DISTRIBUTIONAL DYNAMICS OF SECTORAL CHANGE IN THE LONG RUN

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DISTRIBUTIONAL DYNAMICS OF SECTORAL CHANGE IN THE LONG RUN

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DECLARATION OF ORIGINALITY

I, Fatma Kübra Toka, certify that

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ABSTRACT

Distributional Dynamics of Sectoral Change in the Long Run

This thesis explores the link between structural change and income distribution by using a two sector endogenous growth model with heterogeneous agents. Asymmetric external effects which stem from the “catching up with Joneses” type preferences, create structural transformation between sectors. Analytical and numerical solutions of the model indicate the unbalanced growth among production sectors. Simulation results point out that the conventional preference structure gives the same distributional dynamics.

ÖZET

Uzun Dönemli Sektörel Değişimin Dağılımsal Dinamikleri

Bu tez, iki sektörlü ve heterojen ajanlı bir endojen büyüme modeli kullanarak sektörel değişim ile gelir dağılımı arasındaki ilişkiyi incelemektedir. Modeldeki tercih yapısından kaynaklanan asimetrik dışsal etkiler sektörel değişimlere neden olmaktadır. Modelin analitik ve numerik çözümleri, üretim sektörleri arasında dengesiz büyümenin var olduğunu kanıtlamaktadır. Simulasyon sonuçları konvensiyonel tercih yapısının aynı dağılımsal dinamiklere sebep olduğunu göstermiştir.

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TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

Structural change can be defined by the transfer of employment and output share from agriculture to manufacturing and service sectors. During economic development, it is an important underlying process that is linked to both growth and inequality. The development process of the US economy shows that the share of employment in agriculture is decreasing, while the employment share in service sector is increasing and manufacturing sector has an inverse U-shape during this process. Shifts in British economy and OECD countries are consistent with the US experience. Dennis and İşcan (2009), Alvarez-Cuadrado and Poschke(2011) and Herrendorf, Rogerson and Valentinyi (2013) are the studies including historical evidences for srtructural change. Table 1. is taken form Alvarez-Cuadrado and Poschke(2011) shows the decrease in employment share in agriculture for twelve developed countries.

Table 1. Average Annual Change in Employment Share

	Average annual change in the employment share in agriculture L^A (percentage points)						
Country	All years	1800–1839	1840–1879	1880–1919	1920–1959	1960–	Years covered
Belgium	−0.31		−0.33	−0.45	−0.32	−0.15	1846–2005
Canada	−0.37			−0.39	−0.57	−0.18	1881–2006
Finland	−0.55			−0.06	−0.83	−0.75	1880–2000
France	−0.32		−0.24	−0.12	−0.50	−0.41	1856–2005
Germany	−0.37		−0.23	−0.38	−0.51	−0.34	1849–1990
Japan	−0.61			−0.72	−0.51	−0.67	1872–2000
Netherlands	−0.20	−0.07	−0.08	−0.36	−0.33	−0.16	1800–2005
South Korea	−0.86				−0.09	−1.43	1918–2005
Spain	−0.45		0.02	−0.33	−0.54	−0.69	1860–2001
Sweden	−0.51		−0.32	−0.56	−0.75	−0.31	1860–2000
UK	−0.17	−0.28	−0.28	−0.20	−0.07	−0.06	1801–2005
USA	−0.35	−0.15	−0.48	−0.52	−0.48	−0.15	1800–1999
Average	−0.42	−0.16	−0.24	−0.37	−0.46	−0.44	

Source: Alvarez-Cuadrado and Poschke(2011)

On the other hand, Firebaough (1999), Moller, Piketty and Saez (2003), and Anderson and Nielson (2009) are the empirical studies show the non-monotonic change in income inequality for developed countries. Figure 1. is taken from Piketty and Saez (2003) indicates that income inequality decreases until 1980s then it starts to increase for US economy. The inequality dynamics in industrialized economies consistent with this U-shaped pattern of US.

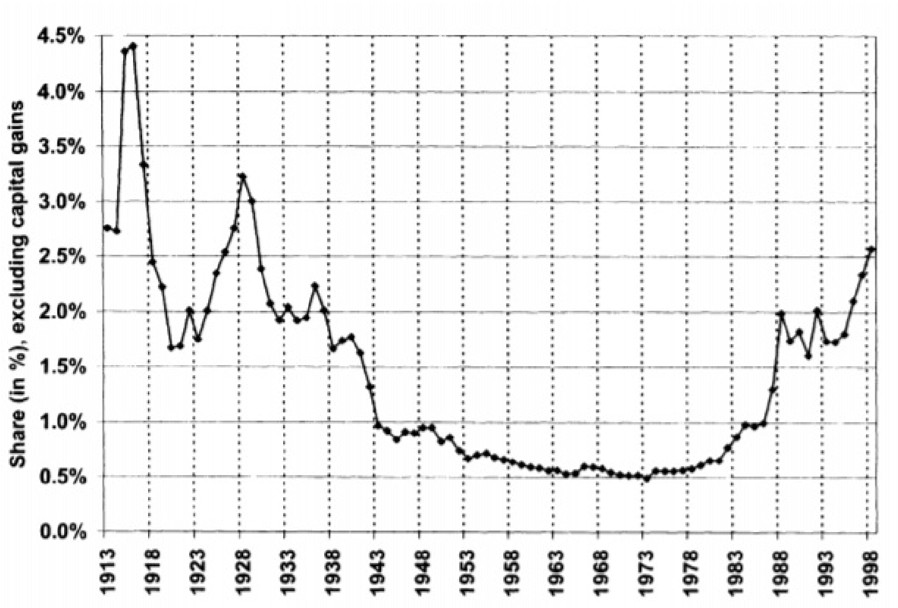

Figure 1. Income inequality in US
Source: Piketty and Saez (2003)

Based on these observations, we investigate the channels structural change affect income inequality.

We explore a two-sector model with interdependent preferences and heterogeneous agents. Heterogeneity stems from the initial wealth endowments of households. Utility of households depends on both their own consumption, and their reference consumption levels. “Catching up with the Joneses” formulation is used for the reference consumption level which is determined by the past consumption levels. This preference structure leads to a social comparison between households and the change in reference consumption levels changes the demand structure of households. Hori, Ikefuji and Mino (2015) examine this issue by using identical agents. We extend this idea using heterogeneous agents and labor-leisure choice. Also, CES labor supply function is used for households and the aggregation of labor supply provide imperfect substitution between sectors. Therefore, there is limited degree of mobility between sectors and wages do not equalize in the equilibrium. We point out that the asymmetric externalities between commodities lead to unbalanced growth between production sectors. The commodity with higher consumption externality grows faster than the other so this alters the output and consumption shares of sectors. The significant part is that the representative agent is present in the macro equilibrium which means all households behave as if the economy has one representative household with average capital endowment. After deriving the aggregate dynamics, we show that the distributional dynamics of capital and income depend on the leisure choice of households.

We set up a simulation based on the calibrated parameters and it indicates that the growth rate of the sector with higher conformism (Sector 1 in the model) is higher than the other sector (Sector 2). Sector 2 only produces consumption goods so it stands for agriculture sector and sector 1 produces both consumption and investment goods so it is for non-agriculture sectors. Thus, the share of production and consumption of sector 1 increase during the transition while the same shares of sector 2 decrease. Simulation results also indicate that aggregate labor therefore the leisure is constant during the transition. Thus, the structural transformation does not lead to any change in distributional dynamics of wealth and income.

This thesis proceeds as follows. In Chapter 2, there is a summary of related literature. In Chapter 3, analytical model takes place. In Chapter 4, there is numerical simulation and in Chapter 5 there is distributional dynamics and finally, in Chapter 6 conclusion takes place.

CHAPTER 2

LITERATURE REVIEW

Existing literature does not study the link between structural change and income inequality, although there exist important studies about each separately. Thus, we analyze these two subjects independently.

Structural transformation is studied through several channels in the literature. We can separate them into two categories: demand driven and supply driven structural change. Studies of Echevarria (1997), Laitner (2000), Kongsamut, Rebelo and Xie (2001) and Hori et. al (2015) can be placed in the first category. Studies based on the second approach include Baumol (1967), Ngai and Pissarides (2007), and Acemoglu and Guerrieri (2008). Our study examines structural change via demand side, so we mostly focus on the demand driven structural change models in this review.

Echevarria (1997) is one of the earliest paper that explains the link between economic growth and changes in the composition of primary, manufacturing and service sectors. In order to focus on changes in sectoral composition which stem from income elasticities, non-homothetic preferences are used. Crucial assumption is that the rate of technical change and labor intensities are taken highest for manufacturing in which capital is produced and primary goods are the most inelastic ones. Results show that if a country is poor, it only consumes primaries and cannot invest much. When the country becomes richer, it starts to invest more, therefore growth rate increase. Then production moves from primary to manufacturing. This causes increase in the growth rate. Thus, non-homothetic preferences lead to the conclusion that structural transformation from primary sector to service has an effect on growth rate.

Laitner (2000) uses two period OLG model to examine the impact of industrialization on the saving tendency of a country. There are two consumption goods in the model: agricultural and manufactured goods. Preferences of consumers are designed according to Engel's law, which indicates that until a threshold income level, agents only consume agricultural goods after that level they can consume manufactured goods. One equilibrium of model is that if agricultural consumption cannot reach the threshold level, consumers invest only in land rather than physical capital. Second equilibrium presents that when income increases by an exogenous technological improvement, consumers are saturated with agricultural goods, then their demand shifts to the second sector. Thus they invest in physical capital and the economy allocates more labor to manufacturing. As a result, rich countries have higher saving rate than poor ones.

Kongsamut et al. (2001) show that structural change dynamics satisfy generalized balanced growth path. They construct three models with different production technologies and price structures for three sectors; agriculture, manufacturing and service. Preferences capture income elasticities of three goods and they are time-separable (Stone-Gear utility function). They show that generalized balanced growth path is satisfied under some restrictions in all three models. The common result of these models is that the labor share falls in agriculture, increases in service and remains constant in manufacturing. They explain variation in labor allocation across sectors is caused by having different income elasticities of demand.

Altough Ngai and Pissarides (2007) study a technology driven structural change model, we benefit from their model. They study the effect of total factor productivity growth rates of sectors on structural transformation. They construct a model with multi sectors which have identical production functions but different TFP growth rates. Their results are consistent with Baumol (1967), imply that if there is a price inelastic demand then sectors with low TFP growth rate have more employment share. They also show that under some restrictions, these results are satisfied in the models with many capital goods sectors and many intermediate goods sectors.

This thesis strongly benefits from Hori et. al (2015) who show that structural change stems from the variation in externalities of different consumption goods. They use "catching up with Joneses" preferences, thus the utility of households depend not only on their individual consumption but also on the external habits. They show that if two sectors have different consumption externalities, good with higher importance for households has higher growth rate than the other and there exists an unbalanced growth across sectors. As a result, the commodity based consumption externalities alters the demand structure of households, therefore, the sectoral composition of the economy.

Chatterjee (1994), Caselli and Ventura (2000), Maliar and Maliar (2001) and Sorger (2002) are some of the papers that introduce income inequality into dynamic macro models with complete asset markets via heterogeneous agents. Caselli and Ventura (2000) introduce several sources of consumer heterogeneity (initial endowments, skills, and tastes) in representative agent models with one sector and they demonstrate the effects of aggregate shocks on the distributional dynamics of consumption, capital, and income.

Garcia-Penalosa and Turnovsky (2006) set out an endogenous growth model with leisure choice and heterogeneity. They show that higher growth rate simultaneously increases the income inequality in the economy. Garcia-Penalosa and Turnovsky (2008) develop a growth model with "keeping up with the Joneses" preferences and heterogeneous agents. Their results present that aggregate equilibrium coincides with representative agent economy and consumption externalities lead to a less unequal economy.

Koyuncu and Turnovsky (2010) is another paper that we heavily benefit from for the inequality dynamics of the model. This paper uses “catching up with Joneses” types of preferences in order to analyze the effect of tax policies on income distribution and growth. They show that consumption externality essentially changes the impact of tax policy and “catching up with Joneses” preferences lead more unequal distribution of income than conventional time-separable preferences.

Boppart (2014) sets up a model with non-Gorman preference to analyze the income and substitution effects on structural change. He shows that these two driving forces of structural change have equal influence. Recently, Liu (2017) investigates the impact of income inequality on structural transformation by using a multi sector model with heterogeneous agents. In the equilibrium, all goods in the market cannot be consumed due to income elastic demand that stems from non-homothetic preferences. It is shown that income inequality negatively affects the aggregate demand for luxury goods which are produced in service sector by assumption of the paper. Hence, unequal distribution of income has an adverse impact on expenditure share of service sector.

CHAPTER 3

MODEL

3.1 General Setup

3.1.1 Firms

The representative firm in sector $j(=1,2)$ produces goods according to the following production technology

$$Y_j = A_j K_j^{\alpha} (B_j L_j)^{1-\alpha}$$

where $0 < \alpha < 1$ is for the share of capital in production and $A_j > 0$ is total factor productivity. Y_j is output, K_j is capital, L_j is labor inputs in sector j and B_j is for the sector-specific externalities. In the equilibrium $K_j = \bar{K}_j$ and $L_j = \bar{L}_j$. $B_j \equiv \bar{K}_j / \bar{L}_j$, so the production technology turns to AK form $Y_j = A_j K_j$ for every sector $j(=1,2)$.

It is assumed that the capital does not depreciate. Imposing equilibrium conditions firms' maximization problems in sectors 1 and 2 give the following:

$$r = \alpha A_1 = p \alpha A_2, \qquad p = \frac{A_1}{A_2} \tag{1}$$

$$\omega_1 = (1-\alpha) A_1 \frac{K_1}{L_1} \tag{2}$$

$$\omega_2 = p(1-\alpha) A_2 \frac{K_2}{L_2} \tag{3}$$

where r and ω_j are factor prices in the equilibrium. Capital market clearing condition: $K_1 + K_2 = K$.

3.1.2 Consumers

There exists a continuum of heterogeneous agents indexed by i. Heterogeneity stems from their initial capital endowments. Utility of households depends on their own consumption, their reference consumption levels for different goods, and labor-leisure choice. The utility of household i at time t is given by the function u_t

$$u_i(t) = \left[\gamma(c_{i,1}(t)h_1(t)^{-\theta_1})^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)(c_{i,2}(t)h_2(t)^{-\theta_2})^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \left[1 - L_i(t)\right]^{\eta} \tag{4}$$

where $c_{i,j}$ is the consumption for good j and $\varepsilon > 0$ is the elasticity of substitution between good 1 and 2. $\gamma \in (0,1)$ is a constant. L_i is the labor supply of household and $L_i + l_i = 1$ where l_i is leisure time. $\eta > 0$ gives the effect of leisure in utility of household i. h_j is the consumption externality for every sector $j(= 1,2)$. It is obtained by taking average of the economy wide average consumption of commodity j, $\bar{c}_{j,u}$, up to time t:

$$h_j(t) = \phi_j \int_{-\infty}^{t} e^{-\phi_j(t-u)} \bar{c}_j(u) du, \qquad \phi_j > 0, \qquad j = 1,2 \tag{5}$$

Taking time derivative of the above equation yields

$$\dot{h}_j(t) = \phi_j(\bar{c}_j(t) - h_j(t)), \qquad j = 1,2. \tag{6}$$

where $\phi_j > 0$ is the rate of adjustment of habit formation.

The lifetime utility maximization problem of agent i is

$$\max \qquad U = \int_0^\infty e^{-\rho t} ln u_i dt \tag{7}$$

subject to

$$\dot{K}_i = rK_i + \omega_1 L_{i,1} + \omega_2 L_{i,2} - E_i \tag{8}$$

$$E_i = c_{i,1} + pc_{i,2} \tag{9}$$

where $\rho > 0$ is subjective discount rate and E_i is the total expenditure for household i. Following Petrella, Rossi and Santoro (2013), we model labor supply of household i as a CES function:

$$L_i = \left[\psi^{-\frac{1}{\sigma}} (L_{i,1})^{\frac{1+\sigma}{\sigma}} + (1-\psi)^{-\frac{1}{\sigma}} (L_{i,2})^{\frac{1+\sigma}{\sigma}}\right]^{\frac{\sigma}{1+\sigma}} \tag{10}$$

where σ is the elasticity of substitution between sectors, for $\sigma = 0$, labor cannot move between sectors, for $\sigma \longrightarrow \infty$, labor mobile between sectors and for $\sigma < \infty$ there is limited degree of labor mobility. ψ is the steady state ratio of labor supply in the sector 1 over total labor supply. $L_{i,j}$ is labor supplied to sector j by household i.

First order conditions with respect to

$$c_{i,1}: \frac{\gamma c_{i,1}^{-1/\varepsilon} h_1^{-\theta \frac{\varepsilon-1}{\varepsilon}}}{\gamma (c_{i,1} h_1^{-\theta_1})^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)(c_{i,2} h_2^{-\theta_2})^{\frac{\varepsilon-1}{\varepsilon}}} = \lambda_i \tag{11}$$

$$c_{i,2}: \frac{(1-\gamma) c_{i,2}^{-1\varepsilon} h_2^{-\theta \frac{\varepsilon-1}{\varepsilon}}}{\gamma (c_{i,1} h_1^{-\theta_1})^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)(c_{i,2} h_2^{-\theta_2})^{\frac{\varepsilon-1}{\varepsilon}}} = p\lambda_i \tag{12}$$

$$L_{i,1}: \frac{\eta \psi^{-\frac{1}{\sigma}} (L_{i,1})^{\frac{1}{\sigma}} L_i^{-\frac{1}{\sigma}}}{1-L_i} = \lambda_i \omega_1 \tag{13}$$

$$L_{i,2}: \frac{\eta (1-\psi)^{-\frac{1}{\sigma}} (L_{i,2})^{\frac{1}{\sigma}} L_i^{-\frac{1}{\sigma}}}{1-L_i} = \lambda_i \omega_2 \tag{14}$$

From (11) and (12),

$$\frac{pc_{i,2}}{c_{i,1}} = \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon} \left(\frac{ph_2^{\theta_2}}{h_1^{\theta_1}}\right)^{1-\varepsilon} \tag{15}$$

When elasticity of substitution between two goods is 1, i.e. $\varepsilon = 1$, the impact of reference level of consumption disappears. Thus, we assume that $\varepsilon \neq 1$. By using (9) and (15) we can get the following,

$$c_{i,1} = E_i \left[1 + p^{1-\varepsilon} \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon} \left(\frac{h_2^{\theta_2}}{h_1^{\theta_1}}\right)^{1-\varepsilon}\right]^{-1} = E_i X \tag{16}$$

$$c_{i,2} = (1-X)E_i \tag{17}$$

where $X = \left[1 + p^{1-\varepsilon} \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon} \left(\frac{h_2^{\theta_2}}{h_1^{\theta_1}}\right)^{1-\varepsilon}\right]^{-1}$. Now, we can rewrite the household i's utility function with total expenditure term;

$$u_t = E_i\left[\gamma(Xh_1^{-\theta_1})^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)(1-X)h_2^{-\theta_2})^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}\left[1-L_i\right]^{\eta} \tag{18}$$

By maximizing (7) subject to (8), and (18), we get the following first order condition with respect to E_i:

$$\frac{1}{E_i} = \lambda_i \tag{19}$$

Euler conditions gives $\frac{\dot{\lambda}_i}{\lambda_i} = -r + \rho$ so

$$\frac{\dot{E}_i}{E_i} = r - \rho \tag{20}$$

Since the right hand side of this equation is independent from i,

$$\frac{\dot{E}_i}{E_i} = \frac{\dot{E}}{E} = r - \rho \qquad \forall i \tag{21}$$

which means growth rate of total expenditure is same for all households.

$$\frac{\dot{c}_{i,1}}{c_{i,1}} = r - \rho + (1-\varepsilon)(1-X)\left(\theta_1\frac{\dot{h_1}}{h_1} - \theta_2\frac{\dot{h_2}}{h_2}\right) \tag{22}$$

$$\frac{\dot{c}_{i,2}}{c_{i,2}} = r - \rho + (1-\varepsilon)X\left(\theta_2\frac{\dot{h_2}}{h_2} - \theta_1\frac{\dot{h_1}}{h_1}\right) \tag{23}$$

Derivation of (22) and (23) is in the Appendix A.

For both (22) and (23), right hand sides are independent from i since X and h_j don not depend on i. So

$$\frac{\dot{c}_{i,1}}{c_{i,1}} = \frac{\dot{c}_1}{c_1} \qquad \text{and} \qquad \frac{\dot{c}_{i,2}}{c_{i,2}} = \frac{\dot{c}_2}{c_2} \qquad \forall i. \tag{24}$$

Hence, all households choose the same growth rate for each commodity, independently from their initial capital endowments.

From (13) and (14), we can derive labor supply decision of households:

$$\frac{\omega_1}{\omega_2} = \left(\frac{L_{i,1}}{L_{i,2}}\right)^{1/\sigma} \left(\frac{\psi}{1-\psi}\right)^{-1/\sigma} \tag{25}$$

where σ is the elasticity of substitution between sectors for labor supply. For $\sigma = 0$, there is labor immobility, for $\sigma \longrightarrow \infty$, there is free labor mobility between sectors so wages are equal; and for $\sigma < \infty$ there is limited degree of labor mobility and wages are not equal across sectors. Thus, the labor supply of household i depends on the elasticity of substitution between sectors.

From (13) and (19), we get

$$E_i = \frac{\omega_1 (1 - L_i) L_i^{1/\sigma}}{\eta \psi^{-1/\sigma} (L_{i,1})^{1/\sigma}} \tag{26}$$

When the system of equation composed of (10), (25) and (26) is solved, labor supply decision of household i can be found.

3.1.3 Goods Market

Sector 1 produces both investment and consumption goods whereas sector 2 produces only consumption goods. Therefore, $\dot{K}(t) = A_1 K_1(t) - c_1(t)$ and $c_2(t) = A_2 K_2(t)$.

From these two equations, we get the aggregate capital accumulation equation:

$$\dot{K}(t) = A_1 K(t) - E(t) \tag{27}$$

3.2 The Aggregate Economy

Let $\xi_i = \frac{E_i}{K_i}$ and $\xi = \frac{E}{K}$. In order to examine the evolution of aggregate variable $K(t)$ and $E(t)$, it will be shown that the ratio $Z = \frac{\xi_i}{\xi}$ is constant in the steady state:

$$\dot{Z} = Z\left(\frac{\dot{\xi}_i}{\xi_i} - \frac{\dot{\xi}}{\xi}\right) = \left(\frac{\dot{E}_i}{E_i} - \frac{\dot{K}_i}{K_i} - \frac{\dot{E}}{E} + \frac{\dot{K}}{K}\right)$$

$$\dot{Z} = Z\left(\frac{\dot{K}}{K} - \frac{\dot{K}_i}{K_i}\right)$$

$$\frac{\dot{Z}}{Z} = -\frac{\dot{k}_i}{k_i}$$

For now, we assume $\dot{k}_i = 0$ at the steady-state. In section 2.4, we will show that it is true. Then

$$\frac{\dot{Z}}{Z} = 0$$

From (21) and (27), we get $\xi(t) = \xi(t)\{\xi(t) - [(1-\alpha)A_1 + \rho]\}$. $\xi(t)$ cannot converge to zero or ∞ since in the former case $\dot{K}(t)/K(t)$ converges to A_1 and to zero in the later, respectively. In both cases, transversality condition is violated. Thus, the economy always stays at $\xi^* \equiv (1-\alpha)A_1 + \rho$ and

$$E(t) = [(1-\alpha)A_1 + \rho]K(t) \qquad \forall t \geq 0 \tag{28}$$

Proposition 1: In equilibrium, $\xi(t)$ stays constant at $\xi^* \equiv (1-\alpha)A_1 + \rho$. $K(t)$ and $E(t)$ grow at $g^* = \alpha A_1 - \rho$. The capital share and the interest rate stay constant.

From (25), we can derive the aggregate labor supply as follows

$$\frac{L_{i,1}}{L_{i,2}} = \left(\frac{\omega_1}{\omega_2}\right)^{\sigma} \left(\frac{\psi}{1-\psi}\right) \tag{29}$$

The above equation can be aggregated as

$$\frac{L_1}{L_2} = \left(\frac{\omega_1}{\omega_2}\right)^{\sigma} \left(\frac{\psi}{1-\psi}\right) \tag{30}$$

Because $\frac{L_{i1}}{L_{i2}}$ is constant, $\frac{L_i}{L_{i1}}$ is also constant and (26) can be aggregated as

$$E = \frac{\omega_1 (1-L) L^{1/\sigma}}{\eta \psi^{-1/\sigma} (L_1)^{1/\sigma}} \tag{31}$$

By using (29), we can write (10) in the following form

$$L_i = L_{i2} \left[\psi^{-\frac{1}{\sigma}} \left(\left(\frac{\omega_1}{\omega_2}\right)^{\sigma} \left(\frac{\psi}{1-\psi}\right) \right)^{\frac{1+\sigma}{\sigma}} + (1-\psi)^{-\frac{1}{\sigma}} \right]^{\frac{\sigma}{1+\sigma}} \tag{32}$$

Above equation is linear in *i* so (10) can be aggregated

$$L = \left[\psi^{-\frac{1}{\sigma}} (L_1)^{\frac{1+\sigma}{\sigma}} + (1-\psi)^{-\frac{1}{\sigma}} (L_2)^{\frac{1+\sigma}{\sigma}} \right]^{\frac{\sigma}{1+\sigma}} \tag{33}$$

Finally, the solution of the system of equations including (30), (31) and (33) gives the aggregate labor supply, and labor supply for each sector.

3.3 The Stationary Equilibrium

In order to analyze the structural change between two sectors, let $x_{1,t} \equiv c_{1,t}/K_t$, $x_{2,t} \equiv pc_{2,t}/K_t$ and $\delta_{j,t} \equiv h_{j,t}/K_t$ for $(j = 1,2)$.

3.3.1 Dynamic System of the Two Sectors

From (9) and (28):

$$x_{2,t} = (1-\alpha)A_1 + \rho - x_{1,t} \tag{34}$$

since in the equilibrium $c_{j,t} = \bar{c}_{j,t}$, following three differential equations can be derived by using (6), (22) and (28)

$$\dot{x}_{1,t} = (1-\varepsilon)\left(1 - \frac{x_{1,t}}{(1-\alpha)A_1+\rho}\right)\left[\theta_1\phi_1\left(\frac{x_{1,t}}{\delta_{1,t}} - 1\right) - \theta_2\phi_2\left(\frac{x_{2,t}}{\delta_{2,t}} - 1\right)\right]x_{1,t} \tag{35}$$

$$\dot{\delta}_{1,t} = \phi_1 x_{1,t} - (\phi_1 + g^*)\delta_{1,t} \tag{36}$$

$$\dot{\delta}_{2,t} = \frac{\phi_2}{p}x_{2,t} - (\phi_2 + g^*)\delta_{2,t} \tag{37}$$

where $x_{2,t}$ comes from (34) and $0 \leq x_{1,t} \leq (1-\alpha)A_1 + \rho$. From (16) and (28), following equation can be derived as

$$x_{1,t} = ((1-\alpha)A_1+\rho)\left[1 + p^{1-\varepsilon}\left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(\frac{h_2^{\theta_2}}{h_1^{\theta_1}}\right)^{1-\varepsilon}\right]^{-1} \tag{38}$$

which is determined by the state variables $h_{1,0}$ and $h_{2,0}$ so $x_{1,0}$ is taken as fixed variable. Given $x_{1,0}$, $\delta_{1,0}$ and $\delta_{2,0}$, while $\theta_1 \neq \theta_2$, the two sector economy is characterized by (35)-(37).

Hori et al. (2015) analyze both the cases of symmetric and asymmetric externalities and they show that the first one leads to balanced growth and does not create structural change, so we focus on the case of asymmetric externalities. Following definition and propositions are highly compatible with Hori et al. (2015).

Definition: A stationary equilibrium is an equilibrium in which $\dot{x}_{1,t} = \dot{\delta}_{1,t} = \dot{\delta}_{2,t} = 0$ holds and $c_{1,t}$, $c_{2,t}$, $h_{1,t}$ and $h_{2,t}$ grow at constant rates.

Proposition 2: Suppose $\varepsilon \neq 1$ and $\theta_1 \neq \theta_2$. If there exists an SE, $\dot{c}_{1,t}/c_{1,t} \neq \dot{c}_{2,t}/c_{2,t}$ must hold in SE.

Proposition 3:

(i) If $(1-\varepsilon)(\theta_1-\theta_2) > 0$, there exists a stationary equilibrium in which followings hold:

$$\frac{\dot{c}_{1,t}}{c_{1,t}} = \frac{\dot{h}_{1,t}}{h_{1,t}} = \frac{\dot{K}_{1,t}}{K_{1,t}} = \alpha A_1 - \rho \equiv g^*, \tag{39a}$$

$$\frac{\dot{c}_{2,t}}{c_{2,t}} = \frac{\dot{h}_{2,t}}{h_{2,t}} = \frac{\dot{K}_{2,t}}{K_{2,t}} = \frac{1+(\varepsilon-1)\theta_1}{1+(\varepsilon-1)\theta_2} g^* \equiv \hat{g}^* \qquad (0 < \hat{g}^* < g^*), \tag{39b}$$

$$\hat{x_1}^* = (1-\alpha)A_1 + \rho, \qquad \hat{\delta}_1^* = \frac{\phi_1[(1-\alpha)A_1+\rho]}{\phi_1 + g^*}, \qquad \hat{\delta}_2^* = 0, \tag{39c}$$

$$\frac{c_{2,t}}{h_{2,t}} = \frac{A_2 k_{2,t}}{\delta_{2,t}} = \frac{(\phi_2 + \hat{g}^*)}{\phi_2} \tag{39d}$$

where, $\hat{x}_1^* \equiv lim_{t\longrightarrow\infty} x_{1,t}$, $\hat{\delta}_1^* \equiv lim_{t\longrightarrow\infty} \delta_{1,t}$ and $\hat{\delta}_2^* \equiv lim_{t\longrightarrow\infty} \delta_{2,t}$

(ii) If $(1-\varepsilon)(\theta_1-\theta_2)<0$, there exists a stationary equilibrium in which followings hold:

$$\frac{\dot{c}_{2,t}}{c_{2,t}}=\frac{\dot{h}_{2,t}}{h_{2,t}}=\frac{\dot{K}_{2,t}}{K_{2,t}}=\frac{\dot{K}_{1,t}}{K_{1,t}}=\alpha A_1-\rho\equiv g^*, \tag{40a}$$

$$\frac{\dot{c}_{1,t}}{c_{1,t}}=\frac{\dot{h}_{1,t}}{h_{1,t}}=\frac{1+(\varepsilon-1)\theta_2}{1+(\varepsilon-1)\theta_1}g^*\equiv\hat{\hat{g}}^* \qquad (0<\hat{\hat{g}}^*<g^*), \tag{40b}$$

$$\hat{\hat{x}}_1^*=0, \qquad \hat{\hat{\delta}}_2^*=\frac{\phi_2[(1-\alpha)A_1+\rho]}{p(\phi_2+g^*)}, \qquad \hat{\hat{\delta}}_1^*=0, \tag{40c}$$

$$\frac{c_{1,t}}{h_{1,t}}=\frac{x_{1,t}}{\delta_{1,t}}=\frac{(\phi_1+\hat{\hat{g}}^*)}{\phi_1} \tag{40d}$$

where, $\hat{\hat{x}}_1^*\equiv lim_{t\longrightarrow\infty}x_{1,t}$, $\hat{\hat{\delta}}_1^*\equiv lim_{t\longrightarrow\infty}\delta_{1,t}$ and $\hat{\hat{\delta}}_2^*\equiv lim_{t\longrightarrow\infty}\delta_{2,t}$.

Proofs of Proposition 2 and 3 are in the Appendix B.

If $(1-\varepsilon)(\theta_1-\theta_2)>0$, growth rates of sector 1 for both consumption and output are higher than sector 2 in the stationary equilibrium. When we assume $\theta_1>\theta_2$, increase in consumption of good 1 leads to an increase in the relative importance of good 1. Thus, agents have higher level of conformism towards good 1. Therefore consumption of good 1 has higher growth rate than good 2. At the asymptotic convergence, the capital share of sector 2 converges to zero.

In the case of $(1-\varepsilon)(\theta_1-\theta_2)<0$, consumption of sector 2 has higher growth rate than sector 1 but capital shares of two sectors grow at the same rate, because sector 1 produces investment goods for sector 2.

Structural change stems from the difference in the degree of consumption externalities, θ_j, rather than ϕ_j for $j(=1,2)$.

3.3.2 Stability of the Dynamic System

To evaluate the stability of dynamic system characterized by (35)-(37), we linearized it around stationary equilibrium

$$\begin{bmatrix} \dot{x}_1 \\ \dot{\delta}_1 \\ \dot{\delta}_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{x}_1}{x_1} & \frac{\partial \dot{x}_1}{\delta_1} & \frac{\partial \dot{x}_1}{\delta_2} \\ \frac{\partial \dot{\delta}_1}{x_1} & \frac{\partial \dot{\delta}_1}{\delta_1} & \frac{\partial \dot{\delta}_1}{\delta_2} \\ \frac{\partial \dot{\delta}_2}{x_1} & \frac{\partial \dot{\delta}_2}{\delta_1} & \frac{\partial \dot{\delta}_2}{\delta_2} \end{bmatrix} \begin{bmatrix} \dot{x}_1(t) - x_1^* \\ \dot{\delta}_1(t) - \delta_1^* \\ \dot{\delta}_2(t) - \delta_2^* \end{bmatrix}$$

For $(1-\varepsilon)(\theta_1 - \theta_2) > 0$, Jacobian matrix of this linearized system is the following

$$\begin{bmatrix} -(1-\varepsilon)(\theta_1 g^* - \theta_2 \hat{g}^*) + (1-\varepsilon)\theta_2(\phi_2 + \hat{g}^*) & 0 & \frac{(1-\varepsilon)p\theta_2}{\phi_2}(\phi_2 + \hat{g}^*)^2 \\ \phi_1 & -(\phi_1 + g^*) & 0 \\ -\phi_2/2 & 0 & -(\phi_2 + g^*) \end{bmatrix}$$

There are three eigenvalues of this matrix. One of them is obviously $\mu_1 = -(\phi + g^*) < 0$. Other two eigenvalues can be found by solving the following equations:

$$\mu_2 + \mu_3 = -(1-\varepsilon)(\theta_1 g^* - \theta_2 \hat{g}^*) - (1 + (\varepsilon - 1)\theta_2)\phi_2 - g^* - (\varepsilon - 1)\theta_2 \hat{g}^*$$

$$\mu_2 \mu_3 = (1-\varepsilon)(\theta_1 g^* - \theta_2 \hat{g}^*)(\phi_2 + g^*) - (1-\varepsilon)\theta_2(\phi_2 + \hat{g}^*)(g^* - \hat{g}^*)$$

We get $\mu_2 = -(1-\varepsilon)(\theta_1 g^* - \theta_2 \hat{g}^*)$ and $\mu_3 = -(1 + (\varepsilon - 1)\theta_2)\phi_2 - g^* - (\varepsilon - 1)\theta_2 \hat{g}^*$.

μ_2 and μ_3 are both negative due to the following assumptions: $(1-\varepsilon)(\theta_1-\theta_2)>0$, $\varepsilon>0$, $0<\theta_2<1$ and $\hat{g}^*<g^*$. Jacobian of the system has three negative eigenvalues so the system is locally stable around stationary equilibrium.

For $(1-\varepsilon)(\theta_1-\theta_2)<0$, Jacobian matrix of this linearized system is the following

$$\begin{bmatrix} (1-\varepsilon)(\theta_1\hat{\hat{g}}^*-\theta_2 g^*)+(1-\varepsilon)\theta_2(\phi_2+\hat{\hat{g}}^*) & \frac{(1-\varepsilon)\theta_1}{\phi_1}(\phi_1+\hat{g}^*)^2 & 0 \\ \phi_1 & -(\phi_1+g^*) & 0 \\ -\phi_2/p & 0 & -(\phi_2+g^*) \end{bmatrix}$$

There are three eigenvalues of this matrix. One of them is obviously $\zeta_1=-(\phi_2+g^*)<0$. Other two eigenvalues can be found by solving the following equations:

$$\zeta_2+\zeta_3=(1-\varepsilon)(\theta_1\hat{\hat{g}}^*-\theta_2 g^*)-(1+(\varepsilon-1)\theta_1)\phi_1-g^*-(\varepsilon-1)\theta_1\hat{\hat{g}}^*$$

$$\zeta_2\zeta_3=-(1-\varepsilon)(\theta_1\hat{\hat{g}}^*-\theta_2 g^*)(\phi_1+g^*)+(1-\varepsilon)\theta_1(\phi_1+\hat{\hat{g}}^*)(\hat{\hat{g}}^*-g^*)$$

We get $\zeta_2=(1-\varepsilon)(\theta_1\hat{\hat{g}}^*-\theta_2 g^*)$ and $\zeta_3=-(1+(\varepsilon-1)\theta_1)\phi_1-g^*-(\varepsilon-1)\theta_1\hat{\hat{g}}^*$. ζ_2 and ζ_3 are both negative due to the following assumptions: $(1-\varepsilon)(\theta_1-\theta_2)<0$, and $\hat{g}^*<g^*$. Jacobian of the system has three negative eigenvalues so the system is locally stable around stationary equilibrium.

CHAPTER 4

NUMERICAL SIMULATIONS

To examine the dynamics of structural change from sector 2 to sector 1, we set up a numerical simulation based on the parameters in Table 2.

Table 2. Parameter Values

α	0.3
$A_1 = A_2$	0.2
ρ	0.04
ε	0.04
θ_1	0.75
θ_2	0
σ	1
ψ	0.98
η	2
$\phi_1 = \phi_2$	0.2

We use the standard values for α, A_1, A_2, and ρ. For ε,θ_1, θ_2, ϕ_1 and ϕ_2, we use the values in Hori et al. (2015). We simulate the case of $(1-\varepsilon)(\theta_1-\theta_2)>0$ since it creates different growth rates for output and capital shares of two sectors. Structural change stems from the difference in the degree of consumption externalities, θ_j, rather than ϕ_j for $j(=1,2)$. So, we take $\phi_1=\phi_2$, $\varepsilon<1$ and $\theta_1>\theta_2$ and θ_2 is normalized to 1 for simplicity.

Elasticity of substitution between sectors in labor supply, σ, is taken as 1 based on Petrella, Rossi and Santoro (2013) in order to get limited degree of labor mobility between sector 1 and sector 2. We take ψ which is the ratio of employment share of non-agricultural sectors to total employment for US, as 0.98 to be consistent with the literature. η gives the importance of leisure in utility is taken 2 as in Koyuncu and Turnovsky (2011).

Transitional dynamics of the economy is obtained by solving the differential equation system defined in (35)-(37). Figures (2-4) show the transitional dynamics of the economy considering three distinct initial points for given K_0.

Figure 2 indicates that if the economy starts initially from $K_0 = 1.5/0.18$, it monotonically converges to the stationary equilibrium. The consumption share of commodity 1 one is

$$c_{1,t}/E_t = x_{1,t}/[(1-\alpha)A_1 + \rho]$$

and output share is

$$Y_{1,t}/Y_t = [\alpha A_1 - \rho + x_{1,t}]/A_1$$

Thus, the monotonic increase in $x_{1,t}$ leads to increase in the consumption and output shares of commodity 1, while the commodity 2 decreases monotonically. Hence, there exists a structural transformation from sector 2 to sector 1.

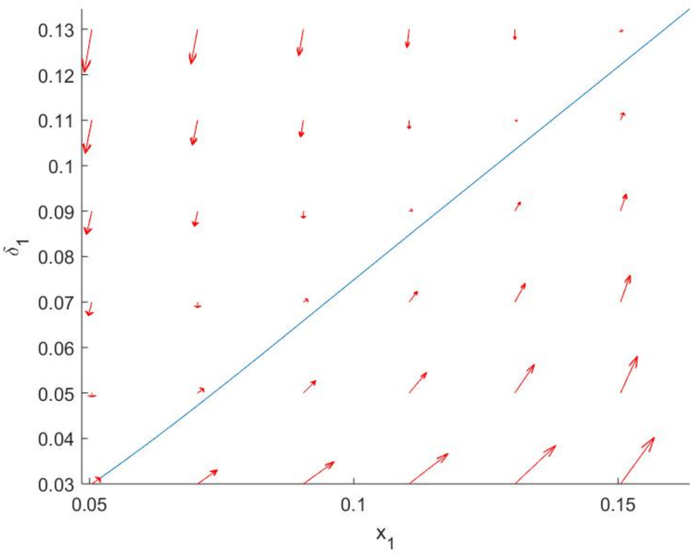

Figure 2. Phase diagram: $K_0 = 1.5/0.18$

Figure 3 and Figure 4 show that economy converges to a stationary equilibrium non-monotonically. Monotonic increase in $x_{1,t}$ starts at later periods of transition. Therefore, the consumption and output shares of commodity 1 increases while the same shares of commodity 2 decreases monotonically at the later periods.

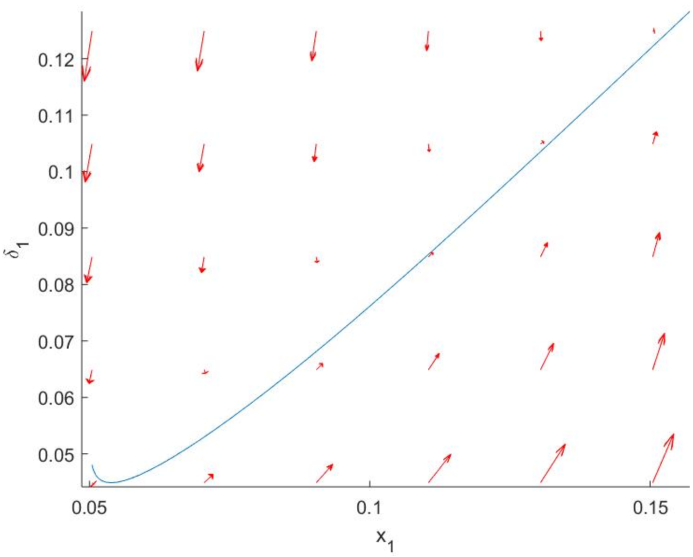

Figure 3. Phase diagram: $K_0 = 5.2$

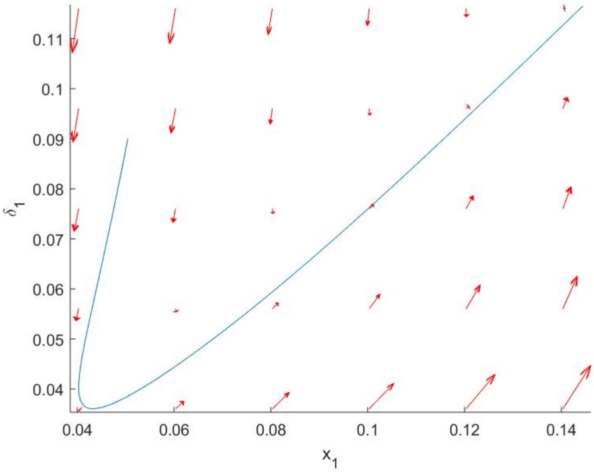

Figure 4. Phase diagram: $K_0 = 0.5/0.18$

First initial value of K gives the monotonic convergence of the economy so we continue our simulation with $K_0 = 1.5/0.18$. Figure 5 demonstrates the transition of economy through the stationary equilibrium in three dimensions. The red arrows show the direction field through the trajectory.

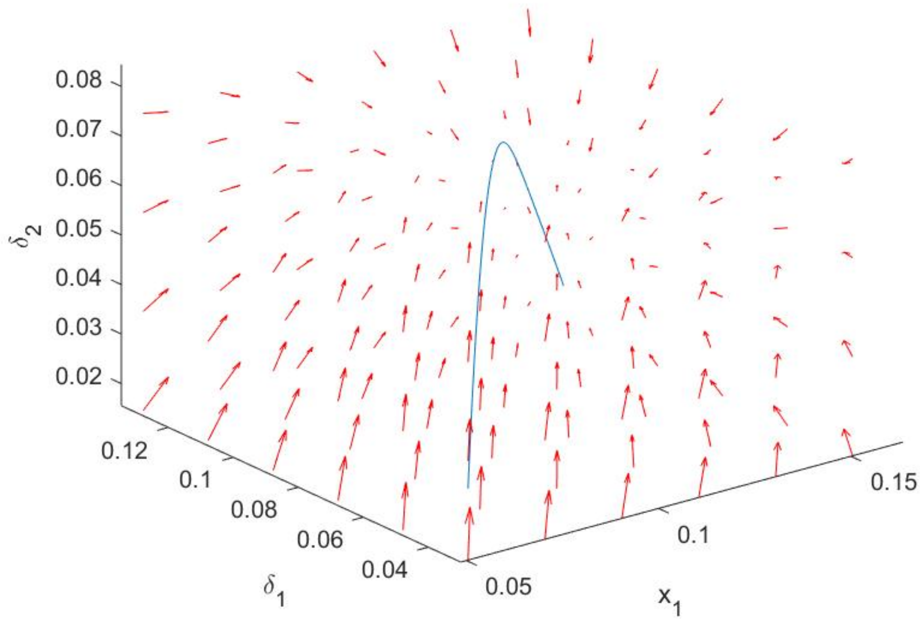

Figure 5. Phase diagram: $K_0 = 1.5/0.18$ in (x_1-$\delta_1 - \delta_2$) space

In Figure 6, we plot the consumption shares for two sectors. Consumption share of sector 1 which is the sector with high consumption externality, increases whereas the consumption share of sector 2 decreases. Our simulation results are consistent with Hori et al. (2015). In our model, sector 1 produces both consumption and investment goods while sector 2 produces only consumption goods. Hence sector 2 stands for agriculture goods and sector 1 is for non-agriculture goods. Thus, the simulation results reflect the empirical fact that expenditure and employment share in agricultural sector has a decline whereas service sector has an increase.

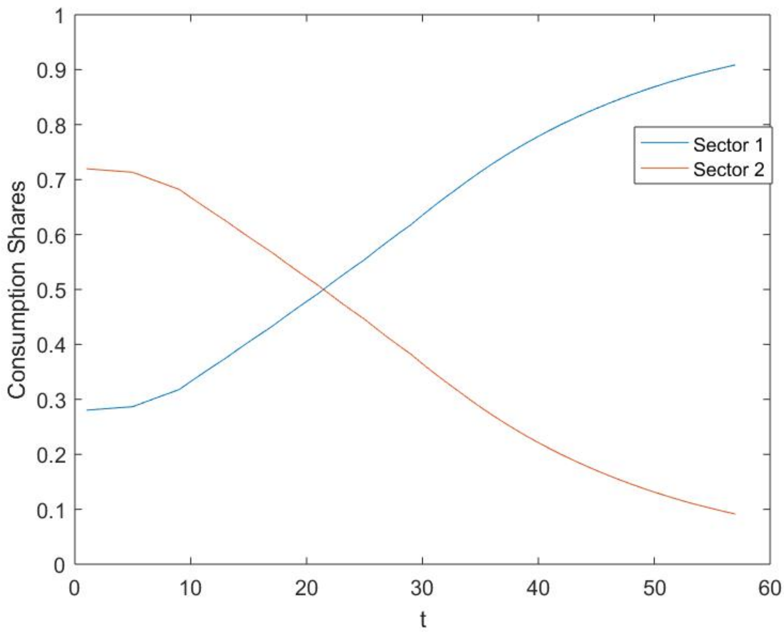

Figure 6. Change in consumption shares for sectors

The CES formulation of aggregate labor supply creates limited mobility between sectors so wages do not equalize in the stationary equilibrium. The ratio of wage 1 over wage 2 increases during the structural transformation since sector 1 is the faster growing sector.

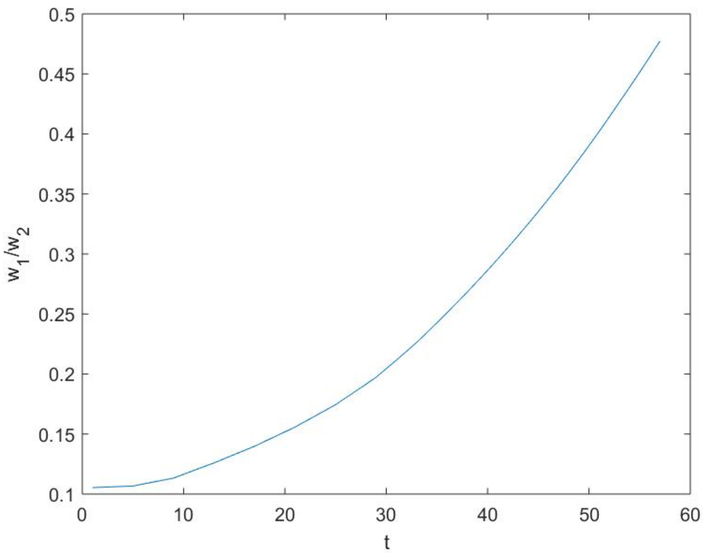

Figure 7. Wage ratio

We also investigate the dynamics of labor supply which can be found from the system of equations including (30), (31) and (33). Simulation results show that employment share in sector 1 increases while it decreases in sector 2. However aggregate labor supply is constant during the transition; therefore leisure also remains unchanged.

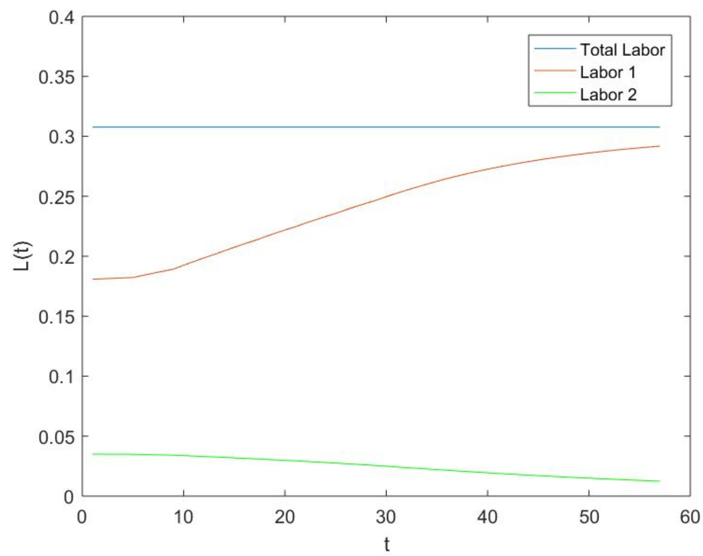

Figure 8. Aggregate labor supply

CHAPTER 5

THE DISTRIBUTION OF WEALTH AND INCOME

5.1 Dynamics of Relative Capital Stock

To derive the dynamics of the relative capital stock of an individual, $k_i = K_i/K$, we first derive the dynamic equation of the aggregate capital stock:

$$\frac{\dot{K}}{K} = r + \frac{\omega_1 L_1 + \omega_2 L_2}{K} - \frac{E}{K} \tag{41}$$

The dynamics of individual i's relative capital stock can be written as

$$\dot{k}_i = rK_i/K + \frac{\omega_1 L_{i,1} + \omega_2 L_{i,2}}{K} - \frac{E_i}{K} - k_i\left(r + \frac{\omega_1 L_1 + \omega_2 L_2}{K} - \frac{E}{K}\right) \tag{42}$$

From (29) and (30), we get

$$\omega_2 = \omega_1 \left(\frac{L_{i,1}}{L_{i,2}}\right)^{-1/\sigma} \left(\frac{\psi}{1-\psi}\right)^{1/\sigma} = \omega_1 \left(\frac{L_1}{L_2}\right)^{-1/\sigma} \left(\frac{\psi}{1-\psi}\right)^{1/\sigma} \tag{43}$$

By plugging (26), (31) and (43) into (42), we obtain

$$\dot{k}_i = \frac{\omega_1}{K}\left(\frac{\eta L_i^{\frac{1+\sigma}{\sigma}} - (1-L_i)L_i^{1/\sigma}}{L_{i,1}^{1/\sigma}\psi^{-1/\sigma}} - k_i\left(\frac{\eta L^{\frac{1+\sigma}{\sigma}} - (1-L)L^{1/\sigma}}{L_1^{1/\sigma}\psi^{-1/\sigma}}\right)\right)$$

$$= \frac{\omega_1}{K}\left[\left(\frac{L_i}{L_{i,1}}\right)^{1/\sigma}\left(1 - l_i\left(\frac{1+\eta}{\eta}\right)\right) - k_i\left(\frac{L_i}{L_{i,1}}\right)^{1/\sigma}\left(1 - l\left(\frac{1+\eta}{\eta}\right)\right)\right]$$

The equation above indicates the evolution of relative capital stock from initial capital endowment k_0. Transversality condition implies the following:

$$\lim_{t \longrightarrow \infty} \lambda K e^{-\rho t} = 0 \tag{44}$$

Constant l implies that growth rate of λ and K is constant so transversality condition is satisfied if and only if

$$l > \frac{\eta}{1+\eta} \tag{45}$$

Therefore, coefficient of $\dot{k}_i$ is positive. Only way of $\dot{k}_i = 0$ is

$$k_i = \left(\frac{L_i}{L_{i,1}} \frac{L_1}{L}\right)^{1/\sigma} \frac{1 - l_i\left(\frac{1+\eta}{\eta}\right)}{1 - l\left(\frac{1+\eta}{\eta}\right)} \tag{46}$$

From (29) and (30), $\frac{L_i}{L_{i,1}} = \frac{L_1}{L}$ so

$$l_i - l = \left(l - \frac{\eta}{1+\eta}\right)(k_i - 1) \tag{47}$$

Therefore, equation (45) yields a positive link between leisure and relative capital. In numerical simulation, we show that leisure is constant during the transition so relative wealth of agent i also remains constant.

5.2 Dynamics of Relative Income

Let $y_i = \frac{Y_i}{Y}$ be relative income of household i

$$y_i = \frac{rK_i + \omega_1 L_{i,1} + \omega_2 L_{i,2}}{rK + \omega_1 L_1 + \omega_2 L_2}$$

We can define the share of leisure of agent i in the economy-wide leisure as

$$l_i = g_i l$$

$$\text{where} \qquad \int_0^1 g_i d_i = 1$$

$$y_i(t) - 1 = \alpha k_i(t) + (1-\alpha)\frac{l(t)}{1-l(t)}(1-g_i) \tag{48}$$

Since both leisure and relative capital are constant during transition, relative income of household *i* does not change. Although there exists a structural transformation from sector 2 to sector 1, it doesn't affect the distributional dynamics of the model economy.

CHAPTER 5

CONCLUSION

We set up a two-sector model with "catching up with Joneses" preferences and heterogenous agents. Utility of households depends on both their own consumption, their reference consumption levels and leisure choice. Our analytical results are compatible with Hori et al. (2015), who demonstrate output and consumption shares of sector with higher consumption externality increase. We show that the relative income and capital endowment of household depend on their leisure choice. However, outcome of our numerical simulation indicates that leisure remains unchanged during the structural transformation. Hence, although there exists a shift in employment and output shares of sectors, distributional dynamics do not change in our model economy.

One reason behind this result is that we use AK type production functions in both sectors so the aggregate labor disappears in dynamic system we solve. Using Ramsey type growth model may solve this problem. Another source of the result is that the ratio of labor supplied to sector 1 to labor supplied to sector 2 is same for all households. Although the wages are different across sectors, it does not affect the income distribution of households. In order to solve this problem, there can be skill heterogeneity across households which differentiates the ratio of labor supplied to sector 1 to sector 2 for each household and this will create income differences between households.

APPENDIX A

DERIVATION OF EQUATIONS (22) AND (23)

Differentiating both sides of (15) with respect to time yields

$$\frac{p\dot{c}_{i2}}{c_{i1}} - \frac{pc_{i2}\dot{c}_{i1}}{c_{i1}^2} = \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(\frac{(1-\varepsilon)p\theta_2\dot{h}_2h_2^{\theta_2-1}}{h_1^{\theta_1}} - \frac{(1-\varepsilon)p\theta_1\dot{h}_1h_2}{h_1^{\theta_1+1}}\right)\left(\frac{ph_2^{\theta_2}}{h_1^{\theta_1}}\right)$$

$$\frac{\dot{c}_{i,1}}{c_{i,1}} - \frac{\dot{c}_{i,2}}{c_{i,2}} = \left(\theta_1\frac{\dot{h}_1}{h_1} - \theta_2\frac{\dot{h}_2}{h_2}\right)(1-\varepsilon) \tag{A.1}$$

$$\text{Let} \qquad G = \left[\gamma(c_{i,1}h_1^{-\theta_1})^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)(c_{i,2}h_2^{-\theta_2})^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Take the time derivative of (11):

$$\frac{-1/\varepsilon\dot{c}_{i1}\gamma c_{i1}^{\frac{-1-\varepsilon}{\varepsilon}}h_1^{-\theta_1\frac{\varepsilon-1}{\varepsilon}} - \frac{\varepsilon-1}{\varepsilon}\theta_1\dot{h}_1\gamma c_{i1}^{\frac{-1}{\varepsilon}}h_1^{-\theta_1\frac{\varepsilon-1}{\varepsilon}-1}}{G^{\frac{\varepsilon-1}{\varepsilon}}} = \dot{\lambda}_i \tag{A.2}$$

Dividing (11) to (A.2) yields:

$$-\frac{1}{\varepsilon}\frac{\dot{c}_{i1}}{c_{i1}} - \frac{\varepsilon-1}{\varepsilon}\theta_1\frac{\dot{h}_1}{h_1} - \frac{\varepsilon-1}{\varepsilon}\frac{\dot{G}}{G} = \frac{\dot{\lambda}_i}{\lambda_i} \tag{A.3}$$

$$\begin{aligned}\frac{\dot{G}}{G} &= G^{\frac{1-\varepsilon}{\varepsilon}}\big(\dot{c}_{i1}\gamma c_{i1}^{\frac{-1}{\varepsilon}}h_1^{-\theta_1\frac{\varepsilon-1}{\varepsilon}} - \theta_1\dot{h}_1\gamma c_{i1}^{\frac{\varepsilon-1}{\varepsilon}}h_1^{-\theta_1\frac{\varepsilon-1}{\varepsilon}-1} + \dot{c}_{i2}(1-\gamma)c_{i2}^{\frac{-1}{\varepsilon}}h_2^{-\theta_1\frac{\varepsilon-1}{\varepsilon}} \\ &\quad - \theta_2\dot{h}_2\gamma c_{i2}^{\frac{\varepsilon-1}{\varepsilon}}h_2^{-\theta_2\frac{\varepsilon-1}{\varepsilon}-1}\big) \\ &= G^{\frac{1-\varepsilon}{\varepsilon}}\left(\left(\frac{\dot{c}_{i1}}{c_{i1}} - \theta_1\frac{\dot{h}_1}{h_1}\right)(\gamma(c_{i,1}h_1^{-\theta_1})^{\frac{\varepsilon-1}{\varepsilon}}) + \left(\frac{\dot{c}_{i2}}{c_{i2}} - \theta_2\frac{\dot{h}_2}{h_2}\right)((1-\gamma)(c_{i,2}h_2^{-\theta_2})^{\frac{\varepsilon-1}{\varepsilon}})\right)\end{aligned} \tag{A.4}$$

By plugging (A.2) into (A.4), we get

$$\frac{\dot{G}}{G} = G^{\frac{1-\varepsilon}{\varepsilon}} \left(\left(\frac{\dot{c}_{i1}}{c_{i1}} - \theta_1 \frac{\dot{h}_1}{h_1} \right) (\gamma (c_{i,1} h_1^{-\theta_1})^{\frac{\varepsilon-1}{\varepsilon}}) \right.$$

$$\left. + \left(\frac{\dot{c}_{i1}}{c_{i1}} - \theta_1 \frac{\dot{h}_1}{h_1} + \varepsilon \left(\theta_1 \frac{\dot{h}_1}{h_1} - \theta_2 \frac{\dot{h}_2}{h_2} \right) ((1-\gamma)(c_{i,2} h_2^{-\theta_2})^{\frac{\varepsilon-1}{\varepsilon}}) \right) \right)$$

$$= \left(\frac{\dot{c}_{i1}}{c_{i1}} - \theta_1 \frac{\dot{h}_1}{h_1} \right) + \frac{(1-\gamma)(c_{i,2} h_2^{-\theta_2})^{\frac{\varepsilon-1}{\varepsilon}}}{G^{\frac{\varepsilon-1}{\varepsilon}}} \varepsilon \left(\theta_1 \frac{\dot{h}_1}{h_1} - \theta_2 \frac{\dot{h}_2}{h_2} \right) \tag{A.5}$$

Combine (A.3) with (A.5):

$$\frac{\dot{\lambda}_i}{\lambda_i} = -\frac{\dot{c}_{i1}}{c_{i1}} + \frac{\varepsilon - 1}{\varepsilon} \frac{(1-\gamma)(c_{i,2} h_2^{-\theta_2})^{\frac{\varepsilon-1}{\varepsilon}}}{G^{\frac{\varepsilon-1}{\varepsilon}}} \varepsilon \left(\theta_1 \frac{\dot{h}_1}{h_1} - \theta_2 \frac{\dot{h}_2}{h_2} \right) \tag{A.6}$$

By plugging (12) and (19) into (A.6)

$$\frac{\dot{c}_{i1}}{c_{i1}} = r - \rho + (1-\varepsilon)(1 - c_{i1}/Ei) \left(\theta_1 \frac{\dot{h}_1}{h_1} - \theta_2 \frac{\dot{h}_2}{h_2} \right) \tag{A.7}$$

From (16), $c_{i1}/Ei = X$ so we get (22) and derivation of (23) is same as (22).

APPENDIX B

PROOF OF PROPOSITIONS 2 AND 3

Proof of proposition 2: Let assume there exists a SE with $\dot{c}_{1,t}/c_{1,t} = \dot{c}_{2,t}/c_{2,t}$.Then, from (A.1), $\theta_1 \dot{h}_{1,t}/h_{1,t} = \theta_2 \dot{h}_{2,t}/h_{2,t}$ due to $\varepsilon \neq 1$. $\dot{c}_{1,t}/c_{1,t} = \dot{c}_{2,t}/c_{2,t}$ implies $\dot{c}_{1,t}/c_{1,t} = \dot{c}_{2,t}/c_{2,t} = g^*$ then $x_{1,t}$ and $x_{2,t}$ become constant. These constants are denoted as $\hat{x}_1$ and $\hat{x}_2$, respectively. From (36) and (37), $\delta_{1,t}$ converges to $\phi_1\hat{x}_1/(\phi_1 + g^*)$ and $\delta_{2,t}$ converges to $\phi_2\hat{x}_2/(\phi_2 + g^*)$. By definition of $\delta_{i,t}$, $\lim_{t\longrightarrow+\infty} \dot{h}_{1,t}/h_{1,t} = g^* = \lim_{t\longrightarrow+\infty} \dot{h}_{2,t}/h_{2,t}$. This is a contradiction because of $\theta_1 \dot{h}_{1,t}/h_{1,t} = \theta_2 \dot{h}_{2,t}/h_{2,t}$ is shown above and $\theta_1 \neq \theta_2$. Thus $\dot{c}_{1,t}/c_{1,t} \neq \dot{c}_{2,t}/c_{2,t}$ in SE.

Proof of Proposition 3:

Case i): By plugging (39c) into (35) - (37), I obtain $\dot{x}_{1,t} = \dot{\delta}_{1,t} = \dot{\delta}_{2t} = 0$. The values in (39c) are candidates for SE. From (34), $\hat{x}_1^* = (1-\alpha)A_1 + \rho$ implies $x_{2,t} = 0$. Since good 2 is pure consumption good, $c_{2,t} = A_2 K_{2,t}$. Then $x_{2,t} = A_1 \frac{K_{2,t}}{K_t} = 0$ so $\frac{K_{2,t}}{K_t} = 0$, $\frac{K_{1,t}}{K_t} = 1$ and $\frac{\dot{K}_{1,t}}{K_{1,t}} = g^*$ in the SE. .Due to $\dot{x}_{1,t} = \dot{\delta}_{1,t} = 0$, and the definition of $x_{1,t}$ and $\delta_{1,t}$, $\frac{\dot{c}_{1,t}}{c_{1,t}} = \frac{\dot{h}_{1,t}}{h_{1,t}} = \frac{\dot{K}_{1,t}}{K_{1,t}} = g^*$ which is (39a). Since $\dot{x}_{2,t} = 0$, $\dot{\delta}_{2,t} = 0$, $x_2^* = 0$ and $\delta_2^* = 0$, $\frac{\dot{c}_{2,t}}{c_{2,t}} = \frac{\dot{h}_{2,t}}{h_{2,t}}$. From (A.1) and (39a):

$$\alpha A_1 - \rho - \frac{\dot{c}_{2,t}}{c_{2,t}} = \left(\theta_1(\alpha A_1 - \rho) - \theta_2 \frac{\dot{h}_{2,t}}{h_{2,t}}\right)(1-\varepsilon)$$

Above equation gives (39b).

$1 + (\varepsilon - 1)\theta_j > 0$ for $j = 1,2$ so $\hat{g}^* > 0$. $(1-\varepsilon)(\theta_1 - \theta_2) > 0$ implies $\hat{g}^* < g^*$.

From (37) and (39b),

$$\frac{\phi_2}{p}x_{2,t} = (\phi_2 + g^*)\delta_2 \qquad \Rightarrow \qquad \frac{c_{2,t}}{h_{2,t}} = \frac{\phi_2 + g^*}{\phi_2}$$

which is (39d).

Case ii): By plugging (40c) into (35) - (37), I obtain $\dot{x}_{1,t} = \dot{\delta}_{1,t} = \dot{\delta}_{2t} = 0$. The values in (40c) are candidates for SE. Since $\hat{\hat{x}}_1^* = 0$, $x_2 = (1-\alpha)A_1 + \rho$ which means $\frac{K_{2,t}}{K_t} = 1 - \alpha + \rho/A_1$ so $\frac{K_{1,t}}{K_t} = \alpha - \rho/A_1$, and $\frac{\dot{c}_{2,t}}{c_{2,t}} = g^*$. Then $\frac{\dot{K}_{1,t}}{K_{1,t}} = \frac{c\dot{K}_{2,t}}{K_{2,t}} = \frac{\dot{K}_t}{K_t} = g^*$. Due to $\dot{\delta}_{1,t} = 0$, and the definition of $\delta_{1,t}$, $\frac{\dot{c}_{1,t}}{c_{1,t}} = \frac{\dot{h}_{1,t}}{h_{1,t}} = g^*$ which is (40a). From (A.1) and (40a), I obtain (40b). $1 + (\varepsilon - 1)\theta_j > 0$ for $j = 1,2$ so $\hat{\hat{g}}^* > 0$. $(1-\varepsilon)(\theta_1 - \theta_2) < 0$ implies $\hat{\hat{g}}^* < g^*$. From (37) and (40b), I obtain (40d).

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