DISRIBUTED AMPLIFIERS FOR WIDE-BAND AMPLIFICATION

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PREFACE

This thesis was prepared in accordance with the requirements of the Engineering Graduate School program of Robert College. The subject of the thesis is Distributed Amplifiers for Wide-band Amplification. This is a new method of design where the grid-to-cathode and plate-to-cathode capacitances are used as the shunting elements of artificial transmission lines. In this manner the tube gains add instead of multiplying as is the case for the cascaded amplifiers. The advantage of this system is that even if the gain of individual tubes is less than unity, the adding of gains will provide a sufficient amplification.

This thesis consists of 10 chapters; the first chapter is an introduction showing the deficiencies of conventional amplifiers for the high frequencies, chapters 2, 3 and 4 deal with the basic principles and different characteristics of the amplifier, chapter 5 introduces the principle of staggering, that is, the cutoff frequencies of the plate and grid lines are made slightly different, chapter 6 deals with the effect of improper termination of the lines, chapter 7 shows the results due to increase in frequency, chapter 8, deals with noise in the amplifiers, chapter 9 introduces the theory of transistorized distributed amplifiers and chapter 10 gives two practical designs of distributed amplifiers.

I would like to express my deep gratitude to Prof. Dr. Mustafa Santur for his valuable suggestions and urgings in the preparation of this thesis.

Josef Kasuto

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Chapter 1

INTRODUCTION

As science is progressing continuously and communication is beginning to be done with frequencies of the order of hundreds and thousands of megacycles, the need for amplifiers that could amplify such frequencies also arises. Conventional amplifiers can achieve amplification at these frequencies only at the expense of gain due to the gain-bandwidth product of an amplification stage. When the stage gain has been made unity or less, this stage is no longer useful for the purpose of amplification. Furthermore, since the gain in cascaded amplifiers is the product of individual stage gains, we get no advantage in cascading any number of such stages. To show this, let us compute approximately the relation between gain, bandwidth and number of stages in a conventional uncompensated amplifier. The gain of n stages is

$$A = (g_m R_L)^n \qquad (1\text{-}1)$$

where g_m is the transconductance of the tube
R_L is the load resistance of a stage
n is the number of stages

We also know that the result of cascading n stages is to decrease the bandwidth by approximately $\sqrt{n}$. The upper 3-db frequency of an amplifier which has n stages will therefore be

$$f = \frac{1}{2\pi R_L C_s \sqrt{n}} \qquad (1\text{-}2)$$

where C_s is the sum of the input and output capacitances of one stage. The "figure of merit" of a tube "F" can be defined as the gain-bandwidth product and is given by [1]

$$\text{Gain} \times \text{bandwidth} = F = \frac{g_m}{2\pi C_s} \qquad (1\text{-}3)$$

(1) F.E. Terman, "Electronic and Radio Engineering" - p. 291

Combining equations (1-1), (1-2) and (1-3) we have

$$f_2 \, A^{1/n} \sqrt{n} = F \tag{1-4}$$

As an example to the limitations of the bandwidth of a conventional amplifier let us calculate the possible bandwidth in an amplifier where the required gain is, for example, $e^2 = (2.72)^2 = 7.4$. Using the tube 6AK5 for which g_m is 5.1×10^{-3} mhos, input capacitance is 4 μμF and output capacitance is 2.8 μμF, we have 6.8 μμF for the sum of the two capacitances. Being a little unrealistic, let us also assume that the stray capacitances are zero. With these considerations we can compute the figure of merit "F" from eq.(1-3) as

$$F = \frac{g_m}{2\pi C_s} = \frac{5.1 \times 10^{-3}}{2\pi \times 6.8 \times 10^{-12}} = 120 \times 10^6 \ \mathrm{sec}^{-1}$$

Computing from equation (1-4) that f_2 will be a maximum for a given value of gain if

$$n = 2 \ln A \tag{1-5}$$

for a gain $A = e^2$, n will be 4, and from equation (1-4) f_2 will be 36.4 Mc. We can conclude that, although we are using a tube having a high figure of merit, and we assume that we reduce all stray capacitances to zero, it is not possible to build a conventional amplifier having a gain of 7.4 with a bandwidth exceeding 36.4 Mc. We can of course remedy this situation to some extent by using some kind of high-frequency compensation but this does not procure some valuable advantage.

To be able to get bandwidths larger than the bandwidths obtained with conventional amplifiers, a new principle, called distributed amplification, has been designed where ordinary electron tubes are placed along artificial transmission lines, thus cancelling the effect of gain-bandwidth product which was a handicap.

Chapter 2

BASIC PRINCIPLES

As shown above, and as shown by Wheeler and others, the frequency limit of a conventional uncompensated amplifier is proportional to the ratio of the transconductance of the tube, g_m, to the square root of the product of the input and output capacitances. We have seen that it is not to our advantage to cascade the different tubes. It is not also to our advantage to simply parallel the tubes, because in that case the increase in g_m is compensated for by the corresponding increase in the combined capacitance of the tubes.

This difficulty is overcomed by combining the tubes in the manner shown in Fig. 2-1. The capacitances of the tubes are separated as the shunting elements of an artificial transmission line while the transconductances

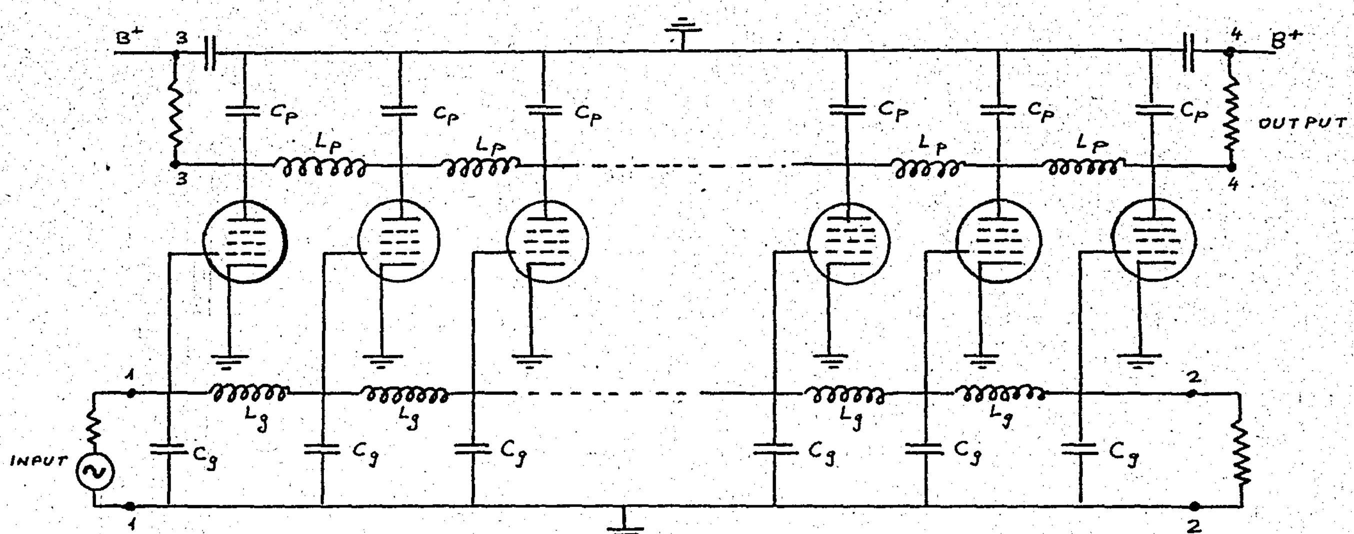


FIG 2-1 — BASIC DISTRIBUTED AMPLIFIER

(g_m) of the tubes may be added almost without limit. Between the input terminals 1-1, and terminals 2-2 there is an artificial transmission line, which is composed of the grid-cathode capacitances of the tubes, C_g, as the shunting elements and of the inductances between tubes (or sections). L_g. In this case the characteristic impedance of the grid line will be as follows:

$$Z_{o_g} = \sqrt{\frac{L_g}{C_g}} \tag{2-1}$$

If the proper terminating impedance, that is, the characteristic impedance of the line is connected across terminals 2-2, there will be no reflections from these terminals. If we assume also that the transmission line is dissipationless, it can be shown that the driving point impedance at terminals 1-1 is independent of the number of sections so connected. In a similar fashion, a second transmission line is formed by making use of the plate-to-cathode capacitances to shunt another set of coils L_p. The characteristic impedance of the plate line is similarly independent of the number of sections and its value is

$$Z_{o_p} = \sqrt{\frac{L_p}{C_p}} \tag{2-2}$$

Impedances connected across terminals 3-3 and 4-4 are made equal to Z_{o_p}. The impedance connected to terminals 2-2 will be called the grid termination; the one connected to terminals 3-3 will be called the reverse termination; and that connected across terminals 4-4 will be called the plate termination. These terminals are at the same time the output terminals.

These two artificial transmission lines are designed so as to have the same velocity of propagation v_p given as

$$v_p = \frac{1}{\sqrt{L_g C_g}} = \frac{1}{\sqrt{L_p C_p}} \tag{2-3}$$

In Fig. 2-1 each tube, with its portion of transmission line is called a section. The combination of n such sections is called a stage.

A signal voltage e_g applied across terminals 1-1, which are the input terminals, will cause a wave to travel along the grid line, applying voltages e_g to the control grids of the successive tubes with a progressive phase difference determined by the phase shift per section of line. The resulting plate current $e_g g_m$ thereby produced in each tube divides, half flowing toward the terminating resistance (reverse termination) at terminals 3-3, half flowing toward the load impedance (plate termination) at terminals 4-4. Because of the phase differences, the currents flowing to the left towards the reverse termination from the various tubes cancel each other to a large extent, and whatever is not cancelled is absorbed by the reverse termination. However, the currents flowing toward the output terminals 4-4 from the various tubes produce waves that all add together at the load. This is because the phase difference in each section of the plate line is the same as in each section of the grid line. Thus, the output voltage is directly proportional to the number of tubes. The net result is that the effective g_m of this distributed "stage" may be increased to any desired limit. We can therefore say that, no matter how low the gain of each tube (section) is, (even if it is less than unity), as long as the gain per section is greater than the transmission-line loss of the section, the signal in the plate line will increase and can be made to be as large as one desires by merely using a sufficient number of tubes. If we denote the load impedance by Z_L, and considering that this impedance is equal to the characteristic impedance Z_{o_p}, the output voltage of such a stage consisting of n sections will be

$$\text{Output Voltage} = \frac{e_g g_m Z_L n}{2} \qquad (2\text{-}4)$$

and the amplification per stage is

$$A = \frac{n g_m Z_L}{2} \qquad (2\text{-}5)$$

Examination of equation (2-5) shows that by making the number of tubes sufficiently large, the output voltage will exceed the input voltage e_g, even when the gain per tube is much less than unity. When sufficient gain has been accumulated in one distributed amplifier stage, then such stages can be cascaded in the normal manner as shown in Fig. 2-2,

The bandwidth of the distributed amplifier is determined by the cutoff frequency of the artificial line. In general, the higher this cutoff frequency, the lower will be the characteristic impedance of the line, and hencethe less the voltage gain. The constancy of the amplification with frequency below the cutoff value will depend on the extent to which the characteristic impedance of the artificial transmission line is constant with the variation in frequency and upon the exactness of the impedance

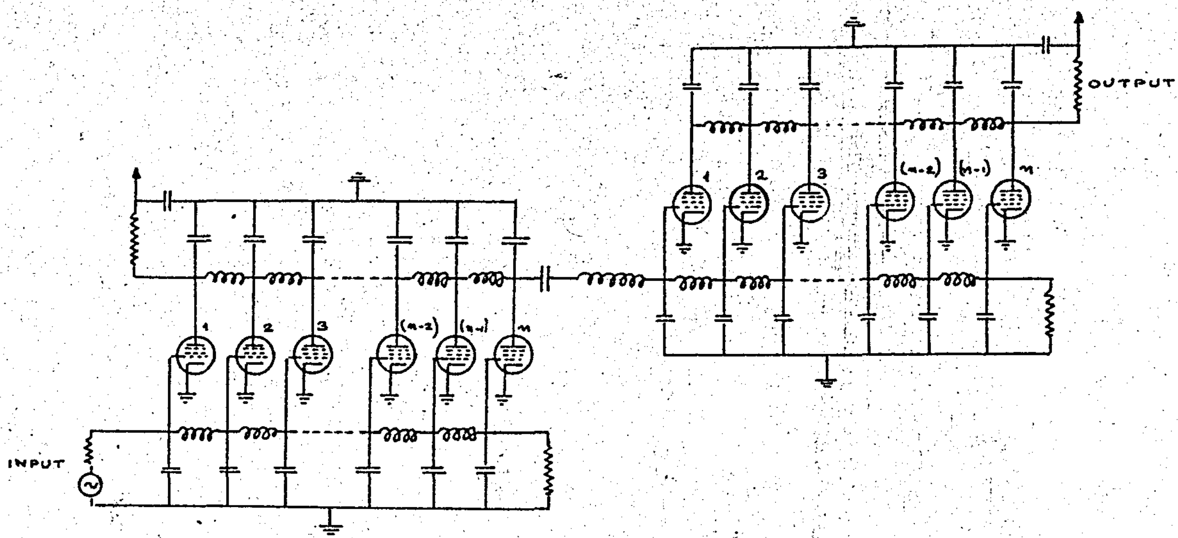


FIG. 2-2 — TWO STAGE DISTRIBUTED AMPLIFIER, HAVING n TUBES PER STAGE.

matches that exist at the line terminals.

Chapter 3

CASCADING OF STAGES

We can easily show that there is an optimum method of dividing the tubes into groups.

Let us assume that we have an amplifier having n tubes per stage, and that it has m such stages. We will have a total of mn tubes in this amplifier. Let us also assume that it is possible to match impedances between the generator and the grid transmission line and between stages.

If the voltage that is applied to the grid line is e_g, then the current that will flow in each plate circuit will be $e_g g_m$. The impedance that appears between the plate and cathode of each tube is $Z_{op}/2$. Thus, the voltage developed by a single tube is $e_g g_m Z_{o_p}/2$. Therefore, the gain of the stage is

$$A = \frac{n g_m Z_{op}}{2} \tag{3-1}$$

However, if such stages are to be cascaded, then, in general, a transformer must be provided to match the plate line to the grid line of the next stage. Thus the voltage at the grid of the next stage will be

$$e_{g_2} = \frac{n e_g g_m Z_{op}}{2} \sqrt{\frac{Z_{og}}{Z_{op}}} \tag{3-2}$$

Hence, the gain of a single stage measured from grid line to grid line is

$$A = \frac{n g_m}{2} \sqrt{Z_{o_p} Z_{o_g}} \tag{3-3}$$

If such stages are cascaded m times, then the resultant gain of the cascaded stages will be

$$G = A^m = \left(\frac{n g_m}{2} \sqrt{Z_{o_p} Z_{o_g}} \right)^m \tag{3-4}$$

We can now make use of the fact that Z_{o_p} and Z_{o_g} are not really independent variables. More fundamental parameters will be the grid-to-cathod capacitance C_g, plate-to-cathode capacitance C_p, and the desired cut-off frequency f_c. Using these, the characteristic impedance of the transmission lines can be written in terms of f_c, C_p and C_g. It then follows that

$$A = \frac{n}{2f_c} \frac{g_m}{\pi\sqrt{C_p C_g}} \tag{3-5}$$

It is convenient to express the high-frequency figure of merit of a tube as a "bandwidth index frequency" or sometimes called the "Wheeler's bandwidth index frequency f_o"(1) and this is defined as

$$f_o = \frac{g_m}{\pi\sqrt{C_g C_p}} \tag{3-6}$$

Using this definition we can rewrite equations (3-4) and (3-5) as

$$A = n\frac{f_o}{2f_c} \tag{3-7}$$

$$G = \left(n\frac{f_o}{2f_c}\right)^m \tag{3-8}$$

The total number of electron tubes in a cascaded amplifier is $N = mn$. We want to determine the least number of tubes required to produce a given gain. We can do this as follows.

If the gain per stage is

$$G^{1/m} = n\frac{f_o}{2f_c} \tag{3-9}$$

Solving for n we get

$$n = \frac{2f_c}{f_o} G^{1/m} \tag{3-10}$$

Hence

$$N = m G^{1/m} \frac{2f_c}{f_o} \tag{3-11}$$

If we differenciate (3-11) with respect to m and set the resultant equal to zero, we find that the smallest N is obtained when

$$m = \ln G \tag{3-12}$$

From this and equation (3-9) it follows that the corresponding

(1) H. A. Wheeler, "Wide-band amplifiers for television", Proc. IRE, vol. 27, pp. 429 - 438, July, 1939.

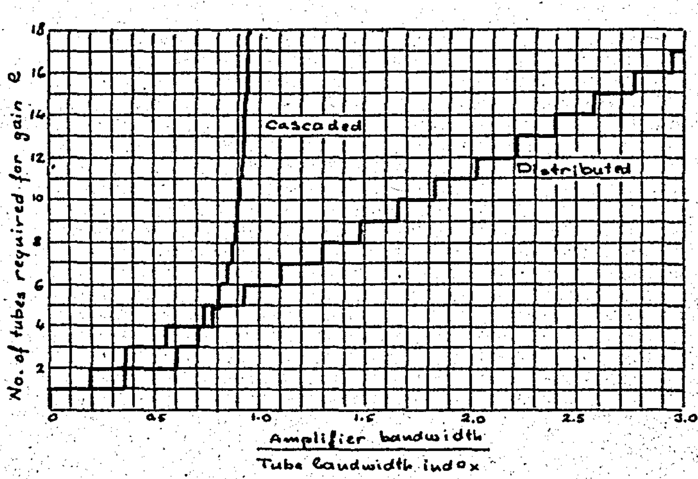


FIG. 3-1 — NUMBER OF TUBES REQUIRED TO PRODUCE A GAIN OF e IN CASCADED AND DISTRIBUTED AMPLIFIERS

number of sections per stage is

$$n = \frac{2f_c}{f_o}\, e \qquad (3\text{-}13)$$

From equation (3-12) it follows that, for optimum utilization of tubes, the gain of each stage should be e .

The number of sections required to produce a gain of e is plotted in Fig. 3-1 for the case under discussion, and also for the conventional cascaded amplifier.

It can be clearly seen from this plot that the distributed amplifier is the only means available for amplification when the maximum frequency desired is greater than the bandwidth index frequency of the tube being used. Furthermore, it is usually found impractical to achieve much more than 50% of the theoretically available bandwidth with conventional circuits. This is so because the theoretical limit requires the use of extremely complex coupling circuits, which can hardly be considered practical and which increase the stray capacitances to ground. This is not so with the distributed amplifier.

The basic principles discussed above are for the low-pass filter structure. But it is obvious that the same principles can be applied to the band-pass filters. The advantage of the distributed amplifier is that it can be made to operate, even in cascaded form, from the very high frequencies, down to very low frequencies, and even DC, by utilising the well-known direct-coupling techniques.

Chapter 4

FREQUENCY-RESPONSE CHARACTERISTICS

To analyse the frequency-response characteristics of the distributed amplifiers, we will carry out the discussions and derivations in terms of the low-pass structures of the types shown in Figs. 2-1 and 2-2. However, as most of the equations are of general type, they can be made applicable to other possible structures by simple modifications.

The voltage gain of an amplifier consisting of n sections per stage and m cascaded stages is, as derived in the previous chapter

$$G = \left(\frac{n g_m}{2} \sqrt{Z_{o_g} Z_{o_p}} \right)^m \qquad (4\text{-}1)$$

where, as before

G = total gain

Z_{o_p} = characteristic impedance of the plate line

Z_{o_g} = characteristic impedance of the grid line.

For the case shown in Figs. 2-1 and 2-2, and assuming that the two transmission lines are identical

$$Z_{o_p} = Z_{o_g} = \frac{R}{\sqrt{1 - x_k^2}} \qquad (4\text{-}2)$$

where

$x_k = \frac{f}{f_c}$

$R = \frac{1}{\pi f_c C}$

f = frequency

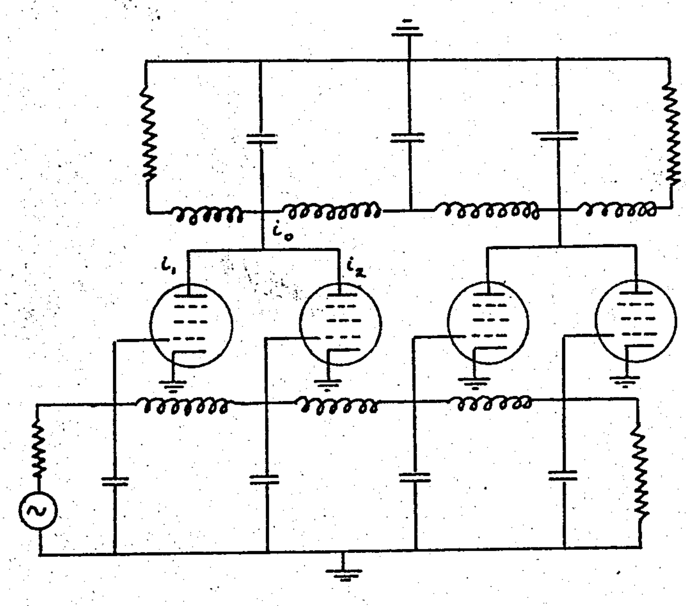
f_c = cutoff frequency of the transmission line

Substituting these in equation (4-1), we get for the gain of the distributed amplifier

$$G = \left[\frac{n g_m}{2} R \right]^m \left(1 - x_k^2 \right)^{-m/2} \qquad (4\text{-}3)$$

The second factor of this equation shows that the gain of the simpl structures shown in Figs. 2-1 and 2-2 will be a function of frequency. This is due to the fact that the mid-shunt characteristic impedance of a constant-k filter section rises rapidly as the cutoff frequency is approached(1). This, in turn, causes the gain of the amplifier to increase sharply near cutoff, producing a large undesired peak. In principle we can equalize this peak, but this becomes very difficult if we want to increase the number of cascaded stages.

There are several methods to eliminate this undesired peak. The



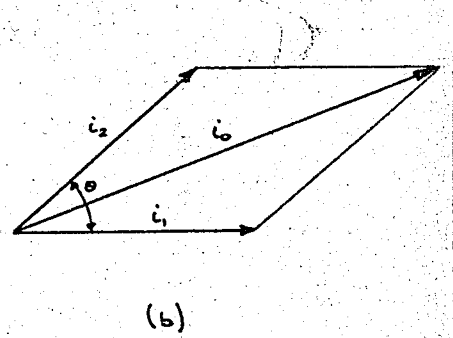


Fig. 4-1 — (a) Paired plate type of distributed amplifier. (b) Current phase relations in paired plate amplifier.

methods we will discuss below are the following: a) Paired-plate or paired-grid connection; b) Negative mutual inductance circuit; c) The bridged-tee connection.

a) Paired-plate or paired-grid connection

In Fig. 4-1 we see a different arrangement of electron tubes along the transmission lines from those discussed previously. The grids of the tubes are still connected periodically along the grid line, but the

(1) W. L. Everitt and G.E. Anner, "Communication Engineering", McGraw-Hill Book Co., pp. 267 - 268, Tokyo, 1956.

plates are paired as shown, with a dummy capacitance being placed at the point where the plate capacitance is missing. This particular arrangement of tubes is called the "paired-plate" connection. We can also pair the grids and leave the plates connected periodically. This connection is called the "paired-grid" connection. As the action of the two circuits is similar, let us analyse the paired-plate connection only.

We can understand the operation of the paired-plate circuit by referring to the vector diagram of the plate currents at the common junction, shown in Fig. 4-1(b). Let i_1 be the current in one of the tubes, and i_2 the current in the other. The phase angle between i_1 and i_2 is determined by the phase shift between the grids of the two tubes, which is the phase shift per section and is given by

$$\theta = 2 \sin^{-1} x_k \tag{4-4}$$

where x_k is the normalized frequency of the section, as defined previously. Supposing that i_1 and i_2 are equal in magnitude, the resultant current vector is a function of x_k and is

$$i_o = 2i\sqrt{1 - x_k^2} \tag{4-5}$$

It can be easily seen that this factor is the reciprocal of the characteristic impedance function, Z_{o_p}, of the section. Thus, the voltage developed in the plate line, being the product of i_o and Z_{o_p} will be constant over the pass-band of the filter.

If we leave some of the plates unpaired, the gain of a stage can be made to have a frequency response which is intermideate between the plate characteristic of the completely paired stage and the rising characteristics of the constant-k sections. The control of the degree of rise in gain is a very valuable feature of this circuit. This increase in gain can be used to compensate for the decrease in gain which is due to attenuation in the transmission line at high frequencies.

Since the plate-to-cathode capacitance of most pentodes is about one half of the grid-to-cathode capacitance, the addition of extra capacitance in the plate line does not reduce materially the design cutoff frequency.

b) Negative mutual inductance circuit[a]

This method, used to improve the frequency response, is slightly

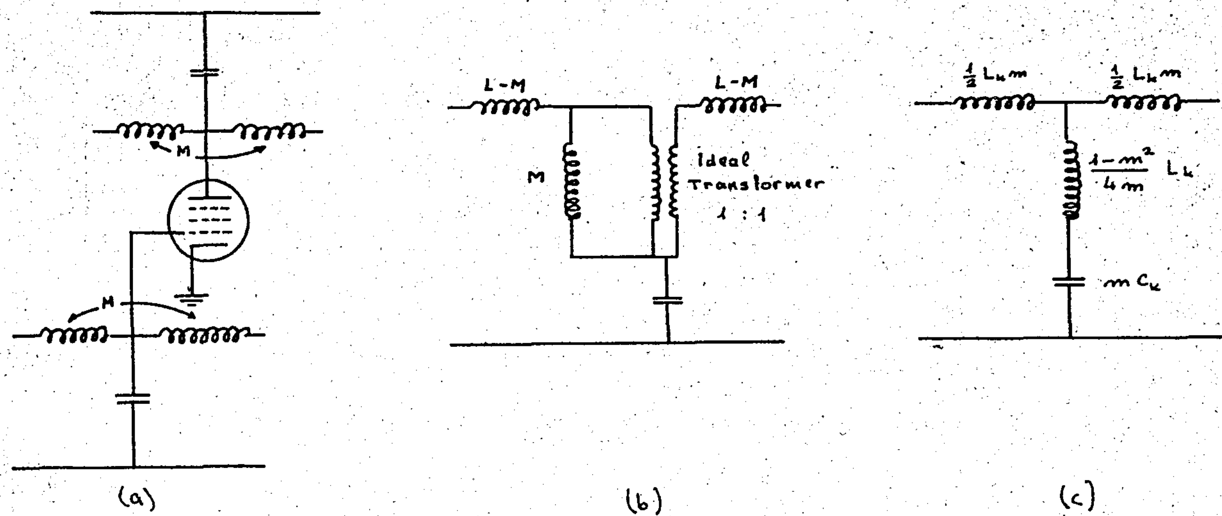


Fig 4-2 — (a) Circuit using mutual coupling between coils; (b) Circuit equivalent to (a) by transformer theory; (c) m-derived filter circuit equivalent to (a) and (b).

more complicated from the original design viewpoint, but has several desirable features which are considered to be of great importance.

The basic connection is shown in Fig. 4-2(a), in which the only difference from Fig. 2-1 is that the adjacent coils are wound on the same form and in the same direction, and have a large coefficient of coupling. We can resolve each section into the form of Fig. 4-2(b) by the conventional transformer theory. By proper design, we can equate this to the usual m-derived section shown in Fig. 4-2(c). If the mutual inductance is negative, as it is in the case we are discussing, the constant m will be larger than unity. This has two advantages. First, m greater than unity leads to a more linear phase shift through the stage. This becomes particularly important if a large number of stages are to be cascaded. Second, m greater than unity leads to a larger value of capacitance C than would be called for if constant-k sections were to be used instead. For a given capacitance C,

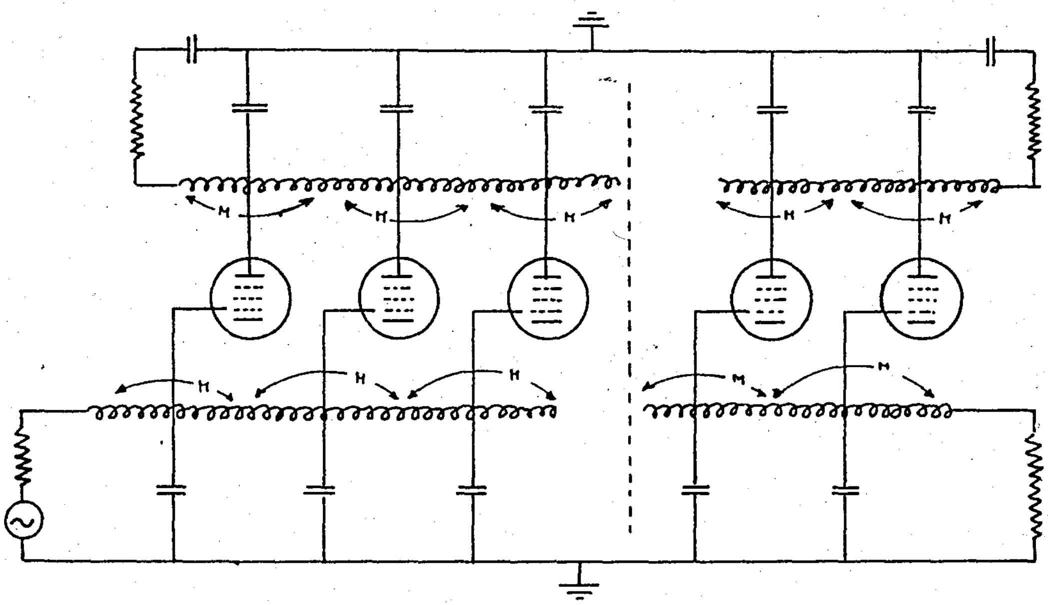


FIG. 4-3 — AN n-STAGE DISTRIBUTED AMPLIFIER USING MUTUAL COUPLING BETWEEN COILS.

then, it is possible either to increase the gain per section for the same bandwidth, or to increase the bandwidth for the same gain.

Let us now compute the grid-to-plate gain, and the phase shift for a stage having n sections connected as shown in Fig. 4-3. To perform this it is necessary to calculate the transfer characteristic of one section, i.e., the voltage developed per section of plate line per volt in grid line. From the circuit for one section of Fig. 4-4 we have for thegrid drive voltage e_g

$$e_g = (i_1 - i_2)\frac{1}{j\omega C_g} = \frac{e_o}{Z_o}\left(1 - e^{-j\theta}\right)\frac{1}{j\omega C_k m} \qquad (4\text{-}6)$$

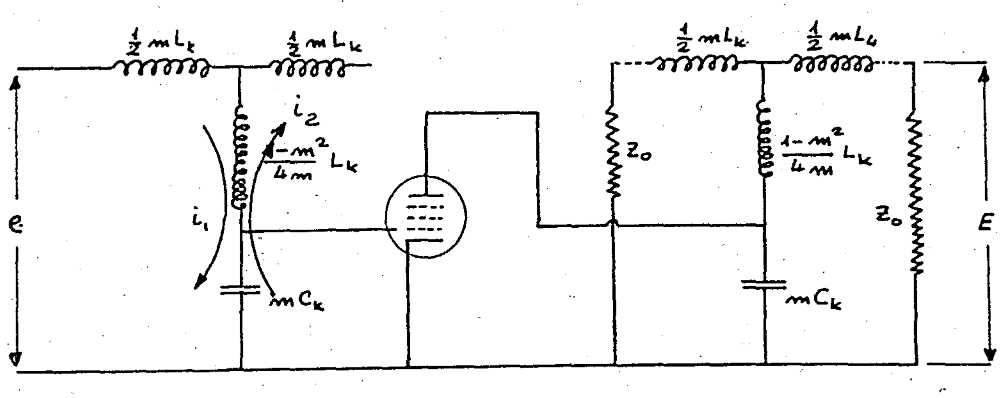


FIG. 4-4 — NEGATIVE MUTUAL INDUCTANCE CONNECTION AND SYMBOLS where

$$Z_o = R_o\sqrt{1 - x_m^2} \qquad x_m = \pi f R_o C_k \tag{4-7}$$

and

$$\theta = 2 \tan^{-1} \frac{m x_m}{\sqrt{1 - x_m^2}}$$

the phase shift per section of line, but

$$(1 - e^{-j\theta}) = \frac{2 j m x_m}{\sqrt{1 - x_m^2} + j m x_m},$$

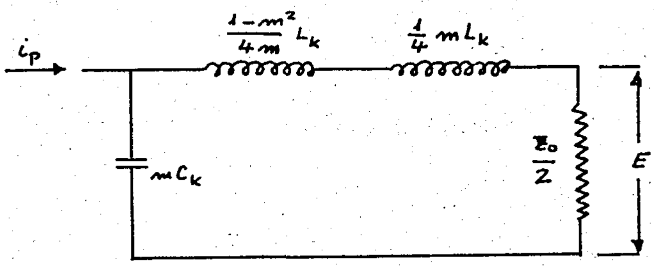


FIG. 4-5 — EQUIVALENT PLATE CIRCUIT OF THE NEGATIVE MUTUAL INDUCTANCE CONNECTION.

or

$$\frac{e_g}{e_o} = \frac{1}{\sqrt{1 - x_m^2}} \cdot \frac{1}{\sqrt{1 - (1-m^2)\, x_m^2}} \angle \tan^{-1} \frac{m x_m}{\sqrt{1 - x_m^2}} \tag{4-8}$$

The voltage developed per section of plate line may be readily calculated from the redrawn plate circuit shown in Fig. 4-5.

$$E = i_p \frac{\frac{Z_o}{2} \cdot \frac{1}{j\omega m C_k}}{\frac{1}{j\omega m C_k} + \frac{Z_o}{2} + j\omega \frac{L_k}{4m}}$$

$$= \frac{i_p R_o}{2} \frac{m x_m}{\sqrt{1 - (1-m^2)\, x_m^2}} \angle -\tan^{-1} \frac{m x_m}{\sqrt{1 - x_m^2}} \tag{4-9}$$

but

$$i_p = e_g g_m$$

Thus the transfer characteristic is given by

$$\frac{E}{e_o} = \frac{g_m R_o}{2} \frac{1}{\sqrt{1 - x_m^2\, [1 - (1-m^2)\, x_m^2]}} \angle -\theta \tag{4-10}$$

The delay per section τ is given by

$$\tau = \frac{d\theta}{d\omega} = \frac{d\theta}{dx_m} \cdot \frac{1}{\omega_c}$$

$$= \frac{2}{\omega_c} \cdot \frac{m}{\sqrt{1 - x_m^2}\left[1 - (1 - m^2) x_m^2\right]} \tag{4-11}$$

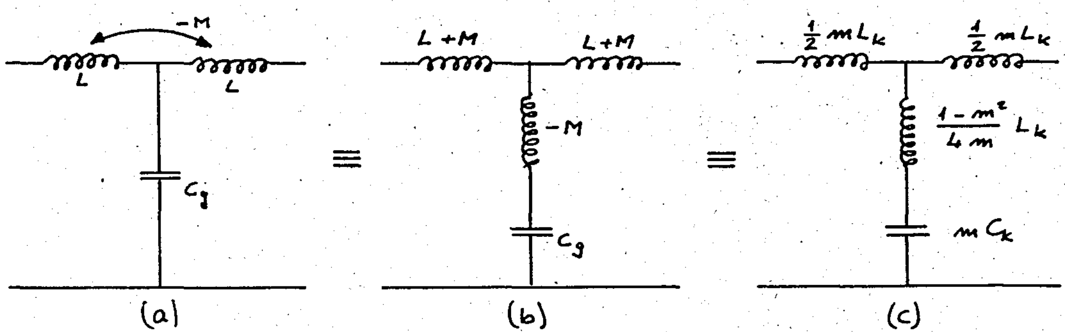


FIG. 4-6 — NEGATIVE MUTUAL INDUCTANCE CONNECTION AND ITS m-DERIVED EQUIVALENTS.

Equating the physical structure against the desired structure as shown in Fig. 4-6, it is evident that

$$M = \frac{m^2 - 1}{4m} L_k \qquad L = \frac{m^2 + 1}{4m} L_k \qquad C_g = m C_k \tag{4-12}$$

In a normal constant-k section, not m-derived

$$x_k = \pi f R_o C_k = \pi f R_o C_g \tag{4-13}$$

In the above m-derived structure

$$x_m = \pi f R_o C_k = \pi f R_o \frac{C_g}{m} = \frac{x_k}{m} \tag{4-14}$$

Then substituting $x_m = \frac{x_k}{m}$ in the equations for amplitude response, phase shift, and phase delay so that the result may be compared to constant-k operation it is found that

$$\frac{E}{e_o} = \frac{g_m R_o}{2} \cdot \frac{m^3}{\left[m^2 - (1 - m^2) x_k^2\right]\sqrt{m^2 - x_k^2}} \angle -\theta_2 \tag{4-15}$$

$$\theta = 2 \tan^{-1} \frac{m x_k}{\sqrt{m^2 - x_k^2}} \tag{4-16}$$

$$\tau = \frac{1}{\pi f_c} \frac{m^3}{\left[m^2 - (1 - m^2) x_k^2\right]\sqrt{m^2 - x_k^2}} \tag{4-17}$$

where f_c is equal to

$$f_c = \frac{1}{\pi R_o C_g}$$

These values being for one section, for the total gain and phase shift of the stage of amplifier having n sections, as shown in Fig. 4-3 we have

$$G = \frac{R_o g_m}{2} \cdot \frac{n m^3}{\left[m^2 - (1-m^2) x_k^2\right] \sqrt{m^2 - x_k^2}} \qquad (4\text{-}18)$$

$$\phi = 2n \tan^{-1} \frac{m x_k}{\sqrt{m^2 - x_k^2}} \qquad (4\text{-}19)$$

where

$$R_o = \frac{K}{\pi f_m C_g} \qquad (4\text{-}20)$$

$$L_k = \frac{C_g R_o^2}{m} \qquad (4\text{-}21)$$

$$L = \frac{1+m^2}{4m} L_k \qquad M = \frac{1-m^2}{4m} L_k \qquad (4\text{-}22)$$

in which

f_m = maximum frequency required with amplitude of phase tolerance e

K = coverage factor to be determined from Fig. 4-9 for a desired value of tolerance e

$x_k = f/f_o = (f/f_m)K$ normalized frequency function

m = Design parameter selected from Fig. 4-9 for desired e

The time delay through the stage will be given by the derivative of the phase shift with respect to angular frequency. This is

$$\tau = \frac{d\phi}{d\omega} = \frac{d\phi}{dx_k} \cdot \frac{1}{\omega_o} = \frac{n\, d\theta}{d x_k} \cdot \frac{1}{\omega_o}$$

$$= \frac{n m^3}{\left[m^2 - (1-m^2) x_k^2\right] \sqrt{m^2 - x_k^2}\, \pi f_o} \qquad (4\text{-}23)$$

It is interesting to observe that both gain and delay functions are the same except for numerical constants. Figs. 4-7 and 4-8 show the relative gain, time delay and phase shift as a function of normalized frequency x_k for four values of m.

Fig. 4-9 is designed to permit the selection of any desired

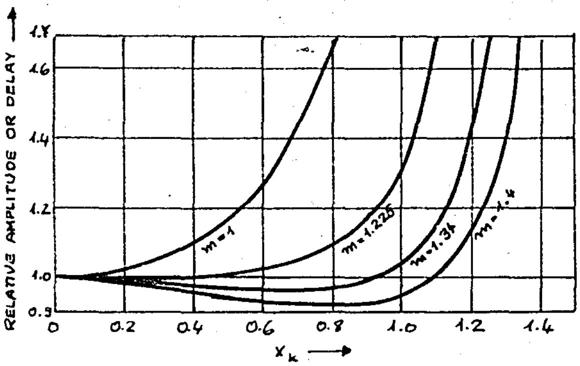


FIG. 4-7 — RELATIVE GAIN AND TIME DELAY OF AMPLIFIER WITH MUTUAL COUPLING VERSUS NORMALIZED FREQUENCY.

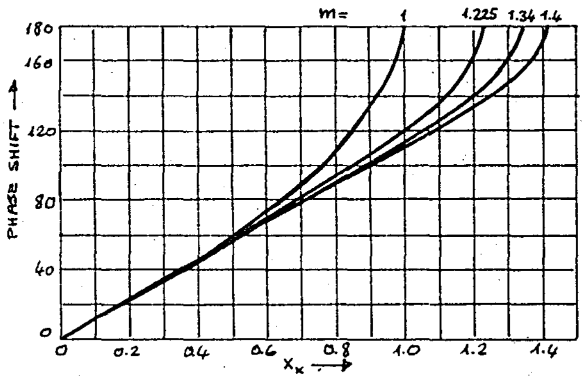


FIG. 4-8 — PHASE SHIFT OF AMPLIFIER USING MUTUAL COUPLING VERSUS NORMALIZED FREQUENCY.

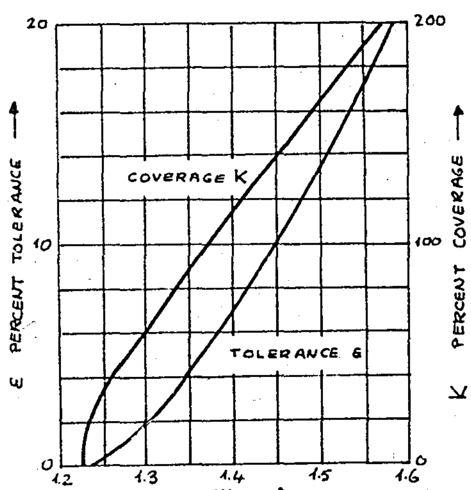


FIG. 4-9 — PERCENT TOLERANCE OR PHASE LINEARITY AND PERCENT OF BAND COVERED FOR AMPLIFIER WITH MUTUAL COUPLING.

tolerance in either phase or amplitude linearity as a function of percent band coverage K over which tolerance may be maintained.

c) The bridged-tee connection

The third method of equalizing frequency response is by means of the bridged-tee connection shown in Figs. 4-10 and 4-11(a). By simple transformer theory this is equivalent to the circuits shown in Figs. 4-11(b) and 4-11(c). Fig. 4-11(c) corresponds to a line having mutual coupling between coils and shunted by an impedance Z_c.

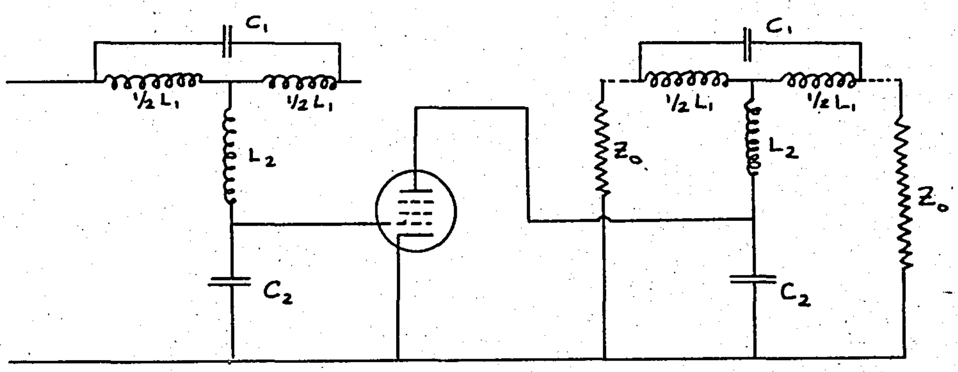


FIG. 4-10 — BRIDGED-TEE CONNECTION

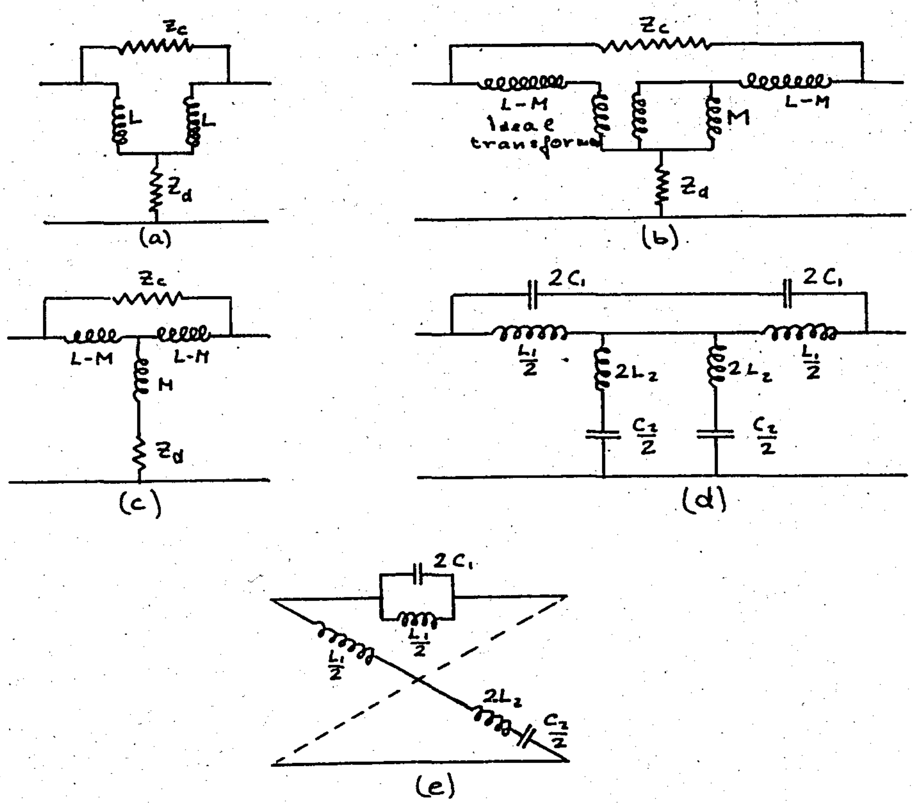


FIG. 4-11 — EQUIVALENT CIRCUITS FOR BRIDGED-TEE CONNECTION.

If Z_c is the capacitance C_1 and Z_d is C_g, the tube capacitance, then, using Fig. 4-11(d), the circuit can be converted into the lattice shown in Fig. 4-11(e), having the arms

$$Z_a = \frac{\frac{1}{4}\frac{L_1}{C_1}}{\frac{1}{2} j\omega L_1 + \frac{1}{2j\omega C_1}}$$

$$Z_b = \frac{1}{2} j\omega L_1 (1+\alpha) + \frac{2}{j\omega C_g} \qquad (4\text{-}24)$$

where

$$\alpha = 4\frac{L_2}{L_1}$$

The characteristic impedance of this lattice network is given by

$$Z_o = \sqrt{Z_a Z_b} = \sqrt{\frac{L_1}{C_g}\,\frac{1-\omega^2\frac{L_1 C_g}{4}(1-\alpha)}{1-\omega^2 L_1 C_g}} \qquad (4\text{-}25)$$

If $C_g(1+\alpha) = 4C_1$, this equation is independent of frequency and the value of Z_o becomes

$$Z_o = \sqrt{\frac{L_1}{C_g}} = R_o \qquad (4\text{-}26)$$

The propagation constant γ of a lattice is defined as

$$\gamma = \ln\left[\frac{1+\sqrt{\frac{Z_a}{Z_b}}}{1-\sqrt{\frac{Z_a}{Z_b}}}\right] = 2\tan^{-1}\sqrt{\frac{Z_a}{Z_b}} \qquad (4\text{-}27)$$

but when

$$C_g(1+\alpha) = 4C_1 \quad ,$$

$$\sqrt{\frac{Z_a}{Z_b}} = \frac{j x_k}{1 - x_k^2(1+\alpha)}$$

where x_k is defined as before as $\pi f R_o C_g$

As $\sqrt{\frac{Z_a}{Z_b}}$ is always imaginary, the propagation constant γ is imaginary and thus represents only a phase shift with no attenuation, i,e., the characteristics of a low-pass filter section. The phase shift θ is then

$$\Theta = 2 \tan^{-1} \frac{X_k}{1 - X_k^2 (1+\alpha)} \qquad (4\text{-}28)$$

and the delay τ is

$$\tau = \frac{d\Theta}{d\omega} = \frac{1}{\pi f_c} \frac{1 + X_k^2 (1+\alpha)}{\left[1 - X_k^2 (1+\alpha)\right]^2 + X_k^2} \quad \text{seconds} \qquad (4\text{-}29)$$

The grid drive is calculated in the same fashion as the case for the negative mutual inductance connection, with the exception that part of the input current flows in the bridging arm as shown in Fig. 4-12. Thus the net current through the capacitor is

$$(i_1 - i_3) - (i_2 - i_3) = (i_1 - i_2)$$

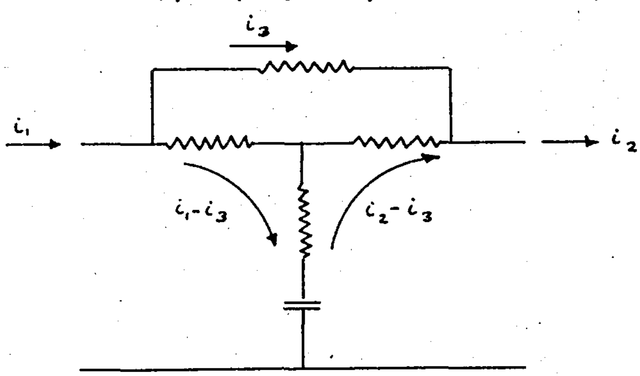


FIG. 4-12 – BRIDGED – TEE CURRENT CONNECTIONS

Therefore

$$e_g = (i_1 - i_2) \frac{1}{j\omega C_2} \qquad (4\text{-}30)$$

or

$$\frac{e_g}{e_o} = \frac{1}{j\omega C_2 Z_o} \left(1 - e^{-j\Theta}\right) \qquad (4\text{-}31)$$

or

$$\frac{e_g}{e_o} = \frac{1}{\sqrt{\left[1 - X_k^2 (1+\alpha)\right]^2 + X_k^2}} \angle -\tan^{-1} \frac{X_k}{1 - X_k^2 (1+\alpha)} \qquad (4\text{-}32)$$

The voltage developed per section of plate line may be readily calculated from the redrawn plate circuit shown in Fig. 4-13. As no voltage difference exists across the two ends of the bridging arm due to the current i_p it may be omitted, allowing the series arms and terminating resistors to be combined in parallel. Thus,

$$E = i_p \frac{\frac{Z_o}{2} \cdot \frac{1}{j\omega C_2}}{\frac{Z_o}{2} + \frac{1}{j\omega C_2} + \frac{1}{4} j\omega L_1 (1+\alpha)}$$

$$= \frac{i_p R_o}{2} \frac{1}{\sqrt{[1 - x_k^2(1+\alpha)]^2 + x_k^2}} \angle \tan^{-1} \frac{x_k}{1 - x_k^2(1+\alpha)}$$

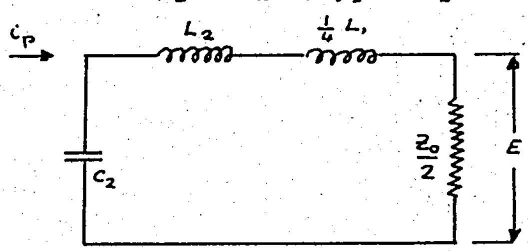


FIG. 4-13 — EQUIVALENT PLATE CIRCUIT OF THE BRIDGED-TEE CONNECTION

but

$$i_p = e_g g_m \tag{4-33}$$

Thus the transfer characteristic is given by

$$\frac{E}{e_o} = \frac{g_m R_o}{2} \frac{1}{[1 - x_k^2 (1+\alpha)]^2 + x_k^2} \angle -\theta \tag{4-34}$$

The stage gain A for n sections will be

$$A = n \frac{R_o g_m}{2} \frac{1}{[1 - x_k^2 (1+\alpha)]^2 + x_k^2} \tag{4-35}$$

Equating the physical structure against the desired structure as shown in Fig. 4-14 it is evident that $L+M = \frac{1}{2} L_1$ and $-M = L_2$. Therefore,

$$L = L_1/2 + L_2 = \frac{L_1}{2}\left(1 + \frac{\alpha}{2}\right) \qquad \text{as} \quad L_2 = \frac{\alpha L_1}{4}$$

the coefficient of coupling k is

$$k = \frac{M}{L} = -\frac{\alpha}{2+\alpha} \tag{4-36}$$

or

$$\alpha + 1 = \left(\frac{1-k}{1+k}\right) \tag{4-37}$$

$$C_1 = \frac{1+\alpha}{4} C_g = \frac{1}{4}\left(\frac{1-k}{1+k}\right) C_g \tag{4-38}$$

Thus the transfer characteristic can be given as

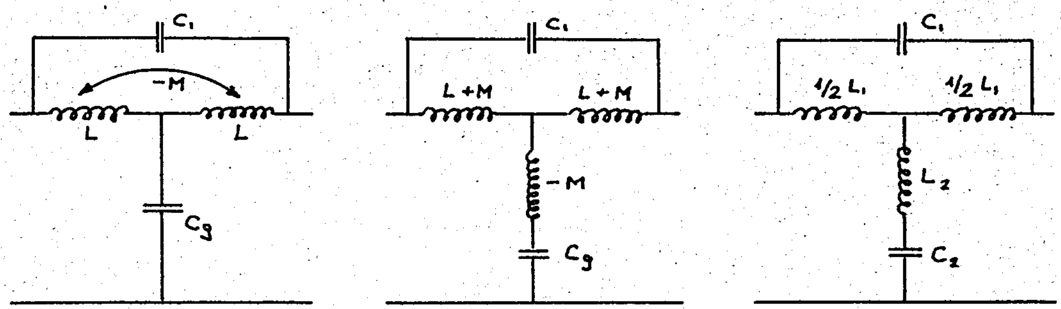


FIG. 4-14 — BRIDGED-TEE CONNECTION AND ITS EQUIVALENTS.

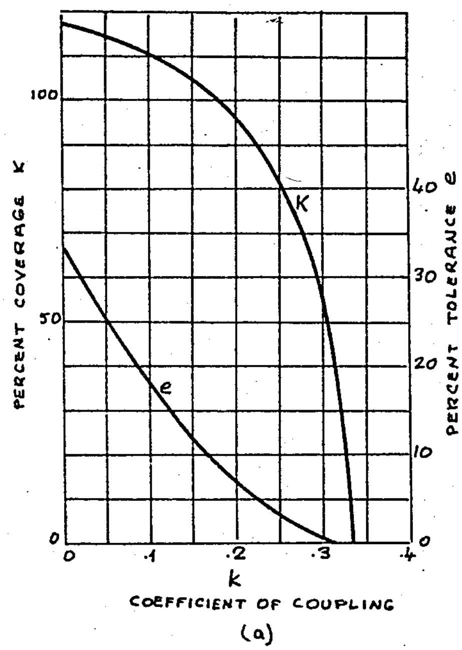
$$\frac{E}{e_o} = \frac{g_m R_o}{2} \frac{1}{\left[1 - x_k^2\left(\frac{1-k}{1+k}\right)\right]^2 + x_k^2} \angle -\theta \tag{4-39}$$

$$\theta = 2\tan^{-1} \frac{x_k}{1 - x_k^2\left(\frac{1-k}{1+k}\right)} \tag{4-40}$$

and the delay τ as

$$\tau = \frac{1}{\pi f_c} \frac{1 + x_k^2\left(\frac{1-k}{1+k}\right)}{\left[1 - x_k^2\left(\frac{1-k}{1+k}\right)\right]^2 + x_k^2} \text{ seconds} \tag{4-41}$$

From these equations we can plot curves as a function of k, the coefficient of coupling and x_k. These curves are given in Fig. 4-15.



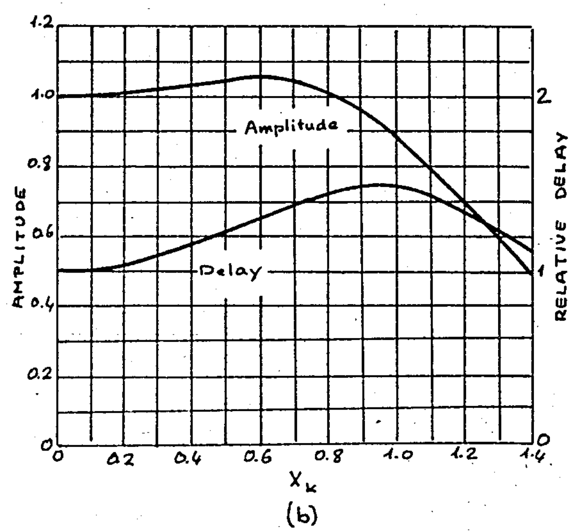


FIG. 4-15 — (a) COVERAGE AND TOLERANCE FOR BRIDGED-TEE AMPLIFIER.
(b) GAIN AND TIME DELAY FOR BRIDGED-TEE AMPLIFIER.

Chapter 5

PRINCIPLE OF STAGGERING

Another method used to improve both gain and phase-shift characteristics at the same time is called "staggering". The lumped lines in the distributed amplifier are so arranged that the anode-line travelling wave and the grid-line travelling wave are not in phase at corresponding points along the lines. It may appear at first that this will produce distortion by spacing the component signals (i.e., the signals that appear at the output through the different tubes) in time. But it is found that the steady-state gain and phase-shift characteristics may be improved considerably by properly adjusting the stagger.

In an amplifier embodying the constant-k LC filter network as the elements of the lumped lines, the stagger is introduced by making the cutoff frequency of the grid line a little higher than that of the plate line.

At a given frequency, a line with a higher cutoff frequency produces a smaller phase shift than one with a lower cutoff frequency. Also, the difference between the phase shifts produced by the two lines increases continuously as the frequency is increased.

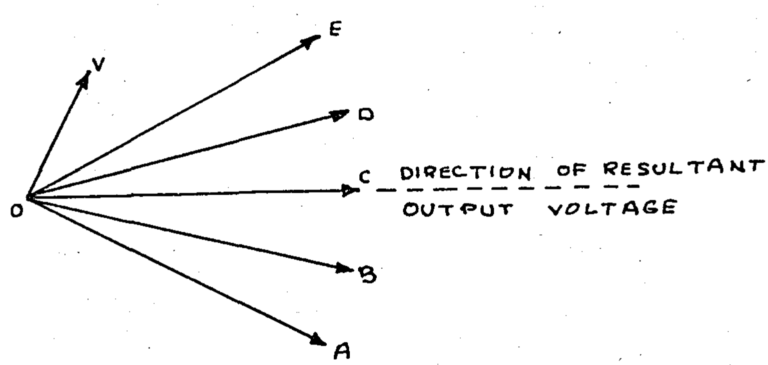


FIG. 5-1 — DIAGRAM TO ILLUSTRATE THE IMPROVEMENT IN PHASE-SHIFT CHARACTERISTIC DUE TO "STAGGERING" IN 5-TUBE AMPLIFIER
OV - INPUT VOLTAGE
OA, OB, OC, OD, OE — OUTPUT VOLTAGES DUE TO TUBES V_1, V_2, V_3, V_4, V_5 RESPECTIVELY.

With the above arrangement, therefore, the anode line will introduce a phase shift larger than that introduced by the grid line. In the amplifier, let the phase shifts introduced by the individual sections of the anode line and the grid line be θ_p and θ_g respectively.

The improvement in the steady-state characteristic of the amplifier can be understood in the following way: let OA in Fig. 5-1 represent the voltage vector at a particular frequency at the output terminals of the amplifier due to the component signal which has travelled through the first tube and then through the anode line. Its phase angle with respect to the input voltage is $\pi+(n-1)\theta_p$, n being the number of tubes in the amplifier. The component signal, which travels along the first section of the grid line and then arrives at the output terminals through the second tube and the remaining portion of the anode line, suffers a phase shift of $\theta_g+(n+2)\theta_p+\pi$, and consequently lags behind the voltage OA by an angle $(\theta_p-\theta_g)$. Let this voltage be represented by the vector OB. In a similar manner, the component signal which arrives at the output through the third tube travels through one section more of the grid line and one section less of the anode line, and consequently falls behind OB by the same phase angle $(\theta_p-\theta_g)$. Thus, the overall output is the vector sum of these and all other such component signals travelling through different portions of the anode and grid lines, and suffers a net phase

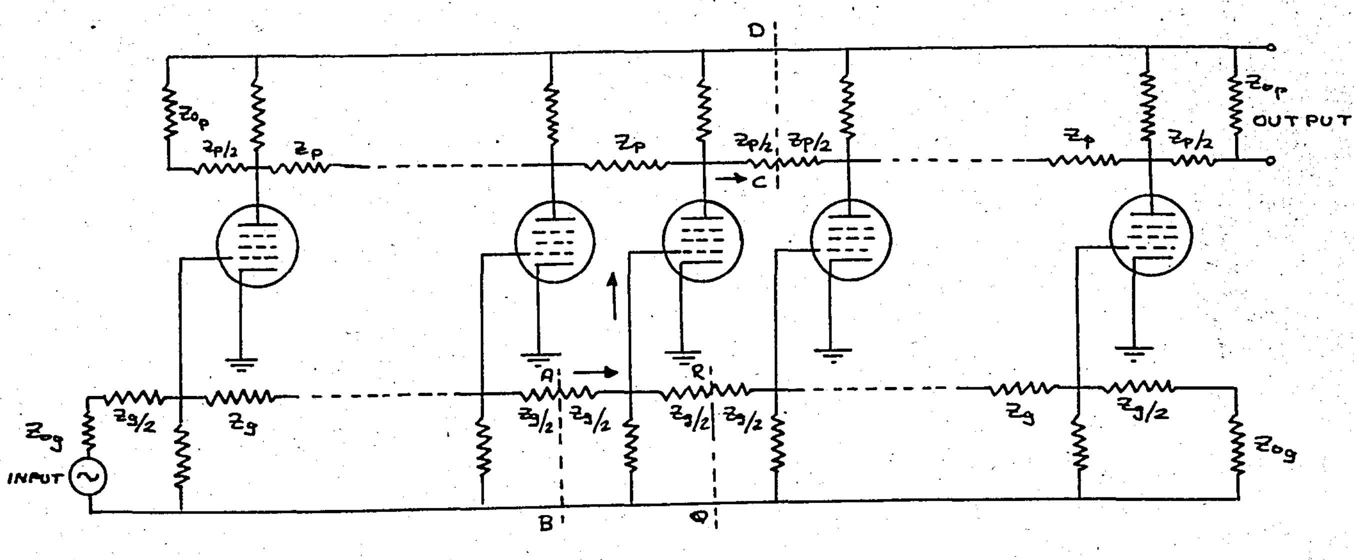


FIG. 5-2 — SIMPLIFIED CIRCUIT DIAGRAM FOR CALCULATING THE EFFECTS OF STAGGERING.

shift which is less than that introduced by the anode line from the anode of the first tube to that of the last tube, and greater than the phase shift introduced by the grid line between the grids of the first and last tubes. The phase shift of the final output is the average of these two phase shifts. Let us show this fact.

Consider the portion of a distributed amplifier between AB and CD in Fig. 5-2. It is the portion between the middle of a series element immediately preceeding the grid of a tube and the middle of the series element immediately following the anode of the same tube. Let the voltage at CD due to a voltage e_o at AB be E, considering only the forward travelling signal. Let $\frac{E}{e_o} = G$, be the gain of the system between AB and CD. Let there be n tubes in all the amplifier and let the phase shifts per section of the anode line and grid line be θ_p and θ_g respectively. Assuming that there is no loss due to attenuation within the pass-band (but only phase shifts of θ_p and θ_g per section) the component signal E_1 at the output terminals (due to the input e_o) passing through the first tube of the amplifier, and all but one section of the anode line, may be written as

$$E_1 = G\, e_o\, e^{-j(n-1)\theta_p} \qquad (5\text{-}1)$$

In a similar manner, the component signal E_2 passing through the second tube to the output terminals is

$$E_2 = G\, e_o\, e^{-j[(n-2)\theta_p + \theta_g]} \qquad (5\text{-}2)$$

since this component passes through one grid line section and (n-2) anode line sections. The other components passing through the third, fourth, n'th tubes are given by

$$E_3 = G\, e_o\, e^{-j[(n-3)\theta_p + 2\theta_g]}$$

$$E_4 = G\, e_o\, e^{-j[(n-4)\theta_p + 3\theta_g]}$$

$$E_n = G\, e_o\, e^{-j(n-1)\theta_g}$$

The resultant output E_0 is

$$E_o = \sum_{m=1}^{n} E_m = G e_o e^{-j(n-1)\theta_p}\left[1 + e^{j\psi} + e^{2j\psi} + \cdots\cdots + e^{j(n-1)\psi}\right] \quad (5\text{-}3)$$

where $\psi = (\theta_p - \theta_g)$

Performing the summation we get

$$E_o = G e_o e^{-j(n-1)\theta_p} \frac{(1 - e^{jn\psi})}{(1 - e^{j\psi})} = G e_o \frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}} e^{-j(n-1)\phi'} \quad (5\text{-}4)$$

where $\phi' = \dfrac{(\theta_p + \theta_g)}{2}$

Since the angle of G is equal to $-\phi'$, the overall phase shift is equal to $-n\phi'$, and is thus equal to the average of the phase shifts introduced by the anode and the grid lines.

Since the gridline cutoff frequency is made higher, the phase-shift characteristic of the grid line is such that its deviation from linearity is less than that of the phase-shift characteristic of the anode line at all frequencies below the anode line cutoff frequency. Consequently the average of the two characteristics is more linear than the phase-shift characteristic

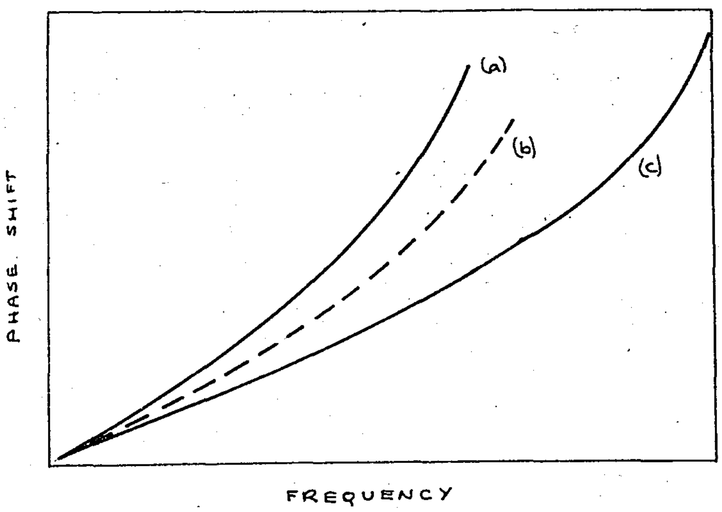


FIG. 5-3 — CURVES ILLUSTRATING THE PRINCIPLE OF PHASE-SHIFT IMPROVEMENT BY STAGGERING.

(a) PHASE-SHIFT CHARACTERISTIC OF PLATE LINE
(b) OVER ALL PHASE-SHIFT CHARACTERISTIC OF AMPLIFIER
(c) PHASE-SHIFT CHARACTERISTIC OF GRID LINE

of the amplifier that would have been obtained by making the cutoff frequencies of the two lines identical and equal to that of the anode line (See Fig. 5-3).

The improvement in the gain characteristics is also easily understood. If the stagger is adjusted in such a manner that the vectors OA, OB, etc., in Fig. 5-1 arrange themselves so that the resultant is zero at some frequency just below cutoff, the peak in the gain characteristic can be avoided.

Let us now analyse the effect of staggering on the gain and phase-shift characteristics of the three different networks studied above, namely, the constant-k filter network, the mutual inductance network, and the bridged-tee network.

a) Amplifier with constant-k LC network

For an amplifier with constant-k LC filter network, the variation in magnitude of G is the same as that of the mid-shunt characteristic impedance of a constant-k LC section. Thus

$$G = \frac{g_m R_o}{2} \frac{1}{\sqrt{1-x_k^2}} e^{-j(\theta_p+\theta_g)/2} = \frac{g_m Z_{op}}{2} e^{-j\phi'} \qquad (5\text{-}5)$$

where $R_o = \sqrt{\frac{L}{C}}$ for the anode line

$Z_{o_p} = R_o \frac{1}{\sqrt{1-x_k^2}}$

$x_k = \frac{f}{f_c}$

n = number of tubes

$\psi = (\theta_p - \theta_g) = 2\left(\sin^{-1} x_k - \sin^{-1} q x_k\right)$

$q = \frac{f_{op}}{f_{og}}$

f_{op} and f_{og} = anode- and grid-line cutoff frequencies respectively

g_m = mutual conductance of the tubes used

The overall gain characteristic is

$$A = g_m \frac{Z_{op}}{2} \frac{\sin n\,\psi/2}{\sin \psi/2} \qquad (5\text{-}6)$$

This gain is measured from the grid of the first tube to the plate

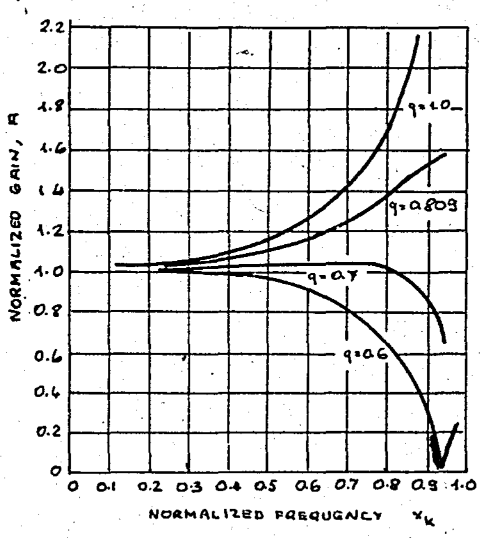


Fig. 5-4 — NORMALIZED GAIN/FREQUENCY CHARACTERISTIC FOR DIFFERENT VALUES OF STAGGERING FACTOR q

q = 1 CORRESPONDS TO THE UNSTAGGERED CASE.

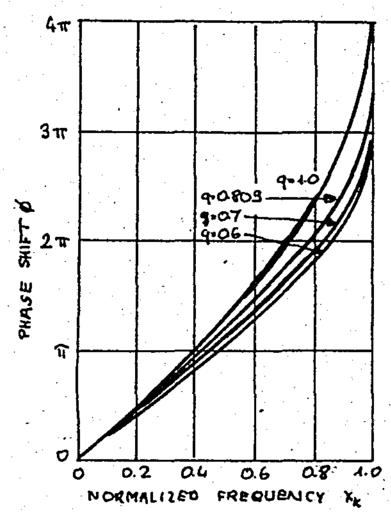


Fig. 5-5 — NORMALIZED PHASE-SHIFT/FREQUENCY CHARACTERISTIC FOR DIFFERENT VALUES OF STAGGERING FACTOR q

q = 1 CORRESPONDS TO THE UNSTAGGERED CASE.

of the final tube. The phase shift is given by

$$\phi = (n-1)\left(\sin^{-1} x_k + \sin^{-1} q x_k\right) \tag{5-7}$$

The gain and phase shift characteristics for the unstaggered amplifier are given by

$$A = n g_m \frac{Z_{op}}{2} \tag{5-8}$$

$$\phi = 2(n-1) \sin^{-1} x_k \tag{5-9}$$

linearization of the phase-shift characteristic, results in an overall improvement of the amplifier response.

The phase shift and gain characteristics of an amplifier with m-derived filter sections can be further improved by introducing staggering. The principle is the same as for the constant-k filter sections case. The stagger in this case is introduced by choosing different m values for the plate and grid lines, the grid line having a larger m.

Since the overall delay characteristics of a staggered distributed amplifier is the average of the delay characteristics of the anode and grid

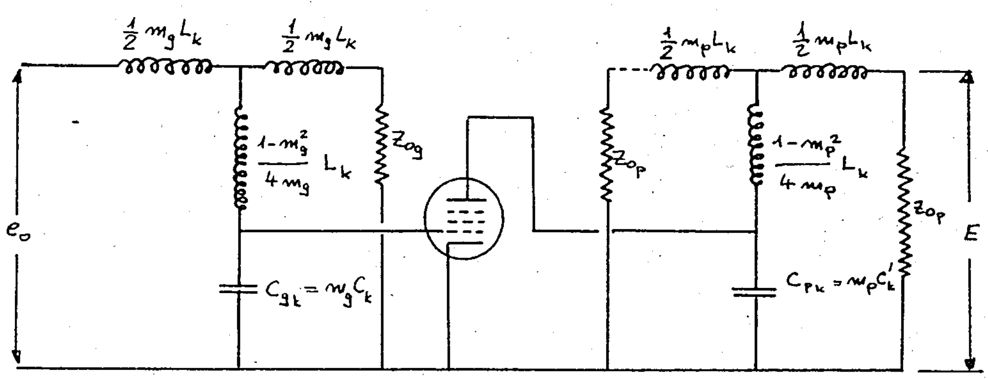


FIG. 5-6 — EQUIVALENT CIRCUIT FOR CALCULATING E/e_o FOR M-DERIVED SECTIONS.

lines, the m values of the lines are so chosen thatthe average of the two delay characteristics may be as flat as possible. An inspection of Fig. 4-7 reveals that there is a certain number of such possibilities.

For the determination of gain in the m-derived staggered amplifier consider the circuit arrangement shown in Fig. 5-6. This is a portion of the equivalent circuit of the m-derived LC filter used in the staggered distributed amplifier. The ratio E/e_o can be calculated as done previously and is

$$G = \frac{E}{e_o} = \frac{R_{op} g_m}{2} \frac{1}{\sqrt{(1-x_g^2)[1-(1-m_g^2)x_g^2][1-(1-m_p^2)x_p^2]}} e^{-j(\theta_p+\theta_g)/2}$$

$$= g_m \frac{Z_o'}{2} e^{-j\phi'} \qquad (5\text{-}12)$$

where $\Theta_g = 2\tan^{-1}\dfrac{m_g x_g}{\sqrt{1-x_g^2}}$ (phase shift per section of grid-line)

$\Theta_p = 2\tan^{-1}\dfrac{m_p x_p}{\sqrt{1-x_p^2}}$ (phase shift per section of anode-line)

$$x_p = \frac{\omega R_{og} C_{gk}}{2 m_g}$$

$$x_g = \frac{\omega R_{op} C_{pk}}{2 m_p}$$

m_g and m_p are m values of grid and anode lines respectively

R_{op} = characteristic impedance of plate line

R_{og} = characteristic impedance of grid line

While combinations of the anode- and grid-line m values are chosen from Fig. 4-7 in order to obtain a flat delay characteristic, it is assumed that the curves for these delay characteristics are normalized with respect to the same frequency. This requires that $x_p m_p$ be equal to $x_g m_g$ for a given value of ω, that is, $R_{og} C_{gk} = R_{op} C_{pk}$

Normalized frequency $= x_k = x_p m_p = x_g m_g = \frac{\omega}{2} R_{op} C_{pk} = \frac{\omega}{2} R_{og} C_{gk}$

The overall gain characteristic of the amplifier becomes

$$A = \frac{g_m Z_o'}{2}\,\frac{\sin n\Psi/2}{\sin \Psi/2} \tag{5-13}$$

where $$Z_o' = \frac{R_{op}\, m_g}{\sqrt{(m_g^2 - x_k^2)\left[1 + x_g^2\left(1 - 1/m_g^2\right)\right]\left[1 + x_k^2\left(1 - 1/m_p^2\right)\right]}} \tag{5-14}$$

and $\Psi = \Theta_p - \Theta_g$

The phase-shift characteristic is given by

$$\phi = \frac{n}{2}\left(\Theta_p + \Theta_g\right) \tag{5-15}$$

Curves for the gain and phase-shift characteristics for three combinations of m values are shown in Figs. 5-7(a), (b) and (c); in Fig. 5-7(a) $m_g = 1.4$ and $m_p = 1.2$; in Fig. 5-7(b) $m_g = 1.375$ and $m_p = 1.175$; and in Fig. 5-7(c) $m_g = 1.4$ and $m_p = 1.14$. The number of tubes is 5. The value of $R_{og}C_g$ is kept equal to $R_{op}C_p$. This means that the ratio of the cutoff

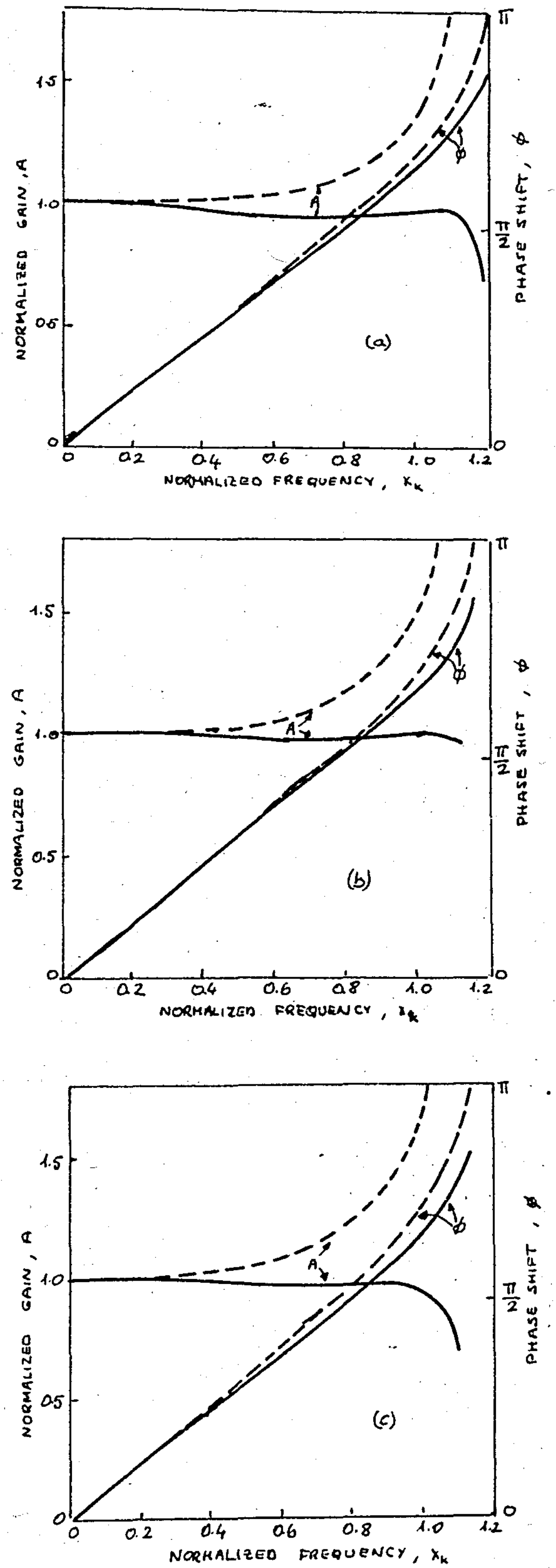


Fig. 5-7 — Normalized gain and phase-shift curves for a 5 tube amplifier

(a) ——— Staggered : $m_g = 1.4$; $m_p = 1.2$
- - - - Unstaggered : $m_g = m_p = 1.2$

(b) ——— Staggered : $m_g = 1.375$; $m_p = 1.175$
- - - - Unstaggered : $m_g = m_p = 1.2$

(c) ——— Staggered : $m_g = 1.4$; $m_p = 1.14$
- - - - Unstaggered : $m_g = m_p = 1.14$

frequencies of the anode and grid lines is made equal to the ratio of their m values.

The gain and phase shift of the corresponding unstaggered distributed amplifier (obtained by putting $m_g = m_p = m$) are

$$A_o = n \frac{g_m R_{op}}{2} \frac{m}{\sqrt{m^2 - x_k^2}\left[1 - (1 - 1/m^2)\, x_k^2\right]} \qquad (5\text{-}16)$$

$$\phi = 2n \tan^{-1} \frac{m\, x_k}{\sqrt{m^2 - x_k^2}} \qquad (5\text{-}17)$$

These are also plotted in Figs. 5-7(a), (b) and (c) in dotted lines. We see that the improvement is worthy in each case.

c) Amplifier with bridged-tee network

As we have seen already in section 4-c bridged-tee networks seen in Fig. 5-8 can be used satisfactorily in distributed amplifiers. Especially if the values are so adjusted that

$$C_g (1 + \alpha) = 4 C_1$$

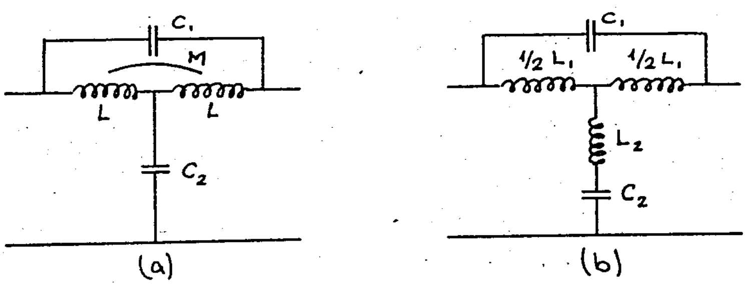


FIG. 5-8 — BRIDGED-TEE NETWORK EQUIVALENT CIRCUITS
(a) ACTUAL
(b) THEORETICAL IN WHICH $L_2 = -M$ AND $\alpha = \frac{4L_2}{L_1}$

the impedance of the network becomes independent of frequency.

The gain, phase shift and delay characteristics of an amplifier with such a circuit were given by

$$A = n \frac{R_o g_m}{2} \frac{1}{\left[1 - x_k^2 (1 + \alpha)\right]^2 + x_k^2} \qquad (4\text{-}35)$$

$$\theta = 2n \tan^{-1} \frac{x_k}{1 - x_k^2 (1 + \alpha)} \qquad (4\text{-}28)$$

$$\tau = \frac{1}{\pi f_c} \frac{1 + x_k^2 (1 + \alpha)}{\left[1 - x_k^2 (1 + \alpha)\right]^2 + x_k^2} \qquad (4\text{-}29)$$

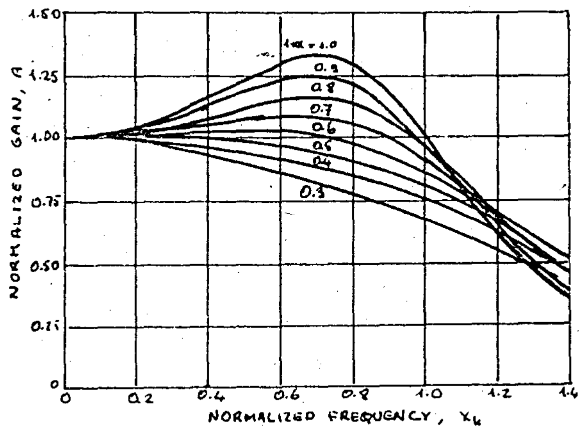


Fig. 5-9 — NORMALIZED GAIN/FREQUENCY CURVE FOR BRIDGED-TEE NETWORK FOR DIFFERENT VALUES OF $(1+\alpha)$

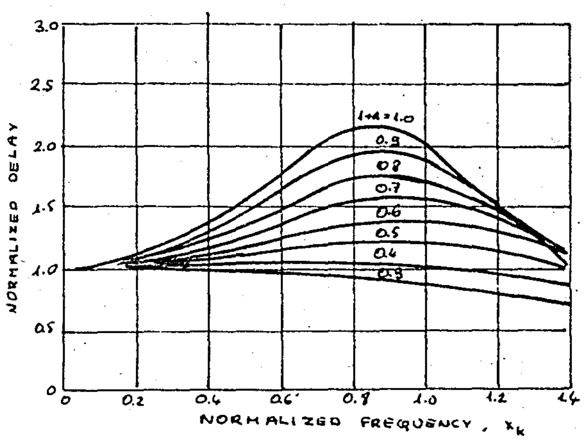


Fig. 5-10 — NORMALIZED DELAY/FREQUENCY CURVE FOR BRIDGED-TEE NETWORK FOR DIFFERENT VALUES OF $(1+\alpha)$

where $X_k = \pi f R_o C_g$

$\alpha = 4 \frac{L_2}{L_1}$

R_o = characteristic impedance

The gain and delay characteristics for different values of $(1+\alpha)$ are plotted in Figs. 5-9 and 5-10 .

A comparison of these curves shows that very flat gain and delay characteristics cannot be obtained simultaneously. If an amplifier is built to give a flat gain characteristic, its delay characteristic will not be very satisfactory. Also, since values of $(1+\alpha)$ smaller than 0.45 require the

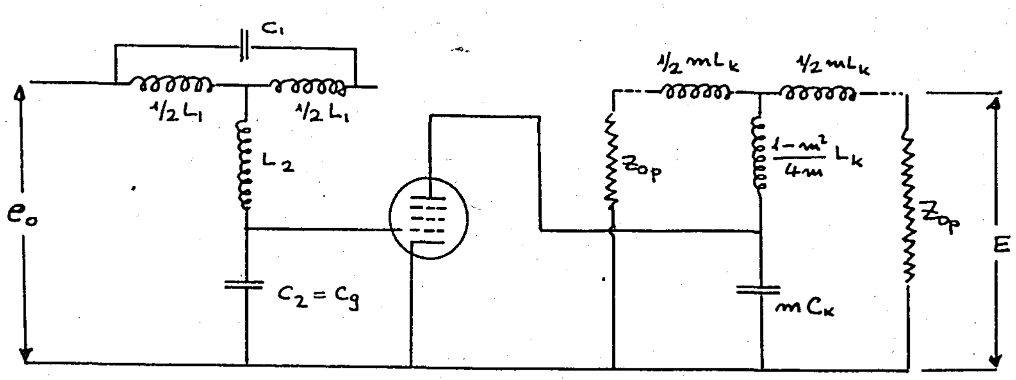


FIG. 5-11 — EQUIVALENT CIRCUIT FOR CALCULATING E/e_o FOR AMPLIFIER WITH BRIDGED-TEE NETWORK IN THE GRID CIRCUIT AND M-DERIVED SECTIONS IN THE ANODE CIRCUIT.

coefficient of coupling to be larger than 0.38, practical difficulties limit the construction of such lines.

The bridged-tee network may however be used very effectively in an amplifier made up of m-derived sections in the plate line, and bridged-tee sections in the grid line and obtain a flat delay characteristic. For this case the value of E/e_o can be calculated in a similar way as the previous cases (see Fig. 5-11), and is

$$\frac{E}{e_o} = \frac{g_m R_{op}}{2} \frac{1}{\sqrt{[1 - x_k^2(1+\alpha)]^2 + x_g^2}\ \sqrt{[1-(1-m^2)x_p^2]}} e^{-j(\theta_p + \theta_g)/2}$$

$$= \frac{g_m Z_o''}{2} e^{-j\phi'} \tag{5-18}$$

where $\theta_p = 2 \tan^{-1} \frac{m x_p}{\sqrt{1 - x_p^2}}$

$\theta_g = 2 \tan^{-1} \frac{x_g}{1 - x_g^2(1+\alpha)}$

$x_g = \pi f R_{og} C_g$

$x_p = \frac{\pi f}{m} R_{op} C_p$

Applying the general relation, equation (5-4), the overall gain characteristic is

$$A = \frac{g_m Z_o''}{2} \frac{\sin n\Psi/2}{\sin \Psi/2} \tag{5-19}$$

where $\Psi = \theta_p - \theta_g$

$$Z_o'' = \frac{1}{\sqrt{[1 - x_k^2 (1+\alpha)]^2 + x_g^2}\ \sqrt{[1-(1-m^2)x_p^2]}} \qquad (5\text{-}20)$$

and the overall phase shift characteristic can be written as

$$\phi = \frac{n(\theta_p + \theta_g)}{2} \qquad (5\text{-}21)$$

The curves obtained for such an amplifier having m = 1.4 for the plate line and $(1+\alpha) = 0.45$ for the grid line are shown in Fig. 5-12

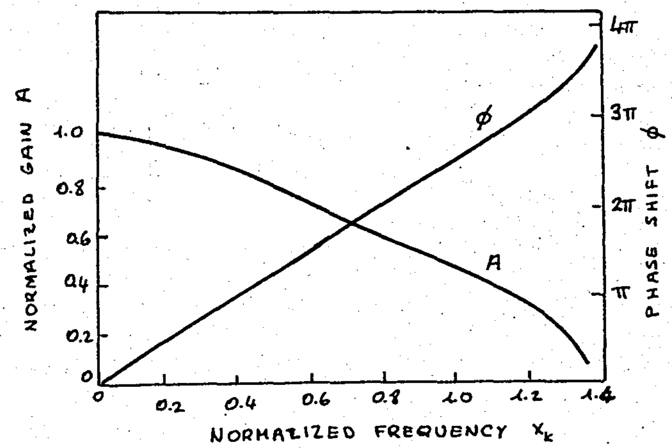


FIG. 5-12 — NORMALIZED GAIN AND PHASE SHIFT CURVES FOR AMPLIFIER WITH BRIDGED-TEE NETWORK IN THE GRID CIRCUIT AND M-DERIVED NETWORK IN THE ANODE CIRCUIT.

Chapter 6

EFFECT OF IMPROPER TERMINATION OF LINES

In all the previous discussions we have assumed that the lines were terminated with their characteristic impedances. It should be pointed out that any artificial line has to be terminated by proper half sections and impedances equal to the characteristic impedance of the lines in order to avoid reflections. This is done in the conventional way, and it will not be discussed here. However, in any practical situation, this is not the case and the terminations cannot be perfect. We can therefore have reflections from all four sets of terminals. The effect of these reflections can be understood by referring at Fig. 6-1(a), which is the schematic diagram of one stage of distributed amplifier. It shall be assumed that the lines are dissipationless, and that all sections are identical. Each stage has a phase shift of ϕ degrees, and each end of each line is terminated by a terminal half-section. These terminal half-sections will be assumed to have a phase shift of $\frac{1}{2}\psi$ degrees. If a signal e is introduced into the grid line, then a portion of that signal will be reflected from the grid terminations. If we denote the reflection coefficient of the line by δ , the reflected wave will have an amplitude equal to δe where

$$\delta = \frac{Z_L - Z_o}{Z_L + Z_o} \qquad (6\text{-}1)$$

For the sake of simplicity we will assume that the secondary reflections from the input and from the plate termination are negligible. The reflected voltage δe will appear at the grids of the various tubes and will add vectorially to the original wave. In a similar fashion, reflections may be expected from the reverse termination in the plate line. The net voltage at the output of the distributed amplifier is then the vector sum of all these voltages. The net voltage due to reflections is

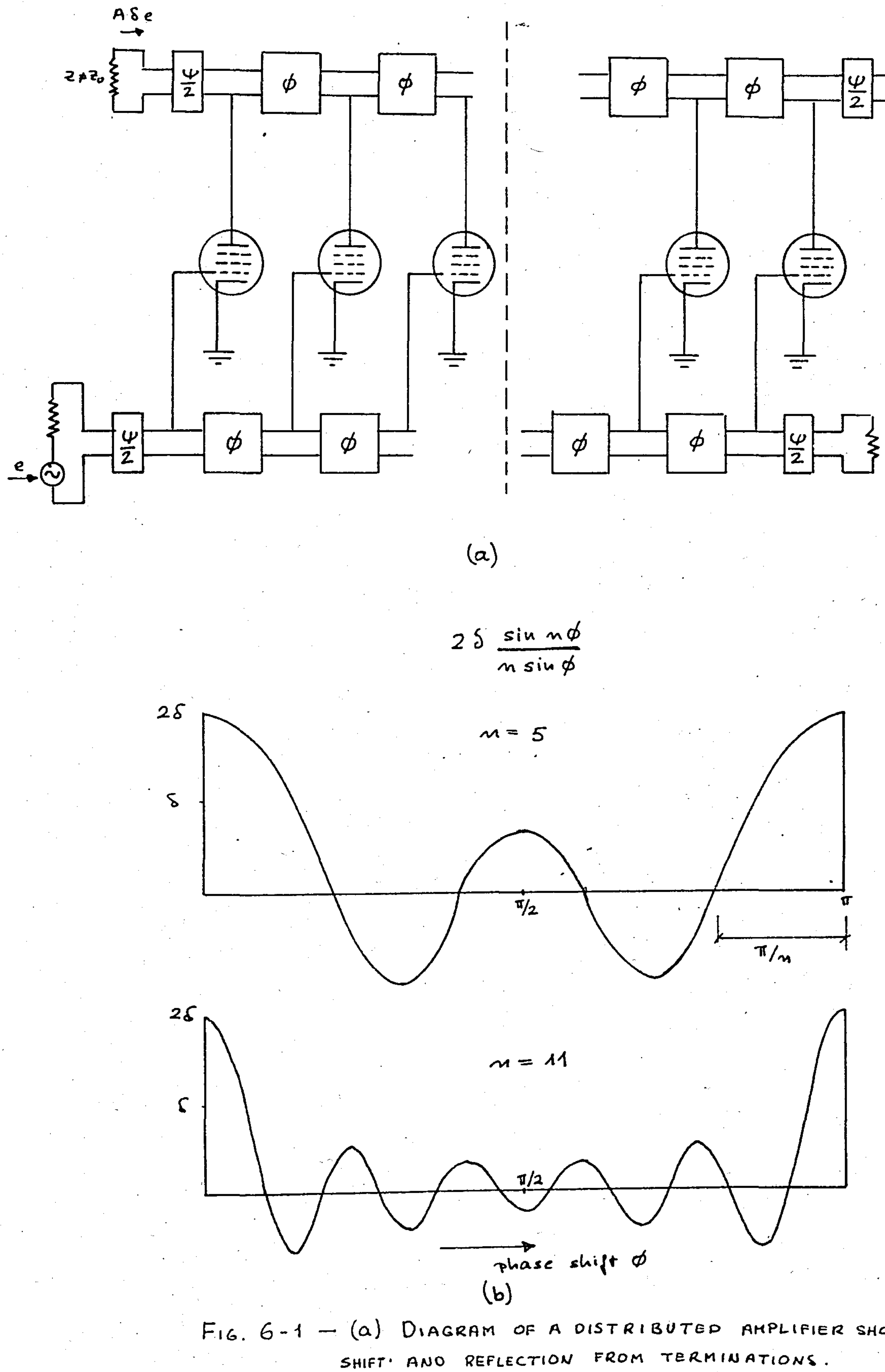


FIG. 6-1 — (a) DIAGRAM OF A DISTRIBUTED AMPLIFIER SHOWING PHASE SHIFT AND REFLECTION FROM TERMINATIONS.
(b) RATIO OF SIGNAL TO REFLECTED VOLTAGES.

$$E_R = 2A_o e\delta\left[e^{j[2\psi+(n-1)\phi]} + e^{j[2\psi+(n+1)\phi]} + \cdots + \cdots + e^{j[2\psi+(2k+n-1)\phi]} + \cdots + e^{j[2\psi+3(n-1)\phi]}\right] \tag{6-2}$$

The voltage due to the signal at the output terminals, neglecting reflections is

$$E_s = A_o e n e^{j[\psi+(n-1)\phi]} \tag{6-3}$$

where A_o = amplification per section

e = input signal

n = number of tubes per stage

The ratio of the reflected voltage to the signal voltage then is given by E_R/E_s and is given by

$$\frac{E_R}{E_s} = \frac{2\delta}{n}\sum_{k=0}^{n-1} e^{j[\psi+2k\phi]} \tag{6-4}$$

$$= 2\delta\,\frac{\sin n\phi}{n\sin\phi}\, e^{j[\psi+(n-1)\phi]} \tag{6-5}$$

This equation shows the importance of the reflections. The magnitude of this equation is plotted in Fig. 6-1(b) for n = 5 and n = 11 . It is evident from equation (6-5) and Fig. 6-1(b) that the relative magnitude of the reflected voltages near the center of the band depends upon $(2\delta/n)$ and that the larger peaks are displaced towards the edges of the band.

For practical purposes, we can say that the reflection factor for low values of ϕ , that is, for low frequencies, may be made nearly zero. The larger peaks as ϕ approaches π tend to move toward the edges of the useful range of the amplifier. Furthermore, the concave phase characteristic of the normal constant-k section will still further crowd these large peaks toward the upper end of the frequency band. It is evident that as the number of sections n is increased, the seriousness of small mismatches is reduced.

The actual output voltage is the vector sum of the nominal output signal and the reflected signal. Fig. 6-2 shows the magnitude of the variations in the output voltage for n = 5 and n = 11 when it is assumed that δ is small compared to 1 and $\phi = \psi$. If ψ is less than ϕ, as is usually the case, the effect on the curve shown in Fig. 6-2 will be to crowd it toward the right and displace it slightly downward in accordance with equation (6-6).

$$E_{out} \approx E_s \left[1 + 2\delta \left(\frac{\sin(2n\phi + \psi - \phi)}{2n \sin\phi} - \frac{\sin(\psi - \phi)}{2n \sin\phi} \right) \right] \qquad (6\text{-}6)$$

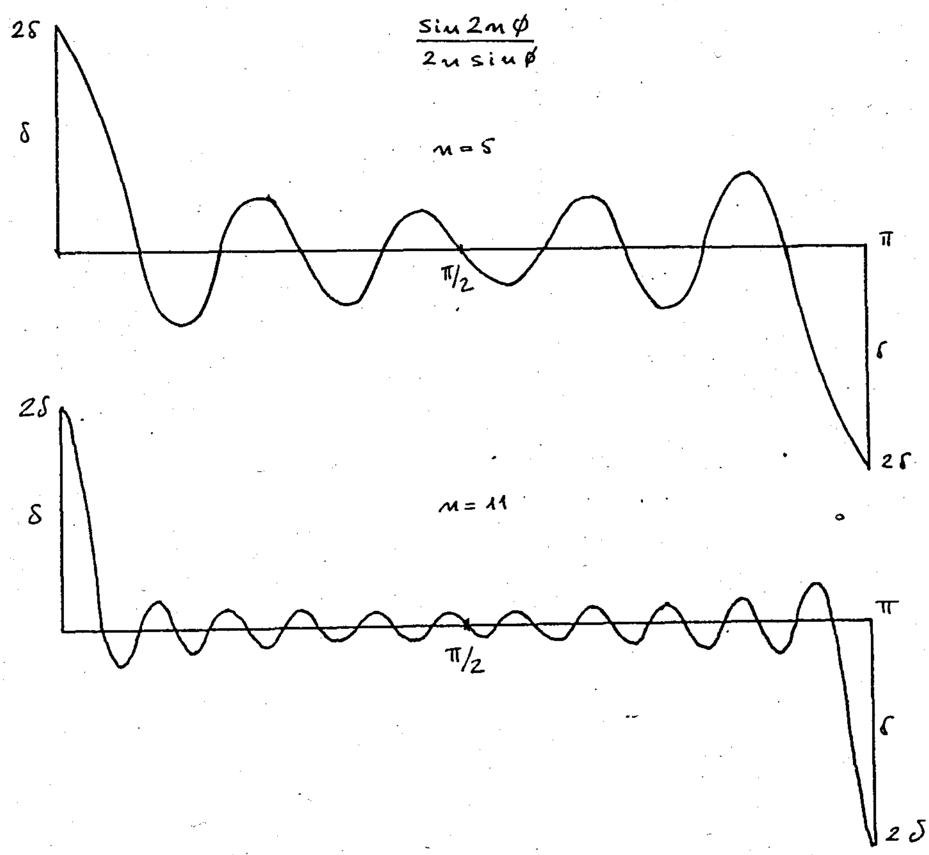


FIG. 6-2 — VARIATIONS IN OUTPUT VOLTAGE.

When the number of sections is small, that is, less than four, the value of m in the terminal half-section may be so selected that the characteristic impedance and the terminating resistance will be equal,

($\delta = 0$), at a frequency coinciding with one of the maxima of Fig. 6-1(b). This will further tend to reduce the reflection effect from an imperfect termination.

Chapter 7

EFFECTS DUE TO INCREASE IN FREQUENCY

When we increase the frequency there are losses introduced in the amplifier. These losses are the following:

a) Incidental dissipation

We know quite well that a series resistance and a shunt conductance produce attenuation in a filter. Equation (7-1) is a good approximation of the effect of such dissipation. We can see from this equation that

$$\alpha = \frac{x_k}{2}\left(\frac{1}{Q_c}+\frac{1}{Q_L}\right)\frac{d\phi}{dx_k} \qquad (7\text{-}1a)$$

$$= \left(\frac{G}{2C}+\frac{R}{2L}\right)\frac{d\phi}{d\omega} \qquad (7\text{-}1b)$$

where α = attenuation in nepers
Q_c = the Q of the condensers
Q_L = the Q of the inductors
x_k = the normalized frequency function
G = the shunt conductance across the capacitance C
R = the resistance in series with the inductance L
ϕ = the phase shift of the section in radians

That dissipation produces an attenuationin the pass band proportional to the sum of the reciprocals of the Q's of the inductors and condensers and proportional to the normalized slope of the phase function times the normalized frequency function x_k. As the phase function of a constant-k section is concave and rises sharply near cutoff, a marked increase in attenuation will occur near the cutoff frequency. The advantage of a linear phase function such as that obtained from section utilizing negative mutual inductance is also immediately evident when considering the effect of incidental dissipation.

b) Lead inductance

Lead inductance in the grid and plate circuits has the effect of reducing the cutoff frequency and producing a peak near cutoff. The use of negative mutual inductance can completely compensate for this effect. The costants L and M of the negative-mutual-inductance circuit as previously discussed need to be modified to correct for the presence of lead inductance. The following equations show how L and M need to be modified to compensate for the grid (or plate) lead inductance

$$L = \left(\frac{m^2+1}{4m} - \gamma\right) L_k \qquad (7\text{-}2)$$

$$M = \left(\frac{m^2-1}{4m} + \gamma\right) L_k \qquad (7\text{-}3)$$

where $$\gamma = \frac{\text{lead inductance}}{L_k}$$

c) The effect of grid losses

The cathode lead inductance, in conjunction with the grid-to-cathode capacitance, produces an input grid conductance which is equal to(1)

$$G = g_m \omega^2 L_c C_g \qquad (7\text{-}4)$$

Including also the conductance due to transit time effects and considering that there is no capacitance in parallel with the inductance in the cathode lead, the input conductance becomes approximately

$$G \cong g_m \omega^2 \left(L_c C_g + K T^2 \right) \text{ mhos} \qquad (7\text{-}5)$$

where L_c = cathode-lead inductance

C_g = grid-cathode capacitance

T = transit time

K = a constant depending on the tube.

(1) F. E. Terman, "Electronic and Radio Engineering", McGraw-Hill Book Co., Tokyo, 1955, p. 433.

If there is appreciable cathode-to-screen and cathode-to-filament capacity shunting the cathode-lead inductance equation (7-5) will be modified to give the following expression

$$G \cong g_m \omega^2 \left[\frac{L_c C_g}{1 - p^2 x_k^2} + K T^2 \right] \qquad (7\text{-}6)$$

where

$$p = \frac{\text{cutoff frequency}}{\text{cathode antiresonant frequency}}$$

The attenuation per section introduced by this conductance will be given by

$$\alpha \approx \frac{G}{4\pi f_c C} \frac{d\phi}{dx_k} \qquad (7\text{-}7)$$

where ϕ is the phase shift per section

f_c is the cutoff frequency

x_k is the normalized frequency function

The most appreciable source of attenuation being the grid loading, the gain of an n-section distributed amplifier with losses may be obtained from the expression for the lossless case by means of the following expression

$$A = A_o e^{-n\alpha/2} \frac{\sinh\left(\frac{n\alpha}{2}\right)}{\sinh\left(\frac{\alpha}{2}\right)} \qquad (7\text{-}8)$$

A_o is the gain of the lossless circuit, and the frequency dependence of A_o and α is a function of the circuit type being considered. In the derivation of equation (7-8) it is assumed that the input half-section offers the same attenuation as the transmission line half-section.

As an illustration, consider the effect of grid loading on the amplitude response of a distributed amplifier using constant-k transmission lines. If the input conductance is assumed to have the frequency variation in equation (7-5) the attenuation factor becomes

$$\alpha = \frac{G_o \omega^2}{\omega_c C} \frac{1}{\sqrt{1 - x_k^2}} = \alpha_o \frac{x_k^2}{\sqrt{1 - x_k^2}} \qquad (7\text{-}9)$$

Using this in conjunction with equation (7-8), normalized curves of gain for various values of $m\alpha_o/2$ may be plotted as a function of frequency. These curves are shown in Fig. 7-1.

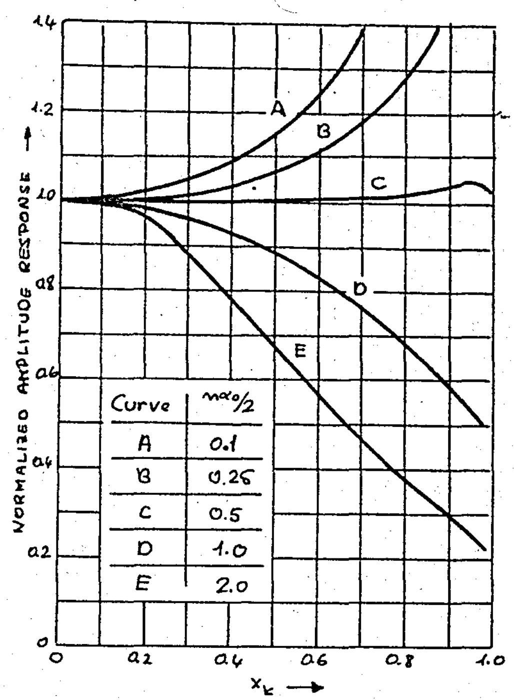


FIG. 7-1 — AMPLITUDE RESPONSE OF CONSTANT-K AMPLIFIER AS A FUNCTION OF $m\alpha_o/2$

The modifications caused by the presence of shunting capacitance in the cathode circuit can be shown by a similar family of curves. If it is assumed that the transit-time effects are small compared to the effect of impedance in the cathode circuit, the attenuation factor may be written

$$\alpha = \alpha_o' \frac{x_k^2}{1-p^2 x_k^2} \frac{1}{\sqrt{1-x_k^2}} \tag{7-9a}$$

where α_o' is distinguished from α_o since the transit-time losses are not included in this equation. Using this relation, equation (7-8) can again be used to plot normalized gain curves. The resultant curves are shown in Figs. 7-2, 7-3, and 7-4 as a function of parameters p and $m\alpha/2$. In Fig. 7-5 is shown the measured amplitude response curves of constant-k circuits having 16 and 8 identical sections. These correspond to $m\alpha_o/2=1$ (curve A) and $m\alpha_o/2=0.5$ (curve B) where $p = 0.5$.

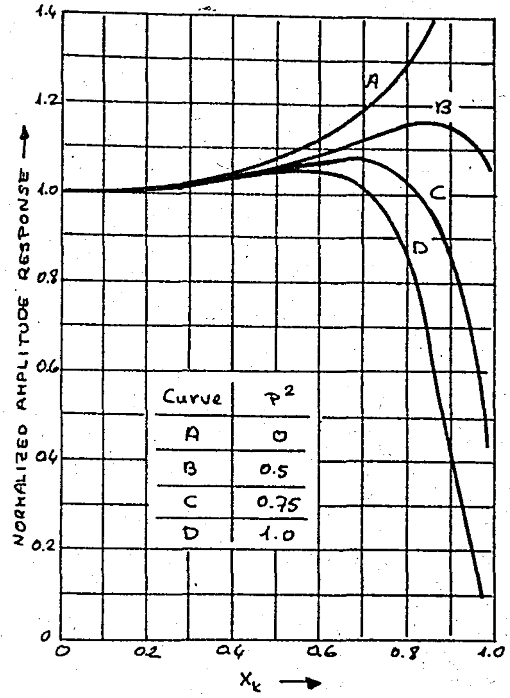


FIG. 7-2 – AMPLITUDE RESPONSE OF CONSTANT-K NETWORK AS MODIFIED BY ANTI-RESONANT CIRCUIT IN EACH CATHODE CIRCUIT $m\alpha_0'/2 = 0.25$

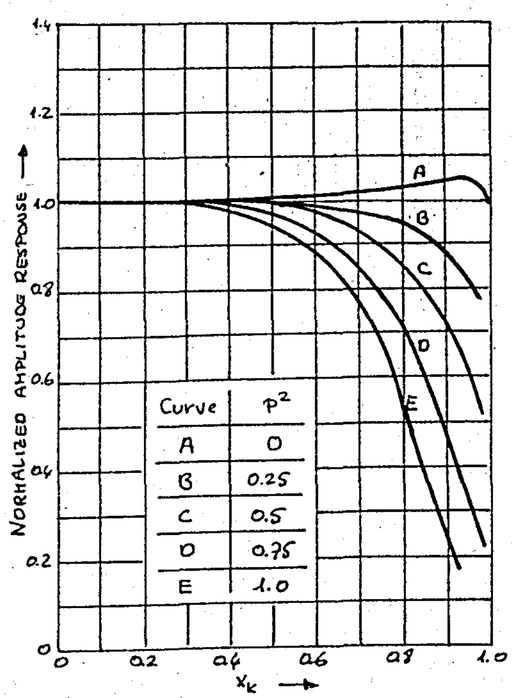


FIG. 7-3 – AMPLITUDE RESPONSE OF CONSTANT-K NETWORK AS MODIFIED BY ANTIRESONANT CIRCUIT IN EACH CATHODE CIRCUIT $m\alpha_0'/2 = 0.5$

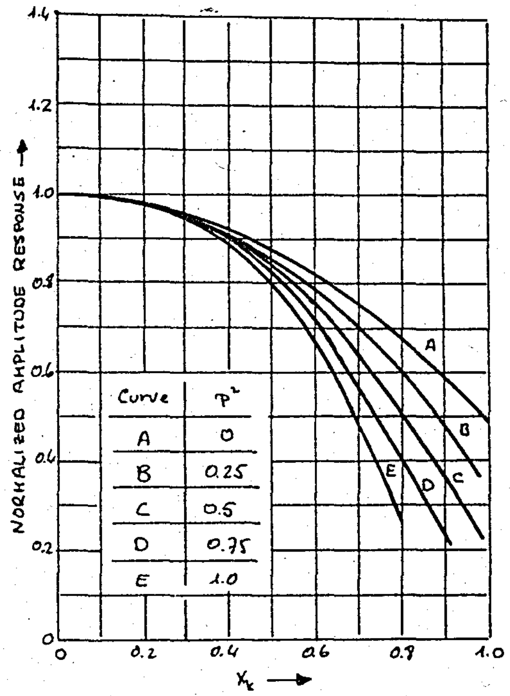


FIG. 7-4 – AMPLITUDE RESPONSE OF CONSTANT-K NETWORK AS MODIFIED BY ANTIRESONANT CIRCUIT IN EACH CATHODE CIRCUIT. $m\alpha_0'/2 = 1$

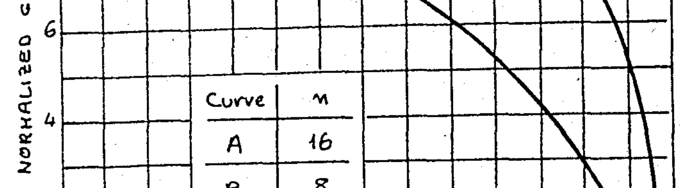


FIG. 7-5 – MEASURED AMPLITUDE RESPONSE CONSTANT-K CIRCUIT FOR $m\alpha_0/2 = 0.5$ AND $m\alpha_0/2 = 1$ AND FOR $p = 0.5$.

As another example of the effect of grid losses on the predicted response of a distributed amplifier, consider the negative mutual inductance circuit. If the effect of cathode-to-ground capacity is neglected, the attenuation due to cathode-lead inductance and transit time becomes

$$\alpha = \alpha_o \frac{x_k^2 \, m^3}{\left[m^2 - (1-m^2)\, x_k^2\right]\sqrt{m^2 - x_k^2}} \qquad (7\text{-}10)$$

The curves plotted in Figs. 7-6, 7-7 and 7-8 show the amplitude response of a negative mutual inductance circuit for various values of m and $m\alpha_o/2$. These curves are again calculated from equation (7-8) using equation (7-10) as the value of α.

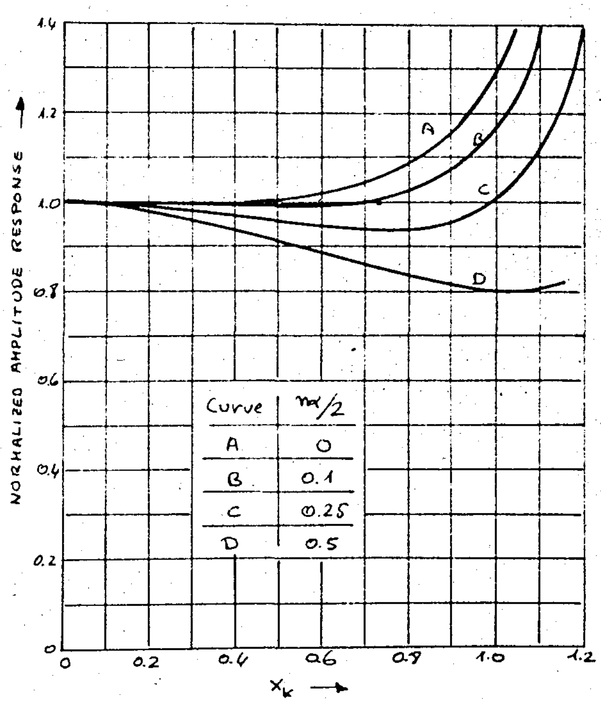


FIG. 7-6 — AMPLITUDE RESPONSE OF NEGATIVE MUTUAL INDUCTANCE CIRCUIT AS A FUNCTION OF $m\alpha_o'/2$, $m = 1.225$

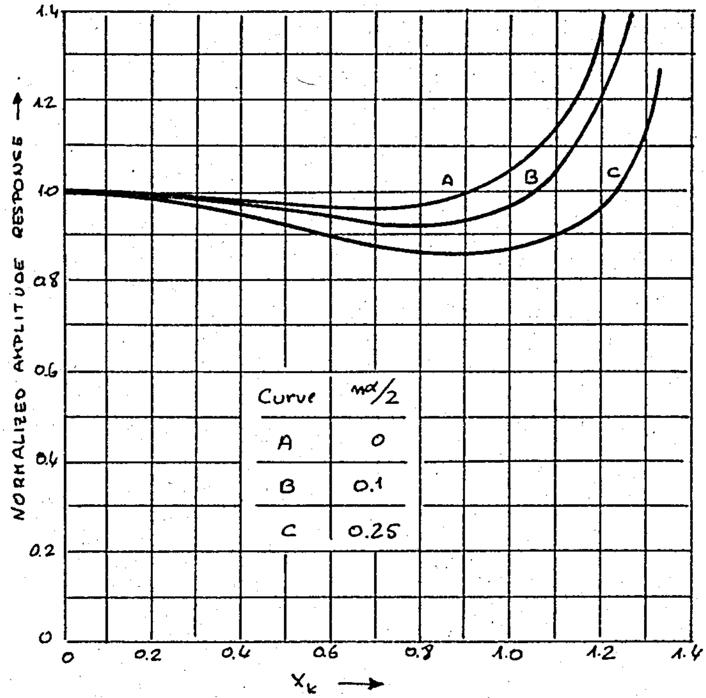


FIG. 7-7 — AMPLITUDE RESPONSE OF NEGATIVE MUTUAL INDUCTANCE CIRCUIT AS A FUNCTION OF $m\alpha_0'/2$, $m = 1.34$

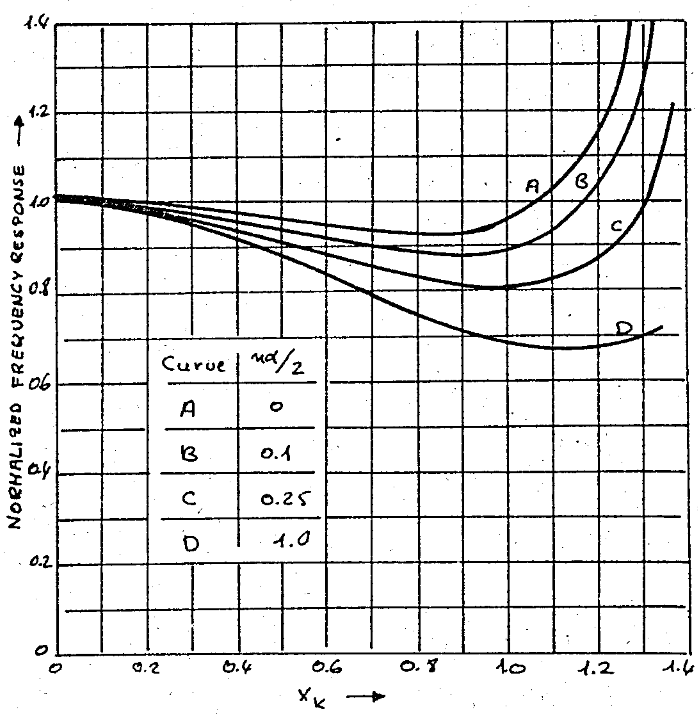


FIG. 7-8 — AMPLITUDE RESPONSE OF NEGATIVE MUTUAL INDUCTANCE CIRCUIT AS A FUNCTION OF $m\alpha_0'/2$, $m = 1.4$

d) Effect of distributed capacitance in the coil windings

The presence of distributed capacitance results in lowering the amplifier cutoff frequency and in altering the impedance of the transmission lines, thus making it difficult to terminate properly. The constant-k and paired plate circuits with distributed capacitance are subject to an approximate analysis. If the distributed capacity in each coil can be considered as equivalent to a lumped capacity connecting the ends of the coil, the transmission-line sections take on the configuration of shunt-derived sections in an m-derived structure and the circuit may be analysed as such.

The negative mutual inductance circuit does not yield readily to analysis when distributed capacitance is included. It has been found experimentally, however, that the negative mutual inductance circuit suffers from the effects mentionned above.

Chapter 8

NOISE IN DISTRIBUTED AMPLIFIERS

There are four sources of noise of basic and unavoidable nature that need to be considered in any amplifier which extend to high frequencies. These are:

a) Thermal noise in the input impedance

b) Shot-effect noise generated by the electron stream in the electron tubes

c) Induced grid noise which is associated with transit time effects at the high frequencies

d) Thermal noise in the equivalent grid-loading impedance which is developed between the cathode and the grid of an electron tube as a result of grid-to-cathode capacitance and the cathode lead inductance.

The ideal amplifier would be one in which the only noise in the output terminals was due to the thermal noise in the input impedance of the amplifier. The thermal noise in the input impedance can be used as a comparison standard and all other noises can be measured in terms of it.

The manner in which all these various noises appear in the output of the distributed amplifier is considered below, the analysis being carried out for a single-stage distributed amplifier as shown in Fig. 2-1 .

a) Thermal noise

The grid line is terminated with resistances on each end, and both of these act as generators of thermal noise. The noise generated in the input termination will cause a noise voltage to appear at the output terminals in exactly the same way as if it were a signal. The noise due to the grid termination produces a noise wave on the grid line which is amplified by the tubes, the noise signals adding in the plate line in a way which depends upon the phase shift per section. The addition of the noise voltages in the plate line due to the backward going wave is the same mathematical problem as was considered in the case of reflections in Chapter 6. Calling the noise power in the output due to the input impedance N_1 and noise power due to the grid termination N_2,

N_T = total thermal noise output in a band Δf cycles wide at frequency f

$$= N_1 + N_2$$

$$= N_0 \frac{Z_{op}}{Z_{og}} A_0^2 n^2 \left[1 + \left(\frac{\sin n\phi}{n \sin \phi} \right)^2 \right] \qquad (8\text{-}1)$$

where $N_0 = 4kT\Delta f$ watts

k = Boltzman's constant

T = temperature of the termination, ^oK

Δf = bandwidth in cps in which noise is to be measured

f = frequency

A_0 = amplification of each section $= g_m Z_{op}/2$

ϕ = phase shift per section

n = number of sections per stage.

The first term in equation (8-1) is the amplified noise arising in the input impedance. The second term is due to the noise originating in the grid termination; it can have a value of unity when $\phi = 0, \pi$, etc., but is in general , smaller than unity. The functional dependence of this noise power on the phase shift per section is identical with the square of the voltage reflections from the grid termination shown plotted for n = 5 and n = 11 in Fig. 6-1(b). As can be seen from equation (8-1) and Fig. 6-1(b), the thermal noise due to the grid termination is usually small compared with the noise due to the input impedance. Only at DC and at cutoff do the two terms become equal.

b) Shot-effect noise

The shot-effect noise is due to the random emission of electrons from the cathode. The effect of this noise can be represented by a resistor in the grid circuit which is assigned a value such that this fictitious resistance generates as much noise as is actually observed in the plate circuit of the tube. If the impedance, looking back from the grid toward the input terminals, can be made much higher than this noise resistance, then the noise due to the shot-effect will be small compared with the thermal noise. At low frequencies and in narrow-band amplifiers, the input impedance can be made high, and consequently, the shot-effect noise can be made to be negligible. In wide-band amplifiers, including the distributed amplifier, the input impedance cannot be made high, and as a result, the noise generated by the shot-effect cannot be neglected.

However, in the case of the distributed amplifier, the shot-effect noise can be made negligibly small in spite of the fact that the grid-to-ground impedance is not high when compared to the equivalent noise resistance. This can be seen from the following considerations. Each tube develops a random noise current in its plate circuit independently of the other tubes used in the distributed amplifier. The noise currents cause voltages to appear on the plate line, and these voltages add in the output terminals in a random manner. The random addition of voltages can be obtained by taking a sum of the noise power produced by the individual tubes; thus if the tubes are alike, the total noise power will be proportional to the number of tubes. On the other hand, the signal at the output terminals is proportional to the number of tubes, and the signal power is proportional to the number of tubes squared. Hence, the signal noise ratio will be proportional to n, where n is the number of tubes. Thus, by using a sufficient number of sections, it is possible to make the signal as large as one desires compared to the shot-effect noise.

The effect of shot noise can be computed in the usual manner. The following results are given without proof. The shot-effect noise power N_S in the output of the distributed amplifier is

$$N_s = m N_o A_o^2 \frac{R_{eq}}{Z_{op}} \tag{8-2}$$

where A_o is the amplification per section. Thus, for a given tube and desired bandwidth, N_o, A_o, R_{eq}, and Z_{op} are known constants.

c) High frequency noise

The transit time effects and cathode lead inductance can both be taken into account by representing them as shunt resistances from grid to ground in each tube. Associated with this equivalent resistance there is a noise, which can be evaluated in a standard manner(1).

The behaviour of this noise in the output of the distributed

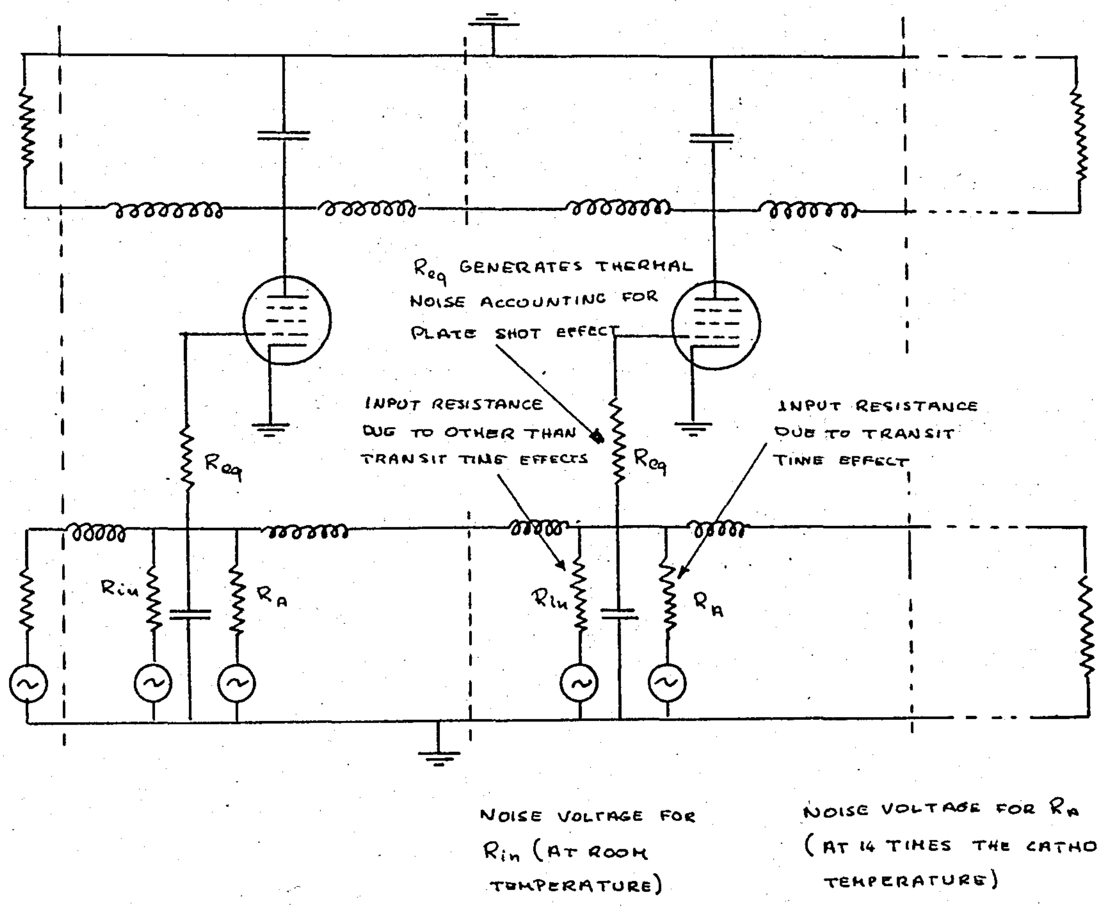


FIG. 8-1 – SOURCES OF NOISE IN SECTION (SYMBOLS AFTER TERMAN).

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(1) F. E. Terman, "Electronic and Radio Engineering", McGraw-Hill Book Co., Tokyo, 1955, pp. 434 - 442.

amplifier is very complicated. In the first place, the magnitude of the noise is a rapid function of frequency (the noise power per cycle is approximately proportional to the frequency squared). In the second place, each tube generates noise voltages which propagate in both directions from the tube. Thus, the noise generated by one tube is amplified by all other tubes. Moreover, the amplification depends upon the particular position of the tube in the distributed amplifier.

Fig. 8-1 shows a single section of distributed amplifier, indicating the sources of high-frequency noise. It should be pointed out that the magnitudes of the two sources of noise are determined by the geometric factors within the tube itself. For the purpose of this discussion, it will be assumed that an equivalent resistance R_A and an accompanying voltage can be found which accounts for the existing noise. If the noise power that R_A can deliver is N_T, then it can be shown that the total noise power N_A due to high-frequency effects in the output is given by

$$N_A = \frac{N_T R_A A_o^2 Z_{og}^2}{Z_{op} (Z_{og} + 2R_A)^2} P \qquad (8\text{-}3)$$

where P is a constant which depends on ϕ and n. Near DC and near cutoff,

$$\phi \to 0 \text{ or } \pi \quad \text{and} \quad P \to n^3 \qquad (8\text{-}4)$$

Near midband

$$\phi \to \pi/2 \quad \text{and} \quad P \to \frac{n^3}{3} \qquad (8\text{-}5)$$

Thus it can be seen that the noise power in the output due to grid loading effects is proportional to n^3, whereas the signal voltage is proportional to n^2. Hence, should the noise from this source be at all appreciable, increasing the number of stages, decreases the signal-to-noise ratio. However, for reasons having to do with attenuation, this noise is not too important. This will be discussed in part (d).

d) The noise factor of the distributed amplifier

The noise in the output of the amplifier is the sum of the three noises given above:

Total noise power = thermal noise + shot noise + grid-loading noise

or $$N_{TOTAL} = N_T + N_S + N_A \qquad (8\text{-}6)$$

The noise factor N.F. can be defined as the ration of the total noise in the output terminals to the noise due to the input impedance. Thus,

$$N.F. = \frac{N_T + N_S + N_A}{N_1} \qquad (8\text{-}7)$$

where the notation is as used above. Substituting values of these terms from equations (8-1), (8-2) and (8-3) and simplifying,

$$N.F. = 1 + \left(\frac{\sin n\phi}{n \sin\phi}\right)^2 + \frac{1}{n}\,\frac{R_{eq}}{Z_{og}} + n\,\frac{Z_{og}}{R_A}\,\frac{\alpha}{4} \qquad (8\text{-}8)$$

in which it has been assumed that

1 - $Z_{op} = Z_{og}$ for reasons of simplicity

2 - $R_A \gg Z_{og}$ for reasons to be explained below

3 - α is a numerical factor, equal to about 5, which takes into account the experimentally observed values of noise associated with R_A.

It should also be remembered that R_A is a function of frequency:

$$R_A \approx \frac{1}{f^2}$$

From equation (8-8) it will be seen that noise factor of the amplifier depends upon competing factors of $(^1/_n)$ and n. Thus, one would think that there should be an optimum value of n for minimum noise. Actually, such a choice would have little physical meaning. In the first place, R_A is a function of frequency; and in the second place, if frequency response is to be at all uniform, one must choose tubes in which $R_A \gg Z_{op}$ at the highest frequency of interest in order to avoid attenuation. Under these conditions, the associated high-frequency noise will also be small. Therefore, by using a sufficient number of sections, the shot noise can be made negligibly small and the resulting noise factor can be made to approach unity except at low and high frequencies where it approches 2 due to the noise arising in the grid termination.

Chapter 9

TRANSISTORIZED DISTRIBUTED AMPLIFIER

Transistorized distributed amplifiers can also be built but they do not offer a great advantage compared to cascaded transistor amplifiers. This has two main reasons. One reason is the base current, which is required for the operation of the transistor, introducing losses along the base line, making this line a lossy line. The second reason is the interaction of the base and collector circuits due to the junction capacitance. Fig. 9-1 shows this capacitance.

The common-emitter current gain of a transistor is shown in Fig. 9-2(a). For amplification to be independent of frequency the collector current, i_c, should be constant. We know that i_c is given as βi_b, where i_b is the base current. If i_b is as in Fig. 9-2(b), having the same slope as β,

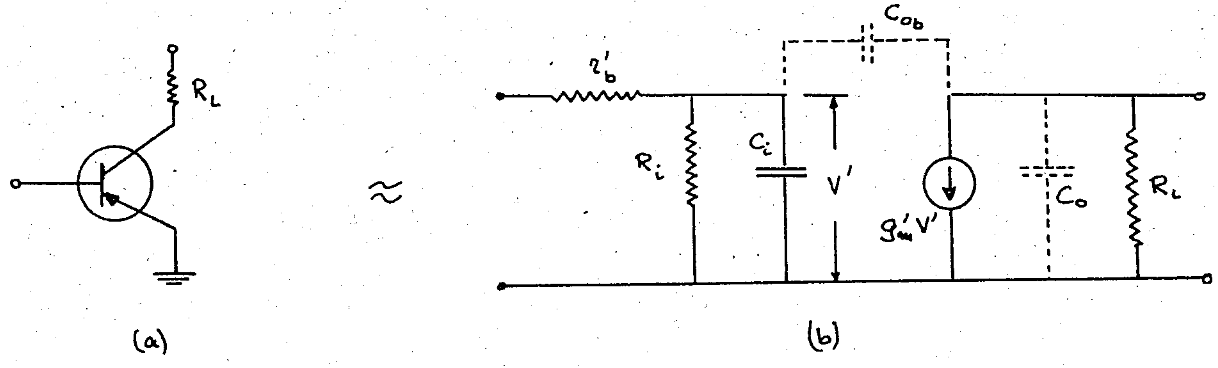

FIG. 9-1 — (a) A CE TRANSISTOR STAGE; (b) AN APPROXIMATE MODEL FOR THE CE STAGE.

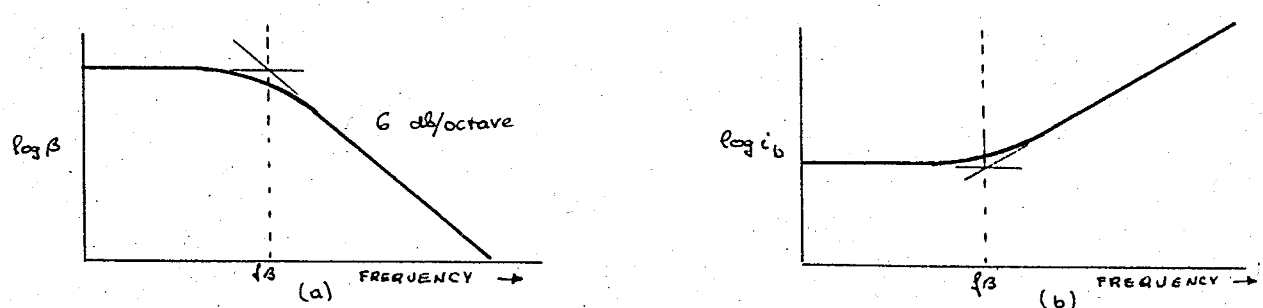

FIG. 9-2 — (a) COMMON EMITTER CURRENT GAIN; (b) BASE CURRENT i_b REQUIRED TO GIVE CONSTANT COLLECTOR CURRENT INDEPENDENT OF FREQUENCY.

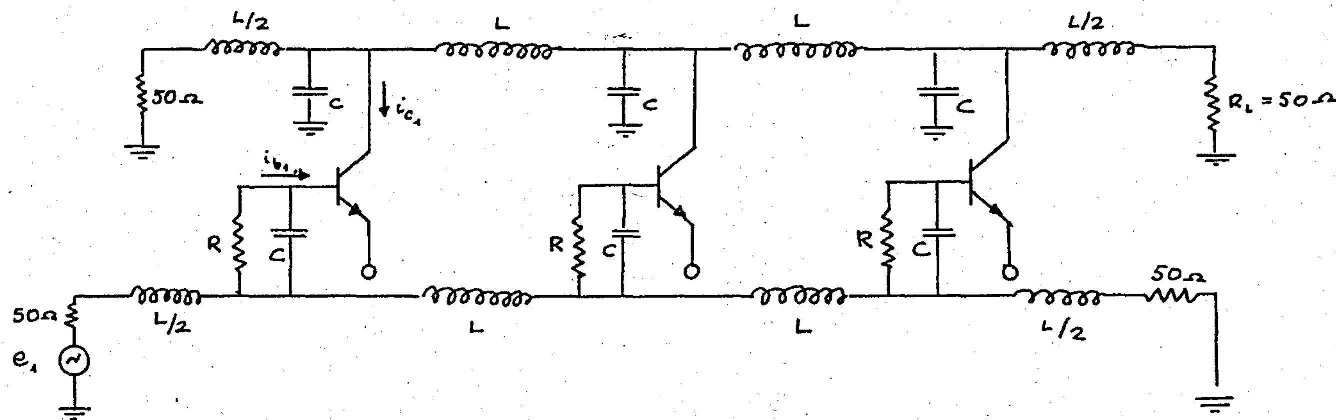

FIG. 9-3 — A THREE-SECTION SIMPLE DISTRIBUTED AMPLIFIER.

i_c will be constant. This can be easily achieved by using an RC network for the base.

If e_1 is the input voltage, the base current will be

$$i_{b_1} \approx \frac{e_1}{R} \tag{9-1}$$

The output voltage will be, considering that we have n sections

$$e_o = n i_c \frac{R_L}{2} = n \beta_o i_b \frac{R_L}{2} = n \beta_o e_1 \frac{R_L}{2R} \tag{9-2}$$

where β_o is the low-frequency current gain

n is the number of sections

R_L is the load resistance

i_b is the base current

i_c is the collector current

We have then for the voltage gain

$$A_v = \frac{e_o}{e_1} = \frac{n \beta_o R_L}{2R} \tag{9-3}$$

With the double-diffused transistor, however, the situation is not so simple and the equivalent circuit of Fig. 9-4 has to be used to be able to analyse the amplifier.

In this circuit we have

r'_{bb} = Base spreading resistance

C_{c_1} = collector capacitance under emitter dot

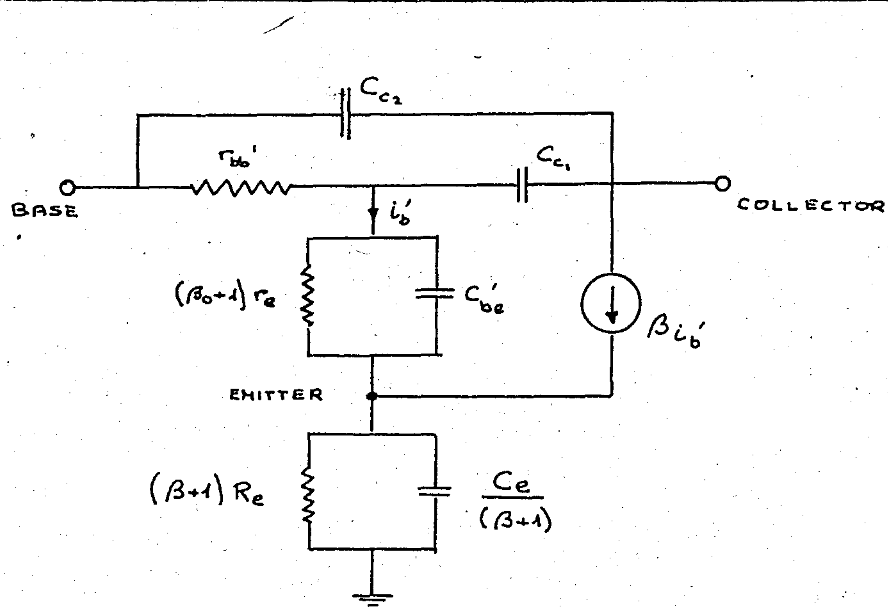

FIG. 9-4 — APPROXIMATE HIGH-FREQUENCY EQUIVALENT CIRCUIT FOR DOUBLE-DIFFUSED TRANSISTOR.

C_{c_2} = collector base capacitance which is not directly under emitter dot plus stray collector base capacitance

r_e = emitter resistance

C_{be}' = diffusion capacitance

$\beta = \dfrac{\beta_o}{1 + j\, f/f_\beta}$ = current gain at frequency f

β_o = low-frequency current gain

f_β = beta cutoff frequency

For the high frequencies under consideration the approximate input circuit is as shown in Fig. 9-5.

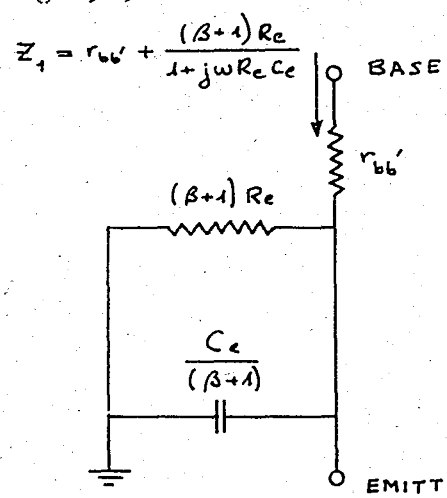

FIG. 9-5 — APPROXIMATE INPUT CIRCUIT FOR $f \gg f_\beta$

The input impedance of this circuit is

$$Z_1 \approx r_{bb'} + \frac{(\beta+1)\,R_e}{1 + j\omega R_e C_e} \tag{9-4}$$

where R_e and C_e are the resistance and capacitance inserted in the emitter. Using the relation

$$\beta = \frac{\beta_o}{1 + j\dfrac{f}{f_\beta}} = \frac{\beta_o}{1 + j\dfrac{\omega}{\omega_\beta}}$$

and considering that the frequencies of interest are below $f_2 = \frac{1}{2\pi R_e C_e}$ this can be written as

$$Z_1 \approx (r_{bb'} + R_e) + \frac{\beta_o R_e}{1 + j\dfrac{\omega}{\omega_\beta}} \tag{9-5}$$

If we insert a parallel RC network in the base of the transistor as is done above the total impedance becomes

$$Z_T \approx \frac{R}{1 + j\omega RC} + (r_{bb'} + R_e) + \frac{\beta_o R_e}{1 + j\dfrac{\omega}{\omega_\beta}} \tag{9-6}$$

The RC combination is chosen to have the same 3 db point as f_β. Therefore

$$f_\beta = \frac{1}{2\pi RC}$$

Equation (9-6) becomes then.

$$Z_T = \frac{R + \beta_o R_e}{1 + j\dfrac{\omega}{\omega_\beta}} + (r_{bb'} + R_e) \tag{9-7}$$

This in combination with the transistor is shown in Fig. 9-6.

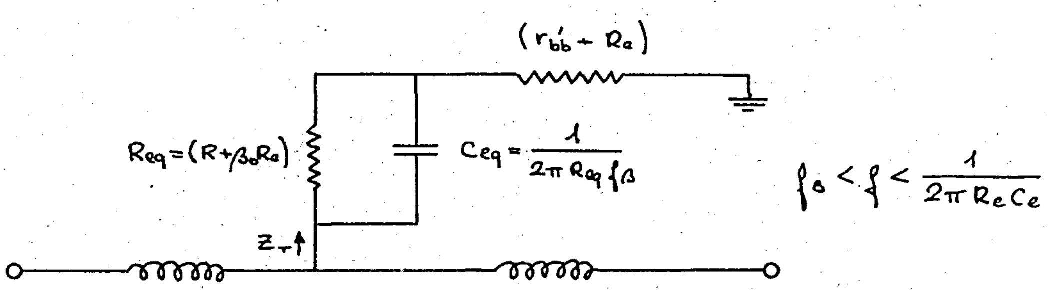

FIG. 9-6 – APPROXIMATE EQUIVALENT INPUT IMPEDANCE OF INPUT RC COMBINATION PLUS TRANSISTOR.

For very high frequencies, from equation (9-4), recalling that $f_\beta = \frac{1}{2\pi RC}$

$$Z_1 = r_{bb}' + \frac{R_e}{1 + j\omega R_e C_e} + \frac{\beta_o R_e}{(1 + j\omega R_e C_e)(1 + j\omega RC)} \qquad (9\text{-}8)$$

An analysis of equation (9-8) shows that at very high frequencies ($f > \frac{1}{2\pi R_e C_e}$), the r_{bb}' term will predominate and the response of the amplifier will fall, while the input current no longer rises. The role of the $R_e C_e$ combination is then to preserve the fall of the input impedance Z_T to as close to the 6 db fall-out curve as possible.

Let us now show the effect of R_e and C_e on the gain-bandwidth product. It can easily be shown that the voltage gain A_v of one section at low frequencies is

$$A_v = \frac{\beta_o R_L}{2\left[(R + \beta_o R_e) + (r_{bb}' + R_e)\right]} \qquad (9\text{-}9)$$

Considering first the uncompensated case where C_e is zero we can write for the bandwidth (Fig. 9-6)

$$BW_u = \frac{1}{2\pi\ (r_{bb}' + R_e)\ C_{eq}}$$

where C_{eq} is as shown in Fig. 9-6, and equal to

$$C_{eq} = \frac{1}{2\pi f_\beta\ (R + \beta_o R_e)} \qquad (9\text{-}10)$$

Using equation (9-10) the expression for the uncompensated bandwidth becomes

$$BW_u = f_\beta \left[\frac{R + \beta_o R_e}{r_{bb}' + R_e}\right] \qquad (9\text{-}11)$$

Hence the gain-bandwidth product for uncompensated case can be written as the product of gain and bandwidth or, equations (9-9) and (9-11)

$$GB_u = \frac{\beta_o R_L}{2\ (r_{bb}' + R_e)}\ f_\beta \qquad (9\text{-}12)$$

We can see ffom equation (9-9), (9-11) and (9-12) that we can somewhat alter the gain-bandwidth product by varying R_e, although an optimum case will be obtained when $R_e = 0$.

Let us now consider the compensated case. From equation (9-8), the input impedance Z_1 of the transistor will be

$$Z_1 = r_{bb'} + \frac{R_e}{1+j\omega R_e C_e} + \frac{\beta_o R_e}{(1+j\,\omega/\omega_\beta)(1+j\omega R_e C_e)}$$

If we add the parallel RC to the base circuit we get a total impedance Z_T equal to

$$Z_T = \frac{R}{1+j\,\omega/\omega_\beta} + r_{bb'} + \frac{R_e}{1+j\omega R_e C_e} + \frac{\beta_o R_e}{(1+j\,\omega/\omega_\beta)(1+j\,\omega R_e C_e)} \qquad (9\text{-}13)$$

As explained above, to have a good compensation R_eC_e should be chosen to have the same break frequency as the uncompensated bandwidth. Therefore

$$f_{2u} = \frac{1}{2\pi R_e C_e} \qquad (9\text{-}14)$$

We should now find the value of Z_T at f_{2c}, where f_{2c} is the upper 3 db point of the compensated amplifier. Equating real and imaginary parts of Z_T at f_{2c} will give us the value of f_{2c}. Since $f_{2c} \gg f_\beta$ we can assume $1 + j\,\omega/\omega_\beta \approx j\,\omega/\omega_\beta$. Furthermore we can also assume $j\omega R_e C_e = j$ as seen from equation (9-14). With these assumptions we have

$$Z_T\Big|_{f=f_{2c}} = -j\frac{\omega_\beta}{\omega_{2c}}\left[R + \frac{\beta_o R_e}{1+j}\right] + r_{bb'} + \frac{R_e}{1+j} \qquad (9\text{-}15)$$

Reducing this we obtain

$$Z_T = \left[r_{bb'} + \frac{R_e}{2} - \frac{f_\beta \beta_o R_e}{2 f_{2c}}\right] - j\left[\frac{f_\beta}{2 f_{2c}}(\beta_o R_e + 2R) + \frac{R_e}{2}\right] \qquad (9\text{-}16)$$

Equating the real and imaginary part of this equation we obtain

$$f_{2c} = f_\beta\left(\frac{R + \beta_o R_e}{r_{bb'}}\right) \qquad (9\text{-}17)$$

By taking the ratio of equations (9-17) and (9-11) we obtain the bandwidth improvement factor K

$$K = \frac{f_{2c}}{f_{2u}} = \left(1 + \frac{R_e}{r_{bb'}}\right) \qquad (9\text{-}18)$$

We can now specify a design procedure. Given the required bandwidth and all the required transistor parameters, it is required to know L_o and C_o of the line, the input RC and R_eC_e of the emitter. We would then use the following procedure:

1 - From the transmission line equations

$$Z_o = \sqrt{\frac{L_o}{C_o}\left(1 - \frac{f}{f_c}\right)^2} \quad \text{and} \quad f_c = \frac{1}{\pi\sqrt{L_o C_o}}$$

we select the cutoff frequency of the line so that the required Z_o does not depart by more than an arbitrary amount from the low frequency value throughout the frequency range of interest. This completely specifies L_o and C_o. Note that this C_o will be the same as C_{eq}.

2 - Arbitrarily select a convenient input C somewhat greater than C_o, say 2 or 3 times.

3 - Select the input R to give $R = \frac{1}{2\pi f_B C}$.

4 - From equation (9-17) determine R_e

5 - From equations (9-11) and (9-14) select C_e.

An example to this procedure will be given in the next chapter.

Chapter 10

PRACTICAL DESIGNS

One of the two designs to be given is a distributed amplifier which covers from 10 to 360 mC. This amplifier has a gain of 8 db flat to ± 2 db over the given range and is constructed of 6AK5 pentodes. The transmission lines are of the constant-k type with m-derived half sections. Two of the sections of the amplifier are shown in Fig. 10-1. The chosen impedances for the grid and plate lines are 50 and 93 ohms respectively.

For the constant-k low-pass lines used in this amplifier we have

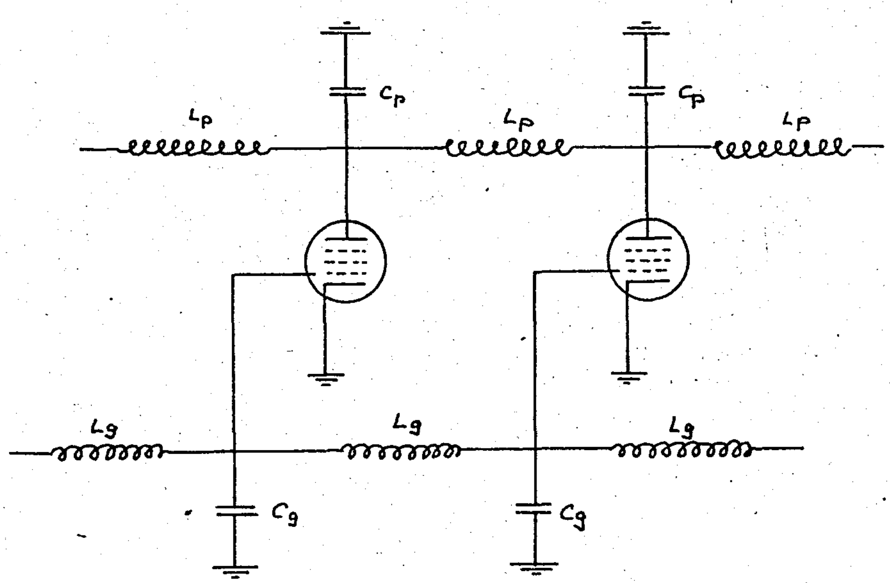

FIG. 10-1 — TWO SECTIONS OF THE CONSTANT-K TYPE CIRCUIT USED IN DISTRIBUTED AMPLIFIER.

$$C/\text{section} = \frac{2}{2\pi R f_c}$$

$$L/\text{section} = \frac{2R}{2\pi f_c}$$

where $R = \sqrt{\frac{L}{C}}$ characteristic impedance

$f_c =$ cutoff frequency.

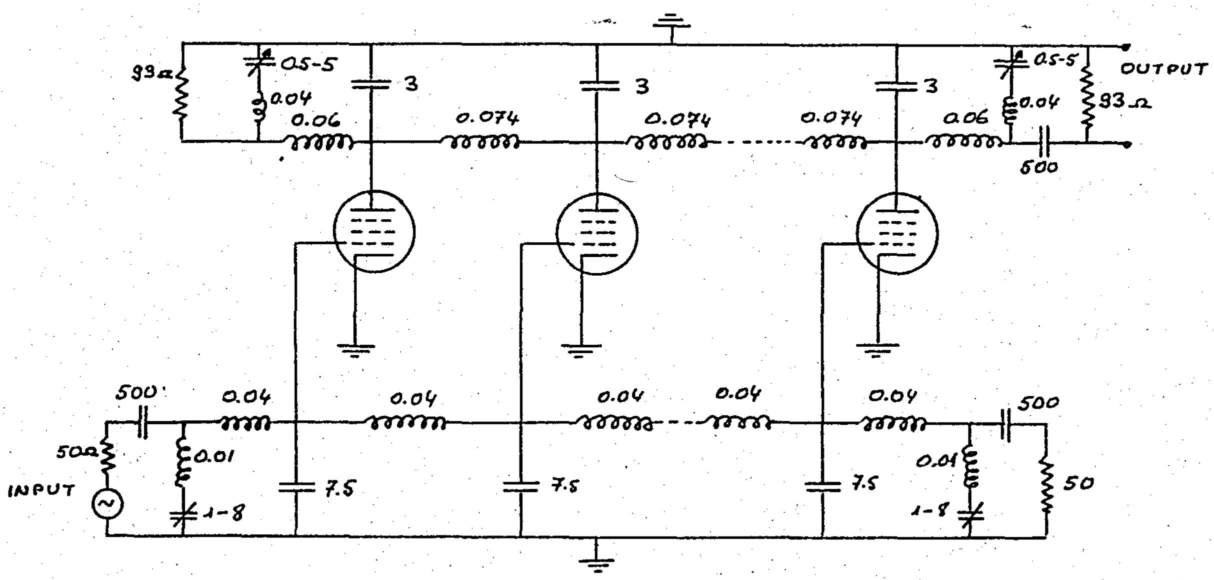

FIG. 10-2 — PARTIAL CIRCUIT SHOWING COMPONENT VALUES FOR THE 360 MC DISTRIBUTED AMPLIFIER. INDUCTANCES ARE IN MICROHENRIES AND CAPACITANCES IN MICRO MICROFARADS.

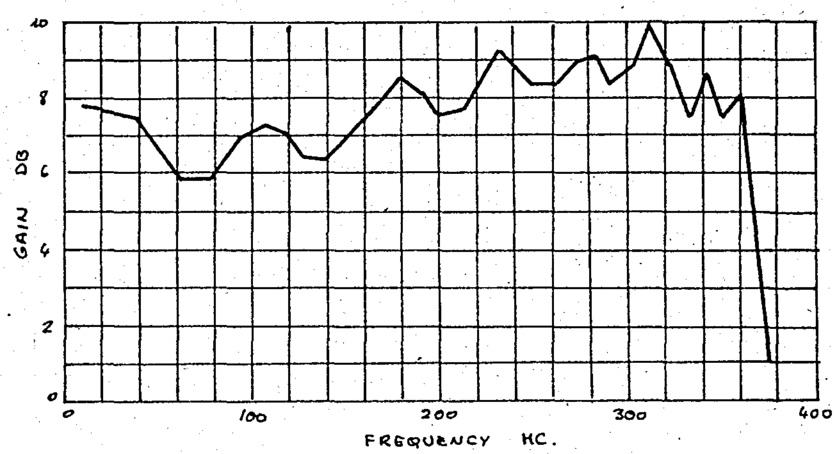

FIG. 10-3 — GAIN-FREQUENCY CURVE SHOWS RESPONSE OF AMPLIFIER TO BE FLAT WITHIN ± 2 DB FROM 10 TO 360 MC.

The amplifier uses a total of 9 tubes. The reason for choosing the 6AK5's is because of its high figure of merit. The tube is operated with $E_b = E_{sc} = 120$ V and $E_c = -1.5$ V. The calculations made, the amplifier is as shown in Fig. 10-2 having a gain $e = 2.718$ or 8.68 db. The gain-frequency characteristic is shown in Fig. 10-3.

The second design is of a transistorized distributed amplifier and its design is done according to the pattern given in chapter 9. The

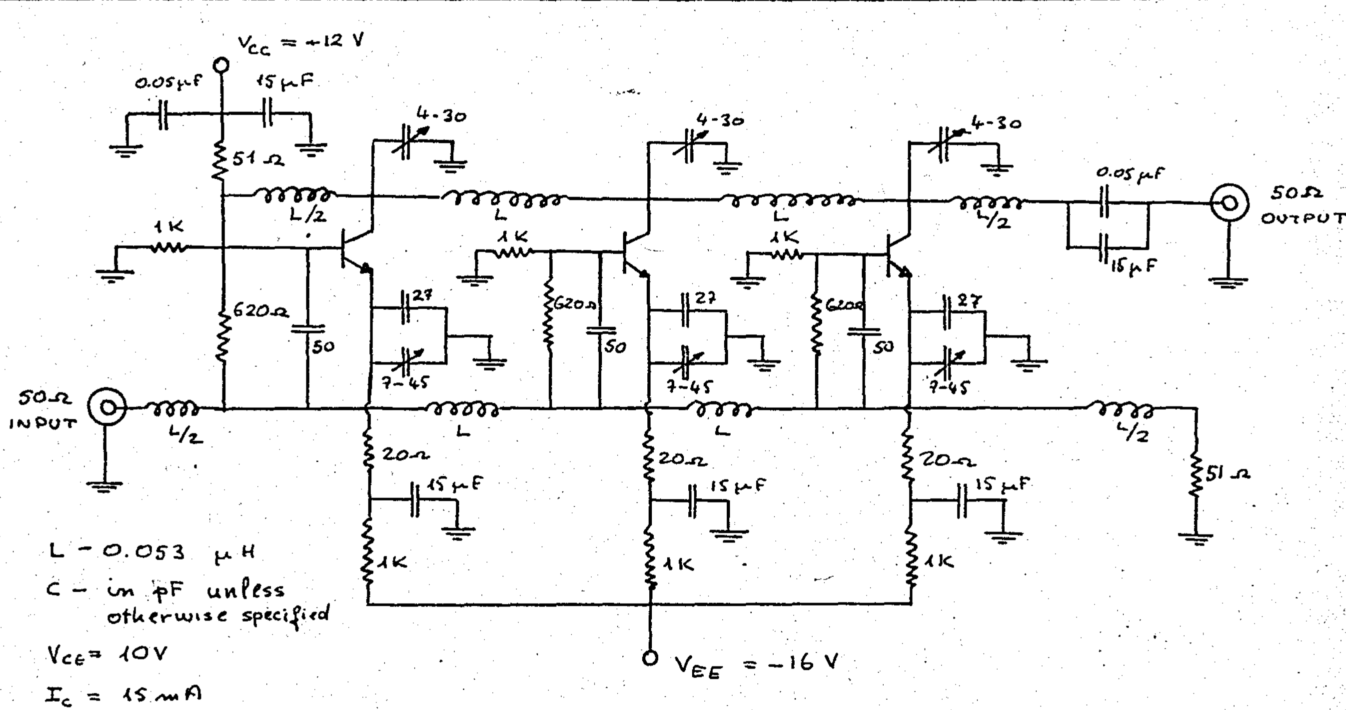

Fig. 10-4 – One stage of a 200 Mc/sec transistor distributed amplifier

transistor used is a Fairchild N104B double-diffused silicon transistor. Its typical values at $V_{CE} = 10$ V and $I_E = 10$ mA are

$r_{bb'} = 70$ ohms

$C_{c_1} = 1$ pF

$C_{c_2} = 2$ pF

$\beta_o = 80$

$f_\beta = 5$ mC

$r_e = 2.5$ ohms

$C_{b'e} = 150$ pF.

Let us assume a low-frequency Z_o of 50 ohms. The design steps are:

1 - Select a cutoff frequency of 300 mC. This will give a Z_o of about 40 ohms at 165 mC which is assumed to be adequate. From the transmission line equations, $L_o = 0.053$ H and $C_o = 21$ pF.

2 - Select C to be 50 pF.

3 - R is calculated to be $\frac{1}{2\pi f_\beta C}$. Select R = 620 ohms.

4 - From equation (9-17) $R \cdot \beta_o = 2300$ ohms. Select $R_e = 20$ ohms.

5 - Equations (9-11) and (9-14) yield C_e. Select $C_e = 62$ pF.

The circuit for one stage of this amplifier is given in Fig. 10-4.

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