ANALYSIS OF A SUPPLY CHAIN UNDER ADVANCE DEMAND INFORMATION

by

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ABSTRACT

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In this study we investigate the effects of incorporating advance demand information in a two-echelon distribution model which consists of a central depot and several retailers. The depot is not allowed to hold inventory. In particular, the benefits of advance demand information on system costs and ordering and allocating policies are investigated. Two types of advance demand information structures are considered. Under perfect and partial advance demand information structures, the approximate optimal ordering and allocating policies and the expected system cost are obtained. A computational study is presented in order to demonstrate the effect of system parameter.

ÖZET

ÖN TALEP BİLGİSİ ALTINDA BİR TEDARİK ZİNCİRİNİN ANALİZİ

Bu çalışmada merkezi bir depo ve birkaç bayiden oluşan iki kademeli bir dağıtım modeline ön talep bilgisinin dahil edilmesinin etkileri incelenmektedir. Merkezi depo envanter bulundurmamaktadır. Ön talep bilgisinin özellikle de sistem maliyetleri, sipariş ve dağıtım politikaları üzerindeki faydaları incelenmektedir. İki tip ön talep bilgisi ele alınmıştır. Tam ve kısmi ön talep bilgisi yapıları altında yaklaşık sipariş ve dağıtım politikaları ile beklenen sistem maliyeti elde edilmiştir. Sistem parametrelerinin etkisini göstermek için sayısal bir çalışma sunulmuştur.

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LIST OF SYMBOLS/ABBREVIATIONS

$A_{i,t}$	Part of the partial demand structure that is observed in advance
a_i	Constant part of the inventory at the end of the planning horizon
$a_{i,t}$	Realization of $A_{i,t}$
$B_{i,t}$	Part of the partial demand structure that is observed when it is needed
b	penalty cost per unit per period
$b_{i,t}$	Realization of $B_{i,t}$
C	Expected one-period system-wide inventory cost
C_i	Expected one-period inventory cost of retailer i
$D_{i,t}$	Demand at retailer i in period t
$\bar{D}_{i,t}$	Demand information vector of retailer i in period t
$d_{i,t}$	Realization of $D_{i,t}$
F	Distribution function
G	Loss function
h	holding cost per unit per period
L_d	Lead time from the depot to retailers
L_s	Lead time from the supplier to the depot
l	Demand lead time
N	Number of retailers
NI_i	Net inventory on hand at retailer i at the beginning of the planning horizon
Q	Size of order placed by the depot in single order model
Q_t	Order amount placed by the depot in period t in multiple order periods model
S	Order-up-to level
u	Standard normal variable
v	Standard normal variable
$X_{i,t}$	Inventory position of retailer i in period t

z_i	Allocation quantity to retailer i in single order model
$z_{i,t}$	Allocation quantity of order amount placed in period t to retailer i in multiple order periods model
β	No stockout probability
Φ	Cumulative distribution function of standard normal variable
μ_i	Mean demand per period at retailer i
$\hat{\mu}_i$	Mean of ς_i
σ_i	Standard deviation in demand per period at retailer i
$\hat{\sigma}_i$	Standard deviation of ς_i
ς_i	Variable part of the inventory at the end of the planning horizon
ADI	Advance demand information
c.d.f.	cumulative distribution function

1. INTRODUCTION

In control of inventory systems, uncertainty is generally considered as the main problem which the systems have to cope with. Holding safety stock is the solution applied for decades. With developments in information technologies, information flow is now faster, cheaper and easier. So, cooperation among supply chain members increases. Developing cooperation among supply chain members offers an alternative solution, information sharing. Information sharing can be realized through point-of-sale data, demand forecasts and production scheduling and it decreases demand variability.

Information on future demand is referred to advance demand information. When advance demand information is available, customers place their orders before their due dates for a future delivery. The time between the period in which customer places order and the period in which customer actually needs it is called demand lead time. If demand lead time is the same for each retailer, then there exists perfect advance demand information. In literature advance demand information is generally assumed to be perfect. However, we consider two types of advance demand information in this study: Perfect and partial ADI. In perfect ADI, full information is obtained and demand for the demand lead time becomes deterministic. In partial ADI, partial information of future demand is obtained. A part of the future demand is known in advance and remaining part of the future demand is known when it is actually needed.

In this thesis we analyze a single item, multi-echelon distribution system where advance demand information is available. The distribution system consists of a depot and several retailers. The depot does not hold inventory, it determines the amount of order and allocates the quantity received from the supplier among retailers. Only the retailers observe customer demands. Customers place orders a number of periods in advance and this is the same for every customer. Using the advance demand information and distribution of demand beyond demand lead time, the depot places an order to the outside supplier. The order arrives after a fixed supply lead time. The depot receives the shipment and allocates it among retailers.

The aim of this research is to investigate the impact of advance demand information on the system costs and inventory policies in a two-echelon distribution system. Our objective is to find optimal ordering and allocating policies that minimizes inventory related costs.

We make two main contributions in this study: First, we introduce and analyze a two-echelon allocation model where perfect and partial advance demand information are incorporated. Second, our computational results demonstrate that advance demand information reduces expected inventory costs.

In Chapter 2, we present a literature review on multi-echelon inventory systems and systems with ADI. General description of the model is presented in Chapter 3. In this chapter we describe the system in details and introduce the notation used in this study. In the next two chapters we analyze our model under two different ADI structures. In Chapter 4, we analyze the model with perfect ADI under two different ordering policies which are single order policy and order every period policy. In Chapter 5, we analyze the model under partial ADI. We establish three different models for partial ADI case. In the first model, the amount of demand information is the same for each retailer. In the second one, the amount of ADI depends on retailers. In the third model, we consider correlated demands among retailers. In this chapter, we analyze these three models and obtain approximate optimal ordering and allocating policies. Besides, we analyze the cost of the partial ADI model where the amount of demand information depends on retailer and the cost of the partial ADI model with correlated demands and provide cost function under approximate optimal order and allocation policies. We also provide a computational study in Chapter 6. Finally, we discuss our findings and give conclusions in Chapter 7.

2. LITERATURE REVIEW

There is a vast literature constituted for decades focusing on a variety of subjects and using various approaches about inventory systems. The studies considering deterministic demand and single echelon are simpler. But the complexity increases when probabilistic demand and management of a whole supply chain are the subjects. Since we consider a single-item, periodic review, single cycle distribution system with available advance demand information in our study, we specially focus on the literature about multi-echelon inventory systems and the literature about advance demand information which became a subject to study with the developing co-operation among supply chain members and information technologies.

2.1. Modelling Multi-Echelon Inventory

Eppen and Schrage [1] analyze a distribution system consisting of a supplier, a depot and several retailers satisfying independent normally distributed customer demands. The depot does not hold inventory but allocates the shipment arriving from the supplier among retailers. This multi-echelon, multi period problem is considered in two ways: In the first model, the depot is allowed to order and allocate every period. Applying a base stock policy, an approximate optimal ordering policy is obtained. In the second model, the depot is allowed to order and allocate every m periods. This difference is motivated by the presence of a fixed ordering cost. Eppen and Schrage provide an approximately optimal (m, y) policy in which the inventory position is raised to y in every m periods. Policies are said to be approximately optimal because of the allocation assumption. The allocation assumption implies that the depot receives sufficient material to allocate among retailers so that equal stockout probability is achieved for each retailer.

Federgruen [2] analyzes a single-item, discrete-time, multi-echelon distribution system with linear shipment costs. He considers systems without central inventories and with central inventories. In systems without central inventories he formulates the problem as a dynamic problem where the decision variables are the size of the order placed by the depot and its allocation among the retailers. Since the exact solution is impractical, he suggests two types of approximation: Relaxation and restriction. In approximation by relaxation, he relaxes the nonnegativity constraint which guarantees nonnegative allocations to retailers. The problem is then decomposed into a dynamic program and a myopic allocation problem. The obtained dynamic problem reduces to a single location inventory problem. Federgruen proposes that an order-up-to policy is optimal for this problem and optimum order amount can be optimally allocated with a myopic allocation policy. In approximation by restriction, the order policy is restricted to the regular-interval critical-number policies in which an order is placed every m periods to bring the system wide inventory position to the critical number. This gives a feasible order policy and it can be complemented with an appropriate allocation policy. In systems with central inventory, Federgruen again suggests two approximation approaches using infinite planning horizon, stationary data and identical cost rates for all retailers. In approximation by relaxation, he first establishes a dynamic problem with three decision variables which are the order amount placed by the depot, the amount to be withdrawn from the depot's inventory and its allocation among the retailers. Relaxing the nonnegativity constraint, the order amount is obtained with an order-up-to policy, the amount to be withdrawn is obtained with a modified base stock policy and this amount is allocated with a myopic allocation policy. In approximation by restriction, he uses the regular-interval critical number policy. Firstly, the quantity withdrawn is determined; secondly, the stock withdrawn is allocated among the retailers.

Federgruen and Zipkin [3] analyze a periodic review, two-echelon inventory system consisting of a depot and a retailer. Formulating the problem as a dynamic program and decomposing it into two seperate single-location problems do not provide a tractable solution. Because of the computational difficulties, Federgruen and Zipkin extend their study to the infinite-horizon case. They first demonstrate a similar decomposition for both the discounted and the average cost infinite-horizon problems, which are easier to solve than the finite-horizon case. They secondly show that normal demands require no integration, which is one of the difficulties in computation. Lastly, they analyze the problem for the several retailers case. They aproximate this problem by pretending the several retailers as a single retailer. Relaxing non negative allocation constraint, a relaxed problem is obtained which reduces to the infinite-horizon problem. A feasible policy for orders and shipments from the depot to the single retailer is obtained for the reduced problem. For the original several retailers problem, an allocation policy is required and the myopic allocation policy is proposed for this. .

Jackson [4] models a periodic review, two-echelon distribution system consisting of a depot and N retailers. This paper is an extension of Eppen and Schrage [1] to allow the depot to hold inventory and make allocations to the retailers in every period of the cycle. The depot places order every m periods but makes allocation in every period of the cycle consisting of m periods . If the depot has sufficient inventory, the allocation is made to raise every retailer to its ship-up-to level. If the depot has insufficient inventory to bring each retailer to its ship-up-to level in any period in the cycle, it is called runout period. In this case a runout allocation rule is used. Jackson suggests an exact model for this problem but because of the complexity in solving the exact model, he also suggests an approximate cost function that is computationally tractable. A simulation is done for the exact model and the empirical results demonstrate that the approximation is an effective substitute for simulation.

Lagodimos [5] models a two-echelon production network to evaluate the service performance under different rationing policies. This system can be viewed as a two-echelon distribution system where the depot supplies the retailers and it is allowed to hold inventory. Lagodimos considers two different rationing policies when the inventory at the depot is insufficient to satisfy retailers: Fair share rationing policy as push policy and priority policy as pull policy. Using these policies, models are developed to determine three different customer service measures. Because of the assumptions related to the rationing policies, the models are approximate but Monte Carlo estimations and simulation results show that these assumptions are quite accurate.

Diks, de Kok and Lagodimos [6] review the literature contributing to control of multi-echelon systems. Firstly, major elements of multi-echelon systems like ordering policies and service measures are introduced. Then, the paper focuses on the analysis of control policies reviewing the most important results of multi-echelon theory from a service measure perspective. While doing this, differences between echelon stock policies and installation stock policies are underlined. Practically useful continuous and periodic ordering policies are reviewed in the content of installation stock policies and rationing policies. Lastly, customer service measures are reviewed in the content of echelon stock policies.

Güllü [7] analyzes a two-echelon distribution system consisting of a depot and several retailers where the depot does not hold inventory and the demand is observed at retailers. In addition to Eppen and Schrage [1] model, this paper considers an information process which models correlation of demands and forecasts through time and among retailers. The paper provides an approximate optimal system-wide order-up-to level under the correlated demand-forecast structure. The model with correlated demand-forecast structure is compared with a standard demand model to demonstrate the contribution of the study. It is seen that the suggested model yields in lower system costs and lower order-up-to level.

Erkip, Hausman and Nahmias [8] establish a multi-echelon inventory system consisting of a depot and several retailers in which the depot does not hold inventory. This study is an extension of the Eppen and Schrage [1] model to the case of correlated demands. It is stated that the demands are correlated among retailers and through time. The correlated demand is modelled using an autoregressive process of demand. Applying the same method with Eppen and Schrage, they obtain an explicit expression for the optimal order-up-to level as a function of correlation coefficient. In the analysis, two assumptions are required. One of them is the allocation assumption which means that the depot receives sufficient material from the supplier to achieve an equal fractile of allocation at each retailer. The other one is the coefficient of variation assumption which means that the coefficients of variations of demands at the retailers are equal. At the end of the study, a numerical analysis is provided to indicate the effect of correlation. This study is important due to being the first correlated demand model for a multi-echelon inventory system.

2.2. Modelling Advance Demand Information

Hariharan and Zipkin [9] analyze a continuous review, single-unit system with customer demands following poisson process. Perfect demand information is obtained since customers place orders in advance for a future delivery and will not take the delivery before due dates. This study is important due to being the first demand lead time model. The analysis assumes that demand lead time is smaller than supply lead time. Both deterministic and stochastic lead times for supply and demand are considered. When there is not order cost, they show that an order-base-stock policy is optimal. But when a fixed order cost is included, an (s, S) policy is followed for optimality. The analysis concludes that demand lead times are the opposite of supply lead times, that is, supply lead time increases uncertainty while demand lead time reduces it.

Özer and Wei [10] study a periodic review, stochastic, capacitated, finite and infinite horizon production system with advance demand information. The authors model ADI in such a way that in period t, the customers can place orders for period t or for delivery in a future period $s \in t+1, ..., t+N$. So the demand for a future period s consists of observed and unobserved parts. The observed part consists of the demands that were placed for period s from period $s-N$ until the current period t. The unobserved part consists of the demands that will be placed for period s from current period until period s. For the model with zero fixed cost, they prove the optimality of a state-dependent modified base stock policy. If the inventory position is below the critical level, the manufacturer should produce to bring the inventory position up to the base stock level. For the model with positive fixed cost, the manufacturer is restricted to produce full capacity or not to produce. They show that a state-dependent threshold policy is optimal for this case. They also prove the optimality of these policies for infinite horizon case. They show that the advance demand information can be a substitute for capacity and inventory.

Zhu and Thonemann [11] analyze a supply chain with a single retailer and N identical customers where customer demands are normally distributed. The retailer has the opportunity to acquire demand information from some customers and to use this information to improve demand forecast. This study models and solves information sharing problem and generates structural insights into the optimal sharing policy and inventory policies. The model is analyzed under correlated demands and imperfect information. The research concludes that contacting all customers is optimal when information cost and demand correlation are small. If the information cost is high or if the cost is medium and demand correlation is low, then contacting no customer is optimal. Contacting some customers and using collected information to estimate the demands of customers that are not contacted is optimal if the information cost and the demand correlation are high.

Dellaert and Melo [12] study a stochastic single product, make-to-stock problem with partial customer-order information. They use partial customer-order information such that arriving orders are divided into categories according to their urgency. In otherwords, in a period customers place orders with different due dates varying from one to N. At the end of period t, the exact demand is known for period $t+1$ and partial information of demand for periods $t+2, ..., t+N$ is known. The authors model this problem using a Markov decision process. Since it is too complex to obtain optimal result, they propose two heuristics for the problem. The first one is the (x, T, w) policy where production is initialized when the required delivery of the period is at least x and the known demand for the next T periods is produced with an additional amount w for buffer stock. The second heuristic is least-cost-per-period (LCP) strategy. In this one they select the production amount that minimizes total relevant costs per period considering the production decision which is characterized by T and w. At the end, they make a numerical study and compare the heuristics with the optimal solution for small problems. They also compare the heuristics with (s, S) and (R, S) inventory policies.

Tan, Güllü and Erkip [13] model an inventory control system with imperfect advance customer demand information and periodic review ordering policies. Imperfect ADI means uncertain prospective future orders. In various ways, the system obtains the information of the number of customers interested in a product and treats each demand information as a prospective demand. So, they introduce notation such as p which is the probability of a prospective demand to be realized as demand in the next period and r that is the probability of an ADI to remain in the system without being realized. Using these probabilities and the number of collected information in incorporating ADI with ordering policies, they develop a dynamic cost model and show that optimal ordering policy is state-dependent order-up-to type where the order-up-to level is a function of ADI size k. In addition to the characterization of optimal ordering policy, they present an algorithm for computing optimal order-up-to level. Emprical tests confirm that imperfect ADI becomes beneficial under increased customer reliability level and increased variability in demand.

Gallego and Özer [14] analyze a discrete-time, single-item, single-location, periodic review inventory problem with advance demand information. Customers have different demand lead times such that at the end of period t, the system observes a demand vector consisting of demand for period t and demand information $D_{t,s}$ representing orders placed by customers during period t for a future period $s \in t+1, ..., t+N$ where N is the information horizon. The authors consider the demand information in two parts: observed part which has already been observed and unobserved part that is not yet known. They show that a state-dependent base stock policy is optimal in the case with zero setup cost and a state-dependent (s, S) policy is optimal for positive setup cost.

Karaesmen, Buzacott and Dallery [15] consider a make-to-stock production system consisting of manufacturing stage and a finished goods inventory where the capacity is limited and advance demand information is available. The supplier receives orders H periods in advance of their due dates. As the consequence of this assumption, the demands which have to be satisfied in the next H periods are known with certainty. They define the vector $D(t) = (D_1(t), D_2(t), ..., D_H(t))$ as the ADI vector. All the elements of this vector are either one or zero as the customer orders exactly one unit with probability q or does not order. An element $D_i(t)$ of the vector is the number of demands that will be satisfied at the end of period $t + i - 1$. The authors model this problem and state the structure of the optimal control policy which is threshold type.

Since it is not simple to obtain optimal result, they propose a simpler control policy which is an extension of the well-known base stock policy. They define a release lead time L that is used to regulate the timing of material release to the production and ADI is integrated to the policy through the usage of L. Then they find the optimal values of parameters L and S, where S is produce-up-to level to optimize inventory related costs. At the end of the study, they present a numerical study concluding that integrating ADI decreases costs in general and the proposed (S, L) policy is effective.

Thonemann [16] models a manufacturer- installer supply chain with N installers and I products where ADI is available. In this study two types of ADI is considered. In the case with A-ADI (aggregated ADI), customers share information with manufacturer whether they will place an order in the next period but to which manufacturer they will place an order for which product remains uncertain. In the case with D-ADI (detailed ADI), customers share information with manufacturer whether they will place an order; in addition to this information, they state which product they will order, but which manufacturer will receive the order remains uncertain. He develops a mathematical model and obtains an optimal inventory policy in which the base-stock policy is adjusted in each period according to the ADI obtained at the end of the previous period. It is first decided what base-stock level should be for period t and secondly how much to order. He develops expressions for the optimal base-stock levels with A-ADI and D-ADI, derives exact expression and closed-form approximations for the expected base-stock level and costs, and uses these approximations to identify under which condition ADI structure performs well.

Our model differs from the papers above by incorporating advance demand information into the Eppen and Schrage model. Eppen and Schrage [1] model is extended to incorporate various cases such as correlated demands and central inventory before, but not to incorporate advance demand information. The differences of our thesis from the literature are listed below:

1. We extend the Eppen and Schrage model into the case with perfect advance demand information, analyzing single order and multiple order policies.

2. We extend the Eppen and Schrage model into the case with partial advance demand information and provide approximate optimal ordering and allocating policies.
3. We extend our partial ADI model into the case with retailer dependent ADI amount and provide expected cost function for this model.
4. We extend our partial ADI model with retailer dependent ADI amount into the case where demand among retailers is assumed to be correlated and provide expected cost function for this model.

3. GENERAL DESCRIPTION OF THE MODEL

We consider a single item, multi-echelon distribution system where advance demand information is available. There are two echelons involved in the system: A depot supplied from an outside supplier and several retailers replenished from the depot. Constant lead times are considered during the shipments between the supplier and the depot and the shipments between the depot and the retailers. The customer demand is observed only at the retailers and the retailers are supplied from only the depot, transshipments among retailers are not allowed. The system we consider is illustrated in Figure 3.1. In this study, the multi-echelon inventory system is a centralized inventory system with a central decision maker. In our problem the depot has the role of making decisions. The depot possesses the demand information of each retailer placed by the customers before their due dates in every period and information about inventories of the product at the retailers. Taking the inventory level information and advance demand information into account, the depot decides to the order amount that will be placed to the supplier and the allocation of the inventory received from the supplier among the retailers. The depot is not allowed to hold inventory, it has a role of making decisions about ordering and rationing on the basis of the information obtained.

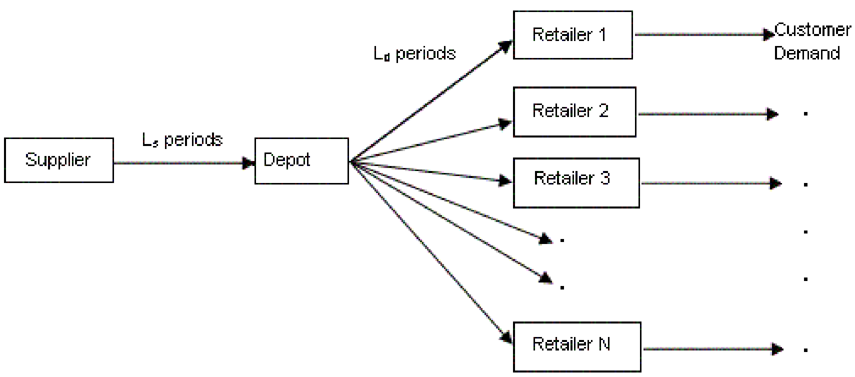

Figure 3.1. Multi-echelon inventory system

Two types of advance demand information structures are considered in our study: Perfect ADI and partial ADI. When perfect advance demand information is available, the customers give their orders in advance and the depot obtains full information about the demand that will be actually needed in the future. The time between the period in which the customer gives order and the period in which the customer actually needs it, is called demand lead time. We will denote demand lead time with l and the demand at retailer i in period t with $D_{i,t}$. When the information of $D_{i,t}$ is observed, we denote the realization of $D_{i,t}$ with $d_{i,t}$ to distinguish the demand notation. Customers place their orders at retailer i l periods before they actually need them and we denote the demand information of l periods with $\bar{D}_{i,t}$, which is a vector. So the demand vector has l entries corresponding to the demands for the next l periods when the system is at the beginning of period t in the planning horizon. We state the demand vector explicitly as below:

$$\bar{D}_{i,t} = (d_{i,t}, d_{i,t+1}, \ldots\ldots\ldots\ldots, d_{i,t+l-1})$$

In period t, the demands for periods $t, t+1, ..., t+l-1$ are known but the demands for periods from $t+l$ until the end of the planning horizon are random. At the beginning of period $t+1$, the information of $D_{i,t+l}$ is observed. Then it is denoted as $d_{i,t+l}$ and the demand vector again has the same number of entries, $(d_{i,t+1}, \ldots\ldots\ldots, d_{i,t+l-1})$ coming from the previous period and $(d_{i,t+l})$ coming from the current period; the new demand vector becomes

$$\bar{D}_{i,t+1} = (d_{i,t+1}, \ldots\ldots\ldots, d_{i,t+l-1}, d_{i,t+l}).$$

Therefore, demand for l periods will always be deterministic for retailer i. We analyze this perfect ADI model in Chapter 4.

In the second type of ADI, partial advance demand information is available and the total demand in period t is denoted as $D_t = \mu + \sqrt{y}A_t + \sqrt{1-y}B_t$. This type of demand information is introduced by Kunnumkal and Topaloğlu [17]. In this demand structure there are three parts. Firstly, μ is the mean demand per period and this is the fixed part of the ADI structure that exists in every period. A_t and B_t are independent normally distributed random variables with mean zero and standard deviation σ. The second part of the structure A_t is observed l periods in advance. This is the part of the demand structure that is known before due date. The third part of the ADI structure is B_t. It becomes known in period t when it is actually needed. This partial ADI model is analyzed in Chapter 5. In this demand structure, y is a parameter indicating the degree of demand knowledge. It takes values between zero and one. When the value of y is zero, the demand structure takes the form $D_t = \mu + B_t$. It means that advance demand information is not obtained. We just know μ in advance and B_t is observed in period t when it is actually needed. When the value of y equals to one, the demand structure takes the form $D_t = \mu + A_t$. In this case, the demand of period t is observed l periods in advance which means perfect advance demand information. This type of demand structure is motivated by the assumption that the customer demand in period t is the sum of the demands generated by K independent sources as expressed by Kunnumkal and Topaloğlu [17]. The information that whether they will generate a unit of demand in period t is acquired from a fraction of the sources in period $t - l$. q is introduced as the probability that a particular source generates demand and y is introduced as the fraction of the sources that the customer acquires ADI. So $D_t = A_t + B_t$ where A_t and B_t are binomially distributed variables with parameters (q, Ky) and $(q, K[1-y])$. If K is large, A_t and B_t are approximated by normally distributed random variables with parameters $(qKy, \sqrt{q[1-q]Ky})$ and $(qK[1-y], \sqrt{q[1-q]K[1-y]})$. Introducing μ as qK and σ as $\sqrt{q[1-q]K}$, $D_t = \mu + \sqrt{y}A_t + \sqrt{1-y}B_t$ is obtained.

When customers place orders to the retailers in advance, the depot also observes the demand in advance and makes the decisions of ordering and allocating according to advance demand information. To see how ADI facilitates supply chain management and effects inventory related costs and inventory control policy, we extend the Eppen and Schrage model incorporating it with perfect and partial advance demand information. Our aim is to solve this model and find optimal quantity ordered by the depot to supply the retailers and optimum allocation of this quantity among the retailers with the objective of minimizing expected total inventory related costs.

The notation needed along with the notation already introduced are summarized below:

N : number of retailers

l : demand lead time (a constant)

L_d : lead time (a constant) from the depot to retailers

L_s : lead time (a constant) from the supplier to the depot

Q : quantity ordered by the depot at the beginning of the cycle (parameter to be optimized)

z_i : stock allocated to retailer i (on hand inventory at retailer i is not included)

NI_i : net inventory on hand at retailer i at the beginning of the cycle

$X_{i,t}$: inventory position of retailer i in period t, after observing the demand information and after allocation of any quantity received by the depot

$D_{i,t}$: demand at retailer i in period t, a normally distributed random variable

$d_{i,t}$: realization of $D_{i,t}$, which is observed in period t-l

$\bar{D}_{i,t}$: demand vector of retailer i in period t having l entries corresponding to the demands for the next l periods

h : holding cost per unit per period

b : penalty cost per unit per period

μ_i : mean demand per period at retailer i

σ_i : standard deviation in demand per period at retailer i

In this thesis we consider a single-cycle system except in one chapter. In Section 4.2, the depot will be assumed to order every period and the model will be analyzed under multiple order periods. In the single-cycle system, the depot places an order only at the beginning of the cycle. At the end of the cycle, inventory related costs consisting of holding and backorder costs are incurred. Since all unsatisfied demand is backordered, the backordered amount at the end of the cycle causes backorder costs and the leftover inventory at the end of the cycle causes holding costs. We ignore ordering costs and other types of costs. We asume linear and fixed holding and backorder costs for each retailer.

We consider that the events occur in a cycle of length $L_s + L_d + 1$ and the depot is allowed to place an order only once. Since the events do not occur with the same order in every period, we divide the cycle into parts to analyze the events occuring in the cycle. Please see Figure 3.2.

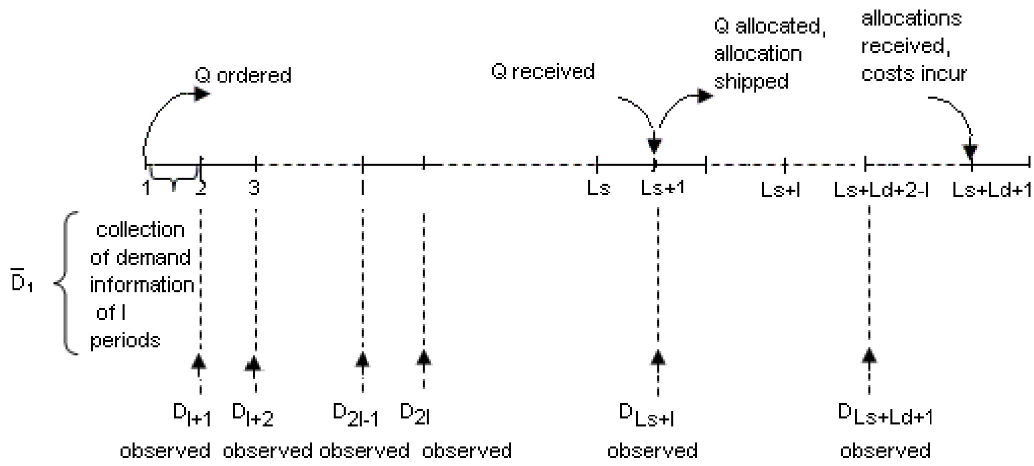

Figure 3.2. The events occur in the planning horizon

In the first period the following events occur:

1. The depot observes advance demand vector.
2. The depot updates the state of the system on the basis of ADI.
3. The depot places an order of size Q with the supplier.

The order of events take place from the second period until period $L_s + 1$:

1. The depot observes new advance demand information.
2. The depot updates the state of the system on the basis of ADI.

The order of events take place during period $L_s + 1$:

1. The depot observes new advance demand information.
2. The depot receives the order amount Q from the supplier and allocates it among the retailers.
3. The depot updates the state of the system on the basis of ADI.

The order of events take place from period L_s+2 until period $L_s+L_d+1-l+1$ is the same with the second part. In period $L_s + L_d + 1 - l + 1$, the last advance demand information is obtained.

The order of events take place during period $L_s + L_d + 1$:

1. The retailers receive the allocated amount.
2. The depot updates the state of the system on the basis of ADI.
3. The inventory related costs incur.

4. ANAYSIS OF THE MODEL WITH PERFECT DEMAND INFORMATION

In this chapter we present the model of the single-item, two-echelon distribution system with perfect demand information, which we introduced in the previous chapter. The model with perfect ADI will be analyzed under two different cases each in seperate sections. In the first one we study single cycle problem with a single order and in the second one we study multiple order problem where order every period policy is available. However, only the single order policy is considered in the partial ADI model.

In our study, the number of periods in the planning horizon is $L_s + L_d + 1$. The depot places an order at the beginning of the cycle, the order amount is received in time period $L_s + 1$ and it is allocated among retailers. The retailers receive the shipments in time period $L_s + L_d + 1$. Then at the end of the cycle in period $L_s + L_d + 1$, the inventory related costs are incurred.

We can express the problem that will be solved as follows:

$$min \sum_{i=1}^{N} \Bigg\{ hE \left[X_{i,L_s} + z_i - d_{i,L_s+l} - \sum_{t=L_s+l+1}^{L_s+L_d+1} D_{i,t} \right]^+ + bE \left[\sum_{t=L_s+l+1}^{L_s+L_d+1} D_{i,t} + d_{i,L_s+l} - X_{i,L_s} - z_i \right]^+ \Bigg\}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} z_i = Q$$
$$z_i \geq 0$$

In the problem above; at the beginning of period $L_s + 1$, the new demand information is observed, the order amount is received by the depot and it is allocated among retailers. After this allocation, the inventory position of retailer is expressed as $X_{i,L_s} + z_i - d_{i,L_s+l}$.

Dropping the demands for periods from $L_s + l + 1$ until the end of the planning horizon from the inventory position at period $L_s + 1$ gives the inventory position of retailer i at the end of the cycle. So it is clear that the objective function is stated as the minimization of holding cost caused by the leftover inventory at the end of the planning horizon and penalty cost caused by the unsatisfied demand at the end of the planning horizon. There are two constraints we have to take into account while minimizing the costs. First one ensures that all of the order amount is allocated among retailers and the other one ensures that nonnegative allocations are achieved which means there is no shipments among retailers. In our solution, we ignore the nonnegativity constraint and assume that it holds. For a discussion of relaxing the nonnegativity constraint please see Eppen and Schrage [1].

We note that this study is trivial for a problem with $L_s + L_d + 1 \leq l$. In such a problem, there is no uncertainty throughout the cycle; because, the customer demand for the whole cycle is obtained with the advance demand information. In single order case, the system can satisfy customer demand by placing an order amount that is equal to the demand of the whole cycle. In order every period case, the system can match customer demand by adjusting the times of orders. Hence we focus on the case of $L_s + L_d + 1 > l$. L_s, L_d and l can take values that are smaller or larger than each other but in this study we consider the case of $L_s > l$ and $L_d > l$ for notational simplicity. This makes sense, as in a real application we would expect the supply lead times to be larger than the demand lead times.

There is one more point to note that the solution of the problem expressed above is not optimum but approximate because we make an assumption called allocation assumption in Eppen and Schrage [1]. Since we extend Eppen and Schrage model incorporating it with different types of advance demand information, we make the same allocation assumption as in Eppen and Schrage. In period $L_s + 1$, the depot receives the amount which is ordered at the beginning of the cycle and allocates it among retailers. We assume that the depot receives a sufficient amount so that the same stockout probability in all retailers is ensured. This assumption enables us to obtain explicit epressions.

Our aim is to solve the model in two steps. In the first step we find optimum allocation policy for a given order amount Q. This means we pretend as the system is in time period L_s+1 having the demand information of the periods from the beginning until period L_s+l (the demand information of the period L_s+l is included). In the second step we will go back to the beginning of the cycle and find the optimum quantity to order using the information of optimal allocation policy. In this step, differently from the first one, demands of the periods from $l+1$ until the end of the cycle are unknown. So, the randomness of the demand expressions depends on the period which we look to the system from. We distinguish the randomness of demand by using uppercase letter for random and lowercase letter for known demand.

4.1. Analysis of the Single Order Model

In our single-cycle problem, the objective is to find optimum order amount and optimum allocation policy minimizing the expected inventory related costs at the end of the planning horizon. For this type of single-cycle problem we consider a finite planning horizon with the length of L_s+L_d+1 periods. The single-cycle problem with perfect advance demand information is illustrated in Figure 4.1.

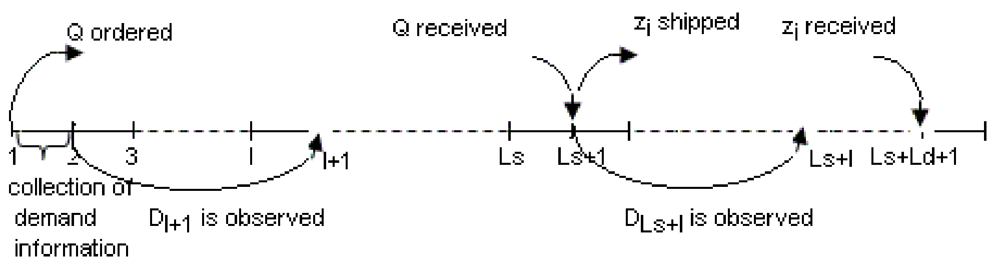

Figure 4.1. Single-cycle problem with perfect ADI

As we can see from Figure 4.1, the events occuring in the cycle begins at the beginning of the first period with observing the demand information for the next l periods. The depot updates the inventory position of the total system using the advance demand information as follows:

$$\sum_{i=1}^{N} X_{i,1} = \sum_{i=1}^{N} NI_i - \sum_{i=1}^{N}\sum_{t=1}^{l} d_{i,t}$$

Taking the demand information into account, the depot places an order of Q. The retailers satisfy customer demands from on hand inventory. In the second period, new demand information is obtained, customer demand of the period is satisfied and the inventory positions of retailers are updated. These events occur in the same order until period $L_s + 1$ and the inventory position of retailer i is updated as follows:

$$
\begin{aligned}
&X_{i,2} = X_{i,1} - d_{i,1+l} \\
&X_{i,3} = X_{i,2} - d_{i,2+l} \\
&\vdots \\
&X_{i,L_s} = X_{i,L_s-1} - d_{i,L_s+l-1}
\end{aligned}
$$

In period $L_s + 1$; firstly, new demand information is observed. After the demand observation, the depot receives the order amount Q from the supplier and allocates it among the retailers. After the allocation, the inventory position of retailer i is updated as below:

$$
X_{i,L_s+1} = NI_i - \sum_{t=1}^{L_s+l} d_{i,t} + z_i \tag{4.1}
$$

In the last period, the retailers receive the allocated amounts, satisfy customer demand and then finally the inventory related costs are incurred. The inventory position of retailer i at the end of the cycle is as follows:

$$
\begin{aligned}
X_{i,L_s+L_d+1} &= X_{i,L_s+1} - \sum_{t=L_s+l+1}^{L_s+L_d+1} d_{i,t} \\
&= NI_i - \sum_{t=1}^{L_s+L_d+1} d_{i,t} + z_i
\end{aligned} \tag{4.2}
$$

In the first step of our solution approach, our aim is to find optimum allocation for a given Q. To achive this, we pretend as the system is in time period $L_s + 1$. In period $L_s + 1$, we have demand information of periods until period $L_s + l$ (the demand information of period $L_s + l$ is included). Here, optimum allocation should be such that a certain no stockout probability is achieved at each retailer. We can express this as follows:

$$P\left(X_{i,L_s+L_d+1} \geq 0\right) = \beta$$
$$P\left(X_{i,L_s+1} - \sum_{t=L_s+l+1}^{L_s+L_d+1} D_{i,t} \geq 0\right) = \beta$$
$$P\left(\sum_{t=L_s+l+1}^{L_s+L_d+1} D_{i,t} \leq X_{i,L_s+1}\right) = \beta$$

where β is the no stockout probability. Since the demands are normally distributed, we are able to write

$$P\left(\sum_{t=L_s+l+1}^{L_s+L_d+1} D_{i,t} \leq X_{i,L_s+1}\right) = \Phi\left(\frac{X_{i,L_s+1} - E\left[\sum_{t=L_s+l+1}^{L_s+L_d+1} D_{i,t}\right]}{\sqrt{Var\left(\sum_{t=L_s+l+1}^{L_s+L_d+1} D_{i,t}\right)}}\right) = \Phi(v)$$

where v is the β-th percentile of standard normal distribution $v = \Phi^{-1}(\beta)$. So we can write

$$X_{i,L_s+1} = NI_i - \sum_{t=1}^{l} d_{i,t} - \sum_{t=l+1}^{L_s+l} d_{i,t} + z_i = E\left[\sum_{t=L_s+l+1}^{L_s+L_d+1} D_{i,t}\right] + v\sqrt{Var\left(\sum_{t=L_s+l+1}^{L_s+L_d+1} D_{i,t}\right)}.$$

Normal demand enables us to write the equation above as

$$NI_i - \sum_{t=1}^{l} d_{i,t} - \sum_{t=l+1}^{L_s+l} d_{i,t} + z_i = \left(L_d + 1 - l\right)\mu_i + v\sigma_i\sqrt{L_d + 1 - l}. \tag{4.3}$$

Since the allocation assumption ensures that sufficient quantity is received by the depot to achieve the same stockout probability in each retailer, we are able to use the same no stockout probability for each retailer. So we can write

$$\sum_{i=1}^{N}\left[NI_i - \sum_{t=1}^{l} d_{i,t} - \sum_{t=l+1}^{L_s+l} d_{i,t}\right] + Q = \sum_{i=1}^{N}\left[\left(L_d + 1 - l\right)\mu_i + v\sigma_i\sqrt{L_d + 1 - l}\right]$$

where $Q = \sum_{i=1}^{N} z_i$. Solving the equation above for v, we obtain

$$v = \frac{\sum_{i=1}^{N} \left[NI_i - \sum_{t=1}^{l} d_{i,t} - \sum_{t=l+1}^{L_s+l} d_{i,t} \right] + Q - \sum_{i=1}^{N} (L_d + 1 - l)\, \mu_i}{\sum_{i=1}^{N} \sigma_i \sqrt{L_d + 1 - l}}.$$

Substituting v in equation (4.3) and solving equation (4.3) for z_i gives optimum allocation policy z_i^* which is below:

$$\begin{aligned} z_i^* = {} & (L_d + 1 - l)\mu_i - NI_i + \sum_{t=1}^{l} d_{i,t} + \sum_{t=l+1}^{L_s+l} d_{i,t} \\ & + \left\{ \sum_{j=1}^{N} \left[NI_j - \sum_{t=1}^{l} d_{j,t} - \sum_{t=l+1}^{L_s+l} d_{j,t} - (L_d + 1 - l)\mu_j \right] + Q \right\} \sigma_i / \sum_{j=1}^{N} \sigma_j \end{aligned}$$

While the amount Q is being allocated in period $L_s + 1$, demands for the periods from $L_s + l + 1$ until the end of the cycle are not observed yet. As it is seen above, the optimum allocation includes the mean demand of this unobserved part of the cycle. If the inventory on hand at the beginning of the cycle is not sufficient to satisfy the observed part of the cycle which refers to the demands for the periods until period $L_s + l + 1$, then the inventory position before the allocation in period $L_s + 1$ is negative. This means that an additional amount is required to satisfy customer demand at the end of the cycle and this amount is included in allocation policy. If the inventory position before the allocation in period $L_s + 1$ is positive, then there is inventory on hand in that period. This means that the requirement to satisfy the mean demand of the unobserved part is as less as the inventory on hand in period $L_s + 1$. Besides these, the allocation policy includes the amount which is the inventory at the end of the cycle allocated among retailers proportional to their standard deviations.

We now know how to allocate the order amount among retailers and from now the question is how much to order to minimize inventory related costs. From now, the system is at the beginning of the cycle and only the $\bar{D}_{i,1} = (d_{i,1}, d_{i,2}, \ldots\ldots\ldots\ldots, d_{i,l})$ is available. Demands for the periods from $l+1$ until the end of the cycle are unknown. At the end of the planning horizon while the inventory related costs are being incurred, we incur one-period holding and penalty costs of period $L_s + L_d + 1$. We consider identical holding and penalty costs for each retailer. Let C_i be the expected one-period inventory cost of retailer i, C_i can be expressed as

$$C_i = \int_0^{\infty} hx dF_{X_{i,L_s+L_d+1}}(x) - \int_{-\infty}^{0} bx dF_{X_{i,L_s+L_d+1}}(x)$$

where $F_{X_{i,L_s+L_d+1}}$ is the distribution function of X_{i,L_s+L_d+1}.

Under the optimum allocation policy; substituting z_i^* into equation (4.2), the inventory level of retailer i at the end of the cycle, X_{i,L_s+L_d+1}, is obtained as

$$\begin{aligned}(L_d + 1 - l)\mu_i - &\sum_{t=L_s+l+1}^{L_s+L_d+1} D_{i,t} \\ &+ \left\{ \sum_{j=1}^{N} \left[NI_j - \sum_{t=1}^{l} d_{j,t} - \sum_{t=l+1}^{L_s+l} D_{j,t} - (L_d + 1 - l)\mu_j \right] + Q \right\} \sigma_i / \sum_{j=1}^{N} \sigma_j .\end{aligned}$$

Note that X_{i,L_s+L_d+1} consists of constant and variable parts. We define $X_{i,L_s+L_d+1} = a_i - \varsigma_i$ where the scalar part is

$$a_i = (L_d + 1 - l)\mu_i + \left\{ \sum_{j=1}^{N} \left[NI_j - \sum_{t=1}^{l} d_{j,t} - (L_d + 1 - l)\mu_j \right] + Q \right\} \sigma_i / \sum_{j=1}^{N} \sigma_j$$

and the variable part is

$$\varsigma_i = \sum_{t=L_s+l+1}^{L_s+L_d+1} D_{i,t} + \sum_{j=1}^{N} \sum_{t=l+1}^{L_s+l} D_{j,t}\sigma_i / \sum_{j=1}^{N} \sigma_j .$$

The C_i can now be written as

$$C_i = \int_0^{a_i} h(a_i - x) dF_{\varsigma_i}(x) + \int_{a_i}^{\infty} b(x - a_i) dF_{\varsigma_i}(x)$$

where F_{ς_i} is c.d.f. of ς_i. Since

$$\frac{\partial C_i}{\partial a_i} = hF_{\varsigma_i}(a_i) - b(1 - F_{\varsigma_i}(a_i)),$$

we can say that the minimum C_i is acquired when $F_{\varsigma_i}(a_i) = b/(b+h)$.

We now find the order amount Q which minimizes C_i. Normal demand enables us to write

$$a_i = E[\varsigma_i] + u\sqrt{Var(\varsigma_i)} \tag{4.4}$$

where u is the $b/(b+h)$th percentile of standard normal distribution. Since the demand at retailer i is normally distributed with parameters μ_i and σ_i, ς_i is normally distributed with parameters

$$\hat{\mu_i} = (L_d + 1 - l)\mu_i + (L_s \sum_{j=1}^{N} \mu_j)\frac{\sigma_i}{\sum\limits_{j=1}^{N} \sigma_j}$$

and

$$\hat{\sigma_i} = \sqrt{(L_d + 1 - l)\sigma_i^2 + (L_s \sum_{j=1}^{N} \sigma_j^2)\left(\frac{\sigma_i}{\sum\limits_{j=1}^{N} \sigma_j}\right)^2}.$$

Solving equation (4.4) for u and substituting the parameters yields

$$u = \frac{Q + \sum\limits_{j=1}^{N}\left[NI_j - \sum\limits_{t=1}^{l} d_{j,t} - (L_s + L_d + 1 - l)\mu_j\right]}{\sqrt{L_s \sum\limits_{j=1}^{N} \sigma_j^2 + (L_d + 1 - l)(\sum\limits_{j=1}^{N} \sigma_j)^2}}.$$

The optimum order amount Q^* which minimizes total inventory cost can be expressed as

$$Q^* = -\sum_{j=1}^{N} NI_j + \sum_{j=1}^{N}\sum_{t=1}^{l} d_{j,t} + \sum_{j=1}^{N}(L_s + L_d + 1 - l)\mu_j$$
$$+ u\sqrt{L_s \sum_{j=1}^{N}\sigma_j^2 + (L_d + 1 - l)(\sum_{j=1}^{N}\sigma_j)^2}.$$

At the beginning of the cycle, demands for the next l periods are observed. The depot considers this advance demand information and the mean demand of the unobserved part of the cycle while it is making the order amount decision. Since the inventory on hand at the beginning of the cycle satisfies a part of the customer demand, the order amount is as less as the inventory the system has at the beginning of the cycle. The optimum order amount also includes the standard deviation of the demand beyond demand lead time multiplied by standard normal variable.

The optimum order amount expression makes sense because as it is seen above, the length of the planning horizon which we have to take the variance of into account decreases from $L_d + 1$ to $L_d + 1 - l$. Because the demand of l periods is known before order is placed at the beginning of the cycle. So the uncertainty of the cycle decreases l periods which is defined as demand lead time.

4.2. Analysis of the Multiple Order Periods Model

Differently from the analysis above, we will now analyze the supply chain under order every period policy. We require some extra notation in this section which are:

$z_{i,t}$: the allocation quantity of the order amount that is ordered in period t, received and allocated by the depot in period $t + L_s$

Q_t : the order amount that is placed by the depot in period t

S : order-up-to level to which an order is placed to bring the inventory position up

In order every period policy, the following events occur in time period t:

1. The depot observes new demand information.
2. The depot receives the replenisment that is ordered L_s periods ago and allocates it among the retailers.
3. The depot updates the state of the system on the basis of ADI.
4. The depot places a replenisment order Q_t with the supplier.
5. The inventory related costs are incurred.

In this model, the planning horizon is not limited with $L_s + L_d + 1$ periods. Suppose that we are looking to the system from period one and considering the effect of the decision made in this period. The depot places an order in period one, the ordered amount Q_1 is received in time period L_s+1 and it is allocated among retailers. The retailers receive the shipments in time period L_s+L_d+1 and the inventory related costs are incurred. So the inventory cost of the decision made in period one is incurred in period $L_s + L_d + 1$. The future costs are not discounted. Here, we note that, besides the events occuring between period one and period $L_s + L_d + 1$, the depot and the retailers keep receiving the amounts that were already ordered. This system is explicitly shown in Figure 4.2.

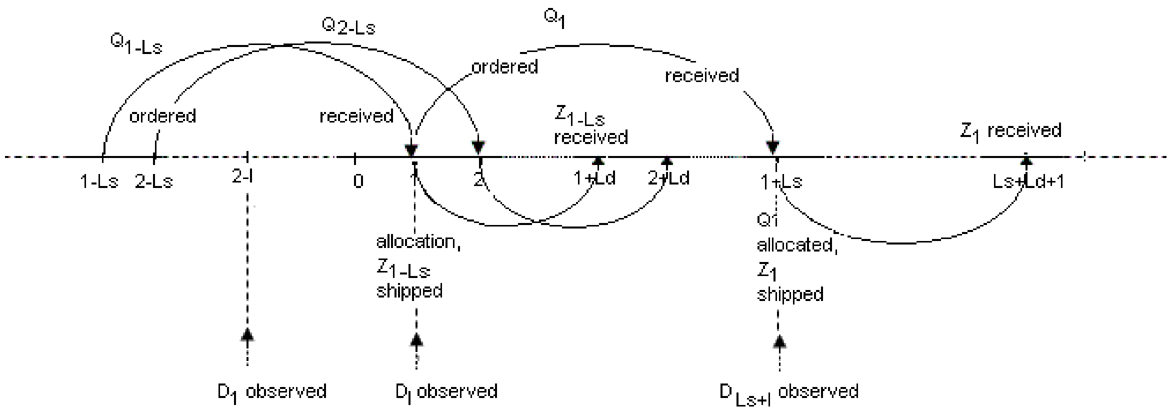

Figure 4.2. Multiple order system

In period one, firstly new advance demand information is obtained. Then the amount ordered in period $1 - L_s$ is received and it is allocated among retailers. So the inventory position of the system in period one can be expressed as follows:

$$\sum_{i=1}^{N} X_{i,1} = \sum_{i=1}^{N} [X_{i,0} + z_{i,1-L_s} - d_{i,l}]$$

The depot places an order to bring the system inventory up to S where $S = Q_1 + \sum_{i=1}^{N} X_{i,1}$. We can update inventory position of the system until period $L_s + 1$ as

$$\sum_{i=1}^{N} X_{i,2} = \sum_{i=1}^{N} [X_{i,1} + z_{i,2-L_s} - d_{i,l+1}]$$
$$\vdots$$
$$\sum_{i=1}^{N} X_{i,L_s} = \sum_{i=1}^{N} [X_{i,L_s-1} + z_{i,0} - d_{i,L_s+l-1}] .$$

The inventory position of the system in period $L_s + 1$ is

$$\sum_{i=1}^{N} X_{i,L_{s+1}} = \sum_{i=1}^{N} [X_{i,L_s} + z_{i,1} - d_{i,L_s+l}]$$

which is equal to

$$\sum_{i=1}^{N} X_{i,L_{s+1}} = \sum_{i=1}^{N} \left[X_{i,0} + \sum_{t=1-L_s}^{1} z_{i,t} - \sum_{t=l}^{L_s+l} d_{i,t} \right]$$

where $\sum_{i=1}^{N} z_{i,1} = Q_1$. So we can write

$$\sum_{i=1}^{N} X_{i,L_{s+1}} = \sum_{i=1}^{N} \left[X_{i,0} + \sum_{t=1-L_s}^{0} z_{i,t} - \sum_{t=l}^{L_s+l} d_{i,t} \right] + Q_1 .$$

Since $Q_1 = S - \sum_{i=1}^{N} X_{i,1}$ and $\sum_{i=1}^{N} X_{i,1} = \sum_{i=1}^{N} [X_{i,0} + z_{i,1-L_s} - d_{i,l}]$,

$$\sum_{i=1}^{N} X_{i,L_{s+1}} = S + \sum_{i=1}^{N} \left[\sum_{t=2-L_s}^{0} z_{i,t} - \sum_{t=l+1}^{L_s+l} d_{i,t} \right] .$$

Normal demands and equal fractile allocation enable us to write

$$\sum_{i=1}^{N} X_{i,L_{s+1}} = \sum_{i=1}^{N} \left[(L_d - l + 1)\mu_i + v\sigma_i \sqrt{L_d - l + 1} \right]$$
$$= S + \sum_{i=1}^{N} \left[\sum_{t=2-L_s}^{0} z_{i,t} - \sum_{t=l+1}^{L_s+l} d_{i,t} \right]. \quad (4.5)$$

Solving the equation above for v we obtain

$$v = \frac{S + \sum_{i=1}^{N} \left[\sum_{t=2-L_s}^{0} z_{i,t} - \sum_{t=l+1}^{L_s+l} d_{i,t} \right] - \sum_{i=1}^{N} (L_d - l + 1)\mu_i}{\sum_{i=1}^{N} \sigma_i \sqrt{L_d - l + 1}}.$$

Substituting v into equation (4.5) yields

$$X_{i,L_{s+1}} = (L_d - l + 1)\mu_i + \frac{\left\{ S + \sum_{j=1}^{N} \left[\sum_{t=2-L_s}^{0} z_{j,t} - \sum_{t=l+1}^{L_s+l} d_{j,t} - (L_d - l + 1)\mu_j \right] \right\} \sigma_i}{\sum_{j=1}^{N} \sigma_j}$$

which includes the items that were already at retailer i and the new items that are allocated to retailer i.

Using the same method with the previous section, we define stock level of retailer i at the end of the period $L_s + L_d + 1$ which is denoted by X_{i,L_s+L_d+1} as

$$(L_d - l + 1)\mu_i - \sum_{t=L_s+l+1}^{L_s+L_d+1} D_{i,t}$$
$$+ \frac{\left\{ S + \sum_{j=1}^{N} \left[\sum_{t=2-L_s}^{0} z_{j,t} - \sum_{t=l+1}^{L_s+l} D_{j,t} \right] - \sum_{j=1}^{N} (L_d - l + 1)\mu_j \right\} \sigma_i}{\sum_{j=1}^{N} \sigma_j}.$$

The scalar part of X_{i,L_s+L_d+1} is denoted with a_i which is

$$(L_d - l + 1)\mu_i + \frac{\left\{S + \sum\limits_{j=1}^{N}\left[\sum\limits_{t=2-L_s}^{0} z_{j,t} - (L_d - l + 1)\mu_j\right]\right\}\sigma_i}{\sum\limits_{j=1}^{N}\sigma_j}$$

and the random part of X_{i,L_s+L_d+1} is denoted with ς_i which is

$$\sum_{t=L_s+l+1}^{L_s+L_d+1} D_{i,t} + \frac{\left\{\sum\limits_{j=1}^{N}\sum\limits_{t=l+1}^{L_s+l} D_{j,t}\right\}\sigma_i}{\sum\limits_{j=1}^{N}\sigma_j}.$$

Since the demand at retailer i is normally distributed with parameters μ_i and σ_i, ς_i is normally distributed with parameters

$$\hat{\mu_i} = (L_d + 1 - l)\mu_i + (L_s \sum_{j=1}^{N}\mu_j)\frac{\sigma_i}{\sum\limits_{j=1}^{N}\sigma_j}$$

and

$$\hat{\sigma_i} = \sqrt{(L_d + 1 - l)\sigma_i^2 + (L_s \sum_{j=1}^{N}\sigma_j^2)\left(\frac{\sigma_i}{\sum\limits_{j=1}^{N}\sigma_j}\right)^2}.$$

The expected one-period inventory cost at retailer i at the end of the cycle, C_i, can be written as

$$C_i = \int_0^{a_i} h(a_i - x)dF_{\varsigma_i}(x) + \int_{a_i}^{\infty} b(x - a_i)dF_{\varsigma_i}(x) \tag{4.6}$$

where F_{ς_i} is cumulative distribution function of ς_i. Normal demand enables us to write $a_i = E[\varsigma_i] + u\sqrt{Var(\varsigma_i)}$ where u is the $b/(b+h)$th percentile of standard normal distribution. Solving the expression, we can find S at which minimum C_i is achieved. Substituting the parameters we obtain

$$u = \frac{S + \sum_{j=1}^{N} \left[\sum_{t=2-L_s}^{0} z_{j,t} - (L_s + L_d + 1 - l)\mu_j \right]}{\sqrt{L \sum_{j=1}^{N} \sigma_j^2 + (L_d + 1 - l)(\sum_{j=1}^{N} \sigma_j)^2}}.$$

Solving the equation above for S, the optimum order-up-to level S^* which minimizes C_i is obtained as follows:

$$S^* = -\sum_{j=1}^{N} \left[\sum_{t=2-L_s}^{0} z_{j,t} - (L_s + L_d + 1 - l)\mu_j \right] + u\sqrt{L \sum_{j=1}^{N} \sigma_j^2 + (L_d + 1 - l)(\sum_{j=1}^{N} \sigma_j)^2}.$$

Since the expression above does not depend on i, the obtained S^* minimizes expected system-wide inventory cost which can be expressed as $\sum_{i=1}^{N} C_i$. In period one, the depot uses the information of inventory position of the system and places Q_1 to bring the inventory position up to optimum order-up-to level S^*. So we can find the optimum order amount of period one dropping the inventory position of the system in period one from S^*, which yields

$$\begin{aligned} Q_1 &= S^* - \sum_{j=1}^{N} X_{j,1} \\ &= \sum_{j=1}^{N} \left[-X_{j,0} - \sum_{t=1-L_s}^{0} z_{j,t} + d_{j,l} + (L_s + L_d + 1 - l)\mu_j \right] \\ &\quad + u\sqrt{L \sum_{j=1}^{N} \sigma_j^2 + (L_d + 1 - l)(\sum_{j=1}^{N} \sigma_j)^2}. \end{aligned}$$

The difference of the expression above from the result of single-cycle problem is the existence of $X_{j,0}$ and $\sum_{t=1-L_s}^{0} z_{j,t}$. We can consider $X_{j,0}$ as the beginning inventory of the single-cycle problem. Also, $\sum_{t=1-L_s}^{0} z_{j,t}$ makes sense. Because the depot keeps receiving previously ordered amounts until period $L_s + 1$; so while making the order decision in period one considering the inventory costs that will be incurred in $L_s + L_d + 1$, the depot have to take $\sum_{t=1-L_s}^{0} z_{j,t}$ into account.

5. ANALYSIS OF THE MODEL WITH PARTIAL DEMAND INFORMATION

In this chapter we will analyze the supply chain model for a different advance demand information structure. We consider partial advance demand information for single-cycle problem where the depot places an order at the beginning of the planning horizon and is not allowed to place one more order. We will find optimum order amount and optimum allocation policy to minimize inventory costs for this problem. The considered advance demand information is denoted as

$$D_{i,t} = \mu_i + \sqrt{y}A_{i,t} + \sqrt{1-y}B_{i,t}$$

where μ_i is the mean demand per period for retailer i, $A_{i,t}$ and $B_{i,t}$ are independent normally distributed random variables with mean zero and standard deviation σ_i and y is a parameter taking values between zero and one. This type of demand structure is introduced by Kunnumkal and Topaloğlu [17]. In this type of customer demand, a part of the demand information is acquired in advance, that is $A_{i,t}$. The value of the random variable $A_{i,t}$ becomes known in period $t-l$ and the value of random variable $B_{i,t}$ becomes known in period t when it is actually needed. The system under such a type of demand information is illustrated in Figure 5.1. In this structure y is the parameter indicating the amount of advance demand information. Here we can express the mean and the standard deviation of this demand structure in two seperate process. If the system is in somewhere before period $t-l$, demand has the mean μ_i and the standard deviation σ_i; if the system is in somewhere between $t-l$ and t, demand has the mean $\mu_i + \sqrt{y}A_{i,t}$ and the standard deviation $\sigma_i\sqrt{1-y}$.

This type of demand structure is motivated by the assumption that the customer demand observed by retailer i in period t is the sum of the demands generated by K_i independent sources as described by Kunnumkal and Topaloğlu [17]. The information that whether they will generate a unit of demand in period t is acquired from a

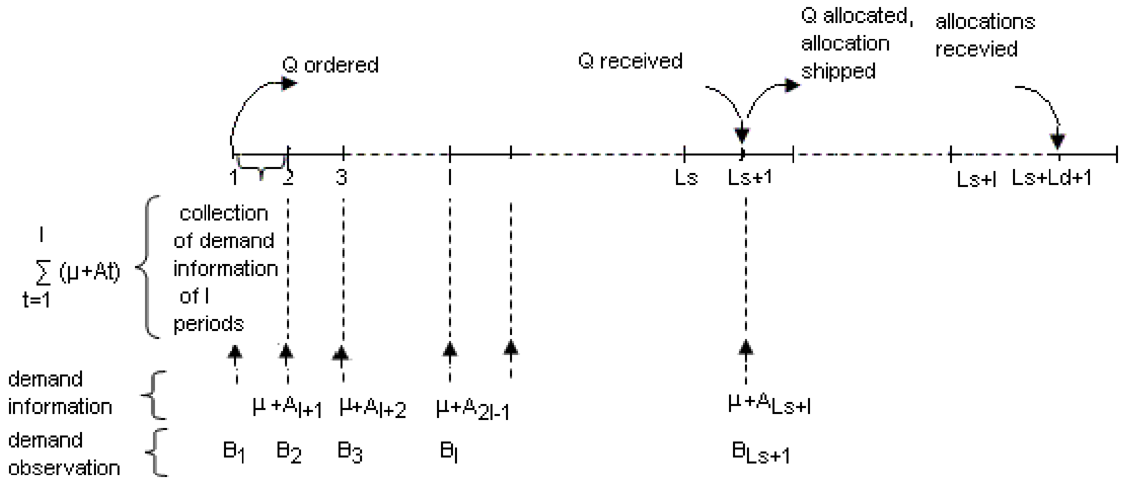

Figure 5.1. Single-cycle problem with partial ADI

fraction of the sources in period $t - l$. q is introduced as the probability that a particular source generates demand and y is introduced as the fraction of the sources that the customer acquires ADI. So $D_i, t = A_{i,t} + B_{i,t}$ where $A_{i,t}$ and $B_{i,t}$ are binomially distributed variables with parameters $(q, K_i y)$ and $(q, K_i[1 - y])$. If K_i is large, $A_{i,t}$ and $B_{i,t}$ are approximated by normally distributed random variables with parameters $(qK_i y, \sqrt{q[1-q]K_i y})$ and $(qK_i[1-y], \sqrt{q[1-q]K_i[1-y]})$. Introducing μ as qK_i and σ as $\sqrt{q[1-q]K_i}$, $D_{i,t} = \mu_i + \sqrt{y}A_{i,t} + \sqrt{1-y}B_{i,t}$ is obtained.

In this model, since a part of the demand is observed in advance and the other part is observed in its due date, the order of events is a bit different from the previous chapter. Here we have to state the order of the observation of $B_{i,t}$. In this respect, we present the order of events occur in the cycle in this model. In the first period the following events occur:

1. The depot observes advance demand vector.
2. The depot updates the state of the system on the basis of ADI.
3. The depot places an order of size Q with the supplier.
4. $B_{i,t}$ is observed.

The order of events take place from the second period until period $L_s + 1$:

1. The depot observes new advance demand information.
2. The depot updates the state of the system on the basis of ADI.
3. $B_{i,t}$ is observed.

The order of events take place during period $L_s + 1$:

1. The depot observes new advance demand information.
2. The depot receives the ordered amount Q from the supplier and allocates it among the retailers.
3. The depot updates the state of the system on the basis of ADI.
4. $B_{i,t}$ is observed.

The order of events take place from period L_s+2 until period $L_s+L_d+1-l+1$ is the same with the second part. In period $L_s+L_d+1-l+1$, the last advance demand information is obtained.

The order of events take place during period $L_s + L_d + 1$:

1. The retailers receive the allocated amount.
2. $B_{i,t}$ is observed.
3. The inventory related costs incur.

5.1. Analysis of the Optimal Policy

In this chapter, we use the same solution approach with the one we followed in the previous chapter. At the beginning of the cycle, the first advance demand information is obtained. So the inventory position of retailer i in period one is

$$X_{i,1} = NI_i - \sum_{t=1}^{l} (\mu_i + \sqrt{y} a_{i,t})$$

Using this information, the depot places an order of Q. After the depot placed an order with size Q, $B_{i,1}$ is observed. At the beginning of the second period new advance demand information is obtained. Up to period L_s+1, the inventory position is updated as below:

$$
\begin{aligned}
X_{i,2} &= X_{i,1} - \sqrt{1-y}b_{i,1} - (\mu_i + \sqrt{y}a_{i,1+l}) \\
X_{i,3} &= X_{i,2} - \sqrt{1-y}b_{i,2} - (\mu_i + \sqrt{y}a_{i,2+l}) \\
&\vdots \\
X_{i,L_s} &= X_{i,L_s-1} - \sqrt{1-y}b_{i,L_s-1} - (\mu_i + \sqrt{y}a_{i,L_s+l-1})
\end{aligned}
$$

At the beginning of period $L_s + 1$; new advance demand information is obtained, the depot receives the order amount Q and allocates it among retailers. After the allocation, the inventory position is updated. It is stated as follows

$$
X_{i,L_s+1} = X_{i,L_s} - \sqrt{1-y}b_{i,L_s} - (\mu_i + \sqrt{y}a_{i,L_s+l}) + z_i
$$

which can be expressed as

$$
X_{i,L_s+1} = NI_i - \sum_{t=1}^{L_s} \sqrt{1-y}b_{i,t} - \sum_{t=1}^{L_s+l} (\mu_i + \sqrt{y}a_{i,t}) + z_i.
$$

After the allocation in period $L_s + 1$, we know all the values in the equation above. Here what we do not know is

$$
\sum_{t=L_s+1}^{L_s+L_d+1} \sqrt{1-y}B_{i,t} + \sum_{t=L_s+l+1}^{L_s+L_d+1} (\mu_i + \sqrt{y}A_{i,t}) \tag{5.1}
$$

with mean $(L_d - l + 1)\mu_i$ and standard deviation $\sigma_i\sqrt{L_d - ly + 1}$. The allocation should be such that inventory position of retailer i at period t satisfies this unobserved demand with a probability. Normal demand enables us to write X_{i,L_s+1} as follows

$$
\begin{aligned}
NI_i - \sum_{t=1}^{L_s} \sqrt{1-y}b_{i,t} - \sum_{t=1}^{L_s+l} (\mu_i + \sqrt{y}a_{i,t}) + z_i& \\
= (L_d - l + 1)\mu_i + v(\sigma_i\sqrt{L_d - ly + 1}).& \qquad (5.2)
\end{aligned}
$$

Equal fractile allocation which we assumed in allocation assumption enables us to write

$$\sum_{i=1}^{N}\left[NI_i - \sum_{t=1}^{L_s}\sqrt{1-y}b_{i,t} - \sum_{t=1}^{L_s+l}(\mu_i + \sqrt{y}a_{i,t})\right] + Q = \sum_{i=1}^{N}\left[(L_d - l + 1)\mu_i + v(\sigma_i\sqrt{L_d - ly + 1})\right]. \quad (5.3)$$

Solving equation (5.3) for v yields

$$v = \frac{\sum_{i=1}^{N}\left[NI_i - \sum_{t=1}^{L_s}\sqrt{1-y}b_{i,t} - \sum_{t=1}^{L_s+l}(\mu_i + \sqrt{y}a_{i,t})\right] + Q - \sum_{i=1}^{N}(L_d - l + 1)\mu_i}{\sum_{i=1}^{N}\sigma_i\sqrt{L_d - ly + 1}}.$$

Solving equation (5.2) for z_i and substituting v gives us how to allocate the order amount Q optimally. Thus, optimum allocation policy is

$$z_i^* = (L_d - l + 1)\mu_i - NI_i + \sum_{t=1}^{L_s}\sqrt{1-y}b_{i,t} + \sum_{t=1}^{L_s+l}(\mu_i + \sqrt{y}a_{i,t}) + \left[\sum_{j=1}^{N}\left\{NI_j - \sum_{t=1}^{L_s}\sqrt{1-y}b_{j,t} - \sum_{t=1}^{L_s+l}(\mu_j + \sqrt{y}a_{j,t}) - (L_d - l + 1)\mu_j\right\} + Q\right]\sigma_i / \sum_{j=1}^{N}\sigma_j.$$

In this case, the optimum allocation policy considers mean demand beyond the allocation period, the demand observed until the allocation period and the inventory at the end of the cycle which is allocated among retailers proportional to their standard deviations. The allocations are as less as the inventory the retailers has at the beginning of the cycle.

Knowing how to allocate Q optimally among retailers, X_{i,L_s+L_d+1}, the stock level of retailer i at the end of the cycle can be expressed as

$$X_{i,L_s+1} - \sum_{t=L_s+1}^{L_s+L_d+1}\sqrt{1-y}B_{i,t} - \sum_{t=L_s+l+1}^{L_s+L_d+1}(\mu_i + \sqrt{y}A_{i,t}).$$

Writing inventory position of retailer i at period t explicitly, we obtain the inventory at the end of the planning horizon as

$$(L_d - l + 1)\mu_i - \sum_{t=L_s+1}^{L_s+L_d+1} \sqrt{1-y}B_{i,t} - \sum_{t=L_s+l+1}^{L_s+L_d+1} (\mu_i + \sqrt{y}A_{i,t}) \\ + \left\{ \sum_{j=1}^{N} \left[NI_j - \sum_{t=1}^{l} (\mu_j + \sqrt{y}a_{j,t}) - (L_d - l + 1)\mu_j \right] + Q \right\} \sigma_i / \sum_{j=1}^{N} \sigma_j \\ + \left\{ -\sum_{t=1}^{L_s} \sqrt{1-y}B_{j,t} - \sum_{t=l+1}^{L_s+l} (\mu_j + \sqrt{y}A_{j,t}) \right\} \sigma_i / \sum_{j=1}^{N} \sigma_j.$$

We seperate the inventory level of retailer i at the end of the planning horizon into two parts which are the scalar part a_i and the variable part ς_i. The scalar part a_i is

$$(L_d - l + 1)\mu_i + \left\{ Q + \sum_{j=1}^{N} \left[NI_j - \sum_{t=1}^{l} (\mu_j + \sqrt{y}a_{j,t}) - (L_d - l + 1)\mu_j \right] \right\} \sigma_i / \sum_{j=1}^{N} \sigma_j$$

and the variable part ς_i is

$$\sum_{t=L_s+1}^{L_s+L_d+1} \sqrt{1-y}B_{i,t} + \sum_{t=L_s+l+1}^{L_s+L_d+1} (\mu_i + \sqrt{y}A_{i,t}) \\ + \sum_{j=1}^{N} \left[\sum_{t=1}^{L_s} \sqrt{1-y}B_{j,t} + \sum_{t=l+1}^{L_s+l} (\mu_j + \sqrt{y}A_{j,t}) \right] \sigma_i / \sum_{j=1}^{N} \sigma_j.$$

Since $A_{i,t}$ and $B_{i,t}$ are normally distributed with parameters zero and σ_i, ς_i is normally distributed with parameters

$$\hat{\mu_i} = (L_d - l + 1)\mu_i + (L_s \sum_{j=1}^{N} \mu_j)\sigma_i / \sum_{j=1}^{N} \sigma_j$$

and

$$\hat{\sigma_i} = \sqrt{(L_d - ly + 1)\,\sigma_i^2 + (L_s \sum_{j=1}^{N} \sigma_j^2) \left(\frac{\sigma_i}{\sum_{j=1}^{N} \sigma_j} \right)^2}.$$

As we explained in the previous chapter, one-period inventory related cost incurring at the end of the planning horizon is expressed as

$$C_i = \int_0^{a_i} h(a_i - x)dF_{\varsigma_i}(x) + \int_{a_i}^{\infty} b(x - a_i)dF_{\varsigma_i}(x) \tag{5.4}$$

where F_{ς_i} is c.d.f. of ς_i. Since

$$F_{\varsigma_i}(a_i) = \Phi\left(\frac{a_i - \hat{\mu}_i}{\hat{\sigma}_i}\right) = \Phi(u) = \frac{b}{b+h}$$

where $\Phi(.)$ is the c.d.f. of the standard normal distribution, we can write

$$u = \frac{\left\{Q + \sum_{j=1}^{N}\left[NI_j - \sum_{t=1}^{l}(\mu_j + \sqrt{y}a_{j,t}) - (L_s + L_d - l + 1)\mu_j\right]\right\}}{\left[L_s \sum_{j=1}^{N}\sigma_j^2 + \left(\sum_{j=1}^{N}\sigma_j\right)^2 (L_d - ly + 1)\right]^{1/2}}.$$

Solving the equation above for Q gives us optimum value of Q which is denoted by Q^*. So

$$\begin{aligned} Q^* = &-\sum_{j=1}^{N}\left[NI_j - \sum_{t=1}^{l}(\mu_j + \sqrt{y}a_{j,t}) - (L_s + L_d - l + 1)\mu_j\right] \\ &+ u\left[L_s \sum_{j=1}^{N}\sigma_j^2 + \left(\sum_{j=1}^{N}\sigma_j\right)^2 (L_d - ly + 1)\right]^{1/2} \end{aligned}$$

where u is $\Phi^{-1}\left(\frac{b}{b+h}\right)$. The policy determining the optimum order amount is the same with perfect ADI model. The depot considers the mean demand of the planning horizon and the advance demand information obtained at the beginning of the cycle. The inventory on hand at the beginning of the cycle is subtracted from the amount required at the end of the cycle. The depot also considers the standard deviation of the demand beyond demand lead time multiplied by standard normal variable. It is easy to see from the expression above that this type of advance demand information can not reduce uncertainty as much as perfect ADI does if y is not equal to one. Because the demand information we obtain at the end of the cycle is less than the information we obtain with perfect ADI. We note that standard deviation of the planning horizon increases as y goes from one to zero.

5.2. Analysis of the Partial Demand Information Model Where the Amount of Information Depends on Retailer

The demand structure we analyze in the previous section is $D_{i,t} = \mu_i + \sqrt{y}A_{i,t} + \sqrt{1-y}B_{i,t}$. As it is known, $\sqrt{1-y}B_{i,t}$ is the part of $D_{i,t}$ that is observed in period t. The other part of the demand information is stated as advance demand information which is observed l periods in advance. The value of parameter y varies between zero and one and it is accepted as the amount of advance demand information. When y equals to zero, the demand information of retailer i of period t which the depot has l periods in advance is just μ_i and in time period t the depot observes $B_{i,t}$. But when y is equal to one, the depot observes $(\mu_i + A_{i,t})$ in advance which means perfect information. In this section our demand structure is

$$D_{i,t} = \mu_i + \sqrt{y_i}A_{i,t} + \sqrt{1-y_i}B_{i,t}$$

which implies that the amount of advance demand information depends on retailer. In the previous section retailers have the same y value, but in this one they can take various values of y independent of each other.

When we apply the same solution approach with the one we used in the previous section, we obtain the same expression with the one we obtain in the analysis of the model in which the amount of ADI is independent of retailers for the inventory position in period $L_s + 1$. It is

$$X_{i,L_s+1} = NI_i - \sum_{t=1}^{L_s} \sqrt{1-y_i}b_{i,t} - \sum_{t=1}^{L_s+l}(\mu_i + \sqrt{y_i}a_{i,t}) + z_i.$$

As we explained before, normally distributed demands and allocation assumption enable us to assume the existence of a v value such that

$$\sum_{i=1}^{N}\left[NI_i-\sum_{t=1}^{L_s}\sqrt{1-y_i}b_{i,t}-\sum_{t=1}^{L_s+l}(\mu_i+\sqrt{y_i}a_{i,t})+z_i\right]= \sum_{i=1}^{N}\left[(L_d-l+1)\mu_i+v(\sigma_i\sqrt{L_d-ly_i+1})\right]. \quad (5.5)$$

Replacing $\sum_{i=1}^{N} z_i$ with Q and solving the equation above for v, we obtain

$$v=\frac{\sum_{i=1}^{N}\left[NI_i-\sum_{t=1}^{L_s}\sqrt{1-y_i}b_{i,t}-\sum_{t=1}^{L_s+l}(\mu_i+\sqrt{y_i}a_{i,t})\right]+Q-\sum_{i=1}^{N}(L_d-l+1)\mu_i}{\sum_{i=1}^{N}\sigma_i\sqrt{L_d-ly_i+1}}.$$

Solving equation (5.5) for z_i yields

$$z_i=(L_d-l+1)\mu_i+v(\sigma_i\sqrt{L_d-ly_i+1})-NI_i+\sum_{t=1}^{L_s}\sqrt{1-y_i}b_{i,t}+\sum_{t=1}^{L_s+l}(\mu_i+\sqrt{y_i}a_{i,t}). \quad (5.6)$$

When we substitute the expression obtained for v in equation (5.6), we obtain optimum allocation policy that is a bit different from the one we obtain in the previous section.

$$\begin{aligned} z_i^* =&\ (L_d-l+1)\mu_i-NI_i+\sum_{t=1}^{L_s}\sqrt{1-y_i}b_{i,t}+\sum_{t=1}^{L_s+l}(\mu_i+\sqrt{y_i}a_{i,t}) \\ &+\frac{\left\{\sum_{j=1}^{N}\left[NI_j-\sum_{t=1}^{L_s}\sqrt{1-y_j}b_{j,t}-\sum_{t=1}^{L_s+l}\left(\mu_j+\sqrt{y_j}a_{j,t}\right)\right]\right\}\sigma_i\sqrt{L_d-ly_i+1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d-ly_j+1}} \\ &+\frac{\left\{Q-\sum_{j=1}^{N}(L_d-l+1)\mu_j\right\}\sigma_i\sqrt{L_d-ly_i+1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d-ly_j+1}} \end{aligned}$$

In this case, the same allocation policy is obtained with the one obtained in the model where y is the same for all retailers. Here, the difference is the dependence of y on retailers. Because of this dependence, expected inventory at the end of the cycle is allocated among retailers proportional to the standard deviations of the unobserved part of the cycle in period L_s+1.

Since z_i^* expression is different from the one in the previous section; from now, the expressions we will obtain using z_i^* will differ from the ones obtained in the previous section. Therefore, although we follow the same solution approach with the one we used in Section 5.1, we will state the analysis explicitly.

When the system pretends as it is in period L_s+1, the stock level of retailer i at the end of the planning horizon is obtained by subtracting the demands for the periods from L_s+1 until the end of the cycle from $X_{i,Ls+1}$. So $X_{L_s+L_d+1}$ is

$$NI_i - \sum_{t=1}^{L_s}\sqrt{1-y_i}b_{i,t} - \sum_{t=1}^{L_s+l}(\mu_i+\sqrt{y_i}a_{i,t}) + z_i^* - \sum_{t=L_s+1}^{L_s+L_d+1}\sqrt{1-y_i}B_{i,t} - \sum_{t=L_s+l+1}^{L_s+L_d+1}(\mu_i+\sqrt{y_i}A_{i,t}).$$

We know how to allocate Q among retailers and we are now in the second step. So the system is at the beginning of the planning horizon. Substituting the explicit expression for optimum allocated amount z_i^* to the expression above, we obtain the stock level of retailer i at the end of the cycle as follows:

$$\begin{aligned}
&(L_d-l+1)\mu_i - \sum_{t=L_s+1}^{L_s+L_d+1}\sqrt{1-y_i}B_{i,t} - \sum_{t=L_s+l+1}^{L_s+L_d+1}(\mu_i+\sqrt{y_i}A_{i,t}) \\
&\quad + \frac{\left\{\sum_{j=1}^{N}\left[NI_j - \sum_{t=1}^{l}(\mu_j+\sqrt{y_j}a_{j,t}) - (L_d-l+1)\mu_j\right]\right\}\sigma_i\sqrt{L_d-ly_i+1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d-ly_j+1}} \\
&\quad + \frac{\left\{\sum_{j=1}^{N}\left[-\sum_{t=1}^{L_s}\sqrt{1-y_j}B_{j,t} - \sum_{t=l+1}^{L_s+l}(\mu_j+\sqrt{y_j}A_{j,t})\right] + Q\right\}\sigma_i\sqrt{L_d-ly_i+1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d-ly_j+1}}
\end{aligned} \tag{5.7}$$

Seperating $X_{L_s+L_d+1}$ into two parts, we get the scalar one as

$$a_i = (L_d - l + 1)\mu_i + \frac{\sum_{j=1}^{N}\left[NI_j - \sum_{t=1}^{l}(\mu_j + \sqrt{y_j}a_{j,t})\right]\sigma_i\sqrt{L_d - ly_i + 1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}} + \frac{\left[\sum_{j=1}^{N} -(L_d - l + 1)\mu_j + Q\right]\sigma_i\sqrt{L_d - ly_i + 1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}}$$

and the variable one

$$\varsigma_i = \sum_{t=L_s+1}^{L_s+L_d+1}\sqrt{1-y_i}B_{i,t} + \sum_{t=L_s+l+1}^{L_s+L_d+1}(\mu_i + \sqrt{y_i}A_{i,t}) + \frac{\sum_{j=1}^{N}\left[\sum_{t=1}^{L_s}\sqrt{1-y_j}B_{j,t} + \sum_{t=l+1}^{L_s+l}(\mu_j + \sqrt{y_j}A_{j,t})\right]\sigma_i\sqrt{L_d - ly_i + 1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}}$$

where $X_{L_s+L_d+1} = a_i - \varsigma_i$. The variable ς_i has the mean

$$\hat{\mu_i} = (L_d - l + 1)\,\mu_i + \frac{\left(L_s\sum_{j=1}^{N}\mu_j\right)\sigma_i\sqrt{L_d - ly_i + 1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}}$$

and the standard deviation

$$\hat{\sigma_i} = \sqrt{(L_d - ly_i + 1)\,\sigma_i^2 + (L_s\sum_{j=1}^{N}\sigma_j^2)\left(\frac{\sigma_i\sqrt{L_d - ly_i + 1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}}\right)^2}.$$

The cost function is the same with equation (5.4). F_{ς_i} is cumulative distribution function of ς_i and $\Phi(.)$ is cumulative distribution function of standard normal variable where $F_{\varsigma_i}(a_i) = \Phi(u)$. Therefore $u = (a_i - \hat{\mu_i})/\hat{\sigma_i}$. When we substitute a_i, $\hat{\mu_i}$ and $\hat{\sigma_i}$ in the equation above, we obtain u as follows:

$$u = \frac{\left\{ \sum_{j=1}^{N} \left[NI_j - \sum_{t=1}^{l} \sqrt{y_j} a_{j,t} - (L_s + L_d + 1)\, \mu_j \right] + Q \right\}}{\sqrt{\left(\sum_{j=1}^{N} \sigma_j \sqrt{L_d - l y_j + 1} \right)^2 + L_s \sum_{j=1}^{N} \sigma_j^2}}$$

Solving the equation above for Q, optimum order amount is obtained such that

$$Q^* = -\sum_{j=1}^{N} \left[NI_j - \sum_{t=1}^{l} \sqrt{y_j} a_{j,t} - (L_s + L_d + 1)\mu_j \right] + u \left[L_s \sum_{j=1}^{N} \sigma_j^2 + \left(\sum_{j=1}^{N} \sigma_j \sqrt{L_d - l y_j + 1} \right)^2 \right]^{1/2}$$

where u is the $\Phi^{-1}\left(\frac{b}{b+h}\right)$. We see that optimum order amount is determined using the same policy with the one in the previous section. In this section, y depends on retailer. This means that each retailer can take a value of y independent of the others. So, Q^* decreases or increases depending on the values y_i takes.

5.2.1. Analysis of the Cost Function under Partial Advance Demand Information Where the Amount of ADI Depends on Retailer

We obtained optimum order amount Q^* and optimum allocation policy z_i^* for the partial ADI model where the amount of ADI depends on retailer. Now; using the optimal ordering and allocating policies, we will state the cost function for this model. We will provide the cost function of retailer i and the expression for system-wide inventory costs. At the end of the analysis, we can explicitly see how the amount of advance demand information acquired by retailer i, which is denoted with y_i, effects the overall system cost.

As we stated before, we denoted cost function of retailer i by C_i. Inventory related costs consist of holding and penalty costs. So one period inventory cost of retailer i

can be expressed as following:

$$C_i = hE\left[X_{i,L_s+L_d+1}\right]^+ + bE\left[-X_{i,L_s+L_d+1}\right]^+$$

We substitute optimum order amount Q^* in equation (5.7) which gives the expression for X_{i,L_s+L_d+1} and obtain

$$\begin{aligned}
X_{i,L_s+L_d+1} = &- \sum_{t=L_s+1}^{L_s+L_d+1} \sqrt{1-y_i} B_{i,t} - \sum_{t=L_s+l+1}^{L_s+L_d+1} \sqrt{y_i} A_{i,t} \\
&+ \frac{\sum_{j=1}^{N} \left[- \sum_{t=1}^{L_s} \sqrt{1-y_j} B_{j,t} - \sum_{t=l+1}^{L_s+l} \sqrt{y_j} A_{j,t} \right] \sigma_i \sqrt{L_d - l y_i + 1}}{\sum_{j=1}^{N} \sigma_j \sqrt{L_d - l y_j + 1}} \\
&+ \frac{u \left[L_s \sum_{j=1}^{N} \sigma_j^2 + \left(\sum_{j=1}^{N} \sigma_j \sqrt{L_d - l y_j + 1} \right)^2 \right]^{1/2} \sigma_i \sqrt{L_d - l y_i + 1}}{\sum_{j=1}^{N} \sigma_j \sqrt{L_d - l y_j + 1}}.
\end{aligned}$$

It can be seen that X_{i,L_s+L_d+1} consists of known and unknown parts. We will denote the known part with a_i which is

$$a_i = \frac{u \left[L_s \sum_{j=1}^{N} \sigma_j^2 + \left(\sum_{j=1}^{N} \sigma_j \sqrt{L_d - l y_j + 1} \right)^2 \right]^{1/2} \sigma_i \sqrt{L_d - l y_i + 1}}{\sum_{j=1}^{N} \sigma_j \sqrt{L_d - l y_j + 1}}$$

and unknown part with ς_i which is

$$\begin{aligned}
\varsigma_i = &+ \sum_{t=L_s+1}^{L_s+L_d+1} \sqrt{1-y_i} B_{i,t} + \sum_{t=L_s+l+1}^{L_s+L_d+1} \sqrt{y_i} A_{i,t} \\
&+ \frac{\sum_{j=1}^{N} \left[\sum_{t=1}^{L_s} \sqrt{1-y_j} B_{j,t} + \sum_{t=l+1}^{L_s+l} \sqrt{y_j} A_{j,t} \right] \sigma_i \sqrt{L_d - l y_i + 1}}{\sum_{j=1}^{N} \sigma_j \sqrt{L_d - l y_j + 1}}.
\end{aligned}$$

So the cost function can be written as

$$C_i = hE\left[a_i - \varsigma_i\right]^+ + bE\left[\varsigma_i - a_i\right]^+.$$

We know that $[a_i - \varsigma_i]^+ = (a_i - \varsigma_i) + [\varsigma_i - a_i]^+$. Hence C_i can be expressed as follows:

$$C_i = hE\left[a_i - \varsigma_i\right] + (h+b)\,E\left[\varsigma_i - a_i\right]^+$$

The expected shortage is expressed as

$$E\left[\varsigma_i - a_i\right]^+ = \sqrt{Var(\varsigma_i)}G\left(\frac{a_i - E\left[\varsigma_i\right]}{\sqrt{Var(\varsigma_i)}}\right)$$

where $G_u(k) = \int\limits_k^\infty (u_0 - k)\, f_u(u_0) du_0$. So the cost function of retailer i takes the form

$$C_i = h\left(a_i - E\left[\varsigma_i\right]\right) + (h+b)\sqrt{Var(\varsigma_i)}G\left(\frac{a_i - E\left[\varsigma_i\right]}{\sqrt{Var(\varsigma_i)}}\right). \tag{5.8}$$

It is easy to see that

$$E\left[\varsigma_i\right] = 0$$

and

$$\begin{aligned} Var(\varsigma_i) &= (L_d+1)\,(1-y_i)\sigma_i^2 + (L_d - l + 1)y_i\sigma_i^2 \\ &\quad + \sum_{j=1}^N \left[L_s y_j \sigma_j^2 + L_s(1-y_j)\sigma_j^2\right]\left(\frac{\sigma_i\sqrt{L_d - l y_i + 1}}{\sum\limits_{j=1}^N \sigma_j\sqrt{L_d - l y_j + 1}}\right)^2 \\ &= \left[L_s\sum_{j=1}^N \sigma_j^2 + \left(\sum_{j=1}^N \sigma_j\sqrt{L_d - l y_j + 1}\right)^2\right]\left(\frac{\sigma_i\sqrt{L_d - l y_i + 1}}{\sum\limits_{j=1}^N \sigma_j\sqrt{L_d - l y_j + 1}}\right)^2. \end{aligned}$$

Substituting these expressions and a_i into equation (5.8) we obtain the cost function as

$$C_i = hu\left[L_s\sum_{j=1}^{N}\sigma_j^2 + \left(\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}\right)^2\right]^{1/2}\frac{\sigma_i\sqrt{L_d - ly_i + 1}}{\sum\limits_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}}$$
$$+ (h+b)\left[L_s\sum_{j=1}^{N}\sigma_j^2 + \left(\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}\right)^2\right]^{1/2}\frac{\sigma_i\sqrt{L_d - ly_i + 1}}{\sum\limits_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}}G(u)$$

where $u = \Phi^{-1}\left(b/(b+h)\right)$.

For the overall system cost we sum the costs of all retailers and obtain $C = \sum\limits_{i=1}^{N} C_i$ as

$$\sum_{i=1}^{N} hu\left[L_s\sum_{j=1}^{N}\sigma_j^2 + \left(\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}\right)^2\right]^{1/2}\frac{\sigma_i\sqrt{L_d - ly_i + 1}}{\sum\limits_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}}$$
$$+ \sum_{i=1}^{N}(h+b)\left[L_s\sum_{j=1}^{N}\sigma_j^2 + \left(\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}\right)^2\right]^{1/2}\frac{\sigma_i\sqrt{L_d - ly_i + 1}}{\sum\limits_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}}G(u)$$

which equals to

$$C = hu\left[L_s\sum_{j=1}^{N}\sigma_j^2 + \left(\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}\right)^2\right]^{1/2}$$
$$+ (h+b)\left[L_s\sum_{j=1}^{N}\sigma_j^2 + \left(\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}\right)^2\right]^{1/2} G(u).$$

How y_i affects the cost function was the question. Now; from the equation above, we can see that y_i, the amount of advance demand information acquired by retailer i, affects the cost function negatively. When y_i takes the maximum value one, the variance and the cost decreases. When it takes the minimum value zero, the variance and the cost increases.

5.3. Analysis of the Model with Correlated Demands

In this section, we analyze the effect of correlation to the optimal order amount under partial advance demand information. The correlation of demands among retailers is considered, the demands are not allowed to be correlated through time.

In this section, we will again apply the same solution approach with the one used in the previous sections. In the first step of our solution approach, we pretend as the system is in period $L_s + 1$. The order amount Q, which has been placed at the beginning of the planning horizon, is received by the depot in period $L_s + 1$. Then, the depot decides how to allocate this amount among retailers, knowing the demand observations of the previous periods. Since there is no uncertainty for the demands of the previous periods, consideration of correlation of demands for these periods is out of question. Therefore we can express the inventory position of retailer i as the same in the previous section, which is

$$X_{i,L_s+1} = NI_i - \sum_{t=1}^{L_s} \sqrt{1-y_i} b_{i,t} - \sum_{t=1}^{L_s+l} (\mu_i + \sqrt{y_i} a_{i,t}) + z_i.$$

Normal distributed demands and equal fractile allocation allow us to write the equation below:

$$\sum_{i=1}^{N} \left[NI_i - \sum_{t=1}^{L_s} \sqrt{1-y_i} b_{i,t} - \sum_{t=1}^{L_s+l} (\mu_i + \sqrt{y_i} a_{i,t}) + z_i \right] = \sum_{i=1}^{N} \left[(L_d - l + 1)\mu_i + v(\sigma_i \sqrt{L_d - l y_i + 1}) \right] \quad (5.9)$$

The expression above means that there is such a v such that the depot achieves equal fractile point in each retailer at the end of the planning horizon.

We note that as it is seen in equation (5.9), correlation of demands is not considered during the allocation decision. Because in period $L_s + 1$, the depot can not affect the order amount. So whatever the customer demand is, the depot can not change the amount received. Hence, we first obtain v, replacing $\sum_{i=1}^{N} z_i$ with Q in (5.9), as follows:

$$\frac{\sum_{i=1}^{N}\left[NI_i - \sum_{t=1}^{L_s}\sqrt{1-y_i}b_{i,t} - \sum_{t=1}^{L_s+l}(\mu_i + \sqrt{y_i}a_{i,t})\right] + Q - \sum_{i=1}^{N}(L_d - l + 1)\mu_i}{\sum_{i=1}^{N}\sigma_i\sqrt{L_d - ly_i + 1}}$$

Substituting v in equation (5.9) and solving it for z_i gives optimum allocation policy which is

$$\begin{aligned} z_i^* &= (L_d - l + 1)\mu_i - NI_i + \sum_{t=1}^{L_s}\sqrt{1-y_i}b_{i,t} + \sum_{t=1}^{L_s+l}(\mu_i + \sqrt{y_i}a_{i,t}) \\ &+ \frac{\left\{\sum_{j=1}^{N}\left[NI_j - \sum_{t=1}^{L_s}\sqrt{1-y_j}b_{j,t} - \sum_{t=1}^{L_s+l}(\mu_j + \sqrt{y_j}a_{j,t})\right]\right\}\sigma_i\sqrt{L_d - ly_i + 1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}} \\ &+ \frac{\left\{Q - \sum_{j=1}^{N}(L_d - l + 1)\mu_j\right\}\sigma_i\sqrt{L_d - ly_i + 1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}}. \end{aligned}$$

We obtain the same allocation policy with the model in which correlation is not considered. Because, correlation of demands is not taken into account while the depot is allocating the order amount among retailers.

The stock level of retailer i at the end of the cycle, which is denoted with X_{i,L_s+L_d+1}, can be expressed as

$$\begin{aligned} X_{i,L_s+L_d+1} &= X_{i,L_s+1} - \sum_{t=L_s+1}^{L_s+L_d+1}\sqrt{1-y_i}B_{i,t} - \sum_{t=L_s+l+1}^{L_s+L_d+1}(\mu_i + \sqrt{y_i}A_{i,t}) \\ &= NI_i - \sum_{t=1}^{L_s}\sqrt{1-y_i}b_{i,t} - \sum_{t=1}^{L_s+l}(\mu_i + \sqrt{y_i}a_{i,t}) + z_i^* \\ &- \sum_{t=L_s+1}^{L_s+L_d+1}\sqrt{1-y_i}B_{i,t} - \sum_{t=L_s+l+1}^{L_s+L_d+1}(\mu_i + \sqrt{y_i}A_{i,t}). \end{aligned} \quad (5.10)$$

When we substitute z_i^* in equation (5.10), we obtain X_{i,L_s+L_d+1} as

$$\begin{aligned}
&-\sum_{t=L_s+1}^{L_s+L_d+1}\sqrt{1-y_i}B_{i,t}-\sum_{t=L_s+l+1}^{L_s+L_d+1}\sqrt{y_i}A_{i,t} \\
&\quad+\frac{\left\{\sum_{j=1}^{N}\left[NI_j-\sum_{t=1}^{l}\sqrt{y_j}a_{j,t}-(L_s+L_d+1)\mu_j\right]\right\}\sigma_i\sqrt{L_d-ly_i+1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d-ly_j+1}} \\
&\quad+\frac{\left\{\sum_{j=1}^{N}\left[-\sum_{t=1}^{L_s}\sqrt{1-y_j}B_{j,t}-\sum_{t=l+1}^{L_s+l}\sqrt{y_j}A_{j,t}\right]+Q\right\}\sigma_i\sqrt{L_d-ly_i+1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d-ly_j+1}}. \qquad (5.11)
\end{aligned}$$

Our one-period expected inventory cost function is

$$C_i=\int_0^{\infty}hx\,dF_{X_{i,L_s+L_d+1}}(x)-\int_{-\infty}^{0}bx\,dF_{X_{i,L_s+L_d+1}}(x)$$

where $F_{X_{i,L_s+L_d+1}}$ is the distribution function of X_{i,L_s+L_d+1}. Since X_{i,L_s+L_d+1} consists of scalar and variable parts, we seperate it into parts where the scalar part is

$$a_i=\frac{\left\{\sum_{j=1}^{N}\left[NI_j-\sum_{t=1}^{l}\sqrt{y_j}a_{j,t}-(L_s+L_d+1)\mu_j\right]+Q\right\}\sigma_i\sqrt{L_d-ly_i+1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d-ly_j+1}}$$

and the variable one is

$$\begin{aligned}
\varsigma_i=&\sum_{t=L_s+1}^{L_s+L_d+1}\sqrt{1-y_i}B_{i,t}+\sum_{t=L_s+l+1}^{L_s+L_d+1}\sqrt{y_i}A_{i,t} \\
&+\frac{\sum_{j=1}^{N}\left[\sum_{t=1}^{L_s}\sqrt{1-y_j}B_{j,t}+\sum_{t=l+1}^{L_s+l}\sqrt{y_j}A_{j,t}\right]\sigma_i\sqrt{L_d-ly_i+1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d-ly_j+1}}.
\end{aligned}$$

Now we can write the cost function as

$$C_i = \int_0^{a_i} h(a_i - x)dF_{\varsigma_i}(x) + \int_{a_i}^{\infty} b(x - a_i)dF_{\varsigma_i}(x)$$

where F_{ς_i} is c.d.f. of ς_i. Since

$$\frac{\partial C_i}{\partial a_i} = hF_{\varsigma_i}(a_i) - b(1 - F_{\varsigma_i}(a_i)),$$

we can say that the minimum C_i is acquired when $F_{\varsigma_i}(a_i) = b/(b+h)$. The normal distribution enables us to write $F_{\varsigma_i}(a_i) = \Phi\left(\frac{a_i - \hat{\mu_i}}{\hat{\sigma_i}}\right) = \Phi(u)$ where $E[\varsigma_i] = \hat{\mu_i}$ and $\sqrt{Var(\varsigma_i)} = \hat{\sigma_i}$. It is easy to see that $\hat{\mu_i} = 0$ and

$$\begin{aligned}
\hat{\sigma_i}^2 &= (L_d + 1)(1 - y_i)\sigma_i^2 + (L_d - l + 1)y_i\sigma_i^2 \\
&+ Var\left(\sum_{t=1}^{L_s}\sum_{j=1}^{N}\sqrt{1-y_j}B_{j,t}\right)\left(\frac{\sigma_i\sqrt{L_d - ly_i + 1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}}\right)^2 \\
&+ Var\left(\sum_{t=l+1}^{L_s+l}\sum_{j=1}^{N}\sqrt{y_j}A_{j,t}\right)\left(\frac{\sigma_i\sqrt{L_d - ly_i + 1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}}\right)^2 \\
&+ 2Cov\left(\sum_{j=1}^{N}\sum_{t=1}^{L_s}\sqrt{1-y_j}B_{j,t}, \sum_{k=1}^{N}\sum_{t=l+1}^{L_s+l}\sqrt{y_k}A_{k,t}\right)\left(\frac{\sigma_i\sqrt{L_d - ly_i + 1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}}\right)^2
\end{aligned}$$

where

$$\begin{aligned}
Var\left(\sum_{t=1}^{L_s}\sum_{j=1}^{N}\sqrt{1-y_j}B_{j,t}\right) &= L_s\sum_{j=1}^{N}(1-y_j)\sigma_j^2 \\
&+ 2\sum_{t=1}^{L_s}\sum_{j=1}^{N}\sum_{k=j+1}^{N} Cov\left(\sqrt{1-y_j}B_{j,t}, \sqrt{1-y_k}B_{k,t}\right),
\end{aligned}$$

$$Var\left(\sum_{t=l+1}^{L_s+l}\sum_{j=1}^{N}\sqrt{y_j}A_{j,t}\right) = L_s\sum_{j=1}^{N} y_j\sigma_j^2 + 2\sum_{t=l+1}^{L_s+l}\sum_{j=1}^{N}\sum_{k=j+1}^{N} Cov\left(\sqrt{y_j}A_{j,t}, \sqrt{y_k}A_{k,t}\right),$$

and

$$Cov\left(\sum_{j=1}^{N}\sum_{t=1}^{L_s}\sqrt{1-y_j}B_{j,t}, \sum_{k=1}^{N}\sum_{t=l+1}^{L_s+l}\sqrt{y_k}A_{k,t}\right) = 0.$$

Here we assume that there is correlation between $A_{i,t}$ and $A_{k,t}$, between $B_{i,t}$ and $B_{k,t}$ and no correlation between $A_{k,t}$ and $B_{j,t}$. In any period t, $\sqrt{1-y_i}B_{i,t}$ and $\sqrt{y_i}A_{i,t+l-1}$ are observed. If we consider a correlation between these demand expressions, then it means that the demands of period t and period $t+l-1$ are correlated. However; in this study, the demands are not allowed to be correlated through time. So we can express $\hat{\sigma_i}^2$ as follows:

$$(L_d - ly_i + 1)\sigma_i^2 + \left[L_s\sum_{j=1}^{N}\sigma_j^2 + 2\sum_{t=1}^{L_s}\sum_{j=1}^{N}\sum_{k=j+1}^{N} Cov\left(\sqrt{1-y_j}B_{j,t}, \sqrt{1-y_k}B_{k,t}\right) + 2\sum_{t=l+1}^{L_s+l}\sum_{j=1}^{N}\sum_{k=j+1}^{N} Cov\left(\sqrt{y_j}A_{j,t}, \sqrt{y_k}A_{k,t}\right)\right]\left(\frac{\sigma_i\sqrt{L_d - ly_i + 1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}}\right)^2$$

Since $u = (a_i - \hat{\mu_i})/\hat{\sigma_i}$, substituting a_i, $\hat{\mu_i}$, $\hat{\sigma_i}$ and solving the obtained equation for Q, we obtain approximate optimal order amount as follows:

$$Q^* = -\sum_{j=1}^{N}\left[NI_j - \sum_{t=1}^{l}\sqrt{y_j}a_{j,t} - (L_s + L_d + 1)\mu_j\right] + u\Bigg\{\left(\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}\right)^2 + L_s\sum_{j=1}^{N}\sigma_j^2 + 2\sum_{j=1}^{N}\sum_{k=j+1}^{N}\left[\sum_{t=1}^{L_s} Cov\left(\sqrt{1-y_j}B_{j,t}, \sqrt{1-y_k}B_{k,t}\right) + \sum_{t=l+1}^{L_s+l} Cov\left(\sqrt{y_j}A_{j,t}, \sqrt{y_k}A_{k,t}\right)\right]\Bigg\}^{1/2}$$

It is seen that the depot places an order amount considering the mean demand of the cycle that is not satisfied by the inventory the system has at the beginning of the cycle, the advance demand information obtained at the beginning of the planning horizon and the standard deviation of the demand beyond demand lead time which is multiplied by the standard normal variable. Here, differently from the model with uncorrelated demands, covariance between the demands exists in optimal order amount. So we can say that correlated demands has an increasing impact on order amount.

5.3.1. Analysis of the Cost Function of the Partial ADI Model with Correlated Demands

We obtained approximate optimal allocating and ordering policies for the partial ADI model with correlated demands. We will now provide the cost function for this model using the obtained policies. Substituting Q^* obtained for the model with correlated demands into equation (5.11) which gives the expression for X_{i,L_s+L_d+1}, we obtain the stock level at the end of the cycle as follows:

$$
\begin{aligned}
&-\sum_{t=L_s+1}^{L_s+L_d+1} \sqrt{1-y_i} B_{i,t} - \sum_{t=L_s+l+1}^{L_s+L_d+1} \sqrt{y_i} A_{i,t} \\
&+ \frac{\sum_{j=1}^{N} \left[-\sum_{t=1}^{L_s} \sqrt{1-y_j} B_{j,t} - \sum_{t=l+1}^{L_s+l} \sqrt{y_j} A_{j,t} \right] \sigma_i \sqrt{L_d - l y_i + 1}}{\sum_{j=1}^{N} \sigma_j \sqrt{L_d - l y_j + 1}} \\
&+ u \Bigg\{ \left(\sum_{j=1}^{N} \sigma_j \sqrt{L_d - l y_j + 1} \right)^2 + L_s \sum_{j=1}^{N} \sigma_j^2 + \\
&\quad 2 \sum_{j=1}^{N} \sum_{k=j+1}^{N} \Bigg[\sum_{t=1}^{L_s} Cov\left(\sqrt{1-y_j} B_{j,t}, \sqrt{1-y_k} B_{k,t} \right) + \\
&\quad\quad \sum_{t=l+1}^{L_s+l} Cov\left(\sqrt{y_j} A_{j,t}, \sqrt{y_k} A_{k,t} \right) \Bigg] \Bigg\}^{1/2} \left(\frac{\sigma_i \sqrt{L_d - l y_i + 1}}{\sum_{j=1}^{N} \sigma_j \sqrt{L_d - l y_j + 1}} \right)
\end{aligned}
$$

It is easy to see that the expression above consists of known and unknown parts. The known part is

$$a_i = u\Bigg\{\left(\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}\right)^2 + L_s\sum_{j=1}^{N}\sigma_j^2 + 2\sum_{j=1}^{N}\sum_{k=j+1}^{N}\Bigg[\sum_{t=1}^{L_s} Cov\left(\sqrt{1-y_j}B_{j,t}, \sqrt{1-y_k}B_{k,t}\right) + \sum_{t=l+1}^{L_s+l} Cov\left(\sqrt{y_j}A_{j,t}, \sqrt{y_k}A_{k,t}\right)\Bigg]\Bigg\}^{1/2}\left(\frac{\sigma_i\sqrt{L_d - ly_i + 1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}}\right)$$

and unknown part is

$$\varsigma_i = +\sum_{t=L_s+1}^{L_s+L_d+1}\sqrt{1-y_i}B_{i,t} + \sum_{t=L_s+l+1}^{L_s+L_d+1}\sqrt{y_i}A_{i,t} + \frac{\sum_{j=1}^{N}\left[\sum_{t=1}^{L_s}\sqrt{1-y_j}B_{j,t} + \sum_{t=l+1}^{L_s+l}\sqrt{y_j}A_{j,t}\right]\sigma_i\sqrt{L_d - ly_i + 1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}}.$$

One period inventory cost of retailer i is stated as

$$C_i = hE\left[X_{L_s+L_d+1}\right]^+ + bE\left[-X_{L_s+L_d+1}\right]^+$$

which can be written as

$$C_i = hE\left[a_i - \varsigma_i\right]^+ + bE\left[\varsigma_i - a_i\right]^+.$$

Since $[a_i - \varsigma_i]^+ = (a_i - \varsigma_i) + [\varsigma_i - a_i]^+$, we can express cost function as follows:

$$C_i = h\left(a_i - E\left[\varsigma_i\right]\right) + (h + b)\,E\left[\varsigma_i - a_i\right]^+$$

Since expected shortage is stated as

$$E[\varsigma_i - a_i]^+ = \sqrt{Var(\varsigma_i)}G\left(\frac{a_i - E[\varsigma_i]}{\sqrt{Var(\varsigma_i)}}\right),$$

we can write the cost function of retailer i as

$$C_i = h(a_i - E[\varsigma_i]) + (h+b)\sqrt{Var(\varsigma_i)}G\left(\frac{a_i - E[\varsigma_i]}{\sqrt{Var(\varsigma_i)}}\right). \tag{5.12}$$

where $G_u(k) = \int_k^\infty (u_0 - k) f_u(u_0) du_0$. It is easy to see that $E[\varsigma_i] = 0$. From the analysis of the model with correlated demands, we know that

$$\begin{aligned} Var(\varsigma_i) &= (L_d - ly_i + 1)\sigma_i^2 \\ &\quad + \Bigg[L_s \sum_{j=1}^{N} \sigma_j^2 + 2\sum_{t=1}^{L_s}\sum_{j=1}^{N}\sum_{k=j+1}^{N} Cov\left(\sqrt{1-y_j}B_{j,t}, \sqrt{1-y_k}B_{k,t}\right) + \\ &\qquad 2\sum_{t=l+1}^{L_s+l}\sum_{j=1}^{N}\sum_{k=j+1}^{N} Cov\left(\sqrt{y_j}A_{j,t}, \sqrt{y_k}A_{k,t}\right)\Bigg]\left(\frac{\sigma_i\sqrt{L_d - ly_i + 1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}}\right)^2 \end{aligned}$$

which is equal to

$$\begin{aligned} Var(\varsigma_i) = \Bigg\{&\left(\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}\right)^2 + L_s\sum_{j=1}^{N}\sigma_j^2 + \\ &2\sum_{j=1}^{N}\sum_{k=j+1}^{N}\Bigg[\sum_{t=1}^{L_s} Cov\left(\sqrt{1-y_j}B_{j,t}, \sqrt{1-y_k}B_{k,t}\right) + \\ &\qquad \sum_{t=l+1}^{L_s+l} Cov\left(\sqrt{y_j}A_{j,t}, \sqrt{y_k}A_{k,t}\right)\Bigg]\Bigg\}\left(\frac{\sigma_i\sqrt{L_d - ly_i + 1}}{\sum_{j=1}^{N}\sigma_j\sqrt{L_d - ly_j + 1}}\right)^2 \end{aligned}$$

Substituting $E[\varsigma_i]$, $\sqrt{Var(\varsigma_i)}$ and a_i into equation (5.12), we obtain the cost function as

$$C_i = [hu + (h+b)G(u)] \left\{ \left(\sum_{j=1}^{N} \sigma_j \sqrt{L_d - ly_j + 1} \right)^2 + L_s \sum_{j=1}^{N} \sigma_j^2 + 2 \sum_{j=1}^{N} \sum_{k=j+1}^{N} \left[\sum_{t=1}^{L_s} Cov\left(\sqrt{1-y_j} B_{j,t}, \sqrt{1-y_k} B_{k,t} \right) + \sum_{t=l+1}^{L_s+l} Cov\left(\sqrt{y_j} A_{j,t}, \sqrt{y_k} A_{k,t} \right) \right] \right\}^{1/2} \left(\frac{\sigma_i \sqrt{L_d - ly_i + 1}}{\sum_{j=1}^{N} \sigma_j \sqrt{L_d - ly_j + 1}} \right)$$

where $u = \Phi^{-1}(b/(b+h))$.

For the overall system cost we sum the costs of all retailers and obtain the system-wide inventory as follows:

$$C = [hu + (h+b)G(u)] \left\{ \left(\sum_{j=1}^{N} \sigma_j \sqrt{L_d - ly_j + 1} \right)^2 + L_s \sum_{j=1}^{N} \sigma_j^2 + 2 \sum_{j=1}^{N} \sum_{k=j+1}^{N} \left[\sum_{t=1}^{L_s} Cov\left(\sqrt{1-y_j} B_{j,t}, \sqrt{1-y_k} B_{k,t} \right) + \sum_{t=l+1}^{L_s+l} Cov\left(\sqrt{y_j} A_{j,t}, \sqrt{y_k} A_{k,t} \right) \right] \right\}^{1/2}$$

In the expression above, we see that the standard deviation of the demand for the whole cycle effects the inventory costs. The variance of the whole cycle is considered in three parts. First, the variance of the demand for L_s periods until the allocation period; second, the variance of the demand after the allocation period and the third, the covariance of the demand for the periods until the allocation policy. The third part comes with the correlated demands and it has increasing impact on the cost function.

6. COMPUTATIONAL STUDY

In Chapter 5, we provided approximate optimal ordering and allocating policies and expected inventory cost for the two-echelon problem where partial advance demand information is available. In this chapter we present numerical examples in order to demonstrate the impact of parameter settings and to observe the value of advance demand information for the models analyzed in Section 5.2 and Section 5.3. We do not conduct a computational study for the perfect ADI model, which is analyzed in Chapter 4. Because perfect ADI model already exists in partial ADI model when (y_1, y_2)=(1,1). In such a case, demand is totally observed in advance and partial ADI model takes the form of perfect ADI model.

We consider a large set of parameter combinations. We construct this numerical study for the problem with two retailers in order to obtain computational simplicity. The parameter set is described in Table 6.1. As it is seen from the table, we consider three cases for the inventory the system has at the beginning of the cycle. These are mean demand for L_s periods, mean demand for the whole cycle and the advance demand information. We consider two cases for the coefficient of variation and two different cases for mean demand. So, we achieve the σ_i/μ_i values under four different σ_i values and therefore we consider 16 combinations of μ_i and σ_i values. (σ_1, μ_1) and (σ_2, μ_2) combinations, which we construct our computational study for, are listed in Table 6.2. Moreover, we test all the values demand lead time can take. As we explained in Chapter 4, we assume that lead time from the supplier to the depot and lead time from the depot to retailers are greater than demand lead time. Thus, we express demand lead time values tested as $\min(L_s, L_d) - 1$, $\min(L_s, L_d) - 2, ..., 1$. For instance, for $(L_s, L_d) = (7, 5)$ combination, the values we consider for demand lead time are four, three, two and one. We test positive and negative correlation coefficients to see the impact of positively and negatively correlated demands. In the study, the values of ADI amount we test begin with zero and end when it takes its maximum value one. Eleven different values are tested for the amount of ADI with 0.1 increase amount .

Table 6.1. Parameter set

Parameter	Symbol	Value tested
Holding cost	h	1
Penalty cost	b	7
Inventory at the beginning of the cycle	NI	$L_s\mu$, $(L_s + L_d + 1)\mu$, ADI
Coefficient of variation for retailer 1	σ_1/μ_1	0.1, 0.5
Coefficient of variation for retailer 2	σ_2/μ_2	0.1, 0.5
Mean demand of retailer 1	μ_1	25, 100
Mean demand of retailer 2	μ_2	25, 100
Lead time from the supplier to the depot	L_s	3, 5, 7
Lead time from the depot to retailers	L_d	2, 4, 5
Demand lead time	l	$min(L_s, L_d) - 1, min(L_s, L_d)$-2,.., 1
Correlation coefficient	$Corr(B_{1,t}, B_{2,t})$	-0.5, 0, 0.5, 0.8
Correlation coefficient	$Corr(A_{1,t}, A_{2,t})$	-0.5, 0, 0.5, 0.8
Amount of ADI obtained by retailer 1	y_1	0, 0.1, 0.2,...., 1
Amount of ADI obtained by retailer 2	y_2	0, 0.1, 0.2,...., 1

For each combination of (σ_1, μ_1) and (σ_2, μ_2), we solved the problem under 21 combinations of L_s, L_d and l, and for each of these 21 combinations we obtained the results for 121 different (y_1, y_2) values. We did the same computations for three different cases of net inventory the system has at the beginning of the planning horizon. So, in total we examined 3x16x21x121 cases just for the model with uncorrelated demands and we observed the changes in optimal order amount, allocations, expected inventory costs of each retailer and system-wide expected inventory cost according to the parameter sets.

We note that we needed values for ADI and for the demand observations of the periods until the allocation period. Therefore, we generated normal random variates with parameters (0,1) using random number generation function in Excel. Since $A_{i,t}$ and $B_{i,t}$ are normally distributed with parameters (0,σ_i), we generated $a_{i,t}$ values for the first $L_s + l$ periods and $b_{i,t}$ values for the first L_s periods using the generated normal variates. We use the same random variate in all (L_s, L_d, l) and (μ_i, σ_i) combinations to be able to make comparison. We present the values generated for the case with $\sigma_1 = \sigma_2 = 2.5$ in Table 6.3. Since the maximum value we test for L_d is five and since

Table 6.2. (σ_i, μ_i) combinations tested

$(\sigma_1/\mu_1, \sigma_2/\mu_2)$	(σ_1, μ_1)	(σ_2, μ_2)	$(\sigma_1/\mu_1, \sigma_2/\mu_2)$	(σ_1, μ_1)	(σ_2, μ_2)
0.1, 0.1	2.5, 25	2.5, 25	0.5, 0.1	12.5, 25	2.5, 25
0.1, 0.1	2.5, 25	10, 100	0.5, 0.1	12.5, 25	10, 100
0.1, 0.5	2.5, 25	12.5, 25	0.5, 0.5	12.5, 25	12.5, 25
0.1, 0.5	2.5, 25	50, 100	0.5, 0.5	12.5, 25	50, 100
0.1, 0.1	10, 100	2.5, 25	0.5, 0.1	50, 100	2.5, 25
0.1, 0.1	10, 100	10, 100	0.5, 0.1	50, 100	10, 100
0.1, 0.5	10, 100	12.5, 25	0.5, 0.5	50, 100	12.5, 25
0.1, 0.5	10, 100	50, 100	0.5, 0.5	50, 100	50, 100

we assume $L_d > l$, we generated four different values for $a_{i,t}$. As we see from the optimal allocation policy, we need $b_{i,t}$ of the first L_s periods and $a_{i,t}$ of the periods from $l+1$ until period $l+L_s$. Since the maximum value we test for L_s is seven, we generate seven different values for $b_{i,t}$ and $a_{i,t}$ of each retailer.

Table 6.3. Values generated for demand information and demand observation until allocation period

t	$a_{1,t}$	$a_{2,t}$	t	$b_{1,t}$	$b_{2,t}$	t	$a_{1,t}$	$a_{2,t}$
1	-0.75058	2.737556	1	-1.41981	-0.21321	$l+1$	4.15364	1.687846
2	-3.19421	-2.71675	2	-1.01012	-0.46539	$l+2$	-4.03099	-0.95331
3	0.610643	-1.72551	3	0.337133	-1.28302	$l+3$	1.347371	1.894028
4	3.191184	-4.22608	4	-0.91373	4.93053	$l+4$	2.255479	-3.61047
			5	-0.81748	2.164182	$l+5$	4.797289	-2.11809
			6	-0.9256	5.939137	$l+6$	-0.21129	-3.80393
			7	3.356604	-1.63727	$l+7$	-1.30949	-0.90719

6.1. Experiments with Uncorrelated Demand

6.1.1. Analysis of Order Amount

We see that approximate optimal order quantity is affected from many of the parameters tested. One of the parameters affecting Q^* is the amount of ADI. Q^* can increase or decrease with y_1 and y_2 depending on the obtained demand information.

Since the partial ADI structure is $D_{i,t} = \mu_i + \sqrt{y_i}A_{i,t} + \sqrt{1-y_i}B_{i,t}$, the system takes mean demand into account while making ordering decision when there is no ADI available. We know that $A_{i,t}$ and $B_{i,t}$ are normally distributed with parameters $(0, \sigma_i)$. When ADI is available; after the observation of $A_{i,t}$ and $B_{i,t}$, the required amount is either more than mean demand or less than μ_i. To demonstrate this observation, we give Figure 6.1, Figure 6.2 and Figure 6.3. In these figures, there are 11 curves each demonstrating the change with y_2 when y_1 is constant. The first curve consists of the results obtained when y_1 is zero and y_2 takes the values varying between zero and one. The second curve demonstrates the results obtained when y_1 is 0.1 and y_2 varying between zero and one. As it is seen in Figure 6.1 demonstrating the change of Q^* with y_i when $(L_s, L_d, l) = (5, 4, 1)$, Q^* decreases as y_1 increases and Q^* increases as y_2 increases. The reason for such a change can be observed from Table 6.3. In Table

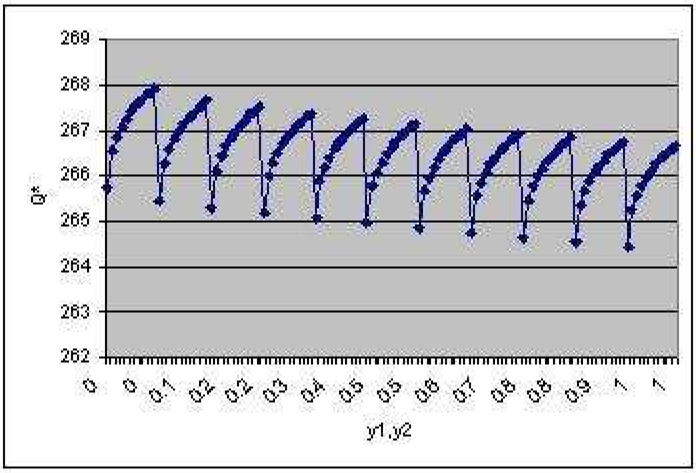

Figure 6.1. Order amount versus amount of ADI of retailers when $NI_i = L_s\mu_i$, $(\sigma_i, \mu_i) = (2.5, 25)$, L_s=5, L_d=4, l=1

6.3, $a_{1,1}$ is a negative value and $a_{2,1}$ is a positive value. As y_1 increases, the impact of $a_{1,1}$ increases and the impact of $a_{2,1}$ increases as y_2 increases. When $l = 2$, sum of $a_{1,1}$ and $a_{1,2}$ is again negative, sum of $a_{2,1}$ and $a_{2,2}$ is nearly zero. So Q^* decreases as y_1 increases, however Q^* decreases as y_2 increases although demand information for the second retailer is nonnegative. Please see Figure 6.2. This figure shows the case with $(L_s, L_d, l) = (5, 4, 2)$. Demand lead time increases in this case so ADI varies. The decrease in standard deviation part of Q^* expression is more than the increase in mean

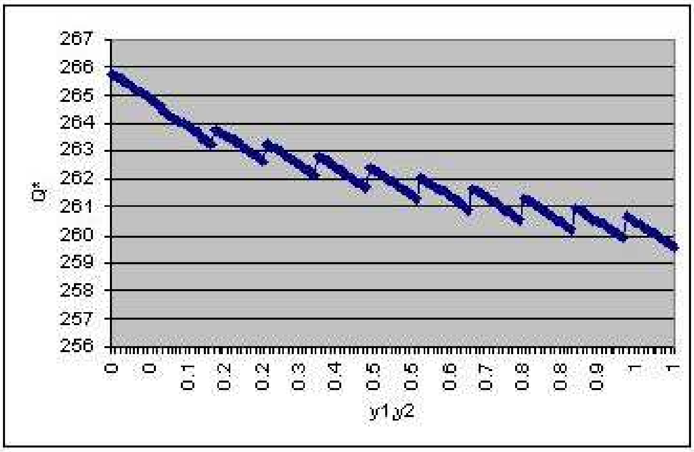

Figure 6.2. Order amount versus amount of ADI of retailers when $NI_i = L_s\mu_i$, $(\sigma_i, \mu_i) = (2.5, 25)$, L_s=5, L_d=4, l=2

part of the Q^* expression. Therefore Q^* decreases as y_2 increases in this case. When we consider the case with $(L_s, L_d, l) = (5, 4, 3)$, we can see from Table 6.3 that total demand information obtained by both of the retailers are negative. Since both of the standard deviation and mean parts of Q^* expression decreases with y_2 and l in this case, Q^* decreases as y_1 and y_2 increases. It is demonstrated in Figure 6.3. So it is possible to say that if $a_{i,t}$ observed for demand lead time is a negatively large value, Q^* decreases as y_i increases; but if ADI is a positively large value, then Q^* increases as uncertainty decreases.

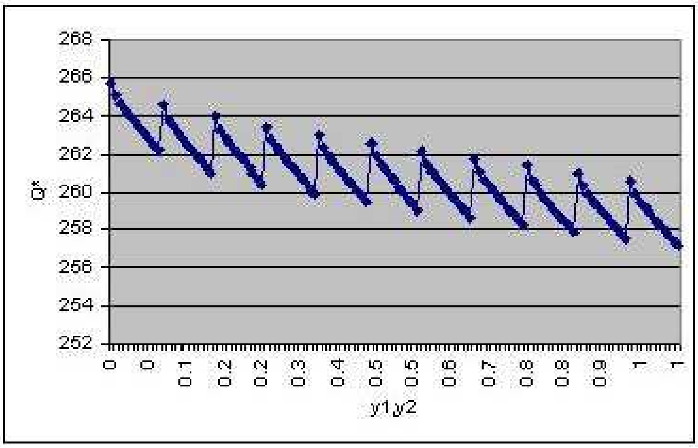

Figure 6.3. Order amount versus amount of ADI of retailers when $NI_i = L_s\mu_i$, $(\sigma_i, \mu_i) = (2.5, 25)$, L_s=5, L_d=4, l=3

Another parameter affecting Q^* is lead time. The results obtained for 21 combinations of L_s, L_d and l enable us to observe that L_s, L_d and l have impact on Q^*. Q^* increases or decreases with l related to the acquired demand information. In Figure 6.1, Q^* obtained when $(y_1, y_2) = (1,1)$ is larger than Q^* obtained when $(y_1, y_2) = (0,0)$. In Figure 6.2 and Figure 6.3, Q^* obtained when $(y_1, y_2) = (1,1)$ is smaller than Q^* obtained when $(y_1, y_2) = (0,0)$. One more point to note that Q^* takes the maximum values when L_d has the largest value under the case with $NI = L_s\mu_i$. For the cases with the same L_d and with the same l, Q^* increases as the length of the cycle increases. For the cases with only the same L_d, Q^* increases as l decreases but the opposite can also be realized related to the demand information. But this observation holds when $NI = L_s\mu_i$. In other NI cases, the impact of L_d is not so straightforward because L_s also affects order amount. The results obtained for different combinations of L_s, L_d and l where $NI_i = L_s\mu_i$, $(\sigma_i, \mu_i) = (2.5, 25)$ and $(y_1, y_2) = (1,1)$ are ordered in Table 6.4, which confirms our observations.

Table 6.4. Increasing order of Q^* obtained for (L_s, L_d, l) cases when $NI_i = L_s\mu_i$, $(\sigma_i, \mu_i) = (2.5, 25)$, $(y_1, y_2) = (1,1)$

Q^*	(L_s, L_d, l)	Q^*	(L_s, L_d, l)	Q^*	(L_s, L_d, l)
162.7475	3,2,1	260.7402	7,4,2	309.5651	3,5,2
164.1883	5,2,1	265.476	3,4,1	309.6253	7,5,3
165.476	7,2,1	266.6511	5,4,1	310.7402	5,5,2
257.1624	5,4,3	267.7388	7,4,1	311.8278	7,5,2
258.2773	3,4,2	306.1275	5,5,4	316.6511	3,5,1
258.4502	7,4,3	307.4153	7,5,4	317.7388	5,5,1
259.5651	5,4,2	308.4502	5,5,3	318.7561	7,5,1

Increase in standard deviation affects Q^*. It increases as standard deviation increases. We demonstrate how Q^* changes with standard deviation under the case with $(L_s, L_d, l) = (3, 2, 1)$, $NI_i = L_s\mu_i$, $(\sigma_1, \mu_1) = (2.5, 25)$ and $y_1 = 0.8$ in Figure 6.4. Here, we notice that not only the standard deviation but also the mean has impact on Q^*. As it is seen in the figure, Q^* with (σ_1, σ_2)=(2.5,10) has larger values than Q^*

with (σ_1, σ_2)=(2.5,12.5). Because the case with (σ_1, σ_2)=(2.5,10) has larger μ_i values than the case with (σ_1, σ_2)=(2.5,12.5). Besides, we can see that coefficient of variation does not have a determining impact. Because the case with $(\sigma_1/\mu_1, \sigma_2/\mu_2) = (0.1, 0.5)$ and $(\sigma_2, \mu_2) = (50, 100)$ has the largest Q^*, the case with $(\sigma_1/\mu_1, \sigma_2/\mu_2) = (0.1, 0.1)$ and $(\sigma_2, \mu_2) = (10, 100)$ has larger Q^* than the case with $(\sigma_1/\mu_1, \sigma_2/\mu_2) = (0.1, 0.5)$ and the case with $(\sigma_1/\mu_1, \sigma_2/\mu_2) = (0.1, 0.1)$ $(\sigma_2, \mu_2) = (2.5, 25)$ has the smallest Q^*.

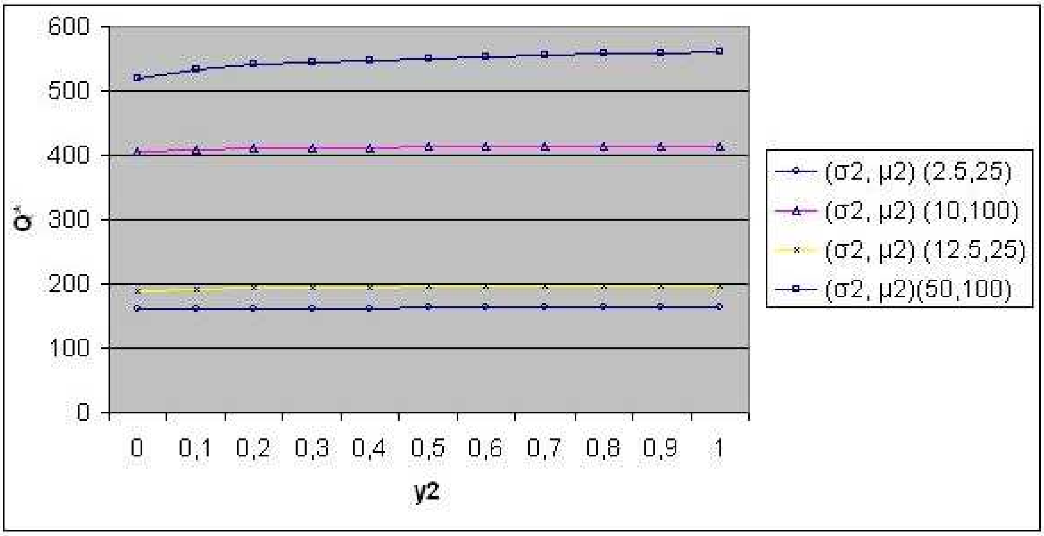

Figure 6.4. Order amount versus ADI amount of retailer 2 when $y_1 = 0.8$, $NI_i = L_s\mu_i$, $(L_s, L_d, l) = (3, 2, 1)$, $(\sigma_1, \mu_1) = (2.5, 25)$

When we analyze our results obtained for the case where NI is equal to the mean demand of the cycle instead of the mean demand of L_s periods, we see that the value of Q^* decreases. When we analyze our results obtained for the case where NI is equal to the advance demand information obtained at the beginning of the cycle, we see that Q^* increases so much. This is what we expect, because the on hand inventory at the beginning of the planning horizon decreases in this case. In Table 6.5, we provide values of Q^* to show how it changes with NI under the case with $(\sigma_i, \mu_i) = (2.5, 25)$ and $(L_s, L_d, l) = (3, 2, 1)$. We denote $NI_i = L_s\mu_i$ with NI1, $NI_i = \sum_{t=1}^{l}(\mu_i + \sqrt{y_i}a_{i,t})$ with NI2 and $NI_i = (L_s + L_d + 1)\mu_i$ with NI3 in the table.

But y_i, σ_i and l affect Q^* in the same direction as we expressed for the case where NI is equal to the mean demand of L_s periods. We do not observe an increase

Table 6.5. Q^* changing with different y_i and NI when $(\sigma_i, \mu_i) = (2.5, 25)$, $(L_s, L_d, l) = (3, 2, 1)$

y_2	y_1	NI1	NI2	NI3	y_1	NI1	NI2	NI3
0	0	162.2013	260.21432	12.201298	0.7	161.07282	259.08584	11.072821
0.1	0	162.99873	261.01175	12.998726	0.7	161.87177	259.8848	11.871772
0.2	0	163.28806	261.30108	13.288056	0.7	162.16268	260.1757	12.162678
0.3	0	163.49292	261.50594	13.492917	0.7	162.36917	260.3822	12.369172
0.4	0	163.6535	261.66652	13.653498	0.7	162.53145	260.54447	12.531447
0.5	0	163.78533	261.79835	13.785329	0.7	162.66504	260.67806	12.665036
0.6	0	163.89634	261.90936	13.896339	0.7	162.77787	260.7909	12.777874
0.7	0	163.99121	262.00423	13.991205	0.7	162.87464	260.88767	12.874642
0.8	0	164.07294	262.08597	14.072942	0.7	162.95836	260.97139	12.958362
0.9	0	164.14361	262.15664	14.143611	0.7	163.0311	261.04412	13.031099
1	0	164.20468	262.2177	14.204677	0.7	163.09433	261.10735	13.094329
0	0.5	161.31884	259.33187	11.318844	1	160.71654	258.72956	10.716541
0.1	0.5	162.11732	260.13034	12.117319	1	161.51629	259.52931	11.516285
0.2	0.5	162.40773	260.42076	12.407732	1	161.80801	259.82104	11.808014
0.3	0.5	162.61371	260.62674	12.613714	1	162.01536	260.02838	12.015359
0.4	0.5	162.77546	260.78848	12.775459	1	162.17852	260.19154	12.178518
0.5	0.5	162.9085	260.92152	12.908498	1	162.31302	260.32605	12.313024
0.6	0.5	163.02076	261.03379	13.020763	1	162.42682	260.43984	12.426816
0.7	0.5	163.11694	261.12996	13.116936	1	162.52458	260.5376	12.524577
0.8	0.5	163.20003	261.21306	13.200034	1	162.60933	260.62236	12.609331
0.9	0.5	163.27212	261.28515	13.272124	1	162.68315	260.69617	12.68315
1	0.5	163.33468	261.3477	13.334676	1	162.74751	260.76053	12.747509

or decrease tendency of Q^* for the case where NI is equal to the mean demand of the cycle but for the other case we can say that Q^* increases as the length of the cycle beyond demand lead time increases. If the length of the cycle beyond demand lead time is the same for some combinations of L_s, L_d and l, then Q^* generally increases as demand lead time increases.

6.1.2. Analysis of Allocation

As we can see from equation (5.2), the allocation policy is affected from a large number of parameters. Demand lead time, the amount of ADI, the obtained demand information and the demand observation of the periods until the allocation period are all interacting. So it is difficult to observe the change in allocation amount with the change in these parameters. But we tested the case where the value of ADI and the demand observation for each retailer and for each time period are equal. If the value is positive, the amount allocated to retailer i first increases then decreases as its ADI amount increases, the allocated amount of the other retailer first decreases then increases. If the value is negative, the opposite case occurs. The allocated amount of retailer i decreases as its ADI amount increases, the allocated amount of the other retailer retailer first increases then decreases. If the ADI and demand observation of one retailer is negative and the other is positive with the equal absolute values, the allocation of retailer with positive demand increases as its ADI amount increases, the allocation of the other retailer first decreases then increases. The allocation of retailer with negative demand decreases as its ADI amount increases, the allocation of the other retailer first increases then decreases. Since $A_{i,t}$ and $B_{i,t}$ take different random values with parameters $(0, \sigma_i)$ in our study, the allocation follows different directions as y_i changes. In addition to these, we observe that z_i/Q^* changes in the same direction with z_i. If z_i increases with y_i, z_i/Q^* increases but if it decreases, z_i/Q^* decreases.

We can say that the major component determining the allocation amount is the standard deviation. The results explicitly show that the retailer with high standard deviation receives the largest share.

The value of net inventory the system has at the beginning of the cycle has an indirect impact on the allocation policy through the opimum order amounts. Here it is noteworthy to say that when the net inventory on hand at the beginning of the cycle is equal to the mean demand of the cycle, we observe negative allocations for the case where the length of the cycle and demand lead time are high and standard deviation is small.

6.1.3. Analysis of Expected Inventory Cost

The parameters affecting system-wide inventory cost are standard deviation, amount of ADI, demand and supply lead times. The results explicitly show that expected inventory cost decreases as amount of ADI and demand lead time increases. In order to understand the value of advance demand information, we find per cent change in the expected inventory cost. In Table 6.6, we provide the per cent decrease in expected system-wide inventory cost for each (y_1, y_2) combination under the case with $(L_s, L_d, l) = (5, 4, 1)$, $(\sigma_i, \mu_i) = (2.5, 25)$ and $NI_i = L_s\mu_i$. It is clear that per cent decrease in total cost increases as uncertainty decreases. In Table 6.7, we provide the per cent decrease in expected cost for each (y_1, y_2) combination under the case with $(L_s, L_d, l) = (5, 4, 2)$, $(\sigma_i, \mu_i) = (2.5, 25)$ and $NI_i = L_s\mu_i$. From Table 6.7, we can see that per cent decrease under the case with $l = 2$ is larger than per cent decrease in the case with $l = 1$. These tables explicitly show that y_i and l has decreasing impact on total cost.

Table 6.6. Per cent decrease in the expected cost with (y_1, y_2) when $(L_s, L_d, l) = (5, 4, 1)$, $(\sigma_i, \mu_i) = (2.5, 25)$

(y_1,y_2)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0		0.335	0.672	1.013	1.356	1.703	2.053	2.406	2.763	3.123	3.487
0.1	0.335	0.669	1.006	1.346	1.689	2.035	2.384	2.737	3.093	3.452	3.815
0.2	0.672	1.006	1.342	1.682	2.024	2.369	2.718	3.07	3.425	3.784	4.147
0.3	1.013	1.346	1.682	2.02	2.362	2.707	3.055	3.406	3.761	4.119	4.481
0.4	1.356	1.689	2.024	2.362	2.703	3.047	3.395	3.745	4.099	4.457	4.818
0.5	1.703	2.035	2.369	2.707	3.047	3.391	3.737	4.087	4.441	4.797	5.158
0.6	2.053	2.384	2.718	3.055	3.395	3.737	4.083	4.433	4.785	5.141	5.5
0.7	2.406	2.737	3.07	3.406	3.745	4.087	4.433	4.781	5.133	5.488	5.847
0.8	2.763	3.093	3.425	3.761	4.099	4.441	4.785	5.133	5.484	5.838	6.196
0.9	3.123	3.452	3.784	4.119	4.457	4.797	5.141	5.488	5.838	6.192	6.549
1	3.487	3.815	4.147	4.481	4.818	5.158	5.5	5.847	6.196	6.549	6.905

We analyze each combination of L_s, L_d and l to see the impact of these parameters on C. Under the cases with the same cycle length, minimum cost is achieved

Table 6.7. Per cent decrease in the expected cost with (y_1, y_2) when $(L_s, L_d, l) = (5, 4, 2)$, $(\sigma_i, \mu_i) = (2.5, 25)$

(y_1,y_2)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0		0.672	1.356	2.053	2.763	3.487	4.226	4.982	5.755	6.548	7.361
0.1	0.672	1.342	2.024	2.718	3.425	4.147	4.883	5.636	6.406	7.195	8.005
0.2	1.356	2.024	2.703	3.395	4.099	4.818	5.551	6.3	7.067	7.853	8.659
0.3	2.053	2.718	3.395	4.083	4.785	5.5	6.231	6.977	7.74	8.522	9.324
0.4	2.763	3.425	4.099	4.785	5.484	6.196	6.923	7.666	8.425	9.203	10
0.5	3.487	4.147	4.818	5.5	6.196	6.905	7.629	8.368	9.123	9.897	10.69
0.6	4.226	4.883	5.551	6.231	6.923	7.629	8.348	9.084	9.836	10.61	11.39
0.7	4.982	5.636	6.3	6.977	7.666	8.368	9.084	9.815	10.56	11.33	12.11
0.8	5.755	6.406	7.067	7.74	8.425	9.123	9.836	10.56	11.31	12.07	12.85
0.9	6.548	7.195	7.853	8.522	9.203	9.897	10.61	11.33	12.07	12.82	13.6
1	7.361	8.005	8.659	9.324	10	10.69	11.39	12.11	12.85	13.6	14.37

with maximum demand lead time. Under the cases with the same demand lead time, minimum cost is achieved with minimum cycle length. Under the cases with the same cycle length but different L_s and L_d values, the case with larger L_d value has larger total cost.

Besides, system-wide inventory cost increases as σ_i increases. We have observed that cost decreases as ADI amount increases. We want to see how decrease amount of cost with respect to the ADI amount changes as standard deviation changes. Table 6.8 demonstrates the per cent decrease in expected inventory cost with selected ADI amounts for each combination of (σ_1, σ_2). We give the results for four (y_1, y_2) combinations. It is easy to see that per cent decrease increases with increasing ADI amount. As we can see from the table, per cent decrease for the same (y_1, y_2) combination does not follow the same direction as standard deviation changes. The reason for this is the effect of decrease amount obtained with ADI and the cost obtained when ADI is not available. For the case with $y_2 = 0$ and y_1 is constant, when σ_1 is constant and $\sigma_1 > \sigma_2$, decrease amount increases as σ_2 gets closer to σ_1. If σ_2 takes a larger value than σ_1, then decrease amount begins to decrease. We can see the impact of this observation when y_i takes large values, as it is demonstrated in Table 6.8.

Table 6.8. Per cent decrease in system-wide expected inventory cost with selected ADI amounts for each combination of (σ_1, σ_2) when $(L_s, L_d, l) = (3, 2, 1)$

	y_1, y_2	y_1, y_2	y_1, y_2	y_1, y_2		y_1, y_2	y_1, y_2	y_1, y_2	y_1, y_2
(σ_1, σ_2)	(0,1)	(0.4,1)	(0.8,1)	(1,1)	(σ_1, σ_2)	(0,1)	(0.4,1)	(0.8,1)	(1,1)
(2.5,2.5)	6.02	8.23	10.6	11.80	(12.5,2.5)	1.76	5.02	8.43	10.19
(2.5,10)	8.45	9.22	10	10.5	(12.5,10)	5.34	7.79	10.4	11.75
(2.5,12.5)	8.57	9.18	9.85	10.2	(12.5,12.5)	6.02	8.23	10.6	11.80
(2.5,50)	8.74	8.89	9.06	9.15	(12.5,50)	8.45	9.22	10	10.46
(10,2.5)	2.17	5.37	8.72	10.46	(50,2.5)	0.46	3.83	7.34	9.14
(10,10)	6.02	8.23	10.6	11.80	(50,10)	1.76	5.02	8.43	10.19
(10,12.5)	6.64	8.59	10.7	11.75	(50,12.5)	2.17	5.37	8.72	10.46
(10,50)	8.57	9.18	9.85	10.19	(50,50)	6.02	8.23	10.6	11.80

The parameter NI, net inventory on hand at the beginning of the cycle, does not have an effect on inventory costs.

There are two parameters affecting the inventory cost of retailer i. These parameters are σ_i and y_i. Net inventory the system has at the beginning of the cycle does not affect C_i. It also does not affect system-wide inventory cost. Moreover, we consider the same net inventory policy for each retailer. The results show that the retailer with higher standard deviation has the higher inventory cost. Also, the cost of retailer one decreases as y_1 increases and C_2 increases as y_1 increases. C_1 increases and C_2 decreases as y_2 increases. Therefore we can generalize this such that decrease in uncertainty of a retailer decreases this retailer's cost and increases the other retailer's cost.

6.2. Experiments with Correlated Demand

We construct the same computational study except one parameter for the partial ADI model with correlated demands. We test net inventory the system has at the beginning of the planning horizon for one value, which is $NI_i = L_s\mu_i$. As it can be seen in Table 6.1, we consider four different values of correlation coefficient for both $B_{i,t}$ and $A_{i,t}$. So we make our observations for 16 combinations of correlation coefficients.

We compare the results obtained for the correlated demands case with the one obtained for the uncorrelated demands case.

6.2.1. Analysis of Order Amount

The results for approximate optimal order quantity are presented in Table 6.9. In the table, for instance $Q(0, 0.5)$ is used to denote Q^* obtained under the case with $Corr(A_{1,t}, A_{2,t}) = 0$ and $Corr(B_{1,t}, B_{2,t}) = 0.5$. The table demonstrates the comparison of Q^* obtained with different values of correlation coefficient with Q^* obtained under the case with uncorrelated demands. We compare the results of correlated and uncorrelated demands in two seperate process; when $y_i = 0$ and $y_i = 1$. We can say that $Corr(A_{1,t}, A_{2,t})$ has no impact when $y_i = 0$ and $Corr(B_{1,t}, B_{2,t})$ has no impact when $y_i = 1$. In addition to these, we can say that $Corr(A_{1,t}, A_{2,t})$ has more impact when $y_i > 0.5$ and $Corr(B_{1,t}, B_{2,t})$ has more impact when $y_i < 0.5$. The reason for this observation is the demand structure with $\sqrt{y_i}$ multiplier of $A_{i,t}$ and $\sqrt{1 - y_i}$ multiplier of $B_{1,t}$. We can say that positive correlation coefficient increases order amount and negative correlation coefficient decreases order amount. Also we note that Q^* is the same for the combinations with the same $Corr(A_{1,t}, A_{2,t})$ when $y_i = 1$ and Q^* is the same for the combinations with the same $Corr(B_{1,t}, B_{2,t})$ when $y_i = 0$. In the model with correlated demands, Q^* follows a similar change with the one in the model with uncorrelated demands as σ_i, L_s, L_d and l change.

6.2.2. Analysis of Allocation

The results obtained for the correlated model imply that correlation affects the allocated amount in the same way as it affects Q^* and C. Positive correlation coefficient increases allocated amount and negative correlation coefficient decreases allocated amount. $Corr(A_{1,t}, A_{2,t})$ has no impact when $y_i = 0$ and $Corr(B_{1,t}, B_{2,t})$ has no impact when $y_i = 1$. Besides, we can say that $Corr(A_{1,t}, A_{2,t})$ has more impact when $y_i > 0.5$ and $Corr(B_{1,t}, B_{2,t})$ has more impact when $y_i < 0.5$.

Standard deviation affects allocation amount. The retailer with larger standard

Table 6.9. Comparison of Q^* obtained for the correlated demand model with Q^* obtained for the uncorrelated demand model

$Q(Corr(A_{1,t}, A_{2,t}), Corr(B_{1,t}, B_{2,t}))$	y_i=0	$y_i = 1$
$Q(0, -0.5)$	$< Q(0, 0)$	$= Q(0, 0)$
$Q(0, 0.5)$	$> Q(0, 0)$	$= Q(0, 0)$
$Q(0, 0.8)$	$> Q(0, 0)$, $> Q(0, 0.5)$	$= Q(0, 0)$
$Q(-0.5, -0.5)$	$< Q(0, 0)$, $< Q(-0.5, 0)$	$< Q(0, 0)$
$Q(-0.5, 0)$	$= Q(0, 0)$	$< Q(0, 0)$
$Q(-0.5, 0.5)$	$> Q(0, 0)$	$< Q(0, 0)$
$Q(-0.5, 0.8)$	$> Q(0, 0)$, $> Q(-0.5, 0.5)$	$< Q(0, 0)$
$Q(0.5, -0.5)$	$< Q(0, 0)$	$> Q(0, 0)$
$Q(0.5, 0)$	$= Q(0, 0)$	$> Q(0, 0)$
$Q(0.5, 0.5)$	$> Q(0, 0)$	$> Q(0, 0)$
$Q(0.5, 0.8)$	$> Q(0, 0)$, $> Q(0.5, 0.5)$	$> Q(0, 0)$
$Q(0.8, -0.5)$	$< Q(0, 0)$	$> Q(0, 0)$
$Q(0.8, 0)$	$= Q(0, 0)$	$> Q(0, 0)$
$Q(0.8, 0.5)$	$> Q(0, 0)$	$> Q(0, 0)$
$Q(0.8, 0.8)$	$> Q(0, 0)$, $> Q(0.8, 0.5)$	$> Q(0, 0)$

deviation receives more allocation than the retailer with smaller standard deviation.

6.2.3. Analysis of Expected Inventory Cost

The results for expected inventory cost are presented in Table 6.10. The observations we stated for approximate optimal order amount obtained under correlated demands case hold for expected inventory cost obtained under correlated demands case. The numerical examples show that positively correlated demands has an increasing impact and negatively correlated demands has a decreasing impact on system costs. In the model with correlated demands, C follows a similar change with the one in the model with uncorrelated demands as σ_i, L_s, L_d and l change.

Table 6.10. Comparison of C obtained for the correlated demand model with C obtained for the uncorrelated demand model

$C(Corr(A_{1,t}, A_{2,t}), Corr(B_{1,t}, B_{2,t}))$	y_i=0	$y_i = 1$
$C(0, -0.5)$	$< C(0, 0)$	$= C(0, 0)$
$C(0, 0.5)$	$> C(0, 0)$	$= C(0, 0)$
$C(0, 0.8)$	$> C(0, 0)$, $> C(0, 0.5)$	$= C(0, 0)$
$C(-0.5, -0.5)$	$< C(0, 0)$, $< C(-0.5, 0)$	$< C(0, 0)$
$C(-0.5, 0)$	$= C(0, 0)$	$< C(0, 0)$
$C(-0.5, 0.5)$	$> C(0, 0)$	$< C(0, 0)$
$C(-0.5, 0.8)$	$> C(0, 0)$, $> C(-0.5, 0.5)$	$< C(0, 0)$
$C(0.5, -0.5)$	$< C(0, 0)$	$> C(0, 0)$
$C(0.5, 0)$	$= C(0, 0)$	$> C(0, 0)$
$C(0.5, 0.5)$	$> C(0, 0)$	$> C(0, 0)$
$C(0.5, 0.8)$	$> C(0, 0)$, $> C(0.5, 0.5)$	$> C(0, 0)$
$C(0.8, -0.5)$	$< C(0, 0)$	$> C(0, 0)$
$C(0.8, 0)$	$= C(0, 0)$	$> C(0, 0)$
$C(0.8, 0.5)$	$> C(0, 0)$	$> C(0, 0)$
$C(0.8, 0.8)$	$> C(0, 0)$, $> C(0.8, 0.5)$	$> C(0, 0)$

Correlation also affects the expected cost of retailer i. Positive correlation coefficient increases C_i and negative correlation coefficient decreases C_i. $Corr(A_{1,t}, A_{2,t})$ has no impact when $y_i = 0$ and $Corr(B_{1,t}, B_{2,t})$ has no impact when $y_i = 1$. $Corr(A_{1,t}, A_{2,t})$ has more impact when $y_i > 0.5$ and $Corr(B_{1,t}, B_{2,t})$ has more impact when $y_i < 0.5$.

Standard deviation, σ_i, is the major component affecting the cost of retailer i positively. The retailer with higher standard deviation has larger inventory costs.

7. CONCLUSIONS

In this thesis; a single-item, multi-echelon distribution system is analyzed under advance demand information. Advance demand information is defined by two different structures. First advance demand structure we consider is perfect ADI and the second one is partial ADI.

For the model with perfect ADI, single order policy and order every period policy are analyzed. We obtain approximate optimal ordering and allocating policies for single order model and approximate optimal order-up-to level and allocation policy for order every period model. We observe that the impact of advance demand information is to decrease or to increase the order amount depending on the acquired advance demand information.

Partial ADI model is analyzed under three different cases. Firstly, the amount of advance demand information is assumed to be identical for each retailer and optimum ordering and allocating policies are obtained for this case. We observe that if the value of ADI amount takes its maximum value one, then the problem takes the form of perfect ADI. Thus, the obtained policy to determine the approximate optimal order amount is the same with the one obtained in perfect ADI model.

In the second partial ADI model, the amount of advance demand information is assumed to be retailer dependent, which means that ADI amount of any retailer is independent of the other retailers. Approximate optimal ordering and allocating policies, expected system-wide inventory cost and retailer cost functions are obtained for this model. Numerical examples show that ADI can decrease or increase order amount, depending on demand lead time, amount of ADI and obtained demand information. It is certain that mean and standard deviation of demand has increasing impact on order amount. Besides, order amount increases as the inventory the system has at the beginning of the cycle decreases. However, the change in order amount with respect to supply lead times and demand lead time is not so straightforward. The observations show that cost decreases as amount of ADI and demand lead time increases. We illustrate that the value of ADI is larger with larger ADI amount and larger demand lead time. Standard deviation has increasing effect on system cost but the change in the value of ADI with respect to the standard deviation of demand is not so clear. Lastly we observe that the major component affecting allocation of order amount and the cost of each retailer is standard deviation. The retailer with larger standard deviation receives the larger share of order amount and causes larger part of inventory cost.

In the third model, the partial ADI problem with retailer dependent ADI amount is analyzed under correlated demands among retailers. Approximate optimal ordering and allocating policies, expected system-wide inventory cost and expected retailer cost functions are obtained for this model. We observe that the allocation policy is the same with the one obtained for the model with uncorrelated demands. But in this model the ordering policy includes the covariance of the demands in addition to the policy obtained for the model with uncorrelated demands. Numerical examples show that positively correlated demands increase order amount and expected cost, negatively correlated demands decrease order amount and expected cost.

We conclude that to achieve minimum inventory cost, the amount of ADI and demand lead time have to be maximized. However; acquiring ADI causes cost. Therefore, it can be too costly to obtain maximum advance demand information. A research is needed to find the ADI amount and the demand lead time that will minimize total system cost by comparing the cost saving obtained with ADI and cost of acquiring ADI. Also smaller expected cost can be achived with smaller length of planning horizon. But smaller cost can be obtained with larger demand lead time when the length of the cycle is large. This again requires to be investigated. There is one more point to note that the approximate optimal order amount does not affect expected inventory cost. So decrease or increase in order amount does not have an impact on inventory cost. But high order amount causes high purchasing costs. So a comparison between purchasing cost and cost of acquiring ADI to decrease order amount is required.

In this study we assume identical demand lead times for each retailer and identical lead times from the depot to each retailer. Further studies can be performed under retailer dependent demand lead times and supply lead times from the depot to retailers. Moreover, a research can be conducted to find optimal system performance considering pricing contracts. In a study with pricing contract under available ADI, a decentralized system has to be considered instead of centralized system. In such a problem, the depot has contracts with the supplier and the retailers. Each retailer has ability to affect the proportion of ADI in total demand. Increasing ADI brings extra cost to retailer. But the depot makes more accurate ordering and allocating decisions with more ADI. So the depot offers some incentives to retailers such as price discounts or favorable allocations in the case of material shortage. The supplier also offers some incentives to the depot to reduce its uncertainty, such as buyback contract or price discount. As it is seen, a detailed research has to be made to analyze such a problem.

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