. If they find it significant they place a buy or a sell order according to the sign of the received information. Conversely, if they find it insignificant then they stay inactive in that period.

This model will be presented deeply in the next section. First, statistical properties or stylized facts, as it is named in literature, of returns that are investigated in that section are explained. And then, description of the model is given. In model description part, model variables are introduced which consist of exogenous and endogenous ones, along with the construction of the model. After the model description part, effects of the key model parameters on stylized facts are investigated in order to extract the actual tasks that are performed by these important parameters. Finally, in the conclusion part results of the study are summarized and conclusion is drawn from the outcome of the parameter analysis part.

In the third section of the study, I will introduce another single asset market model which is based on the study of Farmer and Joshi (2002). This is somewhat more complicated model than the one examined first. In this model agents are grouped into two basic categories. First agent type is the value investors who compare fundamental value with the current price in order to decide on his demand. Second category is the trend followers who seek to extract useful information from the past prices. At each period, they compare past prices with the current one and decide again on whether to buy or sell a specific amount of asset determined by the threshold values of each agent which will be explained in the related section (not one share of an asset in this model), or stay inactive at that period. The aim of this model is again to investigate the crucial parameters and the basic mechanism behind that lead to the observed stylized facts in real financial markets. This model setting is more realistic than the first one since it takes into account the general trader strategies more realistically. However, again these strategies are the simplified versions of the real ones. Similar to the first model, no learning algorithm is included in this model such as genetic algorithms or classifier systems.

There are two main differences present in this second model. First, each agent possesses two threshold values instead of one. Second, no updating scheme for these thresholds is present in this model which is a very crucial in the first proposed model.

So, in this third section, first model construction along with the important parameters is introduced. In the next part where agent types are explained, important parameters belonging to those differing agent types are investigated successively by examining their effects in private. Finally, we mix these two groups and simulate them together similar to a real financial market with logical parameter settings in order to catch those stylized facts common to real financial markets.

Last part of this study is allocated for the conclusion section. In this final section I will interpret the results obtained from the study and explained what should be the next steps in a further study.

2. SIMPLE AGENT-BASED MARKET MODEL WITH A SINGLE TRADER TYPE

In this section of the study, we examine a simple market model with a single trader type, which generates price returns with similar statistical properties of price returns observed in real financial markets.

Agent-based market model proposed here and the ones modeled in the past by other researchers; attempt to explain the origins of the observed macro-statistical properties called stylized facts considering the simple behavioral rules of the market participants. In order to achieve this, financial markets are modeled as a system of heterogeneous and interacting agents by various modelers. The one proposed here attempts to explain these facts with a single type of trader possessing a very simple decision rule. In agent-based market modeling a simple decision rule which is easy to understand is deserved. This is due to the fact that, the results of the complex models are too complicated to interpret and main parameters that are responsible for the observed patterns are difficult to be elucidated. Therefore, complexity weakens the explanatory power of such models which is the main objective of the construction of agent-based models. Even if your model is well constructed and captures the patterns observed in real life, if modeler can not be able to come into a conclusion of which part of the model is responsible for the observed patterns then this model is of no use.

Model constructed here leads to the absence of autocorrelation in returns, volatility clustering, excess volatility, and bursts in market activity that can not be explicable by the external noise. This is attributed to feedback mechanism which is the key structure in most dynamical systems and to the heterogeneity of trading strategies.

In the following subsections, first statistical properties of returns common to most real financial markets that are investigated in this study are explained. Then, description of the model is given, in which model parameters are introduced and construction of the model is presented. And at last model is illustrated by a flow chart. After model description, key model parameters are examined separately in order to understand the effects of these model parameters to the observed stylized facts. This is the most crucial part of the study since this will elucidate the functions and the roles of the model parameters that are responsible for the generation of the common statistical facts. Finally, results are summarized and conclusion that is drawn from the analysis section is given in the last part of this section.

2.1. Statistical Properties Investigated in This Section

Time series of stock returns exhibit common statistical features, called stylized facts, for a wide range of markets and time periods. Ones that are considered in this part of the study are as follows;

1) Excess Volatility: The observed magnitude of the variability in returns is much higher than the expected variability, which is anticipated from fundamental economic variables. In this study, excess volatility is examined through the comparison of the variability seen in asset returns with the one indicated by the fundamental economic variable which is represented by the public signal received by all market traders in the model.

2) Positive Excess Kurtosis (heavy tails): The distribution of the returns exhibits a heavy tail with a positive excess kurtosis. In our model, we are looking for the excess kurtosis of nearly 10.

3) Absence of Autocorrelation in returns: Daily returns of the asset in our model should exhibit insignificant autocorrelations for a wide range of lags. Zero autocorrelation corresponds to the unpredictability of the asset returns.

4) Volatility Clustering: Although returns are uncorrelated, absolute or squared daily returns display a positive, significant and slowly decaying autocorrelation function (which is named as power law decaying). This feature of the asset returns can be explained by the following; 'large changes in prices tend to be followed by the large changes, negative or positive, and small changes in prices tend to be followed by the small ones again of either sign.' This feature is examined by computing the

autocorrelation of the absolute returns $|r_t(\Delta)|$ where Δ is the time lag ranging from one day to several weeks.

2.2. Model Description

This model describes a market where a single asset is traded at discrete periods, which are interpreted as trading days by N agents. Price of the asset is at period t denoted by P_t. At each period, agents trade this asset according to their strategies, which can be explained as follows:

Each agent may perform one of the three actions, namely buying or selling one unit of the asset or taking no action. Demand of each agent is denoted by $d_{i,t}$. If agent decides to buy the asset, then his demand will be $d_{i,t} = 1$. On the other hand, if he wants to sell one share of the asset, it will be $d_{i,t} = -1$. Demand is zero, $d_{i,t} = 0$, when agent decides to be inactive at period t.

All agents receive a common signal modeled by a sequence of IID Gaussian random variables ε_t at each period t where $\varepsilon_t \sim N(0, D^2)$. This signal may be interpreted in two ways. First it may represent the public information for a forecast of the future return r_{t+1}. And second, it may stand for the volatility. Upon receiving this signal, agents decide on whether this signal is significant or not, according to the magnitude of the signal. If this signal is found to be significant, agent places a buy or sell order in accordance with the sign of the signal.

The trading rule or the strategy of an agent can be summarized as follows:

Each agent i = 1, 2, …, N has a time varying decision threshold $\theta_{i,t}$. This threshold can be interpreted as the agent's subjective estimation of the return, or volatility. At any time period an incoming signal with a magnitude less than the agent's threshold leaves the agent inactive at that period. So, if the magnitude of the public signal is not significant with respect to the agent's threshold, that is $|\varepsilon_t| \leq \theta_{i,t}$, then that agent remains inactive for that period and the demand of that agent is zero, $d_{i,t} = 0$. In contrast, if the magnitude of the incoming signal is greater than the agent's threshold, $\varepsilon_t > \theta_{i,t}$, agent places a buy order, $d_{i,t} = 1$ and he places a sell order, $d_{i,t} = -1$, if $\varepsilon_t < -\theta_{i,t}$.

Hence, trading strategy for an agent can be represented as the following partial function;

$$d_{i,t} = \begin{cases} 1, & if\, \varepsilon_t > \theta_{i,t} \\ -1, & if\, \varepsilon_t < -\theta_{i,t} \\ 0, & \text{otherwise} \end{cases} \tag{2.1}$$

After all agents place their orders according to their trading strategies, excess demand is computed as follows;

$$X_t = \sum_{i=1}^{N} d_{i,t} \tag{2.2}$$

Excess demand at period t directly affects the price change (return) at that period by price impact function G(x) as follows;

$$r_t = \ln \frac{P_t}{P_{t+1}} = G\left(\frac{X_t}{N}\right) \tag{2.3}$$

which is an increasing function with G(0) = 0. The market depth λ is defined so that G'(0) = 1/λ. In this model a linear market impact function is considered: G(x) = x/ λ.

As it is mentioned above, thresholds are assigned to each agent specifically. Initial thresholds $\theta_{i,0}$ are drawn from a population distribution as positive IID variables. In the model, uniform distribution is chosen as the initial threshold distribution.

Thresholds are updated at each time period by any agent i = 1, 2, …, N, with a common updating probability S. Value for this probability is determined by the modeler as an important model parameter.

Update frequency S represents the fraction of agents updating their thresholds at any period and the inverse of it, 1/S, represents the average time period during which an agent keeps his threshold unchanged.

Since time periods are interpreted as days, assigning values to 1/S between 10 to 1000 days will be logical. So, S is given values between 10^{-1} and 10^{-3}.

If agent makes a decision on updating his threshold, he sets it to the current absolute return;

$$\theta_{i,t+1} = |r_t| = \left|\ln \frac{P_t}{P_{t+1}}\right| \tag{2.4}$$

So, at each time period, an IID random variable $u_{i,t} \sim U\ [0\ 1]$ is drawn for each agent. This random number is compared with the common updating frequency S. If the random number generated is greater than or equal to the updating frequency, then agent's threshold remains same as the one at the previous period. Conversely, if it is less than frequency S, then agent's threshold is updated to the most recent absolute return.

This scheme can be illustrated as follows;

$$\theta_{i,t+1} = \begin{cases} \theta_{i,t}\ , \text{ if } u_{i,t} \geq S \\ |r_t|\ , \text{ if } u_{i,t} < S \end{cases} \tag{2.5}$$

So, initial threshold values have no significant effect on the observed price behavior since heterogeneity dominates even if initial thresholds are same $\theta_{i,0} = \theta_0$.

In addition, this way of updating scheme adds to the heterogeneity of the system which is believed and expected to be important for the model in order to generate some of the stylized facts that are described in the previous section.

Consequently we may portray the model with the following figure;

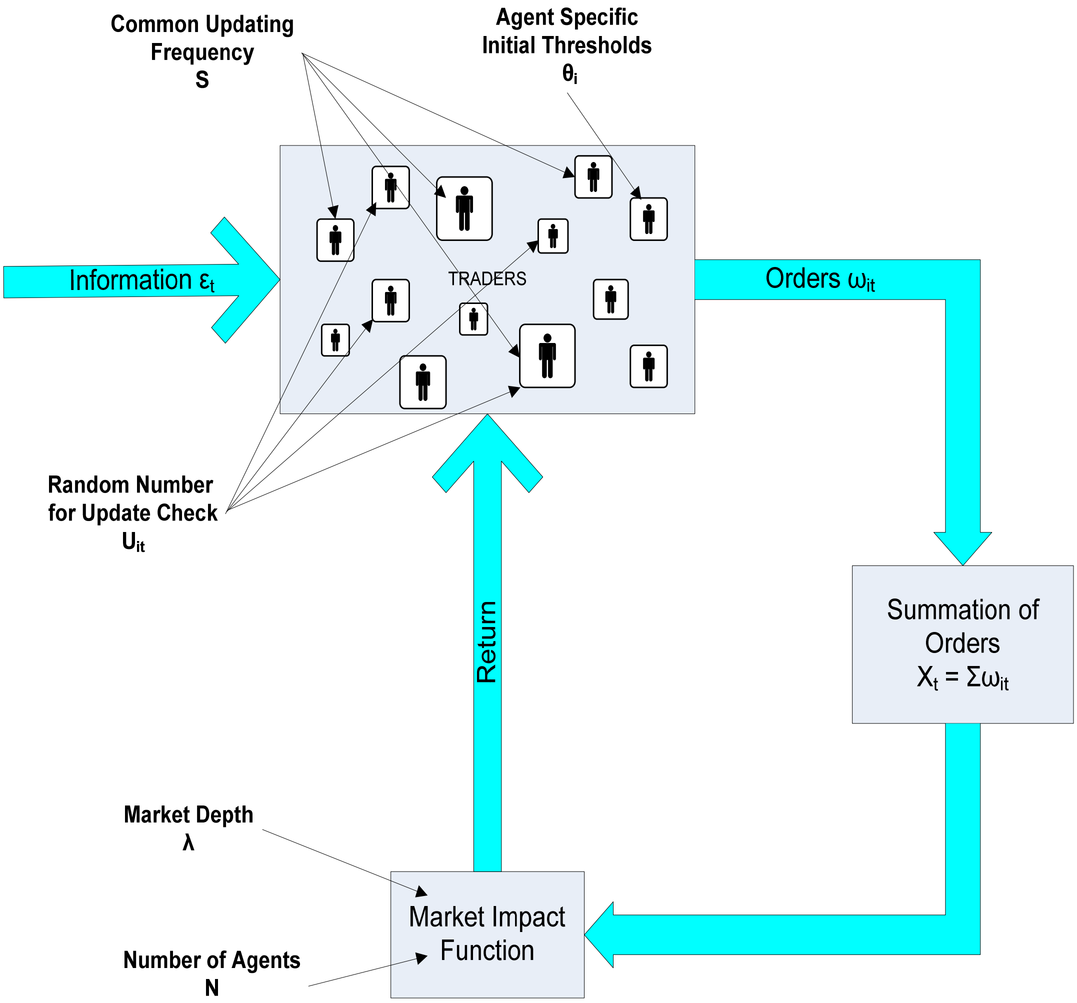

Figure 2.1. Model illustration

2.3. Investigation of the effects of Model Parameters

In this part of the section, I examined the effect of the model parameters one by one, namely;

- market depth λ
- standard deviation D of the common signal
- updating frequency S
- number of agents N

by considering the resulting stylized facts indicated above. All other parameters other than the one examined are kept constant (ceterus paribus).

Similar to the original paper, Ghoulmie et al. (2005), I simulate the prices of a single asset for $T=10^4$ periods. After eliminating the first 10^3 periods in order to decrease the sensitivity of returns to the initial conditions, I generate the time series of returns, their histogram, and standard deviation of returns (volatility), the sample autocorrelation function of returns and the sample autocorrelation function of absolute returns.

2.3.1. Updating Frequency S

In order to begin examining the effects of the parameters on stylized facts, I choose updating frequency first because the effect of S at the extremes can be more easily perceived than any other parameter.

2.3.1.1. Update Frequency S = 1: At the one extreme, S may be chosen as 1 which may correspond to a model possessing feedback structure without heterogeneity among agents since each agent at any time period will update his threshold to the absolute value of the most recent return.

As a result it can be easily seen that all agents will act in the same manner and therefore generate the same order. Then, excess order will take the values N, 0 or –N according to the sign and magnitude of the incoming public signal. This is due to the fact that all agents will select the same one of the three choices of ‘’buy, sell or stay inactive’’ according to their strategies, since all have the same public information and same thresholds. This can be represented as the following partial function;

$$X_{t+1} = \begin{cases} N, \text{ if } \varepsilon_t \geq -|r_t| \\ -N, \text{ if } \varepsilon_t < -|r_t| \\ 0, \quad \text{otherwise} \end{cases} \tag{2.6}$$

Consequently, there will be no heterogeneity among agents since they set their threshold to the same value, absolute value of the return; however feedback structure persists because all agents set their thresholds to an endogenous variable. As a result, return at a period depends on the past only through the absolute value of return of the previous period which can be represented by the following partial function;

$$r_{t+1} = \begin{cases} G(N)\ ,\ \text{if}\ \ \varepsilon_t \geq |r_t| \\ G(-N)\ ,\ \text{if}\ \ \varepsilon_t < -|r_t| \\ 0\ ,\ \text{otherwise} \end{cases} \tag{2.7}$$

As we investigate the literature, it is understood that this dependence structure on the absolute value of the return match with the ARCH models, Engle (1995), which leads to uncorrelated returns and volatility clustering.

All numerical simulations, including for S = 1, of the model for examining the effects of updating frequency are performed with the following parameter values;

- Number of agents N = 1000
- Standard Deviation D of the Common Signal = 0.001
- Market Depth λ = 10

For S = 1, tri-modal distribution of the returns is observed from the simulation. This is also expected from the analytical study of the model mentioned above.

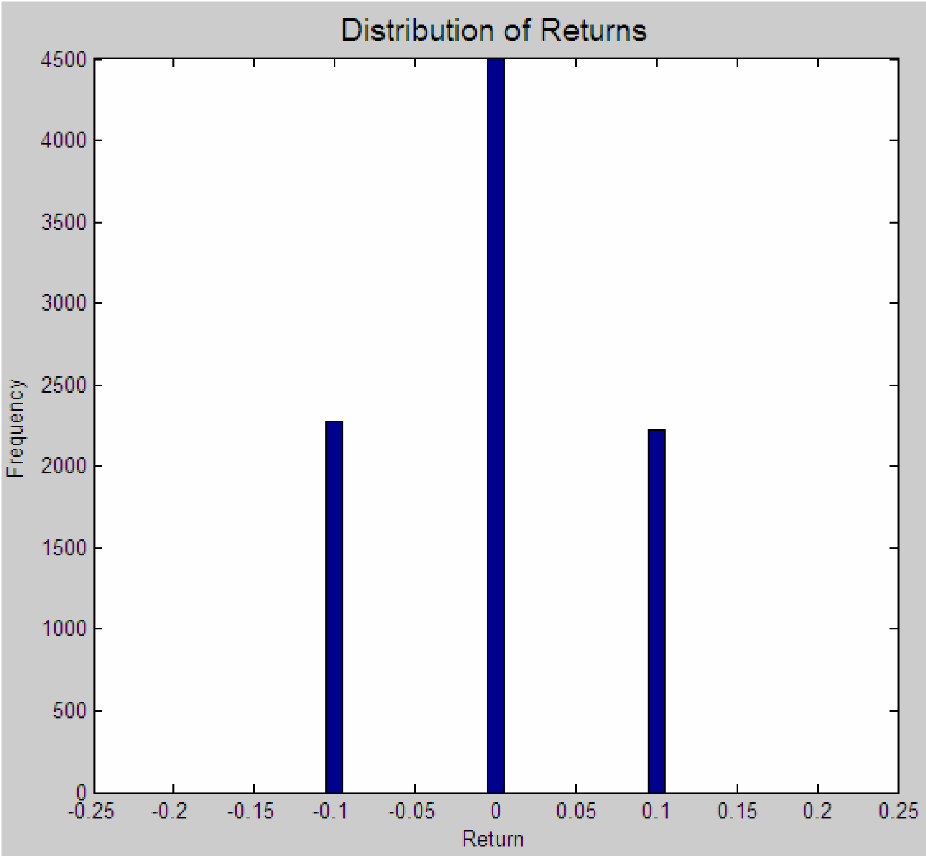

Figure 2.2. Distribution of the returns where S = 1

Three values of return are observed as -0.1, 0 and 0.1 since market depth is taken as 10 which allows for the maximum return of 0.1 and minimum return of -0.1 that can also be seen from the figure.

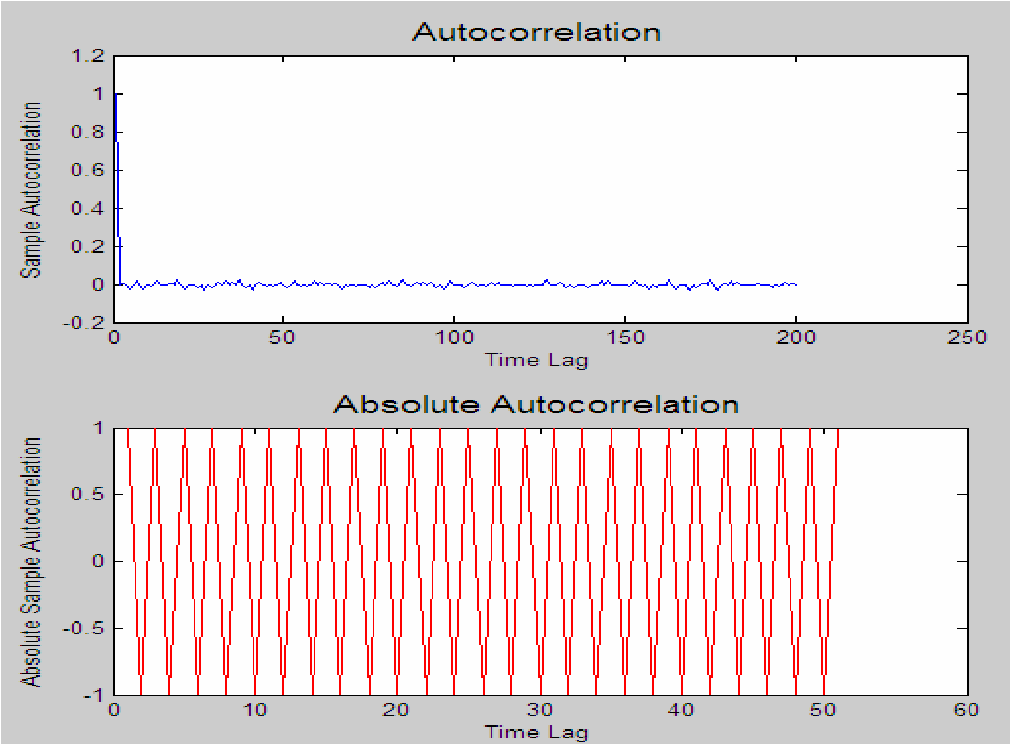

Figure 2.3. Autocorrelation functions of returns and absolute returns where S = 1

In addition to the tri-modal return distribution, permanent perfect autocorrelation of magnitude 1 also confirms the intuition that update frequency of 1 leads to the absence of autocorrelation among returns as well as to the volatility clustering.

To conclude, having an updating frequency that is less than 1 is a must in order to have a logical model since S = 1 or near 1 (which will be presented in the following) produces an illogical tri-modal distribution of returns that is not in accordance with the real financial markets.

2.3.1.2. Update Frequency S = 0: At the other extreme where S = 0, there will be no threshold updating. This corresponds to the model where heterogeneity is present in the model but feedback structure no longer exists since updating does not take place and there is no endogenous variable that is used in the system.

Therefore, trading strategies will no longer be affected by price changes and by reason of this, feedback mechanism is lost. On the other hand, heterogeneity is still present in the model through the differing initial thresholds. Distribution of the thresholds remains same throughout the simulation and all agents keep their initial thresholds and do not change it since no one is going to update his threshold.

The return r_t depends only on the incoming public signal ε_t and the initial distribution of the thresholds. So, we can conclude that returns are IID random variables which are found by transforming the normally distributed IID sequence of public signal ε_t by nonlinear function resulting from the initial distribution of the thresholds. Thus, it is expected that price follows a random walk and model no longer exhibits volatility clustering.

Numerical simulation where S = 0 is again performed with the same parameter settings as it was done for the first extreme case. However, in order to understand the effects of the distribution of the initial thresholds, I simulate the model for two different threshold distributions, namely normal and uniform;

➢ <u>Normally Distributed Thresholds (absolute values are taken)</u>

- Updating Frequency S = 0
- Initial Thresholds ~ abs(N [0, D^2])
- Number of agents N = 1000
- Standard Deviation D of the Common Signal = 0.001
- Market Depth λ = 10

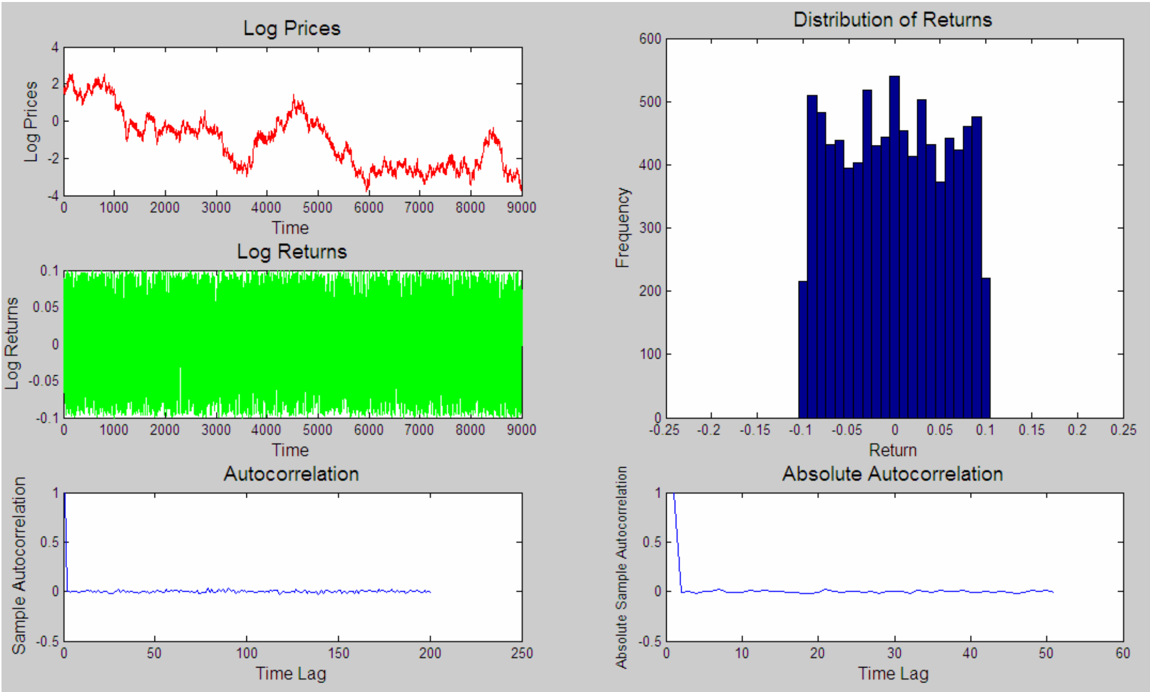

Figure 2.4. Simulation of the model with S = 0 and normally distributed thresholds

Normally distributed thresholds lead nearly uniformly distributed returns. Absence of autocorrelation for absolute returns can be easily figured out. This is the indicator of the diminishing volatility clustering due to the absence of updating.

- <u>Uniformly Distributed Thresholds</u>

Updating Frequency S = 0

Initial Thresholds ~ U [0, D]

Number of agents N = 1000

Standard Deviation D of the Common Signal = 0.001

Market Depth λ = 10

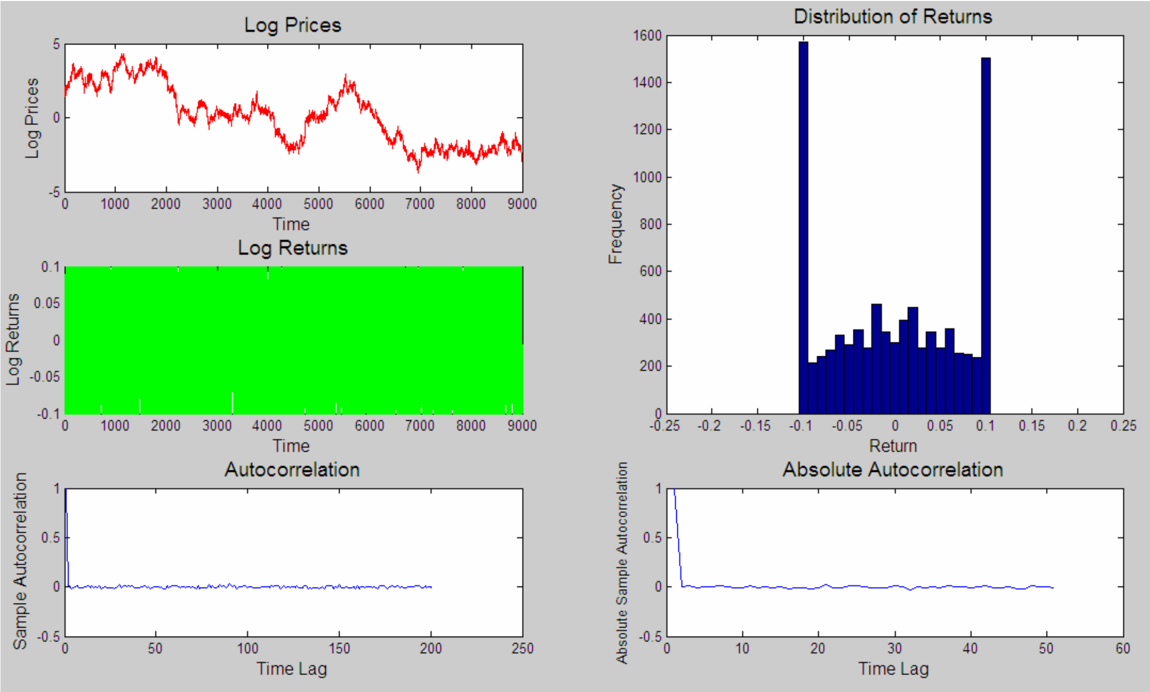

Figure 2.5. Simulation of the model with S = 0 and uniformly distributed thresholds

Uniformly distributed thresholds produce returns mostly having its maximum magnitude with nearly the same frequency of opposite signs and uniformly distributed returns between these two extreme values.

In both simulations, with normally and uniformly distributed initial thresholds, volatility clustering diminishes which can be understood from the autocorrelation figures of the absolute returns. As a result, we must have a logical value for updating frequency which should be greater than zero, that is to say model must possess updating mechanism for thresholds, and less than 1, that is agents should not change their thresholds at specified (by S) time periods. Logical choices for this parameter are determined to be between 10^{-1}-10^{-3}, in the original paper, Ghoulmie et al. (2005), where the model is first introduced.

2.3.1.3. Update Frequency S other than Extreme Values: After investigating the extreme values of the updating frequency which may also be seen as the structural validation of our model, we examine the effects of S on stylized facts including excess volatility and excess kurtosis and determine whether mentioned interval is a valid choice or not.

In order to achieve this, I perform sensitivity analysis for S by simulating the model for values decreased from 0.9 to 0.01. I put important figures in the following.

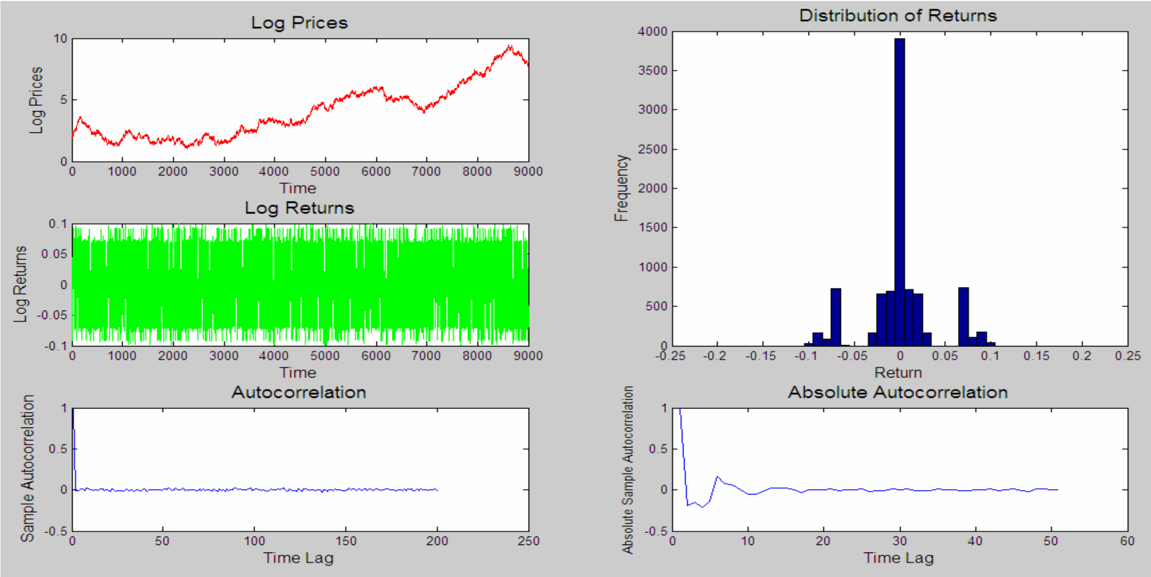

Figure 2.6. Simulation of the model with S = 0.7

As it is seen from the related figures, tri-modal distribution of returns remains as an outcome of the model when update frequency is as high as 0.7. Below this value, this property of the returns slowly diminishes and at S = 0.3 it totally disappears, Figure 2.7.

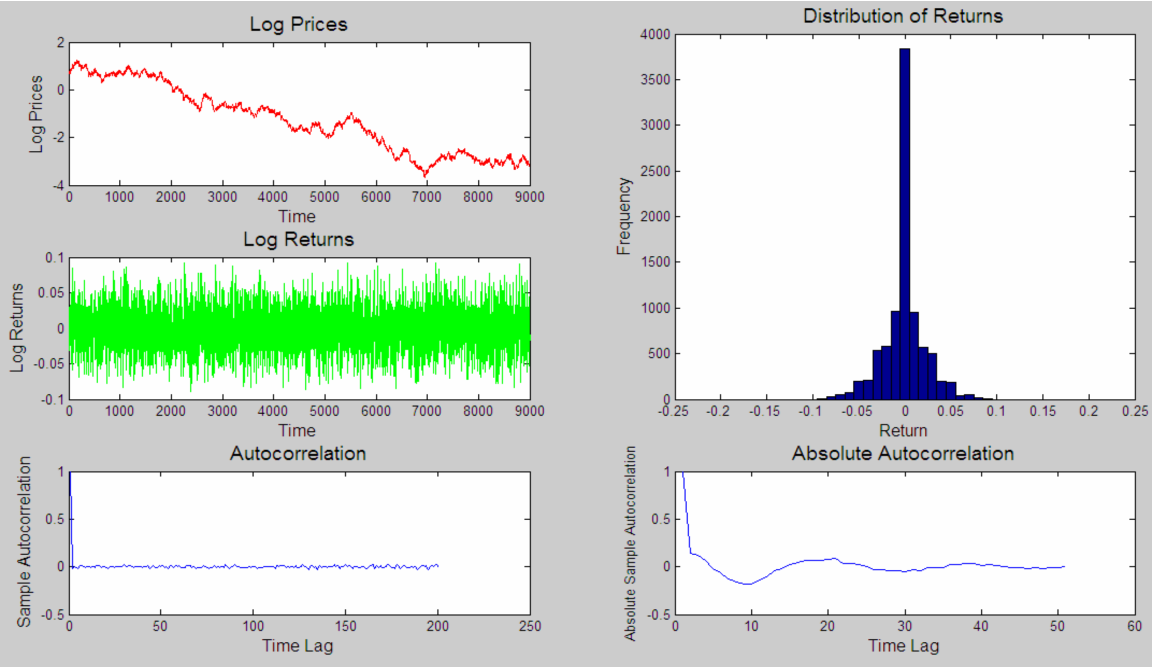

Figure 2.7. Simulation of the model with S = 0.3

In addition to this unrealistic tri-modal return distribution, absolute autocorrelation does not produce desired power law decay where update frequency is above 0.3. This property which is the indicator of volatility clustering remains till S takes the value of 0.05.

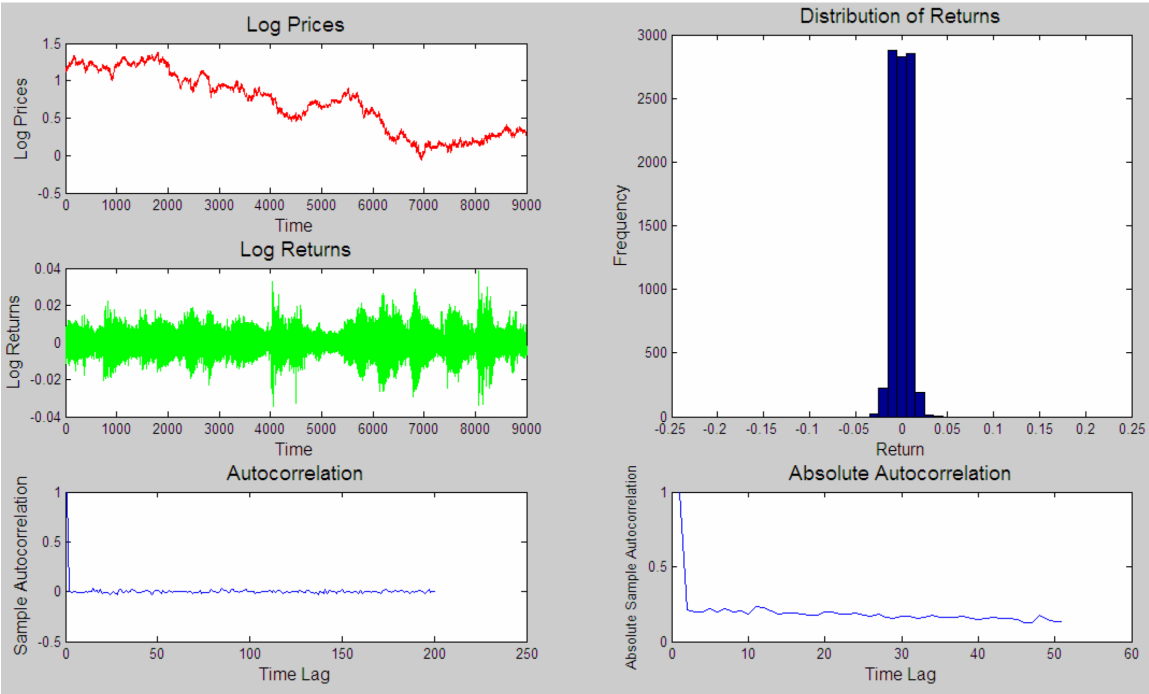

Figure 2.8. Simulation of the model with S = 0.01

Below this value, particularly it can be seen from the result of the simulation with S = 0.01, we can easily observe from the absence of the autocorrelation in absolute returns that volatility clustering ceases to exist. So, we can be confident that for the sake of logical model as well as the model possessing an important stylized fact we could assign S values between 0.05-0.3.

Following Table 2.1 demonstrates the changes in the magnitudes of the other two stylized facts created by update frequency changes.

Table 2.1. Stylized Facts related to the various Update Frequency values

Update Frequency S	***Excess Kurtosis***	***Standard Deviation of Returns***
1.00	-0.9999	0.0707
0.90	0.2407	0.0509
0.80	0.7315	0.0431
0.70	1.0215	0.0376
0.60	1.2491	0.0330
0.50	1.3860	0.0291
0.40	1.5998	0.0257
0.30	1.8774	0.0223
0.20	1.7525	0.0193
0.10	1.9247	0.0165
0.05	1.7232	0.0143
0.01	-0.4709	0.0084
0.00	-1.3688	0.0714

As we examine the Table 2.1 values and the related figure for excess kurtosis, Figure 2.9, we observe inverse parabola shape. For the pre-mentioned values of S between 0.05-0.3, excess kurtosis is nearly 2. Since, it is desired to get positive excess kurtosis for nearly 10, which is the value observed in real markets, we accept this interval. Below 0.05, excess kurtosis decreases steeply to negative values. Above 0.3, it decreases slowly; however for high values, which are more unrealistic because of the tri-modal distribution structure of the returns, excess kurtosis also reaches to negative values.

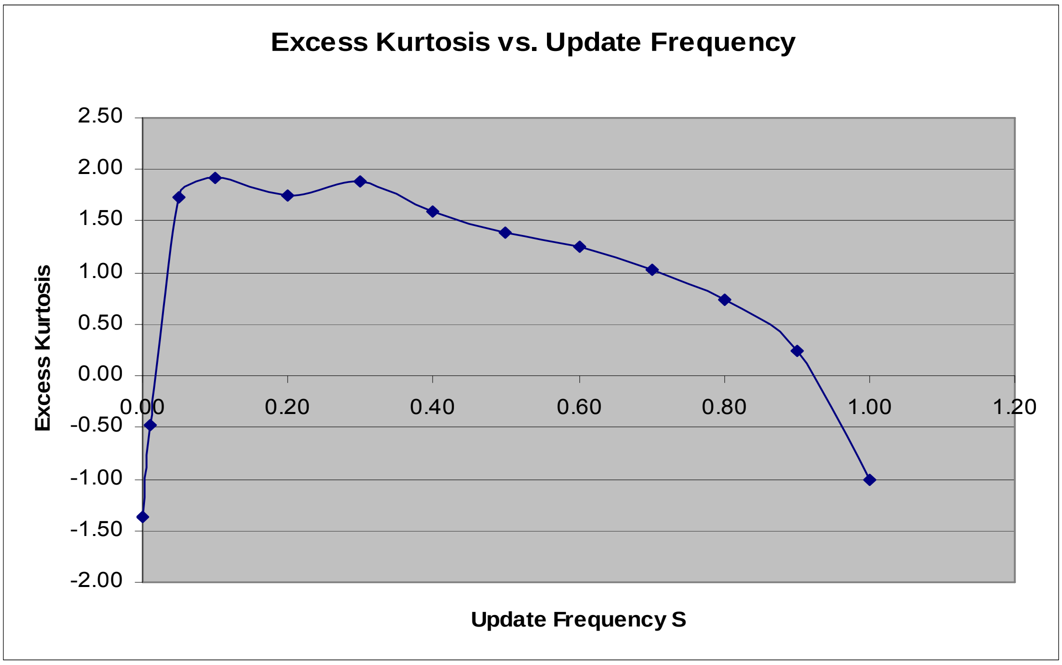

Figure 2.9. Excess Kurtosis versus Update Frequency

This may be due to the fact that, tri-modal distribution appears with the increasing values of S. And this distribution structure leads to lowering of the excess kurtosis. Besides, lower values for S also leads to distributions possessing platykurtic property which can be seen from the Figure 2.4, Figure 2.5 and Figure 2.6.

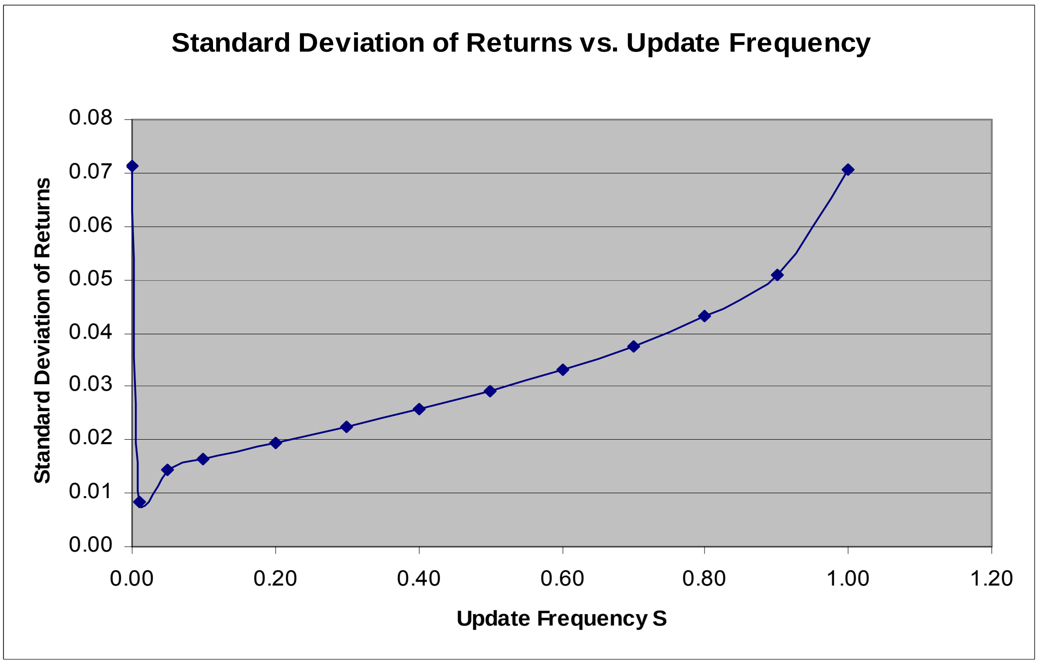

Figure 2.10. Standard Deviation of Returns versus Update Frequency

On the other hand, if we look at the Figure 2.10 demonstrating the excess volatility part, we observe that volatility increases along with an increase in update frequency. This may be again being due to the appearance of tri-modal distribution structure.

As a result, we have to consider these two observed patterns and come into a conclusion that there is a trade of between the excess volatility in returns and excess kurtosis in them.

2.3.2. Market Depth λ

Market depth λ gives us the possibility of adjusting the maximum return value. It is an important parameter for the model to be a logical one. In the original paper of Ghoulmie et al (2005), in order to interpret the trading periods as days and compare the results of the model to the observed real market outcomes, market depth is given values between $5 \leq \lambda \leq 20$ so as to allow a maximal range of daily returns between 5% and 20%.

Therefore, I investigate the effects of the market depth λ on stylized facts performing the sensitivity analysis with the values for λ selected between 1 and 20.

In order to get sensible results, following numerical simulations of the model are performed with the same random number seed along with the following parameter values;

- Updating Frequency S = 0.1
- Number of agents N = 1000
- Standard Deviation D of the Common Signal = 0.001

I put the figures for market depth values of 20, 15, 5 and 1 below in order to depict the sensitivity of stylized facts and other properties of returns and prices to the changing values of market depth.

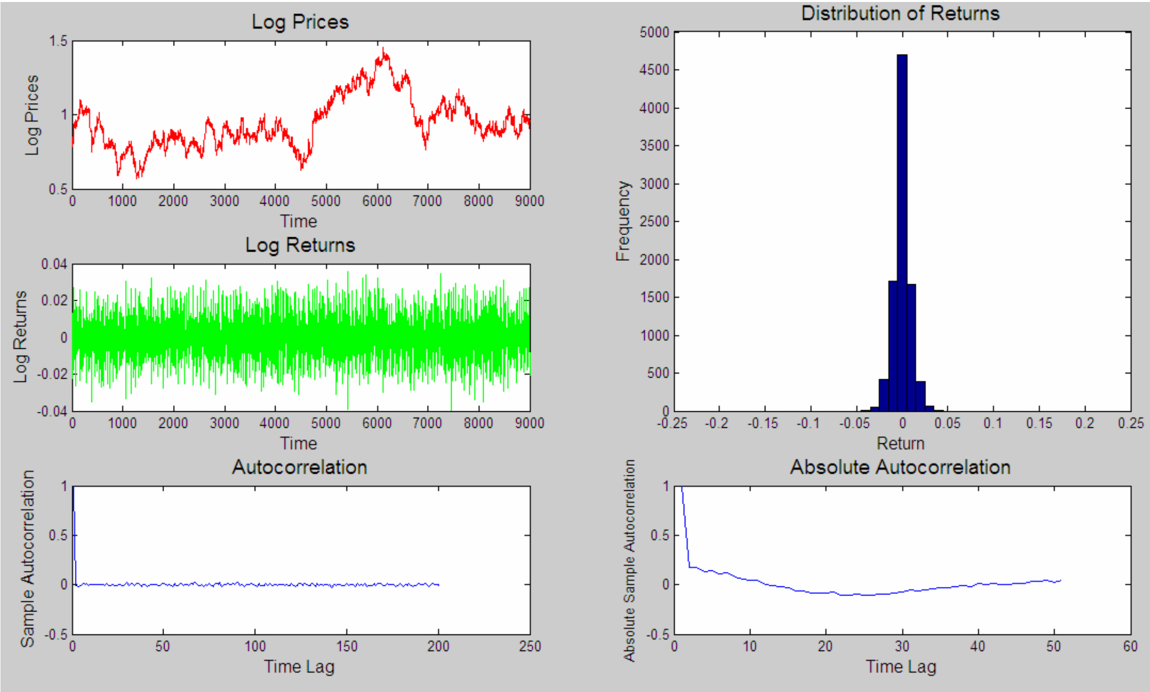

Figure 2.11. Simulation of the model for market depth λ = 20

As we examine the autocorrelation and absolute autocorrelation of the returns resulting from the simulation of the model with changing market depth values, we observe the fact that for all values of market depth, absence of autocorrelation can be observed and the volatility clustering is present for all values and it fluctuates around zero with diminishing magnitudes.

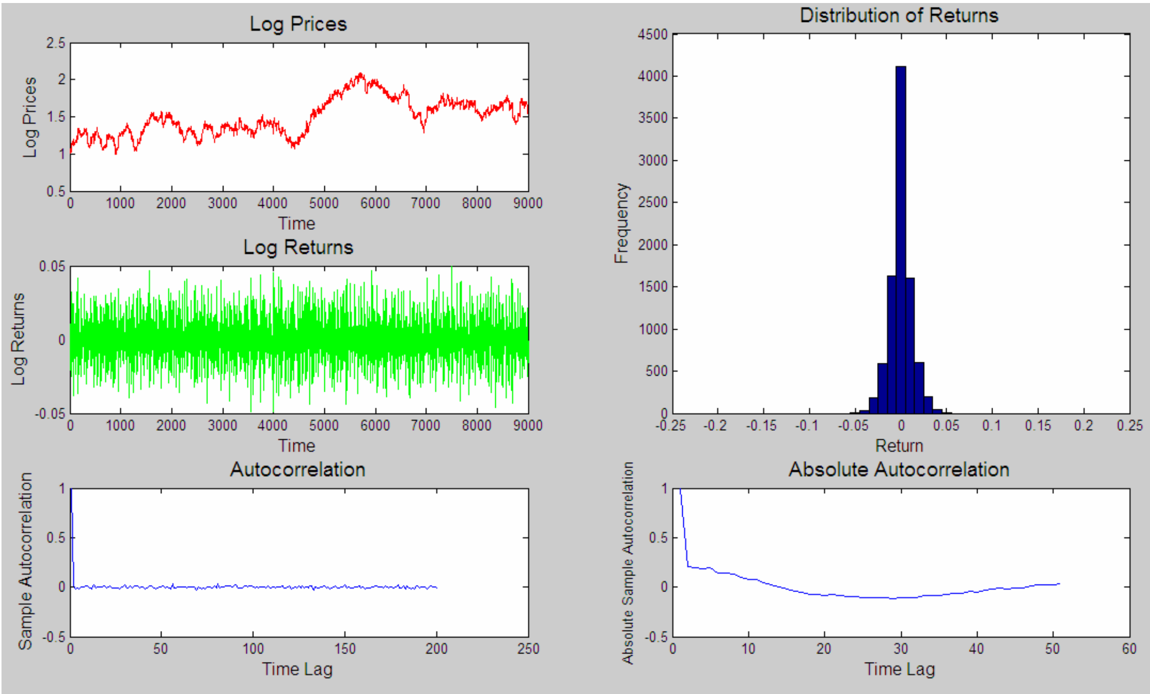

Figure 2.12. Simulation of the model for market depth λ = 15

If we look at the resulting distributions corresponding to the different market depth values, it can be observed that the tails become tighter as we decrease market depth.

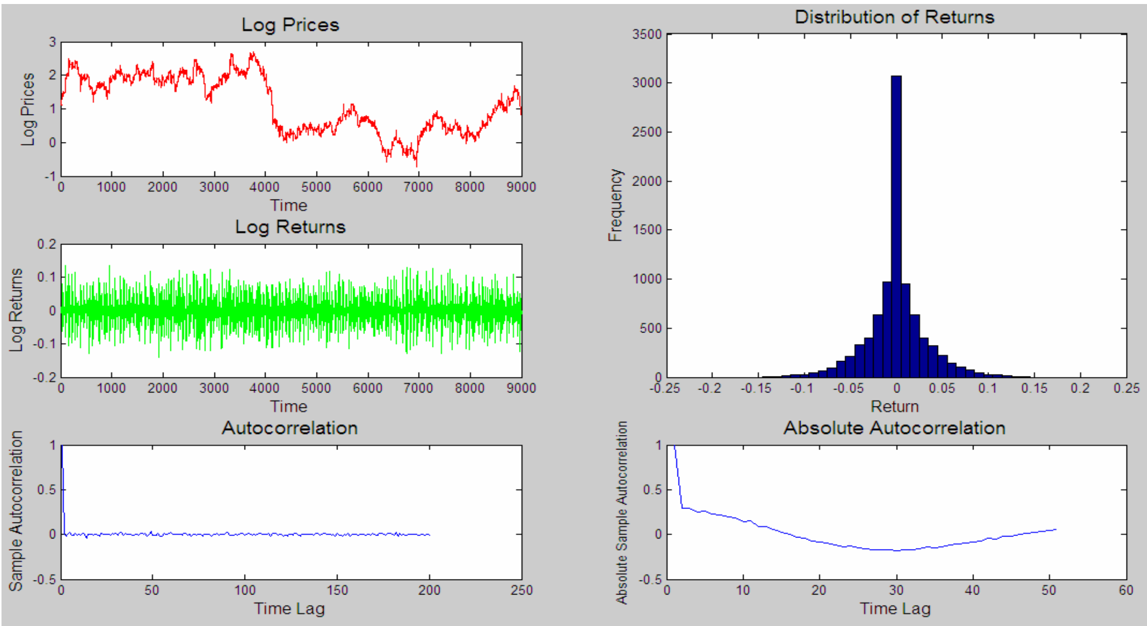

Figure 2.13. Simulation of the model for market depth λ = 5

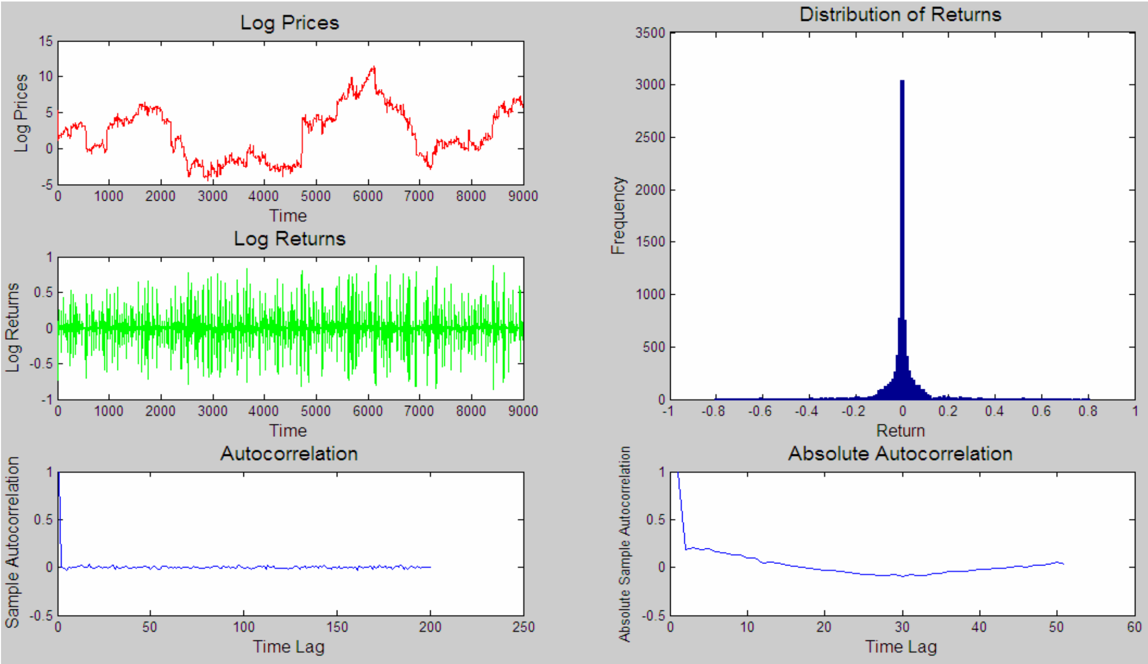

Figure 2.14. Simulation of the model for market depth λ = 1

Since the tightness of the tails of a distribution is related to its kurtosis, excess kurtosis should increase by lowering of the market depth values. This is also seen from Table 2.2 in which the excess kurtosis and excess volatility results are supplied.

Table 2.2. Stylized Facts related to the various Market Depth values

Market Depth	***Excess Kurtosis***	***Standard Deviation of Returns***
20	1.0368	0.0089
18	1.1818	0.0098
15	1.3428	0.0116
12	1.6943	0.0138
10	1.9247	0.0165
8	2.1168	0.0197
5	2.2783	0.0311
3	2.9269	0.0492
2	4.5259	0.0715
1	11.1485	0.1426

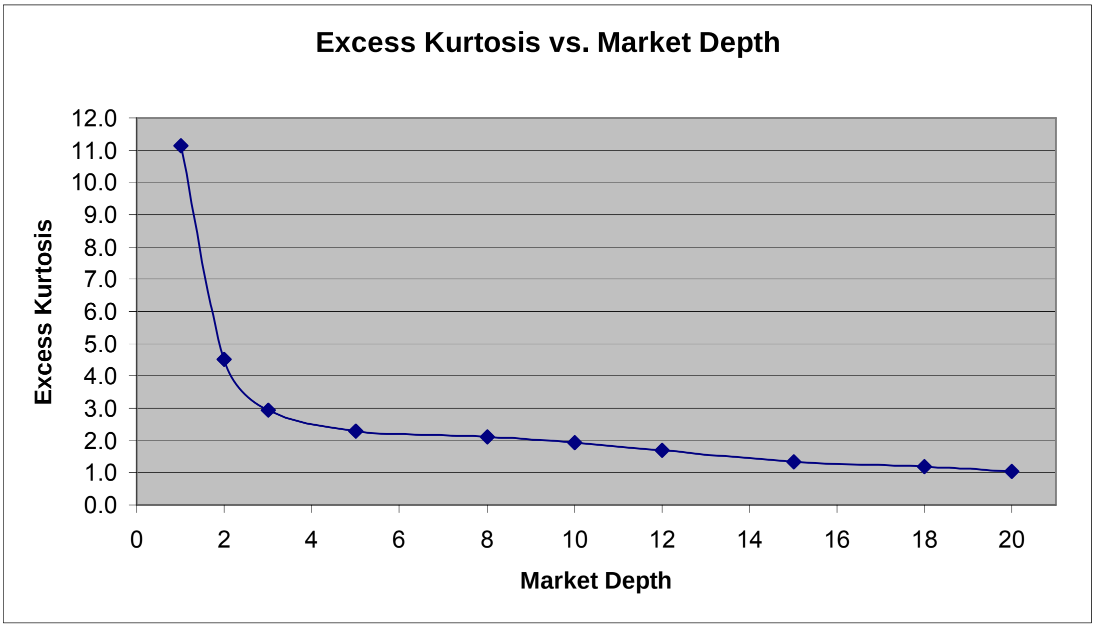

Figure 2.15. Excess Kurtosis versus Market Depth

When we examine the table values and figures, for the change of market depth values, we observe similar behaviors from the results of the analysis of the excess kurtosis and excess volatility. Each shows logarithmic decay together with an increase in the value of the market depth. As a result, for the sake of the two latter mentioned stylized facts, model should possess low values for the market depth.

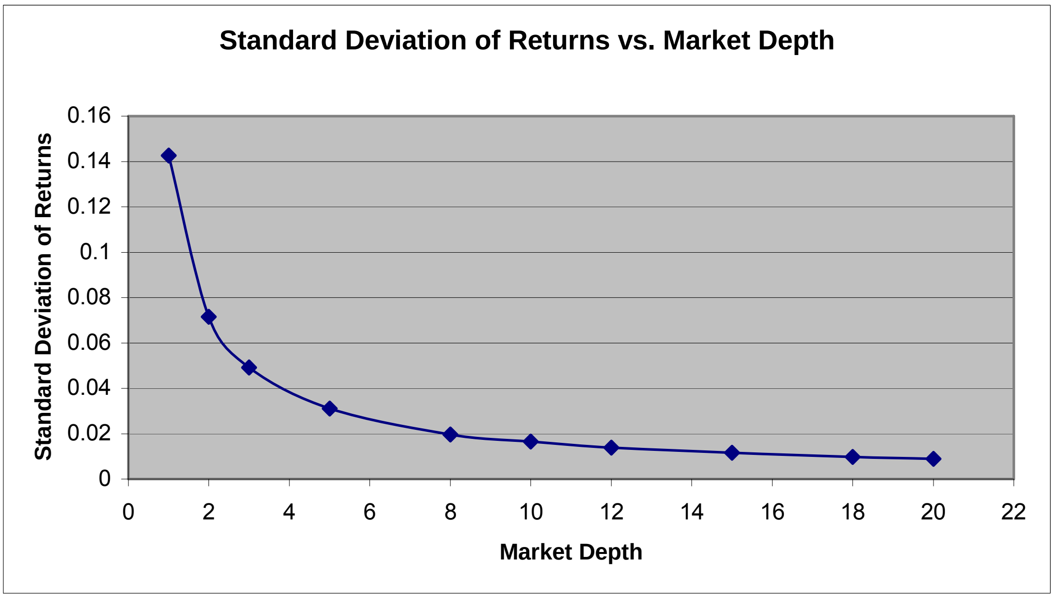

Figure 2.16. Standard Deviation of Returns versus Market Depth

This resulting scheme may be explained as follows. Market depth is used in the computation of the return at a period. After all agents place their order and excess demand is found, it is divided by the market depth. If we increase this value too much, this will leave return values with very low values at each period and due to the updating scheme of the thresholds, this property will decrease the heterogeneity of traders with respect to the common public signal since they will all be less than the incoming signal.

In contrast, if we decrease the value of the market depth, this will increase the resulting returns and due to the updating scheme agents will have high thresholds in one period. In the next period due to high values of thresholds some agents will cease to be active and thus return decreases. This will cause the thresholds to decrease and agents will be active at later periods. As a result, this feedback structure leads to the increasing volatility and excess kurtosis values.

2.3.3. Standard Deviation of the Common Signal D

The magnitude of the public information representing the volatility is chosen in order to produce realistic values for the annual volatility. It is emphasized in Ghoulmie et al. (2005) that this constraint leads us to choose D in the range of 10^{-3}-10^{-2}.

In this section, I am going to study the effects of the standard deviation of the input noise D to the model's observed statistical facts by performing the sensitivity analysis. I choose values for D mostly within the given interval; however I also simulate this model for values outside this range.

Again for these simulations, I stick to the principle of "other things being constant" (ceterus paribus). So, I choose other parameters as follows;

- Updating Frequency S = 0.1
- Number of agents N = 1000
- Market Depth λ = 10

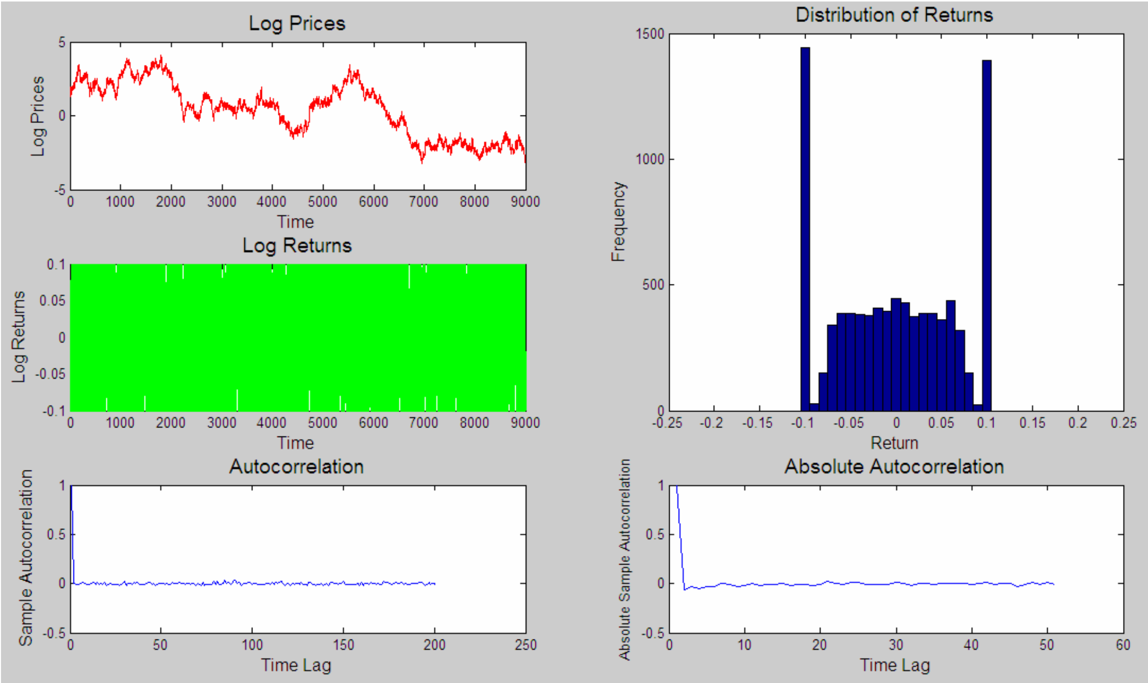

Figure 2.17. Simulation of the model for standard deviation of the signal D=0.1

From the figure of the simulation run for D = 0.1, Figure 2.17, we observe that returns are mostly taken values at their allowed extreme values of -0.1 and 0.1, which is determined by setting market depth to 10. This can be easily understood because most of the time magnitude of the incoming public signal will be greater than the threshold values of all agents since at some periods they will set it to the most recent absolute return, which is going to be less than or equal to 0.1. Besides this unrealistic outcome, absence of volatility clustering forces us to assign D values less than the maximum allowed return which is determined by market depth value.

These results are also obtained from the simulation with D=0.05 where the maximum allowed return is 0.1. The resulting return distribution is nearly uniform. In addition to the above arguments for D=0.1, it can be also seen from the return distributions that, these distributions are platykurtic and this makes excess kurtosis to take negative values for high D values. Moreover, at this high D values, persistence in volatility do not observed in returns which can be seen from the absolute return autocorrelation graphs.

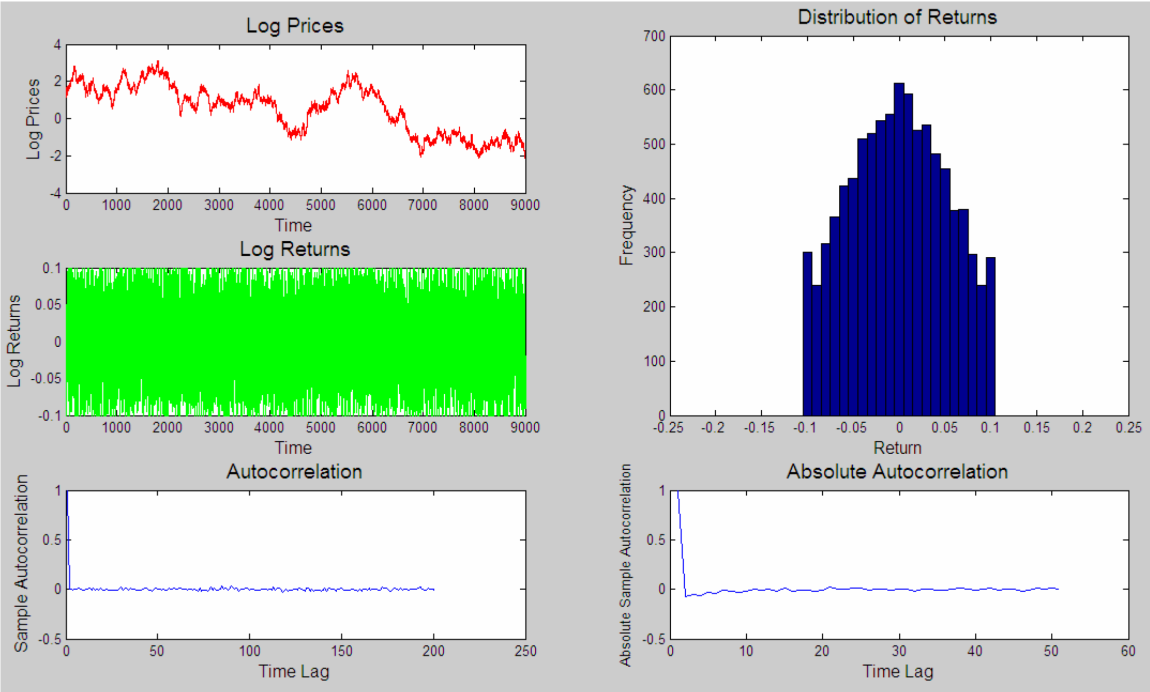

Figure 2.18. Simulation of the model for standard deviation of the signal D=0.05

As we decrease the magnitude of D to values near 0.01, the distribution of the returns changes from a distribution which is between the uniform and normal distributions to normal distribution.

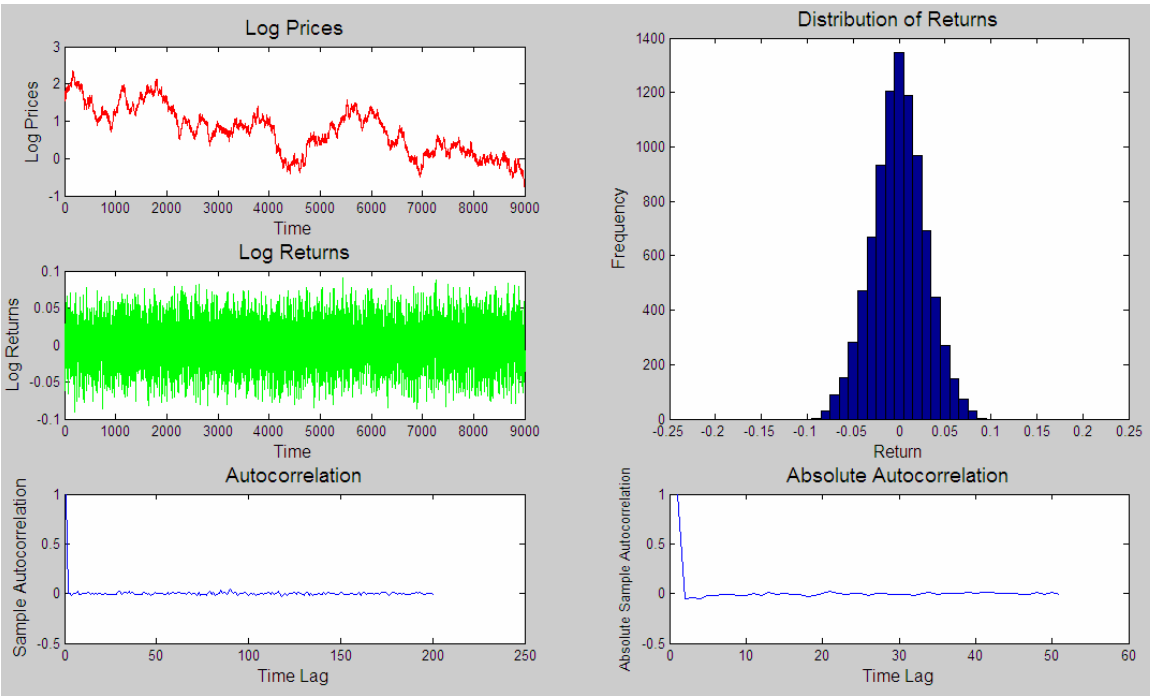

Figure 2.19. Simulation of the model for standard deviation of the signal D=0.01

However, normal distribution is not the desired outcome since we are in search of a leptokurtic distribution of returns. As we come to the level of D=0.003, volatility clustering appears. And most desired solution is obtained for D values near 0.001.

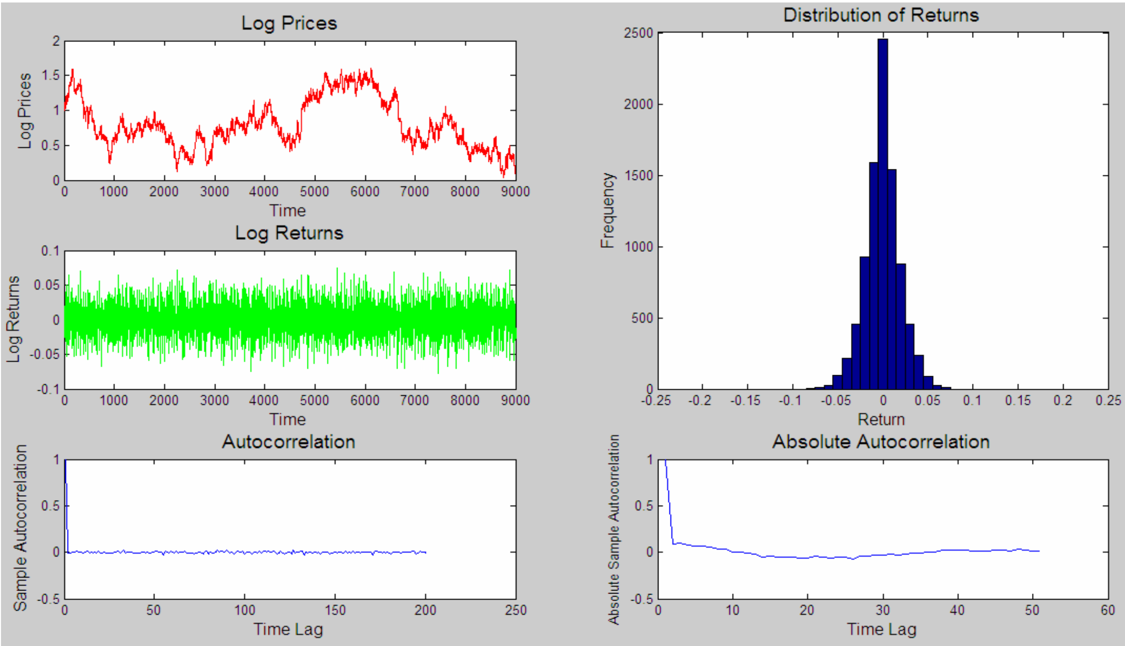

Figure 2.20. Simulation of the model for standard deviation of the signal D=0.003

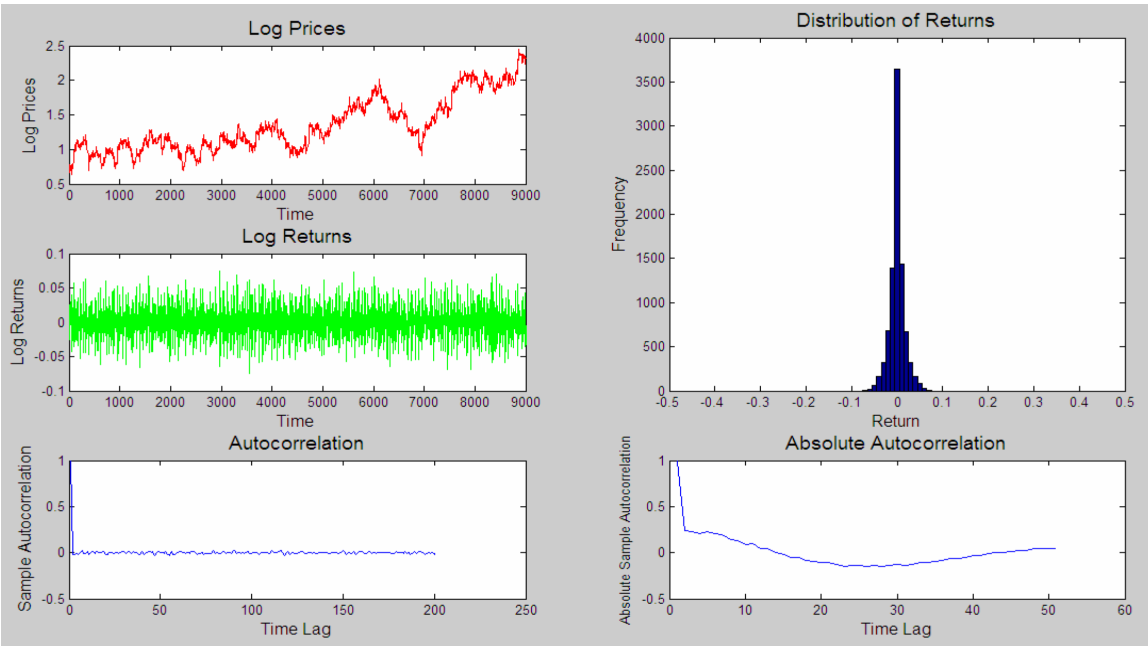

Figure 2.21. Simulation of the model for standard deviation of the signal D=0.001

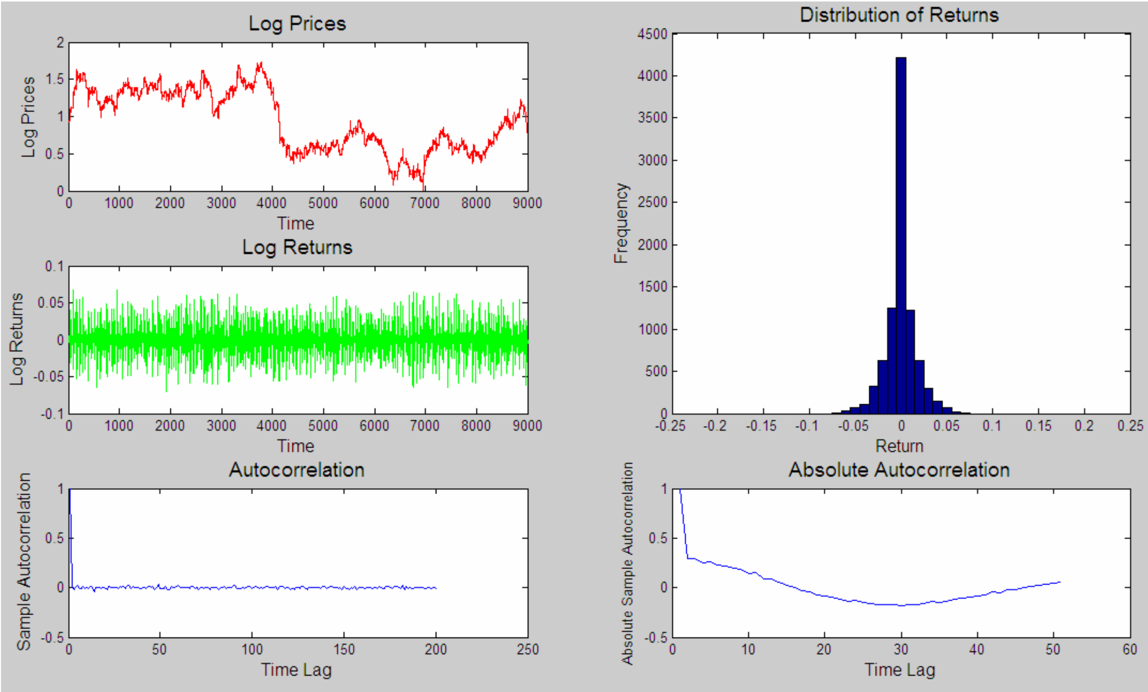

Figure 2.22. Simulation of the model for standard deviation of the signal D=0.0005

Moreover, below D value of 0.0005, power law decay property in autocorrelation of absolute return begins to distort. As a result, model should have a value for D above 0.0005 and below 0.004 in order for the model to possess volatility clustering property.

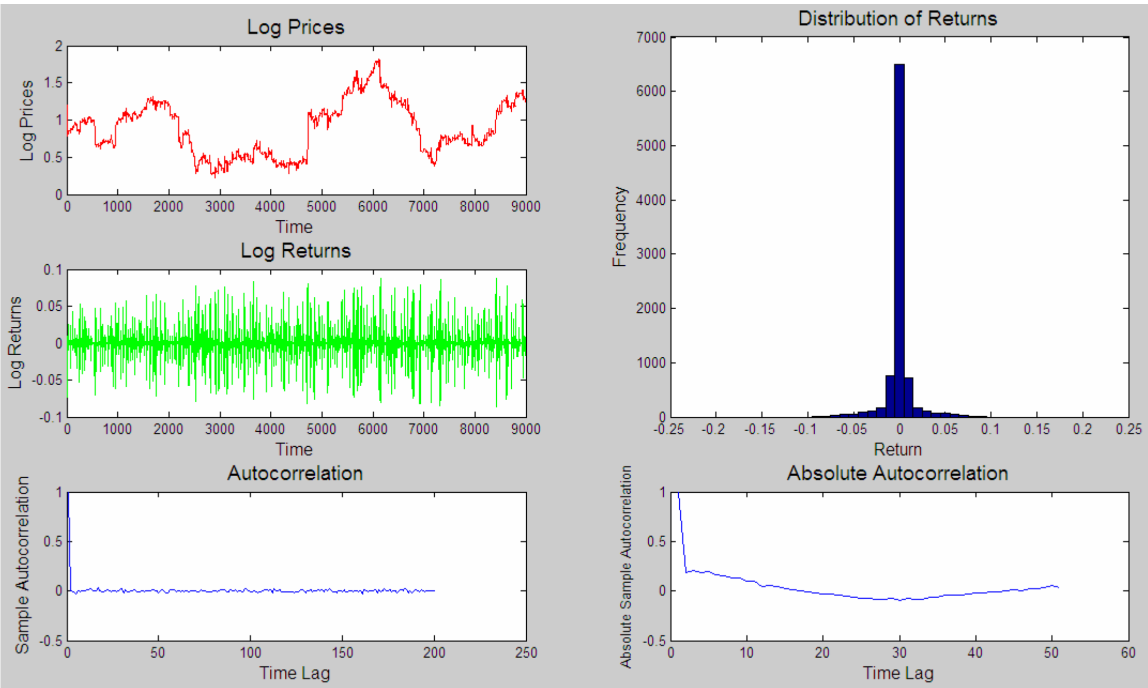

Figure 2.23. Simulation of the model for standard deviation of the signal D=0.0001

What follows is the table of the excess kurtosis and volatility results and figures related to these facts for the differing D values;

Table 2.3. Stylized Facts related to various Standard Deviations of Common Signal

Standard Deviation of Common Signal	***Excess Kurtosis***	***Standard Deviation of Returns***
0.1	-1.2230	0.0675
0.05	-0.8975	0.0533
0.01	-0.0800	0.0284
0.009	-0.0275	0.0272
0.008	0.0743	0.0262
0.007	0.1363	0.0250
0.006	0.2709	0.0236
0.005	0.3803	0.0221
0.004	0.4929	0.0205
0.003	0.7235	0.0191
0.002	1.0368	0.0179
0.001	1.9247	0.0165
0.0005	2.2783	0.0156
0.0001	11.1485	0.0143

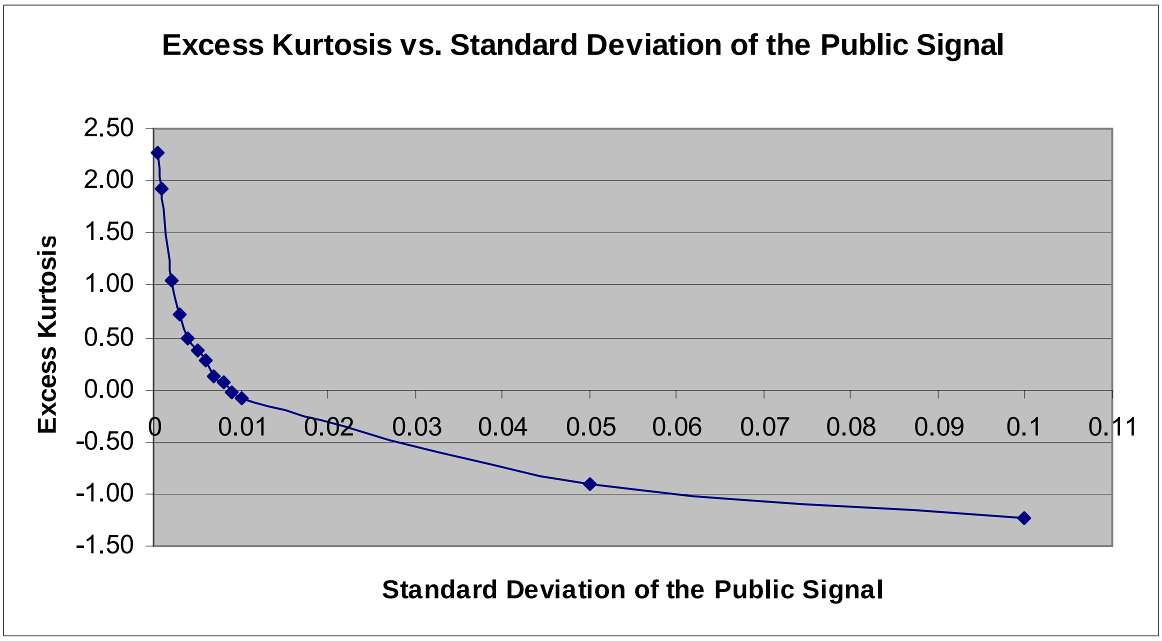

Figure 2.24. Excess Kurtosis versus Standard Deviation of the Public Signal

As we examine the results of Table 2.3, we can reach to a conclusion that in order to get heavy tailed return distribution, we have to assign values for the standard deviation of the public signal below 0.002.

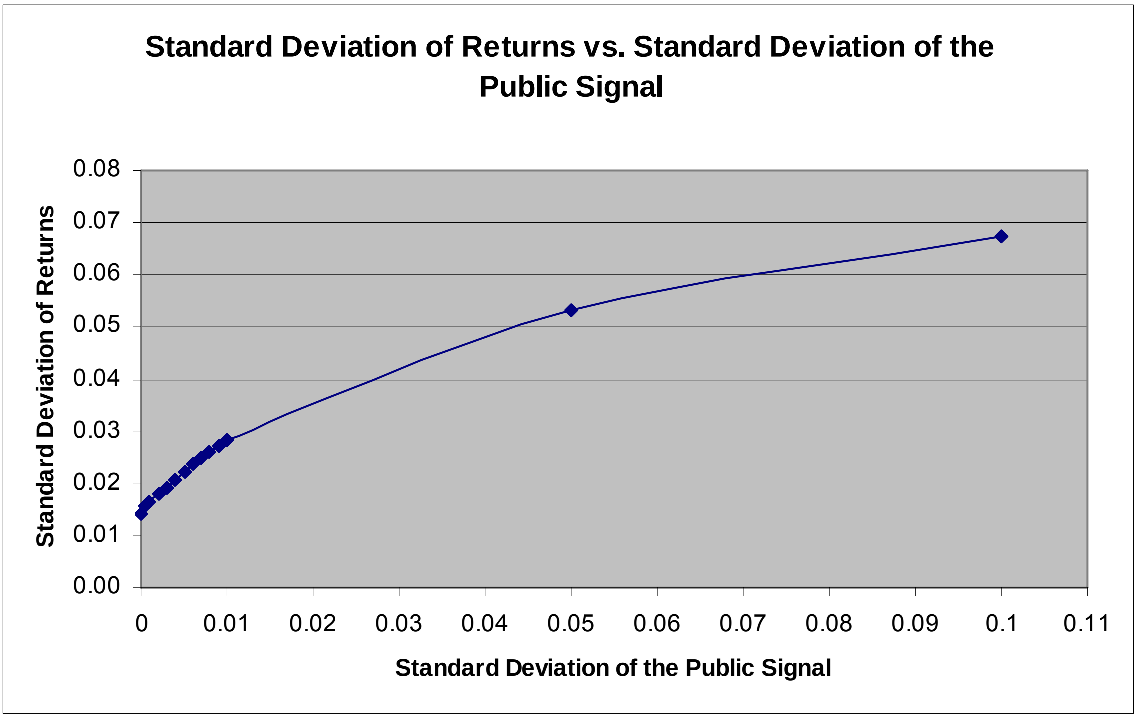

Figure 2.25. Standard Deviation of Returns vs. Standard Deviation of the Public Signal

On the contrary, excess volatility decreases along with a decrease in D value. These results may be attributed to the fact that high values of D leads to a uniform like return distribution which has negative excess kurtosis whereas possesses high standard deviation.

As a result, there is a trade of between these two statistical facts. But, as we examine figures carefully, increase in kurtosis is much more than the decrease in volatility for D values less than 0.002.

2.3.4. Number of agents N

Number of agents is not considered as a significant parameter in literature except that it should be a large number. In order to investigate this, I also examine the effects of this parameter, if any, to stylized facts.

In simulations that are performed with different number of agents, parameter values are chosen as follows;

- Updating Frequency S = 0.1
- Standard Deviation D of the Common Signal = 0.001
- Market Depth λ = 10

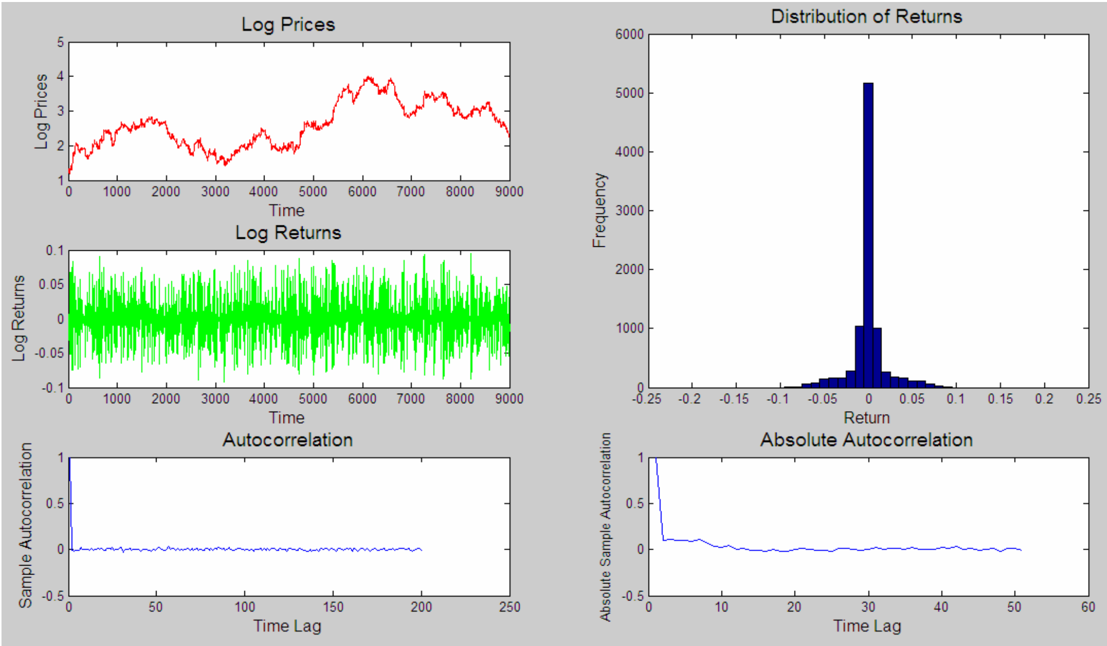

Figure 2.26. Simulation of the model for number of agents N=100

Graphs belonging to the analysis of the effect of the number of agents on stylized facts generate nearly the same results except for N = 100. At this value of N, return distribution is more tailed than the one for the N values greater than 250. However, absence of absolute autocorrelation appears as a result of the simulation.

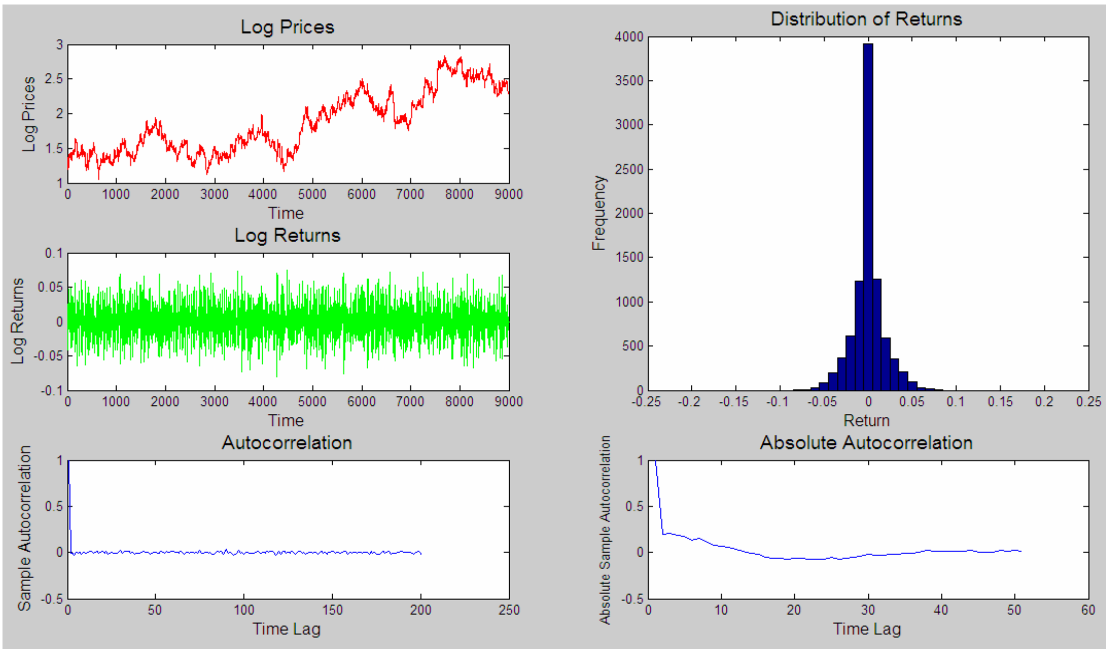

Figure 2.27. Simulation of the model for number of agents N=250

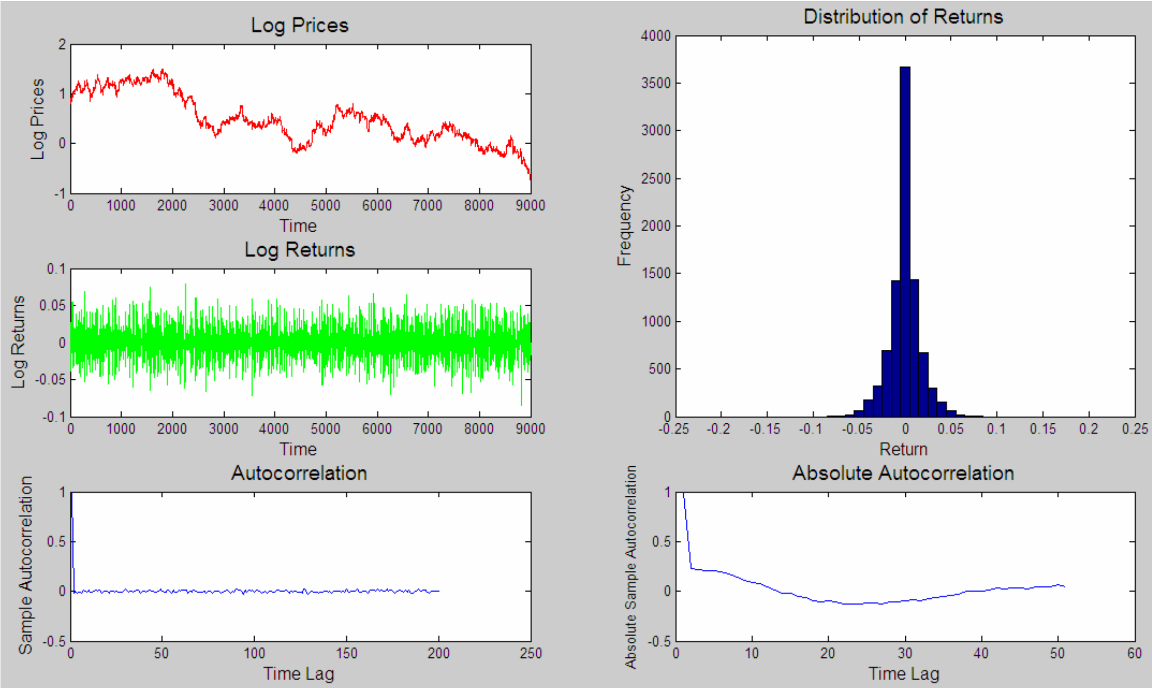

Figure 2.28. Simulation of the model for number of agents N=500

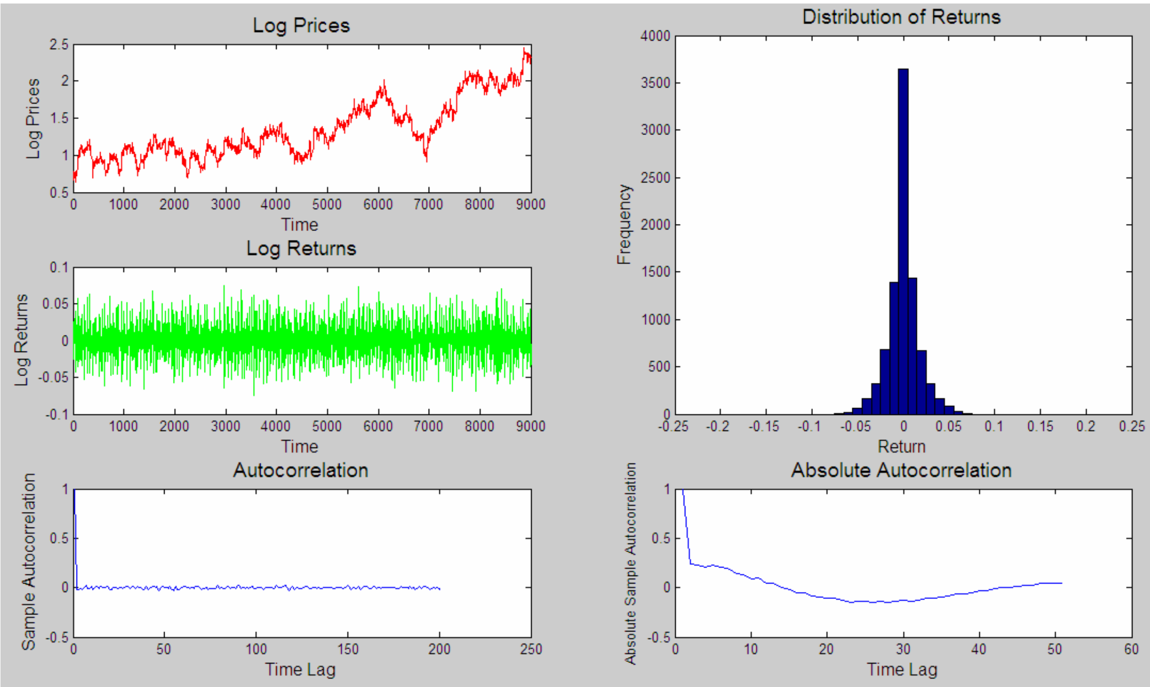

Figure 2.29. Simulation of the model for number of agents N=1000

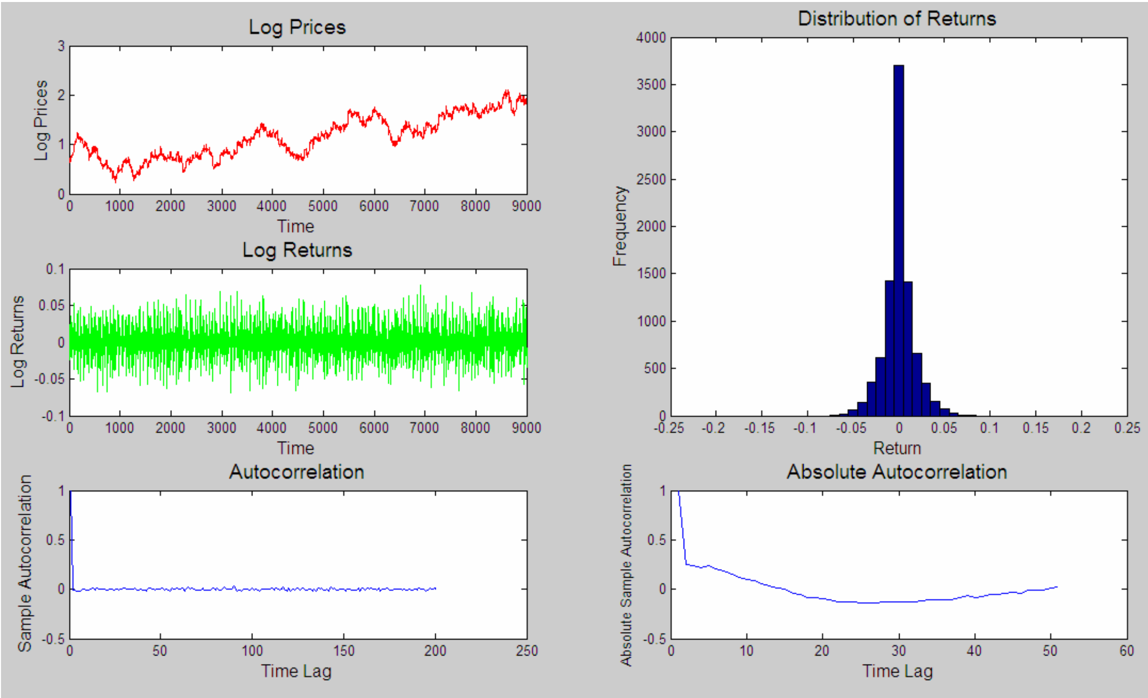

Figure 2.30. Simulation of the model for number of agents N=5000

Absence of absolute autocorrelation in absolute returns for N=100 may be due to the fact that there is not enough number of agents that will incorporate necessary inertia into the return dynamics at successive periods. So, persistence in volatility could not appear.

In addition to this, at this value of N=100, excess kurtosis is twice the nearest value and standard deviation of returns is the greatest at this value which can be seen from the Table 2.4 below. However, absence of autocorrelation in absolute returns and this low number of agents contradicts with reality since there are a large number of agents in real financial markets.

Table 2.4. Stylized Facts related to the different Number of Agents

Number of Agents	***Excess Kurtosis***	***Standard Deviation of Returns***
100	4.8205	0.0195
250	2.0313	0.0175
500	1.9127	0.0161
750	1.8420	0.0161
1000	1.9247	0.0165
1500	2.0129	0.0161
2000	2.0345	0.0162
2500	2.0002	0.0162
3000	2.0232	0.0160
4000	1.8365	0.0162
5000	1.9285	0.0162

At other N values there is not much change in volatility and excess kurtosis which can be observed from Figure 2.31 and Figure 2.32.

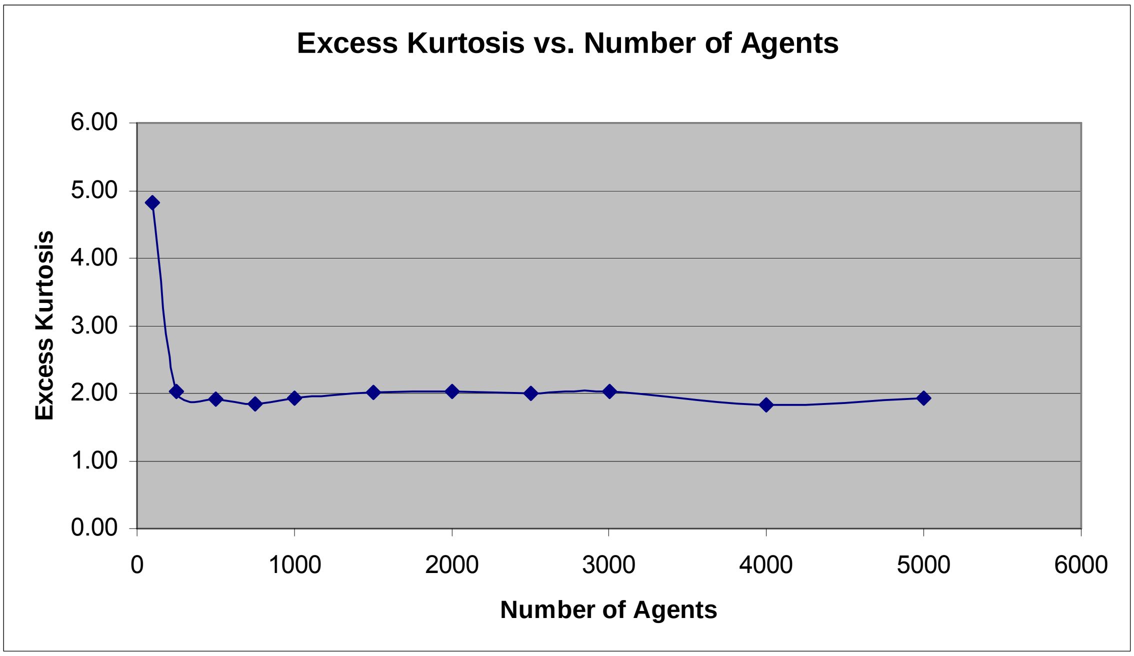

Figure 2.31. Excess Kurtosis versus Number of Agents

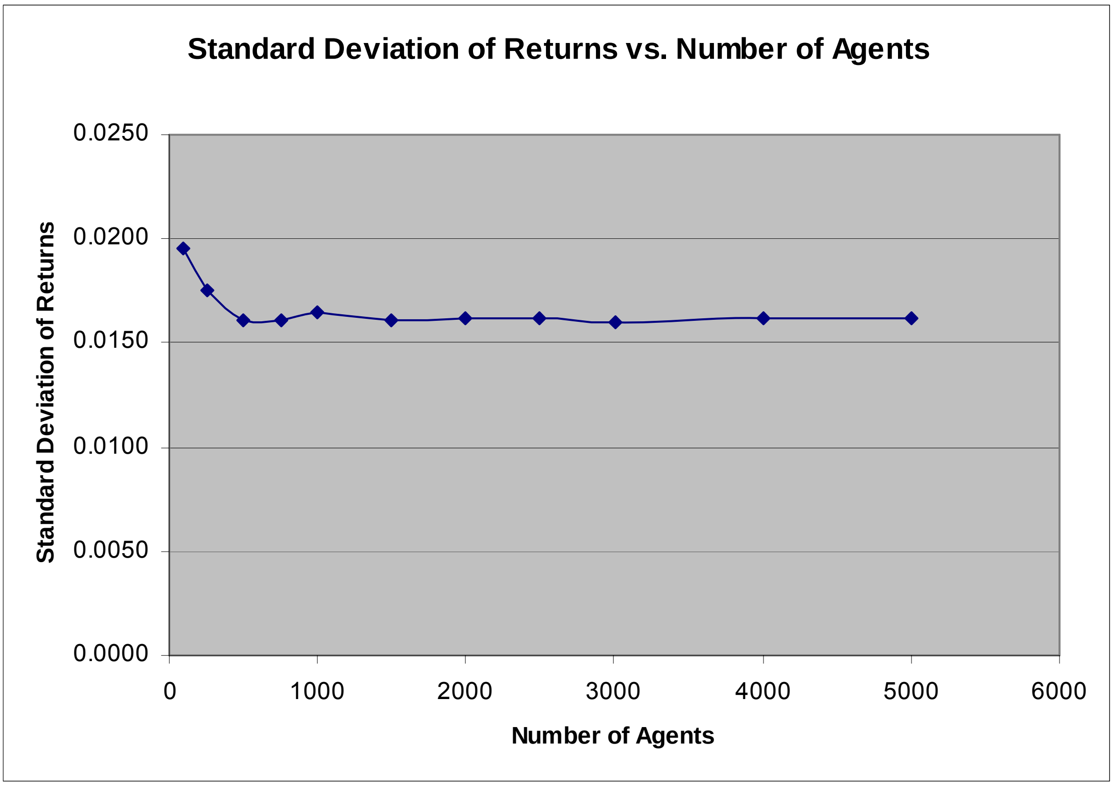

Figure 2.32. Standard Deviation of Returns versus Number of Agents

Consequently, since excess kurtosis and volatility indicates no significant pattern, agent values can be chosen between 1000 and 3000 whenever we consider the excess kurtosis because excess volatility is nearly same for all number of agents.

2.3.5. Concluding Remarks

To conclude, we may summarize what we did and what we obtain from the simulations of the model presented in this section.

In this section we introduce a single asset market model which is capable of producing stylized facts that are observed in real financial markets. These are obtained as a result of the three main property of the model, namely;

- Threshold behavior of the agents
- Feedback provided by equating the thresholds of agents to the absolute return by threshold updating mechanism

- Updating scheme of thresholds which leads to the emergence of heterogeneous agents

This study provides us with the effects of the certain model parameters as well as the main model ingredients of a market trading a single asset. Conclusion derived form the results of the simulations performed in the previous parts of this section can be summarized as follows;

- Threshold updating is needed in order to provide the model with ecology of evolving heterogeneous agents which is a common property of real financial markets. Moreover, for higher updating frequencies model exhibits tri-modal return distribution due to the loss of heterogeneity among agents. On the other hand, very low updating frequencies leave agents remaining with their initial uniformly distributed thresholds and due to this fact resulting distribution of returns are platykurtic.
- Threshold behavior of agents in this model leads to investor inertia which result in a distribution with heavy tails. So, threshold response must be embedded in a market model in order to obtain leptokurtic distribution of asset returns.
- Market depth is an important parameter in this model. Its value should be determined carefully for the model in order to be a logical one. As it is mentioned in the analysis part of the market depth, low values of this parameter is desired based on the stylized fact considerations.
- Standard deviation of the public signal should be determined by considering the maximum allowed return which is set by assigning appropriate market depth values. So, too small values and too high values for D with respect to the maximum allowed return is not desirable in order to have a meaningful model.
- Number of agents should be chosen a large number in order to correctly simulate a market. Low values for N is meaningless when considering the real market conditions. Moreover number of agents has no significant effect on the observed facts for high values of it.

I finish conclusion part by providing possible future works for this model. In this study sensitivity analysis is performed for each variable separately in order to understand their functions and effects in the model. However in a future study, sensitivity can be performed in order to understand the composed effects of the variables that might be present in the model. Moreover, updating scheme could be changed and made more realistic. But this should be done without deteriorating the simplicity of the model.

As a last remark, one very important and unrealistic assumption is made while constructing this model. Agents in this model are allowed to take infinite positions. This should not be allowed since in real life, traders could not take infinite positions due to risk considerations and wealth restrictions.

3. SINGLE ASSET MARKET MODEL WITH TWO TRADER TYPES

In this section of the study, we examine more complicated single asset market model than the one investigated in the previous section. In this model, we separate traders into two broad types, namely trend follower and value investor. This distinction was not present in the previous model.

As the one explained previously, this model is also constructed in order to explain the causes of the observed stylized facts in real financial markets using agent-based modeling technique. The model proposed here attempts to accomplish this objective in more realistic settings, including value investors and trend followers into the model who trade similar to the real market participants.

The model proposed here is also a simple model when compared to those models constructed by many authorities in agent-based market modeling field. There are no complicated decision rules and there is no application of artificial intelligence such as embedding genetic algorithms into the model which makes model outputs nearly impossible to comprehend.

There are three main differences between the model proposed in previous section and the one proposed here. First, there are two thresholds per agent in the model proposed here which makes the model totally different from the previous one. Second, no threshold updating takes place in the current model which should be included in a further study. Third, agents decide on their demands first by considering their current positions. Then, they determine their orders according to their threshold values which together with the position considerations form the trading strategy.

The model developed here leads to the absence of autocorrelation in returns, excess volatility and some other stylized facts mentioned before.

In the following sections, first general price process is introduced and then each one of the two trader (agent) types are described and their crucial parameters are investigated separately. Along with this investigation, particular effects of these agents on the observed price behavior and stylized facts are extracted by analysis and interpretation of the results. Lastly, heterogeneous setting of the agents is examined and resulting graphical behavior is compared qualitatively with the observed price dynamics of the real financial markets.

So, we can start with the description of the model;

3.1. Model Description

In this part, model is described first by providing the price formation mechanism and then explaining the agent types, separately.

3.1.1. Price Formation Model

There are two broad trader types;

1) Directional Traders

They buy and sell by placing market orders, which are always filled. In this model we examine two types of directional traders, namely;

- Trend Followers (Chartists)
- Value Investors (Fundamentalists)

Typically, the buy and sell orders do not match and the excess order is taken up by the second type of trader, market maker.

2) Market Maker

Market maker fills the excess order at a price that is shifted from the previous price by an amount that depends on the net order of the two directional traders mentioned above. It is the market impact function, which is used to set prices by the market maker. In this model, marker maker is risk neutral since he is allowed to take arbitrarily large positions.

3.1.1.1. Market Impact Function: Market impact function is an increasing function of the net order since buying drives price up and selling drives it down.

Let $x_{i,t}$ be the position of the directional trader labeled i at time period t. Position of a directional trader depends on the previous position as well as the previous prices and on any additional external information I_t. So we can say that,

$$x_{i,t+1} = x_{i,t}(P_t, P_{t-1}, \ldots, I_t) \qquad (3.1)$$

$x_{i,t}$ is considered as a strategy or decision rule of agent i. The order $\omega_{i,t}$ of agent *i* is determined from the position through the following relation

$$\omega_{i,t+1} = x_{i,t+1} - x_{i,t} \qquad (3.2)$$

Trading process at any time step can be decomposed into two parts;

- The directional traders observe the most recent prices and information at any time step and submit orders $\omega_{i,t}$ according to their decision strategy.
- The market maker fills the orders at the new price P_{t+1} which is set by using the market impact function.

The market maker bases price formation only on the net order

$$\omega_t = \sum_{i}^{N} \omega_{i,t} \qquad (3.3)$$

The algorithm that is used by the market maker in order to compute the fill price for the net order ω_t, is an increasing function (market impact function) with G(0) = 1 which can be represented as follows;

$$P_{t+1} = G(P_t, \omega_t) \qquad (3.4)$$

Letting p_t = log P_t and providing necessary approximations and calculations we end up with the following formulation for the price formation of the market maker as,

$$p_{t+1} = p_t + \frac{1}{\lambda}\sum_{i}^{N} \omega_{i,t}(p_t, p_{t-1,\ldots,} I_t) + \xi_{t+1} \tag{3.5}$$

where λ is the market depth and ξ_t is the noise term. On the other hand $\omega_{i,t}$ is the agent specific decision rule (or simply the demand of an agent) which will be examined deeply in the following sections. ξ_t can be interpreted as the random perturbations in the price.

3.1.2. Agent Behaviors

In this section trading strategies are described in more detail. In this paper, financial trading strategies are classified based on their information inputs. Decision rules that depend only on the price history are called *technical* or *chartist* strategies. Trend following strategies are a commonly used special case in which positions are positively correlated with recent price changes. *Value* or *fundamental* strategies, in contrast, are based on external information which leads to a subjective assessment of the long-term fundamental value. Investors using these strategies do not believe that this is the same as the current price.

In the following subsections, I introduce these two trader types and investigate their crucial parameters along with the examination of their particular influence on the observed price behavior and statistical facts.

3.1.2.1. Value Investor (Fundamentalist): Value investors make a subjective assessment of value in relation to price in order to decide on their move. They believe that future prices will move toward their perceived value. Fundamentalists take long (positive) positions when they think the market is undervalued (current price of the asset is less than the perceived value) and take short (negative) positions when they think the market is overvalued (current price of the asset is greater than the perceived value).

In our model, estimated value is taken as an exogenous input variable and the logarithm of the value follows a random walk:

$$v_{t+1} = v_t + \eta_{t+1} \tag{3.6}$$

where η_t is a normal, IID noise process with standard deviation σ_η and mean μ_η .

Most of the value investor strategies only depend on the mispricing $m_t = p_t - v_t$ where p_t and v_t is the logarithms of the price and the value, respectively.

In literature value investors are modeled based on two broad strategy types;

1) Position-based value strategies
2) Order-based value strategies

However, these two broad types have specific drawbacks which are overcome by a third strategy type that will be used in our model, namely;

3) Position-based threshold dependent value strategy

We are going to examine these three value strategies respectively.

1) *<u>Position based value strategy</u>*

According to a position based value investor strategy, trader takes a long (positive) position when the asset is underpriced (mispricing is negative). If the asset becomes even more underpriced, the position either stays the same or gets larger. Similarly, if the mispricing is positive (asset is overpriced) agent takes a short (negative) position.

For the simplest class of these strategies, the position of the fundamentalist is of the form

$$x_{t+1} = c(v_t - p_t) \tag{3.7}$$

where c represents the capital scaling parameter.

In this strategy, agent determines his position according to the magnitude of the mispricing. At each period, value investor updates his position and places a buy or sell order, an amount equals to the difference between his successive positions.

There are two important pitfalls of this type of strategy. One is the resulting excessive transaction costs due to the high trading frequency. In real markets, whenever an agent places an order he has to pay transaction cost. Paying this cost whenever mispricing changes, even for a very small amount of a change, seems to be an unrealistic assumption.

Second is the lack of cointegration between price and the value. This is a model property that should be satisfied since in real market, prices track values that is to say they are cointegrated. The reason for this problem can be quoted from Farmer and Joshi (2002) , " …while a trade entering a position moves the price toward value, an exiting trade of the same size moves it away from value by the same amount. Thus, while the negative autocorrelation induced by simple value strategies might reduce the rate at which prices drift from value, this is not sufficient for cointegration…"

Moreover, an unbounded position for a trader may be resulted from the lack of cointegration which implies unbounded risk. This is so because mispricing may become unbounded. As a result this strategy should not be the only strategy present in the model in order to avoid lack of cointegration.

2) *Order-based value strategies*

One way of solving the problem of lack of cointegration, is to change the value strategy. Order-based value strategies solve this problem under the unrealistic allowance of unbounded inventories for agents.

Agents possessing this type of strategy buy the asset whenever the asset is underpriced and sell as long as the asset is overpriced. That is as long as the asset is mispriced, agent places an order of the magnitude according to the level of mispricing.

So, the simplest order value strategy can take the form as follows;

$$\omega_{t+1} = c(v_t - p_t) \tag{3.8}$$

where c again represents the capital scaling parameter.

The problem with this kind of strategy is the unbounded inventory that an agent is allowed to accumulate. This is so because, agent places orders without considering current position. Therefore, if agent is in the long position at period t, and if mispricing persists being negative at succeeding periods, agent continues to buy more and this trading is persisted even when mispricing goes to zero, leading to unbounded inventories.

Consequently, agents in our model should consider their positions when they place orders, whereas orders should not be determined solely according to the position which is mentioned above. So, we have to find a strategy that solves the problems mentioned above.

3) *Position Based Threshold Dependent Value Strategy*

This type of strategy solves the problem of unbounded risk along with the transaction cost problem.

Main problem with the position based strategy is the excessive transaction cost that results from the high frequency of trading activity due to the fact that agents trade whenever their position is changed according to the level of mispricing.

This is prevented by posing threshold values for entering a position and exiting it. So, this threshold dependency reduces the trading frequency which is the unrealistic part of the position based strategies. On the other hand, since this strategy considers the current position of the traders, it does not allow agents to possess unbounded inventories.

According to this type of fundamentalist strategy, which is both non-linear and position dependent, a short (negative) position –c is entered when the mispricing exceeds the entry threshold (T) and exited when it goes below exit threshold (τ). Similarly, a long (positive) position c is entered when the mispricing drops below a threshold $-T$ and exited when it exceeds $-\tau$, where (T) and (τ) are both specific to each trader.

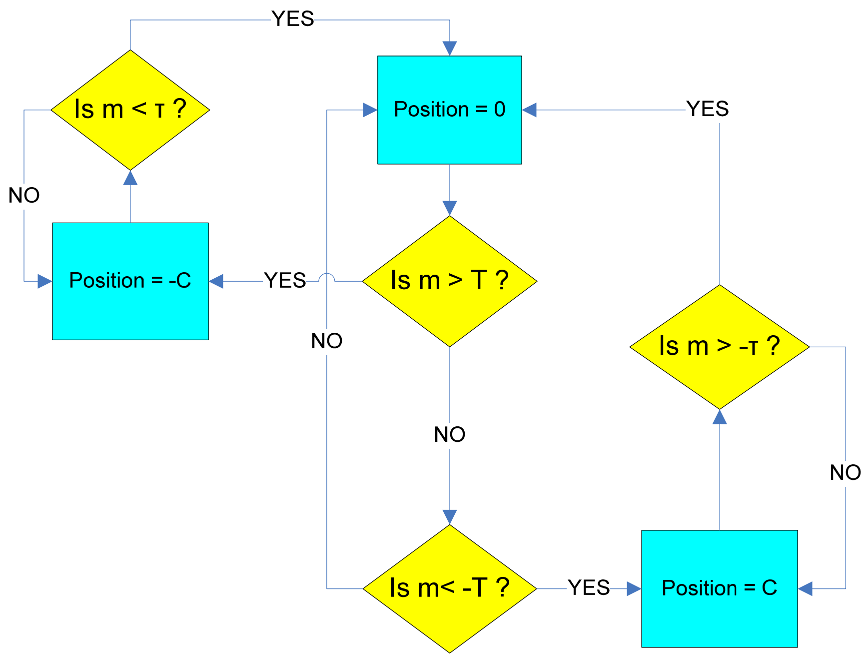

Figure 3.1. Flowchart of the nonlinear threshold dependent value strategy. If position is zero and mispricing is above the entry threshold T, a negative position is entered. This position is exited when m drops the value of the exit threshold. If m is below the negative of the entry threshold, then positive position is taken and it is exited when m exceeds negative of the exit threshold.

Since this strategy depends on its own position as well as the mispricing, and it can be illustrated as a flowchart, as shown in Figure 3.1. Parameter c is chosen so that c = $a\,(T - \tau)$, where a (scale parameter for capital assignment) is a positive constant.

There are several requirements for the model parameters that should be examined and met in order to obtain a valuable and logical value strategy.

First, we discuss the magnitudes and signs of the entry and exit thresholds and then come to the issue of assigning logical values to the capital scaling parameter.

➢ Entry and Exit Thresholds

As it is mentioned above, entry and exit thresholds determine when an agent is going to be active or stay inactive which provides investor inertia.

First of all, as it is also implied in Farmer and Joshi (2002), the entry threshold should be positive and greater than the exit threshold. In order to take an active position, value investor compares the magnitude of the absolute value of mispricing with the entry threshold T. So, assignment of a negative value for an entry threshold contradicts with the general logic of the value strategy. In addition to this, entry threshold should be greater than the exit threshold because after entering a position, if this property does not hold and thus an agent possesses an exit threshold that is greater than the entry threshold; this agent will be forced to leave that position even if the magnitude of the mispricing increases. This is also illogical and again contradicts with the logic of the value investor strategy. So, entry thresholds take greater values than the exit thresholds.

Interpretation of the logical values for the exit threshold can be made intuitively. Exit threshold τ can be assigned both positive and negative values according to real market. Traders who do not want to leave their current position before they extract full profit will choose $\tau < 0$. On the other hand, some may want to leave their positions before mispricing reaches zero because they believe that there will be very little expected profit left and do not want to lose any of the gained return, these traders will choose $\tau > 0$. So, we can allow τ taking both positive and negative values in our simulations.

Besides, for a logical strategy, traders should not exit their positions at a mispricing that is further from zero than the entry point (threshold). So, we can summarize the requirements for the entry and exit thresholds as, $-T < \tau < T$ and $|\tau_{max}| \leq T_{min}$.

In addition, $\tau < 0$ is desirable for cointegration since this property causes price changes that are always in the opposite sign of the mispricing. In order to investigate this effect I simulate my model twice having all parameter values same except exit threshold values.

In the first simulation, I do not allow exit threshold to take positive values.

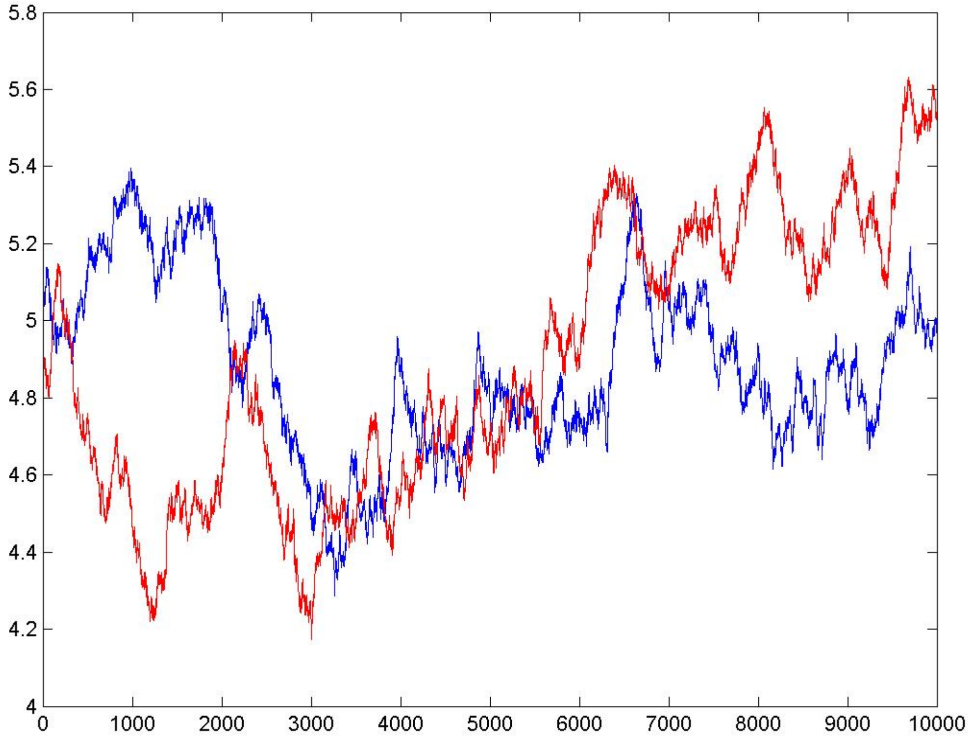

Figure 3.2. The resultant dynamics of the simulation performed with 1000 value investor agents. Other parameter are N=1000, a=0.001, σ_η=0.01, σ_ξ=0.01, T_{min} 0.5, T_{max}=6.0, τ_{min}=-0.5, τ_{max}=0. Blue curve represents the log-price, and red curve represents the log-value.

Mispricing results of the simulations are given in Table 3.1;

Table 3.1. Outputs of two simulations considering mispricing

MISPRICING	***First Run***	***Second Run***
Absolute Mispricing Totals	3428.60	3847.10
Mispricing Mean	-0.0026	0.0216
Mispricing Standard Deviation	0.3943	0.4354

It can easily be seen from the Figure 3.2 and Figure 3.3, and from the results of the simulations, mispricing increases when we allow τ taking positive values.

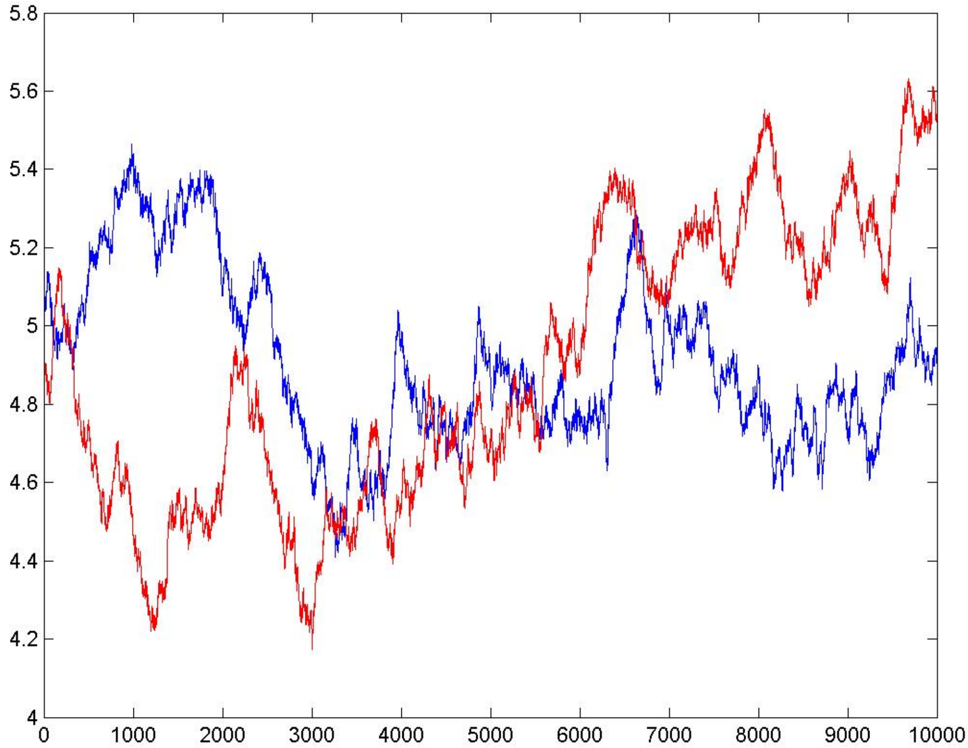

Figure 3.3. The resultant dynamics of the simulation performed with 1000 value investor agents. All parameters and random number seed are same as Figure 3.2 except that τ_{max}=0.5.

Besides from those figures we can conclude that allowing τ taking positive values has a bouncing effect around zero mispricing since some agents leave their positions before mispricing reaches zero which causes it to persist for longer periods. So, we are sure that for the sake of cointegration we should assign negative values to exit threshold.

In addition to the above discussion, it is worthy to investigate the effect of the magnitude of minimum entry threshold, T_{min}.

Intuitively, lowering T_{min} should increase the trading volume and increasing it may have a decreasing effect on that volume. This is so because low T_{min} values allow traders taking active positions even for very low values of mispricing. In order to see whether this intuition is true or not, simulations with different T_{min} values are performed and the resulting Figure 3.4 and Table 3.2 are provided as follows;

Table 3.2. Changes in trading volume with respect to differing T_{min} values

Minimum Entry Threshold Value	***Mean of Trading Volume***
1.0	0.0014
0.9	0.0016
0.8	0.0018
0.7	0.0019
0.6	0.0021
0.5	0.0023
0.4	0.0027
0.3	0.0030
0.2	0.0037
0.1	0.0184

From the results of the simulation, we understand that decreasing the minimum value of the entry threshold leads to an increase in trading volume which is an expected outcome of our model.

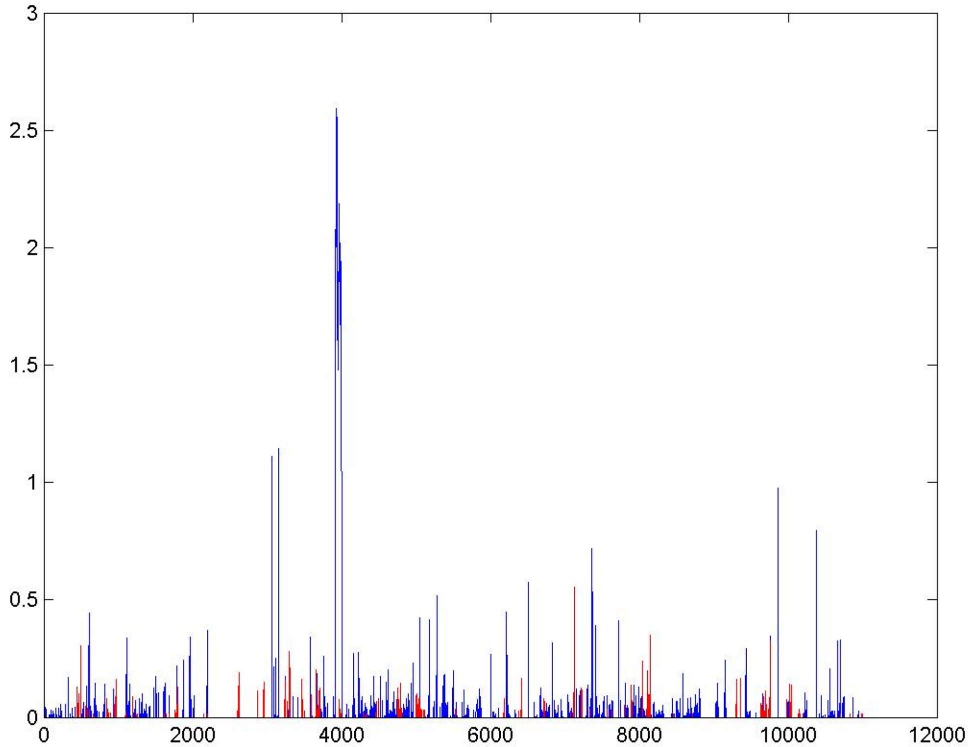

Figure 3.4. Trading Volume Comparison for T_{min}=0.1 and T_{min}=1.0. Blue histogram belongs to T_{min}=0.1 and the red one belongs to T_{min}=1.0. It is easily seen that decreasing the minimum value of the entry threshold leads to the increasing trading volume.

- Capital Scaling Parameter

In order to avoid the emergence of price dynamics driven only by random number sequence and to let this dynamics being driven by the endogenous interactions of the agents, scaling parameter 'a' should be chosen carefully. If 'a' is assigned too small value, agents may not provide enough restoring force for the mispricing because once all N value investors take a positive or negative position, cointegration ceases to exist since no trader will have an effect on price. If a is chosen too big, it may cause instabilities and oscillations in price since a single trader may reverse the mispricing sign.

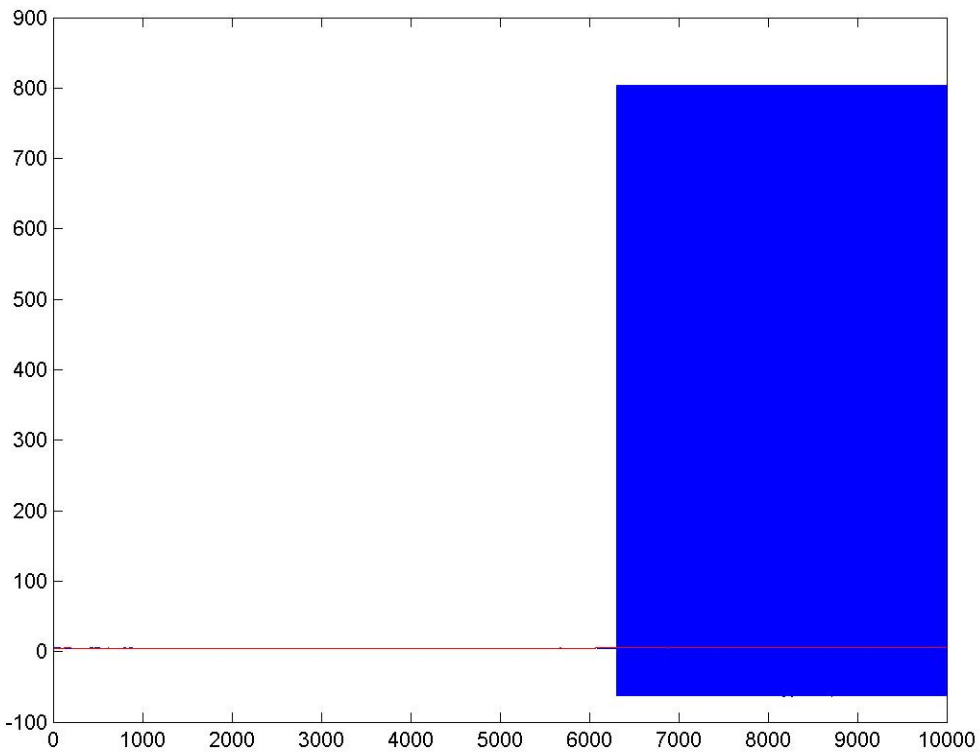

Figure 3.5. The resultant dynamics of the simulation performed with 1000 value investor agents. All parameters and random number seed are same as Figure 3.4 except for the value for a=0.25.

Figure 3.5 is an output of a simulation performed with a very high a value. Initially no trader is excited and price dynamics is stable. However when a single agent is excited at a later period his effect on price causes oscillations around the value which may be seen from the dark area of the oscillating price around value after time period 6000.

Above discussion and simulations are performed by supplying the same information to all traders. However in real life, values are perceived differently. In order to incorporate this property into the model, the dynamics of the values can be modeled as a simple reference value process $\bar{v}_t$ that follows Eq. (3.6) with a fixed random offset v_i which are selected uniformly between v_{min} and v_{max} for each fundamentalist trader. So, the value perceived by the i^{th} trader at time t is given by

$$v_{i,t} = \bar{v}_t + v_i \qquad (3.9)$$

➢ Offset v_i for the perceived Values

Since offset values are assigned in order to represent the fact that agents perceive values differently, this may cause the mispricing to persist in higher magnitudes since agents may not respond together and may not impose necessary impact on price to force it to track the value.

This may be analyzed by applying sensitivity analysis on offsets by considering the total absolute mispricing. Table 3.3 illustrates the effect of offset intervals on mispricing.

Table 3.3. Effect of Offsets on Mispricing

Offset	***Total Absolute Mispricing***
No Offset	*4920*
[0.5 -0.5]	5701
[1.0 -1.0]	7204
[1.5 -1.5]	10430
[2.0 -2.0]	28233
[2.5 -2.5]	7370
[3.0 -3.0]	26281
[3.5 -3.5]	14702
[4.0 -4.0]	196698
[4.5 -4.5]	192686
[5.0 -5.0]	192004

From Table 3.3, we may conclude that introduction of offsets clearly increases the mispricing as it is expected. However, there is no explicit result that concludes us that increasing offsets will increase total absolute mispricing. Moreover, inclusion of offsets that are chosen from an interval having a larger width with respect to the interval [3.5 -3.5] clearly leads to unstable price dynamics. So, we can be sure that offsets should not be assigned high values, larger than nearly 3.5, in order to get stable price dynamics.

3.1.2.2. Trend Follower (Chartist): Trend followers are the positive feedback investors who believe that an upward movement in prices will be followed by an upward movement and a downward movement will be followed by a downward one. So, they take a positive (long) position if prices are recently going up and take a negative (short) position if prices are recently going down.

General trend following strategy is

$$x_{i,t+1} = c(p_t - p_{t-\theta_i}) \tag{3.10}$$

where θ_i is the time delay of the i^{th} chartist and c is the capital scaling parameter.

Due to the same reasons as we discuss for value investing strategies, we may not use simple trend following strategy mentioned above. Trading strategy modeled for a trend follower in our study can be summarized as follows;

First of all, traders' decisions in the model are based on the position of the trader. Each trading day chartist checks whether he is in the long (positive), short (negative) or inactive (zero) position. Considering his current position, chartist compares Excess Price, XP,

$$Excess\,Price_i = Price(t) - Price(t - \theta_i) \tag{3.11}$$

the price change within θ days (θ may be interpreted as subjective time delay in order to compute the excess price for an agent), with two predetermined thresholds specific to each agent. These thresholds are again the Entry (T) and the Exit (τ) thresholds.

According to this strategy, from a zero position, a long position c is entered when the Excess Price (XP) exceeds the entry threshold (T). This position is exited when XP drops below exit threshold (τ). Similarly, a short position –c is entered when XP drops below negative value of the entry threshold ($-T$), and exited when XP exceeds the negative value of the exit threshold ($-\tau$).

Parameter c is chosen so that c = $a\,(T - \tau)$, where a is a scale parameter for capital and it is required to be a positive constant. This strategy is illustrated by the following Figure 3.6;

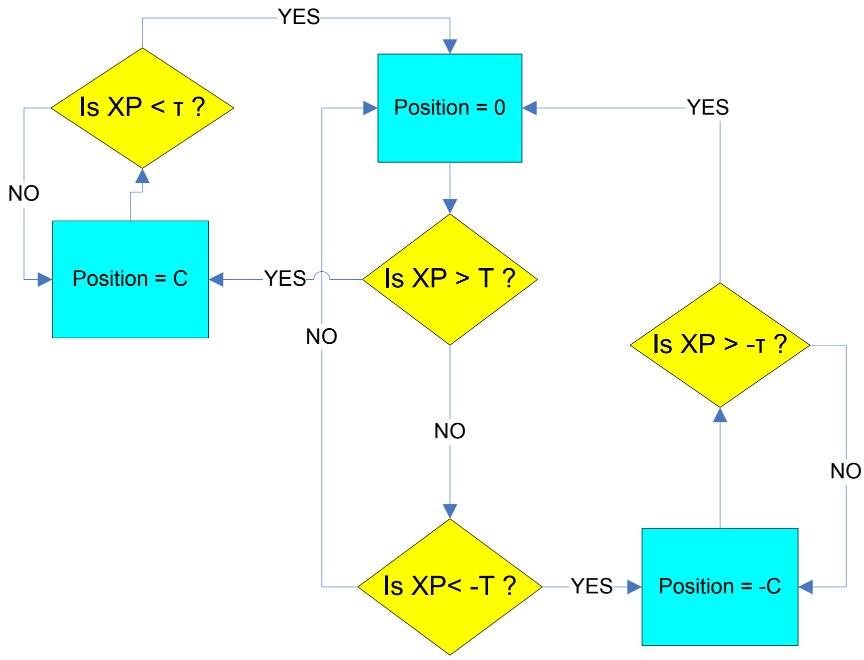

Figure 3.6. Flowchart of the nonlinear threshold dependent trend following strategy. If position is zero and excess price, XP is above the entry threshold T, a long position is entered. This position is exited when XP drops the value of the exit threshold. If XP is below the negative of the entry threshold, then a short position is taken and it is exited when XP exceeds negative of the exit threshold.

This strategy is similar to the one that is considered for the value investing strategy except that the determination of the positions and the variable that is compared with thresholds are differed. So, I will not repeat here the same considerations for threshold values and accept the requirements that are explained in value investor section.

For the trend follower I will examine the effects of the standard deviation of the noise term and the maximum time delay θ_{max}, which is a specific parameter for the trend followers.

- Agent specific time delay θ and Standard Deviation of Noise term σ_ξ

As described once, θ is the agent specific time parameter which corresponds to the time period that an agent uses to compute his subjective value of the excess price. In our model, θ values are assigned by the modeler as uniformly distributed random numbers. The effect of the interval width for this parameter is examined in this part.

The effect of time delay, that is how far they look back while considering the excess price, is important to discuss here. If traders look closer past prices while computing their anticipation for the excess price, only some of the traders will be excited and trade due to the threshold behavior. So, if time delay is a small number then the dynamics of the prices are not affected significantly by trend followers. On the other hand, if they look too far then agent specific excess prices are likely being large to excite most of the agents as time progresses.

In addition, we should also take into account the noise term incorporated into the price formation. This is so because noise term works as an exciter and if the excitement at a single period is high then time delay may have differing effects. The interpretation of the noise term was explained earlier in the study but we should consider it here also by performing sensitivity analysis while discussing the time delay effect.

In order to check our expectations and effect of the noise, we simulate trend follower model for differing θ intervals with different standard deviations of the noise. We change the width of the time interval, and simulate the model for θ ~ U [1 10] and then θ ~ U [1 50] and at last for θ ~ U [1 100].

Table 3.4. Effects of Time Delay and Noise term on trading volume

Trading Volume (Mean)		**Maximum Time Delay (θ)**		
		10	***50***	***100***
Standard Deviation of the Noise (σ_ξ)	***0.01***	0.0000	0.0000	3.1865E-06
	0.05	5.2965E-06	0.0011	0.1463
	0.1	0.0023	0.3945	0.2163
	0.2	1.5645	0.4328	0.2330
	0.35	1.5746	0.4331	0.2461
	0.5	1.5786	0.4527	0.2748

Simulation results for the differing time delay and standard deviation of the noise term are given in Table 3.4. After examining the table, for low values of the standard deviation of the noise term σ_ξ below 0.1 trading volume is insignificant, nearly zero, for maximum time delays of 10 and 50. It is zero for σ_ξ = 0.01. On the other hand, for these low values of σ_ξ, some trades occur when we raise the maximum time delay to 100 and at σ_ξ = 0.05 significant trading is encountered for time delay equals to 100. So, we can conclude that for the low σ_ξ values we have to increase θ in order to get sensible price dynamics. If we assign low values to θ, then the dynamics is maintained by the noise term which is unrealistic and undesirable outcome for our model.

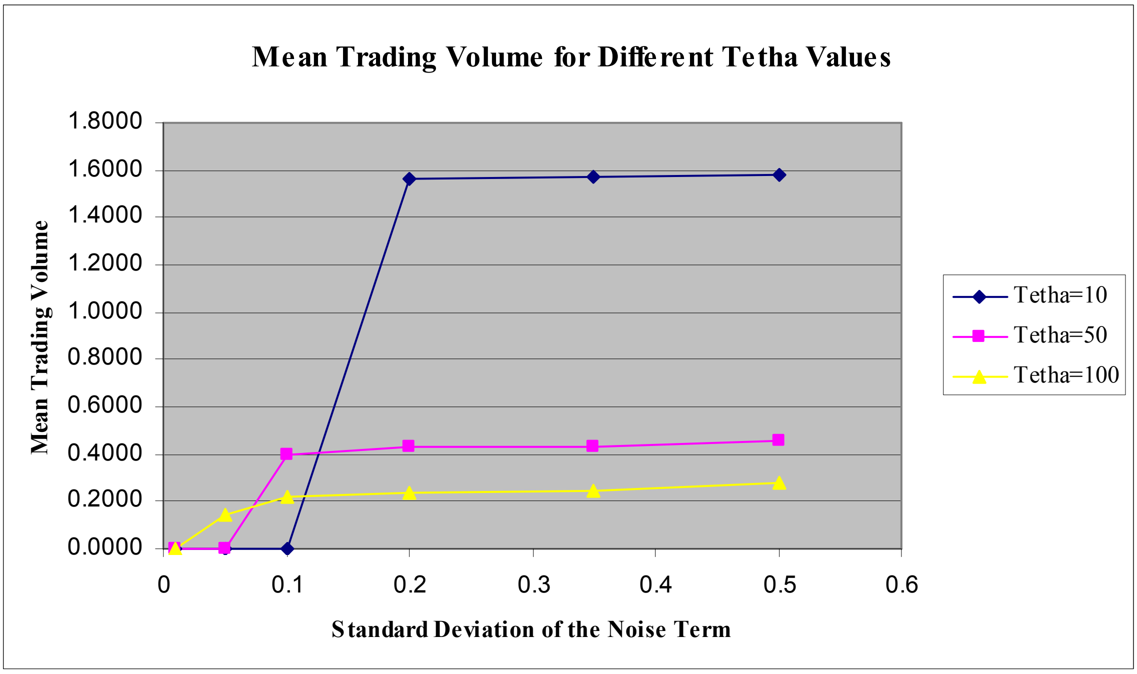

Figure 3.7. Mean trading volume variations with respect to changes in tetha θ and noise term σ_{ξ} values

Besides, for high values of σ_{ξ}, enough impulse is applied by the noise term to excite the traders looking near past in order to determine their subjective assessment of the excess price. So, considerable trading takes place for the low values of the time delay. This is so because high values of σ_{ξ} induce large short-term price changes. Traders looking near past price values are excited more than the one looking far because noise term has a lower effect on the long term price changes since σ_{ξ} has a zero mean.

Trading volume for θ = 10 is unrealistically large (approximately 20% of all traders place orders at each time period) as it is seen from Figure 3.7 and due to this fact corresponding excess kurtosis takes low values.

Table 3.5. Effect of Time Delay and Noise term on Excess Kurtosis

Excess Kurtosis		Maximum Time Delay (θ)		
		10	***50***	***100***
Standard Deviation of the Noise (σ_ξ)	***0.01***	-0.0527	-0.0527	-0.0527
	0.05	-0.0538	-0.0489	60.3332
	0.1	-0.0399	7.8807	18.0568
	0.2	0.8577	6.8071	9.6566
	0.35	0.7931	6.5111	8.2608
	0.5	0.6413	4.7858	3.6425

Table 3.5 illustrates the effect of the two parameters, σ_ξ and θ, on excess kurtosis. It can be clearly seen from this results that for low values of σ_ξ, price dynamics is managed by the normally distributed noise term for which excess kurtosis is zero.

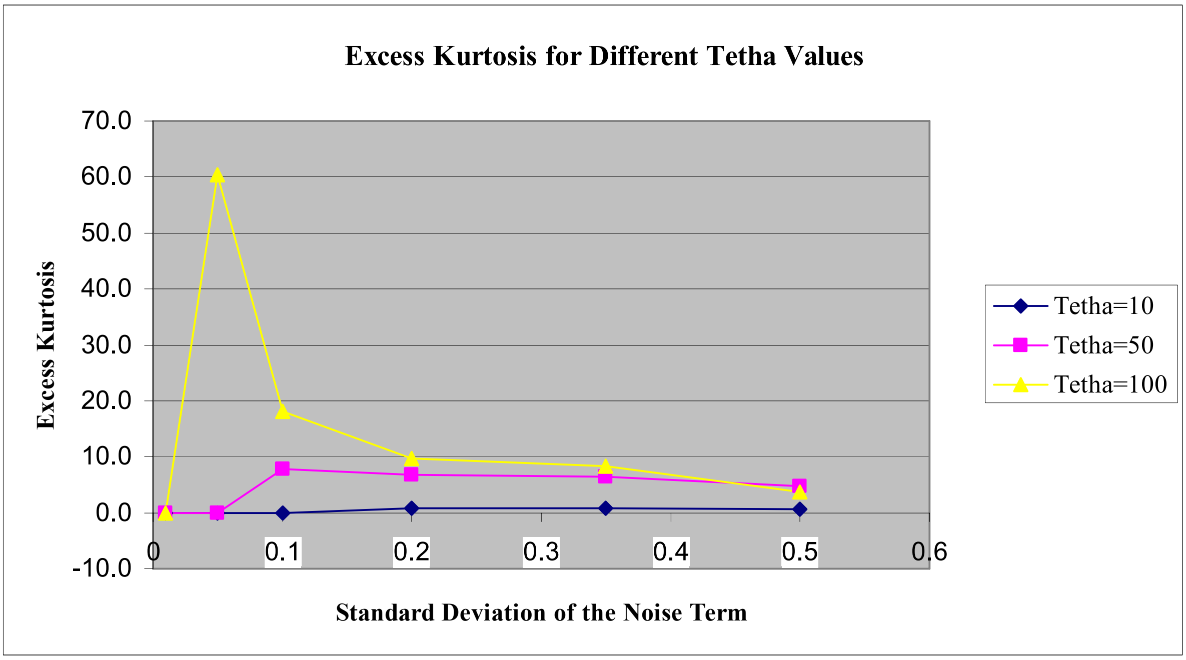

Figure 3.8. Variation of excess kurtosis with respect to changes in theta θ and noise term σ_ξ values

On the other hand, for high values of σ_ξ, excess kurtosis begins to take higher values. It is important to see from the Table 3.5 and Figure 3.8 that, along with the increasing trading volume excess kurtosis decreases. This may be owing to the decreased peakedness of the return distribution (which leads to more normal like distributed returns) which is affected by the excess demand directly and thus by the trading volume indirectly. So, unresponsiveness of the traders to the variations in prices (investor's inertia property) is diminished because their threshold responses are weakened. As a result, decreased inertia of the traders forces excess kurtosis to decrease.

3.1.2.3. Trend Followers and Value Investors Together: In this section we examine the dynamics obtained from simulating the trend followers and value investors together. We use strategies for these two trader types that are investigated in the previous sections. Comparison is made qualitatively with the annual prices and dividends for the S&P index. Dividends in real market correspond to the values in our model.

The parameters for the simulation of the two trader types together are given in Table 3.6. Parameters are chosen in order to catch the similarity between statistical facts observed in real data and to obtain a graphical correspondence with the real market dynamics of S&P index of American stock prices.

Table 3.6. Parameter values of the simulation with both trader types

Parameters	Values
Number of Value Investor Agents	1500
Number of Trend Follower Agents	1500
Minimum Entry Threshold	0.4
Maximum Entry Threshold	4
Minimum Exit Threshold	0.4
Maximum Exit Threshold	4
Scale Parameter for Capital Assignment (value investor)	0.003
Scale Parameter for Capital Assignment (trend follower)	0.003
Minimum Offset for Value	-2
Maximum Offset for Value	2
Maximum Time Delay	100
Standard Deviation of the Noise Term	0.1
Market Depth	1

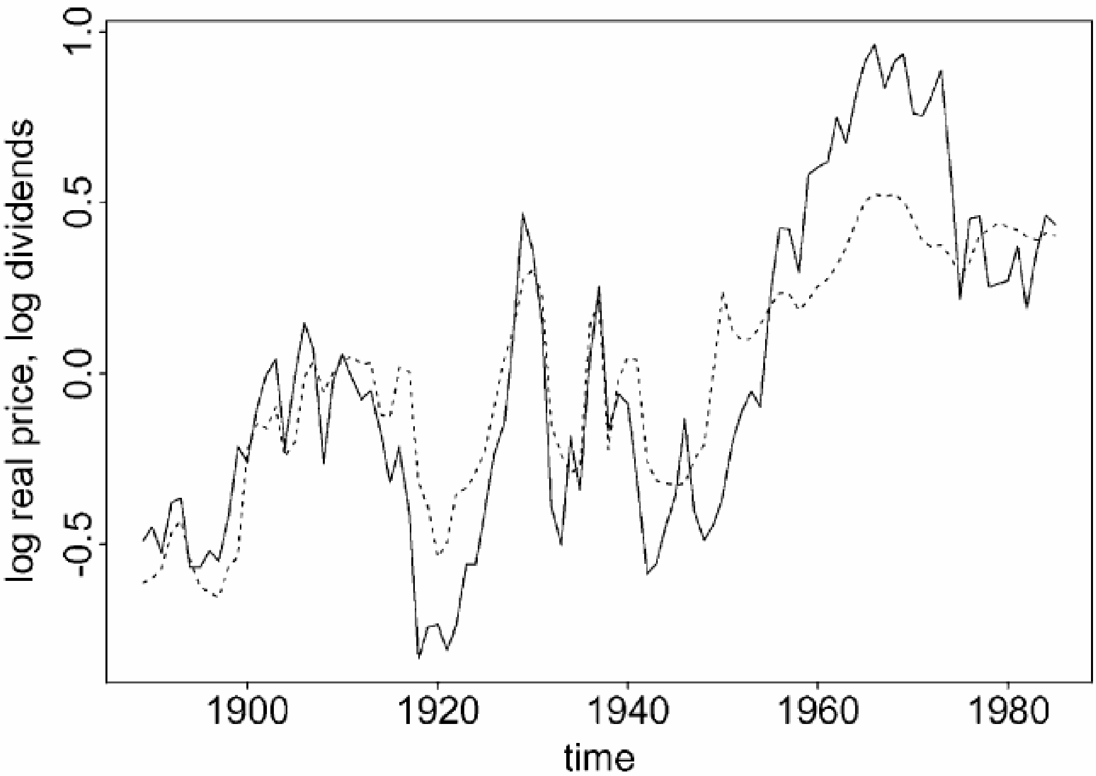

Figure 3.9. Solid curve is the inflation adjusted annual prices and other curve is the dividends for the S&P index of American stock prices

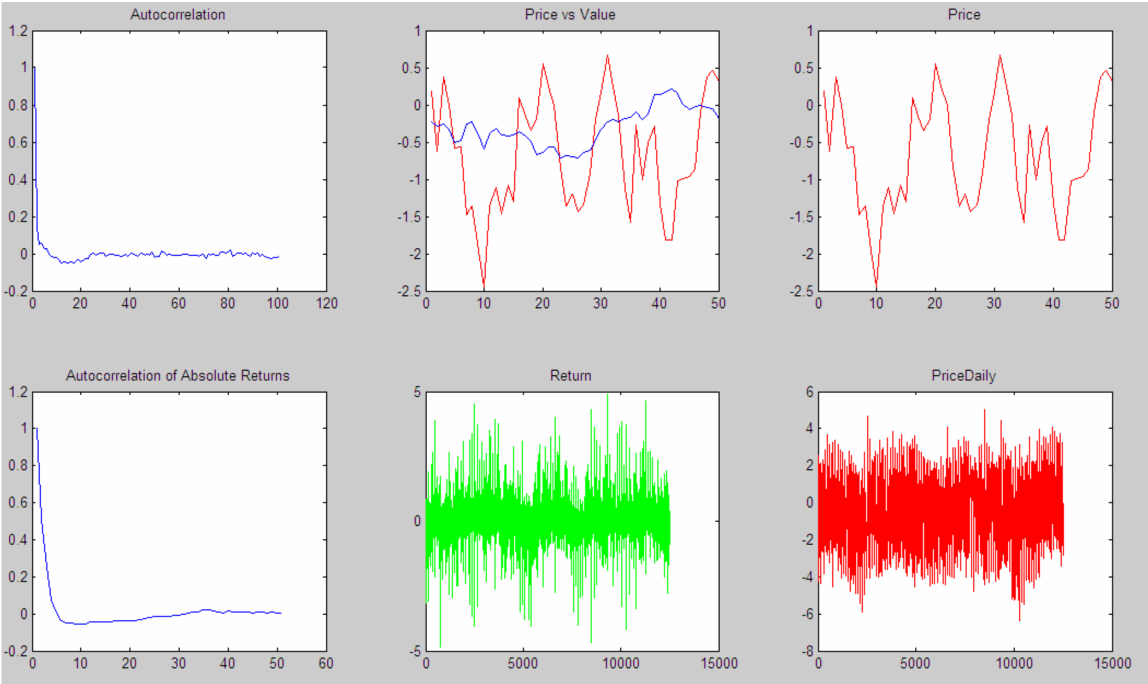

Figure 3.10. Simulation results with both agent types

As we examine above Figure 3.10 with the one for the S&P index, Figure 3.9, we observe qualitative correspondence. In both series price fluctuates around value and mispricing is present for periods of decades. The excess kurtosis is nearly 15 and the volatility of the return is nearly 0.5. So, these statistics are the desired ones which we obtained from this model. On the other hand autocorrelation in returns are not purely zero and volatility clustering is weak. So, further study is needed in order to obtain the desired autocorrelation results.

3.2. Concluding Remarks

In conclusion part of this section, I will summarize the findings I obtained from the simulations of the model constructed in this section.

In this part of the study we propose a single asset market model traded by two different trader types. Main ingredients of the model presented here can be summarized as follows;

- Threshold response of traders. This property is a must if we want to obtain high excess kurtosis values. In this model each agent possesses two thresholds, one entering and the other for exiting a current position.
- There is fundamental value process which represents the dividend process in real markets. Fundamental values are used by value investors while deciding on their position and thus their demands.
- Fundamental values are perceived differently by all agents which is performed by assigning differing offset values to each value investor.
- Heterogeneity is provided by the uniformly distributed threshold values assigned to the agents in addition to the separation of the traders into two types, value investor and trend follower.
- Feedback mechanism is incorporated by supplying all traders with the price dynamics. Prices are perceived same among all agents.

In this section of the study, I examine two different traders separately by investigating the effects of their crucial parameters on the resulting price dynamics. Then, I simulate these two differing traders together in order to imitate the real market and catch the statistical facts common to these real financial markets

Conclusion derived form the results of the simulations performed in the previous parts of this section can be summarized as follows;

- Trend followers induce positive autocorrelations. On the other hand value investors induce negative autocorrelations. So, in a simulation setting, in which these two trader types are simulated together, certain parameters should match in order to achieve zero autocorrelation in returns. In our model a nonzero autocorrelation structure is present in the very first time lags. In order to avoid this problem, this issue should be studied more deeply in a future study.
- While considering the volatility clustering, we could not achieve the desired power low decay scheme of the autocorrelation function of the absolute return. So, this issue also has to be examined more.
- By assigning certain parameters for offset values, we could make the prices to follow value.

- Desired excess kurtosis could be easily obtained by changing the maximum allowed threshold values and threshold intervals.
- Excess volatility is also obtained easily from the simulation of the model presented in this section

I conclude this section with the critical overview of the model presented here. In this model, static nature of the threshold values is one of the most important pitfalls of this model. In real life agents reform their decision strategies or even modify them. So, lack threshold updating is the one contradiction of this model with reality. Another important fact is the static number of agent types. In this model switching between agent strategies is also not allowed.

So, these critical issues should be handled in a future study.

4. CONCLUSIONS

In this study I have examined two different agent based modeling, possessing different sophistications.

In the first model, I modeled a simple single asset market model with a single trader type. Results obtained from investigation of this model are summarized as follows;

- Threshold updating is needed in order to provide the model with ecology of evolving heterogeneous agents which is a common property of real financial markets. Moreover, for higher updating frequencies model exhibits tri-modal return distribution due to the loss of heterogeneity among agents. On the other hand, very low updating frequencies leave agents remaining with their initial uniformly distributed thresholds and due to this fact resulting distribution of returns are platykurtic.
- Threshold behavior of agents in this model leads to investor inertia which results in a distribution with heavy tails. So, threshold response must be embedded in a market model in order to obtain leptokurtic distribution of asset returns.
- Market depth is an important parameter in this model. Its value should be determined carefully for the model in order to be a logical one. As it is mentioned in the analysis part of the market depth, low values of this parameter is desired based on the stylized fact considerations.
- Standard deviation of the public signal should be determined by considering the maximum allowed return which is set by assigning appropriate market depth values. So, too small values and too high values for D with respect to the maximum allowed return is not desirable in order to have a meaningful model.
- Number of agents should be chosen a large number in order to correctly simulate a market. Low values for N is meaningless when considering the real market conditions. Moreover number of agents has no significant effect on the observed facts for high values of it.

In this model, one very important and unrealistic assumption is made while constructing this model. Agents in this model are allowed to take infinite positions. This should not be allowed since in real life, traders could not take infinite positions due to risk considerations and wealth restrictions.

Moreover, sensitivity analysis is performed for each variable separately in order to understand their functions and effects in the model. However in a future study, sensitivity should be performed in order to understand the composed effects of the variables that might be present in the model.

To conclude the critical overview of the first model presented in this study, I am going to add one last remark. Updating scheme in this model could be changed and made more realistic. But this should be done without deteriorating the simplicity of the model. These should be considered in a future study.

In the second model, I constructed a single asset market model with two trader types. Results obtained from investigation of this model are summarized as follows;

- Trend followers induce positive autocorrelations. On the other hand value investors induce negative autocorrelations. So, in a simulation setting, in which these two trader types are simulated together, certain parameters should match in order to achieve zero autocorrelation in returns. In our model a nonzero autocorrelation structure is present in the very first time lags. In order to avoid this problem, this issue should be studied more deeply in a future study.
- While considering the volatility clustering, we could not achieve the desired power low decay scheme of the autocorrelation function of the absolute return. So, this issue also has to be examined more.
- By assigning certain parameters for offset values, we could make the prices to follow value.
- Desired excess kurtosis could be easily obtained by changing the maximum allowed threshold values and threshold intervals.
- Excess volatility is also obtained easily from the simulation of the model presented in this section

Two unrealistic assumptions are made in this second model. Static nature of the threshold is one of the most important unrealistic assumptions of this model. In real life agents reform their decision strategies or even modify them. So, lack of threshold updating is the one contradiction of this model with reality. Another important unrealistic assumption is the static number of agent types. In this model switching between agent strategies is also not allowed.

So, these critical issues should also be considered in a future study as the ones for the first model examined in this study.

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