# DETERMINING OPTIMAL QUALITY LEVELS AND PRICES IN A HYBRID MANUFACTURING / REMANUFACTURING SYSTEM

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*To My Family*

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# ABSTRACT


## DETERMINING OPTIMAL QUALITY LEVELS AND PRICES IN A HYBRID MANUFACTURING / REMANUFACTURING SYSTEM

Setting the quality and price of products are two important issues for firms involved in remanufacturing. The aim of this study is to develop and solve a mathematical model that seeks optimum values for quality levels and prices of remanufactured products as well as prices of manufactured products. A base model and two extensions of it are formulated in order to maximize the total profit of an Original Equipment Manufacturer (OEM) having a hybrid manufacturing / remanufacturing system. It is assumed that a remanufactured product's quality level can not exceed the quality level of its manufactured version and the unit cost of remanufacturing is a linear function of the quality level. Customer preference is modeled by using gravity based approach and total demand is shared by the products of the OEM and a competitor's product according to this preference function.

As the formulated models are nonlinear and nonconvex, the simplex search method developed by Nelder and Mead is used to solve the models. Since the method has originally been proposed for unconstrained problems, some modifications are necessary to handle quality and price constraints. A number of experiments are carried out to obtain the best values of the decision variables and to see how they are affected by the changes in the problem parameters. The effects of changes in the market, in the manufacturing environment and in the quality levels of manufactured products are discussed in detail.

# ÖZET

## MELEZ BİR İMALAT / YENİDEN İMALAT SİSTEMİNDE EN İYİ KALİTE SEVİYESİ VE FİYATLARIN BELİRLENMESİ

Ürünlerin kalite ve fiyatlarının belirlenmesi yeniden üretim yapan firmalar için iki önemli konudur. Bu çalışmanın amacı; yeniden imal edilmiş ürünlerin kalite seviyesi ve fiyatlarının, ayrıca imal edilmiş ürünlerin fiyatlarının en iyi değerlerini araştıran bir matematiksel model geliştirip çözmektir. Melez bir imalat / yeniden imalat sistemi olan bir orijinal parça üreticisinin toplam kârını enbüyüklemek için bir esas model ve iki eklentisi formüle edilmiştir. Yeniden imal edilmiş bir ürünün kalite seviyesinin imal edilmiş versiyonunun kalite seviyesini aşamayacağı ve yeniden imal etmenin birim maliyetinin, kalite seviyesinin doğrusal bir fonksiyonu olduğu varsayılmıştır. Müşteri tercihi yerçekimine dayanan yaklaşım kullanılarak modellenmiştir ve toplam talep, orijinal parça üreticisinin imal edilmiş ve yeniden imal edilmiş ürünleri ile rakip firmanın bir ürünü tarafından bu tercih fonksiyonuna göre paylaşılmaktadır.

Formüle edilen modeller doğrusal ve dışbükey olmadığından, modelleri çözmek için Nelder ve Mead tarafından geliştirilen simpleks arama yöntemi kullanılmıştır. Yöntem orijinal olarak kısıtsız problemler için önerildiğinden, kalite ve fiyat kısıtlarını sağlamak için bazı değişiklikler yapılmıştır. Karar değişkenlerinin en iyi değerlerini bulmak ve problem parametrelerindeki değişikliklerden nasıl etkilendiklerini görmek için bir dizi deney yapılmıştır. Pazardaki, üretim ortamındaki ve imal edilmiş ürünlerin kalitelerindeki değişikliklerin etkileri ayrıntılı olarak irdelenmiştir.

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| | |
|---|---|
| $A_j$ | Amount of product *j* sold to customers |
| $c$ | Cost coefficient |
| $c_r$ | Remanufacturing cost coefficient |
| $C_j$ | Unit cost of producing product *j* |
| $C_M$ | Unit cost of manufactured product for three-product problem |
| $C_R$ | Unit cost of remanufactured product for three-product problem |
| $d$ | Total demand |
| $\mathbf{D}$ | Direction vector used in the formation of simplex |
| $i$ | Index for simplex vertices |
| $j$ | Index for products |
| $k$ | A parameter used in the quality level constraints |
| $n$ | Number of vertices in a simplex |
| $N$ | Set of manufactured and remanufactured products |
| $N_C$ | Set of competitors' products |
| $N_M$ | Set of manufactured products |
| $N_R$ | Set of remanufactured products |
| $P_j$ | Price of product *j* |
| $P_C$ | Price of competitor's product for three-product problem |
| $P_M$ | Price of manufactured product for three-product problem |
| $P_R$ | Price of remanufactured product for three-product problem |
| $Q_j$ | Quality level of product *j* |
| $Q_C$ | Quality level of competitor's product |
| $Q_M$ | Quality level of manufactured product for three-product problem |
| $Q_{Mj}$ | Quality level of manufactured version of remanufactured product *j* |
| $Q_R$ | Quality level of remanufactured product for three-product problem |
| $s$ | Step size parameter |
| $U_j$ | Market share of product *j* |
| $\mathbf{X}$ | A vertex of a simplex |
| $Z$ | Objective function value of a simplex vertex |

| | |
|---|---|
| $\Pi$ | Profit function |
| $\alpha$ | Reflection coefficient in the simplex search algorithm |
| $\beta$ | Contraction coefficient in the simplex search algorithm |
| $\gamma$ | Expansion coefficient in the simplex search algorithm |
| $\varepsilon$ | Parameter for termination condition in the simplex search algorithm |
| $\lambda$ | Exponent of price in the customer preference function |
| $\chi$ | Shrinkage coefficient in the simplex search algorithm |
| $\omega$ | Fixed unit lost profit |

| | |
|---|---|
| OEM | Original Equipment Manufacturer |
| KKT | Karush-Kuhn-Tucker |

# 1. INTRODUCTION

Today's competitive market structure is forcing firms to make use of all the activities that will bring cost advantage. Also, environmentally conscious policies in production receive growing attention because of government legislations, care for the environment and corporate image.

Remanufacturing, which is the process of bringing a used product to as good as new condition, helps manufacturing firms in increasing profit, dealing with environmental legislations and enhancing corporate image. Also, from customer's point of view, remanufacturing provides affordable prices and helps increase environmental awareness.

Remanufacturing has been widening its scope in manufacturing industry rapidly in recent years. Some examples for remanufactured products are automotive parts, vehicle tires, compressors, office furniture, printers, photocopiers, laser toner cartridges, electrical equipments, musical instruments, aircraft parts, etc. Stock et al. (2002) state that over $100 billion worth of products are returned from customers to retailers annually.

Pricing remanufactured products is a crucial decision for the firms. Also, the quality level to which the remanufactured product will be reconditioned is an important decision since it determines both the cost of the remanufactured product and the product's utility to the customer. Although it is assumed that the product is as good as new after remanufacturing by definition, it may be more profitable to remanufacture the used product to a quality level which is less than the quality level of the new product.

So, determining quality levels and pricing are two important issues in remanufacturing which will affect a firm's profit dramatically since they determine both a product's market share and the unit profit gained from the product.

The aim of this study is to formulate and solve a mathematical model that will help remanufacturing firms in making quality level and pricing decisions together. The model seeks optimal quality levels for remanufactured products and optimal prices for all the manufactured and remanufactured products to maximize the total profit.

Since the resulting model is nonlinear and highly nonconvex, obtaining mathematical solutions through derivatives is not practical, so it is necessary to use a direct search method for solving the problems. The simplex search designed for solving multi-dimensional optimization problems is used as the solution procedure. In order to handle the constraints of the problem, some modifications have been made to the basic simplex search procedure.

The remainder of the thesis is organized as follows: The problem description together with the assumptions and motivation behind the study are given in Chapter 2. Chapter 3 includes a brief introduction on remanufacturing and literature review on the quality issues, customer choice and pricing in remanufacturing. Formulation of a base model and two extensions of it are described in detail and objective function plots and optimality conditions are given in Chapter 4. In Chapter 5, simplex search is explained in detail together with the modifications for handling constraints and design of experiments. Results of the experiments, outcomes and insights derived from the results and sensitivity analysis according to problem parameters are provided in Chapter 6. Conclusions of the study and suggestions for further research are contained in the final chapter.

# 2. PROBLEM DESCRIPTION AND MOTIVATION

In this chapter, the problem and its assumptions are described and contribution of the study to the literature is explained. Research questions that we seek answers for are also provided.

## 2.1. Problem Description

An Original Equipment Manufacturer (OEM) with hybrid manufacturing / remanufacturing system is considered where the OEM produces new products and remanufactured versions of them. There are also competitors' products in the market. Although remanufacturing is defined as bringing the used product back to the quality level of the new one, it may be more profitable to remanufacture the products to a lower quality level than their new counterparts. So, the OEM has to decide the quality levels to which remanufactured products will be restored as well as the prices of both remanufactured and manufactured products.

It is assumed that remanufacturing of all manufactured products of the OEM is desired because of legislations or as a strategic decision. So, fixed costs of remanufacturing are not included in the objective function since they should incur in all circumstances. It is also assumed that manufacturing / remanufacturing capacity of the OEM is enough to satisfy the total demand of the market. Unit cost of remanufacturing is taken as a linear function of the quality level of the product, and customer preference is modeled as a function which is increasing in the product's quality level and decreasing in the product's price. If there are two products of the same quality, customers prefer the one with the lower price and if there are two products of the same price, customers prefer the one with the higher quality. Demand is assumed to be constant and shared by the manufactured and remanufactured products of the OEM and the competitors' products according to a preference function. It is also assumed that customer preference function is known by the OEM and it produces exactly the same amount of the products' market share to satisfy demand. All customers are assumed to have the same preference function, so no customer segmentation is considered.

The quality level of a remanufactured product should be less than or equal to its new version. The unit cost of remanufacturing depends only on the quality level that the remanufactured product will be restored. Moreover, its price will be greater than its unit remanufacturing cost. It is also assumed that there are enough used products to satisfy the demand for remanufactured products. Prices of manufactured products must be greater than their unit manufacturing costs, as well.

The aim of the model is to maximize the total profit of the OEM which is the difference between the revenues gained by selling the manufactured and remanufactured products and unit costs of producing them. A nonlinear programming model is constructed to maximize profit for a single period while satisfying quality constraints and lower bound constraints on prices. Since one period is considered, prices and qualities of competitors' products are assumed to be fixed and it is assumed that competitor firms do not react to the policies of the OEM. The input of the model will be prices and quality levels of competitors' products and quality levels of manufactured products of the OEM. The model will determine quality levels of remanufactured products and prices of manufactured and remanufactured products. Market share of each product and expected profit are also calculated.

## 2.2. Motivation

Motivation behind this study is to analyze whether remanufacturing becomes more profitable by choosing appropriate quality levels and prices. The properties of the optimal solutions and model behavior according to problem parameters are also analyzed.

## 2.3. Research Contribution

Studies related with remanufacturing except Mitra (2007) assume that remanufactured products reach a quality level as the new one, but since they contain a remanufactured label on them, they are sold at a lower price. Remanufacturing used products to a lower quality level is considered only in Mitra (2007) where two quality levels are considered: refurbished and remanufactured. However, they are problem parameters rather than decision variables.

This thesis to the best of our knowledge is the first study that seeks optimal quality levels for remanufactured products to maximize the total profit.

## 2.4. Research Questions

Research questions that we seek answers for can be summarized as follows:

i. Are there specific quality levels and prices for remanufactured products which maximize total profit?

ii. If there is more than one optimal value for these variables, do they have common properties?

iii. How do changes in qualities of manufactured products of the OEM affect optimal values of the decision variables and the optimal objective value?

iv. How do changes in manufacturing environment affect optimal values of the decision variables and the optimal objective value?

v. How do changes in market (changes in competitors' products or customer preference) affect optimal values of the decision variables and the optimal objective value?

vi. Does quality management on remanufacturing bring competitive advantage to the OEM; and if so, for which values of problem parameters?

# 3. LITERATURE REVIEW

Remanufacturing has been a popular topic for research in recent years. Studies have widely been focused on acquisition of products, network design with reverse flows, product design for remanufacturing, production control, capacity planning, inventory and supply chain issues, etc. There are also many works on the customer choice and pricing since they have long been considered as crucial managerial problems. In this chapter, a brief introduction to remanufacturing is provided. Moreover, literature on the quality management aspect of remanufacturing and customer choice and pricing are reviewed.

## 3.1. Remanufacturing

Remanufacturing is a manufacturing strategy that makes use of used products utilizing both material values and production energy (value added by production operations of parts) that exist in the product. Remanufacturing brings the product back into an "as new" condition by carrying out the necessary disassembly, overhaul and replacement operations (Fleischmann et al., 1997).

Remanufacturing is ''. . . an industrial process in which worn-out products are restored to like-new condition. Through a series of industrial processes in a factory environment, a discarded product is completely disassembled. Useable parts are cleaned and refurbished. Then the new product is reassembled from the old and, where necessary, new parts to produce a fully equivalent and sometimes superior in performance and expected lifetime to the original new product.'' (Lund, 1983).

Reverse logistics comprises acquisition of used products from customers. Hence, reverse logistics network design has been an important issue in remanufacturing. Fleischmann et al. (1997) give a review on mathematical models for reverse logistics proposed in the literature. Both Bras and McIntosh (1999) and Guide (2000) give an overview of research about remanufacturing.

### 3.2. Quality Issues on Remanufacturing

Studies about quality issues on remanufacturing are limited; especially focusing on quality management of potential used products and quality management of manufactured products that will affect their remanufacturing operations.

Krikke et al. (1998) propose a model that takes the actual condition of the used product into account. There are quality classes for every component of the product; and the quality level of a subassembly depends on the quality class of the parent assembly and transition probabilities between them. The aim of the model is to maximize profit with a stochastic dynamic programming algorithm by selecting a reuse option for each quality class.

Matthews and Lave (1995) considered the option of improving quality levels of manufactured products in order to decrease their remanufacturing costs hence making them more attractive to remanufacturing.

To the author's knowledge, quality management for remanufactured products is studied only by Mitra (2007). He seeks optimum prices for remanufactured products according to their quality levels to maximize total profit. There are two quality levels of remanufactured products: "as good as new" ones named as remanufactured products and "lower quality" items named as refurbished products. A mathematical model is formulated to set the prices of remanufactured and refurbished products such that the total revenue is maximized. The emphasis is on revenue maximization and costs of remanufacturing and refurbishing are not considered. Quality levels are not considered as decision variables, they take part in the model as problem parameters being the available amounts of remanufactured and refurbished products (which are of different quality by definition), and optimum prices are determined in terms of these available amounts. Demand for remanufactured and refurbished products are modeled as linear functions of prices and availabilities such that not all units will be sold. Demand for manufactured products is assumed to be enough such that all of them will be sold, so manufactured products are excluded from the model. Some numerical experiments and sensitivity analysis are carried out and conclusions are drawn accordingly. Cellular phone industry in India is considered and according to conclusions drawn from the model, it is pointed out that since available amount of remanufactured products for cellular phones will increase, remanufacturing will be a huge business opportunity for OEMs and third party remanufacturers.

### 3.3. Customer Choice and Pricing

Pricing of remanufactured products has been studied by various researchers. All of them mention that even though the remanufactured products' qualities and warranty conditions are the same as the new products, they are sold at lower prices (Ferrer, 1997), (Ayres et al., 1997), (Maslennikova and Foley, 2000), (Lebreton and Tuma, 2006).

Matthews and Lave (1995) present economic models aiming to set prices for remanufactured products according to production costs; the prices decline each time the product is used.

Paton (1994) discusses problems and opportunities related to electronic products' reuse and offers methods to integrate reuse into existing business strategies which are integrating refurbished products into the existing product mix, selling used products through alternate channels, and using components in service and support.

Guide et al. (2003) consider both determining quality-dependent acquisition prices of used products and selling prices of remanufactured products to maximize total profit. They conclude that quantity and quality of product returns can be controlled by varying acquisition prices, demand can be influenced by varying the selling price and profit can be maximized by matching supply and demand.

Lebreton and Tuma (2006) propose a mathematical model that aims to maximize profit of the remanufacturing operation for given selling prices and give a case study for tire industry. They introduce quality levels of tires targeting different segments: a high-end tire targeted for the premium segment, a middle-range tire primarily addressing the budget customers and a cheap imported tire for the low-budget segment. Hence, they are considered as different products having separate demands. They conclude that low budget market segment provides a growth potential in an already saturated car tire market, and hence remanufacturing could be a competitive advantage for firms in tire industry.

Inderfurth (2004) seeks an optimum coordinated manufacturing / remanufacturing policy for a hybrid manufacturing / remanufacturing company under product substitution.

Ayres and Ferrer (2000) present an input-output model for discussing the situation where remanufacturing holds a significant share of the economy and apply it to 30-sector aggregation of the French input–output national data. The model assumes that the final demand from remanufacturing and from original manufacturing remains the same and the remanufactured product is sold at a price lower than that of a new one. They conclude that remanufacturing may satisfy the same final demand from all sectors, requiring fewer inputs such as raw materials, semi-finished goods and energy but more labor and transport services.

Vorasayan and Ryan (2005) propose a mathematical model to find the optimal prices of remanufactured products and proportion of incoming products to refurbish that maximize total profit. Prices of new products are not considered as decision variables, since they suggest that prices of new products be determined by market considerations. The producer has control on prices and supply of remanufactured products since it is considered as the sole source of remanufactured products. They conclude that for a manufacturer in a competitive market, introducing remanufactured products to the market can be profitable even when they potentially reduce the demand for new products. According to numerical studies, they suggest that significant proportions of returns should be remanufactured for the cases where high demand for new products and high backorder penalties combine with low remanufacturing costs and high perceived quality of the remanufactured products.

Product line selection and pricing with remanufacturing option is studied in the M.S. thesis by Esenduran (2004). In this study, a mathematical model for product line selection and pricing is formulated and product line selection and pricing subproblems are solved to maximize total profit. Here, a remanufactured product's quality level is fixed to a value which is less than the quality level of its new version, so the unit remanufacturing cost for a product is fixed. Also, two extensions of the base model are introduced. The first one considers availability constraint for the remanufactured products so that the total amount of remanufactured products sold to customers cannot exceed the total available amount of remanufacturable products in the OEM. The second extension includes the cost of lost customers in the objective function.

# 4. MODEL FORMULATION

In this chapter, a base model and two extensions of it are described together with profit function plots that show model behavior according to problem parameters. Also, necessary conditions for optimality are given and are questioned for each model.

## 4.1. Base Model

An OEM producing both new products and remanufactured versions of these in a hybrid manufacturing / remanufacturing facility aims at determining the best quality levels and prices of remanufactured products as well as prices of new products to maximize the total profit.

The constructed nonlinear model maximizes the total profit which is the difference between the revenues gained by selling the manufactured and remanufactured products and unit costs of producing them for a single period subject to pricing and quality constraints.

The OEM produces a set of manufactured products denoted by $N_M$, and remanufactured versions of these denoted by $N_R$. There are competitors' products in the market denoted by $N_C$. The set comprising manufactured and remanufactured products of the OEM is denoted by $N$; namely, $N = N_M \cup N_R$. The products are indexed by $j$ and $Q_j$ denotes quality levels of the products while $P_j$ denotes their prices. Quality levels of remanufactured products and prices of manufactured and remanufactured products are decision variables, while quality levels of manufactured products and competitors' products are parameters as well as prices of competitors' products.

Since the customers' perception of quality of a product depends on a variety of parameters (i.e., configuration, attributes, appearance, functionality, usefulness, performance, robustness, expected lifetime, warranty conditions, etc.), we assumed that $Q_j$ is a function of all these parameters and increasing as they fit the customers' expectations.

The quality level of a remanufactured product should be lower than the quality level of its manufactured version, so the model should have the following constraint:

$$Q_j \leq Q_{Mj} \quad , \quad j \in N_R \, , \tag{4.1}$$

where $Q_{Mj}$ is the quality level of the manufactured version of the remanufactured product *j*. If the quality level of the remanufactured product turns out to be the same as the quality level of its manufactured version, then they both give the same utility to the customer if their prices are also the same.

We have to point out that we did not restrict the model to have $Q_1 < Q_2$ if $Q_{M1} < Q_{M2}$. Namely, a remanufactured product's quality level may be higher than another remanufactured product's quality level even if the ranking of the quality levels of their manufactured versions is in reverse order.

There is a linear relationship between the unit cost of production $C_j$ and its quality level given as

$$C_j = c\,Q_j \quad , \quad j \in N \, , \tag{4.2}$$

where $c$ is an appropriate cost coefficient.

The price $P_j$ of a product should be greater than or equal to its unit cost for both manufactured and remanufactured products:

$$P_j \geq C_j \quad , \quad j \in N \tag{4.3}$$

which can be written as

$$P_j \geq c\,Q_j \quad , \quad j \in N \, . \tag{4.4}$$

There is a demand $d$ for all the products and they get a market share denoted by $U_j$ according to their quality levels and prices by the customer preference function (4.5) given below. This function is formulated by using the gravity based approach proposed by Huff (1964) to location-allocation problems, where the probability that a facility being patronized by a customer is directly proportional to the attractiveness of the facility and inversely proportional to a power of the distance between customer and the facility. The same approach is used for modeling customer preference. Namely, the probability that a customer buys a product is directly proportional to the quality level of the product and inversely proportional to a power of the price of the product.

$$U_j = \frac{Q_j / P_j^{\lambda}}{\sum_{k \in N \cup N_C} Q_k / P_k^{\lambda}} \quad , \quad j \in N \cup N_C \, . \tag{4.5}$$

Customer segmentation is not considered and it is assumed that all customers in the target market have the same preference function.

The numerator of the function (4.5) designates the attractiveness of product $j$ to the customers. The attractiveness is increasing in the quality of the product and decreasing in the price of the product as one might expect. Since $U_j$ denotes the probability of product $j$ to be bought by the customers, the attractiveness is divided by the sum of the attractiveness of all the products that exist in the market. The parameter $\lambda$ is related with the product's industry and the customer segment, but since we assumed that the customers are homogeneous in their purchasing preferences, $\lambda$ turns out to be dependent only on the industry. For some products, the customers' sensitivity in price is higher and for some products, the customers are more sensitive in quality. As $\lambda$ increases, the customers become more sensitive to price and the importance of price in making a buying decision increases. We can say that firms in the industries having a high $\lambda$ value should find a way to decrease the prices whereas firms in the industries having a low $\lambda$ value should supply higher quality products to the customers.

It is seen from the preference function (4.5) that the market share of a product depends on the quality levels and prices of other products existing in the market as well as on its own quality level and price. So, all decision variables, namely, quality levels of remanufactured products, prices of remanufactured and manufactured products are affected from each other and affect profit dramatically.

The amount $A_j$ of products sold is given as

$$A_j = d\,U_j\,, \tag{4.6}$$

where $d$ is the demand parameter. It is assumed that all demand is satisfied by the market share of the OEM's and competitors' products.

The objective function Π is the total profit given as

$$\Pi = \sum_{j \in N} (P_j - C_j)\, A_j \tag{4.7}$$

or equivalently as

$$\Pi = \sum_{j \in N} (P_j - c\,Q_j)\, d\,U_j\,. \tag{4.8}$$

This problem aiming at maximizing Π subject to quality and pricing constraints can be formulated as a nonlinear programming model, which is called the base model:

$$\max \quad \Pi = \sum_{j \in N} (P_j - C_j)\, A_j \tag{4.9}$$

$$\text{such that} \quad Q_j \le Q_{Mj} \qquad j \in N_R \tag{4.10}$$

$$P_j \ge C_j \qquad j \in N \tag{4.11}$$

$$Q_j \ge 0 \qquad j \in N_R \tag{4.12}$$

By substituting $C_j$, $A_j$, and $U_j$, the base model can equivalently be written as

$$\max \quad \Pi = \sum_{j \in N} (P_j - c\,Q_j)\, d\, \frac{Q_j / P_j^{\lambda}}{\sum_{k \in N \cup N_C} Q_k / P_k^{\lambda}} \tag{4.13}$$

$$\text{such that} \quad Q_j \le Q_{Mj} \qquad j \in N_R \tag{4.14}$$

$$P_j \ge c\,Q_j \qquad j \in N \tag{4.15}$$

$$Q_j \ge 0 \qquad j \in N_R \tag{4.16}$$

### 4.1.1. Profit Function Plots

Figure 4.1 shows the profit function as a function of price and quality level of remanufactured product for a problem with one manufactured, one remanufactured and one competitor's product. The price of the manufactured product ($P_M$) is taken as 960, and the parameter values are set as follows: $\lambda = 2$, $c = 16$, the quality level of the manufactured product ($Q_M$) = 30, the quality level of the competitor's product ($Q_C$) = 25 and the price of the competitor's product ($P_C$) = 400.

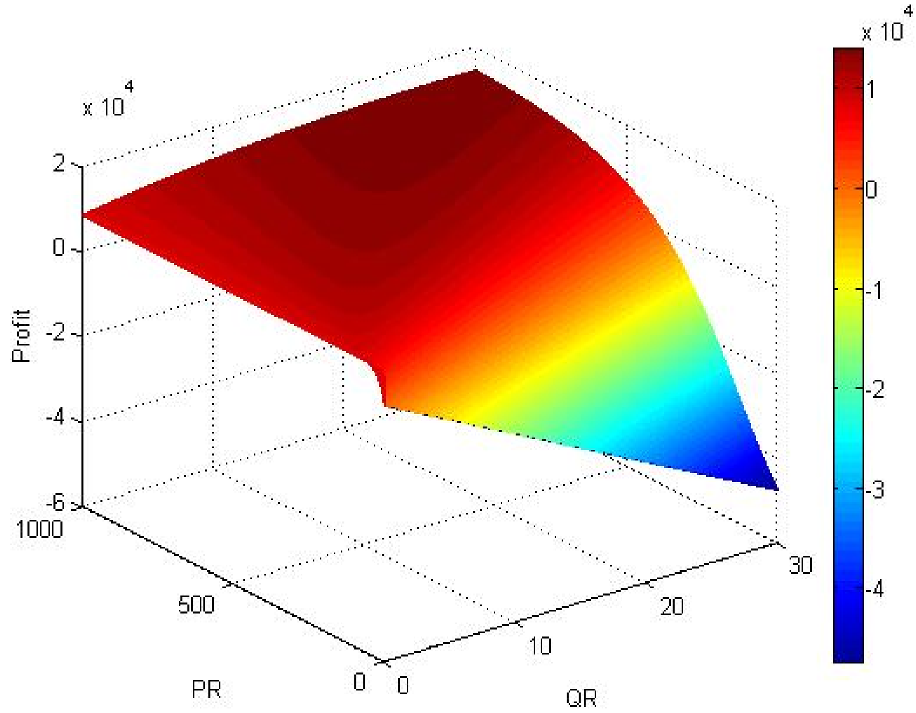


Figure 4.1. Profit of the base model as a function of quality level and price of remanufactured product

Figure 4.2 shows the profit function as a function of prices of manufactured and remanufactured products for a problem with one manufactured, one remanufactured and one competitor's product. The quality level of the remanufactured product ($Q_R$) is taken as 15, while the parameter values are: $\lambda = 2$, $c = 16$, $Q_M = 30$, $Q_C = 25$ and $P_C = 400$.

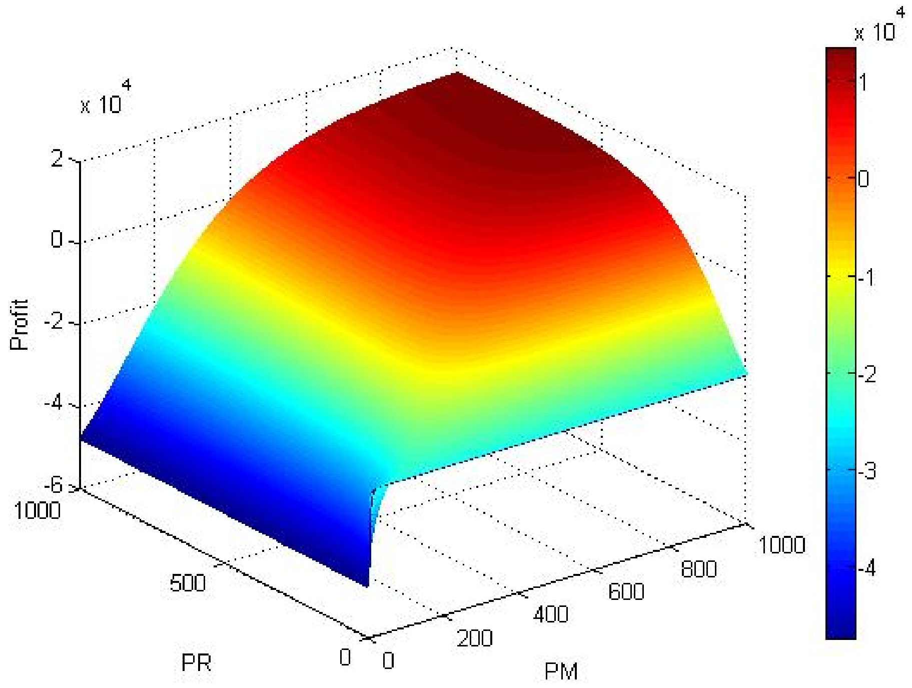


Figure 4.2. Profit of the base model as a function of prices of manufactured and remanufactured products

It seems from the objective function plots of the base model that the profit increases along with the decision variables (quality level of the remanufactured product, prices of the manufactured and remanufactured products). Since the numerical experiments of the base model have set the quality level of the remanufactured product to its upper limit and the prices of the manufactured and remanufactured products to infinity (see section 6.1), it is seen that we should modify the model in an appropriate way to get rid of this unrealistic situation.

### 4.2. Model Including Lost Profit

Since the necessity of adding the model a mechanism that will prevent the model to set the decision variables to infinity is seen, we achieved this by adding a cost item (lost profit) to the profit function for the amount of lost market share. The lost profit is obtained by multiplying the unit lost profit (average profit gained by selling manufactured and remanufactured products) with the amount of competitors' products sold to customers (in other words the amount of lost market share) as it is seen below:

$$\sum_{k \in N_C} A_k \; \frac{\sum_{k \in N} A_k (P_k - C_k)}{\sum_{k \in N} A_k} . \tag{4.17}$$

The new profit function becomes

$$\Pi = \sum_{j \in N} (P_j - C_j)\, A_j - \sum_{k \in N_C} A_k \; \frac{\sum_{k \in N} A_k (P_k - C_k)}{\sum_{k \in N} A_k} \tag{4.18}$$

which simplifies to

$$\Pi = \sum_{j \in N} (P_j - C_j)\, A_j * \left( 1 - \frac{\sum_{k \in N_C} A_k}{\sum_{k \in N} A_k} \right) . \tag{4.19}$$

It can equivalently be written as

$$\Pi = \sum_{j \in N} (P_j - c\,Q_j)\, d \; \frac{Q_j / P_j^{\lambda}}{\sum_{k \in N \cup N_C} Q_k / P_k^{\lambda}} * \left( 1 - \frac{\sum_{k \in N_C} Q_k / P_k^{\lambda}}{\sum_{k \in N} Q_k / P_k^{\lambda}} \right) . \tag{4.20}$$

The extended model where lost profit is considered becomes

$$\max \qquad \Pi = \sum_{j \in N} (P_j - c\,Q_j)\, d \; \frac{Q_j / P_j^{\lambda}}{\sum_{k \in N \cup N_C} Q_k / P_k^{\lambda}} * \left( 1 - \frac{\sum_{k \in N_C} Q_k / P_k^{\lambda}}{\sum_{k \in N} Q_k / P_k^{\lambda}} \right) \tag{4.21}$$

$$\text{such that} \quad Q_j \le Q_{Mj} \qquad j \in N_R \tag{4.22}$$

$$P_j \ge c\,Q_j \qquad j \in N \tag{4.23}$$

$$Q_j \ge 0 \qquad j \in N_R \tag{4.24}$$

### 4.2.1. Profit Function Plots

Figure 4.3 shows the profit function including lost profit with respect to price and quality level of remanufactured product for a problem with one manufactured, one remanufactured and one competitor's product. The price of the manufactured product is taken as 960 which is the optimum value for that decision variable for parameter values $\lambda = 2,\ c = 16,\ Q_M = 30,\ Q_C = 25$ and $P_C = 400$.

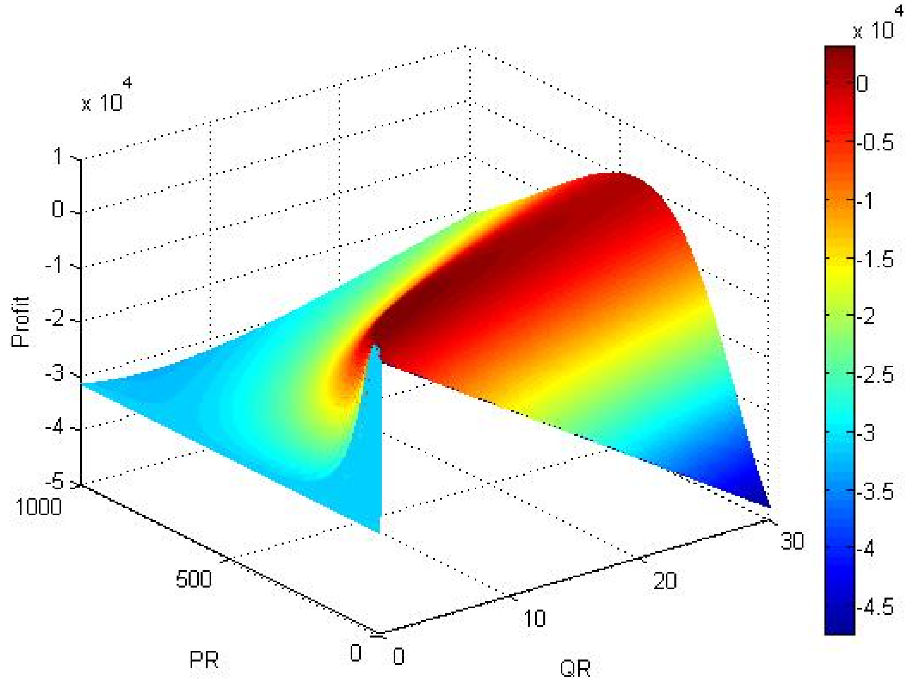


Figure 4.3. Profit of the lost profit case as a function of quality level and price of remanufactured product

Figure 4.4 shows the profit for the lost profit case as a function of prices of manufactured and remanufactured products. The quality level of the remanufactured product is taken as 15, while $\lambda = 2,\ c = 16,\ Q_M = 30,\ Q_C = 25$ and $P_C = 400$.

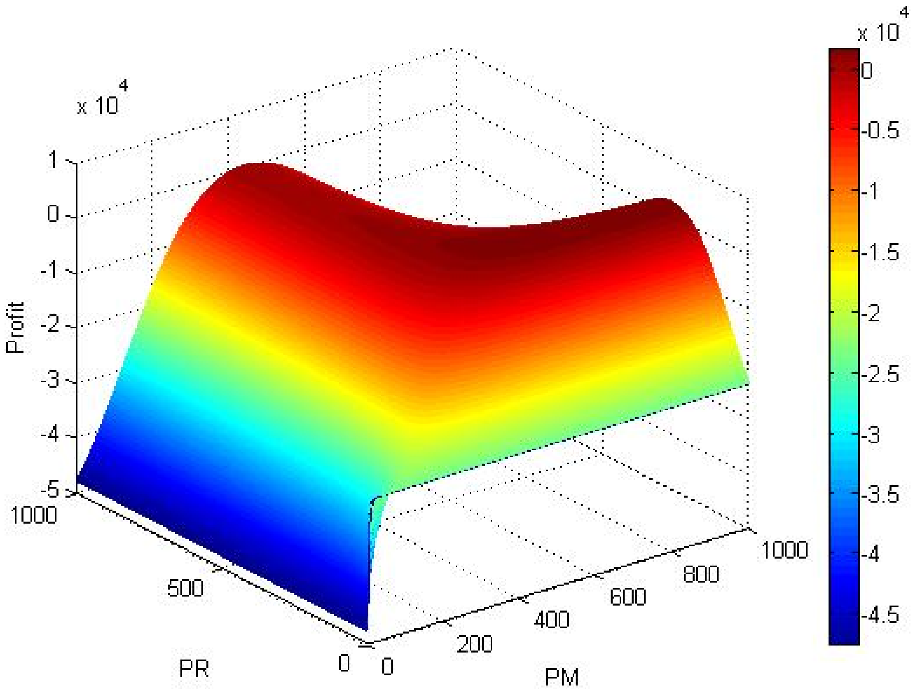


Figure 4.4. Profit of the lost profit case as a function of prices of manufactured and remanufactured products

### 4.3. Model With Fixed Unit Lost Profit

As an extension, unit lost profit is considered as a parameter which is determined by the market instead of being calculated with equation (4.17). Lost profit is calculated as follows:

$$\omega \sum_{k \in Nc} A_k \, , \qquad (4.25)$$

where $\omega$ is the unit lost profit.

The profit function for the extended model becomes

$$\Pi = \sum_{j \in N} (P_j - C_j) \, A_j - \omega \sum_{k \in Nc} A_k \, . \qquad (4.26)$$

The extended model with fixed unit lost profit is given as

$$\max \quad \Pi = \sum_{j \in N} (P_j - C_j)\, A_j - \omega \sum_{k \in N_C} A_k \tag{4.27}$$

$$\text{such that} \quad Q_j \le Q_{Mj} \qquad j \in N_R \tag{4.28}$$

$$P_j \ge c\, Q_j \qquad j \in N \tag{4.29}$$

$$Q_j \ge 0 \qquad j \in N_R \tag{4.30}$$

### 4.3.1. Profit Function Plots

Following figures show the behavior of the objective function with respect to price and the quality level of the remanufactured product for unit lost profit values $\omega = 100$, 200, 300, 400 and 500. Other parameter values are chosen as $\lambda = 2$, $c = 16$, $Q_M = 30$, $Q_C = 25$ and $P_C = 400$ and the price of the manufactured product is taken as 960.

The figures show that when the value of the unit lost profit is small, the model behaves as our base model. When it increases, the model resembles the extended model where unit lost profit is considered as the average profit gained by selling manufactured and remanufactured products. If the unit lost profit parameter increases too much, profit function fails to be positive for all values of decision variables.

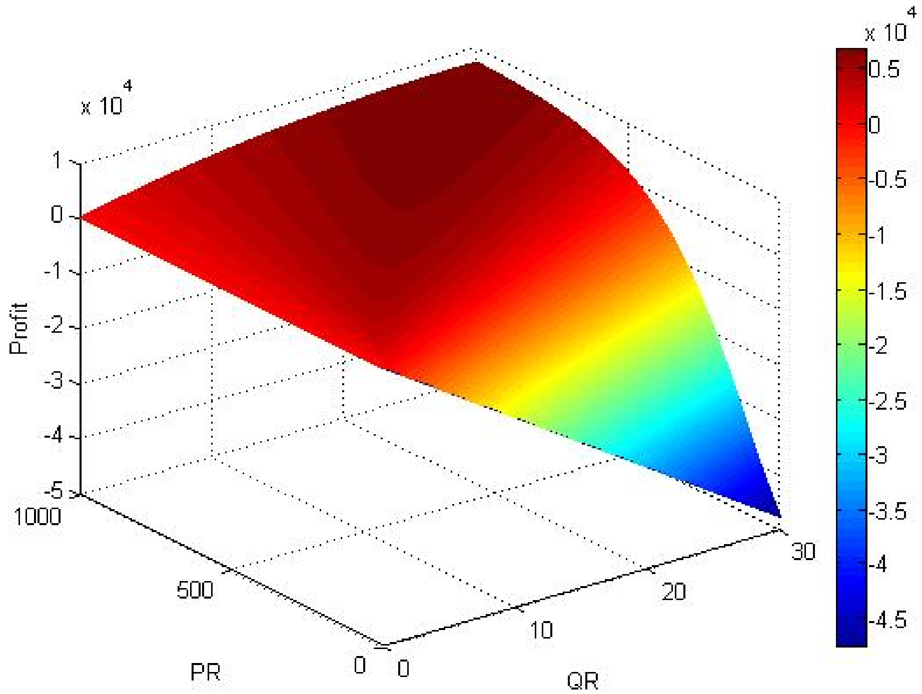


Figure 4.5. Profit of the fixed unit lost profit case for ω =100 as a function of quality level and price of remanufactured product

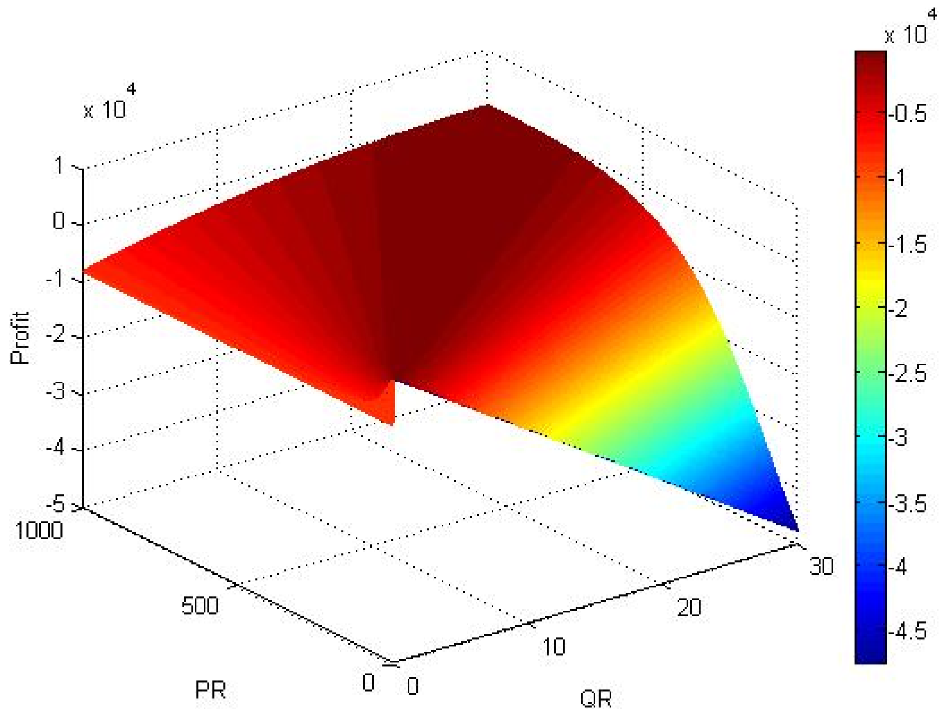


Figure 4.6. Profit of the fixed unit lost profit case for ω=200 as a function of quality level and price of remanufactured product

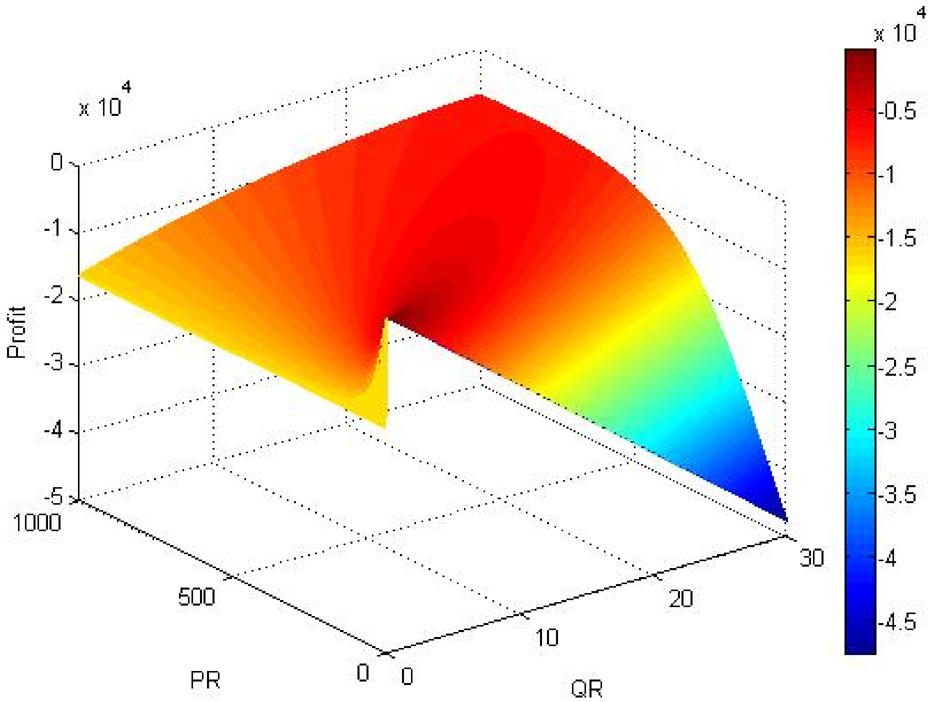


Figure 4.7. Profit of the fixed unit lost profit case for ω=300 as a function of quality level and price of remanufactured product

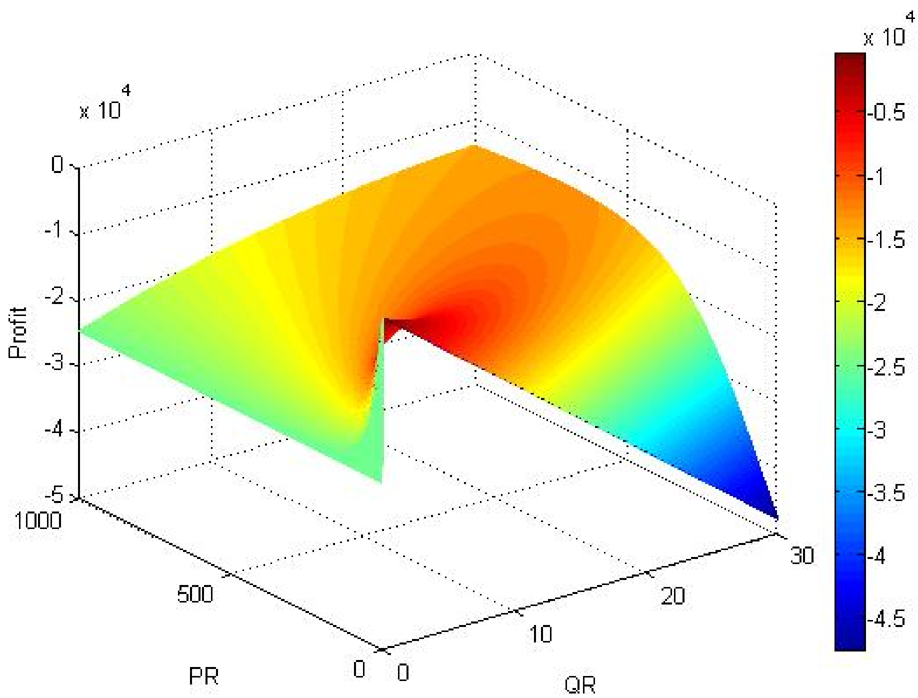


Figure 4.8. Profit of the fixed unit lost profit case for ω=400 as a function of quality level and price of remanufactured product

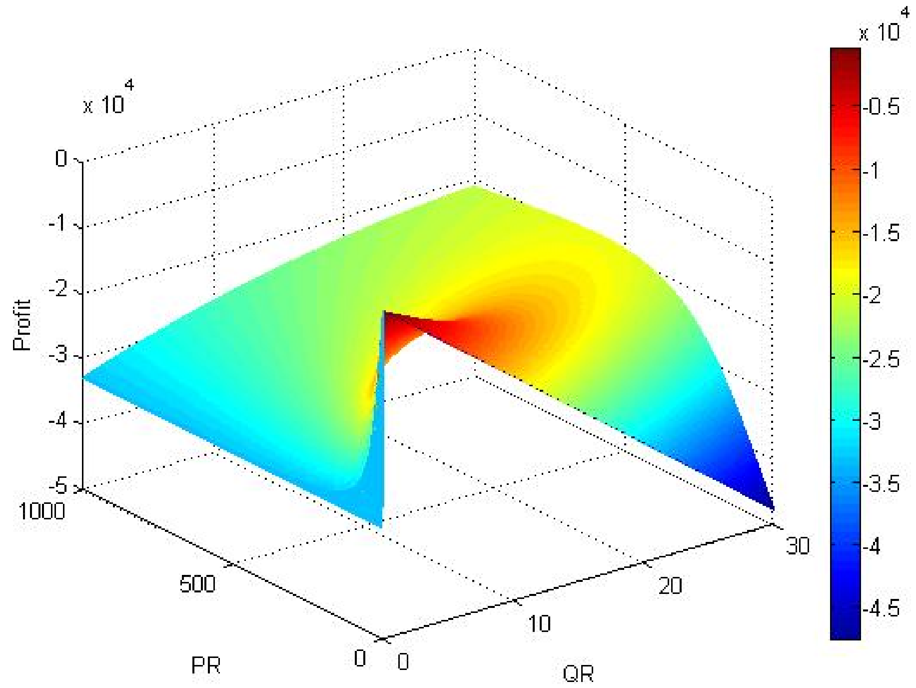


Figure 4.9. Profit of the fixed unit lost profit case for ω=500 as a function of quality level and price of remanufactured product

### 4.4. Optimality Conditions

We analyzed optimality conditions for all the models. Unfortunately, we failed to find a closed form equation for the decision variables that satisfy these conditions.

First order Karush-Kuhn-Tucker necessary conditions for optimality for a problem instance having one manufactured, one remanufactured and one competitor's product are given below. The conditions are valid both for the base model and the two extended models since their constraints are the same.

The three-product model can be written as follows:

$$\min \quad -\Pi \tag{4.31}$$

$$\text{such that} \quad c\,Q_M - P_M \le 0 \tag{4.32}$$

$$c\,Q_R - P_R \le 0 \tag{4.33}$$

$$Q_R - Q_M \le 0 \tag{4.34}$$

$$-Q_R \le 0 \tag{4.35}$$

where $Q_M$, $Q_R$, $P_M$ and $P_R$ are the quality levels and prices of the manufactured and remanufactured products respectively.

Karush-Kuhn-Tucker first order necessary conditions for optimality are as follows:

$$\frac{\partial \Pi}{\partial P_M} - u_1 = 0 \tag{4.36}$$

$$\frac{\partial \Pi}{\partial P_R} - u_2 = 0 \tag{4.37}$$

$$\frac{\partial \Pi}{\partial Q_R} + c\, u_2 + u_3 - u_4 = 0 \tag{4.38}$$

$$u_1 (c\, Q_M - P_M^*) = 0 \tag{4.39}$$

$$u_2 (c\, Q_R^* - P_R^*) = 0 \tag{4.40}$$

$$u_3 (Q_R^* - Q_M) = 0 \tag{4.41}$$

$$u_4 (-Q_R^*) = 0 \tag{4.42}$$

$$u_1, u_2, u_3, u_4 \geq 0 \tag{4.43}$$

where $u_1$, $u_2$, $u_3$ and $u_4$ are the Lagrange multipliers of the constraints (4.32), (4.33), (4.34) and (4.35), respectively.

From equations (4.36) and (4.37), we can make the substitutions $u_1 = \frac{\partial \Pi}{\partial P_M}$ and $u_2 = \frac{\partial \Pi}{\partial P_R}$.

So, equations (4.39), (4.40) and (4.43) can be written as follows:

$$\frac{\partial \Pi}{\partial P_M} (c\, Q_M - P_M^*) = 0 \tag{4.44}$$

$$\frac{\partial \Pi}{\partial P_R} (c\, Q_R^* - P_R^*) = 0 \tag{4.45}$$

$$\frac{\partial \Pi}{\partial P_M} \geq 0 \tag{4.46}$$

$$\frac{\partial \Pi}{\partial P_R} \geq 0 \tag{4.47}$$

Equation (4.41) states that either $u_3 = 0$ or $Q_R^* = Q_M$, and equation (4.42) states that either $u_4 = 0$ or $Q_R^* = 0$.

If we assume that $Q_R^* < Q_M$ and $Q_R^* > 0$, then this implies $u_3 = 0$ and $u_4 = 0$. So, equation (4.38) reduces to

$$\frac{\partial \Pi}{\partial Q_R} + c\frac{\partial \Pi}{\partial P_R} = 0\,. \tag{4.48}$$

If we also assume that prices are strictly greater than unit manufacturing and remanufacturing costs, namely, $P_M^* > c\,Q_M$ and $P_R^* > c\,Q_R^*$, equations (4.44), (4.45), (4.46), (4.47) and (4.48) reduce to

$$\frac{\partial \Pi}{\partial P_M} = 0 \tag{4.49}$$

$$\frac{\partial \Pi}{\partial P_R} = 0 \tag{4.50}$$

$$\frac{\partial \Pi}{\partial Q_R} = 0 \tag{4.51}$$

As a result, if at the optimum solution quality, price and nonnegativity constraints are satisfied as strict inequalities, KKT necessary conditions for optimality require that partial derivatives of the objective function with respect to decision variables are equal to zero. If the constraints are binding, then partial derivatives with respect to decision variables do not need to be equal to zero and KKT conditions consist of the equations (4.36) - (4.43).

### 4.4.1. Base Model

Partial derivatives of the objective function with respect to decision variables for the base model are as follows:

$$\frac{\partial \Pi}{\partial P_M} = \left( \frac{Q_M}{P_M^{\lambda}\ Z} - \frac{\left(P_M - c\ Q_M\right) Q_M \lambda}{P_M^{\lambda}\ Z\ P_M} + \frac{\left(P_M - c\ Q_M\right) Q_M^{2}\ \lambda}{\left(P_M^{\lambda}\right)^2\ Z^2\ P_M} + \frac{\left(P_R - c\ Q_R\right) Q_R Q_M \lambda}{P_R^{\lambda}\ Z^2\ P_M^{\lambda}\ P_M} \right) d \tag{4.52}$$

where $Z = \frac{Q_M}{P_M^{\lambda}} + \frac{Q_R}{P_R^{\lambda}} + \frac{Q_C}{P_C^{\lambda}}$

$$\frac{\partial \Pi}{\partial P_R} = \left( \frac{Q_R}{P_R^{\lambda}\ Z} - \frac{\left(P_R - c\ Q_R\right) Q_R \lambda}{P_R^{\lambda}\ Z\ P_R} + \frac{\left(P_R - c\ Q_R\right) Q_R^{2}\ \lambda}{\left(P_R^{\lambda}\right)^2\ Z^2\ P_R} + \frac{\left(P_M - c\ Q_M\right) Q_M Q_R \lambda}{P_M^{\lambda}\ Z^2\ P_R^{\lambda}\ P_R} \right) d \tag{4.53}$$

where $Z = \frac{Q_M}{P_M^{\lambda}} + \frac{Q_R}{P_R^{\lambda}} + \frac{Q_C}{P_C^{\lambda}}$

$$\frac{\partial \Pi}{\partial Q_R} = \left( -\frac{\left(P_M - c\ Q_M\right) Q_M}{P_M^{\lambda}\ Z^2\ P_R^{\lambda}} - \frac{c\ Q_R}{P_R^{\lambda}\ Z} + \frac{P_R - c\ Q_R}{P_R^{\lambda}\ Z} - \frac{\left(P_R - c\ Q_R\right) Q_R}{\left(P_R^{\lambda}\right)^2\ Z^2} \right) d \tag{4.54}$$

where $Z = \frac{Q_M}{P_M^{\lambda}} + \frac{Q_R}{P_R^{\lambda}} + \frac{Q_C}{P_C^{\lambda}}$

### 4.4.2. Model Including Lost Profit

Partial derivatives of the objective function with respect to decision variables for the model including lost profit are given below:

$$\frac{\partial \Pi}{\partial P_M} = \left( \frac{Q_M}{P_M{}^{\lambda}\ Z} - \frac{(P_M - c\,Q_M)\,Q_M \lambda}{P_M{}^{\lambda}\ Z\ P_M} + \frac{(P_M - c\,Q_M)\,Q_M{}^2\ \lambda}{\left(P_M{}^{\lambda}\right)^2\ Z^2\ P_M} + \frac{(P_R - c\,Q_R)\,Q_R Q_M\,\lambda}{P_R{}^{\lambda}\ Z^2\ P_M{}^{\lambda}\ P_M} \right) d$$
$$* \left( 1 - \frac{Q_C}{P_C{}^{\lambda}\left(\frac{Q_M}{P_M{}^{\lambda}} + \frac{Q_R}{P_R{}^{\lambda}}\right)} \right) - \frac{\left( \frac{(P_M - c\,Q_M)\,Q_M}{P_M{}^{\lambda}\ Z} + \frac{(P_R - c\,Q_R)\,Q_R}{P_R{}^{\lambda}\ Z} \right) d\ Q_C Q_M\,\lambda}{P_C{}^{\lambda}\left(\frac{Q_M}{P_M{}^{\lambda}} + \frac{Q_R}{P_R{}^{\lambda}}\right)^2 P_M{}^{\lambda}\ P_M} \qquad (4.55)$$

where $Z = \frac{Q_M}{P_M{}^{\lambda}} + \frac{Q_R}{P_R{}^{\lambda}} + \frac{Q_C}{P_C{}^{\lambda}}$

$$\frac{\partial \Pi}{\partial P_R} = \left( \frac{Q_R}{P_R{}^{\lambda}\ Z} - \frac{(P_R - c\,Q_R)\,Q_R \lambda}{P_R{}^{\lambda}\ Z\ P_R} + \frac{(P_R - c\,Q_R)\,Q_R{}^2\ \lambda}{\left(P_R{}^{\lambda}\right)^2\ Z^2\ P_R} + \frac{(P_M - c\,Q_M)\,Q_M Q_R \lambda}{P_M{}^{\lambda}\ Z^2\ P_R{}^{\lambda}\ P_R} \right) d$$
$$* \left( 1 - \frac{Q_C}{P_C{}^{\lambda}\left(\frac{Q_M}{P_M{}^{\lambda}} + \frac{Q_R}{P_R{}^{\lambda}}\right)} \right) - \frac{\left( \frac{(P_M - c\,Q_M)\,Q_M}{P_M{}^{\lambda}\ Z} + \frac{(P_R - c\,Q_R)\,Q_R}{P_R{}^{\lambda}\ Z} \right) d\ Q_C Q_R \lambda}{P_C{}^{\lambda}\left(\frac{Q_M}{P_M{}^{\lambda}} + \frac{Q_R}{P_R{}^{\lambda}}\right)^2 P_R{}^{\lambda}\ P_R} \qquad (4.56)$$

where $Z = \frac{Q_M}{P_M{}^{\lambda}} + \frac{Q_R}{P_R{}^{\lambda}} + \frac{Q_C}{P_C{}^{\lambda}}$

$$\frac{\partial \Pi}{\partial Q_R} = \left( -\frac{(P_M - c\,Q_M)\,Q_M}{P_M{}^{\lambda}\ Z^2\ P_R{}^{\lambda}} - \frac{c\,Q_R}{P_R{}^{\lambda}\ Z} + \frac{P_R - c\,Q_R}{P_R{}^{\lambda}\ Z} - \frac{(P_R - c\,Q_R)\,Q_R}{\left(P_R{}^{\lambda}\right)^2\ Z^2} \right) d$$
$$* \left( 1 - \frac{Q_C}{P_C{}^{\lambda}\left(\frac{Q_M}{P_M{}^{\lambda}} + \frac{Q_R}{P_R{}^{\lambda}}\right)} \right) + \frac{\left( \frac{(P_M - c\,Q_M)\,Q_M}{P_M{}^{\lambda}\ Z} + \frac{(P_R - c\,Q_R)\,Q_R}{P_R{}^{\lambda}\ Z} \right) d\ Q_C}{P_C{}^{\lambda}\left(\frac{Q_M}{P_M{}^{\lambda}} + \frac{Q_R}{P_R{}^{\lambda}}\right)^2 P_R{}^{\lambda}} \qquad (4.57)$$

where $Z = \frac{Q_M}{P_M{}^{\lambda}} + \frac{Q_R}{P_R{}^{\lambda}} + \frac{Q_C}{P_C{}^{\lambda}}$

### 4.4.3. Model With Fixed Unit Lost Profit

Partial derivatives of the objective function with respect to decision variables are given below for the model with fixed unit lost profit:

$$\frac{\partial \Pi}{\partial P_M} = \begin{pmatrix} \frac{Q_M}{P_M^{\lambda} Z} - \frac{(P_M - c\,Q_M)\,Q_M \lambda}{P_M^{\lambda} Z\, P_M} + \frac{(P_M - c\,Q_M)\,Q_M^2 \lambda}{\left(P_M^{\lambda}\right)^2 Z^2\, P_M} \\ + \frac{(P_R - c\,Q_R)\,Q_R Q_M \lambda}{P_R^{\lambda} Z^2\, P_M^{\lambda}\, P_M} - \frac{Q_C Q_M\, \omega \lambda}{P_C^{\lambda} Z^2\, P_M^{\lambda}\, P_M} \end{pmatrix} d \tag{4.58}$$

where $Z = \frac{Q_M}{P_M^{\lambda}} + \frac{Q_R}{P_R^{\lambda}} + \frac{Q_C}{P_C^{\lambda}}$

$$\frac{\partial \Pi}{\partial P_R} = \begin{pmatrix} \frac{Q_R}{P_R^{\lambda} Z} - \frac{(P_R - c\,Q_R)\,Q_R \lambda}{P_R^{\lambda} Z\, P_R} + \frac{(P_R - c\,Q_R)\,Q_R^2 \lambda}{\left(P_R^{\lambda}\right)^2 Z^2\, P_R} \\ + \frac{(P_M - c\,Q_M)\,Q_M Q_R \lambda}{P_M^{\lambda} Z^2\, P_R^{\lambda}\, P_R} - \frac{Q_C Q_R^2\, \omega \lambda}{P_C^{\lambda} Z^2\, P_R^{\lambda}\, P_R} \end{pmatrix} d \tag{4.59}$$

where $Z = \frac{Q_M}{P_M^{\lambda}} + \frac{Q_R}{P_R^{\lambda}} + \frac{Q_C}{P_C^{\lambda}}$

$$\frac{\partial \Pi}{\partial Q_R} = \begin{pmatrix} -\frac{(P_M - c\,Q_M)\,Q_M}{P_M^{\lambda} Z^2\, P_R^{\lambda}} - \frac{c\,Q_R}{P_R^{\lambda} Z} + \frac{(P_R - c\,Q_R)}{P_R^{\lambda} Z} \\ -\frac{(P_R - c\,Q_R)\,Q_R}{\left(P_R^{\lambda}\right)^2 Z^2} + \frac{d\,Q_C\, \omega}{P_C^{\lambda} Z^2\, P_R^{\lambda}} \end{pmatrix} d \tag{4.60}$$

where $Z = \frac{Q_M}{P_M^{\lambda}} + \frac{Q_R}{P_R^{\lambda}} + \frac{Q_C}{P_C^{\lambda}}$

Since the partial derivatives given above are very complex, we failed to find a closed form solution for the decision variables that make partial derivatives equal to zero. So, we could not obtain a mathematical solution to our models, but we used these partial derivatives in verifying the solutions obtained through simplex search algorithm (see Section 6.2.1).

# 5. SOLUTION PROCEDURE

In this chapter, simplex search algorithm used in solving our models is described in detail together with the modifications necessary for handling constraints. Furthermore, the outline of the design of experiments is given.

We seek an optimal solution for a constrained nonlinear programming problem. As shown previously, by taking derivatives we failed to find the optimal solutions (see Section 4.4). Moreover, since the models have variables in continuous space, complete enumeration is not possible. Thus, we used a direct search method developed for solving multi-dimensional optimization problems. Direct search methods start with a set of solutions and generate a new candidate at each iteration. A discussion of these methods can be found in (Lewis et al., 2000), (Torczon, 1997), and (Wright, 1996).

Simplex search method developed by Nelder and Mead (1965), is a well known direct search method which can be used to solve the problem in question. Lewis et al. (2000) state that "Of all the direct search methods, the Nelder-Mead simplex algorithm is the one most often found in numerical software packages." Another popular method is the pattern search (Hooke and Jeeves, 1961).

Humphrey and Wilson (2000) propose a revised simplex search procedure consisting of a three phase application of the original Nelder-Mead method in order to avoid the weaknesses of direct search methods.

## 5.1. Simplex Search Algorithm

Simplex search algorithm starts with an initial simplex and works by taking a series of steps to modify the simplex and finally converges to a point which is a local optimum. A simplex is a polygon consisting of $n$+1 points (or vertices) in $n$ dimensions. In two dimensions a simplex is a triangle while it is a tetrahedron in three dimensions.

Each vertex of the simplex is represented by a multi-dimensional vector whose elements are quality levels of the remanufactured products, prices of manufactured and remanufactured products, respectively. At each step of the algorithm the objective values are calculated for each vertex.

Each iteration starts by reflecting the worst vertex of the simplex over the central point by a reflection coefficient ($\alpha$), and accepts or rejects the newly generated point. We also attempt expansion, contraction or shrinkage steps according to the objective value. Reflection provides moving the worst vertex (having minimum objective value for a maximization problem) through the opposite face of the simplex to a better point. Reflection steps are constructed to conserve the volume of the simplex, hence maintaining its nondegeneracy. When the reflected point is better than the best vertex of the previous simplex, an attempt is performed to expand the reflected point by an expansion coefficient ($\gamma$). Otherwise, if the reflected point has an objective value which is smaller than the next to worst vertex of the previous simplex, an attempt is performed for contracting the reflected point by a contraction coefficient ($\beta$). If the contracted point is worse than the worst vertex of the previous simplex, the entire simplex is shrunk by a shrinkage coefficient ($\chi$). After all these steps, termination criterion is questioned and reflection step is repeated unless it is satisfied. The coefficients conform to the following inequalities as stated by Bazaraa et al. (1993).

| | |
|---|---|
| Reflection coefficient: | $\alpha > 0$ |
| Expansion coefficient: | $\gamma > 1$ |
| Contraction coefficient: | $0 < \beta < 1$ |
| Shrinkage coefficient: | $\chi > 0$ |

Detailed description of the algorithm is given below and the visual representation and flowchart of the algorithm for our maximization problem can be seen in Figure 5.1 and Figure 5.2.

Since the method is originally developed for unconstrained optimization, some modifications are needed to handle the constraints of the model. These modifications are described in Section 5.1.2.

### 5.1.1. Steps of the Algorithm

*1. Construction of the initial simplex*

Choose $\mathbf{X}^1$ randomly and construct the rest of the simplex by computing other vertices with

$$\mathbf{X}^{i+1} = \mathbf{X}^1 + \mathbf{D}^i \quad i = 1, ..., n. \tag{5.1}$$

as suggested by Bazaraa et al. (1993), where $\mathbf{D}^i$ is the direction vector whose $i^{\text{th}}$ component is equal to $a$ and all other components are equal to $b$, where

$$a = \frac{s}{n\sqrt{2}}(\sqrt{n+1} + n - 1) \tag{5.2}$$

and

$$b = \frac{s}{n\sqrt{2}}(\sqrt{n+1} - 1) \tag{5.3}$$

where $s$ is a positive scalar being the step size parameter. $s$ is set to 2 for quality level variables and set to 100 for price variables.

Then go to step 2.

The vertices calculated by equation (5.1) form a simplex, since $\mathbf{X}^i$, $i = 1, ..., n+1$ are affinely independent as it is shown by Esenduran (2004).

*2. Comparison of vertices*

Compute profit function (Π) for each vertex as shown in equation (4.21).

Compare vertices according to their profit values and set $\mathbf{X}^{min}$, $\mathbf{X}^{ntw}$, $\mathbf{X}^{max}$ and their profit values $Z^{min}$, $Z^{ntw}$ and $Z^{max}$ where

$$Z^{min} = \min_i \prod(\mathbf{X}^i) \quad i = 1, ..., n+1 \tag{5.4}$$

and $\mathbf{X}^{min}$ is the corresponding vertex.

$$Z^{ntw} = \min_i \prod(\mathbf{X}^i) \quad i = 1,...,n+1, \quad i \neq min \tag{5.5}$$

and $\mathbf{X}^{ntw}$ is the corresponding vertex.

$$Z^{max} = \max_i \prod(\mathbf{X}^i) \quad i = 1,...,n+1 \tag{5.6}$$

and $\mathbf{X}^{max}$ is the corresponding vertex.

Compute centroid $\mathbf{X}^{cent}$ of all the vertices in the current simplex except $\mathbf{X}^{min}$ with:

$$\mathbf{X}^{cent} = \frac{1}{n} \sum_{\substack{i=1 \\ i \neq \min}}^{n+1} \mathbf{X}^i \tag{5.7}$$

Go to step 3.

*3. <u>Reflection</u>*

Compute reflected point $\mathbf{X}^{ref}$ of $\mathbf{X}^{min}$ through $\mathbf{X}^{cent}$ with:

$$\mathbf{X}^{ref} = \mathbf{X}^{cent} + \alpha\,(\mathbf{X}^{cent} - \mathbf{X}^{min}) \tag{5.8}$$

and let its objective value Π ($\mathbf{X}^{ref}$) be $Z^{ref}$.

Then:

If $Z^{ref} \geq Z^{max}$, go to step 4.

If $Z^{ntw} \leq Z^{ref} < Z^{max}$, accept reflection and replace $\mathbf{X}^{min}$ with $\mathbf{X}^{ref}$ to form a new simplex and go to step 7.

If $Z^{min} < Z^{ref} < Z^{ntw}$, accept reflection and replace $\mathbf{X}^{min}$ with $\mathbf{X}^{ref}$ and go to step 5.

If $Z^{ref} < Z^{min}$, go to step 5 without accepting reflection.

4. *Expansion*

Compute expansion point $\mathbf{X}^{exp}$ of $\mathbf{X}^{ref}$ over $\mathbf{X}^{cent}$ with:

$$\mathbf{X}^{exp} = \mathbf{X}^{cent} + \gamma\,(\mathbf{X}^{ref} - \mathbf{X}^{cent}) \tag{5.9}$$

and let its objective value $\Pi\,(\mathbf{X}^{exp})$ be $Z^{exp}$.

Then:

If $Z^{exp} > Z^{ref}$, accept expansion and replace $\mathbf{X}^{min}$ with $\mathbf{X}^{exp}$ and go to step 7.

If $Z^{exp} \leq Z^{ref}$, accept reflection and replace $\mathbf{X}^{min}$ with $\mathbf{X}^{ref}$ and go to step 7.

5. *Contraction*

Compute contraction point $\mathbf{X}^{cont}$ of $\mathbf{X}^{min}$ over $\mathbf{X}^{cent}$ with:

$$\mathbf{X}^{cont} = \mathbf{X}^{cent} + \beta\,(\mathbf{X}^{min} - \mathbf{X}^{cent}) \tag{5.10}$$

and let its objective value $\Pi\,(\mathbf{X}^{exp})$ be $Z^{cont}$.

Then:

If $Z^{cont} \geq Z^{min}$, accept contraction and replace $\mathbf{X}^{min}$ with $\mathbf{X}^{cont}$ and go to step 7.

If $Z^{cont} < Z^{min}$, go to step 6.

6. *Shrinkage*

Replace $\mathbf{X}^{i}$ with $\mathbf{X}^{i} + \chi\,(\mathbf{X}^{max} - \mathbf{X}^{i})$ for $i = 1, \ldots, n$.

Go to step 7.

7. *Termination*

Compute centroid $\mathbf{X}^{cent}$ and its profit value $Z^{cent}$ as described in step 3.

If the termination condition

$$\left\{\frac{1}{n+1}\sum_{i=1}^{n}\left[Z^{i}-Z^{cent}\right]^{2}\right\}^{1/2} < \varepsilon \tag{5.11}$$

is satisfied, set $\mathbf{X}^*$ to $\mathbf{X}^{max}$, $Z^*$ to $Z^{max}$ and stop. Otherwise, go to step 2.

Figure 5.1 shows the steps of the Nelder-Mead simplex search algorithm visually for a maximization problem in three dimensions (Press et al., 1992). Also, flowchart representation of the algorithm can be seen in Figure 5.2.

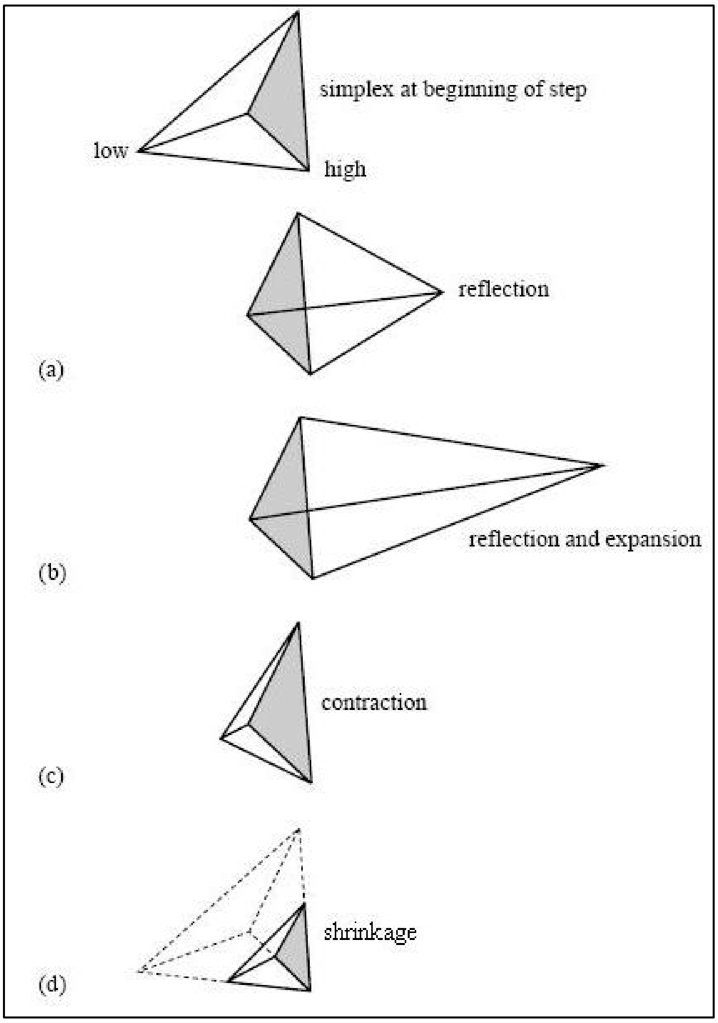


Figure 5.1. Visual representation of the steps of the Nelder-Mead simplex search algorithm in three dimensions

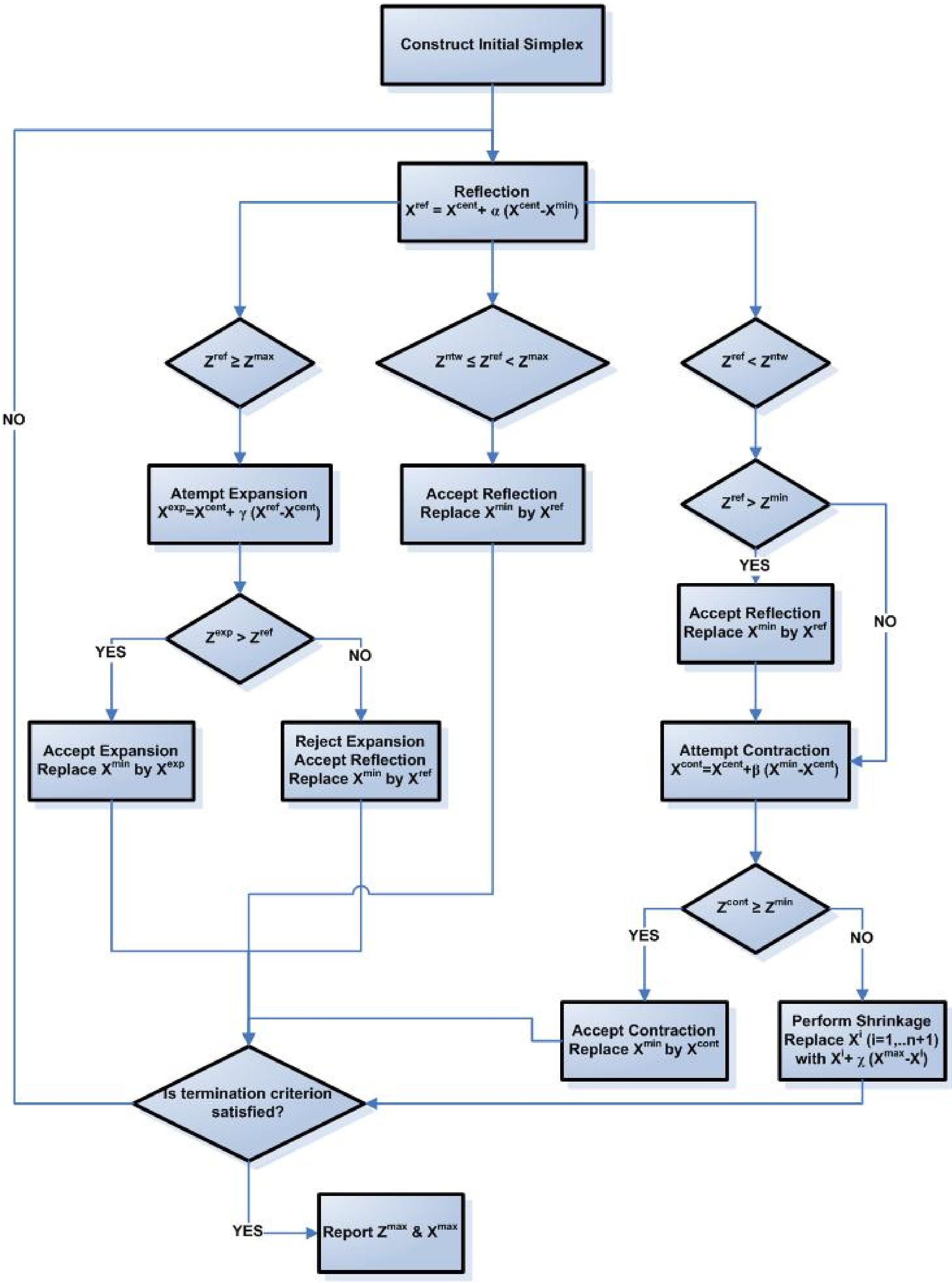


Figure 5.2. Flowchart representation of the Nelder-Mead simplex search algorithm

### 5.1.2. Modifications to Handle Constraints

Original simplex search algorithm works for unconstrained optimization. Since our model has quality and price constraints, we need to make some modifications to the procedure.

Initial simplex is constructed from a randomly chosen initial point which is computed as

$$\mathbf{X}^1 = \{Q_{M1} - 10U, Q_{M2} - 4U, C_{M1} + 200U, C_{M2} + 200U, C_{M1} + 100U, C_{M2} + 100U\} \quad (5.12)$$

where $U$ is a random number uniformly distributed in (0,1).

According to problem parameters given in tables below, it is seen that the first point of the simplex is feasible with respect to constraints (4.22) and (4.24) since quality levels of the manufactured products are all greater than 10, it is also feasible with respect to constraints (4.23) since positive numbers are added to the unit production costs. Since all entries of direction vector **D** are positive, the vertices generated from the initial point $\mathbf{X}^1$ will be feasible with respect to constraints (4.22) and (4.23).

For satisfying constraints (4.24), we need to check if they are violated while generating the vertices of the simplex. If they are violated for some remanufactured product $j$, we reset their quality levels as $Q_j = Q_j - 2(Q_j - Q_{Mj})$ where $Q_{Mj}$ is the quality level of the manufactured version of remanufactured product $j$. In this fashion, feasibility of the initial simplex is guaranteed but we also have to guarantee feasibility of our simplices modified at each step. For doing this, we check feasibility of each candidate point newly generated by reflection, expansion, contraction or shrinkage steps by updating reflection, expansion, contraction or shrinkage coefficients such that feasibility according to constraints (4.22), (4.23) and (4.24) is guaranteed.

For the reflection step, we update $\alpha$ as seen below:

$$\alpha = (Q_{Mj} - \mathbf{X}_j^{cent}) / (\mathbf{X}_j^{cent} - \mathbf{X}_j^{min}) \text{ if constraint (4.22) is violated for } j.$$

$\alpha = (C_j - \mathbf{X}_j^{cent})/(\mathbf{X}_j^{cent} - \mathbf{X}_j^{min})$ if constraint (4.23) is violated for $j$.

$\alpha = (0 - \mathbf{X}_j^{cent})/(\mathbf{X}_j^{cent} - \mathbf{X}_j^{min})$ if constraint (4.24) is violated for $j$.

If more than one of the constraints (4.22), (4.23) and (4.24) are violated, we set $\alpha$ to the minimum of the values computed above.

For the expansion step, we update $\gamma$ as follows:

$\gamma = (Q_{Mj} - \mathbf{X}_j^{cent})/(\mathbf{X}_j^{ref} - \mathbf{X}_j^{cent})$ if constraint (4.22) is violated for $j$.

$\gamma = (C_j - \mathbf{X}_j^{cent})/(\mathbf{X}_j^{ref} - \mathbf{X}_j^{cent})$ if constraint (4.23) is violated for $j$.

$\gamma = (0 - \mathbf{X}_j^{cent})/(\mathbf{X}_j^{ref} - \mathbf{X}_j^{cent})$ if constraint (4.24) is violated for $j$.

If more than one of the constraints (4.22), (4.23) and (4.24) are violated, we set $\gamma$ to the minimum of the values computed above.

For the contraction step, we update $\beta$ as follows:

$\beta = (Q_{Mj} - \mathbf{X}_j^{cent})/(\mathbf{X}_j^{min} - \mathbf{X}_j^{cent})$ if constraint (4.22) is violated for $j$.

$\beta = (C_j - \mathbf{X}_j^{cent})/(\mathbf{X}_j^{min} - \mathbf{X}_j^{cent})$ if constraint (4.23) is violated for $j$.

$\beta = (0 - \mathbf{X}_j^{cent})/(\mathbf{X}_j^{min} - \mathbf{X}_j^{cent})$ if constraint (4.24) is violated for $j$.

If more than one of the constraints (4.22), (4.23) and (4.24) are violated, we set $\beta$ to the minimum of the values computed above.

Since the shrinkage step guarantees feasibility related with quality, price and nonnegativity constraints, a modification is not necessary for this step.

### 5.2. Experimental Design

In this section, we explain the design of experiments that are performed to find the optimal values of the decision variables and to analyze the behavior of the models (lost profit case and fixed unit lost profit case). Firstly, we carried out some three-product problem experiments for the model with lost profit (4.21) – (4.24) consisting of one manufactured product, remanufactured version of it and one competitor's product in order to see which parameter intervals give meaningful and useful results for a wide range of parameter values. These parameter values are given in Table 5.1 and it is seen that a total of 1440 different experiments are performed.

Table 5.1. Parameter set for three-product experiments for the lost profit case

| $s$ | $\lambda$ | $c$ | $Q_M$ | $Q_C$ | $P_C$ |
|---|---|---|---|---|---|
| 2, 100 | 2 | 2 | 5 | 5 | 400 |
| | 3 | 4 | 10 | 10 | |
| | | 6 | 15 | 15 | |
| | | 8 | 20 | 20 | |
| | | 10 | 25 | 25 | |
| | | 12 | 30 | 30 | |
| | | 14 | 35 | | |
| | | 16 | 40 | | |
| | | 18 | 45 | | |
| | | 20 | 50 | | |
| | | | 55 | | |
| | | | 60 | | |

According to the solutions of the three-product experiments, we determined intervals for the problem parameters which give meaningful and useful results. Then we carried out five-product experiments using these new parameter intervals as given in Table 5.2. All combinations of the parameters are used, so a total of 10800 different experiments are carried out with 10 runs.

Two different step sizes ($s$) are used for the experiments of the lost profit case as given in Table 5.2. It is seen that the change in the step size does not affect solutions. So, we can say that the constructed initial simplices are sufficient to cover the solution space.

Table 5.2. Parameter set for five-product experiments for the lost profit case

| $s$ | $\lambda$ | $c$ | $Q_{M1}$ | $Q_{M2}$ | $Q_C$ | $P_C$ |
|---|---|---|---|---|---|---|
| 2, 100 | 2 | 10 | 25 | 10 | 20 | 400 |
| 4, 200 | 3 | 12 | 30 | 15 | 25 | 500 |
| | 4 | 14 | 35 | 20 | 30 | 600 |
| | | 16 | 40 | | 35 | 700 |
| | | 18 | | | 40 | 800 |
| | | 20 | | | | |

We also analyze if changes in the remanufacturing cost coefficient ($c_r$) affect solutions when the cost coefficient of manufacturing ($c$) remains constant. Hence, experiments given in Table 5.2 are performed also for the case where $c_r$ is 60 per cent of $c$ as given in Table 5.3. 5400 different experiments are performed for 10 runs.

Table 5.3. Parameter set for five-product experiments where the cost coefficient of remanufacturing is 60 per cent of the coefficient of manufacturing

| $c_r$ | $s$ | $\lambda$ | $c$ | $Q_{M1}$ | $Q_{M2}$ | $Q_C$ | $P_C$ |
|---|---|---|---|---|---|---|---|
| 0.6 $c$ | 2, 100 | 2 | 10 | 25 | 10 | 20 | 400 |
| | | 3 | 12 | 30 | 15 | 25 | 500 |
| | | 4 | 14 | 35 | 20 | 30 | 600 |
| | | | 16 | 40 | | 35 | 700 |
| | | | 18 | | | 40 | 800 |
| | | | 20 | | | | |

Finally, Table 5.4 gives the experimental design for the fixed unit lost profit case (4.27) - (4.30). A total of 1080 experiments are carried out for 10 runs in this case.

Table 5.4. Parameter set for the fixed unit lost profit case for five-product experiments

| $\omega$ | $s$ | $\lambda$ | $c$ | $Q_{M1}$ | $Q_{M2}$ | $Q_C$ | $P_C$ |
|---|---|---|---|---|---|---|---|
| 80 | 2, 100 | 2 | 10 | 25 | 10 | 20 | 400 |
| 160 | | | 12 | 30 | 15 | 25 | |
| 200 | | | 14 | 35 | 20 | 30 | |
| | | | 16 | 40 | | 35 | |
| | | | 18 | | | 40 | |
| | | | 20 | | | | |

Solution algorithm is coded in Visual C++ 6.0 environment. In order to deal with local minima, the solution algorithm is run 10 times for each combination of the parameter sets above with 10 random initializations. Random initializations are provided by choosing the first point of the simplex randomly with equation (5.19) and then constructing the rest of the simplex with the equation (5.1).

Experiments are performed on a PC with a 2.4 GHz Pentium IV processor and 504 MB RAM. For stopping conditions, $\varepsilon = 10^{-9}$ is used in all the experiments.

It takes about two to five minutes to make 10 runs for each 360 parameter combination set (different values of $c$, $Q_{M1}$, $Q_{M2}$, $Q_C$), namely, 3600 experiments for the problem instance having two manufactured, two remanufactured and one competitor's product for a given value of $\lambda$ and $P_C$. Therefore, we can say that one experiment takes about 0.05 seconds.

Some problems are solved also with Excel Solver in order to test our algorithm. We found that Excel Solver and our algorithm give the same objective values in general. This supports our belief for the optimality of the output of our algorithm although we can never be sure that we have found the optimum solution. But, after a number of experiments Excel Solver failed to find the solution, hence we could not carry out all the experiments that have different parameter sets by Excel Solver.

# 6. EXPERIMENTAL RESULTS AND INSIGHTS

In this chapter, results of the experiments are analyzed in two ways. First, best solutions found by the algorithm are given for each model. Then, the behavior of the solutions for the model with lost profit is discussed as parameters are changed.

One of the main aims of this study is to investigate whether setting quality levels for remanufactured products less than the quality level of new ones increases profitability. Therefore, the emphasis is given to results and outcomes of the solutions where quality levels of remanufactured products are set to values less than the quality levels of their manufactured versions (namely, where the quality level constraints are not binding). It is seen from the results of the experiments that when the value of $\lambda$ is less than two, at least one of the remanufactured products' quality level is set to its upper limit for all values of problem parameters. So, results of the experiments where $\lambda \geq 2$ are taken into consideration.

Since we had 10 runs having different starting solutions for each parameter combination, we had the chance to observe if different runs for the same parameter values give the same values for the decision variables and the objective function. We can say that approximately 90 per cent of the runs of the same parameter set give the same objective value but need not have the same decision variable values. The common characteristic of all alternative optimum solutions is described in Section 6.2. Discussion about model behavior according to problem parameters given below is provided from the average values of decision variables and the market share of products which are calculated from alternative optimum solutions.

For discussion of the five-product instances, we used the average of the quality levels and prices of remanufactured products since there are two remanufactured products. Among the alternative solutions, the quality levels and prices of remanufactured products may show big differences and as it is pointed out before, a remanufactured product's quality level may be higher than another remanufactured product's quality level even if the ranking of the quality levels of their manufactured versions is in reverse order. This is also valid for prices of them. When we take the average of them, we obtain the behavior on average. For prices of the manufactured products, we analyzed the behavior of one of them since they show the same behavior.

### 6.1. Base Model

We constructed the base model and carried out some experiments to see if our formulation makes sense or not. Results of the experiments show that whatever values are assigned to the parameters, decision variables are set to their upper limits for variables that have an upper bound (quality level variables). On the other hand, variables that do not have an upper bound (price variables) are set to infinity. By setting prices to infinity, a small market share is gained by the equality (4.5), but this gives higher profit than by setting a reasonable price and gaining a larger market share. In order to overcome this unexpected behavior, a mechanism that will prevent the model to set prices to infinity should be added to the model. This is achieved by adding a cost item to the profit function for the amount of lost market share, hence obtaining a new model that is given in Section 4.2.

### 6.2. Model Including Lost Profit

By adding the lost profit due to the competitor's market share to the profit function, the model started to behave as expected. A three-product instance (consisting of one manufactured product, its remanufactured version and one competitor's product) served us as a tool for understanding model behavior according to problem parameters. We tried a wide range of parameter values in order to see which values of parameters are meaningful (Table 5.1).

When coefficient $c$ (relating the unit cost to the quality) is at low levels, the model sets the quality level of the remanufactured product to its upper level (makes $Q_R = Q_M$). When the value of $c$ increases, $Q_R$ starts to take values less than $Q_M$. The value of $c$ where $Q_R$ starts to take values less than $Q_M$ depends on the values of $Q_M$ and $Q_C$. As $Q_M$ and $Q_C$ increase, the value of $c$ where $Q_R$ starts to take values less than $Q_M$ decreases. Namely, at smaller values of $c$, $Q_R$ starts to be less than $Q_M$.

The second problem we solve is a five-product instance having two manufactured, two remanufactured and one competitor's product for different meaningful problem parameter sets that are given in Table 5.2 with 10 different random starting solutions for each parameter set. Discussion given below is valid for both three-product and five-product problem instances.

It is seen from the results that, different runs having different starting solutions bring different optimal solutions that have the same objective value. Prices of manufactured products ($P_M$) are the same for each of them, whereas quality levels and prices of remanufactured products ($Q_R$ and $P_R$) vary. Total market share of the remanufactured products are also the same for all the alternative solutions. This also holds for manufactured and competitor's products.

The common point of the optimal solutions (solutions of different parameter combinations and alternative solutions of the same parameter combination) found by the algorithm is that, the ratio of the prices to costs of remanufactured products being equal to $\lambda$ where quality levels of remanufactured products are set to values less than quality levels of the manufactured products. Namely, $P_R / C_R = \lambda$ for all $\lambda$ values at an optimum solution. It is also possible to say that the ratio of prices to quality levels for remanufactured products become equal to $c\lambda$ ($P_R / Q_R = c\lambda$) for all parameter values if the quality level constraints (4.22) are not binding.

Also, the ratio of prices of manufactured products to their costs equals to $\lambda$ when $\lambda = 2$ and when quality levels of remanufactured products are not set to their upper limits. Namely, $P_M / C_M = \lambda$ ($P_M / Q_M = c\lambda$) for $\lambda = 2$. For other values of $\lambda$, this equation does not hold. When $\lambda < 2$, $P_M / C_M > \lambda$ and when $\lambda > 2$, $P_M / C_M < \lambda$ at optimum solutions.

Another interesting outcome realized is that when $\lambda = 2$, total market share of the OEM (manufactured and remanufactured products sold to customers) equals to 70.71 per cent. In other words, the market share of the competitor's product equals to 29.29 per cent. This holds both for three-product and five-product problems for the solutions where quality levels of remanufactured products are set to values less than quality levels of their manufactured versions.

Moreover, profit gained from manufactured and remanufactured products become equal to each other for $\lambda$=2. In other words, the values of $\sum_{j \in N_M} (P_j - C_j)\, A_j$ and $\sum_{j \in N_R} (P_j - C_j)\, A_j$ become equal. For other values of $\lambda$, this situation is not observed.

### 6.2.1. Sample Experiments and Their Output

Table 6.1 shows input and output data for some experiments selected among the 10800 experiments given in Table 5.2. Some general outcomes can be reached from the results.

Partial derivatives of the profit function with respect to prices of manufactured (4.55) and remanufactured products (4.56), and quality levels of remanufactured products (4.57) are zero for all of the experiments given in Table 6.1. For all other experiments where quality levels of the remanufactured products are not set to their upper limits, we have the same situation. For some experiments where quality levels are set to their upper limits, partial derivatives of the profit function with respect to quality levels of remanufactured products does not equal to zero as expected from equation (4.45). These observations confirm that solutions found by the algorithm satisfy KKT first order necessary conditions that are derived in Section 4.4.

Table 6.1. Input and output data for sample experiments

| *INPUT* | | | | | | *OUTPUT* | | | | | | *MARKET SITUATION* | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\lambda$ | *c* | $Q_{M1}$ | $Q_{M2}$ | $Q_C$ | $P_C$ | $Q_{R1}$ | $Q_{R2}$ | $P_{R1}$ | $P_{R2}$ | $P_{M1}$ | $P_{M2}$ | $A_{R1}$ | $A_{R2}$ | $A_{M1}$ | $A_{M2}$ | $A_C$ | *Profit* |
| 2 | 16 | 20 | 25 | 25 | 500 | 20.8 | 8.64 | 664 | 277 | 960 | 640 | 13.8 | 33.1 | 9.53 | 14.3 | 29.3 | 10723 |
| 2 | 16 | 20 | 25 | 25 | 600 | 28.3 | 18.8 | 907 | 603 | 960 | 640 | 14.5 | 21.9 | 13.7 | 20.6 | 29.3 | 15442 |
| 2 | 16 | 20 | 40 | 40 | 500 | 11.1 | 4.5 | 356 | 144 | 960 | 640 | 16.1 | 39.7 | 5.96 | 8.94 | 29.3 | 6702.1 |
| 2 | 16 | 20 | 40 | 40 | 600 | 25.4 | 6.58 | 813 | 211 | 960 | 640 | 10.1 | 39.1 | 8.58 | 12.9 | 29.3 | 9651 |
| 2 | 18 | 20 | 40 | 40 | 600 | 18 | 4.79 | 647 | 172 | 1080 | 720 | 11.3 | 42.4 | 6.78 | 10.2 | 29.3 | 8578.6 |
| 2 | 20 | 20 | 40 | 40 | 600 | 16 | 3.53 | 640 | 141 | 1200 | 800 | 10.3 | 46.7 | 5.49 | 8.24 | 29.3 | 7720.8 |
| 3 | 16 | 20 | 25 | 25 | 500 | 5.39 | 5.39 | 259 | 259 | 849 | 609 | 32.4 | 32.4 | 5.1 | 9.21 | 20.8 | 11593 |
| 3 | 16 | 20 | 25 | 25 | 600 | 7.39 | 7.39 | 355 | 355 | 897 | 657 | 29.6 | 29.6 | 7.43 | 12.6 | 20.7 | 15784 |
| 3 | 16 | 20 | 40 | 40 | 500 | 4.13 | 4.13 | 198 | 198 | 819 | 579 | 34.5 | 34.5 | 3.55 | 6.7 | 20.8 | 8884.1 |
| 3 | 16 | 20 | 40 | 40 | 600 | 5.64 | 5.64 | 270 | 270 | 855 | 615 | 32 | 32 | 5.4 | 9.67 | 20.8 | 12109 |
| 3 | 18 | 20 | 40 | 40 | 600 | 4.61 | 4.61 | 249 | 249 | 934 | 664 | 33.7 | 33.7 | 4.14 | 7.68 | 20.9 | 11152 |
| 3 | 20 | 20 | 40 | 40 | 600 | 3.86 | 3.86 | 232 | 232 | 1016 | 716 | 34.9 | 34.9 | 3.22 | 6.13 | 20.8 | 10362 |

### 6.3. Model With Fixed Unit Lost Profit

When we model the lost profit as a fixed parameter whose value is determined by the market, instead of the average profit gained by the OEM by selling manufactured and remanufactured products; optimum solution occurs where $Q_R = Q_M$ when the value of the lost profit parameter is low. When the value of the lost profit parameter is high, the algorithm generally fails to find a solution having a positive objective value. The figures given in this section are based on the solutions of the experiments given in Table 6.2 below.

Table 6.2. Test data for sample problem of Section 6.3

| $Q_{M1}$ | $Q_{M2}$ | $Q_C$ | $P_C$ | $c$ | $\lambda$ |
|---|---|---|---|---|---|
| 25 | 10 | 20 | 400 | 18 | 2 |

Figure 6.1 and Figure 6.2 respectively show the total profit and the market share of products as unit lost profit (ω) is changed. It is seen that as ω is increased, the total profit of the OEM decreases as one might expect.

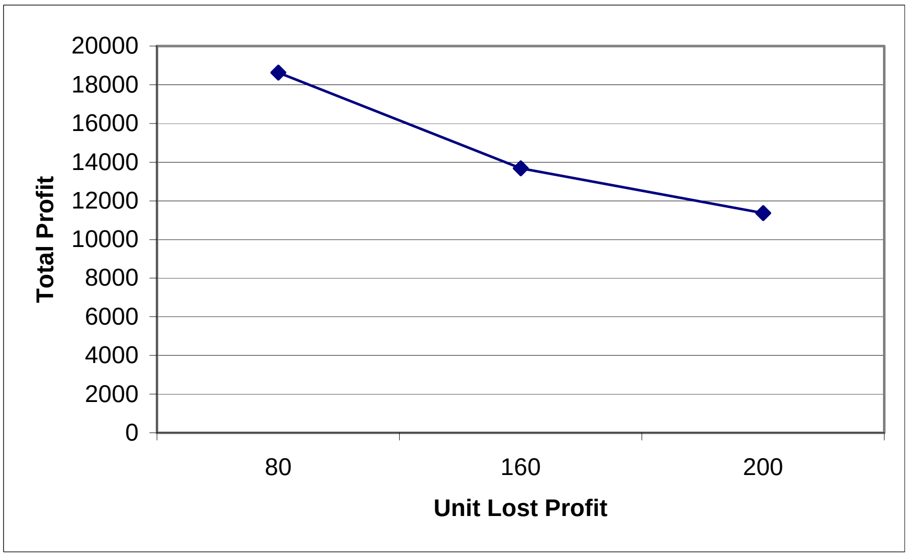


Figure 6.1. Total profit as a function of unit lost profit

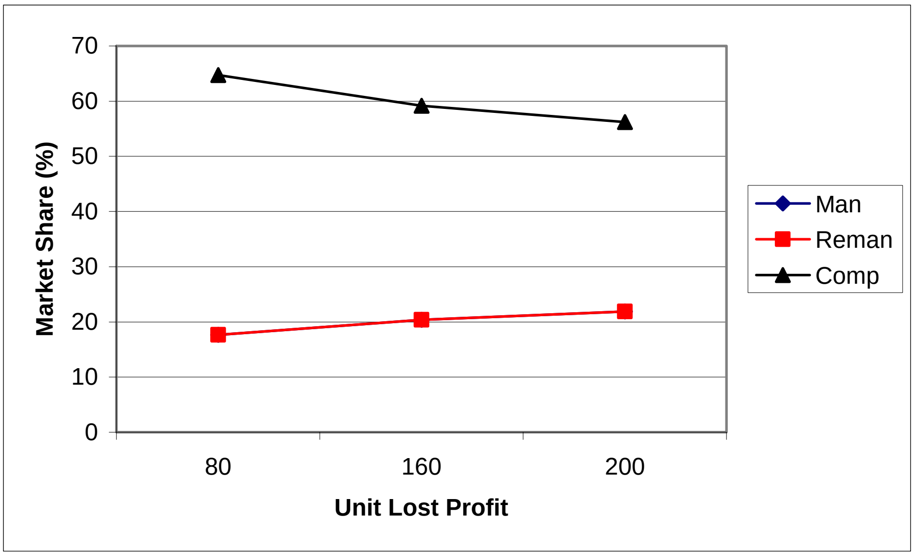


Figure 6.2. Market shares with respect to unit lost profit

Figure 6.2 shows that as unit lost profit increases, the OEM tries to increase its total market share in order to lose less from lost profit. This causes total profit to decrease as shown in Figure 6.1. The increase in market share is achieved by decreasing prices without decreasing quality levels as shown in Figure 6.3. It is seen that market share and prices of manufactured and remanufactured products are the same since quality levels of remanufactured products are set equal to the quality levels of their manufactured versions, so that they are perceived as equal products by customers.

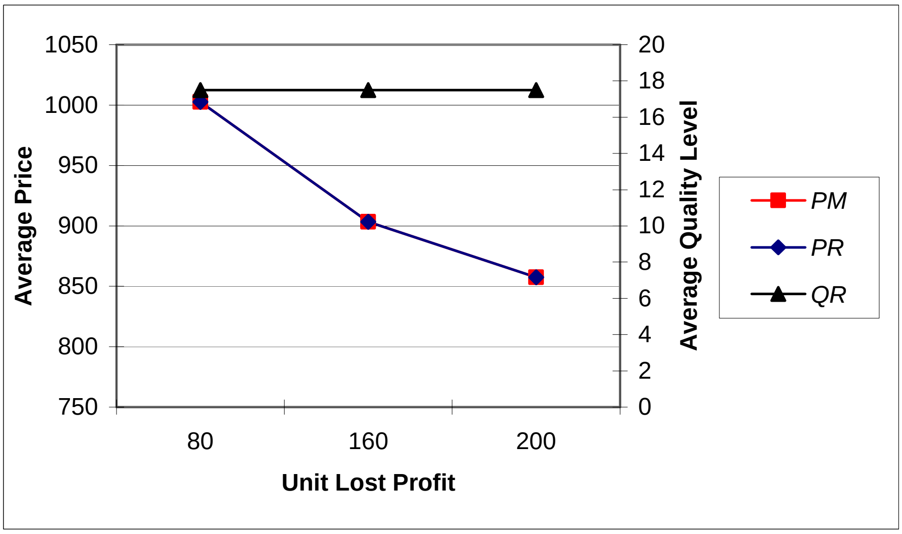


Figure 6.3. Average prices and quality levels of products with respect to unit lost profit

## 6.4. Effect of Problem Parameters

In this section the model behavior is analyzed according to problem parameters in order to derive insights from the results. Discussion given below is reached from the results of the experiments of model (4.21) – (4.24).

### 6.4.1. Effect of Changes in the Quality Level of Competitor's Product

Table 6.3 shows test data for sample experiments.

Table 6.3. Test data for sample problem of Section 6.4.1

| $Q_{M1}$ | $Q_{M2}$ | $P_C$ | $c$ | $\lambda$ |
|---|---|---|---|---|
| 25 | 15 | 500 | 18 | 2 |

As the quality level of competitor's product ($Q_C$) increases, quality levels and prices of the remanufactured products decrease in general. Prices of manufactured products also decrease when $\lambda > 2$ while they remain constant when $\lambda = 2$.

The reason for this behavior is that by offering products with lower quality levels and lower prices, the OEM tries to increase its competitive advantage. By doing this, the OEM preserves its total market share. The decrease in the market share of manufactured products is offset by the increase in the market share of remanufactured products. But since prices decrease, the average profit gained by selling manufactured and remanufactured products decreases, which results in less total profit.

Figure 6.4 shows the total profit as a function of the quality level of competitor's product for different values of price of competitor's product. We see that as the competitor's product becomes more attractive to the customers (i.e., as its quality increases or price decreases or both), the total profit decreases considerably.

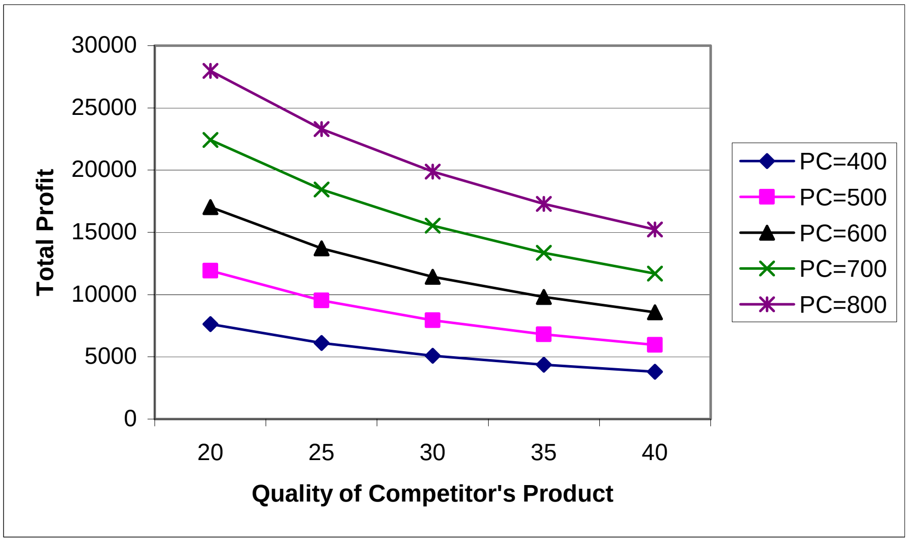


Figure 6.4. The total profit as a function of quality level of the competitor's product for different values of price of competitor's product

It is observed that profit gained from manufactured and remanufactured products are decreasing as the quality of competitor's product increases and as price of competitor's product decreases. This is the same trend of the total profit given in Figure 6.4.

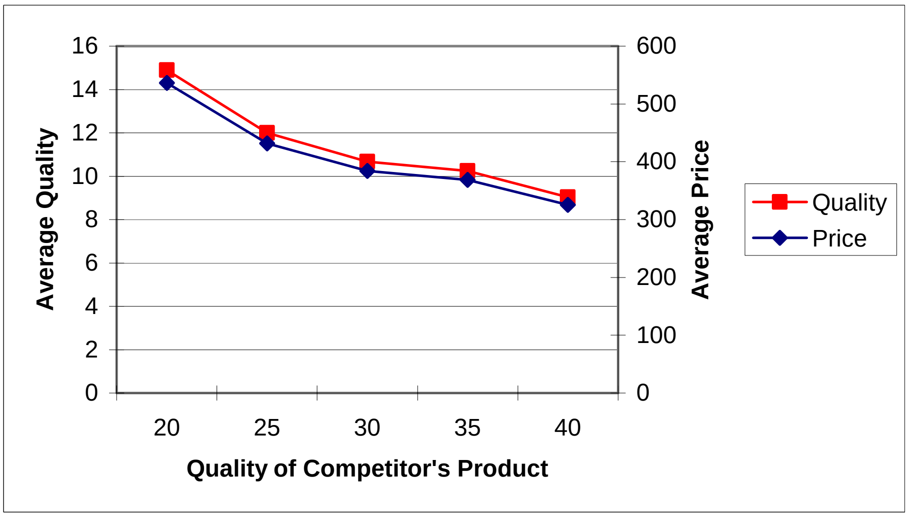


Figure 6.5. Average quality level and price of remanufactured products with respect to the quality level of the competitor's product

Figure 6.5 shows that as quality level of competitor's product increases, quality levels and prices of remanufactured products decrease on average in order to regain market share.

Figure 6.6 shows the market share of products as quality of competitor's product changes. It is seen that the market share of remanufactured products increase with the quality level of the competitor's product, whereas the market share of manufactured products decrease and the total market share of the OEM is conserved. Since manufactured products' prices have not been decreased, their market share decreases.

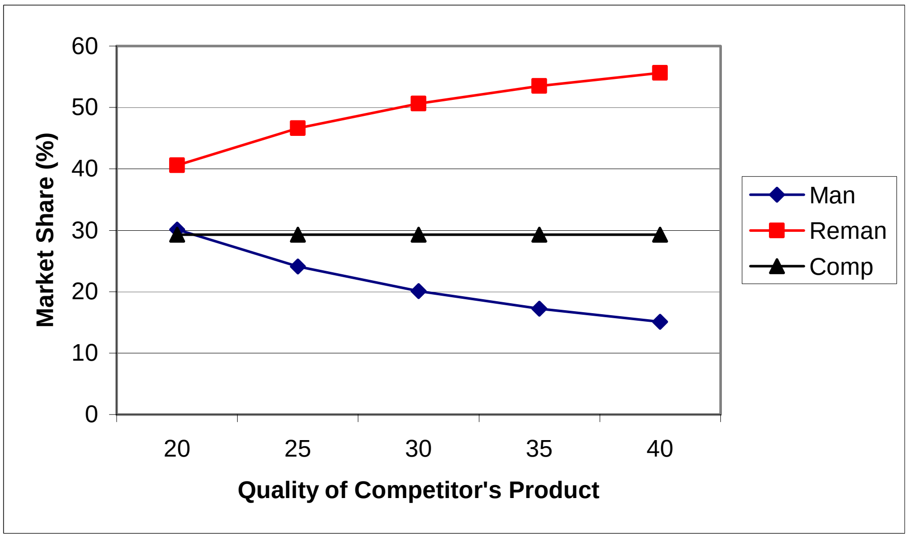


Figure 6.6. Market shares with respect to the quality level of the competitor's product

### 6.4.2. Effect of Changes in the Price of Competitor's Product

The price of competitor's product ($P_C$) affect solutions in an opposite way of its quality level ($Q_C$) as expected. As the price of the competitor's product increases, quality levels and prices of remanufactured products increase. Prices of manufactured products also increase when $\lambda > 2$, whereas they remain constant when $\lambda = 2$.

The decreasing market share of remanufactured products offsets the increasing market share of manufactured products. Hence, the market share of the OEM remains constant. Since prices of products of the OEM increase, average profit increases and as a consequence, the total profit increases as well, as can be seen in Figure 6.4.

Table 6.4 shows test data for sample experiments that the figures in this section are derived from.

Table 6.4. Test data for sample problem of Section 6.4.2

| $Q_{M1}$ | $Q_{M2}$ | $Q_C$ | $c$ | $\lambda$ |
|---|---|---|---|---|
| 25 | 15 | 30 | 18 | 2 |

Figure 6.7 shows the market share of products as the price of the competitor's product changes. As price of the competitor's product increases, the market share of manufactured products increase and the market share of remanufactured products decrease; they come closer to each other and become equal at the end. The reason for this is that quality levels and prices of remanufactured products are set as equal to their manufactured versions for the last two experiments. Figure 6.8 shows the average quality level and price of remanufactured products as a function of the price of the competitor's product.

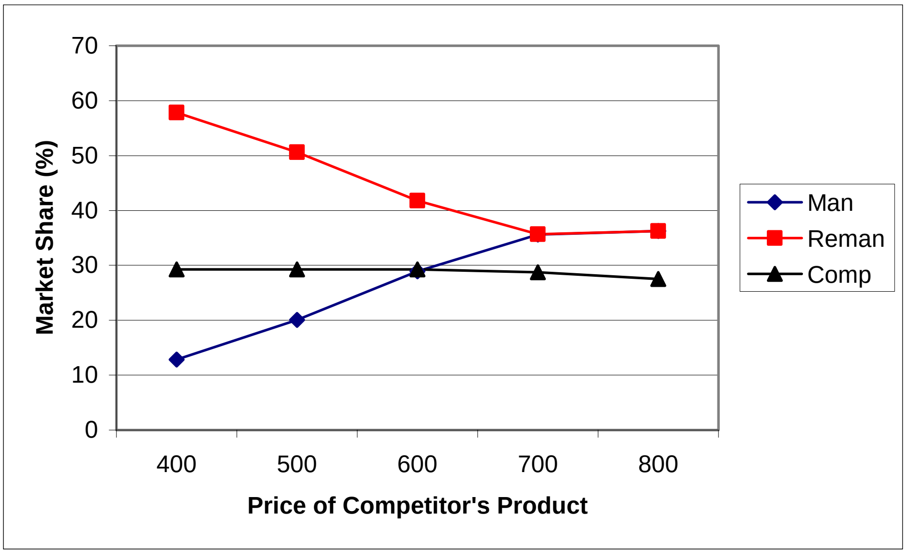


Figure 6.7. Market shares with respect to the price of the competitor's product

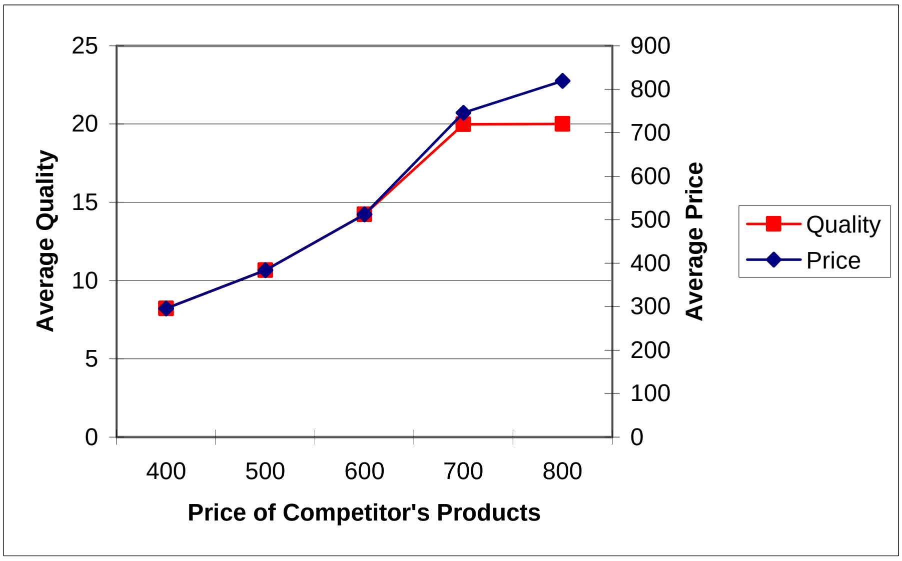


Figure 6.8. Average quality level and price of remanufactured products with respect to the price of the competitor's product

### 6.4.3. Effect of Changes in Quality Levels of Manufactured Products

While the quality level of a manufactured product ($Q_M$) increases, its price is also increased since the cost of manufacturing is increased. As a result, its market share is decreased a little. This decrease in the market share is offset by changing (increasing or decreasing according to values of the parameters) quality levels and prices of remanufactured products, hence increasing their market share.

Table 6.5 shows test data for sample experiments that the figures in this section are based on.

Table 6.5. Test data for sample problem of Section 6.4.3

| $Q_{M2}$ | $Q_C$ | $P_C$ | $c$ | $\lambda$ |
|---|---|---|---|---|
| 15 | 35 | 500 | 18 | 2 |

Figure 6.9 shows the price of a manufactured product as a function of its quality level for different $\lambda$ values. We see that slope of the line decreases as $\lambda$ increases, since an increase in $\lambda$ means that importance of price is increased for the customer in making a buying decision, so the increase in price is diminished.

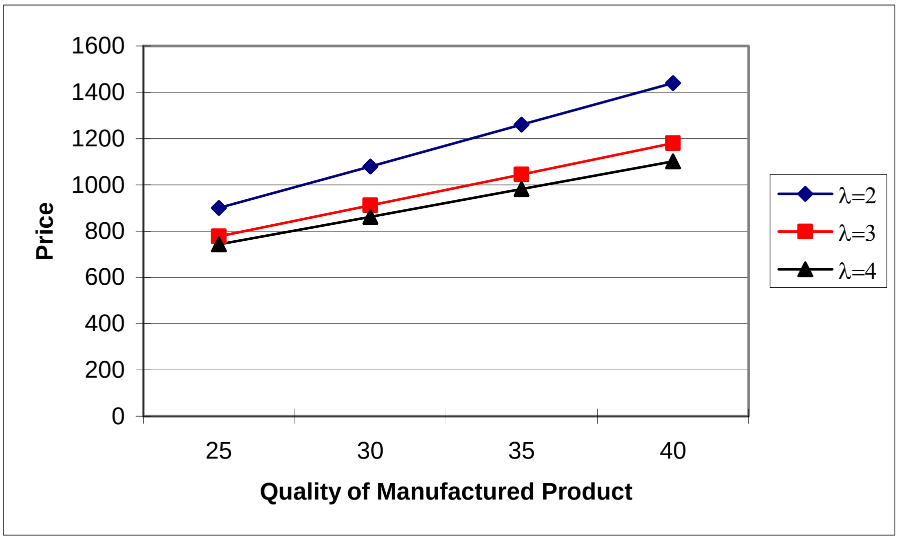


Figure 6.9. Price of a manufactured product as a function of its quality level

For $\lambda = 2$, the OEM conserves its total market share and total profit by increasing market share of the remanufactured products by choosing appropriate quality-price combinations. The price and the market share of the other manufactured product are not affected by this change. Figure 6.10 shows the increase in the market share of remanufactured products and the decrease in the market share of manufactured products with respect to the quality level of a manufactured product for $\lambda = 2$. We see that the market share of the competitor's product remains constant at 29.29 per cent.

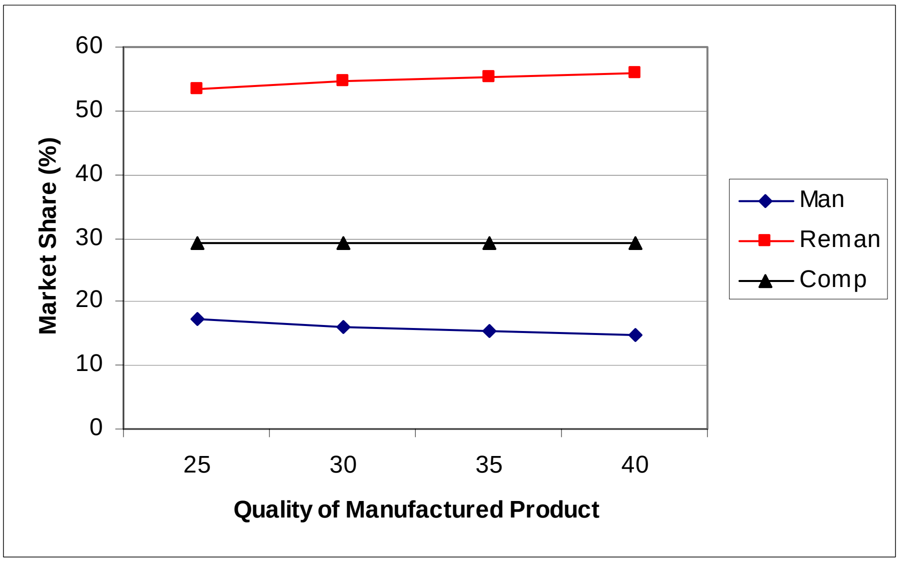


Figure 6.10. Market shares with respect to the quality level of a manufactured product for $\lambda = 2$

For $\lambda > 2$, although total market share of the OEM has not been changed much, total profit could not be conserved, since the market share of the other manufactured product is increased by decreasing its price, hence decreasing total profit.

Figure 6.11 and Figure 6.12 respectively show market share of products with respect to quality level of a manufactured product for $\lambda = 3$ and $\lambda = 4$. We see that as $\lambda$ increases, market share of products show the same trend but the market share of remanufactured products increase considerably. Market share of manufactured and competitor's product decrease as $\lambda$ increases, so total market share of the OEM is increased.

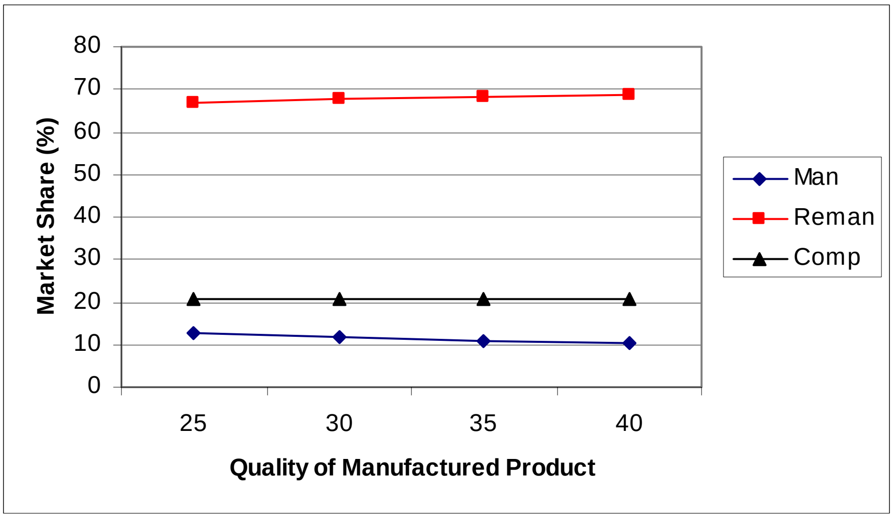


Figure 6.11. Market shares with respect to the quality level of a manufactured product for $\lambda = 3$

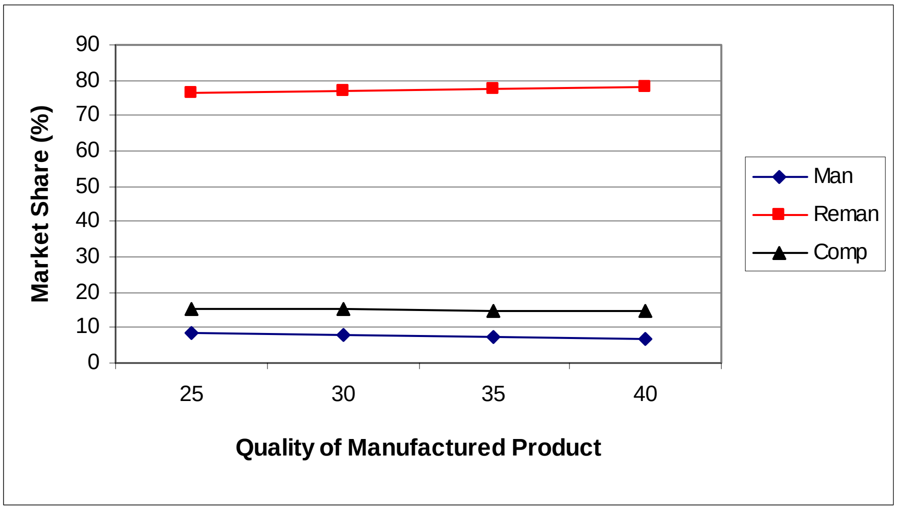


Figure 6.12. Market shares with respect to the quality level of a manufactured product for $\lambda = 4$

Figure 6.13 shows the total profit of the OEM as a function of quality level of a manufactured product for different $\lambda$ values. We see that profit remains constant for $\lambda = 2$ and decreases a little bit for $\lambda > 2$ since importance of price increases as $\lambda$ increases.

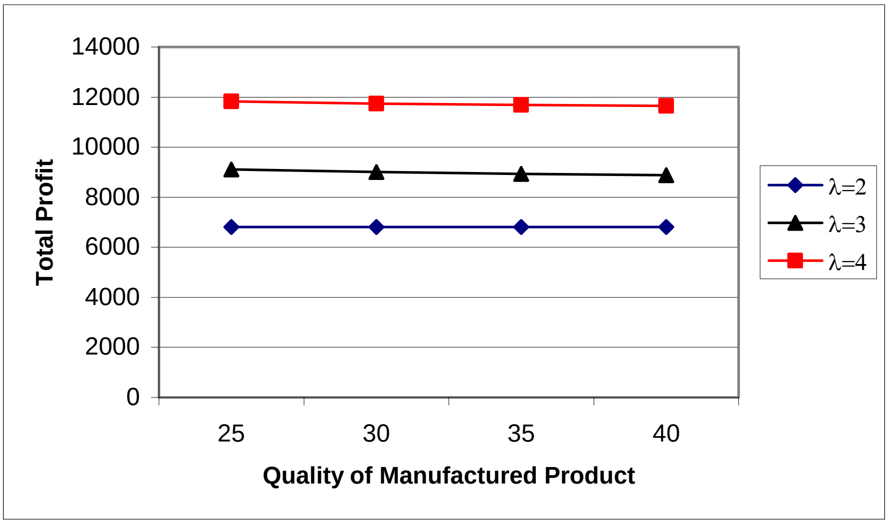


Figure 6.13. Total profit as a function of quality level of a manufactured product

### 6.4.4. Effect of Changes in the Cost Coefficient *c*

As the cost coefficient ($c$) increases, quality levels of remanufactured products decrease in order to regain cost advantage, and prices of remanufactured products also decrease as expected since they are now of lower quality. Prices of manufactured products increase since their unit costs are increased.

Table 6.6 shows test data for sample experiments that the figures in Sections 6.4.4, 6.4.5 and 6.4.6 are based on.

Table 6.6. Test data for sample problem of Sections 6.4.4, 6.4.5 and 6.4.6

| $Q_{M1}$ | $Q_{M2}$ | $Q_C$ | $P_C$ | $\lambda$ |
|---|---|---|---|---|
| 35 | 20 | 35 | 500 | 2 |

Figure 6.14 shows price of a manufactured product as a function of the cost coefficient. Since the other manufactured product also shows the same trend, this figure reflects the behavior on average.

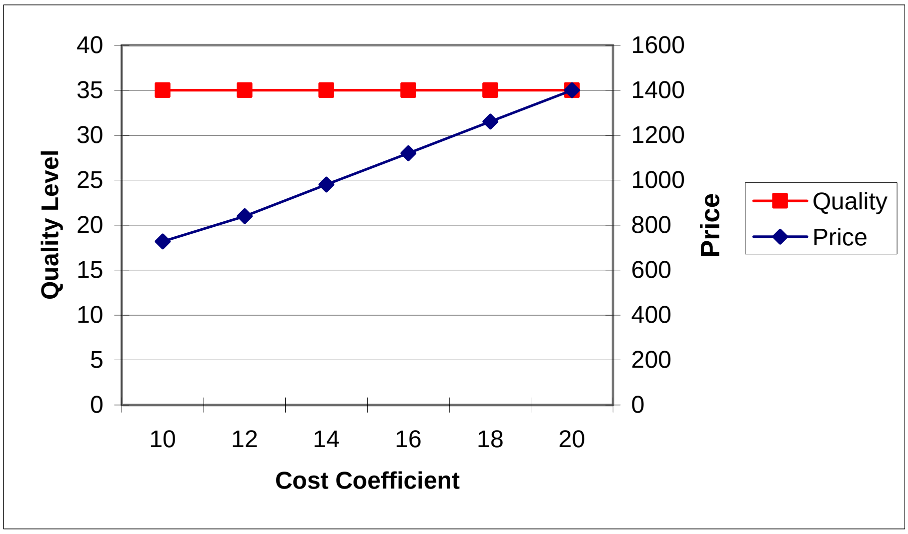


Figure 6.14. Price of a manufactured product as a function of the cost coefficient

Figure 6.15 shows average quality level and price of remanufactured products as a function of the cost coefficient.

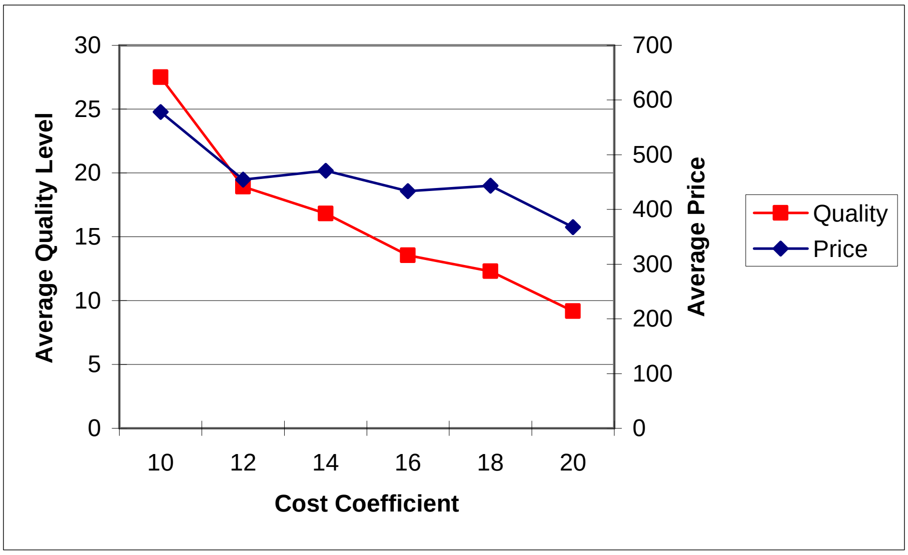


Figure 6.15. Average quality level and price of remanufactured products with respect to the cost coefficient

Figures 6.16 and 6.17 show that the ratio of prices to cost of manufacturing and remanufacturing remain constant being equal to $\lambda$ where quality levels of remanufactured products are not set to their upper limits.

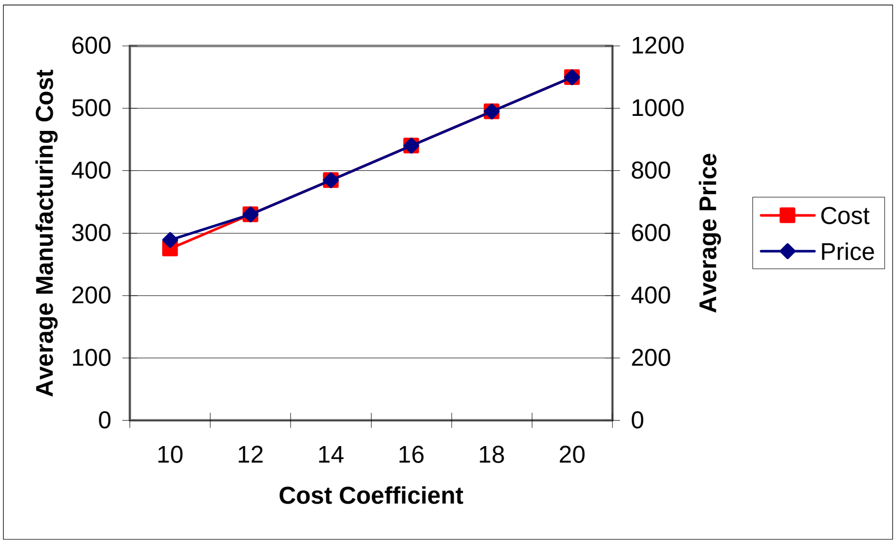


Figure 6.16. Average cost and price of manufactured products with respect to the cost coefficient

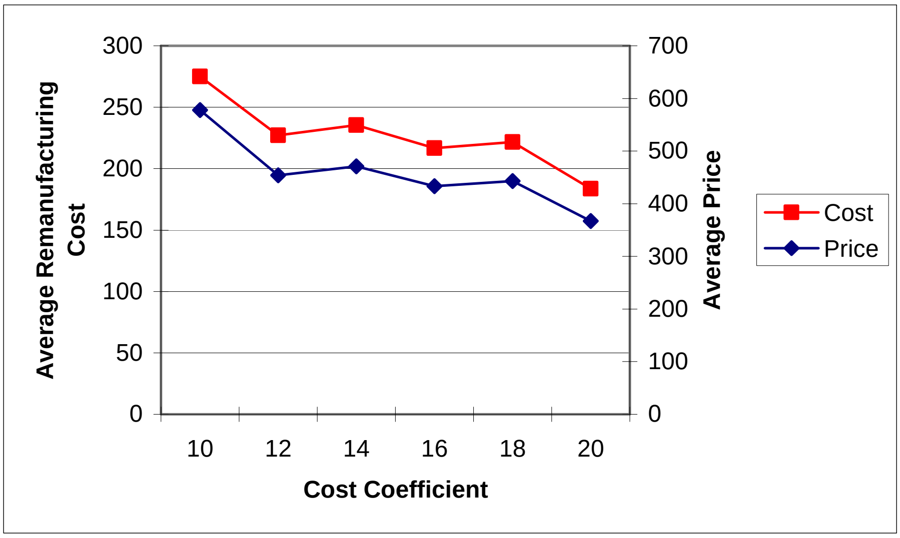


Figure 6.17. Average cost and price of remanufactured products with respect to the cost coefficient

By conserving total market share of the OEM, the remanufactured products' market share increases and the manufactured products' market share decreases as shown in Figure 6.18, since price of manufactured products increase whereas price of remanufactured products decrease. Total profit decreases for all values of $\lambda$ since costs increase as shown in Figure 6.19.

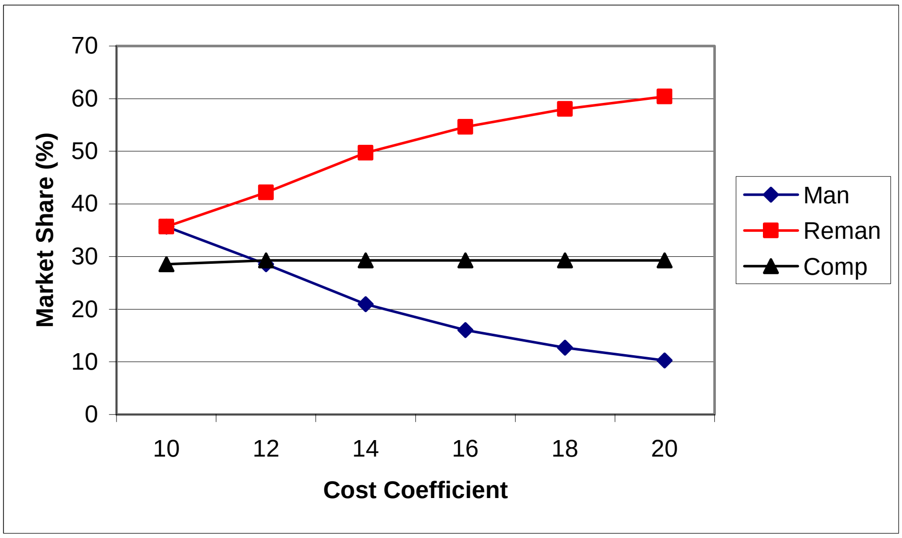


Figure 6.18. Market shares with respect to the cost coefficient

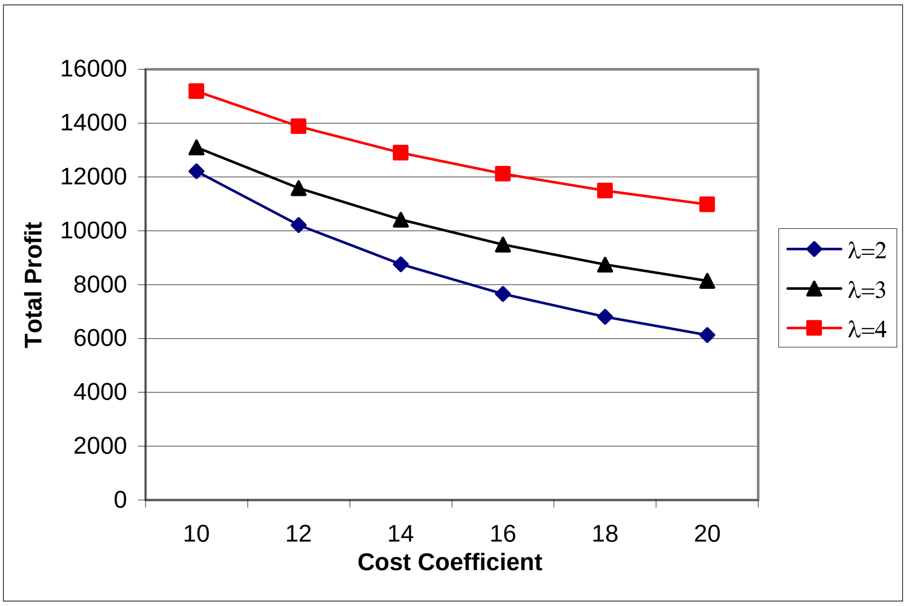


Figure 6.19. Total profit as a function of the cost coefficient

### 6.4.5. Effect of Changes in the Remanufacturing Cost Coefficient $c_r$

As remanufacturing costs show differences industry by industry, we assumed that, as a worst case, the coefficient of cost of remanufacturing is the same as the cost coefficient of manufacturing a new product at the experiments. In other words, we assumed that, if the quality level that remanufactured product will reach is the same as the quality level of its manufactured version; its production cost will be the same as its manufactured version. If the quality level of a remanufactured product turns out to be less than the quality level of its manufactured version, then its unit remanufacturing cost will be less than unit manufacturing cost too.

Since Dowlatshahi (2000) denotes that cost of remanufacturing is on average 40 - 60 per cent of manufacturing a new product, we also carried out the experiments by setting cost coefficient for remanufactured products as 60 per cent of the cost coefficient for manufactured products. Namely, the situation where $C_R = 0.6\, c\, Q_R$ is also analyzed. Here, $c$ is the cost coefficient of manufacturing a new product to be multiplied with its quality level to obtain unit manufacturing cost.

As the cost coefficient of remanufacturing is decreased by 40 per cent, more solutions set quality levels of remanufactured products to their upper limits as expected. For the solutions where the quality level constraints are not binding, all the results and discussion provided in this chapter are also valid.

If we compare the situations where the cost coefficient of remanufacturing is less than manufacturing and where they are the same, we see that a higher total profit is obtained when the remanufacturing cost coefficient is less than the manufacturing cost coefficient as expected and as shown in Figure 6.20.

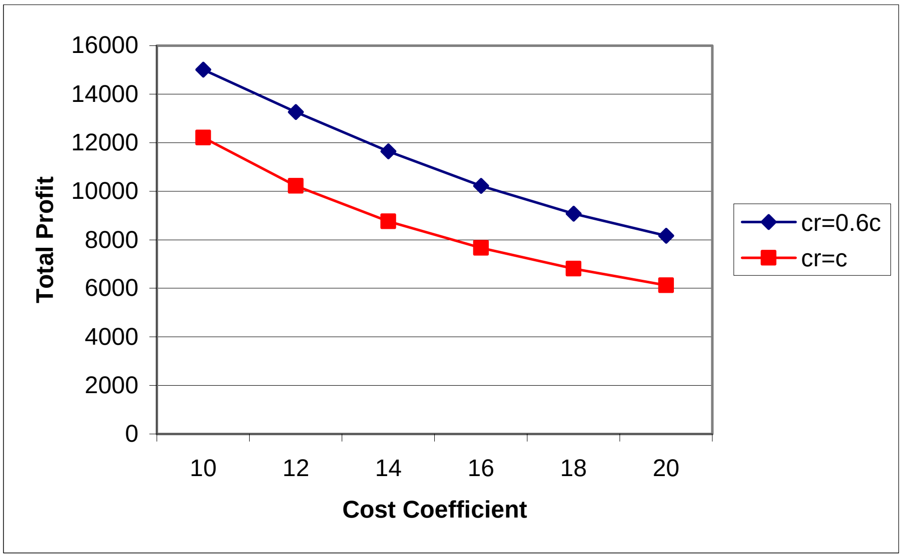


Figure 6.20. Comparison of profit for different values of the cost coefficient of remanufacturing

Figure 6.21 compares average price of manufactured products for the two cases. We see that except the first two points, prices are set to the same values (since for the first two points, for the case where the cost coefficient of remanufacturing is 60 per cent of the cost coefficient of manufacturing, quality levels of remanufactured products are set to their upper limits). So, we can say that prices of manufactured products are not affected by unit remanufacturing cost.

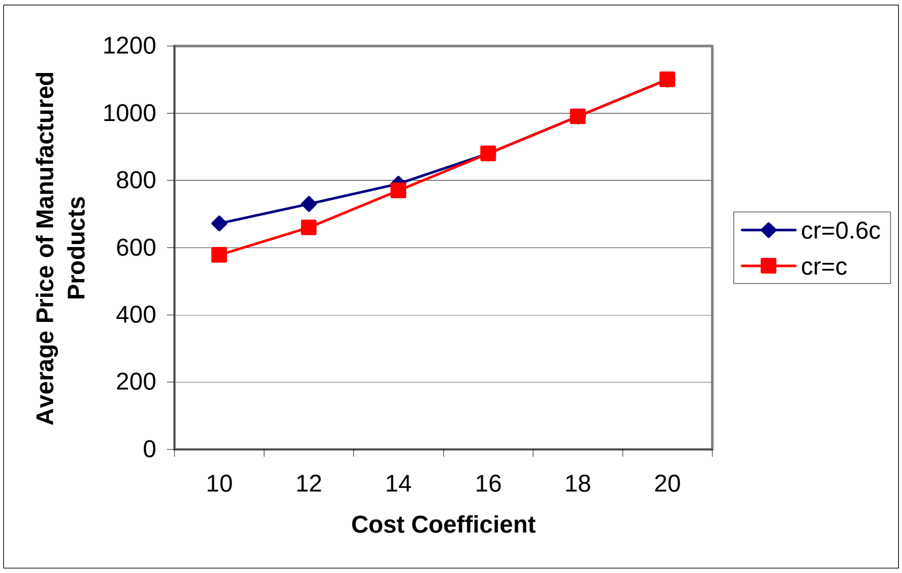


Figure 6.21. Comparison of average price of manufactured products for different values of the cost coefficient of remanufacturing

Figures 6.22 and Figure 6.23 respectively show average quality level and price of remanufactured products for the cases where the cost coefficient of remanufacturing is 60 per cent of coefficient of manufacturing and where they are the same. We see that quality levels are higher for the first case as expected, but prices are nearly the same. It is also seen that the ratio of prices to new costs is equal to $\lambda$ in this case too.

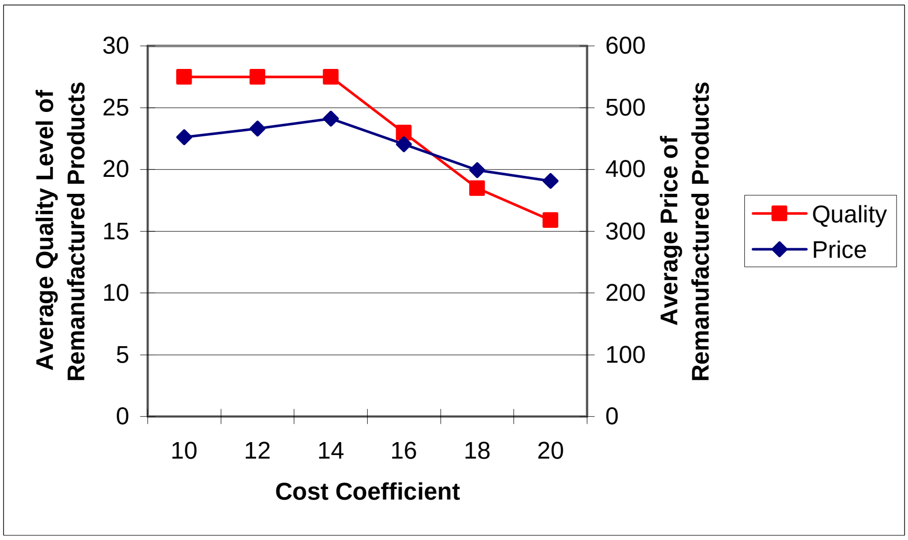


Figure 6.22. Average quality level and price of remanufactured products with respect to the cost coefficient where $c_r = 0.6\ c$

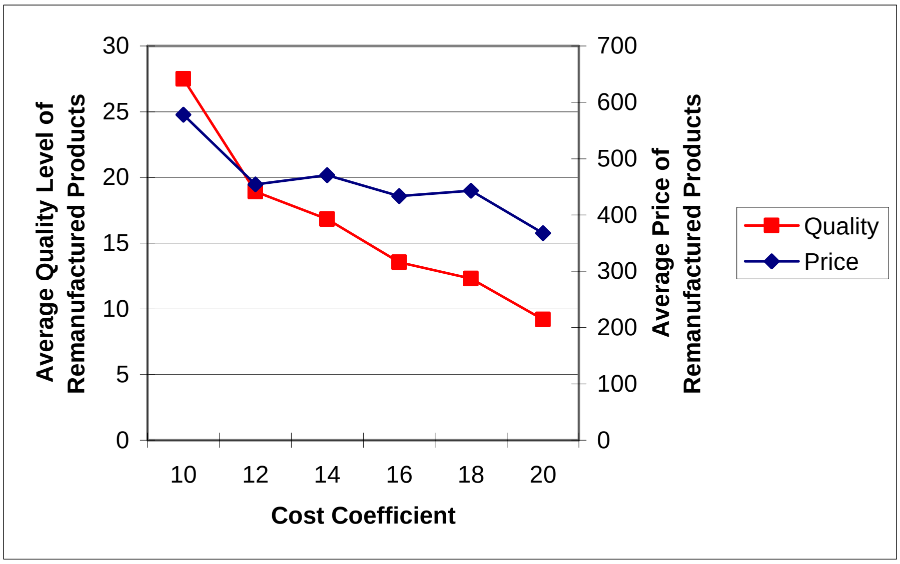


Figure 6.23. Average quality level and price of remanufactured products as a function of the cost coefficient where $c_r = c$

If we compare Figure 6.18 with Figure 6.24 which shows the market share of products as a function of the cost coefficient for the case where the cost coefficient of remanufacturing is 60 per cent of coefficient of manufacturing, we see that the remanufactured products' market share is higher. But as $c$ increases, the difference between the two cases decreases. Also the solution is more robust to the changes in costs for this case.

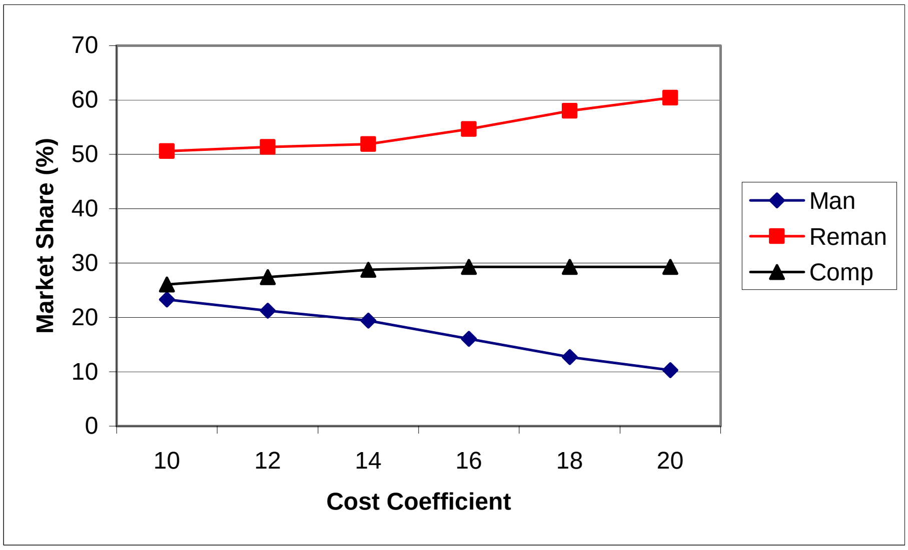


Figure 6.24. Market shares with respect to the cost coefficient where $c_r = 0.6\ c$

Figure 6.25 shows the profit gained from manufactured and remanufactured products with respect to the cost coefficient for the two cases. It is seen that when the cost coefficients are equal, profit gained from manufactured and remanufactured products are also equal (since $\lambda$=2). When the cost coefficient of remanufacturing is 60 per cent of coefficient of manufacturing, profit gained from remanufactured products is significantly higher than manufactured products for all values of the cost coefficient.

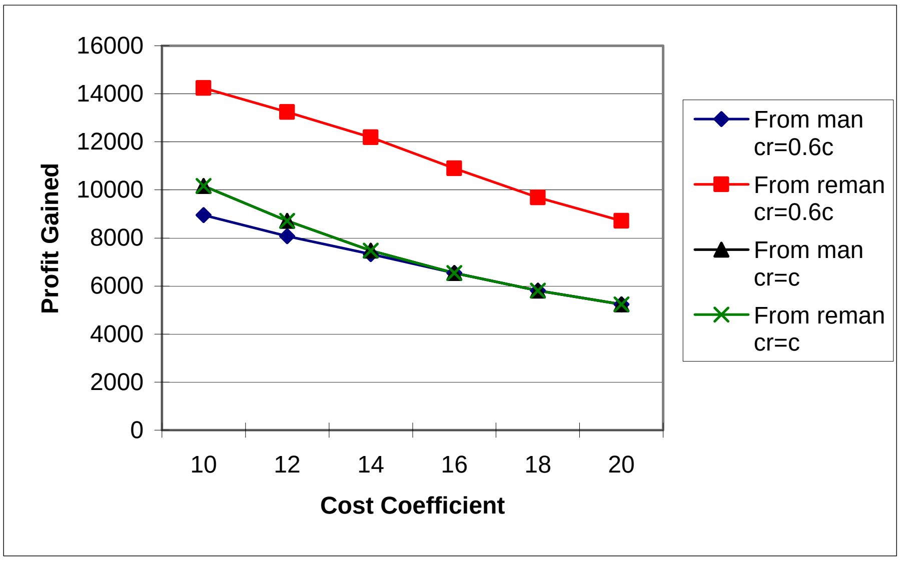


Figure 6.25. Comparison of profit gained from manufactured and remanufactured products for different values of the cost coefficient of remanufacturing

### 6.4.6. Effect of Changes in $\lambda$

Since $\lambda$ is the power of prices in customer preference function (4.5), an increase in $\lambda$ means that the importance of price increases for customers in making a buying decision and the importance of quality level decreases. So, as $\lambda$ increases we expect to have lower quality levels and lower prices for remanufactured products at an optimum solution.

According to the results of our experiments, we observed this expected situation. As $\lambda$ increases while fixing all other parameters constant, quality levels and prices of the remanufactured products decrease as well as prices of manufactured products. By this way the market share of remanufactured products increase and the market share of manufactured and competitor's products decrease. As a consequence, total profit is increased as can be seen in Figure 6.13 and Figure 6.19. So we can say that remanufacturing is more advantageous for industries that have bigger $\lambda$.

Figure 6.26 and Figure 6.27 respectively show average prices of manufactured and remanufactured products for different $\lambda$ values. Figure 6.28 shows average quality level of remanufactured products. It is seen that prices of manufactured and remanufactured products and qualities of remanufactured products decrease on average as $\lambda$ increases. It is also observed that as the value of $\lambda$ increases, the effect of $\lambda$ on the solution decreases.

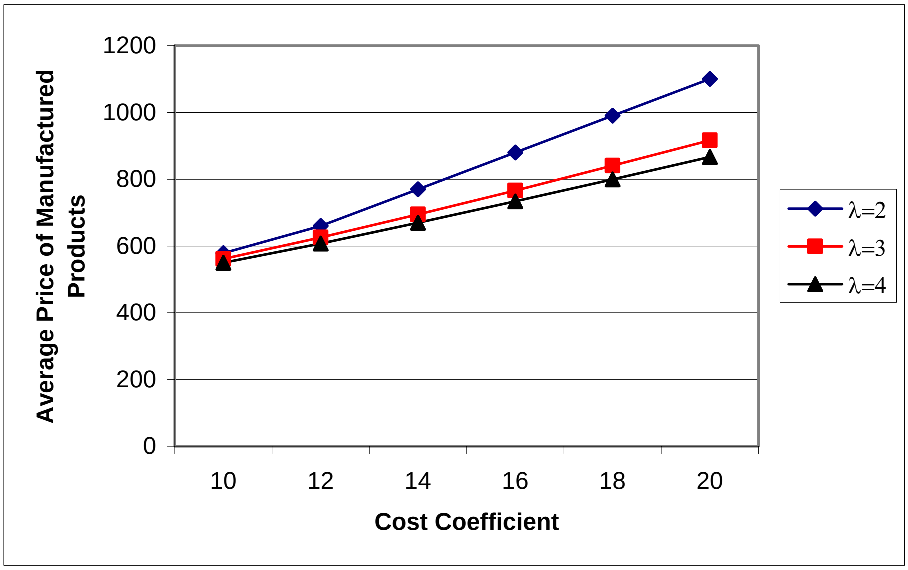


Figure 6.26. Average price of manufactured products as a function of the cost coefficient for different values of $\lambda$

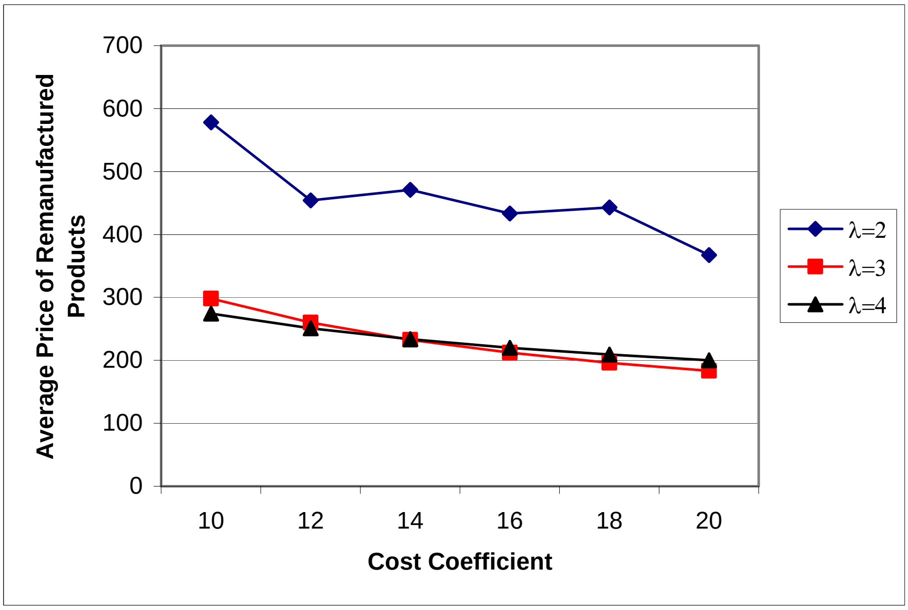


Figure 6.27. Average prices of remanufactured products as a function of the cost coefficient for different values of $\lambda$

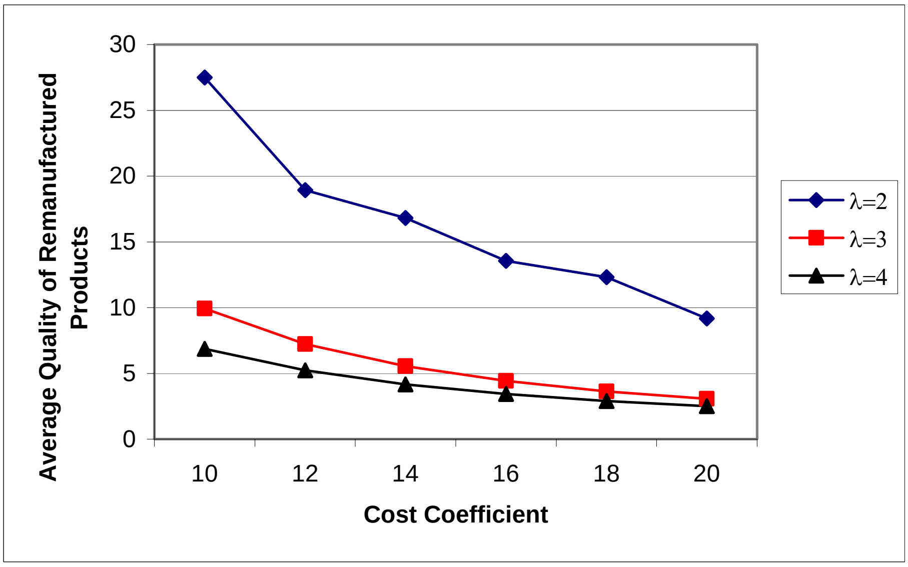


Figure 6.28. Average quality level of remanufactured products as a function of the cost coefficient for different values of $\lambda$

Figure 6.29, Figure 6.30 and Figure 6.31 respectively show the market share of manufactured, remanufactured and competitor's products, with respect to the cost coefficient for different values of $\lambda$. As $\lambda$ increases, total market share of the OEM increases, meaning that the increase in the market share of remanufactured products is much more than the decrease in the market share of manufactured products.

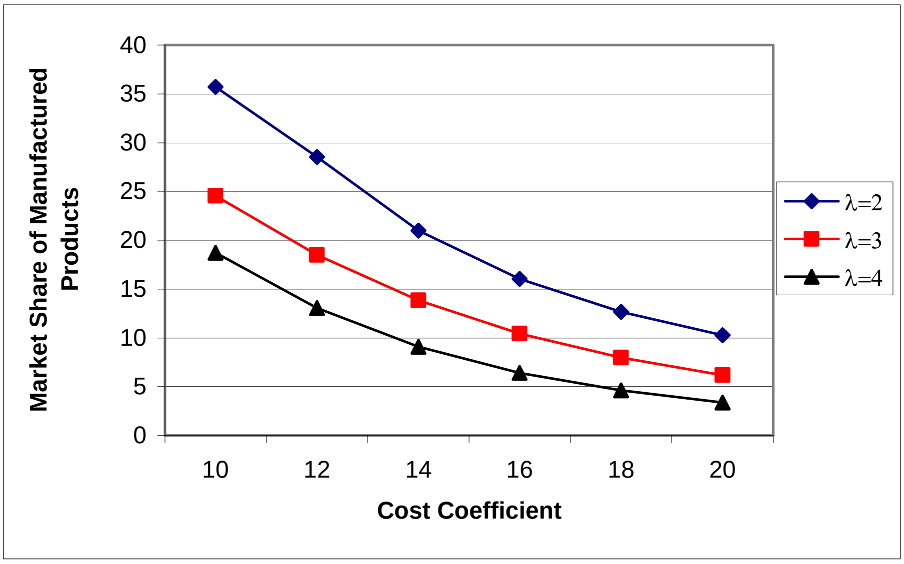


Figure 6.29. Market share of manufactured products as a function of the cost coefficient for different values of $\lambda$

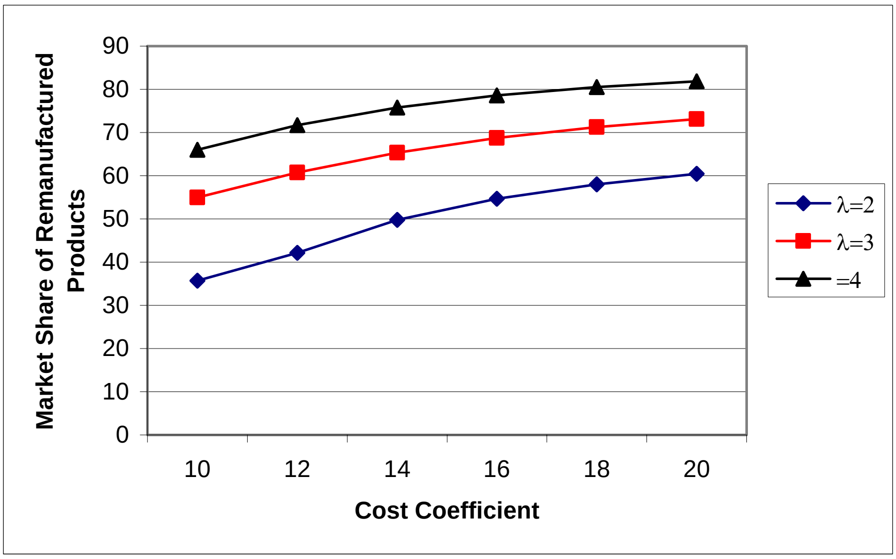


Figure 6.30. Market share of remanufactured products as a function of the cost coefficient for different values of $\lambda$

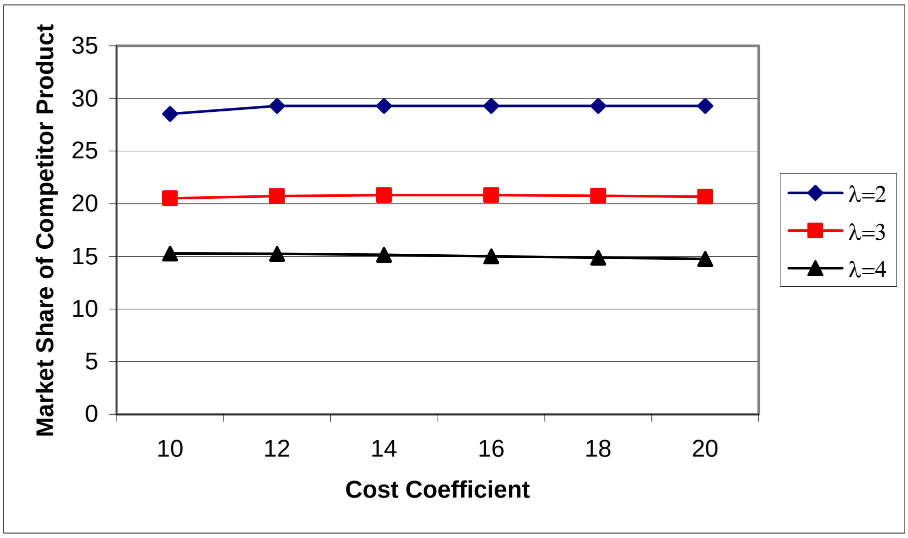


Figure 6.31. Market share of competitor's product as a function of the cost coefficient for different values of $\lambda$

Figure 6.32 and Figure 6.33 respectively show the profit gained from manufactured and remanufactured products, with respect to the cost coefficient for different values of $\lambda$. It is seen that profit gained from manufactured products decreases as $\lambda$ increases whereas profit gained from remanufactured products increases. Total profit of the OEM increases with $\lambda$, meaning that the increase in profit gained from remanufactured products is more than the decrease in profit gained from manufactured products.

Moreover, as it is pointed out before, profit gained from manufactured and remanufactured products become equal to each other for $\lambda$=2. For $\lambda > 2$, the profit gained from manufactured and remanufactured products no longer become equal to each other as can be seen in Figure 6.32 and Figure 6.33.

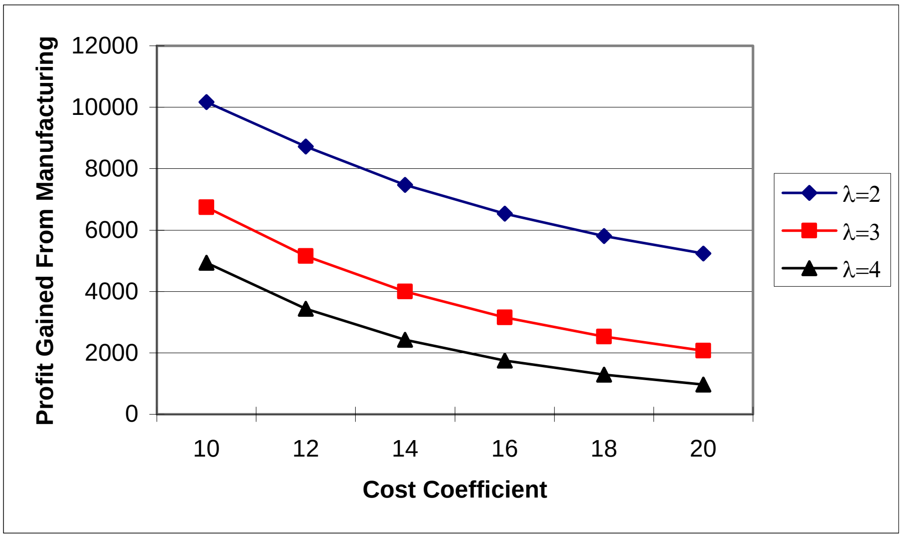


Figure 6.32. Profit gained from manufactured products with respect to the cost coefficient for different values of $\lambda$

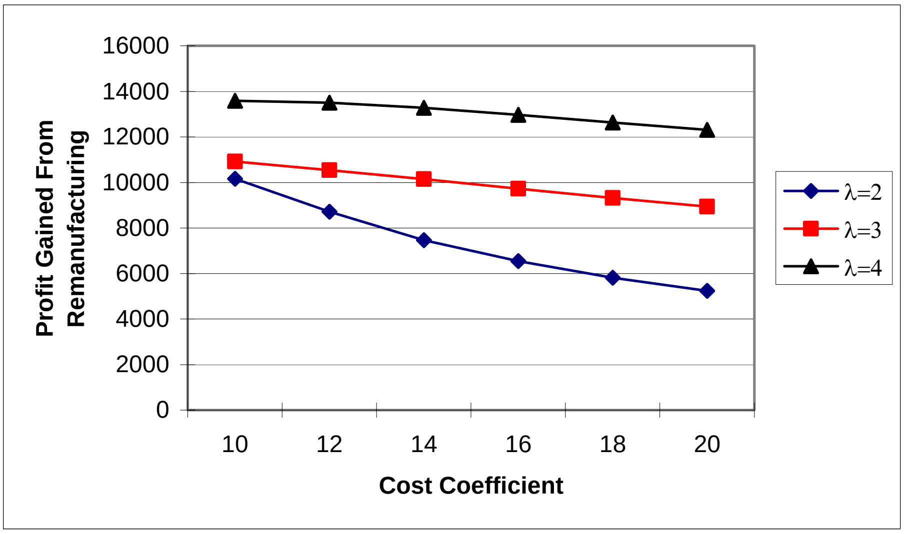


Figure 6.33. Profit gained from remanufactured products with respect to the cost coefficient for different values of $\lambda$

### 6.4.7. Effect of the Quality Level Constraints

As we eliminate quality level constraints (4.22) in the base model, quality levels of remanufactured products are set to infinity as expected. In the lost profit case, however, quality levels are not set to infinity even if we eliminate quality level constraints. Also, the ratio of prices to quality levels of remanufactured products becomes equal to $c\lambda$ in this case too as it is described above (for $\lambda = 2$, this is valid also for manufactured products).

When we modify one of the constraints (4.22) for a remanufactured product *j* by changing the right hand side with a parameter instead of $Q_{Mj}$ (namely, $Q_j \leq k$ for an appropriate parameter *k*), the solution does not change if the constraint is not binding and *k* is greater than $Q_{Mj}$. If *k* is less than $Q_{Mj}$ and less than the optimal value of $Q_j$, new $Q_j$ is set to the value of *k*. Also, if the constraint is binding (i.e., if $Q_j = Q_{Mj}$), the value of $Q_j$ is changed according to the value of *k*.

# 7. CONCLUSION AND FURTHER RESEARCH

In this study, we worked on a problem about determining optimal quality levels and prices in a hybrid manufacturing/remanufacturing system. Assuming that remanufacturing is desired for some reasons (i.e., strategically or as a legislation, etc.) and it is possible to remanufacture the products to a quality level less than the quality level of its manufactured version, we formulated a model that seeks optimal price-quality combinations for remanufactured products and optimal prices for manufactured products that maximize total profit for a single period. Cost of remanufacturing a product is modeled as a linear function of its quality level that is less than or equal to the quality level of its manufactured version which is also on sale. Customer preference is modeled by using the gravity based approach originally proposed for location-allocation problems, and total demand is shared by manufactured and remanufactured products of the OEM and a competitor’s product whose quality level and price are fixed at some predetermined values. Since the results of the base model showed that a cost parameter for the lost market share should be added to the objective function in order to obtain meaningful results; two extensions of the base model considering lost profit are also constructed.

The formulated nonlinear models are solved by the simplex search algorithm originally developed by Nelder and Mead which is a well-known direct search method. Since the algorithm is originally designed for unconstrained problems, some modifications are made to the algorithm to handle quality and price constraints of our models. Some of the problems are also solved with Excel Solver in order to see if they give the same results, and we found out that they both give the same objective value but the solutions may have different decision variables since having multiple optimal solutions is also possible. But Excel Solver failed to find the solution after a series of experiments, so we could not make all the experiments having different parameter combinations with Excel Solver.

Experimental results show that after adding lost profit parameter to the profit function, the model started to behave as expected.

Analyzing the solutions found by the algorithm, it is observed that they have some features that should pointed out. The ratio of prices to costs of remanufactured products is equal to $\lambda$ which is the power of price in customer preference function, where quality constraints are not binding. Moreover, total market share of the OEM remains almost constant for a fixed value of $\lambda$ whereas other parameters are changed. This means that there are different optimal market share values to be gained by the firms for different industries having different $\lambda$ values if qualities and prices of competitors' products are fixed. As the competitor's product becomes more attractive to the customers (i.e., as its quality increases or price decreases or both), average quality levels and average prices of remanufactured products decrease and as a consequence total profit of the OEM decreases considerably but it conserves its total market share. As the quality level of a manufactured product of the OEM increases, its price also increases and market share decreases. This decrease is offset by the increase in the market share of remanufactured products by choosing appropriate quality-price combinations, keeping total market share and total profit almost constant. As unit production costs increase, prices of manufactured products increase whereas quality levels and prices of remanufactured products decrease, the market share of remanufactured products increase whereas the market share of manufactured products decrease, keeping total market share constant but with a less total profit. If remanufacturing the cost coefficient decreases while manufacturing the cost coefficient remains constant, quality levels of remanufactured products increase whereas prices are kept almost constant, so their market share increases. The market share of manufactured products decrease since their quality levels and prices did not changed but remanufactured products are now more attractive to customers. Total market share remains constant but with a higher total profit value. An increase in $\lambda$ means that the importance of price is increased, so quality levels of remanufactured products and prices of all products decrease, the market share of remanufactured products increase and the market share of manufactured products decrease, total market share and profit of the OEM increases.

As a conclusion, we can say that remanufacturing brings a considerable advantage for products that have high $\lambda$ values in their customer preference function and remanufacturing to a lower quality level is more profitable for products having higher production costs. We may conclude that, remanufacturing, together with quality and price management serve as a tool for increasing robustness of the OEM in terms of market share and profit to changes in parameters of the market and the manufacturing environment.

As a further research, it may be interesting to investigate the case where quality levels of incoming products vary and unit remanufacturing costs depend both on the incoming quality levels and the quality level that the product will be reconditioned. It will also be interesting to add availability restriction on remanufactured products since their available amounts are naturally restricted with the amounts that are sold in previous periods, so formulating a multi period model may contribute literature.

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