A FORECASTING IMPLEMENTATION OF BOX-JENKINS TIME SERIES ANALYSIS

by

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ABSTRACT

The ultimate effect of a decision generally depends on the outcome of factors that can not be foreseen at the time decision is made. Among wide variety of forecasting methods, the Box-Jenkins approach is known as the application of the more general and statistical based methods of time series analysis.

This thesis covers the implementation of Box-Jenkins approach by using computer. A computer software is developed for building univariate and bivariate models, and for making the forecasts. The underlying principles of the Box-Jenkins approach are presented, and a methodology of using the approach is suggested.

BOX-JENKINS ZAMAN SERİSİ ANALİZİNİN BİR TAHMİN UYGULAMASI

ÖZET

Bir kararın nihai etkisi genellikle sonuçları önceden kestirilemeyen bir takım sebeplerin sonuçlarına bağlıdır. Geleceği tahmin etmek amacı ile pek çok metod geliştirilmiştir. Bunlar arasında Box-Jenkins yaklaşımı zaman serileri analizinin genel ve istatistiğe dayalı yöntemlerinin tahmin yapmak için uygulanması olarak bilinir.

Bu çalışma Box-Jenkins yaklaşımının bilgisayar aracılığı ile uygulanmasını kapsamaktadır. Bu amaçla tek zaman serileri ve çift zaman serileri modelleri oluşturmak ve tahmin yapmak için bir bilgisayar yazılımı geliştirilmiştir. Box-Jenkins yaklaşımının kullanılabilmesi için bir metodoloji önerilmiştir.

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LIST OF SYMBOLS

AR(p)	Autoregresisve process of order p
ARIMA(p,d,q)	Autoregressive integrated moving average process of order (p,d,q)
ARMA(p,q)	Mixed process of order (p,q)
B	Backward shift operator
B^b	Dead time operator of order b of a transfer function
c_k	Autocovariance coefficient estimate at lag k, k = 0,1,...,K
MA(q)	Moving average process of order q
N_t	Noise series for t = 1,2,...,N
r_k	Autocorrelation coefficient estimate at lag k, k = 0,1,2,...,k
SOE	Sum of errors
SSE	Sum of squares of errors
u_t	White noise process, t = 1,2,...,N
v(B)	Transfer function of the filter between an input and an output
v_k	Impulse response weights, k = 0,1,2,...,K
X_t	Input series of a system, t = 1,2,...,N
$\hat{Z}_t(\ell)$	Forecast made at origin t for lead time ℓ, ℓ = 1,2,...,L
$\delta(B)$	Left hand side linear operator of order r of a transfer function model
$\theta(B)$	Moving average operator of order q
σ_u^2	Variance of white noise series

$\varphi(B)$	Autoregressive operator of order p
$\Psi(B)$	Linear filter operator
$\omega(B)$	Right hand side operator of order s of transfer function model
∇^d	Differencing operator of order d

I. INTRODUCTION

1.1 Forecasting and Planning

Forecasting may be defined as the estimation of the future based on the past by using the methodologies which are developed for this purpose. It is used by decision and policy makers as an aid to determine goals and targets, to understand the environment and causal factors that affect it, and to understand the uncertainities in the future and to force decision about the level of risk appropriate.

Planning on the other hand may be defined as attempts at purposeful, future oriented decision making (1). A firm is generally exposed to uncontrollable external events, and controllable internal events. Forecasting applies directly to uncontrollable external events, while decision making applies directly to controllable internal events. Planning is considered the link that integrates them.

1.2. Current Status of Forecasting

1.2.1. Theory and Applications

As a result of increasing uncertainty of the environment, a number of estimation and prediction methods are developed both in theory and practice for organizational forecasting. The current forecasting literature can be classified into two groups with regard to the theory and practice of the forecasting methods.

In the first group, the works on forecasting have generally been written by the specialists who have accomplished the theoretical formulation and verification of specific methods and who are trying to convey the state of the art knowledge to a group of specialists. In this area, the work of Box and Jenkins (2) does an excellent job in developing and providing statistical properties of specific classes of forecasting methods.

The second group of the literature is concerned on translating what is theoretically possible and computationally feasible into a form that can be easily understood and applied. In this area, the work of Makridakis et al (3) is a complete study on forecasting where they put a broad range of forecasting methods into a form that can easily be understood and applied by nonspecialists. The work of Nelson (4) is related with the essence of the application of Box-Jenkins models and their forecasting.

There are many research articles on forecasting. Two important applications of Box-Jenkins method are worth to mention. First is the univariate application of the method to actual time series. It includes the comparison of the method with the exponentially smoothed method. It is concluded that two methods perform equally well on the given data (5).

Secondly, in a recent study, the procedural steps in the Box-Jenkins transfer function method are demonstrated in an application to the advertising and sales relationship with particular focus on the advertising lag structure (6).

1.2.2. Forecasting Methods

To deal with the increasing variety and complexity of managerial forecasting problems, a wide variety of forecasting methods are developed that originated from several major fields of study. Although various classification schemes are possible depending on the framework chosen, the generally accepted classification is given here for describing the methods. According to this classification, the existing forecasting methods fall into two major categories: quantitative methods, and qualitative methods.

Quantitative methods can be applied when sufficient information about the past is available in the form of numerical data, and it can be assumed that some aspects of the existing patterns will continue into the future. These methods are further divided into two categories.

(1) Causal methods. The objective of forecasting using causal methods is to determine the cause-effect relationship of the factors to be forecasted with one or more independent variables by assuming that this relationship will hold into the future.

(2) Time series methods. The objective of these forecasting methods is to determine the pattern in the historical data series and extrapolate that pattern into the future. The system is treated as a black box and, as a result ,the factors affecting its behavior are not considered.

Qualitative methods require subjective estimation, intuitive thinking, judgement, and accumulated knowledge through the opinions of the experts.

Another useful classification is to divide forecasting approaches into two groups: statistical methods, and filtering methods.

ARIMA schemes which were first introduced by Box and Jenkins are quite sophisticated mathematical models. Their work also includes an extension of ARIMA scheme into multivariate models which is known as transfer function analysis. Regression methods and ARIMA schemes are accepted as statistical approaches to forecasting.

Filtering methods, first introduced by Kalman (7), are engineering approaches. The name of Kalman filter is used synonymously with Bayesian forecasting. In this approach, it is assumed a fixed model with varying parameters and variances.

1.2.3. Selection of the Forecasting Method

The selection of the forecasting method for a given situation is a decision problem. The information supplied from this system is to be used to improve the decision process. There are several useful criteria that are used to select, to compare, and to evaluate the competing methods. The first problem to be solved is to define the variables of the forecast to be analysed and predicted.

An overview of decision criteria will clarify the interrelationships among criteria and need to select a forecasting method that best meets all the requirements of a given situation.

Level of detail is related with the decision on what level the forecast to be made such as regional or market demand, or product or product group demand. An important class of decisions involves the time elements: forecast period, and forecast lead time or forecast horizon.

The form of the final forecast is also an important consideration. Various methods provide different outcomes such as mean, an estimate of the standard deviation of forecast error, or a probability interval. Technical sophistication is a determinant on the applicability of a method for a given situation, because the application of a method will be restricted with the capabilities and interests of the people who will make and use forecasts.

Before choosing a method, the extensiveness, accuracy, currency, and representativeness of the available data must be considered, because the ability of many forecasting methods is limited with the amount of available data. The type of data series is another factor on the selection of a forecasting method. The more general classification is macro series and micro series. The pattern of the data must be taken into consideration, because, there are many methods that can only cope with a certain data pattern. Four types of data patterns can generally exist in the data series: horizontal, trend, seasonal, and cyclical. The variability of data series is a result of the process generating the variable under study. A number of simple methods are available for the stationary series, but more sophisticated methods are developed to handle nonstationary series.

The element of cost depends on the developement, installation, maintenance and operation of the method, data requirements, computer requirements, and human sources requirement.

Forecast accuracy have an important impact on the selection of the forecasting method. Altough there are several statistical measures of accuracy, it is not possible to propose a robust measure of a specific method in common use for all situations. The relative importance of a decision criterion depends on the forecasting situation. The criteria mentioned above have interrelationships, therefore the trade offs for a given situation must be correctly established. The costs and benefits of a forecasting application must be considered in the evaluation of alternative forecasting methods.

Chambers et al propose that the stage of the product life cycle for which it is making the forecast is an important consideration. Their approach is based on matching methods with the forecasting need as determined by product life cycle (8).

In another study, several different criteria for evaluating alternative forecasting methods are described and those criteria are used to match the situation with the most appropriate forecasting method. It is concluded that simple methods can do as well in a wide variety of cases, therefore the mathematically sophisticated methods should not be selected, unless there is a strong evidence that those methods will do better (9).

In a recent study, it is proposed that each method has strengths and weaknesses, every forecasting situation is limited by constraints. An evaluation of twenty common forecasting methods are arrayed against sixteen evaluative dimensions. It is concluded that extrapolations can be improved by combining forecasts, or simulating a range of input assumptions, or selectively applying judgement (10).

The research articles on this subject are numerous, only a few of the studies are included here (11),(12),(13),(14),(15).

1.3. Overview and Framework for Thesis

The need to carry out this study has been arisen from the sales forecasting problem of an existing group of companies. A marketing company, which markets hundreds of glass products, has been chosen within this group in order to define forecasting problem, to establish objectives and determine contents of the study, and to apply resulting model to the chosen sales forecasting situation. Although this study is originated from a specific forecasting situation, the forecasting model developed can be applied to any univariate and bivariate time series forecasting situation.

1.3.1. Definition of the Forecasting Problem

The forecasts are made for product groups which are formed by aggregating the products within the same product class. The unit of measure is determined as units of product sold during a month. Forecasts are made by using only internal data sources.

The data sources are investigated in two dimensions. The first is sales series or as it is frequently called <u>output series</u>. Secondly, the factors that affect sales, which are called input series or leading indicators, are sought out. These are price, advertisement expenditures, and sales promotion expenditures. It is found out that about fifteen years of monthly data available for sales series, price data are also available for the same length of time, but it is not possible to determine the expenditures of advertisement and promotion on product group basis.

At this point, it is better to distinguish the difference between aggregate and point data. Aggregate data represent the value of a variable accumulated over a period of time, while point data indicate the value of a variable at specific points in time. Of the data mentioned above, price data are point data.

The changes in price series do not exactly match with sales series in this forecasting situation, consequently to establish a proper relationship between price and sales will not be efficient.

Time series data must be collected in equispaced time intervals. In this respect, a month can not be assumed to be a period. A trading days adjustment on data is recommended to increase forecast accuracy (16).

Another fact is that the available data do not properly represent actual demand, because it is collected on delivery basis. Actual demand may be best represented by collecting data on order basis.

The investigation on sales data by using tools, such as graph of data, autocorrelations, differencing, etc., has exhibited horizontal, trend or seasonal patterns. Also, many of the series have shown the indications of nonstationarity.

The problem may be stated as to find and implement a forecasting method that meets all the requirements of the given situation.

1.3.2. Objectives of This Thesis

The main objective of this study is to develop an interactive computer software that is used to build, and to forecast with univariate and bivariate Box-Jenkins models.

The other objective is to explain statistical concepts and underlying Box-Jenkins modelling procedures in a comprehensive way, and to provide guidelines for building Box-Jenkins models properly and quickly.

1.3.3. Evaluation of Box-Jenkins Approach

Box and Jenkins have effectively assembled in a comprehensive way the relevant information required to understand and use time series ARIMA (Autoregressive/Integrated/Moving Average) processes, their names have frequently been used as synonymous with the general ARIMA processes applied to time series analysis, forecasting, and control. Their work includes univariate time series analysis, multivariate time series analysis, and design of discrete control schemes. The theoretical aspects of Box-Jenkins time series are quite sophisticated. The relative development time of the method and to build a forecasting model take time, but the forecasts can be made quickly. Box-Jenkins approach to forecasting is apparently the most accurate with regard to mean square error, and the most developed statistical method presently available.

The time series are fitted with a mathematical model which is optimized on parameters in order to assign smaller errors to history than any other model. Box and Jenkins propose a general class of models for forecasting, their approach is appropriate to handle various data patterns.

Box-Jenkins philosophy of model building for time series includes two principles (17). First is the principle of parsimony that can be described as the smallest number of parameters that should be employed for adequate representation of underlying model of a series.

The second principle is to apply iterative procedure in the selection of a model. There are three stages in their approach to model building.

(1) Identification. The methods are proposed to define models which may be good representation of underlying generating mechanism.

(2) Estimation. The model selected is fitted to data and the parameters are estimated by minimizing sum of squares of errors.

(3) Diagnostic checking. Adequacy of the fitted model is tested by using available statistical measures ; and then, causes of lack of fit, if it exists are diagnosed.

These three stages of the approach are iterated until an appropriate representation is found. The selected model is then applied to make the forecasts.

In this study, the basic theory, modelling procedures, and relevant algorithms are largely drawn from the work of Box and Jenkins. The basic notation and terminology are also adopted from their work.

II. BOX-JENKINS MODEL BUILDING AND FORECASTING

In this chapter, first the underlying theory of Box-Jenkins models is presented. Then, the statistical tools used in time series analysis, and methods for building, identifying, fitting and checking models for time series are illustrated in accordance with the three stage Box-Jenkins approach to model building: identification, estimation, and diagnostic checking. Finally, the forecasting version of the developed model is briefly explained.

A phenomenon that evolves through time according to probabilistic laws is called stochastic process, it is simply referred as process in this study. The time series to be analysed may then be considered as one particular realization of a variable, from an infinite population of such realizations of that variable, produced by the underlying probability mechanism which is generally called generating mechanism of the process. The three stage Box-Jenkins procedure is designed to find a model that is a good representation of the unknown underlying process.

2.1. Univariate Box-Jenkins Models

ARIMA is an acronym for Autoregressive/Integrated/Moving Average. An ARIMA process refers to the particular generating mechanism which describes the evolution of observations through time, and the derivation of the conditional distribution of future realizations. The general nonseasonal model is represented by ARIMA (p,d,q), where p denotes order of the autoregressive process, d denotes degree of differencing to achieve a stationary mean, and q denotes order of the moving average process. There is no limit to the variety of ARIMA models; p,d, and q are nonnegative integers.

2.1.1. Fundamentals of ARIMA Processes

The underlying logic in Box-Jenkins approach is to exploit the dependency relationships in successive observations of the time series. Thus, a time series of that type may be considered as generated from a time series of random "shocks" u_t. These shocks are random drawings from a fixed distribution, and they are usually assumed to be normally, independently, identically distributed having a mean zero and a constant variance σ_u^2. The sequence of random variables $u_t, u_{t-1}, u_{t-2}, \ldots$ is often called white noise process.

The white noise process u_t is supposed to be transformed to the process Z_t, which represents time series observations, by a linear filter so that

$$Z_t = \mu + u_t + \Psi_1 u_{t-1} + \Psi_2 u_{t-2} + \cdots \qquad (1)$$

$$= \mu + \Psi(B) u_t$$

where

B is backward shift operator such that $B^m Z_t = Z_{t-m}$, m is nonnegative integer, $\Psi(B) = 1 + \Psi_1 B + \Psi_2 B^2 + \ldots$ is the linear operator, μ is a parameter that represents the level of the process.

If this sequence of $\Psi_1, \Psi_2, \ldots$ weights are finite, or infinite and convergent, the filter is said to be stable and the process Z_t to be stationary. The parameter μ is then the mean about which the process varies. Otherwise, Z_t is nonstationary and μ can be regarded as a reference point for the level of the process. Box-Jenkins models are based on this idea of linear filtering.

An ARIMA(p,0,0) or AR(p) process is called autoregressive process of order p so that the current value of the deviations from mean of the process is represented by a linear combination of previous values of the process and current shock u_t. Let $\tilde{z}_t$ be deviations from μ, $\tilde{z}_t = z_t - \mu$. Then AR(p) process can be written as

$$\tilde{z}_t = \varphi_1 \tilde{z}_{t-1} + \varphi_2 \tilde{z}_{t-2} + \ldots + \varphi_p \tilde{z}_{t-p} + u_t \qquad (2)$$

or

$$\varphi(B)\ \tilde{z}_t = u_t$$

where $\varphi(B) = 1 - \varphi_1 B - \varphi_2 B^2 - \dots - \varphi_p B^p$ is the autoregressive operator of order p.

It is required that $\varphi(B)$ must be stationary.

Another useful representation is moving average process of order q, ARIMA(0,0,q) or MA(q). The current value of $\tilde{z}_t$ is expressed as a finite number of previous shocks plus current shock u_t. That is

$$\tilde{z}_t = u_t - \theta_1 u_{t-1} - \theta_2 u_{t-2} - \dots - \theta_q u_{t-q} \quad (3)$$

or

$$\tilde{z}_t = \theta(B)\ u_t$$

where $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q$ is the moving average operator of order q. It is required that $\theta(B)$ must be invertible.

It is interesting to note that an AR process of infinite order can be represented by an MA process of finite order. Also, an MA process of infinite order can be represented by an AR process of finite order. This characteristic of ARIMA process sufficesto explain how to build parsimonous models.

The general ARIMA process of order (p,d,q) is defined by

$$\varphi(B)\ \nabla^d z_t = \theta_0 + \theta(B)\ u_t \quad (4)$$

where

$\nabla^d = (1-B)^d$ is the differencing operator,

θ_0 is a constant which denotes the deterministic shift in the process.

Letting $\omega_t = \nabla^d z_t$ then Equation (4) can be written as

$$\varphi(B)\ \omega_t = \theta_0 + \theta(B)\ u_t$$

or

$$\varphi(B)\ \tilde{\omega}_t = \theta(B)\ u_t \quad (5)$$

where

$$\tilde{\omega}_t = \omega_t - \mu_\omega .$$

Another useful representation is provided by letting $\zeta(B) = \varphi(B)\nabla^d$. Then the process becomes

$$\zeta(B)\, \tilde{z}_t = \theta(B)\, u_t \quad (6)$$

The process defined by Equation (6) can be represented as a linear filter of infinite order such that

$$\tilde{z}_t = \frac{\theta(B)}{\zeta(B)} u_t \quad (7)$$

or

$$\tilde{z}_t = \zeta^{-1}(B)\, \theta(B)\, u_t$$

2.1.2. Identification of ARIMA Models

The statistical tools for the analysis of time series are proposed as autocorrelation function (acf), partial autocorrelation function (pacf), and differencing. Box and Jenkins also propose that spectral analysis is a useful device of analysing time series (18).

Estimates of autocorrelations of any time series are computed from the sample data. Let c_k be the autocovariance coefficient at lag k, it can be written as

$$c_k = \frac{1}{N} \sum_{t=1}^{N-k} [(z_t - \bar{z})(z_{t+k} - \bar{z})] \quad (8)$$

for $k = 0,1,2,\ldots,K$

where $\bar{z}$ denotes the sample mean

$$\bar{z} = \frac{1}{N} \sum_{t=1}^{N} z_t \quad (9)$$

N is the number of observations in the series. Let r_k be the autocorrelation coefficient at lag k, it is computed as follows :

$$r_k = \frac{c_k}{c_o} \quad , \quad k = 0,1,2,\ldots, K. \tag{10}$$

A graph of autocorrelation coefficients is called autocorrelation function (acf).

Partial autocorrelations are used to measure the degree of association between z_t and z_{t+k}, but the effects of intervening z's are somehow partialled out. A plot of partial autocorrelation coefficients is called partial autocorrelation function (pacf).

ARIMA processes provide a general class of models. The selection of the most appropriate model for the given time series requires experience. In general, there are several useful guidelines for stationary series.

(1) A pure stationary AR process has a theoretical acf that drops off to zero, but it has a pacf that cuts off to zero after lag p, the order of AR process. The standard error of the partial autocorrelation coefficients after lag p is approximated as

$$se_{r_k} = 1/\sqrt{n} \quad . \tag{11}$$

(2) A pure MA process has a theoretical acf that cuts off to zero after lag q, the order of MA process, but it has a theoretical pacf that drops off to zero. The estimated autocorrelations of order q+1, and higher, are approximately, independently distributed with variance 1/n.

(3) A stationary mixed ARMA process has
   a) a theoretical acf that tails off toward zero after the first q-p lags;
   b) a theoretical pacf that tails off toward zero after the first q-p lags.

2.1.3. Estimation of ARIMA Models

At the estimation stage, the values of the parameters, of the model selected at the identification stage that seems promising to provide parsimonous and statistically adequate representation of the given time series, are computed by minimizing conditional sum of square of errors so that

$$SSE(\underline{\varphi},\underline{\theta}) = \sum_{t=1}^{n} [u_t \mid \underline{\varphi},\underline{\theta},\underline{\omega}]^2 \quad (12)$$

where

$$u_t = \omega_t - \sum_{i=1}^{p} \varphi_i \omega_{t-i} + \sum_{j=1}^{q} \theta_j u_{t-j} \quad (13)$$

The minimization of sum of squares function is accomplished on a computer by using a nonlinear least squares estimation method developed by Marquardt (19). A back forecasting procedure is proposed to approximate unconditional sum of squares of erros (20).

2.1.4. Diagnostic Checking of ARIMA Models

After having the estimated parameters of the specified ARIMA model, diagnostic checks are applied to verify the model adequacy. There are two general factors of interest for diagnostic checking.

(1) The residuals or errors left over after fitting an ARIMA model are expected to be a white noise process. Any recognizable pattern in the estimated autocorrelation function of errors could point out the model inadequacy. A lack of fit test is proposed to test whether the autocorrelation estimates $r_k(u)$ for the residuals are significantly different from zero. The statistic for this purpose is called Q statistic and it is computed as follows:

$$Q_u = n \sum_{k=1}^{K} [r_k(u)]^2 \quad (14)$$

where K is maximum time lag considered.

Q_u statistic is approximately distributed as x^2 with K-p-q degrees of freedom (21), (22).

(2) The sampling statistic of the current optimum solution are studied to see if the model is overfitted. There are two summary statistics provided :

(a) standard errors of the estimated parameters;

(b) correlation matrix of the estimated parameters.

High standard errors of the estimated parameters, and high correlation between parameters may be the signs of overfitting.

2.1.5. Seasonal Models

The general seasonal ARIMA model is represented as ARIMA(p,d,q) $(P,D,Q)^S$ where

- P denotes order of seasonal AR process,
- D denotes degree of seasonal differencing,
- Q denotes order of seasonal MA process, and
- S is period of seasonality.

It can be written as

$$\varphi(B)\ \Phi(B)\ \nabla^d\ \nabla^D\ z_t = \theta(B)\ \Theta(B)\ u_t \qquad (15)$$

where $\varphi(B)$ and $\theta(B)$ are nonseasonal operators as previously identified,
$\Phi(B)$ is seasonal AR operator of order P,
$\Theta(B)$ is seasonal MA operator of order Q, and
∇^D is the differencing operator so that $\nabla^D = (1-B^S)^D$.

The general seasonal ARIMA model is in multiplicative form. Thus, it can be represented as a two stage filtering operation on the process. First is the nonseasonal filtering such that

$$a_t = \theta^{-1}(B)\ \varphi(B)\ \omega_t \qquad (16)$$

where $\omega_t = \nabla^d\ \nabla^D\ z_t$

Then, applying seasonal filtering on a_t,

$$\Phi(B)\ a_t = \Theta(B)\ u_t \qquad (17)$$

which has the same meaning with Equation (15).

Although the mathematics of seasonal ARIMA processes seems sophisticated, it should be kept in mind the the fundamentals of seasonal model building are similar to nonseasonal model building, considering the fact that it takes place at the second stage of the filtering procedure. At the model identification stage, the autocorrelations and partial autocorrelations should be axamined for a seasonal pattern at spikes S lags apart.

2.2. Transfer Function Models

A transfer function model describes the dynamic response of input variables or leading indicators of a system on the output of that system. In this study, only the bivariate case is studied, namely there is one input series. After a brief explanation of transfer function models, the model building is described in terms of three main stages: identification, estimation, and diagnostic checking.

2.2.1. Fundamentals of Transfer Function Models

A very general form of the transfer function can be written as

$$Z_t = v_0 X_t + v_1 X_{t-1} + \dots + N_t$$

or (18)

$$Z_t = v(B)\, X_t + N_t$$

where

z_t denotes output series for $t = 1,2,\dots,N$,
X_t denotes input series for $t = 1,2,\dots,N$,
N_t denotes the sum of all effects of all variables other than X_t, usually called noise,
$v(B)$ is called the transfer function of the system, the weights $v_0, v_1, \dots$ are called impulse response function of the system.

The infinite series $v_0 + v_1 B + v_2 B^2 + \dots$ must be convergent for the system to be stable. The stability condition implies that a finite incremental change in the input results in a finite incremental change in the output. Sometimes, there is a delay of the effect of input to output. Impulse response weights for the periods that input effect lags output are theoretically zero.

It is not practical to represent the system with a high order of transfer function. The parsimonious form of the model is represented by the ratio of two polynomials, such that

$$v(B) = \frac{\omega(B)}{\delta(B)} B^b \qquad (19)$$

where

$\omega(B)$	is a polynomial operator of order s,
$\delta(B)$	is a polynomial operator of order r,
B^b	is a dead time operator of order b representing the number of periods before any effect is discernible.

Thus, the model becomes

$$\delta(B)\ z_t = \omega(B)\ x_{t-b} + n_t \qquad (20)$$

where

$$\delta(B) = 1 - \delta_1 B - \delta_2 B^2 - \ldots - \delta_r\ B^r \quad ,$$

$$\omega(B) = \omega_0 - \omega_1 B - \omega_2 B^2 - \ldots - \omega_s\ B^s \quad ,$$

$z_t = \nabla^d Z_t$, the differenced output series,

$x_t = \nabla^d X_t$, the differenced input series,

n_t is the noise of the model, $n_t = \nabla^d N_t$,

r,s,b are nonnegative integer constants.

An ARIMA model is applied to the noise series so that

$$\varphi(B)\ n_t = \theta(B)\ a_t \qquad (21)$$

Then, the output z_t can be written in the following form

$$z_t = \frac{\omega(B)}{\delta(B)}\ x_{t-b} + \frac{\theta(B)}{\varphi(B)}\ u(t) \qquad (22)$$

which is the general form of bivariate transfer function for the transformed series.

2.2.2. Identification of Transfer Function Models

If the estimated autocorrelation and cross correlation functions of x_t and z_t series fail to damp out quickly, then a degree of differencing d is necessary to induce stationary. The stationarity assumption implies that the constituent processes x_t and z_t have constant means and variances. The estimates of cross covariance coefficients $c_{xy}(k)$ are computed by the following formula.

$$c_{xy}(k) = \frac{1}{N} \sum_{t=1}^{n-k} (x_t - \bar{x})(z_{t+k} - \bar{z}) \quad , \quad k=0,1,2,\ldots,K \qquad (23)$$

Let $c_{xx}(k)$ and $c_{zz}(k)$ be the autocorrelations of input and output series respectively, and S_x and S_z be the estimates of σ_x and σ_z respectively. The cross correlation coefficients $r_{xy}(k)$ are estimated as follows:

$$r_{xy}(k) = \frac{c_{xy}(k)}{S_x \; S_y} \quad , \quad k=0,1,2,\ldots,K \qquad (24)$$

where $S_x = \sqrt{C_{xx}(0)}$,

$S_z = \sqrt{C_{zz}(0)}$.

A plot of $r_{xy}(k)$, $k=0,1,\ldots,K$ is called cross correlation function.

The first step at the identification stage is the differencing operation to achieve stationarity in the series.

Secondly, input series x_t and output series y_t are prewhitened. Prewhitening of the series is accomplished by fitting an ARIMA model to input series x_t , such that

$$\theta_x^{-1}(B) \; \Phi_x(B) \; x_t = \alpha_t \qquad (25)$$

which transforms the correlated input series x_t into the uncorrelated white noise series α_t. The same prewhitening operation is applied to output series z_t as follows:

$$\beta_t = \theta_x^{-1}(B)\ \Phi_x(B)\ z_t \tag{26}$$

Then, autocorrelations $r_{\alpha\alpha}(k)$ and $r_{\beta\beta}(k)$, and crosscorrelations $r_{\alpha\beta}(k)$ are computed so that

$$r_{\alpha\beta}(k) = \frac{c_{\alpha\beta}(k)}{S_\alpha\ S_\beta} \quad , \qquad k = 0,1,2,\ldots,K \tag{27}$$

where

$c_{\alpha\beta}(k)$ is cross covariance coefficient at lag k,

$S_\alpha = \sqrt{c_{\alpha\alpha}(0)}$, and

$S_\beta = \sqrt{c_{\beta\beta}(0)}$.

Finally, impulse response weights estimates v_k are computed as

$$v_k = \frac{S_\beta}{S_\alpha}\ r_{\alpha\beta}(k) \quad , \qquad k = 0,1,2,\ldots,K \tag{28}$$

The preliminary estimates v_k are statistically inefficient, but it can provide a rough basis for selecting appropriate operators $\delta(B)$ and $\omega(B)$ of the transfer function model.

Knowing the v_k values, r,s, and b may be guessed by employing the following guidelines:

(1) First b values of impulse response weights, $v_0, v_1, \ldots, v_{b-1}$, will not be significantly different from zero.

(2) If $r \leq s$ then a further s-r+1 values, $v_b, v_{b+1}, \ldots, v_{b+z-r}$, will not show any clear pattern.

(3) Value v_j with $j \geq b+s-r+1$ will follow a fixed pattern.

It is possible to find a rough estimate of number of impulse response weights which are significantly different from zero except first b-1 values. Let h be this estimate, then the noise series n_t can be estimated as

$$n_t = z_t - \sum_{i=0}^{h} v_i \; x_{t-i} \tag{29}$$

or

$$n_t = z_t - v(B) \; x_t$$

An ARIMA model for the noise series is then specified so that

$$\varphi(B) \; n_t = \theta(B) \; u_t \tag{30}$$

2.2.3. Estimation of Transfer Function Models

Having specified transfer function model for the given series, the estimates of parameters can be obtained by minimizing the conditional sum of squares of errors function:

$$SSE(b, \underline{\delta}, \underline{\varphi}, \underline{\theta}) = \sum_{i=a+p+1}^{n} [u_t \mid \underline{x}, \underline{z}, \underline{u}]^2 \tag{31}$$

where a is the larger of r and s+b. Marquardt's nonlinear least squares estimation method is employed to solve iteratively for the best values of the parameters.

The calculation of u's is accomplished in the following way: First, the output y_t from transfer function is computed as

$$y_t = \delta^{-1}(B) \; \omega(B) \; x_{t-b} \tag{32}$$

Secondly, having calculated y_t series, the noise series n_t can be obtained from

$$n_t = z_t - y_t \tag{33}$$

Finally, u's can be obtained from

$$u_t = \theta^{-1}(B) \; \varphi(B) \; n_t \tag{34}$$

2.2.4. Diagnostic Checking of Transfer Function Model

The residuals, u_t, are assumed to be normally, independently, identically distributed having mean zero and variance σ_u^2. If the autocorrelation function $r_{uu}(k)$ shows a clear pattern, this indicates model inadequacy. Q_{uu} statistic to test residual autocorrelations is computed as

$$Q_{uu} = (n - a - p) \sum_{k=1}^{K} r_{uu}^2(k) \tag{35}$$

Q_{uu} is approximately distributed as χ^2 with K-p-q degrees of freedom.

If the cross correlation function $r_{\alpha u}(k)$ shows any significant spikes, then transfer function model is suggested to be inadequate. The statistic to test cross correlations $r_{\alpha u}(k)$ is computed as

$$Q_{\alpha u} = (n - a - p) \sum_{k=0}^{K} r_{\alpha u}^2 (k) \tag{36}$$

$Q_{\alpha u}$ is approximately distributed as χ^2 with K-r-s degrees of freedom (23).

The same summary statistics given for the univariate model checking are also provided to test overfitting.

2.3. Forecasting

The model fitted to the time series may not be the forecast function. The minimum mean square error forecasts are obtained from the difference equation form of the model. Also, probability limits of the forecasts are provided.

2.3.1. Forecasting with ARIMA Models

The exact forecast function is computed from the fitted function in Equation (5). Assuming a deterministic shift θ_0 exists in the process, it is written as

$$\theta_0 = (1 - \varphi_1 - \varphi_2 - \dots - \varphi_p)\,\bar{\omega} \tag{37}$$

The model fitted to the series can be written as

$$(1 - \varphi_1 B - \varphi_2 B^2 - \dots - \varphi_p B^p)(1 - B)^d z_t = \theta_0 + \theta(B)\, u_t \tag{38}$$

which is the final form.

By doing necessary multiplications in Equation (38), the model can be written as

$$(1 - \zeta_1 B - \zeta_2 B^2 - \dots - \zeta_{p+d} B^{p+d})\, z_t = \theta_0 + \theta(B)\, u_t \tag{39}$$

Finally, the forecast function can be written as

$$z_t = \sum_{i=1}^{p+d} \zeta_i\, z_{t-i} + \theta_0 + u_t - \sum_{j=1}^{q} \theta_i\, u_{t-i} \tag{40}$$

On the other hand, the forecast function may also be represented as a linear combination of current and previous shocks $u_t, u_{t-1}, \dots$ That is

$$z_{t+\ell} = (u_{t+\ell} + \Psi_1 u_{t+\ell} + \dots + \Psi_{\ell-1} u_{t-1}) + (\Psi_\ell u_t + \Psi_{\ell+1} u_{t-1} + \dots) \tag{41}$$

$$= e_t(\ell) + \hat{z}_t(\ell)$$

where

$e_t(\ell)$ denotes the forecast error for lead time ℓ,

$\hat{z}_t(\ell)$ denotes forecast made at origin t for lead time ℓ.

From Equation (41) $e_t(\ell)$ can be written as

$$e_t = u_{t+\ell} + \Psi_1 u_{t+\ell-1} + \dots + \Psi_{\ell-1} u_{t+1} \tag{42}$$

Since the expected value of $e_t(\ell)$, $\underset{t}{E}[e_t(\ell)]$, is zero, forecasts are unbiased. The variance of the forecast error is then obtained as

$$v(\ell) = var[e_t(\ell)] = (1 + \Psi_1^2 + \Psi_2^2 + \ldots + \Psi_{\ell-1}^2) \ \sigma_u^2 \qquad (43)$$

The upper and lower probability limits are computed as

$$z_{t+\ell}(\ell) = \hat{z}_t(\ell) \pm c\sqrt{v(\ell)} \qquad (44)$$

where c is the value of standard normal distribution depending on the probibility that a future value lies in the interval.

2.3.2. Forecasting with Bivariate Transfer Function Models

The model fitted to the series in Equation (22) may be reorganized to obtain the following form of the model:

$$\varphi(B)\ \delta(B)\ \nabla^d z_t = \varphi(B)\ \omega(B)\ \nabla^d\ x_{t-b} + \theta(B)\ \delta(B)\ u_t \qquad (45)$$

By doing necessary operations in Equation (45), the final forecast function becomes

$$z_t = \sum_{i=1}^{p+r+d} \Delta_i\ z_{t-i} + \Omega_0\ x_{t-b} - \sum_{j=1}^{p+s+d} \Omega_i\ x_{t-b-i} + \sum_{k=1}^{q+r} \theta_i\ u_{t-i} \qquad (46)$$

The variance of the forecast error for lead time ℓ is given by

$$v(\ell) = \sigma_\alpha^2 \sum_{j=b}^{\ell-1} v_j^2 + \sigma_u^2 \sum_{j=0}^{\ell-1} \Psi_j^2 \qquad (47)$$

The probability limits are then computed in the same way as in Equation (44).

III. METHODOLOGY FOR THE IMPLEMENTATION

The three basic analytical stages of Box-Jenkins approach comprise several procedural steps in the computer implementation of the method. In this chapter, the analysis is presented in terms of procedural steps. Illustrative outlines are given in order to demonstrate how to operate the computer model during the application of the method.

The computer model does not provide automatic model selection, therefore external intervention is necessary at decision points during the execution of the program. The user should strictly follow the procedural steps, which are given in the following sections, during model building and forecasting.

3.1. Methodology for ARIMA Models

The methodology of using ARIMA processes includes six procedural steps. The complete analysis is outlined in Figure 3.1.

Step 1. The user has facilities such as to enter new data series, to access existing data series, or to update existing data series. The data series are saved for later use, the maintenance of data files is the responsibility of the user.

Step 2. The analysis of data is performed in this step. The computer software will provide autocorrelation function and partial autocorrelation function. It is recommended to apply an iterative process to achieve a stationary mean in the series by providing values of d, S, and D. A subclass of models, which are candidates for a good representation of the process under study, is determined in this step.

Step 3. The user will specify p, q and P, Q parameters of ARIMA(p,d,q)$(P,D,Q)^S$ model. If a deterministic shift is recognized in the series, the control parameter M is set to one. The computer program will provide starting values of the parameter $\underline{\varphi}$, $\underline{\theta}$, $\underline{\Phi}$, $\underline{\Theta}$, θ_0 for the optimization.

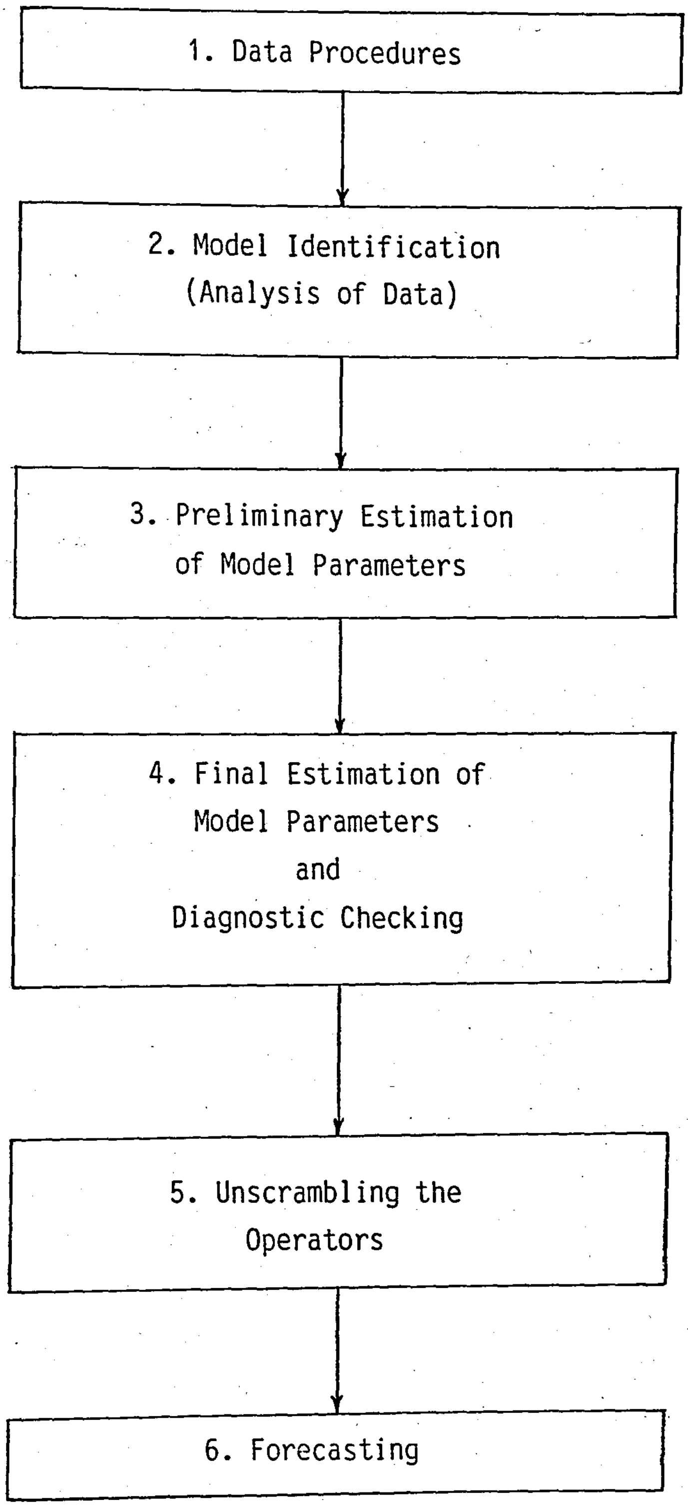

Figure 3.1 Outline of Methodology for ARIMA Processes Applied to Forecasting

Step 4. Having starting points of the parameters, the computer will carry out the minimization procedure to fit the chosen model to the given data series. The stationarity of AR process, and invertibility of MA process should be checked on the resulting values of the parameters.

An ARMA(0,q) process is always stationary. For an ARMA(1,q) process, the absolute value of φ_1 must be less than one. The stationarity requirement for an ARMA(2,q) process consists of three conditions:

$$\begin{aligned} |\varphi_2| &< 1 \\ \varphi_2 - \varphi_1 &< 1 \\ \varphi_2 + \varphi_1 &< 1 \end{aligned} \tag{48}$$

The stationary conditions are mathematically complicated for $p > 2$, at least, a rough check may be made

$$\varphi_1 + \varphi_2 + \ldots + \varphi_p < 1 \quad , \quad p > 2 \tag{49}$$

The invertibility conditions of MA processes are similar to the stationarity requirements of AR processes. An ARMA(p,0) process is always invertible. An ARMA(p,1) process is invertible if $|\theta_1| < 1$. The invertibility requirements of ARMA(p,2) are a set of three conditions:

$$\begin{aligned} |\theta_2| &< 1 \\ \theta_2 - \theta_1 &< 1 \\ \theta_2 + \theta_1 &< 1 \end{aligned} \tag{50}$$

If $q > 2$, then a rough check might be

$$\theta_1 + \theta_2 + \ldots + \theta_q < 1 \quad , \quad q \quad 2 \tag{51}$$

The relevant statistics for diagnostic checking are provided in this step.

Step 5. If the current fitted model is decided to forecast future values of the series, the exact forecast function is computed by unscrambling the operators which are employed during model building stages. The model is saved for later use.

Step 6. The forecasts of future values are provided with the upper and lower probability limits. Sum of errors, SOE, is computed for tracking signal test (24). It is a measure of accuracy of the forecasting model.

Let current time be N, the forecast error at time t be $u_t = z_t - \hat{z}_t$, thus

$$SOE = \sum_{t=1}^{N} u_t \tag{52}$$

If forecast is unbiased, $E[u_t]$ will be zero, that is to say forecast error u_t is considered a random variable having mean zero. A significant departure of SOE from zero may indicate the inadequacy of present model.

The printouts are also designed for each step except step one. They include detailed information about the process.

3.2. Methodology for Transfer Function Models

The analysis of transfer function method for bivariate time series resembles to that of ARIMA method in many respects. It includes eight procedural steps. The complete analysis is outlined in Figure 3.2.

Step 1. The contents of this step is the same with the univariate case, except there are two data series.

Step 2. Prewhitening of input series is accomplished by employing steps two through five of ARIMA case on input series. It aims to remove systematic variation in the series.

Step 3. Prewhitening of output series is accomplished by using the model built in the last step, step 2. The cross correlation function and direct estimate of impulse response function are provided. Initial estimate of noise series n_t are computed depending on direct estimates of impulse response weights. The user should specify candidate models for the representation of the given data series.

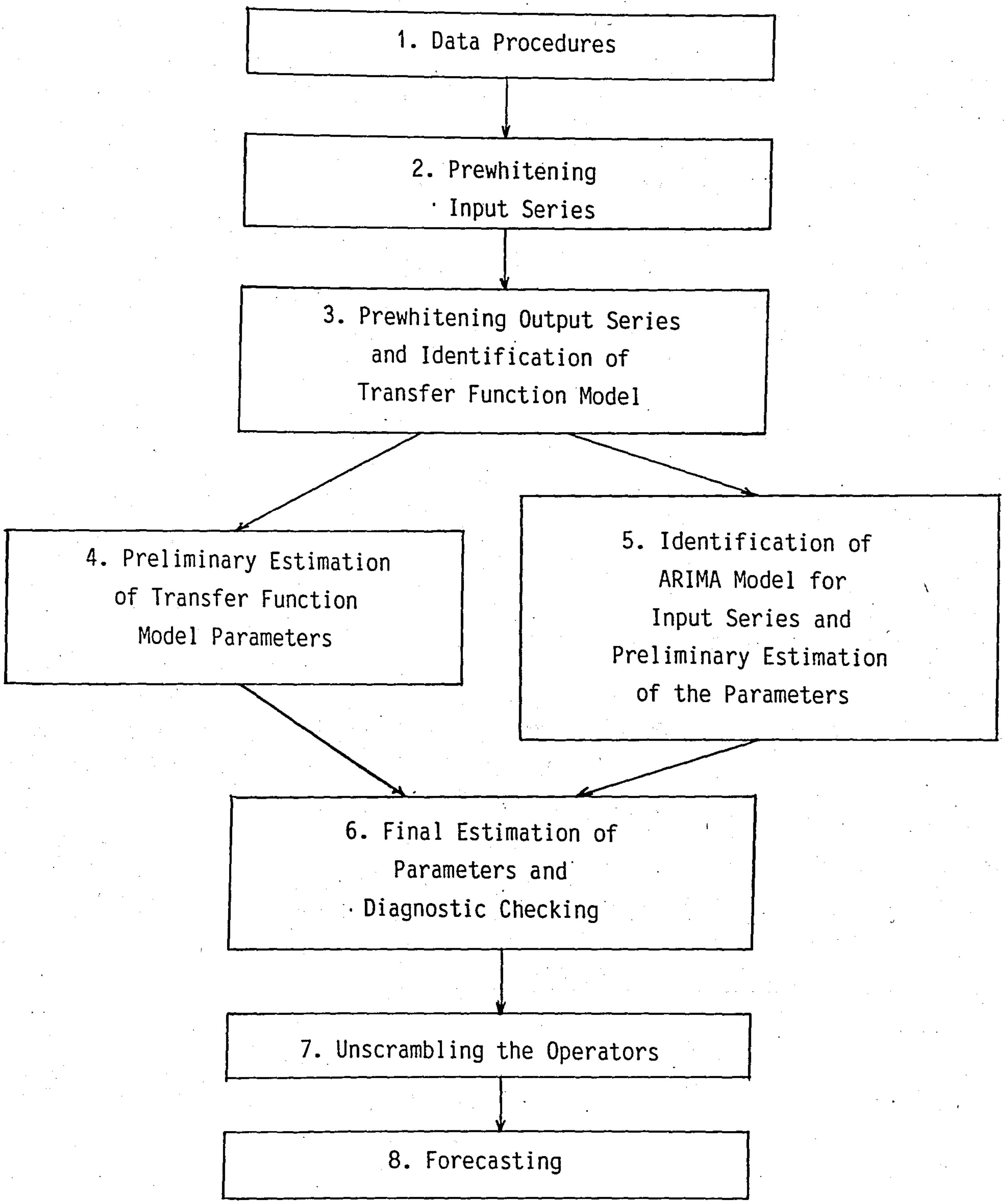

Figure 3.2 Outline of Box-Jenkins Transfer Function Analysis Applied to Forecasting

Step 4. Starting values of the specified transfer function model parameters $\underline{\delta}$ and $\underline{\omega}$ are computed in this step.

Step 5. This step basically includes steps two and three of ARIMA case which are applied to noise series n_t.

Step 6. The minimization procedure is executed to estimate values of parameters $\underline{\delta}$, $\underline{\omega}$, $\underline{\varphi}$, and $\underline{\theta}$ of the specified transfer function model. The summary statistic is provided for diagnostic checking. The stability of the transfer function model should be checked (25). The stability of the first order model requires absolute value of δ_1 must be less than one, and for the second order model, the parameters δ_1,δ_2 satisfy

$$\delta_2 + \delta_1 < 1$$

$$\delta_2 - \delta_1 < 1 \qquad (53)$$

$$-1 < \delta_2 < 1$$

Step 7. The forecast function is computed by unscrambling the operators. The parameter values of the forecast function are saved for making the forecasts later.

Step 8. This step is mainly the same with the Step 6 of ARIMA methodology.

The printouts, which include detailed information about the process, are provided in steps two through eight.

IV. CONCLUSION

The Box-Jenkins time series analysis is theoreticaly and statistically very powerful approach for making the forecast of the future. The forecasting application of the approach provides minimum mean square error forecast that there is no other forecast that conditioned only the same history of the series which can produce sum of squares of errors will on the average be smaller. Contrary to this fact there are a few difficulties hindering widespread adoption of the approach as a basis for forecasting.

The Box-Jenkins methods are not suited to the handling of large number of series if the forecasts are needed quickly and cheaply. The methods perform well on situations where good data exist over a reasonable period of time, and where high accuracy of forecasts are necessary for an important planning issue. In certain cases, the method will be either uneconomic to perform, or inferior to some other time series methods, because it is not suitable for short series, and for longer forecast lead times.

Application of Box-Jenkins time series analysis to operational forecasting requires a substantial amount of computing at each stage of model building and forecasting procedures. A computer program is designed with the objective of offering maximum flexibility and ease of use to nonprogramming users. The program can be operated in a fairly mechanical manner, but time series model building considerably requires significant human judgement. The user of the computer program should understand both the methodology suggested and each aspect of the computer output. The computer model for Box-Jenkins approach can not be applied meaningfully unless the methodology and the underlying principles are well understood. The level of understanding of the various steps in the procedures of applying Box-Jenkins method will be a major determinant in its use. In addition, there is a subtantial need for experience and some trial and error in successfully applying the method.

In this study, Marquardt's nonlinear least squares estimation algorithm is applied for iteratively minimizing sum of squares of residuals. A study of further development of the computer model could be designing an efficient nonlinear optimization method to maximize likelihood function instead of using Marquardt's algorithm. It is generally expected that maximum likelihood estimates of the parameters on the average will be more efficient than least squares estimates of the parameters.

This study may also be further extended in two dimensions:

(1) In this study, it has been dealt only with the bivariate case of transfer function analysis. The forecasting version of multivariate transfer function analysis may be applied for making the use of full capabilities of transfer function method.

(2) Box-Jenkins time series analysis can be applied for process control. The objective to design control schemes is to minimize overall measure of error at the targeted output (26). This model may be developed to design discrete control schemes.

The application of Box-Jenkins time series analysis to forecasting has considerably lagged behind its theoretical formulation and verification. This study should be granted successful if it provides any impetus on the application of Box-Jenkins time series analysis to forecasting, and if it encourages further study on the subject.

APPENDICES

APPENDICES

Appendix A COLLECTION OF TIME SERIES USED FOR APPLICATIONS

A.1. Series 1 : Monthly Sales of Glass Product A
A.2. Series 2 : Monthly Sales of Glass Product B
A.3. Series 3 : Monthly Sales of Glass Product C
A.4. Series 4 : Monthly Sales of Glass Product D

Appendix B APPLICATIONS

B.1. Application 1 : Stochastic Model Building and Forecasting for Series 1
B.2. Application 2 : Stochastic Model Building and Forecasting for Series 2
B.3. Application 3 : Stochastic Model Building for Series 3
B.4. Application 4 : Stochastic Model Building for Series 4

APPENDIX A

COLLECTION OF TIME SERIES USED FOR APPLICATIONS

A.1 Series 1 : Monthly Sales of Glass Product A

Period	Observation	Period	Observation	Period	Observation
1	3829	47	5998	93	5743
2	2592	48	4860	94	5729
3	3498	49	3953	95	9142
4	3028	50	4860	96	9440
5	3414	51	3366	97	1175
6	3104	52	2424	98	2142
7	4627	53	3678	99	3437
8	2698	54	2729	100	6757
9	2422	55	3800	101	9124
10	4196	56	3677	102	4811
11	4627	57	3922	103	2719
12	4762	58	3846	104	6662
13	3295	59	3073	105	3504
14	2521	60	3810	106	6318
15	2988	61	3890	107	5327
16	2878	62	3013	108	3092
17	2846	63	2406	109	2086
18	4809	64	2978	110	4266
19	3128	65	3083	111	3262
20	2495	66	4165	112	5155
21	2906	67	4722	113	4796
22	3453	68	4065	114	3680
23	4905	69	2704	115	7197
24	5302	70	2392	116	7398
25	3384	71	4039	117	14374
26	3014	72	5487	118	8265
27	2350	73	5010	119	6181
28	2295	74	4022	120	4939
29	2655	75	6133		
30	2613	76	4818		
31	3169	77	3589		
32	3845	78	3648		
33	3983	79	3417		
34	2906	80	4247		
35	2471	81	4639		
36	2630	82	6091		
37	2665	83	6005		
38	2459	84	4816		
39	3876	85	2371		
40	3623	86	3302		
41	3559	87	4477		
42	4143	88	4598		
43	3317	89	8998		
44	3586	90	1534		
45	2976	91	6186		
46	4552	92	5987		

A.2 Series 2 : Monthly Sales of Glass Product B

Period	Observation	Period	Observation	Period	Observation
1	1728	41	1830	81	1641
2	1306	42	1662	82	1716
3	1528	43	2344	83	2214
4	1491	44	1502	84	1386
5	1489	45	1400	85	1066
6	1600	46	1825	86	1247
7	1291	47	1702	87	1513
8	1164	48	1931	88	1783
9	1147	49	1915	89	3638
10	1150	50	2453	90	738
11	1884	51	1646	91	818
12	1627	52	1217	92	1647
13	2163	53	1029	93	1250
14	1485	54	1700	94	1704
15	1732	55	1400	95	3355
16	1109	56	1300	96	2335
17	1362	57	1400	97	540
18	1839	58	1301	98	928
19	1395	59	1470	99	1519
20	877	60	2434	100	2262
21	1035	61	1451	101	2786
22	1204	62	1642	102	1303
23	1383	63	1404	103	1213
24	1639	64	1769	104	2447
25	1793	65	1484	105	1442
26	1844	66	1683	106	1676
27	1330	67	1243	107	1718
28	1696	68	2519	108	863
29	1788	69	1906	109	1064
30	1871	70	1418	110	1282
31	1980	71	1570	111	1511
32	1215	72	1437	112	2037
33	729	73	1611	113	1428
34	1240	74	922	114	1153
35	1572	75	2405	115	1670
36	2254	76	2410	116	1539
37	2235	77	2113	117	2935
38	2031	78	1816	118	1974
39	1592	79	1571	119	2241
40	2103	80	1402	120	1469

A.3 Series 3 : Monthly Sales of Glass Product C

Period	Observation	Period	Observation	Period	Observation
1	2790	37	3395	73	3027
2	2143	38	4071	74	2900
3	2809	39	2693	75	3737
4	2513	40	2869	76	3476
5	2910	41	3022	77	3385
6	3962	42	5868	78	3345
7	2100	43	4925	79	2949
8	1960	44	4762	80	3352
9	2031	45	3385	81	3939
10	3305	46	3199	82	4251
11	3358	47	3222	83	5716
12	3365	48	2254	84	3784
13	3060	49	2813	85	2234
14	4102	50	3828	86	3207
15	3492	51	2739	87	4928
16	3072	52	2855	88	5276
17	3198	53	3193	89	8890
18	3509	54	4947	90	4849
19	2763	55	4050	91	3788
20	2489	56	3358	92	9384
21	2606	57	3749	93	6441
22	3242	58	3308	94	4169
23	2429	59	3576	95	7602
24	2880	60	5400	96	6420
25	2732	61	5015	97	1788
26	2973	62	2764	98	2686
27	2453	63	2326	99	4193
28	2354	64	2700	100	9114
29	2966	65	2397	101	8386
30	3188	66	3366	102	4404
31	2928	67	2683	103	3925
32	2490	68	2911	104	7212
33	2589	69	4547	105	4167
34	2827	70	3139	106	5577
35	2811	71	3164	107	5853
36	3111	72	2956	108	4015

A.4 Series 4 : Monthly Sales of Glass Product D

Period	Observation	Period	Observation
1	329	31	25
2	308	32	33
3	262	33	28
4	242	34	28
5	551	35	33
6	200	36	29
7	284	37	225
8	35	38	252
9	140	39	301
10	584	40	512
11	936	41	957
12	594	42	218
13	314	43	400
14	293	44	625
15	258	45	442
16	381	46	473
17	406	47	550
18	618	48	405
19	391	49	155
20	619	50	218
21	709	51	223
22	187	52	343
23	127	53	381
24	52	54	278
25	65	55	649
26	10	56	586
27	29	57	778
28	30	58	437
29	22	59	313
30	31	60	715

APPENDIX B APPLICATIONS

In this appendix four univariate applications of the Box-Jenkins approach are presented for the time series given in Appendix A.

B.1. Application 1 : Stochastic Model Building and Forecasting for Series 1

B.1.1. Model Identification

Number of observations, N : 108

Mean of the observations, $\bar{z}$ = 3975

Degree of nonseasonal differencing, d = 1

Period of seasonality, S = 6

Degree of seasonal differencing, D = 1

Number of observations in the differenced series, n = 101

Mean of the differenced series, $\bar{\omega}$ = -24.9

Variance of the differenced series, $\hat{\sigma}^2_\omega = 6.078 \times 10^6$

```
  ** Autocorrelations of the Series  **
  Approximate standard error : 9.950E-02
Lag -------------------------------------------------- Autocor.
 0                   |******************************** 1.00E+00
 1        ************|                                 -3.9E-01
 2                  **|                                 -9.0E-02
 3                  **|                                 -7.9E-02
 4                    |***                              9.44E-02
 5                    |******                           2.15E-01
 6    ****************|                                 -5.4E-01
 7                    |*******                          2.36E-01
 8                    |*                                6.09E-02
 9                    |**                               7.48E-02
10                   *|                                 -5.6E-02
11                  **|                                 -7.3E-02
12                    |***                              1.05E-01
13                    |                                 -2.9E-02
14                    |                                 -1.8E-02
15                  **|                                 -6.8E-02
16                    |*                                5.84E-02
17                    |*                                5.16E-02
18                   *|                                 -3.1E-02
```

```
  ** Partial Autocorrelations of the Series   **
  Approximate standard error : 9.950E-02
Lag ------------------------------------------------------ Partials
 1        *****************************|               -3.9E-01
 2                **********************|               -2.9E-01
 3              ************************|               -3.1E-01
 4                         ************|               -1.7E-01
 5                                     |************** 1.95E-01
 6  ***********************************|               -4.8E-01
 7                   ******************|               -2.4E-01
 8                          ***********|               -1.5E-01
 9                          ***********|               -1.5E-01
10                                 ****|               -5.8E-02
11                                     |*******        1.00E-01
12                     ****************|               -2.2E-01
13                                 ****|               -6.1E-02
14                                     |               4.35E-03
15                              *******|               -1.0E-01
16                                     |               1.15E-02
17                                     |**********     1.40E-01
18                                 ****|               -5.8E-02
```

ARIMA(1,1,1)(0,1,1)^6 model is specified for Series 1. It can be written as follows:

$$(1 - \varphi_1 B)\ \omega_t = \theta_0 + (1 - \theta_1 B^6)(1 - \theta_1 B)\ u_t$$

or

$$(1 - \varphi_1\ B)(1 - B)(1 - B^6)\ z_t = \theta_0 + (1 - \theta_1 B^6)(1 - \theta_1 B)\ u_t$$

B.1.2. Model Estimation and Diagnostic Checking Results

(a) Preliminary estimates:

$\theta_0 = -19.24$

$\varphi_1 = 0.2280$

$\theta_1 = 0.5913$

$\theta_1 = 0.2398$

SSE $= 3.743 \times 10^8$

$\sigma_u^2 = 4.377 \times 10^6$

(b) Final estimates:

Parameter	Value	Standard Error
θ_0	-5.1219	3.8380
φ_1	0.1751	0.1180
θ_1	0.9060	0.0534
θ_1	0.8039	0.0796

SSE = 2.335×10^8

$\hat{\sigma}^2_u = 2.407 \times 10^6$

(c) Residuals:

```
  ** Residual autocorrelations **
  Aproximate standard error : 9.950E-02
Lag -----------------------------------------------Autocor.
 0        |***************************************** 1.00E+00
 1        |                                          1,38E-02
 2      **|                                          -6.1E-02
 3   *****|                                          -1.3E-01
 4       *|                                          -3.3E-02
 5        |***                                       8.26E-02
 6        |                                          -4.4E-03
 7        |****                                      1.05E-01
 8        |****                                      9.47E-02
 9        |**                                        5.66E-02
10       *|                                          -3.1E-02
11        |                                          6.72E-03
12        |*                                         2.59E-02
13        |*                                         3.51E-02
14       *|                                          -3.2E-02
15        |                                          -6.9E-04
16      **|                                          -5.8E-02
17        |*                                         4.02E-02
18        |*                                         3.05E-02
```

Chi-square statistic for residual autocorrelations,
Q = 6.311, Degrees of freedom = 14

(d) Correlation matrix of the estimated parameters:

	θ_0	φ_1	θ_1	θ_1
θ_0	1.000	0.011	0.057	-0.088
φ_1	0.011	1.000	0.520	-0.170
θ_1	0.057	0.520	1.000	-0.360
θ_1	-0.088	-0.170	-0.360	1.000

(e) Forecast function:

$$z_{N+\ell} = -5.122 + 1.175\, z_{N+\ell-1} - 0.175\, z_{N+\ell-2} + z_{N+\ell-6} - 1.180\, z_{N+\ell-7} + 0.175\, z_{N+\ell-8} - 0.906\, u_{N+\ell-1} - 0.804\, u_{N+\ell-6} + 0.728\, u_{N+\ell-7}$$

for $\ell = 1,2,\ldots,$

B.1.3. Forecasting

Forecast base time : 108

Forecast lead time : 12

Time	Actual	Forecast	90% Probability Limits Lower limit	Upper limit
109	2.086000E+03	3.282533E+03	7.224050E+02	5.842660E+03
110	4.266000E+03	4.432797E+03	1.781555E+03	7.084040E+03
111	3.262000E+03	4.200969E+03	1.525212E+03	6.876727E+03
112	5.155000E+03	5.243843E+03	2.550875E+03	7.936811E+03
113	4.796000E+03	6.388686E+03	3.679726E+03	9.097646E+03
114	3.680000E+03	4.543673E+03	1.819004E+03	7.268342E+03
115	7.197000E+03	3.431161E+03	5.931711E+02	6.269151E+03
116	7.398000E+03	4.348077E+03	1.478152E+03	7.218002E+03
117	1.437400E+04	4.070256E+03	1.177509E+03	6.963004E+03
118	8.265000E+03	5.099952E+03	2.185939E+03	8.013966E+03
119	6.181000E+03	6.237366E+03	3.302477E+03	9.172255E+03
120	4.939000E+03	4.385930E+03	1.430353E+03	7.341506E+03

B.2. Application 2 : Stochastic Model Building and Forecasting for Series 2

B.2.1. Model Identification

Number of observations, N : 108

Mean of the observations, $\bar{z}$ = 1629

Degree of nonseasonal differencing, d = 1

Period of seasonability, S = 6

Degree of seasonal differencing, D = 1

Number of observations in the differenced series, n = 101

Mean of the differenced series, $\bar{\omega}$ = -0.03

Variance of the differenced series, $\hat{\sigma}^2_\omega$ = 5.523 x 10^5

```
   ** Autocorrelations of the Series   **
   Approximate standard error : 9.950E-02
Lag -------------------------------------------------- Autocor.
 0                ¦*********************************** 1.00E+00
 1     ***********¦                                    -3.3E-01
 2          ******¦                                    -2.0E-01
 3                ¦                                    1.69E-02
 4                ¦****                                1.42E-01
 5                ¦*                                   5.72E-02
 6 ***************¦                                    -4.5E-01
 7                ¦********                            2.36E-01
 8                ¦*                                   5.12E-02
 9               *¦                                    -4.2E-02
10               *¦                                    -3.0E-02
11                ¦*                                   5.41E-02
12                ¦*                                   5.63E-02
13               *¦                                    -4.7E-02
14                ¦*                                   5.56E-02
15             ***¦                                    -1.1E-01
16                ¦***                                 1.06E-01
17               *¦                                    -5.6E-02
18                ¦                                    -2.7E-03
```

```
   ** Partial Autocorrelations of the Series  **
   Approximate standard error : 9.950E-02
Lag ---------------------------------------------------- Partials
 1          ******************************¦           -3.3E-01
 2        ********************************¦           -3.5E-01
 3                  **********************¦           -2.4E-01
 4                                      **¦           -3.1E-02
 5                                        ¦********** 1.07E-01
 6  ***************************************¦           -4.3E-01
 7                           *************¦           -1.4E-01
 8                     *******************¦           -2.0E-01
 9                       *****************¦           -1.8E-01
10                               *********¦           -1.0E-01
11                                       *¦           -1.7E-02
12                     *******************¦           -2.1E-01
13                                    ****¦           -4.7E-02
14                                        ¦**         2.42E-02
15                         ***************¦           -1.7E-01
16                                        ¦******     6.97E-02
17                                        ¦**         2.41E-02
18                                      **¦           -2.6E-02
```

ARIMA(1,1,1)$(0,1,1)^6$ model is specified for Series 2. It can be written as follows:

$$(1 - \varphi_1 B)\omega_t = (1 - \theta_1 B^6)(1 - \theta_1 B)\, u_t$$

or

$$(1 - \varphi_1 B)(1 - B^6)(1 - B)\, z_t = (1 - \theta_1 B^6)(1 - \theta_1 B)\, u_t$$

B.2.2. Model Estimation and Diagnostic Checking Results

(a) Preliminary estimates:

φ_1 = 0.6116

θ_1 = 0.3907

θ_1 = 0.4909

SSE = 5.022×10^7

$\hat{\sigma}^2_u$ = 5.415×10^5

(b) Final estimates:

Parameter	Value	Standard Error
φ_1	0.1964	0.1283
θ_1	0.8632	0.0702
θ_1	0.6167	0.088

SSE = 2.748×10^7

$\hat{\sigma}^2_u = 2.804 \times 10^5$

(c) Residuals:

```
  ** Residual autocorrelations **
  Aproximate standard error : 9.950E-02
Lag ------------------------------------------------Autocor.
 0       |******************************************** 1.00E+00
 1       |                                             7.11E-03
 2   ****|                                             -1.1E-01
 3      *|                                             -2.7E-02
 4       |                                             5.06E-03
 5    ***|                                             -8.7E-02
 6      *|                                             -3.6E-02
 7       |***                                          6.97E-02
 8       |*                                            3.05E-02
 9       |                                             -1.1E-02
10       |                                             1.55E-02
11       |**                                           5.12E-02
12       |***                                          7.42E-02
13       |**                                           5.84E-02
14       |**                                           5.35E-02
15      *|                                             -4.0E-02
16      *|                                             -2.5E-02
17       |                                             7.36E-04
18      *|                                             -3.7E-02
```

Chi-square statistic for residual autocorrelations,

Q = 4.612, Degrees of freedom = 15

(d) Correlation matrix of the estimated parameters:

	φ_1	θ_1	θ_1
φ_1	1.000	0.630	-0.230
θ_1	0.630	1.000	-0.350
θ_1	-0.230	-0.350	1.000

(e) Forecast function:

$$z_{N+} = 1.196\, z_{N+\ell-1} - 0.196\, z_{N+\ell-2} + z_{N+\ell-6} - 1.200\, z_{N+\ell-7} + 0.196\, z_{N+\ell-8} - 0.863\, u_{N+\ell-1} - 0.617\, u_{N+\ell-6} + 0.532\, u_{N+\ell-7}$$

B.1.3. Forecasting

Forecast base time : 108

Forecast lead time : 12

Time	Actual	Forecast	90% Probability Limits Lower limit	Upper limit
109	1.064000E+03	7.529725E+02	-1.20757E+02	1.626702E+03
110	1.282000E+03	1.526588E+03	6.056317E+02	2.447545E+03
111	1.511000E+03	1.305219E+03	3.674625E+02	2.242975E+03
112	2.037000E+03	1.663040E+03	7.126843E+02	2.613395E+03
113	1.428000E+03	2.228076E+03	1.265984E+03	3.190169E+03
114	1.153000E+03	1.096733E+03	1.231777E+02	2.070288E+03
115	1.670000E+03	7.425469E+02	-3.44556E+02	1.829650E+03
116	1.539000E+03	1.468209E+03	3.503645E+02	2.586054E+03
117	2.935000E+03	1.237422E+03	9.880754E+01	2.376036E+03
118	1.974000E+03	1.593393E+03	4.359598E+02	2.750826E+03
119	2.241000E+03	2.158067E+03	9.824136E+02	3.333719E+03
120	1.469000E+03	1.026651E+03	-1.66886E+02	2.220189E+03

B.3 Application 3 : Stochastic Model Building for Series 3

B.3.1. Model Identification

Number of observations, N : 108

Mean of the observations, $\bar{z}$ = 3716

Degree of nonseasonal differencing, D: 1

Period of seasonality, S = 6

Degree of seasonal differencing, D = 1

Number of observations in the differenced series, n = 101

Mean of the differenced series, $\bar{\omega}$ = 2.980

Variance of the differenced series, $\hat{\sigma}^2_{\omega} = 3.569 \times 10^6$

```
  ** Autocorrelations of the Series   **
  Approximate standard error : 9.950E-02
Lag ------------------------------------------------------ Autocor.
 0                      |******************************** 1.00E+00
 1                    **|                                 -7.3E-02
 2  ********************|                                 -6.3E-01
 3                      |*                                4.53E-02
 4                      |****************                 4.97E-01
 5                    **|                                 -7.0E-02
 6    ******************|                                 -5.7E-01
 7                      |*                                5.87E-02
 8                      |************                     3.66E-01
 9                      |                                 -1.9E-02
10               *******|                                 -2.5E-01
11                      |*                                6.38E-02
12                      |*****                            1.89E-01
13                     *|                                 -3.9E-02
14                    **|                                 -7.1E-02
15                     *|                                 -4.0E-02
16                      |*                                6.19E-02
17                      |                                 3.89E-03
18                     *|                                 -3.3E-02
```

```
  ** Partial Autocorrelations of the Series  **
  Approximate standard error : 9.950E-02
Lag ----------------------------------------------- Partials
 1                                         ****¦          -7.3E-02
 2   ***************************************¦          -6.4E-01
 3                                     ********¦          -1.3E-01
 4                                             ¦********* 1.52E-01
 5                                             ¦          -8.7E-03
 6                     **********************¦          -3.8E-01
 7                                   **********¦          -1.7E-01
 8                          ******************¦          -3.0E-01
 9                                       ******¦          -1.0E-01
10                                             ¦          -1.2E-03
11                                             ¦*         2.39E-02
12                                 ************¦          -1.9E-01
13                                    *********¦          -1.5E-01
14                                           **¦          -3.2E-02
15                                     ********¦          -1.3E-01
16                                             ¦*         2.02E-02
17                                           **¦          -4.7E-02
18                                        *****¦          -9.0E-02
```

ARIMA(2,1,0)(1,1,0)^6 model is specified for Series 3. It can be written as follows:

$$(1 - \Phi_1 B^6)(1 - \varphi_1 B - \varphi_2 B^2)\ \omega_t = u_t$$

or

$$(1 - \Phi_1 B^6)(1 - \varphi_1 B - \varphi_1 B^2)(1 - B^6)(1 - B)\ z_t = u_t$$

B.3.2. Model Estimation and Diagnostic Checking Results

(a) Preliminary estimates:

φ_1 = -0.1190

φ_2 = -0.6350

Φ_1 = -0.0885

SSE = 1.950×10^8

$\hat{\sigma}^2_u$ = 2.074×10^6

(b) Final estimates:

Parameter	Value	Standard Error
φ_1	-0.322	0.0881
φ_2	-0.533	0.0869
Φ_1	-0.633	0.0910

$SSE = 1.497 \times 10^8$

$\hat{\sigma}^2_u = 1.528 \times 10^6$

```
** Residual autocorrelations **
Aproximate standard error : 9.950E-02
Lag -------------------------------------------------------Autocor.
 0        |****************************************** 1.00E+00
 1      **|                                           -5.5E-02
 2       *|                                           -3.0E-02
 3   *****|                                           -1.2E-01
 4        |                                           -1.5E-02
 5  ******|                                           -1.4E-01
 6      **|                                           -4.9E-02
 7   *****|                                           -1.2E-01
 8        |******                                     1.59E-01
 9        |**                                         5.30E-02
10    ****|                                           -9.5E-02
11        |*                                          3.57E-02
12      **|                                           -4.6E-02
13        |*                                          2.64E-02
14        |**                                         4.84E-02
15     ***|                                           -9.1E-02
16        |                                           -8.3E-03
17        |****                                       9.74E-02
18        |*                                          2.73E-02
```

Chi-square statistic for residual autocorrelations,
Q = 11.879, Degrees of freedom = 15

(d) Correlation matrix of the estimated parameters:

	φ_1	φ_2	Φ_1
φ_1	1.000	0.170	0.230
φ_2	0.170	1.00	-0.120
Φ_1	0.230	-0.120	1.000

(e) Forecast function:

$$z_{N+\ell} = 0.678\, z_{N+\ell-1} - 0.211\, z_{N+\ell-2} + 0.533\, z_{N+\ell-3} + 0.367\, z_{N+\ell-6}$$
$$- 0.249\, z_{N+\ell-7} + 0.078\, z_{N+\ell-8} - 0.195\, z_{N+\ell-9} + 0.633\, z_{N+\ell-12}$$
$$- 0.403\, z_{N+\ell-13} + 0.134\, z_{N+\ell-14} - 0.338\, z_{N+\ell-15}$$

for $\ell = 1,2,\ldots$

B.4. Application 4 : Stochastic Model Building for Series 4

B.4.1. Model Identification

Number of observations, N : 60

Mean of the observations, $\bar{z}$ = 654

Degree of nonseasonal differencing, d = 1

Period of seasonality, S = 1

Degree of seasonal differencing, D = 0

Number of observations in the differenced series, n = 59

Mean of the differenced series, $\bar{\omega}$ = 6.54

Variance of the differenced series, $\hat{\sigma}^2_{\omega} = 4.77 \times 10^4$

```
 ** Autocorrelations of the Series  **
 Approximate standard error : 1.302E-01
Lag ------------------------------------------ Autocor.
 0          |**************************************** 1.00E+00
 1  ********|                                        -2.1E-01
 2  ********|                                        -2.0E-01
 3          |                                        7.88E-03
 4        **|                                        -6.2E-02
 5         *|                                        -2.6E-02
 6          |**                                      4.94E-02
 7          |***                                     8.57E-02
 8    ******|                                        -1.5E-01
 9          |***                                     8.56E-02
10          |**                                      6.19E-02
11          |                                        5.08E-04
12     *****|                                        -1.3E-01
13        **|                                        -5.3E-02
14          |*                                       4.79E-02
15     *****|                                        -1.4E-01
16          |*********                               2.34E-01
17        **|                                        -5.4E-02
18    ******|                                        -1.6E-01
```

```
   ** Partial Autocorrelations of the Series   **
   Approximate standard error : 1.302E-01
Lag -------------------------------------------------------- Partials
 1         ******************************|              -2.1E-01
 2  *************************************|              -2.5E-01
 3                        ***************|              -1.1E-01
 4                 **********************|              -1.6E-01
 5                      *****************|              -1.2E-01
 6                                *******|              -5.0E-02
 7                                       |*******       5.17E-02
 8                   ********************|              -1.4E-01
 9                                       |******        4.25E-02
10                                       |*******       4.82E-02
11                                       |***********   8.04E-02
12                         **************|              -9.9E-02
13                        ***************|              -1.1E-01
14                                 ******|              -4.6E-02
15       ********************************|              -2.2E-01
16                                       |************  8.74E-02
17                           ************|              -8.8E-02
18                ***********************|              -1.6E-01
```

ARIMA(1,1,1) model is specified for Series 4. It can be written as follows:

$$(1 - \varphi_1 B)\ \omega_t = (1 - \theta_1 B)\ u_t$$

or

$$(1 - \varphi_1 B)(1 - B)\ z_t = (1 - \theta_1 B)\ u_t$$

B.4.2. Model Estimation and Diagnostic Checking Results

(a) Preliminary estimates:

φ_1 = 0.9544

φ_2 = -0.7560

SSE = 2.168×10^7

$\hat{\sigma}^2_u$ = 8.430×10^5

(b) Final estimates:

Parameter	Value	Standard Error
φ_1	0.9629	0.0221
θ_1	0.8032	0.0883

$SSE = 3.916 \times 10^6$

$\hat{\sigma}_u^2 = 6.870 \times 10^4$.

(c) Residuals:

```
 ** Residual autocorrelations **
 Aproximate standard error : 1.302E-01
Lag -----------------------------------------------------Autocor.
 0          |****************************************** 1.00E+00
 1     *****|                                           -1.4E-01
 2     *****|                                           -1.3E-01
 3          |                                           1.73E-02
 4          |                                           -1.9E-02
 5          |                                           7.78E-04
 6          |****                                       1.01E-01
 7          |*****                                      1.24E-01
 8    ******|                                           -1.5E-01
 9          |*                                          3.84E-02
10          |*                                          4.40E-02
11          |                                           1.91E-02
12      ****|                                           -1.0E-01
13         *|                                           -3.3E-02
14          |**                                         5.00E-02
15     *****|                                           -1.4E-01
16          |********                                   1.89E-01
17        **|                                           -6.1E-02
18   *******|                                           -1.8E-01
```

Chi-square statistic for residual autocorrelations,

$Q = 11.452$, Degrees of freedom = 16

(d) Correlation matrix of the estimated parameters:

	φ_1	θ_1
φ_1	1.000	0.440
θ_1	0.440	1.000

(e) Forecast function:

$$z_{N+\ell} = 1.963\ z_{N+\ell-1} - 0.963\ z_{N+\ell-2} - 0.803\ u_{N+\ell-1}$$

for $\ell = 1,2,\ldots$

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