INVESTIGATION OF PROSPECTIVE MATHEMATICS TEACHERS' MATHEMATICAL MODELING PROCESSES

by

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ABSTRACT

INVESTIGATION OF PROSPECTIVE MATHEMATICS TEACHERS' MATHEMATICAL MODELING PROCESSES

In this study, prospective mathematics teachers' mathematical modeling processes while they engaged in modeling activities were investigated. The change in their opinions about mathematical model and modeling after they experienced the modeling process was also examined. For that purpose, two modeling activities were implemented to understand how prospective mathematics teachers experience modeling processes, and a pre- and post-questionnaire were applied to find out the change in their opinions. This study was conducted as a phenomenological qualitative study with prospective mathematics teachers. The results showed that some of prospective teachers changed their opinions about mathematical model and modeling after the implementations of the modeling activities. Most of them explained mathematical models by relating real-life and as a visualizing/teaching method whereas they usually explained it by giving examples before the implementations of the modeling activities. Besides, prospective teachers usually explained mathematical modeling by relating real-life while they explained it as a method to visualize/make abstract things more concrete before they engaged in the modeling activities. On the other hand, it was observed that most of the modeling cycle elements were observed throughout the study although most of PTs did not have any experience of modeling activity before this study. All of the groups experienced basically three elements of modeling cycle: formulating a model, computing a solution of the model, and reporting the solution. In addition, some of them experienced also the interpreting the solution, validating the conclusions, and formulating a new or modified model elements of the modeling cycle.

ÖZET

MATEMATİK ÖĞRETMEN ADAYLARININ MATEMATİKSEL MODELLEME SÜREÇLERİNİN İNCELENMESİ

Bu çalışmada matematik öğretmen adaylarının modelleme etkinlikleri üzerinde çalışırken modelleme süreçlerini nasıl deneyimledikleri incelenmiştir. Ayrıca, öğretmen adaylarının matematiksel model ve modelleme hakkındaki düşüncelerindeki değişimler de incelenmiştir. Bu amaçla, modelleme süreçlerinin nasıl deneyimlendiğini anlamak için iki modelleme etkinliği uygulanmıştır. Düşüncelerindeki değişimleri anlamak için de matematiksel model ve modelleme ile ilgili bir ön-test ve son-test uygulanmıştır. Çalışmanın deseni fenomenolojik nitel araştırma çalışması olup öğretmen adayları ile birlikte gerçekleştirilmiştir. Çalışmanın sonuçları bazı öğretmen adaylarının matematiksel model ve modelleme ile ilgili düşüncelerinde değişiklikler olduğunu göstermiştir. Öğretmen adaylarının modelleme etkinlikleri üzerinde çalıştıktan sonra matematiksel modelleri çoğunlukla gerçek hayatla ilişkilendirerek veya görselleştirme ve öğretim yöntemi olarak tanımladıkları, bu etkinliklerden önce ise genellikle örnekler vererek açıkladıkları gözlemlenmiştir. Ayrıca, modelleme etkinliklerinden sonra öğretmen adayları modellemeyi gerçek hayatla ilişkilendirip anlatmışlardır. Bu etkinliklerden önce ise modellemeyi görselleştirme ya da soyut şeyleri somut hale getirme yöntemi olarak açıklamışlardır. Öğretmen adaylarının neredeyse tamamı daha önce bir modelleme etkinliği üzerinde çalışmamış olduklarını belirtmiştir. Buna rağmen, çalışma boyunca modelleme süreci aşamalarının birçoğu gözlemlenmiştir. Tüm grupların model oluşturma, oluşturulan modelin çözümünü hesaplama ve çözümü paylaşma modelleme aşamalarını deneyimledikleri, bazı grupların ise çözümü yorumlama, sonuçları doğrulama ve yeni veya değiştirilmiş bir model oluşturma aşamalarını da deneyimledikleri gözlemlenmiştir.

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LIST OF ACRONYMS/ABBREVIATIONS

CCSSI	Common Core State Standards Initiative
CCSSM	Common Core State Standards for Mathematics
GAIMME	Guidelines for Assessment and Instruction in Mathematical Modeling Education
MoNE	Ministry of National Education
OECD	Organisation for Economic Co-operation and Development
PISA	Programme for International Student Assessment
PT	Prospective Teacher
R	Researcher
T	Teacher

1. INTRODUCTION

Students generally question where and when they can utilize their mathematical knowledge in their daily life. The ability to use mathematics in different contexts and in real-life problem situations can be accepted as an essential requirement in today's world. For that purpose, mathematical models and modeling can be helpful for students to be able to make interpretations about problem situations they have encountered and to find solutions to those problems.

In the literature, there are different definitions of model, mathematical model and mathematical modeling. Lesh and Doerr (2003) defined models as "conceptual systems (consisting of elements, relations, operations, and rules governing interactions) that are expressed using external notation systems" (p. 10). In addition, Greefrath and Vorhölter (2016) explained mathematical model as "representation of the real world, which-although simplified-matches the original and allows the application of mathematics" (p. 9). Sriraman (2005) defined modeling as processes used to model problem situations where the model indicates the outcome of the modeling process. In general, mathematical modeling can be described as a complex process including the application of mathematics to solve real-life problems.

Mathematical modeling provides students rich learning environments where they can use and develop some important mathematical processes such as "constructing, explaining, justifying, predicting, conjecturing, and representing, as well as quantifying, coordinating, and organising data" (English and Watters, 2005). For that purpose, one of the mathematical practices in the Common Core State Standards for Mathematics (CCSSM) states 'Model with mathematics' which shows that CCSSM emphasizes the integration of modeling into the mathematics classes. Mathematical modeling is a competency that students should develop, and when they become mathematically proficient, they can apply their mathematical knowledge to solve various types of problems arising in everyday life, society, and the workplace (Common Core State Standards Initiative (CCSSI), 2010). Moreover, the present Turkish secondary school mathematics education program (Ministry of National Education (MoNE), 2018) and the previous secondary school mathematics education programs (MoNE, 2013; MoNE, 2017) included some objectives suggesting integration of the mathematical modeling activities into the math classes. Mathematical modeling had also been involved as a skill that was aimed to be gained by students in the general purposes of the secondary school mathematics education program published in 2013 (MoNE, 2013).

It is known that, teachers have an important role in students' learning of mathematics and mathematical modeling, (Cai *et al.*, 2014). In addition, experience of mathematical modeling process affects the prospective teachers' (PTs) quality of instruction positively (Anhalt and Cortez, 2016). In this regard, mathematical modeling becomes important for both development of students' important mathematical processes and PTs' quality of instructions in their future classes. Although there have been some studies about integration of mathematical modeling in mathematics education, making PTs and in-service teachers engage in mathematical modeling in order to provide students to experience mathematical modeling process as part of their mathematics education is still needed (Anhalt and Cortez, 2016).

In this regard, the purpose of this research study is to investigate the mathematical modeling processes that PTs experienced when they engaged in the modeling activities. The change in the opinions of PTs about mathematical model and modeling will also be examined. Hence, throughout the study, modeling processes that PTs experienced and their explanations for mathematical model and modeling will be examined.

2. LITERATURE REVIEW

In this section, a review of the literature on particular topics about mathematical modeling in mathematics education will be presented. To address the purposes of the research study, first, definitions of mathematical models and modeling will be discussed in detail. Second, approaches to using mathematical modeling in mathematics education will be explained. Then, characteristics of the mathematical modeling activities will be introduced. Last, studies on mathematical modeling in teacher education will be presented.

2.1. Mathematical Models and Modeling

In everyday language, the word 'model' has been used in many different ways. It can be used for the synonymous of the word 'representation' as well as for the synonymous of the word 'miniature'. In mathematics education literature, it has also various definitions. For instance, Swetz and Hartzler (1991) explained model in two ways; model in the physical sense and model in the theoretical sense. They defined model in the physical sense as "a replication, usually scaled down, of an object", and theoretical model of an object/phenomenon as "a set of rules or laws that accurately representing that object/phenomenon in the mind of an observer" (p.1). On the other hand, Lesh and Doerr (2003) defined models as "conceptual systems (consisting of elements, relations, operations, and rules governing interactions) that are expressed using external notation systems, and that are used to construct, describe, or explain the behaviors of other system(s)" (p.10).

Swetz and Hartzler (1991) defined a mathematical model as "a mathematical structure that approximates the features of a phenomenon" (p.1). According to Lesh and Doerr (2003) "mathematical model focuses on structural characteristics (rather than, for example, physical or musical characteristics) of the relevant systems" (p.10). On the other hand, Cheng (2001) defined mathematical a model as "a simplification or abstraction of a (complex) real world problem or situation into a mathematical form, thereby converting the real-world problem into a mathematical problem" (p.64).

The terms mathematical model and mathematical modeling are different from each other. Sriraman (2005) explained this difference by referring the difference between process and product. In Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME) report (2016), mathematical modeling was explained as "a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena" (p.8). Niss, Blum, and Galbraith (2007) stated that "the term "real world" is often used to describe the world outside mathematics, even though, say, quantum physics or orbitals in chemistry may appear less than real to some refer to the daily life and other disciplines" (p. 3). Moreover, mathematical modeling involves mathematization which was defined as "making symbolic descriptions of meaningful situations" (Lesh *et al.*, 2000, p.594). Mathematizing was also defined as fundamental mathematical activities involved in transforming a real-life problem to a mathematical form such as structuring, conceptualizing, making assumptions, formulating a model, interpreting or evaluating a mathematical outcome in relation to the original problem in PISA draft mathematics framework (OECD, 2015). Hence, it can be understood that mathematization is a key process at the center of the mathematical modeling process.

In literature, there are various definitions of modeling process based on different modeling approaches, different research purposes, different school aims and different modeling activities (Kaiser *et al.*, 2006; Borromeo-Ferri, 2006). According to Swetz and Hartzler (1991) mathematical modeling is a process consisting of four basic stages. First stage includes observing of a phenomenon, defining a problem situation in that phenomenon, and determining the key factors affecting the problem. Second stage consists establishing the relationships among factors and interpreting them mathematically to form a model. Third stage is analyzing the model mathematically. The last stage involves getting results and reinterpreting them considering the original problem situation and drawing conclusions (Swetz and Hartzler, 1991). Similarly, modeling process is represented as a four-step cyclic process involving description, manipulation, translation, and verification in the following Figure 2.1 (Lesh and Doerr, 2003). Description is mapping from the real world to the model world, manipulation includes working on the model to make predictions regarding the initial problem situation, translation (or prediction) is about taking related results back into the real world, and verification is regarding the usefulness of predictions (Lesh and Doerr, 2003). On the other hand, when this modeling cycle is compared with the modeling cycle provided by Swetz and Hartzler (1991), it can be observed that they have similar stages. For instance; the second step in the modeling cycle of Lesh and Doerr (2003), namely, manipulation includes similar process with the third stage of the modeling cycle of Swetz and Hartzler (1991) which is analyzing the model mathematically.

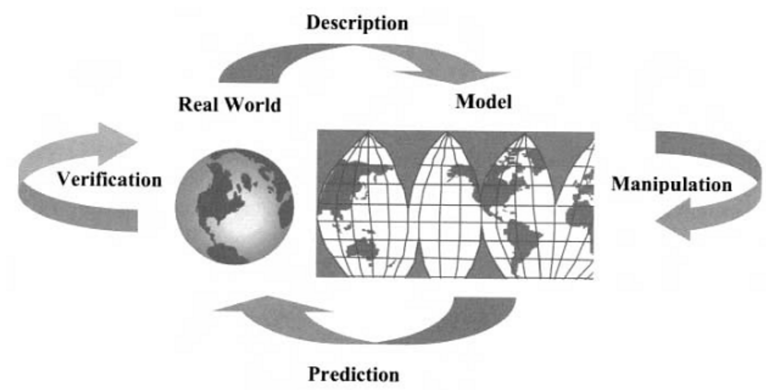


Figure 2.1. Modeling cycles often involve four basic steps (Lesh and Doerr, 2003, p. 17).

Lesh and Doerr (2003) also explained the cyclic nature of the modeling process by sequences of iterative cycles as shown in Figure 2.1. Here, iterative cycles indicate the revision or rejection of descriptions, explanations, and predictions according to the feedback from the trial testing results. Thus, problem solvers can obtain many levels and various types of responses and they can question the relevant usefulness of the other ways of thinking (Lesh and Doerr, 2003).

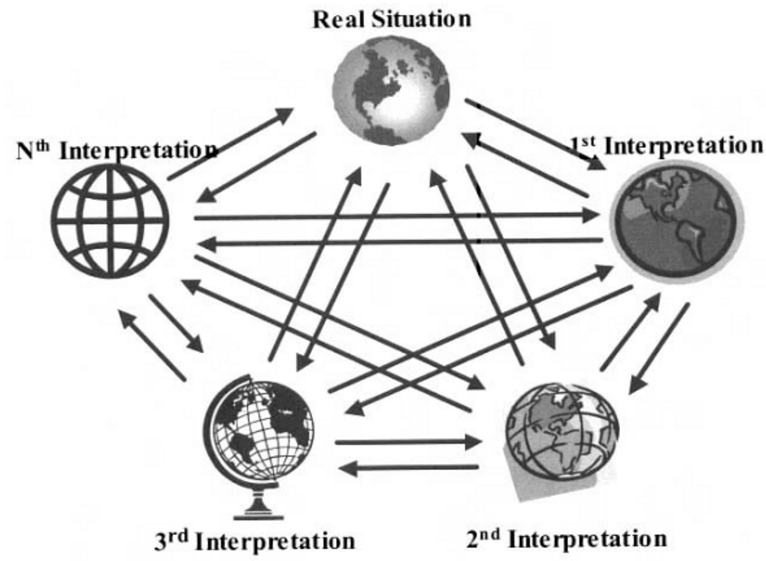


Figure 2.2. Modeling may involve sequences of iterative cycles (Lesh and Doerr, 2003, p. 18).

Blum and Leiss (2007) also explained modeling process as a cyclic process as shown in Figure 2.3. The modeling process starts with understanding of the problem situation and creating a situation model. By making the situation more simplified and definite, a real model of the given problem situation is constructed. Then, this real model is transformed into a mathematical model. After the appropriate mathematical operations performed on the mathematical model, some mathematical results are obtained. After that, those results are interpreted and validated considering them as real results in the real-world conditions. It should be noted that, the cycle might be repeated several times until a final answer for the initial problem situation is found and exposed.

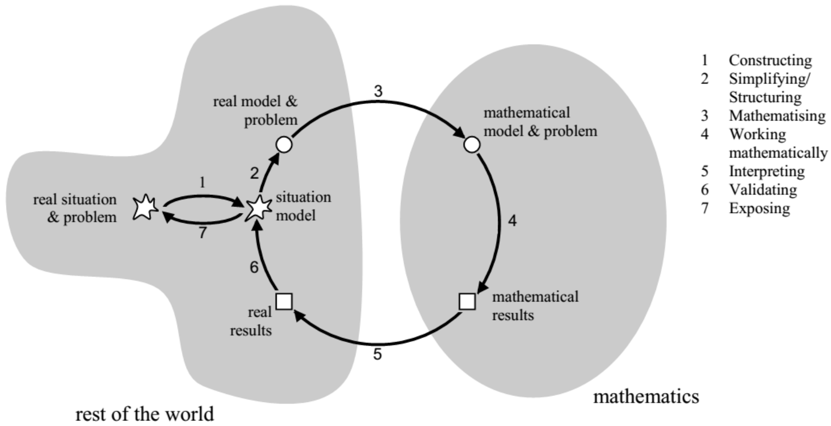


Figure 2.3. Modeling cycle (Blum and Leiss, 2007, p. 225).

CCSSI (2010) defined modeling in the CCSSM as "the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions" (p. 72). The basic modeling cycle in CCSSM was represented in Figure (CCSSI, 2010). First, the variables and the important factors in the problem situation are determined. Second, a model is developed by using various types of representations to demonstrate the relationship between the variables. Third, ne 2.4 cessary operations including mathematical operations are applied on those relationships to get some results. Then, mathematical results are interpreted regarding the original problem situation. Last, the conclusions are validated by comparing the results and the original problem situation. Based on these comparisons, if the model is acceptable, the conclusions are reported. If it is not, then the model is improved or a new model is constructed which requires repetition of the modeling cycle until getting an appropriate model to answer the original problem situation.

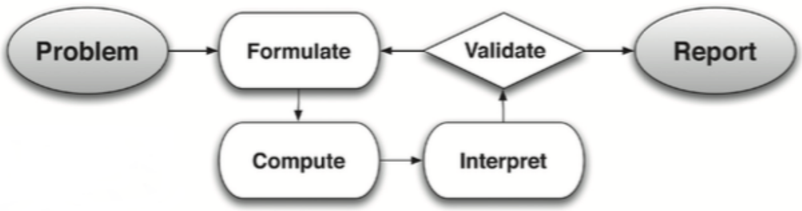


Figure 2.4. The basic modeling cycle (CCSSI, 2010, p. 72).

Furthermore, using mathematical models and involvement of mathematical modeling in mathematics instructions have also been emphasized in the current and the previous Turkish secondary school mathematics education programs (MoNE, 2013; MoNE, 2017; MoNE, 2018). Mathematical modeling was defined as a dynamic method which provides us to understand easily the relationships in the nature of the problems in all fields of life, to state mathematically, to classify, to generalize, and to draw conclusions (MoNE, 2013). The modeling cycle offered by the Turkish secondary school mathematics education program was presented in the following Figure 2.5. According to this cycle, the modeling process starts with a real-life problem and continues with the translation of the given problem situation into the mathematical form. After mathematization of the problem, it is solved mathematically. At the end, the obtained results are interpreted considering the real-life situation.

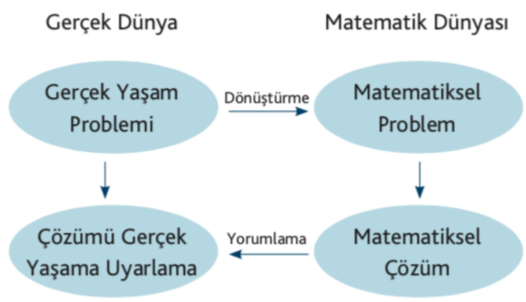


Figure 2.5. Mathematical modeling process is a cyclic process (MoNE, 2013, p. V).

In the Programme for International Student Assessment (PISA) (OECD, 2015), mathematical modeling was emphasized with mathematical literacy. Mathematical literacy was defined as an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts (OECD, 2015). Since mathematical literacy includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena, it can be understood that it was intended to integrate the notion of mathematical modeling. As it was represented in the following figure, mathematical modeling cycle consists of four basic processes: formulating, employing, interpreting, and evaluating. First, the problem solver tries to determine the related mathematics in the problem situation and formulates the situation mathematically. Then, problem solver employs mathematical concepts, procedures, and tools to get mathematical results. After getting the mathematical results, those results are interpreted considering the original problem and this involves the problem solver interpreting, applying, and evaluating mathematical results and their reasonableness in the context of a real-world-based problem.

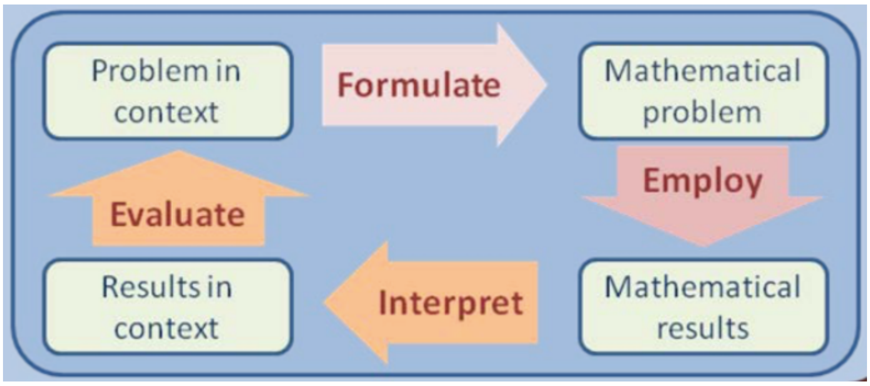


Figure 2.6. The math modeling cycle (OECD, 2015, p. 7).

In summary, there are different definitions of mathematical modeling in the related literature and there is a consensus on the cyclic nature of the modeling process. It can be also observed that, in most of the definitions, modeling process starts with the identification of the problem situation and continues with simplification. After simplification, the problem is interpreted mathematically and proper manipulations are done to get mathematical results. Then, process continues with interpretation of the mathematical results concerning the original problem situation. Last, the solution is validated regarding the original problem situation. Here, it is important to indicate that in the case of the acceptable solution, the result is reported. On the other hand, when the solution is unacceptable, the model is improved or a new model is developed by repeating the modeling process which shows the cyclic nature of the modeling process.

2.2. Approaches to Using Mathematical Modeling in Mathematics Education

There are different approaches to mathematical modeling in the literature. Kaiser and Sriraman (2006) expressed this situation as there is no homogeneous understanding about modeling within the international discussion on modeling. Julie and Mudaly (2007) also explained the existing discussion on how to use mathematical modeling in teaching. The discussion has been based on using mathematical modeling as "a vehicle for the development of mathematics or treated as content in and of itself" (Julie and Mudaly, 2007, p. 504). Using mathematical modeling as a vehicle requires to use mathematics in some contexts with the aim of providing support for the learning of mathematical concepts and procedures (Julie and Mudaly, 2007). On the other hand, using mathematical modeling as content requires to formulate "mathematical models of natural and social phenomena without the prescription that certain mathematical concepts, procedures or the like should be the outcome of the model-building process" (Julie and Mudaly, 2007, p. 504). However, these two approaches should not be seen as antagonistic since new mathematical content may be needed in solving authentic problems, whereas real-world contexts can provide appropriate tools for the needed mathematics (Galbraith, 2012, p.5).

Furthermore, Kaiser and Sriraman (2006) made a different classification about the perspectives on modeling based on the central aims in using mathematical modeling. According to this classification, there are six different perspectives on modeling, namely, realistic/applied modeling, contextual modeling, educational modeling, socio-critical modeling, epistemological modeling, and cognitive modeling (Kaiser and Sriraman, 2006). Briefly, realistic modeling emphasizes the ability of learners to understand the world and solve real world problems by using mathematics; contextual modeling considers subject-related and psychological goals and supports solving word problems; educational modeling emphasizes pedagogical and subject related goals such as constructing learning processes, and developing concepts; cognitive modeling examines the cognitive processes occurring in the modeling process; socio-critical modeling perspective draws attention to the role of mathematics in society, and the function of modeling in society; and epistemological modeling emphasizes generating relations between mathematics and reality, and developing mathematical theory as a part of the mathematization process.

2.3. Characteristics of the Mathematical Modeling Activities

In mathematics education, it is important to provide students learning environments in which they can use both of their mathematical thinking ability and also the computational ability. In this regard, problem-solving activities named as modeling activities in which students produce “sharable, manipulatable, modifiable, and reusable conceptual tools (e.g., models) for constructing, describing, explaining, manipulating, predicting, or controlling mathematically significant systems” (p. 3) can be helpful in teaching mathematics (Lesh and Doerr, 2003).

There is a distinction between the traditional textbook word problems and the modeling activities. In the former one, using some procedures including preestablished constructs to process information is emphasized, while in the latter one, processing constructs is emphasized (Lesh and Doerr, 2003). Students themselves interpret situations mathematically, in other words they mathematize the situations while they engaged in modeling activities. Thus, modeling activities make students to study on “the structural and systemic characteristics of the “thing” that is being designed, constructed, or modeled” (Lesh and Zawojewski, 2007, p. 784). In other words, modeling activities require students to produce models which are complex, reusable and sharable tools in other circumstances for a given situation, instead of to produce brief mathematical answers for questions including pre-mathematized situations (Lesh and Zawojewski, 2007).

Lesh *et al.* (2000) described six principles to design useful modeling activities; a) the model construction principle, b) the reality principle, c) the self-assessment principle, d) the construct documentation principle, e) the construct share-ability and reusability principle, f) the effective prototype principle. Briefly, according to the model construction principle, the activity should involve “development of an explicit construction, description, explanation, or justified prediction" (Lesh *et al.*, 2000, p. 609). In the reality principle, providing students to understand the situation on the basis of their own knowledge is emphasized. In addition, the self-assessment principle suggests that modeling activities should make students to ask themselves whether their solutions need to be revised or not (Lesh *et al.*, 2000). The construct documentation principle, necessitates students to show how they reason about to solve the problem situation. Furthermore, construct share-ability and reusability principle emphasizes making students to be able to create "sharable, transportable, easily modifiable, and reusable" models (Lesh *et al.*, 2000, p.625). The effective prototype principle aims to provide students to produce useful prototype for also interpretation of other similar situations.

In summary, modeling activities are similar to many real-life situations in which mathematics is useful and students make symbolic descriptions of meaningful situations rather than make meaning of symbolically described situations (Lesh and Doerr, 2003). In other words, modeling activities involve problems which require problem solvers deal with real-life problems that they use mathematizing to solve the given problem situation. When problem solvers engaged in modeling activities, they produce answers that go beyond short answers to narrowly specified questions, and they use their mathematical skills that go beyond the computational skills which are emphasized in the traditional textbook word problems (Lesh and Doerr, 2003).

2.4. Mathematical Modeling and Teacher Education

Nowadays, learning to respond the needs of the 21st century and to attain this century's valued skills has become more important than before. Thus, integration of skills like "critical thinking, problem solving, communication, collaboration, creativity, and innovation" (p. 42) in every child's education life should be emphasized (Kay and Greenhill, 2011). Preparing students to have that kind of skills demanded by this century and to respond the needs of this century has been increasingly needed. In this context, using mathematical modeling problems in mathematics instructions can be helpful for students. In mathematical modeling problems different understandings and abilities are emphasized than those on traditional textbook word problems (Lesh and Lehrer, 2003). In other words, during the model development process, students are needed to mathematize which often involves quantifying, organizing, categorizing, and dimensionalizing the relations and patterns given in the problem situation (Lesh and Harel, 2003). Moreover, at the end of the model development process, models that are “sharable (with other people), reusable (in other situations) and modifiable (for other purposes)” (p. 160) are aimed to be produced (Lesh and Harel, 2003). Thus, modeling can be considered as a useful method to bring characteristics of 21st-century problems into the mathematics classroom (English and Gainsburg, 2016). In this context, integrating mathematical modeling to mathematics classrooms has been important for students, teachers and also PTs.

Daher and Shahbari (2015) examined the modeling processes of the PTs when they engaged in the modeling activities from a cognitive perspective. They observed three different cycles of modeling processes from a cognitive perspective and in all of these cycles PTs had discussions on necessary mathematics to build a mathematical model as well as appropriate technology for the model development. It was found that modeling activities support students to have mathematical discussions and communication which can provide students to have rich mathematical learning environments (Daher and Shahbari, 2015).

Moreover, in Turkey, there are also some research studies on mathematical modeling and teacher education. For instance, Kertil (2008) examined the prospective mathematics teachers’ ability of using mathematics and problem solving in modeling activities. In this study, he found that PTs had many difficulties in some phases of modeling process. Hence, he indicated that it can be argued that the modeling skills of PTs are inadequate and they cannot use their mathematical knowledge to interpret the real-life situations (Kertil, 2008). In other study, Türker, Sağlam, and Umay (2010) focused on the performances of prospective mathematics teachers during the mathematical modeling process and their views about modeling process. After the implementation of four modeling activities and semi structured interviews, it was observed that PTs have difficulty in solving real-life problems that PTs explained these problems as having multi-variety, ill-defined structures rather than having simple structures where what is given and wanted are clearly expressed (Türker *et al.*, 2010).

Furthermore, Bukova-Güzel (2011) investigated the approaches of prospective mathematics teachers in their experiences of constructing mathematical modeling problems and the extent to which they perform the modeling process when solving those problems in the mathematical modeling course. It was found that PTs considered various criteria when constructing modeling problems such as relevance to life, and the possibility of being solved by mathematical knowledge (Bukova-Güzel, 2011). It was also observed that PTs had difficulties in the interpreting and validating steps of modeling process. Based on this, Bukova-Güzel (2011) suggested that PTs need to be provided proper learning environments in which they can work on modeling problems, have discussions, and learn how to adjust mathematical results to real life. Korkmaz (2014) also studied on the evolution of prospective mathematics teachers' thinking about mathematical modeling and pedagogical knowledge of mathematical modeling throughout a modeling course. At the end of the study, it was found that PTs developed positive views about mathematical modeling and use of mathematical modeling activities in the classroom, and participating in a modeling course provided PTs to have the knowledge and skills needed for teaching mathematics by using mathematical modeling (Korkmaz, 2014).

On the other hand, Tekin-Dede (2019) investigated the arguments constructed by PTs during the mathematical modeling process regarding their modeling processes. In this research study, it was found that the PTs made different claims based on their assumptions proper to real life and they tried to reach solutions by justifying these claims. In addition, examining the arguments in the modeling process can provide mathematics educators an in-depth understanding of the process of students' construction of claims and justifications (Tekin-Dede, 2019). In their research study, Didiş, Erbaş, Çetinkaya, Çakıroğlu and Alacacı (2016) investigated the prospective mathematics teachers' interpretations of students' thinking established when they engaged in mathematical modeling activities. Didiş *et al.* (2016) found that getting PTs engaged in analyzing students' work on modeling activities would provide a valuable medium for mathematics teacher educators to understand PTs' ways of interpretations. Also, providing documentation of instructional materials from real classrooms would be helpful to prepare PTs for understanding and interpreting students' ways of thinking in their future instructions (Didiş *et al.*, 2016).

Besides, there are some research studies on mathematical modeling and teacher development. For instance, in their research study, Doerr and Lesh (2003) focused on providing teachers more effective ways of understanding and interpreting the complexities of teaching and learning by using modeling perspective. They found that since teachers have different skills, views and interpretations of rich contexts for teaching and learning, teachers are need to be provided opportunities for teachers' learning and for addressing this variability and its development (Doerr and Lesh, 2003). Moreover, in their research study, Schorr and Lesh (2003) studied with teachers and used thought revealing activities which are accepted as mathematical modeling activities. At the end of their study, it was found that teachers have opportunity to generate new ideas about teaching and learning mathematics because the activities used in this study make use of thought-revealing activities for students, where the students' works become source for teachers' development (Schorr and Lesh, 2003).

In summary, mathematical modeling has been important for teaching and learning mathematics. It offers opportunities for teachers to improve themselves as well as students (Lesh and Doerr, 2003). With modeling approach, the teacher will have to enforce the boundaries of her/his own already constructed mental models to develop different solution methods, and to evaluate models constructed by students (Kertil, 2008). Thus, teachers, and PTs as future teachers, can have various types of models which are useful for integration of mathematical modeling activities into mathematics instructions, and helpful for preparing students to have the 21^{st} century's valued skills and to respond the needs of this century.

3. SIGNIFICANCE OF THE STUDY

Mathematical modeling process involves four basic stages: observing a phenomenon and recognizing the important factors; conjecturing the relationships among factors and interpreting them mathematically; using proper mathematical analysis to the model; finding results and reinterpreting them in the context of the phenomenon (Swetz and Hartzler, 1991). This process provides various opportunities to associate and use ideas from different areas (Cheng, 2001). In addition, since modeling problems don't include a road map or an example from a text or instruction notes, students encounter the challenge of decision making at every stage in the modeling process (GAIMME, 2016). Therefore, students evolve mature levels of thinking when they are developing more sharable, more transportable, and more reusable cognitive tools (Lesh and Harel, 2003). In brief, mathematical modeling problems provide a rich learning environment for students to develop powerful mathematical ideas independently (Doerr and English, 2003).

Mathematical modeling offers students to develop a set of transferable skills which has the potential to be more effective on their futures than teaching them a specific set of mathematical skills for example college algebra (GAIMME, 2016). In other words, experiencing mathematical modeling process and facing with real-life problems can enable students develop skills which can be used in other real-life problem situations. Moreover, using modeling activities for students which are also called thought-revealing activities helps providing contexts for thought-revealing activities for teachers (Lesh and Lehrer, 2003). The models and modeling perspective is important for teachers as well as for students since teachers implement these activities in the classroom settings. In this context, it was implied that modeling activities enables teachers to obtain some helpful tools to understand the ways of thinking of students, to identify students' conceptual strengths and weaknesses, and to assess various products of students (Lesh and Lehrer, 2003; Lesh *et al.*, 2000; English and Gainsburg, 2016). Besides, in the report of GAIMME (2016), it was highlighted that by engaging in modeling activities students can improve their confidence and perseverance such that they are equipped better for challenges in their real-life. In this context, it can be indicated that integration of mathematical modeling in the mathematics courses in all grade levels is necessary. In many research studies on mathematical modeling with PTs, it was also suggested that there is a need to get PTs engaged in mathematical modeling activities to facilitate development of their modeling skills and integration of modeling activities into their future mathematics instructions (Anhalt and Cortez, 2016; Bukova-Güzel, 2011; Maaß, 2006; Şen-Zeytun, 2013; Türker *et al.*, 2010). Thus, integration of mathematical modeling activities into the teacher education programs is also needed and becomes important since PTs will implement the modeling activities in their future instructions when they become teachers.

The purpose of this study is to investigate the mathematical modeling processes that PTs experience when they engaged in the modeling activities, and the opinions of PTs about mathematical model and modeling. In other words, providing PTs to experience the modeling process when they engage in modeling activities and understanding their opinions about mathematical model and modeling were emphasized in this study. Also, one can argue that if we want to change something in education, we can support that change starting to study from the prospective teachers. Within this context, studying with PTs is important and this study attempts to make contributions to PTs to involve in modeling process and develop needed modeling skills which can be helpful for the improvement of their future mathematics instructions. The importance of this study is because of the significant contributions of the mathematical modeling approach to PTs and the possibility that they will use mathematical modeling activities in their future mathematics instructions.

4. STATEMENT OF THE PROBLEM

In Turkish mathematics education programs of elementary, middle school, and high school level include some learning goals about use of mathematical modeling to make students experience mathematical modeling process. It can be observed that providing students to experience mathematical modeling processes requires implementations of mathematical modeling activities in mathematics instructions which necessitates mathematical modeling. Therefore, this study aims to examine the mathematical modeling processes of PTs when they engage in modeling activities and provide them to experience the mathematical modeling processes. In addition, another purpose of this study is to investigate the change in the opinions of PTs about mathematical model and modeling.

4.1. Research Questions

The research questions of the present study include examination of the mathematical modeling process that PTs experience and their responses to explain mathematical model and modeling.

There are two main research questions addressed in the current study:

(i) What are the prospective teachers' opinions about mathematical models and mathematical modeling before and after their experience of mathematical modeling processes?

(ii) How do mathematical modeling processes emerge while prospective teachers are engaged in mathematical modeling activities?

5. METHOD

The purpose of this study is to investigate the mathematical modeling processes that PTs experience when they engaged in modeling activities, and the opinions of PTs about mathematical model and modeling. In this section, the methodology of the research study will be shared. First, the design of the study will be presented. After that, participants of the study, data collection, procedure, and data analysis will be explained, respectively.

5.1. Design of the Study

In this study, understanding the mathematical modeling processes that PTs experienced when they were engaged in the modeling activities was aimed to be examined in detail. In addition, uncovering the opinions of the PTs about mathematical model and modeling before and after their experience was aimed to be investigated. The basic purpose of the qualitative research is to understand how people make sense out of their lives, describe the process rather than the outcome of meaning-making, and explain how people interpret what they experience (Merriam and Tisdell, 2016). This study was conducted as a phenomenological qualitative study to get a deep understanding about the modeling processes that PTs experienced, and how their opinions about mathematical model and modeling were shaped through this experience. In addition, according to Merriam and Tisdell (2016), phenomenological study seeks understanding about the essence and the underlying structure of the phenomenon, and the task of the phenomenologists is to depict the essence or basic structure of the experience. Thus, in this study, mathematical modeling process that PTs experienced is the phenomenon which was investigated in detail. Moreover, in qualitative studies, data are collected through interviews, observations, or document analysis which help to uncover meaning, develop understanding, and discover insights relevant to the research problem (Merriam and Tisdell, 2016). In this study, the opinions of PTs were examined in detail through their answers to the pre- and post-questionnaires. The videos of group studies in which they studied by thinking aloud, the transcriptions of that videotaped group studies, their individual and group solution sheets including their solutions for the modeling problems, and the classroom observations were used for the detailed examination of their experiences of mathematical modeling processes.

5.2. Participants

In this research study, the mathematical modeling processes that PTs experienced during the implementation of the modeling activities, and their opinions about the mathematical model and modeling were aimed to be examined in detail. Based on the purpose of the study, purposive sampling method was used for the selection of the participants. Purposeful sampling is "based on the assumption that the investigator wants to discover, understand, and gain insight and therefore must select a sample from which the most can be learned" (Merriam and Tisdell, 2016, p. 96). Hence, selecting participants who "will yield the most relevant and plentiful data" (Yin, 2011, p.88) was necessary to understand the modeling processes that PTs experienced during the modeling activities, and the opinions of PTs about mathematical model and modeling. Because of these reasons, prospective mathematics teachers studying in the secondary school mathematics education program of a state university in İstanbul were selected as participants of this study. Participants enrolled on a departmental content course, namely 'Teaching Algebra and Pre-Calculus for Secondary School'. This study was conducted in two sessions of this course (one session lasts approximately two hours). The purpose of this course was studying selected high school pre-calculus topics and related concepts in depth to teach meaningfully and to discuss possible teaching strategies. Several secondary school mathematics topics for teaching were included such as real numbers, complex numbers and functions (linear, polynomial, exponential, rational and trigonometric functions). It can be understood that this course enables PTs to reflect on the relationship between the advanced mathematics courses that they take in university and the mathematics that is taught in high school. Besides, mathematical modeling activities include real-life problem situations which often require to consider relationships between variables. It might be more understandable that why this course was suitable to conduct this research study when the characteristics of the mathematical modeling activities were considered together with the content of this course. It was also presumed that most of the prospective teachers had not been engaged in modeling activities before this study since this course was one of their first departmental courses in their curriculum.

At the beginning of the study, PTs were informed about the research study, and their consents were taken to participate in this study. Also, the researcher made assurances about the confidentiality. In the classroom, there were 13 PTs and 12 PTs during the implementations of the first and second modeling activities, respectively. Eight of them volunteered to be videotaped while they were studying on the modeling activities. In addition, the PTs who did not volunteer to be videotaped also studied on the modeling activities but the data obtained from these PTs were not use in data analysis. It was also observed that seven of that eight PTs had not participated any modeling activities before this study. Only one of them indicated that she had been engaged in a modeling activity before and she formed a model during that activity. Thus, the data obtained from those eight PTs were used in this research study.

5.3. Data Collection

During the data collection process, various data sources were used to answer the research questions of the study. To collect detailed and comprehensive data, a pre-questionnaire (Appendix A), and the modeling activities (Appendices B, and C) were implemented. At the end of the study, a post-questionnaire (Appendix D) was implemented.

The pre-questionnaire was applied to get the descriptive information about PTs and their initial opinions about mathematical model and modeling before their experience of the modeling activities. It consisted of three questions which were prepared on the purpose of the study. The latest version of this questionnaire was obtained based on the findings of the pilot study. First question required PTs to explain what they know about mathematical model. Similarly, second question asked PTs to explain what they understand from the phrase ‘mathematical modeling’. The last question was asked to get information about whether PTs experienced mathematical modeling process before or not.

The modeling activities implemented during this study were the "Lost Cell Phone" modeling activity (Appendix B) in the study of Anhalt and Cortez (2016) and the "Summer Reading Program" modeling activity (Appendix C) adapted from the study of English and Fox (2005). During the selection of modeling activities, the following criteria were considered:

(i) The mathematical concepts/ideas included in the activities.
(ii) Meaningfulness of the activities (Lesh *et al.*, 2000).
(iii) The number of the assumptions needed to solve the modeling problem, the number of the proper solution methods to solve the modeling problem, and the number of the solutions at the end of the problem-solving process.

Accordingly, modeling activities which enable PTs to use their elementary or high school mathematics knowledge were selected because lack of content knowledge can make difficult to examine the modeling process that PTs experience. In the second criterion, it was emphasized that PTs can make sense of the problem situation based on their personal knowledge and real-life experiences (Lesh *et al.*, 2000). In addition, third criterion put emphasis on making various assumptions to reach a solution, or using multiple solution ways to find a solution, or getting different solutions based on different assumptions.

In this regard, the "Lost Cell Phone" modeling activity was selected and posed as its original version used in the research study of Anhalt and Cortez (2016). In this activity, participants were expected to find the lost cell phone provided with the knowledge of locations of the cell phone towers detecting the signal. On the other hand, this modeling activity was chosen since it provides using different solution methods, and making different assumptions to solve the problem. Also, the problem situation given in the activity was meaningful to PTs because anyone can encounter to this kind of problem situation in real-life. Moreover, the mathematical concepts and ideas which are needed to solve the problem can be listed as system of equations, and basic geometry knowledge.

In addition, the "Summer Reading Program" modeling activity (English and Fox, 2005) was applied as a second modeling activity. The same scenario in the original problem was used with an additional attachment of a list of sample collection of approved books. In this activity, participants were expected to develop a model to assign points to each student for reading the books and writing reports about the books during the summer reading program and they were expected to write a letter which explains their models. On the other hand, this activity was meaningful for PTs because they can encounter this kind of problem situations in real-life such as in their future classes. Also, this modeling activity requires using various mathematical concepts and ideas. For example, ratio, correspondence, sum, range, and functions are the mathematical concepts and ideas included in this modeling activity (Daher and Shahbari, 2015). It also enabled PTs to make various assumptions, use multiple solution methods, and find various different solutions based on their different assumptions.

The post-questionnaire was applied to determine the PTs' final opinions about mathematical model and modeling after the experience of the modeling process. The post-questionnaire included four questions which were prepared in accordance with the purpose of the study. The first and second questions of the post-questionnaire were the same questions as in the pre-questionnaire asking PTs' opinions about the mathematical model and modeling. In this way, the answers to the first and second questions of the both questionnaires were compared to understand whether there was a change in opinions of PTs about mathematical model and modeling, or not. The third question asked participants to explain some of the characteristics of the mathematical modeling problems in the activities that they worked on. In the fourth question, participants were asked about implementation of the modeling activities in the mathematics classes. Also, the last question asked PTs to explain whether they will use modeling activities in their future mathematics classes or not.

5.4. Procedure

This research study was conducted as a part of the course 'Teaching Algebra and Pre-Calculus for Secondary School' offered for high school mathematics teacher education program of a state university in İstanbul in Spring 2018. Prior to the main research study, in Spring 2017 and Fall 2017, a pilot study was conducted to enhance the design of the study. In this part, first the pilot study, after that data collection procedure of the main research study will be explained in detail.

5.4.1. Pilot Study

The pilot study was conducted in Spring 2017 and Fall 2017. It was consisted of implementations of three modeling activities. The participants were prospective secondary mathematics and physics teachers, and prospective primary mathematics teacher from the science and mathematics teacher education program of a state university in Istanbul. First, in Spring 2017, two of the modeling activities of pilot study was implemented in two weeks. Participants were attending to the course named 'Teaching Algebra and Pre-Calculus for Secondary School'. Two activities were implemented in two sessions of that course. Each session lasted approximately two hours. Also, 26 PTs attended to each session. However, in total, 23 PTs attended to the both sessions of the pilot study.

In the first day of the pilot study, there were five groups including five or six PTs and in the second day, there were six groups including four or five PTs. The data collection process for the first day of the pilot study was consisting of implementation of the pre-questionnaire, the first modeling activity, and class discussion about the group solutions. The second day was consisting of implementation of the second modeling activity, class discussion on the group solutions, implementation of the post-questionnaire, and class discussion about mathematical modeling. In addition, there was a final project in which PTs develop their own modeling activities. Considering the observations about the second modeling activity implementation, third modeling activity, "The Sunken Treasure" developed by Gould *et al.* (2012), was implemented at the end of Fall 2017 in an elective course named ‘Mathematical Thinking’. There were 13 PTs participating in the implementation of last modeling activity and four of them had participated in the previous implementation of modeling activities. They were formed into three groups of four or five PTs. Following that, each group started to study on the problem and tried to develop an initial plan for the solution of the problem. Then, groups continued to develop their final solution. Last, when they finished the group work, each group shared their group solution with other groups.

After the pilot study, the pre-questionnaire was revised, and the plan of the implementation of modeling activities was simplified and revised. During the pilot study, it was observed that:

- The third question of the pre-questionnaire asking PTs what kind of methods they use to solve their daily life problems was not understood by PTs. In addition, this question could not be associated with the aim of the research study. Hence, the pre-questionnaire was revised and this question was not included in the pre-questionnaire of the main study.
- During the implementations of the modeling activities PTs were asked to represent their problem solution process while they engaged in the modeling activities. It was aimed to see that how they explain the experience of their own modeling process. However, it was observed that most of PTs tried to represent their solutions rather than the solution process. In addition, the time was limited and representing the solution process requires more time to do it. Thus, the plan of the implementation of modeling activities was revised and it was not included in the main study.
- PTs made limited number of assumptions, used only one method to solve the modeling problem, and obtained one solution at the end of the problem-solving process during the third modeling activity implementation. Hence, it was observed that this modeling activity did not comply with the criteria of being a modeling activity allowing to examine the modeling process in detail. Therefore, in the main study, only two of the three modeling activities, namely, “Lost Cell Phone” and “Summer Reading Program” modeling activities were implemented.

5.4.2. Main Research Study

First, the data collection process started with implementation of the pre-questionnaire given in the Appendix A. Then, mathematical modeling activities "Lost Cell Phone" (Appendix B) and "Summer Reading Program" (Appendix C), were implemented. Last, a post-questionnaire provided in Appendix D was applied. In summary, the data collection procedure consisted of implementation of the pre-questionnaire, videotaped problem solution process of groups, and implementation of the post-questionnaire.

Firstly, pre-questionnaire (Appendix A) was implemented. Participants had approximately fifteen minutes to complete the form. After that, they were formed into groups of three or four. At first, they studied on the given real-life problem situation in the activity individually for approximately fifteen minutes. The reason for the individual study was providing some time to PTs to form their own ideas for the solution before the group study. When they finished individual study, they started to study with their group members. During the group study, first, PTs were expected to share their individual suggestions for the solution of the problem with their group members. After that, they tried to develop a model for the solution as a group for approximately fifty minutes. When all groups finished their group work, each group was expected to share its own solution with other groups. During the sharing of group solutions, the class discussion considering the comparison of each group solution was done for approximately twenty minutes.

Implementation of the second modeling activity started with forming of groups. The process was the similar to the previous implementation of the first activity. First, they worked on the problem individually for approximately twenty minutes, then they continued with group study. After they worked on the problem as groups approximately forty minutes, each group shared its own group solution to other groups. Then, the data collection process continued with the implementation of the post-questionnaire (Appendix D). Last, there was a class discussion about the mathematical models, mathematical modeling activities, and the modeling process that PTs experienced while working on the modeling activities. Group presentations after each modeling activity and class discussion after post questionnaire were also videotaped.

5.5. Data Analysis

Since the study was a phenomenological qualitative study and all of the data were qualitative, qualitative data analysis methods were used to analyze the data. According to Moustakas (1994), in phenomenological qualitative studies, data analysis includes listing of every expressions about the experience, and preliminary grouping of them; eliminating the unrelated expressions, and presenting the repetitive and overlapping expressions in more exact descriptive words; clustering and thematizing the invariant constituents of the experience that are related into a thematic label; identifying the final version of the invariant constituents and themes; and constructing the description of the essences of the experience by using the relevant, validated constituents and themes, and verbatim examples from the transcribed data. In this study, videotaped classroom observations, individual and group solution sheets, the transcriptions of the videotaped groups' studies, and responses to pre- and post-questionnaires, were used as sources for the data analysis.

The framework for the data analysis suggested by Moustakas (1994) was used to analyze the data. First, the data were organized, then each videotaped classroom session was transcribed. The transcriptions of the videotaped group studies were read and read again and examined together with the individual and group solution sheets to determine the related expressions and to eliminate the unrelated expressions. After forming of the initial labels, example quotes related to each label were highlighted. The similar labels were grouped together to establish the themes by using of the preexisting themes developed in the study of Anhalt and Cortez (2015). Figure 5.1 represents the modeling process which Anhalt and Cortez (2015) expanded from the modeling cycle suggested in CCSSM.

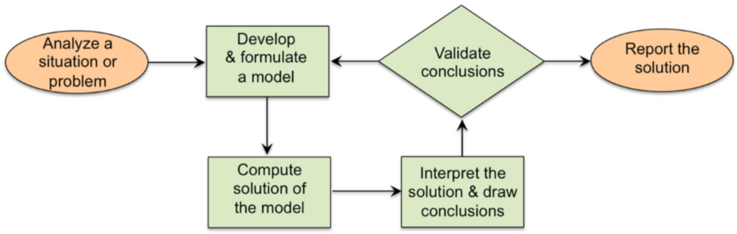


Figure 5.1. Element titles within the modeling cycle (CCSSI 2010, p. 72) have been expanded to be more descriptive (Anhalt and Cortez, 2015, p. 448).

Besides that, the descriptions of the work that each element of the mathematical modeling process entails provided by Anhalt and Cortez (2015) were used during the validation and categorical aggregation (Figure 5.2.). Thus, the themes were formed according to the following framework and the data were represented in text and tabular forms.

Table 5.1. Modeling elements of mathematical modeling and description of the work that they entail.

Modeling Element	What It Entails
1. Analyze the situation or problem	• Identify a problem taken from an external context (often from an everyday life context) that must be solved or a situation that must be understood and explained. • Do background research if necessary. • Make sense of the situation or problem and understand the question.
2. Develop and formutale a model	• Determine all given information. • Determine what assumptions are necessary. • Translate the information given in the problem together with the assumptions into a mathematical problem that can be solved. • Use mathematics appropriate fort he information given and assumed as well as the students' expertise.
3. Compute a solution of the model	• Solve the mathematical problem stated in the model. • Analyze and perform operations in the model • Check for correctness.
4. Interpret the solution and draw conclusions	• Interpret the mathematical solution in terms of the original situation. • Draw conclusions that the solution implies about the original situation.
5. Validate conslusions	• Reflect on whether the mathematical answer makes sense in terms of the original situations (e.g., is the answer within a valid range of values?). • If the conclusions are satisfactory with regard to the accuracy needed, report the solution. If the conclusions are not satisfactory or need to be improved, back to stage 2 ("Develop and formulate a model").
6. Develop and formute a new or modified model	• Revise the assumptions made according to what was learned in the first solution and translate them into a new or modified mathematical problem that can be solved. • The type of mathematics in the current model may be different from the previous one. • Go through these stages again: Compute, Interpret, and Validate.
Report the solution	• Share your conclusions and the reasoning behind them

As it was stated before, the individual and group solutions were also examined carefully together with the transcribed data to get correct understanding of the solution approaches of each group and to verify the themes. On the other hand, in the analysis of the pre- and post-questionnaire, since the first and second questions of both questionnaires were the same questions, they were analyzed together. First, each answer was read and read again. After examination of the given answers, labels were formed carefully. It was observed that some phrases and words were repeated and there occurred some patterns in the data. Based on the patterns in the given answers, the themes were formed considering the PTs' approaches to the mathematical model and modeling. Finally, the data were represented by presenting the categories in text and tabular forms. Besides, remaining questions of the both questionnaires were analyzed independently. As it was in the analysis of the first and second questions, the responses to other questions were examined carefully. Based on the similarities and patterns in the given answers, they were grouped under themes.

5.6. Reliability and Validity Issues

Triangulation is one of the best-known strategies to prop up the validity of a study (Merriam and Tisdell, 2016). In this study, triangulation was used in order to increase the authenticity and trustworthiness of the study. Triangulation using multiple data sources includes comparing and cross-checking the collected data (Merriam and Tisdell, 2016). In this study, transcriptions of the videotaped problem solution processes, videotaped classroom observations, responses to pre- and post-questionnaires, and individual and group solutions for the modeling activities were used in triangulation to enhance the accuracy of the findings.

Besides, rich and thick description was used to enhance the possibility of "transferring" the results of a qualitative study to another setting (Merriam and Tisdell, 2016). Using rich and thick description refers to a description of the setting and participants of the study, as well as a detailed description of the findings with adequate evidence presented in the form of quotes from participant interviews, field notes, and documents. Hence, in this study, to increase the transferability the setting, participants, and findings of the study were explained in detail supported by quotes of participants, individual and group solution sheets of participants, and videotaped classroom observations.

In addition, to enhance the accuracy of the findings, the researcher and her supervisor reviewed and interpreted the data independently. After that, they studied together on the data and confirmed the findings, interpretations, and conclusions of the study.

The researcher usually explores his or her own experiences, in part to examine the experience and to become aware of personal biases, viewpoints, and assumptions (Merriam and Tisdell, 2016). It is known that researchers following a phenomenological approach should set aside prejudgments regarding the phenomenon being investigated in order to launch the study as far as possible free of preconceptions, and knowledge of the phenomenon from prior experience (Moustakas, 1994). This process is called *epoche* which is a Greek word meaning to refrain from judgment and includes setting aside of the everyday understandings, judgments, and knowings, and revisiting the phenomena (Moustakas, 1994). In this study, the researcher implemented the activities together with the teacher of the course that the study was conducted. The researcher was graduated from the secondary school mathematics education program of a state university. Hence, she had read more about and experienced the mathematical modeling process before this study. During the study, the researcher tried to be an objective researcher by trying to hide her feelings, thoughts, and knowings from the participants, and by trying to not interrupt the participants in anyway when they were studying on the modeling activities.

6. FINDINGS

In this section, results of the data analysis will be presented. The research questions will be answered, respectively. First, the results related to PTs' opinions on mathematical models and mathematical modeling in the pre- and post-questionnaire will be shared. The comparison of their answers about the mathematical model and modeling will also be presented to understand the change in their opinions after the experience of mathematical modeling process. Then, their thoughts on characteristics of modeling activities and using this kind of activities in their future classes will be explained. Last, the mathematical modeling processes experienced by PTs through the two modeling activities, namely "Lost Cell Phone" (Anhalt and Cortez, 2016) and "Summer Reading Program" (English and Cortez, 2005) will be reported in detail.

6.1. Prospective Teachers' Opinions about Mathematical Models, Mathematical Modeling, Characteristics of Modeling Activities, and Using Modeling Activities in Their Future Mathematics Instructions

6.1.1. Prospective Teachers' Opinions about Mathematical Models

In both of the pre- and post-questionnaire, the PTs were asked to explain what they understand from the term 'mathematical model'. When the answers of PTs were analyzed, it was found that they explained the mathematical model in four different categories, namely, by giving examples, relating to real-life/daily life, relating with other disciplines, and explaining as a visualizing / teaching method. In the following, Table 6.1 summarizes the PTs' explanations about mathematical model.

Table 6.1. Categories of Prospective Teachers' Mathematical Model Explanations

Categories	**Pre-Questionnaire**	**Post-Questionnaire**
Explaining mathematical model by giving examples	4 (PT1, PT2, PT3, PT6)	1 (PT3)
Explaining mathematical model by relating to real-life/daily life	1 (PT8)	3 (PT1, PT2, PT8)
Explaining mathematical model by relating to other disciplines	1 (PT4)	1 (PT4)
Explaining mathematical model as a visualizing/teaching method	2 (PT5, PT7)	3 (PT5, PT6, PT7)

As it is represented in the Table 6.1, there were four PTs who tried to explain mathematical model by giving examples. PT1 wrote 2D and 3D geometric shapes, graphs, and tables as examples of mathematical model. PT2, PT3 and PT6 also defined mathematical model by giving examples in the pre-questionnaire. They gave similar examples for mathematical models such as graphs, tables, and shapes. It was also noticed that, PT3 defined mathematical model with examples in the post-questionnaire as she did in the pre-questionnaire. Besides the graphs and tables, she gave the coordinate system as an example of the mathematical model.

As it is represented in the Table 6.1, PT1, PT2, and PT8 emphasized using mathematics in daily life. It was noticed that, PT1 tried to relate mathematical model to daily life. As it was stated before, PT1's explained mathematical model by giving examples in the pre-questionnaire, but after the experience of the modeling activities PT1 described it relating with real-life in the post-questionnaire. In both of the questionnaires, PT8 made connections between mathematical models and daily life. She gave roller coaster as an example of daily life model of parabola's arms up and down. In addition, according to her answer in the post-questionnaire, she thought that mathematical model is for associating real-life problems to mathematics.

Moreover, only PT4 explained mathematical model by relating it with other disciplines. He wrote that mathematical model is about using mathematics in other fields such as science, computer science, economics, and physics in the pre-questionnaire. He referred to use of mathematical formulas, schemes, and properties to create a model which is helpful in other disciplines.

Mathematical model was explained as a visualizing or teaching method by PT5, PT6, and PT7. PT5 emphasized making a topic or a case clear by using mathematical models in the post-questionnaire. PT6 also explained that mathematical models are used in solving problems which have no exact solutions. She underlined that by creating mathematical models those problems can be more understandable. It might be interpreted as PT6 put emphasis on visualization of abstract things or problems by using mathematical models to make them more understandable. In addition, PT7 indicated that mathematical models include different teaching methods helping learners to understand concepts easily. In the post-questionnaire, she thought mathematical model itself as a strategy to visualize the concepts.

In summary, PTs approached mathematical model in four different ways. They explained it by giving examples, relating to real-life, relating with other disciplines, and as a visualizing/teaching method. In their pre-questionnaire, PT1, PT2, PT3, and PT6 gave similar examples to explain mathematical model such as two-dimensional and three-dimensional geometric shapes, graphs, and tables. It was also observed that, PT3 explained mathematical model by giving examples in both questionnaires. When the answers of PT1, PT2, and PT8 in the post-questionnaire were examined, it was noticed that they related mathematical model to real or daily life. Besides, PT8 tried to explain mathematical model in the same way, by relating to daily life, in both of the forms. On the other hand, only PT4 related mathematical model to other disciplines such as physics, economics, and computer science in both of the forms. Last, PT5, PT6, and PT7 explained mathematical model as a visualization or teaching method. In their both of the forms, PT5 and PT7 considered mathematical model as a visualization or teaching method to make abstract things more understandable.

6.1.2. Prospective Teachers' Opinions about Mathematical Modeling

The PTs were also asked to explain what they understand from the phrase 'mathematical modeling' in both of the pre- and post-questionnaire. When the answers of PTs were examined, it was observed that they tried to explain the mathematical modeling in three different categories, namely, explaining mathematical modeling by relating it with real-life/daily life, as a method to visualize/make abstract things more concrete, and as a problem-solving method (Table 6.2).

Table 6.2. Categories of Prospective Teachers' Mathematical Modeling Definitions.

Categories	**Pre-questionnaire**	**Post-questionnaire**
Explaining mathematical modeling by relating to real-life/daily life	3 (PT4, PT7, PT8)	5 (PT1, PT2, PT3, PT4, PT8)
Explaining mathematical modeling as a method to visualize/make abstract things more concrete	5 (PT1, PT2, PT3, PT5, PT6)	2 (PT3, PT7)
Explaining mathematical modeling as a problem-solving method	0	2 (PT5, PT6)

PT1, PT2, PT3, PT4, PT7 and PT8 related mathematical modeling to daily or real-life. They put emphasis on using mathematics to solve daily life problems and integrating mathematics into the real-life. In the post-questionnaire, PT1 explained mathematical modeling as it is about using mathematics in our daily life problems. On the other hand, PT2 defined modeling as application of mathematical models in the post-questionnaire where she described mathematical model by giving examples and relating it to real-life. Hence, it might be deduced that she tried to make connection between real-life and mathematical modeling.

Moreover, PT3 emphasized using mathematical modeling for providing students clear understanding. She added that it enables students to transform their abstract thoughts into concrete things which helps them in solving their daily life problems. In both of the forms, PT4 made connection between mathematical modeling and real-life stating that mathematical modeling is using mathematical models in real-life situations which are related to the various areas in real-life such as physics, computer science, and engineering.

In addition, PT8 approached mathematical modeling in the same way in both of the forms and she put emphasis on making relationship between daily life and mathematics while explaining mathematical modeling. She noted that mathematical modeling shows students where they can benefit from mathematics in daily life. Furthermore, in the post-questionnaire, she implied that it is about solving real-life problems with mathematics.

On the other hand, most of the PTs explained mathematical modeling as a method to visualize and to make abstract things more concrete. In their answers, using mathematical modeling to demonstrate abstract things in more concrete and understandable way was underlined. In the pre-questionnaire, PT1 wrote that mathematical modeling enables students understand abstract things easily since it is used in teaching abstract things as concrete things. PT3 also described mathematical modeling as a method to make abstract things more concrete. She also added that mathematical modeling increases students' understanding and helps students in solving daily life problems. PT7 stated that mathematical modeling includes using different materials such as graphs, diagrams which helps students to visualize the concepts and understand them easily.

On the other hand, PT5 and PT6 explained mathematical modeling as a problem-solving method in their post-questionnaires. They focused on the solving problems easily by using mathematical models such as graphs and tables. They also explained what they need to do in the problem-solving process which is implied as mathematical modeling.

When the answers of PTs were examined it was observed that they approached three different ways to mathematical modeling. In the pre-questionnaire, PT4, PT7 and PT8 explained modeling by relating it to real-life. PT1, PT2, PT3, PT4, and PT8 also described it by making connection between daily life and modeling in their post-questionnaire. It was noticed that PT4 and PT8 explained in the same way in both of the forms. Mathematical modeling was explained as a method for visualizing and making abstract things more concrete by PT1, PT2, PT3, PT5 and PT6 in the pre-questionnaire; and by PT3 and PT7 in the post-questionnaire. Lastly, PT5 and PT6 described mathematical modeling as a problem-solving method in their post-questionnaire.

6.1.3. Prospective Teachers' Opinions about the Characteristics of Mathematical Modeling Activities

In the post-questionnaire, besides the opinions of PTs on mathematical model and modeling, their opinions about characteristics of mathematical modeling activities were also asked. Specifically, they were asked to explain the characteristics of the modeling activities they had worked on. Their answers were summarized in the following Table 6.3. It was observed that, most of the PTs noticed and expressed that mathematical modeling activities include daily life or real-life problems. For instance, PT1 and PT3 explained that mathematical modeling activities include daily life problems.

Furthermore, PT1 thought that modeling activities involve effectively difficult questions. Here, she might have tried to point out that those problems can be seen as difficult at first because most of them questioned where they can use mathematics to solve those problems. Also, PT3 indicated that they are not regular mathematical problems which require finding models to solve as those problems are related to daily life.

Table 6.3. Characteristics of Mathematical Modeling Activities.

Characteristics of Mathematical Modeling Activities Stated by Prospective Teachers	**Number of Prospective Teachers**
daily life/real-life problems	7
include mathematical solutions	1
difficult but effective questions	1
increase analytical thinking ability	1
not regular mathematical problems	1
require making/finding a model	2
involve kind of basic characteristics, not detailed information	1
there is no exact solution	1
need to include more mathematical explanations	1
related with geometry	1

On the other hand, some of the PTs wrote that mathematical modeling activities include mathematical solutions, and they increase analytical thinking ability. Furthermore, PT6 stated that there is no exact solution of these problems. Here, with the expression 'there is no exact solution', it might be implied that there is not only one solution, there are many solutions.

Moreover, PT5 noticed that one of the mathematical modeling activities they had engaged in was related to geometry and the other activity needs to involve more mathematical explanations. Here, it might be understood that since the 'Lost Cell Phone' modeling activity requires PTs use their geometrical knowledge to solve the problem, PT5 pointed out this characteristic of the activity. On the other hand, the 'Summer Reading Program' modeling activity involves preparing a rubric to assign points to students for all of the books they have read and written about during this reading program. Therefore, PT5 thought that this modeling problem needs to be provided more mathematical explanations to solve it.

In conclusion, when the answers of the PTs were analyzed, it was seen that various characteristics of the modeling activities were indicated. Involving mathematical solutions, being real-life problems, increasing analytical thinking ability, and being not regular mathematical problems were some of the characteristics that PTs referred. It is noticeable that, being real-life or daily life problems was stated as a common characteristic. Besides, requiring to find a model to solve the problem was specified as a characteristic of modeling activities by some of the PTs.

6.1.4. Prospective Teachers Opinions about Using Mathematical Modeling Activities in Their Future Classes

In the post-questionnaire, PTs were asked to explain what they think about using mathematical modeling activities in their future mathematics classes. Based on the data analysis, it was found that all of the PTs found implementing mathematical modeling activities are useful. Hence, they stated that they will try to implement modeling activities in their future mathematics classes. For instance, PT1 wrote that she will use modeling activities in her future mathematics classes because those activities can help students to understand where their mathematical knowledge can be used in daily life. PT2 also stated that she wants to implement modeling activities in her future classes since she thought modeling activities can increase students' cognitive abilities and make students to like mathematics. Moreover, according to PT7 mathematical modeling activities provide students to visualize the concepts and make connections between the concepts and their mathematical knowledge. For this reason, she indicated that she wants to use mathematical modeling in her future mathematics classes.

In summary, PTs thought that mathematical modeling activities provide students to see the usage areas of their mathematical knowledge in real-life, to increase their cognitive abilities, to understand the relation between their mathematical knowledge and the mathematical concepts and to be able to think in higher levels. Therefore, all of them stated that it is useful for students to implement mathematical modeling activities in their future mathematics classes.

6.2. Mathematical Modeling Processes Experienced by Prospective Teachers

6.2.1. Modeling Process Experienced by 1^{st} Group During the 'Lost Cell Phone' Modeling Activity

In this part, the results related to the modeling process that first group consisting of PT6, PT9 and PT10 experienced during the implementation of the 'Lost Cell Phone' modeling activity will be presented. Based on the data analysis, i.e., analysis of the videotaped individual study, classroom observations, and individual solution sheets, the first element of the modeling process, namely analyzing the situation or problem, was observed during the individual study. In addition, since PTs were asked to develop an individual suggestion for the solution of the problem, they worked on the problem individually and offered some solutions which implies that each PT tried to understand the given problem situation and developed an individual solution.

On the other hand, basically, the second, third, and seventh elements of the modeling cycle in Anhalt and Cortez (2016) were experienced during the group study. It was noticed that those elements of modeling process did not occur in a specific order.

When the data related to the group study of the first group was examined, based on the similarity of their individual solutions, they started to formulate the group solution by drawing the coordinate plane and the circles on the graph paper. It can be noticed that the second element of the mathematical modeling cycle, namely developing and formulating a model, was experienced at the beginning of the group study.

"PT6: Let's draw the coordinates.

PT9: Ok.

PT6: Look, you don't have to go in the direction of minus x-axis.

[...]

PT6: You don't have to go to there [showing the negative side of the x-axis on the coordinate plane], to minus x-axis, you don't have to go further there.

PT6: In my opinion, we can draw it as a draft.

PT9: Exactly. Then, I am starting to draw it."

Here, PT6 offered drawing the coordinates given in the problem. When PT9 was trying to draw those coordinates, PT6 suggested that it is not necessary to draw a long distance in the negative side of the x-axis. They assumed that tower locations as centers and the distances to the cell phone detected by the towers as radii of the centers.

After that, they tried to compute a solution of the model they have developed implying that the third element of the mathematical modeling cycle was observed. As it is shown in the following, PT10 asked whether the cell phone is not at the intersection of those three circles they have drawn. Hereupon, PT6 explained the solution that she computed. She indicated that if it is not at the intersection of those three circles, they can look at the intersection of only two circles at a time.

"PT10: Well, it is ok if the phone is in the intersection point of these three circles, but what if it is not there?

PT6: Then, for example you can look at the intersection of only these two circles and you will subtract this area [the intersection of other circle and this intersection area] from this intersection. Then the area becomes here. If only these two intersect, then you will subtract this area [the intersection of other circle and this intersection area] from the intersection area of these two and you will search here. There is nothing else to do."

They continued to worked on the drawing the circles which implies that they turned back to experiencing the second stage of the modeling cycle.

After they drew the circles, they started to discuss on computing the solution of the model they have formulated. As it is showed below, they focused on the intersection area of the three circles and they colored the intersection area of the three circles with blue.

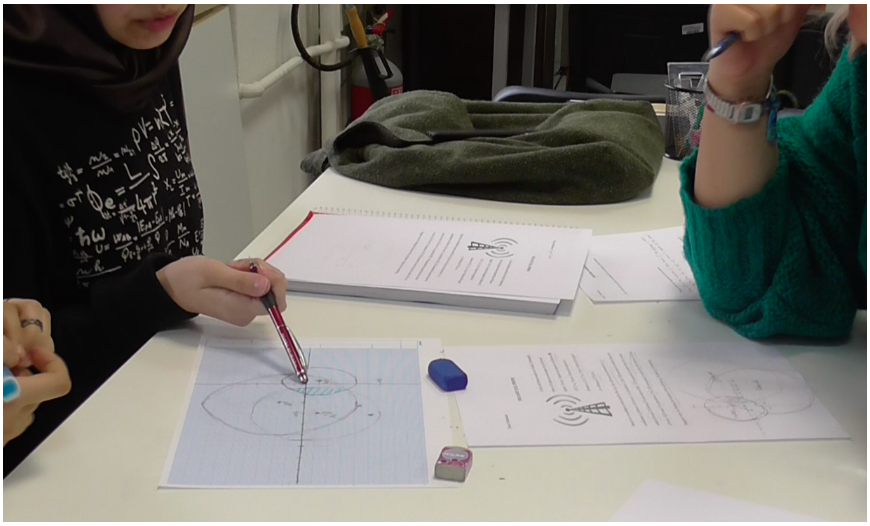


Figure 6.1. Group study of the first group.

"PT9: Look, they all intersect at some point, Ok? Let's show it with a colorful pen.

PT10: Yes.

PT9: Let's draw with this. Where? Here [showing the middle point of that three circles seem to intersect] it is, isn't it? Here, they are all intersect.

PT6: If the phone is at the intersection point of these three circles, then it can be found easily since the area is small enough.

PT10: Yes, the area is small.

PT9: Exactly, if all of them give the signals,

PT6: Yes, from all of them,

PT10: If all of them give signals, the phone is here [pointing the intersection area of the coverage zone of the three towers].

PT9: If all of them give signals, then we say that it is here [the intersection area of the coverage zone of the three towers]

PT10: Yes.

PT9: They should look at here [the intersection area of the coverage zone of the three towers] to find the phone

PT6: It will be easier to look at the intersection area of these three rather than to search the region separately.

PT10: Yes.

PT6: The area is narrower.”

The discussion above can be summarized as follows: PT9 showed the intersection area of three circles by coloring it with blue and PT6 indicated that if the cell phone is at the intersection area of three circles, it can be found easily since the searching area is small. Then, PT9 and PT10 added that if all of the towers give signals, the phone is at the intersection area of these three circles. They all agreed on that solution. In addition, PT6 indicated that it will be easier to examine this intersection area than examining all of the coverage zone of the three towers.

Then, they decided to write their solution on their group solution sheet. While they were writing the solution, they continued to discussion and gave names to each of the intersectional areas.

“PT10: Let write here [the intersection area of three circles] the letter A,

PT9: E1, E2

PT6: Write the equations of all of them if you want,

PT10: I think, write A there [the intersection area of three circles]

PT9: Ok, from the equation of the circle.

PT6: Write A into this [she is pointing the intersection area of three circles] if you want, ok. Write with blue, if you want.

[...]

PT10: Should we say “we give names to the areas”? Should we write it?

PT9: Aren’t they too much? 1 and 2, 2 and 3.

PT10: Then, pairwise. Should I say “two out of three”? Ok, what will we say? Will we say “we look at the intersectional areas”?

PT9: Yes, let’s say “intersectional area”, intersectional area

[...]

PT10: I’ll write “intersectional areas”. Which ones?

PT9: For example, T3 and T1. If T3 and T1, then it is this small area [intersection area of the circles of tower 1 and tower 3].

[...]

PT6: If T1 and T2 give signals and T3 didn’t give signal, we search in the A2 area

PT10: T3,

PT6: If T3 didn't give signal,

PT10: Yes, we,

PT6: Look at area A2. What else we said, T2 andT3. It will be there [showing the intersection area of only towers 2 and 3]."

According to the discussion presented above, they decided to add the intersection areas of two circles at a time in their solution. The intersection area of the circles of tower 1 (T1) and tower 2 (T2) named the A2 which means area 2. In addition, they called A1 the intersectional region of the three circles. They named A3 the intersection area of T2 and T3.

On the other hand, they didn't consider the intersection area of the T1 and T3. Also, the areas where only one tower gets signal called by B1, B2, and B3. In the following the graph that they have drawn is demonstrated. When it is examined, it can be understood that B1 stands for the only area where T1 gets signal, B2 for T2, and B3 for T3.

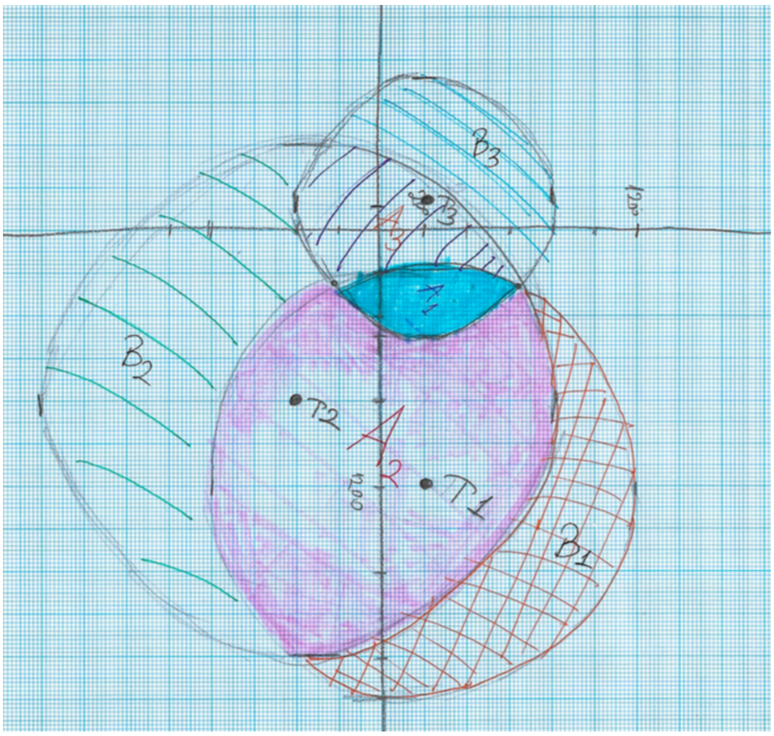
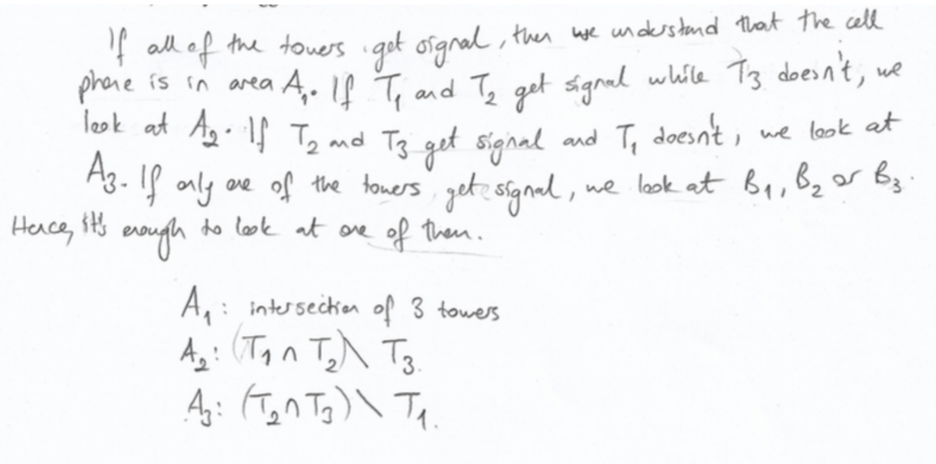


Figure 6.2. Graph of the group solution of the first group.

The written version of the solution is represented in the following Figure 6.3. It involves like the explanation for the graph above.



If all of the towers get signal, then we understand that the cell phone is in area A_1. If T_1 and T_2 get signal while T_3 doesn't, we look at A_2. If T_2 and T_3 get signal and T_1 doesn't, we look at A_3. If only one of the towers get signal, we look at B_1, B_2 or B_3. Hence, it's enough to look at one of them.

A_1: intersection of 3 towers

A_2: $(T_1 \cap T_2) \setminus T_3$

A_3: $(T_2 \cap T_3) \setminus T_1$

Figure 6.3. Written group solution of the first group.

After all groups completed their group study, each group shared its own group solution with other groups. They experienced the reporting stage while they were sharing their group solution with class. To sum up, during the group study, they experienced the second, third, and seventh elements of the modeling process along the implementation of the modeling activity (Figure 6.4).

Figure 6.4. First group's modeling process during the group study.

In Figure 6.4, the modeling process experienced by first group is represented. As it can be noticed that the group study started with the experience of the second element of the modeling process which requires developing and formulating a model and continued with computing solution of the model. After these elements of the modeling process were repeated, the group shared their solution with other groups. Moreover, it might be noted that some of the elements of the modeling process were repeated whereas some of them were not experienced implying that the modeling process was not experienced in a straight order.

6.2.2. Modeling Process Experienced by 2^{nd} Group During the 'Lost Cell Phone' Modeling Activity

The results of the data analysis related to the mathematical modeling process of second group consisting of PT1, PT2, and PT3 will be explained. During the individual study, they worked on the problem individually and offered solutions implying that PTs tried to understand the given problem situation and each of them developed an individual solution. Since PTs have developed almost the same suggestion for the solution, they started to the group study by computing a solution of one of their individual models. In other words, they started to the group study by the third element of the modeling cycle which is computing a solution of the model.

As it is represented in the following, they agreed on solving the equations that they have deal with during the individual study. They discussed on how they can do the computations to solve the equations. PT1 suggested to benefit from the special triangles such as 3-4-5; 6-8-10 and they together with PT3 showed an approximate region that cell phone could be found.

"PT3: We solve the equations

PT1: So hard,

PT3: It comes from the equations,

PT1: I thought that, look, now 3,4,5; 6,8,10, I wonder that we can get the solution

PT3: I don't know if there's any other way,

PT1: How else are we going to do that?

[...]

PT1: We take that square from the special triangle...

PT3: Distance between two points...

PT1: If it's over here, I'm making, directly

PT3: ... It's going to be somewhere out there [pointing an area in the first quarter of the coordinate plane],
PT1: I agree."

After that discussion, they started to write the key points that they have considered for the solution of the problem. Then the second stage of the modeling process, developing and formulating a model, was observed while they were trying to draw coordinate system to the graph paper. It was also noticed that since PT1, PT2, and PT3 developed almost the same solutions during the individual study, they tried to write the same solution as their group solution. In addition, PT2 indicated that each tower scans a circular area. She added that there will be three circles and the phone will be the intersection point of these circles. PT1 and PT3 also agreed with PT2. As it is showed in the following, the group study continued with drawing of the circles. They assumed the locations of the towers as centers and the distances that towers detect the signal as radii of the circles.
"PT1: Now, draw the centers. PT2: Let's see how far away. 1200 and 200. These are 200.
PT1: Ok, it is tower 1.
PT2: Then, 800 and -400. Here is 800, 400, 1, 2, 2.5
PT3: No, it is not there, in the middle,
PT2: Yes, yes, true.
PT1: Ok, there is also 100 and 230 var. -100
PT2: I wish we would draw like this. -100, 1, 2, 3, 4, 5 out there. Is it 230?
PT1: Yes,
PT2: It is somewhere out there"

After that, they tried to compute a solution based on the circles they have drawn. PT1 asked to where they can find the phone and showed that the circles did not intersect at a point. PT3 indicated that the circles didn't intersect because of rounding up the numbers while drawing the circles. In response, PT1 thought that they should intersect, and added if it is needed they can draw from the beginning. In this part, it might be seen that PTs were at the stage of the computing the solution of the model, i.e. the third element of the modeling process was observed.

"PT1: Ok. Where is it going to be?

PT3: Anywhere.

PT1: No, there are three circles, of course they intersect at somewhere. But, is it there?

PT3: You know we didn't do the exact calculations.

In fact, it is obvious they will intersect there. But, since we did round the numbers

PT1: They didn't intersect,

PT3: But, I think it is because we did round up the numbers, because the numbers are,

PT1: So, it is not supposed to be because of the rounding, they should intersect

PT3: Then we drew wrong, it should be true like this

PT1: If not, we'll do it all over again."

The discussion above continued with interpreting the solution. PT3 made a prediction about the location of the cell phone. After this prediction, PT1 drew a new conclusion such that they don't look the intersection point, they look the area in the middle, the intersection set. On the other hand, PT3 kept thinking they should find a single (x, y) point and because of the rounding up the numbers, they couldn't find that point.

"PT3: They should intersect somewhere here

PT1: That is the common point,

PT2: Ok,

PT1: Isn't it? The intersection point of the three circles is there.

PT3: How?

PT1: Namely, we do not look at the intersection point of the lines, we

look at the area that is in the middle. Intersection set.

PT3: We will look at the point where the lines intersect so that they solve the three equations. we need to have a point (x, y), it also needs to be certain, as I said before, it is the unit thing, or, rounding"

As it is seen above, they expected to find the exact point that these three circles intersect. However, they couldn't find the intersection point. They continued to interpret the solution. Hence, the fourth stage of the modeling process continued until they started to write the solution as it is showed below.

"PT1: Yes, our logic is true. Let's say underneath, our point here is a single point that solves three equations, the intersection of three circles, and then we finish.

PT2: Exactly, exactly. What should I write?

PT1: Write, first we determined the position of the three circles on the plane,

PT2: We located the position of three towers

PT1: Position of three towers in our coordinate plane,

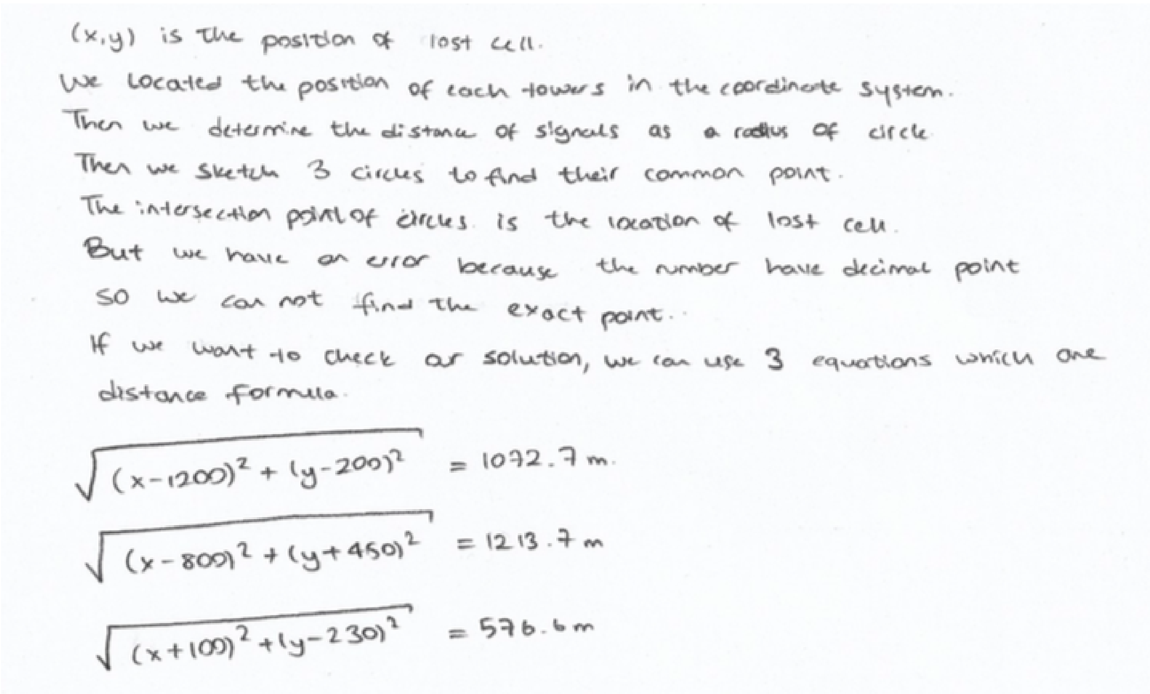
PT2: Cell phone

PT3: What did we do then? Then we? what did we do? We used radii, didn't we?

PT2: Then we use the distance of signals as radius

PT1: Distance of, as Radius, then we say "we draw""

The group solution of the second group is presented below. They summarized briefly what they did to find the solution. It was observed that it includes three equations and an explanation about they couldn't find the exact point which could be found after the solution of those equations.


(x,y) is the position of lost cell.
We located the position of each towers in the coordinate system.
Then we determine the distance of signals as a radius of circle
Then we sketch 3 circles to find their common point.
The intersection point of circles is the location of lost cell.
But we have an error because the number have decimal point
so we can not find the exact point.
If we want to check or solution, we can use 3 equations which one distance formula.

$$\sqrt{(x-1200)^2+(y-200)^2} = 1072.7 \text{ m}$$

$$\sqrt{(x-800)^2+(y+450)^2} = 1213.7 \text{ m}$$

$$\sqrt{(x+100)^2+(y-230)^2} = 576.6 \text{ m}$$

Figure 6.5. The group solution of the second group.

After they wrote the solution to the group solution sheet, they decided to do second trial for drawing the circles. By doing this, they experienced the fifth and sixth elements of the modeling cycle as validating conclusions and formulating a new or modified model.

"PT2: Let's do our second experiment.

PT1: Well, shall we do like this for this time,

PT2: It does not matter,

PT1: Let's do it like this for this time, for example 1 unit will represent 100

PT3: Ok,

PT1: Then, it will be easier to divide"

The explanation of the conversation above can be summarized as PT2 offered drawing again the circles, doing the second trial. PT1 suggested to take 1 unit as 100. In response, PT3 agreed and PT1 indicated that it will be easier to divide the distances on the graph paper. After the discussions, they took 1 unit 10 m. After long calculations, they drew the second graph of the circles. The first and second graphs they had drawn are represented in the following.

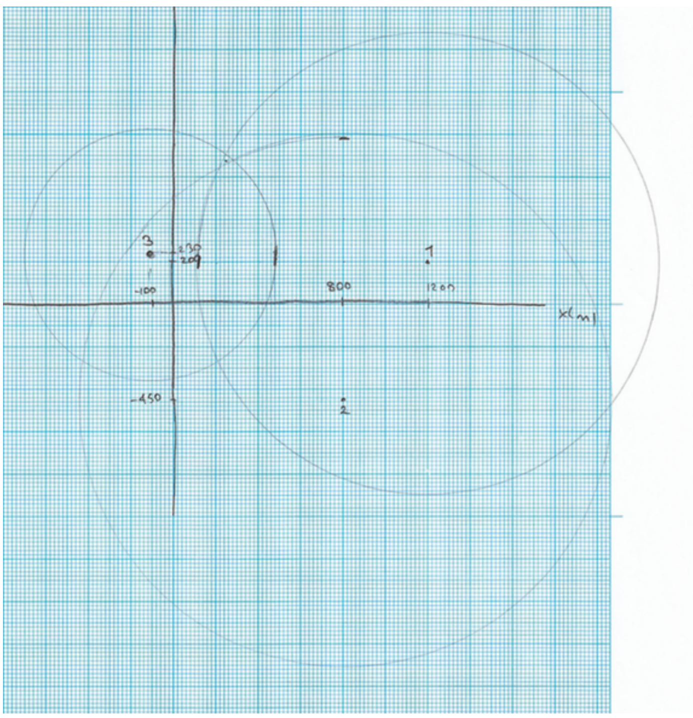


Figure 6.6. First trial for the graphs of the circles.

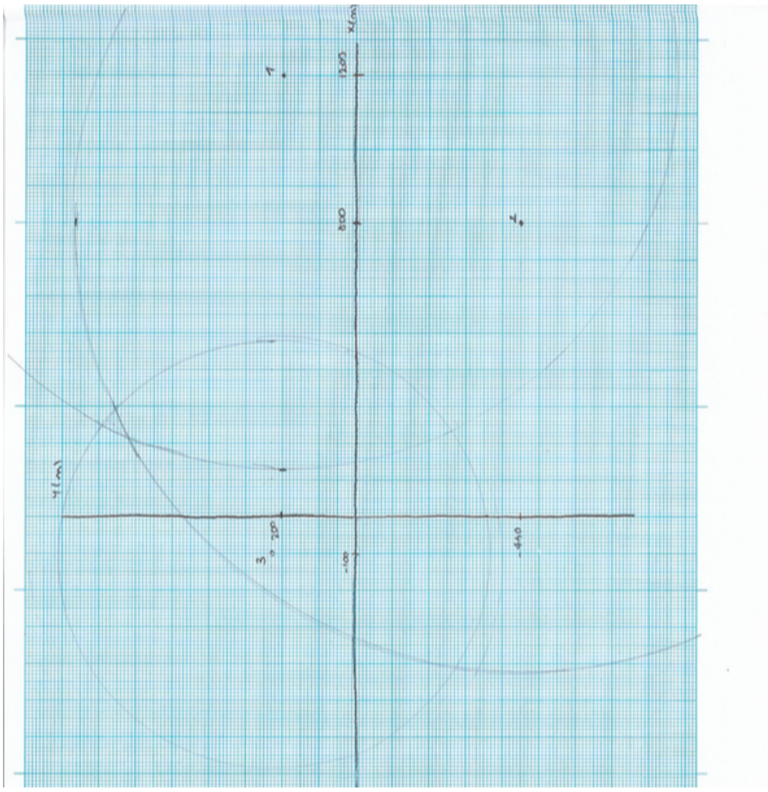


Figure 6.7. Second trial for the graphs of the circles.

As it is presented above, the circles didn't also intersect at a point in the second trial. They thought again that because of the rounding up the numbers, all circles didn't intersect at a point.

"PT1: We can't intersect things here, but the thing is the unit

PT2: They should intersect

PT1: Because we rounded unit squares,

T: All three?

PT2: No, not all three intersect.

PT1: Do they have to intersect? Because we rounding up the numbers,

PT2: Because we can't find it right here, they don't intersect when we rounded up the numbers

[...]

PT1: We're trying for the second time, but there is still a little bit difference

R: Well, have you solved the equation?

PT1: The equation is too long

PT2: The equation is too long

PT1: We

PT2: We said we could make it easier, but we didn't.

PT1: But it seems like they need to intersect. Ultimately, we have one point and that point will solve these three equations."

The conservation between second group, the teacher and the researcher is shown above. To summarize, PTs claimed that three circles should intersect at a single point but they couldn't find that point because they rounded up the numbers. The researcher asked them whether they checked it by solving the equations. In response, they indicated that they couldn't solve those three equations since it is a long process. In the end, they added that they should intersect since there should be a single point which solves those three equations.

When all of the groups finished working on the activity, they shared their own group's solution with class implying that the seventh element of the modeling process was observed at the end of the group study. In conclusion, while the group study, the second group experienced the third, second, fourth, fifth, sixth and the seventh stages of the process (Figure 6.8).

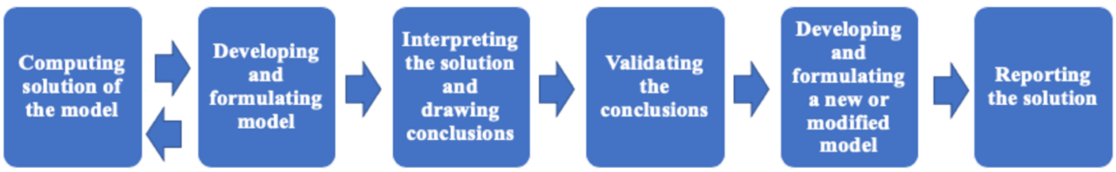


Figure 6.8. Second group's modeling process during the group study.

As it is presented in Figure 6.8, the second group's study started with computing solution of the PT2's model, and continued with developing and formulating of a model. Then the group again computed solution of the model they had developed. After they interpreted the solution they had obtained and made some inferences about their model, they validated the conclusions. Based on this validation, they continued to revise and modify their model. Lastly, they presented their group model to the class.

After all groups explained their group's solution to the class, there was a class discussion about the solution process that PTs experienced. For the individual study, they indicated that first they read the problem, or they just plotted the three towers and find the intersection point of those towers. In addition, they stated that they used models such as graphs, coordinate system to understand the problem and they arranged the solution after using models.

During the implementation of "Lost Cell Phone" activity, the modeling processes experienced by these two groups were different from each other. Although they developed similar models for the solution of the problem, the modeling processes occurred during the group studies were different. In addition, the first group experienced only the modeling cycle elements of formulating the model, doing computations on the model, and reporting the model. Although it seems the cycle involved limited number of elements, there were iterations between those elements. On the other hand, second group experienced most of the elements of the modeling cycle namely; computing the model, developing the model, interpreting the solution, validating the solution, formulating modified model, and reporting the solution. However, there was no straight order in the occurrence of the elements.

6.2.3. Modeling Process Experienced by 1st Group During the 'Summer Reading Program' Modeling Activity

In this part, the results related to the second modeling activity "Summer Reading Program" will be explained. PT4, PT6, and PT7 formed the first group and they worked on together to solve the problem. Based on the data analysis, PTs tried to understand the given problem situation and developed individual suggestions for the solution during the individual study.

The group study started with sharing of the PT7's individual solution to her group members. PT4 also shared his individual solution with his group members and they continued to group study with discussion on his solution.

"PT4: I graded it according to the difficulty of the book. The type of book can be easy, but if it's difficult, students should get more points.

PT6: Level is high but,

PT4: It may be an easy book, but I wanted to students get a high grade if the book is still challenging

[...]

PT4: In the meantime, I made the scoring from 1 to 5, because if I did it between 1 and 10, there would be too many fluctuations.

PT6: So, a total score?

PT4: Exactly, I will combine them according to their weights."

Here, PT4 indicated that he gave points considering not only the variety of the book, but also the difficulty level of the books. He assumed that the variety of a book can be easy but it can be difficult and he added that in that kind of situation he wants students get more points. Moreover, he continued to explain the reason of using a 5-point scale as preventing the fluctuation in the total points. Last, he added the total score will be computed summation of the weighted points from each category that he considers.

Discussion on the PT4's solution was continued for a while. After that, they decided to work on PT4's solution and present it as their group solution. They started to write the solution by relisting the key points that they considered and revising the assumptions they needed. Hence, it might be considered that they were experiencing the element of the modeling process that computing the solution of the model.

"PT4: Yes, how do we write the suggestion? How would you like me to write?

PT6: Then, I think we can write it like you said before, normally the levels of difficulty are determined, but we can sort them by associating with their types, so, maybe we can write this first,

PT7: Yes, exactly, the first point will be given according to this. After you've sorted *them, from 1 to 5, you did it from 1 to 5, right?*

PT4: Yes, I scored from 1 to 5.

PT7: Exactly, this will be the first. What did we say for second, number of pages?

PT4: Yes, let's write the number of pages.

PT7: Let's sort them according to number of pages"

As it is shown above, PT4 asked for how they can write the suggestion. PT6 indicated that they can write as he stated before, namely, relisting the books by relating the difficulty levels to the varieties of the books. PT7 also confirmed this decision by adding that they will give the first point according to this. She asked whether PT4 did the scoring from 1 to 5. After they use a 5-point scale interval, they continued with relisting the books according to the length of the books.

Although PTs focused mostly on computing the solution during the group study, they also tried to focus on interpreting solution of the model. It might be observed in the following that by considering the possible situations based on their assumptions and trying to make decisions on their suggestion for the solution showed that they experienced the fourth stage of the modeling.

"PT4: But, for example, giving 5 points for 24 books to give 5 points is not very little? It seems to me a little, but not a book.

PT7: I thought that one can also read between 17 to 20 easy books to get 5 points, but ultimately, it doesn't matter.

PT4: Hmm, yes.

PT7: Something like that.

PT4: But I don't think there are easy books to read.

[...]

PT4: It might be better if I make it 10.

PT6: Exactly.

PT4: Because there is no chance of closing the difference between the scores with just the report score. I think they should have such kind of chance.

PT6: Then, I think the one will win who reads more books."

Here, PT4 questioned their decision whether giving 5 points for reading 24 books is sufficient. He thought that giving 5 points is less for 24 books. PT7 claimed that it doesn't matter in total, for instance she can read between 17 and 20 books which are easy to read to get 5 points. In addition, PT4 indicated that there are not that much easy books in the list, and suggested to make the total point for this category 10. On the other hand, being able to get more points from the written reports was specified by PT4. In other words, he thought that a student should have the chance the increase his total score by his written reports about the book that he read. PT6 agreed with PT4 since she thought in that situation the one who reads more will win.

"PT7: Yes, there is also reports. But, it seems like someone who doesn't like reading fantastic novels. Or we said reading fantastic books easy, reading realistic is hard. Then, someone who doesn't like reading realistic books will read fantastic books but he will get less points.

PT6: Yes, just a little more

PT7: Actually, this is not fair.

PT6: Exactly. For example, here

PT7: Because he might not want to read it

PT6: Exactly, there might be something like that. For example, they can have the same value. With Grade 5

PT7: Yes, then we can do that.

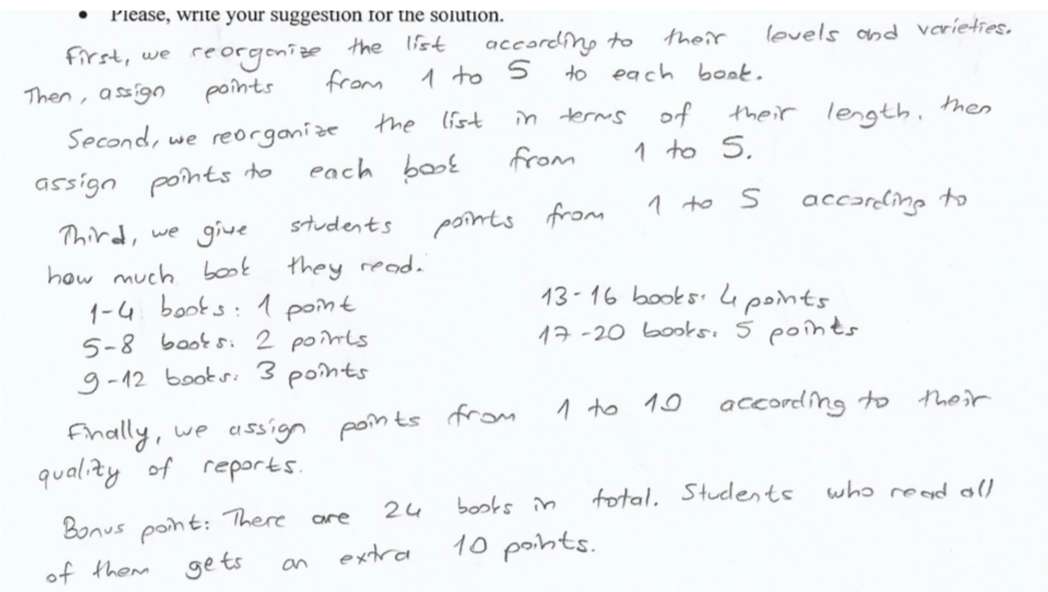
PT6: When we re-list it, I think it would be something like that. For example, if there is a difficulty, first, grade 4 will be fantastic, then grade 5 comes first again. [...]

PT7: Ok."

As it was presented above, PTs consider the possible situations and interpreted their solution. PT7 questioned the fairness of the decision they had made. She pointed out if a student who doesn't like reading realistic books will get less point when he reads fantastic books, and added this situation will not be fair enough. Furthermore, PT6 explained that situation can be balanced if the books relisted according to their grade levels. Then, they continued to interpret their suggestions although they thought that they finished. PT7 kept questioning the fairness of the decision about the relation between the difficulty level and variety of the book. She thought that students don't need to read books they don't like or being difficult to read don't need to make a book high scored.

"PT7: I'm still thinking about it, as I said before, reading fantastic novel can be hard but reading this kind of book doesn't have to be scored high. So, if he doesn't want to read it, he doesn't have to read that book to get a high score. Actually, I wonder that, does it demotivate them?"

Their suggestion for the solution was represented in the following. In summary, they assumed the given sample list of books as the list of all books that students will read during the program. Based on this assumption, they listed the books according to their difficulty level, variety, and length. They assigned points between 1 and 5 to each book. Also, they prepared a scale for the number of the book students read. For instance, if one reads between 9 and 12 books, he gets 3 points. Finally, the reports are graded from 1 to 10 according to their quality. Furthermore, they decided to give bonus points if the student read 24 books in total. If one reads 24 books in total, he gets extra 10 points.



• Please, write your suggestion for the solution.

First, we reorganize the list according to their levels and varieties. Then, assign points from 1 to 5 to each book.

Second, we reorganize the list in terms of their length, then assign points to each book from 1 to 5.

Third, we give students points from 1 to 5 according to how much book they read.

1-4 books: 1 point
5-8 books: 2 points
9-12 books: 3 points
13-16 books: 4 points
17-20 books: 5 points

Finally, we assign points from 1 to 10 according to their quality of reports.

Bonus point: There are 24 books in total. Students who read all of them gets an extra 10 points.

Figure 6.9. The group solution of the first group.

Later, they tried to determine how they grade the written reports. They formulated a model for the quality of the reports. In the following, the conversation about grading the quality of the reports was presented.

"PT7: What is the purpose of this? Making students to read books and understand them.

PT6: Understanding the books.

PT7: Then, we will look at whether students understand the main idea of books they read.

PT6: So, it's important how it's connected the main idea

PT7: Yeah, not the number.

PT4: I need to translate this into a mathematical form. How can we translate it? The simplest way is to look at the number of words.

PT7: But, the number of words for students of this age

PT4: It is not enough,

[...]

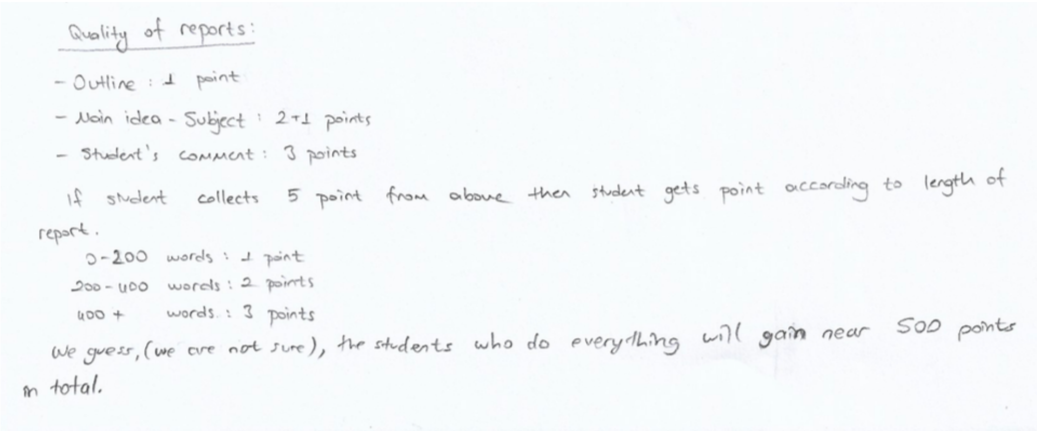
PT6: For instance, first we can look at the general thing, the introduction, body, and the conclusion, and it can be looked at whether he can write paragraphs of introduction, development and conclusion separately.

PT7: Right. Well, after we've looked at them, the topic can be looked. Now the main idea, the topic, student's interpretations

PT6: Did he use his own sentences or sentenced from the books,

PT7: We give all of them, for example, 1 point. No, let's give 2 points each, and there would be 5 things. Then, we can sort them. Or, if outline is not important, we give it 1. The main idea would be 1 point, the topic would be 1 point, what else, comments would be 1 point."

After a long discussion, they decided to consider whether students understand what they read or not. PT4 suggested to mathematize how they determine the quality of the reports. PT6 proposed to look at how introduction, body, and conclusion parts of the reports are written. PT7 agreed with PT6 and added to look at the main idea, topic and interpretations of students in the reports. PT7 also suggested to assign points to each of those things that they will consider. For instance, 1 point can be assigned to the main idea or to the outline of the report. While they were discussing about the grading of the reports, it might be implied that they experienced the second and third stages of the modeling process since they developed a model for grading the written reports and did some computations on it. The final solution for the quality of the reports was showed in the following.


Quality of reports:

- Outline : 1 point
- Main idea - Subject : 2+1 points
- Student's comment : 3 points

If student collects 5 point from above then student gets point according to length of report.

0-200 words : 1 point
200-400 words : 2 points
400 + words : 3 points

We guess, (we are not sure), the students who do everything will gain near 500 points in total.

Figure 6.10. The second part of the group solution of the first group.

In conclusion, at the end of the group study, they shared their solution with class. Based on the data analysis, it might be understood that the stages of the modeling process were not observed in a straight order. During the individual study, they worked on the given problem situation individually, and developed individual solutions. On the other hand, computing the solution of the model, interpreting the solution and drawing conclusions, developing and formulating a model, and reporting the solution were the observed stages of the mathematical modeling process during the group study (Figure 6.11).

Figure 6.11. First group's modeling process during the group study.

In Figure 6.11, the modeling process experienced by first group is represented. As it can be noticed that the group study started with computing solution of the PT4's model. It continued with interpreting the solution and drawing conclusions. After that the second and third elements of the modeling process were repeated, they went back and forth between these stages of the modeling process. Last, the group shared their solution with other groups. Moreover, it might be noted that some of the elements of the modeling process were repeated whereas some of them were not experienced implying that the modeling process was not experienced in a straight order.

6.2.4. Modeling Process Experienced by 2nd Group During the 'Summer Reading Program' Modeling Activity

In this part, findings related to the mathematical modeling process experienced by second group during the application of 'Summer Reading Program' modeling activity will be explained. This group was formed by PT1, PT2, and PT3, and they worked on the problem together as they did before on the problem of Lost Cell Phone modeling activity. Similar to the first group, during the individual study, PTs analyzed the given problem situation, and developed their own individual models.

As it was represented below, the group study started by PT1's sharing of her individual solution with her group. She thought that a student gets A+ if he reads more than 18 books at level 9-10, 4 different types, and more than 400 pages. In addition, for A, he should read more than 15 books at level 8-9, at least 3 types of books. On the other hand, PT3 suggested giving 5 points for each book, 5 points for each 25 pages, 5 points for each grade level that students read. It was noted that she assumed while reading books at grade level 4 brings no point.

"PT1: I thought that, for A+, student reads more than 18 books, 18 or more.

PT3: Yes

PT1: Let the books be at level 9 and 10

PT3: So, from general to specific.

PT1: Yes, I said, at least 4 different types of books, and more than 400 pages. I gave this A+. For A, more than 15 books, and also books at level 8 and 9,

[...]

PT3: [...] *For example, they can read between 10 and 20 books. Student will get 5 points* *for each book that he reads. Then, what else, 'page number', student gets 5 points for each 25 pages that he reads.* [...] *Then, for the grade, each grade, for example, at least at what grade that each grade can read*

PT1: 4

PT3: 4. How many? 3. Student has read 3 books at grade level 4. Also, he has read 5 books from grade level 5. Then, he gets 0 point for the books at grade level 4 that he read while he gets 25 points for the books that he read at grade 5. For example, if he reads books at grade 6, he can get 10 points for each book he read."

Moreover, PT2 suggested giving points between 0 and 100 according the number of books that students read in her individual solution. For the variety of the books, she thought that if the type of books is more scientific or useful in their school life, then those books bring more points. *"PT2: I said, from 0 to 100 for each one, for example, the number of books, for example, according to the number of books, let's give points from 0 to 100. Also, variety. In fact, 'variety' is complex a little. The scientific books or the books which will be useful in school life can be graded higher."*

Then, the group study continued with the discussion on scoring each category. First, they discussed scoring the number of books that students read. For this category, PT2 suggested to give 10 points for each book based on the assumption that students read minimum 10 and maximum 20 books. For instance, if a student read 20 books, he will get 100 points in total for this category.

"PT1: Let's write here, for example, what does it say in the first, number of books. How many books he can read. 20 books

PT3: 'Maximum' between 10 and 20 books

PT1: Between 10 and 20 books.

PT2: Maximum 10 and 20 books. What if one reads less than 10 books

PT3: I think, he can't participate the competition.

PT2: If it is between 10 and 20 books, then 11 books will be given 10 points, for example, 12 books will be given 20 points."

After that, as it was showed below, they discussed on scoring variety of the books.

They assumed that there are 9 different type of books and decided to give 11 points for each different type of the book that students read.

"PT3: Yes, write for 'variety', if he reads one type of book, he gets 10

PT1: I don't think so.

PT3: Yes, it is different. How many types are there?

PT1: You've counted them all, 'historical' 'fiction', right?

PT2: There are 9 types

[...]

PT1: I think, let student get 10 points when he reads a different book. If he read 2 science fiction books, let him get no points.

PT2: When he reads the same type,

PT1: Yes, if he reads the same types of books, let him get no points. If he reads different types of books, let him get 10 points for each type. PT3: But, then he can get max 90.

PT1: Ok, then let's divide 100 by 9

PT2: So, you're saying that, if a student has read 10 books of same type, he gets no points. But if he, hmm, hmm, okay, let's do it like this. [...] *Look, then I've divided 100 by 9 varieties, I got 11. He will get 11 points."*

They also discussed about how they assign points for the length of the books. Since they assumed students will read at least 10 and maximum 20 books, they decided to sum pages of ten books for the minimum point and pages of the twenty books for the maximum point of their scale.

"PT3: I'll collect the pages of the books which have least number of pages, and the pages of book that have the greatest number of pages, and divide the difference between these two by 10. I'm going to do that, aren't I?

PT2: Yes, yes, yes. Let's collect the pages of books that have least number of pages,

PT3: Let's collect 20 books with the highest pages. They can read maximum 20 books."

To assign points for the difficulty of the books, they considered the grade levels of the students and books. At the end of the discussions, they determined a scale with 10-point intervals. According to that scale, if students read books at the same grade level as their grade level, they can get 10 points for each book. For instance, if a 7th grade level student read books at grade level 7, he can get 10 points for each book. In addition, if that student reads books at 8th grade level, he can get 20 points for each book, and if he read books at 9th grade level, he can get 30 points for each book.

"PT3: Then, if a student read books below his grade level, he can get no points. If he reads books at his grade level, he gets lowest score

PT1: He gets 10 points, then

[...]

PT1: What if he reads at grade level 6?

PT3: If he reads at his grade level books, then he gets 10 points for each book.

PT1: Each book? How can we know how many books he will read?

PT3: For example, a student at grade level 6 reads a book at grade level 6, 7, 8, 9, and 10. Theni for grade 6, he gets 10, for 7

PT1: I think we shouldn't give points for each book

PT3: Then, let's choose a total thing, if he reads books at his grade level, he gets 10 points. If he reads one level higher than his grade level, he gets 10 more points. For example, a student at 7th grade level reads books at 7th grade level, he gets 10 points. If he reads books at 8th grade level, he gets 20 points. If he reads at 9th grade level, he gets 30 points."

Later, they discussed about assigning points to the quality of the written reports. As it was demonstrated below, they considered the length of the report, plagiarism, grammar, and quality of the content of the reports. With quality of the content of the reports, they referred that students should write their own comments related to the books. Since they specified four criteria to score the quality of the reports, they decided to evaluate each criterion over 25 points. Up to here, they experienced the second and third elements of the modeling cycle by going back and forth between these elements.

"PT3: Length of the report,

PT1: What if he writes long but irrelevant things?

PT3: Yes, what else, it can be spelling rules, whether student do 'copy-paste' or not,

PT2: 'Plagiarism'

PT1: 'length', 'plagiarism'

PT2: 'grammatical', 'grammar'

PT3: Hı hı, 'grammar' How can we say the context of the report. But it includes interpretations, it depends on the interpretations. How can I determine it?

[...]

PT3: What did we write now? Length.

PT1: 'Length', 'plagiarism', 'grammar', 'quality of the content of reports',

[...]

PT2: Let's give 25 points to each of them

PT1: Ok, let it be 4, each of them 25 points.

PT3: Ok?

PT1: Wait, let's write, each of

PT3: Write 25 points."

Finally, they discussed on the fairness of their suggestion for the solution. They interpreted their solution whether it is fair or not. After the discussion on the fairness represented below, they agreed on that their solution is fair.

"PT3: I wonder that for someone who reads 20 books the difference between the largest and the lowest will probably be too big,

PT1: Ok

PT3: I wonder if it would be fair.

PT1: I think it's very fair.

PT2: No, it makes sense. Let's say 10 books and 20 books

PT1: But student who reads 20 books could be read them from the longest books. We should do it like this

PT2: But everybody will be in the range of the book that everyone reads, you will give points according to this

PT3: But the range is again very large. For example, if a guy reads 198 pages and the other reads 1 page of books, I mean that they will get the same score. I'm saying it's not going to be fair.

PT2: I didn't understand.

PT3: For instance, if we divided it by 10 or it will increase by 10 points in a certain range, I would say that its range would be very wide, such as 200

PT1: Okay, that student gonna win here although he loses out of there. So, there's "Number of books". He'll win what he has lost. I don't think it's very unfair.

PT3: 'Number of books' would also be useful for the ones who read books with less number of pages

PT1: Ok, here, there is nothing beneficial for the ones reading less pages PT3: Someone reading less pages

PT2: If someone reads less pages, then they can't be at the same interval PT1: They can't be

PT2: Gets low points. Namely, when the number of pages collected., it will be a small number

PT1: Then, you wouldn't divide by ten. Consider them as one by one. This one is 1 point. That interval is given 1 point

PT3: Yes, I mean that,

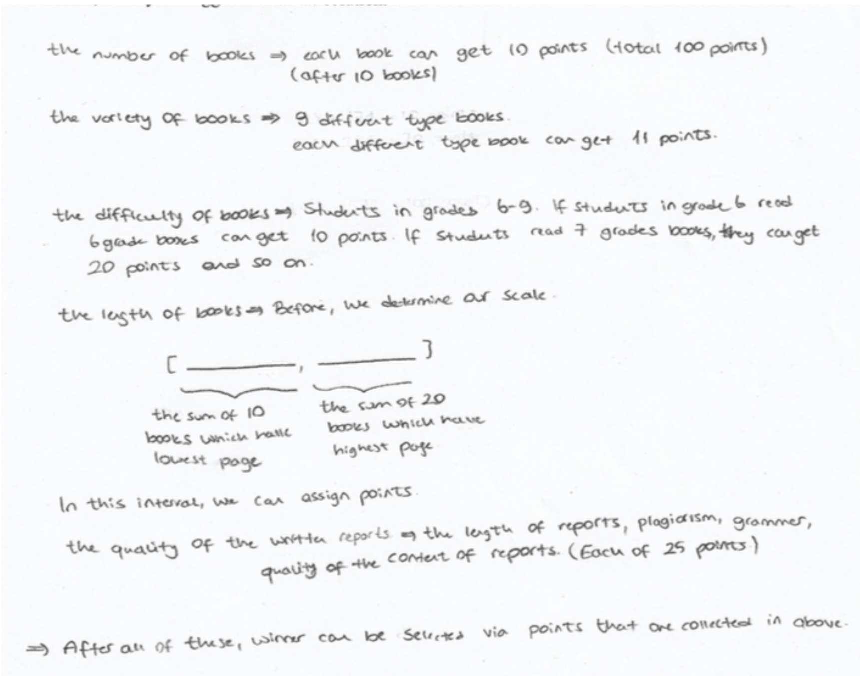
PT1: It depends on her, we didn't write it.

[...]

PT1: We give points to each detail. The winner will have more than 100 points.

PT3: Yes, he will get lots of points."

In summary, as it was represented in the following, they considered five criteria such that number of the books, variety of the books, difficulty of the books, length of the books, and quality of the written reports. Similar to the first group, they also assumed the sample list of books as the list of all books that students will read during the program. Based on this, they decided to give 10 points for each book that students read based on the assumption that they read between 10 and 20 books. To assign points for the variety of the books, they decided to consider the 9 types of books and give 11 points to each type of the book that students read. In addition, they suggested to consider the grade level of the students and books for the difficulty of the books. According to their suggestion, if students read books at the same grade level as their grade level, they can get 10 points. If they read one grade level higher, they can get 20 points. As it is seen in the following, they determined a scale with 10 points intervals for the evaluation of the difficulty of books. They decided to assign points to the length of the books according to the interval determined by the sum of 10 books having minimum number of pages and the sum of 20 books with maximum number of pages. Finally, they concluded to evaluate the quality of the books by considering the length of the reports, plagiarism, grammar, quality of the content of the reports.



the number of books ⇒ each book can get 10 points (total 100 points)
(after 10 books)

the variety of books ⇒ 9 different type books.
each different type book can get 11 points.

the difficulty of books ⇒ Students in grades 6-9. If students in grade 6 read 6 grade books can get 10 points. If students read 7 grades books, they can get 20 points and so on.

the legth of books ⇒ Before, we determine our scale.

[______ , ______]

the sum of 10 books which have lowest page

the sum of 20 books which have highest page

In this interval, we can assign points.

the quality of the written reports ⇒ the legth of reports, plagiarism, grammer, quality of the content of reports. (Each of 25 points)

⇒ After all of these, winner can be selected via points that are collected in above.

Figure 6.12. The group solution of the second group.

At the end of the group study, they shared their solution with their classmates. It might be understood from the data analysis that the stages of the modeling process were not observed in a straight order. During the group study, developing and formulating a model, computing a solution of the model, interpreting the solution and drawing conclusions, and reporting the solution were the observed stages of the mathematical modeling process (Figure 6.13).

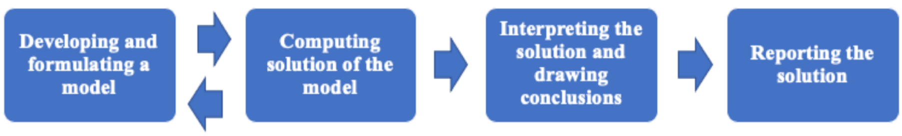


Figure 6.13. Second group's modeling process during the group study.

When the modeling processes experienced by these two groups were compared, it can be observed that, the process included different cycles although they included the same elements of the modeling cycle. First group started with doing some computations on the PT4's individual solution where the second group started with discussions for the formulation of a model. The first group continued to the study with interpretations, and developing a model for the quality of the written reports. It was followed by the computations on the model. This process continued with the iterations of the developing a model and computing the model elements of the modeling cycle. On the other hand, second group continued to group study with computing the model. There were repetitions of the modeling elements of modeling cycle that formulating model and computing the model. After the iterations, they interpreted their solution. Finally, both groups shared their group solution with class at the end of the group study.

When all the groups reported their group solutions, the post-questionnaire was implemented. After that, there was a class discussion including also some of the questions in the post-questionnaire. PTs were asked about the properties and the nature of the problems in the activities they engaged in. Also, the mathematics that they used to solve the problems was also discussed. According to PTs, they used graphs, circles, tables, average, addition, subtraction, multiplication, maximum and minimum as mathematical concepts. Also, they found the "Summer Reading Program" modeling activity more difficult than the "Lost Cell Phone" modeling activity since the former one includes multiple possible solutions, logical thinking, lots of variables and requires lots of assumptions while the latter one has an answer if the graph can be drawn big enough. On the other hand, they indicated that they used coordinate system, equations, and formula as models to solve the given problem situation in the activities.

To sum up, during the "Lost Cell Phone" modeling activity, the groups experienced different modeling cycles although they developed similar solutions. Similarly, both groups experienced different modeling processes during the "Summer Reading Program" modeling activity. The elements of the modeling cycles observed during this activity were the same, but PTs experienced them in different order. In addition, it should be noted here that although most of the PTs did not have any experience of modeling activity before this study, most of the modeling cycle elements were observed throughout the study.

7. CONCLUSION AND DISCUSSION

The purpose of this study was to investigate the PTs' modeling processes when they engaged in mathematical modeling activities, and their opinions about mathematical model and modeling. In this regard, the findings of the study will be discussed in this part focusing on the findings of the pre- and post-questionnaires, and the findings of the data that showed the PTs' mathematical modeling processes when they engaged in modeling activities. Then, the implications, limitations, and suggestions for future research studies will be presented.

7.1. Prospective Teachers' Opinions on Mathematical Model and Modeling

In the previous chapter, PTs' explanations of mathematical model and modeling were provided. PTs explained mathematical models in four different ways: i) giving examples, ii) relating it to real-life, iii) relating it with other disciplines, and iv) as a visualization or teaching method. They gave graphs, tables, geometric shapes, and coordinate system as examples of the mathematical models. It was noticed that, the coordinate system was given as an example of the mathematical model after the implementation of the "Lost Cell Phone" modeling activity in which they used coordinate plane to solve the problem. Furthermore, they explained mathematical modeling by relating to real-life/daily life, as a method to visualize or make abstract things more concrete, and as a problem-solving method.

When these findings were considered, it can be seen that they explained mathematical models and modeling with examples and they thought them as they are used in visualizing some mathematical concepts. Hence, it can be implied that the analysis of the data showed that PTs approached mathematical models and modeling by relating them to using representational and concrete materials to make mathematics more concrete. It can be also stated that prior to implementation, PTs had limited knowledge about the definitions of mathematical models and modeling. This result is consistent with previous research studies that mathematical models and modeling is mostly associated with concrete materials and manipulatives and visualization of mathematical concepts to make them more concrete (Abramovich, 2010; Anhalt and Cortez, 2016; Gould, 2013; Güder, 2013; Korkmaz, 2014). Gould (2013) aimed to determine the conceptions and misconceptions of teachers about mathematical models and modeling in order to aid in the development of teacher education and professional development programs. At the end of this study, it was found that most of the teachers understand mathematical models and modeling as they are physical manipulatives, and types of visual representations such as equations, formula, graphs, or scaled maps (Gould, 2013). Likewise, most of the teachers believed in that modeling activities include concrete manipulatives and materials in the study of Abramovich (2010) which was conducted with prospective K-12 teachers including their reflections on the use of modeling. Also, Korkmaz (2014) investigated the evolution of prospective mathematics teachers? thinking about mathematical modeling and pedagogical knowledge of mathematical modeling throughout a modeling course. It was found that almost all of the prospective mathematics teachers thought mathematical modeling is related to the concrete manipulatives and visualizing the abstract mathematical concepts (Korkmaz, 2014).

Moreover, PTs also related mathematical models and modeling with real-life. They explained mathematical models as they are used in solving real-life problems. Also, most of the PTs changed their opinions about mathematical models from ‘by giving examples’ to ‘relating to real-life’, and mathematical modeling from ‘visualizing method’ to ‘relating real-life’. On the other hand, in the post-questionnaire, PTs were asked to explain the characteristics of the modeling activities that they engaged in during the study. Most of them related mathematical modeling activities with real-life problem situations. These results support the findings of the previous research studies (Anhalt and Cortez, 2016; Korkmaz, 2014; Tekin, Kula, Hıdırog?lu, Bukova-Gu?zel, and Ug?urel, 2012) that PTs associated mathematical modeling with real-life situations. In the study of Anhalt and Cortez (2016), PTs emphasized the role of real-life or everyday life in mathematical modeling. At the end of the study of Korkmaz (2014), PTs changed their conceptions on mathematical modeling from ‘using concrete manipulatives' to 'relating real-life situations'.

In this study, most of the PTs indicated that they did not engage in mathematical modeling activities before this study. Also, in his study Kertil (2008) investigated the prospective mathematics teachers' ability of using mathematics and problem solving in modeling activities. It was observed that PTs had difficulty in solving modeling problems since they were not accustomed to solving mathematical modeling problems and they did not have enough knowledge about mathematical modeling. Thus, it can be indicated that there is a need of involvement of courses on mathematical modeling in teacher education programs to provide PTs the opportunity to experience mathematical modeling (Lingefjard, 2007).

7.2. Prospective Teachers' Opinions on the Characteristics of the Mathematical Modeling Activities

In the post-questionnaire, after their experience of the mathematical modeling activities, PTs explained the characteristics of the modeling activities that they engaged in during the study. Most of them stated that these activities involve daily life problem situations that was also indicated as "reality principle" by Lesh *et al.* (2000). PTs also indicated that modeling activities require making or finding a model to solve it which is one of the characteristics that Lesh et al. (2000) identified as "the model construction principle". Moreover, according to PTs modeling activities help to increase analytical thinking abilities. This was supported by Lesh *et al.* (2000) indicating that while working on the modeling activities students go beyond producing a brief answer to a question since modeling activities require describing, explaining, quantifying, organizing, coordinatizing, and dimensionalizing, i.e. mathematizing, and interpreting the given problem situation. Hence, it can be inferred that experiencing this kind of processes help problem solves to think mathematically and analytically. On the other hand, in the post questionnaire, PTs' opinions about using mathematical modeling activities in their future instructions were also asked. It was found that, all of the PTs indicated that they will use mathematical modeling activities in their future mathematics classroom settings. PTs indicated that they want to enable students to understand where they can use their mathematical knowledge in their daily lives, to develop their cognitive abilities, to understand the relation between their mathematical knowledge and the mathematical concepts and to think in higher levels.

7.3. Mathematical Modeling Processes Experienced by Prospective Teachers

The modeling processes experienced by PTs during the implementation of both activities were presented in the previous chapter. The results showed that although PTs considered different assumptions to solve the problems, they generally approached to the problems in similar ways.

7.3.1. Mathematical Modeling Processes Experienced by Prospective Teachers during the "Lost Cell Phone" Modeling Activity

For the "Lost Cell Phone" activity, both groups assumed the towers scan circular areas. Hence, they took the given coordinates of the towers as the centers and the distances to the cell phone detected by the towers as radii of the circles. They developed similar suggestions for the solution of the problem which is consistent with the findings of the study of Anhalt and Cortez (2016) that PTs plotted three circles with centers at the tower locations and radii given by the distances to the cell phone recorded by the towers. Also, both groups thought that the circles intersect at a point or a region. On the other hand, first group also assumed that the cell phone can also be in the intersection areas of each two circles. Hence, they determined three intersectional regions that each two of three circles intersect and named them as A1, A2, and A3. Furthermore, when the first group didn?t consider solving the equations of the circles, second group focused on solving the equations of the three circles. However, second group indicated that they couldn't find the intersection point of the three circles since they couldn't solve these three equations. They thought that they couldn't solve the equations because they had difficulties in doing the computations with the given numbers. However, it is the fact that those three circles do not intersect at a single point. This result showed that, PTs had some computational difficulties and mistakes, and they considered their intuitive solutions as correct solutions instead of paying attention to doing mathematical computations. This is consistent with that PTs have some difficulties in forming a mathematical model and working on the mathematical model (Bukova-Güzel and Uğurel, 2010; Şen-Zeytun, 2013).

On the other hand, the group study of both groups was based on the individual solutions they developed because they had similar individual solutions during the implementation of "Lost Cell Phone" modeling activity. First group started their group study by drawing the circles directly where second group decided to solve the equations they found during the individual study at the beginning of the group study. Second group assumed that if they solve the three equations, they can find the single point that circles intersect. However, second group also decided to draw the circles after they shared their individual solutions with their group members, and they agreed on the individual solution of PT2. Similar to the first activity, first group started to their group study with sharing of individual solutions in the implementation of the "Summer Reading Program" modeling activity. After PT4 explained his individual solution, they decided to revise his solution, and present it as a group solution. This result was also observed in the study of Şen-Zeytun (2013) that PTs used one of their group members' individual solution, they worked on the solution together, and enhanced it to produce the group solution.

Unlike the first group, the second group reconsidered their group solution and decided to draw the circles for the second time to show the intersection point of the circles. They thought that they could not observe the intersection point because they might have some mistakes stemming from rounding of numbers. They retried to find the exact point of intersection, but they could not find it. In other words, they reconsidered their initial solution once more and tried to revise it. The same finding was observed in the study of Anhalt and Cortez (2016).

When the modeling processes experienced by both groups during the "Lost Cell Phone" activity were compared, there were similar elements of the modeling cycle such as developing a model, computing a solution of the model, and reporting the solution.

Both groups formulated their group models which were similar to each other, made some computations on their models, and shared their group models with classroom. On the other hand, the second group experienced the other elements of the modeling cycle including validation of the conclusions, and development of the new or modified model elements of the modeling process when they reconsider their initial group solution, and tried to revise it. Moreover, it was also observed that none of PTs in both groups assumed that the towers make correct measurements. They did not consider that towers might have some errors in their measurements. Conversely, in the study of Anhalt and Cortez (2016) PTs assumed that there can be errors in measurements of towers and they determined the $\pm 5\%$ margin of error and revised their initial group solution.

7.3.2. Mathematical Modeling Processes Experienced by Prospective Teachers during the "Summer Reading Program" Modeling Activity

For the "Summer Reading Program" modeling activity, PTs considered many factors and several possible situations during the solution process. They made different assumptions and approached to the problem in different ways based on their assumptions. They gave examples, quantified the data, justified their arguments and questioned the fairness of the suggestions they made during the discussion process. In the studies of the English and Fox (2005), and Daher and Shahbari (2015) problem solvers had also comprehensive discussions based on their various assumptions for the solution of the problem.

First group in the "Summer Reading Program" activity started to the group study by explaining individual solutions and they decided to continue with the PT4's individual solution. They worked on that solution and went through a modeling cycle including the elements that working on the PT4's model and doing computations on it, interpreting the solution, developing a model, computing the model, and sharing the group solution. Basically, it can be indicated that they had a cyclic process. On the other hand, second group also started to construct their solution after they explained their individual solutions. They discussed on the given situation and made assumptions. The modeling cycle element of formulation of the model dominated the group study.

After they developed their solution, they reflected on their solution and interpreted it regarding the fairness. A linear modeling process including the modeling elements that are developing a model, interpreting a model, and sharing the group solution was observed. The results showed that some of the groups experienced linear modeling process rather than cyclic process implying that most of the PTs believed in the correctness of their solutions, and they did not check whether their solutions are correct or not. Therefore, it can be inferred that the validating conclusions, and the developing a new or modified model elements of the modeling cycle were not observed in all groups. This confirms the findings of previous research studies that problem solvers have difficulties in validating (Anhalt and Cortez, 2016; Blum and Borromeo-Ferri, 2009; Bukova-Güzel, 2011; Maaß, 2006; Şen-Zeytun, 2013).

Moreover, PTs indicated that although they did not have to use complex mathematical concepts or ideas to solve the "Summer Reading Program" modeling activity, they found it more difficult. Mathematizing requires making symbolic explanations of meaningful situations whereas traditional textbook problems tend to emphasize opposite kind of processes such as making meaning out of symbolic descriptions (Lesh *et al.*, 2000). During the solution process, PTs needed to consider many variables, make several assumptions, and tried to explain the situation mathematically. It might be indicated that as a result of the open-ended, nonroutine nature of the problem situation in this activity, they found it difficult. On the other hand, one of the groups experienced most of the modeling cycle elements while solving "Lost Cell Phone". The problem situation in this activity involves geometrical computations and various ways to solve the problem but it seems like it has one correct answer. Since PTs did more geometrical and mathematical computations while working on this problem, they might have felt the need to validate their computations and revise the solution.

In general, all of the groups experienced basically three elements of modeling cycle: formulating a model, computing a solution of the model, and reporting the solution. During the solution process of modeling activities, there were only iterations of the modeling cycle elements that formulating a model and computing a solution of the model. In addition, some of them experienced the other elements of the modeling process, namely the interpreting the solution, validating the conclusions, and formulating a new or modified model. It was observed that the validation of conclusions, and formulation of a modified model stages were accomplished by only one group. Most of the PTs did not consider to check their solutions and revise their solution. They might have such kind of difficulty since they were inexperienced in modeling.

To sum up, before modeling process experience, most of the PTs approached to the mathematical model and modeling in the same way and they tried to explain them by giving examples. After engaging in the modeling activities, experiencing the modeling process and developing solutions for the problems including complex real-life problem situations, i.e. modeling problems, together with their group members, their opinions on the mathematical model and modeling were affected. They explained the mathematical model and modeling by relating with real-life and emphasized the reality characteristics of the modeling activities. They also indicated that the modeling activities are important in mathematics education since these activities enable them to understand where they can utilize their mathematical knowledge in real-life, and help to improve their cognitive abilities.

7.4. Suggestions, Limitations, and Implications for Future Research Studies

In this part, suggestions for the mathematics teacher education programs and teacher educators, and mathematics education was provided. Limitations and implications for future research studies were also presented.

7.4.1. Suggestions

This study showed that PTs approached to the mathematical model and modeling in different ways. Although most of them had no experience of modeling activities, they successfully worked on the problems and found solutions.

Findings of this study suggests that using modeling activities in the related courses and incorporating modeling courses in the curriculum of teacher education programs would be helpful to provide PTs the opportunity of improving their skills in modeling and pedagogical knowledge for implementing these activities in their future mathematics classes (Bukova-Güzel, 2011; Kertil, 2008; Korkmaz, 2014; Maaß, 2006; Şen-Zeytun, 2013).

As it was indicated before, PTs indicated that modeling activities include real-life contexts and require to construct models to solve the problem. Modeling activities provide students the opportunity to meet mathematical and daily life problem situations (Lesh *et al.*, 2000; Lesh and Doerr, 2003; English and Walters, 2004). In addition, modeling activities involve nonroutine problems and they offer rich learning environments for students to develop powerful mathematical ideas (Doerr and English, 2003; Lesh *et al.*, 2000), and support significant forms of learning since they usually involve mathematizing which is about quantifying, dimensionalizing, categorizing, and systematizing related objects, relationships, patterns, and regularities (Lesh and Doerr, 2003). In this context, integrating modeling activities in the teacher education programs enable PTs to have rich mathematical learning environments and to use their mathematical knowledge appropriately in different real-life situations.

The results revealed that PTs had difficulties in doing some computations to solve the "Lost Cell Phone" modeling activity. They could not find that the circles they have drawn do not intersect at a single point. Since they couldn't solve the system of the equations by hand, they intuitively believed in that they are intersecting at a single point. Using technology may help students to do more with less mathematics (Ang, 2010). Proper utilization of the technology can be a useful tool for learners to model the situation in the modeling activity (Daher and Shahbari, 2015). Therefore, using appropriate technology might be suggested since it would be beneficial for learners engaged in the modeling activities.

7.4.2. Limitations, and Implications for Future Research Studies

This study was aimed to examine, the modeling processes that PTs experienced, and the change in the PTs' opinions about mathematical models and modeling. It was conducted in the 'Teaching Algebra and Pre-Calculus for Secondary School' course in the secondary school mathematics education program. During the study, two modeling activities were implemented to observe the modeling process that they experienced. A pre- and post-questionnaire were also applied to see the change in their opinions about the models and modeling. Although some changes in PTs' opinions were observed, it was impossible to examine all changes in their opinions about model and modeling in detail because of the implementation of limited number of modeling activities. This can be accepted as a limitation of the study. Therefore, more modeling activities could be implemented to get more detailed and accurate results by conducting long-term studies.

The video recordings of group studies and the transcriptions of this recordings, and the individual and group solutions were used to examine the modeling processes that PTs experienced. It might be indicated that although these data sources were helpful in understanding of their experiences of modeling processes, they did not reflect all details about the process. This can be accepted as another limitation of this study. Therefore, after the implementation of each modeling activity, interviews would be applied to understand how they experienced each modeling process.

This study can be considered as an initial study to examine the PTs' experience of modeling process, and the change in their opinions about mathematical model and modeling. Investigations with teachers and students are also needed. In addition, during this study, modeling activities existing in literature were implemented. In future studies, researchers can develop new modeling activities that would be available for classroom use. Also, longer implementations can include allowing PTs to develop their own modeling activities.

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APPENDIX A: PRE-QUESTIONNAIRE

Name-Surname:

Major:

Years in university:

If you took the following courses, please state in which semester you completed those courses:

Sced 320 (Teaching Methods in Science and Mathematics):

Sced 450 (School Experience in Teaching Science and Mathematics):

1) Have you ever heard the phrase "mathematical model" before? What do you understand from that phrase? (You can explain by giving examples).

2) What do you understand from the phrase "mathematical modeling"? Please explain in detail.

3) Have you ever had an experience of involving in mathematical modeling activity(s)? If your answer is 'Yes', please explain what kind of activity(s) that you had experienced and please give details about your experience.

APPENDIX B: THE LOST CELL PHONE MODELING ACTIVITY

Figure B.1. The Lost Cell Phone Modeling Activity.

A lost cell phone needs to be found. Fortunately, three cell phone towers detect the cell phone signal. Tower 1 detects the signal at a distance of 1072.7 m. Tower 2 detects the signal at a distance of 1213.7 m. Tower 3 detects the signal at a distance of 576.6 m.

Based on a coordinate system used by the city, the cell towers are located at (x, y) coordinates as follows: Cell tower 1 is at position (1200, 200) measured in meters from the center of town. Cell tower 2 is at position (800, -450) measured in meters from the center of town. Cell tower 3 is at position (-100, 230) measured in meters from the center of town.

Suggest a way for finding the location of the cell phone. Explain your reasoning.

APPENDIX C: SUMMER READING PROGRAM MODELING ACTIVITY

Figure C.1. Summer Reading Program Modeling Activity.

The Brisbane City Council Library and St. Peters School are sponsoring a summer reading program. Students in grades 6-9 will read books and prepare written reports about each book to collect points and win prizes. The winner in each class will be he student who has earned the most reading points. The overall winner will be the student who earns the most points.

A collection of approved books has already been selected and put on reserve. See the next page for a sample of this collection. Students who enroll in the program often read between ten and twenty books over the summer. The contest committee is trying to figure out a fair way to assign points each student. Margaret Scott, the program director, said, “Whatever procedure is used, we want to take into account: (a) the number of books, (b) the variety of books, (c) the difficulty of the books, (d) the lengths of the books, and (e) the quality of the written reports”.

Note: The students are given grades of A+, A, A-, B+, B, B-, C+, C, C-, D, or F for the quality of their written reports.

Your mission: Write a letter to Margaret Scott to explain mathematically how to assign points to each student for all of the books that the student reads and writes about during the summer reading program.

Table C.1. A Sample Collection of Approved Books.

	Title of the Book	Author(s)	Reading Level	Number of Pages	Description of the Book
1	Harry Potter and the Sorcerer's Stone	J.K. Rowling	Grade 4	309	Fantasy
2	Because of Mr. Terupt	Rob Buyea	Grade 4	304	Realistic
3	The Lion, The Witch, and The Wardrobe	C.S. Lewis	Grade 5	224	Fantasy and Adventure
4	Odd and the Frost Giants	Neil Gaiman	Grade 5	128	Fantasy and Adventure
5	Surrounded By Sharks	Michael Northrop	Grade 5	224	Adventure
6	Heart of a Samurai	Margi Preus	Grade 5	336	Historical Fiction
7	The Jupiter Pirates: Hunt for the Hydra	Jason Fry	Grade 6	256	Fantasy and Adventure
8	Sky Raiders (Five Kingdoms)	Brandon Mull	Grade 6	432	Fantasy and Adventure
9	Keeper of the Lost Cities	Shannon Messenger	Grade 6	512	Mystery
10	The Seventh Wish	Kate Messner	Grade 6	240	Realistic
11	The Book Thief	Markus Zusak	Grade 7	592	Historical Fiction
12	The False Prince	Jennifer Nielsen	Grade 7	355	Fantasy
13	Deep Blue	Jennifer Donnelly	Grade 7	352	Science Fiction
14	Mockingbird	Kathryn Erskine	Grade 7	256	Realistic
15	The London Eye Mystery	Siobhan Dowd	Grade 7	336	Adventure
16	Sabriel (Old Kingdom)	Garth Nix	Grade 8	496	Science Fiction
17	Life of Pi	Yann Martel	Grade 8	326	Adventure
18	The Last Lecture	Randy Pausch	Grade 8	206	Realistic
19	The Eye of Minds	James Dashner	Grade 9	342	Science Fiction
20	A Study in Charlotte	Brittany Cavallaro	Grade 9	336	Mystery
21	Mosquitoland	David Arnold	Grade 9	384	Realistic
22	Orbiting Jupiter	Gary D. Schmidt	Grade 10	192	Realistic
23	The Chess Queen Enigma: A Stoker & Holmes Novel	Colleen Gleason	Grade 10	360	Historical Mystery
24	Red Rising	Pierce Brown	Grade 10	382	Science Fiction

APPENDIX D: POST-QUESTIONNAIRE

Name-Surname:

Major:

Years in university:

If you took the following courses, please state in which semester you completed those courses:

Sced 320 (Teaching Methods in Science and Mathematics):

Sced 450 (School Experience in Teaching Science and Mathematics):

1) Have you ever heard the phrase "mathematical model" before? What do you understand from that phrase? (You can explain by giving examples).

2) What do you understand from the phrase "mathematical modeling"? Please explain in detail.

3) What were the characteristics of the problems stated in these activities that you have worked on?

4) What do you think about the implementation of the mathematical modeling activities in the mathematics classes? When you become a teacher, would you implement modeling activities in your classroom? Why?

APPENDIX E: TABLE OF MODELING PROCESS ELEMENTS USED IN DATA ANALYSIS

Table E.1. Table of Modeling Process Elements Used in Data Analysis.

Modeling Elements within the modeling cycle (CCSSI 2010, p. 72) have been expanded by Anhalt and Cortez (2015)	**Description and Example from the Data**
1. Analyze the situation or problem	• Reading and trying to understand the problem situation
2. Developing and formulating a model	• Determining all the given information • Discussion about the meaning of the variables • Choosing which variables will be used • Determining the relationship between those variables PT6: Let's draw the coordinates. PT9: Ok. PT6: Look, you don't have to go in the direction of minus x-axis. [...] PT6: You don't have to go to there [showing the negative side of the x-axis on the coordinate plane], to minus x-axis, you don't have to go further there. PT6: In my opinion, we can draw it as a draft. PT9: Exactly. Then, I am starting to draw it.
3. Compute a solution of the model	• Trying to formulize and mathematising the problem situation • Mathematical discussion • Doing mathematical computations Example Excerpt(s): PTs tried to compute a solution of the model they have developed: PT10: Well, it is ok if the phone is in the intersection point of these three circles, but what if is not there? PT6: Then, for example you can look at the intersection of only these two circles and you will subtract this area [the intersection of other circle and this intersection area] from this intersection. Then the area becomes here. If only these two intersect, then you will subtract this area [the intersection of other circle and this intersection area] from the intersection area of these two and you will search here. There is nothing else to do.
4. Interpret the solution and draw conclusions	• Interpreting the solution of the problem considering the original problem situation • Drawing conclusions from the solution to make implications about the original situation Example Excerpt(s): PTs tried to interpret their solution considering the possible conclusions: PT4: But, for example, giving 5 points for

Table E.1. Table of Modeling Process Elements Used in Data Analysis (Cont).

Modeling Elements within the modeling cycle (CCSSI 2010, p. 72) have been expanded by Anhalt and Cortez (2015)	**Description and Example from the Data**
	24 books to give 5 points is not very little? It seems to me a little, but not a book. PT7: I thought that one can also read between 17 to 20 easy books to get 5 points, but ultimately, it doesn't matter. PT4: Hmm, yes. PT7: Something like that. PT4: But I don't think there are easy books to read. [...] PT4: It might be better if I make it 10. PT6: Exactly. PT4: Because there is no chance of closing the difference between the scores with just the report score. I think they should have such kind of chance. PT6: Then, I think the one will win who reads more books.
5. Validate conclusions	• Validating the mathematical interpretations with examples from daily life • Comparison of suggestions • Verification of the solution Example Excerpt(s): PTs saw that the circles they drew do not intersect, then they decided to re-draw them. PT2: Let's do our second experiment [implying to re-draw the circles] PT1: Well, shall we do like this for this time,
6. Develop and formulate a new or modified model	• Deciding to revise the model or formulate a new model based on the validation Example Excerpt(s): PTs draw the circles again to be sure about the correctness of their solution: PT2: Let's do our second experiment. PT1: Well, shall we do like this for this time, PT2: Let's do our second experiment. PT1: Well, shall we do like this for this time, PT2: It does not matter, PT1: Let's do it like this for this time, for example 1 unit will represent 100 PT3: Ok, PT1: Then, it will be easier to divide
7. Report the solution	• Sharing the solution