# QUINE ON MODAL LOGIC

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# QUINE ON MODAL LOGIC

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Thesis Abstract

Hanife Bilgili, “Quine on Modal Logic”

This thesis aims at examining and evaluating Quine’s objections to modal logic. In the history of the study of modality there has been an effort to offer a system that is capable of putting the subject in question out formally; however, the most accepted one today is the possible worlds semantics created by Kripke. Hence, the investigation will begin with a short introduction to modal logic in order to continue with a powerful tool at hand. The next step will be an examination of Quine’s objections from his articles, starting with his 1943 article and ending with 1971. After presenting the objections, I will give the accounts of the philosophers who have responded to his objections. And after each objection and response, I discussed whether Quine’s points still have a philosophical weight.

Tez Özeti

Hanife Bilgili, “Quine’ın Modal Mantığa Bakışı”

Bu tezin amacı Quine’ın modal mantığa karşı getirdiği itirazları incelemek ve değerlendirmektir. Modalite çalışmaları tarihinde söz konusu konuyu formal bir şekilde yazmayı çabalayan sistemler önerilmiştir. Fakat bunlar içinde bugün en çok kabul göreni Kripke’nin olası dünyalar semantiğidir. Bu yüzden daha güçlü bir silahla devam edebilmek için, çalışma modal mantığa kısa bir girişle başlamaktadır. Bunun ardından Quine’ın 1943 ve 1971 yılları arasında konuyla ilgili yazdığı makalelerdeki itirazları incelenecektir. İtirazlar sunulduktan sonra bu itirazlara cevap bulmaya çalışan felsefecilerin açıklamaları sunulacaktır. Her itiraz ve cevabın ardından Quine’ın itirazlarının hala felsefi ağırlığının olup olmadığı konusunda kendi fikrimi söyleyeceğim.

## ACKNOWLEDGMENTS

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My special thanks goes to my family, Öznur, Abdullah, Çağın and Alper Bilgili, for their loving, supporting, and believing in me through this project and for all my life. I also want to thank Mine Ö. Çanakçı, who has been a great roommate and a sister for the last four years. And finally I want to thank my geek friend Ertan Doğrultan, who encouraged me a lot, for discussing with me logic, or any other subject.

*For all the possible me's at every accessible world,*

*who I'm sure are studying this subject*

CONTENTS

# CHAPTER 1

# INTRODUCTION

## Aim of Thesis

My thesis will be a study of Willard Van Orman Quine’s objections to modal logic, his arguments and responses directed to these objections by other philosophers.

In order to present these ideas, first I will introduce modal logic, only some basic material that will help us understand possible world semantics.

After that, in the second chapter I will present Quine’s arguments against quantified modal logic through some very important papers, like “On What There Is”, “Notes on Existence and Necessity”, “The Problem of Interpreting Modal Logic”, “Three Grades of Modal Involvement”, and “Two Dogmas of Empiricism.”

After presenting each argument by Quine, with the aid of that tool I will focus on the answers given to him, by philosophers like Rudolf Carnap, Ruth Barcan Marcus, Saul Kripke, Terence Parsons, Kit Fine.

Also, I will present my own ideas on Quine’s objections, whether they are valid or whether they make sense from the vantage point of recent understanding of modal logic; specifically Kripkean semantics, and whether the answers given to his objections fully understand what he is saying and value his objections, and whether they are strong enough to refute the objections.

## Some Terminology

Below, I am presenting some terminology that Quine often uses, essentially with his definitions. I did not want to give these definitions within the chapters in order to prevent any possible disruption of the flow.

### Principle of Substitutivity

Given a true statement of identity, one of its two terms may be substituted for the other in any true statement and the result will be true.

E.g.: “9 = number of planets” is a true statement of identity. And if we replace ‘9’ with ‘the number of planets’ in the sentence “9 is greater than 4”, which is true, the sentence will become “the number of planets is greater than 4” which has the same truth value.

### Purely Designative Occurrence

If the name of an object refers simply to the object designated by that name, that is a purely designative occurrence of that name. Principle of substitutivity fails if the name does not occur purely designatively.

E.g.: Giorgione=Barbarelli is a true statement of identity. And the occurrence of ‘Giorgione’ in the sentence ‘Giorgione was so-called because of his size’ is not a purely designative occurrence of the name. Because when we replace ‘Giorgione’ with ‘Barbarelli’ the sentence becomes ‘Barbarelli was so-called because of his size’ which is not true, which is because the name ‘Giorgione’ did not occur purely designatively. The phrase ‘so-called’ imposes more burden on the name ‘Giorgione’ is supposed to hold, more than just referring to the person.

If we write the sentence as ‘Giorgione was called ‘Giorgione’ because of his size’ the first occurrence of ‘Giorgione’ is a purely designative occurrence, referring simply to the person Giorgione, and the second occurrence is not referring to the person, hence not a purely occurrence of the name.

## Truth-Functional Operator

An operator which assigns the truth-value of the complex proposition, exclusively depending on the truth-values of its component parts.

E.g.: $\neg$, v are truth-functional operators, if P is true we are certain that $\neg$P is false.

‘believes that’, ‘follows from’ are not truth-functional operators, although the sentence “moon is made up of blue cheese” is false, the sentence “I believe that moon is made up of blue cheese” may not be false.

## Intensional Compound

A compound whose truth value is not determined merely by the truth values of the components of the compound.

E.g.: In the sentence “Necessarily no spinster is married” there is a non-truth functional operator, a modal operator, so, the components’ truth values would not be enough to decide the truth value of the sentence.

## Referential Opacity

A context is referentially opaque if it can render a referential occurrence non-referential.

E.g.: the context ‘‘. . .’ contains six letters’ is a referentially opaque one. If it is applied to the name ‘Cicero’ for example, this name would not refer to the person Cicero any more, but becomes a meaningless word containing 6 letters.

## Substitutional Quantification

Having the truth value true as a result of instantiating an existential quantification with a constant for the variable of quantification.

E.g.: For the sentence $\exists$x(x is a mammal $\wedge$ x can fly), we can have an instance by replacing the variable of quantification with a constant, and get the sentence “bat is a mammal and bat can fly” which is true.

## Historical Background

Although the semantics of modal logic was founded in the second half of the 20th century, modal logic was an ancient topic having its roots in Aristotle’s philosophy on the level of a debate on necessity, possibility and contingency. However, Lewis was the first logician who studied modal logic in a modern and a systematic way. Lewis was not the introducer of modal logic, but he might be said to be the modern re-introducer of modal logic.

Originally, Lewis did not intend to formulate modal logic. In order to avoid the paradoxes of material implication, he came with the notion of strict implication for which he has developed a propositional calculus. He was in fact trying to find the appropriate axiom that would best capture the meaning of implication which he thought was misused by Bertrand Russell and Alfred North Whitehead in *Principia Mathematica*. So, he introduced a new connective called strict implication, denoted by the fishhook "⥽" meaning "P strictly implies Q" and shown as "P ⥽Q" in modern notation is turned out to be equal to "□ (P ⊃ Q)."

The notion of strict implication was based on the idea that for a proposition in the form "P ⊃Q" to be strictly true meant it is impossible for P to be true and Q to be false, otherwise it is a contradiction. This system also had its own problems like an impossible proposition's implying every proposition or every proposition's implying a necessary proposition. However, unfortunately for Lewis, the paradoxes of material implication are now replaced by the paradoxes of strict implication.

Lewis gave five formulations of modal logic some of which are still used today, specifically $S_4$ and $S_5$. Lewis is aware of the fact that they are different from one another, but he cannot prove it. And he does not know which notion of necessity corresponds to the necessity that is used in our daily language. Since Lewis's system lacked semantics, there was no interpretation of the formulas either. So he had such problems in interpreting his logic.

The confusion of use-mention is what is claimed by Quine to be responsible for Lewis's introduction of the new connective; "⥽" (Fitting and Mendelsohn 41). When we say 'P strictly implies Q' we are saying something *about* 'P' and 'Q', which means that we are not making a sentence in the object language that contains the symbols 'P' and 'Q'. That is why, we are *mentioning* these propositions and we are not *using* them. And this way ‘necessity’ ceases to be a propositional operator and becomes a sentence predicator. And being a sentence predicator, ‘necessity’ does not work with the sentences themselves but with their very names.

Ignoring the difference between use and mention, Whitehead and Russell wrote ‘P implies Q’ for ‘if P then Q.’ And Whitehead and Russell’s naming ‘if P then Q’ as material implication led Lewis to the notion of strict implication (Quine, Modality 196). Quine presents Lewis’s definition of strict implication as a material implication that has to be not merely true but analytic in order to qualify as implication properly so called (Quine, Modality 196).

Lewis followed them and translated ‘P ⥽Q’ as ‘Necessarily not (P and not Q)’. This way ‘necessarily’ became an operator ranging over sentences. It is Carnap who first referred to the ‘use-mention’ confusion in his book, *Meaning and Necessity*, and hence drew attention to the difference between phrases ‘necessarily’ and ‘is analytic.’ (Quine, Modality 196).

It is not only the similarity in meaning between ‘use-mention’ and ‘Necessarily-is analytic.’ There is more than that, if we consider the usages too. “Use” is more like ‘Necessarily P’ in the sense that it uses ‘P’, and it is saying something using ‘P’; they are at the same level. And “mention” is more like ‘is analytic’ in the sense that it is saying ‘‘P’ is analytic’, it is mentioning ‘P’; so they are not at the same level.

Grounding his dislike of modal logic to strict implication, Quine may be committing the fallacy known as genetic fallacy. Since strict implication did not work well and had its own paradoxes, it does not mean that it cannot lead to a sound system as modal logic. I think modal logic strengthened with possible world semantics do not rely on a faulty context as strict implication any more. It is a whole new system, fruitful if we want to see what it led to.

# CHAPTER 2

# MODAL LOGIC

## What is Modal Logic?

Modal logic can be defined as an attempt to characterize the alternative state of affairs, or how things could have been. Although there are some other attempts to formalize modal logic, Kripke's possible world semantics has been the dominant one. And I will be giving a brief account of modal logic through possible world semantics that will be adequate to present Quine's objections to modal logic.

First of all a language is needed in which it will be possible to express modal propositions. However, this language will be without any meaning, it will consist of a pure syntax. After syntax, in order to give these formulas meaning we need semantics. Semantics is the tool that enables us to pick out valid ones from the set of all formulas.

After having both syntax and semantics at hand, we shall know the formulas and their meanings. And we will need a proof system with which we can prove exactly the valid formulas of our logic. However, we shall not need proof systems, so I will not go into them.

## The System K

Kripke semantics can be built by adding an axiom, an inference rule and an operator to the classical logic.

### The Language of the System

Propositional letters/variables of the language are used to build formulas. Propositional connectives are employed in order to build more complex formulas from the propositional letters.

| | |
|---|---|
| Propositional letters/variables: | P, Q, R… |
| Propositional connectives: | ¬ (unary), ∧ , ∨ , ⊃ , ≡ (binary) |
| (Modal operators: ) | □: necessarily |
| | ◊ : possibly |

We have two new operators, □ and ◊ , in our language in distinction from classical logic. These are modal operators, both monadic. □ stands for necessity, where '□X' is read as "it is necessary that X," and ◊ stands for possibility, where ' ◊ X' is read as "it is possible that X," for a formula X of K. The possibility operator ◊ can be defined in terms of the □ as shown below.

[Def ◊P] = ¬□¬P

## Propositional Modal Formulas

The Propositional Modal Formulas are specified as follows:

1. Every $X$ that is a propositional letter is also a formula.
2. If $X$ is a formula, so is $\neg X$ .
3. If $X$ and $Y$ are formulas, and $\circ$ any binary connective of our language, $(X \circ Y)$ is also a formula.
4. If $X$ is a formula, so are $\Box X$ and $\Diamond X$ .

## Axiomatic System K

The basic system according to Kripke is the system K. It consists of one axiom in addition to axioms of classical logic, which is: $\Box(P \supset Q) \supset (\Box P \supset \Box Q)$

This axiom means that when we have a necessary conditional, if the antecedent is necessary then it follows that the consequent should also be necessary.

New systems can be built up by adding new axioms, such as the system T which is formed by adding the 'Axiom of Necessity' $(\Box P \supset P)$ to the system K.

The system K has 3 basic inference rules, and further some other rules might be derived from these.

Inference Rules:

US: the rule of Uniform Substitution:

The result of uniformly replacing any variable(s) $P_1, \ldots, P_n$ in a theorem by any wff $\beta_1, \ldots, \beta n$, respectively is itself a theorem.

MP: the rule of Modus Ponens (or the rule of detachment)

If $\alpha$ and $\alpha \supset \beta$ are theorems, so is $\beta$.

N: the rule of Necessitation

If $\alpha$ is a theorem, so is $\Box \alpha$.

## Kripke Semantics for the System K

We start with a non-empty set G and a binary relation R on G.

### Accessibility Relation

The accessibility relation, R, is a binary relation between possible worlds. It shows how the worlds are related with one another.

For example take the sentence "If I had never read Nietzsche it would be possible that I might be living a more joyful life." In this sentence I'm talking about a possible situation, in which I had never read any piece by Nietzsche and a possible outcome of this action. Our world would easily be that way, so we say that it is "accessible" from our world. Possible situations are called possible worlds. We conceive the set G consisting of such possible worlds.

I will be using the symbols $\Delta, \Gamma, \Omega$ for possible worlds. And write $\Delta R \Gamma$ to denote $\Gamma$ is accessible from $\Delta$. So for example $\Delta R \Gamma$ could tell the story given above, where $\Delta$

stands for the actual world I'm in and Γ for the possible world in which I had never read Nietzsche.

We may put several conditions on the accessibility relation R depending on the modal system we want to work in; such as

1. Reflexivity: $\forall\Delta\ (\Delta \in G \supset \Delta R \Delta)$, meaning that any world $\Delta$ in G has an access to itself.
2. Anti-symmetry: $\forall\Delta\forall\Gamma(\Delta \in G \wedge \Gamma \in G, \Delta R\Gamma \wedge \Gamma R\Delta \supset \Delta = \Gamma)$, meaning that two distinct possible worlds are not mutually accessible to each other.
3. Transitivity: $\forall\Delta\forall\Gamma\forall\Omega(\Delta \in G \wedge \Gamma \in G \wedge \Omega \in G, \Delta R\Gamma \wedge \Gamma R\Omega \supset \Delta R\Omega)$, meaning that a possible world that is accessible to another possible world it can access is also accessible to it.
4. But in system K there is no condition on R.

Frame

A frame is a pair consisting of a set of possible worlds, G, and an accessibility relation R on set G, shown as <G, R>.

Model

A modal model is formed by adding a valuation function, V. The function V determines for each propositional letter P the set of worlds in which P is assumed to be true in the model.

So, a propositional modal model is a triple <G, R, V>, where <G, R> is a frame and V is a relation between possible worlds and propositional letters. We now define truth relation, $\Vdash$, between the possible worlds and propositions, where we read $\Gamma \Vdash X$ as "the formula X is true at the world Γ" and define $\Gamma \Vdash X$ as follows:

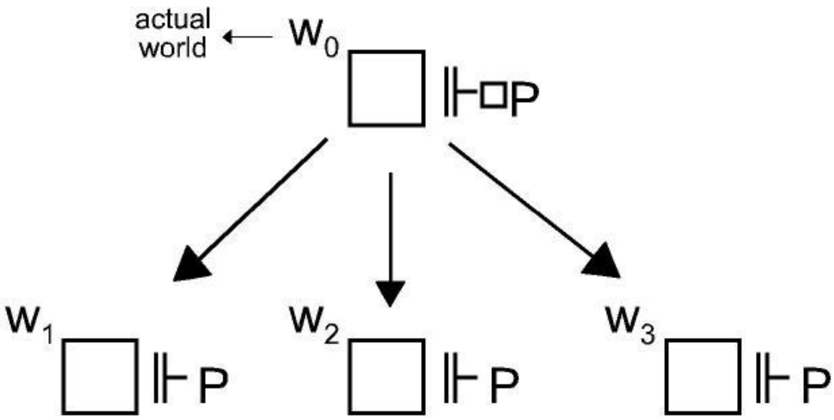
Let <G, R, V> be a model.

For each $\Gamma \in G$:

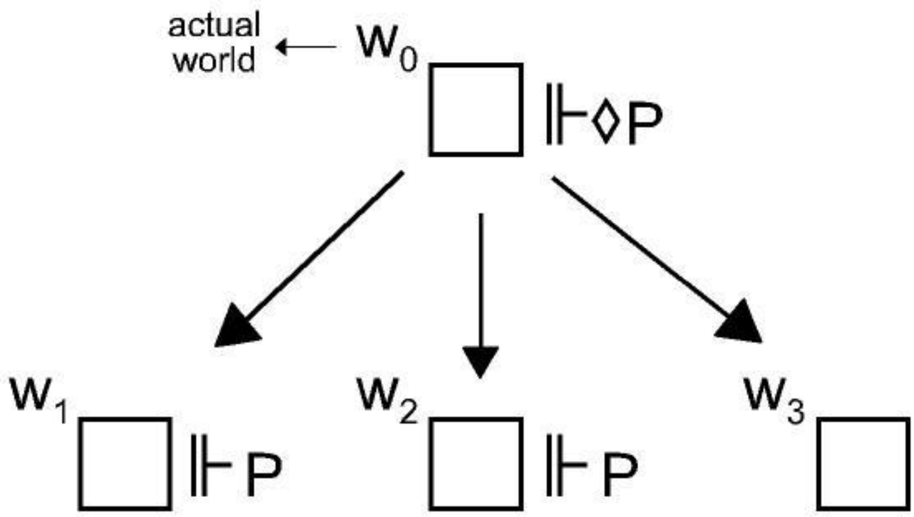
0. $\Gamma \Vdash X \Leftrightarrow V(X) = T$, if $X$ is a propositional letter.
1. $\Gamma \Vdash \neg X \Leftrightarrow \text{not } \Gamma \Vdash X$.
2. $\Gamma \Vdash (X \wedge Y) \Leftrightarrow \Gamma \Vdash X \text{ and } \Gamma \Vdash Y$.
3. $\Gamma \Vdash \Box X \Leftrightarrow \forall\Delta(\text{ if } \Gamma R \Delta \text{ then } \Delta \Vdash X)$
4. $\Gamma \Vdash \Diamond X \Leftrightarrow \exists\Delta(\Gamma R \Delta \text{ and } \Delta \Vdash X)$

The third and the fourth clauses stated above are crucial to the semantics of modal logic, as they set forth the criterion of being a necessary truth and a possible truth. What 3 says is that in order for X to be necessarily true at possible world Γ it should be true in every world that are accessible from Γ. And 4 says that in order for X to be possibly true at possible world Γ it should be true in at least one of the worlds that is accessible from Γ.

According to 3, a necessary truth should be true in every accessible possible world. And another thing about the necessity operator is that, in any interpretation of □, the necessity operator is not a truth-functional one; that is, the truth-value of P itself is not always sufficient to determine the truth-value of □P.




The valuation may assign different truth values to the same proposition in different worlds. That is to say, the truth value of P might differ from world to world. For example my having dark hair is true in the actual world, but in some other possible world I might be blond. This can be explained in terms of modal operators as follows; in order for □P to be true, V should assign the proposition the truth value true in every possible world. For ◊ P to be true on the other hand, V's assigning true to P in at least one possible world is enough.




As I claimed before, modal operators are not truth-functional. Since they are not truth functional, it is a question that under what conditions the propositions are given the truth values T or F.

So, it can be stated that modal propositions involve considerations about how things might have been. And these considerations are about the nature of conceivable possible worlds alternative to the actual one, in such a way that modal propositions say something about all possible worlds that are accessible from our actual world in our model, or about some of them, depending on how our statements are modified by $\Box$ or $\Diamond$.

Finally, to take a last look at modal operators "necessarily p" is T at a world $\Gamma$ if and only if p itself would be T in every possible world $\Delta$ accessible from $\Gamma$, and "possibly p" is T at $\Gamma$ if and only if p itself would be T in at least one possible world $\Delta$ accessible from $\Gamma$.

# CHAPTER 3

# NECESSITY DEFINED IN TERMS OF ANALYTICITY

## On the Concept of Analyticity

In most of his papers on modal logic, Quine's first move is to define necessity in terms of analyticity as we see in the following quotations:

"A statement of the form 'Necessarily…' is true if and only if the component statement which 'necessarily' governs is analytic, …" (Quine, Reference and Modality 21).

"… 'necessarily', this adverb being so construed as to yield truth when and only when applied to an analytic statement (Quine, Two Dogmas of Empiricism 29).

"The result of prefixing '□' to any statement is true if and only if the statement is analytic." (Quine, The Problem of Interpreting Modal Logic 45).

The motive for this move can be explained with an appeal to Carnap, but also it plays into Quine's hands too. It plays into Quine's hands in the sense that he uses analyticity to define necessity in order to weaken necessity context, since analyticity, as he shows it to be, not a secure foundation.  In "Two Dogmas of Empiricism" Quine destroys analyticity to be a valid concept, and his argument is as follows:

Analytic statements are the ones that are true by virtue of meaning alone, independent of facts about the world (Quine, Two Dogmas of Empiricism 23). For example the sentence

(1) All bachelors are bachelors.

is analytic, since without looking at each bachelor, even without knowing the meaning of the word ‘bachelor’ we can know the truth of it.

Quine classifies analytic statements in 2 groups. The first one is the class of logical truths, including sentences which are true only by its logical structure. The sentence

(2) No unmarried man is married.

can be an example to this kind of analytic statements. What bestows analyticity on this sentence is not the meanings of these words, but its structure which can be formulated as ‘No not-A is A.’ Hence any instance of this schema, formed by replacing the words ‘marry’ and ‘man’ would be true and analytically true.

The second type of analytic statements cannot be said to be logically true. They need an appeal to meanings of the non-logical particles. For example the sentence

(3) No bachelor is married.

cannot be said to be analytic by its schema only. But by putting synonyms for synonyms it can be turned in to a logical truth as in the first class (Quine, Two Dogmas of Empiricism 23). So, in order to claim that this sentence is analytic, the meanings of the words are required. For instance in the sentence (3) if we replace ‘bachelor’ with its synonym ‘unmarried man’ we get the sentence

(4) No unmarried man is unmarried man.

which is a logical truth and hence an analytic truth in the form of the first class. Quine says that there is no problem with the ones in the first class, but our problems of analyticity lie in the second type concerning synonyms (Quine, Two Dogmas of Empiricism 24).

So, since we explained the concept of analyticity relying on the notion of synonymy, we now need to know what synonymy is, which is declared by Quine to be in no less need of clarification than analyticity itself (Quine, Two Dogmas of Empiricism 23).

Quine considers the definition of synonymy as the relation between terms which are, in Leibniz's phrase, interchangeable *salva veritate* (Quine, Two Dogmas of Empiricism 27). So, when two terms that are said to be synonyms, they should be interchangeable within sentences *salva veritate*, i.e. preserving the truth value. Then if we are to assume analyticity, the kind of synonymy we want to use here can be explained in this way (Quine, Two Dogmas of Empiricism 28):

Saying that 'bachelor' and 'unmarried man' is synonyms is exactly the same as saying

(5) All and only bachelors are unmarried men

is analytic. But we already assumed analyticity in this argument. Is there a way to do this without assuming analyticity?

Quine suggests a way which does not use analyticity, but presupposes it for another purpose. Presupposing analyticity, Quine construes the adverb 'Necessarily' to apply to analytic sentences.

(6) Necessarily, all and only bachelors are bachelors.

(7) Necessarily, all and only bachelors are unmarried man.

Even if we narrow down the meaning of ‘Necessarily’ to be truly applicable to only analytic sentences, (6) is definitely a true sentence, since it would be a member of the first class, i.e. logically true sentences. And as a result of the criterion of interchangeability *salva veritate* (7) also becomes true. However, if (7) is true, as a necessary result of the way ‘Necessarily’ is constructed, (5) turns out to be analytic (Quine, Two Dogmas of Empiricism 29), meaning that ‘bachelor’ and ‘unmarried man’ are synonyms. But if we suppose that the adverb ‘Necessarily’ makes sense, we will also be supposing that we have a clear definition for the notion of analyticity which was presupposed in the construction of this adverb. So, in Quine’s highly interesting idiom the argument presented here ‘is not flatly circular’, but it ‘has the form of a closed curve in space.’ (Quine, Two Dogmas of Empiricism 29).

In “The Problem of Interpreting Modal Logic” he gives another argument against analyticity’s. He gives a definition for logical truth as follows: every logical truth is deducible by the logic of truth-functions and quantification from true statements containing only logical signs (Quine, The Problem of Interpreting Modal Logic 43). But analyticity is not restricted to the statements which include only logical signs. An analytic statement can be a logical truth though it includes extra-logical primitives (Quine, The Problem of Interpreting Modal Logic 44). Quine defines analyticity as “a statement is analytic if by putting synonyms for synonyms it can be turned into a logical truth” (Quine, The Problem of Interpreting Modal Logic 44).

So in order to know what analyticity is, we need to know what synonymy relation means. Hence, Quine gives definitions of synonymy for statements, names, predicates as follows: if statements are claimed to be synonymous then the biconditional binding them should be analytic, if names are claimed to be synonymous then the identity statement binding them should be analytic, if predicates are claimed to be synonymous then the statement formed by combining their applications to a into a universally quantified biconditional should be analytic. Therefore, we are running around a circle here, by arriving at the analyticity at the end of each case. So, this means that we are back where we started, with no satisfactory foundation at all. Eventually, this is to leave necessity, hence modal logic with no satisfactory ground at all.

### The Source of Quine's Obsession with Analyticity: Carnap

Although he does not state this explicitly in every paragraph wherever he defines necessity in terms on analyticity, obviously Quine is dealing with something or with someone in his head, that is to explain his obsession about his definition of necessity in terms of analyticity. And the following quotation from *Word and Object* reveals the fact that he gets the idea from Carnap with whom he admired and had a teacher-pupil relation: "His (Lewis) interpretation of necessity, sharpened in formulation by Carnap, is that a sentence beginning with 'necessarily' is true if and only if the rest of it is analytic." (Quine, Modality 195).

To study the ideas that are grounded in Quine's arguments I will examine some parts of *Meaning and Necessity.*

In *Meaning and Necessity,* Rudolf Carnap creates a brand new language specific to his formal system. So, before getting into details I want to give definitions of some basic notions/conventions in this system.

A state-description is a class of sentences in a formal system, 'which contains for every atomic sentence either this sentence or its negation, but not both, and no other sentences' (Carnap 9).

And the range of a sentence P is defined as 'the class of all those state-descriptions in which a given sentence P holds' (Carnap 9).

Carnap gives a convention for the explicatum 'L-truth,' which is to stand for Leibniz's necessary or Kant's analytic truth. But before giving an account for what an L-truth is, he equates some notions, for instance he speaks of L-truth 'as an explicatum for the familiar but vague concept of logical or necessary or analytic truths' (Carnap 10). Thus Carnap combines logical, necessary and analytic truth in his artificial system, and defines the concept of L-truth as an explicatum for this system. So, necessity is not defined as analyticity in our daily word language, but only for that specific system of Carnap's as explicatum. After this more vague clarification Carnap defines L-truth as follows:

'A sentence P is L-true in a semantical system *S* if and only if P is true in *S* in such a way that its truth can be established on the basis of the semantical rules of the system *S* alone, without any reference to (extra-linguistic) facts' (Carnap 10). This definition of L-truth goes in a similar way as how Kant's analyticity is defined. And also, he gives the following definition, which goes in a similar way as Leibniz's definition of necessary truth:

‘A sentence P is L-true $=_{Df}$ P holds in every state-description’ (Carnap 10).

In the light of this convention and definition, it would not be wrong to make the claim that for Carnap,

(8) P is analytic if and only if P is necessary

which seems to have strongly influenced Quine in his objections to modality.

The original idea in Carnap’s system is his assigning every entity an extension and also an intension. Extensions and intensions of predicators, designators, sentences or individual expressions are defined separately, but I will give the definition for predicators only.

‘The extension of a predicator is the corresponding class.’ (Carnap 19)

‘The intension of a predicator is the corresponding property.’ (Carnap 19)

Then given the designation ‘Hx’ stands for ‘x is human’; the extension of ‘H’ is the class Human and the intension of ‘H’ is the property Human (Carnap 19).

Carnap’s main motive was to give a formal interpretation for meaning. That is why his state-descriptions differ from Leibniz’s possible worlds in the sense that Carnap’s state-descriptions are given in terms of linguistic entities, like predicators and designators. Hence, there are no possible objects open to discussion about their appearances, essences and identities as in Leibniz’s or Kripke’s worlds. State-descriptions are well-formed linguistic forms, which are hard to object as opposed to Kripke’s possible worlds.

However Quine still objects to Carnap's system in the sense that while assigning an extension and an intension to its linguistic objects, it duplicates the entities. But Carnap explains in his book that in the later chapters (Carnap 145-172) he constructs a metalanguage which is neutral with regards to extension and intension. In this metalanguage a property and a corresponding class are not mentioned as two entities, but one entity only. 'The possibility of this neutral language shows that our distinction between extension and intension does not presuppose a duplication of entities' (Carnap 2).

Besides, if duplication of entities is the way to give a perfect explanation to the problem, which is claimed for Carnap's system, then it is not redundant. There is no reason to apply Occam's Razor to elements of a system, if the multiplication in question is enabling that system work properly.[1]

There is a section dedicated to Quine in *Meaning and Necessity*, named "Quine on Modalities". In this section Carnap gives Quine's well-known examples where principle of interchangeability does not hold between '9' and 'the number of planets' in modal contexts. With Carnap's formulation of the principle, it holds only in extensional contexts and apparently '9 is necessarily greater than 7' is not one (Carnap 51).

Carnap presents another problem raised by Quine was about modal sentences with variables, like 'There is something which is necessarily greater than 7.' In "Notes on Existence and Necessity" Quine claims this sentence to be meaningless (Quine, Notes on Existence and Necessity 124). Because if we are to get a '9'

[1] Conversation with Timothy Williamson

instance of this existential claim the sentence will turn out to be true, whereas if we get a ‘the number of planets’ instance of the claim then it will turn out to be false. For Quine both of these instances are not meaningless, but the occurrences of designators are not purely designative, which means that they do not function as names, so the principle does not apply to them. And because of the same reason existential generalization is not applicable to these occurrences either. So, the sentence

(9) ‘There is something which is necessarily greater than 7’

has no meaning and cannot be admitted as a sentence (Carnap 194). From these Quine concludes that the occurrences which are not purely designative are not subject to law of substitutivity of identity, or to the laws of application, or existential generalization. And no pronoun or variable occurring in this type of context can refer back to an antecedent or a quantifier prior to that context. Hence, according to Quine these facts constitute serious restrictions about the use of modal operators.

Carnap agrees with Quine’s conclusions concerning quotational contexts; however he refuses conclusions concerning modal contexts, since he thinks that it is possible to combine modalities and variables in his system (Carnap 195).

Church finds an answer to this problem which makes use of two different kinds of variables. In the case of Quine’s examples he will use a variable for number concept and a different variable for the number. Carnap thinks this is an unnecessary duplication (Carnap 195). He thinks we can rather use one kind of variable whose value-extensions are classes, and value-intensions are properties. This way it is aimed that any unnecessary duplication is to be avoided.

Although the debate between Quine and Carnap may be differentiated in many subjects, the most important and the most apparent one in my study is necessity-analyticity relation. I believe Quine's taking necessity to mean analytic and logical truths is not a wonder. Carnap equates logical, necessary and analytic truths in such a strong way that Quine starts his objections with assuming that necessity is to mean analyticity, and after destroying the concept analyticity he sticks to the idea that necessity is to be equated with logical truths. It is an open sign that Quine was always struggling with Carnap in his head the whole time in his essays.

P is analytic if and only if P is necessary.

This convention was intended to hold only in the artificial system of Carnap, but Quine interpreted the notions analyticity and necessity in a way as they are done generally. However, these notions are not to be applied in any language, but rather in the artificial language constructed by Carnap in accordance with the conventions he gives.

Quine's war was mostly with Carnap, as can be observed in "Two Dogmas of Empiricism" where he destroys the distinction between analytic and synthetic statements, on the basis that this distinction is circular, right after Carnap constructed a formal semantic approach based on the distinction between analytic and synthetic statements.

This is a weak point for Quine's system as well, in the sense that if one is to give a satisfactory definition of analyticity –although no one achieved yet–, or if one is to define necessity without an appeal to analyticity –like Kripke did– most of his arguments would be wasted. In the fragments above, taken from articles of Quine, he defines necessity in terms of analyticity, and tries to show that defining necessity in terms of analyticity leads us to circularity, and that it is not a secure foundation. However, after studying *Meaning and Necessity* it is not hard to find out Quine's inspiration/source for these definitions.

Surprisingly, in the last paragraph of forty first section of *Word and Object* Quine says something which he never did in his early papers. This is a surprise because it is not customary to witness serious changes throughout his all philosophical ideas, in which field they might be. The following quote is from a paragraph where he mentions various possible senses of the adverb 'necessarily' but he sticks to the sense of analytic necessity:

"Among the various possible senses of the vague adverb 'necessarily', we can single out one−the sense of analytic necessity−according to the following criterion: the result of applying 'necessarily' to a statement is true if, and only if, the original statement is analytic." (Quine, Notes on Existence and Necessity 121).

In this paragraph Quine for the first time considers what would be if Necessity were to mean more than mere analyticity. Actually this is not the first time that he is mentioning an option for necessity to be understood in a way other than analyticity. In "Notes on Existence and Necessity", he also considers physical necessity (Quine, Notes on Existence and Necessity 124), and also in *Word and Object* he considers conditional necessity (Quine, Modality 196), but he always decided to work on necessity understood as analyticity probably because it fits his purposes in the best way. However, here Quine suggests that analyticity cannot be blamed for all the problems of modality, and explaining

(10) Necessarily 9>4

with

(11) ‘9>4’ is analytic.

would not cause repudiation of objects, or making all the ontology with intensional objects on its own. And this is the structure of his argument: the explanation of (10) in terms of (11) at most yields to unquantified modal logic, i.e. Grade 1 and Grade 2, which are harmless degrees. When we write an attribute in the form of an abstract as in ‘A=x[ x has A or x=a ]’ then this also commits us to the problems originated from differentiating between attributes of an object as essential or accidental, which is discussed in detail in the next section.

Since defining necessity in terms of analyticity led to bigger problems than analyticity itself did, analyticity cannot be admitted to be the only problem with modality. Modality is not innocent on its own either, and it has weaknesses apart from those of analyticity.

# CHAPTER 4

# ESSENTIALISM

## Mathematicians and Cyclists

Another paragraph where Quine refers to Aristotelian distinction between essential and accidental attributes of an object is in *Word and Object*, forty first section. When we think at the propositional level, modal sentences tell us whether a proposition is necessary or possible. However, when we want to talk about attributes like 'x[x>4] is necessary of 9' we are stating an attribute that is necessary to '9'. Since modalities yield us to talk of difference between necessary and contingent attributes of an object, we can do this for mathematicians and cyclists. Quine arguably claims that being rational is a necessary attribute of mathematicians and being two-legged is a contingent attribute for them. Whereas, the attribute of being two-legged is necessary to cyclists and being rational is a contingent one.

Now keeping these in mind, assume there is an individual who is a mathematician and also a cyclist. Then, with the conditions given earlier, for this particular individual it can be said that he is both necessarily rational and not necessarily rational, or both necessarily two-legged and not necessarily two-legged.

Quine gives this example just to evoke bewilderment but it became an important question and many modal logicians attempted responding to it.

Marcus gives a summary of Quine's Mathematician and Cyclist argument as follows (Marcus 19):

44. Modalities yield talk of a difference between necessary and contingent attributes.

45. Mathematicians may be said to be necessarily rational and not necessarily two-legged.

46. Cyclists are necessarily two-legged and not necessarily rational.

47. $a$ is a mathematician and a cyclist.

48. Is this concrete individual necessarily rational and contingently two-legged or vice versa?

49. “Talking referentially of the object, with no special bias toward a background grouping of mathematicians as against cyclists … there is no semblance of sense in rating some of his attributes as necessary and others as contingent.”

These sentences can be formulated as:

$(x)\Box(Mx \supset Rx)$ (from 47)

$(x)\neg\Box(Mx \supset Tx)$ (from 47)

$(x)\Box(Cx \supset Tx)$ (from 46)

$(x)\neg\Box(Cx \supset Rx)$ (from 46)

And the individual Quine wants us to consider, say $a$, has the following property:

$Ma \bullet Ca$ (from 47)

And the conclusions that can be derived from these can at most be;

$\Box(Ma \supset Ra)$

$\neg\Diamond(Ma \bullet \neg Ra)$

$\Diamond(Ma \bullet \neg Ta)$

$\neg\Box(Ma \supset \neg Ta)$

$\Box(Ca \supset Ta)$

$\neg\Diamond(Ca \bullet \neg Ta)$

$\Diamond(Ca \bullet \neg Ta)$

$\Diamond(Ca \bullet \neg Ra)$

$\neg\Box(Ca \bullet \neg Ra)$

$Ta$

$Ra$

$Ta \bullet Ra$.

None of these conclusions answer the question, asked in 48, whether $a$ is necessarily rational and contingently two-legged or vice versa. These conclusions do not make sense of Quine's conclusion, represented in 49, that 'there is no semblance of sense in rating some…attributes as necessary and others as contingent.'

Quine told several times that he understands necessity in terms of analyticity. However, as analyticity has been cleared off our system, we are left with logical necessities only. Keeping these in mind Marcus objects to Quine's objection on the basis that these attributes that Quine mentions are not logical necessities. So, modal logic that we are discussing about cannot countenance these attributes, namely M, T, C, and R as logically necessary.

I think mathematician and cyclist argument can be rebuffed by a reformulation of the case stated by Quine. He suggested that all mathematicians are necessarily rational and contingently two-legged whereas all cyclists are necessarily two-legged and contingently rational. These statements can be formulated as;

$$Mx \supset \Box Rx \bullet Tx$$

and

$$Cx \supset \Box Tx \bullet Rx .$$

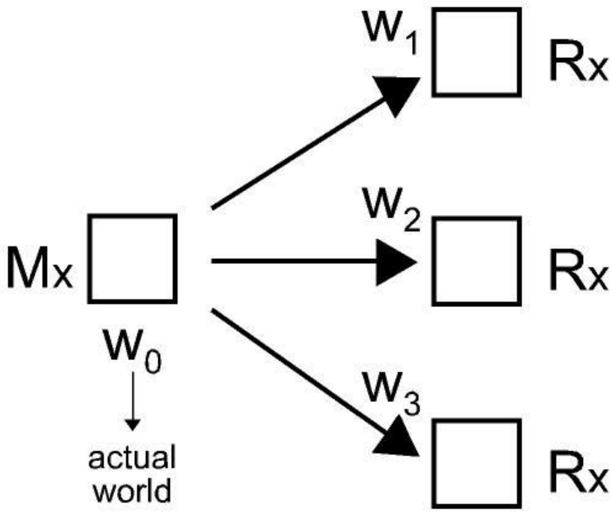
With these conditionals in mind, Quine asked the following question:

What will be the situation when we have an individual who is both a mathematician and also a cyclist? This particular case leads to a/the paradoxical situation where this very individual is both necessarily and contingently rational, also both necessarily and contingently two-legged.

I think this paradox can be rebuffed with the help of modal models.

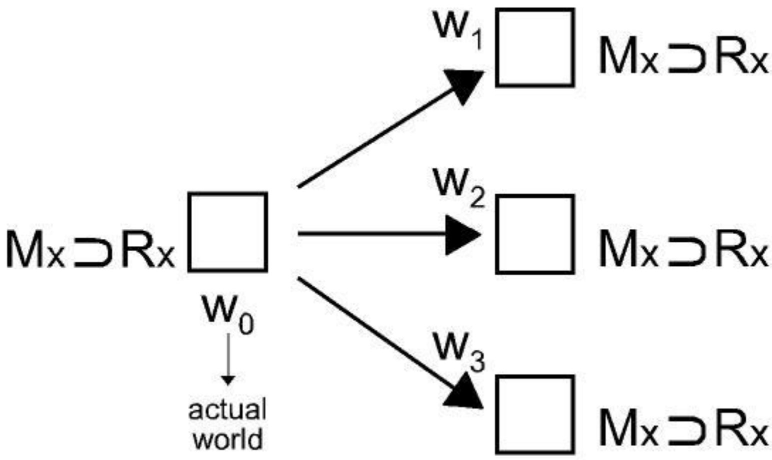
Suppose we create our model according to Quine's proposal of necessary and contingent attributes of mathematicians and cyclists. The model is introduced as $M = \langle G, R, D \rangle$ and $w_0$, $w_1$, $w_2$, $w_3 \in G$. Suppose $w_0$ is related with the relation $R$ to all other three worlds in the model; i.e. $w_1$, $w_2$, $w_3$, and further that the Quinean formula holds in this model.

Then the model will look like:




Now, obviously the formula $Mx \supset \Box Rx$ holds in this model. However, this formula does not capture what we really want to say, although it seems to do so. The formula $Mx \supset \Box Rx$ actually says if $x$ is $M$ in the actual world, then $x$ is $R$ in all of the possible worlds accessible from $w_0$; i.e. $w_1$, $w_2$, $w_3$. So, this formulation says a lot more than what we intend to say.

What we want to say is that $x$ is $R$ in all the possible worlds where $x$ is $M$. We do not want to say something binding $x$ to be $R$ in all the accessible possible worlds even though $x$ is not $M$ in that possible world. Hence, the formula $\Box(Mx \supset Rx)$ captures the idea better. If we are to show it with the same model, it would look like the following diagram:




In this case, if $x$ is $M$ in a possible world, then $x$ should be $R$ only in that very world. Because $x$ is $M$ in the actual world instead of forcing $x$ to be $R$ in every possible world we are saying that $x$ is $R$ in every accessible possible world in which $x$ is $M$. This way we are not burdening being a mathematician more than what it is, that being rational in all accessible possible worlds even though the condition of being a mathematician is not fulfilled in that world.

So, although $\Box(Mx \supset Rx)$ gives the intended sense of the sentence 'every mathematician is necessarily rational', the tendency to write $Mx \supset \Box Rx$ for it is commonsensical. This is similar to the following problematic case that commonsense leads us to.

Suppose we want to say ‘oranges and apples are healthy fruits’, ‘F’ standing for being an apple, ‘G’ being an orange and ‘H’ for being a healthy fruit. Here, ‘Fx’ and ‘Gx’ are bound with the connective ‘and’. Commonsensically we might tend to write it as

$(x)(Fx \wedge Gx \rightarrow Hx)$

However, we know that nothing can be both an orange and an apple; hence, this sentence does not hold. Hence, there is no entity satisfying the condition of being ‘F $\wedge$ G’. What our original sentence wants to say is that anything that is an apple or an orange is a healthy fruit. So, ‘Fx’ and ‘Gx’ should be connected with disjunction rather than conjunction, which is clearly an example to unreliability of common sense.

## Aristotelian Essentialism

I want to define Quine’s three grades before going into the jungle. If we are to line up the degrees of modal involvement from the most innocent to the most dangerous ‘Nec.’ as a semantical predicate, which is the first grade would come out as the first one. It only attaches to the names of statements, behaving like a verb and never interfere with what is inside, so there is no danger of vague sentences whose truth value cannot be assigned properly. It affirms that the statement it attaches is necessarily true as in the example ‘Nec. ‘9>5’’.

The second grade serves as a statement operator. It attaches to the statements themselves like an adverb. This grade is the one that satisfies Quine’s definition of necessity in terms of analyticity. When we say ‘nec(9>5)’ it is to be interpreted as

‘‘9>5’ is analytic’. This grade is not dangerous on it won however if it is extended to cover the open sentences it gives rise to many problems, which brings us to the third grade.

The third grade is a sentence operator. This grade is in no way innocuous when compared to the former two grades. What is crucial about the third grade is its being open to quantification since it includes open sentences; hence they have a variable inside the modal contexts. So a quantifier can be placed prior to the modal context. However, for Quine modal contexts are referentially opaque, hence, no variable can reach an antecedent quantifier within a modal context. And this gives rise to many problems including Aristotelian essentialism.

There is one passage where Quine explicitly gives a definition for Aristotelian essentialism as required by modal logic, the fullest account that I know of. He defines Aristotelian essentialism as ‘the doctrine that some of the attributes of a thing may be essential to the thing, and others accidental.’ (Quine, Three Grades of Modal Involvement 173-174).

Aristotelian essentialism allows us to have a construction in the form,

(12) $(\exists x)(\ \text{nec}\ Fx \bullet Gx \bullet \neg \text{nec}\ Gx\ )$

Since Aristotelian essentialism allows sentences of this form, we can state, as in (12), the essentialist claim that there is an object $x$ which has the property $F$ essentially and the property $G$ accidentally. For example, for number ‘9’ being greater than 4 is an essential property, whereas standing for the number of planets is an accidental one. The number of planets in the actual world is equal to the number '9' but it can be equal to some other number at a possible world.

Quine claims that not only it is possible to put this formulation but also even a stronger version can be shown to be given as an instance of Aristotelian essentialism. And that is the following formulation involving Universal Quantifier:

(13) $(\forall x)(\text{ nec } Fx \bullet Gx \bullet \neg \text{nec } Gx)$.

This statement says that all $x$'s are such that they have the property $F$ essentially and the property $G$ accidentally.

In my opinion, with a formulation in the form of (13) Quine does not intend to assert that these properties are non-trivial, or that they say something new. All he aims at saying is to give something stronger than the one with the formulation with an existential quantifier. What he really wants to say is that the sentence which is true with existential quantifier can be shown to be true for even a stronger version. And this stronger version is the same schema but with a universal quantifier in front: $(x)(\text{ nec } Fx \bullet Gx \bullet \neg \text{nec } Gx)$. In order to show that his argument is true he needs to form a sentence which should hold for any object whatsoever. It is a sure fact that there is no object to satisfy the condition $x \neq x$, which makes self-identity an essential property of every object, hence a necessary condition. That is why the statement $(x)(\text{ nec } Fx \bullet Gx \bullet \neg \text{nec } Gx)$ holds for every object when '*F'* is replaced with self-identity and '*G'* with an arbitrary contingent truth.

I think there is no need to refute this argument by going over the self-identity condition. According to Quine, the idea behind Aristotelian essentialism is being capable of distinguishing between essential and accidental properties of an object. And also in this paper he has this very idea in mind. So, our distinguishing between even only one object's properties as essential or accidental is enough for Quine to declare modal logic to lead us to the metaphysical jungle of Aristotelian essentialism (Quine, Three Grades of Modal Involvement 174).

To explain my point, suppose we accept Quine's doubtful example which says that all mathematicians are necessarily rational and contingently two-legged. Accepting this condition given by Quine for argument's sake, $(\exists x)(\ \Box Rx \bullet Tx \bullet \neg\Box Tx\ )$ would hold for any mathematician. This sentence basically says that $x$ has the property $R$ essentially and the property $T$ accidentally, and this is the core of the argument. If there were some property holding for each and every object other than self-identity that could also work for this argument.

Hence, he is not only showing that modal logic can distinguish between some objects' necessary and contingent properties; rather for all objects. And that is why he suggests replacing '*F*' with '$x = x$', and '*G*' with '$x = x \bullet p$', where $p$ is a contingent truth. So, it is possible to show that Aristotelian essentialism is required by quantified modal logic with these replacements. Here my aim is not to define Quine, but give his motivation for this argument. What is essential to his argument is not the attribute he is using but the reasoning behind, which I think is ingenious and a difficulty modal logicians still write on.

My aim in this section is to present 4 different responses each at different levels. The first one is by Marcus and claims that if essentialist premises are not added to system quantified modal logic does not lead to any essentialist claims. The second and the third positions are by Terence Parsons, the first one follows Marcus in the sense that the argument relies on trivializing essentialist attributes. The third position by Parsons has an interesting claim that quantified modal logic at most enables us to express essentialist claims formally, but never commits us to one necessarily. And the last position by Fine asserts that modal logic is not even adequate to enable us to talk of such essentialist attributes.

Marcus presents two different interpretations of Quine's argument; the first one is shown to be invalid and the second one is shown to be not leading to an essentialist conclusion unless some essentialist premises are added. Because of its more relevant content I will focus on the second interpretation.

Marcus gives three accounts of essentialist statements:

There is some attribute $\hat{y}Ay$ such that

$$(x)((x \in \hat{y}Ay) \supset \Box (x \in \hat{y}Ay))$$

Weak version:

There is some attribute $\hat{y}Ay$ such that

$$(\exists x)(\exists z)(\Box (x \in \hat{y}Ay) \bullet \neg \Box (z \in \hat{y}Ay))$$

Strong version:

There is some attribute $\hat{y}Ay$ such that

$$(\exists x)(\exists z)(\ \Box(x \in \hat{y}Ay\ ) \bullet (\ z \in \hat{y}Ay\ ) \bullet \neg \Box (z \in \hat{y}Ay\ ))$$

Attributes fulfilling the weak or strong version along with the first one is to be called essential attributes. In the light of this characterization Marcus aims at finding an answer to the question whether quantified modal logic is essentialist.

Now suppose we have a statement of self-identity;

$$aIa\,.$$

From this statement the following attributes can be assigned to $a$:

$$\hat{x}\ (xIx)$$

$$\hat{x}\ (aIx)$$

$$\hat{x}\ (xIa).$$

Of these three attributes the first one is different from the other two in the sense that it does not have a reference to $a$ at all. It can be applied to any object whether that object has a relation to $a$ or not. Marcus names this kind of attributes non-referential, since they are represented by an abstract that does not mention $a$. And two different attributes can be strictly equated in the following way:

$$a \in \hat{x}(xIx) \equiv a \in \hat{x}(aIx).$$

Since they are strictly equated, they can be substituted for each other, which means that referential attributes can be replaced by non-referential attributes.

Marcus would not count two-leggedness among provably essential attributes of quantified modal logic. This is because if attributes like $\Box(a \in \hat{x}Tx)$ were categorically true then the attribute $(a \in \hat{x}Tx)$ could be proved for any object (Marcus 51). Hence, an essential attribute can be trivialized. However, this is contrary to the assumption of traditional essentialism, which says that such attributes are necessary to some objects but not to all. So, an attribute which can be proved to hold for any object is not essentialist at all.

Marcus' response to mathematician and cyclist argument can be used as a response against the Aristotelian essentialism argument as well. In this argument which appears at the end of "Three Grades of Modal Involvement" Quine asserts self-identity to be an essential attribute for any object. But with the same line of thought with mathematician and cyclist argument, self-identity cannot be counted among the provably essential attributes of quantified modal logic either. So this property too can be trivialized, which prevents quantified modal logic from being an essentialist system.

For the second level, I will present ideas of Terence Parsons, a student of Marcus, from his 1967 paper, in which he defines four different grades of Aristotelian essentialism which are subclassifications of Quine's third grade of modal involvement. Based on these definitions Parsons claims that Quine shows only the first degree to be required by quantified modal logic, but he only argues the fourth grade to be paradoxical (Parsons, Grades of Essentialism in Quantified Modal Logic 182).

Parsons' aim is to show that Quine's claim on third grades' committing one to Aristotelian essentialism does not hold (Parsons, Grades of Essentialism in Quantified Modal Logic 181).

Parsons gives the structure of Quine's attack as follows:

(i) Quantified Modal Logic permits quantifiers outside of a modal operator to bind the variables within the scope of that operator.

(ii) Therefore quantified modal logic is committed to Aristotelian essentialism.

(iii) But there are insuperable difficulties in making sense of Aristotelian essentialism.

(iv) So there are insuperable difficulties in making sense of quantified modal logic.

So, paraphrased in terms of this structure Parsons argues that the degrees of essentialism that Quine mentions are not the same. For example for the step (ii) his example for essentialist claim is 'Everything is necessarily self-identical', however for the step (iii) that is the insuperability of Aristotelian essentialism his example is 'Something is necessarily rational.' Parsons claims that they are not illustrating the same level of essentialism, so the inference to his conclusion about quantified modal logic is invalid (Parsons, Grades of Essentialism in Quantified Modal Logic 182).

The first grade of Aristotelian essentialism that is mentioned by Quine is:

(G1) For some F, $[\exists x]\Box F$

This formula says that some objects have some properties essentially. And going along with Quine's examples, 'being rational' or 'being greater than 7' can be counted among them. However, Parsons claims that (G1) does not commit one to these kinds of attributes. What Quine has shown (G1) to be true for is essences like 'being self-identical' or 'being A or not-A' and these are not the essences that led to problematic cases when they occurred in a quantified modal sentence.

In the second grade Parsons uses Marcus's formulations of essentialism.

(G2w) There is some attribute $\hat{y}Ay$ such that

$$(\exists x)(\exists z)(\Box(x \in \hat{y}Ay) \bullet \neg\Box(z \in \hat{y}Ay))$$

(G2s) There is some attribute $\hat{y}Ay$ such that

$$(\exists x)(\exists z)(\Box(x \in \hat{y}Ay) \bullet (z \in \hat{y}Ay) \bullet \neg\Box(z \in \hat{y}Ay))$$

This grade is deeper than (G1), in the sense that it says more than some objects' having essences, but some objects have different essences than other objects (Parsons, Grades of Essentialism in Quantified Modal Logic 184). Suppose the attribute we are dealing with in this sentence is 'being two-legged.' So while written in the form of (G2w) the sentence was in the form:

$$(\exists x)(\exists z)(\Box(x \in \hat{y}(y \text{ is two-legged})) \,\&\, \neg\Box(z \in \hat{y}(y \text{ is two-legged})))$$

However, when written in the form

$$(\exists x)(\exists z)(\Box(x \text{ is two-legged}) \,\&\, \neg\Box(z \text{ is two-legged}))$$

where there are no attributes; hence there are no essential attributes, which would not raise any essentialist objections, as was claimed by Marcus (Parsons, Grades of Essentialism in Quantified Modal Logic 185). I do not take this grade seriously, because writing an attribute in the form of a non-attribute does not change the fact that it is still an attribute. Hence, writing it in a different form does not prevent it from being an essential attribute.

Parsons suggests limiting attributes to non-referential ones, in order to give a deeper characterization of essentialism, (G3), which does not depend on the contained singular terms. Here $F$ stands for some formula, and for any formula $F$ there is some $G$ which contains no free variable other than $x_1,...,x_n$, and no constants, and such that $F$ and $\langle s_1,...,s_n \rangle \in \hat{x}_1,,,\hat{x}_n \ (G)$(where $s_1,...,s_n$ include all of the constants and free variables of $F$ ) are intersubstitutable everywhere (Parsons, Grades of Essentialism in Quantified Modal Logic 186). And $F^*$ stands for some of these associated $G$ 's. So, the third grade of essentialism is characterized as follows:

(G3w) For some $F$ ,

$$(\exists z_1)...(\exists z_k) \square F \ \& \ (\exists y_1)...(\exists y_n) \ (\neg \square (\langle y_1,..., y_n \rangle \in \hat{x}_1...\hat{x}_n (F^*)) \ )$$

where $z_1,..., z_k$ include all of the free variables of $F$ .

(G3s) For some $F$ ,

$$(\exists z_1)...(\exists z_k) \square F \ \& (\exists y_1)...(\exists y_n) \ (\langle y_1,..., y_n \rangle \in \hat{x}_1...\hat{x}_n (F^*)) \& \neg \square$$

$$(\langle y_1,..., y_n \rangle \in \hat{x}_1...\hat{x}_n (F^*)) \ )$$

where $z_1,...,z_k$ include all of the free variables of $F$ . This way our formulas have the following form:

referential but not an attribute & attribute but not referential

which clears our system from referential attributes, hence makes modal logic neutral with respect to essentialism of grade three, meaning that neither (G3s) nor (G3w) are provable in this system (Parsons, Grades of Essentialism in Quantified Modal Logic 187).

For example, consider the following quotation from "Reference and Modality":

> Essentialism is abruptly at variance with the idea, favored by Carnap, Lewis, and others, of explaining necessity by analyticity. For the appeal to analyticity can pretend to distinguish essential and accidental traits of an object only relative to how the object is specified, not absolutely. Yet the champion of quantified modal logic must settle for essentialism.

The kind of essentialism Quine is referring to here comes from explaining necessity by analyticity. He says that distinguishing between essential and accidental traits of an object depends on how the object is specified. So, the sentence '□ (9>7)' is true by virtue of '9's meaning, where as '□ (the number of planets>7)' is false since being >7 is not a necessary trait of 'the number of planets.' That is why necessity understood in terms of analyticity devotes us at most essentialism of (G1).

Still we haven't found the grade that Quine argues to be paradoxical, which brings us to the (G4) which presents the grade where some objects have atomic essences which other objects lack.

(G4w) For some *atomic F* ,

$$(\exists z_1)...(\exists z_k)\Box F \;\&\; (\exists y_1)...(\exists y_n)\left(\neg\Box\left(\langle y_1,...,y_n\rangle \in \hat{x}_1...\hat{x}_n\left(F^*\right)\right)\right)$$

(G4s) For some *atomic F* ,

$$(\exists z_1)...(\exists z_k)\Box F \;\&\; (\exists y_1)...(\exists y_n)\left(\left(\langle y_1,...,y_n\rangle \in \hat{x}_1...\hat{x}_n\left(F^*\right)\right) \& \neg\Box\right.$$

$$\left.\left(\langle y_1,...,y_n\rangle \in \hat{x}_1...\hat{x}_n\left(F^*\right)\right)\right)$$

Parsons claims that nothing of the form of this grade is provable in most quantified modal logics, so they are free of essentialism in this sense. However, there is one exception that should be made about the identity-symbol if it occurs as an atomic two-placed relation in the system. In such cases it must be excluded from the scope of the axiom-schemata. If they are not, unacceptable conclusions might occur as in the following case:

The theorem $\Box(Fa \vee \neg Fa)$ yields the conclusion $(\exists y)Fy \supset (x)Fx$ which is definitely not an acceptable theorem. So, in order to avoid such unacceptable conclusions one needs to make some exceptions.

Parsons considers the objection that one might always add defined predicates that are equivalent to molecular formulas. And this way essentialism is being reintroduced. For instance take the predicate “Rxy” to stand for “Fx v – Fy” then if the denial of (G4w) is applied to “Rxy” “(x)(y)□Rxy” can be proved. And together with the definitional equivalent of “Rxy”, i.e. “Fx v –Fy”, it leads to the same paradoxical theorem. He answers such a possible objection on the basis that such definitions can be theoretically eliminable (Parsons, Grades of Essentialism in

Quantified Modal Logic 190). Hence, when they are used, they can be eliminated, and thus we can do without essentialism. However, if they are not theoretically eliminable, the language is to be called essentialist (Parsons, Grades of Essentialism in Quantified Modal Logic 191).

In his 1969 paper however, Parsons presents a different argument. He starts with giving two different accounts for essence, individual and general essences; and sticks to the general essences. He defines the doctrine of general essences as picking certain characteristics necessarily true for certain objects, as an extension of the natural kinds (Parsons, Essentialism and Quantified Modal Logic 36). He explains his motivation for studying essentialism as Quine's argument which says that quantified modal logic is committed to essentialism, and essentialism is a false or at least philosophically suspect doctrine. If both are true this is a serious problem for quantified modal logic, which is the third grade of Quine, i.e. the only grade in which modal operators can be combined with quantifiers. So this article is aimed at finding a precise formulation of the kind of essentialism in which both A and B is true, a project that failed in the first and the second positions I have just presented.

As Parsons searches a schema S, which makes any system of quantified modal logic dangerous version of essentialism if any instance of S holds in that system, he presents two of them.

(14) $(\exists x_1)\ldots(\exists x_n)(\Box F \,\&\, \neg \Box G)$

and

(15) $(\exists x_1)\ldots(\exists x_n)(\ \Box F\ )\ \&\ (\exists x_1)\ldots(\exists x_n)(\ \neg\Box F\ )$

(14) is a reformulation of (12). Quine was worried about quantifying into modal context, because of the sentences such as '□(x>7)' to which it is not possible to assign a truth value. However, Parsons claims that (14) does not endorse one to such controversial sentences, but only claims like 'something is necessarily either-bald-or-not-bald' whose essentialism is open to discussion, or claims which are essentialist at most innocuously, and whose truth condition can be precise. This is a very weak form of essentialism with respect to the form Quine mentions in (12).

(15) on the other hand states a more complex form of essentialism with respect to (14). This is due to the fact that in (14) one quantifier covers the whole formula, which means it distinguishes between the essential property $F$ and the accidental property $G$ for the same object. However, in (15) there is one quantifier specifically for the essential property $F$ and another for the accidental property $F$, so the properties here are not just necessary and not necessary, but necessary for this object and necessary for that one (Parsons, Essentialism and Quantified Modal Logic 38), which requires a complex categorization as opposed to (14).

The source of this perplexity Quine raises lies in the difficulty to define the thing about the object which makes a property necessary with respect to that object. So, in order to find this thing he wants to complicate the formula so that it will cover the sense of essentialism offered by Quine. What we are concerned with is properties, whereas (15) treats relations too. So, Parsons restricts (15) to apply to properties only by suggesting the following formula to define an essential sentence:

$$(16)\ (\exists x_1)...(\exists x_n)(\ \pi_n x_n \ \&\ \Box F\ )\ \&\ (\exists x_1)...(\exists x_n)(\ \pi_n x_n\ \&\ \neg\Box F\ )$$

where F is an open formula who free variables are in $x_1,...,x_n$, and $\pi_n x_n$ is any conjunction of formulas of the form $x_i = x_j$ or $x_i \neq x_j$ for every $1 \leq i < j \leq n$, but not both $x_i = x_j$ and $x_i \neq x_j$ for any $i$, $j$ (Parsons, Essentialism and Quantified Modal Logic 40).

Accepting (16) as the schema characterizing the essentialist account, now Parsons turns to the question whether quantified modal logic is committed to this doctrine. To answer this question two notions are to be clarified: what kind of modal logic is being questioned to commit one to essentialism, and what 'commitment' means. To answer the latter question first, Parsons claims a system is essentialist if it has some essential sentence as its theorem, or if it requires an essential sentence to be true although it is not a theorem of the system, if the system allows the formulation of some essential sentence and hence presupposes its meaningfulness.

Now in order to answer the question, suppose there is a system in which we have a set of possible worlds, and there is a domain for each possible world and an assignment of the extensions to predicates of the language so that the truth-value of formulas can be defined. And call the set of possible worlds with these assignments a model. In this system Parsons defines necessity of a closed sentence A as □A is true at H if and only if A is true at every possible world relative to H. given that a sentence is a theorem of quantified modal logic if it is true in every world in every model; he defines *Theorem 1* as follows (Parsons, Essentialism and Quantified Modal Logic 42):

There are certain models, called maximal models, in which no essential sentence is true in any world in the model.

Hence, by definition this system is not committed to essentialism in the first sense. And for the second sense we have to search if any essential sentences are true as a result of facts. Any set of non-modal facts expressible in our symbolism without any modal signs must be expressible by a consistent set of sentences. Whatever these sets are the sentences expressing them must be true in some possible world by the definition of maximal models. But we also know that no essential sentence is true in a maximal model. Therefore there is a world in which all the facts of this world hold, and no essential sentence is true. Hence the system is not committed to essentialism in the second sense either.

But we need to examine the applications of modal logic, in which the question of essentialism re-arises. There are two ways to do this: by extending the class of necessary sentences to include the truths of a priori discipline; or by extending the class to include sentences which are analytic. While answering the first application Parsons gives two requirements so that we will be endorsing no essential sentences, hence avoid essentialism: (i) the axioms will be closed and contain no constants and (ii) the axioms contain no modal operators, except on the front (Parsons, Essentialism and Quantified Modal Logic 44). (i) is easy to avoid, since any sentence with a constant can be reformulated without a constant. And in (ii) there is an apparent distinction about operators' location. Consider the following case where the theorem is applied to arithmetic. Take the sentence:

(17) Necessarily 9>7

This sentence can be represented in two ways:

(18) $\Box(\exists x)(\exists y)(x \text{ is nine } \& \ y \text{ is seven } \& \ x > y)$

(19) $(\exists x)(\exists y)(x \text{ is nine } \& \ y \text{ is seven } \& \ \Box(x > y))$.

(18) can be satisfied by the existence of two things at each possible world, such that the first one is nine and the second is seven, and the first is greater than the second. However, (19) requires that there are things nine and seven and at each possible world, such that the first one is greater than the second one. So (19) requires something stronger than (18), the two object should exist at each world. So (19) is essentialist while (18) is not. That is why Parsons limits the quantifiers only on the front. I emphasized this point because this was a problem raised by Quine and he was against de re modality. The problem here is avoided by Quine's suggestion from "On What There Is" where he proposes applying modalities to whole statements (Quine, On What There Is 24).

Parsons has one job left, which is to find an answer the question whether quantified modal logic is committed to the third sense of essentialism. His argument is an interesting one and continues as follows: a system of quantified modal logic can assert, deny or be neutral with respect to the truth of essentialism, but cannot be neutral to the meaningfulness of essentialism, since in quantified modal logic systems essentialist sentences are formulable. So, in order to guarantee the meaningfulness of those sentences the system has to provide a meaning for each essential sentence (Parsons, Essentialism and Quantified Modal Logic 49). Hence, quantified modal logic is committed to the third sense of essentialism.

Parsons admits that he is in agreement with Quine in this sense. However, they evaluate the same conclusion in different ways. For Quine this is a weakness, and an ill feature of quantified modal logic, whereas for Parsons this is not that bad a conclusion. Parsons' argument might be right but I do not agree with him about the conclusion. Quine's argument was that quantified modal logic commits one to essentialism. And Parsons showed that it did, which is not a refutation of Quine's point. Also, Parsons used Quinean distinction, and restricted the attributes as well to exclude properties Quine employed in showing that quantified modal logic commits us to essentialism. All these points weaken his argument, and this weakening is damaging the argument if it is particularly directed as a response to Quine's argument.

The fourth position is by Kit Fine, who does not want to save modal logic as Marcus or Parsons did. His aim is to deport modal logic from the study of essences, which might work well for Quine, but he has another reason to justify this move. He starts by defining the job of essences, which can be used to characterize what the subject, or at least part of it is about (Fine). But not all the properties of an object is in question here, only the properties that are essential to their bearers. And these essential properties, or say essences, can be understood in two ways; one conceived on the model of definition and the other elucidated in modal terms (Fine 2). In the second version, the one that I'm interested in, they are expressed with necessity, a concept that may apply to propositions, as one proposition may be claimed to be necessary, or to objects, as an object may be said to be necessarily a certain way. And the de re cases of modal attribution are the one that gives the concept of essence as opposed to de dicto.

Fine claims that his argument is not against modal accounts. Nevertheless, it is not hard to avoid the suspicion that they are somehow based upon a misreading of the standard informal way of expressing essentialist claims (Fine 4). His main aim is to clear metaphysics from modality, so to show that assimilation of essence to modality is fundamentally misguided and because of this the corresponding conception of metaphysics should be given up (Fine 3). He is not saying that the modal account cannot capture the concept of essence, but rather the notion of essence is not to be understood in modal terms, or even regarded as extensionally equivalent to a modal notion.

Fine accepts that if an object essentially has a certain property then it is necessary that it has the property; but he rejects the converse. To start with he gives a basic account; i.e. the modal account takes an object to have a property essentially just in case it is necessary that the object has the property. There are variants that make the necessary possession of the property conditional on something else; one is existence, that an object necessarily has a certain property if it exists and Fine's argument is on existentially conditioned form, and then he generalizes it.

For instance consider the individual Socrates and the singleton set whose sole member is Socrates. So, it is necessary that Socrates belongs to the singleton if he exists, for necessarily the singleton exists if Socrates exists. And singleton necessarily is a member of the singleton if they both exist. So Socrates belongs to the singleton essentially. However, it is no part of the essence that Socrates belongs to the singleton.

Fine gives another example for two entities which are not connected in anyways, and whose existence does not depend on another. Consider, Socrates and the Eiffel Tower. These two entities are both concrete, and they are necessarily distinct. However, their necessary distinctness does not guarantee that it is essential to Socrates that he is distinct from the Tower; they are not connected in such a way. This is an absurd example but I believe it makes Fine's point clear; that necessity does not entail essence.

Fine's aim is to restrict the concept of essence to definitional account instead of the modal one. But his argument works for my discussion as well. This argument can be presented against Parsons who thinks that modal logic allows us express essentialist sentences in our system, but still does not form an answer to Quine.

# CHAPTER 5

# QUANTIFYING IN

Carnap claims that an unquantified modal logic is of interest only to provide a basis for a quantified modal system. So, if it was not possible to create a quantified modal logic system then logicians would abandon the unquantified system entirely (Carnap 196). So, as Carnap stated, quantifying into modal systems is of great interest to logicians. And for Quine quantification is a significant as well, because to him, if we cannot quantify into modal contexts then there is no point of modal logic. Modal logic should give us some advantage to write "Nec 9 > 4" instead of "'9>4' is analytic", since writing "Nec 9 > 4" means merely "'9>4' is analytic". The advantage comes from the possibility of quantification. When we write "Nec 9<4" we have the advantage to quantify into a modal context like $\exists x Nec(x > 4)$. This way it is possible to quantify into the modal position, which, according to Quine, is not possible. This was what Lewis intended to do but he could not, Marcus did that instead (Quine, Modality 197).

Quine claims that modal contexts are not different from quotational contexts when referential opacity is in question (Quine, Reference and Modality 20). I think here is the appropriate place to take some time to explain the significance of opacity for Quine[2].

[2] Marcus is not happy with Quine's labeling these contexts as referentially opaque, and his extensional way of dealing with modality. She wants to examine modal contexts in a systematic and formal manner. And that is why she creates an intensional system in which she is able to present them (Marcus 8).

It would not be wrong to say that for Quine there are three degrees of referential opaqueness: quotational contexts, propositional attitude contexts, modal contexts. Quine’s both 1943 and 1961 papers begin with his introducing the principle of substitutivity. This principle says that ‘given a true statement of identity, one of its two terms may be substituted for the other in any true statement and the result will be true’ (Quine, Reference and Modality 17).

For example, take the true statement of identity to be:

The president of US in 2010=Barack Obama

And take the following to be any true statement:

The president of US in 2010 is the first black president of US.

Then substituting the president of US in 2010 for its equivalent in the identity statement we get

Barack Obama is the first black president of US.

which is a true statement.

However, there are cases where this principle does not hold. To continue with Quine's well-known example take our statement of identity to be

Cicero=Tully

and the sentence

‘Cicero’ contains six letters

as the true statement. When we apply the principle of substitutivity we get

‘Tully’ contains six letters

which is definitely not a true sentence. So what causes this failure? Quine thinks that the principle fails in this example because the occurrence of the name ‘Cicero’ is not purely referential (Quine, Reference and Modality 17), which means ‘the statement depends not only to the object but on the form of the name’ in Quine's words (Quine, Reference and Modality 18). In other words, if the name used does not refer only to the object but to the name instead, or more than just the object as in the case of ‘so-called’ that occurrence is said to be not purely referential.

Another well-known example given by Quine to reveal his point is the following:

The identity statement given in this case is,

Giorgione=Barbarelli

And the sentence to which we are supposed to apply the principle is,

Giorgione was so-called because of his size

When we substitute Giorgione with Barbarelli we have,

Barbarelli was so-called because of his size

which is also false. What is responsible for the falsity of this sentence in how ‘Giorgione’ is used. Here the word is used in a way that it not only burdens to be a name for the person Giorgione but it is also the referent of by the phrase ‘so-called.’ So in this case ‘Giorgione’ is both used and mentioned; used in the sense that it refers to the person that it names, and mentioned in the sense it does not refer to the person. It is easier to see when we replace the word ‘so-called’ with the name:

‘Giorgione was called ‘Giorgione’ because of his size.’

Now it is clear that the first occurrence of the name refers purely to the individual it names, so it is used here. However, the second does not, so it is mentioned.

Keeping these in mind about referential opacity, we can move one step further, towards quantification. Since no object in an opaque context can reach the antecedent quantifier, modal contexts would behave a lot similar to quotational contexts in the case of quantification too. Consider the following case:

When we want to replace ‘9’ with its equivalent ‘the number of planets’ in Quine’s famous example

(20) Necessarily 9>4

we get

(21) Necessarily the number of planets>4

which is false. The falsity of this sentence tells us that the occurrence of ‘9’ in (20) is not purely referential, meaning that this occurrence of ‘9’does not refer simply to the object that it names. And this occurrence’s not being purely referential tells us the fact that the context it appears in is referential opaque, namely the modal one. This is so because even though we have the equivalence

(22) 9=the number of planets

it is not possible to substitute one for another in this context as to preserve the truth value, which makes (20) a referentially opaque context. In this equivalence there are two different ways of referring '9', but they are not connected to each other with necessary equivalence. That is why their equivalence does not hold in a necessity context. Quine names these objects 'stubborn objects'; those which are specifiable in ways that fail of necessary equivalence (Quine, Modality 197).

Now that the reason for failure of the principle of substitutivity is revealed, Quine allows quantification into modal positions if the universe of objects is narrowed down as to exclude stubborn objects. Hence, Quine legitimizes quantifying into modal positions by the following criterion:

"whenever each of the two open sentences uniquely determines one and the same object $x$ the sentences are equivalent by necessity." (Quine, Modality 197).

Even though we still have stubborn objects in our ontology, this way they are filtered out since there is no necessary equivalence between distinct ways of referring them. It is not the huge regimentation Quine has in mind; however, it allows the quantification which is not expected of him either. Although exclusion of stubborn objects does not seem to be a big move, this argument of Quine has a far reaching consequence which is presented in the same paper right after the criterion (Quine, Modality 198):

In accordance with the criterion suppose $Fx$ and $Gx$ are arbitrary open sentences, and '$Fx$ and $x$ only' stands for '$(w)(Fw$ if and only if $w = x)$'.

If $Fx$ and $x$ only and $Gx$ and $x$ only, then (necessarily $(w)(Fw$ if and only if $Gw$ )).

Let $p$ be any arbitrary true sentence, and let $y$ be any object, and let $x = y$ .

Then,

$(p$ and $x = y$ ) and $x$ only

And

$x = y$ and $x$ only.

If $Fx$ is to be defined as ‘ $p$ and $x = y$ ’ and $Gx$ as ‘ $x = y$ ’, then ‘$(p$ and $x = y$ ) and $x$ only’ and ‘ $x = y$ and $x$ only’ leads us to the conclusion that

(23) ‘Necessarily $(w)((p$ and $w = y$ )iff $w = y$ )’.

If a ‘ $y$ ’ for ‘w’ instance of this universal claim is get we have

$(p$ and $y = y$ ) iff $y = y$;

which is simply equal to $p$ . So (23) gives us ‘Necessarily $p$ .’ Although we picked $p$ to be any true sentence, including contingent truths as well as necessary ones, $p$ ended up being a necessary sentence. So this kind of a criterion leads to the conclusion that any true sentence is necessarily true.

The criterion of necessary equivalence between different ways of referring to an object led to a conclusion that defenders of modal logic would want to avoid. So, they need to suggest some new criterion that would serve for the same purpose, that is to tell apart the necessary ways of referring to an object from contingent ones, instead of necessarily equating them.

Kripke's distinction between designators as rigid and nonrigid can be used as a solution to this problem raised by Quine (Vaidya). In *Naming and Necessity* Kripke names something 'a rigid designator if in every possible world it designates the same object, a nonrigid or accidental designator if it is not the case' (Kripke 48). To him proper names and natural kind terms are rigid designators, and some definite descriptions, like 'the square root of 16', are de facto rigid –that is rigid only in virtue of a property that is satisfied by the same object in every possible world – while others, like 'the youngest person in the room', are not rigid. So it is not an appropriate move to equate names and natural kind terms, which are rigid, with definite descriptions used to fix reference. For example, '9' is a rigid designator, meaning that '9' is to pick out '9' at every possible world, whereas 'the number of planets' is nonrigid, because the equality of 'the number of planets' and '9' is a contingent fact about this world, not a necessary condition that should be the case at every possible world.

# CHAPTER 6

# BARCAN FORMULAS

When we take a step further from propositional modal logic, we enter the territory of first-order modal logic. And thus accept the quantifiers $\forall$ and $\exists$ to our system in addition to the operators $\Box$ and $\Diamond$ which were already present in our system. And Ruth Barcan Marcus was one of the logicians who came up with a schema in which combinations of modal operators and quantifiers are present.[3]

In classical first-order logic we are not allowed to switch the places of the quantifiers in a combination of more than one quantifier in all cases. For example, $(\forall x)(\forall y)\varphi$ is equivalent to $(\forall y)(\forall x)\varphi$, whereas $(\forall x)(\exists y)\varphi$ is not equivalent to $(\exists y)(\forall x)\varphi$, since −supposing $\varphi$ is something like $Fxy$ and $F$ stands for successor relation− from 'for all $x$ there exists a $y$ such that $y$ is the successor of $x$' we cannot infer 'there exists a $y$ which is the successor of all $x$'s.'

The attempt to combine modal operators with our first-order quantifiers may lead to similar problems as well.

Barcan Formulas are the ones that are in the following form:

$$(\forall x)\Box\varphi \supset \Box(\forall x)\varphi$$

$$\Diamond(\exists x)\varphi \supset (\exists x)\Diamond\varphi$$

[3] The very same formulas were suggested by R. Carnap in *Meaning and Necessity* in the following forms (Carnap 186):

$$(x)N(fx) \equiv N(x)(fx)$$

$$(\exists x)\Diamond(fx) \equiv \Diamond(\exists x)(fx)$$

And Converse Barcan Formulas are the ones that are in the following form:

$$\Box(\forall x)\varphi \supset (\forall x)\Box\varphi$$

$$(\exists x)\Diamond\varphi \supset \Diamond(\exists x)\varphi$$

The validity of both Barcan and Converse Barcan formulas depends on the type of the frame.

Both Barcan and Converse Barcan formulas can be proved in a formal system appropriate for constant domain models, as shown below.

Proof by tableau method of the Barcan Formulas in S5, with constant domains:

$(\forall x)\Box\varphi \supset \Box(\forall x)\varphi$

1 $\quad \neg[(\forall x)\Box(Fx) \supset \Box(\forall x)(Fx)\ ]$

1 $\quad (\forall x)\Box(Fx)$

1 $\quad \neg\Box(\forall x)(Fx)$

1.1 $\quad \neg(\forall x)(Fx)$

1.1 $\quad \neg Fy$

1 $\quad \Box Fy$

1.1 $\quad Fy$

$\Diamond(\exists x)\varphi \supset (\exists x)\Diamond\varphi$

1 $\neg[\Diamond(\exists x)(Fx) \supset (\exists x)\Diamond(Fx)]$

1 $\Diamond(\exists x)(Fx)$

1 $\neg(\exists x)\Diamond(Fx)$

1.1 $(\exists x)(Fx)$

1.1 $Fa$

1 $\neg\Diamond(Fa)$

1.1 $\neg Fa$

Proof by tableau method of the Converse Barcan Formulas in S5, with constant domains:

$\Box(\forall x)\varphi \supset (\forall x)\Box\varphi$

1 $\neg[\Box(\forall x)(Fx) \supset (\forall x)\Box(Fx)]$

1 $\Box(\forall x)(Fx)$

1 $\neg(\forall x)\Box(Fx)$

1 $\neg\Box(Fy)$

1.1 $\neg Fy$

1.1 $(\forall x)(Fx)$

1.1 $Fy$

$(\exists x)\Diamond\varphi \supset \Diamond(\exists x)\varphi$

1 $\neg[(\exists x)\Diamond(Fx) \supset \Diamond(\exists x)(Fx)]$

1 $(\exists x) \Diamond (Fx)$

1 $\neg\Diamond(\exists x)(Fx)$

1 $\Diamond(Fa)$

1.1 $Fa$

1.1 $\neg(\exists x)(Fx)$

1.1 $\neg Fa$

From my point of view, Barcan and Converse Barcan formulas are contrary to one's intuitions. For example, take one form of the Barcan formulas, namely $\Diamond\exists x \supset \exists x\Diamond$. In this Barcan formula we are saying that 'if it is possible that there exists an $x$ such and such, then there exists an $x$ which is possibly such and such.' So by starting with a possible being with a certain property we arrive at an existent which has that property possibly.

Departing from a merely possible existence and arriving at an existence is a huge and an absurd step even for someone to whom modal models are familiar. So if we are to think in terms of Quine's ideas there is no way for him to accommodate this in his ontology. And that is the reason for Quine's rejection of the formula $\Diamond\exists x \supset \exists x\Diamond$ and its obscure ontological consequences.

Quine is not the only philosopher who wants to reject Barcan formula. Kripke who is a modal logician is not a fan of the Barcan formulas and their consequences either. As shown above Barcan formulas can be shown to be valid in constant domains, and stronger than this it comes as a logical truth of modal logic if we restrict modality to constant domains. But if we expand it to include varying domain models we fail to show their validity in those ones. So, Kripke suggested the varying domain models to clear modal logic from such queer consequences.

Non-proof by tableau method of the Barcan Formulas in S5, with varying domains:

$(\forall x)\Box(Fx) \supset \Box(\forall x)(Fx)$

1 $\neg[\ (\forall x)\Box(Fx) \supset \Box(\forall x)(Fx)\ ]$

1 $(\forall x)\Box(Fx)$

1 $\neg\Box(\forall x)(Fx)$

1.1 $\neg(\forall x)(Fx)$

1.1 $\neg(Fp_{1.1})$

1 $\Box Fq_1$

1.1 $Fq_1$

This proof cannot be completed and here is the reason: the schema $(\forall x)\Box(Fx) \supset \Box (\forall x)(Fx)$ asserts that if all the $x$'s are such that in all the possible worlds they are $F$, then in all the worlds all the $x$'s are $F$. However, we are working in varying domain models and $x$'s are picked from what exists at each world. So, this world's $x$'s being $F$ in all the worlds do not make in all the $x$'s $F$ at all the accessible worlds. This is why Kripke's introducing varying domains avoids the problem to some level.

Repudiation of Concrete Objects

Quine is worried about the Barcan formulas, and their ontological consequences. In the paper "The Problem of Interpreting Modal Logic" he gives an improved version of his argument on repudiation of concrete objects which appeared in his 1946 letter to Carnap. Quine's claim is that in order to make sense of Barcan formulas some partial criterion must be adopted which says 'An existential quantification holds if there is a constant whose substitution for the variable of quantification would render the matrix true' (Quine, The Problem of Interpreting Modal Logic 46). This criterion that should be adopted in order to make sense of Barcan and Converse Barcan formulas leads to queer ontological consequences, that there are no concrete/material objects, but only a multitude of distinguishable entities instead (Quine, The Problem of Interpreting Modal Logic 47). Quine's argument goes like this:

Take C to express a congruence relation which holds between Venus, the Evening Star (abbreviated as ES) and the Morning Star (abbreviated as MS) and each with itself.

Then,

$$MS \mathrm{C} ES \bullet \Box (MS \mathrm{C} MS)$$

So with our partial criterion we have,

$$(\exists x)(x \mathrm{C} ES \bullet \Box (x \mathrm{C} MS)).$$

But we also have

$$ES \mathrm{C} ES \bullet \neg\Box (ES \mathrm{C} MS)$$

Again with partial criterion we have,

$$(\exists x)(x \mathrm{C} ES \bullet \neg\Box (x \mathrm{C} MS)).$$

$(\exists x)(x \mathrm{C} ES \bullet \Box (x \mathrm{C} MS))$ and $(\exists x)(x \mathrm{C} ES \bullet \neg\Box (x \mathrm{C} MS))$ are incompatible sentences and this means there are at least two objects both in C relation with $ES$ and with $MS$, the former necessary and the latter contingent. This number of objects can be increased by using other names, like Hesperus or Phosphorus, for the same ball of matter.

Hence, adopting this criterion, we are committed to an ontology where we have to/are forced to repudiate material objects and have only multiplicities of distinct objects. And this new version is named by Quine as the contemplated version of quantified modal logic (Quine, The Problem of Interpreting Modal Logic 47).

Although there are no explicit relations revealed between this argument and stubbornness by Quine, I think they are somehow related. I think there might be a relation because here too the problem arises from the fact that there are more than one way of referring to the same object and that these ways of referring are not necessarily equivalent to each other. If Evening Star and Morning Star were necessarily equivalent to each other, then in the second sentence formulated they would be in C relation necessarily; eventually the contradiction would have been avoided.

From this argument Quine concludes that modal logic is stuck with only lots of distinct entities in its ontology, like Evening Star-concept or Morning Star-concept, for example, which are kinds of intensional objects.

# CHAPTER 7

# POSSIBILIA

Another objection by Quine has its roots in the discussion between two camps; namely deflationists and inflationists. These two camps' major difference of opinion is on things which do not exist. Deflationists argue that 'things which do not exist cannot be referred to or mentioned; no statement can be about them' (Fitting and Mendelsohn 168). So, if we are to refer to a thing, it should exist. Whereas for inflationists, if we are referring to something, the object of the statement should not be existing necessarily (Fitting and Mendelsohn 176). Their understanding of existence is somehow different from deflationists'. To take a glance at the position, consider early position of Russell. He thinks that anything has a *being,* however *existence* is a condition satisfied only by some of the things (Fitting and Mendelsohn 175).

So, something's having *being* is an adequate condition for that thing to be referred, its having *existence* is not required; which brings us to Quine's complaint about some philosophers 'ruining the good old word 'exist'' (Quine, On What There Is 23). If we are to adopt an inflationist point of view, we would be assigning the property of *being* to any object whatsoever, without questioning and 'limit the word 'existence' to actuality' (Quine, On What There Is 23) only; which means that Pegasus has being although lacks existence. This, Quine dislikes, because this is the motive that gives rise to the kind of entities called unactualized possibles. And this also, Quine dislikes for many reasons. First of all it 'offends his aesthetic taste', since he has 'a taste for desert landscapes'. But more significantly, to present in his own words, this ‘slum of possibles is a breeding ground for disorderly elements’, like possible objects (Quine, On What There Is 23). He is against all these kinds of entities, like the possible fat man in the doorway and the possible bald man in the doorway, for the following reasons: how to decide whether there are two distinct possible men or they are the same man, or how many possible men there are in that doorway, or whether the number of possible thin men are greater than the number of possible fat men, or how many of them look alike, or their being alike means they are the same, or whether is it not possible for two possible things to be alike, and whether this may mean it is impossible for two things to be alike. And most importantly is the concept of identity simply inapplicable to unactualized possibles? This last question is related to the conditions of existence, which are being self-identical and being distinct from others (Quine, On What There Is 24). With the questions he asks, Quine points out that it is not possible to distinguish one possible man from another. We cannot know how many of them are there, or which ones are the same, or which ones are distinct. If we are to claim that there are at least two possible men in the doorway we need to be capable of identifying and distinguishing these men, so that they can fulfill the criterion for being an entity. How are we to talk about the identity of these entities while we cannot even distinguish one from another? Quine is afraid that possible man or any possible entity is going to enter our ontology and just multiply the number of entities. They will not be distinguishable from other entities, so they will not have any identity at all. Having a taste for desert landscapes (Quine, On What There Is 23) intermediary, blurry things at our world tangled with entities of the actual world, of which we are not certain whether they are entities or not. This is the nightmare of Quine.

So, he suggests that we clear this slum ‘and be done with it.’ (Quine, On What There Is 24). Hence, we cannot talk about individual identity of these objects. Sticking to the motto ‘no entity without identity’ we cannot talk about them as entities either.

At the time Quine wrote this article, possible world semantics were not founded yet. And since possible worlds were not present, he thought of possible objects as blurry, ghost-like entities taking place in the actual world, which is the main weakness of this objection of Quine. From a contemporary point of view, with the tools of semantics at hand, the problematic point about this position is the misinterpretation of possible objects that places them in the actual world. If they are to be considered in the way Quine does, then it is perfectly probable that such identity problems may arise. Rather, sending them to possible worlds apart from the actual world, as Kripke suggested, would be a more appropriate way of interpreting them. Unactualized possibles have their existence in a possible world, not in the actual one. That is why they are named ‘unactualized’ possibles, meaning they are not actualized, and that do not take place in the actual world. And that is why Quine need not worry about their blurry existences in this world. There are no shadowy possible men in the doorway other than the ones we can see. There is no fat man in the doorway, there is no bald man in the doorway to be equated or differentiated.

Besides, if the possible worlds are to be stipulated in a way to provide spatio-temporal coordinates of these possible objects, none of the objects would collapse on others, thus they would be separated from one another. This way Quine’s doubts could be relieved with the introduction of definitely separated entities in the world.

In the light of these suggested solutions, one might think that Quine would accept the existence of this type of entities, but I don't see it happening. I do not think that Quine would be satisfied with this newly formulated conception possible world theory. From a deflationist point of view this might seem to be making the ontology more crowded. Having possible worlds, and more horrifyingly having infinitely many of them might drive Quine crazy, thus he might reject them immediately as well.

Now suppose two people are going through the doorway and they hardly get through. And one of them says "if there was a fatter man in this doorway instead of you, we might have got stuck." Now this is a stipulated possible world in which the same person is stuck in the doorway with a possible fat man. Now, is it really compulsory to conduct a thorough study on who that man is? I do not think it is. Possible worlds are mere stipulations, to which we do not spy out with a telescope to examine what is going on at these worlds. So, who this fat man is, whether he is the same man with the bald man, or what his other features are do not matter at all.

Kripke claims that most of the problems concerning possible worlds arise from the way they are interpreted. In *Naming and Necessity* he says that 'possible worlds are not foreign countries' and 'we are not observers.' (Kripke 43) Hence, possible objects which take place in possible worlds are not open to our examining. However, in Quinean way of interpreting possible objects, his questions about the identity of these objects were caused by a similar way of understanding possible objects. But in fact they should be handled as they are not open to any kind studies, because we are not watching and seeing what is happening there. And as a result of this construction there is no way to understand whether these men are equal to each other or not.

If a Quinean understanding of ontology is to be adopted, then it would not be that hard to criticize his attitude towards possible objects. He wants to place any object that is to be mentioned in this world, whatsoever. His doubts concerning possible men in this case are a result of his placing possible men in the actual world. However, there is a reason why possible men are not to be placed in the actual world, and that is because they are not actual men; but unactualized possible men. So, the significant thing about unactualized possibles is that they are not actualized. Hence, there is no need to struggle placing them in the actual world, since they are not in it.

I believe when Quine wrote "On What There Is" he was not thinking of going over modal logic. But rather an ontological problem, or going over the ontological consequences of speech which involves modality. My claim stems from the fact that Quine starts with asking the question "what is there?" He is more concerned with the entities that might take place in the ontology, namely possible objects. He is not concerned with the symbol $\Diamond$, but the entities that it causes eventually. With this kind of an operator we come to accept entities which do not really exist, but still have being at some level, as evidenced by the Barcan Formula in the form $\Diamond\exists Fx \supset \exists\Diamond Fx$, which is a logical truth of modality with constant domain models. This way meaning of existence is duplicated as existence and subsistence, and our ontology gets crowded as we have a new kind of entities which are not actually entities in Quine's opinion. This kind of a multiplication, defended by inflationist philosophers, cannot be one Quine favors, since it forces objects without entities into our ontology.

On the other hand, when Kripke appeared up on the scene, he was definitely talking about something else. When Quine is threatened with the existence of some possible man he rightly wants to place that man somewhere in this world. However, he can not only place that ghost-like, blurry thing anywhere in this world, he also has the problem of distinguishing this man from all these possible men whatever properties they may have different from this man. However, Kripke seems quite confident about the fact that this possible man is nowhere in the actual world, but in some possible world. As Kripke is relieved by sending the possible men to some possible worlds other than ours, for Quine the problem still remains. This is why I think Kripke moved the problem to some other realm, namely the realm of metaphysical necessity, and provided a solution to the problem there. However, since Quine does not consider modality as metaphysical this solution is implausible to him, and I do not think it answers the problem in Quine's head.

# CHAPTER 8

# CONCLUSION

In this study my aim was to see after the development of systems of modal logic, whether Quine's objections had any philosophical weight.

After presenting some introductory information to Kripkean semantics, I started giving Quine's objections to modal logic. After explaining his defining necessity in terms of analyticity with an appeal to Carnap, I claimed that he does this to weaken the concept of necessity, and hence modal logic.

The second objection I presented was essentialism. In the mathematician and cyclist argument Quine defines a case in which an object has a property both necessarily and not necessarily. Quine says that his aim is to evoke the appropriate sense of bewilderment, and I think he does. People tried to defeat this objection and they succeeded. But for a person hearing the problem for the first time it is really confusing. For the Aristotelian essentialism, I agree with Quine. None of the articles I have examined has defeated Quine's claims that Aristotelian essentialism is required by quantified modal logic as a result of open sentences.

Actually Quine's objections regarding Aristotelian essentialism and also Barcan formulas and possibilia in some sense are an outcome of quantifying into modal contexts. If only de dicto modality were allowed then the problem of possibilia would have been avoided. And if modal operators were to apply only closed sentences than the Barcan formulas would have been avoided. But I cannot imagine such a modal logic, where what you can do or say is really restricted, and the only thing you were allowed to do would be a mere talk of analyticity, as also stated by Quine.

I wanted to write my thesis on Quine, as he is one of the greatest philosophers of the $20^{th}$ century. What is interesting about him is that he has no interest in constructing a new system of modal logic and in none of the articles I worked on for this project, he offers a new system, but he criticizes the ones that are being or have been constructed. He serves like a gadfly, whose purpose is to irritate or confuse; evoke the appropriate sense of bewilderment in his own words. Although he is not a modal logician, he has contributed to the study of modality in a different way, by revealing the weaknesses of the system, and hence helping the developing and strengthening.

His gadfly-like behaviors are understandable, at least in the case of modal logic, because the consequences of the study of modality are hard to handle, especially ontologically, which is important to him more than anything.

I believe my work is done here neither with Quine, nor with modal logic. I have my questions in mind about modal logic, which were not covered in the scope of this study.  I believe transworld identity is one of the most interesting topics about possible worlds, and arguments raised about the problem by Chisholm and Kaplan are really interesting and worth studying.

I am also planning on working on the ontological and metaphysical consequences of quantified modal logic, specifically of Barcan Formulas. Since Kripke's possible worlds semantics is the most accepted system, I answered Quine's objections with his account. However, I also want to work on Marcus', Stalnaker's and Williamson's understandings of possibilia. Modal logic is a fruitful and interesting subject where one can specialize in logic, metaphysics or ontology, and I think it owes a lot to Quine.

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