A PROFITABILITY ANALYSIS OF PRODUCT SALES AND LEASING IN SUPPLY CHAINS

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ABSTRACT

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As a result of marketing policies based on cheaper and less durable products with short life cycles, the consumption and disposal rates have increased and therefore many developed countries introduce laws and regulations to minimize the disposal of end-of-life products. Thus, collecting used products, recycling and remanufacturing have become important and profitable operations for many companies. Since components are used without losing their added values for the majority of the multi-component durable products, remanufacturing is a better alternative than recycling. However, product remanufacturing has uncertainties in quality, quantity and timing of returns since the manufacturing systems are generally based on the sales of products, not the sales of services. Our aim in this study is to compare the profitability of systems based on sales of products with or without remanufacturing and examine whether or not the sales of services is more advantageous than sales of goods. For this purpose, three different structures are constructed. The first structure (TSM) is a traditional supply chain where products are sold to customers while the second structure (SRM) includes reverse supply chain where used products are acquired by the manufacturer for remanufacturing and both remanufactured and new products are sold. Finally, the third structure (LM) considers a closed loop supply chain where products are leased and leased products are remanufactured at the end of each lease period. To compare the profitability of supply chains with respect to product life cycle, these supply chains are modeled as queuing networks with $GI/G/1$ and $GI/G/\infty$ stations. Then, these networks are analyzed using parametric decomposition method in order to obtain performance measures. After validating the accuracy of the queuing network models using simulation, profit functions for each model are defined using the performance measures. Then, the profitability of the structures is compared while changing the values of parameters of interest. Also, cannibalization effect of the remanufactured products and effects of proper design are examined. Finally, conclusions are derived.

ÖZET

TEDARİK ZİNCİRLERİNDE ÜRÜN VE HİZMET SATIŞININ KARLILIĞININ İNCELENMESİ

Kısa ömürlü, daha ucuz ve daha az dayanıklı ürünlere dayalı pazarlama politikalarının bir sonucu olarak tüketim ve atılım hızı artmaktadır ve bu yüzden birçok gelişmiş ülke ömrünü tamamlamış ürünlerin atılımını en aza indirgemek için yasalar ve düzenlemeler ortaya koymaktadır. Böylece ürünlerin geri toplanması, geri dönüşümü ve yeniden imalatı işleri birçok şirket için önemli ve kar sağlayan etkinlikler haline gelmektedir. Çok bileşenli dayanıklı ürünlerin çoğunluğunda bileşenler ek değerlerini yitirmeden kullanıldığı için, yeniden imalat geri dönüşümden daha iyi bir alternatiftir. Ancak yine de üretim sistemleri çoğunlukla hizmet satışı değil, ürün satışı üzerine kurulduğu için ürünlerin yeniden imalatı geri dönen ürünlerin kalitesi, sayısı ve zamanlaması bakımından belirsizlikler içermektedir.. Bu çalışmadaki amacımız, yeniden imalat süreçlerini içeren ve içermeyen ürün satış sistemlerini karşılaştırmak ve hizmet satışının ürün satışından daha avantajlı olup olmadığını incelemektir. Bu amaç doğrultusunda, üç farklı yapı kurulmuştur. Birinci yapı ürünlerin tüketicelere satıldığı geleneksel tedarik zinciriyken ikinci yapı hem yeni hem de yeniden imal edilmiş ürünlerin satıldığı ve kullanılan ürünlerin yeniden imalat için geri alındığı tersine tedarik zinciridir. Son olarak, üçüncü yapı kiralanan ürünlerin her kira dönemi sonunda yeniden imal edildiği kapalı döngü tedarik zincirini dikkate almaktadır. Ürün yaşam döngüsünü göz önünde bulundurarak tedarik zincirlerinin karlılığını karşılaştırabilmek için, bu tedarik zincirleri kuyruk ağları kullanılarak modellenmiştir. Daha sonra, bu modeller performans ölçütlerini elde edebilmek için parametrik ayırma methodu kullanılarak analiz edilmiştir. Simulasyon kullanarak kuyruk ağı modellerinin doğruluğunu onayladıktan sonra, performans ölçütlerini kullanarak her model için kar fonksiyonları tanımlanmıştır. Daha sonra yapıların karlılığı ilgilenilen parametre değerleri değiştirilerek karşılaştırılmıştır. Ayrıca, yeni ürünlerin satışlarının yeniden imal edilmiş ürünlerin satışları nedeniyle düşmesinin karlılık üzerindeki etkileri ve tasarım ve yeniden imal edilebilme arasındaki ilişki incelenmiştir. Son olarak, sonuçlar ortaya konmuştur.

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LIST OF SYMBOLS/ABBREVATIONS

b	Unit backorder cost
c_r	Ratio between unit core cost and unit raw material cost
$C_{a,i}^2$	Squared coefficient of variation of the external inter-arrival time at station i
$C_{d,i}^2$	Squared coefficient of variation of departures from station i
$C_{s,i}^2$	Squared coefficient of variation of service time of jobs at station i
d_i	Depreciation for one dollar investment
D	Index for disposal station
D_t	Depreciation payment for lease period t
$E[N_i]$	Expected number of jobs at station i
$E[W_i]$	Expected waiting time at station i
F_t	Finance payment of lease period t
h_i	Unit holding cost per unit time at station i
i	Index for the stations in the network
k	Ratio between the average remanufacturing time and the production time
L	Index for demand station in the leasing model
M	Total number of sales of a firm
M_N	Number of sales of new products
M_R	Number of sales of remanufactured products
m_n	Unit profit margin of a new product
$m_{d.r}$	Unit margin of a disposed product class r
n	Maximum number of times that a product can be remanufactured

N	Total useful life of a product
p	Ratio between price of a remanufactured product and a new product
P_m	Selling price of a new product
P_r	Selling price of a remanufactured product
P_t	Total lease payment for period t
p_{ij}	Transfer probability of jobs from station i to station j
P_{acq}	Unit acquisition cost
P_{raw}	Unit raw material cost
$P^{(r)}$	Return probability of product class r
$P_{i,j}^{(k,l)}$	Transfer probability of k type product at station i to l type product at station j
$\mathrm{PV}(P_t)$	Present value of lease payment for period t
$q^{(r,\theta)}$	Proportion of product class r with grade level θ
r	Index for product classes
R	Index for demand station in the sales with remanufacturing model
r_i	Annual interest rate
$R_{b,t}$	Residual value of the product at the beginning of period t
R_t'	Residual value after remanufacturing at the end of period t
s_i	Processing cost of station i per hour
t	Index for lease periods
T_L	Duration of the lease agreement
y_i	Overall yield of the remanufacturing station i
α	Percent of increase in the total number of sales

β	Grade distribution ratio
γ	Ratio between the number of sales of remanufactured and new products
δ	Geometric depreciation rate
δ_r	Ratio between the proportion of good quality cores and bad quality cores
η	Number of times that a product can be leased during its useful life
θ	Index for grades
λ_i	Arrival rate at station *i*
λ_i^r	Arrival rate of product class *r* at station *i*
$\lambda_{d.m}$	Arrival rate of the demand for new products
$\lambda_{d.r}$	Arrival rate of the demand for remanufactured products
λ_{ij}	Flow rate from station *i* to *j*
π_1	Profit rate of the traditional sales model
π_2	Profit rate of the sales with remanufacturing model
π_3	Profit rate of the leasing model
ρ_i	Utilization of station *i*
τ_i	Mean processing time of all jobs at station *i*
φ_t	Percent of increase in residual value at period *t*

FCFS	First come first served policy
LM	Leasing model
TSM	Traditional sales model
scv	Squared coefficient of variation

SRM	Sales with remanufacturing model

1. INTRODUCTION

In the last four decades increasing competition in major markets of the industrialized world has generated marketing policies based on cheaper and less durable products with decreasing life cycles. As a consequence, consumption and disposal rates have increased to unmanageable levels. Currently most industrialized countries are facing major environmental challenges. This fact has placed great emphasis to the product remanufacturing business and the need for a closed loop supply chain. Today, many companies recognize the residual value of end-of-life products and concentrate on the remanufacturing strategy since it is a profitable business policy.

Remanufacturing can offer many benefits like sustainable profitability, energy savings, significant reduction in carbon emissions and material conservation. Since most of the raw materials of the end-of-life product are used in the remanufacturing process, only a small fraction of material processing is needed for new products. Thus, the volume of the energy consumption and the amount of natural resources required to produce the assets are reduced considerably. As a result of the reduction in the consumption of raw materials, life cycle costs decrease. Considering the energy consumption, only about 15 percent of the energy used in the production of a new product is required to remanufacture a product [1]. Consequently, carbon emissions are reduced significantly. In terms of the carbon emissions, estimated savings are 28 million tons of CO_2 annually [1]. Moreover, from the perspective of the customers, remanufactured products can be purchased for lower prices without compromising quality. To represent the importance of remanufacturing, it is claimed that the size of the remanufacturing sector in the United States is $53 billion, with over 70,000 firms and 480,000 employees [2]. However, remanufacturing has some difficulties and uncertainties. For instance, collection of returned products, design for remanufacturing, uncertainty in product returns are some of them.

In the existing structure with remanufacturing process, the collection rates are still less than manufacturing rates. Also, since products are sold the return time and condition of used products are unknown. This fact complicates the remanufacturing process. On the other hand, in most cases customers prefer to own the right to use instead of owning the product. In order to derive maximum benefit from remanufacturing and cope with its difficulties, sales of services can be employed instead of sales of products. Considering the sales of services, leasing strategy is preferable due to low willingness to buy remanufactured product and short usage periods. In leasing strategy, controlling the product returns reduces uncertainty in the remanufacturing activities and allows us to sustain high profit margins. Moreover, leasing is a widely used business strategy in United States. As an illustration, according to the research done by Equipment Leasing Association in 1999, 80 percent of all US companies lease some or all of their equipment and approximately 226 billion worth of equipment are leased in the US [3].

In this thesis, we examine three different structures. The first structure is a traditional supply chain where products are sold to customers while the second structure includes reverse supply chain where used products are acquired by the manufacturer for remanufacturing and both remanufactured and new products are sold. Finally, the third structure considers a closed loop supply chain where products are leased and leased products are remanufactured at the end of each lease period. We investigate which structure is more sustainable and profitable. For this purpose, we use open and closed queuing networks with *GI/G/1* and *GI/G/∞* stations.

The rest of the thesis is organized as follows. Chapter 2 provides an overview of the product life cycle modeling, remanufacturing and leasing literature. In Chapter 3, the main objectives of the thesis are described. In Chapter 4, after the problem definition, the model formulation and determination of the leasing payments take place. Validation for the proposed method is stated in this chapter, as well, and at the end profitability function is given. Numerical analysis is illustrated in Chapter 5. In the last chapter, conclusions are drawn.

2. LITERATURE REVIEW

The problem we consider falls in the research areas of remanufacturing, leasing and product life cycle. Therefore, in the literature review we consider the studies related to these subjects. Section 2.1 discusses literature on product life cycle modeling, which forms the basis of our research methodology. Section 2.2 discusses literature on remanufacturing where driving factors, benefits and problems of remanufacturing are highlighted. Also literature for our analytical model is given. In Section 2.3, the growing literature on leasing is discussed. The main topics in this section are challenges and benefits of leasing, determination of lease payments and comparison of leasing and selling systems.

2.1. Product Life Cycle Modeling

Since concerns about environmental issues accumulate, developments in industry cause rapid changes on product life cycles. The technological improvement interest of customers triggers the decrease of the total life cycle of a product. Thus, environmental waste increases considerably. To take precautions for this problem, companies focus on the design of the total product life cycle including product design, production, usage, maintenance and recycling/disposal. Companies are realizing that only the effective product design and production is not enough for providing good resource consumption and recycling performance. However, controlling the flow of a product particularly in the usage and disposal phases is difficult. Furthermore, assessing the quality of the product through the whole life cycle is problematic. To cope with these difficulties, Kimura and Suzuki assert that the preplanned and controlled closed cycles in remanufacturing give us the opportunity to make the whole product life cycle visible [4]. In other words, remanufacturing is a good alternative since it allows us to control the life cycle of a product. Issues like remanufacturing, refurbishment, small repairs help us not only to satisfy environmental conditions but also to allow the customers to follow the technological improvements by periodical upgrades and to maintain the products.

In the literature, from the perspective of total life cycle, remanufacturing is compared to manufacturing new products in terms of energy usage and other costs. For instance, it is asserted that when the product fits the necessary production characteristics of remanufacturing, the energy required to remanufacture a product is significantly less than the energy used in recycling [2]. Moreover, Boustani et al. [5] investigate the potential savings of energy and economics through the total life cycle by extending the service life of a product via remanufacturing. They conclude that if the cost of remanufacturing is less than the purchase cost, then remanufacturing becomes economically preferable. Yet, they also state that despite the savings achieved in production, remanufacturing is a net energy-expending end-of-life alternative. They observe that the energy consumption in customer usage period is significantly higher for a remanufactured product than a new product as a result of efficiency improvements in new products. Another consideration is based on the environmental aspects. Lindahl et al. [6] focus on the environmental impacts of remanufacturing by using the life cycle assessment methodology. They look at various cases in the literature and conclude two important issues. One of them is that remanufacturing is more environmental friendly than manufacturing a new product with respect to material resource perspective. The other is that we cannot draw a general conclusion about the advantages of remanufacturing since the benefits from remanufacturing are highly context-related. Furthermore, Smith and Keoleian [7] develop a life cycle model to analyze environmental and economic perspectives on remanufacturing. The life-cycle model show that the remanufactured engine could be produced with 68 to 83 percent less energy and 73 to 87 percent fewer carbon dioxide emissions.

Considering life cycle implications for the remanufacturing system, the disposal rate is very important for profitable remanufacturing. This rate is influenced by the age of the product, the mean product life time, the rate of technological development and the willingness to return products for remanufacturing. In this context, Ostlin et al. [8] investigate the factors that affect product returns from end of use and end of life. The rate of technological development and the expected life of a product classified as the major influencing characteristics. To balance the product returns and demand for remanufactured products, by using product life cycle perspective they propose strategies which helps to forecast the general trends of remanufacturing volumes. Moreover they suggest that the life cycle theory could be effective in remanufacturing industry.

2.2. Remanufacturing

After The Second World War the popularity of remanufacturing has ascended considerably due to various reasons. This increase demonstrates itself especially in automotive sector. In recent years, remanufacturing expanded to several sectors such as electrical apparatus, toner cartridges, household appliances, machinery, cellular phones and single use cameras. Haynesworth and Lyons [9] characterize remanufacturing as the process of bringing a product to like-new condition through replacing and rebuilding component parts. In the following paragraphs, the driving factors, the benefits and the difficulties of remanufacturing are introduced.

When we look at the driving factors of remanufacturing, we see three major issues; ecological factors, legislation and economics. The environmental concerns and legislation due to these concerns seem to be the forcing factors. On the other hand, firms prefer remanufacturing most frequently for its cost reduction properties. In the context of ecological factors, the predictable increase in hazardous materials enforces firms and governments to become conscious. Moreover, decreasing proportions of landfills has brought remanufacturing to life. As it renders a large reduction in industrial waste and in energy consumption, remanufacturing becomes preferable. Boustan, et al. [10] state that the amount of waste generated in remanufacturing is significantly less than the waste generated during the manufacture of new components.

Another driving factor for remanufacturing is legislation. Because of the legislations, firms are forced to take back their products and recover them. This provides firms to manufacture more efficiently and use returned products in their production processes. Brito [11] states that these legislations include two important issues; consumer rights and pro-environmental legislations. As an illustration of these legislations, the European Union publishes a directive called the European Directive on Waste Electronic and Electrical Equipment. The idea is to reduce the waste that results from electronic and electrical products and support the re-use of this type of products. Also, countries employ legislation for environmental concerns. For instance, The German Recycling and Waste Control Act and Swedish take-back legislation are examples for legislations that aims increased producer responsibility. Additionally, the Environmental Protection Agency proposes the

Recovered Material Advisory Notice that supports the idea of reusing and rebuilding motor vehicle parts. All of the legislations put forward the necessity of remanufacturing with respect to both environmental and economic factors.

Finally, economic factors cause firms to concentrate on remanufacturing, as for expensive products, remanufacturing returned products is less costly than producing new ones. In this context, Brito states that generally companies can decrease the material costs, production costs and can make value added recovery by remanufacturing. Also, they can have a green image, comply with the laws, protect their market share and improve customer relations with the help of remanufacturing [11]. Although the costs decrease in remanufacturing, the quality of the remanufactured products can be equal or superior to the new product [9]. Therefore, they have the same warranty conditions as the new ones.

Major benefits of remanufacturing are reduction in environmental impact, energy savings and cost savings. The environmental benefits can be grouped as; energy conservation, raw material conservation, landfill space conservation and air pollution reduce. Reusing the product and reprocessing the waste materials decreases the raw material consumption. Also, remanufacturing reduces the amount of material processing and in this way it differentiates from recycling strategies. Extended life time of products reduces the emissions and energy consumptions in manufacturing new products relatively. Seitz and Peattie [12] give an example from automotive part remanufacturing which reveals that the energy consumption of a new starter is more than eleven times of the energy required to remanufacture the product. When we evaluate the company benefits, we see concentration on life cycle costs. As an initial remark, the usage of returned products decreases the raw material costs. Additionally, processing costs are reduced due to low remanufacturing times. Also, the low price of remanufactured products enables firms to target the second hand market and compete with products at lower price.

Reprocessing of used products consists of collection, grading product recovery and re-assembly. Brito [11] classifies product recovery with respect to the necessary activities to be performed on a returned product as follows:

i. Repair : The product is repaired at the product level without disassembly
ii. Refurbishing : The product undergoes a more extensive operation for upgrading
iii. Remanufacturing: The product undergoes operations at the component level where it is disassembled and its components are refurbished
iv. Part recovery : Parts of the product can be collected for part recovery which is called retrieval
v. Recycling : Recycled material can be used in raw material input with the help of recycling
vi. Incineration : Returned products can be burned for producing energy in the case of incineration

When we look at the whole picture, there are obstacles in remanufacturing. In this context, Amezquita et al. state that products which have a damaged or broken major part cannot be remanufactured [10]. If the major parts of a product are damaged, the cost of remanufacturing increases. In this condition, the firm decides not to remanufacture or the product cannot be remanufactured again due to technologic constraints. Guide [13] defines seven characteristics that complicates remanufacturing as the uncertain timing and quantity of returns, the need to balance returns with demands, the complications in disassembly of returned products, the uncertainty in materials recovered from returned items, the requirement for a reverse logistics network, the complication of material matching restrictions and the problems of stochastic routings for materials for remanufacturing operations with highly variable processing times. The uncertainty and variability in disassembly can be avoided with design for remanufacturing. Amezquita et al. confirm that the design for remanufacturing must contain these attributes; ease of disassembly, ease of cleaning, ease of inspection, ease of part replacement, ease of re-assembly and reusable components [11]. The attributes above decrease the remanufacturing time of a product considerably.

In addition, there are specific problems particularly in collection and reprocessing. Geyer and Jackson [14] state some limitations with respect to these problems. Limited access to end-of-life products leaving the use phase, limited feasibility of end-of-life product reprocessing and limited market demand for the secondary output from reprocessing are those limitations. Moreover, Geyer et al. [15] propose a model remanufacturing system with product take back. The constraints of the model are limited component durability and finite product life cycle. Moreover, they state that remarkable cost savings are achieved as a result of careful coordination of collection rate, product life cycle and component durability. They show that remanufacturing cost is reduced if the yield of remanufacturing is above a critical value.

Particularly, in the remanufacturing area various types of queuing networks are employed to have a method of approach for the uncertainty in quantity, quality and timing of product returns. Similarly, in this research queuing networks are used to model the forward, reverse and closed loop supply chains. To this end, we give related literature to our queuing network model in the following paragraphs.

Souza and Ketzenberg [16] use a *GI/G/1* network to model a remanufacturing facility with a grading station. The remanufacturing facility is modeled as three stations where each station is employed for a different quality grade type to capture the impact of different quality grades for processing times and costs. Here, they try to maximize total profit and find the optimal product mix. In another article, Souza and Ketzenberg [17] consider a make to order manufacturing process where demand can be met with either new or remanufactured product. They model each station as a *GI/G/1* queue. Remanufacturing and manufacturing activities are conducted at different stations. The objective is to find out the optimal long–run production mix where the average order lead-time is restricted by service level constraints

Vorasayan and Ryan [18] examine the optimal price of a refurbished product and proportion to refurbish for a competitive strategy in the market by taking into account the market cannibalization effect of the remanufactured products. The system that they propose consists of 5 station which are manufacturing, customer usage, evaluation, refurbishing and storage. Except for the customer usage station, they model each station as *M/M/1* queue. The customer usage station is modeled as $M/G/\infty$ queue. All the stations are analyzed independently. In another study, Vorasayan and Ryan [19] develop a general queuing network in relation with both market segmentation and uncertainty in manufacturing at the same time without restrictive assumptions on probability distributions of service times. They use a *GI/G/c* open queuing network for this model. They also consider the consumer willingness to buy refurbished products. They examine the variation in profit with respect to product return probabilities, quality grade of returned products, number of times that a product can be refurbished and uncertainty in time of remanufacturing.

Furthermore, Toktay et al. [20] use a closed queuing network to model the life cycle of a single use camera. To find optimal ordering policy for handling the uncertainty in return flows which cannot be observed, they propose a heuristic procedure. The objective is minimizing the total expected procurement, inventory holding and lost sales cost. In another study, Bayindir et al. [21] model the life time of a product as $M/G/\infty$ queuing network to find the optimal return probability of sold items. It is assumed that there is no demand difference between remanufactured and new products due to customer willingness. They form a cost model to investigate the utilization of remanufacturing policy under controllable return rate.

2.3. Leasing

Nowadays, leasing has become a remarkable and consistent strategy as an alternative to retail. Fishbein et al. define leasing as a strategy that improves resource productivity by preventing generation of waste and employing closed loop systems [22]. Particularly, the benefit of closed loop systems arises from the fact that the waste of returned product becomes the source of raw materials for a new product. This fact is achieved by recycling, remanufacturing or direct reusing. Accordingly, leasing strategy is more environmentally friendly than traditional selling strategy, especially for durable products. Furthermore, Desai and Prohit state that lower payments in leasing make the products more affordable [23]. They also state that in lease agreements customers are paying for only a portion of the product’s full value over the lease period and thus they take the advantage of being able to finance more advanced products. In another study, Fishbein et al. [22] examine leasing strategy as an alternative to retail for increasing resource productivity by decreasing waste generation as a result of closed loop material use. Also, they classify the requirements for the extended producer responsibility as taking back the products when consumers discard them, managing them at their own expense and meet specified recycling targets. They state that in order to satisfy these requirements, companies tend to increase reuse and remanufacturing.

For the most of the leasing agreements, at the end of the lease period the ownership of the product remains in the possession of the lessor. This ownership, in a sense, forces manufacturers to manage the product returns and reduce the life cycle impacts. However, collecting the products after leasing period is not sufficient to provide environmental benefits and profitability. To capture the advantage of leasing on profitability and environmental benefits, Mont et al. [24] suggest that the manufacturer should increase the durability of the leased products with continuous upgrade and maintenance of the product, collect the product for remanufacturing and recycling at the end of its useful life and follow the basic waste management hierarchy (reduce, reuse, and recycle).

However, accomplishing these goals is not simple for leasing companies as lack of durable products becomes a problem. At the end of the lease period, the residual value of products which highly depends on the length of the lease period can be very low. In other words, they can be consumed heavily during the customer usage period. Therefore, the companies are obligated to design durable and reusable products to apply leasing as a competitive strategy. Fishbein et al. defined the product design issue as a key factor for closing materials loops and improving the efficiency of resource utilization [22]. The companies have to connect the link between end of life management and product design. Gray and Charter [25] categorize the product design issue for different strategies. According to them, a design strategy can focus on a particular step in the remanufacturing process. As an illustration, design strategies can be based on core collection, disassembly, multiple life cycles, upgrades and eco-design. Each strategy affects different parts in the remanufacturing process.

Another important issue in leasing is the determination of the lease payments. Lease payments consist of two parts; the depreciation part and the finance part. The depreciation part is the payment for the product's value that is lost during the leasing period and the finance part is the interest payment for using the lessor's money that is tied up in the product [3]. Both parts are correlated with the residual value of the product. Pierce [26] states that the residual value of the product cannot be calculated precisely due to the uncertainty about market conditions, general economic conditions and also the complexity of determination of the deterioration rate of the product. However, it is known that as the length of the lease increases, the residual value at the end of the leasing period inevitably decreases. Sharma [27] determines residual value of the electronic products where the residual value of an asset of any age is defined as the current value of an asset as a fraction of the purchase price of the new product. At this point, we can easily say that the determination of the leasing payments is a very complex issue and reflecting the remanufacturing effect to prices further complicates the matter.

Another important factor for the determination of the lease payments is the depreciation rate. Depreciation rate of a product affects the profitability of a leasing system since it has a great impact on the residual value at the end of the lease period. Desai and Purohit [28] try to show that the profitability of selling and leasing strategies is powerfully linked with depreciation rates in contradiction to common belief that leasing dominates selling in every aspect. Thus, they find out that selling is preferred if the depreciation rate of the sold products is higher than the leased products. In the end, with an application on a car model, they demonstrate that leased cars are likely to depreciate less than sold cars. Also, they concentrate on the comparison of selling and leasing strategy of a durable product. For long term decisions they investigate whether both strategies can be applied in a competitive market. In other words, they focus on employing concurrently leasing and selling strategies and determining an optimal strategy for different depreciation rates. In the following paragraphs we continue to discuss some related literature about the comparison of selling and leasing strategies.

Mont et al. [24] focus on the shift from selling products to selling functions and service through leasing. They propose a new business model for baby prams where they combine leasing and remanufacturing. The objective is determining the potential problems and changes in product design to make the supply chain work. Also they organize the reverse logistic system with different levels of refurbishing and remanufacturing. Taking into account the financial analysis, they find out that leasing system is more profitable per pram than traditional sales. Also they suggest that the new model provides customers to have high quality products in exchange of low prices and avoid transaction costs of selling the product.

In another paper, Hanafiah et al. [29] propose a model that concerns with the problem of remanufacturing in both developed and developing countries for selling and leasing conditions. Total life cycle cost of the product and product take-back options for remanufacturing in leasing and selling are considered with this model. Also the socio-economic scenarios are taken into account. It has been found that leasing becomes more economically preferable due to the increasing demand of second hand products and a common nature in developing countries.

Although leasing can be economically preferable, the logistical and environmental decisions affect this implication. In his dissertation, Sharma [27] discusses the logistical and environmental issues in electronic equipment leasing. In an environment where remarkable portion of the business assets is leased, he focuses on short life cycles of electronic equipment and strict environmental legislation on electronic waste. For analysis of decisions about lease lengths, use of logistic facilities and end of life management of assets, he proposes a deterministic, multi-period, mixed integer linear model. He finds out that without appropriate decisions for logistical and environmental issues profitability of a leasing company can decrease. Also he shows that lease lengths and both forward and reverse product flows are influenced considerably by uncertainty in transportation costs. Furthermore, he defines a strong connection between asset purchase costs, asset disposal costs and transportation costs.

We conclude from the literature review that although structures with sales of product and sales of service are investigated separately, only a few studies are carried out to compare them. Also, a few number of studies include life-cycle considerations in the introduction and management of remanufactured products. In this study, we compare the profitability of different supply chains with sales of product and with sales of service with respect to product life cycle. Also, studies in the literature rarely combine remanufacturing and sales of service. Therefore, to the best of our knowledge nobody has considered the determination of lease payments where leased products are remanufactured at the end of each lease term. Moreover, studies about remanufacturing only focus on the specific problems of remanufacturing operations. However, in this research, we concentrate on the profitability of all the operations in a closed loop supply chain rather than studying only the remanufacturing operations. Besides, we use open and closed queuing networks to model our supply chains.

3. OBJECTIVES

In this research, we compare three different supply chain structures with respect to the product life cycle. The first structure considers the traditional supply chain where a product is designed, manufactured, sold, used and disposed off, while the second structure includes a reverse supply chain to recover some of the used products for remanufacturing and sales. The third structure is a closed loop supply chain where leased products are remanufactured at the end of each lease period and new products are manufactured only if a returned product cannot be remanufactured. In this structure, we reflect the remanufacturing effect to lease payments. Our aim is to find out when each structure is more profitable. We examine the parameters of importance that differentiates the profitability of each structure for a single product over a profit function. Since we study the life cycle of durable and remanufacturable products, all life cycle costs are considered in the profit function. We compare the profitability of the structures while changing the values of parameters of interest. In order to compare the structures properly, we use queuing network models. The open and closed queuing network models allow us to track the return of products and model the returned product quality.

Moreover, we consider the market cannibalization effect of the remanufactured products where new and remanufactured products are offered at the same time. We study whether the system profits decline when remanufactured products cannibalize the sales of new products. Also, the impacts of product design to remanufacturability level in the introduction of a remanufacturable product are considered. Here, we investigate the changes in profitability as the maximum number of times that a product can be remanufactured increases and the average remanufacturing time decreases as a result of the increase in the production and design time.

4. MODEL

4.1. Problem Definition

In a traditional supply chain, a product goes through the steps of product design, production, distribution, customer usage and disposal. Here, a product is designed with respect to customer needs. Then the product is manufactured and distributed to the customer. The customer uses the product for some time. Finally, at the end of its useful life, the product goes to disposal. On the other hand, in a supply chain with remanufacturing, a probabilistic amount of used products are remanufactured. Thus, the time to disposal is delayed. Both structures are based on the sales of the product. Yet, a supply chain designed for the sales of the service of the product would have major structural differences. In order to model these three supply chain alternatives and study their differences, we propose three different queuing networks.

Firstly, we consider the sales with remanufacturing model (SRM) as it has the most general network structure. In a supply chain with end-of-life product returns, a certain portion of the demand rate for the new products comprises the return rate. The demand for remanufactured products is satisfied by this rate. However, sometimes it is difficult to balance the return rate and demand. If the demand rate is larger than the return rate, lost sales can be problem. For the exact opposite of this situation, when the return rate is larger, holding costs increase as a result of increased number of finished products waiting in storage.

A further problem, in a supply chain where both new and remanufactured products are sold at the same time, sales of remanufactured products can cannibalize the sales of new products. Remanufacturing systems that focus on the end-of-life returns can have a direct cannibalization threat if remanufactured products still have viable technology and same functionality as a new product. In this situation, when the profit margin of a remanufactured product is lower than that of a new product, cannibalization of the sales of new products can cause the total system profit to decrease.

Furthermore, uncertainties in quality, quantity and timing of returns are major challenges in product remanufacturing. These uncertainties are directly transferred to the remanufacturing step where the variation in the quality of returns causes a high variability in the remanufacturing process. Categorization of returned products (cores) based on the quality can decrease the variability in remanufacturing times. As the quality of a core is a function of the usage duration and behavior, the approach in the categorization should account for them. In the categorization, cores are categorized into levels such as that require less remanufacturing effort or more extensive operations. Cores that cannot be remanufactured are disposed off.

As another problem, products are mainly designed for decreasing the assembly and production times. This fact can cause variability in disassembly times and inefficiencies in remanufacturing operations. In this context, Sundin [30] reveals some problems in remanufacturing that arise from lack of proper product design such as product complexity, increased part fragility and wear resistance, difficulties in identification and handling. Design for remanufacture which considers all of the steps involved in remanufacture can prevent inefficiencies in remanufacture, especially in disassembly, and can decrease remanufacture times.

Secondly, the traditional sales model (TSM) where only manufactured products are sold is considered. This model is a modification of the sales with remanufacturing model where the return rate equals to zero. The major problem in this model is that the end-of-life products directly go to disposal. Therefore, returned products do not provide any benefits and thus raw material consumption increases.

The third structure is the leasing model where both remanufactured and new products are only leased. The fundamental difference between this model and the above models is that in this one demand is for the right of use of the service the product provides. Here, the approach of the demand can be considered more utilitarian. Therefore, there is no demand separation between new and remanufactured products as both products are capable of performing the same tasks. With the help of the known leasing periods, the return times of the leased goods can be determined perfectly. Furthermore, leasing can help companies to forecast the quality of the returned leased products easily. We can control the quality, quantity and timing of the returned products which are the main problems of remanufacturing. Also, raw materials which are used to manufacture new products are consumed only when leased product cannot be remanufactured again. On the other hand, to maximize the profits a leasing company must have a decision on some important topics like length of leases, efficient utilization of logistics facilities for material, flow to and from customer sites, and equipment reuse, refurbishment and disposal actions [27].

4.2. Queuing Network Models

In modeling supply chain systems as a queuing network, stations in the network represent facilities where products are processed through different operations. Possible routings are represented by the arcs between these stations. Also, the order in which the products are processed in these stations is determined by the queue discipline. We use the FCFS discipline (first come, first served) in which products are processed in the order of their arrival. Moreover, it is important to determine how products enter and leave the queuing network. In an open queuing network, products enter the network at one or more stations, goes through the stations of the network and finally leave the network at a certain station. However, in a closed network, population of products remains constant since products never leave the network. In this study, the traditional sales model and the sales with remanufacturing model are represented as open queuing networks. On the other hand, leasing system is modeled as a closed queuing network. In the following sections, each model is interpreted in detail.

4.2.1. Sales with Remanufacturing Model (SRM)

We model the remanufacturing system as a multiclass open queuing network where each class represents a product and how many times it has been remanufactured. Under FCFS policy, we use open queuing network similar to [16], [17] and [19]. The network consists of 12 stations. The queuing network model is depicted in Figure 4.1. We have no distribution assumptions about inter-arrival times and service times for each station. Therefore, we evaluate each station in the queuing network as independent queues with general inter-arrival and service time distribution with parameters $\{\lambda_i, C_{a,i}^2, \tau_i, C_{s,i}^2\}$, where λ_i is the arrival rate at station i, $C_{a,i}^2$ is the squared coefficient of variation (scv) of the external inter-arrival time at station i, τ_i is the mean processing time of all jobs at station i and $C_{s,i}^2$ is the scv of service time of jobs at station i.

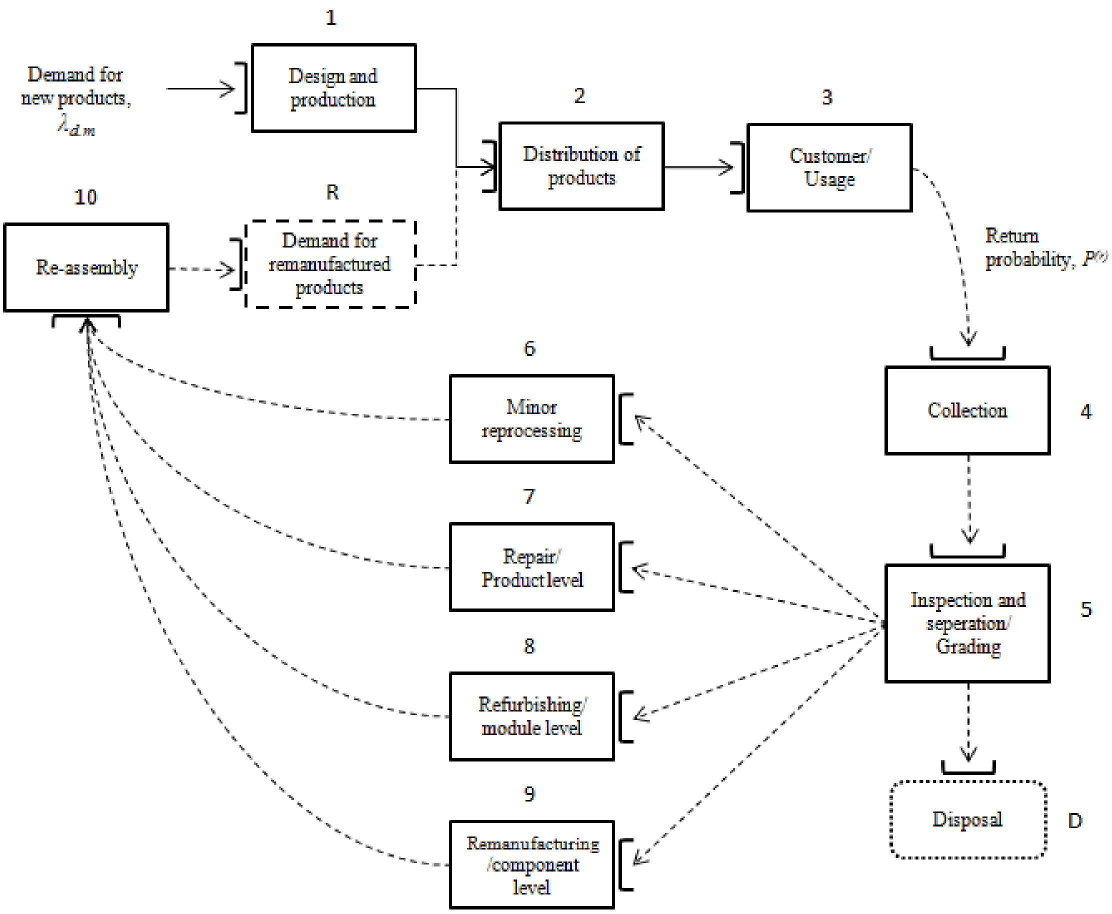

Figure 4.1. The open queuing network for the SRM

Also, we consider product classes which are determined by how many times the product has been returned from customer. Here, n is the maximum number of times that a product can be remanufactured before being disposed off. We assume that the manufacturer tracks the number of returns by keeping the history of the product. Product class *0* represents the new products. If they return from the customer after the usage period, the product's class becomes class *1*. Similarly, if the customer purchased a product class r, it becomes product class *r+1* when it returns to the manufacturer. However, when a product class *n* is returned, it becomes class *n+1* and directly goes to disposal since it cannot be remanufactured again. For product classes $r = 0,1,......,n,n+1$ and all stations in the model, let λ_i^r = the arrival rate of product class *r* at station *i*. Furthermore, $P^{(r)}$ denotes the return probability of product class *r* and $q^{(r,\theta)}$ is the proportion of product class *r* with grade level θ.

Furthermore, SRM is a hybrid manufacturing system, which contains both remanufacturing and manufacturing processes. Customers can purchase either new products or remanufactured products with different prices. In general, the price of remanufactured products is lower than the price of the new products. To model demand differences between new and remanufactured products, the demands of new products are met by production station and the demand for remanufactured products is met by demand station for remanufactured products. The arrivals of the external demand for new products at rate $\lambda_{d.m}$ initiate the production of new products at the production station. This station is modeled as *GI/G/1*. We know that for competitive strategy design for remanufacture has a great significance. By achieving proper product design, we can obtain sustainability in remanufacturing cycles and extend the life cycle of the product. To include the design effect in our models, time of the design period for new products is added to the production time. Therefore, it is assumed that both production design and production occurs at the same station.

On the other hand, we employ the demand station for remanufactured products. At this station, after re-assembly the remanufactured products are matched with the demands where the service time models the demand inter-arrivals. We model this station as *GI/G/1* queue. Also, the demand arrival rate for remanufactured products, $\lambda_{d.r}$ are assumed to be larger than the arrival rate of the demand station. Thus, all remanufactured products are sold. We assume no backorders for remanufactured products. Moreover, this station provides us to consider the lost sales. Since we assume higher demand arrival rate, demand cannot be satisfied when there is no products in the demand station. Therefore, the potential sale is lost.

Both distribution station and usage station have infinite servers. We assume that the new and the remanufactured products are distributed and used by customer immediately after production and remanufacturing. Thus, distribution station and usage station become *GI/G/∞*. All departures from production station and demand station for remanufactured products generate the arrivals for distribution station. At this station, products are transferred to the customers. Likewise, all distributed goods come to usage station. After a product is purchased, the customer keeps it for a certain amount of time which is generally distributed. In order to study the effects of remanufacturing, we assume that duration of the customer usage period is shorter in expectation than the duration of the life cycle of the product. In other words, we allow the products to be purchased multiple times by customers.

After the usage period, at the end of their useful lives products are either returned with probability $P^{(r)}$ due to end-of-use and end-of-life returns. As expected, the return rates increase as the return probabilities increase. Also, the product return rate is generally less than the demand rate of the remanufactured products. Here, the collection stage refers to the activities of taking back returned products available and includes purchasing and transportation of them. The collection station is modeled as a single server with general inter-arrival time and general service time.

In addition, grading issue plays an important role in our system, especially for profitability. The study of Ferguson et al. shows that a proper grading policy enhances profitability by about % 4 [31]. Also, Guide claims that one of the essential problems of remanufacturing is the uncertainty in materials recovered from returned items [13]. With this idea, in our model the collected products are inspected and graded at the grading station after all collected products flow from collection station to grading station. The grading station is modeled as a *GI/G/1* queue. To state precisely whether a returned product is re-usable, grading stage is mandatory. Furthermore, Ferguson et al. find out that the value of the grading system is more considerable when the ratio of return rates to demand rates increases and when there are no more than five quality grades [31]. To this end, we determine five grades which needs; minor re-processing (or sell as-is), repair (product level), refurbishing (module level), remanufacturing (component level), disposal. This grading approach is similar to the determination of the recovery options that is presented by Brito [11]. The grade that needs disposal refers to the products which cannot be remanufactured due to technical reasons. Afterwards, graded products are sent to their stations with probability $q^{(r,\theta)}$ which is the proportion of the r^{th} time returned products with grade level θ. Souza et al. claim that the proportion of used products assigned to each quality grade is stable [16]. On the contrary, in our model, as the number of cycles that a product makes increases, the distribution of the proportion of grades differs. Namely, the proportion of grades is dependent on the number of returns for a product, *r*, and as *r* increases, the proportion of bad quality cores increases. The representation of this part of the model can be seen from Figure 4.2.

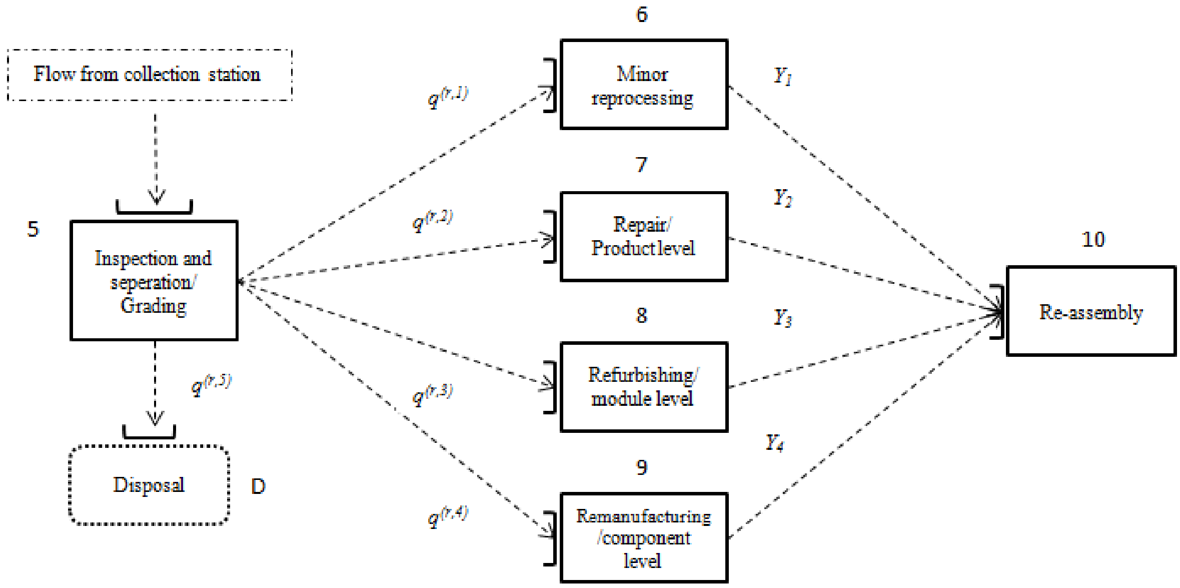

Figure 4.2. Departures from grading station according to grades of the cores

After leaving the grading station, products enter their respective reprocessing station. Our remanufacturing facility is modeled as four parallel stations. Souza et al. state that the variability of processing times and costs for different quality grades are different [16]. To model this variability, each station is modeled as a *GI/G/1* queue. We assume that the processing times in these stations increases as the quality level decreases. Also, as the quality of the core decreases service times become more variable.

Ferrer and Ketzenberg assign a recovery yield y_i for each returned product *i* [32]. Additionally, Souza and Ketzenberg define the overall yield of the remanufacturing process [17]. Likewise, in our model, every station at remanufacturing facility has a recovery yield. Remanufacturing yield follows a Bernoulli process with probability y_i that a core is remanufactured well and probability $1- y_i$ that the products go to disposal from each station *i* which represent remanufacturing stations here.

We assume that disposal activities are conducted by the manufacturer in the SRM. Here, disposal activities refer the operations like recycling, landfilling and incineration for products that cannot be remanufactured due to technical reasons or high costs. For example, products that require heavy repair or that cannot satisfy the demand after remanufacturing are disposed. The disposal station is modeled as a *GI/G/1* queue.

Finally, the reprocessed products are merged at the re-assembly station and tested at the last step of the remanufacturing process. This station is also modeled as a *GI/G/1* queue. Then, these products directly go to the demand station for remanufactured products which is described above. In this way a new cycle starts and a product goes through the same steps mentioned until it is disposed off. The number of cycles that the product can complete which is determined at production design depends on the remanufacturability of the product.

4.2.2. Traditional Sales Model (TSM)

We consider our selling model as a typical forward supply chain. In this model, the customer is at the end of the process and products are directly disposed off at the end of their useful lives. Additionally, in comparison to the SRM, there is no remanufacturing activity. In other words, this model looks like the SRM where $P^{(r)}=0$ for $r=n=0$. When $n=0$, the SRM becomes the TSM which consists of design and production station, distribution station and customer usage station. Another fundamental difference between the TSM and the SRM is that in the TSM only the new products are purchased by the customers. The open queuing network for the TSM is represented in Figure 4.3.

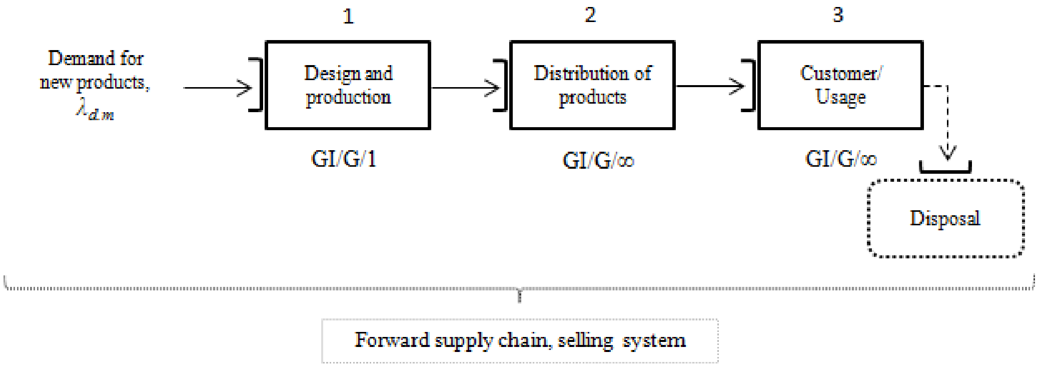

Figure 4.3. The open queuing network for the TSM

4.2.3. Leasing Model (LM)

The leasing model (LM) is the modified version of the remanufacturing model. Both models include forward and reverse supply chains. However, although most of the stations in the LM work the same way as in the SRM, there are some basic differences. These differences are that the return probability equals to one, the return time is fixed and the return quantity is known.

In a leasing system the customer who leases the product has an obligation to return the product. Therefore, the return probability, $P^{(r)}=1$ for $r=1,...,n,n+1$. Also, this generates higher flow rates than in the SRM at the remanufacturing stage. As a second point, by connecting disposal station and production station departures from disposal station become arrivals for production station. Namely, the leasing system produces a new product to replace a used product that is disposed off. This means arrival rates for disposal and production stations are equal ($\lambda_1 = \lambda_D$). Therefore, the products never exit the system and thus a finite population of products circulates in the system. In summary, there are no external arrivals and departures so under FCFS policy the LM becomes a multiclass closed queuing network where every station except distribution and customer usage stations is modeled as *GI/G/1* queue similarly in the SRM. Figure 4.4 depicts our closed network.

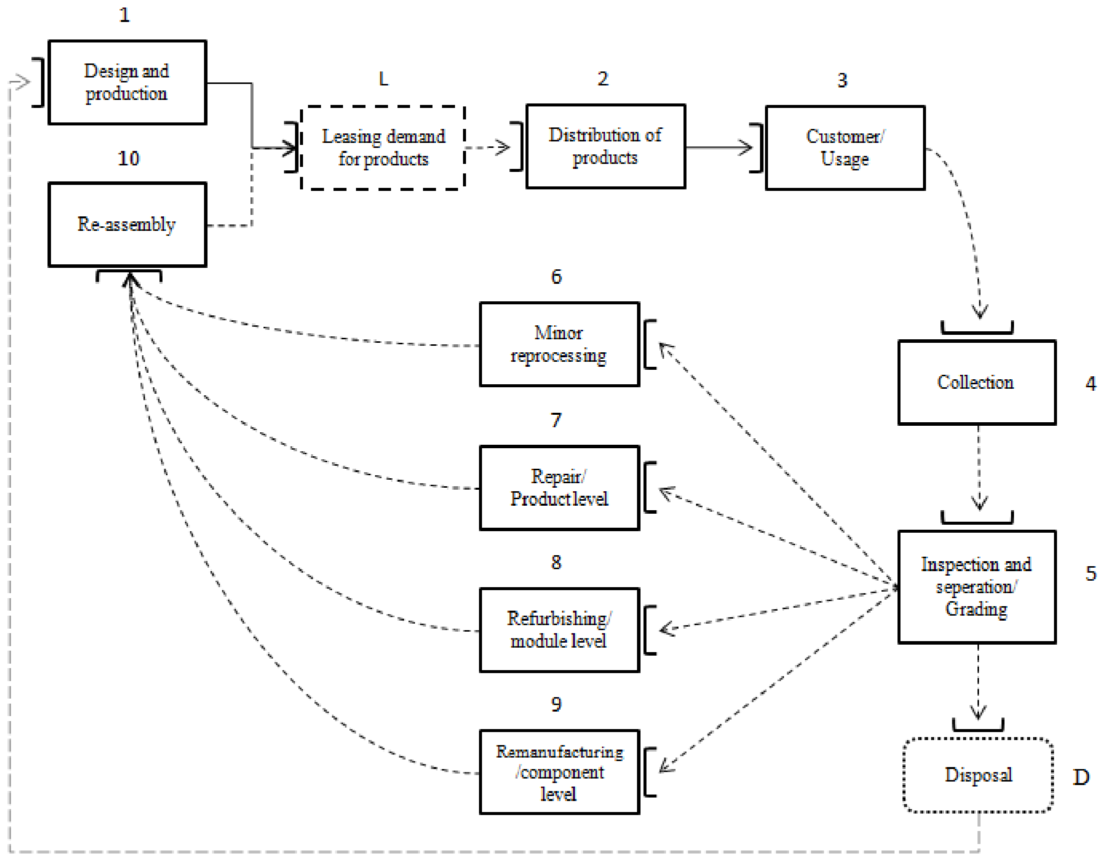

Figure 4.4. The closed queuing network for the LM

Moreover, products remain at the customer for a certain amount of time which depends on the duration of the lease agreement, T_L which is expressed by years. Thus, the service time at customer usage station becomes deterministic. If we depict the total useful life of the product as N, then we can say that T_L cannot exceed N. Also, the product can be leased during its useful life for η periods and η can be written as;

$$\eta = \frac{N}{T_L} \tag{4.1}$$

After each period, the product enters the remanufacturing cycle. In this context, the number of these cycles is equal to η-1 since after the leased product returns at the end of the last lease period in its useful life, it is directly disposed off.

A significant difference from the remanufacturing model is that since the customer is not interested in whether the product is newly manufactured or remanufactured before there is no demand differentiation between remanufactured and new products. All products are leased at the demand station, *L*. All arrivals from remanufacturing part of the system and production station flow directly to demand station. We model the demand station for leased products as a *GI/G/1* queue and at this station products are stored and match with the demand for leased products.

Another difference between the SRM and the LM occurs at the grading station. It is assumed that processing time of the grading station is expected to be considerably shorter for the LM. This is because the uncertainty in the quality of the returned product decreases by keeping the records of product's maintenance during the lease period [31]. Furthermore, due to the short customer usage periods and increased number of remanufacturing cycles, the deterioration of the product is slowed down. Consequently, the grade distribution is affected. In the SRM, we assume that the grade distribution depends only on the number of returns. However, the length of the customer usage period is another important factor for deterioration rates. As the length of the usage period and number of returns increases, the core quality shifts towards bad quality grades. In other words, the grade distributions in the LM and in the SRM must be different due to the short usage periods in leasing. To analyze the effects of these concepts, we define a parameter, δ_r where r is the number of returns. This parameter is the ratio between the proportion of good quality cores, $\theta = \{1, 2\}$, and bad quality cores, $\theta = \{3, 4, 5\}$. We assume that δ_r is larger in the LM in comparison to the SRM for $r = 1, \ldots, n$ and the quality distributions are determined by considering this parameter.

$$\delta_r = \frac{q^{(r,1)} + q^{(r,2)}}{q^{(r,3)} + q^{(r,4)} + q^{(r,5)}} \tag{4.2}$$

4.3. Model Formulation and Performance Evaluation

In this section, the performance measures of the queuing network models such as throughput rate, expected waiting time in a queue, expected number of jobs in the system, average number of busy servers in *GI/G/*∞ queues are computed in order to define a profit function by using the performance measures to compare the profitability of the models. For performance evaluation of the queuing network models, we use parametric decomposition method which is widely used in the analysis of queuing networks. In parametric decomposition method, the network is divided into individual stations and thus, the necessary parameters of each station are analyzed and estimated independent of the rest of the network [33]. In this section,

According to Bitran and Dasu [34], the decomposition approach has three basic steps; characterization of the arrival process, analysis of the queue and determination of the departure process. We follow these steps in our work to apply parametric decomposition approach to our model successfully. Moreover, under the parametric decomposition approach, we know that in addition to the decomposition assumption ,which assumes that all stations are stochastically independent, we can employ the assumption that two parameters, which are mean and variance, of the inter-arrival and service time distributions are sufficient to calculate approximately the performance measures at each station [34]. Under these assumptions, firstly we analyze the flows by estimating arrival and departure processes for each station. In order, routing probabilities, traffic rate equations and scv of both arrivals and departures are determined. Here, the throughput rate of each station is computed. After the flow analysis, we compute the expected waiting time and the expected number of jobs at each station by using the first two moments of the inter-arrival and service times.

4.3.1. Routing Probabilities and Traffic Rate Equations

To determine the routing probabilities, we define $P_{i,j}^{(k,l)}$ and p_{ij} where $P_{i,j}^{(k,l)}$ represents the transfer probability of k type product at station i to l type product at station j and p_{ij} represents the transfer probability of jobs from station i to station j.

The first part of the queuing network acts as a forward supply chain. The new products go directly to the distribution station from the production and design station and the product class remains unchanged.

$$P_{1,2}^{(0,0)} = 1 \tag{4.3}$$

Also, all products flow from distribution station to customer usage station and again the product class does not change. Thus, the transfer probability from distribution station can be given as;

$$P_{2,3}^{(r,r)} = 1 \tag{4.4}$$

where $r = 0,1,.....,n$. Return probability of the new products from customer after first usage is equal to the routing probability between customer usage and collection station. Also the product class becomes $r+1$ time returned product. Subsequently, returned products pass into grading station. Thus, the transfer probabilities from customer usage station to collection station and collection station to grading station can be given as;

$$P_{3,4}^{(r-1,r)} = P^{(r)} \tag{4.5}$$

$$P_{4,5}^{(r,r)} = 1 \tag{4.6}$$

where $r = 1,.....,n,\ n+1$. Routings between remanufacturing stations and grading is determined by the grade proportions of that return. In other words, the grade distribution of r class products defines the transitions at this part of the queuing network. For $r = 1,.....,n$, the routing equations set is below;

$$
\begin{aligned}
P_{5,6}^{(r,r)} &= q^{(r,1)} \\
P_{5,7}^{(r,r)} &= q^{(r,2)} \\
P_{5,8}^{(r,r)} &= q^{(r,3)} \\
P_{5,9}^{(r,r)} &= q^{(r,4)} \\
P_{5,D}^{(r,r)} &= q^{(r,5)}
\end{aligned}
\tag{4.7}
$$

$$
\sum_{\theta=1}^{5} q^{(r,\theta)} = 1 \tag{4.8}
$$

As mentioned before, all remanufacturing stations have an overall yield. With probability y_i, where $i = 1,2,3,4$ and denotes the remanufacturing stations for different grades, the reprocessed products continue their routes to assembly. On the other hand, a fraction $1\text{-}y_i$ of products cannot be reprocessed and is disposed. Thus, the routing probabilities from the remanufacturing stations are given as;

$$
\begin{aligned}
P_{6,10}^{(r,r)} &= y_1, P_{6D}^{(r,r)} = 1 - y_1 \\
P_{7,10}^{(r,r)} &= y_2, P_{7D}^{(r,r)} = 1 - y_2 \\
P_{8,10}^{(r,r)} &= y_3, P_{8D}^{(r,r)} = 1 - y_3 \\
P_{9,10}^{(r,r)} &= y_4, P_{9D}^{(r,r)} = 1 - y_4
\end{aligned}
\tag{4.9}
$$

where $r = 1,.....,n$. The equation (4.9) depicts the flow from re-assembly station to distribution where $r = 1,.....,n$.

$$
P_{10,R}^{(r,r)} = P_{R2}^{(r,r)} = 1 \tag{4.10}
$$

Finally, the last routing probability equation is for the products which directly go to disposal after grading because they cannot be remanufactured again.

$$
P_{5D}^{(n+1,n+1)} = 1 \tag{4.11}
$$

Considering the traffic rates, the only external arrival to our queuing network occurs at production and design station where demands for new products arrive. The arrival rates of new products to production, distribution and customer usage stations is equal to each other and also equal to the demand for new products for our hybrid production system. Equation (4.15) shows the arrivals of one time returned products to collection and grading stations.

$$\lambda_{d.m} = \lambda_1^{(0)} = \lambda_2^{(0)} = \lambda_3^{(0)} \tag{4.12}$$

$$\lambda_4^{(1)} = \lambda_5^{(1)} = \lambda_{d.m} P^{(1)} \tag{4.13}$$

For r class products where $r = 1,....,n$, the arrival rates to distribution, customer usage stations can be expressed as;

$$\lambda_2^{(r)} = \lambda_3^{(r)} = \lambda_{d.m} \prod_{k=1}^{r} P^{(k)} (q^{(k,1)} y_1 + q^{(k,2)} y_2 + q^{(k,3)} y_3 + q^{(k,4)} y_4) \tag{4.14}$$

Likewise, for $r = 2,....,n+1$ the arrival rates to collection and grading stations can be obtained by;

$$\lambda_4^{(r)} = \lambda_5^{(r)} = \lambda_{d.m} \prod_{k=2}^{r} P^{(k)} (q^{(k-1,1)} y_1 + q^{(k-1,2)} y_2 + q^{(k-1,3)} y_3 + q^{(k-1,4)} y_4) \tag{4.15}$$

With the help of the equation sets (4.6) and (4.8), we can easily obtain the arrival rates for the remaining part of the network for $r = 1,....,n$.

$$\begin{aligned} \lambda_6^{(r)} &= \lambda_5^{(r)} q^{(r,1)} \\ \lambda_7^{(r)} &= \lambda_5^{(r)} q^{(r,2)} \\ \lambda_8^{(r)} &= \lambda_5^{(r)} q^{(r,3)} \\ \lambda_9^{(r)} &= \lambda_5^{(r)} q^{(r,4)} \end{aligned} \tag{4.16}$$

$$\lambda_D^{(r)} = \lambda_5^{(r)} q^{(r,5)} + \lambda_6^{(r)} (1-y_1) + \lambda_7^{(r)} (1-y_2) + \lambda_8^{(r)} (1-y_3) + \lambda_9^{(r)} (1-y_4) \tag{4.17}$$

$$\lambda_{10}^{(r)} = \lambda_R^{(r)} = \lambda_6^{(r)} y_1 + \lambda_7^{(r)} y_2 + \lambda_8^{(r)} y_3 + \lambda_9^{(r)} y_4 \tag{4.18}$$

While the above equations depict the arrival rates for each product class to each station, we can express the arrival rates of all products to station i as $\lambda_i = \sum_r \lambda_i^r$.

$$\lambda_1 = \lambda_{d.m} \tag{4.19}$$

$$\lambda_2 = \lambda_3 = \lambda_{d.m} + \lambda_{d.m} \sum_{r=1}^{n} \prod_{k=1}^{r} P^{(k)} (q^{(k,1)} y_1 + q^{(k,2)} y_2 + q^{(k,3)} y_3 + q^{(k,4)} y_4) \tag{4.20}$$

$$\lambda_4 = \lambda_5 = \lambda_{d.m} P^{(1)} + \lambda_{d.m} P^{(1)} \sum_{r=2}^{n+1} \prod_{k=2}^{r} P^{(k)} (q^{(k-1,1)} y_1 + q^{(k-1,2)} y_2 + q^{(k-1,3)} y_3 + q^{(k-1,4)} y_4) \tag{4.21}$$

$$\begin{aligned} \lambda_6 &= \sum_{r=1}^{n} \lambda_5^{(r)} q^{(r,1)} \\ \lambda_7 &= \sum_{r=1}^{n} \lambda_5^{(r)} q^{(r,2)} \\ \lambda_8 &= \sum_{r=1}^{n} \lambda_5^{(r)} q^{(r,3)} \\ \lambda_9 &= \sum_{r=1}^{n} \lambda_5^{(r)} q^{(r,4)} \end{aligned} \tag{4.22}$$

$$\lambda_{10} = \lambda_R = \sum_{r=1}^{n} \lambda_6^{(r)} y_1 + \lambda_7^{(r)} y_2 + \lambda_8^{(r)} y_3 + \lambda_9^{(r)} y_4 \tag{4.23}$$

In order to calculate the average arrival rate to the disposal station, we have to consider the arrivals from three different part of the network. The first part is the products that are separated according to the quality of the returned product during grading process. The second part is the products that cannot be remanufactured again due to n in the grading process. As the last part, we consider the products that are disposed depending on the overall yield of the remanufacturing stations. Taking these parts into consideration, we can obtain the following equation for $r = 1, \ldots, n$;

$$\lambda_D = \sum_r \lambda_D^r + \lambda_5^{n+1} + \sum_r \lambda_6^{(r)} (1 - y_1) + \lambda_7^{(r)} (1 - y_2) + \lambda_8^{(r)} (1 - y_3) + \lambda_9^{(r)} (1 - y_4) \tag{4.24}$$

4.3.2. Squared Coefficient of Variances for Arrivals and Departures

Since our model is a multiclass generalized queuing network with probabilistic routing, we use related approaches in the literature to determine squared coefficient of variances for arrivals and departures. Moreover, in this context we have to deal with superposition process that merges the individual arrivals from other stations and splitting process that decomposes the departures from a station to other stations. In our network, assembly, distribution and disposal stations need the former process and on the other hand grading station requires the latter process.

For probabilistic routing, Whitt [35] proposes a procedure to aggregate all classes in a single one and apply formulations of single class network. In other words, we reduce our multiclass network to aggregate single class network. Let $C_{a,ij}^2$ and $C_{d,ij}^2$ denote the scv of departures and arrivals of aggregate jobs that move from station i to station j with probability p_{ij} respectively while $C_{d,i}^2$ denotes the scv of departures from station i , then $C_{a,ij}^2$ and $C_{d,ij}^2$ can be estimated as;

$$C_{a,ij}^2 = C_{d,ij}^2 = 1 + p_{ij}(C_{d,i}^2 - 1) \tag{4.25}$$

where $i, j \in \{1,2,3,4,5,6,7,8,9,10,R,D\}$. In order to do compute scv of arrivals, we need transition probabilities. We can obtain the transition probabilities from the equations below;

$$p_{ij} = \frac{\lambda_{ij}}{\lambda_i} \tag{4.26}$$

where λ_{ij} is the flow rate from station i to j and equals to sum of flow rates of product class r from station i to j.

$$\lambda_{ij} = \sum_r \lambda_{ij}^r \tag{4.27}$$

$$\lambda_{ij}^{r} = \lambda_i^{r} P_{ij}^{(r,r)} \quad \text{for } r = 0,1,\ldots\ldots,n,n+1 \tag{4.28}$$

Equation (4.28) is valid for all stations except for the collection station since in this station the product class changes. Therefore, the flow rate from customer usage station to collection station of product class *r* can be obtained as;

$$\lambda_{34}^{r} = \lambda_i^{r-1} P_{ij}^{(r-1,r)} \quad \text{for } r = 1,\ldots\ldots,n,n+1 \tag{4.29}$$

To approximate the scv of departures from any station which is depicted in equation (4.25) as $C_{d,i}^2$, we use the formula that is proposed by Whitt [35] . At this point, we have to remark that scv of service time for each product class is equal because they are processed similarly at every station. As a function of inter-arrival and processing time parameters it is estimated for every station by;

$$C_{d,i}^2 = \lambda_i^2 \tau_i^2 (C_{s,i}^2 - C_{a,i}^2) + C_{a,i}^2 \tag{4.30}$$

Furthermore, for distribution, re-assembly and disposal stations we merge the flows departing from different stations to obtain the scv of arrivals. We can write the scv of arrivals at these stations by a convex combination of arrivals from related stations. For this purpose, we use here the approximation by Bitran and Morabito [36] as;

$$C_{a,j}^2 = \sum_i \frac{\lambda_{ij}}{\lambda_j} \cdot C_{a,ij}^2 \tag{4.31}$$

where $j = \{10,D\}$ and for distribution station $i = \{1, R\}$, for re-assembly station $i = \{6, 7, 8, 9\}$ and finally for disposal station $i = \{5, 6, 7, 8, 9\}$.

4.3.3. Estimating the Performance Measures

After the calculations above, we estimate the performance measures which we use in our profit function. At this stage, expected waiting time in each station and expected number of jobs at each station come forward. To estimate these measures, let ρ_i be the utilization of station *i* and it is calculated for every station in the network by;

$$\rho_i = \lambda_i \tau_i \quad \text{, for } i = 1,\ldots\ldots,10, R,D \tag{4.32}$$

Under the FCFS policy the expected waiting time of jobs at each station *i* can be estimated by the approximation of Bitran and Morabito [36]. By employing this approximation and using the parameters $\left\{\rho_i, c^2_{a_i}, \tau_i, c^2_{s_i}\right\}$ obtained from the previous steps, the expected waiting time at each station, $E[W_i]$ is calculated as;

$$E[W_i] = \frac{\rho_i \tau_i (C^2_{a,i} + C^2_{s,i})}{2(1-\rho_i)} g(\rho_i, C^2_{a,i}, C^2_{s,i}) \tag{4.33}$$

where $g(\rho_i, C^2_{a,i}, C^2_{s,i}) = \exp\left[\frac{-2(1-\rho_i)(1-C^2_{a,i})}{3\rho_i (C^2_{a,i} + C^2_{s,i})}\right]$ if $C^2_{a,i} < 1$ and $g(\rho_i, C^2_{a,i}, C^2_{s,i}) = 1$ if $C^2_{a,i} \geq 1$.

Equation (4.33) is valid for all stations except distribution and customer usage stations. Because of being infinite servers, waiting time at these stations is equal to zero. In addition to this, the expected number of jobs at these stations can be obtained as $E[N_i] = \lambda_i \tau_i$ for *i* = {*2,3*} where for the other stations it is obtained by;

$$E[N_i] = \lambda_i \{\tau_i + E[W_i]\} \tag{4.34}$$

4.4. Determination of the Lease Payments

Lease payment is the value of use or occupation of a product for a specified period of time. Payments can be made at the beginning or at the end of each lease period. In our model, we assume that payments are received at the beginning of each period η. In general, the amount of the payment is strongly related to the duration of the lease agreement, T_L . This is because the residual value of the product varies depending on the length of the lease period. For instance, if T_L is increased, the residual value of the product at the end of the lease period decreases. In this context, the residual value can be defined as the estimated market value of the product at the expiration of a lease after it has depreciated. Therefore, the estimation of the residual value is essential for the manufacturers to determine the lease payments.

Lease payments consist of two parts; depreciation payment and finance payment. The depreciation part is the decline in the product's value during the leasing period associated with aging or usage of the product. We can interpret this as the difference between the initial value of the product at the beginning of lease period and the residual value at the end of period. The residual value is estimated as a percentage of the current purchase price in terms of the depreciation rate. There are two forms of depreciation; straight-line and geometric [37]. Straight-line depreciation assumes equal depreciation in value for all lease periods of a product throughout its life cycle. Also, in this form the depreciation rate increases from period-to-period to have equal depreciation in value for all periods. On the other hand, geometric depreciation assumes higher dollar depreciation in early periods of a product and the depreciation rate remains constant over the life time of the product. In other words, the decline in the product's value is higher for the early years of a lease period. Moreover, Fraumeni [37] states that according to Bureau of Economic Analysis report, geometric pattern of depreciation is appropriate for most assets. As an illustration, empirical studies show that electric equipments, motor vehicles, tires, industrial buildings and industrial machines have geometric depreciation rates. Similarly, we use geometric depreciation assumption in our model. During each lease period, assuming that the products depreciate with rate δ which is constant for all periods., the depreciation of one dollar investment can be obtained by;

$$d_i = \delta(1-\delta)^{i-1} \quad \text{, for } i=1,2,\ldots,T_L \tag{4.35}$$

where i is the index for years. Also, depreciation payments per period, D_t, where t is the index for lease periods, can be obtained by equation (4.35). Here, the depreciation part of the lease payment is assumed to be a function of the residual value of the product at the beginning of a lease period, $R_{b,\eta}$.

$$D_t = R_{b,t} \sum_{i=1}^{T_L} \delta(1-\delta)^{i-1} \quad \text{, for } t=1,\ 2,\ldots,\ \eta \tag{4.36}$$

Note that a higher depreciation rate results in a higher depreciation payment. Moreover, the residual value of the product at the beginning of the subsequent lease period, $R_{b,t+1}$ can be written as;

$$R_{b,t+1} = R_{b,t} - D_t \text{ , for } t=1,\ 2,\ldots,\ \eta \tag{4.37}$$

In equation (4.36) we assume that the residual value of a product at the beginning of the first lease period, $R_{b,1}$ is equal to the selling price of a new product, P_m. New products begin to lose value immediately after they are leased and continue to lose value in the other lease periods until they are scrapped. Figure 4.5 is the representation of the geometric pattern of the changes in residual value for the first lease period of a product where $T_L=5$. Each year the product is depreciated by a fixed depreciation rate but the decline in its residual value is reduced from year-to-year. In other words, geometric depreciation implies that the residual value of the product falls by the largest absolute amount in the first year of a lease period and by decreasing amounts at each subsequent year. This exponential decrease leaves a value at the end that is larger than zero.

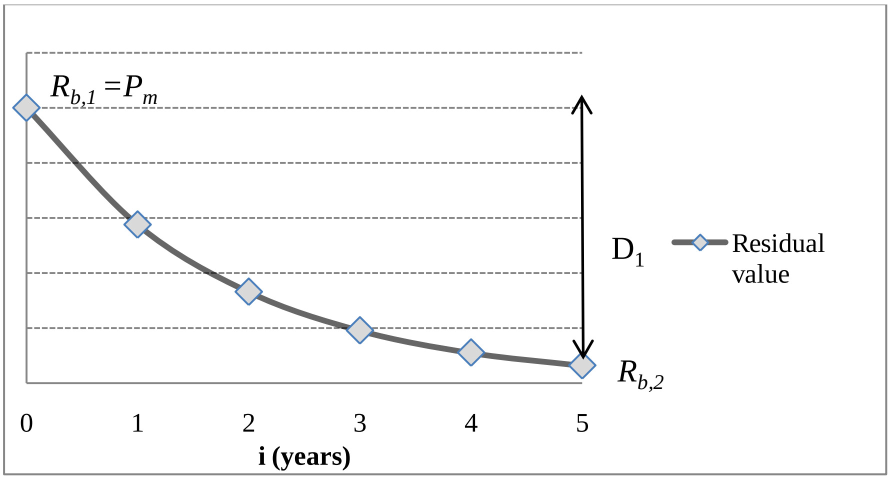

Figure 4.5. Geometric depreciation for the first lease period

We reflect the effect of remanufacturing activities which are performed at the end of each period *t* to the lease payments in our model since the residual value of the product increases remarkably after the remanufacturing activities. We need a modified version of the equation (4.37) for the residual value estimation in our model. Recall the residual value of the product at the beginning of period *t*, $R_{b,t}$ and let R'_t denote the residual value after remanufacturing at the end of period *t*;

$$R'_t = R_{b,t}(1+\varphi_t) \quad \text{, for } t = 2, \ldots, \eta \tag{4.38}$$

where φ_t is the percent of increase in residual value at period η by remanufacturing and it takes different values for η.

Here, we assume that the residual value after remanufacturing at the end of each lease period *t* is equal to the price of a remanufactured product, P_r. In other words, the residual value of the product is increased to the level which is equal to the price of a remanufactured product by remanufacturing at the end of each period. Here, P_r is a predetermined value and by changing it we can increase or decrease the lease payments. Then, φ_t can be obtained by;

$$\varphi_t = \frac{P_r}{R_{b,t}} - 1 \quad \text{, for } t = 2, \ldots, \eta \tag{4.39}$$

This equation implies that as the residual value of the product decreases, the percent increase in the residual value as a result of remanufacturing increases. Thus, residual value of the product continues to have high values, which is important for the finance part of the lease payments. However, the quality of a returned product decreases in the late periods of its useful life as the number of lease periods increases and therefore remanufacturing the product to increase its residual value becomes more costly. Figure 4.6 illustrates the change in residual value where leased products are remanufactured at the end of each lease period for $\eta=5$ and $T_L=2$. For the first lease period, the depreciation is higher since $P_m > P_r$. We can see that a higher residual value can be accomplished as a result of remanufacturing after each period. In this way, we can have higher lease payments.

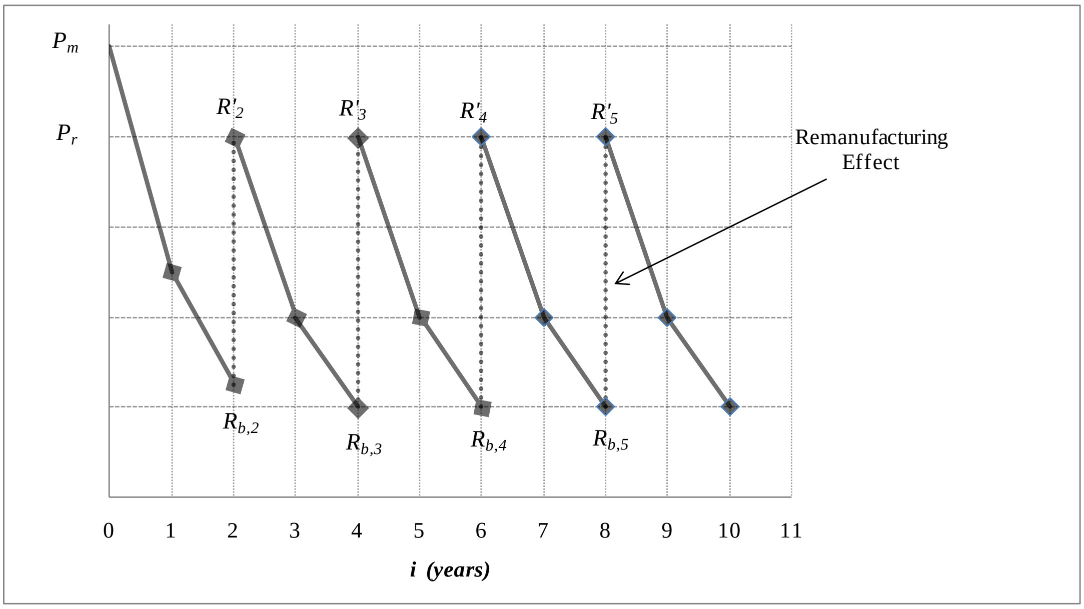

Figure 4.6. The change in residual value with remanufacturing for $\eta=5$

The second part of the lease payment is the finance fee which is the total interest that the lessee has to pay for the use of the manufacturer's money. The lessee ties up the manufacturer's money during the usage of the product. The finance payment is related to the value of the product at the beginning of the lease period, residual value at the end of the period, the interest rate and the duration of the lease period. The total finance payment of each lease period, F_t can be written similarly in [3].

$$F_t = \frac{(R_t' + R_{b,t+1})T_L r_i}{2}, \text{ for } t=1, 2, \ldots, \eta \tag{4.40}$$

where r_i is the annual interest rate. It is important to note that the lower the interest rate, the lower the finance payments of each lease period.

Consequently, we obtain lease payments as a summation of depreciation payment and finance payment. Here, we ignore other charges, fees, rebates, credits and cash down payments and assume that net capitalized cost is equal to the selling price of the new product. The total lease payment for period *t* can be given by;

Lease Payment(t) = Depreciation Payment(t) + Finance Payment(t)

$$P_t = D_t + F_t \text{ , for } t=1, 2, \ldots, \eta \tag{4.41}$$

In summary, the critical factors for determining lease payments are depreciation rate, δ, percentage of increase in residual value by remanufacturing, ϕ_t, interest rate, r_i and the number of periods that the product is leased during its useful life, η. Also, if $\eta=1$, then the LM becomes the TSM since leasing a product for one period throughout its useful life is same as selling a product. Therefore, we assume that lease payment is equal to the price of a new product when $\eta=1$.

After calculating the lease payments, we find the present value of the lease payments collected throughout the product's useful life in order to compare the profitability of the LM with TSM and SRM. Note that we assume that the payments are made at the beginning of each period. As an example for five lease periods, when the manufacturer is receiving payments for a lease agreement, the cash flow schedule would appear as in Figure 4.7.

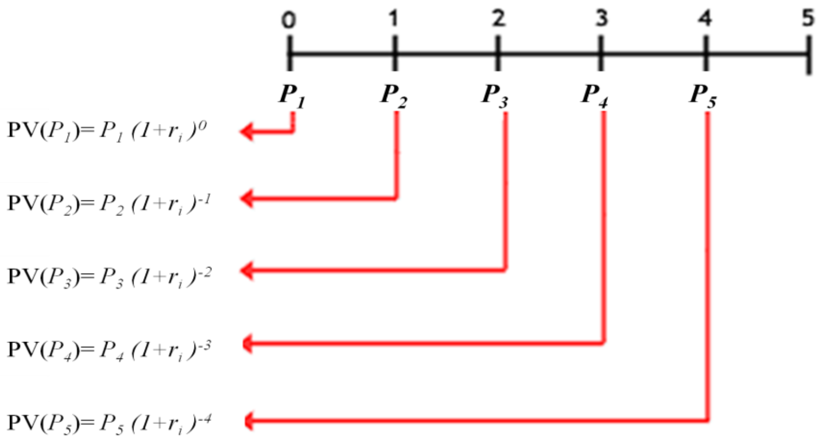

Figure 4.7. The cash flow schedule for $\eta=5$

Assuming the first payment is made today, the formula of the present value of lease payments, PV(P_t) is given by;

$$PV(P_t) = P_t(1+r_i)^{-(t-1)} \quad \text{, for } t=1, 2, \ldots, \eta \tag{4.42}$$

4.5. Model Validation

In this section, we validate the accuracy of the queuing network models using simulation. Several simulation experiments are done using the simulation software, Arena 11.0. Each experiment is replicated 20 times where each replication runs for 100,000 time units. The warm-up period is set as 5000 time units for each replication. The warm-up period is large enough to allow all queues reach their steady-state values. The results for the performance measure values of the queuing network are obtained by Mathematica 6.0.

To show the accuracy of analytical models, we compare the expected values of the throughput rates of each station and the waiting time at each station with the values obtained in the simulation experiments by using experimental designs for each model. For the SRM, we employ $3\times3\times3$ full-factorial design of model parameters $P^{(r)}$, n and $q^{(r,\theta)}$. We consider three different configurations for $P^{(r)}$ and n because these parameters affect the arrival rates of the stations in the remanufacturing cycle. Also, we analyze three cases while determining the experimental values of the grade distributions. In the first case, the grade of the product is related with the number of times the product is remanufactured. Namely, as this number increases, the proportion of bad quality cores reasonably increases. On the other hand, the grade distributions are equal and independent from r in the second case. Lastly, the third case is the extreme case where the grade proportions take extreme values for any two grades related with r. The experimental values of grade distributions are given in Table 4.1. In total, there are 27 experiments for the validation of the SRM.

Table 4.1. Experimental values for grade distributions

	Case 1	**Case 2**	**Case 3**
r = 1	(0.4, 0.25, 0.2, 0.1, 0.05)	(0.2, 0.2, 0.2, 0.2, 0.2)	(0.45, 0.4, 0.05, 0.05, 0.05)
r = 2	(0.25, 0.4, 0.2, 0.1, 0.05)	(0.2, 0.2, 0.2, 0.2, 0.2)	(0.05, 0.45, 0.4, 0.05, 0.05)
r = 3	(0.2, 0.25, 0.4, 0.1, 0.05)	(0.2, 0.2, 0.2, 0.2, 0.2)	(0.05, 0.05, 0.45, 0.4, 0.05)
r > 3	(0.05, 0.1, 0.2, 0.4, 0.25)	(0.2, 0.2, 0.2, 0.2, 0.2)	(0.05, 0.05, 0.05, 0.45, 0.4)

On the other hand, since $P^{(r)}=0$ and $n=0$ in the TSM, we conduct our experiments for the TSM by changing the values of the mean processing time of jobs at design and production station, τ_1. We determine three configurations for τ_1 as $\tau_1 = 1$, $\tau_1 = 1.3$ and $\tau_1 = 1.6$ in order to test the accuracy of the TSM.

For the LM, we have $3 \times 3 \times 2$ full-factorial design of model parameters η, $q^{(r,\theta)}$ and y_i. The same approach in the validation of SRM for grade distributions is employed and the yield of remanufacturing stations is added as an experimental factor instead of the return probability. This is because $P^{(r)}=1$ in all experiments for the LM. Also, experimental values of the yield of remanufacturing stations are set under the assumption that remanufacturing operations of the good quality cores have higher yields. In total, there are 18 experiments for the LM. The experimental values of each factor are presented in Table 4.2 for the SRM and the LM.

Table 4.2. Experimental values for factorial designs

Sales with remanufacturing model

Factor	Experimental Values
n	4, 5, 6
$q^{(r,\theta)}$	Case 1, Case 2, Case 3
$P^{(r)}$	0.6, 0.7, 0.8

Leasing model

Factor	Experimental Values
η	4, 5, 6
$q^{(r,\theta)}$	Case 1, Case 2, Case 3
y_i	(0.9, 0.8, 0.8, 0.7), (0.8, 0.7, 0.7, 0.6)

In all of the experiments, the service time of each station is exponentially distributed except the customer usage station of the LM. Also, we choose the parameter value of the demand arrival rate for new products, $\lambda_{d.m}$ in all experiments that achieves station utilization that is smaller than one. Namely, arrival rate of a station is smaller than the service rate of that station. The results of the approximation and the simulation are compared using the percent relative error defined as in (4.45).

$$Relative\ Error = (\ Approximation - Simulation\ ) / Simulation \qquad (4.43)$$

As a result of the experiments for the TSM, since the throughput rate for each station in the model always equals to $\lambda_{d.m}$, the analytical method calculates the throughputs with a very low relative error . Therefore, there is no need to point out this in the validation tables. On the other hand, relative errors of approximation results of expected waiting times are very small for the TSM. The relative errors are between 0.11 percent and 0.80 percent. Also, since the distribution station and customer usage station have infinite servers, the expected waiting time at these stations is always equal to 0. The results are depicted in Table 4.3.

Table 4.3. Validation of the expected waiting times for the TSM

	$\tau_1 = 1$			$\tau_1 = 1.3$			$\tau_1 = 1.6$		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.4285	0.4251	0.80%	0.8311	0.8386	-0.89%	1.4769	1.4681	0.60%
2	0	0	0.00%	0	0	0.00%	0	0	0.00%
3	0	0	0.00%	0	0	0.00%	0	0	0.00%
D	0.0882	0.0884	-0.23%	0.0882	0.0881	0.11%	0.0882	0.0885	-0.34%

For both the SRM and the LM, the results of the analytical model demonstrate a high degree of correspondence with the results of the simulation model. In fact, generally throughput rates are calculated with a relative error smaller than one percent. The relative errors of the estimation of expected throughputs change from 0.01 percent to 0.70 percent and the average of errors is 0.18 percent for the SRM. On the other hand, for the LM relative errors of throughput rates fluctuate in the interval 0.03 - 1.07 percent and the average of errors is 0.18 percent. Although the relative errors of the expected waiting time are larger than the relative errors of throughput rates, the accuracy level of the estimations are high. In all of the experiments for the SRM, the relative errors of the expected waiting times are smaller than 5 percent. The maximum relative error is 4.89 percent and the average of errors is 0.63 percent. On the other hand, considering the LM, most of the time relative errors are smaller than 5 percent. The maximum relative error is 6.77 percent and the average of errors is 0.61 percent. These results can be seen from the validation tables. The tables can be found in Appendix A.

4.6. Profit Function

In this section, we develop a profit function for each model to compare these models based on profitability and understand the behavior of each model under different parameter values. The total profit structure consists of two parts: revenues and costs. For the TSM, revenues originate only from new product sales while both new and remanufactured products are sold in the SRM. Revenues for the LM accumulate from lease payments. Also, we have a revenue from disposal activities for the last two models since we assume these activities are done by the manufacturer. In the TSM, the manufacturer is not responsible for the disposed product. The common cost parameters for each model are raw material cost, processing cost, backorder cost and holding cost. Additionally, in our closed loop supply chain models the acquisition and disposal costs are considered.

Firstly, the profit function of the TSM is given and the components of it are explained. Then, the profit functions of the SRM and the LM are demonstrated, respectively. Before expressing the formulas of the profit functions and their components, the necessary notations are given below.

Table 4.4. The necessary notations for total profit structures

P_m	Selling price of a new product
P_r	Selling price of a remanufactured product
P_η	Lease payment for period η
P_{raw}	Unit raw material cost
P_{acq}	Acquisition cost of a core
$m_{d,r}$	Unit margin for disposal of a product class r
h_i	Unit holding cost at station i
s_i	Processing cost of station i per hour
B	Unit backorder cost

Revenue from new products is the income from new product sales in the TSM. It is calculated simply by multiplying demand arrival rate for new products with the price of a new product, P_m since all demand is satisfied. Furthermore, raw material cost is the main part of the costs associated with manufacturing new products. For the TSM, it is calculated as a multiplication of the demand arrival rate for new products, $\lambda_{d.m}$ and unit raw material cost which is depicted as P_{raw}. Moreover, the processing cost implies the operational expense of each station. It is obtained as a multiplication of the processing cost per hour for station i, the mean processing time and the expected throughput of station i. Also, we have inventory holding cost for each station except the production and design station. The holding cost of a station is evaluated by multiplying expected number of jobs in station i with the corresponding holding cost per unit. Holding costs per unit time for every station are assumed to be equal. In the production and design station, if the demand for new products is not satisfied, it is backordered until the product is ready. In order to calculate the backorder cost for the TSM, we multiply the expected number of jobs in this station with the unit backorder cost. Also we have to remind that processing cost and holding cost are not considered for customer usage station. Moreover, there is no positive profit from disposal because the products are not collected and disposal activities are not performed by the manufacturer. The components of the profit function are given in Table 4.5 for the TSM and the formula of the profit function can be obtained by the equation 4.44.

Table 4.5. Components of the profit function for the TSM

Revenue from new products	$P_m \lambda_{d.m}$
Raw material cost	$P_{raw} \lambda_{d.m}$
Processing cost	$\sum_i s_i \lambda_i \tau_i$
Holding cost	$\sum_i h_i E[N_i]$
Backorder cost	$bE[N_1]$

$$\pi_1 = (P_m - P_{raw})\lambda_{d.m} - \sum_i s_i \lambda_i \tau_i - \sum_i h_i E[N_i] - bE[N_1] \tag{4.44}$$

For the SRM, both revenue from new and revenue from remanufactured products are considered. Revenue from new products is obtained similarly in the TSM. On the other hand, revenue from remanufactured products is obtained by multiplication of the price of a remanufactured product with the throughput of demand station where remanufactured products are matched with the demand. Also, processing cost, raw material cost, holding cost and backorder cost are calculated similarly in the TSM. Furthermore, acquisition cost is the expense for collecting back the product at the end of its usage period. For the SRM, manufacturer must pay the residual value of the product to customer. This residual value is equal to the acquisition cost of a core which is depicted as P_{acq}. Thus, the acquisition cost for this model is calculated by the multiplication of the return arrival rate with the unit acquisition cost. In this model, the return arrival rate is equal to the throughput of the collection station. Moreover, while disposal cost represents the processing cost of disposal activities, revenue from disposal is the salvage value of the end of life product which can be sold to recycling facilities. Here, we define $m_{d,r}$ as the unit margin for disposal of the product class r. We assume revenue from a disposed product is greater than the processing cost of that product at disposal station. Namely, we determine positive margin for disposed products. We calculate the profit from disposal by multiplying the throughput of disposal station with the unit margin of the disposed product. The components of the profit function are given in Table 4.6 for the SRM and the formula of the profit function can be obtained by the equation 4.45.

Table 4.6. Components of the profit function for the SRM

Revenue from new products	$P_m \lambda_{d.m}$
Revenue from remanufactured products	$P_r \lambda_R$
Raw material cost	$P_{raw} \lambda_{d.m}$
Processing cost	$\sum_i s_i \lambda_i \tau_i$
Holding cost	$\sum_i h_i E[N_i]$
Acquisition cost	$P_{acq} \lambda_4$
Profit from disposal	$\sum_r m_{d.r} \lambda_D^r$
Back order cost	$bE[N_1]$

$$\pi_2 = (P_m - P_{raw})\lambda_{d.m} + P_r \lambda_R + \sum_r m_{d.r} \lambda_D^r - \sum_i s_i \lambda_i \tau_i - \sum_i h_i E[N_i] - P_{acq} \lambda_4 - bE[N_1] \quad (4.45)$$

The profit function of the LM and the SRM is similar. However, there are some differences. One of the differences is that revenues accumulate from lease payments in the LM. We obtain revenue from leased products by multiplying the present value of lease payments with the arrival rate of the demand station for leased products. Also, for determining raw material cost of the LM, unit raw material cost is multiplied with the throughput of production and design station since a new product is only manufactured when a returned product is disposed. Furthermore, the manufacturer does not have to pay any price for the cores in the LM because the customer returns the product at the end of lease period. However, replacement of the leased products generates cost. This issue causes the value of the acquisition cost become positive for the LM but in comparison to the SRM this is a very small amount. The acquisition cost for the LM is obtained by multiplying the throughput of the collection station with the unit acquisition cost, P_{acq}. Also, in order to capture the advantage of high residual values at the end of product's useful life in the LM, we calculate the revenues from disposal by multiplying the throughput of disposal station with the present value of the residual value of the end of life product, *PV(R)*. The components of the profit function are given in Table 4.7 for the LM and the formula of the profit function can be obtained by the equation 4.46.

Table 4.7. Components of the profit function for the LM

Revenue from leased products	$\sum_{r=0}^{\eta-1} PV(P_r)\lambda_L^r$
Raw material cost	$P_{raw}\lambda_1$
Processing cost	$\sum_i s_i \lambda_i \tau_i$
Holding cost	$\sum_i h_i E[N_i]$
Acquisition cost	$P_{acq}\lambda_4$
Profit from disposal	$\lambda_D PV(R)$

$$\pi_3 = \sum_{r=0}^{\eta-1} PV(P_r)\lambda_L^r + \lambda_D PV(R) - P_{raw}\lambda_1 - \sum_i s_i \lambda_i \tau_i - \sum_i h_i E[N_i] - P_{acq}\lambda_4 \qquad (4.46)$$

5. NUMERICAL ANALYSIS

In this section, we perform numerical experiments for each model to explore the following questions;

i. What are the significant parameters for each model?
ii. How are the total profits of each model affected by the significant parameters?

In addition, we investigate the market cannibalization effect of the sales of remanufactured products in the SRM and we examine the effects of the relationship between design time and remanufacturability on profitability.

In the numerical analysis, we assume that equal number of customers is served in all models except for when the cannibalization effect is studied in order to compare them properly. To this end, expected number of jobs at customer usage station is set to be equal for each model. Also, since there are a lot of parameters in the models, some of the parameters are fixed in the numerical experiments using the approximations in the related literature.

We set the mean and the scv of service times of all stations as follows. For the production and design station, we set the mean service time in order to a have higher service rate than the arrival rate of demands for new products. As the observed variability for standard manufacturing and assembly operations are low [17], we set the scv of service time at production and design station and re-assembly station at $C_{s,1}^2 = C_{s,10}^2 = 0.25$. Also, the mean and the scv of service time of the distribution and the collection station are selected to be equal. This is because we assume that the distribution channels are optimized for product delivery and collection such that on the average they take equally long. The mean service time of the customer usage station is very long compared to the other stations because the time that the products are held by customers is much longer than the time to manufacture them. As a matter of convenience, we assume exponential service times for customer usage station and thus we set $C_{s,3}^2 = 1$. On the other hand, when considering the LM, the service time at the customer usage station is deterministic and equals to the duration of the lease period. Therefore, it changes with the number of lease periods. Hence, the variability of the service time is equal to zero. Furthermore, the mean service time at the grading station is fixed as $\tau_5 = 0.5$ in the SRM. However, since the quality of the returned products is known in leasing, grading the products takes a shorter time. Thus, for the LM the mean processing time of the grading station is reduced by half with respect to the time in the SRM. Also, the variability at the grading station is minimal and therefore we set the scv of the service time of this station as $C_{s,5}^2 = 0$ according to the study by [16]. For the remanufacturing stations, Souza and Ketzenberg state that time to remanufacture a product increases due to the decreasing quality level and the scv of service time at remanufacturing stations for different grades varies from 1.5 to 2 [16]. We set the mean and scv of the service times at the remanufacturing stations accordingly. The fixed values of the mean and scv of service times of remanufacturing stations are presented in Table 5.1. Here, as the quality level decreases, both the mean and scv of service time for the corresponding stations increase.

Table 5.1. Mean service times and scvs of remanufacturing stations

Station, i	τ_i	$C_{s,i}^2$
6	1	1.5
7	1.2	1.6
8	1.4	1.7
9	1.6	1.8

At the demand stations, *R* and *L*, the mean service time is equal to the average time between demand arrivals. Namely, $\tau_R = 1/\lambda_{d.r}$ and $\tau_L = 1/\lambda_{d.l}$. We assume Poisson arrivals for the demand and thus service time becomes exponentially distributed. Since the service time of these stations are exponentially distributed, we set $C_{s,R}^2 = C_{s,L}^2 = 1$. Also, we consider that demand for new products follows a Poisson process ($C_a^2 = 1$) with rate $\lambda_{d.m} = 0.1$.

In the cost parameters, the selling price of new products, unit margin from disposed products, unit raw material cost, unit backorder cost and holding cost which is equal for each station are fixed as $P_m=100$, $m_{d,r}$=5, $P_{raw}=40$, $b=0.1$ and $h_i=0.01$, respectively. The holding cost per unit time appears small because it is scaled by the time factor. Namely, monthly holding cost rate is transformed to hourly holding cost rate. Also, for the acquisition cost in the LM, we consider a cost value that is a small fraction of the acquisition cost of the SRM as in the leasing system it is not expected to pay a price for cores. Moreover, to determine the processing cost of stations per hour, we mainly use the values from the study of Souza and Ketzenberg [17]. Also, for the stations that are not considered in that study we use the survey result in the study of Hammond et al. [38]. The survey result represents which operations are most costly in remanufacturing. According to this survey, we set the processing cost of collection, grading and re-assembly stations less costly compared to the processing cost of the remanufacturing stations. Since the mean processing times at the remanufacturing stations are different, hourly processing costs of these stations are same. Although assigned processing costs are equal for each model, the processing cost of collection station is not considered, since the customer has an obligation to return the leased product to the manufacturer in the leasing model. The fixed values of processing costs are given in Table 5.2.

Table 5.2. The processing cost of stations per hour

Station	**Name**	c_i
1	Production	5
2	Distribution	0.5
3	Customer Usage	0
4	Collection	0.25
5	Grading	0.25
6	Minor Reprocess	5
7	Repair	5
8	Refurbish	5
9	Remanufacture	5
10	Assembly	0.5

We now discuss the experimental designs which are performed to determine the significant parameters for each model. We do not perform experiments for the TSM because all of the parameters of this model are set to constant values before. The profit rate of the TSM is obtained using these values. We conduct experiments for both the SRM and the LM. There are six parameters of interest for the former model. Three of the parameters that are presented in previous chapters are the maximum number of times that a product can be remanufactured, overall yield of remanufacturing stations and the return probability. In addition to these, we present three new parameters. One of them is the ratio between average remanufacturing time and the production and design time. This ratio is important to reflect the effect of design to the remanufacturing time of a product. As expected, design for remanufacture provides that a product can be remanufactured easily. However, a proper design for remanufacture requires more time at the stage of production and design of a new product. To consider this fact, we obtain the ratio by using the equation (5.1).

$$k = \frac{\tau_6 + \tau_7 + \tau_8 + \tau_9}{4\tau_1} \tag{5.1}$$

Another parameter of interest is the ratio, c_r, of unit acquisition cost to unit raw material cost. We define this ratio in order to examine the advantages of remanufacturing models where raw material consumption is considerably low in comparison to the TSM. When we compare the SRM with the LM, this ratio becomes important since the unit acquisition cost is very small for the LM. Considering the last parameter, we define a ratio, p, between the price of remanufactured product, P_r and the price of new product, P_m to see the effects of price differences and customer willingness to buy remanufactured product.

The parameters used for the leasing model are the number of periods that a product can be leased during its useful life, the increase in residual value at the end of each lease period by remanufacturing, grade distributions, overall yield of remanufacturing stations and the ratio between production time and average remanufacturing time. The values of the last two parameters are set to the same values as in the SRM. For the increase in residual value, we determine the experimental values by changing p where P_m is held constant since we assumed in Section 4.4 that the residual value of the product is increased to the level which is equal to P_r at the end of each lease period by remanufacturing. The experimental values for grade distributions are determined using the ratio $\beta = \delta_{r,R} / \delta_{r,L}$ where $\delta_{r,L}$ and $\delta_{r,R}$ are the ratios between the proportion of good quality cores, $\theta = \{1, 2\}$, and bad quality cores, $\theta = \{3, 4, 5\}$ of product class r for the LM and the SRM, respectively. Here, the grade distributions for the SRM are constant. In order to capture the advantage of the leasing model on core qualities, the grade distributions for the LM are calculated with respect to the values of the SRM by changing β. The summary of the experimental factors and their values are represented in Table 5.3.

Table 5.3. Experimental values for the factors

Experimental design for the SRM

Factor	Experimental Values
n	1, 2, 3
$P^{(r)}$	0.6, 0.7, 0.8
y_i	(0.8, 0.7, 0.7, 0.6),(0.9, 0.8, 0.8, 0.7), (1.0, 0.9, 0.9, 0.8)
k	0.5, 0.8, 1.0
c_r	0.4, 0.25, 0.1
p	0.6, 0.7, 0.8

Experimental design for the LM

Factor	Experimental Values
η	4, 5, 10
β	0.25, 0.5, 1.0
y_i	(0.8, 0.7, 0.7, 0.6),(0.9, 0.8, 0.8, 0.7), (1.0, 0.9, 0.9, 0.8)
k	0.5, 0.8, 1.0
p	0.6, 0.8, 1.0

In summary, we perform 3^6 full factorial design for the SRM and in total there are 729 experiments. Also, 243 experiments are done as a result of the 3^5 full factorial design for the LM. The statistical analysis software package Design Expert 7.0 is used to analyze experimental results. The ANOVA for the SRM is summarized in Table 5.4. As the p-values less than 0.05 indicate, all of the factors are significant for this model. However, more attention is focused on four factors which are the maximum number of times that a product can be remanufactured, the ratio between the average remanufacturing time and the production time, the ratio of unit core cost to unit raw material cost and the ratio of the price of the remanufactured products to the price of new products. We conclude that these factors affect the profitability of the model notably since sum of squares of these factors are larger. In addition, we ignore the interactions between these factors since the influence from interactions is small as the small sum of squares of residual indicates.

Table 5.4. Analysis of variance for the SRM

Response π_2

ANOVA for selected factorial model

Analysis of variance table [Classsical sum of squares - Type II]

Source	Sum of Squares	df	Mean Square	F value	p-value Prob > F	
p	66.43	2	33.21	5907.79	< 0.0001	
c_r	62.99	2	31.49	5601.78	< 0.0001	
k	38.37	2	19.19	3412.64	< 0.0001	
n	9.61	2	4.81	855.09	< 0.0001	
y_i	2.01	2	1.01	178.8	< 0.0001	
$P^{(r)}$	0.062	2	0.031	5.53	0.0041	
Model	179.47	12	14.96	2660.27	< 0.0001	significant
Residual	4.03	716				
Cor Total	183.5	728				

To assist in the practical interpretation of this experiment, Figure 5.1, Figure 5.2 and Figure 5.3 present plots of three main effects. The main effect plots are just graphs of the response averages at the levels of three factors while the maximum number of times that a product can be remanufactured, n changes. Notice that p has a positive main effect; that is, increasing the value of this factor moves the profitability of the SRM upward for all values of n. On the other hand, as k and c_r increases, profitability of the model declines. Therefore, the graphs of these factors have the similar shape. Moreover, it can be seen from the three figures that higher profits are obtained with large value of n. In other words, n has a positive effect on profitability.

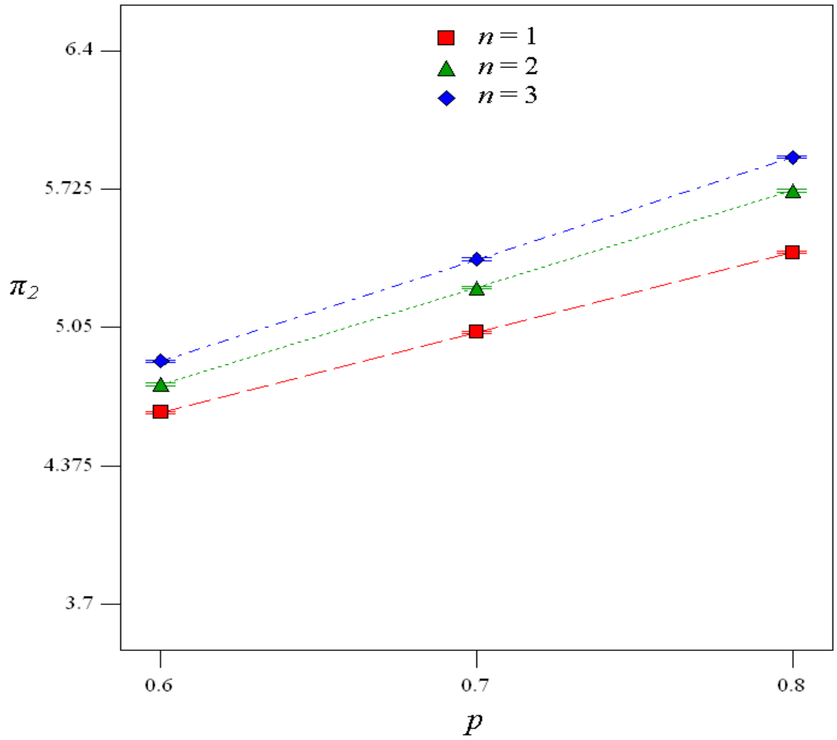

Figure 5.1. Effect of p for different n values

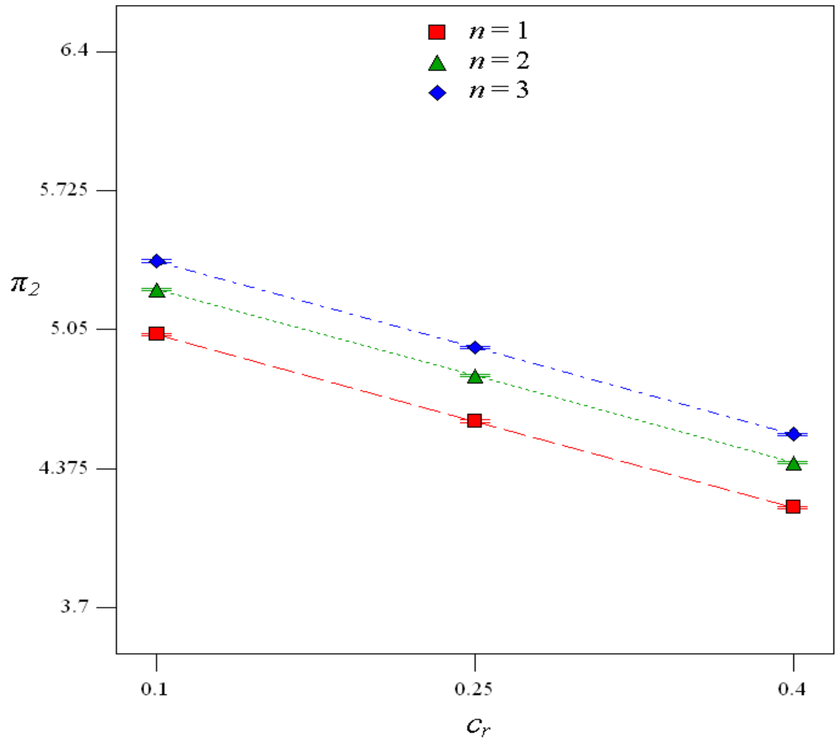

Figure 5.2. Effect of c_r for different n values

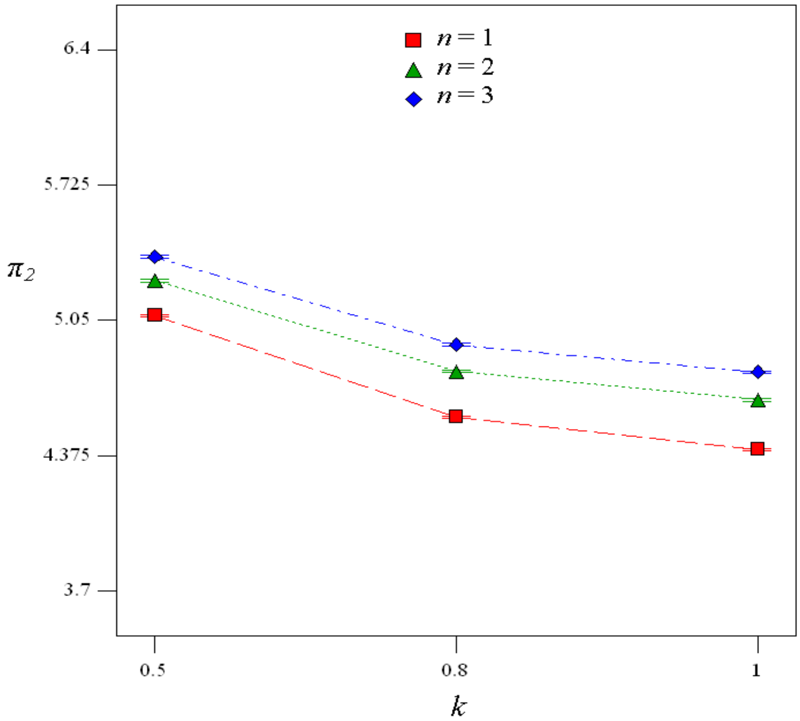

Figure 5.3. Effect of *k* for different *n* values

Considering the experiments for the LM, the results in Table 5.5 show that the most important effect is the increase in residual value at period η by remanufacturing since *p* has the largest sum of squares. Also, the number of lease periods during product's useful life has a great importance for the profitability of the LM. Similar to the SRM, the ratio between the average remanufacturing time and the production time significantly affect the profitability of this model. However, in this experiments we find that the difference in response between the levels of main factors is not the same at all levels of the factors. This implies that there is an interaction between the factors. We figure this out by observing that the sum of squares of the residual is very large when we ignore the interaction effects. In this context, interaction effects are taken into consideration and we focus on two interaction effects. We discover that the interaction between the increase in residual value and η is highly considerable and the interaction between *k* and η is significant since these interaction effects have larger sum of squares.

Table 5.5. Analysis of variance for the LM

Response π_3

ANOVA for selected factorial model

Analysis of variance table [Classsical sum of squares - Type II]

Source	**Sum of Squares**	**df**	**Mean Square**	**F value**	**p-value Prob > F**	
p	208.2	2	104.1	3044.3	< 0.0001	
k	123.13	2	61.57	1800.46	< 0.0001	
η	37.71	2	18.86	551.43	< 0.0001	
β	1.02	2	0.51	14.92	< 0.0001	
y_i	0.16	2	0.081	2.36		
Interaction Effects						
p - k	2.52E-03	4	6.30E-04	0.018	0.9993	
p - y_i	1.34	4	0.33	9.76	< 0.0001	
p - β	0.79	4	0.2	5.77	0.0002	
p - η	122.09	4	30.52	892.59	< 0.0001	
k - y_i	3.81	4	0.95	27.84	< 0.0001	
k - β	1.99	4	0.5	14.58	< 0.0001	
k - η	13.03	4	3.26	95.26	< 0.0001	
y_i - β	0.38	4	0.095	2.78	0.0279	
y_i - η	0.045	4	0.011	0.33	0.8604	
β - η	0.49	4	0.12	3.55	0.008	
Model	514.8	50	10.28	300.74	< 0.0001	significant
Residual	6.57	192	0.034			
Cor Total	520.75	242				

The fact that some interaction effects have great importance makes the analysis of how the parameters affect the profitability of the LM more complicated. It means that not only the parameter value itself, but also the levels of the other factors must be taken into consideration. The interaction effects are shown in Figure 5.4, Figure 5.5, Figure 5.6. In Figure 5.4, we see that the increase in residual value by remanufacturing affects the profitability more drastically as the number of lease periods increases. As expected, the increase in residual value has a positive effect for all levels of η. Namely, increasing the value of this parameter moves profits upward. Moreover, we observe in Figure 5.5 that increasing the number of lease periods is not effective if the increase in residual value by remanufacturing is low. However, if the increase in residual value is high, the large number of lease periods increases profitability. Finally, high profits can be accomplished by decreasing *k*. Also, we see in Figure 5.6 that the effect of this ratio is more evident for large values of η.

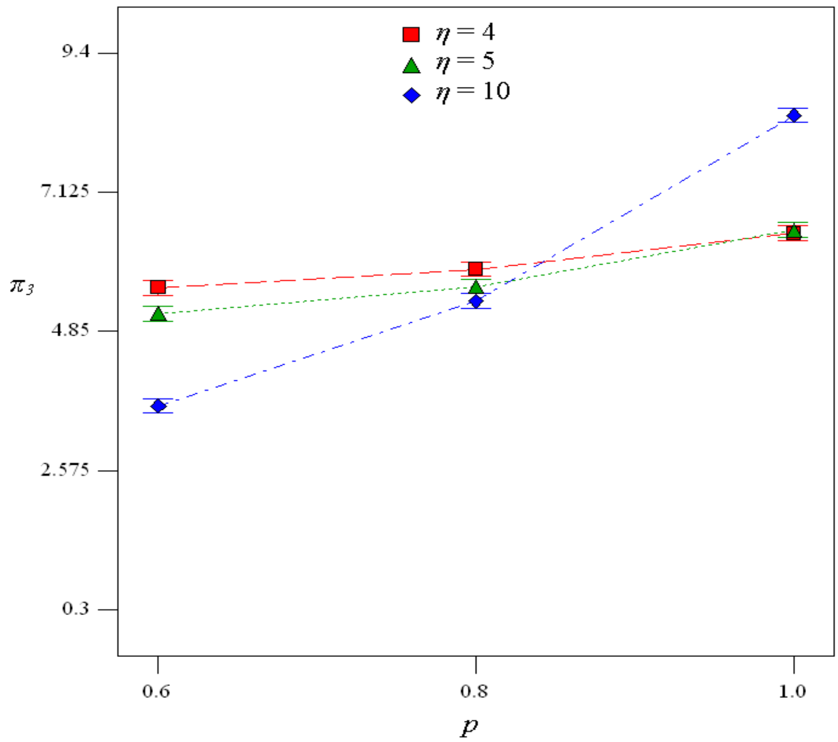

Figure 5.4. Interaction effect between *p* and η

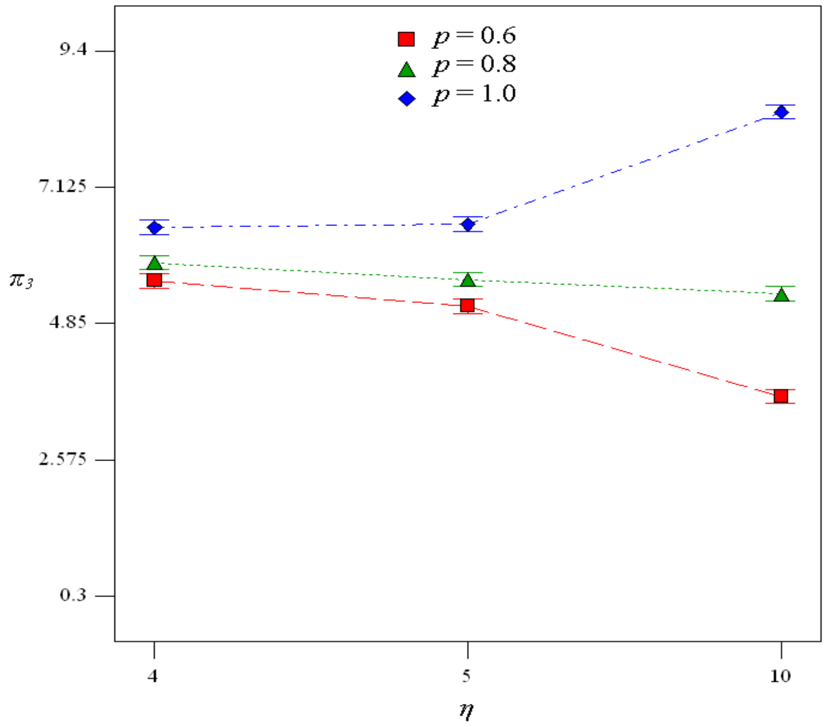

Figure 5.5. Interaction effect between η and p

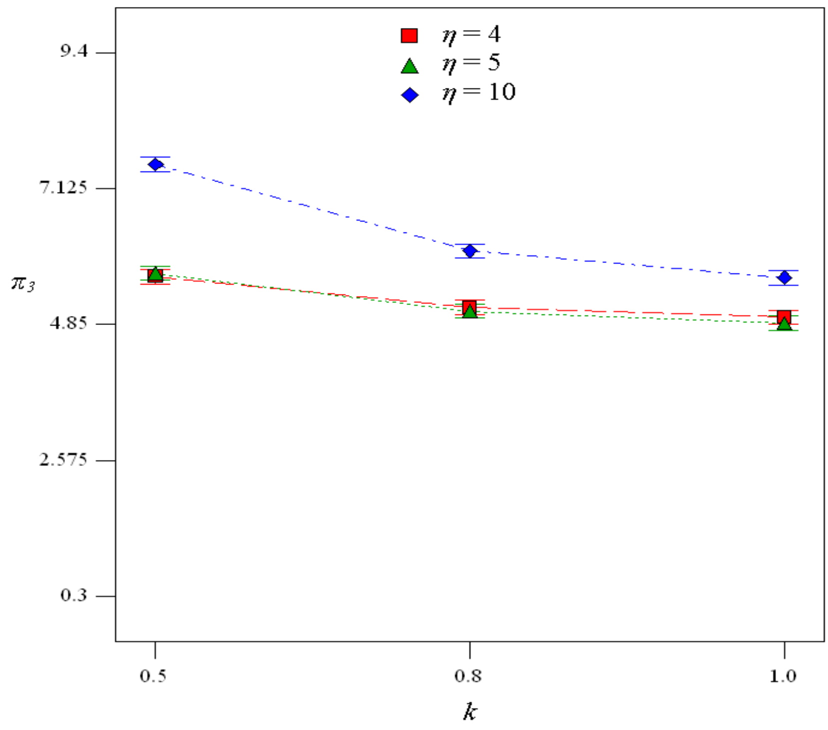

Figure 5.6. Interaction effect between of k and η

5.1. Sensitivity Analysis for the Parameters of Interest

In the previous section, we determined the significant factors and discussed their effects on the profitability of each model. We now compare the profitability of our models and search for the critical values that change the ranking of profitabilities. To this end, we perform sensitivity analysis for the following parameters; the profit margin of a new product, the ratio between price of a remanufactured product and a new product, the number of lease periods and the increase in residual value at the end of each lease period by remanufacturing. In the sensitivity analysis, while these parameters are changed, other significant parameters (k and c_r) are held constant. In order to examine the changes in profitability easier, constant values of parameters are determined due to the experiment sets whose calculated profits are high.

In these experiments, to observe the effects under different throughput rates we consider three different throughput rate configurations. For the SRM, we arrange this by changing the yield of remanufacturing stations and return probability of the products. On the other hand, since the return probability is equal to one for the LM, we employ a different strategy to determine the throughput rate configurations. We arrange low, average and high throughput rates by changing the yield of remanufacturing stations and also the values of β. Notice that, for the large values of β the throughput rates decrease because the proportion of products going to disposal increases. In Table 5.6, Case 1, Case 2 and Case 3 represent low, average and high throughput rates for the SRM and LM, respectively.

Table 5.6. Throughput rate configurations for the SRM and LM

	SRM			LM	
	$P^{(r)}$	y_i		β	y_i
Case 1	0.5	(0.8, 0.7, 0.7, 0.6)	**Case 1**	1	(0.8, 0.7, 0.7, 0.6)
Case 2	0.6	(0.9, 0.8, 0.8, 0.7)	**Case 2**	0.5	(0.9, 0.8, 0.8, 0.7)
Case 3	0.7	(1, 0.9, 0.9, 0.8)	**Case 3**	0.25	(1, 0.9, 0.9, 0.8)

5.1.1. Change in the Unit Profit Margin of a New Product

In this section, we study the effect of the unit profit margin of a new product on profitability. We investigate whether it is profitable to remanufacture a product while unit profit margin of a new product is low. Also, we compare the profitability of the SRM and the TSM while varying this parameter. To this end, we define the unit profit margin of a new product by finding the net profit as a percentage of the revenue. Here, net profit is obtained simply by subtracting the marginal cost from marginal revenue. For new products, marginal revenue is equal to the unit selling price of a new product and marginal cost is equal to the sum of the unit raw material cost and unit production cost, c_p. Then, the unit profit margin of a new product, m_n can be obtained as;

$$m_n = \frac{P_m - P_{raw} - c_p}{P_m} \times 100 \tag{5.2}$$

In order to observe the effect of the unit profit margin, m_n is changed from 5% to 50% by varying the selling price of a new product while P_{raw} and c_p are held constant in the experiments. Here, the question is for which values of m_n the SRM is more profitable than the TSM. Also, for each experiment we compute the profitability of the SRM for three different values of the ratio between the price of a remanufactured and a new product, p.

In Table 5.7, we present the results for $p=0.5$, $p=0.7$ and $p=1$ as m_n changes. Also, the results are computed for different throughput configurations that are proposed in Section 5.1. In all experiment, the number of times that a product can be remanufactured is selected to be one, $n=1$. Moreover, the results represented under the title "π_1" are the profit rates of the TSM. Notice that the profit rates of the TSM changes with different values of m_n. This is because the price of a new product changes in the experiments. However, π_1 is same for Case 1, Case 2 and Case 3 since different throughput rate configurations affect only the throughput rates of the SRM. The results are depicted in Figure 5.7, Figure 5.8 and Figure 5.9.

As a result of these experiments, it is observed that both the profitability of the SRM and the TSM exponentially increase as the unit profit margin of a new product linearly increases. This is because P_m is increased exponentially in order to have linear increase in m_n . Also, when the ratio between the price of a remanufactured and a new product is high enough, the SRM performs better than the TSM for all values of m_n. This implies that it is profitable to remanufacture a product although m_n is very low. For instance, this situation occurs when $p=0.7$ and $p=1$ for our cost structure. High profit rates are obtained in the SRM because remanufactured goods have significantly higher profit margins than new when p is high. As an illustration, when new and remanufactured products are sold at the same price, namely $p=1$, selling more remanufactured parts with a higher profit margin further increases total system profit. As can be seen from Table 5.7, when $p=0.7$ and $p=1$, the profit rates of the SRM increase as the throughput rates increase for the remanufactured products.

On the other hand, when p is low, the TSM becomes more profitable than the SRM after a certain value of m_n for all throughput rate configurations. This implies that it is not profitable to remanufacture a product if the unit profit margin of a new product is high enough and p is low. For instance, when $p=0.5$, if m_n is greater than 35%, TSM is profitable than the SRM. In this situation, because of the high profits made by selling new products, we choose not to offer remanufactured products. This is because we can cannibalize this profitable market. If we increase the number of sales of remanufactured products, the profitability of the SRM decreases since new product sales are cannibalized. It can be seen from Table 5.7, when we compare the profit rates of Case 1, Case 2 and Case 3 for $p=0.5$, the profit rates of the SRM decreases after a certain value of m_n.

Table 5.7. Profit rates for Case 1, Case 2 and Case 3 as m_n changes

Case 1: y_i=(0.8, 0.7, 0.7,0.6) , $P^{(r)}$=0.5

			Profit rate, π_2			
Ex.	P_m	m_n	p = 0.5	p = 0.7	p = 1	π_1
1	52.65	5%	0.5324	0.7937	1.1858	0.2403
2	56.00	10%	0.8259	1.1040	1.5210	0.5753
3	58.80	15%	1.0713	1.3632	1.8011	0.8552
4	63.00	20%	1.4394	1.7522	2.2213	1.2753
5	67.00	25%	1.7899	2.1225	2.6215	1.6752
6	72.00	30%	2.2281	2.5855	3.1217	2.1753
7	77.00	35%	2.6663	3.0485	3.6220	2.6753
8	84.00	40%	3.2797	3.6967	4.3223	3.3754
9	91.00	45%	3.8931	4.3449	5.0226	4.0754
10	100.00	50%	4.6818	5.1783	5.9230	4.9753

Case 2: y_i=(0.9, 0.8, 0.8,0.7) , $P^{(r)}$=0.6

			Profit rate, π_2			
Ex.	P_m	m_n	p = 0.5	p = 0.7	p = 1	π_1
1	52.65	5%	0.7131	1.1257	1.7444	0.2403
2	56.00	10%	0.9828	1.4216	2.0798	0.5753
3	58.80	15%	1.2083	1.6690	2.3601	0.8552
4	63.00	20%	1.5464	2.0400	2.7805	1.2753
5	67.00	25%	1.8685	2.3934	3.1809	1.6752
6	72.00	30%	2.2710	2.8351	3.6814	2.1753
7	77.00	35%	2.6736	3.2769	4.1818	2.6753
8	84.00	40%	3.2371	3.8953	4.8825	3.3754
9	91.00	45%	3.8007	4.5137	5.5832	4.0754
10	100.00	50%	4.5253	5.3088	6.4841	4.9753

Case 3: y_i=(1, 0.9, 0.9,0.8) , $P^{(r)}$=0.7

			Profit rate, π_2			
Ex.	P_m	m_n	p = 0.5	p = 0.7	p = 1	π_1
1	52.65	5%	0.8908	1.4360	2.2537	0.2403
2	56.00	10%	1.1391	1.7189	2.5887	0.5753
3	58.80	15%	1.3466	1.9554	2.8687	0.8552
4	63.00	20%	1.6579	2.3102	3.2886	1.2753
5	67.00	25%	1.9543	2.6480	3.6886	1.6752
6	72.00	30%	2.3248	3.0703	4.1886	2.1753
7	77.00	35%	2.6954	3.4926	4.6885	2.6753
8	84.00	40%	3.2141	4.0839	5.3885	3.3754
9	91.00	45%	3.7329	4.6751	6.0884	4.0754
10	100.00	50%	4.3998	5.4353	6.9884	4.9753

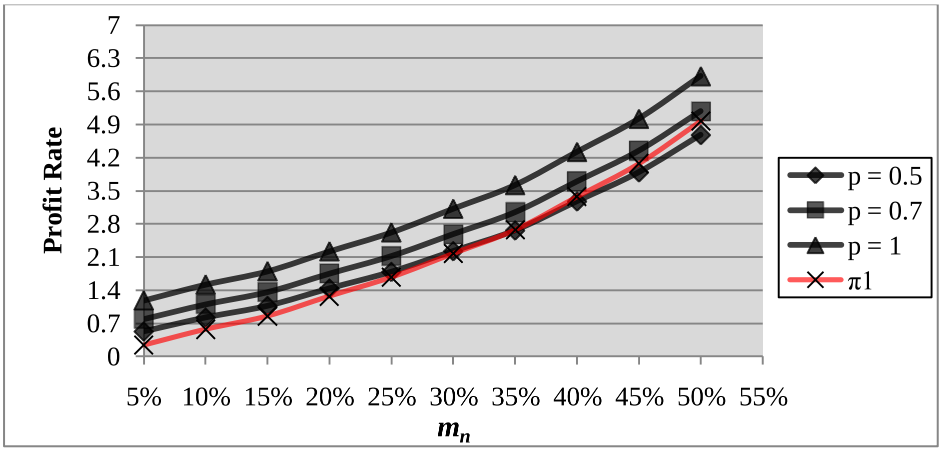

Figure 5.7. Comparison of profit rates for Case 1 as m_n changes

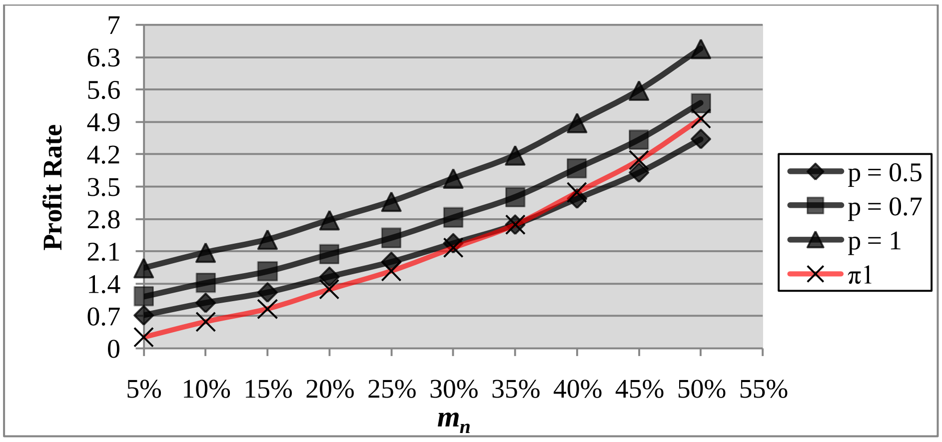

Figure 5.8. Comparison of profit rates for Case 1 as m_n changes

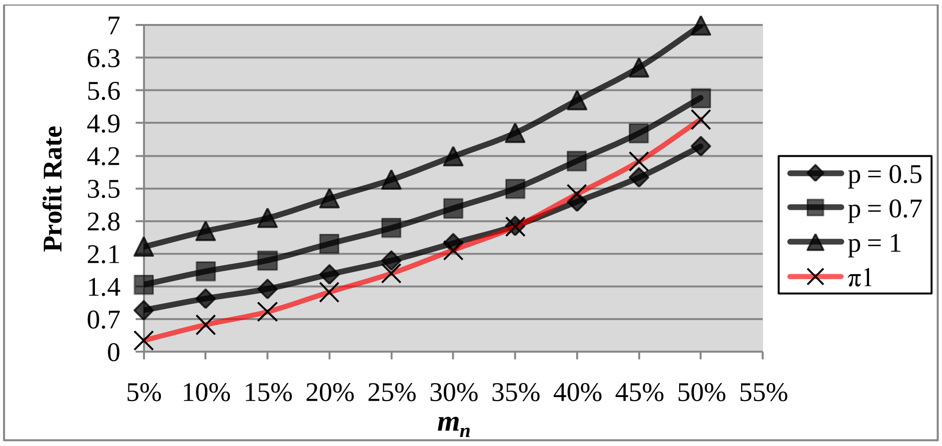

Figure 5.9. Comparison of profit rates for Case 1 as m_n changes

5.1.2. Change in the Ratio between the Price of Remanufactured and New Products

One of the important parameters that affect the profitability of the SRM is the ratio between the price of remanufactured and new products, *p*. In the ANOVA analysis, we concluded that increasing this ratio affects profitability positively. As expected, a higher price obtained for remanufactured products increases the profitability of the SRM. Here, we investigate for which values of *p* the SRM is more profitable than the TSM. Also we examine whether it is profitable to remanufacture when *p* is low. To this end, *p* is changed from 0.35 to 0.80 with 0.05 increments by varying the price of a remanufactured product, P_r while price of a new product, P_m is held constant since Lund and Hauser [39] state that the price of a remanufactured product commonly is between 45% and 65% of the price of a comparable new product. The constant value of P_m is chosen to provide 50% unit profit margin of a new product since we observed in the previous section that high profit margins change the ranking of profitabilities. Also, in order to capture the effect of *n* while changing *p*, we compute profit rates for different values of *n* for each experiment.

In Table 5.8, we present the results for $n=1$, $n=2$ and $n=3$ as *p* changes. Also, the results are represented under the title "π_1" are the profit rates of the TSM. Notice that this value remains constant for all experiments in sensitivity analysis because the changes in the price of remanufactured products do not affect the profitability of the TSM. Also, the results are depicted in Figure 5.10, Figure 5.11 and Figure 5.12.

As a result of these experiments, we observe that as *p* increases, the profit rates of the SRM linearly increases with constant increments. The reason for the constant increments is that the price of the remanufactured products is always multiplied with the same throughput rate for a certain throughput rate configuration. However, the amount of increments differs as a result of different *n* values and different throughput rate configurations. For instance, the highest amount of increments eventuates for $n=3$ under Case 3. When we compare the profitability of the SRM and the TSM, we observe from the results that the SRM is more profitable than the TSM for high values of *p*. This is because selling a remanufactured product becomes more profitable than selling a new product as the price of a remanufactured product increases. For instance, in our experiments if $p \geq 0.65$, the profit rates of the SRM is larger for all throughput rate configurations and for all values of *n*. However, this critical value is only applicable for our cost structure. Another result is that it is not profitable to remanufacture a product if the selling price of a remanufactured product is low. This implies that selling remanufactured products with a low profit margin decreases the total system profit by discouraging the sales of new products with higher profit margins.

Moreover, it is observed from the results that *n* considerably affects the profitability of the SRM. Here, let us define remanufacturability of a product as the maximum number of times that a product can be remanufactured, *n*. The results show us that profit rates can be increased by high remanufacturability when high price ratio between remanufactured and new products is provided. On the contrary, if the price ratio is low, increasing the remanufacturability of the product decreases the total system profits. For instance, if $p \leq 0.60$, the SRM with *n=1* performs better than the models with *n=2* and *n=3* for our cost structure. On the other hand, if $p \geq 0.65$, the profit rates take higher values as *n* increases. The results described above can be seen in all throughput rate configurations.

Moreover, throughput rate configurations allow us to observe the profit changes clearly. As the throughput rates increases, the difference in profit between the models becomes more evident. In other words, the difference in maximum and minimum profit values increases.

Table 5.8. Profit rates for *n=1*, *n=2* and *n=3* as *p* changes

Case 1: y_i=(0.8, 0.7, 0.7,0.6) , $P^{(r)}$=0.5

			Profit rate, π_2			
Ex.	***p***	P_r	**n=1**	**n=2**	**n=3**	π_1
1	0.35	35	4.3053	4.1449	4.1062	4.9753
2	0.40	40	4.4293	4.2961	4.2636	4.9753
3	0.45	45	4.5533	4.4472	4.4211	4.9753
4	0.50	50	4.6773	4.5983	4.5785	4.9753
5	0.55	55	4.8013	4.7494	4.7360	4.9753
6	0.60	60	4.9253	4.9005	4.8935	4.9753
7	0.65	65	5.0493	5.0516	5.0509	4.9753
8	0.70	70	5.1733	5.2027	5.2084	4.9753
9	0.75	75	5.2973	5.3538	5.3658	4.9753
10	0.80	80	5.4213	5.5049	5.5233	4.9753

Case 2: y_i=(0.9, 0.8, 0.8,0.7) , $P^{(r)}$=0.6

			Profit rate, π_2			
Ex.	***p***	P_r	**n=1**	**n=2**	**n=3**	π_1
1	0.35	35	4.1682	3.9338	3.8623	4.9753
2	0.40	40	4.3234	4.1295	4.0701	4.9753
3	0.45	45	4.4785	4.3252	4.2780	4.9753
4	0.50	50	4.6337	4.5208	4.4858	4.9753
5	0.55	55	4.7889	4.7165	4.6937	4.9753
6	0.60	60	4.9440	4.9122	4.9015	4.9753
7	0.65	65	5.0992	5.1079	5.1093	4.9753
8	0.70	70	5.2544	5.3036	5.3172	4.9753
9	0.75	75	5.4096	5.4993	5.5250	4.9753
10	0.80	80	5.5647	5.6950	5.7329	4.9753

Case 3: y_i=(1, 0.9, 0.9,0.8) , $P^{(r)}$=0.7

			Profit rate, π_2			
Ex.	***p***	P_r	**n=1**	**n=2**	**n=3**	π_1
1	0.35	35	4.0389	3.7342	3.6235	4.9753
2	0.40	40	4.2240	3.9737	3.8824	4.9753
3	0.45	45	4.4091	4.2132	4.1413	4.9753
4	0.50	50	4.5943	4.4527	4.4002	4.9753
5	0.55	55	4.7793	4.6922	4.6590	4.9753
6	0.60	60	4.9645	4.9317	4.9179	4.9753
7	0.65	65	5.1496	5.1712	5.1768	4.9753
8	0.70	70	5.3347	5.4107	5.4356	4.9753
9	0.75	75	5.5199	5.6502	5.6945	4.9753
10	0.80	80	5.7050	5.8897	5.9534	4.9753

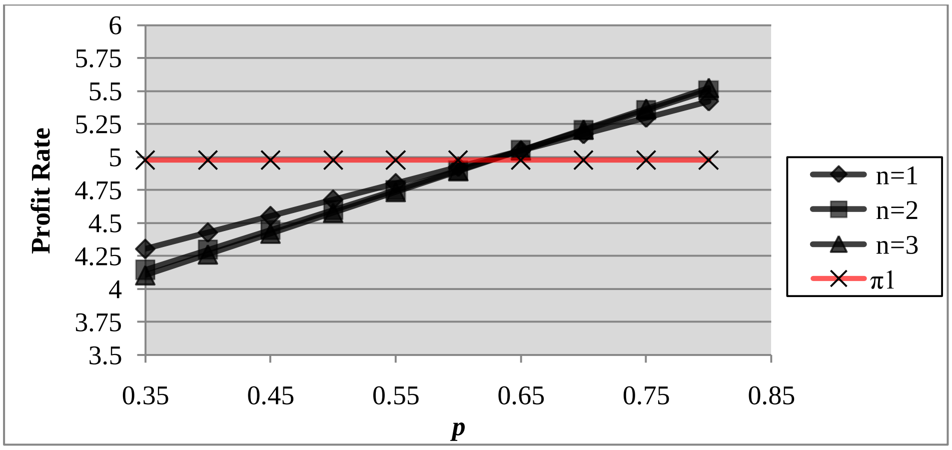

Figure 5.10. Comparison of the profit rates of the SRM for Case 1 as *p* changes

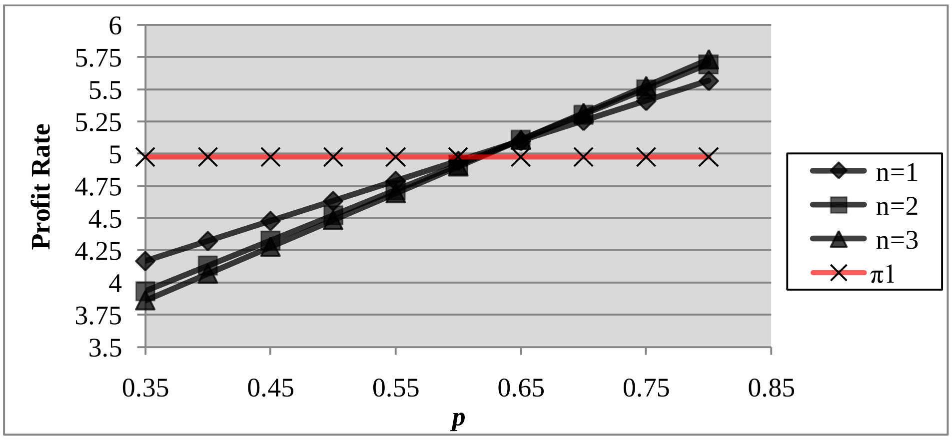

Figure 5.11. Comparison of the profit rates of the SRM for Case 2 as *p* changes

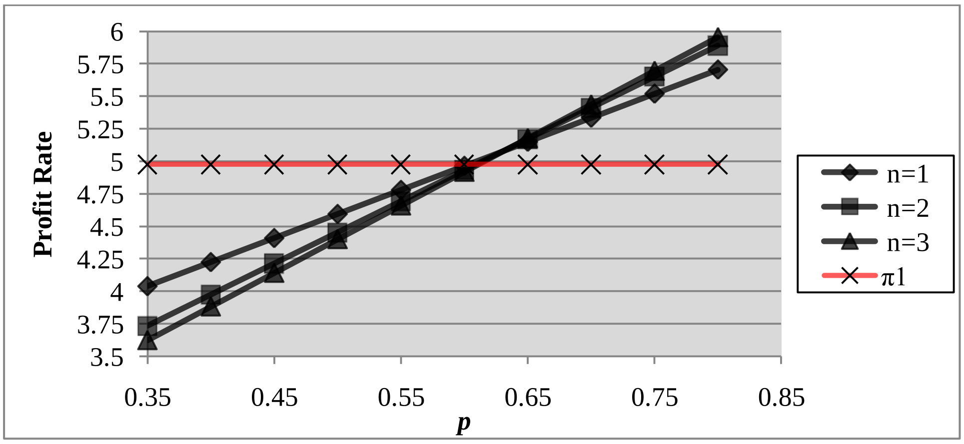

Figure 5.12. Comparison of the profit rates of the SRM for Case 3 as *p* changes

5.1.3. Change in the Number of Lease Periods

In this section, we compare the profitability of the LM with the SRM and the TSM while changing the number of lease periods throughout the product's useful life. Also, we investigate whether increasing the number of lease periods, η is profitable for the LM since η is an important factor that affects the profitability of the LM. Increasing the number of lease periods can increase or decrease the profitability because the changes in the number of lease periods influence both the revenues and the processing costs. It is obvious that the processing costs of all stations become larger when the number of lease periods increases as a result of increase in the throughput rates of each station. The throughput rates increase because the number of returns that products made increases as the duration of the lease periods decrease. On the other hand, the effect of the increase in η on the revenues is related to the increase in residual value after remanufacturing. If the increase in residual value is large enough, the lease payments continue to be high and thus increase in revenues can be larger than the increase in the costs.

We change η from 1 to 10 in order to draw some insight about the effects of this parameter. Profit rates for these experiments are computed for different values of the increase in residual value and for different throughput rate configurations that are given in Section 5.1. Since we assumed in Section 4.4 that the residual value of the product is increased to the level which is equal to P_r at the end of each lease period by remanufacturing, we determine the levels of the increase in residual value by changing p where P_m is held constant. The constant value of P_m is chosen to provide 50% unit profit margin similarly in the previous section. The levels for p are selected to be $p=0.6$, $p=0.8$ and $p=1$. Here, the largest value of p implies that the residual value of the product becomes equal to the selling price of a new product after remanufacturing and this value is the maximum value that can be achieved by remanufacturing. Also, to compare the profitability of the LM with the other models as η changes, we present the profit rates of the other models.

In Table 5.9, we present the results for $p=0.6$, $p=0.8$ and $p=1$ as the number of lease periods changes. Also, the results are represented under the title "π_1" and "π_2" are the profit rates of the TSM and the SRM respectively. The profit rates for the TSM are computed with 50% unit profit margin of a new product and remain constant for different values of *η* because the changes in *η* do not affect the profitability of the TSM. Also, the profit rates of the SRM are computed for different values of *p* and for different throughput rate configurations For the SRM, *n* is selected to be 3 since highest profits are achieved when *n=3*. The results are depicted in Figure 5.13, Figure 5.14 and Figure 5.15.

As a result of these experiments, first we observe that the profitability of the LM increases as the number of lease periods increases up to a certain value. When the number of lease periods is equal to this critical value, profit rate of the LM is maximized. This critical value changes for different values of *p*, e.g. for *p=0.6*, *p=0.8* and *p=1*, the critical value of *η* is equal to 3, 4 and 8 respectively. This is because high profit rates are sustained for larger number of lease periods a result of higher lease payments. However, after this value profit rates decline as the number of lease periods increases in all throughput rate configurations for different values of *p*. This is because increase in the revenues becomes smaller than the increase in costs after a certain value of *η* since the changes in the number of lease periods influence both the revenues and the processing costs.

When we compare the LM with the TSM and the SRM, we observe that the LM performs better than the other models in terms of profitability for high lease payments accomplished by high increase in residual value after remanufacturing. On the other hand, for low lease payments, e.g. *p=0.6*, the profitability of the LM can be lower than the profitability of the other models as we increase *η* after the maximum profits are achieved. These results can be seen for all throughput rate configurations.

In summary, we conclude that if the increase in residual value after remanufacturing at the end of each lease period is large enough, high profitability can be achieved by increasing the number of lease periods up to a certain value. As *η* increases, the product is frequently remanufactured in its useful life. Thus, residual value of the product remains at high values. This fact provides high lease payments for each lease period. However, high remanufacturability is required to increase the number of lease periods. This requirement is mandatory for the LM to become more profitable than the other models.

Table 5.9. Profit rates for *p=0.6*, *p=0.8* and *p=1* as η changes

Case 1: y_i=(0.8, 0.7, 0.7,0.6) , β = 1 Case 1: y_i=(0.8, 0.7, 0.7,0.6) , $P^{(r)}$=0.5

		Profit rate, π_3			Profit rate, π_2			
Ex.	η	***p =0.6***	***p =0.8***	***p =1***	***p =0.6***	***p =0.8***	***p =1***	π_1
1	1	5.7502	5.7625	5.7854	4.8935	5.5233	6.1531	4.9753
2	2	7.0115	8.5446	10.0600	4.8935	5.5233	6.1531	4.9753
3	3	7.0262	9.3468	11.6782	4.8935	5.5233	6.1531	4.9753
4	4	6.8243	9.4296	12.1767	4.8935	5.5233	6.1531	4.9753
5	5	5.2222	8.7085	12.2556	4.8935	5.5233	6.1531	4.9753
6	6	4.6060	8.6306	12.4042	4.8935	5.5233	6.1531	4.9753
7	7	3.7692	8.3540	12.8192	4.8935	5.5233	6.1531	4.9753
8	8	3.0691	8.1757	13.0262	4.8935	5.5233	6.1531	4.9753
9	9	1.5504	7.1005	12.3957	4.8935	5.5233	6.1531	4.9753
10	10	-0.0729	5.9037	11.6809	4.8935	5.5233	6.1531	4.9753

Case 2: y_i=(0.9, 0.8, 0.8,0.7) , β = 0.5 Case 2: y_i=(0.9, 0.8, 0.8,0.7) , $P^{(r)}$=0.6

		Profit rate, π_3			Profit rate, π_2			
Ex.	η	***p =0.6***	***p =0.8***	***p =1***	***p =0.6***	***p =0.8***	***p =1***	π_1
1	1	5.7648	5.7822	5.8021	4.9015	5.7329	6.5642	4.9753
2	2	6.9896	8.5699	10.1495	4.9015	5.7329	6.5642	4.9753
3	3	7.0439	9.4265	11.8100	4.9015	5.7329	6.5642	4.9753
4	4	6.7495	9.5167	12.2458	4.9015	5.7329	6.5642	4.9753
5	5	5.4918	8.7967	12.3626	4.9015	5.7329	6.5642	4.9753
6	6	5.0739	8.7542	12.4740	4.9015	5.7329	6.5642	4.9753
7	7	4.6444	8.6781	12.9128	4.9015	5.7329	6.5642	4.9753
8	8	4.2258	8.5315	13.2409	4.9015	5.7329	6.5642	4.9753
9	9	3.0877	7.8137	12.5572	4.9015	5.7329	6.5642	4.9753
10	10	1.5161	6.4612	11.8884	4.9015	5.7329	6.5642	4.9753

Case 3: y_i=(1.0, 0.9, 0.9,0.8) , β = 0.25 Case 3: y_i=(1, 0.9, 0.9,0.8) , $P^{(r)}$=0.7

		Profit rate, π_3			Profit rate, π_2			
Ex.	η	***p =0.6***	***p =0.8***	***p =1***	***p =0.6***	***p =0.8***	***p =1***	π_1
1	1	5.7749	5.7978	5.8221	4.9179	5.9534	6.9889	4.9753
2	2	7.0040	8.6297	10.2548	4.9179	5.9534	6.9889	4.9753
3	3	7.0950	9.5083	11.9225	4.9179	5.9534	6.9889	4.9753
4	4	6.7842	9.6163	12.4504	4.9179	5.9534	6.9889	4.9753
5	5	5.7524	8.9544	12.4675	4.9179	5.9534	6.9889	4.9753
6	6	5.4626	8.8368	12.5045	4.9179	5.9534	6.9889	4.9753
7	7	5.1440	8.7824	12.9422	4.9179	5.9534	6.9889	4.9753
8	8	4.8863	8.6623	13.4217	4.9179	5.9534	6.9889	4.9753
9	9	3.9610	8.1658	12.6548	4.9179	5.9534	6.9889	4.9753
10	10	2.6006	6.9538	11.9897	4.9179	5.9534	6.9889	4.9753

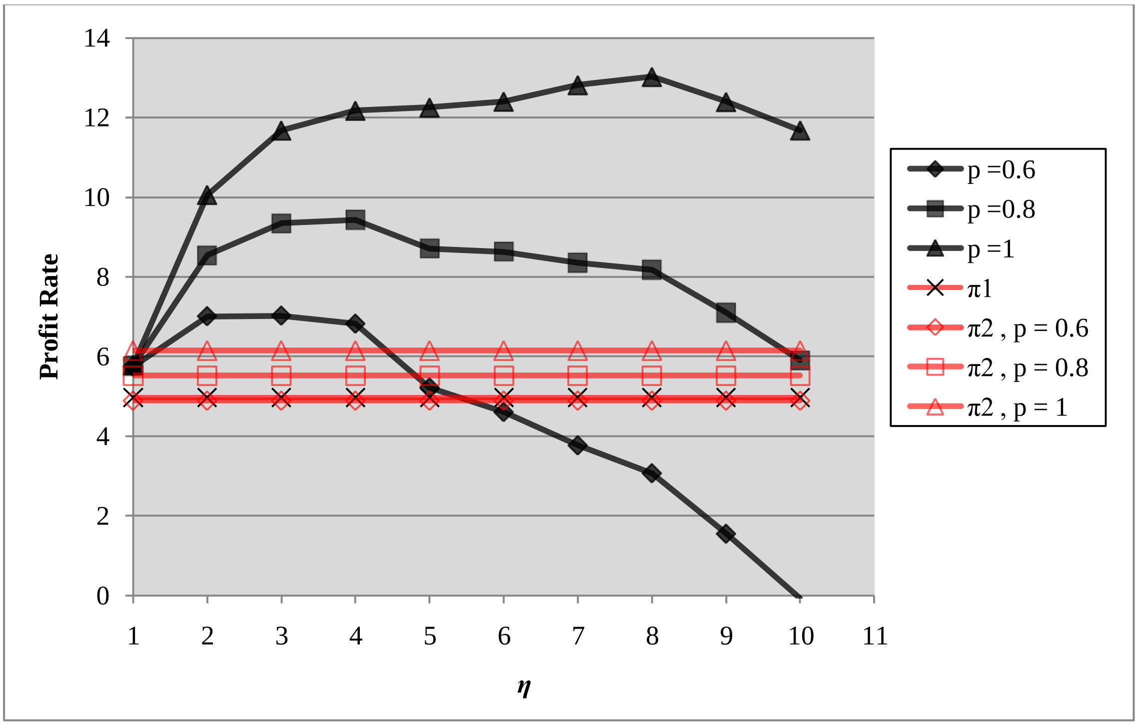

Figure 5.13. Comparison of profit rates for Case 1 as η changes

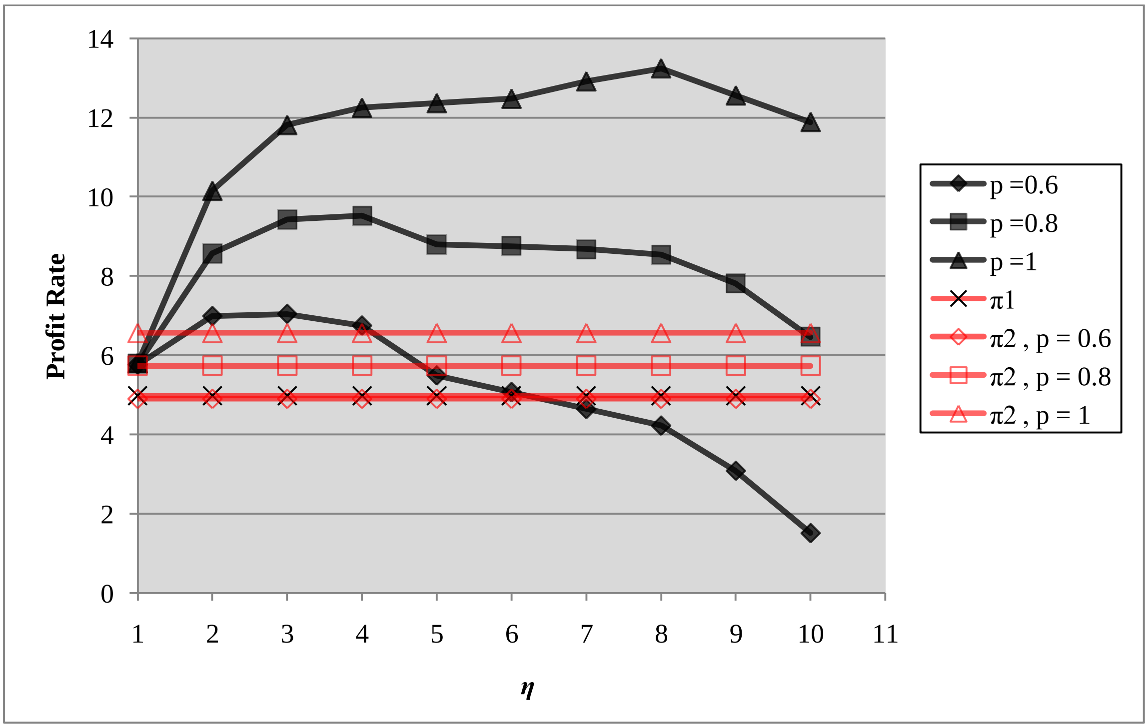

Figure 5.14. Comparison of profit rates for Case 2 as η changes

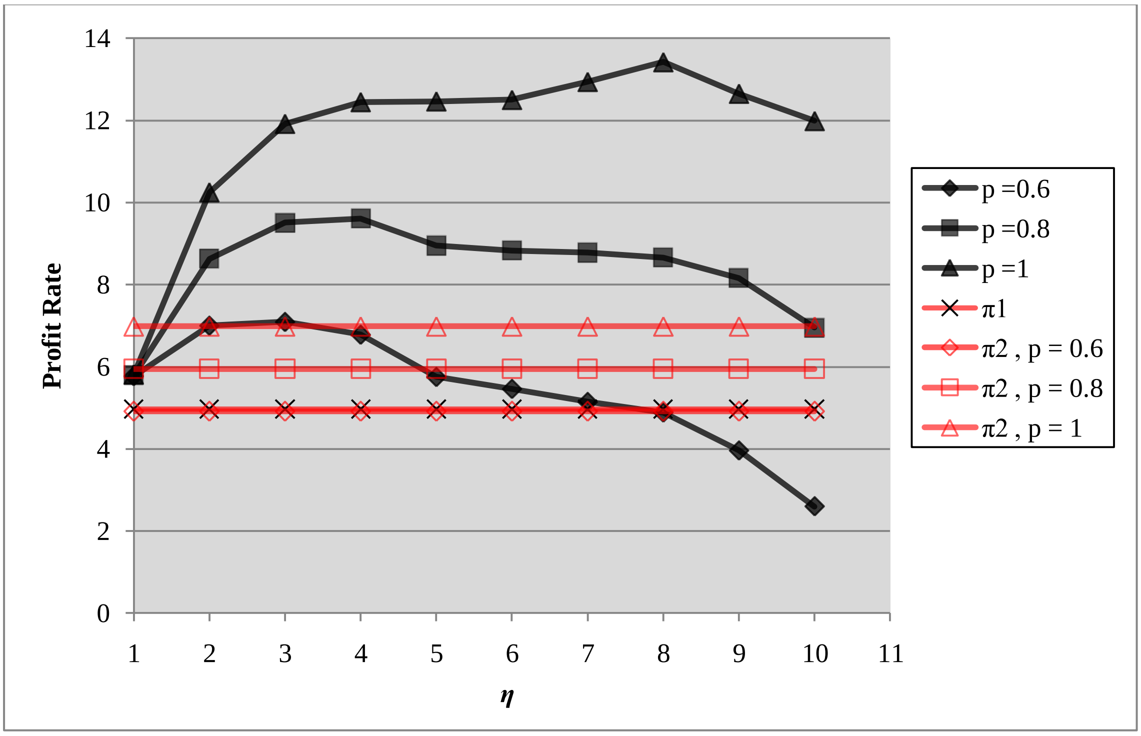

Figure 5.15. Comparison of profit rates for Case 3 as η changes

5.1.4. Change in the Increase in Residual Value after Remanufacturing

In the previous section, we observed that different values of the increase in residual value influence the effect of η. This shows us that the increase in residual is a significant factor that affects the profitability of the LM. This implication is also shown in the ANOVA analysis. Mainly, the increase in residual value affects the finance part of the lease payments. As the value of the increase in residual value increases, finance part of the lease payments becomes larger since the residual value of the product continues to be high. Since we assumed that the residual value of the product is increased to the level which is equal to P_r at the end of each lease period by remanufacturing, if we increase P_r, the increase in residual value increases. In this section, in order to investigate the behavior of the LM as the increase in residual value by remanufacturing changes, we determine the levels of the increase by changing p where P_m is held constant similarly in the previous section. We vary p from 0.55 to 1.00 with 0.05 increments. Here, we search for the values where the LM becomes profitable than the TSM and the SRM and examine whether the LM is profitable when lease payments are low as a result of low increase in residual value by remanufacturing.

Profit rates for the LM are computed for different values of η and for different throughput rate configurations that are given in Section 5.1. The levels for η are selected to be $\eta=2$, $\eta=5$ and $\eta=10$ to compare the effects of the increase in residual value on profitability for low, average and high number of lease periods. Also, to compare the profitability of the LM with the other models, we include their profit rates. The profit rates for the TSM are computed with 50% unit profit margin of a new product and remain constant for different values of p since P_m is held constant. Also, the profit rates of the SRM are computed for different values of p and for different throughput rate configurations similar to the previous section. For the SRM, n is selected to be 3 since highest profits are achieved when $n=3$. The profit rates of the SRM increase as p increases because P_r takes larger values. In Table 5.10, profit rates are presented for $\eta=2$, $\eta=5$ and $\eta=10$ as p changes. Also, the results are represented under the title "π_1" and "π_2" are the profit rates of the TSM and the SRM respectively. The results are depicted in Figure 5.16, Figure 5.17 and Figure 5.18.

As a result of these experiments, it can be stated that the profitability of the LM increases for all levels of η as the increase in residual value by remanufacturing increases. This is because high lease payments provide high profits. Also, the ranking of the profitabilities for different values of η changes as the lease payments increases as a result of increase in p. For the low values of p, e.g. $p \leq 0.75$, leasing the product for low number of periods by increasing the duration of the lease periods is profitable. However, if p is high, e.g. $p \geq 0.80$, the LM where η=5 performs better. Moreover, it can be observed that the profitability of the LM can decrease to the negative values for low values of p if the number of lease periods is high, e.g. η=10.

Another result reported in the table and graphs is that the LM is more profitable than the TSM and the SRM for high values of p. On the other hand, the TSM and the SRM can perform better than the LM when the lease payments are low as a result low values of p. This is because when the lease payments are low, increase in the cost of remanufacturing decreases the profits. These results can be seen for all throughput rate configurations.

In summary, the LM is more profitable than the other models if the increase in residual value is large enough. However, if the lease payments are low, it is not profitable to lease the products for large number of lease periods. Also, sensitivity to the changes in the lease payments is high for the large values of η. This is because the revenues are affected more by the changes as the number of lease periods increase.

Table 5.10. Profit rates for η=1, η=5 and η=10 as p changes

Case 1: y_i=(0.8, 0.7, 0.7,0.6) , β = 1

		Profit rate, π_3				
Ex.	***p***	**η=2**	**η=5**	**η=10**	π_1	π_2
1	0.55	6.5893	4.3500	-1.5661	4.9753	4.7360
2	0.60	7.0115	5.2222	-0.0729	4.9753	4.8935
3	0.65	7.3386	6.0928	1.4227	4.9753	5.0509
4	0.70	7.7870	6.9661	2.9158	4.9753	5.2084
5	0.75	8.1654	7.8371	4.4109	4.9753	5.3658
6	0.80	8.5446	8.7085	5.9037	4.9753	5.5233
7	0.85	8.9232	9.5791	7.3996	4.9753	5.6807
8	0.90	9.3024	10.4524	8.8926	4.9753	5.8382
9	0.95	9.6808	11.3234	10.3854	4.9753	5.9957
10	1.00	10.0600	12.2556	11.6809	4.9753	6.1531

Case 2: y_i=(0.9, 0.8, 0.8,0.7) , β = 0.5

		Profit rate, π_3				
Ex.	***p***	**η=2**	**η=5**	**η=10**	π_1	π_2
1	0.55	6.5943	4.6648	0.2809	4.9753	4.6937
2	0.60	6.9896	5.4918	1.5161	4.9753	4.9015
3	0.65	7.4383	6.3170	2.7542	4.9753	5.1093
4	0.70	7.7802	7.1448	3.9894	4.9753	5.3172
5	0.75	8.1746	7.9707	5.2271	4.9753	5.5250
6	0.80	8.5699	8.7967	6.4612	4.9753	5.7329
7	0.85	8.9645	9.6219	7.7002	4.9753	5.9407
8	0.90	9.3598	10.4697	8.9341	4.9753	6.1485
9	0.95	9.7542	11.3457	10.4677	4.9753	6.3564
10	1.00	10.1495	12.3626	11.8884	4.9753	6.5642

Case 3: y_i=(0.8, 0.7, 0.7,0.6) , β = 0.25

		Profit rate, π_3				
Ex.	***p***	**η=2**	**η=5**	**η=10**	π_1	π_2
1	0.55	6.5973	4.9510	1.5134	4.9753	4.6590
2	0.60	7.0040	5.7524	2.6006	4.9753	4.9179
3	0.65	7.4785	6.5518	3.6907	4.9753	5.1768
4	0.70	7.8173	7.3538	4.7781	4.9753	5.4356
5	0.75	8.2230	8.1541	5.8679	4.9753	5.6945
6	0.80	8.6297	8.9544	6.9538	4.9753	5.9534
7	0.85	9.0358	9.7539	8.0451	4.9753	6.2123
8	0.90	9.4424	10.5559	9.1291	4.9753	6.4711
9	0.95	9.8481	11.3562	10.5129	4.9753	6.7300
10	1.00	10.2548	12.4675	11.9897	4.9753	6.9889

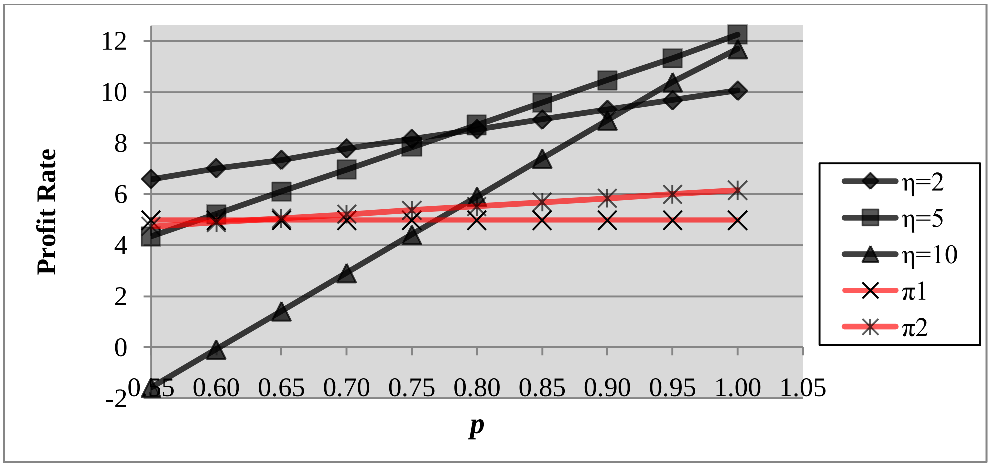

Figure 5.16. Comparison of profit rates for Case 1 as *p* changes

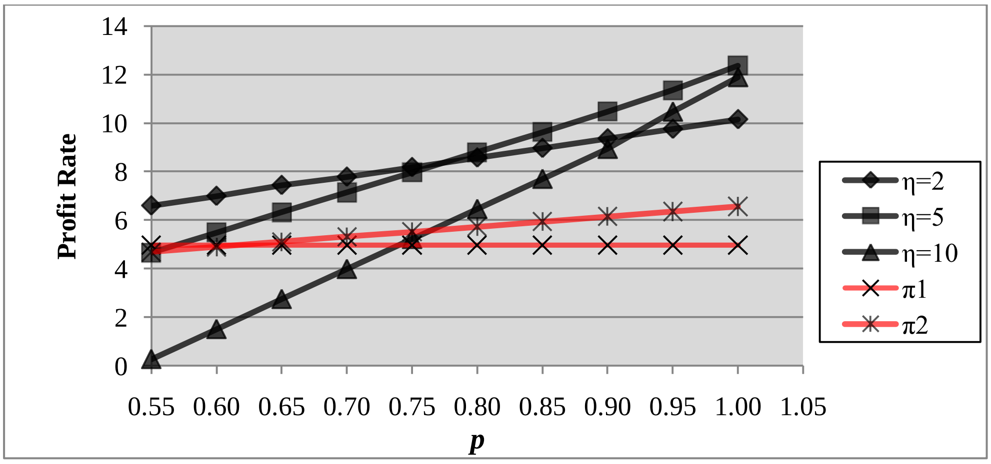

Figure 5.17. Comparison of profit rates for Case 2 as *p* changes

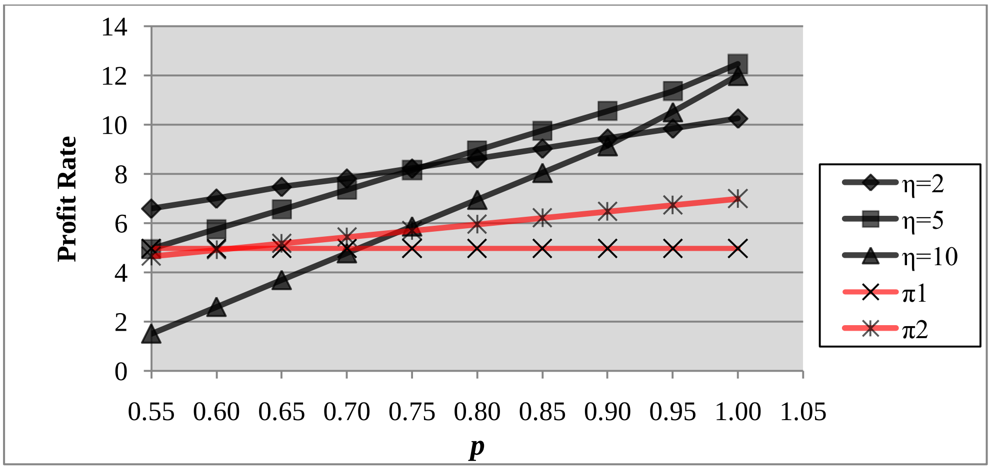

Figure 5.18. Comparison of profit rates for Case 3 as *p* changes

5.2. Market Cannibalization Effect

In Section 5.1, for the comparison of models we assumed that equal number of customers is served in each model. In the SRM, we provide this by decreasing the arrival rate of demands for new products as the proportion of the customers that buy remanufactured products increases. This implies that we allow cannibalization of new product sales. In this section, we present a more detailed discussion of the market cannibalization of new products sales by remanufactured products. We investigate whether cannibalization actually decreases the overall profitability of the firms. The cannibalization effect is considered under two situations. One of them is that the market share of the firm remains constant when cannibalization occurs. In the second situation, we assess the effect of cannibalization when the market share of the firm increases as a result of the sales of remanufactured products.

Let M_0 denote the total number of sales of a firm which sells both the new and remanufactured products. Also, let $M_{N,0}$ and $M_{R,0}$ denote the number of sales of new products and remanufactured products, respectively. Then, M_0 can be written as:

$$M_0 = M_{N,0} + M_{R,0} \tag{5.3}$$

Initially, we assume $M_{R,0}=0$ and the firm sells only new products. Let α denote the percent of increase in the total number of sales when remanufactured products are introduced. Then, the total number of sales of a firm after the remanufactured products are introduced can be written as;

$$M_1 = M_{N,0}(1+\alpha) \tag{5.4}$$

where $\alpha \geq 0$. Let $M_{N,1}$ and $M_{R,1}$ denote the number of sales of new products and remanufactured products after the remanufactured products are introduced, respectively. M_1 can be written as;

$$M_1 = M_{N,1} + M_{R,1} \tag{5.5}$$

where $M_{N,1} > 0$ and $M_{R,1} > 0$. Introducing remanufactured products cannibalizes the sales of new products if $M_{N,1} < M_{N,0}$. Namely, the number of sales of new products decreases when the remanufactured products are introduced. The possible situations for the sales of new products are presented in Table 5.11.

Table 5.11. Cannibalization of the sales of new products

$M_{N,1} - M_{N,0} > 0$	Increase in market size
$M_{N,1} - M_{N,0} = 0$	No cannibalization
$M_{N,1} - M_{N,0} < 0$	Cannibalization

We denote the proportion of the sales of remanufactured products in the market by β_1 and it can be written as:

$$\beta_1 = \frac{M_{R,1}}{M_1} \tag{5.6}$$

Then, the number of sales of new products after the remanufactured products are introduced can be obtained as;

$$M_{N,1} = M_1(1-\beta_1) = M_{N,0}(1+\alpha)(1-\beta_1) \tag{5.7}$$

We denote the ratio between $M_{N,1} - M_{N,0}$ and $M_{N,0}$ as γ and it can be calculated by;

$$\gamma = (1+\alpha)(1-\beta_1) - 1 \tag{5.8}$$

Here, if $\gamma = 0$ and $\gamma > 0$, there is no cannibalization . On the other hand, if $\gamma < 0$, cannibalization occurs. The extent of cannibalization increases as γ decreases. In order to decrease γ, we increase β_1.

We construct our experiments for different values of *p* and c_r. In Table 5.12, profit rates are presented for *p=0.6, p=0.7* and *p=0.8* as the extent of cannibalization increases when *α=0*. The profit rates for *α=0.3* are computed similarly and presented in Table 5.13. Also, the results are depicted in the figures below.

As a result of these experiments, we conclude that when core cost is high and price of the remanufactured products is low, cannibalization affects profitability negatively for both *α=0* and *α>0*. It can be observed from Figure 5.19 and 5.22 that as the extent of cannibalization increases, the profit rates decreases for *p=0.6, p=0.7* and the model with no cannibalization performs better in comparison to models where cannibalization occurs with ratio γ. This is because selling a new product is more profitable than selling a remanufactured product when c_r is large and therefore as the sales of remanufactured products cannibalize the sales of new products, profitability decreases remarkably.

However, the effect of cannibalization differs when core cost is low. Allowing cannibalization causes a significant increase in profit rates for both *α=0* and *α>0* in this situation. It can be observed from Figure 5.21 and 5.24 that as the extent of cannibalization increases, profitability increasess for all price adjustments. Also, models with cannibalization perform better than the model with no cannibalization for all values of γ. This is because the unit margin for a remanufactured product increases with low core cost and high price. Thus, increasing the sales of remanufactured products in the market provides better profitability since selling a remanufactured product is more profitable in comparison to selling a new product.

In summary, the issue whether cannibalization decreases the overall profitability of the firms depends on the core cost and price of a remanufactured product. If the core cost is low enough and price of the remanufactured products is high enough, selling remanufactured products can be profitable although the sales of remanufactured products cannibalize the sales of new products. The results mentioned above are applicable for both *α=0* and *α>0*.

Table 5.12. Profit rates for $\alpha=0$ as the extent of cannibalization increases

$c_r = 0.4$

		Profit rate, π_2			
Ex.	β_1	$p = 0.6$	$p = 0.7$	$p = 0.8$	π_1
1	0.34	4.1040	4.3555	4.6070	4.4702
2	0.38	4.0548	4.3291	4.6033	4.4702
3	0.42	4.0060	4.3017	4.5975	4.4702
4	0.46	3.9560	4.2719	4.5879	4.4702
5	0.50	3.9060	4.2411	4.5761	4.4702
6	0.54	3.8561	4.2092	4.5624	4.4702
7	0.58	3.8055	4.1757	4.5460	4.4702
8	0.62	3.7547	4.1412	4.5277	4.4702
9	0.66	3.7036	4.1055	4.5074	4.4702
10	0.70	3.6519	4.0684	4.4850	4.4702

$c_r = 0.25$

		Profit rate, π_2			
Ex.	β_1	$p = 0.6$	$p = 0.7$	$p = 0.8$	π_1
1	0.34	4.3440	4.5955	4.8470	4.4702
2	0.38	4.3248	4.5990	4.8733	4.4702
3	0.42	4.3060	4.6017	4.8975	4.4702
4	0.46	4.2859	4.6019	4.9179	4.4702
5	0.50	4.2660	4.6010	4.9361	4.4702
6	0.54	4.2461	4.5992	4.9524	4.4702
7	0.58	4.2255	4.5957	4.9660	4.4702
8	0.62	4.2047	4.5912	4.9777	4.4702
9	0.66	4.1836	4.5855	4.9874	4.4702
10	0.70	4.1618	4.5784	4.9949	4.4702

$c_r = 0.1$

		Profit rate, π_2			
Ex.	β_1	$p = 0.6$	$p = 0.7$	$p = 0.8$	π_1
1	0.34	4.5840	4.8355	5.0870	4.4702
2	0.38	4.5947	4.8690	5.1433	4.4702
3	0.42	4.6060	4.9017	5.1975	4.4702
4	0.46	4.6159	4.9319	5.2478	4.4702
5	0.50	4.6259	4.9610	5.2960	4.4702
6	0.54	4.6360	4.9892	5.3423	4.4702
7	0.58	4.6455	5.0157	5.3860	4.4702
8	0.62	4.6547	5.0412	5.4277	4.4702
9	0.66	4.6636	5.0655	5.4675	4.4702
10	0.70	4.6718	5.0884	5.5049	4.4702

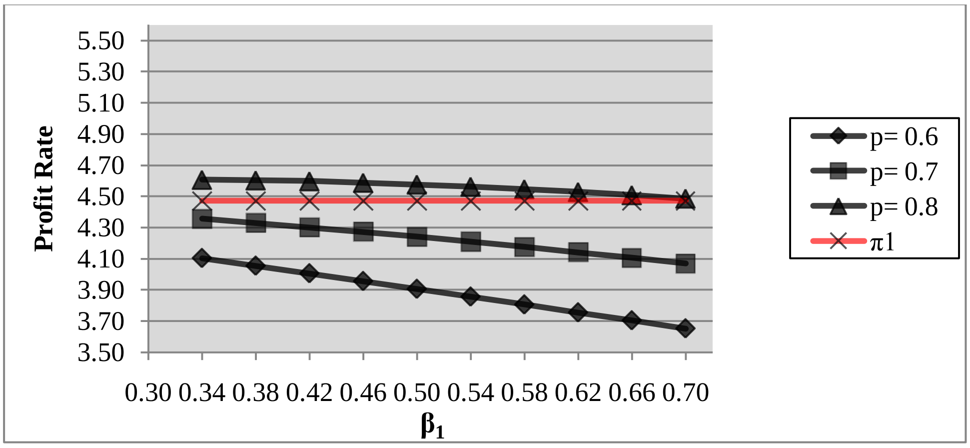

Figure 5.19. Comparison of profit rates for $\alpha=0$ and $c_r=0.4$ as β_1 changes

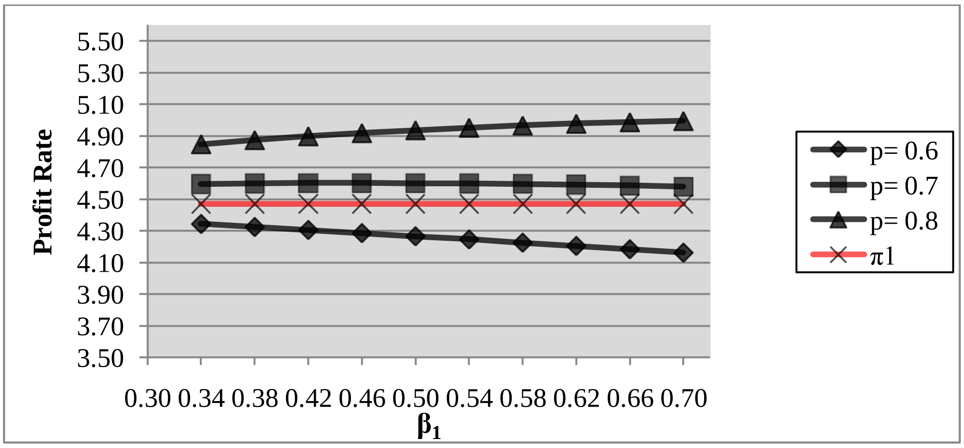

Figure 5.20. Comparison of profit rates for $\alpha=0$ and $c_r=0.25$ as β_1 changes

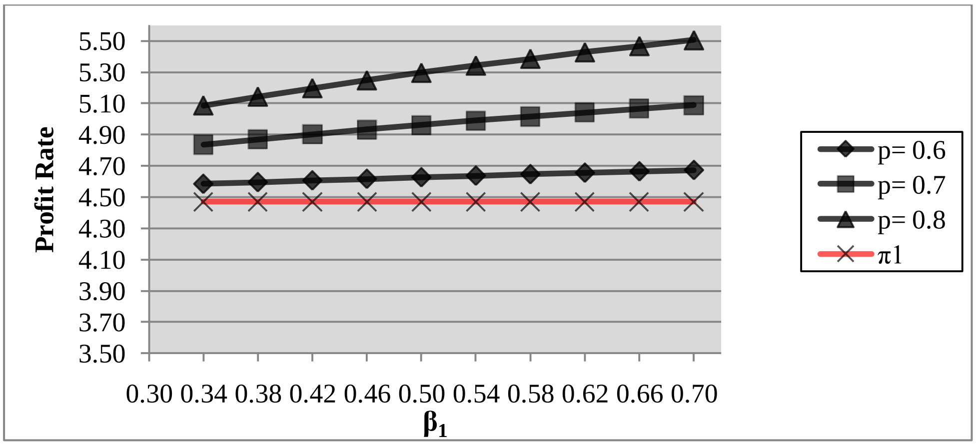

Figure 5.21. Comparison of profit rates for $\alpha=0$ and $c_r=0.1$ as β_1 changes

Table 5.13. Profit rates for *α=0.30* as the extent of cannibalization increases

$c_r = 0.4$

Ex.			Profit rate, π_2 $p = 0.6$	$p = 0.7$	$p = 0.8$
1	π_1	$\alpha = 0.3$	5.8100	5.8100	5.8100
2	β_1	0.34	5.3347	5.6616	5.9886
3		0.38	5.2717	5.6283	5.9849
4		0.42	5.2078	5.5923	5.9768
5		0.46	5.1433	5.5542	5.9650
6		0.50	5.0786	5.5142	5.9499
7		0.54	5.0125	5.4716	5.9306
8		0.58	4.9469	5.4282	5.9096
9		0.62	4.8807	5.3831	5.8855
10		0.66	4.8143	5.3368	5.8593

$c_r = 0.25$

Ex.			Profit rate, π_2 $p = 0.6$	$p = 0.7$	$p = 0.8$
1	π_1	$\alpha = 0.3$	5.8100	5.8100	5.8100
2	β_1	0.34	5.6467	5.9736	6.3005
3		0.38	5.6227	5.9793	6.3359
4		0.42	5.5978	5.9823	6.3668
5		0.46	5.5723	5.9832	6.3940
6		0.50	5.5466	5.9823	6.4179
7		0.54	5.5194	5.9785	6.4376
8		0.58	5.4929	5.9742	6.4556
9		0.62	5.4656	5.9681	6.4705
10		0.66	5.4383	5.9608	6.4833

$c_r = 0.1$

Ex.			Profit rate, π_2 $p = 0.6$	$p = 0.7$	$p = 0.8$
1	π_1	$\alpha = 0.3$	5.8100	5.8100	5.8100
2	β_1	0.34	5.9587	6.2856	6.6125
3		0.38	5.9737	6.3302	6.6869
4		0.42	5.9878	6.3723	6.7568
5		0.46	6.0013	6.4122	6.8230
6		0.50	6.0146	6.4503	6.8859
7		0.54	6.0264	6.4855	6.9445
8		0.58	6.0389	6.5202	7.0015
9		0.62	6.0506	6.5530	7.0554
10		0.66	6.0623	6.5848	7.1073

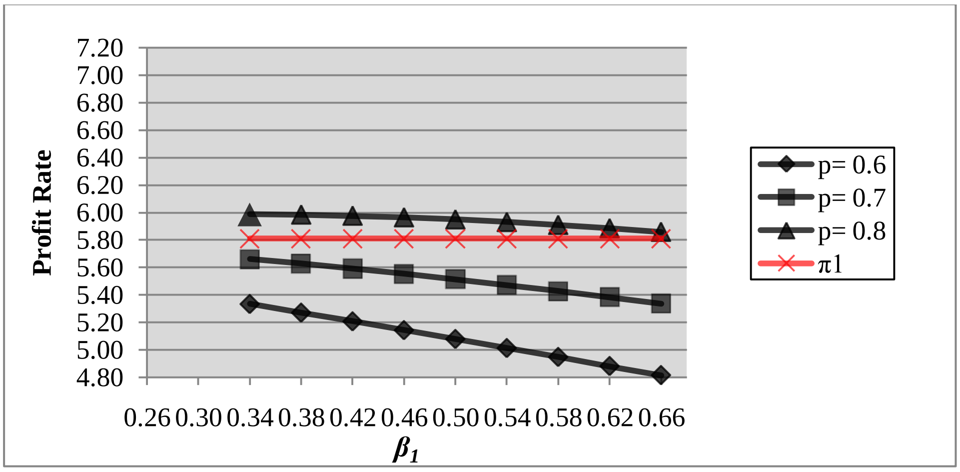

Figure 5.22. Comparison of profit rates for $\alpha=0.30$ and $c_r=0.4$ as β_1 changes

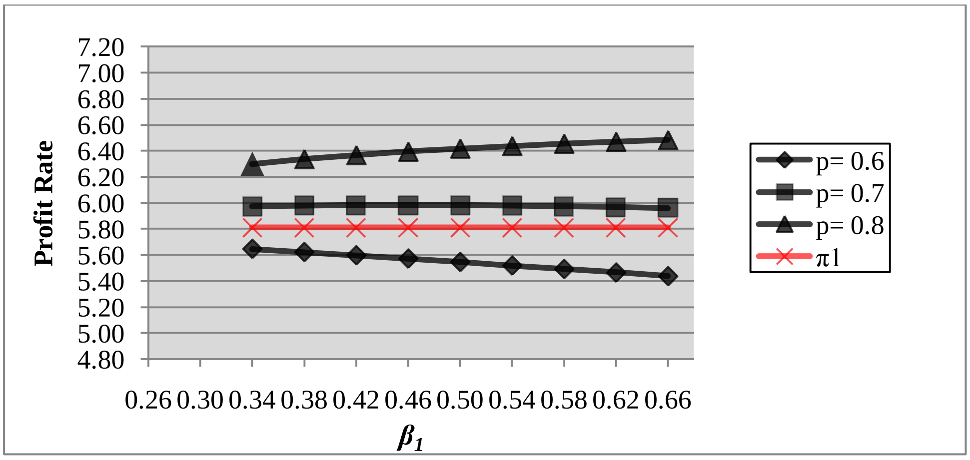

Figure 5.23. Comparison of profit rates for $\alpha=0.30$ and $c_r=0.25$ as β_1 changes

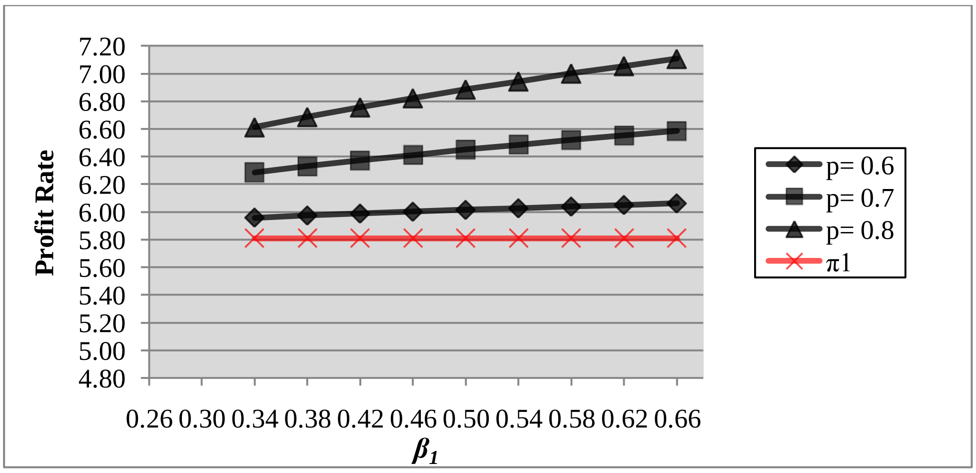

Figure 5.24. Comparison of profit rates for $\alpha=0.30$ and $c_r=0.1$ as β_1 changes

5.3. Design for Remanufacture

This section focuses on the design issue which plays an important role in improving the efficiency of remanufacturing. Design for remanufacture can reduce the constraining issues and sustain the facilitating properties for remanufacturing [25]. This is because remanufacturability of a product is influenced by the physical characteristics specified during the design phase. To increase the remanufacturability of a product, specific stages of the remanufacturing process can be considered depending on the product. Here, we investigate whether the profitability is increased for the remanufacturing models when the remanufacturability of a product is improved by proper design. To this end, we consider the effects of design for remanufacture on two factors that affect the profitability of the remanufacturing system; the maximum number of times that a product can be remanufactured, n and average remanufacturing time.

5.3.1. Design Effect on Multiple Remanufacturing Cycles

Here, we concentrate on the design for multiple remanufacturing cycles and define the remanufacturability level as the maximum number of times that a product can be remanufactured, denoted by n. This is because the ANOVA analyses showed that increasing n results in better profitability for both the SRM and the LM. However, in these analyses we assumed that production and design time of the product is constant as n increases. In reality, achieving both higher remanufacturability and durability requires more time in the design phase. Considering our model, mean service time of production and design station, τ_1 increases for a higher remanufacturability level. For this purpose, we increase n as τ_1 increases for two cases. Case 1 and Case 2 refer to logarithmic and linear increase in τ_1, respectively. The experimental values of τ_1 and n are presented in Table 5.14 and the linear and logarithmic increase patterns can be seen in Figure 5.25.

Table 5.14. Experimental values for τ_1 and n

	Case 1		Case 2	
Ex	τ_1	n	τ_1	n
1	1.30	0	1.3	0
2	1.70	1	2.04	1
3	2.23	2	2.78	2
4	2.92	3	3.52	3
5	3.82	4	4.26	4
6	5.00	5	5	5

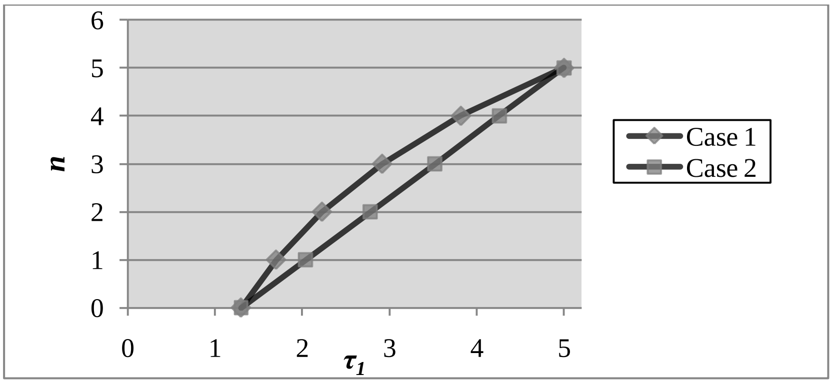

Figure 5.25. Logarithmic and linear increase in n

We construct our experiments for different values of p and c_r. Also, in the experiments, we consider the relation between design and remanufacturability for two situations. Firstly, we assume the market size remains constant when the remanufactured products are introduced. Namely, the sales of new products are cannibalized. The profit rates for this situation are presented in Table 5.15. The results are depicted in Figure 5.26, Figure 5.27 and Figure 5.28. Secondly, we assume that the market size expands as the sales of remanufactured products increase as a result of the large values of n. The profit rates for this situation are presented in Table 5.16. The results are depicted in Figure 5.29, Figure 5.30 and Figure 5.31.

For constant market size, it is observed from the numerical results reported in the tables and graphics that increasing n decreases the profit rates for low price and high core cost. Namely, the profitability of products with $n=0$, is higher than the profitability of products with *n>0*. This is because increasing the design time causes the unit profit margin of a remanufacturable product to decrease. However, for high price and low core cost , higher *n* values causes a higher increase in the revenues from remanufactured products than the increase in the processing costs due to the larger design times. For instance, when $p=0.8$ and $c_r=0.1$, we obtain the highest profit rate for $n=2$. This situation can be seen from Figure 5.28. Also, these results are similar for both Case 1 and Case 2. The only difference is that linear increase results in lower profits than the logarithmic increase.

On the other hand, if the market share expands due to increase in the number of sales of the remanufactured products, increasing *n* has a positive effect on profitability. As we increase *n* up to a specific value where the increase in the processing costs due to the larger design times is smaller than the increase in the revenues from remanufactured products, we obtain higher profitability. This critical value changes for different values of *p*, e.g. for $p=0.6$, $p=0.8$ and $p=1$, the critical value of *n* is equal to 3, 3 and 4 respectively. This result is applicable for all adjustments of *p* and c_r. Also, the results are similar for both Case 1 and Case 2. However, logarithmic increase provides higher profit rates.

Table 5.15. Profit rates as τ_1 and n changes when market size remains constant

$c_r = 0.4$		Profit rate, π_2					
		Case 1			Case 2		
Ex.	***n***	$p = 0.6$	$p = 0.7$	$p = 0.8$	$p = 0.6$	$p = 0.7$	$p = 0.8$
1	0	5.3230	5.3230	5.3230	5.3230	5.3230	5.3230
2	1	4.2158	4.5861	4.9563	4.1085	4.4788	4.8490
3	2	4.0283	4.5072	4.9862	3.8846	4.3636	4.8426
4	3	3.8520	4.3697	4.8874	3.7070	4.2247	4.7423
5	4	3.6446	4.1786	4.7125	3.5418	4.0757	4.6097
6	5	3.3782	3.9195	4.4608	3.3782	3.9195	4.4608

$c_r = 0.25$		Profit rate, π_2					
		Case 1			Case 2		
Ex.	***n***	$p = 0.6$	$p = 0.7$	$p = 0.8$	$p = 0.6$	$p = 0.7$	$p = 0.8$
1	0	5.3230	5.3230	5.3230	5.3230	5.3230	5.3230
2	1	4.6358	5.0060	5.3763	4.5285	4.8987	5.2690
3	2	4.4483	4.9272	5.4062	4.3046	4.7836	5.2626
4	3	4.2719	4.7896	5.3073	4.1269	4.6446	5.1623
5	4	4.0647	4.5986	5.1326	3.9618	4.4958	5.0298
6	5	3.7984	4.3397	4.8810	3.7984	4.3397	4.8810

$c_r = 0.1$		Profit rate, π_2					
		Case 1			Case 2		
Ex.	***n***	$p = 0.6$	$p = 0.7$	$p = 0.8$	$p = 0.6$	$p = 0.7$	$p = 0.8$
1	0	5.3230	5.3230	5.3230	5.3230	5.3230	5.3230
2	1	5.0558	5.4260	5.7963	4.9485	5.3187	5.6890
3	2	4.8682	5.3472	5.8262	4.7246	5.2036	5.6826
4	3	4.6919	5.2096	5.7272	4.5469	5.0646	5.5822
5	4	4.4847	5.0187	5.5527	4.3819	4.9159	5.4499
6	5	4.2186	4.7599	5.3012	4.2186	4.7599	5.3012

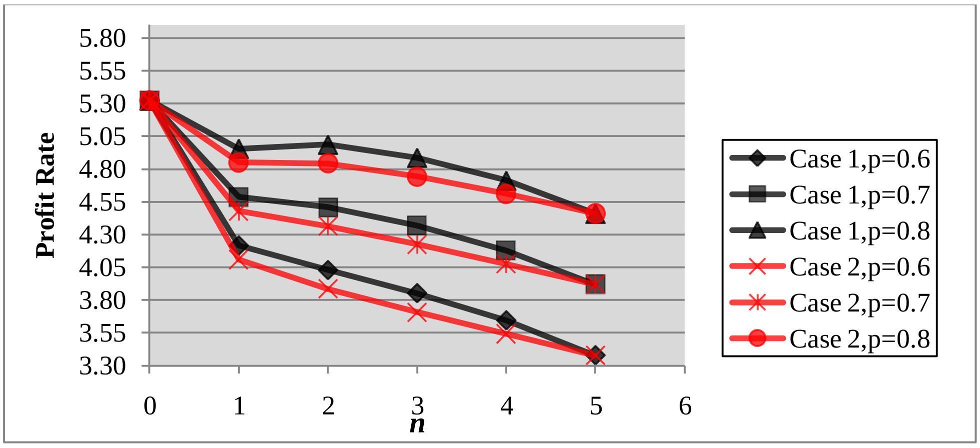

Figure 5.26. Comparison of profits rates for constant market size and c_r=*0.4*

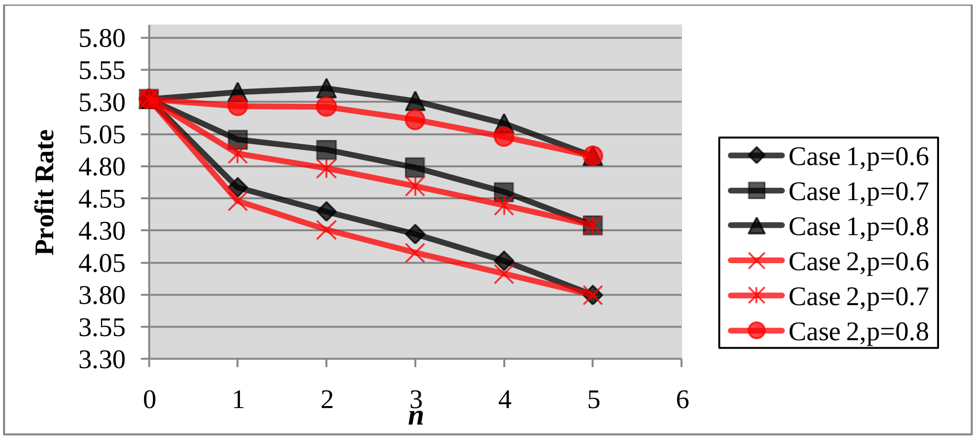

Figure 5.27. Comparison of profits rates for constant market size and c_r=*0.25*

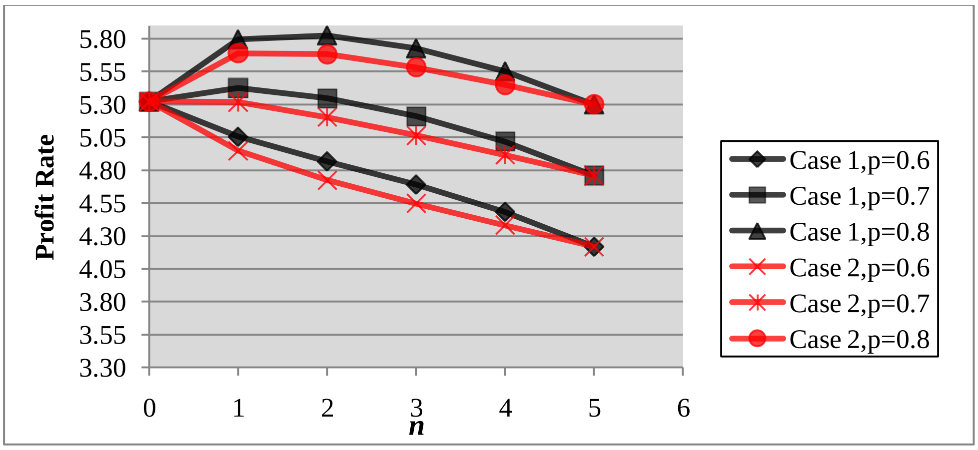

Figure 5.28. Comparison of profits rates for constant market size and c_r=*0.1*

Table 5.16. Profit rates as τ_1 and n changes when market size increases

$c_r = 0.4$		**Profit rate, π_2**					
		Case 1			**Case 2**		
Ex.	***n***	***p* = 0.6**	***p* = 0.7**	***p* = 0.8**	***p* = 0.6**	***p* = 0.7**	***p* = 0.8**
1	0	5.3230	5.3230	5.3230	5.3230	5.3230	5.3230
2	1	6.6948	7.2828	7.8708	6.5243	7.1123	7.7003
3	2	7.7314	8.6507	9.5701	7.4556	8.3749	9.2942
4	3	7.9876	9.0612	10.1348	7.6866	8.7602	9.8338
5	4	7.8164	8.9618	10.1071	7.5955	8.7408	9.8862
6	5	7.3541	8.5329	9.7117	7.3541	8.5329	9.7117

$c_r = 0.25$		**Profit rate, π_2**					
		Case 1			**Case 2**		
Ex.	***n***	***p* = 0.6**	***p* = 0.7**	***p* = 0.8**	***p* = 0.6**	***p* = 0.7**	***p* = 0.8**
1	0	5.3230	5.3230	5.3230	5.3230	5.3230	5.3230
2	1	7.3617	7.9497	8.5377	7.1913	7.7793	8.3673
3	2	8.5375	9.4568	10.3762	8.2617	9.1810	10.1004
4	3	8.8585	9.9321	11.0057	8.5575	9.6311	10.7047
5	4	8.7175	9.8628	11.0082	8.4965	9.6419	10.7873
6	5	8.2692	9.4480	10.6268	8.2692	9.4480	10.6268

$c_r = 0.1$		**Profit rate, π_2**					
		Case 1			**Case 2**		
Ex.	***n***	***p* = 0.6**	***p* = 0.7**	***p* = 0.8**	***p* = 0.6**	***p* = 0.7**	***p* = 0.8**
1	0	5.3230	5.3230	5.3230	5.3230	5.3230	5.3230
2	1	8.0287	8.6167	9.2047	7.8583	8.4463	9.0343
3	2	9.3436	10.2630	11.1823	9.0678	9.9871	10.9065
4	3	9.7294	10.8030	11.8766	9.4284	10.5020	11.5756
5	4	9.6185	10.7639	11.9093	9.3975	10.5430	11.6883
6	5	9.1843	10.3630	11.5419	9.1843	10.3630	11.5419

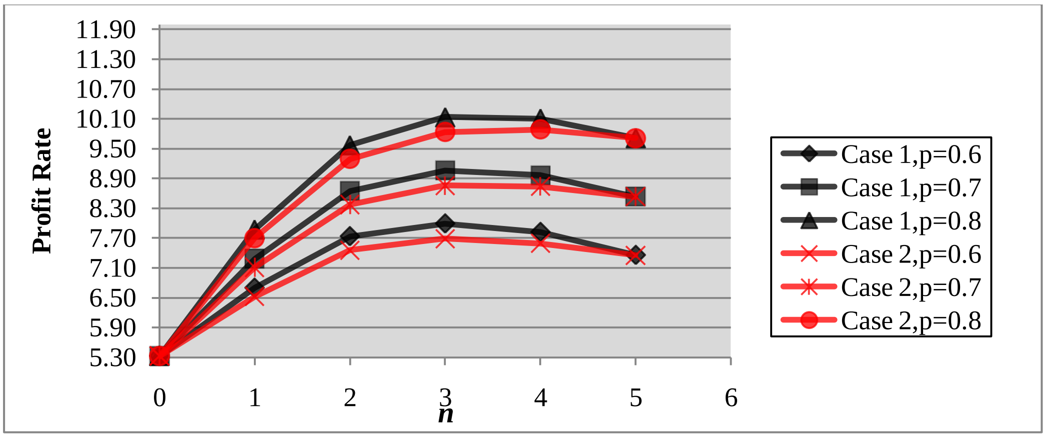

Figure 5.29. Comparison of profits rates for increased market size and $c_r=0.4$

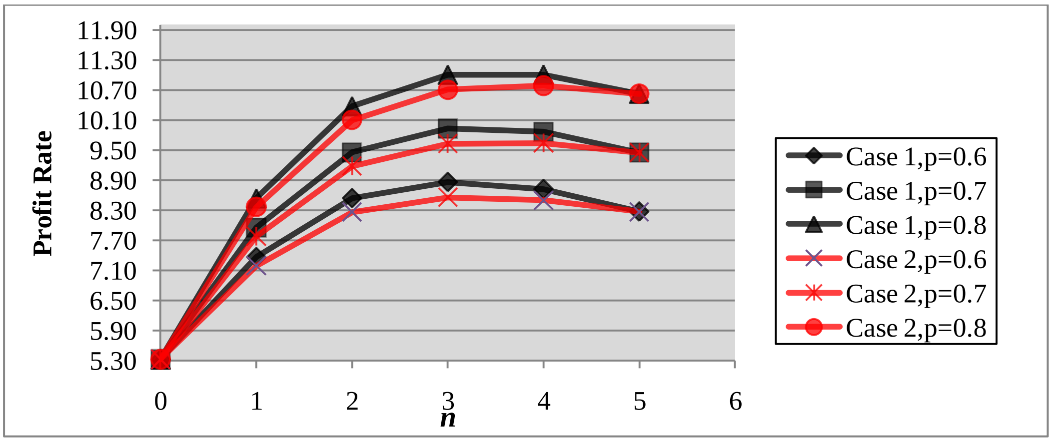

Figure 5.30. Comparison of profits rates for increased market size and $c_r=0.25$

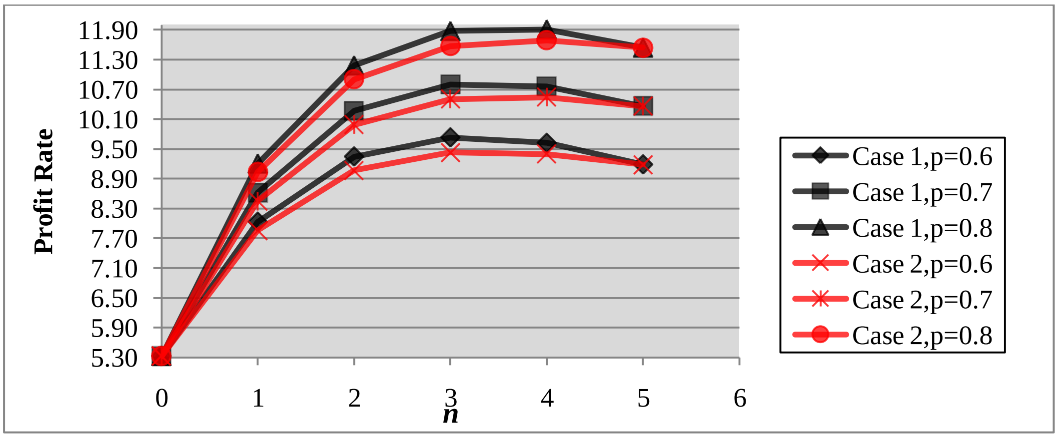

Figure 5.31. Comparison of profits rates for increased market size and $c_r=0.1$

5.3.2. Design Effect on the Average Remanufacturing Time

In the ANOVA analyses, we observed that the ratio between the average remanufacturing time and the production and design time, *k* is a significant factor for the profitability and as *k* increases, the profitability of the SRM and the LM decreases. However, in those experiments, we varied *k* by decreasing the average remanufacturing time while the production and design time was held constant. In this section, we study the effects of proper design on profitability of the SRM and the LM when the average remanufacturing time decreases as a result of the increase in the production and design time. Namely, we concentrate on the ease of remanufacturing achieved by proper design. We investigate whether proper design for remanufacture increases the profits. To this end, we decrease average remanufacturing time, τ_R logarithmically and linearly as the production and design time, τ_1 increases. The experimental values for τ_1 and τ_R are given in Table 5.17. Here, Case 1 and Case 2 represent logarithmic and linear decrease in τ_R, respectively. Also, the logarithmic and linear decrease patterns can be seen in Figure 5.32.

Table 5.17. Experimental values for τ_1 and τ_R

	Case 1		Case 2	
Ex	τ_1	τ_R	τ_1	τ_R
1	1.70	1.7	1.70	1.7
2	1.80	1.4	1.80	1.59
3	1.90	1.2	1.90	1.48
4	2.00	1.05	2.00	1.37
5	2.10	0.95	2.10	1.26
6	2.20	0.86	2.20	1.15
7	2.30	0.79	2.30	1.04
8	2.40	0.74	2.40	0.93
9	2.50	0.71	2.50	0.82
10	2.60	0.7	2.60	0.7

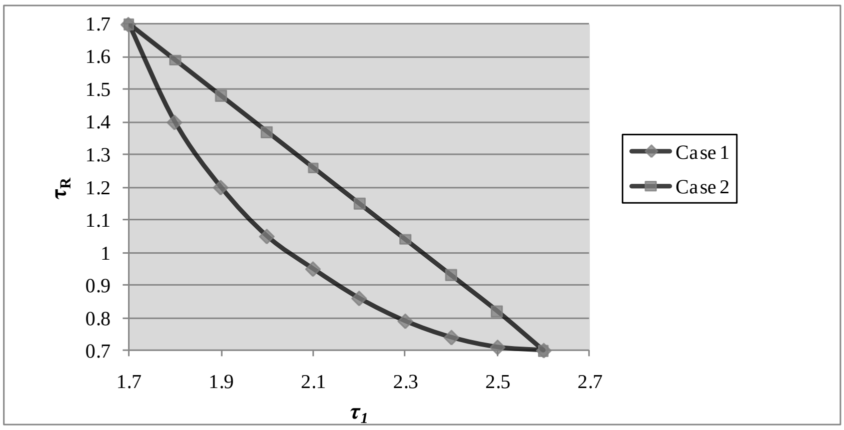

Figure 5.32. Logarithmic and linear increase in *n*

We construct our experiments for the SRM and the LM in order to observe the effects of proper design for both models. The results are computed for different *p* values to observe the changes in profitability for different price of a remanufactured product and lease payments. The levels for *p* are selected to be *p=0.6*, *p=0.8* and *p=1*. Also, we compute the results for different *n* values for the SRM, e.g. *n=1*, *n=2* and *n=3* and η for the LM, e.g. $\eta=2$, $\eta=5$ and $\eta=8$. In this way, we observe the changes in profitability for different throughput rates in the remanufacturing cycle as τ_R decreases. The profit rates for the SRM are presented in Table 5.18 and the results are depicted in Figure 5.33, Figure 5.34 and Figure 5.35. The profit rates for the LM are presented in Table 5.19 and the results are depicted in Figure 5.36, Figure 5.37 and Figure 5.38.

As a result of these experiments, we observe that for the logarithmic decrease in τ_R, the profitability of the SRM and the LM increases as the design and production time increases up to a certain value. When the design and production time is equal to this critical value, profit rate of the SRM and the LM is maximized. This critical value changes for different values of *n* for the SRM and η for the LM. For our cost structure, when *n=1*, *n=2* and *n=3*, the critical value of τ_1 is equal to 1.90, 2.00 and 2.20 respectively for the SRM and when $\eta=2$, $\eta=5$ and $\eta=8$, the critical value of τ_1 is equal to 2.00, 2.40 and 2.40 respectively for the LM. These results can be seen for all *p* values. However, after this value profit rates decline as the design time increases. This is because decrease in the costs as a result of the shorter remanufacturing times becomes smaller than the increase in the design and production costs after a certain value of τ_1 . Therefore, the total costs increases and thus the profit rates of the SRM and the LM decrease.

On the other hand, for the linear decrease in τ_1, we observe that the profit rates of the SRM and the LM linearly increase as the production and design time increases when *n* and η is large, e.g. $n=2$ and $n=3$, $\eta=5$ and $\eta=8$. This is because the decrease in the remanufacturing costs becomes larger than the increase in the design and production costs as the throughput rates in the remanufacturing cycle increase. However, the profit rates of the SRM and the LM linearly decrease if the throughput rates are low, e.g. $n=1$ and $\eta=2$. These results can be seen for all *p* values.

Also, when we compare the profit rates of Case 1 and Case 2, we observe that if the average remanufacturing decreases logarithmically as the production and design time increases, higher profit rates are obtained for all configurations. This is because the average remanufacturing time takes smaller values in Case 1 than in Case 2 as can be seen in Figure 5.32.

Table 5.18. Profit rates of the SRM as τ_1 and τ_R changes

p=0.6	Profit rate, π_2					
	Case 1			Case 2		
Ex.	***n= 1***	***n= 2***	***n= 3***	***n= 1***	***n= 2***	***n= 3***
1	4.9595	4.8889	4.8620	4.9595	4.8889	4.8620
2	4.9869	4.9406	4.9225	4.9491	4.8910	4.8686
3	4.9944	4.9662	4.9546	4.9387	4.8931	4.8751
4	4.9919	4.9787	4.9725	4.9284	4.8951	4.8817
5	4.9796	4.9782	4.9762	4.9180	4.8972	4.8882
6	4.9653	4.9750	4.9771	4.9076	4.8993	4.8948
7	4.9469	4.9666	4.9723	4.8972	4.9013	4.9013
8	4.9246	4.9530	4.9618	4.8868	4.9034	4.9079
9	4.8983	4.9342	4.9456	4.8764	4.9055	4.9144
10	4.8680	4.9101	4.9238	4.8680	4.9101	4.9238

p=0.8	Profit rate, π_2					
	Case 1			Case 2		
Ex.	***n= 1***	***n= 2***	***n= 3***	***n= 1***	***n= 2***	***n= 3***
1	5.7000	5.8469	5.8975	5.7000	5.8469	5.8975
2	5.7274	5.8986	5.9580	5.6897	5.8489	5.9041
3	5.7350	5.9242	5.9901	5.6793	5.8510	5.9106
4	5.7325	5.9367	6.0080	5.6689	5.8531	5.9172
5	5.7202	5.9361	6.0117	5.6585	5.8552	5.9237
6	5.7058	5.9330	6.0126	5.6482	5.8572	5.9303
7	5.6875	5.9246	6.0077	5.6378	5.8593	5.9368
8	5.6651	5.9110	5.9973	5.6274	5.8614	5.9433
9	5.6388	5.8922	5.9811	5.6170	5.8634	5.9499
10	5.6085	5.8681	5.9592	5.6085	5.8681	5.9592

p=1.0	Profit rate, π_2					
	Case 1			Case 2		
Ex.	***n= 1***	***n= 2***	***n= 3***	***n= 1***	***n= 2***	***n= 3***
1	6.4406	6.8048	6.9330	6.4406	6.8048	6.9330
2	6.4680	6.8565	6.9935	6.4302	6.8069	6.9395
3	6.4755	6.8821	7.0256	6.4199	6.8090	6.9461
4	6.4731	6.8947	7.0435	6.4095	6.8111	6.9527
5	6.4607	6.8941	7.0472	6.3991	6.8132	6.9592
6	6.4464	6.8910	7.0480	6.3887	6.8152	6.9657
7	6.4280	6.8826	7.0432	6.3783	6.8173	6.9723
8	6.4057	6.8690	7.0327	6.3679	6.8193	6.9788
9	6.3794	6.8501	7.0166	6.3575	6.8214	6.9854
10	6.3491	6.8261	6.9947	6.3491	6.8261	6.9947

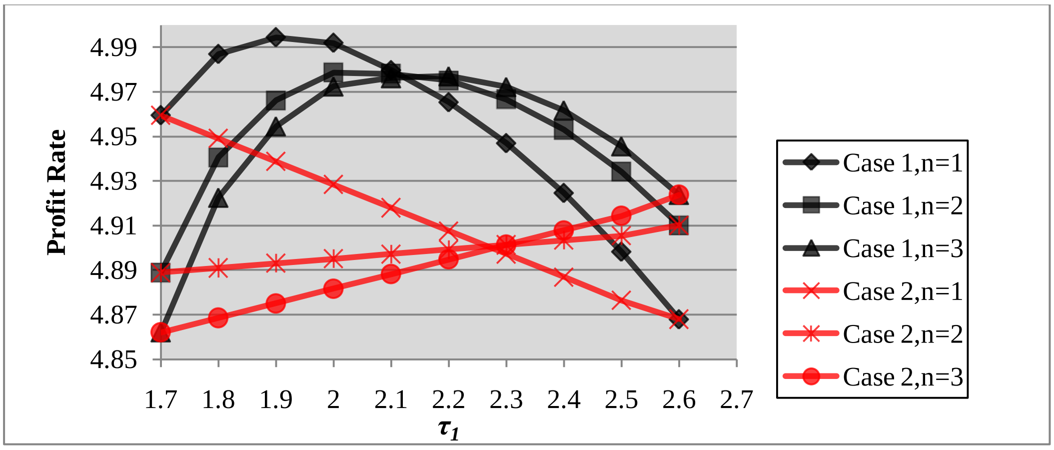

Figure 5.33. Comparison of profits rates of the SRM when *p=0.6*

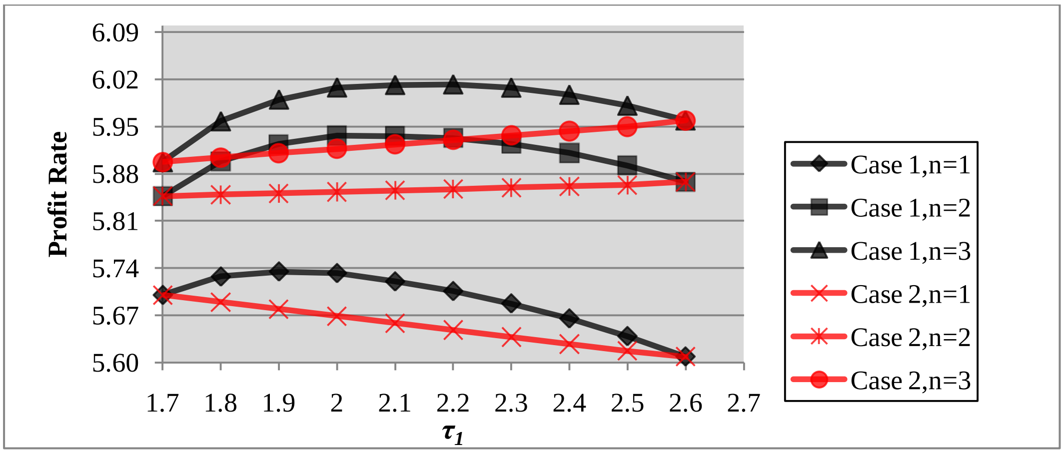

Figure 5.34. Comparison of profits rates of the SRM when *p=0.8*

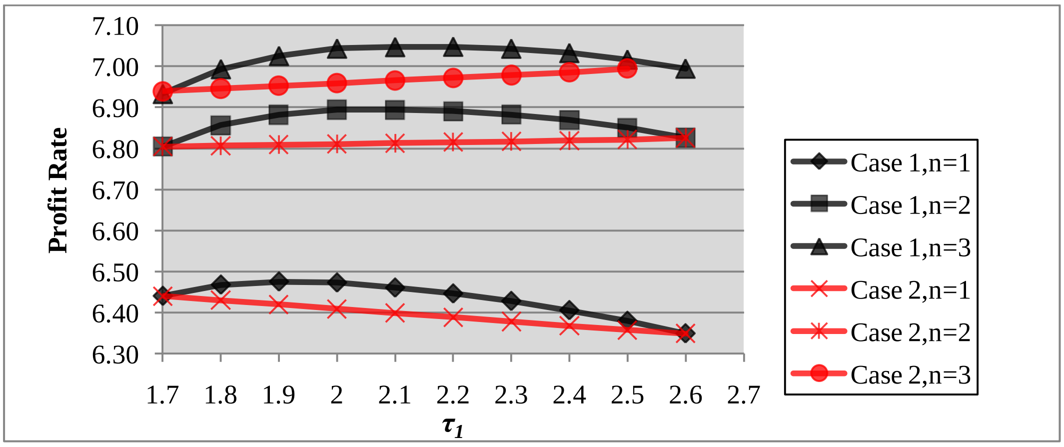

Figure 5.35. Comparison of profits rates of the SRM when *p=1.0*

Table 5.19. Profit rates of the LM as τ_1 and τ_R changes

p=0.6	**Profit rate, π_3**					
	Case 1			**Case 2**		
Ex.	**η=2**	**η=5**	**η=8**	**η=2**	**η=5**	**η=8**
1	6.9346	5.2163	3.9920	6.9346	5.2163	3.9920
2	7.0097	5.6118	4.6413	6.9284	5.3214	4.1816
3	7.0419	5.8541	5.0469	6.9221	5.4263	4.3705
4	7.0527	6.0200	5.3313	6.9158	5.5312	4.5591
5	7.0421	6.1094	5.4948	6.9095	5.6360	4.7473
6	7.0272	6.1834	5.6340	6.9031	5.7407	4.9351
7	7.0037	6.2269	5.7249	6.8967	5.8453	5.1227
8	6.9716	6.2398	5.7675	6.8903	5.9498	5.3100
9	6.9309	6.2221	5.7618	6.8838	6.0542	5.4970
10	6.8816	6.1738	5.7079	6.8816	6.1738	5.7079

p=0.8	**Profit rate, π_3**					
	Case 1			**Case 2**		
Ex.	**η=2**	**η=5**	**η=8**	**η=2**	**η=5**	**η=8**
1	8.5603	8.4184	8.1080	8.5603	8.4184	8.1080
2	8.6354	8.8138	8.7573	8.5541	8.5234	8.2975
3	8.6676	9.0562	9.1629	8.5478	8.6284	8.4865
4	8.6784	9.2220	9.4473	8.5415	8.7333	8.6751
5	8.6678	9.3114	9.6108	8.5352	8.8380	8.8632
6	8.6529	9.3855	9.7500	8.5288	8.9427	9.0511
7	8.6294	9.4289	9.8409	8.5224	9.0473	9.2386
8	8.5973	9.4418	9.8835	8.5160	9.1518	9.4260
9	8.5566	9.4241	9.8778	8.5095	9.2563	9.613
10	8.5073	9.3758	9.8239	8.5073	9.3758	9.8239

p=1.0	**Profit rate, π_3**					
	Case 1			**Case 2**		
Ex.	**η=2**	**η=5**	**η=8**	**η=2**	**η=5**	**η=8**
1	10.1854	11.6214	12.2274	10.1854	11.6214	12.2274
2	10.2605	12.0169	12.8767	10.1792	11.7265	12.4170
3	10.2927	12.2593	13.2823	10.1729	11.8315	12.6059
4	10.3036	12.4251	13.5667	10.1666	11.9363	12.7945
5	10.2929	12.5145	13.7302	10.1603	12.0411	12.9826
6	10.2780	12.5886	13.8694	10.1539	12.1458	13.1705
7	10.2545	12.6320	13.9603	10.1475	12.2504	13.3580
8	10.2224	12.6449	14.0029	10.1411	12.3549	13.5453
9	10.1817	12.6272	13.9972	10.1347	12.4593	13.7324
10	10.1324	12.5789	13.9433	10.1324	12.5789	13.9433

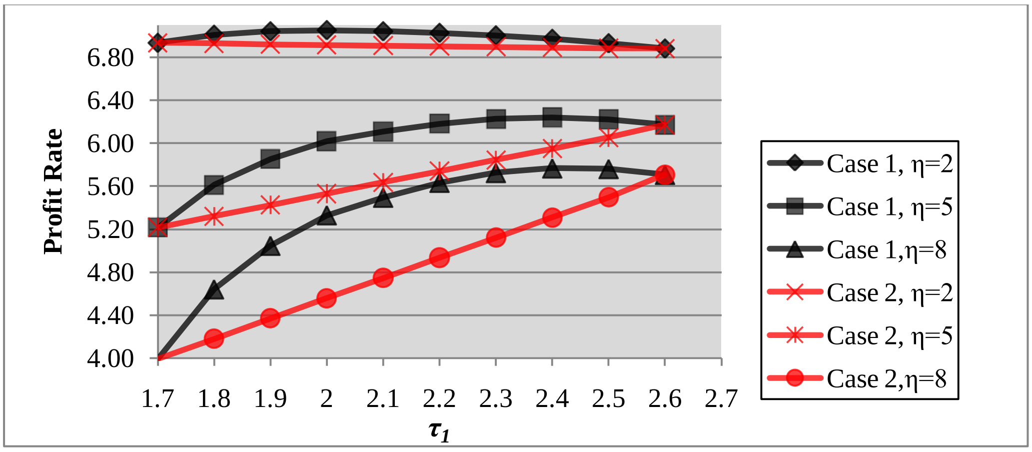

Figure 5.36. Comparison of profits rates of the LM when *p=0.6*

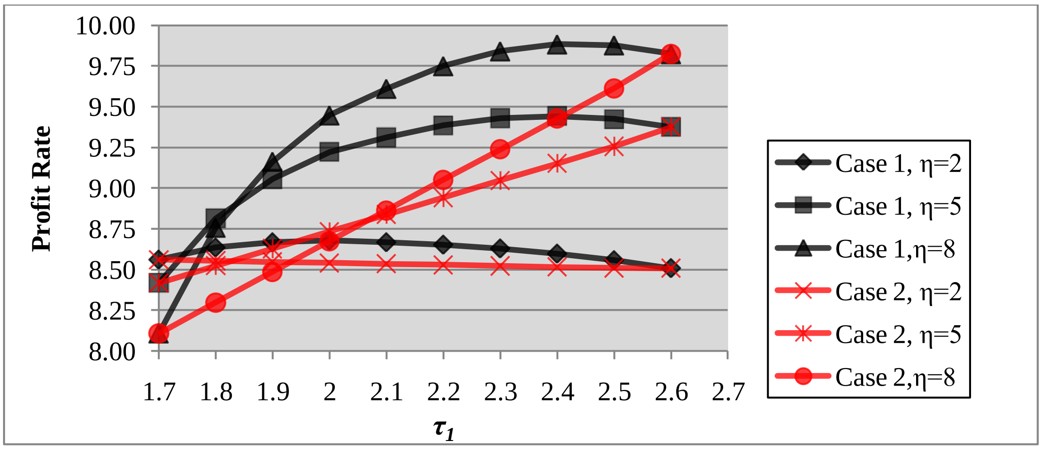

Figure 5.37. Comparison of profits rates of the LM when *p=0.8*

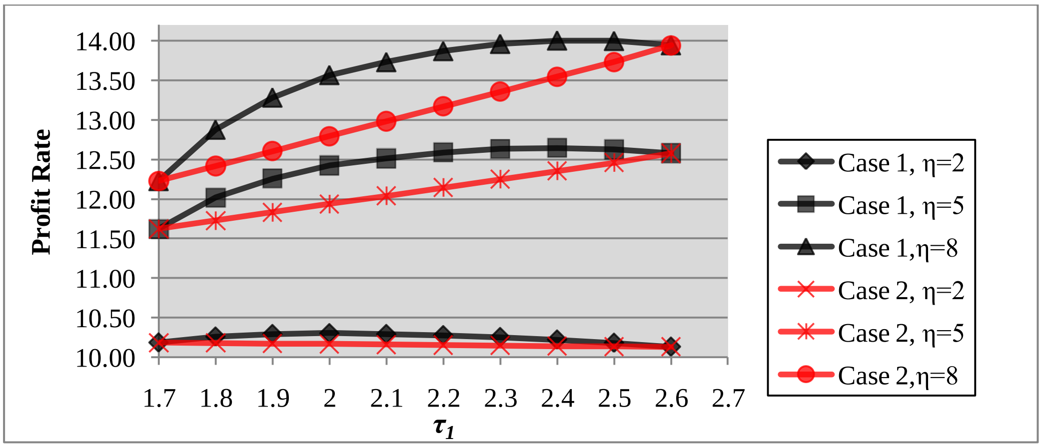

Figure 5.38. Comparison of profits rates of the LM when *p=1.0*

6. CONCLUSIONS

In this study, we explored the idea of sales of services as a sustainable alternative to the existing supply chain structure. To this end, we compared three different structures in terms of profitability. The first structure is a traditional supply chain where products are sold to customers while the second structure includes reverse supply chain where used products are acquired by the manufacturer for remanufacturing and both remanufactured and new products are sold. Finally, the third structure considers a closed loop supply chain where products are leased and leased products are remanufactured at the end of each lease period. For comparing these structures, we modeled them as queuing networks with *GI/G/1* and *GI/G/∞* stations. We analyzed these networks using the parametric decomposition method to obtain performance measures. We also proposed a method to determine the lease payments for the remanufactured products. Then, using the performance measures, we defined a profit function for each model. In the numerical analysis, first we obtained the most significant parameters of each model using ANOVA. Then, we examined the effects of the changes in these parameters by comparing the models. In these analyses, we assumed that equal number of customers is served in each model. Thus, we allowed cannibalization of sales of new products in the SRM. After these analyses, cannibalization effect was considered in detail. Finally, in the last part of the numerical analysis, the relation between design and remanufacturability was examined.

As a result, when the unit profit margin of a new product is changed by varying the price of a new product, we observed that if the ratio between the price of a remanufactured and a new product is high enough, e.g. $p \geq 0.7$, the SRM performs better than the TSM for all values of the unit profit margin of a new product. On the other hand, when the price of a remanufactured product is low ($p = 0.5$), the TSM becomes more profitable than the SRM when the unit profit margin of a new product is greater than 35%. This implies that it is not profitable to remanufacture a product if the unit profit margin of a new product is high enough and the price of a remanufactured product is low. Moreover, when we varied the price of a remanufactured product and held the unit profit margin of a new product constant, we observed that if the ratio between the price of a remanufactured product and a new product is smaller than 0.65, the TSM performs better. This implies that selling remanufactured products with a low profit margin decreases the total system profit by discouraging the sales of new products with higher profit margins.

Also, we observed that the SRM is more profitable in comparison to the TSM if the price of a remanufactured product is high although sales of remanufactured products cannibalize the sales of new products. Also, when introducing remanufactured products increases the market share of the manufacturer, the profitability of the SRM increases. Here, sales of the remanufactured products cannibalize the sales of new products offered by the other manufacturers in the market. However, the price of the remanufactured products and the acquisition cost of the used products affect the profitability of the SRM considerably.

When we considered the LM, which is offered as an alternative to the SRM, we observed that the LM provides the best performance in terms of profitability. This fact is a consequence of the low acquisition cost, the low cost in the grading process and high lease payments achieved by remanufacturing in the LM. Acquisition cost is low in the LM because the manufacturer does not have to pay any core cost since the customer has an obligation to return the product at the end of the lease period. Moreover, in the grading process the products are graded in less time because the uncertainty of the quality of the products is decreased due to the known history of the product in leasing. This results in less cost in the grading process for the LM. Also, remanufacturing the product at the end of each lease period provides high lease payments throughout the useful life of the product. This is because the residual value, which is very important in determining the lease payments, is increased by remanufacturing at each lease period. Since we assumed that the residual value of the product is increased to the level which is equal to P_r at the end of each lease period by remanufacturing, if we increase P_r, the increase in residual value increases. In this context, we observed that the LM is more profitable than the other models if the increase in residual value by remanufacturing is large enough, e.g. $p \geq 0.80$, for all number of lease periods configurations. However, if the lease payments are low, it is not profitable to lease the products for large number of lease periods. For the low values of p, e.g. $p \leq 0.75$, leasing the product for low number of periods by increasing the duration of the lease periods is profitable. This is because if the unit profit margin is low as a result of low lease payments, decreasing the remanufacturing costs by decreasing the number of lease periods is the best idea in order to increase the total system profit.

Furthermore, we concluded that the remanufacturability of a product is essential for the profitability of the SRM and the LM. We defined the remanufacturability level of a product as the number of times that a product can be remanufactured, *n*. For the SRM, increasing the remanufacturability level of the product provides us to keep the product in the market for a longer time period. The product is sold multiple times with the same price and the revenues from the sales of that product increases. Also, since the remanufacturing cost of a product is lower than the production cost of a new product, a higher unit profit margin is obtained for the remanufactured product. Thus, the profitability of the SRM increases as *n* increases when the unit profit margin of a remanufactured product is higher than the unit profit margin of a new product. On the other hand, for the LM, a high remanufacturability level allows us to increase the number of times that a product can be leased in its useful life. Therefore, the duration of the lease periods is decreased. Decreasing the duration of the lease periods gives higher profit rates for the LM since the added value in remanufacturing at the end of each lease period is high. We observed that the profitability of the LM increases as the number of lease periods increases up to a certain value. When the number of lease periods is equal to this critical value, profit rate of the LM is maximized. This critical value changes for different values of *p*, e.g. for $p=0.6$, $p=0.8$ and $p=1$, the critical value of η is equal to 3, 4 and 8 respectively. This is because high profit rates are sustained for larger number of lease periods a result of higher lease payments. In the opposite case where added value in remanufacturing is low ($p=0.6$), decreasing the number of lease periods is a more appropriate decision because of high remanufacturing cost. The TSM and the SRM performs better than the LM if we increase the number of lease periods when lease payments are low.

In the last part of the thesis we examined the fact that longer design time is required to increase the remanufacturability of a product To this end, we considered the effects of design for remanufacture on two factors that affect the profitability of the remanufacturing system; the maximum number of times that a product can be remanufactured, *n* and the average remanufacturing time. First we concentrated on the design for multiple remanufacturing cycles. For the SRM, we observed that up to a certain value of the remanufacturability level the increase in the profits compensates the increase in the costs due to long production and design time. This critical value changes for different values of *p*, e.g. for *p=0.6*, *p=0.8* and *p=1*, the critical value of *n* is equal to 3, 3 and 4 respectively. However, beyond these values, the profit rates decrease with an increase in the remanufacturability level. Furthermore, we studied the effects of proper design on the profitability of the SRM and the LM when the average remanufacturing time decreases as a result of the increase in the production and design time. Namely, we concentrated on the ease of remanufacturing achieved by proper design. We observed that the profitability of the SRM and the LM increases as the design time increases since decrease in the costs as a result of the shorter remanufacturing times is larger than the increase in the design and production costs. However, when the decrease in the remanufacturing cost cannot compensate the increase in the production and design cost, the system profits decrease for both the SR and the LM.

In conclusion, we see that the sales of service model with remanufacturing displays a better performance in terms of profitability in comparison to sales of products models with or without remanufacturing. Also, it is observed that the sales of products model with remanufacturing is more profitable than the traditional sales model although cannibalization occurs. However, these results are applicable for the specific values of the significant parameters set in the numerical analysis. For further research, these models can be investigated in terms of the environmental concerns. The comparison of models with respect to energy consumptions can be of interest. Also, since we assumed that the demand arrival rate for remanufactured products is higher than the arrival rates of the demand station, this assumption can be relaxed in another study. Moreover, further research can be conducted where the price changes as the demand changes. We hope to have provided a starting point for the research on the sales of service systems with remanufacturing and cannibalization of new product sales by remanufactured goods.

APPENDIX A: MODEL VALIDATION TABLES

Table 0.1. Validation of expected throughput rates for the SRM with *n=4*

	$P^{(r)}=0.6,\ q^{(r,\theta)}$ = Case 1			$P^{(r)}=0.6,\ q^{(r,\theta)}$ = Case 2			$P^{(r)}=0.6,\ q^{(r,\theta)}$ = Case 3		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.4	0.4	0.00%	0.4	0.4	0.00%	0.4	0.3996	0.10%
2	0.7323	0.7334	-0.15%	0.6439	0.6436	0.05%	0.7277	0.7269	0.11%
3	0.7323	0.7334	-0.15%	0.6439	0.6436	0.05%	0.7277	0.7269	0.11%
4	0.4394	0.44	-0.14%	0.3863	0.3856	0.18%	0.4366	0.4359	0.16%
5	0.4394	0.44	-0.14%	0.3863	0.3856	0.18%	0.4366	0.4359	0.16%
6	0.1362	0.1362	0.00%	0.0762	0.0763	-0.13%	0.1175	0.1174	0.09%
7	0.1212	0.1214	-0.16%	0.0762	0.0759	0.40%	0.1516	0.1514	0.13%
8	0.0968	0.0971	-0.31%	0.0762	0.0761	0.13%	0.0829	0.0827	0.24%
9	0.0504	0.0506	-0.40%	0.0762	0.0759	0.40%	0.0491	0.0489	0.41%
10	0.3323	0.3328	-0.15%	0.2439	0.2436	0.12%	0.3277	0.3272	0.15%
D	0.1071	0.1072	-0.09%	0.1424	0.1467	-2.93%	0.1089	0.1087	0.18%
R	0.3323	0.3328	-0.15%	0.2439	0.2436	0.12%	0.3277	0.3272	0.15%

	$P^{(r)}=0.7,\ q^{(r,\theta)}$ = Case 1			$P^{(r)}=0.7,\ q^{(r,\theta)}$ = Case 2			$P^{(r)}=0.7,\ q^{(r,\theta)}$ = Case 3		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.4	0.4	0.00%	0.4	0.3999	0.03%	0.4	0.4007	-0.17%
2	0.8314	0.8319	-0.06%	0.7116	0.7122	-0.08%	0.8223	0.8234	-0.13%
3	0.8314	0.8319	-0.06%	0.7116	0.7122	-0.08%	0.8223	0.8234	-0.13%
4	0.582	0.5823	-0.05%	0.4981	0.4986	-0.10%	0.5756	0.5763	-0.12%
5	0.582	0.5823	-0.05%	0.4981	0.4986	-0.10%	0.5756	0.5763	-0.12%
6	0.1697	0.17	-0.18%	0.0974	0.0983	-0.92%	0.1401	0.1405	-0.28%
7	0.1575	0.1575	0.00%	0.0974	0.0973	0.10%	0.1888	0.189	-0.11%
8	0.1296	0.1298	-0.15%	0.0974	0.0972	0.21%	0.1164	0.1165	-0.09%
9	0.0699	0.0698	0.14%	0.0974	0.0975	-0.10%	0.0742	0.0741	0.13%
10	0.4315	0.4318	-0.07%	0.3116	0.3123	-0.22%	0.4223	0.4228	-0.12%
D	0.1506	0.1505	0.07%	0.1865	0.1863	0.11%	0.1533	0.1535	-0.13%
R	0.4315	0.4318	-0.07%	0.3116	0.3123	-0.22%	0.4223	0.4227	-0.09%

	$P^{(r)}=0.8,\ q^{(r,\theta)}$ = Case 1			$P^{(r)}=0.8,\ q^{(r,\theta)}$ = Case 2			$P^{(r)}=0.8,\ q^{(r,\theta)}$ = Case 3		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.4	0.3997	0.08%	0.4	0.4	0.00%	0.4	0.4002	-0.05%
2	0.9497	0.9488	0.09%	0.7908	0.7895	0.16%	0.9337	0.9332	0.05%
3	0.9497	0.9488	0.09%	0.7908	0.7895	0.16%	0.9337	0.9332	0.05%
4	0.7598	0.7581	0.22%	0.6327	0.6305	0.35%	0.7469	0.7458	0.15%
5	0.7598	0.7581	0.22%	0.6327	0.6305	0.35%	0.7469	0.7458	0.15%
6	0.2075	0.2076	-0.05%	0.1221	0.1218	0.25%	0.1641	0.164	0.06%
7	0.1999	0.1994	0.25%	0.1221	0.1216	0.41%	0.2299	0.2296	0.13%
8	0.17	0.1698	0.12%	0.1221	0.1215	0.49%	0.1576	0.157	0.38%
9	0.0956	0.0952	0.42%	0.1221	0.1217	0.33%	0.1085	0.1084	0.09%
10	0.5497	0.5492	0.09%	0.3908	0.3892	0.41%	0.5337	0.533	0.13%
D	0.21	0.2089	0.53%	0.2418	0.2412	0.25%	0.2132	0.2128	0.19%
R	0.5497	0.5492	0.09%	0.3908	0.3892	0.41%	0.5337	0.533	0.13%

Table 0.2. Validation of expected throughput rates for the SRM with $n=5$

	$P^{(r)} = 0.6,\ q^{(r,\theta)} = Case\ 1$			$P^{(r)} = 0.6,\ q^{(r,\theta)} = Case\ 2$			$P^{(r)} = 0.6,\ q^{(r,\theta)} = Case\ 3$		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.4	0.4006	-0.15%	0.4	0.3996	0.10%	0.4	0.4003	-0.07%
2	0.7369	0.738	-0.15%	0.6473	0.6469	0.06%	0.7303	0.7309	-0.08%
3	0.7369	0.738	-0.15%	0.6473	0.6469	0.06%	0.7303	0.7309	-0.08%
4	0.4422	0.4425	-0.07%	0.3884	0.3879	0.13%	0.4382	0.4383	-0.02%
5	0.4422	0.4425	-0.07%	0.3884	0.3879	0.13%	0.4382	0.4383	-0.02%
6	0.1367	0.1371	-0.29%	0.0773	0.0773	0.00%	0.1178	0.1178	0.00%
7	0.122	0.1216	0.33%	0.0773	0.0772	0.13%	0.1519	0.1519	0.00%
8	0.0984	0.0989	-0.51%	0.0773	0.0772	0.13%	0.0832	0.0834	-0.24%
9	0.0537	0.0537	0.00%	0.0773	0.0769	0.52%	0.0518	0.0518	0.00%
10	0.3369	0.3374	-0.15%	0.2473	0.2473	0.00%	0.3303	0.3306	-0.09%
D	0.1052	0.1051	0.10%	0.1411	0.1406	0.36%	0.1079	0.1077	0.19%
R	0.3369	0.3374	-0.15%	0.2473	0.2473	0.00%	0.3303	0.3306	-0.09%

	$P^{(r)} = 0.7,\ q^{(r,\theta)} = Case\ 1$			$P^{(r)} = 0.7,\ q^{(r,\theta)} = Case\ 2$			$P^{(r)} = 0.7,\ q^{(r,\theta)} = Case\ 3$		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.4	0.3999	0.03%	0.4	0.3997	0.08%	0.4	0.4007	-0.17%
2	0.8416	0.8407	0.11%	0.7188	0.7183	0.07%	0.828	0.8284	-0.05%
3	0.8416	0.8407	0.11%	0.7188	0.7183	0.07%	0.828	0.8284	-0.05%
4	0.5891	0.588	0.19%	0.5031	0.5026	0.10%	0.5796	0.579	0.10%
5	0.5891	0.588	0.19%	0.5031	0.5026	0.10%	0.5796	0.579	0.10%
6	0.1707	0.1705	0.12%	0.0996	0.0996	0.00%	0.1408	0.1409	-0.07%
7	0.1592	0.1588	0.25%	0.0996	0.0996	0.00%	0.1895	0.1893	0.11%
8	0.1332	0.1331	0.08%	0.0996	0.0995	0.10%	0.117	0.1169	0.09%
9	0.0771	0.0771	0.00%	0.0996	0.0995	0.10%	0.0801	0.0798	0.38%
10	0.4416	0.4409	0.16%	0.3188	0.3186	0.06%	0.428	0.4276	0.09%
D	0.1475	0.1471	0.27%	0.1844	0.1841	0.16%	0.1516	0.1514	0.13%
R	0.4416	0.4409	0.16%	0.3188	0.3186	0.06%	0.428	0.4276	0.09%

	$P^{(r)} = 0.8,\ q^{(r,\theta)} = Case\ 1$			$P^{(r)} = 0.8,\ q^{(r,\theta)} = Case\ 2$			$P^{(r)} = 0.8,\ q^{(r,\theta)} = Case\ 3$		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.4	0.3999	0.03%	0.4	0.3996	0.10%	0.4	0.3999	0.03%
2	0.9695	0.9687	0.08%	0.8049	0.8027	0.27%	0.9449	0.9446	0.03%
3	0.9695	0.9687	0.08%	0.8049	0.8027	0.27%	0.9449	0.9446	0.03%
4	0.7756	0.7742	0.18%	0.6439	0.6412	0.42%	0.7559	0.755	0.12%
5	0.7756	0.7742	0.18%	0.6439	0.6412	0.42%	0.7559	0.755	0.12%
6	0.2092	0.2095	-0.14%	0.1265	0.1262	0.24%	0.1653	0.1658	-0.30%
7	0.2035	0.2024	0.54%	0.1265	0.1259	0.48%	0.2313	0.2311	0.09%
8	0.177	0.1768	0.11%	0.1265	0.1261	0.32%	0.1588	0.1585	0.19%
9	0.1096	0.1097	-0.09%	0.1265	0.1261	0.32%	0.1199	0.1196	0.25%
10	0.5694	0.5688	0.11%	0.4049	0.4031	0.45%	0.5449	0.5447	0.04%
D	0.2061	0.2055	0.29%	0.239	0.2381	0.38%	0.211	0.2103	0.33%
R	0.5694	0.5688	0.11%	0.4049	0.4031	0.45%	0.5449	0.5447	0.04%

Table 0.3. Validation of expected throughput rates for the SRM with $n=6$

	$P^{(r)}=0.6,\ q^{(r,\theta)}=Case\ 1$			$P^{(r)}=0.6,\ q^{(r,\theta)}=Case\ 2$			$P^{(r)}=0.6,\ q^{(r,\theta)}=Case\ 3$		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.4	0.4008	-0.20%	0.4	0.4003	-0.07%	0.4	0.4001	-0.02%
2	0.7386	0.7395	-0.12%	0.6485	0.6483	0.03%	0.731	0.7305	0.07%
3	0.7386	0.7395	-0.12%	0.6485	0.6483	0.03%	0.731	0.7305	0.07%
4	0.4431	0.4434	-0.07%	0.3891	0.3886	0.13%	0.4386	0.4375	0.25%
5	0.4431	0.4434	-0.07%	0.3891	0.3886	0.13%	0.4386	0.4375	0.25%
6	0.1368	0.1367	0.07%	0.0777	0.0776	0.13%	0.1179	0.1177	0.17%
7	0.1223	0.1224	-0.08%	0.0777	0.0774	0.39%	0.1519	0.1516	0.20%
8	0.099	0.0991	-0.10%	0.0777	0.0777	0.00%	0.0832	0.0832	0.00%
9	0.0549	0.055	-0.18%	0.0777	0.0779	-0.26%	0.0525	0.0521	0.77%
10	0.3386	0.3387	-0.03%	0.2486	0.2481	0.20%	0.331	0.3304	0.18%
D	0.1046	0.1047	-0.10%	0.1406	0.1405	0.07%	0.1076	0.107	0.56%
R	0.3386	0.3387	-0.03%	0.2486	0.2481	0.20%	0.331	0.3304	0.18%
	$P^{(r)}=0.7,\ q^{(r,\theta)}=Case\ 1$			$P^{(r)}=0.7,\ q^{(r,\theta)}=Case\ 2$			$P^{(r)}=0.7,\ q^{(r,\theta)}=Case\ 3$		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.4	0.4005	-0.12%	0.4	0.3993	0.18%	0.4	0.4007	-0.17%
2	0.8456	0.8465	-0.11%	0.722	0.7206	0.19%	0.8298	0.8315	-0.20%
3	0.8456	0.8465	-0.11%	0.722	0.7206	0.19%	0.8298	0.8315	-0.20%
4	0.5919	0.5918	0.02%	0.5054	0.5042	0.24%	0.5808	0.5818	-0.17%
5	0.5919	0.5918	0.02%	0.5054	0.5042	0.24%	0.5808	0.5818	-0.17%
6	0.171	0.1711	-0.06%	0.1006	0.1006	0.00%	0.1409	0.1416	-0.49%
7	0.1599	0.1599	0.00%	0.1006	0.1006	0.00%	0.1897	0.1901	-0.21%
8	0.1346	0.1349	-0.22%	0.1006	0.1002	0.40%	0.1172	0.1173	-0.09%
9	0.0799	0.0796	0.38%	0.1006	0.1003	0.30%	0.0819	0.0819	0.00%
10	0.4456	0.4459	-0.07%	0.322	0.3213	0.22%	0.4298	0.4308	-0.23%
D	0.1463	0.1459	0.27%	0.1834	0.1829	0.27%	0.1511	0.151	0.07%
R	0.4456	0.4459	-0.07%	0.322	0.3213	0.22%	0.4298	0.4308	-0.23%
	$P^{(r)}=0.8,\ q^{(r,\theta)}=Case\ 1$			$P^{(r)}=0.8,\ q^{(r,\theta)}=Case\ 2$			$P^{(r)}=0.8,\ q^{(r,\theta)}=Case\ 3$		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.4	0.3998	0.05%	0.4	0.3995	0.13%	0.4	0.3996	0.10%
2	0.9784	0.9771	0.13%	0.8121	0.8108	0.16%	0.9488	0.947	0.19%
3	0.9784	0.9771	0.13%	0.8121	0.8108	0.16%	0.9488	0.947	0.19%
4	0.7827	0.7808	0.24%	0.6497	0.6479	0.28%	0.759	0.7569	0.28%
5	0.7827	0.7808	0.24%	0.6497	0.6479	0.28%	0.759	0.7569	0.28%
6	0.21	0.2106	-0.28%	0.1288	0.1288	0.00%	0.1658	0.1653	0.30%
7	0.205	0.2043	0.34%	0.1288	0.1283	0.39%	0.2317	0.2314	0.13%
8	0.1802	0.1797	0.28%	0.1288	0.1285	0.23%	0.1593	0.1591	0.13%
9	0.1159	0.1151	0.70%	0.1288	0.1282	0.47%	0.124	0.1232	0.65%
10	0.5784	0.5774	0.17%	0.4121	0.4114	0.17%	0.5488	0.5475	0.24%
D	0.2043	0.2034	0.44%	0.2376	0.2365	0.47%	0.2102	0.2094	0.38%
R	0.5784	0.5774	0.17%	0.4121	0.4114	0.17%	0.5488	0.5475	0.24%

Table 0.4. Validation of expected waiting times for the SRM with $n=4$

	$P^{(r)} = 0.6,\ q^{(r,\theta)} = Case\ 1$			$P^{(r)} = 0.6,\ q^{(r,\theta)} = Case\ 2$			$P^{(r)} = 0.6,\ q^{(r,\theta)} = Case\ 3$		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.6667	0.6678	-0.16%	0.6667	0.6712	-0.67%	0.6667	0.6637	0.45%
2	0	0	0.00%	0	0	0.00%	0	0	0.00%
3	0	0	0.00%	0	0	0.00%	0	0	0.00%
4	0.1408	0.1408	0.00%	0.1197	0.119	0.59%	0.1396	0.1397	-0.07%
5	0.1408	0.1414	-0.42%	0.1197	0.1196	0.08%	0.1396	0.1398	-0.14%
6	0.1577	0.1604	-1.68%	0.0825	0.0839	-1.67%	0.1332	0.1321	0.83%
7	0.2041	0.2056	-0.73%	0.1208	0.1218	-0.82%	0.2669	0.2636	1.25%
8	0.2195	0.2209	-0.63%	0.1672	0.1658	0.84%	0.1837	0.1803	1.89%
9	0.1405	0.1424	-1.33%	0.2222	0.2185	1.69%	0.1363	0.1347	1.19%
10	0.4977	0.4978	-0.02%	0.3226	0.3208	0.56%	0.4874	0.4885	-0.23%
D	0.0283	0.0279	1.43%	0.0383	0.0381	0.52%	0.0288	0.0287	0.35%
R	2.0689	2.0883	-0.93%	1.1418	1.1346	0.63%	2.0056	2.005	0.03%
	$P^{(r)} = 0.7,\ q^{(r,\theta)} = Case\ 1$			$P^{(r)} = 0.7,\ q^{(r,\theta)} = Case\ 2$			$P^{(r)} = 0.7,\ q^{(r,\theta)} = Case\ 3$		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.667	0.6629	0.62%	0.6667	0.6657	0.15%	0.6667	0.672	-0.79%
2	0	0	0.00%	0	0	0.00%	0	0	0.00%
3	0	0	0.00%	0	0	0.00%	0	0	0.00%
4	0.2052	0.2037	0.74%	0.1658	0.1669	-0.66%	0.202	0.2017	0.15%
5	0.2052	0.205	0.10%	0.1658	0.1669	-0.66%	0.202	0.203	-0.49%
6	0.2045	0.2072	-1.30%	0.1079	0.1098	-1.73%	0.1629	0.1619	0.62%
7	0.2796	0.2795	0.04%	0.1587	0.1576	0.70%	0.3516	0.3584	-1.90%
8	0.3104	0.3077	0.88%	0.2209	0.2193	0.73%	0.2725	0.2687	1.41%
9	0.2018	0.1993	1.25%	0.2952	0.2954	-0.07%	0.2157	0.2159	-0.09%
10	0.7589	0.7557	0.42%	0.4526	0.4543	-0.37%	0.7309	0.738	-0.96%
D	0.0407	0.0406	0.25%	0.0514	0.0514	0.00%	0.0415	0.0419	-0.95%
R	4.2667	4.3088	-0.98%	1.8	1.8262	-1.43%	3.9599	4.0391	-1.96%
	$P^{(r)} = 0.8,\ q^{(r,\theta)} = Case\ 1$			$P^{(r)} = 0.8,\ q^{(r,\theta)} = Case\ 2$			$P^{(r)} = 0.8,\ q^{(r,\theta)} = Case\ 3$		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.6667	0.6666	0.02%	0.6667	0.6704	-0.55%	0.6667	0.6651	0.24%
2	0	0	0.00%	0	0	0.00%	0	0	0.00%
3	0	0	0.00%	0	0	0.00%	0	0	0.00%
4	0.3063	0.3053	0.33%	0.2313	0.2295	0.78%	0.2981	0.2967	0.47%
5	0.3063	0.3054	0.29%	0.2313	0.2298	0.65%	0.2981	0.297	0.37%
6	0.2618	0.2617	0.04%	0.1391	0.1383	0.58%	0.1963	0.1967	-0.20%
7	0.3788	0.3784	0.11%	0.2061	0.2067	-0.29%	0.4574	0.4575	-0.02%
8	0.4374	0.4391	-0.39%	0.2888	0.2868	0.70%	0.3962	0.3939	0.58%
9	0.2891	0.284	1.80%	0.3886	0.3883	0.08%	0.3363	0.3404	-1.20%
10	1.2208	1.221	-0.02%	0.6416	0.6419	-0.05%	1.1445	1.1586	-1.22%
D	0.058	0.0583	-0.51%	0.0688	0.0684	0.58%	0.0597	0.0591	1.02%
R	18.2242	18.6067	-2.06%	3.1142	3.0949	0.62%	13.4165	13.2697	1.11%

Table 0.5. Validation of expected waiting times for the SRM with n=5

	$P^{(r)}$ = 0.6, $q^{(r,\theta)}$ = Case 1			$P^{(r)}$ = 0.6, $q^{(r,\theta)}$ = Case 2			$P^{(r)}$ = 0.6, $q^{(r,\theta)}$ = Case 3		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.6667	0.6709	-0.63%	0.6667	0.6699	-0.48%	0.6667	0.6726	-0.88%
2	0	0	0.00%	0	0	0.00%	0	0	0.00%
3	0	0	0.00%	0	0	0.00%	0	0	0.00%
4	0.1419	0.1417	0.14%	0.1205	0.1192	1.09%	0.1403	0.1402	0.07%
5	0.1419	0.1432	-0.91%	0.1205	0.1201	0.33%	0.1403	0.1406	-0.21%
6	0.1583	0.1585	-0.13%	0.0837	0.0845	-0.95%	0.1336	0.1305	2.38%
7	0.2058	0.2028	1.48%	0.1226	0.121	1.32%	0.2675	0.2654	0.79%
8	0.2238	0.2209	1.31%	0.1698	0.1714	-0.93%	0.1845	0.1811	1.88%
9	0.1506	0.1493	0.87%	0.2257	0.2246	0.49%	0.1445	0.1453	-0.55%
10	0.5083	0.5094	-0.22%	0.3285	0.3306	-0.64%	0.4933	0.4929	0.08%
D	0.0278	0.0271	2.58%	0.0379	0.0376	0.80%	0.0285	0.0283	0.71%
R	2.1354	2.1544	-0.88%	1.1683	1.1769	-0.73%	2.0418	2.0439	-0.10%

	$P^{(r)}$ = 0.7, $q^{(r,\theta)}$ = Case 1			$P^{(r)}$ = 0.7, $q^{(r,\theta)}$ = Case 2			$P^{(r)}$ = 0.7, $q^{(r,\theta)}$ = Case 3		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.6667	0.6726	-0.88%	0.6667	0.6737	-1.04%	0.6667	0.6699	-0.48%
2	0	0	0.00%	0	0	0.00%	0	0	0.00%
3	0	0	0.00%	0	0	0.00%	0	0	0.00%
4	0.2088	0.206	1.36%	0.1681	0.1664	1.02%	0.204	0.2024	0.79%
5	0.2088	0.2078	0.48%	0.1681	0.1682	-0.06%	0.204	0.2039	0.05%
6	0.2058	0.2035	1.13%	0.1106	0.1105	0.09%	0.1638	0.1661	-1.38%
7	0.2835	0.2777	2.09%	0.1629	0.162	0.56%	0.3532	0.3535	-0.08%
8	0.3209	0.3168	1.29%	0.2269	0.2253	0.71%	0.2743	0.2758	-0.54%
9	0.2253	0.2248	0.22%	0.3034	0.3083	-1.59%	0.2353	0.2285	2.98%
10	0.7908	0.7822	1.10%	0.4679	0.465	0.62%	0.7483	0.747	0.17%
D	0.0398	0.0394	1.02%	0.0508	0.051	-0.39%	0.041	0.0411	-0.24%
R	4.6457	4.683	-0.80%	1.8892	1.9317	-2.20%	4.1476	4.1889	-0.99%

	$P^{(r)}$ = 0.8, $q^{(r,\theta)}$ = Case 1			$P^{(r)}$ = 0.8, $q^{(r,\theta)}$ = Case 2			$P^{(r)}$ = 0.8, $q^{(r,\theta)}$ = Case 3		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.6667	0.6663	0.06%	0.6667	0.6652	0.23%	0.6667	0.6698	-0.46%
2	0	0	0.00%	0	0	0.00%	0	0	0.00%
3	0	0	0.00%	0	0	0.00%	0	0	0.00%
4	0.3167	0.3166	0.03%	0.2374	0.2355	0.81%	0.3038	0.302	0.60%
5	0.3167	0.316	0.22%	0.2374	0.2367	0.30%	0.3038	0.3024	0.46%
6	0.2646	0.2641	0.19%	0.1449	0.1427	1.54%	0.1981	0.2014	-1.64%
7	0.3876	0.3856	0.52%	0.2148	0.2145	0.14%	0.4609	0.4601	0.17%
8	0.4613	0.4564	1.07%	0.3014	0.3034	-0.66%	0.4003	0.4012	-0.22%
9	0.3403	0.3373	0.89%	0.4062	0.4041	0.52%	0.3801	0.38	0.03%
10	1.3226	1.3253	-0.20%	0.6804	0.6749	0.81%	1.1973	1.1903	0.59%
D	0.0575	0.0568	1.23%	0.0678	0.0667	1.65%	0.0589	0.058	1.55%
R	31.0678	30.2289	2.78%	3.459	3.4008	1.71%	16.4777	17.2818	-4.65%

Table 0.6. Validation of expected waiting times for the SRM with $n=6$

	$P^{(r)}=0.6,\ q^{(r,\theta)}=$ Case 1			$P^{(r)}=0.6,\ q^{(r,\theta)}=$ Case 2			$P^{(r)}=0.6,\ q^{(r,\theta)}=$ Case 3		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.6667	0.6719	-0.77%	0.6667	0.6651	0.24%	0.6667	0.6687	-0.30%
2	0	0	0.00%	0	0	0.00%	0	0	0.00%
3	0	0	0.00%	0	0	0.00%	0	0	0.00%
4	0.1423	0.1422	0.07%	0.1208	0.1207	0.08%	0.1405	0.1412	-0.50%
5	0.1423	0.1417	0.42%	0.1208	0.1214	-0.49%	0.1405	0.1402	0.21%
6	0.1585	0.1579	0.38%	0.0842	0.0832	1.20%	0.1337	0.133	0.53%
7	0.2064	0.2107	-2.04%	0.1233	0.125	-1.36%	0.2677	0.2681	-0.15%
8	0.2253	0.2286	-1.44%	0.1708	0.1722	-0.81%	0.1847	0.1845	0.11%
9	0.154	0.1573	-2.10%	0.2271	0.2254	0.75%	0.1467	0.1456	0.76%
10	0.5119	0.5121	-0.04%	0.3308	0.3313	-0.15%	0.4949	0.4918	0.63%
D	0.0276	0.0275	0.36%	0.0378	0.0384	-1.56%	0.0284	0.0284	0.00%
R	2.1585	2.1731	-0.67%	1.1787	1.1706	0.69%	2.0515	2.0304	1.04%
	$P^{(r)}=0.7,\ q^{(r,\theta)}=$ Case 1			$P^{(r)}=0.7,\ q^{(r,\theta)}=$ Case 2			$P^{(r)}=0.7,\ q^{(r,\theta)}=$ Case 3		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.6667	0.6691	-0.36%	0.6667	0.6656	0.17%	0.6667	0.6682	-0.22%
2	0	0	0.00%	0	0	0.00%	0	0	0.00%
3	0	0	0.00%	0	0	0.00%	0	0	0.00%
4	0.2102	0.2099	0.14%	0.1691	0.1681	0.59%	0.2046	0.2036	0.49%
5	0.2102	0.2093	0.43%	0.1691	0.1692	-0.06%	0.2046	0.2051	-0.24%
6	0.2063	0.2052	0.54%	0.1119	0.1124	-0.44%	0.1641	0.1664	-1.38%
7	0.2851	0.2872	-0.73%	0.1648	0.1646	0.12%	0.3537	0.3574	-1.04%
8	0.3251	0.324	0.34%	0.2296	0.2243	2.36%	0.2749	0.2754	-0.18%
9	0.2348	0.2326	0.95%	0.307	0.3044	0.85%	0.2414	0.2425	-0.45%
10	0.8037	0.8014	0.29%	0.4749	0.4678	1.52%	0.7537	0.7567	-0.40%
D	0.0395	0.0394	0.25%	0.0505	0.0499	1.20%	0.0408	0.0413	-1.21%
R	4.8092	4.8841	-1.53%	1.9306	1.9163	0.75%	4.2079	4.2623	-1.28%
	$P^{(r)}=0.8,\ q^{(r,\theta)}=$ Case 1			$P^{(r)}=0.8,\ q^{(r,\theta)}=$ Case 2			$P^{(r)}=0.8,\ q^{(r,\theta)}=$ Case 3		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.6667	0.668	-0.19%	0.6667	0.6692	-0.37%	0.6667	0.6665	0.03%
2	0	0	0.00%	0	0	0.00%	0	0	0.00%
3	0	0	0.00%	0	0	0.00%	0	0	0.00%
4	0.3215	0.3202	0.41%	0.2406	0.2389	0.71%	0.3058	0.3034	0.79%
5	0.3215	0.3199	0.50%	0.2406	0.2397	0.38%	0.3058	0.303	0.92%
6	0.2659	0.2631	1.06%	0.1478	0.1452	1.79%	0.1987	0.198	0.35%
7	0.3916	0.3894	0.56%	0.2193	0.2157	1.67%	0.4622	0.4609	0.28%
8	0.4723	0.4676	1.01%	0.3079	0.3075	0.13%	0.4018	0.4004	0.35%
9	0.3644	0.3564	2.24%	0.4152	0.4181	-0.69%	0.3961	0.3974	-0.33%
10	1.3717	1.3653	0.47%	0.701	0.6978	0.46%	1.2164	1.209	0.61%
D	0.0568	0.0562	1.07%	0.0674	0.0673	0.15%	0.0587	0.0585	0.34%
R	44.559	46.85	-4.89%	3.6556	3.7059	-1.36%	17.8737	17.8726	0.01%

Table 0.7. Validation of expected throughput rates for the LM

with $y_i = (0.9, 0.8, 0.8, 0.7)$

	$\eta = 4,\ q^{(r,\theta)} = Case\ 1$			$\eta = 4,\ q^{(r,\theta)} = Case\ 2$			$\eta = 4,\ q^{(r,\theta)} = Case\ 3$		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.3	0.3005	-0.17%	0.3	0.3005	-0.17%	0.3	0.3001	-0.03%
L	0.942	0.9429	-0.10%	0.7439	0.7445	-0.08%	0.9128	0.9107	0.23%
2	0.942	0.9429	-0.10%	0.7439	0.7445	-0.08%	0.9128	0.9119	0.10%
3	0.942	0.9429	-0.10%	0.7439	0.7445	-0.08%	0.9128	0.9107	0.23%
4	0.942	0.9419	0.01%	0.7439	0.7437	0.03%	0.9128	0.9107	0.23%
5	0.942	0.9419	0.01%	0.7439	0.7437	0.03%	0.9128	0.9107	0.23%
6	0.2231	0.2233	-0.09%	0.1387	0.1391	-0.29%	0.1627	0.1622	0.31%
7	0.2299	0.2305	-0.26%	0.1387	0.1384	0.22%	0.2437	0.2445	-0.33%
8	0.2092	0.2093	-0.05%	0.1387	0.1385	0.14%	0.1997	0.1989	0.40%
9	0.1286	0.1283	0.23%	0.1387	0.1387	0.00%	0.1595	0.1588	0.44%
10	0.642	0.6425	-0.08%	0.4438	0.4439	-0.02%	0.6128	0.6118	0.16%
D	0.3	0.2994	0.20%	0.3	0.2997	0.10%	0.3	0.2989	0.37%
	$\eta = 5,\ q^{(r,\theta)} = Case\ 1$			$\eta = 5,\ q^{(r,\theta)} = Case\ 2$			$\eta = 5,\ q^{(r,\theta)} = Case\ 3$		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.3	0.3004	-0.13%	0.3	0.3002	-0.07%	0.3	0.3001	-0.03%
L	0.9871	0.9873	-0.02%	0.7761	0.7757	0.05%	0.9384	0.9386	-0.02%
2	0.9871	0.9865	0.06%	0.7761	0.7756	0.06%	0.9384	0.9385	-0.01%
3	0.9871	0.9865	0.06%	0.7761	0.7756	0.06%	0.9384	0.9385	-0.01%
4	0.9871	0.9851	0.20%	0.7761	0.7747	0.18%	0.9384	0.9372	0.13%
5	0.9871	0.9851	0.20%	0.7761	0.7747	0.18%	0.9384	0.9372	0.13%
6	0.2271	0.2274	-0.13%	0.1488	0.1489	-0.07%	0.1656	0.166	-0.24%
7	0.2378	0.2377	0.04%	0.1488	0.1483	0.34%	0.2466	0.2465	0.04%
8	0.2251	0.2245	0.27%	0.1488	0.1485	0.20%	0.2026	0.2023	0.15%
9	0.1606	0.1603	0.19%	0.1488	0.1483	0.34%	0.1856	0.1851	0.27%
10	0.6872	0.6869	0.04%	0.4761	0.4755	0.13%	0.6384	0.6385	-0.02%
D	0.3	0.2982	0.60%	0.3	0.2992	0.27%	0.3	0.2987	0.44%
	$\eta = 6,\ q^{(r,\theta)} = Case\ 1$			$\eta = 6,\ q^{(r,\theta)} = Case\ 2$			$\eta = 6,\ q^{(r,\theta)} = Case\ 3$		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.3	0.2999	0.03%	0.3	0.2997	0.10%	0.3	0.3004	-0.13%
L	1.0127	1.0109	0.18%	0.7967	0.7944	0.29%	0.9497	0.9486	0.12%
2	1.0127	1.0109	0.18%	0.7967	0.7944	0.29%	0.9497	0.9484	0.14%
3	1.0127	1.0109	0.18%	0.7967	0.7944	0.29%	0.9497	0.9484	0.14%
4	1.0127	1.0093	0.34%	0.7967	0.7932	0.44%	0.9497	0.9469	0.30%
5	1.0127	1.0093	0.34%	0.7967	0.7932	0.44%	0.9497	0.9469	0.30%
6	0.2293	0.2299	-0.26%	0.1552	0.1552	0.00%	0.1669	0.1671	-0.12%
7	0.2424	0.2417	0.29%	0.1552	0.1541	0.71%	0.2479	0.2474	0.20%
8	0.2342	0.2331	0.47%	0.1552	0.1547	0.32%	0.2039	0.2037	0.10%
9	0.1787	0.1778	0.51%	0.1552	0.1547	0.32%	0.1972	0.1958	0.72%
10	0.7127	0.7109	0.25%	0.4967	0.4947	0.40%	0.6497	0.6482	0.23%
D	0.3	0.2984	0.54%	0.3	0.2985	0.50%	0.3	0.2987	0.44%

Table 0.8. Validation of expected throughput rates for the LM

with $y_i = (0.8, 0.7, 0.7, 0.6)$

	$\eta = 4,\ q^{(r,\theta)} = Case\ 1$			$\eta = 4,\ q^{(r,\theta)} = Case\ 2$			$\eta = 4,\ q^{(r,\theta)} = Case\ 3$		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.3	0.3001	-0.03%	0.3	0.3003	-0.10%	0.3	0.3003	-0.10%
L	0.7929	0.7924	0.06%	0.6442	0.6448	-0.09%	0.7744	0.7751	-0.09%
2	0.7929	0.7924	0.06%	0.6442	0.6448	-0.09%	0.7744	0.7751	-0.09%
3	0.7929	0.7924	0.06%	0.6442	0.6448	-0.09%	0.7744	0.7751	-0.09%
4	0.7929	0.7924	0.06%	0.6442	0.6448	-0.09%	0.7744	0.7752	-0.10%
5	0.7929	0.7924	0.06%	0.6442	0.6448	-0.09%	0.7744	0.7752	-0.10%
6	0.2053	0.2056	-0.15%	0.1229	0.1231	-0.16%	0.157	0.1572	-0.13%
7	0.2034	0.2031	0.15%	0.1229	0.1228	0.08%	0.2266	0.2268	-0.09%
8	0.1775	0.1776	-0.06%	0.1229	0.1231	-0.16%	0.1673	0.1676	-0.18%
9	0.1033	0.1032	0.10%	0.1229	0.1232	-0.24%	0.1217	0.1215	0.16%
10	0.4929	0.4923	0.12%	0.3443	0.3445	-0.06%	0.4744	0.4748	-0.08%
D	0.3	0.3001	-0.03%	0.3	0.3002	-0.07%	0.3	0.3004	-0.13%
	$\eta = 5,\ q^{(r,\theta)} = Case\ 1$			$\eta = 5,\ q^{(r,\theta)} = Case\ 2$			$\eta = 5,\ q^{(r,\theta)} = Case\ 3$		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.3	0.3	0.00%	0.3	0.3005	-0.17%	0.3	0.2997	0.10%
L	0.8158	0.8155	0.04%	0.6608	0.6623	-0.23%	0.7872	0.7867	0.06%
2	0.8158	0.8155	0.04%	0.6608	0.6623	-0.23%	0.7872	0.7867	0.06%
3	0.8158	0.8155	0.04%	0.6608	0.6623	-0.23%	0.7872	0.7867	0.06%
4	0.8158	0.8152	0.07%	0.6608	0.6622	-0.21%	0.7872	0.7867	0.06%
5	0.8158	0.8152	0.07%	0.6608	0.6622	-0.21%	0.7872	0.7867	0.06%
6	0.2076	0.2073	0.14%	0.1288	0.1291	-0.23%	0.1587	0.1591	-0.25%
7	0.2081	0.2081	0.00%	0.1288	0.129	-0.16%	0.2283	0.2277	0.26%
8	0.1869	0.1872	-0.16%	0.1288	0.1293	-0.39%	0.169	0.1691	-0.06%
9	0.1221	0.1217	0.33%	0.1288	0.1291	-0.23%	0.1369	0.1366	0.22%
10	0.5158	0.5155	0.06%	0.3608	0.3618	-0.28%	0.4872	0.4869	0.06%
D	0.3	0.2997	0.10%	0.3	0.3003	-0.10%	0.3	0.2997	0.10%
	$\eta = 6,\ q^{(r,\theta)} = Case\ 1$			$\eta = 6,\ q^{(r,\theta)} = Case\ 2$			$\eta = 6,\ q^{(r,\theta)} = Case\ 3$		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.3	0.3001	-0.03%	0.3	0.3004	-0.13%	0.3	0.3007	-0.23%
L	0.8271	0.8273	-0.02%	0.67	0.6721	-0.31%	0.7921	0.7944	-0.29%
2	0.8271	0.8273	-0.02%	0.67	0.6721	-0.31%	0.7921	0.7944	-0.29%
3	0.8271	0.8273	-0.02%	0.67	0.6721	-0.31%	0.7921	0.7944	-0.29%
4	0.8271	0.8269	0.02%	0.67	0.6682	0.27%	0.7921	0.7837	1.07%
5	0.8271	0.8269	0.02%	0.67	0.6682	0.27%	0.7921	0.7837	1.07%
6	0.2088	0.2087	0.05%	0.1322	0.1332	-0.75%	0.1594	0.1601	-0.44%
7	0.2104	0.2104	0.00%	0.1322	0.1324	-0.15%	0.2289	0.2294	-0.22%
8	0.1915	0.1916	-0.05%	0.1322	0.1323	-0.08%	0.1696	0.1703	-0.41%
9	0.1312	0.1313	-0.08%	0.1322	0.1325	-0.23%	0.1426	0.1426	0.00%
10	0.5271	0.5272	-0.02%	0.37	0.3724	-0.64%	0.4921	0.4936	-0.30%
D	0.3	0.2996	0.13%	0.3	0.3002	-0.07%	0.3	0.3007	-0.23%

Table 0.9. Validation of expected waiting times for the LM

with $y_i = (0.9, 0.8, 0.8, 0.7)$

	$\eta = 4,\ q^{(r,\theta)} = Case\ 1$			$\eta = 4,\ q^{(r,\theta)} = Case\ 2$			$\eta = 4,\ q^{(r,\theta)} = Case\ 3$		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.4286	0.434	-1.24%	0.4286	0.4303	-0.40%	0.4286	0.4263	0.54%
L	16.245	17.4238	-6.77%	2.904	2.9165	-0.43%	10.4713	10.3293	1.37%
2	0	0	0.00%	0	0	0.00%	0	0	0.00%
3	0	0	0.00%	0	0	0.00%	0	0	0.00%
4	0.4452	0.4419	0.75%	0.2961	0.2982	-0.70%	0.4198	0.4157	0.99%
5	0.4452	0.4435	0.38%	0.2961	0.2945	0.54%	0.4198	0.4162	0.86%
6	0.2871	0.2894	-0.79%	0.161	0.1616	-0.37%	0.1944	0.1975	-1.57%
7	0.4571	0.4527	0.97%	0.2396	0.2356	1.70%	0.4961	0.4985	-0.48%
8	0.5797	0.5757	0.69%	0.3374	0.3383	-0.27%	0.5433	0.5442	-0.17%
9	0.4147	0.417	-0.55%	0.4564	0.4525	0.86%	0.5481	0.548	0.02%
10	1.7934	1.7878	0.31%	0.7981	0.7955	0.33%	1.5828	1.5656	1.10%
D	0.0882	0.0879	0.34%	0.0882	0.0877	0.57%	0.0882	0.087	1.38%

	$\eta = 5,\ q^{(r,\theta)} = Case\ 1$			$\eta = 5,\ q^{(r,\theta)} = Case\ 2$			$\eta = 5,\ q^{(r,\theta)} = Case\ 3$		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.4286	0.4268	0.42%	0.4286	0.4305	-0.44%	0.4286	0.4313	-0.63%
L	71.2578	67.3198	5.85%	3.4656	3.429	1.07%	15.241	15.0194	1.48%
2	0	0	0.00%	0	0	0.00%	0	0	0.00%
3	0	0	0.00%	0	0	0.00%	0	0	0.00%
4	0.4873	0.4868	0.10%	0.317	0.3172	-0.06%	0.4419	0.4375	1.01%
5	0.4873	0.483	0.89%	0.317	0.3167	0.09%	0.4419	0.4411	0.18%
6	0.2937	0.2949	-0.41%	0.1748	0.1744	0.23%	0.1985	0.1998	-0.65%
7	0.4793	0.4816	-0.48%	0.2608	0.2604	0.15%	0.5045	0.5053	-0.16%
8	0.6444	0.6452	-0.12%	0.3683	0.3672	0.30%	0.5543	0.5539	0.07%
9	0.5533	0.5528	0.09%	0.4998	0.5014	-0.32%	0.6761	0.6729	0.48%
10	2.1965	2.1897	0.31%	0.9086	0.9124	-0.42%	1.7657	1.7574	0.47%
D	0.0882	0.0877	0.57%	0.0882	0.0881	0.11%	0.0882	0.0871	1.26%

	$\eta = 6,\ q^{(r,\theta)} = Case\ 1$			$\eta = 6,\ q^{(r,\theta)} = Case\ 2$			$\eta = 6,\ q^{(r,\theta)} = Case\ 3$		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.4286	0.4281	0.12%	0.4286	0.4307	-0.49%	0.4286	0.4281	0.12%
L	10.5416	10.382	1.54%	3.9184	3.8172	2.65%	18.8776	18.1658	3.92%
2	0	0	0.00%	0	0	0.00%	0	0	0.00%
3	0	0	0.00%	0	0	0.00%	0	0	0.00%
4	0.5128	0.5103	0.49%	0.331	0.3274	1.10%	0.4521	0.4523	-0.04%
5	0.5128	0.5112	0.31%	0.331	0.3288	0.67%	0.4521	0.45	0.47%
6	0.2975	0.2983	-0.27%	0.1837	0.183	0.38%	0.2004	0.2025	-1.04%
7	0.4921	0.4904	0.35%	0.2747	0.2727	0.73%	0.5082	0.5018	1.28%
8	0.6828	0.6754	1.10%	0.3887	0.3873	0.36%	0.5592	0.56	-0.14%
9	0.6404	0.6381	0.36%	0.5286	0.5233	1.01%	0.7374	0.7297	1.06%
10	2.4803	2.4514	1.18%	0.9868	0.9832	0.37%	1.8546	1.8408	0.75%
D	0.0882	0.0865	1.97%	0.0882	0.0874	0.92%	0.0882	0.0885	-0.34%

Table 0.10. Validation of expected waiting times for the LM

with $y_i = (0.8, 0.7, 0.7, 0.6)$

	$\eta = 4,\ q^{(r,\theta)} = Case\ 1$			$\eta = 4,\ q^{(r,\theta)} = Case\ 2$			$\eta = 4,\ q^{(r,\theta)} = Case\ 3$		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.4286	0.4292	-0.14%	0.4286	0.4267	0.45%	0.4286	0.4301	-0.35%
L	3.828	3.8584	-0.79%	1.8111	1.7995	0.64%	3.4331	3.4575	-0.71%
2	0	0	0.00%	0	0	0.00%	0	0	0.00%
3	0	0	0.00%	0	0	0.00%	0	0	0.00%
4	0.3284	0.3276	0.24%	0.2376	0.236	0.68%	0.3159	0.3179	-0.63%
5	0.3284	0.3258	0.80%	0.2376	0.2361	0.64%	0.3159	0.317	-0.35%
6	0.2583	0.2564	0.74%	0.1402	0.1418	-1.13%	0.1863	0.185	0.70%
7	0.3875	0.3856	0.49%	0.2077	0.207	0.34%	0.4483	0.4546	-1.39%
8	0.4631	0.4698	-1.43%	0.2911	0.2964	-1.79%	0.4243	0.4256	-0.31%
9	0.3168	0.3109	1.90%	0.3918	0.3855	1.63%	0.3869	0.3802	1.76%
10	0.9719	0.9725	-0.06%	0.525	0.5287	-0.70%	0.9027	0.903	-0.03%
D	0.0882	0.0876	0.68%	0.0882	0.0885	-0.34%	0.0882	0.0885	-0.34%

	$\eta = 5,\ q^{(r,\theta)} = Case\ 1$			$\eta = 5,\ q^{(r,\theta)} = Case\ 2$			$\eta = 5,\ q^{(r,\theta)} = Case\ 3$		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.4286	0.4262	0.56%	0.4286	0.4262	0.56%	0.4286	0.4251	0.82%
L	4.4304	4.4072	0.53%	1.948	1.9474	0.03%	3.6997	3.6963	0.09%
2	0	0	0.00%	0	0	0.00%	0	0	0.00%
3	0	0	0.00%	0	0	0.00%	0	0	0.00%
4	0.3445	0.3459	-0.40%	0.2467	0.247	-0.12%	0.3245	0.3256	-0.34%
5	0.3445	0.3433	0.35%	0.2467	0.2494	-1.08%	0.3245	0.3246	-0.03%
6	0.262	0.2608	0.46%	0.1479	0.148	-0.07%	0.1887	0.1897	-0.53%
7	0.3994	0.3978	0.40%	0.2195	0.2198	-0.14%	0.4529	0.4545	-0.35%
8	0.4962	0.4975	-0.26%	0.3081	0.3067	0.46%	0.4339	0.4403	-1.45%
9	0.3883	0.3882	0.03%	0.4155	0.4181	-0.62%	0.4486	0.4445	0.92%
10	1.065	1.071	-0.56%	0.5644	0.5705	-1.07%	0.9501	0.9494	0.07%
D	0.0882	0.0881	0.11%	0.0882	0.0888	-0.68%	0.0882	0.0876	0.68%

	$\eta = 6,\ q^{(r,\theta)} = Case\ 1$			$\eta = 6,\ q^{(r,\theta)} = Case\ 2$			$\eta = 6,\ q^{(r,\theta)} = Case\ 3$		
Node	App.	Sim.	R.Error	App.	Sim.	R.Error	App.	Sim.	R.Error
1	0.4285	0.4292	-0.16%	0.4286	0.4265	0.49%	0.4286	0.4319	-0.76%
L	2.7554	2.7592	-0.14%	2.0307	2.039	-0.41%	3.8096	3.8998	-2.31%
2	0	0	0.00%	0	0	0.00%	0	0	0.00%
3	0	0	0.00%	0	0	0.00%	0	0	0.00%
4	0.3526	0.352	0.17%	0.2519	0.2542	-0.90%	0.3279	0.3312	-1.00%
5	0.3526	0.354	-0.40%	0.2519	0.2542	-0.90%	0.3279	0.3307	-0.85%
6	0.2638	0.2637	0.04%	0.1523	0.1532	-0.59%	0.1896	0.1901	-0.26%
7	0.4053	0.4079	-0.64%	0.2262	0.2264	-0.09%	0.4546	0.4521	0.55%
8	0.5129	0.5109	0.39%	0.3178	0.3184	-0.19%	0.4361	0.4389	-0.64%
9	0.4253	0.4159	2.26%	0.429	0.4299	-0.21%	0.473	0.4876	-2.99%
10	1.1147	1.1208	-0.54%	0.5874	0.5991	-1.95%	0.9688	0.9774	-0.88%
D	0.0882	0.0874	0.92%	0.0882	0.0889	-0.79%	0.0882	0.0868	1.61%

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