A STUDY ON A THERMAL POWER PLANT MODELLING AND CONTROL

by

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ABSTRACT

In this study, a deterministic mathematical model of Anbarlı Thermal Power Plant is developed from physical laws. Resultant model is used to design the steady state regulator for 100.5 MW load level using linear optimal control theory. Although the resultant model is primarily designed for steady state control, it can be easily adapted for other Thermal Power Plant configurations and control studies.

Computer simulations are carried out to investigate the dynamic behaviour of the controlled system under disturbances in generation.

The simulation results indicate the feasibility of applying linear optimal control to a Thermal Power Plant based on minimization of a quadratic cost functional related to state variables and control inputs.

BİR TERMİK GÜÇ SANTRALININ MODELLENMESİ VE KONTROLU ÜZERİNE ÇALIŞMA

ÖZET

Bu çalışmada, fiziksel kanunlar yardımıyla Anbarlı Termik Güç santralının matematiksel modeli çıkarıldı.

Elde edilen bu model, lineer optimal kontrol teorisi yardımıyla 100.5 MW yük seviyesi için kararlı hal regülatörünün tasarımını yapmakta kullanıldı. Bu model öncelikle kararlı hallerin kontrolü için tasarlanmasına rağmen, bu model diğer termik güç santrallerine ve kontrol çalışmalarına kolaylıkla uygulanabilir.

Üretimdeki denge bozucu etkiler altında kontrol edilen sistemin dinamik davranışlarını gözlemek amacıyla bilgisayarda benzeşim çalışmaları yapıldı.

Bu benzeşim çalışmaları sonunda, hal değişkenleri ve kontrol girdilerine bağlı olan quadratik değer fonksiyonunun minimizasyonuna dayanan lineer optimal kontrol teorisinin kolaylıkla bir termik güç santraline uygulanabilirliği ortaya çıktı.

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$\underline{\underline{A}}$	Linear plant model system matrix
A_d, A_r	Total cross sectional area of downcomer and riser tubes respectively
A_v	Total governor valve area
A_{rr}, A_{ps}, A_{ss}	Cross sectional area of reheater tubes, primary superheater tubes, secondary superheater tubes respectively
$\underline{\underline{B}}$	Linear plant model control matrix
C	Nozzle coefficient
C_F	Calorific value of fuel
$C_r, C_{ps}, C_{ss}, C_{rr}$	Heat capacitance for riser tubes, primary superheater tubes, secondary superheater tubes and reheater tubes respectively
D_d, D_r	Total diameter of downcomer and riser tubes respectively
D_{ps}, D_{rr}, D_{ss}	Diameter of primary superheater tubes, reheater tubes, secondary superheater tubes
f_d, f_r	Friction coefficients of downcomer and riser tubes respectively
f_2, f_3, f_4	Friction loss factor of primary superheater, secondary superheater and reheater respectively
$\underline{\underline{G}}$	Feedback gain matrix
h_c	Spray water enthalpy
h_o	Mixture enthalpy in the riser
h_w	Liquid phase enthalpy in the drum
h_{ex}	L.P.T exhaust enthalpy
h_{fw}	Feedwater enthalpy

h_s, h_{ws}	Vapour and liquid enthalpy in the drum respectively
h_{ps}, h_{ro}, h_{ss}	Outlet steam enthalpy of primary superheater, reheater, secondary superheater respectively
h_{hp}	H.P.T exhaust enthalpy
h_{ds}	Desuperheater outlet steam enthalpy
J	Cost functional
K_{hp}	H.P.T power output proportional constant
K_{lp}	L.P.T flow coefficient
k_e	Evaporation constant
k_1, k_2, k_3, k_4	Tube to working fluid heat transfer coefficien[t] for riser, primary superheater, secondary superheater, reheater respectively
L_d, L_r	Total length of downcomer and riser tubes respectively
L_{dl}, L_{rl}	Length of one downcomer and riser tube respectively
L_{ps}, L_{ss}, L_{rr}	Length of primary superheater tubes, secondar[y] superheater tubes and reheater tubes respectively
$M_r, M_{ps}, M_{rr}, M_{ss}$	Total metal mass of riser, primary superheater, reheater, secondary superheater
MWhp	H.P.T power output
MWlp	L.P.T power output
m_c	Spray mater flowrate
m_e	Evaporation mass flow rate
m_f	Fuel flow rate
m_o	Flow rate through the riser
m_s	Drum steam flow rate
m_t	Steam flow through the governor valve
m_w	Water flow through the downcomer
m_{ex}	L.P.T exhaust flow

m_{ps}, m_{ss}	Steam flow through primary superheater, secondary superheater
m_{ri}, m_{ro}	Steam inlet flow of reheater and L.P.T
m_{hp}	Steam outlet flow of H.P.T
$\underline{\underline{P}}$	Solution of matrix Riccati differential equations
P_d, P_{md}	Pressure in the steam drum and mud drum respectively
$P_{ps}, P_{ss}, P_{hp} P_{ro}$	Pressure at the outlet of primary superheater, secondary superheater, H.P.T, reheater respectively
$\underline{\underline{Q}}$	State penalization matrix
$\underline{\underline{R}}$	Control penalization matrix
T_s, T_w	Drum saturation and liquid temperature
$T_{m1}, T_{m2}, T_{m3}, T_{m4}$	Metal temperatures of riser, primary superheater, secondary superheater, reheater respectively
T_{ps}, T_{ro}, T_{ss}	Steam temperatures at the outlet of primary superheater, secondary superheater, reheater
t_f	Terminal time
u	Control input vector
$\underline{U}_{ss}$	Values of control inputs at steady state
$\underline{X}$	Plant state vector
X_o	Steam quality through the riser
$\underline{X}_{ss}$	Values of state variables at steady state
V_d	Drum volume
V_w	Liquid volume in the drum
$V_r, V_{ps}, V_{ss}, V_{rr}, V_{hp}$	Storage volume of riser tubes, primary superheater tubes
W_t	Total power output of turbines
$\gamma_1, \gamma_2, \gamma_3, \gamma_4$	Rate of heat absorbtion of riser metal, primary superheater metal, secondary superheater metal and reheater metal respectively

$\theta_{g1}, \theta_{g2}, \theta_{g3}\ \theta_{g4}$	Heat absorbed by riser metal, primary superheater metal, secondary superheater metal and reheater metal
$\theta_{r}, \theta_{ps}, \theta_{ss}, \theta_{rr}$	Tube to flowing steam heat transfer rate in riser, primary superheater, secondary superheater and reheater
$\rho_{o}, \rho_{s}, \rho_{dw}$	Density of water-steam mixture, saturated vapor and saturated liquid respectively
$\rho_{ps}, \rho_{ss}, \rho_{hp}, \rho_{ro}$	Steam density at the outlet of primary superheater, secondary superheater, H.P.T and reheater respectively

I. INTRODUCTION

In the recent years, Hydro electric power plants have been running as load followers while Thermal Power Plants have been used to meet the base load. A drastic change of these conditions is predicted in the coming years. This is mainly due to fact that nearly all available water power has now been utilized in some parts of the world. However demand for power, continue to rise and the dominating role of the Hydro Electric Power Plants in power systems will rapidly diminish. It is inevitable to use thermal power plants as load followers although they were not designed for this purpose.

Dynamic model of these plants must be known

- to be able to judge the possible contribution of thermal power plants to the existing grid system,
- the indicate the limitations of the existing plants from the point of view of the new operating requirements,
- to design regulators for steady state and large load changes.

There are mainly two approaches to the model building

i) Physical laws,

ii) Process identification.

Models in works (1,2,3) have been developed by using the physical laws while in (4) two different models have been built by using both modelling techniques and it has been proposed to combine the resultant models to get the improved model.

Dynamic analysis of a drum type marine boiler has been considered in (1). This study can be considered the first valuable attempt to the power plant modelling although the boiler studied is too simple to be répresentative of a power plant.

Both works in (2,3) represent conmplete dynamic models of a thermal power plant. Furthermore, step responses of dynamic models to main inputs such as to fuel flow and governor valve position have been studied.

In these works, only a power plant mathematical model is developed. No attempt is made to study the dynamic control of the power plant.

Dynamic control of a thermal power plant is intended to minimize transient fluctuations in plant variables such as pressures and temperatures which are caused by variations in load demand or other disturbances in generation. As the operating temperatures and pressures and already high, it is essential that these transients should be kept at a minimum level so as to avoid any plant component damage and shut down.

In the existing thermal power plants, the dynamic control of Boiler-Turbine unit is based on the use of independent conventional controllers to maintain the plant at a set point. These regulators are inadequate for load tracking, even for steady state control, due to seperate time constants. The optimally designed regulators based on multivariable control theory result in superior performance as reported in (5) which compares the performance of the power plant nonlinear model with conventional and optimal controllers related to the structure of the equivalent linearized model. The deterministic optimal linear controllers require

i) Perfect process model,

ii) Measurable state variables and process disturbances.

However, recent developments in optimal control theory make it possible to design regulators in the absence of a perfect model, and for the cases of unmeasurable states and disturbances.

For the case of unmeasurable process disturbances, an observer can be used as in (6) to reconstruct the state from measured outputs or the problem can be formulated as a stochastic control problem.

An approach to control system design has been offered in (7) based on optimal linear regulator theory and which includes the model inaccuracies. This approach has been applied to controller design of a Coal Fired Thermal Power Plant of which mathematical model has been developed in (8). This control system produces an integral type action which guarantees zero steady state error not requiring complete state feedback.

In this study, as an extension of the earlier research, a deterministic mathematical model of Anbarlı Thermal Power Plant is developed and the resultant model is used to design steady state controllers based on linear optimal control theory.

In Chapter II, the characteristics of the unit no.3 of Anbarlı Thermal Power Plant and the process are described.

In Chapter III, the describing equations of each plant subsection are obtained by using the physical laws, state relations and Acceptance Test Data of the unit with the general assumptions made in power plant modelling literature. The resultant model is not only applicable for steady state control of Anbarlı Thermal Power Plant but also other applications and different power plant configurations.

The state variables are selected and the dynamic nonlinear plant model is developed accordingly in Chapter IV.

In Chapter V, energy optimal time invariant state regulator problem is encountered having a deep insight of different power plant control strategies and conventionaly designed existing control system of unit no.3 of Anbarlı. A solution algorithm for the linear regulator problem is developed.

A flexible and convenient computer program is developed to design and analyze Anbarlı Thermal Power Plant controls. Description of this computer program and the computation algorithm with the system main inputs and outputs are given in Chapter VI.

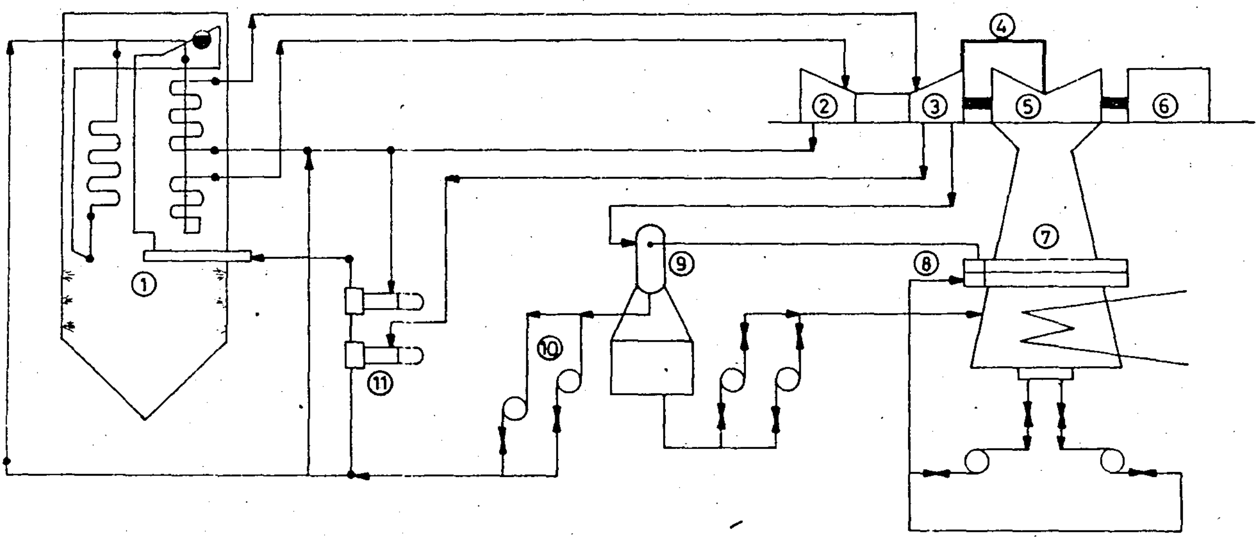
In order to investigate the dynamic performance of the controlled system due to changes in throttle density and enthalpy, a series of computer simulations are carried out. The simulation results are presented and discussed in Chapter VII.

Chapter VIII, concluding this study, resumes the basic results and gives recommendations for further studies to improve the modelling and control of Thermal Power Plants.

II. PLANT DESCRIPTION

2.1 DESCRIPTION OF THE UNIT NO.3 OF ANBARLI THERMAL POWER PLANT

Oil fired Anbarlı Thermal Power Plant consists of 5 separate units with total nated power of 630. MW. In this study, unit no.3 is considered for the modelling and control purposes. This unit is a 110 MW. system with rated steam conditions of 12.56 MPa, 540°C at the throttle and 540°C reheat. The Boiler is rated at 362,000 kg of steam per hour and is of the natural circulation drum type. The turbine is a tandem compound, single reheat, double exhaust condensing type with a rated capacity of 110 MW. at 3000 rpm. The following figure shows the basic components and the circuit diagram of the unit under consideration.



① Boiler
② High pressure turbine (H.P.T.)
③ Intermediate pressure turbine (I.P.T.)
④ Cross-over piping
⑤ Low pressure turbine (L.P.T.)
⑥ Generator
⑦ Condenser
⑧ Low pressure heaters
⑨ Deaerator
⑩ Boiler feed pumps
⑪ High pressure heaters

FIGURE :- 2.1 - The basic plant components and the circuit diagram of Anbarlı unit no:3

The unit operates in the combined reheat-regenerative cycle. Fig (2.2) shows the ideal T-s diagram of the process. Since the mass of the steam flowing through the various components are not the same, T-s diagram only shows the state of the working fluid at the various points.

During the process, feedwater from High Pressure Heater I, at state 17, is heated in the Boiler to superheated state 1. Steam, at state 1, enters High Pressure Turbine and expands to state 2 doing some amount of work. After expansion to state 2, some amount of steam is extracted for High Pressure Heater I. The amount of steam that is not extracted enters again into the boiler and is reheated to state 3.

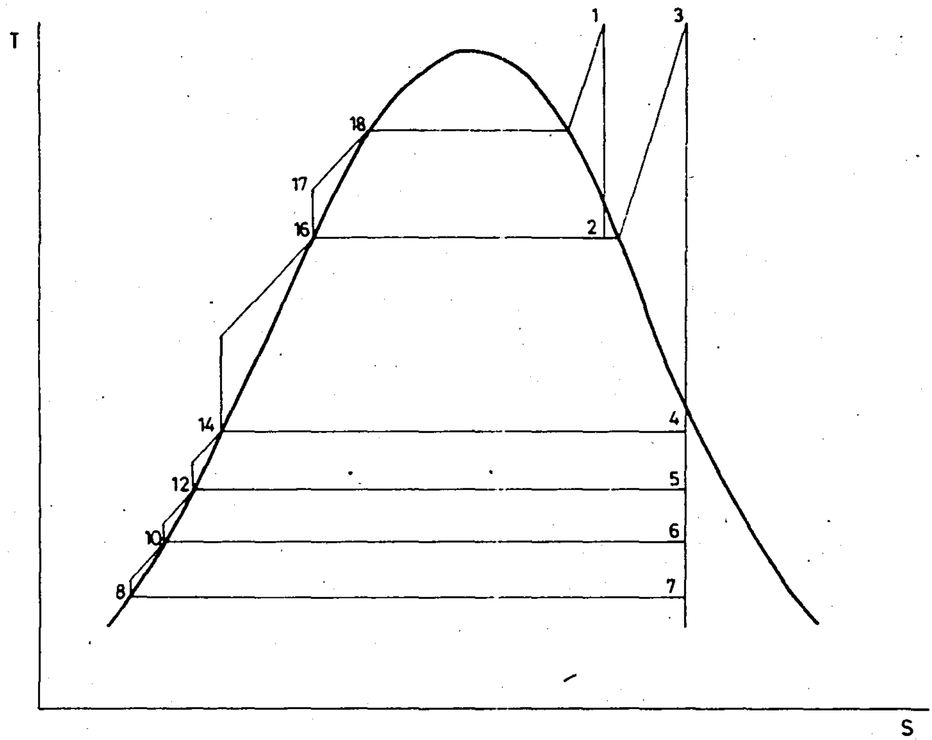


FIGURE : 2.2_ T_s diagram of the process.

Reheating of the steam is necessary to decrease the moisture content in the Intermediate and Low Pressure Turbine Stages. Steam, at state 3, expands to state 7 doing work in both Intermediate and Low Pressure Turbines. During the final expansion from state 3 to state 7, steam is extracted from two points in the I.P.T. for the High Pressure Heater II and for the dearator where the heating and removing of air from feedwater takes place, and from two points in the L.P.T for low Pressure Heater. Steam that is not extracted is condensed in the Condenser to state 8 and is pumped into low Pressure Heater. Points 10,12, 14 and 16 on the T-s diagram show the state of the mixture of the condensate and extracted steam in low Pressure Heater, Deaerator, High Pressure Heater II and High Pressure Heater I respectively. With this type of power cycle, the unit thermal efficiency is increased up to % 89.6.

2.2 BOILER CHARACTERISTICS AND OPERATION

Drum type natural circulation boiler consists of a single cylindrical steam drum, a mud drum, three superheaters of which two of them in the convective zone, whereas the other in the radiant zone and two desuperheater. At the normal operating conditions, the reheat desuperheater does not perform any control action. The simplified diagrams of the boiler with the main and reheat steam lines, and the boiler drum system are given in Fig. (2.3) and Fig. (2.4) respectively.

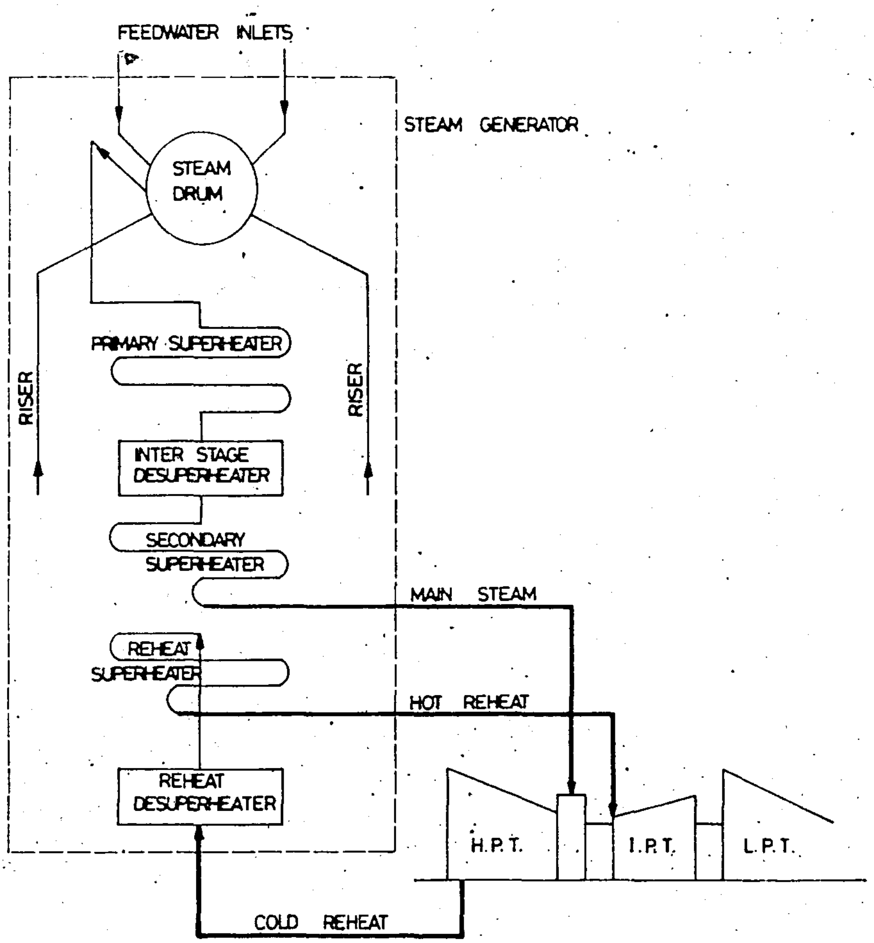


FIGURE: 2.3 The simplified diagram of the boiler with the main and reheat steam lines.

slightly subcooled feedwater from the High Pressure Heater I flows through the economizer and enters the Steam Drum where the saturated conditions exist.

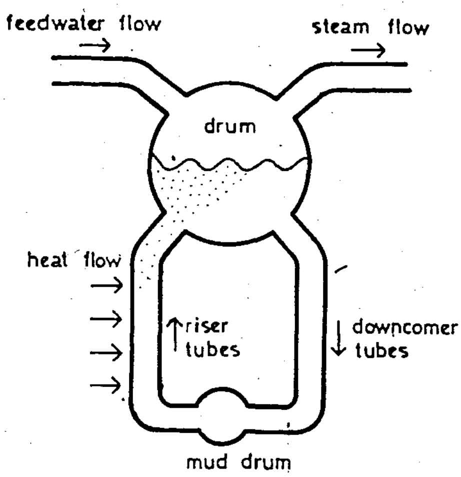


FIGURE: 2.4_ The simplified diagram of the drum system.

Water at saturated conditions flows from the Steam Drum to Mud Drum through the unheated downcomer tubes. The water in the Riser tubes, on its return to the steam Drum, absorbs heat and becomes a mixture of water and vapour. The density of the mixture is less than the density of the water in the downcomer tubes. The difference in the densities sets up a constant natural circulation that discharges water-vapor mixture into the steam Drum where the saturated steam is separated from the water. Saturated steam from the steam Drum absorbs heat, first in the Primary superheater then in the Secondary Superheater which are placed in the convective zone of the Boiler, and leaves the Boiler at a temperature of about 540°C. Between the two superheaters, there exists a desuperheater which controls the secondary superheater inlet temperature by spraying water in to steam in order to avoid the overheating of the overheating of the secondary superheater tubes and High Pressure Turbine front stages.

The flow of steam from secondary superheater passes through normally open throttle valves and governing valves and enters the H.P.T. In the H.P.T, steam is expanded to a certain intermediate pressure and fed again in to the Boiler where it absorbs heat while passing through the reheater placed in the radiant zone of the Boiler. From the Reheater, steam enters the I.P.T and L.P.T turbine group for final expansion.

2.3 TURBINE CHARACTERISTICS

The unit is a 110. MW, tandem compound, two casing reheat, double flow exhaust condensing turbine directly connected to a main generator rotating at 3000 rpm. The Turbine unit has been designed to operate with 12.56 mPa-540°C throttle steam with single reheat to 540°C and exhausti at 1.5"Hg. Abs.

The unit consists of a High pressure turbine, an Intermediate Pressure Turbine and a double flow low pressure Turbine operating on a regenerative cycle with five stages of feedwater heating. The High Pressure Turbine is of a impulse type turbine whereas the Intermediate and low Pressure Turbines are of reaction type turbines.

In the next chapter, each plant section is considere in detail.

III. MATHEMATICAL MODEL BUILDING

3.1 GENERAL REMARKS ON MODEL BUILDING

In this study, the mathematical model of unit no.3 of Anbarlı Thermal Power Plant is developed. The model is primaril designed for steady state control of Boiler-Turbine group. However the model can be conveniently adapted to other applications and to different power plant configurations. For the wide range of applicability of the resultant model and for the convenience of the analysis, the unit is divided into 9 subsections. For each subsection, the laws governing the transfer of energy, momentum, mass and heat transfer equations are applied with the properly selected state relations for the required thermal variables.

The unit Acceptance Test Data (A.T.D) (9) is taken into account during the modelling whenever a model parameter can not be determined from the physical laws.

The plant subsections are;

1) Boiler

i) Downcomer

ii) Riser

iii) Steam drum

iv) Primary superheater

v) Desuperheater

vi) Secondary superheater

vii) Reheater

2) High Pressure Turbine (H.P.T.)

3) Intermediate and Low Pressure Turbine (L.P.T)

During the modelling, the following general assumptions are made in addition to the specific assumptions corresponding to each individual subsections, expressed in the related subsections;

i) Flue gas dynamics is neglected. Because the time constants involved are shorter than the other process lags,

ii) The economizer and the feedwater heaters dynamics are neglected in order to have low order model and to avoid computational difficulties,

iii) The properties of the working fluid is assumed to be variable only in the axial direction. This assumption reduces the P.D.E's of motion to O.D.E's and simplifies the numerical analysis of the system,

iv) Steam flow is considered to be incompressible,

v) Dynamics of actuators and sensors are neglected.

3.2 THE DOWNCOMERS

The component under consideration is shown in Fig (3.1) and the variables used are indicated.

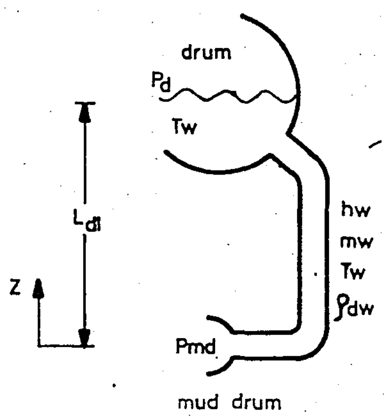


FIGURE: 3.1_ The simplified diagram of the downcomer tubes.

The downcomers are several parallel coupled tubes located on the exterior of the furnace walls and can be treated as one tube.

The following assumptions are made;

i) No heat is supplied to the downcomers,

ii) Liquid temperature is the same as the drum liquid temperature, T_w,

iii) the downcomer flow is incompressible.

Assumptions (i) and (iii) lead us to apply only the momentum equation with friction losses due to pipe, bends and mud drum entrance.

That is;

$$\frac{1}{g}(P_d - P_{md}) = (f_d \frac{L_d}{D_d} + \varepsilon_1 + \varepsilon_2 + 1)\frac{m_w^2}{2g A_d^2 \rho_{dw}} - \rho_{dw} L_{d1} + \frac{L_{d1}}{g A_d}\frac{dm_w}{dt} \quad (3.1)$$

3.3 RISER

In figure (3.2), the component under consideration is shown and the variables used are indicated.

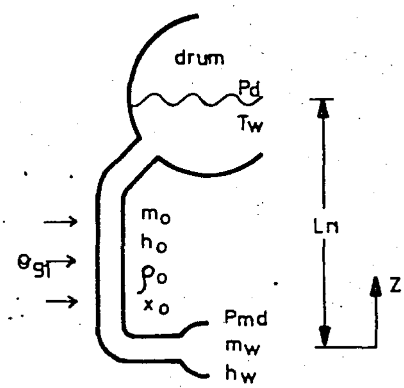


FIGURE: 3.2. The simplified diagram of the riser tubes.

Again the parallel tubes which create the furnace walls are treated as one tube. The following assumptions are made;

i) There is always saturation state in the risers,

ii) Steam quality, x_o, and the density of the water-vapour mixture, ρ_o, are assumed to be constant within the riser tubes,

iii) There is no velocity difference between the vapour and liquid phases in the risers,

iv) Drum system circulation is natural.

Thus, the following equations may be written;

continuity equation:

$$m_w - m_o = V_r \frac{d}{dt}(\rho_o) \qquad (3.2)$$

where ρ_o, mixture density, is given by

$$\rho_o = \frac{\rho_{dw}\rho_s}{(\rho_{dw} - \rho_s)X_o + \rho_s} \qquad (3.3)$$

Energy equation applied to the flow in the risers yields:

$$\theta_r + m_w h_w - m_o h_o = V_r \frac{d}{dt}(\rho_o h_o) \qquad (3.4)$$

where h_o, mixture enthalpy, is given by

$$h_o = h_{ws} + X_o(h_s - h_{ws}) \qquad (3.5)$$

and θ_r which is the heat transfer rate from risers walls to 2- phase flow, is given by the following empirical relation (10)

$$\theta_r = k_1(T_{m1} - T_s)^3 \qquad (3.6)$$

And the energy equation applied to the riser tubes yields:

$$\theta_{g1} - \theta_r = M_r C_r \frac{d}{dt}(T_{m1}) \qquad (3.7)$$

where θ_{g1} is the heat absorbed by the riser tubes due to convection and radiation. It can be taken as proportional to the heat supplied to the boiler. Therefore

$$\theta_{g1} = \gamma_1 C_f m_f \tag{3.8}$$

Applying the momentum equation and adding the friction losses due to the pipe, bends, entrance and exit; one can obtain

$$\frac{1}{g}(P_{md} - P_d) = (f_r \frac{L_r}{D_r} + 1)\frac{m_o^2}{2A_r^2 g \rho_o} - \frac{m_w^2}{2gA_d^2 \rho_{dw}} + \varepsilon_3 \frac{m_o^2}{2gA_r^2 \rho_o}$$

$$+ \varepsilon_4 \frac{m_w^2}{2gA_r^2 \rho_{dw}} + \rho_o l_{r1} + \frac{l_{r1}}{gA_r}\frac{dm_o}{dt} \tag{3.9}$$

The proportional constants k_1, γ_1 will be obtained from steady state values of θ_r and θ_{g1} respectively.

3.4 THE STEAM DRUM

Fig (3.3) illustrates the simplified diagram of the drum and indicates the variables used.

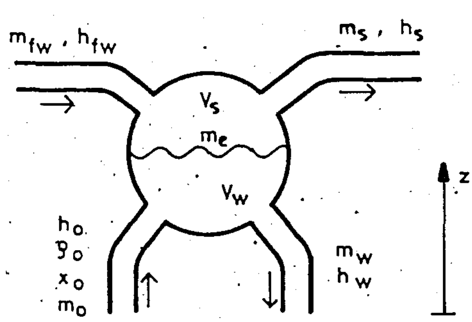


FIGURE : 3.3 The simplified diagram of the steam drum.

Saturated liquid leaves the drum and enters the downcomers while a saturated liquid-vapor mixture enters the drum from the risers. Saturated steam leaves the drum and enters the primary superheater.

The temperatuie ou the steam in the drum is assumed to be equal to the saturation temperature, T_s, corresponding to the drum pressure, P_d. Whereas T_w, the temperature of the liquid phase in the drum, is determined by the coming feedwater temperature accordingly. The feedwater temperature can be assumed to be constant.

Since the water in the drum exists in two phases, namely vapor and liquid, both the energy and continuity equations are applied to the liquid phase and only the continuit equation is applied to the vapour phase.

It was possible to define the control volume to be consisting of the whole steam drum, but the resultant model would be far from to represent evaporation and condensation mechanisms in the drum which considerably affects the drum steam outlet flow.

Continuity equation for the liquid phase;

$$(1-X_o)m_o + m_{fw} - m_w - m_e = \frac{d}{dt}(V_w \rho_{dw}) \tag{3.10}$$

Energy equation for the liquid phase;

$$(1-X_o)m_o h_{ws} + h_{fw} m_{fw} - h_w m_w - h_s m_e = \frac{d}{dt}(V_w \rho_{dw} h_w) \tag{3.11}$$

And continuity equation for the vapor phase yields

$$X_o m_o + m_e - m_s = \frac{d}{dt}(V_s \rho_s) \tag{3.12}$$

The steam drum contains both saturated vapor and subcooled liquid with changing vapor pressure and saturation temperature. Evaporation and condensation will take place at the drum liquid phase to maintain a temperature balance at the liquid surface. m_e represents this evaporation or condensation rate in the drum and it can be taken as proportional to the temperature difference between the liquid temperature and the saturation temperature (5), Thus;

$$m_e = k_e(T_w - T_s) \tag{3.13}$$

Proportional constant, ke, will be found from the steady state valves of m_e.

The state of the steam-water mixture in the riser and of the vapour phase in the drum, is the saturation state. Therefore only one indepent variable is sufficient to define any thermal property. Choosing saturated steam density, ρ_s, as independentvariable, the drum pressure, p_d, saturation temperature, T_s, saturated vapor enthalpy, h_s, saturated liquid enthalpy, h_{ws} and saturated liquid density, ρ_{dw} can be obtained from the following state relations in the forms of;

$$T_s = f(\rho_s) \tag{3.14}$$

$$h_s = f(\rho_s) \tag{3.15}$$

$$h_{ws} = f(\rho_s) \tag{3.16}$$

$$\rho_{dw} = f(\rho_s) \quad (3.17)$$

$$P_d = f(\rho_s) \quad (3.18)$$

The liquid phase in the drum is slightly undercooled. Neglecting the pressure dependence of the enthalpy, the liquid phase enthalpy can be expressed as a function of the drum liquid temperature. Hence

$$h_w = f(T_w) \quad (3.19)$$

3.5 PRIMARY SUPERHEATER

This convection type superheater can be assumed to be a cross flow heat exchanger. The tubes are represented by one tube with constant dimensions throughout the whole length. The following figure shows the component and indicates the variables used.

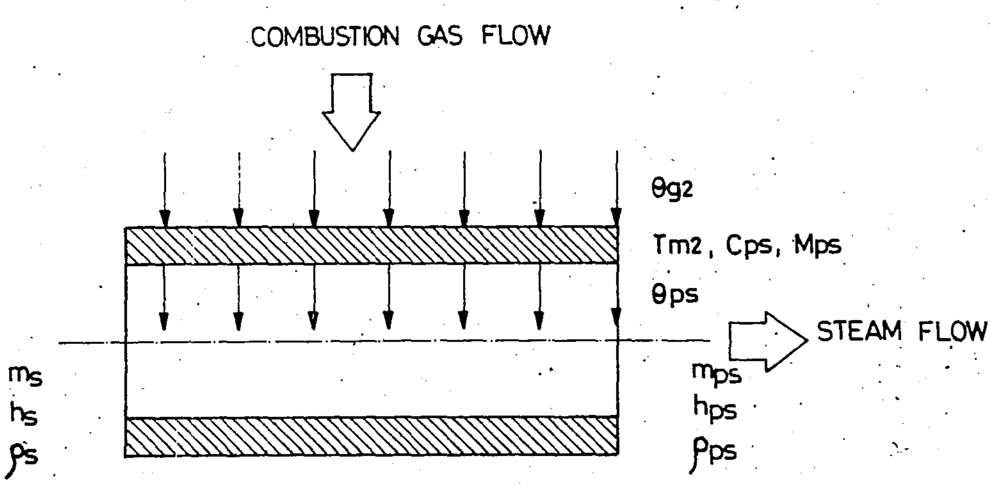


FIGURE: 3.4_ One tube representation of the primary superheater.

The simplifying assumptions considered here are;

i) There is no heat transportation in the axial direction neither in the tube nor in the steam,

ii) Heat conductivity of the tube material is infinite,

iii) The pressure drop takes place before the heat transfer section.

The following equations may be written under these assumptions;

momentum equation:

$$P_d - P_{ps} = f_2 \frac{m_s^2}{\rho_s} \tag{3.20}$$

continuity equation:

$$m_s - m_{ps} = V_{ps} \frac{d}{dt} (\rho_{ps}) \tag{3.21}$$

Energy equation for the steam side:

$$\theta_{ps} + m_s h_s - m_{ps} h_{ps} = V_{ps} \frac{d}{dt} (\rho_{ps} h_{ps}) \tag{3.22}$$

And energy equation for the tube material:

$$\theta_{g2} - \theta_{ps} = M_{ps} C_{ps} \frac{d}{dt} (T_{m2}) \tag{3.23}$$

where θ_{ps} is the turbulent heat transfer rate from the tube material to flowing steam in the primary superheater, expressed by (3)

$$\theta_{ps} = k_2 (m_s)^{0.8} (T_{m_2} - T_{ps}) \quad (3.24)$$

and θ_{g2} is the heat transfer rate from the combustion products to the tube material. It can be related to the heat input to the boiler as follows,

$$\theta_{g2} = \gamma_2 C_F m_F \quad (3.25)$$

The constants f_2, k_2, γ_2 will be obtained from eqn's (3.20), (3.24) and (3.25) at the specified steady state operating conditions. The purpose of defining such constants is to avoid themodel complexity and to represent the real process at steady state.

Since the flowing steam is in the superheat region, any intensive properties of the steam can be expressed by two independent thermal variables. The operating variables T_{ps} and P_{ps} can be determined from the state relations in the forms of,

$$P_{ps} = f(\rho_{ps}, h_{ps}) \quad (3.26)$$

$$T_{ps} = f(\rho_{ps}, h_{ps}) \quad (3.27)$$

3.6 DESUPERHEATER

The temperature control is accomplished by varying the desuperheater flow which removes the energy from the superheated steam to avoid the overheating of the secondary superheater tubes and High Pressure Turbine front stages. Water supplied to the desuperheater is taken from the outlet of the deareator.

In Fig (3.5) the component under consideration is shown and the variables used are indicated.

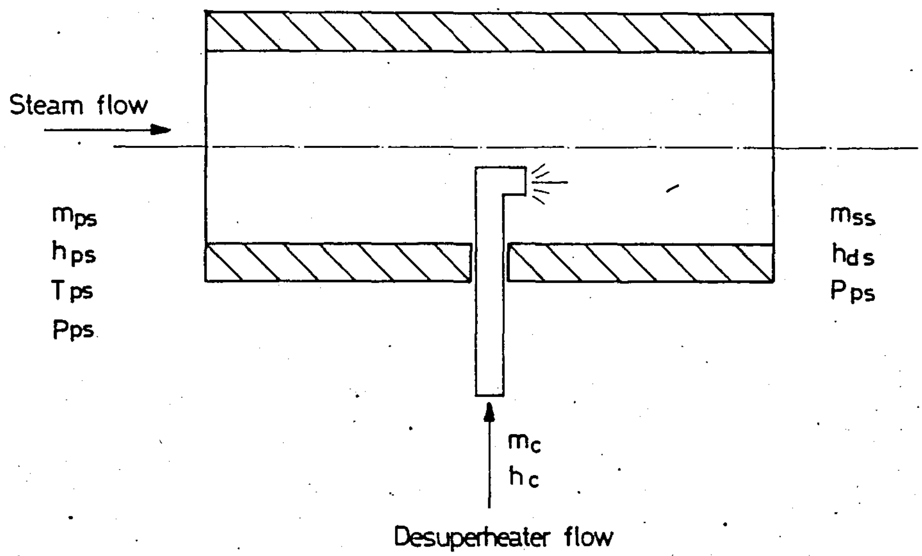


FIGURE: 3.5_ A schematic diagram of the desuperheater.

The simplifying assumptions, considered here, are:

i) The desuperheater have no storage volume,

ii) The pressure loss across the desuperheater is negligible,

iii) Desuperheater flow temperature does not vary,

iv) The process can be considered to be adiabatic mixing of the two streams.

Thus, the following equations may be written;

continuity equation:

$$m_{ps} + m_c = m_{ss} \tag{3.28}$$

Energy equation:

$$m_{ps} h_{ps} + m_c h_c = m_{ss} h_{ds} \tag{3.29}$$

where

m_c : desuperheater flow

h_c : coolant water enthalpy which is assumed to be constant.

3.7 SECONDARY SUPERHEATER

This horizontal type superheater, placed at the convective zone of the boiler, is treated in a similiar way to the modeling of the primary superheater.

Again, the superheater is assumed to be a heat exchanger of cross flow type and tubes are represented by one tube. The Fig. (3.6) shows the simplified diagram of the secondary superheater tubes and indicates the variables used.

All assumptions used for the modeling purpose of the primary superheater are also valid for this subsection.

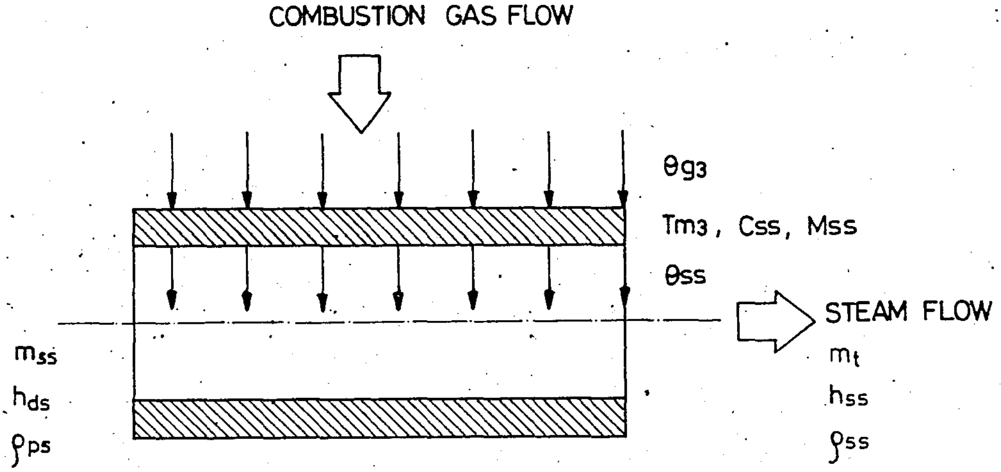


FIGURE : 3.6_ One tube representation of the secondary superheater.

The describing equations of the secondary superheaters are;

continuity equation:

$$m_{ss} - m_t = V_{ss} \frac{d}{dt} (\rho_{ss}) \tag{3.30}$$

Momentum equation:

$$P_{ps} - P_{ss} = f_3 \frac{m_{ss}^2}{\rho_{ps}} \tag{3.31}$$

Enegy equation for the steam side:

$$\theta_{ss} + m_{ss} h_{ds} - m_t h_{ss} = V_{ss} \frac{d}{dt} (\rho_{ss} h_{ss}) \tag{3.32}$$

Energy equation for the tube material:

$$\theta_{g3} - \theta_{ss} = C_{ss} M_{ss} \frac{d}{dt} (T_{m3}) \tag{3.33}$$

Where θ_{ss} is the turbulent heat transfer rate from tube material to the flowing steam. It is expressed as (3)

$$\theta_{ss} = k_3 (m_{ss})^{0.8} (T_{m3} - T_{ss}) \tag{3.34}$$

and θ_{g3} is the heat transfer rate from the combustion products to the tube material. It can be related to the heat input to the boiler as follows;

$$\theta_{g3} = \gamma_3 c_f m_f \tag{3.35}$$

The unknown operating variables T_{ss}, P_{ss} can be obtained from the state relations in the forms of:

$$P_{ss} = f(\rho_{ss}, h_{ss}) \tag{3.36}$$

$$T_{ss} = f(\rho_{ss}, h_{ss}) \tag{3.37}$$

The constants f_3, k_3 and γ_3 will be determined from the equations (3.31), (3.34) and (3.35) at the specified steady state operating conditions.

3.8 THE HIGH PRESSURE TURBINE (H.P.T)

The flow of steam from the secondary superheater bifurcates and each half after passing through the normally open throttle valve enters a steam chest on either side of the H.P.T which contains 6 impulse stages and a curtis stage. Each steam chest contains two governing valves that are opened Sequentially to control the flow of steam to H.P.T.

The valves are of the single seated plug type and are operated by a hydraulic servomotor which is positioned by the turbine governor. The servomotor opens and closes the valves in a predetermined sequence. The valves are set so that No.s 1 and 2, No.s 3 and 4 are opened together.

It is assumed that;

- The turbine throttle valves and the turbine stages can be considered as a steam nozzle,
- Sonic flow conditions exist throughout the H.P.T,
- Governor valves are ideal restrictions which preserves the enthalpy of the flowing steam.

The continuity and energy equations are applied to the H.P.T. as stated below.

continuity equation:

$$m_t - m_{hp} = V_{hp}\frac{d}{dt}(\rho_{hp}) \qquad (3.38)$$

Energy equation:

$$\dot{m}_t h_{ss} - m_{hp} h_{hp} - MW_{hp} = V_{hp}\frac{d}{dt}(\rho_{hp} h_{hp}) \qquad (3.39)$$

Turbine steam flow rate, m_t , in the existence of the sonic flow conditions may be obtained from

$$m_t = CA_v \frac{P_{ssk}}{\sqrt{T_{ssk}}} \qquad (3.40)$$

where Av, the total flow area, is controlled by the governor valve position and P_{ssk}, T_{ssk} represents the absolute values of the throttle pressure and temperature respectively.

The output of the H.P.T, MW_{hp}, can be taken as directly proportional to the steam flow for small perturbations. Therefore

$$MW_{hp} = K_{hp} m_t \qquad (3.41)$$

The H.P.T. exhaust pressure, P_{hp}, may related to the H.P.T exhaust flow, m_{hp}, using the A.T.D of the turbine group in order not to use a state relation for p_{hp}.

$$m_{hp} = f(P_{hp}) \qquad (3.42)$$

The proportional constant K_{hp} and the sonic valve coefficient, C, can be obtained from the steady state values of power output of H.P.T and Throttle flow respectively.

3.9 REHEATER

From the H.P.T, steam enters the boiler and absorbs heat while passing through the reheater placed at the radiant zone of the boiler. Fig (3.7) shows the component and indicates the variables used.

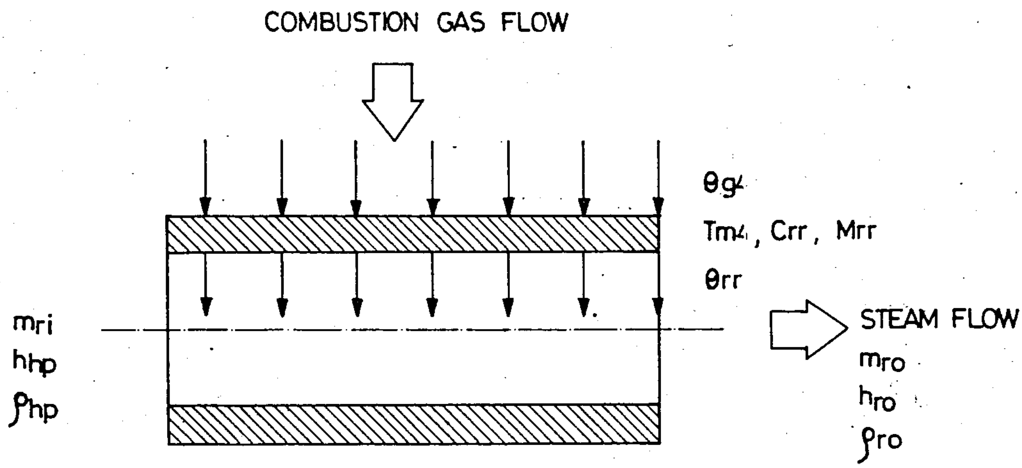


FIGURE: 3.7_ One tube representation of the reheater.

The mathematical model of the Reheater is similiar to that of primary and secondary superheater. The describing equations are;

continuity equation:

$$m_{ri} - m_{ro} = V_{rr}\frac{d\rho_{ro}}{dt} \tag{3.43}$$

Energy equation for the steam side:

$$\theta_{rr} + m_{ri}h_{hp} - m_{ro}h_{ro} = V_{rr}\frac{d}{dt}(\rho_{ro}h_{ro}) \tag{3.44}$$

Energy equation for the tube material:

$$\theta_{g4} - \theta_{rr} = M_{rr}C_{rr}\frac{d}{dt}(T_{m4}) \tag{3.45}$$

Momentum equation:

$$P_{hp} - P_{ro} = f_4 \frac{m_{ri}^2}{\rho_{hp}} \tag{3.46}$$

θ_{rr} is the turbulent heat transfer rate from tube material to the flowing steam. It is expressed as (3);

$$\theta_{rr} = k_4 (m_{ri})^{0.8} (T_{m4} - T_{ro}) \tag{3.47}$$

θ_{g4} is the heat transfer rate from the combustion products to the tube material. It can be related to heat input rate as,

$$\theta_{g4} = \gamma_4 m_f c_f \tag{3.48}$$

Before the Reheater, some amount of steam is extracted for the High Pressure Heater I. The reheater inlet flow rate, m_{ri}, can be related to the H.P.T steam flow rate, m_t. The A.T.D of the unit may be then used to determine the reheater inlet steam flow as follows,

$$m_{ri} = f(m_t) \tag{3.49}$$

The Reheater outlet steam conditions, T_{ro} and P_{ro}, can be obtained using the state relations in the forms of:

$$T_{ro} = f(\rho_{ro}, h_{ro}) \tag{3.50}$$

$$P_{ro} = f(\rho_{ro}, h_{ro}) \tag{3.51}$$

The proportional constants f_4, k_4, γ_4 can be obtained from the equations (3.46), (3.47) and (3.48) at the specified steady state operating conditions.

3.10 THE LOW PRESSURE TURBINE (L.P.T)

The reaction type Low Pressure and Intermediate Pressure Turbines are considered as the low Pressure Turbine. The following assumptions are made;

i) L.P.T can be considered as a steam nozzle,
ii) Sonic flow conditions exist thoughout the L.P.T,
iii) L.P.T exhaust conditions are constant,
iv) Dynamic of the L.P.T may be neglected.

Four steam extraction points are present in the L.P.T which must be accounted in the modelling. There are basically two approaches to this problem. The first requires the identification of the pressures in the L.P.T at the entrance to the each extraction points which is very difficult. The second approach to the extraction problem, adopted here, lumps all of the extraction points into one.

The power output of the L.P.T is given by

$$MW_{lp} = m_{lp}(h_{ro} - h_{ex}) \tag{3.52}$$

where hex is the exhaust saturated steam enthalpy corresponding to the condenser pressure, 1.5"Hg. m_{lp} is the average steam flow to consider the four extraction points as one extraction point expressed as,

$$m_{lp} = \frac{1}{2}(m_{ro} + m_{ex}) \tag{3.53}$$

m_{ro} is the inlet flow rate and can be calculated using the inlet steam conditions from the following equation;

$$m_{ro} = K_{lp} \frac{P_{rok}}{\sqrt{T_{rok}}} \tag{3.54}$$

where P_{rok} and T_{rok} represents the absolute values of the L.P.T inlet pressure and temperature respectively.

m_{ex} is the flow to the condenser and can be found as a function of the inlet steam flow, m_{ro}, using the result of the A.T.D of the turbine group. Hence

$$m_{ex} = f(m_{ro}) \tag{3.55}$$

The proportional constant K_{lp} can be obtained from eqn. after determining the values of P_{ro} and T_{ro} from A.T.D.

The total power of the Turbine group is then equal to the sum of the L.P.T and H.P.T outputs,

$$W_t = MW_{lp} + MW_{hp} \tag{3.56}$$

IV. DYNAMIC EQUATIONS OF THE PLANT

4.1 STRUCTURE OF THE NONLINEAR PLANT

In the previous chapter, the describing equations of each subsection are obtained using physical laws, undetermined state relations and suggested functional relations for some of the plant parameters.

In this chapter, dynamic nonlinear plant model will be derived considering ρ_s, V_w, h_o, h_w, Tm1, ρ_{ps}, Tm2, ρ_{ss}, h_{ss}, Tm3, ρ_{hp}, h_{hp}, ρ_{ro}, h_{ro}, Tm4, as state variables and m_{fw}, m_{fu}, Av and m_c as the control inputs.

All the state variables are best process informative plant variables and well represent the dynamics of the controlled variables Vw, Pd, Tss and Power output.

The dynamic equations for these selected state variables can be found rearranging the scattered describing equations of each subsections properly. When each section is considered, the related state relations will be given. These state relations were obtained using data from Van Wylen and Sonntag steam tables (11) for independent variables. The Multivariable Regression subprogram of the SPSS package was used to determine the state relations. The application range of these relations were chosen according to temperature and pressure measurement results in A.T.D.

The functional relations defining the plant parameters m_{ex}, m_{hpo} and m_{ri} were also determined using data from A.T.D. They are given in the respective subsections.

4.2 DRUM SYSTEM

Drum system is considered to be consisting of downcomer tubes, riser tubes and steam drum. It's found that state variables, V_w, ρ_s, h_w, h_o and T_{ml}, can represent the dynamic of the drum system.

In the derivation of the dynamic equations for above state variables, it is assumed that the acceleration terms in the momentum equations for downcomer and riser can be neglected as proposed in (4). This is quite reasonable since the downcomer and riser are the fastest modes of the boiler dynamic.

The dynamic relations;

$$\frac{d}{dt}(V_w) = \frac{2{,}0132\, V_w (m_o x_o + m_e - m_s) + (V_D - V_w)((1-x_o)m_o - m_w - m_e + m_{fw})}{(V_D - V_w)\,\rho_{ow} - 2.0132\,\rho_s} \quad (4.1)$$

$$\frac{d}{dt}(\rho_s) = \frac{1}{(V_D - V_w)}\left(\dot{V}_w \rho_s + m_o x_o + m_c - m_s\right) \quad (4.2)$$

$$\frac{d}{dt}(h_w) = \frac{1}{\rho_{Dw} V_w}((1-x_o)m_o(h_{ws}-h_w)+m_{fw}(h_{fw}-h_w)-m_e(h_s-h_w)) \quad (4.3)$$

$$\frac{d}{dt}(h_o) = \frac{1}{V_r \rho_o}(\theta_r + m_w(h_w - h_o)) \quad (4.4)$$

$$\frac{d}{dt}(T_{m1}) = \frac{1}{M_r C_r}(\gamma_1 C_f m_f - \theta_r) \quad (4.5)$$

are obtained by using:

- the continuity equation for vapor phase in the steam drum, eq. (3.12)
- the continuity for liquid phase in the steam drum with the state relation for ρ_{dw}, eqn's (3.10) and (4.8)
- the continuity and energy equations for the liquid phase in the steam drum, eqn's (3.10) and (3.11),
- the continuity and energy equations for the riser flow, eqn's (3.2) and (3.4),
- the energy equation applied to riser tubes, eqn. (3.7)

respectively.

The downcomer flow, m_w, and theriser flow, m_o, can be determined from the following static relations;

$$\frac{A_d}{L_{d1}}(P_d - P_{md}) = -gA_d\rho_{dw} + \left(f_d\frac{L_d}{D_d} + \varepsilon_1 + \varepsilon_2 + 1\right)\frac{m_w^2}{2L_{d1}A_d\rho_{dw}} \qquad (4.6)$$

$$\frac{A_r}{L_{r1}}(P_d - P_{md}) = -gA_r\rho_o - \left(f_r\frac{L_r}{D_r} + \varepsilon_3 + 1\right) - \varepsilon_4\frac{m_w^2}{2A_rL_{r1}\rho_o} + \frac{A_r m_w^2}{2A_d^2L_{r1}\rho_{dw}} \qquad (4.7)$$

In both equations pressure differential $(P_d - P_{md})$ is an unknown quantity. However, it can be related to a state variable after the evaluation of the steady state values of the m_o and m_w as in Appendix A.

The operating variables $\rho_{dw}, T_s, h_{ws}, h_s, T_w$ and P_d can be evaluated from the following state relations;

$$\rho_{dw} = -2.0132\,\rho_s + 794.4534 \quad (kg/m^3) \qquad (4.8)$$

$$T_s = 0.711342\,\rho_s + 274.4 \quad (°C) \qquad (4.9)$$

$$h_{ws} = 4.72178\,\rho_s + 1159.2 \quad (kJ/kg) \qquad (4.10)$$

$$h_s = -2.753\,\rho_s + 2881.68 \quad (kJ/kg) \qquad (4.11)$$

$$P_d = 117727.53\,\rho_s + 3720292 \quad (kg/m^2) \qquad (4.12)$$

which are satisfactory for use in the drum system and were determined for saturated steam temperature between 311°C and 342°C using data from Van Wylen and Sonntag steam tables. ρ_o, θ_r, and m_e can now be evaluated using the eqn's 3.3, 3.6 and 3.13 respectively.

m_f, m_{fw} and h_{fw} are inputs to drum system whereas the drum outlet flow, m_s, may be considered as the perturbated variable, which is determined by the primary superheater flow conditions.

4.3 PRIMARY SUPERHEATER

The state variables ρ_{ps}, h_{ps} and Tm2 represent the dynamic of the primary superheater.

The dynamic relation for ρ_{ps} is obtained directly from the continuity eqn. (3.21)

$$\frac{d}{dt}(\rho_{ps}) = \frac{1}{V_{ps}}(m_s - m_{ps}) \tag{4.13}$$

The continuity eqn. (3.21) and the energy equation for steam side (3.22) are manipulated to obtain the dynamic relation for h_{ps},

$$\frac{d}{dt}(h_{ps}) = \frac{1}{V_{ps}\rho_{ps}}(\theta_{ps} + m_s(h_s - h_{ps})) \tag{4.14}$$

And the energy equation applied to the primary superheater tube material eqn. (3.23) directly yields the dynamic relation for Tm2.

$$\frac{d}{dt}(T_{m2}) = \frac{1}{M_{ps}C_{ps}}(\gamma_2 c_f m_f - \theta_{ps}) \qquad (4.15)$$

The following state relations can be used to determine the operating variables ρ_{ps} and T_{ps},

$$P_{ps} = -6424.4 h_{ps} + 955.64 \rho_{ps} T_{ps} - 219360.8 \rho_{ps} + 23.882\times10^{6} \ (Pa) \qquad (4.16)$$

$$T_{ps} = 1.09\times10^{-4} h_{ps}^{2} - 0.37 h_{ps} + 5.46\times10^{-6} P_{ps} + 441.23 \quad (^\circ C) \qquad (4.17)$$

where the units of h_{ps} and ρ_{ps} are kJ/kg and kg/m^3 respectively. These state relations are determined using data from Van Wylen and Sonntag sonntag steam tables and are valid in the range of

$$394^\circ C \leqslant T_{ps} \leqslant 570^\circ C$$

$$12.4\ MPa \leqslant P_{ps} \leqslant 13.1\ MPa.$$

The variables h_s, P_d, m_s, and θ_{ps} can be obtained from eqn's (4.11), (4.12), (3.20) and (3.21) respectively. Whereas the primary superheater outlet flow, m_{ps}, can be determined from eqn. (3.28)

4.4 SECONDARY SUPERHEATER

The state variables are ρ_{ss}, h_{ss} and Tm3. The dynamic relations are obtained by manipulating the describing equations of the secondary superheater in a similiar way to that of the primary superheater. These are;

$$\frac{d}{dt}(\rho_{ss}) = \frac{1}{V_{ss}}(m_{ss} - m_t) \tag{4.18}$$

$$\frac{d}{dt}(h_{ss}) = \frac{1}{V_{ss}\rho_{ss}}(m_{ss}(h_{ds} - h_{ss}) + \theta_{ss}) \tag{4.19}$$

and

$$\frac{d}{dt}(T_{m3}) = \frac{1}{M_{ss}C_{ss}}(\gamma_3 C_f m_f - \theta_{ss}) \tag{4.20}$$

where;

h_{ds} = secondary superheater inlet enthalpy determined by desuperheater from eq. (3.29)

m_{ss} = secondary superheater inlet steam flow obtainable from eqn. (3.31)

m_t = Throttle flow determined by governing valve

θ_{ss} = Heat transfer rate to the flowing steam obtainable from eqn. (3.34)

The state relations to determine the throttle conditions, P_{ss} and T_{ss} are;

$$P_{ss} = -6424.4\, h_{ss} + 955.64\, \rho_{ss} T_{ss} - 219360.8\, \rho_{ss} + 23.882\times 10^{6} \quad (Pa.) \qquad (4.21)$$

$$T_{ss} = 1.09\times 10^{-4} h_{ss} - 0.37 h_{ss} + 5.46\times 10^{-6} P_{ss} + 441.28 \qquad (^{\circ}C) \qquad (4.22)$$

These state relations are valid in the range of

$$394\,^{\circ}C \leq T_{ss} \leq 570\,^{\circ}C$$

$$12.41\, MPa \leq P_{ss} \leq 13.1\, MPa$$

4.5 HIGH PRESSURE TURBINE

The state variables are ρ_{hp} and h_{hp}. The dynamic relation for ρ_{hp} is directly obtained from eqn. (3.38)

$$\frac{d}{dt}(\rho_{hp}) = \frac{1}{V_{hp}}(m_t - m_{hp}) \qquad (4.23)$$

whereas the dynamic relation for h_{hp} is found by combining the energy eqn. (2.39) and continuity eqn. (2.38)

$$\frac{d}{dt}(h_{hp}) = \frac{1}{V_{hp}\,\rho_{hp}}\left(\dot{m}_t(h_{ss}-h_{hp}) - MW_{hp}\right) \tag{4.24}$$

A.T.D measurements show that the pressure ratio between the inlet and outlet of the H.P.T is below the critical ratio. i.e.

$$\frac{P_{hp}}{P_{ss}} < 0.545$$

This justifies the validity of sonic flow assumption for H.P.T. Then throttle flow, $\dot{m}_t$, can be obtained from eqn. (3.40). The H.P.T exhaust flow $\dot{m}_{hp}$ can be obtained from following relation which was determined using data from A.T.D.

$$m_{hp} = 3.000948 \times 10^{-5} P_{hp} + 0.76 \quad (kg/s) \tag{4.25}$$

4.6 REHEATER

The dynamics of the reheater is well represented by the state variables ρ_{ro}, h_{ro} and Tro. The dynamic relations

$$\frac{d}{dt}(\rho_{ro}) = \frac{1}{V_{rr}}(m_{ri} - m_{ro}) \tag{4.26}$$

$$\frac{d}{dt}(h_{ro}) = \frac{1}{V_{rr}\,\varrho_{ro}}\left(\theta_{rr} + m_{ri}\,(h_{hp} - h_{ro})\right) \qquad (4.27)$$

and

$$\frac{d}{dt}(T_{m4}) = \frac{1}{M_{rr}C_{rr}}\left(\gamma_4\, c_f\, m_f - \theta_{rr}\right) \qquad (4.28)$$

are obtained considering,

- the continuity eqn. (3.43)
- the energy equation for steam side (3.44) and the continuity eqn. (3.43)
- the energy equation for reheater tube material, eqn. (3.45) respectively.

The operating variables P_{ro} and T_{ro} can be evaluated from the following state relations.

$$T_{ro} = (0.474\,h_{ro} + 4.01\,\varrho_{ro} - 1153.)/(1 + 4.3\times 10^{-3}\varrho_{ro}) \quad (^\circ C) \qquad (4.29)$$

$$P_{ro} = -29.88\,h_{ro} + 99336.3\,\varrho_{ro} + 496.57\,\varrho_{ro}T_{ro} + 120427 \quad (Pa.) \qquad (4.30)$$

which were determined using data from Van Wylen and Sonntag steam tables and are valid in the range of

$$371\,^{\circ}C \leqslant T_{ro} \leqslant 549\,^{\circ}C$$

$$0.414\ MPa. \leqslant P_{ro} \leqslant 2.76\ MPa.$$

θ_{rr} and m_{ro} can be evaluated from eqn's (3.47) and (3.54) respectively whereas the reheater steam inlet flow, m_{ri}, can be obtained from following relation which was determined using data from A.T.D.

$$m_{ri} = 0.873\, m_t + 1.73 \qquad (kg/s) \tag{4.31}$$

This relation was determined with the aid of the least square curve fitting method assuming a linear relation between m_{ri} and m_t, and using the valves of m_{ri} and m_t at 70.5MW, 100.5 MW and 120 MW load levels from A.T.D.

4.7 LOW PRESSURE TURBINE (L.P.T)

A.T.D. shows that sonic flow conditions exist across the L.P.T, i.e. inlet steam flow can be calculated from eqn. (3.54) and the L.P.T. exhaust flow, m_{ex}, can be evaluated from

$$m_{ex} = 0.72\, m_{ro} + 5.46 \qquad (kg/s) \tag{4.32}$$

which was determined using the valves of m_{ex} and m_{ro} at 70.5 MW, 100 MW and 120 MW load levels from A.T.D with the aid of the least square curve fitting method.

4.8 COMPLETE NONLINEAR MODEL

The deterministic nonlinear plant model is described by a sector differential equation

$$\dot{x}(t) = f(x(t), u(t)) \quad (4.33)$$

where the function f(x(t),U(t))contains the plant parameters which are obtainable from 37 nonlinear equations, x(t) is the plant state vector, 16-dimensional vector with components.

X_1 drum liquid volume, Vw (m^3)
X_2 saturated steam density in the steam drum, ρ_s (kg/m^3)
X_3 drum liquid enthalpy, h_w (kJ/kg)
X_4 saturated mixture enthalpy, h_o (kJ/kg)
X_5 riser tube temperature, Tm1 (°C)
X_6 primary superheater steam outlet density, ρ_{ps} (kg/m^3)
X_7 primary superheater steam outlet enthalpy, h_{ps} (kJ/kg)
X_8 primary superheater tube temperature, Tm2 (°C)
X_9 secondary superheater steam outlet density, ρ_{ss} (kg/m^3)
X_{10} secondary superheater steam outlet enthalpy, h_{ss} (kJ/kg)
X_{11} secondary superheater tube temperature, Tm3 (°C)
X_{12} H.P.T steam exhaust density, ρ_{hp} (kg/m^3)
X_{13} H.P.T steam exhaust enthalpy, h_{hp} (kJ/kg)
X_{14} reheater steam outlet density, ρ_{ro} (kg/m^3)
X_{15} reheater steam outlet enthalpy, h_{ro} (kJ/kg)
X_{16} reheater tube temperature, Tm4 (°C)

and U(t) is the plant control vector, 4 dimensional vector with components;

U_1 feedwater flow, m_{fw} (kg/s)

U_2 fuel flow, m_{fu} (kg/s)

U_3 desuperheater flow, m_c (kg/s)

U_4 total governing valve area, Av (m^2)

The nonlinear model represents the dynamics of the each subsection rather than the assembled plant. The purpose of doing so is to save the model generality. Hence the resultant nonlinear model can be easily adapted for other power plant configurations e.g. having different drum circulation characteristics and heat transfer mechanisms of superheaters.

The open loop responses of the system defined by eqn. (4.34) with the specified initial conditions can be obtained by integrating the nonlinear model. For this purpose, a nonlinear simulation program is developed in Fortran language. In this computer program each subsection is represented by a subprogram. The following diagram illustrates the interaction between the subsections (or subprograms) with the main inputs and intermediate plant variables and also indicates the solution logic followed.

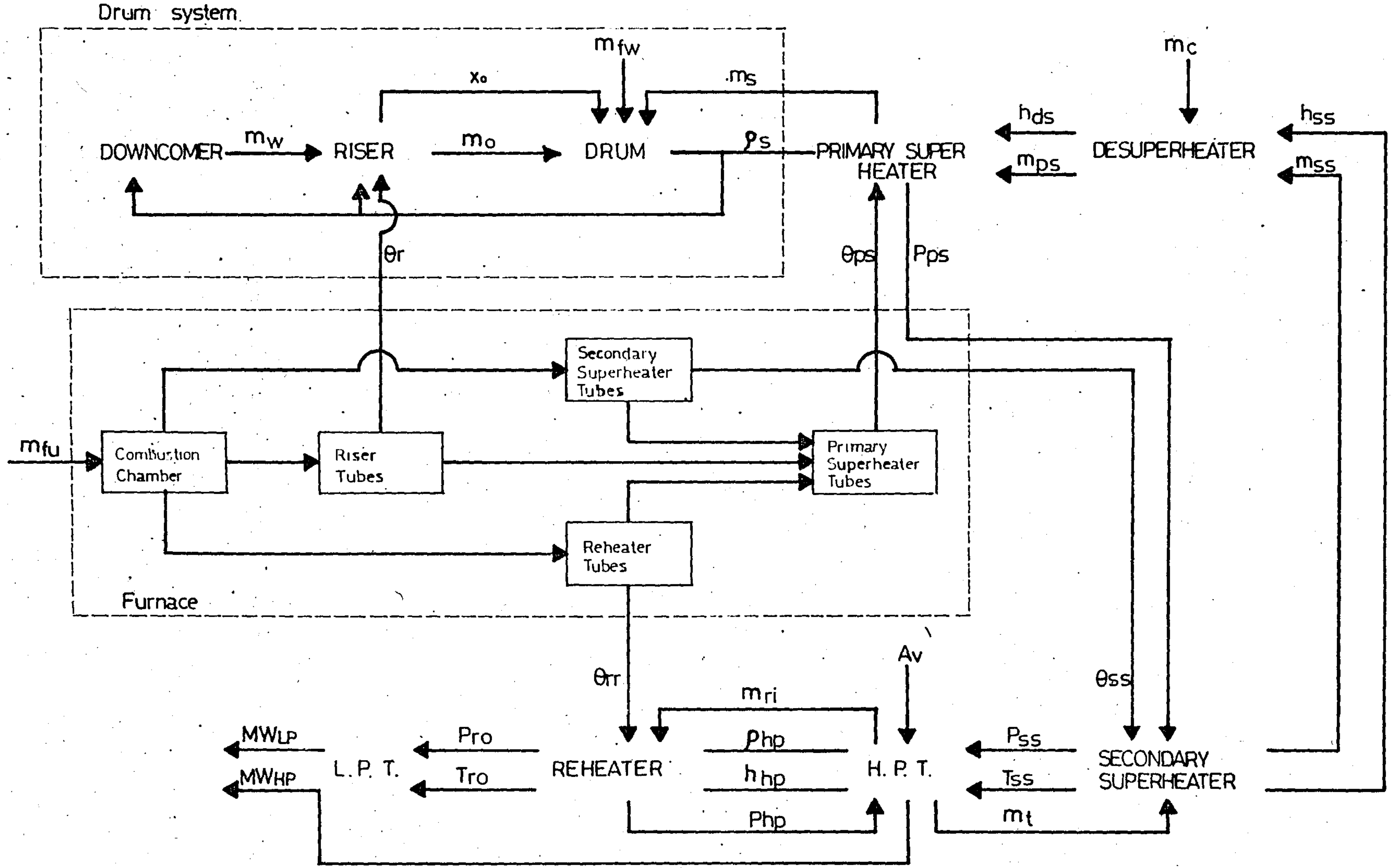


FIGURE: 4.1_ The solution diagram

V. THERMAL POWER PLANT CONTROL

5.1 AN OVERVIEW TO THERMAL POWER PLANT CONTROL STRATEGIES

Generally Thermal power plants are subject to;

- track power generation demands supplied by load schedule,
- maintain the process variables at a set point to maximize the plant efficiency and to avoid the components damage.

Two control strategies were developed to meet the above requirements. These are;

i) Turbine following,
ii) Boiler following.

In the turbine following mode, An operator or an automatic control system adjusts the boiler firing rate while the turbine governor valves are automatically adjusted to maintain throttle pressure. Therefore Turbine output follows the Boiler output. The main disadvantage of this strategy is the slow generation response.

Whereas in the Boiler following mode which is used in Anbarlı Thermal Power Plant, An operator or an automatic control system adjusts the turbine governor valves to maintain desired generation while the boiler firing rate is automatically adjusted. Therefore Boiler output follows the Turbine output. The main advantage of this mode is that Turbine responds quickly while the main disadvantage is the undesirable fluctuations in the throttle pressure and temperature.

In the recent years, a coordinated control system which uses both Boiler and Turbine following philosophies has been developed. This type of control system develops control signals simultaneously for the boiler firing rate and the Turbine governor valves and includes feedforward features which will change set points as a function of power demand input. It is proved in (12) that Coordinated control system yields faster and more stable responses than earlier control strategies.

5.2 EXISTING CONTROL SYSTEM AND CONTROL OBJECTIVE

The existing Anbarlı Thermal Power Plant Control System consists of 5 distinct feedback control loops. These include;

- Turbine speed control,
- Drum pressure control,
- Throttle temperature control,
- Drum liquid level control,
- Combustion control.

The regulators act on:

- Turbine governer valve,
- Fuel flow,
- Desuperheater flow,
- Feedwater flow.

The regulators were designed to hold some important process variables at a fixed desired level. In order to show the main weakness of the existing control system, consider the decrease in the power demand consequently.

- Resistive torque will decrease,
- Turbine speed will increase,
- Turbine governer valve will be closed to decrease the Turbine flow,
- The drum pressure will increase due to less demand of steam,
- Heat input to the boiler will be decreased acting on fuel flow.

This highly interactive system can not be controlled adequately by the conventional control systems since the system components offer serious problems due to separate time constants of the independent control loops.

However, the plant responses become quicker and more stable with the optimally designed regulators using the same control inputs as conventional controllers and variations in state variables or controlled outputs.

Control objective considered here, is the design of the controller which is driven by the observed deviations of the state variables from their steady state valves and generates the control correction signals such that the deviation of the actual state variables from their ideal values vanish in a tolerable time interval without using the excessive control energy expenditure. This is so called energy optimal linear state regulator problem.

5.3 ANALYSIS OF THE STATE REGULATOR PROBLEM AND ITS APPLICATION TO STEADY STATE CONTROL OF ANBARLI THERMAL POWER PLANT

The nonlinear plant model has been described by a vector differential equation

$$\dot{\underline{x}} = \underline{f}(\underline{x}(t), \underline{u}(t)) \tag{5.1}$$

where,

$\underline{X}$: 16 - dimensional plant state vector,

$\underline{U}$: 4 - dimensional control input vector.

Defining the deviation of state variables from their steady state values

$$\delta\underline{x}(t) = \underline{x}(t) - \underline{x}_{ss} \qquad \text{(state perturbation vector)}$$

and the deviation of control inputs from their steady state values

$$\delta \underline{u}(t) = \underline{u}(t) - \underline{u}_{ss}$$ (control correction vector)

the relationship between $\delta \underline{x}(t)$ and $\delta \underline{u}(t)$ is determined by the linear perturbation model

$$\delta \dot{\underline{x}}(t) = \underline{\underline{A}}\, \delta \underline{x}(t) + \underline{\underline{B}}\, \delta \underline{u}(t) \tag{5.2}$$

which is obtained by expanding (3.1) about the steady state values of state variables, $\underline{X}ss$ and control inputs, $\underline{U}ss$. Where; $\underline{\underline{A}}$ is a (16x16) time ivariant matrix which is obtained by evaluating the elements of the jacobian matrix $\frac{\partial f}{\partial \underline{x}}$ along the pre-computed steady state valves of $\underline{X}(t)$ and $\underline{U}(t)$.

$$\underline{\underline{A}} = \left.\frac{\partial f}{\partial \underline{x}}\right|_{\substack{u_{ss} \\ x_{ss}}} \tag{5.3}$$

and $\underline{\underline{B}}$ is a (4x16) time invariant matrix which is obtained by evaluating the plements of the jacobian matrix $\frac{\partial f}{\partial \underline{u}}$ along the pre-computed steady state valves of $\underline{X}(t)$ and $\underline{U}(t)$.

$$\underline{\underline{B}} = \left.\frac{\partial f}{\partial \underline{u}}\right|_{\substack{u_{ss} \\ x_{ss}}} \tag{5.4}$$

The elements of $\underline{\underline{A}}$ and $\underline{\underline{B}}$ matrices were found as a function of steady state values of the state variables and control inputs to take the advantage of applicability of linear model to any specified steady state profile. The nonzero elements of $\underline{\underline{A}}$ and $\underline{\underline{B}}$ matrices are given in Appendix (E).

The steady state values of the state variables, control inputs and proportional constants defined in model development are given for 70.5MW, 100.5 MW and 120 MW. load levels in Appendix (B). These values, mostly, were picked up from A.T.D. for the unit. The steady state values of some state variables defining the drum system and all proportional constants were calculated. The steady state values of the state variables and proportional constants of the drum system were determined by solving the nonlinear simultaneous algebraic equations obtained by setting the derivative terms to zero in the describing equations of the drum system. In Appendix (A) the calculation of the steady state values of drum system state variables and proportional constants are given for 70.5 MW. Load level.

Control objective is to bring the plant, approximated by eqn. (5.2), from an initial state $\delta \underline{x}(t_o) = \underline{x}(t_o)$ to a terminal state $\delta \underline{x}(t_f) = \underline{0}$ using acceptable levels of control $\delta \underline{U}(t)$ and not exceeding acceptable levels of state on the way without using the excessive control energy expenditure.

This can be achieved by minimizing the quadratic cost functional

$$J = \int_{t_o}^{t_f} (\delta \underline{x}^T(t) \underline{\underline{Q}} \delta \underline{x}(t) + \delta \underline{u}^T(t) \underline{\underline{R}} \delta \underline{u}(t)) dt \qquad (5.5)$$

where;

$\underline{\underline{Q}}$ is a 16x16 dimensional symmetric, at least positive semi definite matrix.

$\underline{\underline{R}}$ is a 4x4 dimensional symmetric positive definite matrix.

t_f is the terminal taken to be infinite or for the computation purpose, much larger than the largest system time constant.

Under these conditions, the value of the cost functional is never negative.

$\underline{\underline{Q}}$ and $\underline{\underline{R}}$ matrices are specified by the designer. By a proper selection of the weighting matrices $\underline{\underline{Q}}$ and $\underline{\underline{R}}$, any desired response can be obtained either far all of the state variables or for only one or more of selected variables and also state and control constraints may be indirectly included to the model.

The uniqueness and the existence of the time invariant state regulator problem is guaranted under the condition of controllability of the system. An n-order system is controllabl if the augmented matrix

$$[B \vdots AB \vdots \ldots \vdots A^{n-1}B]$$

have n linearly independent column vector.

If the system is both uncontrollable and unstable then the cost functional will be infinite for all controls since the control interval is infinite. As a result some or all of the state variables will not reach to the steady state.

Assuming the system defined by eqn. (5.2) is controllabl the optimal control correction vector $\delta \underline{U}(t)$ can be found as a linear function of the actual state perturbation vector, $\delta \underline{X}(t)$

$$\delta \underline{u}(t) = -\underline{\underline{G}}\,\delta \underline{x}(t) \qquad t \in (t_o, t_f) \tag{5.6}$$

where $\underline{\underline{G}}$ is constant 4x16 dimensional feedback gain matrix

$$\underline{\underline{G}} = \underline{\underline{R}}^{-1}\underline{\underline{B}}^{T}\underline{\underline{P}} \tag{5.7}$$

and $\underline{\underline{P}}$ is a constant symmetric positive definite 16x16 dimensional matrix which is the steady state solution of the Riccati Matrix Differential Equation (MRE)

$$\frac{d}{dt}(\underline{\underline{P}}(t)) = -\underline{\underline{P}}(t)\underline{\underline{A}} - \underline{\underline{A}}^{T}\underline{\underline{P}}(t) - \underline{\underline{Q}} + \underline{\underline{P}}(t)\underline{\underline{B}}\,\underline{\underline{R}}^{-1}\underline{\underline{B}}^{T}\underline{\underline{P}}(t) \tag{5.8}$$

subject to the boundary condition at the terminal time tf,

$$\underline{\underline{P}}(t_f) = \underline{\underline{O}} \tag{5.9}$$

The optimal trajectory is the solution of the homogenous equation

$$\dot{\underline{x}}(t) = \underline{\underline{K}}\,\underline{x}(t) \tag{5.10}$$

where,

$$\underline{\underline{K}} = \underline{\underline{A}} - \underline{\underline{B}}\,\underline{\underline{R}}^{-1}\underline{\underline{B}}^{T}\underline{\underline{P}} \tag{5.11}$$

If all the eigenvalues of the matrix $\underline{K}=\underline{A}-\underline{B}\underline{R}^{-1}\underline{B}^{T}\underline{P}$ have negative real parts then optimal system is stable. In other words if the controlled system is unstable, the optimal system can be stable. These results are proved in Appendix (C).

There are mainly four numerical methods to compute the steady state solution of time invariant Matrix Riccati Differential Equation. These are;

i) Direct Integration,

ii) The Kalman-Englar method,

iii) Diagonalization method,

iv) Newton Raphson method.

The first two methods are usefull when a complete solution is required. However long computing times may result and symmetric characteristic of the solution may be destroyed by the accumulation of round-off and chopping errors.

The last two methods give only the steady state solution. In this study, diagonalization method given in (13) will be used to compute steady state solution of the time invariant MRE. A brief explaination of this method is given in Appendix D.

5.4 CONTROLLER DESIGN ALGORITHM

The solution of the time invariant state regulator problem provides us with a deterministic feedback design that attempts to vanish deviations of the true state x(t), from its steady state response, $\underline{X}ss$. Fig. (5.1) shows the structure of the time invariant deterministic control system

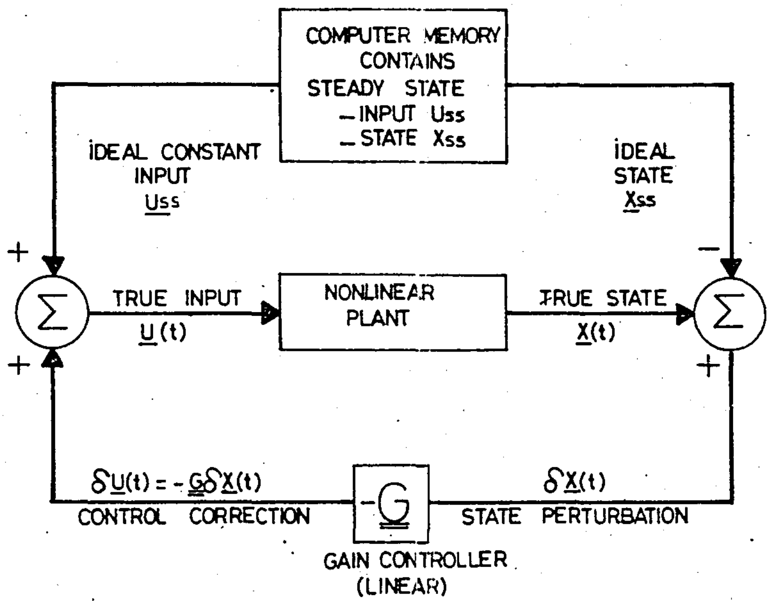


FIGURE: 5.1– The structure of the time invariant deterministic control system for steady state control.

The steps that must be followed to arrive the solution of the time invariant state regulator problem are;

1) Modelling

Step 1: Derivation of the nonlinear deterministic plant model,

$$\dot{\underline{x}}(t) = f(\underline{x}(t), \underline{u}(t))$$

Step 2: Specification of the desired steady state operating profile. This can be done with the aid of the simulation program or by evaluation of the system's dynamic equations as described in Appendix A.

Step 3: Selection of the weighting matrices $\underline{\underline{Q}}$ and $\underline{\underline{R}}$ in the quadratic cost functional (5.5). Weighing is done according to the relative importance of individual state errors and constraints both on the states and control inputs.

2) Off-line Computations:

Step 4: Determination of the linearized perturbation model. That is computation of system matrices $\underline{\underline{A}}$ and $\underline{\underline{B}}$ according to eqn's (5.3) and (5.4) respectively.

Step 5: The matrices $\underline{\underline{A}}$, $\underline{\underline{B}}$, $\underline{\underline{Q}}$ and $\underline{\underline{R}}$ are the coefficients of the Matrix Riccati differential eqn. (5.8). Matrix Riccati with the initial condition $\underline{\underline{P}}(t_f)=\underline{\underline{O}}$ has to be solved for $t \in (to, t_f)$. Then steady state solution is used in eqn. (5.7) to determine the feedback gain matrix $\underline{\underline{G}}$.

3) On-line Computations:

Step 6: Computation of the state perturbation vector

$\delta X(t)$ from $\delta \underline{x}(t) = \underline{X}(t) - \underline{X}_{ss}$. This step requires the determination of the true states. Since $\underline{X}ss$ is precomputed in Step 2. The true states can be obtained by integrating the nonlinear plant model (5.1). An alternative way of obtaining state perturbation vector $\underline{X}(t)$ is to integrate the linear plant model (5.2)

Step 7: Determination of the control correction vector, $\delta\underline{U}(t)$ from eqn.(5.6)

Step 8: Computation of the true control input, $\underline{U}(t)$, by $\underline{U}(t) = \underline{U}ss + \delta\underline{U}(t)$

A digitial computer program is developed under the light of above controller design algorithm to design and analyze Ańbarlı Thermal Power Plant Controls. Next section describes the structure of this computer program in detail.

VI. DESCRIPTION OF THE COMPUTER PROGRAMS AND COMPUTATION ALGORITHM

A flexible and convenient computer is developed in Fortran language to simulate Anbarlı Thermal Power Plant at any predescribed steady state profile with optimally designed state regulator. During the development of the computer program, it is desired that the resultant program is applicable not only for Anbarlı Thermal Power Plant case but also for any Thermal Power Plant simulation and steady state control studies.

The computer program includes;

- main program,
- linear plant model generation subprogram,
- controller design program,
- nonlinear plant model simulation program,
- linear plant model simulation subprogram,
- Runge-Kutta IV integration subprogram.

With the main inputs;

- $\underline{X}ss$: vector containing the steady state values of the state variables,
- $\underline{U}ss$: vector containing the steady state values of

the control inputs; feedwater flow, fuel flow, governing valve area, desuperheater flow,

- $\underline{XI}$: vector containing the perturbated state variables
- $\underline{Q}$,$\underline{R}$: Penalization matrices
- HFW,HEXH,HC : Feedwater enthalpy,L.P.T exhaust enthalpy and desuperheater flow enthalpy respectively.
- ho : Integration step size
- steady state values of the proportional constants, kl, δl,fl
- Plant physical data (in Appendix F.)

and the main outputs;

- the optimal trajectories of the state variables
- the values of the controlled variables

Drum pressure, p_d

Throttle temperature, Tss

Drum liquid volume, Vw

Active power output, Wt

The following diagram illustrates the computer computation steps in sequence with the main inputs and outputs of each step.

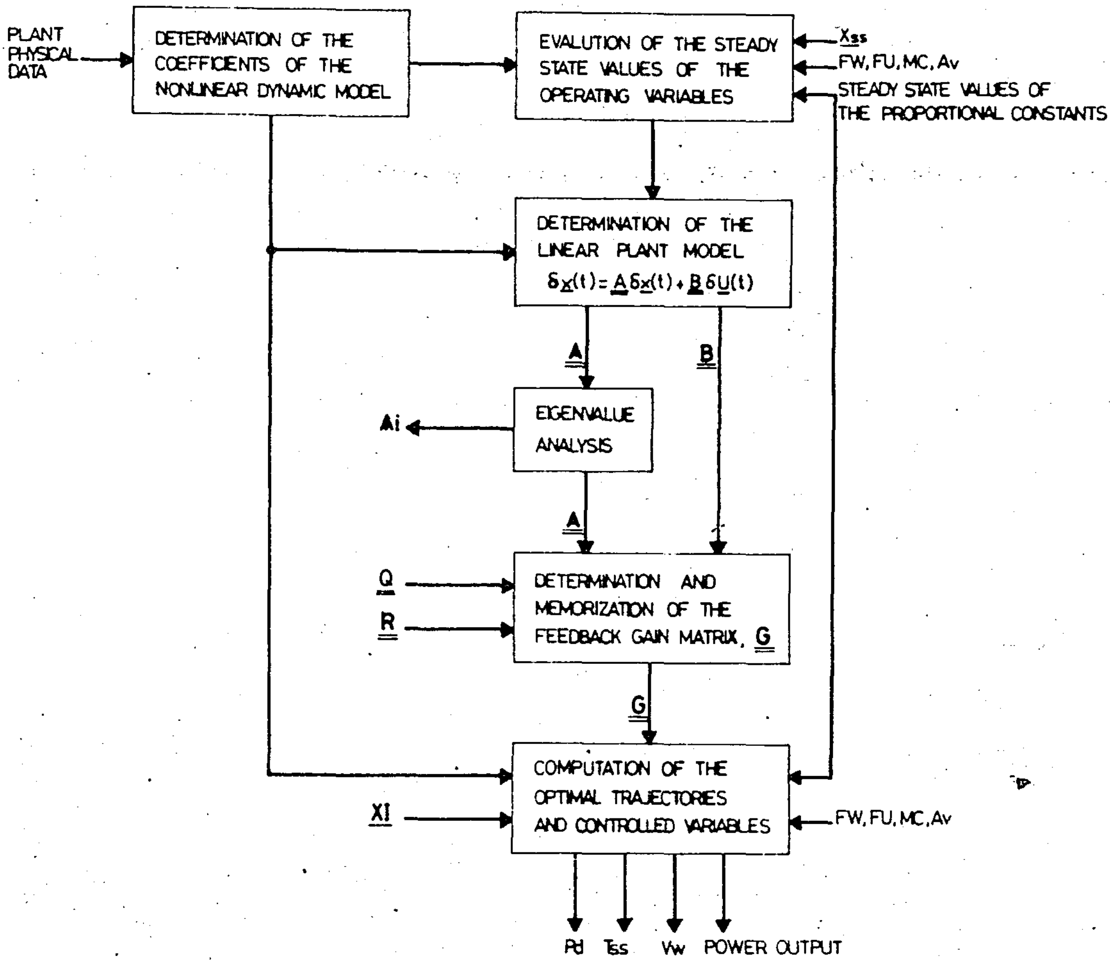


FIGURE : 6.1 _ The computer program computation sequencies.

In the following paragraphs, the structure of the subprograms and their functions are stated under the light of Fig (6.1).

Main program is written mainly for input-output routines but it also computes the constant coefficients of dynamic equations of the nonlinear plant model.

The linear plant model is obtained by the evaluation of the system matrices $\underline{\underline{A}}$ and $\underline{\underline{B}}$ with the aid of the subprogram MATRI. This subprogram is enable to generate linear system matrices for any specified steady state profile since it contains the elements of the system matrices as a function of the steady state values of the state variables, control inputs, proportional constants ant the operating variables. Only the steady state values of some operating variables are not a priori known. These valves are obtained by one step numerical integration of the nonlinear plant model with the method of Runge-Kutta IV.

With the linear plant model simulation program, the optimal trajectories of the state variables and control inputs can beeasily computed, Subprogram LINEAR contains the linear plant model which consist of 16 first order simultaneous differential equations of which all the coefficients are precomputed by subprogram MATRI.

The steady state solution of Matrix Riccati Differential Equations with the diagonalization method and constant feedback gain matrix are computed with the aid of the controller design program. Following diagram illustrates the structure of this program with the related subprograms and their functions.

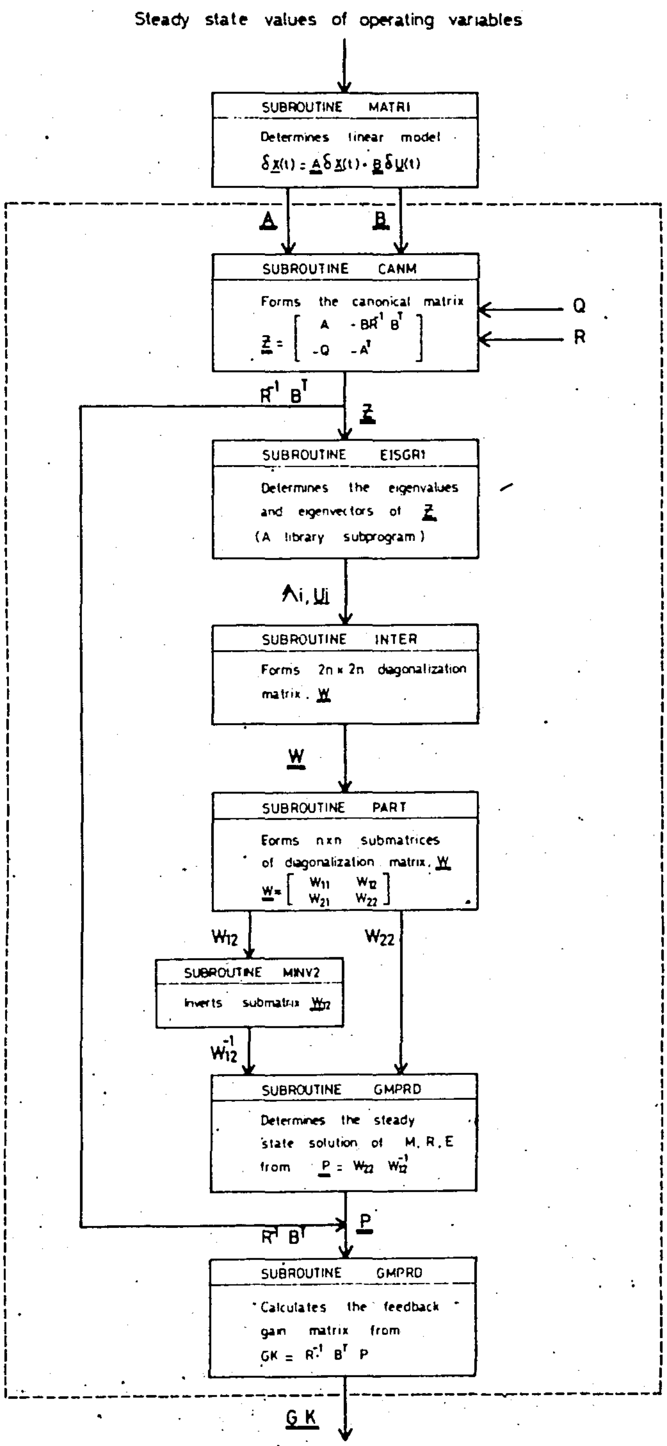


FIGURE 6.2- The structure of the controller design program.

With the aid of the nonlinear plant simulation program, the optimal trajectories and the values of the controlled variables, p_d, V_w, T_{ss} and Active Power can also be determined from the integration of the dynamic equations of the nonlinear plant model using the XI, initial state vector, as imposed initial conditions and precomputed optimal feedback gain matrix by subprogram GAIN.

Subprogram RKV performs the integration using the Runge-Kutta IV method with a properly selected integration step size, ho.

Nonlinear plant simulation program is intentionally represented by 8 subprograms which of each represents the dynamics of one or more than one plant subsection defined in modelling step, rather than one very complex subprogram. Thefore any power plant configuration different than Anbarlı Thermal Power Plant can be easily simulated using the slightly modified version of the existing subpragram s.

The figure (4.1) illustrates the interaction between the subprograms and solution routines for nonlinear simulation program.

The following figure shows the overall structure of the computer program written to design and analyze Anbarlı Thermal Power Plant controls with nonlinear plant simulation subprograms and related subsections.

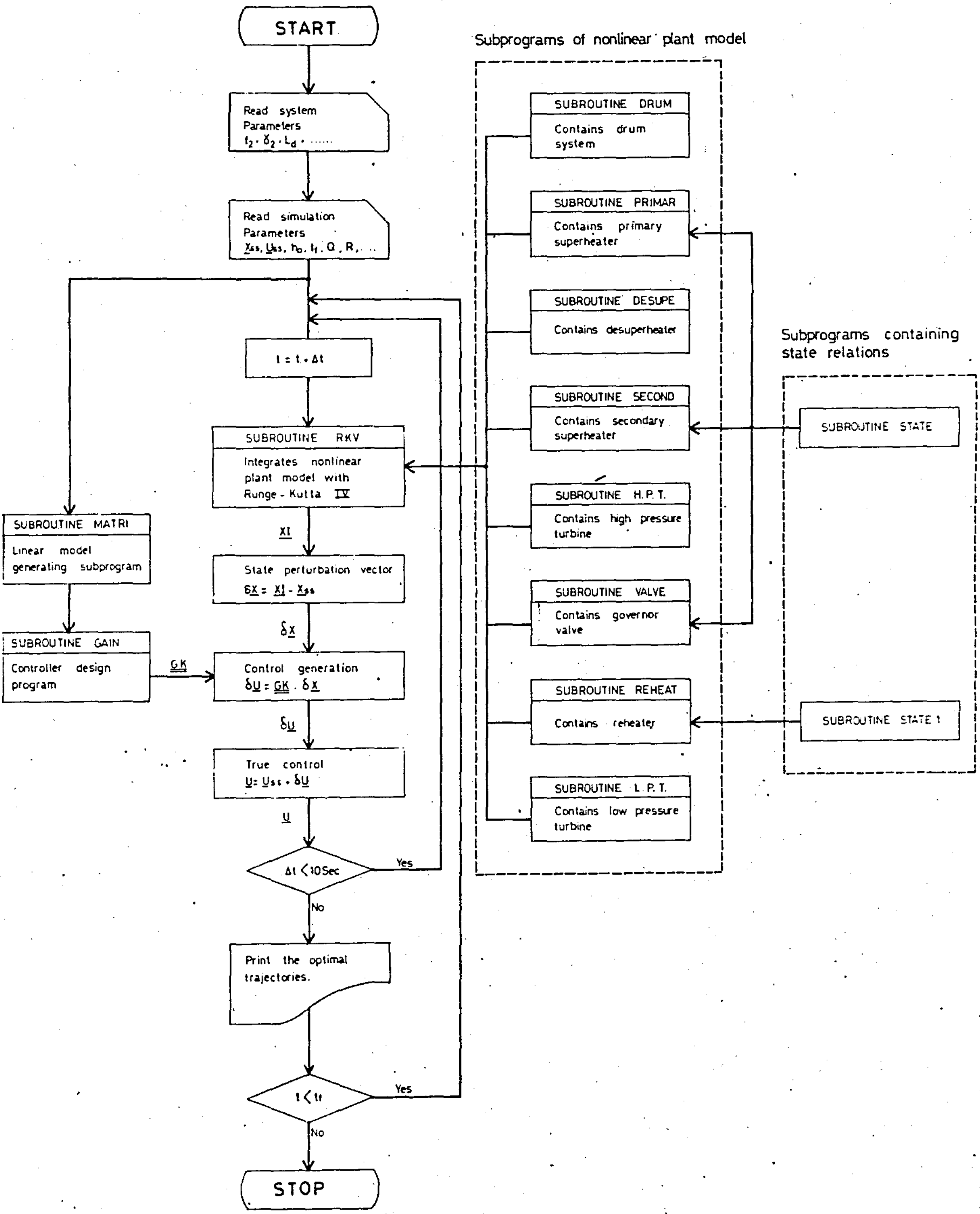


FIGURE : 6.3 _ Main program and subroutines arrangements.

VII. SIMULATION AND RESULTS

In order to investigate the dynamic performance of the controlled system, a series of computer simulation were carried out with the aid of the nonlinear plant simulation program.

The open loop characteristics of the system were determined by eigenvalue analysis. Eigenvalue analysis was performed by using the subprogram EISGR1 from Cern library. The eigenvalues of the open-loop system, given in table 7.1, were found to be all real and distinct. 14 of 16 eigenvalues have negative real parts whereas the remaining 2 eigenvalues have positive real parts. Furthermore, there exist a zero eigenvalue which corresponds to the drum liquid volume.

Eigenvalue analysis indicates;

- the open loop system is unstable,
- drum liquid volume can be affected by every mode of the system and has to be tightly regulated.

Open loop system	Controlled system
0.	-4.90
-4.91	-1.65
-1.65	-0.90
$-.91$	-0.49
$-.51$	-0.366
$-.37$	-0.26
$-.27522$	-0.28
$-.20131$	-0.19331
$.27782$	-0.695×10^{-1}
3.66×10^{-2}	-0.446×10^{-1}
-7.75×10^{-2}	-0.545×10^{-1}
-6.25×10^{-2}	-0.336×10^{-1}
-4.16×10^{-2}	-0.80×10^{-2}
-3.601×10^{-3}	-0.44×10^{-2}
-8.1×10^{-3}	-0.104×10^{-3}
-7.254×10^{-3}	-7.46×10^{-4}

TABLE 7.1 Eigenvalues of open loop and controlled systems

The weighting matrices $\underline{\underline{Q}}$ and $\underline{\underline{R}}$ were selected after successive runs. The first case study was made with all weightings, both on states and control inputs, set to unity and the weightings were then adjusted taking into account the constraints on state variables and control inputs. All weightings on state variables were set to 10^{-5} except the weighting corresponding to drum liquid volume which was set to 10. for the tight regulation. The values $(10^4,10^8,10^8,10^8)$ were selected as diagonal elements of the $\underline{\underline{R}}$ matrix accordingly.

The closed loop eigenvalues were found to be all real, distinct and negative. This indicates that although the open loop system is unstable, the controlled system is asymptotically stable. However one of the eigenvalues of the controlled system was very close to origin. After introducing a degree of stability, $\alpha = 0.008$ (Change in statement of the problem and the role of predescribed degree of stability is briefly given in Appendix C) this eigenvalue was moved considerably to the left of the origin. As a result, closed loop system responses became quicker. The eigenvalues of the controlled given in Table. 7.1.

After the determination of the weighting factors, the optimal trajectories and the values of controlled variables were determined from the integration of the nonlinear model using the precomputed feedback gain matrix. Integration is initiated from a perturbated state with integration step size $h_o = 0.02$ which was determined considering the fastest mode of the system. The perturbated state vector was taken to be corresponding to that of 100.5 MW load level with change in ρ_{ss} and h_{ss}. The values at 70.5 MW load level were assigned for ρ_{ss} and h_{ss}.

During the first simulation run, it was observed that nonlinear system behaviour diverged from the steady state profile. Hand solution used for determining the proportional constants was found to be unsuitable for nonlinear model integration routine. Computer solutions of steady state algebraic equations for the nonlinear model were then used to determine the proportional constants e.g. the friction coefficients f_2, f_3 and heat transfer coefficients k_1, k_2 etc.

Simulation results are illustrated in figures (7.1)-(7.3). It is observed that the controlled variables, P_d, V_w, T_{ss} and Power output reach the steady state in a real time period and the fluctuations die out approximately at the same time.

That is coordinated control of Boiler-Turbine group is achieved. This coordination is inherent in the selection of the weighting matrix $\underline{\underline{Q}}$. Another interesting result of this simulation is the tight regulation of the throttle temperature, T_{ss}, without significiant change in control action. This can be considered as a good coordination of fuel flow and desuperheater flow.

A second simulation run was carried out to investigate the controlled system responses to a change in weighting factor corresponding to that of drum liquid volume. For this purpose, weighting matrix $\underline{Q}$ was rearrenged taking 10^{-1} as the weighting factor for the drum liquid volume and 10^{-5} for the rest. While the weighting matrix $\underline{\underline{R}}$ was taken to be corresponding to that of first simulation run. This change produced unsatisfactory results from the point of view of dynamic control of a Thermal Power Plant. Although good control of Power output and Throttle temperature were achieved. Drum liquid volume and drum pressure could not be controlled. However stable trend in these variables toward the origin was observed in a simulation time interval of 1700 sec. These results are important in order to show

- the interaction between the drum liquid volume and the drum pressure or superiority of the optimal controllers over the conventional controllers,
- essence of tight regulation of drum liquid volume.

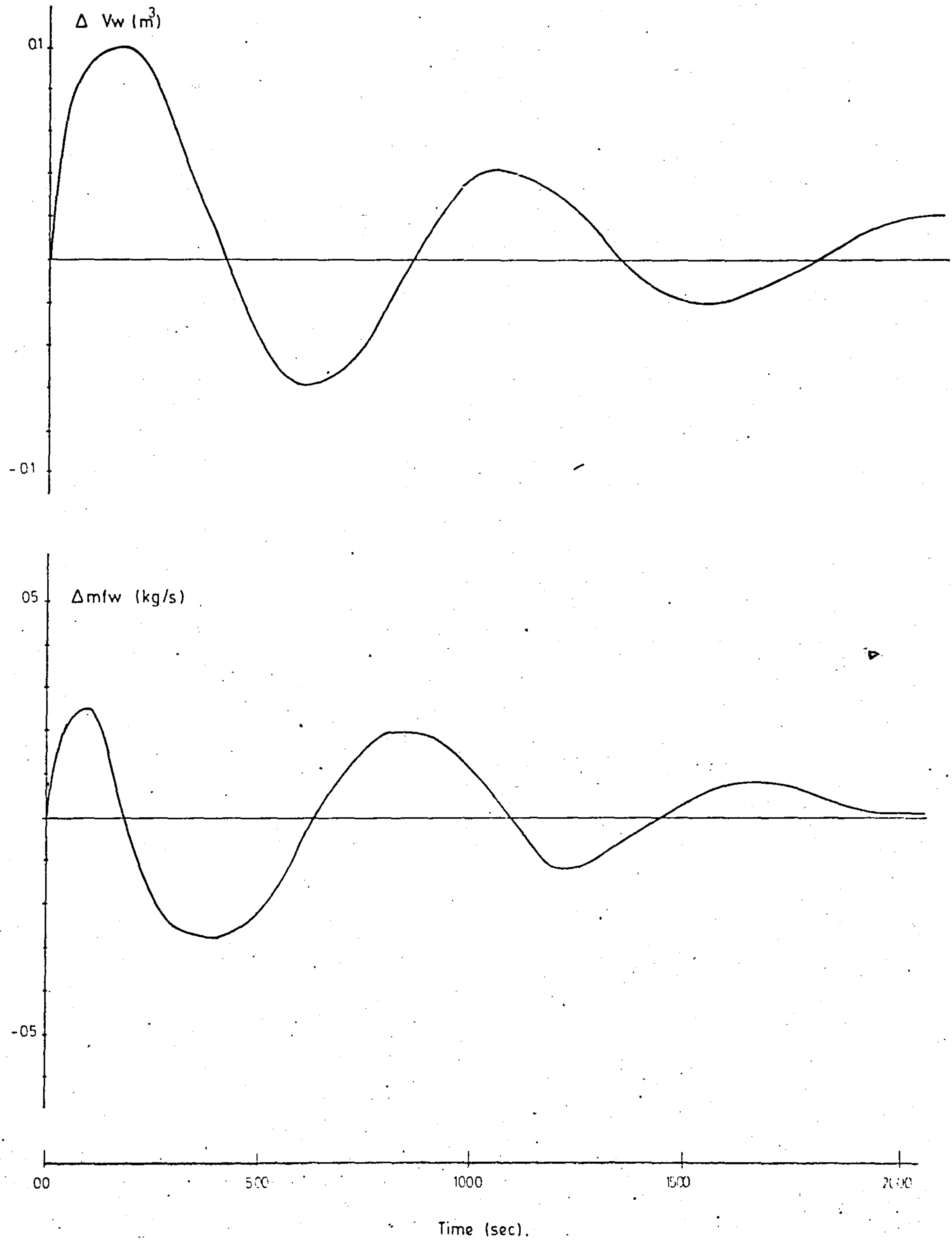


FIGURE : 7.1. Drum liquid volume and feedwater flow responses to changes in throttle density and ethalpy.

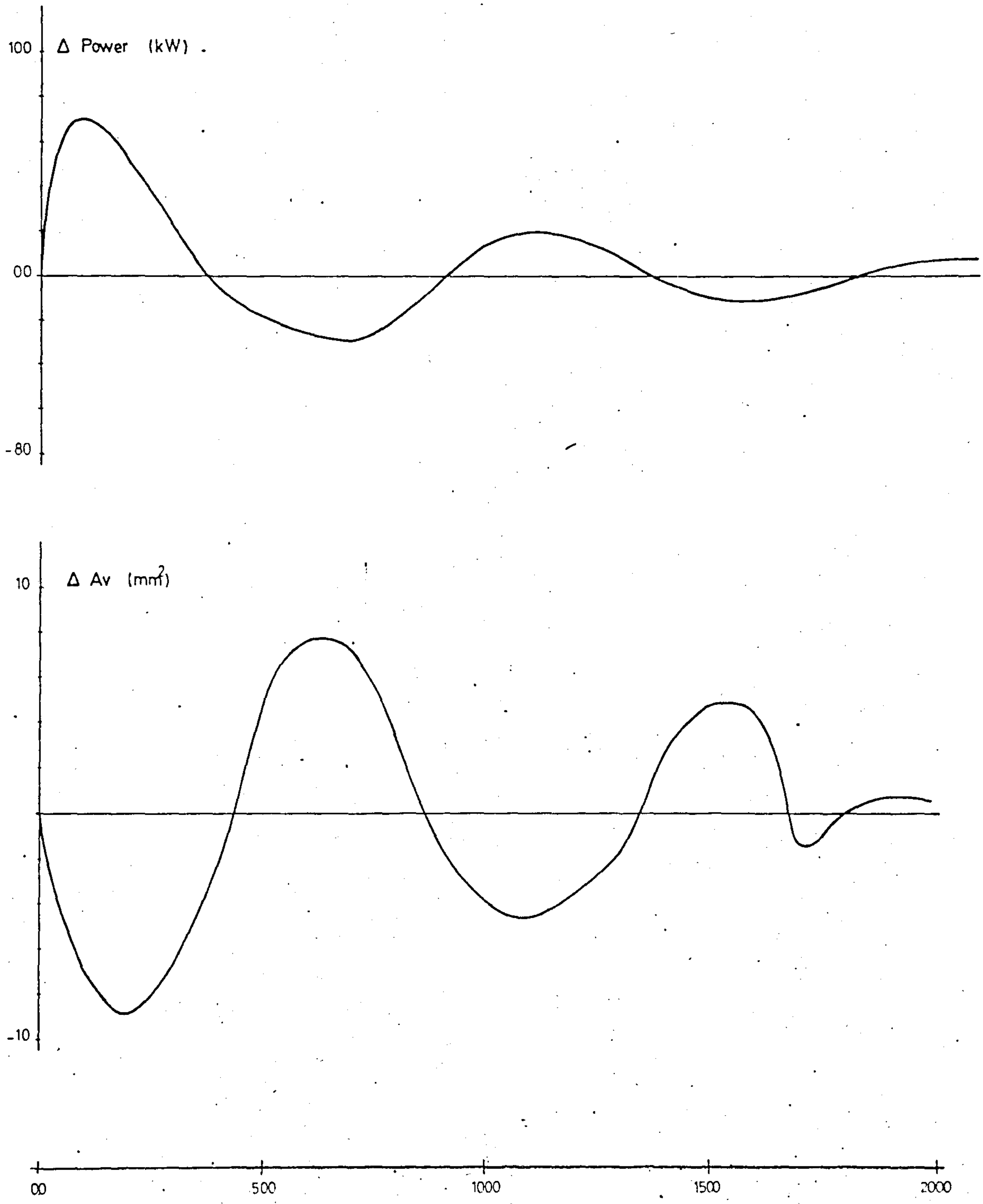


FIGURE : 7. 2- Power output and governing valve responses to changes in throttle density and enthalpy.

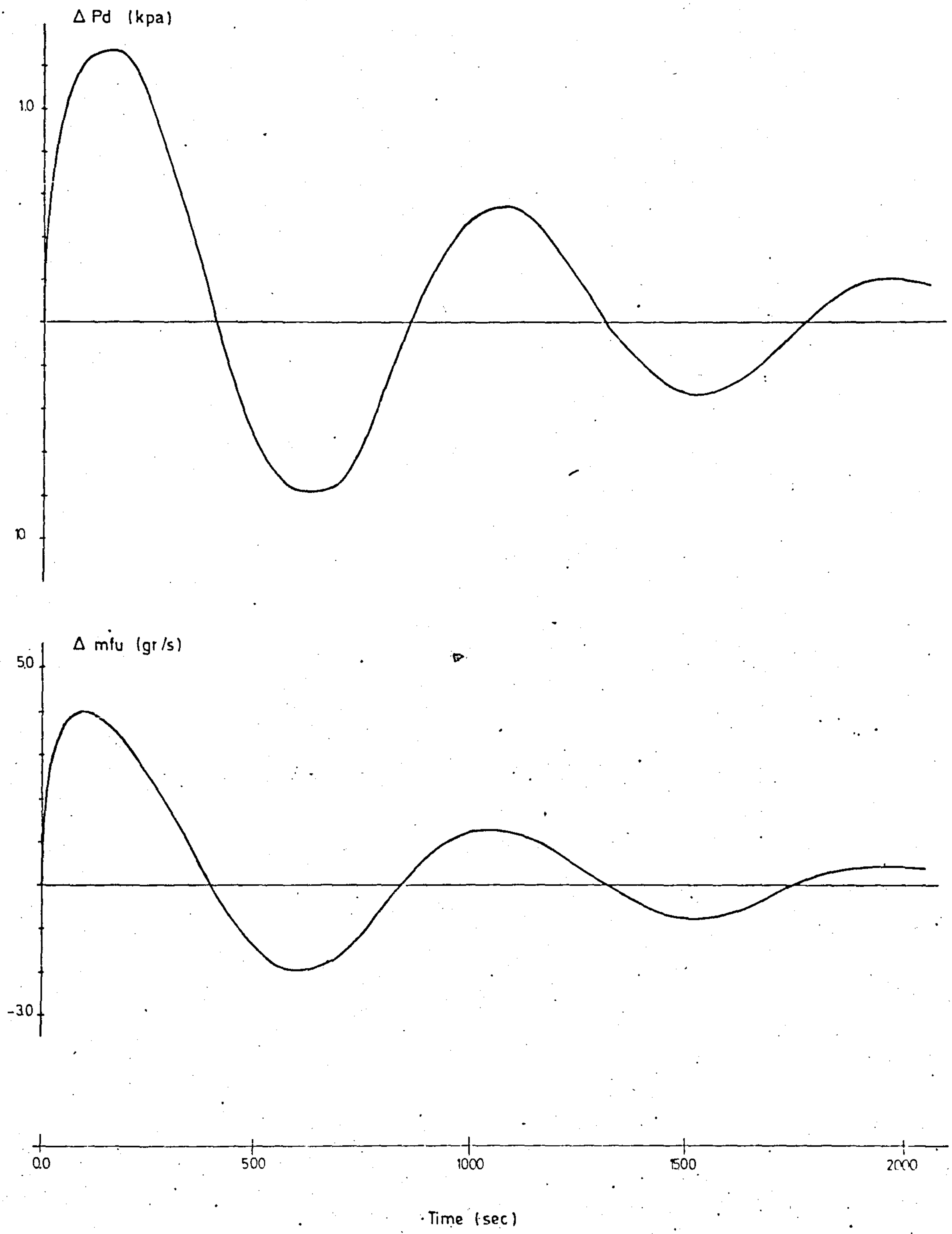


FIGURE: 7.3_ Drum pressure and fuel flow responses to changes in throttle density and enthalpy.

VIII. CONCLUSIONS

Although the model is based on a particular Boiler-Turbine unit, the model is a representative of a large power plant, As a result of deterministic model development, all parameters are Physical quantities which are easily obtainable from design data or unit acceptance test data. Thus the model is conveniently adapted to other power plant configurations.

The following important assumptions of model building have been verified and can be recommended as an initial set of approximation

- One tube representation of superhaters is sufficient,
- Treating the H.P.T as a steam nozzle is particularly good for control analysis.

However, the mathematical model can be significantly improved by;

- Including the flue gas dynamics to improve the steam temperature responses to fuel flow perturbations,
- Modelling the governor valve overlap to increase the model sensitivity for governor valve perturbations.

Furthormore, the study indicates the feasibilty of applying linear optimal control leading coordinated control of Boiler-Turbine group based on minimisation of a cost functional related to state variables and control inputs.

To handle easily steady state control of a thermal power plant;

- System local observability and controllability has to be studied,
- Constraints have to be directly included to the model. This is especially important in design of regulators for large load changes.

APPENDIX A

Evaluation of the steady state values of drum system state variables and proportional constants for 70.5 MW load level. Drum system is considered to be consisting of downcomer, riser and steam drum.

i) Calculation of heat transfer coefficient, k_1;

k_1 can be obtained form eqn. (2.6) after determining the steady state values of T_{m1}, T_s and θ_r. T_{m1} and T_s are known from A.T.D. The steady state value of θ_r, heat transfer rate from riser walls to 2- phase flow, is obtained by considering the heat balance for the boiler.

Usefull energy added to the boiler $= \sum_i$ Heat absorbed by the each components of the Boiler

or

$$\theta = \theta_r + \theta_{ps} + \theta_{ss} + \theta_{rr} + \theta_{eco} \qquad (1)$$

The following figures are calculated using the steady state values of the operating variables in A.T.D. Usefull energy added to the boiler,

$$\theta = \eta_b m_f c_f = 1.73 \times 10^5 \ kJ/s \qquad (2)$$

Heat absorbed by;

primary superheater, $\theta_{ps} = m_{ps}(h_{ps} - h_s) = 3.07 \times 10^4 \ kJ/s$

Secondary superheater, $\theta_{ss} = m_{ss}(h_{ds} - h_{ss}) = 2.375 \times 10^4 \ kJ/s$

reheater, $\theta_{rr} = m_{ro}(h_{hp} - h_{r\acute{o}}) = 2.23 \times 10^4 \ kJ/s$

economizer, $\theta_{eco} = m_{eco}(h_{eco} - h_{eci}) \doteq 1.93 \times 10^4 \ kJ/s$

Consequently heat absorbed by the riser tubes is found from (1) as

$$\theta_r = 7.7 \times 10^4 \ kJ/s$$

Inserting $T_{m1} = 450°C$, $T_s = 329°C$ and $\theta_r = 7.7 \times 10^4 \ kJ/s$ into eqn. (2.6), one obtains,

$$k_1 = 0.0418785 \ kJ/s\,(°C)^3$$

Similarly other heat transfer coefficients are calculated using the steady state values of θ_{ps}, θ_{ss} and θ_{rr}.

ii) Steady state values of the m_o, m_w, x, and ho. consider the following describing equations of the drum system;

The physical plant data and the steady state values of the operating variables hws, hs, ρs, ρdw and hw are obtained from A.T.D for the boiler unit. The values of these operating variables are;

ρ_s : 76.74 (kg/m^3)

ρ_{dw} : 639.96 (kg/m^3)

h_{ws} : 1521.5 (kJ/s)

h_s : 2670.4 (kJ/s)

h_w : 1470.00 (kJ/s)

Inserting these numerical values into eqn's (5) and (6), one obtains

$$\rho_o = \frac{87.196}{(X_o + 0.1363)} \tag{7}$$

$$h_o = 1521.5 + 1148.9X_o \tag{8}$$

At steady state operating conditions, all derivative terms in eqn's (1), (2), (3) and (4) will drop. Combining new forms of the eqn's (3), (4) and inserting eqn. (8) with the steady value of θ_r calculated previously, one get an expression for m_o and m_w,

$$m_w = m_o \frac{67.}{(0.0521 + X_o)} \tag{9}$$

Equating the new forms of the eqn's (1) and (2) one gets

$$\left(f_d \frac{L_d}{D_d} + \varepsilon_1 + \varepsilon_2 + 1\right) \frac{m_w^2}{2gA_d^2 \rho_{dw}} - \rho_{dw} L_{d1} = -\left(f_r \frac{L_r}{D_r} + 1\right) \frac{m_o^2}{2gA_r^2 \rho_o} + \frac{m_w^2}{2gA_d^2 \rho_o}$$

$$- \varepsilon_3 \frac{m_o^2}{2gA_r^2 \rho_o} - \varepsilon_4 \frac{m_w^2}{2gA_r^2 \rho_{dw}}$$

$$- \rho_o L_{r1} \qquad (10)$$

Inserting physical plant data, the values of the friction loss factors, numerical value of the ρ_{dw} eqn. (7) for ρ_o and eqn. (9) for mw and mo into eqn. (10), one obtain an expression in terms of steam quality, xo,

$$4033.7435 X_o^3 - 9.186 X_o^2 - 70.22554 X_o - 4.7547512 = 0 \qquad (11)$$

Eqn. (11) is solved for steam quality x_o. It is found that 2 roots of the above equation is complex and the other is real. i.e the steam quality at the outlet of the riser tubes is

$$X_o = 0.1588$$

now one can determine the circulation mass flow rate from eqn. (9) as

$$m_w = m_o = 317.6861 \quad (kg/s)$$

- momentum equation for downcomer tubes,

$$\frac{1}{g}(P_d - P_{md}) = (f_d \frac{L_d}{D_d} + \varepsilon_1 + \varepsilon_2 + 1)\frac{m_w^2}{2gA_d^2\rho_{dw}} - \rho_{dw}L_{d1} + \frac{L_{d1}}{gA_d}\frac{dm_w}{dt} \tag{1}$$

- momentum equation for riser tubes,

$$\frac{1}{g}(P_{md} - P_d) = (f_r \frac{L_r}{D_r} + 1)\frac{m_o^2}{2A_r^2 g\rho_o} - \frac{m_w^2}{2gA_d^2\rho_{dw}} + \varepsilon_3\frac{m_o^2}{2gA_r^2\rho_o} + \varepsilon_4\frac{m_w^2}{2gA_r^2\rho_{dw}} + \rho_o l_{r1} + \frac{l_{r1}}{gA_r}\frac{dm_o}{dt} \tag{2}$$

- continuity equation for riser tubes,

$$m_w - m_o = V_r \frac{d}{dt}(\rho_o) \tag{3}$$

- heat transfer equation for steam side in the riser tubes,

$$\theta_r + m_w h_w - m_o h_o = V_r \frac{d}{dt}(\rho_o h_o) \tag{4}$$

where mixture density, ρ_o is

$$\rho_o = \frac{\rho_{dw}\rho_s}{(\rho_{dw} - \rho_s)X_o + \rho_s} \tag{5}$$

and mixture enthalpy, ho is

$$h_o = h_{ws} + X_o(h_s - h_{ws}) \tag{6}$$

and the mixture enthalpy, ho, from eqn. (8) as

$$h_o = 1703.44 \quad (kJ/kg)$$

The pressure differential can be calculated from eqn. (1) dropping the accelaration term. Inserting the plant physical data, the numerical values of friction losses, ρ_{dw}, mw, one gets

$$P_d - P_{md} = 201006.08 \quad kg/m^2$$

It is better to think ($P_d - P_{md}$) as a function of ρ_s rather than consant. The following relation can be used:

$$P_d - P_{md} = 1855.4\rho_s + 58631 \quad (kg/m^2)$$

iii) Steady state values of the evaporation mass flow, m_e. consider describing equations of the steam drum,

$$(1-X_o)m_o + m_{fw} - m_w - m_e = \frac{d}{dt}(V_w \rho_{dw}) \qquad (1)$$

$$(1-X_o)m_o h_{ws} + h_{fw} m_{fw} - h_w m_w - h_s m_e = \frac{d}{dt}(V_w \rho_{dw} h_w) \qquad (2)$$

$$X_o m_o + m_e - m_s = \frac{d}{dt}(V_s \rho_s) \qquad (3)$$

Using the above equations with time derivatives set equal to zero with the expression for evaporation mass flow

$$m_e = k_e (T_w - T_s) \quad (4)$$

give the evaporation constant ke,

$$k_e = \frac{m_o (x_o (h_{ws} - h_{fw}) - (h_{ws} - h_w))}{(h_s - h_{fw})(T_s - T_w)} \quad (5)$$

Since all the steady state values of the variables in eqn. (5) are known, ke is easily found as

$$k_e = -0.525 \quad (kg/s\,°C)$$

then steady state value of the evaporation mass flow is evaluated from eqn. (4) as

$$m_e = 4.861 \quad (kg/s)$$

APPENDIX B

Steady state values of the operating variables and constants at 70.5 MW load level:

m_w^*	:	317.907 (kg/s)	Tm4	:	650°C
m_o^*	:	317.214 (kg/s)	m_f	:	4.5 (kg/s)
V_w	:	6.92 (m^3)	m_c	:	4.527389 (kg/s)
ρ_s	:	76.74 (kg/m^3)	m_{fw}	:	53.8 (kg/s)
h_w	:	1470.00 (kj/kg)	h_{fw}	:	1275.606 (kj/kg)
h_o	:	1703.44 (kj/kg)	h_c	:	632.17 (kj/kg)
Tm1	:	450°C	h_{ex}	:	2406.48 (kj/kg)
ρ_{ps}	:	42.16 (kg/m^3)	A_v	:	$1.57 \times 10^{-2}\ m^2$
h_{ps}	:	3251.7 (kj/kg)	f_2^*	:	3985.0726 (s/m^3)
Tm2	:	600°C	f_3^*	:	1057.7098 (s/m^3)
ρ_{ss}	:	36.22 (kg/m^3)	f_4^*	:	185.07358 (s/m^3)
h_{ss}	:	3459.68 (kj/kg)	γ_1^*	:	0.3821553
Tm3	:	700°C	γ_2^*	:	0.158914
ρ_{hp}	:	7.08 (kg/m^3)	γ_3^*	:	0.1227788
h_{hp}	:	3094.26 (kj/kg)	γ_4^*	:	0.1167458
ρ_{ro}	:	5.005 (kg/m^3)	C^*	:	$8.3728 \times 10^{-3}\ (°K)^{1/2} m^{-2}$
h_{ro}	:	3530.115 (kj/kg)	Klp^*	:	$8.059 \times 10^{-4}\ 1/(°K)^{1/2}$

Khp^*	:	365.4195 kj/s kg
k_1^*	:	0.0418785 kj/s $(^\circ C)^3$
k_2^*	:	9.7631282 kj/$(kg)^{0.8}$$(^\circ C)$
k_3^*	:	5.9511408 kj/$(kg)^{0.8}$$(^\circ C)$
k_4^*	:	7.9872948 kj/$(kg)^{0.8}$$(^\circ C)$

* These values were calculated

Steady state values of the operating variables and constants at 100.5MW load level.

m_w^*	: 332.141 (kg/s)	f_2^*	: 4059.126 (s/m^3)
m_o^*	: 331.754 (kg/s)	f_3^*	: 1279.0331 (s/m^3)
V_w	: 6.92 (m^3)	f_4^*	: 149.99272 (s/m^3)
ρ_s	: 79.557 (kg/m^3)	γ_1^*	: 0.3966733
h_w	: 1470.3 (kj/kg)	γ_2^*	: 0.1621778
h_o	: 1789.55 (kj/kg)	γ_3^*	: 0.124947
Tm1	: 470. ˚C	γ_4^*	: 0.1182666
ρ_{ps}	: 43.7 (kg/m^3)	C^*	: 7.9894×10^{-3} $(˚K)^{1/2}/m^2$
h_{ps}	: 3223.48 (kj/kg)	Klp^*	: 7.969×10^{-4} $1/(˚K)^{1/2}$
Tm2	: 625 ˚C	Khp^*	: 330.7230 kj/kg.s
ρ_{ss}	: 36.516 (kg/m^3)	k_1^*	: 3.94762×10^{-2} $kg/s(˚C)^3$
h_{ss}	: 3453.4 (kj/kg)	k_2^*	: 8.091134 $kj/(kg)^{0.8}$˚C
Tm3	: 725. ˚C	k_3^*	: 5.3051268 $kj/(kg)^{0.8}$˚C
ρ_{hp}	: 9.94 (kg/m^3)	k_4^*	: 7.8912559 $kj/(kg)^{0.8}$˚C
h_{hp}	: 3122.677 (kj/kg)		
ρ_{ro}	: 7.166 (kg/m^3)		
h_{ro}	: 3549.223 (kj/kg)		
Tm4	: 675. ˚C		
m_f	: 6.1944 (kg/s)		
h_c	: 684.7744 (kj/kg)		
m_{fw}	: 77.7 (kg/s)		
h_{fw}	: 1292.557 (kj/kg)		
m_c	: 5.63716 (kg/s)		
A_v	: 2.355×10^{-2} m^2		
h_{ex}	: 2400.1994 (kj/kg)		

* These values were calculated

Steady state values of the operating variables and constants at 120 MW load level.

m_w^* : 332.573 (kg/s)

m_o^* : 331.99703 (kg/s)

V_w : 6.92 (m^3)

ρ_s : 82.017 (kg/m^3)

h_w : 1494.75 (kj/kg)

h_o : 1882.29 (kj/kg)

Tm1 : 490°C

ρ_{ps} : 48.84 (kg/m^3)

h_{ps} : 3131.47 (kj/kg)

Tm2 : 650°C

ρ_{ss} : 38.19 (kg/m^3)

h_{ss} : 3442.2 (kj/kg)

Tm3 : 750°C

ρ_{hp} : 12.04 (kg/m^3)

h_{hp} : 3147.837 (kj/kg)

ρ_{ro} : 8.787 (kg/m^3)

h_{ro} : 3554.586 (kj/kg)

Tm4 : 700°C

m_f : 7.444 (kg/s)

h_c : 687.66524 (kj/kg)

m_{fw} : 98.3 (kg/s)

h_{fw} : 1338.9379 (kj/kg)

m_c : 4.1 (kg/s)

A_v : 3.14×10^{-2} m^2

h_{ex} : 2400.69 (kj/kg)

f_2^* : 1393.6888 (s/m^3)

f_3^* : 551.40243 (s/m^3)

f_4^* : 144.3814 (s/m^3)

γ_1^* : 0.401214

γ_2^* : 0.1446353

γ_3^* : 0.1299698

γ_4^* : 0.1147262

C^* : 7.0596×10^{-3} $(°K)^{1/2}/m^2$

Klp^* : 7.925×10^{-4} $1/(°K)^{1/2}$

Khp^* : 294.3632 kj/kg.s

k_1^* : 3.3141×10^{-2} $kj/s(°C)^3$

k_2^* : 5.4551018 $kj/(kg)^{0.8}(°C)$

k_3^* : 4.9210281 $kj/(kg)^{0.8}(°C)$

k_4^* : 6.5547812 $kj/(kg)^{0.8}(°C)$

* These values were calculated

APPENDIX C.

FORMULATION OF THE STATE REGULATOR PROBLEM

Consider the time invariant control system described by

$$\dot{\underline{X}}(t) = \underline{\underline{A}}\,\underline{X}(t) + \underline{\underline{B}}\,\underline{u}(t) \qquad (1)$$

with the initial condition

$$\underline{X}(t_o) = \underline{X}_o$$

where;

$\underline{X}$ is an n-dimensional state perturbation vector,

$\underline{U}$ is an m-dimensional control correction vector,

$\underline{\underline{A}}$ is an nxn dimensional constant system matrix,

$\underline{\underline{B}}$ is an nxm dimensional constant control matrix.

It is desired to bring the plant defined by eqn. (1) from an initial state $\underline{X}(to) = \underline{X}_o$ to terminal state $\underline{X}(tf) = \underline{O}$. using acceptable levels of control $\underline{U}(t)$ and not exceeding acceptable levels of state on the way. This can be achieved by the minimization of the cost functional made up of a quadratic form in the terminal state plus an integral of quadratic forms in state and the control.

$$J = \frac{1}{2}\underline{X}^T(t_f)\underline{\underline{S}}_f\underline{X}(t_f) + \frac{1}{2}\int_{t_o}^{t_f}(\underline{X}^T(t)\underline{\underline{Q}}\underline{X}(t) + \underline{U}^T(t)\underline{\underline{R}}\underline{u}(t))dt \qquad (2)$$

where, $\underline{\underline{S}}f$ is an nxn dimensional positive semidefinite matrix,

$\underline{\underline{Q}}$ is an nxn dimensional positive semidefinite matrix,

$\underline{\underline{R}}$ is an mxm dimensional positive definite matrix,

tf is the terminal time which will be taken larger than the largest time constant of the system or infinite.

In the steady state control problem, $\underline{\underline{S}}f$ is always choosen as zero matrix since the contribution of the terminal state to cost functional is zero.

Applying the variational method, Hamiltonian of the cost functional can be obtained as

$$H = \frac{1}{2}(\underline{X}^T(t)\underline{\underline{Q}}\underline{X}(t) + \underline{U}^T(t)\underline{\underline{R}}\underline{u}(t)) + \underline{\lambda}^T(\underline{\underline{A}}\underline{X} + \underline{\underline{B}}\underline{u}) \qquad (3)$$

where λ^T is the co-state vector. The co-state vector λ^T is the solution of the vector differential equation

$$\dot{\lambda}^T(t) = -\frac{\partial H}{\partial x(t)} \qquad (4)$$

which can be determined from (3) as,

$$\dot{\lambda}(t) = -\underline{\underline{Q}}x(t) - \underline{\underline{A}}^T\lambda(t) \qquad (5)$$

Suppose that x(t) and λ(t), the solutions of the canonical equations (11), are related by the equation,

$$\underline{\lambda}(t)=\underline{\underline{P}}(t)\underline{x}(t) \qquad t\in[t_o,t_f] \tag{12}$$

By differentiating (12), with respect to time, we have

$$\dot{\underline{\lambda}}(t)=\dot{\underline{\underline{P}}}(t)\underline{x}(t)+\underline{\underline{P}}(t)\dot{\underline{x}}(t) \tag{13}$$

substitution of (10) into (13) yields,

$$\dot{\underline{\lambda}}(t)=\dot{\underline{\underline{P}}}(t)\underline{x}(t)+\underline{\underline{P}}(t)\left[\underline{\underline{A}}\,\underline{x}(t)-\underline{\underline{B}}\,\underline{\underline{R}}^{-1}\underline{\underline{B}}^{T}\underline{\lambda}(t)\right]$$

or

$$\dot{\underline{\lambda}}(t)=\underline{\underline{P}}(t)\underline{x}(t)+\underline{\underline{P}}(t)\left[\underline{\underline{A}}\,\underline{x}(t)-\underline{\underline{B}}\,\underline{\underline{R}}^{-1}\underline{\underline{B}}^{T}\underline{\underline{P}}(t)\dot{\underline{x}}(t)\right] \tag{14}$$

Replacing $\dot{\underline{\lambda}}(t)$ in eqn.(14) with (9) we finally have,

$$-\underline{\underline{Q}}\,\underline{x}(t)-\underline{\underline{A}}^{T}\underline{\underline{P}}(t)\underline{x}(t)=\dot{\underline{\underline{P}}}(t)\underline{x}(t)+\underline{\underline{P}}(t)\left[\underline{\underline{A}}\,\underline{x}(t)-\underline{\underline{B}}\,\underline{\underline{R}}^{-1}\underline{\underline{P}}(t)\underline{x}(t)\right]$$

$$-\dot{\underline{\underline{P}}}(t)\dot{\underline{x}}(t)=\left[\underline{\underline{P}}(t)\underline{\underline{A}}-\underline{\underline{P}}(t)\underline{\underline{B}}\,\underline{\underline{R}}^{-1}\underline{\underline{B}}^{T}\underline{\underline{P}}(t)+\underline{\underline{Q}}+\underline{\underline{A}}^{T}\underline{\underline{P}}(t)\right]\underline{x}(t)$$

or

$$-\dot{\underline{\underline{P}}}(t)=\underline{\underline{P}}(t)\underline{\underline{A}}-\dot{\underline{\underline{P}}}(t)\underline{\underline{B}}\,\underline{\underline{R}}^{-1}\underline{\underline{B}}^{T}\underline{\underline{P}}(t)+\underline{\underline{Q}}+\underline{\underline{A}}^{T}\underline{\underline{P}}(t) \tag{15}$$

Equation (15) is so-called Riccati Matrix differential equation (M.R.E)

Along the optimal trajectory

$$\frac{\partial H}{\partial \underline{u}(t)} = 0 \qquad (6)$$

which implies that

$$\frac{\partial H}{\partial \underline{u}(t)} = \underline{\underline{R}}\,\underline{u}(t) + \underline{\underline{B}}^T \underline{\lambda}^T(t) = 0 \qquad (7)$$

From eqn. (7), it is deduced that control vector along the optimal trajectory is given by;

$$\underline{u}(t) = -\underline{\underline{R}}^{-1} \underline{\underline{B}}^T \underline{\lambda}(t) \qquad (8)$$

The assumption that $\underline{\underline{R}}$ is positive definite for $t \in (t_o, t_f)$ quarantees the existence of the $\underline{\underline{R}}^{-1}$ and $\underline{U}(t)$ for $t \in (t_o, t_f)$

Substituting (8) into (1) and considering with (5), one get reduced canonical equations;

$$-\dot{\underline{\lambda}}(t) = \underline{\underline{Q}}\,\underline{x}(t) + \underline{\underline{A}}^T \underline{\lambda}(t) \qquad (9)$$

$$\dot{\underline{x}}(t) = \underline{\underline{A}}\,\underline{x}(t) - \underline{\underline{B}}\,\underline{\underline{R}}^{-1} \underline{\underline{B}}^T \underline{\lambda}(t) \qquad (10)$$

or in matrix form;

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\lambda}(t) \end{bmatrix} = \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix} \begin{bmatrix} x(t) \\ \lambda(t) \end{bmatrix} \qquad (11)$$

The boundary condition of (15) can be obtained considering the final time condition of the co-state

$$\lambda(T) = \frac{\partial}{\partial x} < X(t_f), X^T(t_f), S_f > = 0 \qquad (16)$$

with (12). It is deduced that $\underline{\underline{P}}$ (tf) can be choosen arbitrarily or simply

$$\underline{\underline{P}}(t_f) = \underline{\underline{O}}. \qquad (17)$$

M.R.E can be then solved for $\underline{\underline{P}}$(t), $t \in$[to,tf] , using (17) as the boundary condition. Thus it is possible to determine the co-state vector λ(t) for $t \in$[to,tf] from (12).

The optimal control correction vector can be rewritten inserting value of λ(t), in (8) as,

$$\underline{U}(t) = -\underline{\underline{G}}(t)\underline{X}(t) \qquad (18)$$

where the feedback gain matrix, $\underline{\underline{G}}$(t), is

$$\underline{\underline{G}}(t) = \underline{\underline{R}}^{-1}\underline{\underline{B}}^T\underline{\underline{P}}(t) \qquad (19)$$

The optimal trajectories may be determined from (11) using the initial conditions,

$$\underline{X}(t_o) = \underline{X}_o$$

and

$$\underline{\lambda}(t_o) = \underline{\underline{P}}(t_o)\underline{X}(t_o)$$

Consider the linear regulator problem with quadratic cost functional

$$J=\frac{1}{2}\int_0^{\infty} e^{2\alpha t}\left(\delta \underline{X}^T(t)\underline{\underline{Q}}\,\delta\underline{X}(t)+\delta\underline{U}^T(t)\underline{\underline{R}}\,\delta\underline{U}(t)\right)dt \qquad (20)$$

rather than (2).

In order to have bounded value for J, $(\delta\underline{X}^T(t)\underline{\underline{Q}}\delta\underline{X}(t)+\delta\underline{U}^T(t)\underline{\underline{R}}\delta\underline{U}(t))$ term should decrease as fast as $e^{-2\alpha t}$ or $\delta\underline{X}(t)$ and $\delta\underline{U}(t)$ should decrease as fast as $e^{-\alpha t}$ i.e all of the eigenvalues of the controlled system should be at the left of the vertical line at $-\alpha$ in the complex plane(absolute damping condition or predescribed degree of stability).

Letting $\delta\hat{\underline{X}}(t)=e^{\alpha t}\delta\underline{X}(t)$ and $\delta\hat{\underline{U}}(t)=e^{\alpha t}\delta\underline{U}(t)$, the system defined by (1) becomes

$$\delta\dot{\hat{\underline{X}}}(t)=(\underline{\underline{A}}+\alpha\underline{\underline{I}})\delta\hat{\underline{X}}+\underline{\underline{B}}\delta\hat{\underline{U}}(t) \qquad (21)$$

and quadratic cost functional (20) reduces to

$$J=\frac{1}{2}\int_0^{\infty}\left(\delta\hat{\underline{X}}^T(t)\underline{\underline{Q}}\,\delta\hat{\underline{X}}(t)+\delta\hat{\underline{U}}^T(t)\underline{\underline{R}}\,\delta\hat{\underline{U}}(t)\right)dt \qquad (22)$$

The optimal control law for the linear regulator problem defined by (21) and (22) is given by

$$\delta\hat{\underline{U}}(t)=-\underline{\underline{R}}^{-1}\underline{\underline{B}}^T\underline{\underline{\bar{P}}}\,\delta\hat{\underline{X}}(t) \qquad (23)$$

Where $\underline{\underline{\bar{P}}}$ is the steady state solution of

$$-\dot{\underline{\underline{P}}}=(\underline{\underline{A}}+\alpha\underline{\underline{I}})^T\underline{\underline{P}}+\underline{\underline{P}}(\underline{\underline{A}}+\alpha\underline{\underline{I}})-\underline{\underline{P}}\,\underline{\underline{B}}\,\underline{\underline{R}}^{-1}\underline{\underline{B}}^T\underline{\underline{P}}+\underline{\underline{Q}} \qquad (24)$$

Hence the optimal control law for the linear optimal problem defined by (1) and (20) is given by

$$\delta\underline{U}(t)=-\underline{\underline{R}}^{-1}\underline{\underline{B}}^T\underline{\underline{P}}\,\delta X(t) \qquad (25)$$

This is the control correction vector of a control system with predescribed degree of stability, α.

APPENDIX D.

DIAGONALIZATION METHOD

This method gives the steady state solution of Matrix Riccati Equation as

$$\underline{\underline{P}} = \underline{\underline{W}}_{22}\,\underline{\underline{W}}_{12}^{-1} \qquad (1)$$

where W22 and W12 are the nxn submatrices of the 2n x 2n matrix W which diagonalizes the canonical matrix, (refer to Appendix C. eqn. 11)

$$Z = \begin{bmatrix} A & -BR^{-1}B^{T} \\ -Q & -A^{T} \end{bmatrix} \qquad (2)$$

Diagonalization method of obtaining P includes;

- Formation of 2nx2n canonical matrix, Z,
- Evaluation of the all eigenvalues of the Z. The eigenvalues may be pure real or complex and if λ is an eigenvalue of Z,

$-\lambda$ also is an eigenvalue. Eigenvalues with negative real parts are of the controlled system.

- Evaluation of the eigenvectors corresponding to only eigenvalues with negative real parts,
- Formation of a 2nxn matrix, W1, from these n eigenvectors as follows. If $\underline{e}$ is a real characteristic vector, let $\underline{e}$ be one of the columns of W1. If $\underline{e}$ and $\underline{e}^*$ form a complex conjugate pair, let Re($\underline{e}$) be one column of W1 and Im($\underline{e}$) another.
- Partitioning of 2nxn matrix W1, into two (nxn) submatrices,

$$W_1 = \begin{bmatrix} W_{12} \\ W_{22} \end{bmatrix} \qquad (3)$$

- Computation of $\underline{\underline{P}}$ from eqn. (1).

APPENDIX E.

The nonzero elements of $\underline{\underline{A}}$ matrix:

$$A(1,2) = \frac{1}{(V_d \rho_{dw} - 794.45 X_1)} \Big\{ -0.71 k_e (3.0132 X_1 - V_d) - \frac{1.006 X_1 X_2 A_{ps}^2}{2 f_2 m_s} C$$

$$- \frac{V_s AD2 \rho_{dw}}{2 C_{dd} m_w} C_{12} + m_o (3.0132 X_1 - V_d) C_{11} + (2.0132 X_o X_1 - V_s (1 - X_o))$$

$$\Big[\frac{AR2 \rho_o C_{13}}{2 C_{dr} m_o ((\rho_{dw} - X_2) X_o + X_2)^2} \Big(- \rho_{dw} X_2 (\rho_{dw} - X_2) C_{11}$$

$$+ (\rho_{dw} - 2.132 X_2)((\rho_{dw} - X_2) X_o + X_2) + \rho_{dw} X_2 + 1.0132 \rho_{dw} X_2 X_o \Big)$$

$$+ \frac{AR2 \rho_o}{2 C_{dr} m_o} \Big(\frac{E \cdot AD2}{C_{dd}} \cdot C_{12} + \frac{2.0132 E \cdot m_w^2}{\rho_{dw}^2} - RALRC_4 \Big) \Big] \Big\}$$

$$A(1,3) = \frac{k_e}{6.27} \frac{(3.0132 X_1 - V_d)}{C_{14}}$$

$$A(1,4) = \frac{1}{C_{14}} \Big\{ \frac{m_o (3.0132 X_1 - V_d)}{(h_{ss} - h_{ws})} + \frac{(2.0132 X_o X_1 - V_s (1 - X_o)) AR2 \cdot \rho_o}{2 C_{dr} m_o}$$

$$\left(-gA_r+\frac{C_{dr}m_o^2}{\varrho_o^2 AR2}\frac{(\varrho_{dw}-X_2)\varrho_{dw}X_2}{(h_s-h_{ws})((\varrho_{dw}-X_2)X_o+X_2)^2}\right)\Bigg\}$$

$$A(1,6)=-\frac{1.006X_1X_2A_{ps}^2}{f_2 m_s C_{14}}C_3$$

$$A(1,7)=-\frac{1.006X_1X_2A_{ps}^2}{f_2 m_s C_{14}}C_2$$

$$A(2,6)=\frac{1}{V_s}\left(A(1,6)X_2+\frac{X_2A_{ps}^2}{2f_2m_s}C_3\right)$$

$$A(2,7)=\frac{1}{V_s}\left(A(1,7)X_2+\frac{X_2A_{ps}^2}{2f_2m_s}C_2\right)$$

$$A(3,2)=\frac{1}{\varrho_{dw}X_1}\Bigg\{\frac{2(1-X_o)(h_{ws}-X_3)C_{dr}m_o}{AR2\,\varrho_o}\left(C_{13}+\frac{E\cdot AD2}{C_{dd}}C_{12}\right.$$

$$\left.+\frac{2.0132\,E\cdot m_w^2}{\varrho_{dw}^2}+RALRC_4\right)+0.711\,k_e(h_s-X_3)-X_2(h_{ws}-X_3)C_{11}$$

$$+4.72\,m_o(1-X_o)+2.753\,m_e\Bigg\}$$

$$A(3,3)=\frac{1}{\varrho_{dw}X_1}\left\{-(1-X_o)m_o-m_{fw}+m_e-\frac{k_e}{6.27}(h_s-X_3)\right\}$$

$$A(3,4)=-\frac{(h_{ws}-X_3)(1-X_o)AR2\varrho_o}{2(h_s-h_{ws})\varrho_{dw}X_1C_{dr}m_o}\left\{\frac{(\varrho_{dw}-X_2)\varrho_{dw}}{((\varrho_{dw}-X_2)X_o+X_2)}C_{13}-m_o\right\}$$

$$A(4,2)=\frac{1}{V_r\rho_o}\left\{\frac{AD2}{2C_{dd}}\frac{X_3\rho_{dw}}{m_w}C_{12}-\frac{2C_{dr}m_o}{AR2\rho_o}X_4\left(C_{13}\frac{E\cdot AD2}{C_{dd}}C_{12}\right.\right.$$

$$\left.\left.+\frac{2.0132\,E\cdot m_w^2}{\rho_{dw}^2}-RALRC_4\right)-2.134\,k_1(X_5-T_s)^2\right\}$$

$$A(4,3)=\frac{m_w}{V_r\rho_o}$$

$$A(4,4)=\frac{1}{V_r\rho_o}\left\{m_o-\frac{AR2\,\rho_o X_4}{2C_{dr}m_o}\,\frac{\rho_{dw}X_2(\rho_{dw}-X_2)}{(h_s-h_{ws})((\rho_{dw}-X_2)X_o+X_2)}C_{13}\right\}$$

$$A(4,5)=\frac{3k_1(X_5-T_s)^2}{V_r\rho_o}$$

$$A(5,2)=\frac{2.134026\,k_1(X_5-T_s)^2}{C_rM_r}$$

$$A(5,5)=-\frac{3k_1(X_5-T_s)^2}{C_rM_r}$$

$$A(6,2)=\frac{1}{V_{s1}}\left\{\frac{X_2A_{ps}^2}{2f_2m_s}C_1\right\}$$

$$A(6,6)=\frac{1}{V_{s1}}\left\{\frac{X_2A_{ps}^2}{2f_2m_s}C_3-\frac{A_{ss}^2X_6}{2f_3m_{ss}}C_3-\frac{m_{ss}}{2X_6}\right\}$$

$$A(6,7)=-\frac{C_2}{V_{s1}}\left(\frac{X_2A_{ps}^2}{2f_2m_s}+\frac{X_6A_{ss}^2}{2f_3m_{ss}}\right)$$

$$A(6,9)=\frac{1}{V_{s1}}\frac{A_{ss}^2 X_6}{2f_3 m_{ss}}C_7$$

$$A(6,10)=\frac{1}{V_{s1}}\frac{A_{ss}^2 X_6}{2f_3 m_{ss}}C_6$$

$$A(7,2)=\frac{1}{V_{s1}X_6}\left\{-2.753\,m_s+\frac{A_{ps}^2}{2f_2 m_s}X_2C_1C_5\right\}$$

$$A(7,6)=-\frac{1}{V_{s1}X_6}\left\{\frac{k_2(m_s)^{0.8}}{(1-5.218\times10^{-3}X_6}(1.786-2.1\times10^{-3}X_7+5.687\times10^{-7}X_7^2)\right.$$

$$\left.-\frac{X_2A_{ps}^2}{2f_2m_s}C_3C_5\right\}$$

$$A(7,7)=\frac{1}{V_{s1}X_6}\left\{m_s-\frac{k_2(m_s)^{0.8}}{(1-5.218\times10^{-3}X_6)}(2.18\times10^{-4}X_7-0.4026)\right.$$

$$\left.-\frac{A_{ps}^2}{2f_2m_s}X_2C_2C_5\right\}$$

$$A(7,8)=\frac{k_2(m_s)^{0.8}}{V_{s1}X_6}$$

$$A(8,2)=\frac{X_2A_{ps}^2}{2f_2m_sC_{ps}m_{ps}}\left(\frac{0.8k_2(X_8-T_{ps})}{(m_s)^{0.2}}C_1\right)$$

$$A(8,6)=\frac{1}{C_{ps}M_{ps}}\left\{\frac{k_2(m_s)^{0.8}}{(1-5.218\times10^{-3}X_6)^2}(1.786-2.1\times10^{-3}X_7+5.687\times10^{-7}X_7^2)\right.$$

$$\left.+\frac{A_{ps}^2X_2}{2f_2m_s}\frac{0.8k_2(X_8-T_{ps})}{(m_s)^{0.2}}C_3\right\}$$

$$A(8,7)=\frac{1}{C_{ps}M_{ps}}\left\{\frac{k_2(m_s)^{0.8}}{(1-5.218\times10^{-3}X_6)}(2.18\times10^{-4}X_7-0.40257)\right.$$

$$\left.+0.4\frac{k_2A_{ps}^2}{f_2(m_s)^{1.2}}X_2(X_8-T_{ps})C_2\right\}$$

$$A(8,8)=-\frac{k_2(m_s)^{0.8}}{C_{ps}M_{ps}}$$

$$A(9.6)=\frac{A_{ss}^2X_6}{2f_3m_{ss}V_{s2}}\left\{\frac{f_3m_{ss}^2}{A_{ss}^2X_6^2}+C_3\right\}$$

$$A(9,7)=\frac{1}{V_{s2}}\frac{A_{ss}^2X_6}{2f_3m_{ss}}C_2$$

$$A(9,9)=\frac{1}{V_{s2}}\left\{-\frac{A_{ss}^2X_6}{2f_3m_{ss}}C_7-\frac{cA_v}{(T_{ss}k)^{1/2}}C_9\right\}$$

$$A(9,10)=-\frac{1}{V_{s2}}\left\{\frac{A_{ss}^2X_6}{2f_3m_{ss}}C_6-\frac{cA_v}{(T_{ss}k)^{1/2}}C_8\right\}$$

$$A(10,6)=\frac{C_{10}}{V_{s2}X_9}\left(\frac{C_2A_{ss}^2X_6}{2f_3m_{ss}}+\frac{m_{ss}}{X_6}\right)$$

$$A(10,7)=\frac{1}{V_{s2}X_9}\left\{\frac{A_{ss}^2X_6}{2f_3m_{ss}}C_2C_{10}+m_{ps}\right\}$$

$$A(10,9)=-\frac{1}{V_{s2}X_9}\left\{\frac{A_{ss}^2X_6}{2f_3m_{ss}}C_7C_{10}+\frac{k_3(m_{ss})}{(1-5.218\times10^{-3}X_9)^2}(1.786-2.1\times10^{-3}X_{10}\right.$$

$$\left.+5.687\times10^{-7}X_{10}^2)\right\}$$

$$A(10,10)=\frac{1}{V_{s2}X_9}\left\{-m_{ss}-\frac{k_3\,(m_{ss})^{0.8}}{(1-5.218\times10^{-3}X_9)}\,(2.18\times10^{-4}X_{10}-0.4025)+\frac{A_{ss}^2X_6}{2f_3m_{ss}}C_6C_{10}\right\}$$

$$A(10,11)=\frac{1}{V_{s2}}\,\frac{k_3\,(m_{ss})^{0.8}}{X_9}$$

$$A(11,6)=-\frac{0.8A_{ss}^2k_3}{2C_{ss}M_{ss}f_3}\,\frac{X_6(X_{11}-T_{ss})}{(m_{ss})^{1.2}}\left\{\frac{f_3m_{ss}^2}{A_{ss}^2X_6^2}+C_3\right\}$$

$$A(11,7)=-\frac{0.8A_{ss}^2k_3}{2f_3C_{ss}M_{ss}}\,\frac{X_6(X_{11}-T_{ss})}{(m_{ss})^{1.2}}C_2$$

$$A(11,9)=\frac{k_3}{C_{ss}M_{ss}}\left\{(m_{ss})^{0.8}\frac{(1.786-2.1\times10^{-3}X_{10}+5.687\times10^{-7}X_{10}^2)}{(1-5.218\times10^{-3}X_9)^2}\right.$$

$$\left.+\frac{A_{ss}^2X_8}{2f_3m_{ss}}\,\frac{0.8(X_{11}-T_{ss})}{(m_{ss})^{0.2}}C_7\right\}$$

$$A(11,10)=\frac{k_3}{C_{ss}M_{ss}}\left\{(m_{ss})^{0.8}\frac{(2.18\times10^{-4}X_{10}-0.40257)}{(1-5.218\times10^{-3}X_9)}+\frac{0.4A_{ss}^2(X_{11}-T_{ssh})X_6}{f_3(m_{ss})^{1.2}}C_6\right\}$$

$$A(11,11)=-\frac{1}{C_{ss}M_{ss}}\,k_3\,(m_{ss})^{0.8}$$

$$A(12,9)=\frac{1}{V_{hp}}\,\frac{cA_v}{(T_{ssk})}C_{15}\cdot C_9$$

$$A(12,10)=\frac{1}{V_{hp}}\,\frac{cA_v}{(T_{ssk})^{1/2}}C_{15}\cdot C_8$$

$$A(12,12)=\frac{1}{V_{hp}}\frac{3.001\times10^{-5}f_4\,m_{ri}^2}{X_{14}^2}$$

$$A(12,14)=\frac{3.001\times10^{-5}}{V_{hp}}\left\{496.57T_{rok}+99336.38\right.$$

$$\left.+496.57X_{14}\frac{(-2.041\times10^{-3}X_{15}+8.967}{(1+4.3036\times10^{-3}X_{14})^2}\right\}$$

$$A(12,15)=-\frac{3.001\times10^{-5}}{V_{hp}}\left\{-29.88+\frac{(-2.041\times10^{-3}X_{15}+8.967)}{(1+4.3036\times10^{-3}X_{14})^2}\right\}$$

$$A(13,9)=\frac{(X_{10}-X_{13}-K_{hp})}{V_{hp}X_{12}}\frac{cA_v}{(T_{ssk})^{1/2}}C_9$$

$$A(13,10)=\frac{1}{V_{hp}X_{12}}\left\{m_t+(X_{10}-X_{13}-K_{hp})\frac{cA_v}{(T_{ssk})^{1/2}}C_8\right\}$$

$$A(13,13)=-\frac{m_t}{X_{12}V_{hp}}$$

$$A(14,9)=\frac{0.8728}{V_{rr}}\frac{cA_v}{(T_{ssk})^{1/2}}C_9$$

$$A(14,10)=\frac{0.8728}{V_{rr}}\frac{cA_v}{(T_{ssk})^{1/2}}C_8$$

$$A(14,14)=-\frac{K_{lp}}{V_{rr}(T_{rok})^{1/2}}\left\{\frac{-2.041\times10^{-3}X_{15}+8.97}{(1+4.3036\times10^{-3}X_{14})^2}\left(496.57X_{14}-\frac{P_{ro}}{2T_{rok}}\right)\right.$$

$$\left.+496.57T_{rok}+99336.38\right\}$$

$$A(14,15) = -\frac{K_{lp}}{V_{rr}(T_{rok})^{1/2}}\left\{\frac{1}{(1+4.3036\times10^{-3}X_{14})}\left(235.5X_{14}-0.23712\frac{P_{ro}}{T_{rok}}\right)-29.88\right\}$$

$$A(15,9) = \frac{0.8728}{V_{rr}X_{14}}\left(X_{13}-X_{15}+\frac{0.8k_4(X_{16}-T_{ro})}{(m_{ri})^{0.2}}\right)\frac{cA_v}{(T_{ssk})^{1/2}}C_9$$

$$A(15,10) = \frac{0.8728}{V_{rr}X_{14}}\left(X_{13}-X_{15}-\frac{0.8k_{rr}(X_{16}-T_{ro})}{(m_{ri})^{0.2}}\right)\frac{cA_v}{(T_{ssk})^{1/2}}C_8$$

$$A(15,12) = \frac{m_{ri}}{V_{rr}X_{14}}$$

$$A(15,14) = -\frac{k_4(m_{ri})^{0.8}}{V_{rr}X_{14}}\left\{\frac{-2.04\times10^{-3}X_{15}+8.97}{(1+4.3036\times10^{-3}X_{14})^2}\right\}$$

$$A(15,15) = -\frac{1}{V_{rr}X_{14}}\left\{m_{ri}+\frac{0.474\,k_4(m_{ri})^{0.8}}{(1+4.3036\times10^{-3}X_{14})}\right\}$$

$$A(15,16) = \frac{k_4(m_{ri})^{0.8}}{V_{rr}X_{14}}$$

$$A(16,9) = -\frac{0.696\,k_4(X_{16}-T_{ro})}{C_{rr}M_{rr}(m_{ri})^{0.2}}\frac{cA_v}{(T_{ssk})^{1/2}}C_9$$

$$A(16,10) = -\frac{0.696\,k_{rr}(X_{16}-T_{ro})}{C_{rr}M_{rr}(m_{ri})^{0.2}}\frac{cA_v}{(T_{ssk})^{1/2}}C_8$$

$$B(8,2) = \frac{\gamma_2 C_f}{C_{ps} M_{ps}}$$

$$B(9,4) = -\frac{C}{V_{s2}} \frac{P_{ssk}}{(T_{ssk})^{1/2}}$$

$$B(10,3) = \frac{(h_c - X_7)}{V_{s2} X_9}$$

$$B(11,2) = \frac{\gamma_3 C_f}{C_{ss} M_{ss}}$$

$$B(12,4) = \frac{C_{15} C}{V_{hp}} \frac{P_{ss}}{(T_{ssk})^{1/2}}$$

$$B(13,4) = (X_{10} - X_{13} - K_{hp}) \frac{C P_{ssh}}{V_{hp} X_{14}}$$

$$B(14,4) = \frac{0.8728 C}{V_{rr}} \frac{P_{ssh}}{(T_{ssk})^{1/2}}$$

$$B(15,4) = \frac{0.8728 C}{V_{rr} X_{14}} \left(X_{13} - X_{15} + \frac{0.8 K_4 (X_{16} - T_{ro})}{(m_{ri})^{0.8}} \right) \frac{P_{ssk}}{(T_{ssk})^{1/2}}$$

$$B(16,2) = \frac{\gamma_4 C_f}{C_{rr} M_{rr}}$$

$$B(16,4) = -\frac{0.696 C}{C_{rr} M_{rr}} \frac{K_4 (X_{16} - T_{ro})}{(m_{ri})^{0.2}} \frac{P_{ssk}}{(T_{ssk})^{1/2}}.$$

$$A(16,14) = \frac{k_4 (m_{ri})^{0.8}}{C_{rr} M_{rr}} \left(\frac{-2.041 \times 10^{-3} X_{15} + 8.967}{1 + 4.3036 \times 10^{-3} X_{14}} \right)$$

$$A(16,15) = \frac{0.4742\, k_4 (m_{ri})^{0.8}}{C_{rr} M_{rr}} \frac{1}{(1 + 4.3036 \times 10^{-3} X_{14})}$$

$$A(16,16) = -\frac{k_4 (m_{ri})^{0.8}}{C_{rr} M_{rr}}$$

The nonzero elements of B matrix:

$$B(1,1) = \frac{V_s}{C_{14}}$$

$$B(2,1) = \frac{X_2}{C_{14}}$$

$$B(3,1) = \frac{(h_{fw} - X_3)}{\rho_{dw} X_1}$$

$$B(5,2) = \frac{\gamma_1 C_f}{C_r M_r}$$

$$B(6,3) = \frac{1}{V_{s1}}$$

Where:

$$V_s = V_d - X_1$$

$$AD2 = 2L_{d1}A_d$$

$$AR2 = 2L_{r1}A_r$$

$$C_{dd} = f_d \frac{L_{d'}}{D_{d'}} + \varepsilon_1 + \varepsilon_2 + 1$$

$$C_{dr} = f_r \frac{L_r}{D_r} + \varepsilon_3 + 1$$

$$E = \frac{A_r}{2L_{r1}A_d^2}$$

$$RALD = \frac{A_{d'}}{L_{d1}}$$

$$RALR = \frac{A_r}{L_{r1}}$$

$$C_1 = 117727.53 + f_2 \frac{m_s^2}{X_2^2 A_{ps}^2}$$

$$C_2 = 955.6375 X_6 \frac{(2.18\times10^{-4} - 0.40257)}{(1-5.218\times10^{-3}X_6)} - 6424.436$$

$$C_3 = 955.6375 X_6 \frac{(1.786 - 2.1\times10^{-3}X_7 + 5.68\times10^{-7}X_7^2)}{(1-5.218\times10^{-3}X_6)^2}$$

$$+ 955.6375\, T_{ps} - 219360.82$$

$$C_4 = -63.175X_2 + 5777.8$$

$$C_5 = h_s - X_7 + \frac{0.8\,k_2(X_8 - T_{ps})}{(m_s)^{0.2}}$$

$$C_6 = -6424.436 + \frac{955.638X_9(2.118\times10^{-4}X_{10} - 0.4025)}{(1 - 5.218\times10^{-3}X_9)}$$

$$C_7 = \frac{955.63X_9(1.7869 - 2.1\times10^{-3}X_{10} + 5.6873\times10^{-7}X_{10})}{(1 - 5.218\times10^{-3}X_9)^2} + 955.638T_{ss}$$

$$-219360.82$$

$$C_8 = \frac{2.1611\times10^{-4}X_{10} - 0.4025}{(1 - 5.218\times10^{-3}X_9)}\left(955.638X_9 - \frac{1}{2}\frac{P_{ssk}}{(T_{ssk})^{1/2}}\right) - 6424.436$$

$$C_9 = \frac{1.786 + 2.1\times10^{-3}X_{10} + 5.687\times10^{-7}X_{10}^2}{(1 - 5.218\times10^{-3}X_9)^2}\left(955.638X_9 - \frac{1}{2}\frac{P_{ssk}}{T_{ssk}}\right)$$

$$+955.638T_{ssk} - 219360$$

$$C_{10} = h_{s2} - X_{10} + \frac{0.8\,k_3(X_{11} - T_{ss})}{(m_{ss})^{0.2}} - m_c\frac{(h_c - X_7)}{m_{ss}}$$

$$C_{11} = \frac{-4.728(h_s - h_{ws}) + 7.475(X_4 - h_{ws})}{(h_s - h_{ws})^2}$$

$$C_{12} = -2.0132gA_d - \frac{2.0132C_{dd}m_w^2}{\rho_{dw}^2 AD2} + RALDC_4$$

$$C_{13} = -g\,A_r + \frac{C_{dr}}{AR2}\,\frac{m_o^2}{\rho_o^2}$$

$$C_{14} = V_d\,\rho_{dw} - 794.45\,X_1$$

$$C_{15} = \left(1 - \frac{5.238\times10^{-5} f_4\, m_{ri}}{X_{12}}\right)$$

APPENDIX F.

Physical data:

l_d : 131.6 m

D_d : 0.62 m.

L_{dl} : 32.9 m

A_d : 0.30175 m^2

L_r : 19350 m

D_r : 1.08187 m

Lrl : 47 m

A_r : 0.91876 m^2

V_d : 22.373 m^3

M_r : 141525 kg

C_r : 0.47 kJ/kg-$^\circ$C

V_r : 43.143 m^3

L_{ps} : 11726 m

D_{ps} : 4.064×10^{-2} m

M_{ps} : 67072 kg

V_{ps} : 15.2 m^3

C_{ps} : 0.47 kJ/kg-$^\circ$C

A_{ps} : $1.29 \times 10^{-3} m^2$

V_{ss} : 9.7 m^3

A_{ss} : 1.17×10^{-3} m^2

L_{rr} : 5170 m

D_{rr} : 5.59×10^{-2} m

M_{rr} : 25000 kg

V_{rr} : 15.503 m^3

A_{rr} : 2.453×10^{-3} m^2

V_{hp} : 1.7 m^3

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