# COMPETITION IN TECHNOLOGY ADOPTION WITH REVENUE AND TECHNOLOGY UNCERTAINTY

by

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# ABSTRACT


## COMPETITION IN TECHNOLOGY ADOPTION WITH REVENUE AND TECHNOLOGY UNCERTAINTY

The thesis analyzes the investment decision on technology adoption under revenue and technology uncertainty. In a duopoly market, the firms make a decision on investing on a technology at the beginning of the time horizon. That is an irreversible investment, and as a consequence of their investment, firms invent the technology in an uncertain time in the future. This time depends on the amount of investment and the characteristics of the firms. When they adopt the technology, they start to collect revenue with that technology or technology related service. It is advantageous to be the first to adopt the technology, as the leaders get big pie usually. Also in this model, the revenue rates decrease by time with discrete market shocks, which causes the revenue uncertainty and limited economical life of the market. The leader company is affected less from those shocks so that the competition is about being the leader.

This model is analyzed under three different cases where the firms compete and set decisions simultaneously, they decide sequentially and a single decision maker decides on behalf of them in order to increase the total profit. It is shown that there is a single equilibrium of this model. Computational results and parametric analysis are fulfilled as well. It is shown that the companies get the highest expected profits in the first best case, simultaneous decision case and sequential decision case respectively.

# ÖZET


## BELİRSİZ KOŞULLAR ALTINDA TEKNOLOJİ REKABETİ

Bu tezde, belirsiz koşullar altında teknoloji yatırımı konusu incelenmektedir. Belirsizliğe yol açan hususlar, yatırım yapıldığı zaman projenin ne zaman sonlanarak teknolojinin edinileceği ve teknolojinin getireceği gelir miktarıdır. İki firmalı bir pazarın analiz edildiği çalışmada, firmalar ilk başta yatırım miktarlarını kararlaştırmalıdırlar. Yatırım yapıldıktan sonra vazgeçmek ve yatırılan tutarı geri almak mümkün değildir. Firmaların teknolojiye ne zaman sahip olacakları ise yatırdıkları tutara ve verimlilik gibi kendilerine özgü bazı özelliklere bağlıdır. Zaman geçtikçe pazarın gelir getirme oranının rastsal bir şekilde düştüğü varsayılmaktadır. Ancak, teknolojiyi daha önce edinip pazara ilk giren firma, bu durumdan daha az etkilenmektedir. Dolayısıyla, firmalar pazara ilk giren olmak için rekabet etmektedirler. Pazara ilk giren olmak ise daha fazla yatırım gerektireceği için, bazı koşullarda beklenen faydayı getirmeyebilmektedir.

Yapılan analiz sonucunda, firmaların aynı zamanda karar verdiği ve sırayla karar verdiği modeller analiz edilmiştir. Ayrıca, eğer iki firma toplam karlarını artırmak için çalışırlarsa ortaya çıkacak sonuç da incelenmiştir. Bunun sonucunda, problemin firmaların karını en çoklayan bir denge çözümü olduğu bulunmuştur. Ayrıca, belirtilen modeller için sayısal çözümler bulunmuş ve parametrelerin sonuca etkisi incelenmiştir.

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# LIST OF SYMBOLS/ABBREVIATIONS

| | |
|---|---|
| $\alpha$ | Interest rate |
| $\beta_{ij}$ | Multiplying factor at each market arrival for company $i$ |
| $\lambda$ | Arrival rate for market shocks |
| $\Pi_i$ | Expected total profit function for company $i$ |
| $i$ | Index for the companies |
| $j$ | State of being leader or follower |
| $t$ | Time |
| $c_i$ | Investment made at the beginning for company $i$ |
| $e_i$ | Efficiency factor for company $i$ |
| $r_i$ | Revenue rate at the beginning for company $i$ |
| $R_{ij}(t)$ | Revenue rate for company $i$ at time $t$ |
| $K_{ij}$ | Effective discount rate for company $i$ |
| $M(t)$ | No of arrivals occur by time $t$ |
| $x$ | Arrival time of the technology for the first company |
| $y$ | Arrival time of the technology for the second company |
| $f_1(x)$ | Density function for the arrival of the technology for the first company |
| $f_2(y)$ | Density function for the arrival of the technology for the second company |

# 1. INTRODUCTION

In the last century, technology has improved greatly. New technologies, therefore new products, have been introduced to the markets constantly. Technology developers make huge investments and the companies, using those technologies, have to decide carefully in which technology and when to invest.

Technology related markets have grown up greatly and reached huge amounts worldwide. Especially, telecommunication and information technology markets became the leading and steadily growing markets. As a consequence, investment decisions, research and development processes and demand management issues get to become extremely important.

Also, the technology related products are changing rapidly compared to the traditional products and sometimes replaced with the new developed products in quite short times. This makes investing in the right technologies and responding quickly to the demand the key points of the business.

In this study, a version of the investment problems in technology related fields is analyzed. This problem can also be considered as a kind of a research and development problem. Growing technology related markets and hot competition in those markets inspired us to analyze such models. Those problems, and the environments originated from, have some common properties. In general, return of the investment and the demand- thus, the revenue- is stochastic. Hence, competition between the agents becomes tougher.

In this thesis, firstly a basic model is set, which includes both technology and revenue related uncertainties (See Chapter 3). In this model, there are two companies which will invest on the same or substitutable technology. Number of the companies are limited to two so as to make the analysis deeply and decrease the level of algebraic complexity. As those companies operate in the same market regarding that specific technology, they compete with each other. All the information are common to both parties in this model and in the other models. Companies decide on the amount to invest on that technology at the beginning of the time horizon simultaneously, and no further investment is allowed in the future times. As the decisions are made simultaneously, companies will not know the investment level of the other company exactly. According to a probabilistic distribution, companies will invent the technology at a point in time successfully. In this case, a probabilistic distribution is used, which is directly linked with the amount of money invested and a company-specific efficiency coefficient; since companies may have different efficiency properties and the arrival time of the technology is directly affected by efficiency. According to the model, companies will get the technology successfully sooner or later, as exponential distribution structure is used.

After inventing the technology successfully, companies will start to accumulate revenue with the operations related to that technology. The rate of revenue obtained by a company depends on the conditions of the market (which is modeled through a stochastic structure as well), being leader or the follower in the market and company specific initial revenue rate.

It is modeled in the way that the revenue rates of the companies decrease by the external shocks introduced to the market. Those outside effects can be explained as the new foreign players entering into the market or development of some other substitute products or technologies which shrinks the size of the market analyzed. Both companies are exposed to the effects of those external shocks, but they are affected with respect to their own characteristics. That is, one of the companies may loose more share by those external shocks; or it may have more loyal customers. As a result, those characteristics affect the decision levels as well. Also it is assumed in the model that, the first inventor of the technology, which is the leader, loses less revenue at the time of revenue shrinks compared to being the second inventor. That is related with the fact that, if one enters into the market earlier, she gets more market share and more loyal customers rather than entering into the market after other players.

Therefore, the competition between those two companies will be on the issue of being the leader. In order to get the technology first, companies should invest more money, but that may be also more costly. So the tradeoff is between the leadership and cost of investment. This problem is captured by the basic model.

In Chapter 3, this basic model is analyzed and extended. After the basic model is analyzed, first best solution of the system is characterized (Chapter 4). First best solution is the solution, in which the companies do not compete with each other and try to maximize their total profits. It is just like there is a single decision maker on top of those companies and makes the decisions on behalf of them in order to maximize the total expected profits. By treating in this way, one of them may sacrifice some of its profit, in order to increase total benefit. By this model, such a case is explored. Through this analysis, the total expected profit lost by the competition in the original model can be characterized and presented.

Then in Chapter 5, the basic model is analyzed in a principal agent setting such that one of the companies sets its investment level first and the other party sets its investment level later knowing the decision of the former one. In this environment, the first mover determines the market. Because, the first one knows the reaction of the follower company against its decision, as all the information is common in the problem. Therefore, the first mover chooses an investment level which maximizes its own profit knowing the response of the second mover. This analysis characterizes the environments in which those type of decisions are made in a leader-follower manner. Some markets with a big and natural leader resembles this model.

After those three models are characterized, the results and effects of parameters on the results are analyzed (Chapter 6). Computational analysis is made in this chapter with different parameter sets. Both single effects of the parameters are displayed, and the combined effect of some different parameters together analyzed. Also by this way, different effects of different models on the solutions can be presented clearly. First best solution naturally leads to the highest total profit. By numerical results, the total expected profit yielded by the other models are calculated and those results are compared with the first best solution. Thus, the effect of competition on the optimum results can be characterized.

# 2. LITERATURE SURVEY

Our study can be classified under different related topics in the literature. This is an irreversible investment problem basically. Also it is a technology adoption problem and a research and development problem in a sense. Because, there is a decision of technology adoption and the companies do not get the technology as they make investment. There is a time gap between the time of investment and handling the technology, which can be introduced as a research and development process. Similar studies can be seen widely in this point of view.

In the literature, there is a respected number of studies about technology adoption. Since this topic is finding more application areas in the real life, different versions of such problems are characterized and analyzed.

Technology adoption is mainly an investment problem, where the investment is irreversible; that is, when one invests, she cannot take her money back. Also there are some uncertainties in the problem nature due to the revenues, costs or time of availability which make it harder to solve. In this type of investment problems, almost in all cases, revenue stream is considered as stochastic (Kamien and Schwartz 1972, Reinganum 1981, Fudenberg and Tirole 1985). In our study, the revenue schemes are assumed to be stochastic as well with stochastic jumps downwards.

Dixit and Pindyck (1994) made respected contributions to the literature regarding investment with revenue related uncertainties. Based on a real options approach, they developed a solution procedure to the problem of irreversible investment with uncertainty. In this approach, the present value of making investment immediately or the present value of waiting for a while to adopt is compared. In our study, no further investment is allowed and the parties set their investment levels at the beginning. Therefore, only the present values of the total expected profit is maximized in our model.

In Kamien and Schwartz (1972) Model, revenue rates are assumed to decrease by time as it is the case in our model; but it decreases in a deterministic way in their model, which has an exponential shape. Whereas, in our model, revenue rates decrease by discrete jumps of which timing is stochastic, and representing the external effects on the system.

McDonald and Siegel (1986) have studied investment problem as well, and they developed a real option based approach similar to Dixit and Pindyck. In their model, both revenue and the investment cost is stochastic and the agent can invest at any time and get the revenue. McDonald and Siegel compares the benefits and costs at any time and decide to invest when the benefits exceed the costs at a certain level. In our model, parties make the investment at the beginning and start getting benefit in a future time which is probabilistic. Our model is more like a research and development problem from this point of view. There is a time gap between the time of investment and the time of getting revenue.

Moretto (2008) also studied the effect of uncertainty on the investment decisions in a competitive market. In his model, there are many identical companies to invest. Moretto also considered the externalities introduced to the market. At the beginning, positive externalities dominate the market when its size is small. That is, when the market is small, entry of a new firm increases the profitability of the others. On the other hand, for a large market size, negative externalities dominate the market, which means the entry of a new firm lowers the profitability of others. In this model, firms may enter to the market at any time and their decision is irreversible, so this is just an investment problem. Moretto found out that in the first case, under the positive externalities, firms invest simultaneously after the profitability of the market has developed sufficiently. In the second case, firms invest sequentially, as the market profitability develops. Also, it is concluded that the optimal start-up size of the market is determined by the profit level of an initial investment under negative externalities.

Nielsen (2002) also studied the effect of competition on the investment decision in negative and positive externalities cases. He has set up an oligopoly model and shown that at the beginning, firms invest sequentially as the market develops; whereas, they invest simultaneously when the market has developed enough. Conclusions are very same with the study of Moretto.

As a different version of the investment problems, there is also considerably high number of studies regarding technology investment or research and development processes in the literature.

Reinganum (1981) has made a research on technology adoption under competition. In this paper, there are two firms to adopt a new cost reducing technology. Cost of the technology decreases by the time and if one of the firms adopt it before the other, it is expected to make substantial profits at the expense of the other firm. Therefore, there is a tradeoff between the foregone profits due to the delaying adoption of the technology and cost of the adoption. This setting is very similar to our model. In two identical firms case, it is shown that there exists a pair of equilibria in which one of the firms adopts the technology relatively early. In non-identical firms case, it is shown that there always exists a Nash equilibrium in which the net value of being first is positive.

Murto (2006) made a research on technology adoption where there are technological uncertainties along with revenue related uncertainties as well. In this study, technology progress always in the positive direction, that is, the best available technology never gets worse. So, Murto researched the evolution of a technology over the generations which is not the case in our model. Revenue is uncertain here but it may increase or decrease stochastically. Murto suggests a kind of optimal stopping type solution: When the ratio of the present values of revenue stream and investment cost is higher than a determined constant, the investment is done. In this thesis, we are interested in the single technology and the companies decide on to invest on that technology or not, and if they invest, how much to invest. Therefore, we can develop an equilibrium solution for both companies.

Krankel *et al.* (2006) and Alvarez and Stenbacka (2001) have related studies including both types of technology and revenue related uncertainties as well. In Krankel *et al.* successive products are introduced to the market where both the revenue stream and the technological improvements are stochastic like it is the case in Murto (2006) model. They develop a threshold policy, in which the new technology is introduced to the market when the incumbent technology is under the threshold level. In the Alvarez and Stenbacka model, there is a multi-stage technology project which evolves according to a stochastic process. Authors propose a solution to this problem based on the real options approach as well.

There are also some different types of studies about technology adoption and innovation. Cohen *et al.* (1996) studied on a model about new product development in order to determine the optimal time to enter the market. They characterized the new product development process as an additive multistage process, which has mainly design and process stages in their model. At each stage, product performance increases depending on the length of the time at that stage, size of the team and input performance level from the prior stage. In this model, the tradeoff is between the performance of the product and the timing to enter the market. If development process lengthens in time, a higher performance product is obtained but some profit is lost due to the delay. They found that it is possible to set the optimal time-to-market and the product performance target, both depending on the firm's cost structure and market characteristics. Also, it is shown that it is optimal to concentrate efforts on the most productive stage.

Rahman and Loulou (2001) also studied on a technology investment problem which has a multistage decision process. They analyzed the effect of expectations regarding technological progress on a firm's technology adoption decision. They developed a duopoly model with two stages. At the beginning of each stage, a new cost reducing technology is available with a charge. Firms either keep the current technology or upgrade it at those times. Consequently, they found that deterministic expectations of a better future technology delay the adoption of the current available technology in duopoly. In cases of asymmetric equilibrium, it is seen that the firm acquiring more technology gets better profit compared to the rival.

Lukach *et al.* (2007) made a research to analyze the effect of an entrant threat on research and development investments in a monopoly market with a two period decision model. For this analysis, they developed a model with technological uncertainty and threat of a new entry with a new technology. In their model, there is an incumbent company producing with a unit cost. Also there is another company outside the market which produces with a lower cost using a new technology, but the entry to the market has a cost. There are two periods in the model, and at the beginning, the incumbent company decides whether to start or not an irreversible research and development project. If he decides to start, he invests some money at the beginning, and some more at the second stage. The investment amount in the second stage is uncertain, which can be high or low depending on the first stage project success. As a result, authors found that under the threat of a new technology entry, a greater technical uncertainty positively affects the decision to start an research and development project.

In the literature above multi-stage and multi-product technology investment problems have been studied. Our study differs in the way that no multi-decision structure is involved in it. There is only one technology or technology related product and the companies make their decisions just at the beginning of the time horizon by considering their expected future profit. And another different point of our model is revenue rate characterization. We assume that the revenue rates decrease by the time and it decreases with stochastic jumps. Those jumps also can be considered as newcomers to the market or emerging substitute markets which shrinks the market of interest.

Related to the research and development or technology adoption, there are some other kind of researches analyzing the problem with different point of views as well. Afonso *et al.* (2008) have studied the effects of time-to-market along with the target costing issues on the new production development success. Target cost is determined based on a decided target price and target margin. In order to do this analysis, they made a research including an electronic questionnaire among Portugese manufacturing companies. As a result of this research, it is seen that both target costing and time-to-market are correlated to new product development success. Therefore, by target costing and reduction of time-to-market together, companies may have considerable advantages. Also , in this paper, it is seen that time-to-market is not significantly correlated with target costing, which means that time-to-market can be affected by other practices or techniques as well.

Silipo and Weiss (2005) studied cooperation and competition issues in research and development processes. They developed a model of the firms in research and development projects with spillovers and uncertainty. According to this setting, homogeneous firms may cooperate in three ways. They can jointly agree on the level of the project, they can share information, they can set up joint research facilities. It is shown that, in a clear pattern, incremental spillovers lead to cooperation; whereas, when the spillovers are offsetting, competition is preferable. In our thesis, no cooperation is considered; but the first best solution is analyzed by only changing the objectives of the firms.

Boldrin and Levine (2008) studied on the cases where the innovation can be done under the perfect competition, while most of the studies about the topic focus on the monopoly and oligopoly environments. In the research, Fruits, mp3's, books and pharmaceuticals are some examples of perfectly competitive innovation. They analyzed different cases of such markets and as a consequence, it is asked if copyright or patents are a good idea from a social point of view.

Theotoky (1998) analyzed the competition in research and development processes under uncertainty where imitation is easy. She has developed a mixed duopoly model in which one of the firms- public one- tries to maximize social welfare and the other- private one- tries to maximize his own profit. In this model, once one of the firms invents the technology, the other one imitates it easily with a quite low cost. She found that the public firm invests more in mixed duopoly and the private firm invests less also compared to the private duopoly. Thus, relative to social optimum, public firm overinvests while private firm underinvests in research and development in the mixed duopoly.

As a result, our study is a combination of irreversible investment problems and technology development problems. It is separated from pointed studies in the revenue structure, availability of the technology and influence of being the first entrant to the market. Revenue rates in our model reduces with discrete stochastic jumps and the amount of that decrease depends on being leader or follower in the market, which is a quite sensible assumption. Also, the companies handle the technology in a continuous stochastic time which depends on the amount of investment as well. In chapter 3, our model is characterized.

# 3. BASIC MODEL

In the main model of the thesis, there are two companies competing in the same market, which uses a certain technology related product or service. At the beginning of the time horizon, none of the companies has the technology, and they will make an investment in order to get it in the future. Timing of the arrival of the technology is probabilistic. Mean time for the arrival time depends on the amount of investment made, which is the decision variable, and the efficiency of the companies, which is a parameter. This probabilistic structure is characterized by the exponential distribution. Therefore, the companies will get the technology at a point in time successfully, if they invest on it.

When the companies obtain the technology, they will start to collect revenue from that technology or technology related service; but the rate of this revenue is stochastic as well. In the models throughout the thesis, an external effect is introduced to the models regarding the revenue rates of the companies as well. According to this assumption, with random timings, there occurs some natural market shocks, which decrease the revenue rates of the companies. Amount of the decrease depends on the company itself, and the state of being leader or the follower in the market. Here, being leader means getting the technology first; because the companies actively take place in the market after they get the technology. Occurrence of the market shocks is characterized by the Poisson Process in the model.

In the following section, this main model is described as well as the model parameters and the decision variables.

### 3.1. Model Description and Assumptions

Table 3.1 presents the notation used in the model.

Table 3.1. Parameters and Variables

| | |
|---|---|
| $i$ : | index of the firms, $i = 1, 2$ |
| $c_i$ : | investment made at $t = 0$ by the firm $i$ (Decision variable) |
| $\alpha$ : | interest rate |
| $r_i$ : | revenue rate for the firm $i$ at $t = 0$. It will decrease by a multiplier at each natural market shock |
| $j$ : | state of being leader or follower in getting the technology, $j = 1, 2$ |
| $R_{ij}(t)$ : | revenue rate at time $t$ for company $i$. $R_{ij}(t) = r_i$ at $t = 0$ |
| $\beta_{ij}$ : | revenue multiplier at each market shock for company $i$. $\beta_{ij} \in [0, 1]$. $\beta_{ij} = \beta_{i1}$ if company $i$ is the leader in getting the technology; $\beta_{ij} = \beta_{i2}$ if company $i$ is the follower in getting the technology. $\beta_{11}$ and $\beta_{21}$ cannot occur at the same time. |
| $M(t)$ : | number of market shocks occur by time $t$. (Poisson Process) |
| $\lambda$ : | rate of market shocks |
| $x$ : | arrival time of the technology for the first company |
| $y$ : | arrival time of the technology for the second company |
| $e_i$ : | efficiency coefficient for company $i$ |
| $f_1(x)$ : | density function for x. (Exponential) |
| $f_2(y)$ : | density function for y. (Exponential) |
| $\Pi_i$ : | expected profit for company $i$ (Objective function) |

In this basic model, both companies choose their $c_i$'s, which is amount of investment, simultaneously. Two of the parties are assumed to be rational, risk neutral and all the information (parameters etc.) are accepted as common knowledge in the basic model. Hence, there is a sort of perfect competition among those two. Both of the companies are going to try to maximize their expected profits accordingly. The decision will be made at time $t = 0$, and no further investment opportunity is available. After making the decision on the investment, they will wait for the arrival of the technology and start to accumulate profit when it is done. That is like making investment on technical and human resources and waiting for the result of the research and development project. Investment cost here is a sunk cost; when it is made, it cannot be recovered. In reality, some of the investments can be recovered partially or fully if it is broken up. For example, in some Research and Development projects, money invested can be held back, such as resalable technical stuff. However, most of the investments cannot be recovered, or it is not reasonable to try to recover. In this model, also in most of the literature, it is assumed that after the investment is made, it is not withdrawn and the money cannot be taken back, investment is irreversible.

Also, according to this model, since the arrival time of the technology is characterized by the exponential distribution, each company will get the technology at some time; and this time depends on the money invested and efficiency of the company. $x$ is the arrival time of the technology for the first company, and $y$ is the arrival time of the technology for the second company as mentioned above. Those are exponential variables which have the following density functions:

$$f_1(x) = \frac{c_1}{e_1}\, e^{\frac{-c_1}{e_1}x}, \qquad x \geq 0 \tag{3.1}$$

$$f_2(y) = \frac{c_2}{e_2}\, e^{\frac{-c_2}{e_2}y}, \qquad y \geq 0 \tag{3.2}$$

In this structure, mean time for the arrival of the technology is $\left(\frac{e_1}{c_1}\right)$ for the first company and $\left(\frac{e_2}{c_2}\right)$ for the second company respectively. It means that, expected time for the technology availability decreases exponentially by the amount of money invested, and increases by the efficiency coefficient. First part of this comment is highly reasonable. When the amount of investment is increased, it is usual that the expected time of getting technology successfully decreases (Kamien and Schwartz 1972). Also that decrease should not be linearly proportional to the money, that is, every incremental money invested brings a lower decrease on the expected time. Expected time depends on the efficiency coefficient as well. Lower $e_i$ means that company $i$ is more efficient, that is, with the same amount of investment, it is more likely to invent the technology first. This parameter is a little tricky one. Its unit is the reverse of the monetary unit. It is a measure for a company to convert money into a successful investment. Therefore it is not an easy one to measure. However, by considering former experiences of the companies and through benchmarking, this coefficient can be estimated.

For the rest of the derivations, two new terms will be defined at this point in order to simplify the equations:

$$C_i = \frac{c_i}{e_i}, \qquad i = 1, 2 \tag{3.3}$$

Using this term makes the derivations look neater. But also, those $C_i$ can be interpreted as the effective value of investments of the companies. Companies invest an amount of $c_i$ as mentioned before. However, this amount shortens the expected arrival time of the technology depending on the efficiency factor $e_i$. Here, it can be said that $e_i$ normalizes the effect of investment $c_i$ on the expected arrival time of the technology. For example, if the company is as inefficient as double in comparison, it has to make double investment as well, to get the same probability of the success.

When the companies get the technology, they will start to earn revenue. Each company will get revenue by the rate of $R_{ij}(t)$ at time $t$ as mentioned before. Also, at time $t = 0$, $R_{ij}(t)$ is a known constant, which is $r_i$.

$$R_{ij}(0) = r_i, \qquad i = 1, 2 \qquad j = 1, 2 \tag{3.4}$$

At $t = 0$, companies do not have the technology; but if they do, they would get $r_i$ revenue at a time. After the clock begins to work and time $t$ differs from 0, $R_{ij}(t)$

gets a random value. $R_{ij}(t)$ is characterized as follows at any time $t$:

$$R_{ij}(t) = r_i \, \beta_{ij}^{M(t)}$$

In this equation, $i$ stands for the company, and $j$ stands for the state of being the leader or the follower for company $i$ as it is the usual case.

$M(t)$ in this structure is the number of natural market shocks, which is an integer. $M(t)$ is a Poisson Process with rate $\lambda$. That is, revenue rate of company $i$ is assumed to be diminished by $\beta_{ij}$ at each market shock. This assumption is reasonable in the sense that, each market has new opportunities at the beginning, especially technology related markets, and as time goes on, profitability of the market decreases because of reasons like the market starts to saturate; substitute products emerge; or the products become less attractive.

Also, by this structure, outside effects are being included into the model. $M(t)$ can also be interpreted as the newcomers to the market, so that they take away a share of the market, and therefore revenue rates decrease.

Another important point which affects the revenue rate is the condition of $\beta_{ij}$. For company $i$, its revenue rate is multiplied by $\beta_{i1}$ if it is the leader and by $\beta_{i2}$ if it is the follower. In other words, $(1 - \beta_{ij})$ is the proportion of lost revenue, or lost customers, due to the market shocks. It is assumed that;

$$\beta_{i1} \geq \beta_{i2} \tag{3.5}$$

That is, a company loses less proportion of its revenue because of the external market shocks, if it is the leader compared to being the follower. This is also the key element which makes being leader or follower different in the issue of profitability.

This assumption can be interpreted in this way as well: Customers of market's leader generally become more loyal, as it is more experienced in the market. Therefore, at the time of market shocks, e.g. newcomers into the market, the older the company is, it is more likely to keep higher proportion of its customers compared to being follower. Besides, $\beta_{ij}$ also depends on the company itself. Some companies may have relatively higher rate of keeping their customers compared to others, maybe they have a better customer relationship service, although it is the leader or the follower. However, at any time, being leader is more advantageous rather than being follower for a specific company in terms of getting higher revenue. But sometimes, to become the leader may be more costly, and this is the key strategy of the competition in this model.

As a result of these assumptions, expected revenue rate of company $i$ gets to be as follows:

$$E[R_{ij}(s)] = r_i \; E[\beta_{ij}^{M(s)}], \qquad s \geq 0 \tag{3.6}$$

$E[\beta_{ij}^{M(s)}]$ can be written in an open form as it is known that $M(s)$ is Poisson($\lambda$) process. When the expectation operation is applied, it gets to be as follows:

$$\begin{aligned} E[\beta_{ij}^{M(s)}] &= \sum_{n=0}^{\infty} \beta_{ij}^{n} \, \frac{\lambda s^{n} e^{-\lambda s}}{n!} \\ &= e^{-\lambda} \sum_{n=0}^{\infty} \frac{[\beta_{ij} \lambda s]^{n}}{n!} \\ &= e^{-s(\lambda(1-\beta_{ij}))} \end{aligned} \tag{3.7}$$

Accordingly, expected revenue rate at any time for company $i$ will be in the following form:

$$E[R_{ij}(s)] = r_i \; e^{-s(\lambda(1-\beta_{ij}))}, \qquad s \geq 0 \tag{3.8}$$

Figure 3.1 shows an instance of the revenue rate for the first company ($R_{1j}(t)$). As can be seen in the figure, at time $t = 0$, revenue rate of the first company equals

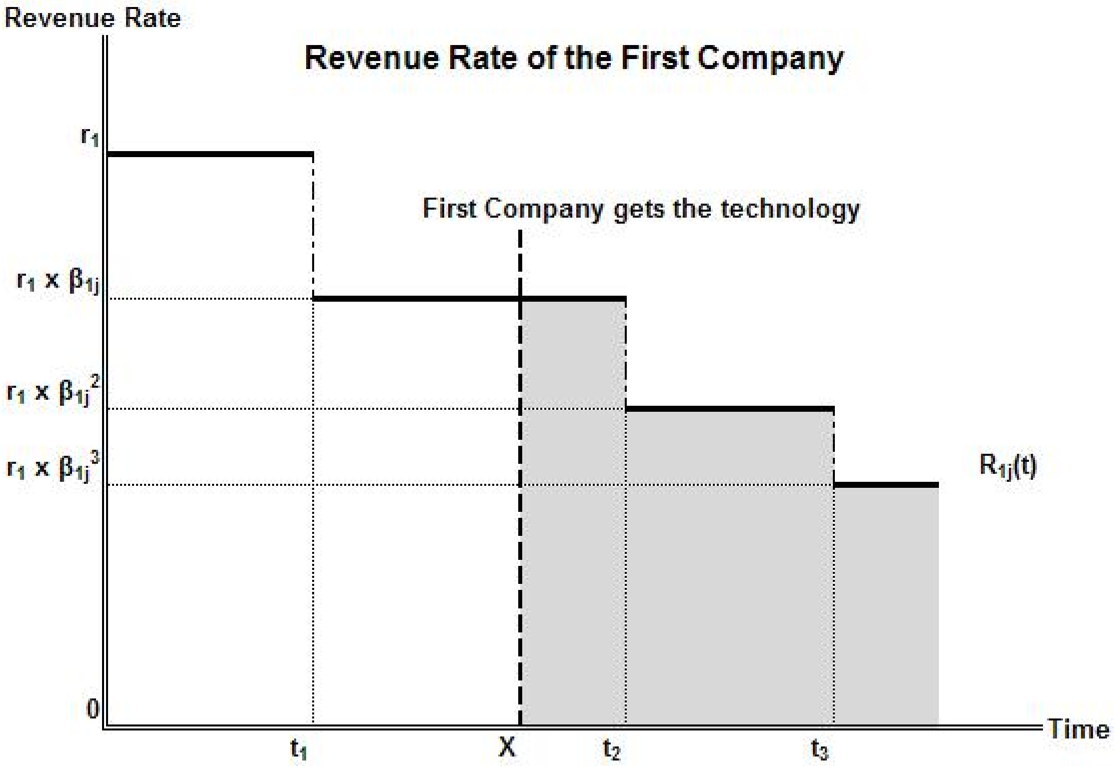


Figure 3.1. An Instance of the Revenue Rate for the First Company

to $r_1$. At $t_1$, $t_2$ and $t_3$ in the figure, there occur external market arrivals. Those arrivals are described as the outside effects which decreases the revenue rates (maybe the possible market shares) of the companies. At each market arrival, revenue rate of the first company is discriminated by the factor $\beta_{1j}$. Here $j$ refers to the state of being leader or not. It equals 1 if the first company gets the technology first, and equals 2 if the first company gets the technology after than the second company. Revenue rate diminishes by $\beta_{1j}$ at each market arrivals constantly. If the first company invents the technology and enters into the market at time $X$, as it is shown in the figure, it starts to collect revenue starting at that time $X$. Also, the total amount of revenue collected by the first company equals to the shaded area in the figure. As a result, in order to get higher revenue, companies should invent the technology firstly and as early as possible. Of course that is costly and the tradeoff is between this cost and the revenue.

Knowing all the information above, companies will primarily try to maximize their expected profits, which is basically the difference of revenues and costs. Both parties know their own expected profit function and the other company's expected profit functions, which are going to be derived in the next section. Then, they will decide on their investment level simultaneously, considering the reaction of their competitors. Hopefully, there will be an equilibrium level, at which both companies want to invest as much as the equilibrium solution found, not a different amount.

### 3.2. Derivation of the Expected Profit Functions

According to the assumptions and derivations in section 3.1, profit functions can be derived in a straightforward manner. Basically, as both companies have the same structure mathematically, their profit functions will be symmetrical, only differing in the parameters and variables.

Present value of the expected profit function for the first company, which is basically the difference between expected total discounted revenue and the money invested, is as the following. Notice that all the values are discounted to the present value with interest rate $\alpha$.

$$\begin{aligned} \Pi_1(c_1, c_2) \;=\; & \int_0^\infty f_2(y) \;\; \left\{ \int_0^y \left( \int_x^\infty e^{-\alpha s} E\left[R_{11}(s)\right] \, ds \right) f_1(x)\, dx \right. \\ & \left. + \int_y^\infty \left( \int_x^\infty e^{-\alpha s} E\left[R_{12}(s)\right] \, ds \right) f_1(x)\, dx \right\} dy \; - c_1 \qquad (3.9) \end{aligned}$$

where

$R_{11}(s)$ is the revenue rate for the first company if it is the leader,

$R_{12}(s)$ is the revenue rate for the first company if it is the follower.

Equation (3.9) is derived by taking expectation on $x$ and $y$, which are the times of the technology availability to the companies respectively. In the outer integral, time of the availability is considered between 0 and $\infty$ for the second company, as this is from the first company's point of view. Then, for $x$, which is the time of the availability for the first company, expectation operation is applied. $x$ may be either equal or smaller than $y$, or greater than $y$. The first case means the leadership of the first company and vice versa.

The most inner integral, which is $(\int e^{-\alpha s} E\left[R_{1j}(s)\right]\ ds)$, is the expected discounted revenue of the company. It is discounted with the constant interest rate $\alpha$. Here, $j$ equals 1 if the company is the leader, that is $x < y$; and equals 2 if the company is the follower. The only cost, therefore the negative term in the profit function, is the investment cost, $c_1$ in the case of the first company.

In the same way, expected total profit for the second company can be derived as follows:

$$\begin{aligned} \Pi_2(c_1, c_2) &= \int_0^\infty f_1(x) \quad \left\{ \int_0^x \left( \int_y^\infty e^{-\alpha s} E\left[R_{21}(s)\right]\ ds \right) f_2(y)\ dy \right. \\ &\quad \left. + \int_x^\infty \left( \int_y^\infty e^{-\alpha s} E\left[R_{22}(s)\right]\ ds \right) f_2(y)\ dy \right\} dx \ - c_2 \qquad (3.10) \end{aligned}$$

In equation (3.8), it is shown that $E\left[R_{ij}(s)\right] = r_i e^{-s(\lambda(1-\beta_{ij}))}$. Consequently, expected discounted total revenue for the first company, given a $x$ value is as the following:

$$\begin{aligned} ER_{1j}(x) &= \int_x^\infty e^{-\alpha s} E\left[R_{1j}(s)\right]\ ds \\ &= \int_x^\infty e^{-\alpha s}\ r_1 E\left[\beta_{1j}^{M(s)}\right]\ ds \qquad (3.11) \end{aligned}$$

By utilizing equation (3.8) and after some algebra, expected discounted total revenue for the first company, given a $x$ value turns out to be as the following:

$$\begin{aligned} ER_{1j}(x) &= \int_x^\infty e^{-\alpha s}\, r_1 e^{-s(\lambda(1-\beta_{1j}))}\, ds \\ &= \int_x^\infty r_1 e^{-s(\alpha+\lambda(1-\beta_{1j}))}\, ds \\ &= \frac{r_1}{\alpha+\lambda(1-\beta_{1j})}\, e^{-x(\alpha+\lambda(1-\beta_{1j}))} \end{aligned} \tag{3.12}$$

In the above equation, $j$ equals 1 if the company is the leader, and equals 2 if the company is the follower. In the same way, expected discounted total revenue for the second company, given a $y$ value can be found to be:

$$ER_{2j}(y) = \frac{r_2}{\alpha+\lambda(1-\beta_{2j})}\, e^{-y(\alpha+\lambda(1-\beta_{2j}))} \tag{3.13}$$

For simplicity in the other derivations, a new notation can be defined as the following:

$$K_{ij} = \alpha + \lambda(1-\beta_{ij}) \tag{3.14}$$

Here, $i$ labels the company, and $j$ labels the state of being leader or follower. Although the term $K_{ij}$ has been used to simplify the equations, it has an interpretation as well. If the equation above is searched out carefully, it is seen that $K_{ij}$ is the total discount rate of the revenue. $\alpha$ in the equation is the interest rate, and naturally the revenue is discounted by this rate steadily. On the other hand, the term $\lambda(1-\beta_{ij})$ is the rate of revenue lost due to the natural market shocks. Those market shocks, which are the external elements of the environment, occur with rate $\lambda$. And at each market shock, revenue rate of the companies decreases by $(1-\beta_{ij})$. Therefore, at a unit time, the revenue rate diminishes by $\lambda(1-\beta_{ij})$ due to the market shocks only. As a result, the real discount rate that is applied to the revenue is $K_{ij}=\alpha+\lambda(1-\beta_{ij})$ in total.

Based on this notation, expected total discounted revenues will take the following form:

$$ER_{1j}(x)=\frac{r_1}{K_{1j}}e^{-xK_{1j}} \tag{3.15}$$

$$ER_{2j}(y)=\frac{r_2}{K_{2j}}e^{-yK_{2j}} \tag{3.16}$$

Also, as a result of this simplification, expected profit functions in (3.9) and (3.10) can be written in the following way:

$$\begin{aligned}\Pi_1(c_1,c_2) &= \int_0^\infty C_2e^{-C_2y}\left\{\int_0^y C_1e^{-C_1x}\,\frac{r_1}{K_{11}}e^{-xK_{11}}\,dx\right.\\ &\left.+\int_y^\infty C_1e^{-C_1x}\,\frac{r_1}{K_{12}}e^{-xK_{12}}\,dx\right\}dy\;-c_1\end{aligned} \tag{3.17}$$

$$\begin{aligned}\Pi_1(c_1,c_2) &= \int_0^\infty C_1e^{-C_1x}\left\{\int_0^x C_2e^{-C_2y}\,\frac{r_2}{K_{21}}e^{-yK_{21}}\,dy\right.\\ &\left.+\int_x^\infty C_2e^{-C_2y}\,\frac{r_2}{K_{22}}e^{-yK_{22}}\,dy\right\}dx\;-c_2\end{aligned} \tag{3.18}$$

After some algebra as below, expected profit function can be written in the following form, which is easier to make derivations on it:

$$\Pi_1(c_1,c_2) = \int_0^\infty C_2e^{-C_2y}\quad\left\{\quad\frac{r_1C_1}{K_{11}}\left[-\frac{1}{K_{11}+C_1}e^{-x(K_{11}+C_1)}\right]_0^y\right.$$

$$+ \frac{r_1 C_1}{K_{12}} \left[ - \frac{1}{K_{12} + C_1} e^{-x(K_{12}+C_1)} \right]_y^{\infty} \Bigg\} dy \; - c_1$$

$$= \int_0^{\infty} C_2 e^{-C_2 y} \quad r_1 C_1 \quad \Bigg\{ \quad \frac{1}{K_{11}\left(K_{11}+C_1\right)} - \frac{1}{K_{11}\left(K_{11}+C_1\right)} e^{-y(K_{11}+C_1)}$$

$$+ \frac{1}{K_{12}\left(K_{12}+C_1\right)} e^{-y(K_{12}+C_1)} \quad \Bigg\} dy \; - c_1 \tag{3.19}$$

If the integral operator is applied to the terms in the parentheses separately;

$$\begin{aligned} \Pi_1(c_1, c_2) \quad = \quad & \frac{r_1 C_1 C_2}{K_{11}\left(K_{11}+C_1\right)} \int_0^{\infty} e^{-C_2 y} \, dy \\ & - \frac{r_1 C_1 C_2}{K_{11}\left(K_{11}+C_1\right)} \int_0^{\infty} e^{-y(K_{11}+C_1+C_2)} \, dy \\ & + \frac{r_1 C_1 C_2}{K_{12}\left(K_{12}+C_1\right)} \int_0^{\infty} e^{-y(K_{12}+C_1+C_2)} \, dy \quad - c_1 \end{aligned} \tag{3.20}$$

When the integrals above are solved, expected profit function evolves to its final form as follows:

$$\Pi_1(c_1, c_2) \quad = \quad \frac{r_1 C_1}{K_{11}(K_{11}+C_1)} + \; r_1 C_1 C_2 \left\{ \frac{1}{K_{12}(K_{12}+C_1)(K_{12}+C_1+C_2)} \right.$$

$$\left. - \frac{1}{K_{11}(K_{11}+C_1)(K_{11}+C_1+C_2)} \right\} - c_1$$

$$\Pi_1(c_1, c_2) = r_1 C_1 \left\{ \frac{C_2}{K_{12}\left(K_{12}+C_1\right)\left(K_{12}+C_1+C_2\right)} + \frac{1}{K_{11}\left(K_{11}+C_1+C_2\right)} \right\} - c_1 \tag{3.21}$$

In the very similar way, expected total profit for the second company can be found as the following;

$$\Pi_2(c_1, c_2) = r_2 C_2 \left\{ \frac{C_1}{K_{22}\left(K_{22}+C_2\right)\left(K_{22}+C_2+C_1\right)} + \frac{1}{K_{21}\left(K_{21}+C_2+C_1\right)} \right\} - c_2 \tag{3.22}$$

By this way, expected profit functions have been characterized in terms of variables and parameters clearly. Expected profit functions depend on the decision variables $c_1$ and $c_2$ as can be seen above. Hence, both companies will set a decision on their investment level, given the decision of the other.

This problem can be solved if both companies set such $c_i$'s that, given the choice of the other, no one should deviate from its choice. That is, for this problem to be solved, there must be $(c_1, c_2)$ pairs, which are optimal for both of the companies at the same time; so that they pick this solution values simultaneously and know that this is their best choice. These $(c_1, c_2)$ pairs are the equilibrium solutions of the problem.

In order to solve this problem, firstly, the response functions- the optimal reaction $c_i$ curve of the company $i$ given the other one's decisions- has to be found. In the next section, response functions are derived based on the above profit functions (3.21) and (3.22).

### 3.3. Derivation of the Best Response Functions

Expected total profit functions for the companies have been found as can be seen in (3.21) and (3.22). It is mentioned that the expected profit functions depend on $c_1$ and $c_2$ both. Response function for the first company is the set of $c_1$'s, which are the optimal reactions to the given $c_2$'s; and vice versa for the second company.

Given a $c_2$ value, expected profit function for the first company is a single variable function in $c_1$. Therefore, in the classical way, $c_1$ values which satisfy $\frac{d}{dc_1}\Pi_1(c_1, c_2) = 0$, are the critical ones. These are candidates for the optimal solution. After this step, if it can be shown that $\Pi_1(c_1, c_2)$ is a concave function in $c_1$, it means that those candidate points are the optimum response values for the first company.

Response function for the first company can be derived as in the following sequence of equations:

$$c_1^*(c_2) = arg \max_{c_1} \Pi_1(c_1, c_2)$$

Hence:

$$\begin{aligned}
\frac{d}{dc_1}\Pi_1(c_1, c_2) &= \frac{r_1}{e_1}\left\{\frac{C_2}{K_{12}(K_{12}+C_1)(K_{12}+C_1+C_2)} + \frac{1}{K_{11}(K_{11}+C_1+C_2)}\right\} \\
&+ r_1 C_1 \left\{\frac{-C_2}{K_{12}(K_{12}+C_1)^2(K_{12}+C_1+C_2)}\frac{1}{e_1}\right. \\
&+ \frac{-C_2}{K_{12}(K_{12}+C_1)(K_{12}+C_1+C_2)^2}\frac{1}{e_1} \\
&+ \left.\frac{-1}{K_{11}(K_{11}+C_1+C_2)^2}\frac{1}{e_1}\right\} - 1 \\
&= \frac{r_1}{e_1}\left\{\frac{C_2}{K_{12}(K_{12}+C_1)(K_{12}+C_1+C_2)}\right.
\end{aligned}$$

$$- \frac{C_1\, C_2}{K_{12}(K_{12}+C_1)^2(K_{12}+C_1+C_2)^2} - \frac{C_1\, C_2}{K_{12}(K_{12}+C_1)(K_{12}+C_1+C_2)^2}$$

$$+ \frac{1}{K_{11}(K_{11}+C_1+C_2)} - \frac{C_1}{K_{11}(K_{11}+C_1+C_2)^2} \Bigg\} - 1 \quad (3.23)$$

Finally, after some algebraic simplifications in the above equation, response function of the first company takes the following form:

$$\begin{aligned} \frac{d}{dc_1}\Pi_1(c_1,c_2) &= \frac{r_1 C_2}{e_1}\left\{\frac{K_{12}^2+K_{12}C_2-C_1^2}{K_{12}(K_{12}+C_1)^2(K_{12}+C_1+C_2)^2}\right\} \\ &\quad + \frac{r_1}{e_1}\left\{\frac{K_{11}+C_2}{K_{11}(K_{11}+C_1+C_2)^2}\right\} - 1 \\ &= 0 \end{aligned} \quad (3.24)$$

The set of points $(c_1, c_2)$, that makes the first derivative of the expected profit function $\left(\frac{d}{dc_1}\Pi_1(c_1,c_2) = 0\right)$ as in the equation (3.24), is the best response set for the first company given $c_2$ values, if it is proven that the second order conditions hold and the expected profit function is concave.

As both of the companies have the same structure of expected profit functions, response function for the second company can be derived in the similar way with the equation (3.24) above.

$$\begin{aligned} \frac{d}{dc_2}\Pi_2(c_1,c_2) &= \frac{r_2 C_1}{e_2}\left\{\frac{K_{22}^2+K_{22}C_1-C_2^2}{K_{22}(K_{22}+C_2)^2(K_{22}+C_2+C_1)^2}\right\} \\ &\quad + \frac{r_2}{e_2}\left\{\frac{K_{21}+C_1}{K_{21}(K_{21}+C_2+C_1)^2}\right\} - 1 \end{aligned}$$

$$= \quad 0 \tag{3.25}$$

An illustration of the response functions can be seen in Figure 3.2.

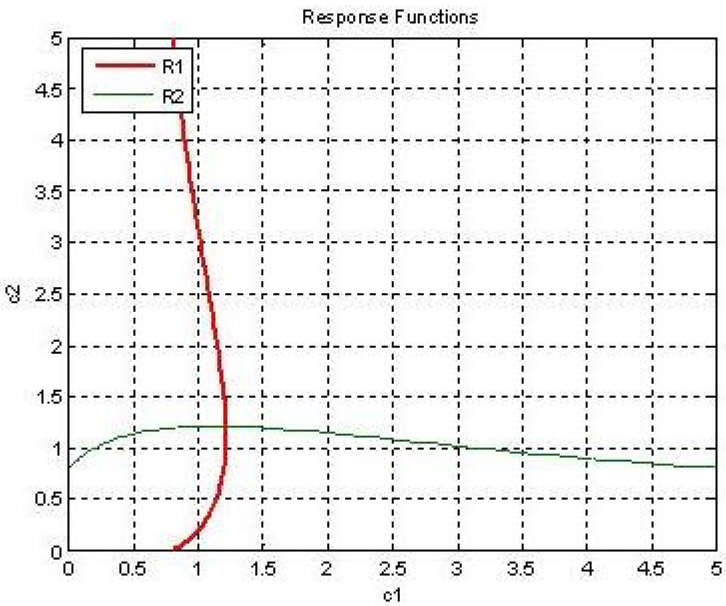


Figure 3.2. An Illustration of the Response Functions

In the response function equations, e.g. equation (3.24), $c_1$ is a function of $c_2$ and it can be written as a function of $c_2$. ($c_1 = h(c_2)$) However, the response function derived is in a highly complicated form and the function $c_1 = h(c_2)$ can not be written explicitly. Yet, $c_1(c_2)$ exists in the response function implicitly. Also, those equations can be solved easily by using numerical methods. Throughout the thesis, some examples of the numerical solutions is presented.

In addition, in order for the solution found using equations (3.24) and (3.25) to be the optimum responses, it has to be shown that expected profit functions are concave. If they are concave, the response values that make the first derivatives 0 are optimum solutions to maximize the expected total profits. Then the $(c_1, c_2)$ pairs found by the first order condition equations form the best response function.

In the next section, it is proven that the expected profit functions are concave, so that the solutions $(c_1(c_2))$ and $(c_2(c_1))$ found through first order conditions are optimum reactions (best responses) if they exist. Surely, some other proofs are needed to show that this system has at least one equilibrium. Existence of the equilibrium is proven in Section 3.4 and uniqueness of the equilibrium is proven in Section 3.5.

### 3.4. Concavity of the Expected Profit Functions

If it can be shown that the expected profit functions are concave, first order conditions found in section 3.3 are the optimum decision levels given the decision of the other party. In the next step, if it can also be shown that there are any $(c_1, c_2)$ pairs which is optimum for both of the companies, it means that this problem has a solution, namely an equilibrium.

Proposition 1: *The expected profit functions are concave and the first order conditions are the best response functions.*

Proof: In order to show that the expected profits are concave, following conditions must hold:

$$\frac{d^2}{dc_1^2}\,\Pi_1(c_1, c_2) \;\leq\; 0, \tag{3.26}$$

$$\frac{d^2}{dc_2^2}\,\Pi_2(c_1, c_2) \;\leq\; 0 \tag{3.27}$$

First order condition for the first company has been shown in equation (3.24). Also, the second order condition, which is the concavity condition of the expected total profit, can be proven in the following sequence of equations:

$$\frac{d}{dc_1}\Pi_1(c_1, c_2) \;=\; \frac{r_1 C_2}{e_1}\left\{\frac{K_{12}^2 + K_{12}C_2 - C_1^2}{K_{12}\,(K_{12}+C_1)^2\,(K_{12}+C_1+C_2)^2}\right\}$$

$$+ \frac{r_1}{e_1} \left\{ \frac{K_{11} + C_2}{K_{11} \ (K_{11} + C_1 + C_2)^2} \right\} - 1$$

$$= \ 0$$

$$\begin{aligned} \frac{d^2}{dc_1^2} \Pi_1(c_1, c_2) \ &= \ \frac{r_1 C_2}{e_1} \left\{ \frac{-2C_1}{K_{12} \ (K_{12} + C_1)^2 \ (K_{12} + C_1 + C_2)^2} \ \frac{1}{e_1} \right. \\ &+ \frac{-2 \, ( \ K_{12} \ (K_{12} + C_1)^2 \ (K_{12} + C_1 + C_2)^2 \ )}{K_{12} \ (K_{12} + C_1)^3 \ (K_{12} + C_1 + C_2)^2} \ \frac{1}{e_1} \\ &+ \left. \frac{-2 \, ( \ K_{12} \ (K_{12} + C_1)^2 \ (K_{12} + C_1 + C_2)^2 \ )}{K_{12} \ (K_{12} + C_1)^2 \ (K_{12} + C_1 + C_2)^3} \ \frac{1}{e_1} \right\} \\ &+ \frac{r_1}{e_1} \left\{ \frac{-2 \ (K_{11} + C_2)}{K_{11} \ (K_{11} + C_1 + C_2)^3} \ \frac{1}{e_1} \right\} \end{aligned} \tag{3.28}$$

After proper algebraic simplifications, equation (3.28) take its following final form:

$$\begin{aligned} \frac{d^2}{dc_1^2} \Pi_1(c_1, c_2) \ &= \ -2\frac{r_1}{e_1^2} \ \left\{ \frac{K_{11} + C_2}{K_{11} \ (K_{11} + C_1 + C_2)^3} \right. \\ &+ \frac{C_1 \ C_2}{K_{12} \ (K_{12} + C_1)^2 \ (K_{12} + C_1 + C_2)^2} \\ &+ \frac{C_2 \ (K_{12}^2 + K_{12} C_2 - C_1^2)}{K_{12} \ (K_{12} + C_1)^3 \ (K_{12} + C_1 + C_2)^2} \\ &+ \left. \frac{C_2 \ (K_{12}^2 + K_{12} C_2 - C_1^2)}{K_{12} \ (K_{12} + C_1)^2 \ (K_{12} + C_1 + C_2)^3} \right\} \end{aligned} \tag{3.29}$$

In a similar sequence of equations, second order derivative of the expected function in $c_2$ for the second company can be shown in the following form:

$$\begin{aligned}\frac{d^2}{dc_2^2}\,\Pi_2(c_1,c_2) &= -2\frac{r_2}{e_2^2}\left\{\frac{K_{21}+C_1}{K_{21}\,(K_{21}+C_2+C_1)^3}\right.\\ &+\frac{C_2\,C_1}{K_{22}\,(K_{22}+C_2)^2\,(K_{22}+C_2+C_1)^2}\\ &+\frac{C_1\,\,(K_{22}^2+K_{22}C_1-C_2^2)}{K_{22}\,(K_{22}+C_2)^3\,(K_{22}+C_2+C_1)^2}\\ &\left.+\frac{C_1\,\,(K_{22}^2+K_{22}C_1-C_2^2)}{K_{22}\,(K_{22}+C_2)^2\,(K_{22}+C_2+C_1)^3}\right\} \qquad (3.30)\end{aligned}$$

In order to show the concavity of the expected profit functions, the conditions in (3.26) and (3.27) must hold. As $r_i$ and $e_i$ are nonnegative terms by their definition, one must show that the terms between curly brackets in the above equations (3.29) and (3.30) are nonnegative.

Second order condition for the first company, which is the equation (3.29) above, can be written in the following way as well by grouping the terms with similar denominator in a single term; so that it can be seen whether it is negative or not more easily:

$$\begin{aligned}\frac{d^2}{dc_1^2}\,\Pi_1(c_1,c_2) &= -2\frac{r_1}{e_1^2}\left\{\frac{K_{11}+C_2}{K_{11}\,(K_{11}+C_1+C_2)^3}\right.\\ &+\frac{C_1\,C_2\,(K_{12}+C_1)\,(K_{12}+C_1+C_2)}{K_{12}\,(K_{12}+C_1)^3\,(K_{12}+C_1+C_2)^3}\\ &\left.+\frac{C_2\,(K_{12}^2+K_{12}C_2-C_1^2)\,(2K_{12}+2C_1+C_2)}{K_{12}\,(K_{12}+C_1)^3(K_{12}+C_1+C_2)^3}\right\} \qquad (3.31)\end{aligned}$$

After grouping negative and positive terms together, the above equation turns out to be as follows:

$$\frac{d^2}{dc_1^2}\,\Pi_1(c_1,c_2) \quad = \quad -2\frac{r_1}{e_1^2} \quad \left\{ \frac{(K_{11}+C_2)\;(K_{11}+C_2)^3}{K_{11}\;(K_{11}+C_2)^3\;(K_{11}+C_1+C_2)^3} \right.$$

$$- \quad \frac{C_1^2\;C_2\;(K_{12}+C_1)}{K_{12}\;(K_{12}+C_1)^3\;(K_{12}+C_1+C_2)^3} \tag{3.32}$$

$$+ \quad \left. \frac{C_2\;(K_2+C_1)\;(2K_2^2+3K_2C_2+K_2C_1)\;+\;K_2C_2^3}{K_{12}\;(K_{12}+C_1)^3\;(K_{12}+C_1+C_2)^3} \right\} \tag{3.33}$$

In the above equation, only negative term in the curly brackets is the middle part which is labeled as (3.32). With a careful look, it can be seen that the first part of the equation contains this negative portion, with only a difference of $K_{11}$ replacing with the $K_{12}$.

Now, let a new function be defined very similar to the (3.32).

$$g(K) = \frac{C_1^2\;C_2\;(K+C_1)}{K\;(K+C_1)^3\;(K+C_1+C_2)^3} \tag{3.34}$$

In the function definition above, as long as the parameters are positive, $g(K)$ is positive as well.

According to this definition, the second order condition for the first company can also be written as the following:

$$\frac{d^2}{dc_1^2}\,\Pi_1(c_1,c_2) = -2\frac{r_1}{e_1^2} \quad \left\{ \cdots \;+\; g(K_{11}) \;-\; g(K_{12}) \right\} \tag{3.35}$$

where " $\cdots$ " is a positive portion. Also, it is known that the functions $g(K_{11})$ and $g(K_{12})$ are positive as well. Then, if one can show that $g(K_{11}) \geq g(K_{12})$, it is proven that the expected profit function is concave and the solution set found by the first order condition in section (3.3) is optimum.

In the function (3.34), it is apparent that $g(K)$ decreases if $K$ increases. It is mentioned in section (3.1) that;

$$K_{ij} = \alpha + \lambda(1 - \beta_{ij})$$

Also, in this model, it is assumed that $\beta_{i2} \geq \beta_{i1}$ as mentioned in the same section (3.1) by the equation (3.5).

As a result of this definition and the assumption, it can be seen that

$$K_{11} < K_{12}$$

Therefore, it holds that

$$g(K_{11}) > g(K_{12})$$

and as a result, it is proven that

$$\frac{d^2}{dc_1^2}\,\Pi_1(c_1, c_2) \;<\; 0$$

hence, the expected function is concave.

Since, the second company has the same structure of profit function and first order condition, all the derivation made for $\Pi_1(c_1, c_2)$ hold for $\Pi_2(c_1, c_2)$ as well. Thus, the solution found by first order condition of the second company is also optimum, as the profit function is found to be concave.

It is mentioned above that, proving first order solutions to be optimum is not necessary for solving this problem. Also, it must be shown that this system has a unique equilibrium; which is a single $(c_1, c_2)$ pair, optimal for both companies. In the following section, uniqueness of the solution found by the first order conditions is going to be proven.

### 3.5. Uniqueness of the Equilibrium

Throughout the thesis, it is shown that the expected profit functions are concave and companies can find their best responses $c_i(c_j)$ by first order condition functions. However, for this problem to have a solution, the best response functions, also shown in (3.24) and (3.25), have to intersect at least once. That is, there has to be at least one $(c_1, c_2)$ pair that satisfies the best response functions (3.24) and (3.25) simultaneously.

If there are more than one such $(c_1, c_2)$ pairs, or the best response functions intersect more than once, then it means that the problem has multiple equilibria.

In this section, it is shown that this problem has a unique equilibrium, meaning that the best response functions intersect only once. There are several ways to show that. (See Cachon and Netessine (2003), Fudenberg and Tirole (1995))

One of those widely used methods is the algebraic methods approach. If it can be shown that, slope of the one of the response function is higher than that of the other always, it means that they can only intersect once if they do. In this model, as $c_i(c_j)$ cannot be written explicitly, so as to find the slope of the best response function, implicit differentiation has to be utilized. Nevertheless, as the best response function is a high-degree and complicated in $c_i$'s, this method could not be used.

Another method in order to show the uniqueness of the equilibrium of two functions is supermodularity (Topkis 1998). In the game theory area, this method is used commonly as well. According to the supermodularity approach, two functions have a unique equilibrium if the following conditions hold simultaneously;

$$\frac{\partial^2}{\partial c_1 \partial c_2} \Pi_1(c_1, c_2) > 0$$

$$\frac{\partial^2}{\partial c_1 \partial c_2} \Pi_2(c_1, c_2) > 0$$

.

Similar to the supermodularity approach, Cachon and Netessine (2003) show that, if two functions with two variables satisfy the following conditions, this system has a unique equilibrium.

$$-1 \; < \; \frac{\partial^2}{\partial c_1 \partial c_2} \Pi_1(c_1, c_2) \; < \; 1 \tag{3.36}$$

$$-1 \; < \; \frac{\partial^2}{\partial c_1 \partial c_2} \Pi_2(c_1, c_2) \; < \; 1 \tag{3.37}$$

Proposition 2: *The conditions in (3.36) and (3.37) hold for this system and there exists a unique equilibrium.*

Proof: In order to show that the conditions above hold, firstly, cross partial of the expected profit functions should be derived. This derivation can be done in the following way for the first company. Since the companies have common structure of expected profit functions, the same derivation can be applied to the second company symmetrically as it is the case throughout this thesis.

First order derivative of the expected profit function has been found in section (3.3) above. On top of that equation, second order partial derivative in $c_2$ can be found as below:

$$\begin{aligned} \frac{d}{dc_1}\Pi_1(c_1, c_2) \quad &= \quad \frac{r_1 C_2}{e_1} \left\{ \frac{K_{12}^2 + K_{12}C_2 - C_1^2}{K_{12}\,(K_{12} + C_1)^2\,(K_{12} + C_1 + C_2)^2} \right\} \\ &+ \frac{r_1}{e_1} \left\{ \frac{K_{11} + C_2}{K_{11}\,(K_{11} + C_1 + C_2)^2} \right\} - 1 \end{aligned}$$

$$\frac{\partial^2}{\partial c_1 \partial c_2}\,\Pi_1(c_1,c_2) = \frac{r_1}{e_1 e_2}\left\{\frac{K_{12}^2 + K_{12}C_2 - C_1^2}{K_{12}\,(K_{12}+C_1)^2\,(K_{12}+C_1+C_2)^2}\right\}$$

$$+ \frac{r_1 C_2}{e_1}\left\{\frac{K_{12}}{K_{12}\,(K_{12}+C_1)^2\,(K_{12}+C_1+C_2)^2}\,\frac{1}{e_2}\right.$$

$$\left. + \frac{-2\,(K_{12}^2 + K_{12}C_2 - C_1^2)}{K_{12}\,(K_{12}+C_1)^2\,(K_{12}+C_1+C_2)^3}\,\frac{1}{e_2}\right\}$$

$$+ \frac{r_1}{e_1}\left\{\frac{1}{K_{11}\,(K_{11}+C_1+C_2)^2}\,\frac{1}{e_2} + \frac{-2\,(K_{11}+C_2)}{K_{11}\,(K_{11}+C_1+C_2)^3}\,\frac{1}{e_2}\right\}$$

$$\frac{\partial^2}{\partial c_1 \partial c_2}\,\Pi_1(c_1,c_2) = \frac{r_1}{e_1 e_2}\left\{\frac{K_{12}^2 + K_{12}C_2 - C_1^2}{K_{12}\,(K_{12}+C_1)^2\,(K_{12}+C_1+C_2)^2}\right. \tag{3.38}$$

$$+ \frac{K_{12}C_2}{K_{12}\,(K_{12}+C_1)^2\,(K_{12}+C_1+C_2)^2} \tag{3.39}$$

$$- \frac{2C_2\,(K_{12}^2 + K_{12}C_2 - C_1^2)}{K_{12}\,(K_{12}+C_1)^2\,(K_{12}+C_1+C_2)^3} \tag{3.40}$$

$$\left. + \frac{1}{K_{11}\,(K_{11}+C_1+C_2)^2} - \frac{2\,(K_{11}+C_2)}{K_{11}\,(K_{11}+C_1+C_2)^3}\right\}$$

There are two group of fractions in the last equation above: the ones with $K_{12}$ terms in the denominator and the ones with $K_{11}$ in the denominator.

If the equations in the first three lines above, (3.39), (3.39) and (3.40), which are the ones with $K_{12}$ terms in the denominator, are grouped in a single fraction, the result will be as the following after some algebra:

$$= \frac{(K_{12}^2 + K_{12}C_2 - C_1^2)\,(K_{12}+C_1+C_2-2C_2) \;+\; K_{12}C_2\,(K_{12}+C_1+C_2)}{K_{12}\,(K_{12}+C_1)^2\,(K_{12}+C_1+C_2)^3}$$

$$= \frac{K_{12}^2 + K_{12}C_2 + C_1C_2 - C_1^2}{K_{12}\,(K_{12}+C_1)\,(K_{12}+C_1+C_2)^3} \tag{3.41}$$

Also, terms with $K_{11}$ in the denominator in the above function, which is the last line, can be written as a single fraction which is similar to the (3.41):

$$\begin{aligned} &= \frac{(K_{11}+C_1+C_2)\; - \; 2(K_{11}+C_2)}{K_{11}\,(K_{11}+C_1+C_2)^3} \\ &= \frac{(C_1-K_{11}-C_2)\,(K_{11}+C_1)}{K_{11}\,(K_{11}+C_1)\,(K_{11}+C_1+C_2)^3} \\ &= -\frac{K_{11}^2 + K_{11}C_2 + C_1C_2 - C_1^2}{K_{11}\,(K_{11}+C_1)\,(K_{11}+C_1+C_2)^3} \end{aligned} \tag{3.42}$$

Notice that the equations (3.41) and (3.42) have the same structure. And finally, cross partial of the expected profit function for the first company gets out to be as the following:

$$\begin{aligned} \frac{\partial^2}{\partial c_1 \partial c_2}\,\Pi_1(c_1,c_2) &= \frac{r_1}{e_1e_2}\left\{ \frac{K_{12}^2 + K_{12}C_2 + C_1C_2 - C_1^2}{K_{12}\,(K_{12}+C_1)\,(K_{12}+C_1+C_2)^3} \right. \\ &\qquad \left. - \frac{K_{11}^2 + K_{11}C_2 + C_1C_2 - C_1^2}{K_{11}\,(K_{11}+C_1)\,(K_{11}+C_1+C_2)^3} \right\} \end{aligned} \tag{3.43}$$

With the same method applied above, cross partial of the expected profit for the second company company can be written as following as well:

$$\begin{aligned} \frac{\partial^2}{\partial c_1 \partial c_2}\,\Pi_2(c_1,c_2) &= \frac{r_2}{e_2e_1}\left\{ \frac{K_{22}^2 + K_{22}C_1 + C_2C_1 - C_2^2}{K_{22}\,(K_{22}+C_2)\,(K_{22}+C_2+C_1)^3} \right. \\ &\qquad \left. - \frac{K_{21}^2 + K_{21}C_1 + C_2C_1 - C_2^2}{K_{21}\,(K_{21}+C_2)\,(K_{21}+C_2+C_1)^3} \right\} \end{aligned} \tag{3.44}$$

At this point, in order to show that the conditions (3.36) and (3.37) above hold, value the function (3.43) must be between −1 and 1.

It is algebraically apparent that the equations (3.41) and (3.42) are smaller than 1 with different signs. Absolute value of the some of two numbers smaller than 1 with different signs is always smaller than 1 as well. Therefore, in the function (3.43) the value of the terms in the curly bracket sums up to a number which is smaller than 1 in absolute value. This situation is also valid for the equation (3.44). All in all, it can be said that if the conditions below hold, conditions in (3.36) and (3.37) automatically hold; and those are the sufficient conditions to prove the uniqueness of the solution found by the first order conditions in section (3.3).

$$\frac{r_1}{e_1 e_2} \leq 1$$

$$\frac{r_2}{e_1 e_2} \leq 1$$

These assumptions can be made with no limitation to the model by scaling the parameters $r_i$ and $e_i$ properly.

As it is also proven in this section that if this problem has a solution, it is unique; one can easily solve this model by solving following best response functions simultaneously:

$$\begin{aligned} \frac{d}{dc_1}\Pi_1(c_1, c_2) &= \frac{r_1 C_2}{e_1}\left\{\frac{K_{12}^2 + K_{12}C_2 - C_1^2}{K_{12}(K_{12}+C_1)^2(K_{12}+C_1+C_2)^2}\right\} \\ &\quad + \frac{r_1}{e_1}\left\{\frac{K_{11}+C_2}{K_{11}(K_{11}+C_1+C_2)^2}\right\} - 1 \\ &= 0 \end{aligned} \tag{3.45}$$

$$\frac{d}{dc_2}\Pi_2(c_1,c_2) = \frac{r_2C_1}{e_2}\left\{\frac{K_{22}^2+K_{22}C_1-C_2^2}{K_{22}(K_{22}+C_2)^2(K_{22}+C_2+C_1)^2}\right\} + \frac{r_2}{e_2}\left\{\frac{K_{21}+C_1}{K_{21}(K_{21}+C_2+C_1)^2}\right\} - 1 = 0 \quad (3.46)$$

Those functions are very hard to solve algebraically, but it can be solved easily by numerical methods. The solution will be a $(c_1, c_2)$ pair, which are the decision levels of the companies. This solution is an equilibrium, which means that any of the companies can not get better by changing its decision level, given that the other one's decision is not changed.

By this way, the solution procedure of the basic model is analyzed throughout this chapter. In the next chapter, first best solution of the same model will be analyzed. That is, how the solution changes if the companies only try to maximize their total profit.

# 4. FIRST BEST SOLUTION

In the previous chapter, a solution procedure to the basic model described has been developed. According to the model, companies decide on their investment levels simultaneously and they compete each other.

In this chapter, the solution in the case of the companies trying to maximize their total profit and not competing with each other is analyzed. Here, it is just like there is a single decision maker and determines decisions on behalf of the companies in order to maximize the total profit. All other properties are assumed to be the same here as well. Again, the companies (or the single decision maker) set the investment levels at the same time. The difference in this model that their objective is not to maximize their own expected profits separately, but the total profit of the market. By this analysis, the effect of the competition on the profitability of the market can be understood more clearly.

Expected total profit functions have been derived in the previous chapter, (3.21) and (3.22). Consequently, the total profit, which is characterized by $E\Pi(c_1, c_2)$ will be as follows:

$$\begin{aligned}
\Pi(c_1, c_2) \;&=\; \Pi_1(c_1, c_2) \;+\; \Pi_2(c_1, c_2) \\
&=\; r_1 C_1 \left\{ \frac{C_2}{K_{12}\,(K_{12}+C_1)\,(K_{12}+C_1+C_2)} \;+\; \frac{1}{K_{11}\,(K_{11}+C_1+C_2)} \right\} \\
&\quad + r_2 C_2 \left\{ \frac{C_1}{K_{22}\,(K_{22}+C_2)\,(K_{22}+C_2+C_1)} \;+\; \frac{1}{K_{21}\,(K_{21}+C_2+C_1)} \right\} \\
&\quad - c_1 \;-\; c_2 \qquad (4.1)
\end{aligned}$$

In order to maximize the total channel profit in $c_1$ and $c_2$, first order conditions may be utilized. Those are the partial derivatives of the expected total profit according to $c_1$ and $c_2$ separately, which equal to 0.

First order conditions can be written in closed form as the following:

$$\frac{\partial}{\partial c_1}\Pi(c_1,c_2) = \frac{\partial}{\partial c_1}\Pi_1(c_1,c_2) \; + \; \frac{\partial}{\partial c_1}\Pi_2(c_1,c_2) = 0 \tag{4.2}$$

$$\frac{\partial}{\partial c_2}\Pi(c_1,c_2) = \frac{\partial}{\partial c_2}\Pi_1(c_1,c_2) \; + \; \frac{\partial}{\partial c_2}\Pi_2(c_1,c_2) = 0 \tag{4.3}$$

In the above conditions, $\frac{\partial}{\partial c_1}\Pi_1(c_1,c_2)$ and $\frac{\partial}{\partial c_2}\Pi_2(c_1,c_2)$ have been found in the previous chapter, which are the best response functions of the companies in the basic model. But, the terms $\frac{\partial}{\partial c_1}\Pi_2(c_1,c_2)$ and $\frac{\partial}{\partial c_2}\Pi_1(c_1,c_2)$ have not been derived yet. Those terms also can be derived in the following way:

$$\Pi_1(c_1,c_2) = r_1 C_1 \left\{ \frac{C_2}{K_{12}\;(K_{12}+C_1)\;(K_{12}+C_1+C_2)} + \frac{1}{K_{11}\;(K_{11}+C_1+C_2)} \right\} - c_1$$

$$\begin{aligned} \frac{\partial}{\partial c_2}\Pi_1(c_1,c_2) \;&=\; r_1 C_1 \left\{ \frac{1}{K_{12}\;(K_{12}+C_1)\;(K_{12}+C_1+C_2)}\,\frac{1}{e_2} \right. \\ &\left. + \frac{-C_2}{K_{12}\;(K_{12}+C_1)\;(K_{12}+C_1+C_2)^2}\,\frac{1}{e_2} + \frac{-1}{K_{11}\;(K_{11}+C_1+C_2)^2}\,\frac{1}{e_2} \right\} \end{aligned}$$

After some algebra, this function turns out to be as the following:

$$\frac{\partial}{\partial c_2}\Pi_1(c_1,c_2) = \frac{r_1 C_1}{e_2} \left\{ \frac{1}{K_{12}\;(K_{12}+C_1+C_2)^2} - \frac{1}{K_{11}\;(K_{11}+C_1+C_2)^2} \right\} \tag{4.4}$$

Also, in the same way, $\frac{\partial}{\partial c_1}\Pi_2(c_1, c_2)$ can be derived as follows:

$$\frac{\partial}{\partial c_1}\Pi_2(c_1, c_2) = \frac{r_2 C_2}{e_1}\left\{\frac{1}{K_{22}\,(K_{22}+C_1+C_2)^2} - \frac{1}{K_{21}\,(K_{21}+C_1+C_2)^2}\right\} \tag{4.5}$$

By inserting the solutions of the functions (4.4) and (4.5) into the expected total channel profit function (4.1), first order condition for the first best solution gets out to be in the following form:

$$\begin{aligned}\frac{\partial}{\partial c_1}\Pi(c_1, c_2) &= \frac{r_1 C_2}{e_1}\left\{\frac{K_{12}^2 + K_{12}C_2 - C_1^2}{K_{12}(K_{12}+C_1)^2(K_{12}+C_1+C_2)^2}\right\} \\ &+ \frac{r_1}{e_1}\left\{\frac{K_{11}+C_2}{K_{11}(K_{11}+C_1+C_2)^2}\right\} \\ &+ \frac{r_1 C_1}{e_2}\left\{\frac{1}{K_{12}\,(K_{12}+C_1+C_2)^2} - \frac{1}{K_{11}\,(K_{11}+C_1+C_2)^2}\right\} - 1 \\ &= 0 \end{aligned} \tag{4.6}$$

$$\begin{aligned}\frac{\partial}{\partial c_2}\Pi(c_1, c_2) &= \frac{r_2 C_1}{e_2}\left\{\frac{K_{22}^2 + K_{22}C_1 - C_2^2}{K_{22}(K_{22}+C_2)^2(K_{22}+C_2+C_1)^2}\right\} \\ &+ \frac{r_2}{e_2}\left\{\frac{K_{21}+C_1}{K_{21}(K_{21}+C_2+C_1)^2}\right\} \\ &+ \frac{r_2 C_2}{e_1}\left\{\frac{1}{K_{22}\,(K_{22}+C_2+C_1)^2} - \frac{1}{K_{21}\,(K_{21}+C_2+C_1)^2}\right\} - c_2 \\ &= 0 \end{aligned} \tag{4.7}$$

Simultaneous solutions of the equations (4.6) and (4.7) gives the critical points.

Those points may be maximum, minimum or saddle points. In order to ensure that the solution found is a maximum, second order conditions have to be examined as well. However, as the functions are in a complicated form, the second order conditions could not be derived in an open form in this thesis.

Although we do not know the expected profit function is concave or quasi-concave so that the results found are the first best solutions or not, those first order conditions can be used to solve this problem numerically. It is very easy numerically to solve the equations (4.6) and (4.7), and to test the solution whether it is a maximum or not. In Chapter 6, numerical solutions of this model is analyzed.

# 5. PRINCIPAL AGENT MODEL

In game theory literature, Principal Agent models are widely studied. In this kind of a model, the players take their decision in a leader-follower manner. That is, the leader set her decision first, and knowing the decision of the leader, the follower makes her decision later.

In the basic model analyzed in Chapter 3, companies make their decision simultaneously at the beginning of the time horizon and compete with each other. However, in this chapter, the companies make their decisions one by one. For example, the first company sets its investment level; and with this knowledge, second company makes its decision. Nevertheless, although the companies give their decisions in a leader-follower manner, it is assumed that they invest money at the same time, just at the beginning of the time horizon, so that the solution structure developed above can be utilized here as well. Assumption of making decisions one by one but investing at the same time is reasonable practically. Because, in some markets, one of the players is the natural leader decision maker; and just after they decide on their actions, they are followed by the other parties. Maybe those leader decision makers are the ones with grater sources.

At this point, notion of being leader should not be confused. Here, one of the companies are normally accepted as the leading decision maker and the other follows the first in decision making. It can be defined that the first company sets its investment level ($c_1$) first, and the second company sets its decision level ($c_2$) later with the knowledge of $c_1$ in this context. But, it is assumed that they take those decisions in a very short time period- practically at the same time, and invest money at time $t = 0$ as in the previous models. Also, the time of getting the technology is again probabilistic, and entering the market first does not depend on whether being the leader decision maker or not in this model. Anyway, both companies are assumed to start the investment process at the same time.

As it is mentioned, in the principal agent model, it is defined in this context that, firstly the first company sets its decision $c_1$; and knowing $c_1$, the second company makes its decision $c_2$. But, when making the decision, the first company will know the best response of the second company, so that it will make a decision by this consideration.

As a result, solution of this problem will develop in this procedure. Best response functions of both companies are known, also characterized in Section 3.3. The first company, knowing the response of the second company against its any decision, will pick the most profitable point for itself on the best response curve of the second company.

Best response functions have been found as below:

$$\begin{aligned}\frac{d}{dc_1}\Pi_1(c_1,c_2) &= \frac{r_1C_2}{e_1}\left\{\frac{K_{12}^2+K_{12}C_2-C_1^2}{K_{12}(K_{12}+C_1)^2(K_{12}+C_1+C_2)^2}\right\}\\ &\quad+\frac{r_1}{e_1}\left\{\frac{K_{11}+C_2}{K_{11}(K_{11}+C_1+C_2)^2}\right\}-1\\ &= 0 \end{aligned} \tag{5.1}$$

$$\begin{aligned}\frac{d}{dc_2}\Pi_2(c_1,c_2) &= \frac{r_2C_1}{e_2}\left\{\frac{K_{22}^2+K_{22}C_1-C_2^2}{K_{22}(K_{22}+C_2)^2(K_{22}+C_2+C_1)^2}\right\}\\ &\quad+\frac{r_2}{e_2}\left\{\frac{K_{21}+C_1}{K_{21}(K_{21}+C_2+C_1)^2}\right\}-1\\ &= 0 \end{aligned} \tag{5.2}$$

By the equation (5.1), $c_1^*(c_2)$, which is the optimum decision $c_1$ of the first company given a $c_2$ value can be found. In this model, it is hard to show it explicitly but included implicitly. In the similar way, $c_2^*(c_1)$ can be found by the equation (5.2).

As the first company is the leading decision maker, it also determines the solution of the system in this procedure: It takes the best response of the second company, which is $c_2^*(c_1)$, and insert this result in its profit function. Then, it maximizes it s profit, and pushes the second company to its solution in a sense, by utilizing the best response function of the second company.

Therefore, the problem takes this form: The first company tries to maximize its profit function, which can be written in $c_1$ only:

$$\begin{aligned}\Pi_1(c_1) &= r_1 C_1 \left\{ \frac{C_2^*(c_1)}{K_{12}\left(K_{12}+C_1\right)\left(K_{12}+C_1+C_2^*(c_1)\right)} \right. \\ &\qquad \left. + \frac{1}{K_{11}\left(K_{11}+C_1+C_2^*(c_1)\right)} \right\} - c_1 \end{aligned} \tag{5.3}$$

where $C_2^*(c_1)$ is obtained from the following equation:

$$\begin{aligned}\frac{d}{dc_2}\Pi_2(c_1,c_2) &= \frac{r_2 C_1}{e_2}\left\{ \frac{K_{22}^2 + K_{22}C_1 - C_2^2}{K_{22}(K_{22}+C_2)^2(K_{22}+C_2+C_1)^2} \right\} \\ &\qquad + \frac{r_2}{e_2}\left\{ \frac{K_{21}+C_1}{K_{21}(K_{21}+C_2+C_1)^2} \right\} - 1 \\ &= 0 \end{aligned} \tag{5.4}$$

By this way, $c_2$ is a function of $c_1$ by the best response function of the second company; and the solution of the problem is characterized by the two equations above. $c_1$ is found by maximizing the equation (5.3) and $c_2$ is found by the equation (5.2).

This problem is very hard to solve algebraically. However, those resulting equations (5.3 and 5.2) can be solved by numerical methods easily. In Chapter 6, all three models developed in Chapters 3, 4 and 5 are analyzed numerically, so that it is shown how the optimum solutions change between those models.

# 6. NUMERICAL ANALYSIS

In this chapter, all three models developed in Chapters 3, 4 and 5 are analyzed and compared numerically. In the following sections, the way of solving those models are explained.

## 6.1. Numerical Solution of the Basic Model

In the Section 3.3 of Chapter 3, the solution of the model has been characterized. Shortly, it is the simultaneous solution of the following best response functions in $c_1$ and $c_2$.

$$\begin{aligned} \frac{d}{dc_1}\Pi_1(c_1,c_2) &= \frac{r_1C_2}{e_1}\left\{\frac{K_{12}^2+K_{12}C_2-C_1^2}{K_{12}(K_{12}+C_1)^2(K_{12}+C_1+C_2)^2}\right\} \\ &+ \frac{r_1}{e_1}\left\{\frac{K_{11}+C_2}{K_{11}(K_{11}+C_1+C_2)^2}\right\} - 1 \\ &= 0 \end{aligned} \quad (6.1)$$

$$\begin{aligned} \frac{d}{dc_2}\Pi_2(c_1,c_2) &= \frac{r_2C_1}{e_2}\left\{\frac{K_{22}^2+K_{22}C_1-C_2^2}{K_{22}(K_{22}+C_2)^2(K_{22}+C_2+C_1)^2}\right\} \\ &+ \frac{r_2}{e_2}\left\{\frac{K_{21}+C_1}{K_{21}(K_{21}+C_2+C_1)^2}\right\} - 1 \\ &= 0 \end{aligned} \quad (6.2)$$

Numerical solution to this system has been found using MATLAB. It is processed in this way: Firstly an initial $c_1$ point is picked. Given that $c_1$, correspondent $c_2$ is found by the second function and this $c_2$ is inserted into the first function again. By doing this operation back and forth, optimal $(c_1, c_2)$ pair is found easily. It is proven in Chapter 3 that the profit functions are concave in $c_1$ and $c_2$ respectively and the system has a unique equilibrium.

This $(c_1, c_2)$ pair is the optimum solution of the system. Upon this solution, the optimum total profit is found using following profit functions, which are derived in Section 3.2 of Chapter 3.

$$\begin{aligned}\Pi_1(c_1, c_2) &= r_1 C_1 \left\{ \frac{C_2}{K_{12}\,(K_{12} + C_1)\,(K_{12} + C_1 + C_2)} \right. \\ &\quad \left. + \frac{1}{K_{11}\,(K_{11} + C_1 + C_2)} \right\} - c_1 \end{aligned} \tag{6.3}$$

$$\begin{aligned}\Pi_2(c_1, c_2) &= r_2 C_2 \left\{ \frac{C_1}{K_{22}\,(K_{22} + C_2)\,(K_{22} + C_2 + C_1)} \right. \\ &\quad \left. + \frac{1}{K_{21}\,(K_{21} + C_2 + C_1)} \right\} - c_2 \end{aligned} \tag{6.4}$$

## 6.2. Numerical Analysis of the First Best Solution

In order to find the first best solution, the companies try to maximize their total profit only, without any concern of their single profits, as it is mentioned before. Result of the first best solution is analyzed by using MATLAB as well. First order conditions for the first best solution have been derived in Chapter 4. Those conditions are:

$$\begin{aligned}\frac{\partial}{\partial c_1}\Pi(c_1, c_2) &= \frac{r_1 C_2}{e_1} \left\{ \frac{K_{12}^2 + K_{12} C_2 - C_1^2}{K_{12}(K_{12} + C_1)^2 (K_{12} + C_1 + C_2)^2} \right\} \\ &\quad + \frac{r_1}{e_1} \left\{ \frac{K_{11} + C_2}{K_{11}(K_{11} + C_1 + C_2)^2} \right\} \\ &\quad + \frac{r_1 C_1}{e_2} \left\{ \frac{1}{K_{12}\,(K_{12} + C_1 + C_2)^2} - \frac{1}{K_{11}\,(K_{11} + C_1 + C_2)^2} \right\} - 1 \end{aligned}$$

$$= \quad 0 \tag{6.5}$$

$$\begin{aligned}
\frac{\partial}{\partial c_2}\Pi(c_1, c_2) &= \frac{r_2 C_1}{e_2}\left\{\frac{K_{22}^2 + K_{22}C_1 - C_2^2}{K_{22}(K_{22}+C_2)^2(K_{22}+C_2+C_1)^2}\right\} \\
&+ \frac{r_2}{e_2}\left\{\frac{K_{21}+C_1}{K_{21}(K_{21}+C_2+C_1)^2}\right\} \\
&+ \frac{r_2 C_2}{e_1}\left\{\frac{1}{K_{22}\left(K_{22}+C_2+C_1\right)^2} - \frac{1}{K_{21}\left(K_{21}+C_2+C_1\right)^2}\right\} - c_2 \\
&= 0
\end{aligned} \tag{6.6}$$

By solving these two equations simultaneously, critical points for the first best solution are found. Solution procedure of those equations is the same as the procedure in Section 6.1. However, $(c_1, c_2)$ pairs obtained from those conditions are not guaranteed to be optimum. Therefore, it is numerically justified by searching that the result is optimum.

After getting the optimum $(c_1, c_2)$, optimum profit levels are found by the expected total profit functions again:

$$\begin{aligned}
\Pi_1(c_1, c_2) &= r_1 C_1\left\{\frac{C_2}{K_{12}\left(K_{12}+C_1\right)\left(K_{12}+C_1+C_2\right)}\right. \\
&\left. + \frac{1}{K_{11}\left(K_{11}+C_1+C_2\right)}\right\} - c_1
\end{aligned} \tag{6.7}$$

$$\begin{aligned}
\Pi_2(c_1, c_2) &= r_2 C_2\left\{\frac{C_1}{K_{22}\left(K_{22}+C_2\right)\left(K_{22}+C_2+C_1\right)}\right. \\
&\left. + \frac{1}{K_{21}\left(K_{21}+C_2+C_1\right)}\right\} - c_2
\end{aligned} \tag{6.8}$$

### 6.3. Numerical Solution of the Principal Agent Model

The principal agent model is characterized in Chapter 5. In this model, the first company takes its decision first, and with that knowledge, second company takes its decision.

As mentioned in the related chapter, the first company determines the solution as it is the first mover. In summary, the first company chooses the most profitable $(c_1, c_2)$ pair on the best response function of the second company for itself. In other words, the first company pushes the second decision maker to the $(c_1, c_2)$ point on the best response function of the second company, at which $\Pi_1(c_1, c_2)$ is maximum.

As a result, the new model for the solution takes the following form:

$$max \quad \Pi_1(c_1, c_2) = r_1 C_1 \left\{ \frac{C_2}{K_{12}\ (K_{12} + C_1)\ (K_{12} + C_1 + C_2)} + \frac{1}{K_{11}\ (K_{11} + C_1 + C_2)} \right\} - c_1 \tag{6.9}$$

s.t.

$$\frac{d}{dc_2}\Pi_2(c_1, c_2) = \frac{r_2 C_1}{e_2} \left\{ \frac{K_{22}^2 + K_{22}C_1 - C_2^2}{K_{22}(K_{22} + C_2)^2 (K_{22} + C_2 + C_1)^2} \right\} + \frac{r_2}{e_2} \left\{ \frac{K_{21} + C_1}{K_{21}(K_{21} + C_2 + C_1)^2} \right\} - 1 = 0 \tag{6.10}$$

In MATLAB, the result of this model is found searching the $(c_1, c_2)$ point, which maximizes the $\Pi_1(c_1, c_2)$ above, on the the second equation- the best response function of the second company.

### 6.4. Numerical Results and Comments

In the previous sections, solution procedures have been explained. In this section, numerical results of those three models will be analyzed with different parameter sets. By this way, both the effects of the different models and the effects of the parameters can be seen.

In the first part of the computational analysis, effect of the single parameters on the equilibria is analyzed. In order to do that, a basic parameter set is determined and the results are compared to that basic set by changing one parameter level at a time. Firstly the goal is to see how the parameter levels change the optimum results, so that the analysis is made for identical companies. Then, the total expected profit of the market is shown with varying parameter levels both in table form and graphically. By this way, three different models can be compared under different parameter conditions.

As identical companies are analyzed to see the single effects of the parameters, in simultaneous move models, which are the basic model and the first best solution, companies have the same optimum decisions and expected profits. Whereas, in the principal agent model, their optimum decisions change as one of them decides before the other.

In the basic parameter set, the following parameter levels are used:

Table 6.1. Parameter Levels in the Basic Set

| | | | |
|---|---|---|---|
| $\lambda$ | 1,0 | $e_2$ | 1,0 |
| $\alpha$ | 0,1 | $\beta_{11}$ | 0,9 |
| $r_1$ | 1,0 | $\beta_{12}$ | 0,5 |
| $r_2$ | 1,0 | $\beta_{21}$ | 0,9 |
| $e_1$ | 1,0 | $\beta_{22}$ | 0,5 |

In Table 6.2, it can be seen that the expected profit is the highest in first best solution as it is expected, and the results are close in the other two models with a slight difference. But, for identical firms, the simultaneous model always yields more expected profit compared to the principal agent model.

Then, by changing $\lambda$, $\alpha$, $r$, $e$ and $\beta$ values one by one and keeping the others constant at the base set levels, effect of different characteristics of the model are analyzed.

### 6.4.1. Effect of the Arrival Rate of Market Shocks ($\lambda$) on the Optimum Results

In this part, $\lambda$ values are changed from 1 to 2 and the total expected profit levels in both three models are calculated. At each run, the first best result is accepted as 100%, and the percentage of the other results are calculated in comparison.

Table 6.2. Effect of $\lambda$ on the Optimum Results

| | **First Best** | **Simultaneous Game** | | **Principal Agent Game** | | | |
|---|---|---|---|---|---|---|---|
| $\lambda$ | **Total Profit** | **Total Profit** | **%** | **P1** | **P2** | **Total Profit** | **%** |
| **1,0** | 3,647 | 3,089 | 84,7 | 1,545 | 1,533 | 3,077 | 84,4 |
| **1,1** | 3,343 | 2,805 | 83,9 | 1,403 | 1,389 | 2,791 | 83,5 |
| **1,2** | 3,076 | 2,559 | 83,2 | 1,280 | 1,265 | 2,545 | 82,7 |
| **1,3** | 2,840 | 2,344 | 82,5 | 1,172 | 1,151 | 2,323 | 81,8 |
| **1,4** | 2,629 | 2,155 | 81,9 | 1,078 | 1,058 | 2,135 | 81,2 |
| **1,5** | 2,440 | 1,987 | 81,4 | 0,994 | 0,969 | 1,964 | 80,5 |
| **1,6** | 2,269 | 1,838 | 81,0 | 0,920 | 0,898 | 1,818 | 80,1 |
| **1,7** | 2,115 | 1,705 | 80,6 | 0,853 | 0,829 | 1,682 | 79,5 |
| **1,8** | 1,974 | 1,585 | 80,3 | 0,794 | 0,767 | 1,561 | 79,1 |
| **1,9** | 1,845 | 1,477 | 80,1 | 0,740 | 0,706 | 1,446 | 78,4 |
| **2,0** | 1,726 | 1,379 | 79,9 | 0,691 | 0,657 | 1,348 | 78,1 |

In Table 6.2, it is shown that, as $\lambda$ increases, the total expected profit of the firms decrease convexly in all of the models. Higher $\lambda$ means that the market shocks which reduce the revenue rates are expected to occur more often. Therefore, those results are sensible. When $\lambda$ increases, the firms tend to invest less. As a consequence, they are expected to get the technology later and collect less revenue. As a bigger $\lambda$ value means a faster diminishing revenue rate, the companies does not spend more money for the investment.

Also, when the percent total profits compared to the first best solution are considered, the total expected profits decrease in percentage with increasing $\lambda$. That is, the market loses efficiency if $\lambda$ grows.

The total expected profit levels can be seen in Figure 6.1 for three models as well. The total profits decrease with $\lambda$.

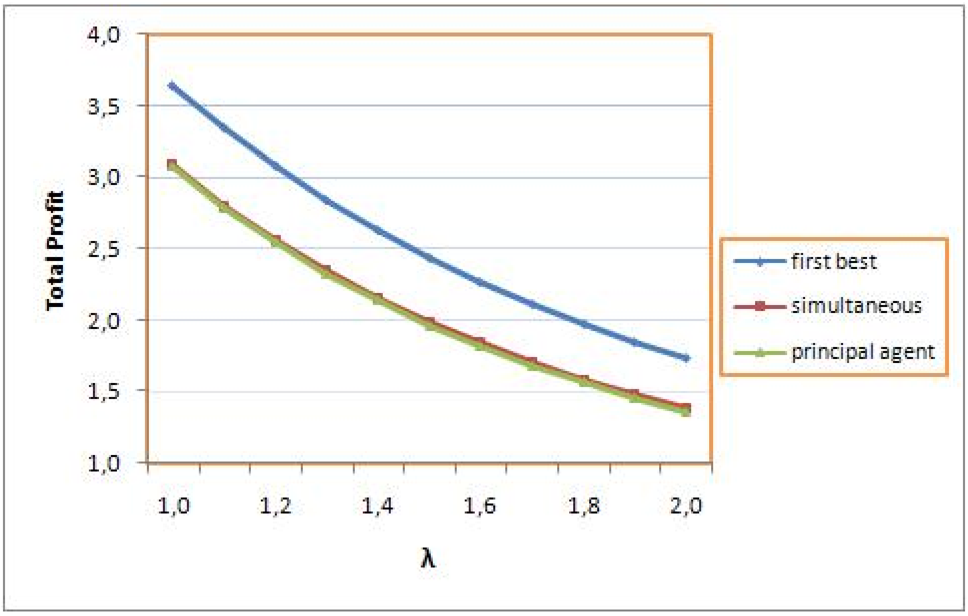


Figure 6.1. Optimum Results with Changing $\lambda$

### 6.4.2. Effect of the Discount Rate ($\alpha$) on the Optimum Results

In this section, $\alpha$ level is increased from 0,05 to 0,15. $\alpha$ is the discount rate of the future profits, therefore, it is expected that the total profit decreases if $\alpha$ increases. In Table 6.3, the results are shown.

Naturally, the total expected profits decrease with increasing $\alpha$. The firms tend

Table 6.3. Effect of $\alpha$ on the Optimum Results

| | **First Best** | **Simultaneous Game** | | **Principal Agent Game** | | | |
|---|---|---|---|---|---|---|---|
| $\alpha$ | **Total Profit** | **Total Profit** | **%** | **P1** | **P2** | **Total Profit** | **%** |
| **0,05** | 5,377 | 4,371 | 81,3 | 2,186 | 2,177 | 4,362 | 81,1 |
| **0,06** | 4,945 | 4,060 | 82,1 | 2,030 | 2,015 | 4,045 | 81,8 |
| **0,07** | 4,564 | 3,781 | 82,8 | 1,891 | 1,877 | 3,767 | 82,5 |
| **0,08** | 4,224 | 3,528 | 83,5 | 1,764 | 1,751 | 3,515 | 83,2 |
| **0,09** | 3,920 | 3,299 | 84,1 | 1,649 | 1,635 | 3,284 | 83,8 |
| **0,10** | 3,647 | 3,089 | 84,7 | 1,545 | 1,533 | 3,077 | 84,4 |
| **0,11** | 3,399 | 2,897 | 85,2 | 1,449 | 1,436 | 2,884 | 84,9 |
| **0,12** | 3,173 | 2,720 | 85,7 | 1,360 | 1,350 | 2,710 | 85,4 |
| **0,13** | 2,968 | 2,557 | 86,2 | 1,279 | 1,267 | 2,546 | 85,8 |
| **0,14** | 2,779 | 2,406 | 86,6 | 1,203 | 1,192 | 2,395 | 86,2 |
| **0,15** | 2,606 | 2,266 | 87,0 | 1,133 | 1,124 | 2,257 | 86,6 |

to invest less as the money is more valuable in time with a higher $\alpha$ value. That is, the effective cost of the investment with the same money increases. But, the results of the simultaneous model and the principal agent model gets closer to the first best result in percentage. That is, the firms are affected less due to the competition if $\alpha$ rises. Also, this result can be seen in Figure 6.2. The gap between the profit functions decreases as $\alpha$ increases. This is because the investment is less attractive under high $\alpha$ values, and the competition weakens in relation with that.

Again, the total expected profit is slightly higher in simultaneous model compared to the principal agent model. Besides, it is seen that there is a convex relationship between $\alpha$ and the total profit.

### 6.4.3. Effect of Initial Revenue Rate ($r_i$) on the Optimum Results

In this section, the initial revenue rates $r_i$ are increased from 1 to 2. Table 6.4 shows that the optimum results increase with $r_i$ as it is expected.

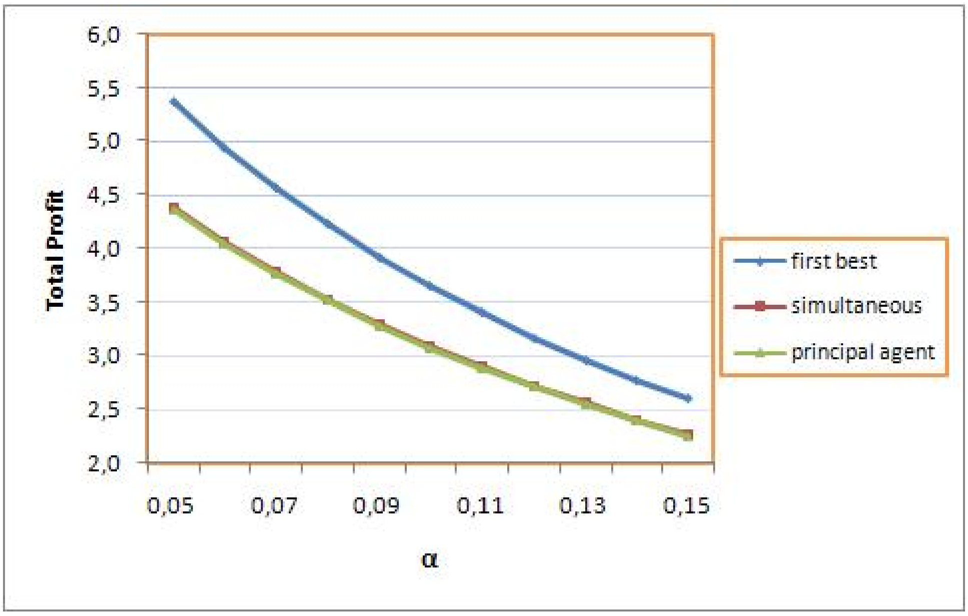


Figure 6.2. Optimum Results with Changing $\alpha$

$r_i$ is the possible revenue rates of the companies at the beginning and it is diminished with natural market shocks. Therefore, unless they get the technology later, the companies have higher profits with increasing $r_i$ when the other conditions are the same. The results also confirm that comment.

Also, the results of simultaneous game and principal agent model increase slightly in percentage with respect to the first best solution; which means that the competition diminishes total profit less in simultaneous game and principal agent model if $r_i$ increases.

In Figure 6.3, it is seen that the total profits increase quite linearly in $r_i$. However, when the initial revenue rates are doubled, the total profits increase more than double. Because, the companies invest more when the initial revenue rate $r_i$ rises, and they are expected to get the technology earlier compared to the base parameter set. If they get the technology at the same time, they are expected to have double profit with doubled $r_i$. Therefore, as they invest more, the total profit level increases with a higher rate with respect to $r_i$.

Table 6.4. Effect of $r_i$ on the Optimum Results

| | | First Best | Simultaneous Game | | Principal Agent Game | | | |
|---|---|---|---|---|---|---|---|---|
| $r_1$ | $r_2$ | **Total Profit** | **Total Profit** | % | **P1** | **P2** | **Total Profit** | % |
| **1,0** | **1,0** | 3,647 | 3,089 | 84,7 | 1,545 | 1,533 | 3,077 | 84,4 |
| **1,1** | **1,1** | 4,125 | 3,500 | 84,8 | 1,750 | 1,737 | 3,487 | 84,5 |
| **1,2** | **1,2** | 4,612 | 3,918 | 85,0 | 1,959 | 1,950 | 3,909 | 84,8 |
| **1,3** | **1,3** | 5,105 | 4,342 | 85,1 | 2,171 | 2,158 | 4,329 | 84,8 |
| **1,4** | **1,4** | 5,605 | 4,771 | 85,1 | 2,386 | 2,374 | 4,760 | 84,9 |
| **1,5** | **1,5** | 6,110 | 5,205 | 85,2 | 2,602 | 2,592 | 5,194 | 85,0 |
| **1,6** | **1,6** | 6,620 | 5,642 | 85,2 | 2,821 | 2,811 | 5,632 | 85,1 |
| **1,7** | **1,7** | 7,134 | 6,083 | 85,3 | 3,042 | 3,032 | 6,073 | 85,1 |
| **1,8** | **1,8** | 7,653 | 6,528 | 85,3 | 3,264 | 3,253 | 6,517 | 85,2 |
| **1,9** | **1,9** | 8,175 | 6,975 | 85,3 | 3,488 | 3,475 | 6,963 | 85,2 |
| **2,0** | **2,0** | 8,701 | 7,425 | 85,3 | 3,713 | 3,698 | 7,411 | 85,2 |

### 6.4.4. Effect of Efficiency Factor ($e_i$) on the Optimum Results

In this section, the effect of efficiency factor on the total expected profit is analyzed. In Table 6.5, it is shown that $e_i$ is increased from 1 to 2 for both firms. As the higher $e_i$ means that the firms get less efficient, the total expected profits decrease in $e_i$.

When the firms get inefficient, it is seen that they make higher investments in order to get the technology. Doubled $e_i$ means that expected time of the arrival of the technology doubles unless a higher amount invested. Therefore, the firms choose to invest more in optimum solution but this results in a lower total profit as the initial cost is increased. Another result is that, when $e_i$ increases, the gap between the total profit levels of first best solution and the other two models grows. That is, the firms lose more in percentage due to the competition with increasing $e_i$.

In Figure 6.4, shapes of the total expected profits can be seen as well.

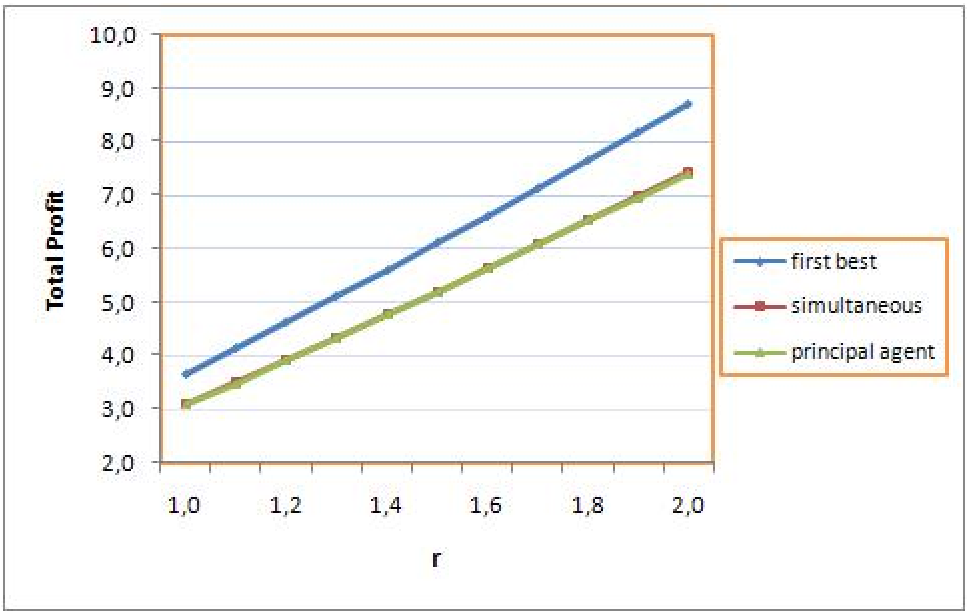


Figure 6.3. Optimum Results with Changing $r_i$

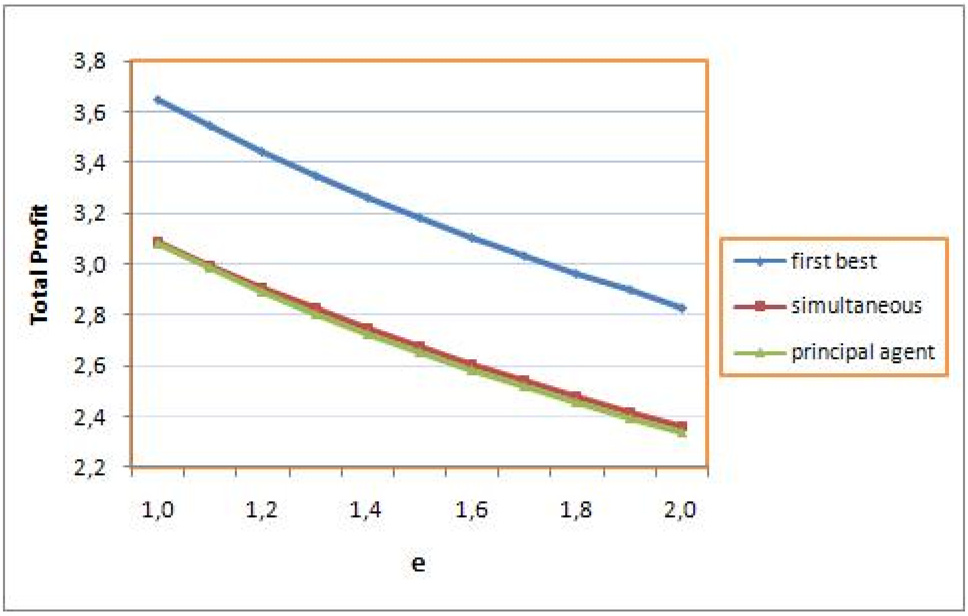


Figure 6.4. Optimum Results with Changing $e_i$

Table 6.5. Effect of $e_i$ on the Optimum Results

| | | **First Best** | **Simultaneous Game** | | **Principal Agent Game** | | | |
|---|---|---|---|---|---|---|---|---|
| $e_1$ | $e_2$ | **Total Profit** | **Total Profit** | **%** | **P1** | **P2** | **Total Profit** | **%** |
| **1,0** | **1,0** | 3,647 | 3,089 | 84,7 | 1,545 | 1,533 | 3,077 | 84,4 |
| **1,1** | **1,1** | 3,540 | 2,994 | 84,6 | 1,497 | 1,486 | 2,983 | 84,3 |
| **1,2** | **1,2** | 3,442 | 2,905 | 84,4 | 1,453 | 1,435 | 2,888 | 83,9 |
| **1,3** | **1,3** | 3,350 | 2,823 | 84,3 | 1,412 | 1,393 | 2,805 | 83,7 |
| **1,4** | **1,4** | 3,263 | 2,745 | 84,1 | 1,373 | 1,353 | 2,726 | 83,5 |
| **1,5** | **1,5** | 3,181 | 2,672 | 84,0 | 1,337 | 1,314 | 2,651 | 83,3 |
| **1,6** | **1,6** | 3,104 | 2,604 | 83,9 | 1,302 | 1,283 | 2,585 | 83,3 |
| **1,7** | **1,7** | 3,030 | 2,539 | 83,8 | 1,270 | 1,247 | 2,517 | 83,1 |
| **1,8** | **1,8** | 2,960 | 2,477 | 83,7 | 1,239 | 1,212 | 2,451 | 82,8 |
| **1,9** | **1,9** | 2,893 | 2,418 | 83,6 | 1,210 | 1,185 | 2,394 | 82,8 |
| **2,0** | **2,0** | 2,829 | 2,362 | 83,5 | 1,182 | 1,152 | 2,334 | 82,5 |

### 6.4.5. Effect of the Revenue Multiplier at the Market Shocks ($\beta_{ij}$) on the Optimum Results

In the base set, $\beta_{i1}$ and $\beta_{i2}$ are set to 0,9 and 0,5 respectively for both firms. In this section, $\beta_{i1}$'s are kept constant, and $\beta_{i2}$'s are increased to 0,9. As the $\beta_{i2}$ level gets higher relatively, the total expected profit is expected to rise, since the downward jump of the revenue rate at the market shocks will decrease. In Table 6.6, the optimum results with changing $\beta_{i2}$ can be seen.

As $\beta_{i2}$ rises, the total profit levels rise for all models as well. However, the investment level increases in the first best solution, but decreases in the other two competitive models. And when $\beta_{i2}$ equals to $\beta_{i1}$, which means that there is no advantage or disadvantage of getting the technology first, the optimum results are the same in both three models. That is, when entering into the market before the other company has no competitive advantage, both firms invests as much as the first best solution in the simultaneous game and the principal agent model as well. This result can be shown in Figure 6.5 as well.

Table 6.6. Effect of $\beta_{ij}$ on the Optimum Results

| | | **First Best** | **Simultaneous Game** | | **Principal Agent Game** | | | |
|---|---|---|---|---|---|---|---|---|
| $\beta_{i1}$ | $\beta_{i2}$ | **Total Profit** | **Total Profit** | % | **P1** | **P2** | **Total Profit** | % |
| **0,90** | **0,50** | 3,647 | 3,089 | 84,7 | 1,545 | 1,533 | 3,077 | 84,4 |
| **0,90** | **0,54** | 3,718 | 3,203 | 86,2 | 1,602 | 1,589 | 3,190 | 85,8 |
| **0,90** | **0,58** | 3,805 | 3,337 | 87,7 | 1,669 | 1,659 | 3,328 | 87,5 |
| **0,90** | **0,62** | 3,913 | 3,496 | 89,4 | 1,748 | 1,740 | 3,488 | 89,2 |
| **0,90** | **0,66** | 4,047 | 3,686 | 91,1 | 1,843 | 1,840 | 3,683 | 91,0 |
| **0,90** | **0,70** | 4,217 | 3,917 | 92,9 | 1,959 | 1,957 | 3,915 | 92,8 |
| **0,90** | **0,74** | 4,435 | 4,201 | 94,7 | 2,100 | 2,096 | 4,196 | 94,6 |
| **0,90** | **0,78** | 4,721 | 4,556 | 96,5 | 2,278 | 2,276 | 4,554 | 96,5 |
| **0,90** | **0,82** | 5,104 | 5,009 | 98,1 | 2,504 | 2,504 | 5,008 | 98,1 |
| **0,90** | **0,86** | 5,634 | 5,602 | 99,4 | 2,801 | 2,801 | 5,602 | 99,4 |
| **0,90** | **0,90** | 6,400 | 6,400 | 100,0 | 3,200 | 3,200 | 6,400 | 100,0 |

Also, in the principal agent model, to have the priority in decision making has not an advantage when $\beta_{i2}$ equals to $\beta_{i1}$, that is, there is no competitive relationship any more between the companies.

### 6.4.6. Combined Effect of the Parameters on the Optimum Results

In this section, a broader computational analysis is done with different parameter sets. Here, the aim is to see the combined effect of some parameters as well as for non-identical firms case. Arrival rate of the market shocks ($\lambda$) and the discount rate ($\alpha$) are set to 1 and 0,1 respectively, throughout the analysis of this section. Those parameters are analyzed in sections 6.4.1 and 6.4.2; and as they are outside components of the model, they are not included into the variables in this section.

In Table 6.7, different parameter sets and their results is shown. For stated parameters, the decision levels and the expected profits can be seen in the table.

The first setting in the table is the base setting. Both parties have identical parameters. Therefore, they choose to invest the same amount of money in the First Best Solution (see Chapter 4) and Simultaneous Game Model (see Chapter 3). On

Table 6.7. Computational Results with Different Parameter Sets and Non-identical Firms

| | | | | | | | | | First Best Solution | | | | | Simultaneous Game (Basic Model) | | | | | Principal Agent Model | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Set No** | $r_1$ | $r_2$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $e_1$ | $e_2$ | $c_1$ | $c_2$ | **Profit 1** | **Profit 2** | **Total Profit** | $c_1$ | $c_2$ | **Profit 1** | **Profit 2** | **Total Profit** | $c_1$ | $c_2$ | **Profit 1** | **Profit 2** | **Total Profit** |
| **1** | **1** | **1** | **0,9** | **0,5** | **0,9** | **0,5** | **1** | **1** | **0,551** | **0,551** | **1,823** | **1,823** | **3,647** | **1,212** | **1,212** | **1,545** | **1,545** | **3,089** | **1,230** | **1,211** | **1,545** | **1,533** | **3,077** |
| **2** | 1 | 1 | **0,8** | **0,7** | **0,8** | **0,7** | 1 | 1 | 0,625 | 0,625 | 1,297 | 1,297 | 2,593 | 0,774 | 0,774 | 1,277 | 1,277 | 2,554 | 0,780 | 0,774 | 1,277 | 1,276 | 2,552 |
| **3** | 1 | 1 | 0,9 | 0,5 | 0,9 | 0,5 | **3** | **3** | 0,661 | 0,661 | 1,153 | 1,153 | 2,307 | 1,263 | 1,263 | 0,958 | 0,958 | 1,915 | 1,350 | 1,255 | 0,960 | 0,913 | 1,872 |
| **4** | **2** | **2** | 0,9 | 0,5 | 0,9 | 0,5 | 1 | 1 | 0,927 | 0,927 | 4,351 | 4,351 | 8,701 | 2,231 | 2,231 | 3,713 | 3,713 | 7,425 | 2,250 | 2,230 | 3,713 | 3,698 | 7,411 |
| **5** | 1 | 1 | 0,9 | 0,5 | 0,9 | 0,5 | 1 | **3** | 0,782 | 0,088 | 3,103 | 0,101 | 3,204 | 1,069 | 0,955 | 2,469 | 0,359 | 2,828 | 1,290 | 0,840 | 2,501 | 0,266 | 2,767 |
| **6** | 1 | 1 | 0,9 | 0,5 | 0,9 | 0,5 | **3** | 1 | 0,088 | 0,782 | 0,101 | 3,103 | 3,204 | 0,955 | 1,069 | 0,359 | 2,469 | 2,828 | 0,820 | 1,045 | 0,364 | 2,547 | 2,911 |
| **7** | 1 | 1 | **0,8** | **0,7** | 0,9 | 0,5 | 1 | 1 | 0,243 | 0,948 | 0,864 | 2,598 | 3,461 | 0,763 | 1,192 | 1,195 | 1,905 | 3,100 | 0,750 | 1,191 | 1,195 | 1,918 | 3,113 |
| **8** | 1 | 1 | 0,9 | 0,5 | **0,8** | **0,7** | 1 | 1 | 0,948 | 0,243 | 2,598 | 0,864 | 3,461 | 1,192 | 0,763 | 1,905 | 1,195 | 3,100 | 1,230 | 0,762 | 1,906 | 1,189 | 3,095 |
| **9** | **2** | 1 | **0,8** | **0,7** | 0,9 | 0,5 | 1 | 1 | 0,776 | 0,871 | 3,285 | 1,828 | 5,113 | 1,316 | 1,208 | 3,375 | 1,479 | 4,854 | 1,330 | 1,208 | 3,375 | 1,470 | 4,845 |
| **10** | **2** | 1 | 0,9 | 0,5 | **0,8** | **0,7** | 1 | 1 | 1,335 | 0,070 | 7,065 | 0,341 | 7,405 | 1,974 | 0,735 | 5,380 | 1,107 | 6,487 | 2,050 | 0,732 | 5,383 | 1,101 | 6,484 |

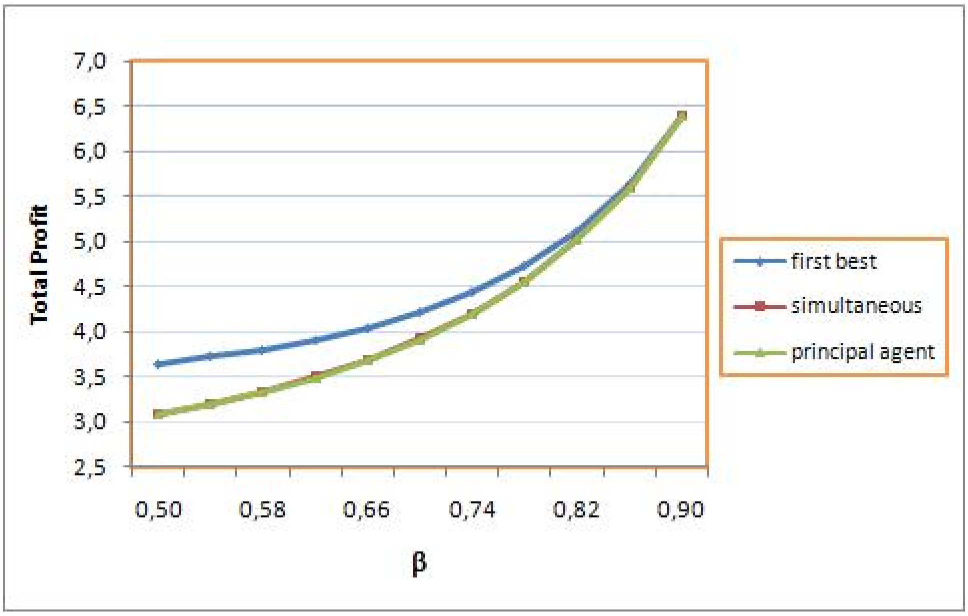


Figure 6.5. Optimum Results with Changing $\beta_{ij}$

the other hand, in the Principal Agent Model (see Chapter 5), the first decision maker has an advantage and gets a little bit higher profit. In the First Best Solution, total channel profit is the maximum. As there is no competition, companies do not invest much and get the highest total expected profit. Whereas, in the Simultaneous Game, because of the competition, companies invest relatively higher amounts in the equilibrium solution and they get lower profits compared to the First Best Solution. In the Principal Agent Model, the equilibrium solution is very close to the equilibrium of the Simultaneous Game. However, in this model, first company has the advantage of being first decision maker; and it invests a higher amount compared to the Simultaneous Game. By this decision, first decision maker increases the probability of inventing the technology before. Although the expected profit of the first company is the same as the Simultaneous Game, the second company gets lower expected profit in the Principal Agent Model as it loses some competence advantage in this model.

In the second parameter set, the difference between $\beta_{ij}$ values are decreased compared to the basic set. By this change, being leader or follower will be less important since the $\beta_{ij}$ values are closer now. Again all parameters are common for both companies. In this setting, it can be analyzed that how the result changes if leadership is less of importance. As in the base set, in the First Best Solution and Simultaneous Game, companies have the same decision in equilibrium; and in the Principal Agent Model, first company takes advantage of being the first decision maker. In the First Best Solution, companies invest more in this time, but they get a lower total revenue. This is resulted from the decrease of $\beta_{i1}$'s. In the Simultaneous Game, companies do not invest as much as in the base set, and they get lower revenue as well. This stems from the decrease in $\beta_{i1}$'s. In the Principal Agent Models, companies invest less compared to the base set as well, because the same reason stated. As a result, in this parameter set, companies invest less in the competing models as being leader is not so advantageous as in the base set.

In the third parameter set, efficiency factors in the base set are increased for both companies. By this way, the companies become less efficient in reaching the technology and they are expected to invent the technology three times slower according to the base set. Therefore, in all three models, companies invest more and get lower profits according to the base set since they loose efficiency. Again the channel profit is maximum in the First Best Solution and minimum in the Principal Agent Model. Besides, the difference of total profits in the Simultaneous Game and the Principal Agent Model increases with higher $e_i$ values. It can be said that, with increasing $e_i$, competitive advantage of being the first mover strengthens.

In the fourth parameter set, initial revenue rates are doubled for both companies in comparison with the base set. The companies have the same properties, so they make the same optimum decisions in the First Best Solution and the Simultaneous Game. On the other hand, in the Principal Agent Model, first decision maker sets a slightly higher investment level, and get a little higher expected profit as well. Increasing the initial revenue rates- also the revenue stream- pushes the companies to invest more. Because, with a doubled revenue stream, they want to get the technology earlier. Therefore, in this parameter set, total channel profits are more than double of the profits in the base parameter set for all three models. As in the previous parameter sets, the biggest total profit is in the First Best Solution followed by the Simultaneous Game and the Principal Agent Model respectively.

In the fifth parameter set, only one parameter in the base set is changed. The second company is made three times inefficient in this setting. In the First Best solution, the second company (inefficient one) makes a very low investment and gets a very low profit as well. On the other hand, the first company makes a higher investment and earns a higher expected profit than earned in the base set. In the First Best Solution, it is the case just like a single decision maker is maximizing the total profit. Results show that, when one of the companies is inefficient, the single decision maker reserves most of the effort to the efficient one. However, total profit is lower in this setting because of the efficiency lost in the system. In the Simultaneous Game and the Principal Agent Model, inefficient one gets quite lower profits and the other one has higher expected profits as well. This makes sense as the efficient party most probably invents the technology first. (Three times faster on the average with the same investment.) But, in those models the total expected profit decreases also due to the lower efficiency of the system. In the Simultaneous Game solution of this setting, the efficient party sets a lower investment decision compared to the base set; whereas in the Principal Agent Model, it makes a higher investment decision than made in the base set. The inefficient one gets the lowest expected profit in the Principal Agent Model as can be seen in the table.

In the sixth parameter set, the first company is made to be three times inefficient. That is, the expected time of getting the technology with the same investment becomes three times longer. The results of the First Best Solution and the Simultaneous Game are symmetric with the fifth parameter set explained above. However, in the Principal Agent Model, the inefficient one is the first to set decision. When the inefficient party has the deterministic power by being the first mover, both of the parties get higher profits in comparison with the case that the efficient one is the first mover (fifth case). Also the total profit of the system is higher in this case. So, the results show that the system is more profitable when the inefficient company is the first mover in the Principal Agent model.

In the seventh parameter set, the $\beta_{1j}$ values are made to be more conservative by increasing $\beta_{12}$ and decreasing $\beta_{11}$. By this change from the base set, for the first company, being the leader inventor of the technology is not as important as to the second company. Therefore, the first company invests less in this setting and gets lower revenue compared to the base setting. In the First Best Solution, the total profit of the system decreases in this setting. The single decision maker of the system reserves most of the effort to the second company, for which being the leader inventor is important due to higher $\beta_{21}$ level. Since, $\beta_{21}$ is bigger than $\beta_{11}$, leadership of the second company is more profitable for the system. Whereas, in the Simultaneous Game and the Principal Agent Model, the total expected profit of the system is a bit higher than that of the base setting. In this models, the total expected profit increases since the companies do not compete with each other as harsh as the case in the base setting. Whereas, in the First Best Solution, the total expected profit decreases because of the conservative revenue structure of one of the companies.

The eighth parameter setting is similar to the seventh setting. In this case, the $\beta_{ij}$ values are made to be more conservative by increasing $\beta_{22}$ and decreasing $\beta_{21}$ for the second company. The First Best Solution and the Simultaneous Game solution are same with the seventh case just by replacing the companies. Yet, in the Principal Agent Model, companies invest a little higher and the total expected profit decreases compared to the seventh setting. This is resulted from the characteristics of the companies, where the first mover in decision making has more competitive characteristics with higher $\beta_{11}$ value.

In the ninth setting, parameters are kept same as in the seventh setting but the revenue rate of the first company is doubled. By this way, the first company has a higher revenue system and less sensitive to the leadership in inventing the technology compared to the other company. Therefore, in the Simultaneous Game, the first company makes higher investment and also gets higher profit than the seventh setting. Moreover, the expected profit increases more than one times, while the revenue rate only doubles. However, in the First Best Solution, the first company does not invest as much as the second company although it has a higher revenue rate. This is because of the fact that leadership does not change the revenue stream of the first company as much as it affects the second one.

The last setting is very similar to the eighth setting by only doubling the revenue rate of the first company. In this case, the first company has a higher revenue rate and the second company is not so sensitive to the leadership in inventing the technology as the first company. Therefore, in all of the models, the first company invests more and the total expected profit of the system exceeds the double of the total expected profit in the eighth setting.

In conclusion, the results of the First Best Solution yields always higher total expected profits for the system. If the companies are identical, they get the same expected profit in the First Best Solution and the Simultaneous Game. However, in the Principal Agent Model, the first mover has a higher expected profit if they have equal characteristics. Higher revenue rates and efficiency levels get always higher profits to the companies if other parameters are kept the same. Total expected profit of the system is close each other in the Simultaneous Game and the Principal Agent Model. In most of the cases, the Simultaneous Game brings a higher total expected profits. But, when the first decision maker is more inefficient or less sensitive to the leadership, the Principal Agent Model gives a higher total expected profit.

# 7. CONCLUSION

Throughout this thesis, competition of two companies investing in a new technology is analyzed. Both companies make investment at the beginning of the time horizon and wait for the arrival of the technology. No further investment is allowed and the arrival time of the technology is a random value depending on the amount of the investment and the efficiency of the company. When the companies invent the technology, they start to collect revenue by that technology or the service related with that technology. Revenue rates of the companies here depend on the characteristics of the companies and the external effects introduced to the market. Revenue rate is assumed to diminish stochastically by time. The company which invents the technology first and enters into the market as the leader will take advantage as the revenue rate of the leader decreases slower. Therefore, the core tradeoff is between getting the technology earlier to become the leader and its investment cost.

In Chapter 3, the companies make their decisions simultaneously. This is the basic model of the study. Throughout this chapter, the model is analyzed and it is shown that this model has a unique equilibrium, which is the unique solution at which both of the companies do not deviate from their decisions unless the other one does not.

In Chapter 4, the first best solution of the system is analyzed. That is, how the total expected profit changes if a single decision maker sets decisions of the companies in order to maximize the total profit. All other properties and assumptions of the system are kept the same with the model in Chapter 3. By this way, the effect of the competition in the first model can be analyzed. It is shown that there is a unique solution of this model as well. Although, it could not be shown algebraically, it can be solved by numerical methods easily.

In Chapter 5, the same basic model is analyzed what if the companies takes their decisions sequentially. That is, the first mover sets its decision, and knowing that, the second one takes a decision. This is also called principal agent model. It is again shown that this problem has a solution as well and it can be solved numerically.

In Chapter 6, procedures for the numerical solutions of both three models stated above are developed. Then, the effects of single parameters are analyzed and displayed also graphically. Company characteristics are set to be the same in order to see the difference in optimum results by changing parameters at a time. Also with different parameter sets and non-identical firms, the results are found and presented. It is shown that the first best solution yields always the highest total expected profit as it is expected. On the other hand, the results of the basic model and the principal agent model are relatively close. In general, the basic model yields higher total expected profits. However, when the first mover is more inefficient or less sensitive to the leadership, the principal agent model yields a bit higher total expected profit. Also, in all of the cases, the expected profit increases with the revenue rate and efficiency level as it is usual.

As a future work, those models can be analyzed for more than two agents. Of course it will be a more complex analysis. In that model, the sequence of inventing the technology will be important. The early entering will cause a higher market share, therefore a higher revenue rate. Also, those models can be analyzed if multiple investments in time is allowed. Then, a different approach is needed as the companies can make investment decisions at times other than the beginning.

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