A METHODOLOGY FOR STATISTICAL SENSITIVITY ANALYSIS OF SYSTEM DYNAMICS MODELS

by

Mustafa Hekimoğlu

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ABSTRACT

A METHODOLOGY FOR STATISTICAL SENSITIVITY ANALYSIS OF SYSTEM DYNAMICS MODELS

Simulation models consist of many different components that analysts cannot estimate in perfect precision and results are always subject to some level of uncertainty. So, sensitivity analysis of system dynamics models should be conducted in order to reach more reliable conclusions.

The selection of sensitivity type should be appropriate for the purpose of the model. Specifically, system dynamics modeling is a behavior pattern-oriented methodology and sensitivity analysis of such models should consider the changes in behavior patterns rather than the numerical values of variables. In this thesis, a new approach, called behavior pattern sensitivity analysis, is suggested for dealing with the parameter uncertainty. In this approach pattern characteristics of output behavior are subject to statistical analysis. Changes in pattern measures, such as inflection point, period or amplitude, as a result of different parameter values are measured in order to determine possible sensitivity points of the model.

In this thesis, two statistical analysis procedures are utilized for measuring sensitivities to model parameters. Firstly, sensitivity data is evaluated with regression method, which is a convenient way of multi-variate analysis. Secondly, ANOVA of clusters (Kleijnen and Helton, 1999[a]) is used on the same sensitivity data to make comparison with the regression results. This approach is applied to three different system dynamics models, which are the project management model by Taylor and Ford (2006), a generic supply line model and the inventory workforce model by Sterman (2000).

The project management model is a medium size system dynamics model which is subject to sensitivity analysis with correlation-based screening method in the study of Taylor et al. (2007). In this thesis, this model is analyzed with pattern sensitivity approach. The other two models consist of supply line structures which are very common in many social and managerial systems. In particular, this type of model is prone to oscillate under specific circumstances. In this thesis, pattern sensitivity of oscillations is analyzed through these two system dynamics models. Our method provides promising results for statistical analysis of oscillatory system dynamics models which are difficult to analyze with other standard approaches such as screening.

In conclusion, analysis of the project management model indicates that regression method applied to behavior pattern measures can identify the most important parameters of the model. Furthermore, we find out that different behavior modes of this model have different parameter sensitivities, so separation of behavior modes before statistical analysis is necessary. Moreover, in all of our analyses regression results are successfully confirmed with ANOVA of clusters even in nonlinear cases. Therefore, we conclude that regression is a convenient method for sensitivity analysis of system dynamics method. Our experiments with the oscillatory models show that the pattern sensitivity approach can reliably rank the parameters of the models according to their effects on the selected pattern measures of oscillations.

ÖZET

SİSTEM DİNAMİĞİ MODELLERİNİN DAVRANIŞ DUYARLILIĞI ANALİZİ İÇİN BİR YÖNTEM

Simulasyon modelleri farklı ve kesin olarak tahmin edilemeyecek bileşenlerden oluşur. Dolayısıyla benzetim sonuçları her zaman için bir miktar belirsizlik ihtiva eder. Bu sebeple sistem dinamiği modelleri duyarlılık testine tabi tutularak belirsizliklerin sonuçlar üzerindeki etkisi araştırılmalıdır.

Uygulanacak olan duyarlılık analizi yöntemi modelin amacına uygun olarak seçilmelidir. Sistem dinamiği davranış odaklı bir metodololojidir. Dolayısıyla bu tür modellerin duyarlılık analizinde davranışın şeklindeki değişimler dikkate alınmalıdır. Davranış odaklı duyarlılık analizi, bu tür modellerin analizine yeni bir yaklaşım olarak önerilmiştir. Bu yaklaşımda, model çıktısının belirli karakteristikleri istatistiksel analize tabi tutulmaktadır.

Duyarlılığı ölçmek için iki tür istatistiksel analiz kullanılmıştır. İlk olarak duyarlılık verileri regresyon analizi ile değerlendirilmiş, daha sonra grupların ANOVA test metodu ile analizi yoluna gidilmiştır. Bu iki farklı analiz metodu üç farklı sistem dinamiği modeline uygulanmıştır. Bu modeller sırasıyla; Taylor ve Ford (2006)'un proje yönetimi modeli, temel tedarik hattı modeli ve envanter-işgücü modelleridir (Sterman, 2000).

Proje yönetimi modeli orta büyüklükte bir sistem dinamiği modelidir. Bu model daha önceki çalışmalarda istatistiksel korelasyon taraması (screeening) metodu ile analiz edilmiştir (Taylor et al., 2007). Bu tezde ise aynı model, önerilen duyarlılık analizi metodu ile incelenmiştir. Diğer iki model ise tedarik hattı yapılarından oluşmaktadır ve bu yapılar dalgalanma göstermeye oldukça yatkın modellerdir. Dalgalanmaların

davranış odaklı duyarlılık analizi, iki sistem dinamiği modeli vasıtasıyla gerçekleştirilmiştir. Bu iki modelin analizden, dalgalanma gösteren sistemlerin istatistiksel analizinde oldukça olumlu sonuçlar elde edilmiştir.

Sonuç olarak, proje yönetimi modelinin regresyon yöntemi ile analizi, davranış bakımından modelin en önemli parametrelerini bulmuştur. Ayrıca farklı model davranışı türlerinin modelin farklı parameterelerine duyarlılık göstermesi, analizin başında farklı davranışların birbirinden ayrılması gerekliliğini göstermiştir. Tüm analizlerde regresyon sonuçları ANOVA yöntemi ile de teyid edilmiştir. Dalgalanma davranışı gösteren modeller üzerindeki testler, davranış odaklı duyarlılık analizinin model parameterelerini, çeşıtli dalga ölçütleri üzerindeki etkilerine göre sıralayabildiğini göstermiştir.

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LIST OF SYMBOLS/ABBREVIATIONS

b_j	Standardized regression coefficient for parameter j
C_{it}	The value of a variable at time t for simulation run i
DT	Step size of simulation runs
$F(n_x - 1, N - n_x)$	F statistic for ANOVA of clusters
f_{rw-s}	Rework fraction due to schedule pressure in the project management model by Taylor and Ford(2006)
I_i	Inflection level from simulation run i
i	Index for the sensitivity simulation run
j	Index for model parameter
N	Number of sensitivity simulations taken in each sensitivity analysis
n_{x_q}	Number of response values in each cluster
n_x	Number of cluster in ANOVA of clusters
s_{rw-s}	Sensitivity of rework fraction to schedule pressure in the project management model
T_i	Peak of tipping point behavior from simulation run i
t	Index for simulation time
t_r	Required time variable for project completion in the project management model by Taylor and Ford(2006)
t_a	Available time variable in the project management model
q	Index for cluster number in ANOVA of clusters
R^2	Coefficient of determination of a regression model
$\overline{x}_j$	Mean of value of parameter j
x_{ij}	The value of Parameter j in simulation run i
W	Transformed dependent variable
y_i	The value of dependent variable in simulation run i
$\overline{y}$	Mean value of dependent variable in sensitivity simulation
$\hat{y}_i$	Dependent variable estimated by regression equation for simulation run i
$\bar{y}_q$	Average of response variable in each cluster

α_0	Initial guess for value of transformation exponent
α_j	Transformation exponent for dependent variable j
β_j	Regression coefficient for parameter j
λ	Transformation exponent for Box-Cox transformation
ν	Expected value of dependent variable y
$\rho_{x_j y}$	Correlation coefficient between parameter j (x_j) and dependent variable (y)
σ_y	Standard deviation for dependent variable
σ_{x_j}	Standard deviation for parameter j
τ_i	Inflection time in s-shaped growth behavior
ω	Vector of transformed dependent variable

1. INTRODUCTION

Simulation models consist of various type of information such as parameters, probability distributions and feedback loops which include some level of uncertainty. Namely, values of parameters cannot be estimated precisely due to data availability or time constraints. Since this uncertainty makes simulation models less reliable, results should be tested for their sensitivity to the changes in model components.

Furthermore, sensitivity tests on simulation results not only increase the reliability, but they also “guide” information collection efforts and indicate the validity of the model (Sterman, 2000). Components, to which simulation results are sensitive, need more attention than other parts of the model. Researchers should collect more information on this type of components in order to have a more reliable simulation model (Sterman, 2000). In fact, parameter sensitivity of the model can be compared with the information from real system. If any parameter is found to be important in the model analysis although it is an ignorable point of the real system, the model should be examined for its structural validity.

System dynamics can be seen as a branch of control theory that deals with socio-economic problems (Ford, 1999). In this methodology, the problematic behavior is tried to be explained by system structure. The behavior pattern of the system is the main interest of analysts rather than the specific values of the variables. Therefore, a behavior-pattern-oriented approach should be applied to sensitivity analysis of system dynamics models which can be conducted in various ways. For instance, changes of behavior mode in response to changing model parameters can be tested. Such an analysis aims to determine the model components that change the behavior mode. Also, the sensitivity of specific behavior characteristics can be analyzed against uncertainty in parameter values. This approach, which is called behavior pattern sensitivity, constitutes the main focus of this thesis.

We define behavior pattern sensitivity as the changes in specific characteristics of output behavior, such as oscillation period or inflection level of s-shaped growth, in response to changing values of parameters. Such sensitivity information not only indicates the important parameters of the model, but also provides clues for leverage points of the system. Furthermore, behavior pattern sensitivity has great deal of importance for oscillatory system dynamics models since it is difficult to analyze oscillations with correlation based statistical methods using the values of model variables. In this thesis, two oscillatory system dynamics models are subject to sensitivity analysis with pattern sensitivity approach.

Literature review on sensitivity analysis of system dynamics models is presented in Chapter 2 of the thesis and the comparison of different sensitivity analysis methods is presented in Chapter 3. In Chapter 4, application of the pattern sensitivity to a project management model is discussed. The project management model by Taylor and Ford (2006) is a nonlinear, medium size system dynamics model that focuses on the project failures due to deadline stress and ripple effect.

Furthermore, sensitivity analysis of oscillatory system dynamics models is discussed in Chapter 5. Oscillations are special behavior modes which follow cyclic patterns in state space. This feature of oscillatory models makes standard analysis methods, such as screening, inconclusive. In this thesis, applications of behavior pattern sensitivity to two different oscillatory system dynamics models are presented in Chapter 6 and 7 respectively.

2. SENSITIVITY ANALYSIS OF SYSTEM DYNAMICS MODELS

Model parameters are subject to uncertainty in system dynamics models. This uncertainty may yield unreliable simulation results especially since these models usually have nonlinear and complex structures. Thus, in system dynamics projects, sensitivity of results to the different model components should be analyzed after the modeling phase. Different researchers consider sensitivity of system dynamics models in various studies. Detailed literature review on sensitivity analysis is given in Section 2.1 and a new sensitivity analysis approach, called behavior pattern sensitivity, is suggested in Section 2.2.

2.1. Sensitivity Analysis Studies in Literature

Sensitivity analysis is a natural successor of the modeling process since it both guides the modeling effort and provides information on structural validity. A well known sensitivity analysis study is given by Ford (1990). The author explores the sensitivity of a large scale system dynamics model to its parameters and employs an algorithm in order to fulfill the assumptions of latin hypercube sampling strategy. In the study, sensitivity of results are measured by partial correlation coefficients which indicate the strength of linear relationship between two variables after the effects of other variables are removed. Then, Ford and Flynn (2005) propose Pearson correlation coefficients instead of the partial ones for simpler sensitivity analysis procedure. This method, named screening, is discussed in Section 3.1. Furthermore, Taylor et al.(2007) present an application of screening method to a project management model by Taylor and Ford (2006). Pearson correlation is an easy way for measuring the strength of linear relationship between two variables. However, it may create problems in multi-variate sensitivity analysis since the effects of other parameters are ignored in this statistic. In such cases, regression may be a convenient way for quantification of relationship strength between variables. In system dynamics literature, regression is used by Kleijnen (1995) in order to compute sensitivity coefficients from sampling data obtained through design of experiments (DOE).

Latin hypercube sampling (LHS) is common for sensitivity analysis of simulation models. In fact, McKay et al.(1976) conclude that this strategy is the most suitable sampling strategy for sensitivity analysis purposes. Moreover, Clemson et al.(1995) compare LHS with Taguchi method and conclude that LHS is convenient when Taguchi method is inefficient because of large number of parameters or nonlinearities existing in the model structure. Furthermore, Özbaş et al.(2008) compare DOE with LHS and they conclude that both measures have advantages in different situations. Particularly, LHS is a convenient way if the parameters values have continuous distribution within their ranges, while DOE is efficient for determining interactions between parameters (Özbaş et al., 2008).

One can identify more than one type of sensitivity for simulation models. Sensitivity type should be selected according to the purpose of the model. In system dynamics literature three different types of sensitivities, named numerical sensitivity, policy sensitivity, behavior mode sensitivity, are defined. Specifically, numerical sensitivity can be used for simulation models which work with great numerical precision such as models in physics or flight simulators (Sterman, 2000). Policy sensitivity is defined as the changes in the optimal policy when parameters values change (Moxnes, 2005; Sterman, 2000). Finally, behavior mode sensitivity is defined as change in mode of output behavior as the values of parameters vary (Sterman, 2000). These changes can be analyzed with behavior mode sensitivity approach in system dynamics studies. Another sensitivity type, called behavior pattern sensitivity, should be added to this list in order to cover all possible sensitivity types for system dynamics models. Behavior pattern sensitivity can be defined as changes in behavior pattern measures, such as peak of boom and bust or inflection of s-shaped growth, as model inputs are altered. This sensitivity type is the main topic of this thesis and discussed in great detail in the following section.

2.2. Behavior Pattern Sensitivity

Behavior measures are the subject matter of system dynamics studies in which the problematic behavior is analyzed with respect to its structure. For instance a population in an isolated area, which follows a boom and bust behavior, can be explained by depleting resources and vanishing birth rate. In such situation, sensitivity of the behavior to the model parameters can be analyzed by using peak or equilibrium level of the behavior pattern and this approach is called behavior pattern sensitivity.

Behavior pattern sensitivity aims to explore the effect of varying model inputs on the specific behavior measures. In fact, behavior measures are used for various purposes in system dynamics literature. Particularly, Coyle (1978) use pattern characteristics for a measure, called performance index, in order to summarize the dynamic behavior for comparison of different simulation runs. Moizer et al.(2001) present a sensitivity analysis method using these performance indexes. Furthermore, Özbaş et al.(2008) do the sensitivity analysis of an oscillatory system dynamics model through the oscillation measures, which are period and amplitude. In this thesis, a formal procedure for behavior pattern sensitivity analysis is suggested and this approach is applied to three system dynamics models .

Like other sensitivity analysis procedures, behavior pattern sensitivity analysis starts with the selection of parameters to be included (Figure 2.1). Some of the parameters may be certain in simulation models while many of them are not. Therefore, the model builder should make a distinction among parameters especially in the analysis of large size simulation models. After determination of parameters, their distribution functions and ranges can be identified according to the prior knowledge about the system. Parameters and their distribution information should be entered to Vensim's Sensitivity Simulation module which is described in great detail in the study by Ford and Flynn (2005). In this thesis, ± 20 per cent of each parameter value is taken for the boundries of distribution ranges and we assumed that each parameter value has uniform distribution within these ranges.

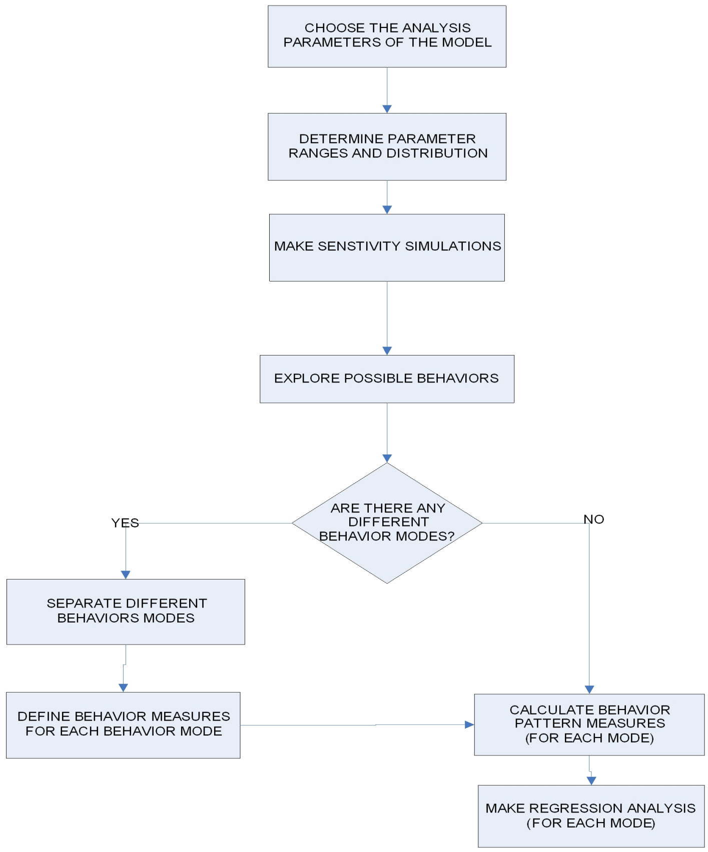


Figure 2.1. Sensitivity Analysis Methodology for Behavior Pattern Sensitivity

The next step in behavior pattern sensitivity is separation of different output behaviors, if necessary. Different structures of the model are important for different behavior modes. After separation, behavior measures of each mode should be identified and estimated for each simulation run. Estimation of these measures may necessitate different statistical procedures according to their complexity. Namely, peak of boom and bust behavior can be estimated with ordinary maximum function in Excel while oscillation period may require autocorrelation function. These estimated values constitute the dependent variable in statistical analysis phase of the study. In order to examine the importance of different parameters, various analysis methods, such as regression, correlation or ANOVA, can be used. In this thesis, regression and ANOVA of clusters are utilized for statistical analysis of pattern sensitivity. These methods for statistical analysis of sensitivity data are discussed in the next chapter.

3. SENSITIVITY ANALYSIS METHODS

Sensitivity analysis of any simulation model consists of some major phases. The first phase includes identification of model output. The second phase is obtaining the sensitivity data using a sampling strategy or statistical design and the final phase is the statistical analysis. For statistical analysis of sensitivity data obtained from a system dynamics model, various methods are suggested in previous studies. Ford and Flynn (2005) propose Pearson correlation coefficients for such analysis and Kleijnen (1995) utilizes a regression model. In this chapter, statistical methods that can be used for sensitivity analysis will be described briefly. Screening method is discussed in Section 3.1 and then regression is described in Section 3.2. Finally in Section 3.3, statistical analysis with ANOVA of clusters is reviewed briefly.

3.1. Correlation and Screening Method

Analysis methods can be classified according to their assumptions on functional relationship between dependent and independent variables. Some methods, such as correlation and regression, assume linear relationship between variables while more general ones, such as nonlinear regression or graphic based methods, have no linearity assumption. Specifically, Pearson correlation coefficient, given in Equation 3.1, is a simple way of determining the strength of the linear relationship between two variables. This statistic takes values within -1 and 1 range in which 1 indicates perfect positive correlation while 0 indicates no linear relationship.

$$\hat{\rho}_{x_j y} = \frac{\sum_{i=1}^{N}(x_{ij} - \overline{x}_j)(y_i - \overline{y})}{\left[\sum_{i=1}^{N}(x_{ij} - \overline{x}_j)^2\right]^{1/2}\left[\sum_{i=1}^{N}(y_i - \overline{y})^2\right]^{1/2}} \tag{3.1}$$

Ford and Flynn (2005) suggest Pearson correlation in order to conduct quick sensitivity analysis, called screening. In this method, correlation coefficients between the output and each parameter are calculated and plotted against simulation time (See

Figure 4.3). Parameters that have high correlation with output variable are concluded to be the high sensitivity ones. Taylor et al. (2007) present an application of screening method to a tipping point project management model by Taylor and Ford (2006). In this thesis, the same model is analyzed with pattern sensitivity approach in Section 4.1. Statistical analysis of pattern measure sensitivity is conducted with regression method which is discussed in next section.

3.2. Regression Method

Another convenient method assuming linear relationship between variables is regression. In this method, an equation that minimizes the sum of squares of residual terms is calculated by using ordinary least squares algorithm (Draper and Smith, 1998). In a standard regression model, given in Equation 3.2, dependent variable (y) is tried to be explained by k independent variables (x_i). This method is commonly used for emprical analysis in economy and we use it for sensitivity analysis purposes. Standardized regression coefficients, given in Equation 3.3, give the importance of independent variables for the dependent one in a regression equation (Saltelli et al., 2000). In other words, the simulation model is more sensitive to the parameters that have larger-magnitude standardized regression coefficients in the regression equation.

$$y_i = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k + e_i \tag{3.2}$$

$$b_j = \frac{\beta_j \hat{\sigma}_{x_j}}{\hat{\sigma_y}} \tag{3.3}$$

In regression methodology, another important issue is addition of interaction terms to the regression model. In linear regression, multiplication of regressors may be included in the equation in order to represent the interaction between dependent variables. In this thesis, when interaction terms were added to the regression model, we realized that these terms make the sensitivity analysis extremely complex. Thus, sensitivity analysis of regression models including interaction terms is suggested as further research, as the issue is beyond the scope of this thesis. However some preliminary trials are performed in order to guide any future research. The result of regression models including interaction terms are discussed briefly in Section 6.2.3 and the results are given in Appendix B.

Furthermore, regression method not only provides information about the parameter sensitivity but also indicates the convenience of linear statistical model to the data set at hand. For this purpose, analysis tools, named coefficient of determination, residual plots and PRESS statistic, are suggested in regression literature. Particularly, coefficient of determination (R^2) is a statistic that indicates the portion of variation in dependent variable explained by independent variables. The formulation of R^2 is given in Equation 3.4. In this statistic, variance explained by regression equation is divided to the variance of independent variable. Therefore, it points out the power of regression equation for explaining the dependent variable.

$$R^2 = \frac{\sum_i^N (\hat{y}_i - \overline{y})^2}{\sum_i^N (y_i - \overline{y})^2} \tag{3.4}$$

Moreover, residual plots are other common tools in regression literature which indicate the fulfillment of regression assumptions. Specifically, in regression method it is assumed that the residual terms have normal distribution with zero mean and constant variance (Draper and Smith, 1998). If a non-random pattern appears in residual plots, one can conclude that linear regression is not appropriate for the data set at hand. In such cases, nonlinear regression models or transformation algorithms can be used for the data set at hand. Another analysis tool for suitability of the linear regression model is PRESS statistic. Detailed description of the application of this statistic is given in Draper and Smith (1998). In this thesis, coefficient of determination and residual plots are employed for the diagnosis of regression model.

In behavior pattern sensitivity, the estimation of pattern measures is followed by regression analysis. After regression coefficients are calculated, the coefficient of determination and residual plots should be evaluated in order to check the fulfillment of linearity and normality assumptions. If the linearity assumption of regression is satisfied, the analysis results can be used with reliance. However, if any non-linear functional relationship is detected, one should employ more general methods such as transformation on regression variables or ANOVA of clusters. The former method is discussed in Section 3.2.1 while the latter one is the topic of Section 3.3.

3.2.1. Transformation on Dependent and Independent Variables

When a nonlinear relationship is detected at the diagnosis of regression model, one can utilize transformation on either dependent or independent variables. Once the transformation is applied, the diagnosis tests should be applied to this nonlinear regression model. In regression method, two different transformation algorithms are suggested for dependent and independent variables. Namely, Box-Cox transformation is suggested for transformation on dependent variable (Draper and Smith, 1998) whereas Box and Tidwell (1962) propose an algorithm for transformation on independent ones.

Box-Cox transformation exists in the family of power transformations, i.e. y^λ. In this method, it is aimed to find λ value that maximizes the likelihood function of regression equation under the assumption that the residual terms are normally distributed ($e \sim N(0, \sigma^2)$) (Draper and Smith, 1998). In this search, as λ approaches 0, all y_i values become 1 eventually and regression analysis becomes inconclusive. Therefore, natural logarithm of y is taken instead of transformation with zero. This transformation is formulated in Equation 3.5.

$$W = \begin{cases} (y^\lambda - 1)/\lambda & \text{for } \lambda \neq 0 \\ ln(y) & \text{for } \lambda = 0 \end{cases} \tag{3.5}$$

In this formula, additional divisor for non-zero λ values is added to solve the discontinuity problem as λ approaches zero (Draper and Smith, 1998). W goes to ln(y) as λ approaches zero. Therefore, the divisor makes the transformation formulation more consistent. On the other hand, main drawback of this formula is high variability of

W as λ changes (Draper and Smith, 1998). An extended version of this transformation is suggested for dealing with this problem in the book of Draper and Smith (1998). However, since high variance problem creates minor difference in the analysis phase (Draper and Smith, 1998), we prefer to employ the transformation in Equation 3.5 for sensitivity analysis applications.

Another appropriate method for dealing with the nonlinearity in sensitivity data is transformation on independent variables. This transformation algorithm, which is suggested by Box and Tidwell (1962), is a Taylor approximation to the true functional relationship between dependent and independent variable. This method can be explained as follows:

$$E[y] = \nu = f(\omega, \beta) \tag{3.6}$$

Let the expectation of dependent variable (y) is equal to ν, ω is the vector of transformed independent variables $\{x_i, i \in 1 \dots k\}$ with a transformation constant α and β is the regression coefficient of transformed variables. Let's assume that the first guess on transformation constant is $\alpha^{(0)}$ and $\omega^{(0)}$ is the first transformed variable. Once we apply Taylor Expansion around $\alpha^{(0)}$, resulting formulation is given in Equation 3.7 (Box and Tidwell, 1962).

$$\nu = f(\omega, \beta) + \sum\nolimits_{j}^{k} (\alpha_j - \alpha_0) \left\{ \frac{\delta f(\omega, \beta)}{\delta \alpha_j} \right\}_{\omega=\omega^{(0)}} \tag{3.7}$$

In this expansion, partial derivative of $f(\epsilon, \beta)$ with respect to α can be written as follows:

$$\left\{\frac{\delta f(\omega,\beta)}{\delta\alpha_j}\right\}_{\omega=\omega^{(0)}} = \left\{\frac{\delta f(\omega,\beta)}{\delta\omega_j}\right\}_{\omega=\omega^{(0)}} \left\{\frac{\delta\omega_j}{\delta\alpha_j}\right\}_{\alpha=\alpha^{(0)}} \tag{3.8}$$

The partial derivative of ϵ with respect to α can be calculated from the specific form of the transformation. On the other hand $\delta f(\epsilon,\beta)/\delta\epsilon$ must be estimated. This estimation can be done with fitting observations (y) to $f(\epsilon^{(0)},\beta)$ by least squares method. Then the approximate $(\delta f(\epsilon,\beta)/\delta\alpha)$ values can be calculated. They "provide a set of independent variables from which we can obtain the adjustments to the constants of transformations by refitting the observations to the expression in" Equation 3.7 (Box and Tidwell, 1962).

Both of these transformation procedures are commonly used in regression literature and they are tried for sensitivity analysis purposes in this thesis. We conclude that transformation on response is an efficient way for dealing with the nonlinear cases of sensitivity analyses while transformation on independent variables make the analysis more complex. Thus, Box-Cox transformation is utilized for the nonlinear cases of sensitivity analyses in this thesis.

3.3. Statistical Analysis (ANOVA) of Clusters in Scatter Plots

Another efficient way of dealing with nonlinearity between system output and parameters is using statistical analysis of clusters in which output variable y is plotted against each parameter x_j and this plot is subject to statistical analysis after it is divided into several clusters (Kleijnen and Helton, 1999[a]). This method is a more general way of one-variate sensitivity analysis since it does not have the linearity assumption between dependent and independent variables, i.e. this is a "statistical model independent method" (Saltelli et al., 2000). The scatter plot for each parameter is subject to statistical analysis in order to detect any non-random pattern. However, major drawbacks of this method are; great number of scatter plots requirement for large models and incapability of multi-variate sensitivity analysis with this method.

In the literature, there are various test statistics that compare different features of clusters such as mean, median and variance. Kleijnen and Helton(1999[a]) review six of these statistics which focus on a different feature of clustered values. In this thesis, we use the statistic that compares means of different clusters with each other using ANOVA test. This statistic, given in Equation 3.9, is "the most powerful" one for equality of means under the normality assumption (Kleijnen and Helton, 1999[a]). Furthermore, since ANOVA test is robust to the deviations from normality, this test is also applicable to many sensitivity analysis cases (Kleijnen and Helton, 1999[a]).

$$F(n_x - 1, N - n_x) = \frac{\left[\sum_{q=1}^{n_x} n_{x_q} \bar{y_q}^2 - N\bar{y}^2\right]/(n_x - 1)}{\left[\sum_{k=1}^{N} y_k^2 - \sum_{q=1}^{n_x} n_{x_q} \bar{y_q}^2\right]/(N - n_x)} \tag{3.9}$$

In this formulation, n_x, n_{x_q} and N represent number of clusters, number of variable in each cluster and sample size. Furthermore, $\bar{y}_q$ is the average of response variable in each cluster and $\bar{y}$ is the common average of response. The computer program for this statistic, which is developed in R Gui (Venables and Smith, 2002), is given in Appendix C.

Other tests reviewed in Kleijnen and Helton's study (1999[a]) compare the locations of values in each cluster using ranked transformation (Kruskal-Wallis Tets), or spread of response values in different clusters. Moreover, these different tests are compared through the sensitivity analysis of a dynamic model by Kleijnen and Helton (1999[b]). They conclude that in sensitivity analysis of simulation models, "tests based on location, such as the ones using mean or ranked transformation, appear to be more effective in identifying important variables" (Kleijnen and Helton, 1999[b]). Therefore we use ANOVA F test (Equation 3.9) in order to compare regression results with a more general statistical method.

Linear regression is a convenient way of multi-variate statistical analysis with its diagnosis statistics. However due to dissatisfied assumptions on linearity and normality, it may elicit erroneous results if there is a nonlinear relationship between dependent and independent variables. On the other hand, ANOVA of clusters gives the relationship strength between dependent variable and each independent variable. So, this method only allows one-variate sensitivity analysis whereas regression method is capable of multi-variate analysis.

Furthermore, we should note that ANOVA of clusters is more advantageous for one-variate sensitivity analysis than correlation since it uses clusters to detect possible relationship between variables. In Figure 3.1, ANOVA of clusters and correlation test are applied to a quadratic equation. In this figure, yellow asterix indicate the means of clusters whereas the blue line shows the general average of variable y. Obviously, x is found to be significant by ANOVA test whereas Pearson correlation coefficient between x and y is very close to zero and insignificant. Therefore, we conclude that ANOVA of cluster method is more appropriate for one-variate sensitivity analysis than Pearson correlation coefficient.

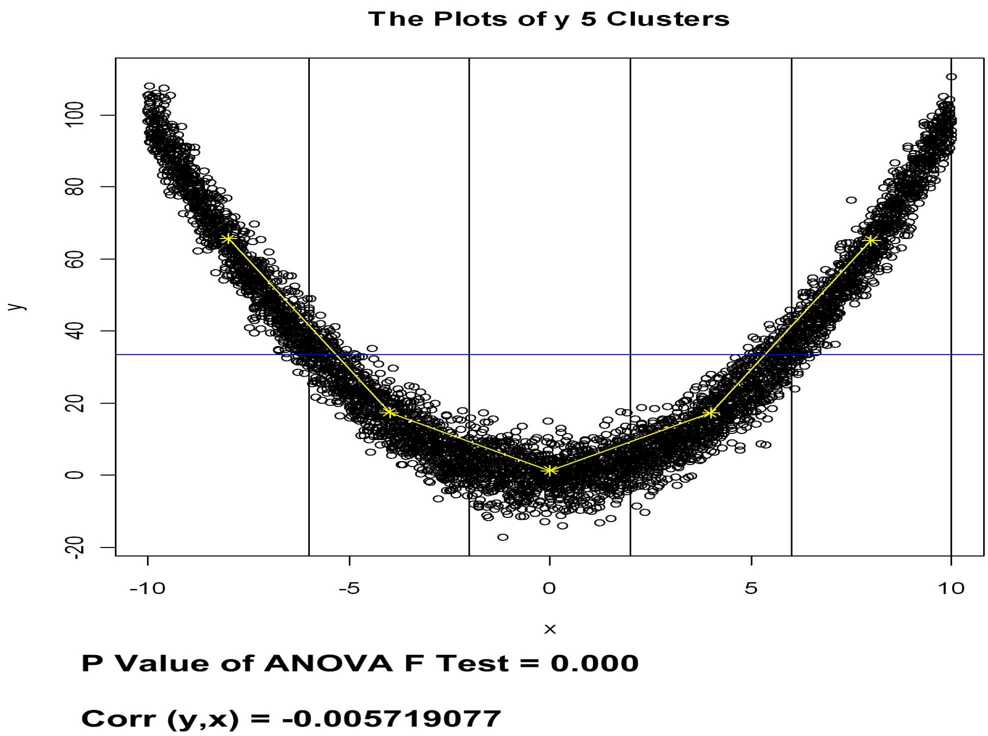


Figure 3.1. Anova of Clusters for Quadratic Form Relationship

In this thesis, regression method is utilized for behavior pattern sensitivity analysis of system dynamics models. When nonlinear relationship is detected between parameters and pattern measures, Box-Cox transformation is applied to the response variable in order to obtain a better regression model. Furthermore, regression results are compared with the results of ANOVA of clusters. In other words, we compare multi-variate sensitivity analysis results (regression) with one-variate analysis (ANOVA of clusters) in this thesis. This approach is applied to the sensitivity analysis of three different system dynamics models in the following chapters.

4. SENSITIVITY ANALYSIS OF A PROJECT MANAGEMENT MODEL

Project management is one of the most interesting and challenging research fields in management science. In project management, it is widely known that there may be some changes in project specifications before the completion of project. Namely in software development projects, the client may change software specifications during the project and these changes create additional works for the project team. Furthermore, this additional work may have some “secondary or tertiary” impacts on project performance such as motivation loss or deadline stress (Taylor and Ford, 2006). Such kinds of side effects are called ripple effect in project management literature. Due to additional works and the ripple effect, project scope may begin to increase as time proceeds. This phenomenon is an example of “bifurcating behaviors” that can be well explained with tipping point structures (Taylor and Ford, 2006) in system dynamics literature.

Tipping point is a typical threshold “below which the system tends to remain stable” (Taylor and Ford, 2006). For project management cases; unless the conditions cross the tipping point, the completed percentage of project scope keeps rising until the end of the project. Taylor and Ford (2006) present a system dynamics model including tipping point structure in order to explain the failure of a project due to ripple effect and deadline stress.

In this chapter of the thesis, the behavior pattern sensitivity analysis of the project management model is presented. In the following section, a brief description of the model takes place whereas pattern measures of output behaviors and their estimation procedures are discussed in Section 4.2. The sensitivity analysis of this model is discussed in Section 4.3.

4.1. Model Description

Project failures may stem from many different factors such as schedule pressure, mismanagement or unrealistic goals. Usually combined effects of these factors are responsible for the system behavior. Taylor and Ford (2006) present a system dynamics model that focuses on schedule pressure and ripple effect in order to explain the failure of a simple project. This model consists of seven stocks in “three sectors, named workflow, resource allocation and schedule” (Taylor and Ford, 2006).

The stock flow diagram of workflow sector is given in Figure 4.1. At the beginning of the project, all tasks exist in initial completion backlog. As simulation time proceeds, task packages pass to quality assurance backlog through initially complete work rate. This flow is determined by initial completion productivity and minimum initial completion duration.

Quality assurance backlog has two outflows. One of them, named discover rework rate , goes to rework backlog representing tasks that require reprocessing due to errors. This rate is determined by a function of project complexity and schedule pressure. Project complexity is taken as a constant whereas schedule pressure is a function of required time for project backlog, available time and sensitivity of rework to schedule pressure. The mathematical formulation of this function is given in Equation 4.1. In this equation f_{rw-s} represents rework fraction due to schedule pressure, t_r is required time, t_a represents available time and s_{rw-s} is sensitivity variable.

On the other hand, the “complement” of the rework fraction determines approve work rate that goes to work released stock (Taylor and Ford, 2006).

$$f_{rw-s} = \big((t_r/t_a) - 1\big)s_{rw-s} \tag{4.1}$$

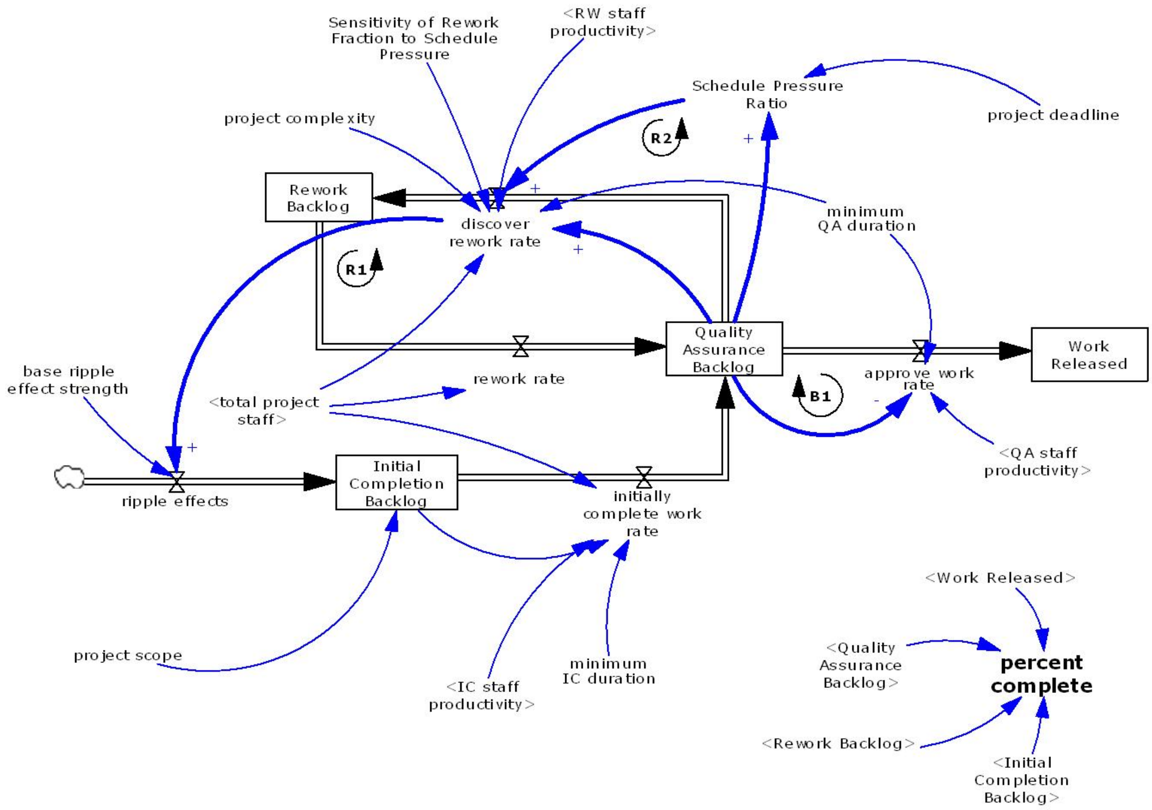


Figure 4.1. Stock Flow Structure of The Project Management Model by Taylor and Ford (2006)

Each flow of workflow sector is constrained by three factors named minimum duration and productivity of each process and labor resource that comes from the resource allocation sector of the model. In resource allocation sector, total staff is assigned to initial completion, quality assurance or rework processes according to the amount of tasks in each stock.

In this model, the main concern is on percent complete variable which represents the percent of project tasks completed. The behavior pattern sensitivity analysis is conducted using the behavior measures of this variable. Furthermore, among various feedback loops that affect percent complete, three of them play active role in the model. These are balancing loop 1 (B1), reinforcing loop 1 and 2 (R1 and R2).

Feedback loop B1 “draws the project tasks from rework cycle” and this loop leads the completion of all tasks unless feedback loop R1 exists (Taylor and Ford, 2006). Through reinforcing loop R1, new tasks are added to initial backlog which represents the additional tasks due to the ripple effect described above. Additional tasks are affected by rework rate and base strength of ripple effect, which represents the interdependence between project tasks. Additional project tasks are also increased by R2 loop that includes the schedule pressure components of the model. All of these feedback loops constitute a tipping point structure that can follow either s-shaped growth or tipping point behavior. For this model s-shaped growth represents the successful projects whereas tipping point behavior indicates the project failure.

In this model there are 14 parameters that are subject to uncertainty and included in the sensitivity analysis. The parameter values and distributions are given in Table 4.1. Further explanations on the project management model can be found in the articles by Taylor and Ford (2006) and Taylor et al. (2005).

Table 4.1. Parameter Distributions of Project Management Model by Taylor and Ford (2006)

Parameters	**Range**	**Distribution**
Project Complexity	[0.24- 0.36]	Uniform
Base ripple effect strength	[0.8 - 1.2]	Uniform
Project Deadline	[240 - 360]	Uniform
Scope Initial	[28000 - 42000]	Uniform
Sensitivity to Schedule Pressure	[0.32 - 0.48]	Uniform
Total Staff	[1200 - 1800]	Uniform
Staff Adjustment Time	[3.2 - 4.8]	Uniform
IC staff Productivity	[0.8 - 1.2]	Uniform
RW staff Productivity	[0.8 - 1.2]	Uniform
QA staff Productivity	[0.8 - 1.2]	Uniform
Minimum IC Duration	[0.8 - 1.2]	Uniform
Minimum RW Duration	[0.8 - 1.2]	Uniform
Minimum QA Duration	[0.8 - 1.2]	Uniform
Release Productivity Adjustment	[0.4 - 0.6]	Uniform

Moreover, Taylor et al. (2007) apply screening method to this project management model. In order to asses the power of this method, we replicate their studies and we propose following criticisims.

In screening, the execution of sensitivity simulations is followed by the correlation analysis directly. However, the project management model presents two different output behaviors which are tipping point behavior (Figure 4.5) and S-shaped growth (Figure 4.4). The estimation of correlation coefficients using all simulation runs, including both of two behavior patterns, like in Figure 4.2, makes correlation coefficients weaker. Therefore, different behavior modes should be separated from each other at the beginning of the analysis.

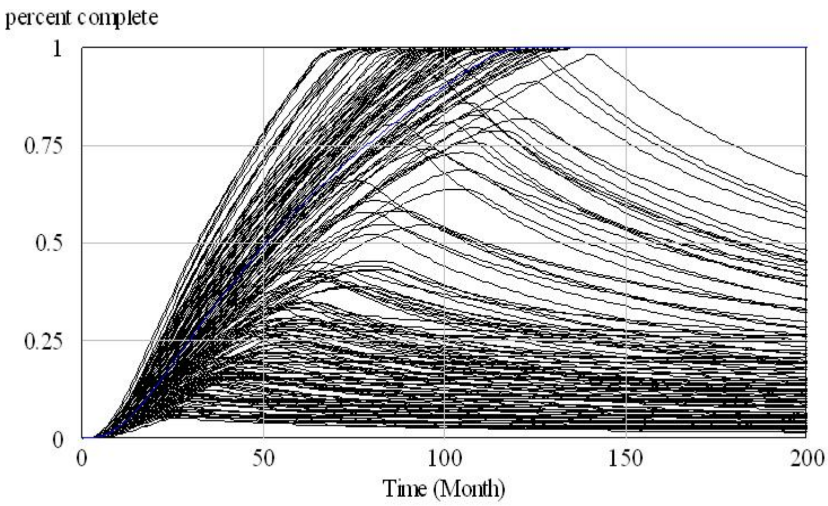


Figure 4.2. Sensitivity Graphs of Project Management Model (Taylor and Ford, 2006)

Furthermore, screening method uses values of variables for correlation estimation. However, this approach to measurement of sensitivity is not appropriate for system dynamics models which are behavior pattern-oriented rather than forecasting or prediction. In other words, numeric values of variables are out of concern in system dynamics so using variable values in calculation of correlation coefficients may create misleading results about the parameter sensitivity of the model. In order to evaluate this weakness of screening method, sensitivity simulations are executed with different seed values and screening analysis are applied to these simulation runs.

We explore the sensitivity of screening results to different seed values in sensitivity simulation in Table 4.2 and 4.3. In Table 4.2 the parameter rankings for different seed values at simulation time 20 are given. Top three parameters vary a little bit for different seeds. However, after the fourth parameter, the ranking begins to differ significantly in each column. For instance, the rank of complexity parameter changes for each seed values. This indicates that numerical values of variables in system dynamics models are subject to significant variation. This variation can also be observed from screening graph of complexity parameter in Figure 4.3.

Table 4.2. Table of Parameter Ranking From Screening with Different Seed Values at Time 20

Rank	Different Seed Values			
	Seed 1234	Seed 4423	Seed 1025	Seed 7865
1	Scope	Total staff	Totalstaff	Scope
2	Total staff	Scope	Scope	Totalstaff
3	Icprod	Icprod	Icprod	Icprod
4	StaffAdjust	QAprod	Complexity	QAprod
5	Complexity	Deadline	QAprod	Complexity
6	Release	StaffAdjust	minIC	StaffAdjust
7	RWprod	minQA	minRW	RWprod
8	minQA	minRW	Ripple	Ripple
9	Ripple	SensePress	SensePress	SensePress
10	QAprod	Release	Deadline	Release
11	SensePress	Complexity	minQA	minIC
12	minIC	RWprod	StaffAdjust	Deadline
13	minRW	Ripple	RWprod	minQA
14	Deadline	minIC	Release	minRW

The same ranking comparison for time point 150 indicates more interesting results. Parameter rankings for that point are given in Table 4.3. First parameter is the same for all seeds in Table 4.3. However, second and third parameters vary in each column. Interestingly, *ripple* parameter is the third one for seed 1025 and 7865 while it is at rank nine for seed 1234. This ranking variation, which can also be seen

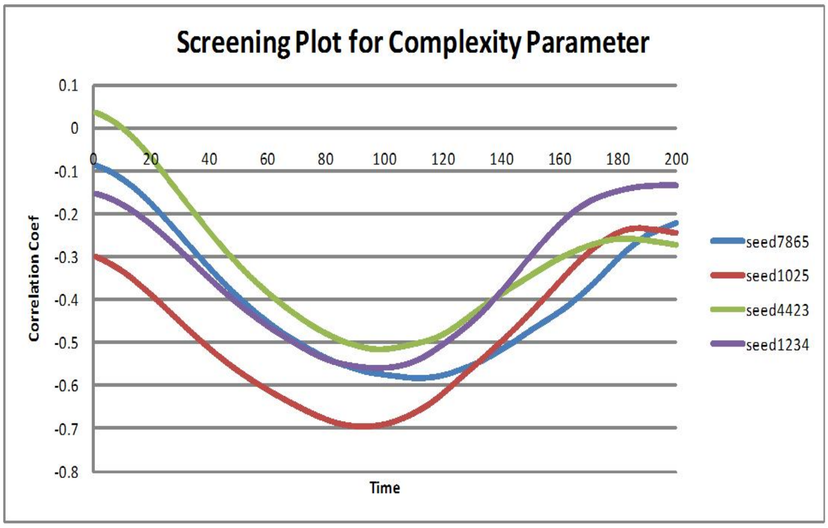


Figure 4.3. Screening Graph for Complexity Parameter from Sensitivity Runs with Different Seed Values

in Figure 4.3, indicates that sensitivity results become less reliable towards the end of simulation. Furthermore, sample size of sensitivity simulations are 200 in these analyses. Screening method is also tried with different sample sizes and it is concluded that very different parameter rankings can be obtained using correlation coefficients even with 500 runs-sensitivity simulations.

To sum up, screening analysis is a quick way of sensitivity analysis using Pearson Correlation coefficients. However, this approach does not seem very appropriate for sensitivity analysis of system dynamics models due to high variability of correlation coefficients. Moreover, Pearson Correlation focuses on the correlation between only two variables and ignores the effects of other ones. However, there may be strong dependence between the parameters of system dynamics models and ignorance of this relationship may yield erroneous results.

In this thesis, the pattern sensitivity analysis of the project management model is conducted using parameter ranges and distributions specified in Table 4.1. In pattern sensitivity, one should identify and estimate specific measures of behavior pattern for each simulation run. Difficulty of this process varies according to complexity of the behavior mode under analysis. Behavior measures and their estimation procedures are discussed in the following section.

Table 4.3. Table for Parameter Order from Sensitivity Simulations with Different Seed Values at Time 150

Rank	Parameter Order for Different Seed Values			
	seed1234	seed4423	seed1025	seed7865
1	Complexity	Complexity	Complexity	Complexity
2	Scope	Totalstaff	Totalstaff	Totalstaff
3	Totalstaff	Scope	Ripple	Ripple
4	Ripple	QAprod	Scope	Scope
5	QAprod	Deadline	QAprod	QAprod
6	RWprod	SensePress	RWprod	Icprod
7	StaffAdjust	Ripple	minQA	Deadline
8	Icprod	Icprod	StaffAdjust	SensePress
9	Deadline	minRW	Deadline	minQA
10	minQA	StaffAdjust	SensePress	RWprod
11	Release	Release	Icprod	Release
12	minIC	RWprod	Release	StaffAdjust
13	SensePress	minIC	minIC	minRW
14	minRW	minQA	minRW	minIC

4.2. Behavior Measures and Estimation Procedures

During the pattern sensitivity analysis of system dynamics models, measures of output behavior, such as period and amplitude of oscillation or equilibrium level of s-shaped growth, should be considered. This process requires identification of the measures and their estimation procedures at the beginning of the analysis. As stated above, the project management model may follow two different behavior modes which are s-shaped growth (Figure 4.4) and tipping point (Figure 4.5) patterns.

In s-shaped growth behavior (Figure 4.4), project tasks are completed rapidly at the initial phases of simulation. Then, increasing quality assurance backlog causes higher rework rate and growing *initial completion backlog* through ripple effect. Thus, increment rate of *percent complete* variable, begins to decline, i.e. second derivative of the behavior pattern becomes negative. This measure is called inflection time which is an important measure of s-shaped behavior. Like inflection time, inflection level of this pattern is also important for such kind of analysis. Furthermore, the time value at which the system reaches equilibrium is another important measure for s-shaped growth. All behavior measures of s-shaped growth pattern can be listed as follows:

- Inflection Level
- Inflection Time
- Equilibrium Time

Other behavior of project management model is tipping point pattern (Figure 4.5) which represents the project failure due to increasing incomplete tasks backlog. Inflection level and time are useful measures for sensitivity analysis of this pattern too. Additionally, tipping point level and tipping time are very characteristic measures for this behavior mode. Tipping point is the time at which the project scope begins to inflate as opposed to initial phases of the simulation. After this time, the project goes out of control and initial completion and rework backlogs keep growing. Thus, behavior measures of tipping point behavior can be summarized as follows:

- Inflection Level
- Inflection Time
- Tipping Point Level (Peak of Pattern)
- Tipping Point Time

In this thesis, inflection level of s-shaped growth and peak of tipping point pattern are used for sensitivity analysis of project management model. Other behavior measures are omitted since the complete sensitivity analysis of this model is out of scope. The estimation procedures of these two measures can be explained as follows:

Inflection level of the s-shaped growth is the value of percent complete variable at the inflection time. Hence, in order to estimate inflection point level, its time should be determined first.

Mathematically, the output behavior is a function of time and other model constant. Inflection time is the time point at which the second derivative of the pattern

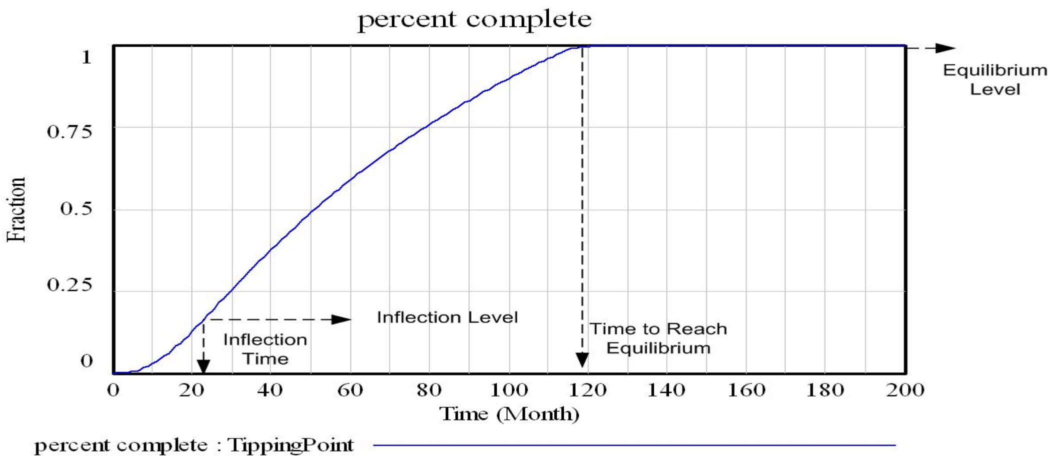


Figure 4.4. S-Shaped Growth Behavior of The Project Management Model by Taylor and Ford (2006)

becomes negative when we take the derivative of the pattern with respect to time. Also we know that when the second derivative of a function shifts to negative, the first derivative is at maximum. Therefore, we calculate differences between successive time points and take the maximum of these differences as inflection time. The formulation of the inflection point estimation is given in Equation 4.2 and 4.3.

$$\tau_i = \{t : (C_{i(t+DT)} - C_{it}) \text{ is at maximum for all } t \in [0, T)\} \tag{4.2}$$

$$I_i = C_{i\tau_i} \tag{4.3}$$

In these equations, τ_i, I_i and C_{it} represent inflection time, inflection level and the value of *percent complete* variable at time t for simulation run i. Furthermore, T represents the simulation length while DT gives the step size of the simulation.

Other behavior measure, which we use in this study, is peak of tipping point behavior. At this point, the behavior pattern reaches its maximum and then begins to decline. Therefore, this behavior measure can be estimated using simple maximum function in an Excel spreadsheet. The mathematical expression of this estimation

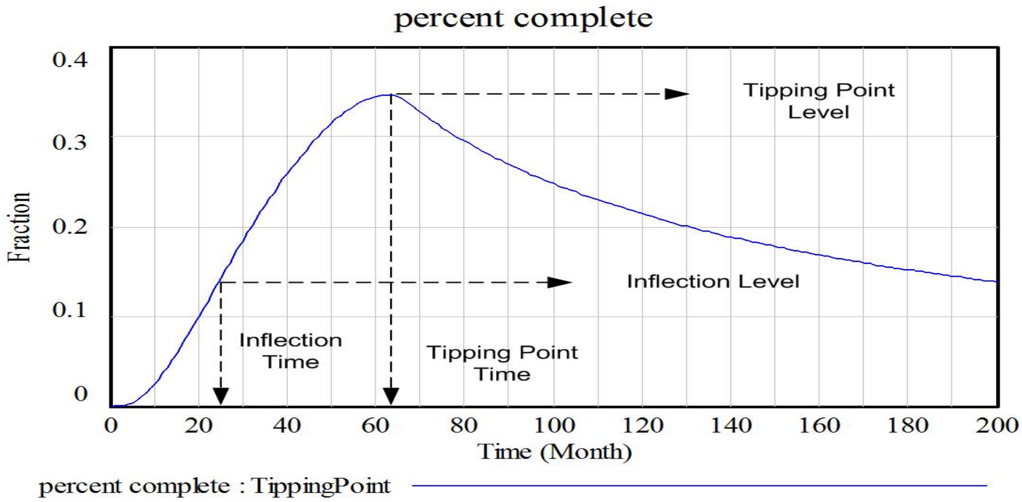


Figure 4.5. Tipping Point Behavior of The Project Management Model

procedure is given in Equation 4.4. In this equation T_i is the peak of *percent complete* variable at simulation run i.

$$T_i = \max C_{it} \tag{4.4}$$

These estimation procedures are utilized for every simulation run after sensitivity simulations completed. Then, statistical analysis of the sensitivity data is conducted using regression method. Discussion of statistical analysis and interpretation of results are given in the following section.

4.3. Behavior Pattern Sensitivity Analysis of Project Management Model

As discussed above, behavior pattern sensitivity of system dynamics models starts with identification of parameters and their distributions. The parameter ranges and distributions are given in Table 4.1 for the project management model. Then these specifications are entered to the Vensim's Sensitivity Simulation module. Parameter ranges, distributions, sampling strategy and, if important, seed value is entered as input to this software which runs the simulations. For the analysis of project management model, 200 sensitivity runs are used with latin hypercube sampling strategy and seed value 1234. These simulation runs are exported to an Excel spreadsheet in order to conduct behavior measure estimation and statistical analysis.

Before the estimation procedures, which are described above, one should diagnose all simulations to check whether there is any different behavior mode than expected ones. For project management model, s-shaped growth and tipping point behavior exist in sensitivity simulation runs and these two different patterns are separated from each other. It is found that 107 of 200 runs are s-shaped growth while 93 runs are tipping point pattern. In the following section, the sensitivity of s-shaped growth pattern to the model parameters is analyzed via regression analysis and ANOVA test of clusters.

4.3.1. Sensitivity Analysis of S-Shaped Growth Pattern

As discussed above, s-shaped growth behavior indicates the project success for this model and in this analysis the only inflection point of s-shaped growth is subject to sensitivity analysis. Inflection points are estimated according to the procedure described above and these values are used as dependent variable in regression analysis. Coefficients of this regression equation are given in Table 4.4.

In Table 4.4, like all regression result tables in this thesis, parameters are ordered according to the magnitude of standardized regression coefficients. Regression results point out that, *project scope* and *total staff* parameters are the most influential parameters for inflection level of s-shaped growth. However, both low R square value (Table 4.5) and non-random pattern in residual plot (Figure 4.6) indicate inappropriateness of this model for the data set at hand. Thus, transformation on dependent variable is applied to this analysis.

In order to obtain transformation exponent Box-Cox search function in R Gui is utilized. This function takes the linear regression model and calculates log-likelihood

Table 4.4. Regression Results for the Inflection Level Measure

Coefficients					
	Regression Coefficients			t value	Sig.
Model	beta (β)	std. error	std. coef. (b)		
(Constant)	1.006	.170		5.902	.000
Icprod	-.337	.042	-.572	-8.059	.000
scope	.000	.000	.448	5.423	.000
totalstaff	.000	.000	-.322	-3.721	.000
complex	-.565	.179	-.269	-3.154	.002
deadline	-.002	.001	-.246	-3.027	.003
ripple	-.119	.037	-.221	-3.208	.002
QAprod	-.089	.044	-.145	-2.030	.046
minRW	.065	.040	.119	1.639	.105
minQA	.058	.037	.104	1.544	.127
StaffAdjust	.011	.011	.078	1.068	.289
minIC	-.031	.037	-.058	-.826	.411
RWprod	.025	.038	.047	.655	.514
Release	-.010	.078	-.009	-.129	.897
SensePress	-.002	.091	-.001	-.020	.984

Table 4.5. Regression Summary for Inflection Level of S-Shaped Pattern

Model Summary		
R Square	Adjusted R Square	Std. Error of the Estimate
.674	.616	.039485012

function of the regression equation for different exponent values (λ). Box-Cox search algorithm gives 95 per cent confidence interval for transformation exponent since it is necessary to try different exponent values within this range in order to get the best result. In this study, however, the exponent that gives the maximum of log-likelihood function is used directly. Specifically, -1.35353535 is used as transformation exponent for inflection level of s-shaped growth.

Transformed values are entered to the regression equation of which results are given in Table 4.6 and 4.7. Higher value of coefficient of determination (R square in Table 4.7) and better residual plots (Figure 4.7) indicate that this model is more appropriate than the first one.

After the application of transformation, initial completion productivity remains as the top important parameter while the ranking of the remaining parameters changes significantly. Specifically, project complexity and scope are concluded as second and third important parameters whereas total staff is at the fourth rank. These results indicate that initial productivity, complexity and project scope parameters are really efficient points for controlling the inflection level of s-shaped growth pattern.

Despite its convenience, regression methodology requires the fulfillment of three fundamental assumptions on residual terms. However, in this case these necessary conditions are not satisfied and transformation on dependent variable is utilized. Conceptually, sensitivity of transformed dependent is more difficult to interpret than the sensitivity of original variable. So, we need to compare regression results with ANOVA test of clusters, which is more appropriate to nonlinear cases. This method focuses on the scatter plots of dependent variable against each parameter. Each plot is divided

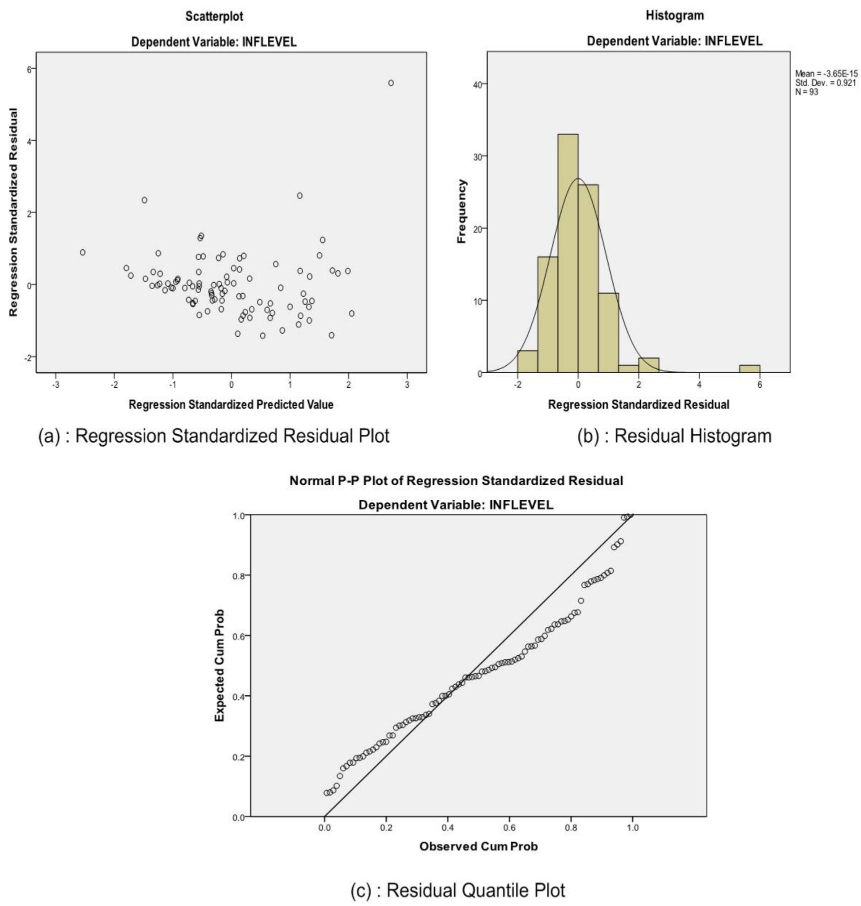


Figure 4.6. Regression Plots for Inflection Level of S-Shaped Growth

Table 4.6. Regression Results for Transformed Inflection Level

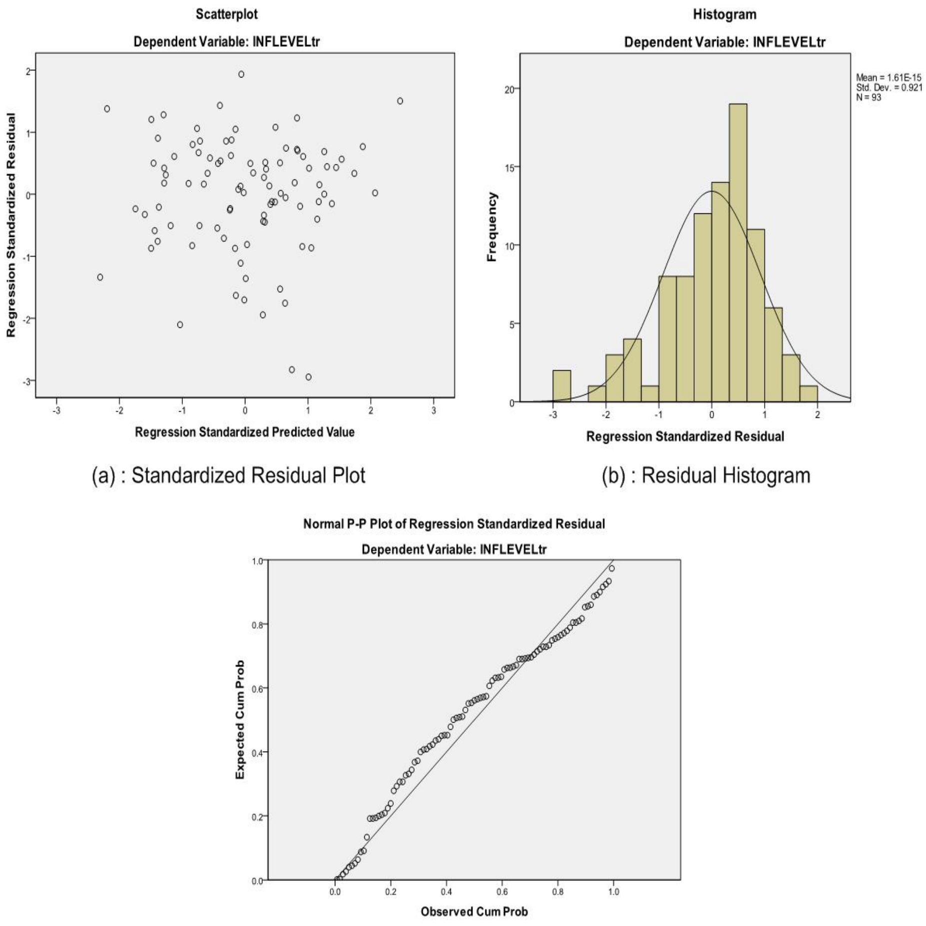
Coefficients					
	Regression Coefficients			t value	Sig.
Model	beta (β)	std. error	Std. Coef. (b)		
(Constant)	-17.340	3.744		-4.631	.000
Icprod	9.234	.919	.556	10.045	.000
complex	25.254	3.937	.428	6.415	.000
scope	.000	.000	-.415	-6.439	.000
totalstaff	.004	.001	.322	4.772	.000
ripple	4.412	.816	.290	5.405	.000
deadline	.049	.013	.245	3.868	.000
minRW	-1.607	.875	-.104	-1.836	.070
QAprod	1.141	.965	.066	1.183	.241
StaffAdjust	-.243	.235	-.059	-1.038	.302
minIC	.739	.823	.049	.898	.372
Release	-1.422	1.704	-.046	-.835	.406
RWprod	-.681	.835	-.046	-.816	.417
minQA	-.593	.822	-.038	-.722	.472
SensePress	-.180	2.003	-.005	-.090	.929

into several clusters and means of these clusters are compared with each other in order to detect any non-random pattern. For instance, the scatter plots of inflection level against project complexity and sensitivity to schedule pressure parameters are given in Figure 4.8.

In these plots, the mean value of each cluster is marked with asterix (red). These cluster means are linked with a red line and the mean of all data points are given with a horizontal blue line. In ANOVA analysis, difference between the mean of each cluster and general mean is summed and divided by the sum of variances in clusters. Therefore, this statistic (Equation 3.9) shows if the cluster means significantly differ

Table 4.7. Summary of Regression for Transformed Inflection Level

Model Summary		
R Square	Adjusted R Square	Std. Error of the Estimate
.802	.766	.86751



(a) : Standardized Residual Plot

(b) : Residual Histogram

(c) : Residual Quantile Plot

Figure 4.7. Regression Plots for Transformed Inflection Level of S-Shaped Growth

from the general mean of dependent variable. Furthermore, this statistic follows F distribution under the normality assumption so, the significance can be measured with the P-value. The more significant difference indicates a higher sensitivity parameter. For inflection point of s-shaped growth, analysis results are given in Table 4.8.

According to the ANOVA analysis, initial completion productivity and project complexity are the most important parameters for inflection level of s-shaped growth. Nevertheless, this method yields a different parameter, called *quality assurance productivity*(QAprod), at third rank. This parameter is at rank eight in regression analysis. Therefore, we can conclude that top two parameters are really important for inflection level of s-shaped growth whereas the rank of other parameters may change in different analysis. In Figure 4.8 there is an obvious trend in the plot of complexity (rank two) where as it is more difficult to detect a non-random trend in the plot ripple effect

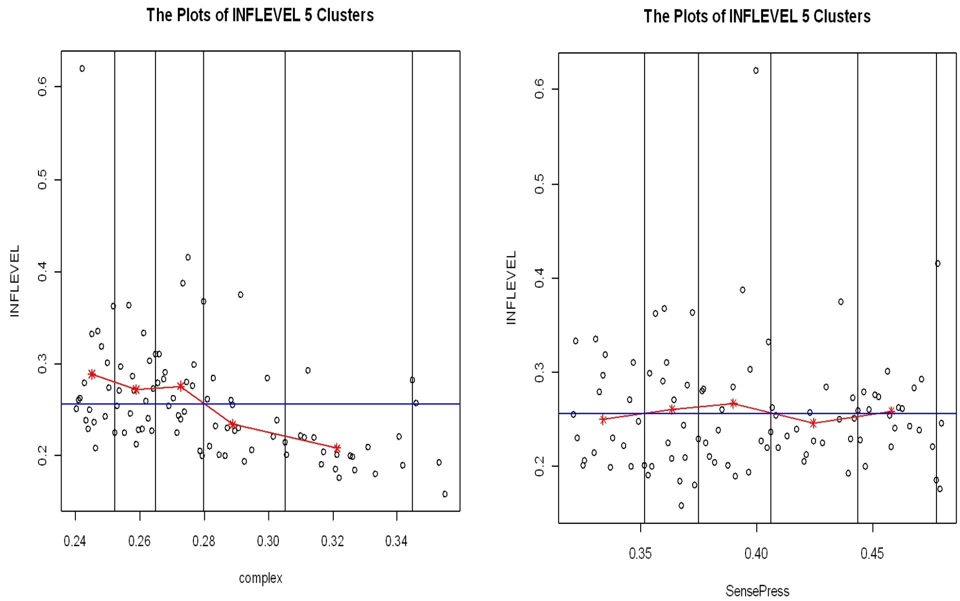


Figure 4.8. Some Clustered Scatter Plots Used in Sensitivity Analysis of Inflection Level

parameters which take place at 14.

The results of ANOVA and regression methods are compared in Table 4.9. We can conclude that transformed regression and ANOVA method give the same parameters for top two ranking. So, we can conclude that these two parameters have great priority for inflection level. Furthermore, both inappropriate linear regression and the difference between results of two methods make us suspect that interactions between parameters may be significantly important for this pattern measure. However, since analysis of parameter interactions are kept beyond the scope of this thesis, we leave this issue to the further studies. Tipping point pattern of this model is analyzed for parameter sensitivity in the following section.

4.3.2. Sensitivity Analysis of Tipping Point Behavior Pattern

Another output behavior of project management model is tipping point pattern (Figure 4.5). In this behavior, the system follows an s-shaped pattern initially until it reaches the tipping point. After that time an exponential decay takes place. Possible measures that can be used for the sensitivity analysis of this pattern are given above.

Table 4.8. Results of ANOVA Test of Clusters for Inflection Level

Parameter	F VALUES	P Values
Icprod	3.7782426	0.006975
complex	3.4257793	0.011889
QAprod	1.5881025	0.184527
ripple	1.4668815	0.219097
scope	1.4639112	0.220014
totalstaff	0.9736765	0.426141
minIC	0.7839083	0.538641
Release	0.7609801	0.553454
RWprod	0.6049602	0.660084
StaffAdjust	0.5059918	0.731416
minRW	0.4174723	0.795633
minQA	0.3926557	0.813397
deadline	0.2179392	0.9278
SensePress	0.2087415	0.93295

Among these measures, tipping point level is used for sensitivity analysis of this pattern.

As discussed above, the peak of a tipping point pattern is estimated by simple maximum function in Excel. Those estimated peaks constitute the dependent variable in regression equation. Regression coefficients and summary statistics are given in Table A.1 and Table A.2 in Appendix A. According to the regression results, *total staff* and *project scope* are two most important parameters of the model. However, R square value (Table A.2), non-random pattern of residual plot and abnormal distribution in histogram (Figure A.1) indicate problems in regression assumptions. So, we conclude that transformation on dependent variable using Box-Cox method is necessary.

The transformation exponent that is obtained from the Box-Cox search algorithm in R Gui is -0.4242. Regression results for transformed dependent variable are given in Table 4.10 while diagnostics are given in Table 4.11 and Figure 4.9.

Transformed regression model is more appropriate than the original regression

Table 4.9. Summary Table for Inflection Level Measure

	Regression Results		ANOVA of CLUSTERS
Rank	Untransformed Dependent	Transformed Dependent	
1	Icprod	Icprod	Icprod
2	scope	complexity	complexity
3	totalstaff	scope	QAprod
4	complexity	totalstaff	ripple
5	deadline	ripple	scope
6	ripple	deadline	totalstaff
7	QAprod	minRW	minIC
8	minRW	QAprod	Release
9	minQA	StaffAdjust	RWprod
10	StaffAdjust	minIC	StaffAdjust
11	minIC	Release	minRW
12	RWprod	RWprod	minQA
13	Release	minQA	deadline
14	SensePress	SensePress	SensePress

model due to better residual plots and higher R square value. These diagnostics indicate that parameters are capable of explaining more variability in transformed variable than the untransformed one. According to the regression results, *total staff* and *project scope* are the most important parameters of the model. Then, *initial completion productivity* and *project complexity* parameters hold the third and fourth place.

In order to check the reliability of regression results, the same analysis is conducted with ANOVA test of clusters. Results that are obtained with this approach are given in Table 4.12. ANOVA analysis indicates that project scope and project deadline parameters have top importance for this measure whereas initial completion productivity takes third place in parameter order. These results are a little bit different from the regression analysis given in Table 4.10. Some of clustered scatter plots, used in ANOVA analysis, are given in Figure A.2. Obviously, there are significant trends in the plots of initial completion productivity and project scope. On the other hand, it is difficult to detect a non-random pattern from the plot of minimum initial completion duration (minIC).

Table 4.10. Regression Results for Transformed Tipping Level

Coefficients					
	Regression Coefficients			t value	Sig.
Model	beta (β)	Std. Error	Std. Coef. (b)		
(Constant)	.721	.055		13.115	.000
totalstaff	.000	.000	.716	18.500	.000
scope	.000	.000	-.652	-17.350	.000
Icprod	.165	.012	.497	13.935	.000
complex	-.632	.055	-.453	-11.585	.000
deadline	.002	.000	.440	12.515	.000
ripple	-.087	.013	-.243	-6.552	.000
QAprod	.070	.012	.214	5.862	.000
SensePress	-.185	.032	-.207	-5.772	.000
Release	.107	.026	.152	4.188	.000
RWprod	.027	.013	.074	2.033	.045
minQA	.015	.012	.042	1.177	.242
minRW	.011	.013	.031	.861	.392
StaffAdjust	-.001	.003	-.007	-.183	.855
minIC	.001	.012	.003	.095	.924

Results of three analyses for peaks of tipping point patterns are summarized in Table 4.13. According to this table, the transformation does not create significant alterations for the ranking of top three parameters whereas ANOVA points a different importance ranking. This indicates that it is necessary to compare the results of a statistical analysis with another method before having any conclusion. In this analysis, total staff, project scope and project deadline are found to be important parameters of the model for tipping point pattern. Furthermore, the difference between the results of multi-variate (regression) and one-variate analysis (ANOVA of clusters) indicate that the nonlinearities in model structure and interactions among parameters are important for this pattern measure.

Table 4.11. Summary of Regression for Transformed Tipping Level

Model Summary		
R Square	Adjusted R Square	Std. Error of the Estimate
.897	.882	.01404

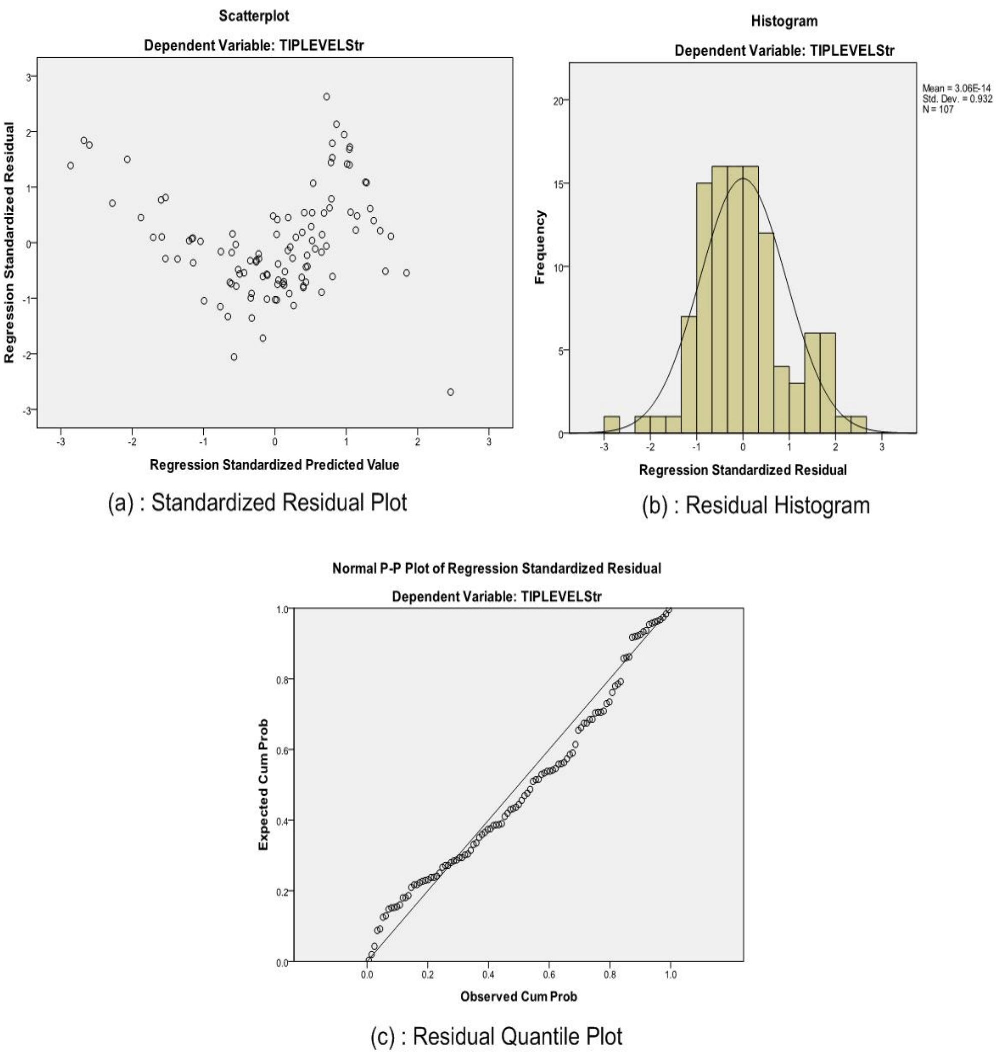


Figure 4.9. Regression Plots for Transformed Tipping Point Measure

Regression is a useful method for behavior pattern sensitivity analysis. This method not only gives the importance ranking of parameters but also includes some measures that indicate the appropriateness of the statistical model to the data set. On the other hand, regression has some certain assumptions which are normality of residual terms, zero expectation and constant variance. Especially after detection of any nonlinearity, the results of regression should be confirmed with a more general method like ANOVA of clusters. In our analyses, both statistical approaches agree with each other only for top parameters and the rankings of remaining ones significantly differ. Therefore, we conclude that parameter interactions may be significant for this system dynamics model.

Table 4.12. Results of ANOVA Test of Clusters for Tipping Point Levels

Parameter	F Value	P Value
scope	5.1830164	0.000767
deadline	3.324247	0.013292
Icprod	2.5340498	0.044723
Release	2.2912379	0.064661
QAprod	2.263322	0.067446
totalstaff	2.1763479	0.076891
complex	1.5652129	0.189276
minQA	0.775745	0.543461
SensePress	0.624592	0.646016
minRW	0.6216839	0.648072
StaffAdjust	0.6098569	0.656458
ripple	0.5667467	0.687331
RWprod	0.5474144	0.701303
minIC	0.521682	0.719984

Table 4.13. Summary Table for Tipping Point Level

	Regression Results		ANOVA of CLUSTERS
RANK	Untransformed Dependent	Transformed Dependent	
1	totalstaff	totalstaff	scope
2	scope	scope	deadline
3	Icprod	Icprod	Icprod
4	deadline	complexity	Release
5	complexity	deadline	QAprod
6	ripple	ripple	totalstaff
7	Release	QAprod	complexity
8	QAprod	SensePress	minQA
9	SensePress	Release	SensePress
10	RWprod	RWprod	minRW
11	minQA	minQA	StaffAdjust
12	StaffAdjust	minRW	ripple
13	minRW	StaffAdjust	RWprod
14	minIC	minIC	minIC

Another interpretation of our results is that the order of top parameters does not alter very much with transformation on dependent variable. So, we conclude that first order regression model may indicate some sensitivity information to the end user without any further analysis. Furthermore, our approach to sensitivity analysis is found to be convenient for the project management model. In the following chapters, pattern sensitivity analysis of oscillatory model is discussed.

5. SENSITIVITY ANALYSIS OF OSCILLATORY MODELS

Oscillation is a common behavior mode which we usually deal with in real systems. For instance, inventory cycles in supply chains, or “business cycles” in national economies are chronic problems which are intensified with mismanagement of the decision maker. So, understanding the nature of oscillations is important task for policy makers or managers. However, oscillatory systems have nonlinear structure and they follow cyclic behavior pattern. Therefore, obtaining an analytic solution or complete understanding of model behavior is really difficult for these systems. Likewise, sensitivity analysis of oscillatory systems is difficult with standard statistical procedures such as screening.

There are several attempts for sensitivity analysis of oscillatory models in system dynamics literature. Ford and Flynn (2005) try screening method for an oscillatory model and report that this method fails to detect the important parameters even for simple models because of cyclic nature of the pattern. Furthermore, Moizer et al.(2001) conduct a sensitivity analysis of an oscillatory model using performance indexes suggested by Coyle (1978). Ozbas et al.(2008) use some behavior measures of oscillations for sensitivity analysis with using design of experiments for sampling. In this thesis, sensitivity analysis of oscillatory system dynamics models are conducted with pattern sensitivity approach described in Chapter 3.

As discussed in previous sections, pattern sensitivity analysis requires clear identification of the behavior mode and its measures. Sterman (2000) classifies oscillatory patterns under four different categories named stable, unstable, limit cycle and chaotic. Among these oscillation patterns, first three are considered in this thesis. Chaotic behavior is ignored because of its rarity and extreme difficulty.

Stable (damping) oscillations tend to come back its equilibrium after a perturbation. In state space, the radius of the circular pattern decreases as time proceeds. This type of behavior is the most common oscillation type that one can face in real systems. The pattern measures of this behavior mode can be listed as follows:

- Period
- Amplitude Slope
- Maximum Amplitude

Period of any oscillation can be defined as the time interval between two successive peaks (Figure 5.1), i.e. the reciprocal of the frequency. This pattern measure depends on the duration of time lags in the structure and interactions between these delays. For instance, oscillations in national economy are long period while biological systems oscillate with shorter periods. The estimation of this behavior measure can be conducted via autocorrelation or spectral density functions which are discussed in the studies of Barlas (1990) and Barlas et al.(1997). In this study, autocorrelation function in BTSII is utilized for period estimations. BTSII is a validation software that conducts statistical tests between output behavior of the model and real data (Barlas et al., 1997).

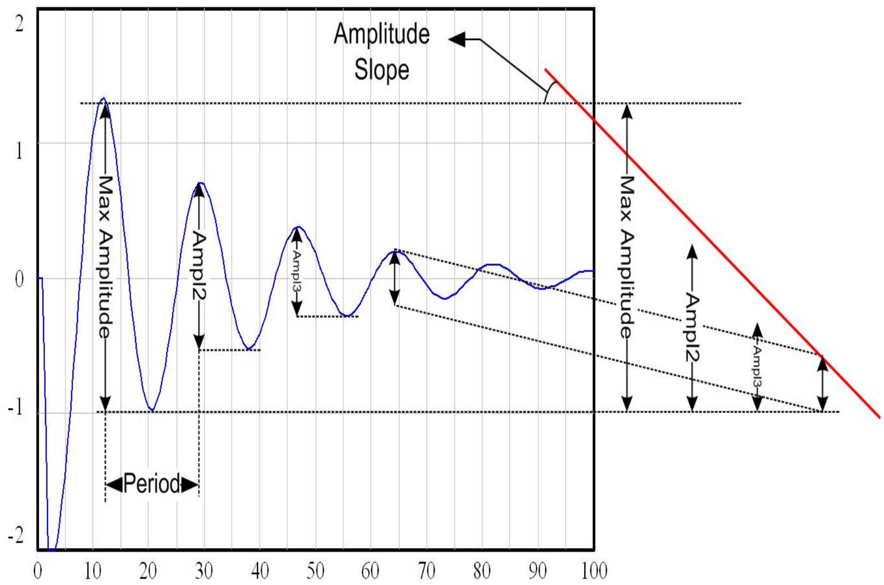


Figure 5.1. Pattern Measures of Damping Oscillation

Second pattern measure of a damping oscillation is amplitude slope. Amplitude can be defined as the discrepancy between a successive peak and trough of an oscillation. The stability character of an oscillatory system can be measured with the slope of a straight line fitted to successive amplitudes (Figure 5.1). On the other hand, sometimes amplitudes may follow a nonlinear pattern. In such cases, a transformation should be applied to the amplitudes in order obtain a linear pattern.

In our analyses the nonlinear pattern of amplitudes is exponential decay and we linearize this pattern by taking its natural logarithm. In other words, log-amplitude slope is used as a stability measure (See Figure 7.4) in this thesis. The log-amplitude slopes for two different oscillatory behaviors are given in Figure 5.2. Obviously, large value of this measure indicates a system that tends to reach the equilibrium faster whereas smaller amplitude slopes point out less stable oscillations. The estimation of this measure necessitates calculation of successive amplitudes in an oscillation. The estimation procedure, given in Appendix C, is developed in R-Gui environment (Venables and Smith, 2002).

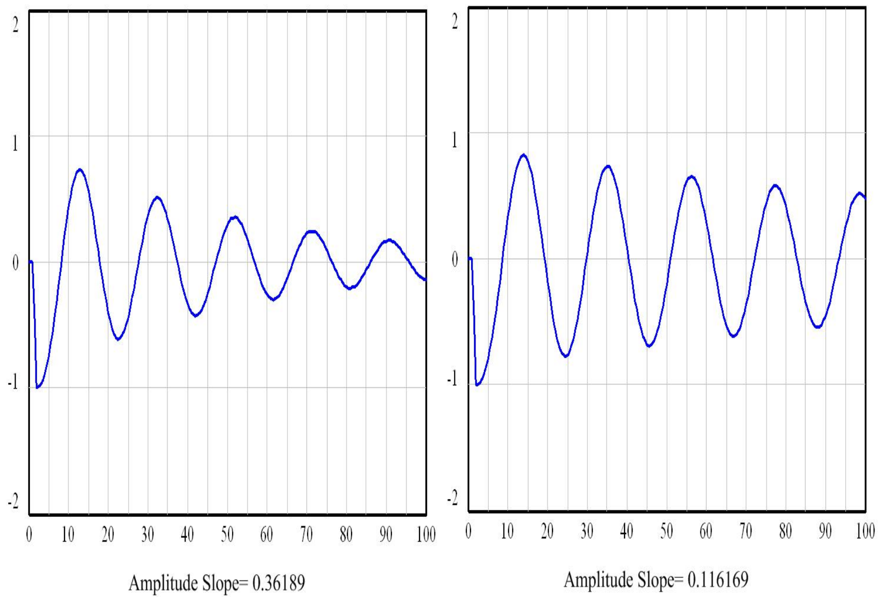


Figure 5.2. Amplitude Slopes of Two Different Stable Oscillations

The last behavior measure of a damping oscillation is the maximum amplitude that indicates the first response of the system to the incoming perturbation (Figure 5.1). This behavior measure shows the stability character of the system like the previous one. Specifically, more stable systems give smaller responses to incoming shocks while larger responses emerge from unstable ones. Since incoming noise never ends in real systems (Sterman, 2000), the initial response to each shock is a very critical indicator for stability of the system.

Another oscillation type is unstable (growing) oscillations. This pattern follows a trajectory that diverges infinity once it is perturbed from its equilibrium. The behavior measures of this pattern are the same with stable oscillations except the maximum amplitude. For unstable oscillations, this pattern measure depends on not only the model structure but also simulation length since the heights of successive peaks increase as time proceeds. Therefore, the initial response of the model can be tested using minimum amplitude which can be estimated with the minimum function in Excel after estimation of all amplitudes.

Limit cycle is the third oscillation type that will be discussed in this chapter. This behavior pattern emerges when nonlinear model structure limits growing amplitudes (Sterman, 2000). Therefore, amplitudes remain constant during the entire simulation after an initial growth. The pattern measures of this behavior mode are period and the height of constant amplitudes.

Pattern measures constitute the main focus of model builder while (s)he is analyzing the behavior of oscillatory systems. Period of "inventory cycles" or heartbeat strength (amplitude) are several examples of oscillation measures that we observe in real systems. In the following chapters, applications of pattern sensitivity analysis to two oscillatory system dynamics models are explained in great detail.

6. ANALYSIS OF A BASIC OSCILLATORY SUPPLY LINE MODEL

Supply chains are one of the most common structures in business world. Raw material procurement of a manufacturing facility or financing of a project are some examples of supply chains from business context. From the modeling point of view "supply chains consist of a stock and flow structure for the acquisition, storage and conversion of inputs into outputs and the decision rules governing these flows." (Sterman, 2000). The fundamental purpose of the supply chain managers is maintaining the stock at its desired value. Thus, supply chain structures include negative feedback loops that create corrective action once discrepancy arises between the stock and its desired level.

Furthermore, the transformation process in each supply chain takes some amount of time, i.e. there is a time delay in every supply chain structure. Interaction between negative feedback loops and the time lag may yield oscillations (Sterman, 2000). In this thesis, parameter sensitivity of a modified version of simple supply line model by Sterman (2000) is analyzed. In the following section, a brief description of this model is given and pattern sensitivity analysis of this model is discussed in Section 6.2.

6.1. Model Description

Supply chains are good examples of material delay formulations that are rigorously discussed in system dynamics literature. They consist of stocks and flows that represent the processes in the system. Furthermore, in order to obtain a complete model, the decision rules that govern this stock flow structure should be added to the model (Sterman, 2000).

The fundamental managerial decision of supply chain structures is the amount of order to be placed for keeping the *stock* at the desired level. Once the inventory level drops because of a demand spike, the amount of new order should be decided by the production manager. A popular decision heuristic for such situations is called anchor and adjustment (Sterman, 2000). Experimental studies indicate that decision makers adjust the amount of loss from the stock according to the levels of stock and supply line. The stock flow diagram of resulting model is given in Figure 6.1.

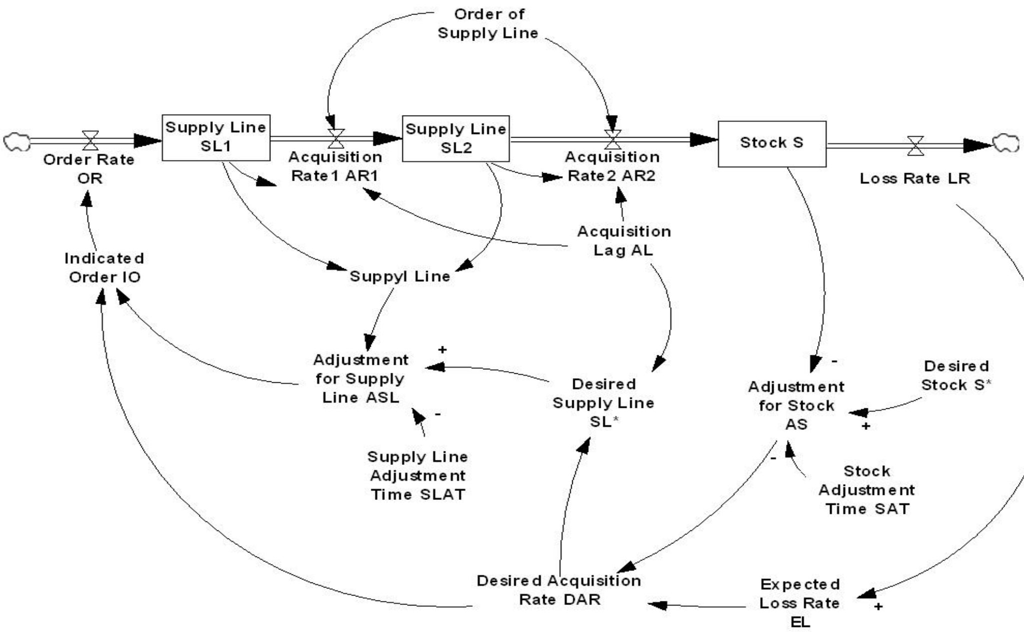


Figure 6.1. Stock Flow Structure of Simple Supply Line Model by Sterman (2000)

The supply chain structure aims to keep the *stock S* at *desired stock* . For this purpose, the expectation of loss is adjusted according to the level of stock (*Adjustment for Stock AS*). Then the summation of stock adjustment and loss expectation constitutes *desired acquisition rate* which is multiplied with time delay for desired value of supply line (*Desired Supply Line SL**).

At this point we should note that the formulation of desired supply line is different from the model given in Yasarcan (2003). In Yasarcan's study, desired supply line is equal to the multiplication of loss rate and acquisition lag. On the other hand in this model, the multiplication of acquisition lag and desired acquisition rate, which is the sum of stock adjustment and expected loss rate, constitutes the desired supply line (Sterman, 2000). The detailed description of this model formulation and underlying assumptions are given in the book of Sterman (2000).

This model has three stocks with generic names which are *stock*, *supply line1*, *supply line2*. In reference book by Sterman (2000), this model has only one stock for *supply line*. We add one more supply line stock in order to obtain more oscillatory model. The relationship between the number of stocks and system behavior can be seen in Yasarcan (2003).

Furthermore, there are three parameters, namely stock adjustment time, supply line adjustment time and acquisition delay, in this model. Stock adjustment time represents the willingness of decision maker for giving response to the changes in stock value. Lower values of this adjustment time indicate a decision maker who is more responsive to the immediate changes in demand. A similar logic applies for supply line adjustment time parameter. Higher values of this parameter indicate that the decision maker gives less weight to the condition of supply line in their orders.

The third parameter of this simple model is acquisition delay which is average amount of time that transformation takes after the related order is placed. In sensitivity analysis of simple supply line model, ±20 per cent of base values of these parameters are used for 200 sensitivity simulation runs. The parameter ranges and distributions are given in Table 6.1.

Table 6.1. Parameter Distributions of Simple Supply Line Model

Parameter Name	Actual Val	Min Value	Max Value	Distribution
Stock Adjust. Time (SAT)	2.5	2	3	Uniform
Supply Line Adjust. Time (SLAT)	3.75	3	4.5	Uniform
Acquisition Lag (AL)	11	8.8	13.2	Uniform

The pattern sensitivity analysis of simple supply line model is conducted according to the procedure given in Figure 2.1. After the execution of sensitivity simulations in Vensim, runs are exported to an Excel sheet and diagnosed virsually for detection of any other behavior modes but damping oscillation. At the end of these checks, we found out that the model does not follow any other oscillation type but damping oscillations for these parameter ranges. Then, the behavior measures, which are period, amplitude slope and maximum amplitude, are estimated for each simulation run.

These behavior measures constitute the dependent variable in the regression analyses that are discussed in the following sections.

6.2. Pattern Sensitivity Analysis of Simple Supply Line Model

Oscillations are cyclic behavior patterns which are difficult to analyze with standard statistical techniques, such as screening. So, sensitivity analysis of oscillatory models should focus on the pattern measures of these behavior modes. Common measures of an oscillatory pattern are period, first amplitude and amplitude slope. In this study, sensitivity analysis of the simple supply line model is conducted through these measures of oscillations. In the following sections, analyses of these pattern measures are given in great detail.

6.2.1. Sensitivity Analysis of Oscillation Period

Period of an oscillation is amount of time between two successive peaks or troughs. This pattern measure indicates how much the system oscillates under certain circumstances. Generally, higher period oscillations are more preferable than the lower period ones since it is more difficult to tolerate high frequency waves (Think about the heartbeat of the body). Sensitivity analysis for period indicates the possible leverage point of the system in order to control this feature of oscillations.

One of the critical steps in pattern sensitivity analysis procedure is the estimation of pattern measures for each simulation run. In this thesis, periods are estimated by autocorrelation function in BTSII which is a validation tool for behavior pattern tests of system dynamics models. Estimated values of periods are subject to regression analysis of which coefficients are given in Table 6.2.

According to the results of regression analysis, stock adjustment time is the most important parameter of the model. The second important parameter is acquisition lag which is the average time lag in supply line. Increasing values of this parameter yields longer-period oscillatory systems.

Table 6.2. Regression Results for Oscillation Period of Simple Supply Line Model

Coefficients					
	Regression Coefficients			t value	Sig.
Model	beta (β)	Std. Error	Std. Coef (b)		
(Constant)	.199	.339		.589	.557
StockAdjst	6.899	.078	.683	87.944	.000
AcqLag	1.510	.018	.657	84.671	.000
SupLineAdjst	2.026	.052	.301	38.722	.000

Table 6.3. Summary Statistics of Regression Model for Period

Model Summary		
R Square	Adjusted R Square	Std. Error of the Estimate
.988	.988	.320

The suitability of regression model can be checked via the coefficient of determination given in Table 6.3 and residual plots in Figure 6.2. Very high value of R square indicates that linear regression model is very successful for explaining the variability of periods. Moreover, residual plots and histogram indicate that the assumption on independence and normality of residuals are satisfactory. In other words, this model provides a good approach to the true functional relationship between period and parameters.

Table 6.4. Results of ANOVA Test of Clusters for Period

ANOVA Results for Period			
Ranking	Parameter Name	F Value	Significance
1	AcqLag	38.874446	0
2	StockAdjst	36.302405	0
3	SupLineAdjst	5.511978	0.00031754

The results of regression analysis are compared with the ANOVA test of clusters. According to the results of ANOVA test, given in Table 6.4, acquisition lag parameter is the most efficient point of the model for period. The second important parameter is found to be stock adjustment time. Although the parameter rankings of two methods

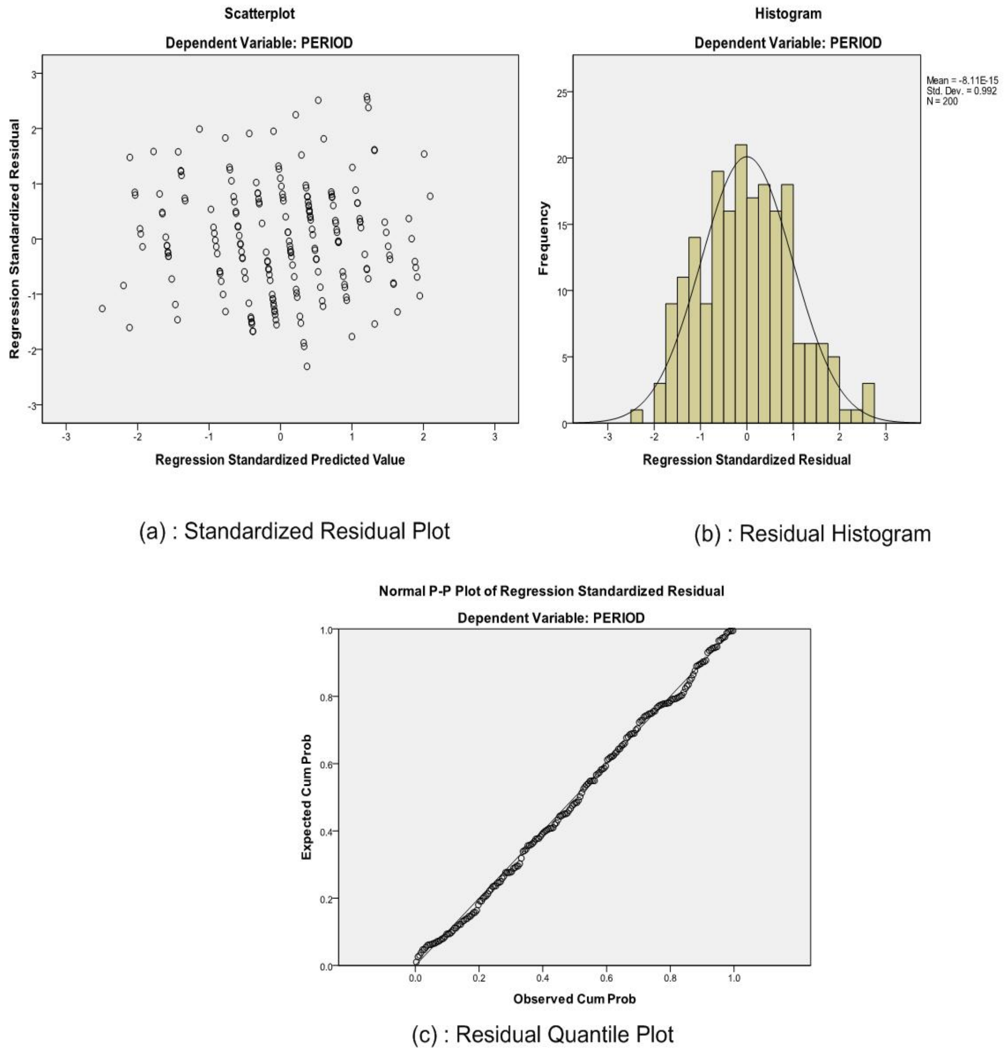


Figure 6.2. Residual Plots of Regression for Period of Supply Line Model

are different, we should point out that p-values of acquisition lag and stock adjustment time are very close to each other in both methods. Visual analysis of clustered scatter plots of these parameters also indicate the similarity of sensitivities (Figure A.3).

Concisely, linear regression is not a bad approximation to the period from simple supply line model. The results of regression coincide with ANOVA test of clusters. Moreover, we can conclude that stock adjustment time and acquisition lag parameters are very efficient points of the system for oscillation period. In the following section, the sensitivity analysis of amplitude slope is discussed in great detail.

6.2.2. Sensitivity Analysis of Amplitude Slope

Amplitude slope is an oscillation measure that indicates the stability character of the system. Namely, higher-slope amplitudes imply that the system tends to reach its equilibrium faster (Figure 5.2). For this measure, the successive amplitudes of an oscillatory behavior are estimated using their periods and then a straight line is fit to these amplitudes. However, sometimes the amplitudes may follow nonlinear patterns (generally exponential) and the amplitude slope may indicate erroneous sensitivity results in such situations. Therefore, taking the logarithm of amplitudes is a useful way in order to analyze the sensitivity of this pattern measure (Figure 7.4). The log-amplitude slope constitute the dependent variable of regression equation of which the coefficients are given in Table 6.5.

Table 6.5. Regression Results for Log-Amplitude Slopes

Coefficients					
	Regression Coefficients			t value	Sig.
Model	beta(β)	Std. Error	Std. Coef (b)		
(Constant)	0.612	0.02		30.885	0
StockAdjst	0.709	0.005	0.808	154.589	0
AcqLag	-0.092	0.001	-0.459	-87.813	0
SupLineAdjst	-0.193	0.003	-0.33	-63.228	0

According to the regression results, stock adjustment time is the most important parameter of the model for log-amplitude slopes. As stated above, higher values of this parameter indicate a less responsive system which tends to reach its equilibrium quicker. Furthermore, the second important parameter is acquisition lag. The negative sign of the regression coefficient indicates that increasing values of this parameter make the system longer-oscillating. Like acquisition lag, regression coefficient of supply line adjustment time has a negative sign which indicates that as the decision maker gives less weight to the condition of supply line, the supply line tends to oscillate longer.

The appropriateness of the linear regression model to this data set can be checked with coefficient of determination and regression plots given in Table 6.6 and Figure A.4.

The R square value of this regression model indicates that all variation in log-amplitude slope can be explained by the model parameters successfully. Moreover, the linearity and normality assumptions seem to be acceptable in Figure A.4 (See histogram and q-q plots of residual terms). Although there is a curvature in residual plot, this problem is ignorable due to high R square value.

Table 6.6. Summary Statistics of Regression for Log-Amplitude Slope

Model Summary		
R Square	Adjusted R Square	Std. Error of the Estimate
.995	.995	.01871270473

ANOVA test of clusters is conducted for log-amplitude slope in order to compare the regression results. According to the results of this test, given in Table 6.7, stock adjustment time is the most efficient parameter of the model. The second important one is acquisition lag. These results are completely the same with the regression model given in Table 6.5. The scatter plots of log-amplitude slopes against each parameter can be seen in Figure A.5. Obviously, stock adjustment time parameter explains the most of variability in log-amplitude slope measure.

Table 6.7. Results of ANOVA Test of Clusters for Log-Amplitude Slope

ANOVA Results for Log-Amplitude Slope			
Ranking	Parameter Name	F Value	Significance
1	StockAdjst	90.04041	0.00E+00
2	AcqLag	15.20013	7.79E-11
3	SupLineAdjst	6.89673	3.27E-05

We can conclude that linear regression model is a useful tool for log-amplitude slope of oscillatory behavior. Regression diagnostics are found to be very good and both of regression and ANOVA of clusters results agree with each other. In other words, one-variate and multi-variate sensitivity analysis applications give the same results for this pattern measures.

For modeling purposes, stock adjustment time parameter is found to be the most critical parameter of the model. Higher values of this parameter increase the stability character of the model while smaller values yield more oscillatory system. These results make significant contribution to our intuition about the stability of supply chain structures that we face in real life. In addition to the amplitude slope, another important stability measure is maximum amplitude of which sensitivity analysis is discussed in the following section.

6.2.3. Sensitivity Analysis of Maximum Amplitude

Maximum amplitude is the last behavior measure that is analyzed for parameter sensitivity of simple supply line model. As stated above, when a perturbation arrives to the system, the corrective action emerges in order to close the resulting gap. Due to inherent time delays, corrective action makes the system overshoot its goal and so oscillations begin. The height of this first overshoot indicates the responsiveness of the system.

Maximum amplitude is the first peak of a damping oscillation since the oscillations tend to disappear as simulation time proceeds. Therefore, the estimation of amplitudes, which is conducted in the analysis of amplitude slope, provides this pattern measure. The regression results for this measure are given in Table 6.8.

According to the regression analysis, stock adjustment time is the most efficient parameter for the maximum amplitude measure. This parameter determines the amount of first corrective action emerges after the perturbation arrives. Like previous analysis, acquisition lag is the second important parameter of the model. Higher values of time delay yield greater corrective action that makes the system less stable and harder to control.

The regression summary statistics and residual plots are given in Table 6.9 and Figure A.6. Very good R square value indicates that this regression model is very successful in explanation of the variance of this pattern measure and normality assumption of residuals terms seems to be acceptable. Like the previous analysis, there

Table 6.8. Regression Results for Maximum Amplitude of Simple Supply Line Model

Coefficients					
	Regression Coefficients			t value	Sig.
Model	beta(β)	Std. Error	Std. Coef (b)		
(Constant)	2.299	0.042		54.51	0
Stock Adjst	-1.088	0.01	-0.807	-111.454	0
Acq Lag	0.144	0.002	0.47	64.926	0
SupLine Adjst	0.278	0.007	0.31	42.757	0

is a slight curvature in the residual plot in Figure A.6. However, the convenience of other diagnosis makes the results of this regression model acceptable.

Table 6.9. Summary Statistics of Regression for Maximum Amplitude Measure

Model Summary		
R Square	Adjusted R Square	Std. Error of the Estimate
.996	.995	.010614737

Analysis of parameter sensitivity with a first order regression model ignores the effect of interactions between model parameter. In order to cover these parameter interactions, the first order regression model should be extended with two-way multiplicative terms. In order to exemplify this analysis, maximum amplitude measure of simple supply line model is used since this model is simple and includes only three parameters.

For simple supply line model there are three interaction terms to be included in the regression equation. Results of this regression analysis are given in Table B.1. According to these results, *stock adjustment time* and *acquisition lag* are found to be two most important parameters like first order regression model. Furthermore, interaction of acquisition lag and supply line adjustment time is significant and the third important parameter. We should note that interpretation of the regression results is difficult when an interaction term is found to be more important than the parameter itself. This issue is beyond the scope of this thesis. Also, results from a regression model including three-way interaction term is given in Table B.2, as a starting point for future research.

Furthermore, the regression results (Table 6.8) are compared with ANOVA test of clusters of which the results are given in Table 6.10. According to this analysis, stock adjustment time is found to be the most efficient parameter for initial responsiveness of the model. The second parameter seems to be acquisition lag of which its higher values make the model more responsive to the incoming shocks. Clustered scatter plots of all parameters are given in Figure A.7.

Table 6.10. Results of ANOVA Test of Clusters for Maximum Amplitude

ANOVA Results for Maximum Amplitude			
Ranking	Parameter Name	F Value	Significance
1	StockAdjst	90.067003	0.000
2	AcqLag	16.962646	5.95E-12
3	SupLineAdjst	6.008482	1.40E-04

Results of both analyses indicate that linear regression is a convenient tool for maximum amplitude. The diagnosis on residual terms and the agreement between the results of both statistical methods indicate the appropriateness of this approach for sensitivity analysis. Besides, like previous analyses, stock adjustment time is found to be the most efficient leverage point of the model.

Table 6.11. Summary of Regression Analysis for Simple Supply Line Model

Pattern Measures of Oscillations			
Rank	Period	Log-Amplitude Slope	Maximum Amplitude
1	StockAdjst (+)	StockAdjst (+)	StockAdjst (-)
2	AcqLag (+)	AcqLag (-)	AcqLag (+)
3	SupLineAdjst (+)	SupLineAdjst (-)	SupLineAdjst (+)

Summary of regression analysis for all pattern measures is given in Table 6.11. In all analyses, stock adjustment time is found to be the most important parameter of the model. Furthermore, the signs in the brackets imply the direction of correlation between the pattern measures and the parameters. For instance, there is negative correlation between stock adjustment time and maximum amplitude measure for the parameter range given in Table 6.1. Furthermore, the results of regression are almost the same with the results of ANOVA given in Table 6.12. Therefore, we can conclude that this model can be analyzed with one-variate sensitivity analysis easily. Interactions among parameters are not significant enough to effect pattern measures of output behavior.

Table 6.12. Summary of ANOVA Analyses for Simple Supply Line Model

Pattern Measures of Oscillations			
Rank	Period	Log-Amplitude Slope	Maximum Amplitude
1	AcqLag	StockAdjst	StockAdjst
2	StockAdjst	AcqLag	AcqLag
3	SupLineAdjst	SupLineAdjst	SupLineAdjst

To sum up, linear regression is a convenient tool for the pattern sensitivity of oscillatory system dynamics models. Important parameters of the model are detected in the analyses of all pattern measures. Furthermore, the results of regression coincide with ANOVA test of clusters so, all interactions among parameters are not very significant for the model output.

In order to evaluate the performance of this approach in more complex oscillatory models, inventory-workforce model by Sterman (2000) is analyzed for behavior pattern sensitivity in the following chapter.

7. ANALYSIS OF A MEDIUM SIZE OSCILLATORY INVENTORY - WORKFORCE MODEL

The most common example of the supply chain structures is production systems in which the production planner aims to keep inventory at some certain value. Once shipments occur, the inventory level falls and a production order is set in order to build it up. However, manufacturing process takes time before the finished goods inventory reaches its desired level even if all other factors, which constrain the production process, are ample. Furthermore, since there are various constraints on manufacturing systems , this time lag is always longer than the expected duration. For instance, lack of raw material or insufficient workforce may stop or slow the manufacturing process. Therefore, the interactions between production and other processes are always important for profitability of a manufacturing firm.

Many subsystems that interacts with production process can be modeled with a supply line structure in manufacturing companies. One of these subsystems is labor management process. Once new employee requirement arises, labor department completes the required procedures, such as announcement, interviews or orientation, and new employee joins the workforce. So, there is time lag between the labor requirement and workforce increase and this delay constrains the output of manufacturing system. Sterman (2000) presents a system dynamics model that includes inventory and workforce supply line structures in order to analyze the oscillations that manufacturing firms usually face during their operations. In this study, pattern sensitivity of inventory workforce model is analyzed using the sensitivity analysis algorithm proposed. In Section 7.1, a brief description of inventory workforce model is presented. Then, pattern sensitivity analysis of this model takes place in Section 7.2.

7.1. Model Description

Workforce of manufacturing firms is one of the main factors that constrain the production output of the manufacturing firms. Sterman (2000) presents a model for interaction of manufacturing and labor management processes. This model consists of two respective sectors named workforce and inventory. Both sectors consist of two-stock supply chain structures respectively. Namely, work in process inventory is the supply line stock of inventory whereas vacancies is the supply line of labor stock (Figure 7.1 and 7.2).

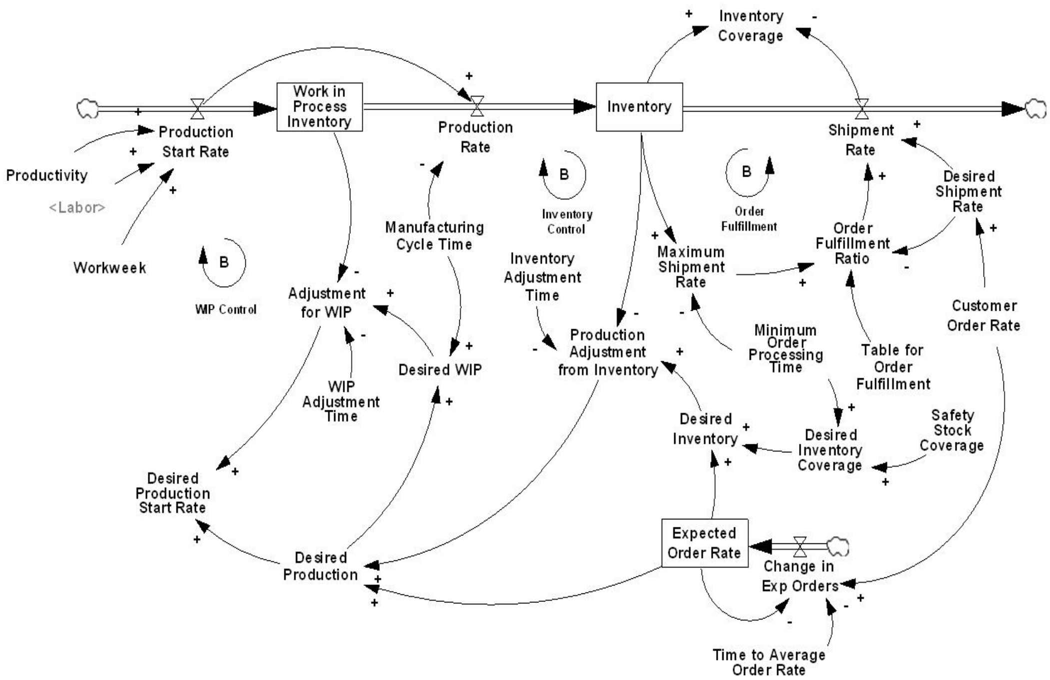


Figure 7.1. Inventory Sector of Inventory Workforce Model by Sterman (2000)

Customer order rate is the main input of the entire model. Desired shipment rate and expected order rate are calculated using this variable in the inventory sector of the model. In addition to the desired shipment rate, shipment rate is affected by minimum order processing time that determines the required amount of time for shipping an order to the customer.

The decision heuristic in the inventory sector of the model is initialized with expected order rate which is smoothed version of customer order rate with first order information delay. The multiplication of this smoothed value with desired inventory coverage, which is the sum of minimum order rate and safety stock coverage, gives the desired inventory variable. Moreover, production adjustment from inventory, which is calculated from desired inventory, is added to expected order rate to obtain desired production. Desired production represents the amount of finished goods that the production manager seeks to obtain. The adjustment variable coming from work in process is added to the desired production to calculate desired production start rate that is the main input of labor sector of the model.

Desired value of labor stock is calculated from desired production start rate using standard workweek and expected productivity. Specifically the calculation of required number of employee is conducted using the standard amount of working time and average labor productivity of the firm. In labor sector, the time delay parameter in workforce supply chain is average time to fill vacancies (Figure 7.2). More detailed discussion on the model formulations and assumptions are presented in Sterman (2000). In this thesis, the sensitivity analysis of this model is conducted through the behavior pattern of inventory stock.

The parameter names, ranges and distributions, which are used in our analysis, are given in Table 7.1.

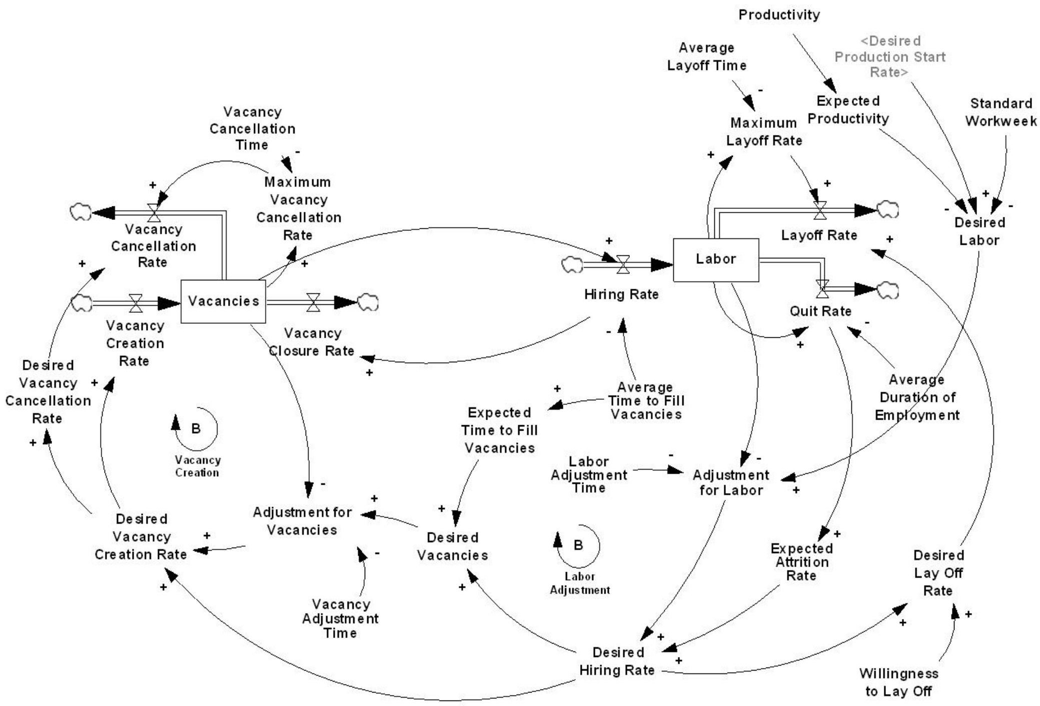


Figure 7.2. Labor Sector of Inventory Workforce Model by Sterman (2000)

Table 7.1. Parameter Ranges and Distributions of Inventory Workforce Model by Sterman (2000)

PARAMETER NAME	Actual Value	Min Value	Max Value	Distribution
Productivity	0.25	0.2	0.3	Uniform
WIP Adjustment Time	6	4.8	7.2	Uniform
Manufacturing Cycle Time	8	6.4	9.6	Uniform
Inventory Adjustment Time	12	9.6	14.4	Uniform
Minimum Order Processing Time	2	1.6	2.4	Uniform
Safety Stock Coverage	2	1.6	2.4	Uniform
Time to Average Order Rate	8	6.4	9.6	Uniform
Vacancy Cancellation Time	2	1.6	2.4	Uniform
Average Layoff Time	8	6.4	9.6	Uniform
Standard Workweek	40	32	48	Uniform
Average Duration of Employment	100	80	120	Uniform
Average Time to Fill Vacancies	8	6.4	9.6	Uniform
Labor Adjustment Time	13	10.4	15.6	Uniform
Vacancy Adjustment Time	4	3.2	4.8	Uniform

7.2. Pattern Sensitivity Analysis of Inventory Workforce Model

Inventory workforce model is a medium size, oscillatory system dynamics model that includes two supply line structures and 14 parameters. As discussed above, pattern sensitivity analysis starts with the determination of parameters and their distributions given in Table 7.1. Then, sensitivity simulations are executed and exported to Excel for the remaining phases of the analysis procedure. After visual check of each simulation run, behavior measures of oscillations are estimated and statistical analysis of these measures are conducted using two different methods described in Chapter 3. In the following sections, sensitivity analysis through pattern measures is discussed in great detail.

7.2.1. Sensitivity Analysis of Oscillation Period

Period is the fundamental feature of an oscillation since other measures such as amplitude or amplitude slope are estimated using it. Moreover, this pattern measure indicates the frequency of oscillations that can be related with the length of time delays and their interactions. Estimated values of this measure constitute the dependent variable of the regression model of which the important results are presented in Table 7.2 and 7.3. We should note that only significant parameters of regression model are given in Table 7.2 whereas full results are presented in Table A.3.

Table 7.2. Significant Parameters of Regression Model for Oscillation Period

Coefficients					
	Regression Coefficients			t value	Sig.
Model	beta	Std. Error	Std. Coef (b)		
(Constant)	38.468	4.286		8.976	.000
StandardWorkweek	-.854	.038	-.530	-22.345	.000
LaborAdjstTime	1.617	.076	.522	21.137	.000
WIPAdjst	2.928	.160	.436	18.266	.000
SafetyStockCoverage	3.542	.499	.176	7.100	.000
MinimumOrderProcssing	-1.539	.473	-.076	-3.256	.001
VacancyAdjstTime	.547	.244	.054	2.241	.026
InvntAdjstTime	.140	.080	.042	1.752	.081

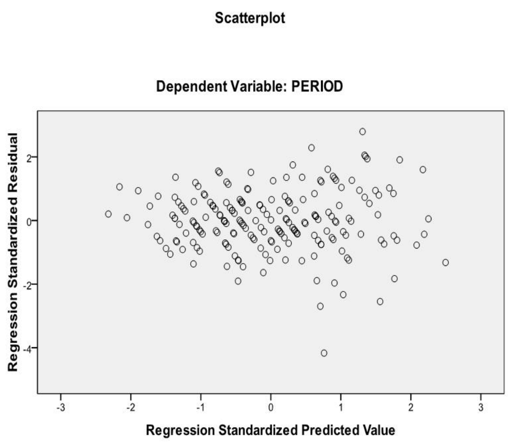
According to the regression results, standard workweek and labor adjustment time are the most important parameters for oscillation period. Increasing values of standard workweek shortens the period while larger values of labor adjustment time lead long period oscillations. Standard workweek parameter is the expectation of production manager on average workhour in an ordinary week. So, this parameter takes place in the formulation of desired labor with desired production start rate. On the other hand, labor adjustment time represents the responsiveness of labor manager to the changes in desired labor value. Positive sign of regression coefficient indicate that larger adjustment time values make the oscillation periods longer.

Interestingly, inventory adjustment time and productivity parameters are resulted as insignificant in the regression equation (See, Table A.3). Inventory adjustment time represents the responsiveness of production manager to the changing demand and this parameter is found to be the most important parameter in the analysis of

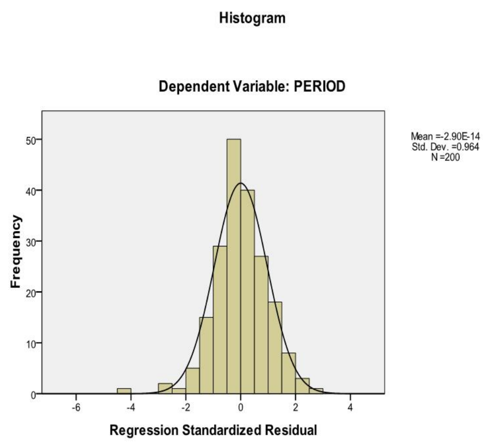
Table 7.3. Summary Statistics of Regression for Log-Amplitude Slope of Inventory Workforce Model

Model Summary		
R Square	Adjusted R Square	Std. Error of the Estimate
.901	.894	1.517

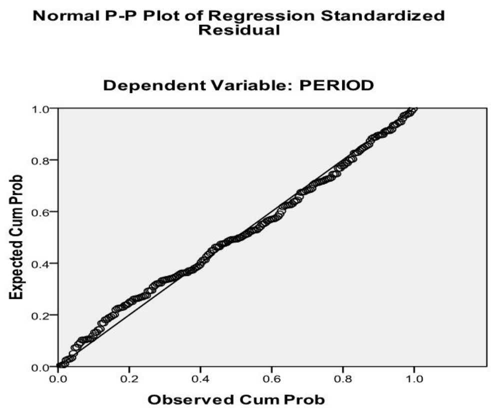
generic supply line model. Furthermore, insignificance of productivity parameter looks very counterintuitive and points out an open question about the structural validity of inventory workforce model.



(a) : Standardized Residual Plot



(b) : Residual Histogram



(c) : Residual Quantile Plot

Figure 7.3. Residual Plots of Regression for Period of Inventory Workforce Model

R square value of this regression equation seems acceptable (Table 7.3). Furthermore, the histogram and quantile-quantile plots of residual terms look very good in Figure 7.3. Although plots of residuals indicate a small problem about the constant variance assumption, this is an ignorable fallacy. Thus, we conclude that regression results are acceptable for sensitivity analysis of oscillation period. Then ANOVA test of clusters is conducted in order to make comparison with these regression results.

ANOVA test of clusters starts with the separation of each plot into different clusters. Then, mean values of clusters are subject to ANOVA analysis with the formulation given in Equation 3.9. Important parameters, obtained with this analysis, are given in Table 7.4 while some of the clustered scatter plots are presented in Figure A.8. Also, one can check the P-values of all parameters in Table A.4 in Appendix A.

Table 7.4. Significant Parameters in ANOVA of Clusters for Period

ANOVA Test Results			
Ranking	Parameter Name	F Value	P Value
1	StandardWorkweek	30.0007569	0.00
2	LaborAdjstTime	19.8905591	0.00
3	WIPAdjst	18.0960062	0.00
4	SafetyStockCoverage	3.7622818	0.01
5	MnfctrngCycleTime	1.6626198	0.16

According to the results of ANOVA test, standard workweek is the most important parameter while labor adjustment time is the second one. The importance of these two parameters is discussed above in sufficient detail. Like regression, ANOVA method gives work in process adjustment time and safety stock coverage parameters at third and fourth rank. The agreement of two different techniques indicates the consistency of regression results for sensitivity analysis purposes.

Comparison of regression results with ANOVA test of clusters is made in Table 7.5. Both techniques conclude the same results for the first four parameters while the ranking of remaining parameters differ. Thus, we conclude that regression analysis is a good approximation for detection of high sensitivity parameters for oscillation periods. In behavior pattern sensitivity of oscillatory models, analysis of oscillation period should be followed by amplitude slope which is discussed in the following subsection.

Table 7.5. Comparison of Regression Results with ANOVA for Period of Inventory-Workforce Model

Rank	Regression	ANOVA of Clusters
1	StandardWorkweek	StandardWorkweek
2	LaborAdjstTime	LaborAdjstTime
3	WIPAdjst	WIPAdjst
4	SafetyStockCoverage	SafetyStockCoverage
5	MinimumOrderProcssing	MnfctrngCycleTime
6	VacancyAdjstTime	Productivity
7	InvntAdjstTime	AvgDuratnEmploy
8	Time2AvgOrderRate	Time2AvgOrderRate
9	Time2FillVacancy	VacancyAdjstTime
10	MnfctrngCycleTime	Time2FillVacancy
11	AvgLayoffTime	MinimumOrderProcssing
12	VacancyCancelTime	VacancyCancelTime
13	Productivity	AvgLayoffTime
14	AvgDuratnEmploy	InvntAdjstTime

7.2.2. Sensitivity Analysis of Amplitude Slope

Another pattern measure of oscillations is amplitude slope which indicates the stability character of the system. More stable oscillations have faster decreasing amplitudes and the system reaches its equilibrium earlier (Figure 5.2). Like the analysis of the generic supply line model, natural logarithms of amplitudes are used in order to estimate amplitude slope measure. The appropriateness of log-amplitude slope can be understood from Figure 7.4.

After estimation of log-amplitude slopes, these values are subject to regression analysis of which the important results are given in Table 7.6 and 7.7 whereas full results are presented in Table A.5. According to the regression results, inventory adjustment time is the most important parameter for log-amplitude slope while standard workweek and manufacturing cycle time take second and third places. The importance of inventory adjustment time is equivalent with the conclusions from the analysis of generic supply line model in previous chapter. Inventory adjustment time is the re-

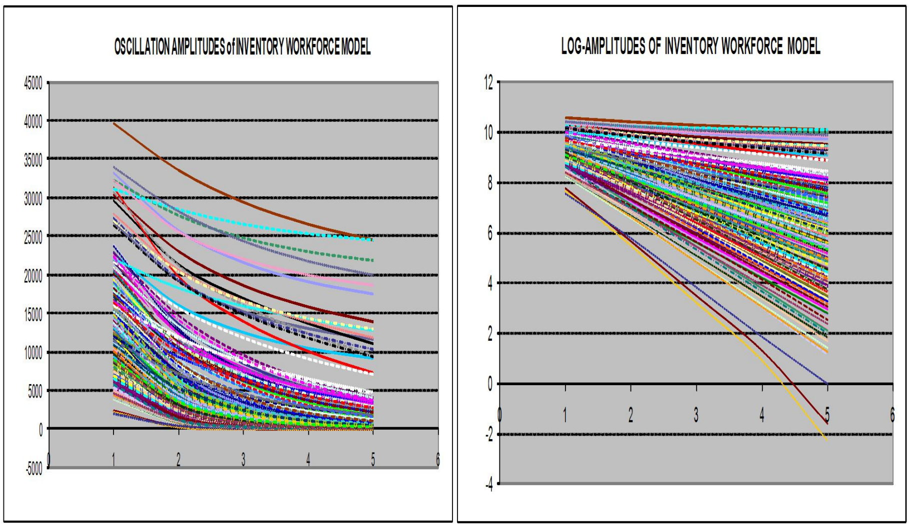


Figure 7.4. Amplitude Patterns for Damping Oscillations of Inventory Workforce Model

sponsiveness of manager to the changing demand and larger adjustment time values yield more stable oscillations. Furthermore, the importance of standard workweek is an interesting result of this analysis. The positive sign of its regression coefficient indicates that greater workhour expectation in a week make the oscillations more stable.

Table 7.6. Significant Parameters of Regression Model for Log-Amplitude Slope

Coefficients					
	Regression Coefficients			t value	Sig.
Model	beta	Std. Error	Std. Coef (b)		
(Constant)	-2.077	.319		-6.519	.000
InvntAdjstTime	.206	.006	.668	34.854	.000
StandardWorkweek	.076	.003	.512	26.745	.000
MnfctrngCycleTime	-.218	.009	-.470	-23.958	.000
VacancyAdjstTime	-.173	.018	-.187	-9.542	.000
WIPAdjst	.083	.012	.134	6.968	.000
SafetyStockCoverage	-.192	.037	-.103	-5.172	.000
Time2FillVacancy	-.046	.009	-.099	-5.037	.000

The appropriateness of this linear regression model to the data set at hand is checked via regression statistics and residual plots given in Table 7.7 and Figure A.9. Coefficient of determination of regression shows that 93 per cent of variance in dependent variable is explained by model parameters (See Table 7.7). In addition to

Table 7.7. Summary Statistics of Regression for Log-Amplitude Slope of Inventory Workforce Model

Model Summary		
R Square	Adjusted R Square	Std. Error of the Estimate
.936	.931	.11277508488

this large R square value, residual plot and histogram seem to be acceptable. So, we conclude that this regression equation is useful for sensitivity analysis of log-amplitude slopes.

Table 7.8. Significant Parameters in ANOVA Test of Clusters for Log-Amplitude Slope

ANOVA Test Results			
Ranking	Parameter Name	F Value	P-Value
1	InvntAdjstTime	31.40281	0.00
2	StandardWorkweek	17.22058	0.00
3	MnfctrngCycleTime	13.57143	0.00
4	SafetyStockCoverage	2.364011	0.05
5	Time2AvgOrderRate	2.240358	0.07

Like other pattern measures, regression results for log-amplitude slope are compared with ANOVA test of clusters. The significant parameters of this analysis approach are given in Table 7.8 whereas full results of analysis are given in Table A.6. Top three parameters in regression, namely inventory adjustment time, standard workweek and manufacturing cycle time, are found to be the most important parameters in ANOVA. The agreement of two methods on the top parameters indicates that these parameters are efficient points for controlling the stability character of oscillations. The clustered scatter plots of first four parameters are given in Figure A.10. Obviously, inventory adjustment time and standard workweek parameters have very strong effect on log-amplitude slope measure while the effect of safety stock coverage is more ambiguous.

Like log-amplitude slope, other pattern measure that indicates the stability character of the model is maximum amplitude of which the sensitivity analysis is discussed

Table 7.9. Results of Regression and ANOVA for Log-Amplitude Slope

Rank	Regression Analysis	ANOVA of Clusters
1	InvntAdjstTime	InvntAdjstTime
2	StandardWorkweek	StandardWorkweek
3	MnfctrngCycleTime	MnfctrngCycleTime
4	VacancyAdjstTime	SafetyStockCoverage
5	WIPAdjst	Time2AvgOrderRate
6	SafetyStockCoverage	AvgDuratnEmploy
7	Time2FillVacancy	LaborAdjstTime
8	Productivity	Time2FillVacancy
9	LaborAdjstTime	MinimumOrderProcssing
10	AvgDuratnEmploy	VacancyCancelTime
11	MinimumOrderProcssing	WIPAdjst
12	AvgLayoffTime	VacancyAdjstTime
13	VacancyCancelTime	Productivity
14	Time2AvgOrderRate	AvgLayoffTime

in the next section.

7.2.3. Sensitivity Analysis of Maximum Amplitude

Maximum amplitude is the first amplitude of damping oscillations and it points out the initial response of the system to the incoming perturbation. This pattern measure is very important for real systems which work in a noisy environment. For instance, customer demand that comes to a manufacturing firm is never as smooth as the simulation input. It always includes some amount of noise which creates continuous perturbation from equilibrium point. Therefore, the initial response to an incoming shock is a useful indicator for the stability character of real system. The estimation of this pattern measure is conducted as we described in Chapter 5. Then we apply regression analysis of which the significant parameters and summary statistics are given in Table 7.10 and 7.11. The full results including all regression coefficients are given in Table A.7.

According to the regression results, standard workweek and inventory adjust-

Table 7.10. Significant Parameters of Regression for Maximum Amplitude

Coefficients					
	Regression Coefficients			t value	Sig.
Model	beta(β)	Std. Error	Std. Coef (b)		
(Constant)	65740.679	5156.739		12.748	.000
StandardWorkweek	-1612.233	45.974	-.657	-35.068	.000
InvntAdjstTime	-2229.345	95.893	-.436	-23.248	.000
MnfctrngCycleTime	2286.739	147.310	.298	15.523	.000
SafetyStockCoverage	8708.900	600.223	.284	14.509	.000
Time2AvgOrderRate	-1311.570	143.782	-.171	-9.122	.000
LaborAdjstTime	647.400	92.048	.137	7.033	.000
VacancyAdjstTime	1473.119	293.909	.096	5.012	.000
Time2FillVacancy	508.314	147.511	.066	3.446	.001
AvgDuratnEmploy	-33.555	11.505	-.055	-2.917	.004

ment time are the most important parameters of the model for initial response. The importance of inventory adjustment time is very intuitive while the ranking of standard workweek is interesting. Apparently, larger values of expected workhour in a week make the system less responsive. Furthermore, manufacturing cycle time and safety stock coverage are the third and fourth parameters that have effect on maximum amplitude. Manufacturing cycle time, which is the time delay of inventory supply chain, is discussed in the previous analysis. Safety stock coverage takes place in the formulation of desired inventory in the model and there is a positive correlation between this parameter and initial response of the system.

R square value of this regression model, given in Table 7.11, indicates that independent variables manage to explain great amount of variance in maximum amplitude measure. On the other hand, the non-random pattern in residual plots, slightly inappropriate histogram and quantile-quantile plot indicate that linear regression model is not very good approach to this data set at hand (Figure 7.5). In order to get more satisfactory regression model, Box-Cox transformation approach is applied to this pattern measure.

Box-Cox transformation is a convenient way of dealing with nonlinearity prob-

Table 7.11. Summary Statistics of Regression for Maximum Amplitude of Inventory Workforce Model

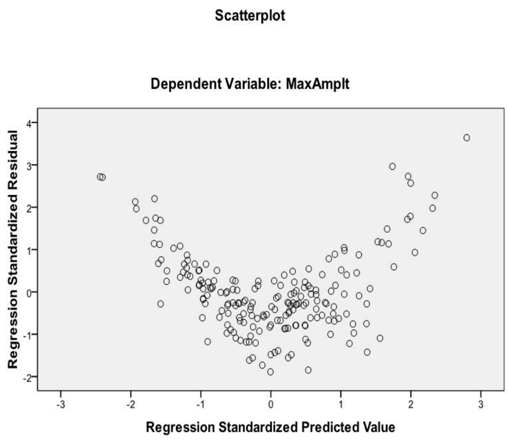
Model Summary		
R Square	Adjusted R Square	Std. Error of the Estimate
.939	.934	1825.5713

lems in regression models. In this approach the transformation exponent, which gives the best log-likelihood value of the regression model for a given data set, is selected and applied to the dependent variable. The calculation of this exponent is conducted in R Gui, which is a common statistical software for statisticians. For sensitivity data, including maximum amplitude and parameters, transformation exponent is calculated as 0.3838 and maximum amplitude is transformed. The significant parameters of regression with transformed maximum amplitude are given in Table 7.12 whereas full results are presented in Table A.8.

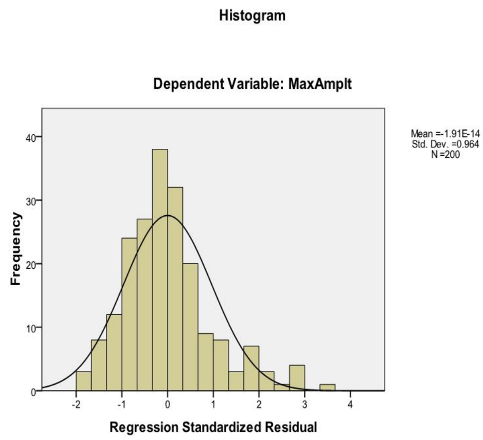
Table 7.12. Significant Parameters of Regression for Transformed Maximum Amplitude

Coefficients					
	Regression Coefficients			t value	Sig.
Model	beta(β)	Std. Error	Std. Coef (b)		
(Constant)	98.507	3.538		27.846	.000
StandardWorkweek	-1.805	.032	-.687	-57.223	.000
InvntAdjstTime	-2.430	.066	-.444	-36.946	.000
MnfctrngCycleTime	2.615	.101	.319	25.878	.000
SafetyStockCoverage	8.938	.412	.272	21.706	.000
Time2AvgOrderRate	-1.403	.099	-.171	-14.220	.000
LaborAdjstTime	.569	.063	.113	9.008	.000
VacancyAdjstTime	1.785	.202	.109	8.855	.000
Time2FillVacancy	.541	.101	.066	5.349	.000
AvgDuratnEmploy	-.025	.008	-.038	-3.131	.002

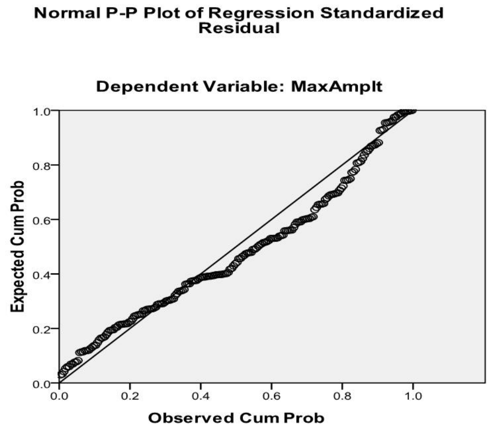
Transformation on dependent variable does not alter the first four parameters of the model. Standard workweek and inventory adjustment time are found to be the most important parameters for maximum amplitude. On the other hand, coefficient of determination of transformed model is higher than the previous regression equation



(a) : Standardized Residual Plot



(b) : Residual Histogram



(c) : Residual Quantile Plot

Figure 7.5. Residual Plots of Regression for Maximum Amplitude of Inventory Workforce Model by Sterman (2000)

(Table 7.13). Furthermore, residual plots, histogram and quantile diagrams seem very good for transformed model. They all indicate that the new model is more appropriate for this data set at hand (Figure 7.6).

Sensitivity of maximum amplitude measure is also analyzed with ANOVA test of clusters in this part of the study. Scatter plots of maximum amplitude against each parameter is divided into five clusters and test statistic, given in Equation 3.9, is calculated for each parameter. Significant parameters obtained with this method are given in Table 7.14. In this analysis, standard workweek and inventory adjustment time are found to be the most important parameters like previous analyses. However,

Table 7.13. Summary Statistics of Regression for Transformed Maximum Amplitude

Model Summary		
R Square	Adjusted R Square	Std. Error of the Estimate
.975	.973	1.25236

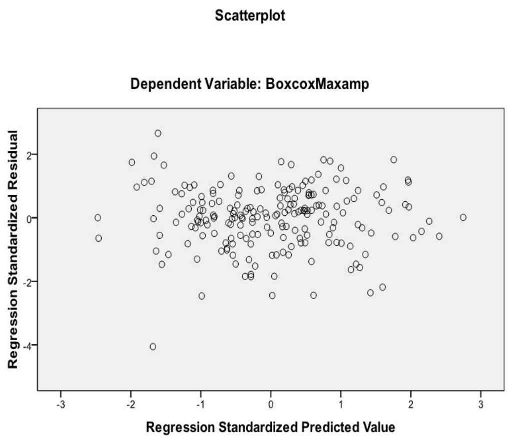
the rankings of safety stock coverage and manufacturing cycle time are different in that table. One can analyze the full ANOVA test results in Table A.9 and scatter plots of top-four parameters are given in Figure A.11 both in Appendix A. Apparently, these parameters have strong effect on maximum amplitude of damping oscillations.

Table 7.14. Significant Parameters in ANOVA Test of Clusters for Maximum Amplitude

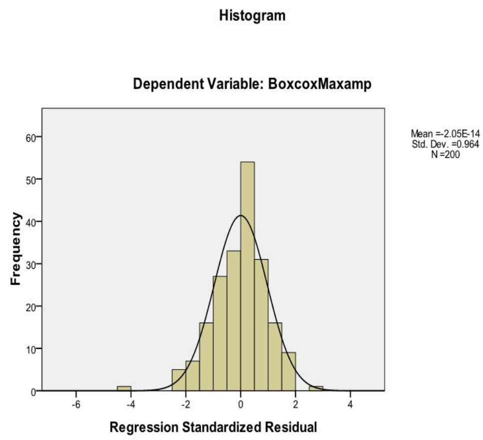
ANOVA Test Results		
Parameter Name	F Value	P-Value
StandardWorkweek	41.3079228	0.00
InvntAdjstTime	10.230399	0.00
SafetyStockCoverage	9.7092925	0.00
MnfctrngCycleTime	7.601395	0.00
LaborAdjstTime	3.8225064	0.01
Time2AvgOrderRate	3.0480612	0.02

The results of different approaches for maximum amplitude are compared with each other in Table 7.15. Apparently, transformation on dependent variable does not create any alteration on the ranking of first 9 parameters. This indicates that Box-Cox transformation is a convenient way for sensitivity analysis via regression method. Moreover, regression and ANOVA analyses result the same ranking for top four parameters. We conclude that regression is useful for detection of parameter importance even if its assumptions are not fulfilled.

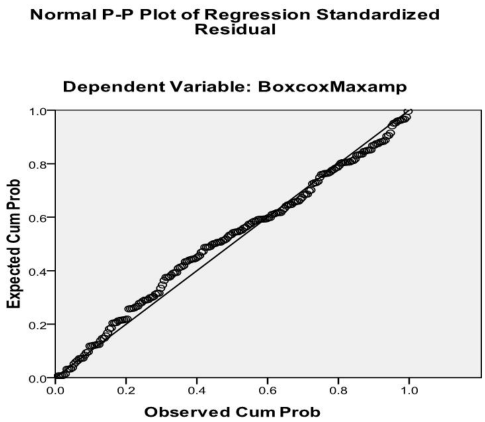
Moreover, comparison of regression results for different pattern measures are given in Table 7.16. Analysis on different pattern measures indicates that standard workweek is an efficient leverage point of this model for controlling the output pattern. Furthermore, inventory adjustment time and manufacturing cycle time are found to be the key points for stability of oscillations. In fact, the importance of these parameters



(a) : Standardized Residual Plot



(b) : Residual Histogram



(c) : Residual Quantile Plot

Figure 7.6. Residual Plots of Regression for Transformed Maximum Amplitude

coincides with the analysis of generic supply line model. Another important result of regression analysis is the ranking of safety stock coverage for different pattern measures. This parameter seems to be an efficient point for period and maximum amplitude of oscillatory pattern. Moreover, the signs in brackets in each cell indicates the direction of correlation between the pattern measure and parameter. For instance, increasing the value of standard workweek yields smaller period and more stable system (larger amplitude slope and smaller maximum amplitude).

Results of ANOVA test of clusters for different pattern measures are given in Table 7.17. Like regression analysis, standard workweek and inventory adjustment time parameters are found to be two most efficient parameters of the model.Then, we

Table 7.15. Comparison of Results of Regression and ANOVA Test for Maximum Amplitude

Rank	Regressio	Trans. Regression	ANOVA
1	StandardWorkweek	StandardWorkweek	StandardWorkweek
2	InvntAdjstTime	InvntAdjstTime	InvntAdjstTime
3	MnfctrngCycleTime	MnfctrngCycleTime	SafetyStockCoverage
4	SafetyStockCoverage	SafetyStockCoverage	MnfctrngCycleTime
5	Time2AvgOrderRate	Time2AvgOrderRate	LaborAdjstTime
6	LaborAdjstTime	LaborAdjstTime	Time2AvgOrderRate
7	VacancyAdjstTime	VacancyAdjstTime	WIPAdjst
8	Time2FillVacancy	Time2FillVacancy	VacancyCancelTime
9	AvgDuratnEmploy	AvgDuratnEmploy	Time2FillVacancy
10	VacancyCancelTime	AvgLayoffTime	MinimumOrderProcssing
11	Productivity	WIPAdjst	AvgDuratnEmploy
12	AvgLayoffTime	Productivity	Productivity
13	WIPAdjst	VacancyCancelTime	VacancyAdjstTime
14	MinimumOrderProcssing	MinimumOrderProcssing	AvgLayoffTime

can name labor adjustment time, safety stock coverage and manufacturing cycle time as the other important ones.

Inventory workforce model by Sterman (2000) is a medium size system dynamics model including 14 parameters and two supply chain structures. The sensitivity analysis of such kind of oscillatory model is a cumbersome task with other statistical approaches, such as screening. On the other hand, behavior pattern measures are convenient tools for sensitivity analysis of system dynamics models. Analysis of these measures points out the high sensitivity parameters and possible leverage points. Furthermore, all regression results for inventory workforce model are compared with ANOVA test of clusters. As discussed in previous chapters, ANOVA method is a one-variate sensitivity analysis method whereas regression is multi-variate one. The agreement between these two methods indicate that Box-Cox transformation is successful in dealing with nonlinearity problems and the interactions among parameters may be not significant. However, analysis on effect of interactions are beyond the scope of thesis and this issue is left to the future studies.

Table 7.16. Summary of Regression Analyses for Inventory Workforce Model

RANK	PERIOD	LOG-AMPLITUDE	MAX AMPLITUDE
1	StandardWorkweek (-)	InvntAdjstTime (+)	StandardWorkweek (-)
2	LaborAdjstTime (+)	StandardWorkweek (+)	InvntAdjstTime (-)
3	WIPAdjst (+)	MnfctrngCycleTime (-)	MnfctrngCycleTime (+)
4	SafetyStockCoverage (+)	VacancyAdjstTime (-)	SafetyStockCoverage (+)
5	MinOrderProcssing (-)	WIPAdjst (+)	Time2AvgOrderRate (-)
6	VacancyAdjstTime (+)	SafetyStockCoverage (-)	LaborAdjstTime (+)
7	InvntAdjstTime (+)	Time2FillVacancy (-)	VacancyAdjstTime (+)
8	Time2AvgOrderRate (-)	Productivity (-)	Time2FillVacancy (+)
9	Time2FillVacancy (+)	LaborAdjstTime (+)	AvgDuratnEmploy (-)
10	MnfctrngCycleTime (+)	AvgDuratnEmploy (-)	AvgLayoffTime (+)
11	AvgLayoffTime (+)	MinOrderProcssing (+)	WIPAdjst (-)
12	VacancyCancelTime (-)	AvgLayoffTime (-)	Productivity (+)
13	Productivity (+)	VacancyCancelTime (-)	VacancyCancelTime (+)
14	AvgDuratnEmploy (-)	Time2AvgOrderRate (+)	MinOrderProcssing (+)

Table 7.17. Summary of ANOVA Analyses for Inventory Workforce Model

RANK	PERIOD	LOG-AMPLITUDE	MAX AMPLITUDE
1	StandardWorkweek	InvntAdjstTime	StandardWorkweek
2	LaborAdjstTime	StandardWorkweek	InvntAdjstTime
3	WIPAdjst	MnfctrngCycleTime	SafetyStockCoverage
4	SafetyStockCoverage	SafetyStockCoverage	MnfctrngCycleTime
5	MnfctrngCycleTime	Time2AvgOrderRate	LaborAdjstTime
6	Productivity	AvgDuratnEmploy	Time2AvgOrderRate
7	AvgDuratnEmploy	LaborAdjstTime	WIPAdjst
8	Time2AvgOrderRate	Time2FillVacancy	VacancyCancelTime
9	VacancyAdjstTime	MinimumOrderProcssing	Time2FillVacancy
10	Time2FillVacancy	VacancyCancelTime	MinimumOrderProcssing
11	MinimumOrderProcssing	WIPAdjst	AvgDuratnEmploy
12	VacancyCancelTime	VacancyAdjstTime	Productivity
13	AvgLayoffTime	Productivity	VacancyAdjstTime
14	InvntAdjstTime	AvgLayoffTime	AvgLayoffTime

8. CONCLUSION

Parameters of simulation models always include some level of uncertainty since it is impossible to have sufficient data for perfectly precise estimation. Therefore, the results of system dynamics models should be tested against uncertainty in parameter values. In other words, sensitivity of the simulation model should be analyzed in order to have more reliable results.

In sensitivity analysis studies, the type of sensitivity can differ according to the purpose of the model. Namely, models dealing with physical phenomenon, like the ones that are used by NASA, should be tested for numerical sensitivity while behavior pattern oriented sensitivity analysis is more appropriate for system dynamics models (Sterman, 2000). In this thesis, changes in specific features of output pattern are analyzed against parameter uncertainty and this approach is called behavior pattern sensitivity analysis.

Behavior pattern sensitivity analysis starts with the identification of model parameters and their distribution ranges for the sampling process. After specification of parameter ranges and distributions, sampling strategy should be selected. Among various alternatives for sampling strategy, Latin Hypercube Sampling is utilized since it is more appropriate for sensitivity analysis of simulation models (McKay et al., 1976).

After the execution of sensitivity simulations, behavior modes and their pattern measures are identified. Pattern measures are estimated for each simulation run using appropriate tools. Next, these measures are subject to statistical analysis using linear regression and ANOVA test of clusters.

Linear regression is a convenient way for explaining the variability in a dependent variable in terms of independent ones. It is a suitable method for sensitivity analysis, with its diagnosis tools that measure the appropriateness of the statistical model. In this thesis, residual plots, histogram and quantile-quantile plots are used for checking regression assumptions, such as normality, linearity and constant variance. Furthermore the coefficient of determination measures the amount of variability explained by the model. Once these assumptions are fulfilled, we can interpret the regression results for sensitivity analysis purpose. Otherwise, Box-Cox transformation is applied on pattern measures in order to obtain a better regression model. For all pattern measures, we compare the regression results with the results of ANOVA test of clusters.

In ANOVA test of clusters, each scatter plot is divided into several clusters and means of all clusters are subject to ANOVA analysis in order to detect any non-random pattern. The test statistic, given in Equation 3.9, is calculated for each parameter and then parameters are ordered according to the magnitude of this test statistic. Actually, this method corresponds to a uni-variate sensitivity analysis. Therefore, the comparison of regression results with ANOVA method also means the comparison of uni-variate sensitivity analysis with multi-variate one.

Three different system dynamics models, namely the project management, simple supply line and inventory workforce models, are analyzed with both methods. The project management model focuses on the dynamics of a simple project and it has two different output behavior modes, which are tipping point behavior and s-shaped growth. Pattern sensitivity of the project management model is analyzed through one measure of each behavior mode, which are the inflection of s-shaped growth and the peak of tipping point pattern. Analysis results indicate that project complexity and total staff are the most important parameters of the model. Furthermore, the partial disagreement between regression and ANOVA results points out that there may be strong interactions among parameters.

We apply behavior pattern sensitivity to oscillatory system dynamics models since the analysis of such models are difficult with standard statistical approaches, such as screening (Ford and Flynn, 2005). Our first oscillatory model is a simple supply line model by Sterman (2000). Pattern sensitivity analysis of this model indicates that the stock adjustment time is the most important parameter for supply line structures. The second important parameter is acquisition lag, which is the time delay in material flow. Results of both analysis methods indicate that pattern sensitivity is a convenient approach for the sensitivity analysis of small oscillatory system dynamics models.

In order to try this approach on more complex oscillatory systems, we conduct sensitivity analysis of an inventory-workforce model. Pattern sensitivity analysis of inventory-workforce model indicates that standard workweek and inventory adjustment time parameters have great importance for all pattern measures. Furthermore, labor adjustment time is another effective leverage point for oscillation period, whereas manufacturing cycle time is important for the stability character of the oscillations. The agreement between results of regression and ANOVA analysis indicates that parameter interactions are probably not very significant.

In summary, pattern sensitivity is particularly appropriate for system dynamics models since it focuses on the shape of output behavior pattern. Our analysis indicates that linear regression is appropriate for multi-variate sensitivity analysis even for nonlinear relationships like the one between peaks of tipping point and parameters.

Furthermore, the comparison of regression with ANOVA test of clusters points out possible significant interactions between model parameters. The analysis of these interactions and interpretation of the results are very difficult and beyond the scope of this thesis. For future research, one can utilize regression models including interaction terms in order to assess the importance of interactions on the pattern measures of dynamic behavior. Our preliminary trials indicate that interpretation of sensitivity results may be very challenging even for medium size simulation models when the interaction terms are included.

APPENDIX A: MORE REGRESSION RESULTS, RESIDUAL AND SCATTER PLOTS

Table A.1. Regression Results for Tipping Point Level Measure

Coefficients					
	Regression Coefficients			t value	Sig.
Model	beta (β)	std. error	Std. Coef. (b)		
(Constant)	-1.5	0.612		-2.45	0.016
totalstaff	0.001	0	0.669	10.691	0
scope	-4.87E-05	0	-0.622	-10.243	0
Icprod	0.992	0.132	0.432	7.503	0
deadline	0.012	0.002	0.398	7.016	0
complex	-3.658	0.608	-0.38	-6.017	0
ripple	-0.447	0.149	-0.18	-3.008	0.003
Release	0.769	0.285	0.158	2.699	0.008
QAprod	0.319	0.132	0.143	2.411	0.018
SensePress	-0.757	0.357	-0.123	-2.123	0.036
RWprod	0.223	0.149	0.089	1.5	0.137
minQA	0.166	0.138	0.069	1.201	0.233
StaffAdjust	0.032	0.034	0.055	0.954	0.343
minRW	0.108	0.144	0.044	0.753	0.453
minIC	0.066	0.138	0.027	0.477	0.634

Table A.2. Regression Summary Statistics for Tipping Level Measure

Model Summary		
R Square	Adjusted R Square	Std. Error of the Estimate
.732	.691	.1563857663

Table A.3. Full Regression Results for Oscillation Period of Inventory Workforce Model

Coefficients					
	Regression Coefficients			t value	Sig.
Model	beta(β)	Std. Error	Std. Coef (b)		
(Constant)	38.468	4.286		8.976	.000
StandardWorkweek	-.854	.038	-.530	-22.345	.000
LaborAdjstTime	1.617	.076	.522	21.137	.000
WIPAdjst	2.928	.160	.436	18.266	.000
SafetyStockCoverage	3.542	.499	.176	7.100	.000
MinimumOrderProcssing	-1.539	.473	-.076	-3.256	.001
VacancyAdjstTime	.547	.244	.054	2.241	.026
InvntAdjstTime	.140	.080	.042	1.752	.081
Time2AvgOrderRate	-.196	.119	-.039	-1.641	.102
Time2FillVacancy	.132	.123	.026	1.080	.281
MnfctrngCycleTime	.129	.122	.026	1.057	.292
AvgLayoffTime	.094	.123	.019	.767	.444
VacancyCancelTime	-.290	.481	-.014	-.603	.547
Productivity	1.180	3.887	.007	.304	.762
AvgDuratnEmploy	-.003	.010	-.006	-.262	.793

Table A.4. Full Results of ANOVA of Clusters for Period of Inventory Workforce Model

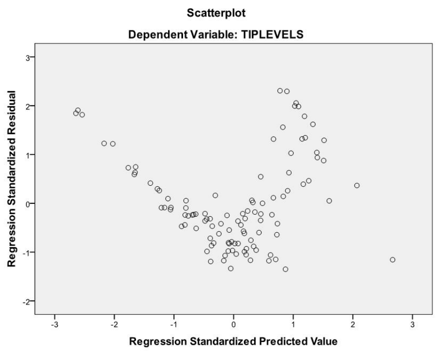
ANOVA Test Results			
Ranking	Parameter Name	F Value	P Value
1	StandardWorkweek	30.0007569	0.00
2	LaborAdjstTime	19.8905591	0.00
3	WIPAdjst	18.0960062	0.00
4	SafetyStockCoverage	3.7622818	0.01
5	MnfctrngCycleTime	1.6626198	0.16
6	Productivity	1.303907	0.27
7	AvgDuratnEmploy	1.1988851	0.31
8	Time2AvgOrderRate	1.0030788	0.41
9	VacancyAdjstTime	0.9619902	0.43
10	Time2FillVacancy	0.798312	0.53
11	MinimumOrderProcssing	0.7662875	0.55
12	VacancyCancelTime	0.6866971	0.60
13	AvgLayoffTime	0.3714502	0.83
14	InvntAdjstTime	0.1366484	0.97

Table A.5. Full Results of Regression Model for Log-Amplitude Slope of Inventory Workforce Model

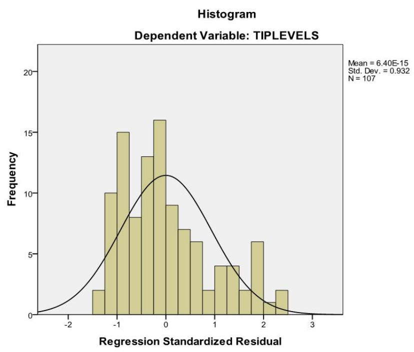
Coefficients					
	Regression Coefficients			t value	Sig.
Model	beta	Std. Error	Std. Coef (b)		
(Constant)	-2.077	.319		-6.519	.000
InvntAdjstTime	.206	.006	.668	34.854	.000
StandardWorkweek	.076	.003	.512	26.745	.000
MnfctrngCycleTime	-.218	.009	-.470	-23.958	.000
VacancyAdjstTime	-.173	.018	-.187	-9.542	.000
WIPAdjst	.083	.012	.134	6.968	.000
SafetyStockCoverage	-.192	.037	-.103	-5.172	.000
Time2FillVacancy	-.046	.009	-.099	-5.037	.000
Productivity	-.444	.289	-.030	-1.536	.126
LaborAdjstTime	.009	.006	.030	1.495	.137
AvgDuratnEmploy	-.001	.001	-.021	-1.108	.269
MinimumOrderProcssing	.038	.035	.020	1.073	.285
AvgLayoffTime	-.008	.009	-.018	-.918	.360
VacancyCancelTime	-.006	.036	-.003	-.166	.868
Time2AvgOrderRate	.001	.009	.001	.065	.948

Table A.6. Full Results of ANOVA of Clusters for Log-Amplitude Slope of Inventory Workforce Model

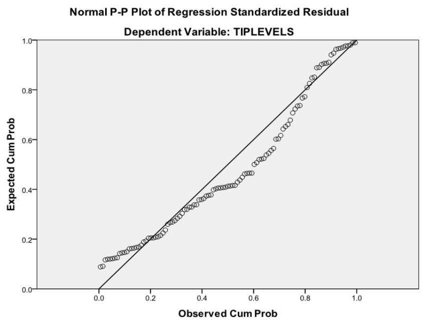
ANOVA Test Results			
Ranking	Parameter Name	F Value	P-Value
1	InvntAdjstTime	31.40281	0.00
2	StandardWorkweek	17.22058	0.00
3	MnfctrngCycleTime	13.57143	0.00
4	SafetyStockCoverage	2.364011	0.05
5	Time2AvgOrderRate	2.240358	0.07
6	AvgDuratnEmploy	1.407796	0.23
7	LaborAdjstTime	1.353445	0.25
8	Time2FillVacancy	1.004352	0.41
9	MinimumOrderProcssing	0.965335	0.43
10	VacancyCancelTime	0.763827	0.55
11	WIPAdjst	0.386211	0.82
12	VacancyAdjstTime	0.215373	0.93
13	Productivity	0.152097	0.96
14	AvgLayoffTime	0.146951	0.96



(a) : Standardized Residual Plots



(b) : Residual Histogram



(c) : Residual Quantile Quantile Plot

Figure A.1. Regression Plots for Tipping Point Measure

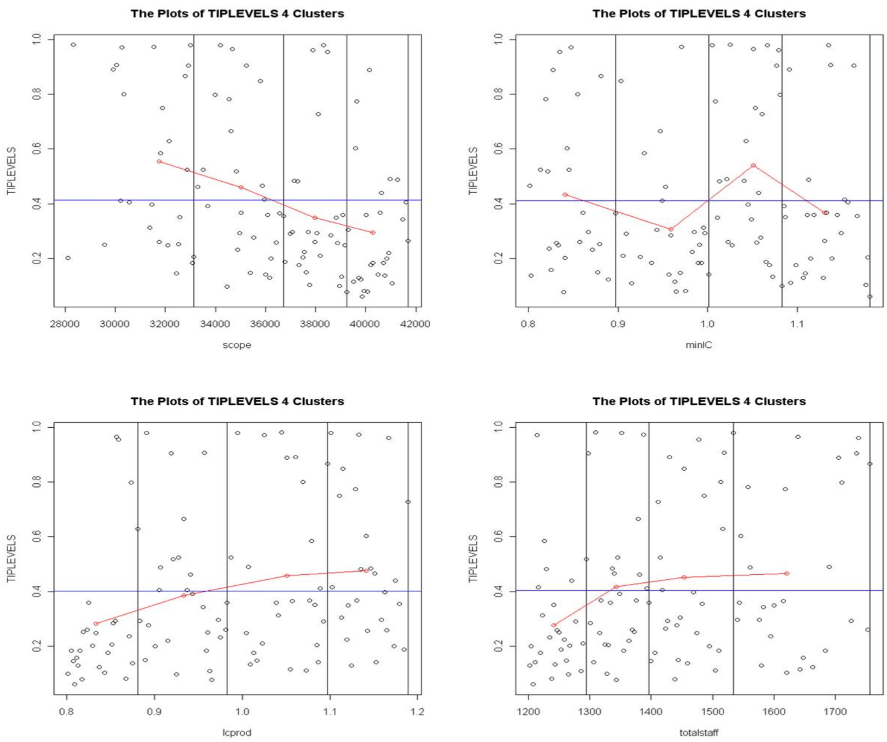


Figure A.2. Some Clustered Scatter Plots Used in Sensitivity Analysis of Tipping Level

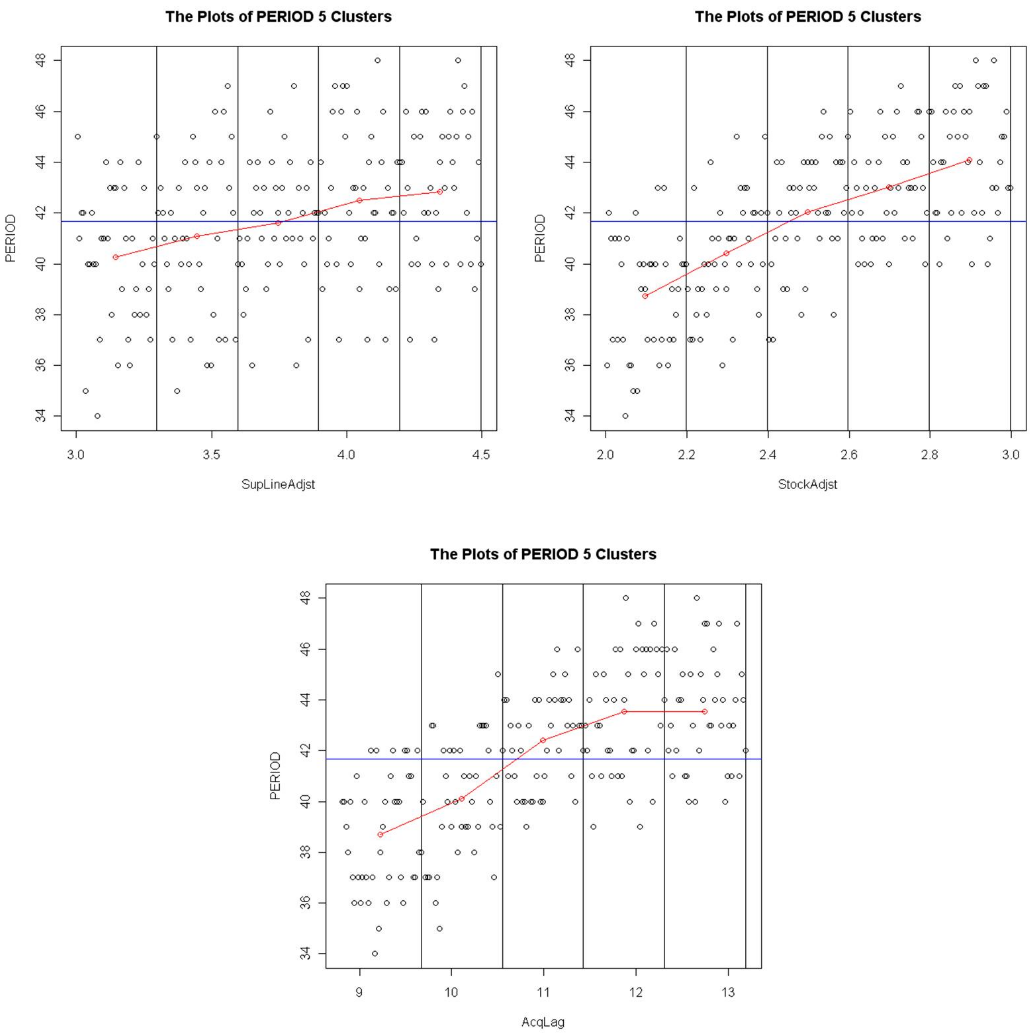
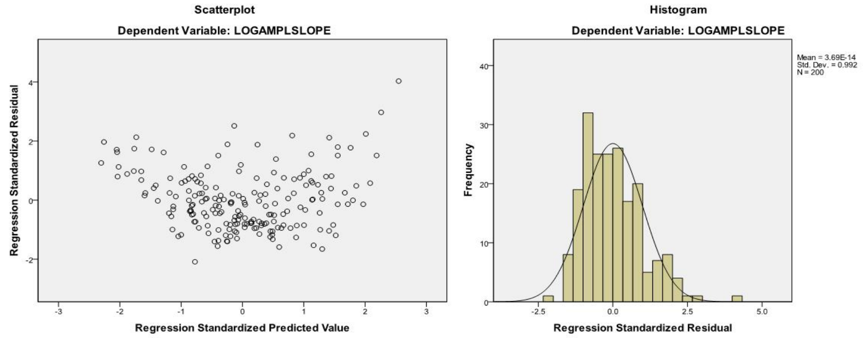
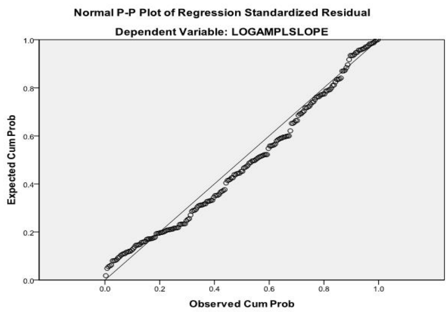


Figure A.3. Clustered Scatter Plots for Period and Parameters of Simple Supply Line Model by Sterman (2000)



(a) : Standardized Residual Plot

(b) : Residual Histogram



(c) : Residual Quantile Plot

Figure A.4. Residual Plots of Regression for Log-Amplitude Slope of Simple Supply Line Model

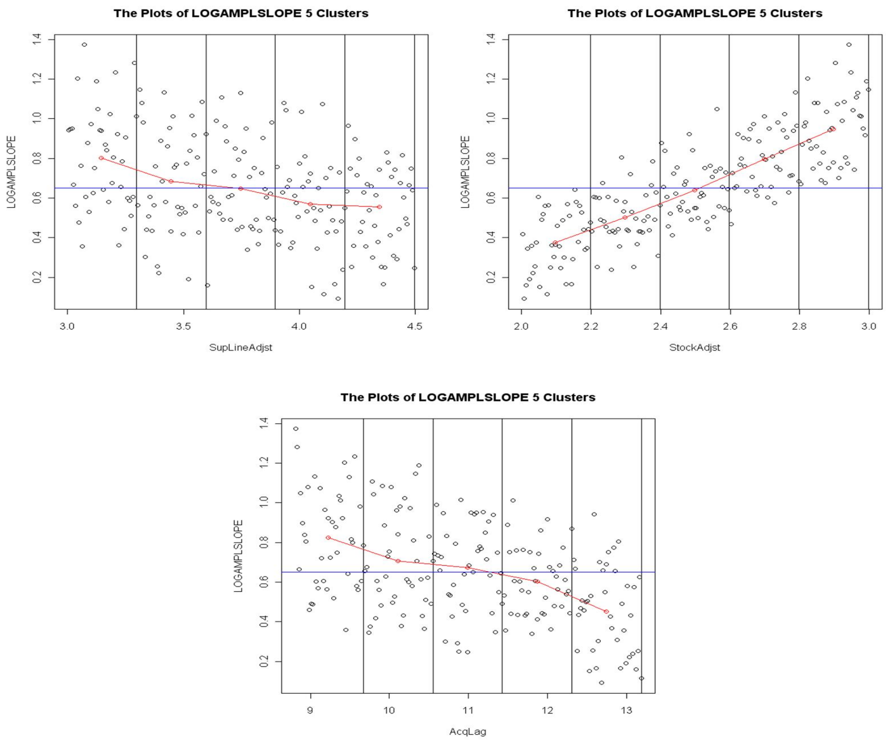
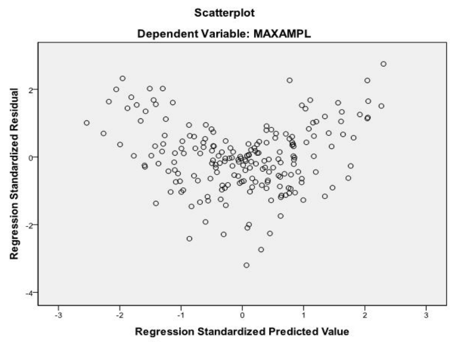
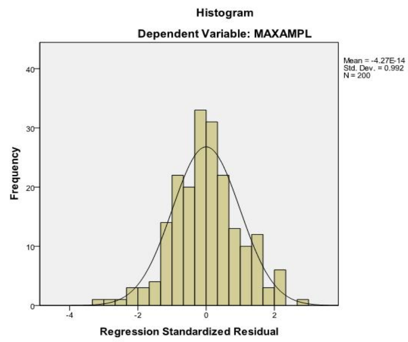


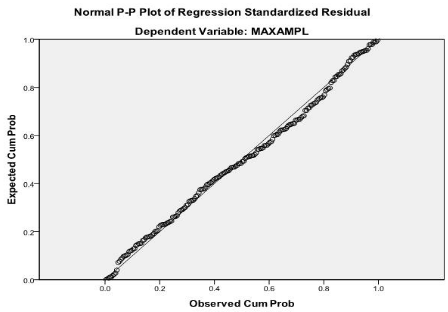
Figure A.5. Clustered Scatter Plots for Log-Amplitude Slope and Parameters of Simple Supply Line Model by Sterman (2000)



(a) : Standardized Residual Plot



(b) : Residual Histogram



(c) : Residual Quantile Plot

Figure A.6. Regression Residual Plots for Maximum Amplitude of Supply Line Model

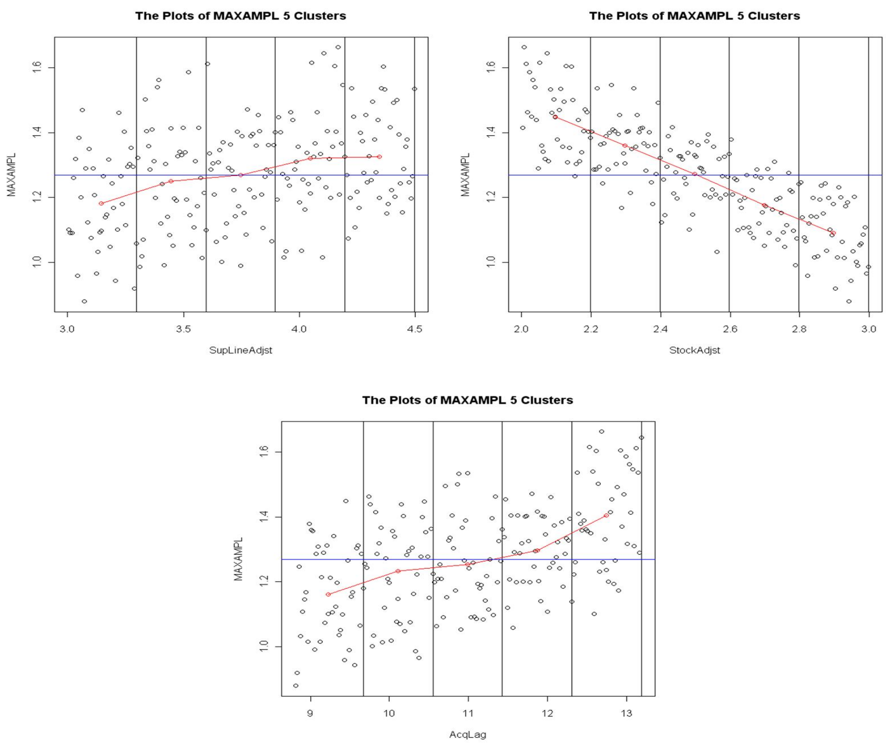


Figure A.7. Clustered Scatter Plots for Maximum Amplitude and Parameters of Simple Supply Line Model by Sterman (2000)

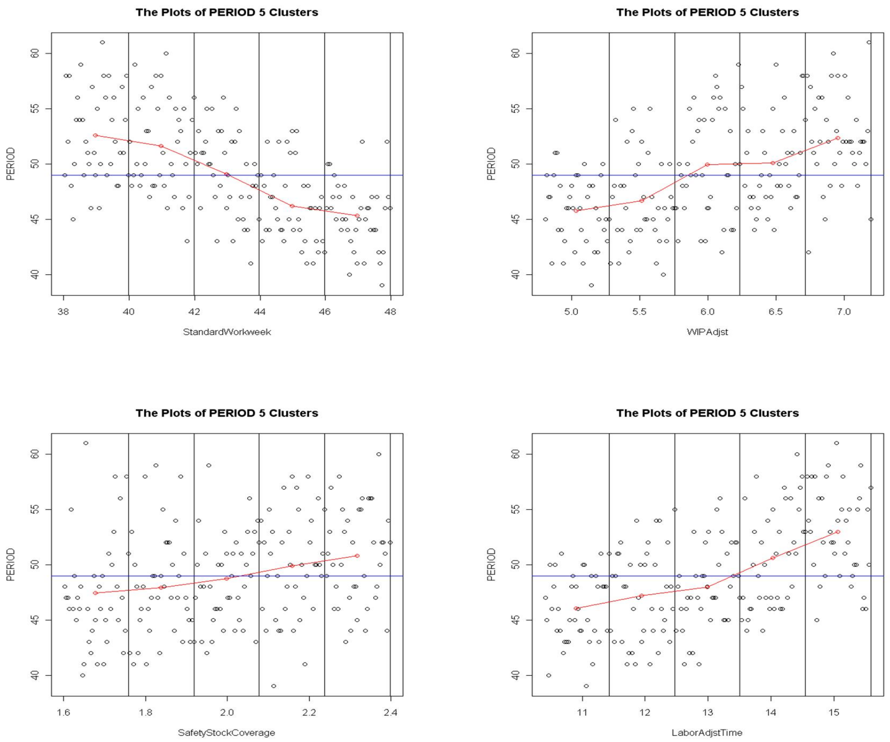
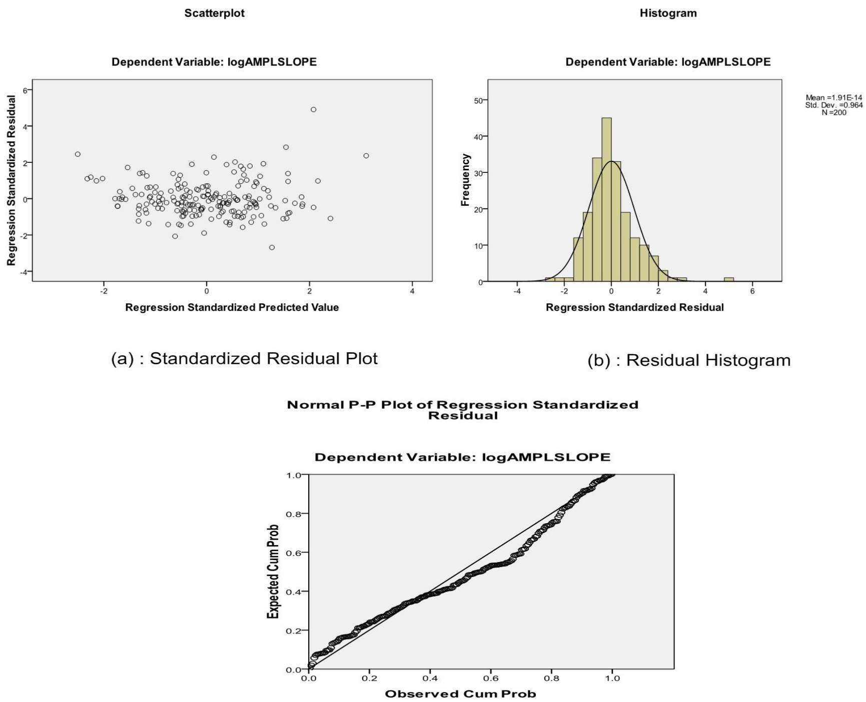


Figure A.8. Some Clustered Scatter Plots for Period and Parameters of Inventory Workforce Model by Sterman (2000)



(a) : Standardized Residual Plot

(b) : Residual Histogram

(c) : Residual Quantile Plot

Figure A.9. Residual Plots of Regression for Log-Amplitude Slope of Inventory Workforce Model

Table A.7. Full Results Regression for Maximum Amplitude of Inventory Workforce Model

Coefficients					
	Regression Coefficients			t value	Sig.
Model	beta(β)	Std. Error	Std. Coef (b)		
(Constant)	65740.679	5156.739		12.748	.000
StandardWorkweek	-1612.233	45.974	-.657	-35.068	.000
InvntAdjstTime	-2229.345	95.893	-.436	-23.248	.000
MnfctrngCycleTime	2286.739	147.310	.298	15.523	.000
SafetyStockCoverage	8708.900	600.223	.284	14.509	.000
Time2AvgOrderRate	-1311.570	143.782	-.171	-9.122	.000
LaborAdjstTime	647.400	92.048	.137	7.033	.000
VacancyAdjstTime	1473.119	293.909	.096	5.012	.000
Time2FillVacancy	508.314	147.511	.066	3.446	.001
AvgDuratnEmploy	-33.555	11.505	-.055	-2.917	.004
VacancyCancelTime	448.973	578.465	.015	.776	.439
Productivity	3568.191	4676.931	.015	.763	.446
AvgLayoffTime	100.060	147.731	.013	.677	.499
WIPAdjst	111.808	192.902	.011	.580	.563
MinimumOrderProcssing	298.836	568.678	.010	.525	.600

Table A.8. Full Results of Regression for Transformed Maximum Amplitude of Inventory Workforce Model

Coefficients					
	Regression Coefficients			t value	Sig.
Model	beta(β)	Std. Error	Std. Coef (b)		
(Constant)	98.507	3.538		27.846	.000
StandardWorkweek	-1.805	.032	-.687	-57.223	.000
InvntAdjstTime	-2.430	.066	-.444	-36.946	.000
MnfctrngCycleTime	2.615	.101	.319	25.878	.000
SafetyStockCoverage	8.938	.412	.272	21.706	.000
Time2AvgOrderRate	-1.403	.099	-.171	-14.220	.000
LaborAdjstTime	.569	.063	.113	9.008	.000
VacancyAdjstTime	1.785	.202	.109	8.855	.000
Time2FillVacancy	.541	.101	.066	5.349	.000
AvgDuratnEmploy	-.025	.008	-.038	-3.131	.002
AvgLayoffTime	.131	.101	.016	1.294	.197
WIPAdjst	-.125	.132	-.011	-.948	.345
Productivity	2.179	3.208	.008	.679	.498
VacancyCancelTime	.257	.397	.008	.648	.518
MinimumOrderProcssing	.255	.390	.008	.654	.514

Table A.9. Full Results of ANOVA of Clusters for Maximum Amplitude of Inventory Workforce Model

ANOVA Test Results		
Parameter Name	F Value	P-Value
StandardWorkweek	41.3079228	0.00
InvntAdjstTime	10.230399	0.00
SafetyStockCoverage	9.7092925	0.00
MnfctrngCycleTime	7.601395	0.00
LaborAdjstTime	3.8225064	0.01
Time2AvgOrderRate	3.0480612	0.02
WIPAdjst	1.9001941	0.11
VacancyCancelTime	1.6288866	0.17
Time2FillVacancy	1.3109803	0.27
MinimumOrderProcssing	0.8177879	0.52
AvgDuratnEmploy	0.6499838	0.63
Productivity	0.3459979	0.85
VacancyAdjstTime	0.3363572	0.85
AvgLayoffTime	0.1653198	0.96

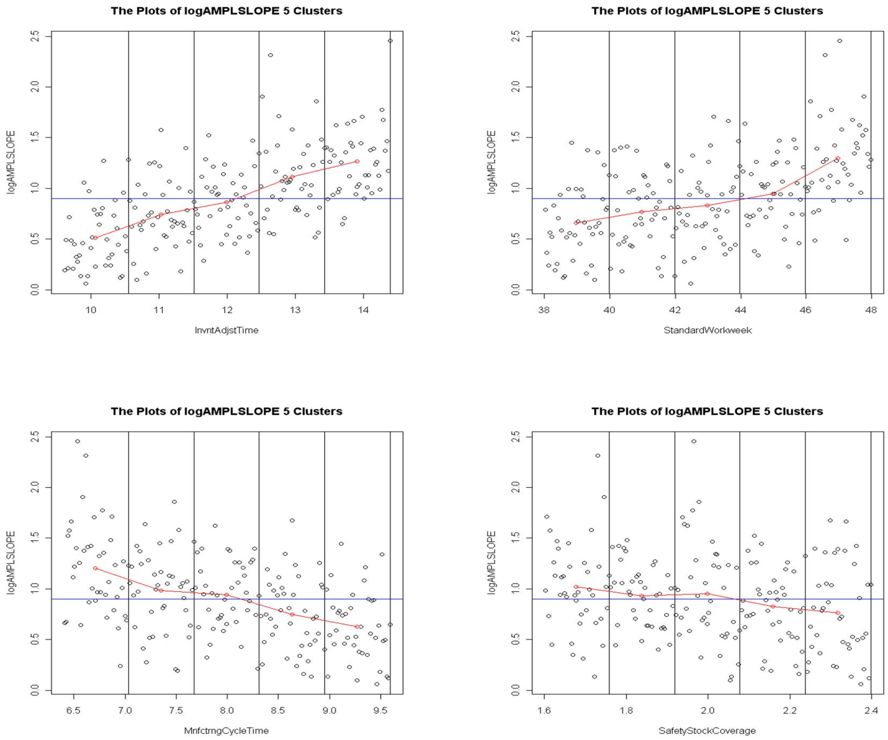


Figure A.10. Some Clustered Scatter Plots for Log-Amplitude Slope and Parameters of Inventory Workforce Model by Sterman (2000)

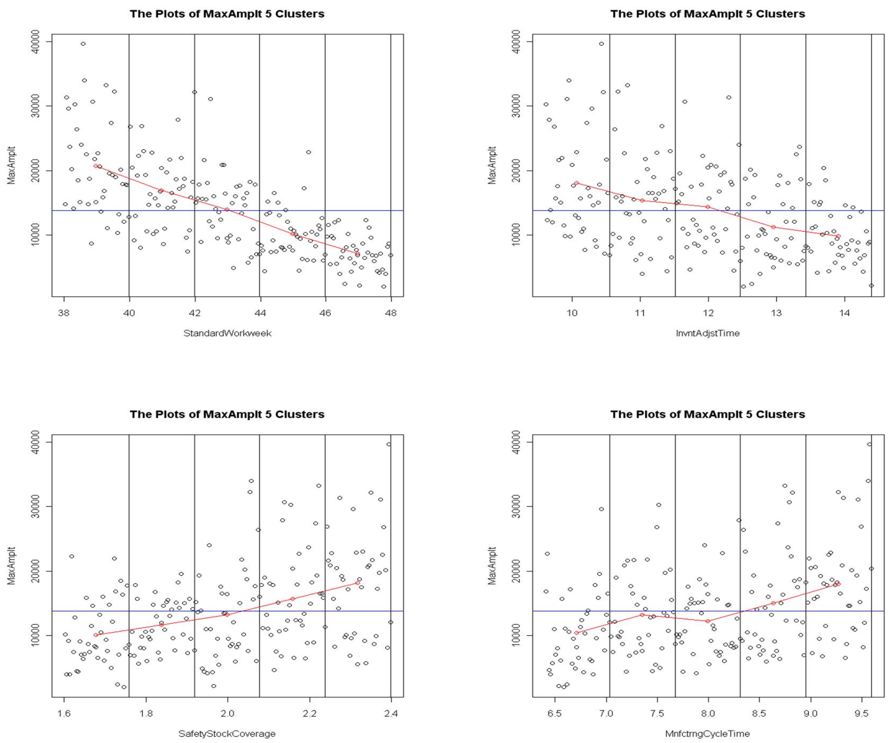


Figure A.11. Some Clustered Scatter Plots for Maximum Amplitude and Parameters of Inventory Workforce Model

APPENDIX B: REGRESSION MODELS INCLUDING INTERACTION TERMS

Table B.1. Results of Regression Model Including Two-Way Interaction Terms for Maximum Amplitude of Simple Supply Line Model

Coefficients					
	Regression Coefficients			t value	Sig.
Model	beta (β)	Std. Error	Std. Coef (b)		
(Constant)	1.304	.087		15.012	.000
StockAdjst	-.338	.028	-.618	-12.212	.000
AcqLag	.043	.006	.347	6.695	.000
Acqlag.Suplineadjst	.006	.001	.277	5.107	.000
SupLineAdjst	.089	.020	.244	4.462	.000
Stockadjst.Suplineadjst	-.019	.006	-.181	-3.317	.001
Acqlag.Stockadjst	-.003	.002	-.088	-1.729	.085

Table B.2. Results of Regression Model Including Three-Way Interaction Term for Maximum Amplitude of Simple Supply Line Model

Coefficients					
	Regression Coefficients			t value	Sig.
Model	beta (β)	Std. Error	Std. Coef (b)		
(Constant)	1.473	.071		20.614	.000
StockAdjst	-.407	.020	-.745	-20.477	.000
Acqlag.Suplineadjst	.010	.002	.453	5.922	.000
AcqLag	.028	.008	.226	3.455	.001
Acq.Stckadjst.Suplinadjt	-.002	.001	-.218	-3.216	.002
SupLineAdjst	.043	.014	.118	3.090	.002
Acqlag.Stockadjst	.003	.003	.084	1.092	.276

APPENDIX C: R GUI CODES THAT ARE USED IN ANALYSES

C.1. Code for ANOVA of Clusters

```
Ftst<- function(x1,m,plt=FALSE,dname,opt=F){
#This function make F test for each variable pair.
#First Column includes independent variable

#m: number of cluster
#dname: name of the dependent variable
#plt: option for plotting the clustered scatter plots
#opt: option for getting F statistic value as output

rss<- 0
avgcls<- 0 #vector keeping cluster averages
clstrnum<- 0 #vector keeping number of values in each cluster
x2<- as.data.frame(x1) #ensures x2 is in data frame format
n<- length(x1[,1]) #number of observations in data set
k<- floor(n/m) #number of observations in each cluster
resid<- n%%m #residual terms to be divided into several clusters
z<- 1 #starting index for clustering

nm<- names(x2) #Take original name to another vector

names(x2)<- c("V1","V2") #Give generic names to vector columns
x2<- x2[order(x2$V1),]

for(i in 1:m){
if(resid==0)
{
avgcls[i]<- mean(x2[c(z:(z+k-1)),2]) #find cluster means
clstrnum[i]<- length(x2[c(z:(z+k-1)),2]) #find var. numbers in clusters
z<- z+k #varaible index for clustering
}
if(resid!=0)
{
avgcls[i]<- mean(x2[c(z:(z+k)),2])
clstrnum[i]<- length(x2[c(z:(z+k)),2])
resid<- resid-1
z<- z+k+1
}
}
```

```
F1<- (sum(clstrnum*(avgcls^2))-n*mean(x2[,2])^2)/(m-1)
F2<- (sum(x2[,2]^2)-k*sum(avgcls^2))/(n-m)

F<- F1/F2 #F statistic

if(plt)
{
tx<- paste(paste("The Plots of",dname),paste(m,"Clusters"))
plot(x2[,1],x2[,2], xlab=nm[1],main=tx,ylab=dname)
abline(v=x2[c(1:m)*k,1])
points(x2[(c(1:m)*k-k/2),1],avgcls,col="red",type="o",pch=8)
abline(h=mean(x2[,2]),col="blue")
}

rss<- 1-pf(F,(m-1),(n-m))
if(opt==T)
rss<- c(1-pf(F,(m-1),(n-m)),F)

rss
}



testall<- function(X,m=5,plt=FALSE,opt=F){
#This function uses the Ftst function above and make F test for all parameters
#FIRST COLUMN SHOULD BE DEPENDENT VARIABLE
#m keeps the number of clusters that is used
#plt: option for plotting the clustered scatter plots
#opt: option for getting F statistic value as output
#m: number of clusters to be used
#X: data table including all parameters and dependent

X<- as.data.frame(X)
ord<- 0
n<- length(X)
nm<- names(X) #KEEPS THE NAME OF VARIABLES
res<- 0 #KEEPS THE RESULTS OF TESTS
output<- 0 #KEEPS RESULTS TABLE
print(paste("Test for variable",nm[1]))
print(paste("Cluster number",m))

resmat<- matrix(c(rep(0,2*(n-1))),ncol=2) #KEEPS RESULTS OF F TEST

for(i in 2:n)
{
x<- data.frame(X[,i],X[,1])
names(x)<- nm[c(i,1)]
res[(i-1)]<- Ftst(x,m,plt,nm[1],opt=F)
```

```
if(opt==T)
resmat[(i-1),]<- Ftst(x,m,plt,nm[1],opt)
if((plt==T)&(i!=n))
windows()
}
if(opt==F)
ord<- order(res) #SORT RESUTLS ACCORDING TO P VALUES
nm<- nm[-1]
print(data.frame(nm[ord],res[ord]))
if(opt==T)
{
ord<- order(resmat[,2],decreasing=T) #SORT RESULTS ACCORDING TO F STATISTICS
print(data.frame(nm[ord],res[ord],resmat[ord,2]))
output<-data.frame(nm[ord],res[ord],resmat[ord,2])
}
output
}
```

C.2. Log-Amplitude Slope Estimation Procedure For Oscillatory Models

```
remov<- function(x){
n<- length(x)
z<- x[n]
i=1
while(x[i+1]<x[i]) i=i+1
while(x[i+1]<z) i=i+1
x[-c(1:i)]
}

period<- function(y,m=250,ploty=F,rmv=F){
yx<- remov(y)
if(rmv)
acrr<- 0
acrr<- acf(yx,lag.max=m,plot=F)$acf
i<-1
while(yx[i+1]>yx[i]) i<-i+1
amppoint<-i

i=1
while(acrr[i+1]<acrr[i]) i<-i+1
per<- (i-1)*2

if(ploty){
plot(yx)
abline(v=amppoint,col="red")
for(i in 1:10)
```

```
abline(v=(i*per+amppoint),col="red")}
c(per,amppoint)
}

amplseq<- function(x){
per<- period(x)[1]
amp<- period(x)[2]
ampl<- 0
tt<- remov2(x)
vl<- which(min(remov2(x)[1:(amp+per)])==tt)
for(i in 1:5)
ampl[i]= max(tt[(1+per*(i-2)*(i>1)+(i>1)*vl):(vl+per*(i-1))])
ampl
}
```

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