OPTIMAL SCHEDULING IN SMART GRIDS WITH PREDICTION ERROR PROBABILITY LIMITATION

by

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ABSTRACT

OPTIMAL SCHEDULING IN SMART GRIDS WITH PREDICTION ERROR PROBABILITY LIMITATION

Smart grid is an electrical grid that uses information and communications technology to improve efficiency, reliability, economics, and sustainability of production and distribution of electricity, which results in much more utilization of this technology in the future. Renewable sources will be utilized more in the future. However, due to its random nature, suppliers are suspicious while integrating these sources into grid. In this thesis, we study demand response, supply management, and power scheduling in a smart grid in the presence of renewable energy sources aiming to increase efficiency and reliability of the grid by limiting the probability of error coming from the prediction of renewable sources. This novel method brings improvement in terms of welfare compared to other methods, which are already available in the literature. In our case, the energy provider bounds the value of maximum prediction error via error probability of prediction. In this decision, the provider compares the cost of spillage and deficit cost of energy production, which are greater than scheduled power production. The goal is to maximize the total social welfare, defined in terms of consumer utility. Furthermore, it has been shown in this research that adding battery to the grid brings an extra improvement in terms of total welfare.

ÖZET

AKILLI ŞEBEKELERDE KESTİRİM HATA KISITLAMA YÖNTEMİYLE ENİYİ PLANLAMA

Akıllı şebeke, bilgi ve haberleşme teknolojilerini kullanan, bu teknolojilerle güç sağlayıcısı ve kullancıların davranışları hakkında bilgi toplayan ve bu bilgileri elektrik üretiminin verimliliğini, güvenilirliğini ve dağıtımnın sürekliliğini sağlamak için kullanan bir elektrik şebekesidir. Yenilenebilir enerji kaynakları gelecekte çok daha fazla kullanılacaktır. Rastgele doğası, şebeke entegre edilirken üreticilerin şüphelenmesine neden olmaktadır. Bu tezde, yenilenebilir enerji kaynakları içeren akıllı şebekelerde verimliliği ve güvenilirliği artırmak için talep müdahalesi, tedarik planlaması ve güç planlaması çalışıyoruz. Bu yenilikçi metod, literatürdeki diğer metodlarla mukayese edildiğinde, refahı iyileştirmektedir. Bizim çalışmamızda, enerji sağlayıcı, kestirim hata ihtimalini değerlendirerek, kestirim hatasını sınırlandırmaktadır. Bu karar, sağlayıcı tarafından fazlalık veya yetersizlik durumu maliyeti göz önune alınarak yapılmaktadır. Toplam sosyal refahın en yüksek olması hedeflenmektedir. Bataryanın sisteme dahil edilmesi de iyileştirme getirmektedir.

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LIST OF SYMBOLS

a	Cost Parameter of scheduled production
b	Cost Parameter of scheduled production
$C(.)$	Cost function
$C_{F_k}(.)$	Cost function of spillage power
$C_{O_k}(.)$	Cost function of electric bought from outside sources
$C_{R_k}(.)$	Cost function of renewable power production
$C_{S_k}(.)$	Cost function of scheduled power production
$C_{Z_k}(.)$	Cost function of battery source
E_k	Amount of power error at time slot k
E_{max}	Maximum error
E_{min}	Minimum error
E_{up}	Upper bound of maximum error
F_k	Amount of power spilled at time slot k
i	user identity
k	time slot
K	Set of time slots
$L(.,.,..)$	Lagrange function
M_{min}	Amount of minimum consumption
M_{max}	Amount of maximum consumption
N	Number of users in the grid
O_k	Amount of power deficit at time slot k as a random variable
P_{max}	Maximum error probability
$Q(.)$	Q function
R_k	Output power of renewable source at time slot k as a random variable
$\hat{R}_k$	Amount of predicted power of renewable source at time slot k
S_k	Produced scheduled power amount at time slot k
S_{max}	Maximum produced scheduled power amount

S_{min}	Minimum produced scheduled power amount
S^*	Optimal produced scheduled power amount
$U(.,.)$	Utility function
$W(.,.)$	Welfare function
w_i	Utility parameter of user i
$w_{i,k}$	Utility parameter of user i at time slot k
x_i^*	Optimal power consumption of user i
$x_{i,k}$	Power consumption of user i at time slot k
Z_{cap}	Capacity of storage
Z_k	Amount of power charged/discharged at time slot k
η	Efficiency of solar array
γ	Parameter between minimum error and standard deviation of distribution
λ_n	Multiplier of the nth constraint
μ	Mean value of distribution
σ	Standard deviation of distribution

LIST OF ACRONYMS/ABBREVIATIONS

ANN	Artificial Neural Network
ARMA	Auto Regressive Moving Average
ARIMA	Auto Regressive Integrated Moving Average
CAES	Compressed Air Energy Storage
EU	European Union
ICT	Information and communication technologies
KKT	Karush Kuhn Tucker
PDF	Probability density function
PEPL	Prediction error probability limitation
PHS	Pumped Storage Hydroelectricity
PV	Photovoltaic
RES	Renewable energy source
SO	System Operator
ZEA	Zero error assumption

1. INTRODUCTION

1.1. Background Information

Smart grids are in fact electricity grids that enable a smart provision of electricity throughout implementing autonomous meters and sensors in its infrastructure [10]. Therefore, by utilizing the information acquired from smart and autonomous meters and sensors, providers can easily monitor the power level and the demand from the consumers, which results in a smart management operation [11]. Current electricity network infrastructure is inadequate in terms of modern challenges such as renewable energy sources, electricity demand and supply methods. On the other hand, information and communication technologies (ICT) have reached certain levels of reliability in technical, management, security, and optimization aspects, which enable them to incorporate new concept of smart grids into conventional electricity networks [12]. According to the United States Department of Energy's Modern Grid Initiative report [13], smart grids have functions that are consumer participation, high quality power, support for different types of storage and generation, higher efficiency and self healing. Therefore, connection and operation of generators of all sizes and technologies is improved. Consumer plays a part of an optimization of the consumption and scheduled power levels of the grid system with receiving greater information from supply sources and send back information to supply sources. Environmental impact of the whole electricity supply system is significantly reduced by preventing overproduction and utilizing renewable energy sources (RES) more and more thanks to the smart grid concept. Renewable energy sources have begun being a substantial part of our energy sources. Due to global climate change, many countries have policy to exchange conventional energy sources into renewable energy sources. The European Union (EU) has planned a 20% share of energy from RES by 2020 [14]. Decreasing the cost of renewable energy sources and subsidy policy of governments lead to attract more investors to implement renewable energy sources. Levelized cost of electricity of photo voltaic panels (PV) have significantly declined after 2010 [15]. Wind turbine cost has also reduced after 2012. European Commission's "Energy road map 2050" report says that by 2050, the share of RES in electricity consumption will be greater than 50% [16]. Smart grids lead to increase utilization of power sources. System operator (SO) manages wholesale electricity market of the power system in real time and also coordinates the supply and demand for electricity. The SO function may be owned by the transmission grid company, or may be fully independent. They are often owned by governments. The SO is required to maintain a continuous balance between electricity supply from power stations and demand from consumers. It also ensures the provision of reserves in case of contingencies, which is operated by determining the optimal combination of energy generation and reserve supplies for each market trading period, through instructing generators' required loads, and managing any contingent events causing disruption in the balance between supply and demand. Increasing load demand and fluctuations in power sources complicates the SO's decisions due to the fact that electricity should be supplied permanently with defined satisfaction levels. This is a challenging task in the case of blackouts, nightmare of the power production process, which affects the daily life of citizens and also causes additional cost to stabilize the power system [17].

High renewable energy penetration is a goal for many countries to increase energy level and reduce carbon emissions from conventional power plants. Wind and solar energy are two of the leading sources among different renewable resources. However, renewable energy usage brings new challenges to the electric power system due to its variable and stochastic nature. In cases where conventional and renewable generation is more than consumption, the excess of energy needs to be spilled or stored to keep the balance between demand and supply. This challenge can be mitigated by increasing flexible resources in the system, such as energy storage technologies and demand response resources. In cases where total generation does not satisfy the demand, the SO must buy deficit power from other operators. The SO periodically (i.e. per 1 hour) communicates with subscribers to give price information and subscribers respond back to the SO to declare their power demand. The SO tries to calculate necessary load with decreasing total cost imposed on. Electricity cannot be stored directly and requires to be converted to other forms of energy. These forms of energy include chemical energy (batteries), kinetic energy (flywheels and compressed air), gravitational potential energy (pumped hydroelectric) and magnetic field (capacitors). Note that, each energy storage application requires a specific type of energy storage technology. These applications could be divided as short and long duration categories by considering the discharge time.

Uncertainty of renewable power output poses a new challenge to power system operation, which is the renewable forecast error. Solutions to overcome this problem are improving quality of forecasting, system reserve increase, scheduling and deployment of energy storage technologies. In the literature, there are numerous approaches to predict renewable sources' output power [18–20]. As mentioned, energy storage technologies give an opportunity of energy saving, when the energy production from RES exceeds the power consumption.
The first objective of this thesis is to propose a novel planning method for electric utility operators, in which the renewable source penetration is high in the smart grid. The second goal of this research is to show that implementing energy storage systems, such as batteries, in the network increases the total social welfare. We also propose algorithms to schedule power system with demand response management.

1.2. Demand Response

Federal Energy Regulatory Commission defines the demand response as electric usage changes by customers from their normal consumption patterns in response to changes in the price of electricity over time or induce lower electricity use, when wholesale market prices are high [21]. Since the nature of renewable sources is stochastic, the output is random. As shown in Figure 1.1, global wind power capacity has significantly increased in the past decade and it will continue to increase. Turkey has increased the cumulative word turbine installations according to Figure 1.2. It is seen that it has reached the production power of 5.5 GW, and due to geographical situations and governmental subsidies, it will continue to increase.

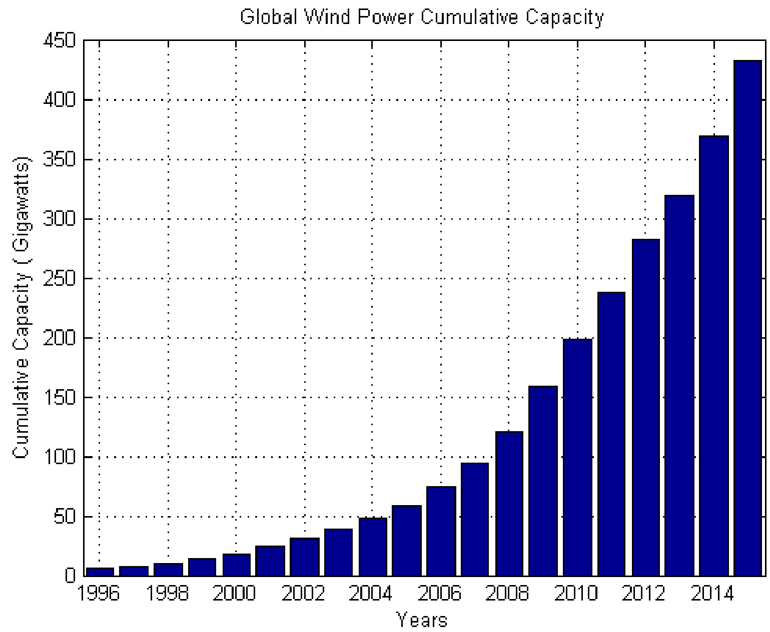

Figure 1.1. Global Wind Power Capacity. Taken from [1].

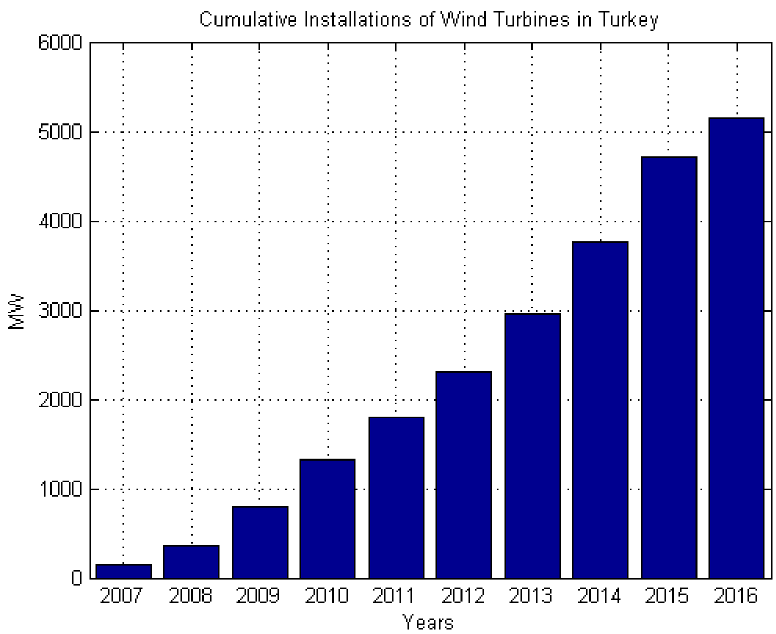

Figure 1.2. Wind Power Capacity in Turkey. Taken from [2].

Figure 1.3 indicates that the cost of wind power production has reduced continuously.

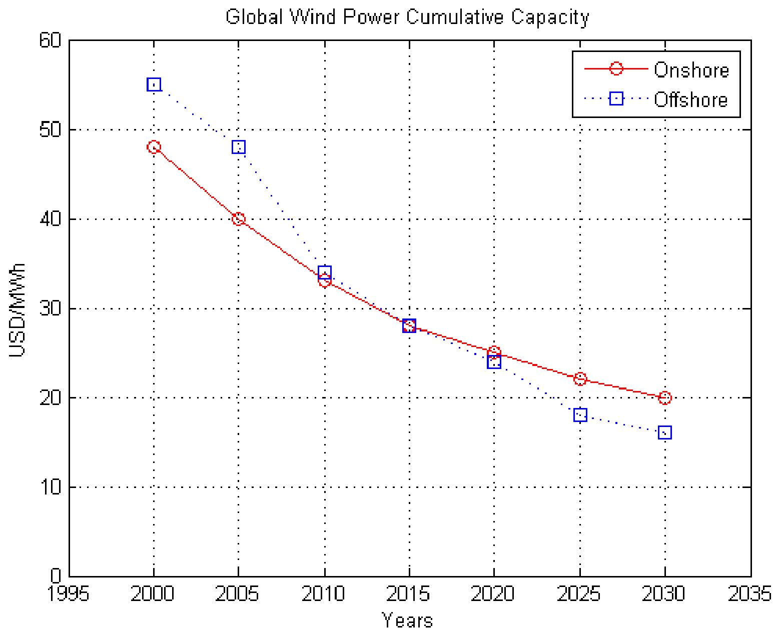

Figure 1.3. Wind Power Cost. Taken from [3].

Similarly as seen in Figure 1.4, cumulative installed solar PV has significantly increased in past decade. Following the global pattern, Turkey has also increased the cumulative PV installations, which is illustrated in Figure 1.5. Currently, the produced power has reached the capacity of 300 MW, it will continue to increase thanks to geographical conditions and governmental subsidies.

Regarding the production cost, it is expected that solar PV cost would be 0.1$ per kWh at the end of 2016 as seen from Figure 1.6.

Near future, suppliers will construct hybrid combinations such as a wind power source with battery banks. This example indicates a landmark scheme for the future energy production, in which batteries and other storage properties will be largely utilized along with power sources.

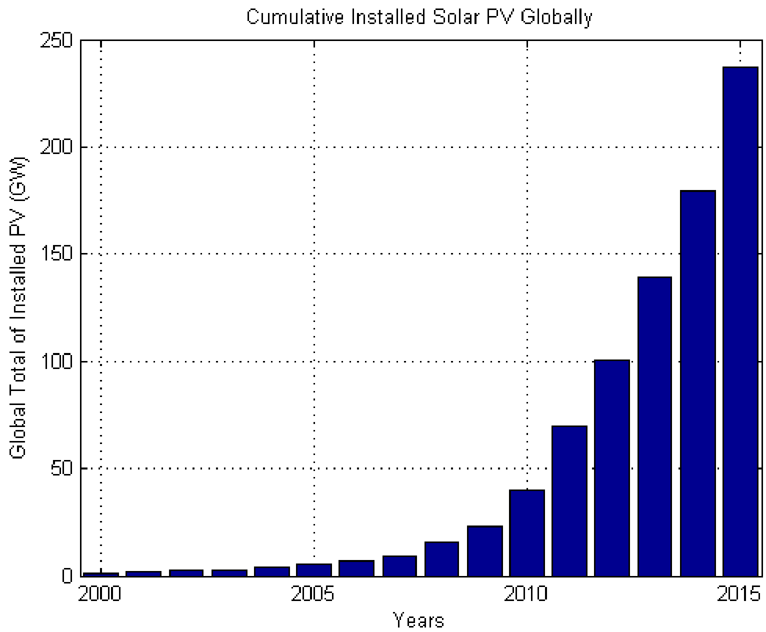

Figure 1.4. Global Solar Power Capacity. Taken from [4].

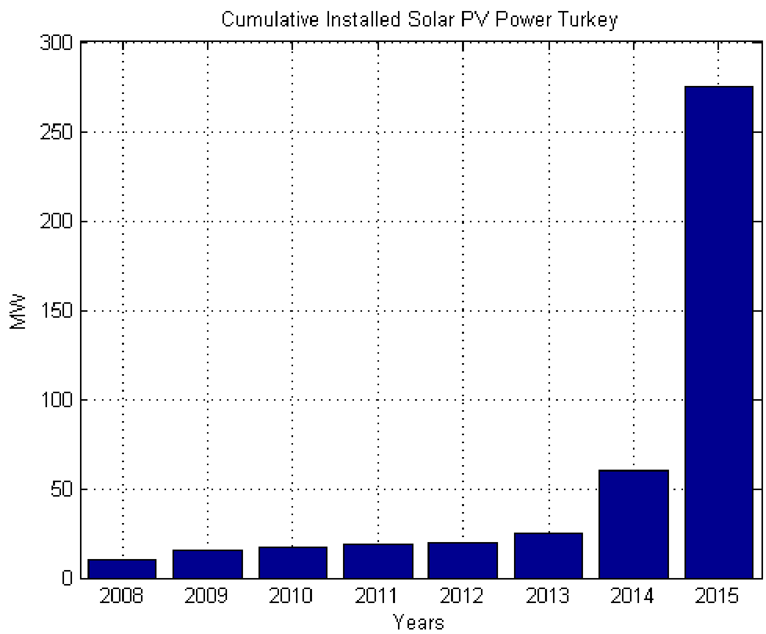

Figure 1.5. Solar Power Capacity in Turkey. Taken from [5].

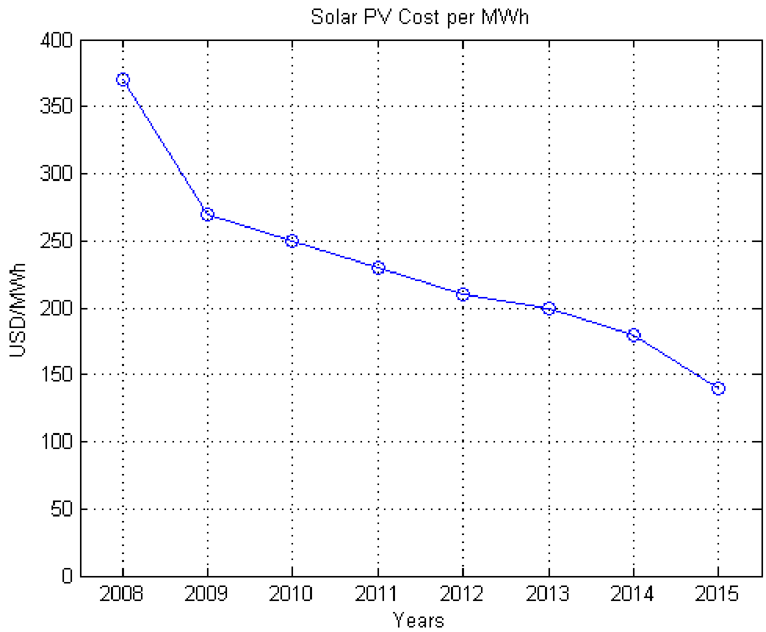

Figure 1.6. Solar Power Cost. Taken from [5].

In this thesis, α-MW-wind turbine output power is modeled as the normal distribution, where its mean is set to α MW and standard deviation $\sigma = 0.1\alpha$ MW. For instance, if we select 1-MW-wind turbine, it follows a Gaussian distribution with parameters $\mu = 1\ MW$ and $\sigma = 0.1\ MW$ as Figure 1.7. If we choose 10 MW power capacity wind turbines, we conclude that expected output power would be 10 MW and standard deviation is 1 MW with normal distribution [6].

Similarly, 90 W/m^2-solar panel output power is also modeled as normal distribution, where the mean is 90 W/m^2 and $\sigma = 6\ W/m^2$ [7]. For instance, if we choose $1km^2$-solar panels, we conclude that expected output power would be 90MW and standard deviation is 6MW with normal distribution. In some models, both wind and solar power sources are utilized. Since both follow the normal distribution, their sum also follows the Gaussian distribution as summation of normal distributions has also a normal distribution. As an example, let us say that the wind and solar power capacities are 1 MW per each source. Therefore, the total output power follows a Gaussian distribution with mean of 2 MW and standard deviation set to $\sigma = 0.12$ MW.

Note that, there are also many approaches to model the output power of renewable energy sources in the literature. For instance, wind speed has already proven to follow the Weibull distribution as Figure 1.10. There are many works which utilize speed probability to model wind power [18, 22–24]. There are also some researches, which utilize solar irradiance distribution to model the solar power output. For instance, it has been reported that it follows the beta distribution as Figure 1.11 [8]. In Table 1.1, we summarize the total system levelized cost for different energy sources. In fact, it subjects to changes in location, time and needs. By using this table, we construct cost functions in Section 3. In Table 1.2, we incorporate different battery costs to be implemented in future power sources, as the levelized costs are currently becoming more inexpensive.

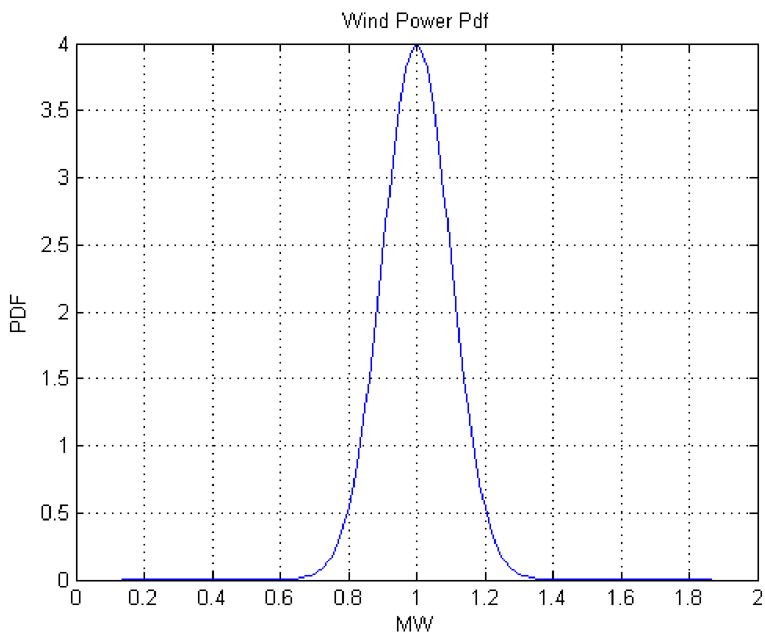

Figure 1.7. 1MW Capacity Wind Power Turbine Output Power PDF. Taken from [6](Rebuplic of Ireland).

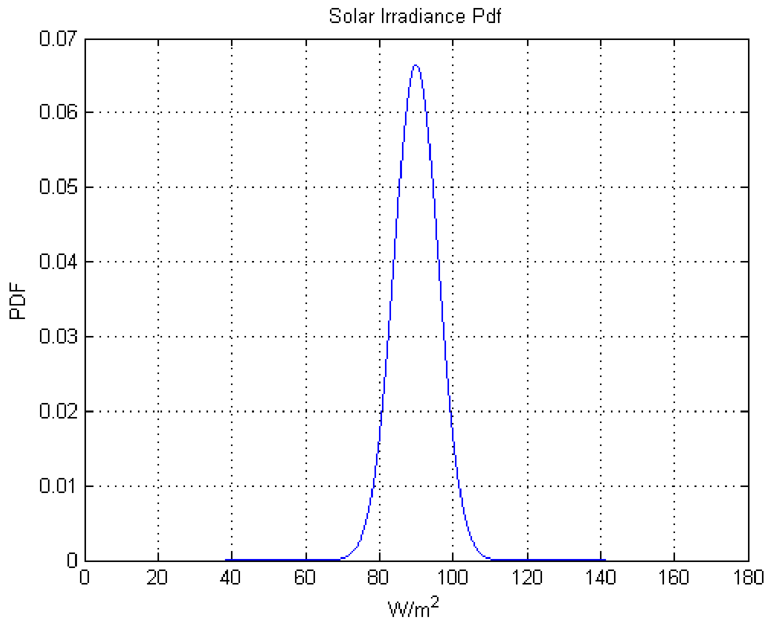

Figure 1.8. Solar Irradiance PDF. Taken from [7](Sweden, during summer).

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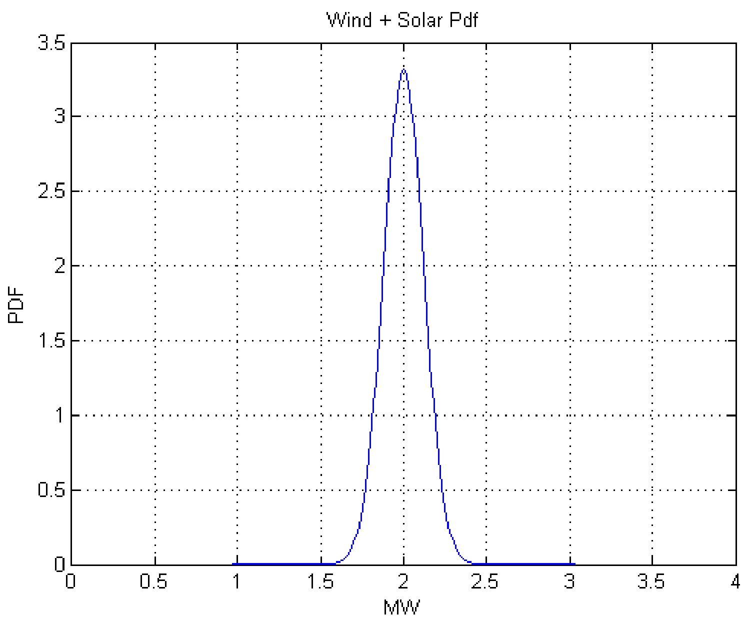

Figure 1.9. 1MW Capacity Wind Power and 1MW Capacity Solar Power System Output Power Distribution.

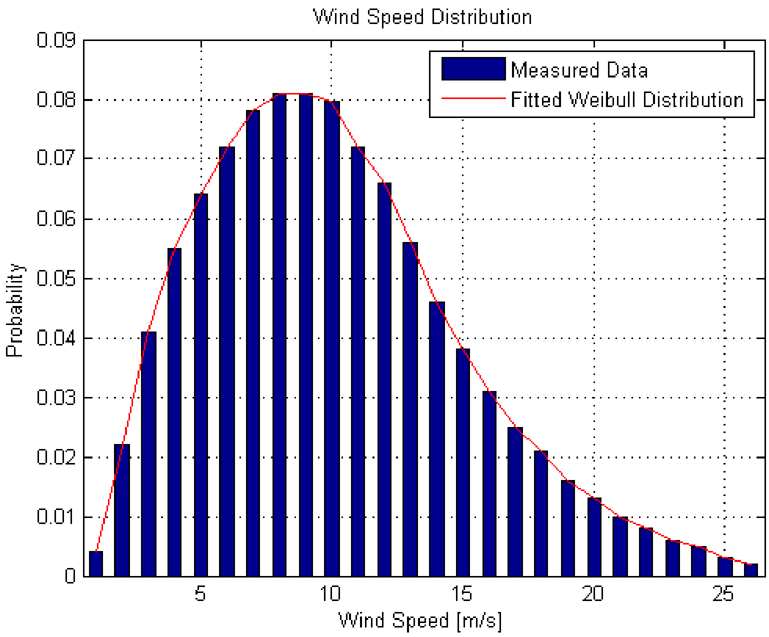

Figure 1.10. Wind Speed Distribution as Weibull Distribution. Taken from [6].

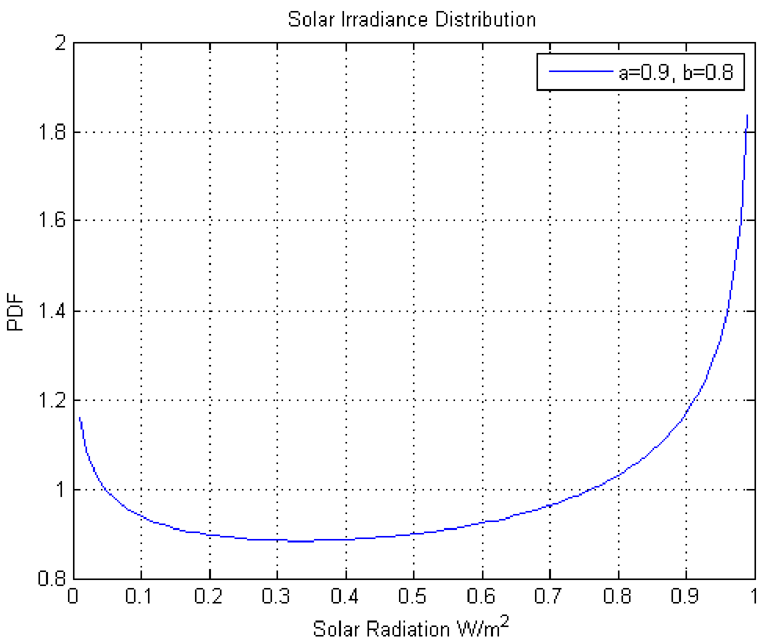

Figure 1.11. Solar Irradiance Distribution as Beta Distribution. Taken from [8].

Table 1.1. Levelized Cost of Different Sources. Taken from [9].

Plant Type	Total System Levelized Cost in 2016 ($/MWh)
Conventional Coal	94.8
Natural Gas CCS	66.1
Wind	97
Wind Offshore	243.2
Solar PV	210.7
Hydro	86.4

Table 1.2. Levelized Cost of Different Storage Sources. Taken from [9].

Storage Options	Total System Levelized Cost in 2015 ($/kWh)
Li-ion	1000-2000
CAES	1600-2200
PHS	1200-2100
NaS Battery	3500-6000
Flywheel	2100-2600

Electricity is supplied permanently and the foremost difficulty is the intermittency of RESs. Since energy production from RES depends heavily upon different parameters, e.g. cloud momentarily occluding sunlight, the power level profile of PV panel contains numerous sporadic changes. Not only does this engender problems for the electricity grid, but the intermittency of RES also makes it an complicated task to fully predict the energy production on a daily basis. Moreover, for a typical household, power consumption profile of electric appliances may not fully match with the expected power generated in RES; therefore, in order for households to utilize more of the energy produced from RES, it is indispensable to incorporate an energy storage unit, such as a Lithium-Ion battery.

The integration of distributed renewable energy sources is also relatively easier with a smart grid infrastructure, because high penetration levels of renewable energy sources can create voltage rises and drops depending on the weather conditions and external factors. In such conditions, the electricity provision from the main conventional power sources can be managed, by monitoring the power production from renewable energy resources through the sensors and smart meters. Smart grids are also more reliable, because the voltage and frequency level throughout the grid can be monitored, and when a failure occurs in the branch of a grid, its location can be detected via sensors and smart meters.

2. SYSTEM MODEL

As mentioned, a smart grid system includes different elements, namely conventional power plant, RESs, and subscribers. The renewable power source creates randomness on the amount of scheduled power production delivered from conventional power plants. The produced scheduled and renewable power at time slot k are denoted as S_k, R_k, respectively, where $k \epsilon K$, K denotes the all time slots.

Let $x_{i,k}$ being the amount of power consumed by subscriber i in time slot k, where $i \epsilon N$, the set of subscribers. In order to represent customer's tendency of usage, we use the same utility function in [25]. Utility function U(x,w) is a concave function and is described for each user. Since the subscriber changes its own consumption character, w is defined as a parameter that represents type of each user.

$$U(x,w) = \begin{cases} wx - \frac{x^2}{4}, & 0 \le x \le 2w, \\ w^2, & x \ge 2w, \end{cases} \tag{2.1}$$

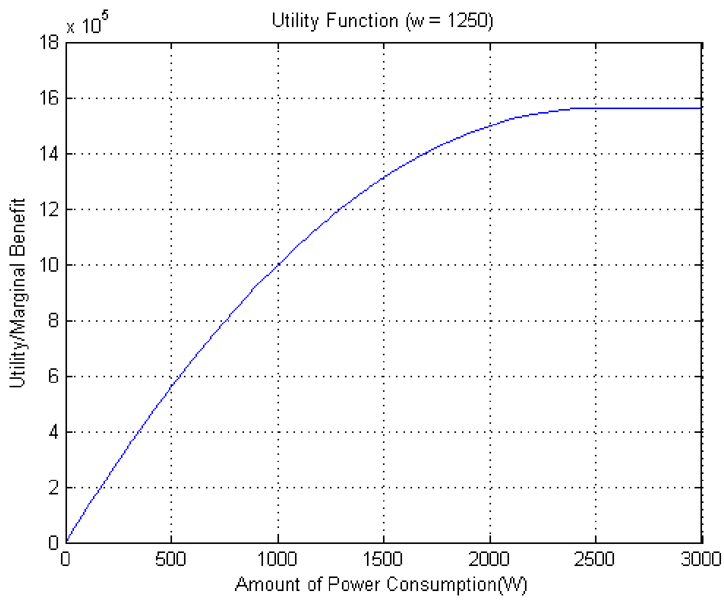

Figure 2.1. Utility Function w=1250

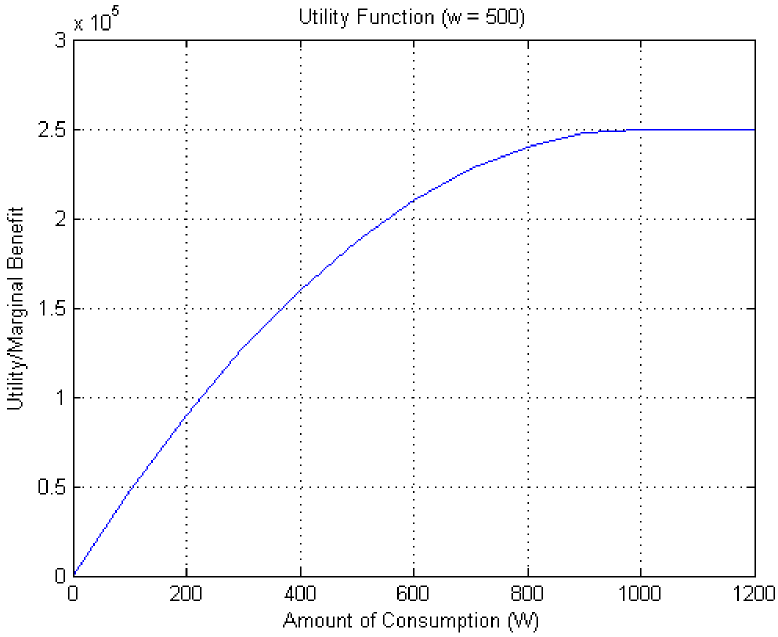

Figure 2.2. Utility Function w=500

Integrating renewable sources into grid brings power deficit and power surplus cases. Correspondingly, let us define the random variable O_k in case of deficit and F_k in case of surplus in time slot k.

Let us denote the predicted renewable power amount as $\hat{R}_k$. Thus, error is the difference between produced renewable power and predicted renewable power i.e. $E_k = R_k - \hat{R}_k$. If E_k is greater than zero, there is surplus, and if E_k is lower than zero, it indicates a deficit. Let N denote the number of subscribers in time slot k.

$$E_k = |E_k| \cdot \underline{\mathrm{sgn}}(E_k) \tag{2.2}$$

$$\underline{\mathrm{sgn}}(E_k) = \begin{cases} -1, & E_k < 0 \\ 1, & E_k > 0 \end{cases} \tag{2.3}$$

Let us define the mathematical form of deficit power, in which the difference between total consumption the total production in case of prediction error leads to deficit in the system.

$$O_k = f(\hat{R}_k) = \sum_{i=1}^{N} x_{i,k} - S_k - \hat{R}_k + |E_k| \geq 0 \tag{2.4}$$

Surplus power is formed, in which the difference between total consumption and the total production in case of prediction error causes surplus in the system.

$$F_k = f(\hat{R}_k) = \sum_{i=1}^{N} x_{i,k} - S_k - \hat{R}_k - |E_k| \leq 0 \tag{2.5}$$

We assume that the power produced from renewable sources is always given into grid regardless of the amount of power produced. At the supply side, we define four cost functions that correspond to scheduled power production, random power production of RES, the power spilled in case of surplus and the power bought from outside sources in case of power deficit, which are denoted as C_{S_k}, C_{R_k}, C_{F_k}, C_{O_k} respectively. In literature, there are many different representations of cost functions such as linear cost function [26] and uniform cost function [27] and quadratic cost function [28]. In order to meet both assumptions, we have chosen second order cost functions,

$$\begin{aligned}
C_{S_k}(z) &= a_k z^2 + b_k z + c_k \\
C_{R_k}(z) &= v_k z^2 + y_k z + z_k \\
C_{F_k}(z) &= e_k z^2 + f_k z + g_k \\
C_{O_k}(z) &= d_k z^2 + q_k z + u_k
\end{aligned}$$

where $d_k > e_k > v_k > a_k \geq 0$, $q_k > f_k > y_k > b_k \geq 0$ and $u_k > g_k > z_k > c_k \geq 0, k \in \mathcal{K}$. We have assumed these parameters according to Table 1.1 and [29].

It is assumed that total welfare of society is maximized, when the sum of all utility functions are increased and total cost is decreased. Therefore, the total welfare function is

$$W(S_k, x_{i,k}) = \sum_{i=1}^{N} U(x_{i,k}, w_{i,k}) - C_{T,k} \tag{2.6}$$

where $C_{T,k} = C_{S_k}(s_k) + C_{R_k}(r_k) + C_{O_k}(o_k) + C_{F_k}(f_k)$ is the total cost in the time slot k.

Our goal is to maximize total welfare function which is formulated in Equation 2.6.

It is assumed that amount of power delivered from conventional source to the system lies between S_{min} and S_{max}. S_{min} is chosen in a way that it satisfies the declared condition, in which maximum % 25 of total capacity comes from renewable sources [30]. S_{max}, on the other hand, should comply with the nominal maximum power capacity of the conventional source. It is also assumed that amount of power consumption of ith subscriber lies between M_{min} and M_{max}. Since some of electrical appliances in house use electricity permanently, we assume that there is minimum consumption. The capacity of residence is limited. Therefore, we assume the consumption is upper bounded.

$$\max_{S_k \in [S_{\min}, S_{\max}]} \max_{x_{i,k} \in [M_{\min}, M_{\max}], i \in N} W(S_k, x_{i,k}) \tag{2.7}$$

2.1. Prediction Error of Renewable Energy Sources

Researchers have considered different types of distributions for Wind power prediction error, such as normal distribution [31], [32], [33], [34], [6], Beta distribution [35], and Cauchy distribution [36]. In our study, we consider the error distribution follows the normal distribution, where the standard deviation of wind power forecast error is

0.1 MW per MW installed capacity, when forecast horizon is one hour [6].

According to [7], global solar irradiation forecasting error distribution is normal distribution with a mean value of zero and a standard deviation of $40W/m^2$. Authors in [37] consider the solar power output in Watt to be equal to the global horizontal irradiance in W/m^2 multiplied by the peak PV efficiency and the array area, where according to [38], PV efficiency is set to 15%. Solar power prediction error is normal distribution with a mean value of zero and a standard deviation $6W/m^2$. Solar panels can produce $85 - 110W/m^2$ on average [39].

Solar power prediction error= Global horizontal irradiance error $\times$ PV efficiency

$$(40W/m^2) \times (15/100) = 6W/m^2 \tag{2.8}$$

2.2. Ramp Constraints

2.2.1. Conventional Source Ramp Constraints

The amount of output power of conventional sources can not be altered instantaneously, and therefore scheduled power production for the current hour must depend on the previous time slot's prediction. To comply with this observation, we include the ramp constraints as follows

$$S_{k-1} - S_D \leq S_k \leq S_{k-1} + S_U, \forall k, \tag{2.9}$$

where S_D and S_U represent ramp-down and ramp-up corresponded power levels [40]. We use the box constraint given in Equation 2.9 by applying the optimal value into the following set for S_k as $[S_{k-1} - S_D, S_{k-1} + S_U]$. In case where S_k^* may be outside of this box, we insert an operation function such that

$$f(X) = \begin{cases} W_{max} & X > W_{max} \\ W_{min} & X < W_{min} \\ X, & otherwise \end{cases} \tag{2.10}$$

2.2.2. Storage Ramp Constraints

The amount of power in the battery is upper bounded by capacity of the battery and the amount of charging and discharging is bounded to the battery, in which depends on the previous time slot's stored energy. The authors of [41] characterizes battery ramp constraints as the time of charging and discharging. In order to simplify the idea of them, we include ramp constraints as follows

$$Z_{k-1} - Z_D \leq Z_k \leq Z_{k-1} + Z_U, \forall k, \tag{2.11}$$

where Z_D and Z_U represent ramp-down and ramp-up corresponded power levels. We use the box constraint given in Equation 2.11 by applying the optimal value into the following set for Z_k as $[Z_{k-1} - Z_D, Z_{k-1} + Z_U]$. In case where Z_k may be outside of this box, we insert an operation function such that

$$f(Y) = \begin{cases} V_{max} & Y > V_{max} \\ V_{min} & Y < 0 \\ Y, & otherwise \end{cases} \tag{2.12}$$

2.3. Total Social Welfare Maximization With Constraint On Prediction Error

Deficit causes more challenging problems compared to the surplus, therefore the cost function of the deficit is greater than the spillage. The SO maximizes the total utility by determining the amount of scheduled power by limiting the probability of prediction error. Therefore, our novel method is based on prediction error probability limitation, which is referred to as PEPL. Our objective is :

$$\max_{S_k \in [S_{\min}, S_{\max}]} \max_{x_{i,k} \in [M_{\min}, M_{\max}], i \in N} W(S_k, x_{i,k}) \tag{2.13}$$

subject to

$$P_{error} \leq P_{max}, \tag{2.14}$$

where P_{error} is the probability of error prediction of renewable sources and P_{max} is the maximum probability of error prediction of renewable sources, which is chosen by the SO.

In grid systems, the SO gives regular offers periodically. In general, period is one hour. Suppose that, the SO has given offer but due to RES output error, it does not meet the requirement. In this case, the SO purchases power deficit from other operators or it must get rid of excess power via spillage. By limiting probability of error, the SO decides more accurately.

Since the distribution of error is known, necessary values are readily found. E_{min} is defined as the minimum value between predicted renewable power and produced renewable power, when the maximum error probability is determined. The SO already know that the renewable energy source production is a stochastic process, therefore, the SO defines the tolerance value. For instance, if the SO declares the production power as 1 MW, but the output does not match with this value, there is a tolerance margin up to a specific value (E_{min}), given that the resulting difference should not be large. In some ranges, the operator can compensate the power deficit from outsourcing. E_{max} is defined as a maximum value between predicted renewable power and produced renewable power. Beyond this value, the SO will be punished by Department of Energy. Therefore, the operator have an error between E_{min} and E_{max}. P_{max} is defined as a maximum probability of error between produced (actual) and predicted renewable power. The operator increases P_{max}, if more risks are acceptable, and vice versa.

E_{min} is defined as a constant multiplied by standard deviation of output power of renewable source.

$$E_{min} = \gamma\sigma. \tag{2.15}$$

SO determines the value of γ, along with the maximum probability of error P_{max}. Since parameters such as the distribution of error, E_{min} and P_{max} are available, determining the maximum error E_{max} is straightforward.

$$Pr\left[E_{min} < \quad X \quad < E_{max}\right] = \int_{E_{min}}^{E_{max}} f_x(x)dx \tag{2.16}$$

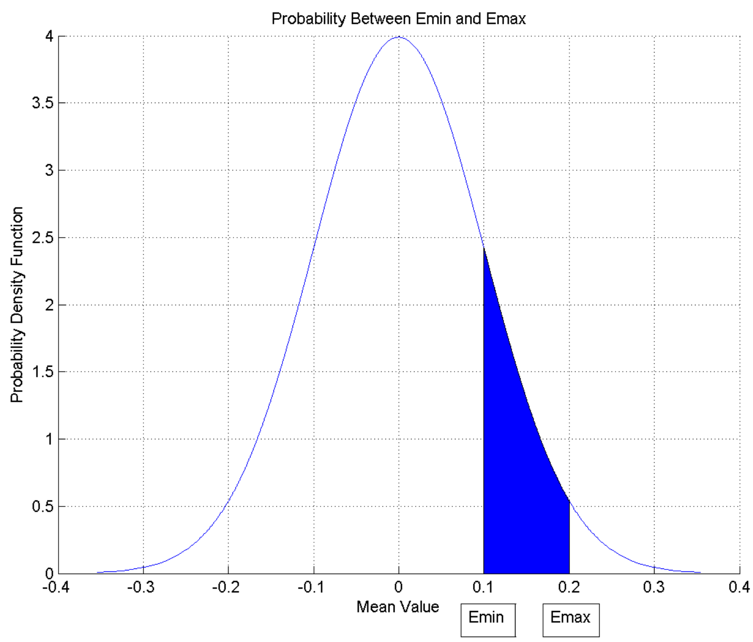

Figure 2.3. Surplus Case : E_{min} and E_{max}.

$$\int_{E_{min}}^{E_{max}} f_x(x)dx \leq P_{max} \tag{2.17}$$

$$\int_{E_{min}}^{E_{max}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \leq P_{max} \tag{2.18}$$

Since the error distribution is normal, mean value is 0 i.e. $\mu = 0$

$$\int_{E_{min}}^{E_{max}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x)^2}{2\sigma^2}} dx \leq P_{max} \tag{2.19}$$

Change the variable $\frac{x}{\sigma}$ as y.

$$\int_{\frac{E_{min}}{\sigma}}^{\frac{E_{max}}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{-\frac{(y)^2}{2}} dy \leq P_{max} \tag{2.20}$$

$$Q(\frac{E_{min}}{\sigma}) - Q(\frac{E_{max}}{\sigma}) \leq P_{max} \tag{2.21}$$

Since we use $E_{min} = \gamma\ \sigma$,

$$Q(\gamma) - P_{max} \leq Q(\frac{E_{max}}{\sigma}) \tag{2.22}$$

Take inverse Q function of both sides. Inequality changes.

$$Q^{-1}(Q(\gamma) - P_{max}) \geq (\frac{E_{max}}{\sigma}) \tag{2.23}$$

$$E_{max} \leq \sigma Q^{-1}(Q(\gamma) - P_{max}) \tag{2.24}$$

Let's consider deficit case.

$$Pr\left[-E_{max} < \quad X \quad < -E_{min}\right] = \int_{-E_{max}}^{-E_{min}} f_x(x)dx \tag{2.25}$$

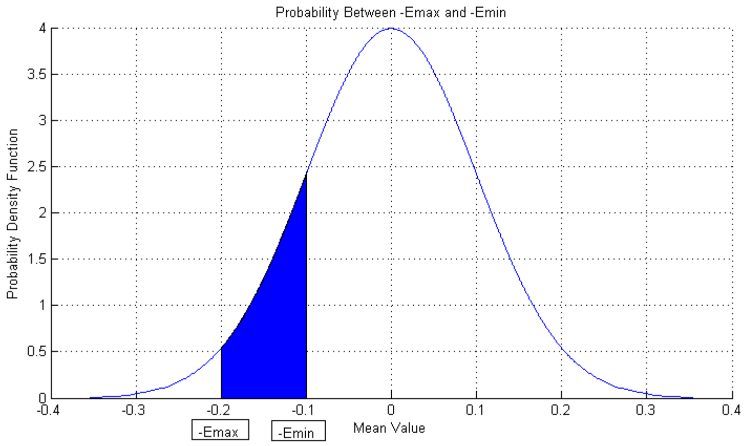

Figure 2.4. Deficit Case : -E_{max} and -E_{min}.

$$\int_{-E_{max}}^{-E_{min}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \leq P_{max} \tag{2.26}$$

Since the error distribution is normal, mean value is 0 i.e. $\mu = 0$

$$\int_{-E_{max}}^{-E_{min}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x)^2}{2\sigma^2}} dx \leq P_{max} \tag{2.27}$$

Change the variable $\frac{x}{\sigma}$ as y.

$$\int_{\frac{-E_{max}}{\sigma}}^{\frac{-E_{min}}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{-\frac{(y)^2}{2}} dy \leq P_{max} \tag{2.28}$$

$$Q(\frac{-E_{max}}{\sigma}) - Q(\frac{-E_{min}}{\sigma}) \leq P_{max} \tag{2.29}$$

$$1 - Q(\frac{E_{max}}{\sigma}) - (1 - Q(\frac{E_{min}}{\sigma})) \leq P_{max} \tag{2.30}$$

$$Q(\frac{E_{min}}{\sigma}) - Q(\frac{E_{max}}{\sigma}) \leq P_{max} \tag{2.31}$$

Since we use $E_{min} = \gamma\ \sigma$,

$$Q(\gamma) - P_{max} \leq Q(\frac{E_{max}}{\sigma}) \tag{2.32}$$

Take inverse Q function of both sides. Inequality changes.

$$Q^{-1}(Q(\gamma) - P_{max}) \geq (\frac{E_{max}}{\sigma}) \tag{2.33}$$

$$E_{max} \leq \sigma Q^{-1}(Q(\gamma) - P_{max}) \tag{2.34}$$

Maximum error value is bounded and it depends on the choice of maximum error probability. E_{up} is defined as the upper value of E_{max}. In other words, it is set up to worst case of error. $|E_{up}|$ is defined as the absolute value of E_{up}. In Figure 2.5, we can see the behavior of $|E_{up}|$ with respect maximum prediction error probability.

$$E_{up} = (\sigma Q^{-1}(Q(\gamma) - P_{max})) \cdot \operatorname{sgn}(E_k), \tag{2.35}$$

$$E_{up} = |E_{up}| \cdot \operatorname{sgn}(E_k) \tag{2.36}$$

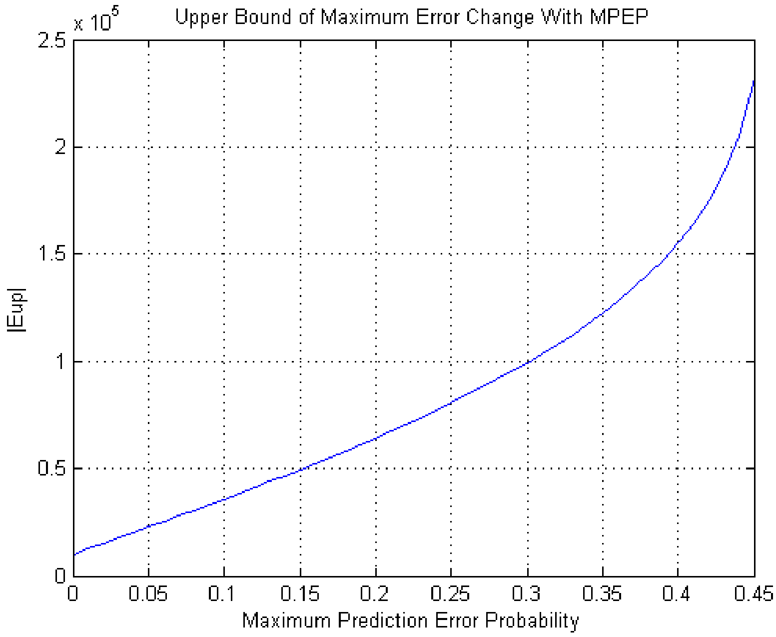

Figure 2.5. $\sigma = 10^5$, $\gamma = 0.1$

2.4. Constraints

First constraint stems from utility function, which is a concave function, as at some points, utility will not increase even though consumption increases.

Second constraint stems from surplus power condition. Since we use error function $|E_{up}|$ as the worst case scenario, surplus must be lower or equal to zero.

Third constraint stems from deficit power condition. Since we use error function $|E_{up}|$ as the worst case scenario, deficit must be greater or equal to zero.

$$\textbf{C1}.\ x_{i,k} - 2w_{i,k} \leq 0, \forall i = 1, \ldots, N. \tag{2.37}$$

$$\textbf{C2}.\ F_k = f(S_k) = \sum_{i=1}^{N} x_{i,k} - S_k - \hat{R}_k - |E_{up}| \leq 0. \tag{2.38}$$

$$\textbf{C3}.\ O_k = f(S_k) = \sum_{i=1}^{N} x_{i,k} - S_k - \hat{R}_k + |E_{up}| \geq 0. \tag{2.39}$$

C1-C2-C3, we have a minimization problem with three inequality-constraints and it can be solved by Karush-Kuhn-Tucker (KKT) conditions [42]. The total cost function is a convex function. Therefore, the objective function is also convex.

Use KKT conditions to find the optimal solutions and construct the Lagrangian L as below. Since price and demand information are updated in each time slot, the problem can be solved independently for each time slot $k \in K$. Hence, without loss of generality, time index k is dropped for simplicity.

Let us denote x_i^* as the optimal power consumption for user i and S^* as the optimal production level. In order to represent the functions as simple as possible, let us to define two functions :

$$\begin{aligned} f_1\big(S^*, \{x_i^*\}_{i=1}^N\big) &\stackrel{def}{=} \textstyle\sum_{i=1}^N x_{i,k} - S_k - \hat{R}_k - |E_{up}| \\ f_2\big(S^*, \{x_i^*\}_{i=1}^N\big) &\stackrel{def}{=} \textstyle\sum_{i=1}^N x_{i,k} - S_k - \hat{R}_k + |E_{up}| \end{aligned}$$

The KKT conditions for the ith subscriber are as below :

(i) $\partial L/\partial x_i|_{x_i^*} \leq 0, x_i^* \geq 0, x_i^*(\partial L/\partial x_i|_{x_i^*}) = 0;$

(ii) $\partial L/\partial S|_{S^*} \leq 0, S^* \geq 0, S^*(\partial L/\partial S|_{S^*}) = 0;$

(iii) $f_1\big(S^*, \{x_i^*\}_{i=1}^N\big) \leq 0, \lambda_1 \geq 0, \lambda_1 f_1\big(S^*, \{x_i^*\}_{i=1}^N\big) = 0;$

$f_2\big(S^*, \{x_i^*\}_{i=1}^N\big) \leq 0, \lambda_2 \geq 0, \lambda_2 f_2\big(S^*, \{x_i^*\}_{i=1}^N\big) = 0;$

(iv) $x_i^* - 2w_i \leq 0, \lambda_{2+i} \geq 0, \lambda_{2+i}(x_i^* - 2w_i) = 0;$

$$
\begin{aligned}
L\left(S_k, x_{i,k}, \lambda_1, \lambda_2, \ldots, \lambda_{2+N}\right) = & \sum_{i=1}^{N} U(x_{i,k}, w_{i,k}) - C_{S_k}(S_k) \\
& - C_{R_k}(R_k) - C_{O_k}(O_k) - C_{F_k}(F_k) \\
& - \lambda_1 \left(\sum_{i=1}^{N} x_{i,k} - S_k - \hat{R}_k - |E_{up}| \right) \\
& - \lambda_2 \left(- \sum_{i=1}^{N} x_{i,k} + S_k + \hat{R}_k - |E_{up}| \right) \\
& - \sum_{i=1}^{N} \lambda_{2+i}(x_{i,k} - 2w_{i,k})
\end{aligned}
$$

Since the consumption of each user and scheduled power production emerges, we can assume as $x_i^* > 0$ and $S^* > 0$, which leads to $\partial L/\partial x_i|_{x_i^*}$ and $\partial L/\partial S|_{S^*}$ are zero. Then,

$$
\frac{x_i^*}{2} = w_i - \lambda_1 + \lambda_2 - \lambda_{2+i}, \tag{2.40}
$$

$$
S^* = \frac{-b + \lambda_1 - \lambda_2}{2a}, \tag{2.41}
$$

where a and b are already defined the constant coefficients of the scheduled power production's cost function. Then, combining Equation 2.40 and Equation 2.41, we obtain

$$
\frac{x_i^*}{2} + 2aS^* + b = w_i - \lambda_{2+i}. \tag{2.42}
$$

From Equation 2.42, we conclude that $x_i^* \neq 2w_i$, because of the fact that S^*, $a > 0$ and $b > 0$. Thus, $\lambda_{2+i} = 0$ by the KKT condition (iv). Then, we have

$$
\frac{x_i^*}{2} = w_i - \lambda_1 + \lambda_2. \tag{2.43}
$$

$$
\frac{x_i^*}{2} + 2aS^* + b = w_i. \tag{2.44}
$$

When we apply KKT condition (iv) ($x_i^* - 2w_i \leq 0$) into Equation 2.43, we conclude that $\lambda_1 > \lambda_2$. Now, by utilizing KKT Condition (iii), we derive Equations 2.45 and 2.46.

$$\lambda_1 \ f_1\big(S^*, \{x_i^*\}_{i=1}^N\big) = 0 \text{ and } \lambda_2 \ f_2\big(S^*, \{x_i^*\}_{i=1}^N\big) = 0.$$

Applying the results: $\lambda_1 > \lambda_2$, $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$.So $\lambda_1 > 0$.

$$\lambda_1(\sum_{i=1}^{N} x_{i,k}^* - S_k^* - \hat{R}_k - |E_{up}|) = 0 \tag{2.45}$$

$$\lambda_2(-\sum_{i=1}^{N} x_{i,k}^* + S_k^* + \hat{R}_k - |E_{up}|) = 0 \tag{2.46}$$

($\sum_{i=1}^N x_{i,k}^* - S_k^* - \hat{R}_k - |E_{up}|$) must be zero in order to satisfy KKT Condition (iii), since $\lambda_1 > 0$. Therefore, $\sum_{i=1}^N x_{i,k}^* = S_k^* + \hat{R}_k + |E_{up}|$. In order to satisfy Equation 2.46, $\lambda_2 = 0$.

$$x_i^* = 2(w_i - \lambda_1). \tag{2.47}$$

$$S^* = \frac{-b + \lambda_1}{2a}. \tag{2.48}$$

$$f_1\big(S^*, \{x_i^*\}_{i=1}^N\big) = 0, \tag{2.49}$$

where $w_i > \lambda_1$ and $\lambda_1 > b$. Substitute Equations 2.47 and 2.48 into 2.49,

$$\lambda_1 = \frac{2\sum_{i=1}^N w_i + \frac{b}{2a} - \hat{R}_k - |E_{up}|}{\frac{1}{2a} + 2N}. \tag{2.50}$$

$$x_i^* = \frac{\frac{w_i}{a} + 4w_iN - 4\sum_{i=1}^{N} w_i - \frac{b}{a} + 2\hat{R}_k + 2|E_{up}|}{\frac{1}{2a} + 2N}. \tag{2.51}$$

$$S^* = \frac{-2bN + 2\sum_{i=1}^{N} w_i - \hat{R}_k - |E_{up}|}{1 + 4aN} \tag{2.52}$$

$$\lambda_2 = 0, \lambda_{2+i} = 0 \tag{2.53}$$

The result indicates that the optimal conditions depend on the selection of P_{max}, which can be chosen between 0 and 0.5.

for each k

Select P_{max}, E_{min} and determine the cost parameters;

Set the cost parameters

Decide E_{min}

Determine P_{max}

Give these parameters to all subscribers;

Receive the utility parameters from all subscribers;

Calculate the predicted power from renewable sources;

Compute the optimal S_k as Equation 2.52;

Check the conventional source ramp constraints

Share the information with subscribers;

Figure 2.6. Optimization algorithm for the provider, renewable source

2.5. Adding Battery to the System

As we have discussed, we consider the case, in which battery banks are integrated to the system to store the surplus power. Let us introduce the random variable Z_k is

for each k

Share the information of utility parameters to all subscribers and the power source;

Take the cost parameters from power source;

Receive the utility parameters from other subscribers

Compute the optimal x_k as Equation 2.51;

Figure 2.7. Optimization algorithm for ith subscriber, renewable source

the stored power at time slot k. Since it depends on the previous time slot k, let us introduce another random variable Z_{k-1} is the stored power at time slot $(k-1)$. At the supply side, we now define five cost functions, instead of four cost functions at the previous batteryless case, which correspond to scheduled power production, renewable power production, the stored power, spillage power and finally, power bought from outside sources in the case of power deficit which are denoted as C_{S_k}, C_{R_k}, C_{Z_k}, C_{F_k} and C_{O_k} respectively. We have assumed these parameters according to Table 1.1, Table 1.2 and [29]. Since the charge and discharge speed of the battery is also bounded, we must get a constraint as Equation 2.54 [43]. Z_{cap} is defined as a capacity of battery.

$$|Z_k - Z_{k-1}| < (0.75) \cdot Z_{cap} \tag{2.54}$$

$$\begin{aligned}
C_{S_k}(z) &= a_k z^2 + b_k z + c_k \\
C_{R_k}(z) &= v_k z^2 + y_k z + z_k \\
C_{Z_k}(z) &= h_k z^2 + j_k z + t_k \\
C_{F_k}(z) &= e_k z^2 + f_k z + g_k \\
C_{O_k}(z) &= d_k z^2 + q_k z + u_k
\end{aligned} \tag{2.55}$$

where $d_k > e_k > h_k > v_k > a_k \geq 0$ and we assume $q_k > f_k > j_k > y_k > b_k \geq 0$ and $u_k > g_k > t_k > z_k > c_k \geq 0, k \in \mathcal{K}$.

$$O_k = f(\hat{R}_k) = \sum_{i=1}^{N} x_{i,k} - S_k - \hat{R}_k - Z_{k-1} + E_k \geq 0 \tag{2.56}$$

$$F_k = f(\hat{R}_k) = \sum_{i=1}^{N} x_{i,k} - S_k - \hat{R}_k + Z_k - Z_{k-1} - E_k \leq 0 \tag{2.57}$$

$$W(S_k, x_{i,k}) = \sum_{i=1}^{N} U(x_{i,k}, w_{i,k}) - C_{T,k}, \tag{2.58}$$

where $C_{T,k} = C_{S_k}(S_k) + C_{R_k}(r_k) + C_{O_k}(o_k) + C_{Z_k(z_k)} + C_{F_k(f_k)}$ is the total power cost in the time slot k. As same with previous case, our goal is :

$$\max_{S_k \in [S_{\min}, S_{\max}]} \max_{x_{i,k} \in [M_{\min}, M_{\max}], i \in N} W(S_k, x_{i,k}) \tag{2.59}$$

Note that, surplus will now be stored in the battery and therefore spillage cost is reduced significantly.

$$Pr\left[-E_{max} < \quad X \quad < E_{max}\right] = \int_{-E_{max}}^{E_{max}} f_x(x)dx \tag{2.60}$$

Since the rest of calculation is the same as Section 2.4, we can reuse Equation 2.34.

Because we have introduced a storing battery to the system, our minimum error condition can be relaxed into 0. Therefore, we can modify E_{max} as

$$E_{max} \leq \sigma Q^{-1}(0.5 - P_{max}). \tag{2.61}$$

$$E_{up} = (\sigma Q^{-1}(0.5 - P_{max})) \cdot \text{sgn}(E_k), \tag{2.62}$$

$$E_{up} = |E_{up}| \cdot \mathrm{sgn}(E_k) \tag{2.63}$$

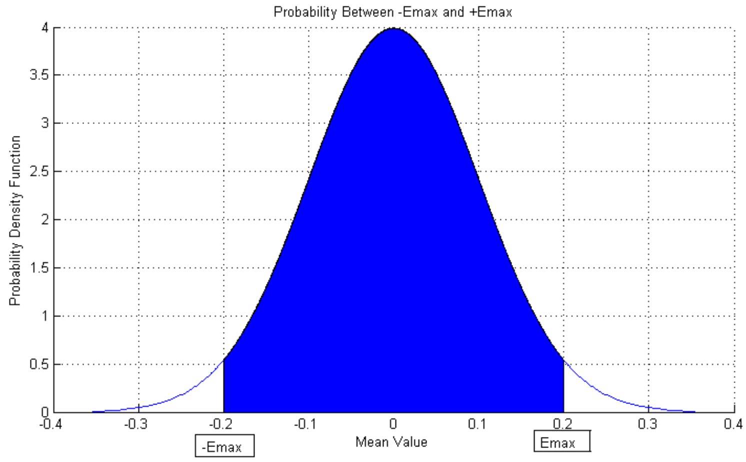

Figure 2.8. Surplus and Deficit Case : -E_{max} and E_{max}.

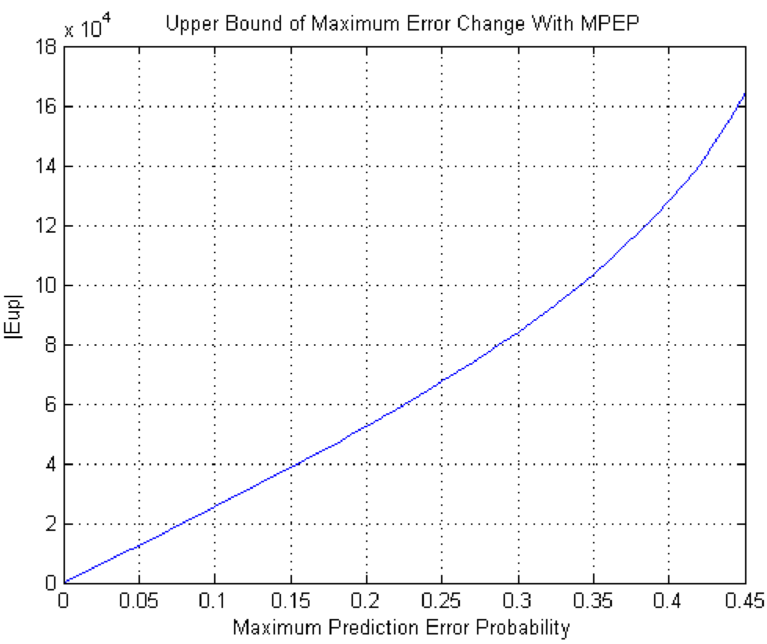

Figure 2.9. $\sigma = 10^5$

2.6. Constraints with Storage

First constraint comes from utility function, which is a concave function.

Second constraint originates from surplus power condition. As mentioned in the previous section, using $|E_{up}|$ as the worst case scenario, dictates surplus to be lower or equal to zero.

Third constraint stems from deficit power condition. Employing $|E_{up}|$ as the worst case scenario, subjects deficit to be greater or equal to zero. Let us define the following functions.

$$f_1\big(S^*, \{x_i^*\}_{i=1}^N\big) \overset{def}{=} \textstyle\sum_{i=1}^N x_{i,k} - S_k - \hat{R}_k + Z_k - Z_{k-1} - |E_{up}|$$
$$f_2\big(S^*, \{x_i^*\}_{i=1}^N\big) \overset{def}{=} \textstyle\sum_{i=1}^N x_{i,k} - S_k - \hat{R}_k - Z_{k-1} + |E_{up}|$$

$$\textbf{C1}.\ x_{i,k} - 2w_{i,k} \leq 0, \forall i = 1, \dots, N. \tag{2.64}$$

$$\textbf{C2}.\ F_k = f(S_k) = \sum_{i=1}^{N} x_{i,k} - S_k - \hat{R}_k + Z_k - Z_{k-1} - |E_{up}| \leq 0 \tag{2.65}$$

$$\textbf{C3}.\ O_k = f(S_k) = \sum_{i=1}^{N} x_{i,k} - S_k - \hat{R}_k - Z_{k-1} + |E_{up}| \geq 0 \tag{2.66}$$

Lagrangian of optimization problem could be written as below. In order not to complicate Lagrangian form, we have not inserted ramp constraints here but we have considered them while applying computer simulations.

$$\begin{aligned} L\left(S_k, x_{i,k}, \lambda_1, \lambda_2, \ldots, \lambda_{2+N}\right) = & \sum_{i=1}^{N} U(x_{i,k}, w_{i,k}) - C_{S_k}(S_k) \\ & - C_{R_k}(R_k) - C_{O_k}(O_k) - C_{F_k}(F_k) \\ & - \lambda_1 \left(\sum_{i=1}^{N} x_{i,k} - S_k - \hat{R}_k + Z_k - Z_{k-1} - |E_{up}| \right) \\ & - \lambda_2 \left(- \sum_{i=1}^{N} x_{i,k} + S_k + \hat{R}_k + Z_{k-1} - |E_{up}| \right) \\ & - \sum_{i=1}^{N} \lambda_{2+i}(x_{i,k} - 2w_{i,k}) \end{aligned}$$

C1-C2-C3, similar to previous cases, this is a minimization problem with three inequality constraints and can be solved via KKT conditions. Note that, time index k is similarly dropped for simplicity.

Defining x_i^* as the optimal power consumption for user i and S^* as the optimal production level. In order to represent the functions as simple as possible, let us to define two functions :

$$\begin{aligned} f_1\big(S^*, \{x_i^*\}_{i=1}^{N}\big) &\stackrel{def}{=} \textstyle\sum_{i=1}^{N} x_{i,k} - S_k - \hat{R}_k + Z_k - Z_{k-1} - |E_{up}| \\ f_2\big(S^*, \{x_i^*\}_{i=1}^{N}\big) &\stackrel{def}{=} \textstyle\sum_{i=1}^{N} x_{i,k} - S_k - \hat{R}_k - Z_{k-1} + |E_{up}| \end{aligned}$$

The KKT conditions for the ith subscriber are as below :

(i) $\partial L/\partial x_i|_{x_i^*} \leq 0, x_i^* \geq 0, x_i^*(\partial L/\partial x_i|_{x_i^*}) = 0;$

(ii) $\partial L/\partial S|_{S^*} \leq 0, S^* \geq 0, S^*(\partial L/\partial S|_{S^*}) = 0;$

(iii) $f_1\big(S^*, \{x_i^*\}_{i=1}^{N}\big) \leq 0, \lambda_1 \geq 0, \lambda_1 f_1\big(S^*, \{x_i^*\}_{i=1}^{N}\big) = 0;$
$f_2\big(S^*, \{x_i^*\}_{i=1}^{N}\big) \leq 0, \lambda_2 \geq 0, \lambda_2 f_2\big(S^*, \{x_i^*\}_{i=1}^{N}\big) = 0;$

(iv) $x_i^* - 2w_i \leq 0, \lambda_{2+i} \geq 0, \lambda_{2+i}(x_i^* - 2w_i) = 0;$

Because the consumption and scheduled power production is present, we strictly say $x_i^* > 0$ and $S^* > 0$, which makes $\partial L/\partial x_i|_{x_i^*}$, and $\partial L/\partial S|_{S^*}$ are zero. Then,

$$\frac{x_i^*}{2} = w_i - \lambda_1 + \lambda_2 - \lambda_{2+i}, \tag{2.67}$$

$$S^* = \frac{-b + \lambda_1 - \lambda_2}{2a}, \tag{2.68}$$

where a and b are constant coefficients of the cost function. Then, combining Equation 2.67 and Equation 2.68, we obtain

$$\frac{x_i^*}{2} + 2aS^* + b = w_i - \lambda_{2+i} \tag{2.69}$$

From Equation 2.69, we conclude that $x_i^* \neq 2w_i$ because of the fact that S^*, $a > 0$ and $b > 0$. Thus, we get $\lambda_{2+i} = 0$ by the KKT condition (iv). Then,

$$\frac{x_i^*}{2} = w_i - \lambda_1 + \lambda_2, \tag{2.70}$$

$$\frac{x_i^*}{2} + 2aS^* + b = w_i, \tag{2.71}$$

putting KKT condition (iv) ($x_i^* - 2w_i \leq 0$) into Equation 2.70, gives us $\lambda_1 > \lambda_2$. Now, based on KKT condition (iii), Equations 2.72 and 2.73 are derived.
$\lambda_1 \; f_1\big(S^*, \{x_i^*\}_{i=1}^N\big) = 0$ and $\lambda_2 \; f_2\big(S^*, \{x_i^*\}_{i=1}^N\big) = 0$.
Apply results, $\lambda_1 > \lambda_2$, $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$. Thus, $\lambda_1 > 0$.

$$\lambda_1(\sum_{i=1}^{N} x_{i,k}^* - S_k^* - \hat{R}_k + Z_k - Z_{k-1} - |E_{up}|) = 0 \tag{2.72}$$

$$\lambda_2(-\sum_{i=1}^{N} x_{i,k}^* + S_k^* + \hat{R}_k - Z_{k-1} - |E_{up}|) = 0 \tag{2.73}$$

$(\sum_{i=1}^{N} x_{i,k}^* - S_k^* - \hat{R}_k + Z_k - Z_{k-1} - |E_{up}|)$ must be zero in order to satisfy KKT condition (iii), since $\lambda_1 > 0$. Therefore, $\sum_{i=1}^{N} x_{i,k}^* = S_k^* + \hat{R}_k - Z_k + Z_{k-1} + |E_{up}|$. In order to satisfy Equation 2.73, $\lambda_2 = 0$.

$$x_i^* = 2(w_i - \lambda_1), \tag{2.74}$$

$$S^* = \frac{-b + \lambda_1}{2a}, \tag{2.75}$$

$$f_1\big(S^*, \{x_i^*\}_{i=1}^{N}\big) = 0 \tag{2.76}$$

where $w_i > \lambda_1$ and $\lambda_1 > b$. Substituting Equations 2.74 and 2.75 into 2.76,

$$\lambda_1 = \frac{2\sum_{i=1}^{N} w_i + \frac{b}{2a} - \hat{R}_k + Z_k - Z_{k-1} - |E_{up}|}{\frac{1}{2a} + 2N} \tag{2.77}$$

$$x_i^* = \frac{\frac{w_i}{a} + 4w_i N - 4\sum_{i=1}^{N} w_i - \frac{b}{a} + 2\hat{R}_k - 2Z_k + 2Z_{k-1} + 2|E_{up}|}{\frac{1}{2a} + 2N} \tag{2.78}$$

$$S^* = \frac{-2bN + 2\sum_{i=1}^{N} w_i - \hat{R}_k + Z_k - Z_{k-1} - |E_{up}|}{1 + 4aN} \tag{2.79}$$

$$\lambda_2 = 0, \lambda_{2+i} = 0 \tag{2.80}$$

The result relates optimal conditions with dependency on the P_{max}, which can be selected between 0 and 0.5.

for each k

Take information from battery Z_{k-1}

Consider Storage Source Ramp Constraints

Set the cost parameters

Decide the value of E_{min};

Determine the value of P_{max}

Give these parameters to all subscribers;

Receive the utility values from all subscribers;

Calculate the predicted power which comes from renewable sources;

Compute the optimal S_k as given in Equation 2.79;

Consider Conventional Source Ramp Constraints

Share the information with subscribers;

Figure 2.10. Optimization algorithm for the provider, renewable source and battery.

for each k

Share the information of utility parameters to all subscribers and the power source;

Take the cost parameters from power source;

Receive the utility values from other subscribers

Compute the optimal x_k as Equation 2.78;

Figure 2.11. Optimization algorithm for the subscriber, renewable source and battery.

3. NUMERICAL ANALYSIS

We have found the optimal solutions in the previous chapter. For the sake of comprehensive comparison, in this chapter, we analyze optimal consumption and scheduled power production in different scenarios. In the first scenario, we analyze optimal total scheduled production and consumption versus different values of maximum prediction error. In the second scenario however, we fix $P_{max} = 0.2$ and One-day-long scheduled produced/consumed power behavior is analyzed via computer simulations. These scenarios are presented in the following different sections: Wind power, solar power, wind and solar power, wind power with battery, solar power with battery, and finally wind and solar power with battery. Note that, we have defined time slots as one hour for all cases. The utility parameter of each user is selected randomly from the interval $[1000, 1500]$ meeting $w_i > \lambda_1$ for all $i \epsilon N$. We assume that total subscribers are $N = 3000$. Then, we assume that the average hourly consumption of a subscriber, 1.25 kW per hour [9]. The total power of RES is assumed to be 1 MW. As indicated in regulations [4], 25% of total capacity could be supplied by renewable source, and therefore the total consumption power is 4 MW. We have chosen cost parameters of scheduled power as to satisfy 41 kurus/ kWh^2 [29]. We also assume that there is a conventional source of capacity of 4 MW due to the fact that, if there is no energy coming from renewable source, conventional source will suffice the consumption solely. We have found optimal values applying ramp constraints as we have already defined.

3.1. Wind Power

In this section, numerical analysis will show the optimal power consumption and production are presented for the smart grid, when wind turbine of 1 MW power capacity is introduced.

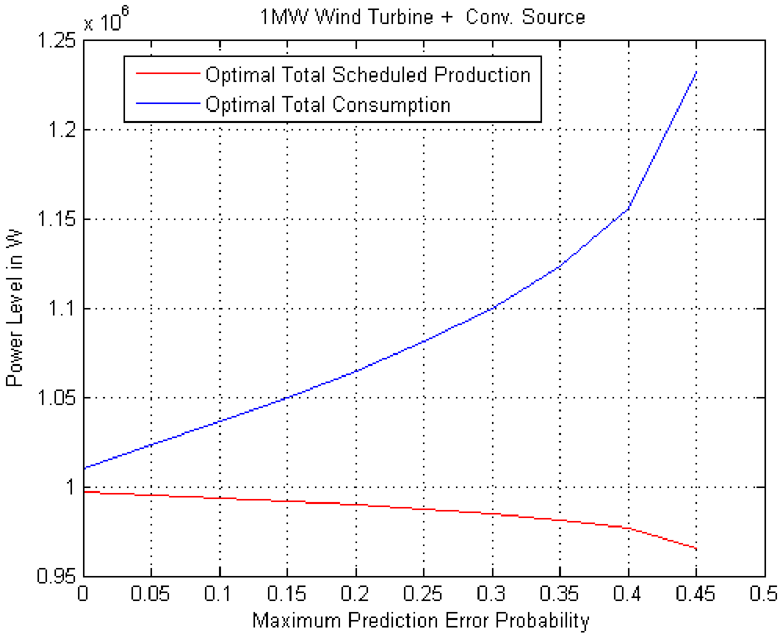

Figure 3.1. 1 MW Wind Turbine in 4 MW capacity residential

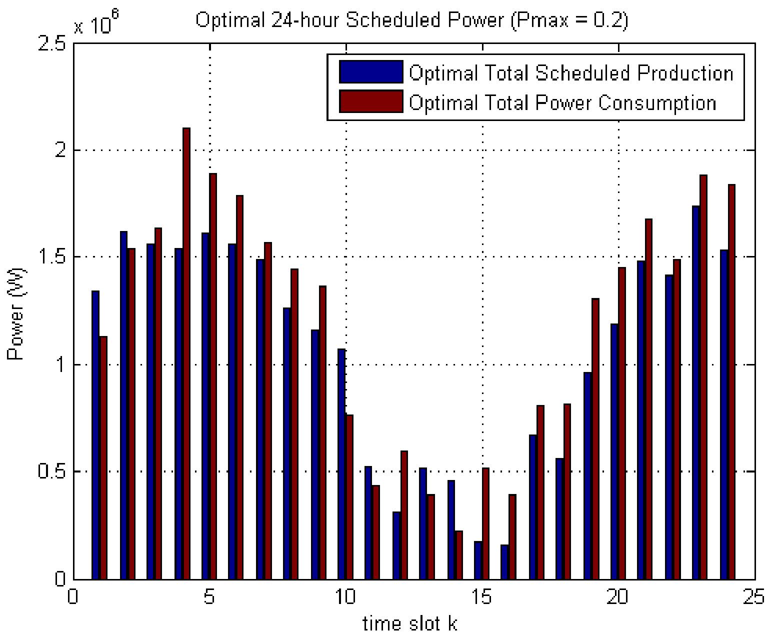

Figure 3.2. 1MW Wind Turbine in 4MW capacity residential

3.2. Solar Power

In this section, numerical analysis will show the optimal consumption and production are presented for the smart grid when we introduce solar arrays of 1 MW power capacity.

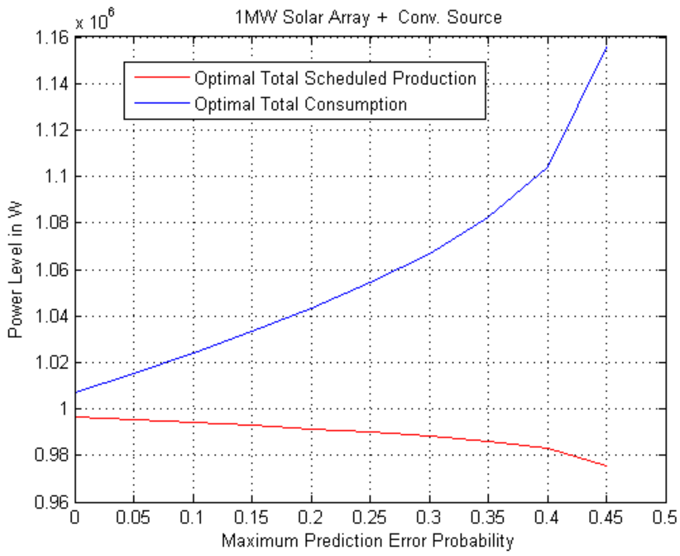

Figure 3.3. 1MW Solar Array in 4MW capacity residential

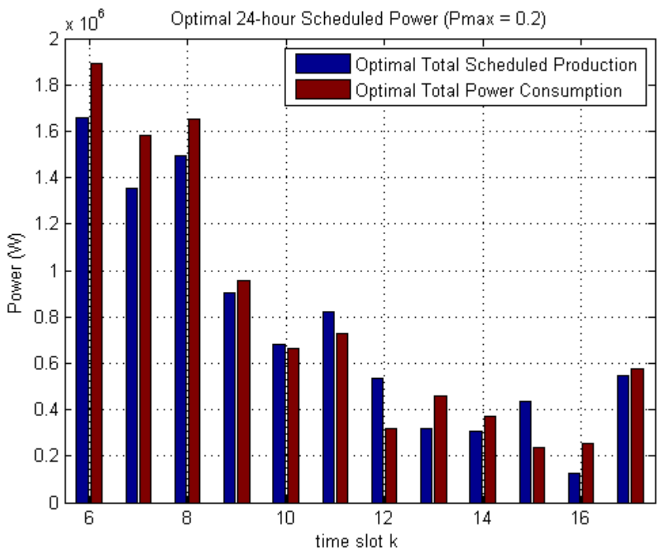

Figure 3.4. 1MW Solar Array in 4MW capacity residential

3.3. Wind and Solar Power

In this section, numerical analysis will show the optimal consumption and production are presented for the smart grid when we introduce wind turbine and solar arrays which has each 0.5 MW power capacity.

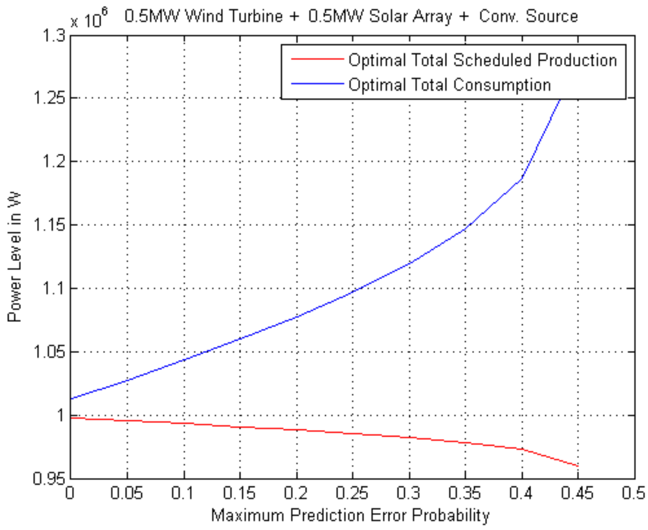

Figure 3.5. 0.5MW Wind Turbine 0.5MW Solar Arrays in 4MW capacity residential

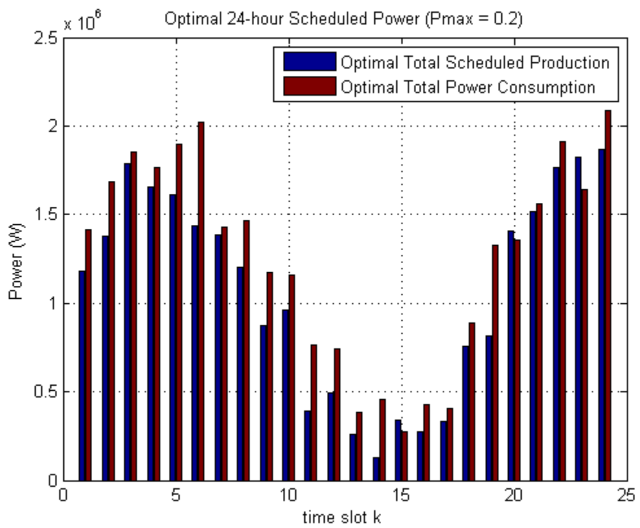

Figure 3.6. 0.5MW Solar Array and 0.5 MW Wind Turbine in 4 MW capacity residential

3.4. Wind Power with Battery

In this section, numerical results showing the optimal consumption and production are presented for the smart grid when we introduce wind turbine which has 1MW power capacity and battery which has 150 KW capacity.

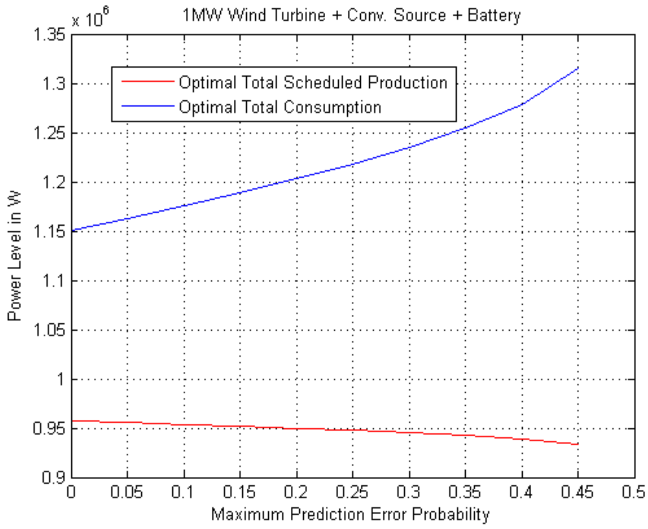

Figure 3.7. 1MW Wind Turbine and 150KW Battery in 4MW capacity residential

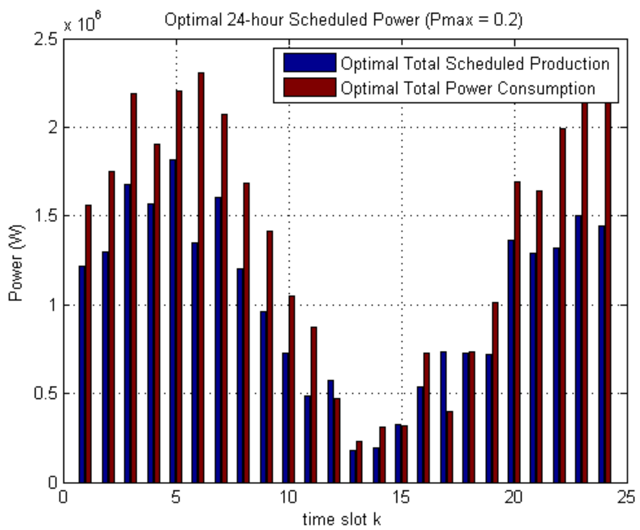

Figure 3.8. 1MW Wind Turbine and 150KW Battery in 4MW capacity residential

3.5. Solar Power with Battery

In this section, numerical results showing the optimal consumption and production are presented for the smart grid when we introduce solar arrays which has 1 MW power capacity with 150 KW power battery capacity.

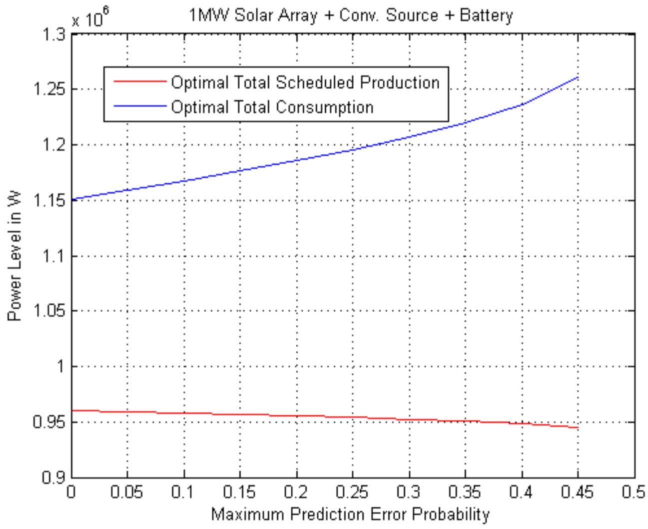

Figure 3.9. 1MW Solar Array and 150KW Battery in 4MW capacity residential

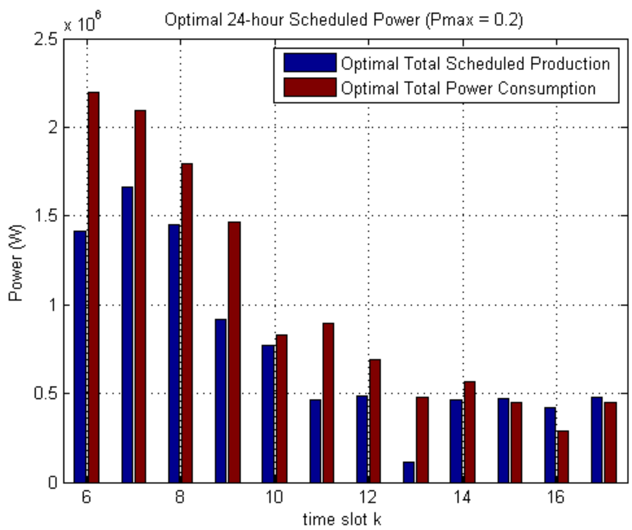

Figure 3.10. 1MW Solar Array and 150KW Battery in 4MW capacity residential

3.6. Wind and Solar Power with Battery

In this section, numerical results showing the optimal consumption and production are presented for the smart grid when we introduce wind turbine and solar arrays which has each 0.5 MW power capacity. The integrated battery has 150 KW power capacity.

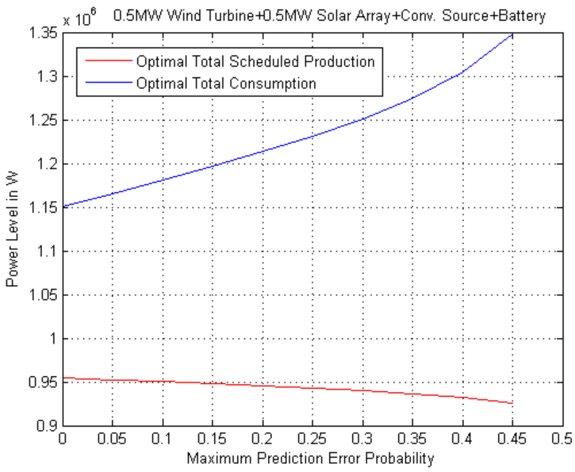

Figure 3.11. 0.5MW Wind Turbine, 0.5 MW Solar Array and 150 KW Battery in 4MW capacity residential

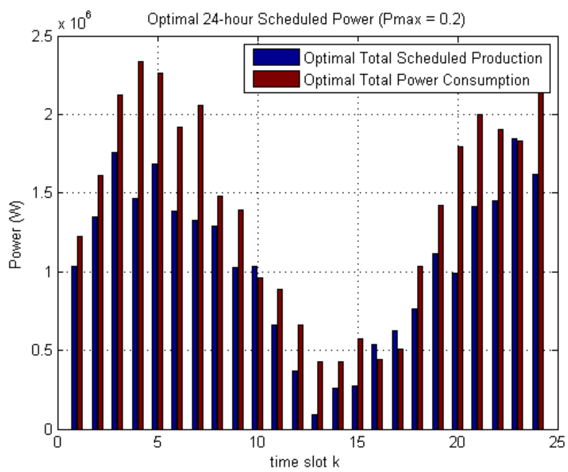

Figure 3.12. 0.5MW Wind Turbine, 0.5MW Solar Array and 150KW Battery in 4MW capacity residential

4. RESULTS

For comparison purposes, we consider a straightforward method, in which the total power consumption and the total power production, which comes from conventional source and the predicted renewable source, are equal, which we refer to as zero error assumption (ZEA). Researchers in the literature measure their method's performance by comparing with ZEA. Similarly, the results from our method PEPL have been compared to this straightforward method, and it has been shown that our method outperforms the ZEA one in terms of total welfare function.

4.1. Renewable Source With Conventional Source

In this section, we present comparisons between our approach PEPL and the ZEA, which indicates that our novel method brings improvement to the system in terms of total welfare function. Note that, the objective function is the same for both methods. The constraint $\sum_{i=1}^{N} x_{i,k} - S_k - \hat{R}_k = 0$ is applied, when the ZEA is in use. We have proven that our method PEPL leads to larger total welfare than that of the ZEA.

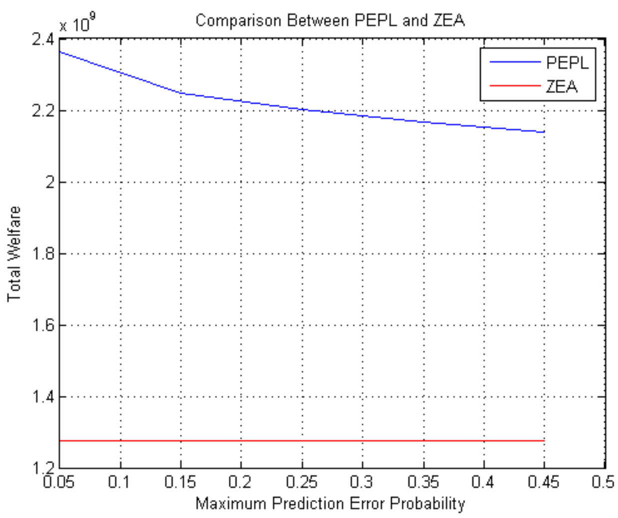

Figure 4.1. 1MW Wind Turbine in 4MW capacity residential

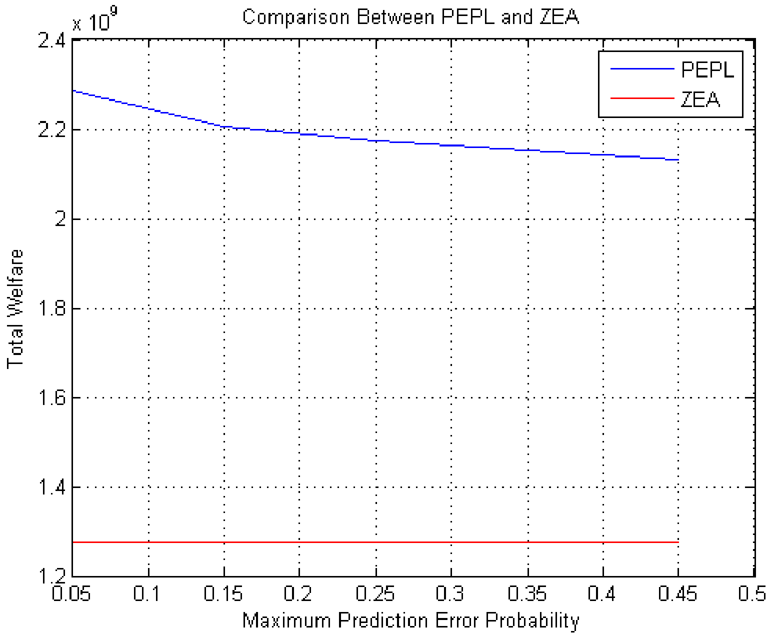

Figure 4.2. 1 MW Solar Array in 4 MW capacity residential

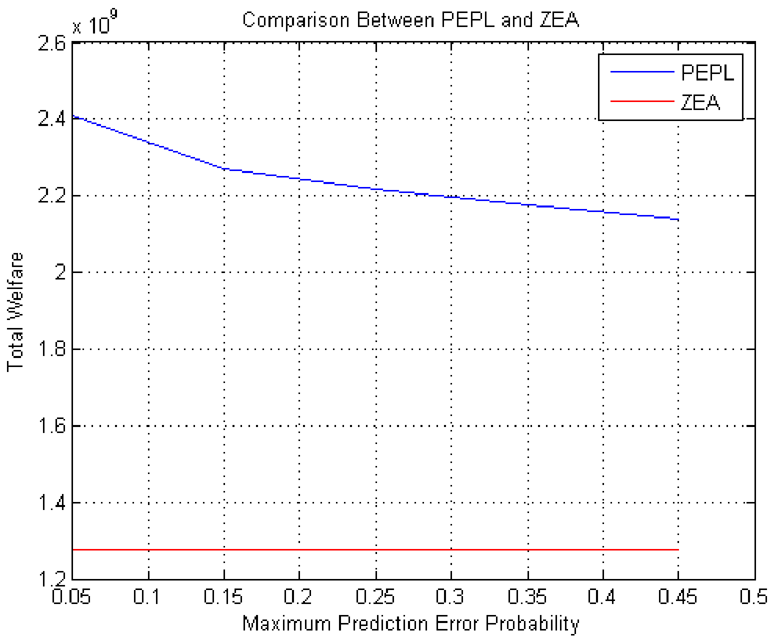

Figure 4.3. 0.5 MW Wind Turbine and 0.5 MW Solar Array in 4 MW capacity residential

4.2. Renewable Source, Battery and Conventional Source

Similar to previous case, the same objective function is employed, however the constraint is modified as $\sum_{i=1}^{N} x_{i,k} - S_k - \hat{R}_k - Z_{k-1} = 0$. It is shown that PEPL results in a greater total welfare than ZEA.

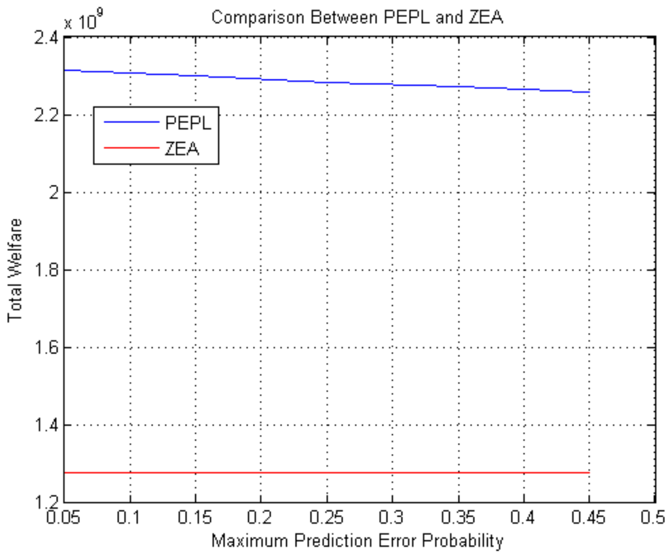

Figure 4.4. 1 MW Wind Turbine 150 kW Battery in 4 MW capacity residential

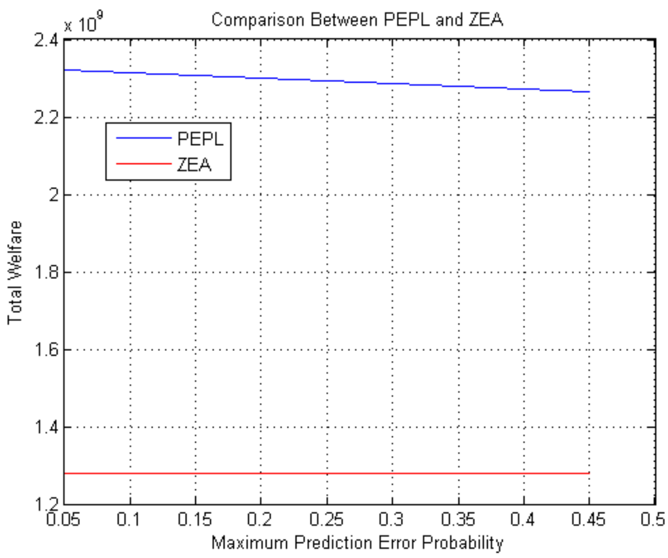

Figure 4.5. 1 MW Solar Array 150 kW Battery in 4 MW capacity residential

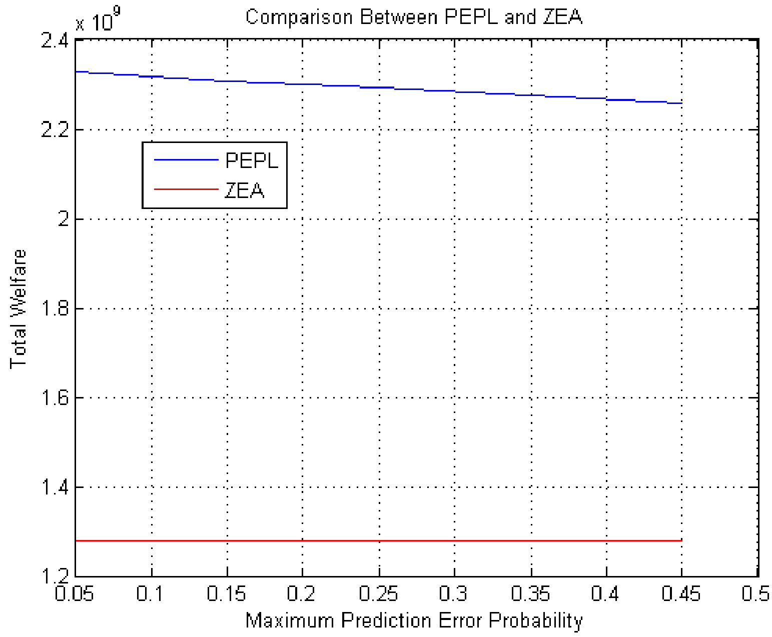

Figure 4.6. 0.5 MW Wind Turbine, 0.5 MW Solar Array and 150 kW Battery in 4MW capacity residential

4.3. Battery Improvement

In this section, it is shown that implementing battery into the grid system increases the total welfare function, compared to the scenario in which there is no integrated battery bank.

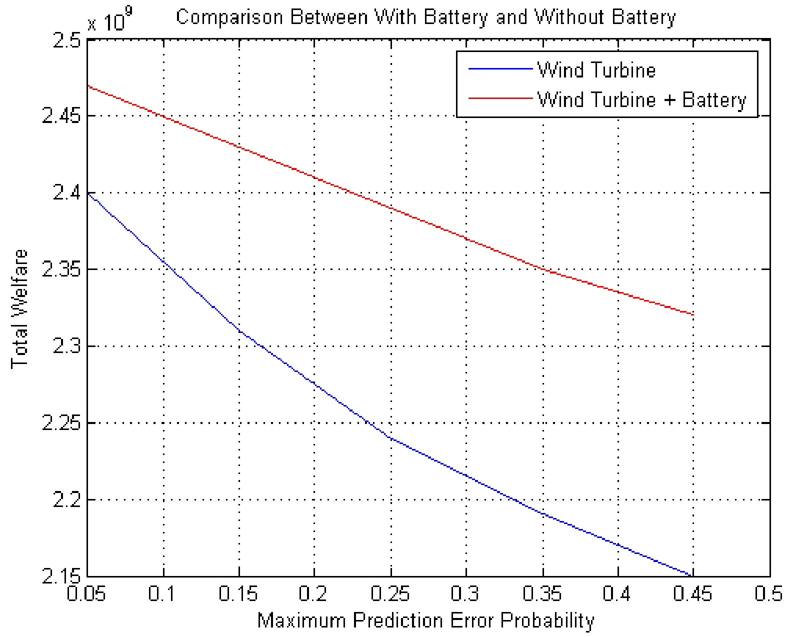

Figure 4.7. 1 MW Wind Turbine and 1 MW Turbine + 150 kW Battery Comparison

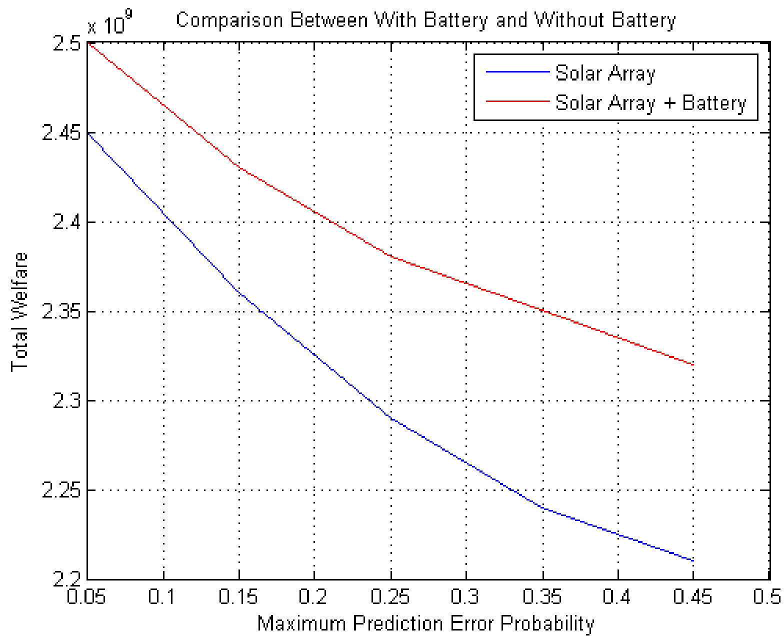

Figure 4.8. 1 MW Solar Array and 1 MW Solar Array + 150 kW Battery Comparison

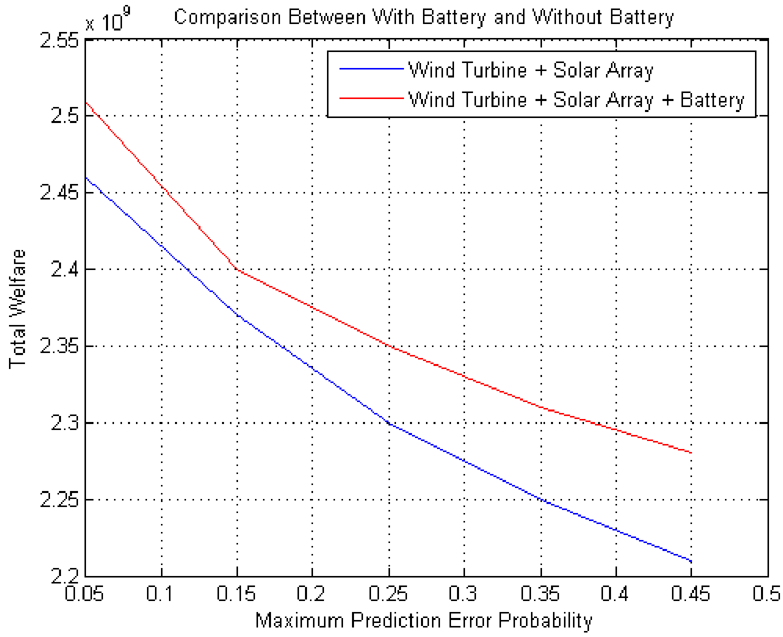

Figure 4.9. 0.5MW Wind Turbine- 0.5MW Solar Array and 0.5MW Wind Turbine-0.5MW Solar Array + 150kW Battery Comparison

5. CONCLUSION

In this thesis, we have studied the behavior of both supply and demand sides in a smart grid system consisting of conventional and renewable energy sources. Our method, which is based on prediction error limitation, has been shown to outperform the method 1, which assumes there is a equality in the amount of consumed power and produced power from conventional energy source and the predicted renewable source. In other words, method 1 assumes that renewable source produces exactly the expected values of its prediction, which is basic due to random nature of renewable sources. To improve, we introduced novel models, which considers the error probability limitation via utilizing probability distributions of the renewable sources. Afterwards, we maximize the total welfare function by calculating optimal consumption and production schedules by limiting error probability. In contrast to several studies in the literature, We have not only considered the deficit case, but also surplus case are studied. Furthermore, ramp constraints are taken into consideration in order to make the problem more realistic. Both analytic and computer simulation results have shown that our method outperforms the method 1 in terms of total welfare function. Furthermore, we have examined another case, in which battery banks are integrated into the grid system to reduce the effects of deficit and surplus, through storing output power. Both analytic and computer simulation results show that integrating battery into the grid system increases the total welfare function. Moreover, it is inferred that our novel method PEPL shows a larger total welfare under battery scenario compared to the method 1 with battery bank. In other words, utilizing our method, penetration rate of renewable energy sources can increases due to the increased total welfare function.

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