ANALYSIS OF A TWO STAGE SUPPLY CHAIN WITH INVESTMENT IN PRODUCT AND PROCESS INNOVATION

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ABSTRACT

ANALYSIS OF A TWO STAGE SUPPLY CHAIN WITH INVESTMENT IN PRODUCT AND PROCESS INNOVATION

We analyze a two-stage supply chain consisting one contract manufacturer (CM) and one original equipment manufacturer (OEM). The CM provides a single technology related product to the OEM who faces effort-dependent random demand. CM and OEM will make two different investments for product and process research and development (R&D). In this model, it is assumed that process innovation decreases the average cost of production and product innovation increases the demand stochastically. The centralized solution is characterized in which a single decision maker controls the whole system and an algorithm is proposed to find the centralized optimal solution.The effect of the parameters on decision variables is monitored by a numerical study. Then we analyze the model from a decentralized point of view. Furthermore, we analyze a revenue sharing contract under different settings and focus on different decision makers in the supply chain while searching for coordination. It is shown that the revenue sharing contract fails to coordinate the supply chain actions under different scenarios where the decision of investment given by the OEM or the CM. We also study the same problem under a budget constraint on investments. We propose an algorithm to find the centralized optimal solution for the budget constrained problem. Finally, we analyze the same problem under decentralized settings to find a Nash equilibrium investments and provide a computational study to gain more insight about the problem.

ÖZET

ÜRÜN VE SÜREÇLERDE YENİLİKÇİLİĞE YATIRIM YAPILABİLEN İKİ AŞAMALI BİR TEDARİK ZİNCİRİ PROBLEMİNİN ANALİZİ

Bu çalışmada bir sözleşmeli üretici (CM) ve bir orijinal malzeme üreticisinden oluşan bir tedarik zinciri problemi analiz edilmiştir. Sözleşmeli üretici orijinal malzeme üreticisine teknolojiye dayalı bir ürün tedarik etmektedir. Orijinal malzeme üreticisin teknoloji yatırımına bağımlı belirli bir kümülatif dağılım fonksiyonuna sahip bir stokastik pazar talebiyle karşı karşıyadır. Sözleşmeli üretici ve orijinal malzeme üreticisi ürün ve süreçlerdeki yenilikçilik için yapılan araştırma ve geliştirme (Ar&Ge) çalışmalarına iki farklı yatırım yapmaktadır. Süreç yenilikçiliği için yapılan yatırımın sözleşmeli üretici firmanın üretim maliyetini düşürdüğü ve ürün yenilikçiliği için yapılan yatırımın ise talebi stokastik olarak arttırdığı varsayılmıştır. Çalışmada, önce sistemi tek bir karar vericinin yönettiği düşünülmüş ve merkezi çözüm için bir algoritma önerilmiştir. Paremetrelerin karar değişkenlerine olan etkisi sayısal bir örnek ile incelenmiştir. Sonra kararların ilgili elemanlarca verildiği dağıtık durumlarda yatırım paylaşım kontratı incelenmiştir. Firmaların araştırma ve geliştirme (Ar&Ge) çalışmaları için yapılan yatırımları paylaşmaları durumunda kar paylaşımı kontratlarının tedarik zincirini koordine etmediği gösterilmiştir. Aynı problem bütçenin kısıtlığı olduğu durum için de incelenmiştir ve merkezi en iyi çözüm için bir algoritma önerilmiştir. Son olarak bütçe kısıtlı problem kararların değişik aktörler verildiği dağıtık durumlarda Nash dengesini bulabilmek için de incelenmiştir.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

1. INTRODUCTION

In today's fast changing highly competitive market environment, most of the firms have focused their attention to the effectiveness and efficiency of separate business functions and management of supply chain. As new ways of doing business, however, a growing number of firms have begun to realize the strategic importance of planning, controlling, and designing a supply chain as a whole. Furthermore, in today's competitive environment firms have to innovate to defend their competitive position as well as to seek competitive advantage. Many firms are seeking to reduce the cost of new products by production process innovations and facilitate the smooth launch of new products by product innovations. Firms innovate in order to reduce costs or to increase demand. Process innovations affect the productivities of labor and capital through efficiency, and product innovations affect the demand curve [1].

Recent trends in outsourcing have extended contract manufacturers' (CM) responsibility to new areas, such as sourcing, design, and even research and development. Historically, large U.S. computer manufacturers such as DEC, HP, and IBM were highly vertically integrated, manufacturing everything from the silicon logs from which silicon wafers were sliced and semiconductor chips produced to the final computer assemblies themselves. In recent years, however, the industry has vertically dis-integrated, as companies such as Apple Computer, Sun Microsystems, and Silicon Graphics Inc. have chosen not to vertically integrate from their inception, and the larger companies have shed many of their manufacturing operations, outsourcing them to hundreds of external vendors. The computer industry is now composed of a very extensive and complex network of interdependent firms supplying one another. The supply chain is made up not only of the end product manufacturers and the firms that provide parts, components, process equipment, and other inputs used to manufacture those products, but also companies that produce the packaging materials used to ship product to market, as well as firms that provide waste management services to companies throughout the supply chain. Increasingly, the chain also includes firms engaged in activities relating to computer end-of-life, such as product retrieval, resale, disassembly, shredding, and asset recovery [2].

Manufacturers increasingly rely on innovation from their suppliers to improve the cost, quality, and timeliness of their products. Manufacturing capabilities are enhanced by supplier innovativeness directly, because of the embedded nature of the supplied component, and indirectly, as the manufacturer learns from its suppliers. Reliance on suppliers for design or manufacturing has become commonplace. Outsourcing has helped companies such as IBM, HP and Motorola reduce production costs using lower-priced manufacturing labor and allowing them to freeze their R&D budgets using specialty design shops. In some cases component suppliers are being required to incorporate design tasks within the scope of their partnership. In response, some manufacturing service suppliers are investing in R&D capabilities to attract more OEM customers. The outsourcing of manufacturing and design have been a much discussed topic in both supply chain research [3].

The importance of product and process innovation in a supply chain and the interaction between the innovators and sellers under competition motivate us to search and analyze a two-echelon supply chain with product and process innovation. Furthermore, for this study, we are motivated by an original equipment manufacturer (OEM) that works with a contract manufacturer (CM) for the production of one of its products. The OEM and the CM invest in the technology and expend a certain amount of effort for the production of this technology related product. A larger investment in technology or a higher level of effort improves the quality of the product and a higher quality product results in an increased market potential (demand) for the product [4]. Critical questions are how much the companies should invest for product and process innovation activities, how a CM and a OEM should respond to each other in a competitive business environment when each of them prefers to invest in different kind of innovation and how a budget constraint on R&D investments affects the companies' investment strategies. Briefly, our motivation originally stems from the R&D expenditures incurred by the innovators and sellers of the new technology related products which attract the technology users and result in an increase in demand and a reduction in production cost.

Coordination among the supply chain agents is another important issue in supply chain management. Moreover, coordination is an important criterion that is used to measure the performance of the supply chain. Contracts also provide the system-wide performance improvement. The objective is to bring decentralized expected profit closer to centralized expected profit. This is also referred to as the channel coordination objective. If decentralized expected profit is equal to centralized expected profit then the channel efficiency is said to be equal to one. One complication for supply chain coordination is that companies involved often have different business logic in terms of their revenues and cost generation way, firm's size and specificity.

In this study, a-two echelon supply chain composed of one contract manufacturer (CM) with R&D and one original equipment manufacturer (OEM) are considered. The CM in this supply chain provides a single technology related product to the OEM who faces effort dependent demand. This work considers both product and process innovation effort in characterizing a two stage supply chain. In our model, demand is assumed to be effort dependent and stochastic. Furthermore, we focus on the relation between the process innovation effort and the production cost. We analyze the effect of process innovation effort on a two stage supply chain with effort dependent demand under the assumption that process innovation effort reduces the production cost.

In the first model given in Chapter 3, we characterize the optimal solution of the supply chain with three decision variables (product innovation effort, process innovation effort and order quantity). The model is analyzed both from a centralized point of view which assumes a single agent controls the entire supply chain profit and from a decentralized point of view which assumes the players make choices with the objective of maximizing their own profit. We propose a model to find a centralized solution for a two-echelon supply chain with product and process innovation investment. We also concentrate on the coordination of supply chain for different cases in which the OEM and the CM share the R&D investment or make their own investment decisions. We analyze a revenue sharing contract under different settings. Furthermore, we focus on different decision makers in the supply chain while searching for the coordination. In this work, it is shown that the revenue sharing contract fails to coordinate the supply chain actions under different scenarios where the decision of investment given by the OEM or the CM.

In the second model given in Chapter 4, we analyze the model under a budget constraint. In this model, product and process innovation investments are limited and have to be allocated between product and process innovation investments optimally. The second model is also evaluated both from a centralized point of view and a decentralized point of view. Nash equilibrium characteristics of the R&D investment game are stated and an illustrative example is given for it. In this work, we propose a model to find a centralized solution and optimal budget allocation between product and process innovation investments for a two-echelon supply chain under a budget constraint. Nash game approach is employed to formulate the structure of the model and we analyze interactions between the echelons. In a competitive supply chain neither agent dominates the other and the two echelons are independent firms. In the game's move, the agents simultaneously make their decisions.

The rest of the thesis is organized as follows: The literature survey on supply chain management, supply chain coordination, contracts, product and process innovation are given in Chapter 2. Chapter 3 presents the mathematical model of a two echelon supply chain with product and process innovation without a budget constraint. Centralized and decentralized solutions as well as the coordination are analyzed. In chapter 4, the model is expanded with a budget constraint on R&D investments and effects of the budget constraint on a-two echelon supply chain are analyzed. Finally, Chapter 5 states concluding remarks and possible extensions for future research.

2. LITERATURE REVIEW

Total R&D effort has long been viewed in both popular and academic literatures as a key indicator of the technological progressiveness of firms and industries. In this study, the papers are selected basically regarding two stage supply chain analysis, supply contratcs, product and process innovation investments and models with budget constraint papers.

Innovation is the process of making change, difference and novelty in the products, services and business manner to create economic and social benefit [5]. Furthermore, it is an important driver of economic and productivity growth. Firms innovate in order to reduce costs or to increase demand. Process innovations affect the productivities of labor and capital through effciency, and product innovations affect the demand curve. A successful product innovation implies that the quality of the product increases and demand increases. In addition, new and better products are probably more specialized which protects the firm from competition. Process innovations require firms to apply technology to improve the efficiency of product development and it can be intended to decrease unit costs of production or delivery, to increase quality, or to produce or deliver new or significantly improved products. Both product and process innovation lead to enormous productivity gains and cost savings [1].

Productivity is an important economic factor which has a key role in evaluating the economic growth. Furthermore, productivity growth arises from the development of new work methods based on new technology and production techniques. It is evident that technological progress has significantly contributed to growth in productivity together with a substantial increase in research and development [6].

In the economic literature, it is well recognized that R&D makes a vital contribution to the level of productivity. However, the innovation and growth literature still lack robust empirical evidence on a possible reverse link from productivity to the R&D activities. A priori it can be assumed that such a relationship is very likely to exist [7].

Moreover, in many studies, it is accepted that the innovation output is conceptualized as cost-reducing in the case of new processes or demand-creating for new products [7].

In this context, Utterback and Abernathy [10] analyze the links between productivity, innovation and research at the firm level. In their model, productivity, innovation and research are endogenously determined, research investment and capital are truncated variables, patents are count data and innovative sales are interval data. They found that firm productivity correlates positively with a higher innovation output even when controlling for the skill composition of labor as well as physical capital intensity. The firm innovation output as measured by patent numbers or innovative sales, rises with its research effort and with the demand pull and technology indicators both directly and indirectly through their effects on research [11].

In recent years, the relation between innovation and performance at various levels of aggregation has been the focus of attention in a number of studies. Loof and Heshmati [12, 13] focus on the relationship between firm size, R&D effort, productivity and growth. They use a model recently developed by Crpon, Duguet and Mairesse [14]. It is referred to as the CDM which is a four-equation knowledge production function model, includes three relationships: the productivity equation relating innovation output to productivity, the knowledge production function relating investment in research to innovation output, and the research investment equation linking research to its determinants. An additional equation concerns investment decisions. By using this model, they examine sensitivity of the estimated relationship between innovation and firm performance. The study gives indications of what factors cause variations in the estimated effects of interest and the direction of changes [5, 10].

Cohen and Klepper [17, 18] have introduced a model related R&D effort devoted to product and process innovation. In this model, the effect of an innovation is expressed by a certain increase of the price-cost margin. For each product, process and product R&D are assumed to increase firms price cost margin on the output sold to current and prospective buyers. It is assumed that process innovation lowers the firms average cost of production and product innovation increases the price buyers willing to pay by adding or improving product features. In the case of a process innovation this increase of a firms price-cost margin results from lower production costs. The firm will benefit from the higher price-cost margin as long as its competitors have not imitated the innovation. In the case of a product innovation the price-cost margin increases because buyers are willing to pay more for new product features and the firm yields monopoly rents until these new features are imitated [8]. In our study, we also assume that process R&D effort lowers the contract manufacturer's unit production cost and we use a decrease rate which is similar with the rate given in Zhou's study [15]. Zhou *et al.* study inventory level dependent rate and we adopt this rate into our model.

Mansfield [6] found that for a given size, there is a close relation between the rate of R&D spending and total number of important inventions in the long run. Pawitt *et al.* [11] found a high correlation between R&D intensity (R&D expenditure) and rates of technical innovation measured by expected annual rate of introduction of new products as a percent of sales. They also found across ten Organizations for Economic Cooperation and Development countries high correlation between national industrial R&D expenditures and national performance in technological innovation after correcting for population differences. The importance of cost saving as an objective for process innovation leads to an increase of process R&D expenditure and also results in a rise of total R&D expenditure. In short, there seems direct relation between innovational effort and innovational output exists [6].

Canakoglu and Bilgic [16] model a two-stage supply chain with technology dependent demand under a multiple period setting. The model consists of one operator which faces a stochastic market demand which depends on technology investment level. Operator decides on the capacity levels through the periods and one time R&D expenditure for the new technology. An algorithm is given in order to find the centralized solution. Then, they study the decentralized system and propose two different coordinated contracts such as profit sharing and quantity discount contract. A profit sharing contract where firms share both the revenue and operating costs generated throughout the periods along with initial technology investment is suggested. Also a coordinating quantity discount contract where the discount on the price depends on the total installed capacity is designed. Furthermore, they enhance the results by proving the Nash equilibrium for the case where the operator decides on her network capacity for each period and the vendor decides on his one time R&D investment at the beginning.

Cachon [24] analyze the supply chain coordination with contracts such as wholesale price contract, buyback contract, sales rebate contract, revenue sharing contract, quantity flexibility contract and quantity discount contract. Cachon and Lariviere [19] compare the revenue sharing contract with buy back, quantity flexibility and sales rebate contracts and they indicate that revenue sharing contract is more capable than the others to coordinate a wide range of supply chains. In our study, we analyse the revenue and investment cost sharing contract under different cases while trying to coordinate agents of supply chain. Furthermore, Cachon [24] stated studies the supply chain coordination by changing the demand structure of the newsvendor model such as price dependent demand, effort dependent demand, and demand updating opportunity. In the effort dependent demand structure, firms exert some effort in order to increase their demand such as sale promotion, advertising and so on. The companies that produce technology related products, can allocate some portion of their R&D investments to create extra demand via various marketing strategies for their new technology. Cachon [24] stated in his review that a retailer can increase a products demand by lowering his price, but the retailer can take other actions to spur demand for instance the retailer can hire more sales people, improve their training, increase advertising, better maintain the attractiveness of the products display, enhance the ambiance of the store interior and he can give the product a better stocking location within the store. All of those activities are costly. As a result, a conflict exists between the supplier and the retailer. Under these conditions the supplier always prefers that the retailer exerts even more effort. Sharing the cost of effort is one solution to the effort coordination problem. Coordination with the effort dependent demand model is complex when the firms are not allowed to contract on the retailer's effort level directly. Cachon [24] prooves that buy backs, revenue sharing, quantity flexibility and sales rebate contracts all fail to coordinate the retailer's action because they all distort the retailer's marginal incentive to exert effort. The quantity discount contract does coordinate this system because the retailer incurs the entire cost of effort but also receives the entire benefit of effort.

Petruzzi and Dada [21] analyze a problem where a decision maker facing random demand for a perishable product decides how much of it to stock for a single selling period. They work on a price dependent demand and they model the demand distribution as addition of a random variable and a piece dependent term. In our model, we also assume that demand is addtive and demand function consists of a real valued function which depends on product innovation investment and a random variable.

Cachon and Netessine [20] reviews the game theoretic approach to supply chain management and the common newsvendor model. Furthermore, their study surveys the applications of game theory to supply chain analysis and outlines game-theoretic concepts that have potential for future application techniques for demonstrating existence and uniqueness of Nash equilibrium are discussed and examined.

Since the mid-1960s when Hadley and Whitin [9] presented a single-period multiproduct inventory control model, several extensions have been proposed in the literature to make the model more applicable to real-world situations. They first presented a formulation for the constrained multi-product newsvendor problem and developed a solution method for the problem. Zhang and Shi [22] study the joint acquisition and pricing problem where the retailer sells multiple products with uncertain demands and the suppliers provide all unit quantity discounts. They determine the optimal acquisition quantities and selling prices so as to maximize the retailer's expected profit, subject to a budget constraint. Furthermore they develop a Lagrangian-based solution approach. The bisection algorithm is applied to solve the Lagrangian dual problem to obtain an upper bound. This research provides the retailer an effective way to use both acquisition and pricing decisions as levers to better match demand and supply, and increase the profit under the circumstance. We also analyze our model under a budget constraint. In our model, product and process innovation investment budgets are limited and have to be allocated optimally. In this work, we propose a model to find a centralized solution and optimal budget allocation for a two-echelon supply chain with product and process innovation investment under a budget constraint.

Abdel-Malek *et al.* [23] consider a single-period problem with budget constraint and proposes models for solving the classical multi-product Newsboy problem with budget constraint. The models can consider random demands that may have different probability distribution functions. In case of uniform probability density functions, the formulae developed for the optimum size is exact, while those for the exponential are approximate with known values of error. For other probability distributions, generic iterative method is proposed to find the optimum or near optimum solution for general continuous density functions of the demand has been introduced. Despite its iterative nature, it is easy to apply and fast in reaching good results if not optimum ones.

This work considers both product and process innovation effort in characterizing a two stage supply chain with effort dependent production cost and demand. In earlier papers, they have considered a two stage supply chain with only effort dependent demand. In our model, we also focus on the relation between the process innovation effort and the production cost. The major difference is that, we analyze the effect of process innovation effort on a two stage supply chain with effort dependent demand under the assumption that process innovation effort reduces the production cost. We characterize the optimal solution to the integrated supply chain with three decision variables (product innovation effort, process innovation effort and order quantity). We propose an algorithm to find a centralized solution for a two-echelon supply chain with product and process innovation investment. We also concentrate on the coordination of supply chain for different cases in which the original equipment manufacturer (OEM) and the contract manufacturer (CM) share the R&D investment or make their own investment decisions. We analyze a revenue sharing contract under different settings. Furthermore, we focus on different decision makers in the supply chain while searching for the coordination. In this work, it is shown that revenue sharing contract fails to coordinate the supply chain actions although different scenarios are applied to the model. We also analyze the model under a budget constraint on R&D investments. We propose a model to find optimal budget allocation for a two-echelon supply chain with product and process innovation investments.

3. TWO-STAGE SUPPLY CHAIN MODEL WITHOUT BUDGET CONSTRAINT

3.1. Model Description

In this study, we are motivated by an original equipment manufacturer (OEM) that works with a contract manufacturer (CM) for the production of one of its technology related products. We consider a two-stage supply chain composed of one CM with R&D and one OEM. The CM in this supply chain provides a single technology related product to the OEM who faces effort dependent random demand. CM and OEM make two different kinds of investment for product and process R&D. In this chapter, first we derive the centralized solution of the system. Then the system is decentralized and a revenue sharing contract is discussed under six different cases in order to see whether they can coordinate the chain or not. In the first three cases, the CM and the OEM share both product and process innovation investment whereas in the last three cases, the agents do not share R&D investments. In the first and fourth cases, the OEM makes the decision of product innovation investment level and the CM makes the decision of process innovation investment level. In the second and fifth cases, the OEM decides on both investments. In the third and sixth cases, the CM is the only decision maker of R&D investment levels.

In our model, process innovation lowers the CM's production cost and product innovation stochastically increases the OEM's demand. Furthermore, there is a direct relation between the investment for product R&D and demand. We assume that product and process innovation will always be successful in the long term. Therefore, we analyze the model with assuming that product and process innovation will always materialize.

The following sequence of events occurs between the CM and the OEM. The CM offers the OEM a contract. The OEM accepts or rejects the contract. Assuming the OEM accepts the contract, the decisions on the level of product and process R&D investment level are made and then OEM submits an order quantity to the CM; the CM produces and delivers to the OEM; the demand is realized; finally transfer payments are made between the firms based upon the agreed contract. See Figure 3.1 for the description of the model.

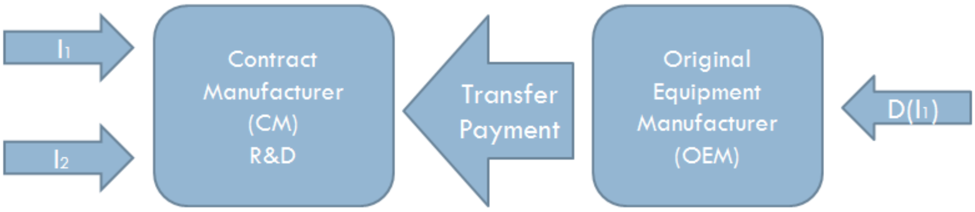


Figure 3.1. Model Description.

All cost parameters and the price of the OEM are stationary. For the centralized system, the objective is to maximize the whole chain's expected profit whereas for the decentralized system, each firm optimizes its own expected profit.Table 3.1 presents the notation used in the model.

Table 3.1. Model Parameters and Decision Variables.

$D(I_1)$	Random demand as a function of product innovation investment, I_1
$c_1(I_2)$	Unit production cost of CM as a function of I_2
c_2	OEM's cost per unit product
g	Penalty cost of unsatisfied demand per unit product
v	Salvage value per unit product
p	The OEM's unit selling price
μ	Mean of demand
Π	Total expected profit of the supply chain.
Q	The order quantity (Decision variable)
I_1	Product Innovation Investment (Decision variable)
I_2	Process Innovation Investment (Decision variable)

3.2. Model Assumptions

We focus on the proven technologies that are known to increase the demand and it is assumed that the innovation is always successful. Furthermore, we assume that demand always increases with product innovation. $D(I_1)$ denotes random demand which depends on product innovation investment. It is assumed that demand is additive and it is expressed as:

$$D(I_1) = \theta(I_1) + \xi$$

where $\theta(I_1)$ is a real valued function and ξ is a random variable such that $D(0) \geq 0$. Under this assumption only mean demand depends on I_1 and the uncertanity is captured by ξ. The form of the demand function specified in this assumption is used extensively in the operations and economic literature. For instance, Petruzzi and Dada [18] suggest this demand form in one of the models to simulate price dependent demand.

$G(x)$ denotes the distribution function and $g(x)$ denotes the density function of demand without the effect of product innovation investment. $F(x|I_1)$ denotes the distribution function and $f(x|I_1)$ denotes the density function of demand. It is assumed that $F(x|I_1)$ is twice differentiable and strictly increasing. Under the additive demand assumption:

$$F(x|I_1) = G(x - \theta(I_1))$$

The additive demand assumption implies that:

$$\frac{dE[D(I_1)]}{dI_1} = \frac{dD(I_1)}{dI_1}$$

Furthermore, it is assumed that expected demand is increasing in product innovation and it is always profitable to spend a non-zero amount on product innovation investment:

$$\frac{dE[D(I_1)]}{dI_1} > 0$$

In our model, this means that product innovation investment never decreases the demand.
Expected demand is diminishingly concave in product innovation investment:

$$\frac{d^2E[D(I_1)]}{dI_1^2} < 0$$

It is assumed that the demand function is strictly positive at every point in the demand set. This implies that $F(x|I_1)$ is strictly increasing and $f(x|I_1) > 0$ for all values of $I_1 \geq 0$.

In this study, we assume that process innovation investment lowers the CM's production cost [15] and the decrease rate is given by:

$$k(I_2) = \frac{dc_1(I_2)}{dI_2} = -\alpha c_1(I_2)^{\beta}$$

Where α is a scale parameter and $\alpha \geq 0$. Furthermore, β is a shape parameter and $0 < \beta < 1$.
From which, $c_1(I_2)$ can be written as:

$$c_1(I_2) = (c_{max}^{1-\beta} - (1-\beta)\alpha I_2)^{\frac{1}{1-\beta}} \tag{3.1}$$

It is assumed that without process innovation investment, production cost is equal to its maximum level which is denoted as c_{max} . The detalied derivation and the steps to obtain $c_1(I_2)$ are given in Appendix A.

The first and the second derivative of $c_1(I_2)$ with respect to process innovation investment, I_2, are given as follows:

$$\frac{\mathrm{d}\, c_1(I_2)}{\mathrm{d}\, I_2} = -\alpha(c_{max}^{1-\beta} - (1-\beta)\alpha I_2)^{\frac{\beta}{1-\beta}} < 0 \tag{3.2}$$

$$\frac{\mathrm{d}^2\, c_1(I_2)}{\mathrm{d}\, I_2^2} = \alpha^2\beta\left[c_{max}^{1-\beta} - (1-\beta)\alpha I_2\right] > 0 \tag{3.3}$$

Every term in the second derivative given in Equation 3.3 is positive. This implies that the CM's production cost given in Equation 3.1 is convex in I_2 . Furthermore, the equations given in Equation 3.2 and Equation 3.3 indicate that CM's production cost is always decreasing in process innovation investment.

Furthermore, we can graphically illustrate the relation between the CM's production cost $c_1(I_2)$ and the process innovation investment I_2. For c_{max}=150, β=0.5 and α=0.01, Figure 3.2 shows that CM's production cost decreases with process innovation investment.

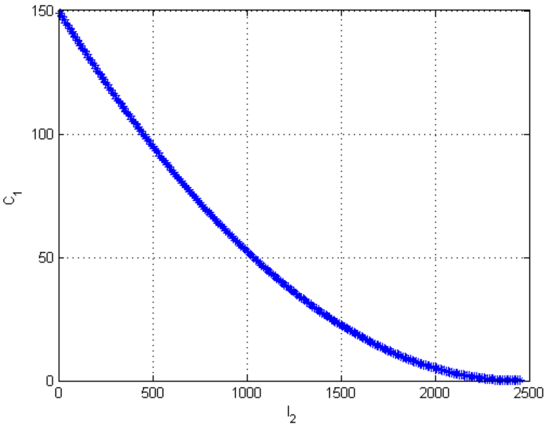


Figure 3.2. Production Cost with Changing I_2.

It is obvious that the production cost given in Equation 3.1 has to be always positive for all I_2 values. Therefore, we can derive an upper bound for process innovation investment. The values for I_2 have to be less than this value. The upper bound is given by:

$$I_2 < \frac{c_{max}^{1-\beta}}{(1-\beta)\alpha} = \bar{I}_2 \tag{3.4}$$

It is assumed that in order for the value chain to earn a positive revenue, the price taken for unit product is larger than the sum of CM's production cost and the OEM's marginal cost:

$$p > c_1 + c_2$$

We assume that the OEM earns $v < c_1 + c_2$ per unit unsold product, where v is net of any salvage expenses.

3.3. Analysis of the Centralized Model

In the centralized setting, the decisions are given by a single decision maker. It is assumed that both CM and OEM belong to the same firm or they are assumed to be an integrated firm. In this system, the central decision maker is going to decide the total product and process innovation investment and order quantity to maximize the expected profit of the chain.

Let $S(Q, I_1)$ be the expected sales:

$$\begin{aligned} S(Q, I_1) &= E[min(Q, D(I_1))] \\ &= \int_0^Q x(F(x|I_1))dx + Q\bar{F}(Q|I_1) \\ &= Q - \int_0^Q F(x|I_1)dx \end{aligned} \tag{3.5}$$

where $\bar{F}(\cdot)$ denotes $1 - \bar{F}(\cdot)$.

Let $In(Q, I_1)$ be the expected left over inventory:

$$In(Q, I_1) = Q - S(Q, I_1) \tag{3.6}$$

Let $L(Q, I_1)$ be the expected lost sales function:

$$L(Q, I_1) = \mu - S(Q, I_1) \tag{3.7}$$

The expected profit for the centralized system is given by:

$$\Pi(Q, I_1, I_2) = pS(Q, I_1) - gL(Q, I_1) + vIn(Q, I_1) - (c_1(I_2) + c_2)Q - I_1 - I_2 \tag{3.8}$$

where $c_1(I_2) = (c_{max}^{1-\beta} - (1 - \beta)\alpha I_2)^{\frac{1}{1-\beta}}$.
The first term in $\Pi(Q, I_1, I_2)$ is revenue from the expected sales, second is the expected cost of lost customers, third is expected revenue from left over inventory, fourth is expected production costs and the last terms are product and process innovation investments.

If we replace Equation 3.5, 3.6 and 3.7 in Equation 3.8, the expected profit for the centralized system can be written as:

$$\Pi(Q, I_1, I_2) = (p - v + g)S(Q, I_1) - (c_1(I_2) + c_2 - v)Q - I_1 - I_2 - g\mu \tag{3.9}$$

where $c_1(I_2)$ is given in Equation 3.1.

The optimal solution of the centralized system is the maximum expected profit that the chain can make. So before decentralizing the chain, centralized solution of the system must be evaluated in order to analyze performance of the decentralized solution.

We first characterize the expected profit function then we determine the centralized decisions about the chain's optimal order quantity and optimal investment levels.

Proposition 3.1. *The expected profit for the centralized system as given in Equation 3.9 is jointly concave in (Q,I_1,I_2), and the solutions obtained from the first order conditions maximize the total expected profit of the value chain.*

Proof. See section B.1 in Appendix B. □

The expected profit given in Equation 3.9 is jointly concave in order quantity, Q, product innovation investment, I_1, and process innovation investment, I_2. The optimal order quatity Q^*, optimal product innovation investment I_1^* and optimal process innovation investment I_2^* satisfy the first order conditions:

$$\frac{\mathrm{d}\,\Pi(Q, I_1, I_2)}{\mathrm{d}\,I_2} = \alpha Q (c_{max}^{1-\beta} - (1-\beta)\alpha I_2)^{\frac{\beta}{1-\beta}} - 1 = 0$$

from which the optimal process innovation investment can be found:

$$I_2^* = \frac{c_{max}^{(1-\beta)} - \left[\frac{1}{\alpha Q}\right]^{\frac{1-\beta}{\beta}}}{(1-\beta)\alpha} \tag{3.10}$$

Note that optimal process innovation investment I_2^* depends only on the order quantity level Q when the parameters are kept constant and it is inversely proportional to Q.

The unit production cost $c_1(I_2)$, which depends on process innovation investment, can be found from Equation 3.1. If we substitute Equation 3.11 into Equation 3.1, we derive $c_1(I_2)$ as:

$$\begin{aligned} c_1(I_2) &= \left[c_{max}^{(1-\beta)} - (1-\beta).\alpha.\frac{\left[\frac{1}{\alpha.Q}\right]^{\frac{1-\beta}{\beta}} - c_{max}^{(1-\beta)}}{(\beta-1).\alpha} \right]^{\frac{1}{1-\beta}} \\ &= \left[\frac{1}{\alpha.Q}\right]^{1/\beta} \end{aligned} \tag{3.11}$$

The production cost at optimal process innovation investment level can be found by using Equation 3.13. Note that the production cost given in Equation 3.13 depends on only order quatity level and it is inversely proportional to Q.

The optimal order quantity Q^* satisfies the first order condition:

$$\frac{\mathrm{d}\,\Pi(Q, I_1, I_2)}{\mathrm{d}\,Q} = (p - v + g)\bar{F}(Q|I_1) - (c_1(I_2) - v + c_2) = 0$$

from which

$$\bar{F}(Q|I_1, I_2) = \frac{c_1(I_2) - v + c_2}{p - v + g}$$

We know that $\bar{F}(Q|I_1) = 1 - F(Q|I_1)$. Then $F(Q|I_1)$ can be written as:

$$F(Q|I_1, I_2) = \frac{p + g - c_1(I_2) - c_2}{p - v + g} \tag{3.12}$$

Q is undefined if the ratio $\frac{p+g-c_1(I_2)-c_2}{p-v+g}$ is negative or greater than one, since $F(.)$ is a cumulative distribution function. Therefore, the following conditions have to be satisfied and these imposed conditions have an action variable (I_2) :

$$v < c_1(I_2) + c_2$$
$$c_1(I_2) + c_2 < p + g$$

When we substitute Equation 3.11 into Equation 3.12:

$$F(Q|I_1, I_2^*) = \frac{p + g - \left[\frac{1}{\alpha Q}\right]^{\frac{1}{\beta}} - c_2}{p - v + g} \tag{3.13}$$

Note that equation Equation 3.13 is enough to find the order quantity for a given product innovation investment.

The optimal product innovation investment I_1 satisfies the first order conditions:

$$\frac{\mathrm{d}\,\Pi(Q, I_1, I_2)}{\mathrm{d}\,I_1} = \frac{\mathrm{d}\,E[D(I_1)]}{\mathrm{d}\,I_1}[(p - v + g)F(Q|I_1)] - 1 = 0 \tag{3.14}$$

If we substitute $F(Q|I_1, I_2^*)$ given in Equation 3.13 into Equation 3.14, we can derive optimal product innovation investment I_1^* from:

$$\left.\frac{\mathrm{d}\,E[D(I_1)])}{\mathrm{d}\,I_1}\right|_{I_1^*} = \frac{1}{p + g - \left[\frac{1}{\alpha Q}\right]^{\frac{1}{\beta}} - c_2} \tag{3.15}$$

Note that the optimal product innovation investment level depends only order quantity level and can be found by using Equation 3.15.

These results enable us to develop an algorithm to solve the centralized problem. Since $\Pi(Q, I_1, I_2)$ is concave in $(Q,\, I_1,\, I_2)$, first order conditions are necessary and sufficient for the solution of the centralized problem. If we know the distribution function of demand and real valued function $\theta(I_1)$, the following algorithm finds the optimal order quatity Q^*, optimal product innovation investment I_1^* and optimal process innovation investment I_2^*:

(i) *Find I_1^* and Q^* simultaneously from Equation 3.13 and Equation 3.15:*

$$F(Q|I_1, I_2^*) = \frac{p + g - \left[\frac{1}{\alpha Q}\right]^{\frac{1}{\beta}} - c_2}{p - v + g}$$

$$\left.\frac{\mathrm{d}\,E[D(I_1)])}{\mathrm{d}\,I_1}\right|_{I_1^*} = \frac{1}{p + g - \left[\frac{1}{\alpha Q}\right]^{\frac{1}{\beta}} - c_2}$$

(ii) *Substitute optimal order quantity Q^*, found in the first step, into Equation 3.10. This gives the optimal product innovation investment I_2^*.*

(iii) *By substituting optimal process innovation investment I_2^*, found in step 2, into Equation 3.1 or substituting optimal order quantity Q^* into Equation 3.11, the CM's production cost at optimal process innovation level can be found.*

3.3.1. Numerical Analysis

We analyze the centralized system in terms of closed form functions in the previous sections. Further insight can be obtained by using numerical examples. Numerical examples are helpful to see the behavior of the parameters over the decision variables and the expected profit. In this section, we illustrate the solution to the centralized system. We use MATLAB R2009a to prepare this numerical study. The demand distribution is determined as $D(I_1) = \theta(I_1) + \xi$ where ξ is distributed uniformly on a finite interval $[L, U]$. Furthermore, expected sales are calculated from:

$$S(Q, I_1) = \begin{cases} Q & \text{if } Q < L + U + \sqrt{I_1} \\ Q - \int_{L+U+\sqrt{I_1}}^{Q} F(x|I_1)dx & \text{if } L + U + \sqrt{I_1} \leq x < 2U + \sqrt{I_1} \\ 2U + \sqrt{I_1} & \text{if } Q \geq 2U + \sqrt{I_1} \end{cases}$$

In this illustration, one parameter is changed gradually while keeping others constant and the integrated chain's expected profit, optimal order quantity, optimal product innovation and optimal process innovation investment are reported.

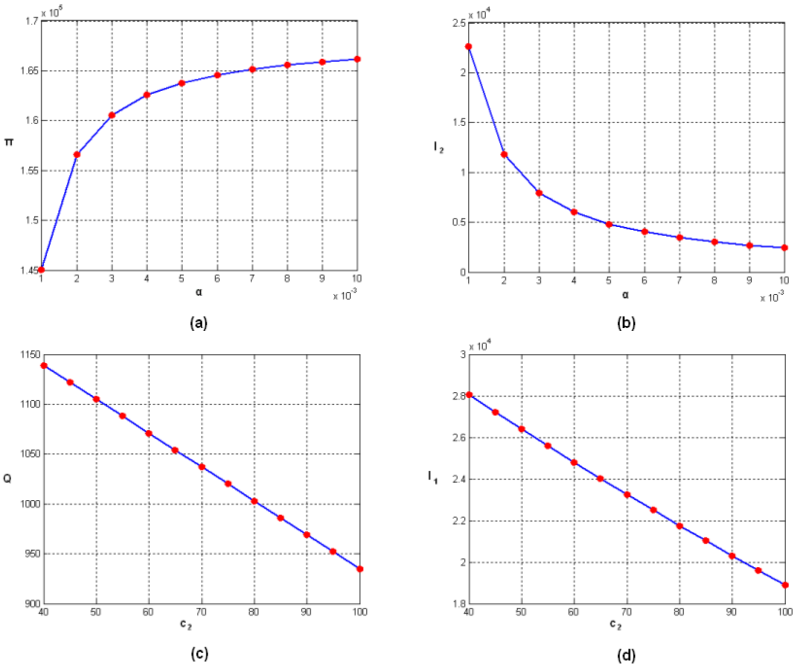
We perform computations 54 times for different values of α, c_2, c_{max} and g to gain intuition about how the model parameters influence the decision variables and the expected profit. We determine the impacts of model parameters on expected profit, order quantity and R&D investment decisions. We determine three levels for α, c_2, c_{max} and two levels for g. In the following example ,the parameters are set as p=300, β=0.5, v=30. The random part of the demand is taken as uniformly distributed between $L \leq \xi \leq U$. In this example, U=$-L$ and U is set as as 500. The impact of product innovation investment I_1 is reflected in demand with $\theta(I_1) = \sqrt{I_1} + U$. By this way, we make sure that $D(I_1) \geq 0$. Table 3.1 presents how the decision variables Q, I_1, I_2 change along with expected profit for different choices of model parameters. During each computation, we always check the condition given in Equation 3.4 which denotes the upper bound of process innovation investment.

First, we analyze the effect of α parameter on Q^*, I_1^*, I_2^* and optimal expected

Table 3.2. The Numerical Illustration of Responses For The Centralized Model (α=0.005, 0.003, 0.001).

Parameters			Q (α)			I_2 (α)			c_1 (α)			I_1 (α)			Expected Sales (α)			π (α)		
c_2	c_{max}	g	0.005	0.003	0.001	0.005	0.003	0.001	0.005	0.003	0.001	0.005	0.003	0.001	0.005	0.003	0.001	0.005	0.003	0.001
80	200	125	1046	1046	1046	5580.40	9215.60	26372.00	0.0366	0.1016	0.9141	29750.00	29739.00	29599.00	664	664	664	174800	171100	153090
80	200	75	1003	1003	1003	5577.10	9206.40	26289.00	0.0398	0.1105	0.9949	21750.00	21740.00	21610.00	637	637	637	142260	138560	120590
80	150	125	1046	1046	1046	4822.50	7952.50	22583.00	0.0366	0.1016	0.9141	29750.00	29739.00	29599.00	664	664	664	175560	172360	156880
80	150	75	1003	1003	1003	4819.20	7943.30	22500.00	0.0398	0.1105	0.9949	21750.00	21740.00	21610.00	637	637	637	143020	139830	124380
80	100	125	1046	1046	1046	3923.50	6454.20	18088.00	0.0366	0.1016	0.9141	29750.00	29739.00	29599.00	664	664	664	176460	173860	161380
80	100	75	1003	1003	1003	3920.20	6445.00	18005.00	0.0398	0.1105	0.9949	21750.00	21740.00	21610.00	637	637	637	143920	141320	128880
60	200	125	1107	1107	1107	5584.60	9227.30	26477.00	0.0327	0.0907	0.8167	33300.00	33290.00	33157.00	680	680	680	196320	192620	174560
60	200	75	1071	1071	1071	5582.10	9220.50	26416.00	0.0349	0.0970	0.8726	24801.00	24791.00	24669.00	654	654	654	162990	159290	141260
60	150	125	1107	1107	1107	4826.70	7964.10	22687.00	0.0327	0.0907	0.8167	33300.00	33290.00	33157.00	680	680	680	197080	193880	178350
60	150	75	1071	1071	1071	4824.30	7957.40	22627.00	0.0349	0.0970	0.8726	24801.00	24791.00	24669.00	654	654	654	163750	160550	145050
60	100	125	1107	1107	1107	3927.70	6465.80	18193.00	0.0327	0.0907	0.8167	33300.00	33290.00	33157.00	680	680	680	197980	195380	182850
60	100	75	1071	1071	1071	3925.30	6459.10	18132.00	0.0349	0.0970	0.8726	24801.00	24791.00	24669.00	654	654	654	164650	162050	149540
40	200	125	1167	1167	1167	5588.30	9237.70	26571.00	0.0294	0.0816	0.7340	37051.00	37041.00	36915.00	692	692	692	219060	215350	197260
40	200	75	1139	1139	1139	5586.60	9232.90	26528.00	0.0309	0.0857	0.7715	28051.00	28042.00	27927.00	667	667	667	185080	181370	163300
40	150	125	1167	1167	1167	4830.40	7974.60	22781.00	0.0294	0.0816	0.7340	37051.00	37041.00	36915.00	692	692	692	219820	216610	201040
40	150	75	1139	1139	1139	4828.70	7969.80	22738.00	0.0309	0.0857	0.7715	28051.00	28042.00	27927.00	667	667	667	185840	182630	167080
40	100	125	1167	1167	1167	3931.50	6476.30	18286.00	0.0294	0.0816	0.7340	37051.00	37041.00	36915.00	692	692	692	220720	218110	205540
40	100	75	1139	1139	1139	3929.70	6471.50	18243.00	0.0309	0.0857	0.7715	28051.00	28042.00	27927.00	667	667	667	186740	184130	171580

profit. In our example, α has three levels, 0.001, 0.003 and 0.005. When α increases and the other parameters are kept constant, both optimal process innovation investment I_2^* and the CM's production cost at optimal investment level $c_1(I_2^*)$ decreases, both expected sales and optimal order quantity Q^* do not change. Furthermore, optimal product innovation investment I_2^* increases. This implies that a reduction in process innovation investment level makes the company invest more in product R&D. In other words, when α increases, CM's production cost decreases with lower process innovation investment and that yields a higher profit for the centralized system. Figure 3.3a illustrates the relation between α and expected profit. Figure 3.3b also illustrates relation between α and optimal process innovation investment. See also Figure 3.4, 3.5 and 3.6 for the responses of decision variables and expected profit when α changes with c_{max}, c_2 and g.



(a) (b) (c) (d)

Figure 3.3. Optimal Results with Changing Parameters (a) α vs π, (b) α vs I_2, (c) c_2 vs Q, (d) c_2 vs I_1.

Second, we analyze the effect of OEM's cost per unit product, c_2, on Q^*, I_1^*, I_2^* and optimal expected profit. In our example, c_2 has three levels, 40, 60 and 80. When c_2 decreases and the other parameters are kept constant, optimal product innovation investment I_1^* and optimal order quantity Q^* increases. This implies that a decrease in OEM's cost per unit product makes the company invest more in product R&D. Decrease in c_2 makes a slight increase in optimal process innovation investment I_2^* which makes a reduction in CM's production cost $c_1(I_2^*)$. Therefore, a decrease in CM's and OEM's cost per unit product yield a higher profit for the integrated supply chain's expected profit. Figure 3.3c illustrates the linear relation between c_2 and optimal order quantity.Figure 3.3d illustrates the relation between c_2 and I_1. See also Figure 3.4 for the responses of decision variables and expected profit when c_2 changes with α.

Third, we analyze the effect of the highest level of CM's production cost, c_{max}, on Q^*, I_1^*, I_2^* and optimal expected profit. In our example, c_{max} has three levels, 100, 150 and 200. For lower values of c_{max}, there is no need to invest more in process innovation. This implies that a decrease in value of c_{max} makes I_2^* decrease. As a result, this decrease in process innovation investment level yields a higher profit for the centralized system. See also Figure 3.5 for the responses of decision variables and expected profit when c_{max} changes with α.

Finally, we analyze the effect of penalty cost per unit product, g, on Q^*, I_1^*, I_2^* and optimal expected profit. In our example, g has two levels, 75 and 125. When penalty cost per unit product decreases, optimal product innovation investment I_1^* decreases and also expected sales decrease. As a result, a decrease in g and expected sales yields a lower profit. See also Figure 3.6 for the responses of decision variables and expected profit when g changes with α.

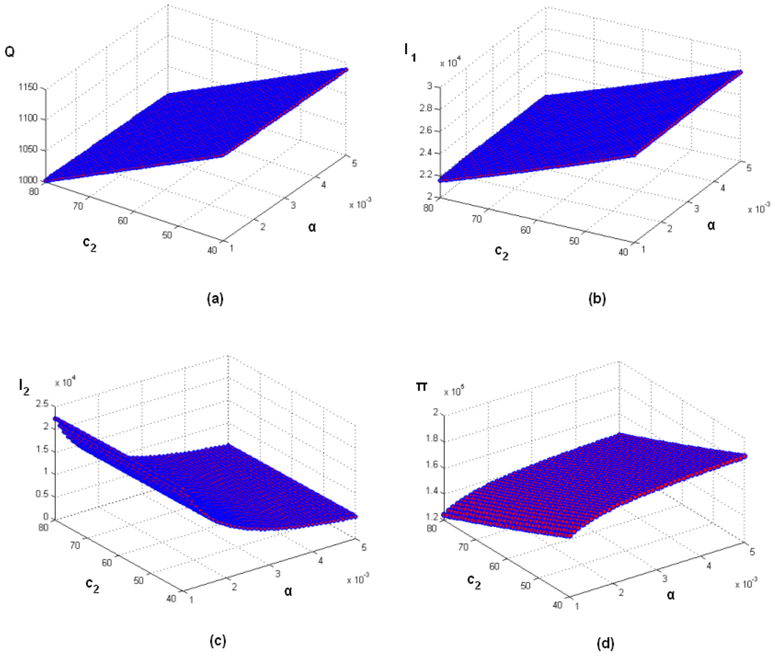


Figure 3.4. Responses of decision variables and expected profit when c_2 and α changes (a)c_2 vs α vs Q, (b)c_2 vs α vs I_1, (c)c_2 vs α vs I_2 (d) c_2 vs α vs π.

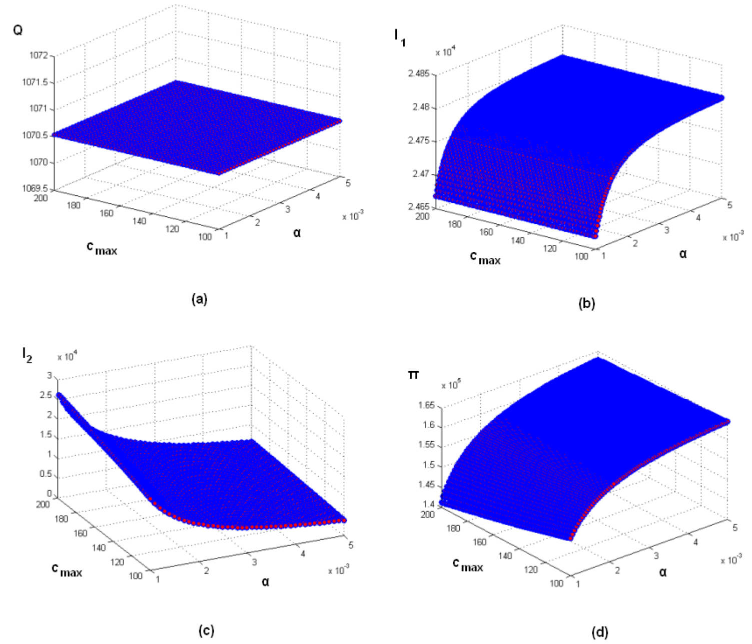


Figure 3.5. Responses of decision variables and expected profit when c_{max} and α changes (a)c_{max} vs α vs Q, (b)c_{max} vs α vs I_1, (c)c_{max} vs α vs I_2 (d)c_{max} vs α vs π.

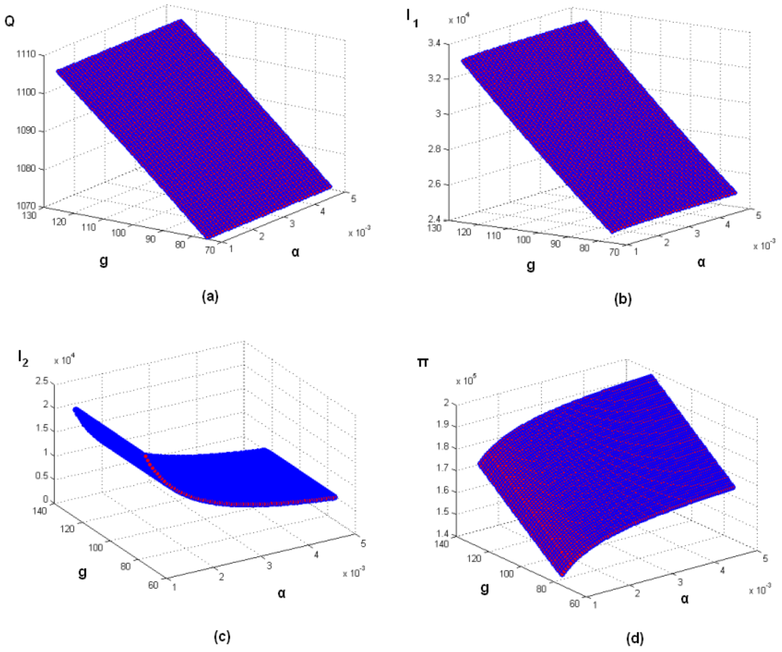


Figure 3.6. Responses of decision variables and expected profit when g and α changes (a)g vs α vs Q, (b)g vs α vs I_1, (c)g vs α vs I_2 (d) g vs α vs π.

3.4. Analysis of the Decentralized Model

In decentralized systems, the CM and the OEM act as independent decision makers and they try to maximize their own profits. There is a transfer payment, T, between two parties paid by the OEM to the CM. We assume that each agent is an independent, risk neutral and self interested agent.

The following sequence of events occurs between the CM and the OEM. The CM offers the OEM a contract. The OEM accepts or rejects the contract. Assuming the OEM accepts the contract, decisions on the level of product and process R&D investment are made by OEM or CM and then OEM submits an order quantity to the CM. The CM produces and delivers to the OEM and the demand is observed. Finally transfer payments are made between the firms based upon the agreed contract.

This section also studies the coordination in a two-echelon supply chain with product and process innovaion investment. The objective is to solve the CM's and

OEM's problem, to decide the optimal order quantity, optimal process and product R&D investment levels when demand and the CM's production cost depend on R&D effort and to analyze whether varriants of a revenue sharing contract coordinates the supply chain or not.

The OEM's objective is to maximize its own expected profit. Let γ be the fraction of product innovation investment and Θ be the fraction of process innovation investment the OEM pays. Furthermore, γ and Θ are exogenously given to the model. Then the OEM's expected profit is given as:

$$\begin{aligned} \Pi_r(Q, I_1, I_2) &= pS(Q, I_1) + vIn(Q, I_1) - gL(Q, I_1) - c_2Q - \gamma I_1 - \Theta I_2 - T_R \\ &= (p + v - g)S(Q, I_1) - (c_2 - v)Q - \gamma I_1 - \Theta I_2 - g\mu - T_R \qquad (3.16) \end{aligned}$$

where T_R is the transfer payment paid by the OEM to the CM and changes according to the contract type.

The CM's objective is to maximize its own expected profit and it can be written as:

$$\Pi_m(Q, I_1, I_2) = T_R - c_1(I_2)Q - (1 - \gamma)I_1 - (1 - \Theta)I_2 \qquad (3.17)$$

where $c_1(I_2) = (c_{max}^{1-\beta} - (1 - \beta)\alpha I_2)^{\frac{1}{1-\beta}}$.

In the following section, we will discuss the coordination of supply chain. We consider revenue sharing contract under different cases and the main issue is to discuss whether they can coordinate the chain or not.

3.5. Coordination under Revenue Sharing

Under decentralized settings, the supply chain members are primarily concerned with optimizing their own objectives. It is commonly seen in the literature that when the supply chain is decentralized and each party tries to maximize its own profit, a considerable decrease in the total supply chain profit may be observed and the supply chain suffers from inefficiencies. Supply chain members use contracts to eliminate these inefficiencies in the supply chain and to achieve the first best solution [4]. The aim of the contracts is to establish transfer payments between the players so that the OEM chooses the order quantity and R&D investment level that maximizes the chain's expected profit. Optimal performance is achievable if the firms are coordinated by contracting on a set of transfer payments such that each firm's objective becomes aligned with the supply chain's objective. In that case, a contract is called coordinating if the decisions made by the different players, which maximize their individual expected profit based on the contract, also maximize the expected total profit of the entire supply chain. This section studies the challenge of coordinating two-step supply chain with product and process innovation effort. It is shown that most of the coordinating contracts with the standard newsvendor model no longer coordinate. In the following parts, we will analyze revenue sharing contract under different settings. Furthermore, we will discuss whether these cases with revenue sharing contract can coordinate the chain or not.

In this section, we analyze the revenue sharing contract. Under this contract type, the CM charges a wholesale price ω for each unit purchased, receives a percentage of the revenue the OEM generates and the agents share their investment costs.

Let ϕ be the fraction of supply chain revenue that the OEM keeps. Under these conditions, the transfer payment can be written as:

$$T(Q,\omega,\phi) = (\omega + (1-\phi)v)Q + (1-\phi)(p-v)S(Q,I_1) \tag{3.18}$$

In the following sections, we will discuss six different cases. See also Table 3.3 for the description of the cases. In the first three cases, the CM and the OEM share both product and process innovation investment whereas in the last three cases, the agents do not share the investments. In the first and fourth cases, the OEM makes the decision of product innovation investment level and the CM makes the decision of process innovation investment level. In the second and fifth cases, the OEM decides on

Table 3.3. Description of the Cases.

Agents Share R&D investments	Agents do not share R&D investments
Case 1. Each Agent Decides on Own Investment	Case 4. Each Agent Decides on Own Investment
• OEM decides on the product innovation investment level	• CM makes only process innovation investment
• CM decides on the process innovation investment level	• OEM makes only product innovation
Case 2. OEM Decides on both Investments	Case 5. OEM Decides on both Investments
• OEM chooses both the product and process innovation investment levels	•OEM makes both product and process innovation investments
Case 3. CM Decides on both Investments	Case 6. CM Decides on both Investments
• CM chooses both product and process innovation investment levels	• CM makes both product and process innovation investment

both investments. In the third and sixth cases, the CM is the only decision maker of R&D investment levels. In this chapter, it is assumed that the order quantity operates with forced compliance. It is supposed that the OEM decides on the order quantity and the CM complies with the decision of OEM. In this chapter, the main question is whether these different cases can coordinate the chain or not.

3.5.1. Case 1. Each Agent Decides on Own Investment: Sharing of R&D investments

In this case, the agents of the supply chain always share both product and process innovation investment cost. Therefore, the fraction of product innovation investment γ and the fraction of process innovation investment Θ are always greater than zero. Suppose that the OEM decides on the product innovation investment level at the same time the order quatity and the CM decides on the process innovation investment level. Then the OEM's expected profit function can be written as:

$$\Pi_r(Q, I_1; I_2, \omega, \phi) = (\phi(p-v)+g)S(Q, I_1) - (\omega + c_2 - \phi v)Q - g\mu - \gamma I_1 - \Theta I_2 \quad (3.19)$$

Proposition 3.2. *The OEM's expected profit given in Equation 3.19 is jointly concave in (Q, I_1) and the solution obtained from the first order conditions maximize the expected profit.*

Proof. The structure of the OEM's expected profit given in Equation 3.19 is similar with the one of the integrated chain's expected profit given in Equation 3.9. Therefore, the proof is also similar.

The concavity of $S(Q, I_1)$ is shown in Appendix B.1. Since $\phi(p-v)+g > 0$, $(\phi(p-v)+g)S(Q, I_1)$ is concave in Q and I_1.

$(-(\omega+c_2-\phi v)Q-\gamma I_1-\Theta I_2)$ is a linear function of Q, I_1 and I_2 and it can be concluded that this function is also jointly concave in Q, I_1 and I_2.

So all parts are concave. Since sum of the functions are concave, the OEM's expected profit is jointly concave in (Q, I_1). See Appendix B.1 for the full proof. □

The expected CM's profit funtion is given by:

$$\begin{aligned}\Pi_m(I_2; Q, I_1, \omega, \phi) &= (\omega + (1-\phi)v)Q + (1-\phi)(p-v)S(Q, I_1) \\ &\quad -c_1(I_2)Q - (1-\gamma)I_1 - (1-\Theta)I_2 \qquad (3.20)\end{aligned}$$

where $c_1(I_2)$ is given in Equation 3.1.

Proposition 3.3. *The expected CM's profit as given in Equation 3.20 is jointly concave in* I_2*, and the solution obtained from the first order conditions maximize the expected profit.*

Proof. Similar to the proof of Proposition 3.1. See Appendix B.1 for the full proof. □

Let I_1^* and I_2^* be optimal product innovation investment and optimal process innovation investment of the integrated channel respectively.

The optimal product innovation investment for a given order quantity I_1^*(Q), maximizes the whole supply chain's revenue when:

$$\frac{\mathrm{d}\,\Pi(Q, I_1^*(Q), I_2^*(Q))}{\mathrm{d}\,I_1} = (p - v + g)\frac{\mathrm{d}\,S(Q, I_1^*(Q))}{\mathrm{d}\,I_1} - 1 = 0 \qquad (3.21)$$

The optimal product innovation investment for a given order quantity, maximizes the OEM's profit when:

$$\frac{\mathrm{d}\,\Pi_r(Q, I_1^*(Q), I_2^*(Q))}{\mathrm{d}\,I_1} = (\phi(p - v) + g)\frac{\mathrm{d}\,S(Q, I_1^*(Q)}{\mathrm{d}\,I_1} - \gamma = 0 \qquad (3.22)$$

When we compare Equation 3.21 with Equation 3.22, we cannot say whether $\frac{d\Pi_r(Q,I_1,I_2)}{dI_1}$ is greater or less than $\frac{d\Pi(Q,I_1,I_2)}{dI_1}$. It depends on ϕ and γ values. If ϕ is equal to 1 and $0 < \gamma < 1$, we can say that the OEM exerts too much product innovation effort. Furthermore, if γ is equal to 1 and $0 < \phi < 1$, the CM and the OEM no longer share the product innovation investment cost and the OEM exerts less product innovation effort.

The optimal process innovation investment for a given order quantity I_2^*(Q), maximizes the whole supply chain's revenue when:

$$\frac{\mathrm{d}\,\Pi(Q, I_1^*(Q), I_2^*(Q))}{\mathrm{d}\,I_2} = -c_1'(I_2^*(Q))Q - 1 = 0 \tag{3.23}$$

where $c_1(I_2)$ is given in Equation 3.1 and $c_1'(.)$ is its first derivative.
The optimal process innovation investment for a given order quantity, maximizes the CM's expected profit when:

$$\frac{\mathrm{d}\,\Pi_r(Q, I_1^*(Q), I_2^*(Q))}{\mathrm{d}\,I_2} = -c_1'(I_2^*(Q))Q - (1 - \Theta) = 0 \tag{3.24}$$

Proposition 3.4. *Under revenue sharing contract settings, when the OEM and the CM share both product and process innovation investments, for* $0 < \Theta < 1$*, it is always true that:*

$$\frac{d\Pi_m(Q, I_1, I_2)}{dI_2} > \frac{d\Pi(Q, I_1, I_2)}{dI_2} \tag{3.25}$$

Proof. When we compare Equation 3.23 and Equation 3.24, for $0 < \Theta < 1$, it is obvious that:

$$-c_1'(I_2^*(Q))Q - (1 - \Theta) > -c_1'(I_2^*(Q))Q - 1 \tag{3.26}$$

Equation 3.26 proves the Proposition 3.4 and shows that the CM's optimal process innovation investment is greater than the centralized system's optimal process innovation investment. Note that if Θ is equal to zero, the CM's optimal process innovation investment becomes equal to the optimal process innovation investment of the centralized system. This shows that the OEM does not prefer to share process innovation investment. □

In short, Proposition 3.4 shows that the CM exerts too much process innovation effort. So it follows that revenue sharing contract under the settings of case 1 when OEM and CM share both product and process innovation investment, cannot coordinate the the supply chain.

3.5.2. Case 2. OEM Decides on both Investments: Sharing of R&D investments

In this case, the agents of the supply chain always share both product and process innovation investment cost. Therefore, the fraction of product innovation investment γ and the fraction of process innovation investment Θ are always greater than zero. Suppose that the OEM chooses both the product and process innovation investment levels at the same time the order quatity.

The OEM's expected profit function is:

$$\Pi_r(Q, I_1, I_2, \omega, \phi) = (\phi(p-v)+g)S(Q, I_1) - (\omega + c_2 - \phi v)Q - g\mu - \gamma I_1 - \Theta I_2 \quad (3.27)$$

Proposition 3.5. *The OEM's expected profit given in Equation 3.27 is jointly concave in (Q,I_1,I_2), and the solution obtained from the first order conditions maximize the expected profit.*

Proof. Similar to the proof of Proposition 3.1. See Appendix B.1 for the full proof. □

Let I_1^* and I_2^* be the optimal product innovation investment and optimal process innovation investment of the integrated supply chain respectively.

The optimal product innovation investment for a given order quantity I_1^*(Q), maximizes the whole supply chain's revenue when:

$$\frac{\mathrm{d}\,\Pi(Q, I_1^*(Q), I_2^*(Q))}{\mathrm{d}\,I_1} = (p - v + g)\frac{\mathrm{d}\,S(Q, I_1^*(Q))}{\mathrm{d}\,I_1} - 1 = 0 \quad (3.28)$$

The optimal product innovation investment for a given order quantity, maximizes the OEM's profit when:

$$\frac{\mathrm{d}\,\Pi_r(Q, I_1^*(Q), I_2^*(Q))}{\mathrm{d}\,I_1} = (\phi(p-v)+g)\frac{\mathrm{d}\,S(Q, I_1^*(Q)}{\mathrm{d}\,I_1} - \gamma = 0 \tag{3.29}$$

When we compare Equation 3.28 and Equation 3.29, we cannot say whether $\frac{d\Pi_r(Q,I_1,I_2)}{dI_1}$ is greater or less than $\frac{d\Pi(Q,I_1,I_2)}{dI_1}$. This result is also found in Case 1. The equality depends on ϕ and γ values. If ϕ is equal to 1 and $0 < \gamma < 1$, we can say that the OEM exerts too much product innovation effort. Furthermore, if γ is equal to 1 and $0 < \phi < 1$, the CM and the OEM no longer share the product innovation investment cost and the OEM exerts less product innovation effort .

The optimal process innovation investment for a given order quantity I_2^*(Q), maximizes the whole supply chain's revenue when:

$$\frac{\mathrm{d}\,\Pi(Q, I_1^*(Q), I_2^*(Q))}{\mathrm{d}\,I_2} = -c_1'(I_2^*(Q))Q - 1 = 0 \tag{3.30}$$

where $c_1(I_2)$ is given in Equation 3.1.
The optimal process innovation investment for a given order quantity, maximizes the OEM's profit when:

$$\frac{\mathrm{d}\,\Pi_r(Q, I_1^*(Q), I_2^*(Q))}{\mathrm{d}\,I_2} = -\Theta = 0 \tag{3.31}$$

When we compare Equation 3.30 and Equation 3.31, we cannot say whether $\frac{d\Pi_r(Q,I_1,I_2)}{dI_2}$ is greater or less than $\frac{d\Pi(Q,I_1,I_2)}{dI_2}$. It depends on the values of $c_1'(I_2)$ and Θ. Therefore, under revenue sharing contract settings, when the OEM chooses both product and process innovation effort level, we cannot make any conclusion about the coordination of the supply chain.

3.5.3. Case 3. CM Decides on both Investments: Sharing of R&D investments

In this case, the agents of the supply chain always share both product and process innovation investment cost. Therefore, the fraction of product innovation investment γ and the fraction of process innovation investment Θ are always greater than zero. Suppose that the CM chooses both product and process innovation investment levels and the OEM chooses the order quatity.

The expected OEM's total profit function can be written as:

$$\Pi_r(Q; I_1, I_2, \omega, \phi) = (\phi(p-v)+g)S(Q, I_1) - (\omega + c_2 - \phi v)Q - g\mu - \gamma I_1 - \Theta I_2 \quad (3.32)$$

Proposition 3.6. *The OEM's expected profit as given in Equation 3.32 is jointly concave in Q, and the solution obtained from the first order condition maximizes the expected profit.*

Proof. Similar to the proof of Proposition 3.1. See Appendix B.1 for the full proof. □

The CM's expected profit funtion is given by:

$$\begin{aligned} \Pi_m(I_1, I_2; Q, \omega, \phi) &= (\omega + (1-\phi)v)Q + (1-\phi)(p-v)S(Q, I_1) \\ &\quad - c_1(I_2)Q - (1-\gamma)I_1 - (1-\Theta)I_2 \end{aligned} \quad (3.33)$$

where $c_1(I_2)$ is given in Equation 3.1.

Proposition 3.7. *The CM's expected profit as given in Equation 3.33 is jointly concave in (I_1,I_2), and the solution obtained from the first order conditions maximize the expected profit.*

Proof. Similar to the proof of Proposition 3.1. See Appendix B.1 for the full proof. □

Let I_1^* and I_2^* be optimal product innovation investment and optimal process innovation investment respectively.

The optimal product innovation investment for a given order quantity I_1^*(Q), maximizes the whole supply chain's revenue when:

$$\frac{\mathrm{d}\,\Pi(Q, I_1^*(Q), I_2^*(Q))}{\mathrm{d}\,I_1} = (p - v + g)\frac{\mathrm{d}\,S(Q, I_1^*(Q))}{\mathrm{d}\,I_1} - 1 = 0 \tag{3.34}$$

The optimal product innovation investment for a given order quantity, maximizes the CM's profit when:

$$\frac{\mathrm{d}\,\Pi_m(I_1^*(Q), I_2^*(Q); Q)}{\mathrm{d}\,I_1} = ((1 - \phi)(p - v)\frac{\mathrm{d}\,S(Q, I_1^*(Q)}{\mathrm{d}\,I_1} - (1 - \gamma) = 0 \tag{3.35}$$

When we compare Equation 3.34 and Equation 3.35, we cannot say whether $\frac{d\Pi_m(I_1,I_2;Q)}{dI_1}$ is greater or less than $\frac{d\Pi(Q,I_1,I_2)}{dI_1}$. It depends on ϕ and γ values. If γ is equal to zero or both ϕ and γ are equal to 0, we can say that the CM exerts less product innovation effort.

The optimal process innovation investment for a given order quantity I_2^*(Q), maximizes the whole supply chain's revenue when:

$$\frac{\mathrm{d}\,\Pi(Q, I_1^*(Q), I_2^*(Q))}{\mathrm{d}\,I_2} = -c_1'(I_2^*(Q))Q - 1 = 0 \tag{3.36}$$

where $c_1(I_2)$ is given in Equation 3.1.
The optimal process innovation investment for a given order quantity, maximizes the CM's profit when:

$$\frac{\mathrm{d}\,\Pi_m(I_1^*(Q), I_2^*(Q); Q)}{\mathrm{d}\,I_2} = -c_1'(I_2^*(Q))Q - (1 - \Theta) = 0 \tag{3.37}$$

Proposition 3.8. *Under revenue sharing contract settings, when the CM is the only* *decision maker of product and process innovation investment levels, for* $0 < \Theta < 1$*, it is always true that:*

$$\frac{d\Pi_m(Q, I_1, I_2)}{dI_2} > \frac{d\Pi(Q, I_1, I_2)}{dI_2} \tag{3.38}$$

Proof. When we compare the equations Equation 3.36 and Equation 3.37, for $0 < \Theta < 1$, it is obvious that :

$$-c_1'(I_2^*(Q))Q - (1 - \Theta) > -c_1'(I_2^*(Q))Q - 1 \tag{3.39}$$

Equation 3.39 proves the Proposition 3.8 and shows that the CM's optimal process innovation investment is greater than the one found from the centralized solution. □

Proposition 3.8 shows that the CM exerts too much process innovation effort. So it follows that revenue sharing contract when the OEM and the CM share both product and process innovation investment and when the CM is the only decision maker of product and process innovation investment level, cannot coordinate the the supply chain.

3.5.4. Case 4. Each Agent Decides on Own Investments: No Sharing of R&D investments

This section studies the challenge of coordinating the chain when the agents do not share the total R&D investment. In this case, the agents of the supply chain make their own investment decisions. Thus, the CM makes only process innovation investment and the OEM makes only product innovation investment. Suppose that the OEM chooses the order quantity at the same time. Therefore, the fraction of product innovation investment (γ) the OEM pays is equal to 1 and the fraction of process innovation investment the OEM pays (Θ) is equal to zero.

The OEM's expected profit function is:

$$\Pi_r(Q, I_1, \omega, \phi) = (\phi(p - v) + g)S(Q, I_1) - (\omega + c_2 - \phi v)Q - g\mu - I_1 \tag{3.40}$$

Proposition 3.9. *The expected OEM's profit given in Equation 3.40 is jointly concave in (Q, I_1) and the solution obtained from the first order conditions maximize the expected profit.*

Proof. Similar to the proof of Proposition 3.1. See Appendix B.1 for the full proof. □

The CM's expected profit funtion is given by:

$$\begin{aligned} \Pi_m(I_2; Q, I_1, \omega, \phi) &= (\omega + (1 - \phi)v)Q + (1 - \phi)(p - v)S(Q, I_1) \\ &\quad -c_1(I_2)Q - I_2 \end{aligned} \tag{3.41}$$

where $c_1(I_2)$ is given in Equation 3.1.

Proposition 3.10. *The CM's expected profit as given in Equation 3.41 is jointly concave in (I_2), and the solution obtained from the first order condition maximizes the expected profit.*

Proof. Similar to the proof of Proposition 3.1. See Appendix B.1 for the full proof. □

Let I_1^* and I_2^* be optimal product innovation investment and optimal process innovation investment respectively.

The optimal product innovation investment for a given order quantity I_1^*(Q), maximizes the chain's revenue when:

$$\frac{\mathrm{d}\,\Pi(Q, I_1^*(Q), I_2^*(Q))}{\mathrm{d}\,I_1} = (p - v + g)\frac{\mathrm{d}\,S(Q, I_1^*(Q))}{\mathrm{d}\,I_1} - 1 = 0 \tag{3.42}$$

The OEM's optimal product innovation investment for a given order quantity can be found from:

$$\frac{\mathrm{d}\,\Pi_r(Q, I_1^*(Q), I_2^*(Q))}{\mathrm{d}\,I_1} = (\phi(p-v)+g)\frac{\mathrm{d}\,S(Q, I_1^*(Q)}{\mathrm{d}\,I_1} - 1 = 0 \tag{3.43}$$

Proposition 3.11. *Under revenue and investment cost sharing contract settings, when the OEM and the CM make their own investment decisions, when* $0 < \phi < 1$, *it is always true that:*

$$\frac{d\Pi_r(Q, I_1, I_2)}{dI_1} < \frac{d\Pi(Q, I_1, I_2)}{dI_1} \tag{3.44}$$

Proof. When we compare Equation 3.42 and Equation 3.43, it is obvious that $(\phi(p-v)+g) <$ (p-v+g) when $0 < \phi < 1$. So $\frac{d\Pi_r}{dI_1} < \frac{d\Pi}{dI_1}$ and it indicates that the OEM exerts less product innovation effort. ☐

According to Proposition 3.11, I_1^* cannot be OEM's optimum product innovation investment and this shows that the OEM choses a lower effort than the optimal one. Coordinating contract can be defined as the one that makes the OEM choose the same R&D investment level found from the centralized solutions. So it follows that revenue sharing contract when the OEM and the CM make their own investment decisions, cannot coordinate the whole chain.

Let compare the CM's optimal process innovation effort with the one obtained from the centralized solution.

The optimal process innovation investment for a given order quantity I_2^*(Q), maximizes the whole supply chain's revenue when:

$$\frac{\mathrm{d}\,\Pi(Q, I_1^*(Q), I_2^*(Q))}{\mathrm{d}\,I_2} = -c_1'(I_2^*(Q)) - 1 = 0 \tag{3.45}$$

The CM's optimal process innovation investment for a given order quantity can be found from:

$$\frac{\mathrm{d}\,\Pi_m(Q, I_1^*(Q), I_2^*(Q))}{\mathrm{d}\,I_2} = -c_1'(I_2^*(Q)) - 1 = 0 \tag{3.46}$$

When we compare Equation 3.45 with Equation 3.46, it is obvious that:

$$\frac{d\Pi_m(Q, I_1, I_2)}{dI_2} = \frac{d\Pi(Q, I_1, I_2)}{dI_2} \tag{3.47}$$

This indicates that I_2^* is the CM's optimal process innovation investment and the CM chooses the same effort level with the one obtained from the centralized solution. However, this does not mean that this contract type can coordinate the chain. Since Proposition 3.11 shows that the OEM exerts less product innovation effort than the integrated chain's optimal product innovation effort. Under revenue sharing contract where the total revenue is shared whereas the investments are not shared between the supply chain agents, the chain is not coordinated.

3.5.5. Case 5. OEM Decides on both Investments: No Sharing of R&D investments

In this case, the agents of the supply chain do not share the R&D investment. The OEM makes both product and process innovation investment. Therefore, the fraction of product innovation investment γ and the fraction of process innovation investment Θ is equal to 1. Suppose that the OEM chooses both the product and process innovation effort level at the same time the order quatity.

Then the OEM's expected profit function is:

$$\Pi_r(Q, I_1, I_2, \omega, \phi) = (\phi(p - v) + g)S(Q, I_1) - (\omega + c_2 - \phi v)Q - g\mu - I_1 - I_2 \tag{3.48}$$

Proposition 3.12. *The OEM's expected profit as given in Equation 3.48 is jointly concave in (Q,I_1,I_2), and the solution obtained from the first order conditions maximize* *the expected profit.*

Proof. Similar to the proof of Proposition 3.1. See Appendix B.1 for the full proof. □

Let I_1^* and I_2^* be optimal product innovation investment and optimal process innovation investment respectively.

The optimal product innovation investment for a given order quantity I_1^*(Q), maximizes the whole supply chain's revenue when:

$$\frac{\mathrm{d}\,\Pi(Q, I_1^*(Q), I_2^*(Q))}{\mathrm{d}\,I_1} = (p - v + g)\frac{\mathrm{d}\,S(Q, I_1^*(Q))}{\mathrm{d}\,I_1} - 1 = 0 \tag{3.49}$$

The optimal product innovation investment for a given order quantity, maximizes the OEM's expected profit when:

$$\frac{\mathrm{d}\,\Pi_r(Q, I_1^*(Q), I_2^*(Q))}{\mathrm{d}\,I_1} = (\phi(p - v) + g)\frac{\mathrm{d}\,S(Q, I_1^*(Q)}{\mathrm{d}\,I_1} - 1 = 0 \tag{3.50}$$

Proposition 3.13. *Under revenue sharing contract settings, when the agents do not share the R&D investment and the OEM decides on both investments, it is always true that if* $0 < \phi < 1$*:*

$$\frac{d\Pi_r(Q, I_1, I_2)}{dI_1} < \frac{d\Pi(Q, I_1, I_2)}{dI_1} \tag{3.51}$$

Proof. When we compare Equation 3.49 and Equation 3.50, it is obvious that $(\phi(p - v) + g) <$ (p-v+g) when $0 < \phi < 1$. □

Proposition 3.13 indicates that the OEM exerts less product innovation effort. So it follows that revenue sharing contract when the OEM decides on both investments, cannot coordinate the whole chain.

The optimal process innovation investment for a given order quantity I_2^*(Q), maximizes the whole supply chain's revenue when:

$$\frac{\mathrm{d}\,\Pi(Q, I_1^*(Q), I_2^*(Q))}{\mathrm{d}\,I_2} = -c_1'(I_2^*(Q))Q - 1 = 0 \tag{3.52}$$

where $c_1(I_2)$ is given in Equation 3.1.
The optimal process innovation investment for a given order quantity, maximizes the OEM's profit when:

$$\frac{\mathrm{d}\,\Pi_r(Q, I_1^*(Q), I_2^*(Q))}{\mathrm{d}\,I_2} = -\Theta = 0 \tag{3.53}$$

When we compare Equation 3.52 and Equation 3.53, we cannot say whether $\frac{d\Pi_r(Q,I_1,I_2)}{dI_2}$ is greater or less than $\frac{d\Pi(Q,I_1,I_2)}{dI_2}$. It depends on the values of $c_1'(I_2)$ and Θ. Therefore, under revenue sharing contract settings, when the agents do not share the investments and the OEM decides on both investment levels, the contract does not coordinate the supply chain.

3.5.6. Case 6. CM Decides on both Investments: No Sharing of R&D investments

In this case, the CM makes both product and process innovation investment. Therefore, the fraction of product innovation investment the OEM pays, γ and the fraction of process innovation investment the OEM pays, Θ, is equal to 0. Suppose that the CM decides on both product and process innovation effort level and the OEM chooses the order quantity.

The OEM's expected profit function can be written as:

$$\Pi_r(Q, I_1, \omega, \phi) = (\phi(p - v) + g)S(Q, I_1) - (\omega + c_2 - \phi v)Q - g\mu \tag{3.54}$$

Proposition 3.14. *The expected OEM's profit as given in Equation 3.54 is jointly* *concave in Q and the solution obtained from the first order conditions maximize the expected profit.*

Proof. Similar to the proof of Proposition 3.1. See Appendix B.1 for the full proof. □

The CM's expected profit funtion is given by:

$$\begin{aligned}\Pi_m(I_2; Q, I_1, \omega, \phi) &= (\omega + (1-\phi)v)Q + (1-\phi)(p-v)S(Q, I_1) \\ &\quad -c_1(I_2)Q - I_1 - I_2 \end{aligned} \tag{3.55}$$

where $c_1(I_2)$ is given in Equation 3.1.

Proposition 3.15. *The expected CM's profit as given in Equation 3.55 is jointly concave in (I_2, I_2), and the solution obtained from the first order condition maximizes the expected profit.*

Proof. Similar to the proof of Proposition 3.1. See Appendix B.1 for the full proof. □

Let I_1^* and I_2^* be optimal product innovation investment and optimal process innovation investment respectively.

The optimal product innovation investment for a given order quantity I_1^*(Q), maximizes the chain's revenue when:

$$\frac{\mathrm{d}\,\Pi(Q, I_1^*(Q), I_2^*(Q))}{\mathrm{d}\,I_1} = (p - v + g)\frac{\mathrm{d}\,S(Q, I_1^*(Q))}{\mathrm{d}\,I_1} - 1 = 0 \tag{3.56}$$

The optimal product innovation investment for a given order quantity, maximizes the CM's profit when:

$$\frac{\mathrm{d}\,\Pi_m(I_1^*(Q), I_2^*(Q); Q)}{\mathrm{d}\,I_1} = ((1-\phi)(p - v)\frac{\mathrm{d}\,S(Q, I_1^*(Q)}{\mathrm{d}\,I_1} - 1 = 0 \tag{3.57}$$

Proposition 3.16. *Under revenue sharing contract settings, when the agents do not share the R&D investment and the CM decides on both investments, it is always true that if* $0 < \phi < 1$*:*

$$\frac{d\Pi_m(Q, I_1, I_2)}{dI_1} < \frac{d\Pi(Q, I_1, I_2)}{dI_1} \tag{3.58}$$

Proof. When we compare Equation 3.56 and Equation 3.57, it is obvious that $((1 - \phi)(p - v)) <$ (p-v+g) when $0 < \phi < 1$. ☐

Proposition 3.16 shows that the CM exerts less product innovation effort and , I_1^* cannot be CM's optimal product innovation investment So it follows that revenue sharing contract when the CM decides on both investments, cannot coordinate the supply chain.

Let compare the CM's optimal process innovation effort with the one obtained from the centralized solution.

The optimal process innovation investment for a given order quantity I_2^*(Q), maximizes the whole supply chain's revenue when:

$$\frac{\mathrm{d}\,\Pi(Q, I_1^*(Q), I_2^*(Q))}{\mathrm{d}\,I_2} = -c_1'(I_2^*(Q)) - 1 = 0 \tag{3.59}$$

The CM's optimal process innovation investment for a given order quantity can be found from:

$$\frac{\mathrm{d}\,\Pi_m(Q, I_1^*(Q), I_2^*(Q))}{\mathrm{d}\,I_2} = -c_1'(I_2^*(Q)) - 1 = 0 \tag{3.60}$$

When we compare Equation 3.59 and Equation 3.60, it is obvious that:

$$\frac{d\Pi_m(Q, I_1, I_2)}{dI_2} = \frac{d\Pi(Q, I_1, I_2)}{dI_2} \tag{3.61}$$

This shows that I_2^* is the CM's optimal process innovation investment and the CM chooses the same process innovation effort level with the one obtained from the centralized solution. However, this does not mean that this contract type can coordinate the chain. Since Proposition 3.16 indicates that the CM exerts less product innovation effort than the integrated chain's optimal product innovation effort. Therefore, it follows that under revenue sharing contract where the total revenue is shared whereas the investments are not shared between the supply chain agents and when the CM decides on both investments, cannot coordinate the supply chain.

To summary, we concentrate on the coordination of supply chain for different cases in which the original equipment manufacturer (OEM) and the contract manufacturer (CM) share the R&D investments or make their own investment decisions. We analyze revenue sharing contract under different settings within the different cases. Furthermore, we focus on different decision makers in the supply chain while searching for the coordination. In this work, it is shown that revenue sharing contract fails to coordinate the supply chain actions although different scenarios are applied to the model. See Table 3.4 for the summary of the case results.

Table 3.4. Summary of the Cases.

Agents Share R&D investments	Agents do not share R&D investments
Case 1. Each Agent Decides on Own Investment • The manufacturer exerts too much process innovation effort. (not coordinate) $\frac{d\Pi_m(Q,I_1,I_2)}{dI_2} > \frac{d\Pi(Q,I_1,I_2)}{dI_2}$	Case 4. Each Agent Decides on Own Investment • The retailer exerts less product innovation effort. (not coordinate) $\frac{d\Pi_r(Q,I_1,I_2)}{dI_1} < \frac{d\Pi(Q,I_1,I_2)}{dI_1}$
Case 2. OEM Decides on both Investments • If Φ is equal to 1 and $0 < \gamma < 1$, the retailer exerts too much product innovation effort. (not coordinate) • Otherwise, no conclusion.	Case 5. OEM Decides on both Investments • The retailer exerts less product innovation effort. (not coordinate) $\frac{d\Pi_r(Q,I_1,I_2)}{dI_1} < \frac{d\Pi(Q,I_1,I_2)}{dI_1}$
Case 3. CM Decides on both Investments • The manufacturer exerts too much process innovation effort. (not coordinate) $\frac{d\Pi_m(Q,I_1,I_2)}{dI_2} > \frac{d\Pi(Q,I_1,I_2)}{dI_2}$	Case 6. CM Decides on both Investments • The manufacturer exerts less product innovation effort. (not coordinate) $\frac{d\Pi_r(Q,I_1,I_2)}{dI_1} < \frac{d\Pi(Q,I_1,I_2)}{dI_1}$

4. TWO-STAGE SUPPLY CHAIN MODEL WITH BUDGET CONSTRAINT

4.1. Model Description and Assumptions

In this chapter, we focus on how a budget constraint affects the optimal product and process innovation investment decisions. In traditional study of supply chain management, people mainly consider the decisions from the perspective of operations management and often ignore to consider the impact of a budget constraint within the supply chain. The purpose of this chapter is to investigate the effect of a budget constraint on the optimal order quantity and optimal product & process innovation investment levels. In chapter 3, we are searching for coordination whereas in Chapter 4, we are not interested in coordination.

When companies make investment in product and process innovation R&D, budget usually presents a constraint reducing the investment level. In this chapter, we discuss a two-stage supply chain model in which the agents have limited investment availability because of their budget constraint and their budgets have to be allocated between product and process innovation optimally. In our model, a two-stage supply chain is modelled with budget constraint and it is composed of one CM with R&D and one OEM. The CM provides a single technology related product to the OEM who faces effort dependent random demand. The model is same with the one described in Chapter 3 that there are two kinds of investment as product and process R&D investment. We only add a budget constraint to the previous model. Furthermore, the assumptions described in Chapter 3 are also valid for this chapter.

In this chapter, we analyze the problem under both centralized and decentralized settings. In centralized systems, the agents are assumed to be an integrated firm and the model is analyzed under a common budget constraint. The problem is to determine the optimal order quantity and investment level so as to maximize the integrated channel's expected profit, subject to a budget constraint. We develop a Lagrangian based solution algorithm to solve the problem. A numerical example is presented to verify the approach. In decentralized systems, the CM and the OEM act as independent firms and they try to maximize their own profits under a budget constraint.

4.2. Analysis of the Centralized Model

The optimal solution of the centralized system is the maximum profit that the chain can make. So before decentralizing the chain, centralized solution of the system must be evaluated in order to analyze performance of the decentralized solution. The problem of this section is to determine the optimal order quantity and R&D investment decisions to maximize the whole chain's expected profit. In this section the model is subject to a budget constraint and we will analyze the effect of this constraint.

The mathematical model for the centralized system is given as:

$$Max\, \Pi(Q, I_1, I_2) = (p - v + g)S(Q, I_1) - (c_1(I_2) + c_2 - v)Q - I_1 - I_2 - g\mu$$

$$subject\, to$$

$$I_1 + I_2 \leq B$$

where $c_1(I_2) = (c_{max}^{1-\beta} - (1-\beta)\alpha I_2)^{\frac{1}{1-\beta}}$.

Proposition 4.1. *The expected profit for the centralized system is jointly concave in* (Q, I_1, I_2)*, and the solutions obtained from the first order conditions maximize the total expected profit of the integrated chain.*

Proof. Similar to the proof of Proposition 3.1. See Appendix B.1 for the full proof. □

To solve the optimization problem, the budget constraint is initially relaxed.

Then using the algorithm given in Chapter 3, the optimal values of investment levels, I_1^* and I_2^*, can be found. By substituting these values in the constraint, it can be found whether it is binding or not. If it is the latter, then I_1^* and I_2^* are the optimal values and the problem is solved. Otherwise, the constraint is set to equality and the Lagrangian function is introduced as in the following:

$$\begin{aligned} L(Q, I_1, I_2, \lambda_B) &= (p-v+g)S(Q, I_1) - (c_1(I_2) + c_2 - v)Q \\ &\quad -I_1 - I_2 - g\mu + \lambda_B(B - I_1 - I_2) \end{aligned} \tag{4.1}$$

where λ_B is the Lagrangian multiplier.
The method of Lagrange multipliers provides us a strategy to find the maxima of our profit function when it is subject to budget constraint. Lagrangian relaxation has provided the best existing algorithm for the problem and has enabled the solution of problems of practical size [25].

Proposition 4.2. *The Lagrangian function as given in Equation 4.1 is jointly concave in (Q,I_1,I_2,λ_B), and the solutions obtained from the first order conditions maximize the function.*

Proof. See section B.2 in Appendix B. □

All analysis can be carried through the same way in the previous chapter with the only addition being that the lagrange multiplier, λ_B, becomes a decison variable. As the profit function is jointly concave, first order conditions are necessary and sufficient to find the optimum decision variables. The optimal order quatity Q^*, optimal product innovation investment I_1^*, optimal process innovation investment I_2^* and optimal Langrangian multiplier λ_B^* satisfy the first order conditions:

$$\frac{\mathrm{d}\, L(Q, I_1, I_2, \lambda_B)}{\mathrm{d}\, I_2} = (c_{max}^{1-\beta} - (1-\beta)\alpha I_2)^{\frac{\beta}{1-\beta}} \alpha Q - 1 - \lambda_B = 0 \tag{4.2}$$

from which the optimal process innovation investment can be found under a budget constraint:

$$I_2 = \frac{\left[\frac{1+\lambda_B}{\alpha Q}\right]^{\frac{1-\beta}{\beta}} - c_{max}^{1-\beta}}{(\beta-1)\alpha} \tag{4.3}$$

The CM's production cost per unit product, which depends on process innovation investment, is given in Equation 3.1. If we substitute Equation 4.3 into Equation 3.1, we derive $c_1(I_2)$ under budget constraint as:

$$\begin{aligned} c_1(I_2) &= \left[c_{max}^{(1-\beta)} - (1-\beta)\alpha \frac{\left[\frac{1+\lambda_B}{\alpha Q}\right]^{\frac{1-\beta}{\beta}} - c_{max}^{(1-\beta)}}{(\beta-1)\alpha}\right]^{\frac{1}{1-\beta}} \\ &= \left[\frac{1+\lambda_B}{\alpha Q}\right]^{1/\beta} \end{aligned} \tag{4.4}$$

The production cost at optimal process innovation investment level under budget constraint can be found by using Equation 4.4.

The optimal order quantity of the integrated channel under the effect of a budget constraint satisfies the first order conditions and can be found from:

$$\frac{\mathrm{d}\, L(Q, I_1, I_2, \lambda_B)}{\mathrm{d}\, Q} = (p - v + g)\bar{F}(Q/I_1) - (c_1(I_2) - v + c_2) = 0 \tag{4.5}$$

We know that $\bar{F}(Q|I_1) = 1 - F(Q|I_1)$. Then $F(Q|I_1)$ can be written as:

$$F(Q|I_1, I_2) = \frac{p + g - c_1(I_2) - c_2}{p - v + g} \tag{4.6}$$

Q is undefined if the ratio $\frac{p+g-c_1(I_2)-c_2}{p-v+g}$ is negative or greater than one, since $F(.)$ is a cumulative distribution function. Therefore, the following conditions, which are also given in Chapter 3, have to be satisfied for $0 < I_2 < \frac{(c_{max})^{1-\beta}}{(1-\beta)\alpha}$ which is given in Equation

3.4 and these imposed conditions have an action variable (I_2) :

$$
\begin{aligned}
&v < c_1(I_2) + c_2 \\
&c_1(I_2) + c_2 < p + g
\end{aligned}
$$

If we substitute Equation 4.4 into Equation 4.6,

$$F(Q|I_1, I_2) = \frac{p + g - \left(\frac{1+\lambda_B}{\alpha Q}\right)^{\frac{1}{1-\beta}} - c_2}{p - v + g} \tag{4.7}$$

The optimal process innovation investment, maximizes the whole supply chain's revenue when:

$$\frac{\mathrm{d}\, L(Q, I_1, I_2, \lambda_B)}{\mathrm{d}\, I_1} = \frac{\mathrm{d}\, E(D(I_1))}{\mathrm{d}\, I_1}((p - v + g)F(Q|I_1)) - 1 - \lambda_B = 0 \tag{4.8}$$

If we substitute $F(Q|I_1, I_2)$ given in Equation 4.7 into Equation 4.8, we can derive optimal product innovation investment I_1 from:

$$\left.\frac{\mathrm{d}\, E(D(I_1))}{\mathrm{d}\, I_1}\right|_{I_1^*} = \frac{1 + \lambda_B}{p + g - \left[\frac{1+\lambda_B}{\alpha Q}\right]^{\frac{1}{1-\beta}} - c_2} \tag{4.9}$$

For all $\lambda_B \neq 0$ cases, we get the binding budget constraint condition:

$$\frac{\mathrm{d}\, L(Q, I_1, I_2, \lambda_B)}{\mathrm{d}\, \lambda_B} = B - I_1 - I_2 = 0 \tag{4.10}$$

In which case, the constraint B=I_1+I_2 must be satisfied to find the optimal values.

These results enable us to develop an algorithm to solve the centralized problem under a budget constraint. If we know the distribution function of demand and real valued function $\theta(I_1)$, by solving Equation 4.3, 4.7, 4.9 and 4.10 simultaneously, the optimal order quatity Q^*, optimal product innovation investment I_1^* and optimal process innovation investment I_2^* can be found by searching over λ_B. Since $L(Q, I_1, I_2, \lambda_B)$ is concave in Q,I_1,I_2 and λ_B, first order conditions are necessasry and sufficient for the solution of the centralized problem.

4.2.1. Numerical Analysis

We analyze the model with a budget constraint under the centralized settings in terms of closed form functions. Further insight can be obtained by using numerical examples. Moreover, numerical illustrations are helpful to see the behavior of parameters and different budget values over the decision variables and expected profit. In this example, we use MATLAB R2009a and prepare four tables for different budget values. The values of α and c_2 change gradually while other parameters are kept constant for each different budget value. Since the numerical illustration given in Chapter 3 indicates that changes in α and c_2 values affect the results more than the change in other parameter values. In each table, we can compare the calculated values with the ones obtained from the centralized solution of the model without the effect of budget constraint given in Chapter 3.

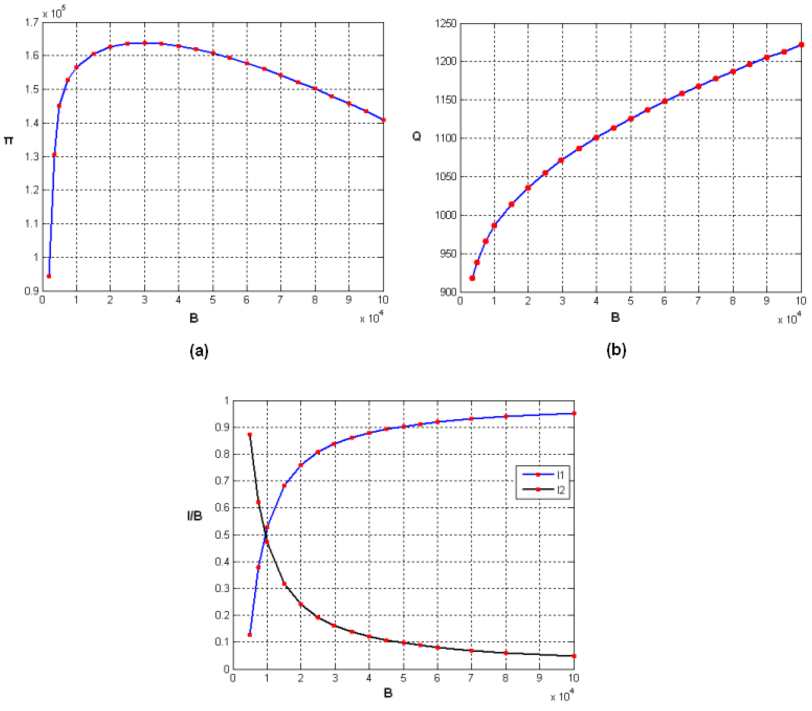
We observe the impact of model parameters and budget constraint on the expected profit, order quantity and R&D investment decisions. We determine three levels for α and c_2, four levels for the budget. In the following example ,the parameters are set as $p = 300$, $\beta = 0.5$, $v = 30$. The random part of the demand is taken as uniformly distributed between $-L \leq \xi \leq U$. In this example, $U = L$ is set as as 500. The impact of product innovation investment I_1 is reflected in demand with $\theta(I_1) = \sqrt{I_1} + U$. By this way, we make sure that $D(I_1) \geq 0$. Expected sales are calculated as given in Chapter 3. Table 4.1, 4.2, 4.3 and 4.4 present how the decision variables (Q^*, I_1^*, I_2^* and λ_B) change along with the expected profit for different choices of model parameters and budget values. Furthermore, the values are calculated separately for the models with and without budget constraint.

First, we compare the results of two different models in order to observe the impact of the budget constraint on decision variables and the expected profit. In this numerical example, budget has four values, 2000, 5000, 7500 and 15000. These values are determined according to the results of centralized solution without budget constraint given in Table 3.1 and in this example, the budget values are selected as less than the sum of product and process innovation investment levels found in numerical illustration part of Chapter 3. According to Table 4.1, 4.2, 4.3 and 4.4, under the effect of budget constraint, when budget becomes tighter product innovation investment sharply decreases while process innovation investment, order quantity and expected profit slightly decrease. Figure 4.1a shows the responses of supply chain's profit when the budget changes. When $\alpha = 0.005$, $c_2 = 60$, $g = 75$ and c_{max}=150, the optimal budget ($B = I_1 + I_2$) is equal to 30000. Figure 4.1b illustrates the responses of optimal order quantity when budget changes. Furthermore, CM's production cost slightly increases. These results indicate that under the budget constraint, the system continues to make investment in process innovation to decrease the production cost. However, firms do not prefer to make product innovation investment when the budget becomes tighter. Furthermore, when budget is high, the company allocates a great portion of budget to product innovation R&D. Thus, we obtain lower product innovation investment values while the budget becomes tighter as illustrated in Figure 4.1c.

When budget changes from 15000 to 2000, the values of product and process innovation investment and expected profit sharply decrease whereas λ_B and CM's production cost sharply increases. Especially, the value of the product innovation investment decreases while the budget value decreases from 15000 to 2000. The difference between the optimal values of two different models also sharply increases. These results show that when budget becomes tighter, the impact of a budget constraint on the given model increases.

For each budget value, we analyze the effect of α and c_2. In this example, α has three values, 0.001, 0.003 and 0.005. When α increases and other parameters are kept constant, CMs production cost, process innovation investment and λ_B decrease. This is an expected result since we obtain the same results from the nemerical illustration given in Chapter 3. This result shows that production cost decreases more with a lower process innovation investment when α gets higher. Therefore, it is obvious that budget constraint does not impact the relation between α and process innovation investment. Furthermore, expected profit, product innovation investment and order quantity slightly increases when alfa changes from 0.005 to 0.001.

In this numerical example, c_2 has three values, 40, 60 and 80. When c_2 decreases and other parameters are kept constant, λ_B and the optimal order quantity increase. However, change in c_2 does not affect the process and product innovation investment whereas we observe in Chapter 3 that product innovation investment increases with a decrease in c_2 without the impact of the budget constraint. Furthermore, it is obvious that expected profit increases while c_2 decreases.



(c)

Figure 4.1. Optimal Results with Changing Budget (a) B vs π, (b) B vs Q, (c) B vs (I/B).

Table 4.1. The Numerical Illustration of Supply Chain Model with Budget Constraint: B=15000.

Parameters		Without Budget					With Budget					
α	c_2	Q	c_1	I_2	I_1	π	λ	Q	c_1	I_2	I_1	π
0.005	80	1003	0.0398	4819.20	21750.00	143020	0.4584	956	0.0930	4777.00	10223.00	140860
0.005	60	1071	0.0349	4824.30	24801.00	163750	0.5572	1014	0.0943	4776.00	10224.00	160560
0.005	40	1139	0.0309	4828.70	28051.00	185840	0.6560	1072	0.0954	4775.40	10225.00	181420
0.003	80	1003	0.1105	7943.30	21740.00	139830	0.7308	940	0.3765	7755.90	7244.10	135880
0.003	60	1071	0.0970	7957.40	24791.00	160550	0.8480	998	0.3808	7753.60	7246.40	155240
0.003	40	1139	0.0857	7969.80	28042.00	182630	0.9651	1056	0.3846	7751.50	7248.50	175760
0.001	80	1003	0.9949	22500.00	21610.00	124380	3.5906	887	26.7725	14146.00	854.00	95988
0.001	60	1071	0.8726	22627.00	24669.00	145050	3.8965	945	26.8363	14134.15	865.85	112710
0.001	40	1139	0.7715	22738.00	27927.00	167080	4.2025	1003	26.8932	14123.17	876.83	130580

Table 4.2. The Numerical Illustration of Supply Chain Model with Budget Constraint: B=10000.

Parameters		Without Budget					With Budget					
α	c_2	Q	c_1	I_2	I_1	π	λ	Q	c_1	I_2	I_1	π
0.005	80	1003	0.0398	4819.20	21750.00	143020	1.0294	928	0.1914	4724.00	5276.00	137370
0.005	60	1071	0.0349	4824.30	24801.00	163750	1.1668	986	0.1933	4723.10	5276.90	156500
0.005	40	1139	0.0309	4828.70	28051.00	185840	1.3043	1044	0.1950	4722.40	5277.60	176780
0.003	80	1003	0.1105	7943.30	21740.00	139830	1.9101	906	1.1470	7451.00	2549.00	129980
0.003	60	1071	0.0970	7957.40	24791.00	160550	2.1067	964	1.1546	7448.60	2551.40	148600
0.003	40	1139	0.0857	7969.80	28042.00	182630	2.3032	1022	1.1614	7446.50	2553.50	168390
0.001	80	1003	0.9949	22500.00	21610.00	124380	5.5104	878	55.0164	9660.30	339.70	73268
0.001	60	1071	0.8726	22627.00	24669.00	145050	5.9454	936	55.0935	9649.90	350.09	88143
0.001	40	1139	0.7715	22738.00	27927.00	167080	6.3804	994	55.1627	9640.60	359.41	104180

Table 4.3. The Numerical Illustration of Supply Chain Model with Budget Constraint: B=7500.

Parameters		Without Budget					With Budget					
α	c_2	Q	c_1	I_2	I_1	π	λ	Q	c_1	I_2	I_1	π
0.005	80	1003	0.0398	4819.20	21750.00	143020	1.7623	908	0.3698	4655.70	2844.30	134020
0.005	60	1071	0.0349	4824.30	24801.00	163750	1.9493	966	0.3725	4654.90	2845.10	152750
0.005	40	1139	0.0309	4828.70	28051.00	185840	2.1363	1024	0.3749	4654.10	2845.90	172630
0.003	80	1003	0.1105	7943.30	21740.00	139830	4.4568	882	4.2521	6790.20	709.80	122730
0.003	60	1071	0.0970	7957.40	24791.00	160550	4.8239	940	4.2643	6788.30	711.70	140700
0.003	40	1139	0.0857	7969.80	28042.00	182630	5.1909	998	4.2751	6786.50	713.50	159830
0.001	80	1003	0.9949	22500.00	21610.00	124380	6.5267	875	74.0490	7284.60	215.40	58181
0.001	60	1071	0.8726	22627.00	24669.00	145050	7.0301	933	74.1307	7275.10	224.90	71886
0.001	40	1139	0.7715	22738.00	27927.00	167080	7.5336	991	74.2044	7266.50	233.50	86751

Table 4.4. The Numerical Illustration of Supply Chain Model with Budget Constraint: B=2000.

Parameters		Without Budget					With Budget					
α	c_2	Q	c_1	I_2	I_1	π	λ	Q	c_1	I_2	I_1	π
0.005	80	1003	0.0398	4819.20	21750.00	143020	30.3166	860	53.0679	1985.10	14.90	79636
0.005	60	1071	0.0349	4824.30	24801.00	163750	32.4330	918	53.0833	1984.60	15.40	94321
0.005	40	1139	0.0309	4828.70	28051.00	185840	34.5495	976	53.0971	1984.30	15.70	110160
0.003	80	1003	0.1105	7943.30	21740.00	139830	22.9661	861	86.0436	1981.00	19.00	51306
0.003	60	1071	0.0970	7957.40	24791.00	160550	24.5836	919	86.0718	1980.00	20.00	64096
0.003	40	1139	0.0857	7969.80	28042.00	182630	26.2011	977	86.0973	1979.10	20.90	78045
0.001	80	1003	0.9949	22500.00	21610.00	124380	8.8181	870	127.3266	1927.10	72.90	15849
0.001	60	1071	0.8726	22627.00	24669.00	145050	9.4757	928	127.4085	1919.80	80.20	26373
0.001	40	1139	0.7715	22738.00	27927.00	167080	10.1335	986	127.4840	1913.10	86.90	38055

4.3. Analysis of the Decentralized Model

In this section, we analyze the problem under decentralized settings and we focus on how a budget constraint affects the optimal product and process innovation investment decisions. In the decentralized system, the CM and the OEM act as independent decision makers and the agents are primarily concerned with maximizing their own profits under the effect of a budget constraint. Furthermore, we assume the CM and the OEM are risk neutral.

This section studies the revenue sharing contract under a two-echelon supply chain system and we are not interested in the coordination of supply chain actions in this chapter. Nash game approach is employed to formulate the structure of supply chain and we analyze interactions between the echelons. In a competitive supply chain neither agent dominates the other, that is, the two echelons are independent firms. In the game's move, the agents simultaneously make their decisions. In this section, the OEM makes the decision on order quantity and also he is the player who makes the product innovation investment. The OEM who makes the product innovation investment to increase the demand in the market, buys the technology related product from the CM under a revenue sharing contract. Furthermore, the CM is the player in the game who makes both the product and process innovation investment.

Let the product innovation investment cost the OEM pays be denoted by I_1^R, the product innovation investment cost the CM pays be denoted by I_1^M and the process innovation investment cost that the CM pays be denoted by I_2^M. Let best response function of the OEM be denoted by $R_r(I_1^M, I_2^M)$ and the best response function of the CM be denoted by $R_m(I_1^R)$.

The OEM's expected profit function is:

$$\Pi_r(Q, I_1^R | I_1^M, I_2^M) = (\phi(p-v)+g)S(Q, I_1) - (\omega + c_2 - \phi v)Q - g\mu - I_1^R \qquad (4.11)$$

where $I_1 = I_1^R + I_1^M$.

The OEM's response function is:

$$R_r(I_1^M, I_2^M) = \underset{Q, I_1^R}{\operatorname{argmax}} \, \Pi_r(Q, I_1^R | I_1^M, I_2^M)$$

Proposition 4.3. *The OEM's expected profit function as given in Equation 4.11 is jointly concave in I_1^R and Q.*

Proof. Similar to the proof of Proposition 3.1. See Appendix B.1 for the full proof. □

For given I_1^M and I_2^M the OEM's optimization problem is:

$$Max\, \Pi_r(Q, I_1^R | I_1^M, I_2^M) = (\phi(p-v)+g)S(Q, I_1) - (\omega + c_2 - v)Q - I_1^R - g\mu$$

subject to

$$I_1^R + I_1^M + I_2^M \leq B$$

OEM's objective is to maximize his profit for given I_1^M and I_2^M under the effect of a budget constraint. To solve the problem budget constraint is set to equality and the Lagrangian function is introduced as following:

$$\begin{aligned} \text{Ł}_R(Q, I_1^R | I_1^M, I_2^M) &= (\phi(p-v)+g)S(Q, I_1) - (\omega + c_2 - v)Q - I_1^R - g\mu \\ &\quad + \lambda_{B_1}(B - I_1^M - I_2^M - I_1^R) \end{aligned} \tag{4.12}$$

where λ_{B_1} is the Lagrangian multiplier of OEM's best response function.

Proposition 4.4. *The Lagrangian function as given in Equation 4.12 is jointly concave in (I_1^R, Q and λ_{B_1}), and the solutions obtained from the first order consitions maximize the function.*

Proof. Similar to the proof of Proposition 4.2. See section B.2 in Appendix B. □

The Lagrangian function is jointly concave in (I_1^R, Q, λ_{B_1}), therefore first order conditions are necessarry and sufficient to find the optimal values of order quantity and OEM's product innovation investment level for given I_1^M and I_2^M.

The optimal order quantity Q^* satisfies the first order condition:

$$\frac{\mathrm{d}\,\Pi_r(Q, I_1^R | I_1^M, I_2^M)}{\mathrm{d}\,Q} = (\phi(p-v)+g)\bar{F}(Q|I_1) - (\omega + c_2 - \phi v) = 0$$

from which

$$\bar{F}(Q|I_1) = \frac{\omega + c_2 - \phi v}{\phi(p-v)+g}$$

We know that $\bar{F}(Q|I_1) = 1 - F(Q|I_1)$. Then $F(Q|I_1)$ can be written as:

$$F(Q|I_1) = \frac{\phi p + g - \omega - c_2}{\phi(p-v)+g} \tag{4.13}$$

Q is undefined if the ratio $\frac{\phi p+g-\omega-c_2}{\phi(p-v)+g}$ is negative or greater than one, since $F(.)$ is a cumulative distribution function. Therefore, the following conditions have to be satisfied:

$$\begin{aligned} &\phi v < c_2 + \omega \\ &c_2 + \omega < p + g \end{aligned}$$

The OEM's product innovation investment for given I_1^M and I_2^M, maximizes the OEM's expected profit when:

$$\frac{\mathrm{d}\,\Pi_r(Q, I_1^R | I_1^M, I_2^M)}{\mathrm{d}\,I_1^R} = \frac{\mathrm{d}\,E[D(I_1)]}{\mathrm{d}\,I_1^R}[(\phi(p-v)+g)F(Q|I_1)] - 1 - \lambda_{B_1} = 0 \tag{4.14}$$

If we substitute $F(Q|I_1)$ given in Equation 4.13 into Equation 4.14, we can derive the

OEM's optimal product innovation investment I_1^{R*} from:

$$\frac{\mathrm{d}\, E[D(I_1)])}{\mathrm{d}\, I_1^R}\bigg|_{I_1^{R*}} = \frac{1+\lambda_{B_1}}{\phi p + g - \omega - c_2} \tag{4.15}$$

For all $\lambda_{B_1} \neq 0$ cases, we get the binding budget constraint condition:

$$\frac{\mathrm{d}\, L_R((Q, I_1^R | I_1^M, I_2^M))}{\mathrm{d}\, \lambda_{B_1}} = B - I_1^M - I_2^M - I_1^R = 0 \tag{4.16}$$

In which case, the constraint $B = I_1^M + I_2^M + I_1^R$ must be satisfied to find the optimal values.

The CM's expected profit function is:

$$\Pi_m(I_1^M, I_2^M | Q, I_1^R) = (1-\phi)(p-v)S(Q, I_1) - (\omega + (1-\phi)v)Q - c_1(I_2^M)Q - I_1^M - I_2^M \tag{4.17}$$

where $I_1 = I_1^R + I_1^M$.

The CM's response function is:

$$R_m(Q, I_1^R) = \underset{I_1^M, I_2^M}{\operatorname{argmax}}\, \Pi_m(I_1^M, I_2^M | Q, I_1^R)$$

Proposition 4.5. *The CM's expected profit function as given in Equation 4.17 is jointly concave in* I_1^M *and* I_2^M *.*

Proof. Similar to the proof of Proposition 3.1. See Appendix B.1 for the full proof. □

For given I_1^R and Q the CM's optimization problem is:

$Max\, \Pi_m(I_1^M, I_2^M | Q, I_1^R) = (1-\phi)(p-v)S(Q, I_1) - (\omega + (1-\phi)v)Q - c_1(I_2^M)Q - I_1^M - I_2^M$

subject to

$$I_1^R + I_1^M + I_2^M \leq B$$

CM's objective is to maximize own profit for given I_1^R and Q under the effect of a budget constraint. To solve the problem, the budget constraint is set to equality and the Lagrangian function is introduced as following:

$$\begin{aligned} Ł_M(I_1^M, I_2^M|Q, I_1^R) &= (1-\phi)(p-v)S(Q, I_1) - (\omega + (1-\phi)v)Q - c_1(I_2^M)Q \\ &\quad -I_1^M - I_2^M + \lambda_{B_1}(B - I_1^M - I_2^M - I_1^R) \end{aligned} \tag{4.18}$$

where λ_{B_2} is the Lagrangian multiplier of CM's best response function.

Proposition 4.6. *The Lagrangian function as given in Equation 4.18 is jointly concave in (I_1^M, I_2^M and λ_{B_2}), and the solutions obtained from the first order consitions maximize the function.*

Proof. Similar to the proof of Proposition 4.2. See section B.2 in Appendix B. □

The Lagrangian function is jointly concave in (I_1^M, I_2^M and λ_{B_2}), therefore first order conditions are necessarry and sufficient to find the CM's optimal product and process innovation investment levels for given I_1^R and Q.

The CM's product innovation investment for given I_1^R and Q, maximizes the CM's expected profit when:

$$\frac{\mathrm{d}\,\Pi_m(I_1^M, I_2^M|Q, I_1^R)}{\mathrm{d}\,I_1^M} = \frac{\mathrm{d}\,E[D(I_1)]}{\mathrm{d}\,I_1^M}[(1-\phi)(p-v)F(Q|I_1)] - 1 - \lambda_{B_2} = 0 \tag{4.19}$$

If we substitute $F(Q|I_1)$ given in Equation 4.13 into Equation 4.19, we can derive the

CM's optimal product innovation investment I_1^{M*} from:

$$\frac{\mathrm{d}\, E[D(I_1)]}{\mathrm{d}\, I_1^M}\bigg|_{I_1^{M*}} = \frac{(1+\lambda_{B_2})(\phi(p-v)+g)}{(1-\phi)(p-v)(\phi v+g-\omega-c_2)} \tag{4.20}$$

The CM's process innovation investment for given I_1^R and Q, maximizes the CM's expected profit when:

$$\frac{\mathrm{d}\, \Pi_m(I_1^M, I_2^M|Q, I_1^R)}{\mathrm{d}\, I_2^M} = \alpha Q(c_{max}^{1-\beta} - (1-\beta)\alpha I_2)^{\frac{\beta}{1-\beta}} - 1 - \lambda_{B_2} = 0 \tag{4.21}$$

from which the optimal process innovation investment can be found:

$$I_2^{M*} = \frac{[\frac{1+\lambda_{B_2}}{\alpha Q}]^{\frac{1-\beta}{\beta}} - c_{max}^{(1-\beta)}}{(\beta-1)\alpha} \tag{4.22}$$

For all $\lambda_{B_2} \neq 0$ cases, we get the binding budget constraint condition:

$$\frac{\mathrm{d}\, L_R((Q, I_1^R|I_1^M, I_2^M))}{\mathrm{d}\, \lambda_{B_2}} = B - I_1^M - I_2^M - I_1^R = 0 \tag{4.23}$$

In which case, the constraint $B = I_1^M + I_2^M + I_1^R$ must be satisfied to find the optimal values.

Theorem 4.1. *Suppose that for each player the strategy space is compact and convex and the payoff function is continuous and quasi-concave with respect to each players own strategy. Then there exists at least one pure strategy NE in the game [20].*

Theorem 4.1 given in Cachon and Netessine study [20] is widely used in Game Theory to show existence of a Nash equilibrium and the original of this theorem is also given in Debreu's study [26]. When we check the concavity of OEM's and CM's profit functions, we can see that they are jointly concave with respect to own strategies. Since they are concave there is at least one Nash equilibrium in the game (Theorem 4.1). If a large enough Q, I_1^M, I_2^M, I_1^R are imposed then the strategy space is compact.

There exists a Nash equilibrium between the OEM and the CM in investments,

I_1^M, I_2^M and I_1^R; the order quantity Q is decided by the OEM according to Equation 4.13 where $I = I_1^R + I_2^R$, the model game is characterized by the following system of equations:

$$\left.\frac{\mathrm{d}\,E[D(I_1)]}{\mathrm{d}\,I_1^R}\right|_{I_1^{R*}} = \frac{1+\lambda_{B_1}}{\phi p + g - \omega - c_2}$$

$$\left.\frac{\mathrm{d}\,E[D(I_1)]}{\mathrm{d}\,I_1^M}\right|_{I_1^{M*}} = \frac{(1+\lambda_{B_2})(\phi(p-v)+g)}{(1-\phi)(p-v)(\phi v + g - \omega - c_2)}$$

$$I_2^{M*} = \frac{\left[\frac{1+\lambda_{B_2}}{\alpha Q}\right]^{\frac{1-\beta}{\beta}} - c_{max}^{(1-\beta)}}{(\beta-1)\alpha}$$

$$B = I_1^M + I_2^M + I_1^R$$

Then the solution I_1^M, I_2^M, I_1^R is a Nash equilibrium of the game.

4.3.1. Numerical Analysis

We analyze the model with budget constraint under decentralized settings in terms of closed form functions. In this example, we have shown that for the model we proposed in this section there is a Nash equilibrium. We use MATLAB R2009a and prepare a table which consists of centralized and decentralized systems solutions. By this way, we can compare the budget allocation strategy under centralized and decentralized settings.

We observe the impact of model parameters on CM's and OEM's expected profi, and their R&D investment decisions. The values of α and c_2 change gradually while other parameters are kept constant when budget is equal to 2000 and 15000. In this example, α and c_2 are the changing parameters. Since the numerical illustration given in Chapter 3 indicates that changes in α and c_2 values affect the results more than the change in other parameter values. For each different parameter values, we have shown that there is a nash equilibrium and we compare the results with the ones obtained from centralized solutions.

We determine three levels for α and c_2, whwn budget is equal to 2000 and 15000. In the following example ,the parameters are set as $p = 300$, $\beta = 0.5$, $v = 30$. The random part of the demand is taken as uniformly distributed between $-L \leq \xi \leq U$. In this example, $U = L$ is set as as 500. The impact of product innovation investment I_1 is reflected in demand with $\theta(I_1) = \sqrt{I_1} + U$. By this way, we make sure that $D(I_1) \geq 0$. Expected sales are calculated as given in Chapter 3.

For each different parameter set, the centralized system solutions are calculated in previous section. In order to compare the budget allocation ways of centralized and decentralized systems, we set order quantity as the optimal order quantity of centralized solution. In this example, we fix ϕ as 0.4 and calculate ω for each different I_1^R and Q^* (centrally optimal quantity) values by using Equation 4.13 given as:

$$F(Q|I_1) = \frac{\phi p + g - \omega - c_2}{\phi(p - v) + g}$$

First I_1^M and I_2^M are given, then we calculate I_1^R by using the following equation:

$$B = I_1^M + I_2^M + I_1^R$$

After the calculation of I_1^R, ω is calculated from the following equation:

$$\left.\frac{\mathrm{d}\, E[D(I_1)]}{\mathrm{d}\, I_1^R}\right|_{I_1^{R*}} = \frac{1 + \lambda_{B_1}}{\phi p + g - \omega - c_2}$$

Finally, for given I_1^R, we find I_1^M, I_2^M and λ_{B2} simultaneously from the following equations:

$$\left.\frac{\mathrm{d}\,E[D(I_1)]}{\mathrm{d}\,I_1^M}\right|_{I_1^{M*}} = \frac{(1+\lambda_{B_2})(\phi(p-v)+g)}{(1-\phi)(p-v)(\phi v+g-\omega-c_2)}$$

$$I_2^{M*} = \frac{\left[\frac{1+\lambda_{B_2}}{\alpha Q}\right]^{\frac{1-\beta}{\beta}} - c_{max}^{(1-\beta)}}{(\beta-1)\alpha}$$

$$B = I_1^M + I_2^M + I_1^R$$

This calculation is performed until we assure the convergence of I_1^M and I_2^M. This shows that for each parameter set there is a Nash equilibrium between I_1^M, I_2^M and I_1^R. Table 4.5 presents these convergence results with the centralized solutions. Table 4.5 indicates that for both centralized and decentralized systems, more budget is allocated to process R&D when budget is equal to 2000. Under decentralized settings, the OEM's product R&D investment is always greater than the CM's product R&D investment and total optimal product innovation investment level under decentralized settings is always greater than the one under centralized settings when budget is equal to 2000. However, when budget is equal 15000, the opposite result is observed.

When α increases from 0.001 to 0.005, we observe that the budget allocation to process innovation investment increases whereas the budget allocation to product innovation investment decreases. Since the CM's production cost decreases when α increases, this makes companies invest more in process innovation and less in product innovation.

Furthermore, when α decreases, the CM's production cost sharply increases. This makes the CM's expected profit negative which implies that for some data set and for lower α values, the CM cannot make any profit when budget becomes tighter. However when α and the OEM's product cost c_2 decreases, the OEM earns more profit. Note that the sum of OEM's and CM's profit is nearly equal to the optimal profit of centralized system for each data set.

Table 4.5. The Numerical Illustration of Supply Chain Model with Budget Constraint, Centralized and Decentralized Solutions: B=2000.

Parameters		Centralized						Decentralized								
α	c_2	λ_b	Q	c_1	I_2	I_1	π	w	λ_{b_1}	λ_{b_2}	c_1	${I_1}^R$	${I_1}^M$	${I_2}^M$	π^R	π^M
0.005	80	30.3166	860	53.0679	1985.10	14.90	79636	-41.4523	16.1322	30.4404	53.4611	20.8532	4.8513	1974.30	67650	11992
0.005	60	32.4330	918	53.0833	1984.60	15.40	94321	-32.0772	17.5676	32.5538	53.4391	20.2449	4.8576	1974.90	77087	17239
0.005	40	34.5495	976	53.0971	1984.30	15.70	110160	-22.7152	19.421	34.6579	53.3914	18.9303	4.8664	1976.20	87140	23030
0.003	80	22.9661	861	86.0436	1981.00	19.00	51306	-41.0684	9.2013	23.1446	87.3759	58.5214	8.1737	1933.30	67768	-16470
0.003	60	24.5836	919	86.0718	1980.00	20.00	64096	-31.7233	10.2517	24.7561	87.2747	54.8867	8.2092	1936.90	77218	-13122
0.003	40	26.2011	977	86.0973	1979.10	20.90	78045	-22.3664	11.2543	26.3707	87.2046	52.3664	8.2300	1939.40	87283	-9233
0.001	80	8.8181	870	127.3266	1927.10	72.90	15849	-40.1649	2.6927	8.9978	132.0604	441.4239	47.1783	1511.40	68770	-53027
0.001	60	9.4757	928	127.4085	1919.80	80.20	26373	-30.8719	3.0521	9.6540	131.8051	418.8967	47.5081	1533.60	78337	-52052
0.001	40	10.1335	986	127.4840	1913.10	86.90	38055	-21.5637	3.4116	10.3109	131.5962	400.4372	47.7591	1551.80	88514	-50537

Table 4.6. The Numerical Illustration of Supply Chain Model with Budget Constraint, Centralized and Decentralized Solutions: B=15000.

Parameters		Centralized						Decentralized								
α	c_2	λ_b	Q	c_1	I_2	I_1	π	w	λ_{b_1}	λ_{b_2}	c_1	${I_1}^R$	${I_1}^M$	${I_2}^M$	π^R	π^M
0.005	80	0.4584	956	0.0930	4777.00	10223.00	140860	-41.4818	0.0929	0.0261	0.0415	5125.00	5057.50	4817.50	78498	64106
0.005	60	0.5572	1014	0.0943	4776.00	10224.00	160560	-32.0952	0.1670	0.0699	0.0421	5125.00	5058.10	4816.90	88953	73348
0.005	40	0.6560	1072	0.0954	4775.40	10225.00	181420	-22.7088	0.2412	0.1059	0.0426	5125.00	5058.60	4816.40	100028	83135
0.003	80	0.7308	940	0.3765	7755.90	7244.10	135880	-41.5850	0.2940	0.1795	0.1749	3661.00	3452.90	7886.10	77178	59870
0.003	60	0.8480	998	0.3808	7753.60	7246.40	155240	-32.1974	0.3818	0.2590	0.1768	3660.00	3455.40	7884.60	87465	68944
0.003	40	0.9651	1056	0.3846	7751.50	7248.50	175760	-22.8100	0.4698	0.3384	0.1785	3659.00	3457.70	7883.30	98366	78560
0.001	80	3.5906	887	26.7725	14146.00	854.00	95988	-41.9295	2.1026	3.5959	26.8469	639.50	228.40	14132.10	71250	24710
0.001	60	3.8965	945	26.8363	14134.15	865.85	112710	-32.5590	2.3290	3.8941	26.8215	633.40	229.60	14137.00	81008	31691
0.001	40	4.2025	1003	26.8932	14123.17	876.83	130580	-23.1886	2.5574	4.1920	26.7961	627.30	230.70	14142.00	91333	39226

5. CONCLUSION

Supply chain management is an effective tool for increasing profits of firms. Most of the companies try to find the optimal production quantity in order to increase their market shares and to compete with other CMs without damaging their profit margin. Furthermore, many firms are seeking to reduce the cost of new products by production process innovations and facilitate the smooth launch of new products by product innovations. Firms innovate in order to reduce costs or to increase demand. Therefore, there are many studies analyzing the product innovation effect onthe demand curve and the process innovation effect on the production cost separately. This thesis is an attempt to do both in a single model.

In this study, a-two echelon supply chain composed of one CM with R&D and one OEM is considered. The CM in this supply chain provides a single technology related product to the OEM who faces effort dependent random demand. This work considers both product and process innovation effort in characterizing a two stage supply chain. Furthermore, we focus on the relation between the process innovation effort and the production cost. We analyze the effect of process innovation effort on a two stage supply chain with effort dependent demand under the assumption that process innovation effort reduces the production cost. The main objectives are to provide insights to conditions where such product and process innovation R&D investments take place, to coordinating contracts and to budget constraint on R&D investments.

First, the problem is analyzed without a budget constaint. The model is analyzed both from a centralized and a decentralized point of view. We propose a model to find a centralized solution for a two-echelon supply chain with product and process innovation investment which is given in Chapter 3. A numerical study illustrates the behavior of the central solution for various parameter levels for the two echelon model without a budget constraint. We also concentrate on the coordination of supply chain for different cases in which the OEM and the CM share the R&D investment or make their own investment decisions. Revenue sharing contract is analyzed for different cases. It is shown that revenue sharing contract fails to coordinate the supply chain actions although different scenarios are applied to the model.

In the literature, the impact of a budget constraint within the supply chain is often ignored. One of the aims this thesis is to investigate the effect of a budget constraint on the optimal order quantity and optimal product & process innovation investment levels.When companies make investment in product and process innovation R&D, budget usually presents a constraint reducing the investment level. Therefore, in Chapter 4 we focus on how a budget constraint affects the optimal product and process innovation investment decisions. We discuss a-two-stage supply chain model in which the agents have limited investment availability because of their budget constraint and their budgets have to be allocated between product and process innovation investment optimally. We analyze the problem under both centralized and decentralized settings. We develop a Lagrangian based solution and propose an algorithm to solve the centralized problem and to find the optimal budget allocation for a two-echelon supply chain with product and process innovation investment under a budget constraint. A numerical example is presented to verify the approach. We show that when the budget becomes tighter, the company allocates a great portion of budget to process innovation R&D to make a reduction in production cost.

In Chapter 4, a Nash game approach is employed to formulate the interactions between the echelons. Nash equilibrium characteristics of the R&D investment game is stated and an illustrative example is given for it. We have shown that there exists a Nash equilibrium for different parameter sets and we compare the results with the centralized solutions. Although we have not encountered otherwise in our numerical study the equilibrium may not be unique. It is shown that total optimal product innovation investment level under decentralized settings is always greater than the optimal centralized product innovation investment level. In the decentralized setting, the supply chain agents increase the product innovation R&D to make an increase in their own profits. Firms make R&D investment when the marginal profit of the new technology is attractive enough. Because of this fact, firms engage in innovation activities to reduce cost and to increase demand.

To summarize, we model a-two stage supply chain with effort dependent random demand and analyze R&D investment decisions with and without a budget constraint. Some insights are provided to light the phenomenon up. As a future work, the decision variables can be determined sequentially. In that model, solving sequential problems will be important. The product and process innovation investment levels will be determined first and the order quantity will be determined later.

APPENDIX A: DERIVATION OF $c_1(I_2)$

In our study, it is assumed that process innovation investment lowers the CM's unit production cost and the decrease rate is given by:

$$\frac{\mathrm{d}\, c_1(I_2)}{\mathrm{d}\, I_2} = -\alpha c_1(I_2)^{\beta}$$

where α is a scale parameter and $\alpha \geq 0$. Furthermore, β is a shape parameter and $0 < \beta < 1$.

We take the integral of both sides then the equation is obtained as follows:

$$\frac{[c_1(I_2)]^{-\beta+1}}{-\beta+1} = -\alpha I_2 + constant \tag{A.1}$$

It is assumed that without process innovation investment, production cost is equal to its maximum level which is denoted as c_{max}. Then when $(I_2 = 0)$, we can find the constant as:

$$\frac{c_{max}^{-\beta+1}}{-\beta+1} = constant \tag{A.2}$$

Let substitute (A.2) into (A.1), the following function is derived as:

$$\frac{c_1(I_2)^{-\beta+1}}{-\beta+1} = -\alpha I_2 + \frac{c_{max}^{(1-\beta)}}{1-\beta}$$

Finally, $c_1(I_2)$ can be rewritten as:

$$c_1(I_2) = (c_{max}^{1-\beta} - (1-\beta)\alpha I_2)^{\frac{1}{1-\beta}}$$

APPENDIX B: PROOFS

B.1. Proof of Proposition 3.1

The integrated chain's expected profit function is:

$$\Pi(Q, I_1, I_2) = (p - v + g)S(Q, I_1) - (c_1(I_2) + c_2 - v)Q - I_1 - I_2 - g\mu \qquad \text{(B.1)}$$

We analyze the expected profit function one by one and divide the profit function into three:

(i) $\underline{(p - v + g)S(Q, I_1)}$

The concavity of $S(Q, I_1)$ w.r.t. I_1, and Q can be shown by checking the Hessian matrix of $S(Q, I_1)$:

$$H = \begin{vmatrix} \frac{\partial^2 S(Q,I_1)}{\partial Q^2} & \frac{\partial^2 S(Q,I_1)}{\partial I_1 \partial Q} \\ \frac{\partial^2 S(Q,I_1)}{\partial I_1 \partial Q_2} & \frac{\partial^2 S(Q,I_1)}{\partial I_1^2} \end{vmatrix}$$

The second order and cross partial derivatives are as follows:

$$\begin{aligned} \frac{d^2 S(Q, I_1)}{dQ^2} &= -f(Q|I_1) \\ \frac{d^2 S(Q, I_1)}{dI_1^2} &= F(Q|I_1)\frac{d^2 E(D(I_1)}{dI_1^2} - f(Q|I_1)[\frac{dE(D(I_1)}{dI_1}]^2 \\ \frac{d^2 S(Q, I_1)}{dI_1 dQ} &= f(Q|I_1)\frac{dE(D(I_1)}{dI_1} \end{aligned}$$

When we check the Hessian matrix, we see that the determinant of the first principal submatrix is negative such that $H_1 = -f(Q|I_1) < 0$. Since $f(x|I_1)$ is a probability density function and $f(x|I_1)$ is positive for every $x \geq 0$ which is given in model assumptions.

The determinant of the second principal submatrix is:

$$H_2 = -f(Q|I_1)F(Q|I_1)\frac{d^2E(D(I_1)}{dI_1^2} > 0$$

The determinant of the second principal submatrix, $|H_2|$ is strictly positive since $f(x|I_1)$ is positive for every $x \geq 0$ and $F(x|I_1)$ is stricly increasing function. Furthermore, it is known that $\frac{d^2E(D(I_1)}{dI_1^2}$ is strictly negative. Therefore, we can cocnlude that $|H_2|$ is strictly positive. Hence, since $(p-v+g) > 0$, $(p-v+g)S(Q, I_1)$ is jointly concave in Q and I_1.

(ii) $\underline{-(c_1(I_2)Q}$

Let $K_1(Q, I_2)$ denote $-c_1(I_2)Q$. Therefore, to prove the concavity $K_1(Q, I_2)$, it is necessary to show the convexity of $c_1(I_2)Q$ at I_2^* which is obtained from:

$$\frac{\mathrm{d}\,\Pi(Q, I_1, I_2)}{\mathrm{d}\,I_2} = \left[c_{max}^{1-\beta} - (1-\beta)\alpha I_2\right]^{\frac{\beta}{1-\beta}} \alpha Q - 1 = 0$$

Optimal process innovation investment, I_2^* is:

$$I_2^* = \left[\frac{\left[\frac{1}{\alpha Q}\right]^{\frac{1-\beta}{\beta}} - c_{max}^{1-\beta}}{(\beta-1)\alpha}\right] \tag{B.2}$$

Substitute Equation B.2 into $c_1(I_2)$, then we obtain c_1 as:

$$c_1(Q) = \left(\frac{1}{\alpha Q}\right)^{\frac{1}{\beta}}$$

Now $K_1(Q, I_2)$ function depends on only Q and can be rewritten as:

$$K_1(Q) = \left[\frac{1}{\alpha Q}\right]^{\frac{1}{1-\beta}} Q \tag{B.3}$$

$K_1(Q)$ given in Equation B.3 can be derived as:

$$\begin{aligned} K_1(Q) &= \left[\frac{1}{\alpha Q}\right]^{\frac{1}{\beta}} Q \\ &= \left[\frac{Q^{\beta}}{\alpha Q}\right]^{\frac{1}{\beta}} \\ &= \left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\beta}} Q^{\left[\frac{\beta-1}{\beta}\right]}\right] \end{aligned}$$

To show $K_1(Q)$ is a convex function, we check the second derivative:

$$\begin{aligned} \frac{\mathrm{d}\, K_1(Q)}{\mathrm{d}\, Q} &= \left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\beta}} \left[\frac{\beta-1}{\beta}\right] \left(Q^{\frac{-1}{\beta}}\right)\right] \\ \frac{\mathrm{d}^2\, K_1(Q)}{\mathrm{d}\, Q^2} &= \left(\frac{1}{\alpha}\right)^{\frac{1}{1-\beta}} \left[\frac{-\beta}{1-\beta}\right] (Q)^{\frac{-\beta}{\beta-1}} > 0 \end{aligned}$$

since the second derivative is positive, $K_1(Q)$ is convex in Q at optimal process innovation level, I_2^*. This proofs the concavity of $-c_1(I_2)Q$.

(iii) $\underline{-(c_2 - v)Q - I_1 - I_2}$

This function is a linear function of Q, I_1 and I_2. So it is concave.

So all parts are concave. Since sum of functions are concave the integrated chain's expected profit function is concave in Q, I_1 and I_2.

B.2. Proof of Proposition 4.2

The Lagrangian function is given as:

$$\begin{aligned} L(Q, I_1, I_2, \lambda_B) &= (p - v + g)S(Q, I_1) - (c_1(I_2) + c_2 - v)Q \\ &\quad - I_1 - I_2 - g\mu + \lambda_B(B - I_1 - I_2) \end{aligned}$$

We analyze the function one by one:

(i) $\underline{(p - v + g)S(Q, I_1)}$

The concavity of $S(Q, I_1)$ w.r.t. Q and I_1 is shown in Appendix B.1. Therefore, $(p - v + g)S(Q, I_1)$ is jointly concave in Q and I_1.

(ii) $\underline{-(c_1(I_2)Q)}$

Let $K_2(Q, I_2)$ denote $-c_1(I_2)Q$. To prove the concavity $K_2(Q, I_2)$, it is necessary to show the convexity of $c_1(I_2)Q$ atoptimal process innovation investment level, I_2^*. The CM's production cost at optimal process innovation investment is:

$$c_1(Q, \lambda_B) = \left[\frac{1 + \lambda_B}{\alpha Q}\right]^{\frac{1}{\beta}} \tag{B.4}$$

If we substitute Equation B.4 into $K_2(Q, I_2)$ which is rewritten as:

$$K_2(Q, \lambda_B) = \left[\frac{1 + \lambda_B}{\alpha Q}\right]^{\frac{1}{1-\beta}} Q \tag{B.5}$$

The convexity of $K_2(Q, \lambda_B)$ w.r.t. I_1 and λ_B can be shown by checking the Hessian matrix of $K_2(Q, \lambda_B)$:

$$H = \begin{vmatrix} \frac{\partial^2 K_2(Q,\lambda_B)}{\partial Q^2} & \frac{\partial^2 \; K_2(Q,\lambda_B)}{\partial \lambda_B \partial Q} \\ \frac{\partial^2 K_2(Q,\lambda_B)}{\partial I_1 \partial Q_2} & \frac{\partial^2 K_2(Q,\lambda_B)}{\partial \lambda_B^2} \end{vmatrix}$$

The second order and cross partial derivatives are as follows:

$$
\begin{aligned}
\frac{d^2K_2(Q,\lambda_B)}{dQ^2} &= (1+\lambda_B)^{\frac{1}{\beta}}\left[\frac{1-\beta}{\beta^2}\right]Q^{\frac{-(1+\beta)}{\beta}}\\
\frac{d^2K_2(Q,\lambda_B)}{d\lambda_B^2} &= (1+\lambda_B)^{\frac{1-2\beta}{\beta}}\left[\frac{1-\beta}{\beta^2}\right]Q^{\frac{(\beta-1)}{\beta}}\\
\frac{d^2K_2(Q,\lambda_B)}{d\lambda_B dQ} &= (1+\lambda_B)^{\frac{1-\beta}{\beta}}\left[\frac{\beta-1}{\beta^2}\right]Q^{\frac{-(\beta+1)}{\beta}}
\end{aligned}
$$

When we check the Hessian matrix, we see that the determinant of the first principal submatrix is positive such that $H_1 = (1+\lambda_B)^{\frac{1}{\beta}}\left[\frac{1-\beta}{\beta^2}\right]$. Since λ_B and Q are always positive and $0 < \beta < 1$.

The determinant of the second principal submatrix is:

$$
H_2 = (1+\lambda_B)^{\frac{2(1-\beta)}{\beta}}\left[\frac{(1-\beta)^2}{\beta^4}\right]\left[Q^{\frac{-2}{\beta}} - Q^{\frac{-2(\beta+1)}{\beta}}\right] > 0
$$

The determinant of the second principal submatrix, $|H_2|$ is strictly positive since $0 < \beta < 1$ and therefore $Q^{\frac{-2}{\beta}} > Q^{\frac{-2(\beta+1)}{\beta}}$. We can cocnlude that $-(c_1(I_2).Q)$ is jointly concave in Q and I_2.

(iii) $\underline{\lambda_B(B - I_1 - I_2)}$

Since the Hessians of $\lambda_B I_1$ and $\lambda_B I_2$ are zero for all variables that shows that they are both negative semi-definite and positve semidefinite. That implies that $\lambda_B I_1$ and $\lambda_B I_2$ are linear functions. Therefore $\lambda_B(B - I_1 - I_2)$ is jointly concave in λ_B, I_1 and I_2.

(iv) $\underline{-(c_2 - v)Q - I_1 - I_2}$

This function is a linear function of Q, I_1 and I_2. So it is concave.

So all parts are concave. Since sum of functions are concave the Langrangian function is concave in Q, I_1, I_2 and λ_B.

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